INVENTORY CONTROL MODELS FROM BUYER'S AND VENDOR'S PERSPECTIVES

by

© Sandeepa Goel

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TO MY HUSBAND AND MY PARENTS

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Sandeepa Goel

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TABLE OF CONTENTS

ABSTRACT

Inventories play an important role in the efficient functioning of an organization. A variety of models for inventory management, most of which assume that the unit cost of purchasing is independent of time and free from inflation, are available in the literature. In this thesis with the aid of several simplifying assumptions, such as

i) the supplier has multi-item to supply,

ii) supplies are subject to constant inflation, and

iii) the unit price is subject to the same inflation as other inventory related costs, a model has been developed which allows the determination of the optimal quantity and the optimal quantity discount schedule for a supplier under inflationary conditions. This model suggests that the optimal order quantity increases as the rate of inflation increases.

The effect of inflation on a generalized EOQ formulation from both a buyer's and a vendor's perspectives has been analyzed by considering two common price policies - a) fixed mark-up on purchased cost, and b) fixed amount over purchase cost.The analysis shows that the calculations of model related to the first pricing policy (fixed mark-up)is simpler than the one related to the second price

policy (fixed amount over purchase cost).In addition to the inflation, effects of advertizing, price elasticity and economies of scale have also been incorporated in the formulation.

Most of the buyer's and the vendor's inventory models assume the ideal conditions in which vendor maximize his profits and buyer minimizes his inventory related costs. However, such an ideal situation is far from being real. More realistically, vendor can maximize his profits subject to the maximum costs buyer is prepared to pay, and by the same token buyer can minimize his inventory related costs subject to the minimum profits acceptable to the vendor. In addition to these constraints, there are other constraints such as the floor space available to the buyer and the maximum number of orders vendor can handle. Also, there are situations where holding/set-up costs of buyer as well as vendor and some values of buyer's costs and vendor's profits are known. A generalized model, which incorporates these various constraints, has been developed by using two separate approaches i) lagrangian multiplier, and ii) linear programming.

Both approaches yield identical results and provide a functional relationship between the buyer's and vendor's EOQs.

Following a similar approach, a generalized quantity discount model has been developed to increase vendor's profits and decrease buyer's inventory related costs at the same time .

The effect of announced price increases on optimal ordering quantity has also been analysed.The analysis shows that due to price increases, the optimal ordering quantity also increases creating cost savings for the buyer.

CHAPTER-1

INTRODUCTION

Inventory management has been hailed to be one of the most important management functions in the present business climate. It plays a very important role in the national economy. The report of Time magazine (1982) on U.S economy may be cited as an example.

"Inventory liquidation has been driving the economy steadily lower since last December.....Companies have been emptying warehouses, trimming stock piles and cutting back orders from suppliers. This was the major cause of the 3.9% drop in the gross national product during the first quarter........ with interest rates high and sales projections dismal......few businesses seem eager to start hastily rebuilding their stocks to former levels".

The impact of inventory levels on management performance measures, such as financial ratios, is immediate. Therefore, inventory is usually classified as one of the current assets of an organization. A slight reduction in inventory lowers assets and affects the financial ratio which is normaly used as a measure of liquidity. Changes in the inventory level may also affect revenues and operating expenses which, in turn, may cause a perturbation in the operating profits of an organization and affect its return on investment. Consequently companies are allocating more resources to controlling inventory.

A survey of 1700 companies in 1981 showed that 64% were already using some form of inventory control systems and many others were considering doing so. ("Fortune" magazine march(1981)).

The recent report in "Fortune" Magazine (May 25(1987)),

"While almost everything else declined, swelling business inventories pushed GNP growth to a 43% annual rate." High lights the importance of inventory control.

Insufficient inventories hamper production and fail to generate adequate sales, whereas excessive inventories adversely affect the cash flow and the liquidity position. From either perspective, poor inventory management can present a serious threat to the viability of an organization and can have disastrous effects on its solvency(Monks(1977)). The sales department sees inventory control as fundamental to good customer service and feels that manufacturing is inadequate if any item is not available at the time it is due to be shipped. From the financial perspective, inventories are a necessary evil that ties up capital which could be better used elsewhere. Operating managers have difficulty in understanding the costs associated with carrying inventories. They look upon inventory control measures with dismay because of the apparent inefficiency forced on the plant. From the manufacturer's point of view, inventories should be an unlimited resource.

Inventories in a business serve very much like the suspension system of an automobile. Ups and downs in sales can be absorbed by inventory. Without inventories, production would have to respond directly to sales if service to customers is not to suffer.

Inventories also disengage manufacturing operations from varying production rates. Lot size inventories make possible fewer machine setups and higher machine utilization(Plossl (1985)).

Inventories are necessary to give good customer service, to run the plant more efficiently by keeping production at fairly leveled rates and to run reasonable sized manufacturing lots. Inventory is thus, not a necessary evil but, instead, a very useful shock absorber.

Inventories also make possible smooth and efficient operation of an organization by decoupling individual segments from the total operation, thus allowing flexibility in planning, production and marketing. As a result production costs, material handling costs, purchasing costs etc., which would contribute substantially to a firm's profits,may be considerably reduced e.g.("Fortune" magazine 1981):

"The Minnesota study provides the first statistical evidence of the big payoff companies can wring from sophisticated inventory control. Installation costs ranged from less than $100,000 for small companies to more than $1 million for large ones. But the average increase in annual inventory turnover was an astounding 50.3%. For the typical company with $65 million in annual sales, that made possible an inventory reduction of about $8 million, and a saving of $1.8 million per year in carrying costs calculated at recent interest rates."

Inventory management broadly comprises developing, implementing and reviewing inventory policies relating to procurement, storage, use and disposal of inventories to achieve the requisite service level while keeping the investment in inventories within appropriate financial constraints.

The concept of the economic lot-size was first published in 1915 and a statistical approach to determining order points was presented by Wilson in 1934. However, these fairly sophisticated techniques of inventory management had very little application. Perhaps this was because of lack of encouragment of scientific management in the 1930's and 1940's. During the depression of the 1930's the most important objective of most companies was survival. During the late 1940's when pent-up demand provided a ready market for every article that could be produced, inventory control, leveling workload and competition on the basis of customer service - were not important in most business operations.

In the meanwhile the scientific theory of inventory control has developed gradually. The scientific management movement from the early 1900's to World War II, had helped to provide a basis for production and inventory control. Management scientists started to direct their attention to production and inventory control in those cases where the essentials of the problem could be expressed numerically and statistical probability theories could be applied and many of the decisions could be made as a result of balancing alternative solutions. Considerable progress was made in forecasting, inventory control and mathematical programming.

There have been two streams of development in the field of inventory operations. One is represented by the mathematical abstraction of the inventory system in which the major effort has gone into modeling the process and searching for optimal policies in terms of minimizing relevant costs. The other stream is primarily concerned with practical issues such as demand and cost measurement, system design, relations among logistics and other industrial management functions and system management, (e.g, Magee (1968) and Brown (1967,1977)).

An inventory system can be defined as a coordinated set of rules and procedures that allow for routine decisions to be made on the quantity and timing of order of each item needed in the procurement manufacturing and processes to meet customer demand. The question regarding how much to order is answered by the "economic order quantity" (EOQ). The EOQ provides an optimal order or lot size by minimizing the cost components such as the ordering costs, the inventory carrying cost and the stock-out cost (if shortages are permitted). EOQ formulas apply to individual items and indicate an optimal condition for each item based on some definite assumptions regarding costs. The EOQ approach has some significant advantages for example, for a family of items for which set-up and inventory carrying costs are about the same, the EOQ approach provides a much simplified method of calculating optimal order quantity for each item. When there is a restriction on the number of orders that can be handled by this approach to obtain the least total lot size inventory for the family of items, a very important point is often overlooked: The application of EOQ's is much more effective when items are grouped together.

Unfortunately, the EOQ formulas have little or no practical application, mainly because they are based on many assumptions, such as the amount of inventory carried is a direct result of the number of orders placed and the inventory will be withdrawn at a fairly uniform rate. EOQ further assume that the only factors significant in the calculation of the most economical lot size are those included in the formulas and that costs relating to ordering and to carrying inventory vary uniformly with the size of the lot ordered.

For the more realistic results from the EOQ approach it is necessary to modify the EOQ calculations. For example, SCRAP LOSSES can be offset by inflating the lot size by the percentage of average loss expected. MINIMUM QUANTITIES can be established to place a floor under the calculated EOQ's to reflect a vendor's minimum purchase of quantities or batches of items made from one unit of raw material. MAXIMUM FIGURES are set as a ceiling on calculations for bulky items where space limitations exist. Calculated EOQ's are also adjusted to even multiples of packaged lots(such as dozens, pallet loads), container batches in which the items are moved or units of raw material(coils, bundles,drums etc) from which the item is made.

OBJECTIVES

The EOQ formulas contain many assumptions. Several attempts have been made in the past to relax the assumptions involved in the classical EOQ model by, for example, introducing multi-items under constrained systems, quantity discount schedules, consideration of inflationary trends etc. In most of the work involving these modifications the classical EOQ model is considered only from the buyer's perspective, such as how much to order in order to minimize inventory-related costs and how a buyer should react to a quantity- discount schedule(Hadley and Whitin (1963),Chase and Aquilano(1981) and Silver and Peterson(1985)). The theory approaching the problem from the vendor's perspective,i.e. how to order to maximize profit and how a vendor or seller should develop a discount pricing structure is much less established.

Inflation is a world-wide phenomenon; in particular the 1970's may be rightly called the decade of inflation. In the business world of industrial change and complexity of business activity when the world is affected by double digit inflation and the size of inventory is becoming very large, most of the inventory projections based upon historical cost may provide misleading information and, therefore, could be detrimental to the interests of an enterprise. All of the literature concerning economic lot size calculations under inflationary conditions(constant and variable rate of inflation) consider the situation from the buyer's side (e.g, Buzacott(1977),Gupta(1987) and Aggrawal(1981)). No work has been done which treats the inflationary effect from the vendor's perpsective. Inflation affects the economy as a whole so it is worthwhile to see the effect from the vendor's point of view as well.

Quantity discounts are also among the important issues of inventory control because quantity discounts may give buyers cost-lowering opportunities beyond those explicit in the discount schedule itself. To lower cost per unit, a buyer may order quantities larger than necessary and enter prearranged resale agreements or adhoc brokerage situations. Many examples are available in the literature to show how quantity discounts are beneficial to the buyer (e.g,Hadley and Whitin(1963), Hax and Candea(1984)).But it is also beneficial for sellers in the sense that sellers save in several ways by selling fewer, larger orders to their customers. One such saving arises from lower sales costs in that fewer sales calls are made, fewer orders are processed and so on. A second saving arises from lowered costs for raw materials because quantity discounts are often available to the seller. Third, the time value of money is taken in-to account because larger revenues are available for reinvestment for longer periods. Finally, longer production runs without attendant increases in holding costs are possible(Monahan(1984), Crowther(1964)).

With the help of discounts a seller can maximize profits by modifying the buyer's order policy (Lal and Staelin(1984)). But there is a need for discount policies which not only maximize the vendor's(seller's) profits but at the same time minimize the buyer's inventory-related costs. In this area, some work has been done (for example Banerjee(1986 b) with a joint economic lot size approach, Dada and Shrikanth(1987)). But they still overlooked one important area in that their models are not applicable in different markets where the aimed-for profits and costs are preassigned.

The main objectives of the present thesis are:

(i) To develop a more generalized inventory model from both buyer's and vendor's perspectives under constrained system.

(ii) To analyse the vendor's inventory problem under an inflationary environment with a constant rate of inflation for multi-products under constrained system.

(iii) To analyse both buyer's and vendor's perspective with inflation under a different pricing policy.

(iv) The development of generalized quantity discount schedules from buyer's and vendor's perspectives.

(v) The optimal ordering quantity for an announced price increase for the buyer.

(vi) Application of sensitivity analysis to inventory systems in different areas with the help of numerical examples.

SCOPE OF THESIS

The scope of the thesis is as follows:

(i) The emphasis is mainly on the development of theoretical aspects of inventory system. This needs demonstration through actual case studies.

(ii) The proposed models are illustrated by illustrative examples. However, real-life data not being available, these could not be used for testing these models.

(iii) Only deterministic models are considered. Demand and lead time uncertainties are not covered.

(iv) The models under inflationary condition are studied in relation to cost minimization under a constant inflation rate with the condition that the inflation rate should be less than the inventory carrying costs.

(v) Most of the inventory models are analysed without shortages.

(vi) An attempt is made to study one buyer's and one vendor's case.

(vii) No specific case is considered either at the national or organizational level , due to the constraint of time and limited scope of the study.

LAYOUT OF THE THESIS

Chapter 1

This introductory chapter contains general literature, objectives and scope of the thesis.

Chapter 2

In this chapter, an inventory control vendor model in terms of an economic quantity discount (EQD) schedule under the effect of inflation has been developed and analysed.

Chapter 3

The effect of inflation on generalized economic order quantity modelled from the buyer's and the vendor's perspectives has been analysed in this chapter. The model is examined using two pricing policies given by Buzacott(1975) under three scenarios given by Lee and Rosenblatt(1986) and the assumptions provided by Subramanyam and Kumaraswamey (1981).

Chapter 4

In this chapter, a new approach to inventory control is provided. The objective has been to develop a generalized inventory control model which considers the perspectives of both the buyer and the vendor. The model uses two different approaches, namely (i) the application of Lagrangian multipliers in conjunction with the Kuhn-Tucker conditions and (ii) the application of linear programming. It is shown that both approaches yield the same results. The approaches developed are illustrated by numerical example.

Chapter 5

In this chapter a generalized optimal quantity discount approach, which minimizes buyer's costs and maximizes vendor's profit, has been developed. Through the dual variables a relationship between vendor and buyer models and related pricing schemes has been established.

Chapter 6

This chapter deals with a situation in which a buyer has an opportunity to place a special order before an announced price increase takes effect.The proposed model is examined under three scenarios which take into consideration the effects of factors such as advertisement and damage during shipment.

CHAPTER 7

Conclusions and suggestions for further work have been listed under this chapter.

NOTATION

$\overline{A}_j(t)$ = The vendor's order processing and manufacturing set-up cost for the j^{th} item (\$/order) at time "t".

