THE UNIVERSITY OF MANITOBA

THE MEASUREMENT OF FORCES AND MOMENTS DELIVERED BY DENTAL APPLIANCES

JEAN-PIERRE PAQUIEN

A THESIS SUBMITTED TO THE FACULTY OF GRADUATE STUDIES

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR

THE DEGREE OF MASTER OF SCIENCE OF MECHANICAL ENGINEERING

DEPARTMENT OF MECHANICAL ENGINEERING

WINNIPEG, MANITOBA

JUNE, 1978

THE MEASUREMENT OF FORCES AND MOMENTS DELIVERED BY DENTAL APPLIANCES

BY

JEAN-PIERRE PAQUIEN

A dissertation submitted to the Faculty of Graduate Studies of the University of Manitoba in partial fulfillment of the requirements of the degree of

MASTER OF SCIENCE

I hereby declare that I am the sole author of this thesis. I authorize the University of Manitoba to lend it to other institutions or individuals for the purpose of scholarly research.

J. P. Paquien

ABSTRACT

In orthodontic treatment, the movement of teeth is performed with the aid of small springs. These springs, called orthodontic appliances, are made of stainless steel wire formed into complex, convoluted shapes. The present lack of quantitative information concerning their performance is a hindrance in the continuing attempts to improve the efficiency and efficacy of orthodontic treatment. Thus the development of an instrument for measuring the forces and moments delivered by orthodontic appliances was undertaken. Such an instrument should, ideally, provide three-dimensional force measurement in the range ±250 gf (±1 gf) and three-dimensional moment measurement in ±2000 mm gf (±10 mm gf).

The instrument developed uses strain gage force transducers and has the capability of measuring the force and moment characteristics relevant to the orthodontic needs. A mini-computer controls the acquisition of data from the instrument and manipulates these data into a form suitable for assessment by an orthodontist researcher. The major problem of stiction occurring at some point of the instrument was overcome but due to a limitation of time

and of the performance of the designed testing equipment, only the first hemisphere of the space was tested. Imperfection in performance due to buckling in some transducers was observed but methods for removing these imperfections have been given. However, results from the prototype instrument were encouraging and show that an improved version can be built which will satisfy all of the orthodontic requirements.

ACKNOWLEDGEMENTS

As a cultural Franco-Canadian exchange, the author wishes to express his deep gratitude to the Canadian Council of Arts for their complete support. He would also like to sincerely thank Doctor Thornton-Trump from Mechanical Engineering (thesis advisor) for his guidance in the preparation and presentation of this thesis, Doctor K. R. McLachlan from Civil Engineering, without whose teaching, comments and criticism, this study would not have been possible, and Doctor A. Storey and Doctor J. Windburn from Preventive Dental Science, for their assistance in the orthodontic field.

In addition, the author would like to thank those associated with Mechanical and Civil Engineering and Preventive Dental Science Departments of the University of Manitoba who so willingly offered their help during the course of these studies, in particular the technicians, J. Clark, W. Wiles and J. Sewell, and the secretary, R. Hanley. Finally, the author extends his thanks to the typist, A. M. Phillips.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

NOMENCLATURE

[A]	Conversion matrix
A1	Longitudinal angle
A2	Angle of latitude
a	Slope of the regression line
[B]	Stiffness matrix
bij	Coefficient of the matrix
b	Transducer width
C_i	Transducer response
d	Transducer deflection
d_1	Distance between the forces $\vec{f}_1$, $\vec{f}_2$ and $\vec{f}_3$
d_2	Distance between the forces $\vec{f}_4$, $\vec{f}_5$ and $\vec{f}_6$
E	Transducer elastic modulus
$\vec{F}$	Force
$\vec{Fh}$ ($\vec{Fx}$, $\vec{Fy}$)	Horizontal force
$\vec{Fz}$	Vertical force
$\vec{f}_i$	Forces transduced
g	Gage factor
h	Transducer thickness
I	Transducer inertia
k_i	Calibration coefficients
L	Transducer length
L1	Distance from the transducer root to the strain gages

$\vec{M}$	Moment
Mf	Bending moment
$\vec{Mv}$ ($\vec{Mx}$, $\vec{My}$)	Azimuthal moment
$\vec{Mz}$	Pivoting moment
$\vec{n}$	Vectorial position
P	Maximum transducer load
R	Resistance of the strain gage
r	Coefficient of correlation
$\{\vec{T}\}$	Primary assembly vector
U	Output bridge voltage
V	Input bridge voltage
V1	Output of the L.V.D.T. force transducer
$\{\vec{V}\}$	Secondary assembly vector
$\{\vec{W}\}$	Activation vector assembly
Z	Transducer section modulus
δ	Spring deflection
λ_i	Assymetry ratio
ε	Strain
T	Stress

Subscripts

c	Computed
i	Transducer number
j	Observation
k	Experiment conditions
o	Reference
T	Under test
Δ	Variation

Units

gf	Gram force
N	Newton
mm	Millimeter
mm gf	Moment unit
V	Volt
°	Degree

Symbols

$\|\| \ \|\|$	Absolute value
$\rightarrow$	Vectorial quantity
[ ]	Matrix
{ }	Assembly vector

Abbreviations

D.A.S.	Data Acquisition System
L.V.D.T.	Linear Voltage Differential Transformer
H.P.	Hewlett Packard

1. INTRODUCTION

From the earliest days of dentistry, orthodontics has always been part of it and has played an important role in dental treatment. The orthodontic treatment deals with malocclusion and provides a means for effecting substantial changes in the position of teeth thereby improving occlusion.

Although the problems in orthodontics are mechanical in nature due to the fact that no motion can take place without the application of force, there is an important biological factor which differentiates the orthodontic problem from other problems in mechanics. The change of position of a tooth in response to an applied force is made possible by the inherent ability of the bone tissue to resorb and rebuild under the influence of pressure stimuli providing the pressure is kept within physiological limits, consequently limiting the magnitude of the applied force to a range of a few grams-force (gf) to a maximum of 200 gf. The motion of a particular tooth is in the order of a few millimeters (mm), and can take place in the three spatial dimensions, so the force-system must be applied with a lot of care and controlled with respect to the neighboring teeth.

The uses of different techniques and the complexity of the oral environment as well as the limited area of the crown of a tooth make the mechanical means to generate forces very complex in structure. These "generators" of force called orthodontic appliances are all based upon spring structures and will vary from a single arch wire or a coil spring to very complex springs as can be seen in Figure 1-1. Up to now, only the orthodontist's experience based upon his empirical knowledge of the mechanical and biological uncertainties allows him to choose the most suitable and appropriate structure of an orthodontic appliance to move a particular tooth or row of teeth within the context of the oral environment. Due to tremendous development in the orthodontic field in the last decade the need for a better understanding of the phenomena involved has developed.

A brief review of the general literature in the field of orthodontics (1, 2) indicates that the fundamental behavior of orthodontic appliances is not well understood. However, such an understanding of behavior is particularly critical in the application of the segmented arch technique. A precise knowledge of the performances of the orthodontic appliances is therefore considered of great importance in the development of techniques to be used by the orthodontists. Hence the main thrust of the work here is that of developing techniques and instruments to provide a fundamental knowledge of orthodontic appliances. Since the last decade many orthodontist re-

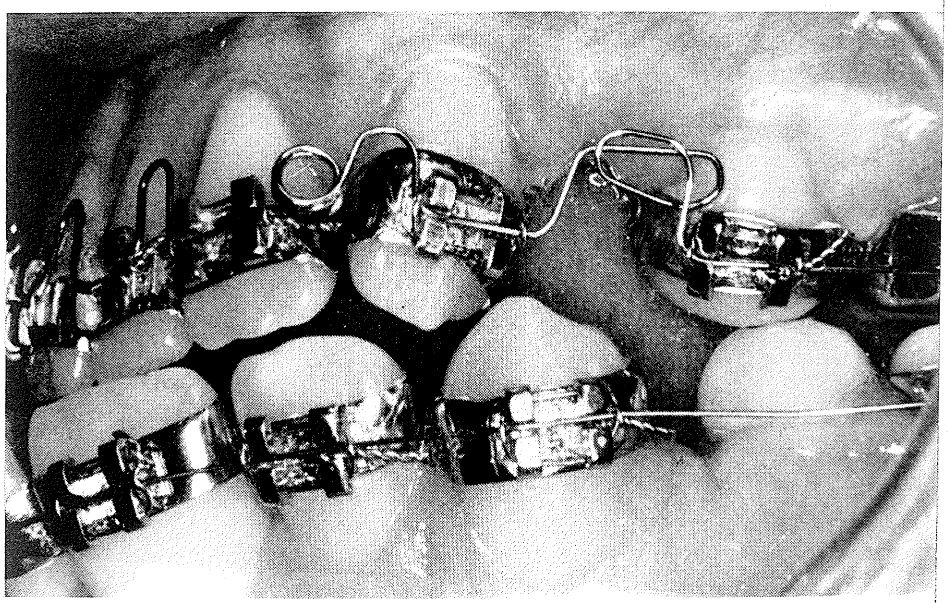


FIGURE 1-1 EXAMPLE OF APPLIANCES IN ORTHODONTIC TREATMENT

searchers have seriously investigated, with little success, the mechanical response of orthodontic appliances. None of them have succeeded in designing an instrument which could measure the three dimensional force-system developed by these appliances (3, 4). This could be due to a lack of cooperation between orthodontists and engineers.

Owing to several personal talks with the orthodontist researcher, Doctor G. Winburn, who works closely with Doctor K. R. McLachlan from Civil Engineering, the author became interested in the problem of designing a measurement instrument which could provide a way to select and classify the present orthodontic appliances with respect to their mechanical characteristics.

2. INSTRUMENT REQUIREMENTS

By far the two most important criteria that this instrument should meet are that it be easy for the orthodontist researchers to use and that it enable them to measure in three dimensions the force and moment delivered by an orthodontic appliance. To satisfy these two major criteria, one must develop a statement of performance requirements that the instrument must meet.

First of all the instrument must reflect the oral environment situation as closely as possible and the results must be displayed in a form easily interpreted by the orthodontist. Thus the instrument must be kept as small as possible but must allow enough room to place a model tooth at its center of resistance and must, at least, leave a free solid angle of 180 degrees. Due to the large amount of data to be treated, there should also be an automatic collection and reduction of data. It is worth noting that the technology needed to perform this last requirement offers many ways to organize and to display the final results.

Secondly with the help of the existing literature dealing with the force-systems delivered by the orthodontic appliances (5, 6) and with many talks with orthodontists (10, 11) the main mechanical characteristics of the instrument such as its range, its sensitivity and its constraints, could be defined. A change of geometrical configuration (never more than a few millimeters in orthodontics) causes the orthodontic appliance to generate a three dimensional force and moment system. Depending upon the dimensional characteristics of the appliance structures, the force generated can range from a few gf to about 200 gf and the moment can range from a few hundred millimeter gram-force (mm gf) to about 2,000 mm gf. Thus it follows that the instrument should have a sensitivity at least equal to the units used, that is, 1 gf and 100 mm gf and have the capacity to measure a maximum force magnitude of 200 gf and a maximum moment magnitude of 2,000 mm gf in any direction. The instrument must also have a means of activating any orthodontic appliance attached to the model tooth within a range of 1 mm to 10 mm for the linear change of configuration and from a few degrees to 180 degrees for the angular change. From these last requirements one can clearly see that any measurement of a force and moment should never geometrically distort any part of the instrument more than 0.1 mm in order that no significant interference with the activation of an appliance could take place.

3. INSTRUMENT DEVELOPMENT

3.1 Discussion of Methods and Techniques

Many methods of investigating the performance of orthodontic appliances are possible. Some of them do not even involve the use of an instrument. For instance a statistical technique could have been used in a clinical situation but the lack of reliability of the mechanical and biological responses and the massive data required due to the great number of appliances would not have provided a way of characterizing new appliances before application. An approach based only on theoretical analysis would not have permitted the investigation of all appliances due to the great difficulty of analysing their structures even using the finite element method (7). Thus in light of the above difficulties the need for a practical approach appears necessary. One direct and practical method could be that of idealizing the gum surrounding the model tooth by an elastic base and to measure the displacement of the tooth under the orthodontic forces. But in reference to the values given in the discussion of the instrument requirement, the size of the displacement of the model tooth would require an exotic and not commonly used technology to measure it. Therefore the next practical method was to measure directly the force and moment components delivered by orthodontic appliances. Direct measurement is possible by using a weightbridge technique for each of the six components of the force-system. Since the moment component is a scalar product of force component and distance, clearly the measurement of moment becomes a problem of measuring force. It must be kept in mind that several methods are available to determine the six components from force measurements. Two such methods are the servomechanical method and the strain bridge method. The devices using these methods require six force transducers placed in an appropriate geometrical configuration in order that the computation of the six components involves a linear combination of the six transducer responses. Servomechanical and strain bridge devices are often used for studies in wind tunnels (8) or the measurement of the rocket thrust characteristics (9), therefore they represent no major theoretical development although there is a problem of scale. The system of forces developed by the transducers of these devices must satisfy the laws of equilibrium; that is, in our case, the instrument must be in equilibrium and must also support the model tooth so that the whole system provides an equilibrium support with known constraints.

An attempt to design such a device based upon the servomechanical method was made by Doctor K. R. McLachlan but due to many circumstances, such as the requirement of extreme tolerances and some technical problems, the idea was left for the strain bridge method. The principle schematically shown in Figure 2-1 employed the latter method. This principle is used to test rocket thrust (9) and consists of breaking down the six Cartesian components of the force-system mathematically represented by $\{\vec{T}\} = \{\vec{F}, \vec{M}\}$ into a new system $\{\vec{V}\}$ of six forces $\vec{f}_i$ such as $\{\vec{V}\} = \{\sum_{i=1}^{6} \vec{f}_i\}$ capable of being measured by six force transducers. In this principle the six force transducers are placed in accordance with the equilateral triangle laws to provide a linear split of the force-system $\{\vec{T}\}$ and an easy computation to reconstitute the force-system from the six force transducer responses. The split can be mathematically represented by a matrix [A] and we can see how the instrument performed mathematically according to the following equation:

$$\{\vec{T}\} = [A]\ \{\vec{V}\} \quad \text{or} \quad \{\vec{V}\} = [A]^{-1}\ \{\vec{T}\} \tag{3.1}$$

For the reader who is not familiar with the concept of matrix algebra, a traditional approach to the vectorial quantity representing the force-system can give the set of the six equations following:

$$Fx = (f_2 - f_3)\ \sqrt{3}/2 \tag{3.2}$$

$$Fy = ((f_2 - f_1) - (f_1 - f_3))/2 \tag{3.3}$$

$$Fz = f_4 + f_5 + f_6 \tag{3.4}$$

$$Mx = d_2\ (f_5 - f_4)/2 \tag{3.5}$$

$$My = d_2\ ((f_6 - f_4) - (f_5 - f_6))/2\quad 3 \tag{3.6}$$

$$Mz = -d_1\ (f_1 + f_2 + f_3)/2\ \sqrt{3} \tag{3.7}$$

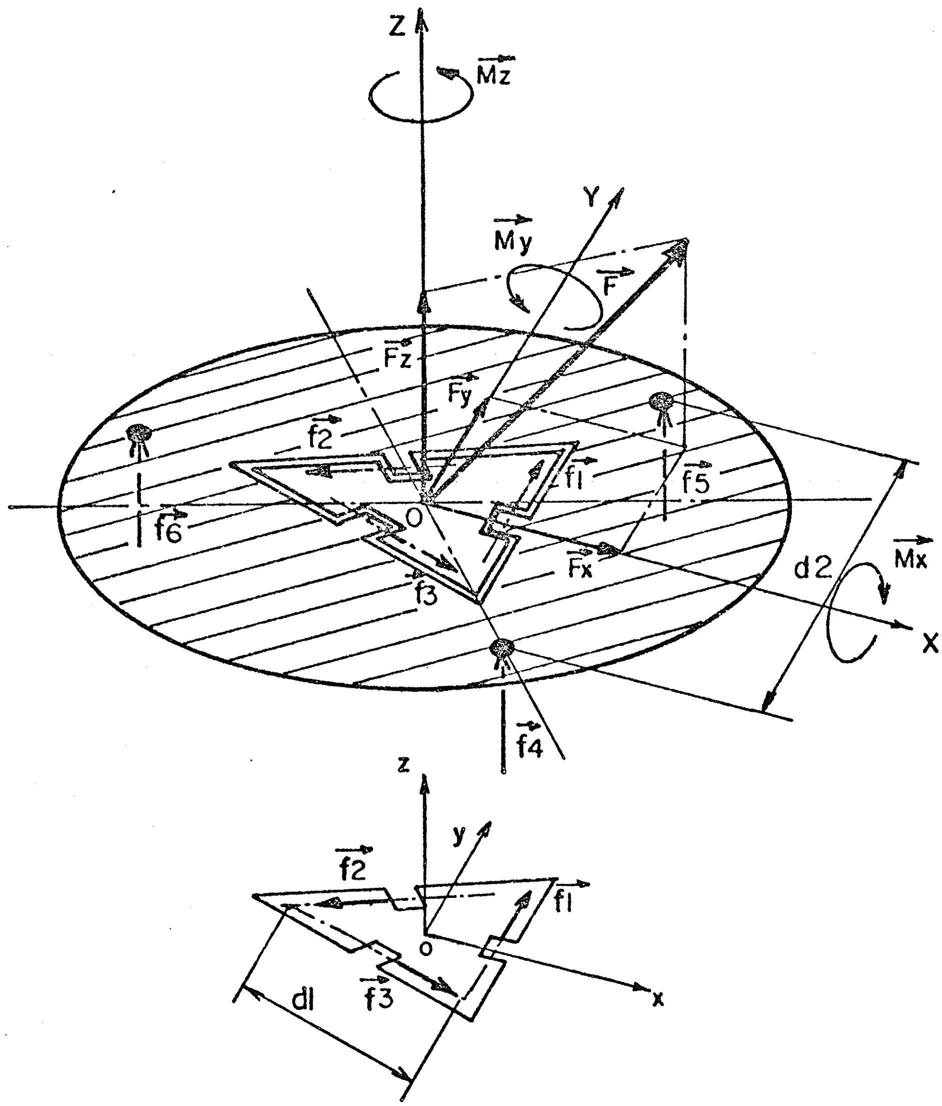


FIGURE 3-1   FORCE AND MOMENT RELATIONSHIPS

The test of an appliance will involve a great number of loading geometries and each observation will be represented by the six component quantities (equations 3.2 to 3.7) so the use of an automatic means of data collection and of computation is almost imperative. In light of the last point it becomes clear that the use of an electrical type of force transducer as well as a digital computer is strongly desirable. Knowing that all force transducers begin with the measurement of a displacement and that an electrical type of transducer is preferred, an electromechanical device must be used. Moreover, according to the instrument requirements already discussed, the transducers will have to meet the following requirements:

a. It should be linear for ease of computation.

b. The transducer should have a small displacement (< 1 mm) and a high sensitivity (1 gf).

c. It must permit uncomplicated associated electrical signal conditioning.

d. It should be directionally sensitive and constrained in the unique direction of the force that it transduces.

e. It should be fairly small in size in order to have good sensitivity in the measurement of moment.

f. It should have a zero frequency response and be stable in its behavior.

In spite of the great advantages of the fluidic transducer, it happens that the method had to be rejected due to the limitations of resources and a more conventional electromechanical method had to be employed. Among all of the transducer types available two were retained, the piezo-electric and the resistive type. In many respects the piezo-electric type entailed greater difficulties in reaching the requirements d) and f) than the resistive spring type. Thus the resistive spring type was selected, but this type dictated the use of a cantilever with strain gages in order to satisfy the sensitivity and constraints requirements. The most commonly used method to convert a change of resistance to an electrical signal is to mount the strain gages in a constant voltage four-arm Wheatstone bridge. In this particular case the Wheatstone bridge will be used as a direct read-out device due to the availability of the high sensitivity digital voltmeter in the laboratory. It will be demonstrated later that the use of a pair of strain gages is imperative for having a linear conversion between the change of resistance and the voltage output of the bridge. But these strain gages must be arranged and placed in such a manner that they permit the transducers to be sensitive only in bending. Therefore one gage will be cemented on top of the cantilever and the other placed symmetrically on the bottom and they will be located as closely as possible to the position of the maximum bending moment.

In order to apply the proper constraints we end up by using two types of cantilever, type A and type B, as shown in Figure 3-2. The force transducers of type A which measure the forces $\vec{f}_1$, $\vec{f}_2$, $\vec{f}_3$ (Fig. 3-1) must be kept free longitudinally in order to be constrained only in the direction where they measure the force. The type B which measure $\vec{f}_4$, $\vec{f}_5$, $\vec{f}_6$ (Fig. 3-1) must be constrained longitudinally to some extent.

No additional restrictions on the dimensions of the cantilever result from the use of strain gages since some strain gages as small as 2 mm in length are available. By making the cantilever out of high quality stainless steel we ensure that the linearity as well as the stability and the zero frequency response

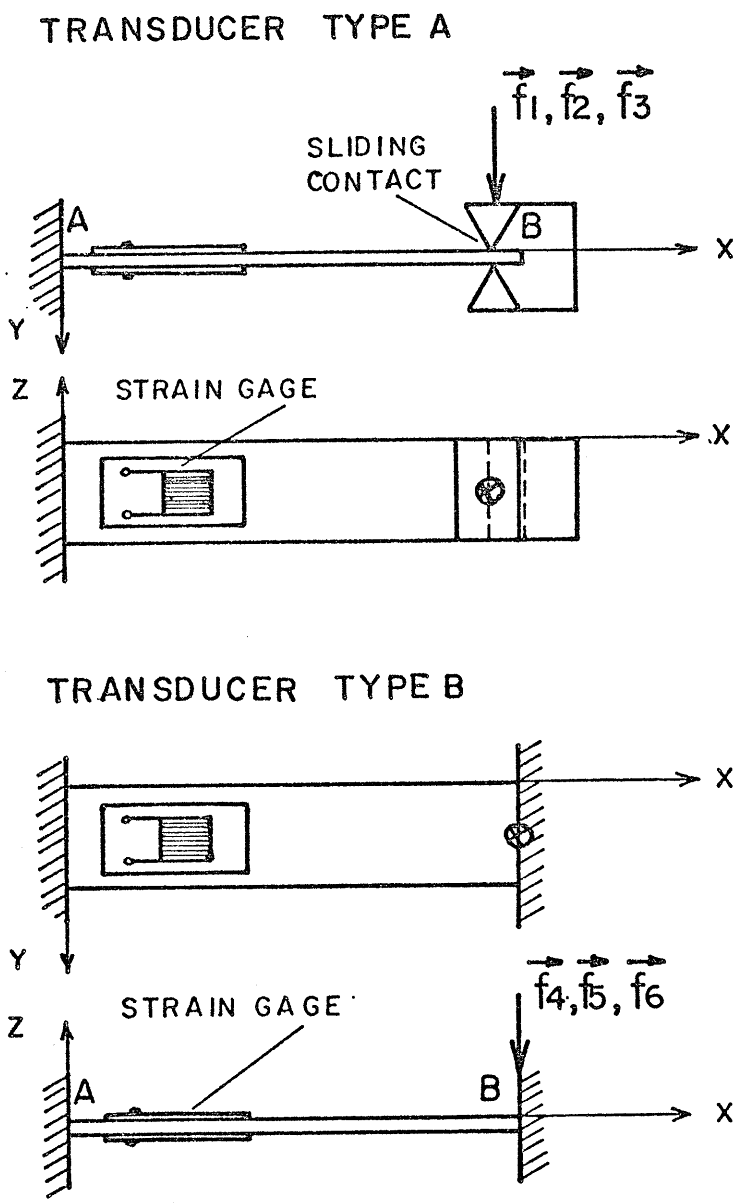


FIGURE 3-2 TRANSDUCER TYPES

requirements are obtained, however, the cantilever method must have its validity checked. This check has been performed and is covered in detail in section 3.3, page 19.

