A MATHEMATICAL INVESTIGATION OF THE RELATIVE IMPORTANCE OF MUTUAL IMPEDANCE IN SHORT CIRCUIT CALCULATIONS FOR TRANSMISSION NETWORKS

A Thesis

Presented to

the Department of Electrical Engineering
of the Faculty of Engineering and Architecture
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In Partial Fulfilment

of the Requirements for the Degree

Master of Science

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by

Leonard Arthur Bateman

April, 1948

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Note:

In order to facilitate typewriting, subscripts throughout this thesis have been aligned with the symbols to which they refer.

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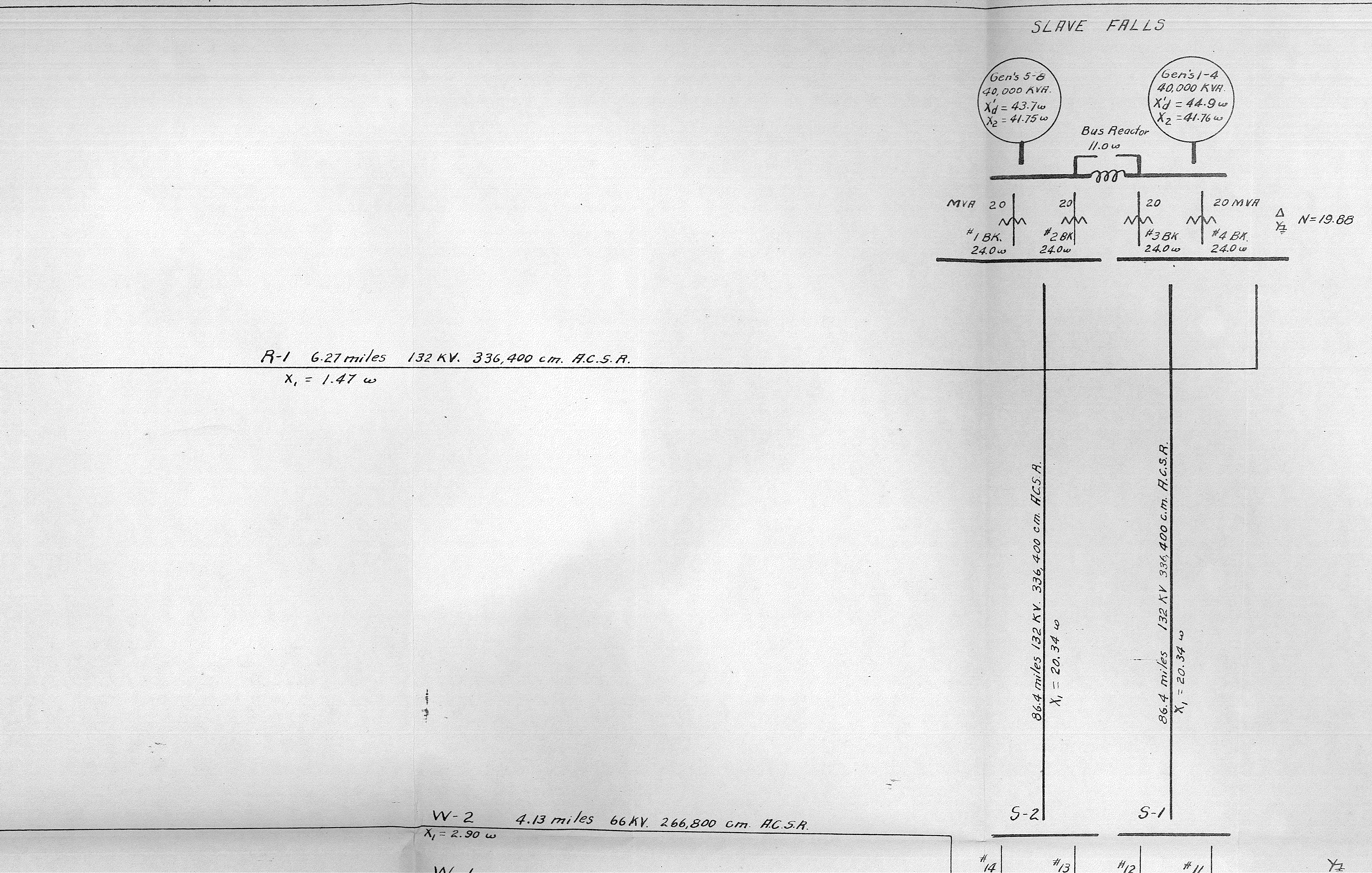
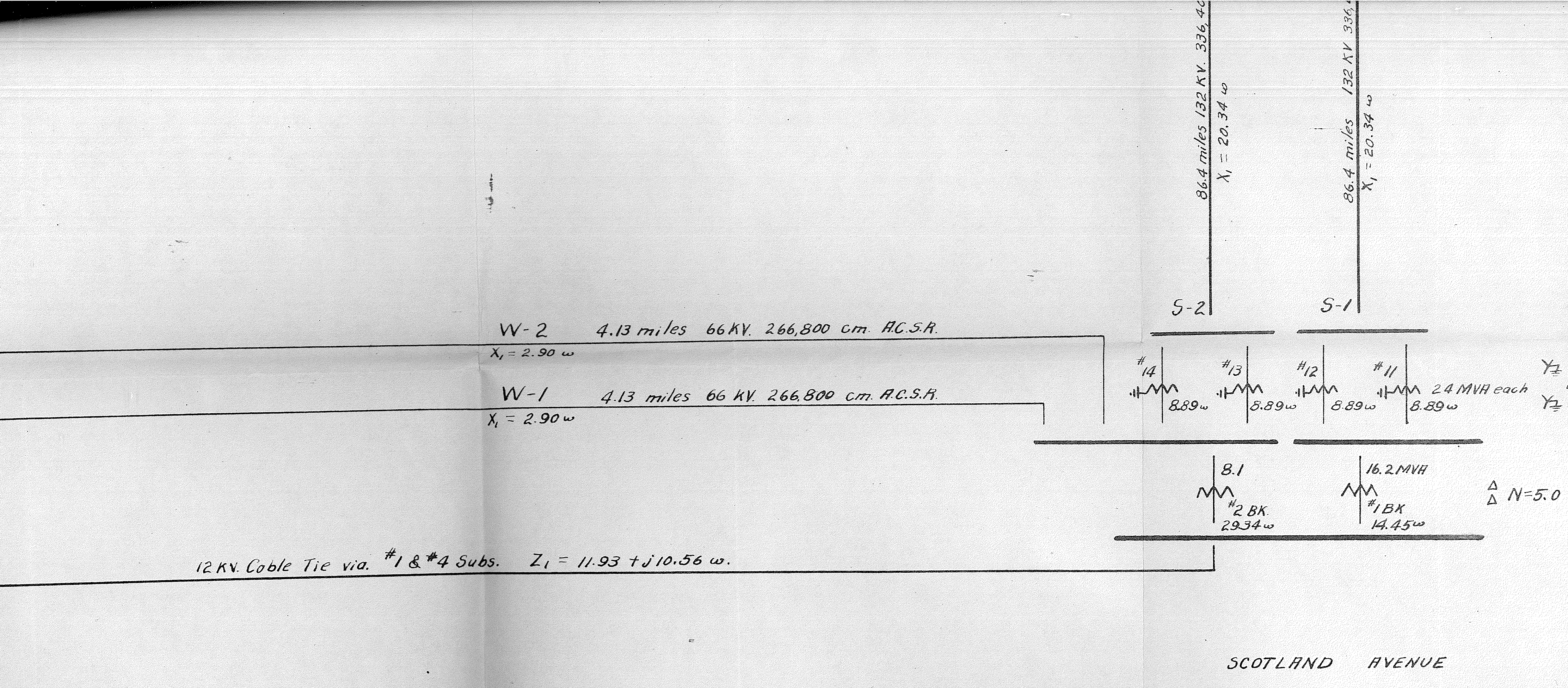


