Economic Base and Input-Output Analysis: The Techniques and Their Application to a Mining Community's Economy

by

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ABSTRACT

The economic structure of most mining communities is often a relatively simple one, consisting mainly of one or more mining operations along with a group of support industries. These communities, because of their dependence on the exploitation of a natural resource which is subject to sharp fluctuations in demand and to physical and economic depletion, are often susceptible to periods of "boom and bust". How to quantitatively assess the impact of periods of boom and bust on total employment in a mining community is a problem faced by those involved in the formulation of community development plans.

Economic base analysis and input-output analysis are two techniques which can be used to assess the impact of an industry on a community's economy. The focus of this practicum is directed to the application of these techniques to the mining community of Leaf Rapids, Manitoba.

The study shows that both techniques can be used to assess the impact of a rise or fall in the level of mine employment on total community employment. Both techniques, however, have advantages and disadvantages. The main advantage of economic base analysis over input-output analysis is that the former's data requirements are not as great as the latter's. Consequently, the time, effort, and cost involved in an input-output study are much greater than those involved in an economic base study. Input-output analysis, on the other hand, has two main advantages over economic base analysis. First, it can provide a more

detailed description of the internal workings of a community's economy than can the latter. Second, unlike economic base analysis, input-output analysis can yield employment multipliers which not only apply to specific economic activities, but which also distinguish the short-term impact on total community employment from the long-term impact.

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CHAPTER 1

THE PROBLEM AND ITS SETTING

1.1 Preamble

1.1.1 *The Life Cycle of a Mine*

Individuals are born, grow in both physical and psychological stature, mature, age, and die. Mines, like people, also pass through the stages of youth, maturity, and old age. The four stages in the life cycle of a mine are: (1) discovery and take-off; (2) growth and maturity; (3) peak; and (4) decline and closure. These stages are shown in Figure 1.1.

Figure 1.1 portrays a static situation wherein the mine's discoverable and mineable ore reserves[1] are finite and predictable. This is a somewhat oversimplified illustration of the life cycle of a mine. In reality, the situation is a dynamic one, and the mine's ore reserves, while continuously being depleted because of mining, are at the same time being replenished by two processes (as shown in Figure 1.2).

The first of these is the discovery and delineation of additional ore reserves. In practice, a mining company with a large mineral deposit will rarely completely delineate the

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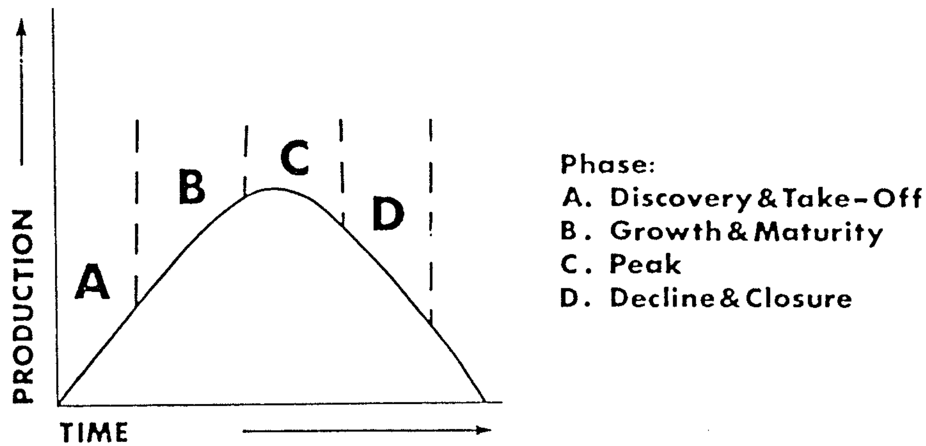
[1]The term "ore reserves" refers to that part of a mineral deposit which: (1) has been reasonably well delineated with respect to quality and quantity; (2) can be extracted with current mining technology; and (3) can be delivered at competitive market prices.

Figure 1.1

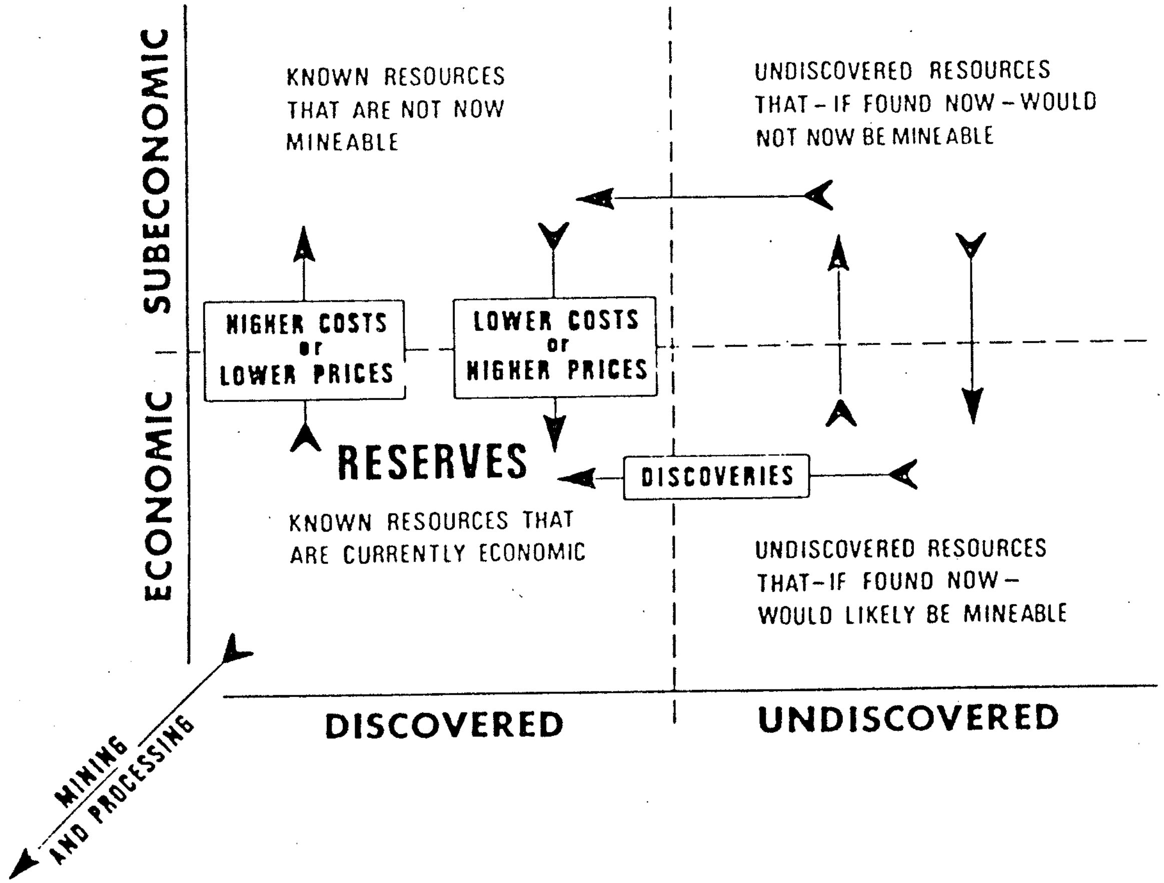
The Life Cycle of a Mine



From: Elver (1976, p. 3)

Figure 1.2

The Flow of Resources Over Time



Modified from: Findlay et al. (1976, p. 18) deposit's reserves.[2] Instead, it will delineate sufficient reserves to allow for the efficient planning of future mining and investment,[3] and will maintain an ongoing program of exploration and development in order to find just enough reserves to replace what has been mined. As a result, unless a mine is at or near the end of its productive life, its reserves, while continuously being depleted by mining, are at the same time being replenished by the discovery of new reserves.

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[2]There are a number of reasons why a mining company with a large mineral deposit will not completely delineate the deposit's reserves. First, although it is possible to estimate a deposit's total reserves under today's economic and technologic conditions, it is impossible to estimate how much of this material would be mineable under the conditions of the future (McDivitt, 1957, p.31).

Secondly, when evaluating a mineral deposit, it is necessary to discount the future profits of mining to their present value (McAllister, 1976, p.40). The present value (PV) of a sum of money S_n to be received n years from now is determined using the formula:

$$PV = S_n/(1+r)^n$$

where r is the discount rate. For example, if r is 15 percent, $1.00 of profit received one year from now has a present value of $0.87; similarly, $1.00 of profit received twenty years from now has a present value of only $0.06. There is therefore, a point where the cost of delineating reserves which will not be mined for a number of years exceeds the present value of the future profits derived from mining these reserves.

Thirdly, some governments tax reserves in the ground held by mining companies (Brobst and Pratt, 1973, p. 6; Tilton, 1977, p. 18), making it economically advantageous, and sometimes necessary, for a mining company not to completely delineate a deposit's reserves.

[3]In the past, mining companies maintained reserves to last about twenty years at some anticipated rate of production. In recent years, however, this figure has dropped to about ten to fifteen years because of rising interest rates (Govett and Govett, 1976, p.18).

The second process of replenishing ore reserves involves the reduction of costs relative to prices. For a metallic mineral deposit, the cutoff grade[4] effectively defines the deposit's reserves. Any material whose grade is greater than the cutoff grade is included in the reserves estimate while any material whose grade is lower than the cutoff grade is excluded from the estimate. According to Mackenzie (1977, p. 112), for any mining operation:

> ...where the objective is maximization of total profit over mine life, the cut-off grade is determined by the equation of marginal cost and marginal revenue. At the [economic] margin, grade is just sufficient to offset cost: MC=MR=PGR/(1+D), or G=MC (1+D) /PR, where MC= cost of mining and processing a marginal ton of ore ($); MR = revenue realized from a marginal ton of ore ($); P= net price of metal recovered ($ per ton); G= cut-off grade, percentage content of mineral product in marginal ton of ore; R= mill recovery factor (metal recovered/metal content of ore reserves); and D= dilution factor (waste mined/ore mined).

The cutoff grade, then, varies directly with the cost of producing a saleable mineral product and inversely with the price received for the product. As a result, any favourable change in the economic, political, and technological environment which reduces the cost of mining and processing a ton of ore relative to the price received for that ton of ore will, in most cases lower a mineral deposit's cutoff grade.[5] This, in turn,

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[4]The cutoff grade is the minimum grade of ore which can be economically mined.

[5]For example, in the early 1960's, the cutoff grade for porphyry copper deposits was about 0.8 percent copper; by 1970, this was reduced to 0.3 to 0.4 percent copper (Bosson and Varon, 1977, pp. 34-35). This reduction in the cutoff grade of porphyry copper deposits was primarily due to the reduction of mining and processing costs brought about by various breakthroughs in mining technology.

generally leads to an increase in the deposit's ore reserves.[6]

It is important to note that the second process of replenishing ore reserves, as shown in Figure 1.2, also works in reverse. Any unfavourable change in the economic, political, and technological environment which induces costs to rise relative to prices will cause a mine's ore reserves to diminish. This "economic" depletion of ore reserves proceeds in two ways. First of all, rising costs or falling prices may deter investment in exploration and development; and if there is no exploration and development work, production from mineral deposit will inevitably reduce the level of reserves to zero. Secondly, rising costs or falling prices will raise the mineral deposit's cutoff grade which in turn will reduce the deposit's reserves.

If the cost of mining and processing a ton of ore increases to the point where it exceeds the revenue realized from that ton of ore, the mineral deposit's ore reserves will be "economically" exhausted and all mining activity associated with the deposit will cease.[7] In some instances, the cessation of mining activity

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[6]For metallic mineral deposits, the relationship between cutoff grade and ore reserves is exponential rather than linear. That is, ore reserves increase geometrically as the cutoff grade declines arithmetically (Mackenzie, 1977, p. 109). Consequently, a reduction in cutoff grade by one-half may, for example, lead to more than a doubling of ore reserves.

[7]In some instances, a mining operation may continue even though costs exceed the value of production. According to Barchyn (1978, p.13), this may occur when:

> "...factors other than economic ones come to bear on the closure decision. The economy of the mining community and the livelihood of the people dependent on the industry is often at stake in such decisions. These factors may result in pressure on the company to continue producing beyond the point where purely economic considerations would suggest closure."

This, however, would strictly be a short-run phemomenon since no mining company could operate at a loss for any great length of time.

will be permanent; in others, it may be temporary and mining activity will resume once favourable changes in the operating environment have reduced costs to the point where they are once again less than prices.

The above discussion illustrates one of the most important characteristics of a mining operation: the exhaustibility of its chief asset, the mineral deposit. All mineral deposits are finite bodies which are subject to physical and economic depletion. The time scale in the life of a mining operation may vary from a few years to several hundreds of years, but in the end, its ore reserves will be exhausted and all mining activity with the deposit will have to cease.

1.1.2 *The Cyclical Nature of Mineral Commodity Prices*

Mineral commodity prices, with a few exceptions, show pronounced cyclical movements over long periods of time. Over the years, they rise and fall in a continuous wavelike pattern.

The cyclical nature of mineral commodity prices can be attributed to the following characteristics of mineral supply and demand. The first of these is that the demand for mineral commodities is cyclical. The demand for mineral commodities is derived largely from their use as industrial raw materials in industries whose output is greatly affected by changes in the level of overall economic activity: construction, capital equipment, transportation, and consumer durables (Tilton, 1977, p.67). The level of economic activity in market economies rarely stands still; it is instead characterized by periods of prosperity, during which the demand for the above industries' output rises, followed by periods of depressed economic activity, during which the demand for their output falls. Consequently, the demand for mineral commodities rises and falls in conjunction with the level of economic activity in the market economies. Since mineral commodity prices in these economies are, for the most part, established by the "law of supply and demand", it then follows that when the demand for mineral commodities rises during periods of prosperity, so do mineral commodity prices; similarly, when mineral demand falls during a recession, prices also fall.

The second important characteristic is the low short-run price elasticity of supply, once available capacity is being fully used. When there is an upward shift in mineral demand, prices rise and act as an incentive for the expansion of productive capacity. If excess capacity exists, mineral supply can be easily and quickly expanded. However, as output approaches full capacity, supply becomes inelastic (i.e., becomes unresponsive to further changes in price), and until existing mines, smelters and refineries are expanded or new facilities are brought into production, temporary shortages can occur, with prices rising even more.

Upon a downward shift in demand, prices begin to fall and producers reduce production in order to bring supply into balance with demand. If prices remain too low for too long, producers will also curtail exploration and development, and as mineral deposits are exhausted, new ones will not be brought into production. Upon an upward shift in demand and rising prices, capacity shortages result again, followed by even stronger price rises, and the cycle will begin once again (Chambers et al., 1976, pp. 18-19; Tilton, 1977, pp. 64-65).

The third important characteristic of mineral supply and demand is the low short-run price elasticity of demand. In the short-run, the demand for mineral commodities is relatively unresponsive to changes in price. There are two reasons for this. First of all, mineral commodities are generally used as intermediate goods in the production of consumer and capital goods. Since the cost of the mineral commodities used in the production of such goods is often a small part of the latter's cost, mineral commodity prices can rise sharply and yet only cause a small increase in the final cost of consumer and capital goods.[8] Second, while producers of most final and consumer goods can substitute one mineral commodity for another,[9] rarely can this be done on short notice because the use of alternative materials often requires ordering new equipment, changing the existing layout of a fabricating facility, and retraining workers. In addition, given the costs involved, producers of final or consumer goods may hesitate to substitute one mineral commodity for another until they are sure that the price change will last (Tilton, 1977, p.65).

[8]For example, the price of the cold rolled steel sheet used in refrigerators or of the copper used in an outboard motor can rise sharply and yet cause only a small increase in the final cost of the refrigerator or of the outboard motor.

[9]Aluminum, for example, has replaced steel in bus and railroad coach bodies, copper in electrical transmission cables and in electrical wiring, and tin as well as steel in containers.

Consequently, in the short-run, rising prices during a period of shortages will not encourage a reduction in the demand for mineral commodities thereby exacerbating the situation; similarly, during a period when supply exceeds demand, falling prices will not, in the short-run, cause mineral demand to rise.

1.2 The Problem

At the present time, there are five mining communities in northern Manitoba. These communities are: (1) Lynn Lake; (2) Leaf Rapids; (3) Snow Lake; (4) Flin Flon; and (5) Thompson (see Figure 1.3). The first three are relatively small and their economic structure is a relatively simple one, consisting mainly of one or more copper-zinc mines along with a group of support industries in the trade, service, transportation, and communications sectors. The latter two are larger and have a more complex economic structure. Both serve a cluster of mines (copper-zinc in the case of Flin Flon and nickel-copper in the case of Thompson) and are centres for further processing (i.e., smelting and refining). One of them, Thompson, is also an administrative and transportation centre for northern Manitoba.

The central fact of life in all five communities is their dependence on the economic viability of one or more mining operations. In the previous section, it was stated that: (1) mineral deposits are finite bodies which are subject to physical and economic exhaustion; and (2) the demand for and the price of mineral commodities generally rise and fall over time in a

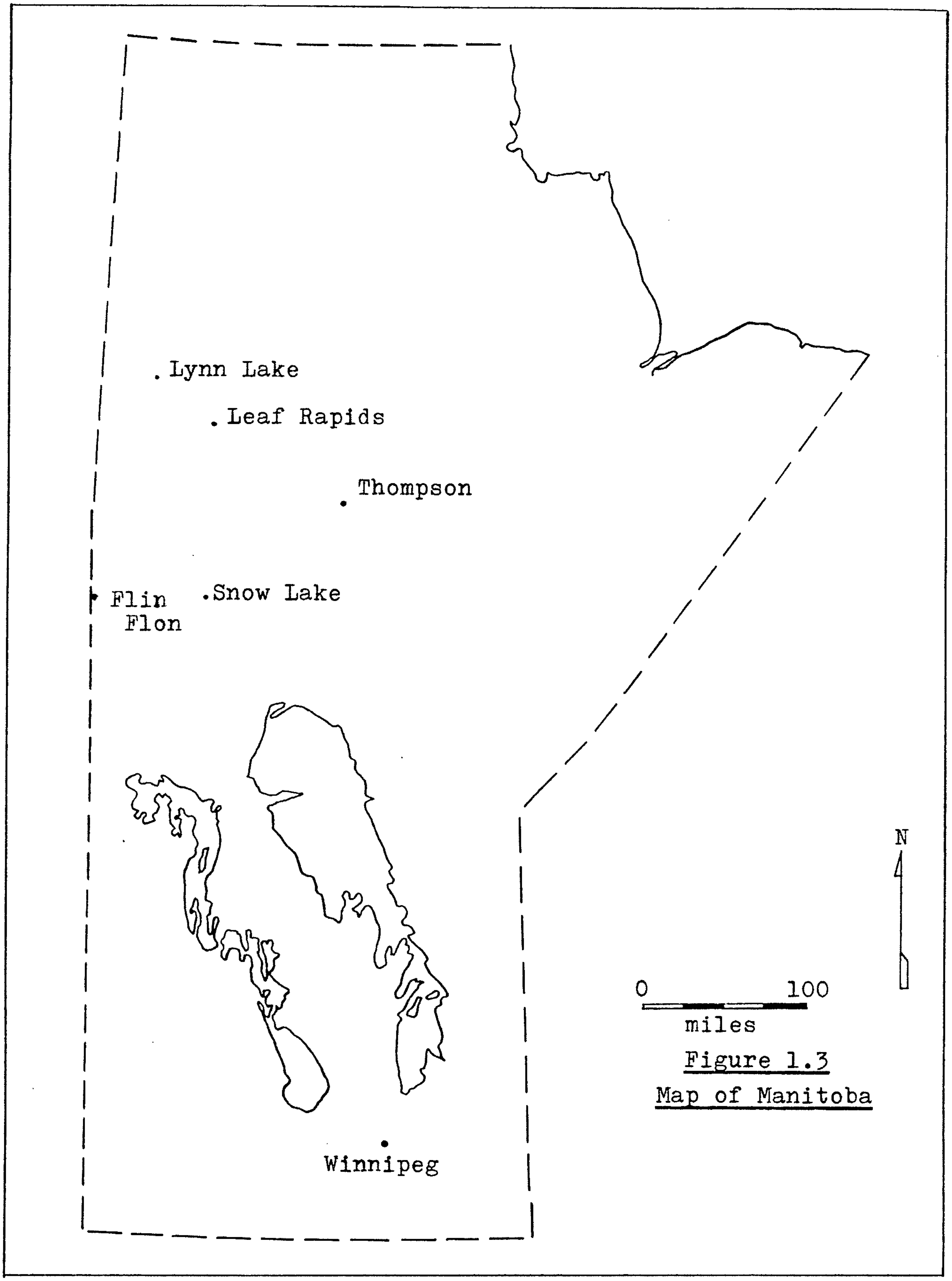

Figure 1.3

Map of Manitoba

continuous wavelike pattern. Consequently, these communities, because of their dependence on an industry engaged in the exploitation of mineral resources, are often susceptible to periods of "boom" and "bust". How to quantitatively assess the impact of periods of boom and bust on total employment in a mining community is a problem faced by persons involved in the formulation of community development plans.

1.3 Focus of the Practicum

There are two analytical techniques commonly used by regional economists and planners to quantitatively assess the impact of an industry on a community's economy. These are: (1) economic base analysis; and (2) input-output analysis.

The focus of this practicum is directed to the application of these techniques to the economy of Leaf Rapids, Manitoba. The practicum describes: (1) the theoretical framework of both techniques; (2) the various steps involved in the empirical implementation of both techniques; and (3) the advantages and disadvantages of both techniques as a means of assessing the impact of a rise or fall in the level of mine employment on total employment in the community.