A_{0j} = The vendor's order processing and manufacturing set-up cost for the j^{th} item (\$/order).

$A_1(t)$ = The buyer's cost of processing and procuring an order at time t.

$A_2(t)$ = The vendor's set up cost per set up at time t.

A_1 = The buyer's ordering or processing cost per order.

A_2 = The supplier's (vendor's) production setup cost per setup.

B = Amount of demand unsatisfied and put on back order list.

$\overline{C}_j(t)$ = The vendor's unit manufacturing cost exclusive of order processing for j^{th} items at time "t".

C_{0j} = The vendor's unit manufacturing cost exclusive of order processing, manufacturing set-up and inventory carrying cost (\$/unit) for the j^{th} items.

C_j = The number of inventory cycle over the planning period 'L' for the j^{th} item.

$C_1(t)$ = The cost per unit at time t incurred by the buyer.

$C_2(t)$ = The unit production cost at time t incurred by the vendor.

C_1 = The buyer's unit purchase cost.

C_2 = The vendor's total variable production cost per unit.

C_1' = Cost per unit.

C_2' = Cost per unit after the announced price increase becomes effective($C_2' > C_1'$).

D_j = The total yearly number of units of item 'j'demanded by the customer; $j = 1, 2, \ldots, N$.

D = The total yearly number of units demanded by buyer.

$d(K)$ = Per unit dollar discount.

f = Upper limit on the floor space available.

f_j = Floor space required by one unit of the j^{th} item.s/year).

H_j = The vendor's holding cost of the j^{th} item expressed as an annual percentage (%/year).

$h_j(t,t+w)$ = $H_j\overline{C}_j(t)w$, the inventory holding cost of item j produced at time t and held in stock until time t+w.

$h(t,t+1)$= Inventory holding cost in \$/\$/ unit time at time t and held in stock until t+1. It is usually assumed that $h(t,t+1) = H$ where H is the inventory holding cost in \$/\$/ unit time.

H_1 = The buyer's inventory holding cost in dollar per dollar per unit of time.

H_2 = The vendor's inventory holding cost in dollar per dollar per unit of time.

K_j = Inflation rate for the j^{th} item.

K = Rate of inflation (percentage/ unit time).

K^* = Optimal discount rate.

k = Positive integer, greater than or equal to one.

L = Planning period.

M_2 = The vendor's gross profit on sales.

M = Gross profit expressed as a percent.

N = Number of times the product is advertised.

$p(t)$ = Selling price per unit at time t.

Q_j = Order quantity for the j^{th} item.

Q_j^* = Optimal order quantity for the j^{th} item.

Q_v & Q_b= Order quantity(production batch size) per order from vendor and buyer's perspective.

Q = Current order quantity before price increase

Q_{opt} = Special order quantity.

R = Vendor's annual production rate for the item per unit of time.

R_j = The vendor's annual inventory production rate of the j^{th}item (units/year).

s = Shortage cost for each unit short.

T_j = Inventory cycle for the j^{th} item.

TC = Total system cost.

$TC_1{}'$ = Total inventory related cost before the price increase.

$TC_2{}'$ = Total inventory related cost after the price increase.

TC_1 = The buyer's total relevant cost per period.

U = Fraction of number of items that are damaged or are defective.

v = Unit cost of selling a defective item.

Z_v & Z_b= Net profit per unit time from vendor and buyer's perspective, and Z_1, Z_2 and Z_3 are net profit for scenario's 1, 2, and 3 respectively.

αpD = Advertising expenditure ($0 < \alpha < 1$) which is a fraction of the total revenue or we can express it as $\alpha MC_2{}'D$.

Π_2 = The vendor's total profit per period;

$F(Q)$ = $TC_2{}'(Q) - TC_1{}'(Q)$

CHAPTER-2

IMPACT OF INFLATION ON ECONOMIC QUANTITY DISCOUNT SCHEDULE TO INCREASE VENDOR PROFITS

Inventories play an extremely important role in a nation's economy. In Canada, the total inventories held by a typical Canadian manufacturer represent on the average 34% of the current asset and 90% of the working capital (Silver & Peterson,1985). Herron(1979) suggested that for many firms inventory cost could be as much as pre-tax operating profits. Thus a small decrease in inventory cost could result in substantial gains in an organizations profitability (Hall,1983). Over the past several years the literature dealing with inventory control systems has been exploding. Several models have been suggested that are applicable under a variety of conditions (Buffa and Miller(1979), Chase and Aquilano (1981), Hadley and Whitin (1963), Silver & Peterson(1985)). Recently, Gupta (1987) has developed an inventory control model with inflation from buyer's perspective. Most often, the main objective in these models is to minimize the buyer's total cost related to inventory. It is the buyer who is advised to best respond to the fixed quantity discount schedule of the supplier.

The literature dealing with supplier's perspective is extremely sparse (Dada and Srikanth, 1987; Lee and Rosenblatt, 1986). Lal and Staelin (1984) suggested that the discount pricing structure implicitly or explicitly assume that discounts are given by the seller in response to pressures from a large buyer. They further contend that there are numerous situations in which a large seller offers a discount to a large number of small buyers who lack the economic power to demand such a price discount.

Goyal (1977) was perhaps the first researcher who, under the assumption of an infinite production rate, formulated a model to determine the supplier's economic production policy in response to the buyer's purchase order. Banerjee(1986a) generalized this model by incorporating a finite supplier production rate.

A price discount approach for encouraging the customer (buyer) to deviate from his economic policy in order to increase or maximize the supplier's profits was initially suggested by Monahan (1984). Lal and Staelin (1984) in their approach similar to that of Monahan, extended the model to incorporate variable ordering and shipping costs and situations in which the supplier faces numerous groups of buyers, each having different ordering policies.

More recently Banerjee(1986 b) has pointed out that Monahan's model is essentially correct when the supplier is only an intermediatory between the producer and the retailer and, in the process, incurs only a negligible inventory carrying cost or none at all. He developed a generalized version of Monahan's model and demonstrated its equivalence with the joint economic lot size approach.

Joglekar's(1985) approach is different in the sense that he gives more importance to marketing goals and then develops a quantity discount model aimed at maximizing vendor profits.

All models reviewed above for determining the supplier's optimal inventory policy have assumed that the unit cost of purchasing is independent of time and free from inflation. Therefore, the purpose of this chapter is to determine the optimal supply quantity and the optimal quantity discount schedule for a supplier under inflationary condition. The model presented in this chapter is a generalized form of Monahan's(1984) and Banerjee's (1986b) models. In this chapter, we have made the same assumptions as Monahan(1984) and Banerjee(1986 b), in addition to the following: (i) the supplier has multiple items to supply, (ii) supplies are subjected to constant inflation, and (iii) the unit price is subject to the same inflation rate as other inventory related costs.

Economic Order Quantities with inflation for multi-items under constraints system

This section deals with the situation in which N items are stocked. The problem is to determine the vendor's optimal quantities Q_j's ($j = 1, 2, \ldots, N$) which will minimize the total system costs (and maximize vendor's profits) with inventory over a planning horizon under inflation and under certain constraints such as (i) floor space constraints, and (ii) constraints on the number of orders. However, it should be possible to include more constraints. In this chapter our objective is to minimize total cost in order to maximize vendor's profit.

A simple way to consider inflation in the development of inventory models is to assume that there is a constant inflation rate of K_j for the j^{th} item ($j = 1, 2, \ldots, N$).

Let the unit cost of the j^{th} item at time 't' be $X_j(t)$, which becomes $X_j(t + \delta t)$ at time $t + \delta t$ through inflation.

This cost structure has been employed in the development of the inventory model in this section to incorporate the effect of inflation for various inventory-related cost parameters.

It will be assumed here that the vendor is distributor as well as manufacturer. $\bar{A}_j(t)$ and $\bar{C}_j(t)$ being assumed constant, we can write

$$\overline{A}_j(t) = A_{0j}\, e^{K_j t} \qquad j = 1, 2, \ldots, N \tag{1}$$

$$\overline{C}_j(t) = C_{0j}\, e^{K_j t} \qquad j = 1, 2, \ldots, N \tag{2}$$

Total system costs over a given planning period 'L' = $\rho_j T_j$ (where ρ_j is an integer) are equal to ordering costs + purchasing costs + inventory carrying costs

The ordering cost for j^{th} item is given by:

$$A_{0j} + \overline{A}_j(T_j) + \overline{A}_j(2T_j) + \ldots + \overline{A}_j(\rho_j - 1)T_j. \tag{3}$$

$$= A_{0j} \;\; (e^{K_j L} - 1)/(e^{K_j T_j} - 1) \tag{4}$$

The purchasing cost for j^{th} item is

$$D_j T_j [C_{0j} + \overline{C}_j(T_j) + \overline{C}_j(2T_j) + \ldots\ldots + \overline{C}_j(\rho_j - 1)T_j]$$

$$= D_j T_j C_{0j} (e^{K_j L} - 1)/(e^{K_j T_j} - 1) \tag{5}$$

The inventory carrying cost for the j^{th} item is:

$$= \sum_{m=0}^{\rho_j - 1} \int_{w=0}^{T_j} \frac{D_j^2}{R_j}\, h_j(t, t+w)\; dw$$

$$= \sum_{m=0}^{\rho_j - 1} \int_{w=0}^{T_j} D_j^2\; H_j\; \frac{\overline{C}_j}{R_j}\, (mT_j)\, w\, dw$$

$$= \sum_{m=0}^{\rho_j - 1} \frac{D_j^2}{R_j} H_j \bar{C}_j (mT_j) \frac{T_j^2}{2}$$

$$= \frac{D_j^2 H_j C_{0j} T_j^2 \left(e^{K_j L} - 1\right)}{2R_j \left(e^{K_j T_j} - 1\right)} \qquad (6)$$

Assuming that K_j is small enough such that K_j^3 and its higher powers are negligible, we have

$$e^{K_j T_j} - 1 = 1 + K_j T_j + \frac{K_j^2 T_j^2}{2} - 1$$

$$= K_j T_j + \frac{K_j^2 T_j^2}{2} .$$

Thus the average total system variable cost for all items is

$$TC\ (L, T_1, T_2, \ldots, T_N) = TC = \sum_{j=1}^{N} \left(A_{0j} + D_j T_j C_{0j} + D_j^2 H_j\ C_{0j}\ \frac{T_j^2}{2R_j} \right)$$

$$\left(\frac{e^{K_j L} - 1}{K_j T_j + K_j^2 T_j^2 / 2} \right) \qquad (7)$$

with,

$$D_j T_j = Q_j \qquad (8)$$

FLOOR SPACE CONSTRAINT:

Let $f_j (j = 1, 2, \ldots, N)$ be the floor space required by one unit of the j^{th} item and let 'f' be the upper limit on the total floor space available. If the floor space constraint is not to be violated by any order quantity Q_j (of the j^{th} item) at any time, the following condition must be satisfied:

$$\sum_{j=1}^{N} f_j Q_j \leq f \tag{9}$$

By substitution of the value of Q_j from (8), equation (9) becomes

$$\sum_{j=1}^{N} f_j D_j T_j \leq f \tag{10}$$

Initially the problem is solved ignoring the constraint (10), i.e. we minimize over each T_j's separately. This yields

$$TC = \sum_{j=1}^{N} \left(A_{0j} + D_j T_j C_{0j} + \frac{D_j^2 H_j C_{0j} T_j^2}{2R_j} \right) \left(\frac{e^{K_j L} - 1}{K_j T_j + K_j^2 T_j^2 / 2} \right) \tag{11}$$

By differentiating (11) w.r.t. T_j for the optimal value of the T_j's, we get

$$\frac{dTC}{dT_j} = 0 \quad (j = 1, 2, \ldots, N)$$

$$\frac{dTC}{dT_j} = \left(e^{K_jL} - 1\right) \left[\frac{\left(K_jT_j + \frac{K_j^2T_j^2}{2}\right)\left(D_jC_{0j} + \frac{D_j^2}{R_j} H_j T_jC_{0j}\right) - \left(A_{0j} + D_jT_jC_{0j} + \frac{D_j^2H_j C_{0j} T_j^2}{2R_j}\right)\left(K_j + K_j^2T_j\right)}{\left(T_jK_j + \frac{K_j^2T_j^2}{2}\right)^2} \right] = 0 \qquad (12)$$

so that

$$- A_{0j} (1 + K_jT_j) + D_jC_{0j} \frac{T_j^2}{2}\left(D_j\frac{H_j}{R_j} - K_j\right) = 0 \qquad (13)$$

Since $D_jT_j = Q_j$, putting the value of T_j in (13) we can get:

$$C_{0j} Q_j^2 \left(D_j\frac{H_j}{R_j} - K_j\right) = 2 A_{0j} (D_j + K_jQ_j) \qquad (14)$$

so that

$$Q_j^* = \sqrt{\frac{2A_{0j}\left(D_j + K_jQ_j\right)}{C_{0j}\left(D_j\frac{H_j}{R_j} - K_j\right)}} \qquad (15)$$

Solving Equation (14):

$$Q_j^* = \frac{2 A_{0j}K_j + \sqrt{4 A_{0j}^2K_j^2 + 8 C_{0j}\left(D_j\frac{H_j}{R_j} - K_j\right) A_{0j}D_j}}{2 A_{0j}\left(D_j\frac{H_j}{R_j} - K_j\right)} \qquad (16)$$

PARTICULAR CASES:

(i) If there is no inflation i.e. if $K_j = 0 \quad \forall$ items and $j=1$, then the optimal quantity given by equation (15) is the same as that in the Banerjee's model (1986a,b).

(ii) If there is no inflation, i.e. if $K_j = 0 \quad \forall$ items, there is no carrying cost (i.e. $H_j = 0 \quad \forall$ items) and $j=1$, then equation (11) is the same as given by Monahan.

(iii) From equation (16) we see that the Q_j's are real only when the following holds:

$$A_{0j}^2 K_j^2 + 8\, C_{0j}\left(D_j \frac{H_j}{R_j} - K_j\right) A_j\, D_j \geq 0 \qquad \text{for } \forall\, H_j \geq K_j$$

If the Q_j's of (16) satisfy (9) the Q_j's are optimal. Otherwise, if the Q_j's of (16) do not satisfy (9), the constraint is violated and the Q_j's (16) are not optimal. To find the optimal values of Q_j's we then apply the Lagrangian multiplier technique, using the Kuhn-Tucker(1951) conditions approach.