SLAVE FALLS
Gen's 5-8
40,000 KVA.
$X_d' = 43.7\omega$
$X_2 = 41.75\omega$
Gen's 1-4
40,000 KVA.
$X_d' = 44.9\omega$
$X_2 = 41.76\omega$
Bus Reactor
11.0ω
MVA 20
20
20
20 MVA
#1 BK.
24.0ω
#2 BK
24.0ω
#3 BK
24.0ω
#4 BK.
24.0ω
N=19.88
R-1 6.27 miles 132 KV. 336,400 cm. A.C.S.R.
$X_1 = 1.47\omega$
86.4 miles 132 KV. 336,400 cm. A.C.S.R.
$X_1 = 20.34\omega$
86.4 miles 132 KV 336,400 c.m. A.C.S.R.
$X_1 = 20.34\omega$
S-2
S-1
W-2 4.13 miles 66 KV. 266,800 cm. A.C.S.R.
$X_1 = 2.90\omega$
#14
#13
#12
#11



Notes:- All quantities shown in ohms as on 66 KV. System.

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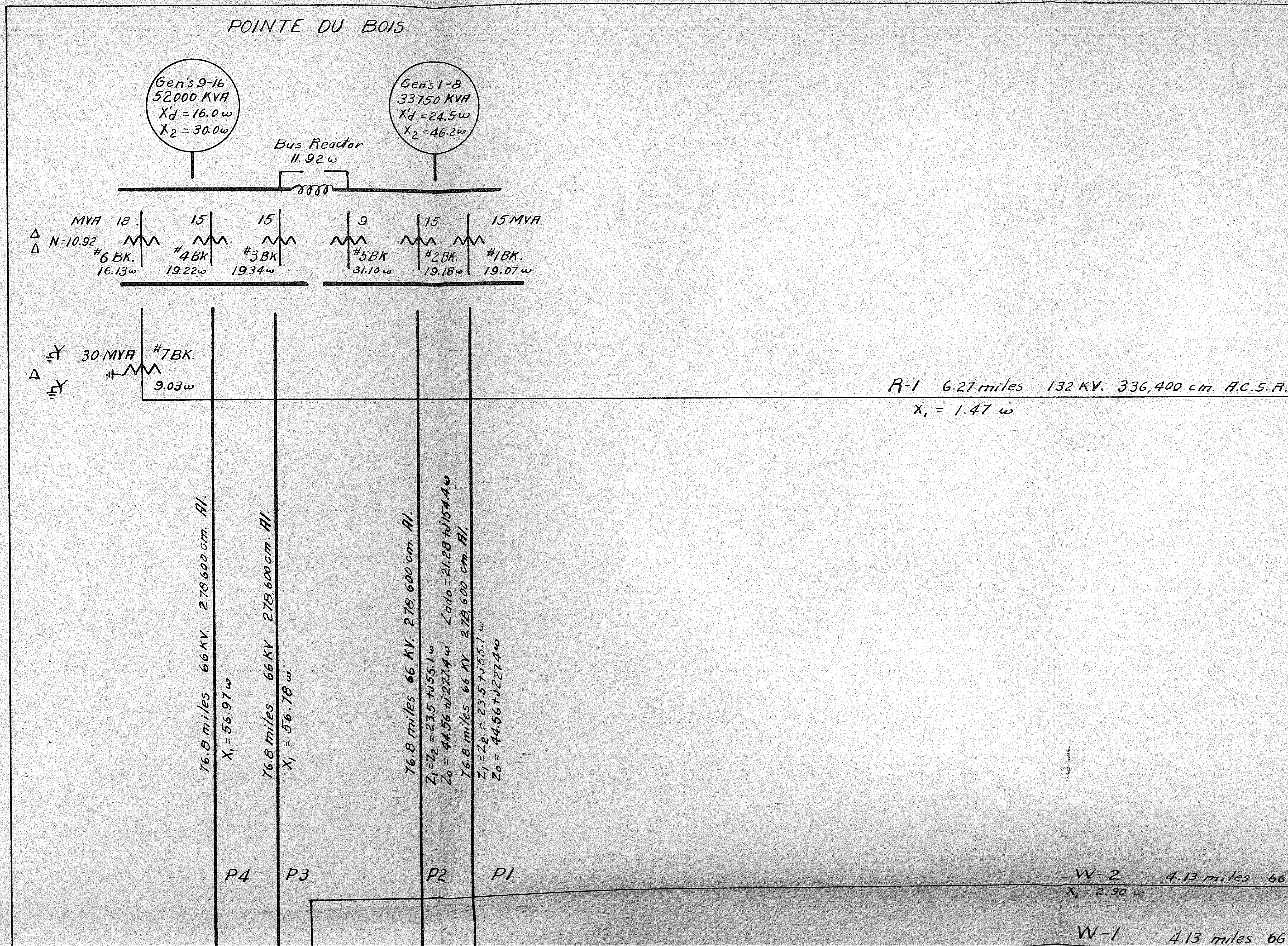
A mathematical investigation of the relative importance of mutual impedance in short circuit calculations for transmission networks