1.4 Importance of the Study

It was stated earlier that mining communities are often susceptible to periods of boom and bust. During a boom period, a mining community, especially a small one, must often face a variety of serious and interrelated problems stemming from the rapid growth of population and economic activity. For example, housing may be inadequate and the price of various goods and services may skyrocket out of reach of average families. The local government may find itself financially hamstrung despite the booming prosperity, and public services and facilities may fail to keep pace with the increasing population. There may be a lack of cultural and social integration of the new population with the existing population, worker productivity may decline, labour turnover may increase, and the level of "community stress"[10] may increase (Ford, 1977, p.1).

During a bust period, a mining community may face the problems of stagnation and decline, the severity of which depends on the circumstances surrounding the community. If the community is located near a large urban area, the consequences of a loss of mine employment are not too severe since alternative employment can usually be found in the same area (Lajzerowicz et al., 1976, p. 9). On the other hand, if the community is located in a remote locality, the consequences of a loss of mine employment can be serious. Since alternative employment is usually unavailable in the area, most, if not all of the unemployed miners will leave the community. The resulting loss of this part of the population and its expenditures will bring about a reduction in the number of service establish-

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[10]According to Lajzerowicz et al. (1976, p.7), community stress symptons include: (1) mental health problems, especially depression; (2) alcoholism and drunkeness; (3) extreme loneliness; (4) feelings of insecurity and uncertainty; (5) promiscuity; and (6) suicide, accident, injury, and violence.

ments required in the community. Labour and movable capital employed in the service industries will follow in the footsteps of the miners, and the local economy will contract (Graham, 1978, p.124).

As the local economy contracts, so will the tax base. The local government will be hard-pressed to maintain the existing level of community services and many amenities of town life, such as elaborate educational and medical facilities and specialized personnel, will disappear (DREE, 1978, p.7). Where individuals have invested in private housing, commercial enterprises, and community facilities, the decline of the community will inevitably result in a great loss of private wealth (Parker, 1960, p.28), both for those who have moved and for those that remain behind. The community will also become aesthetically depressed as homes and commercial buildings become vacant (Lajzerowicz et al., 1976, p.9).

The above shows that the economic and social impacts of periods of boom and bust on a mining community and its residents can be severe. If the residents of a mining community which is about to undergo a period of boom or bust are to develop a set of plans to reduce the severity of the associated impacts, they must first of all be able to assess these impacts. Before they can do so, however, they must have some way of quantitatively assessing the impact of the rise and fall in the level of mine employment on total employment in the community. Economic base analysis and input-output analysis are two techniques which could provide them with a way of doing this.

CHAPTER 2

THEORETICAL FRAMEWORK

2.1 Economic Base Analysis

2.1.1 *Economic Base Theory: A Brief Introduction*[11]

The theoretical underpinning of economic base analysis rests upon " ... the proposition that the rate and direction of growth of a region ... is determined by its function as an exporter to the rest of the world" (Bendavid, 1972, p. 103).

Economic base theory holds that a region's economy can be subdivided into two sectors: (1) a basic (or primary) sector; and (2) a non-basic (or service) sector. The basic sector consists of that section of the region's economy whose output, be it goods or services, is sold to firms and individuals outside the region. The nonbasic sector, on the other hand, consists of that section of the region's economy which serves the local market, producing goods and services for firms and individuals within the region. The fundamental difference between the two sectors is that the basic sector causes income to flow into the region from the outside world, mainly by

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[11]For a more detailed discussion of economic base theory (also referred to in the literature as staples theory and as export base theory) see North (1955), Watkins (1963), and Stabler (1968).

producing goods and services for export,[12] whereas the nonbasic sector is concerned only with the recirculation of income within the region (Lane, 1966, pp. 344-345; Morrison and Smith, 1977, p. 66).

Implicit in such a division of economic activity is the premise:

> ... that the reason for the existence and growth of a region -- whether it is a community or a small resource area at one extreme or a huge metropolitan or resource region at the other extreme -- lies in the goods and services it produces locally but sells beyond its borders. These "basic" activities not only provide the means of payment for raw materials, foods, and manufactured products which the region cannot produce itself but also support the ... ["nonbasic"] activities which are principally local in productive scope and market area (Isard, 1960, p. 190).

In other words, the basic sector in effect leads and determines a region's overall development whereas the nonbasic sector, because it is dependent upon the basic sector, is simply a consequence of the region's overall development.

2.1.2 *The Economic Base Multiplier*

Because the basic sector is viewed as the prime mover of a region's economy, it follows that an increase in the level of activity in the basic sector will lead to a corresponding increase in the level of activity in the nonbasic

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[12]Note that the term "exporting" as used in economic base analysis does not necessarily imply that the goods and services are physically shipped out of the region by their producers; they may instead be consumed in the region by outsiders who occasionally come to the region for that purpose (Hoover, 1975, p. 219). The rule of thumb which is generally used to determine whether or not a particular economic activity exports its output is that any economic activity which causes income to flow into the region from outside sources in effect exports its output.

sector, and total economic activity in the region will rise by some multiple of the initial increase in basic activity. This will be illustrated with the use of the following example.[13]

Imagine a community where all income accruing to residents can be subdivided into basic and nonbasic income.[14] Imagine also that nonbasic income is derived via the following two-step process: (1) the residents spend part of their income locally on goods and services; and (2) part of the income which is spent locally remains within the local economy and becomes nonbasic income while the remainder "leaks out" of the local economy in one form or another. Lastly, imagine that the community's residents have a marginal propensity to consume locally of 0.5. That is, every time they receive some additional income, they immediately spend 50 percent of this income locally on goods and services and withdraw the remaining 50

[13] This example is modified from Tiebout (1962, pp. 58-59).

[14] Basic income would consist of income accruing to residents from outside sources. This could include, for example, the wages and salaries paid to local residents by a local mining operation which sells all of its copper and zinc concentrates to a non-local smelter and refinery, the income received by local residents from the renting of cottages to tourists, etc. Non-basic income would consist of income accruing to residents from local sources. This would include, for example, the local dentist's income, the wages and salaries paid to local residents by local retail and service establishments, etc.

percent for taxes, savings, and a trip to the Bahamas.

Now suppose that the level of basic income in the community were to rise by one dollar. Total income in the community would clearly rise by one dollar. But that is not the end of it. The recipients of this income, after putting aside $0.50 for taxes, savings, and their trip to the Bahamas, would spend the remaining $0.50 locally on goods and services. Does this mean that from the extra dollar of basic income, an additional $0.50 of non-basic income would be created through local spending? The answer is no, because as was pointed out earlier, part of this $0.50 would leak out of the local economy in one form or another.

Suppose that the local economy's marginal propensity of local consumption expenditures to become non-basic income is 0.4. That is, suppose that 40 percent of local consumption expenditures remain in the community as non-basic income while the remaining 60 percent leaks out in the form of wages and salaries paid to out-of-town residents, rent paid to non-local landlords, freight charges paid to trucking firms to bring goods into the community from the outside world, etc. Consequently, out of every new dollar of basic income received by local residents, 50 percent, or $0.50, would be spent locally, and 40 percent of this amount, or $0.20 would remain in the community as non-basic income. A one dollar rise in basic income, then, would create $0.20 of non-basic income, and total community income would rise by $1.20.

It should be pointed out that the newly created $0.20 of non-basic income is not the total expansion of non-basic income. Part of this $0.20 would in turn be spent locally by its recipients. In fact, 50 percent of this amount, or $0.10, would be spent locally, and 40 percent of the $0.10, or $0.04, would remain in the community as non-basic income. Non-basic income would now be up by an additional $0.04, and some of this amount would in turn be spent locally, And so it would continue, with each successive round of spending causing non-basic income to increase in a continuing but diminishing chain.

Fortunately, it is not necessary to trace our each consecutive round of spending and the non-basic income created in each round in order to estimate by how much total community income would rise due to the one dollar increase in the level of basic income. The following multiplier formula can be used to obtain the answer:

$$Y_t = k_{eb} \times Y_b \qquad (2.1)$$

where Y_t is the change in total community income, k_{eb} is the economic base multiplier, and Y_b is the change in basic income. The numerical value of the economic base multiplier can be determined using the following formula:[15]

$$k_{eb} = \frac{1}{1 - s} \qquad (2.2)$$

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[15]The derivation of this formula is shown in Appendix A.

where s is the marginal propensity to consume locally multiplied by the marginal propensity of local consumption expenditures to become non-basic income.

For our hypothetical community, the economic base multiplier is:

$$k_{eb} = \frac{1}{1 - (0.5 \times 0.4)} = 1.25 \qquad (2.3)$$

Substituting the above value of the economic base multiplier into equation (2.1), we see that a one dollar increase in the level of basic income would cause total community income to rise by \$1.25, as shown below:

$$Y_t = 1.25 \times \$1.00 = \$1.25 \qquad (2.4)$$

Similarly, if the level of basic income were to fall by one dollar, total community income would fall by \$1.25, as shown below:

$$Y_t = 1.25 \times (-\$1.00) = -\$1.25 \qquad (2.5)$$

An alternative formula which can be used to compute the numerical value of the economic base multiplier is:[16]

$$k_{eb} = \frac{Y_t}{Y_b} \qquad (2.6)$$

where Y_t is total income and Y_b is basic income. Thus, if total income in our hypothetical community before any changes took place amounted to \$1 million, of which \$0.8 million was basic income and \$0.2 million was non-basic income, then:

$$k_{eb} = \frac{\$1 \text{ million}}{\$0.8 \text{ million}} = 1.25 \qquad (2.7)$$

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[16]The derivation of this formula is shown in Appendix A.

The economic base multiplier can also be used to estimate the impact on total employment of a change in the level of employment in the basic sector. If we assume that employment is proportional to income, then total employment can be be used as a proxy for total income and basic employment can be used as a proxy for basic income. Equation (2.1) can therefore be rewritten as:

$$E_t = k_{eb} \times E_b \qquad (2.8)$$

where E_t is the change in total employment and E_b is the change in basic employment, while equation (2.6) can be rewritten as:

$$k_{eb} = \frac{E_t}{E_b} \qquad (2.9)$$

where E_t is total employment and E_b is basic employment. Thus, for example, if total employment in a region before any changes took place amounted to one thousand, of which 750 was basic employment and 250 was non-basic employment, the economic base multiplier would be:

$$k_{eb} = \frac{1000}{750} = 1.33 \qquad (2.10)$$

Substituting the above value of the economic base multiplier into equation (2.8), we see that if the level of employment in the region's basic sector were to rise by, say, 150, this would cause the level of total employment in the region to rise by 199.5, as shown below:

$$E_t = 1.33 \times 150 = 199.5 \qquad (2.11)$$

Similarly, if the level of employment in the basic sector were to fall by 150, this would cause the level of total employment in the region to fall by 199.5, as shown below:

$$E_t = 1.33 \times (-150) = -199.5 \qquad (2.12)$$

2.1.3 *The Limitations of Economic Base Analysis*

Economic base analysis is not without its shortcomings, and there are several aspects of the technique which can limit its potential for impact analysis. These shortcomings fall into two categories: (1) the weakness of the initial assumption that the basic sector is the prime mover of a region's economy; and (2) the limitations of the technique as a forecasting device.

Most of the critics of economic base analysis disagree with the proposition that the basic sector is the prime mover of a region's economy. Those who support the proposition argue that a region,

> ... like a household or a business firm, must earn its livelihood by producing something that others will pay for. Activities that simply serve the regional market are there as a result of whatever level of income and demand the region may have achieved: They are passive participants in growth but not prime movers. A household, a neighborhood, a firm, or a region cannot get richer by simply "taking in its own washing", but must sell something to others in order to get more income. Consequently ... [the basic sector] provides the economic base of a region's growth (Hoover, 1975, p. 219).

The argument put forward by the critics of economic base analysis is that the:

> ... assumption that the size of the ... [base sector] is the single determinant of the level of regional income is heroic. A significant proportion of regional investment is determined autonomously, and changes in consumption, private investment or government expenditure could stimulate regional income quite as easily as ... expansion [of the basic sector]. The theory dismisses residentiary (i.e. non-basic)

> sectors as passive elements in regional income change. Improvements in efficiency in residentiary activities may reduce factor costs of possible regional exports, so that the causal sequence runs both ways. Similarly, since a region must make the best use of its resources between ... [the basic] and residentiary sectors, a decline in export activity may be associated with rising regional income if a transfer of factors to local activity approximates more closely to the optimal allocation of resources. More important, residentiary sectors may expand independently of changes in the ... [basic sector] and raise regional income. For example, migration into the region induced by non-economic factors may lead to more residentiary activity (increased local investment in social capital, for instance) without an expansion of the ... [basic sector] (Richardson, 1969, p. 250).

Which of the two arguments are correct? Well, that depends to a great extent on the size of the region. Consider, for instance, a large region,

> ... such as a whole country, that comprises several economic regions. Let us assume that these regions trade with one another, but the country as a whole is self-sufficient. We might explain the growth of each of these regions on the basis of its exports to the others and the resulting multiplier effects upon activities serving the internal demand of the region. But if all the regions grow, then the whole country or "super-region" must also be growing, despite the fact that it does not export at all. The world economy has been growing for a long time, though our exports to outer space have just begun and we have yet to locate a paying customer for them. It appears, then, that internal trade and demand can generate regional growth: A region really can get richer by taking in its own washing (Hoover, 1975, p. 221).

For large regions (such as a city, a province, or a country), then, the basic sector is not the only source of growth. Other factors, such as business investment, government expenditures, local consumption expenditures, and changes in technology and productivity can also be important sources of regional economic growth.

On the other hand, at the small community level, especially in the case of a small mining community located in a remote area, the proposition that the basic sector provides the basis for the growth of the local economy is a valid one. A good example of this is the town of Elliott Lake, Ontario, where the:

> The discovery of a rich ore body ... led to the establishment of a thriving town providing goods to the mining community. The subsequent collapse of the mining boom, and the cessation of the basic activity of mining and exporting the ore, led to the concomitant collapse of the non-basic activities that had grown up around it. The dependence of these activities on the mining industry was little less than that of the miners themselves (Brewis, 1969, p.74).

It would seem, then, that the importance of the basic sector as the basis for the growth of a region increases directly with the degree of isolation and specialization of the region and inversely with its size. Consequently, economic base analysis should only be used at the small-area or community level where economic base theory appears to provide a reasonable explanation of regional economic growth.

The limitations of economic base analysis as a forecasting tool are as follows. First of all, some of the critics of economic base analysis point out that in order to use the economic base multiplier as a means of assessing the impact of a change in the level of basic activity on total economic activity in a region, it must be assumed that the base ratio, and hence the multiplier, remains constant over the period of time under consideration.[17] This assumption, in their opinion, is a tenuous one, since in the long-run, a region's base ratio will generally tend to change over time due to changes in social, technological, and economic conditions, the influence of which is often quite difficult to crudely, let alone precisely, estimate. For example, as the population of a region grows, it provides a constantly growing local market for various goods and services, some of which are produced locally and some of which must be imported into the region. As the population grows, it may be that when it reaches a certain size, it may become economical to produce within the region some of the goods or services which formerly had to be imported. Such structural change in the region over time could very well cause the base ratio, and hence the economic base multiplier, to change over time (Isard, 1960, p. 200; Morrison and Smith, 1977, p. 67).

Another example of what could cause a region's base ratio to change over time is the following. Suppose total income

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[17]The base ratio is the ratio of basic income (or employment) to non-basic income (or employment). If, in a particular region, total income amounted to $125 million, of which $100 million was basic income and $25 million was non-basic income, the base ratio would be 1:0.25.

The economic base multiplier was shown earlier to be equal to total income (or employment) divided by basic income (or employment). The economic base multiplier for this region, then, would be 1.25. Note that the economic base multiplier is equal to the sum of the two components of the base ratio when the basic component is set equal to 1 (ie., 1+0.25-1.25). Hence, if the region's base ratio were to change, it follows that its economic base multiplier would also change.

in a region were to double. If this increase was due to a doubling of the number of people in the region, and if these new residents have the same spending patterns as the original residents, then the base ratio would most probably remain the same. However, if the new residents have radically different spending patterns, or if the doubling of total regional income is due to a doubling of per capita income which in turn leads to a change in the spending patterns of the original residents, then the region's overall spending pattern would change. For example, the residents' propensity to spend may decline resulting in taxes and savings taking a bigger bite out of the income "pie". Or, the residents may now spend more money on items such as vacations in the Bahamas or Japanese cameras, thus reducing the proportion of income spent locally. The result of this type of change in the region's overall spending pattern is a situation where income spent locally would be an increasing or decreasing proportion of total regional income, thereby causing the base ratio, and hence the economic base multiplier, to change (Tiebout, 1962, pp. 65-66).

It should be pointed out that just because changes in social, technological, and economic conditions may cause a region's base ratio to change over time, this does not mean that economic base analysis should not be used as a forecasting device. After all, even sophisticated analytical models cannot take such changes into account. What it means, however, is that when an economic base multiplier is used to make a forecast, some attempt must also be made to determine if the region in question is about to undergo (or has undergone) some kind of structural change. If the region is expected to undergo major structural change, than a certain amount of caution must be used when interpreting the projection.

A second factor which may limit the usefulness of economic base analysis as a forecasting technique is the time-lag problem. According to Isard (1960, p. 200), this problem is due to the fact that there is usually a time-lag between the response of the non-basic sector to a change in the basic sector. Consequently, it is quite possible that at any one point in time, a region's base ratio, and hence its economic base multiplier, may be distorted by recent changes in the level of activity in the basic sector which have yet to work their way through the region's economy.

There are three ways of dealing with this problem. The first is to ignore it, on the assumption that in the long-run, whatever time-lag may exist is inconsequential. A second approach is to calculate the economic base multiplier from data showing changes in the level of basic and non-basic activity over a certain period of time (Bendavid, 1972, pp. 111-112). The third approach is that when data for more than two time periods is available, linear regression techniques can be used to compute the economic base multiplier (Weiss and Gooding, 1968, p. 237).

A third limitation of economic base analysis is the fact that the economic base multiplier does not necessarily apply to any specific basic industry (Isard, 1960, p. 203; Morrison and Smith, 1977, p. 67). It is quite possible that in any one region, some basic industries may have a greater impact on the regional economy than others, mainly because of such factors as productivity differentials, differences in the industries' propensity to import goods and services, or differences in prevailing wage rates between industries. For example, industry A may purchase all of its intermediate inputs locally whereas industry B may import all if its intermediate inputs; or, the prevailing wage rate in industry A may be higher than industry B's prevailing wage rate. Clearly, a change in the level of activity in industry A would have a greater impact on the region's economy than a similar change in industry B's level of activity, and the appropriate multiplier to apply to an increase in A's level of activity should be larger than the one to apply to an identical increase in B's level of activity. The economic base multiplier, however, would not take this into account, and a change in A would be viewed as having the same impact on the region as a change in B.

2.2 Input-Output Analysis

2.2.1 *The Input-Output Transactions Table*

The concept of an input-output transactions table can be illustrated with the use of the following example.[18] Imagine a region whose economy can be subdivided as follows: (1) an intermediate sector[19] comprising of three industries

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[18]This example is modified from Masser (1972, pp. 69-76).

[19]In input-output analysis, the intermediate sector comprises of industries which purchase inputs from other industries and process these into goods and services which they sell to other industries or to final users.

Table 2.1

A Hypothetical Input-Output Transactions Table

Sales to: / Purchases by:		Industry			Final Demand		Gross Output
		Agriculture	Manufacturing	Services	Local Consumption	Exports	
Industry	Agriculture	0	200	100	100	100	500
	Manufacturing	300	0	300	100	300	1000
	Services	0	200	0	300	0	500
Primary Inputs	Labour	100	300	100	0	0	500
	Imports	100	300	0	0	0	400
Gross Input		500	1000	500	500	400	2000

(representing agriculture, manufacturing, and services); (2) two final demand sectors (one representing local consumption and one representing exports); and (3) two primary input sectors (one representing payments for labour and one representing imports). The basic data concerning the flow of goods and services between the different sectors in this economy are shown in Table 2.1. The flow of goods and services is measured in money terms, and is viewed as sales transactions between sellers and purchasers during a specific period of time, say a year.

Going across the first row of Table 2.1, we see that the agricultural industry produced $500 worth of output during the year. Of this output: (1) $200 worth was sold to the manufacturing industry; (2) $100 worth was sold to the service industry; (3) $100 worth was sold to local consumers; and (4) the remaining $100 worth was exported to customers outside the region. Now, going down the first column of Table 2.1, we see that to produce this output, the agricultural industry: (1) purchased $300 worth of inputs from the manufacturing sector; (2) paid out $100 in wages and salaries; and (3) imported $100 worth of goods and services from outside the region. Similarly, the manufacturing industry produced $1000 worth of output during the year, of which: (1) $300 worth was sold to the agricultural industry; (2) $300 worth was sold to the service industry; (3) $100 worth was sold to local consumers; and (4) the remaining $300 worth was exported to outside customers. To produce this output, the manufacturing industry:

(1) purchased $200 worth of inputs from the agricultural industry; (2) purchased $200 worth of inputs from the service industry; (3) paid out $300 in wages and salaries; and (4) imported $300 worth of goods and services from outside the region.

Thus, each row in Table 2.1 shows how each industry's output was distributed among the other industries and the final demand sectors; similarly, each column shows the distribution of each industry's purchases of inputs from the other industries and from the primary input sectors. Table 2.1, because it depicts the transactions which take place between the different sectors in the region's economy, is referred to as an input-output transactions table.

As you can see, an input-output transactions table is an extremely useful descriptive tool which can be used to provide a rather complete view of the inner workings of a region's economy. Such a table, however, has an even more important use: namely, it can be transformed into an analytical tool which can be used to predict the impact of a given change in the final demand for an industry's output on the regional economy as a whole.

