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AGGREGATE VERSUS DISAGGREGATE ACREAGE SUPPLY RESPONSE MODELS FOR PRAIRIE WHEAT AND BARLEY

by

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ABSTRACT

The usefulness of a knowledge of agricultural supply relationships cannot be overestimated. This is a study in agricultural supply relationships. It is also an empirical study in aggregation.

Aggregation has been defined as a process whereby a part of the information available for the solution of a problem is sacrificed for the purpose of making the problem more easily manageable. Thus an aggregate quantity may be defined to represent a set of simple quantities, or a set of relationships among many simple quantities may be replaced by a single equation involving aggregate quantities.

Attention has been drawn by Theil to the nature of the parameters of the resulting aggregate equations. This study investigates the nature of the estimated macroparameters of aggregate wheat acreage response equations with a view to obtaining some knowledge of the effects of aggregation on macroparameters in general, and on the macroparameters of the wheat acreage response equations estimated in this study in particular. The explanatory and predictive performances of aggregate and disaggregate models are also compared in analysis involving wheat and barley acreage response.

The analysis relates to the Canadian Prairies (the macrounit). The microunits used in this study are the crop

districts in the Canadian Prairies. Acreage response relationships are estimated for original values of variables only. The time series data used in the wheat study cover the period 1953-54 through 1968-69, while the period 1950-51 through 1968-69 is used in the barley study.

The components of the estimated macroparameters of two wheat acreage response models are calculated, using data for the period specified above. The temporal composition of the macroparameters of one of these models is also investigated. The results indicate that the magnitude of aggregation bias and implied sampling error components of the estimated macroparameters differ for the different macroparameters, and for the same macroparameters at different time periods. Whilst these components of some of the estimated macroparameters are small (under 10 per cent of the magnitude of the estimated macroparameters), these same components assume relatively larger magnitudes in some other cases (about 200 per cent of the estimated macroparameters).

The results also indicate that it is possible for the sign and the magnitude of a macroparameter to be completely distorted by the sign and the magnitudes of these components (of an estimated macroparameter). Thus the results indicate that in practice we can sometimes expect aggregation to lead to misleading results.

Comparisons of the statistical significance of the estimated macroparameters in the wheat acreage equations with the statistical significance of the corresponding estimated microparameters in the crop district equations give another indication, namely, that aggregation can sometimes also lead to a distortion in our vision with respect to the relationships between dependent and independent variables at the level of the constituent microunits. In particular, we may be led to conclude that significant relationships exist between certain microvariables when in fact this may not be the case.

It is shown formally that we cannot decide a priori which will have a higher explanatory power, whether an aggregate or a disaggregate model. It is also shown formally that sometimes a semi-aggregate model can have a higher explanatory power than either an aggregate or a disaggregate model. If the empirical results in this study are any guide to what can always be expected, then they indicate that as far as wheat and barley in the Canadian Prairies are concerned, no significant gain in the explanation of aggregate acreages (in terms of R^2) can be obtained by disaggregation.

There have not been any empirical studies in aggregation using the functional approach, which compare the predictive performances of aggregate and disaggregate models. All previous comparisons between aggregate and disaggregate models have been confined to explanatory powers only. Similarly, there have been no comparisons made of the results of using the coefficients of an aggregate equation for policy purposes with those obtained by using the corresponding coefficients of microequations. These comparisons are also made in this study.

If the results of these comparisons are any guide to what can always be expected, then the indication here also is that as far as wheat acreage response in the Canadian Prairies is concerned, no significant gains can be had by disaggregation. The importance of this conclusion stems from the fact that disaggregation involves extra costs in terms of data collection and estimation for several crop districts or regions.

In the case of barley, the comparison of the performances of the aggregate and the disaggregate models is limited to their predictive abilities only. Here, if the results are any guide to what we can always expect, then they show that barley acreage response analysis at the level of the Provinces (semi-aggregate level) gives better results than analysis at either the level of the crop districts (disaggregate level), or at the aggregate (Canadian Prairies) level.

One cannot make any generalisations with respect to the relative performances of aggregate and disaggregate approaches. The results of this study do not allow us to make any such generalisation. If anything at all, the results confirm the statement by Grunfeld and Griliches that aggregation is not necessarily bad.

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CHAPTER I

INTRODUCTION

AGRICULTURAL SUPPLY ANALYSIS: OBJECTIVES, PROBLEMS, IMPORTANCE

A decade ago Nerlove and Bachman[1] stated the objectives of supply analysis as follows;

> ...to improve (1) our understanding of the mechanism of supply response, (2) our ability to forecast supply changes and (3) our competence to prescribe solutions to problems related to agricultural supply.

These objectives still hold.

Agricultural supply or production response analyses are essential to providing effective "outlook" information and to the development of sound agricultural policies both in the developing and developed countries. In developing economies which are largely based on agriculture, there is an increasing need for development. More detailed knowledge of agricultural supply relations is required in order to coax out the much needed agricultural output which is essential to the overall development of the countries. In the developed nations, on the other hand, improved information relating to supply responses is necessary because of the continuing problems faced in adjusting supplies to market

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[1]M. Nerlove and K. L. Bachman, "The Analysis of Changes in Agricultural Supply: Problems and Approaches," Journal of Farm Economics, Vol. XLII, No. 3 (August, 1960), pp. 531-554.

demand.

Examples of specific situations in which knowledge of quantitative supply relations in agriculture is useful are many. One example will be given. Suppose a government is considering the possibility of guaranteeing some fixed price for wheat. The government may, therefore, wish to know in advance how many bushels of wheat will have to be purchased for storage at public expense at any guaranteed price, so that a firm decision could be made. This is only possible if the demand and supply functions which are likely to determine the price of wheat in a free market are known.

Recognizing that the major problems of agriculture revolve around supply functions and supply relationships, Heady[2] has articulated the importance of research in this area as follows:

> If we know more about supply functions which will prevail over the future or the relative changes which will take place among regions, we could do much better in economic policy formulation and in counselling farmers on investment and occupational choices.

A better knowledge of farm supply patterns would also help in improving the mutual understanding of agricultural problems among nations reaching similar stages of economic development.

But supply analysis is beset with many gaps. Nerlove and Bachman have listed some important ones. On the

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[2]Earl O. Heady, "Uses and Concepts in Supply Analysis," in Heady, et al., <u>Agricultural Supply Functions</u> (Ames, Iowa: Iowa State University Press, 1961), p. 7.

theoretical side these are:[3]

(1) lack of an adequate theory of aggregation of firm supply functions;

(2) lack of an adequate theory of behaviour under uncertainty;

(3) lack of an adequate operational theory of investment for the firm; that is, an empirically useful theory of how so-called fixed factors are varied over time in response to economic stimuli; and

(4) lack of a theory of, or at least techniques for measuring, the diffusion of technological changes and their specific effects on the production possibilities open to the firm.

On the empirical side the gaps include:

(1) lack of reliable data;

(2) poorly estimated and unrealistic production functions;

(3) insufficient producer surveys providing a testing ground for more comprehensive studies of behaviour under uncertainty and variation of fixed factors over time;

(4) lack of sufficient knowledge of the markets for inputs in agriculture; and

(5) difficulty in capturing the dynamic aspects of supply in meaningful relationships.

These gaps still exist today although increasing

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[3]Nerlove and Bachman, op. cit., p. 532.

attempts are being made to bridge them and many important contributions have been made. Recognizing that approaches in all areas of supply analysis are complementary, not competitive, Nerlove and Bachman have drawn attention to the need for capitalizing on this complementarity by attempting to bring together all relevant findings to form a picture of the total supply or production response.[4] This is a study in aggregation which makes empirical contributions to Canadian Prairie wheat and barley acreage responses and to the aggregation problem in general.

AGGREGATION

Even though at the heart of supply analysis lies the individual firm, aggregation of individual firm supply responses is necessary before many results useful for policy and forecasting purposes can be derived. Aggregation is, therefore, as important in supply analysis as it is in many other areas of economics. Since this study focuses on aggregation (within the framework of supply analysis) it is reasonable that we take a general look at the "aggregation problem" in economics.

Generally the name aggregation is given either to the operation of defining a global quantity to represent a set of simple quantities, or to that of representing a set of relationships among many simple quantities by one or more

[4] Ibid., p. 534.

equations involving global quantities. The "aggregation problem" as discussed in economic methodology and in econometrics is concerned with the relationships between such elements as "microvariables" (the variables which impinge directly on the individual decision maker), "microrelations" (the results of individuals' actions), "macrovariables" (eg., totals, averages, indexes of the microvariables), and "macrorelations" (relationships between macrovariables).

On the one hand macrovariables are "constructs" rather than "natural" variables although some constructs by virtue of usage and a standard processing of data may almost come to be accepted as natural variables (eg., Gross National Product).[5] On the other hand, in descriptions of macrounits (such as an area or a nation) we generally are free to postulate relationships between macrovariables (usually but not always on the analogy of microtheory). Given, therefore, the freedom of construction of macrovariables and freedom in postulation of macrorelations, aggregation theory has been concerned with "consistency" between variables and relationships. Suppose, for example, that a microtheory tells us that each agricultural firm's output of a certain commodity is an increasing function of the firm's land fallowed in the previous year. Suppose also that the fallowed acreage of all firms moves up and down simultaneously. Then total

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[5]K. Lancaster, "Economic Aggregation and Additivity," in S. Krupp (ed.), <u>The Structure of Economic Science</u> (Englewood Cliffs: Prentice-Hall, Inc., 1966), pp. 201-213.

output must increase whenever total acreage fallowed does, so that a macrotheory in terms of these aggregates which tells us that total output is a decreasing function of total acreage fallowed is certainly not consistent with the microtheory.

Four approaches to aggregation theory have been suggested as follows:

(1) given the microrelation and microvariables, we may postulate a macrorelation by analogy to the microrelation and then search for relationships between the macrovariables and microvariables which make the macrorelation consistent with the microrelation. This approach has been championed by Klein.[6]

(2) given the microrelation and the microvariables, define the macrovariables and then search for a consistent macrorelation. This approach has been recommended by May.[7]

(3) given the macrotheory and macrovariables defined from some microvariables, one can try to find what kind of microtheory is consistent with the macrotheory.[8]

These three approaches to the aggregation problem have been collectively called the "consistency approach".

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[6] L. R. Klein, "Macroeconomics and the Theory of Rational Behaviour," Econometrica, Vol. 14, No. 2 (April, 1946), pp. 93-108.

[7] K. May, "The Aggregation Problem for a One-industry Model," Econometrica, Vol. 14, No. 2 (April, 1946), pp. 285-298.

[8] Henri Theil, Linear Aggregation of Economic Relations (Amsterdam: North Holland Publishing Company, 1954), p. 5.

Another approach--the "analogy approach" has been pioneered by Theil.[9] This approach recognizes that for one reason or another we usually have our macrovariables and our macro-relations postulated on the analogy of the microrelations--this is what we almost always do in empirical descriptions of macrounits.[10]

This procedure, however, raises some important questions which the "analogy approach" examines. For example, what is the significance of the macroparameters? How much information do they give concerning the relationships between the dependent and the independent variables?[11]

The procedures for studying agricultural supply relations may be put into two braod categories; those using functional analysis and those using mathematical programming. Theil's "analogy approach"--which may be called "statistical aggregation"--relates to the functional method. In the programming approach to supply analysis, considerable

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[9] Ibid., p. 6.

[10] Doubtless, this procedure is often followed out of mere expediency. For example, how could one possibly handle the relations involving thousands and indeed millions of microunits that may constitute the macrounit? Also, in many cases, information is available only for the macrounits rather than for the microunits. Green has defined aggregation as "a process whereby a part of the information available for the solution of a problem is sacrificed for the purpose of making the problem more easily manageable." See H. A. J. Green, Aggregation in Economic Analysis: An Introductory Survey (Princeton: Princeton University Press, 1964), p. 1.

[11] See Chapter II for a detailed theoretical discussion of this approach.

interest has been displayed in recent years in the synthetic estimation of aggregate commodity supply response by the operating unit method of estimation. This consists of defining a universe of farms (comprising an area), selecting a number of benchmark farms to represent the universe, and developing optimum plans for the benchmark farms by programming at variable prices for the commodity. The programmed results are then expanded (by some factor) to derive the aggregate supply response for the universe. One problem in this procedure is in the selection of a manageable number of benchmark farms for programming so that the expanded results give a supply function similar to that obtained if all farms in the universe were programmed. This is an aspect of the "consistency approach" to aggregation. The deviation of the estimate based on benchmark farms from programming all farms in the universe is defined as "aggregation bias".[12]

OBJECTIVES OF THE STUDY

This study is based on Theil's approach. It is

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[12]"Aggregation Bias" in a statistical sense is defined in Chapter II, p. 19. The problem of selection of benchmark farms to minimize the aggregation bias is discussed in a number of articles, see for example: S. J. Sheely and R. H. McAlexander, "Selection of Representative Benchmark Farms for Supply Estimation," Journal of Farm Economics, Vol. 47, No. 3 (August, 1965), pp. 681-695; G. E. Frick and R. A. Andrews, "Aggregation Bias and Four Methods of Summing Farm Supply Functions," Journal of Farm Economics, Vol. 47, No. 3 (August, 1965), pp. 696-700; and also, R. Barker and B. F. Stanton, "Estimation and Aggregation of Firm Supply Functions," Journal of Farm Economics, Vol. 47, No. 3 (August, 1965), pp. 701-712.

concerned with supply analysis using the functional approach (time series). Its purpose is to empirically evaluate the performances of disaggregate and aggregate models in explaining and forecasting aggregate output of two Prairie crops, namely, wheat and barley, and also to find the effect of aggregation on the parameters of aggregate wheat acreage response models.[13] The specific objectives of the study may be stated as follows;

(1) to obtain estimates of the parameters with respect to the micro and macrorelations; and

(2) to use these estimates to find (a) the relationship between the micro and the macroparameters of the acreage response functions; (b) how the disaggregate and the aggregate models compare in explanatory power; and (c) the forecasting performances of the disaggregate and the aggregate models.

GENERAL PROCEDURE

Three alternative single equation linear regression models are considered in the analysis of acreage response functions for wheat and barley using data for the Canadian Prairies. Data at three levels of aggregation are utilised.

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[13]This study is not concerned with how individual supply functions should be aggregated to obtain the aggregate supply function. For a theoretical approach to this problem of aggregation in the context of supply functions see M. Nerlove, "Time Series Analysis of the Supply of Agricultural Products," in E. O. Heady et al., Agricultural Supply Functions (Ames, Iowa: Iowa State University Press, 1961), pp. 54-58.

These three models are specified as follows:

Model I--a set of acreage response functions are specified and estimated one for each microunit (the crop district).

Model II--corresponding macrovariables are defined by aggregating the microvariables and these are then used in the estimation.

Model III--acreage response functions are derived from semi-aggregate variables obtained by dividing the set of all microvariables into non-overlapping groups and adding over all the microunits in each group. Unlike the previous models, this model is used only in the investigation of explanatory powers and forecasting performances; that is, it is not used in the study of the relationships between microparameters and macroparameters.

ORGANIZATION OF THE STUDY

The rest of the study is organized as follows. In Chapter II we discuss some theoretical aspects of Theil's approach to the aggregation problem which are of relevance to this study. The conceptual and the statistical aspects relating to a study of output response models for Canadian Prairie wheat and barley are considered in Chapter III. The statistical results which are subsequently obtained are presented and discussed in Chapter IV. The relationship between the microparameters and the macroparameters are taken up next in Chapter V. This same chapter presents and discusses the explanatory powers and the forecasting performances of the three models. The study is summarised and concluded in Chapter VI.

CHAPTER II

AGGREGATION: SOME THEORETICAL ASPECTS

INTRODUCTION

Linear aggregation of economic relations of relevance to time series analysis has been discussed from a theoretical standpoint by Theil[1] and examined both theoretically and empirically by Grunfeld and Griliches,[2] Boot and DeWit,[3] Green,[4] and Gupta.[5] This chapter surveys some of these researchers' theoretical approaches to the aggregation problem. It is hoped that this survey will place the empirical section of this thesis in clear perspective. The relationship between macroparameters and microparameters can be established either directly or by means of what Theil has

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[1]Theil, op. cit.

[2]Y. Grunfeld and Zvi Griliches, "Is Aggregation Necessarily Bad?" The Review of Economics and Statistics, Vol. XLII, No. 1 (February, 1960), pp. 1-13.

[3]J. C. G. Boot and G. M. DeWit, "Investment Demand: An Empirical Contribution to the Aggregation Problem," International Economic Review, Vol. I, No. 1 (January, 1960), pp. 3-30.

[4]Green, op. cit.

[5]K. L. Gupta, "Aggregation: A Theoretical and Empirical Study." Unpublished Ph.D. Dissertation, University of Toronto, Department of Economics, 1967. Partial summaries of Theil's book plus some additional applications, are also available in R. G. D. Allen, Mathematical Economics (London: MacMillan Book Company, 1956), Chapter 20.

called the "auxiliary equations". The latter approach has always been used in the literature without sufficient explanation of why they are introduced. This is made clear in this chapter.

RELATIONSHIP BETWEEN MACROPARAMETERS AND MICROPARAMETERS

Consider some variable A_t, which is a summation of microvariables A_{it} so that we have;

(2.2.1) $$A_t = \sum_{i}^{I} A_{it} \qquad \begin{matrix} i = 1,\ldots\ldots, I \\ t = 1,\ldots\ldots, T \end{matrix}$$

Let each microvariable A_{it} depend linearly on a micro-independent variable X_{it}, so that we have,

(2.2.2) $$A_{it} = \alpha_{oi} + \alpha_{1i}X_{it} + \varepsilon_{it}$$

where α_{oi} and α_{1i} are microparameters, and ε_{it} is a micro-disturbance in the i-th microequation. Relations (2.2.1) and (2.2.2) imply that

(2.2.3) $$A_t = \sum_{i} (\alpha_{oi} + \alpha_{1i}X_{it} + \varepsilon_{it})$$

that is,

(2.2.4) $$A_t = \alpha_o + \sum_{i} \alpha_{1i}X_{it} + \varepsilon_t$$

where,

(2.2.5) (a) $$\alpha_o = \sum_{i} \alpha_{oi}$$

(b) $$\varepsilon_t = \sum_{i} \varepsilon_{it}$$

It is seen from (2.2.4) that the macrodependent variable depends upon all the microindependent variables and not on the macroindependent variable X_t, where,[6]

(2.2.6) $$X_t = \sum_i X_{it}.$$

Yet, in much empirical work, the macrodependent variable is stated in terms of the macroindependent variable, that is, we postulate the following relation:

(2.2.7) $$A_t = \alpha_o^* + \alpha_1 X_t + \varepsilon_t^*$$

where α_o^* and α_1 are macroparameters and ε_t^* is a macrodisturbance.

This procedure of postulating a relationship between the macrovariables, however, raises a number of questions. One obvious question relates to the nature of these macroparameters and their relationship with the microparameters.

Two Variable Case

To examine the relationship between the macroparameters and the microparameters, we will consider a simple case as follows:

(2.2.8) $$A_{it} = \alpha_{oi} + \alpha_{1i} X_{it} + \varepsilon_{it}, \quad E(\varepsilon_{it}) = 0$$

(2.2.9) (a) $$A_t = \sum_i A_{it}$$

(b) $$X_t = \sum_i X_{it}$$

(2.2.10) $$A_t = \alpha_o^* + \alpha_1 X_t + \varepsilon_t^*$$

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[6] If, however, the microparameters are the same for each i, then a direct summation of all the macrorelations over i truly expresses the aggregate A_t in terms of the macroindependent variable X_t. In general, however, these microparameters will be different for each i.

where all the terms are as previously defined. The least squares estimate of α_1 can be written as,

$$(2.2.11) \qquad \hat{\alpha}_1 = \frac{\sum_t (A_t - \bar{A})(X_t - \bar{X})}{\sum_t (X_t - \bar{X})^2}$$

Using (2.2.8) and (2.2.9) we can express (2.2.11) as,

$$(2.2.12) \qquad \hat{\alpha}_1 = \frac{\sum_t \sum_i (\alpha_{oi} + \alpha_{1i}X_{it} + \varepsilon_{it} - \alpha_{oi} - \alpha_{1i}\bar{X}_i - \bar{\varepsilon}_i)(X_t - \bar{X})}{\sum_t (X_t - \bar{X})^2}$$

$$= \frac{\sum_t \sum_i \alpha_{1i}(X_{it}-\bar{X}_i)(X_t-\bar{X})}{\sum_t (X_t-\bar{X})^2} + \frac{\sum_t \sum_i (\varepsilon_{it} - \bar{\varepsilon}_i)(X_t-\bar{X})}{\sum_t (X_t-\bar{X})^2}$$

By interchanging the order of summation, we get,

$$(2.2.13) \qquad \hat{\alpha}_1 = \frac{\sum_i \alpha_{1i} \sum_t (X_{it}-\bar{X}_i)(X_t-\bar{X})}{\sum_t (X_t-\bar{X})^2} + \frac{\sum_t [\sum_i \varepsilon_{it} - \sum_i \bar{\varepsilon}_i](X_t-\bar{X})}{\sum_t (X_t-\bar{X})^2}$$

or,

$$(2.2.14) \qquad \hat{\alpha}_1 = \sum_i \alpha_{1i}B_i + \xi$$

where,

$$(2.2.15)\text{(a)} \qquad B_i = \frac{\sum_t (X_{it}-\bar{X}_i)(X_t-\bar{X})}{\sum_t (X_t-\bar{X})^2}$$

$$\text{(b)} \qquad \xi = \frac{\sum_t [\sum_i \varepsilon_{it} - \sum_i \bar{\varepsilon}_i](X_t-\bar{X})}{\sum_t (X_t-\bar{X})^2}$$

It can be shown that the sum of the B_i over all i is 1. The least squares estimate of α_o^* is, (2.2.16) $$\hat{\alpha}_o^* = \bar{A} - \hat{\alpha}_1 \bar{X}$$

Using (2.2.8) and (2.2.9) we can write,

(2.2.17) $$\hat{\alpha}_o^* = \sum_i (\alpha_{oi} + \alpha_{1i}\bar{X} + \bar{\varepsilon}) - \hat{\alpha} \quad \bar{X}$$

and using (2.2.13) we can write (2.2.17) as,

(2.2.18) $$\hat{\alpha}_o^* = \sum_i \alpha_{oi} + \sum_i \alpha_{1i}\bar{X}_i - \frac{\sum_i \alpha_{1i} \sum_t (X_{it}-\bar{X}_i)(X_t-\bar{X})\bar{X}}{\sum_t (X_t-\bar{X})^2} - \frac{\sum_t [\sum_i \varepsilon_{it} - \sum_i \bar{\varepsilon}_i] \; (X_t-\bar{X})}{\sum (X_t-\bar{X})^2}$$

By rearranging (2.2.18) we obtain,

(2.2.19) $$\hat{\alpha}_o^* = \sum_i \alpha_{oi} + \sum_i \alpha_{1i}\bar{X}_i - \sum_i \alpha_{1i}\bar{X}B_i - \frac{\sum_t [\sum_i \varepsilon_{it} - \sum_i \bar{\varepsilon}_i](X_t-\bar{X})\bar{X}}{\sum_t (X_t-\bar{X})^2}$$

where B_i is as defined in equation (2.2.15a). Collecting terms we can write (2.2.19) as,

(2.2.20) $$\hat{\alpha}_o^* = \sum_i \alpha_{oi} + \sum_i \alpha_{1i}(\bar{X}_i - \bar{X}B_i) - \frac{\sum_t [\sum_i \varepsilon_{it} - \sum_i \bar{\varepsilon}_i] \; (X_t-\bar{X})\bar{X}}{\sum_t (X_t-\bar{X})^2}$$

or,

(2.2.21) $$\hat{\alpha}_o^* = \sum_i \alpha_{oi} + \sum \alpha_{1i}\Gamma_i - \eta$$

where,

(2.2.22)(a) $$\Gamma_i = \bar{X}_i - \bar{X} \frac{\sum_t (X_{it}-\bar{X}_i)(X_t-\bar{X})}{\sum_t (X_t-\bar{X})^2}$$

(b) $$\eta = \frac{\sum_t \sum_i \varepsilon_{it} - \sum_i \bar{\varepsilon}_i] \; (X_t - \bar{X}) \; \bar{X}}{\sum_t (X_t-\bar{X})^2}$$

It can be shown that the sum of the Γ_i over all i is zero. Thus, it can be seen from (2.2.14) that the estimated macro-parameter, $\hat{\alpha}_1$ is a weighted average of the microparameters plus a random term. Since the microdisturbances have zero means,

$$E(\hat{\alpha}_1) = \alpha_1 = \sum_i \alpha_{1i} B_i \tag{2.2.23}$$

It can also be seen that the weights attached to the microparameters are the regression coefficients in regressions of the microindependent variables on the macroindependent variable, while the random term, ξ, is the coefficient term in a regression of the sum of the microdisturbances on the macroindependent variable, X_t.

Similarly, it follows from (2.2.21) that the estimated macroconstant term $\hat{\alpha}_o^*$ is the sum of the microconstant terms, $\sum_i \alpha_{oi}$, plus a weighted sum of the microparameters, minus a random term η. Since $E(\eta) = 0$,

$$E(\hat{\alpha}_o^*) = \sum_i \alpha_{oi} + \sum_i \alpha_{1i} \Gamma_i \tag{2.2.24}$$

From (2.2.22a) it can be seen that the weights attached to the microefficients are equal to the constant terms in the least squares regressions of the microindependent variables on the macrodependent variable, while the error term in (2.2.21) may also be seen from (2.2.22b) as the constant term in a regression of the sum of the microdisturbances on the macro-independent variable X_t.

It will prove convenient in expressing the macroparameters in terms of the microparameters to make direct use of the regressions of the microvariables on the macrovariables. These regressions have been called "auxiliary regressions" by Theil.[7]

Three Variable Case

In order to show the relationship between the macroparameters and the microparameters where there are more than one independent variables, we will consider the following relations:

$$(2.2.25) \qquad A_{it} = \alpha_{oi} + \alpha_{1i} X_{it} + \alpha_{2i} S_{it}, \qquad \begin{array}{l} i = 1,2,\ldots.,I \\ t = 1,2,\ldots.,T \end{array}$$

$$(2.2.26)\,(a) \qquad A_t = \sum_i A_{it}$$

$$(b) \qquad X_t = \sum_i X_{it}$$

$$(c) \qquad S_t = \sum_i S_{it}$$

$$(2.2.27) \qquad A_t = \alpha_o + \alpha_1 X_t + \alpha_2 S_t.$$

We assume that the microvariable A_{it} and the macrovariable A_t are determined without error. We now introduce the auxiliary regressions as follows:[8]

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[7]Theil, op. cit., p. 13.

[8]The relationships involved in equations (2.2.28), i.e., the auxiliary equations have no economic meaning. They are merely introduced because their coefficients help to explain the way in which the macroparameters are related to the microparameters. These relationships between the macroparameters and microparameters can also be established directly as was done in the two variable case. However, the less laborious approach is used here instead.

$$(2.2.28)\quad (a)\quad X_{it} = \Gamma_{1i} + B_{1i}X_t + C_{1i}S_t$$

$$(b)\quad S_{it} = \Gamma_{2i} + B_{2i}X_t + C_{2i}S_t$$

We assume that the auxiliary regressions also hold exactly, that is, they have no error terms.

Using (2.2.25) and (2.2.28) in (2.2.26) (a), we can write,[9]

$$A_t = \sum_i [\alpha_{oi} + \alpha_{1i}\ (\Gamma_{1i} + B_{1i}\ X_t + C_{1i}S_t) + \alpha_{2i}\ (\Gamma_{2i} + B_{2i}\ X_t + C_{2i}S_t)]$$

Gathering terms from the above equation and by reference to (2.2.27), we have,

$$(2.2.29)\quad (a)\quad \alpha_o = \sum_i \alpha_{oi} + \sum_i \alpha_{1i}\ \Gamma_{1i} + \sum_i \alpha_{2i}\ \Gamma_{2i}$$

or,

$$(b)\quad \alpha_o = \sum_i \alpha_{oi} + I[\mathrm{cov}(\alpha_{1i}, \Gamma_{1i}) + \mathrm{cov}(\alpha_{2i}, \Gamma_{2i})],$$

$$(2.2.30)\quad (a)\quad \alpha_1 = \sum_i \alpha_{1i}B_{1i} + \sum_i \alpha_{2i}B_{2i}$$

or,

$$(b)\quad \alpha_1 = \bar{\alpha}_{1i} + I[\mathrm{cov}(\alpha_{1i}, B_{1i}) + \mathrm{cov}(\alpha_{2i}, B_{2i})],$$

$$(2.2.31)\quad (a)\quad \alpha_2 = \sum_i \alpha_{1i}C_{1i} + \sum_i \alpha_{2i}C_{2i}$$

or,

$$(b)\quad \alpha_2 = \bar{\alpha}_{2i} + I[\mathrm{cov}(\alpha_{1i}, C_{1i}) + \mathrm{cov}(\alpha_{2i}, C_{2i})]$$

where I is the number of microrelations, and

$$\mathrm{cov}(\alpha_{1i}, B_{1i}) = \frac{1}{I}\sum_i (\alpha_{1i} - \frac{1}{I}\sum_i \alpha_{1i})(B_{1i} - \frac{1}{I}\sum B_{1i});$$

the other covariance terms are similarly defined.

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[9]In deriving the covariance expressions use should be made of the fact that

$$\sum_i \Gamma_{1i} = \sum_i \Gamma_{2i} = \sum_i B_{2i} = \sum_i C_{1i} = 0;\ \sum B_{1i} = \sum_i C_{2i} = 1$$

These relationships can easily be established.

(in the case of the coefficients α_{1i} and α_{2i}).[10] For the constant term the bias is a deviation from the sum of the microconstants. Aggregation bias is defined in this way because the terms from which the bias is defined are what one would "naturally" expect for the parameters of one aggregate relationship which is introduced to represent a number of microrelations.[11]

The aggregation bias in (2.2.29b), (2.2.30b) and (2.2.31b) can be seen to vanish under any one of the following two conditions;

(2.2.32) $\alpha_{1i} = \alpha_{1j}$; $\alpha_{2i} = \alpha_{2j}$ for all $i,j = 1,2,\ldots,I$

and

(2.2.33) $\Gamma_{1i} = \Gamma_{1j}$; $\Gamma_{2i} = \Gamma_{2j}$; $B_{1i} = B_{1j}$; $B_{2i} = B_{2j}$; $C_{1i} = C_{1j}$; $C_{2i} = C_{2j}$

for all $i,j = 1,2,\ldots,I$

In general, however, we may not expect conditions (2.2.32) or (2.2.33) to hold, which means that we can in general expect aggregation bias to be present in the macroparameters.

We can generalise the conclusions represented by

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[10] Theil, op. cit., p. 122.

[11] Allen, op. cit., p. 700. It must be pointed out, however, that the aggregation bias depends on the way in which we form the macrovariables. For example, if these are weighted sums of the corresponding microvariables then the aggregation bias would be defined as a discrepancy from the weighted averages of the corresponding microparameters.

(2.2.29b), (2.2.30b), and (2.2.31b) by stating that in general a macroparameter depends upon all microparameters. Thus, for example, if A_i represents wheat acreage, and X_i represents price of wheat, and S_i represents on-farm wheat stocks at planting time, then the macro price parameter depends not only upon the microprice parameters which correspond to it in economic meaning (corresponding parameters"), but also upon the micro on-farm wheat stocks which do not correspond to it in economic meaning ("non-corresponding parameters"). Thus if we wish to describe wheat production in a macro formulation, a low coefficient for the price variable does not necessarily imply that the microparameters are low (or that the price coefficient of some of the microrelations are positive while others are negative) for it may be that the individual reactions to price are all positive and considerable, but that the macro-price parameter has been "disturbed" by the behaviour pattern of the macro entities with respect to the farm stocks.[12]

But there is another aspect, namely, the question of the stability of the macroparameter. With regard to this Theil has written:

> Another interesting point is that the macroparameters may change in the course of time, even if the underlying microparameters remain constant. This is due to the fact that the weights which connect macro and microparameters depend on the behaviour over time of the

[12]Theil, op. cit., p. 23.

> explanatory microvariables . . . if this behaviour changed, one must generally expect that the macroparameters will change too.[13]

The question of parameter stability is important in the context of their use in policy formulation. This is because one would expect that policy would be based only on a parameter that is stable.

The conclusion that macroparameters depend on corresponding and non-corresponding microparameters is an important one. This dependence makes one wonder how much information concerning the relationship between the dependent variable and the appropriate independent variable is conveyed by the macroparameter.

But, does the existence of a bias in the macroparameter affect the efficiency of macromodels in predicting the values of the macro dependent variable? This aspect of aggregation theory is discussed in the next section.

AGGREGATION AND PREDICTION[14]

In this section we shall compare the explanatory power of an all microequations model (Model I), a macroequation

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[13]Theil, op. cit., p. 135. Another aspect of the instability of macroparameters is that different statistical methods yield different estimates of same macroparameters. Ibid., p. 135.

[14]"Prediction" as used in this section refers to the predicted values of the sample values of the endogenous variable. Its strict use is in connection with future values. The word "explanation" is sometimes used interchangeably with prediction in this sense in this section.

model (Model II) and a semi-macroequations model (Model III). The first comparison, that is, Model I versus Model II has been made by Grunfeld and Griliches[15] who postulated "true" microequations of the form,

$$Y_{it} = b_{y1i\cdot 2}X_{1it} + b_{y2\cdot 1i}X_{2t} + U_{it}, \quad i = 1,\ldots.,k \tag{2.3.1}$$

where $X_{2t} = \sum_i X_{1it}$; $b_{y1i\cdot 2}$ and $b_{y2\cdot 1i}$ are microparameters and U_{it} is a microdisturbance.

However, instead of estimating these microequations we estimate the following relationship:

$$Y_{it} = b_{y1i}X_{1it} + W_{it} \tag{2.3.2}$$

where b_{y1i} is a microparameter and W_{it} is a microdisturbance. At the same time we also estimate the following aggregate equation:

$$\sum_i Y_{it} = b_2X_{2t} + U_t \tag{2.3.3}$$

where b_2 is a macroparameter and U_t is a macrodisturbance.

