Optimal Semi-Foldover Plans for Two-Level Fractional Factorial Designs

by

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A Thesis submitted to the Faculty of Graduate Studies of

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Abstract

Two-level fractional factorial (FF) designs are commonly used at the early stage of investigation in industrial experiments. They can often identify which factor effects are significant by running only a fraction (subset) of a full factorial experiment (Wu and Hamada (2000), Box, Hunter and Hunter (2005), Montgomery (2005), Ryan (2007)). However, there will be aliasing of effects in a FF design, which may lead to ambiguities in interpreting the results of an experiment.

One strategy for de-aliasing effects of interest is to run a follow-up experiment, such as a foldover design. One cost-conscious alternative strategy for de-aliasing low-order effects is to augment the initial FF design with only one-half of the runs from a foldover plan. This approach is known as semi-folding.

The primary objective of this thesis is to select semi-foldover plans, that have appealing projection properties. In this thesis, we rank non-regular, orthogonal, combined designs (ie., initial plus semi-foldover) based on the number of estimable models containing a subset of main effects and their corresponding two-factor interactions. With this objective in mind, we use the projection estimation capacity (PEC) and projection information capacity (PIC) criteria (Loeppky, Sitter and Tang (2007)) to rank the combined designs.

A second objective of this thesis is to assess the alias structures of the combined designs using the generalized minimum aberration (minimum G-aberration) criterion (Deng and Tang (1999)). Generally speaking, a design possessing minimum aberration will minimize, or come close to minimizing, the aliasing of low-order effects.

Our research concludes that combined designs possessing desirable projection properties are often non-minimum aberration designs. We also observe that the semi-foldover approach can produce combined designs possessing superior projection properties than the foldover approach.

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Chapter 1

Introduction and Summary

Two-level full factorial designs and two-level fractional factorial designs (FF) are commonly used for screening for significant effects in industrial and agricultural experiments (Wu and Hamada (2000), Box, Hunter and Hunter (2005), Montgomery (2005), and Ryan (2007)). The full factorial "2^n design" denotes a factorial design with n factors each varied at two levels. Such a design is comprised of all possible 2^n observations, or treatment combinations, of the n factors. A 2^{n-p} design is a two-level FF design with n factors, where p is the number of *added* (generated) factors. The added factors are assigned to the interactions of the $n-p$ *basic* (independent) factors in the design matrix of a full factorial design in $n-p$ factors.

Example 1.1. Consider a 2^{7-3} FF design, as displayed in Table 1.1. This design requires 16 runs whereas a 2^7 full factorial design requires 128 runs. There are $7-3=4$ basic factors, whose corresponding columns are denoted by 1, 2, 3 and 4. The three added factors are generated by $5 = 123, 6 = 124$ and $7 = 234$, respectively. By taking all possible products of the generators, we obtain $I = 1235 = 1246 = 2347 = 3456 =$

Table 1.1: A 2^{7-3} Design

RUN	1	2	3	4	5=123	6=124	7=234
1	−	−	−	−	−	−	−
2	+	−	−	−	+	+	−
3	−	+	−	−	+	+	+
4	+	+	−	−	−	−	+
5	−	−	+	−	+	−	+
6	+	−	+	−	−	+	+
7	−	+	+	−	−	+	−
8	+	+	+	−	+	−	−
9	−	−	−	+	−	+	+
10	+	−	−	+	+	−	+
11	−	+	−	+	+	−	−
12	+	+	−	+	−	+	−
13	−	−	+	+	+	+	−
14	+	−	+	+	−	−	−
15	−	+	+	+	−	−	+
16	+	+	+	+	+	+	+

$1457 = 1367 = 2567$, which is called the complete defining relation of the design. Note that all two-factor interactions in this 2^{7-3} design are completely aliased with other two-factor interactions (eg. $12 = 35 = 46$). Effects that are aliased with one another are completely indistinguishable from one another in the subsequent data analysis. This aliasing is a consequence of running only a fraction (subset) of the runs of the 2^7 design.

If sufficient resources exist, various follow-up strategies may be used for de-aliasing effects of interest after an initial 2^{n-p} FF design has been run. The $\mathcal{D}$-optimality criterion (Atkinson and Donev (1992)) provides one algorithmic approach for selecting follow-up runs. A more computationally-demanding Bayesian approach may also be used (Meyer, Steinberg and Box (1996)).

The use of foldover designs would be another follow-up strategy. A foldover design is a 2^{n-p} FF design obtained by reversing the signs of one or more factors (columns) in the initial design (Box and Wilson (1951), Li and Lin (2003)). Foldover designs are useful when an experimenter is faced with one of the two following situations. In the first situation, only a few effects appear to be significant, after the data from the initial experiment has been analyzed. Here it may be possible to discern the key effects based on the initial experiment; otherwise, choose a foldover strategy that will de-alias the few aliased effects. In the second situation, a larger group of effects appear to be significant in the initial experiment. It may be impossible to identify exactly which effects one wishes to de-alias. Here the objective should be to select a foldover plan that minimizes, in some sense, the amount of aliasing in the *combined* (initial plus foldover) design.

Consider reversing the added factors 5, 6 and 7 in the initial design provided in Table 1.1. The foldover design that results is shown in Table 1.2. Note that the foldover design is of equal size (that is, requires 16 runs) to that of the initial 2^{7-3} experiment. It can be shown (Chapter 2) that this foldover design will de-alias all two-factor interactions involving 2.

On occasion, it may not be possible to run another 2^{n-p} experiment, because of constraints upon resources (for example, time, money, etc.). One alternative approach for de-aliasing low-order effects is to augment the initial FF design with only one-half of the runs from a foldover design. This procedure is known as *semi-folding* (Barnett, Czitrom, John and Leon (1997), John (2000), Mee and Peralta (2000)).

As discussed in Mee and Peralta (2000), the use of semi-foldover designs is an

Table 1.2: A 16-run Foldover Design Obtained by Folding on Factors 5, 6 and 7 in the Initial 2^{7-3} Design

Run	1	2	3	4	5	6	7
17	−	−	−	−	+	+	+
18	+	−	−	−	−	−	+
19	−	+	−	−	−	−	−
20	+	+	−	−	+	+	−
21	−	−	+	−	−	+	−
22	+	−	+	−	+	−	−
23	−	+	+	−	+	−	+
24	+	+	+	−	−	+	+
25	−	−	−	+	+	−	−
26	+	−	−	+	−	+	−
27	−	+	−	+	−	+	+
28	+	+	−	+	+	−	+
29	−	−	+	+	−	−	+
30	+	−	+	+	+	+	+
31	−	+	+	+	+	+	−
32	+	+	+	+	−	−	−

attractive alternative follow-up strategy because:

- semi-foldover designs are simple to construct: we obtain a foldover plan and select one-half of the runs from it, no software is required;
- semi-foldover designs are often more "degree-of-freedom-efficient" than foldover designs (that is, semi-folding can be a run-frugal strategy for estimating additional low-order effects);
- semi-foldover designs can, if necessary, be followed by the remaining foldover runs to complete the $2^{n-(p-1)}$ design.

Given the proceeding 2^{7-3} design, one possible semi-foldover plan is obtained by first folding on all three added factors (5, 6 and 7), and then selecting the eight runs for which the effect 127 is "+". The resulting 24-run combined design is shown in Table 1.3, where the 8 runs of the semi-foldover design are in bold.

In this thesis, we consider the determination of optimal semi-foldover plans, given an initial 2^{n-p} FF design, where $5 \leq n \leq 10$ and $1 \leq p \leq 6$. This thesis is organized as follows. Chapter 2 provides a brief review of 2^{n-p} FF designs, including their construction and use in industrial applications. The approach for enumerating all possible semi-foldover plans is reviewed in Chapter 3. Projection estimation capacity (PEC), projection information capacity (PIC) (Loeppky, Sitter and Tang (2007)) and generalized minimum aberration (Fries and Hunter (1980), Deng and Tang (1999), (2002), Li, Lin and Ye (2003)) are also discussed in Chapter 3, and are the three optimality criteria we use for ranking the semi-foldover plans. Catalogs of optimal semi-foldover plans, ranked according to the PEC, PIC and the generalized minimum aberration criteria, are provided in Appendices A.1 and A.2. Sample R code, illustrating the ranking of semi-foldover plans for an initial 2^{4-1} design, is provided in Appendix B.

Table 1.3: A 24-run Design: The 8-Run Semi-foldover Combined with the 16-Run 2^{7-3} Initial Design

RUN	1	2	3	4	5	6	7	127
1	−	−	−	−	−	−	−	−
2	+	−	−	−	+	+	−	+
3	−	+	−	−	+	+	+	−
4	+	+	−	−	−	−	+	+
5	−	−	+	−	+	−	+	+
6	+	−	+	−	−	+	+	−
7	−	+	+	−	−	+	−	+
8	+	+	+	−	+	−	−	−
9	−	−	−	+	−	+	+	+
10	+	−	−	+	+	−	+	−
11	−	+	−	+	+	−	−	+
12	+	+	−	+	−	+	−	−
13	−	−	+	+	+	+	−	−
14	+	−	+	+	−	−	−	+
15	−	+	+	+	−	−	+	−
16	+	+	+	+	+	+	+	+
17	−	−	−	−	+	+	+	+
	+	−	−	−	−	−	+	−
18	−	+	−	−	−	−	−	+
	+	+	−	−	+	+	−	−
	−	−	+	−	−	+	−	−
19	+	−	+	−	+	−	−	+
	−	+	+	−	+	−	+	−
20	+	+	+	−	−	+	+	+
	−	−	−	+	+	−	−	−
21	+	−	−	+	−	+	+	+
	−	+	−	+	−	+	−	−
22	+	+	−	+	+	−	+	+
23	−	−	+	+	−	−	+	+
	+	−	+	+	+	+	+	−
24	−	+	+	+	+	+	−	+
	+	+	+	+	−	−	−	−

Chapter 2

Fractional Factorial Designs

2.1 Two-Level Full Factorial and FF Designs

In many industrial experiments, factorial designs are used as a systematic method for assessing the significance of main effects and low-order interactions of some number, say n, of factors. Suppose that each of the n factors are varied at two-levels, for example, at a "low" (or "$-$") and a "high" (or "$+$") value. In this setting the experimenter could consider all 2^n possible treatment combinations. For large n, full factorial designs require many runs to be performed. It is typically not possible to run a full factorial experiment due to the constraints on resources. In this case, one may consider using two-level FF designs to reduce the run size. In such designs we assign p of the factors to interactions amongst the $n-p$ factors in a 2^{n-p} full factorial design.

Example 2.1. Consider again the 2^{7-3} FF design introduced in Chapter 1. We denote the 7 factors using the integers 1 thru 7. As a full factorial design would

Table 2.1: A 2^{7-3} Initial Design

RUN	1	2	3	4	5=123	6=124	7=234
1	−	−	−	−	−	−	−
2	+	−	−	−	+	+	−
3	−	+	−	−	+	+	+
4	+	+	−	−	−	−	+
5	−	−	+	−	+	−	+
6	+	−	+	−	−	+	+
7	−	+	+	−	−	+	−
8	+	+	+	−	+	−	−
9	−	−	−	+	−	+	+
10	+	−	−	+	+	−	+
11	−	+	−	+	+	−	−
12	+	+	−	+	−	+	−
13	−	−	+	+	+	+	−
14	+	−	+	+	−	−	−
15	−	+	+	+	−	−	+
16	+	+	+	+	+	+	+

require $2^7 = 128$ runs, we may reduce the number of runs by generating the factors 5, 6 and 7 by assigning their levels to select interaction columns of the 2^4 full factorial design. One possible assignment is $5 = 123, 6 = 124$ and $7 = 234$. The resulting $2^{-3} = \frac{1}{8}th$ fraction, or 2^{7-3} FF design, is displayed in Table 2.1. (This design is also displayed in Table 1.1

Let I denote the identity element, which is the column of all positive levels "+" (or 1's). Consider the added factor (or generator), $5 = 123$. If both sides are multiplied by 5, we obtain

$$5 \times 5 = 123 \times 5$$
$$5^2 = 1235$$

$$I = 1235.$$

Therefore, the product of columns 1, 2, 3 and 5 will yield all runs at the "+" level. That is, this product yields the identity column, I. Similarly, we have $I = 1246$ and $I = 2347$ from the added factors $6 = 124$ and $7 = 234$. Note that if $I = 1246$ and $I = 2347$, then $I = I^2 = 1246 \times 2347 = 12^2 34^2 67 = 1367$. Here, the exponents in the products are formed by using modulus 2 arithmetic, so that any even power of a factor is equal to I and any odd power is equal to the factor itself. By taking the product of the generators, it will imply another relation in the group. The group formed by the p defining words (generators) is called the defining contrast subgroup (DCS) (Wu and Hamada (2000)) or complete defining relation (Montgomery (2005)). If we multiply 1235, 1246 and 2347 together two at a time and three at a time, then we obtain the DCS of the proceeding example as

$$I = 1235 = 1246 = 2347 = 3456 = 1457 = 1367 = 2567.$$

There is a total of $2^p = 2^3 = 8$ elements, including the identity I, in the DCS and each element is referred to as a "word".

The DCS enables us to determine the *alias structure* of the 2^{7-3} design. For example, if we multiply every element in the DCS by 1, we can determine which effects are aliased (indistinguishable) with 1 in the subsequent data analysis. The alias chain associated with 1 is given by

$$1 = 235 = 246 = 12347 = 13456 = 457 = 367 = 12567.$$

Using the preceding approach we can display a list of alias chains for every factor and two-factor interaction in the experiment.

Table 2.2: The Alias Structure of the 2^{7-3} Design having Generators $5 = 123$, $6 = 124$ and $7 = 234$ (Ignoring Four-factor and Higher-order Interactions)

Defining relation: $I = 1235 = 1246 = 2347 = 3456 = 1457 = 1367 = 2567$
1= 235= 246= 457= 367
2= 135= 146= 567= 347
3= 125= 247= 456= 167
4= 237= 126= 356= 157
5= 123= 345= 137= 267
6= 124= 345= 137= 257
7= 234= 145= 137= 256
12= 35= 46
13= 25= 67
14= 26= 57
15= 23= 47
16= 24= 37
17= 45= 36
27= 34= 56
127= 357= 467= 245= 236= 156

The list of all alias chains is known as the alias structure of the design. There are 15 possible alias chains (one for each degree-of-freedom) for this 2^{7-3} design, as shown in Table 2.2. If a main effect or a two-factor interaction is not aliased with other main effects or two-factor interactions, we say that the effect is *clear*. Table 2.2 shows that all main effects (numbered 1 thru 7) are aliased with three-factor interactions and all two-factor interactions are aliased with other two-factor interactions. Therefore, all two-factor interactions for this 2^{7-3}_{IV} design are not clear although all main effects are clear.