Figure No. 1

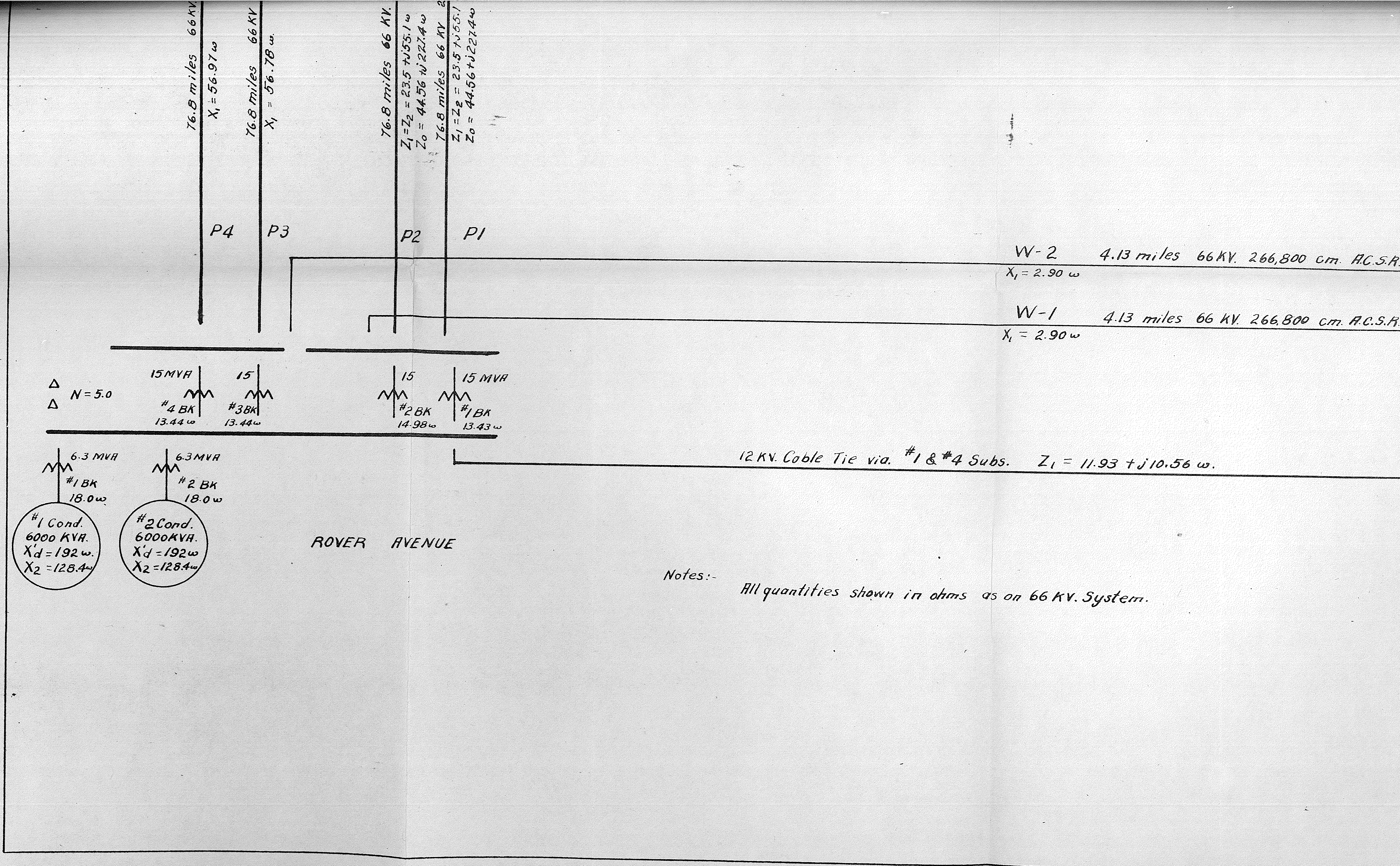
City of Winnipeg Hydro Electric System
High Tension Network

April 1948

L.A. Bateman.



POINTE DU BOIS
Gen's 9-16
52000 KVA
X'd = 16.0 ω
X2 = 30.0ω
Gen's 1-8
33750 KVA
X'd = 24.5 ω
X2 = 46.2ω
Bus Reactor
11.92 ω
MVA 18
N=10.92
#6 BK.
16.13ω
15
#4 BK
19.22ω
15
#3 BK
19.34ω
9
#5 BK
31.10ω
15
#2 BK.
19.18ω
15 MVA
#1 BK.
19.07ω
30 MVA
#7 BK.
9.03ω
R-1 6.27 miles 132 KV. 336,400 cm. A.C.S.R.
X1 = 1.47 ω
76.8 miles 66 KV. 278 600 cm. Al.
X1 = 56.97ω
76.8 miles 66 KV 278,600 cm. Al.
X1 = 56.78 ω.
76.8 miles 66 KV. 278,600 cm. Al.
Z1=Z2 = 23.5 +j55.1 ω
Zo = 44.56 +j227.4ω Zado = 21.28 +j154.4ω
76.8 miles 66 KV 278,600 cm. Al.
Z1=Z2 = 23.5 +j55.1 ω
Zo = 44.56 +j227.4ω
P4
P3
P2
P1
W-2 4.13 miles 66 K
X1 = 2.90 ω
W-1 4.13 miles 66 K


