

TRANSISTOR NOISE IN THE VERY  
HIGH FREQUENCY (VHF) RANGE

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by  
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## ABSTRACT

The purpose of this thesis is to investigate the validity of present noise theory of drift transistors at the frequency of 100 MHz., using the hybrid- $\Pi$  model in a common emitter configuration.

The thesis contains the derivation of the noise figure for the hybrid- $\Pi$  model, taking into account the cross-correlation between the noise generators. The derivation is based on VAN DER ZIEL's definition of the noise generators for the T model representation. (Ref.# 3).

## ACKNOWLEDGEMENTS

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To my brother, Ilyas

## TABLE OF CONTENTS

CHAPTER		PAGE
I	INTRODUCTION. . . . .	1
	An Introduction to the Theory of Noise in Transistors . .	1
	Noise Sources in Transistors. . . . .	2
	Derivation of the Hybrid- $\pi$ Noise Model. . . . .	4
II	EQUIVALENT NOISE RESISTANCE OF A TRANSISTOR IN A COMMON EMITTER CONFIGURATION . . . . .	8
III	EXPERIMENTAL PROCEDURE. . . . .	16
	Noise Jig . . . . .	16
	Noise Measurement Method. . . . .	17
	Correction for Background Noise . . . . .	19
	Calculation of Intrinsic $g_m$ from Extrinsic $y_m$ Measurements. . . . .	20
IV	EXPERIMENTAL RESULTS. . . . .	21
V	CONCLUSION. . . . .	30
	BIBLIOGRAPHY . . . . .	31

## LIST OF FIGURES

FIGURE		PAGE
1.	REPRESENTATION OF RESISTOR THERMAL NOISE. . . . .	2
2.	REPRESENTATION OF A COMMON-EMITTER TRANSISTOR MODEL . . .	4
3.	HYBRID- $\pi$ MODEL OF A TRANSISTOR. . . . .	5
4.	HIGH FREQUENCY HYBRID- $\pi$ MODEL . . . . .	6
5.	REPRESENTATION OF A T MODEL WITH NOISE CURRENT GENERATORS SUPERIMPOSED. . . . .	8
6.	REPRESENTATION OF A HYBRID- $\pi$ MODEL WITH NOISE CURRENT GENERATORS SUPERIMPOSED.. . . .	8
7.	REPRESENTATION OF A HYBRID- $\pi$ MODEL WITH NOISE GENERATORS SUPERIMPOSED. . . . .	10
8.	TRANSFORMATION OF NOISE CURRENT GENERATOR $i_b$ TO THE INPUT . . . . .	11
9.	TRANSFORMATION OF NOISE CURRENT GENERATOR $i_c$ TO THE INPUT . . . . .	11
10.	TRANSFORMATION OF NOISE CURRENT GENERATOR $i_a$ TO THE INPUT . . . . .	12
11.	FICTITIOUS NOISE GENERATORS OF A TRANSISTOR . . . . .	13
12.	NOISE JIG . . . . .	16
13.	REGULATOR . . . . .	17
14.	BLOCK DIAGRAM OF THE NOISE MEASURING APPARATUS. . . . .	18
15.	MEASURED TRANSADMITTANCE ( $y_m$ ) for $I_C = 100\mu A$ . . . . .	21
16.	MEASURED TRANSADMITTANCE ( $y_m$ ) for $I_C = 300\mu A$ . . . . .	22
17.	MEASURED TRANSADMITTANCE ( $y_m$ ) for $I_C = 1 \text{ mA}$ . . . . .	23

## LIST OF FIGURES (Continued...)

FIGURE	PAGE
18. MEASURED INPUT IMPEDANCE FOR $I_C = 100\mu A$ . . . . .	24
19. MEASURED INPUT IMPEDANCE FOR $I_C = 300\mu A$ . . . . .	25
20. MEASURED INPUT IMPEDANCE FOR $I_C = 1 \text{ mA}$ . . . . .	26
21. CONTOURS OF CONSTANT NOISE FIGURE FOR 2N917 TRANSISTOR ( $I_C = 1 \text{ mA}$ ). . . . .	27
22. CONTOURS OF CONSTANT NOISE FIGURE FOR $I_C = 300\mu A$ . . . .	28
23. CONTOURS OF CONSTANT NOISE FIGURE FOR $I_C = 100\mu A$ . . . .	29

## CHAPTER I

### INTRODUCTION

Electrical noise is due to the discrete nature of electron flow and to the randomness of particle motion. This electrical noise can be divided into two groups: (1) noise caused by man (which can be controlled); and, (2) noise due to spontaneous fluctuations (which is beyond control). Spontaneous fluctuations of stationary nature can be described by two basic theorems, namely Nyquist's and Schottky's theorems.

Nyquist's Theorem:<sup>6</sup> In a conductor, the random vibration of ions about a normal or average position is a function of temperature. There is a continuous energy transfer due to collisions between the vibrating ions and the free electrons. Even though the average current is zero, random fluctuations still exist. At temperature  $T$ , the thermal noise of a purely resistive circuit can be represented by noise generators. In a frequency range  $\Delta f$ , the generators deliver a maximum power ( $P$ ) (called the available power) of

$$P = kT\Delta f \quad \dots (1)$$

where  $k$  is Boltzmann's constant,  $T$  is the temperature of the impedance in degrees Kelvin, and  $P$  is the power in watts. It was shown by Nyquist that the mean-square thermal noise voltage  $\overline{e_s^2}$  generated by an impedance  $Z$  is given by:

$$\overline{e_s^2} = 4kTR\Delta f \quad \dots (2)$$

where  $R$  is the resistive part of the impedance  $Z$  in ohms. This noise generator may be represented by a Norton or a Thevenin-equivalent circuit:

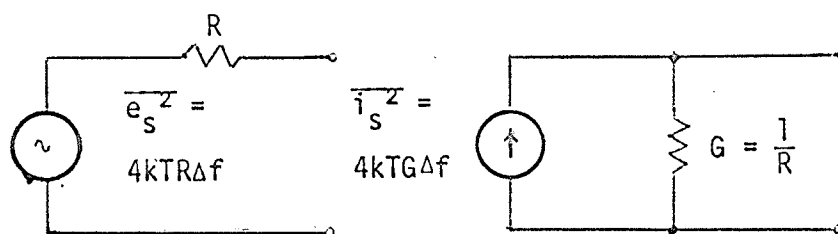


Figure 1

## REPRESENTATION OF RESISTOR THERMAL NOISE

Schottky's Theorem:<sup>6</sup> This theorem deals with the random emission of current carriers in temperature-limited vacuum diodes. The individual current carriers are composed of a series of independent random events. Small variations in current may be represented by a noise-current generator in parallel with the noiseless incremental admittance of the diode. In a frequency interval  $\Delta f$ , the mean-square value of this generator is

$$\overline{i^2} = 2qI\Delta f \quad \dots (3)$$

where  $I$  is the average plate current in amperes, and  $q$  is the electronic charge ( $1.6 \times 10^{-19}$  coulombs).

In a transistor, shot noise is experienced in both junctions. Since the majority of the charge carriers pass through both the emitter-base junction and the collector-base junction, the two noise sources are correlated.

NOISE SOURCES IN TRANSISTORS

According to VAN DER ZIEL<sup>5</sup>, all charge carriers which contribute noise to a npn transistor may be subdivided into eight groups:

- (1) Electrons injected by the emitter and collected by the collector.
- (2) Electrons injected by the emitter and recombining in the base region.
- (3) Electrons injected into the base and returning to the emitter.

- (4) Electrons trapped in the emitter space-charge region and recombining with holes coming from the base.
- (5) Electrons trapped in the emitter space-charge region and returning to the emitter after being detrapped thermally.
- (6) Electrons generated in the base and collected by the emitter.
- (7) Electrons generated in the base and collected by the collector.
- (8) Thermal fluctuation due to the extrinsic base resistance,  $r_{bb}$ .

Each of the eight processes contribute noise in accordance with Schottky's or Nyquist's theorems. The noise contributions are:

From groups (1) and (2) the shot noise due to the emitter current  $I_E$  and due to the current  $I_{EE}$  :

$$2q (I_E + I_{EE}) \Delta f$$

where  $I_{EE}$  is the current due to the electrons injected by the emitter and recombining in the base region.

From group (6) the shot noise due to the current  $I_{EE}$  :

$$2q I_{EE} \Delta f$$

and from groups (3) and (5) the thermal noise due to the real value of the emitter conductance  $g_e$  and due to the low frequency value of the emitter conductance  $g_{eo}$  :

$$4kT(g_e - g_{eo})\Delta f.$$

Thus the total noise current caused by the emitter current can be expressed as

$$\overline{i_1^2} = 4kT(g_e - g_{eo})\Delta f + 2q(I_E + 2I_{EE})\Delta f \quad \dots (4)$$

Fluctuations in the d.c. current carried by the electrons of groups (1) and (7) can be represented by a noise current generator  $i_2$  connected in parallel with the emitter-collector terminals with a mean square

value of

$$\overline{i_2^2} = 2q \alpha_{dc} (I_E + I_{EE}) \Delta f + 2q I_{CC} \Delta f \quad \dots (5)$$

where  $\alpha_{dc}$  is the dc emitter efficiency and  $I_{CC}$  is the current due the electrons generated in the base and collected by the collector.

Since 
$$I_C = \alpha_{dc} (I_E + I_{EE}) + I_{CC}$$

then

$$\overline{i_2^2} = 2q I_C \Delta f. \quad \dots (6)$$

#### DERIVATION OF THE HYBRID- $\pi$ NOISE MODEL

The electrical behaviour of the transistor, especially when it is used in a common-emitter configuration over a wide range of frequencies, can be represented by the widely used hybrid- $\pi$  model of Giacolletto-Johnson<sup>16</sup>. Thus, the noise performance of a common emitter npn transistor amplifier is best represented by a hybrid- $\pi$  model.