2.2.2 *The Direct Requirements Coefficients*

To transform an input-output transactions table into a predictive device, one must first of all develop a table which shows the quantity of inputs that a given industry must purchase from other industries in order to produce one unit of output. This is done by dividing each entry in a given industry's column by that industry's gross output. The coefficients derived in this manner are called direct requirements coefficients, and are calculated only for the industries in the intermediate sector of the transactions table.

For example, to produce $500 worth of output, the agricultural industry purchased $300 worth of inputs from the manufacturing industry, thus the respective direct requirements coefficient is 0.6 (ie., $300/$500). Similarly, to produce $1000 worth of output, the manufacturing industry purchased $200 worth of inputs from the agricutural industry, $0 worth of inputs from itself, and $200 worth of inputs from the service industry, thus the direct requirements coefficients for the manufacturing industry would be 0.2 (ie., $200/$1000), 0, and 0.2 (ie., $200/$1000), respectively. The complete table of direct requirements coefficients derived in this manner is shown in Table 2.2.

The direct requirements coefficients can be used to compute the direct inputs required for any level of final demand for the output of any of the industries in the intermediate sector. For example, if the final demand for agricultural products were to increase by $400, to meet this increase in the final demand for its output, the agricultural industry would have to purchase an additional $240 worth of direct inputs from the manufacturing industry. Similarly, if the final demand for manufactured products were to increase by $400, then to meet this increase in the final for its output, the manufacturing industry would have to purchase an additional $80

Table 2.2

Table of Direct Input Requirements Coefficients

Sales to: \ Purchases By:		Industry		
		Agriculture	Manufacturing	Services
Industry	Agriculture	0.0	0.2	0.2
	Manufacturing	0.6	0.0	0.6
	Services	0.0	0.2	0.0

worth of direct inputs from the agricultural industry and an additional $80 worth of direct inputs from the service industry. In both of these examples, the additional direct inputs required were obtained by multiplying each entry in the industry in question's column in Table 2.2 by the change in the final demand for that industry's output.

2.2.3 *The Direct and Indirect Requirements Coefficients*

From the point of view of the economy as a whole, direct inputs are only part of the picture. Direct inputs must be produced, and their production requires a whole set of additional inputs, usually called indirect inputs. Furthermore, these indirect inputs must also be produced, and their production will require even more indirect inputs. An increase in the final demand for the output of one of the industries in the intermediate sector, then, will lead to both direct and indirect increases in the output of all industries. For example, it was shown above that if the final demand for manufactured products were to increase by $400, then to satisfy this increase in the final demand for its output, the manufacturing industry would have to purchase an additional $80 worth of direct inputs from the agricultural industry and an additional $80 worth of direct inputs from the service industry. Now, according to Table 2.2, the agricultural industry would in turn have to purchase $48 (ie., 0.6 x $80) worth of inputs from the manufacturing industry if it is to satisfy the $80 increase in the demand for its output. Similarly, the service industry would have to purchase $16 (ie., 0.2 x $80) worth of inputs from the agricultural industry and $16 worth of inputs from the manufacturing industry if it is to satisfy the $80 increase in the demand for its output. These inputs must also be produced; and so it continues, with each succeeding round of expansion requiring additional inputs which in turn leads to another round of expansion.

It should be pointed out that even though the chain reaction described above is in principle endless, this does not mean that the initial $400 increase in the final demand for manufactured products will "snowball" into an infinitely large expansion of the region's economy. The reason for this is that there are demand leakages from the regional economy. Each time an industry expands its output, it has to allocate part of the extra revenue generated by the sale of this output to the purchasing of inputs from the primary input sectors. Money paid for imported goods and services leaves the region, and this money's stimulus to the regional economy is ended. Similarly, disbursements for wages and salaries simply drop out of the stream of money that is being circulated among the industries in the intermediate sector. Consequently, the increased demand generated by each round of expansion gets smaller and smaller, and finally just "fizzles" out altogether.

There are two methods which can be used to compute the combined direct and indirect effects on the regional economy of a change in the final demand for an industry's output. The first of these is the iterative method.[20] This method is illustrated in Table 2.3 for an increase of $400 in the

[20]The discussion of the iterative method draws heavily from Bendavid (1972, pp. 135-138).

final demand for manufactured products. The first step in the iterative method is to determine the direct inputs that the manufacturing industry must purchase if it is to satisfy the $400 increase in the demand for its output. As was pointed out earlier, this is done by multiplying each entry in the manufacturing industry's column in Table 2.2 by $400. This computation is shown in the third row of Table 2.3. The total direct inputs required to satisfy the $400 increase in the final demand for manufactured products is shown in the fifth row of Table 2.3.

The total direct inputs are then multiplied through their respective columns in Table 2.2 in order to determine the second round of inputs required. The totals for the second round of inputs are, in turn, multiplied through their respective columns in Table 2.2 in order to determine the third round of inputs required. This procedure can, in theory, continue endlessly; but in Table 2.3, it was terminated after the ninth round. After nine rounds, the numbers in this example are relatively small, and it is likely that the error involved in terminating the procedure after nine rounds is minimal.

The recapitulation at the bottom of Table 2.3 shows that a $400 increase in the final demand for manufactured products would cause: (1) the agricultural industry's output to rise by $129.91; (2) the manufacturing industry's output to rise by $542.02; and (3) the service industry's output to rise by $108.28. In sum, a $400 increase in the final demand for

Table 2.3

Illustration of the Iterative Method of Computing the Direct and Indirect Requirements Coefficients

Change in the Level of Final Demand			By Agriculture	By Manufacturing	By Services
			0	400	0
Sales as Direct Inputs		To Agriculture	0	0	0
		To Manufacturing	400x0.2=80	0x0=0	400x0.2=80
		To Services	0	0	0
		Total	80	0	80
Sales as Indirect Inputs	2nd Round	To Agriculture	80x0=0	80x0.6=48	80x0=0
		To Manufacturing	0x0.2=0	0x0=0	0x0.2=0
		To Services	80x0.2=16	80x0.6=48	80x0=0
		Total	16	96	16
	3rd Round	To Agriculture	16x0=0	16x0.6=9.60	16x0=0
		To Manufacturing	96x0.2=19.20	96x0=0	96x0.2=19.20
		To Services	0x0.2=0	0x0.6=0	0x0=0
		Total	19.20	9.60	19.20
	4th Round	To Agriculture	19.20x0=0	19.20x0.6=11.52	19.20x0=0
		To Manufacturing	9.60x0.2=1.92	9.60x0=0	9.60x0.2=1.92
		To Services	19.20x0.2=3.84	19.20x0.6=11.52	19.20x0=0
		Total	5.76	23.04	1.92
	5th Round	To Agriculture	5.76x0=0	5.76x0.6=3.50	5.76x0=0
		To Manufactufing	23.04x0.2=4.60	23.04x0=0	23.04x0.2=4.60
		To Services	1.92x0.2=0.38	1.92x0.6=1.15	1.92x0=0
		Total	4.98	4.65	4.60
	6th Round	To Agriculture	4.98x0=0	4.98x0.6=2.99	4.98x0=0
		To Manufacturing	4.65x0.2=0.93	4.65x0=0	4.65x0.2=0.93
		To Services	4.60x0.2=0.92	4.60x0.6=2.76	4.60x0=0
		Total	1.85	5.75	0.93
	7th Round	To Agriculture	1.85x0=0	1.85x0.6=1.11	1.85x0=0
		To Manufacturing	5.75x0.2=1.15	5.75x0=0	5.75x0.2=1.15
		To Services	0.93x0.2=0.19	0.93x0.6=0.56	0.93x0=0
		Total	1.34	1.67	1.15
	8th Round	To Agriculture	1.34x0=0	1.34x0.6=0.80	1.34x0=0
		To Manufacturing	1.67x0.2=0.33	1.67x0=0	1.67x0.2=0.33
		To Services	1.15x0.2=0.23	1.15x0.6=0.69	1.15x0=0
		Total	0.56	0.77	0.33
	9th Round	To Agriculture	0.56x0=0	0.56x0.6=0.34	0.56x0=0
		To Manufacturing	0.77x0.2=0.15	0.77x0=0	0.77x0.2=0.15
		To Services	0.33x0.2=0.07	0.33x0.6=0.20	0.33x0=0
		Total	0.22	0.54	0.15
Change in the Level of Final Demand			Recapitulation		
			0	400	0
Requires Direct Inputs of			80	0	80
Requires Indirect Inputs of			49.91+	142.02+	28.28+
Total Inputs Required			129.91+	542.02+	108.28+

manufactured products would cause the region's gross output to rise by $780.21. Note that as was pointed out earlier, the $400 increase in the final demand for manufactured products would not snowball into an infinitely large expansion of the region's economy. As you can see in Table 2.3, because of demand leakages, the increased demand generated by each round of expansion gets smaller and smaller, until it essentially dies out altogether.

The iterative method can be somewhat cumbersome when direct and indirect requirements computations are needed for a number of alternative levels and compositions of final demand. It would therefore be useful to have a set of direct and indirect requirements coefficients which could be used to compute the direct and indirect requirements for any level and composition of final demand.

Such a set of coefficients can be derived by applying the iterative procedure outlined in Table 2.3 for a one dollar change in the final demand for each industry in the intermediate sector, <u>in</u> <u>turn</u>. That is, the iterative procedure would be carried out for a one dollar change in the final demand for agricultural products, then for a one dollar change in the final demand for manufactured products, and so on. The results of doing this are shown in Table 2.4. As was the case with the computations in Table 2.3, the procedure was terminated after the ninth round.

The direct and indirect requirements coefficients can be used to estimate the direct and indirect effects on the

Table 2.4

Table of Direct and Indirect Input Requirements Coefficients (Iterative Method)

Purchases by: / Sales to:		Industry: Agriculture	Industry: Manufacturing	Industry: Services
Industry	Agriculture	1.195	0.326	0.433
Industry	Manufacturing	0.813	1.358	0.976
Industry	Services	0.162	0.271	1.195
Total Direct and Indirect Inputs		2.170	1.955	2.604

region's economy of a change in the final demand for the output of any of the industries in the intermediate sector. For example, if the final demand for manufactured products were to increase by \$400, this would cause: (1) the agricultural industry's output to rise by \$130.40; (2) the manufacturing industry's output to rise by \$543.20; and (3) the service industry's output to rise by \$108.40. In sum, an increase of \$400 in the final demand for manufactured products would cause the region's gross output to rise by \$782.00. In this example, the direct and indirect inputs required were obtained by multiplying each entry in the manufacturing industry's column in Table 2.4 by the change in the final demand for that industry's output.

The iterative method for computing the direct and indirect requirements coefficients is relatively easy to use in the case of our hypothetical economy with only three industries in the intermediate sector. However, when this method is used in the case of an economy with more than a handful of industries in the intermediate sector, the computation of the direct and indirect requirements coefficients can be extremely tedious and time-consuming. For this reason, the iterative method is not used when the intermediate sector contains more than a handful of industries; instead, the matrix inverse method is used.

Before turning to a discussion of the matrix inverse method, let us first have a look at the algebraic notation that will be used in the discussion. Let:

X_i = industry i's gross output

X_j = industry j's total inputs

x_{ij} = industry i's sales to industry j

Y_i = industry i's sales to the final demand sector

V_j = industry j's purchases from the primary input sector

Note that the symbol i stands for any row in the input-output transactions table while the symbol j stands for any column in the table.

The generalized form of our hypothetical economy's input-output transactions table (ie., Table 2.1), using the above algebraic notation, is shown in Table 2.5. If we sum across the first row of Table 2.5 and rearrange the terms, we find that:

$$Y_1 = X_1 - x_{11} - x_{12} - x_{13} \qquad (2.13)$$

That is, the portion of the agricultural industry's output which was sold to the final demand sector is equal to that industry's gross output, less the amount consumed by that industry, less the amount consumed by the manufacturing industry, less the amount consumed by the service industry. If we do the same for the manufacturing and service industries, we obtain the following set of equations:

$$\begin{aligned} Y_1 &= X_1 - x_{11} - x_{12} - x_{13} \\ Y_2 &= X_2 - x_{21} - x_{22} - x_{23} \qquad (2.14) \\ Y_3 &= X_3 - x_{31} - x_{32} - x_{33} \end{aligned}$$

Table 2.5

Generalized Form of an Input-Output Transactions Table

Purchases by: / Sales to:		Industry			Final Demand	Gross Output
		Agriculture	Manufacturing	Services		
	Agriculture	x_{11}	x_{12}	x_{13}	Y_1	X_1
	Manufacturing	x_{21}	x_{22}	x_{23}	Y_2	X_2
	Services	x_{31}	x_{32}	x_{33}	Y_3	X_3
Primary Inputs		V_1	V_2	V_3	-	V
Gross Input		X_1	X_2	X_3	Y	X

The direct requirements coefficients for our hypothetical economy were obtained by dividing each entry in a given industry's column in the input-output transactions table by the column industry's gross output. Note, however, that since an industry's gross output is equal to its gross input, an alternative way of obtaining these coefficiencts is to divide the entries by the industry's gross input. That is:

$$a_{ij} = \frac{x_{ij}}{X_j} \tag{2.15}$$

where a_{ij} is the direct requirements coefficient. Since equation (2.15) can be rewritten as:

$$x_{ij} = a_{ij}X_j \tag{2.16}$$

it follows that every x_{ij} entry in the three equations in (2.14) can be rewritten in this form. That is, x_{11} can be rewritten as $a_{11}X_1$, similarly, x_{23} can be rewritten as $a_{23}X_3$, etc. Thus:

$$\begin{aligned} Y_1 &= X_1 - a_{11}X_1 - a_{12}X_2 - a_{13}X_3 \\ Y_2 &= X_2 - a_{21}X_1 - a_{22}X_2 - a_{23}X_3 \\ Y_3 &= X_3 - a_{31}X_1 - a_{32}X_2 - a_{33}X_3 \end{aligned} \tag{2.17}$$

The above equations are simultaneous linear equations and can be expressed in matrix form,[21] as follows:

$$y = x - Ax \tag{2.18}$$

---

[21] A brief introduction to matrix algebra is provided in Appendix B for those who are unfamiliar with the subject.

where:

$$y = \text{the column vector of final demand} = \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix}$$

$$x = \text{the column vector of gross input} = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}$$

A = the matrix of direct requirements coefficients

$$= \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

If an identity matrix is introduced, equation (2.18) can be rewritten as:

$$y = (I - A)x \qquad (2.19)$$

The simplest solution to this system of simultaneous linear equations is the open solution, which can be obtained by multiplying both sides of equation (2.19) by the inverse matrix $(I - A)^{-1}$ so that:

$$(I - A)^{-1} y = x \qquad (2.20)$$

The matrix $(I - A)^{-1}$ is the matrix of direct and indirect requirements coefficients.

The steps involved in the computation of the direct and indirect requirements coefficients using the matrix inverse method are as follows. First, the direct requirements coefficients are written in matrix form. Thus, for our hypothetical economy:

$$A = \begin{bmatrix} 0 & 0.2 & 0.2 \\ 0.6 & 0 & 0.6 \\ 0 & 0.2 & 0 \end{bmatrix}$$

where the coefficients were obtained from Table 2.2.

Secondly, the matrix of direct requirements coefficients is subtracted from the identity matrix, as follows:

$$(I - A) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 0.2 & 0.2 \\ 0.6 & 0 & 0.6 \\ 0 & 0.2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -0.2 & -0.2 \\ -0.6 & 1 & -0.6 \\ 0 & -0.2 & 1 \end{bmatrix}$$

Thirdly, the (I - A) matrix is inverted using the procedure outlined in Appendix B, which gives:

$$(I - A)^{-1} = \begin{bmatrix} 1.196 & 0.326 & 0.435 \\ 0.815 & 1.359 & 0.978 \\ 0.163 & 0.272 & 1.196 \end{bmatrix}.$$

The direct and indirect requirements coefficients obtained with the matrix inverse method are shown in Table 2.6. Note that the coefficients obtained with the matrix inverse method are more or less the same as those which were obtained with the iterative method. The former, however, are more accurate than the latter because they "... represent the actual limits of the iterative process, limits that can never be fully attained through the iterative method" (Bendavid, 1972, p. 154).

Table 2.6

Table of Direct and Indirect Input Requirements Coefficients (Matrix Inverse Method)

Purchases by: / Sales to:		Indusry		
		Agriculture	Manufacturing	Services
Industry	Agriculture	1.196	0.326	0.435
	Manufacturing	0.815	1.359	0.978
	Services	0.163	0.272	1.196
Total Direct and Indirect Inputs		2.174	1.957	2.609

2.2.4 *The Direct, Indirect, and Induced Requirements Coefficients*

In the previous section, it was shown how an increase in the final demand for an industry's output can have both a direct and an indirect effect on a region's economy. Such an increase, however, can also have what is referred to as an induced effect on the region's economy.

If an industry were to expand its output in order to satisfy an increase in the final demand for its output, not only would it have to purchase additional inputs from other industries in the region, it would also have to purchase additional labour inputs from the primary input sector, either by hiring more employees or by making its employees work longer hours. As a result, the level of personal income in the region would rise. Since it is unlikely that all of this additional income would be saved, taxed away, or spent outside the region, it follows that this increase in the level of personal income would induce consumer demand in the region to increase. Furthermore, the industries from which the first industry would purchase additional inputs would also purchase additional labour inputs thus causing consumer demand to increase even more. The result: when an industry expands its output in order to satisfy an increase in the final demand for its output, the regional economy expands not only because of the additional direct and indirect inputs it purchases from other industries, but also because of the increase in consumer demand in the region which is triggered off by that industry's expansion.

The induced effect on the regional economy of an increase in the final demand for an industry's output can be taken into account by "closing" the input-output transactions table with respect to households. In the case of our hypothetical economy, this is done by treating households as an industry[22] and thus moving the household consumption column and labour inputs row into the intermediate sector.

The transactions table in Table 2.7 is the same as the one in Table 2.1 except that it is closed with respect to households. The direct requirements coefficients obtained by dividing the entries in each industry's column by that industry's gross output are shown in Table 2.8. Writing these coefficients in matrix form gives:

$$A = \begin{bmatrix} 0 & 0.2 & 0.2 & 0.2 \\ 0.6 & 0 & 0.6 & 0.2 \\ 0 & 0.2 & 0 & 0.6 \\ 0.2 & 0.3 & 0.2 & 0 \end{bmatrix}$$

Subtracting the A matrix from the identity matrix gives:

$$(I - A) = \begin{bmatrix} 1 & -0.2 & -0.2 & -0.2 \\ -0.6 & 1 & -0.6 & -0.2 \\ 0 & -0.2 & 1 & -0.6 \\ -0.2 & -0.3 & -0.2 & 1 \end{bmatrix}$$

The inverse of the (I - A) matrix is:

$$(J - A)^{-1} = \begin{bmatrix} 1.891 & 1.036 & 1.269 & 1.347 \\ 2.073 & 2.643 & 2.487 & 2.435 \\ 1.153 & 1.282 & 2.383 & 1.917 \\ 1.231 & 1.257 & 1.477 & 2.383 \end{bmatrix}$$

[22] Treating households as an industry is not as farfetched as it may sound. Households, after all,

> "...sell labour, managerial skills, and privately owned resources; they receive in payment wages and salaries, dividends, rents, proprietor's income, etc. And to produce these resources, they buy food, clothing, automobiles, housing, services, and other consumer goods"(Schaffer, 1976, p.9)

Table 2.7

Hypothetical Input-Output Transactions Table
Closed with Respect to Households

Purchases by:		Industry				Final Demand (Exports)	Gross Output
		Agriculture	Manufacturing	Services	Local Consumption		
Industry	Agriculture	0	200	100	100	100	500
	Manufacturing	300	0	300	100	300	1000
	Services	0	200	0	300	0	500
	Labour	100	300	100	0	0	500
Primary Inputs (Imports)		100	300	0	0	0	400
Gross Input		500	1000	500	500	400	2000

Table 2.8

Table of Direct Input Requirements Coefficients
(Closed with Respect to Households)

Sales to: \ Purchases by:		Industry			
		Agriculture	Manufacturing	Services	Labour
Industry	Agriculture	0.0	0.2	0.2	0.2
	Manufacturing	0.6	0.0	0.6	0.2
	Services	0.0	0.2	0.0	0.6
	Labour	0.2	0.3	0.2	0.0

The table of direct, indirect, and induced requirements coefficients is shown in Table 2.9. Note that these coefficients are much greater than the direct and indirect requirements coefficients in Table 2.6. This is because not only the direct and indirect effects, but also the induced effects of an increase in final demand on the regional economy have been taken into account.

The direct, indirect, and induced requirements coefficients can be used to determine the total effect on the regional economy of an increase in the final demand for the output of one or more of the industries in the intermediate sector. For example, if the final demand for manufactured products were to increase by $400, this would cause: (1) the agricultural industry's output to rise by $414.40; (2) the manufacturing industry's output to rise by $1057.20; (3) the service industry's output to rise by $512.80; and (4) the households "industry's" output to rise by $502.80. A $400 increase in the final demand for manufactured products, then, would cause the region's gross output to rise by $2487.20. In this example, the direct, indirect, and induced effects were obtained by multiplying each entry in the manufacturing industry's column in Table 2.9 by the change in the final demand for manufactured products.

