

THEORY OF ECONOMIC VOLTAGE IMPROVEMENT

FOR A

RURAL DISTRIBUTION SYSTEM

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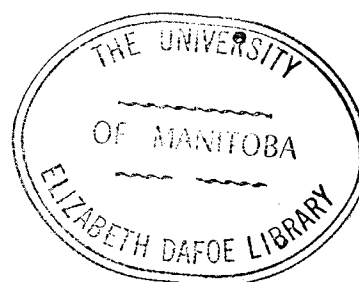
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BY

HEINZ SPERBER

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LIST OF SYMBOLS

A	capital investment in a system improvement, dollars
B	the voltage boost from a capacitor or regulator, volts on 120 volt base
C	rating of a capacitor installation, kvar
D	loss growth ratio, equation A-7
E	line to neutral voltage, kv
F	minimum cost of line improvement, dollars per volt
G	cost of line improvement, dollars per volt
I	line current, amps
L	reduction in power loss due to a system improvement, kw
P	power loss during the first year, kw
Q	the lagging load on a system before capacitors are applied, kvar
R	resistance, ohms/mile
S	line load during the initial year before the system improvement, kva
T	net total cost of one system improvement, dollars
V	voltage at a point on the distribution system, volts on 120 volt base
W	wire factor, equation B-2
X	reactance, ohms/mile
d	annual load growth, percent
g	maximum possible error in G, dollars per volt
i	annual rate of money, percent
m	cost of losses, dollars per kw
n	the length of the time period the distribution system voltage is improved, years



$v$	voltage drop, volts on 120 volt base
$x$	subscript referring to line improvements
$y$	subscript referring to regulators
$z$	subscript referring to capacitors
$\phi$	number of line phases
$\delta V$	improvement in regulation at the end of a time period due to a system change, volts on 120 volt base

## CHAPTER 1

### DEFINITIONS

The following definitions are presented to clarify the presentation:

Refer to drawing 1.

Distribution System - the electrical system between the sub-transmission system and the customer's distribution transformer.

Distribution Station - receives power from the subtransmission circuits and transforms it to the distribution voltage.

Trunk - circuits that emanate from the distribution station and provide paths of power flow to the distribution transformers.

Branch - a portion of the distribution system not part of the trunk but joined to it.

Junction - the point where a branch is joined to the trunk.

Branch End Point - the point on a branch farthest removed from the distribution station.

Line Section - a portion of the trunk terminated by two junctions or the distribution station.

Line Subsection - portion of a line section or branch. Between its extreme ends no load exists, the number of wires is the same and the same conductor is used.

Wire System - 2, 3 and 4-wire systems are possible on a grounded Y-connected 3-phase system. An ungrounded Y-connected 3-phase system will have one wire less than the corresponding grounded wire system. Reference will only be made to grounded systems to eliminate confusion. Refer to drawing 2.

Wire Addition - an increase in the number of wires within a

line subsection.

The following wire additions are possible:

3-wire to 4-wire

2-wire to 4-wire

2-wire to 3-wire

The number of wires is not normally reduced.

Wire Change - a replacement of each wire within a line subsection with a wire of larger conductivity.

Line Improvement - a comprehensive term referring to a wire addition and/or wire change within a line subsection.

Regulator Installation - the addition of a single regulator or bank of regulators to a distribution system and installation of such regulators in one location.

Capacitor Installation - the addition and installation of a bank of capacitors in a distribution system.

System Improvement - a comprehensive term referring to a line improvement, regulator or capacitor installation or any combination of these.

## CHAPTER 2

### THE PROBLEM

#### 2-1 The Challenge

(1)"The field of electric power distribution offers great challenge in many ways. Because it represents nearly half of the total system investment, it is a good place to look to obtain increased savings. It has innumerable apparatus components which offer fruitful ground for improvements or new developments and inventions. Because of its many components and variables which make for many possible choices of designs for the distribution of power, the complexity of the engineering and economics leaves much room for improvement. The repetitive calculations and engineering application of small components offers a new and interesting field for the use of digital computers. Automated engineering of the power distribution system may some day be more than just an ideal."

#### 2-2 The Distribution System

The distribution system is the electrical system between the subtransmission system and the customer's distribution transformer.

Broadly speaking, there are two fundamental types of distribution systems: (1) radial, and (2) network. By the simplest definition, a radial system has a single path of power flow to the load; a network has more than one simultaneous path of power flow to the load. Drawing 3 illustrates simple forms of radial and network systems from a power source with a trunk circuit supplying step-down transformers.

The radial distribution system is frequently subdivided into (1) residential and (2) rural systems.

Residential areas are normally compact, have high load

density, and show comparatively small load increases after their development. The distribution system in residential areas is invariably "load-limited", i.e. the circuit ampacity is exceeded before voltage regulation problems are encountered.

In the initial design of a residential distribution system the circuit wire size is dictated by the "final" predicted load. A number of papers have appeared in the recent literature concerning the design of residential distribution systems. (2-6)

The rural station areas are normally large, have small load density, and show comparatively large rates of load growth. The distribution system in rural areas is invariably "voltage-limited", i.e. voltage regulation becomes excessive before the circuit ampacity is exceeded. It is common practice to design rural distribution systems with adequate regulation for a few years load growth and when the regulation becomes excessive add auxiliary equipment. (7)

These two different design practices are both practical, since the cost of improvement is high in a residential distribution system, while it is relatively low in a rural distribution system.

This paper will only be concerned with the radial type, "voltage-limited", rural distribution system.

### 2-3 Methods of Improving Rural Distribution System Voltage Regulation

To maintain voltage regulation within acceptable limits (8), the distribution engineer has engaged a variety of corrective methods to improve voltages. Short-term solutions are employed where voltage conditions are not extremely critical. These include:

- (1) Wire addition - an increase in the number of phase wires within a line subsection improves regulation since the same load is divided over a greater number of wires.
- (2) Wire change - a replacement of each wire within a line subsection with a wire of larger conductivity reduces the voltage drop over the line subsection.
- (3) Voltage regulator installation - the voltage regulator regulates the voltage on its output side within a prescribed voltage band width. It may be installed wherever it is required on the distribution system.
- and, (4) Shunt capacitor installation - shunt capacitors provide a constant voltage boost. When the capacitor is coupled with a voltage sensing device which switches it into the system during peak demand and out of the system during low demand, the capacitor also reduces the voltage regulation.

Several more expensive and less frequently used solutions, are also available. These include:

- (1) providing a new circuit;
- (2) locating a new distribution station closer to a load center;
- and, (3) converting an area to a higher primary distribution voltage (9).

#### 2-4 Scope of the Paper

This paper concerns itself primarily with the practices of the lightly loaded rural distribution system.

Previously it was mentioned that it is normal practice not to design the rural distribution system for the "ultimate" system load (if, it is at all possible to speak of such a load) but, as

the load grows, to make system changes so that system operation is at all times at a satisfactory level. System operation is satisfactory if voltage regulation is within the prescribed voltage limits.

It is the object of this paper to develop a basic routine, applicable to computers, which reviews system operation and recommends system changes to maintain the prescribed voltage regulation. The routine will be confined to economic selections between one or more of the "short-term" solutions, i.e.

- (1) wire addition
- (2) wire change
- (3) voltage regulator installation
- and, (4) shunt capacitor installation.

The last three corrections mentioned in the previous section are neglected in this study.

## 2-5 The Criterion of Minimum Cost

The economic selection of a short-term system improvement is based upon the following factors: 7, 10

### 1) The annual cost of money.

A postponement of an investment represents a realistic saving.

### 2) The cost of losses.

Power dissipated in transit on an electrical system must be generated.

Reducing the losses reduces operating cost.

### 3) Time.

Loads are invariably dynamic, normally increasing at a constant percentage from one annual load peak to the next. As a result the branch end voltage and the distribution system loss will change with time. It

is normal practice to correct the distribution system voltage regulation for several years operation, the number of years being more or less arbitrarily chosen.

4) Voltage regulation.

Voltage regulation implies that an upper and a lower voltage limit must be maintained on the distribution system. In practice, however, it is necessary only to consider the lower voltage limit.

If the use of large static shunt capacitors is avoided, and if voltage regulators operate within the prescribed voltage limits, then, since there is always some voltage loss during low demand between the distribution station and the branch end points of the distribution system, the upper voltage limit must occur first at the station. If the station is properly regulated the upper voltage limit will not be exceeded on the distribution system.

In this study only the lower limit of voltage regulation will be considered (maximum load condition).

The most economical system improvement will be the one which will yield the largest voltage improvement at the end of the designated time period for the least net total cost accumulated during the designated time period, including the cost of losses.

Since power companies are legally (or otherwise) obligated to provide voltage at customer equipment only to within the prescribed voltage limits, an overcorrection to within a smaller band width of voltage cannot economically be justified, unless a longer time period of satisfactory system operation is to be considered. Sufficient system improvement will be recommended to



provide satisfactory regulation only until the end of the designated time period.

## CHAPTER 3

### LINE IMPROVEMENTS - PART I

#### 3-1 Introduction

Chapters 3 and 4 present a method to correct the system regulation by line improvements. The application of regulators and capacitors is reserved for Chapters 5 and 6.

First, the procedure to determine the economic line improvement of a line subsection is presented; then follows a similar technique for the evaluation of the economic line improvement of a line section.

It should be noted that the method presented in Chapter 3 is approximate. The method will, nonetheless, still be useful since equations are presented which will determine the maximum error in calculations. The approximate method, has the advantage of speed, but will not always determine the absolute minimum. It may however eliminate a number of the more expensive proposals, thereby giving an opportunity to concentrate on the remaining possibilities with a more accurate method to be presented in Chapter 4.

#### 3-2 Preparation of Data

A list of line improvements and the net capital cost (capital investment minus salvage value) of each line improvement is compiled.

Unreasonably expensive line improvements should be avoided to keep to a minimum the length of time required to identify the economic line improvement.

#### 3-3 Cost of Line Improvement

The net cost of a line improvement consists of the difference between the interest charges of the capital expenditure and the kw loss saving resulting from the line improvement, both

summed over the same time period.

