A STUDY OF THE FLEXURAL RIGIDITY OF REINFORCED CONCRETE BEAMS

by

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ABSTRACT

This thesis presents the method and results of an experimental investigation of the flexural ridigity properties of simply supported reinforced concrete beams subjected to short-term loading.

Six beam specimens were tested. The tensile reinforcement of the beams varied from 0.6 per cent to 3.1 per cent; the concrete quality being kept constant.

In general, it was found that as the applied loading increased, the measured value of the flexural rigidity initially decreased rapidly, then levelled off to a nearly constant value, and finally, prior to the applied moment reaching the ultim ate capacity of the test beams, again decreased rapidly. A survey is made of the present practices and codes relating to the subject, and comparisons made between the measured values of flexural rigidity with values calculated according to the generally accepted analytical methods.

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PART 1

INTRODUCTION

1.1 Object

The object of this work is to study the flexural rigidity properties of reinforced concrete beams. The specific problem treated is the variation of flexural rigidity due to variations in the applied moments in simply supported beams subjected to short-term loading. Previous work done on this subject has been examined and the experimental procedures and findings of other investigators summarized.

It is hoped that the experimental work performed in connection with this thesis, together with the data that has been adapted from other investigations, will dispel some of the controversy presently existing on the subject and will provide a basis for future investigations involving other types of structural members and different types of loading such as impact, repeated and sustained loads.

1.2 Present Methods for Estimating Flexural Rigidity

The flexural rigidity of a structural member is normally thought of as the product of the modulus of elasticity E, which is a property of the material of which the member is fabricated and the moment of inertia, I, which is a property dependent upon the physical shape of the member.

There is general agreement as to how the E and I values are to be determined if a material such as steel is utilized - a material which, for structural design purposes, may be considered to be homogeneous and to obey the laws of linear elasticity within the range of the normal working stresses. This agreement, however, does not extend into the field of reinforced concrete.

It is recognized, although perhaps not quite as widely as one might at first expect, that the EI concept in its normal sense, when applied to reinforced concrete is really nothing but an artifice utilized to fit this material into theories which have been formulated for homogeneous, ideally elastic members.

a) Modulus of Elasticity

It is well known that the stress-strain relationship for concrete is not linear. To circumvent this difficulty, the E value is generally taken as the slope of a line secant or tangent to the stress-strain curve at some more or less arbitrary point. The experimental determination of an E value is a problem in itself, and the results may vary depending upon the method used.[9, 10, 17, 63] The designing engineer, not having available the experimental data of the concrete that will be used in his structure may estimate the E value from any one of a number of widely recognized formulae, such as

$E = 1.8 \times 10^6 + 500\, f'_c$ [46]; $E = \dfrac{6 \times 10^6}{1 + 2000/f'_c}$ [66]; $E = 1000\, f'_c$ [55]

$E = \dfrac{30 \times 10^6}{5 + \dfrac{10,000}{f'_c}}$ [49]; $E = 60,000\sqrt{f'_c}$ [66]; $E = 1.8 \times 10^6 + 460\, f'_c$ [49] ETC.

or he may obtain the modular value directly from a code of practice.[11, 14] Often some modifications are necessary to account for long-term effects.[3, 5, 14, 25, 46]

Each of the above methods may have something to commend it; perhaps some particular application in which close agreement is obtained with experimental results. However, the undesirable aspect in using a formula or looking up an E value for a certain "quality" of concrete[11] as one would do for a certain grade of steel is that the basic properties of the material are frequently forgotten - the E value thus obtained being regarded as a constant, applicable for the whole range of magnitude and duration of the stresses that will be induced in the structure. To the writer's experience, limited though it may be, surprisingly many practicing engineers do not fully realize the gulf that exists between the flexural stress-strain concept of the ideal material assumed in the derivations presented in the standard strength of materials texts and that of reinforced concrete.

The problem was pointed out by Freyssinet[6] in 1951, when in closing his paper on the deformation of concrete, he stated:

"However, I believe I have said enough to convince you that the classical strength of materials built up in the light of other materials and other methods than concrete and prestressing must be completely re-built....

"Applied to concrete the concept of mean stress loses all physical significance. Even the word "stress" has two completely different interpretations: on the one hand the ratio of a force to an area, which, if too great, will obviously be very dangerous; on the other hand, the product of a strain and an elastic modulus, a considerable excess of which in 99 times out of a 100 has absolutely no effect. In combining the two under the same term "stress" we are committing the same error as the chef who said he had made a pie containing equal parts of a horse and a rabbit; one horse, one rabbit."

b) Moment of Inertia

The methods recommended for determining the moment of inertia of reinforced concrete sections are indeed varied. For rectangular sections, some authorities recommend that the moment of inertia be based on the full transformed section, others neglect the reinforcement and/or the concrete cover; many favour the so-called "cracked section" including the transformed area of the steel, and some have suggested $\frac{1}{12} A_c H^{1.5}$ as a basis, H being the total depth of the member.[52] The Swedish State Specifications [11] require that "due allowance must be made for the influence of cracking within the tensile zone". However, it is apparently left up to the individual designer just how this allowance is to be made. Jain[64] has suggested that the effective value of I, for reinforced concrete arches, be taken "equal to the lowest value for the cracked section plus one-third the difference between this lowest value and the highest value for the uncracked section". Eppes[2] pointed out the differences of opinion amongst the better-known authorities. Fig. 1.1 is a graphical summary of the various recommended methods of determining the moment of inertia, reproduced from Eppes' paper.

1.3 Importance of Study

On the previous pages, the writer has indicated the rather wide range of basic assumptions upon which the designer may base his EI computations. It is obvious that the absolute stiffness values computed for a given structural member could vary appreciably depending upon the assumptions used. The question to be answered then is whether or not an estimate of the actual effective EI value is of practical importance. As one might expect at this stage, there is considerable controversy on this point also.

A number of designers including some authorities in the field feel that, except in estimating deflections which are of minor importance in most structures analysed on the basis of conventional theory, the actual stiffness value is more or less of academic interest - the relative value being the important one.

Upon hearing of this thesis project, a prominent professor of Civil Engineering at a large university in the United States wrote to the author: "... Insofar as structural analysis is concerned, the more important quantity is the relative EI value and not the absolute EI value. Consequently, if you are equally wrong in all parts of the structure, you may end up being right after all..." This thinking also appears to be reflected in the A.C.I. Building Code requirements.[55]

Nevertheless, even in ordinary frame analysis, it would appear that in a number of cases the beams could be expected to show some cracking in the tensile zone under working loads, while the columns do not. Therefore, if the stiffness of both beams and columns has been computed on the same basis, as specified by the A.C.I. code, the actual stiffness ratios and hence the distribution of moments will be different from that which was anticipated.

Eppes[2] and Marshall[1] have shown that the theoretical moments may vary 10 to 30 per cent depending upon variations in the ratio of column stiffness to girder stiffness. Further, Tichý and Vorliček of the Czechoslovak Academy of Sciences by means of a theoretical solution, not yet published, had found that owing to the variability of rigidity the maximum possible difference of the elastic bending moments in reinforced concrete continuous beams would be in the order of 8%, and recommended that this variability be allowed for. These investigators felt that "... the bending moments in a continuous beam should not, in practice be distributed according to the (ordinary) computation, even in the elastic region of loading. The non-homogeniety of material, fabrication, etc. caused the rigidity EI of reinforced or prestressed concrete sections to be non-uniform, even though the shape and the reinforcement of sections did not change along the beam. The rigidity of sections followed the laws of probability. Owing to the variability, the distribution of moments differed from the calculated moments, the kind of loading having some influence."

Undoubtedly, the practical significance of such possible discrepancies as mentioned above has been minimized by the fact that reinforced concrete can adjust itself to unexpected conditions. In discussing the capacity of reinforced concrete to withstand overloads, Freyssinet[6] stated: "This is only possible because concrete, like a living being, has a great ability to adapt itself to circumstances, as long as sufficient time is allowed for it to adjust itself. Without this adaptability, no concrete structure could exist."

There are, however, numerous other applications, although perhaps not as common as conventional frame analysis and deflection computations, where the flexural rigidity is an important factor. Some of these are lateral stability problems of isolated members[27,37] and the analysis of structures or structural components where the stresses depend upon the ratio of torsional to flexural rigidity.[1, 34, 47, 50] Also, in the various procedures of design based upon the ultimate strength concept[4, 56, 57, 64] - a concept which is presently gaining wider acceptance - the flexural rigidity properties of the structural members are of much greater importance.

Ferguson[46] in his recent book on reinforced concrete fundamentals, summed up the present situation as: "Further study is needed to establish what EI is really effective." This thesis is an attempt to provide further information on this subject.

1.4 Outline of Tests

The six 8" x 10" x 6'-0" long beam specimens were divided into three groups of two. The concrete quality was kept constant, the amount of tensile reinforcement being varied as shown below:

Beam designation	A_s, per cent
1, 2	1.4
3, 4	0.6
5, 6	3.1

All specimens were tested in 11 to 20 increments of load to failure. Deflections were measured by means of an Ames dial graduated to 0.001", and 6-8" standardizing strain gauges were utilized to measure strains on the concrete surface at three levels on each side of the beam - two inches from the extreme tension and compression fibres and at the centre line of the specimen. Readings were taken after each increment of load.

1.5 Acknowledgement

The investigation reported herein was carried out at the Materials Testing Laboratory of the Department of Civil Engineering, the University of Manitoba, under the general guidance of Professor C. Berwanger and Professor W. F. Riddell.

The author wishes to thank Mr. J. Wiebe of the University Staff for his most competent operation of the testing machine and the general assistance and encouragement that he gave; Mr. A. Reinart for assisting in preparing the test specimens and recording test data; the Highways Branch, Department of Public Works of the Province of

Manitoba for making available the particular type of strain gauges and part of the strain reading equipment used in the tests, and the Templeton Engineering Company for typing this report.

The writer feels especially indebted to Professor C. Berwanger who suggested the problem and provided encouragement and guidance throughout the tests and the analysis of the test results.

Thanks is also extended to Mr. S. Ödman, Techn. Dr. Concrete Research Institute, Royal Institute of Technology, Stockholm, Sweden, for his suggestions and assistance in supplying reference material.

1.6 Notation

The most common symbols are listed below for easy reference. Infrequently-used symbols are defined in the text of the paper when they are first introduced.

A = area

A_c = concrete gross area

A_s = area of tension reinforcement

b = width of rectangular member

c = distance from neutral axis to compression edge of beam

d = distance from centroid of tension reinforcement to compression edge of beam (effective depth of beam)

E = modulus of elasticity

E_c = modulus of elasticity of concrete (secant modulus, unless otherwise noted)

E_s = modulus of elasticity of reinforcing steel in the elastic region

EI = flexural rigidity or stiffness of a cross section

f_c = compressive unit stress in extreme fibre of concrete in flexure

f'_c = compressive strength of concrete as determined from tests of 6" x 12" cylinders

f'_{ct} = tensile strength of concrete

f_s = stress in tensile reinforcement

f_u = ultimate strength of tensile reinforcement

f_y = yield point stress in tensile reinforecment

H = total depth of a rectangular member

I = moment of inertia

I_c = moment of inertia of concrete, computed on assumptions as noted where symbol is used

I_s = moment of inertia of the transformed area of reinforcing steel

k = ratio of depth of neutral axis to effective depth of beam

L = length, as noted where symbol is used

M = bending moment

M_f = bending moment causing cracking of concrete in the tensile zone

M_u = ultimate moment capacity of beam

n = E_s/E_c = modular ratio

p = A_s/bd

P = concentrated load

R = radius of curvature of beam

δ = deflection

ϵ = strain

ϵ_c = strain in concrete

ϵ_s = strain in steel

ϕ = curvature of beam = $\frac{1}{R}$

PART 2

PREVIOUS INVESTIGATIONS

2.1 General Outline of Investigations

The majority of investigators examining phenomena where the flexural rigidity plays an important role have concerned themselves mainly with measuring deflections and deriving "correction factors", which, when used in conjunction with the particular method advanced by the investigator, would yield reasonable agreement between measured and computed deflections. Yu and Winter (3), for example, proposed that the moment of inertia be computed on the basis of the cracked section allowing for the transformed area of steel and the modulus of elasticity be estimated from the relationship $E = 1000\, f'_c$. For more accurate results a correction factor was proposed, the factor being $(1 - \frac{bM_1}{M})$ where M_1 is defined as $0.1(f'_c)^{2/3}\, H(H-kd)$. To account for creep effects, a different factor is proposed for estimating long-term deflections. The recommendations were based on results from a total of 175 tests. Blakey (25) on the basis of 33 beam tests, proposed to evaluate the flexural rigidity - for the purpose of deflection estimates - on a basis similar to that proposed by Yu and Winter, without the use of a correction factor. On the other hand, the Portland Cement Association (40), on the basis of deflection tests performed by Bach and Graf, concludes that "...the cracked section theory is not applicable to the common type of deflection problem. The use of the uncracked cross-section is more convenient and is believed to be a better approximation".

In these and other investigations probing the aspects of deflections (12, 15, 41) the flexural rigidity concept itself has played a minor role. The emphasis has been on the development of empirical methods for predicting deflections. In the process, the fundamental measure of flexural rigidity, the moment-curvature relationship, has received, by comparison, little consideration.

More recently, with the advent of ultimate design procedures for reinforced concrete, the fundamental aspects of flexural rigidity have been receiving more attention. At the symposium on the strength of concrete structures held in London in 1956(4), Baker, Guyon and Hajnal-Kónyi presented some theoretical approaches to the moment-curvature relationship, computations of rotations and probable variation of EI along the length of span. Jain (64) in a paper evaluating the ultimate strength of reinforced concrete arches proposed that the effective value of I be taken equal to the lowest value for the cracked section plus 1/3rd the difference between this lowest value and the highest value for the uncracked section. This empirical relationship was based upon experimental work done in connection with a PhD thesis submitted to London University in 1956. Unfortunately, this paper was not available for the writer's examination. Berwanger in his investigation of the application of ultimate design theory to reinforced concrete continuous beams (56) used the product E_c $(bd^3/12)$ as the expression for flexural rigidity, E_c being the initial tangent modulus. He found that this expression agreed reasonable well with the slope of the initial straight-line portion of the moment-curvature curve. A portion of Berwanger's experimental data has been adapted for this study of the flexural rigidity properties and is included in the section discussing the author's own test results.

Two investigators, Marshall and Eppes, have previously presented papers on the subject of flexural rigidity, while two others, Johnson and Nylander, in reporting on other phenomena have also made some study of this subject. A brief resumé of their work is presented on the following few pages.

a) MARSHALL. In April 1945, W. T. Marshall, PhD, published a paper entitled "The Flexural Rigidity of Reinforced Concrete Beams"(1) in which he criticized the vagueness of the British Code of Practice on this subject, pointed out some problems in which a realistic estimate of the effective EI is required and presented the results of flexural tests on 8 singly and 8 doubly reinforced simply supported beams. Fig. 2.1 is reproduced from Marshall's paper and shows the loading and strain reading arrangement employed and the EI vs P variation found.

The tests were made on 5-1/2 x 3" beams, 3' - 6" long tested in a 3'-0" span with third-point loading. The strains were measured by means of mirror extensometers fixed to the tension and compression sides on both faces of the test specimen. To gain some idea of the short-term effects of creep, the loading was maintained for a 10 minute period before applying the next increment; extensometer readings being taken at the beginning and the end of the 10 minute interval. This explains the discontinuities of the EI vs P curves shown in Fig. 2.1. No strain readings were taken beyond the computed working load of the beams. The flexural rigidity was computed from the relationship EI = MR, M and R being the total values of moment and radius of curvature at the end of each increment of load.

Marshall made no definite recommendations regarding the procedure to be followed in estimating the flexural rigidity, but did conclude that "... The experiments show the fallacy of calculating the EI on the uncracked section, for at the working loads the flexural rigidity is much less than that of the uncracked section. Also, the flexural rigidity varies with the amount of reinforcement whereas the value calculated on the uncracked section does not".

b) EPPES. The findings of this investigator were reported in the October 1959 issue of the A.C.I. Journal(2). A total of 9 - 6" x 6" x 36" beams were tested on a 32" span. An 8" Berry strain gauge was used to measure strains at two levels on the tension and compression sides on both faces of the test specimens. Fig. 2.2 shows the testing arrangement employed and the EI vs M relationship found. The theoretical basis upon which the EI values were determined was the same as Marshall's i.e. EI = MR. As shown in Fig. 2.2, the measured values of flexural rigidity decreased materially as the applied loading was increased.

In closing, Eppes felt that it was not possible to draw formal conclusions on the basis of his test results, however he did present the following general observations:

1. There appears to be two general values of EI, a value in the upper range comparing somewhat with the value calculated on the basis of the gross section of concrete with the transformed area of the reinforcing steel being included and a value in the lower range, comparable to the value computed on the basis of the cracked section.

2. The percentage of steel reinforcement affects the range of values of EI that a beam might have. The stiffness of a beam for a low percentage of steel would vary more with respect to moment than for a beam with a high percentage of steel.

C) JOHNSON. In 1950, this Swedish investigator published a largely theoretical paper dealing with the problem of flexural rigidity and deformations after the formation of cracks[5]. His approach to the problem was fundamentally different from that employed first by Marshall and later by Eppes. As a basis, Johnson studied a number of reinforced concrete prisms submitted to pure tension in order to establish the stress and strain pattern in the concrete and the reinforcing steel at and between tensile cracks. Then, assuming that the phenomena observed in the tension specimens would also be applicable to the tensile zone of a reinforced concrete beam, and that plane sections will remain plane after bending, he derived the expression

$$\frac{1}{EI} = \beta \frac{1}{bd^3 pE_s} \left[1 - \gamma \frac{M_f}{M}\right] \text{ - - - - - - - - - - - - } 2.1$$

where M_f = moment causing cracking of concrete in the tensile zone

and β and γ are exponential functions of the product np

Realizing that this equation would be too cumbersome for practical applications, Johnson simplified it into two equations:

$$\frac{1}{EI} = \frac{1}{bd^3 pE_s} \left[2.60 \sqrt[6]{np} - \frac{0.60}{\sqrt[6]{np}} \cdot \frac{M_f}{M}\right] \text{ - applicable for } 0.015 < np < 0.15 \text{ - - } 2.2$$

and
$$\frac{1}{EI} = \frac{1}{bd^3 pE_s} \left[5.0 \sqrt{np} - 0.9 \frac{M_f}{M}\right] \text{ - applicable for } 0.15 < np < 2 \text{ - - } 2.3$$

The approximations involved in making these formulae applicable for a range of p and n values are in the order of 5 - 12 per cent.

In order to make his formulae more palatable to the Swedish designer, who is used to compute deformations on the basis of the gross concrete section and an arbitrarily reduced E_c value to allow for the effects of cracking, Johnson substitutes $I = \frac{bH^3}{12}$ and $d = 0.92H$ into equations 2.2 and 2.3 and obtains

$$E_{jd} = \frac{pE_s}{0.28 \sqrt[6]{np} - \frac{0.082}{\sqrt[6]{np}} \cdot \frac{M_f}{M}} \text{ for the range } 0.015 \leq np \leq 0.15 \text{ - - } 2.4$$

and
$$E_{jd} = \frac{pE_s}{0.53 \sqrt{np} - 0.12 \frac{M_f}{M}} \text{ for the range } 0.15 \leq np \leq 2 \text{ - - - - } 2.5$$

The quantity E_{jd} being called the "idealized modulus of elasticity".

Thus, the designer can follow his accustomed procedure for estimating I, and then, instead of taking an arbitrarily reduced value of E, he would compute it from equation 2.4 or 2.5 for the particular moment value he is interested in.

Fig. 2.3 is reproduced from Johnson's paper. It represents the general shape of the curve of the idealized modulus of elasticity - i.e. a plot of equations 2.4 or 2.5. It is interesting to note that since in this method I is treated as a constant through the whole range of loading the shape of the theoretical E_{jd} vs M curve should in fact be similar to the EI vs M curve determined experimentally.

d) NYLANDER The main subject matter of Professor Nylander's papers reviewed by the author (58, 61, 65) was not flexural rigidity but such other structural problems as transfer of moments due to formation of cracks and torsional restraint by concrete structures. The flexural rigidity, however, did enter as a secondary consideration in several instances, and received considerable attention. Fig. 2.4 is reproduced from ref. 58 and shows the measured variation of flexural rigidity with applied bending moment for 10 beams with varying degree of tensile reinforcement. The beams tested had a cross-section of 20 cm by 20 cm and a constant moment span of 13 m, the moment being applied by means of weights cantilevered outside the vertical supports. The flexural rigidity values were based on deflections measured by means of dial gauges at 3 locations along the constant moment section.

Professor Nylander noted that "... it is characteristic for the flexural rigidity to fall off rapidly with the appearance of the first flexural cracks, or to be true even somewhat before the first cracks can be observed".

2.2 Discussion

Professor Marshall's pioneering work demonstrated that some reduction in the EI value occurs as loading is applied. However, his investigation was limited in that it was not carried beyond the working loads computed on the basis of a contemporary code of practice.

Eppes reported on a more extensive investigation and presented a number of EI vs M curves, however, his computations of the flexural rigidity, like those of Marshall, were based on the ratio of <u>total</u> moment to <u>total</u> curvature at the instant strain measurements were taken. The author of this thesis, however, feels as was proposed by Professor Berwanger, that when the moment-curvature relationship is not linear over the whole range of loading in question, a better estimate of the effective EI value would be obtained from the ratio of <u>change</u> in moment to the corresponding <u>change</u> in curvature. In other words, the effective EI could be visualized as the slope of the moment-curvature curve in much the same manner as the E value is visualized as the slope of the stress-strain curve. As long as the moment-curvature relationship is linear, there is really no difference between the two approaches, but when it becomes non-linear, or when sudden changes in slope occur, marked differences are found as will be shown later.

The author found Johnson's theoretical approach to the problem very interesting. A number of authorities feel that the phenomena of cracking, crack spacing and depth, the distribution of stresses at and between cracks, and other factors, are so complex as to defy a theoretical treatment of the problem of variations in flexural rigidity with applied loading. Johnson made certain simplifying assumptions to permit the derivation of his equations for the "idealized modulus of elasticity". The main difficulty in the application of his method is that an estimate of the tensile strength of the concrete is required - a property dependant upon a large number of factors. A comparison of Johnson's method with values measured during the course of these experiments will be made later.

Nylander's EI vs M curves shown in Fig. 2.3 were based on deflection measurements. It would appear to the author that deflections are not as sensitive and representative a measurement of the flexural rigidity as the moment-curvature relationship. The general shape of the EI vs M curve, however, should be the same.

PART 3

THEORETICAL CONSIDERATIONS

3.1 General Differential Equation of the Elastic Curve

In calculus it is shown that the expression for curvature is

$$\phi = \frac{d^2y/dx^2}{\left[1 + (dy/dx)^2\right]^{3/2}} \quad \text{------------} \quad (3.1)$$

If the slope dy/dx of the particular curve in question is small at any given point, as is the case of the elastic curve for beams whose deflections are small in comparison to their span lengths, the higher power of this small quantity $(dy/dx)^2$ will be extremely small and may therefore be disregarded, thus simplifying the expression for curvature to

$$\phi = \frac{d^2y}{dx^2}$$

Further, in the standard strength of materials texts it is shown that the moment-curvature relationship for a beam is defined by

$$EI\,\frac{d^2y}{dx^2} = M \quad \text{--------------} \quad (3.2)$$

This moment-curvature relationship, derived on the basis of more or less idealized assumptions, has been extensively utilized in structural analysis regardless of the material used. In order to facilitate the analysis of reinforced concrete structures, which by the nature of their fabrication are normally highly indeterminate, various authorities have prepared extensive tables of moment coefficients and evaluated integrals for variable section members,[14, 40, 43] all based on this basic moment-curvature relationship. It may be of interest here to review the approximations involved in applying equation 3.2 to reinforced concrete. The fundamental assumptions underlying this relationship are:

1. The slope of the elastic curve is small
2. Plane transverse sections before bending will remain plane during bending.
3. The proportional elastic limit of the material is not exceeded.
4. The modulus of elasticity is the same for both tension and compression.
5. The beam has a longitudinal plane of symmetry and the loads are acting in this plane.
6. The material is homogeneous.
7. The loads are gradually applied.