As Grunfeld and Griliches put it

> what we are contrasting are the results of leaving one macrovariable out of K microequations with the results of leaving out K microvariables from the macroequation.[16]

The objective of this contrast was to find out how the models will perform in prediction when the microequations and the macroequations are mispecified.

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[15]Grunfeld and Griliches, op. cit., pp. 1-13.

[16]Ibid., p. 7.

Gupta[17] has compared Model III with Models I and II using basically the Grunfeld and Griliches equations that is when both the microequations and the macroequations have different mispecification errors. Our approach in comparing the relative predictive powers of Model I, II and III will be to consider microequations (whether they have a specification error or not)[18] and equations which are exactly like these microequations except that the variables in these latter equations are either aggregates (Model II) or semi-aggregates (Model III). This is the approach taken by Theil. Generally, however, the discussion here follows the lines taken by Grunfeld and Griliches and by Gupta.

All Microequations Model Versus a Macroequation Model

In order to compare the "explanatory power" of an all microequations model (Model I) with a macroequation model (Model II), we shall consider the following types of relations:

(2.3.4) $$A_{it} = \alpha_{oi} + \alpha_{1i}X_{it} + \varepsilon_{it}, \quad E(\varepsilon_{it}) = 0$$

(2.3.5) $$A_t = \alpha_o + \alpha_1 X_t + \varepsilon_t$$

(2.3.6) (a) $$A_t = \sum_i A_{it}$$

(b) $$X_t = \sum_i X_{it}$$

(2.3.7) $$X_{it} = \Gamma_i + B_i X_t + V_{it}$$

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[17]Gupta, op. cit., p. 13.

[18]For a discussion of specification errors see Chapter III, p. 49. See also H. Theil, Economic Forecast and Policy (Amsterdam: North Holland Publishing Company, 1958), p. 327.

where ε_{it} and ε_t are microdisturbances and macrodisturbances, respectively, and α_{oi}, α_{1i}, α_o, and α_1 are micro- and macro-parameters. Equation (2.3.7) is the i-th auxiliary equation which expresses a relationship between X_{it} and the macro-independent variable X_t; Γ_i and B_i are parameters and V_{it} is a residual in the auxiliary equation.

After some manipulations involving the above equations we can obtain,[19]

$$(2.3.8) \qquad \varepsilon_t = \sum_i \varepsilon_{it} + \sum_i \alpha_{1i} V_{it}$$

From (2.3.8) we can write,

$$(2.3.9) \qquad E(\varepsilon_t)^2 = E(\sum_i \varepsilon_{it})^2 + E(\sum_i \alpha_{1i} V_{it})^2$$

$$+ 2E(\sum_i \varepsilon_{it})(\sum_i \alpha_{1i} V_{it})$$

If as is often assumed in regression analysis,

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[19] Equations (2.3.6) and (2.3.4) imply that

$$(2.19.1') \quad A_t = \sum_i (\alpha_{oi} + \alpha_{1i} X_{it} + \varepsilon_{it})$$

Using (2.3.7) in the above equation, we get,

$$(2.19.2') \quad A_t = \sum_i \alpha_{oi} + \alpha_{1i}(\Gamma_i + B_i X_t + V_{it}) + \varepsilon_{it}$$

$$= \sum_i \alpha_{oi} + \sum_i \alpha_{1i}\Gamma_i + \sum_i \alpha_{1i} B_i X_t + \sum_i \alpha_{1i} V_{it} + \sum_i \varepsilon_{it}$$

By reference to (2.3.5) we see that

$$(2.19.3') \quad \varepsilon_t = \sum_i \varepsilon_{it} + \sum_i \alpha_{1i} V_{it}.$$

the X_i's are nonstochastic;[20] then (2.3.9) reduces to,

$$(2.3.10) \qquad E(\varepsilon_t)^2 = E(\sum_i \varepsilon_{it})^2 + E(\sum_i \alpha_{1i}V_{it})^2$$

It can be seen from (2.3.10) that $E(\varepsilon_t)^2$ is greater than $E(\sum_i \varepsilon_{it})^2$ unless the second term on the right hand side of (2.3.10) vanishes.[21]

However, the econometrician is often faced with data in which the X_i's (i.e., the independent variables) are generated by some random process, which means that the X_i's are stochastic. In this case, if the X_i's are correlated with the ε_i's, the last term on the right hand side of (2.3.9) does not vanish.[22] In fact, there is the possibility of a negative correlation between $\sum_i \varepsilon_{it}$ and $\sum_i \alpha_{1i}V_{it}$ which could make $E(\varepsilon_t)^2$ smaller than $E(\sum_i \varepsilon_{it})^2$.

Grunfeld and Griliches have used "comparable R^2's" as measures of the predictive powers of Model I and Model II.

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[20]The term non-stochastic as used here means that the X_i's are fixed. If this is so, then the Γ_i's and B_i's as well as the V_{it}'s in (2.3.7) are also non-stochastic.

[21]For example this will be so when V_{it} is zero.

[22]If the X_i's are stochastic, then so are the V_{it}'s, but if the ε_i's and the V_i's are independently distributed, then the last term in (2.3.9) will still vanish.

This is a "composite R^2" value (R_c^2) for Model I, and an "aggregate R^2" (R_a^2) for Model II. These are defined as follows:

(2.3.11) $R_c^2 = 1 - \dfrac{S_c^2}{S_A^2}$ and, (2.3.12) $R_a^2 = 1 - \dfrac{S_a^2}{S_A^2}$

where S_c^2 is the variance of the sum of the residuals in Model I, and S_a^2 is the variance of the residuals in Model II and S_A^2 is the variance of the macrodependent variable A_t. Since the denominators are the same in (2.3.11) and (2.3.12), all we need do in order to compare the predictive powers of Model I and Model II is to compare the variances of their residuals. If following Grunfeld and Griliches, and Gupta, we regard the disturbances in (2.3.9) as calculated residuals, we can interpret the terms in (2.3.9) as variances and co-variances. In this case, we note that unless the sum of the last two terms in (2.3.9) vanishes, Model I will predict better than Model II. However, if the last term is negative there is the possibility that both models will predict with equal accuracy, or Model II will outperform Model I.

Relative Predictive Power of a Semi-macroequations Model (Model III)

Consider microrelations of the forms of (2.3.4), where i = 1,....,I, and t = 1,....,T. Let us partition the I individuals into m groups of sizes I(1),....,I(m), respectively. Now consider for each group a relation of the form of (2.3.4), but involving the sum of the variables in that group so that we have,

$$(2.3.4)\,(a) \qquad A_{irt} = \alpha_{oir} + \alpha_{1ir}X_{irt} + \varepsilon_{irt}\,, \qquad r = 1,2,\ldots\ldots,m\,, \quad i = 1,2,\ldots\ldots,I(r).$$

$$(2.3.13)\,(a) \qquad A_{rt} = \sum_{i}^{I(r)} A_{irt}$$

$$(b) \qquad X_{rt} = \sum_{i=1}^{I(r)} X_{irt}$$

$$(c) \qquad X = \sum_{r}^{m} X_{rt}$$

$$(d) \qquad \sum_{r} I(r) = I$$

$$(2.3.14) \qquad A_{rt} = \alpha_{or} + \alpha_{1r}X_{rt} + W_{rt}$$

$$(2.3.15) \qquad X_{irt} = \Gamma_{ir} + B_{ir}X_{rt} + V_{irt}$$

where W_{rt} are semi-macrodisturbances and all other variables are as previously defined. It can be shown that,[23]

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[23] From (2.3.13) and (2.3.4a) we have:

$$(2.23.1') \qquad A_{rt} = \sum_{i} A_{irt} = \sum_{i}[\alpha_{oir} + \alpha_{1ir}X_{irt} + \varepsilon_{irt}]$$

Using (2.3.15) we can write,

$$(2.23.2') \qquad A_{rt} = \sum_{i} [\alpha_{oir} + \alpha_{1ir}(\Gamma_{ir} + B_{ir}X_{rt} + V_{irt}) + \varepsilon_{irt}]$$

$$= \sum_{i} \alpha_{oir} + \sum_{i}\alpha_{1ir}\Gamma_{ir} + \sum_{i} \alpha_{1ir}B_{ir}X_{rt} + \sum_{i} \alpha_{1ir}V_{irt} + \sum_{i} \varepsilon_{irt}$$

By reference to (2.3.14) we see that,

$$(2.23.3') \qquad W_{rt} = \sum_{i} (\alpha_{1ir}V_{irt} + \varepsilon_{irt})$$

$$(2.3.16) \qquad W_{rt} = \sum_{i}^{I(r)} (\alpha_{1ir} V_{irt} + \varepsilon_{irt})$$

which means that the sum of the disturbances of Model III is,

$$(2.3.17) \qquad \sum_{r}^{m} W_{rt} = \sum_{r}^{m} \sum_{i}^{I(r)} (\alpha_{1ir} V_{irt} + \varepsilon_{irt}).$$

Hence we can define the predictive power of Model III as,[24]

$$(2.3.18) \qquad R_p^2 = 1 - \frac{S^2_{\sum\limits_r W_{rt}}}{S_A^2}$$

where the numerator of the last term on the right hand side of (2.3.18) is the variance of the sum of the disturbances of Model III. We wish to compare R_p^2 with R_c^2 and R_a^2. To do this, we will assume that there are only four individuals (i.e., i=1,....,4). We will also assume that each group contains two individuals (i.e., I(r)=2, m=2 and r=1,2), and that variables are measured by their deviation from their mean.[25] Consider now the following equations;

(2.3.19)

(a)	$A_1 = A_{11} + A_{21}$	(f)	$A_2 = A_{32} + A_{42}$
(b)	$X_1 = X_{11} + X_{21}$	(g)	$X_2 = X_{32} + X_{42}$
(c)	$X_{11} = B_1 X_1 + V_{11}$	(h)	$X_{32} = B_3 X_2 + V_{32}$
(d)	$X_{21} = B_2 X_1 + V_{21}$	(i)	$X_{42} = B_4 X_2 + V_{42}$
(e)	$A_1 = C_1 X_1 + W_1$	(j)	$A_2 = C_2 X_2 + W_2$

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[24]This definition has been given by Gupta, op. cit., p. 19.

[25]For convenience, the subscript t has been dropped.

Using (2.3.4a) and (2.3.19) we can show that,[26]

$$(2.3.20) \qquad (W_1+W_2) = \sum_{r=1}^{2} W_r = \sum_{i=1}^{4} \varepsilon_i + \sum_{i=1}^{2} \alpha_{1i}V_{i1} + \sum_{j=3}^{4} \alpha_{1j}V_{j2}$$

From (2.3.20) we can write,

$$(2.3.21) \qquad E(\sum_{r=1}^{2} W_r)^2 = E(\sum_{i=1}^{4} \varepsilon_i)^2 + E(\sum_{i=1}^{2}\alpha_{1i}V_{i1})^2 + E(\sum_{j=3}^{4}\alpha_{1j}V_{j2})^2 + 2E(\sum_{i=1}^{2}\alpha_{1i}V_{i1})(\sum_{j=3}^{4}\alpha_{1j}V_{j2}) + 2E(\sum_{i=1}^{2}\alpha_{1i}V_{i1})(\sum_{i=1}^{4}\varepsilon_i) + 2E(\sum_{j=3}^{4}\alpha_{1j}V_{j2})(\sum_{i=1}^{4}\varepsilon_i).$$

If as before, we assume that the X_i's are fixed, then (2.3.21) reduces to,

$$(2.3.22) \qquad E(\sum_{r=1}^{2} W_r)^2 = E(\sum_{i=1}^{4}\varepsilon_i)^2 + E(\sum_{i=1}^{2}\alpha_{1i}V_{i1})^2 + E(\sum_{j=3}^{4}\alpha_{ij}V_{j2})^2 + 2E(\sum_{i=1}^{2}\alpha_{1i}V_{i1})(\sum_{j=3}^{4}\alpha_{1j}V_{j2})$$

We note from (2.3.22) that Model I will predict better than Model III unless the sum of the last three terms on the right hand side of (2.3.22) vanishes. It is possible that the last term in (2.3.22) is negative, in which case

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[26]See footnote 23, p. 28.

there is the possibility that Model III may outperform Model I. These conclusions hold if the X_i's are fixed. If they are stochastic, however, and they are correlated with the ε_i's, then the outcome will depend on the signs and the magnitudes of the terms in (2.3.21).

Once again, following Grunfeld and Griliches, if we interpret the disturbances in (2.3.21) as calculated residuals, we can regard the terms in (2.3.21) as variances and covariances. On the basis of the simplifying assumption that the last two covariance terms in (2.3.21) vanish, we can write that equation as,

$$(2.3.23) \qquad S^2_{\underset{r}{\Sigma} W_r} = S^2_{\underset{i}{\Sigma}\varepsilon_i} + \sum_{i}^{2} \alpha^2_{1i} S^2_{v_{i1}} + 2\alpha_{11}\alpha_{12} S_{v_{11}v_{21}} + \sum_{j=3}^{4} \alpha^2_{1j} S^2_{v_{j2}} + 2\alpha_{13}\alpha_{14} S_{v_{32}v_{42}} + 2\sum_{i}\underset{i\neq j}{\sum_{j}} \alpha_{1i}\alpha_{1j} S_{v_{i1}v_{j2}}$$

To compare Model III with Model I in terms of explanatory power, we can write (2.3.23) as follows:[27]

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[27]It should be noted from (2.3.19c) and (2.3.19d) that,

$$(2.27.1') \qquad V_{11} = X_{11} - B_1 X_1$$

and,

$$(2.27.2') \qquad V_{21} = X_{21} - B_2 X_1$$

Adding the above two equations, we get,

$$(2.27.3') \qquad V_{11} + V_{21} = X_{11} + X_{21} - (B_1 + B_2) X_1 = X_1 - X_1 = 0, \text{ (since } B_1 + B_2 = 1)$$

Hence,

$$(2.27.4') \qquad V_{11} = -V_{21}, \text{ and } S^2_{v_{11}} = S^2_{v_{21}} = - S_{v_{11}v_{12}}.$$

$$(2.3.24)\qquad S^2_{\underset{r}{\Sigma W_r}} = S^2_{\underset{i}{\Sigma \varepsilon_i}} + (\alpha_{11}-\alpha_{12})^2 S^2_{v_{11}} + (\alpha_{13}-\alpha_{14})^2 S^2_{v_{32}}$$

$$+2\sum_{i\neq j}\sum \alpha_{1i}\alpha_{1j} S_{v_{i1}v_{j2}}$$

If we follow Grunfeld and Griliches and make new definitions for α_{1i} and α_{1j}, we can express (2.3.23) also as,[28]

$$(2.3.25)\qquad S^2_{\sum_{r=1}^{2} W_r} = S^2_{\sum_{i=1}^{4} \varepsilon_i} + \sum_{i=1}^{2} S^2_{A_{i1}} \alpha^2_{A_{i1}} (1-\gamma_{x_{i1}x_1})$$

$$+ 2\alpha_{A_{11}} \alpha_{A_{21}} [\gamma_{x_{11}x_{21}} - \gamma_{x_{11}x_1}\gamma_{x_{21}x_1}] S_{A_{11}} S_{A_{21}}$$

$$+ \sum_{j=3}^{4} S^2_{A_{j2}} \alpha^2_{A_{j2}} (1 - \gamma^2_{x_{j2}x_2})$$

$$+ 2\alpha_{A_{32}}\alpha_{A_{42}} [\gamma_{x_{32}x_{42}} - \gamma_{x_{32}x_2}\gamma_{x_{42}x_2}] S_{A_{32}} S_{A_{42}}$$

$$+ 2\sum_{i\neq j}\sum \alpha_{1i}\alpha_{1j} S_{v_{i1}v_{j2}}$$

We can now draw the following conclusions,

(i) If all the expressions on the right hand side of (2.3.23) are positive, then Model I will predict better than Model III. As can be seen from (2.3.24), however, if the partitioning of the microunits with respect to Model III

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[28] See the mathematical notes to this chapter, Appendix A. These relationships have been shown to hold by Grunfeld and Griliches and by Gupta. See Grunfeld and Griliches, op. cit., pp. 12-13 and Gupta, op. cit., pp. 101-108. The reason for repeating these proofs in this study is to provide a quick reference for this chapter.

is such that the coefficients belonging to the microunits in each group are close to one another, then the difference in predictive performance between Model I and Model III will not be great.

(ii) In (2.3.25), the second and fourth terms on the right hand side indicate that, if the correlation between the X_i's of the same group is very high (close to 1), then the predictive performance of Model III may come closer to that of Model I. In (2.3.25), the signs of the third and fifth terms on the right hand side can be shown to be negative. Also, the last term on the right hand side of (2.3.25) may be negative. Consequently, there is the possibility that Model III may turn out to outperform Model I in prediction.

These conclusions hold if the last two terms on the right hand side of (2.3.21) vanish. If, however, these two terms do not vanish, then the result will depend on the signs and the order of magnitudes of the terms involved.

In order to compare Model III with Model II while still assuming that we have calculated residuals, and on the basis of the simplifying assumption that the last term in (2.3.9) vanishes, we can write that equation as,[29]

$$(2.3.26) \qquad S^2_{\varepsilon_t} = S^2_{\sum\limits_i^4 \varepsilon_i} + \sum_{i=1}^{4} S^2_{A_i} \left(1 - \gamma_{x_i x}\right) \alpha^2_{A_i}$$

[29]This can be seen from following the procedure outlined in Appendix A.

$$+ \sum_{i \neq j}\sum S_{A_i} S_{A_j} [\gamma_{x_i x_j} - \gamma_{xx_i}\gamma_{xx_j}] \alpha_{A_i} \alpha_{A_j}$$

In comparing (2.3.25) to (2.3.26) it seems reasonable to conclude that if the X_i's within each group are highly correlated then,

$$[\sum_{i=1} S^2_{A_{i1}} \alpha^2_{A_{i1}} (1 - \gamma^2_{x_{i1}x_1}) + \sum_{j=3}^{4} S^2_{A_{j2}} \alpha^2_{A_{j2}} (1 - \gamma_{x_{j2}x_2})]$$

$$< \sum_{i=1}^{4} S^2_{A_i} (1 - \gamma_{x_1 x}) \alpha^2_{A_i}$$

While the third and fifth terms of (2.3.25) and the third term of (2.3.26) are each negative,[30] it is difficult to say what are their relative magnitudes. The last term in (2.3.25) has no counterpart in (2.3.26). If this term is negative and its magnitude relatively large, then Model III may predict better than Model II but the result may well be reversed if its sign is positive.

To summarise, we have shown formally that aggregation bias exists in macroparameters. Whether this bias is large or small in any particular situation is an empirical question. In this study, aggregation bias is calculated for the parameters of the aggregate Canadian Prairie wheat acreage supply response function. The temporal compositions of the macroparameters are also investigated.

The "predictive" performances of aggregate and disaggregate models are also compared empirically. It was seen

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[30]This has been shown by Gupta, op. cit., p. 22.

from the theoretical discussion that a partitioned approach may sometimes have a higher explanatory power than either an all-microequations model or a macroequation model. Consequently, a semi-macroequations model (Model III) is included in the comparison of explanatory powers. The measure of explanatory power used is the multiple correlation coefficient, R^2.

It is recognized that a high multiple correlation coefficient does not necessarily imply low forecast errors.[31] Consequently a comparison that is also made among the three models is their performances in forecasting the aggregate Canadian Prairie wheat and barley acreages.

The smallest units on which data are readily available are crop districts. These are used as the micro-units in this study.

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[31] Theil, Economic Forecast and Policy, op. cit., pp. 211-215.

CHAPTER III

CONCEPTUAL AND STATISTICAL MODELS AND PROBLEMS ASSOCIATED WITH ESTIMATION

INTRODUCTION

This chapter is devoted to a discussion of the conceptual and statistical matters relating to the estimation of output response functions for wheat and barley in the Canadian Prairies. It also contains a discussion of the methodology for the calculation of aggregation bias where the macrovariables are weighted averages of the corresponding microvariables. There are three sections. Section one is concerned with developing conceptual models for these crops. Econometric topics including the choice of variables and equations, and estimation techniques are discussed in section two. In section three we discuss the methodology for the calculation of aggregation bias in the macroparameters and the macrodisturbances. Microeconomic theory assumes that entrepreneurs act rationally, implying that entrepreneurs' adjustments to economic changes are guided by the criterion of profit maximisation. This assumption is taken as the point of departure in the estimations in this study. Also, whether or not production and input markets are perfectly or imperfectly competitive is not germane to this study. This is because, the objective of this study is not to explain the determination of product and input prices, but rather to study farmers' reactions, given these product and input prices.

CONCEPTUAL MODELS FOR WHEAT AND BARLEY PRODUCTION

For expositional tidiness the discussion will be in terms of a single commodity--"grains". However, when the specific models are presented, distinction will be made between the two different categories of grains considered in this study.

Grain production in the Canadian Prairies can be separated into two components--acreage and yield. Though farmers can influence yields to some extent, year to year changes in yields are considered to be mainly due to technological and weather factors which are outside the control of farmers. Therefore, in considering grain production decisions it seems reasonable to assume that in any year the acreage that the farmer plans to plant is a measure of his intended supply.[1] Acreage rather than production is, therefore, taken as the variable to be explained in this study. The variables which influence the desired acreage of grains at time t are the expected price level of grains, the price of other factors used in conjunction with land, and the opportunity cost of the land used in grain production. Thus

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[1]This assumption has been made in many studies, see B. Oury, "A Tentative Production Model for Wheat and Feedgrains in France." Unpublished Ph.D. Dissertation, University of Wisconsin, Department of Agricultural Economics, 1963, p. 142; Timothy E. Josling, The United Kingdom Grains Agreement (1964): An Economic Analysis, Institute of International Agriculture, Michigan State University, 1967, p. 32.

for any given state of technology, desired acreage may be written as,

(3.1.1) $$A_g^* = F(P_g^*, P_o, P_c)$$

where:

A_g^* = desired acreage of grains,

P_o = the price of inputs other than land,

P_g^* = the expected price level of grains, and

P_c = the opportunity cost of land devoted to grains.

But there are other technical constraints which farmers take into consideration in formulating acreage plans. Problems of disease control and soil fertility impose limitations on the rotational pattern that farmers may adopt and consequently affect the desired acreage. Various institutional factors also exert an important influence on the acreage devoted to grains. These institutional factors show up in various forms, a significant one as far as grains in the Prairies are concerned is stocks of grains held on farms. As stocks of wheat owned by farmers on farms accumulate due to the imposition of quotas and other restraints by the Canadian Wheat Board, farmers may plan to divert production from wheat to other commodities such as flax, rapeseed, barley or some non-grain commodity. Other influences on desired grain acreage are the improvements in technology such as better varieties and improved chemicals. Therefore, we may express desired acreage as,

(3.1.2) $$A_g^* = F(P_g^*, P_o, P_c, S_w, I, T, C)$$

where:

S_w = stocks of grains on farms,

I = institutional factors,
T = technological changes, and
C = cultural factors, and
all other variables are as previously described.

Even though farmers may formulate specific plans as to the acreage they intend to sow to grains, the acreage that is actually seeded is influenced to a large extent by the condition of the land prior to and during the planting season. Bad conditions may prevent preparation of the land for grains and this land may subsequently be used for some other non-grain enterprise. Therefore, actual seeded acreage can be expressed as,

$$(3.1.3) \qquad A_g = F(P^*_g, P_o, P_c, S_w, I, T, C, W)$$

where:

W = weather,
A_g = actual seeded acreage, and
all other variables are as previously described.

STATISTICAL MODELS AND APPROACHES USED TO ESTIMATE ACREAGE RESPONSE FUNCTIONS

In the previous section, we derived a functional expression relating seeded acreage to certain other variables (both economic and non-economic). The relationship was stated in a deterministic form. Doubtlessly, some other relevant variables may have been ignored in the above formulation partly as a result of ignorance of the theoretical basis for their inclusion. If these omitted variables are assumed to be random, that is, if they can be represented by a disturbance term with a random distribution, then we can write, (3.1.4) $A_g = F(P^*_g, P_o, P_c, T, W, S_w, C, I, U)$

where:

U = the disturbance term, and
all other variables are as previously described.

Price Expectation Models

The prices entering the acreage response function, it may be noted, are those expected by grain producers. But farmers' expectations are not directly observable. It is required, therefore, to replace these expectational variables by observable variables. Toward this end, Nerlove has proposed a general price expectation model which states that the "normal" price expected by farmers in the current period is the expected "normal" price in the immediate past period plus some factor which "is proportional to the difference between the actual and the expected "normal" price."[2]

In symbols, this hypothesis can be expressed as,

(3.1.5) $P^*_t = P^*_{t-1} + \beta [P_{t-1} - P^*_{t-1}], \quad 0 < \beta \leq 1,$

where:

P^*_t = the expected "normal" price in period t,

P^*_{t-1} = the expected "normal" price in period t-1,

P_{t-1} = the actual price received in period t-1, and

β = the coefficient of expectations.

It can be seen from equation (3.1.5) that if $\beta = 1$, we can write,

(3.1.6) $P^*_t = P_{t-1}$

[2]Nerlove, The Dynamics of Supply, op. cit., pp. 52-53.

If, therefore, farmers' supply response involves a price expectation model such as is represented by equation (3.1.6), we can rewrite equation (3.1.4) as,

$$(3.1.7) \qquad A_G = F(P_{g,t-1}, P_o, P_c, T, W, S_w, C, I, U)$$

where:

$P_{g,t-1}$ = the actual price received for grains in t-1, and

all other variables are as previously described. This function is the first model examined in this study and is equivalent to testing the hypothesis that farmers plan production upon the basis that this year's price will be the same as the price realized last year. This specification of the response model has been variously referred to as a "simple cobweb model", "the model of extrapolative expectations" and as the "static model."[3] This static model may be regarded as a limiting case of the more general Nerlove model of price expectation formation--the "model of adaptive expectations."

The general price expectation model expressed in equation (3.1.5) implies that the expected "normal" price to which producers respond is a weighted average of past prices with the weights declining from recent to less recent

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[3]For example, see J. H. Duloy and A. S. Watson, "Supply Relationships in the Australian Wheat Industry: New South Wales," Australian Journal of Agricultural Economics, Vol. 8, No. 1 (June, 1964), pp. 36-7; and T. J. Mules and F. G. Jarrett, "Supply Responses in the South Australian Potato Industry," Australian Journal of Agricultural Economics, Vol. 10 (June, 1966), p. 54.

periods. The rate of decline of the weights depends upon the value of β, the coefficient of expectation. If β is close to one, then the weights decline sharply and only a few past prices are important. If β is close to zero, a great many past prices are given consideration. Clearly an estimate of β would shed a great deal of light on the price response pattern.

To obtain an estimate of β, however, it is first necessary to assume that there is no lag in the adjustment of current acreage to "desired" or "long-run equilibrium acreage."[4] This is a reasonable assumption in the case of grains in the Prairies. Within the confines of a given planting period, agronomic limitations such as rotation requirements, and technological rigidities such as limiting machinery capacity are not likely to seriously restrict the farmer making a complete and "instantaneous" adjustment to changes in expected "normal" price.

Making this assumption and assuming a simple supply response equation such as,

$$(3.1.8) \qquad A_t = \alpha_o + \alpha_1 P^*_t + U_t,$$

where all symbols are as previously described, we can by simple manipulations, transform equations (3.1.5) and

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[4]See Nerlove, The Dynamics of Supply, op. cit., pp. 61-62. This assumption is necessary to enable β to be unambiguously interpreted as the coefficient of expectation.

(3.1.8) into the following equation:[5]

(3.1.9) $$A_t = \alpha_o\beta + \alpha_1\beta P_{t-1} + (1 - \beta)A_{t-1} + V_t$$

where,

(3.1.10) $$V_t = U_t - (1 - \beta)U_{t-1} .$$

Therefore, if the general price expectation model is used, we can combine it with (3.1.4) to obtain,

(3.1.11) $$A_g = F(P_{g,t-1}, P_o, P_c, T, W, S_w, C, I, A_{g,t-1}, V)$$

This model is also examined in this study.

Choice of Estimation Technique

It seems appropriate to indicate approaches to statistical estimation which have been used in connection with farm supply models. Many research workers have used single equation regression models. These include studies

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[5]The derivation of equation (3.1.9) can be achieved by lagging equation (3.1.8) by one period to obtain,

(3.5.1') $$A_{t-1} = \alpha_o + \alpha_1 P^*_{t-1} + U_{t-1}$$

From (3.5.1') we can write,

(3.5.2') $$P^*_{t-1} = (A_{t-1} - \alpha_o - U_{t-1})^{1}/\alpha_1$$

Substituting (3.5.2') in (3.1.5) above, we get,

(3.5.3') $$P^*_t = (1 - \beta)(A_{t-1} - \alpha_o - U_{t-1})^{1}/\alpha_1 + \beta P_{t-1}$$

Using (3.5.3') in (3.1.8) above we get,

(3.5.4') $$A_t = \alpha_o + (1-\beta)(A_{t-1}-\alpha_o-U_{t-1}) + \alpha_1\beta P_{t-1} + U_t .$$

i.e.,

(3.5.5') $$A_t = \alpha_o\beta + \alpha_1\beta P_{t-1} + (1 - \beta)A_{t-1} + V_t .$$

by Bowlen,[6] Candler,[7] Oury,[8] Duloy and Watson,[9] Pluta,[10] Josling,[11] and Schmitz.[12] However, since Haavelmo[13] discussed the implications of simultaneous equation estimation and applied the technique with Girschick[14] in determining the structural parameters of a system including a demand and supply equation for food in aggregate, some studies have also used this approach in connection with supply estimation. For example, Foote[15] and Cromarty[16] have used the simultaneous

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[6]B. J. Bowlen, "The Wheat Supply Function," Journal of Farm Economics, Vol. 37 (December, 1955), pp. 1177-1185.

[7]Wilfred Candler, "An Aggregate Supply Function for New Zealand Wheat," Journal of Farm Economics, Vol. 39 (December, 1958), pp. 1732-1741.

[8]B. Oury, op. cit.

[9]Duloy and Watson, op. cit.

[10]L. A. Pluta, "A Study of Acreage Allocation Decisions in Alberta and Saskatchewan, 1921-1962." Unpublished Ph.D. Dissertation, Queen's University, Department of Economics, 1967.

[11]Josling, op. cit.

[12]A. Schmitz, "An Economic Analysis of the World Wheat Economy in 1980." Unpublished Ph.D. Dissertation, The University of Wisconsin, Department of Economics, 1968.

[13]T. Haavelmo, "The Statistical Implications of a System of Simultaneous Equations," Econometrica, Vol. 11, No. 1 (January, 1943), pp. 1-12.

[14]M. A. Girschick and T. Haavelmo, "Statistical Analysis of the Demand for Food: Examples of Simultaneous Estimation of Structural Relations," Studies in Econometric Method, W. C. Hood and T. C. Koopmans (eds.) (New York: Wiley, 1953), pp. 92-111.

[15]R. J. Foote, "A Four Equation Model of the Feed and Livestock Economy and its Endogenous Mechanism," Journal of Farm Economics, Vol. XXXV, No. 1 (February, 1953), pp.44-61.

[16]W. A. Cromarty, "An Econometric Model for U. S.

equation approach involving supply and demand of agricultural products in the United States.

The simultaneous equation approach was not considered for adoption in this study. If, equation (3.1.11) which contains a lagged value of the endogenous variable is estimated by least squares, biased and inconsistent estimates will result.[17] It is possible, however, to obtain asymptotically unbiased and consistent estimates of the parameters of (3.1.11) by using an estimator such as Liviatan's.[18] Despite the bias, however, least squares can be used to obtain the parameters of the model of adaptive expectations. For as Duloy and Watson have pointed out, "the bias of classical least squares estimates of distributed lag models compared with asymptotically unbiased estimators, can be considered in the same way as have similar problems arising with small sample estimates of simultaneous equations."[19]

Choice of Functional Form

Given the hypothesised existence of an acreage response function, the next step is to determine the

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Agriculture," Journal of American Statistical Association, Vol. 54 (September, 1959), pp. 556-574.

[17]This is so because A_{t-1} can be shown to be correlated with the disturbance, V_t in equation (3.1.11).

[18]N. Liviatan, "Consistent Estimation of Distributed Lags," International Economic Review, Vol. 4, No. 1, (January, 1963), pp. 44-52. See also L. Klein, "The Estimation of Distributed Lags," Econometrica, Vol. 26 (October, 1958), p. 5.

[19]Duloy and Watson, op. cit., p. 37.

appropriate functional form. One rule that appears to be relevant in choosing among alternative forms is to refer to the logic or the basic mechanics of the decision process involved. In this sense, a "true" functional representation underlies a given acreage response function. However, we know very little about the mechanics of entrepreneurial decision processes to be able to establish both theoretically and empirically that a particular functional form adequately describes the logic and mechanics of an acreage response decision process. Yet, research of nearly four decades in the field of supply analysis using time series has established a strong presumption that two functions are realistic initial approximations of the "true" form. These are the Cobb-Douglas type functions which give a linear relationship between the logarithms of the variables, and functions which are linear in the original variables. These two functions are utilized in this study.

SOME MORE COMMON ECONOMETRIC PROBLEMS ASSOCIATED WITH ESTIMATION

Although the variables to be included in the estimated acreage functions are based on the theoretical models which we have developed, not all the variables are included in any equation for three reasons, namely, (1) when the number of regressors are large relative to the length of the time series available, the variance of the estimated coefficients increases and, hence, reliable estimates of the coefficients of the included variables cannot be obtained.

This has an important bearing on hypothesis testing; (2) variables may be omitted either because they cannot be measured or data for them are unavailable or difficult to quantify (e.g., disease and insect attack). The omission of variables gives rise to the problem of specification bias. It may also produce autocorrelated errors; (3) some variables may be omitted from an estimated equation because of the presence of multicollinearity.

Specification bias, autocorrelation and multicollinearity are econometric problems which very often show up in statistical estimation of parameters. Below is a brief account of these more common problems.[20]

Multicollinearity

There is a tendency for many economic series to move together over time as a result of the general interdependence of economic phenomena. This usually results in the appearance of approximate linear relationships in time series of regressors. This phenomenon is known as multicollinearity or intercorrelation.