For small a.c. signals, the T-equivalent model of a common-emitter transistor configuration is as follows:

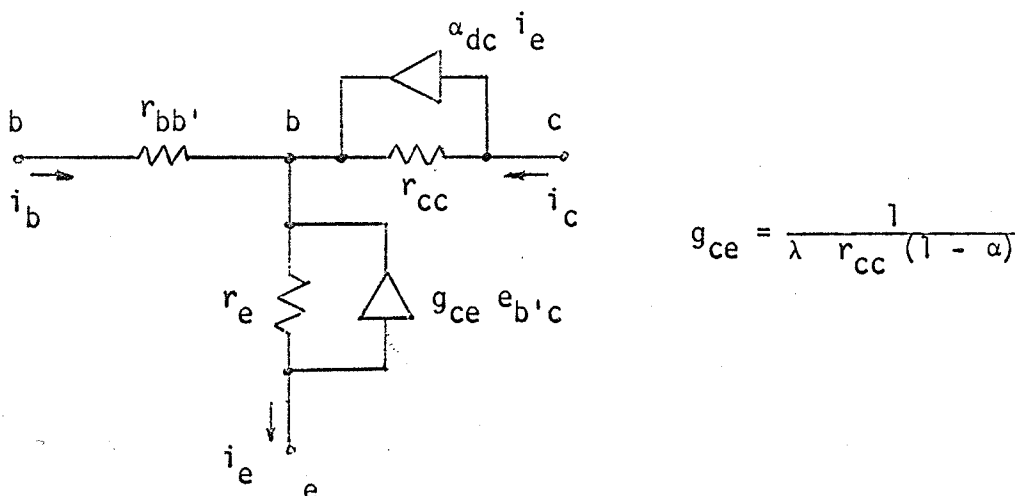


Figure 2

REPRESENTATION OF A COMMON-EMITTER TRANSISTOR MODEL

where  $b$ ,  $b'$ ,  $c$  and  $e$  are the base, intrinsic base, collector and emitter terminals of a transistor, respectively;  $r_{bb'}$  is the extrinsic base resistance,  $r_{cc}$  is the reverse junction resistance,  $g_{ce}$  is the feedback conductance which is a function of the emitter current  $I_E$  and the voltage  $V_{bc}$ , while  $\lambda$  is a constant which ranges from 2 to 5 for most transistors.<sup>11</sup>

The current generator  $\alpha i_e$  in Figure 2 will yield several current generators when  $i_e = -g_{ce} e_{b'c} + \frac{e_{b'e}}{r_e}$  is substituted. Now the current generator may be split into two parts. A few simplifications such as  $g_m = \frac{\alpha}{r_e}$ ,  $r_{b'e} = \frac{r_e}{1 - \alpha}$  and  $e_{cb'} \approx e_{ce}$  will result in the following model

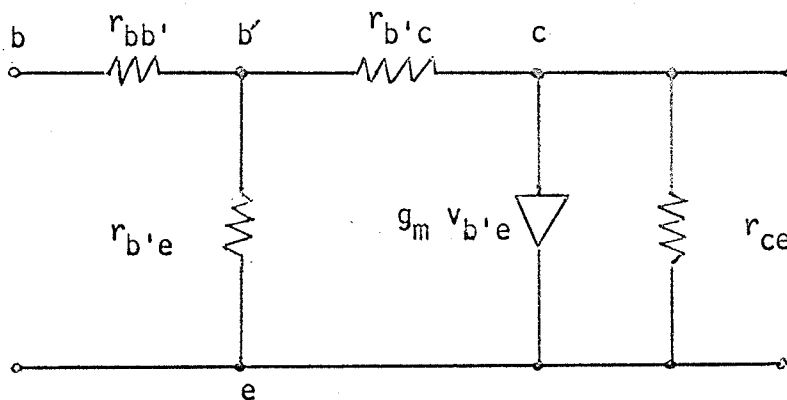


Figure 3

#### HYBRID- $\pi$ MODEL OF A TRANSISTOR

where

$$r_{ce} = \frac{1}{g_{ce}} \quad \text{and}$$

$$r_{b'c} = \frac{r_{cc}}{1 - g_{ce} r_{cc} (1 - \alpha)} .$$

The model of Figure 3 may be applicable over a wide range of frequencies. As the frequency increases, the effect of the capacitances should be considered. Therefore the model is modified to that of Figure 4.

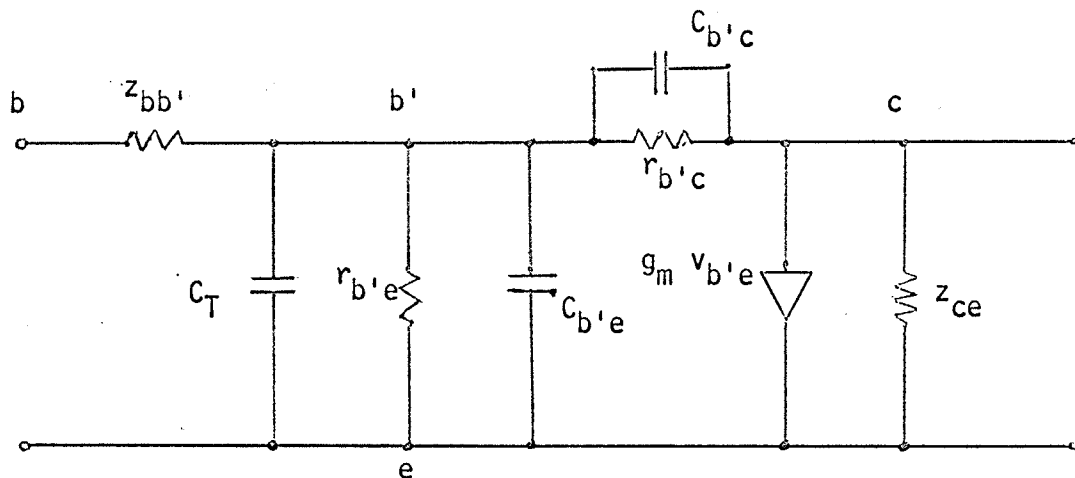


Figure 4

HIGH FREQUENCY HYBRID  $\pi$  MODEL

At high frequencies,  $r_{bb'}$  must be replaced by  $z_{bb'}$ , and as well  $r_{b'e}$  by  $z_{b'e}$  or  $z_{\pi}$ ;  $r_{b'c}$  by  $z_{b'c}$  and  $r_{ce}$  by  $z_{ce}$ . The resistances  $r_{b'c}$  and  $r_{ce}$  are large enough to be ignored at high frequencies. Ignoring  $r_{ce}$  implies a load impedance that is small compared to  $r_{ce}$ , whereas ignoring  $r_{b'c}$  is a recognition of the fact that  $r_{b'c}$  is large compared to the reactance of  $C_{b'c}$ . The capacitance  $C_{b'e}$  is large enough at low frequencies to swamp the effects of the emitter-base transition capacitance  $C_T$  and, hence,  $C_T$  is ignored at low frequencies. But at high frequencies, the capacitance  $C_{b'e}$  is not large enough to swamp the effect of  $C_T$ . Therefore, both capacitances  $C_{b'e}$  and  $C_T$  should normally be kept separate.

Since the capacitance  $C_{b'e}$  is directly proportional to the emitter current  $I_E$ , a low  $I_E$  will result in small values of  $C_{b'e}$ , which

will enable the emitter-base transition capacitance  $C_T$  to be of major importance. But for large values of emitter current,  $C_T$  is normally ignored.

Various relationships that should be kept in mind are:

- (a)  $g_m r_{b'e} = \beta$  ( $\beta$  is the d.c. current gain and normally assumed independent of the Q point).
- (b)  $g_m = \frac{\alpha_{dc}}{r_e}$  (Hence  $g_m$  increases as  $I_E$  increases).
- (c)  $r_{b'e} = \frac{r_e}{1 - \alpha_{dc}}$  (Hence  $r_{b'e}$  decreases as  $I_E$  increases).
- (d)  $C_{b'e} r_{b'e} = \text{Constant}$  (Hence  $C_{b'e}$  increases as  $I_E$  increases).

In the  $\pi$ -model, forward transfer is indicated by the current generator  $g_m v_{b'e}$ . The transconductance  $g_m$ , which at high frequencies actually becomes  $y_m$ , is considered independent of frequency and equal to the intrinsic low frequency transconductance  $g_m = \frac{\alpha}{r_e}$  of the transistor. The generator  $g_m v_{b'e}$  is not constant with increasing frequency because of the low-pass filter effect of  $r_{bb'}$  and  $C_{b'e}$  at the input circuit. If a comparison is made between the h-parameters type circuit model and the hybrid- $\pi$ , it can be concluded that  $g_m v_{b'e}$  is not frequency dependent. But this analogy<sup>16</sup> breaks down as the frequency increases and  $g_m$  becomes complex, and takes the form of:

$$g_m = \frac{q I_C / kT}{1 - j \omega / 2.4 \omega_\alpha} \quad \dots (7)$$

where  $\omega_\alpha$  is the upper radian frequency. The transconductance  $g_m$  is no longer in the form shown for higher frequencies than alpha cut-off frequency  $f_\alpha$ ; but it is considered in the very complex form of  $y_m$ <sup>16</sup>.

# CHAPTER II EQUIVALENT NOISE RESISTANCE OF A TRANSISTOR IN A COMMON-EMITTER CONFIGURATION

The noise current generators  $i_1$  and  $i_2$  are superimposed on a T-model representation of the common-emitter configuration:

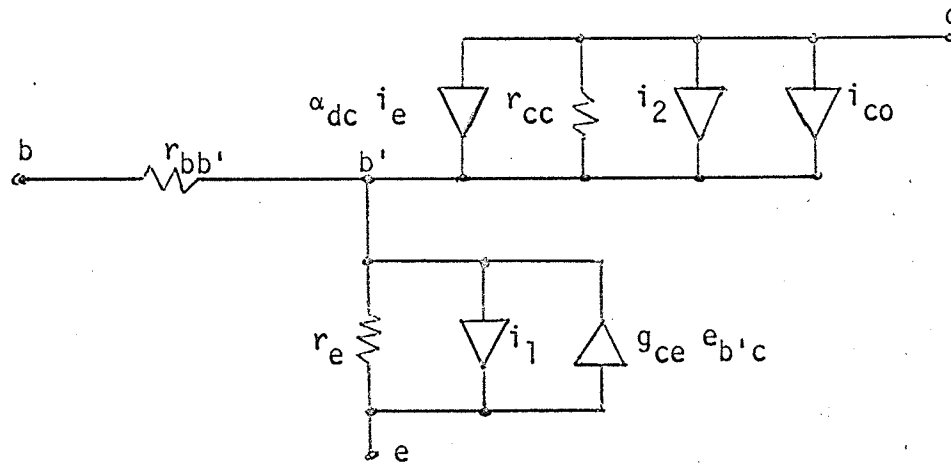


Figure 5

## REPRESENTATION OF A T-MODEL WITH NOISE CURRENT GENERATORS SUPERIMPOSED

These noise current generators  $i_1$  and  $i_2$  are superimposed on the hybrid- $\pi$  model of Figure 4. Now splitting the  $i_2$  current generator into two parts, the model becomes

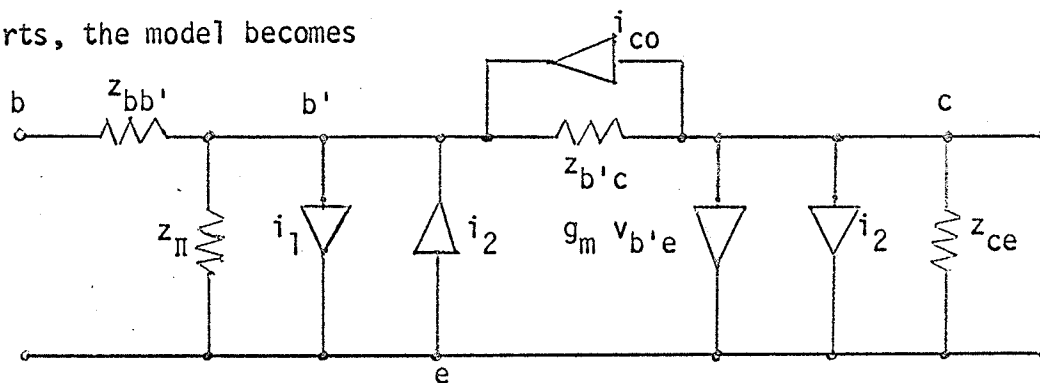


Figure 6

## REPRESENTATION OF A HYBRID- $\pi$ MODEL WITH NOISE CURRENT GENERATORS SUPERIMPOSED

The mean square values of the noise current  $i_1$  and  $i_2$  in the T-model are defined by VAN DER ZIEL<sup>5</sup> thus:

$$\overline{i_1^2} = 4kT(g_e - g_{eo})\Delta f + 2q(I_E + 2I_{EE})\Delta f \quad \dots (4)$$

$$\overline{i_2^2} = 2q I_C \Delta f \quad \dots (6)$$

$$\overline{i_1 i_2} = 2kT\alpha y_e \Delta f \quad \dots (8)$$

where  $\alpha$  is the a.c. intrinsic current gain and  $y_e$  is the emitter admittance.