Let μ denote a Lagrangian multiplier and let

$$TC = F(L, T_1, T_2, \ldots, T_N, \mu)$$

$$= \sum_{j=1}^{N} \left(A_{0j} + D_j T_j C_{0j} + \frac{D_j^2 H_j C_{0j} T_j^2}{2R_j} \right) \left(\frac{e^{K_j L} - 1}{K_j T_j + \frac{K_j^2 T_j^2}{2}} \right) + \mu \left(\sum_{j=1}^{N} f_j D_j T_j - f \right) \qquad (17)$$

For optimal T_j's the conditions :

$$\frac{\partial F}{\partial T_j} = 0 \qquad j = 1, 2, \ldots, N \quad \text{and} \quad \frac{\partial F}{\partial \mu} \leqq 0 \ .$$

must hold.

Thus

$$\frac{\partial F}{\partial T_j} = \left(e^{K_j L} - 1 \right) \left[\left(K_j T_j + \frac{K_j^2 T_j^2}{2} \right) \left(D_j C_{0j} + \frac{D_j^2}{R_j} H_j T_j C_{0j} \right) - \left(K_j + K_j^2 T_j \right) \cdot \frac{\left(A_{0j} + D_j T_j C_{0j} + \frac{D_j^2 H_j C_{0j} T_j^2}{2 R_j} \right)}{\left(T_j K_j + \frac{K_j^2 T_j^2}{2} \right)^2} \right] + \mu f_j D_j \qquad (18)$$

and

$$\frac{\partial F}{\partial \mu} = \sum_{j=1}^{N} f_j D_j T_j - f \leqq 0 \qquad (19)$$

From the Kuhn-Tucker(1951) condition if $\mu = 0$ then $\Sigma f_j D_j T_j - f < 0$ which is not true so if $\mu > 0$ then $\Sigma f_j D_j T_j - f = 0$ should satisfy.

After simplification of equation (18) we get:

$$A_{0j}\,(1+K_jT_j) + D_jC_{0j}\,\frac{T_j^2}{2}\left(D_j\,\frac{H_j}{R_j} - K_j\right) + \frac{\mu f_j D_j K_j T_j^2}{\left(e^{K_jL} - 1\right)}\left(1 + \frac{K_jT_j}{2}\right)^2 = 0 \qquad (20)$$

After substituting $\bar{K}_j = e^{K_jL} - 1$ and the value of T_j from equation (8), the optimal value would be:

$$Q_j^* = \sqrt{\frac{2\,A_{0j}\,(D_j + K_jQ_j)}{C_{0j}\left(D_j\frac{H_j}{R_j} - K_j\right) + \frac{2\mu^* f_j K_j}{\bar{K}_j}\left(1 + \frac{K_jQ_j}{2D_j}\right)^2}} \qquad (21)$$

CONSTRAINT ON THE TOTAL NUMBER OF ORDERS:

Let h be the maximum number of orders that can be placed during the planning period L. This requires that the following condition must be satisfied

$$\sum_{j=1}^{N}\frac{D_j}{Q_j} \le h \quad \Rightarrow \quad \sum_{j=1}^{N}\frac{1}{T_j} \le h \qquad (22)$$

We assume that there is no fixed cost per order. The only costs are inventory carrying costs and cost of purchases. Thus the total variable cost for all the items, with Lagrangian multipliers is: $TC = E(L, T_1, T_2, \ldots, T_N, \eta)$

$$= \sum_{j=1}^{N} \left(D_j T_j C_{0j}\right)\left(e^{K_j L} - 1\right) \Big/ \left(K_j T_j + \frac{K_j^2 T_j^2}{2}\right) + \eta \left(\sum_{j=1}^{N} \frac{1}{T_j} - h\right) \tag{23}$$

The essential conditions of optimality are:

$$\frac{\partial E}{\partial T_j} = \frac{\left(e^{K_j L} - 1\right)\left(K_j T_j + \frac{K_j^2 T_j^2}{2}\right)\left(D_j C_{0j}\right) - \left(D_j T_j C_{0j}\right)\left(K_j + K_j^2 T_j\right)}{\left(K_j T_j + \frac{K_j^2 T_j^2}{2}\right)^2}$$

$$- \eta \frac{1}{T_j^2} = 0 \tag{24}$$

$$\frac{\partial E}{\partial \eta} = \sum_{j=1}^{N} \frac{1}{T_j} - h \leqq 0$$

$$= \sum_{j=1}^{N} \frac{D_j}{Q_j} - h \leqq 0$$

$$\eta \geqq 0$$

Using (8), equation (24) becomes

$$- \left(e^{K_j L} - 1\right) \frac{D_j C_{0j}}{2} = \eta \left(\frac{1}{T_j} + \frac{K_j}{2}\right)^2 \tag{25}$$

and, if η^* is the value of η such that Q_j*'s of (26) satisfy (25),

$$\Rightarrow \quad Q_j^* = \sqrt{\frac{-2\eta^* \left[1 + \frac{K_j Q_j}{2D_j}\right]^2 D_j}{C_{0j}\left[e^{K_j L} - 1\right]}} \tag{26}$$

NUMERICAL EXAMPLE

Parameter/Item(i)	1	2
D_j	2000.00	500.00
C_{0j}	20.00	40.00
A_{0j}	50.00	100.00
H_j	0.40	0.42
K_j	0.10	.05
f_j	1.05	2.05
R_j	2667.00	800.00

Total floor space area available = 400.00 m^2. The optimal lot sizes in the absence of the space constraint are given by:

$$Q_1 = 224.86$$

$$Q_2 = 112.430$$

If these Q_j's were used, the maximum floor space required for inventory would be:

$$(1.05)(224.86) + (2.05)(112.430) = 466.5 \text{ m}^2$$

This is greater than the maximum available floor space(i.e. 400 m^2) for inventory. Hence the constraint is violated and, on introduction of a Lagrangian multiplier μ, the optimal Q_j's are

$$= (1.05)\left[\frac{10 + \sqrt{100 + 1600000(2 + \mu^*)}}{4(2 + \mu^*)}\right]$$

$$+ (2.05)\left[\frac{10 + \sqrt{100 + 1600000(2 + \mu^*)}}{8(2 + \mu^*)}\right] = 400 \text{ s.q. m.}$$

$$\Rightarrow \quad \left(1 + \sqrt{1 + 16000(2 + \mu^*)}\right) = \frac{1600}{20.75}(2 + \mu^*)$$

$$\Rightarrow \quad \sqrt{1 + 16000(2 + \mu^*)} = \frac{1600}{20.75}(2 + \mu^*) - 1$$

$$\mu^* = .72$$

Consequently, the optimal Q_j's are

$$Q_1^* = 192.555$$

$$Q_2^* = 96.277$$

The minimum total system cost for the two items in the presence of the space constraint is:

$$TC = 46912.73266$$

PARTICULAR CASE:

If there is no inflation, i.e.

$$K_1 = K_2 = 0,$$

the optimal order quantities in the absence of the space constraint are given by:

$$Q_1 = \sqrt{\frac{2(2667)(50)}{(.4)(20)}} = 182.59$$

$$Q_2 = \sqrt{\frac{2(800)(100)}{(.42)(40)}} = 97.59$$

If these Q_j's were used, the maximum floor space required in inventory would be:

$$f = (182.59)(1.05) + (97.59)(2.05)$$
$$= 391.78 \text{ (sq. m.)}$$

This is less than the maximum allowable space in inventory. Hence the constraint is inactive.

Thus

$$Q_1^* = Q_1 = 182.59$$

$$Q_2^* = Q_1 = 97.59$$

Economic Quantity Discount (EQD):

The unit cost of an item is not always independent of the quantity produced. Discounts are generally offered for the purchase of larger quantities. These discounts take the form of price breaks of the following type:

Let quantities $q_0 = 0, q_1, q_2, \ldots, q_j, \ldots, q_m, q_{m+1}$, ... $(q_j < q_{j+1}, j = 1, 2, \ldots, m)$, be given such that, if a quantity Q at time t is purchased and $q_j \le Q < q_{j+1}$, the cost per unit is

$$C_j(t) = C_{0j}e^{Kt}$$

clearly, C_{0j} is the per unit purchase cost for Q items at time t = 0,

$$q_j \le Q < q_{j+1}$$

and $C_{0j+1} < C_{0j+1} < C_{0j}$.

Let $TC_j(L, Q)$ be the average cost over L for $q_j \le Q < q_{j+1}$. Then

$$TC_j(L, q) = \left(A_0 + Q\,C_{0j} + \frac{H\,C_{0j}Q^2}{2\,R}\right)\left(\frac{e^{KL} - 1}{2\,DKQ + K^2Q^2}\right)2D^2$$

where $j = 0, 1, 2, \ldots, n$

In this way one can obtain m + 1 cost curves, one for each

C_{0j}. These cost curves do not intersect, since $TC_{j+1}(L, Q) < TC_j(L, Q)$ for all Q.

The solution procedure can be explained with the help of a numerical example. Let

$D = 500.00 \qquad A_0 = 20 \qquad H = .3 \qquad K = .12$

$q_1 = 200 \qquad q_2 = 250 \qquad C_{01} = 2.4 \qquad C_{02} = 2$

$R = 800$

<u>CASE I</u> No Inflation:

To determine the optimal Q, first compute $Q^{(2)}$

$$Q^{(2)} = \sqrt{\frac{2A_0R}{HC_{02}}} = 230.94$$

$Q^{(2)}$ does not satisfy $Q^{(2)} \geq 250$; hence $Q^{(2)}$ is not physically realizable. Therefore one should compute the following:

$$TC(q_2) = DC_{02} + \frac{A_0D}{q_2} + \frac{H\ C_{02}q_2D}{2\ R} = 1086.875$$

The second stage is begun by computing

$$Q^{(1)} = \sqrt{\frac{2A_0R}{HC_{01}}} = 210.8185$$

$Q^{(1)}$ is allowable as $200 \leq Q^{(1)} \leq 250$.

Then

$$TC\left(Q^{(1)}\right) = DC_{01} + \frac{A_0 D}{Q^{(1)}} + \frac{H\, C_{01} Q^{(1)} D}{2R} = 1294.868$$

$$\hat{TC} = \text{Min}\left[TC\ (q_2),\ TC\ (Q^{(1)})\right] = TC\ (q_2)$$

Thus $Q^* = q_2 = 250$ is optimal and the optimal value of Q occurs at the first price break itself.

CASE II

Inflation exists i.e., $K > 0$. For the given value of K compute $Q^{(2)}$ as:

$$Q^{(2)} = \frac{2A_0K \pm \sqrt{4A_0^2K^2 + 8C_{02}\left(\frac{D}{R}H - K\right)A_0D}}{2\,C_{02}\left(\frac{D}{R}H - K\right)} = 403.088$$

$Q^{(2)}$ is allowable as $Q^{(2)} = 403.1$ is optimal and the optimal value of Q occurs at $Q^{(2)} = 403.1$.

The table below represents the optimal order quantity at different values of inflation rate.

K	0.000	.025	.050	.075	0.125	.15
Q^*	230.94	249.61	273.34	304.88	420.2	557.94
TCQ^*	1086.8	1048.63	1050.25	1054.45	1037.82	1015.07

CONCLUSION

The results obtained in this chapter demonstrate that the economic quantity discount is affected by inflation, and the effect becomes more significant at higher values of inflation rates. The vendor's optimal quantity shows how the constraint is active under inflationary condition. This study illustrates the importance of taking into account inflation and discounting.

CHAPTER 3

THE EFFECTS OF INFLATION ON GENERALIZED ECONOMIC ORDER QUANTITIES

A major concern of inventory management is to know when and how much to order(or to manufacture) so that the total cost per unit time is minimized. The total cost includes the costs of carrying, shortage and common replenishment or set up and the purchase or production cost. Since the early work of Hadley (1964) several significant contributions have been made in determining the economic order quantity(EOQ), (Chase and Aquilano (1981)). However, the studies incorporating explicitly the effect of inflation and time value are sparse. Brown (1967) considered minimizing the present value of all the future costs by considering the step increase in price due to inflation. Bierman and Thomas (1977) proposed a model to determine EOQ by minimizing the present value of all future costs under inflation. A similar approach was taken by Jesse et.al.(1983). Buzacott (1975) showed that with inflation the choice of the inventory carrying charges used in the EOQ formula depends on the company's pricing policy. He determined EOQ under the following conditions:

(a) The price includes a fixed mark up on the purchase cost.

(b) The price includes a fixed cash amount over the purchase cost.

(c) The price is determined by the purchase cost plus all interval costs allocatable to them.

Aggarwal (1981) developed methods of calculating EOQ when a bulk order could be placed before an impending step increase in purchase cost. Misra (1979) developed a model which considered time discounting and two different inflation rates. His approach was similar to the one suggested by Bierman and Thomas (1977).

Subramanyam and Kumaraswamey (1981) developed a model (S-K model) for determining the economic order quantity (without inflation) that takes into account the effects of factors such as advertising and the possibility of some of the ordered items being damaged. Lee and Rosenblatt (1986) re-examined and re-formulated the S-K model under three different scenarios.

In this chapter the objective is to analyze the effect of inflation on a generalized EOQ formulation from a buyer and a vendor's perspective by considering the first two common pricing policies considered by Buzacott(1975) under the three scenarios given by Lee and Rosenblatt (1986) and the assumptions of the S-K model.

The effect of inflation on a vendor lot size model under various constraints is analyzed by Bector et al.(1987). However they do not consider the effects of damage and advertisement cost as given in the S-K model. This work presents the effects of a constant inflation under given planning horizon on a generalized buyer's and vendor's model.

Lee and Rosenblatt's(1986) Three Scenarios:

Scenario 1: To satisfy all the demand for D units, more than D units would have to be ordered per period so that, after the damaged or defective items have been discarded, enough items will be left to satisfy all the demand.

Scenario 2: The damaged or defective items are discarded and not sold. Hence, if the firm has ordered D units per period, less than D units of demand would be actually satisfied. The unsatisfied demand would incur a certain shortage cost ($s per unit) to the firm.