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[20] The discussion here of econometric problems associated with the estimation of quantitative relationships between variables is not meant to be exhaustive. The purpose is to provide a quick reference for the more common problems associated with econometric estimation. More exhaustive and detailed reference to the material discussed here can be obtained from a large number of textbooks such as J. Johnston, Econometric Methods (London: McGraw-Hill Book Company, 1960), and A. S. Goldberger, Econometric Theory (New York: John Wiley & Sons, Inc., 1964). Other sources some of which are cited in this chapter also exist.

The problem presented by multicollinearity is that the variance of the regression coefficients become blown up and thereby reduce our confidence in these coefficients.

Multicollinearity also introduces the problem of "identification" of regression coefficients. In general, the coefficient of the i-th regressor measures the change we can expect to occur in the regressand if the i-th regressor changes by one unit, while all other regressors remain unchanged. However, in multicollinear data, there can be no independent movement of regressors and, hence, we do not know how much information concerning the relationship between regressand and a regressor is contained in its coefficient. Suggestions for dealing with multicollinearity include omitting some of the explanatory variables from the equation, and the use of extraneous information, if any.[21]

There is no concensus on how great the intercorrelation among regressors should be before the problems of multicollinearity mentioned above become serious. In this study, a correlation among regressors with a modulus in excess of 0.8 is taken as prima facie evidence of serious multicollinearity between the two variables. Where two or more variables are multicollinear in this sense one or more of them is suppressed.

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[21] See Goldberger, op. cit., pp. 255-262.

Specification Bias

Excluding relevant variables biases the estimated parameters, unless the regressors are orthogonal (i.e., completely independent of each other). To illustrate this, consider a true model:

$$(3.2.1) \qquad A = X\beta + U$$

where, A is an nxl vector. X is an nxk, matrix, β is an kxl vector, and U is an nxl vector. Suppose that the kth variable is omitted, and we instead estimate the following relation:

$$(3.2.2) \qquad A = \bar{X}\bar{\beta} + \bar{U}$$

where $\bar{X}$ is identical to X except that the k-th column is not represented in $\bar{X}$, (i.e., $\bar{X}$ is nxk-l; $\bar{\beta}$ and $\bar{U}$ are of appropriate order). In this case we can show that,

$$(3.2.3) \qquad E(\bar{\beta}) = \beta + \hat{\Pi}_k\beta_k$$

where $\hat{\Pi}_k\beta_k$ is the bias due to the error in specification.[22] However, if all the variables in X are uncorrelated then $\hat{\Pi}_k$ is a null vector and the specification bias vanishes. The X_i's are unlikely to be orthogonal so that we can in general expect specification bias when one or more relevant regressors have been excluded.

Autocorrelation

One of the most important assumptions of the

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[22] Π_k is a vector of regression coefficients in what Theil has called the "auxilliary regression." See Thiel, Economic Forecast and Policy, op. cit., p. 327.

classical linear regression model is that the disturbances are randomly and independently distributed. However, there are several reasons why this assumption may be invalid. One common cause of autocorrelated errors is the omission of variables from the analysis. These omitted variables subsequently find their place in a "composite" disturbance term so that, if these omitted variables are serially correlated, the "composite" disturbance may also be serially correlated, even though the disturbance term in the "true" relation may be non-autocorrelated. However, serial correlation in individual omitted variables need not result in autocorrelated disturbances since individual components may well cancel one another out. Autocorrelated errors are likely to result if the serial correlation in the omitted variables is pervasive and if the omitted variables tend to move together in the same direction. A second cause of serially correlated errors is mispecification of the functional form. If for instance, a linear form of equation is specified when the true form is quadratic, then the quadratic term will find its way into the disturbance term. A third possible cause of autocorrelation is errors in the variables.

If we use least squares when there are autocorrelated errors, the estimates of the parameters obtained are unbiased. However, the variances of these estimates are biased--in particular they are biased downwards if errors are positively autocorrelated, which is usually the case in economic time series. Estimation of regression relationships in the presence of autocorrelated errors are discussed in the textbooks footnoted.[23]

Because of the possible serious consequences of unwittingly applying least squares to relationships containing autocorrelated errors, it is important that we test for the presence of autocorrelation, and if necessary adopt alternative estimating procedures to circumvent it.

A widely used statistic to test for autocorrelation is the Durbin-Watson[24] statistic. This statistic is defined as,

$$(3.2.4) \qquad d = \frac{\Sigma_{t=2}^{T} (\hat{U}_t - \hat{U}_{t-1})^2}{\Sigma_{t=1}^{T} U_t^2}$$

Durbin and Watson have tabulated upper and lower bounds for different numbers of explanatory variables and observations. When conducting a one-tailed test for positive autocorrelation, we will reject the hypothesis of random disturbances in favour of that of positive autocorrelated disturbances, if the calculated d is less than d_L. If $d > d_U$, the null

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[23] See also W. J. Craddock, Autocorrelated Errors in Single Equation Least Squares Regression, Dept. of Agricultural Economics, University of Manitoba, Technical Bulletin No. 9 (January, 1968); and P. S. Dhruvarajan, "Problems of Serial Correlation and Missing Values in Econometric Models," Dept. of Economics (Mimeographed), University of Manitoba. More advanced material is contained in P. S. Dhruvarajan, "Maximum Likelihood Estimation of Simultaneous Equation Systems with Temporally Dependent Disturbances." Unpublished Ph.D. Dissertation, NorthWestern University, 1967.

[24] J. Durbin and C. S. Watson, "Test for Serial Correlation in Least Squares Regression," Biometrica II, 38 (June, 1951), pp. 159-178.

hypothesis will not be rejected, while the test is inconclusive if $d_L < d < d_U$.

The Durbin-Watson test has two notable shortcomings. First, the inconclusive range tends to be relatively large when the number of observations is about 25 or less and there are only a few explanatory variables. This means that we may tend not to reject many false, null hypotheses. Secondly, the test is strictly applicable only when the regressors in the model are truly fixed. Consequently, when lagged values of the regressand are included as predetermined variables, the test only becomes an approximate one.

Other tests which are also used to detect autocorrelation in the disturbances have been developed by Von-Neuman,[25] Theil and Nager,[26] and Theil.[27]

METHODOLOGY FOR THE CALCULATION OF AGGREGATION BIAS

In Chapter II, we introduced the relationship between the macro and micro parameters. In the specific example used in the three variable case, we represented

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[25] J. Von-Neuman, "Distribution of the Ratio of the Mean Square Successive Difference to the Variance," Annals of Mathematical Statistics, Vol. 12 (December, 1941), pp. 367-395.

[26] H. Theil and A. L. Nager, "Testing the Independence of Regression Disturbances," Journal of American Statistical Association, Vol. 56, No. 296 (December, 1961), pp. 793-806.

[27] H. Theil, "The Analysis of Disturbances in Regression Analysis," Journal of American Statistical Association, Vol. 60, No. 312 (December, 1965), pp. 1067-1079.

these relationships by the following equations:

(2.2.29b) $$\alpha_o = \sum_i \alpha_{oi} + I[\text{cov}(\alpha_{1i},\Gamma_{1i}) + \text{cov}(\alpha_{2i},\Gamma_{2i})],$$

(2.2.30b) $$\alpha_1 = \bar{\alpha}_{1i} + I[\text{cov}(\alpha_{1i},B_{1i}) + \text{cov}(\alpha_{2i},B_{2i})],$$

(2.2.31b) $$\alpha_2 = \bar{\alpha}_{2i} + I[\text{cov}(\alpha_{1i},C_{1i}) + \text{cov}(\alpha_{2i},C_{2i})],$$

where the terms in square brackets constitute the aggregation bias in each case, I is the total number of microrelations and all other symbols are as previously described.

In this section, we develop formulae for the calculation of aggregation bias for the four-variable case where all macroindependent variables are weighted averages of corresponding microindependent variables.

Consider the following micromodel,

(3.3.1) $$A_{it} = \alpha_{oi} + \alpha_{1i}P_{it} + \alpha_{2i}X_{it} + \alpha_{3i}T_t + \varepsilon_{it}\ ,\qquad i = 1,2,\dots,I$$

Let the corresponding macromodel be,

(3.3.2) $$A_t = \alpha_o + \alpha_1 P_t + \alpha_2 X_t + \alpha_3 P_t + \varepsilon_t\ .$$

where,

(3.3.3)(a) $$A_t = \sum_i A_{it}\ ,$$

(b) $$P_t = \sum_i \lambda_i P_{it}\ ,$$

(c) $$X_t = \sum_i \delta_i X_{it}\ ,$$

(d) $$T_t = \sum_i \omega_i T_{it}$$

A_{it}, P_{it}, X_{it} and T_{it} are microvariables, while A_t, P_t, X_t

and T_t are macrovariables. λ_i, δ_i and ω_i are weights used in constructing the macrovariables. All other terms carry the same meaning as before.

To facilitate the calculation of the aggregation bias, let us also set up auxiliary equations as follows:

$$(3.3.4) \qquad P_{it} = \Gamma_{1i} + B_{1i}P_t + C_{1i}X_t + D_{1i}T_t + V_{1it} ,$$

$$(3.3.5) \qquad X_{it} = \Gamma_{2i} + B_{2i}P_t + C_{2i}X_t + D_{2i}T_t + V_{2it} ,$$

$$(3.3.6) \qquad T_{it} = \Gamma_{3i} + B_{3i}P_t + C_{3i}X_t + D_{3i}T_t + V_{3it} .$$

Using (3.3.1), (3.3.4), (3.3.5) and (3.3.6) in (3.3.3a) we get,

$$(3.3.7) \qquad A_t = \sum_i A_{it} = \sum_i [\alpha_{oi} + \alpha_{1i}(\Gamma_{1i}+B_{1i}P_t+C_{1i}X_t+D_{1i}T_t+V_{1it})$$
$$+\alpha_{2i}(\Gamma_{2i}+B_{2i}P_t+C_{2i}X_t+D_{2i}T_t+V_{2it})$$
$$+ \alpha_{3i}(\Gamma_{3i}+B_{3i}P_t+C_{3i}X_t+D_{3i}T_t+V_{3it})+\varepsilon_{it}]$$

Gathering terms from (3.3.7) and by reference to (3.3.2) we have,

$$(3.3.8)(a) \quad \alpha_o = \sum_i \alpha_{oi} + \sum_i \alpha_{1i}\Gamma_{1i} + \sum_i \alpha_{2i}\Gamma_{2i} + \sum_i \alpha_{3i}\Gamma_{3i}$$

$$(b) \quad \alpha_1 = \sum_i \alpha_{1i}B_{1i} + \sum_i \alpha_{2i}B_{2i} + \sum_i \alpha_{3i}B_{3i}$$

$$(c) \quad \alpha_2 = \sum_i \alpha_{1i}C_{1i} + \sum_i \alpha_{2i}C_{2i} + \sum_i \alpha_{3i}C_{3i}$$

$$(d) \quad \alpha_3 = \sum_i \alpha_{1i}D_{1i} + \sum_i \alpha_{2i}D_{2i} + \sum_i \alpha_{3i}D_{3i}$$

$$(e) \quad \varepsilon_t = \sum_i \alpha_{1i}V_{1it} + \sum_i \alpha_{2i}V_{2it} + \sum_i \alpha_{3i}V_{3it} + \sum_i \varepsilon_{it}$$

Thus in (3.3.8), we have expressed the macroparameters and macrodisturbances as functions of the microparameters and microdisturbances with certain weights--the coefficients of the auxiliary equations. These weights obey the following rules, namely;[28]

$$\text{(3.3.9)(a)} \quad \sum_i \lambda_i B_{1i} = 1, \ \sum_i \lambda_i \Gamma_{1i} = \sum_i \lambda_i C_{1i} = \sum_i \lambda_i D_{1i} = \sum_i \lambda_i V_{1it} = 0$$

$$\text{(b)} \quad \sum_i \delta_i C_{2i} = 1, \ \sum_i \delta_i \Gamma_{2i} = \sum_i \delta_i B_{2i} = \sum_i \delta_i D_{2i} = \sum_i \delta_i V_{2it} = 0$$

$$\text{(c)} \quad \sum_i \omega_i D_{3i} = 1, \ \sum_i \omega_{i\ 3i} = \sum_i \omega_i B_{3i} = \sum_i \omega_i D_{3i} = \sum_i \omega_i V_{3it} = 0$$

With a slight change in the formulation of (3.3.1), we can express relations (3.3.8) in a form which will allow us to calculate aggregation bias for all the macroparameters and macrodisturbances.

Thus, (3.3.1) may be written

$$\text{(3.3.1a)} \qquad A_{it} = \alpha_{oi} + \frac{\alpha_{1i}}{\lambda_i}(\lambda_i P_{it}) + \frac{\alpha_{2i}}{\delta_i}(\delta_i X_i) + \frac{\alpha_{3i}}{\omega_i}(\omega_i T_{it}) + \varepsilon_{it}$$

or

$$\text{(3.3.1b)} \qquad A_{it} = \alpha_{oi} + \alpha'_{1i}(\lambda_i P_{it}) + \alpha'_{2i}(\delta_i X_{it}) + \alpha'_{3i}(\omega_i T_{it}) + \varepsilon_{it}$$

where

$$\text{(3.3.10)} \qquad \frac{\alpha_{1i}}{\lambda_i} = \alpha'_{1i} \ ; \ \frac{\alpha_{2i}}{\delta_i} = \alpha'_{2i} \ ; \ \frac{\alpha_{3i}}{\omega_i} = \alpha'_{3i}$$

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[28] The proof is given in Appendix B.

Therefore, we can use (3.3.10) to write (3.3.8) as follows:

(3.3.11) (a) $$\alpha_o = \sum_i \alpha_{oi} + \sum_i \alpha'_{1i}\lambda_i\Gamma_{1i} + \sum_i \alpha'_{2i}\delta_i\Gamma_{2i} + \sum_i \alpha'_{3i}\omega_i\Gamma_{3i}$$

(b) $$\alpha_1 = \sum_i \alpha'_{1i}\lambda_i B_{1i} + \sum_i \alpha'_{2i}\delta_i B_{2i} + \sum_i \alpha'_{3i}\omega_i B_{3i}$$

(c) $$\alpha_2 = \sum_i \alpha'_{1i}\lambda_i C_{1i} + \sum_i \alpha'_{2i}\delta_i C_{1i} + \sum_i \alpha'_{3i}\omega_i C_{3i}$$

(d) $$\alpha_3 = \sum_i \alpha'_{1i}\lambda_i D_{1i} + \sum_i \alpha'_{2i}\delta_i D_{2i} + \sum_i \alpha'_{3i}\omega_i D_{3i}$$

(e) $$e_t = \sum_i \alpha'_{1i}\lambda_i V_{1it} + \sum_i \alpha'_{2i}\delta_i V_{2it} + \sum_i \alpha'_{3i}\omega_i V_{3it} + \sum_i \varepsilon_{it}$$

Now,

(3.3.12) $$\text{cov}(\alpha'_{1i},\lambda_i\Gamma_{1i}) = \frac{1}{I}\sum_i(\alpha'_{1i}-\bar{\alpha}'_{1i})(\lambda_i\Gamma_{1i}-\frac{1}{I}\sum_i\lambda_i\Gamma_{1i})$$

$$= \frac{1}{I}\sum_i(\alpha'_{1i}-\bar{\alpha}'_{1i})(\lambda_i\Gamma_{1i}),$$

since $\Sigma\lambda_i\Gamma_{1i} = 0$.

$$= \frac{1}{I}\sum_i \alpha'_{1i}\lambda_i\Gamma_{1i}$$

Therefore,

(3.3.13) $$\sum_i \alpha'_{1i}\lambda_i\Gamma_{1i} = I\ \text{cov}(\alpha'_{1i},\lambda_i\Gamma_{1i})\ .$$

Similarly,

(3.3.14) (a) $$\sum_i \alpha'_{2i}\delta_i\Gamma_{2i} = I\ \text{cov}(\alpha'_{2i},\delta_i\Gamma_{2i})$$

(b) $$\sum_i \alpha'_{3i}\omega_i\Gamma_{3i} = I\ \text{cov}(\alpha'_{3i},\omega_i\Gamma_{3i})$$

Thus in (3.3.11a), α_o can be expressed as,

$$(3.3.15) \qquad \alpha_o = \sum_i \alpha_{oi} + I[\mathrm{cov}(\alpha'_{1i},\lambda_i\Gamma_{1i}) + \mathrm{cov}(\alpha'_{2i},\delta_i\Gamma_{1i}) + \mathrm{cov}(\alpha'_3,\ \omega_i\Gamma_{3i})]$$

Now,

$$(3.3.16) \qquad \mathrm{cov}(\alpha'_{1i},\lambda_i B_{1i}) = \frac{1}{I}\sum_i(\alpha'_{1i}-\bar{\alpha}'_{1i})(\lambda_i B_{1i}-\frac{1}{I}\sum_i\lambda_i B_{1i})$$

$$= \frac{1}{I}\sum_i(\alpha'_{1i}-\bar{\alpha}'_{1i})(\lambda_i B_{1i}-\frac{1}{I}) \ ,$$

$$\text{since } \sum_i \lambda_i B_{1i} = 1$$

$$= \frac{1}{I}\sum_i(\alpha'_{1i}\lambda_i B_{1i}) - \frac{1}{I}\bar{\alpha}'_{1i}$$

Therefore,

$$(3.3.17) \qquad \sum_i \alpha'_{1i}\lambda_i B_{1i} = \bar{\alpha}'_{1i} + I\ \mathrm{cov}(\alpha'_{1i},\lambda_i B_{1i})$$

Similarly it can be shown that

$$(3.3.18)(a) \qquad \Sigma\alpha'_{2i}\delta_i B_{2i} = I[\mathrm{cov}(\alpha_{2i},\delta_i B_{2i})],$$

$$(b) \qquad \Sigma\alpha'_{3i}\omega_i B_{3i} = I[\mathrm{cov}(\alpha'_{3i},\omega_i B_{3i})].$$

Thus in (3.3.11b), α_1 can be expressed as,

$$(3.3.19) \qquad \alpha_1 = \bar{\alpha}'_{1i} + I[\mathrm{cov}(\alpha'_{1i},\lambda_i B_{1i}) + \mathrm{cov}(\alpha'_{2i},\delta_i B_{2i}) + \mathrm{cov}(\alpha'_{3i},\omega_i B_{3i})]$$

We can express the macroparameters α_2, α_3 and the macrodisturbance ε_t similarly using the above procedure. Thus we obtain,

$$(3.3.20)\ (a)\quad \alpha_o = \sum_i \alpha_{oi} + I[\mathrm{cov}(\alpha'_{1j}, \lambda_i \Gamma_{1j}) + \mathrm{cov}(\alpha'_{2i}, \delta_i \Gamma_{2i}) + \mathrm{cov}(\alpha'_{3i}, \omega_i \Gamma_{3i})]$$

$$(b)\quad \alpha_1 = \bar{\alpha}'_{1i} + I[\mathrm{cov}(\alpha'_{1i}, \lambda_i B_{1i}) + \mathrm{cov}(\alpha'_{2i}, \delta_i B_{2i}) + \mathrm{cov}(\alpha'_{3i}, \omega_i B_{3i})]$$

$$(c)\quad \alpha_2 = \bar{\alpha}'_{2i} + I[\mathrm{cov}(\alpha'_{1i}, \lambda_i C_{1i}) + \mathrm{cov}(\alpha'_{2i}, \delta_i C_{2i}) + \mathrm{cov}(\alpha'_{3i}, \omega_i C_{3i})]$$

$$(d)\quad \alpha_3 = \bar{\alpha}'_{3i} + I[\mathrm{cov}(\alpha'_{1i}, \lambda_i D_{1i}) + \mathrm{cov}(\alpha'_{2i}, \delta_i V_{2it}) + \mathrm{cov}(\alpha'_{3i}, \omega_i D_{3i})]$$

$$(e)\quad \varepsilon_t = \sum_i \varepsilon_{it} + I[\mathrm{cov}(\alpha'_{1i}, \lambda_i V_{1it}) + \mathrm{cov}(\alpha'_{2i}, \delta_i V_{2it}) + \mathrm{cov}(\alpha'_{3i}, \omega_i V_{3it})]$$

The terms in the square brackets constitute the aggregation bias.

We must point out that in practice we do not have the macroparameters (and macrodisturbances)--we have only their estimates. In general, therefore, to obtain the estimates of these macroparameters (and macrodisturbances), we have to add the implied sampling error to each expression in equation (3.3.20). The implied sampling errors are obtained by regressing the sum of the microresiduals on the macrovariables.[29] Of course we do not have the microparameters also.

[29]See Chapter II, p. 15 equation (2.2.14) and also p. 17 second paragraph for the simple case considered in that chapter. In the case of more than one variable, see Boot and De Wit, op. cit., p. 12; T. Kloek, "Note on Convenient Matrix Notations in Multivariate Statistical Analysis and in the Theory of Linear Aggregation," International Economic Review, Vol. 2, No. 3 (September, 1961), pp. 351-359.

These are assumed to coincide with their least squares estimates.

To conclude this chapter, we have considered the conceptual and statistical models and problems associated with the estimation of wheat and barley acreage response models. The discussion was in terms of a single commodity called "grains". We also briefly reviewed the more common econometric problems associated with the estimation of quantitative relationships among variables. We have also considered the methodology for the calculation of aggregation bias in the estimated macroparameters where the macrovariables are weighted averages of the corresponding microvariables. The results of the estimation of the relationships form the subject of Chapter IV. In Chapter V, aggregation bias and the implied sampling errors of the estimated macroparameters are calculated along the lines considered in this chapter. Also considered in Chapter V are the explanatory powers, and the forecasting abilities of the aggregate and the disaggregate models mentioned in Chapter I.

CHAPTER IV

THE ESTIMATED ACREAGE RESPONSE FUNCTIONS FOR WHEAT AND BARLEY

INTRODUCTION

Several wheat and barley equations were estimated for the macrounit (Canadian Prairies), and because each of these equations was not used for the stated objectives of this study, it was necessary to select among them. Thus one of the purposes of this chapter is to present and discuss the several macroequations which were estimated in this study for wheat and barley. This is done to show how the equations which were selected for further study stand in relation to other estimated macroequations. To the best knowledge of this author, there have been no functions fitted for barley in the Canadian Prairies. This provides the second justification for this chapter. The chapter also considers the variables which were used in the statistical estimations in this study.

In Chapter V we calculate the components of the macroparameters (and macrodisturbances) of the selected wheat equations. This is done in order to provide information about the nature of these macroparameters (and disturbances). The questions of the explanatory powers and the forecasting abilities of the aggregate and the disaggregate wheat and barley models are also taken up in the next chapter.

DESCRIPTION OF THE VARIABLES CONSIDERED IN THE ESTIMATION OF THE ACREAGE RESPONSE EQUATIONS

Wheat and Barley Price Variables

Each year the Canadian Wheat Board (C.W.B.) announces the initial prices to be paid for the different grades of wheat to be delivered to the licensed elevators. Before a producer can deliver his wheat to any licensed elevator, he must obtain a delivery permit book from the Board. However, when and how much wheat may be delivered by the individual producer is determined by a system of delivery quotas. Wheat farmers deliver most of their grain to the Board through these quotas. The remainder of their crop is either carried over on the farm, or used as feed on the farm, or sold to other farmers. The Wheat Board pays farmers in two installments; an initial payment at the time the grain is delivered to the elevator and subsequently a final payment.[1] The latter payment is usually not available to producers until about eighteen months from the harvest period.

The problem posed to supply response analysis by this characteristic of wheat marketing is that when

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[1] In certain years interim payments have also been made by the C.W.B. prior to the final payment being made.

intentions for the season in year t are formulated, farmers are not aware of the wheat price obtained for year t-1.

Since price lagged one year is unknown it is, therefore, not a relevant variable in making decisions on how much acreage to seed in the current year. It follows from this that expected "normal" price for the year in which wheat is planted, is not the relevant variable on the basis of which acreage decisions for that year are made. This is because farmers have no additional information on the basis of which to revise their previous expectations at the time when they make their acreage decisions. It has been suggested that in a situation such as this, the expected "normal" price lagged one year rather than the current expected "normal" price is the relevant variable.[2] This hypothesis provides the basis for introducing wheat price with a two year lag into the wheat equations.[3]

When faced with the same situation, Schmitz[4] suggested that while the total payments for the previous year's crop may not be available to the farmers at seeding, farmers are able to calculate final prices on the basis of the initial payments. This hypothesis is also incorporated in this study for comparison. Another price series which has also been suggested is the March average of the Wheat

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[2]Nerlove, The Dynamics of Supply, op. cit., p. 212.

[3]See p. 79.

[4]Schmitz, op. cit., p. 112.

Board International Wheat Agreement price for Number 2 Northern Wheat at Thunder Bay. It has been suggested that this price is correlated with the Wheat Board's price expected by farmers.[5] This suggestion is also incorporated in this study for comparison with the other hypotheses.

The above remarks on wheat marketing also apply to barley. Accordingly, time lags of one and two periods were experimented with in connection with barley prices. The price series used for barley, however, is the estimated average farm price for barley. This series is used instead of the C.W.B. final payment for barley. The reason for this is that the bulk of the barley produced on the Prairies is not delivered to the C.W.B., but it is disposed of in various other ways.[6] A considerable quantity of barley is sold by one farmer to another and to feedlot operators within the same province on a non-quota basis. Also since 1960, individual grain producers have been permitted to deliver non-quota barley to feedmills which have been designated as non-quota mills by the Canadian Wheat Board. Prices received for these non-quota barley sales are generally lower than those set by the C.W.B., and this is especially so during

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[5] R. E. Capel, "Predicting Wheat Acreage in the Prairie Provinces," Canadian Journal of Agricultural Economics, Vol. XVI, No. 2 (June, 1968), pp. 87-89.

[6] This statement is made strictly on the basis of information prior to the 1969-70 crop year.

years of surplus.[7] The aggregate Prairie price of barley was obtained as a weighted average of the barley prices for each of the three Provinces, the weights being the proportion of the total Prairie barley production accounted for by each Province.

Prices of Competing Commodities

In the theoretical model discussed in Chapter III, a variable was specified to represent the prices of competing commodities. In the Canadian Prairies, the major crop and livestock enterprises in addition to wheat are: oats, barley, rapeseed, flax and beef. The prices of these commodities were used in the wheat equations. In taking account of them in the wheat regression equations, the prices of these competing commodities were either introduced as ratios with wheat prices or they were introduced separately. The prices of the competing commodities used in the barley equations are those of wheat, oats, flax, rapeseed and beef. These prices were introduced in the regression equations in the same way as for wheat.

Input Prices

Also specified in the theoretical discussion was a variable representing input prices. An index of prices of inputs used by farmers was considered for inclusion in the wheat and barley equations. As a result of the very high

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[7] Canadian Agriculture in the Seventies, Report of the Federal Task Force on Agriculture, Ottawa (December, 1969), p. 70.

correlation between this index and the trend variable (mentioned later), this index was omitted from the analysis.[8] The remark by Schmitz on input prices in connection with wheat acreage response should, however, be noted:

> . . . capital has continually replaced labour as an input. Also, increased input costs (e.g., price of machinery) coupled with relatively stable wheat prices have expedited the increase in farm consolidation in order to take advantage of economies of size; this reduction in farm numbers appears to have had little effect on aggregate wheat acreage. The switch from wheat to other crops such as barley and oats has not been made since they require the same inputs as does wheat production.[9]

Other Variables

In general, the greater the rainfall, the larger the acreage seeded to wheat and barley. However, rainfall may also cause water-logging conditions in soils and this may hinder land preparation and seeding operations, and consequently may restrict seeded acreages. Water-logging conditions depend on the soil type and the amount of the rainfall beyond a certain critical level. Ideally, therefore, to take into account its water-logging effect on seeded acreages, rainfall should be included in an acreage response function in both its absolute amount and the amount of the rainfall beyond a certain critical level. Because of the difficulty of obtaining data on these critical levels either for each crop district or for the Prairies as a whole, only

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[8] For the period 1953-54 to 1968-69, the correlation coefficient between these two variables was 0.95.

[9] Schmitz, op. cit., p. 120.

the absolute amount of rainfall was included in the regression equations in this study.

Stocks of wheat and barley were included at their levels as on March 31, of every year. A dummy variable was also specified for wheat stocks on the basis of the belief that farmers respond differently to changes in wheat stocks beyond a specified level.[10] The specified level used in this study is the average level of wheat stocks in the five year period preceding any particular year being considered. Thus if the period t is being considered, the dummy variable takes the value 1, if the level of stocks in t is greater than the average level of wheat stocks in the five year period preceding t, and 0 otherwise.

To account for the effect of improvements in technology, a trend variable was included. The trend variable may also account for secular influences on seeded acreages that are not explicitly taken account of by other variables, such as, the cumulative effect of decreased livestock, forage, oats, and barley production on seeded wheat acreages, and institutional factors operating in the Canadian Prairies.

The variables included in the estimation and the units of measurement are:

A_t = wheat acreage seeded at time t (1,000 acres),

B_t = barley acreage seeded at time t (1,000 acres),

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[10] A discussion of dummy variables is contained in several textbooks on econometrics. See for example Johnston, op. cit., pp. 221-228.

A_{t-1} = wheat acreage seeded at time t-1 (1,000 acres),

B_{t-1} = barley acreage seeded at time t-1 (1,000 acres),

P_{w-1} = Canadian Wheat Board payments to farmers for Number 2 Northern Wheat in t-1 (dollars per bushel),

P_{w-2} = Canadian Wheat Board payments to farmers for Number 2 Northern Wheat in t-2 (dollars per bushel),

P'_t = Canadian Wheat Board International Wheat Agreement average price for February, March, and April for Number 2 Northern Wheat at Thunder Bay (dollars per bushel),

P_{b-1} = the estimated farm price for barley in t-1 (dollars per bushel),

P_{b-2} = the estimated farm price for barley in t-2 (dollars per bushel),

P_{f-1} = the estimated farm price for flax in t-1 (dollars per bushel),

P_{r-1} = the estimated farm price for rapeseed in t-1 (dollars per bushel),

$P_{e,t}$ = the average beef price for April (dollars per pound),

P_{o-1} = the estimated farm price for oats in t-1 (dollars per bushel),

Smf_{t-1} = summerfallow acreage at time t-1 (1,000 acres),

M_t = rainfall prior to seeding (six month average in inches),

T_t = time--a gross variable accounting for technological, institutional and unspecified secular influences on seeded acreages,

$S_{w,t}$ = the Prairie total stocks of wheat on farms on March 31 (1,000 bushels),

$S_{b,t}$ = the Prairie total stocks of barley on farms on March 31 (1,000 bushels),

DS_t = dummy variable for stocks. DS_t takes the value 1 at time t if the level of stocks in t is greater than the average level of stocks in the five year period preceeding t, and 0 otherwise.

In the following statistical presentation of the acreage response equations, both the multiple correlation coefficient, R^2, and the Durbin-Watson (D.W.) statistics are given. The former measures the power of all the regressors in explaining the values taken by the regressand. The latter measures the degree of autocorrelation in the disturbances. For each coefficient the standard error is bracketed. The bracketed figure below the constant term is the standard error of estimate. The level of statistical significance is also indicated for each coefficient. Thus a two-asterisk superscript indicates that the regression coefficient is statistically significant at the one per cent level of confidence, a one-asterisk superscript indicates statistical significance at the five per cent level and "°"indicates significance at the ten per cent level of confidence. No other levels of statistical significance are indicated. The presence of positive serial correlation is indicated by a two-asterisk superscript. One-asterisk indicates an inconclusive test, and no superscript indicates the absence of serial correlation among the calculated residuals. The test for serial correlation is at the one per cent level of confidence.

RESULTS OF THE ESTIMATION OF THE ACREAGE EQUATIONS FOR WHEAT

The wheat acreage response equations are presented in Tables 4.1 and 4.2. Equations (4.1.1) to (4.1.11) in Table 4.1 are based on the traditional model, and equations (4.1.12), to (4.1.17) are based on Nerlove's general price expectation model, hereafter referred to as the distributed lag model.[11] These were discussed in Chapter III. The difference between Tables 4.1 and 4.2 lies in the fact that the wheat price variable in Table 4.1 has a lag of two periods while the corresponding variable in Table 4.2 has a lag of one period.

In Table 4.1, the coefficient of the price of wheat is estimated with the theoretically correct positive sign throughout. The coefficient of stocks of wheat is estimated with the theoretically correct sign in all equations except (4.1.10). The coefficient of rainfall has a positive sign in all equations in which it is present except equation

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[11] Sometimes "dynamic model" is used interchangeably with "distributed lag model" in this study. The results from the Cobb Douglas functions are not presented, because they were not better than those from the functions linear in the original variables.