By reviewing the design requirements it appears that an instrument can be built in two stages. The first stage would measure horizontal forces, thus using type A cantilever supports (Fig. 3-2 type A). The second stage will support the first stage as well as measuring the vertical forces and thus will employ type B transducer (Fig. 3-2 type B).

3.2 Instrument Configuration

The physical realisation of the device that can be seen in Figure 3-3 shows how the two transducer types explained in section 3.1 have been incorporated with respect to the instrument requirements. The two types of transducers briefly discussed at the end of the last chapter can be easily distinguished in this picture.

The type of transducer referred to as type B, page 14, measures the vertical force component (Fz) and the azimuthal moment (Mv) and is rigidly attached on one end to the frame of the instrument and to the other end to the body of the part suspended on them. These three transducers 4, 5, 6, type B, define a hor-

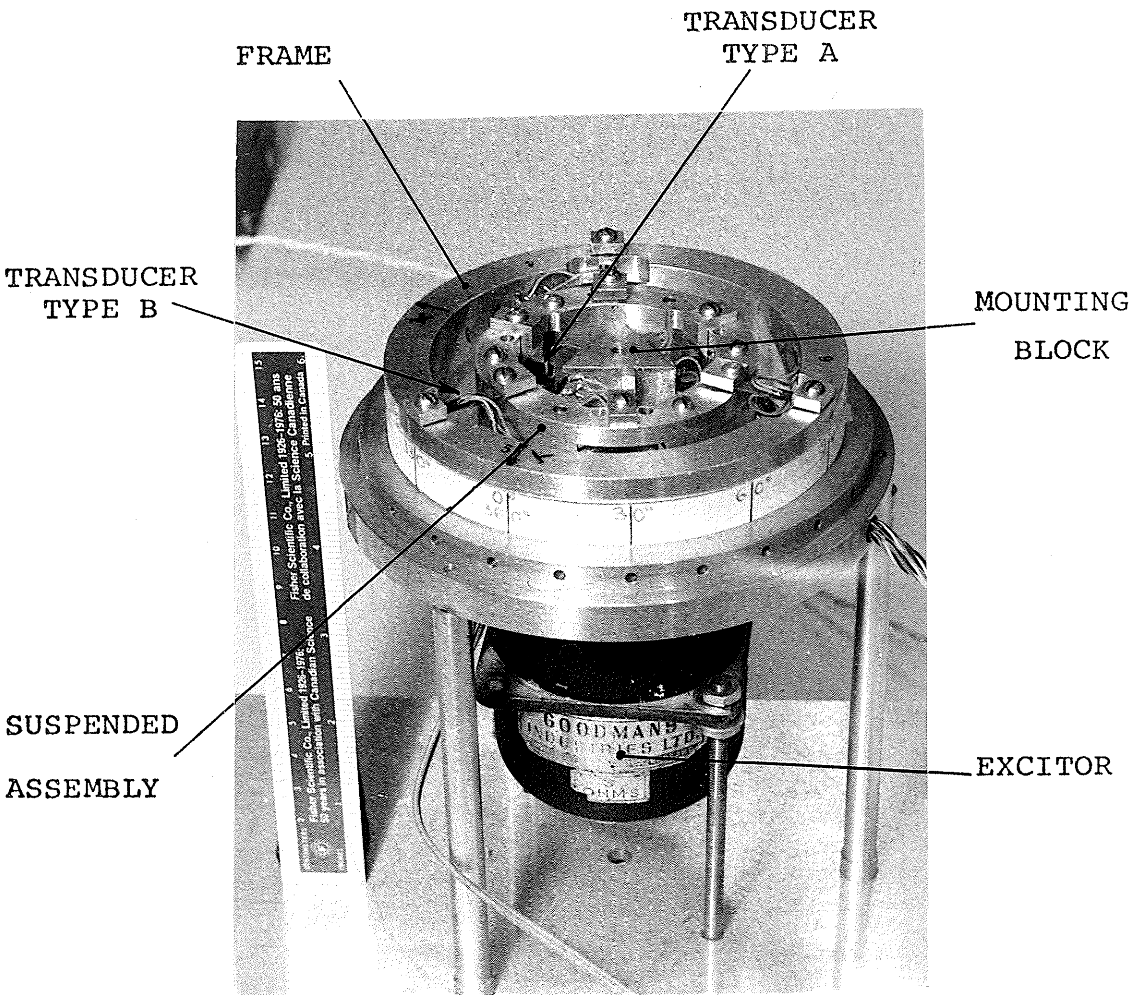
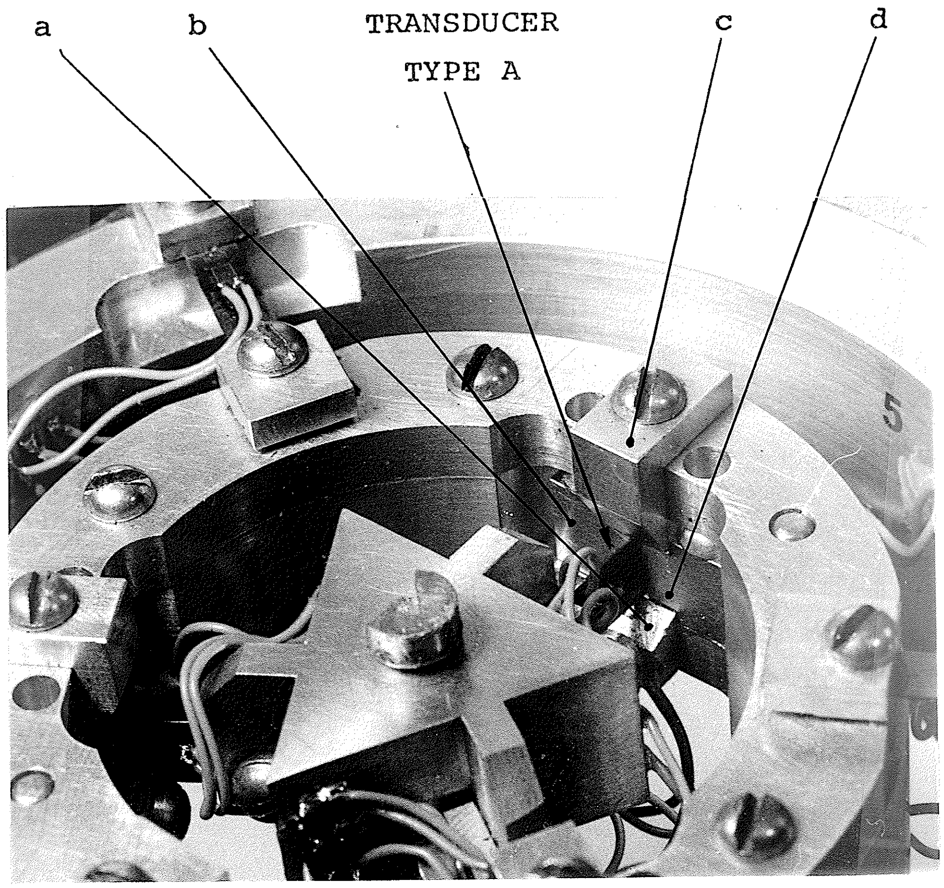


FIGURE 3-3 GENERAL VIEW OF THE INSTRUMENT

izontal plane which contains the center of resistance of the instrument in order that no undesirable moments can be generated by the instrument geometry. The constraints on the cantilevers rigidly position the central body in the horizontal plane. The central body is made of an aluminum alloy in order to minimize the preload on the transducers, and yet leave them sensitive in a vertical direction.

The other transducers 1, 2, 3, referred to as type A, page 14, measure the horizontal force (Fh) and the pivoting moment (Mz). They are rigidly attached to the triangular shaped mounting block, in the center of which a model tooth will later be implanted, and are free to slide horizontally through their respective contacts located in the body of the suspended ring. The ring of the suspended part has been designed in order to receive the four adjustable parts which constitute each contact and in doing so provide a way to minimize the geometrical distortion of the central mounting block caused by their respective gaps. A picture of such a contact can be seen, Figure 3-4.

On the bottom of the instrument an electromagnetic exciter can be partly seen. It permits, as will later be demonstrated in section 3.3, an induced



a - b - c - d. The four adjustable parts of a contact

FIGURE 3-4 CLOSE-UP VIEW OF THE SLIDING CONTACT vibration of the mounting block in order to overcome the stiction problem bound to occur at any sliding contact.

3.3 Engineering Design

In this section the validity of the resistive spring method will be shown by presenting the way in which the calculation defining the transducer characteristics was made with respect to the instrument and transducer requirements.

3.3.1 Dimensional Characteristics

The dimension of the cantilever was found by applying beam theory under the set of the following constraints implicitly mentioned in the previous sections:

a. The maximum deflection that any of these transducers can undergo should not be greater than 0.1 mm because the accuracy of the cantilever technique depends upon having a small deformation of the beam ends and, moreover, no significant geometrical distortion of the instrument under the application of an orthodontic force-system should take place.

b. The length of the cantilever shall remain within the limits which permit the detection of the minimum moment, therefore length "L" should not be greater than 30 mm.

c. The minimum sensitivity should allow detection of a force equal to 1 gf.

d. Type B transducers, which have both ends rigidly attached, should contribute equally to the support of the weight (113 gf) of the suspended system supported on their extremities.

e. The position, the size and the minimum strain reliably detectable by the strain gages should be designed to give the highest sensitivity.

The physical definition of each type of transducer with their respective formulae used for the calculation are displayed in Table 3.1, page 21.

First an estimate of the cantilever size was made to allow selection of the type of strain gages * to be used. The strain gage size in return defined

* EA 06 - 062 AP120

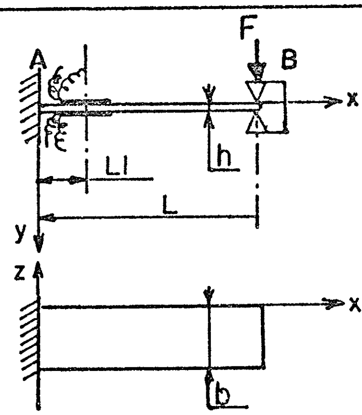
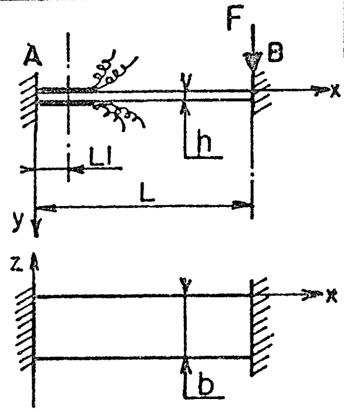
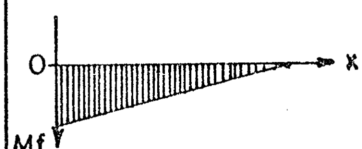
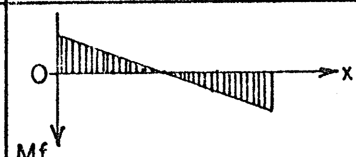
<table>
<tr><td></td><td colspan="2">TYPE</td><td colspan="4">A</td><td colspan="4">B</td></tr>
<tr><td rowspan="3">DEFINITION OF TYPE</td><td colspan="2">GEOMETRICAL CHARACTERISTICS</td><td colspan="4"></td><td colspan="4"></td></tr>
<tr><td colspan="2">CONSTRAINTS</td><td colspan="4">A : FIXED
B : FREE IN X AND Y DIRECTION</td><td colspan="4">A : FIXED
B : FREE ONLY IN Y DIRECTION</td></tr>
<tr><td colspan="2">BENDING MOMENT DIAGRAM Mf</td><td colspan="4"></td><td colspan="4"></td></tr>
<tr><td rowspan="5">FORMULAE</td><td>1</td><td>DEFLECTION d</td><td colspan="4">$FL^3/3EI$</td><td colspan="4">$FL^3/12EI$</td></tr>
<tr><td>2</td><td>INERTIA $I = bh^3/12$</td><td colspan="4">$FL^3/3Ed$</td><td colspan="4">$FL^3/12Ed$</td></tr>
<tr><td>3</td><td>BENDING MOMENT Mf</td><td colspan="4">$F(L-x)$</td><td colspan="4">$F(L/2-x)$</td></tr>
<tr><td>4</td><td>STRAIN $\epsilon = \sigma/E$</td><td colspan="4">$E\,Mf/Z$</td><td colspan="4">$E\,Mf/Z$</td></tr>
<tr><td>5</td><td>SECTION MODULUS $Z = bh^2/6$</td><td colspan="4">$E(L-LI)\,F/\epsilon$</td><td colspan="4">$EF(L/2-LI)/\epsilon$</td></tr>
<tr><td rowspan="4">RESULTS</td><td colspan="2" rowspan="2">DIMENSIONAL CHARACTERISTICS (M·M)</td><td>h</td><td>b</td><td>L</td><td>LI</td><td>h</td><td>b</td><td>L</td><td>LI</td></tr>
<tr><td>0.25</td><td>6.</td><td>8.5</td><td>3.</td><td>0.25</td><td>4.</td><td>14.</td><td>2.</td></tr>
<tr><td colspan="2">TRANSDUCER SENSITIVITY (G·F)</td><td colspan="4">1.</td><td colspan="4">1.</td></tr>
<tr><td colspan="2">TRANSDUCER DETECTION RATIO Fmax/Fmin</td><td colspan="4">75</td><td colspan="4">45</td></tr>
</table>

TABLE 3-1 TRANSDUCER CALCULATION SUMMARY

the distance L1 (see Table 3-1) giving the highest transducer sensitivity (at L1 = 2.5 mm). Knowing that the minimum reliable strain ε_{min} for strain gages was estimated at 10 micro strain (μs) and that, as said previously, the material used for the cantilever is a stainless steel which has an elastic modulus E of about $E = 2 \times 10^5$ N/mm^2 and that the maximum deflection d_{max} acceptable by the cantilever is equal to 0.1 mm, the formulae 2 and 5 in Table 3-1 were worked out. After several iterations for the value L, in both formulae, in order to reach the highest ratio F_{max}/F_{min} and to keep F_{min} to 1 gf, we end up for each type of transducer with a system of two equations with two unknowns (h and b) shown below.

$$I = bh^3/12 \tag{3.8}$$

$$Z = bh^2/6 \tag{3.9}$$

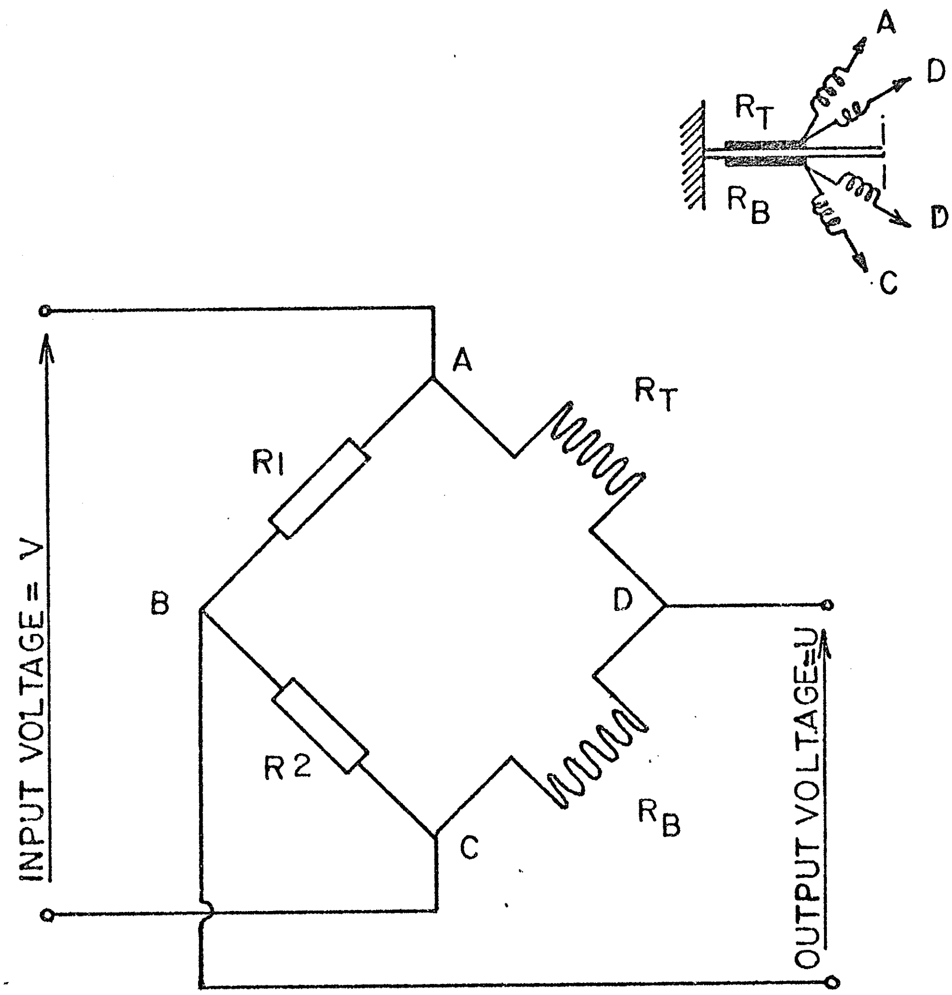
By solving this system of equations finally we get the results shown at the bottom of Table 3-1, page 21.

Notice that a compromise was made in regard to the ratio F_{max}/F_{min} which we would like to have been twice as big as they are. There is still a method available to extend the range of the instrument to that of the force-system delivered by orthodontic springs by covering the range not only with one instrument but with two similar instruments of different capacities. From the dimensional results of these beam transducers one can clearly see that great care had to be taken when cementing the strain gages on the beams. For example, in order to avoid altering the beam response, the strain gages were cemented with the thinnest possible film of glue and waterproof protection, and the most pliable electrical wires, running from them, were coiled and soldered to allow minimum mechanical resistance to motion. However, within the workshop facilities the strain gages were positioned on the beams with a dimensional tolerance of 0.5 mm.

3.3.2. Constraints Characteristics

From the point of view of the constraint requirements, it must be demonstrated that the transducers are sensitive only to bending and that their means of attachment do not generate any auxiliary source of non-linearity.

With respect to bending, it must be demonstrated that by cementing two strain gages symmetrically on the two opposite flats of the transducer cantilever and by using the Wheatstone bridge method we end up by interpreting only the bending strains.



$U = V_B - V_D$

$V = V/2$

<u>Legend</u>

R1, R2 : Unstrained resistances

R_T, R_B : Strain gages

FIGURE 3-5 STRAIN GAGE CIRCUIT AND ARRANGEMENT

The Wheatstone bridge method with its associated derived formula $U = V/2 - V_D$ (3.10) in the case where it is used as a direct read-out device is shown in Figure 3-5. One can clearly see that when the cantilever is subjected to bending strains R_T becomes $R_T + \Delta R$ and R_B becomes $R_B - \Delta R$ and leads to the equation

$$V_D = V/2 + V\Delta R/2R \qquad (3.11)$$

By substituting this equation into the equation (3.10) we come to the following equation

$$U = V\Delta R/2R \qquad (3.12)$$

When the cantilever is subjected to tensile or compressive strain, it can be seen that U remains equal to zero, thus the transducer is only sensitive to bending.

With respect to auxiliary source of nonlinearity generated by the constraints put on the transducers we will only investigate at this stage the stiction constraint inherent to the presence of sliding contacts at one end of each type A transducer. However, it should be mentioned that the type B transducers, during the testing of the instrument, exhibited a buckling phenomenon. This latter point will be fully discussed in the discussion of the results in Chapter 5.