The first assumption is true if one considers the slope of the average deflected curve and neglects local sudden variations which may occur at tensile cracks. The second assumption is not strictly correct for a reinforced concrete beam, since the strain in the concrete on the tension side of the neutral axis will vary considerably at any given level due to the formation of cracks. This was demonstrated by A. H. Mattock, PhD, in his paper on the strength of singly reinforced concrete beams in bending.[4] Figure 3.1 showing the probable distribution of strains at the level of the tensile reinforcement is reproduced from his paper. If, however, in a beam not failing by excessive yield of steel, one measures the extension per unit length of a gauge length including several cracks, the apparent tensile strain will be found to vary very nearly linearly with the distance from the neutral axis. This was shown in experiments by Hajnal-Kőnyi, Lewis[4] and Baker,[53] the latter finding that this relationship held even close to the ultimate moment capacity of the test beams.

Modifications have to be made to the general moment-curvature relationship if conditions 5 and 7 are not met, regardless of the material used, these need therefore not be considered here.

Assumptions 3, 4 and 6 involve the greatest approximations. A reinforced concrete section, of course, is not homogeneous, nor is its modulus of elasticity (using this term loosely) after the formation of cracks the same for tension and compression, and the limits of the integral $\int_A y^2 da$, which defines the moment of inertia of the cross section, cannot be established at all sections along the length of the beam.

These discrepancies, however, do not invalidate the basic approach to the moment-curvature relationship. Equation 3.2 could be written $M = X\phi$ where X is a parameter relating moment to the average curvature. In effect, this is what has been done when the EI concept has been applied to reinforced concrete. However, the fact that the symbols EI have been used to denote X often has clouded the fact that in the case of reinforced concrete this parameter is really a complex variable rather than the product of two well-defined properties of the material and the cross-section of the structural member in question. Thus, as long as the limitations of the basic properties and definitions are realized, there should really be no objection to applying the relationship $EI = \frac{M}{\phi}$ to reinforced concrete. The important points to be kept in mind are:

(a) The effective EI value is not exclusively defined by the material and geometric properties of the cross section but is also influenced by the type and intensity of loading, and

(b) The moment-curvature relationships are greatly modified by time due to the phenomena known as creep. Thus, values obtained from short-term tests cannot be indiscriminately applied to problems involving sustained loadings.

3.2 Flexural Rigidity Considerations

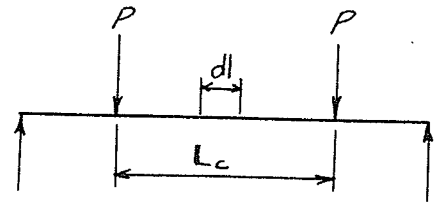
Experimentally, the flexural rigidity of a beam could be determined in two ways:

1. On the basis of deflection measurements, utilizing the relationship $EI = \frac{KM}{\delta}$ where K is a constant depending upon the loading and conditions of support.

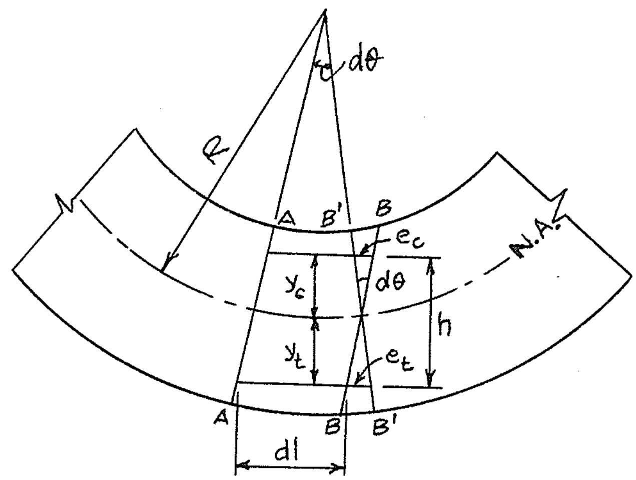
2. On the basis of strain measurements to establish the curvature, then obtaining the flexural rigidity from the relationship $EI = \frac{M}{\phi}$

As was discussed in part 2, both methods have been used previously. It was felt, however, that the second method, involving the fundamental measure of flexural rigidity - the moment-curvature relationship - provides a more accurate basis and yields a truer picture, and was therefore adopted for this investigation. Deflection measurements were obtained for the purpose of comparison. Even the method adopted, however, can be treated in two ways: the EI can be considered as the ratio of the total moment to the total curvature, or as the ratio of change in moment to the corresponding change in curvature. The former method was used by two previous investigators[1,2] the latter was proposed for this investigation. As was mentioned previously in Part 2, there is no difference between the two approaches as long as the moment-curvature relationship is linear, but marked differences are found when this relationship becomes non-linear.

3.3 Measurement of Curvature



Consider a straight beam subjected to two concentrated loads of equal magnitude and symmetrically positioned on the beam. The central portion of the beam, L_c, will then have zero shear and be subjected to pure bending; thus deforming into a circular arc of radius R. Referring to the sketch, let A-A and B-B represent two plane transverse sections a finite distance dl apart, parallel to one another before the beam was loaded. After loading is applied, section B-B will rotate, with reference to section A-A, through an angle $d\theta$ into position B'-B'. Now if we let the distance between the plane sections, dl, be the gauge length, and consider two gauges a vertical distance h apart, then these gauges would be measuring the tensile and compression strains, e_t and e_c resp., which result from the bending of the beam. Let y_c and y_t be the distances from the gauge lines to the neutral axis as indicated,



Then, $y_c = \dfrac{e_c}{d\theta}$ and $y_t = \dfrac{e_t}{d\theta}$

also, $y_c + y_t = h$ - - - - - - - - - - - - - - - (3.3) substituting values of y_c and y_t into equation 3.3 yields

$$h = \frac{e_c + e_t}{d\theta} \quad \text{- - - - - - - - - - - - - - - -} \quad (3.4)$$

But $d\theta = \frac{dl}{R}$; substituting this value of $d\theta$ into equation 3.4 and rearranging, yields $R = \frac{dl\ h}{e_c + e_t}$ - - - - - - - - - - (3.5)

Since the unit strains are $\epsilon_c = \frac{e_c}{dl}$ and $\epsilon_t = \frac{e_t}{dl}$,

equation 3.5 can be re-written as

$$R = \frac{h}{\epsilon_c + \epsilon_t} \quad \text{or} \quad \frac{1}{R} = \phi = \frac{\epsilon_c + \epsilon_t}{h} \quad \text{- -} \quad (3.6)$$

Thus, the curvature can be measured by having two gauges a known distance h apart, measuring the tensile and compressive strains. In these tests, the curvature was established in the above manner. In order to achieve more accurate results, a total of 6 gauges was used, 3 on each face of the test beam as shown in Fig. 4.1b With this arrangement average strains could be computed and a check on the gauge readings also obtained. Thus, increments of curvature corresponding to loading (moment) increments were computed on the basis of equation 3.6 and the flexural rigidity then obtained from the relationship $EI = \frac{\Delta M}{\Delta \phi}$.

The validity of this method of establishing the curvature of a reinforced concrete beam, depends upon the validity of the fundamental assumptions discussed in Section 3.1. However, as was then pointed out, a gauge length sufficiently large to include several tension cracks, as was the case in these tests, would yield average results which closely agree with the elastic theory upon which equation 3.6 is based.

PART 4

MATERIALS, FABRICATION AND TESTING PROCEDURE

4.1 Materials

(a) Cement

Ordinary Type I Portland Cement, manufactured by the Canada Cement Company, was used in all mixes. The cement was supplied in bags and stored in the Materials Testing Laboratory.

(b) Fine and Coarse Aggregate

Both aggregates were obtained from the normal laboratory supply. The fine aggregate was a sand having a fineness modulus of 2.30 with a gradation which followed closely the upper limit permitted by the Standard Specifications for Concrete Aggregates, A.S.T.M. Designation C33. The coarse aggregate was un-washed crushed limestone, 3/4" maximum size. Since considerable amounts of rock dust was observed adhering to the crushed limestone aggregate, it was decided to run a wash test on same. This revealed a surprising 10.4 per cent passing the No. 200 sieve. The aggregate sieve analyses are given in Table 4.1

TABLE 4.1

SIEVE ANALYSIS OF AGGREGATES

SAND		CRUSHED LIMESTONE	
Sieve No.	Percent Retained (cumulative)	Sieve Size	Percent Retained (cumulative)
4	0.7	1-1/2 inch	0
8	6.8	1 inch	0
16	21.0	3/4 inch	0.6
30	41.0	1/2 inch	53.6
50	68.5	3/8 inch	80.0
100	92.3	No. 4	96.7

WASH TEST:

Passing No. 200 sieve = 2.8 per cent   Passing No. 200 sieve = 10.4 per cent

(c) Concrete Mixture

The concrete mixture was designed to have a 28-day cylinder strength of about 4000 psi, due consideration being given to the anticipated curing conditions. The average strength actually attained was 4280 psi.

The average proportions of cement to sand to crushed limestone were 1: 2.57: 2.57 by weight and the cement-water ratio was 1.76. The average slump was 2 inches. Mixing of the concrete was done in a tilting drum mixer of 2 cu. ft. capacity. It was necessary to keep a careful check on the mixing process in order to ascertain that the rock dust on the coarse aggregate would not prevent bond between cement and aggregate. Owing to the relatively small capacity of the mixer, two batches were required for each beam. One 6" x 12" control cylinder was taken from each batch. The results of compression tests of these cylinders are listed with the results of the beam tests.

(d) Reinforcing Steel

All of the reinforcing steel required for this investigation was received in one shipment. The longitudinal tensile reinforcement in the test beams consisted of #4 and #5 deformed bars. Stirrups were bent from #3 bars. Tension tests were performed on two samples of #4 and #5 bars. The average cross sectional area of the test-samples was determined by dividing the weight of the sample by the product of its length and density; an average of 3 measurements being used for the density. The strains were measured by means of an Ames dial and a 2-inch gauge length extensometer attachment enabling strains to be read to the nearest 1/10,000 -th of an inch. In each case, a definite yield plateau was noticed. The results of the tension tests are shown in Table 4.2.

TABLE 4.2

TENSION TESTS OF REINFORCEMENT

Bar Size	A_s Sq. In.	f_u av. psi	f_y max. and min. psi	f_u av. psi	Av. elongation in 2'' at ult. load per cent	E_s aver. psi x 10^6
#4	0.196	45,900	no measurable spread	72,900	14.0	29.6
#5	0.317	42,900	43,200 42,600	74,200	18.8	29.0

4.2 Fabrication and Curing

The design of the test beams was based on the following considerations:

1. The size of the specimens was limited to an 8'' width and a 6'- 0'' length by the steel forms available. It was felt that the 14'' depth of these forms would make the beams too rigid for the purpose of this investigation, consequently it was decided to screed the test specimens off to a depth of10''

2. In order to permit an investigation of flexural rigidity values up to the ultimate moment capacity, a bond or diagonal tension failure could not be permitted. Also, the constant moment section within the fixed span length was to be as long as the bond and diagonal tension considerations permitted, in order to assure the greatest possible separation between the load and gauge points.

3. The quality of the concrete was to remain uniform and the percentage of tensile reinforcement was to vary in order for the tests to indicate the effect of such steel variations on the flexural rigidity.

∴ A total of 6 beams were cast, a set of two at a time. Figure 4.1 (a) shows the principal dimensions of the beams and the reinforcement provided. The reinforcing steel was tied into a rigid unit prior to being placed in the forms. Two batches of concrete was required for the casting of each beam. In order to assure a uniform quality of concrete within the section where the strains were to be measured, the central portion was cast from one batch, the ends being filled in from the second batch. One control cylinder was taken from each batch. The concrete was consolidated through the vibration of forms, this being accomplished by means of an electric motor driven inertial vibrator, permanently attached to the under side of the steel forms. The mix was screeded to a uniform depth of 10''. Figure 4.2 shows the steel forms and the vibrator drive. The dimensions were found to be true to within approximately 0.2'' at the time of testing. Actual measured dimensions have been used in the computations.

The forms were removed three to four days after casting and the beams stored at 70 - 75° F in the air of the laboratory until testing took place at an age of 28 days. The control cylinders were also air cured so that they would be representative of the concrete in the test beams.

4.3 The Standardizing Strain Gauge

A modified version of a recently developed unbonded wire strain gauge, known as the Standardizing Strain Gauge, was used for strain measurements in the beam tests. Originally, this gauge was devised primarily for measurements requiring long-time stability. However, it is quite versatile, being adapted to different measuring problems simply by changing the method of attachment. Reference 62 presents a more complete discussion of the fundamentals of this gauge and the various applications to which it is suited.

Professor Berwanger of the University of Manitoba made several modifications to this gauge and devised a number of new methods for installation and attachment. Figure 4.3 illustrates diagrammatically the particular modification which was used in these tests. In principle, the essential parts of the gauge are a tube, a piston fitted into one end of the tube and a small diameter unbonded Advance wire stretched inside the tube from the piston to the other end of the tube. The wire is attached in such a manner that it is under a slight tension when the piston is in the normal position, i.e. when there is no pressure applied to it. Some means are then devised which would provide for the measurement of strains through the measurement of the length of travel of the piston.

In this application, the gauge was provided with a pair of mechanical gauge points - one attached to a brass tube freely sliding on the piston end of the gauge, the other to a second tube which was attached to the other end of the gauge and provided with an opening for the passage of the lead wires and for the application of pneumatic pressure to the piston. The whole assembly was then clamped to the special gauge plates. The clamping procedure is discussed in the next section.

The change in resistance per unit resistance of the Advance wire has been found experimentally to be proportional to the strain of the wire. Knowing the initial resistance of the wire and its proportionality constants, the various strains may be determined by measuring the changes in electrical resistance of the wire with conventional strain bridges. Referring to Figure 4.3, an initial gap d is set between the invar screw and the pin attached to the piston. By applying pneumatic pressure to the piston it can be made to move forward to touch the invar screw. This movement will strain the Advance wire and by measuring the resultant changes in electrical resistance, this initial gap can be measured. This would constitute a "zero" reading. When the gauge length L then undergoes a change ΔL due to strains induced in the test specimen, an identical change Δd will be reflected in the gap d. By again extending the piston, the new gap $d \pm \Delta d$ can be measured. The strain Δd is then the difference between this reading and the initial reading. This procedure would be repeated throughout the test - the piston being extended each time a strain reading is to be obtained. The maximum value which $d + \Delta d$ may have is fixed by the elastic properties of the wire as the wire strain must not exceed the elastic limit. For the Advance wire used in these gauges, this limit was approximately 5000 micro-inches per inch. The next section will describe in detail the attachment of the gauges.

4.4 Attachment of Strain Gauges

In order to measure strains with this Standardizing Strain Gauge assembly, special gauge plates had to be fastened to the beams (Figure 4.3). The following procedure was used:

A grid, as shown in Figure 4.1 (b) was accurately laid out on both sides of the beam to straddle the centre line of the span. The lower gauge line was measured 2" from the smooth underside of the beam and the central and top gauge lines laid out 3" and 6" resp. from this bottom line. The remaining distance between the top gauge line and the screeded surface of the beam was found to be within the limits $2''^{+1/8}_{-1/16}$. Only in one case, on one side, this dimension was found to be 2-1/4". With the grid as a guide, the gauge plates were then attached to the beams by means of "Seal-All" brand cement. This method of attachment was found to be very satisfactory - the cement formed so good a bond with the concrete that when the gauge plates were removed after testing, chunks of concrete invariably came off together with the plates. The horizontal spacing of the gauge plates was checked by means of an 8" set bar and the vertical spacing by means of a steel ruler graduated to 1/128". It was originally thought that the clamping brackets would allow considerable flexibility, however it was found that for satisfactory overall alignment the gauge plates should not deviate from the grid by more than about 1/32". The glue hardened in approximately 2 to 3 hours, however in order to preclude any possibility of movement between the concrete surface and the gauge plate, some 16 to 20 hours were allowed to elapse prior to clamping on the gauges and testing.

The clamping of strain gauges to the gauge plates was accomplished by means of brackets especially designed for this purpose. A total of two sets of three brackets was required to attach the 6 gauges. Figure 4.4 shows details of one set of brackets and illustrates the method of clamping one end of the gauges. A second, identical set of brackets was used to clamp the gauge points at the other end.

The brackets were fabricated from 1-1/2" x 1/2" plate stock - the 1/2" thickness being required for the method of fabrication. 4" x 1/4" stove bolts with rounded ends were used for supporting and levelling the brackets and 2" x 1/4" bolts for clamping the gauges. The bolts used for this purpose had special machined ends to fit into holes provided in the invar plates at the gauge points of the strain gauges. The steel straps by means of which the bottom bracket was suspended from the top bracket was free to rotate and was provided with large holes through which the clamping screws could pass freely so that each bracket could act independently of the others.

In order to avoid damage, the gauges were not attached until the beam had been positioned in the testing machine. Following that, the brackets were placed around the test specimen and aligned so that the clamping screws would line up with the gauge plates; the gauges inserted one at a time, adjustments being made in the alignment of the clamps as required, and finally the clamping screws tightened.

4.5 Testing Procedure

All beams were tested in a 200,000 lb. Riehle testing machine. The location of the position of the supports and loading points was marked on the beam before it was introduced into the testing machine. The end supports were positioned according to marks on the testing machine and the beam then lowered onto the supports and positioned correctly transversely and longitudinally. Next, the loading plates and rollers were centered over the marks on the beam and the loading beam lowered on the load supports and centered on the beam. It was found necessary to build the loading beam supports up somewhat to provide clearance over the gauge brackets. With the loading beam in place, the overall arrangement was checked to ascertain that everything was still positioned correctly. An Ames dial, cantilever supported by a laboratory stand, was then introduced under the beam at the centre of the span for the purpose of deflection measurements. The Standardizing Strain Gauges were finally clamped to the test beam according to the procedure outlined previously. An initial gap was set by opening the invar screw 1/4 turn for the gauges on the tension side, 1/2 turn for the central gauges and 1-3/4 turn for the top gauges - the larger gap being required on the compression side since the strains to be measured would be negative, i.e. the gap would be closing.

When all the gauges were in place and the beam ready for testing, the electrical circuits were checked. Frequently it was found that one or two soldered connections had broken loose thus necessitating last-minute repairs. An average gauge factor setting of 2.16 was set on the strain indicator and an "in clamps" reading taken on all 6 gauges. This reading gave the resistance of the gauges and lead wires; the values being set on the variable resistance box to compensate for the initial resistance of the gauges. i.e. a variable resistance box was used instead of a dummy gauge. The "in clamps" reading also served as a check on the gauge and the installation, as it had to agree reasonably well with the value measured during the calibration of the gauge - allowance being made for the additional resistances of the longer leads and the multi-terminal box. Major differences usually indicated a partial short-circuit or poor connections.

Prior to the application of the first load increment, a "zero" reading was taken on all gauges by applying air pressure and forcing the piston to extend against the invar screw, thus establishing the magnitude of the initial gap. The air pressure was supplied through a system of plastic tubing connected to the permanent air line in the laboratory. This air line, however, was fed from a tank which was kept at pressures ranging between 50 and 90 psi - pressures too high to be applied directly to the gauge. Pressure reduction was obtained by throttling through the valve located at the take-off from the laboratory air line. A set of needle valves was installed at the other end of the plastic tube which fed the air from the permanent line to the instrument bench. These valves were kept partially open to maintain the pressure in the feed line at the desired level. Air pressure to the 6 gauges was supplied through an equal number of plastic tubes, attached to a brass header located at the end of the main feed line. Each gauge was isolated through a needle valve which was opened only when a reading was taken. Figures 4.5 and 4.6 illustrate the general arrangement of air hoses, valves and gauges. Figures 4.7 - 4.10 are photographs of the beam test set-up.

Due to the tight schedule established for the beam tests, it was found that the control cylinders had to be tested some 16 - 20 hours prior to the beam tests. However, it was felt that at an age of 28 days any possible changes in the properties of the concrete attributable to the few hour differential between the beam and control cylinder tests would be negligible. The cylinders were also tested in the 200,000 lb. machine. Strains were measured on two diametrically opposite surfaces by means of 8" Berry gauges. In the first tests, the same gauge plates were used in both beam and cylinder tests. For the last two tests, however, special gauge plates were provided for the cylinder tests, as it was felt that better accuracy would be obtained if the taper of the holes in the gauge plates was identical to the taper of the mechanical gauge points of the Berry gauge.

PART 5

PRESENTATION AND ANALYSIS OF TEST RESULTS

5.1 Results of Beam Tests

The strain readings recorded during the flexural tests of the 6 beam specimens are reproduced in tables 5.1 to 5.6 inclusive. The flexural rigidity vs. moment and moment vs. curvature relationships for the beams were computed according to the procedure outlined on pages 3.4 to 3.7 on the basis of these strain readings, and are shown in figures 5.4 to 5.14.

The strain readings obtained from tests of the balanced design and over-reinforced beams, i.e. beams No. 1, 2, 5 & 6, were on the whole quite satisfactory and could be applied directly for the computation of curvatures. An average of the readings of the two tension and compression gauges was used in the computations. In the case of beam No. 2, though, too large an initial gap had been set for gauge #1, as a result of which the Advance wire in the gauge was over-strained requiring the replacement of this gauge for the succeeding tests. Further, under initial low strains the #2 gauge provided rather erratic readings, and it was therefore necessary to discard the first four readings. Consequently, no M vs. ϕ curve was prepared for beam No. 2, and the EI vs. M curve, shown in figure 5.6, is not complete over the whole range of loading.

The analysis of the under-reinforced beams, No. 3 and No. 4, proved to be somewhat more complex. As can be seen from the recorded strains (tables 5.3 and 5.4) the behaviour of the compression gauges #1 and #2 was quite erratic and not at all compatible with the strains recorded by the central and bottom gauges. After considerable study of the problem it was decided to discard the readings of the top gauges and to compute the compressive strains by straight line interpolation between the strains recorded by the central and bottom gauges. Considering that the strains had been measured over an 8-inch gauge length, this straight line interpolation may be expected to yield reasonably valid results, as was discussed in section 3.1. In addition, strain relationships examined for the other beams provided further arguments for the validity of this method of interpolation. (See Figure 5.16)

Attempts were made during the tests to visually trace the crack development pattern in order to possibly relate it to curvature variation phenomena. However, due to the somewhat inadequate lighting and the limited number of observers, this was not too successful as in a number of cases the first crack was observed under loads considerably in excess of the value indicated by test results.

Although somewhat of an approximation, the average moment-curvature relationship can be represented throughout most of the loading range by a series of straight lines, as shown in figures 5.5, 5.8, 5.10, 5.12 and 5.14. Similar relationships were also found in a study of data from 6 continuous beams, tested by Professor Berwanger in connection with an ultimate strength design investigation. The moment-curvature relationships for two of the beams are shown in figures 5.17 and 5.18.

In examining figures 5.5 to 5.14 it will be noted that the M vs ϕ relationship changed suddenly when the applied moment reached a value ranging between 60 and 90 inch kips. The theoretical stress in the extreme tension fibre under this applied moment would be in the order of 10 to 16 per cent of f'_c - a value approximately equal to the tensile strength of the concrete.[46] In other words, when the tensile strength of the concrete is exceeded and cracking develops, a sudden change occurs in the moment-curvature relationship. In the cases where strain readings were obtained close to the ultimate moment capacity of the test specimen, a second major change in the moment-curvature relationship will be noticed - this change occurring just prior to the ultimate load when general yielding is beginning to take place. Since slope of the M vs. ϕ curve represents the flexural rigidity it would appear that the beams have constant EI values over certain ranges of loading, sudden transitions occurring when cracking occurs and sometimes also near the ultimate load. In reality, however, an instantaneous transition from one value to another, as would be indicated by a break in a straight line, does not occur, as can be seen from the EI vs. M curves in figures 5.4 to 5.13. Eppes[2] and Johnson[5] predicted a definite initial plateau in the EI vs. M relationship, which presumably would correspond to the initial slope of the M vs. ϕ curve. The author's experimental results did not indicate such a definite plateau thus reinforcing Professor Nylander's contention that the flexural rigidity begins to fall off even before the first cracks can be observed. However, the number of readings obtained prior to cracking was rather limited and the measurement of the small initial strains more subject to error. In theory, under short term loading conditions it would be reasonable to expect the initial EI value to be fairly constant prior to cracking and to correspond to the value of the full transformed cross section.