Scenario 3: In scenarios 1 and 2, damaged or defective items are assumed to be visible or easily identifiable, so that they can be discarded and not be sold to customers. In some circumstances, however, defective items cannot be identified without an extensive inspection procedure. Without inspection, the firm can be assured to order and sell D units per period. A cost is, however, incurred for each defective item sold.

MODEL

Due to inflation it will be assumed that

$$A_1(t) = A_{1o}\, e^{Kt} \quad \text{and} \quad A_2(t) = A_{2o}\, e^{Kt}$$

where A_{1o} and A_{2o} are the set up costs at time zero for the buyer and the vendor

Similarly,

$$C_1(t) = C_{1o}\, e^{Kt} \quad \text{and} \quad C_2(t) = C_{2o}\, e^{Kt}$$

$$p(t) = p_o\, e^{Kt}$$

Where C_{1o} is cost per unit at time zero and C_{2o} is unit production cost at time zero and p_o is the selling price at time zero.

As in the S-K model, the following demand function is assumed to reflect the advertising and price effects:

$$D = a^N/p^e \tag{1}$$

(here e is the elasticity $e \geq 1$ and $a > 0$).

Also, the pricing policies for the buyer and the vendor can be written as Buzacott (1975):

$$\text{(i)}\ p(t) = \lambda_1\, C_1(t) = \lambda_2\, C_2(t), \tag{2}$$

with $\lambda_1 \neq \lambda_2$ and $C_1(t) \neq C_2(t)$.

$$\text{(ii)}\quad p(t) = C_1(t) + P_1 = C_2(t) + P_2\ ; \tag{3}$$

here P_1 and P_2 are constants.

$$C_1 = b_1 + {}^{d_1}/_{Q^\gamma} \tag{4}$$

$$C_2 = b_2 + {}^{d_2}/_{Q^\gamma} \tag{5}$$

with $b_1, b_2 > 0$; $\frac{d_1}{Q^\gamma}$ and $\frac{d_2}{Q^\gamma}$ represent the economies of scale effects for buyer and vendor; d_1, d_2 and γ are constants of the same sign.

For the sake of simplicity it is assumed that the holding cost h(t,t+1) is identical for the buyer and the vendor. Also, note that $p \le D$, $C_2 \le C_1$ and $e, \gamma = 1$.

The net profit functions of the S-K model for buyer and vendor are given below where both the buyer and vendor are advertising their product

$$Z_b = D(p - C_1) - \alpha pD - UC_1D - \left[{}^{Q}/_{2}\ C_1h + {}^{D}/_{Q}\ A_1\right] \tag{6}$$

and

$$Z_v = D(\lambda_1 C_1 - C_2) - \alpha pD - UC_2D - \left[{}^{Q}/_{2}\ {}^{D}/_{R}\ C_2h + {}^{D}/_{Q}\ A_2\right] \tag{7}$$

The optimal quantities would be

$$Q_b = \frac{-\ d_1 + \sqrt{2A_1aN/h\ \lambda_1}}{b_1}$$

and

$$Q_v = \frac{-\ d_2 + \sqrt{2RA_2b_2/h}}{b_2}$$

Lee and Rosenblatt (1985) recognized the problems and inconsistencies associated with the S-K model and re-formulated it in terms of three different scenarios. The three different scenarios from the buyer's and the vendor's perspective are summarized in Table I.

THE BUYER AND VENDOR MODELS WITH INFLATION

The effect of inflation on different models [e.g.the S-K model and the three scenarios above] are presented in Tables II, III and IV under two common price policies.

(a) Price with a fixed mark-up on purchase cost:

If C(t) is the purchase cost of an item bought at time t then this guideline requires that the selling price of the item if sold during the period (t,t+1) is to be set at

$$p(t,t+1) = (1+m)C(t)$$
$$= \lambda C(t)$$

($\lambda > 0$; m is the allowable mark up).

The net profit function over the interval (0,L) of the S-K model with inflation would be, e.g. by substituting e.g. (1), (2) and (4):

$$Z_b = \frac{aN}{\lambda}(\lambda-1) - \alpha aN - U\frac{aN}{\lambda} - \left[\frac{1}{2}Q\left(b_1 + \frac{d_1}{Q}\right)H\left(e^{KL}-1\right)^2 + \frac{aN\ A_{1o}}{\left(b_1 + \frac{d_1}{Q}\right)Q}\right]$$

Similarly we can have net profit function for different scenario from buyer and vendor perspective. The optimal order quantities are summarized in Table II.

(b) Price with a fixed amount added over purchase cost:

In this case the price p(t,t+1) is set at p(t+1) = C(t) + P.

The optimal quantities are summarized in Table III and IV.

CONCLUSIONS

In this chapter we summarized models from the buyer's and vendor's perspectives that incorporate inflationary trends, effects of advertising, price elasticity and economies of scale. The models are presented under two price policies in which the models related to the first price policy (fixed mark-up on purchase cost) are simpler than the models related to the second price policy (fixed amount over purchase cost). Special cases can also be considered, e.g., if the inflationary trend is zero the S-K model and the three different scenarios lead to their original economic order quantities.

Table I: Optimal order quantity (without inflation) from the buyer's and the vendor's perspectives.

Buyer's Case	**Vendor's Case**
Scenario 1:	
$Z_1=D\left[\frac{p-c_1}{1-U}\right]-\alpha pD-\left\{\frac{1}{2}Qc_1h+\frac{DA_1}{Q(1-U)}\right\}$	$Z_1=D\left[\frac{p-c_2}{1-U}\right]-\alpha pD-\left\{\frac{1}{2}Q\frac{D}{R}c_2h+\frac{DA_2}{Q(1-U)}\right\}$
$Q_b=\frac{-d_1+\sqrt{2A_1aN/h\,\lambda_1(1-U)}}{b_1}$	$Q_v=\frac{-d_2+\sqrt{2RA_2b_2/h(1-U)}}{b_2}$
Scenario 2:	
$Z_2=D(1-U)p-Dc_1-\alpha pD-UsD-\left[\frac{1}{2}Qc_1h+\frac{DA_1}{Q}\right]$	$Z_2=D(1-U)p-Dc_2-\alpha pD-UsD-\left[\frac{1}{2}Q\frac{D}{R}c_2h+\frac{DA_2}{Q}\right]$
$Q_b=\frac{-d_1+\sqrt{2aN(A_1b_1-Usd_1)/h\,\lambda_1b_1}}{b_1}$	$Q_v=\frac{-d_2+\sqrt{2\lambda_2R(A_2b_2-Usd_2)/h}}{b_2}$
Scenario 3:	
$Z_3=D(p-c_1)-\alpha pD-UvD-\left[\frac{1}{2}Qc_1h+\frac{DA_1}{Q}\right]$	$Z_3=D(p-c_2)-\alpha pD-UvD-\left[\frac{1}{2}Q\frac{D}{R}c_2h+\frac{DA_2}{Q}\right]$
$Q_b=\frac{-d_1+\sqrt{2aN(A_1b_1-Uvd_1)/h\lambda_1b_1}}{b_1}$	$Q_v=\frac{-d_2+\sqrt{2\lambda_2R(A_2b_2-Uvd_2)/h}}{b_2}$

Table II: Optimal order quantity from the buyer's and the vendor's perspectives

Buyer's Case	**Vendor's Case**
S-K Model:	
$Q_b = \dfrac{-d_1 + \sqrt{2A_{1o}\, aN/H\ \lambda_1 S^2}}{b_1}$	$Q_v = \dfrac{-d_2 + \sqrt{2A_{2o}\, Rb_2/H}\ S}{b_2}$
Scenario 1:	
$Q_1 = \dfrac{-d_1 + \sqrt{2A_{1o}\, aN/H\ \lambda_1 (1-U) S^2}}{b_1}$	$Q_1 = \dfrac{-d_2 + \sqrt{2RA_{2o}\, b_2/H\ S(1-U)}}{b_2}$
Scenario 2:	
$Q_2 = \dfrac{-d_1 + \sqrt{\dfrac{2aN(A_{1o}\, b_1 - Usd_1)}{H\lambda_1 b_1 S^2}}}{b_1}$	$Q_2 = \dfrac{-d_2 + \sqrt{2\lambda_2 R(A_{2o}\, b_2 - Usd_2)/HS}}{b_2}$
Scenario 3:	
$Q_3 = \dfrac{-d_1 + \sqrt{\dfrac{2aN(A_{1o}\, b_1 - Uvd_1)}{H\lambda_1 b_1 S^2}}}{b_1}$	$Q_3 = \dfrac{-d_2 + \sqrt{2\lambda_2 R(A_{2o}\, b_2 - Uvd_2)/HS}}{b_2}$

where $S = (e^{KL} - 1)$

Table III: Economic Order Quantity from the **Buyer's Perspective**

S-K Model:

$$Q_b^* = \frac{-\,d_1M + \sqrt{d_1^2M^2 - 0\left[d_1^2b_1^2S^3 - \frac{2aN}{H}\,(P_1d_1 + A_{10}b_1S + A_{10}P_1)\right]}}{b_10}$$

Scenario 1:

$$Q_1^* = \frac{-\,d_1M + \sqrt{d_1^2M^2 - 0\left[d_1^2b_1S^3 - \frac{2aN}{(1-U)H}\,(P_1d_1 + A_{10}b_1S + A_{10}P_1)\right]}}{b_10}$$

Scenario 2:

$$Q_2^* = \frac{-\,d_1M + \sqrt{d_1^2M^2 - 0\left[d_1^2b_1S^3 - \frac{2aN}{H}\,(P_1d_1 - Usd_1S + A_{10}b_1S + A_{10}P_1)\right]}}{b_10}$$

Scenario 3:

$$Q_3^* = \frac{-\,d_1M + \sqrt{d_1^2M^2 - 0\left[d_1^2b_1S^3 - \frac{2aN}{H}\,(P_1d_1 - Uvd_1S + A_{10}b_1S + A_{10}P_1)\right]}}{b_10}$$

where

$$M = b_1S^3 + P_1S^2$$

$$0 = b_1S^3 + \frac{P_1^2}{b_1}S + 2P_1S^2$$

$$S = (e^{KL} - 1)$$

Table IV: Economic Order (Lot Size) from the **Vendor's Perspective**

S-K Model:

$$Q_v^{\#} = \frac{-d_2S^2 + \sqrt{S^4d_2^2 - T\left[d_2^2S^2 - \frac{2R}{H}(P_2d_2 + A_{2o}b_2S + A_{2o}P_2)\right]}}{b_2T}$$

Scenario 1:

$$Q_1^{\#} = \frac{-d_2S^2 + \sqrt{S^4d_2^2 - T\left[d_2^2S^2 - \frac{2R}{H(1-U)}(P_2d_2 + A_{2o}b_2S + A_{2o}P_2)\right]}}{b_2T}$$

Scenario 2:

$$Q_2^{\#} = \frac{-d_2S^2 + \sqrt{S^4d_2^2 - T\left[d_2^2S^2 - \frac{2R}{H}(P_2d_2 + P_2A_{2o} + A_{2o}b_2S - Usd_2S)\right]}}{b_2T}$$

Scenario 3:

$$Q_3^{\#} = \frac{-d_2S^2 + \sqrt{S^4d_2^2 - T\left[d_2^2S^2 - \frac{2R}{H}(P_2d_2 + P_2A_{2o} + A_{2o}b_2S - Uvd_2S)\right]}}{b_2T}$$

where

$$T = \frac{P_2}{b_2}S + S^2$$

$$S = (e^{KL} - 1)$$

CHAPTER 4

A NEW COMBINED APPROACH TO INVENTORY CONTROL FROM BUYER'S AND VENDOR'S PERSPECTIVE

There is extensive literature available on how a buyer should develop policies for replenishment of stock in order to minimize his inventory related-costs. For example Hadley and Whitin (1963); Chase and Aquilano (1977); Silver and Perterson (1985). Since the early work of Goyal (1977) and Dolan (1978) a number of researchers have formulated models from a vendor's perspective to determine the optimal order quantity. These include Monahan (1984), Banerjee (1986 a, b), Rosenblatt and Lee (1985), Lal and Staelin (1984), Dada and Shrikanth (1987) and Goyal (1987).

Monahan (1984) argued that the vendor's order processing and manufacturing set-up cost per order is larger than the purchaser's fixed order processing cost i.e. if the buyer adopts his Q as the order quantity for minimizing his total inventory-related costs, the supplier incurs a significant cost penalty resulting primarily from too frequent orders. Monahan (1984) suggested that by reducing the price of the items through quantity discount, the vendor can entice his major customers to increase his present order size by a factor of K.

Banerjee (1986)showed that Monahan's (1984) model is limited to the case that the vendor buys from another supplier. He extended this model by incorporating vendor's inventory carrying cost and demonstrated its equivalence with a joint economic lot size(JELS) approach suggested earlier by Banerjee(1986 a). Lee and Rosenblatt(1986) examined the joint problem (buyer and supplier)of ordering and offering price discount. They generalized Monahan's(1984) model by imposing constraints on the amount of price discount that can be offered to the buyer and by introducing a link where the potential benefit to the supplier in offering quantity discount is to alter a pattern of orders placed by the buyer which may reduce the supplier's opportunity cost of holding inventory.

Lal and Staelin (1984) also proposed a pricing discount structure in the situations in which (1) the seller's product does not represent a major component of the buyer 's final product, (2) the demand for the product is derived, and (3) the price is only one of the many factors to consider in making a purchase decision . Recently Dada and Srikanth (1987) extended Lal and Staelin's model by relating the assumption on the relative values of the parameter. Their model allows the buyer inventory carrying cost and then let the joint cost of the system depend on the pricing scheme.

Banerjee (1986) developed a joint economic lot size (JELS) model for a special case where a vendor produces two orders for a purchaser on a lot per lot basis under deterministic conditions. He proposed a joint optimal ordering policy which can be beneficial for both parties (the buyer and the supplier) and, at least, does not place either at a disadvantage. Banerjee overlooked in his formulation the following issues. First there should be some constraints imposed in order to determine economic lot size from the vendor's or from the buyer's perspective Banerjee (1986 a,b) developed his model under the condition that the vendor's objective is to maximize profits and the buyer's objective is to minimize costs. However, in the real world this situation is unlikely to occur. Generally vendor's objective is to maximize profits subject to the maximum cost the buyer may be prepared to incur and the buyer's objective should be to minimize costs subject to the minimum profits acceptable to the vendor. Banerjee and others ignored thses and other constraints such as floor space available and the maximum number of orders allowed. This is a special case of the model discussed in this chapter.