76.8 miles 66 KV
X1 = 56.97 ω
76.8 miles 66 KV
X1 = 56.78 ω
76.8 miles 66 KV.
Z1 = Z2 = 23.5 + j55.1 ω
Zo = 44.56 + j227.4 ω
76.8 miles 66 KV
Z1 = Z2 = 23.5 + j55.1
Zo = 44.56 + j227.4 ω
P4
P3
P2
P1
W-2
4.13 miles 66 KV. 266,800 cm. A.C.S.R.
X1 = 2.90 ω
W-1
4.13 miles 66 KV. 266,800 cm. A.C.S.R.
X1 = 2.90 ω
15 MVA
#4 BK
13.44 ω
15
#3 BK
13.44 ω
15
#2 BK
14.98 ω
15 MVA
#1 BK
13.43 ω
Δ
Δ
N = 5.0
6.3 MVA
#1 BK
18.0 ω
6.3 MVA
#2 BK
18.0 ω
#1 Cond.
6000 KVA.
X'd = 192 ω.
X2 = 128.4 ω
#2 Cond.
6000 KVA.
X'd = 192 ω
X2 = 128.4 ω
12 KV. Cable Tie via. #1 & #4 Subs. Z1 = 11.93 + j10.56 ω.
ROVER AVENUE
Notes:-
All quantities shown in ohms as on 66 KV. System.

Chapter I

The Problem

and

Definitions of Terms Used

CHAPTER I

THE PROBLEM AND DEFINITIONS OF TERMS USED

The problem. The voltages and currents in a symmetrical three phase system are balanced under normal conditions. However, during fault conditions when the system is not symmetrical by reason of the fault, the voltages and currents are not symmetrical and the sequence impedances of the three phases when carrying positive-negative-and zero-sequence currents will cause unbalanced voltage drops to appear at the terminals of the circuit. In addition to the positive-sequence component of voltage drop caused by positive-sequence current flowing in an unsymmetrical circuit, a negative-and zero-sequence component of voltage drop can be produced. The positive-sequence network is therefore coupled with the other two sequence networks. The mutual or coupling impedances between the positive-negative-sequence network and the positive-zero-sequence network and other mutual impedances are the subject of special study in this thesis.

Statement of the problem. It is the purpose of this study to investigate the relative importance of mutual impedance between the various sequence networks when sequence components of current are flowing in transmission networks due to unbalance caused by fault conditions.

Importance of the study. The thesis is undertaken, using the transmission line network of the City of Winnipeg Hydro Electric System, because of the fundamental difference established in the system network when the change was made from an ungrounded delta system to a grounded wye system. This occurred when the Slave Falls transmission lines were raised from a nominal 66 kv voltage level to a nominal 132 kv voltage level by reconnecting the transformers at Slave Falls from delta-delta to delta-wye, with solidly grounded neutral. The method of interconnecting the 66 and 132 kv systems is shown in Fig. 1. The outstanding point to notice is the installation of autotransformers, four banks of 24,000 kva capacity at Scotland Avenue and one bank of 30,000 kva on the tie line between Slave Falls and Pointe du Bois at Pointe du Bois.

The addition of these grounds on the system, provided a source of zero sequence current to the 132 and 66 kv systems. It is the zero sequence current which is new to the 66 kv system and with which mutual impedance between transmission lines plays such an important part.

Definitions of terms used. The impedances offered to positive-sequence currents in the three phases of a circuit will be defined as the ratios of the voltage drops in the three phases to the corresponding phase currents, with only positive-sequence currents flowing in the circuit. The impedances to positive-sequence currents in phases a, b, and c will be designated by Za1, Zb1, and Zc1 respectively. The negative-sequence impedances of the three phases will be designated by Za2, Zb2, and Zc2, and the zero-sequence impedances by Zao, Zbo, and Zco.

The definitions of the sequence impedances of the three phases of an unsymmetrical circuit given here are based on the flow of positive, -negative -, and zero-sequence currents in the circuit and the unbalanced voltage drops resulting from them.

Self impedances are indicated by Z with two like subscripts, and the sequence designation numeral. For example Zaal is the self impedance of phase a for positive-sequence currents.

Sequence impedances of the three phases refer to the impedances in the three phases to the sequence currents, when currents of each sequence are applied separately (Za1, Zb1, Zc1; Za2, Zb2, Zc2; Zao, Zbo, Zco).

Circuit or conductor self and mutual impedances refer to the self impedances of the elements or conductors of the actual circuit (Zaa, Zbb, Zcc---------Znn) and the mutual impedances between elements (Zab, Zbc,----------Zcn).

Sequence self and mutual impedances refer to the self impedances in the sequence networks (Z11, Z22, and Zoo) and the mutual impedances between these networks (Z12, Z21, Z10, Z01, Z20, Z02). These are defined on pages 229 and 230 of E. Clarke's book Vol. I Circuit Analysis of AC Power Systems.

Detailed considerations. The relative importance of Mutual Impedance in short-circuit calculations for transmission networks can be demonstrated adequately by considering mathematically a portion only of the ultimate network.

The effects of physical arrangement, including transpositions of the conductors, ground wires, and adjacent circuits, upon the self and mutual impedance of the circuit and upon the mutual impedances of the sequence networks of the circuit, can be thoroughly investigated for a pair of adjacent circuits on the same steel towers.

Once the various elements of impedance are developed with their related effects, those which appreciably change the value of current flowing in the circuit or adjacent circuits under fault conditions can be incorporated as circuit constants and used in determining the fault currents.

The lines chosen for this detailed investigation will be two actual circuits operating at 69 kv between Pointe du Bois and Rover Avenue Terminal Station of the City of Winnipeg Hydro Electric System. These lines were constructed in 1910 and have an equivalent 6'-0" delta spacing with no overhead ground wire protection. Adjacent to these lines and on the same 100 foot right-of-way are lines P3 and P4. These were constructed in 1921 and have vertical conductor configuration with overhead ground wire protection. (See Fig. 30).

These lines have a source of zero-sequence current at Pointe du Bois if #7 transformer bank is metallically connected to the tie bus at the same time as these lines. They have a source of zero-sequence current from Scotland Avenue autotransformers over the W1 and W2 lines if these lines are metallically connected to the 69 kv tie bus at Rover Avenue.

However, since our investigation will be confined to two lines and the values used therein would be heavily shaded by conditions outside the circuit, principally when network reductions are made, we shall assume that these two lines are connected to delta-wye transformer banks backed by sufficient generating capacity, to adequately feed the lines. Conditions at Rover Avenue will be assumed as follows: Both lines connected to the 60 kv tie bus, connections beyond the tie bus being neglected since a symmetrical non-rotating load is assumed.