On the whole, in these tests the general flexural rigidity vs. moment relationship was found to have an S shape - i.e. the EI values initially decreased sharply, attaining a more or less constant value for moments in the range of 0.38 M_u to 0.80 M_u on the average, and beyond the latter value of moments again decreased rapidly. In the case of beam No. 6, the first heavily reinforced beam to be tested, it turned out that strain readings were not obtained in that stage of loading which produced the later drop in EI values. The strain gauges were removed after the loading reached 90% of the theoretical ultimate moment capacity, because it was feared that the gauges might be damaged if a sudden compression failure occurred. In the case of this beam, however, the preliminary estimate of the ultimate moment capacity turned out to be considerably less than the actual value. Nevertheless, the EI vs. M curve obtained does cover the full working load range. In the next section, comparisons will be made between these experimental values of flexural rigidity and those computed according to various recommended analytical methods.

In order to compute the flexural rigidity according to any of the recognized methods, an estimate of E_c is required. The experimental determination of the E_c value for the test beams was not in all cases entirely satisfactory. For example, the measured values of the secant modulus as obtained from compression tests of control cylinders, appears to be too high for beams 2, 5 and 6, being 7.0 x 10^6 psi, 6.5 x 10^6 psi and 6.7 x 10^6 psi respectively, whereas the ultimate strengths were 4360 psi, 4470 psi and 3700 psi resp. A study of the stress-strain relationship measured for control cylinders from beams 3 and 4 revealed that the ASCE - ACI Joint Committee recommendation for estimating the secant modulus of concrete, $E_c = 1.8 \times 10^6 + 500 f'_c$, closely agreed with these test results (see figure 5.15) and since basically the same mix had been used in all test beams, it was decided to utilize the above mentioned relationship to estimate E_c in all cases. Also, it was felt that it would be desirable to use such an analytical approach as might be employed by a designing engineer in practice.

5.2 Comparison of Measured and Calculated Values of Flexural Rigidity

Table 5.0 presents a comparison of measured and calculated EI values at the working loads of the beams. The working load was determined as follows: the ultimate moment capacity of the beam, M_u, was computed according to the ACI - ASCE Joint Committee recommendation for rectangular beams: $M_u = bd^2 f'_c q\,(1-0.59q)$ where $q = p\,\frac{f_y}{f'_c}$ and a load factor of 1.8 was then applied to estimate the working moment M_w i.e. $M_w = \frac{M_u}{1.8}$. The aforementioned expression for M_u was applicable for all test beams, as the limiting value of q(55) was not exceeded. In every case, the computed M_u was somewhat lower than the test value, leaving an actual factor of safety against total collapse to be 2.1 - 2.7. For purposes of comparison, both calculated and measured values of M_u are indicated in figures 5.4 - 5.13. Thus, the measured EI values tabulated in table 5.0 represent the flexural rigidity of the beams under design conditions, assuming an ultimate strength procedure of design is followed. The EI gross section was computed on the basis of the gross concrete section neglecting the tensile reinforcement; for the cracked section the transformed area of steel was allowed for and all the concrete on the tensile side of the neutral axis was discarded, as is customarily done. The Joint Committee recommended that "the load factor shall be taken equal to 1.8 for beams and girders subjected to bending only".(55) This factor was therefore adopted for computing M_w for the test beams. In actual structures, wind and earthquake loadings may be a consideration in which case somewhat different load factors may be desired(55). As a matter of interest, working loads were also computed on the basis of the usual straight-line theory. These ranged 9 - 17 per cent less than those based on the ultimate design method. For practical EI considerations, however, the difference is negligible.

TABLE 5.0

COMPARISON OF MEASURED AND CALCULATED EI VALUES

Beam No.	Calculated EI lb-in^2 x 10^6 Gross sect.	Calculated EI lb-in^2 x 10^6 Cracked sect.	Measured EI (at working load) lb-in^2 x 10^6 From EI vs M curve	Measured EI (at working load) lb-in^2 x 10^6 From slope of M vs ϕ curve
1	2580	1130	800	750
2	2860	1080	1280	-
3	2790	550	480	300
4	2970	580	1100	330
5	2270	1350	1680	1670
6	2670	1450	1350	1110

A number of interesting observations can be made from figures 5.4 - 5.14 and table 5.0. From the curves, it will be seen that by the time the working load is applied, the flexural rigidity has reduced to the level where it will remain more or less constant until close to failure. This appears to be the case with all beams, except No. 4. Even in this case, the EI value is reducing rapidly and small changes in M_W would produce large changes in the apparent EI. In comparing the measured flexural rigidities at working loads, it will be seen that the slope of the M vs. ϕ curve in a number of cases yields a somewhat lower value than that obtained directly from the EI vs M curve, the reason being that the straight-line interpolation of the former does not reflect the transition curves of the latter. The secondary slope of the M vs. ϕ curve agrees closely with the flat portion of the EI vs M curve. From a practical point of view, however, the simpler M vs. ϕ relationship appears to be of more use. From the comparison of calculated and measured values of flexural rigidity, it appears that the values computed on the basis of the fully cracked section agree best with the test results.

As is verified by Johnson's theoretical approach - the solutions of his equations 2.4 and 2.5 are reproduced in form of curves in figure 2.3 - the effective value of flexural rigidity should be somewhat higher than that computed on the basis of the cracked section. This is only reasonable, since the concrete between the tensile cracks will add to the rigidity of the beam. Eppes, see figure 2.2, found this to be the case. However, as was mentioned previously, his experimental values of EI were apparently based upon the ratio of total moment to total curvature, rather than the change in moment to the corresponding change in curvature, and therefore would be somewhat high for all values of moment beyond that which causes the first break in the M vs. ϕ relationship.

The measured values of EI for beams 1, 3 and 6 were somewhat below those computed on the basis of the cracked section. This could be due to a combination of experimental error, interpolation of curves, over estimating the E_c value at working load in the analytical method, and effects of creep. The latter encompasses a very extensive field in itself and usually enters as a major consideration in problems involving sustained loadings. Marshall[1] however, in tests of one-half hour duration found creep to lower the apparent EI value by as much as 7% from the initial value. Considering that these tests lasted over more than twice that time, and that there is a host of factors which influence the creep characteristics of a given concrete mix[13, 19, 45, 48] this aspect alone may account for the apparent slight discrepancy.

5.3 Errors in Tests and Analysis

The tests and the analysis reported herein are both subject to errors. For one, the basic assumptions upon which the analysis is based may not be strictly correct. The basic approximations involved were discussed in section 3 when it was shown that they were not of sufficient magnitude to invalidate the method of approach employed. However, there is some controversy as to the question of the applicability of this method for carrying out analysis in the loading range just prior to failure when general yielding begins to take place. Nevertheless, as the cross section approaches the "plastic" condition, deformations increase with little or no increase in the applied load, and hence the flexural rigidity of the section in question approaches zero, as was found in these tests and was further verified by the nearly horizontal M vs. ϕ relationship near the ultimate moment values found from Professor Berwanger's data.

As far as the tests themselves are concerned, it is quite apparent that all the physical components of variation in the tests could not be perfectly controlled. The reinforcing cage was carefully centred in the steel forms, but could quite conceivably have moved over to one side during the vibration of the forms, thus inducing an aspect of assymmetry. The moment computations are affected by the accuracy of the positioning of the test specimens in the testing machine. Even though the utmost care was used in setting up the tests, the positioning was done by eye thus undoubtedly inducing some error. In a number of cases there was a definite variation in observed strains between the two faces of the test beams, and although part of this variation was undoubtedly due to the non-homogenity of the material, it is also likely that part was due to eccentricities in the applied loading. In the computations, of course, average strains were used thus reducing the magnitude of the possible error due to this cause.

The gauge readings themselves may have been subject to error due to a number of causes. The clamping arrangement was devised to allow, in theory, the parallel, free movement of the gauge plates and the invar set screw, while keeping the gauges securely attached. In practice, however, the brackets turned out to be rather clumsy and heavy and may possibly have affected the readings. Any such possible restrictions, however, should reduce the measured strains and thus increase the apparent flexural rigidity. In these tests, some measured values were, if anything, on the low side. Nevertheless, since glue was found to work so satisfactorily in the attachment of the gauge point plates, it is suggested that in future investigations using this type of gauge assembly to measure concrete surface strains, the gauge plates and clamping device should be combined into one light unit and glued on to the surface. This would provide for a more flexible installation and would remove any possibility of restriction. Further, the readings of this type of gauge are affected by vibration of the strain gauge wires. This could have been caused by occasional pressure surges in the air lines, occurring when the compressor cut in. However, in all cases time was allowed for the gauge to stabilize. Whenever readings were taken, the potentiometer needle of the strain indicator was always steady. The standardizing strain gauges are also fairly sensitive to temperature changes. However, the laboratory temperature was carefully observed throughout the tests and found to vary no more than 1° F. Finally, an average gauge factor setting of 2.16 was used for all gauges. The maximum variation in the actual gauge factor values from this average was in the order of 1% and as a consequence, no gauge factor correction was applied.

The gauge brackets were fabricated to clamp the gauges on lines 2 inches from the top and the bottom and at the centre line of the beam. On second thought it would appear that the accuracy could have been increased if the gauges had been located a little closer to the extreme tension and compression fibres thus increasing the magnitude of the strains measured. Also, the central gauges were provided primarily for the purpose of providing a check on the readings of the outside gauges. However, being located close to the neutral axis, the central gauges measured relatively small strains, especially during the first few loading increments, and therefore were subject to relatively greater error. Although agreement on the whole was good, a better check would likely have been obtained had two gauges been utilized on each face both above and below the horizontal centre line of the beam. This, however, would have required a total of 8 gauges and necessitated a different method of clamping the gauges.

As was discussed on page 5.9, aspects of creep may also have modified the results. This, however, could not rightly be termed an error as far as the experimental results are concerned because creep is a factor in all concrete construction and therefore measured EI values modified by this factor would be closer to the true rigidity values. Creep effects on the M vs ϕ relationship under sustained loading would, of course, be much more pronounced and would involve a separate study.

In spite of the numerous approximations used and the number of possible sources of error the results on the whole appear quite reasonable. The object of these tests was to study the general flexural rigidity properties of the test specimen in order to gain more knowledge of this aspect of reinforced concrete. Had the object been to provide exact numerical data rather than a picture of the general trend, the magnitudes and effects of these various errors would have been considered much more critical. Concrete can, or should, be considered essentially a statistical material, and therefore any investigation attempting to formulate a new theory or provide a set of new equations would require the testing of a sufficiently large number of specimens to permit a statistical analysis of each of the variables which may be a consideration.

usable in that form. This scatter could no doubt be explained by the fact that the readings had been obtained by the use of a gauge having a gauge length of only one inch.

Some difficulty was experienced in checking Johnson's theoretical solution. However, as can be seen in the E_{jd} vs. M curve shown in figure 2.3, the flexural rigidity, on the basis of his method, may be expected to fall off rapidly once cracking occurs and then level off to a value somewhat greater than that computed on the basis of the fully cracked section. Thus, in general, the shape of this "theoretical" curve agrees reasonably well with the experimental values, with the exception of values just prior to the ultimate load. Then again, as Johnson himself pointed out, his relationships are not valid for very far advanced loadings.

It was thought to be of interest to compare the basis of computing flexural rigidity values used in this investigation with those used by previous investigators. The data from beams No. 1 and 5, covering the most complete range of loading, was chosen for the comparison. Figures 5.19 and 5.20 show EI values computed on the basis of test data from these two beams using the different basic assumptions discussed earlier in Section 3.2. As was expected, defining the flexural rigidity as the ratio of change in moment to the corresponding change in curvature rather than the ratio of total moment to total curvature, yielded lower values and indicated a more radical drop as the ultimate load was approached. It was extremely surprising, however, to find that the EI values based on measurements of deflections at the centre of the span yielded, in the case of these two beams, a nearly constant value. Furthermore, in the case of beam No. 5, this value was considerably below the more or less level plateau found by the other methods. The deflection measurements for the other beams were then studied as well and it was found that the characteristic S shape was reflected in those measurements. The author cannot offer an explanation as to why the deflection measurements from beams 1 and 5 should reflect a nearly constant EI value. One would certainly expect variations such as found by Nylander (see figure 2.4) and also in deflection measurements of beams 2, 3 and 4. Also, there seems to be no explanation as to why in the case of beam No. 5 the EI values based on deflection measurements should be so much less than those based on curvature measurements and the value computed by the cracked section. It may be concluded that deflections, as measured by one gauge only, are not as reliable a basis for establishing the flexural rigidity.

6.2 Conclusions

On the basis of the tests reported herein and the material examined in connection with this work, the following general conclusions may be drawn:

1. Allowances should be made for the effects of cracking when estimating the flexural rigidity of reinforced concrete beams under design load conditions. Of the analytical methods in common use on this continent, the "cracked section" method, which neglects all the concrete on the tension side of the neutral axis, agrees best with the measured values, although it may be expected to yield results which are somewhat on the conservative side.

2. The flexural rigidity of a reinforced concrete beam is not only a function of the shape of the cross section and the material properties as determined from the usual control tests, but is also a function of the type, intensity and duration of loading.

3. The amount of reinforcement provided affects the rigidity of the beam. The stiffness of a beam having a low percentage of tensile reinforcement may be expected to vary more with respect to applied loading than that of one having a high percentage. This aspect is not reflected in estimates based on the concrete gross section.

4. The moment-curvature relationship within the range of working loads may be approximated by a series of straight lines. The slope of these lines would indicate two fairly well defined values of flexural rigidity: one corresponding roughly

to the product of the initial tangent modulus of concrete and the moment of inertia of the full transformed section, this being effective up to the cracking load; and a second, considerably reduced value, corresponding approximately to the value computed on the basis of the cracked section and being effective up to the vicinity of loading producing initial yielding. In measurements of EI, however, the initial value does not appear too well defined, the measured rigidity having fallen off considerably even before the cracking load, as indicated by the break in the M vs. ϕ relationship, is reached.

5. The results reported herein and the conclusions drawn are based on tests of isolated, laboratory sized specimens. Whereas they are of value in indicating a general trend, when extending the conclusions to actual structures, the following should be recognized:

(a) In full scale structural members shearing forces and sometimes torsional moments and axial forces will modify the EI values.

(b) Creep will greatly influence deflections and rotations and due allowance for this phenomena should be made. Neville[19] made a fairly thorough study of this aspect of concrete and listed some 60 references on the subject. In addition, Yu and Winter,[3] Johnson,[5] Hanson,[36] 5.(b) Continued.

Backmark[41] and Nylander[61] have investigated the problem of long-term creep effects as related to deflections and re-distribution of moments.

(c) Concrete itself is a non-homogeneous material, hence for a specified 28-day strength, mixes using different types of aggregates and cements may usually be expected to exhibit different stress-strain characteristics and crack patterns.

(d) When reinforced concrete beams are subjected to repeated loadings, cracking develops at a maximum load smaller than the load causing cracking under static loading.[4] Therefore, modifications to the EI vs. M relationship will be required if the structural member is to be subjected to dynamic or repeated loadings of fair intensity.

In spite of the rather large number of differences that exist between the more or less ideal conditions of the laboratory and the actual conditions at the construction site, it is felt that the information gathered in this investigation is valid for the formation of a basis upon which the flexural rigidity of a structural framework or isolated member may be predicted, due cognizance being given to those of the aforementioned factors, that are applicable to the particular problem at hand.

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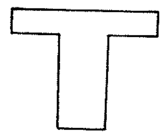
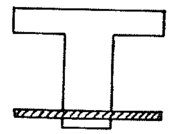
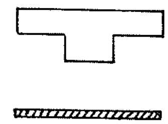
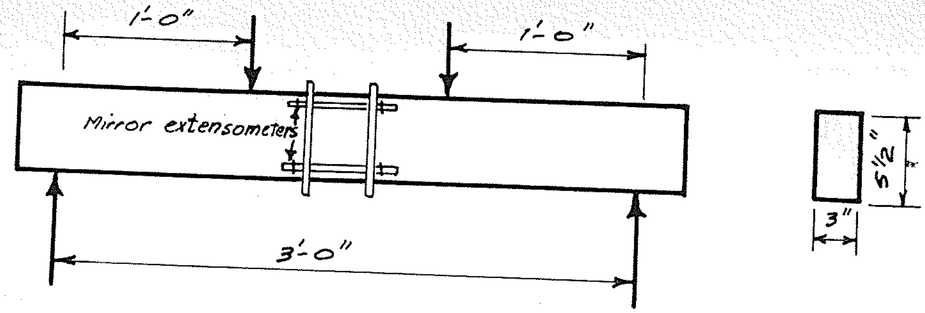
SOURCE	RECOMMENDATIONS FOR CALCULATED MOMENT OF INERTIA					
						
	GROSS SECTION		GROSS SECTION, INCL. TRANSF. AREA OF STEEL		CRACKED SECT. INCL. TRANSF. AREA OF STEEL	
	BEAM	COL.	BEAM	COL.	BEAM	COL.
TAYLOR, THOMPSON & SMULSKI	X	X				
C.R.S.I. DESIGN HANDBOOK	X	X OR	X	X		
ASCE-ACI JOINT COMMITEE REPORT	X			X		
BRITISH STANDARD CODE	X	X OR	X	X OR	X	X
ACI CODE	X ANY REASONABLE ASSUMPTION MAY BE ADAPTED FOR COMPUTING RELATIVE STIFFNESS OF COLUMNS AND OF FLOOR SYSTEMS. THE ASSUMPTIONS MADE SHALL BE CONSISTENT THROUGHOUT THE ANALYSIS.					
PORTLAND CEM. ASSOCIATION	X	X OR	X	X		
SWEDISH STATE SPECIFICATIONS		DEPENDING UPON LOADING	X	X OR	X	X

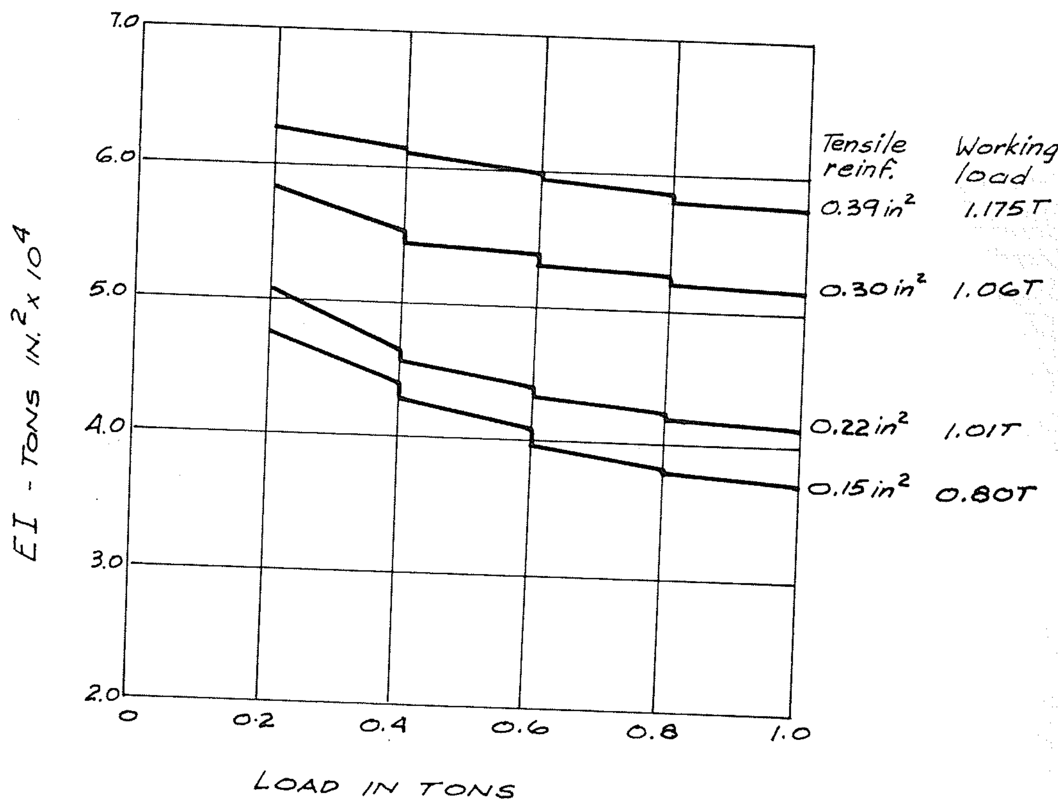
METHODS FOR CALCULATING MOMENT OF INERTIA, AS RECOMMENDED BY VARIOUS AUTHORITIES