Second, if there is a situation where the holding and set up costs for the buyer and the vendor are fixed, and some values of costs and profits are also known to them, then according to JELS the optimal order quantity for buyer or vendor would not change even if there is a shift in the costs and profits.

The purpose of this chapter is to generalize Banerjee's model by addressing the two issues mentioned above. The above stated constraints are imposed on the model and we show that the optimal order quantity depends on the holding cost and set up cost of the vendor and the buyer respectively. First, using Kuhn-Tucker (1951) conditions, we develop a buyer/ vendor relationship by which the buyer's optimal order quantity can be found if the vendor's optimal production quantity is known or vice versa. Then we develop a buyer/vendor inventory frontier by using a linear programming approach. Finally, we provide a numerical example to illustrate the relationship. We also apply sensitivity analysis to establish a functional relationship through the Lagrangian multipliers.

Using Khun-Tucker conditions

Let us assume that there is one buyer and one supplier (vendor) in the market. The buyer follows an economic order quantity policy and all the relevant assumptions, as described below, hold: uniform deterministic demand, no shortages, constant lead time for the supplier and the buyer, and a supplier's production rate which is greater than the demand rate for the product [see Silver and Peterson (1985) and Banerjee (1986 b)].

BUYER'S PERSPECTIVE

The buyer's objective is to minimize his total inventory related cost with respect to the supplier's profits (Banerjee (1984 b)) (there may be some other constraints such as available floor space and maximum order quantity). This can be expressed as the following non linear program(NLP):

Minimize TC_1

subject to $\Pi_2 \geqq B$,

Here B is the minimum profit acceptable to the supplier, whose aim is that this profit (Π_2) should be greater than or equal to minimum profit (B),

$$TC_1 = C_1D + \frac{D}{Q}A_1 + \frac{Q}{2}H_1C_1 \tag{1}$$

is a nonlinear strictly convex function and

$$\Pi_2 = DM_2C_1 - A_2D/Q - DH_2C_2Q/2R - DC_2 \qquad (2)$$

is a nonlinear and strictly concave function, i.e. the local minimum is a global minimum. Therefore, in order to solve the above NLP problem, we can apply the Lagrangian multiplier approach. We write accordingly:

$$TC_1 = L(Q, \mu) = C_1D + DA_1/Q + Q\ H_1C_1/2 - \mu\ [DM_2C_1 - A_2D/Q - DC_2 - DQ\ H_2C_2/2R - B] \qquad (3)$$

(here μ is the Lagrangian multiplier or dual variable or marginal value). Using the Kuhn-Tucker (1951) conditions we get :

$$\frac{\partial L}{\partial Q} = -\frac{D}{Q^2}A_1 + \frac{1}{2}H_1C_1 - \mu\frac{D}{Q^2}A_2 + \mu\frac{D}{2R}(H_2C_2) = 0 \qquad (4)$$

and

$$\frac{\partial L}{\partial \mu} = DM_2C - \frac{D}{Q}A_2 - D\frac{Q}{2R}H_2C_2 - DC_2 \geq B \qquad (5)$$

From the complementary condition we see that, if $\mu = 0$,then

$$DM_2C_1 - \frac{D}{Q}A_2 - \frac{DQ}{2R}H_2C_2 - B > 0$$

is not true, so that we must have $\mu > 0$ and

$$D_1M_2C_1 - \frac{D}{Q}A - \frac{DQ}{2R}H_2C_2 - DC_2 - B = 0 \qquad (6)$$

Equation (4) leads to

$$Q_b = \sqrt{\frac{2D\,(A_1 + \mu A_2)}{H_1 C_1 + \mu \frac{D}{R} H_2 C_2}} \tag{7}$$

The value of μ can be found from equation (7) by substituting the value of Q_b obtained by solving the quadratic equation(6).

$$Q_b = \frac{(DM_2C_1 - B - DC_2) \pm \sqrt{(DM_2C_1 - B - DC_2)^2 - 2DA_2\frac{D}{R} H_2C_2}}{\frac{D}{R} H_2C_2} \tag{8}$$

PARTICULAR CASES

(i) If we consider $\mu = 0$, equation (7) is equivalent to the standard EOQ formula [Chase and Aquilano (1981)].

(ii) If A_1 and H_1 are both zero, then equation (7) is equivalent to Banerjee's [1986 a, b] formula.

(iii) If H_1, A_1 and H_2 are all zero, equation (3) represents Monahan's [1984] formula.

(iv) From equation (8) we see that Q_b would be positive only if

$$(DM_2C_1 - B - DC_2) > \sqrt{(DM_2C_1 - B - DC_2)^2 - 2DA_2H_2C_2 \frac{D}{R}}$$

(v) If $\mu = 1$, then equation (7) is equivalent to Banerjee's JELS

VENDOR'S PERSPECTIVE

The vendor's objective is to maximize his total profit subject to the buyer's cost. The NLP model from the vendor's perspective can be expressed as:

Maximize Π_2

subject to $TC_1 \leqq K$,

Here K is the maximum cost which is acceptable to the buyer ,whose aim is that the total inventory related costs (TC_1) should be less than or equal to maximum cost (K).

The functions TC_1 and Π_2 are the same as given in the buyer's perspective. We can apply again the Lagrangian multiplier technique and obtain:

$$\Pi_2 = F(Q, b) = DM_2C_1 - \frac{DA_2}{Q} - D\frac{Q}{2R} H_2C_2 - DC_2 - \beta \left[C_1D + \frac{D}{Q} A_1 + \frac{Q}{2} H_1C_1 - K\right] \tag{9}$$

(here β is the Lagrangian multiplier). Using the Kuhn-Tucker (1951) conditions we get

$$\frac{\partial F}{\partial Q} = \frac{D}{Q^2} A_2 - \frac{D}{2R} H_2C_2 + \beta \frac{D}{Q^2} A_1 - \beta \frac{1}{2} H_1C_1 = 0 \tag{10}$$

and

$$\frac{\partial F}{\partial \beta} = C_1 D + \frac{D}{Q} A_1 + \frac{Q}{2} H_1 C_1 - K \leqq 0 \tag{11}$$

From the complementary condition we see that, if $\beta = 0$,

$$C_1 D + \frac{D}{Q} A_1 + \frac{Q}{2} H_1 C_1 - K < 0 \tag{12}$$

is not true.

So $\quad \beta > 0 \quad$ and

$$C_1 D + \frac{D}{Q} A_1 + \frac{Q}{2} H_1 C_1 - K = 0 \tag{13}$$

should satisfy equation (6).

From equation (8) we get :

$$Q_v = \sqrt{\frac{2D(A_2 + \beta A_1)}{\frac{D}{R} H_2 C_2 + \beta H_1 C_1}} \tag{14}$$

The value of β can be found from equation (14) by substituting the value of Q_v obtained by solving the quadratic equation (12)

$$Q_v = \frac{(K - C_1 D) + \sqrt{(C_1 D - K)^2 - 2DA_1 H_1 C_1}}{H_1 C_1} \tag{15}$$

All particular cases discussed above for equations (3), (7) and (8) for the buyer's case are applicable for equations (9), (14) and (15) also.

RELATIONSHIP BETWEEN MODELS FOR BUYER'S AND VENDOR'S PERSPECTIVE

A comparison of equations (2) and (8) above shows that, for positive value of μ and β, Q_b and Q_v will be equal if μ and β are reciprocal, that is:

$$\text{If} \quad \beta = \frac{1}{\mu} \quad \left(\text{or } \mu = \frac{1}{\beta}\right), \quad Q_b = Q_v \tag{16}$$

This implies that if we know the value of β we can find the value of μ and then, with equation (7), we can determine the optimal value of the buyer's order quantity. Similarly if we know the value of μ, we can get the vendor's optimal production quantity from equation (15).

BUYER/VENDOR INVENTORY FRONTIER

An inventory frontier (FF') represent the trend of the relationship between buyer's costs and vendor's profits for fixed holding, set-up and purchasing costs. It can be presented graphically by Figure-1. Each point on this frontier shows the buyer's minimum total costs and the vendor's maximum profits in relation to other points which are not on the frontier (i.e., on the R.H.S. of the frontier). An inventory frontier is helpful in making decisions in the sense that, with given holding, set-up and purchasing costs, the buyer and the vendor can decide about their minimum and maximum possible costs and profits under a given market situation.

PROPERTIES

(i) The inventory frontier is a strictly concave curve (as shown in Figure-I). Because of the quadratic nature of the vendor's profit and the buyer's cost functions.

(ii) The relationship $\beta = 1/\mu$ holds on the frontier.

(iii) Like the production frontier [Baker et al (1984)], the inventory frontier has regions of increasing cost and decreasing return to scale.

(iv) An inventory frontier defines constant holding, purchasing and set up costs for both vendor and buyer.

In Figure-1 FF' is an inventory frontier. Point A below this frontier is associated with some amounts of the buyer's inventory related cost and the vendor's profits. The objective of the buyer is to minimize his costs and that of the vendor is to maximize his profits. To do so, the buyer would tend to move towards the frontier in the direction AC' under the assumption that the vendor's profits would be constant. This can be expressed as linear programming problem. Let H_A denote the amount of the buyer's costs which can be minimized by keeping the vendor's profit constant. T_{1A} and T_{1j} correspond to the buyer's actual inventory related costs on the frontier. Π_{2A} and Π_{2j} correspond to vendor's actual profits on the inventory frontier. Then, with relative weights a_j associated with the points on the frontier, we have the linear program:

$$
\begin{aligned}
&\text{Minimize} \quad H_A \\
&\text{subject to} \\
&\quad a_j T_{1j} \leqq H_A T_1 \\
&\quad a_j \Pi_{2j} \leqq \Pi_{2A} \qquad\qquad (17) \\
&\quad \sum_{j=1}^{n} a_j = 1 \\
&\quad H_A, \ \forall a_j \geqq 0 \qquad j = 1, 2, \ldots, n,
\end{aligned}
$$

Similarly, the vendor will tend to move from point A towards the frontier in the direction AC in order to maximize his profit subject to the buyer's constant inventory related costs. This can also be expressed in the form of an L.P. problem by letting H_B denote the amount of vendor's profits, which can be maximized by keeping the buyer's cost constant (as will be shown later, these points C' and C can also be obtained using the equations derived in the last section):

$$
\begin{aligned}
&\text{Maximize} && H_B \\
&\text{subject to} && \\
& && a_j T_{1j} \leq T_{1A} \\
& && a_j \Pi_{2j} \geq H_B \Pi_{2A} \qquad (18) \\
& && \sum_{j=1}^{n} a_j = 1 \\
& && H_B,\ \forall a_j \geq 0 \quad j = 1, 2, \ldots, n.
\end{aligned}
$$

A NUMERICAL EXAMPLE:

For the purpose of illustration, suppose that a supplier produces a certain product to fulfil periodic orders from a single buyer. The following data are available:

D = 500 units, R = 800 units

H_1 = \$ 0.4/\$/year, H_2 = \$ 0.3/\$/year

A_1 = \$ 65.00, A_2 = \$ 60.00

C_1 = \$ 40.0 / unit, C_2 = \$ 25.0 / unit

M_2 = 0.8

B = \$ 2926.1

K = \$ 21034.79

We first use the formulation derived using Khun-Tucker conditions. Substituting the above data in equation (8), we can calculate Q_b = 75.64 which on substitution in equation (7), gives μ = 0.8. Similarly, using equation (15), we can calculate Q_v = 75.65 which, on being sustituted in equation (14) yields β = 1.25. We find that these values of μ and β satisfy the relationship $\beta = 1/\mu$ (equation (16) which was derived earlier), showing the validity of the formulation.

It is, however, important to note that this formulation yields meaningful results only for certain ranges of values of the Lagrangian multipliers μ and β. We have calculated the values of B, K, the optimal economic quantities for buyer and vendor (i.e., Q_b and Q_v) for a variety of values of μ and β between -1.5 and 15.0. A critical examination of raw data revealed that for the same optimal economic quantities both for buyer and vendor, an efficient inventory frontier (a plot between K and B as shown in Figure-2) is obtained only for positive values of μ and β.

Now, consider a point A (Figure- 2) on the R.H.S. of the inventory frontier at which the buyer's inventory related costs (K) and the vendor's profits (B) are known (B = 2914.46, K = 21073.59). By using the above data and the formulation of last section, we can calculate the values of the buyer's inventory related costs (K) at point C' on the frontier (keeping the vendor's profits constant) as well as the vendor's profits (B) at point C on the frontier (keeping the buyer's inventory related costs constant). These values are given in Table-1 and are highlighted.

Using the data (K, B) on the inventory frontier (Figure-2) and the formulation of this section, solution of the two linear programming problems using LINDO yields the same values of the buyer's inventory related costs at point C' and the vendor's profits at point C as obtained by the formulation of section I (Table -1).

IMPLEMENTATION AND SENSITIVITY ANALYSIS:

One of the major problems for the supplier (vendor) in implementing a policy of keeping an optimal inventory, while maximizing profits as outlined above, is the difficulty of properly estimating the relevant cost parameters pertaining to the customer (buyer). While the values of the parameters D_1, R, A_2, H_2 and C_2 are usually known to the vendor, the buyer may be reluctant to reveal information concerning his cost parameters A_1 and H_1. However, to use the model presented above individual knowledge of these two parameters is not really necessary for the supplier . Some estimate of buyer's inventory related costs which are reflected in K is usually sufficient. If the supplier has dealt with the buyer in the recent past, he may take the buyer's inventory related costs from the buyer's recent economic ordering behavior. Otherwise, the supplier may attempt to extract from the buyer some information concerning his EOQ by quoting a hypothetical or trial price as a part of the normal negotiation process, so that the value of A_1 and H_1 , and hence the value of K , can then be computed from the hypothetical order size.