The net total cost over a time period of  $n$  years using straight line depreciation is partly derived in Appendix A and is given by equation 3-3-1.

$$T_x = n i A_x - m D L_x \quad 3-3-1$$

$T_x$  net total cost of line improvement, dollars  
 $n$  the length of the time period the distribution system voltage is improved, years  
 $i$  annual rate of money, percent  
 $A_x$  capital investment in line improvement minus salvage value, dollars  
 $m$  cost of losses, dollars per kw  
 $D$  loss growth ratio, defined by equation A-7  
 $L_x$  reduction in power loss during initial year due to line improvement, kw

The improvement in regulation at the end of the time period as a result of the line improvement is partially derived in B-4.

$$\delta V = S(1+d)^n (W_0 - W_n) \quad 3-3-2$$

where

$\delta V$  improvement in voltage during last year due to line improvement, volts on 120 volt base.  
 $S$  line load during initial year before the system improvement, kva  
 $W_0$  wire factor of line before system improvement, equation B-2  
 $W_n$  wire factor of line after system improvement  
 $d$  annual load growth, percent

The per volt cost of the line improvement will be given by the ratio  $G = \frac{T_x}{\delta V} \quad 3-3-3$

Given a choice of two or more alternative methods of improving a line's performance, the most economical method is the one which gives the largest voltage improvement at the end of the time period considered for the least net total cost during that period. Thus the most economical method of improvement of a line subsection is given by the smallest ratio,  $G$ , of equation 3-3-3. The amount of improvement will be determined later.

### 3-4 Cost of Line Improvement with a Changing Distribution System

The accurate calculation of power loss saving by the formula of Appendix A is complicated by three auxiliary effects:

#### 1) Preceding Elements

Each kw of power loss must be carried through all the preceding elements of the system. The kw loss saving in the preceding elements of the system has been neglected by the formula in Appendix A.

#### 2) Changing Line Load

To calculate the power loss saving the formula in Appendix A uses the line load before any system improvement is applied. The correct calculation requires that the element's line load immediately prior to the system improvement should be used. (The two line loads are different unless the particular line improvement is the first to be recommended.)

#### and 3) Voltage

Deviations in the line load result in deviations in the voltage.

These factors may be taken into account by an iterative solution (11-12) but the solution is relatively lengthy.

It may be profitable to examine the degree of error in equation 3-3-1. Experience has shown (13) that:

- (a) losses on a distribution system represent less than 12% of the circuit load;
- (b) losses are not reduced by more than 50% through line improvements.

If there were no dependency between the reduction of losses in one line subsection and the losses elsewhere, then the net total cost of the line improvement would be given by equation 3-3-1.

$$T_x = \sum n_i A_x - m D L_x$$

First, consider the effect a line improvement has on the losses of all line subsections between the distribution station and the line subsection where the line improvement is to occur. From (a) it is concluded that for every one unit of loss reduction in one line subsection, losses are reduced by less than  $u_1$  units in all preceding line subsections. This leads to an inequality

$$\sum n_i A_x - m(1+u_1) D L_x < T_x < \sum n_i A_x - m D L_x \quad 3-4-1$$

$u_1$  largest ratio by which losses are reduced in the preceding line subsections by the reduction of losses in one line subsection (normally less than 12%).

The reduction in the line current and the effect on losses may be expressed by another inequality.

$$\sum n_i A_x - m D L_x < T_x < \sum n_i A_x - m(1-u_2) D L_x \quad 3-4-2$$

where

$u_2$  largest ratio by which losses in one line subsection are reduced as a result of line improvements occurring elsewhere. (normally less than 12%).

Including both effects

$$n i A_x - m(1+u_1) DL_x < T_x < n i A_x - m(1-u_2) DL_x \quad 3-4-3$$

thus

$$n i A_x - m DL_x < T_x < n i A_x - m DL_x + (u_1 + u_2) m DL_x \quad 3-4-3$$

$$\text{let } u = u_1 + u_2$$

$$n i A_x - m DL_x < T_x < n i A_x - m DL_x + u m DL_x \quad 3-4-5$$

It is shown here that  $u$  will not exceed 24%. It is believed that this figure is overly pessimistic. Further study will probably indicate a smaller numeric value.

Dividing both sides of 3-4-5 by  $\delta V$

$$G < \frac{T_x}{\delta V} < G + g \quad 3-4-6$$

where

$$g = \frac{u m D L_x}{\delta V} \quad 3-4-7$$

It is assumed that equation 3-4-6 also makes allowance for errors in voltage, which are normally considerably smaller than  $u$ .

### 3-5 Selection of Economic Line Improvements in a Line Subsection

If two line improvements are proposed for one line subsection and it is desired to determine the more economical line improvement (1) calculate for both the cost per volt,  $G$ , from equation 3-3-3 and (2) their corresponding error,  $g$ , from equation 3-4-7.

The cost of the first line improvement will be between  $G_1$  and  $G_1 + g_1$ , dollars per volt.

The cost of the second line improvement will be between  $G_2$  and  $G_2 + g_2$ , dollars per volt.

The following conditions are possible. Examine drawing 4.

- a) The first line improvement is clearly more economical

$$G_1 + S_1 < G_2$$

- b) The second line improvement is clearly more economical

$$G_2 + S_2 < G_1$$

- and c) It is not possible to determine which line improvement is more economical. The approximate solution is inadequate.

Wherever the approximate solution does not suffice an iterative solution must be employed.

Every proposed line improvement must be examined and the  $G + g$  value calculated. Where more than one line improvement is proposed the lowest  $G + g$  value is found. If one line improvement is found to be clearly more economical its parameters are stored and the other line improvements are discarded; where it is not possible to recognize the economic line improvement, both are stored. When the economical choice is in doubt it is left to a later development to determine the economic line improvement for the line subsection.

### 3-6 Selection of Economic Line Improvement in a Line Section or Branch

Line sections and branches are composed of one or more line subsections. The previous section provided a means to calculate the least expensive line improvement and its cost, within a maximum calculable error, for each line subsection. If several line subsections occur within a line section or branch whose unit costs per volt improvement are all different, then the most economic choice must be found. In order that the cost of alternatives can be compared it is essential that

the same improvement in regulation should be provided by all alternatives. For this purpose the G ratios of equation 3-3-3 are suitable.

Line improvements are proposed for two line subsections within one line section. The cost of the two line improvements is between  $G_1$  and  $G_1 + \delta_1$  for the first line improvement;

$G_2$  and  $G_2 + \delta_2$  for the second line improvement.

The following conditions are possible. Examine drawing 4.

a) The first line improvement is clearly more economical

$$G_1 + \delta_1 < G_2$$

b) The second line improvement is clearly more economical

$$G_2 + \delta_2 < G_1$$

and c) It is not possible to determine which line improvement is more economical. The approximate solution is inadequate.

This argument may be extended to line sections and branches with any number of line subsections.

The analysis is essentially the same as that of the previous section. There is, however, one very important difference. It is only possible to implement one line improvement in a line subsection - the most economic. After the most economic line improvement has been found, all other suggested line improvements of the line subsection may be discarded.

This is not the situation in the selection of line improvements for a line section or branch. It is natural that the most economic line improvement should be recommended first, but if that one line improvement does not provide sufficient regulation the next most economic (and so forth) may also be required.



It is convenient to arrange and store all the line improvements of a branch or line section in increasing sequential order, according to their cost, until either the voltage regulation is satisfied or all line improvements have been stored.

It will not always be possible to obtain a clear sequential order of line improvements. This will be the case when condition (c) occurs. Here the two items are placed side-by-side and are given the same sequence number. It is left to a later portion of this study to determine the true sequence.

### 3-7 Summary

The cost per volt of each line improvement is calculated by an approximate method. The maximum error in calculation is also determined.

From a suggested list of line improvements the most economical line improvement is determined for each line subsection and line section.

Only one line improvement is required per line subsection. The line improvements within each line section or branch are arranged according to their cost.

If the approximate solution is not capable of determining the sequence of line improvements, the accurate procedure in Chapter 4 will be required.

## CHAPTER 4

### LINE IMPROVEMENTS - PART II

#### 4-1 Introduction

The most economic line improvements are found by examining the total cost of each possible combination of line improvements.

The process is repeated until the system regulation requirements are satisfied.

#### 4-2 Possible Combinations of Line Improvements

It will be observed from figure 5 that a line improvement executed in the line section between the station and junction 1 will reduce the regulation at all branch ends; a line improvement in the line section between junctions 3 and 4 will improve the voltage at the branch ends of branches d and e; a line improvement on a branch reduces the regulation of that branch. In general it is observed that a reduction in regulation occurs only on the load side of the line improvement.

The branch end peak load voltages  $V_a$ ,  $V_b$ ,  $V_c$ ,  $V_d$  and  $V_e$  of the respective branches a, b, c, d and e are in general not of the same magnitude. Some of the branches will have peak load voltages below the minimum acceptable voltage level. To be perfectly general it is assumed that voltages at the end of all branches fall below the acceptable voltage level. A minimum cost method must be found to improve the regulation at all the branch ends. It is necessary to consider only the regulation at the branch ends since the minimum voltage of a system without regulators or capacitors occurs at the branch ends.

Since a line improvement reduces the regulation on its load side only, it can be concluded that those branches which

do not have excessive regulation do not require any line improvement and may be excluded from the study.

Before actually proceeding to determine the most economical manner in which the distribution voltage can be improved, let us find all the possible combinations from which the economic choice can be determined.

As a first step the branch end with the lowest peak load voltage and the branch end with the next lowest voltage is found. The difference between these two branch end voltages represents the smallest amount (through the use of line improvements) which could make both branch end voltages equal.

Let us illustrate this last point with drawing 5. Let the lowest voltage occur at branch end e and the next lowest at d, the difference between the two being  $V_{de}$ . It is the object now to select at minimum cost line improvements such that the minimum voltage at e may be elevated. If the line improvement should be chosen between the station and junction 4 then branch ends e and d experience the same improvement in regulation (since the improvement in regulation occurs on the load side of a line improvement); if the line improvement should be chosen between junction 4 and branch end e, then branch e experiences an improvement in regulation but branch end d does not. In the first solution the difference in voltage at branches e and d remains the same, in the second situation the difference decreases.

It is important to notice that the line improvements are chosen in this instance with the primary purpose that the voltage at branch end e (only) is elevated most economically.

To improve the voltage at branch end point e the line improvements may be selected among the line subsections between

node point 0 and the end point of branch e.

The most economic of all possible line improvements should be chosen first.

The reader should note here that as long as branch e has the lowest voltage all branches other than e are neglected - only the trunk and branch e are considered.

When two branches share the lowest voltage then the selection of line improvements must be oriented to benefit both branch end voltages equally. In the situation where branches d and e share the lowest voltage line selections may be made:

- 1) between node points 0-4, which improves voltage at both branch end points;
- or 2) on branch e which improves voltage at branch point e only; and on branch d which improves voltage at branch point d only.

The most economical should be employed.

It is not difficult to obtain the combinations for other branches. The number of possible combinations is equal to the number of branches with equal low voltage. If, however, one of the line sections has employed all the line improvements which are suggested then one of the possible combinations is eliminated.