2.2 Resolution and Minimum Aberration

Box and Hunter (1961) introduced the notion of the *resolution* of a FF design. The resolution is the length of the shortest word in the DCS, where the length of a word is defined to be equal to the number of letters in the word. The resolution is used to distinguish between two competing 2^{n-p} designs. The shortest word in the DCS of this 2^{7-3} design is of length 4; therefore, we say that this design is a "resolution IV" 2^{7-3} or 2^{7-3}_{IV} design. All other things being equal, designs with higher resolution are preferred, in order to minimize aliasing of low-order effects. To distinguish between two or more 2^{n-p} designs having the same resolution, Fries and Hunter (1980) introduced the minimum aberration (MA) criterion. The MA criterion selects a "good" 2^{n-p} design by choosing a design that sequentially minimizes the occurrence of short words in its DCS.

Before formally defining the MA criterion, we need to introduce the notion of a design's word length pattern (WLP). Recall that the number of letters in a word is its word length. The vector $W = (A_1, A_2, \ldots, A_n)$ is called the WLP of the design, where A_i are the number of words of length i, $(i = 1, 2, \ldots, n)$ in the design. An MA design may now be defined as follows:

Definition 2.2.1. For any two 2^{n-p} designs d_1 and d_2, let r be the smallest integer such that $A_r(\mathrm{d}_1) \neq A_r(\mathrm{d}_2)$, where A_i denotes the number of words of length i in its DCS, $1 \leq i \leq n$. Then d_1 is said to have less aberration than d_2 if $A_r(\mathrm{d}_1) < A_r(\mathrm{d}_2)$. If there is no design with less aberration than d_1, then d_1 is the MA FF design.

For the 2^{7-3}_{IV} FF design in Example 2.1, all 7 words in the DCS are of length 4.

Therefore, this design has WLP $W = (0, 7, 0, 0, 0)$. It turns out that this design is the MA 2^{7-3} design (Chen, Sun and Wu (1993)), since there is no other 2^{7-3} FF design having less aberration than this design. Note that due to the likely significance of main effects and two-factor interactions, we do not consider designs having main effects or tow-factor interactions aliased with the identity (overall mean) I. Therefore, we do not consider designs have non-zero entries for A_1 and A_2. We then write a design's WLP by beginning with A_3.

The following example illustrates how the MA criterion may be used to distinguish between two designs have the same resolution.

Example 2.2. Consider two 2^{7-2} designs, say d_1 and d_2. We denote the 7 factors using the integers 1 thru 7. Factors 6 and 7 are the two added factors. Let d_1 have generators $6 = 1234$ and $7 = 1245$, and d_2 have $6 = 123$ and $7 = 145$. The DCS of d_1 is $I_1 = 12346 = 12457 = 3567$ with WLP $W_1 = (0, 1, 2, 0, 0)$. The DCS of d_2 is $I_2 = 1236 = 1457 = 234567$ with WLP $W_2 = (0, 2, 0, 1, 0)$.

By comparing W_1 and W_2, we observe that both 2^{7-2} designs have resolution IV. However, note that d_1 has only 1 four-letter word, whereas d_2 has 2 four-letter words. Thus, d_1 has less aberration than d_2. It turns out that d_1 is the MA design.

Generally speaking, the MA criterion provides a useful approach for selecting "good" FF designs when all factors are of equal importance. Note that Deng and Tang (1999) generalized the MA criterion to *non-regular* designs. Non-regular designs have a more complicated alias structure than "regular" FF designs in that effects may also be partially aliased with one another. Also, the run size of non-regular designs need not be a power of 2. We reserve discussion concerning "generalized" MA until

Chapter 3.

2.3 Foldover Plans

A standard follow-up strategy for de-aliasing effects from an initial 2^{n-p} design is achieved by means of conducting a foldover design (Box and Wilson (1951), Li and Mee (2002), Box, Hunter and Hunter (2005), Montgomery (2005)). This approach adds a second design of equal size, by reversing the signs of one or more of the n columns (factors) in the initial design.

Li and Lin (2003) proved that any non-trivial foldover *plan* (the set of factors to be sign-reversed) is *equivalent* to one of the $2^p - 1$ possible non-trivial *core foldover plans*, where a core foldover plan is a foldover plan consisting only of added factors. We say that two foldover plans are equivalent if they produce the same foldover runs.

Example 2.3. The preceding result implies that there are $2^p - 1 = 2^3 - 1 = 7$ non-trivial core foldover plans for the 2^{7-3}_{IV} FF design in Example 2.1. One possible core foldover plan is obtained by reversing the signs of all three added factors (5, 6 and 7). That is, column 5 becomes "-5", 6 becomes "-6" and 7 become "-7". The 32-run combined (initial plus foldover) design is shown in Table 2.3.

The DCS of the initial MA 2^{7-3} design (Example 2.1) is $I = 1235 = 1246 = 2347 = 3456 = 1457 = 1367 = 2567$. The DCS of the foldover design is $I = -1235 = -1246 = -2347 = 3456 = 1457 = 1367 = -2567$. Combining the two DCS's yields $I = 3456 = 1457 = 1367$. Although the combined design has the same resolution (IV) as the initial design, it has 4 fewer four-letter words than the initial design. In

Table 2.3: A 32-run Combined Design Obtained by Folding on Added Factors 5, 6 and 7 in the Initial 2^{7-3} Design

RUN	1	2	3	4	5	6	7
1	−	−	−	−	−	−	−
2	+	−	−	−	+	+	−
3	−	+	−	−	+	+	+
4	+	+	−	−	−	−	+
5	−	−	+	−	+	−	+
6	+	−	+	−	−	+	+
7	−	+	+	−	−	+	−
8	+	+	+	−	+	−	−
9	−	−	−	+	−	+	+
10	+	−	−	+	+	−	+
11	−	+	−	+	+	−	−
12	+	+	−	+	−	+	−
13	−	−	+	+	+	+	−
14	+	−	+	+	−	−	−
15	−	+	+	+	−	−	+
16	+	+	+	+	+	+	+
17	−	−	−	−	+	+	+
18	+	−	−	−	−	−	+
19	−	+	−	−	−	−	−
20	+	+	−	−	+	+	−
21	−	−	+	−	−	+	−
22	+	−	+	−	+	−	−
23	−	+	+	−	+	−	+
24	+	+	+	−	−	+	+
25	−	−	−	+	+	−	−
26	+	−	−	+	−	+	−
27	−	+	−	+	−	+	+
28	+	+	−	+	+	−	+
29	−	−	+	+	−	−	+
30	+	−	+	+	+	+	+
31	−	+	+	+	+	+	−
32	+	+	+	+	−	−	−

the initial design, recall that all two-factor interactions are aliased with other two-factor interactions, although all main effects are are clear (Table 2.2). After folding on added factors 5, 6 and 7, we observe that all two-factor interactions involving 2 are de-aliased in the combined design, since 2 does not appear in any of the four-letter words in the DCS of the combined design.

Chapter 3

Optimal Semi-Foldover Plans

3.1 Semi-Foldover Designs

Most books on "experimental design" mention various strategies for augmenting an initial 2^{n-p} design with follow-up runs. One of the most popular follow-up strategies is to run a foldover design; however, the primary argument against conducting foldovers is that they are degree-of-freedom inefficient. Mee and Peralta (2000) point out that for 16- and 32-run initial 2^{n-p} FF designs, a foldover plan typically provides no more than one-half of the degrees-of-freedom for de-aliasing two-factor interactions. Barnett et al.. (1997) described a 2^{8-4}_{IV} semi-conductor experiment in which they used only one-half of the runs from a foldover design to estimate the 7 two-factor interactions involving one of the factors. This approach was named "semi-folding" by the authors.

Assuming that three-factor and higher-order interactions are negligible, semi-foldover designs and foldover designs may de-alias the same number of low-order effects. Since a semi-foldover design requires only one-half of the runs of a foldover design, this may allow for considerable cost-savings.

Example 3.1. Recall the 2^{7-3}_{IV} design given in Examples 2.1 and 2.3. One possible semi-foldover plan is obtained by first folding on all three added factors (5, 6 and 7), and then selecting the eight runs for which the three-factor interaction 127 is " + ". The resulting 24-run combined (initial plus semi-foldover) design is shown in Table 3.1, where the runs of the semi-foldover design come from the foldover design displayed in Table 2.3. (This 24-run design is also provided in Table 1.3.)

Example 3.2. Consider semi-folding the 32-run MA design, d_1, in Example 2.2. One possible semi-foldover plan is obtained by first folding on added factor 6, and then selecting the 16 runs for which the effect 135 is "−". The resulting 48-run combined design is shown in Table 3.2.

3.2 Enumerating the Semi-Foldover Plans

Li and Lin (2003) showed that for a given 2^{n-p} design, any (non-trivial) foldover plan is equivalent to one of $2^p - 1$ core foldover plans where a core foldover plan is constructed by reversing the signs of one or more of the added factors. To construct a semi-foldover plan, we select one-half of the runs of a core foldover plan by subsetting on one of the $2^{n-p} - 1$ effects that are in distinct alias chains in the alias structure of the foldover design. (For example, consider Table 3.3 which displays the alias

Table 3.1: The 24-run Combined Design Obtained by Folding on Added Factors 5, 6 and 7 in the Initial 2^{7-3}_{IV} Design and Subsetting on 127^{+} in the Foldover Design

RUN	1	2	3	4	5=123	6=124	7=234	127
1	−	−	−	−	−	−	−	−
2	+	−	−	−	+	+	−	+
3	−	+	−	−	+	+	+	−
4	+	+	−	−	−	−	+	+
5	−	−	+	−	+	−	+	+
6	+	−	+	−	−	+	+	−
7	−	+	+	−	−	+	−	+
8	+	+	+	−	+	−	−	−
9	−	−	−	+	−	+	+	+
10	+	−	−	+	+	−	+	−
11	−	+	−	+	+	−	−	+
12	+	+	−	+	−	+	−	−
13	−	−	+	+	+	+	−	−
14	+	−	+	+	−	−	−	+
15	−	+	+	+	−	−	+	−
16	+	+	+	+	+	+	+	+
17	−	−	−	−	+	+	+	+
18	−	+	−	−	−	−	−	+
19	+	−	+	−	+	−	−	+
20	+	+	+	−	−	+	+	+
21	+	−	−	+	−	+	+	+
22	+	+	−	+	+	−	+	+
23	−	−	+	+	−	−	+	+
24	−	+	+	+	+	+	−	+

Table 3.2: The 48-run Combined Design Obtained by Folding on Added Factor 6 in the Initial 2^{7-2} Design and Subsetting on 135^- in the Foldover Design

RUN	1	2	3	4	5	6=1234	7=1245	135
1	−	−	−	−	−	+	+	−
2	+	−	−	−	−	−	−	+
3	−	+	−	−	−	−	−	−
4	+	+	−	−	−	+	+	+
5	−	−	+	−	−	−	+	+
6	+	−	+	−	−	+	−	−
7	−	+	+	−	−	+	−	+
8	+	+	+	−	−	−	+	−
9	−	−	−	+	−	−	−	−
10	+	−	−	+	−	+	+	+
11	−	+	−	+	−	+	+	−
12	+	+	−	+	−	−	−	+
13	−	−	+	+	−	+	−	+
14	+	−	+	+	−	−	+	−
15	−	+	+	+	−	−	+	+
16	+	+	+	+	−	+	−	−
17	−	−	−	−	+	+	−	+
18	+	−	−	−	+	−	+	−
19	−	+	−	−	+	−	+	+
20	+	+	−	−	+	+	−	−
21	−	−	+	−	+	−	−	−
22	+	−	+	−	+	+	+	+
23	−	+	+	−	+	+	+	−
24	+	+	+	−	+	−	−	+
25	−	−	−	+	+	−	+	+
26	+	−	−	+	+	+	−	−
27	−	+	−	+	+	+	−	+
28	+	+	−	+	+	−	+	−
29	−	−	+	+	+	+	+	−
30	+	−	+	+	+	−	−	+
31	−	+	+	+	+	−	−	−
32	+	+	+	+	+	+	+	+
33	−	−	−	−	−	−	+	−
34	−	+	−	−	−	+	−	−
35	+	−	+	−	−	−	−	−
36	+	+	+	−	−	+	+	−
37	−	−	−	+	−	+	−	−
38	−	+	−	+	−	−	+	−
39	+	−	+	+	−	+	+	−
40	+	+	+	+	−	−	−	−
41	+	−	−	−	+	+	+	−
42	+	+	−	−	+	−	−	−
43	−	−	+	−	+	+	−	−
44	−	+	+	−	+	−	+	−
45	+	−	−	+	+	−	−	−
46	+	+	−	+	+	+	+	−
47	−	−	+	+	+	−	+	−
48	−	+	+	+	+	+	−	−

Table 3.3: The Alias Structure of the 2_{IV}^{7-3} Foldover Design Obtained by Folding on Added Factors 5, 6 and 7 (Ignoring Four-factor and Higher-order Interactions)

Defining relation: $I = -1235 = -1246 = -2347 = 3456 = 1457 = 1367 = -2567$

$$
\begin{gathered}
1 = -235 = -246 = 457 = 367 \\
2 = -135 = -146 = -567 = -347 \\
3 = -125 = -247 = 456 = 167 \\
4 = -237 = -126 = 356 = 157 \\
-5 = 123 = -345 = -137 = 267 \\
-6 = 124 = -345 = -137 = 257 \\
-7 = 234 = -145 = -137 = 256 \\
12 = -35 = -46 \\
13 = -25 = 67 \\
14 = -26 = 57 \\
-15 = 23 = -47 \\
-16 = 24 = -37 \\
-17 = -45 = -36 \\
-27 = 34 = 56 \\
-127 = 357 = 467 = -245 = -236 = 156
\end{gathered}
$$

structure of the foldover design obtain by folding the 2_{IV}^{7-3} design (Example 2.3) on the added factors 5, 6 and 7. From Table 3.3 we note that the main effect, 1, is aliased with the three-factor interactions -235, -246, 457 and 367. This alias chain indicates that we will obtain the same 8 follow-up runs regardless if we subset on 1, -235, -246, 457 or 367 . Similarly for the 14 remaining alias chains.) By keeping the effect on which we subset constant at either " $-$" or "+", this implies that for a given 2^{n-p} initial design, there are $(2^p - 1) \times (2^{n-p} - 1) \times 2$ distinct semi-foldover plans to consider for optimality.