However, as the supplier's estimate of A_1 and H_1 and, hence, K may be in error, the sensitivity of the inventory model to such errors must be examined. For the example cited in the last section, we calculated the vendor's profit B is for various values of K obtained by varying the values of A_1, H_1 and A_2, H_2 (the buyer's as the well as vendor's costs). The results are plotted in Figures 3 and 4. In Figure-3, the holding costs of the buyer (H_1) and of the vendor (H_2) are varied while A_1 and A_2 are kept constant. In Figure-4, the ordering (A_1) and production set up (A_2) costs, of buyer and vendor are varied while H_1 and H_2 are kept constant. The following important observations can be made from Figure-3:

i) The vendor's actual gross profits far exceed his goal (curve 1, for $A_1 = 65$, $H_1 = 0.4$, $A_2 = 60$, $H_2 = 0.3$) if he underestimates the buyer's holding costs as well as his own holding costs by 10% (curve 2, $A_1 = 65$, $H_1 = 0.3$, $A_2 = 60$, $H_2 = 0.2$). In this case the entire inventory frontier moves up in a parallel fashion.

ii) The vendor's actual profits fall far below his goal if he overestimates the holding costs (H_1, H_2) by 10% (notice the parallel down movement of the inventory frontier represented by curve 3 in this case).

iii) By over estimating the buyer's holding costs and underestimating his own holding costs both by 10%, the vendor overestimates his actual profits (compare curve 4 with curve 1) and the shape of the inventory frontier is changed.

iv) By underestimating buyer's holding costs by 10% and keeping his own holding costs constant, the vendor only marginally increases his profits above his goal (compare curve 5 with curve 1) and the shape of the inventory frontier is somewhat changed.

v) The above four observations suggest that, given the vendor's holding costs, his estimated profits are inversely related to the buyer's holding costs, i.e., his profits are overestimated if he underestimates the buyer's holding costs and vice versa.

A critical examination of figure 4 shows that the variations in the vendor's profits are exactly similar in nature to the variations in the buyer's ordering (A_1) or the vendor's set-up (A_2) costs and to variations in their holding costs H_1 and H_2. However, the extent of these variations A_1, A_2 is much smaller compared to that of the variations with H_1, H_2.

In general, for the vendor's constant inventory related costs the model is more sensitive to the buyer's holding costs (H_1) than to his ordering costs (A_1). We may also comment here that ,if the vendor is not careful in correctly estimating buyer's inventory related costs, not only will he wrongly estimate his profits but he may grossly overprice or underprice his selling costs.

In order to see the trend of deviation of Q_V and

Q_b, we can use equation (2), and express the vendor's percentage profit penalty (PPP) as

$$PPP = \frac{\Pi_2(Q_v') - \Pi_2(Q_v)}{\Pi_2(Q_v)} \times 100$$

Here we use a quantity Q_v' which deviates from Q_v according to the following relation:

$$Q_v' = (1+dp)Q_v,$$

that is, the percentage deviation of Q_v' from Q_v is dp . Also, if we define Q_b' similar to Q_v', the buyer's percentage cost penalty (PCP) is

$$PCP = \frac{TC_1(Q_b') - TC_1(Q_b)}{TC_1(Q_b)} \times 100$$

Figure 5 shows how the plot of the vendor's(PPP) and the buyer's(PCP) against the dp(percentage deviation) indicates that even for values of dp significantly different from zero the associated profit and cost penalties are quite small; especially, when the deviations are positive, the penalties are negligible.

CONCLUSION

In this chapter a more generalized theory of economic order quantity is developed which takes into account the buyer's as well as the vendor's perspective. Also the classical EOQ, Banerjee's (1986 b, a) and Monahan's (1984) formulas are shown to be special cases of the generalized economic order quantities. Moreover, a relationship is also developed between the vendor's and buyer's economic order quantities by using two different approaches. This is demonstrated by considering a numerical example and, also, by implementing sensitivity analysis.

TABLE - 1

At point C'	At point C
$Q_v = Q_b = 71.96$	$Q_v = Q_b = 88.07$
$B = 2914.0$	**$B = 2952.95$**
$K = 21027.33$	$K = 21073.59$
$\mu = 0.5$, $\beta = 2.0$ $\mu = {}^{1}/_{\beta}$ holds	$\mu = 2.5$, $\beta = 0.4$ $\mu = {}^{1}/_{\beta}$ holds

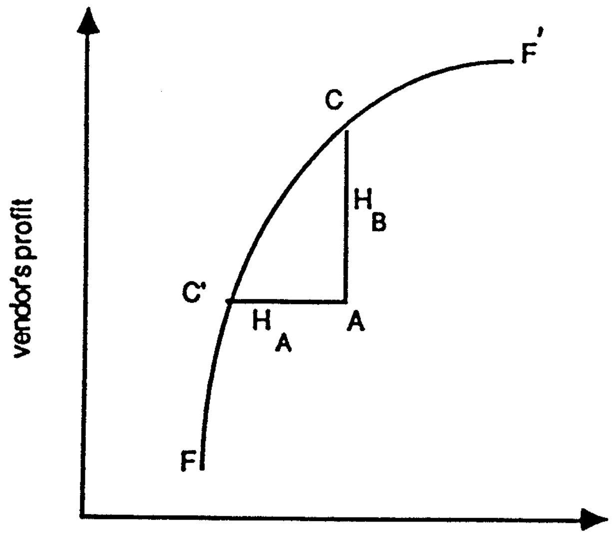
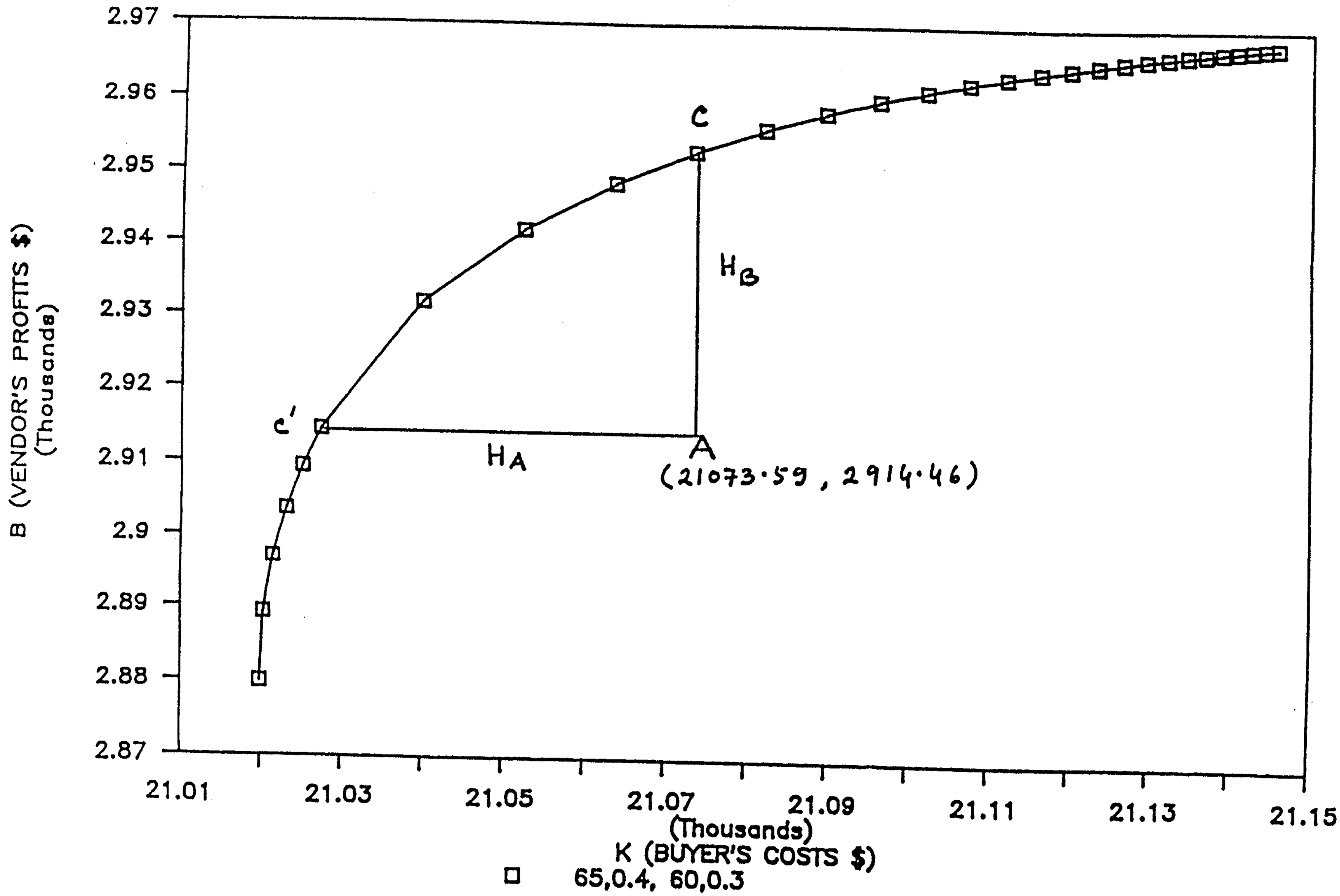