ADAPTED IN PART FROM REF. 2

FIGURE 1.1



METHOD OF TEST EMPLOYED BY MARSHALL

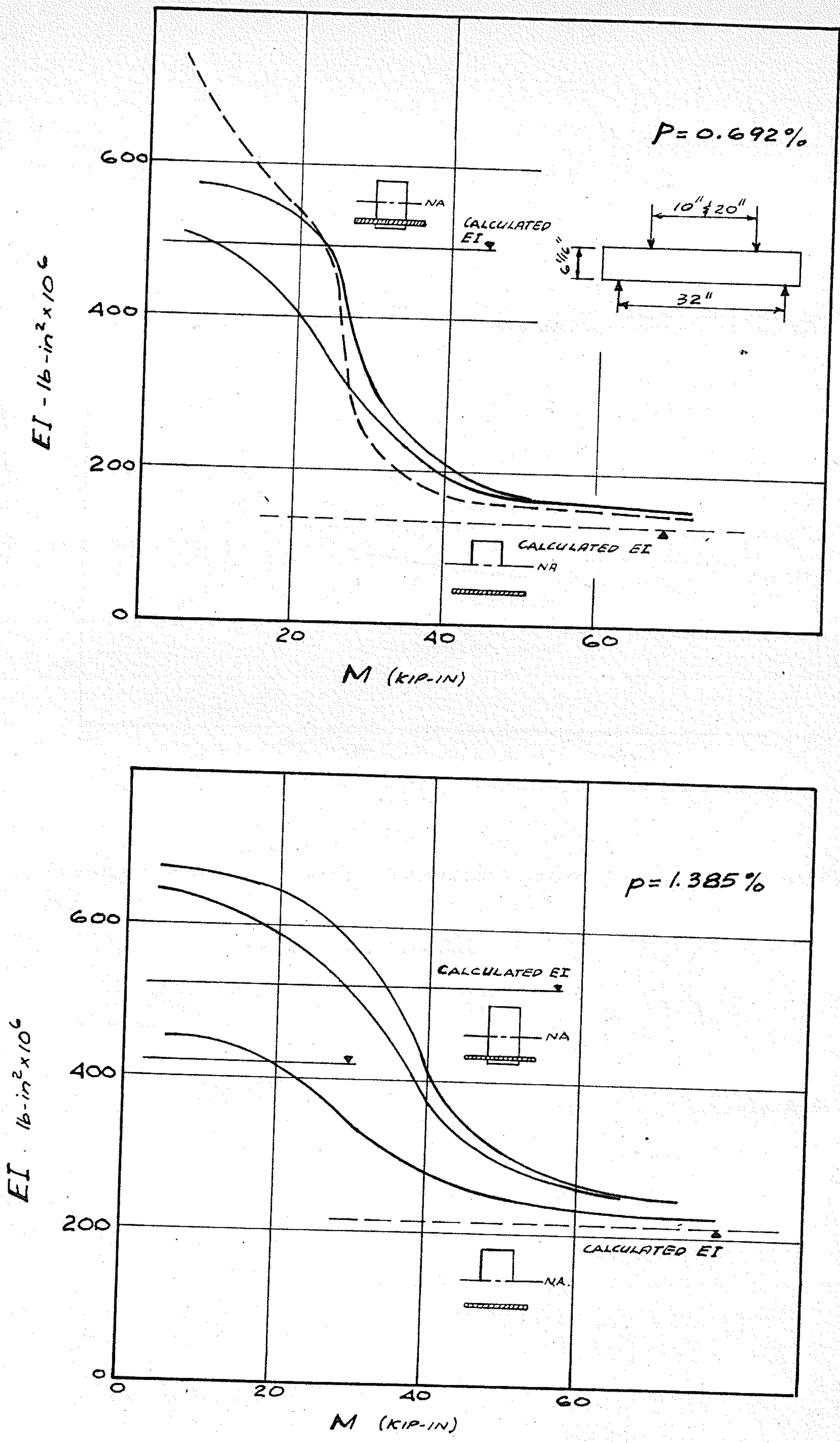


EI vs. LOAD VARIATION
FOUND BY MARSHALL

TESTS BY MARSHALL

Adapted From Ref. 1

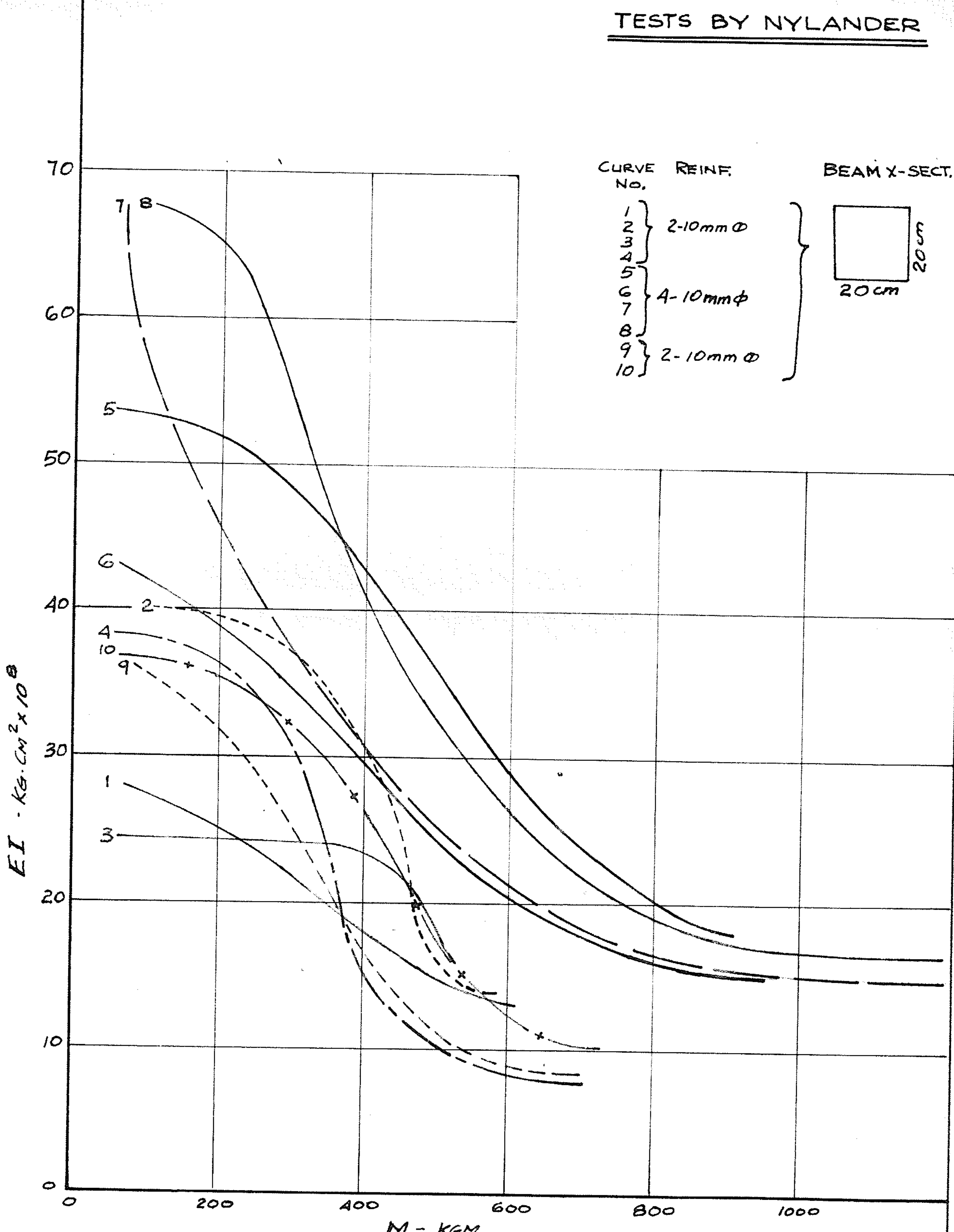
FIGURE 2.1



FLEXURAL RIGIDITY vs. MOMENT AS MEASURED BY EPPES

ADAPTED FROM REF. 2

FIGURE 2.2

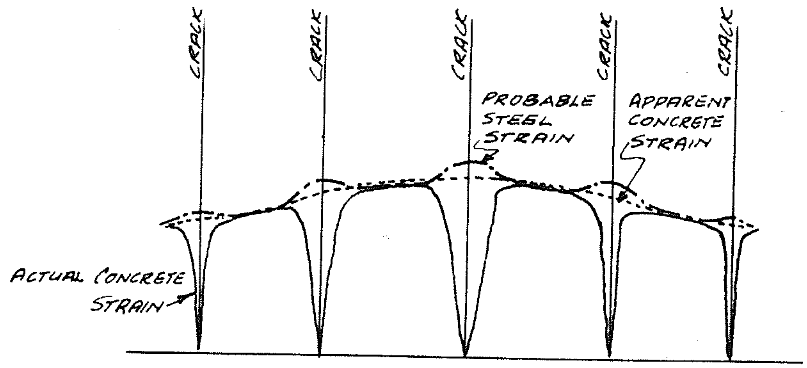


FLEXURAL RIGIDITY vs. MOMENT

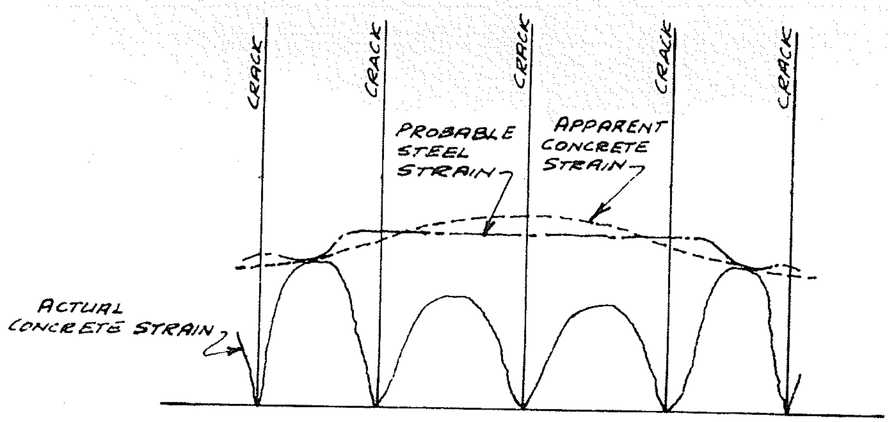
FOR VARIOUS AMOUNTS OF TENSILE REINFORCEMENT

ADAPTED FROM REFERENCE 58.

FIGURE 2.4



(a) PROBABLE DISTRIBUTION OF STRAIN WITH GOOD BOND



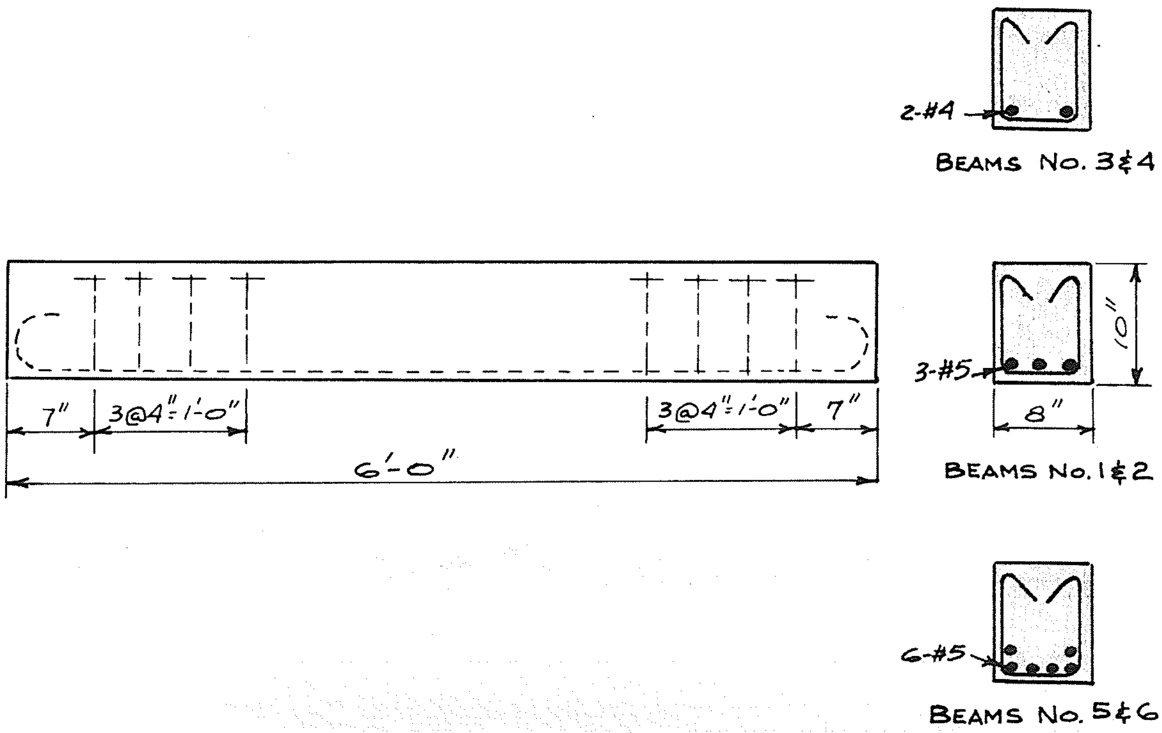
(b) PROBABLE DISTRIBUTION OF STRAIN WITH POOR BOND

DISTRIBUTION OF STRAINS AT THE LEVEL OF TENSILE REINFORCEMENT

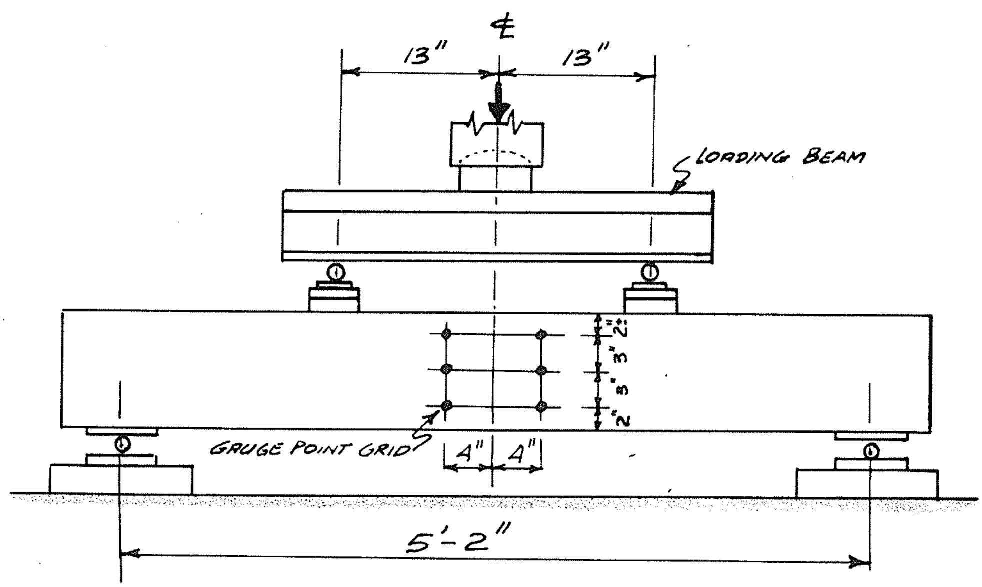
AFTER A.H. MATTOCK, Ph.D.

REPRODUCED FROM REF. 4

FIGURE 3.1

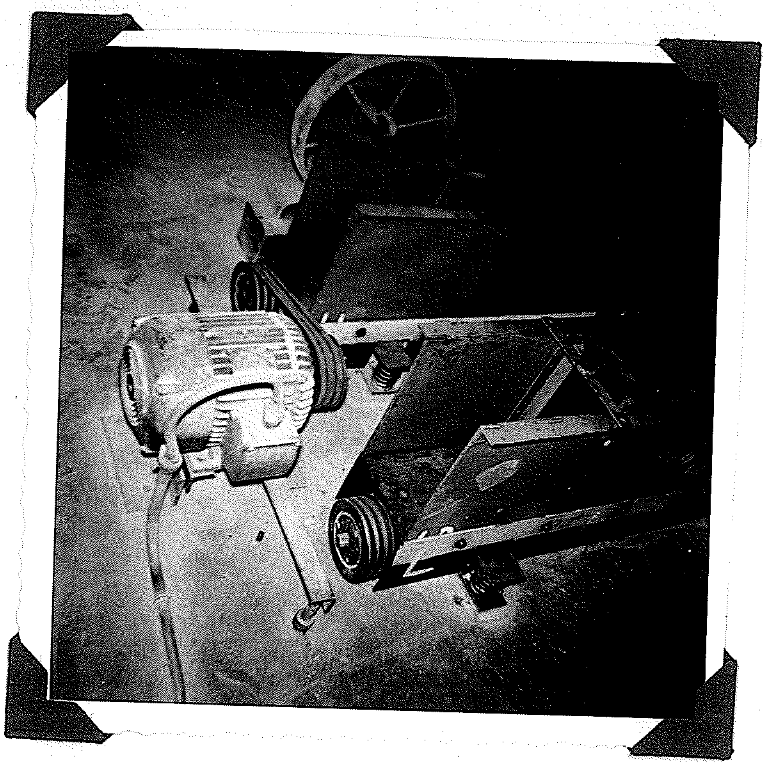


(a) DETAILS OF TEST BEAMS

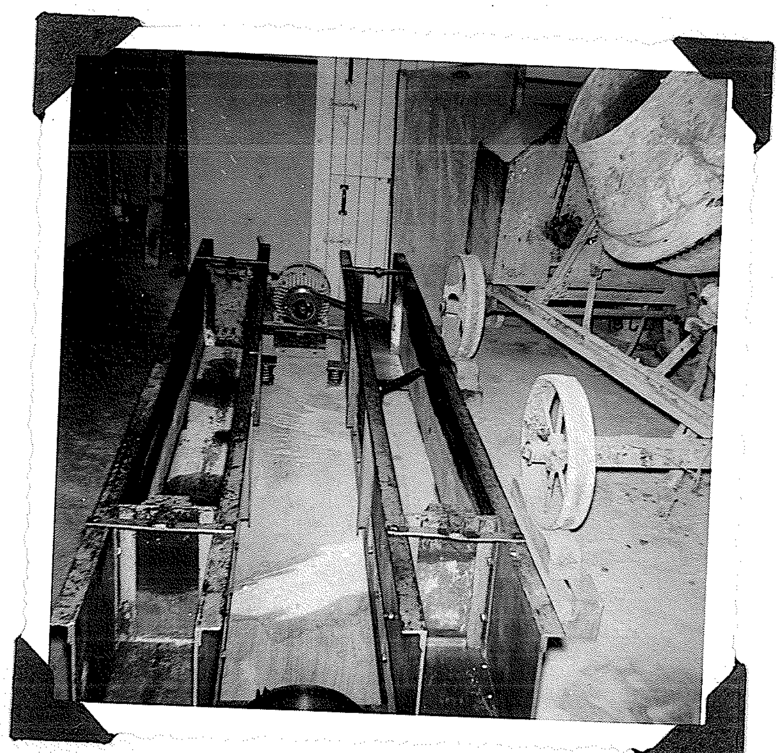


(b) TESTING ARRANGEMENT

FIGURE 4.1

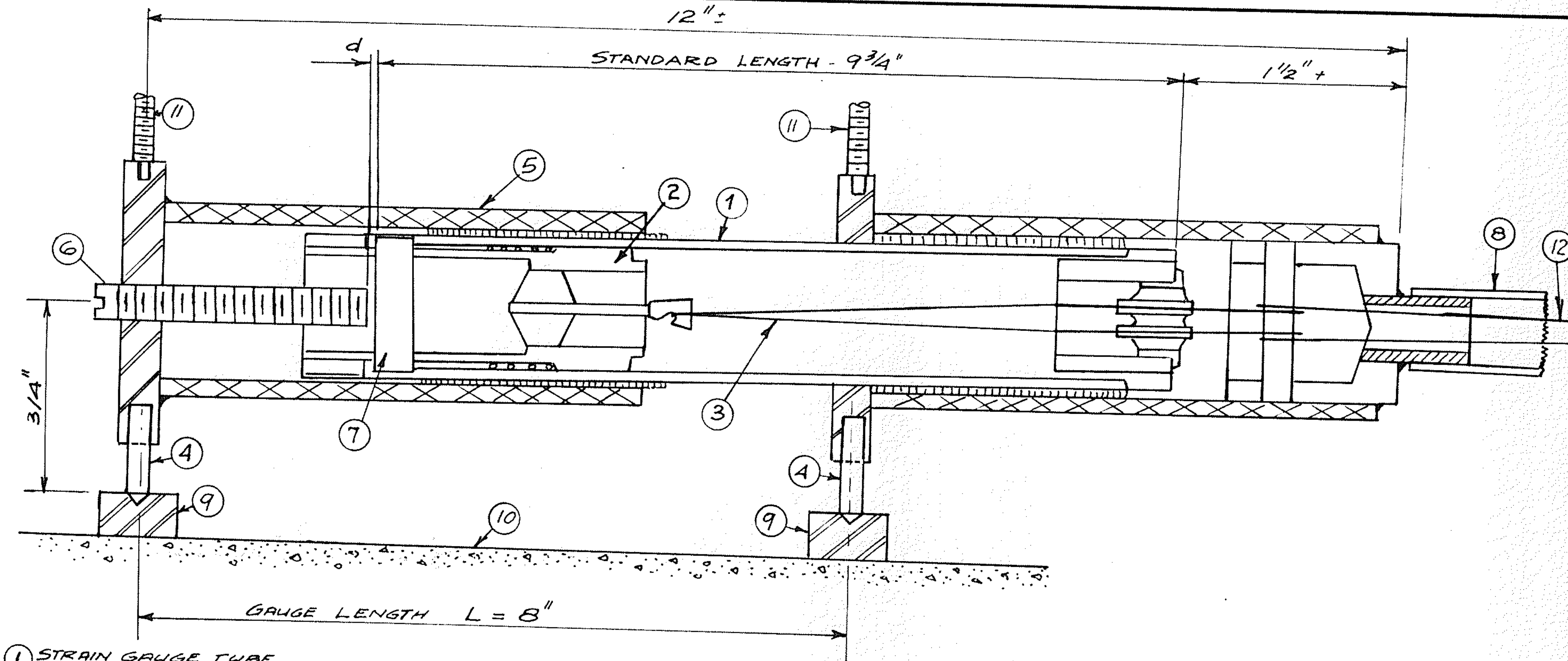


(a) Electric Form Vibrator Drive



(b) Steel Forms Used for
Casting Test Beams

FIGURE 4.2



(1) STRAIN GAUGE TUBE

(2) PISTON

(3) 0.001" ADVANCE WIRE

(4) MECHANICAL GAUGE POINTS

(5) BRASS TUBE, FREELY SLIDING ON GAUGE

(6) INVAR SCREW FOR SETTING INITIAL GAP "d"

(7) PIN, FASTENED TO GAUGE PISTON

(8) PLASTIC TUBE FOR TRANSMITTING AIR PRESSURE

(9) SPECIAL GAUGE PLATES, GLUED TO THE SURFACE OF THE TEST BEAM

(10) TEST BEAM SURFACE

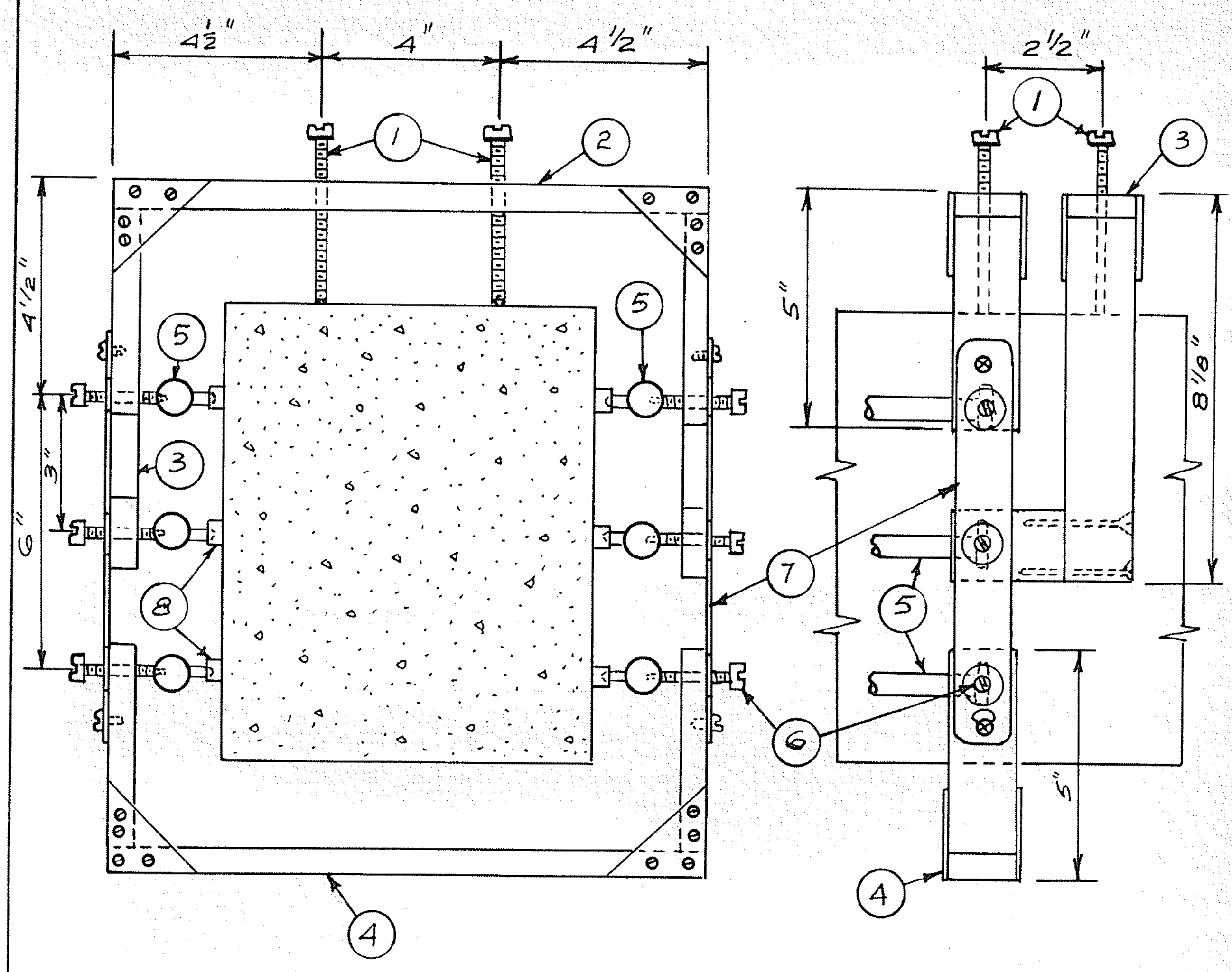
(11) CLAMPING SCREWS (SEE FIG. 4.4)

(12) LEAD WIRES TO STRAIN INDICATOR

SCHEMATIC SECTION
ILLUSTRATING ASSEMBLY OF
STANDARDIZING STRAIN GAUGE

(NOT TO SCALE)

FIGURE 4.3



(1) SCREWS FOR SUPPORTING & LEVELLING BRACKETS

(2) UPPER BRACKET (ALL BRACKETS 1 1/2" x 1/2" PLATE)