Refer to the initial 16-run MA 2_{IV}^{7-3} design in Example 2.3. By the preceding "counting rule", there will be $(2^3 - 1) \times (2^{7-3} - 1) \times 2 = 210$ possible semi-foldover plans to assess. One possible semi-foldover plan has already been described in Table 3.1. For the MA 2^{7-2}_{IV} design in Example 3.2, there will be $(2^2 - 1) \times (2^{7-2} - 1) \times 2 = 186$ possible semi-foldover plans. One possible semi-foldover plan has already been described in Table 3.2.

3.3 Ranking Semi-Foldover Plans Using the PEC and PIC Criteria

The combined (initial plus semi-foldover) design is a non-regular design. Although non-regular designs have a more complicated alias structure than FF designs (Section 2.2), non-regular designs possess some very useful *projection properties*.

A design is said to "projected" when we consider the design composed of the subset of significant main effects. This procedure allows us to consider a design with fewer factors and a higher resolution than the original design from which we projected. To illustrate the projection approach we first consider a simple example from the FF context.

Example 3.3. (Montgomery (2005), pp. 287–289.) Consider a 2^{4-1}_{IV} design, as shown in Table 3.4, with added factor $D = ABC$, (here $A = 1$, $B = 2$, $C = 3$ and $D = 4$). For this design, all two-factor interactions are aliased with other two-factor interactions. Suppose that main effect B is deemed not significant after analyzing the main effects. If we discard B from any subsequent analysis, the 2^{4-1}_{IV} design yields (i.e., "projects" into) a 2^3 full factorial in A, C and D. This projection is displayed

Table 3.4: A 2^{4-1} Design with $D = ABC$

RUN	A	B	C	D=ABC	Run Label
1	$-$	$-$	$-$	$-$	(1)
2	$+$	$-$	$-$	$+$	ad
3	$-$	$+$	$-$	$+$	bd
4	$+$	$+$	$-$	$-$	ab
5	$-$	$-$	$+$	$+$	cd
6	$+$	$-$	$+$	$-$	ac
7	$-$	$+$	$+$	$-$	bc
8	$+$	$+$	$+$	$+$	abcd

in Figure 3.1. Note that all two-factor interactions amongst A, C and D are clear in the (projected) 2^3 design whereas they are not clear in the 2_{IV}^{4-1} design. The 2^3 design in A, C and D is displayed in Table 3.5.

In this chapter we use the *projection estimation capacity* (PEC) and *projection information capacity* (PIC) criteria (Loeppky, Sitter and Tang (2007)) to systematically select combined designs with good projection properties.

Definition 3.3.1. Given an $N \times n$ non-regular design, d, where n is the number of factors, let $\rho_k(\mathrm{d})$ be the number of estimable models containing k main effects and their associated two-factor interactions. Also, let

$$p_k(\mathrm{d}) = \frac{\rho_k(\mathrm{d})}{\binom{n}{k}},$$

so that $0 \leq p_k \leq 1$, for all k, $k = 1, \ldots, n$. The sequence $(p_1, p_2, \ldots, p_n)$ is called the PEC sequence of the design d. It is desirable to sequentially maximize the coordinates

Figure 3.1: Projection of the 2_{IV}^{4-1} Design to a 2^3 Design in Factors A, C and D

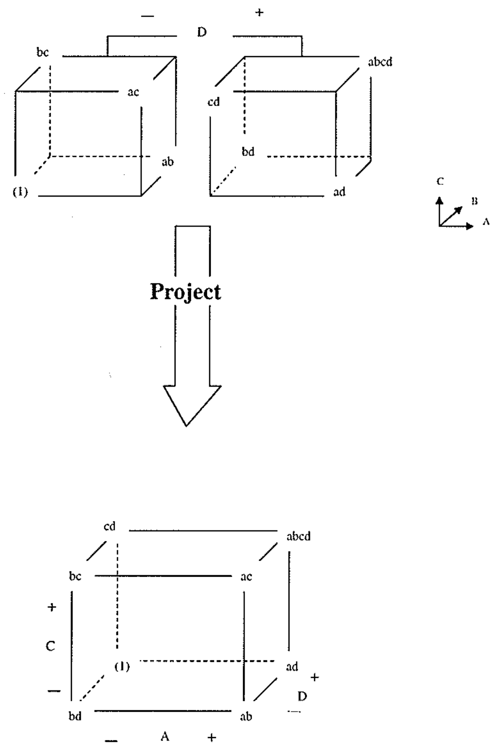

Table 3.5: The Projected 2^3 Design in Factors A, C and D

RUN	A	C	D	Run Label
1	−	−	−	(1)
2	+	−	+	ad
3	−	−	+	d
4	+	−	−	a
5	−	+	+	cd
6	+	+	−	ac
7	−	+	−	c
8	+	+	+	acd

of the PEC sequence.

Before we discuss the PEC criterion any further, it is useful to describe what we mean by an "estimable model". We will do this by considering a FF design, although our emphasis in this chapter will be upon (non-regular) combined designs.

Example 3.4. Consider a 2^{4-1}_{IV} design, where $4 = 123$ such that $I = 1234$. Suppose that the design matrix is given by

$$\mathbf{X} = \begin{array}{c} \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 12 & 23 & 34 \end{array} \\ \left[\begin{array}{ccccccc} - & - & - & - & + & + & + \\ + & - & - & + & - & + & - \\ - & + & - & + & - & - & - \\ + & + & - & - & + & - & + \\ - & - & + & + & + & - & + \\ + & - & + & - & - & - & - \\ - & + & + & - & - & + & - \\ + & + & + & + & + & + & + \end{array}\right]_{8\times 7} \end{array}.$$

The design matrix, $\mathbf{X}$, consists of main effects 1, 2, 3, 4 and the two-factor interactions 12, 23 and 34. Note that the columns of $\mathbf{X}$ are not linearly independent since $12 = 34$. A consequence of this dependence is that $\det(\mathbf{X}'\mathbf{X})$ will equal 0, and we say that the model (consisting of 1, 2, 3, 4, 12, 23 and 34) is not estimable.

The PEC criterion implies that a given design matrix, $\mathbf{X}$, will contain k main effects and their corresponding $\binom{k}{2}$ two-factor interactions. $\mathbf{X}$ will be an $N \times p$ matrix, where $p = 1 + k + \binom{k}{2}$, and includes a term for the overall mean (denoted by β_0).

To obtain the PEC sequence of the 24-run combined design in Table 3.1, we first observe that for $k = 1, 2, 3, 4, 5, 6$ and 7, there are $\binom{7}{1} = 7$, $\binom{7}{2} = 21$, $\binom{7}{3} = 35$, $\binom{7}{4} = 35$, $\binom{7}{5} = 21$, $\binom{7}{6} = 7$ and $\binom{7}{7} = 1$ possible models, respectively, to consider.

Suppose that $k = 3$. We use $\mathbf{X}_1$ thru $\mathbf{X}_{35}$ to denote the 35 possible design (model) matrices. One of the 35 possible design matrices is given by

$$\mathbf{X}_1 = \begin{array}{c} \begin{array}{ccccccc} \beta_0 & 1 & 2 & 3 & 12 & 13 & 23 \end{array} \\ \begin{bmatrix} 1 & -1 & -1 & -1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 \\ 1 & 1 & 1 & -1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & 1 & -1 & 1 & -1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}_{24 \times 7} \end{array},$$

where the entries for columns (factors) 1, 2 and 3 are taken from Table 3.1. Note that $\det(\mathbf{X}_1'\mathbf{X}_1) = 4586471424$ (non-zero). Therefore, we say that the model comprised of the main effects 1, 2 and 3 and their corresponding two-factor interactions is estimable.

Another possible design matrix is give by $\mathbf{X}_{35}$, where the three factors under consideration are 5, 6 and 7. Using columns 5, 6 and 7 from Table 3.1, we have

$$\mathbf{X}_{35} = \begin{bmatrix} \beta_0 & 5 & 6 & 7 & 56 & 57 & 67 \\ 1 & -1 & -1 & -1 & 1 & 1 & 1 \\ 1 & 1 & 1 & -1 & 1 & -1 & -1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & -1 & 1 & 1 & -1 & -1 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & -1 & -1 & -1 & 1 & 1 & 1 \end{bmatrix}_{24\times 7} .$$

Here, $\det(\mathbf{X}'_{35}\mathbf{X}_{35})$ is 786432000. We conclude that the model comprised of the main effects 5, 6 and 7 (and their corresponding two-factor interactions) is also estimable.

For the combined design in Table 3.1, it turns out that for $k = 3$, all 35 models have a non-zero determinant. We conclude that $p_3 = 1$ for this 24-run combined design. We may proceed in a similar fashion to obtain p_3 for the 209 (remaining) semi-foldover plans. This procedure must also be performed for $p_4, \ldots, p_7$ for all 210 semi-foldover plans.

Note that all combined designs are orthogonal arrays in that the columns are pairwise orthogonal. (However, the combined designs are unbalanced due to the fact that we subset on a given effect.) Therefore, for a given initial 2^{n-p} design, all PEC-optimal semi-foldover plans have $p_1 = p_2 = 1$. Consequently, we begin our PEC sequences with p_3, rather than p_1 or p_2.

Table 3.6: Given the Initial MA 2^{7-3}_{IV} Design: Two Semi-Foldover Plans Ranked According to the PEC Criterion

	Core Foldover	Subset On	(p_3, p_4, p_5, p_6)
an optimal semi-foldover plan	5, 6, 7	127^+	(1, 0.914, 0.571, 0)
a poorer semi-foldover plan	5, 6, 7	27^+	(1, 0.857, 0.286, 0)

Example 3.5. Recall that the MA 2^{7-3}_{IV} initial design has 210 possible semi-foldover plans. Table 3.6 displays two of these plans ranked according to the PEC criterion. An optimal 24-run combined design is obtained by first folding on columns 5, 6 and 7 and then subsetting on 127^+. The values of p_4 and p_5 for the optimal semi-foldover plan exceed those for the poorer semi-foldover plan, the latter plan being constructed by first folding on columns 5, 6 and 7 and then subsetting on 27^+.

From Table 3.6 we infer that the design in Table 3.1 allows for the estimation of all models containing any 3 of the 7 main effects (along with their corresponding two-factor interactions). We similarly conclude that the optimal semi-foldover plan results in 91.4% of the models containing four main effects, along with their corresponding two-factor interactions, to be estimable. This is superior to the poorer semi-foldover plan in which only 85.7% of models involving four factors, and their corresponding two-factor interactions, are estimable.

It is interesting to note that the optimal semi-foldover plan in Table 3.6 has a PEC sequence identical to that obtained by folding on factors 5, 6 and 7, despite the semi-foldover design being only one-half the run size of the foldover.

It turns out that there are 126 semi-foldover plans that have the optimal PEC sequence in Table 3.6. The remaining 84 plans possess the poorer PEC sequence in Table 3.6. One reasonable question to ask is "can one further distinguish between the 126 optimal semi-foldover plans?" In this section we use the PIC sequence as a secondary criterion for choosing between designs with the same PEC sequences.

Definition 3.3.2. Given an $N \times n$ non-regular design, d, let $\mathcal{F}$ be the class of models containing k main effects and their corresponding two-factor interactions and define

$$d_k(\mathrm{d}) = \sum_{i \in \mathcal{F}} \frac{[\det(\mathbf{X}_i'\mathbf{X}_i/N)]^{1/p}}{\binom{n}{k}},$$

where $\mathbf{X}_i$ is the ith model matrix and $p = 1 + k + \binom{k}{2}$ is the number of parameters in the model. Note that $0 \leq d_k \leq 1$, for all k, $k = 1, \ldots, n$. The sequence $(d_1, d_2, \ldots, d_n)$ is called the PIC sequence of d. As with the PEC criterion, it is also desirable to sequentially maximize the entries of the PIC sequence.

Consider the semi-foldover plan in Example 3.1. We may obtain d_3, for example, by summing up the 35 entries in the column entitled "$\frac{[\det(\mathbf{X}_i'\mathbf{X}_i/24)]^{1/7}}{\binom{7}{3}}$" in Table 3.7. It turns out that $d_3 = 0.9901549$. The values of d_4, d_5 d_6 and d_7 for the semi-foldover plan in Example 3.1 are obtained in a similar fashion.

It turns out that 14 of the 126 PEC-optimal semi-foldover plans share the optimal PIC sequence displayed in Table 3.8. The remaining $126 - 14 = 112$ PEC-optimal semi-foldover plans have an inferior PIC sequence, which is not displayed in Table 3.8. The remaining $210 - 126 = 84$ plans possess PEC and PIC sequences identical to those

Table 3.7: Calculating d_3 for the Semi-Foldover Plan in Example 3.1

Design Matrices	$\det(\mathbf{X}_i'\mathbf{X}_i)$	$\frac{[\det(\mathbf{X}_i'\mathbf{X}_i/24)]^{1/7}}{\binom{7}{3}}$
X_1	4586471424	0.02857143
X_2	4586471424	0.02857143
X_3	786432000	0.02220902
X_4	4586471424	0.02857143
X_5	3221225472	0.02716498
X_6	3221225472	0.02716498
X_7	4586471424	0.02857143
X_8	4586471424	0.02857143
X_9	786432000	0.02220902
X_{10}	4204265472	0.02857143
$\vdots$	$\vdots$	$\vdots$
X_{30}	3221225472	0.02716498
X_{31}	4586471424	0.02857143
X_{32}	4586471424	0.02857143
X_{33}	4586471424	0.02857143
X_{34}	3221225472	0.02716498
X_{35}	786432000	0.02220902
		$\sum = 0.9901549$

Table 3.8: Given the Initial MA 2_{IV}^{7-3} Design: Semi-Foldover Plans Ranked Sequentially According to the PEC and PIC Criteria

	Core Foldover	Subset On	(p_3, p_4, p_5, p_6)	(d_3, d_4, d_5, d_6)	No. of Semi-Foldover Plans
an optimal semi-foldover plan	5, 6 & 7	127^+	(1, 0.914, 0.571, 0)	(0.990, 0.885, 0.529, 0)	14
a poorer semi-foldover plan	5, 6 & 7	27^+	(1, 0.857, 0.286, 0)	(0.979, 0.825, 0.283, 0)	84

Table 3.9: Optimal Semi-Foldover Plans for the Five Non-Isomorphic Initial 2^{7-3} Designs Assessed According to the PEC and PIC Criteria

ID	Design Generators	Core Foldover	Subset On	(p_3, p_4, p_5, p_6)	(d_3, d_4, d_5, d_6)
7.3.1*	5=123, 6=124, 7=234	5,6,7,56, 57,67,567	(127)±	(1, 0.914, 0.571, 0)	(0.990, 0.885, 0.529, 0)
7.3.2	**5=12, 6=23,7=234**	**567**	**(4,7,14,17,27)±**	**(1, 0.971, 0.857, 0)**	**(1, 1, 0.857, 0.571)**
7.3.3	5=12, 6= 23, 7=14	567	(13,16,24,27,34, 37, 46, 67)±	(1, 0.943, 0.714, 0)	(1, 0.943, 0.714, 0.143)
7.3.4	5=12, 6=123, 7=124	5	(13,14,16,17,34, 37, 134, 137)±	(1, 0.914, 0.571, 0)	(1, 0.914, 0.571, 0)
7.3.5	5=12, 6=13 7=14	567	(4,14,16,24,34, 45, 46, 47, 146)±	(1, 0.914, 0.571, 0)	(1, 0.914, 0.571, 0)

Note:
1. * Denotes the MA 2^{7-3} design.
2. The optimal semi-foldover plans are in bold

of the poorer semi-foldover plan in Table 3.8.