Table 4.1

Results of the Estimation of the Aggregate Wheat Acreage Response Equations When Wheat Price is Lagged by Two Periods, Canadian Prairies, 1953-54--1968-69[a]

Equation Number	Constant Term	P_{w-2}	S_w	T	M	DS	Smf_{t-1}	P_{b-1}	P_{f-1}	P_{o-1}	P_e	$\frac{P_{w-2}}{P_{f-1}}$	A_{t-1}	R^2	D.W.
4.1.1	446.63 (1259.44)	13668.415** (2722.868)	-0.003967 (0.00345)	284.688** (89.445)										.8632	2.67
4.1.2	-7849.44 (1665.21)	18451.557** (3051.310)	-0.003330 (0.00464)		2788.839 (3435.156)									.7959	1.66*
4.1.3	-2566.45 (1069.95)	13152.106** (2323.413)	-0.005901* (0.00304)	335.641** (78.965)	5440.826* (2293.078)									.9227	2.69
4.1.4	-4177.92 (1112.72)	13977.328** (3135.632)	-0.004756 (0.00421)	310.550** (102.157)	5819.817* (2555.851)	-497.042 (1203.671)								.9240	2.79
4.1.5	-3776.33 (1304.30)	14195.929** (3070.155)	-0.004114 (0.00359)	236.801^{o} (143.979)			0.1502 (0.3459)							.8852	2.65
4.1.6	-5209.34 (1063.28)	12249.773** (2458.951)	-0.005601* (0.00304)	320.420** (78.626)	5031.692* (2345.699)				1304.480 (1222.641)					.9306	2.69
4.1.7	-3498.92 (11[illegible].61)	13300.728** (2769.844)	-0.003723 (0.00348)	260.624* (93.751)								5988.315 (6511.422)		.8915	2.56
4.1.8	-3045.00 (1302.92)	14533.752** (3594.923)	-0.002051 (0.00305)	205.021* (76.243)	4850.382* (2631.536)			-30.056 (70.342)						.8843	2.39
4.1.9	-7639.33 (1394.05)	12326,138** (3483.059)	-0.004303^{o} (0.00300)	283.011^{o} (162.531)					1109.754 (1403.312)		203.516 (129.113)			.8636	1.94
4.1.10	506.42 (1253.13)	10322.034** (2842.003)	0.002000 (0.00441)	193.981^{o} (110.502)				-29.918 (72.451)		-7.506 (60.034)				.8944	2.69

Table 4.1 (continued)

Equation Number	Constant Term	P_{w-2}	S_w	T	M	DS	Smf_{t-1}	P_{b-1}	P_{f-1}	P_{o-1}	P_e	$\frac{P_{w-2}}{P_{f-1}}$	A_{t-1}	R^2	D.W.
4.1.11	-2318.17 (1291.12)	15216.618** (2972.381)		302.197* (132.299)			0.1650 (0.3351)							.8773	1.57*
4.1.12	910.36 (1235.81)	9356.192* (4455.014)	-0.00446^{o} (0.00341)	250.451* (92.218)									0.3018^{o} (0.2126)	.8969	2.98
4.1.13	1567.42 (1271.82)	8605.498* (4546.592)		239.411* (94.507)									0.2629 (0.2550)	.8809	2.38
4.1.14	-2584.53 (1127.16)	13201.359** (4559.602)	-0.005903* (0.00320)	336.262* (95.999)	5463.500* (2989.058)								-0.0036 (0.2815)	.9227	2.69
4.1.15	-3698.09 (1529.27)	10068.967* (5503.343)	-0.003612 (0.00420)										0.5039* (0.2939)	.8278	2.12
4.1.16	-2624.90 (1317.25)	8913.984** (4763.141)		189.407 (151.746)			0.1507 (0.3496)						0.2749 (0.2655)	.8829	2.40
4.1.17	-6989.04 (1584.60)	9663.332^{o} (6363.036)	-0.003120 (0.00446)		-683.778 (3802.432)							8943.974 (8276.235)	0.4623^{o} (0.3243)	.8460	2.14

[a]The variables appearing in Table 4.1 have been defined on pp. 66-68. For explanations of symbols, see pp. 68-69.

Table 4.2

Results of the Estimation of the Aggregate Wheat Acreage Response Equations When Wheat Price is Lagged by One Period, Canadian Prairies, 1953-54--1968-69[a]

Equation Number	Constant Term	P_{w-1}	S_w	T	M	DS	Smf_{t-1}	P_{b-1}	P_{f-1}	P_{o-1}	P_e	$\frac{P_{w-1}}{P_{f-1}}$	A_{t-1}	R^2	D.W.
4.1.1a	-450.06	13036.394**	0.003563	182.404^{o}										.8413	1.42*
	(1467.93)	(3323.876)	(0.00373)	(123.981)											
4.1.2a	-6429.93	16404.303**	0.005184^{o}		1237.926									.8149	1.64*
	(1535.36)	(2553.20)	(0.00387)		(3263.815)										
4.1.3a	-1206.52	11998.001**	0.002109	236.058^{o}	3431.136									.8559	1.55*
	(1461.13)	(3451.890)	(0.00397)	(133.482)	(3253.675)										
4.1.4a	2148.79	10410.203**	-0.001773	304.878	2774.388	1183.000								.8655	1.39*
	(1480.58)	(3971.448)	(0.00611)	(157.926)	(3367.547)	(1401.285)									
4.1.5a	5108.87	12644.109**	0.003383	243.673^{o}			-0.2138							.8459	1.48*
	(1511.09)	(3490.245)	(0.00386)	(171.531)			(0.3754)								
4.1.6a	-2614.15	10991.920*	0.001852	248.922^{o}					938.766					.8595	1.64*
	(1513.58)	(4101.476)	(0.00414)	(140.840)					(1874.495)						
4.1.7a	-195.42	13173.911**	0.003565	182.911^{o}								-807.188		.8415	1.42*
	(1532.56)	(3755.733)	(0.00390)	(129.547)								(8430.891)			
4.1.10a	-943.00	12933.100**	0.001381	250.745^{o}				-18.746		-9.300				.8496	1.63*
	(1489.31)	(3338.124)	(0.00403)	(153.830)				(80.334)		(80.938)					

Table 4.2 (continued)

Equation Number	Constant Term	P_{w-1}	S_w	T	M	DS	Smf_{t-1}	P_{b-1}	P_{o-1}	P_e	$\frac{P_{w-1}}{P_{f-1}}$	A_{t-1}	R^2	D.W.
4.1.11a	-7398.83 (1555.65)	13051.110** (3301.597)		245.897* (108.460)			-0.2210 (0.2456)						.8351	1.80
4.1.13a	-2605.73 (1063.41)	8598.328** (2680.617)		118.779° (89.123)								0.4730** (0.1332)	.9167	2.31
4.1.14a	-2498.95 (1154.56)	8263.452* (3044.832)	-0.001168 (0.00335)	140.405 (111.024)	826.063 (2738.803)							0.4769* (0.1728)	.9182	2.30
4.1.15a	-5960.00 (1139.38)	10257.565** (2593.075)	-0.000051 (0.00317)									0.5335** (0.1572)	.9044	2.17
4.1.17a	-5461.98 (1243.44)	10150.942** (3212.317)	-0.000012 (0.00347)		-708.668 (2792.187)						-120.543 (7236.004)	0.5450** (0.1772)	.9015	2.29

[a]The variables appearing in Table 4.2 have been defined on pp. 66-68. For explanations of symbols, see pp. 68-69.

(4.1.17). Trend, summerfallow, and the ratio of wheat price to flax price have positive coefficients in all equations in which they are featured. Similarly, the dummy wheat stock variable (DS) has the theoretically correct negative sign in the only equation (4.1.4) in which it is used. However, the price of flax and the price of beef have the theoretically wrong signs on their coefficients, while the price of barley, and price of oats have the theoretically right signs. Generally the coefficient of the lagged dependent variable is positive; however, it has a negative sign in equation (4.1.14). As can be seen from that equation, however, the magnitude of the coefficient is very close to zero.

It can be seen that in general the price of wheat shows significance at the 10 per cent level or better in every equation. This is also true of trend except in equation (4.1.16). The coefficient of stocks of wheat is generally larger than its standard deviation, but it shows statistical significance at the 10 per cent level or better in only five equations. Similarly, the coefficient of lagged wheat acreage is larger than its standard deviation in every equation, but it is significant at the 10 per cent level or better in equations (4.1.12), (4.1.15) and (4.1.17) only.

In general, the magnitudes of the coefficients are larger in the static models than in the distributed lag models. Serial correlation in the residuals generally does not seem to be a problem in these estimations, and the

Durbin Watson test shows no serial correlation except in equations (4.1.2) and (4.1.11) for which the tests are inconclusive.

Alternative Ways of Incorporating the Price of Wheat

Of the three different ways previously discussed of taking the price of wheat into account in the wheat acreage response functions, P_{w-2} seems to be the most adequate from the statistical point of view. Equations in which P_{w-1} are used are shown in Table 4.2. Several difficulties arise when P_{w-1} is used. One difficulty is the frequency of the theoretically wrong sign on some coefficients. Thus in one or more equations theoretically wrong signs are obtained for the coefficients of the following variables: wheat stocks on farms, flax price in t-1, summerfallow acreage, the ratio of wheat price in t-1 to flax price in t-1, the dummy variable, rainfall, and the lagged wheat acreage. Equations in which the C.W.B. International Agreement prices (averages for February, March and April) were used, generally turned out poorer than those with P_{w-1}. These equations have not been tabulated in this study.

The Distributed Lag Hypothesis

The distributed lag hypothesis is that the value of the coefficient of expectation, β, is constrained as follows: $1 > \beta > 0$. As can be seen from equation (3.1.9) the coefficient of the lagged dependent variable (α_2) is $1 - \beta$. If the value of α_2 is 0, then the value of β is 1.

A value of 1 for β implies that the expected "normal" price for t is the price obtained last year. If, on the other hand, the value of α_2 is 1, then the value of β is 0 implying that only one "normal" price is expected every time. Being constant, therefore, the expected "normal" price does not affect acreage decisions. Thus to provide support for the distributed lag hypothesis, the value of β must be significantly different from both 0 and 1.

In all the equations containing P_{w-2} (Table 4.1), the coefficients of the lagged dependent variable [except in (4.1.14)] are larger than their standard deviations, but in only three of the six equations in which the lagged dependent variable is present are these coefficients significantly different from 0 at the 10 per cent level of confidence or better. These three coefficients are also significantly different from 1 at least at the 10 per cent level of confidence.

In the equations with P_{w-1}, however, most of the coefficients of the lagged dependent variable are significantly different from both 0 and 1 at the one per cent level of confidence.

The tendency of the coefficient of the lagged dependent variable to fail to show significance when P_{w-2} is used, is probably due to the relatively high intercorrelation between P_{w-2} and the lagged dependent variable, A_{t-1}.[12] As

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[12]The correlation coefficient, r, between A_{t-1} and P_{w-2} is 0.78. The correlation coefficient, r, between P_{w-1} and A_{t-1} is 0.70.

was mentioned in Chapter III, very high intercorrelation among variables increases the size of the standard deviations relative to the coefficients. The consequence of this is that the coefficients may fail to reach significance.

The R^2's tend to be larger for the distributed lag models than for the static models. Thus it can be concluded that the distributed lag models have a higher explanatory value than the static models. However, caution must be exercised when interpreting this statistic. For as pointed out by Brandow[13] and Griliches,[14] if economic variables have a high degree of inertia, lagging the dependent variable and using it as a regressor will automatically give high R^2.

The Durbin Watson test shows that autocorrelation in the residuals in the distributed lag models is weaker than in the static models. This is particularly so with the equations in which P_{w-1} is used. It should be pointed out, however, that the Durbin Watson test becomes a weak one when we have the lagged endogenous variable as a regressor. For as pointed out by Griliches,[15] the lagged dependent variable may be removing the serial correlation in the residuals for

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[13] George E. Brandow, "A Note on the Nerlove Estimate of Supply Elasticity," Journal of Farm Economics, XL (August, 1958), pp. 719-722.

[14] Zvi Griliches, "The Demand for Fertilizer: An Economic Interpretation of a Technical Change," Journal of Farm Economics, XL (August, 1958), pp. 591-606.

[15] Zvi Griliches, "A Note on Serial Correlation Bias in Estimates of Distributed Lag," Econometrica, XXIX (January, 1961), pp. 65-73.

the wrong reasons. If this is the case then, the lack of autocorrelation among the residuals does not provide support for the adequacy of the distributed lag hypothesis as we might otherwise be led to conclude.

Economic Interpretation

The interpretation of the estimated acreage equations is made with reference to equations (4.1.3) and (4.1.12) in Table 4.1; the former is the traditional model while the latter is the distributed lag model. These two equations are chosen because the signs on all their coefficients are consistent with a priori expectations about them, also their coefficients are statistically significant, and they have relatively high explanatory power.

With regard to equation (4.1.3), a 10 cent change in farm price in t-2 results in a 1.315 million acres change in the same direction in the seeded acreage at time t. Similarly, seeded wheat acreage will change by 0.590 million acres in the opposite direction as a result of a 100 million bushels change in stocks; the time variable (T_t) indicates that seeded wheat acreage increases by 0.336 million acres each year, while for each one inch change in the six month average rainfall (M_t) the seeded wheat acreage changes by 5.441 million acres in the same direction. These changes assume that all other variables remain constant.

The model underlying equation (4.1.12) in Table (4.1) is:

$$A_t = \alpha_o + \alpha_1 P^*_{w-1} + \alpha_2 S_{w,t} + \alpha_3 T_t + U_t$$

and

$$P^*_{w-1} = P^*_{w-2} + \beta(P_{w-2} - P^*_{w-2})$$

It can be shown that these above two equations lead to

$$A_t = \alpha_o\beta + (1-\beta)A_{t-1} + \alpha_1\beta P_{w-2} - (1-\beta)\alpha_2 S_{w,t-1} - (1-\beta)\alpha_3 T_{t-1} + \alpha_2 S_{w,t} + \alpha_3 T_t + V_t \quad (4.2.0)$$

Where V_t is as defined in (3.1.10).[16] By reference to equation (4.2.0) it can be seen that in equation (4.1.12) in Table 4.1, the constant term is $\alpha_o\beta$, the coefficient of P_{t-2} is $\alpha_1\beta$ and the coefficient of A_{t-1} is $1-\beta$. The value obtained for the coefficient of P_{t-2} is 9.356 million. The value obtained for the coefficient of the lagged dependent variable is 0.3018. This implies that $\beta = 0.6982$. Therefore, α_1 is 13.400 million which indicates that a 10 cent change in the expected farm price for wheat will change the aggregate seeded wheat acreage by 1.350 million acres in the same direction. The coefficient for stocks of wheat, α_2, is 4,460 which indicates that a 100 million bushels change in wheat stocks will change the aggregate seeded wheat acreage by 0.446 million acres, in the opposite direction. The coefficient of the trend variable, α_3, indicates that seeded acreage

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[16] $S_{w,t-1}$ and T_{t-1} were excluded in (4.1.12) because of the very high intercorrelation between these variables and some of the variables included in (4.1.12). Generally these correlations were greater than 0.8 and their inclusion resulted in the coefficients of P_{w-2} and $S_{w,t}$ having the theoretically wrong signs.

increases by 0.250 million acres yearly.

In the model of adaptive expectations, the expected "normal" price is a weighted average of past prices with the weights declining from recent to less recent periods. The sum of the weights over the first t observed past prices is,

$$S_t = 1 - (1 - \beta)^t$$

which can be made as close to one as we may please by making t large enough.[17] S_t depends on the size of the coefficient of expectation (β); for a value of β close to one only relatively few past prices influence the acreage seeded through their effect on the expected "normal" price. The number of years which need to be taken into account so that the sum of the weights differs from 1 by 0.05 is two to three years. Thus the effects of a particular price change can be expected to last for a period of two to three years due to its (declining) influence on the expected "normal" price.

RESULTS OF THE ESTIMATION OF THE ACREAGE EQUATIONS FOR BARLEY

The barley acreage response equations are presented in Tables 4.3 and 4.4. Equations (4.2.1) to (4.2.6) in Table 4.3 are based on the traditional (static) model, while equations (4.2.7) to (4.2.11) in the same table are based on the distributed lag model. The difference between Tables 4.3 and 4.4 lies in the length of the lag of the barley

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[17]Nerlove, op. cit., p. 187.

Table 4.3

Results of the Estimation of the Aggregate Barley Acreage Response Equations When Barley Price is Lagged by One Period, Canadian Prairies, 1950-51--1968-69[a]

Equation Number	Constant Term	P_{b-1}	S_w	T	$\frac{S_w}{S_b}$	P_{o-1}	P_{f-1}	$\frac{P_{b-1}}{P_{o-1}}$	$\frac{P_{b-1}}{P_{f-1}}$	B_{t-1}	S_b	R^2	D.W.
4.2.1	8183.17 (920.91)	-2698.306^{o} (1652.980)	0.011361** (0.00244)	-244.763** (51.742)								.6440	1.53*
4.2.2	11159.54 (705.19)	-3262.998* (1276.622)	0.014278** (0.00206)	-199.837** (41.762)	-1504.056** (441.985)							.8051	2.39
4.2.3	9085.44 (941.01)	-2083.142 (1971.487)	0.010970** (0.00258)	-238.246** (53.952)		-2280.022 (3768.552)						.6531	1.41*
4.2.4	10250.39 (897.46)	-1115.879 (1981.555)	0.010195** (0.00251)	-244.800** (51.629)		615.415 (4052.583)	-1194.946^{o} (772.693)					.7070	1.38*
4.2.5	130[illegible].53 (1332.[illegible]8)			-147.384* (83.043)	538.111 (901.049)			-3805.325^{o} (2441.893)				.2545	0.881**
4.2.6	10122.02 (659.85)		0.00961** (0.00217)	-233.001** (46.969)				-3426.438* (1603.383)	3793.142 (6356.747)			.6898	1.84
4.2.7	2400.23 (743.07)	7[illegible].442 (1[illegible]21.941)	0.007278** (0.00239)	-139.151* (54.558)						0.5019** (0.1669)		.7837	2.55
4.2.8	-3[illegible]03.75 ([illegible]4.37)	2295.544^{o} (1518.732)	0.002534 (0.00176)						6127.083^{o} (5704.533)	0.8217** (0.1545)		.7073	2.79

Table 4.3 (continued)

Equation Number	Constant Term	P_{t-1}	S_w	T	$\frac{S_w}{S_b}$	P_{o-1}	P_{f-1}	$\frac{P_{b-1}}{P_{o-1}}$	$\frac{P_{b-1}}{P_{f-1}}$	B_{t-1}	S_b	R^2	D.W.
4.2.9	-2713.33 (950.12)		0.00367* (0.00193)				290.281 (665.235)	1409.329 (1615.129)		0.7978** (0.1827)		.6463	2.22
4.2.10	5373.[illegible] (703.[illegible])	-2145.164 (1934.461)	0.029832** (0.00863)	-144.769** (50.786)						0.1872 (0.2174)		.8063	2.41
4.2.11	-1264.76 (930.20)	2894.888* (1609.338)								0.8806** (0.1763)	-0.009720 (0.01100)	.6500	2.58

[a]The variables appearing in Table 4.3 have been defined on pp. 66-68. For explanations of symbols, see pp. 68-69.

Table 4.4

Results of the Estimation of the Aggregate Barley Acreage Response Equations When Barley Price is Lagged by Two Periods, Canadian Prairies, 1950-51--1968-69[a]

Equation Number	Constant Term	P_{b-2}	S_w	T	$\frac{S_w}{S_b}$	P_{o-1}	P_{f-1}	$\frac{P_{b-2}}{P_{o-1}}$	$\frac{P_{b-2}}{P_{f-1}}$	B_{t-1}	S_b	R^2	D.W.
4.2.1a	9499.05 (828.21)	-3578.402* (1366.174)	0.009544** (0.00202)	-230.757** (43.401)								.7121	1.66*
4.2.2a	10[illegible]2[illegible].87 (725.29)	-2372.753* (1234.982)	0.011468** (0.00195)	-189.179** (41.900)	-1095.315* (464.572)							.7939	2.34
4.2.3a	8744.18 (844.19)	-4632.646* (2118.374)	0.009513** (0.00206)	-235.053** (44.714)		2908.909 (4399.771)						.7208	1.88
4.2.4a	9809.77 (822.38)	-3691.168° (2132.750)	0.009402** (0.00200)	-244.873* (44.186)		4467.233 (4444.802)	-940.745° (710.632)					.7540	1.88
4.2.7a	2367.51 (742.76)	-231.281 (1978.699)	0.007429** (0.00206)	-144.862* (55.695)						0.4771* (0.2213)		.7838	2.50
4.2.8a	-3731.21 (895.07)	2386.648 (2190.727)	0.003574* (0.00182)						3769.569 (6885.555)	0.8824** (0.1867)		.6861	2.73
4.2.10a	4278.33 (72[illegible].90)	-1002.578 (2010.826)		-121.527* (49.923)						0.2705 (0.2459)	0.023829** (0.00632)	.7930	2.43
4.2.11a	-3408.24 (904.28)	2909.120 (2254.074)	0.003746* (0.00182)					182.663 (1838.124)		0.8897** (0.1884)		.6797	2.72

[a]The variables appearing in Table 4.4 have been defined on pp. 66-68. For explanations of symbols, see pp. 68-69.

price variable used.

In the static model in Table 4.3, the estimated coefficient for barley price has the theoretically wrong sign in all the equations. Stocks of wheat have a positive coefficient in every equation in which they are used, indicating that as wheat stocks on the farm increase, farmers increase their barley acreages. The ratio of the stocks of wheat to the stocks of barley was introduced as a regressor in two of the equations but it has the theoretically wrong sign on its coefficient in one of these equations. Oats and flax prices are also introduced in some of the equations (sometimes as ratio with barley prices). The signs of the coefficients obtained are often not consistent with the theoretically expected ones. On the other hand the trend variable has a consistently negative sign in every equation in which it is used. This indicates that changes in technology, institutional factors and other secular factors are having a consistently decreasing influence on seeded barley acreage.[18] Similarly, the coefficient of the lagged dependent variable is consistently positive in every equation in which it is used.

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[18] The operations of the Canadian Wheat Board encourage wheat production, at the expense of feed grains such as barley. While this statement may not be true for all periods of time, for example, it does not apply to the period after the 1969 crop year, it is certainly true for the period considered in this study. See the Report of the Federal Task Force on Agriculture, Canadian Agriculture in the Seventies, op. cit., Chapter V.

The coefficient of stocks of wheat on farms is significant at the five per cent level of confidence or better in every equation except in (4.2.8). This is also true of the coefficients of trend and the lagged dependent variable, B_{t-1}. The latter, however, is not significant at the 10 per cent level in (4.2.10). The other variables which are used in addition to these basic variables (and sometimes as substitutes for them) generally fail to reach significance at the 10 per cent level of confidence. In a number of instances, the coefficients of barley price are significant at the 10 per cent level of confidence or better even though generally they have the theoretically wrong signs.

The Durbin Watson test is generally inconclusive for the static models. In one case (equation (4.2.5), there is definite evidence of positive serial correlation in the residuals.

The Distributed Lag Hypothesis

When considering the distributed lag as a maintained hypothesis, reference will be made to equation (4.2.7) only. This equation is chosen because on the basis of explanatory power and significance of its coefficients, it is better than all the other equations. The signs of all the coefficients of that equation are as theoretically expected. The static equivalent of equation (4.2.7) is equation (4.2.1).

The coefficient of the lagged dependent variable, B_{t-1}, is significantly different from zero and one, each at the one per cent level of significance. This lends strong support for the use of a distributed lag hypothesis in the barley study.

The distributed lag model has a higher R^2 than the static model, indicating that it has a higher explanatory power than the static model. However, it has been mentioned that the high R^2's obtained for the distributed lag models may be due to some other cause.[19]

The coefficient of P_{b-1} has the theoretically correct sign in the distributed lag model. This contrasts with the negative sign in the static model. It was mentioned in Chapter III that omitted relevant variables may introduce bias in the coefficients of variables which are retained in an equation. This bias may sometimes lead to theoretically wrong signs in the retained coefficients. This fact may partly account for the negative sign on the coefficients of $P_{b=1}$ (and P_{b-2}) in the static model (which does not include B_{t-1}).

Economic Interpretation

The economic interpretation of the estimated barley equations is made with reference to equation (4.2.7) in Table (4.3) again. As explained for equation (4.1.12) for wheat, we may, similarly, represent the coefficient of barley price lagged one period by $\alpha_1\beta$, and the coefficient

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[19]See footnotes 13 and 14 on p. 77.

of the lagged dependent variable by $1-\beta$. The value obtained for $1-\beta$ is 0.5019, hence $\beta = 0.4981$. The value obtained for $\alpha_1\beta$ is 0.076 million implying that $\alpha_1 = 0.153$ million. This indicates that a dollar change in barley price will result in a change of 0.153 million acres in barley acreage in the same direction. The coefficient of wheat stocks indicates that a 100 million bushels change in wheat stocks will change seeded barley acreage by 0.728 million acres in the same direction. The coefficient of trend indicates that seeded barley acreage decreases yearly by 0.139 million acres. All these changes assume that there are no changes in other variables in the equation.

To summarize, economic theory stresses the role of prices in determining supply response. The results from this study suggest that for barley, prices as specified in this analysis, are almost irrelevant in the explanation of the aggregate Canadian Prairie barley acreage. Perhaps one reason why statistically poor results are generally associated with the price variables in the barley equations, is the fact that the relevant prices may not have been correctly identified. However, while the appropriateness of the price variables used in the analysis may be open to

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[20]Since $1-\beta = .5019$, the coefficient of expectation, $\beta = .4981$. As explained in connection with wheat, this means that the number of years which need to be taken into account so that the sum of the weights differed from 1 by 0.05 is four to five years. That is, the effects of a particular price change can be expected to last for a period of four to five years.

question, the result that barley producers are unresponsive to barley prices may not be altogether surprising, if in fact Canadian Prairie grain farmers regard barley (and other feed grains) as a residual, as has sometimes been suggested. That is, grain farmers may make their decisions on how much wheat to produce before they allocate the rest of the land to other grains on the basis of their on-the-farm feed requirements and their expectations regarding opportunities for marketing.[21]

While farm prices received for barley appear to be irrelevant in determining barley acreage, stocks of wheat appear to be quite relevant in explaining the aggregate barley acreage. As already mentioned in this chapter, the coefficient of stocks of wheat is generally significant at the five per cent level of confidence or better in every equation in Table 4.3. During the period investigated in this study, year to year fluctuations of the order of 100 million bushels were common in stocks of wheat, and these fluctuations alone could account for about 75 per cent of the yearly changes in the aggregate barley acreage in the Canadian Prairies.

In contrast to barley, the results of this study suggest that grain producers are responsive to wheat price changes. As previously mentioned also in this chapter, the coefficient of wheat price is estimated in this study with

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[21]Schmitz, op. cit., p. 119.

the theoretically correct sign and is generally significant at the five per cent level of confidence or better. But the prices of oats and barley appear to be irrelevant in explaining the aggregate Canadian Prairie wheat acreage. A possible reason for this situation was given above. Similarly, the farm prices received for flax do not appear to be important in explaining the aggregate wheat acreage seeded in the Canadian Prairies. This statement is made on the basis of the frequency of the theoretically wrong signs on the coefficient of the flax price variable when it is included in an equation, and its inability to show statistical significance.

On the other hand, rainfall as a proxy for soil moisture, and trend as a gross variable accounting for the influences of technology, and institutional and secular factors, are quite relevant in explaining the aggregate Prairie wheat acreage.

CHAPTER V

THE NATURE OF ESTIMATED MACROPARAMETERS AND THE EXPLANATORY AND PREDICTIVE POWERS OF AGGREGATE AND DISAGGREGATE ACREAGE SUPPLY RESPONSE MODELS

This chapter is divided into three parts. Part I investigates the nature of the estimated macroparameters (and disturbances) of the Canadian wheat acreage response functions. Part II compares the explanatory and predictive powers of aggregate and disaggregate wheat and barley acreage response models, to see if there is any gain in disaggregation. Macroparameters may be used as ingredients in policy formulation. These parameters are decomposed to gain some knowledge about their nature. In Part III the results obtained are compared with the results of other similar studies in aggregation.

The microunits used in the wheat study are 42 crop districts comprising the 14 crop districts in Manitoba, the 20 in Saskatchewan and the eight in Alberta. In an attempt to limit the size of the study, it was decided to use a smaller number of divisions in the case of barley. A result of this was the arbitrary decision to consider all Saskatchewan as one region only. Thus for barley, the Prairies were assumed to be divided into only 23 regions including the 14 crop districts in Manitoba, and the eight crop districts in Alberta.

PART I

THE NATURE OF THE ESTIMATED MACROPARAMETERS

The composition of the estimated macroparameters and disturbances is discussed in this part of Chapter V. The discussion is, however, confined to the static and the distributed lag wheat acreage response models. There are three sections. In the first section, we consider the equations on which the calculation of the components of the extimated macroparameters is based. The second section presents the numerical results. In the third section, we examine the temporal structure of the estimated macroparameters.

Equations Needed for the Calculation of the Components of the Estimated Macroparameters

The nature of the estimated macroparameters are investigated for two of the estimated macroequations. One static and one distributed lag wheat acreage response model are used for this purpose. The equations utilised are (4.1.3) and (4.1.12) in Table 4.1. Taking the static model first, equation (4.1.3) may be considered in the context of the following set of relations;[1]

$$(5.1.1) \qquad A_{it} = \alpha_{oi} + \alpha_{1i}P_{w-2} + \alpha_{2i}S_{w,it} + \alpha_{3i}M_{it} + \alpha_{4i}T_{t} + \varepsilon_{it}$$

[1]The purpose of considering this set of relationships is to be able to calculate the values of the components of the macroparameters of the aggregate equation (5.1.3).

where: $i = 1,2....,\ I$ ($I = 42$ and represents the total number of microunits (crop districts) used in the wheat study).

Defining

(5.1.2)(a) $$A_t = \sum_i A_{it}$$

(b) $$S_{w,t} = \sum_i S_{w,it}$$

(c) $$M_t = \sum_i \lambda_i M_{it}$$

we can write the macroequation as,

(5.1.3) $$A_t = \alpha_o + \alpha_1 P_{w-2} + \alpha_2 S_{w,t} + \alpha_3 M_t + \alpha_4 T_t + \varepsilon_t$$

The variables appearing in equation (5.1.3) were defined in the previous chapter as follows:

A_t = wheat acreage seeded at time t,

P_{w-2} = the Canadian Wheat Board payments to farmers for Number 2 Norther Wheat in t-2,

$S_{w,t}$ = the Prairie total stocks of wheat on farms on March 31, of year t,

M_t = rainfall prior to seeding (six month average), and

T_t = time--a gross variable accounting for technological, institutional and unspecified secular influences on seeded wheat and barley acreages.

The variables A_t, $S_{w,t}$ and M_t are formed by the aggregation process defined in equation (5.1.2), while the variables T_t

and P_{w-2} are the same for the microequations and the macroequations. The subscript i has been used to indicate variables which refer to the crop districts.

Of the other symbols in the above set of equations, α_{oi}, α_{1i}, α_{2i}, α_{3i} and α_{4i} are microparameters. The corresponding macroparameters are α_o, α_1, α_2, α_3, and α_4 while λ_i is the weight used in constructing the aggregate rainfall variable M_t.[2] In constructing the aggregate rainfall variable, the weight attached to each microunit's rainfall, is the ratio of that microunit's average wheat acreage for the period considered in this study, to the Prairie average wheat acreage for the same period, while ε_{it} and ε_t are microdisturbance and macrodisturbance, respectively.

The stock and the price of wheat variables call for further comment. Data are not available for stocks of wheat on farms at the crop district level. Data on stocks of wheat on farms, however, are available for each of the three Provinces. Since the total stock of wheat on farms for each Province is, on the average, proportional to the seeded acreage in each Province, in the absence of any better information, it was necessary to assume that this proportionality also apply to each microunit.[3] This assumption was

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[2]The weight used for each crop district is as shown in Table A.8(b').

[3]Prairie average wheat acreage for the period used in this study is 25.097 million. The corresponding figures for Manitoba, Saskatchewan and Alberta are 2.798 million, 16.471 million, and 5.828 million, respectively. Expressed as a

used to obtain the time series on stocks of wheat for each crop district. Prices, however, were assumed to be the same for all the microunits. In practice, there are differences in the farm price received for wheat in the different crop districts. These differences, which arise from differences in transportation charges have not been taken into consideration in this study. However, since grain transportation costs did not change during the period of analysis, the use of one price for all regions would have no effect on the results.

Equation (4.1.3) is now represented as equation (5.1.3) in the above set of equations. It is a macromodel in the sense that it involves variables which have been aggregated over microunits. Equation (5.1.1) is the micro-relation of the i-th unit.

In order to facilitate the calculation of the components of the macroparameters in the static wheat model, let us consider the following auxiliary equations.[4]

(5.1.4) $$S_{w,it} = \Gamma_{1i} + B_{1i}P_{w-2} + C_{1i}S_{w,t} + D_{1i}M_t + E_{1i}T_t + V_{1it}$$

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proportion of the Prairie figure, these are .1115, .6564, and .2321, respectively.

The Prairie average stocks of wheat for the same period is 383.468 million bushels. The corresponding figures for Manitoba, Saskatchewan and Alberta are 35.593 million, 260.250 million, and 87.625 million, respectively. Expressed as a proportion of the Prairies figure for stocks these are .0928, .6787 and .2285, respectively.

[4]Auxiliary equations were discussed in Chapter II, p. 18.

$$(5.1.5) \qquad M_{it} = \Gamma_{2i} + B_{2i}P_{w-2} + C_{2i}S_{w,t} + D_{2i}M_t + E_{2i}T_t + V_{2it}$$

where Γ_{1i}, Γ_{2i}, B_{1i}, B_{2i}, C_{1i}, C_{2i}, D_{1i}, D_{2i}, E_{1i} and E_{2i} are parameters in the auxiliary regressions, and V_{1it} and V_{2it} are residuals. All the variables appearing in equations (5.1.4) and (5.1.5) are as previously defined.

From the above set of equations (i.e., (5.1.1) through (5.1.5)) we can express the macroparameters of equation (4.1.3) as follows:

$$(5.1.6)\,(a) \quad \alpha_o = \sum_i \alpha_{oi} + I[\mathrm{cov}(\alpha_{2i},\Gamma_{1i}) + \mathrm{cov}(\alpha'_{3i},\lambda_i\Gamma_{2i})]$$

$$(b) \quad \alpha_1 = \sum_i \alpha_{1i} + I[\mathrm{cov}(\alpha_{2i},B_{1i}) + \mathrm{cov}(\alpha'_{3i},\lambda_i B_{2i})]$$

$$(c) \quad \alpha_2 = \bar{\alpha}_{2i} + I[\mathrm{cov}(\alpha_{2i},C_{1i}) + \mathrm{cov}(\alpha'_{3i},\lambda_i C_{2i})]$$

$$(d) \quad \alpha_3 = \bar{\alpha}'_{3i} + I[\mathrm{cov}(\alpha_{2i},D_{1i}) + \mathrm{cov}(\alpha'_{3i},\lambda_i D_{2i})]$$

$$(e) \quad \alpha_4 = \sum_i \alpha_{4i} + I[\mathrm{cov}(\alpha_{2i},E_{1i}) + \mathrm{cov}(\alpha'_{3i},\lambda_i E_{2i})]$$

$$(f) \quad \varepsilon_t = \sum_i \varepsilon_{it} + I[\mathrm{cov}(\alpha_{2i},V_{1it}) + \mathrm{cov}(\alpha'_{3i},\lambda_i V_{2it})]$$

Where:

$$(5.1.6a)\,(i) \quad \bar{\alpha}_{2i} = \frac{1}{I}\sum_i^I \alpha_{2i}$$

$$(ii) \quad \alpha'_{3i} = \frac{\alpha_{3i}}{\lambda_i}$$

$$(iii) \quad \bar{\alpha}'_{3i} = \frac{1}{I}\sum_i \alpha'_{3i}$$

The derivation of the relationships in equation (5.1.6) follow the lines considered in Chapter III, pp. 53-58. We have refrained from repeating the derivation here.