2.2.5 *The Limitations of Input-Output Analysis*

As was pointed out earlier, input-output analysis can be used both: (1) as a descriptive device which describes the flow of goods and services in an economy during a

Table 2.9

Table of Direct, Indirect, and Induced Input Requirements Coefficients

Purchases by: / Sales to:		Industry: Agriculture	Industry: Manufacturing	Industry: Services	Industry: Labour
Industry	Agriculture	1.891	1.036	1.269	1.347
Industry	Manufacturing	2.073	2.643	2.487	2.435
Industry	Services	1.153	1.282	2.383	1.917
Industry	Labour	1.231	1.282	2.383	1.917
Total Direct, Indirect, and Induced Inputs		6.348	6.243	8.522	7.616

particular period of time; and (2) as a predictive device to draw inferences about the effect on an economy of an increase in the final demand for an industry's output. Most of the critics of input-output analysis do not object to its use as a descriptive device; instead, most of their objections revolve around the fact that the use of input-output analysis for predictive purposes involves a number of assumptions which can limit its predictive capabilities.

The first of these assumptions is that all economic activities in the region under study do not experience economies or diseconomies of scale. That is, each economic activity in the region has a linear production function. Thus, for example, if the production of one refrigerator requires ten pounds of steel, then the production of ten refrigerators requires one hundred pounds of steel and the production of one hundred refrigerators requires one thousand pounds of steel. Similarly, if the production of one refrigerator requires two man-hours of labour, then the production of ten refrigerators will require twenty man-hours of labour and the production of one hundred refrigerators will require two hundred man-hours of labour. In sum, it is assumed that whenever an economic activity's output changes (increases or decreases) by some factor, then its input requirements will change by the same factor.

For some economic activities, this assumption is a valid one; for others, it is not. Many economic activities, both in the private and public sector, experience economies of scale. For example,

> An integrated iron and steel works achieves lowest average costs when it operates at full capacity and produces at least ten million tons of steel annually. Likewise an electric power plant, an oil refinery, or a deep coal mine operation operates at lowest average costs when its scale is very large. Many important functions provided by a city, region, or central government are also subject to scale economies. These functions range from cultural activities such as a symphony or national park to basic services in providing law and order (such as police and fire protection), highway maintenance, and water and sewage disposal (Isard, 1975, p. 67).

On the other hand, many economic activities experience diseconomies of scale. For some economic activities, after a certain scale of operations is reached,

> ... certain diseconomies develop. Administration becomes unwieldy, bureaucracy develops, and workers get in each others' way. Moreover, environmental costs may appear when an industry or region or population cluster becomes so large that its pollution emissions exceed the natural capacity of the environment to absorb them.
>
> The presence of diseconomies is often reflected in increases in specific variable costs per unit. For example, more and more management services per unit of output may be required as operations expand, more and more inputs of administrative personnel per unit of output may be needed in running an educational system, and more and more paperwork and secretarial services may be required to reach a decision.
>
> Or, while raw material costs per unit may decrease up to a point because of economies in buying raw materials in volume (say in carload lots), these costs may then rise. Beyond a certain scale the demand may exceed the supply capacity of a nearby source of that raw material. It may become necessary to tap a more distant and more expensive source, or a source able to provide only a lower quality of raw material (Isard, 1975, pp. 67-68).

Consequently, the assumption that all economic activities in the region under study do not experience economies or diseconomies of scale may, in some instances, limit the predictive capability of input-output analysis.

The second assumption which may limit the predictive capability of input-output analysis is that the economic system under study is in stationary equilibrium. The standard procedure in input-output analysis is to investigate the economy's flow of goods and services during a specific period of time, usually a calendar year, and to compute the direct requirements coefficients (from which the direct and indirect as well as the direct, indirect, and induced requirements coefficients are derived) without referring to events which have taken place in the immediately preceding time period or which are about to take place in the immediately subsequent time period.

This assumption may limit the predictive capability of input-output analysis for two reasons. First of all,

> Any particular year may involve "irregularities" that bring the reliability of the coefficients derived from the transactions data into question. Such irregularities may include major strikes, passing fads, unusually large inventories and other temporary influences on the regional economy (Bendavid, 1972, p.164).

Secondly, this assumption:

> ... implies that all inputs are produced in the same period as the output to which they contribute, and ... that there is a precise functional relationship between the output of a raw material in ... [a certain period of time] and the output of the finished good in the same time period.
>
> In reality, current output in any industry is

> related to "previous" output in the supplying industries and expected "future" demand by consumers. For technological, institutional and geographic reasons, the time which elapses between "previous" and "future" may be much longer than the period on which the [input-output transactions] table is based (Wright, 1965?, p. 46).

The last assumption is that the direct requirements coefficients, from which the direct and indirect requirements coefficients and the direct, indirect, and induced requirements coefficients are derived, remain constant over time. There are a number of factors which could cause the direct requirements coefficients to change over time. One factor which could cause these coefficients to change is a change in the price of certain inputs relative to the price of other inputs. This could induce some substitution among inputs. For example, if the price of copper relative to the price of aluminum were to rise, building contractors might substitute aluminum wiring for copper wiring in homes and office buildings. Similarly, if the price of labour relative to the price of capital equipment were to rise, firms might alter their production processes by substituting capital equipment for labour.

Technical change is another factor which could cause the direct requirements coefficients to change. For example, technological change could reduce the number of man-hours of labour or the number of pounds of steel required to produce one refrigerator.

Yet another factor which could cause the direct requirements coefficients to change over time is population growth.

As a region's population grows, it provides a constantly growing local market for a variety of goods and services, and once the market reaches a certain size, it may become economical to produce within the region some of the goods and services which formerly had to be imported into the region.

It should be pointed out that the fact that the direct requirements coefficients may not remain constant over time does not mean that input-output analysis should not be used as a predictive device. All it means is that:

> ... it seems reasonable to view the use of the ... input-output technique for projection purposes as an approximative procedure. The more we can foresee changes in the structure of the ... inter-industry system and alter the coefficients accordingly, the firmer the resulting projections. It likewise follows that the accuracy of the projections will tend to be greater the closer the projection year is to the base year, in time as well as in size and composition of the variables to be measured. As a consequence, the more frequently an input-output table is revised and brought up to date, the greater the applicability of the technique (Isard, 1960, p. 343).

CHAPTER 3

METHODOLOGY

3.1 Delineating the Study Area

In both an economic base study and an input-ouput study, goods and services which are sold to customers outside the region must be distinguished from those which are sold to local customers. As a result, some kind of geographic and/or economic boundary between the region and the "rest of the world" must be established.

The easiest way of delineating the study area involves the adoption of political boundaries, so that the area is bounded in geographic space (Morrison and Smith, 1977, p.105). Political boundaries, however, are not always suitable for use in economic analysis because they do not always coincide with economic boundaries. As a result, placing the emphasis on political rather than economic boundaries can often lead to a situation where only half a crop-growing area or half a coalfield is included in the economic base or input-output study. Since economic base analysis and input-output analysis are techniques which were designed for the analysis of economic systems, the area included in either type of study should not cut across the local economic structure. When political boundaries, or for that matter any type of geographical boundaries, cut across the local economic structure, the area under study should be delineated on the basis of economic rather than geographical space. This, however, is easier said than done, and the task of determining an area's economic boundaries can be a difficult one.

Fortunately, in the case of Leaf Rapids, Manitoba, the study area could be delineated on the basis of political boundaries since these do not cut across the local economic structure. The study area, therefore, was viewed as comprising of the area within the community's municipal boundaries (as shown in Figure 3.1).

3.2 The Economic Base Study

The data used to compute the economic base multiplier for Leaf Rapids, Manitoba, were obtained from the community's input-output transactions table. However, since one of the objectives of this practicum is the description of the steps involved in an economic base study, the steps that would have been followed if an economic base study of the community would have been carried out are described below.

3.2.1 *The Unit of Measure*

The first step in the economic base study would have been the selection of a unit of measure. There is no single most appropriate unit of measure for an economic base study. Employment, payrolls, income accruing to residents, sales and value added,[23] all can be appropriate units of measure. Which of these is chosen for a particular economic base study

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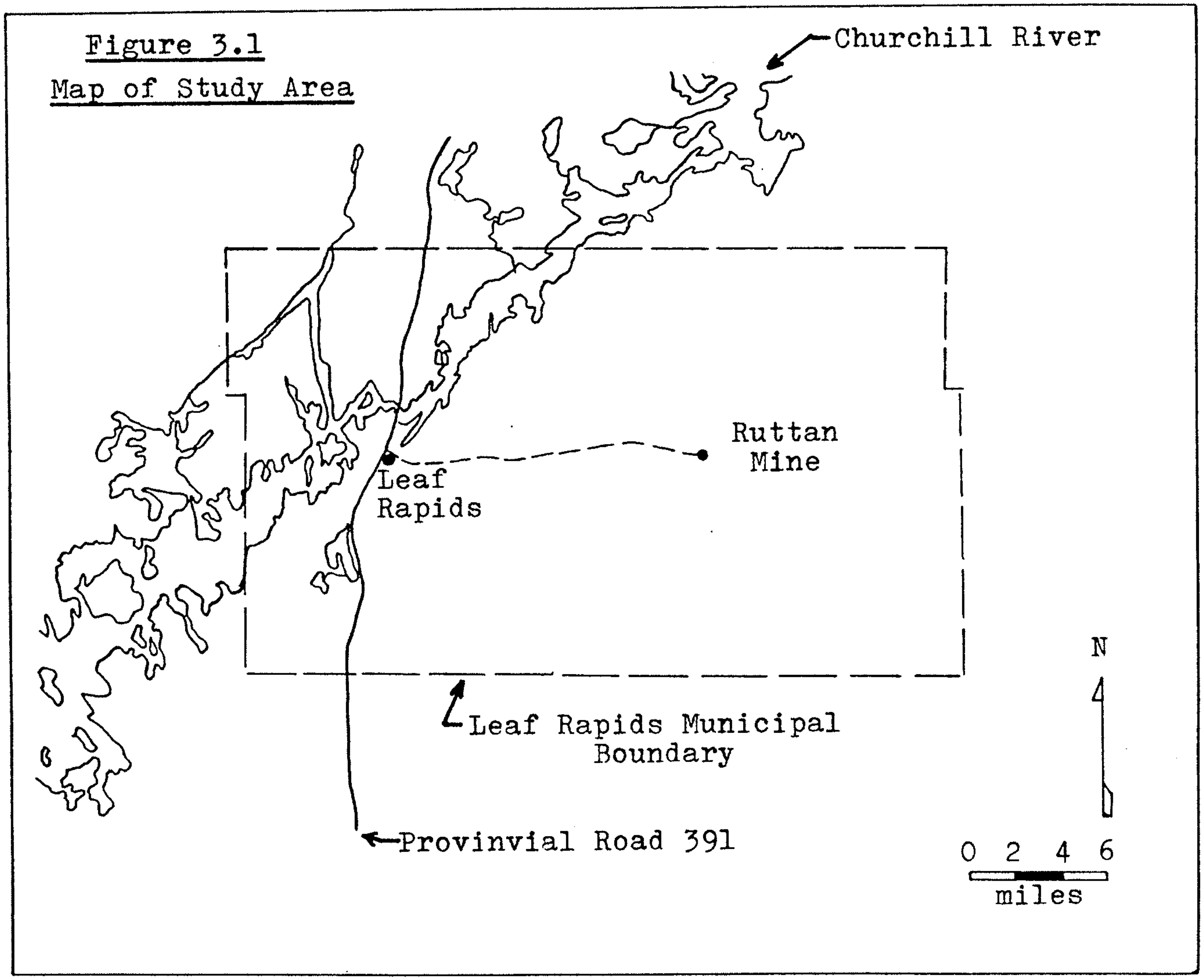
[23]Value added is the value of a business establishment's output less the cost of inputs that the establishment purchased from other establishments and individuals.

Figure 3.1

Map of Study Area



Modified from: Clarke and Grimes (1975, p. 85).

depends primarily on the objective of the study and the availability of data (Tiebout, 1962, p.45).

Since the purpose of the economic base study of Leaf Rapids, Manitoba was the computation of an employment multiplier which could be used to assess the impact on total employment of a rise or fall in the level of mine employment, the logical unit of measure to select would have been employment.

3.2.2 *The Method of Allocating Employment to the Basic and Non-basic Sectors.*

The next step would have involved the selection of a method of allocating employment to the basic and non-basic sectors. There are four methods which can be used to do this. These are: (1) the assumptions approach; (2) the location quotient method; (3) the minimum requirements method; and (4) the direct survey method.

By far the simplest of the four methods is the assumptions approach. In this approach, an arbitrary assumption is made as to which industries in the study area are basic and which are non-basic. A common assumption that is made by the users of this approach is that all manufacturing, agricultural, and extractive (i.e., mining, logging, etc.) industries are basic while all remaining industries are non-basic. Thus, all employment in the first three industries is classified as basic employment while all remaining employment is non-basic.

This approach is attractive because of its simplicity, and in certain small and isolated communities, it can be used with reasonable accuracy (Bendavid, 1972, p.108). Its use, however, is not recommended because in most cases, the error involved in using this approach can be large since most industries have both a basic and non-basic component.

The second method of allocating employment to the basic and non-basic sectors is based on the formula for the location quotient.[24] The main idea behind this method is that if a region is more specialized than the nation as a whole in the production of a particular good or service, then that region must be exporting part of its production of that good or service (Tiebout, 1962, p.47).

The location quotient method makes use of the following formula:

$$A_r = \frac{A_n \times E_r}{E_n} \qquad (3.1)$$

where A_r is the number of people in the region employed in industry A, A_n is the number of people employed in industry A at the national level, E_r is total regional employment, and E_n is total national employment. Solving for A_r determines the number of people in the region which would be employed in industry A if that industry produced just enough of whatever good or service it produces to satisfy regional demand for that good or service. For example, if for a particular

[24]The location quotient is a statistical device which is used to gauge the relative specialization of a region in selected industries. For a detailed discussion of the concept of the location quotient, see Isard (1960, pp. 123-126) and Bendavid (1972, pp. 93-98).

region A_r = 5000, and if the number of people actually employed in industry A was 9000, this would mean that 4000 of these people owed their jobs to the export market while the remaining 5000 owed their jobs to the regional market. Consequently, 4000 of the industry A's employees would be allocated to the basic sector while the remaining 5000 would be allocated to the non-basic sector.

The location quotient method has a number of conceptual weaknesses. The method, as formulated in equation (3.1), assumes that: (1) there are no major differences between the region's and the nation's productivity and consumption patterns; (2) there is no significant crosshauling of commodities between the region and the nation;[25] (3) exports or imports of a particular commodity at the national level do not constitute a significant proportion of that commodity's total output; and (4) regional employment in any industry is not a significant proportion of national employment in that industry (Davis, 1975, p.2).

In addition to the above, there is also the problem of "product mix." The term product mix suggests that within any one industry, many different products are involved. Even such a simple commodity as cigarettes has a very large

---

[25]That is, all of a region's consumption of a particular good or service comes from the region's production. Thus, if a region produces more of a particular good or service, it may export some of that good or service; it cannot, however, also import some of that good or service.

variety of different brands. Thus, if a region's cigarette industry produced just enough cigarettes to meet the region's needs, chances are they would be all of one brand. If so, it would be likely that a good portion of these cigarettes would be exported from the region while a variety of other brands would be imported into the region. If this were the case, the location quotient method would allocate all employment in the cigarette industry to the non-basic sector, even though some of that industry's output was exported from the region. The location quotient method, then, can sometimes understate the level of basic employment.

The third method of allocating employment to the basic and non-basic sectors is the minimum requirements method. This method involves the selection of a large number of regions which are "similar" to the region under study. For each region, the percentage distribution of total employment among the various industries in the region is computed. Then, for each industry, the percentages attributed to it in the various regions are ranked in decreasing order of magnitude. Lastly, a list comprising of the lowest ranked value for each industry is drawn up. This list is referred to as a minimum requirements profile.

The assumption behind this approach is that for each industry, the lowest percentage of total employment attributed to it represents the minimum required in that industry to satisfy a region's needs. Consequently, any employment in excess of this minimum requirement must be basic employment.

There are a number of problems associated with the minimum requirements method. First of all, there is the problem of deciding how many regions to include in the sample. Secondly, there is the problem of deciding what constitutes a "similar" region. Thirdly, there is the problem of dealing with "oddball" regions. The sample of regions from which the minimum requirements profile is drawn up may contain some regions in which truly unusual and irrelevant factors are at play and, unless these oddball regions are somehow taken into account, the resulting minimum requirements profile may be somewhat distorted.

The fourth method of allocating employment to the basic and non-basic sectors is the direct survey method. In this method, information regarding each business establishment's proportion of sales to customers from outside the region and to local customers is obtained, usually by means of a questionnaire, and these proportions are applied to the establishment's total employment. Thus, for example, if 60 percent of a business establishment's sales are to customers from outside the region and 40 percent of its sales are to local customers, then 60 percent of that establishment's total employment is allocated to the basic sector while the remaining 40 percent is allocated to the non-basic sector. The underlying assumption here is that employment is proportional to sales. As for government establishments, all employment in local government establishments is allocated to the non-basic sector while all employment in non-local government establishments is allocated to the basic sector.

Of the four methods of allocating employment to the basic and non-basic sectors, the direct survey method is the one which yields more precise data, mainly because its implementation does not involve the use of simplifying and sometimes erroneous assumptions about the nature of a region's economy. Unfortunately, while this method yields more precise data, it is not used in many economic base studies because for anything but small communities, it is tedious and time-consuming as well as expensive. However, since Leaf Rapids is a relatively small community with a limited number of business and government establishments, the direct survey method would have been used.

3.2.3 *The Questionnaire*

As was pointed out above, the direct survey method requires that the following data be obtained from each business establishment in the study area: (1) a breakdown, on a percentage basis, between sales to customers located outside the area and sales to customers located inside the study area; and (2) total employment. As for government establishments, only employment data are required.

The questionnaire which would have been used to collect the above data is shown in Appendix C. Note that the questionnaire contains some hypothetical answers. These will be used later to illustrate how the questionnaire would have been processed.

As you can see, the questionnaire is relatively short and straightforward. In section A, the respondent is asked to indicate the name and address of the business or government establishment in question, as well as his own name and his position in the establishment's hierarchy. In Section B, he is asked to provide a breakdown, on a percentage basis, between the establishment's sales to customers located outside the study area and sales to customers located inside the study area. In section C, the respondent is asked to indicate the number of people his establishment employs on a full-time, part-time, and temporary basis.

3.2.4 *The Conduct of the Survey*

Two approaches can be used in a questionnaire survey. The first involves mailing the questionnaires to all business and government establishments in the study area and requesting that they fill them out and mail them back. The second involves directly interviewing a representative of each business and government establishment in the study area with the questionnaire.

The direct approach is by far the better of the two approaches because: (1) the response rate is higher; (2) any difficulties which the respondents may encounter while filling out the questionnaire can be resolved during the interview thereby preventing the respondents from misinterpreting one or more questions and subsequently providing incorrect information; and (3) it allows the person carrying out the study to familiarize himself with the general character of the study area. Consequently, the direct interview approach would have been used in the economic base study.

3.2.5 *Processing the Questionnaire Data*

Suppose a hypothetical business establishment filled out the questionnaire as shown in Appendix C. The data would be processed as follows.

Part-time and temporary employment would be converted to full-time job equivalents using the following formula:

$$FTJE=\frac{PT \times HR}{STD} \tag{3.2}$$

where FTJE is full-time job equivalents, PT is the number of part-time or temporary employees, HR is the average number of hours per week worked by these employees, and STD is the number of hours per week in the establishment's standard work week. For example, during the month of February, the two part-time employees worked an average of twenty hours per week. Thus:

$$FTJE= \frac{2 \times 20}{40} = 1 \tag{3.3}$$

That is, the two part-time employees would be converted to one full-time job equivalent. Now, since five people worked full-time during the month of February, total employment during February was six full-time job equivalents. Following the same procedure for the months of May, July and November, total employment during these months would be computed as being equal to six, eight, and six full-time job equivalents, respectively.

By summing total employment during the four months (each of which represents a specific season) and dividing by four, average annual employment could be determined. In the case of our hypothetical business establishment, average annual employment amounts to 6.5 full-time job equivalents. Now, since 60 percent of this establishment's sales was to outside customers and the remaining 40 percent of sales was to local customers, then 60 percent of its average annual employment, or 3.9 full-time equivalents, would be allocated to the basic sector, while the remaining 40 percent, or 2.6 full-time equivalents, would be allocated to the non-basic sector.

By following the above procedure for every business establishment in the study area, all employment in these establishments could be allocated to the basic and non-basic sectors. As for government establishments, all employment in non-local government establishments would be allocated to the basic sector while all employment in local government establishments could be allocated to the non-basic sector.

3.3 The Input-Output Study

3.3.1 *The Intersectoral Flows Approach: A Simplified Form of Input-Output Analysis*

From an operational point of view, a serious shortcoming of input-output analysis is the extremely large amount of data required for the construction of an input-output transactions table. Some of the data which must be obtained includes:

> ...annual expenditures on material and intermediate inputs, heat, light and power, transport costs, rents and rates, taxes, wages, profits and miscellaneous expenditures; annual sales to various types of purchasers, such as

> local businesses, businesses in other regions, ...wholesalers and retailers, and households; annual expenditures on new capital plant and equipment, plus changes in inventories and depreciation expenses on plant and equipment ...Wherever appropriate, the distribution of sales and purchases between local and out-of-region receipts and expenditures is also requested (Richardson, 1972, p.101).

These data, especially at the small area level, are generally unavailable from published sources and must therefore be collected by means of a detailed survey. Not only can this be extremely tedious and time-consuming, it can also be very expensive. Furthermore, much of this information may be of a confidential nature, and consequently may not be easy to obtain.