For example, if all five branches have equal low voltage then the line improvements may be made in the following combinations:

- 1) line section 0-1
- 2) line section 1-2 and branch a
- 3) line section 2-3 and branches a, b
- 4) line section 3-4 and branches a, b, c
- and 5) no line section and branches a, b, c, d, e

If all the line improvements of line section 1-2 have been employed then combination 2 is not possible. The same applies to branches.

The selection of line changes continues until the voltage is satisfactory. If this is not possible additional forms of regulation are required i.e. voltage regulators, capacitors, etc.

#### 4-3 Selection of Economic Line Improvements in a Distribution System

Let  $F_{ab}$  be the minimum per volt cost of line improvement in a line section between the points "a" and "b".

Let  $F_i$  be the minimum per volt cost of one combination.

Consider as an example the case where all five branch ends of a distribution system have equal low voltage. The minimum cost of each combination as it is listed on the previous page will be

$$\begin{aligned} F_1 &= F_{01} \\ F_2 &= F_{12} + F_a \\ F_3 &= F_{23} + F_a + F_b \\ F_4 &= F_{34} + F_a + F_b + F_c \\ F_5 &= F_a + F_b + F_c + F_d + F_e \end{aligned} \quad 4-3-1$$

The minimum cost by which the regulation on the distribution system can be improved is represented by the smallest per volt cost,  $F_1$ .

The components,  $F_{ab}$ , in the line improvement combination with the smallest incremental cost,  $F_1$ , are now known. The improvement in regulation provided by the combination can be found.

The maximum improvement in regulation is determined:

- 1) by the regulation of the line improvement with the minimum voltage regulation;
- and 2) by the regulation on the distribution system being satisfied.

If condition (1) applies further line improvements are necessary to correct the regulation of the distribution system. The entire process of finding the line improvements with minimum incremental cost must be repeated.

If condition (2) applies the solution is terminated.

Of course, this entire approach rests on the fact that the per volt cost of a line improvement can be calculated. The determination of that cost was shown to be quite complicated. An approximate technique was developed, which allowed the determination of the unit cost of a line improvement within a maximum calculable error. From 3-4-6.

$$C < F < C + G \quad 4-3-2$$

Since each unit cost of a line improvement does in fact represent a band width of cost, the precise minimum,  $F_1$ , of equations 4-3-1 will not always be determinable.

This approximate method does, however, still have merit:

- 1) It does lead to a much faster solution - if a solution is possible - than the iterative procedure soon to be introduced;
- 2) It will often eliminate the most expensive alternatives and allow concentration on the more reasonable prospects.

In order that the minimum cost,  $F_1$ , of equations 4-3-1 can be found, each term,  $F_{ab}$ , must be the minimum cost line improvement of the line section between the junctions "a" and

"b".

If such a minimum cost cannot be found, as was shown to be the case in section 3-6 when condition (c) applied, then all the alternatives (c) should be substituted into equations 4-3-1 to obtain the maximum benefit from the approximate solution.

When the approximate solution does not identify the most economic combination of line improvements, those combinations, which are shown to be reasonable prospects for the economic solution, should be isolated for further study with the accurate solution.

The accurate solution employs the nodal iterative solution. (11, 12) This method calculates the distribution system loss and voltage accurately. The saving in losses and improvement in the lower limit of regulation due to a combination of line improvements may be calculated by a comparison of the load flow solution before and after the line improvements are implemented. A load flow solution may be obtained for each desired alternative and the losses calculated.

The alternatives may be compared by equation 4-3-1 to determine the most economical line improvements.

#### 4-4 Wire Addition

In the analysis up to this point it has been assumed that, if one were to examine the recommendations after the voltage had been improved to a satisfactory level, one would find a 4-wire system feeding into a 3-wire system which in turn supplies a 2-wire system. This result is not guaranteed since the most economic line selection within a line section was made without regard to the number of wires in the adjacent line section.

It is pointless to select a wire addition in a line subsection as the most economic improvement when the number of wires on the source-side is less.

The following rule is required to prevent improper recommendations:

A line improvement may only be a wire addition when it does not exceed the number of wires of the line subsection preceding it; otherwise it must be a wire change. If in the subsequent calculation a wire addition is recommended for the preceding line subsection, then it will be feasible to recommend a wire addition for the particular line subsection as well.

#### 4-5 Summary

The economic line improvement or combination of line improvements are found:

- 1) by exploring all the possible combinations in which the line improvements may be arranged;
- 2) determining the combination of line improvements with the smallest per volt cost;
- and 3) finding the smallest improvement in voltage regulation by any member of the economic combination.

If the improvement in regulation was not sufficient to provide adequate system regulation the entire procedure is repeated.

Where the calculation in losses by the approximate method (Chapter 3) is not sufficiently precise to give a distinct solution, the calculation of losses is performed by the nodal iterative procedure.



## CHAPTER 5

### NEW REGULATORS

#### 5-1 Introduction

The cost of a new regulator installation is determined by the ampere rating of the voltage regulator and the number of phase wires which have to be regulated. As these two requirements are determined by the line load and the line construction, the cost of a regulator installation is determined by the nature of the distribution system at the regulator location. Since the cost of a regulator installation is independent of the regulator boost it is desirable to locate a regulator at a point where it will deliver the maximum boost. This location will represent the minimum per volt cost.

For purposes of this study it will be assumed that the voltage on the output side of the regulator is held at a constant value.

#### 5-2 Line Improvements and One Regulator

To obtain the minimum cost provide the maximum regulator boost--the difference between the input and output voltage. The regulator is located at the point where the lower limit of voltage regulation will exist on the distribution system at the end of the time period.

With the new regulator on the system the branch end voltages will change from their previous values. The new branch end voltages will be approximately the initial voltages at the branch ends minus the regulator boost.

$$\begin{array}{llll} V_a' & = & V_a & - & E_y \\ V_b' & = & V_b & - & E_y \\ \text{Etc.} & & & & \end{array} \quad 5-2-1$$

$V_a'$       modified voltage at the end of branch a, volts  
 $V_a$       original voltage at the end of branch a, volts  
 $B_y$       regulator boost, volts

The remaining voltage requirements must be provided by line improvements. These line improvements are chosen at minimum cost according to the method explained in Chapter 4. The voltage difference of the two lowest modified branch voltages is found and then the minimum cost combination of line improvements is selected. This procedure is repeated until the voltage on the distribution system is satisfied. The procedure is identical so far to that of Chapter 4 with the exception that the modified branch voltages are used.

Equation 5-2-1 places two conditions on the procedure of Chapter 4:

- 1) the regulator will always boost at its maximum value regardless of the line improvement proposed and,
- 2) the regulator boost is applied at all branch ends.

The regulator will boost at its maximum limit as long as its input is at the minimum voltage limit. Thus if the regulator is at all times placed at the point on the distribution system where the minimum voltage limit occurs on the input side of the regulator, then the regulator will always boost at its maximum value. One modification from the method of Chapter 4 therefore will be to locate the regulator after each selection of line improvements at the point where the lower regulation limit occurs.

Once the regulator position can be tracked, it is relatively easy to determine whether the regulator is boosting on all branches, since it is possible for a regulator to boost

only on its load side. If the regulator is moved past a branch by a recommended line improvement the branch will no longer experience the voltage boost of the regulator and one of the assumptions is violated.

### 5-3 Moving a Regulator Past a Junction

If the regulator is at all times placed at the point on the distribution system where the minimum voltage limit occurs then it will occasionally happen that a regulator is relocated to the load side of a junction due to the manner in which line improvements are selected. The recommended line improvements represent the most economical choice of line improvements. However, relocating a regulator to the load side of a junction means that the voltage on one branch has been lowered. If the voltage on that branch should be outside the allowable voltage limit then additional line improvements will be required to restore the voltage.

It is clear that in this situation the most economical selection of line improvements does not lead to the best application of the regulator. In order to arrive at the most economical system improvement the reverse situation - the best application of the regulator - must also be studied.

Thus, in this situation one is always faced with two possible ways in which the economic system solution can be obtained:

- 1) Line improvements are selected in the most economical manner and the regulator is placed where the minimum voltage limit occurs on its input side;
- and 2) the regulator is fixed at the source side of the junction. The remaining voltage requirement is

provided by line improvements.

Whether it is economical to relocate a regulator to the load side of a junction may be determined by the inequality

5-3-1. Refer to drawing 6.

$$T_{(s-n)x} + T_{(n-m)x} + T_{(n-t)x} - \delta Ay = T_{(n-m)xf} + T_{(n-t)xf} \quad 5-3-1$$

where

$T_x$  net total cost of line improvement, dollars, see equation 3-3-1. The subscripts in brackets refer to the portion of the system where the line improvements are chosen; the subscript "f" requires that the regulator is fixed at a junction.

$\delta Ay$  the difference in the cost of the regulator installation when it is permitted to relocate regulators and when it is not, dollars.

The derivation of inequality 5-3-1 will now be explained.

The cost on the right-hand side of the inequality represents the cost of the line improvements when the regulator location is fixed at the junction, n. (see drawing 6). The output voltage of a regulator operating within its rating is not affected by line improvements on its input side. Thus if a regulator is fixed at a location where the minimum voltage limit occurs on its input side, all line improvements must be chosen on its output side.

The  $T_x$  terms of inequality 5-3-1 may be solved by the approximate or accurate method described in the two previous Chapters.

If the regulator location is not fixed to any one point then the line improvements may be selected on the input and

output side of the regulator. Line improvements selected on the input side of the regulator elevate the input voltage and the regulator is relocated to the point where the minimum voltage limit occurs on its input side. Relocating the regulator elevates the voltage at the branch end. Line improvements selected on the output side of the regulator elevate the voltage at the branch end but do not require a relocation of the regulator.

Since the number of phase wires decreases towards the end of a distribution system it may happen that when a regulator installation is constantly relocated by line improvements that the number of regulators in the final location is less than it would have been had the regulator installation been fixed at a previous junction. The difference in cost of a regulator installation at the two points is given by  $\delta Ay$ .

It is important to realize that inequality 5-3-1 involves all the expenditures for both schemes from the time that the regulator is first placed at the junction to the time that the distribution system voltage regulation is satisfied.

Fortunately it will not be necessary to solve both system schemes independently since for each branch or line section the line improvements are chosen in an economic sequence.

Consider the following illustration. Refer to drawing 6. The first line improvement which is selected in line section n-m when the regulator location is permitted to move will also be the first line improvement which is selected in line section n-m when the regulator is fixed at junction, n. The same will be true for the second line improvement in line section n-m, and so forth, until the regulation for one of the two schemes will be satisfied.