When 
$$g_{eo} = \frac{q(I_E + I_{EE})}{kT}$$

is substituted into equation (4), it yields

$$\overline{i_1^2} = (4kT g_e - 2q I_E)\Delta f. \quad \dots (9)$$

Now, for the hybrid-II model of Figure 6, the noise current generators become:

$$\overline{i_b^2} = (\overline{i_1} - \overline{i_2})(\overline{i_1} - \overline{i_2})^* \quad \dots (10)$$

$$\overline{i_a^2} = 2q I_{CC} \Delta f \quad \dots (11)$$

$$\overline{i_c^2} = 2q I_C \Delta f. \quad \dots (12)$$

The noise currents  $\overline{i_a^2}$ ,  $\overline{i_b^2}$  and  $\overline{i_c^2}$  are due to the d.c. current fluctuations.

A convenient way of rating the relative noisiness of circuits is by relating signal to noise power ratios of input and output. Therefore, a noise figure can be defined:

$$F = \frac{\text{source-available signal power}}{\text{source-available noise power}} \div \frac{\text{output-available signal power}}{\text{output-available noise power}} \quad \dots (13)$$

The noise figure  $F$  may be evaluated several ways. One way to calculate  $F$  is by transferring all the noise sources to the input. The

equivalent input noise generator will produce exactly the same noise at the output.

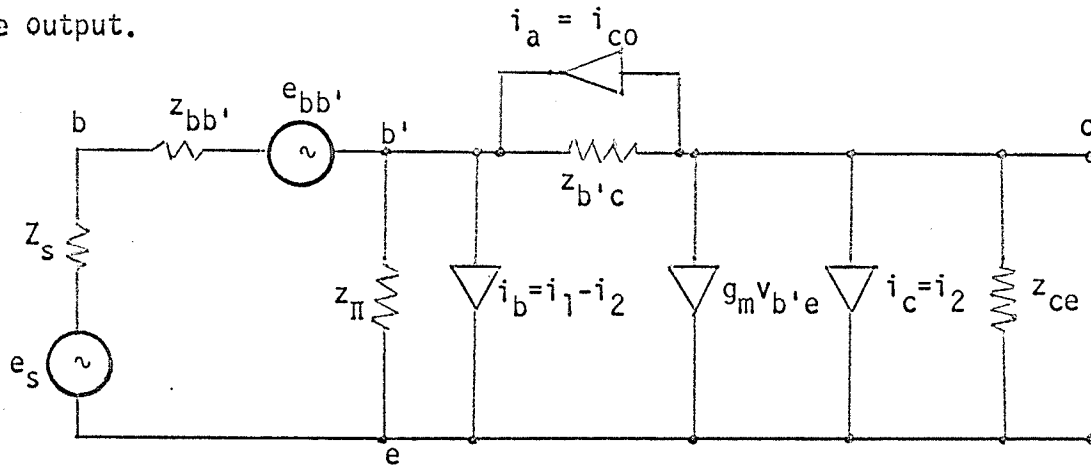


Figure 7

#### REPRESENTATION OF A HYBRID- $\pi$ MODEL WITH NOISE GENERATORS SUPERIMPOSED

The thermal noise generators due to the real part of the source impedance  $Z_s$  and the base impedance  $z_{bb'}$ , are

$$\overline{e_s^2} = 4kT \operatorname{Re} \{Z_s\} \Delta f \quad \dots (14)$$

$$\overline{e_{bb'}^2} = 4kT \operatorname{Re} \{z_{bb'}\} \Delta f. \quad \dots (15)$$

Since  $\overline{e_s^2}$  and  $\overline{e_{bb'}^2}$  are already at the input, only  $\overline{i_b^2}$ ,  $\overline{i_c^2}$  and  $\overline{i_a^2}$  need to be transferred to the input.

The feedback term  $z_{b'c}$  can be neglected because it does not affect the noise figure.

$i_b$  is replaced by an equivalent generator  $e_y$  at the input,

where

$$e_y = i_b (Z_s + z_{bb'}). \quad \dots (16)$$

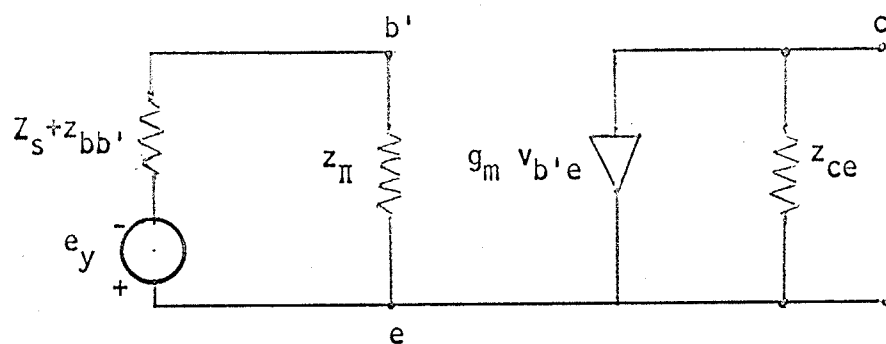


Figure 8

TRANSFORMATION OF NOISE-CURRENT GENERATOR  $i_b$  TO THE INPUT

$i_c$  is replaced by an equivalent generator  $e_z$  at the input:

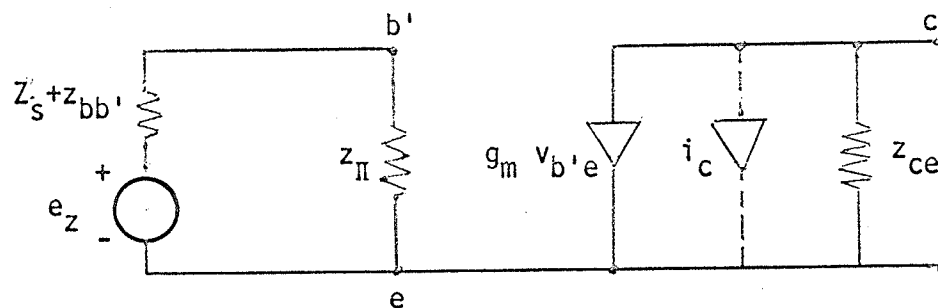


Figure 9

TRANSFORMATION OF NOISE-CURRENT GENERATOR  $i_c$  TO THE INPUT

Since

$$v_{b'e} = \frac{e_z}{Z_s + z_{bb'} + z_{\Pi}} \cdot z_{\Pi}$$

and

$$i_2 = g_m v_{b'e}$$

then

$$e_z = i_c \left( \frac{Z_s + z_{bb'} + z_{\Pi}}{g_m z_{\Pi}} \right) \quad \dots (17)$$

For the noise current generator  $i_a$  in the model, an equivalent generator  $e_w$  is placed at the input:

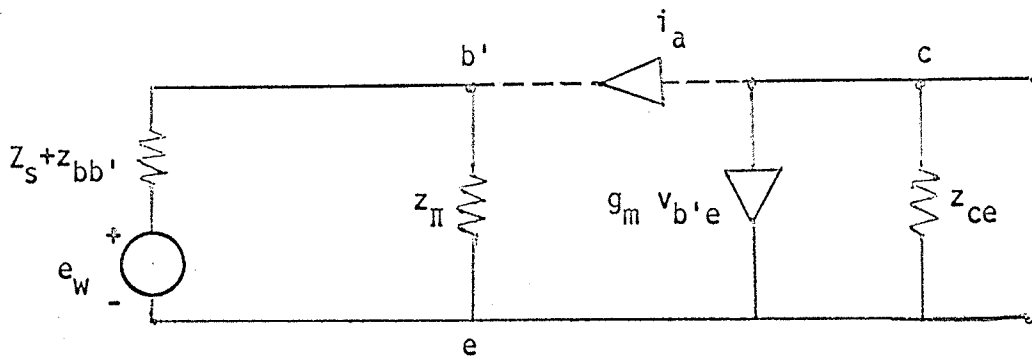


Figure 10

TRANSFORMATION OF NOISE-CURRENT GENERATOR  $i_a$  TO THE INPUT

Since

$$\frac{v_{b'e}}{Z_s + z_{bb'}} + \frac{v_{b'e}}{z_{II}} = i_a = -(g_m v_{b'e} + \frac{v_c}{z_{ce}})$$

then

$$v_{b'e} = i_a \left[ \frac{1}{\frac{1}{z_{II}} + \frac{1}{Z_s + z_{bb'}}} \right]$$

Now, if  $i_a$  is considered open circuit in Figure 10, then

$$v_{b'e} = \frac{e_w}{Z_s + z_{bb'} + z_{II}} z_{II}$$

Noting that

$$-\frac{v_c}{z_{ce}} = g_m v_{b'e}$$

it follows that

$$e_w = i_a \left[ \frac{(Z_s + z_{bb'})(1 + g_m z_{II}) + z_{II}}{g_m z_{II}} \right] \quad \dots (18)$$

All the equivalent noise generators now appear at the input.