We wish to point out that Table 3.8 displays "an optimal semi-foldover plan" and a "poorer semi-foldover plan" for just one possible initial 2^{7-3} design. We need to also consider ranking 24-run combined designs using the four remaining *non-isomorphic* 2^{7-3} designs listed in Chen, Sun and Wu (1993). (Wu and Hamada (2000, pg. 311) state that "two designs or arrays are said to be isomorphic if one design can be obtained from the other by row permutations, column permutations, or relabeling of levels.") The non-isomorphic designs in essence represent the entire class of designs for a given value of n and p. Table 3.9 displays the results when all five non-isomorphic 2^{7-3} designs are considered. The five non-isomorphic initial 2^{7-3} FF designs are

Table 3.10: Optimal Foldover Plans for the Five Non-Isomorphic Initial 2^{7-3} Designs Assessed According to the PEC Criterion

ID	Design Generator	Core Foldover	(p_3, p_4, p_5, p_6)
7.3.1*	5=123, 6=124, 7=234	5,6,7,56,	(1, 0.914, 0.571, 0)
7.3.2	**5=12, 6=23,7=234**	**567**	**(1 0.914, 0.571, 0)**
7.3.3	5=12, 6= 23, 7=14	567	(1, 0.943, 0.714, 0.143)
7.3.4	5=12, 6=123, 7=124	5	(1 0.914, 0.571, 0)
7.3.5	5=12, 6=13 7=14	567	(1, 1, 0.857, 0.571)

Note:
1. * Denotes the MA 2^{7-3} design.
2. The optimal foldover plan is in bold

labeled 7.3.1–7.3.5, which follows the notation of Chen, Sun and Wu (1993). Given a non-isomorphic initial design, the semi-foldover plans are assessed according to the PEC and PIC criteria.

Table 3.9 shows that the combined design that uses the initial MA 2^{7-3} design does not have the optimal PEC sequence. Using the PEC criterion, we rank the 5 combined designs in the following descending order: (1) 7.3.2 (bold in the table), (2) 7.3.3, (3) 7.3.1, 7.3.4 and 7.3.5 (three-way tie). We use the PIC sequence in an attempt to distinguish between designs 7.3.1, 7.3.4 and 7.3.5. In doing so, we observe that designs 7.3.4 and 7.3.5 are also tied with respect to the PIC criterion but are superior to design 7.3.1.

Given the initial designs 7.3.1–7.3.5, we also rank their foldover plans according to the PEC criterion. The results are displayed in Table 3.10. It is useful to point out that the optimal semi-foldover plan (in bold) in Table 3.9 has a PEC sequence superior to that of the PEC-optimal foldover plan in Table 3.10, (1, 0.971, 0.857, 0) vs. (1, 0.914, 0.571, 0). The implication is that superior projection properties, if of interest to an experimenter, may be obtained by run-frugal design construction strategies, such as semi-folding.

3.4 Ranking Semi-Foldover Plans Using the Generalized Minimum Aberration Criterion

A non-regular design is an orthogonal array whose columns do not form an Abelian group. One consequence is that main effects may be partially aliased with two-factor interactions. An appealing feature of non-regular designs is that they possess more flexible run-sizes than regular FF designs. Whereas FF design run-sizes must be a power of 2, non-regular orthogonal designs can have run-sizes that, for example, are a multiple of 4. The combined designs in this thesis are all non-regular designs.

Deng and Tang (1999) generalized the resolution and MA criteria as a means for ranking non-regular designs. The *minimum G-aberration criterion* is a generalization of the MA criterion introduced by Fries and Hunter (1980) for ranking FF designs. Let D denote a 2^n full factorial design. Any n-factor regular or non-regular design, d, is a collection of points in D, such that d $\subseteq$ D. Therefore, D represents the "design space" of the n factors. Li, Lin and Ye (2003) defined $X_J(\mathbf{x}) = \prod_{j \in J} x_j$ on D, where $\mathbf{x} \in$ d, $J \in P$ and P is the collection of all subsets of $\{1, \ldots, n\}$. Then the *indicator function* of d can be written as $F(\mathbf{x}) = \sum_{J \in P} b_J X_J(\mathbf{x})$, where $b_J = \frac{1}{2^n} \sum_{\mathbf{x} \in \mathrm{d}} X_J(\mathbf{x})$ denotes the coefficients of this polynomial function. Note that $b_0 = \frac{N}{2^n}$. (Additional details can be found in Ye (2003) and Li, Lin and Ye (2003)).

Example 3.6. Consider the 2_{IV}^{7-3} design from Example 2.1. The indicator function of this regular design is

$$\begin{aligned}F(\mathbf{x}) =& \tfrac{1}{8}+\tfrac{1}{8}x_1x_2x_3x_5+\tfrac{1}{8}x_1x_3x_6x_7+\tfrac{1}{8}x_1x_2x_4x_6+\tfrac{1}{8}x_1x_4x_5x_7\\ &+\tfrac{1}{8}x_2x_3x_4x_7+\tfrac{1}{8}x_2x_5x_6x_7+\tfrac{1}{8}x_3x_4x_5x_6.\end{aligned}$$

The term $x_1x_2x_3x_5$, for example, in the preceding indicator function represents the four-factor interaction between main effects 1, 2, 3 and 5. Note that all of the terms in the indicator function of the 2_{IV}^{7-3} design are exactly those words in its DCS. We defer discussion concerning the interpretation of the b_J until after the next example.

Example 3.7. Consider the 24-run combined design from Example 3.1. The indicator function of the design is

$$\begin{aligned}F(\mathbf{x}) =& \tfrac{3}{16}-\tfrac{1}{16}x_1x_2x_7+\tfrac{1}{16}x_1x_3x_4+\tfrac{1}{16}x_1x_5x_6-\tfrac{1}{16}x_2x_3x_6-\tfrac{1}{16}x_2x_4x_5\\ &+\tfrac{1}{16}x_3x_5x_7+\tfrac{1}{16}x_4x_6x_7+\tfrac{1}{16}x_1x_2x_3x_5+\tfrac{1}{16}x_1x_2x_4x_6+x_1x_3x_6x_7\\ &+x_1x_4x_5x_7+\tfrac{1}{16}x_2x_3x_4x_7+\tfrac{1}{16}x_2x_5x_6x_7+x_3x_4x_5x_6-\tfrac{1}{16}x_1x_2x_3x_4x_5x_6x_7.\end{aligned}$$

The term $x_1x_2x_7$, for example, in the preceding indicator function denotes the three-factor interaction between factors 1, 2 and 7.

The coefficients of an indicator function are useful for illustrating the alias structure of the corresponding design. In particular, b_J/b_0 measures the degree of aliasing (i.e., correlation) associated with a word, X_J. For example, consider $b_{127}/b_0 = -\frac{1}{16}/\frac{24}{2^7} = -\frac{1}{3}$, in Example 3.7. This implies that factor 1 is partially aliased (having correlation of $-\frac{1}{3}$) with the two-factor interaction 27. Similarly, factor 2 is partially aliased with 17 (having correlation of $-\frac{1}{3}$) and 7 is partially aliased with 12 (also having correlation of $-\frac{1}{3}$).

The *generalized word length* of a word, X_J, in the indicator function is defined to be the number of letters in the word + $(1 - |b_J/b_0|)$ (Li, Lin and Ye (2003)). For example, the word-length of the three-factor interaction $x_1x_2x_7$ in Example 3.7 is $3+(1-1/3) = 3\frac{2}{3}$. As in the MA criterion for FF designs, longer words are preferred. Here, the generalized word length definition penalizes words with larger correlations. Finally, recall that for a regular design partial aliasing does not occur. Therefore, in a FF design the word length of a word reduces to the number of letters that comprise the word.

Definition 3.4.1. Let d be an $N \times n$ design, and let $f_{i+l/t}$ be the number of words of fractional length, $i+l/t$, in the indicator function, where $i = 1, \ldots, n$, $l = 0, \ldots, t-1$ and $t = N/4$. The *extended word length pattern* (EWLP) of d is defined to be

$$(f_1, \ldots, f_{1+(t-1)/t}, f_2, \ldots, f_{2+(t-1)/t}, \ldots, f_n, \ldots, f_{n+(t-1)/t}).$$

Furthermore, the *generalized resolution* is the length of the smallest word in the EWLP.

Example 3.8. Recall the 2^{7-3}_{IV} initial design having 210 possible semi-foldover plans. Consider the two semi-foldover plans listed in Table 3.11. For both semi-foldover plans, we rank the resulting combined designs according to the generalized MA criterion. Here we may denote the entries of the EWLP by (f_2, $f_{2.167}$, $f_{2.333}$, $f_{2.5}$, $f_{2.667}$, $f_{2.833}$, f_3,..., $f_{7.833}$), where $t = N/4 = 24/4 = 6$. However, note that Ingram and Tang (2005) state that, if the number of runs is a multiple of 8, then $\sum_{\mathbf{x}\in \mathrm{d}} X_J(\mathbf{x})$ can only take on values from the set $0, 8, \ldots, N-8, N$. Therefore, in this example we

Table 3.11: Given the Initial MA 2^{7-3}_{IV} Design: Two Semi-Foldover Plans Ranked According to the Generalized MA Criterion

	Core Foldover	Subset On	$(f_2, f_{2.333}, f_{2.667}, f_3, \dots, f_5, f_{5.333}, f_{5.667})$
Semi-foldover plan 1	5, 6, 7	127^+	(0, 0, 0, 0, 0, 7, 3, 0, 4, 0, 0, 0)
Semi-foldover plan 2	5, 6, 7	27^+	(0, 0, 3, 0, 0, 0, 3, 0, 8, 0, 0, 0)

can modify (i.e., shorten) the EWLP to be of the form $(f_2, f_{2.333}, f_{2.667}, f_3, \dots, f_{7.667})$. Generally speaking, in this thesis we will truncate EWLPs beginning at words of length 6 in order to save space. This is acceptable since we assume that higher-order interactions are negligible. Table 3.11 displays the general EWLP as $(f_2, f_{2.333}, f_{2.667}, f_3, \dots, f_5, f_{5.333}, f_{5.667})$. Semi-foldover plan 1 has 7 words of length 3.667, 3 words of length 4 and 4 words length of 4.667. Semi-foldover plan 2 has 3 words of length 2.667, 3 words of length 4 and 4 words of length 4.667.

For given N and n, a generalized MA design is one that results from sequentially minimizing the EWLP. From Table 3.11, we observe that the (generalized) resolution of semi-foldover plans 1 and 2 are 3.667 and 2.667, respectively. Therefore, semi-foldover plan 1 has higher resolution. It turns out that semi-foldover plan 1 is the generalized MA semi-foldover plan for $N = 24$ and $n = 7$.

3.5 Overview of the Catalog of Optimal Semi-Foldover Plans

The previous sections utilized the PEC, PIC and generalized MA criteria for ranking combined designs. The tables in Appendix A contain 12-, 24- and 48-run combined designs ranked according to the three preceding criteria. The 24- and 48-run combined designs begin with an initial 16- and 32-run non-isomorphic 2^{n-p} FF design, respectively, for $n = 5,\ldots,10$ and $p = 1,\ldots,6$. The non-isomorphic 16- and 32-run initial FF designs are obtained from the catalog provided by Chen, Sun and Wu (1993). The 8-run MA initial FF designs are obtained from Wu and Hamada (2000, pg. 193).

In Appendix A.1, all 12-, 24- and 48-run semi-foldover plans (or equivalently, combined designs) are assessed sequentially with respect to the PEC and PIC criteria. Note that the first column heading, "ID", indicates the initial design. All MA initial designs are labeled with a $*$. The columns labeled "Core fo" and "SS" record the core foldover plan(s) and the effect(s) upon which we should subset, for a given initial design.

Example 3.9. Consider the 12-run combined designs. Table A.1 in Appendix A.1 only considers 8-run MA initial designs. Consider semi-folding the MA 2^{4-1}_{IV} design. The optimal PEC sequence is achieved by folding on (added) factor 4 and then by subsetting on one of the factors, 1, 2, 3 or 4, at either their low or high levels. The resulting optimal PEC sequence is $(p_3, p_4, p_5) = (1, 1, 0)$. The PIC sequence is $(d_3, d_4, d_5) = (0.951, 0.858, 0)$.

Example 3.10. Consider the 24-run combined designs. Table A.2 uses the non-isomorphic 16-run initial designs from Chen, Sun and Wu (1993). For example, the notation 7.3.1, 7.3.2,...,7.3.5 in Table A.2 implies that there are five non-isomorphic 2^{7-3} designs. This notation follows that of Chen, Sun and Wu (1993). For $N = 24$ and $n = 7$, the optimal semi-foldover plan is achieved by first using the initial design labeled 7.3.2. We then fold on the three-factor interaction 567, and choose the semi-foldover runs by subsetting on 4, 7, 14, 17 or 27 at either their low or high levels. The PEC sequence of the corresponding design is $(p_3, p_4, p_5, p_6) = (1, 0.971, 0.857, 0)$. We use the PIC criterion to distinguish between designs 7.3.1, 7.3.4 and 7.3.5, which are tied with respect to the PEC criterion. Here, $(d_3, d_4, d_5, d_6) = (1, 0.914, 0.571, 0)$ for designs 7.3.4 and 7.3.5 and $(d_3, d_4, d_5, d_6) = (0.990, 0.885, 0.529, 0)$ for design 7.3.1. All entries in the row corresponding to design 7.3.2 are displayed in bold to indicate their optimality with respect to the PEC criterion.