Let us apply this principle to inequality 5-3-1. Assume that permitting the regulator location to move is the economical solution. In order to determine the most economical line improvements for the left-hand side every permissible line improvement throughout the distribution system - and, of course, in the line sections which appear on the right-hand side of inequality 5-3-1 - must be examined. The economical line improvements of the line sections appearing on the right-hand side are stored in a storage area set aside for the solution with the regulator location fixed at the junction. These economical line improvements are then compared against the line improvements throughout the distribution system to determine the economical line improvements for the left-hand side. These are also stored.

The economical line improvements are determined from 4-3-1 until the voltage regulation with the scheme where the regulator is permitted to be relocated, is satisfied.

The difference in the cost of the regulator installation,  $\delta A_y$ , when it is permitted or not permitted to relocate the regulator can now be calculated.

The left-hand side of inequality 5-3-1 has now been completely solved; the right-hand side has at least been partially solved.

Proceed to test inequality 5-3-1. If

- 1) the inequality is satisfied, relocating the regulator location is the more economic scheme;
- and if 2) the inequality is not satisfied complete the solution of the right-hand side. Fixing the regulator location at the node may be the more economic scheme. Proceed to test inequality 5-3-1.

#### 5-4 Variable Regulator Cost

Assume a regulator is located not too far from the end of a four wire line section. It will, therefore, have to regulate all three line phases. But if it is moved towards the branch end only a small distance an abrupt change in the number of line wires occurs, and the regulator installation cost decreases. The previous sections do not allow for this contingency.

The cost for line improvements must increase when the regulator is moved. Only if the regulator cost decreases by a larger amount can the overall cost be less. It can therefore be concluded that a cheaper solution is only possible when the regulator is moved from the position determined in the previous section to a location where there is a decrease in the number of phase wires. By fixing the regulator at this location the approximate line improvements can be found. It can now be decided if this solution is more economic.

Regulator cost is also affected by the regulator ampere rating. Since this in turn is related to line load the regulator cost can decrease when the regulator is placed in a line section with less load. The same procedure may be employed to determine whether a lesser cost solution is possible. Difference in cost due to regulator ampere ratings is normally neglected.

#### 5-5 Existing Regulators

Existing regulators may be treated in two possible ways:

- a) A policy is adopted whereby no regulators are moved from their existing location.
- b) Regulators are moved on the distribution system at

a price.

If it is assumed that regulators hold a fixed output voltage then a regulator which is not to be moved may be viewed equivalent to a distribution station. The system may be broken into two parts: the first reaching from the station to the regulator, the second extending beyond the regulator. Each part may be solved by the techniques outlined in the preceding pages to give a minimum cost. The total recommendation will consist of the sum of recommendations for each part.

In the second situation the existing regulator will be moved about by the most economic line changes as explained. Instead of employing the cost of a new regulator installation only the moving charges of the regulator are required.

This technique will also permit fixing some existing regulators while permitting others to move.

#### 5-6 How Many New Regulators?

It is seen that the program does not determine the economic number of new regulators directly but solves only for the economic line improvements when the number of regulators is given. By changing the number of regulators from zero to the largest reasonable number the facility for determining the economic number of regulator installations is provided. With some experience in distribution problems the economic number of regulator installations can normally be guessed very closely.

#### 5-7 Summary

The economic regulator location is found by:

- 1) modifying the original branch end voltages by the boost of the regulator;



- 2) determining the economic combination of line improvements from equation 4-3-1;
- and 3) locating the regulator input at the point where the minimum acceptable voltage limit occurs.

If the regulator has to be relocated to the load side of a junction, where the branch end voltage drops below the minimum acceptable voltage without the regulator boost, a solution must be developed with the regulator fixed at the junction and also with the regulator on the load side of the junction. It was shown that the two solutions are partially common.

Whether the regulator should be moved past the junction can be determined from inequality 5-3-1.

## CHAPTER 6

### NEW CAPACITORS

#### 6-1 Introduction

Only three-phase capacitor installations are considered. It is assumed that all possible combinations of capacitor size and location are specified. The most economic capacitor size and location is identified.

#### 6-2 Method 1

The first of the permissible capacitors is installed on the distribution system and by the method of Chapter 5 the most economic solution is determined. The entire procedure is repeated with the next permissible installation, and so forth, until the economic solution with each installation has been found. The economic capacitor installation will correspond to the solution with the lowest over-all cost.

This method provides the easiest approach though not necessarily the fastest solution.

#### 6-3 Method 2

Chapters 4 and 5 developed a technique to improve the regulation with the most economical line improvements.

When a capacitor is installed on the rectified distribution system some of the line improvements become superfluous due to the voltage boost of the capacitor. It is the object of section 6-3 to determine the most expensive superfluous line improvements.

Several observations on the nature of the solution will be helpful:

- 1) The most economic line improvement is always chosen from the available line improvements. In other words,

the remaining line improvements can never be quite as economical.

- 2) A capacitor installation is intended to replace the most expensive line improvement (including losses) which would have been required had the capacitor not been installed.
- 3) The boost provided by a capacitor installation is not the same at every junction. It decreases towards the distribution station proportionally to the decrease in short circuit kva.
- and 4) The line improvements which are replaced by the capacitor must not provide a larger improvement in regulation than the capacitor boost.

These considerations lead to the following procedure to determine the most expensive aggregate of line improvements which would have been required had the capacitor not been installed.

- 1) The line improvements are numbered in ascending order as they are selected by the procedure of Chapters 4 and 5. Any combination of line improvements chosen simultaneously is given the same number.
- 2) The line improvements which should be replaced by the capacitor installation are those with the largest numbers.
- 3) The capacitor boost effective at the first branch end is calculated. Line improvements are replaced until the improvement in regulation equals the capacitor boost.
- 4) The effective capacitor boost between the first and second branch end is calculated. Line improvements are replaced until the improvement in regulation equals the capacitor boost. If the combination of line improvements which is

to be replaced by the capacitor involves terms which are located between the station and the first junction, all but these terms, may be replaced. If all terms were replaced the first branch end's regulation limit would be exceeded.

Also refer to Appendix D.

and 5) Repeat (4) until the capacitor location is encountered.

The procedure to determine the most expensive line improvements which may be replaced by the addition of a capacitor is similar if the distribution system is regulated by a voltage regulator. Three situations are possible:

If the most expensive combination of line improvements which is to be replaced by the capacitor installation involves line improvements which are located only on the regulator input side then the solution proceeds in the same fashion as if no regulator were present.

If the most expensive combination of line improvements which is to be replaced by the capacitor installation involves line improvements which are also situated on the output side of the regulator, then the regulator must be relocated to maintain the voltage at the branch ends. Refer to 5-2. It is assumed that the regulator does not cross a junction when it is relocated.

Two solutions are possible if the regulator crosses a junction during relocation, and if the branch end of the branch joined to the junction experiences less than the specified minimum voltage. This situation was discussed in 5-3. Since the boost provided by a capacitor is normally fairly small it will, most probably, not be economical to relocate the regulator past the junction. However, an accurate solution can be obtained

by repeating the procedure presented in 5-3.

If all the line improvements which are obviated by one capacitor are restored with the original line configuration then the distribution system is obtained as it would be with the capacitor.

Proceed to the calculation of the saving of power losses. The formula for the saving of power losses due to the addition of one capacitor is given in Appendix C. The cost of a capacitor installation will be

$$T_z = n i A_z - \sum_k m L_z \quad 6-3-1$$

$T_z$  net total cost of capacitor installation, dollars  
 $n$  time period for which distribution system voltage is improved, years  
 $i$  annual rate of money, percent  
 $A_z$  capital investment in capacitor  
 $k$  total number of line subsections where capacitor is effective  
 $m$  cost of losses, dollars per kw  
 $L_z$  defined in Appendix C. reduction in losses per line subsection over total time period due to capacitor, kw.

A capacitor installation will be more economical than the line improvements which were replaced, if inequality 6-3-2 applies.

$$T_z < \sum T_x \quad 6-3-2$$

where

$T_z$  defined by equation 6-3-1.  
 $T_x$  aggregate cost of capital investment in line improvement not required due to the installa-

tion of the capacitor, dollars.

#### 6-4 Economic Capacitor Installation

The procedure of section 6-3 may be applied in succession to each combination of capacitor size and location.

If it is found from inequality 6-3-2 that several capacitor installations are more economical than line improvements then the most economical single capacitor installation is given by the smallest value obtained from equation 6-3-1 from amongst all suggested capacitor installations.

## CHAPTER 7

### SUMMARY AND CONCLUSION

A method has been presented for the rural, radial distribution system whereby the short term economics of

- (1) wire change
- (2) wire addition
- (3) shunt capacitor installation
- (4) regulator installation

may be evaluated and automatically selected to give the minimum cost, while providing the desired improvement in regulation.

In addition the following corrective measures are available to the distribution engineer:

- (1) locating a new distribution station closer to a load center
- (2) providing new circuits
- (3) conversion to a higher primary distribution voltage.

These have been omitted from this study because it is believed that these corrective measures should be evaluated on a much broader scale than is required in this analysis. Once the major decision has been made, however, this paper may be useful to determine supplementary regulation.

With a dynamic, growing system the short-term solution will always be inadequate at some time in the future and it will be necessary to establish continuity in planning between successive short-term plans. To derive the maximum value from the distribution dollar, it is necessary to formulate a long-term plan, and establish the short-term time intervals, which may then be analyzed economically by the method of this paper.

Maximum allowance has been made for judgment in the preparation of the data since experience and human judgment

can detect impossible or unreasonable solutions. The "reasonable" solutions are entered into the computer, which then evaluates the most efficient solution.

It is not intended that the method as presented here is the end of the road. Much work remains to be done in extending and improving the method. These will include consideration of those items of system improvements that were omitted from the present analysis, and the inclusion of others such as an area load forecast, reliability and continuity of service, etc.

It is hoped that this presentation will initiate further research along the above outlines and in its own small way become one of the stepping stones to the advancement of the art and science of engineering.



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APPENDIX



# APPENDIX A

## Power Loss Saving from Line Improvement

The power loss over a line section equals the line load squared times a line constant

$$P = \frac{(\phi)(I)^2 (R)}{1000} \quad A-1$$

$$I = \frac{S}{(\phi) (E)} \quad A-2$$

$$P = (S)^2 \frac{R}{1,000 (\phi) (E)^2} \quad A-3$$

where

P power loss, kw

$\phi$  number of line phases

I line current, amps.

R resistance, ohms/mile

E line to neutral voltage, kv

S total line load, kva

Equation A-3 may be rewritten in terms of a constant, K.

$$K = \frac{R}{1,000 (\phi) (E)^2} \quad A-4$$

$$P = S^2 K \quad A-5$$

Postulate that the load, S, increases by a constant percentage, d, from one year to the next. Employing equation, A-5, it is possible to determine the loss for every year.

The accumulated losses to year n can be obtained by summing all terms in the loss series. Since, however, the last load which is available is last year's load, it is necessary to neglect the first term.