As was mentioned in Chapter III, however, we do not have the macroparameters in the set of equations (5.1.6). In considering the estimates of these macroparameters, therefore, we have to add the implied sampling error to the right hand side of each of the expressions in the set of equations (5.1.6). Of course we do not have the microparameters also. These are assumed to coincide with their estimates.[5]

For the distributed lag model (i.e., equation (4.1.12)), the microequations are,

$$(5.1.7) \qquad A_{it} = \alpha_{oi} + \alpha_{1i}P_{w-2} + \alpha_{2i}S_{w,it} + \alpha_{3i}T_t + \alpha_{4i}A_{it-1} + \varepsilon_{it}$$

where, $i = 1,2,\ldots.,I$ ($I=42$ and represents the total number of crop districts used in the wheat study),

Defining

$$(5.1.8)(a) \qquad A_t = \sum_i A_{it}$$

$$(b) \qquad S_{w,t} = \sum_i S_{w,it}$$

$$(c) \qquad A_{t-1} = \sum_i A_{it-1}$$

the macroequation and the corresponding auxiliary equations are,

$$(5.1.9) \qquad A_t = \alpha_o + \alpha_1 P_{w-2} + \alpha_2 S_{w,t} + \alpha_3 T_t + \alpha_4 A_{t-1} + \varepsilon_t .$$

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[5]Implied sampling error was defined in Chapter III, p. 58.

$$(5.1.10) \qquad S_{w,it} = \Gamma_{1i} + B_{1i}P_{w-2} + C_{1i}S_{w,t} + D_{1i}T_t + E_{1i}A_{t-1} + V_{1it}$$

$$(5.1.11) \qquad A_{it-1} = \Gamma_{2i} + B_{2i}P_{w-2} + C_{2i}S_{w,t} + D_{2i}T_t + E_{2i}A_{t-1} + V_{2it}$$

Equation (4.1.12) is now represented as equation (5.1.9). All symbols are as previously indicated. Equations (5.1.7) through (5.1.11) lead to a representation of the macro-parameters of the dynamic model (equation 4.1.12) as follows;

$$(5.1.12)(a) \qquad \alpha_o = \sum_i \alpha_{oi} + I[\mathrm{cov}(\alpha_{2i},\Gamma_{1i}) + \mathrm{cov}(\alpha_{4i},\Gamma_{2i})]$$

$$(b) \qquad \alpha_1 = \sum_i \alpha_{1i} + I[\mathrm{cov}(\alpha_{2i},B_{1i}) + \mathrm{cov}(\alpha_{4i},B_{2i})]$$

$$(c) \qquad \alpha_2 = \bar{\alpha}_{2i} + I[\mathrm{cov}(\alpha_{2i},C_{1i}) + \mathrm{cov}(\alpha_{4i},C_{2i})]$$

$$(d) \qquad \alpha_3 = \sum_i \alpha_{3i} + I[\mathrm{cov}(\alpha_{2i},C_{1i}) + \mathrm{cov}(\alpha_{4i},D_{2i})]$$

$$(e) \qquad \alpha_4 = \bar{\alpha}_{4i} + I[\mathrm{cov}(\alpha_{2i},E_{1i}) + \mathrm{cov}(\alpha_{4i},E_{2i})]$$

$$(f) \qquad \varepsilon_t = \sum_i \ it + I[\mathrm{cov}(\alpha_{2i},V_{1it}) + \mathrm{cov}(\alpha_{4i},V_{2it})]$$

where,

$$(5.1.13) \qquad \bar{\alpha}_{4i} = \frac{1}{I}\sum_i^I \alpha_{4i}$$

Once again, in considering the estimates of the macropara-meters in the set of equations (5.1.12), we should add the implied sampling error to each of the expressions on the right hand side of that set of equations.

Numerical Results

Tables 5.1 and 5.2 respectively, specify the results for the static wheat model (i.e., equation (4.1.3)) and the dynamic wheat model (i.e., equation (4.1.12)). The corresponding microequations and the auxiliary equations are presented in the appendix to this study, the former as Tables A.1 and A.2 and the latter as Tables A.8 and A.9. The equations (4.1.3) and (4.1.12) for whose macroparameters aggregation bias are calculated were presented in Tables 4.1 and 4.2.

Before presenting the results, however, some terms which are used in connection with aggregation theory and which are also used in this chapter should be explained. Each expression in the square brackets in the set of equations (5.1.6) is the aggregation bias. This was mentioned in Chapter II. The aggregation bias has two components. Where one of these components is a function of microparameters which have the same economic meaning as the macroparameter being considered, this component of the aggregation bias is called the bias due to "corresponding parameters" otherwise, the bias is due to "non-corresponding parameters". Thus with respect to the macroconstant, α_o, in (5.1.6) the total aggregation bias is due to non-corresponding parameters only, while in the case of α_2 in the same equation, part of the total bias, namely, I $\mathrm{cov}(\alpha_{2i}, C_{1i})$, is due to corresponding parameters. This distinction between the components of the total aggregation bias is made in this study, in describing the components of the estimated macroparameters, so as to give an idea of the influence of non-corresponding parameters on the magnitudes of the estimated macroparameters. As was mentioned in Chapter II, the aggregation bias is defined as the discrepancy between the macroparameter and the sum (or average) of the corresponding microparameters. This sum or average is referred to as the "true value" of the macroparameter.

From Table 5.1 the following points may be noted:

1. The aggregation bias due the corresponding parameters is larger in magnitude than the bias due to the non-corresponding parameters in each of the cases where these components of the aggregation bias exist together.

2. The magnitude of the total aggregation bias is small by comparison with the estimated macroparameters for each of the following variables: wheat price, stocks of wheat, rainfall, and trend. Expressed as percentages of the estimated macroparameters, these are 3.2, 1.0, 0.1, and 8.3 per cent, respectively. On the other hand, the magnitude of the aggregation bias in the constant is quite considerable. The corresponding percentage is 26.0.

3. The magnitude of the implied sampling error is small by comparison with the estimated macroparameters of the price of wheat and the trend variables, but is quite considerable for the constant term, stocks of wheat and rainfall.

4. The true value of the macroparameter is a large

TABLE 5.1

Aggregation Bias and Implied Sampling Errors of the Least Squares Macroparameters of the Static Wheat Acreage Response Equation, (Equation (4.1.3), 1953-54--1968-69[1]

Constant Term		
$\sum_i \alpha_{oi}$		751.24
Aggregation Bias (total)[2]		-667.46
$42\ \mathrm{cov}(\alpha_{2i}, \Gamma_{1i})$	-111.98	
$42\ \mathrm{cov}(\alpha'_{3i}, \lambda_i\Gamma_{2i})$	-555.48	
Implied Sampling Error		-2591.67
Estimated Macroparameter		-2566.45
Coefficient of Price of Wheat Lagged Two Periods		
$\sum_i \alpha_{1i}$		12,574.68
Aggregation Bias (total)[2]		428.26
$42\ \mathrm{cov}(\alpha_{2i}, B_{1i})$	-23.58	
$42\ \mathrm{cov}(\alpha'_{3i}, \lambda_i B_{2i})$	451.84	
Implied Sampling Error		-5.99
Estimated Macroparameter		13,152.11
Coefficient of Stocks of Wheat		
$\bar{\alpha}_{2i}$		-0.003168
Aggregation Bias (total)[2]		-0.000056
$42\ \mathrm{cov}(\alpha_{2i}, C_{1i})$	-0.000370	

Table 5.1 (Continued)

42 $\text{cov}(\alpha'_{3i}, \lambda_i C_{2i})$	0.000314	
Implied Sampling Error		-0.002629
Estimated Macroparameter		-0.005901
<u>Coefficient of Rainfall</u>		
$\bar{\alpha}'_{3i}$		1262.40
Aggregation Bias (total)[2]		-3.48
42 $\text{cov}(\alpha_{2i}, D_{1i})$	0.86	
42 $\text{cov}(\alpha'_{3i}, \lambda_i D_{2i})$	-4.34	
Implied Sampling Error		4077.9
Estimated Macroparameter		5440.8
<u>Coefficient of Trend</u>		
$\sum_i \alpha_{4i}$		320.086
Aggregation Bias (total)[2]		-28.082
42 $\text{cov}(\alpha_{2i}, E_{1i})$	-6.114	
42 $\text{cov}(\alpha'_{3i}, \lambda_i E_{2i})$	-21.968	
Implied Sampling Error		41.126
Estimated Macroparameter		335.641

[1]There are some discrepancies (due to rounding of figures) between the directly estimated macroparameters on the one hand, and the sum (or average) of the corresponding microparameters, plus the aggregation bias, plus the implied sampling error. The following are the order of magnitude of the discrepancies for the various macroparameters:

Constant	2.2 per cent
Coefficient of price of wheat	1.1 per cent
Coefficient of stocks of wheat	0.8 per cent
Coefficient of rainfall	1.9 per cent
Coefficient of trend	0.7 per cent

Table 5.1 (continued)

[2]The figure 42 represents the total number of crop districts used. The components of the aggregation bias (total) are as specified in the set of equations (5.1.6).

TABLE 5.2

Aggregation Bias and Implied Sampling Errors of the Least Squares Macroparameters of the Dynamic Wheat Acreage Response Equation, (Equation (4.1.12)), 1953-54--1968-69[1]

Constant Term		
$\sum_i \alpha_{oi}$		2572.02
Aggregation Bias (total)[2]		-1498.29
42 $\mathrm{cov}(\alpha_{2i},\Gamma_{1i})$	-39.33	
42 $\mathrm{cov}(\alpha_{4i},\Gamma_{2i})$	-1458.96	
Implied Sampling Error		-139.36
Estimated Macroparameter		910.36
Coefficient of Price of Wheat Lagged Two Periods		
$\sum_i \alpha_{1i}$		10012.27
Aggregation Bias (total)[2]		-542.13
42 $\mathrm{cov}(\alpha_{2i},B_{1i})$	22.03	
42 $\mathrm{cov}(\alpha_{4i},B_{2i})$	-564.16	
Implied Sampling Error		-285.71
Estimated Macroparameter		9356.19
Coefficient of Wheat Stocks		
$\bar{\alpha}_{2i}$		-0.003523
Aggregation Bias (total)[2]		-0.000114
42 $\mathrm{cov}(\alpha_{2i},C_{1i})$	-0.000752	
42 $\mathrm{cov}(\alpha_{4i},C_{2i})$	0.000638	
Implied Sampling Error		-0.000878
Estimated Macroparameter		-0.004460

Table 5.2 (continued)

Coefficient of Trend		
$\sum_i \alpha_{3i}$		369.975
Aggregation Bias (total)[2]		-119.570
42 $\text{cov}(\alpha_{2i}, D_{1i})$	5.315	
42 $\text{cov}(\alpha_{4i}, D_{2i})$	-124.885	
Implied Sampling Error		-1.198
Estimated Macroparameter		250.451
Coefficient of Lagged Acreage		
$\bar{\alpha}_{4i}$		0.1535
Aggregation Bias (total)[2]		0.0220
42 $\text{cov}(\alpha_{2i}, E_{1i})$	-0.0283	
42 $\text{cov}(\alpha_{4i}, E_{2i})$	0.0503	
Implied Sampling Error		0.1195
Estimated Macroparameter		0.3018

[1]There are some discrepancies (due to rounding of figures) between the estimated macroparameters on the one hand, and the sum of the corresponding microparameters, plus the aggregation bias, plus the implied sampling error. The following are the orders of magnitude of the discrepancies for the various macroparameters:

Constant	2.6 per cent
Coefficient of price of wheat	1.8 per cent
Coefficient of stocks of wheat	1.2 per cent
Coefficient of trend	0.5 per cent
Coefficient of lagged acreage	2.2 per cent

[2]The figure 42 represents the total number of crop districts used. The components of the aggregation bias (total) are as specified in the set of equations (5.1.11).

part of the estimated macroparameter for the price of wheat, the stocks of wheat, and the trend variables. In percentage terms, these are 95.6, 54.0, and 95.3 per cent, respectively.

From Table 5.2, the following may also be noted;

1. The bias due to the corresponding parameters is larger in magnitude than the bias due to the non-corresponding parameters in each of the cases where both components of the aggregation bias exist.

2. The magnitude of the total aggregation bias is small by comparison with the estimated macroparameters for each of the following variables: wheat price, stocks of wheat, and the lagged dependent variable. The corresponding percentages are 5.7, 2.5, and 7.2 per cent, respectively. On the other hand, the magnitudes of the aggregation bias in the constant term and in the coefficient of the trend variable are quite considerable. The corresponding percentages are 164.6, and 47.9 per cent, respectively.

3. The magnitude of the implied sampling error is generally a very small part of the estimated macroparameter in all cases except for the lagged dependent variable.

4. The true value of the macroparameter is a sizeable part of the estimated macroparameter for the stocks of wheat and the lagged dependent variable. The percentages are 79.0 and 52.0 per cent, respectively. The true value of the macroparameter, however, exceeds the estimated macroparameter in the case of the price of wheat, the constant term and the trend variable. Expressed as a percentage of the

estimated value of the macroparameter, the figures are 107.0, 282.0 and 147.7, respectively.[6]

Temporal Composition of the Estimated Macroparameters

In order to examine the temporal composition of the macroparameters, the total data interval was divided into two periods of equal length, the first running from 1953-54 through 1960-61, and the second from 1961-62 through 1968-69. This study of the temporal structure of the macroparameters is limited to the static wheat equation only. The results are presented in Tables 5.3 and 5.4.

The most interesting findings from the study of the temporal behaviour of the macroparameters are the changes in the structure that take place in the macroparameters over time. In both the first half and second half estimates of the price of wheat macroparameter, the true values still form the dominant part of the estimated macroparameters. As percentages, the true values are 80.0 and 93.7 of the macroparameter in the first half and second half, respectively. This may be compared with a value of 95.6 per cent obtained for the total data interval (see Table 5.1).

Similar shifts in the magnitude of the true value relative to the estimated macroparameter occur in the other macroparameters. For stocks of wheat, the proportions are

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[6]The estimates of the macrodisturbances were also partitioned into similar components. The results are included in the appendix as Tables A.10 and A.11.

TABLE 5.3

Aggregation Bias and Implied Sampling Errors of the Least Squares Macroparameters of the Static Wheat Acreage Response Equation (4.1.3), 1953-54--1960-61[1]

Constant Term		
$\sum_i \alpha_{oi}$		-7875.88
Aggregation Bias (total)[2]		-19961.40
42 $\text{cov}(\alpha_{2i}, \Gamma_{1i})$	-3326.67	
42 $\text{cov}(\alpha'_{3i}, \lambda_i \Gamma_{2i})$	-16634.73	
Implied Sampling Error		8976.27
Estimated Macroparameter		-19384.39
Coefficient of Price of Wheat Lagged Two Periods		
$\sum_i \alpha_{1i}$		15878.08
Aggregation Bias (total)[2]		3503.95
42 $\text{cov}(\alpha_{2i}, B_{1i})$	-50.65	
42 $\text{cov}(\alpha'_{3i}, {}_i B_{2i})$	3554.60	
Implied Sampling Error		-28.93
Estimated Macroparameter		19847.61
Coefficient of Stocks of Wheat		
$\bar{\alpha}_{2i}$		0.000272
Aggregation Bias (total)[2]		0.000244
42 $\text{cov}(\alpha_{2i}, C_{1i})$	0.000390	
42 $\text{cov}(\alpha'_{3i}, \lambda_i C_{2i})$	-0.000146	

TABLE 5.3 (continued)

Implied Sampling Error		-0.000140
Estimated Macroparameter		0.000381
<u>Coefficient of Rainfall</u>		
$\bar{\alpha}^*_{3i}$		2499.64
Aggregation Bias (total)[2]		1778.48
42 cov(α_{2i}, D_{1i})	-5.92	
42 cov$(\alpha'_{3i}, \lambda_i D_{2i})$	1784.40	
Implied Sampling Error		3495.75
Estimated Macroparameter		7973.20
<u>Coefficient of Trend</u>		
$\sum_i \alpha_{4i}$		600.98
Aggregation Bias (total)[2]		-42.07
42 cov(α_{2i}, E_{1i})	-12.68	
42 cov$(\alpha'_{3i}, \lambda_i E_{2i})$	-29.39	
Implied Sampling Error		74.38
Estimated Macroparameter		718.88

[1]There are some discrepancies (due to rounding of figures) between the estimated macroparameters on the one hand and the true value of the macroparameter, plus the total aggregation bias, plus the implied sampling error. The following are the order of magnitude of the discrepancies for the various macroparameters:

Constant term	2.7 per cent
Coefficient of price of wheat	2.1 per cent
Coefficient of stocks of wheat	1.4 per cent
Coefficient of trend	0.2 per cent
Coefficient of rainfall	2.5 per cent

The results of the microregressions and the auxiliary equations from which the figures in Table 5.3 have been calculated are not presented.

[2]See footnote 2 for Table 5.1.

TABLE 5.4

Aggregation Bias and Implied Sampling Errors of the Least Squares Macroparameters of the Static Wheat Acreage Response Equation (4.1.3), 1961-62--1968-69[1]

<u>Constant Term</u>		
$\sum_i \alpha_{oi}$		-6711.59
Aggregation Bias (total)[2]		-1788.74
42 $\mathrm{cov}(\alpha_{2i},\Gamma_{1i})$	-357.75	
42 $\mathrm{cov}(\alpha'_{3i},\lambda_i\Gamma_{2i})$	-1430.99	
Implied Sampling Error		1400.63
Estimated Macroparameter		-7311.74
<u>Coefficient of Price of Wheat Lagged Two Periods</u>		
$\sum_i \alpha_{1i}$		14053.29
Aggregation Bias (total)[2]		665.50
42 $\mathrm{cov}(\alpha_{2i},B_{1i})$	33.21	
42 $\mathrm{cov}(\alpha'_{3i},\lambda_i B_{2i})$	632.29	
Implied Sampling Error		-60.36
Estimated Macroparameter		15003.51
<u>Coefficient of Stocks of Wheat</u>		
$\bar{\alpha}_{2i}$		-0.004656
Aggregation Bias (total)[2]		0.019083
42 $\mathrm{cov}(\alpha_{2i},C_{1i}$	-0.010050	
42 $\mathrm{cov}(\alpha'_{3i},\lambda_i C_{2i})$	0.029113	
Implied Sampling Error		-0.004513

TABLE 5.4 (continued)

Estimated Macroparameter		0.010024
Coefficient of Rainfall		
$\bar{\alpha}'_{3i}$		4311.96
Aggregation Bias (total)[2]		137.51
42 $\text{cov}(\alpha_{2i}, C_{1i})$	4.91	
42 $\text{cov}(\alpha'_{3i}, \lambda_i C_{2i})$	132.60	
Implied Sampling Error		2749.26
Estimated Macroparameter		7130.50
Coefficient of Trend		
$\sum \alpha_{4i}$		431.62
Aggregation Bias (total)[2]		-36.34
42 $\text{cov}(\alpha_{2i}, E_{1i})$	-7.47	
42 $\text{cov}(\alpha'_{3i}, \lambda_i E_{2i})$	-28.87	
Implied Sampling Error		69.93
Estimated Macroparameter		466.62

[1]There are some discrepancies (due to rounding of figures) between the estimated macroparameters on the one hand, and the true value of the macroparameter, plus the total aggregation bias, plus the implied sampling error. The following are the order of magnitude of the discrepancies for the various macroparameters:

Constant Term	2.9 per cent
Coefficient of price of wheat	2.3 per cent
Coefficient of stocks of wheat	1.1 per cent
Coefficient of trend	0.3 per cent
Coefficient of rainfall	1.9 per cent.

The results of the microregressions and the auxiliary equations from which the figures in Table 5.4 have been calculated are not presented.

[2]See footnote 2, for Table 5.1.

71.3 and per cent of the macroparameter in the first half and second half, respectively. These may be compared with 53.0 per cent for the total interval. The true value is 40.1 per cent of the macroparameter for rainfall in the first half and 53.5 per cent in the second half. These may be compared with 23.2 per cent in the total interval. The corresponding figures for trend are 83.6 per cent and 92.5 per cent in the first half and second half, respectively. These may be compared with 95.3 per cent in the total interval estimate.

Changes are also evident in the other components of the estimated macroparameters. The total aggregation bias changes in magnitude relative to the estimated macroparameters for each of the following variables in both the first and second half estimates: wheat price, stocks of wheat, rainfall, and trend. Expressed as percentages of the estimated values of the corresponding macroparameters these are, 18.0, 64.0, 22.0, 6.0 per cent, respectively, for the first half data. The corresponding figures for the second half data are, 4.0, 19.0, 2.0, 8.0 per cent, respectively. These may be compared with 3.2, 1.0, 0.1 and 8.3 per cent, respectively obtained for the total interval. The change in this component for stocks of wheat in the second half is particularly noteworthy. As can be seen from Table 5.4 the estimated macroparameter of wheat stocks is positive despite the fact that the average of the corresponding microparameters is negative. This is due to the presence of a relatively large positive aggregation bias. Some changes are also evident in the implied sampling error component of the estimated macroparameters. For example, in the parameters of wheat price, stocks of wheat, rainfall, and trend, the magnitude of the implied sampling error expressed as percentages of the first half value of the estimated macroparameters are, 0.1, 44.0, 75.0, 12.0 per cent respectively. The corresponding figures for the second half are, 0.2, 37.0, 44.0, 10.0, per cent, respectively. These may be compared with 0.4, 45.0, 39.0, 15.0 per cent, respectively, obtained for the total interval.

These figures provide information on the composition of the same macroparameters as these macroparameters take on various values over time. The results indicate that the relationship between the macroparameters and the microparameters changes over time. When the results are applied in particular to the true value, they indicate that it is not always that the same macroparameter may adequately reflect (or fail to reflect) the average relationship existing at the microunit level. A summary of the true value as a percentage of the estimated macroparameter is given in Table 5.5.

PART II

EXPLANATORY AND PREDICTIVE POWERS OF AGGREGATE AND DISAGGREGATE ACREAGE SUPPLY RESPONSE MODELS

In order to compare the relative performance of

TABLE 5.5

True Value of Estimated Macroparameter Expressed as Percentage of the Estimated Macroparameter, Wheat Acreage Response Models

	Static Model	Distributed Lag Model	Temporal 1953-54 to 1960-61	Temporal 1961-62 to 1968-69
Constant	29.3	282.0	40.6	91.8
Price of Wheat	95.6	107.0	80.0	93.7
Stocks of Wheat	54.0	79.0	71.3	53.0
Trend	95.3	147.7	83.6	92.5
Rainfall	23.2		40.1	53.5
Lagged Acreage		52.0		

aggregate and disaggregate models in explaining the aggregate acreages of wheat and barley seeded in the Canadian Prairies, three models were defined in this study as follows: Model I--separate acreage response functions were fit to the data of each microunit (crop district); Model II--an acreage response function was fit to the variables aggregated over all the microunits; Model III--acreage response models were fit to the variables obtained by adding over the microunits for each of non-overlapping groups, formed from all the microunits. In this study, the groups were arbitrarily defined. Three groups were defined such that the microunits in each group shared the common characteristic of belonging to the same Province.

We should, however, explain the sense in which the grouping of the microunits in this study is arbitrary. In the theoretical discussion on the explanatory power of Model III in Chapter II, we indicated the conditions which, all other things being equal, will lead to an increase in the R_p^2.[7] In particular it was found that this will be so, if partitioning is done in a way such that the coefficients of the members in each group are close to one another, and also, if the correlation between the independent variables of the members in the same group is high. (See equations (2.3.25) and (2.3.26) in Chapter II.) Unfortunately, this conclusion is the result of analysis based on one explanatory

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[7]See Chapter II, p. 33.

variable in each microrelation and where each group is made up of two members only. To the best knowledge of this author, no generalization of this approach has yet been developed. This study, on the other hand, included four explanatory variables in each microrelation. Also, in order to limit the number of groups defined from the 42 microunits considered in this study, each group contained more than two members. Consequently no attempt was made to follow the partitioning suggested by the theoretical discussion in Chapter II. In this sense the partitioning followed in this study is arbitrary--it was done in such a way that we could compare analysis at the level of the Prairie Provinces with Models I and II.

Explanatory Power of Models I and II

First the performances of Models I and II in explaining the aggregate acreage response are compared. This is then followed by a comparison of the performances of Models I and II with that of the semi-aggregate model, that is, Model III.

In the case of wheat, the macroequations used in these comparisons are equations (4.1.3) and (4.1.12). The corresponding microequations are presented in the appendix as Tables A.1 and A.2. The microequations which correspond to the macroequations (4.1.3) and (4.1.12) are not necessarily the "best" for each microunit ("best" in terms of explanatory power and theoretically correct signs and statistical significance of the estimated coefficients). Thus several other output response equations were also fit to the data of each microunit and the "best" equation (as defined above) for each microunit was also used in the comparison of the explanatory power of the aggregate and the disaggregate models. The "best" equation (as defined above) for each microunit is presented in the appendix as Table A.3. This set of equations is hereafter referred to as equation (4.0.0).

The barley macroequation used in the comparison of the relative explanatory powers is equation (4.2.7). The corresponding microequations are presented in the appendix as Table A.4.[8]

In order to compare the relative performances of Model I and Model II in explaining the aggregate acreage response, a "composite R^2", (R_c^2), value of the 42 micro acreage response models was computed in the manner of Grunfeld and Griliches.[9] To obtain the R_c^2, residuals $(Y-\hat{Y})$ for each of the 42 micro functions were obtained for each year separately. These were summed over microunits for each year and the sum of the microresiduals squared. Then these squared residuals were summed over the entire sample period 1953-54--1968-69, and this sum was divided by the sum of

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[8]It turned out that each micromodel in Table A.4 is also the "best" (as defined above) equation obtained for barley acreage for each microunit.

[9]Grunfeld and Griliches, op. cit.

squared deviations of the aggregate dependent variable about their mean. The result was subtracted from one to give the "composite R^2" (R_c^2). The R_c^2 of Model I is comparable to the R^2 value from the aggregate acreage response model (Model II) which is designated here as R_a^2.[10]

The results of these computations are presented in Table 5.6.

TABLE 5.6

"Aggregate" and "Composite" R^2 for Wheat and Barley Acreage Response Equations, when the Macrounit is the Canadian Prairies

Equation		R_c^2 (Model I)	R_a^2 (Model II)
(4.1.3)	Wheat	.9249	.9227
(4.0.0)	Wheat	.9411	.9227
(4.1.12)	Wheat	.9041	.8969
(4.2.7)	Barley	.7922	.7837

As is clear from Table 5.6, the multiple correlation coefficient for Model I is larger than that for Model II in all cases. This result implies that the micro acreage response functions (Model I) explain the aggregate acreage response of the microunits in the sample better than one aggregate acreage response equation (Model II).

[10]See Chapter II, p. 27.

The R^2 values in Table 5.6 are suggestive of the relative performance of the models. The question that arises, however, is whether the differences between the R^2's are statistically significant. This is equivalent to enquiring whether the differences between the sum of squared residuals from the two models are statistically significant. Hence, to test whether the differences between the R^2's are significantly different, squared residuals were obtained for each model for each year. The differences between the squared residuals of the two models were calculated for each year separately. These differences were then valued (ranked) according to their absolute values and the sign of the difference was attached to the ranks. The sum of the positive ranks was computed and used as a test statistic.[11]

The results of these computations are given in Table 5.7 The significance level used is 0.05. It can be seen from Table 5.7 that for equation (4.1.3) the sum of the

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[11]This is the Wilcoxon Matched Pair Signed Rank Test for testing whether there is a significant difference between two distributions. The test may be briefly described as follows. Given a set of n pairs of observations, we compute the differences $X_i - Y_i$ and rank them according to their absolute values. The sign of the difference is then attached to the ranks and the sum of the positive ranks T_n computed. The $P_r(T_n \leq x)$ has been tabulated by Wilcoxon. In using the test, the hypothesis tested is that there is no difference between the distributions of X and Y, against the alternative that $P(X > Y) = b$, where $0 < b < 1$. If x is the value of T_n calculated from the sample, the hypothesis is rejected if $P_r(T_n \leq x) \geq 1 - \alpha$, where α is the significance level for the test. Reference to this material may be obtained from D. Owen, Handbook of Statistical Tables (London: Addison-Wesley Publishing Co. Inc., 1962), p. 325.

positive ranks is 72, and the tabulated probability is .590. This means that the probability of obtaining a sum of positive ranks T_n of 72 or less is .590, if the null hypothesis is true. That is, the probability of observing a value of T_n greater than 72 is .410 if the null hypothesis is true. Consequently we accept the conclusion that there is no significant difference between the squared residuals of Models I and II as far as the static wheat model (equation (4.1.3)) is concerned. That is, there is no significant difference between the explanatory powers of Models I and II. This is also the case for the dynamic wheat model (equation (4.1.12)) and the dynamic barley model (equation (4.2.7)). The difference is, however, significant for equation (4.0.0)) for wheat. We may conclude, therefore, that whilst differences exist in the R^2's of Models I and II, these differences are generally not significant at the five per cent level of confidence.

Explanatory Power of Model III Relative to Models I and II

In order to examine the predictive power of the semi-aggregate model (Model III), semi-macrovariables were obtained by adding the corresponding variables for the crop districts in each of the three Canadian Prairie Provinces, separately. The resulting regression equations which were obtained by using these variables are presented in Table A.6.

For each semi-aggregate model the multiple correlation

TABLE 5.7

Wilcoxon Test for the Difference Among the Squared Residuals of Models I and II when the Macrounit is the Canadian Prairies

Equation	Hypothesis[1]	Sum of Positive Ranks	Critical Probability	Tabulated Probability	Result
(4.1.3)	S.R.I = S.R.II	72	0.95	0.590	Accept Hypothesis
(4.0.0)	S.R.I = S.R.II	127	0.95	1.000	Reject Hypothesis
(4.1.12)	S.R.I = S.R.II	94	0.95	0.912	Accept Hypothesis
(4.2.7)	S.R.I = S.R.II	96	0.95	0.928	Accept Hypothesis

[1]The null hypothesis tested is that the sum of squared residuals from Model I (S.R.I) is not different from the sum of the squared residuals from Model II (S.R.II).

coefficient, R_p^2, which corresponds to the R_c^2 (of Model I) and the R_a^2 (of Model II) was calculated. In order to obtain the R_p^2 for the semi-aggregate model, residuals $(Y-\hat{Y})$ for each of the three semi-macroequations representing Manitoba, Saskatchewan, and Alberta were obtained for each year, separately. These residuals were summed over the Provinces (semi-macrounits) for each year and this sum of residuals squared. Then these squared sums of the residuals were again summed over the entire sample period 1953-1968, the resulting figure being divided by the sum of the squared deviations of the aggregate dependent variable about their mean. This final result was then subtracted from one to give the R_p^2.

These R_p^2's are shown in Table 5.8 together with the corresponding R_a^2's and the R_c^2's. As can be seen from that table, the explanatory power of Model III is higher than those of Models I and II in the static wheat equation. Model III is not so successful in the other equations. Generally the R_p^2 is inferior to the R_c^2 but superior to the R_a^2. The Wilcoxon test, however, shows that these differences are not statistically significant.[12]

Predictive Powers of Models I, II and III

The picture that emerges out of the comparisons of the multiple correlation coefficients of Models I, II and

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[12]The only exceptions are the R_c^2 - R_a^2 and R_c^2 - R_p^2 comparisons for equation (4.0.0). These are significant at the five per cent level of confidence.

TABLE 5.8

R_a^2, R_c^2, R_p^2 for Wheat and Barley Acreage Response Equations, when the Macrounit is the Canadian Prairies

Equation		R_c^2 (Model I)	R_a^2 (Model II)	R_p^2 (Model III)
(4.1.3)	Wheat	.9249	.9227	.9301
(4.0.0)	Wheat	.9411	.9227	.9297
(4.1.12)	Wheat	.9041	.8969	.8949
(4.2.7)	Barley	.7922	.7837	.7893

III is not clear cut. While the results indicate that Model I has a relatively higher explanatory power in all cases than the fully aggregate model (Model II), it is not superior to the semi-aggregate model (Model III) in all cases. Similarly, the semi-aggregate model is better than the fully aggregate model in all cases. However, except for the differences between R_c^2 and R_a^2 and between R_c^2 and R_p^2, with respect to (equation (4.0.0)) for wheat, whatever differences that are observed between the comparable multiple correlations are not statistically significant.

The performances of the three models were also evaluated by attempting to predict the aggregate wheat and barley acreages for 1967-68, 1968-69, and 1969-70. Since the first two prediction periods had been used in obtaining all the equations already referred to, Models I, II, and III were refit using data up to 1966-67 only. The resulting equations were then used to predict the aggregate wheat and barley acreages for 1967-68, 1968-69 and 1969-70. In the study of predictive performances of the three models, only equations (4.1.3), (4.0.0) and (4.2.7) were used, that is, the dynamic wheat equation (equation (4.1.12)) was omitted. The results are presented in Table 5.9.[13]

For the static wheat model, that is, equation (4.1.3), it can be seen that Model I has the "best" performance, (that

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[13]The re-estimated microequations and macroequations using data for the period 1953-54--1966-67 for wheat, and for the period 1950-51--1966-67 for barley have not been recorded in this study.