In order to reduce the magnitude of the data collection task, a number of shortcuts have been developed. One of these is the intersectoral flows approach which was developed by Hanson and Tiebout (1963).[26]

The intersectoral flows approach differs from the standard input-output approach in two respects. First of all, in the standard input-output approach, both input and output data are collected from business establishments. However, in the intersectoral flows approach, only output data are collected; <u>no input data are collected</u>. The rationale for only collecting output data is twofold. First, it

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[26] Other shortcuts which have been developed include: (1) the TAP method; (2) the use of hybrid transactions tables with fewer rows than columns; and (3) reducing the number of rows and columns in the transactions table. A brief description of the TAP method is provided in Bendavid (1972, pp. 171-173). A brief description of the TAP method as well as the other two shortcuts is provided in Richardson (1972, pp. 133-138).

is assumed that:

> ...firms know the destination of their outputs far better than the origin of their inputs, especially where regional breakdowns are required. In other words, in terms of their input-output flows, information for the "rows" is easier to obtain than for the "columns." The reason for this is that the bundle of inputs is usually so varied and complex that their origins are difficult even for firms involved to track down accurately. However, the same firms are especially concerned with where and to whom they sell their output (Hansen and Tiebout, 1963, p. 411).

Secondly, when all of the rows in the transactions table are filled in, the columns are filled in; thus the output data in effect generate the input data.

Secondly, in the standard input-output approach, all of the entries in the transactions table are expressed in dollar terms whereas in the intersectoral flows approach, all of the entries in the transactions table are expressed in terms of employment. In a transactions table which has been constructed using the intersectoral flows approach, each industry's total employment is distributed among the other industries and the final demand sectors in proportion to the distribution of that industry's sales to the other industries and the final demand sectors. The underlying assumption here is that employment is proportional to sales. The rows in the transactions table, then, show how total employment in each industry is directly related to that industry's sales to other industries and to the final demand sectors.

From an operational point of view, the intersectoral flows approach has a number of important advantages over the standard input-output approach. The first of these is that by not seeking input data, the quantity of data which must be collected is greatly reduced. This, in turn, greatly reduces the: (1) time required to collect and process the data and set up the transactions table; and (2) cost of conducting the study. This is especially important in the case of a small-scale regional study where whoever must carry out the study must do so with a modest budget and with little or no staff.

The second advantage of the intersectoral flows approach is that since it

> ...utilizes the percentage sales distribution, it is possible to fill the rows for different industries in different measurement units. For example, the trading industry row may be filled with the flow of sales dollars rather than with the distribution charges by the trading industry, whereas the transportation...industry row may be filled with the physical volume of sales rather than with transportation charges (Lee et al., 1971, p.55).

This further reduces the difficulty and cost of obtaining the necessary data.

The third advantage of the intersectoral flows approach is that because all of the entries in the transactions table are expressed in terms of employment, employment multipliers can be obtained directly from the data in the transactions table. When all of the entries in the trans-actions table are expressed in dollar terms, as is the case in the standard input-output approach, employment multipliers can only be obtained through the use of employment-production functions.[27] This is an important advantage when the primary objective of the input-output study is forecasting the effect on an economy's total employment of a change in the final demand for an industry's output.

The fourth advantage of the intersectoral flows approach is that the chances of successfully collecting all of the data required from business establishments are much greater than when the standard input-output approach is used. There are a number of reasons for this. First of all, because the quantity of data that must be collected from individual establishments is much greater in the standard input-output approach than in the intersectoral flows approach, it follows that the questionnaire which must be used in the former approach is longer and more complicated than the questionnaire which must be used in the latter approach. Now, since most people, especially "busy" businessmen, do not like to answer long, complicated questionnaires, the level of cooperation from the business community will usually be greater when the shorter questionnaire is used. Secondly, in the intersectoral flows approach, businessmen are not required to reveal sales information in dollar terms; the whole approach is in percentages. This avoids any concerns that they may have about revealing information on their establishments' sales.

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[27] For a discussion of the use of employment-production functions in input-output analysis, see Richardson (1972, pp. 34-35).

The intersectoral flows approach, because it is a form of input-output analysis, suffers from the same limitations as the latter.[28] In addition to these limitations, the intersectoral flow approach also suffers from the limitation that it sacrifices the data discipline which the standard input-output approach requires in the balancing of row and column data. In the standard input-output approach, both sales and purchases data are collected, and these provide some check on each other since one industry's sale is another industry's purchase. On the other hand, in the intersectoral flows approach, only sales data are collected. Consequently, the system of cross-checking is sacrificed, and the magnitude and direction of any resulting errors are unknown (Bramhall, 1962, pp. 103-104). Lee et al. (1973, p.105), however, point out that while this limitation

> ...cannot be dismissed lightly, it is tempered somewhat by the fact that, in the practical application of the input-output model at the regional level, reconciliation between row and column estimates for cells is based largely on informed judgement. Furthermore, the additional cost incurred in developing column information must be weighted against the resulting improvements in the accuracy of the estimates for cells in each column.

The insectoral flows approach was the approach used for the input-output study of Leaf Rapids for the following reasons. First of all, because the study was to be conducted by one person working on a limited budget, it was felt

[28] These limitations were described in the previous chapter.

that the only way that such a study could be carried out was via this approach.

Secondly, because of the limited number of business establishments in Leaf Rapids, it was felt that a very high response rate was necessary if the study was to be a success. In a region with a large number of business establishments in each industry, the response rate is not as critical a factor in the success of an input-output study as in the case of a small community with a limited number of business establishments in each industry. When each industry contains a large number of establishments, it is usually not feasible, because of the time and expense involved, to survey each and every establishment. Instead, the typical procedure is to select for each industry a sample of establishments to be surveyed, and the survey is "inflated" to the industry level by means of control totals.[29] If a number of establishments in an industry's sample refuse to answer the questionnaire, all one has to do is add a few more establishments to the sample. However, in a small community, there may only be a few establishments in an industry, and, if all or most of these establishments refuse to answer the questionnaire, one cannot simply add more establishments to the sample because there are no other establishments to add to the sample. As a result, in a small community, it is essential that all

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[29] For a description of this procedure, see Richardson (1972, pp. 94-100).

business establishments in an industry answer the questionnaire; and, it was felt that the easiest way of ensuring this was to use the intersectoral flows approach.

3.3.2 *Listing the Population of Business and Government Establishments*

The first step in the input-output study of Leaf Rapids involved the listing of all business and government establishments in the community. The main source of information for this was the 1977 Manitoba Provincial Phone Directory. Other sources of information included a small brochure published by the Leaf Rapids Chamber of Commerce and the 1977 community report for Leaf Rapids published by the Manitoba Department of Industry and Commerce.

3.3.3 *Subdivision of the Intermediate Sector*

A study of the list of business establishments indicated that Leaf Rapids' intermediate sector could be subdivided into the following ten distinct industries:[30]

1. metal mining - all establishments primarily engaged in the mining, milling and beneficiation of metal ores.

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[30] The industries' definitions draw heavily from DBS (1970).

2. services incidental to mining - all establishments solely engaged in the provision of goods and services incidental to the operation of a metal mine.

3. fishing and trapping - all establishments[31] primarily engaged in commercial fishing and in the trapping of wild animals for commercial purposes.

4. construction - all general contractors primarily engaged in the construction and repairing of buildings, highways, or heavy construction; and all special trade contractors such as plumbers, carpenters, electricians, etc., primarily engaged in construction and repair work.

5. transportation - all establishments primarily engaged in the transportation of passengers, freight, or mail by air, and in operating fixed-wing aircraft; the provision of local and long distance trucking and related services; the operation of bus or coach lines; and the operation of a taxicab service.

6. communications and utilities - all establishments, whether or not owned by a government, primarily engaged in the provision of telephone service and in the generation, transmission, and distribution of electricity.

7. wholesale trade - all establishments primarily engaged in the buying of merchandise for resale to retailers and to other wholesalers as well as to industrial, commercial, institutional, and professional users.

8. retail trade - all establishments primarily engaged in the buying of merchandise for resale to the general public for household or personal consumption as well as the provision of related services such as installation and repair.

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[31] Note that the term "establishments" as used here includes individuals working on their own account or with the help of members of their own families.

9. finance, insurance and real estate - all establishments primarily engaged in the operation of a banking service; the selling of insurance; the buying, selling, renting, managing and appraising of real estate; and the developing and operating of real estate (i.e., apartment buildings, trailer courts, commercial buildings, etc.)

10. services- all establishments primarily engaged in the provision of full-year and/or seasonal lodging; the preparation and serving of meals and/or beverages; the operation of trailer campsites and campgrounds; the provision of facilities for holding religious services or for promoting religious activities; and the provision of miscellaneous personal services such as barber and beauty shops, car and truck rental agencies, dentists' offices, etc.

3.3.4 *Subdivision of Final Demand*

Final demand was subdivided into the following seven sectors:

1. household consumption - representing all sales of consumer and personal goods and services to local residents.

2. local government - representing all sales of capital and non-capital goods and services to the local government and to establishments, such as the cinema and the exhibition centre, which are owned and/or operated by the local government.

3. exports: metal ores and concentrates - representing all sales of metal ores and concentrates to non-local smelters and refineries.

4. exports: non-local government - representing all sales of capital and non-capital goods and services to non-local government agencies.

5. exports: fish and fur - representing all sales of fish and animal pelts to non-local purchasers.

6. exports: tourists - representing all sales of goods and services to tourists.

7. exports: miscellaneous - representing all sales to non-local purchasers of capital and non-capital goods and services not classified as belonging to any of the above export sectors.

While most input-output studies have a gross capital formation sector in final demand, such a sector was not included in this study. The reason for this is that because the intersectoral flows approach is concerned only with the collection of output (or row) data, its ability to deal with capital accumulation is quite limited. In the intersectoral flows approach, the only way of obtaining data on capital accumulation is to ask each business establishment to indicate the proportion of its sales which consisted of goods going into capital formation as opposed to current use. However, because it is difficult, if not impossible, for a firm to determine with any degree of confidence whether its goods are bought on capital account or on current account, the accuracy of an answer to such a question would, at the best of times, be questionable.

3.3.5 *The Questionnaire*

The questionnaire that was used to collect the necessary data from all business and government establishments in Leaf Rapids is shown in Appendix D. Note that the questionnaire contains some hypothetical answers. These will be used later to illustrate how the questionnaire data was processed.

As you can see, the questionnaire is relatively short and straightforward. In Section A, the respondent is asked to indicate the name and address of the business establishment in question as well as his own name and his position in the establishment's hierarchy. In Section B, the respondent is first asked to break down his establishment's total sales, on a percentage basis, between sales within the community and sales outside the community. The respondent is then asked to break down sales outside the community, on a percentage basis, between sales to the federal and provincial governments and sales to customers other than these governments. Next, he is asked to break down sales inside the community, on percentage basis, between sales to local consumers, sales to tourists, sales to the local government, and sales to local business establishments. In section C, the respondent is asked to indicate the number of people his establishment employs on a full-time, part-time, and temporary basis.

3.3.6 *The Survey*

As was pointed out in an earlier section, there are two approaches which can be used in a questionnaire survey. The first of these involves the mailing of the questionnaires to all business and government establishments in the community; the second involves the interviewing of a representative of each business and government establishment in the community.

Because the latter is the better of the two approaches, it was the one that was used in the study.

The survey was conducted during the latter part of July, 1978. All in all, the cooperation of the respondents was excellent; only two of the fifty-one people contacted refused to answer the questionnaire.

It was important to note that because of the nature of the information sought and the memory estimates required, a certain amount of response error was unavoidable. No attempt to gauge the extent of this error was made. However, it is likely that this error was minimal because of the fact that for the most part, the respondents occupied key positions that were mainly connected with sales and managerial functions.

3.3.7 *Processing the Questionnaire Data*

Suppose a hypothetical business establishment answered the questionnaire as shown in Appendix D. The data would be processed as follows:

From question B.1, it can be seen that 30 percent of the establishment's total sales was to customers located outside the community while the remaining 70 percent was to customers located inside the community. From question B.2, it can be seen that 50 percent of the establishment's sales to customers located outside the community was to the federal and provincial governments while the remaining 50 percent was to customers other than the two governments. Thus 15 percent (i.e. 0.30 x 0.50) of the establishment's total sales was to the federal and provincial governments and 15 percent (i.e., 0.30 x 0.50) of its total sales was to other customers outside the community.

From question B.3, it can be seen that 20 percent of the establishment's sales to customers located inside the community was to local consumers, 10 percent was to tourists, 20 percent was to the local government, and the remaining 50 percent was to local business establishments. Thus, 14 percent (i.e., 0.70 x 0.20) of the establishment's total sales was to local consumers, 7 percent (i.e., 0.70 x 0.10) was to tourists, 14 percent (i.e., 0.70 x 0.20) was to the local government, and the remaining 35 percent (i.e., 0.70 x 0.50) was to local business establishments.

From question B.4, it can be seen that 10 percent of the establishment's sales to local business establishments was to the construction industry, 40 percent was to the retail trade industry, and the remaining 50 percent was to the service industry. Thus 3.5 percent (i.e., 0.35 x 0.10) of the establishment's total sales was to the construction industry, 14 percent (i.e., 0.35 x 0.40) was to the retail trade industry, and 17.5 percent (i.e., 0.35x0.50) was to the service industry. The resulting sales distribution is shown in Appendix E.

The data in section C of the questionnaire is the same as the data in section C of the questionnaire in Appendix C. How this data is processed was described in an earlier section (see pp. 59-60).

The sales distribution is converted to an employment distribution by multiplying each entry in the sales distribution by the establishment's annual average employment, which in this case is 6.5 full-time job equivalents. The resulting employment distribution is shown in Appendix E.

By following the above procedure for each business establishment surveyed, it was possible to obtain the employment distribution for each of the ten industries in the intermediate sector. As for the government establishments, all employment in non-local government establishments was allocated to the exports: non-local government sector while all employment in the local government establishments was allocated to the local government sector.

CHAPTER 4

THE RESULTS

4.1 The Economic Base Study

The results of the economic base study of Leaf Rapids are shown in Table 4.1, which shows the breakdown of total employment in the community by industry, and in Table 4.2, which shows the allocation of each industry's employment to the basic and non-basic sectors. Two observations are worth noting concerning these tables. First of all,it can be seen in Table 4.1 that Leaf Rapids is truly a single-industry community, with one industry, metal mining, accounting for almost 65 percent of total community employment. In Chapter 1, it was stated that: (1) mineral deposits are finite bodies which are subject to economic as well as physical exhaustion; and (2) the demand for and price of mineral commodities rise and fall over time in a continuous wavelike pattern. With this in mind, it can be seen that Leaf Rapids, because it is highly dependent on the metal mining industry, is quite susceptible to periods of boom and bust.

Secondly, it can be seen in Table 4.2 that the basic sector, which accounts for 73.5 percent of total community employment, is indeed the prime mover of the community's economy. As a result, Leaf Rapids' economic well-being depends upon factors over which the community and its residents have little or no control. These include the demand for metal ores and concentrates by non-local smelters and

Table 4.1

Employment by Industry Group
Leaf Rapids, Manitoba, 1977

Industry Group	Employment Number[a]	Employment Percentage of Total[b]
Metal Mining	643.5	64.9
Local Government	84.6	8.5
Retail Trade	53.8	5.4
Non-local Government	50.0	5.0
Services	47.8	4.8
Services Incidental to Mining	34.0	3.4
Finance, Insurance, and Real Estate	28.1	2.8
Transportation	15.0	1.5
Construction	14.2	1.4
Wholesale Trade	10.0	1.0
Fishing and Trapping	5.0	0.5
Communications and Utilities	5.0	0.5
TOTAL ALL INDUSTRIES	991.0	100.0

[a]In full-time job equivalents.

[b]May not add exactly to total because of rounding.

Table 4.2

Allocation of Employment to the Basic and Non-Basic Sectors
Leaf Rapids, Manitoba, 1977

INDUSTRY GROUP	Employment[a] Total			Distribution of Employment Basic	
	Total	Basic	Non-Basic	Basic	Non-Basic
Metal Mining	643.5	643.5	0	100.0%	0%
Local Government	84.6	0	84.6	0	100.0
Retail Trade	53.8	7.6	46.2	14.1	85.9
Non-Local Government	50.0	50.0	0	100.0	0
Services	47.8	11.0	36.8	23.0	77.0
Services Incidental to Mining	34.0	0	34.0	0	100.0
Finance, Insurance, and Real Estate	28.1	0.5	27.6	1.8	98.2
Transportation	15.0	4.1	10.9	27.3	72.7
Construction	14.2	0.6	13.6	4.2	95.8
Wholesale Trade	10.0	4.9	5.1	49.0	51.0
Fishing and Trapping	5.0	5.0	0	100.0	0
Communications and Utilities	5.0	0.9	4.1	18.0	82.0
TOTAL ALL INDUSTRIES	991.0	728.1	262.9	73.5	26.5

[a]In full-time job equivalents.

refineries, the level of non-local government spending, the level of tourist spending, etc.

The data in Table 4.2 can be used to compute an employment multiplier for Leaf Rapids. Earlier, the economic base multiplier formula was given as:

$$k_{eb} = \frac{E_t}{E_b} \qquad (4.1)$$

where k_{eb} is the economic base multiplier, E_t is total employment, and E_b is basic employment. According to Table 4.2, total employment and basic employment were equal to 991.0 and 728.1 full-time job equivalents, respectively. Substituting these values into equation (4.1) yields:

$$k_{eb} = \frac{991.0}{728.1} = 1.36 \qquad (4.2)$$

The economic base multiplier for Leaf Rapids,then, is 1.36. This implies that if the level of basic employment in the community were to rise by one, a total of 1.36 new jobs (ie., one in the basic sector and 0.36 in the non-basic sector) would be created. Similarly, if the level of basic employment were to decline by one, a total of 1.36 jobs would be lost. By multiplying the expected change in the level of basic employment by the economic base multiplier, an estimate of the impact on total community employment can be obtained.

4.2 The Input-Output Study

4.2.1 *The Input-Output Transactions Table*

The input-output transactions table for Leaf Rapids is shown in Table 4.3. Each row in this table shows how much employment in the row industry was generated by sales to each industry and to each final demand sector. For example, looking at the fourth row, construction, it can be seen that 83.1 percent of total employment in the construction industry (11.8 full-time job equivalents) was generated by sales to the metal mining industry, 2.8 percent (0.4 full-time job equivalents) by sales to itself, 4.2 percent (0.6 full-time job equivalents) by sales to the transportation industry, and so on across the row. For the construction industry, then, some 90.1 percent of total employment (12.8 full-time job equivalents) was generated by sales to other industries while the remaining 9.9 percent (1.4 full-time job equivalents) was generated by sales to final demand.

A number of observations are worth noting concerning Table 4.3. First of all, it can be seen that only 7.5 percent of total community employment was generated by inter-industry sales; the remaining 92.5 percent was generated by sales to final demand. This low percentage of inter-industry sales indicates that the level of structural interdependence in the community's economy is very low. Most of the industries in the community do not depend on one another for their economic well-being since they import most of their inputs and sell most of their output to final demand.[32]

[32]Note that this statement does not apply to all industries in Leaf Rapids. There are several industries in the community in which the percentage of total employment generated by sales to other industries is more than, say, 40 percent. These industries are: (1) services incidental to mining; (2) construction; (3) transportation; and (4) wholesale trade. Each of these industries' economic well-being depends to a large extent on sales to other industries.

Table 4.3

Distribution of Total Employment to Local Industries and Final Demand Sectors, Leaf Rapids, Manitoba, 1977

Industries From \ Industries and Final Demand Sectors To	Metal Mining	Services Incidental to Mining	Fishing and Trapping	Construction	Transportation	Communications and Utilities	Wholesale Trade	Retail Trade	Finance, Insurance, and Real Estate	Services	Local Consumption	Local Government	Exports: Metal Ores and Concentrates	Exports: Fish and Fur	Exports: Nonlocal Government	Exports: Tourists	Exports: Miscellaneous	Total Employment[b]
Metal Mining													100 (643.5)					100 (643.5)
Services Incidental to Mining	100 (34.0)																	100 (34.0)
Fishing and Trapping														100 (5.0)				100 (5.0)
Construction	83.1 (11.8)			2.8 (0.4)	4.2 (0.6)						0.7 (0.1)	4.9 (0.7)			4.2 (0.6)			100 (14.2)
Transportation	57.3 (8.6)			3.3 (0.5)			3.3 (0.5)	2.7 (0.4)		1.3 (0.2)	4.7 (0.7)				2.7 (0.4)		24.7 (3.7)	100 (15.0)
Communications and Utilities	26.0 (1.3)							2.0 (0.1)	4.0 (0.2)	4.0 (0.2)	40.0 (2.0)	6.0 (0.3)			2.0 (0.1)		16.0 (0.8)	100 (5.0)
Wholesale Trade	34.3 (3.4)				3.0 (0.3)		2.0 (0.2)	7.0 (0.7)			3.0 (0.3)	2.0 (0.2)		46.5 (4.6)	2.0 (0.2)			100 (10.0)
Retail Trade	0.9 (0.5)	0.2 (0.1)		0.7 (0.4)			0.2 (0.1)	0.6 (0.3)	0.2 (0.1)	0.4 (0.2)	79.4 (42.7)	3.2 (1.7)			2.6 (1.4)	4.6 (2.5)	7.1 (3.8)	100 (53.8)
Finance, Insurance, and Real Estate								24.9 (7.0)	2.5 (0.7)	5.7 (1.6)	61.2 (17.2)	3.6 (1.0)					1.8 (0.5)	100 (28.1)
Services	1.3 (0.6)										75.5 (36.1)				7.1 (3.4)	2.5 (1.2)	13.4 (6.4)	100 (47.8)
Government												100 (84.6)			100 (50.0)			100 (84.6)
Total Employment	6.1 (60.2)	0 (0.1)		0.1 (1.3)	0.1 (0.9)		0.1 (0.8)	0.9 (8.5)	0.1 (1.0)	0.2 (2.2)	10.0 (99.1)	8.9 (88.5)	64.9 (643.5)	1.0 (9.6)	5.7 (56.1)	0.4 (3.7)	1.5 (15.2)	100 (991.0)

[a] Percentage distribution shown as the top entry; employment distribution, in full-time job equivalents, enclosed in parentheses.