Any stiction at a contact generates a force hysteresis cycle, sketched in Figure 3-6, as long as static contact is maintained. The theory of this phenomenon can be very well summarized by Figure 3-7 shown on page 27. Now if a contact is idealized as shown in Figure 3-8 when static contact is maintained between the two parts, a force F_s at the contact O (Fig. 3-8) is required to move the mass with respect to the x or y axis. But as soon as a dynamic contact is established at the contact O by giving a velocity to the plane P (Fig. 3-8), only a force F_c is required to move the mass even if the displacement of the mass is perpendicular to the velocity and by making reference to the diagram Figure 3-7, it can be seen that in this situation the friction force has passed into the linear domain of the viscous friction. Obviously the change of the contact characteristics has little effect upon the ratio F_s/F_c unless a pressurized bearing is used but the use of such a bearing was out of the question due to the resource limitations. It is second nature for any user of a deflection instrument to tap it gently before taking a reading, therefore the idea of externally applying a vibration below the mounting block of the device was developed. The virtue of the vibration is not only that it initiates a

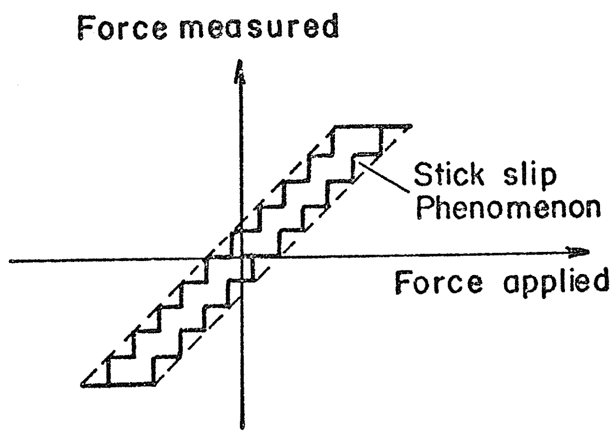


FIGURE 3-6 HYSTERESIS PHENOMENON

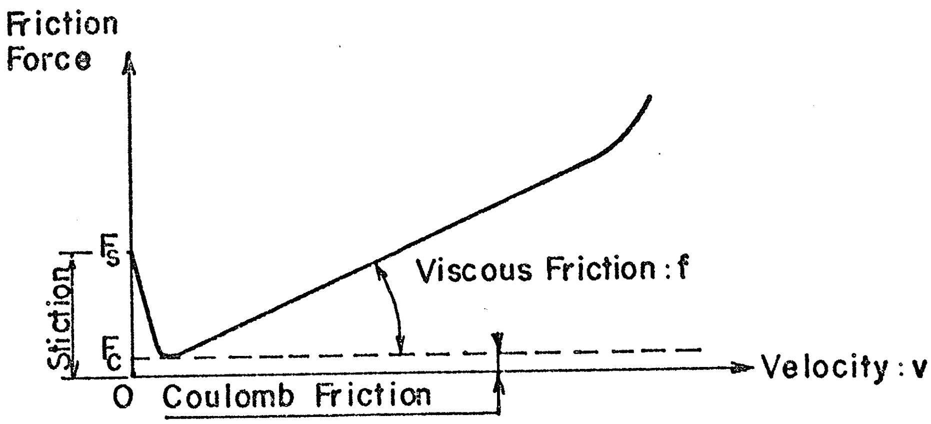


FIGURE 3-7 TYPICAL FORCE - VELOCITY RELATIONSHIP FOR PARTS IN CONTACT

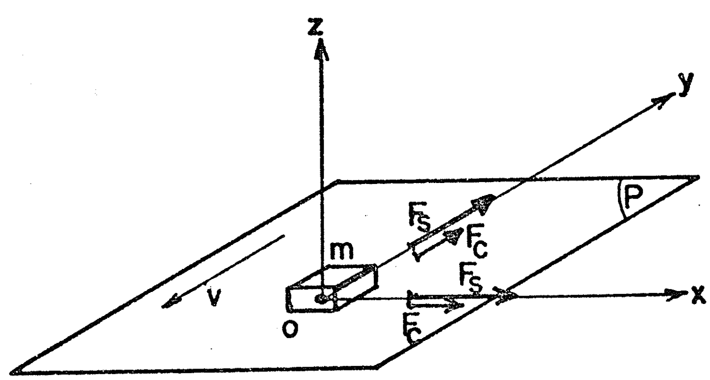


FIGURE 3-8 SKETCH OF FORCES IN AN IDEALIZED CONTACT

relative motion at the contact, but also that it reduces the contact friction force for an instant within the gap domain. For the purpose of testing the efficacy of the vibration method, the central portion of the instrument was mounted on a steel cantilever. An electromechanical excitation of the fundamental mode of the cantilever permitted a wide range of vibration amplitudes and frequencies to be applied. Tests* with this system showed clearly that vibration was a satisfactory way of eliminating the effects of stiction. A number of methods of vibrating the instrument contacts were considered and some were tried but all were considered inferior to the chosen method. For example, an unbalanced mass on the end of a flexible shaft was made to reverberate within the central part of the device. Apart from the fact that this method causes an acceleration perpendicular to the desired direction, it exhibited a distressing inconsistency in its performance. In the final solution to the problem of stiction a method of vibration involving an electromagnetic hammer was selected. A small mass was made to knock against the base of the mounting block to cause vertical acceleration of that block. The fine control and the total lack of unwanted con-

* See Figure D-1 (Appendix D)

straint on the mechanical system provided by this method were primary factors in its selection.

3.4 System Summary

The techniques previously discussed which are used to measure the force system $\{\vec{T}\}$ may now be summarized. The behavior of the force transducers, of the bridge system and the mathematical operations performed has been proven to be linear. The first linear transformation of $\{\vec{T}\}$ into $\{\vec{V}\}$ was expressed by $\{\vec{V}\} = [A]^{-1} \{\vec{T}\}$ and was due to the geometrical properties of the instrument. Then the six new forces $\vec{f}_i$ composing $\{\vec{V}\} = \{\sum_{i=1}^{6} \vec{f}_i\}$ were converted respectively by six electromechanical transducers to six output voltages noted c_i. The combination of the formulae involved in the transducer calculation in section 3.4 is represented by the next equation, 3.13, which describes the complete transduction.

$$c_i = U_i - U_{io} = f_i \; tgV/2EZ \qquad (3.13)$$

where c_i is the transducer response of the transducer measuring f_i, μ volts

U_i is the output voltage of the bridge(volts) (Figure 3-5)

U_{io} is the output voltage of the bridge(volts) at rest (Figure 3-5)

f_i is force measured, N

$t = (L - L1)$ for the transducers of type A (table 3-1), mm

$t = (L/2 - L1)$ for the transducers of type B (table 3-1), mm

g is the gage factor

V is the input voltage of the Wheatstone bridge(volts)(equation 3.12)

E is the modulus of elasticity of the transducer material, N/mm^2

Z is the section modulus, mm^3

Although the factor $tgV/2EZ$ is considered as a constant, it should be pointed out that the coefficients t and V of the factor can be subjected to some variation, t can have a variation only in the case of the transducers of type A since depending on the gap allowance at the contact, L can become $L \pm \Delta L$. The least significant variation in L was obtained by the use of adjustable contacts. V can also have some variation $\pm\Delta V$ due to a change of voltage of the power supply but the error due to this variation can be avoided by normalizing each U_i by the respective value V. In this way we ensure that no error can come from the power supply. From the formula 3.13 we can see that one must take as referential, before starting an experiment, the six bridge output voltages at rest (U_{io}). Knowing that a mini-computer with its auxiliary equipment is used to read, normalize and compute the outputs of the transducers with respect to an experiment procedure and that it is imperative to set up a vibration before any reading of the outputs, the vibration timing was made automatic. The electrical circuit which permits automatic acquisition of data is shown in Appendix A, Figure 1-A.

As a result of the combination of system requirements and the compromises necessary to obtain a real physical system, the instrument shown in Figure 3-9 with its surrounding equipment and on its own in Figure 3-3, was built. The vibration system can be

FIGURE 3-9 THE INSTRUMENT
WITH SURROUNDING EQUIPMENT

From left to right: The D.A.S.
The computer
The plotter
The instrument with
the vibration equipment

seen partially below the block and the two types of transducers can be seen in Figure 3-3, page 16. The sliding contact at points 1, 2, 3, can be seen in Figure 3-4, page 18. The buckling phenomena occurred on the type B transducers, thus limiting the present instrument to the forces and torque in the domain tested, and discussed in Chapter 5.

4. INSTRUMENTATION AND EVALUATION TECHNIQUES

4.1 Methods and Techniques to Evaluate the Instrument Performances

Having designed the instrument, it was necessary also to develop an evaluation procedure not only for the instrument as a whole, but also for the individual components of the instrument. From previous descriptions and from the statement of criteria, page 11, the evaluation procedure should show that:

a. The instrument will behave linearly in any direction under the necessary condition that none of the transducers will exceed their loading capacity, and that a vibration period will take place before any reading is taken. Before the instrument was assembled the transducers were checked to see at what load the maximum deflection of 0.1 mm was obtained. The maximum load for the type A transducers was found to be 70 gf and for type B transducers, 40 gf.

b. The instrument will have equal sensitivity in all directions.

c. The instrument will have a degree of accuracy of sensitivity dictated by the 1 gf sensitivity that each transducer is assumed capable of producing.

Of course it is well understood that any interpretation of the instrument performance will only be made from results which do not include any significant offset errors coming from the testing equipment, the means of data assessment or the environmental conditions. Of particular concern are the points of dynamic contact since these must be kept extremely clean. In the following discussion the significance of the above factors are dealt with in greater detail.

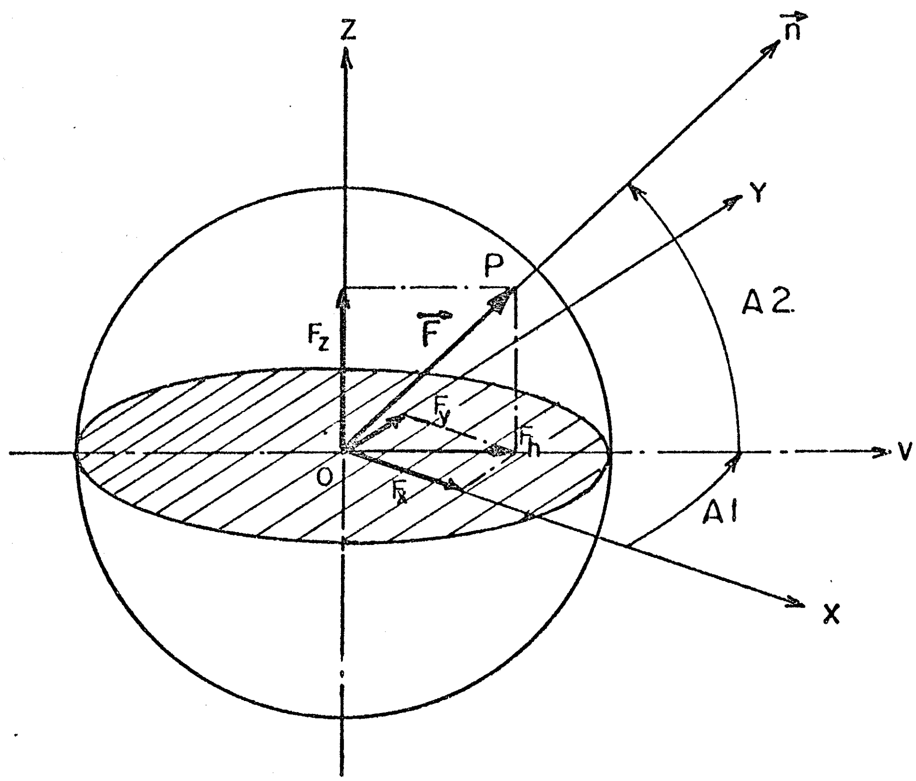
4.1.1 Type of Tests and Analysis

Although the instrument must simultaneously measure the force and moment delivered by an orthodontic appliance, under the assumption of linearity, the evaluation of the instrument performance could be made by conducting separate tests applying pure force and pure moment, respectively. This method of separate tests provides the great advantage of decreasing the sophistication of the testing equipment required and of permitting easier interpretation of the results.

The first set of tests must establish the linearity of the instrument for all directions. From these initial results, the consistency and accuracy of the instrument sensitivity with respect to the direction of the force or moment can be evaluated. In this manner we check the requirements in order of priority.

Each experiment performed will be defined by a spatial position $\vec{n}$ characterized by the longitudinal angle A1 in the xy plane and the angle of latitude A2 in a plane perpendicular to the xy plane, as shown in Figure 4-1, of the vectorial quantity (force or moment) under test characterized by the vector $\vec{OP}$, and will consist of changing the magnitude of $\vec{OP}$ with respect to $\vec{n}$.

For characterizing the linearity of the instrument in any direction $\vec{n}$, a statistical analysis using the regression method is employed. By convention, the known change of magnitude of $\vec{OP}$ which defines either a force magnitude or a moment magnitude will be represented by x_j and be placed on the abscissa axis, and the variable computed from the output of the transducers represented by the letter y_j will be placed on the ordinate axis. The subscript j under x and y defines a particular value of each variable at an observation. From the summary section 3.4 one must re-



OXYZ: CARTESIAN COORDINATE SYSTEM FIXED TO THE CENTER O OF THE INSTRUMENT.

$\overrightarrow{OP} = \vec{F} =$ FORCE.

A1 = ANGLE OF LONGITUDE.

A2 = ANGLE OF LATITUDE.

FIGURE 4-1 SPHERICAL DEFINITION OF A VECTOR IN SPACE member that the linearity of the instrument depends upon the transducer linearity and upon its geometry which affects the mathematics used in resolving the force-system $\{\vec{T}\}$ into the system $\{\vec{V}\}$, consequently we have to check the linearity of the instrument not only in all directions $\vec{n}$ for a force test and a moment test, but also for many values of the output variable y's. These variable y's will be either any transducer response c_i or any force or moment component computed from the c_i in any coordinate system (Cartesian, cylindrical, spherical). If we store the c_{ij} of each observation characterized by the value x_j of each experiment defined by its specification (that is, force test or moment test) and by the direction $\vec{n}$ given by A1 and A2 in which the vectorial quantity $\vec{OP}$ of the test is put, one has to perform only one experiment for each type of test at each direction $\vec{n}$ in order to analyse each variable y. A legitimate part of the data processing can be done during the recording of the data in order to monitor the data. The monitoring can be as simple as a visual check through plotting each point defined by x_j, y_j of each experiment. For the check, the variable y will be either the total computed magnitude force F or moment M under which the instrument is tested. It is also worth noting that the number of observations (x_j, y_j) of each experiment should be about 20 in order to allow the statistical regression method to be applied. By applying this method we gain the great advantage of reducing the amount of data to be used for further analysis. The regression method not only permits a test of linearity, but also allows the quantification of the sensitivity of the instrument for each experiment and each variable y by the slope "a" of the regression line and by the coefficient of regression "r".

Before going any further, the validity attributes of the statistical analysis should be examined. First one must make sure that the independent variable x described experimentally by its x_j values is considered to be without appreciable error, and secondly that the curve which best fits and interprets the values y_j is a straight line. Only then can the regression line of y on x be taken as the best estimate of y_j for a given x_j and then it can be concluded that y_j is linearly dependent on x_j with random variation about the regression line. Then the slope "a" and the coefficient of regression "r" of each analysis can be used in the further analysis of the instrument performance. The performance analysis will consist of gathering all the slopes "a" and coefficient "r" from either all force or moment test experiments for each analysed variable y and to plot them with respect to their respective experimental specifications, that is, with respect to their A1 and A2. By convention A1 will be taken as a variable along the abscissa axis, A2 will be a parameter and the variable along the ordinate axis will be the slopes "a". These plots are in Appendix D, page 101, with their tables of values. One must realize that for a good interpretation of these results one needs not only to have a good understanding of how the instrument works for each component of the force-system $\{\vec{T}\}$, but also to have a mathematical model of, at least, the response of each transducer for the conditions of any experiment under which the instrument is tested.

4.1.2. The Mathematical Model

Knowing that the transducers 1, 2, 3 only define the horizontal force $\vec{Fh}$ in magnitude as well as in direction and that the transducers 4, 5, 6 only define the azimuthal moment $\vec{Mv}$ in magnitude as well as in direction, the mathematical analysis will be limited to these two cases. It is well understood that for Fz and Mz respectively there is no problem of definition because they are constant in direction. As the transducers 1, 2, 3 and 4, 5, 6 are respectively arranged in the same geometrical similitude seen in Figure 4-2, only one demonstration of the analysis is necessary as there is no basic difference between the two cases. The case dealing with the variation of the transducer response C1, C2, C3 with respect to a horizontal force Fh therefore will be treated.

Consider the three measured forces $\vec{f}_1$, $\vec{f}_2$, $\vec{f}_3$ shown in Figure 4-3 (respectively detected by the transducers 1, 2, 3) with their axes 120° apart. When a constant force $\vec{Fh}$ is positioned at an angle A1 as represented in Figure 4-3, the transducer responses are respectively

$$C1 = f_{1\ max.} \sin A1 \qquad (4.1)$$

$$C2 = f_{2\ max.} \sin \left(A1 + \frac{2\pi}{3}\right) \qquad (4.2)$$

$$C3 = f_{3\ max.} \sin \left(A1 + \frac{4\pi}{3}\right) \qquad (4.3)$$

The value of Fh due to the joint action of the force transducers at any particular angle A1 may be obtained from a knowledge of their components at this angle along the axis x and y. The sum of the components along y at A1 is:

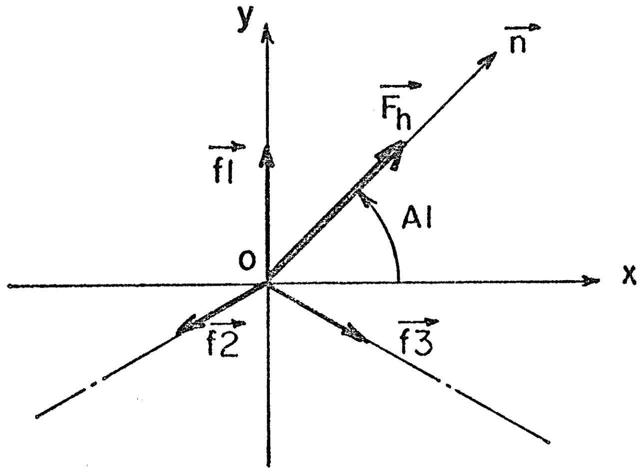
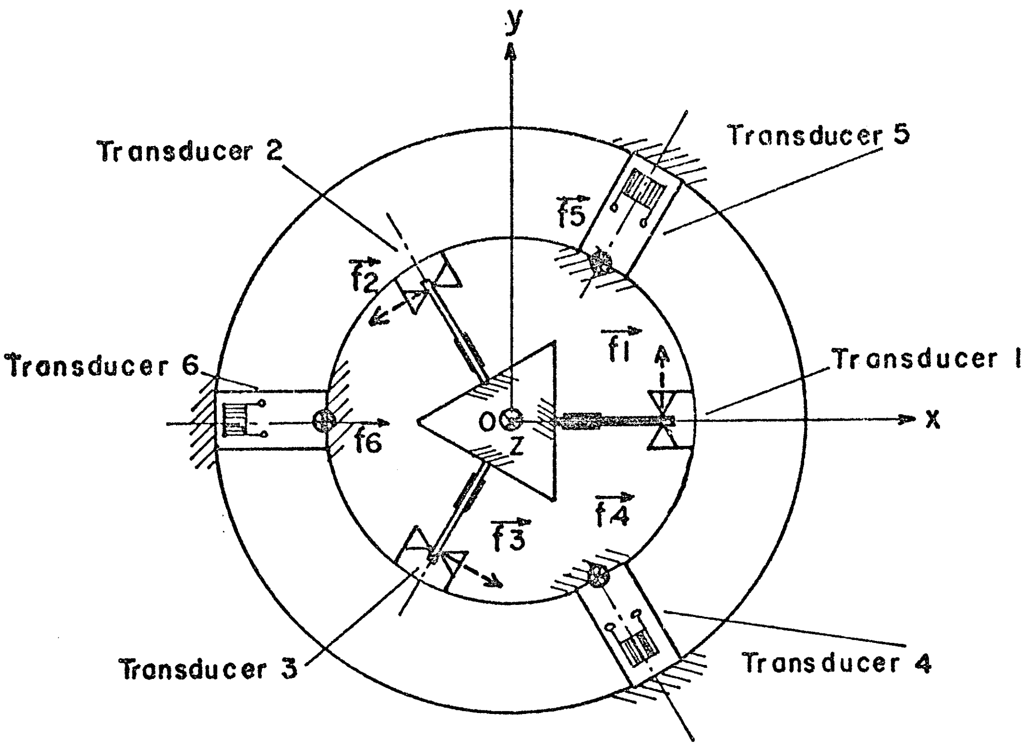


FIGURE 4-3 RESOLUTION OF THE FORCE SYSTEM $\vec{f}_1$, $\vec{f}_2$, $\vec{f}_3$



Symbol: ▨ = Fixed attachment

FIGURE 4-2 TRANSDUCER ARRANGEMENT

$$Fy = C1 \sin 90^\circ + C2 \sin 210^\circ + C3 \sin 330^\circ \qquad (4.4)$$

and along x at A1 is:

$$Fx = C1 \cos 90^\circ + C2 \cos 210^\circ + C3 \cos 330^\circ \qquad (4.5)$$

By substituting the equations (4.1), (4.2) and (4.3) into equations (4.4) and (4.5) and by making some trigonometric simplifications and knowing that

$f_{1\ max.} = f_{2\ max.} = f_{3\ max.} = P$ (4.4) becomes:

$$Fy = (3/2)P \sin A1 \qquad (4.6)$$

and (4.5) becomes:

$$Fx = (3/2)P \cos A1 \qquad (4.7)$$

Therefore the magnitude of the resultant

$$Fh = (Fy^2 + Fx^2)^{1/2} \qquad (4.8)$$

ends up after substitution of (4.6) and (4.7) into (4.8) to find

$$Fh = 3P/2 \qquad (4.9)$$

where P represents the maximum loading capacity of the transducers 1, 2, 3.

The same analysis could have been done for the transducers 4, 5, 6 under a constant azimuthal moment and we would have found

$$C4 = f_{4\ max.} \sin A1 \qquad (4.10)$$

$$C5 = f_{5\ max.} \sin (A1 + \frac{2\pi}{3}) \qquad (4.11)$$

$$C6 = f_{6\ max.} \sin (A1 + \frac{4\pi}{3}) \qquad (4.12)$$

Now one can refer back to the discussion at the end of section 4.11 and can see that the theoretical curve of the plots put in Appendix D, page 101, could be predicted. We can clearly see that the plots in Figures D-4 and D-5 should behave sinusoidally with respect to A1 and that the plots D-3 and D-6 should be a straight line.

4.1.3. Testing Equipment

In the light of previous discussion, the main requirements that the testing equipment should meet are as follows:

a. The vectorial quantities ($\vec{F}$ and $\vec{M}$) should be defined in a way compatible with the analysis and should offer some technical convenience.

b. The method of defining the vectorial quantities should be accurate and reliable enough to avoid generating appreciable error and must permit as far as possible an automatic assessment of their variables.

c. The capacity of the equipment should permit covering the entire range of magnitude and direction over which the instrument must be tested.

Among the three conventional coordinate systems available to define a force or a moment in space (Cartesian, cylindrical, spherical), the spherical definition was chosen for defining the force and the cylindrical definition for the moment. In this manner each force test will define the force under test by its magnitude F and its direction $\vec{n}$ characterized by its angle of longitude A1 and its angle of latitude A2, whereas in the moment test the moment will be defined by its magnitude M characterized by F and a distance D (if we consider again the moment magnitude to be a scalar product) and its direction A1. To define the magnitude of these vectorial quantities only two methods appeared practical, one using the deadweight technique (by far the most accurate when it does not involve the presence of a pulley) and the other technique using the load spring. As it will be seen in the section dealing with the description of this equipment, only the load spring technique used for the force test permitted an automatic assessment of the magnitude force through the means of a linear voltage differential transformer (L.V.D.T.).