(3) CENTRAL BRACKET

(4) LOWER BRACKET

(5) STANDARDIZING STRAIN GAUGES, CLAMPED IN POSITION

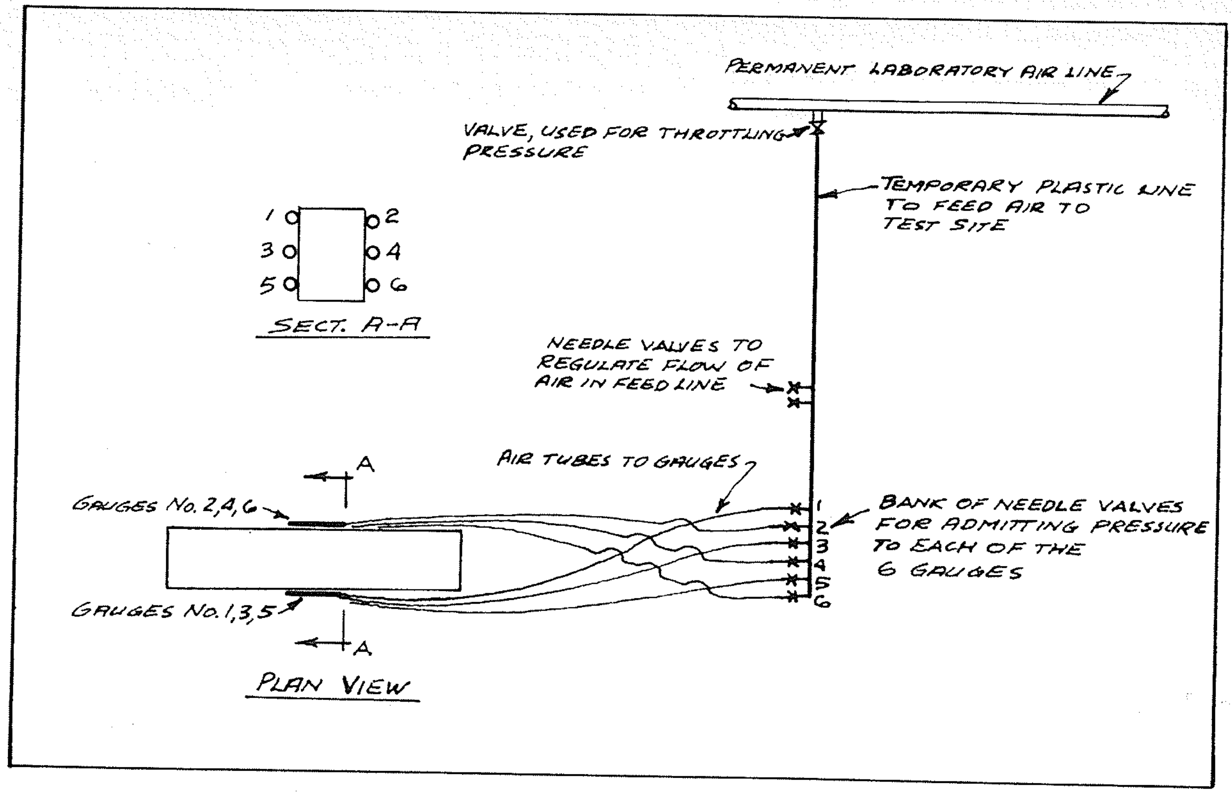
(6) SCREWS FOR TIGHTENING GAUGES AGAINST GAUGE POINT PLATES

(7) SWIVEL STRAP FOR SUPPORTING LOWER BRACKET (1 1/4" x 1/8" ℄)

(8) GAUGE POINT PLATES

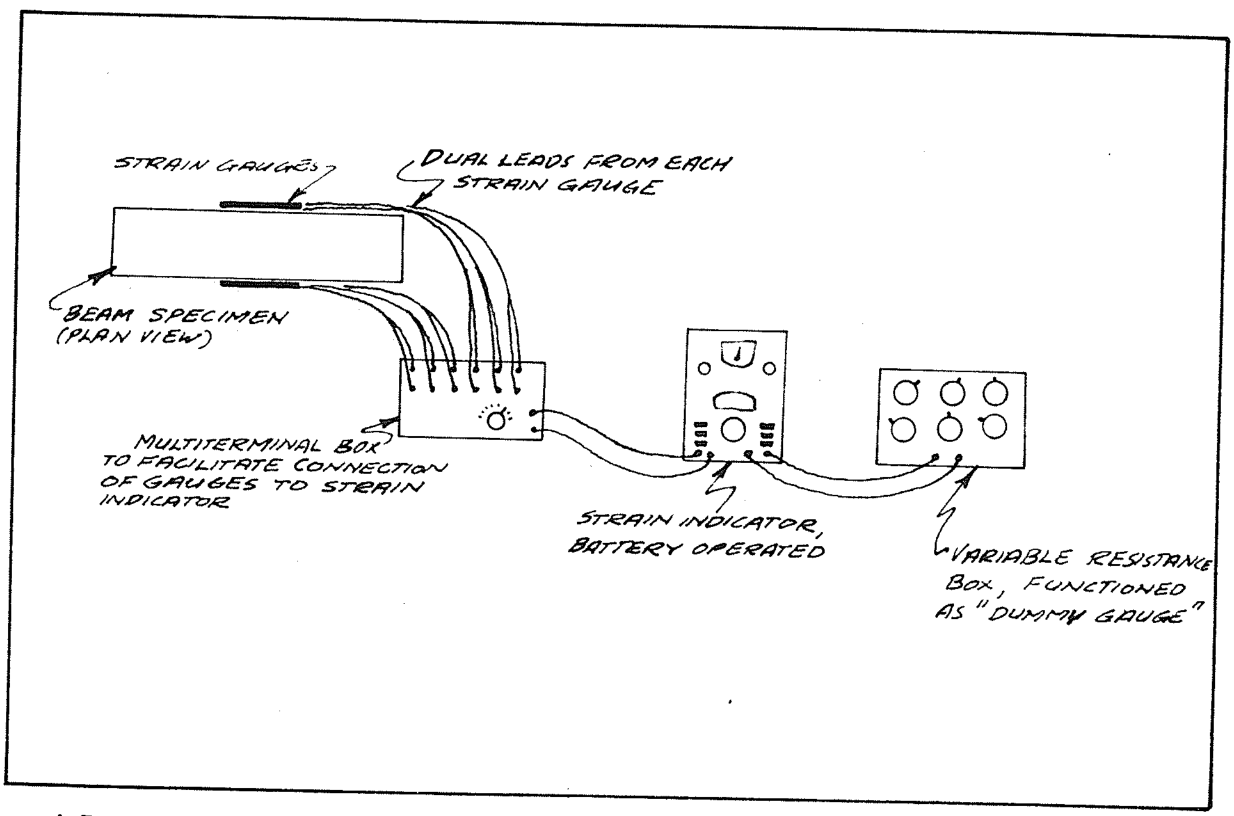
DETAILS OF GAUGE CLAMPS

FIGURE 4.4



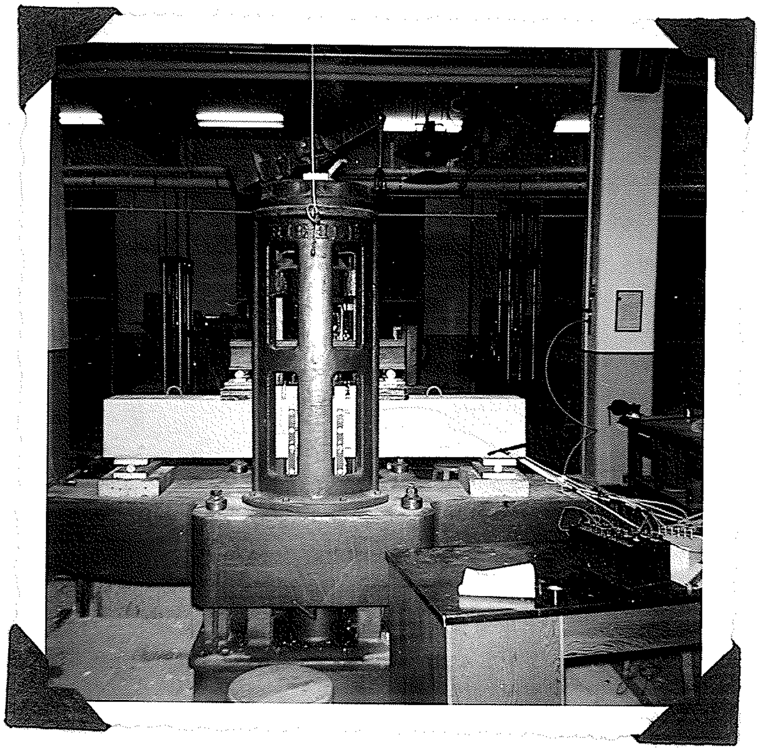
GENERAL ARRANGEMENT OF AIR HOSES
(SCHEMATIC)

FIGURE 4.5



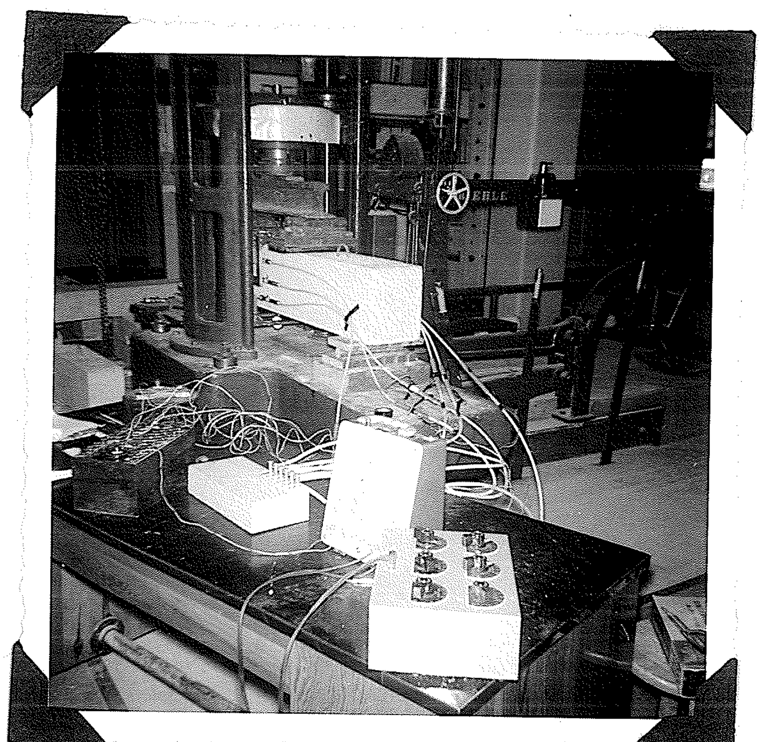
ARRANGEMENT OF STRAIN MEASURING APPARATUS
(SCHEMATIC)

FIGURE 4.6



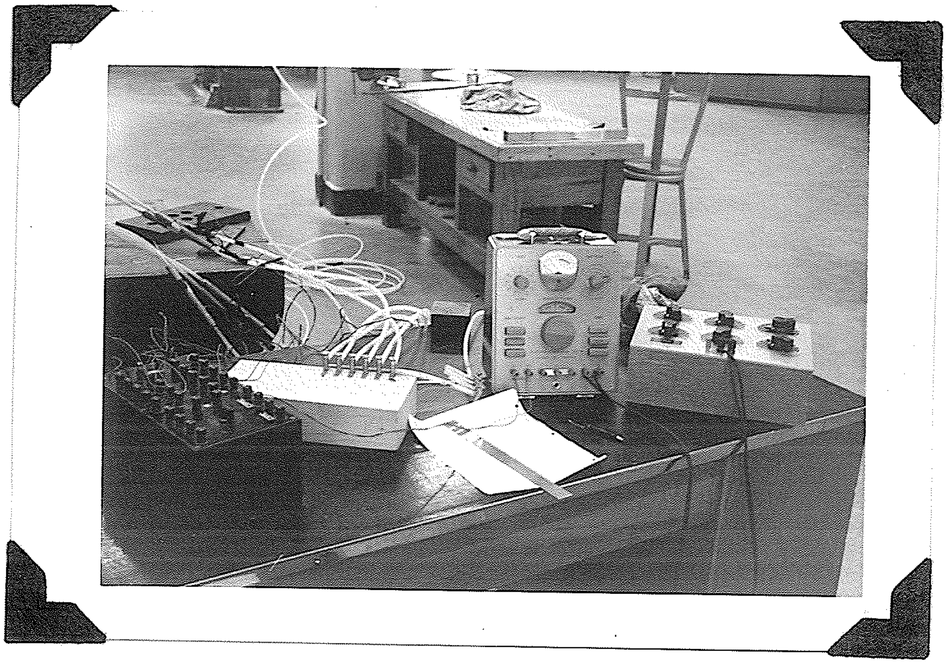
Beam Ready for Testing

FIGURE 4.7



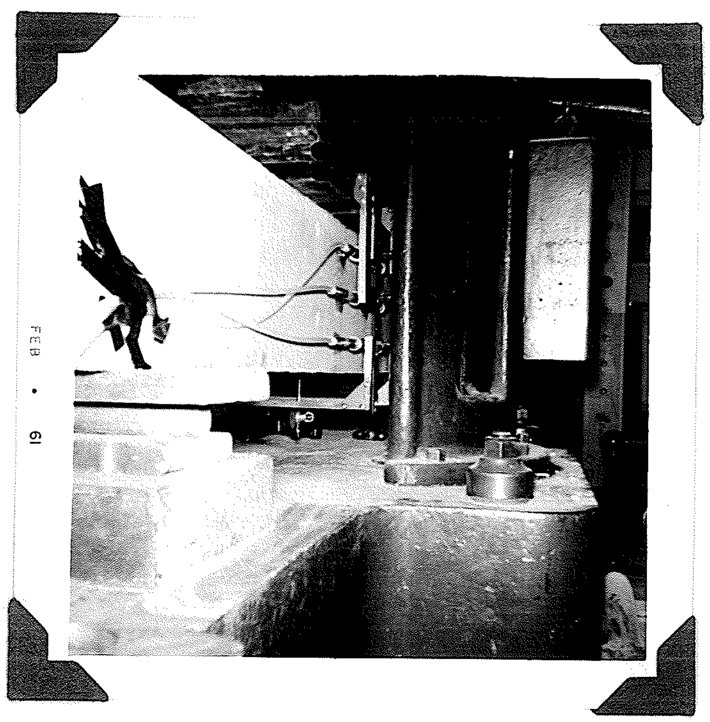
General Arrangement of
Strain Measuring Equipment

FIGURE 4.8



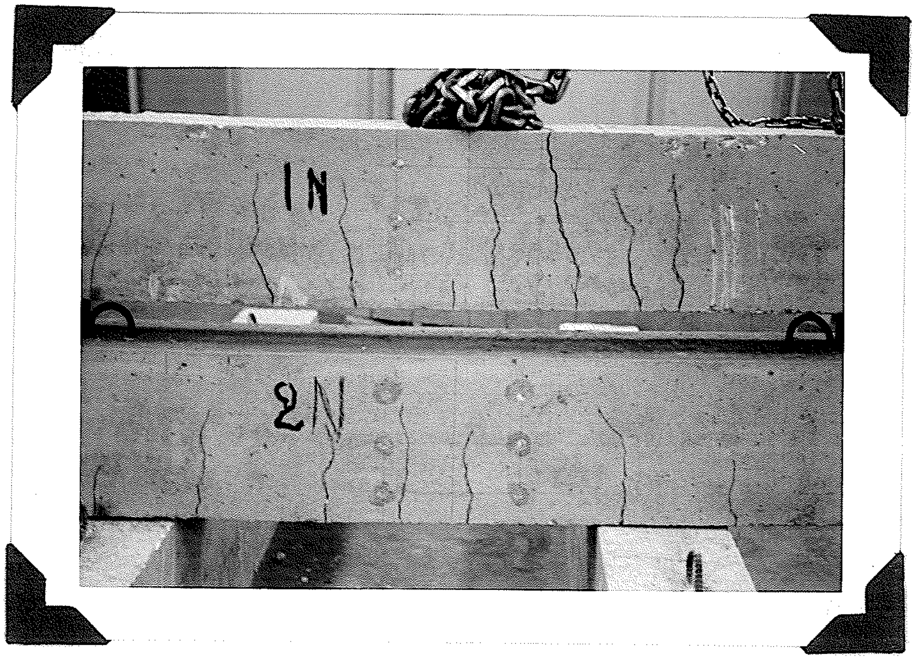
Close-up of Air Line Valving and
Strain Measuring Equipment