Accumulated Power Loss

$$= P \left[ (1+d)^2 + (1+d)^4 + (1+d)^6 \dots (1+d)^{2n} \right] \quad A-6$$

$$= P \left[ \frac{(1+d)^{2(n+1)} - (1+d)^2}{(1+d)^2 - 1} \right]$$

define a loss growth ratio

$$D = \left[ \frac{(1+d)^{2(n+1)} - (1+d)^2}{(1+d)^2 - 1} \right] \quad A-7$$

and

$$\text{accumulated power loss} = PD \quad A-8$$

Similarly the accumulated power loss saving from a line improvement for a time period of n years will be

$$= L_x D \quad A-9$$

where

$L_x$  reduction in power loss during the first year resulting from a line improvement, kw.

## APPENDIX B

### Voltage Drop Calculations

The voltage drop calculation for each mile section is based on the approximate formula given in ref. (14).

$$V = \frac{S (R \cos \theta + X \sin \theta) 120}{1000 \phi E^2} \quad B-1$$

where

- V voltage drop, volts/mile on 120 volt base
- S line load, kva
- R resistance, ohms/mile
- X reactance, ohms/mile
- E line to neutral voltage, kv
- $\phi$  number of line phases
- $\theta$  phase angle between voltage and current

Defining a wire factor

$$W = \frac{(R \cos \theta + X \sin \theta) 120}{1000 \phi E^2} \quad B-2$$

the voltage drop may be written as

$$V = SW \quad B-3$$

A line improvement will produce a new wire factor.

The improvement in voltage from a line improvement will be,  $\delta V$ .

$$\delta V = S(W_o - W_n) \quad B-4$$

where

- $\delta V$  the improvement in voltage resulting from a line change, volts on 120 volt base
- $W_o$  wire factor of the line before the system improvement

$W_n$

wire factor of the line after the system improvement

## APPENDIX C

### Power Loss Saving from Capacitor

The power loss during the  $t$ 'th year on a line without a capacitor is

$$P_t = \left[ (1+d)^{2t} I_r^2 + (1+d)^{2t} I_x^2 \right] R$$

where

$P_t$	power loss during year $t$ , kw
$d$	annual load growth, percent
$I_r$	real component of line current
$I_x$	complex component of line current
$R$	resistance, ohms/mile

When a capacitor is added the line current during year,  $t$ , will be

$$I_t = (1+d)^t I_r + j \left[ (1+d)^t I_x - I_2 \right]$$

$$I_2 = \text{capacitor current}$$

The power loss during the  $t$ 'th year with the capacitor is

$$P_t = (1+d)^{2t} I_r^2 R + (1+d)^{2t} I_x^2 R + I_2^2 R - 2(1+d)^t I_x I_2 R$$

The saving in losses due to the capacitor in the year  $t$  is

$$2(1+d)^t I_x I_2 R - I_2^2 R$$

( $I_2$  is positive)

The total saving in losses over a number of successive years will be the sum of the savings in each individual year.

To make the formula useful we only wish to consider the year 1 to  $n$  in the future. Savings due to a recommended installation of capacitors cannot be realized until the next peak, or the year 1.

The sum of the second term over  $n$  years is

$$- n I_2^2 R$$



The sum of the first term is

$$\frac{2(1+d)}{d} \left[ (1+d)^n - 1 \right] I_x I_z R$$

Loss saving over n years

$$2 \frac{(1+d)}{d} \left[ (1+d)^n - 1 \right] I_x I_z R - n I_z^2 R$$

$$I_z = \frac{C}{3E}$$

$$I_x = \frac{C}{3E}$$

The loss saving in kw over n years for three phases

$$L_z = \frac{CR}{3000E^2} \left\{ 2 \frac{(1+d)}{d} \left[ (1+d)^n - 1 \right] - nC \right\}$$

APPENDIX D

Economic Line Improvements with New Capacitor

It may be asked if the most economic solution is still obtained when some terms of a combination of line improvements are neglected.

Refer to figure 5.

The combinations of line improvements are numbered in ascending order corresponding to ascending cost.

A capital cost of line improvement, dollars  
subscript refers to the location of line improvement

total cost of combination	sequence number
$A_a + A_b + A_c + A_d + A_e$	56
$A_a + A_{12}$	55
-	-
-	-
-	-
	1

According to step (4) of section 6-3 the most expensive combination of line improvements which may be replaced by the capacitor boost setting at the end point of branch b is

$$A_b + A_c + A_d + A_e$$

Since

$$A_a + A_{12} < A_a + A_b + A_c + A_d + A_e$$

and

$$A_a \text{ of } 55 < A_a \text{ of } 56$$

then

$$A_{12} < A_b + A_c + A_d + A_e$$

# APPENDIX E

## Flow Diagram

Read (1) suggested list of line improvements  
(2) net cost of each line improvement,  $A_x$   
(3) system data, etc.

eq. A-7

Calculate D

For each suggested line improvement

Calculate

eq. 3-3-1	Net total cost of line improvement	$T_x$
3-3-2	Improvement in regulation due to line improvement	V
3-3-3	Per volt cost of line improvement	G
3-4-7	Error in G	$\epsilon$

How many suggested line improvements are there in the line subsection:

3-5

None ?

Yes

One ?

Yes

Two ?

Yes

Is the first line improvement clearly more economical?

Yes

No

Is the second line improvement clearly more economical?

Yes

No

Store  $G_1 + g_1$ ;  $V_1$ ; location on system

Store  $G_2 + g_2$ ;  $V_2$ ; location on system

Store  $G_1 + g_1$ ;  $V \neq$  location on system

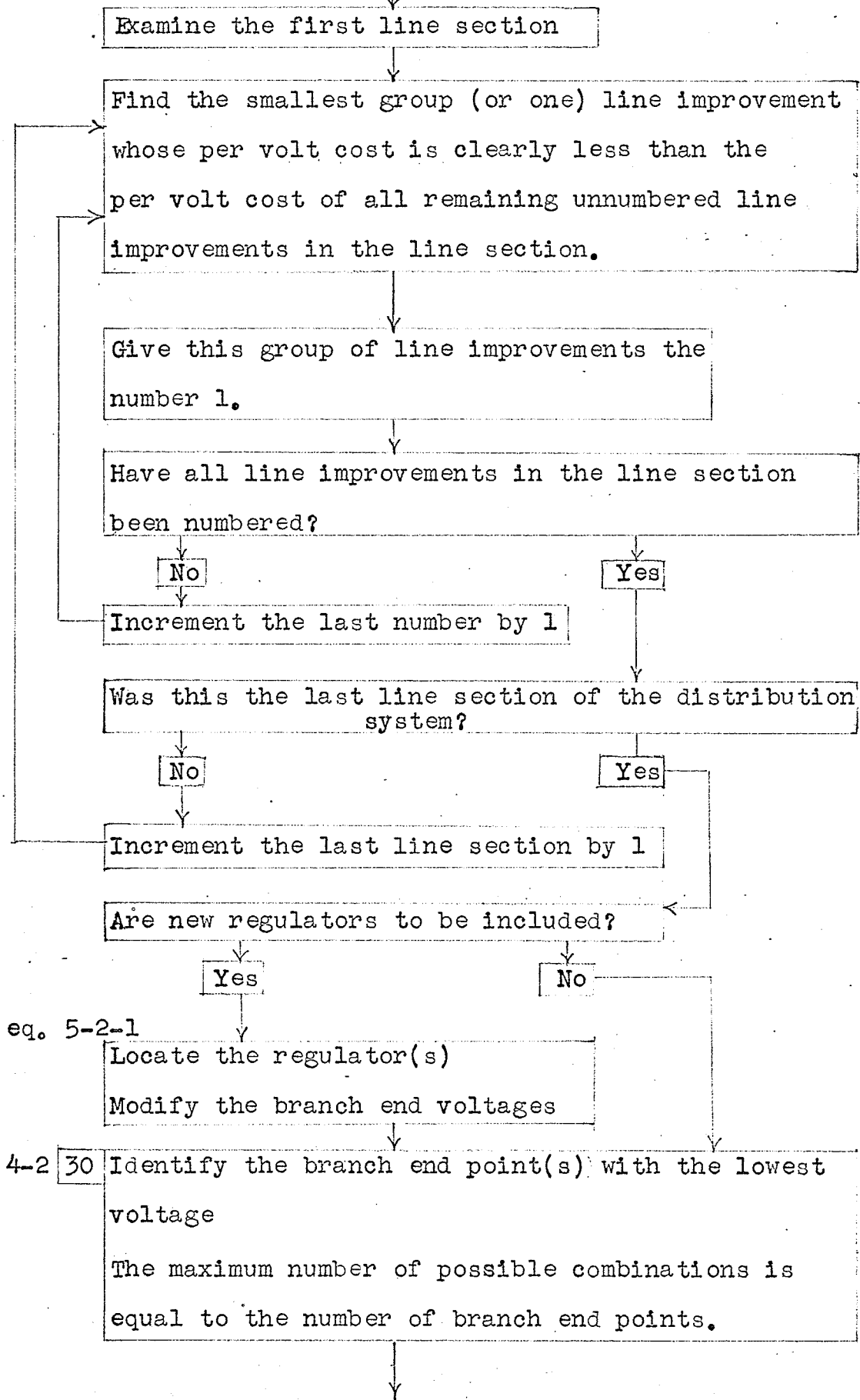
Is this the last line subsection?