4.1.4. Data Assessment and Storage

At this level of the discussion it should be clear to the reader that the author had at his disposition a mini-computer with its data acquisition system (D.A.S.) and all its auxiliary equipment such as a printer, a plotter and a recording system, so that the acquisition of data could be performed either automatically through the D.A.S. or manually through the keyboard of the desktop computer and is entirely controlled through a sequence of commands, prescribed by the experimental procedure explained on page 48 listed under the form of a program written in the BASIC language. The computer is capable of executing the experimental procedure by using an interface BUS which provides a powerful means of communication between instruments involved in the measurement system and a way of transferring data between devices. The data collected by the computer are of two kinds, the outputs of the instrument which are the six force transducer responses c_i plus the input voltage of the bridge (V) and the outputs of the testing equipment

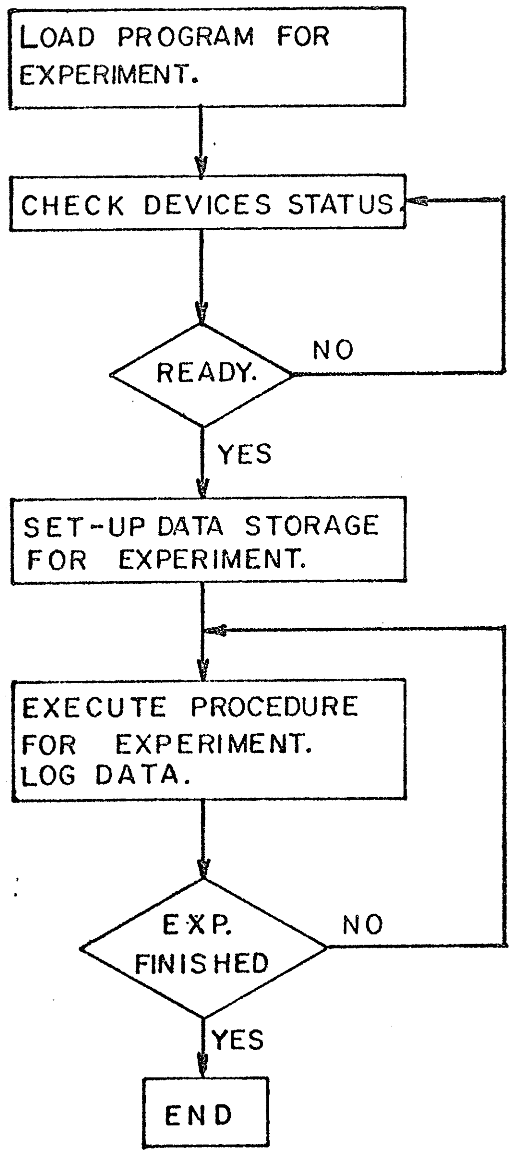


FIGURE 4-4 FLOW DIAGRAM OF DATA PROCESSING PROGRAM

which are either F, A1, A2 for a force test or M, A1, A2 for a moment test. Thus the storage data capacity of the computer should permit storage of the nine data of each of the 20 possible observations per experiment.

It is assumed that the data assessment program displayed in Appendix B-1 can be fully understood only if the reader is already familiar with the HP 9830A mini-computer. Therefore a summary of the functions performed by the stages of the program is shown in the flow diagram, page 47.

4.1.5. Experiment Procedure

It follows from the analysis requirement that the experiment should have the following procedure:

a. Select the type of experimental test, that is, a force test or a moment test.

b. Set the position A1 and A2 of the selected vectorial quantity ($\vec{F}$ or $\vec{M}$) under test.

c. Record the normalized outputs of the instrument transducers when no force or moment is applied to the instrument after having vibrated it.

d. Load the instrument under a force or moment magnitude xj.

e. Record the normalized outputs of the transducers under a load xj after vibration.

f. Store xj and the change of the transducer outputs c_{ij}.

g. Monitor from the c_{ij} the yj value of the vectorial quantity y which is under test.

h. Plot the set (xj, yj).

i. Check for linear validity.

j. Set a new value xi then repeat e), f), g), h), i), j) until the number of experimental observations (xj, yj) is large enough to allow a statistical analysis.

k. Repeat procedure for a new experimental test.

4.1.6. Treatment of Data

The assessment and the treatment of data will be done separately. Although the storage of data as well as the programs were done on commercial tape, at the end of the work the author found it necessary to store the data in rapid access discs of the mass memory system available at the laboratory in order to save appreciable time in the analysis of the data. The program used for the statistical treatment of the experimental data is shown in Appendix B. The program is in the H.P. computer library and it has been slightly altered in order to have an automatic assessment of the stored data and to compute any variable y related to the vectorial quantities so that any y could be treated as well as to allow an independent plotting function. The degree of precision that the program analysis allows is up to the 9th degree for the best fit curve. Also, the program can print out the slope coefficients "a" and the coefficients of correlation "r^2" of the regression line but also the mean and standard deviation value of the experimental and estimated values. This program uses the key functions of the computer and so permits the experimenter to interrupt the assessment program to initiate the regression analysis program as he wishes. Unfortunately, due to a lack of time, the program was not altered to allow fully automatic storage of the results "a" and "r^2" of each experiment. Thus the analysis of these results was by hand.

4.2 Description of the Testing Equipment

4.2.1. The Force Test Equipment

The equipment used for the test of force is shown in operation in Figure 4-5. It consists of two parts. One part has the function of positioning the L.V.D.T. force transducer in space, the other part is the force transducer itself. The former part permits the selection of 12 positions equally spaced at 30° in longitude around the instrument frame and of 5 positions in latitude equally spaced at 15° from the vertical plane. The L.V.D.T. force transducer can slide in the guide which can be moved and fixed in any of the latitude positions. Due to the technical difficulty of making a ball and socket joint at the contact between the instrument and the tip of the L.V.D.T. force transducer, the joint was simplified to a conic contact. The simplification limits, unfortunately, the force test to a hemisphere whereas a total sphere would be desirable. Also due to the overall dimensions of the instrument, only the first latitude position of 30° could be reached by the L.V.D.T. force transducer, requiring the use of the deadweight technique for the equatorial plane in order to investigate a complete hemisphere. This tech-

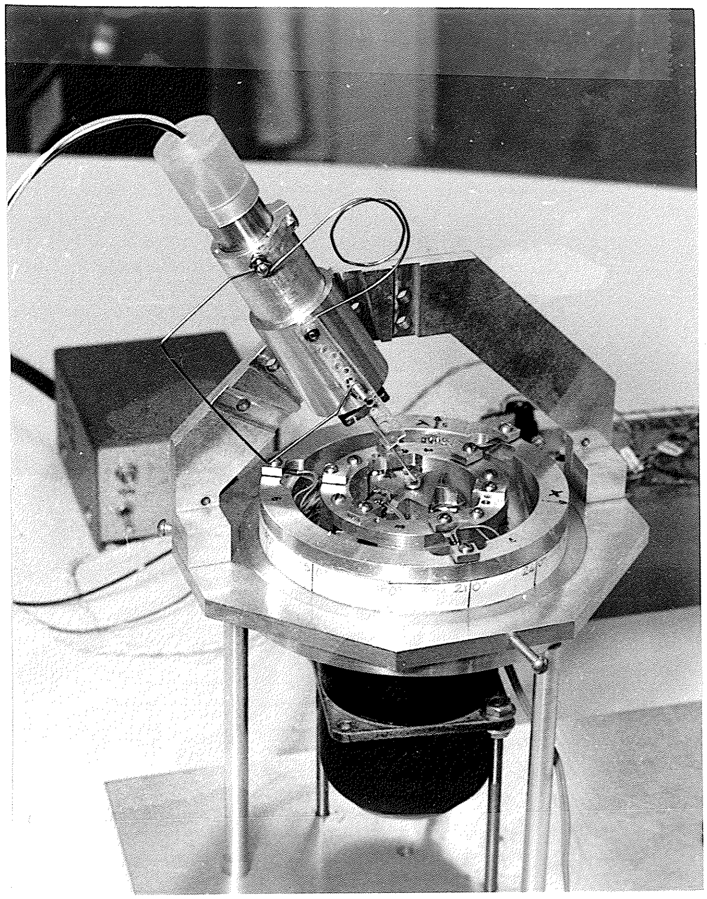


FIGURE 4-5 THE FORCE TEST EQUIPMENT IN POSITION

nique required the fixing of a pulley on the rotating ring of the force test equipment. Also as a result of the difficulty already mentioned which has arisen from the use of a conic contact, it has not been possible to apply a force right to the center of resistance o of the instrument, therefore a slight amount of moment will be generated under the test of force.

In summary, the force test equipment permits the application of a force magnitude ranging from 1.5 gf to 100 gf with the L.V.D.T. force transducer performance fully described in Appendix C and from 2.4 gf* to 96 gf with the deadweight technique in 72 equally spaced positions in the hemisphere.

4.2.2. The Moment Test Equipment

The moment test equipment is made of two devices, one to apply the pivoting moment $\vec{Mz}$, and the other to apply the azimuthal moment $\vec{Mv}$. These two devices can respectively be seen in Figures 4-7 and 4-6. They both use the deadweight techniques in conjunction with some levers; their mechanism is simple

* This value is the weight of the standard nuts used as deadweight. Their accuracy proved to be ±1%.

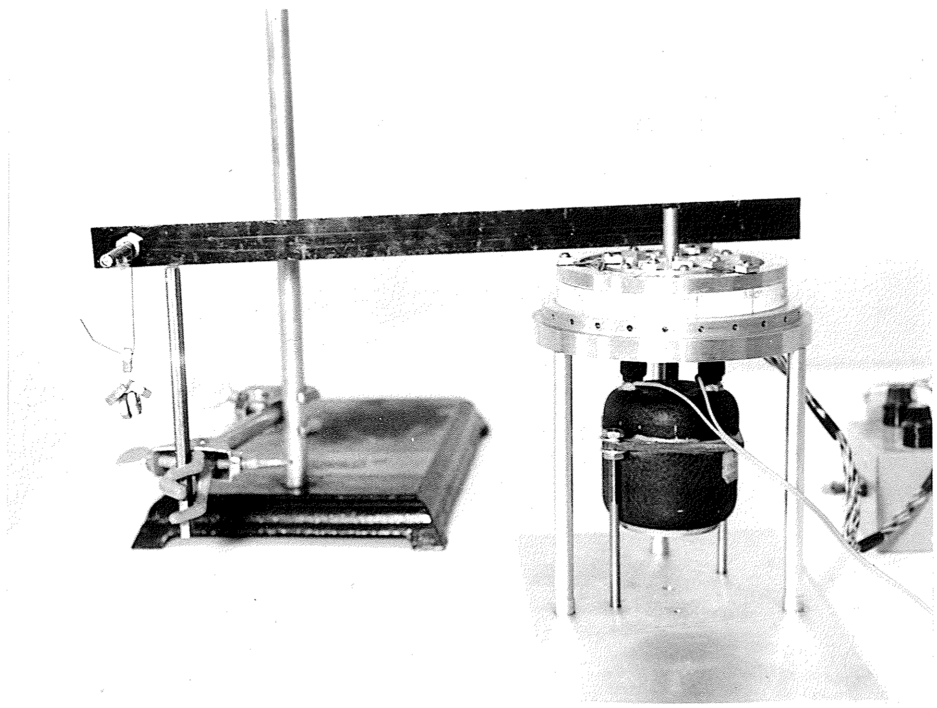


FIGURE 4-6 THE AZIMUTHAL MOMENT TESTING EQUIPMENT

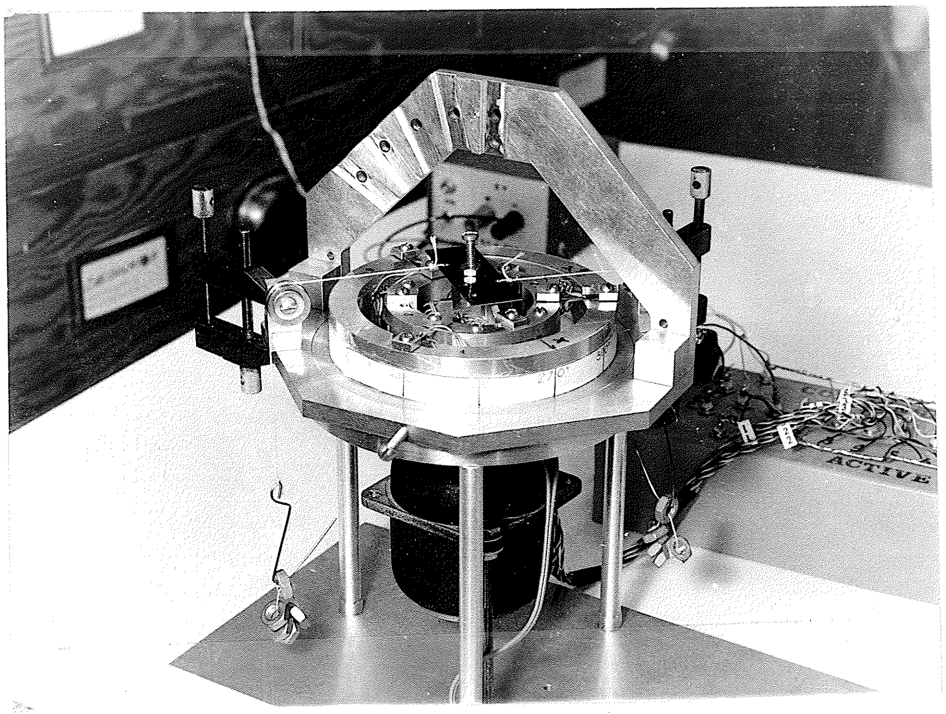


FIGURE 4-7 THE PIVOTING MOMENT TESTING EQUIPMENT

enough to be understood from the two Figures 4-6 and 4-7. The moment test equipment allows application of a moment from 50 mm gf to the maximum capacity of the instrument with an accuracy of about ±1%. It is interesting to note that the analysis of the source of errors in the application of both moments shows that the most significant error is due to the accuracy of the deadweight.

In summary, a test and an analysis procedure have been developed for testing and evaluating the linearity, the sensitivity and the accuracy of the instrument and of its main components, the transducers. A testing apparatus was built under the usual constraints of limitation of resources and of workshop availability which have limited the instrument investigation to a hemisphere. In general, the equipment has good accuracy which allows satisfactory statistical analysis and has a load capacity which is great enough to cover the normal dental appliance. The sensitivity of the D.A.S. digital voltmeter of 1μV and the precision of the mini-computer resolution are more than required thus no significant additional error is introduced. During the experiment a great deal of care was taken to avoid sources of error caused by such variables as temperature sensitivity of the transducers.

In terms of the experiments, the buckling phenomenon referred to in Chapter 3, that the transducers of type B exhibit could not practically be avoided due to time limitations. The effect of the problem was not anticipated as being as disruptive as later tests show. Although the device developed here has successfully met most criteria set out in Chapters 2 and 3, the transducers of type B require redesigning to permit the general testing of dental appliances. Stiction was anticipated as the cause of the most difficulty and, indeed, required a great deal of time, design effort and experimentation. The stiction problem was, however, almost completely removed as a concern or limitation in the operation of the instrument. In general, the basic design principles and the mathematical methods of data analysis have been shown here to be valid.

5. RESULTS AND DISCUSSION

5.1 Results

5.1.1 Calibration Results

The force test was employed to calibrate the six instrument transducers. The transducers 1, 2, 3 were calibrated using horizontal forces and transducers 4, 5, 6 were calibrated using vertical forces. However, due to the testing equipment limitations, the transducers 4, 5, 6 could only be tested under a pushing force, whereas the transducers 1, 2, 3 could be tested in both directions. The calibration of transducers 1, 2, 3 was achieved by comparing the results of three experiments made at the three particular angles $A1 = 0^{\circ}$, $A1 = 120^{\circ}$, and $A1 = 240^{\circ}$, where respectively in each experiment one transducer would have zero response and the two others would have a symmetric response as it can be worked out from Figure 5-1.

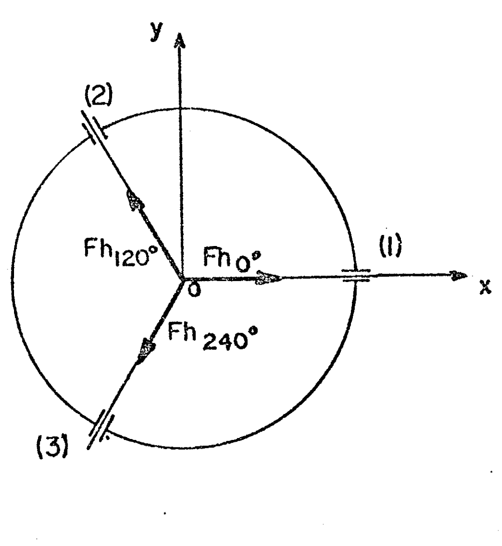


FIGURE 5-1

THE THREE PARTICULAR POSITIONS OF THE HORIZONTAL FORCE TEST

For each experiment, the response of the three transducers 1, 2 and 3 can be plotted against the magnitude of $\vec{Fh}$, and in theory a straight line should join all the observation points of each transducer responses if the transducers have behaved linearly during the experiment and if the experimental procedure has been strictly observed. Therefore, each transducer response could be characterized by a slope "a_{ik}" of the regression line which defines the sensitivity of the transducer i during the experiment k. Then the following table can be constructed:

Experimental Conditions	A2 = 0^o		
	A1 = 0^o	A1 = 120^o	A1 = 240^o
Transducer 1	a_{11}	a_{12}	a_{13}
Transducer 2	a_{21}	a_{22}	a_{23}
Transducer 3	a_{31}	a_{32}	a_{33}

Table 5-1

With what has already been said, one can easily find out by simulating the three experiments through the help of Figure 5-1 that, theoretically,

$$a_{11} = a_{22} = a_{33} = 0 \qquad (5.1)$$

$$a_{12} = -a_{13} \quad (5.2)$$

$$a_{21} = -a_{23} \quad (5.3)$$

$$a_{31} = -a_{32} \quad (5.4)$$

Also it is well understood that no two single cantilever strain transducers under the same load will have exactly the same response due to the dimensional and material tolerances of the beam, the positional tolerance and the bonding variation of the strain gages on the beam. Thus we have

$$a_1 \# a_2 \# a_3 \quad (5.5)$$

where

$$a_1 = (||a_{12}||+||a_{13}||)/2 \quad (5.6)$$

$$a_2 = (||a_{21}||+||a_{23}||)/2 \quad (5.7)$$

$$a_3 = (||a_{31}||+||a_{32}||)/2 \quad (5.8)$$

are respectively the mean sensitivity value a_i of the transducers $i = 1$, 2 and 3. Knowing the a's in Table 5-1 are the product of a statistical estimation, there is little doubt that there will be a difference between the a's of the equations (5.2),(5.3) and (5.4) as shown in Figure 5-2. Therefore, the sensitivity of the transducer will be defined by the mean value of the equation (5.6), (5.7) or (5.8) corresponding to the transducer. The results of the three experiments are shown in Table 5-2.

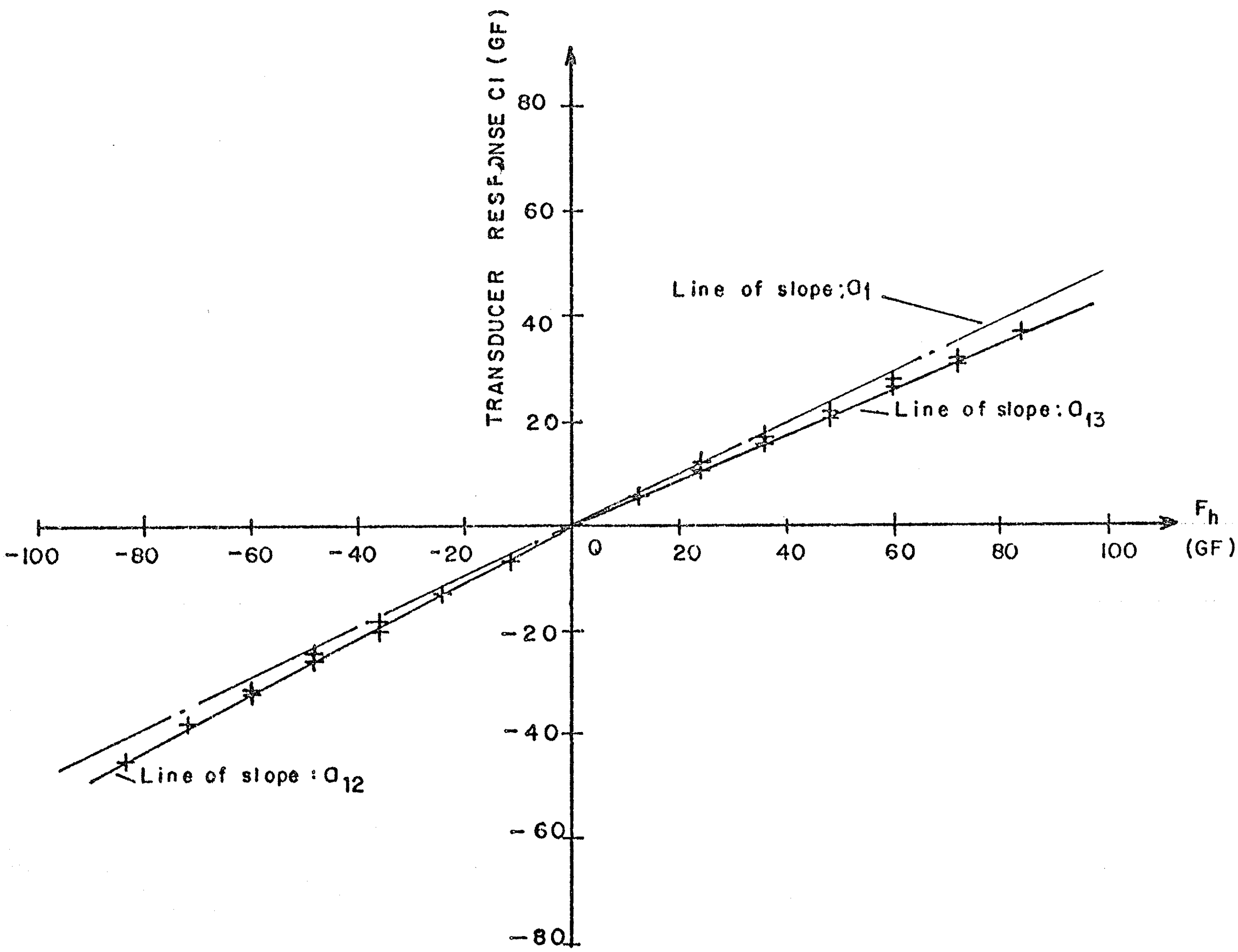


FIGURE 5-2 EXAMPLE OF ASSYMETRY IN HORIZONTAL FORCE TRANSDUCTION

Experimental Conditions	A2 = 0^o			a_i
	A1 = 0^o	A1 = 120^o	A1 = 240^o	
Transducer 1	0.000	-0.442	0.486	0.464
Transducer 2	0.423	-0.023	-0.418	0.420
Transducer 3	-0.461	0.423	0.006	0.442

Table 5-2

From Table 5-2, with regard to the a_i value, a calibration of transducers 1, 2 and 3 can be performed. If the transducer response C2 is taken as the reference, the transducer response C1 must be multiplied by a calibration coefficient $k_1 = 0.420/0.464 = 0.905$ and the transducer response C3 by a coefficient $k_3 = 0.420/0.442 = 0.950$ in order to calibrate each transducer to the same sensitivity. With respect to the difference of 10% between the value a_{ik} mentioned in equations (5.2), (5.3), (5.4), an analysis had to be done to find the cause of this variation. Thus a test not only limited to the three mentioned angles but covering the twelve possible A1 positions was conducted. From these twelve experiments, twelve sets of three "a" values defining the three transducer responses were found by using the regression method with the factor of calibration k_2 and k_3. The results of these analyses are listed in Table D-1, page 104, and are plotted in Figure 5-3. From the plot it was found that the behavior of the transducer responses was not entirely corresponding to that given by the three equations (4.1), (4.2) and (4.3). Instead, the three following equations could be deduced from the plot:

$$a_1 = 2/3 \sin (A1) + \lambda_1 \tag{5.9}$$

$$a_2 = 2/3 \sin (A1 + 2\pi/3) + \lambda_2 \tag{5.10}$$

$$a_3 = 2/3 \sin (A1 + 4\pi/3) + \lambda_3 \tag{5.11}$$

where $a_i = C_i/Fh$

C_i/Fh max. = 2/3 with respect to equation (4.9)

λ_i is a factor of non symmetrical response with respect to a zero reference.