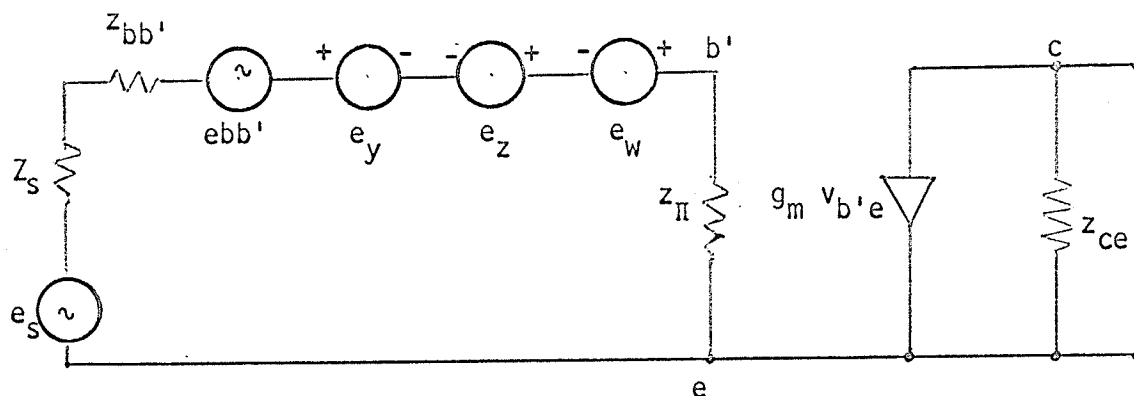


Figure 11

## FICTITIOUS NOISE GENERATORS OF A TRANSISTOR

From Figure 11,

$$e_n = e_s + e_b - e_y + e_z + e_w \quad \dots (19)$$

Now, if  $\overline{e_n^2} = 4kT R_n \Delta f$ , where  $R_n$  is the equivalent noise resistance; the noise figure  $F$  may be found by the ratio of  $\overline{e_n^2}$  to  $\overline{e_s^2}$ .

$$F = \frac{\overline{e_n^2}}{\overline{e_s^2}} = \frac{R_n}{R_s} \quad \dots (20)$$

A significant cross-correlation  $\overline{i_b^* i_c}$  results at high frequencies. (Ref. #17).

where 
$$\overline{i_b^* i_c} = (\overline{i_1 - i_2})^* \overline{i_2}$$

or 
$$\overline{i_b^* i_c} = 2kT \alpha_y \Delta f - 2q I_C \Delta f.$$

If 
$$g_{mo} = \frac{qI_C}{kT} \quad \text{and} \quad g_m = \alpha_y e$$

then 
$$\overline{i_b^* i_c} = 2kT (g_m - g_{mo}) \Delta f. \quad \dots (21)$$

Only the fictitious noise generators  $e_y$  and  $e_z$  are correlated. The multiplication of these generators results the cross-correlation  $\overline{i_b^* i_c}$  term. From the quadratic addition of independent generators results

$$\overline{e_n^2} = \overline{e_s^2} + \overline{e_{bb'}^2} + \overline{e_y^2} + \overline{e_z^2} - \overline{e_y^* e_z} - \overline{e_y e_z^*} + \overline{e_w^2}. \quad \dots (22)$$

For silicon transistors,  $I_{CC}$  is very low and therefore the contribution of  $\overline{e_w^2}$  to the equivalent noise generator may be neglected:

$$\begin{aligned} \overline{e_n^2} &= \overline{e_s^2} + \overline{e_{bb'}^2} + \overline{i_b^2} |Z_s + z_{bb'}|^2 + \\ &\quad \overline{i_c^2} \left| \frac{Z_s + z_{bb'} + z_{\Pi}}{g_m z_{\Pi}} \right|^2 - 2 \operatorname{Re} \{ \overline{e_y^* e_z} \} \quad \dots (23) \end{aligned}$$

If the  $\overline{i_b^2} |Z_s + z_{bb'}|^2$  term is considered,

$$\begin{aligned} \text{then } |Z_s + z_{bb'}|^2 &\{ (\overline{i_1 - i_2})(\overline{i_1 - i_2})^* \} \\ &= |Z_s + z_{bb'}|^2 \{ 2q | -I_B | \Delta f + 2kT(2g_e - \alpha^* y_e^* - \alpha y_e) \} \end{aligned}$$

where the term  $(2g_e - \alpha^* y_e^* - \alpha y_e)$  is negligible.

Considering the  $-2 \operatorname{Re} \{ \overline{e_y^* e_z} \}$  term:

$$\begin{aligned} &- 2 \operatorname{Re} \{ \overline{i_b^* i_c} (Z_s + z_{bb'})^* \left( \frac{Z_s + z_{bb'} + z_{\Pi}}{g_m z_{\Pi}} \right) \} \\ &= - 4kT \Delta f \operatorname{Re} \{ (g_m - g_{mo})(Z_s + z_{bb'})^* \left( \frac{Z_s + z_{bb'} + z_{\Pi}}{g_m z_{\Pi}} \right) \} \end{aligned}$$

$$\text{Using } F = \frac{\overline{e_n^2}}{\overline{e_s^2}} = \frac{R_n}{R_s},$$

it follows that

$$\begin{aligned} F &= 1 + \frac{r_{bb'}}{R_s} + \frac{q I_B}{2kT R_s} |Z_s + z_{bb'}|^2 \\ &\quad + \frac{q I_C}{2kT R_s} \left| \frac{Z_s + z_{bb'} + z_{\Pi}}{g_m z_{\Pi}} \right|^2 \\ &\quad - \frac{R_e}{R_s} \{ (g_m - g_{mo})(Z_s + z_{bb'})^* \left( \frac{Z_s + z_{bb'} + z_{\Pi}}{g_m z_{\Pi}} \right) \} \quad \dots (24) \end{aligned}$$

The noise-figure equation is a function of  $R_s$  and  $X_s$ . Minimized values of  $F$  may be found by differentiating the noise-figure equation to

obtain  $\frac{\partial F}{\partial R_s}$  and  $\frac{\partial F}{\partial X_s}$ . After a few simplifications, it can be shown that:

$$R_{s_{opt}}^2 \approx (r_{bb'} + r_{\Pi})^2 + (X_s + x_{\Pi})^2 + 2g_m[r_{bb'}(r_{\Pi}^2 + x_{\Pi}^2) + r_{\Pi}(r_{bb'}^2 + X_s^2)] \quad \dots (25)$$

and

$$X_{s_{opt}} \approx \frac{-x_{\Pi}}{1 + 2g_m r_{\Pi}}. \quad \dots (26)$$

If the reactance  $x_{\Pi} \gg r_{\Pi}$ , and the optimum source reactance is used,

$$X_{s_{opt}} = \frac{-x_{\Pi}}{1 + 2g_m r_{\Pi}}$$

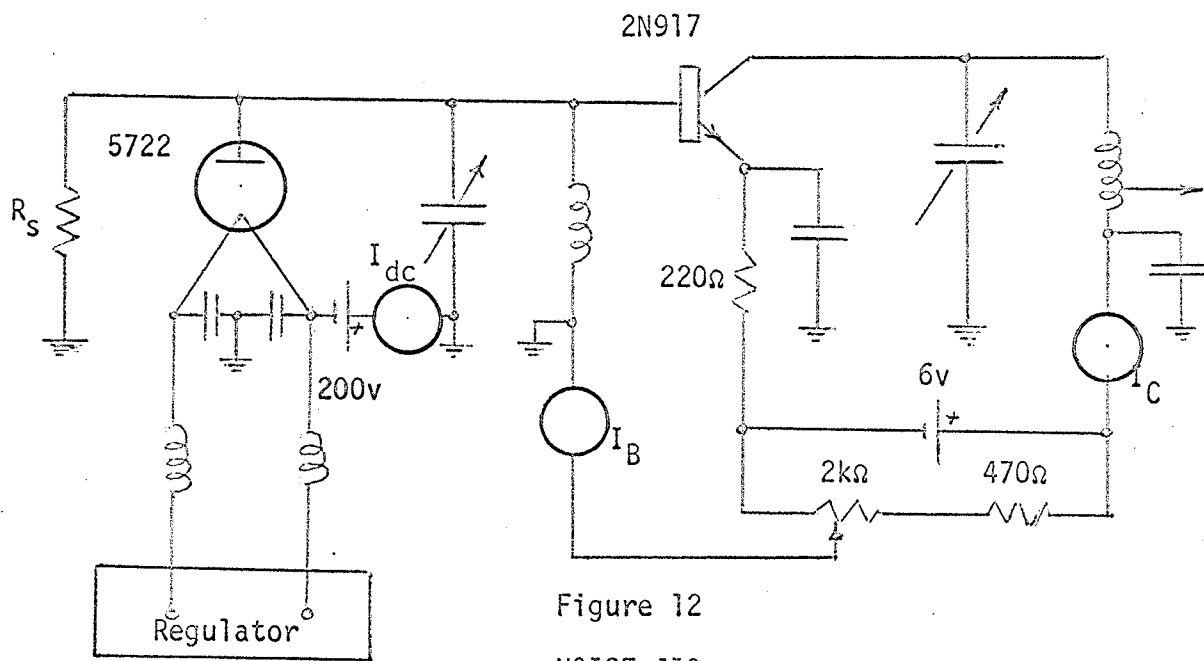
and

$$R_{s_{opt}} = -x_{\Pi} \sqrt{2g_m(r_{bb'} + \frac{r_{\Pi}}{1 + 2g_m r_{\Pi}})} \quad \dots (27)$$

## EXPERIMENTAL PROCEDURE

NOISE JIG

To perform the noise measurements, the following noise jig was designed:



At 100 MHz, the inductance of the leads is very critical and therefore careful lay-out must be considered. The source impedance  $Z_S$  was simulated by a parallel-tuned LC circuit damped with metal-film resistances. Metal-film resistances were not available beyond 500 ohms; therefore a carbon-film rod resistance was used. At low frequencies, the base of the transistor is shorted to ground by the coil of the tank circuit. The coil suppresses low frequency flicker-noise components which could interact with high frequency noise (cross-modulation). The 5722 noise diode was placed at the

input of the transistor. The d.c. path for the noise diode is provided through the RF choke. To minimize variations in plate current, the filament of the noise diode, which has a non-linear resistance with respect to the temperature, was fed from a constant current source:

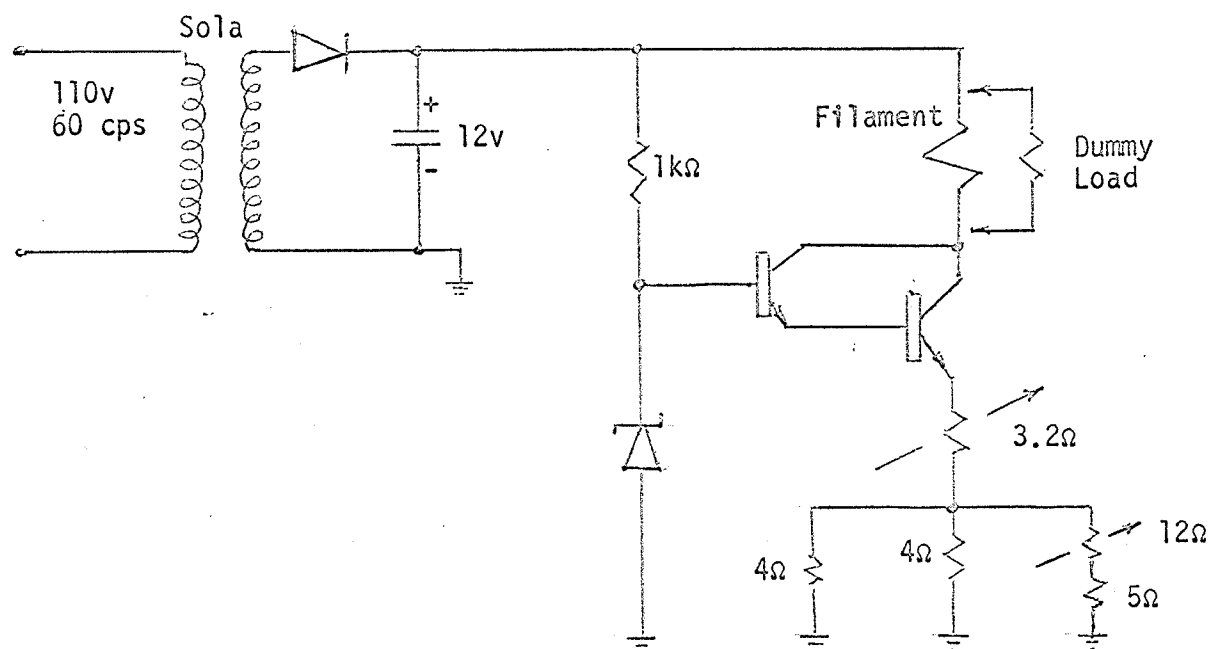


Figure 13  
REGULATOR

A Sola transformer was used to compensate for line voltage variations. Corrective feedback was obtained by the emitter bias resistance  $R_E$  and large gain was obtained by the Darlington connection of the transistors.