FIGURE 4.9

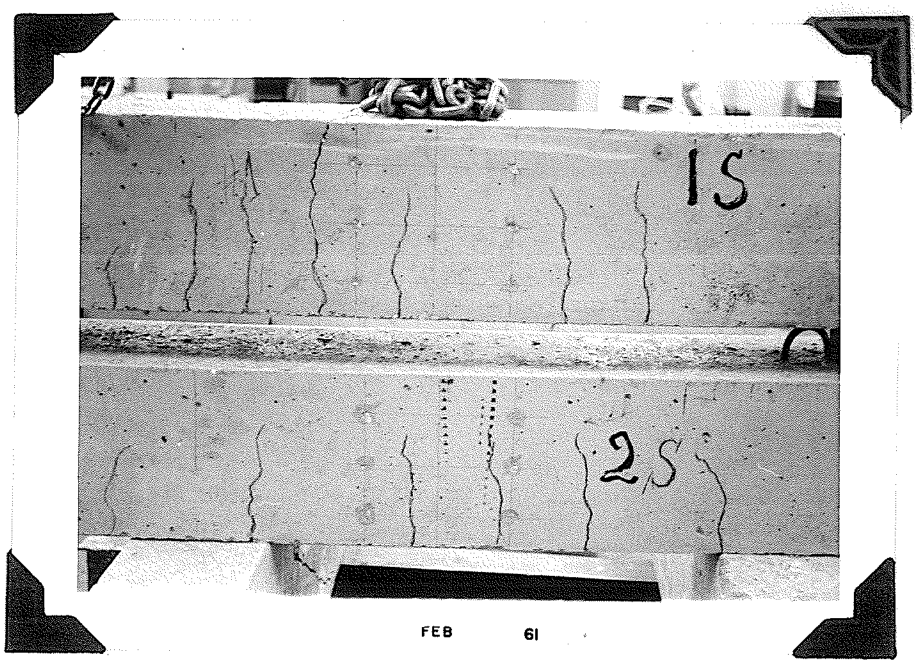


Strain Gauges Clamped in Position

FIGURE 4.10



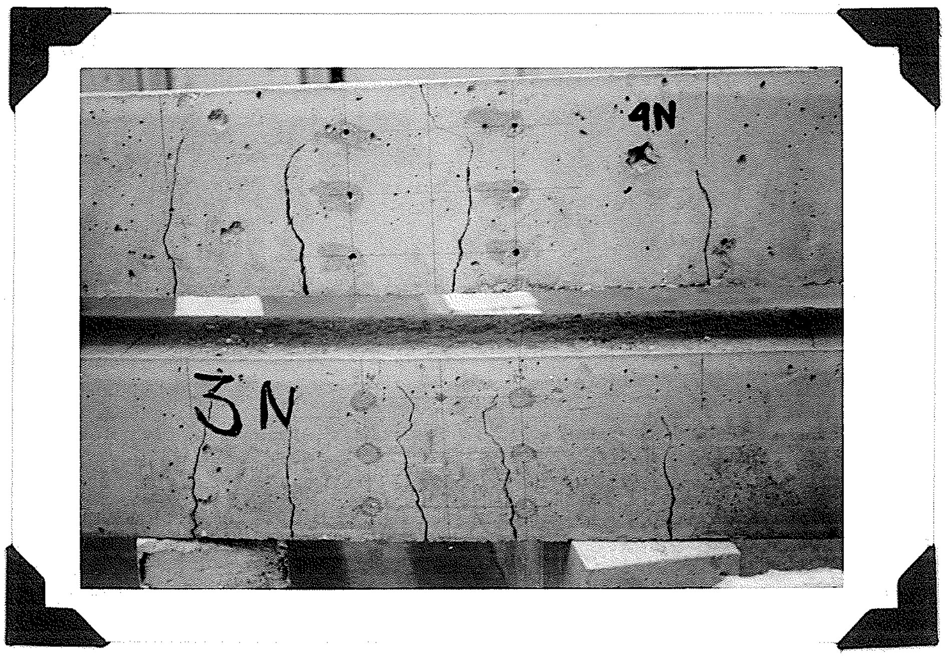
(a) North Face



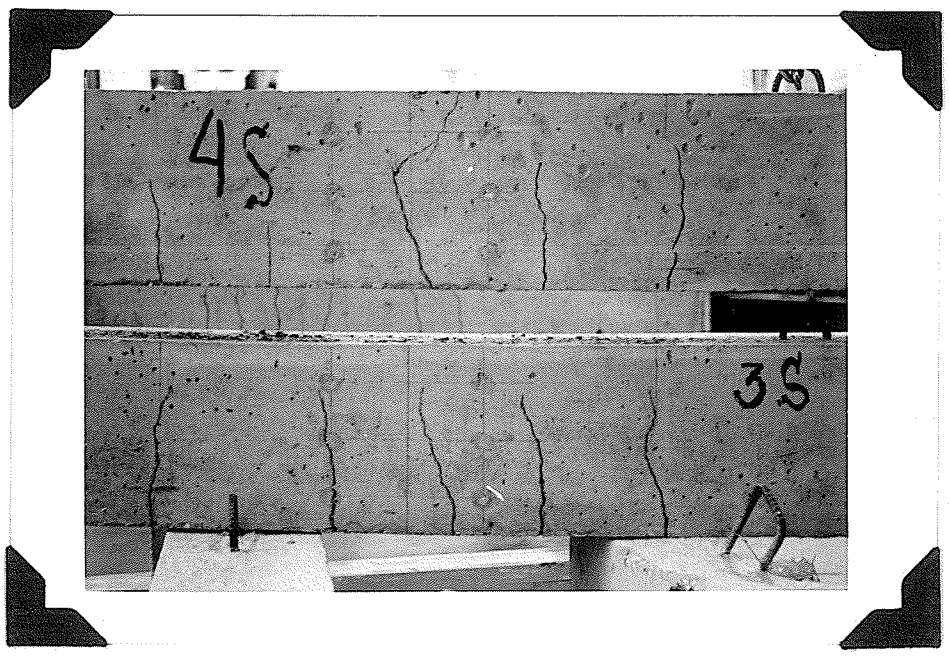
(b) South Face

Beams No. 1 & 2 - Crack Pattern After Testing

FIGURE 5.1



(a) North Face



(b) South Face

Beams No. 3 & 4 - Crack Pattern After Testing

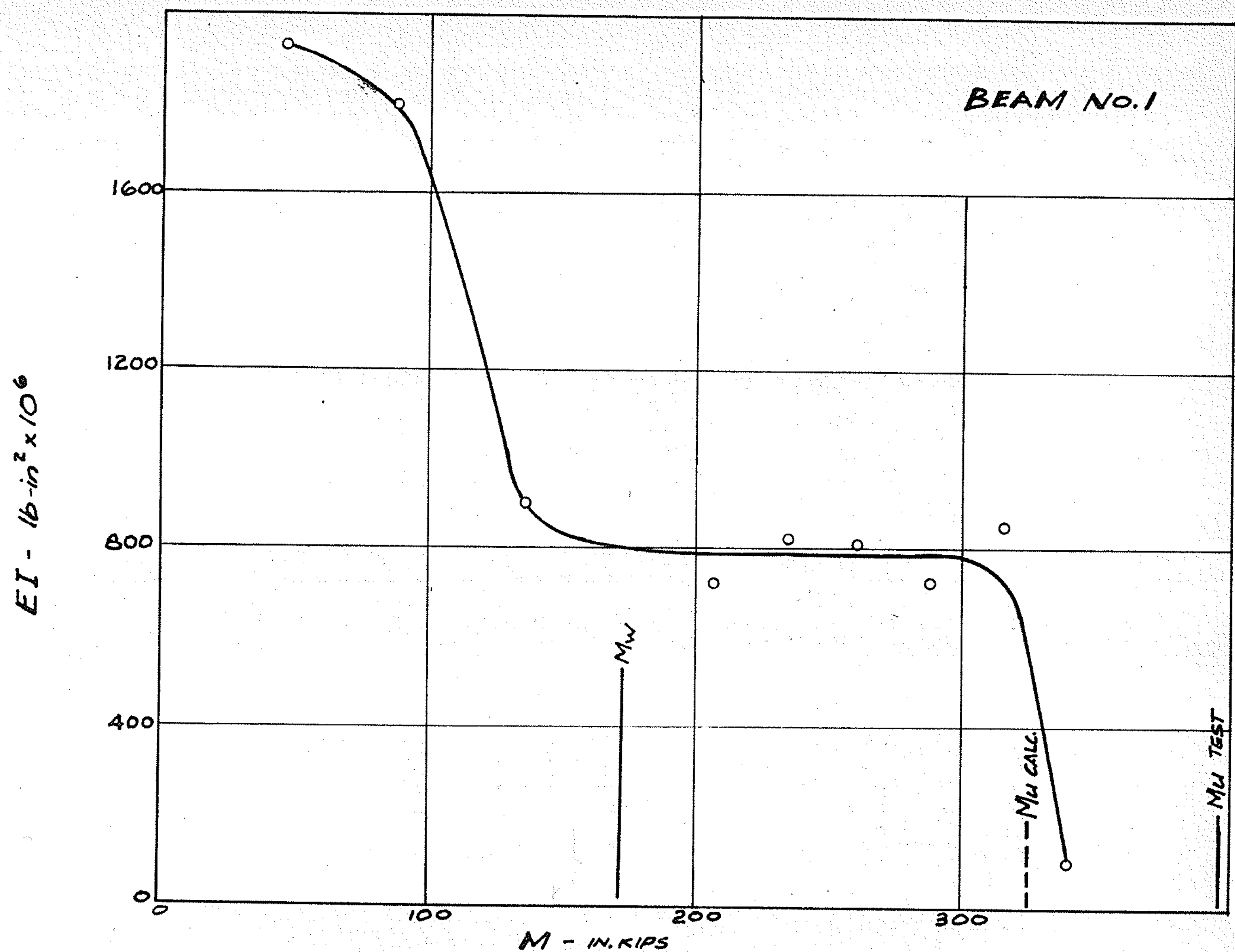
FIGURE 5.2

(a) North Face

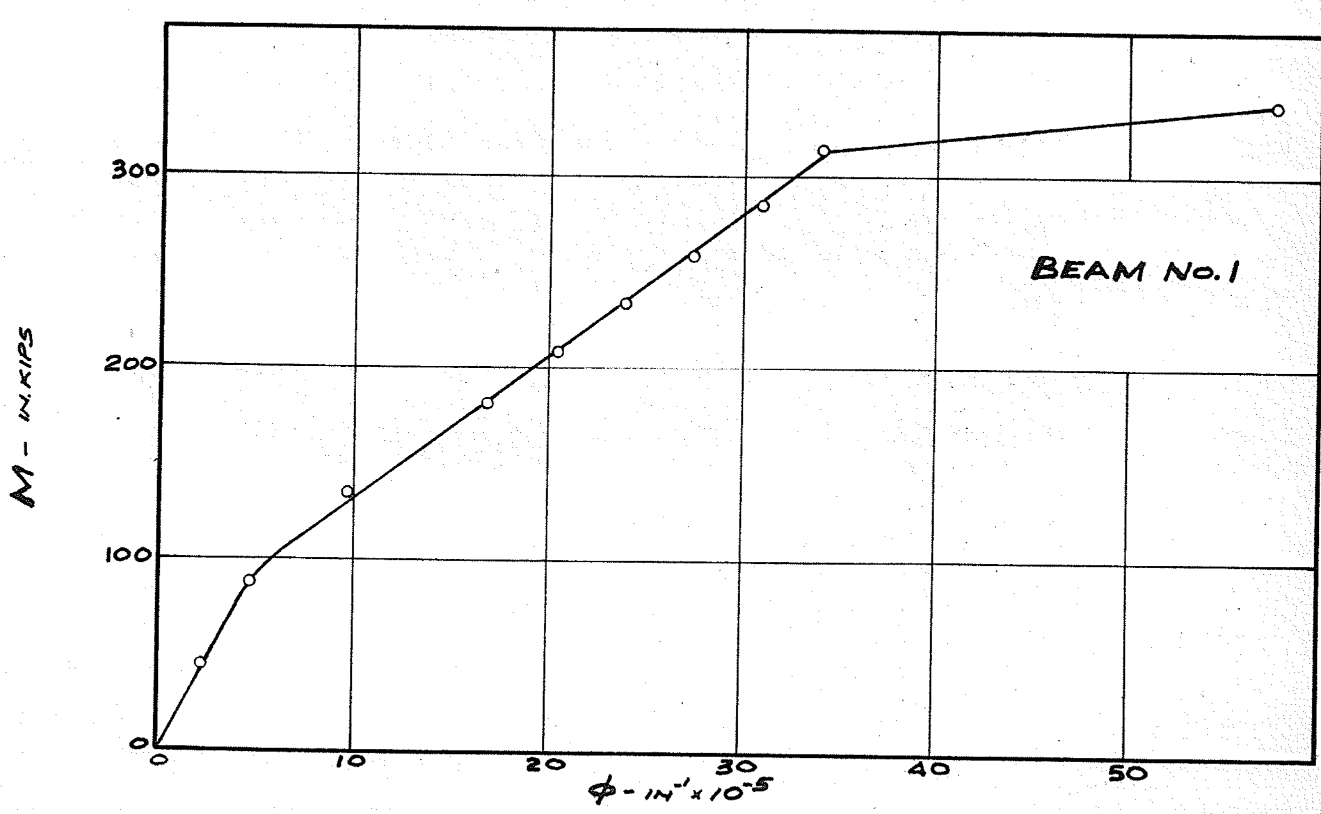
(b) South Face

Beams No. 5 & 6 - Crack Pattern After Testing

FIGURE 5.3

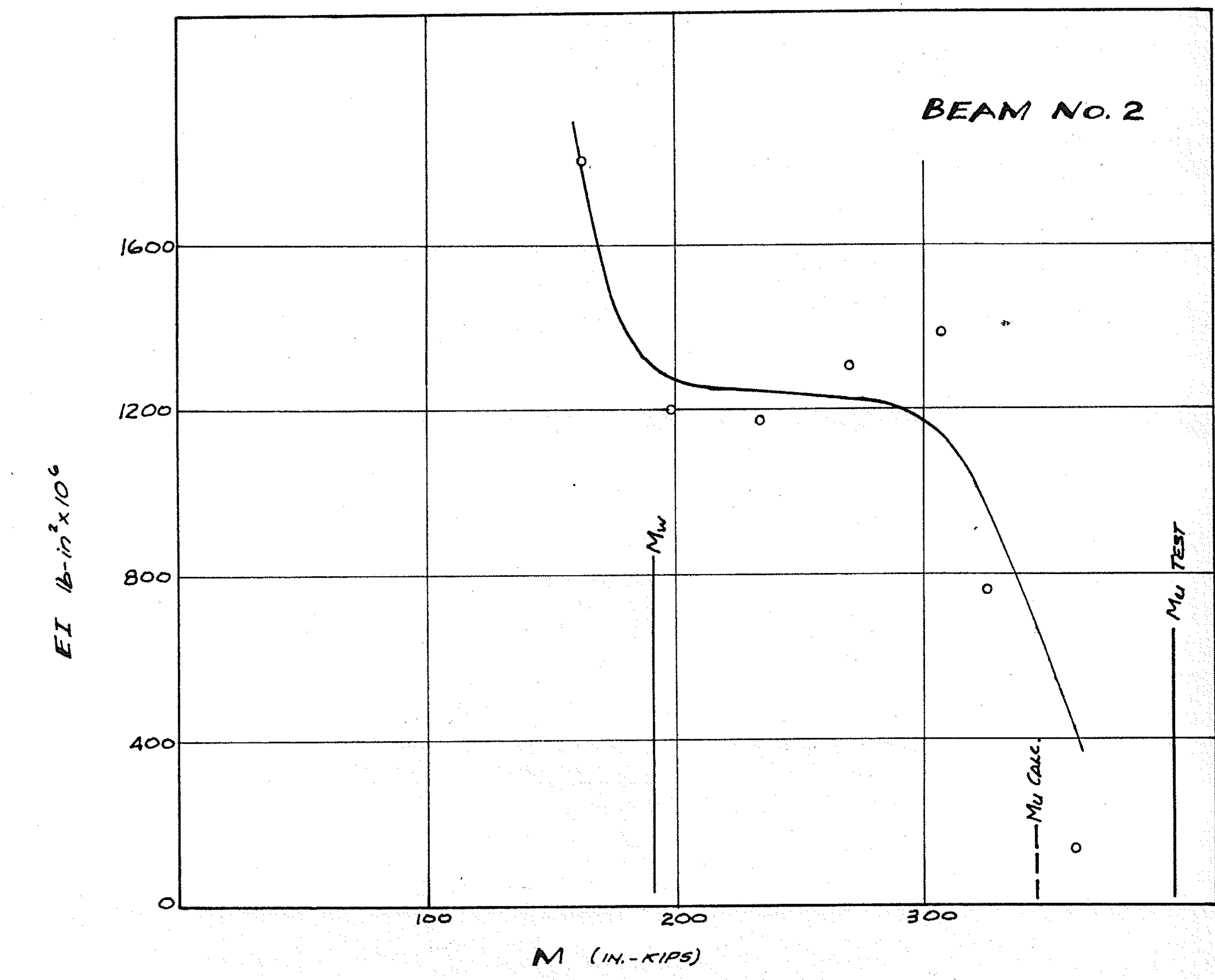


FLEXURAL RIGIDITY vs. MOMENT FIGURE 5.4



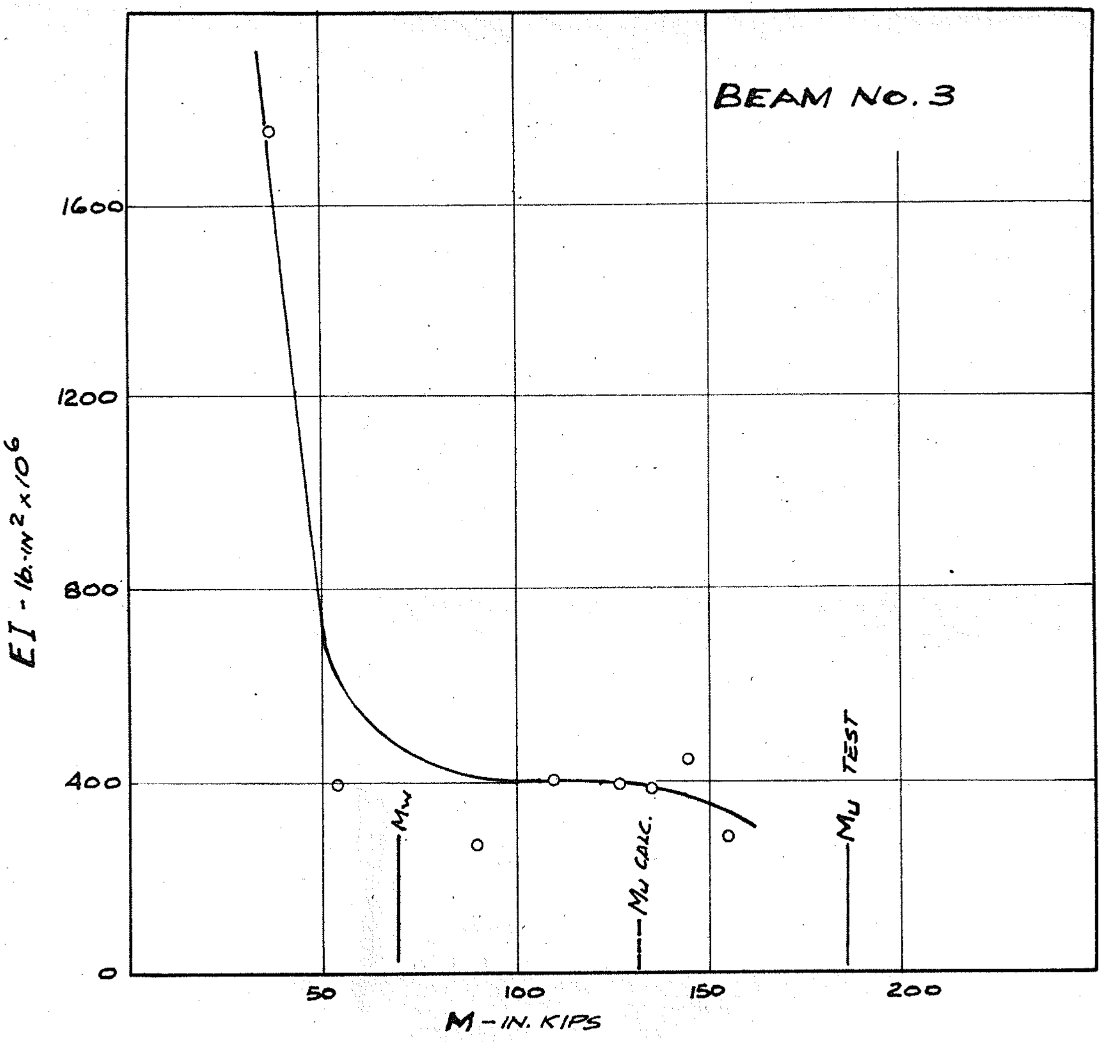