Chapter II

Review of Related Theory Applied

CHAPTER II

REVIEW OF RELATED THEORY APPLIED

Introduction. In developing the art of high voltage transmission, considerable attention has been paid to simplification of calculations required to properly solve each individual problem. This has progressed along with the physical improvements in line reliability determined by observing the performance of lines under all conditions of service, and assessing these results with mathematical treatment.

The biggest aid to the electrical engineer engaged in line design and high tension transmission of power is the method of symmetrical components. This method can be applied equally well to balanced and unbalanced circuits providing the circuit constants remain linear over the entire range of potential and currents encountered during fault or normal conditions.

It will be assumed that the preliminary work on symmetrical components is thoroughly familiar to the reader of the work to follow. The fundamental relations between the various sequence components of current, voltage, and impedance are outlined very thoroughly in Chapter III of E. Clarke's book Vol. I "Circuit Analysis of A.C. Power Systems". It is necessary to make this condition, in view of the continual recurrence of these equations.

Inductance and Resistance of Transmission Lines. The following expression[1] for the potential drop per mile of conductor will be used in the work developed hereafter. (This will be adapted to a three phase system).

$$Va = Iar + j0.00466\, f \left(Ia \log \frac{1}{d} + Ib \log \frac{1}{ab} + + + In \log \frac{1}{an}\right) \tag{1}$$

vector volts per mile

where Va = vector volts drop per mile
Ia = current flowing in conductor a
Ib = current flowing in conductor b
In = current flowing in conductor n
ab = distance between conductor a and conductor b
an = distance between conductor a and conductor n
r = resistance of conductor a per mile
f = frequency in cycles per second
d = G.M.R. of the cross-section of the conductor with respect to itself

This expression assumes that the conductor has a self impedance which is independent of the position of the conductor with respect to other conductors and a mutual impedance with each of the other conductors in the circuit or circuits considered. However, since the current in any conductor must have a return path the self or mutual impedance of a conductor has but little meaning unless associated with some other conductor or conductors.[2]

[1]Applications of the Method of Symmetrical Components, W.V. Lyon, p.113. New York, McGraw Hill Book Co. 1937.

[2]Transmission Line Formula, H.B. Dwight. D. Van Nostrand, 1925.

The Bulletin of the Bureau of Standards Vo. 4 No.2 (American) derives expressions for self and mutual inductance of transmission line conductors.[1]

The conductors are assumed to be infinitely long and of uniform cross-sectional area and permeability throughout their entire length, (i.e. uniform current in each conductor). The vector sum of all currents is zero and it is assumed that no magnetically induced currents are in the earth.

<u>Methods of Calculating sequence self-and mutual impedances.</u> There are two ways of determining the sequence self-and mutual impedances of the simple series circuit shown in Fig. 21.

Let Va, Vb, and Vc be the phase voltages at P referred to ground.

Let Va^1, Vb^1, and Vc^1 be the voltages referred to ground at Q.

Let voltage drops between P and Q in the three phases be va, vb, and vc.

Then $va = Va - Va^1$ (2)

$vb = Vb - Vb^1$ (3)

$vc = Vc - Vc^1$ (4)

The lower case letter v is used here to indicate a series voltage drop as distinguished from the voltages to ground designated by capital V.

[1] E. Clarke Vol. I. p.364 Loc. cit.

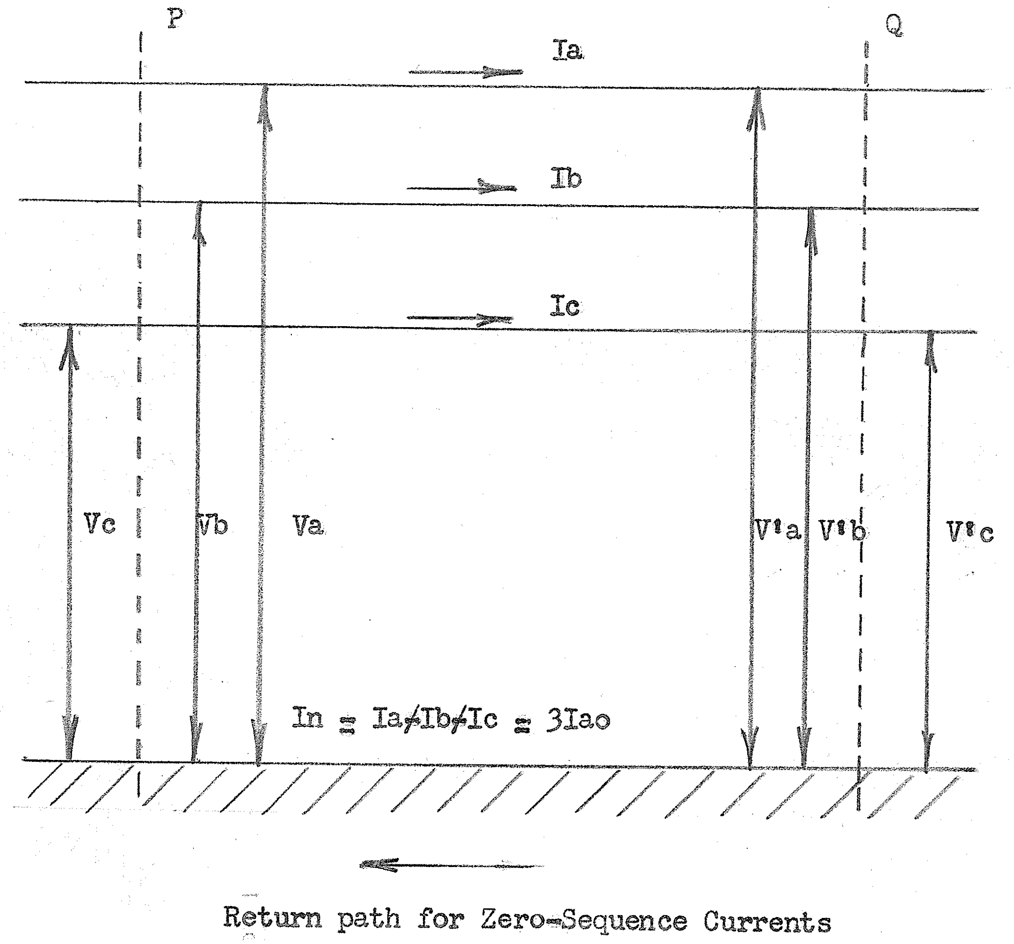


Fig. 21 Three-phase Series Circuit between P and Q with negligible Capacitance and no Internal Voltage.