It is useful to compare the PEC sequences (for our PEC-optimal 24-run combined designs) with the PEC sequences obtained by Loeppky, Sitter and Tang (2007). To construct 24-run designs, Loeppky, Sitter and Tang (2007) developed a procedure for efficiently searching through all design projections arising from a catalog of non-isomorphic Hadamard matrices. (Wu and Hamada (2000, pg. 309) describe a Hadamard matrix as follows: "A Hadamard matrix of order N, denoted by H_N, is an $N \times N$ orthogonal matrix with entries 1 or -1. We can assume without loss of generality that its first column consists of 1's. Then the remaining $N - 1$ columns are orthogonal to the first column and must have half 1's and half -1's". It is also useful to note that for Hadamard matrixes, N is always a multiple of 4.) Table 3.12 compares our five 24-run PEC-optimal combined designs (from Table A.2) with those

Table 3.12: Comparison of Select 24-Run PEC-Optimal Semi-Foldover Plans with Corresponding 24-Run PEC-Optimal Designs from Loeppky, Sitter and Tang (2007)

Design from Loeppky et al.	(p_3, p_4, p_5, p_6)	Our ID	(p_3, p_4, p_5, p_6)
6.1	(1, 1, 1, 1)	6.2.2	(1, 1, 1, 0)
7.1	(1, 1, 1, 1)	7.3.2	(1, 0.971, 0.857, 0)
8.1	(1, 1, 1, 0.786)	8.4.2	(1, 0.957, 0.786, 0)
9.1	(1, 1, 1, 0)	9.5.4	(1, 0.929, 0.643, 0)
10.1	(1, 1, 1, 0)	10.6.3	(1, 0.929, 0.643, 0)

in Table 5 of Loeppky, Sitter and Tang (2007).

By comparing our 24-run PEC-optimal designs with those in Table 5 of Loeppky, Sitter and Tang (2007), we note that all of our semi-foldover plans possess inferior PEC sequences. This is not surprising since Loeppky, Sitter and Tang (2007) begin with a 24-run design whereas we take a 16-run FF and then semi-fold it.

Example 3.11. Consider the 48-run combined designs. Table A.3 in Appendix A.1 lists the non-isomorphic 32-run initial designs from Chen, Sun and Wu (1993). Consider semi-folding an initial 2^{7-2} design. The optimal semi-foldover plan is achieved by using the initial designs labeled 7.2.1 (the MA initial design) or 7.2.6. Note that an identical PEC sequence ($(p_3, p_4, p_5, p_6, p_7) = (1, 1, 1, 1, 1)$) is obtained when beginning with initial designs 7.2.1, 7.2.2, 7.2.4, 7.2.5, 7.2.6 and 7.2.7. Using the PIC criterion as a secondary criterion for distinguishing between the PEC-optimal semi-foldover plans we note that semi-folding the initial designs 7.2.1 and 7.2.6 results in the optimal PIC sequence; namely, $(d_1, d_2, d_3, d_4, d_5, d_6, d_7) = (0.997, 0.990, 0.978, 0.959, 0.930)$. All entries in the rows corresponding to initial designs 7.2.1 and 7.2.6 are displayed in bold to indicate their optimality.

For a given non-isomorphic initial design, the tables in Appendix A.2 lists 12-, 24- and 48-run semi-foldover plans ranked according to the generalized MA criterion. Note that to save space we truncate EWLPs beginning at words of length 6.

Example 3.12. Table A.4 assesses the various 12-run combined designs according to the generalized MA criterion. For example, consider the 2_{IV}^{4-1} initial design. By folding on (added) factor 4 and subsetting on the two-factor interactions, 12, 13 or 14, at either the low or high levels, we obtain the minimum G-aberration 12-run combined design. The corresponding EWLP has entries $(f_1, f_{1.333}, f_{1.667}, f_2, \ldots, f_{4.667}) = (0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 1)$. Therefore, the minimum G-aberration combined designs have 2 words of length 2.667, 1 word of length 4.667, and possess a generalized resolution (R) of 2.667.

Example 3.13. Table A.5 assesses the various 24-run combined designs according to the generalized MA criterion. Consider semi-folding an initial 2^{7-3} design. The MA initial design, 7.3.1, produces the minimum G-aberration 24-run combined design having EWLP $(f_2, f_{2.333}, f_{2.667}, \ldots, f_{5.667}) = (0, 0, 0, 0, 0, 7, 3, 0, 4, 0, 0, 0)$ and $R = 3.667$. This is achieved by folding on either 5, 6, 7, 56, 57, 67 or 567, and then by subsetting on the three-factor interaction 127 at either the low or high levels. All entries in the row corresponding to design 7.3.1 are displayed in bold to indicate that the suggested semi-foldover plans are optimal with respect to the generalized MA criterion.

Example 3.14. Table A.6 assesses the various 48-run combined designs according to the generalized MA criterion. Consider semi-folding an initial 2^{7-2} design. Here, the MA initial design, 7.2.1, does not produce the minimum G-aberration combined design. Rather, the optimal semi-foldover plan is achieved by using the initial design 7.2.3. By folding on effects 6, 7 or 67 and by subsetting on the four-factor interactions 1345 or 1357 at either their low or high levels we obtain the minimum G-aberration 48-run combined design. The optimal combined designs have EWLP (f_3, $f_{3.167}$, $f_{3.333}$, $f_{3.5}$, $f_{3.667}$, $f_{3.833}$, $f_4, \ldots, f_{5.833}$) = (0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 6, 0, 0, 0, 0, 0, 0, 0) and $R = 4$. All entries in the row corresponding to design 7.2.3 are displayed in bold to indicate that the given semi-foldover plans are optimal with respect to the generalized MA criterion.

Chapter 4

Conclusion and Future Work

This thesis has primarily focused upon the selection of semi-foldover plans that have desirable projection properties. To assess such properties we have used the PEC and PIC criteria (Loeppky, Sitter and Tang (2007)). We have also used the generalized MA criterion (Deng and Tang (1999, 2002)) in this thesis to select semi-foldover plans that sequentially minimize the presence of short words in their corresponding EWLPs.

One avenue for future research is to again use the semi-foldover approach for constructing the follow-up runs but rather select the runs using criteria other than PEC, PIC and generalized MA. For example, one might deem an optimal semi-foldover plan to be one that de-aliases the largest number of low-order effects in the initial design. Although the generalized MA criterion will likely perform well according to this criterion, it is unlikely to be optimal in all situations.

An interesting feature of semi-foldover plans is that they may be superior to foldover plans when assessed with respect to the PEC criterion. Examples of such occurrences were noted in Chapter 3. Future research might seek to determine if the projection properties of semi-foldover plans (compared to foldover plans) are frequently superior. If so, this would be another argument in favor of running a semi-foldover plan rather than a foldover plan which requires more runs.

Semi-folding is but one approach proposed in the literature for constructing follow-up runs. Another possibility for future research is to investigate different approaches for constructing the follow-up runs. For example, $\mathcal{D}$-optimal or Bayesian strategies may also be used. Mee and Peralta (2000) highlight the general pros and cons of such competing follow-up strategies, although they do not investigate these competing follow-up strategies in any detail. They conclude that:

1. $\mathcal{D}$-optimal designs can de-alias more low-order effects than semi-foldover designs but are less appealing when taking into account other useful design criteria (for example, robustness to model mis-specification and suitability of further augmentation);

2. A Bayesian follow-up strategy is highly flexible and can take into account multiple design criteria and model uncertainty but is considerably more tedious to implement than semi-folding.

Appendices

Appendix A

Optimality Criteria

A.1 12-, 24- and 48-Run Combined Designs Assessed Sequentially with Respect to the PEC and PIC Criteria

Table A.1: 12-Run Combined Designs Assessed Sequentially with Respect to the PEC and PIC Criteria

Initial design (ID)	Core fo	SS	(p_3, p_4, p_5)	(d_3, d_4, d_5)
4.1	4	$(1,2,3,4)\pm$	(1, 1, 0)	(0.951, 0.858, 0)
5.2	4,5	$(23,24)\pm$	(1, 0.8, 0)	(0.951, 0.686, 0)
6.3	456	$(16)\pm$	(1, 0.8, 0)	(0.951, 0.686, 0)
7.4	45,46,47,56, 57,67,4567	(1,2,3,4, $5,6,7)\pm$	(0.914, 0.571, 0)	(0.869, 0.490, 0)

Note: All 4 designs are MA designs.

Table A.2: 24-Run Combined Designs Assessed Sequentially with Respect to the PEC and PIC Criteria

ID	Core fo	SS	(p_3, p_4, p_5, p_6)	(d_3, d_4, d_5, d_6)
5.1.1*	5	(12,13,14,15, 23,24,25, 34, 35, 45)±	(1, 1, 1, 0)	(0.980, 0.962, 0.943, 0)
5.1.2	5	(12, 13, 15)±	(1, 0.8, 0, 0)	(0.970, 0.766, 0, 0)
5.1.3	**5**	**(134,234,345)±**	**(1, 1, 1, 0)**	**(0.990, 0.968, 0.926, 0)**
6.2.1*	5,6,56	(5, 6, 124, 134)±	(1, 0.933, 0.667, 0)	(0.981, 0.835, 0.330, 0)
6.2.2	**56**	**(23,24,26,35, 45,56)±**	**(1, 1, 1, 0)**	**(0.983, 0.958, 0.912, 0)**
6.2.3	56	(13,14,16,23,24, 26, 35, 45, 56)±	(1, 1, 1, 0)	(0.980, 0.951, 0.904, 0)
7.3.1*	5,6,7,56, 57,67,567	(127)±	(1, 0.914, 0.571, 0)	(0.990, 0.885, 0.529, 0)
7.3.2	**567**	**(4,7,14,17,27)±**	**(1, 0.971, 0.857, 0)**	**(1, 1, 0.857, 0.571)**
7.3.3	567	(13,16,24,27,34, 37, 46, 67)±	(1, 0.943, 0.714, 0)	(1, 0.943, 0.714, 0.143)
7.3.4	5	(13,14,16,17,34, 37, 134, 137)±	(1, 0.914, 0.571, 0)	(1, 0.914, 0.571, 0)
7.3.5	567	(4,14,16,24,34, 45, 46, 47, 146)±	(1, 0.914, 0.571, 0)	(1, 0.914, 0.571, 0)

Note:
1. * denotes the initial MA design
2. Entries in bold depict semi-foldover plans that are optimal with respect to the PEC criterion

(Cont'd) Optimal 24-Run Combined Designs

ID	Core fo	SS	(p_3, p_4, p_5, p_6)	(d_3, d_4, d_5, d_6)
8.4.1*	5678	(4,5,6,7, 8,15)±	(1, 0.914, 0.571, 0)	(0.975, 0.875, 0.521, 0)
8.4.2	**5678**	**(8,18)±**	**(1, 0.957, 0.786, 0)**	**(0.975, 0.912, 0.712, 0)**
8.4.3	5678	(58)±	(1, 0.929, 0.643, 0)	(0.982, 0.889, 0.588, 0)
8.4.4	**567**	**(48)±**	**(1, 0.957, 0.786, 0)**	**(0.975, 0.912, 0.712, 0)**
8.4.5	5678	(48, 78)±	(1, 0.929, 0.643, 0)	(0.986, 0.893, 0.589, 0)
8.4.6	567	(14,24,34,45, 46, 47, 48)±	(1, 0.9, 0.5, 0)	(0.986, 0.865, 0.457, 0)
9.5.1*	567,678	(17)±	(0.988, 0.873, 0.492, 0)	(0.968, 0.836, 0.466, 0.041)
9.5.2	5678	(17)±	(1, 0.921, 0.603, 0)	(0.984, 0.882, 0.551, 0)
9.5.3	5678	(19)±	(1, 0.921, 0.603, 0)	(0.984, 0.882, 0.551, 0)
9.5.4	**56789**	**(38,39,69)±**	**(1, 0.929, 0.643, 0)**	**(0.984, 0.890, 0.588, 0)**
9.5.5	5679	(24,29,34,39, 47, 48)±	(1, 0.921, 0.603, 0)	(0.984, 0.882, 0.551, 0)
10.6.1*	5670,6780	(17)±	(0.983, 0.867, 0.524, 0)	(0.961, 0.826, 0.489, 0.042)
10.6.2	56789	(20, 39)±	(1, 0.924, 0.619, 0)	(0.982, 0.884, 0.565, 0)
10.6.3	**567890**	**(17,19,110, 20,50)±**	**(1, 0.929, 0.643, 0)**	**(0.982, 0.887, 0.586, 0)**
10.6.4	56790	(34, 39, 30, 48)±	(1, 0.924, 0.619, 0)	(0.982, 0.884, 0.565, 0.010)

Table A.3: 48-Run Combined Designs Assessed Sequentially with Respect to the PEC and PIC Criteria

ID	Core fo	SS	$(p_3, p_4, p_5, p_6, p_7)$	$(d_3, d_4, d_5, d_6, d_7)$
6.1*	6	123,124,125,126, 134,135,146,145, 146, 156)±	(1, 1, 1, 1, 0)	(0.995, 0.987, 0.978, 0.968, 0)
7.2.1*	**6, 7**	**(135,137,235,237, 345,347)±**	**(1, 1, 1, 1, 1)**	**(0.997, 0.990, 0.978, 0.959, 0.930)**
7.2.2	67	(124,125,127,134, 135,137,146,156, 167)±	(1, 1, 1, 1, 1)	(0.996, 0.987, 0.974, 0.955, 0.930)
7.2.3	6,7,67	(1345, 1356,)±	(1, 0.971, 0.857, 0.571, 0)	(1, 0.966, 0.838, 0.540)
7.2.4	6,67	(234,235,237,245, 247, 257)±	(1, 1, 1, 1, 1)	(0.996, 0.987, 0.974, 0.955, 0.930)
7.2.5	67	(13,14,15,17,23, 24,25,27,36,46, 56, 67)±	(1, 1, 1, 1, 1)	(0.990, 0.978, 0.959, 0.931, 0.894)
7.2.6	**67**	**(235,245,257,356, 456,567)±**	**(1, 1, 1, 1, 1)**	**(0.997, 0.990, 0.978, 0.959, 0.930)**
7.2.7	67	(135,145,157,235, 245,257,356,456, 567)±	(1, 1, 1, 1, 1)	(0.996, 0.987, 0.974, 0.955, 0.930)
7.2.8	67	(2345, 2357)±	(1, 0.971, 0.857, 0.571, 0)	(0.997, 0.962, 0.841, 0.553, 0.261)