TABLE 5.9

Actual and Predicted Wheat and Barley Acreages for 1967-68, 1968-69 and 1969-70 using Models I, II and III, '000 Acres

		1967-68 Equation			1968-69 Equation			1969-70 Equation		
		4.0.0	4.1.3	4.2.7	4.0.0	4.1.3	4.2.7	4.0.0	4.1.3	4.2.7
		Wheat	Wheat	Barley	Wheat	Wheat	Barley	Wheat	Wheat	Barley
Model I	Forecast	29,301	29,895	6,059	29,247	29,322	7,360	25,132	25,644	9,028
	Actual	29,570	29,570	7,600	28,860	28,860	8,330	24,400	24,400	9,000
	Percentage Error[a]	-0.91	1.1	-20.3	1.3	1.6	-11.6	3.0	5.1	0.3
Model II	Forecast	29,126	29,126	5,146	29,447	29,447	6,618	25,702	25,702	9,182
	Actual	29,570	29,570	7,600	28,860	28,860	8,330	24,400	24,400	9,000
	Percentage Error[a]	-1.5	-1.5	-32.3	2.0	2.0	-20.6	5.3	5.3	2.0
Model III	Forecast	29,244	29,186	6,104	29,457	29,457	7,391	25,206	26,144	9,054
	Actual	29,570	29,570	7,600	28,860	28,860	8,330	24,400	24,400	9,000
	Percentage Error[a]	-1.1	-1.3	-19.6	2.1	2.3	-11.2	3.3	7.1	0.6

[a]The percentage error is calculated as:

$$\frac{\text{Forecast Value - Actual Value}}{\text{Actual Value}} \times 100$$

is, Model I has the smallest percentage forecast errors in each of the three years for which forecasts were made). Model II also gives better predictions for 1968-69 and 1969-70 than Model III and, therefore, the fully aggregate model (Model II) has a better overall performance than the semi-aggregate model.

As was explained above, equation (4.0.0) differs from equation (4.1.3) by not having necessarily the same set of explanatory variables for the acreage response function for each crop district. According to this latter equation, that is, equation (4.0.0), Model I has the "best" predictions (as explained above) for each year for which forecasts were obtained. However, Model III has better predictions for 1967-68 and 1969-70 than Model II and, therefore, the semi-aggregate model (Model III) has better overall performance than the fully aggregate model (Model II). This contrasts with the results obtained for equation (4.1.3) for these two models.

Generally, the predictions for barley are poorer than those for wheat. The equation used for the predictions is equation (4.2.7). According to this equation, Model III has better overall performance by having the smallest forecast errors in two of the three years for which forecasts were obtained. Once again the disaggregate model (Model I) has better predictions than the fully aggregate model (Model II) for each of the three years.

PART III

COMPARISON WITH THE RESULTS OF OTHER STUDIES

There have not been many attempts to empirically study the relative performances of alternative models of the type considered in this study in explaining variation in an aggregate dependent variable. The few studies that have been done include those by Grunfeld and Griliches,[14] Ali Ismail Medani,[15] Gupta,[16] Houck and Subotnik[17] and Heady and Rao.[18]

Grunfeld and Griliches compared the performance of disaggregate and aggregate models (i.e., Model I and Model II) in explaining the variation in macrodependent variables by drawing on the results of their previous studies.[19]

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[14] Grunfeld and Griliches, op. cit.

[15] Ali Ismail Medani, "Constant versus Non-constant Production Elasticities Among Firms." Unpublished Ph.D. Dissertation, University of California, Berkeley, 1965.

[16] Gupta, op. cit.

[17] James P. Houck and Abraham Subotnik, "The U. S. Supply of Soybeans: Regional Acreage Functions," Agricultural Economics Research, Vol. 21, No. 4 (October, 1969), pp. 99-108.

[18] Earl O. Heady and V. Y. Rao, <u>Acreage Response and Production Supply Functions for Soybeans</u>, Iowa Agriculture and Home Economics Experiment Station, Iowa State University of Science and Technology, Research Bulletin 555, September, 1967.

[19] Y. Grunfeld, "The Determinants of Corporate Investment". Unpublished Ph.D. Dissertation, University of Chicago, 1958; Zvi Griliches, "Distributed Lags, Disaggregation and Regional Demand Functions for Fertilizer," <u>Journal of Farm Economics</u>, Vol. XLI, No. 1, (February, 1959), pp. 90-102.

In the comparison based on the study by Grunfeld, the R_c^2 and R_a^2 were 0.906 and 0.926, respectively. That is the explanatory power of the aggregate model was better than that of the disaggregate model. Grunfeld and Griliches computed two R_a^2 and R_c^2 each from their study based on the work by Griliches.[20] Using the first aggregation procedure, the R_c^2 and the R_a^2 turned out to be 0.982 and 0.983, respectively, while the corresponding values obtained for the second aggregation procedure were 0.987 and 0.986, respectively.

Ali Ismail Medani's study was concerned among other things with investigating the gain, if any, to be had by predicting output for each firm with a separate function for each firm and then aggregating the result as compared with a direct prediction of output from a single function fitted to the aggregate data from all the firms. His analysis showed that the R_c^2 was 0.93 as opposed to an R_a^2 value of 0.91. The difference between the R_c^2 and the R_a^2 though small in an absolute sense was found to be significant at the one per cent level of confidence.

Similarly, Gupta posed the question,

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[20]In this latter study, Grunfeld and Griliches used two aggregation procedures. The first procedure was to aggregate the microvariables linearly and then to take the logarithms of the sums. The second procedure was to add the logarithms of all the variables and use their sums as the macrovariables. Since there were two different aggregates, two different kinds of aggregate (R_a^2) and composite (R_c^2) multiple correlations were computed.

> if our aim is the prediction of the rate of change of money wages in the manufacturing sector--narrowly defined--would we gain anything by studying the rate of change of money wages in the industries constituting this sector?[21]

Gupta estimated five alternative equations to reflect various specification biases and found that the R_c^2 was larger than the R_a^2 in every case. The R_c^2 values actually obtained ranged between 0.9247 and 0.9899. In contrast, the R_a^2 was 0.8143.

Gupta also compared the performance of a semi-aggregate model (i.e., Model III in our terminology) with the performances of disaggregate and aggregate models (i.e., Models I and II, respectively, in our terminology). To do this, he defined semi-macrovariables ("partitioned" macrovariables in his terminology) from the microvariables by combining two industries at a time, thus obtaining three differen partitions from the four industries. Once again, each of the three partitions was used in conjunction with the five equations reflecting various specification biases. Thus fifteen R_p^2's were computed. Nine of the R_p^2's were better than the R_c^2's while thirteen of the fifteen R_p^2's were better than the R_a^2. However, since no statistical tests were obtained it is not known whether the R^2's were significantly different from each other.

Heady and Rao obtained R^2 and t values for their "national response equations" which were "almost universally

[21]Gupta, op. cit., p. 89.

larger than for parallel coefficients for regions and states."[22] Even though Heady and Rao's study used both aggregate and disaggregate models, their work differs from this study and the other studies above by the fact that their objective was not to compare the aggregate and disaggregate models. Consequently, no "composite R^2" was obtained.

Houck and Subotnik also estimated six regional soybean acreage supply equations from which a national acreage supply function was developed by summing the six regional functions and collecting terms where appropriate. An aggregate R^2 was obtained for the national model by weighting the computed multiple correlation coefficient for each region by the proportion of that region's acreage variance to the total acreage variance for the United States. The resulting value of R_a^2 of 0.96 turned out to be larger than three of the six micro-R^2's. However, this is not the kind of comparison to make between Models I and II. No composite R^2 was obtained. Houck and Subotnik's study differs from this study and the others mentioned above by the fact that no independent macrovariables were formed.

The only empirical investigations regarding the structure of estimated macroparameters are studies by Boot and De Wit[23] and by Gupta.[24] In the study by Boot and

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[22]Heady and Rao, op. cit., p. 1044.

[23]Boot and De Wit, op. cit.

[24]Gupta, op. cit.

De Wit the magnitude of the implied sampling error in each estimated macroparameter was not greater than one per cent of the aggregation bias, in each estimated macroparameter, however, was about 10.0 per cent of the magnitude of the estimated macroparameter. In one of the estimated macro-constant terms, the bias was about 30.0 per cent. Except for the constant term, the order of magnitudes of the aggregation bias and the implied sampling errors in the estimated macrocoefficients showed that these macrocoefficients were close to their true values.

The study by Gupta on the other hand, showed that while in some estimated macroparameters, the true value was at least 90 per cent of the estimated macroparameter, aggregation bias and implied sampling error could be quite considerable. Thus in one case aggregation bias was four times larger than the estimated macroparameter.

The magnitudes of the aggregation bias and the implied sampling errors obtained in this study were also generally larger than those obtained by Boot and De Wit, and in many cases they were even greater than the appropriate macroparameter, though none was even as large as three times the estimated macroparameter. As Table 5.5 indicated, the effect of magnitudes of these components was to create an appreciable discrepancy between the estimated macroparameter and its true value in some cases.

To summarise, this chapter was concerned with an investigation of the extent of aggregation bias and implied sampling errors in the estimated macroparameters of Canadian Prairie wheat acreage response functions. The explanatory and predictive abilities of aggregate and disaggregate wheat and barley acreage response models were also compared. The results were compared with the results of other studies which fall within the general area of aggregation.

With respect to the macroparameters of the wheat acreage response functions estimated in this study, aggregation was observed to lead to parameters which differed appreciably from their true values. This was the case with stocks of wheat, rainfall, lagged wheat acreage and the constant term. The difference was less marked in the other cases. Unfortunately, these results are meaningless when interpreted in the context of wheat acreage supply response in the Canadian Prairies. This is so because to be able to calculate the true value, the aggregation bias, and the implied sampling error components of estimated macroparameters, the macroequation and the microequations should have the same kinds of variables. The result of using the same kind of variables in every microequation was that many of the microcoefficients obtained had theoretically wrong signs--this is mentioned in the next chapter. Thus the true values obtained for the estimated macroparameters are the "wrong" types. However, these form important components of the estimated macroparameters. This result, however, serves to point out what we can sometimes expect of macroparameters. The results obtained in this chapter are summarised along with other material in the next chapter.

CHAPTER VI

SUMMARY AND CONCLUSIONS

It is appropriate, in bringing this study to a close that we review some of the points made in the preceding chapters and also make some concluding remarks.

GENERAL

1. We showed formally that aggregation generally leads to aggregation bias, and that a macroparameter can be decomposed into its true value and the aggregation bias. The true value of a macroparameter is defined as the average (or sum) of the corresponding microparameters. The true value of a macroparameter is so defined because this is a desirable value (natural value) that a macroparameter in an aggregate relationship introduced to represent a number of microrelationships should have.[1] The average is the relevant desirable value, where the corresponding individual variables assume values different from microunit to microunit at time t, and where the value of the corresponding macrovariable at time t is a summation of the values of the microvariables. In the case where the values of the variables at time t are the same for each microunit and the

[1]See Allen, op. cit., p. 700.

macrounit, the sum of the microparameters is the relevant desirable value that the macroparameter should have.

The involvement of an aggregation bias in a macroparameter introduces the possibility that a macroparameter may not adequately reflect the average (or sum) of the corresponding microparameters. This may be so because of the possibility of the aggregation bias being quite large. There is the possibility also of the sign of a macroparameter being opposite to the sign of the average (or sum) of the corresponding microparameters.

Another implication of the nature of macroparameters is that they may be unstable. This may be so because of the possibility of the aggregation bias varying, the microparameters themselves remaining constant.

In empirical investigation of the composition of macroparameters, however, we can only have estimates of the macroparameters and the microparameters. Assuming that the microparameters coincide with their least squares estimates, the least squares estimates of the macroparameters can be shown to be equal to the true value, plus the aggregation bias, plus an error term (implied sampling error).

Whether macroparameters can in fact give wrong information regarding the composite magnitudes (average or sum) of the corresponding microparameters because of the presence of aggregation bias and error term in them, depends on the values that these components can assume in practice.

In this study, the estimated macroparameters of wheat acreage response models were decomposed into their true values, aggregation bias and implied sampling error components. This kind of decomposition has been reported in two previous studies by other researchers.

Using investment demand equations for 13 manufacturing firms, Boot and De Wit's study showed that the implied sampling error component of the macroparameters estimated were generally, relatively small (under 1.0 per cent of the magnitude of the estimated macroparameter). The aggregation bias component in the macrocoefficients of the two independent variables included in their study was generally about 10.0 per cent of the magnitude of the estimated macroparameter. The corresponding magnitude of the bias in the constant term was 30.0 per cent.

The study by Gupta involving wage determination in four British industries, showed that these components of the estimated macroparameters can be quite large. In one macroparameter the magnitude of the aggregation bias was four times larger than the magnitude of the estimated macroparameter.

In this study, wheat acreage response functions for 42 crop districts in the Canadian Prairies were used. The results indicate that for some variables, the magnitude of the aggregation bias was large. Thus, for example, it was 47.9 per cent of the magnitude of the estimated macroparameter of trend in the distributed lag model.
Its value in the estimated macrocoefficient of wheat stocks

was 190.0 per cent of the estimated macroparameter, using data for the period 1961-62 through 1968-69. The implied sampling error component in the macrocoefficients was sometimes as large as 75.0 per cent of the magnitude of the estimated macrocoefficient. It was larger in some macroconstant terms.

The sign obtained for the macrocoefficient of stocks of wheat using data for the period 1961-62 through 1968-69 was positive, despite the fact that the average of the corresponding microcoefficients was negative. This situation resulted from a relatively large aggregation bias component.

The results of this study, therefore, corroborate those of previous studies in confirming Theil's fears that macroparameters may be unreliable measures of the corresponding microparameters. They also lend support to his warning that we should be cautious as to the magnitudes of the coefficients in macrorelations, even if their standard errors suggest that their statistical estimates are quite reliable.

2. In investigating the possible effects of aggregation, the following points are noteworthy also. In equation (4.1.3) in Table 4.1, all the coefficients have the theoretically correct signs, and each coefficient is significant at the five per cent level of confidence or better. The corresponding microequations in Table A.1, however, indicate that in only 17 of the 42 equations do all the

coefficients have the theoretically correct signs.

The coefficient of wheat price lagged two periods in the macroequation, is significant at the one per cent level of confidence. The coefficient of the same variable, however, fails to show significance at the 10 per cent level of confidence or better in 17 of the corresponding microequations. Similarly, the coefficient of stocks of wheat in the macroequation is significant at the five per cent level of confidence. The coefficient of the same variable, however, fails to show statistical significance at the 10 per cent level of confidence or better in 28 of the 42 microequations. The coefficient of rainfall is also significant at the five per cent level of confidence in the macroequation, whereas the coefficient of the same variable fails to show significance at the 10 per cent level of confidence or better in 29 equations. The coefficient of the trend variable, on the other hand, shows significance at the 10 per cent level of confidence or better in 31 of the 42 microequations (24 of these are at the one per cent level of confidence).

The same picture emerges from the distributed lag wheat equations. In equation (4.1.12), each coefficient of the variables in the macroequation is significant at the 10 per cent level of confidence or better. Many of the corresponding coefficients in the microequations, however, fail to show statistical significance at the 10 per cent level of confidence or better. Thus, the number of the microcoefficients which fail to show statistical significance at the 10 per cent level of confidence or better, for wheat price lagged two periods is 18 from a total of 42. The corresponding figures for stocks of wheat, and lagged wheat acreage are 30 and 32, respectively.[2]

The conclusion to be drawn from this comparison of the statistical significance of the coefficients of the variables in the aggregate equations, vis-a-vis the significance of the coefficients of the variables in the micro-equations, is that aggregation can lead us to place too much confidence on the relationship existing between variables than is actually the case at the microunit level. While this conclusion relates to acreage response models for wheat in the Canadian Prairies, it may very well apply in other cases also.

3. It was demonstrated, however, that if our objective is to explain the variations in the aggregate dependent variable, then aggregation is not necessarily bad. In fact, as was demonstrated theoretically, aggregation may even improve the explanation of aggregate dependent variables, compared with the performance of a completely disaggregate approach. Again, it was formally shown for a very simplified case that stratification of the microdata into groups which are then used to define semi-macrovariables and equations fit

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[2]The number of microequations for which all the coefficients have theoretically correct signs and which show significance at the 10 per cent level of confidence or better is even smaller than the figures cited above for each case.

to these variables, may sometimes give better results in terms of explanatory power.

The empirical results with wheat and barley confirmed these theoretical results on explanatory power. It was observed that the most disaggregate model (Model I) did not always have the highest explanatory power of the three models compared. Similarly, the most aggregate model (Model III) did not always have the worst explanatory power.

The empirical results from this study thus confirm the theoretical conclusions that macroparameters differ from the average (or sum) of the corresponding microparameters and that this difference can in practice be quite large. Two conclusions that come out of the empirical results in this study, and which have not been evident in the results of previous studies are, that it is possible in practice for the sign of the estimated macroparameter to have a sign opposite that of the average value of the microparameters; and that the statistical results of aggregate equations, as regards the statistical significance of their estimated macroparameters, may sometimes give a distorted view of the relationships existing between variables at the microunit level. We may thus on occasion be led to conclude from the results of aggregate equations that significant relationships exist between certain variables at the microunit level, when in fact no significant relationships exist between the microvariables.

The above statements regarding possible dangers inherent in aggregation do not lead to the general conclusion that aggregation is always bad, and hence disaggregation should be resorted to in every situation. They merely serve to indicate what aggregation can sometimes lead to. Whether in any particular situation aggregation is worthwhile or not, can be determined empirically for that particular situation only. Aggregation and disaggregation were evaluated in connection with wheat and barley acreage response in the Canadian Prairies. A summary of the results follow.

PRAIRIES WHEAT AND BARLEY ACREAGES

Aggregation and Explanation

This study has evaluated three alternative time series regression models, with respect to their ability to explain variations in the aggregate wheat and barley acreages cultivated in the Canadian Prairies, and to predict output variations outside the sample period used in the statistical estimations of the regression relationships. The three models were: Model I--separate acreage response functions were fit to the data of each microunit (crop district); Model II--an acreage response function was fit to the variables aggregated over all the microunits; Model III--acreage response models were fit to the variables obtained by adding over the microunits for each of non-overlapping groups, formed from all the microunits. The three groups which were formed coincided with the three Prairie Provinces.

The evaluations were made for wheat, within the frameworks of the traditional static model and the distributed lag model. The analysis showed that for the static model the "comparable R^2" values were 0.9249, 0.9227 and 0.9301 for Models I, II and III, respectively (when the set of variables in each microequation was the same). The results were somewhat different when the set of variables in each microequation and the macroequation was not necessarily the same (equation 4.0.0). The "comparable R^2" values in this case were 0.9411, 0.9227 and 0.9297 for Models I, II and III, respectively. The corresponding figures for the distributed lag wheat model were 0.9041, 0.8969 and 0.8949 for Models I, II and III, respectively.

In the investigation with barley, the differences among the "comparable R^2" values were also small in an absolute sense, and the Wilcoxon test indicated that they were not significant at the five per cent level of confidence. The values were 0.7922, 0.7837 and 0.7893 for Models I, II and III, respectively.

Aggregation and Prediction

With regard to the performances of the models in predicting the aggregate Prairie wheat acreage, Model I had the best predictive performance for the years for which predictions were made. This was the case for equations (4.1.3) and (4.0.0). Neither Model II nor Model III appeared to have any distinct advantage in prediction over the other. Thus, using equation (4.0.0) the semi-aggregate model (Model

III) had better predictions than the fully aggregate model (Model II), in two of the three years for which predictions were made. On the other hand Model III had the overall better predictions than Model II when equation (4.1.3) was used.

In the investigation with barley, Model I once again had better predictions than Model II for each of the three years for which predictions were made. The predictions from Model I were, however, inferior to those from Model III in two of the three years.

The results of this study, therefore, show that there are differences between the explanatory powers of Models I, II and III with regard to the Prairie wheat and barley acreages. In particular, the disaggregate model (Model I) has an overall higher explanatory power, followed by the semi-aggregate and the completely aggregate model, in that order. However, the differences between the three models in terms of explanatory powers are generally very slight, and are generally insignificant at the five per cent level of confidence. Similarly, in predicting the aggregate Prairie wheat acreage, the model with the lowest degree of aggregation (Model I) gives better predictions than the more aggregate models (Models II and III). Here again, whatever differences that were obtained are not particularly striking. With regard to barley, however, the differences between the disaggregate model and the completely aggregate model are somewhat appreciable.

Macroparameters Versus Microparameters as Policy Instruments

This section compares the results of using macroparameters and microparameters in formulating policy with respect to wheat acreage in the Canadian Prairies. The macroequation (4.1.3) is used for this comparison. This equation is selected for this purpose over other macroequations estimated in this study because it has a relatively higher explanatory power (see Table 4.1), all the coefficients of this equation have the theoretically correct signs, and preliminary investigation showed that it predicts better than other equations in Table (4.1), whose coefficients had theoretically expected signs and which were also significant.

Only 17 of the 42 corresponding microequations had the theoretically correct signs on all their coefficients. As mentioned in Chapter V, however, other equations were fit to the data of each crop district. The best of these equations for each microunit was selected and is presented in Table A.3. In each equation in Table A.3, the coefficients of all the variables have the theoretically correct signs and many of these coefficients are statistically significant at the 10 per cent level of confidence or better. These equations have relatively high explanatory power and their predictions are more reliable than comparable microequations. These are the equations compared with the macroequation (4.1.3) in formulating policy with respect to wheat acreage.

Of the variables that are included in the static and the dynamic wheat models, the wheat price and the stocks of wheat variables are of special interest because these variables may be used as policy instruments. The following discussion is, therefore, confined to these two variables only.

The coefficient of wheat price lagged two periods in equation (4.1.3) is 13.152 million. This indicates that a 10 cent change in farm wheat price will result in a 1.315 million acres change in the aggregate seeded acreage in the same direction, given that there is no change in any other variable. On the other hand, the change according to the microequations in Table A.3 is 1.551 million acres.[3]

These results indicate that when wheat price is varied, all other variables (except the dependent variable) remaining unchanged, the results differ according to whether the macroparameter is used or the microparameters. In particular, the magnitude of the change in the aggregate wheat acreage according to the micromodel is larger than the change according to the macromodel. This indicates that any policy on acreage control based on the macromodel would lead to the application of the policy instrument (wheat price in this case) to a greater degree than is actually

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[3]The figure 1.551 was obtained by the addition of all the coefficients of the price variables in the equations in Table A.3. Eighteen of the microequations in Table A.3 are based on Nerlove's general price expectation model. The coefficients in these equations were accordingly divided by the magnitude of the respective coefficients of expectation.

required to achieve the target. For example, if the target is a decrease of five million acres in the aggregate wheat acreage, and it is proposed to achieve this solely by a cut in wheat price, the price cut required to achieve this target is 32 cents according to the micromodel. The corresponding figure according to the macromodel is 38 cents, which is greater than is actually required on the basis of the micromodel. That is to say, the target decrease of five million will be exceeded if the recommendation from the macromodel is used. Similarly, the target will be exceeded, if an increase rather than a decrease is contemplated in the aggregate wheat acreage and the macromodel is again used.

Wheat stocks can also be used as policy variables. In this regard it is recognised, however, that any such use of wheat stocks can only be indirect, the direct instrument being delivery quotas. The macrocoefficient in the static wheat model (equation (4.1.3)) indicates that, if aggregate wheat stocks on farms decrease by 100 million bushels in the Canadian Prairies, aggregate wheat acreage will increase by 0.590 million acres, provided all other variables remained unchanged. We will assume that the 100 million bushels decrease in wheat stocks will result in proportional decreases in wheat stocks on farms in each crop district, these decreases being proportional to the contribution of each crop district's acreage to the aggregate wheat acreage in the Canadian Prairies, for the period used in this study. On the basis of this assumption, the microequations indicate that the increase in the aggregate wheat acreage as a result of 100 bushels decrease in total wheat stocks on farms is 0.446 million acres.[4] These results also apply in reverse, when increases rather than decreases occur in wheat stocks on farms.[5]

Thus the results obtained in considering wheat price and stocks of wheat as policy variables show that the conclusions by the macromodel differ from those by the micromodel. However, these differences are not particularly striking.

Thus as regards wheat acreage in the Canadian Prairies, the results of this study indicate that as far as explanatory power, predictive ability and usefulness for making policy recommendations are concerned, there are only very minor differences between the three models examined in this study. These differences do not appear to make disaggregation worthwhile, given the costs of disaggregation.

Similarly, for barley acreage, the results indicate that there are no appreciable differences in the explanatory powers of the three models investigated. With regard to forecasting future aggregate barley acreages in the Canadian

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[4] Thirteen of the 42 microequations in Table A.3 do not have variables for stocks of wheat. Hence any increase in wheat stocks in the respective crop districts will not affect wheat acreages in these crop districts and consequently will not contribute to changes in aggregate wheat acreages in the Canadian Prairies.

[5] The proportions used in these calculations are as tabulated in Table A(8b').

Prairies, however, there are appreciable differences between Models I and III on the one hand and Model II on the other.

In the case of barley, where it appears that disaggregation either to the level of the crop districts or to the level of the Provinces gives better results, in terms of predicting aggregate barley acreage, the differences between the predictions of Models I and III on the one hand and Model II on the other, narrows considerably with time. So that here also it cannot be concluded that the two disaggregate models are significantly superior to the aggregate model (Model II).

The results obtained in this study do not allow any generalisations to be made in connection with the relative performances of aggregate and disaggregate approaches in acreage studies (of the functional type) for wheat and barley in the Canadian Prairies. If the past, however, is representative of the future, then disaggregation does not appear to be worthwhile, given that in general disaggregation also involves greater costs in terms of data collection and statistical computations. Similarly, no generalisation can be made in terms of aggregate versus disaggregate studies in general. This is because no discernible tendencies were discovered either in the results of this study or in similar other studies in aggregation. If any generalisation can be made at all, it is that aggregation is not necessarily bad.

SHORTCOMING AND SUGGESTIONS FOR FURTHER RESEARCH

While the results obtained in estimating the acreage response functions appear adequate, the resulting equations could benefit from some improvements. For example, in estimating the acreage response for each crop district, the level of stocks of wheat in the individual crop districts was used. However, the total level of wheat stocks on hand for the entire Prairies has a large influence on the number of bushels of wheat which can be sold through quotas. Therefore, the total stocks of wheat for the Prairies should have been included as a variable in addition to the wheat stocks in the individual crop districts. Also, deflation of the price of wheat by the appropriate weighted indexes of the prices of competing commodities, may improve the results of the estimations of the wheat acreage response functions. Similarly, deflation of barley prices by the appropriate weighted indexes of the prices of commodities which compete with barley for land may improve the results of the estimation of the barley acreage response functions.

This study considered the acreages of wheat and barley only. The analysis must be extended to include the yields of wheat and barley. That is, the total production of wheat and barley, rather than the acreages of these crops should be used. This should be done in order to gain a complete picture of the performances of aggregate and disaggregate approaches to the study of wheat and barley production in the Canadian Prairies.

APPENDIXES

APPENDIX A

(i) MATHEMATICAL NOTES TO CHAPTER II

We want to show that

$$(A) \quad \sum_{i}^{2} \alpha^{2}{}_{1i} S^{2}_{v_{i1}} + 2\alpha_{11}\alpha_{12} S_{v_{11}v_{12}} = \sum_{i}^{2} S^{2}_{A_{i1}} \, {}^{2}_{A_{i1}} (1 - \gamma^{2}_{x_{i1}x_{1}})$$

$$+ 2\alpha_{A_{11}}\alpha_{A_{21}} [\gamma_{x_{11}x_{21}} - \gamma_{x_{11}x_{1}}\gamma_{x_{21}x_{1}}] \, S_{A_{11}} S_{A_{21}}$$

$$(B) \quad \sum_{j=3}^{4} \alpha^{2}_{1j} S^{2}_{v_{j2}} + 2\alpha_{13}\alpha_{14} S_{v_{32}v_{42}} = \sum_{j=3}^{4} S^{2}_{A_{j2}} \alpha^{2}_{A_{j2}} (1-\gamma^{2}{}_{x_{j2}x_{2}})$$

$$+ 2\alpha_{A_{32}}\alpha_{A_{42}} [\gamma_{x_{32}x_{42}} - \gamma_{x_{32}x_{2}}\gamma_{x_{42}x_{2}}] \, S_{A_{32}} S_{A_{42}}.$$

In order to do this we first show that

$$S^{2}_{v_{i1}} = S^{2}_{x_{i1}} \, (1 - \gamma^{2}_{x_{i1}x_{1}})$$

$$S^{2}_{v_{j2}} = S^{2}_{x_{j2}} \, (1 - \gamma^{2}_{x_{j2}x_{2}})$$

We then find similar expressions for $S_{v_{11}v_{12}}$ and $S_{v_{32}v_{42}}$. Following Grunfeld and Griliches we also make new definitions for α_{1i} and α_{1j}. Finally these components of the expressions on the left hand side of (A) are brought together to derive the right hand side of (A). This is also done for B.

We note from (2.3.19 c), p. 29 that

$$V_{i1} = X_{i1} - B_{i}X_{1}$$

Therefore,

$$S^2_{v_{i1}} = S^2_{x_{i1}} + B_i^2 S^2_{x_1} - 2B_i S_{x_1 x_{i1}}$$

Now,

$$B_i = \frac{S_{x_1 x_{i1}}}{S_{x_1^2}}$$

Therefore,

$$S^2_{v_{i1}} = S^2_{x_{i1}} - \frac{(S_{x_1 x_{i1}})^2}{S^2_{x_1}}$$

$$= S^2_{x_{i1}} \quad (1 - \frac{(S_{x_1 x_{i1}})^2}{S^2_{x_1} S^2_{x_{i1}}}$$

or,

$$S^2_{v_{i1}} = S^2_{x_{i1}} (1 - \gamma^2_{x_1 x_{i1}})$$

where,

$$\gamma^2_{x_1 x_{i1}} = \frac{(S_{x_1 x_{i1}})^2}{S^2_{x_1} S^2_{x_{i1}}}$$

Similarly,

$$S^2_{v_{j2}} = S^2_{x_{j2}} (1 - \gamma^2_{x_2 x_{j2}}).$$

From (2.3.19c) and (2.3.19d) we also note that

$$V_{11}V_{21} = (X_{11}-B_1X_1)(X_{21}-B_2X_1) = X_{11}X_{21}-X_{11}X_1B_2-X_{21}X_1B_1 + (X_1)^2B_1B_2$$

Since $B_1 = \dfrac{S_{x_{11}x_1}}{S^2_{x_1}}$ and $B_2 = \dfrac{S_{x_{21}x_1}}{S^2_{x_1}}$

$$S_{v_{11}v_{21}} = S_{x_{11}x_{21}} - \frac{S_{x_{11}x_1} S_{x_{21}x_1}}{S^2_{x_1}} - \frac{S_{x_{21}x_1} S_{x_{11}x_1}}{S^2_{x_1}} + \frac{S^2_{x_1} S_{x_{11}x_1} S_{x_{21}x_1}}{(S^2_{x_1})^2}$$

$$= S_{x_{11}x_{21}} - \frac{S_{x_{11}x_1} S_{x_{21}x_1}}{S^2_{x_1}}$$

Similarly,

$$S_{v_{32}v_{42}} = S_{x_{32}x_{42}} - \frac{S_{x_{32}x_2} S_{x_{42}x_2}}{S^2_{x_2}}$$

Following Grunfeld and Griliches,[1] let us make the following definitions[2]

$$\alpha^2_{A_{i1}} = \alpha^2_{1i} \frac{S^2_{x_{i1}}}{S^2_{A_{i1}}} \qquad \text{ie.,} \quad \alpha^2_{1i} = \frac{\alpha^2_{A_{11}} S^2_{A_{i1}}}{S^2_{x_{i1}}}$$

$$\alpha^2_{A_{j2}} = \alpha^2_{1j} \frac{S^2_{x_{j2}}}{S^2_{A_{j2}}} \qquad \text{ie.,} \quad \alpha^2_{1j} = \frac{\alpha^2_{A_{j2}} S^2_{A_{j2}}}{S^2_{x_{j2}}}$$

Using all the appropriate expressions from above we note, therefore, that,

$$\sum_i^2 \alpha^2_{1i} S^2_{v_{i1}} = \sum_i \alpha^2_{A_{i1}} S^2_{A_{i1}} (1 - \gamma^2_{x_i x_1})$$

and that,

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[1]Grunfeld and Griliches, op. cit., p. 12.

[2]$S^2_{x_i}$, $S^2_{x_j}$ are the variances of the microindependent variables and $S^2_{A_i}$, $S^2_{A_j}$ are the variances of the micro-dependent variables.

$$2\alpha_{11}\alpha_{12}S_{v_{11}v_{21}} = 2\left[\frac{\alpha_{A_{11}}S_{A_{11}}}{S_{x_{11}}}\ \frac{\alpha_{A_{21}}S_{A_{21}}}{S_{x_{21}}}\right]\left[S_{x_{11}x_{21}} - \frac{S_{x_{11}x_1}S_{x_{21}x_1}}{S^2_{x_1}}\right]$$

$$= 2\alpha_{A_{11}}\alpha_{A_{21}}S_{A_{11}}S_{A_{21}}\left[\frac{S_{x_{11}x_{21}}}{S_{x_{11}}S_{x_{21}}} - \frac{S_{x_{11}x_1}S_{x_{21}x_1}}{S_{x_1}S_{x_{11}}S_{x_{21}}}\right]$$

$$= 2\alpha_{A_{11}}\alpha_{A_{21}}S_{A_{11}}S_{A_{21}}[\gamma_{x_{11}x_{21}} - \gamma_{x_{11}x_1}\gamma_{x_{21}x_1}]$$

Therefore,

$$\sum_i^2 \alpha^2_{1i}S^2_{v_{i1}} + 2\alpha_{11}\alpha_{12}S_{v_{11}v_{21}} = \sum_i \alpha^2_{A_{i1}}S^2_{A_{i1}}\ (1 - \gamma^2_{x_ix_1})$$

$$+ 2\alpha_{A_{11}}\alpha_{A_{21}}S_{A_{11}}S_{A_{21}}[\ \gamma_{x_{11}x_{21}} - \gamma_{x_{11}x_1}\gamma_{x_{21}x_1}]$$

Similarly,

$$\sum_{j=3}^{4} \alpha^2_{1j}S^2_{v_{j2}} + 2\alpha_{13}\alpha_{14}S_{v_{32}v_{42}} = \sum_{j=3}^{4} S^2_{A_{j2}}\ (1 - \gamma^2_{x_{j2}x_2})$$

$$+ 2\alpha_{A_{32}}\alpha_{A_{42}}[\gamma_{x_{32}x_{42}} - \gamma_{x_{32}x_2}\gamma_{x_{42}x_2}]S_{A_{32}}S_{A_{42}}\ .$$

APPENDIX A

(ii) AGGREGATION BIAS

In Chapter II, we defined aggregation bias for the macroparameters as the discrepancy between the macroparameters and the means of the corresponding microparameters. The bias for the macroconstant term, however, was defined as a discrepancy between the macroconstant term and the sum of the corresponding microconstant terms. It is not immediately obvious why there should be this difference in the definition of aggregation bias. We show in this appendix why this difference arises. The treatment here is also an extension of the three-variable case considered in Chapter II. In contrast to the relationships discussed on p. 19, the relationships considered here have disturbance terms.