From the plot Figure 5-3, λ_1 was estimated to 8% of the $A_{i\ max.}$, λ_2 to 7%, and λ_3 to 6% confirming that the 10% variation observed between the a_{ik}'s involved in equations (5.2), (5.3) and (5.4) was due to a non symmetrical response of the transducers. If one refers back to equation (3.13), it can be seen that the only coefficient of this equation which can change when f_i changes to $-f_i$ is the coefficient $t = L - L1$, Figure 5-4.

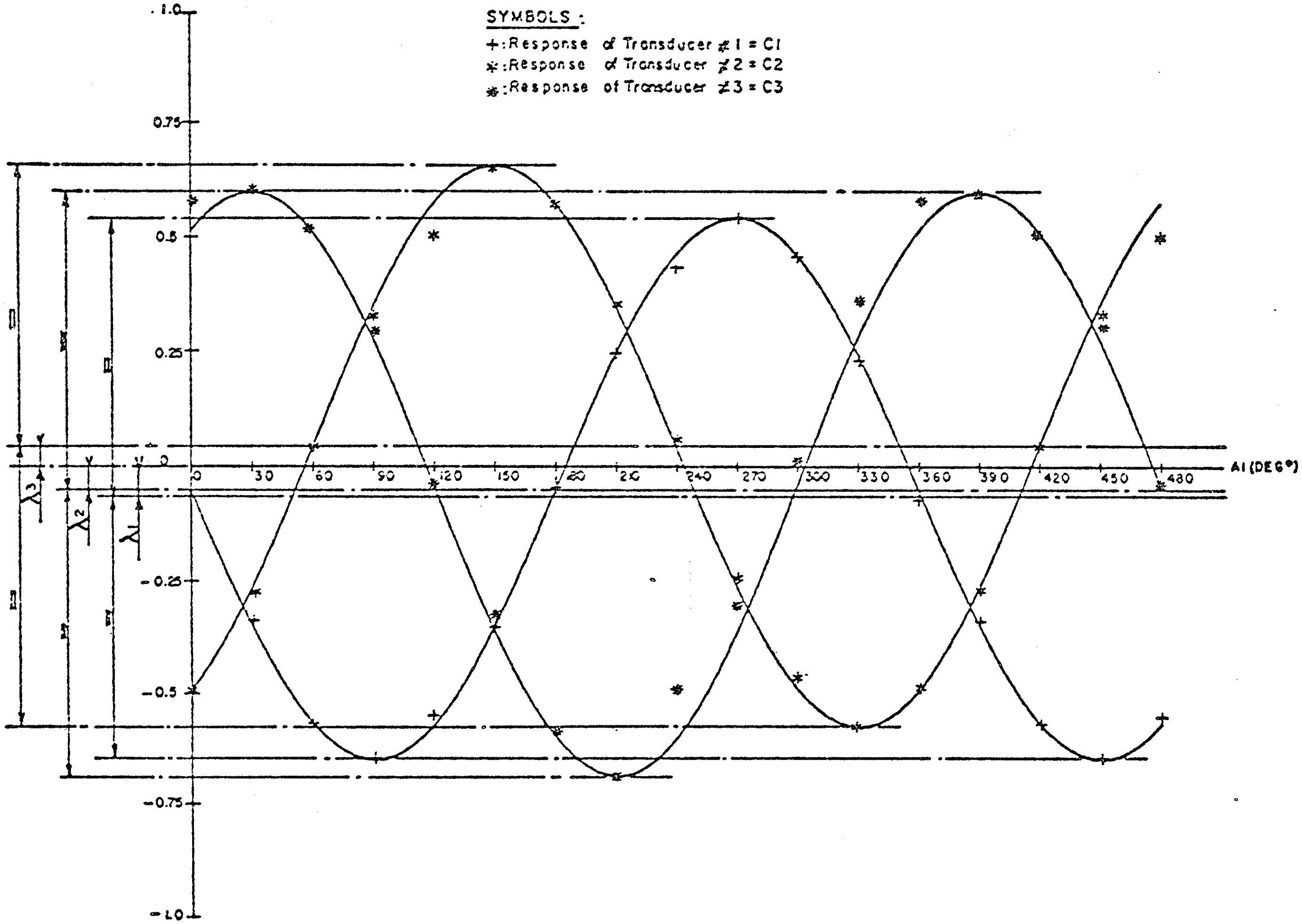


FIGURE 5-3 PLOT OF $a_{ik} = C_i/Fh_k$ VERSUS THE LONGITUDINAL ANGLE A1

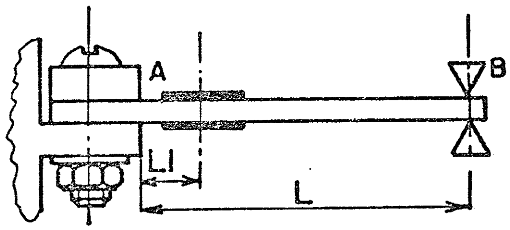


FIGURE 5-4 TYPE A TRANSDUCER

Mathematically, it is straightforward to evaluate the variation Δt which gives rise to a 10% variation of C_i. By applying the error theory the following equation can be written:

$$\Delta(L - L1) = \Delta C_i \; (L - L1)/C_i \qquad (5.12)$$

where $\Delta C_i/C_i = 0.1$ and $L - L1 = 8.5 - 3 = 5.5$ mm (see results in Table 3-1, page 21).

Thus $\Delta(L - L1) = 0.1 \times 5.5 = 0.55$ mm

This variation can be easily accounted for the three following errors.

a. A relative malposition A'A" of clamp I and II as it is shown next page in Figure 5-5.

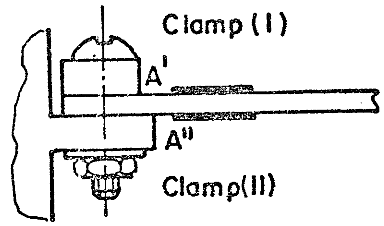


FIGURE 5-5 MALADJUSTMENT OF THE TRANSDUCER CLAMP

b. An imperfect clamping due to the surface of the clamps, as the example shown in Figure 5-6.

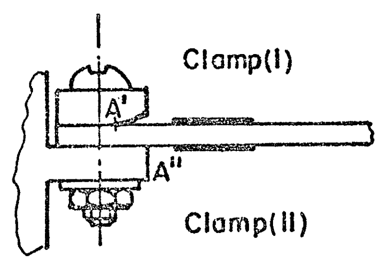


FIGURE 5-6 IMPERFECTION IN THE SURFACE OF THE CLAMP

c. A difference in stiffness of the wires which run from both sides of the cantilever as can be understood from Figure 5-7.

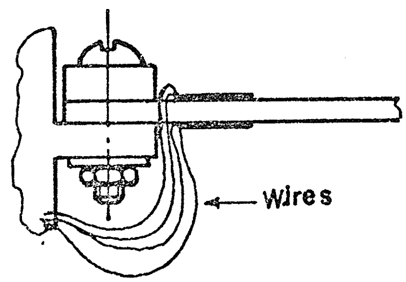


FIGURE 5-7 WIRING ARRANGEMENT FROM THE TRANSDUCER

Only the source of error a) could be evaluated from a visual inspection, and a malposition of the clamps from 0.1 mm to 0.3 mm was observable at each transducer. However, the sources of error b) and c) can together account for the remaining error. Due to a lack of time, no improvements could be made to the means of attachment of these transducers, thus all the following results which involved C1, C2, C3, will exhibit an error within 10%. Concerning the calibration of the transducers 4, 5, 6, nothing particular may be mentioned because responses C4, C5, C6, could not be investigated in detail due to the direction loading limitation of the testing equipment. For the calibration of transducers 4, 5 and 6, C6 was taken as a reference and C4 was multiplied by a coefficient $k_4 = 0.905$ and C5 by a coefficient

$k_5 = 0.952$. All the calibration coefficients can be seen in the subprogram in Appendix B-2 from lines 160 to 210, where E (i, j) is the corrected response of the transducer i at the experimental observation j and where D (i, j) is the non-corrected one. The reader will note that all the responses of transducers 1, 2, 3 have been multiplied by a coefficient $k_7 = 1.17 \times 10^5$ and that all the responses of transducers 4, 5, 6 have been multiplied by a coefficient $k_8 = 0.85 \times 10^5$ in order to scale the instrument response.

In summary, transducers 1, 2, 3, 4, 5, 6 have been respectively calibrated with the following coefficients:

$$k_1 = 1.05 \times 10^5$$
$$k_2 = 1.17 \times 10^5$$
$$k_3 = 1.11 \times 10^5$$
$$k_4 = 0.77 \times 10^5$$
$$k_5 = 0.81 \times 10^5$$
$$k_6 = 0.85 \times 10^5$$

The limitations of time did not permit improving the means of attachment of transducers 1, 2, 3 since this would require some redesign. Thus the results will show at least an error of 10%.

5.1.2. Force Test Results

At each of the sixty-one possible spatial positions (A1, A2) at which the force testing equipment allows an applied load, an experiment was done. By convention, all the quantities derived from the computation of the six transducer responses will bear a subscript c whereas the known quantity F tested will bear a subscript T. Theoretically, from Figure 4-1, where $||\vec{OP}|| = F_T$ (magnitude of the tested force), one can write the following equations:

$$F_c = F_T \tag{5.13}$$

$$Fx_c = F_T \cos (A2) \cos (A1) \tag{5.14}$$

$$Fy_c = F_T \cos (A2) \sin (A1) \tag{5.15}$$

$$Fz_c = F_T \sin (A2) \tag{5.16}$$

For a given experiment characterized by a fixed A2 and A1 the quantities F_c, Fx_c, Fy_c, Fz_c should be linearly dependent upon F_T if the instrument is behaving linearly so that a valid regression analysis can be done for each experiment and for the force and its components. Therefore, from each experiment four values of slope and of regression coefficient can be derived, such as:

$$a_F = F_c/F_T \quad (5.17)$$

$$a_{Fx} = Fx_c/F_T \quad (5.18)$$

$$a_{Fy} = Fy_c/F_T \quad (5.19)$$

$$a_{Fz} = Fz_c/F_T \quad (5.20)$$

From these results it is easy to interpret the general behavior of the instrument under a force test, within the testing equipment limitation and the calibration accuracy, by respectively plotting the slopes "a" with respect to A1 and A2. For convenience, interpretation A1 was chosen for the variable and A2 for the parameter, consequently each of the four plots include as many curves as A2 has values. The complete results are displayed in Tables D-2, D-3, D-4, D-5 and are plotted in Figures D-3, D-4, D-5, D-6. Although a discrepancy of about 10% is expected on the a_F, a_{Fx} and a_{Fy} plots, the curve of parameter A2 = 0 in the a_F plot (Figure D-6) exhibits a discrepancy of about 20%. It must be pointed out that during the assessment of data most plots of F_c against F_T used for monitoring the outputs and inputs instrument have shown a breakup of linearity in the neighborhood of 10 gf due to a buckling phenomenon occurring in transducers 4, 5, 6. Such breakup of linearity can be seen in Figure D-2. Thus only the results involving the computation of the responses of these transducers will be subjected to this error, i. e., the a_F and a_{Fz} plots (Figures D-3 and D-6) will therefore exhibit the buckling error. The a_{Fz} plot, Figure D-6, is the most appropriate plot to interpret the buckling error. Thus by plotting the transducer responses C4, C5, C6 respectively where the discrepancies of the a_{Fz} plot are the least and where the discrepancies are the largest and by comparing the responses, it is apparent that the instrument is the source of an error due to a buckling phenomenon. An example of the discrepancy plots is given in Figure D-7. To understand why the buckling phenomenon appears during the force test it must be remembered that:

a. The transducers 4, 5, 6 hold the suspended ring on one of their ends, so it follows that transducers 4, 5, 6 are deflected due to the weight of the suspended assembly as shown in the sketch in Figure 5-8.

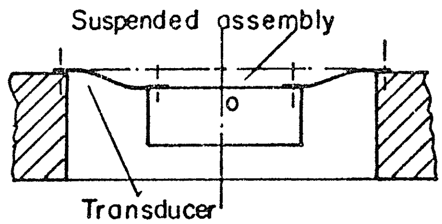


FIGURE 5-8 DEFLECTION OF TYPE B TRANSDUCERS DUE TO THE WEIGHT OF THE SUSPENDED ASSEMBLY

b. The force test generates a little moment which can be as much as 600 mm gf when a horizontal force of 100 gf is applied at the center o.

The simplified sketch below (Figure 5-9) shows how the transducers buckled under the effects of a) and b). In many respects, the transducers behave diaphragmatically.

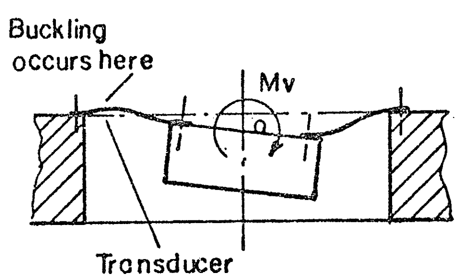


FIGURE 5-9 "DIAPHRAGMATIC" BEHAVIOR OF THE TYPE B TRANSDUCERS

In order to release the transducers 4, 5 and 6 from the critical Eulerian compressive force causing the buckling, the following attempt sketched below, in Figure 5-10, was made:

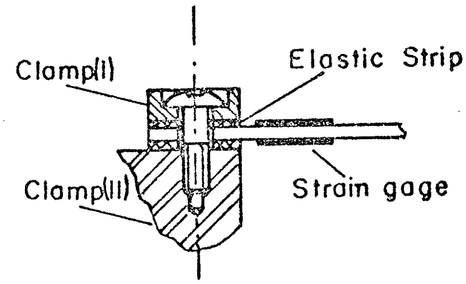


FIGURE 5-10 ALTERNATIVE CLAMPING ARRANGEMENT

At each clamp a thin strip made of an elastic material of low hysteresis was placed. But the results come out as much in error as the previous results. Therefore, a complete redesign of these transducers is necessary but due to a lack of time, the redesign could not be done here.

5.1.3. Moment Test Results

Although the buckling phenomenon was easily interpretable from the force test results, several experiments were made under the application of an azimuthal moment, and it turns out that these results too exhibit a buckling problem. A plot of these results is shown in Figure D-8.

The pivoting moment test results show that there is a real uncoupling between the moment $\vec{Mz}$ and the force $\vec{F}_h$ because no component F_h appears in the plot in Figure D-9. It also appears that the effect of the non symmetrical responses of transducers 1, 2 and 3 were merely cancelled by the computation so, in effect, the slope $a_{Mz} = Mz_C/Mz_T$ is a constant value whether the moment is positive or negative as it can be seen in Figure D-9.

5.2 Discussion

5.2.1. Problem Remedies

Although the unexpected buckling phenomenon destroys the usefulness of the particular arrangement of the transducers 4, 5 and 6, the sensitivity of the principle employing the cantilever transducers has been shown to be sufficient to allow the measurement of forces in the range of those encountered in dental appliances. It appears possible that only small changes in the geometry of the cantilever of the transducers 4, 5 and 6 need to be made to allow buckling to be avoided, thus yielding an instrument design capable of meeting most of the requirements stated in Chapter 3 of the thesis.

The problem of non symmetrical response which arose with transducers 1, 2 and 3 was shown to be primarily a problem of accurate assembly and machining. Sufficient care and the use of high accuracy machining techniques should be able to reduce errors from this source. Thus there is every reason to believe that if:

a. The transducers 4, 5 and 6 are redesigned without reducing their bending sensitivity in such a way that they are not subjected to a compression force in the buckling range as the type shown below in Figure 5-11 would permit

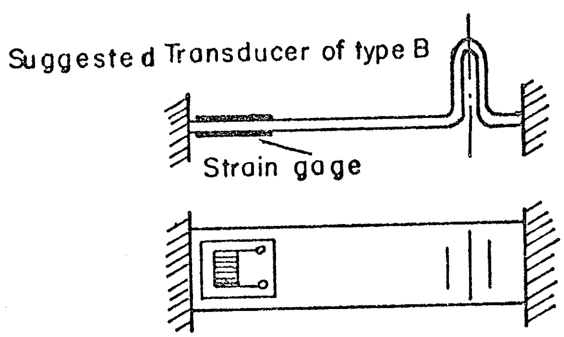


FIGURE 5-11 PROPOSED MODIFICATION TO TYPE B TRANSDUCER

b. The transducer clamps are pinned and are machined with great tolerance and have a non-slip clamping surface.

The total discrepancies of about 20% observed in the force test (see plot in Figure D-3) should become in the order of an acceptable accuracy, hopefully under 5%.

By measuring the "floating" zone displacement of the center o of the instrument due to the gap present at each of the three contacts, it was found that the maximum geometrical distortion under a maximum load was 0.2 mm in the horizontal plane instead of being 0.1 mm. This amount of distortion at the level of the required appliance test accuracy is still very satisfactory and therefore the use of adjustable contacts for transducers 1, 2 and 3 is not necessary. It is important here to mention that limitations of the availability of machining facilities and manpower did not permit achieving the design proposal, and did not allow the evaluation of the instrument in the second hemisphere. It must be kept in mind that in the final development, instrument testing equipment permitting coverage of the entire sphere is necessary.

In summary, by the excellent value of the coefficients of correlation in general, the transducers of the instrument have proved to be of a good and accurate linearity, and the vibration method as well as the means of vibrating the instrument show that the stiction problem has successfully been overcome.

5.2.2. Recommendations for the Appliance Testing

Before any test of appliance could be done with this instrument, a device for activating any appliance must be designed and an approach for treating and analysing the appliance test data must be developed. In view of this requirement, a number of recommendations may be made to define the main design requirements of the activating device and to have an idea of how the appliance test data may be treated.

The activating device must allow an appliance to be activated in the three spatial directions, i. e.: x, y and z, and in the three spatial rotations, i. e.: Θ, Ψ and Ω. However, for technical convenience, a simultaneous excitation of the six components of the activation which can be expressed by an assembly-vector $\{\vec{W}\}$ is not necessary, knowing that any appliance will be tested in its linear range. As already mentioned in Chapter 2, the device must permit activating an appliance with an accuracy of 0.1 mm in displacement and of a few degrees in rotation and must have a means of automatically assessing the component magnitude of the activation vector $\{\vec{W}\}$. At this point it must be emphasized that the accuracy of the whole instrument depends upon how well the activating device requirements are achieved.

With regard now to the treatment of appliance test data, in view of the massive and cumbersome amount of data due to the great variety of appliances, treatment similar to the one used for testing the instrument performance could be employed to define the performances and the reliability of the appliances. Before going any further, it must be mentioned that all the appliances must be tested on a standard model tooth placed at the center of resistance o of the instrument in order to allow a comparison between the appliances. Thus, in light of Chapter 4, an appliance will be successively tested by activating one by one each of the six components of activating vector $\{\vec{W}\}$ and from each of these tests a regression analysis will be done for each of the six computed output components of the force-system $\{\vec{T}\}$. By checking the coefficient of the regression line for each analysis, one knows, within the accuracy of the instrument with its associated activating device, with what degree of confidence the results can be entrusted. Then by gathering all the slope values from the thirty-six different analyses, under a matrix form, one obtains the stiffness matrix [B] of any tested appliance. The matrix [B] can be taken as the transfer function of the appliance if an appliance is considered as a system Σ which can be characterized by:

$$\{\vec{T}\} = [B]\ \{\vec{W}\}$$

By testing many appliances of the same type and of the same dimension within the manufacturing tolerance, one can see within what tolerance each matrix coefficient b_{ij} varies and therefore can quantitatively describe the reliability of the tested appliances in general. The orthodontist researchers may thus interpret the coefficients of the tested appliances and display those results in a form designed to aid practicing orthodontists in the selection of an appliance to be used to correct a particular malocclusion.

6. CONCLUSION

Although the discussion in Chapter 5 has mentioned the difficulties encountered and given remedies for these difficulties, the work done in this thesis has shown satisfactorily that the principle upon which the instrument is based is workable. Due to the successful overcoming of the stiction problem at the contact of the transducers 1, 2 and 3 and to the straightforward remedies to the problem of buckling in the transducers 4, 5 and 6, and of the non symmetrical response of the transducers in general, the technique used in this principle has proved to be more appropriate to the testing of dental appliances than other approaches to the problem because it employs only known technology and requires less development.

The present instrument measures the forces up to a maximum of 100 gf to ±10% within the investigated hemisphere and has a geometrical distortion at the maximum load of 0.2 mm. Since the limitations of the workshop facilities have not permitted achievement of the required redesign, the moments could not be tested.

In view of this work and of the recommendation made concerning the testing of appliances, it is apparent that a computer with its associated equipment defined in Chapters 3 and 4 is required to control the test procedure, to assess the input to output data, to compute and check the validity of the experimental results, and to analyse and display the final results in a suitable orthodontic form.

In summary, for the further studies aiming to develop this instrument, not only a complete investigation of the whole sphere must be done, but also an activating device within the requirements mentioned in Chapter 5 must be designed and an analysis of the treatment and display of the appliance test data must be developed before the instrument becomes operational and of general usefulness to the orthodontist researchers.

REFERENCES

1. C. J. Burstone, H. A. Koenig, "Force Systems from an Ideal Arch", Am. J. Orthodontics, Vol. 65, No. 3, pp. 270-289, March 1974.

2. C. J. Burstone, J. J. Baldwin, D. T. Lawless, "The Application of Continuous Forces to Orthodontics", Angle Orthodontics, Vol. 31, No. 1, pp. 1-14, January 1961.