Note:
1. ∗ denotes the initial MA design
2. Entries in bold depict semi-foldover plans that are optimal with respect to the PEC criterion

(Cont'd) Optimal 48-Run Combined Designs

ID	Core fo	SS	$(p_3, p_4, p_5, p_6, p_7)$	$(d_3, d_4, d_5, d_6, d_7)$
8.3.1*	6,7,67, 68,78,678	(125)±	(1, 0.986, 0.929, 0.786)	(0.997, 0.976, 0.908,0.753)
8.3.2	78	(145, 148, 178)±	(1, 0.986, 0.929, 0.786, 0.5)	(0.996, 0.975, 0.906, 0.751, 0.465)
8.3.3	67,68,78	(1345, 1348)±	(1, 0.971, 0.857, 0.571, 0)	(1, 0.966, 0.838, 0.540, 0)
8.3.4	6,7,8,67, 68,78,678	(1258)±	(1, 0.957, 0.786, 0.429, 0)	(1, 0.952, 0.769, 0.405, 0)
8.3.5	**678**	**(145,148,157,178)±**	**(1, 1, 1, 1, 1)**	**(0.996, 0.989, 0.976, 0.957, 0.930)**
8.3.6	67,678	(145, 148, 158)±	(1, 0.986, 0.929, 0.786, 0.5)	(0.996, 0.972, 0.903, 0.7495, 0.465)
8.3.7	678	(148, 248, 456, 468)±	(1, 0.986, 0.929, 0.786, 0.5)	(0.997, 0.976, 0.908, 0.753, 0.465)
8.3.8	**678**	**(145,148,157)±**	**(1, 1, 1, 1, 1)**	**(0.996, 0.989, 0.976, 0.957, 0.930)**
8.3.9	67	(35,38,57,78, 135, 138, 157, 178)±	(1, 0.971, 0.857, 0.571, 0.125)	(0.992, 0.954, 0.829, 0.541, 0.115)
8.3.10	678	(234,235,238, 247, 257, 278)±	(1, 0.986, 0.929, 0.786, 0.5)	(0.996, 0.974, 0.906, 0.753, 0.465)

(Cont'd) Optimal 48-Run Combined Designs

ID	Core fo	SS	$(p_3, p_4, p_5, p_6, p_7)$	$(d_3, d_4, d_5, d_6, d_7)$
9.4.1*	67, 68, 69, 78, 79, 89	(156)±	(1, 0.984, 0.921, 0.762)	(0.998, 0.975, 0.900, 0.729)
9.4.2	6,7,8,67,68, 69,78,79,89, 678,679,689, 789,6789	(15,19,25,29, 35,39,45,49, 57, 58, 79, 89)±	(1, 0.976, 0.881, 0.667, 0.333)	(0.994, 0.963, 0.857, 0.635, 0.309)
9.4.3	67,68,78,689, 789,6789	(169)±	(1, 0.976, 0.881, 0.643, 0.25)	(0.996, 0.966, 0.861, 0.616, 0.234)
9.4.4	89	(135,139,145, 149, 158, 189)±	(1, 0.976, 0.881, 0.667, 0.333)	(0.996, 0.965, 0.859, 0.637, 0.310)
9.4.5	67 68 69 78 79 89 6789	138± 157± 145± 156± 125± 135± 159±	(1, 0.952, 0.762, 0.381, 0)	(0.998, 0.944, 0.747, 0.369, 0)
9.4.6	**6789**	**(148)±**	**(1, 1, 0.976, 0.667, 0.333)**	**(0.981, 0.981, 0.920, 0.315, 0)**
9.4.7	6	(239, 369)±	(1, 0.944, 0.722, 0.333, 0)	(0.998, 0.937, 0.707, 0.320, 0)
9.4.8	678,679,6789	(139)±	(1, 0.992, 0.960, 0.881, 0.722)	(0.996, 0.981, 0.938, 0.843, 0.672)
9.4.9	678,6789	(159)±	(1, 0.976, 0.881, 0.667, 0.333)	(0.995, 0.962, 0.856, 0.636, 0.310)
9.4.10	6789	(56, 69)±	(1, 0.984, 0.921, 0.762, 0.472)	(0.993, 0.968, 0.892, 0.724, 0.437)

(Cont'd) Optimal 48-Run Combined Designs

ID	Core fo	SS	$(p_3, p_4, p_5, p_6, p_7)$	$(d_3, d_4, d_5, d_6, d_7)$
10.5.1*	67,68,69,60,78, 79,70,89,80,90, 678, 679,670,689, 680,690,789,780, 790,890	(12,13,14,15,16, 17,18,19,23,24,26, 27, 28, 34, 37, 45, 46)±	(1, 0.981, 0.905, 0.724)	(0.992, 0.964, 0.877, 0.688)
10.5.2	678,679,689,690, 789,780	(125,128,129, 120, 137)±	(1, 0.976, 0.881, 0.683, 0.25)	(0.996, 0.964, 0.859, 0.614, 0.232)
10.5.3	70,89,789,780, 790,890	(120, 145, 149, 170)±	(1, 0.971, 0.857, 0.590, 0.2)	(0.996, 0.960, 0.836, 0.564, 0.186)
10.5.4	890	(135,136,145, 140, 158, 159)±	(1, 0.971, 0.857, 0.610, 0.267)	(0.997, 0.961, 0.836, 0.582, 0.248)
10.5.5	678 679 670 689 680 690 67890	(29, 69)± (28, 68)± (25, 56)± (27, 67)± (24, 46)± (23, 36)± (20, 60)±	(1, 0.971, 0.857, 0.610, 0.267)	(0.995, 0.958, 0.833, 0.579, 0.246)
10.5.6	67890	(110)±	(1, 0.986, 0.929, 0.781, 0.367)	(0.995, 0.972, 0.901, 0.739, 0.339)
10.5.7	6789 6780 6790	50± 24± 13±	(1, 0.986, 0.929, 0.790, 0.5)	(0.989, 0.964, 0.896, 0.747, 0.461)
10.5.8	**67890**	**(140)±**	**(1, 0.991, 0.952, 0.857, 0.667)**	**(0.997, 0.980, 0.930, 0.820, 0.620)**
10.5.9	67890	(135, 130, 157, 150)±	(1, 0.981, 0.905, 0.724, 0.4)	(0.996, 0.970, 0.883, 0.692, 0.372)
10.5.10	6789	(25,20,35,30,45, 40, 56, 57, 58, 60, 70, 80)±	(1, 0.986, 0.929, 0.791, 0.517)	(0.994, 0.971, 0.901, 0.752, 0.478)

A.2 12-, 24- and 48-Run Combined Designs Ranked According to the Generalized Minimum Aberration Criterion

Table A.4: 12-Run Combined Designs Ranked According to the Generalized Minimum Aberration Criterion

ID	Core fo	SS	R	EWLP $(f_1, f_{1.333}, f_{1.667}, \ldots, f_4, f_{4.333}, f_{4.667})$
4.1	4	$(12, 13, 14)\pm$	2.667	(0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 1)
5.2	45	$(23, 25)\pm$	2.667	(0, 0, 0, 0, 0, 2, 0, 0, 4, 1, 0, 0, 0, 0, 0)
6.3	456	$(16)\pm$	2.667	(0, 0, 0, 0, 0, 3, 0, 0, 8, 3, 0, 0, 0, 0, 0)
7.4	456	(1,2,3, $4, 5, 6, 7)\pm$	1.667	(0, 0, 1, 0, 0, 3, 0, 0, 11, 7, 0, 4, 0, 0, 3)

Note: $(f_1, \ldots)$ denotes EWLPs starting with word length equal to 1

Table A.5: 24-Run Combined Designs Ranked According to the Generalized Minimum Aberration Criterion

ID	Core fo	SS	R	EWLP (f_2, $f_{2.333}$, $f_{2.667}$,..., f_5, $f_{5.333}$, $f_{5.667}$)
5.1.1*	5	(12,13,14,15, 23,24,25,34, 35, 45)±	2.667	(0, 0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 1)
5.1.2	**5**	**(124,134,145)±**	**3.667**	**(0, 0, 0, 0, 0, 2, 0, 0, 1, 0, 0, 0)**
5.1.3	**5**	**(134,234,345)±**	**3.667**	**(0, 0, 0, 0, 0, 2, 0, 0, 1, 0, 0, 0)**
6.2.1*	**5,6,56**	**(124,134)±**	**3.667**	**(0, 0, 0, 0, 0, 4, 1, 0, 2, 0, 0, 0)**
6.2.2	56	(23,24,26,35, 45, 56)±	2.667	(0, 0, 1, 0, 0, 3, 0, 0, 2, 1, 0, 0)
6.2.3	56	(13,14,16,23, 24,26,35, 45, 56)±	2.667	(0, 0, 1, 0, 0, 4, 0, 0, 1, 0, 0, 0)
7.3.1*	**5,6,7,56,57,67,567**	**(127)±**	**3.667**	**(0, 0, 0, 0, 0, 7, 3, 0, 4, 0, 0, 0)**
7.3.2	567	(14, 17)±	2.667	(0, 0, 1, 0, 0, 6, 1, 0, 4, 2, 0, 0)
7.3.3	567	(13, 16)±	2.667	(0, 0, 1, 0, 0, 6, 1, 0, 4, 2, 0, 0)
7.3.4	567	(34, 37, 134, 137)±	2.667	(0, 0, 1, 0, 0, 6, 2, 0, 3, 0, 0, 2)
7.3.5	567	(14,24,34,45, 46, 47, 146)±	2.667	(0, 0, 1, 0, 0, 6, 3, 0, 2, 0, 0, 2)

Note:
1. * denotes the initial MA design
2. (f_2,...) denotes EWLPs starting with word lengths equal to 2 and truncated at f_6
3. For given n and p, entries in bold denote semi-foldover plans that are optimal with respect to the generalized MA criterion

(Cont'd) 24-Run Combined Deisgns Ranked According to the Generalized Minimum Aberration Criterion

ID	Core fo	SS	R	EWLP (f_2, $f_{2.333}$, $f_{2.667}$, …, f_5, $f_{5.333}$, $f_{5.667}$)
8.4.1*	56,57,58,67, 68,78,5678	(12,13,14,15, 16, 17, 18)±	2.667	(0, 0, 4, 0, 0, 0, 6, 0, 16, 0, 0, 0)
8.4.2	567,5678	(18)±	2.667	(0, 0, 1, 0, 0, 10, 7, 0, 4, 0, 0, 4)
8.4.3	5678	(58)±	2.667	(0, 0, 2, 0, 0, 8, 5, 0, 5, 0, 0, 8)
8.4.4	567	(14,18,24,28, 34, 38)±	2.667	(0, 0, 2, 0, 0, 8, 3, 0, 7, 4, 0, 4)
8.4.5	**5678**	**(48,78)±**	**2.667**	**(0, 0, 1, 0, 0, 10, 5, 0, 6, 0, 0, 4)**
8.4.6	567	(14,24,34,45, 46, 47, 48)±	2.667	(0, 0, 1, 0, 0, 10, 7, 0, 4, 0, 0, 4)
9.5.1*	567,578	(17)±	2.667	(0, 0, 4, 1, 0, 7, 5, 0, 17, 6, 0, 10)
9.5.2	5678	(17)±	2.667	(0, 0, 2, 0, 0, 14, 10, 0, 8, 0, 0, 12)
9.5.3	56789	(19)±	2.667	(0, 0, 2, 1, 0, 13, 5, 0, 13, 6, 0, 6)
9.5.4	**56789**	**(38,39,69)±**	**2.667**	**(0, 0, 2, 0, 0, 14, 9, 0, 9, 0, 0, 12)**
9.5.5	5679	(24,29,34, 39, 47, 48)±	2.667	(0, 0, 2, 0, 0, 14, 10, 0, 8, 0, 0, 12)

(Cont'd) 24-Run Combined Designs Ranked According to the Generalized Minimum Aberration Criterion

ID	Core fo	SS	R	EWLP (f_2, $f_{2.333}$, $f_{2.667}$, …, f_5, $f_{5.333}$, $f_{5.667}$)
10.6.1*	5	(13,14,15, 17, 18)±	2.667	(0, 0, 5, 6, 0, 10, 10, 0, 16, 8, 0, 24)
10.6.2	567890	(20, 39)±	2.667	(0, 0, 3, 5, 0, 14, 7, 0, 22, 6, 0, 21)
10.6.3	567890	(18,19,110, 20, 50)±	2.667	(0, 0, 3, 0, 0, 20, 15, 0, 13, 0, 0, 24)
10.6.4	**56890**	**(34,39,30,48)±**	**2.667**	**(0, 0, 3, 0, 0, 19, 16, 0, 13, 0, 0, 27)**

Table A.6: 48-Run Combined Designs Ranked According to the Generalized Minimum Aberration Criterion

ID	Core fo	SS	R	EWLP (f_2, $f_{2.167}$, $f_{2.33}$, $f_{2.5}$, $f_{2.667}$, $f_{2.833}$,..., $f_{5.833}$)
6.1.1*	6	(123,124,125, 126,134,135, 136,145,146, 156)±	3.667	(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
7.2.1*	6,7	(135,137,235, 237, 345, 347)±	3.667	(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 3, 0, 1, 0, 0, 0, 1, 0)
7.2.2	6,7	(124,125,127, 134,135,137, 146, 156, 167)±	3.667	(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 3, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0)
7.2.3	**6,7,67**	**(1345,1357)±**	**4**	**(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 6, 0, 0, 0, 0, 0, 0, 0)**
7.2.4	6	(134,135,137, 245, 247, 257)±	3.667	(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 3, 0, 0, 0, 0, 0, 2, 0, 1, 0, 0, 0, 0, 0)
7.2.5	67	(134,135,137, 234,235,237, 346, 356, 367)±	3.667	(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 3, 0, 0, 0, 0, 0, 3, 0, 0, 0, 0, 0, 0, 0)
7.2.6	67	(135, 145, 157)±	3.667	(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 3, 0, 0, 0, 0, 0, 2, 0, 1, 0, 0, 0, 0, 0)
7.2.7	67	(135,145,157, 235,245,257, 356, 456, 567)±	3.667	(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 3, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 1, 0)
7.2.8	67	(2345, 2357)±	3.667	(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 1, 0, 0, 0, 2, 0, 0, 0, 0, 0, 2, 0)
8.3.1*	**6,7,67, 68,78,678**	**(125)±**	**3.667**	**(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 3, 0, 1, 0, 0, 0, 6, 0, 2, 0, 0, 0, 2, 0)**
8.3.2	78	(145, 148, 178)±	3.667	(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 4, 0, 1, 0, 0, 0, 4, 0, 0, 0, 0, 0, 4, 0)
8.3.3	67,68,78	(1345, 1348)±	4	(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 0, 12, 0, 0, 0, 0, 0, 0, 0)