Figure-1

Relationship between vendor's profit and buyer's cost

FIGURE 2

Relationship between Vendors' Profit and Buyer's Cost



Relationship between Vendors' Profit and Buyer's Cost

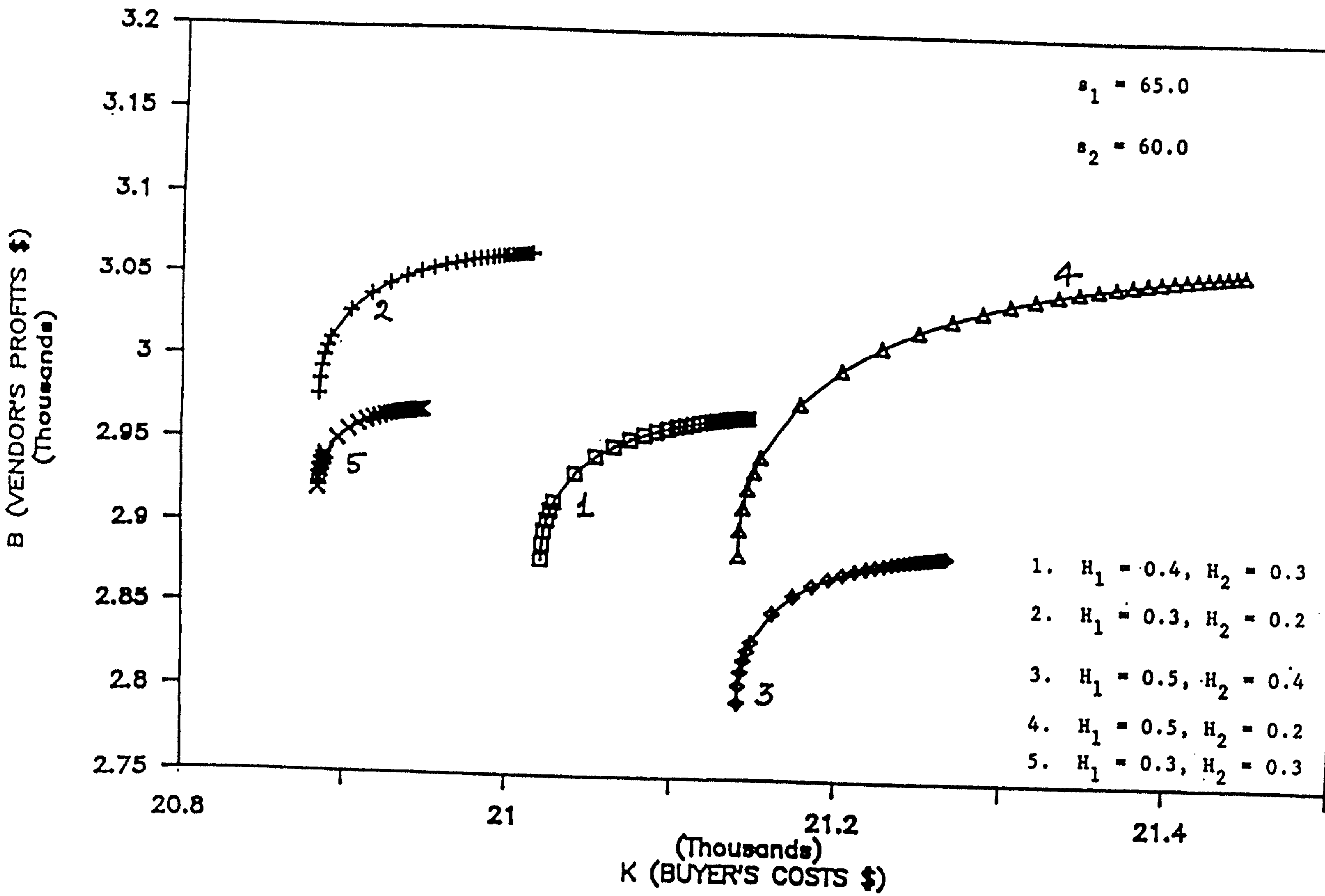


FIGURE 4

Relationship between Vendors' Profit and Buyer's Cost

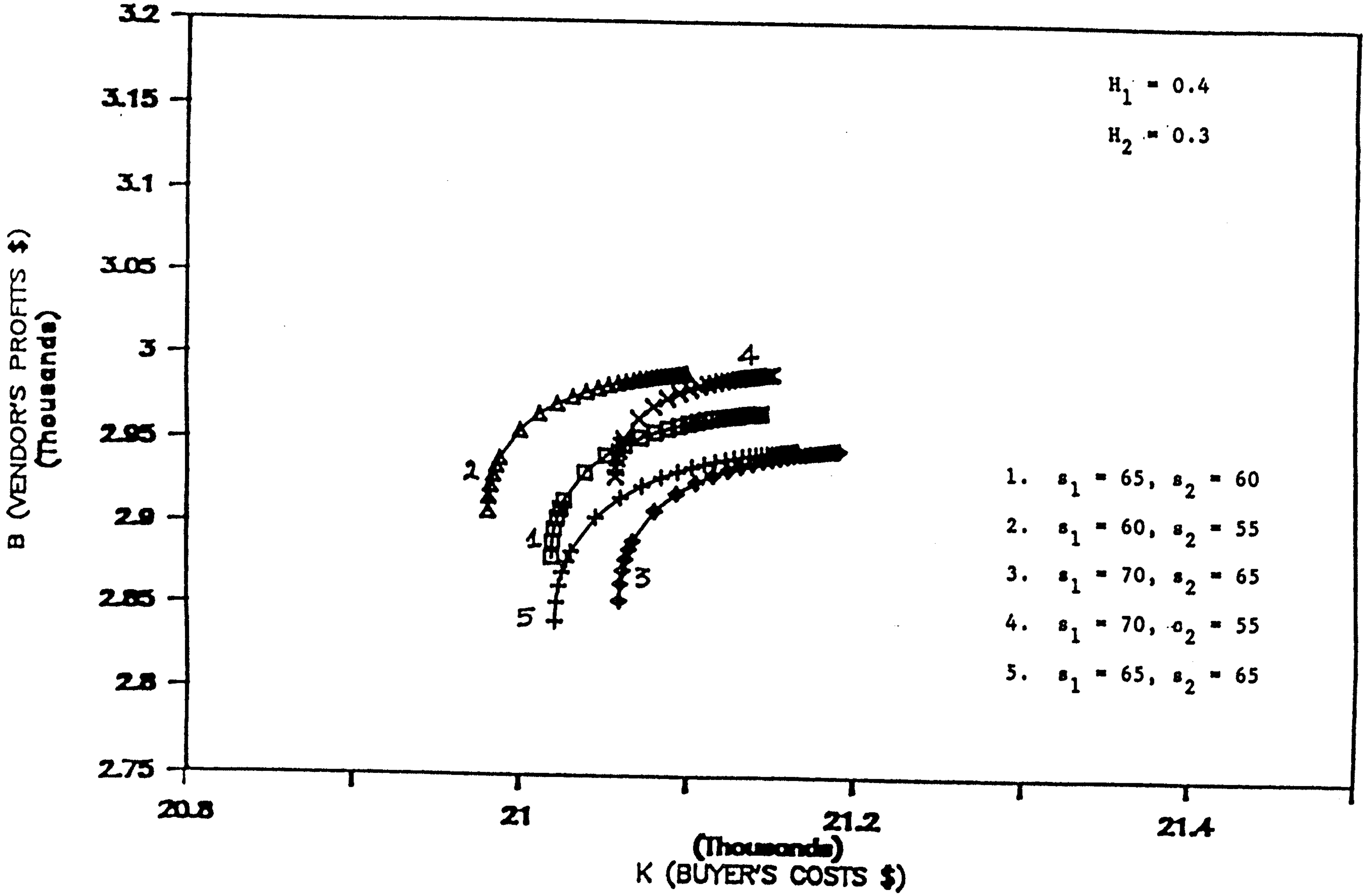
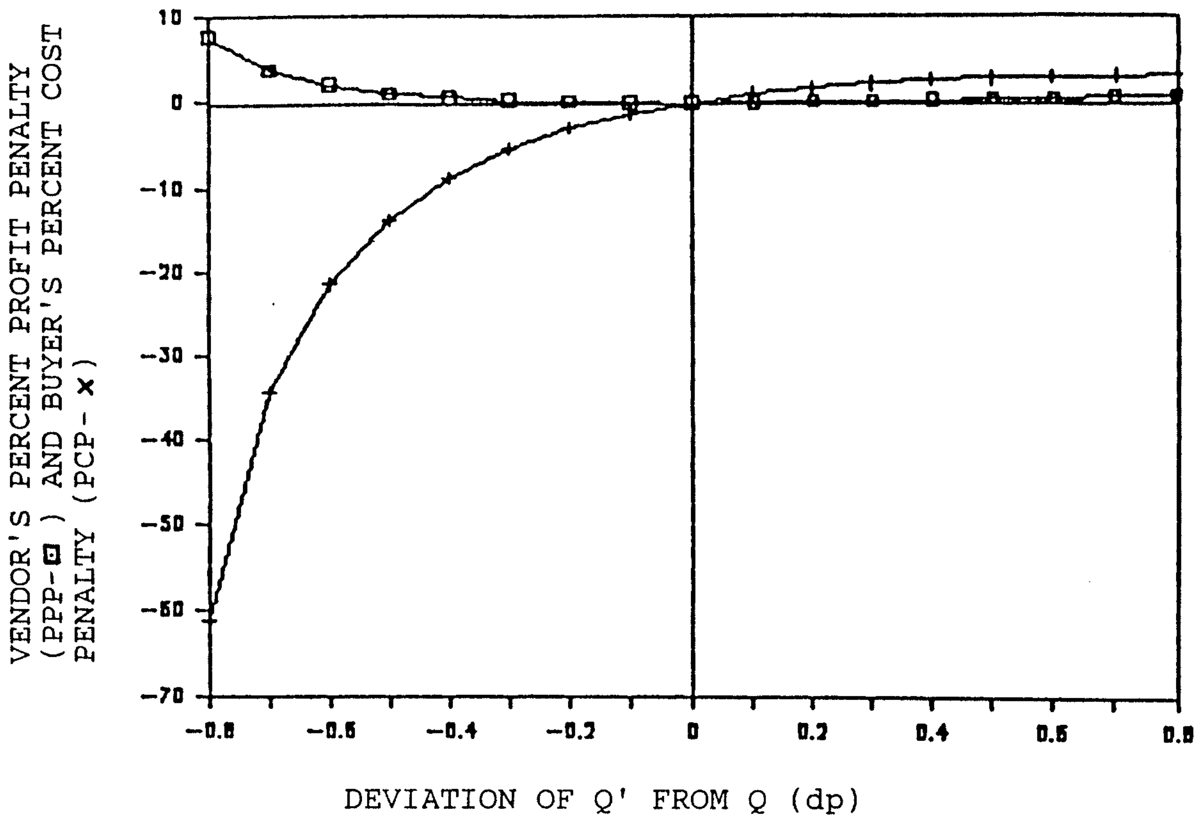


FIGURE 5

VARIATION OF VENDOR'S PERCENT PROFIT PENALTY
(PPP) AND BUYER'S PERCENT COST PENALTY(PCP)
AS A FUNCTION OF DEVIATION OF Q' FROM Q



CHAPTER-5

A QUANTITY DISCOUNT MODEL TO INCREASE VENDOR'S PROFITS AND DECREASE BUYER'S COSTS

In the past few years the research in inventory control systems has concentrated on developing quantity discount schedules from the vendor's perspective. Monahan's (1984) model provides an optimal quantity pricing discount schedule when the buyer is a sole or a major buyer of a supplier. He showed that, by appropriately setting the price, the supplier can always improve his profit. Lee and Rosenblatt (1986 a) generalized Monahan's(1984) model by (i) explicitly incorporating constraints imposed on the amount of discount that can be offered to a buyer and (ii) relaxing the implicit assumption of a lot for lot policy adopted by the supplier. Banerjee(1986 a) also extended Monahan's(1984) model by incorporating the vendor's inventory-carrying cost and showed his model is equivalent to a joint economic lot size model suggested by him earlier (Banerjee 1986b).

Lal and Staelin (1984) suggested a model for an optimal discount pricing policy for the case that the seller's product does not represent a major component of the buyer's final product, that the discount for the product is derived, or that the price is only one of many factors considered in making a purchase decision. Their model is based on minimizing joint (buyer-seller) system cost. They also provide a mechanism for the seller to pass on as much of the cost saving to the buyer as desired. Dada and Srikanth (1987) generalized Lal and Staelin's (1984) model by relaxing their assumptions regarding the relative value of the parameters and allowing the inventory-carrying cost to be a function of price.

Recently Goyal(1987) criticized Lee and Rosenblatt's(1986) work. He suggested that the restriction on the amount of price discount offered by a supplier to his sole buyer seems unreasonable when, in all likelihood, the objective of the supplier is to increase his own profit. He suggested that a higher discount and purchase price may possibly lead to greater economy for the supplier. In reviewing the above literature it appears that nobody has considered the more plausible situation in which the model is viewed from the supplier(vendor's) perspective as well as from the buyer's perspective.

In order to develop the buyer-supplier models, the balance of power between the two group must be understood. For example, although the increase in price or the reduction in quality gives suppliers a bargaining power which allows them to extract greater profits, the buyer cannot simply pass on the additional costs. On the other hand, if the buyers have alternate sources of supply or can substitute materials, they have an effective means of applying counter-pressure. When the supplier is dependent on only a few customers or has high fixed costs, he may find himself at a disadvantage. In such a situation, his best policy should be to maximize his profits with respect to an acceptable buyer's cost level. Similarly, the buyer can also minimize his cost with respect to an acceptable vendor's profit level.

This chapter is a first step in this direction.

FROM VENDOR'S PERSPECTIVE.

We begin our model with the assumption that the vendor's objective is to maximize his profit subject to the buyer's previously agreed maximum inventory-related cost.

The vendor's profit with discount would be the same as that determined by Lee and Rosenblatt's equation (3) except for the holding cost if we consider the vendor to be a manufacturer also. The buyer's inventory-related cost with discount is similar to that given by Monahan's equation (4). If we denote the mutually agreed on maximum acceptable buyer's cost by l, the non-linear program (NLP) would be

$$\text{Maximize} \quad \Pi_2 = DM_2C_1 - Dd(K) - \frac{DA_2}{kKQ} - (k-1)\frac{KQC_2H_2}{2R}$$

subject to (1)

$$C_1D + \sqrt{2A_1C_1H_1D}\left[1 + \left((K-1)^2/2K\right)\right] \leq 1\text{-}Dd(k)$$

If d(K) is the per unit dollar discount offered when the buyer orders K times his current order size,

$$d(K) = \sqrt{2A_1H_1C_1/D}\ (K-1)^2/2K \tag{2}$$

Monahan refers to this value as the "practical" break-even discount, as opposed to the "exact" discount that should be given, which is slightly smaller (equation (7) of Monahan). After substituting the value of d(K) the above NLP problem becomes:

$$\text{Max} \quad \Pi_2 = DM_2C_1 - \left[\sqrt{2A_1H_1C_1D}\,(K-1)^2/2K\right] - \frac{DA_2}{kKQ} - D(k-1)\ \frac{KQC_2H_2}{2R}$$

s.t. (3)

$$C_1D + \sqrt{2A_1C_1H_1D}\left[1 + \left((K-1)^2/2K\right)\right] \leqq 1-\sqrt{2A_1H_1C_1D}\ (K-1)^2/2K$$

Let ϕ be a dual variable or marginal value. Then, after applying the Lagrangian multiplier approach to solve the NLP we get:

$$F(K,\phi) = DM_2C_1 - \left[\sqrt{2A_1H_1C_1D}\,(K-1)^2/2K\right] - \frac{DA_2}{kKQ} - (k-1)D\frac{KQC_2H_2}{2R}$$

$$-\phi[C_1D+\sqrt{2A_1H_1C_1D}\left\langle 1+\left((K-1)^2/2K\right)\right\rangle-1+\sqrt{2A_1H_1C_1D}\ (K-1)^2/2K \tag{4}$$

With the help of Kuhn-Tucker conditions (1951) we observe that

$$\frac{dF}{dK}(K,\phi) = -\sqrt{A_1H_1C_1D/2}\left[1 - \frac{1}{K^2}\right] + \frac{DA_2}{kQK^2} - (k-1)\frac{QH_2C_2D}{2R}$$

$$- \phi\left[2\sqrt{A_1H_1C_1D/2}\left[1 - \frac{1}{K^2}\right]\right] = 0 \qquad (5)$$

The optimal value of K, for a given k, is found to be

$$K^* = \left[\frac{\sqrt{A_1H_1C_1D/2}\,(1+2\phi) + \frac{DA_2}{kQ}}{D(k-1)QH_2C_2/\ 2R + \sqrt{A_1H_1C_1D\ /2}\,(1+2\phi)}\right]^{\frac{1}{2}} \qquad (6)$$

which can be simplified to:

$$K^* = \sqrt{\left(1 + 2\phi + \frac{A_2}{kA_1}\right) \Big/ \left[1 + 2\phi + (k-1)\ DH_2C_2/RH_1C_1\right]} \qquad (7)$$

From (7) we observe that the optimal value K^* of K, for a given k, is a function of i) the set up cost and ii) the carrying cost of both the buyer and the vendor. It also depends on the value of ϕ, the dual variable or shadow price of 1, which represents the maximum additional price the vendor would be willing to pay to increase the discount rate to maximize profit subject to the buyer's inventory relate-cost constraints. It can be shown that, $\phi = \partial\Pi_2 / \partial l$. This expression basically shows If Π_2 is the vendor's optimal profit, the applicability of sensitivity analysis on l which will increase the vendor's profit. Using Kuhn Tucker (1951) conditions again we conclude that the dual variable (or Lagrangian multiplier) is always non-negative:

$$\frac{\partial F}{\partial \phi} = C_1D+\sqrt{2A_1C_1H_1D}\left(1+(K-1)^2/2K\right)-1+\sqrt{2A_1H_1C_1D}\,(K-1)^2/2K\leq 0$$

if $\phi = 0$, then, according to the complimentary condition of Kuhn-Tucker (1951)

$$C_1D + \sqrt{2A_1C_1H_1D}\left[1 + \left((K-1)^2/2K\right)\right]- 1+\sqrt{2A_1H_1C_1D}\,(K-1)^2/2K \geq 0 \quad (8)$$

which is a contradiction and shows the non-binding constraint. This means that, if we increase or decrease the value of L(the buyer's previously agreed cost), the vendor's profit will be constant which is not true.

If $\phi > 0$, then

$$C_1D + \sqrt{2A_1C_1H_1D}\left[1 + \left((K-1)^2/2K\right)\right]- 1+\sqrt{2A_1H_1C_1D}\,(K-1)^2/2K=0 \quad (9)$$

satisfies the non-linear program (3) and shows that the binding constraint (i.e.that the solution is bound by a cost constraint) means the increase or decrease in value of l will directly affect the vendor's profit. So we can conclude that the sign of the dual variable should always be positive.

We find the optimal value of ϕ by solving equation (9) for K:

$$K = \left[\frac{\sqrt{A_1 H_1 C_1 D/2}}{\sqrt{A_1 H_1 C_1 D/2} - 1} \right]^{\frac{1}{2}} \tag{10}$$

Then substituting the value of K in equation (7), we obtain the optimal value of ϕ.

PARTICULAR CASES

(i) If $\phi = 0$, $C_1 = C_2$ and $R = D$ i.e., production is equal to the number of units demanded, which is an unrealistic and a non-economical situation for the supplier. Equation (7) is similar to Lee and Rosenblatt's (1986) optimal discount rate.