3. G. H. Teasley, W. Penley and E. Morrison, "The Design and Fabrication of an Electromechanical Instrument Utilized to Analyse Orthodontic Forces", Am. J. Orthodontics, 49:868, 1963.

4. D. J. Solonche, C. J. Burstone, J. R. Vanderby, "A Device for Determining the Mechanical Behavior of Orthodontic Appliances", IEEE Transactions on Biomedical Engineering, Vol. BME 24, No. 6, Nov. 1977.

5. R. L. Neuger, "The Measurement and Analysis of Moment Applied by a Light Wire Torquing Auxiliary and How These Moments Change Magnitude with Respect to Various Changes in Configuration and Application", Am. J. Orthodontics, Vol. 53, No. 7, pp. 492, July 1967.

6. J. Ray Vanderby, C. J. Burstone, D. J. Solonche, J. Ratches, "Experimentally Determined Force Systems from Vertically Activated Orthodontic Loops", Angle Orthodontics, Vol. 47, No. 4, pp. 272-279, Oct. 1977.

7. T. Y. Yang and J. J. Baldwin, "Analysis of Space Closing Springs in Orthodontics", J. Biomechanics, Vol. 7, pp. 21-28, 1974.

8. C. C. Perry and H. R. Lissner, "The Strain Gage Primer", p. 212, McGraw-Hill Book Company, New York, 1955.

9. E. O. Doebelin, "Measurement Systems - Application and Design", p. 347, McGraw-Hill Book Company, New York, 1966.

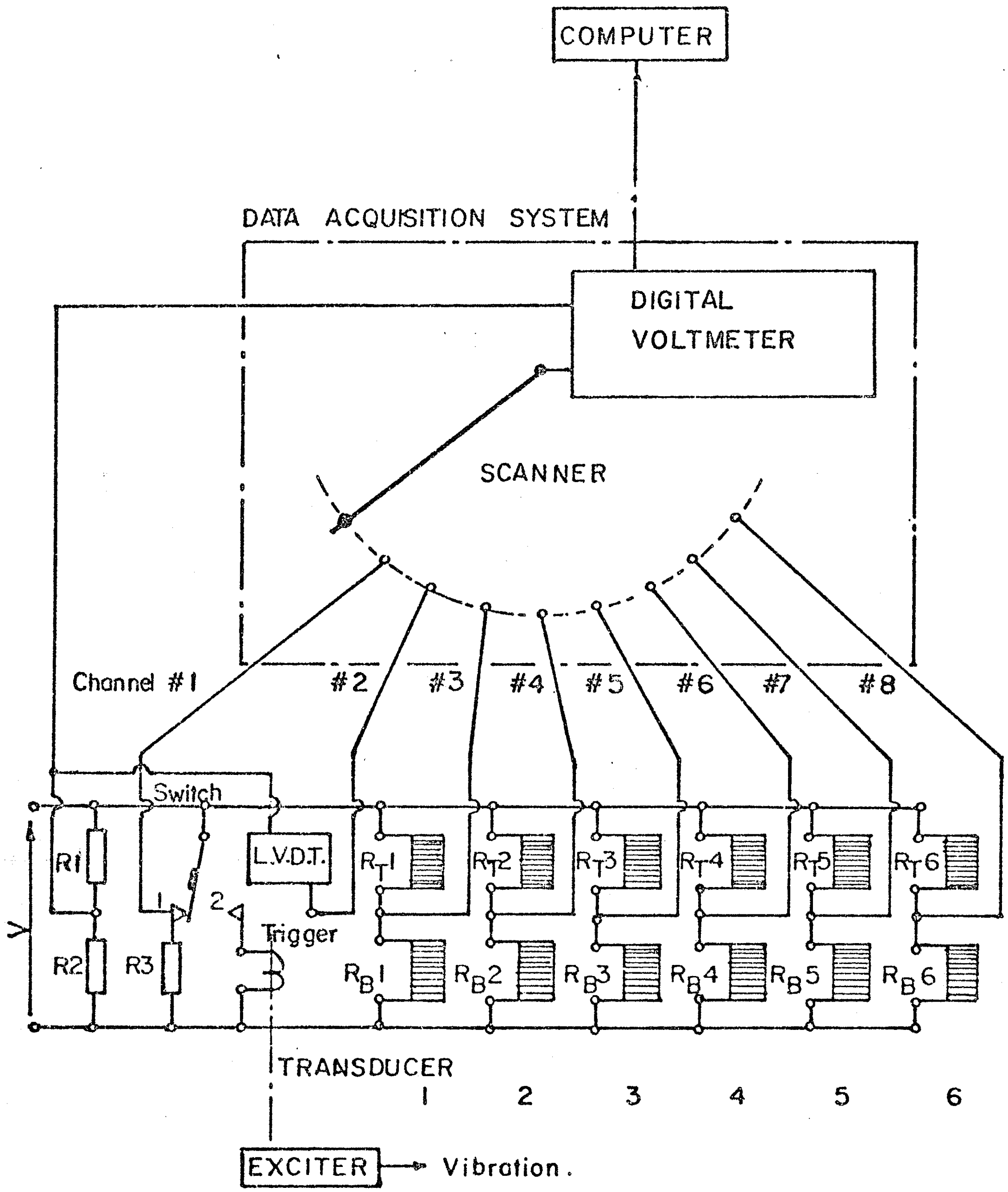
10. A. T. Storey, Head of the Preventive Dental Science Department of the University of Manitoba, Winnipeg.

11. G. T. Windburn, Academic Staff of the Preventive Dental Science Department of the University of Manitoba, Winnipeg.

APPENDIX A

DATA ACQUISITION

Among the six hundred channels that the Data Acquisition System (D.A.S.) has available, only eight are used for the complete drive of the experiments. Six channels (#3 to #8 in Figure A-1) are provided for monitoring the six instrument transducer outputs. One channel (#1 in Figure A-1) records the input voltage V of the bridges when the switch is on position 1 and triggers the vibration when the switch is on position 2 before taking any reading. One channel (#2 in Figure A-1) is used for assessing the outputs of the testing force transducer as mentioned in Chapter 4. The accuracy of the D.A.S. is up to 1μV when the scanner speed is lower than 50 ms per channel, thus permitting us to use the Wheatstone bridges as direct read-outs. The resistances R1 and R2 are the unstrained resistances of the six bridges mounted in parallel as Figure A-1 shows. The entire control of the D.A.S. scanner is accomplished by the computer through its B.U.S. with respect to the program commands defining all the experimental procedure steps shown in Chapter 4.



Legends:

R1,2,3 : Pure Resistances.

R_T and R_B 1,2,3,4,5,6 : Strain-gages.

V: Input voltage.

FIGURE A-1 TRANSDUCER AND CONTROL CIRCUITS

APPENDIX B.1

DATA ASSESSMENT PROGRAM

Only if the reader is familiar with the BASIC language and with the commands used by the HP9830 computer will he fully understand the program displayed on page 87. The program has six subroutines; two deal with the vibration command (subroutine 1690 and 1780), one with the power supply check (subroutine 1860), one with the testing force transducer control (subroutine 1980), another one with the angular computation (subroutine 2050) and one with the plotting function (subroutine 2210). The lines from 780 to 860 show what data are stored on tape and what their assigned positions are within the matrix [D]. The matrix [D] can store twenty experimental sets of data. The string variables A$ and D$ select with respect to the experimental specifications required in lines 140 and 240 the type of test (force or moment test), the experimental conditions (A1, A2) and the testing technique using the deadweight (line 660) or the load spring technique (subroutine 1980). The program possesses a selection of computed variables (line 520) which during the moni-

```
10 REM**PROG.TO TEST THE ORTHO. DEVICE JAN.1978
20 DIM A$[100],B$[10],AS[6],BS[6],CS[6],DS[9,21],TS[20],D$[10],C$[10
30 MAT D=ZER
40 DEG
50 P1=P2=P3=F=A1=A2=A3=A4=I=D1=D2=D3=D4=0
60 A$[1,3]="1   "
70 FORMAT 15B
80 IF STAT4>0 THEN 140
90 BEEP
100 DISP "SN. ON DAS"
110 WAIT 200
120 GOTO 80
130 GOTO 180
140 DISP "INPUT EXP. DETAILS                                  ";
150 INPUT A$[4]
160 DISP "INPUT FILE NUMBER                                       ";
170 INPUT F1
180 DISP "READY";
190 INPUT B$
200 IF B$[1,1]="G" THEN 230
210 GOTO 180
220 P1=P2=P3=F=A1=A2=I=0
230 GOSUB 1870
240 DISP "ZERO LOAD AND SET ANGLES A1,A2";
250 INPUT A1,A2
260 GOSUB 1690
270 GOSUB 1780
280 WAIT 1500
290 WAIT 100
300 OUTPUT (4,70)"@7I@2E@20"
310 ENTER (4,*)T1
320 IF ABS(T1)>1E-04 THEN 370
330 BEEP
340 DISP "SN.ON OR CHECK TRANSDUCER"
350 WAIT 200
360 GOTO 300
370 FOR Q=1 TO 6
380 OUTPUT (4,70)"@7I@3E"
390 WAIT 100
400 OUTPUT (4,70)"@7I@2E@20"
410 ENTER (4,*)A[Q]
420 A[Q]=A[Q]/ABS(B)
430 NEXT Q
440 IF STAT14<9 THEN 490
450 BEEP
460 DISP "PLOTTER NOT HAPPY"
470 WAIT 200
480 GOTO 440
490 SCALE -10,100,-10,100
500 XAXIS 0
510 YAXIS 0
520 DISP "WHAT PLOT C,P3,P4,F,M1,M2,M3";
530 INPUT D$
```

```
540 I=I+1
550 GOSUB 1690
560 GOSUB 1780
570 WAIT 1500
580 GOSUB 1870
590 GOSUB 1980
600 IF A$[8,11]="D.W." THEN 620
610 GOTO 700
620 DISP "NUMBER J OF D.W.";
630 INPUT J
640 P=J*12
650 IF D$[1,1]="M" THEN 670
660 GOTO 700
670 DISP "D.W. DISTANCE IN MM.";
680 INPUT D
690 P=J*12*D/10
700 IF P>100 THEN 2400
710 FOR Q=1 TO 6
720 OUTPUT (4,70)"@7I@3E"
730 WAIT 100
740 OUTPUT (4,70)"@7I@2E@2U"
750 ENTER (4,*)B[Q]
760 C[Q]=(B[Q]/ABS(B))-A[Q]
770 NEXT Q
780 D[1,I]=P
790 D[2,I]=A1
800 D[3,I]=A2
810 D[4,I]=C[1]
820 D[5,I]=C[2]
830 D[6,I]=C[3]
840 D[7,I]=C[4]
850 D[8,I]=C[5]
860 D[9,I]=C[6]
870 C[1]=(C[1])*0.9*1.17
880 C[2]=(C[2]*1.17)
890 C[3]=(C[3]*0.95*1.17)
900 C[4]=C[4]*0.90481*0.854
910 C[5]=C[5]*0.95177*0.854
920 C[6]=C[6]*0.854
930 P1=(C[2]-C[3])*0.866025
940 P2=-C[1]+(C[2]+C[3])/2
950 P3=C[4]+C[5]+C[6]
960 IF D$[1,2]#"P3" THEN 1000
970 Y=(C[4]+C[5]+C[6])*1E+05
980 GOSUB 2210
990 GOTO 1470
1000 IF D$[1,2]#"P4" THEN 1040
1010 Y=SQR(P1↑2+P2↑2)*1E+05
1020 GOSUB 2210
1030 GOTO 1470
1040 IF D$[1,1]#"F" THEN 1080
1050 Y=SQR(P1↑2+P2↑2+P3↑2)*1E+05
1060 GOSUB 2210
1070 GOTO 1470
```

```
1080 IF D$[1,2]#"M1" THEN 1120
1090 Y=32*((C[6]-C[4])-(C[5]-C[6]))*(1E+04)/3.4641
1100 GOSUB 2210
1110 GOTO 1470
1120 IF D$[1,2]#"M2" THEN 1160
1130 Y=32*(C[5]-C[4])*(1E+04)/2
1140 GOSUB 2210
1150 GOTO 1470
1160 IF D$[1,2]#"M3" THEN 1200
1170 Y=-23.9*(C[1]+C[2]+C[3])/0.98
1180 GOSUB 2210
1190 GOTO 1470
1200 IF D$[1,1]#"C" THEN 520
1210 IF I >= 2 THEN 1240
1220 DISP "WHAT BEAM C(1,2,3,4,5,6)";
1230 INPUT C$
1240 IF C$[1,4]#"C(1)" THEN 1280
1250 Y=C[1]*1E+05
1260 GOSUB 2210
1270 GOTO 1470
1280 IF C$[1,4]#"C(2)" THEN 1320
1290 Y=C[2]*1E+05
1300 GOSUB 2210
1310 GOTO 1470
1320 IF C$[1,4]#"C(3)" THEN 1360
1330 Y=C[3]*1E+05
1340 GOSUB 2210
1350 GOTO 1470
1360 IF C$[1,4]#"C(4)" THEN 1400
1370 Y=C[4]*1E+05
1380 GOSUB 2210
1390 GOTO 1470
1400 IF C$[1,4]#"C(5)" THEN 1440
1410 Y=C[5]*1E+05
1420 GOSUB 2210
1430 GOTO 1470
1440 IF C$[1,4]#"C(6)" THEN 520
1450 Y=C[6]*1E+05
1460 GOSUB 2210
1470 D1=P2
1480 D2=P1
1490 GOSUB 2050
1500 A3=D3
1510 D2=SQR(P1↑2+P2↑2)
1520 D1=P3
1530 GOSUB 2050
1540 A4=D3
1550 GOTO 1660
1560 PRINT "LOAD(KP) [illegible]
```

```
1570 D[1,I]=P
1580 D[2,I]=A1
1590 D[3,I]=A2
1600 D[4,I]=Y
1610 D[5,I]=A3
1620 D[6,I]=A4
1630 D[7,I]=P1*1E+05
1640 D[8,I]=P2*1E+05
1650 D[9,I]=P3*1E+05
1660 IF I >= 20 THEN 2460
1670 GOTO 540
1680 STOP
1690 OUTPUT (4,70)"@7I@5N2036"
1700 OUTPUT (4,70)"@7I@5N1036"
1710 DISP "LOAD AND VIBRATION ON"
1720 OUTPUT (4,70)"@7I@2E@20"
1730 ENTER (4,*)R
1740 IF R<0.7 THEN 1710
1750 WAIT 500
1760 RETURN
1770 STOP
1780 OUTPUT (4,70)"@7I@5N2036"
1790 OUTPUT (4,70)"@7I@5N1036"
1800 DISP "VIBRATION OFF"
1810 OUTPUT (4,70)"@7I@2E@20"
1820 ENTER (4,*)R
1830 IF R>0.1 THEN 1800
1840 OUTPUT (4,70)"@7I@3E"
1850 RETURN
1860 STOP
1870 OUTPUT (4,70)"@7I@5N2036"
1880 OUTPUT (4,70)"@7I@5N1036"
1890 WAIT 100
1900 OUTPUT (4,70)"@7I@2E@20"
1910 ENTER (4,*)B
1920 IF ABSB >= 0.6 THEN 1960
1930 BEEP
1940 DISP "BATTERY VOLTAGE LOW"
1950 GOTO 1890
1960 RETURN
1970 STOP
1980 OUTPUT (4,70)"@7I@3E"
1990 WAIT 2000
2000 OUTPUT (4,70)"@7I@2E@20"
2010 ENTER (4,*)T[I]
2020 P=A2*(T[I]-T1)*1E+03/(0.1576*9.9807)
2030 RETURN
```

```
2040 STOP
2050 IF D2#0 THEN 2070
2060 D2=1E-90
2070 D3=ATN(D1/D2)
2080 IF D1>0 THEN 2100
2090 GOTO 2120
2100 IF D2>0 THEN 2140
2110 GOTO 2160
2120 IF D2>0 THEN 2180
2130 GOTO 2160
2140 D3=D3
2150 GOTO 2190
2160 D3=180+D3
2170 GOTO 2190
2180 D3=360+D3
2190 RETURN
2200 STOP
2210 PLOT ABS(P),ABS(Y)
2220 IPLOT -0.5,0
2230 IPLOT 1,0
2240 IPLOT -0.5,0
2250 IPLOT 0,-0.5
2260 IPLOT 0,1
2270 IPLOT 0,-0.5
2280 PEN
2290 IF A$[12,15]#"PLOT" THEN 2390
2300 PLOT P,ABS(C[1])*100000
2310 LABEL (*)"*"
2320 PEN
2330 PLOT P,ABS(C[2])*100000
2340 LABEL (*)"+"
2350 PEN
2360 PLOT P,ABS(C[3])*100000
2370 LABEL (*)"X"
2380 PEN
2390 RETURN
2400 D[1,21]=I
2410 STORE  DATA #5,F1,D
2420 PRINT A$,A1,A2
2430 PRINT "FILE NUMBER";F1
2440 DISP "EXPERIMENT COMPLETED.";
2450 INPUT B$
2460 IF B$[1,1]="Y" THEN 2480
2470 GOTO 140
2480 DISP "SWITCH ME OFF PLEASE.
2490 END
```

toring process can be plotted against the value defined by P on the program page (line 220 or 640 or 690) characterizing the force or moment magnitude under test.

The remainder of the program is just for the apparatus check and the B.U.S. commands for sending and receiving messages to or from the devices connected to the computer.

APPENDIX B.2

THE DATA TREATMENT PROGRAM

The complete program will not be shown for the reasons mentioned in Chapter 4. However, the part of the program which has been altered and which deals with the calibration, the computation and the selection of variables to be analysed by the regression method using the least square technique are displayed on lines 320, 400, 560 and 750 of the subprogram, page 94. These variables are stored in two matrices [E] and [P]. The matrix [E] is reserved for the raw data, i. e.: the transducer responses, whereas the matrix [P] is reversed to the computed variables. The calibration coefficients can be seen on lines from 160 to 210 and the computation on lines from 220 to 300 with respect to the set of equations (page 9). A plotting function is inserted on line 1130 to permit a visual check.

```
10 FORMAT F4.0,2F12.4
20 FORMAT 2F12.4
30 IF N=0 THEN 60
40 DISP "NOT ALLOWED"
50 END
60 GOTO 90
70 IF N#0 THEN 250
80 PRINT "  NO."TAB12"X"TAB24"Y"
90 DISP "INPUT DATA TAPE FILE NO.";
100 INPUT N1
110 IF N1=0 THEN 130
120 LOAD DATA #5,N1,D
130 N3=0
140 WAIT 100
150 N3=N3+1
160 E[1,N3]=D[4,N3]*0.9*1.17*1E+05
170 E[2,N3]=D[5,N3]*1.17*1E+05
180 E[3,N3]=D[6,N3]*0.95*1.17*1E+05
190 E[4,N3]=D[7,N3]*0.90481*0.854*1E+05
200 E[5,N3]=D[8,N3]*0.95177*0.854*1E+05
210 E[6,N3]=D[9,N3]*0.854*1E+05
220 P[1,N3]=(E[2,N3]-E[3,N3])*0.866025
230 P[2,N3]=-E[1,N3]+(E[2,N3]+E[3,N3])/2
240 P[3,N3]=E[4,N3]+E[5,N3]+E[6,N3]
250 P[4,N3]=SQR(P[1,N3]↑2+P[2,N3]↑2)
260 P[5,N3]=SQR(P[1,N3]↑2+P[2,N3]↑2+P[3,N3]↑2)
270 P[6,N3]=54*((E[6,N3]-E[4,N3])-(E[5,N3]-E[6,N3]))/(3.4641)
280 P[7,N3]=54*(E[5,N3]-E[4,N3])/(2)
290 P[8,N3]=-23.9*(E[1,N3]+E[2,N3]+E[3,N3])/0.98
300 P[9,N3]=SQR(P[6,N3]↑2+P[7,N3]↑2)
310 IF N3>1 THEN 340
320 DISP "F(1),M(2),C(3)";
330 INPUT C
340 B[2]=D[1,N3]
350 IF C=1 THEN 540
360 IF C=3 THEN 730
370 E=0
380 B=0
390 IF N3>1 THEN 420
400 DISP "MX(1),MY(2),MZ(3),MV(4)";
410 INPUT A
420 IF A>4 THEN 400
430 IF A=4 THEN 520
440 IF A=3 THEN 500
450 IF A=2 THEN 480
460 Y=P[7,N3]
470 GOTO 940
480 Y=(P[6,N3])
490 GOTO 940
500 Y=P[8,N3]
510 GOTO 940
520 Y=P[9,N3]
530 GOTO 940
```

```
540 A=E=0
550 IF N3>1 THEN 580
560 DISP "FX(1),FY(2),FZ(3),FP(4),F(5)";
570 INPUT B
580 IF B>5 THEN 560
590 IF B=5 THEN 710
600 IF B=4 THEN 690
610 IF B=3 THEN 670
620 IF B=2 THEN 650
630 Y=P[1,N3]
640 GOTO 940
650 Y=P[2,N3]
660 GOTO 940
670 Y=P[3,N3]
680 GOTO 940
690 Y=P[4,N3]
700 GOTO 940
710 Y=P[5,N3]
720 GOTO 940
730 A=E=0
740 IF N3>1 THEN 770
750 DISP "C(1),C(2),C(3),C(4),C(5),C(6)";
760 INPUT E
770 IF E>6 THEN 750
780 IF E=6 THEN 930
790 IF E=5 THEN 910
800 IF E=4 THEN 890
810 IF E=3 THEN 870
820 IF E=2 THEN 850
830 Y=E[1,N3]
840 GOTO 940
850 Y=E[2,N3]
860 GOTO 940
870 Y=E[3,N3]
880 GOTO 940
890 Y=E[4,N3]
900 GOTO 940
910 Y=E[5,N3]
920 GOTO 940
930 Y=E[6,N3]
940 IF N3 >= D[1,2] THEN 1320
950 DISP N+1;B[2];Y
960 WAIT 200
970 IF N3#1 THEN 1250
980 DISP "DO YOU WANT A PLOT";
990 INPUT D$
1000 IF D$[1,1]#"Y" THEN 1300
1010 IF STAT14<9 THEN 1050
1020 DISP "PLOTTER NOT HAPPY";
1030 WAIT 200
1040 GOTO 1010
```

```
1040 GOTO 1010
1050 IF C=1 THEN 1090
1060 IF C=3 THEN 1090
1070 SCALE -100,100,-500,500
1080 GOTO 1110
1090 SCALE 0,130,0,130
1100 OFFSET 20,20
1110 XAXIS 0,10,0,100
1120 YAXIS 0,10,0,100
1130 LABEL (*,2,1.7,0)
1140 FOR Y9=0 TO 100 STEP 20
1150 PLOT 0,Y9,1
1160 CPLOT -4,-0.3
1170 LABEL (*)Y9
1180 NEXT Y9
1190 LABEL (*,2,1.7,0)
1200 FOR X=0 TO 100 STEP 20
1210 PLOT X,0,1
1220 CPLOT -2,-1.5
1230 LABEL (*)X
1240 NEXT X
1250 IF D$[1,1]#"Y" THEN 1300
1260 PLOT B[2],Y
1270 CPLOT -0.3,-0.3
1280 LABEL (*)"+"
1290 PEN
1300 Y=FNX!
1310 GOTO 150
1320 BEEP
1330 DISP "DATA INPUT COMPLETED.
1340 PLOT -15,20,1
1350 LABEL (*,2,1.7,90)
1360 LETTER
1370 PLOT 0,-15,1
1380 LABEL (*,2,1.7,0)
1390 LETTER
1400 DISP "DATA INPUT COMPLETED.
1410 END
```

APPENDIX C

THE TESTING FORCE TRANSDUCER

As already mentioned, a force $\vec{F}$ requires the measurement of the deflection δ of a spring, that is:

$$F = k\delta \qquad \text{(C-1)}$$

A Linear Voltage Differential Transformer (L.V.D.T.) was used in order to convert linearly the displacement δ to a difference of potential V1 as the following formula expresses:

$$\delta = \lambda V1 \qquad \text{(C-2)}$$

By this means, one has a way to automatically assess the magnitude of the force $\vec{F}$ if the L.V.D.T. is coupled to a channel of the D.A.S. (channel #2, Figure A-1). The spring in our case is a cantilever coupled to the L.V.D.T. shaft as Figure C-1, below, shows.