Yes

No

Go to the next line subsection

↓



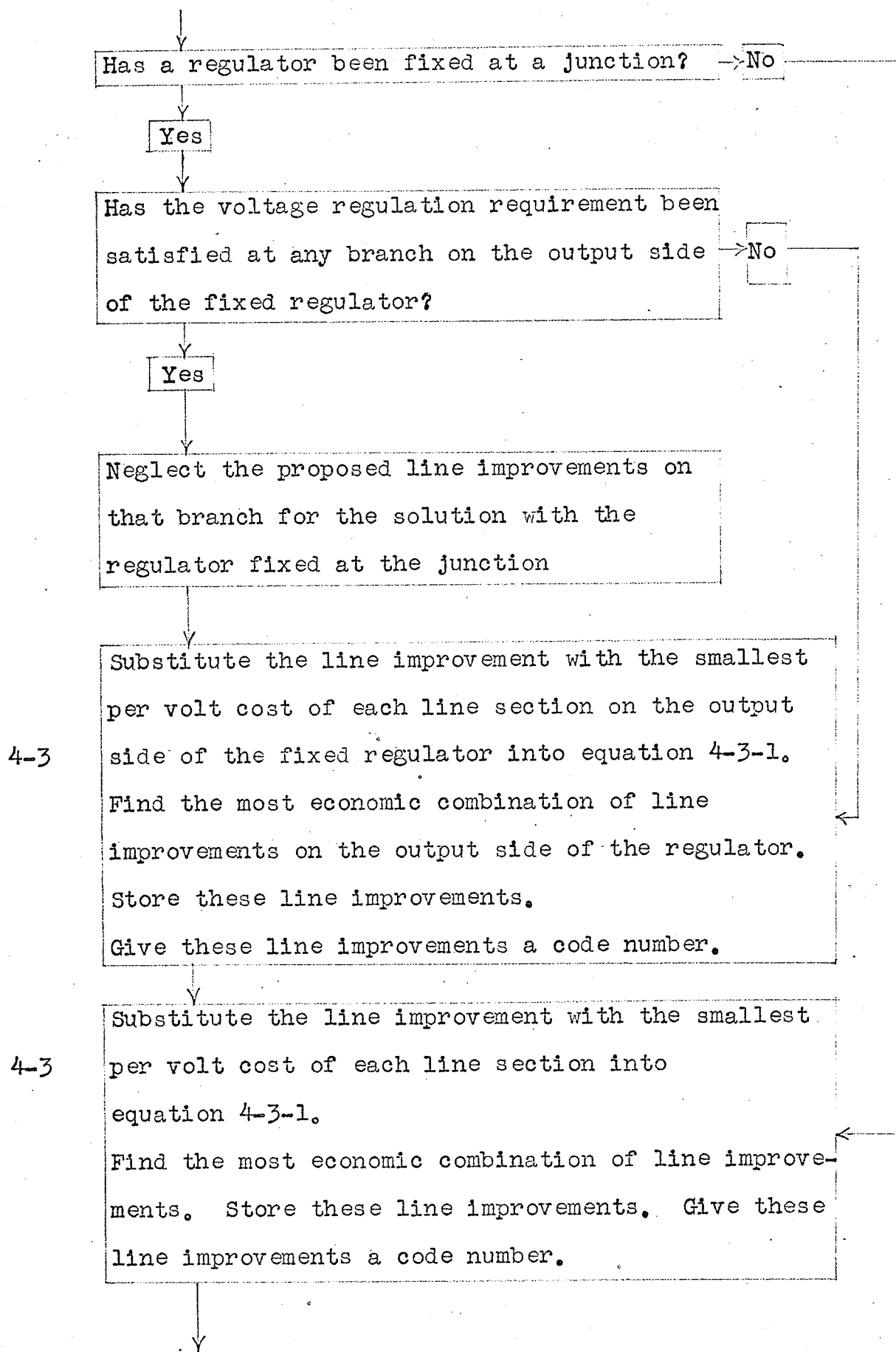
↓  
Y  
Locate the branch end point with the next lowest voltage. Find the voltage difference,  $V_{xy}$ , between the lowest and second lowest voltage.

↓  
Y  
Neglect for a time all branches whose branch end points are not at the lowest voltage.

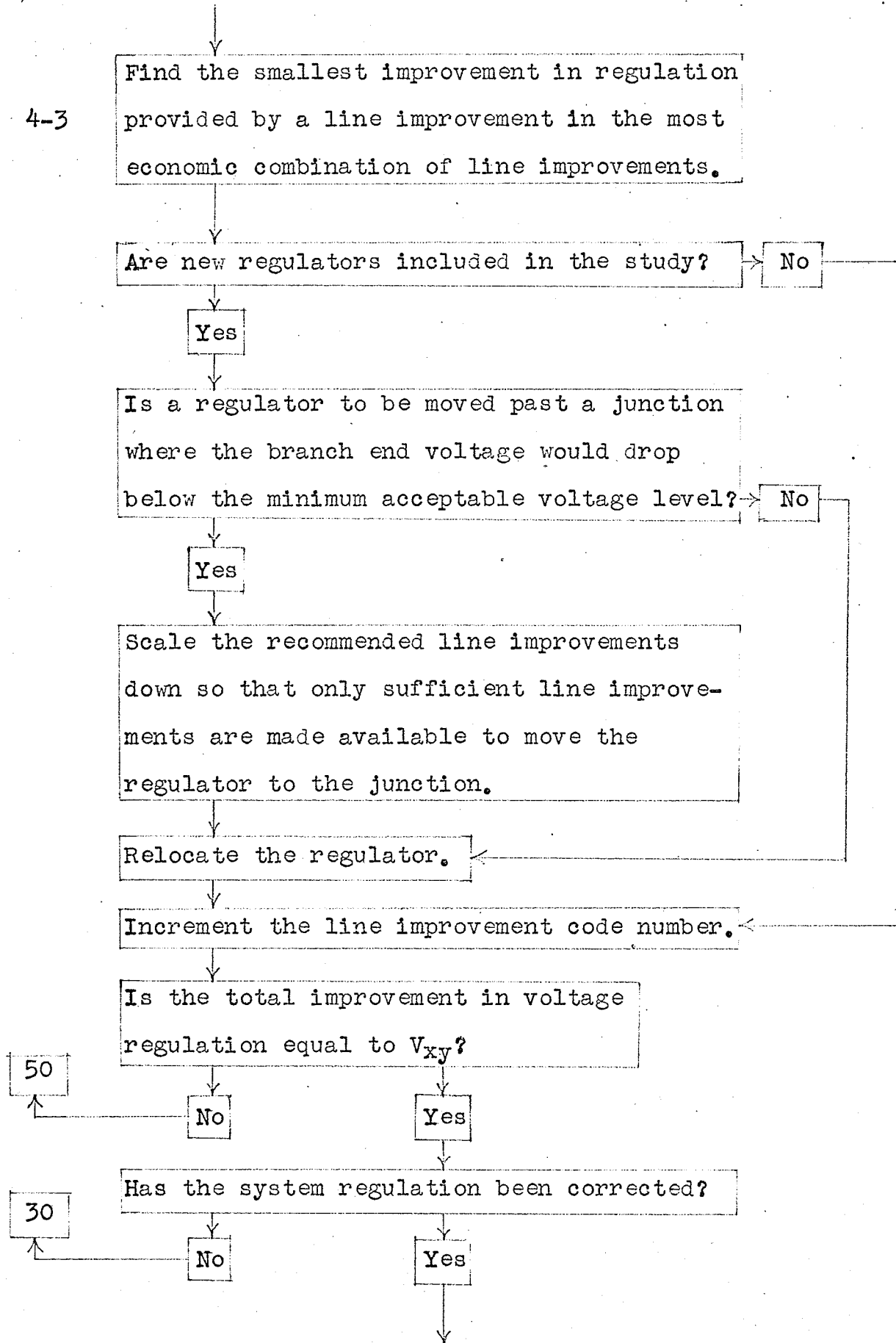
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Y  
3-6 50 Find the line improvement with the smallest per volt line improvement cost in each line section. The number of wires of the line improvement must not exceed the number of wires of the previous line subsection  
4-4

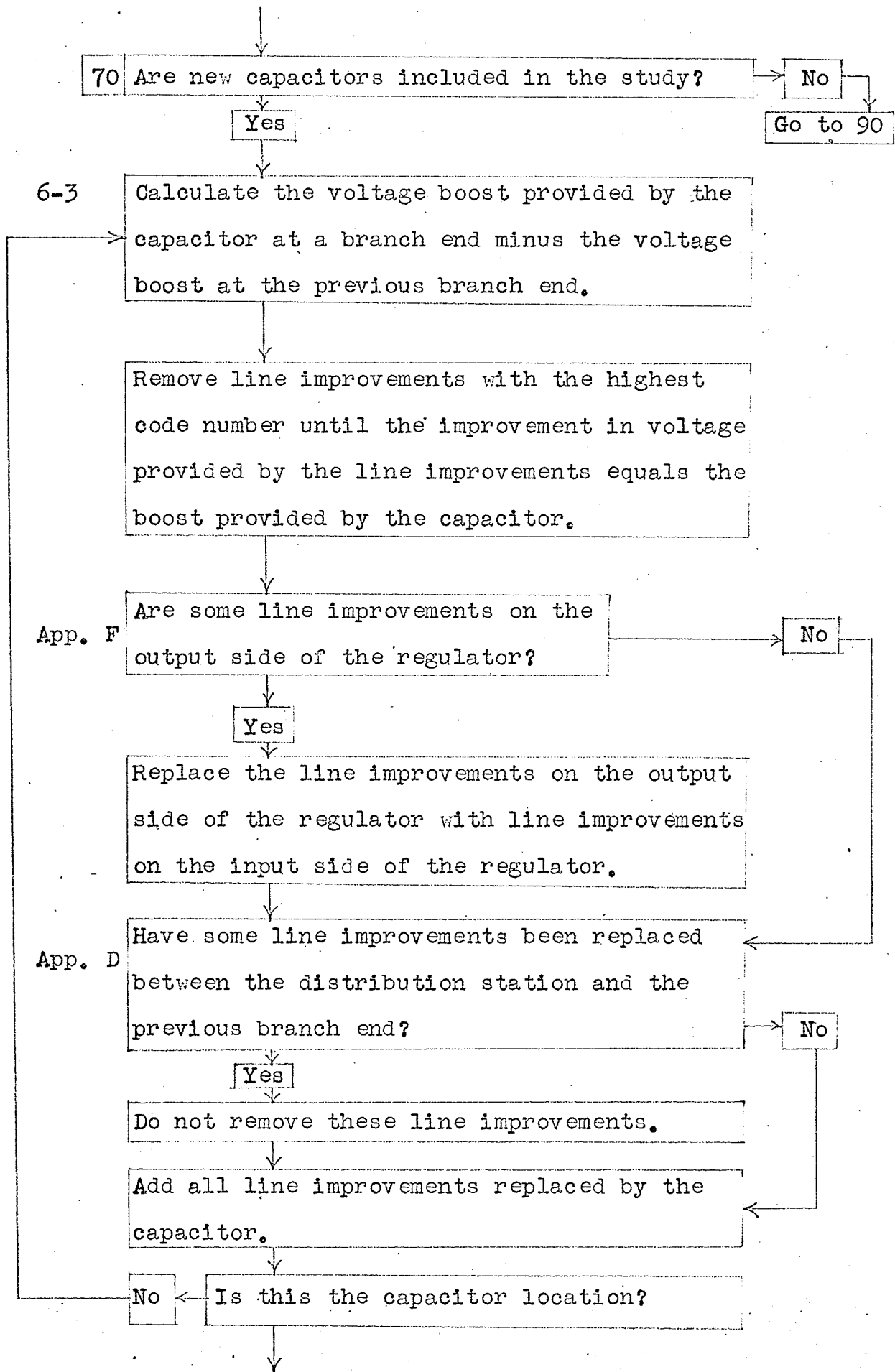
↓  
Y  
4-2 Find all permissible combinations of line improvement