#### NOISE MEASUREMENT METHOD

The source impedance was measured using a Wayne-Kerr Model B-801 bridge. The noise jig was connected to a converter, followed by a pre-amplifier and a 30 MHz. receiver with a precision attenuator.

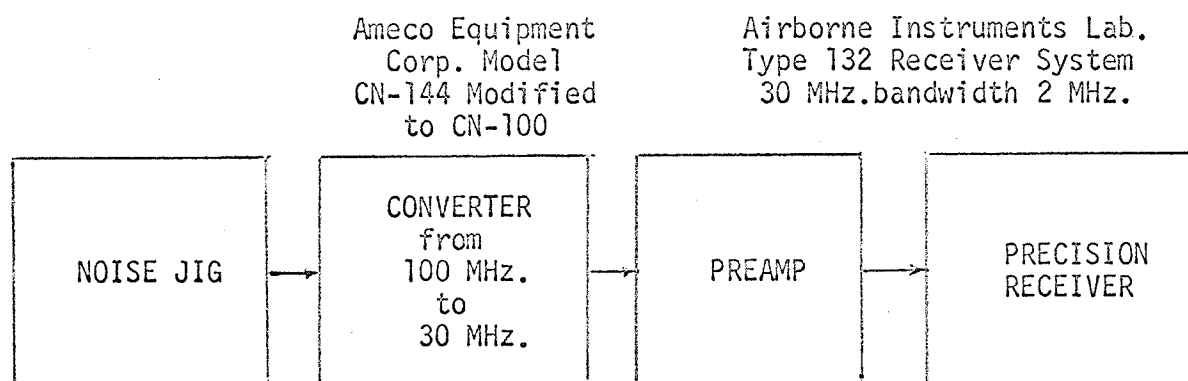


Figure 14

## BLOCK DIAGRAM OF THE NOISE-MEASURING APPARATUS

Drift in the converter gain contributed some error to the measurements, but was minimized by the use of a large Sola transformer.

Noise measurements were made by doubling the output noise power. Now if  $P_o$  represents the output noise power, when the noise diode filament current is turned off, and the d.c. diode current  $I_{dc}$  is equal to zero, then

$$P_o = (4kT R_s \Delta f)(AF) \quad \dots (28)$$

where  $A$  is the power gain of the system and  $F$  is the noise figure of the system.

If  $P_d$  is the output noise power for a given d.c. diode current  $I_{dc}$ , then by superposition

$$P_d = (4kT R_s \Delta f)(AF) + 2q I_{dc} \Delta f |Z_s|^2 A \quad \dots (29)$$

If the output noise power is doubled

ie.  $P_d = 2 P_o$ ,

and 
$$\frac{P_d}{P_o} = 1 + \frac{q I_{dc} |Z_s|^2}{2kT F R_s}$$

from which 
$$F = \frac{q}{2kT} I_{dc} \frac{|Z_s|^2}{R_s} \quad \dots (30)$$

It can also be seen that

$$e_n^2 = 4kT R_n \Delta f = 2q I_{dc} \Delta f |Z_s|^2$$

$$F = \frac{R_n}{R_s} = \frac{q}{2kT} I_{dc} \frac{|Z_s|^2}{R_s}$$

For the case of the source impedance  $Z_s$  equal to the source resistance  $R_s$

$$F = 19.35 I_{dc} R_s \quad \dots (31)$$

at  $T = 293^\circ K$  (room temperature).

#### CORRECTION FOR BACKGROUND NOISE

The output noise power contains a background noise power, therefore a compensation for the background noise must be considered when the output noise power is doubled.

If  $M$  is the output noise-power reading and  $M_b$  is the background noise-power reading, then

$$\frac{M_b}{M_1} = \frac{1}{k}$$

where  $k$  is the ratio of the first output noise-power reading to the background noise-power reading. If the first output noise-power reading  $M_1$  is doubled then the ratio of the second output noise-power reading  $M_2$  to  $M_1$  becomes

$$\frac{M_2}{M_1} = \frac{2 M_1 - M_b}{M_1}$$

or 
$$\frac{M_2}{M_1} = 2 - \frac{1}{k} \quad \dots (32)$$

# CALCULATION OF INTRINSIC $g_m$ FROM EXTRINSIC $y_m$ MEASUREMENTS

Upon examination of the hybrid- $\Pi$  model, it can be seen that the extrinsic transadmittance  $y_m$  as obtained from measurements is not the intrinsic  $g_m$  as defined in the model. This is due to the base impedance  $z_{bb'}$ , which forms the low-pass filter with  $z_{\Pi}$  in parallel with the feedback impedance  $z_{b'c}$ . Therefore, the following calculation has to be made:

The output is short-circuited in the  $y_m$  measurements.

Now

$$y_m = \frac{i_c}{v_{in}} .$$

But from the  $\Pi$ -model, it can be shown that

$$v_{b'e} = \frac{v_{in}}{z_{bb'} + \frac{z_{\Pi}}{z_{b'c}}} \cdot \frac{z_{\Pi}}{z_{b'c}} .$$

Now, since  $g_m v_{b'e} = i_c$

$$\text{then } g_m = y_m \left( 1 + \frac{z_{bb'}}{z_{\Pi} // z_{b'c}} \right) . \quad \dots (33)$$

$g_m$  of equation (33) was still complex; therefore, the magnitude of  $g_m$  is considered. Since the input impedance  $z_{in} = z_{bb'} + z_{\Pi} // z_{bb'}$ , when the output is short-circuited, then

$$g_m = \left| y_m \left( \frac{z_{in}}{z_{in} - z_{bb'}} \right) \right| . \quad \dots (34)$$

# CHAPTER IV EXPERIMENTAL RESULTS

The input impedance  $z_{in}$  and the transadmittance  $y_m$  of the transistor (2N917) were measured, using the high-frequency G.R. Transfer-Function and Immittance Bridge Type 1607-A. The measurements were done with three different collector currents  $100\mu A$ ,  $300\mu A$  and  $1\text{ mA}$ .