Applying the principle of superposition (since all constants are linear) the voltage drops can be written in terms of symmetrical components of current meeting respective impedances in each phase.

$$v_a = V_a - V_a^1 = I_{a1}Z_{a1} + I_{a2}Z_{a2} + I_{ao}Z_{ao} \tag{5}$$

$$v_b = V_b - V_b^1 = a^2 I_{a1}Z_{b1} + aI_{a2}Z_{b2} + I_{ao}Z_{bo} \tag{6}$$

$$v_c = V_c - V_c^1 = aI_{a1}Z_{c1} + a^2 I_{a2}Z_{c2} + I_{ao}Z_{co} \tag{7}$$

where v_a, v_b, and v_c are series voltage drops in phases, a, b, and c respectively in the direction PQ.

Using $V_{ao} = 1/3(V_a + V_b + V_c)$ (8)

$$V_{a1} = 1/3(V_a + aV_b + a^2V_c) \tag{9}$$

$$V_{a2} = 1/3(V_a + a^2V_b + aV_c) \tag{10}$$

The voltage drops v_a, v_b, and v_c can be written in terms of their symmetrical components; grouping as we write

$$v_{ao} = V_{ao} - V^1_{ao} = \frac{I_{a1}}{3}(Z_{a1} + a^2Z_{b1} + aZ_{c1}) + \frac{I_{a2}}{3}(Z_{a2} + aZ_{b2} + a^2Z_{c2}) + \frac{I_{ao}}{3}(Z_{ao} + Z_{bo} + Z_{c0}) \tag{11}$$

$$v_{a1} = V_{a1} - V^1_{a1} = \frac{I_{a1}}{3}(Z_{a1} + Z_{b1} + Z_{c1}) + \frac{I_{a2}}{3}(Z_{a2} + a^2Z_{b2} + aZ_{c2}) + \frac{I_{ao}}{3}(Z_{ao} + aZ_{bo} + a^2Z_{co}) \tag{12}$$

$$v_{a2} = V_{a2} - V^1_{a2} = \frac{I_{a1}}{3}(Z_{a1} + aZ_{b1} + a^2Z_{c1}) + \frac{I_{a2}}{3}(Z_{a2} + Z_{b2} + Z_{c2}) + \frac{I_{ao}}{3}(Z_{ao} + a^2Z_{bo} + aZ_{co}) \tag{13}$$

To demonstrate the proof of the statement made (Chapter I The Problem) in connection with positive-sequence current meeting voltage drops of all three sequences, the negative and zero-sequence currents in 11, 12, and 13 can be made zero. The equations then become:

$$v_{ao} = \frac{I_{a1}}{3}(Z_{a1} + a^2 Z_{b1} + aZ_{c1}) \tag{14}$$

$$v_{a1} = \frac{I_{a1}}{3}(Z_{a1} + Z_{b1} + Z_{c1}) \tag{15}$$

$$v_{a2} = \frac{I_{a1}}{3}(Z_{a1} + aZ_{b1} + a^2 Z_{c1}) \tag{16}$$

which show that positive-sequence currents flowing in the unsymmetrical circuit will cause voltage drops of all three sequences between P and Q unless the coefficients of I_{a1} are zero. This is also true for negative- or zero-sequence currents, as can readily be seen from equations (11), (12) and (13) if the other two sequence currents are made zero and the voltage drops written for the sequence current desired individually.

Equations (11) to (13) can be simplified by replacing the coefficients of the currents by Z's with subscripts as suggested by Dr. E.W. Kimbark in his letter to the Editor of "Electrical Engineering" in October, 1938, page 431. The first subscript in Dr. Kimbark's notation refers to the sequence of the voltage drop given by the equation and the second to the sequence of the current associated with the coefficient.

$$v_{a1} = V_{a1} - V^{1}_{a1} = I_{a1}Z_{11} + I_{a2}Z_{12} + I_{ao}Z_{10} \tag{17}$$

$$v_{a2} = V_{a2} - V^{1}_{a2} = I_{a1}Z_{21} + I_{a2}Z_{22} + I_{ao}Z_{20} \tag{18}$$

$$v_{ao} = V_{ao} - V^{1}_{ao} = I_{a1}Z_{01} + I_{a2}Z_{02} + I_{ao}Z_{00} \tag{19}$$

where the coefficients of the currents are defined below from equations (11), (12) and (13).[1]

[1] E. Clarke Vol. I Circuit Analysis of A.C. Power Systems. p. 229. New York, John Wiley & Sons, 1943.

$Z11 = 1/3(Za1 + a^2Zb1 + aZc1)$ = self-impedance to positive-sequence currents (20)

$Z22 = 1/3(Za2 + Zb2 + Zc2)$ = self-impedance to negative-sequence currents (21)

$Zoo = 1/3(Zao + Zbo + Zco)$ = self-impedance to zero-sequence currents (22)

$Z12 = 1/3(Za2 + a^2Zb2 + aZc2)$ = ratio of positive-sequence voltage drop produced by Ia2 to Ia2 (23)

$Z10 = 1/3(Zao + aZbo + a^2Zco)$ = ratio of positive sequence voltage drop produced by Iao to Iao (24)

$Z21 = 1/3(Za1 + aZb1 + a^2Zc1)$ = ratio of the negative-sequence voltage drop produced by Ia1 to Ia1 (25)

$Z20 = 1/3(Zao + a^2Zbo + aZco)$ = ratio of the negative-sequence voltage drop produced by Iao to Iao (26)

$Z01 = 1/3(Za1 + a^2Zb1 + aZc1)$ = ratio of the zero-sequence voltage drop produced by Ia1 to Ia1 (27)

$Z02 = 1/3(Za2 + aZb2 + a^2Zc2)$ = ratio of the zero-sequence voltage drop produced by Ia2 to Ia2 (28)

The second method of determining the sequence self-and mutual impedances is to replace the phase currents (including sum of phase currents for neutral current) by their symmetrical components of current.

$$Ia = Ia1 + Ia2 + Iao \qquad (29)$$

$$Ib = a^2Ia1 + aIa2 + Iao \qquad (30)$$

$$Ic = aIa1 + a^2Ia2 + Iao \qquad (31)$$

then to resolve va, vb, and vc into their symmetrical components of voltage [1] by equations (8) - (10), and finally to equate the coefficients of Ia1, Ia2 and Iao in the resultant equations for va1, va2, and vao to the corresponding coefficients in equations (17) to (19), obtaining the sequence self-and mutual impedances in terms of the self-and mutual impedances of the conductors. The second of these two methods expresses the self-and mutual impedances of the sequence networks directly in terms of the conductor self-and mutual impedances. While the equations are somewhat longer, the notation is simpler and the number of equations considerably less. This is the method employed in Chapter III wherein the self and mutual impedances of the transmission lines are developed.