↓  
Y

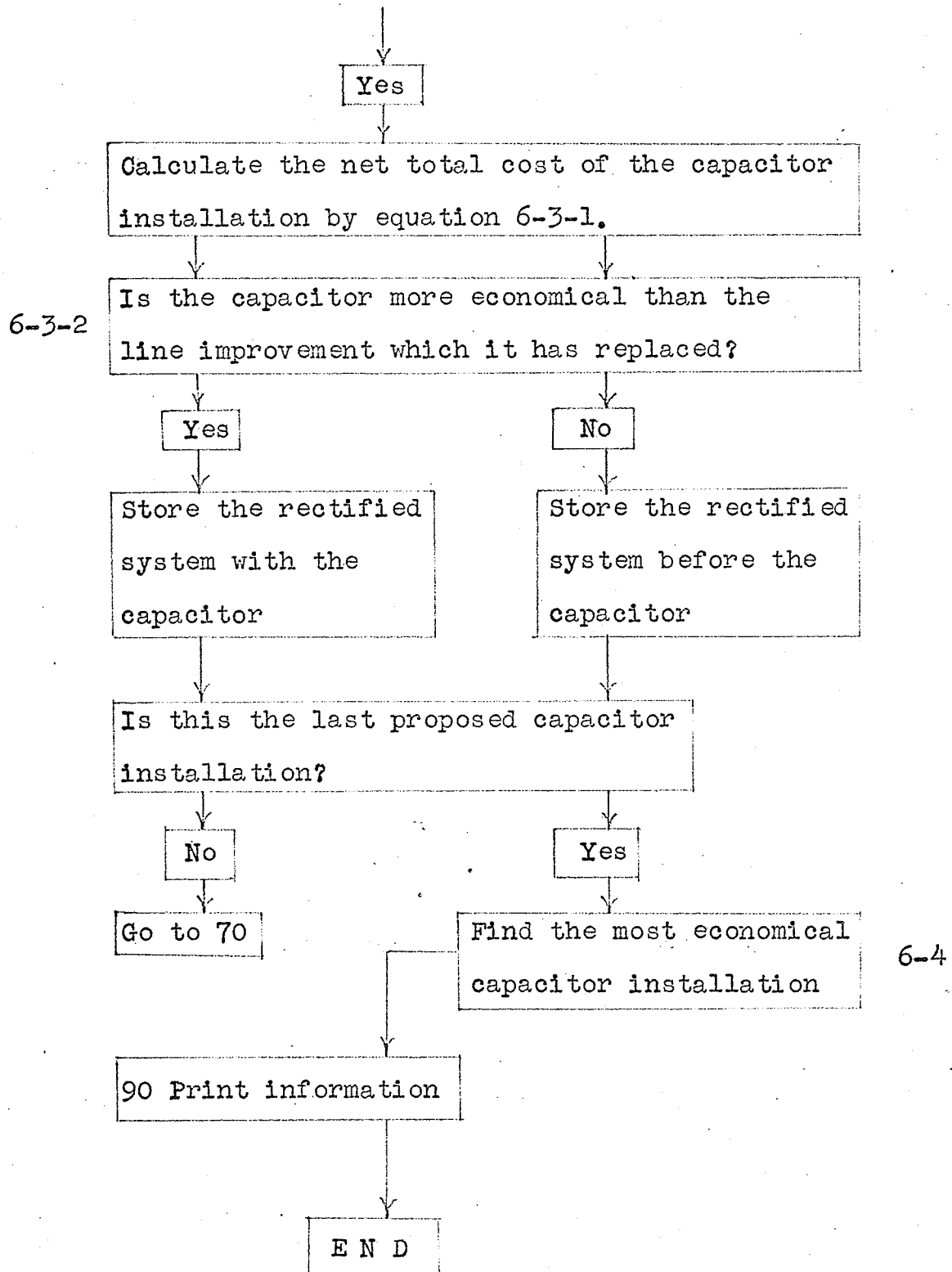


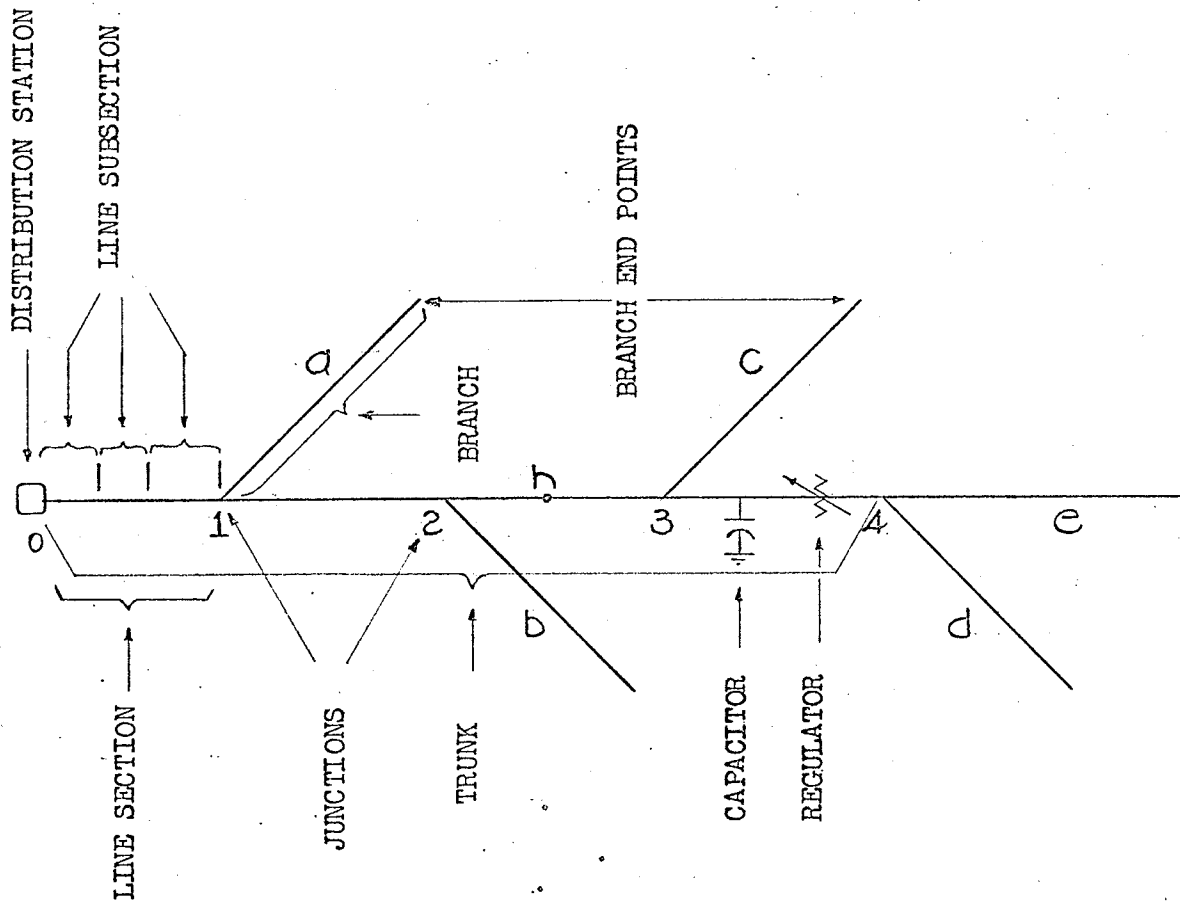
4-3



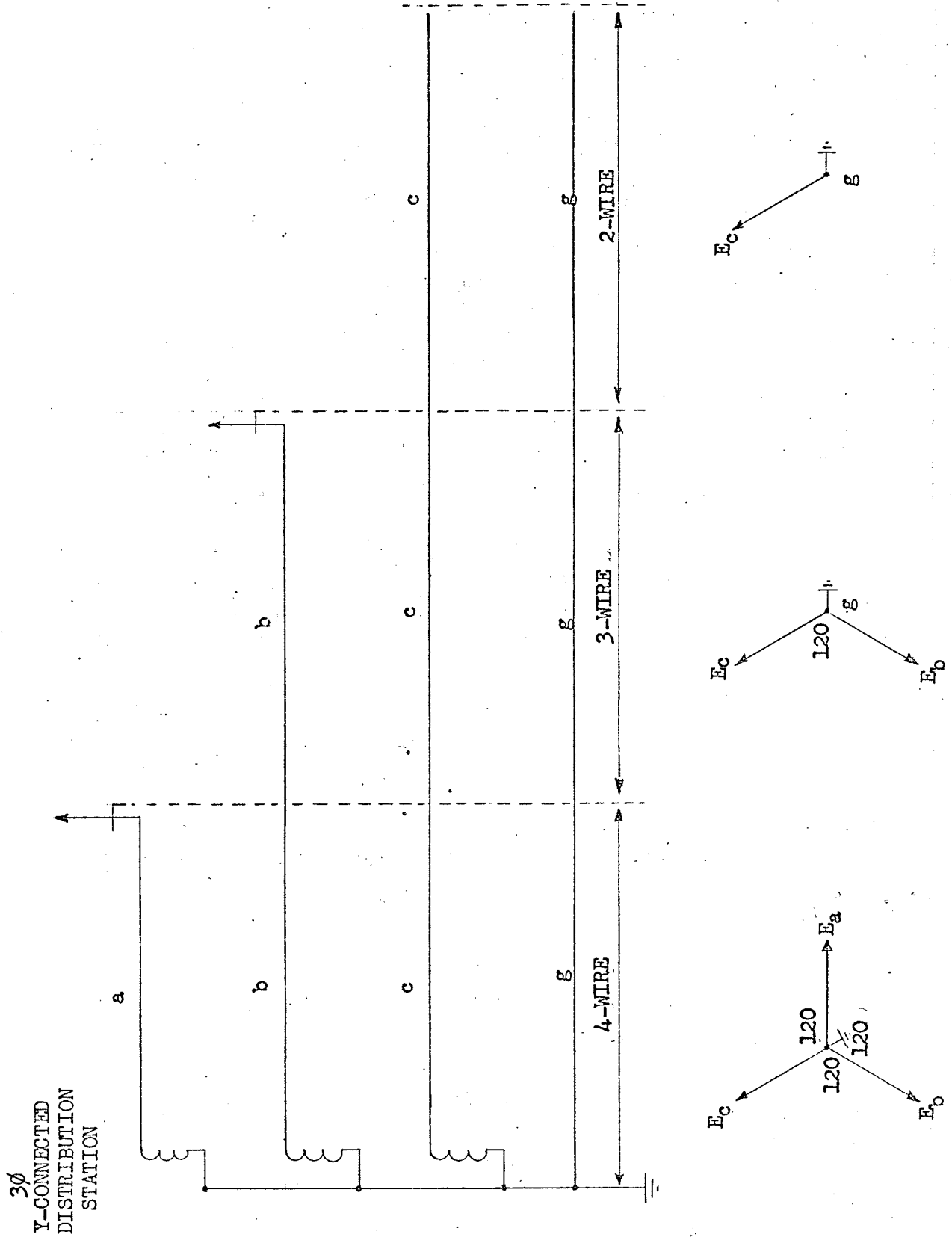




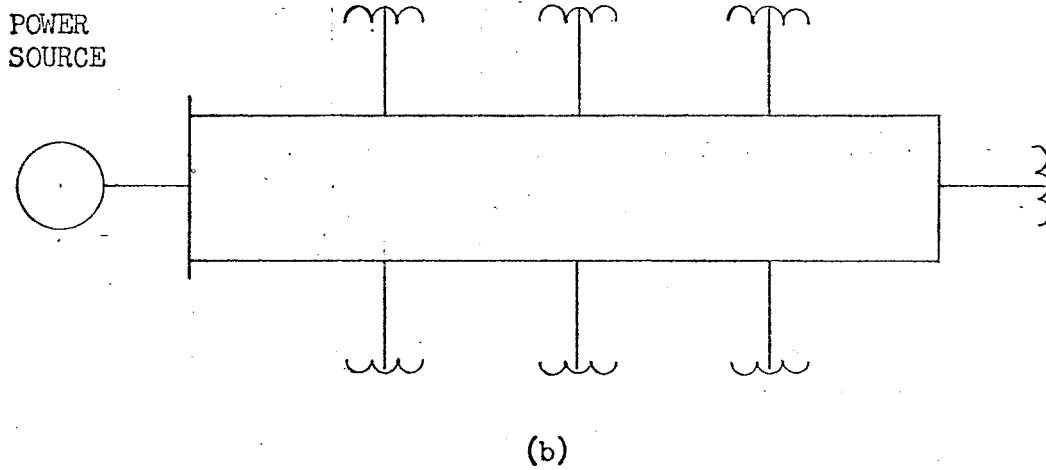
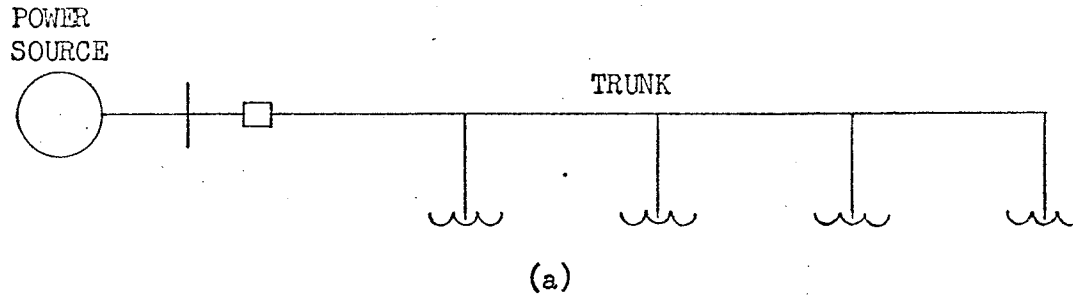




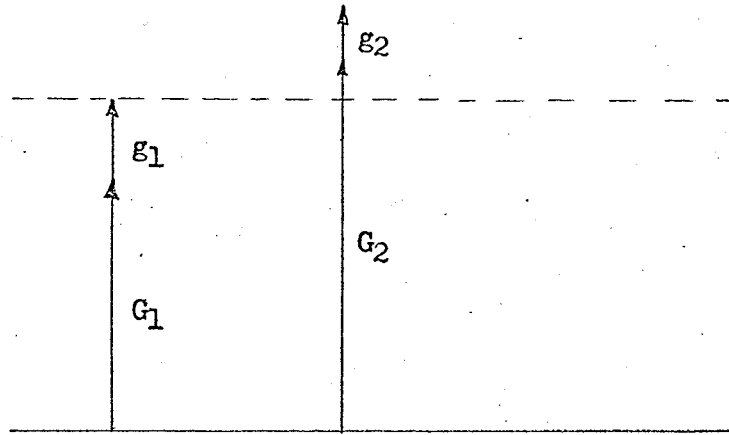
DRAWN B.P.M.	MANITOBA HYDRO ENGINEERING DIVISION	SCALE N.T.S.
CHECKED		DATE JAN. 1966
APPROVED	TYPICAL DISTRIBUTION SYSTEM	DRAWING No. 1



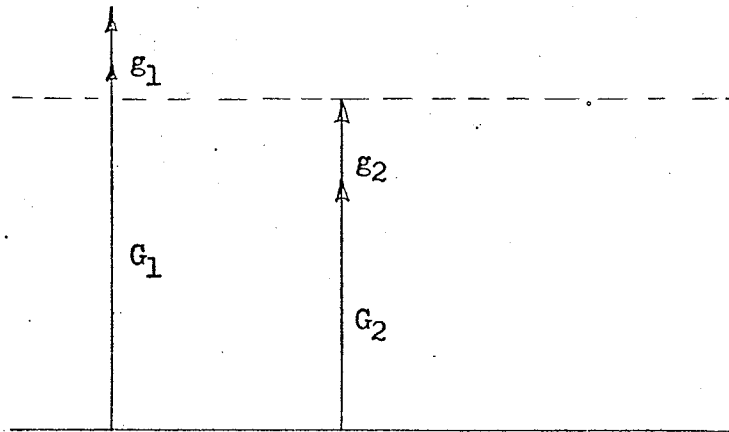
DRAWN E.C.O.	<b>MANITOBA HYDRO</b> ENGINEERING DIVISION  Possible Wire Arrangements On a 3 $\phi$ Y-Grounded Distribution System	SCALE
CHECKED		DATE July, 1966
APPROVED		DRAWING No. 2



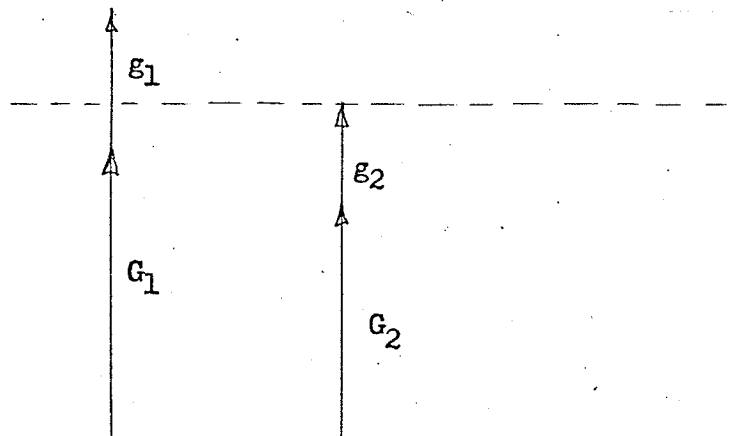