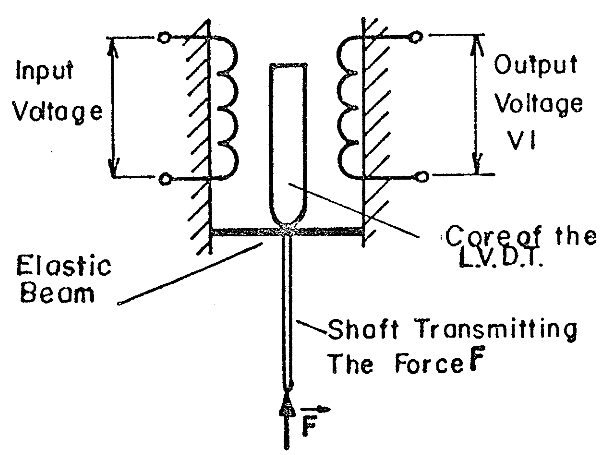


FIGURE C-1 THE L.V.D.T. FORCE TRANSDUCER

If one works in the linear range of the cantilever material and of the L.V.D.T., one can see that by substituting equation C-1 into C-2, we have:

$$F = \lambda k V I \qquad (C-3)$$

where λk is the coefficient of transduction. By the calibration made by means of a scale accurate to 1/10 of a gram, it was found that λk was equal to 157 thus 1 gf corresponds to 1 m.V and by examining the scatter of the plot of F against Vl, shown in Figure C-2, the transducer sensitivity was found to be 1.5 gf, limiting the test of the instrument to that degree of sensitivity. However, a better sensitivity may have been achieved if the cantilever were made with more accuracy, but within the workshop facilities and the limitation of resources, the best was achieved.

It is worth noting that before arriving at this final solution which uses a L.V.D.T. sensitive to 10^{-6} mm deflection, an attempt was made to use a L.V.D.T. sensitive to 0.1 mm. Within this sensitivity, and in order to cover the range of force desired, $F_{min}/F_{max} = 100$, a motion of $\delta = 0.1 \times 100 = 10$ mm was required, thus requiring the use of a coil spring and it follows that a preload of the spring was nec-

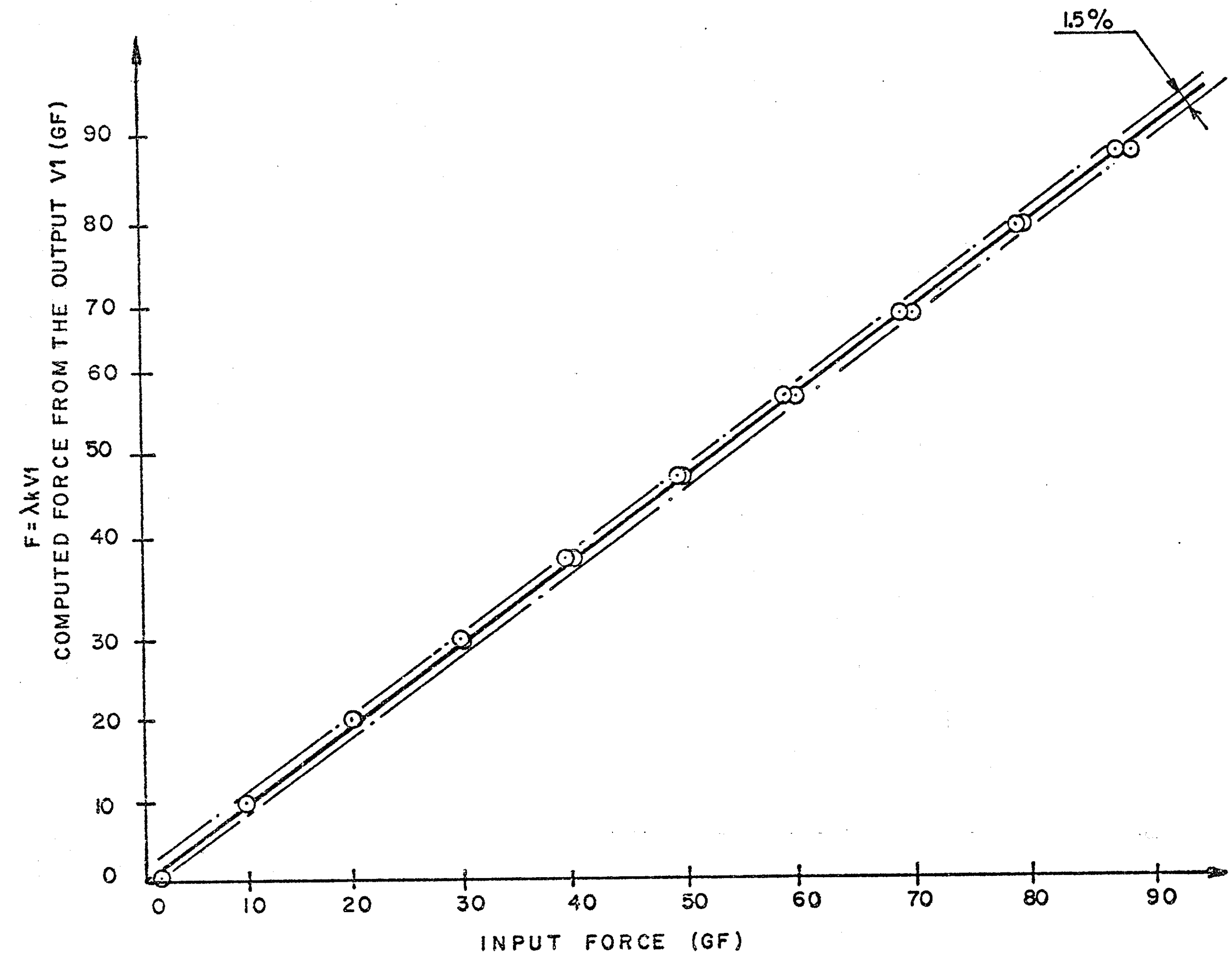


FIGURE C-2 CALIBRATION OF THE L.V.D.T. FORCE TRANSDUCER essary. The calibration of this form of transducer exhibits a dead zone force due to a 12 gf of necessary preload and exhibited a very inconsistent sensitivity due partly to the large motion of the spring. Thus, in order to extend the test range and to increase accuracy of results, the change of type of transducer was made.

APPENDIX D

RESULTS

The following pages present the tables and the plots of the results of the vibration efficacy and of the instrument performances in the investigated hemisphere, taking into consideration the calibration coefficients k_i, page 68.

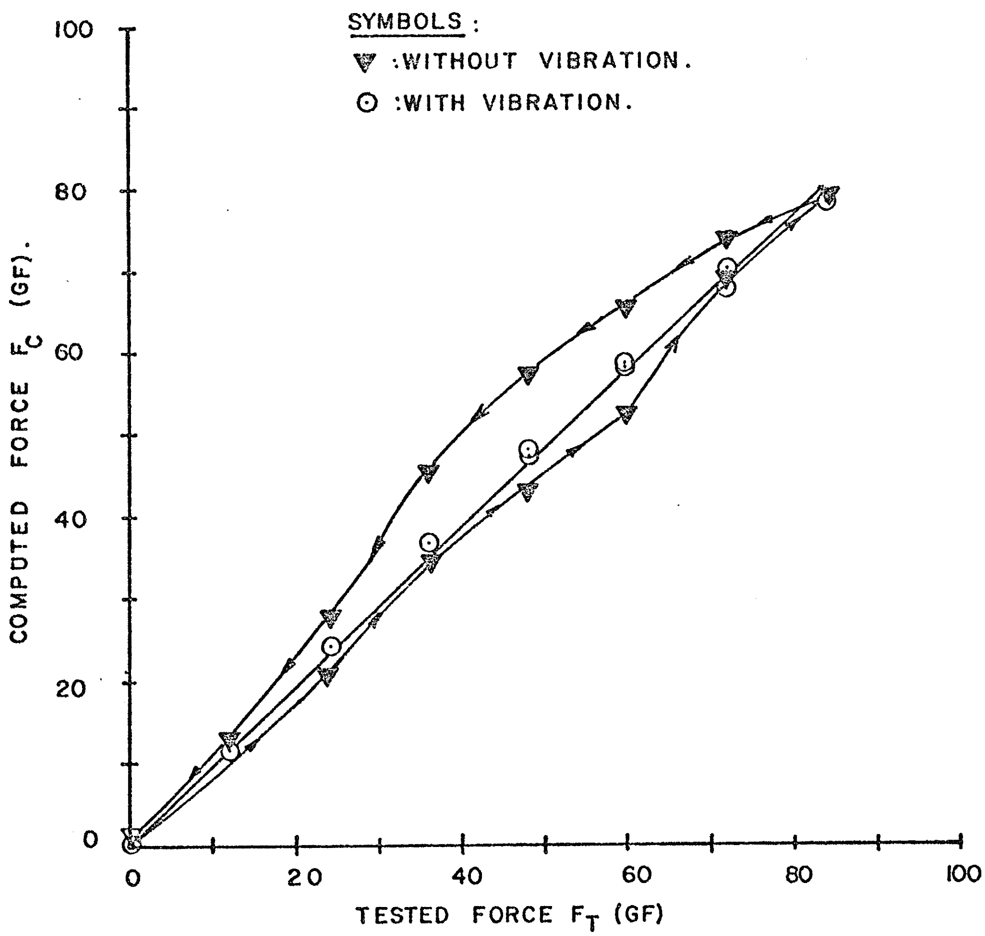


FIGURE D-1 ILLUSTRATION OF THE EFFECTIVENESS OF VIBRATION

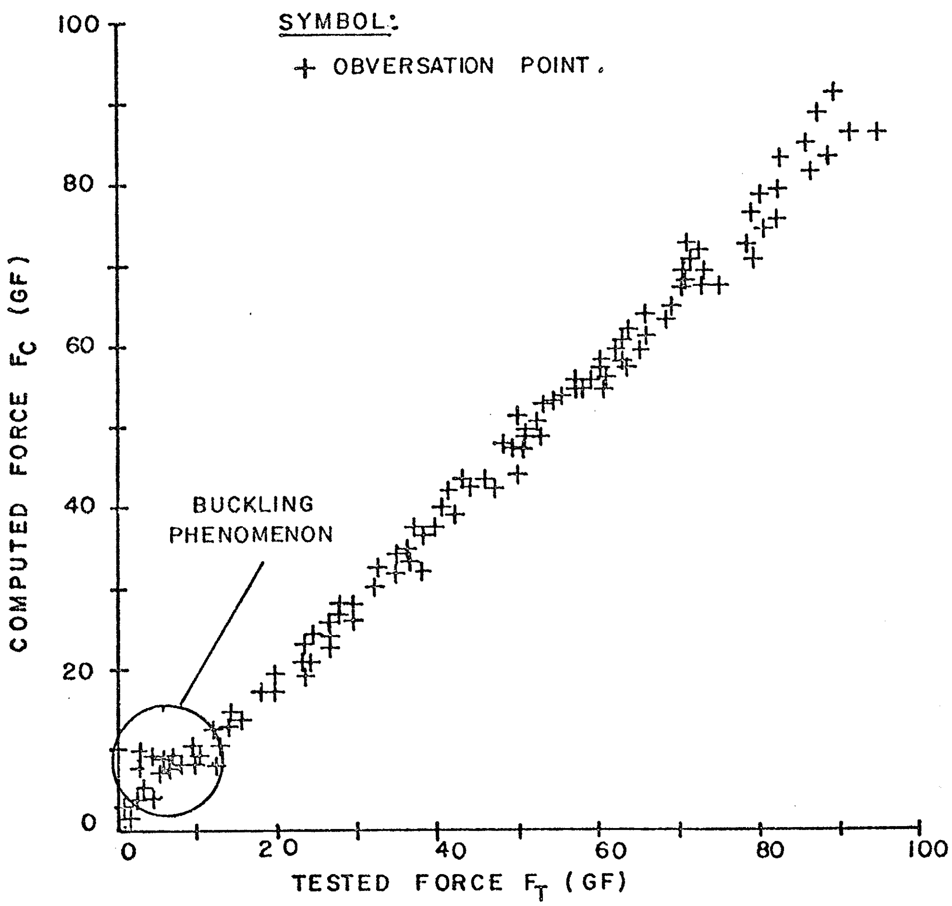


FIGURE D-2 OVERALL FORCE CALIBRATION SHOWING EFFECT OF BUCKLING AT A2 = 30°

TABLE D-1

Slope "a" of the regression analysis of the force transducer responses on the test force F_T

$$a = C_i/F_T$$

Transducer Response A2	A1	C1	C2	C3
0°	0°	-0.060	0.588	-0.495
	30°	-0.339	0.603	-0.288
	60°	-0.574	0.522	0.045
	90°	-0.652	0.294	0.329
	120°	-0.551	-0.004	0.494
	150°	-0.337	-0.353	0.662
	180°	-0.004	-0.599	0.569
	210°	0.250	-0.689	0.351
	240°	0.457	-0.483	0.046
	270°	0.546	-0.297	-0.239
	300°	0.460	0.007	-0.465
	330°	0.230	0.363	-0.578

TABLE D-2

Slopes "a" and coefficients of correlation "r^2" of the regression line of the computed force F_c on the test force F_T

A1 \ A2		0°	30°	45°	60°	75°	90°
0°	a	0.946	1.002	0.996	0.919	0.984	0.978
	r^2	0.998	0.998	0.997	0.999	0.999	0.997
30°	a	0.921	0.915	0.992	0.930	0.988	
	r^2	0.997	0.999	0.997	0.999	0.999	
60°	a	0.950	0.917	0.936	0.926	0.992	
	r^2	0.998	0.999	0.999	0.999	0.999	
90°	a	0.965	0.930	0.916	0.918	0.990	
	r^2	0.997	0.999	0.998	0.999	0.999	
120°	a	0.900	0.964	0.910	0.932	1.007	
	r^2	0.998	0.999	0.998	0.999	0.999	
150°	a	1.002	0.953	0.941	0.944	1.031	
	r^2	0.997	0.999	0.999	0.998	0.999	
180°	a	1.010	0.919	0.918	0.979	0.994	
	r^2	0.997	0.999	0.999	0.999	0.999	
210°	a	0.990	0.930	0.940	0.964	0.990	
	r^2	0.999	0.999	0.999	0.999	0.997	
240°	a	0.815	0.939	0.951	0.917	0.970	
	r^2	0.998	0.999	0.999	0.987	0.995	
270°	a	0.814	0.958	0.940	0.933	0.957	
	r^2	0.998	0.997	0.999	0.978	0.995	
300°	a	0.800	0.959	0.961	0.919	0.983	
	r^2	0.998	0.998	0.988	0.985	0.994	
330°	a	0.883	1.008	1.007	0.973	0.968	
	r^2	0.999	0.999	0.996	0.995	0.999	

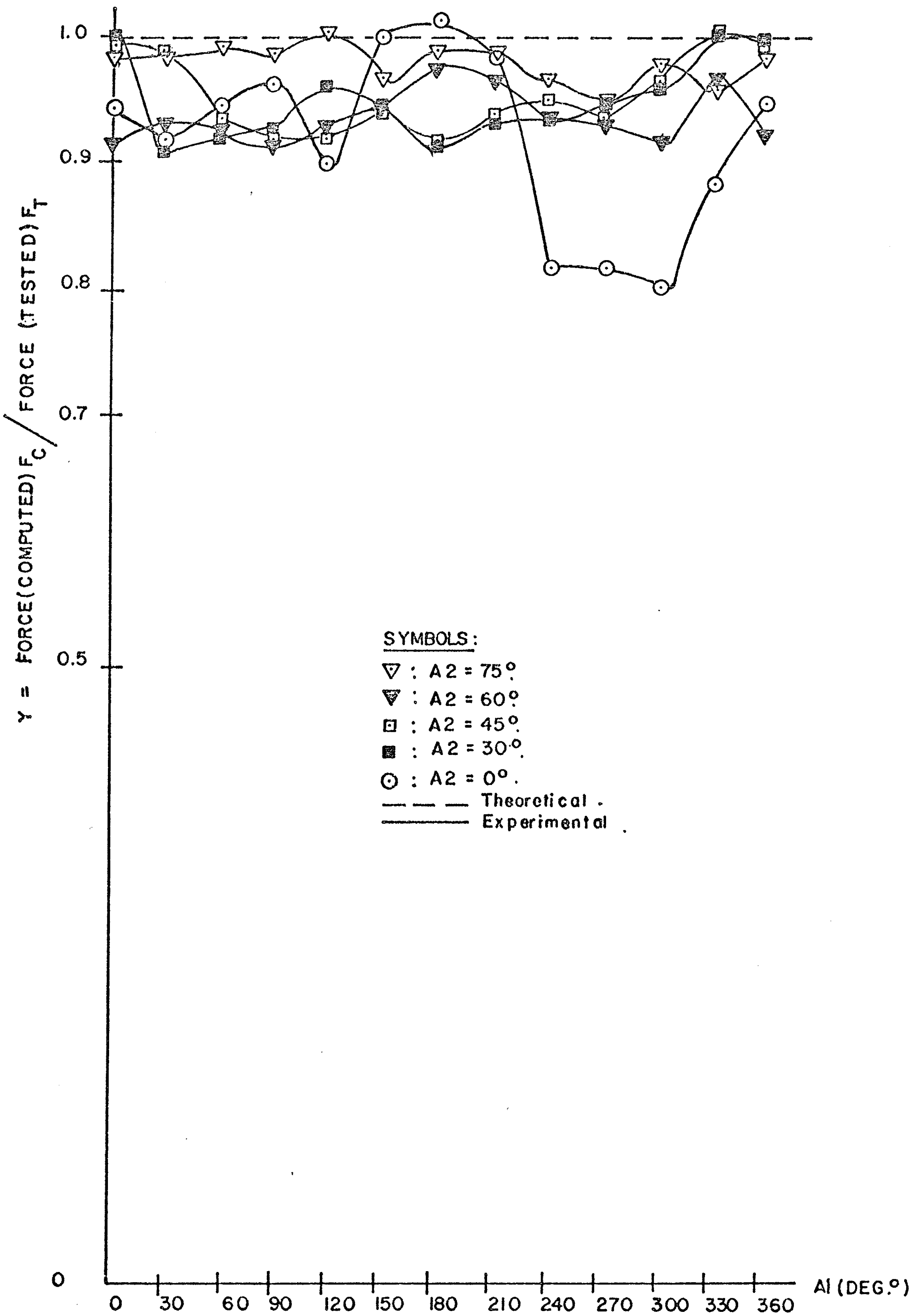


FIGURE D-3 PLOT OF THE RESULTS IN TABLE D-2

TABLE D-3

Slopes a_{Fx} of the regression analysis of the computed force Fx_c on the test force F_T

$$a_{Fx} = Fx_c/F_T$$

A1 \ A2	0^o	30^o	45^o	60^o	75^o	90^o
0^o	0.938	0.900	0.725	0.492	0.203	-0.022
30^o	0.771	0.709	0.595	0.434	0.167	
60^o	0.413	0.388	0.312	0.227	0.087	
90^o	-0.030	-0.012	-0.008	-0.010	-0.088	
120^o	-0.432	-0.475	-0.360	-0.238	-0.130	
150^o	-0.879	-0.734	-0.591	-0.396	-0.254	
180^o	-1.012	-0.806	-0.669	-0.515	-0.238	
210^o	-0.901	-0.679	-0.609	-0.483	-0.242	
240^o	-0.458	-0.398	-0.337	-0.265	-0.189	
270^o	-0.050	0.102	-0.015	-0.032	-0.035	
300^o	0.408	0.485	0.404	0.237	0.103	
330^o	0.816	0.805	0.675	0.437	0.192	