Consider a set of microrelations which we can write using the matrix notation as,

$$\text{(A.1)} \quad A_i = X_i \beta_i + U_i \qquad i = 1, \ldots, I$$

where A_i is a Tx1 column vector, X_i is a Txk matrix, β_i is a kx1 column vector of microparameters, and U_i is a Tx1 column vector of microdisturbances. Let the corresponding macrorelation be,

$$\text{(A.2)} \quad A = X\beta + U$$

where

$$\text{(A.3)} \qquad A = \sum_i A_i \; ; \qquad X = \sum_i X_i$$

and β is a Kx1 matrix of macroparameters, and U is a Tx1 column vector of macrodisturbances. In order to be consistent with our definition in (A.3), we take the first column of X_i not as ℓ (a column vector of T unit elements) but as (1/I) ℓ, and the first element of β_i not as the constant term, but as I times the constant term, (that is if there is a constant term in A.1). So that we have,

$$\text{(A.3a)} \qquad X_i = \begin{pmatrix} \frac{1}{I} & X_{i2(1)} \ldots\ldots & X_{ik(1)} \\ & \cdot & \cdot \\ & \cdot & \cdot \\ & \cdot & \cdot \\ \frac{1}{I} & X_{i2(T)} \ldots\ldots & X_{ik(T)} \end{pmatrix} \qquad \beta_i = \begin{pmatrix} I\beta_{i,1} \\ \cdot \\ \cdot \\ \cdot \\ \beta_{i,k} \end{pmatrix}$$

Now the least squares estimate of β in (A.2) is,

$$\text{(A.4)} \qquad \hat{\beta} = (X'X)^{-1} X' A$$

Using (A.1) and (A.3) in (A.4) we can write,

$$\text{(A.5)} \qquad \hat{\beta} = (X'X)^{-1} X' \left(\sum X_i\beta_i + \sum_i U_i\right)$$

$$= \sum_i (X'X)^{-1}X'X_i\beta_i + (X'X)^{-1}X'\sum_i U_i$$

$$= \sum_i (X'X)^{-1}X'X_i\beta_i + \eta$$

where $\eta = (X'X)^{-1}X'\sum_i U_i$ and is called the implied sampling error.

Now consider the following auxiliary relations.

$$\text{(A.6)} \qquad X_i = X\delta_i + V_i$$

where δ_i is a kxk matrix of parameters in the auxiliary relations and V_i is a Txk matrix of disturbances in the auxiliary relations.

From (A.6) we can write,

(A.7) $\hat{\delta}_i = (X'X)^{-1}X'X_i$

Also,

(A.8) $\sum_i \hat{\delta}_i = M$, a kxk indentity matrix.

Using (A.7), we can now express (A.5) as,

(A.8) $\hat{\beta} = \sum_i \hat{\delta}_i \beta_i + \eta$

From (A.8), we can write,

(A.9) $E(\hat{\beta}) = \beta = \sum_i \delta_i \beta_i$, since $E(U_i) = o$

Now, define the mean of the microparameters, $\bar{\beta}$, as the true value of $\hat{\beta}$, where

$$\bar{\beta} = \frac{1}{I} \sum \beta_i, \text{ and } \bar{\beta} = \frac{1}{I} \begin{pmatrix} I\beta_{i1} \\ \beta_{i2} \\ \vdots \\ \beta_{ik} \end{pmatrix}$$

From (A.9) we can write,

(A.10) $\beta - \bar{\beta} = \sum \delta_i \beta_i - \bar{\beta} = \sum_i \delta_1 (\beta_i - \bar{\beta})$

and we can define (A.10) as the aggregation bias.

Now,

$$\text{(A.11)} \quad \text{cov } (\delta_i, \beta_i) = \frac{1}{I}\sum [(\delta_i - \bar{\delta})(\beta_i - \bar{\beta})]$$

$$= \frac{1}{I}\sum [(\delta_i - \frac{1}{I} M)(\beta_i - \bar{\beta})]$$

$$= \frac{1}{I}\sum \delta_i \beta_i - \frac{1}{I} (\sum \delta_i) \bar{\beta} - \frac{1}{I}\frac{1}{I}\sum \beta_i + \frac{1}{I}\frac{1}{I}\sum \bar{\beta}$$

$$= \frac{1}{I}\sum \delta_i \beta_i - \frac{1}{I} \bar{\beta}$$

Multiplying each side of (A.11) by I, we get,

$$\text{(A.12)} \qquad I\ \mathrm{Cov}\ (\delta_i,\ \beta_i) = \sum \delta_i \beta_i - \bar{\beta} = \sum \delta_i (\beta_i - \bar{\beta})$$

Hence the aggregation bias is I times the covariance between δ_i and β_i, and this is the same for each macroparameter irrespective of whether it is a constant term or a coefficient. Thus the difference in the definition of aggregation bias arises from the fact that the first column of the matrix X in (A.3) is a vector of unit elements and hence to make (A.3) hold, each element in the first column of X_i had to be divided by I. This in turn meant that we redefine the first element of β_i (all i) as in (A.3a).

APPENDIX B

MATHEMATICAL NOTES TO CHAPTER III

We have,

$$A_{it} = \alpha_{oi} + \alpha_{1i}P_{it} + \alpha_{2i}X_{it} + \alpha_{3i}T_t + \varepsilon_{it}$$

where,

$$i = 1,2,\ldots.,I$$

Let the corresponding macromodel be,

$$A_t = \alpha_o + \alpha_1 P_t + \alpha_2 X_t + \alpha_3 T_t + \varepsilon_t$$

where,

$$A_t = \sum_i A_{it}$$

$$P_t = \sum_i \lambda_i P_{it}$$

$$X_t = \sum_i \delta_i X_{it}$$

$$T_t = \sum_i \omega_i T_{it}$$

Let the auxiliary equations be represented as follows:

$$P_{it} = \Gamma_{1i} + B_{1i}P_t + C_{1i}X_t + D_{1i}T_t + V_{1it}$$

$$X_{it} = \Gamma_{2i} + B_{2i}P_t + C_{2i}X_t + D_{2i}T_t + V_{2it}$$

$$T_{it} = \Gamma_{3i} + B_{3i}P_t + C_{3i}X_t + D_{3i}T_t + V_{3it}$$

We wish to show that,

$$\sum_i \lambda_i B_{1i} = 1, \ \sum_i \lambda_i \Gamma_{1i} = \sum_i \lambda_i C_i = \sum_i \lambda_i D_{1i} = \sum_i \lambda_i V_{1it} = 0;$$

$$\sum_i \delta_i C_{2i} = 1, \ \sum_i \delta_i \Gamma_{2i} = \sum_i \delta_i B_{2i} = \sum_i \delta_i D_{2i} = \sum_i \delta_i V_{2it} = 0;$$

$$\sum_i \omega_i D_{3i} = 1, \ \sum_i \omega_i \Gamma_{3i} = \sum_i \omega_i B_{3i} = \sum_i \omega_i C_{3i} = \sum_i \omega_i V_{3it} = 0;$$

In order to do this let us rewrite the micro- and the macro-relations and the auxiliary relations in the matrix notation as follows:

(B.5) $\quad A_i = X_i F_i + E_i \qquad\qquad i = 1,2,\ldots.,I$

where A_i is a txl column vector of the values taken by the endogenous microvariable; X_i is a txk matrix of the values taken by the exogenous microvariables (k=4 in this case); F_i is a kxl column vector of microparameters; E_i is a txl column vector of microdisturbances. Let the corresponding macrorelations be

(B.6) $\quad A = XF + E$

where,

$$A = \sum_i A_i \quad ; \quad X = \sum_i \rho_i X_i$$

and F is a kxl matrix of macroparameters, and E is a txl column vector of macrodisturbances and ρ_i is a scaler used in weighting the columns of X_i.

Let the auxiliary equations be

(B.7) $\quad X_i = XC_i + V_i$

where C_i is a kxk matrix of auxiliary coefficients and V_i is a txk vector of residuals in the auxiliary equations. We

We can write the matrix X explicitly as,

$$X = \begin{pmatrix} 1 & P_1 & X_1 & T_1 \\ 1 & P_2 & X_2 & T_2 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & P_t & X_t & T_t \end{pmatrix} = \begin{pmatrix} 1 & \sum_i \lambda_i P_{i1} & \cdots & \sum_i \omega_i T_{i1} \\ 1 & \vdots & & \vdots \\ \cdot & \cdot & & \cdot \\ 1 & \sum_i \lambda_i P_{it} & \cdots & \sum_i \omega_i T_{it} \end{pmatrix}$$

Now, the matrix $(X'X)^{-1}X'X$ is an identity matrix. Consider the matrix $(X'X)^{-1}X'$ which is of order kxt (K=4). Let η_{1t} represent a typical element of the first row of $(X'X)^{-}X'$. Similarly, let η_{2t}, η_{3t} and η_{4t}, respectively, represent typical elements of the second, third and fourth rows. $\sum_t \eta_{1t}P_t$ is the element in the first row and second column of the identity matrix, $(X'X)^{-1}X'X$, and is, therefore, equal to zero. That is,

$$\text{(B.8)} \qquad \sum_t \eta_{1t}P_t = \sum_t \eta_{1t} \sum_i \lambda_i P_{it} = \sum_i \lambda_i \sum_t \eta_{1t}P_{it} = 0$$

Now, $\sum_t \eta_{1t}P_{it}$ is the constant term in the regression of P_{it} on the independent macrovariables. By reference to (B.4(a)), we can, therefore, write

$$\sum_t \eta_{1t}P_{it} = \Gamma_{1i}$$

That is,

$$\sum_i \lambda_i \sum_t \eta_{1t}P_{it} = \sum_i \lambda_i \Gamma_{1i} = 0 \quad .$$

We can similarly show that

$$\sum_i \lambda_i C_{1i} = \sum_i \lambda_i D_{1i} = 0$$

$$\sum_i \delta_i \Gamma_{2i} = \sum_i \delta_i B_{2i} = \sum_i \Delta_i D_{2i} = 0$$

$$\sum_i \omega_i \Gamma_{3i} = \sum_i \omega_i B_{3i} = \sum_i \omega_i C_{3i} = 0$$

We can also show similarly that

$$\sum_i \lambda_i B_{1i} = \sum_i \delta_i C_{2i} = \sum_i \omega_i D_{3i} = 1$$

It can easily be shown that $\sum_i \rho_i V_{jit} = 0$, where j indicates that the disturbance terms being referred to belong to the j^{th} auxiliary equation (j = 1, 2, 3). From (B.7) and using the least squares definition for C_i, we can write

$$V_{ji} = X_i - XC_i = X_i - X(X'X)^{-1}X'X_i$$

That is,

$$\sum_i \rho_i V_{ji} = \sum_i \rho_i X_i - X(X'X)^{-1}X'\sum_i \rho_i X_i = \sum_i \rho_i X_i - I\sum_i \rho_i X_i = 0$$

That is,

$$\sum_i \lambda_i V_{ijt} = \sum_i \delta_i V_{2it} = \sum_i \omega_i V_{3it} = 0$$

Table A.1

Micro Acreage Response Equations for Wheat, Canadian Prairies, 1953-54 - 1968-69, Regression Coefficients and Standard Errors, Static Models[a]

Crop district	Constant term	P_{W-2}	S_W	T	M	R^2	D.W.
MANITOBA							
1	240.22 (23.21)	6.079 (51.241)	-0.004954 (0.00537)	10.292** (2.255)	-20.295 (38.721)	.8340	0.95*
2	223.51 (22.48)	9.849 (46.434)	-0.012793 (0.00495)	17.534** (2.087)	-28.915° (19.484)	.9298	1.85
3	405.17 (42.76)	15.343 (86.528)	-0.000658 (0.00589)	12.138** (4.201)	1.280 (56.620)	.7162	2.06
4	31.46 (7.77)	10.676 (15.903)	-0.001863 (0.00954)	1.732* (0.716)	-0.384 (4.161)	.6490	1.99
5	213.03 (25.81)	14.270 (51.836)	-0.005885 (0.00758)	6.776** (2.350)	-31.027 27.590	.6777	2.71
6	2.93 (4.36)	10.162 (8.768)	-0.000000 (0.01333)	0.488 (0.421)	-0.726 (5.492)	.4845	2.23
7	95.91 (12.67)	44.428* (25.402)	-0.003103 (0.00423)	7.053** (1.154)	-7.425 (12.700)	.9162	1.30*
8	219.68 (11.61)	-33.042° (23.295)	-0.005895° (0.00381)	9.549** (1.104)	-12.818° (9.292)	.9267	1.83
9	135.77 (11.10)	8.872 (22.470)	-0.005908° (0.00415)	7.275** (1.020)	+15.092° (8.911)	.9129	2.10

Table A.1 (continued)

Crop district	Constant term	P_{w-2}	S_w	T	M	R^2	D.W.
10	13.27 (22.64)	126.409* (46.353)	-0.010504* (0.00532)	15.320** (2.104)	-22.342 (27.498)	.9410	2.05
11	72.79 (21.04)	42.157 (42.902)	-0.011690° (0.00716)	11.765** (1.918)	-7.773 (24.403)	.8969	1.87
12	41.58 (14.22)	23.829 (42.043)	-0.000700 (0.01950)	-0.026 (1.589)	-22.605 (24.274)	.3132	2.80
13	-40.77 (10.96)	41.624* (22.197)	0.017181* (0.00964)	3.357** (0.999)	-0.132 (9.852)	.8894	2.19
14	-0.058 (6.49)	23.455* (13.656)	-0.003560 (0.00720)	3.880** (0.669)	0.120 (11.025)	.9301	1.79
SASKATCHEWAN							
1A	-229.44 (38.82)	428.446** (84.480)	-0.008586* (0.00390)	27.601** (2.770)	9.709 (56.035)	.9640	2.05
1B	-133.85 (33.83)	320.459** (74.387)	-0.007464° (0.00500)	22.615** (2.626)	-131.068* (57.217)	.9625	2.15
2A	108.77 (44.11)	273.920** (96.768)	-0.004605 (0.00418)	15.788** (3.405)	48.120 (70.938)	.8777	1.88
2B	-136.80 (57.51)	713.308** (125.466)	-0.003031 (0.00369)	5.810 (4.792)	-44.324 (73.644)	.8805	2.54
3AS	255.98 (45.52)	409.490** (99.850)	-0.002851 (0.00283)	10.170* (3.513)	128.265* (73.196)	.8639	1.97

Table A.1 (continued)

Crop district	Constant term	P_{w-2}	S_w	T	M	R^2	D.W.
4B	572.54 (54.07)	73.266 (118.664)	-0.010485* (0.00409)	29.342** (3.968)	-79.434 (109.231)	.9113	2.67
5	-653.18 (83.30)	478.794 (181.706)	0.013187 (0.01729)	-21.933** (6.090)	299.493* (116.981)	.6964	1.88
6	25.19 (33.78)	150.851* (74.071)	-0.007967° (0.00566)	5.198° (2.522)	61.460* (29.875)	.7071	2.43
7	129.07 (74.61)	-8.725 (189.597)	0.013089 (0.01149)	14.087* (6.199)	81.903 (101.021)	.5449	1.67*

[a]The symbols P_{w-2}, S_w, T and M represent wheat price lagged two periods, wheat stocks, trend, and rainfall, respectively. Figures in parentheses under the coefficients represent the standard errors of these coefficients, while under the constant terms, they represent standard errors of estimates.

A two-asterisk implies significance at the one percent level, a one-asterisk implies significance at the five percent level, while ° means significance at the 10 percent level of confidence. No other level of significance is indicated. A two-asterisk by the Durbin-Watson (D.W.) statistic indicates the presence of serial correlation, a one-asterisk indicates an inconclusive test, while no-asterisk implies absence of serial correlation.

Table A.1 (continued)

Crop district	Constant term	P_{w-2}	S_w	T	M	R^2	D.W.
3AN	207.67 (27.28)	187.393** (59.359)	-0.006865* (0.00297)	7.309** (1.958)	93.075* (36.365)	.8593	2.11
3BS	599.95 (33.92)	56.166 (73.836)	0.004000° (0.00286)	8.208** (2.443)	-15.470 (29.263)	.7726	2.84
3BN	267.29 (74.85)	477.487** (162.903)	0.000159 (0.00435)	0.047 (5.372)	41.304 (99.799)	.6241	2.13
4A	359.48 (15.92)	-16.601 (34.974)	0.002783 (0.00296)	2.559* (1.147)	22.010* (12.599)	.6460	1.79
4B	3.13 (48.05)	310.856** (104.706)	0.001500 (0.00526)	-5.271° (3.549)	195.747* (86.117)	.6762	2.46
5A	-409.09 (67.41)	705.441** (146.870)	-0.005768 (0.00482)	27.084** (4.853)	-39.473 (77.644)	.93.6	2.42
5B	-663.38 (86.25)	798.273** (211.177)	-0.008074 (0.00648)	32.883** (6.164)	12.213 (133.387)	.9160	2.42
6A	-642.39 (103.01)	1260.750** (226.069)	-0.003755 (0.00455)	6.093 (7.452)	-34.025 (100.102)	.8568	2.32
6B	-121.46 (81.37)	743.553** (212.744)	-0.000535 (0.00465)	1.490 (6.519)	4.020 (114.995)	.7703	2.04
7A	-407.10 (125.03)	818.740** (286.102)	-0.008539 (0.00793)	3.067 (8.923)	242.592° (172.141)	.6860	1.96

Table A.1 (continued)

Crop district	Constant term	P_{W-2}	S_W	T	M	R^2	D.W.
7B	-64.44 (51.91)	454.945** (112.988)	-0.000102 (0.00449)	3.300 (4.243)	75.494 (64.191)	.8179	1.97
8A	-360.82 (61.01)	444.354** (140.213)	-0.006536 (0.00928)	8.132° (4.876)	29.725 (89.892)	.7560	2.46
8B	-151.74 (44.86)	521.505** (111.207)	-0.005542° (0.00395)	3.913 (4.671)	78.578 (67.750)	.8207	2.78
9A	-25.26 (67.64)	584.503** (147.394)	-0.005240 (0.00540)	-1.187 (6.611)	-104.764 (113.683)	.7421	2.07
9B	-18.91 (34.19)	330.699** (75.243)	-0.007770* (0.00451)	0.882 (2.672)	15.717 (38.608)	.7384	2.35
ALBERTA							
1	629.13 (46.96)	64.356 (106.353)	-0.002255 (0.00297)	22.294** (3.834)	76.315° (50.365)	.8764	2.42
2	1155.31 (152.85)	-32.874 (337.706)	-0.001223 (0.00622)	52.481** (11.147)	45.774 (167.142)	.7845	1.84
3	-1883.76 (282.29)	1668.149* (623.689)	-0.007074 (0.02518)	-81.663** (26.887)	764.157° (550.402)	.7464	1.86
4A	684.85 (25.15)	13.065 (54.975)	-0.003195° (0.00195)	12.655** (1.851)	-49.832 (42.076)	.8984	2.08

Table A.2

Micro Acreage Response Equations for Wheat, Canadian Prairies, 1953-54 - 1968-69, Regression Coefficients and Standard Errors, Dynamic Models[a]

Crop district	Constant term	P_{w-2}	S_w	T	A_{t-1}	R^2	D.W.
MANITOBA							
1	173.02 (18.24)	-45.930 (39.561)	-0.004528 (0.00416)	6.505* (2.277)	0.5775* (0.2143)	.8975	1.62*
2	170.68 (18.68)	-68.309° (50.069)	-0.006964° (0.00468)	6.729 (3.962)	0.7574 (0.2657)	.9515	2.03
3	406.91 (42.76)	15.081 (88.628)	-0.000705 (0.00552)	12.175* (4.760)	-0.0001 (0.2867)	.7162	2.06
4	30.10 (7.76)	9.508 (16.508)	-0.001045 (0.00900)	1.629° (0.918)	0.0518 (0.3165)	.6495	2.03
5	193.91 (26.23)	32.273 (54.826)	-0.003103 (0.00735)	7.874* (2.668)	-0.2630 (0.2818)	.6670	2.35
6	3.17 (4.34)	10.523 (8.832)	0.000000 (0.01111)	0.529 (0.439)	-0.0913 (0.2948)	.4881	2.15
7	51.97 (11.22)	11.044 (28.861)	-0.000013 (0.00410)	3.077 (2.383)	0.5649 (0.3042)	.9342	1.28*
8	153.40 (11.29)	-39.449° (22.818)	-0.006316° (0.00368)	5.615* (2.386)	0.4689° (0.2892)	.9306	2.28

Table A.2 (continued)

Crop district	Constant term	P_{w-2}	S_w	T	A_{t-1}	R^2	D.W.
9	122.67 (12.43)	1.591 (26.274)	-0.005262 (0.00461)	6.577* (2.186)	0.0772 (0.3141)	.8907	2.01
10	24.41 (21.94)	60.944 (75.666)	-0.008065° (0.00523)	10.457* (4.296)	0.3811 (0.3208)	.9446	2.09
11	72.04 (20.20)	7.159 (54.743)	-0.006838 (0.00797)	8.116° (4.025)	0.3575** (0.1147)	.9049	2.07
12	-38.71 (13.88)	67.352* (30.396)	0.007800 (0.01700)	-1.253 (1.300)	-0.3314 (0.2746)	.3456	2.08
13	24.73 (10.84)	26.874 (36.410)	0.015681° (0.01000)	2.947* (1.278)	0.1944 (0.3961)	.8919	2.52
14	21.17 (6.15)	-0.372 (24.599)	-0.004000 (0.00600)	2.894* (1.045)	0.4287 (0.3824)	.9373	2.34
SASKATCHEWAN							
1A	-121.76 (35.43)	281.996* (124.606)	-0.006927* (0.00365)	21.249** (4.931)	0.2965° (0.1981)	.9700	2.09
1B	-160.74 (40.69)	253.207* (125.889)	-0.008357° (0.00607)	22.388** (6.175)	0.1214 (0.2535)	.9457	1.66*
2A	124.62 (44.93)	266.138* (121.946)	-0.003581 (0.00418)	14.516** (3.659)	0.0454 (0.2068)	.8731	2.10
2B	-160.80 (58.32)	683.621** (169.643)	-0.003554 (0.00353)	7.227° (4.173)	0.0393 (0.1819)	.8771	2.56

Table A.2 (continued)

Crop district	Constant term	P_{w-2}	S_w	T	A_{t-1}	R^2	D.W.
3AS	155.06 (41.34)	203.196° (129.187)	-0.001239 (0.00238)	7.119* (2.961)	0.5052* (0.2052)	.8877	2.28
3AN	211.94 (32.81)	119.366 (95.763)	-0.004000 (0.00378)	6.529* (2.663)	0.2781 (0.2624)	.7963	2.21
3BS	812.15 (31.04)	103.273° (74.190)	0.004408° (0.00265)	10.748** (2.812)	-0.4203° (0.2679)	.8095	2.23
3BN	181.84 (70.77)	310.884° (205.495)	-0.001667 (0.00449)	2.623 (5.392)	0.3708 (0.3031)	.6640	2.46
4A	341.85 (17.98)	-8.955 (39.796)	0.003739 (0.00304)	2.800* (1.375)	0.0315 (0.2945)	.5483	1.88
4B	31.24 (55.90)	221.791° (159.924)	-0.001079 (0.00763)	0.647 (4.757)	0.3611 (0.3715)	.5618	2.66
5A	-404.56 (63.68)	631.812* (240.848)	-0.005143 (0.00482)	25.475** (6.811)	0.1010 (0.2492)	.9341	2.38
5B	-660.05 (86.26)	809.264* (321.293)	-0.008093 (0.00648)	33.365** (8.480)	-0.0209 (0.2747)	.9160	2.44
6A	-589.87 (102.32)	1124.423** (331.648)	-0.003811 (0.00455)	6.897 (7.333)	0.1040 (0.2022)	.8586	2.34
6B	-103.95 (76.84)	555.749* (235.668)	-0.001225 (0.00436)	2.238 (5.528)	0.2700 (0.2336)	.7952	2.25

Table A.2 (continued)

Crop district	Constant term	P_{w-2}	S_w	T	A_{t-1}	R^2	D.W.
7A	-242.83 (123.18)	543.625^{o} (372.929)	-0.010715^{o} (0.00809)	5.008 (8.832)	0.4475 (0.2902)	.6951	2.71
7B	-52.95 (49.82)	323.324* (138.037)	-0.004082 (0.00428)	3.796 (3.764)	0.3993^{o} (0.2555)	.8323	2.49
8A	-379.12 (60.71)	507.858* (214.536)	-0.004357 (0.00892)	7.860* (4.440)	-0.1705 (0.3675)	.7583	2.35
8B	-137.96 (50.38)	529.513** (148.209)	-0.005375 (0.00437)	0.531 (3.692)	0.0874 (0.2345)	.8012	2.73
9A	-94.55 (66.07)	409.150* (209.827)	-0.006960^{o} (0.00540)	3.241 (4.716)	0.3335 (0.2798)	.7540	2.30
9B	-4.08 (33.88)	282.788* (113.882)	-0.007806* (0.00451)	0.919 (2.544)	0.1532 (0.2535)	.7430	2.56
ALBERTA							
1	806.00 (47.51)	189.983^{o} (118.942)	-0.001127 (0.00284)	26.286** (5.853)	-0.4054^{o} (0.2877)	.8735	2.26
2	1137.29 (153.36)	-18.359 (335.172)	-0.000536 (0.00573)	52.638* (21.866)	0.0035 (0.3391)	.7830	1.83
3	-894.83 (242.91)	889.093^{o} (641.066)	-0.016933 (0.02133)	-28.509 (35.173)	0.6649* (0.2616)	.8122	2.58
4A	787.51 (26.05)	25.184 (60.855)	-0.004070* (0.00234)	15.941** (4.320)	-0.2388 (0.3199)	.8910	1.82

Table A.2 (continued)

Crop district	Constant term	P_{w-2}	S_w	T	A_{t-1}	R^2	D.W.
4B	610.89 (54.06)	134.480 (152.654)	-0.014598* (0.00636)	36.251** (10.835)	-0.2854 (0.3912)	.9112	2.15
5	-156.54 (92.73)	203.652 (255.698)	-0.009771 (0.02208)	-6.073 (12.121)	0.5904* (0.3319)	.6239	2.53
6	36.50 (39.40)	219.807° (147.950)	-0.007383 (0.00700)	3.835° (2.866)	-0.1842 (0.4145)	.6016	2.65
7	90.23 (76.19)	128.112 (210.917)	0.009612 (0.01134)	11.855° (5.538)	-0.2092 (0.4696)	.5263	1.66*

[a] A_{t-1} is wheat acreage lagged one period. All other variables and symbols are as previously described for Table A.1.

Table A.3

Micro Acreage Response Equations for Wheat, Canadian Prairies, 1953-54 - 1968-69, Regression Coefficients and Standard Errors, Equation (4.0.0)[a]

Crop district	Constant term	P_{w-1}	P_{w-2}	P'_t	P_{f-1}	S_w	T	M	Smf_{t-1}	A_{t-1}
MANITOBA										
1	214.08 (22.46)	12.439 (58.585)				-0.004843 (0.00583)	10.113** (2.817)			
				R^2 = .8343	D.W. = 1.84					
2	223.51 (22.48)		9.848 (46.434)			-0.012793* (0.004955)	17.534** (2.086)	28.915° (19.484)		
				R^2 = .9298	D.W. = 1.85					
3	405.17 (42.76)		15.343 (86.528)			-0.000658 (0.00589)	12.138* (4.201)	1.280 (56.620)		
				R^2 = .7162	D.W. = 2.06					
4	-31.76 (6.14)		22.879° (14.034)				1.540* (0.536)		0.2682* (0.1042)	0.2409 (0.2458)
				R^2 = .7810	D.W. = 2.41					
5	-17.05 (22.48)		62.408 (49.157)			-0.000448 (0.00643)	7.366** (2.063)		0.3706* (0.1630)	
				R^2 = .7554	D.W. = 2.43					

Table A.3 (continued)

Crop district	Constant term	P_{w-1}	P_{w-2}	P'_t	P_{f-1}	S_w	T	M	Smf_{t-1}	A_{t-1}
6	-6.31 (4.18)		8.964 (8.650)				0.3059 (0.324)			0.1870 (0.4083)
				R^2 = .5264	D.W. = 2.26					
7	90.09 (12.32)		44.610* (24.694)			-0.003833 (0.00384)	7.076** (1.121)			
				R^2 = .9136	D.W. = 1.77					
8	16.13 (14.10)			1.802 (30.312)		-0.003921 (0.00447)				0.9804** (0.1648)
				R^2 = .8819	D.W. = 1.98					
9	135.77 (11.10)		8.872 (22.470)			-0.005908° (0.00415)	7.275** (1.020)	15.092° (8.911)		
				R^2 = .9129	D.W. = 2.10					
10	-72.81 (22.45)		116.583* (48.837)			-0.008131° (0.00542)	13.312** (2.724)		0.2290 (0.2487)	
				R^2 = .9420	D.W. = 1.86					
11	72.04 (20.20)		7.159 (54.743)			-0.006838 (0.00797)	8.116° (4.025)			0.3575** (0.1147)
				R^2 = .9049	D.W. = 2.07					

Table A.3 (continued)

Crop district	Constant term	P_{w-1}	P_{w-2}	P'_t	P_{f-1}	S_w	T	M	Smf_{t-1}	A_{t-1}
12	-14.13 (12.76)	17.684 (42.094)					-0.286 (1.257)		0.2053 (0.2617)	0.3371 (0.6885)
				R^2 = .3564	D.W. = 1.96					
13	-94.40 (10.53)			41.784° (29.415)			2.083° (1.311)		0.3958° (0.2385)	0.4143* (0.2290)
				R^2 = .8978	D.W. = 1.94					
14	-31.25 (5.66)		8.223 (23.079)				1.581° (1.160)		0.3240° (0.2050)	0.6101° (0.3720)
				R^2 = .9468	D.W. = 2.38					
SASKATCHEWAN										
1A	-121.77 (35.43)		281.996* (124.611)			-0.006927* (0.00365)	21.249** (4.931)			0.2965° (0.1981)
				R^2 = .9700	D.W. = 2.09					
1B	-133.85 (33.83)		320.459** (74.387)			-0.007464° (0.00500)	22.615** (2.626)	-131.068* (52.170)		
				R^2 = .9625	D.W. = 2.15					
2A	126.98 (43.11)		282.137** (93.823)			-0.003744° (0.00220)	14.905** (3.074)			
				R^2 = .8726	D.W. = 2.03					

Table A.3 (continued)

Crop district	Constant term	P_{w-1}	P_{w-2}	P'_t	P_{f-1}	S_w	T	M	Smf_{t-1}	A_{t-1}
2B	-161.88 (55.95)		707.954** (121.780)			-0.003538 (0.00338)	7.302* (3.990)			
				R^2 = .8766	D.W. = 2.52					
3AS	155.06 (41.34)		203.196° (129.187)			-0.001239 (0.00238)	7.119* (2.961)			0.5052* (0.2052)
				R^2 = .8877	D.W. = 2.28					
3AN	194.07 (25.13)	181.573* (73.080)				-0.005973* (0.00324)	6.664* (2.408)	103.200* (40.400)		
				R^2 = .8682	D.W. = 1.98					
3BS	482.55 (30.95)	146.154* (65.890)					8.114* (2.028)			
				R^2 = .7762	D.W. = 3.28					
3BN	-15.42 (62.96)	1.302* (.879)					-0.704 (4.976)			0.5132** (0.1791)
				R^2 = .7099	D.W. = 2.50					
4A	309.52 (18.29)	6.219 (41.365)					2.922* (1.602)			0.1034 (0.2856)
				R^2 = .4896	D.W. = 1.96					

Table A.3 (continued)

Crop district	Constant term	P_{w-1}	P_{w-2}	P'_t	P_{f-1}	S_w	T	M	Smf_{t-1}	A_{t-1}
4B	-160.93 (45.52)	296.900** (106.766)					-3.941 (3.602)			0.4877** (0.1722)
				R^2 = .6830	D.W. = 2.57					
5A	-394.55 (64.81)		455.481** (147.880)				12.911* (6.194)			0.4664** (0.1430)
				R^2 = .9320	D.W. = 1.81					
5B	-641.15 (82.61)		789.414** (179.787)			-0.008019° (0.00611)	32.920** (5.891)			
				R^2 = .9160	D.W. = 2.44					
6A	-427.56 (114.49)		686.830* (280.823)				3.589 (9.089)			0.4664** (0.1583)
				R^2 = .8070	D.W. = 2.45					
6B	-234.12 (70.56)	479.607** (173.199)					-1.452 (5.593)			0.5015** (0.1535)
				R^2 = .8116	D.W. = 2.21					
7A	-710.49 (98.92)	901.651** (269.825)				-0.009857° (0.00634)				0.2653° (0.2020)
				R^2 = .7980	D.W. = 2.44					

Table A.3 (continued)

Crop district	Constant term	P_{w-1}	P_{w-2}	P'_t	P_{f-1}	S_w	T	M	Smf_{t-1}	A_{t-1}
7B	-195.76 (41.38)	578.038* (100.294)				-0.004531* (0.00244)	7.109* (2.758)			
				R^2 = .8738	D.W. = 2.14					
8A	-360.82 (61.01)		444.354** (140.213)			0.006536 (0.00928)	8.132° (4.875)	29.750 (89.890)		
				R^2 = .7560	D.W. = 2.46					
8B	-269.60 (47.42)		652.133** (107.654)			-0.007500* (0.00395)	2.624* (2.960)			
				R^2 = .8116	D.W. = 2.74					
9A	-267.10 (65.60)		669.765** (158.999)			-0.008500° (0.00540)	5.609 (4.373)			
				R^2 = .7354	D.W. = 1.80					
9B	-80.20 (31.87)	380.049** (77.249)				-0.010355** (0.00322)	1.925 (2.124)			
				R^2 = .7520	D.W. = 2.16					
ALBERTA										
1	629.13 (46.95)		64.356 (106.353)			-0.002255 (0.00297)	22.294** (3.834)	76.315° (50.365)		
				R^2 = .8764	D.W. = 2.42					

Table A.3 (continued)

Crop district	Constant term	P_{w-1}	P_{w-2}	P'_t	P_{f-1}	S_w	T	M	Smf_{t-1}	A_{t-1}
2	881.58 (145.60)	150.807 (330.232)				-0.000457 (0.00501)	48.583** (11.552)			
				R^2 = .7868	D.W. = 1.79					
3	-663.83 (202.53)		1047.152* (431.746)		-348.735° (238.023)	-0.048370* (0.01911)		861.090* (367.050)		0.6521** (0.1341)
				R^2 = .8813	D.W. = 2.68					
4A	635.41 (25.47)	23.095 (66.158)				-0.003461° (0.00218)	12.848** (1.772)			
				R^2 = .8863	D.W. = 2.11					
4B	484.70 (52.50)		99.326 (135.242)			-0.010833** (0.00371)	28.056** (4.163)			
				R^2 = .9087	D.W. = 2.42					
5	-112.12 (90.31)		162.805 (183.842)		-53.900 (108.706)	-0.011792 (0.02354)		154.900° (105.270)		0.6371** (0.1971)
				R^2 = .7015	D.W. = 2.28					
6	25.19 (33.78)		150.851* (74.071)			-0.007967° (0.00566)	5.198° (2.522)	61.460* (29.875)		
				R^2 = .7071	D.W. = 2.43					

Table A.3 (continued)

Crop district	Constant term	P_{w-1}	P_{w-2}	P'_{t-1}	P_{f-1}	S_w	T	M	Smf_{t-1}	A_{t-1}
7	76.48 (80.25)		130.473 (146.676)				10.423[c] (5.154)			

R^2 = .4816 D.W. = 2.00

[a]P_{w-1}, P'_t, P_{f-1}, and Smf_{t-1} are C.W.B. total payments for wheat lagged one period; C.W.B. International Wheat Agreement Prices (Ft. William) and Pt. Arthur) average for February, March, and April, lagged one period; estimated farm price for flax lagged one period; and summerfallow acreage lagged one period, respectively. All other variables and symbols are as previously described for Table A.1 footnote a.