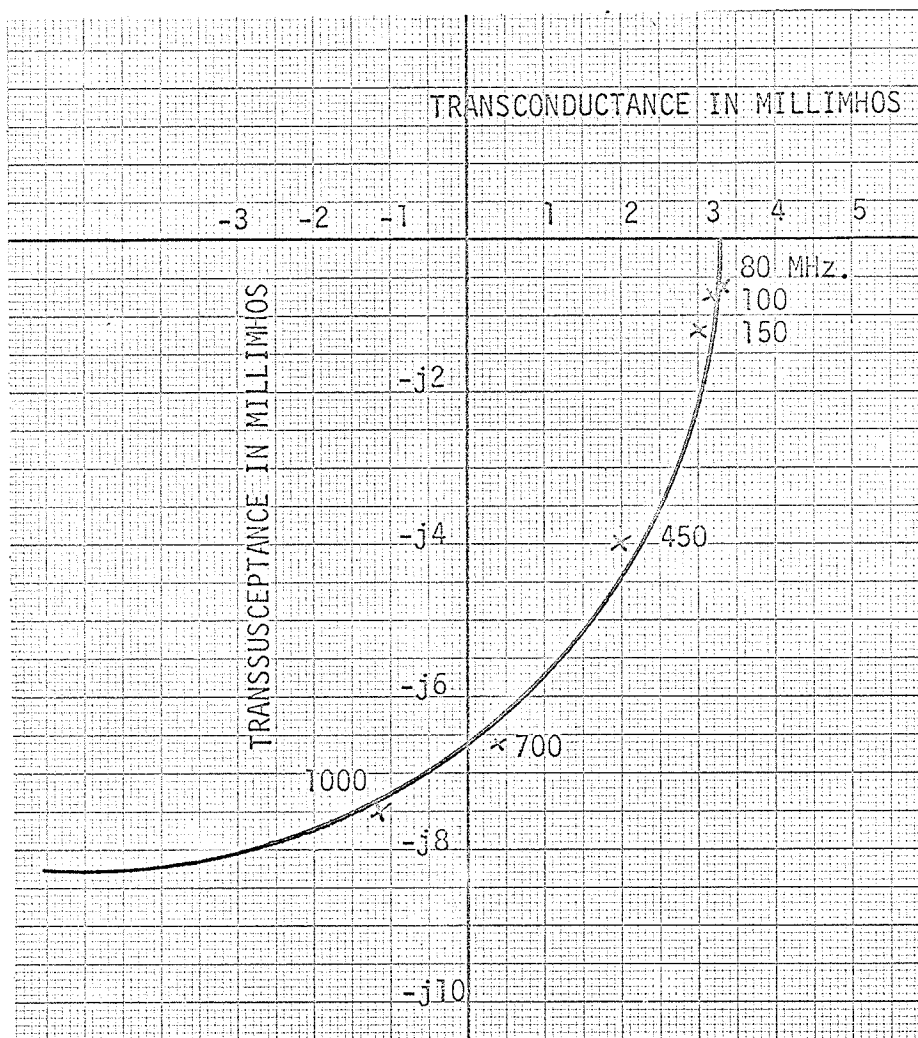


Figure 15

MEASURED TRANSADMITTANCE ( $y_m$ ) for  $I_C = 100\mu A$

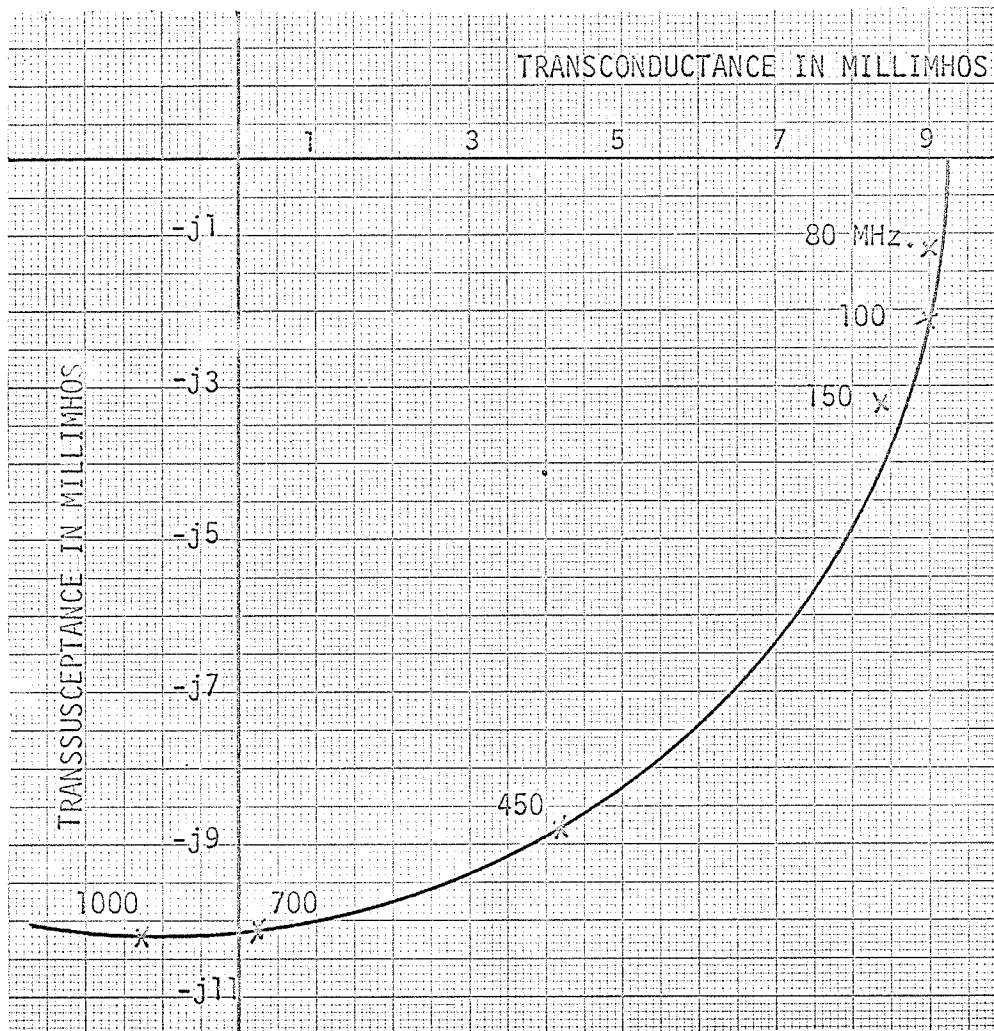


Figure 16

MEASURED TRANSADMITTANCE ( $y_m$ ) for  $I_C = 300 \mu A$

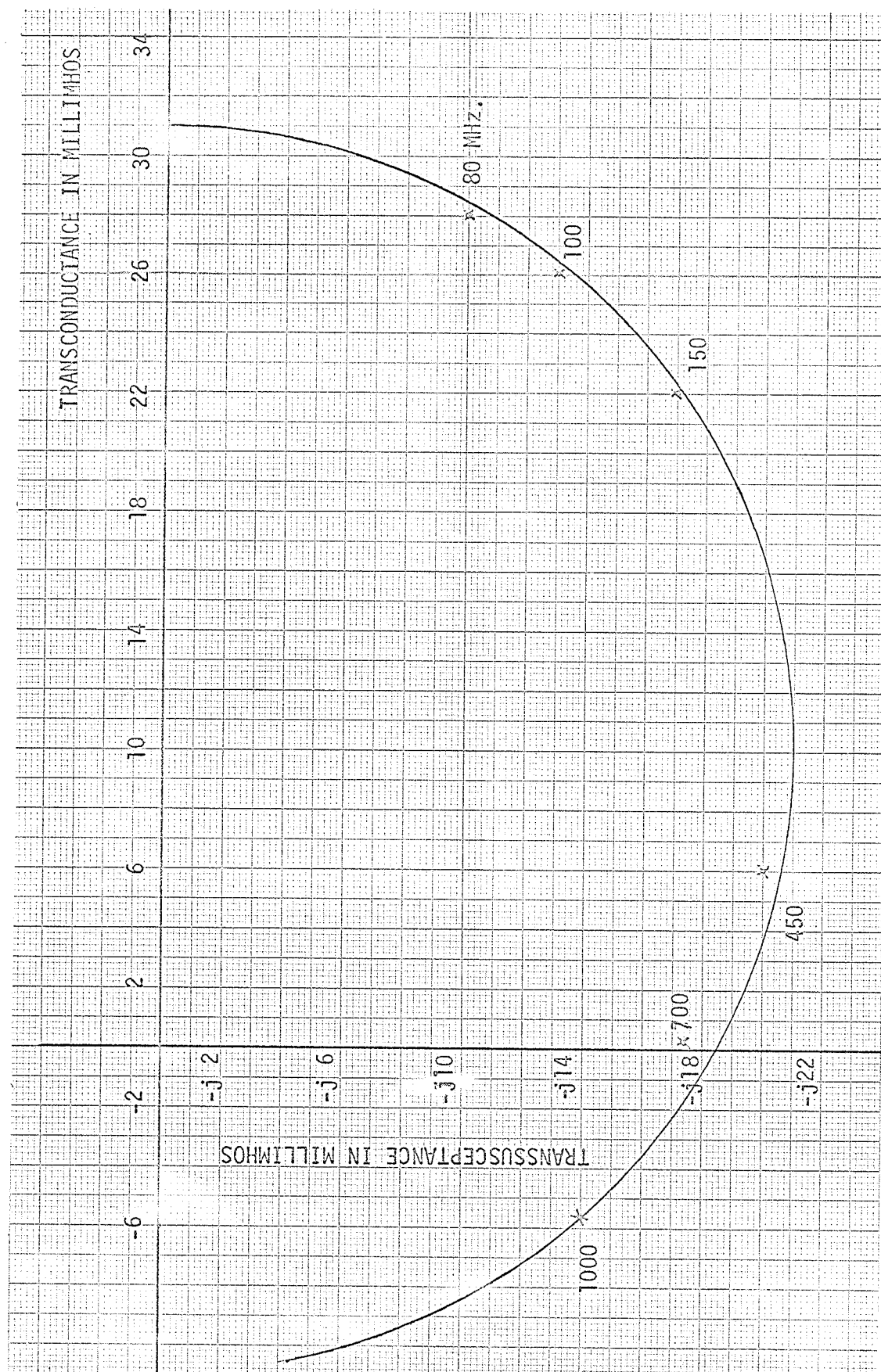


Figure 17

MEASURED TRANSADMITTANCE ( $y_m$ ) for  $I_C = 1 \text{ mA}$

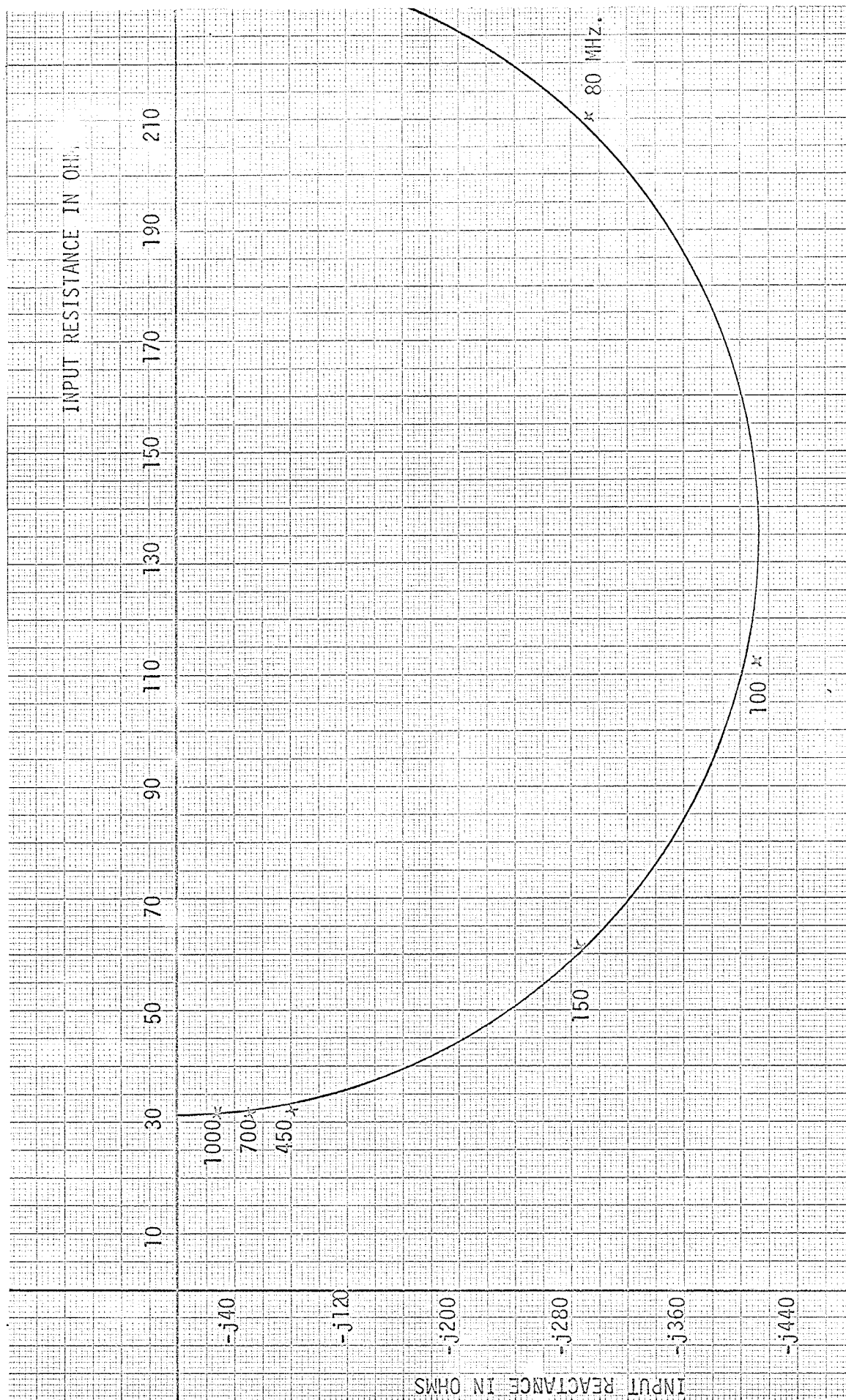


Figure 18  
MEASURED INPUT IMPEDANCE FOR  $I_C = 100 \mu A$

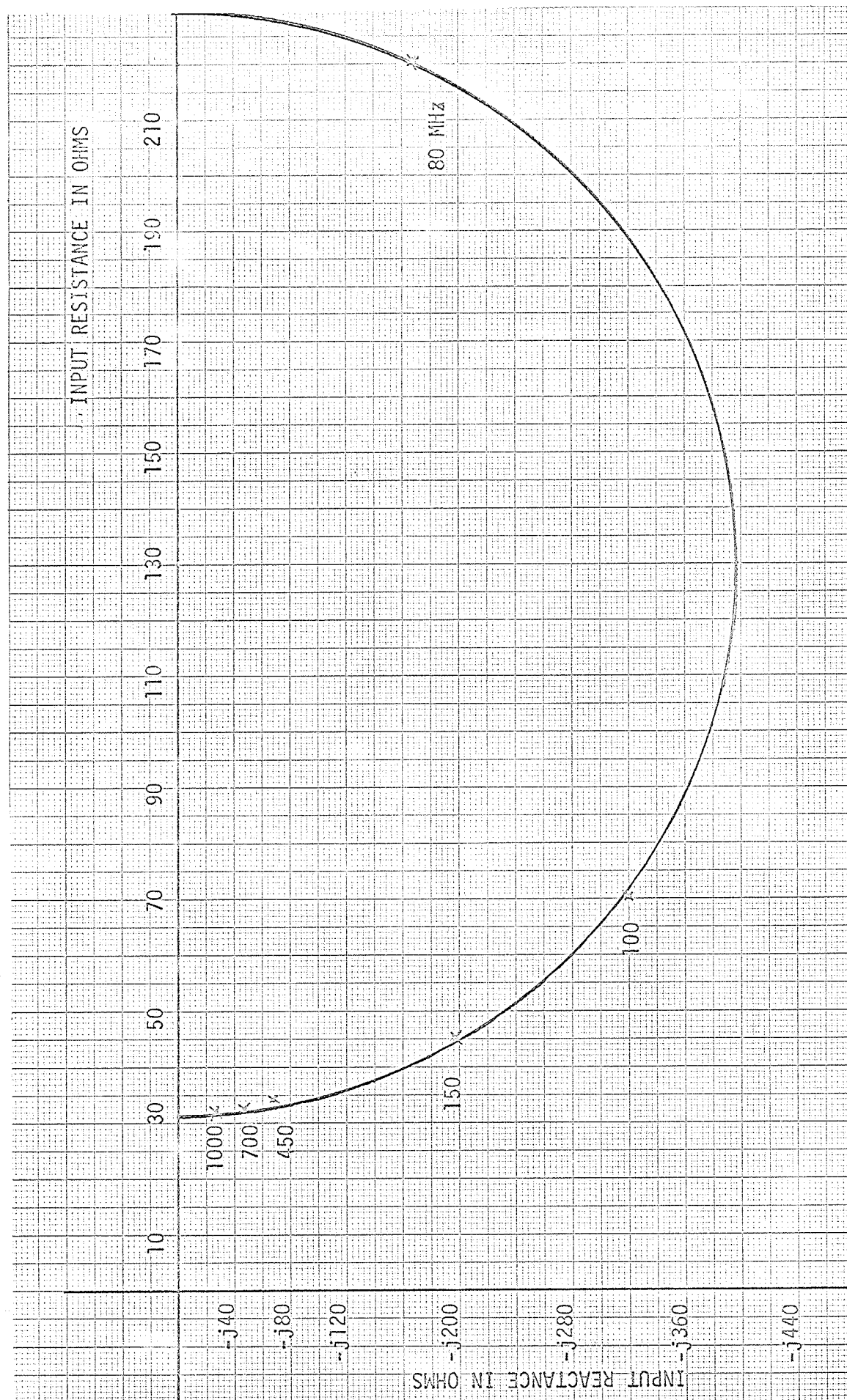


Figure 19