[1] W.V. Lyon, Applications of the Method of Symmetrical Components p. 89, New York, McGraw-Hill Book Co. 1937.

Zero-Sequence Impedance of Conductors. The zero-sequence network includes the ground path. Therefore any method of calculating currents and voltages during faults involving one or two phases and ground on transmission lines requires an understanding of the impedance of the circuit involved, including its return path through the earth. Tests [1] have been conducted to determine the effect of the earth on single phase short circuits involving ground. When Symmetrical Component theory was developed the results of these tests were used to develop empirical formulae [2] giving the zero-sequence impedance of a transmission line as a function of the depth of the ground return.

The method in use today to calculate the self-and mutual impedance of conductors with common earth return is based on an analysis by Dr. John Carson [3]. This work is clearly presented in Vol. I of E. Clarke's book, Circuit Analysis of A.C. Power Systems, p.373. The charts developed in the above mentioned book will be used in determining the values of zero-sequence self and mutual impedance as the accuracy of this method is within 0.1% for the P1 and P2 transmission lines. [4]

[1] Single-Phase Short-Circuit Calculations - W.W. Lewis, General Electric Review, July, 1925.

[2] Calculation of Single-Phase Short-Circuits by the Method of Symmetrical Components - A.P. Mackerras, General Electric Review, Vol. 29, April., 1926, pp.218-231, July, 1926, pp.468-481.

[3] Wave Propagation in Overhead Wires with Ground Return, - John R. Carson, Bell System Technical Journal, Volume 5, 1926, pp.539-554.

[4] E. Clarke Vol. I p.375 Loc. cit.

Zero-Sequence Self-and Mutual Impedances. It will be noted that the equations used herein, see Chapter III page 36, for zero-sequence self and mutual impedance, are based on Carson's equations.[1] The equivalent circuits for zero-sequence quantities, do not give differences in potential of ground points. If zero-sequence voltages at various points along a line were referred to a common ground point, the self and mutual impedances of the "ground-return" circuit would have to be separated. This would require an equivalent self-and mutual impedance of the earth return as well as of the circuit external to the earth. However, the equivalent circuits do give zero-sequence voltages referred to ground at any specific point along the line which are useful in short-circuit studies.

The zero-sequence network can be thought of as a simple loop circuit similar to a single phase circuit feeding a load Z_L with a voltage V applied at the circuit terminals. The current then flows in both conductors of this circuit which has self impedances Zaa and Zbb and mutual impedance Zab. The equivalent impedance of this circuit is $Z = Zaa + Zbb - 2Zab$. This is the impedance of an earth return circuit given by Carson's equations, and if the voltage of any point is required in reference to any other point in the circuit then the separate impedances Zaa, Zbb and Zab, or the equivalent impedances Zaa -Zab and Zbb -Zab must be known.

[1] E. Clarke, Vol. I p.372. Loc.cit.

However, if the voltage to ground, at any point in the circuit, is required, then the equivalent impedance as given by $Z = Z_{aa} + Z_{bb} - 2Z_{ab}$ is satisfactory. This is the impedance given by Carson's equations and this point is noted here for future reference.

Chapter III

Determination of Impedance Values

CHAPTER III

DETERMINATION OF IMPEDANCE VALUES

Data to Accompany Figure 30.

P1 and P2 Lines

Average span 600' Sag 23' at 60°F.

Average height of Conductor say 40 feet

Conductors 278,600 circular mils stranded aluminum

Diameter 0.605" Radius 0.0252 ft.

Resistance at 0°C = 0.2992 ohms per mile, at 23.9°C (77°F) = 0.306 ohms per mile

G.M.R. of Conductor 0.1911"

P3 and P4 Lines

Average Span 400' Sag 8½' at 60°F.

Average height of Conductor say 35 feet

Conductors 278,600 circular mils stranded aluminum

Diameter 0.605" Radius 0.052 feet.

Resistance at 0°C = 0.2992 ohms per mile, at 23.9°C (77°F) = 0.306 ohms per mile

G.M.R. of Conductor 0.1911"

Ground Wire on P3 and P4 Line Towers

Conductor 7 strand steel (Siemens-Martin)

Diamter 0.360"

Resistance at 0°C with 15 amperes flowing = 5.49 ohms per mile

G.M.D. of Conductor 0.00132"