[b] Rows do not always add exactly to totals, due to rounding.

Secondly, it can be seen that 73.5 percent of total community employment was generated by sales to non-local final demand; the remaining 26.5 percent was generated by local sales (ie., sales to local industries and sales to local final demand). This indicates that Leaf Rapids' economy is an open one which in turn implies that the community's economic well-being depends upon factors over which the community and its residents have little or no control. These include the demand for metal ores and concentrates by non-local smelters and refineries, the level of non-local government spending, the level of tourist spending, etc.

Thirdly, almost 65 percent of total community employment is generated by the sale of metal ores and concentrates to non-local smelters and refineries. This, combined with the fact that 6.1 percent of total community employment was generated by sales to the metal mining industry, indicates that the community is highly dependent on the exploitation of mineral resources. As was pointed out earlier, this renders the community quite susceptible to periods of boom and bust.

4.2.2 *The Computation of Employment Multipliers*

The level of economic activity within a region's economy can be viewed as a function of forces operating within and without that economy. In the case of Leaf Rapids, the external forces, by virtue of the fact that almost 75 percent of total community employment is generated by sales to non-local final demand, give rise to greater fluctuations in the level of economic activity in the community. For this reason, the focus here is on the computation of employment multipliers which measure the effect on total community employment of shifts arising in the various non-local final demand sectors.

The methodology used to compute the employment multipliers is that developed by Hansen and Tiebout (1963) in their input-output study of the California economy. The first step in the computation of the multipliers is the tracing of the inter-industry output flows to the seven final demand sectors. Essentially, this involves collapsing the entries in the first ten columns of Table 4.3 into the seven final demand columns by directly and indirectly assigning all employment in each industry to the final demand sectors.

The technique used to trace the inter-industry output flows involved three steps.[33] The first of these was the computation of direct employment requirements coefficients. These coefficients were calculated by dividing the entries in each industry's column in Table 4.3 by that industry's total employment. Each coefficient derived in this manner indicates the number of jobs in the row industry required

[33]It should be pointed out that the technique used in this study to trace the inter-industry output flows differs from the one used by Hansen and Tiebout (1963), mainly because theirs is extremely tedious and time-consuming. Instead, a modified version of the one used by Lee et al. (1973) in their input-output study of the Tennessee economy was used.

per employee in the column industry.[34] The direct employment requirements coefficients obtained in this manner are shown in Table 4.4.

The second step involved the computation of the direct and indirect employment requirements coefficients. This was done by: (1) writing the direct employment requirements coefficients in Table 4.4 in matrix form; (2) subtracting this matrix from the identity matrix; and (3) inverting the resulting matrix.[35] The direct and indirect employment requirements coefficients derived in this manner are shown in Table 4.5.

The last step involved the assigning of all employment in each industry, either directly or indirectly, to each of the final demand sectors. The procedure used to accomplish this will be illustrated using the data in Tables 2.1 and 2.6. Note that for the purpose of this illustration, it will be assumed that the data in these tables are expressed in terms of employment rather than dollars.[36]

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[34] Note that except for the unit of measure, employment, this is akin to the usual input-output direct requirement coefficient.

[35] The matrix was inverted using a built-in (or "canned") computer program at the University of Manitoba's computer facility. A description of this program is provided in Appendix F.

[36] This procedure is not illustrated using the data in Tables 4.3 and 4.5 as the computations involved are too unwieldy for this purpose.

Table 4.4

Table of Direct Employment Requirements
Coefficients, Leaf Rapids, Manitoba, 1977

	Metal Mining	Services Incidental to Mining	Fishing and Trapping	Construction	Transportation	Communications and Utilities	Wholesale Trade	Retail Trade	Finance, Insurance, and Real Estate	Services
Metal Mining	0	0	0	0	0	0	0	0	0	0
Services Incidental to Mining	0.0528	0	0	0	0	0	0	0	0	0
Fishing and Trapping	0	0	0	0	0	0	0	0	0	0
Construction	0.0183	0	0	0.0282	0.0400	0	0	0	0	0
Transportation	0.0134	0	0	0.0352	0	0	0.0500	0.0074	0	0.0042
Communications and Utilities	0.0020	0	0	0	0	0	0	0.0019	0.0071	0.0042
Wholesale Trade	0.0053	0	0	0	0.0200	0	0.0200	0.0130	0	0
Retail Trade	0.0008	0.0029	0	0.0282	0	0	0.0100	0.0056	0.0036	0.0042
Finance, Insurance, and Real Estate	0	0	0	0	0	0	0	0.1301	0.0249	0.0355
Services	0.0009	0	0	0	0	0	0	0	0	0

Table 4.5

Table of Direct and Indirect Employment Requirements Coefficients, Leaf Rapids, Manitoba, 1977

	Metal Mining	Services Incidental to Mining	Fishing and Trapping	Construction	Transportation	Communications and Utilities	Wholesale Trade	Retail Trade	Finance, Insurance, and Real Estate	Services
Metal Mining	1.0000	0	0	0	0	0	0	0	0	0
Services Incidental to Mining	0.0528	1.0000	0	0	0	0	0	0	0	0
Fishing and Trapping	0	0	1.0000	0	0	0	0	0	0	0
Construction	0.0194	0	0	1.0305	0.0143	0	0.0021	0.0003	0	0.0002
Transportation	0.0144	0	0	0.0365	1.0025	0	0.0512	0.0081	0	0.0042
Communications and Utilities	0.0020	0	0	0.0001	0	1.0000	0	0.0029	0.0073	0.0045
Wholesale Trade	0.0057	0	0	0.0011	0.0205	0	1.0216	0.0315	0	0.0001
Retail Trade	0.0016	0.0029	0	0.0292	0.0014	0	0.0103	1.0063	0.0037	0.0044
Finance, Insurance, and Real Estate	0.0002	0.0004	0	0.0039	0.0002	0	0.0014	0.1343	1.0260	0.0349
Services	0.0009	0	0	0	0	0	0	0	0	1.0000

The employment directly assigned to a given final demand sector consists of the entries in that sector's column in the input-output transactions table. Thus, for the agricultural industry, the employment directly assigned to the local consumption sector is 100 while the employment directly assigned to the export sector is also 100. Similarly, for the manufacturing sector, the employment directly assigned to the local consumption and export sectors is 100 and 300, respectively.

The employment directly and indirectly assigned to a given final demand sector is obtained by multiplying the matrix of direct and indirect employment requirements coefficients by a column vector comprising of the employment directly assigned to the final demand sector in question. For example, in the case of the local consumption sector:

$$\begin{bmatrix} 1.196 & 0.326 & 0.435 \\ 0.815 & 1.359 & 0.978 \\ 0.163 & 0.272 & 1.196 \end{bmatrix} \begin{bmatrix} 100 \\ 100 \\ 300 \end{bmatrix} = \begin{bmatrix} 282.7 \\ 510.8 \\ 402.3 \end{bmatrix}$$

Thus, the employment in the agricultural industry directly and indirectly assigned to the local consumption sector is 282.7. Similarly, the employment in the manufacturing and service industries directly and indirectly assigned to the local consumption sector is 510.8 and 402.3, respectively.

The employment indirectly assigned to a given final demand sector is obtained by subtracting the employment directly assigned to the final demand sector in question from the employment directly and indirectly assigned to that final demand sector. For example, the employment in the agricultural industry indirectly assigned to the local consumption sector is 182.7 (ie., 282.7 - 100). Similarly, the employment in the manufacturing industry indirectly assigned to the local consumption sector is 410.8 (ie., 510.8 - 100).

By carrying out the above process for each final demand sector, all employment in Table 4.3 was assigned, either directly or indirectly, to the seven final demand sectors. The results of this process are shown in Table 4.6. Note that this table is nothing more than a more sophisticated version of Table 4.2 In Table 4.2, total community employment was allocated to either the basic sector or the non-basic sector whereas in Table 4.6, it was allocated to seven final demand sectors.[37] Moreover, the inter-industry relationships were taken into account when allocating each industry's employment to the local and non-local final demand sectors.[38]

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[37] The two local final demand sectors in Table 4.6 correspond to the non-basic sector in Table 4.2; the five non-local final demand sectors in Table 4.6 correspond to the basic sector in Table 4.2.

[38] In Table 4.2, only the direct linkages were taken into account when allocating each industry's employment to the basic and non-basic sectors. For example, since all of the services incidental to mining industry's output was sold to the metal mining industry, then all of the former's employment was allocated to the non-basic sector. In Table 4.6 on the other hand, both the direct and non-direct linkages were taken into account when allocating each industry's employment to the local and non-local final demand sectors. For example, going back to the services incidental to mining industry, since all of this industry's output was sold to the metal mining industry and since all of the latter industry's output was sold to the exports: metal ores and concentrates sector, then all of the services incidental to mining industry's employment was allocated to the exports: metal ores and concentrates sector.

Table 4.6

Total Direct and Indirect Employment[a] to Demand Sectors, Leaf Rapids, Manitoba, 1977

From Industry Groups \ To Final Demand Sectors	Local Consumption	Local Government	Exports: Metal Ores and Concentrates	Exports: Fish and Fur	Exports: Nonlocal Government	Exports: Tourists	Exports: Miscellaneous	Total[b]
Metal Mining								
Direct			643.5					643.5
Indirect								
Total			643.5					643.5
Services Incidental to Mining								
Direct								
Indirect			34.0					34.0
Total			34.0					34.0
Fishing and Trapping								
Direct				5.0				5.0
Indirect								
Total				5.0				5.0
Construction								
Direct	0.1	0.7			0.6			1.4
Indirect	0.1		12.5				0.2	12.8
Total	0.2	0.7	12.5		0.6		0.2	14.2
Transportation								
Direct	0.7				0.4		3.7	4.8
Indirect	0.5		9.3	0.2	0.1		0.1	10.2
Total	1.2		9.3	0.2	0.5		3.8	15.0
Communications and Utilities								
Direct	2.0	0.3			0.1		0.8	3.2
Indirect	0.4		1.3					1.8
Total	2.4	0.3	1.3		0.1		0.8	5.0
Wholesale Trade								
Direct	0.3	0.2		4.6	0.2			5.3
Indirect	0.6		3.7	0.1			0.1	4.7
Total	0.9	0.2	3.7	4.7	0.2		0.1	10.0
Retail Trade								
Direct	42.7	1.7			1.4	2.5	3.8	52.1
Indirect	0.5		1.0				0.1	1.7
Total	43.2	1.7	1.0		1.4	2.5	3.9	53.8
Finance, Insurance, and Real Estate								
Direct	17.2	1.0					0.5	18.7
Indirect	7.4	0.3	0.1		0.3	0.4	0.7	9.4
Total	24.6	1.3	0.1		0.3	0.4	1.2	28.1
Services								
Direct	36.1				3.4	1.2	6.4	47.1
Indirect								0.7
Total	36.1				3.4	1.2	6.4	47.8
Government								
Direct		86.4			50.0			134.6
Indirect								
Total		86.4			50.0			134.6
Total Employment								
Direct	99.1	88.5	643.5	9.6	56.1	3.7	15.2	915.7
Indirect	9.5	0.3	62.5	0.3	0.4	0.4	1.2	75.3
Total	108.6	88.8	706.0	9.9	56.5	4.1	16.4	991.0

[a] In full-time job equivalents.

[b] Rows do not always add exactly to totals, due to rounding.

The second step in the computation of employment multipliers involves the computation of a multiplier which measures the direct and indirect impact on total community employment of a change in the level of employment directly assigned to a given final demand sector. The formula used to compute this multiplier is:

$$k = \frac{E_{d+1}}{E_d} \qquad (4.3)$$

where k is the multiplier in question, E_{d+i} is the employment directly and indirectly assigned to a given final demand sector, and E_d is the employment directly assigned to that final demand sector. Thus, for the exports: metal ores and concentrates sector:

$$k = \frac{706.0}{643.5} = 1.10 \qquad (4.4)$$

where the values of E_{d+i} and E_d were obtained from Table 4.6.

The third step in the computation of employment multipliers involves the computation of a multiplier which measures the direct, indirect, and induced impact on total community employment, <u>in the short-run</u>, of a change in the level of employment directly assigned to a given final demand sector. This multiplier is computed for all final demand sectors except the local consumption sector. The formula used to compute this multiplier is:[39]

$$k_{sr} = \frac{k}{1 - \frac{E_c}{E_t}} \qquad (4.5)$$

where k_{sr} is the short-run multiplier in question, k is the multiplier which measures the direct and indirect impact on total employment of a change in the level of employment directly assigned to the final demand sector in question, E_c is the employment directly and indirectly assigned to the local consumption sector, and E_t is total employment. For the exports: metal ores and concentrates sector, then:

$$k_{sr} = \frac{1.10}{1 - \frac{108.6}{991.0}} = 1.24 \qquad (4.6)$$

where the values of E_c and E_t were obtained from Table 4.6 and the value of k was computed above.

The last step in the computation of employment multipliers involves the computation of a multiplier which measures the direct, indirect, and induced impact on total employment, <u>in the long-run</u>, of a change in the level of employment directly assigned to a given final demand

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[39]The derivation of this formula is shown in Appendix G.

sector.[40] This multiplier is only computed for the non-local final demand sectors. The formula used to compute

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[40]The difference between this multiplier and the previous one is as follows. When the level of employment directly assigned to a given final demand sector increases, total community employment expands, both directly and indirectly. As the level of total community employment expands, so does the level of community income (due to the wages and salaries paid to the new employees). Unless all of the additional community income is saved, taxed away, or spent outside the community, the increase in the level of community income should lead to a roughly proportional increase in consumer demand for the output of the community's intermediate sector, which in turn should lead to a further expansion of total community employment. This is the direct, indirect, and induced impact on total employment measured by the short-run multiplier.

In the long-run, however, the original increase in the level of employment directly assigned to the final demand sector in question should not only lead to an expansion of consumer demand, it should also lead to an expansion of the local government. As total employment in the community expands, so would the population. This would lead to an increase in the demand for roads, schools, municipal facilities, and other such services provided by the local government. This increase in the demand for services provided by the local government should induce the local government to expand, thereby increasing the number of people it employs as well as increasing the amount of goods and services it purchases from the community's intermediate sector. This in turn should induce a further increase in the level of total community employment. This is the direct, indirect, and induced impact on total employment measured by the long-run multiplier.

this multiplier is:[41]

$$k_{1r} = \frac{k}{1 - \frac{E_c}{E_t} - \frac{E_{1g}}{E_t}} \qquad (4.7)$$

where k_{1r} is the long-run multiplier and E_{1g} is the employment directly and indirectly assigned to the local government sector. For the exports: metal ores and concentrates sector, then:

$$k_{1r} = \frac{1.10}{1 - \frac{108.6}{991.0} - \frac{88.8}{991.0}} (1.10) = 1.37 \qquad (4.8)$$

where the values of E_c, E_{1g}, and E_t were obtained from Table 4.6 and the value of k was computed above.

The overall values of the multipliers for each of the final demand sectors are summarized in Table 4.7. These multipliers can be used to assess the impact on total community employment of a change in the level of employment directly assigned to a given final demand sector. This can be illustrated with the following example. Suppose that the level of employment in the metal mining industry were to increase by 100. This, because of the fact that all of this industry's output is sold to the exports: metal ores and concentrates sector, would in turn cause the level of employment directly assigned to this particular final demand sector to rise by the same amount. Using the appropriate multiplier from Table 4.7, it can be seen that this increase in employment in the metal mining industry would, in the short-run, create

[41]The derivation of this formula is shown in Appendix G.

Table 4.7

Impact of Final Demand Changes on Employment[a] Leaf Rapids, Manitoba, 1977

	Local Consumption	Local Government	Exports: Metal Ores and Concentrates	Exports: Fish and Fur	Exports: Nonlocal Government	Exports: Tourists	Exports: Miscel-laneous
D + In	1.10	1.00	1.10	1.03	1.00	1.11	1.08
D + In Id-S		1.12	1.24	1.16	1.12	1.25	1.21
D + In Id-L			1.37	1.29	1.25	1.39	1.35

[a] The following abbreviations are used:

D + In — direct plus indirect effect
D + In + Id-S — direct plus indirect plus short-run induced effect
D + In + Id-L — direct plus indirect plus long-run induced effect

124 (i.e., 100 x 1.24) new jobs in the community while in the long-run, this increase would create 137 (i.e. 100 x 1.37) new jobs. By the same token, if the level of employment in the metal mining industry were to decline by 100, this would, in the short-run, cause total community employment to decline by 124 while in the long-run, this would cause total community employment to decline by 137.

4.3 Points to Keep in Mind When Using Employment Multipliers

It is imperative that the following points be kept in mind when using the employment multipliers obtained from both the economic base study and the input-output study of Leaf Rapids. First of all, as forecasting devices, both economic base analysis and input-output analysis are subject to a number of limitations. These were discussed in an earlier chapter and will therefore not be repeated here.

Secondly, it should be kept in mind that employment, the unit of measure in both studies, is in some instances not a sensitive indicator of changes in the level of economic activity. Suppose, for example, that with an unaltered employment level, the metal mining industry were to experience an increase in productivity. Since productivity increases generally result in higher wages, consumer spending in the community would likely rise, and this in turn could lead to an increase in the level of employment in a number of local industries. Such a change in the level of total community employment, however, would not be measured by an employment multiplier since the level of employment in the metal mining industry would not have changed.

Thirdly, it should be kept in mind that both economic base analysis input-output analysis take "supply" for granted. That is, when considering the effects of demand on regional activity, both techniques implicitly assume:

> ... that supplies of inputs would automatically be forthcoming, at no increase in per-unit cost, to support any additional activity responding to increased demand. In other words, supplies of inputs, such as labor, capital, imports, and public services, ... [are] taken to be perfectly elastic and consequently imposing no constraint on regional growth (Hoover, 1975, p. 231).

For example, if the demand for labour exceeds the community's labour force, it is implicitly assumed that more workers will move in from other areas.

CHAPTER 5

DISCUSSION

5.1 The Advantages of Economic Base Analysis over Input-Output Analysis

Economic base analysis has two main advantages over input-output analysis. First of all, the former's data requirements are not as great as the latter's. For example, in the economic base study of Leaf Rapids, representatives of each business establishment in the community would have been asked to: (1) break down their establishment's total sales between those to customers from outside the community and those to customers from within the community; and (2) indicate the approximate number of people employed by their establishment. As for the representatives of government establishments in the community, these would have been asked to indicate the approximate number of people employed by their establishment.

The data requirements of the input-output study of Leaf Rapids, on the other hand, were much greater. The representatives of each business establishment in the community were asked to: (1) break down their establishment's total sales between those to customers, excluding tourists, from outside the community and those to customers, including tourists, from within the community; (2) break down their establishment's sales to customers from outside the community between those to the federal and provincial governments and those to customers other than those governments;

(3) break down their establishment's sales to customers from within the community between those to local consumers, those to tourists, those to the local government, and those to local businesses; (4) allocate their establishment's sales to local businesses to each of the ten industry groups in the community; and (5) to indicate the approximate number of people employed by their establishment. As for the representatives of government establishments in the community, these were asked to indicate the approximate number of people employed by their establishment.

Because an economic base study involves the collecting and processing of less data than an input-output study, the former can therefore be carried out in less time than the latter. Furthermore, since the old saying that "time is money" holds true, it follows that the cost of carrying out an economic base study is less than the cost of carrying out an input-output study.

The second main advantage of economic base analysis over input-output analysis is that the computation of an economic base multiplier requires substantially less time and effort than the computation of employment multipliers using the data in an input-output transactions table. For example, in the case of the economic base study of Leaf Rapids, the economic base multiplier was computed simply by dividing total community employment by that portion of total community employment assigned to the basic sector.

The computation of employment multipliers from the data in Leaf Rapids' input-output transactions table, on the other hand, required substantially more time and effort. It should be pointed out that the computation of the multipliers as such was relatively easy, involving the use of three simple formulas. It's just that before the multipliers could be computed, all employment in the transactions table had to be assigned, both directly and indirectly, to the seven final demand sectors.

It is important to note that the importance of the above advantages is a function of the size of the community under study. In the case of a small community, such as Leaf Rapids, these advantages are not overly important, for the following reasons. First of all, because the number of business and government establishments in a small mining community is generally small, the amount of data which must be collected and processed in an input-output study is not all that great. Secondly, because the number of industry groups in such a community is not large, the task of directly and indirectly assigning all of the employment in the transactions table to the final sectors is not all that difficult and time-consuming. Thus, the time, effort, and cost involved in an input-output study of a small community are not much greater than those involved in an economic base study.

On the other hand, if the community under study is large, the above advantages can be important. Because the number of business and government establishments in such a community is large, the amount of data which must be collected and processed in an input-output study can be quite large. Secondly, because the number of industry groups in such a community can be large, the task of directly and indirectly assigning all of the employment in the transactions table to the final demand sectors can be somewhat time-consuming. Consequently, the time, effort, and cost inolved in an input-output study of a large community can be substantially greater than those involved in an economic base study.

5.2 The Advantages of Input-Output Analysis Over Economic Base Analysis

Input-output analysis has two main advantages over economic base analysis. First of all, an input-output transactions table provides a far more detailed description of the internal workings of a community's economy than does a table in which total community employment is broken down between basic and non-basic employment.