MEASURED INPUT IMPEDANCE for  $I_C = 300 \mu A$

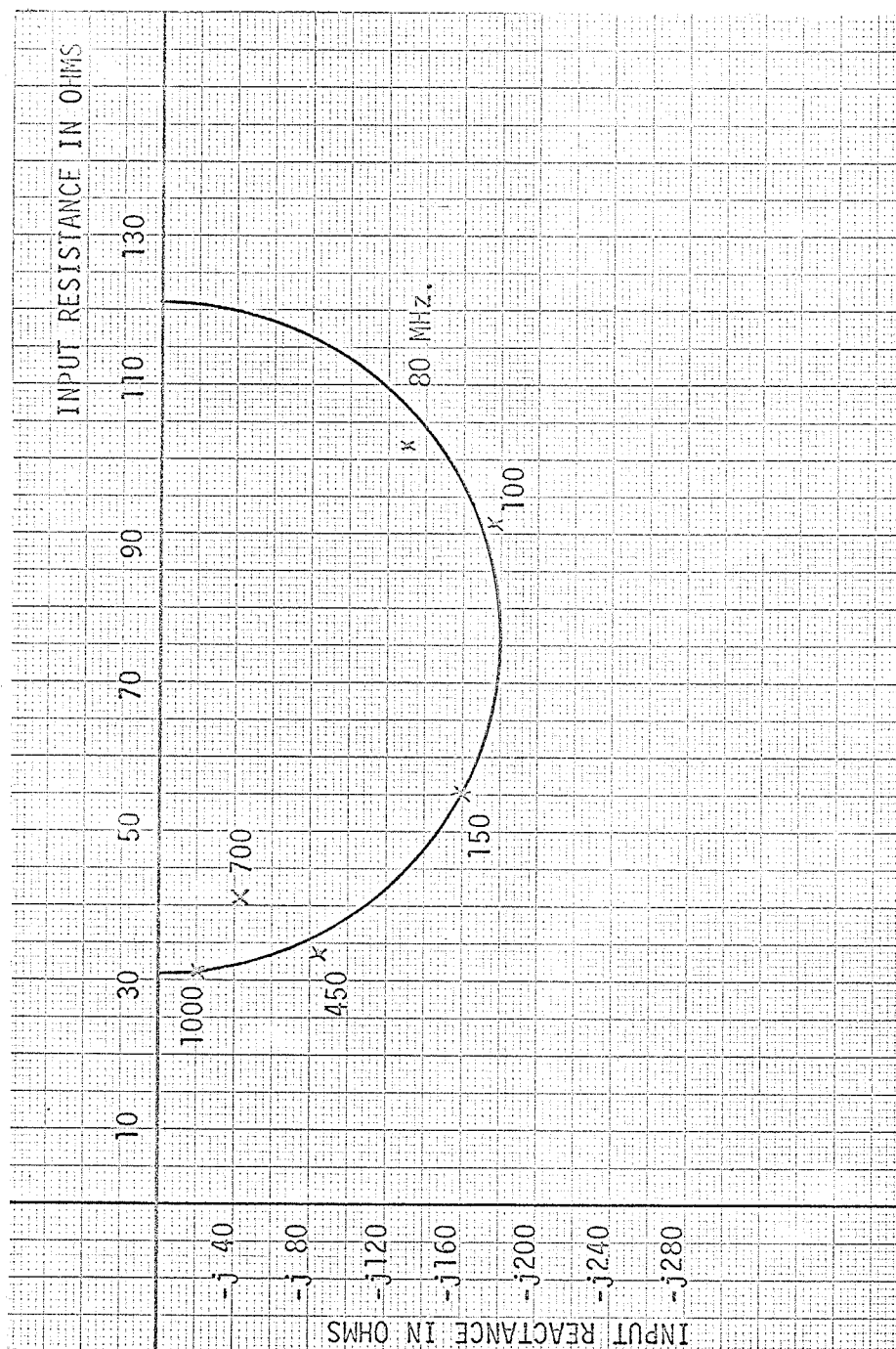


Figure 20

MEASURED INPUT IMPEDANCE for  $I_C = 1 \text{ mA}$

It can be shown that for a constant noise figure  $F$ , equation (24) yields a circle in  $X_S - R_S$  plane. Circled regions converge upon a point, where  $F$  is a minimum. Measured and calculated noise-figure values are within 0.5 db of difference.

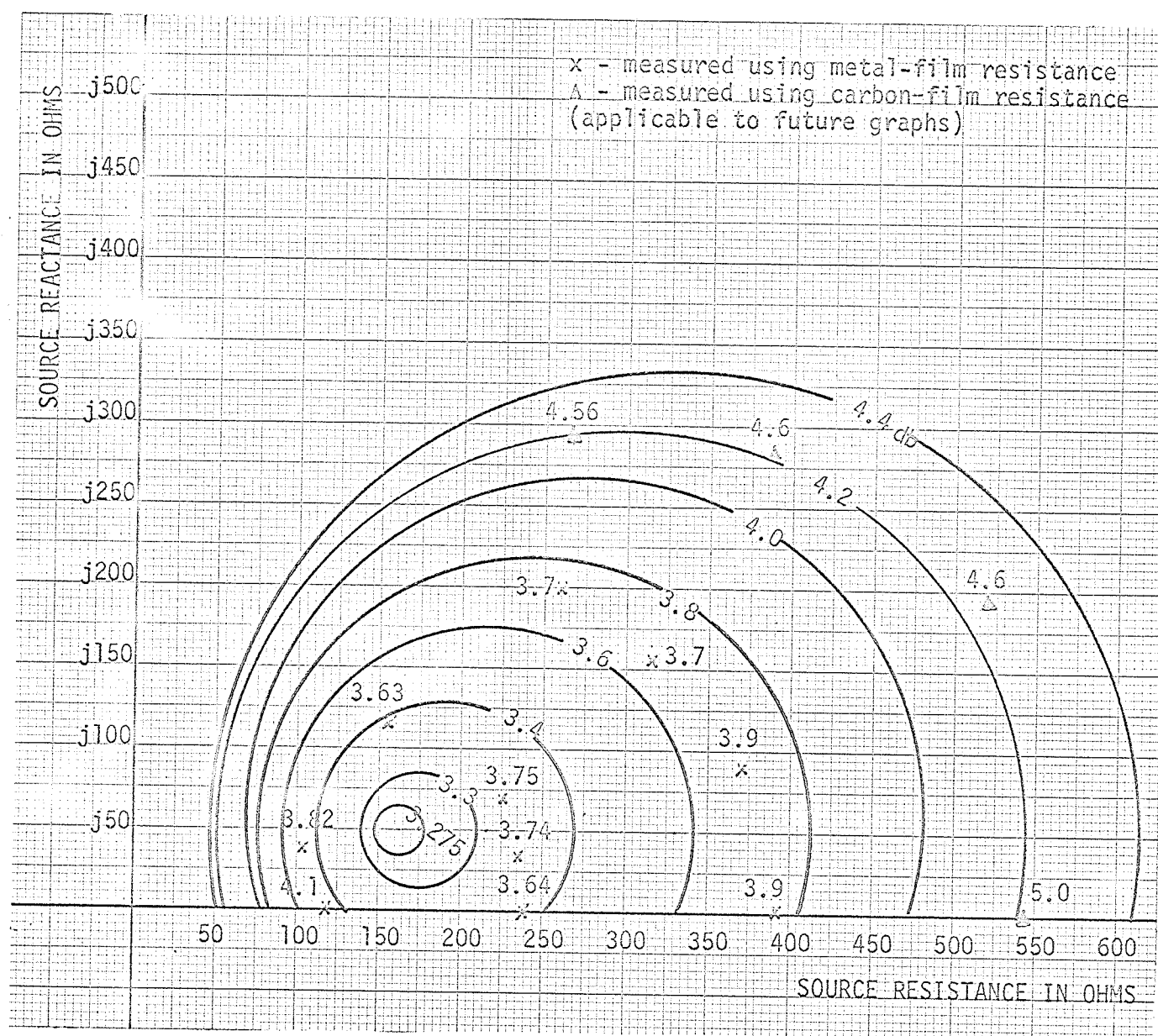


Figure 21

CONTOURS OF CONSTANT NOISE FIGURE FOR 2N917

TRANSISTOR ( $I_C = 1\text{mA}$ )