(ii) If $\phi = 0$ and $k = 1$, i.e. an order-for-order policy is used by the supplier, K^* in (7) is reduced to equation (2), which is the simple relationship that Monahan has proposed. On the other hand, if $R \to \infty$ with the above conditions, i.e.the vendor himself does not produce but buys from another supplier, then equation (7) is reduced to that for Monahan's optimal discount rate.

(iii) If $\phi = 0$ and the inventory holding costs are constant after discount, then the average holding and setup cost would be $KQD/2R$ and A_2D/KQ

respectively, and equation (7) becomes equivalent to that for Banerjee's(1986) optimal discount rate. This situation is not possible once discounts are introduced, since the inventory holding costs are no longer constant.

For determining the pricing policy for the vendor which maximizes his own profits, Goyal's (1987) method is quite interesting. In this chapter, to establish the optimal integer value of k, we will use Goyal's method. To simplify our notation we, let

$$A = 1 - \frac{DH_2C_2}{RH_1C_1}$$

$$B = \frac{DH_2C_2}{RH_1C_1}$$

$$C = \frac{A_2}{A_1}$$

Then for a given value of k the economic value of $K = K(k)$ is obtained:

$$K(k) = \sqrt{\frac{1 + 2\phi + (C/k)}{2\phi + A + kB}} \tag{11}$$

Maximization of Π_2 is achieved by minimizing the function

$$Z = K(2\phi + A + kB) + \frac{1}{K}\left(2\phi + 1 + \frac{C}{k}\right) \tag{12}$$

After substitution of equation (10), with $K = K(k)$, in equation (11) we get $Z = Z(k)$:

$$Z(k) = 2\sqrt{\left(2\phi + 1 + \frac{C}{k}\right)(2\phi + A + kB)} \qquad (13)$$

If we let

$$X = \frac{[Z(k)]^2}{2} = (2\phi + A)(1 + 2\phi) + kB(1 + 2\phi) + \frac{C}{k}(2\phi + A) \qquad (14)$$

minimization of X is achieved by minimizing

$$r = kB(1+2\phi) + \frac{C}{k}(2\phi + A) \qquad (15)$$

which is a convex function of k. We can get the following condition

$$k(k-1) < \frac{C(2\phi + A)}{B(1+2\phi)} \leq k(k-1) \qquad (16)$$

It is interesting to observe here that every value of k obtained from (16) will yield a global minimum of r. If $\phi = 0$ then equation (16) becomes to Goyal's (1987) condition. The algorithm for determining the optimal policy is given in Goyal's (1987) paper.

FROM THE BUYER'S PERSPECTIVE

If we look at the problem from the buyer's perspective then the objective would be to minimize inventory related costs subject to the vendor's previously agreed upon maximum profit. If we denote the vendor's mutually agreed upon minimum acceptable profit by Y, the NLP for this case is:

$$\text{Minimize} \qquad C_1D + \sqrt{2A_1C_1H_1D}\left[1 + \frac{(K-1)^2}{2K}\right]$$

subject to (17)

$$\Pi_2 = DH_2C_1 - \left[\sqrt{2A_1C_1H_1D}\ \frac{(K-1)^2}{2K}\right] - \frac{DA_2}{kKQ}$$

$$- (k-1)\ K\ \frac{Q}{2}\ C_2H_2\ \frac{D}{R} \geqq Y$$

Using Kuhn-Tucker(1951) conditions, the buyer's optimal discount rate is:

$$K^* = \sqrt{\left[\frac{1}{\gamma} + 1 + \frac{A_2}{kA_1}\right] \Big/ \left[\frac{1}{\gamma} + 1 + (k-1)\ \frac{DH_2C_2}{RH_1C_1}\right]} \qquad (18)$$

here γ is a Lagrangian multiplier or shadow price for 'Y'.

The interpretation and sign are similar to those for ϕ as we discussed above for the discount rate policy from the vendor's perspective.

Using Kuhn-Tucker (1951) conditions, we can derive the following relationship between the Lagrangian multipliers for the policies viewed from the vendor and buyer perspectives at the optimal solution.

$$2\phi = \frac{1}{\gamma} \quad \text{where } \phi \text{ and } \gamma > 0$$

All particular cases as established for equation (7) hold for equation (17).

The condition for optimal integer value of k is

$$k(k-1) < \frac{C(\frac{1}{\gamma} + A)}{B(1 + \frac{1}{\gamma})} \leq k(k+1)$$

Note that k must be at least equal to 1.

CONCLUSION

In this chapter we considered a more realistic situation from the vendor's and buyer's perspective in order to develop a generalized optimal quantity discounts that can lower the buyer's costs and maximizes the vendor's profit. We also present optimal pricing schemes suggested by Goyal(1987).

CHAPTER-6

OPTIMAL ORDERING QUANTITY FOR ANNOUNCED PRICE INCREASES

In many situations, a buyer is provided with an opportunity to place a special order for additional stock before impending price increase takes effect. In such situations, the buyer is faced with a problem of determining the optimal size of this special order.

Subramanyam and Kumaraswamey(1981) developed a model (S-K model) for determining the economic order quantity (when no price increase was impending) that takes into account the effects of factors such as advertising and the possibility of some of the order items being damaged. Lee and Rosenblatt(1986) re-examined and reformulated the S-K model under three different scenarios (described later). The purpose of this note is to propose a model to determine the optimal size of the special order when price increase is announced under the conditions considered by Subramanyam and Kumaraswamy(1981) and Lee and Rosenblatt(1986).

In this chapter we have assumed that the buyer has an opportunity at the end of the next EOQ cycle to make a purchase at the current price and the future orders will reflect the price increase (as in Naddor,1966).

Using S-K Model:

The total inventory related cost after the price increase is given by:

$$TC_2' = DC_2' + \alpha MDC_2' + DA_1 + \frac{1}{2} QH_1C_2' \tag{1}$$

and the economic order quantity can be is as follows:

$$EOQ_2 = \sqrt{\frac{2A_1D}{C_2' H_1}} \tag{2}$$

From equation (1) the cost per unit time can be written as:

$$TC_2' (EOQ_2) = \alpha MC_2' D + UC_2' D + DC_2' + \sqrt{2A_1DH_1C_2'} \tag{3}$$

Similarly the total inventory related cost before the price increase is:

$$TC_1' = DC_1' + \alpha MDC_1' + UDC_1' + \frac{A_1D}{Q} + \frac{1}{2} QH_1C_1' \tag{4}$$

If the current order quantity is of size Q, then it will last for Q/D year. The average inventory during this period is Q/2.In order to select the value of Q we will maximize the following expression which obtained by using equations (3) and (4)

$$F(Q) = UQ(C_2' - C_1') + Q(C_2' - C_1') + \alpha Q(C_2' - C_1')M + \frac{Q^2C_1' H_1}{2D} + \frac{Q}{D}\sqrt{2A_1DC_2' H_1} \tag{5}$$

and

$$Q_{opt} = \frac{C_2' - C_1'}{C_1'} \frac{D}{H_1} + \frac{DU}{C_1' H_1}(C_2' - C_1') + \frac{MD\alpha}{C_1' H_1}(C_2' - C_1') + \frac{\sqrt{2A_1 D H_1 C_2'}}{C_1' H_1} \quad (6)$$

Now the model and optimal quantity for the three scenarios using Lee and Rosenblatt approach are presented as follows:

Scenario 1:

To satisfy demand for D units, more than D units would have to be ordered per period so that, after the damaged or defective items have been discarded, enough items will be left to satisfy all the demand. As a result, the TC_2' becomes:

$$TC_2' = \alpha C_2' DM + \frac{1}{2} QC_2' H_1 + \frac{DA_1}{Q(1-U)} + \frac{DC_2'}{(1-U)} \quad (7)$$

and

$$EOQ_2 = \sqrt{\frac{2DA_1}{C_2' H_1 (1-U)}} \quad (8)$$

the optimal quantity with announced price is:

$$Q_{opt} = \frac{C_2' - C_1'}{C_1'} \frac{D}{H_1(1-U)} + \frac{\alpha MD}{H_1} \frac{(C_2' - C_1')}{C_1'} + \frac{1}{C_1' H_1} \sqrt{\frac{2DA_1 C_2' H_1}{(1-U)}} \quad (9)$$

Scenario 2:

The damaged or defective items are discarded and not sold. Hence if the firm has ordered D units per period, less than D units of demand would be actually satisfied. The unsatisfied demand would incur a certain shortage cost to the firm. The total cost TC_2' with price increase and shortage cost or backorder, in this case is:

$$TC_2' = \alpha C_2' DM + DC_2' + UsC_2' D + \frac{1}{2} sC_2' \frac{(Q - B)^2}{Q} + \frac{1}{2} \frac{H_1 C_2' B^2}{Q} + \frac{DA_1}{Q} \qquad (10)$$

which gives

$$EOQ_2 = \sqrt{\frac{2A_1 D}{H_1 C_2'}} \sqrt{\frac{H_1 + s}{s}} \qquad (11)$$

and

$$B = Q \frac{H_1}{H_1 + s} \quad \text{or} \quad EOQ_2 \frac{H_1}{H_1 + s} \qquad (12)$$

The optimal quantity

$$Q_{opt} = \frac{\sqrt{2A_1 DH_1 C_2'} \sqrt{\frac{s}{H_1 + s}} \left[\frac{H_1 + s}{H_1} - s + \frac{2s}{H_1} + \frac{H}{s} \frac{H_1}{s + H_1} + 1 \right] + (C_2' - C_1') D (1 + Us + \alpha M)}{sC_1' \left[1 - \frac{H_1}{H_1 + s} \right]^2 + H_1 C_1' \left[\frac{H_1}{H_1 + s} \right]^2} \qquad (13)$$

if we consider $s=0$ and $\alpha=0$ then the equation (13) is similar to Silver and Peterson's(1985) expression of optimal quantity.

Scenario 3:

In scenarios 1 and 2 damaged or defective items are assumed to be visible or easily identifiable, so that they can be discarded and not be sold to customers. In some circumstances, however, defective items cannot be identified without an extensive inspection procedure. Without inspection, the firm can be assumed to order and sell D units per year. A cost is, however, incurred for each defective item sold (for repair, servicing, compensation or liability). The unit cost of selling a damaged or defective item assumed to be repair cost and act as backorder in the total cost function and $v \geq M$. The total cost function TC_2', then becomes:

$$TC_2' = \alpha C_2' DM + DC_2' + UvC_2' D + \frac{1}{2} vC_2' \frac{(Q - B)}{Q} + \frac{1}{2} \frac{H_1 C_2' B^2}{Q} + \frac{DA_1}{Q} \tag{14}$$

we can get

$$EOQ_2 = \sqrt{\frac{2A_1 D}{H_1 C_2'}} \sqrt{\frac{H_1 + v}{v}} \tag{15}$$

and

$$B = \sqrt{\frac{2A_1 D}{vC_2'}} \sqrt{\frac{H_1}{H_1 + v}} \tag{16}$$

the optimal quantity would be:

$$Q_{opt}=\frac{\sqrt{2A_1DH_1C_2'}\sqrt{\frac{v}{H_1+v}\left[\frac{H_1+v}{H_1}v+\frac{2v}{H_1}+\frac{H_1}{v}\frac{H_1}{v+H_1}+1\right]+(C_2'-C_1')D(1+Uv+\alpha M)}}{vC_1'\left[1-\frac{H_1}{H_1+v}\right]^2+H_1C_1'\left[\frac{H_1}{H_1+v}\right]^2} \tag{17}$$

NUMERICAL EXAMPLE

Let

$D=500$/year $A_1=1.5$ $H_1=0.4$\$/\$/year

$C_1'=27$\$

$C_2'=30$\$ $s=0.3$\$/\$/year $v=0.8$\$/\$/year

$\alpha=0.3$ $M=0.5\%$ $U=0.35\%$

then

	S-K model	Scenario 1	Scenario 2	Scenario 3
EOQ_2	11.18	13.87	17.078	13.693
Q_{opt}	220.756	249.92	457.961	273.331

CONCLUSION

From the above results it is clear that due to increase in price the optimal quantity also increases and creates cost saving for the buyer, e.g, 10% increase in unit price causes a one-time procurement quantity of to increase by almost 95% to 96%.

CHAPTER-7

CONCLUSIONS

In this investigation a new dimension in the analysis of inventory models for vendor and buyer in an inflationary environment has been introduced. The major findings of the thesis are as follows:

(i) inventory models from the vendor's perspective have been developed in an inflationary environment for (a) multi-items under resource constraints and (b) discounts on all units. Optimal quantities are determined with respect to minimization of total system cost subject to the condition that $H > K$.

(ii) Optimal quantities with inflation from buyer's and vendor's perspective have been determined for generalized inventory models as considered by Lee and Rosenblatt(1986) as different scenarios.

(iii) A generalized inventory control model which considers the perspective of both the buyer and vendor has been developed by using two approaches; Lagrangian multipliers and linear programming.

(iv) A quantity discount model has been developed to increase vendor's profits and decrease buyer's costs.

(v) An optimal ordering quantity for announced price increases has been developed for the different scenarios considered by Lee and Rosenblatt(1986).

It is hoped that the work reported here would open new frontiers of inventory research and lead to further investigations.

SUGGESTIONS FOR FURTHER WORK

In this investigation an attempt has been made to study various aspects, of inventory management both from buyer's and vendor's perspective in an inflationary environment, with discount schedules under deterministic demand conditions. Due to the limited scope of this study, it has not been possible to treat all aspects of inventory management.There is scope for extension in the following areas:

(i) Development of inventory models under variable rates of inflation and in a more general inflationary environment.

(ii) There is Scope for further refinement and modifications of deterministic models used in this work by including stochastic demand and lead time assumptions.

(iii)Market situations other than duopoly or bilateral monopoly for one buyer and one vendor considered in this investigation should be explored. For example, perfect competition situation (n buyers and m vendors), monopoly situations (one vendor and n buyers(identical and non-identical buyer)), monopolistic competition situations (more than one vendor(producing a substitutable commodity) and n buyers).

(iv) Development of inventory control models from a buyer-vendor perspective using multi-objective programming.

(v) Study of the impact of different demand-elasticity functions on Lee and Rosenblatt's (1986) three scenarios in analyzing the feasible order quantity from the buyer's and vendor's perspective by maximizing vendor's profits and minimizing buyer's costs.

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