Table 4.2 shows the percentage of employment in each industry and in the community as a whole which was generated by sales to outside customers and the percentage generated by sales to local customers. For example, according to Table 4.2, some 73.5 percent of total community employment was generated by sales to outside customers while the remaining 26.5 percent was generated by sales to local customers. But what proportion of total community employment was generated by sales to the federal and provincial governments? What proportion was generated by sales to local consumers? How many jobs in the community were generated by tourist expenditures? By expenditures by local business establishments? And so on. The answers to these and similar questions simply cannot be obtained from Table 4.2.

The answer to these questions, on the other hand, can readily be obtained from an input-output transactions table. For example, referring to Table 4.3, it can be seen that 5.7 percent of total community employment was generated by sales to the federal and provincial governments, 10 percent was generated by sales to local consumers, 0.4 percent was generated by sales to tourists, and 7.5 percent by sales to local business establishments. In sum, then, an input-output transactions table provides a far more comprehensive picture of the internal workings of Leaf Rapids' economy than does a table which only shows the proportion of employment allocated to the basic sector and the proportion allocated to the non-basic sector.

The second main advantage of input-output analysis over economic base analysis is that unlike the latter, which yields only one multiplier, input-output analysis can be used to compute a variety of multipliers. For example, in the economic base study of Leaf Rapids, only one multiplier, the economic base multiplier, could be computed. This multiplier, which measures the impact on total community employment of a change in the level of employment in the basic sector, has a number of shortcomings. First of all, the economic base multiplier does not apply to any specific basic activity. As a result, a change in the level of employment in, say, the metal mining industry is viewed as having the same impact on total community employment as a similar change in the level of, say, non-local government employment. Secondly, the economic multiplier does not distinguish the short-term impact on total community employment of a given change in the level of basic employment from the long-term impact of such a change.

The employment multipliers computed in the input-output study of Leaf Rapids, on the other hand, do not suffer from the above shortcomings. First of all, it was possible to compute employment multipliers which apply to specific economic activities (eg., local consumption, exports: metal ores and concentrates, etc.). Secondly, for each of these activities, it was possible to compute multipliers which distinguish the short-term impact from the long-term impact.

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APPENDIX A

THE DERIVATION OF THE ECONOMIC BASE MULTIPLIER FORMULAS

The derivation of the economic base multiplier formulas will be illustrated with the use of a simple economic model. Imagine a region where all of the income accruing to the region's residents can be subdivided into basic income and nonbasic income. Thus:

$$Y_t = Y_b + Y_n \qquad (A.1)$$

where Y_t is total regional income, Y_b is basic income, and Y_n is nonbasic income. Furthermore, imagine that non-basic income is generated via the following two-step process: (1) the residents spend part of their income locally on goods and services; and (2) part of these consumption expenditures remain within the regional economy and become nonbasic income while the remainder "leaks" out of the regional economy. Thus:

$$Y_n = a\ b\ Y_t \qquad (A.2)$$

where a is the region's marginal propensity to consume[42] and b is the marginal propensity of regional consumption expenditures to become nonbasic income.[43]

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[42]The marginal propensity to consume is the amount of extra consumption expenditures generated by an extra dollar of income.

[43]The marginal propensity of regional consumption expenditures to become nonbasic income is the amount of extra nonbasic income generated by a one dollar increase in consumption expenditures.

Substituting equation (A.2) into equation (A.1) yields:

$$Y_t = Y_b + a\ b\ Y_t \tag{A.3}$$

Solving equation (A.3) for Y_t yields:

$$Y_t = \frac{Y_b}{1 - a\ b} \tag{A.4}$$

An equation which illustrates how the level of regional income changes with respect to a change in the level of basic income can be obtained by differentiating equation (A.4) with respect to basic income, as follows:

$$\frac{dY_t}{dY_b} = \frac{1}{1 - ab} \tag{A.5}$$

where dY_t is the change in the level of regional income and dY_b is the change in the level of basic income. Rearranging equation (A.5) as follows:

$$dY_t = \frac{1}{1 - ab}\ dY_b \tag{A.6}$$

yields an equation which can be used to assess the change in the level of regional income resulting from a given change in the level of basic income.

It can be seen in equation (A.6) that a change in the level of basic income causes the level of regional income to change by some multiple of the initial change in the level of basic income. This multiple is:

$$k_{eb} = \frac{1}{1 - ab} \tag{A.7}$$

where k_{eb} is the economic base multiplier.

The marginal propensity to consume can be computed using the formula:

$$a = \frac{\Delta C}{\Delta Y_t} \tag{A.8}$$

where ΔC is the change in regional consumption expenditures and ΔY_t is the change in regional income. The marginal propensity of regional consumption expenditures to become nonbasic income can be computed using the formula:

$$b = \frac{\Delta Y_n}{\Delta C} \tag{A.9}$$

where ΔY_n is the change in nonbasic income. If we assume that, in the case of our hypothetical regional economy, marginal propensities are equal to average propensities, then equations (A.8) and (A.9) can be rewritten as:

$$a = \frac{C}{Y_t} \tag{A.10}$$

where C is the level of regional consumption expenditures, and,

$$b = \frac{Y_n}{C} \tag{A.11}$$

Substituting equations (A.10) and (A.11) into equation (A.6) yields:

$$k_{eb} = \frac{1}{1 - \left(\frac{C}{Y_t}\frac{Y_n}{C}\right)} = \frac{1}{1 - \frac{Y_n}{Y_t}} \tag{A.12}$$

Manipulating equation (A.12) as follows:

$$k_{eb} = \frac{1}{1 - \frac{Y_n}{Y_t}} = \frac{1}{\frac{Y_t - Y_n}{Y_t}} = \frac{1}{\frac{Y_b}{Y_t}} = \frac{Y_t}{Y_b} \qquad \text{(A.13)}$$

yields a second equation which can be used to compute the economic base multiplier.

APPENDIX B

A REVIEW OF MATRIX ALGEBRA[44]

B.1 Introduction

The following is a brief review of the fundamentals of matrix algebra. For a more detailed discussion, see any standard first-year university level algebra text.

B.2 Determinants

Before turning to a discussion of matrices, it will be necessary to discuss briefly the concept of a determinant as a prerequisite to a later discussion of matrix inversion.

A determinant comprises of a number of quantities, hereafter referred to as elements, arranged in rows and columns to form a square. These quantities, or elements, may represent numbers, constants, variables, or anything which can take on a single numerical value.

The order of a determinant depends on the number of rows and columns in the determinant. A second order determinant has two rows and two columns; a third order determinant has three rows and three columns, and so on. The example below shows a third order determinant:

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 3 & 1 \\ 1 & 5 & 6 \end{vmatrix}$$

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[44] This review draws heavily from Miernyk (1965, pp. 132-143) and Masser (1972, pp. 134 - 137).

The elements of a third or higher-order determinant can be expressed in terms of minors and cofactors. In defining these terms, the following notation will be used. The example below shows a third order determinant:

$$\begin{vmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{vmatrix}$$

The subscripts in the above determinant are used to denote the location of any element in the determinant. The first subscript identifies the row; the second identifies the column. For example, b_{13} represents the element in the first row and the third column of the determinant.

The minor of a given element of a third order determinant comprises of the second order determinant which remains when the row and the column of the element in question are deleted. Minors will be denoted by the symbol Δ. Appropriate subscripts will indicate the minor of a given element. For example, the minor of element b_{11} is:

$$\Delta_{11} = \begin{vmatrix} b_{22} & b_{23} \\ b_{32} & b_{33} \end{vmatrix}$$

(i.e., the rows and columns which remain after row 1 and column 1 are deleted). Similarly, the minor of element b_{33} is:

$$\Delta_{33} = \begin{vmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{vmatrix}$$

The cofactor of an element consists of that element's minor with the appropriate sign attached. The symbol B will be used to denote cofactors. If the sum of the subscripts is an <u>even</u> number, such as B_{11}, the cofactor will have a <u>plus</u> sign; if the sum is an <u>odd</u> number, such as B_{32}, the cofactor will have a <u>minus</u> sign. The cofactors of the above determinant are:

$$B_{11} = + \begin{vmatrix} b_{22} & b_{23} \\ b_{32} & b_{33} \end{vmatrix}$$

$$B_{12} = - \begin{vmatrix} b_{21} & b_{23} \\ b_{31} & b_{33} \end{vmatrix}$$

$$B_{13} = + \begin{vmatrix} b_{21} & b_{22} \\ b_{31} & b_{32} \end{vmatrix}$$

and so on. Each of the cofactors is evaluated as follows:

$$B_{11} = (b_{22}b_{33}) - (b_{23}\ b_{32})$$

$$B_{12} = -(b_{21}b_{33}) - (b_{23}\ b_{31})$$

$$B_{13} = (b_{21}b_{32}) = (b_{22}\ b_{31})$$

Note that cofactors for only three of the above third order determinant's elements have been written out, to illustrate the <u>rule of signs</u>. Similar cofactors can be written for each of the remaining six elements in this determinant. These however, will not be shown here.

B.3 Matrices

A matrix is a rectangular array of numbers with m rows and n columns. The example below shows a matrix with two rows and three columns:

$$\begin{bmatrix} 0 & 1 & 2 \\ 4 & 6 & 9 \end{bmatrix}$$

Note that unlike a determinant, a matrix need not be square (i.e., it is not necessary for the number of rows to be equal to the number of columns).

A vector is a matrix with one row or one column. The examples below show vectors consisting of one row and three columns and three rows and one column, respectively:

$$\begin{bmatrix} 4 & 5 & 6 \end{bmatrix} \qquad \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

A standard notation is generally used to represent matrices, vectors, and the elements they contain. The use of a capital letter denotes a matrix while a subscripted lowercase letter is used to denote an element of this matrix. The use of a lowercase letter denotes a vector while a subscripted lowercase letter is used to denote an element of this vector. Thus:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$c = \begin{bmatrix} c_1 & c_2 & c_3 \end{bmatrix}$$

$$d = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

Before turning to a discussion of the basic laws of matrix algebra, let us first have a look at a number of definitions.

The principal diagonal of a square matrix (i.e., a matrix where the number of rows equals the number of columns) comprises of the elements running from the upper left hand corner to the bottom right hand corner. That is, the principal diagonal comprises of all of the elements in a square matrix whose row subscript equals the column subscript. Thus:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

where the principal diagonal is contained between the two diagonal lines.

A square matrix is nonsingular if its determinant is not equal to zero. If a matrix is singular (i.e., if its determinant is equal to zero), its inverse cannot be defined.

A matrix which consists of 1's along the principal diagonal with all of the other elements equal to zero is referred to as an identity matrix. Thus:

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

where the symbol I is used to denote the identity matrix.

If the rows and column of matrix have been interchanged, the result is referred to as a transposed matrix. Thus, if:

$$C=\begin{bmatrix} 1 & 2 & 3 \\ 5 & 4 & 1 \\ 1 & 3 & 7 \end{bmatrix}$$

then:

$$C^T=\begin{bmatrix} 1 & 5 & 1 \\ 2 & 4 & 3 \\ 3 & 1 & 7 \end{bmatrix}$$

where the symbol C^T denotes the transpose of matrix C.

Let us now turn to a discussion of some of the basic laws of matrix algebra. The sum of two matrices is obtained by adding the corresponding elements of the two matrices. This is illustrated in the following example:

$$A=\begin{bmatrix} 3 & 5 & 8 \\ 4 & 1 & 0 \end{bmatrix} \qquad B=\begin{bmatrix} 4 & 5 & 6 \\ 2 & 1 & 2 \end{bmatrix}$$

$$A+B=\begin{bmatrix} 3+4=7 & 5+5=10 & 8+6=14 \\ 4+2=6 & 1+1=2 & 0+2=2 \end{bmatrix} = \begin{bmatrix} 7 & 10 & 14 \\ 6 & 2 & 2 \end{bmatrix}$$

Similarly, the difference between two matrices is obtained by subtracting the corresponding elements of the two matrices. Thus, for matrices A and B above:

$$A-B=\begin{bmatrix} 3-4=-1 & 5-5=0 & 8-6=2 \\ 4-2=2 & 1-1=0 & 0-2=-2 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 2 \\ 2 & 0 & -2 \end{bmatrix}$$

Note that a necessary prerequisite for the addition and subtraction of matrices is that the matrices have the same number of rows and columns.

Matrices can only be multiplied when the number of columns in the first matrix equals the number of rows in the second matrix. If this condition is satisfied, then the matrices are said to be conformable. Conformable matrices are multiplied as follows:

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \qquad B = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{bmatrix}$$

$$AB = \begin{bmatrix} (a_{11}\ b_{11} + a_{12}\ b_{21}) & (a_{11}\ b_{12} + a_{12}\ b_{22}) & (a_{11}b_{13} + a_{12}b_{23}) \\ (a_{21}\ b_{11} + a_{22}\ b_{21}) & (a_{21}\ b_{12} + a_{22}\ b_{22}) & (a_{21}b_{13} + a_{22}b_{23}) \end{bmatrix}$$

Consider now the following numerical example. Let:

$$A = \begin{bmatrix} 3 & 5 \\ 4 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 4 & 5 & 6 \\ 2 & 1 & 2 \end{bmatrix}$$

Then:

$$AB = \begin{bmatrix} (3x4) + (5x2) = 22 & (3x5)+(5x1) = 20 & (3x6)+(5x2) = 28 \\ (4x4) + (1x2) = 18 & (4x5)+(1x1) = 21 & (4x6)+(1x2) = 26 \end{bmatrix}$$

$$= \begin{bmatrix} 22 & 20 & 28 \\ 18 & 21 & 26 \end{bmatrix}$$

The multiplication of a matrix by a vector or a vector by a vector follows the same basic rules. For instance:

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \qquad c = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

$$Ac = \begin{bmatrix} (a_{11}c_1 + a_{12}c_2) \\ (a_{21}c_1 + a_{22}c_2) \end{bmatrix}$$

and:

$$d = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} \qquad e = \begin{bmatrix} e_1 & e_2 & e_3 \end{bmatrix}$$

$$de = \begin{bmatrix} d_1e_1 & d_1e_2 & d_1e_3 \\ d_2e_1 & d_2e_2 & d_2e_3 \\ d_3e_1 & d_3e_2 & d_3e_3 \end{bmatrix}$$

It should be pointed out at this time that matrix multiplication does not always follow the commutative law of mathematics whereby the product of a x b is equal to the product of b x a. This can be illustrated as follows.

Let:

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$$

Then:

$$AB = \begin{bmatrix} (1x2) + (3x1) = 5 & (1x4) + (3x3) = 13 \\ (2x2) + (0x1) = 4 & (2x4) + (0x3) = 8 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 13 \\ 4 & 8 \end{bmatrix}$$

whereas:

$$BA = \begin{bmatrix} (2x1)+(4x2) = 10 & (2x3)+(4x0) = 6 \\ (1x1)+(3x2) = 7 & (1x3)+(3x0) = 3 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 6 \\ 7 & 3 \end{bmatrix}$$

In ordinary arithmetic, two numbers are said to be inverses of each other if, and only if, their product is equal to 1 (eg., 2x½ = 1 where ½ is the inverse of 2 and 2 is the inverse of ½), Similarly, two square matrices are said to be inverses of each other if, and only if, their product is equal to the identity matrix. That is, $AA^{-1} = I$ where A^{-1} is the symbol used to denote the inverse of matrix A.

The procedure used to compute the inverse of a matrix will be illustrated using the following numerical example. Let:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 3 \\ 1 & 2 & 4 \end{bmatrix}$$

The first step in the computation of the inverse matrix involves the evaluation of the determinant of the matrix by expanding along the cofactors of row 1, as follows:

$$D = \begin{vmatrix} 1 & 2 & 3 \\ 1 & 3 & 3 \\ 1 & 2 & 4 \end{vmatrix}$$

$$= 1 \begin{vmatrix} 3 & 3 \\ 2 & 4 \end{vmatrix} - 2 \begin{vmatrix} 1 & 3 \\ 1 & 4 \end{vmatrix} + 3 \begin{vmatrix} 1 & 3 \\ 1 & 2 \end{vmatrix}$$

$$= 1(12-6) - 2(4-3) + 3(2-3) = 1$$

The value of the determinant in this example is unity.

The second step in the computation of the inverse matrix involves the identification of all of the cofactors of the determinant, as follows:

Cofactors of D

$$A_{11} = \begin{vmatrix} 3 & 3 \\ 2 & 4 \end{vmatrix} \; (6) \qquad A_{12} = -\begin{vmatrix} 1 & 3 \\ 1 & 4 \end{vmatrix} \; (-1) \qquad A_{13} = \begin{vmatrix} 1 & 3 \\ 1 & 2 \end{vmatrix} \; (-1)$$

$$A_{21} = -\begin{vmatrix} 2 & 3 \\ 2 & 4 \end{vmatrix} \; (-2) \qquad A_{22} = \begin{vmatrix} 1 & 3 \\ 1 & 4 \end{vmatrix} \; (1) \qquad A_{23} = -\begin{vmatrix} 1 & 2 \\ 1 & 2 \end{vmatrix} \; (0)$$

$$A_{31} = \begin{vmatrix} 2 & 3 \\ 3 & 3 \end{vmatrix} \; (-3) \qquad A_{32} = -\begin{vmatrix} 1 & 3 \\ 1 & 3 \end{vmatrix} \; (0) \qquad A_{33} = \begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix} \; (1)$$

The numbers in parentheses above each of the cofactors are the values of the cofactors with the appropriate signs taken into account.

The third step in the computation of the inverse matrix involves the arranging of the cofactors in matrix form and the transposing of this matrix. Thus:

$$\text{Matrix of cofactors} = \begin{vmatrix} 6 & -1 & -1 \\ -2 & 1 & 0 \\ -3 & 0 & 1 \end{vmatrix}$$

and:

$$\text{Adjoint matrix} = \begin{vmatrix} 6 & -2 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix}$$

Note that to avoid confusion with a transposed matrix as such, the transposed matrix of cofactors is here referred to as the adjoint matrix.

The fourth and last step in the computation of the inverse matrix involves the dividing of each element in the adjoint matrix by the value of the original determinant. Since, in the example, the value of the original determinant is 1, the numbers in the adjoint matrix are not changed. Thus:

$$A^{-1} = \begin{bmatrix} 6 & -2 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

Before ending the discussion of matrix inversion, it should be pointed out that a matrix multiplied by its inverse is equal to the identity matrix. That is:

$$AA^{-1} = I$$

This provides us with a means of ensuring that no errors have been made in the computation of the inverse matrix.

Multiplying the original matrix by the inverse matrix yields:

$$AA^{-1}=\begin{bmatrix}(1x6)+(2x-1)+(3x-1) & (1x-2)+(2x1)+(3x0) & (1x-3)+(2x0)+(3x1)\\(1x6)+(3x-1)+(3x-1) & (1x-2)+(3x1)+(3x0) & (1x-3)+(3x0)+(3x1)\\(1x6)+(2x-1)+(4x-1) & (1x-2)+(2x1)+(4x0) & (1x-3)+(2x0)+(4x1)\end{bmatrix}$$

$$=\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}=I$$

The above indicates that no errors have been made in the computation of the inverse matrix.

APPENDIX C

QUESTIONNAIRE FOR THE ECONOMIC BASE

STUDY OF LEAF RAPIDS

Interviewed by: ____________________

Date of Interview: ____________________

Section A: General

A.1 Name of establishment ____________________

A.2 Address of establishment ____________________

A.3 Respondent ____________________

A.4 Respondent's Title ____________________

In sections B and C, information is requested for the calendar year 1977. If it is not possible to provide data for 1977, please use the latest available year. If this questionnaire is completed for any period other than the calendar year 1977, please indicate the year used. 197__

If it is not possible to give exact answers for all questions, please use your best estimate. Please indicate where a best estimate was used.

Section B: Distribution of Sales

B.1 Consider your total sales for the calendar year 1977 as going to two types of customers:

(i) those located outside of the Leaf Rapids area (including tourists) plus all sales to the federal and the provincial governments;

(ii) those located inside of the Leaf Rapids area (excluding tourists) plus all sales to the local government.

Allocate your sales on a percent basis between these two groups:

B.1a percentage sold to outside customers plus the federal and the provincial governments 60%

B.1b percentage sold to inside customers plus the local government 40%

TOTAL:100%

Section C: Employment

In the following questions:

(a) The standard work week is the number of hours per week normally scheduled for the majority of your full-time employees.

(b) Full-time employees are those who regularly work the equivalent of the standard work week of your establishment.

(c) Part-time employees are those who regularly work fewer than the standard number of hours per work week of your establishment.

(d) Temporary employees are those who are hired for discontinuous periods of time or on an irregular basis and who worked at least seven hours during the survey reference period.

When estimating the number of people employed by your establishment, please include all persons on paid personal leave (ie., bereavement leave, jury duty, etc.), paid sick leave (including industrial injury leave), paid holidays and paid vacations. Persons who worked for part of the reference period and who were unemployed or on strike for the remainder of the reference period should also be included in your estimates. Owners and partners of unincorporated businesses and professional practices should be excluded from your estimates if they did not actually work in the establishment in question.

C.1 How many hours per week does your establishment's standard work week consist of? 40 hrs/wk

C.2 During the calendar year 1977, how many people were employed in your establishment during the last pay periods of the months of February, May, July, and November. Please remember to account for all persons who drew pay for services rendered or for paid absence during all or part of these pay periods.

	February	May	July	November
Full-time	5	5	5	5
Part-time	2	2	4	2
Temporary	-	-	-	-

C.3 On the average, how many hours per week did the part-time employees in C.2 work during the survey reference period?

February 20 hrs/wk   July 30 hrs/wk
May 20 hrs/wk   November 20 hrs/wk

C.4 How many of the temporary employees in C.2 worked fewer hours than the standard work week of your establishment?