Note:
1. $*$ denotes the initial MA design
2. (f_2,...) denotes EWLPs starting with word lengths equal to 2 and truncated at f_6
3. For given n and p, entries in bold denote semi-foldover plans that are optimal with respect to the generalized MA criterion

(Cont'd) 48-Run Combined Designs Ranked According to the Generalized Minimum Aberration Criterion

ID	Core fo	SS	R	EWLP (f_2, $f_{2.167}$, $f_{2.33}$, $f_{2.5}$, $f_{2.667}$, $f_{2.833}$, …, $f_{5.833}$)
8.3.4	**6,7,8,67, 68,78,678**	**(12580±**	**4**	**(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 3, 0, 0, 0, 11, 0, 0, 0, 0, 0, 0, 0**
8.3.5	678	(145, 148, 157, 178)±	3.667	(0, 0, 0, 0, 0, 0, 0, 0, 0, 4, 0, 0, 0, 0, 0, 5, 0, 2, 0, 0, 0, 2, 0)
8.3.6	67,678	(145, 148, 158)±	3.667	(0, 0, 0, 0, 0, 0, 0, 0, 0, 5, 0, 1, 0, 0, 0, 4, 0, 2, 0, 0, 0, 0, 0)
8.3.7	67,68	(148, 248, 456, 468)±	3.667	(0, 0, 0, 0, 0, 0, 0, 0, 0, 3, 0, 1, 0, 0, 0, 6, 0, 1, 0, 0, 0, 3, 0)
8.3.8	678	(245, 248, 257)±	3.667	(0, 0, 0, 0, 0, 0, 0, 0, 0, 4, 0, 0, 0, 0, 0, 5, 0, 2, 0, 0, 0, 2, 0)
8.3.9	678	(35,38,57,78, 135, 138, 157, 178)±	2.667	(0, 0, 0, 0, 1, 0, 0, 0, 0, 3, 0, 1, 0, 0, 0, 4, 0, 1, 0, 0, 0, 3, 0)
8.3.10	678	(234,235,238, 247, 257, 278)±	3.667	(0, 0, 0, 0, 0, 0, 0, 0, 0, 4, 0, 1, 0, 0, 0, 5, 0, 2, 0, 0, 0, 2, 0)
9.4.1*	**67,68,69, 78,79,89**	**(156)±**	**3.667**	**(0, 0, 0, 0, 0, 0, 0, 0, 0, 4, 0, 2, 0, 0, 0, 12, 0, 4, 0, 0, 0, 4, 0**
9.4.2	9	(15,19,15,29, 35,39,45,49, 56,57;58,67, 79, 89)±	2.667	(0, 0, 0, 0, 1, 0, 0, 0, 0, 3, 0, 7, 0, 0, 0, 4, 0, 0, 0, 0, 0, 11, 0)
9.4.3	67,69,78, 689,789,6789	(169)±	3.667	(0, 0, 0, 0, 0, 0, 0, 0, 0, 6, 0, 3, 0, 0, 0, 6, 0, 0, 0, 0, 0, 9, 0)
9.4.4	89	(135,139,145, 149, 158, 189)±	3.667	(0, 0, 0, 0, 0, 0, 0, 0, 0, 6, 0, 3, 0, 0, 0, 7, 0, 0, 0, 0, 0, 8, 0)
9.4.5	67,68,69, 78,79,89, 6789	(125,135,145,156, 157, 158, 159)±	3.667	(0, 0, 0, 0, 0, 0, 0, 0, 0, 4, 0, 6, 0, 0, 0, 8, 0, 0, 0, 0, 0, 8, 0)
9.4.6	67,68,69,678, 689,789	(148)±	3.667	(0, 0, 0, 0, 0, 0, 0, 0, 0, 8, 0, 3, 0, 0, 0, 7, 0, 2, 0, 0, 0, 2, 0)

(Cont'd) 48-Run Combined Designs Ranked According to the Generalized Minimum Aberration Criterion

ID	Core fo	SS	R	EWLP $(f_2, f_{2.167}, f_{2.33}, f_{2.5}, f_{2.667}, f_{2.833}, \ldots, f_{5.833})$
9.4.7	6	(239)±	3.667	(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 4, 0, 7, 0, 0, 0, 7, 0, 0, 0, 0, 0, 8, 0)
9.4.8	678,679,6789	(139)±	3.667	(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 6, 0, 1, 0, 0, 0, 8, 0, 4, 0, 0, 0, 5, 0)
9.4.9	6789	(159)±	3.667	(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 8, 0, 3, 0, 0, 0, 7, 0, 0, 0, 0, 0, 4, 0)
9.4.10	6789	(158, 189)±	3.667	(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 7, 0, 2, 0, 0, 0, 7, 0, 3, 0, 0, 0, 3, 0)
10.5.1*	6,7,8,9,0, 6789,6780,6790, 6890,7890	(12,13,14,15, 16,17,18,19, 23,24,25,26, 27,28,34,35, 36, 37, 45, 46)±	2.667	(0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 4, 0, 6, 0, 0, 0, 10, 0, 8, 0, 0, 0, 16, 0)
10.5.2	678,679,689, 689,789,780	(125,128,129, 120, 137)±	3.667	(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 10, 0, 5, 0, 0, 0, 10, 0, 0, 0, 0, 0, 12, 0)
10.5.3	70,89,789, 780,790,890	(120,145,149, 170)±	3.667	(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 9, 0, 6, 0, 0, 0, 10, 0, 0, 0, 0, 0, 15, 0)
10.5.4	890	(135,136,145, 140, 158)±	3.667	(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 8, 0, 6, 0, 0, 0, 12, 0, 0, 0, 0, 0, 16, 0)
10.5.5	678,679,670, 689,680,690, 67890	(23,24,25,27, 28,29,20,36, 46,56,67,68, 69, 60)±	2.667	(0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 5, 0, 6, 0, 0, 0, 15, 0, 4, 0, 0, 0, 11, 0)
10.5.6	789	(110)±	2.667	(0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 4, 0, 3, 0, 0, 0, 18, 0, 4, 0, 0, 0, 11, 0)
10.5.7	678,679,670, 6789,6780,6790, 67890	(14,17,23,27, 36, 46)±	2.667	(0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 8, 0, 3, 0, 0, 0, 11, 0, 6, 0, 0, 0, 10, 0)
10.5.8	**67890**	**(149)±**	**3.667**	**(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 8, 0, 2, 0, 0, 0, 16, 0, 8, 0, 0, 0, 8, 0)**
10.5.9	67890	(235, 230, 257, 250)±	3.667	(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 9, 0, 4, 0, 0, 0, 12, 0, 6, 0, 0, 0, 9, 0)
10.5.10	6789	(25,20,35,30, 45,40,56,57, 58, 60, 70, 80)±	2.667	(0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 7, 0, 3, 0, 0, 0, 12, 0, 7, 0, 0, 0, 12, 0)