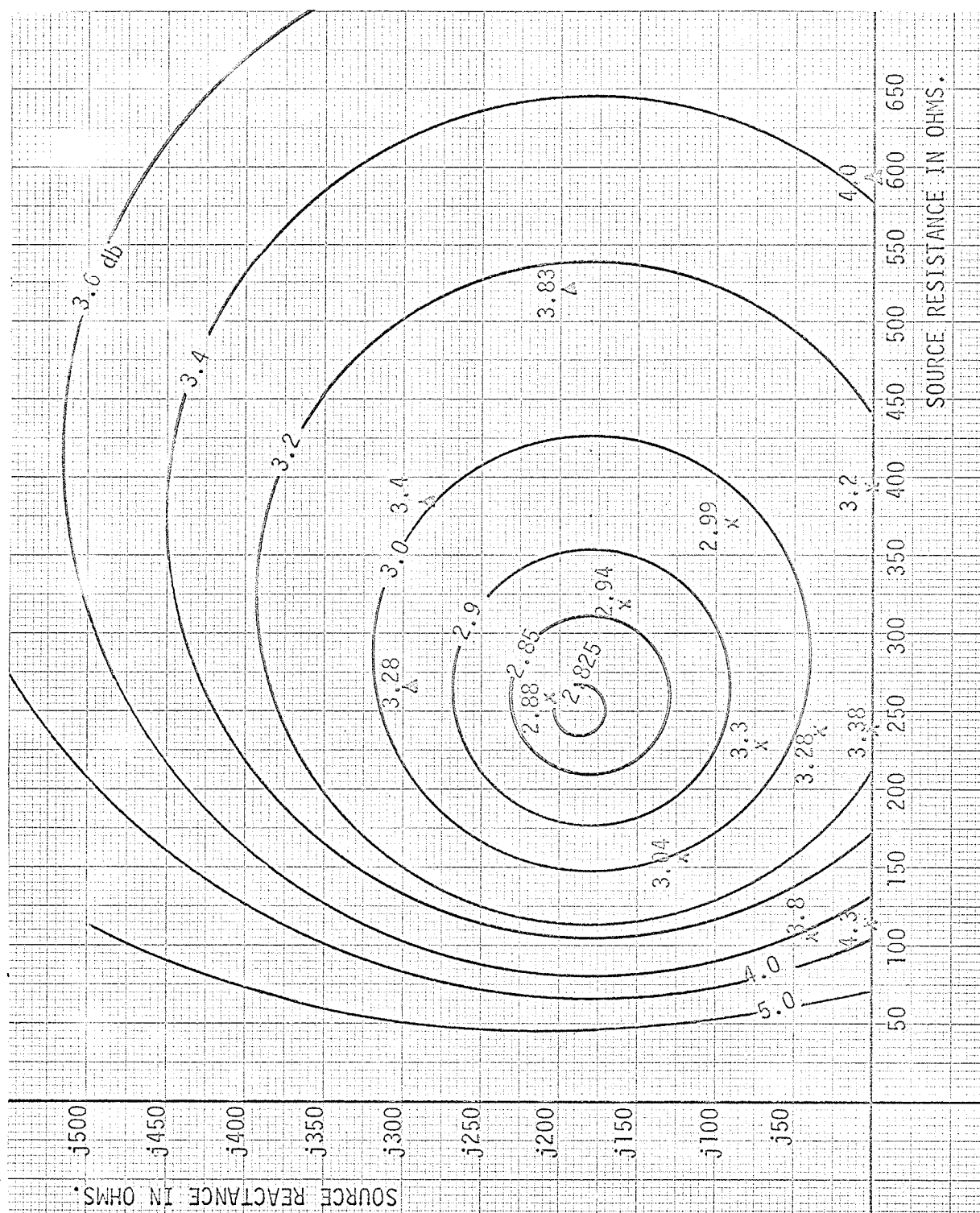


Figure 22

CONTOURS OF CONSTANT NOISE FIGURE FOR  $I_C = 300 \mu A$

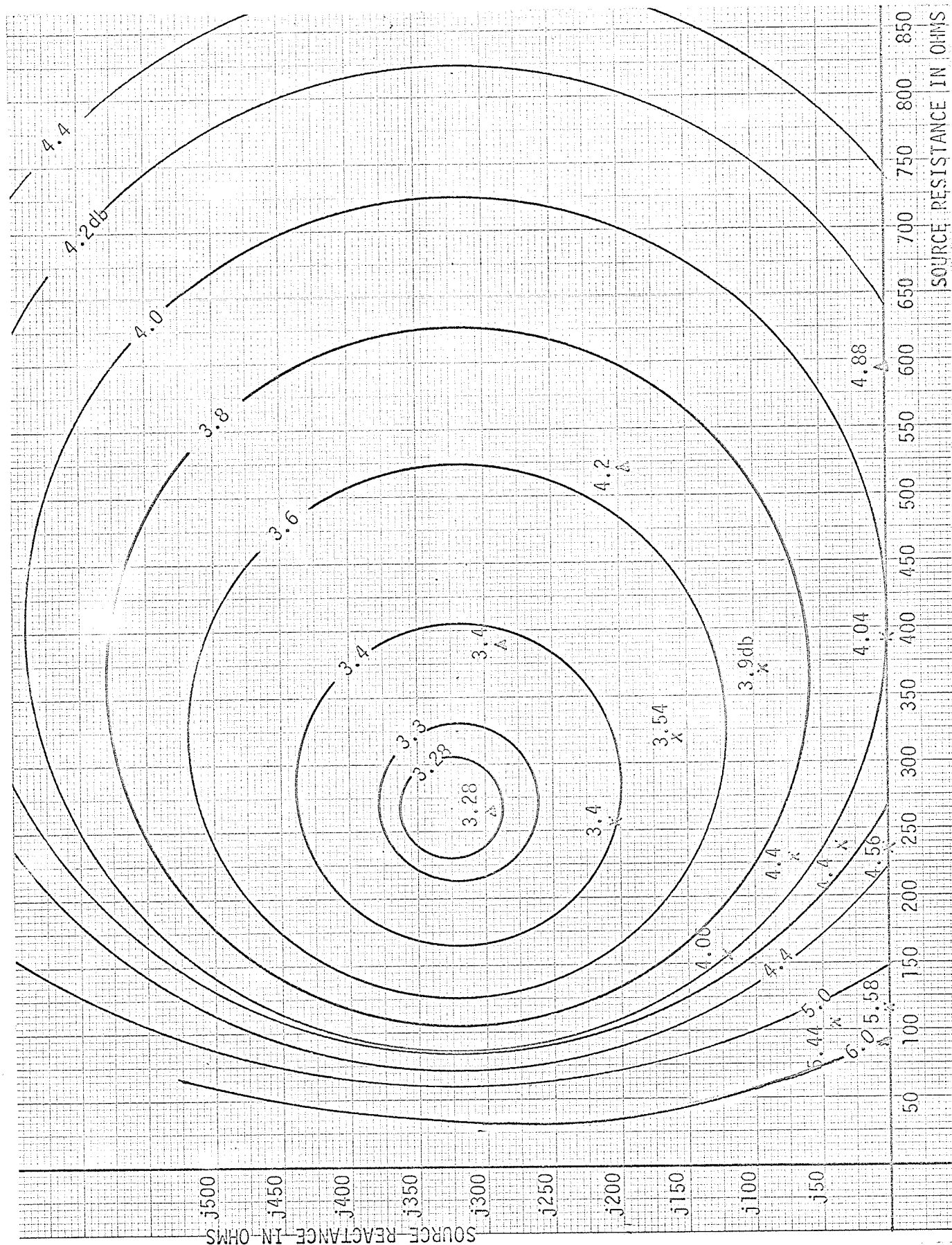


Figure 23

CONTOURS OF CONSTANT NOISE FIGURE FOR  $I_C = 100 \mu A$ .

## CHAPTER V

### CONCLUSION

The high-frequency noise behaviour of the UHF silicon drift transistor 2N917 used in the investigation obeys the theory within reasonable experimental error. Since the calculated and the measured noise figure values of this transistor are within 0.5 db., it can be concluded that the effects of  $x_{bb'}$ ,  $I_{CC}$ , and recombination current  $I_R$  are negligible for this transistor. At this frequency, the contribution of the cross-correlation to the noise figure is quite significant, especially at high collector currents.

It was expected that the intrinsic transconductance  $g_m$  calculated from extrinsic transadmittance  $y_m$  measurements would become complex at 100 MHz. Calculations based on extrinsic transadmittance measurements showed that  $g_m$  had phase angles between  $3^\circ$  and  $12^\circ$  which could be within experimental errors. However, at high  $R_s$  - values ( $2k\Omega$ ), a complex  $g_m$  in the noise figure expression produces a negative noise-figure value which is physically impossible. Thus, only the magnitude of  $g_m$  is considered for the calculations.

The input impedance of the transistor in the common-emitter configuration is capacitive. In order to have a power match for minimum noise figure, an inductive source impedance was introduced. It is interesting to note that introducing inductive source reactance at low currents reduces the noise figure as  $R_{s_{opt}}$  is kept approximately the same.

In the hybrid- $\pi$  model,  $z_{\pi}$  is a function of emitter current  $I_E$ ; thus, the noise figure is a function of  $I_E$ . A collector current value of approximately 450  $\mu A$  gives the minimum noise figure for this transistor.

## BIBLIOGRAPHY

1. Hanson, "Shot Noise in Transistors", Proc. IRE, Vol. 45- 1957, pp. 1538-1542.
2. Nielsen, E.G., "Behaviour of the Noise Figure in Junction Transistors", Proc. IRE., Vol.45, 1957, July, pp. 957-963, pp. 839-857 with correction 1509.
3. Van Der Ziel, "Noise in Junction Transistors", Proc. IRE., Vol. 46, 1958, March, pp. 1019-1038, pp. 589-594.
4. Van Der Ziel, Chennette, "Accurate Noise Measurements on Transistors", Proc. IRE., ED-9, 1962, pp. 123-128.
5. Van Der Ziel, "Shot Noise in Transistors", Proc. IRE., Vol. 48, Jan. 1960, pp. 114-115.
6. Schneider & Strutt, "Shot and Thermal Noise in Silicon and Germanium Transistors", Proc. IRE., Vol. 48, 1960, pp. 1731-1739.
7. Van Der Ziel, "Noise", Prentice Hall, 1954.
8. Gradner & Flyd, IEEE, Transactions, C-T, 1963, March, pp. 45-48.
9. Chennette, "Inductive Sources", Proc. IRE., Vol. 47, 1959, pp. 448-449.
10. Pritchard, Angell, Adler, Early and Webster, "Transistor Internal Parameters for Small Signal Representation", Proc. IRE, Vol. 49, No. 4, 1961, April, p. 725.
11. Joyce & Clarke, "Transistor Circuit Analysis", Addison-Wesley Pub. Co. Inc., copy Oct. 1961.
12. Schwartz, Mischa, "Information, Transmission, Modulation and Noise," McGraw-Hill Book Co., Inc. 1959.
13. Jakobschuk, A., Graduate Thesis, "HF Noise in Transistors", University of Minnesota, August 1964.
14. Swiontek, M.C. & Hassun, R., "The Influence of Transistor Parameters on Transistor Noise Performance", A Simplified Presentation. HP Journal.
15. Hyde, F., "The Physical Basis of Noise", "Noise in Electronic Devices", The Institute of Physics and Physical Society, by Chapman and Hall Ltd., London, Reinhold Publishing Corp., New York.

16. Nanavati, "An Introduction to Semiconductor Electronics", McGraw-Hill Book Co. Inc., 1963.
17. Thornton, DeWitt, Chenette, Lin, "CLT - Characteristics and Limitations of Transistors", SEEC Notes, John Wiley & Sons Inc., 1963.