February	-	hrs/wk	July	-	hrs/wk
May	-	hrs/wk	November	-	hrs/wk

On the average, how many hours per week did these employees work during the survey reference period?

February	-	hrs/wk	July	-	hrs/wk
May	-	hrs/wk	November	-	hrs/wk

Section D: Comments

APPENDIX D

QUESTIONNAIRE FOR THE INPUT-OUTPUT STUDY OF LEAF RAPIDS

Interviewed by: ______________________________

Date of Interview: ______________________________

Section A: General

A.1 Name of establishment ______________________

A.2 Address of establishment ______________________

A.3 Respondent ______________________

A.4 Respondent's Title ______________________

In sections B and C, information is requested for the calendar year 1977. If it is not possible to provide data for 1977, please use the latest available year. If this questionnaire is completed for any period other than the calendar year 1977, please indicate the year used. 197__

If it is not possible to give exact answers for all questions, please use your best estimate. Please indicate where a best estimate was used.

Section B: Distribution of Sales

B.1 Consider your total sales for the calendar year 1977 as going to two types of customers:

(i) those located outside of the Leaf Rapids area (excluding tourists) plus all sales to the federal and provincial governments

(ii) those located inside of the Leaf Rapids area (including tourists) plus all sales to the local government

Allocate your sales on a percent basis between these two groups:

B.1a percentage sold to outside customers plus the federal and the provincial governments 30%

B.1b percentage sold to inside customers plus the local government 70%

TOTAL: 100%

B.2 Taking a typical dollar of sales estimated in B.1a (outside of the Leaf Rapids area plus the federal and provincial governments), how would you distribute this dollar of sales among the following two groups?

B.2a sales to the federal and provincial governments	50%
B.2b sales to customers other than the federal and provincial governments	50%
TOTAL:	100%

B.3 Taking a typical dollar of sales estimated in B.1b (inside of the Leaf Rapids area plus the local government), how would you distribute this dollar of sales among the following groups?

B.3a sales directly to local consumers. Please note that the term "local consumers" refers to individuals and families who live in the Leaf Rapids area and who purchase goods and services for their own personal use	20%
B.3b sales directly to tourists	10%
B.3c sales to the local government	20%
B.3d sales to business establishments located in the Leaf Rapids area. Please note that the term "business establishments" includes self-employed individuals who purchase goods and services for business purposes	50%
TOTAL:	100%

B.4 Taking a typical dollar of sales estimated in B.3d (to local business establishments), how would you distribute this typical dollar of sales among the following industries?

(a)	metal mining	0%
(b)	services incidental to mining	0%
(c)	fishing and trapping	0%
(d)	construction	10%
(e)	transportation	0%
(f)	communications and utilities	0%
(g)	wholesale trade	0%
(h)	retail trade	40%
(i)	finance, insurance, and real estate	0%
(j)	services (business and personal)	50%
(k)	other (please specify) ________________	0%
	TOTAL:	100%

Section C: Employment

In the following questions:

(a) The standard work week is the number of hours per week normally scheduled for the majority of your full-time employees.

(b) Full-time employees are those who regularly work the equivalent of the standard work week of your establishment.

(c) Part-time employees are those who regularly work fewer than the standard number of hours per work week of your establishment.

(d) Temporary employees are those who are hired for discontinuous periods of time or on an irregular basis and who worked at least seven hours during the survey reference period.

When estimating the number of people employed by your establishment, please include all persons on paid personal leave (ie., bereavement leave, jury duty, etc.), paid sick leave (including industrial injury leave), paid holidays and paid vacations. Persons who worked for part of the reference period and who were unemployed or on strike for the remainder of the reference period should also be included in your estimates. Owners and partners of unincorporated businesses and professional practices should be excluded from your estimates if they did not actually work in the establishment in question.

C.1 How many hours per week does your establishment's standard work week consist of? 40 hrs/wk

C.2 During the calendar year 1977, how many people were employed in your establishment during the last pay periods of the months of February, May, July, and November. Please remember to account for all persons who drew pay for services rendered or for paid absence during all or part of these pay periods.

	February	May	July	November
Full-time	5	5	5	5
Part-time	2	2	4	2
Temporary	-	-	-	-

C.3 On the average, how many hours per week did the part-time employees in C.2 work during the survey reference period?

February 20 hrs/wk July 30 hrs/wk

May 20 hrs/wk November 20 hrs/wk

C.4 How many of the temporary employees in C.2 worked fewer hours than the standard work week of your establishment?

February ___-___ hrs/wk   July ___-___ hrs/wk

May ___-___ hrs/wk   November ___-___ hrs/wk

On the average, how many hours per week did these employees work during the survey reference period?

February ___-___ hrs/wk   July ___-___ hrs/wk

May ___-___ hrs/wk   November ___-___ hrs/wk

Section D: Comments

APPENDIX E

THE DATA PROCESSING SHEET FOR THE INPUT-OUTPUT STUDY

Section A: General

A.1 Name of establishment ______________________

A.2 Adress of establishment ______________________

A.3 Date ____________

Section B: Distribution of Sales

B.1 Allocation of total sales to:

(a)	customers located outside of the Leaf Rapids area (excluding tourists) plus all sales to the federal and provincial governments	30%
(b)	customers located inside of the Leaf Rapids area (including tourists) plus all sales to the local government	70%
	TOTAL:	100%

B.2 Percentage of total sales allocated to:

(a)	the federal and provincial governments	15%
(b)	customers outside of the Leaf Rapids area other than the federal and provincial governments	15%
(c)	local consumers	14%
(d)	tourists	7%
(e)	the local government	14%
(f)	local business establishments	35%
	TOTAL:	100%

B.3 Percentage of total sales allocated to:

(a)	metal mining	0%
(b)	services incidental to mining	0%
(c)	fishing and trapping	0%
(d)	construction	3.5%
(e)	transportation	0%
(f)	communications and utilities	0%
(g)	wholesale trade	0%
(h)	retail trade	14%
(i)	finance, insurance, and real estate	0%
(j)	services (business and personal)	17.5%
(k)	others ______________________	0%
	TOTAL:	35%

Section C: Annual Average Employment

C.1 Number of full-time employees:

February 5 July 5
May 5 November 5

C.2 Part-time employment converted to full-time job equivalents:

February 1 July 3
May 1 November 1

C.3 Number of temporary employees who worked the equivalents of the standard work week:

February - July -
May - November -

C.4 Temporary part-time employment converted to full-time job equivalents:

February - July -
May - November -

C.5 Total employment (in full-time job equivalents):

February 6 July 8
May 6 November 6

C.6 Annual average employment = 6.5 full-time job equivalents.

Section D: Distribution of Employment

D.1 Employment generated by sales to:

(a)	the federal and provincial governments	0.98
(b)	customers outside of the Leaf Rapids area other than the federal and provincial governments	0.98
(c)	local consumers	0.91
(d)	tourists	0.46
(e)	the local government	0.91
(f)	metal mining	0
(g)	services incidental to mining	0
(h)	fishing and trapping	0
(i)	construction	0.23
(j)	transportation	0
(k)	communications and utilities	0
(l)	wholesale trade	0
(m)	retail trade	0.91
(n)	finance, insurance, and real estate	0
(o)	services (business and personal)	1.14
(p)	others ____________________	0
	TOTAL:	6.52

APPENDIX F

THE COMPUTER PROGRAM USED TO INVERT THE MATRIX OF DIRECT EMPLOYMENT REQUIREMENTS COEFFICIENTS

THE PROGRAM

```
$JOB    WATFIV    ,NOEXT
C
C  PROGRAM FUNCTION — THE INVERSION OF A 10 BY 10 MATRIX USING
C      INTERNATIONAL MATHEMATICAL AND STATISTICAL LIBRARIES, INC.
C      (IMSL) SUBROUTINE LINV2F
C
C  PROGRAM LANGUAGE — FORTRAN
C
          REAL A(10,10), AINV(10,10), WKAREA(130)
          IA = 10
          N = 10
          IDGT = 0
          DO 1 I=1,N
    1     READ, (A(I,J), J=1,N)
          WRITE (6,102)
  102     FORMAT ('1','THE MATRIX TO BE INVERTED')
          DO 12 I=1,N
   12     WRITE (6,110) (A(I,J), J=1,N)
  110     FORMAT ('0',10F10.4)
          CALL LINV2F (A,N,IA,AINV,IDGT,WKAREA,IER)
          IF (IER.GT.128) GO TO 3
          WRITE (6,101)
  101     FORMAT ('1','THE INVERTED MATRIX')
          DO 2 I=1,N
    2     WRITE (6,107) (AINV (I,J), J=1,N)
  107     FORMAT ('0',10F10.4)
          WRITE (6,200)
  200     FORMAT ('1')
    4     STOP
    3     WRITE (6,108)
  108     FORMAT ('0','TERMINAL ERROR =', I5)
          GO TO 4
          END
$ENTRY
```

PROGRAMMING NOTES

1) The program's parameters are as follows:
   a) A - input matrix containing the matrix to be inverted
   b) AINV - output matrix containing the inverse of matrix A

c) N - the number of rows and columns in matrices A and AINV

d) IA - the number of rows in the dimension statement for matrices A and AINV

e) WKAREA - work area of dimension greater than or equal N^2+3N

f) IDGT - an input option. If IDGT is specified as being greater than zero, the elements of matrix A are assumed to be correct to IDGT decimal digits. The solution will be the exact solution (without any roundoff error) to a matrix $\bar{A}$ whose elements agree with the elements of matrix A in the first IDGT decimal digits. The program first attempts such a solution without iterative improvement. If this fails, then iterative improvement is performed. If this also fails, solution is not possible and the program exits. Upon exit, the first colums of matrix AINV will have been replaced by the best solution that the computer can generate and IDGT is set to the approximate number of digits in the answer which were unchanged by the improvement. The other columns of matrix AINV are left unchanged in this case and IER is set to 131. If IDGT is specified as being equal to zero, iterative improvement is automatically performed

g) IER - an error parameter. If the value of IER is 129, this indicates that the matrix to be inverted is algorithmically singular. If the value of IER is 131, this indicates that the matrix to be inverted is too ill-conditioned for iterative improvement

2) Matrices A and AINV must be mutually exclusive

3) The computer program shown above is written specifically for the inversion of a 10 by 10 matrix. The program, however, can be used for the inversion of any square matrix simply by changing the first three statements of the program. For example, the computer program can be used for the inversion of a 5 by 5 matrix by changing the first three statements as follows:

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1          REAL A(5,5), AINV(5,5), WKAREA(40)
2          IA = 5
3          N = 5
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APPENDIX G

THE DERIVATION OF THE EMPLOYMENT MULTIPLIER FORMULAS USED IN THE INPUT-OUTPUT STUDY[45]

The derivation of the employment multiplier formulas used in the input-output study will be illustrated with the use of a simple economic model. Let the equilibrium income of a region be given by the condition:

$$Y = C + I + G + X \quad (G.1)$$

where:

Y = the sum of all income (before taxes and depreciation) that accrues to the region's residents

C = consumption expenditures on goods and services produced in the region as well as that part of consumption expenditures on imported goods and services that creates income in the region

I = expenditures on investment goods produced in the region as well as that part of expenditures on imported investment goods that creates income in the region

G = non-local and local government expenditures on goods and services produced in the region as well as that part of government expenditures on imported goods and services that creates income in the region

X = the value of exported goods and services that accrues to the region

In the short-run, only consumption expenditures are assumed to be responsive to changes in the level of regional income. The levels of investment and government expenditures

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[45]The derivation draws heavily from Dhruvarajan et al. (1969, pp. 552-556) and Nourse (1968, pp. 174-175).

and the level of exports are independent of the level of regional income. The level of investment expenditures depends more on expectations and interest rates while the level of government expenditures depends more on contracted activities and political decisions. As for the level of exports, it is a function of the demand by non-local customers for goods and services produced in the region.

If we assume that total consumption expenditures on domestically produced as well as imported goods and services is a constant proportion of regional income, then:

$$C_t = c_t Y \qquad (G.2)$$

where C_t is total consumption expenditures and c_t is the marginal propensity to consume. The regional component of consumption expenditures[46] is given by:

$$C = \left(\frac{C}{C_t}\right) C_t = qC_t = qc_t Y = cY \qquad (G.3)$$

where q is the regional component of consumption expenditures as a proportion of total consumption expenditures and c is the regional component of the marginal propensity to consume. Substituting equation (G.3) into equation (G.1) yields:

$$Y = cY + I + G + X \qquad (G.4)$$

Solving equation (G.4) for Y yields:

$$Y = \frac{I + G + X}{1 - c} \qquad (G.5)$$

[46] That is, consumption expenditures on goods and services produced in the region as well as that part of consumption expenditures on imported goods and services that creates income in the region.

An equation which illustrates how the level of regional income changes with respect to a change in, say, the level of exports can be obtained by differentiating equation (G.5) with respect to exports, as follows:

$$\frac{dY}{dX} = \frac{1}{1 - c} \qquad (G.6)$$

where dY is the change in the level of regional income and dX is the change in the level of exports. Rearranging equation (G.6) as follows:

$$dY = \frac{1}{1 - c} dX \qquad (G.7)$$

yields an equation which can be used to assess the change in the level of regional income resulting from a given change in the level of exports. Using the same procedure for the other two terms in the numerator of equation (G.5) yields:

$$dY = \frac{1}{1 - c} dI \qquad (G.8)$$

where dI is the change in the level of investment expenditures, and:

$$dY = \frac{1}{1 - c} dG \qquad (G.9)$$

where dG is the change in the level of government expenditures.

It can be seen in equations (G.7) to (G.9) that a change in the level of investment expenditures, government expenditures, or exports causes the level of regional income to change by some multiple of the initial change in the level of investment expenditures, government expenditures, or exports. This multiple, which in all three equations is the same, is:

$$k_{sr} = \frac{1}{1 - c} \qquad (G.10)$$

where k_{sr} is a multiplier which measures the impact on regional income, in the short-run, of a given change in the level of investment expenditures, government expenditures, or exports.

In the long-run, investment expenditures are assumed to be dependent on the level of regional income. For example, as the region, and hence regional income, grows, firms are induced to expand if they are to meet the local demand for their output. Similarly, as the region grows, the level of housing investment will rise as new houses and apartment buildings are built to house the increasing population. If we assume that total investment expenditures on domestic as well as imported goods and services are a constant proportion of regional income, then:

$$I_t = i_t Y \qquad (G.11)$$

where I_t is total investment expenditures and i_t is the marginal propensity to invest. The regional component of investment expenditures[47] is given by:

$$I = (\frac{I}{I_t}) I_t = mI_t = mi_t Y = iY \qquad (G.12)$$

where m is the regional component of investment expenditures

[47]That is, expenditures on investment goods and services produced in the region as well as that part of investment and expenditures on imported goods and services that creates income in the region.

as a proportion of total investment expenditures and i is the regional component of the marginal propensity to invest.

Total government expenditures in the region are the sum of expenditures by local and non-local governments. That is:

$$G = G_l + G_{nl} \qquad (G.13)$$

where G_l is local government expenditures on goods and services produced in the region as well as that part of local government expenditures on imported good and services that creates income in the region and G_{nl} is non-local government expenditures on goods and services produced in the region as well as that part of non-local government expenditures on imported goods and services that creates income in the region.

In the long-run, local government expenditures, like investment expenditures, are assumed to be dependent on the level of regional income. For example, as the region, and hence regional income, grows, the demand for roads, schools, municipal facilities, and other such services provided by the local government will rise. This, in turn, should induce the local government to increase its expenditures. As for non-local government expenditures, these are assumed to be unresponsive to the level of regional income. As pointed out by Dhruvarajan et al. (1969, p. 554), while these expenditures may have some relation to regional income, the relation is rather tenuous and it is better to assume that they are independent of the level of regional income.

If we assume that total local government on domestic as well as imported goods and services is a constant proportion of regional income, then:

$$G_{1_t} = g_t Y \qquad \text{(G.14)}$$

where G_{1_t} is total local government expenditures and g_t is the marginal propensity to spend of the local government. The regional component of local government expenditures[48] is given by:

$$G_1 = \left(\frac{G_1}{G_{1_t}}\right) G_{1_t} = nG_{1_t} = ng_t Y = gY \qquad \text{(G.15)}$$

where n is the regional component of local government expenditures as a proportion of total local government expenditures and g is the regional component of the marginal propensity to spend of the local government. Substituting equation (G.15) into equation (G.13) yields:

$$G = gY + G_{n1} \qquad \text{(G.16)}$$

As for the level of exports, these are assumed in the long-run, like in the short-run, to be independent of the level of regional income.

Substituting equations (G.3), and (G.12) into equation (G.16) yields:

$$Y = cY + iY + gY + G_{n1} + X \qquad \text{(G.17)}$$

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[48]That is, local government expenditures on goods and services produced in the region as well as that part of local government expenditures on imported goods and services that creates income in the region.

Solving the above equation for Y yields:

$$Y = \frac{G_{nl} + X}{1 - (c + i + g)} \qquad (G.18)$$

An equation which illustrates how the level of regional income changes with respect to a change in, say, the level of exports can be obtained by differentiating equation (G.18) with respect to exports, as follows:

$$\frac{dY}{dX} = \frac{1}{1 - (c + i + g)} \qquad (G.19)$$

Rearranging equation (G.19) as follows:

$$dY = \frac{1}{1 - (c + i + g)} dX \qquad (G.20)$$

yields an equation which can be used to assess the change in the level of regional income resulting from a given change in the level of exports. Using the same procedure for the other term in the numerator of equation (G.19) yields:

$$dY = \frac{1}{1 - (c + i + g)} dG_{nl} \qquad (G.21)$$

where dG_{nl} is the change in the level of non-local government expenditures.

It can be seen in equations (G.20) and (G.21) that a change in the level of non-local government expenditures or in the level of exports causes the level of regional income to change by some multiple of the initial change in the level of non-local government expenditures or exports. This multiple, which in both equations is the same, is:

$$k_{lr} = \frac{1}{1 - (c + i + g)} \qquad (G.22)$$

where k_{1r} is a multiplier which measures the impact on regional income, in the long-run, of a given change in the level of non-local government expenditures or in the level of exports.

The regional component of the marginal propensity to consume can be computed using the formula:

$$c = \frac{\Delta C}{\Delta Y} \qquad (G.23)$$

where ΔC is the change in the regional component of consumption expenditures and ΔY is the change in regional income. Similarly, the regional component of the marginal propensity to invest can be computed using the formula:

$$i = \frac{\Delta I}{\Delta Y} \qquad (G.24)$$

where ΔI is the change in the regional component of investment expenditures, while the regional component of the marginal propensity to spend of the local government can be computed using the formula:

$$g = \frac{\Delta G_1}{\Delta Y} \qquad (G.25)$$

where ΔG_1 is the change in the regional component of local government expenditures.

If we assume that in the case of our hypothetical economy, marginal propensities are equal to average propensities, then equation (G.23) can be rewritten as:

$$c = \frac{C}{Y} \qquad (G.26)$$

Similarly, equations (G.24) and (G.25) can be rewritten as:

$$i = \frac{I}{Y} \tag{G.27}$$

and

$$g = \frac{G_1}{Y} \tag{G.28}$$

Substituting the above three equations into equations (G.10) and (G.23) yields:

$$k_{sr} = \frac{1}{1 - \frac{C}{Y}} \tag{G.29}$$

and

$$k_{1r} = \frac{1}{1 - \left(\frac{C + I + G}{Y} \right)} \tag{G.30}$$

Now, if we assume that, in our hypothetical economy, employment is proportional to income, then equation (G.29) can be rewritten as follows:

$$k_{sr} = \frac{1}{1 - \frac{E_c}{E_t}} \tag{G.31}$$

where E_c is total employment generated directly and indirectly by the regional component of consumption expenditures and E_t is total regional employment. This formula yields a multiplier which measures the impact on total regional employment, in the short-run, of a change in the level of employment in the investment, government (both local and non-local), and export sectors of our hypothetical economy.

Similarly, equation (G.30) can be rewritten as follows:

$$k_{1r} = \frac{1}{1 - \left(\frac{E_c + E_i + E_{1g}}{E_t}\right)} \qquad (G.32)$$

where E_i is total employment generated directly and indirectly by the regional component of investment expenditures and E_{1g} is total employment generated directly and indirectly by the regional component of local expenditures. This formula yields a multiplier which measures the impact on total regional employment, in the long-run, of a change in the level of employment in the non-local government and export sectors of our hypothetical economy.

The multipliers in equations (G.31) and (G.32) focus on the effects on our hypothetical economy of a change in the level of employment in the investment, government, and export sectors as worked through the system via consumer spending in the case of the short-run multiplier and via consumer spending, investment expenditures, and local government expenditures in the case of the long-run multiplier. In other words, the two multipliers measure the direct and induced impact on total employment of a change in the level of employment in these sectors.

It is important to note that in addition to an induced impact on total employment, a direct change in the level of employment in any one of the above-mentioned sectors has, because of inter-industry linkages, an indirect impact on total employment. This stems from the fact that firms and industries generally make purchases from other firms and industries, which in turn make purchases from other firms and industries, and so on. Thus, for example, a direct increase in employment from exports in a given industry causes, through its inter-industry linkages, the level of income and employment in its supplying industries to increase.

The indirect impact on total employment can be taken into account by modifying equations (G.31) and (G.32) as follows:

$$k_{sr} = \frac{k}{1 - \frac{E_c}{E_t}} \qquad (G.33)$$

and

$$k_{1r} = \frac{k}{1 - \left(\frac{E_c + E_i + E_{1g}}{E_t}\right)} \qquad (G.34)$$

where k is total employment generated directly and indirectly by the regional component of expenditures in the sector for which the multiplier is being computed divided by total employment generated directly by the regional component of expenditures in the sector in question.