MOMENT vs. CURVATURE FIGURE 5.5

/

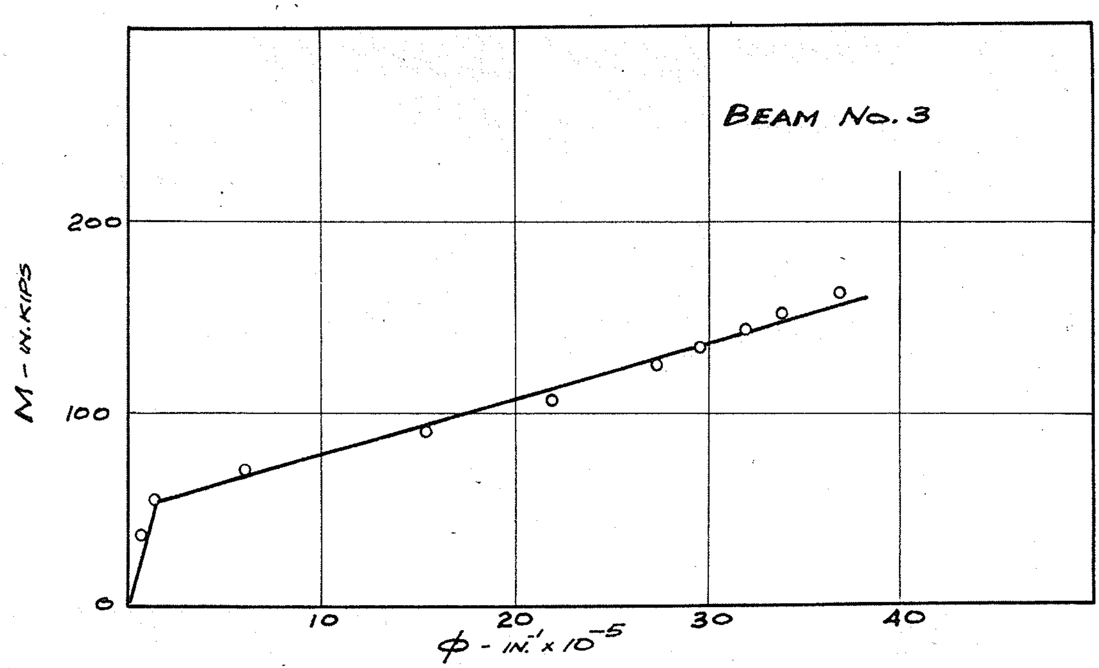


FLEXURAL RIGIDITY vs. MOMENT

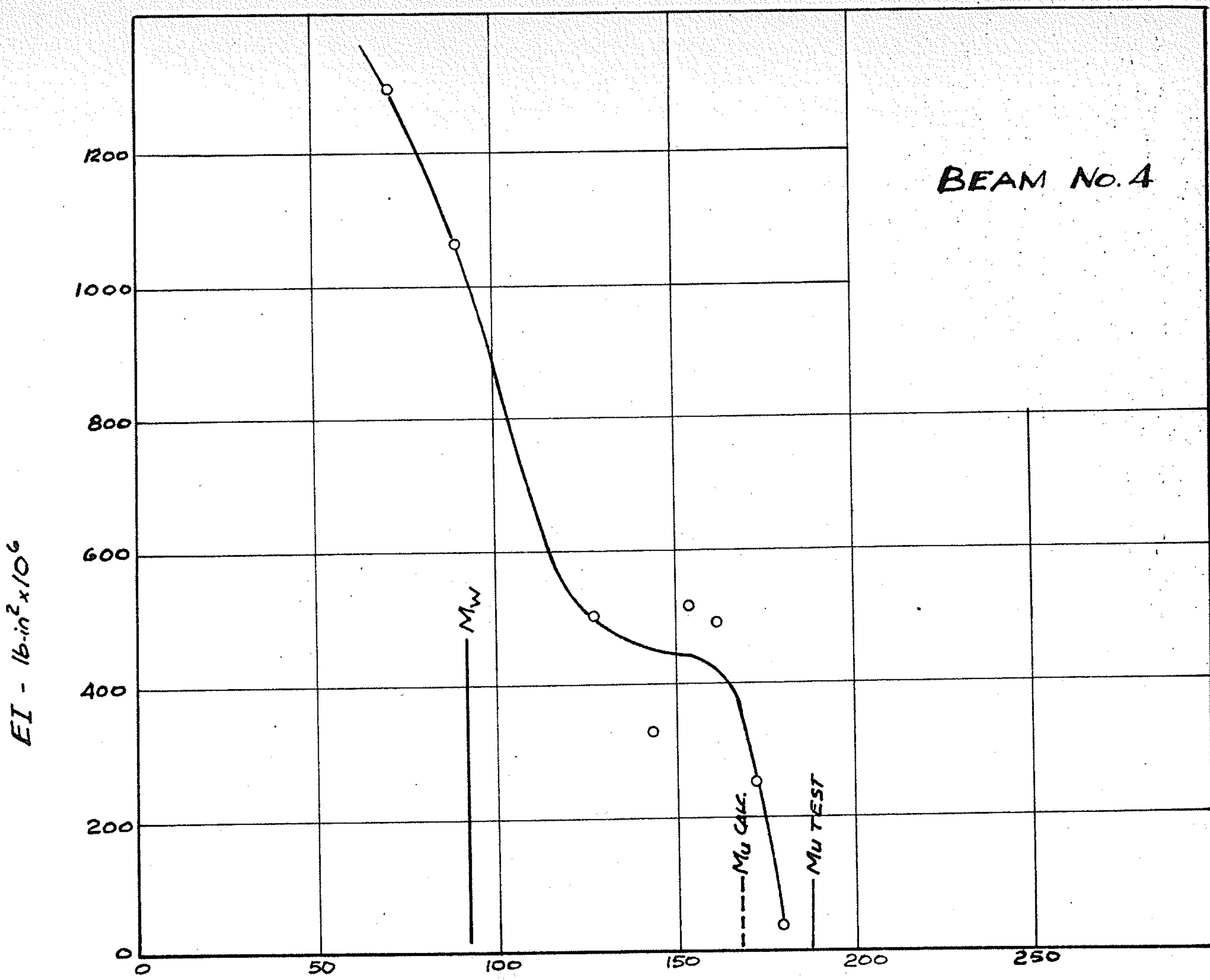
FIGURE 5.6



FLEXURAL RIGIDITY vs. MOMENT FIGURE 5.7

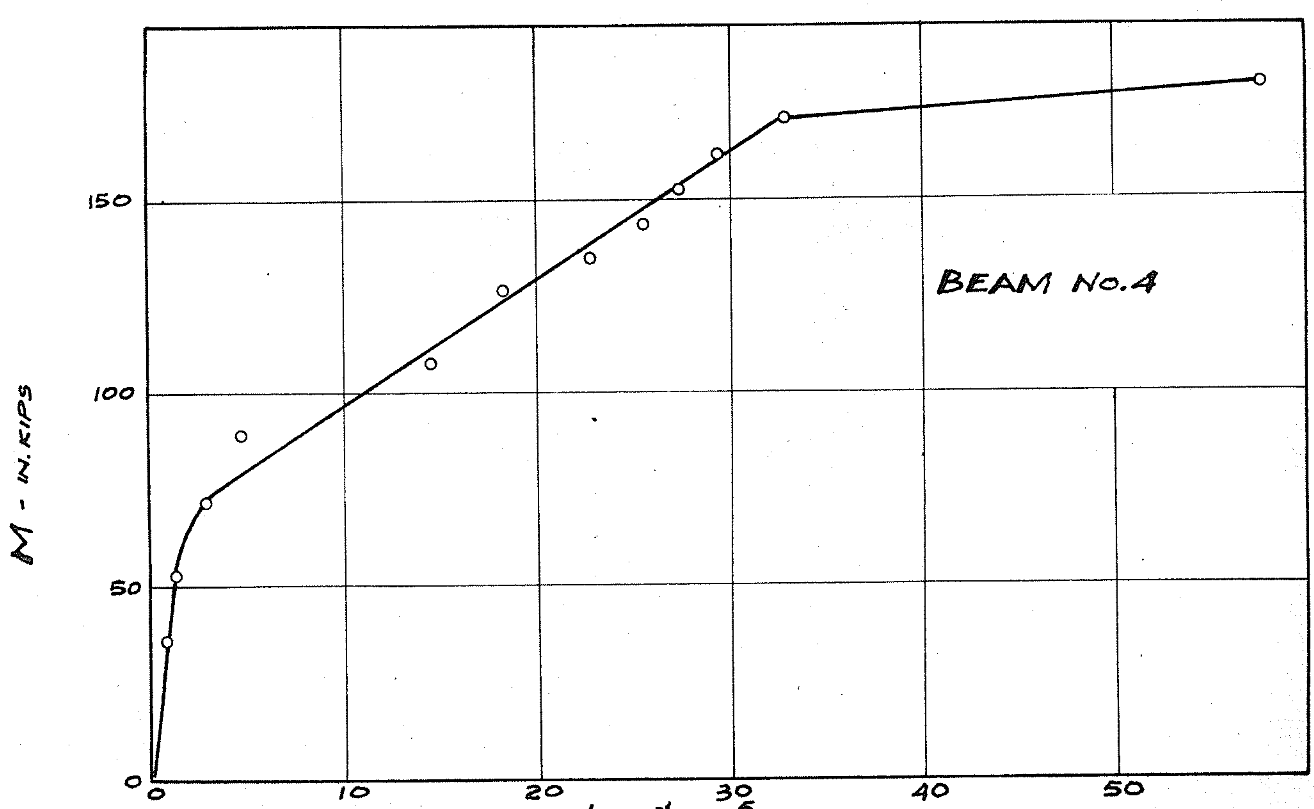


MOMENT vs. CURVATURE FIGURE 5.8



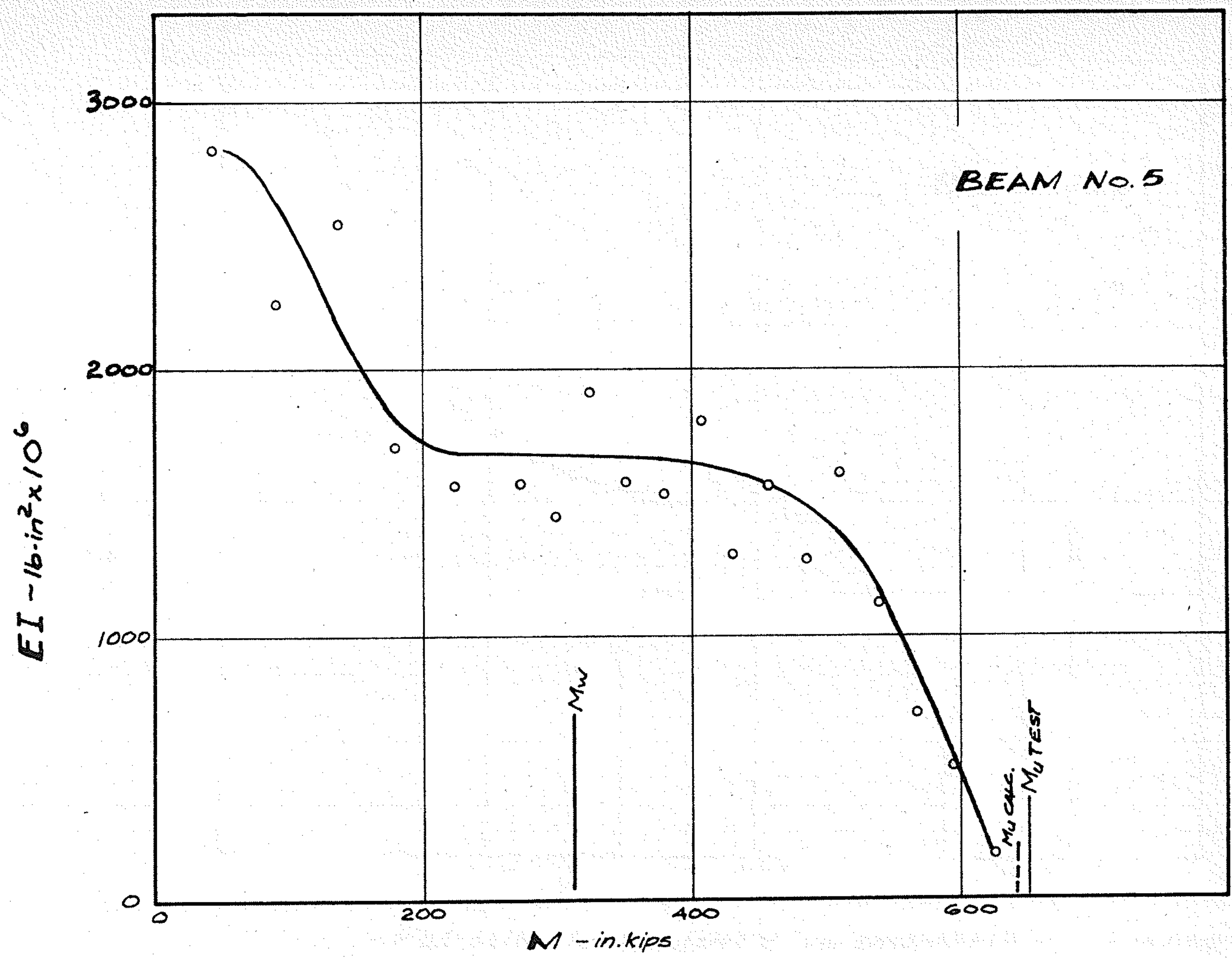
FLEXURAL RIGIDITY vs. MOMENT

FIGURE 5.9

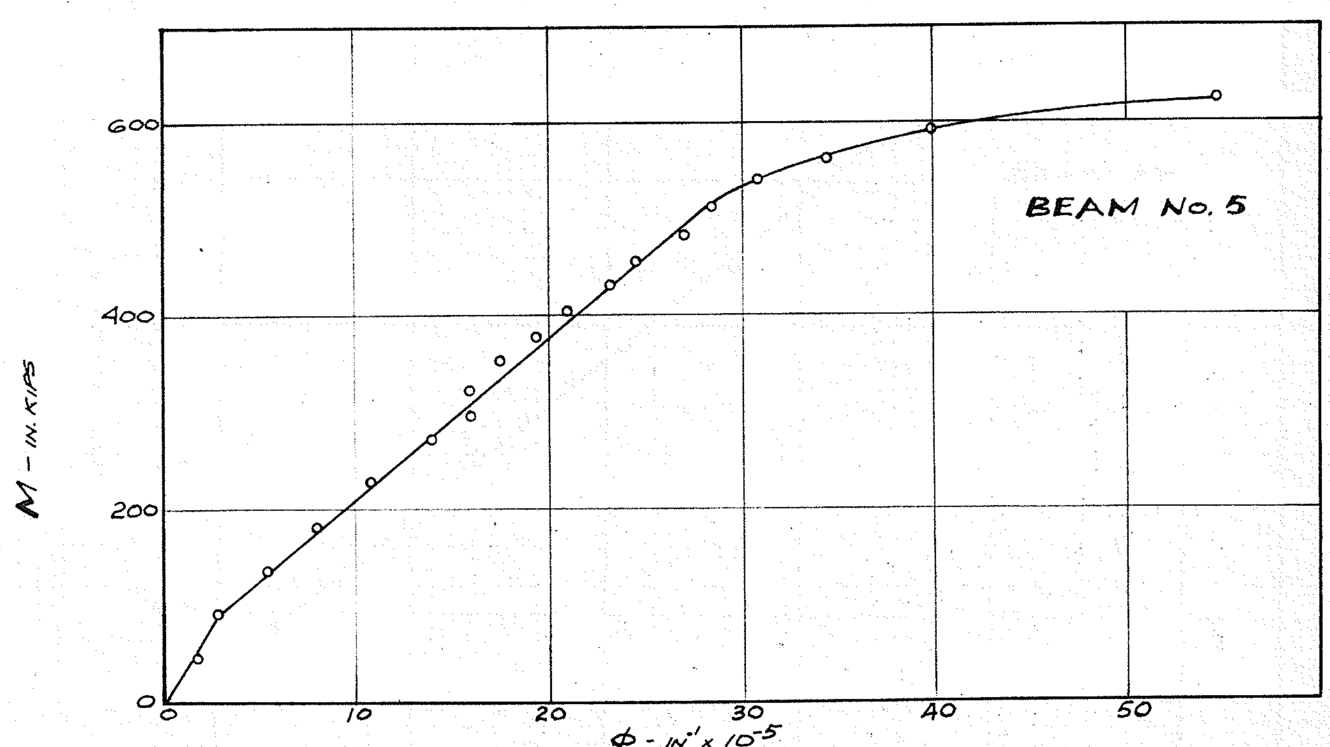


MOMENT vs. CURVATURE

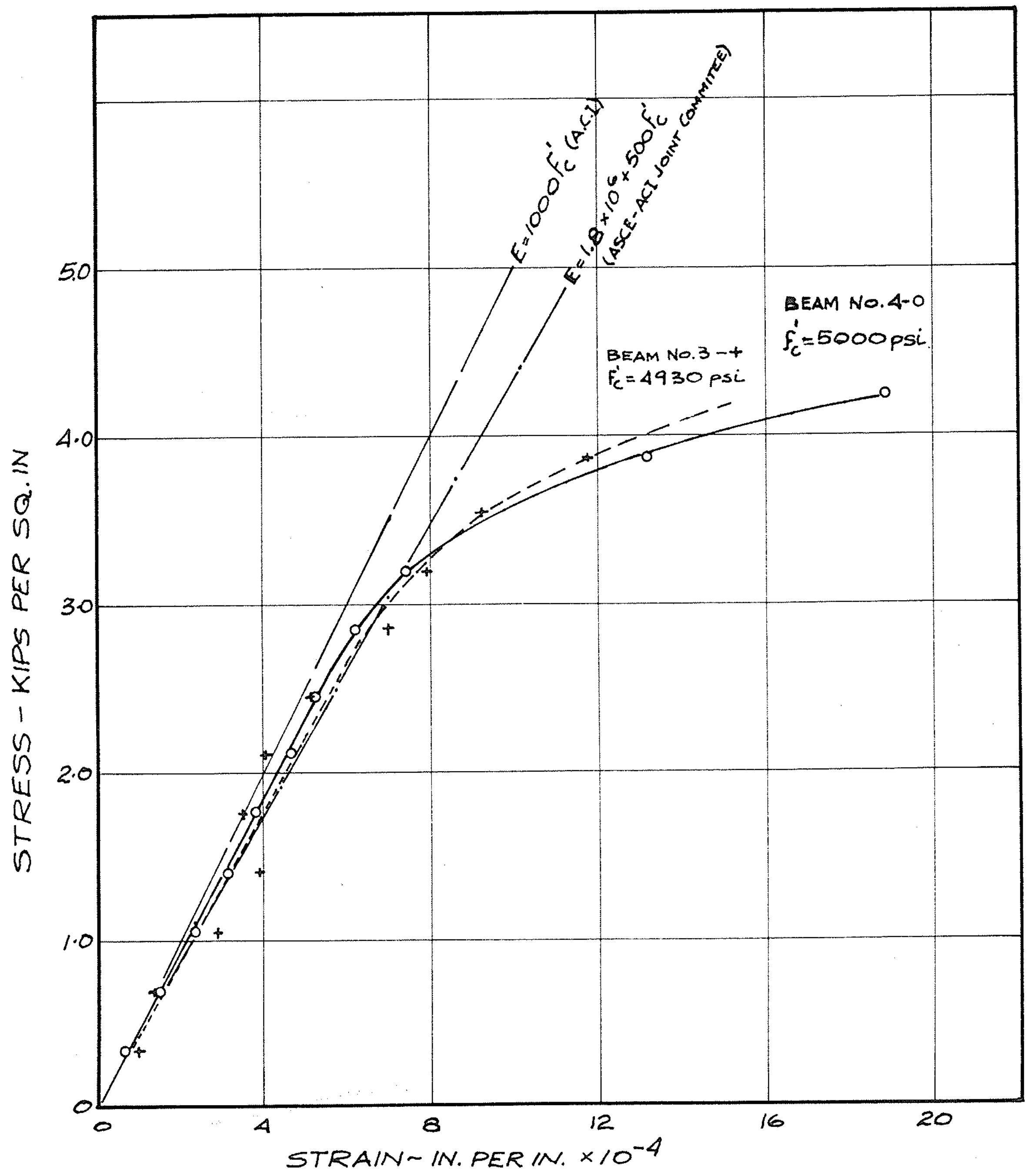
FIGURE 5.10



FLEXURAL RIGIDITY vs. MOMENT FIGURE 5.11

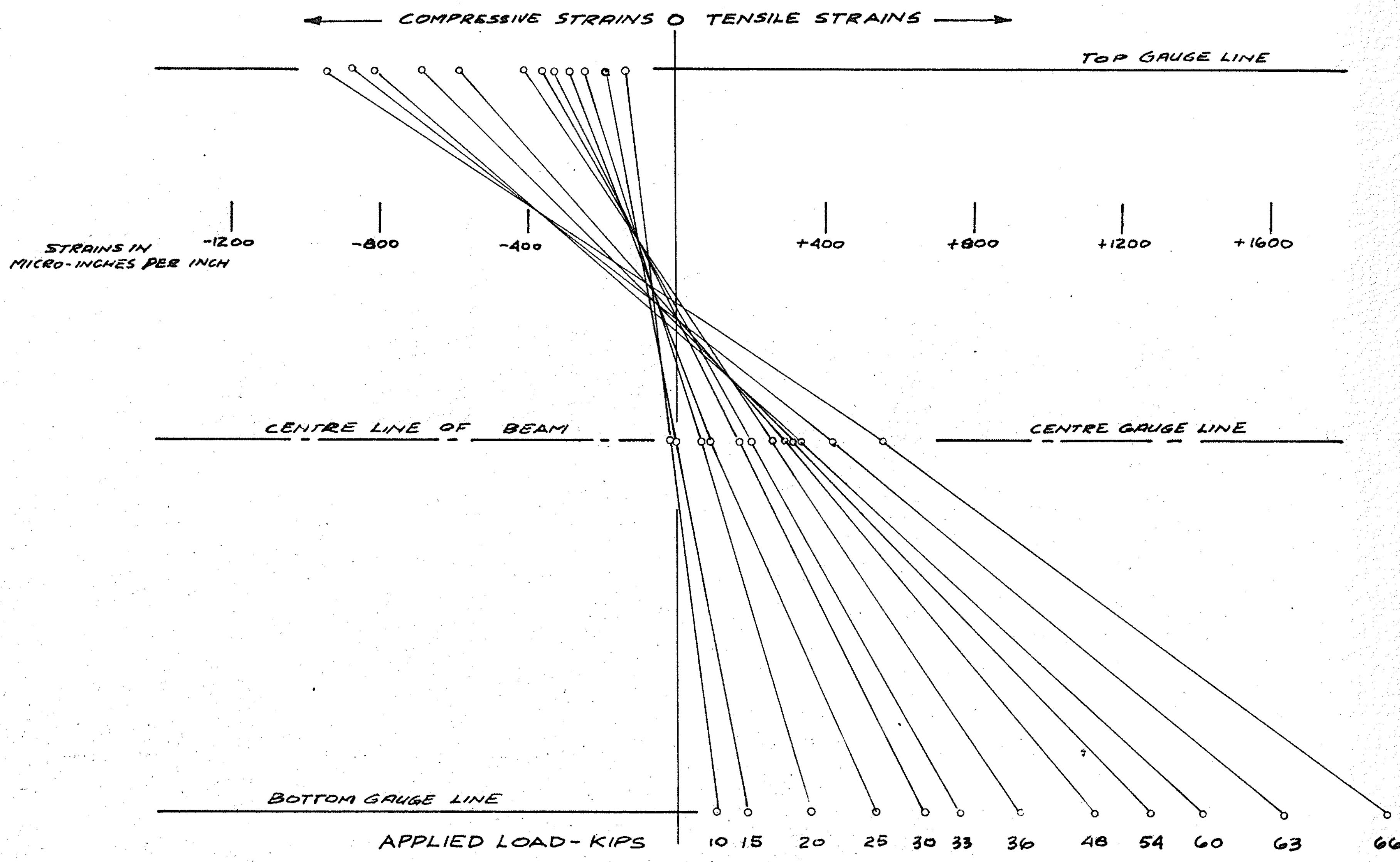


MOMENT vs. CURVATURE FIGURE 5.12



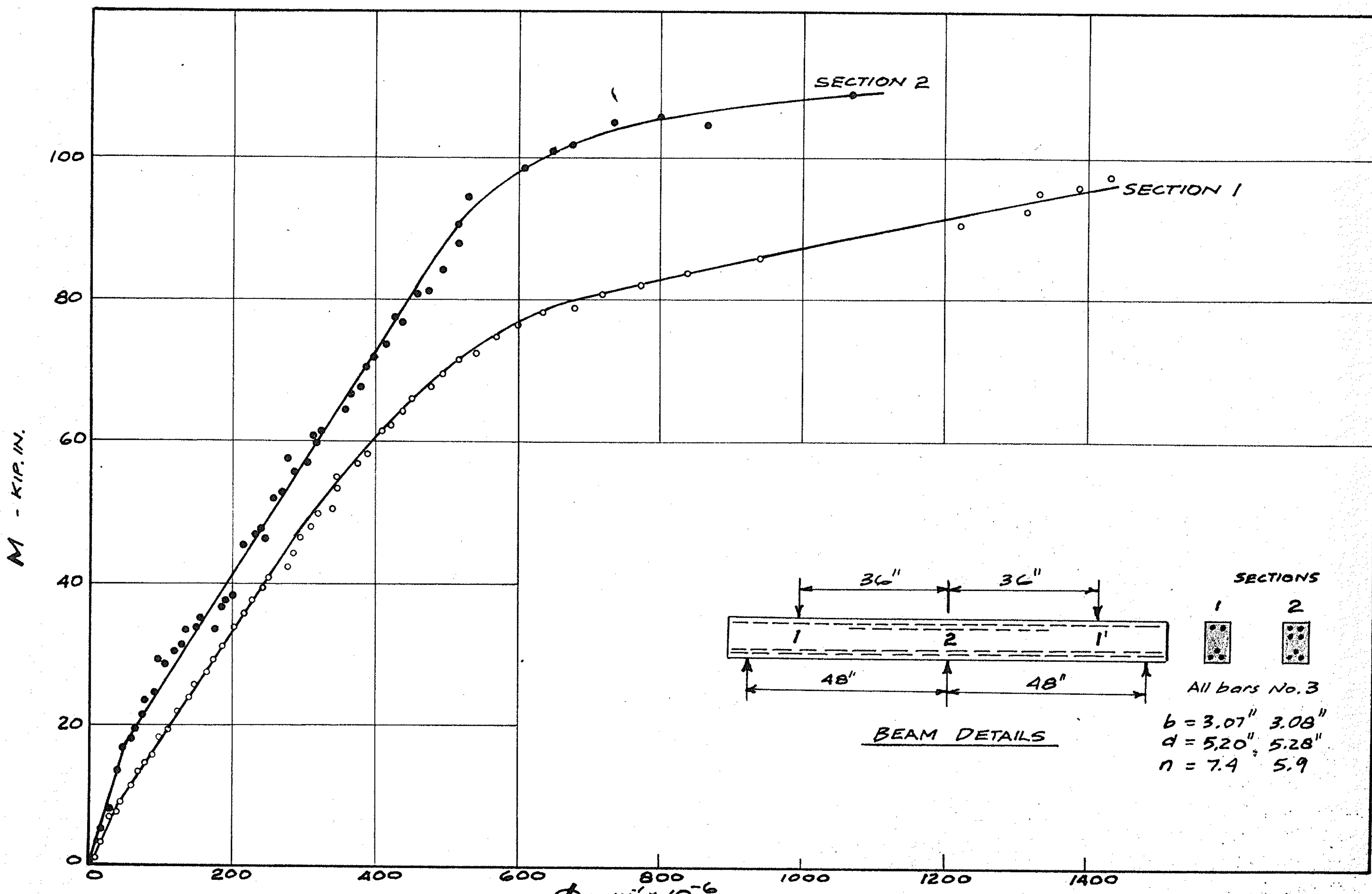
STRESS-STRAIN CURVE FOR CONCRETE
(BEAM No. 4 & No. 3)