Appendix B

R Code

Sample Code: R Code for Ranking an Initial MA 2^{4-1}_{IV} Design According to the PEC, PIC and Generalized Minimum Aberration Criteria

```
getPECandPIC<-function(n_f,p,d,n_r,m_r,k3,k4,k5) {
getsemi<-function(n_f,p,d,n_r,m_r,k3,k4,k5) {
    x1<-c(-1,1,-1,1,-1,1,-1,1,0,0,0,0)
    x2<-c(-1,-1,1,1,-1,-1,1,1,0,0,0,0)
    x3<-c(-1,-1,-1,-1,1,1,1,1,0,0,0,0)
    getdesign<-function(n_f,p,d,n_r,m_r,k3,k4,k5){
        getnonadd<-function(n_f,p,d,n_r,m_r,k3,k4,k5){
            x<-vector(length=(m_r*d))
            X<-matrix(data=x, nrow=m_r, ncol=d, byrow=TRUE)
            getcol<-function(a) {
                c<-vector(length=m_r)
                if(a==1) {
                    c=x1 }
                if(a==2) {
                    c=x2}
                if(a==3) {
                    c=x3 }
            c
            }
            for(i in 1:d) {
                if(i<=(n_f-p)){
                    c<-getcol(i)
                    X[,i]<-c
                }
            }
        X
        }
        Y<-getnonadd(n_f,p,d,n_r,m_r,k3,k4,k5)
        Y
        y1<-vector(length=m_r)
```

```
y2<-vector(length=m_r)
y3<-vector(length=m_r)
y4<-vector(length=m_r)
y5<-vector(length=m_r)
y6<-vector(length=m_r)
y7<-vector(length=m_r)
for(i in 1:d){
    if(i==1){
        y1<-Y[,1] }
    if(i==2){
        y2<-Y[,2]}
    if(i==3) {
        y3<-Y[,3]}
    if(i==4) {
        y4<-Y[,4]}
    if(i==5) {
        y5<-Y[,5]}
    if(i==6){
        y6<-Y[,6]}
    if(i==7){
        y7<-Y[,7]}
}
addgen<-function(n_f,p,d,n_r,m_r,g1,g2,g3,g4,g5,g6,g7){
    getgen<-function(b) {
        g<-vector(length=n_r)
        if(b==1){
            g<-g1}
        if(b==2) {
            g<-g2}
        if(b==3){
            g<-g3}
        if(b==4){
            g<-g4}
        if(b==5){
            g<-g5}
        if(b==6){
            g<-g6}
        if(b==7){
            g<-g7}
    g
    }
    for(i in 1:d){
        if(i>(n_f-p)&& i<=n_f){
            c<-getgen(i)
            Y[,i]<-c
        }
    }
    Y
}
Z1<-addgen(n_f,p,d,n_r,m_r,g1,g2,g3,y1*y2*y3,g5,g6,g7)
y1<-vector(length=m_r)
y2<-vector(length=m_r)
y3<-vector(length=m_r)
y4<-vector(length=m_r)
y5<-vector(length=m_r)
```

```
    y6<-vector(length=m_r)
    y7<-vector(length=m_r)
    for(i in 1:d) {
        if(i==1) {
            y1<-Z1[,1] }
        if(i==2) {
            y2<-Z1[,2]}
        if(i==3) {
            y3<-Z1[,3]}
        if(i==4) {
            y4<-Z1[,4]}
        if(i==5){
            y5<-Z1[,5]}
        if(i==6){
            y6<-Z1[,6]}
        if(i==7){
            y7<-Z1[,7]}
    }
    addint<-function(n_f,p,d,n_r,m_r,g1,g2,g3,g4,g5,g6,g7){
        y<-vector(length=m_r*d)
        Y<-matrix(data=y, nrow=m_r, ncol=d, byrow=TRUE)
        getint<-function(b){
            g<-vector(length=m_r)
            if(b==1){
                g<-g1}
            if(b==2){
                g<-g2}
            if(b==3){
                g<-g3}
            if(b==4){
                g<-g4}
            if(b==5){
                g<-g5}
            if(b==6){
                g<-g6}
            if(b==7) {
               g<-g7}
        g
        }
        for(i in 1:d){
            if(i>n_f && i<=d){
                c<-getint(i)
                Y[,i]<-c
            }
        }
        Y
    }
    Z2<-addint(n_f,p,d,n_r,m_r,g1,g2,g3,g4,y1*y2,y1*y3,y1*y4)
    Z<- Z1 $+$ Z2
    Z
}
getsemidesignoneadded<-function(n_f,p,d,n_r,m_r,k3,k4,k5,g){
        getnonadd<-function(n_f,p,d,n_r,m_r,k3,k4,k5,g){
            x<-vector(length=(m_r*d))
            X<-matrix(data=x, nrow=m_r, ncol=d, byrow=TRUE)
```

```
    getcol<-function(a){
        c<-vector(length=m_r)
        if(a==1){
            c=x1}
        if(a==2){
            c=x2}
        if(a==3){
            c=x3}
    c
    }
    for(i in 1:d){
        if(i<=(n_f-p)){
            c<-getcol(i)
            X[,i]<-c
        }
    }
X
}
Y<-getnonadd(n_f,p,d,n_r,m_r,k3,k4,k5,g)
Y
y1<-vector(length=m_r)
y2<-vector(length=m_r)
y3<-vector(length=m_r)
y4<-vector(length=m_r)
y5<-vector(length=m_r)
y6<-vector(length=m_r)
y7<-vector(length=m_r)
for(i in 1:d){
    if(i==1){
        y1<-Y[,1]}
    if(i==2){
        y2<-Y[,2]}
    if(i==3){
        y3<-Y[,3]}
    if(i==4){
        y4<-Y[,4]}
    if(i==5){
        y5<-Y[,5]}
    if(i==6){
        y6<-Y[,6]}
    if(i==7){
        y7<-Y[,7]}
}
addgen<-function(n_f,p,d,n_r,m_r,g1,g2,g3,g4,g5,g6,g7){
    getgen<-function(b){
        g<-vector(length=n_r)
        if(b==1){
            g<-g1}
        if(b==2){
            g<-g2}
        if(b==3){
            g<-g3}
        if(b==4){
            g<-g4}
        if(b==5){
```

```
                g<-g5}
            if(b==6){
                g<-g6}
            if(b==7){
                g<-g7}
        g
        }
        for(i in 1:d){
            if(i>(n_f-p)&& i<=n_f){
                c<-getgen(i)
                Y[,i]<-c
            }
        }
        Y[,g]<-(-1*Y[,g])
        Y
    }
Z1<-addgen(n_f,p,d,n_r,m_r,g1,g2,g3,y1*y2*y3,g5,g6,g7)
    y1<-vector(length=m_r)
    y2<-vector(length=m_r)
    y3<-vector(length=m_r)
    y4<-vector(length=m_r)
    y5<-vector(length=m_r)
    y6<-vector(length=m_r)
    y7<-vector(length=m_r)
    for(i in 1:d){
        if(i==1){
            y1<-Z1[,1]}
        if(i==2){
            y2<-Z1[,2]}
        if(i==3) {
            y3<-Z1[,3]}
       if(i==4){
            y4<-Z1[,4]}
        if(i==5){
            y5<-Z1[,5]}
        if(i==6){
            y6<-Z1[,6]}
        if(i==7){
            y7<-Z1[,7]}
    }
    addint<-function(n_f,p,d,n_r,m_r,g1,g2,g3,g4,g5,g6,g7){
        y<-vector(length=m_r*d)
        Y<-matrix(data=y, nrow=m_r, ncol=d, byrow=TRUE)
        getint<-function(b) {
            g<-vector(length=m_r)
            if(b==1){
                g<-g1}
            if(b==2){
                g<-g2}
            if(b==3){
                g<-g3}
            if(b==4){
                g<-g4}
            if(b==5){
                g<-g5}
```

```
                if(b==6){
                    g<-g6}
                if(b==7){
                    g<-g7}
            g
            }
            for(i in 1:d){
                if(i>n_f && i<=d){
                    c<-getint(i)
                    Y[,i]<-c
                }
            }
            Y
        }
   Z2<-addint(n_f,p,d,n_r,m_r,g1,g2,g3,g4,y1*y2,y1*y3,y1*y4)
        Z<- Z1 $+$ Z2
        Z
}
Y<-getdesign(n_f,p,d,n_r,m_r,k3,k4,k5)
count<-1
makesemi<-function(n_f,p,d,n_r,m_r,k3,k4,k5){
s<-vector(length=((2^p-1)*2*d*m_r*d))
S<-matrix(data=s, nrow=((2^p-1)*2*d*m_r), ncol=d, byrow=TRUE)
    for(g in (n_f-p$+$1):n_f){
        for(m in 1:d) {
                Y<-getdesign(n_f,p,d,n_r,m_r,k3,k4,k5)
                X<-getsemidesignoneadded(n_f,p,d,n_r,m_r,k3,k4,k5,g)
                    for(j in 1:n_r){
                        S[count,]<-Y[j,]
                        count<-count$+$1
                    }
                    l<-1
                for(r in (n_r$+$1):m_r){
                    while(X[l,m]!=1){
                        l<-l$+$1
                    }
                    Y[r,]<-X[l,]
                    S[count,]<-Y[r,]
                    count<-count$+$1
                    l<-l$+$1
                }
        }
        for(m in 1:d){
                Y<-getdesign(n_f,p,d,n_r,m_r,k3,k4,k5)
                X<-getsemidesignoneadded(n_f,p,d,n_r,m_r,k3,k4,k5,g)
            l<-1
                    for(j in 1:n_r){
                        S[count,]<-Y[j,]
                        count<-count$+$1
                    }
                for(r in (n_r$+$1):m_r){
                    while(X[l,m]==1){
                        l<-l$+$1
                    }
                    Y[r,]<-X[l,]
```

```
                    S[count,]<-Y[r,]
                    count<-count$+$1
                    l<-l$+$1
                }
            }
            F<-F$+$1
        }
S
}
S<-makesemi(n_f,p,d,n_r,m_r,k3,k4,k5)
}
    S<-getsemi(n_f,p,d,n_r,m_r,k3,k4,k5)
    X_Allinitals1<-S[,1:d]
    X_Allinitals<-S[,1:n_f]
    X_inital<-matrix(0,m_r,n_f)
    M_d<-matrix(0,((2^p-1)*2*(2^(n_f-p)-1)),1)
    Pk<-matrix(0,((2^p-1)*2*(2^(n_f-p)-1)),1)
    determine_2<-matrix(0,((2^p-1)*2*(2^(n_f-p)-1)),1)
    p_p3=1$+$k3$+$choose(k3,2)
    p_p4=1$+$k4$+$choose(k4,2)
    p_p5=1$+$k5$+$choose(k5,2)
    nck3=choose(n_f,k3)
    nck4=choose(n_f,k4)
    nck5=choose(n_f,k5)
    determine3<-matrix(0,nck3,1)
    determine3_1<-matrix(0,nck3,1)
    determine4<-matrix(0,nck4,1)
    determine4_1<-matrix(0,nck4,1)
    determine5<-matrix(0,nck5,1)
    determine5_1<-matrix(0,nck5,1)
    x0=matrix(1,m_r,1)
    for(t in 1:((2^p-1)*2*(2^(n_f-p)-1))){
        Y_nck3<-matrix(0,(m_r*nck3),k3)
        Y_kc23<-matrix(0,(m_r*nck3),choose(k3,2))
        Y3<-matrix(0,(m_r*nck3),p_p3)
        Y_nck4<-matrix(0,(m_r*nck4),k4)
        Y_kc24<-matrix(0,(m_r*nck4),choose(k4,2))
        Y4<-matrix(0,(m_r*nck4),p_p4)
        Y_nck5<-matrix(0,(m_r*nck5),k5)
        Y_kc25<-matrix(0,(m_r*nck5),choose(k5,2))
        Y5<-matrix(0,(m_r*nck5),p_p5)
        Dk=0
        pk=0
        count=0
        X_inital1<-X_Allinitals1[(((t-1)*m_r$+$1):(t*m_r)),]
        X_inital<-X_Allinitals[(((t-1)*m_r$+$1):(t*m_r)),]
        print(X_inital1)
        t2=1
        if(k3==3){
            for(i in 1:(n_f-2)){
                for(j in (i$+$1):(n_f-1)){
                    for(b in (j$+$1):n_f){
                        aa1=(t2-1)*m_r$+$1
                        aa2=t2*m_r
                        Y_nck3[(aa1:aa2),] <- array(c(X_inital[,i],X_inital[,j],X_inital[,b]),dim=c(m_r,k3))
```

```
                        Y3[(aa1:aa2),(1:(k3$+$1))] <- array(c(x0,Y_nck3[(aa1:aa2),]),dim=c(m_r,k3$+$1))
                        t1=1
                        for(i_1 in 1:(k3-1)) {
                            for(j_1 in (i_1$+$1):k3){
                                Y_kc23[(aa1:aa2),t1]<-array(c(Y3_nck[(aa1:aa2),i_1]*Y3_nck[(aa1:aa2),j_1]),
                                dim=c(m_r,1))
                                Y3[(aa1:aa2),((k3$+$1)$+$t1)] <- array(c(Y_kc23[(aa1:aa2),t1]),dim=c(m_r,1))
                                t1=t1$+$1
                            }
                        }
                        determine3[t2,1]<-det(t(Y3[(aa1:aa2),])%*%Y3[(aa1:aa2),]/(m_r))
                        determine3_1[t2,1]<-det(t(Y3[(aa1:aa2),])%*%Y3[(aa1:aa2),])
                        Dk<-Dk$+$((determine3[t2,1]^(1/p_p3))/nck3)
                        if(determine3_1[t2,1]!=0){
                            count=count$+$1
                        }
                           pk<-(count/choose(n_f,k3))
                        t2=t2$+$1
                    }
                }
            }
        MAKE=matrix(0,((2^p-1)*2*(2^(n_f-p)-1)),12)
count1=0 count11=0 count12=0 count2=0 count21=0 count22=0 count3=0
count31=0 count32=0  count4=0 count41=0 count42=0 CFV1=0 CFV11=0
CFV12=0  CFV2=0 CFV21=0 CFV22=0 CFV3=0 CFV31=0 CFV32=0  CFV4=0
CFV41=0 CFV42=0 t11=1 t22=1 t3=1 t4=1
X_nc31<-matrix(0,m_r*choose(n_f,1),1)
X_nc32<-matrix(0,m_r*choose(n_f,2),1)
X_nc33<-matrix(0,m_r*choose(n_f,3),1)
X_nc34<-matrix(0,m_r*choose(n_f,4),1)
X_sum1<-matrix(0,choose(n_f,1),1) X_sum2<-matrix(0,choose(n_f,2),1)
X_sum3<-matrix(0,choose(n_f,3),1) X_sum4<-matrix(0,choose(n_f,4),1)
     for(i in 1:n_f){
        aa1=(t11-1)*m_r$+$1
        aa2=t11*m_r
        X_nc31[(aa1:aa2),]<- X_inital[1:m_r,i]
        X_sum1[t11,]<-sum(X_nc31[(aa1:aa2),])
        if(abs(X_sum1[t11,])==m_r){
            count1=count1$+$1}
        if(abs(X_sum1[t11,])==m_r-4){
            count11=count11$+$1}
        if(abs(X_sum1[t11,])==m_r-8){
            count12=count12$+$1}
        CFV1<- count1
        CFV11<- count11
        CFV12<- count12
            t11=t11$+$1
     }
     for(i in 1:(n_f-1)) {
        for(j in (i$+$1):n_f){
        aa1=(t22-1)*m_r$+$1
        aa2=t22*m_r
        X_nc32[(aa1:aa2),]<- array(c(X_inital[1:m_r,i]*X_inital[1:m_r,j]),dim=c(m_r,1))
        X_sum2[t22,]<-sum(X_nc32[(aa1:aa2),])
        if(abs(X_sum2[t22,])==m_r){
```

```
        count2=count2$+$1}
    if(abs(X_sum2[t22,])==m_r-4){
        count21=count21$+$1}
    if(abs(X_sum2[t22,])==m_r-8){
        count22=count22$+$1}
    CFV2<- count2
    CFV21<- count21
    CFV22<- count22
     t22=t22$+$1
    }
}
 for(i in 1:(n_f-2)){
    for(j in (i$+$1):(n_f-1)){
        for(b in (j$+$1):n_f){
            aa1=(t3-1)*m_r$+$1
            aa2=t3*m_r
            X_nc33[(aa1:aa2),]<- array(c(X_inital[1:m_r,i]*X_inital[1:m_r,j]*X_inital[1:m_r,b]),
            dim=c(m_r,1))
            X_sum3[t3,]<-sum(X_nc33[(aa1:aa2),])
                if(abs(X_sum3[t3,])==m_r){
                    count3=count3$+$1}
                if(abs(X_sum3[t3,])==m_r-4){
                    count31=count31$+$1}
                if(abs(X_sum3[t3,])==m_r-8){
                    count32=count32$+$1}
                CFV3<- count3
                CFV31<- count31
                CFV32<- count32
               t3=t3$+$1
        }
    }
}
for(a in 1:(n_f-3)){
        for(i in (a$+$1):(n_f-2)){
                for(j in (i$+$1):(n_f-1)){
                for(b in (j$+$1):n_f){
                aa1=(t4-1)*m_r$+$1
                aa2=t4*m_r
                X_nc34[(aa1:aa2),]<- array(c(X_inital[1:m_r,a]*X_inital[1:m_r,i]*X_inital[1:m_r,j]
                *X_inital[1:m_r,b]),dim=c(m_r,1))
                X_sum4[t4,]<-sum(X_nc34[(aa1:aa2),])
                if(abs(X_sum4[t4,])==m_r){
                count4=count4$+$1}
                if(abs(X_sum4[t4,])==m_r-4){
                    count41=count41$+$1}
                if(abs(X_sum4[t4,])==m_r-8){
                    count42=count42$+$1}
                CFV4<- count4
                CFV41<- count41
                CFV42<- count42
                t4=t4$+$1
                }
            }
        }
    }
```

```
    }
MAKE=matrix(c(CFV1,CFV11,CFV12,CFV2,CFV21,CFV22,CFV3,CFV31,CFV32,CFV4,CFV41,CFV42),1,12)
write(MAKE,file="2^(4-1)EWLP.txt",12,(2^p-1)*2*(2^(n_f-p)-1))
          }
    if(k4==4){
        for(a in 1:(n_f-3)){
            for(i in (a$+$1):(n_f-2)){
                  for(j in (i$+$1):(n_f-1)){
                    for(b in (j$+$1):n_f){
                            aa1=(t2-1)*m_r$+$1
                            aa2=t2*m_r
                            Y_nck4[(aa1:aa2),] <- array(c(X_inital[,a],X_inital[,i],X_inital[,j],
                            X_inital[,b]),dim=c(m_r,k4))
                            Y4[(aa1:aa2),(1:(k4$+$1))] <- array(c(x0,Y_nck4[(aa1:aa2),]),dim=c(m_r,k4$+$1))
                            t1=1
                            for(i_1 in 1:(k4-1)){
                                for(j_1 in (i_1$+$1):k4){
                                    Y_kc24[(aa1:aa2),t1]<-array(c(Y_nck4[(aa1:aa2),i_1]*Y_nck4[(aa1:aa2),j_1]),
                                    dim=c(m_r,1))
                                    Y4[(aa1:aa2),((k4$+$1)$+$t1)] <- array(c(Y_kc24[(aa1:aa2),t1]),dim=c(m_r,1))
                                    t1=t1$+$1
                                }
                            }
                            determine4[t2,1]<-det(t(Y4[(aa1:aa2),])%*%Y4[(aa1:aa2),]/(m_r))
                            determine4_1[t2,1]<-det(t(Y4[(aa1:aa2),])%*%Y4[(aa1:aa2),])
                            Dk<-Dk$+$((determine4[t2,1]^(1/p_p4))/nck4)
                            if(determine_14[t2,1]!=0){
                                count=count$+$1}
                          pk<-(count/choose(n_f,k4))
                            t2=t2$+$1
                    }
                }
            }
        }
    }
    if(k5==5){
        for(h in 1:(n_f-4)){
            for(a in (h$+$1):(n_f-3)){
                for(i in (a$+$1):(n_f-2)){
                    for(j in (i$+$1):(n_f-1)){
                        for(b in (j$+$1):n_f){
                            aa1=(t2-1)*m_r$+$1
                            aa2=t2*m_r
                            Y_nck5[(aa1:aa2),] <- array(c(X_inital[,h],X_inital[,a],X_inital[,i],X_inital[,j]
                            ,X_inital[,b]),dim=c(m_r,k5))
                            Y5[(aa1:aa2),(1:(k$+$1))] <- array(c(x0,Y_nck5[(aa1:aa2),]),dim=c(m_r,k5$+$1))
                            t1=1
                            for(i_1 in 1:(k5-1)){
                                for(j_1 in (i_1$+$1):k5){
                                    Y_kc25[(aa1:aa2),t1]<-array(c(Y_nck5[(aa1:aa2),i_1]*Y_nck5[(aa1:aa2),j_1]),
                                    dim=c(m_r,1))
                                    Y5[(aa1:aa2),((k5$+$1)$+$t1)] <- array(c(Y_kc25[(aa1:aa2),t1]),dim=c(m_r,1))
                                    t1=t1$+$1
                                }
                            }
```

```
                        determine5[t2,1]<-det(t(Y5[(aa1:aa2),])%*%Y5[(aa1:aa2),]/(m_r))
                        determine_15[t2,1]<-det(t(Y5[(aa1:aa2),])%*%Y5[(aa1:aa2),])
                        Dk<-Dk$+$((determine5[t2,1]^(1/p_p5))/nck5)
                        if(determine5_1[t2,1]!=0){
                            count=count$+$1}
                            pk<-(count/choose(n_f,k5))
                        t2=t2$+$1
                    }
                }
            }
        }
    }
  }
}
#####type in getPECandPIC(4,1,7,8,12,3,4,5)######
```

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