TABLE A.4

Micro Acreage Response Equations for Barley, Canadian Prairies, 1950-51 - 1968-69, Regression Coefficients and Standard Errors[a]

Crop district	Constant term	P_{b-1}	$\frac{P_{b-1}}{P_{o-1}}$	$\frac{P_{b-1}}{P_{r-1}}$	S_W	$\frac{P_{b-1}}{P_{f-1}}$	P_{f-1}	T	B_{t-1}	R^2	D.W.
MANITOBA											
1	-41.43 (17.17)	62.223* (29.666)			0.001963 (0.00416)			-0.986 (1.572)	0.7689** (0.1773)	.7781	2.46
2	-148.01 (26.77)	162.917** (44.447)			0.002811 (0.00657)			-0.805 (3.047)	0.8727** (0.1343)	.9269	2.63
3	-135.43 (34.44)	208.015** (55.370)			0.006726° (0.00505)			-5.217 (4.736)	0.7078** (0.1612)	.9386	2.22
4	-35.27 (10.82)	56.590** (18.409)			0.014091 (0.01454)			-1.021 (1.309)	0.6556** (0.1915)	.8811	2.25
5	-66.85 (26.79)	144.438** (42.709)			0.009805 (0.00793)			-5.167° (3.152)	0.6627** (0.1471)	.9346	1.88
6	0.23 (2.57)	5.109 (4.229)			0.006444 (0.00597)			-0.432* (0.238)	0.5878** (0.2075)	.8405	1.79
7	-41.63 (14.95)	53.143* (23.903)			0.001782 (0.00525)			-0.323 (1.615)	0.8309** (0.1766)	.8573	2.29
8	-78.96 (12.95)	98.888** (20.714)			0.002408 (0.00447)			-0.889° (0.507)	0.7739** (0.1216)	.9500	1.95

Table A.4 (Continued)

Crop district	Constant term	P_{b-1}	$\frac{P_{b-1}}{P_{o-1}}$	$\frac{P_{b-1}}{P_{r-1}}$	S_w	$\frac{P_{b-1}}{P_{f-1}}$	P_{f-1}	T	B_{t-1}	R^2	D.W.
MANITOBA											
9	-28.62 (15.75)	79.595** (25.912)			0.000378 (0.00040)			-2.993° (1.756)	0.5925** (0.1592)	.9181	2.36
10	36.86 (26.36)				0.000221* (0.000123)	139.857 (137.722)		-3.345* (1.662)	0.6633** (0.1705)	.8062	2.35
11	13.59 (18.84)	41.026° (25.093)			0.007892° (0.00567)			-3.478° (2.113)	0.6189** (0.1976)	.9084	1.95
12	8.51 (12.13)	37.494* (19.327)			0.023600° (0.01500)			-2.633* (1.069)	0.1638 (0.2273)	.6734	2.13
13	51.56 (21.53)	16.659 (36.695)			0.032143* (0.01964)			-3.889* (1.921)	0.4411* (0.2129)	.6786	2.71
14	2.97 (7.64)	23.296* (12.171)			0.016600* (0.00920)			-2.046* (1.045)	0.4748* (0.2174)	.8715	2.27
SASKATCHEWAN											
Total	6.486 (473.32)				0.004339* (0.00168)	1694.184 (1536.256)		-63.583* (26.006)	0.6228** (0.1750)	.6681	2.40
ALBERTA											
1	-45.59 (19.133)		57.741° (42.534)		0.000228° (0.00017)		-10.816° (7.202)		0.7702** (0.2117)	.6237	1.78

Table A.4 (Continued)

Crop district	Constant term	P_{b-1}	$\frac{P_{b-1}}{P_{o-1}}$	$\frac{P_{b-1}}{P_{r-1}}$	S_w	$\frac{P_{b-1}}{P_{f-1}}$	P_{f-1}	T	B_{t-1}	R^2	D.W.
ALBERTA											
2	-142.69 (50.02)				0.001294** (0.00043)	263.392 (305.054)		0.197 (1.067)	0.9369** (0.0012)	.8983	1.71
3	-35.67 (98.30)	424.318* (180.749)			0.010326° (0.00718)			-2.990 (5.192)	0.2056 (0.2861)	.5892	2.29
4A	6.12 (18.47)	10.541 (37.087)			0.004938** (0.00148)			0.805 (0.901)	0.5053** (0.1582)	.8132	1.78
4B	391.04 (62.88)	284.504* (124.596)			0.001612** (0.00054)			10.548** (2.848)		.6921	1.72
5	308.67 (58.95)	290.533* (105.765)			0.020667° (0.01250)			9.123** (3.262)	0.1637 (0.1881)	.7475	1.86
6	268.88 (49.52)			35.770° (20.058)	0.001938** (0.00048)			-6.648* (2.481)	0.2556° (0.1534)	.7405	2.31
7	-34.62 (48.30)	54.146 (121.329)			0.017612* (0.00850)			14.549* (7.173)	0.4750* (0.2704)	.9430	2.02

[a] P_{t-1}, P_{f-1}, P_{r-1} and P_{o-1} are the prices of barley, flax, rapeseed and oats in t-1. B_{t-1} is barley acreage lagged in t-1. All other variables and symbols are as previously defined for Table A.1.

Table A.5

Acreage Response Equations for Wheat, 1953-54 - 1968-69,
Regression Coefficients and Standard Errors, Static and Dynamic Models[a]

Constant term	P_{w-2}	S_w	M	T	A_{t-1}
MANITOBA (Static model)					
1667.58 (131.27)	368.628° (263.430)	-0.005592* (0.00296)	-227.259 (176.958)	108.100** (12.173)	
		R^2 = .9510	D.W. = 1.71		
SASKATCHEWAN (Static model)					
-3236.76 (849.99)	9844.538** (1854.128)	-0.004281 (0.00344)	2571.157 (1775.531)	212.206** (62.668)	
		R^2 = .9040	D.W. = 2.57		
ALBERTA (Static model)					
841.14 (238.69)	2032.753** (542.587)	-0.003829° (0.00256)	2110.360** (493.141)	25.070° (18.256)	
		R^2 = .7240	D.W. = 2.94		
MANITOBA (Dynamic model)					
1159.91 (125.35)	-48.2450 (358.2407)	-0.003590 (0.00317)		66.037* (25.725)	0.4713° (0.2780)
		R^2 = .9553	D.W. = 2.06		

Table A.5 (continued)

Constant term	P_{w-2}	S_w	M	T	A_{t-1}
SASKATCHEWAN (Dynamic model)					
-1225.89 (900.79)	8054.271* (3118.258)	-0.003673 (0.00361)		172.444* (67.436)	0.0195 (0.2406)
		R^2 = .8922	D.W. = 2.80		
ALBERTA (Dynamic model)					
844.57 (345.32)	1087.325 (1191.085)	-0.002038 (0.00369)		19.432 (27.408)	0.5335° (0.3078)
		R^2 = .7240	D.W. = 2.94		

[a]All variables and symbols are as previously described.

Table A.6

Acreage Response Equations for Wheat, 1953-54 - 1968-69, Regression Coefficients and Standard Errors, Equation (4.0.0)[a]

Constant term	P_{w-2}	S_w	T	M	Smf_{t-1}	P_{f-1}
MANITOBA						
1034.82	427.504°	-0.004379*	103.426**		0.1025	
(126.00)	(290.194)	(0.00250)	(13.049)		(0.1934)	
		R^2 = .9550	D.W. = 1.98			
SASKATCHEWAN						
-3236.76	9844.538**	-0.004281	212.206**	2571.206°		
(849.99)	(1854.128)	(0.00344)	(62.668)	(1775.531)		
		R^2 = .9040	D.W. = 2.57			
ALBERTA						
1042.53	2132.560**	-0.004350*	26.370°	2135.600**		127.276
(247.96)	(607.843)	0.00216)	(19.195)	(515.500)		(290.090)
		R^2 = .8706	D.W. = 1.99			

[a]P_{f-1} and Smf_{t-1} represent estimated farm price received for flax lagged one period, and summerfallow acreage lagged one period, respectively. All other variables and symbols are as previously described.

Table A.7

Acreage Response Equations for Barley, 1950-51 - 1968-69, Regression Coefficients and Standard Errors[a]

Constant term	P_{b-1}	S_w	T	B_{t-1}	$\frac{P_{b-1}}{P_{f-1}}$
Manitoba					
-740.98 (164.18)	1054.781** (262.778)	0.004374 (0.00445)	-18.528 (20.005)	0.7933** (0.1267)	
	R^2 = .9484		D.W. = 2.45		
Saskatchewan					
6.486 (473.32)		0.004339* (0.00168)	-63.583* (26.006)	0.6228* (0.1750)	1694.184 (1536.256)
	R^2 = .6681		D.W. = 2.40		
Alberta					
909.68 (226.62)	377.015 (458.108)	0.008493** (0.00275)	20.302* (10.413)	0.3855* (0.1755)	
	R^2 = .8383		D.W. = 1.98		

[a] B_{t-1} is barley acreage lagged one period. All other variables and symbols are as previously described.

Table A.8 (a)

Coefficients of the Auxiliary Equations for the Static Wheat Model, when the Dependent Variable is Wheat Stocks[a]

Crop district	Γ_{1i}	B_{1i}	C_{1i}	D_{1i}	E_{1i}
MANITOBA					
1	4983.5942	-2494.6281	.008675	-24.1208	204.4656
2	5122.0274	-2563.9233	.008916	-24.7909	210.1452
3	8767.4343	-4388.6975	.015261	-42.4348	359.7080
4	1015.1766	- 508.1650	.001767	- 4.9135	41.6504
5	4014.5620	-2009.5615	.006988	-19.4307	164.7084
6	415.2995	- 207.8857	.000723	- 2.0101	17.0388
7	3599.2625	-1801.6758	.006265	-17.4206	147.6696
8	3506.9737	-1755.4790	.006105	-16.9739	143.8832
9	2999.3854	-1501.3965	.005221	-14.5172	123.0580
10	4937.4498	-2471.5297	.008595	-23.8975	202.5724
11	3414.6849	-1709.2822	.005944	-16.5272	140.0968
12	922.8878	- 461.9682	.001606	- 4.4668	37.8640
13	1292.0429	- 646.7554	.002249	- 6.2535	53.0096
14	1153.6098	- 577.4602	.002008	- 5.5835	47.3300
SASKATCHEWAN					
1A	- 703.5721	624.5710	.027808	- 6.5793	15.9232
1B	- 480.4883	426.5363	.018991	- 4.4932	10.8744
2A	- 737.8927	655.0379	.029165	- 6.9002	16.6999
2B	-1115.4192	990.1735	.044086	-10.4306	25.2441
3AS	-1149.7398	1020.6404	.045442	-10.7515	26.0208
3AN	- 634.9309	563.6372	.025095	- 5.9374	14.3697
3BS	- 840.8545	746.4385	.033234	- 7.8630	19.0301
3BN	-1184.0604	1051.1073	.046799	-11.0724	26.7975
4A	- 394.6868	350.3691	.015600	- 3.6908	8.9325
4B	- 652.0912	578.8707	.025773	- 6.0979	14.7581
5A	- 960.9765	853.0726	.037982	- 8.9863	21.7487
5B	- 926.6559	822.6057	.036625	- 8.6654	20.9720
6A	-1544.4266	1371.0095	.061042	-14.4423	34.9533
6B	-1218.3809	1081.5742	.048155	-11.3934	27.5743
7A	-1081.0986	959.7067	.042729	-10.1096	24.4673
7B	- 840.8545	746.4385	.033234	- 7.8630	19.0301
8A	- 480.4883	426.5363	.018991	- 4.4932	10.8744
8B	- 823.6942	731.2051	.032558	- 7.7026	18.6418
9A	- 858.0148	761.6720	.033912	- 8.0235	19.4185
9B	- 531.9691	472.2366	.021026	- 4.9746	12.0395

Table A.8 (a) (continued)

Crop district	Γ_{1i}	B_{1i}	C_{1i}	D_{1i}	E_{1i}
ALBERTA					
1	-4782.1679	1297.5592	.039836	63.3295	-376.4553
2	-7680.4515	2083.9587	.063979	101.7110	-604.6100
3	-3912.6829	1061.6393	.032593	51.8150	-308.0088
4A	-3709.8030	1006.5914	.030903	49.1283	-292.0380
4B	-3825.7544	1038.0473	.031869	50.6636	-301.1642
5	-1391.1761	377.4718	.011589	18.4231	-109.5143
6	-1738.9702	471.8397	.014486	23.0289	-136.8928
7	-1941.8500	526.8877	.016176	25.7156	-152.8636
$\sum_{i}^{42}$	1.2600	-0.9740	.999999	0.0048	0.0232

[a]These coefficients were obtained by regressing the wheat stocks in each crop district on the macrovariables P_{w-2}, S_w, M, and T (these variables are as previously described). The constant terms in these regressions are the Γ_{1i}, while B_{1i}, C_{1i}, D_{1i}, and E_{1i} are the coefficients of P_{w-2}, S_w, M, and T respectively.

As mentioned in chapter III, the constant terms Γ_{1i} and the coefficients B_{1i}, D_{1i}, and E_{1i} should each sum to zero over all i (i.e. the 42 crop districts). Similarly, the coefficients C_{1i} should sum to 1 over all i. The figures in the last row indicate the deviations from these values as a result of rounding errors.

Table A.8 (b)

Coefficients of the Auxiliary Equations for the Static Wheat Model, When the Dependent Variable is Moisture[a]

Crop district	Γ_{2i}	B_{2i}	C_{2i}	D_{2i}	E_{2i}
MANITOBA					
1	- 46.23	27.85	0.000025	0.95	-0.677
2	121.93	-106.98	0.000072	1.39	3.212
3	39.54	- 36.84	-0.000037	1.47	2.458
4	- 13.67	109.08	-0.000089	0.09	-1.638
5	73.12	- 14.05	-0.000056	1.09	-0.537
6	14.37	6.56	-0.000049	1.15	1.567
7	25.15	- 59.40	0.000071	1.65	2.603
8	- 15.68	- 26.84	0.000030	1.52	5.131
9	- 70.79	- 14.12	0.000015	2.26	2.727
10	108.31	- 39.34	-0.000042	0.95	1.477
11	95.46	- 52.67	-0.000021	1.40	-0.446
12	273.74	-117.76	-0.000069	0.68	2.966
13	7.57	- 12.22	0.000083	1.06	-0.465
14	- 57.42	44.18	-0.000083	1.05	2.134
SASKATCHEWAN					
1A	5.92	- 13.16	0.000011	1.11	0.866
1B	1.62	5.76	0.000013	0.86	-1.049
2A	- 14.82	2.68	0.000038	0.98	-0.954
2B	- 7.84	- 7.06	0.000037	1.21	-2.292
3AS	- 14.82	2.68	0.000038	0.98	-0.954
3AN	3.59	- 10.06	0.000000	1.02	1.591
3BS	- 25.28	- 4.20	0.000036	1.13	2.266
3BN	3.59	- 10.06	0.000000	1.02	1.591
4A	-149.18	19.80	0.000003	2.02	3.273
4B	- 15.44	0.77	-0.000002	0.74	1.807
5A	56.76	- 21.23	-0.000088	1.34	0.389
5B	149.14	- 79.38	0.000004	0.60	0.865
6A	- 55.84	13.84	-0.000018	1.43	0.076
6B	-127.61	90.10	-0.000045	0.81	-1.896
7A	- 96.68	42.71	-0.000007	0.90	1.069
7B	14.21	- 8.08	-0.000095	0.96	4.078
8A	121.65	- 59.07	0.000049	0.91	-1.555
8B	- 38.56	47.14	0.000001	1.00	-3.767
9A	95.80	- 11.16	0.000017	0.34	-3.660
9B	- 13.59	17.06	-0.000057	1.19	-1.736

Table A.8 (b) (continued)

Crop district	Γ_{2i}	B_{2i}	C_{2i}	D_{2i}	E_{2i}
ALBERTA					
1	-65.28	48.83	0.000052	0.92	-3.461
2	-94.97	19.86	0.000081	1.14	0.711
3	11.00	16.69	0.000079	0.43	-3.714
4A	10.52	8.85	-0.000061	0.82	0.129
4B	11.85	12.30	-0.000004	0.45	0.799
5	23.47	3.78	-0.000092	1.01	0.531
6	-43.71	18.90	-0.000041	1.68	-0.827
7	-66.98	85.40	-0.000087	0.84	-1.623
$\sum_i^{42}$	14.64	4.77	0.000002	1.02	-0.0716

[a]See footnote (a) for Table A8 (a). The dependent variable used in each of these auxiliary regressions is rainfall; the independent variables in each case were the macrovariables P_{w-2}, S_w, M, and T for which B_{2i}, C_{2i}, D_{2i}, and E_{2i} are the coefficients respectively, while Γ_{2i} is the constant term in each case. Since the macroparameter M was obtained as a weighted average of the rainfall microvariables (M_i), Γ_{1i}, B_{1i}, C_{1i} and E_{1i} when accordingly weighted should each sum to zero over all i (i.e. the 42 crop districts). The weights are included as Table A.8(b'). Similarly, when accordingly weighted, D_{1i} should sum to 1. The figures in the last row indicate the deviations from these values as a result of rounding errors.

Table A.8 (b')

Weights Used in Constructing the Rainfall Macrovariable M^a

Crop district	Weight
MANITOBA	
1	.010022
2	.010301
3	.017632
4	.002042
5	.008074
6	.000835
7	.007238
8	.007053
9	.006032
10	.009930
11	.006867
12	.001856
13	.002598
14	.002320
SASKATCHEWAN	
1A	.027827
1B	.019004
2A	.029184
2B	.044116
3AS	.045473
3AN	.025112
3BS	.033256
3BN	.046830
4A	.015610
4B	.025791
5A	.038007
5B	.036650
6A	.061083
6B	.048188
7A	.042758
7B	.033256
8A	.019004
8B	.032578
9A	.033935
9B	.021040

Table A.8 (b') (continued)

Crop district	Weight
ALBERTA	
1	.037703
2	.060553
3	.030848
4A	.029248
4B	.030162
5	.010968
6	.013710
7	.015310

[a]Table A8 (b') is to be used in conjunction with Table A.8 (b). See footnote (a) under Table A.8 (b). These weights are explained in chapter IV, p.

Table A.9(a)

Coefficients of the Auxiliary Equations for the Dynamic Wheat Model, when the Dependent Variable is Wheat Stocks[a]

Crop district	Γ_{1i}	B_{1i}	C_{1i}	D_{1i}	E_{1i}
MANITOBA					
1	3541.3642	-1539.8542	.007944	236.6188	-.0836
2	3639.7354	-1582.6279	.008165	243.1915	-.0860
3	6230.1777	-2709.0027	.013976	416.2737	-.1471
4	721.3890	- 313.6740	.001618	48.2001	-.0170
5	2852.7656	-1240.4381	.006400	190.6096	-.0674
6	295.1137	- 128.3212	.000662	19.7182	-.0070
7	2557.6519	-1112.1169	.005738	170.8913	-.0604
8	2492.0711	-1083.6011	.005591	166.5095	-.0589
9	2131.3766	- 926.7641	.004781	142.4094	-.0503
10	3508.5738	-1525.5963	.007871	234.4278	-.0829
11	2426.4903	-1055.0853	.005443	162.1277	-.0573
12	655.8082	- 285.1582	.001471	43.8183	-.0155
13	918.1315	- 399.2215	.002060	61.3456	-.0217
14	819.7602	- 356.4477	.001839	54.7729	-.0194
SASKATCHEWAN					
1A	-1180.4055	1812.9653	.027709	32.1915	-.0884
1B	- 806.1306	1238.1227	.018923	21.9844	-.0604
2A	-1237.9863	1901.4027	.029061	33.7618	-.0927
2B	-1871.3746	2874.2134	.043929	51.0353	-.1401
3AS	-1928.9553	2962.6507	.045281	52.6056	-.1444
3AN	-1065.2440	1636.0907	.025006	29.0508	-.0798
3BS	-1410.7285	2166.7147	.033116	38.4727	-.1056
3BN	-1986.5361	3051.0880	.046632	54.1759	-.1487
4A	- 662.1787	1017.0293	.015544	18.0586	-.0496
4B	-1094.0344	1680.3093	.025682	29.8360	-.0819
5A	-1612.2612	2476.2454	.037846	43.9688	-.1207
5B	-1554.6804	2387.8080	.036495	42.3985	-.1164
6A	-2591.1340	3979.6800	.060825	70.6642	-.1940
6B	-2044.1168	3139.5254	.047984	55.7462	-.1531
7A	-1813.7938	2785.7760	.042577	49.4650	-.1358
7B	-1410.7285	2166.7147	.033116	38.4727	-.1056
8A	- 806.1306	1238.1227	.018923	21.9844	-.0604
8B	-1381.9381	2122.4960	.032440	37.6876	-.1035
9A	-1439.5189	2210.9334	.033792	39.2579	-.1078
9B	- 892.5017	1370.7787	.020951	24.3399	-.0668

Table A.9 (a) (continued)

Crop district	Γ_{1i}	B_{1i}	C_{1i}	D_{1i}	E_{1i}
ALBERTA					
1	- 659.7642	-4943.6894	.041347	-491.0485	.4835
2	-1059.6213	-7939.8648	.066406	-788.6537	.7765
3	- 539.8071	-4044.8368	.033830	-401.7670	.3956
4A	- 511.8171	-3835.1045	.032076	-380.9346	.3751
4B	- 527.8114	-3954.9515	.033078	-392.8388	.3868
5	- 191.9314	-1438.1642	.012028	-142.8505	.1406
6	- 239.9143	-1797.7052	.015035	-178.5631	.1758
7	- 267.9043	-2007.4375	.016790	-199.3955	.1963
$\sum_{i}^{42}$	1.4601	-0.9960	.999981	0.0205	.0001

[a]These coefficients were obtained by regressing the wheat stocks in each crop district on the macrovariables P_{w-2}, S_w, T, and A_{t-1} (these variables are as previously described). The constant terms in these regressions are the Γ_{1i}, while B_{1i}, C_{1i}, D_{1i} and E_{1i} are the regression coefficients of P_{w-2}, S_w, T, and A_{t-1}, respectively.

As mentioned in chapter III, Γ_{1i}, B_{1i}, D_{1i}, and E_{1i} should each sum to zero over all i (i.e. the 42 crop districts). Similarly, C_{1i} should sum to 1 over all i. The figures in the last row indicate the deviations from these values as a result of rounding errors.

Table A.9 (b)

Coefficients of the Auxiliary Equations for the Dynamic Wheat Model, When the Dependent Variable is Lagged Acreage[a]

Crop district	Γ_{2i}	B_{2i}	C_{2i}	D_{2i}	E_{2i}
MANITOBA					
1	102.2701	44.7769	-0.000122	6.7659	0.00431
2	-32.3432	169.0380	-0.000082	11.7817	-0.00111
3	311.9061	-86.9286	-0.000102	8.3089	0.01341
4	20.6111	-10.8113	-0.000008	1.1215	0.00215
5	96.2365	96.6222	-0.000051	4.7663	-0.00182
6	9.7377	1.0372	-0.000002	0.5190	0.00031
7	52.8219	2.3123	-0.000069	5.4161	0.00544
8	120.0737	33.2920	-0.000040	7.6989	-0.00077
9	79.7088	18.3713	-0.000022	5.8569	0.00101
10	-131.1521	31.3724	-0.000066	9.5577	0.01244
11	- 62.8779	60.7131	-0.000085	7.9070	0.00500
12	- 14.1386	31.9940	-0.000012	- 1.1054	0.00110
13	- 74.6701	45.2970	-0.000010	2.3334	0.00211
14	- 50.7758	54.7521	-0.000014	2.3713	0.00035
SASKATCHEWAN					
1A	-291.2990	19.4113	-0.000173	17.4289	0.03300
1B	-254.7860	- 2.8477	-0.000186	18.5259	0.02489
2A	94.3446	- 50.7983	-0.000135	5.2096	0.02800
2B	52.0297	-186.0630	-0.000032	- 4.6433	0.05577
3AS	351.5192	- 88.3289	-0.000021	- 3.0505	0.03712
3AN	225.6973	- 88.4153	-0.000167	2.0747	0.02306
3BS	534.6201	- 52.4689	0.000061	5.1414	0.01149
3BN	340.7127	- 50.0976	0.000264	-10.1850	0.03456
4A	321.8132	- 59.8097	0.000028	1.0119	0.00581
4B	85.8568	25.7150	0.000337	- 8.8553	0.01779
5A	-483.5195	70.3632	-0.000125	13.5663	0.04801
5B	-838.3030	261.3319	-0.000216	15.6232	0.04862
6A	-445.8016	-138.6910	-0.000109	-14.6885	0.09398
6B	2.8562	-178.9715	0.000056	-10.3066	0.06229
7A	-550.5880	15.7555	0.000235	-10.0640	0.06251
7B	132.8126	-131.9547	0.000199	1.1943	0.03249
8A	-395.1455	212.1186	0.000063	0.7655	0.01804
8B	44.5187	-159.7853	0.000190	-8.2514	0.04080
9A	-107.2498	- 11.6366	0.000031	-5.2158	0.03975
9B	- 8.0354	- 21.7968	-0.000055	-6.1258	0.02539

Table A.9 (b) (continued)

Crop district	Γ_{2i}	B_{2i}	C_{2i}	D_{2i}	E_{2i}
ALBERTA					
1	474.4931	135.8299	0.000012	16.1290	0.00374
2	964.7754	258.1830	0.000158	57.6317	-0.01831
3	-956.2818	-564.9013	0.000759	-147.9969	0.15374
4A	563.7410	5.1409	-0.000142	13.0156	0.00422
4B	274.8550	71.1082	-0.000407	28.8695	0.01052
5	-389.0791	-26.6541	0.000213	-38.6965	0.03942
6	-104.1949	141.7519	-0.000107	- 0.7898	0.01008
7	- 80.3465	114.9403	-0.000068	0.3798	0.01116
$\sum_{i}^{42}$	- 62.5741	- 10.2674	-0.000025	0.9977	1.00205

[a]These coefficients were obtained by regressing the lagged wheat acreage in each crop district on the macrovariables P_{w-2}, S_w, T, and A_{t-1} (these variables are as previously described). The constant terms in these regressions are the Γ_{2i}, while B_{2i}, C_{2i}, D_{2i}, and E_{2i} are the regression coefficients of P_{w-2}, S_w, T, and A_{t-1} respectively.

As mentioned in chapter III, Γ_{1i}, B_{1i}, C_{1i}, and D_{2i} should each sum to zero over i. Similarly, E_{1i} should sum to 1 over all i (i.e. the 42 crop districts. The figures in the last row indicate the deviations from these values as a result of rounding errors.

Table A.10

Aggregation Bias and Implied Sampling Errors of the Least Squares Estimated Macrodisturbances of the Static Wheat Acreage Response Equation[a]

	$\sum_i^I e_{it}$ (1)	$\sum_i^I \alpha_{2i} V_{1it}$ (2)	$\sum_i^I \alpha_{3i} V_{2it}$ (3)	I.S.E. (4)	E.M.D. (1+2+3+4)
1953	1252.62439	317.58692	204.2366	-163.48002	1610.96789
1954	- 725.83838	-1220.61662	- 65.1869	598.95513	-1412.68677
1955	1100.19049	-1521.15985	- 82.5319	758.27536	254.77410
1956	- 820.81571	745.30667	-229.2210	-352.63860	- 657.36864
1957	-1316.02720	1521.29553	165.1190	-761.61806	- 391.23073
1958	-1397.11893	870.77480	- 79.9532	-415.23715	-1021.53448
1959	1050.24835	474.78712	145.4559	-227.60310	1442.88827
1960	307.27351	296.42356	- 5.5638	-202.19979	395.93348
1961	101.58474	-1019.12617	- 7.4735	521.96970	- 403.04523
1962	119.11604	- 212.16837	9.4505	94.12709	10.52526
1963	- 524.18621	890.00174	-206.4155	-423.66158	- 264.26155
1964	1251.59392	- 396.94148	201.3219	208.83069	1264.80503
1965	- 980.73790	- 223.41544	-167.5617	103.30092	-1268.41412
1966	1296.17782	-1823.77789	75.0663	927.06611	474.53234
1967	- 4.77902	932.83175	- 33.7622	-462.69812	431.59241
1968	- 709.30590	368.19789	77.0190	-203.38825	- 467.47726

[a]The figures in the last column of the Table are the estimated least squares macrodisturbances (E.M.D.). They are made up of the sum of the estimated least squares microdistrubances ($\sum_i e_{it}$), the aggregation bias due, respectively to the coefficient of stocks of wheat ($\sum_i \alpha_{2i} V_{1it}$) and the coefficients of moisture ($\sum_i \alpha_{3i} V_{2it}$), and the implied sampling errors (I.S.E.). The microdisturbances and the microparameters α_{2i} and α_{3i} were obtained from the least squares regressions reported in Table A.1 the residuals V_{1it} and V_{2it} were obtained from the auxiliary regressions reported in Tables A.8 (a) and A.8 (b); the I.S.E. were obtained as the predicted values of $\sum_i e_{it}$ by regressing $\sum_i e_{it}$ on the macrovariables P_{t-2} S_w, M, and T. These variables are as previously defined for Table A.1.

Table A-11

Aggregation Bias and Implied Sampling Errors of the Least Squares Estimated Macrodisturbances of the Dynamic Wheat Acreage Response Equation[a]

	$\sum_i^I e_{it}$ (1)	$\sum_i^I \alpha_{2i} V_{1it}$ (2)	$\sum_i^I \alpha_{4i} V_{2it}$ (3)	ISE (4)	EMD (1+2+3+4)
1953	1416.66743	- 16.87740	-115.3360	15.28034	1299.73437
1954	- 851.22017	- 239.32119	9.2172	117.95460	- 963.36956
1955	550.78555	10.04588	145.9665	8.98763	715.78556
1956	-1343.04265	290.90813	472.5146	-138.06068	- 717.68060
1957	- 536.04162	271.30937	-399.9210	-162.13813	- 826.79138
1958	-1155.01135	114.91137	- 93.2124	- 53.97971	-1187.29209
1959	1582.32814	110.67890	47.8594	- 57.29744	1683.56900
1960	- 9.19478	- 210.94915	-272.6495	70.98827	- 421.80516
1961	- 245.01399	- 409.40130	93.9719	223.07850	- 337.36489
1962	361.96745	- 97.42712	102.2620	48.70384	415.50617
1963	- 534.29951	93.24564	3.7452	- 38.21672	- 475.52539
1964	1430.45669	- 90.69474	18.2672	54.61104	1412.64019
1965	-1086.60794	- 130.64874	-172.0480	39.76428	-1349.54040
1966	1521.16989	222.52778	104.1236	- 65.70330	1782.11797
1967	- 399.13044	- 141.56958	59.6257	49.25419	- 431.82013
1968	- 704.81273	225.26216	- 4.3863	-114.22679	- 598.16366

[a]See footnote (a) under Table A-10. The microdisturbances and the microparameters α_{2i} and α_{4i} used in the calculation of the figures in this Table were obtained from the least squares regressions reported in Table A2; the residuals V_{1it} and V_{2it} used here were obtained from the auxiliary regressions reported in Tables A9 (a) and A9 (b); the I.S.E. were obtained as the predicted values of $\sum_i e_{it}$ by regressing $\sum_i e_{it}$ on the macrovariables P_{t-2}, S_w, T, and A_{t-1}. These variables are as previously described.

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