FIGURE 5.15



BEAM No. 5 - MEASURED LOAD-STRAIN RELATIONSHIP

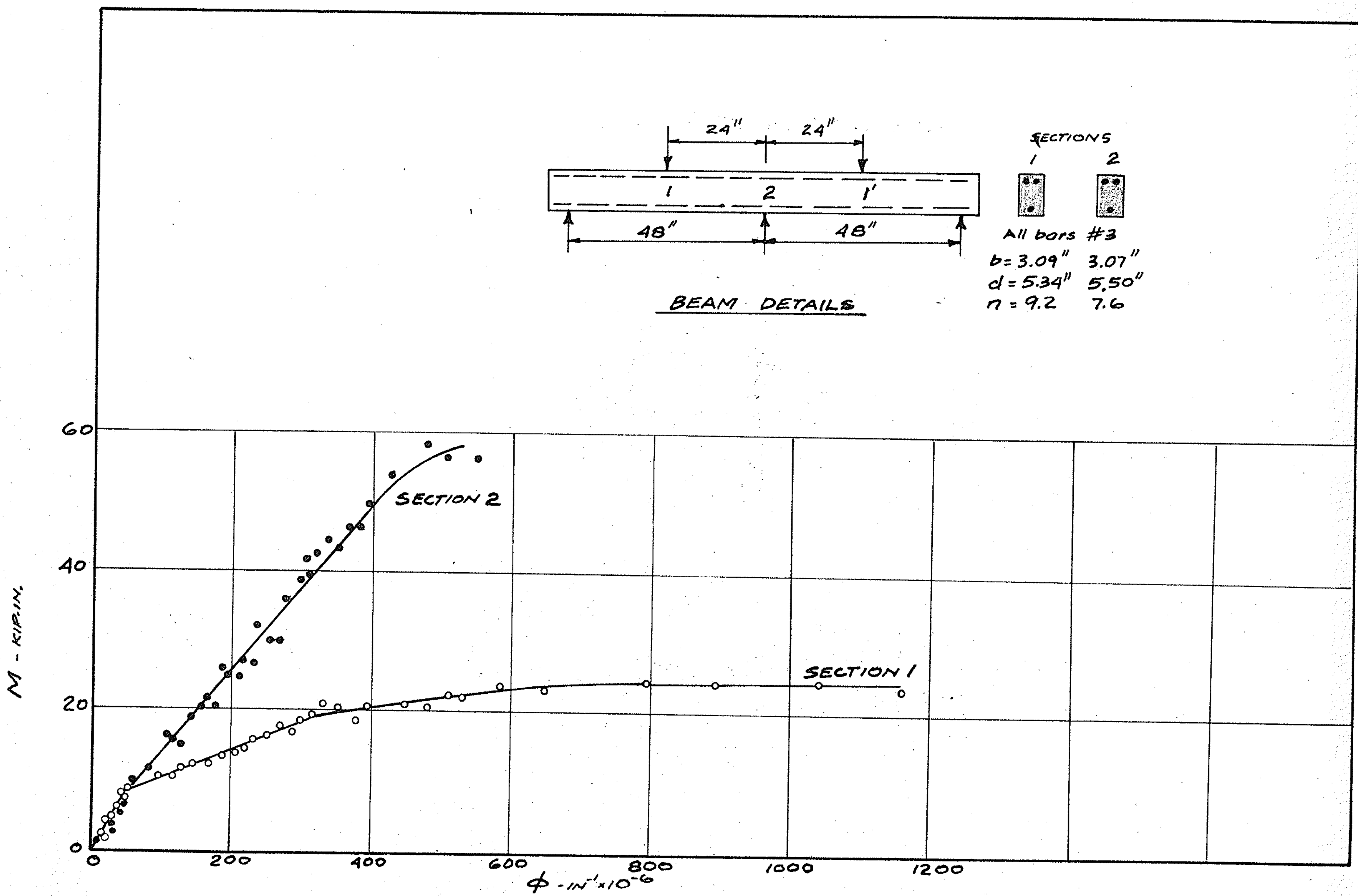
FIGURE 5.16



MOMENT-CURVATURE RELATIONSHIPS FOR CONTINOUS BEAM

BASED ON EXPERIMENTAL DATA BY BERWANGER - REF. 56

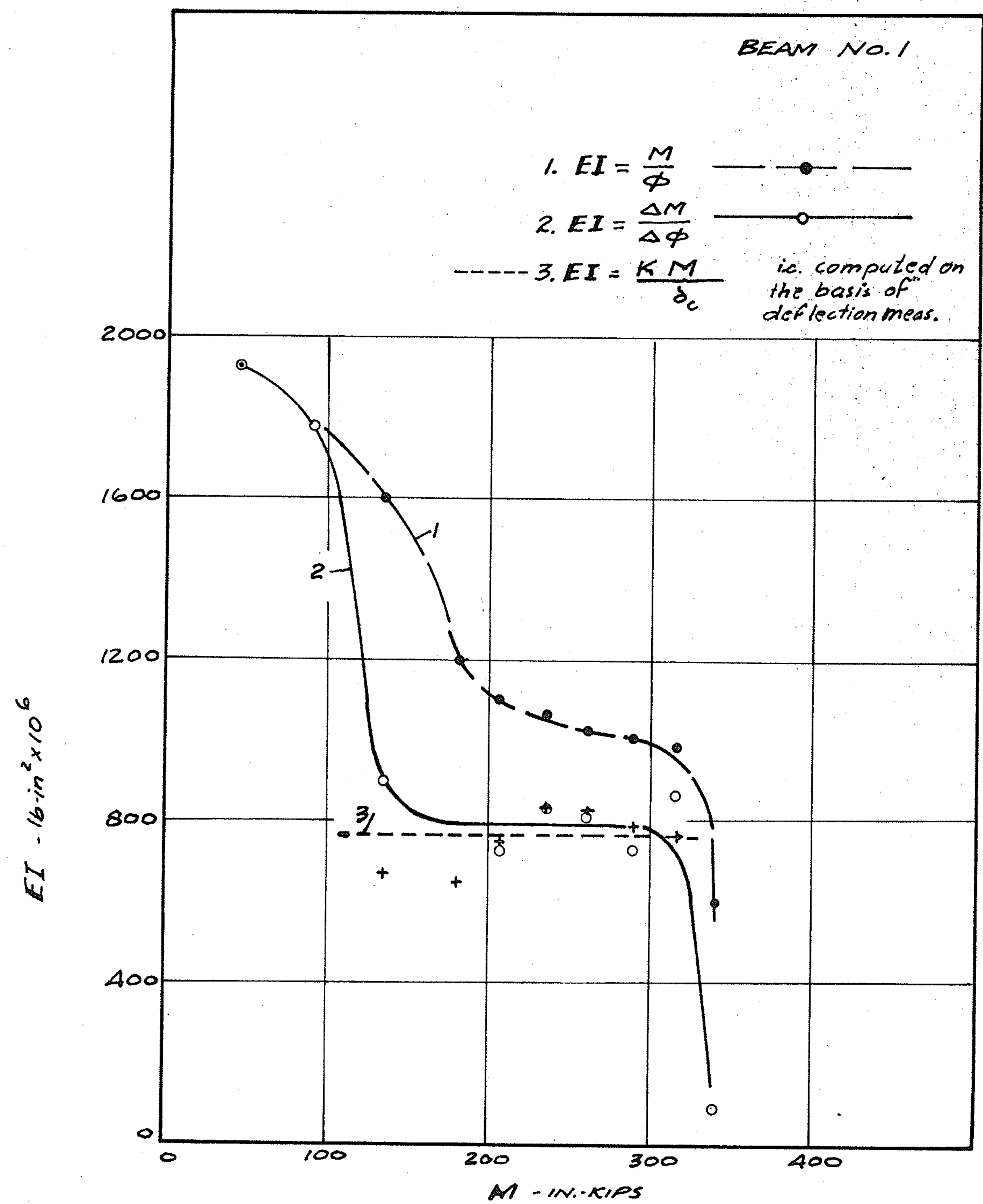
FIGURE 5.17



MOMENT - CURVATURE RELATIONSHIPS FOR CONTINOUS BEAM

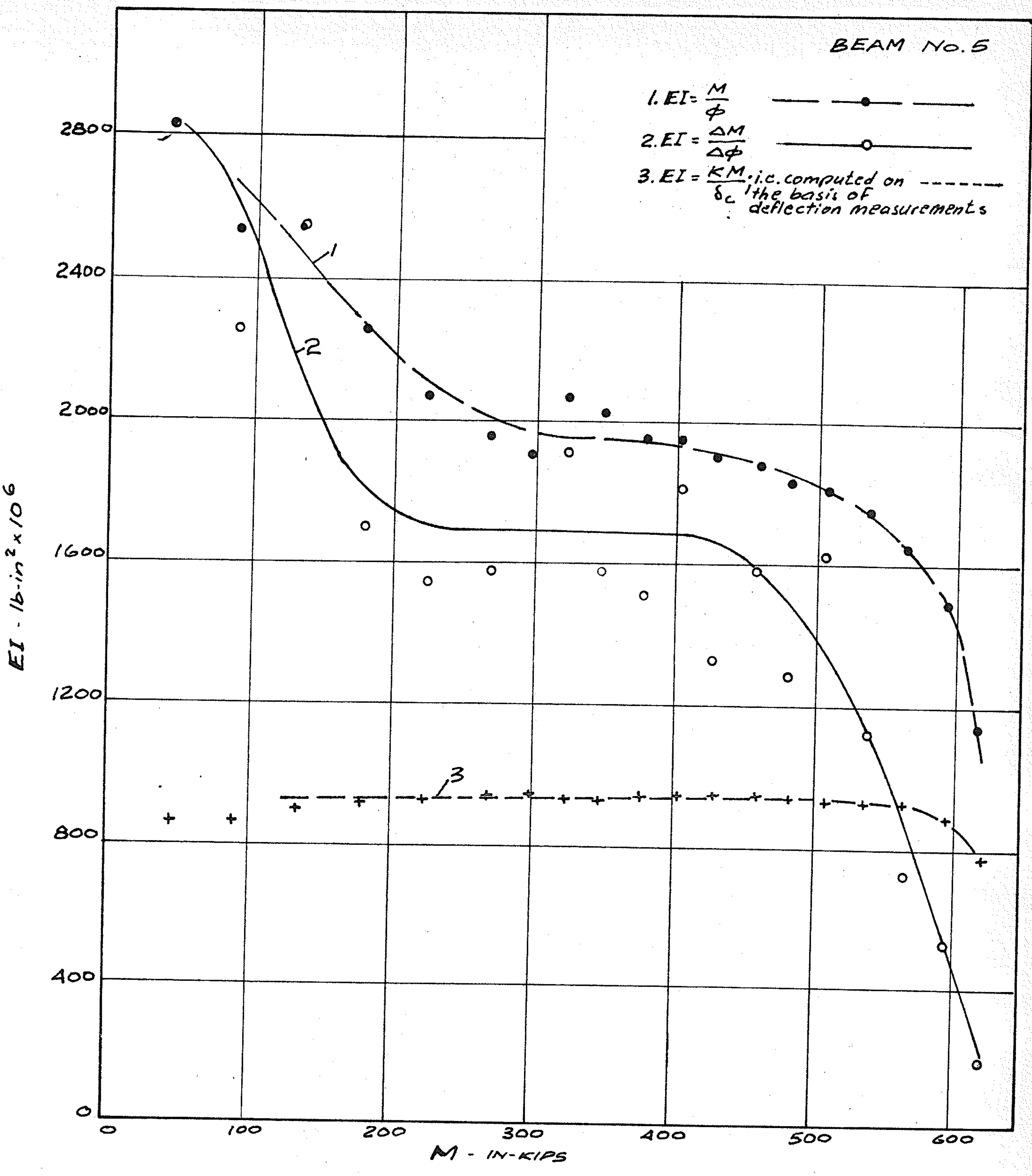
BASED ON EXPERIMENTAL DATA BY BERWANGER - REF. 56.

FIGURE 5.18



COMPARISON OF VARIOUS BASIS FOR DETERMINING FLEXURAL RIGIDITY EXPERIMENTALLY

FIGURE 5.19



COMPARISON OF VARIOUS BASIS FOR DETERMINING FLEXURAL RIGIDITY EXPERIMENTALLY

FIGURE 5.20

FLEXURE TEST OF BEAM No. 1

DATE OF CASTING Nov. 19, 1960

DATE OF TEST Dec. 17, 1960
TIME 10^{15} AM – 11^{15} AM
AVERAGE TEMPERATURE 75°F
GAUGE FACTOR SETTING 2.16

LOAD IN KIPS	GAUGE READINGS IN MICROINCHES No. 1	No. 2	No. 3	No. 4	No. 5	No. 6	MIDSPAN DEFLECTION INCHES x 10^{-3}	REMARKS
GAUGES IN CLAMPS	855	857	506	585	530	722	—	
0	2100	3450	1660	1720	1180	1710	0	
5.0	2080	3390	1670	1715	1280	1740	1.5	
10.0	2001	3355	1670	1640	1310	1800	0	Deflection gauge re-zeroed at 10^k load
15.0	1930	3300	1680	1700	1570	2010	28.5	
20.0	1850	3240	1730	1780	1910	2320	59.5	First crack observed at 20^k
23.0	1800	3210	1800	1850	2080	2510	77.0	
26.0	1725	3180	1830	1920	2210	2660	93.0	
29.0	1685	3150	1860	2000	2340	2860	110.0	
32.0	1620	3115	1875	2065	2480	3000	127.0	
35.0	1580	3090	1920	2135	2620	3170	146.0	
37.5	1520	35	2570	2920	3920	4550	—	
44.66								Ultimate load - Yielding of tens. reinf.

[illegible]

LOCATION OF GAUGES:

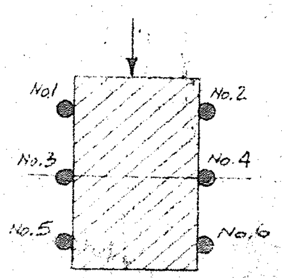


TABLE 5.1

FLEXURE TEST OF BEAM No. 2

DATE OF CASTING Nov. 19 1960

DATE OF TEST DEC. 16, 1960
TIME 1.00 P.M — 4.00 P.M
AVERAGE TEMPERATURE 75°F
GAUGE FACTOR SETTING 2.16

LOAD IN KIPS	GAUGE READINGS IN MICROINCHES						MIDSPAN DEFLECTION INCHES×10⁻³	REMARKS
	No.1	No.2	No.3	No.4	No.5	No.6		
GAUGES IN CLAMPS	839	858	506	587	533	704	—	
0	3650	3600	1985	1425	1520	1260	0	
4.0	5315	3610	2110	2425	1235	1335	13.5	
6.0	5935	4205	2075	3845	2335	1300	20.0	
8.0	5600	3980	2075	1910	3380	1300	28.0	
10.0	5174	3980	2095	1925	2460	1570	36.5	
12.0	5400	3760	2320	3120	3660	1880	46.5	
14.0	5420	3730	2350	2420	2880	1600	57.0	
18.0	5390	3670	2335	2430	2910	1730	78.5	
22.0	5430	3610	2350	2430	3000	1880	98.0	
26.0	5340	3550	2365	2410	3070	2030	117.0	
30.0	5300	3490	2355	2440	3160	2170	137.5	
34.0	5335	3430	2140	2470	3200	2320	158.0	
36.0	5270	3390	2410	2470	3310	2410	175.0	
38.0	5350	3390	2440	2500	3380	2480	186.0	
41.0	5330	3400	3420	3310	3870	4500	214.0	
45.0								← Ultimate load ~ Yielding of tensile reinforcement

Gauge No.1 over-stressed – inoperative.

LOCATION OF GAUGES :

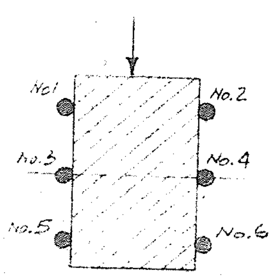


TABLE 5.2

FLEXURE TEST OF BEAM No. 3

DATE OF CASTING Nov. 22, 1960

DATE OF TEST DEC. 20, 1960
TIME 9:30 AM – 10:15 AM
AVERAGE TEMPERATURE 73.5 °F
GAUGE FACTOR SETTING 2.16

LOAD IN KIPS	GAUGE READINGS IN MICROINCHES No. 1	No. 2	No. 3	No. 4	No. 5	No. 6	MIDSPAN DEFLECTION INCHES x 10^{-3}	REMARKS
GAUGES IN CLAMPS	858	860	514	588	531	732	—	
0	3060	3160	1820	1770	650	790	0	
4.0	3100	3260	1850	1770	680	810	19.0	
6.0	3070	3200	1850	1765	740	835	32.0	
8.0	3060	3150	1830	1765	870	970	46.0	
10.0	2980	3120	1965	1980	1310	1415	74.0	
12.0	2970	3120	2040	2150	1590	1780	105.0	First crack observed at 12^K
14.0	2860	3120	2120	2290	1830	2030	126.0	
15.0	3000	3130	2175	2360	1960	2175	136.0	
16.0	2900	3135	2190	2435	2060	2300	148.0	
17.0	2960	3135	2220	2455	2140	2380	210.5	
18.0	3060	3130	2320	2570	2340	2560	263.0	
19.0	3060	3830	4030	4530			364.0	
20.7								← Ultimate load ~ Yielding of tensile reinforcement; secondary compression failure

J. Reinert

LOCATION OF GAUGES:

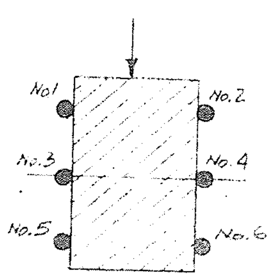


TABLE 5.3

FLEXURE TEST OF BEAM No. 4

DATE OF CASTING Nov. 22 1960

DATE OF TEST DEC. 20 1960
TIME 11:30 AM — 12:20 PM
AVERAGE TEMPERATURE 74.5°F
GAUGE FACTOR SETTING 2.16

LOAD IN KIPS	GAUGE READINGS IN MICROINCHES						MIDSPAN DEFLECTION INCHES x 10^{-3}	REMARKS
	No. 1	No. 2	No. 3	No. 4	No. 5	No. 6		
GAUGES IN CLAMPS	853	853	504	585	524	784	—	
0	3120	3480	1090	1245	1040	725	0	
4.0	3120	3600	1100	1255	1075	745	17.0	
6.0	2620	3040	1100	1255	1095	765	27.5	
8.0	3150	2980	1120	1280	1145	840	38.0	
10.0	3165	3575	1110	1300	1215	905	55.0	First crack observed at 10k
12.0	3160	3530	1235	1385	1665	1260	77.0	
14.0	3030	3500	1310	1480	1875	1450	95.0	
15.0	2960	3500	1365	1550	2050	—	105.0	GAUGE No 6 INOPERATIVE
16.0	3160	3490	1410	1600	2180	—	114.0	
17.0	3170	3490	1460	1655	2285	—	124.0	
18.0	3165	3480	1490	1675	2365	-	142.0	
19.0	3165	3480	1565	1765	2555	-	160.0	
20.0	3165	3650	2960	3230	4050	-	225.0	
21.38								Ultimate load - yielding of tensile reinforcement

J. Rinast

LOCATION OF GAUGES :

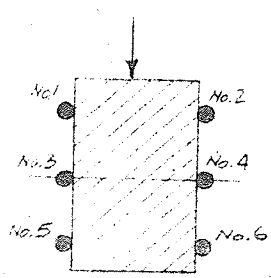


TABLE 5.4

FLEXURE TEST OF BEAM No. 5

DATE OF CASTING Nov. 26 1960

DATE OF TEST DEC. 23, 1960
TIME 11^{00} AM – 12^{30} PM
AVERAGE TEMPERATURE 75°F
GAUGE FACTOR SETTING 2.16

LOAD IN KIPS	GAUGE READINGS IN MICROINCHES						MIDSPAN DEFLECTION INCHES x 10^{-3}	REMARKS
	No. 1	No. 2	No. 3	No. 4	No. 5	No. 6		
GAUGES IN CLAMPS	858	854	514	583	529	563	–	
0	3080	3250	1300	1580	700	700	0	
5.0	3050	3175	1240	1565	750	735	22.5	
10.0	2965	3111	1245	1575	775	800	45.0	
15.0	2930	3060	1255	1600	815	885	64.0	
20.0	2910	3000	1260	1655	880	1060	85.0	
25.0	2810	2955	1270	1671	915	1230	105.0	
30.0	2720	2915	1285	1755	1000	1360	124.0	First observed crack (30^K)
33.0	2660	2885	1285	1785	1040	1460	136.0	
36.0	2785	2850	1295	1815	1075	1525	149.0	
39.0	2750	2820	1295	1835	1125	1620	162.0	
42.0	2685	2775	1295	1855	1165	1685	174.0	
45.0	2645	2735	1285	1860	1200	1750	187.0	
48.0	2570	2665	1275	1865	1235	1815	199.0	
51.0	2530	2620	1270	1875	1275	1895	212.5	
54.0	2455	2555	1275	1895	1320	1965	226.0	
57.0	2415	2505	1265	1895	1360	2035	240.0	
60.0	2330	2430	1265	1905	1420	2105	255.0	
63.0	2280	2370	1295	1995	1540	2335	278.0	
66.0	2145	2305	1340	2135	1710	2610	317.5	
69.0	2055	2235	1600	2455	2375	3485	349.0	
72.49								Ultimate load. Bending Failure ~ yielding of steel with secondary compressive failure in concrete.

I. Reinert

LOCATION OF GAUGES:

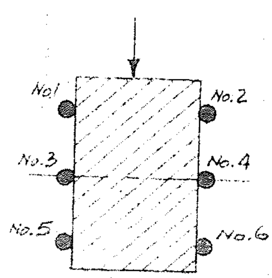


TABLE 5.5

FLEXURE TEST OF BEAM No. 6

DATE OF CASTING Nov. 26, 1960

DATE OF TEST DEC. 23, 1960
TIME 9^{00}AM — 10^{00}AM
AVERAGE TEMPERATURE 75°F
GAUGE FACTOR SETTING 2.16

LOAD IN KIPS	GAUGE READINGS IN MICROINCHES						MIDSPAN DEFLECTION INCHES $\times 10^{-3}$	REMARKS
	No. 1	No. 2	No. 3	No. 4	No. 5	No. 6		
GAUGES IN CLAMPS	856	854	513	585	527	563	—	
0	3080	3275	1365	1430	890	615	0	
5.0	2965	3220	1365	1425	920	675	19.5	
10.0	3065	3170	1380	1440	1015	805	41.5	
15.0	2990	3110	1395	1460	1140	975	67.0	
20.0	2980	3045	1430	1495	1305	1150	91.0	
23.0	2940	3005	1450	1510	1420	1255	104.0	
26.0	2845	2950	1465	1520	1525	1345	117.0	First observed crack (26^K)
29.0	2800	2905	1480	1535	1610	1440	128.5	
32.0	2720	2850	1490	1545	1695	1535	141.5	
35.0	2650	2800	1500	1550	1775	1615	154.0	
38.0	2510	2745	1515	1565	1870	1715	166.0	
41.0	2345	2690	1520	1570	1965	1820	180.0	
44.0	2520	2640	1550	1595	2060	1910	192.0	
47.0	2485	2600	1575	1610	2165	2015	204.0	
70.53								Ultimate load – compression failure

I. Reinart

LOCATION OF GAUGES:

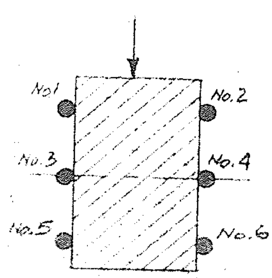


TABLE 5.6

COMPRESSION TESTS ON CONTROL CYLINDERS
BEAM NO. 1

DATE OF CASTING Nov. 19, 1960 DATE OF TESTING DEC. 16, 1960

LOAD IN KIPS	STRESS P.S.I.	GAUGE READINGS INCHES x10⁻⁴		UNIT STRAINS INCHES PER INCH x10⁻⁴		REMARKS
		No. 1	No. 2	GAUGE No. 1	GAUGE No. 2	
0	0	0	0	0	0	
11	390	6.2	6.5	.78	.81	
20	708	14.5	14.0	1.81	1.75	
30	1062	23.5	21.0	2.94	2.62	
40	1416	33.5	27.5	4.19	3.44	
50	1772	44.9	34.5	5.61	4.31	
60	2126	54.3	41.0	6.79	5.12	
70	2480	61.6	47.5	7.70	5.94	
80	2836	70.0	55.0	8.75	6.88	
90	3190	79.5	65.5	9.94	8.19	
100	3540	90.0	79.0	11.25	9.88	
116.7	4130	–	–			Ultimate Load

J. Reinsart

THE ABOVE STRAIN READINGS WERE TAKEN ON CONTROL CYLINDER FROM CENTRE BATCH

CYLINDER FROM END BATCH: ULTIMATE LOAD 121,200 lb
ULTIMATE STRESS 4290 p.s.i.

TABLE 5.7

COMPRESSION TESTS ON CONTROL CYLINDERS
BEAM NO. 2

DATE OF CASTING Nov. 19, 1960 DATE OF TESTING DEC. 16, 1960

LOAD IN KIPS	STRESS P.S.I.	GAUGE READINGS INCHES x10^{-4}		UNIT STRAINS INCHES PER INCH x10^{-4}		REMARKS
		No. 1	No. 2	GAUGE No. 1	GAUGE No 2	
0	0	0	0	0	0	
10	354	1.0	20.5	.12	2.56	
20	708	4.0	14.0	.50	1.75	
30	1062	7.0	2.5	.88	–	GAUGE No. 2 RE-ZEROED.
40	1416	11.1	5.0	1.34	.63	
50	1772	—	8.0	–	1.0	
60	2126	14.0	12.0	1.75	1.5	
70	2480	16.5	17.0	2.06	2.12	
80	2836	40.0	22.0	5.00	2.75	
90	3190	45.0	28.5	5.62	3.56	
100	3540	51.0	36.0	6.38	4.50	
110	3880	56.3	47.0	7.04	5.88	
120	4240	63.0	67.5	7.88	8.44	
123.41	4360	–	–	-	-	Ultimate Load
						J. Reinart

THE ABOVE STRAIN READINGS WERE TAKEN ON CONTROL CYLINDER FROM CENTRE BATCH

CYLINDER FROM END BATCH: ULTIMATE LOAD 124,300 lb
ULTIMATE STRESS 4390 p.s.i.

TABLE 5.8

COMPRESSION TESTS ON CONTROL CYLINDERS
BEAM NO. 3

DATE OF CASTING Nov. 22, 1960 DATE OF TESTING Dec. 19, 1960

LOAD IN KIPS	STRESS P.S.I.	GAUGE READINGS INCHES x 10^{-4} No. 1	No. 2	UNIT STRAINS INCHES PER INCH x 10^{-4} GAUGE No. 1	GAUGE No. 2	REMARKS
0	0	0	0	—		
10	354	12.5	8.0	1.56	1.00	
20	708	17.5	11.0	2.19	1.38	
30	1062	23.0	14.0	2.88	1.75	
40	1416	31.0	19.0	3.88	2.38	
50	1772	28.0	24.0	3.50	3.00	
60	2126	31.0	32.0	3.88	4.00	
70	2480	36.0	41.0	4.50	5.12	
80	2836	42.0	56.0	5.25	7.00	
90	3190	51.0	63.0	6.38	7.88	
100	3540	56.0	74.0	7.00	9.25	
110	3880	74.0	95.0	9.25	11.88	
120	4240	92.5	99.0	11.56	12.38	
131.5	4630	–	-			Ultimate Load
						J. Reimat

THE ABOVE STRAIN READINGS WERE TAKEN ON
CONTROL CYLINDER FROM CENTRE BATCH

CYLINDER FROM END BATCH: ULTIMATE LOAD 129,110 lb
ULTIMATE STRESS 4570 p.s.i.

TABLE 5.9

COMPRESSION TESTS ON CONTROL CYLINDERS
BEAM NO. 4

DATE OF CASTING NOV. 22, 1960 DATE OF TESTING DEC. 19, 1960

LOAD IN KIPS	STRESS P.S.I.	GAUGE READINGS INCHES x 10^{-4} No. 1	No. 2	UNIT STRAINS INCHES PER INCH x 10^{-4} GAUGE No. 1	GAUGE No. 2	REMARKS
0	0	0	0	0	0	
10	354	5.0	8.5	.62	1.06	
20	708	12.5	12.0	1.56	1.5	
30	1062	19.0	17.0	2.38	2.12	
40	1416	25.5	21.0	3.19	2.62	
50	1772	30.5	27.0	3.81	3.38	
60	2126	37.5	33.0	4.69	4.12	
70	2480	42.5	36.0	5.31	4.50	
80	2836	53.0	40.0	6.62	5.00	
90	3190	59.5	46.0	7.44	5.75	
100	3540	95.0	51.0	11.88	6.38	
110	3887	106.0	54.0	13.25	6.75	
120	4240	151.0	59.0	18.88	7.38	Ultimate Load
141.6	5000	—	—	—	—	
						T. Reinsart

THE ABOVE STRAIN READINGS WERE TAKEN ON CONTROL CYLINDER FROM CENTRE BATCH

CYLINDER FROM END BATCH: ULTIMATE LOAD 96,010 lb

ULTIMATE STRESS 3500 psi.

TABLE 5.10

COMPRESSION TESTS ON CONTROL CYLINDERS

BEAM NO.5

DATE OF CASTING NOV. 26, 1960 DATE OF TESTING DEC. 22, 1960

LOAD IN KIPS	STRESS P.S.I.	GAUGE READINGS INCHES $\times 10^{-4}$ No. 1	No. 2	UNIT STRAINS INCHES PER INCH $\times 10^{-4}$ GAUGE No. 1	GAUGE No. 2	REMARKS
0	0	0	0	–	–	
10	354	1.0	.125	.125	.625	
20	708	2.5	8.5	.315	1.06	
30	1062	6.5	13.7	.813	1.72	
40	1416	10.5	18.5	1.315	2.32	
50	1772	15.0	22.5	1.875	2.82	
60	2126	20.0	27.5	2.50	3.44	
70	2480	25.0	32.5	3.13	4.06	
80	2836	30.0	40.0	3.77	5.01	
90	3190	37.5	50.0	4.69	6.25	
100	3540	55.0	67.0	6.88	8.38	
104.73	3710	—	–			Ultimate Load

T. Rimart

THE ABOVE STRAIN READINGS WERE TAKEN ON CONTROL CYLINDER FROM CENTRE BATCH

CYLINDER FROM END BATCH: ULTIMATE LOAD 120, 810 lb

ULTIMATE STRESS 4260 psi.

TABLE 5.11

COMPRESSION TESTS ON CONTROL CYLINDERS

BEAM NO. 6

DATE OF CASTING Nov. 26, 1960

DATE OF TESTING DEC. 22, 1960

LOAD IN KIPS	STRESS P.S.I	GAUGE READINGS INCHES x10^-4 No. 1	No. 2	UNIT STRAINS INCHES PER INCH x10^-4 GAUGE No. 1	GAUGE No 2	REMARKS
0	0	0.0	0	0	0	
10	354	2.5	4.5	.31	.563	
20.3	718	5.0	8.5	.62	1.075	
30	1062	8.0	13.5	1.00	1.688	
40	1416	11.0	18.0	1.38	2.25	
50	1772	13.0	22.5	1.62	2.81	
60	2126	16.5	27.0	2.06	3.38	
70	2480	19.5	32.0	2.44	4.00	
80	2830	22.5	38.0	2.81	4.75	
90	3190	31.0	46.5	3.87	5.81	
100	3540	34.5	53.0	4.31	6.62	
110	3880	39.5	64.0	4.94	8.00	
126.58	4470	-	-			Ultimate Load.
						T. Rinaut

THE ABOVE STRAIN READINGS WERE TAKEN ON CONTROL CYLINDER FROM CENTRE BATCH

CYLINDER FROM END BATCH: ULTIMATE LOAD 115,910 lb

ULTIMATE STRESS 4090 p.s.i.

TABLE 5.12