DRAWN	<b>MANITOBA HYDRO</b> ENGINEERING DIVISION  (a) Simple form of radial system. (b) Simple form of network system.	SCALE
CHECKED		DATE
APPROVED		DRAWING No. 3



(a)

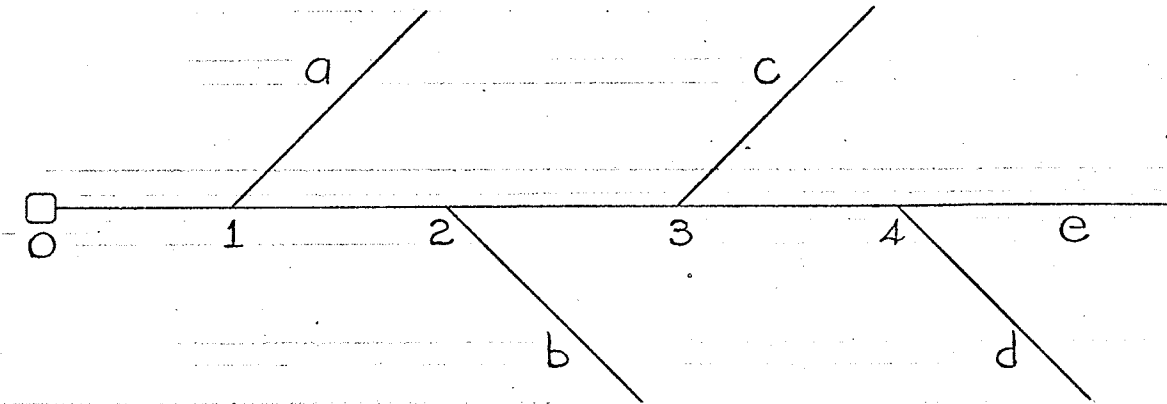


(b)

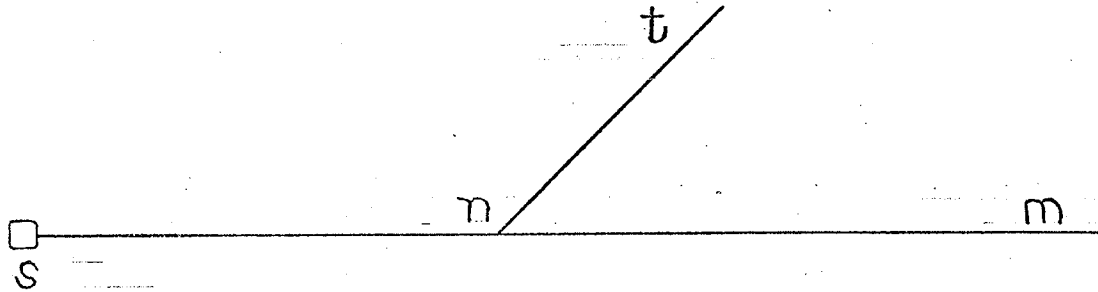


(c)

DRAWN E.C.O.	MANITOBA HYDRO ENGINEERING DIVISION	SCALE
CHECKED		DATE July 20, 1966
APPROVED	(a) Line Improvement #1 most economical. (b) Line Improvement #2 most economical. (c) Intermediate Case.	DRAWING No. 4



DRAWN R.P.M.	MANITOBA HYDRO ENGINEERING DIVISION	SCALE N.T.S.
CHECKED		DATE JAN. 1966
APPROVED		DRAWING No. 5
TYPICAL DISTRIBUTION CIRCUIT		



DRAWN R.P.M.	MANITOBA HYDRO ENGINEERING DIVISION	SCALE N.T.S.
CHECKED		DATE JAN. 1966
APPROVED		DRAWING No. 6
MOVING REGULATOR PAST JUNCTION "n"		