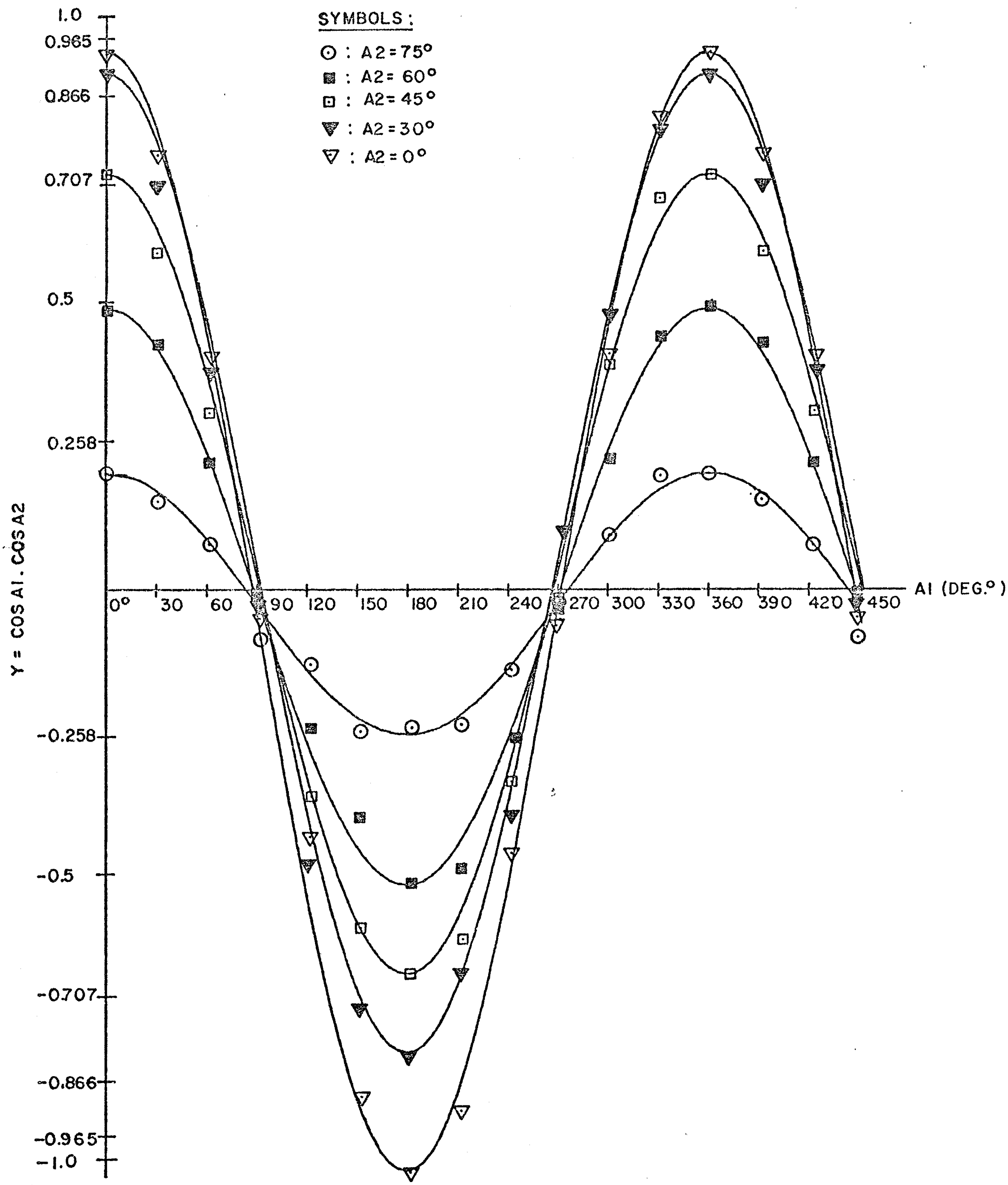


FIGURE D-4 PLOT OF THE RESULTS IN TABLE D-3

TABLE D-4

Slope a_{Fy} of the regression analysis of the computed force Fy_c on the test force F_T

$$a_{Fy} = Fy_c/F_T$$

A1 \ A2	0°	30°	45°	60°	75°	90°
0°	0.107	0.013	-0.012	-0.014	0.004	-0.007
30°	0.496	0.364	0.376	0.192	0.143	
60°	0.857	0.698	0.580	0.390	0.248	
90°	0.964	0.813	0.641	0.417	0.262	
120°	0.796	0.707	0.543	0.362	0.255	
150°	0.492	0.419	0.366	0.248	0.185	
180°	-0.010	0.037	0.010	-0.023	-0.033	
210°	-0.419	-0.460	-0.326	-0.261	-0.125	
240°	-0.675	-0.721	-0.612	-0.425	-0.235	
270°	-0.815	-0.835	-0.677	-0.528	-0.255	
300°	-0.689	-0.693	-0.587	-0.457	-0.228	
330°	-0.338	-0.417	-0.318	-0.216	-0.122	

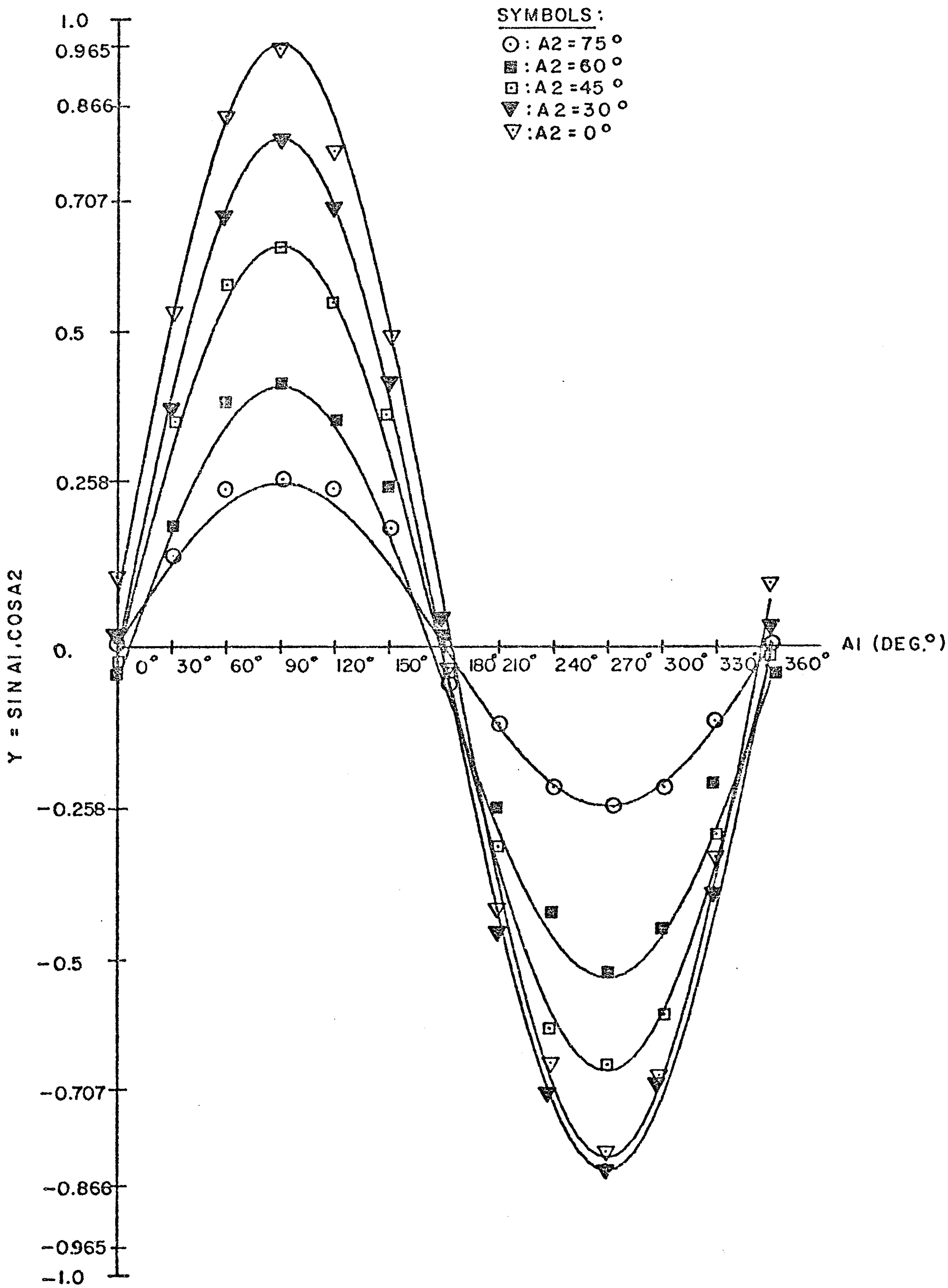


FIGURE D-5 PLOT OF THE RESULTS IN TABLE D-4

TABLE D-5

Slope a_{Fz} of the regression analysis of the computed force Fz_c on the test force F_T

$$a_{Fz} = Fz_c/F_T$$

A2 / A1	0°	30°	45°	60°	75°	90°
0°	0.085	0.457	0.687	0.781	0.969	0.964
30°	0.101	0.457	0.707	0.798	0.969	
60°	0.080	0.458	0.666	0.812	0.959	
90°	0.060	0.460	0.655	0.820	0.955	
120°	0.035	0.451	0.646	0.828	0.970	
150°	-0.001	0.445	0.641	0.824	0.988	
180°	-0.023	0.451	0.638	0.836	0.968	
210°	-0.034	0.456	0.650	0.797	0.952	
240°	-0.027	0.456	0.648	0.765	0.928	
270°	-0.023	0.567	0.655	0.770	0.922	
300°	-0.005	0.508	0.737	0.815	0.951	
330°	0.043	0.444	0.678	0.909	0.948	

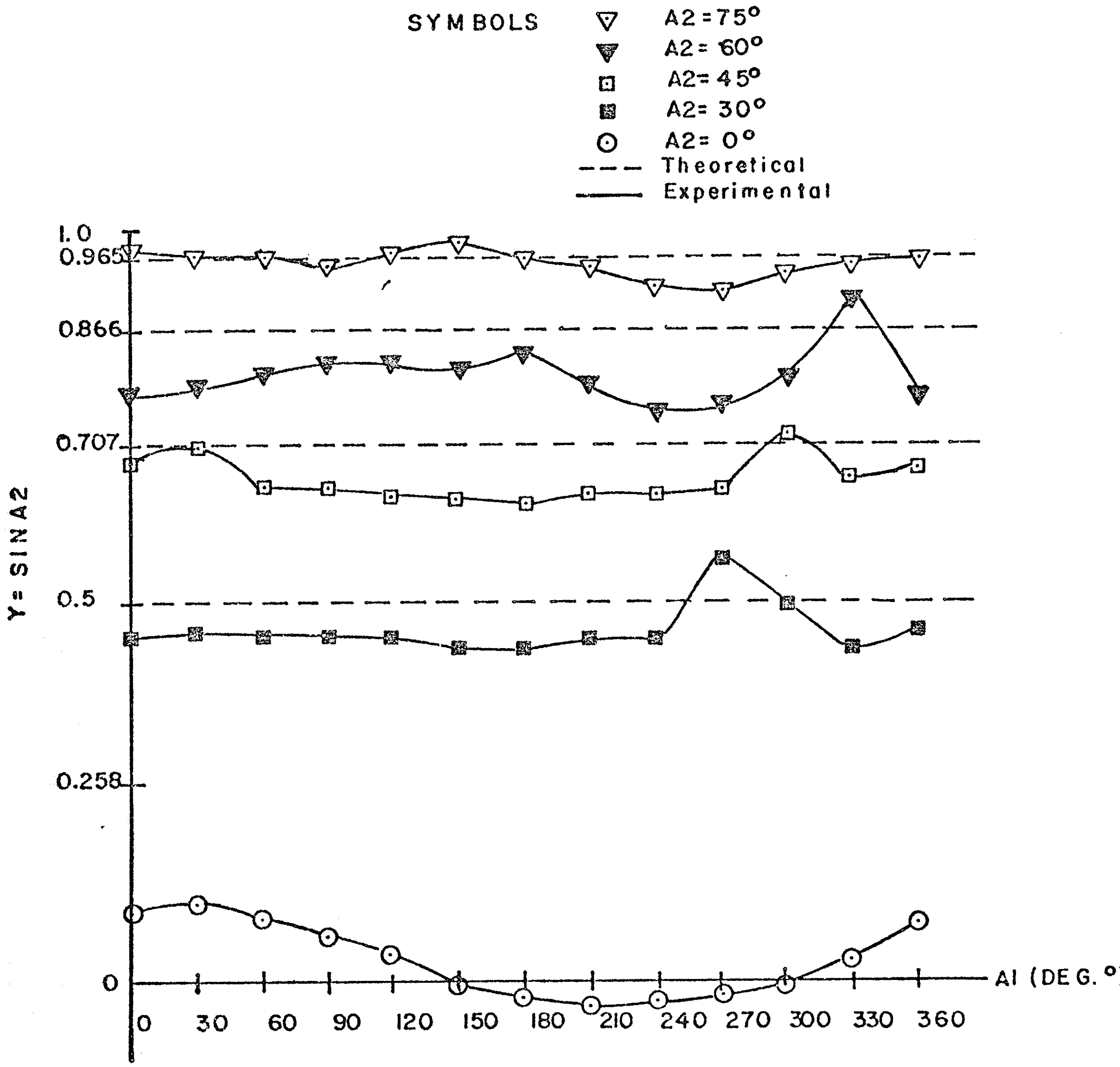


FIGURE D-5 PLOT OF THE RESULTS IN TABLE D-5

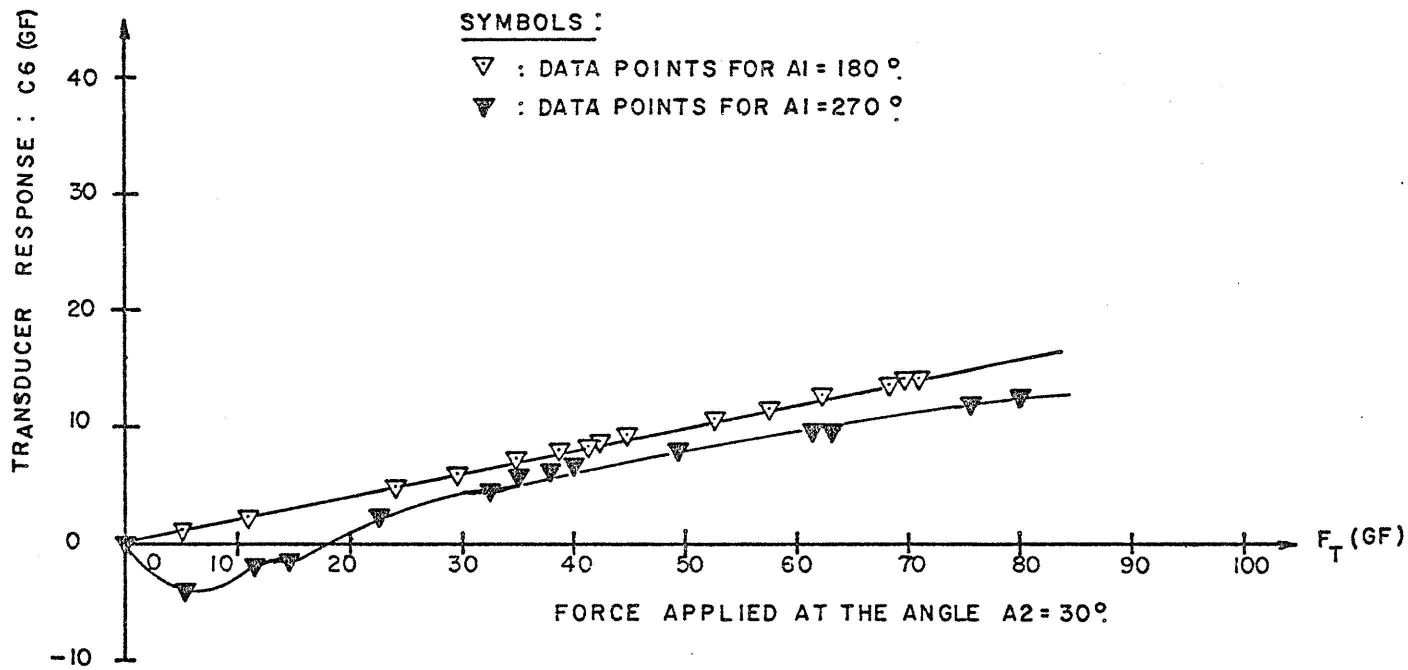


FIGURE D-7 EFFECT OF LONGITUDINAL ANGLE ON BUCKLING IN TRANSDUCER NO. 6

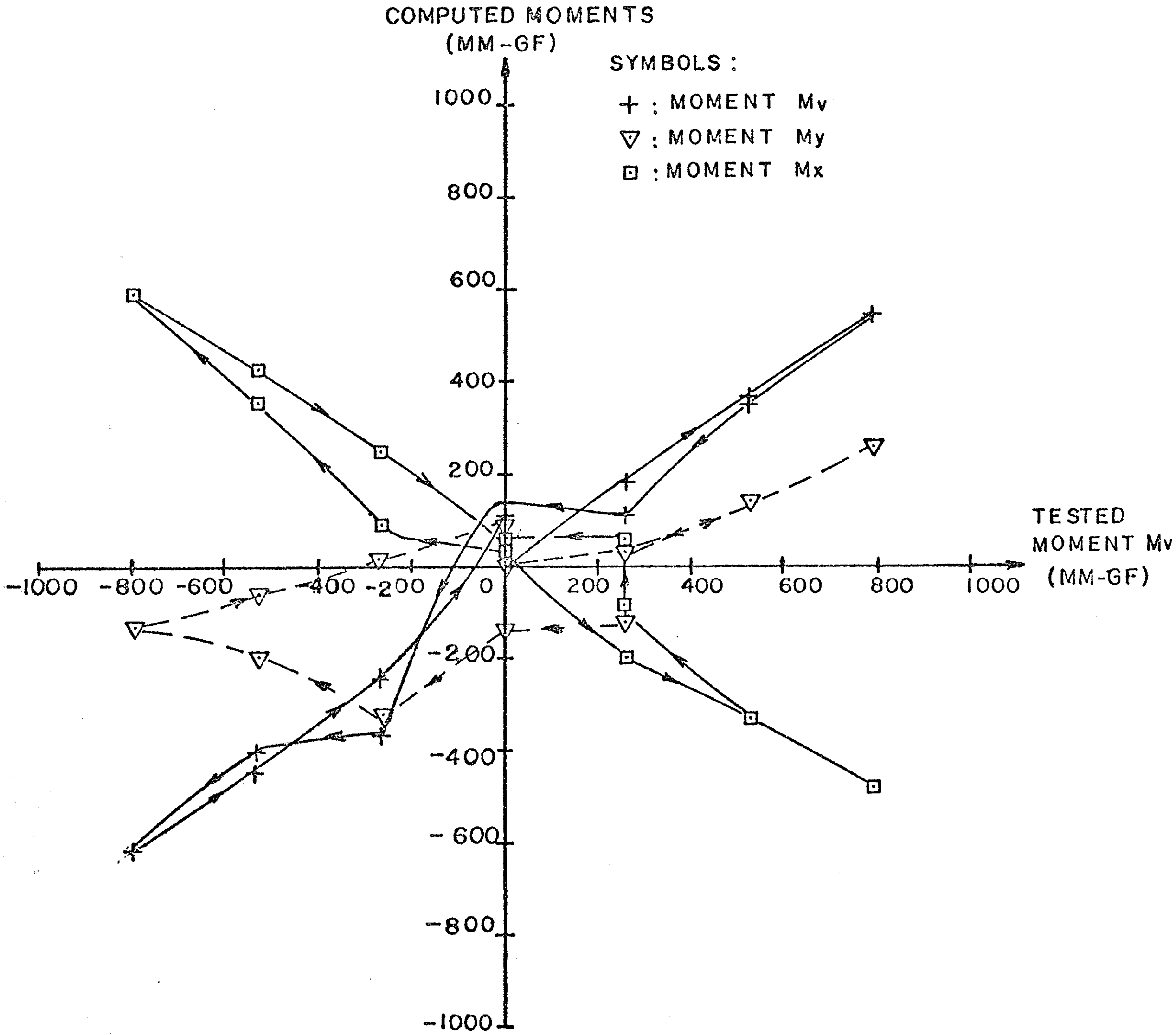


FIGURE D-8 EFFECT OF BUCKLING
ON AZIMUTHAL MOMENT MEASUREMENT

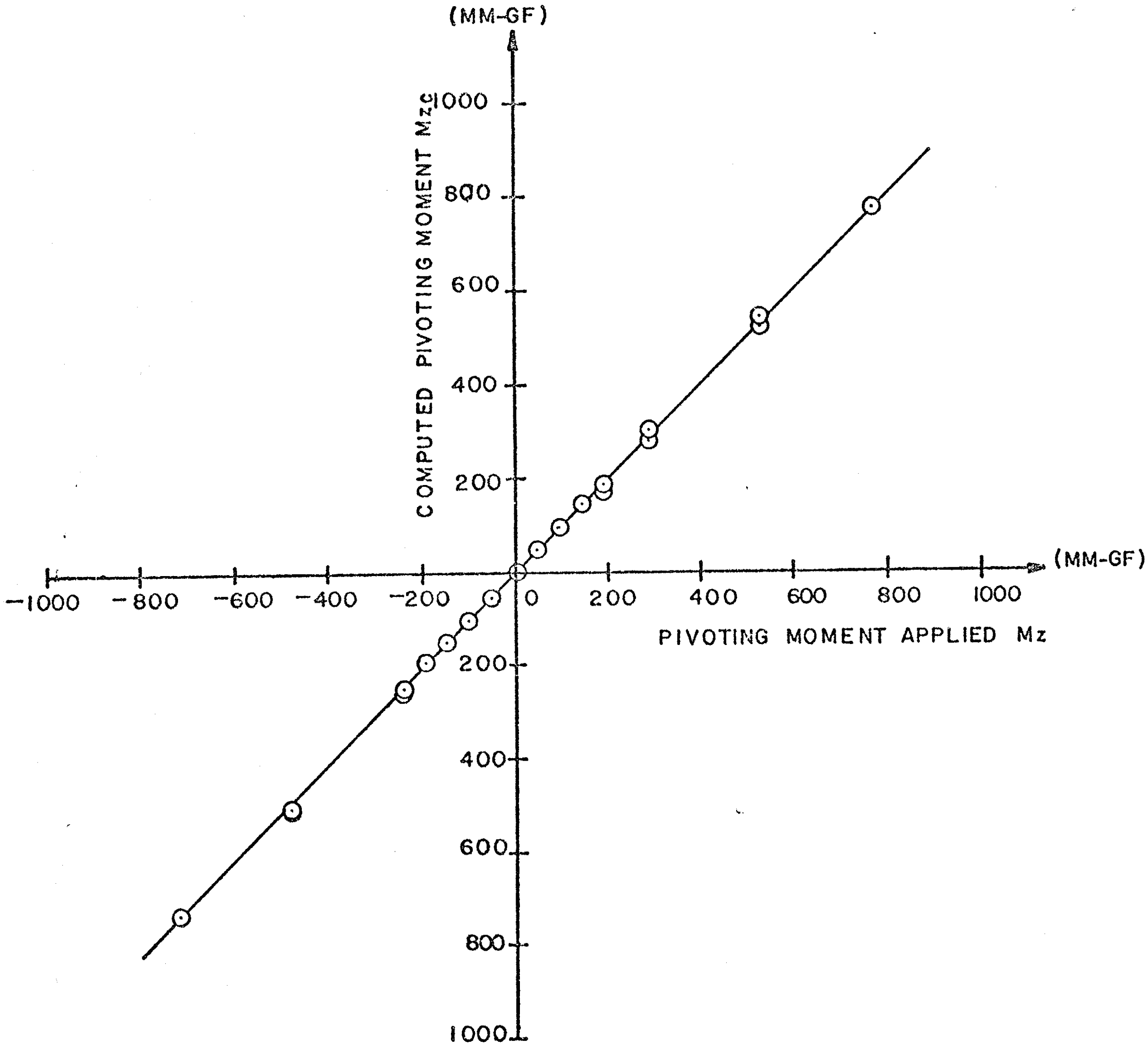


FIGURE D-9 PIVOTING MOMENT CALIBRATION CURVE