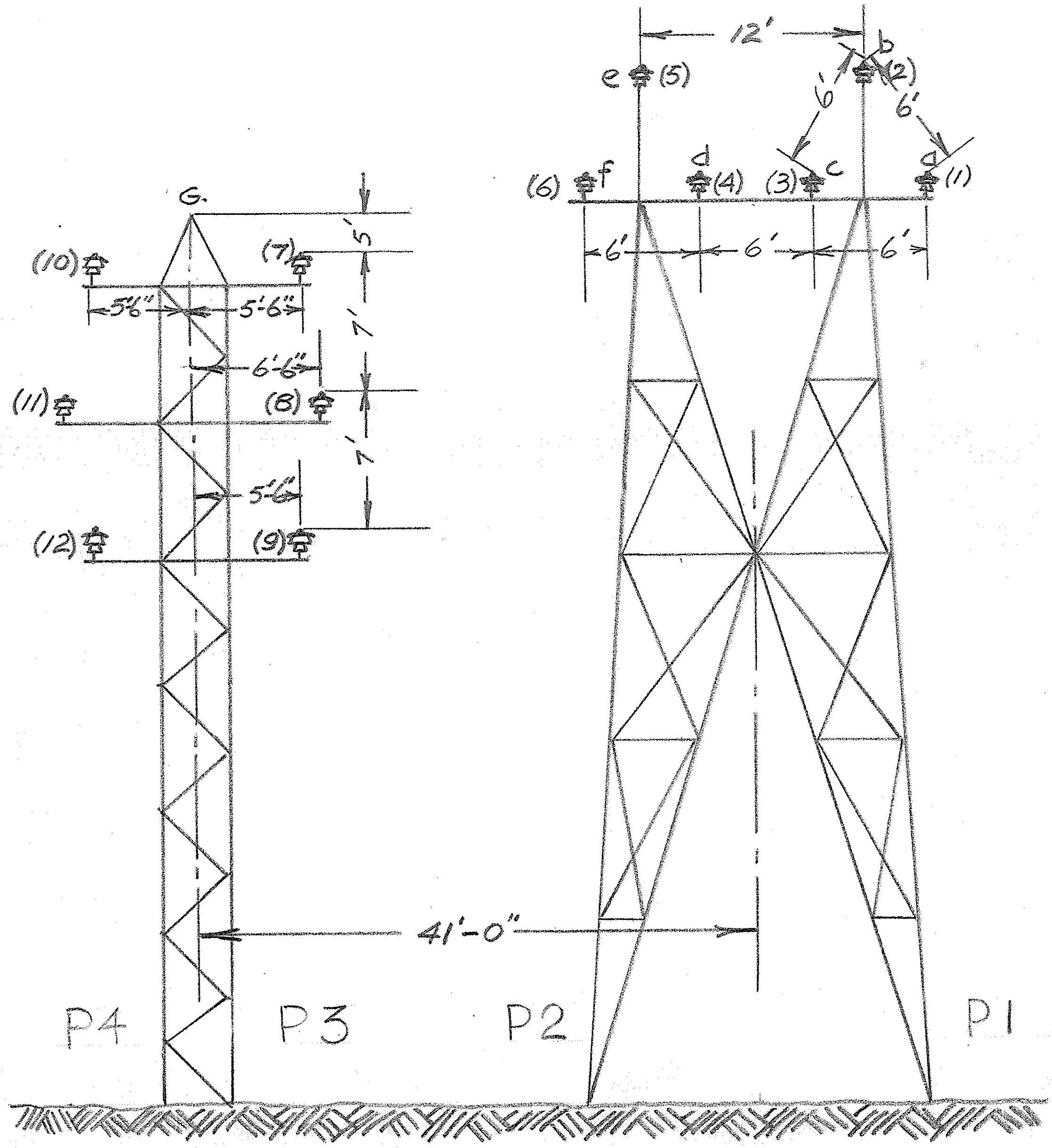


FIGURE 30. TYPICAL CROSS SECTION OF POINTE TRANSMISSION LINES LOOKING TOWARDS POINTE DU BOIS.

Note: The position number on all towers remains unaltered throughout the entire line. For example, the right hand end looking toward Pointe du Bois of the middle crossarm of the north tower line is always position number 8. For the phase letters at any location along line, see transposition drawing Fig. 31.

<u>Distances between Conductors for P1 and P2 Lines.</u> Let the conductors of P1 line be called a, b, and c and the conductors of P2 line be called d, e, f. The equations for these lines will be worked out in general terms so that they may be used for subsequent lines. The positions of the conductors in succeeding sections of the line will be detailed when transpositions are considered.

Sab will designate distance from a conductor to b conductor and will be abbreviated ab.

ab = 6	bd = 10.4
ac = 6	be = 12.
bc = 6	bf = 15.9
ad = 12.	cd = 6.
ae = 15.9	ce = 10.4
af = 18	cf = 12.

In order to calculate the characteristics of this line, the quantities desired will be worked out from first principles where necessary and the theory developed to explain these principles.

<u>Positive-and Negative-Sequence Impedance (either circuit).</u>

$$Z11 = r + j0.00466\, f \log \frac{\sqrt[3]{ab \cdot bc \cdot ca}}{d} \quad \text{vector ohms per mile} \qquad (1)$$

r = resistance per mile of conductor = 0.306 ohms
d = G.M.R. of the conductor

$Zaa1 = Zdd1 = Zaa2 = Zdd2$

$$Z = 0.306 + j\,0.2794 \log \frac{\sqrt[3]{6 \times 6 \times 6}}{0.01592}$$

$$= 0.306 + j\,0.717 \text{ ohms per mile(actual length 76.8 miles.)}$$

The line P1 has identical phase conductors spaced at the vertices of an equilateral triangle ab = bc = ca = 6 feet. Therefore in any relationships involving $\log_{10} \frac{\sqrt[3]{6 \times 6 \times 6}}{6} = \log 1 = 0$ there can be no value of impedance. This is true of the single circuit cross-sequence mutual impedances. The cross-circuit cross-sequence mutual impedances will be developed hereafter.

Mutual Impedances between P1 & P2 Transmission Lines. The mutual impedance between the two lines may represent a considerable portion of the total drop of either line. In the general case there are mutual impedances between the sequence networks between lines as well as between conductors of each line.

Consider first the flow of only positive-sequence currents in circuit d, e, f (P2). Then the fall of potential per mile in conductor a of circuit a, b, c (P1) will be in vector volts per mile for the first section of line untransposed.

$$Va = j\,0.00466\,f\left(\log \frac{1}{ad} + a^2 \log \frac{1}{ae} + a \log \frac{1}{af}\right) Idl \tag{2}$$

$$Vb = j\,0.00466\,f\left(\log \frac{1}{bd} + a^2 \log \frac{1}{be} + a \log \frac{1}{bf}\right) Idl \tag{3}$$

$$Vc = j\,0.00466\,f\left(\log \frac{1}{cd} + a^2 \log \frac{1}{ce} + a \log \frac{1}{cf}\right) Idl \tag{4}$$

The positive-, negative-and zero-sequence components of voltage in conductor a can be determined from the above three equations, and equations 8, 9 and 10 from Chapter II

$$Vao = j\,\frac{0.00466\,f}{3}\left(\log\frac{1}{ad.bd.cd} + a^2\log\frac{1}{ae.be.ce.} + a\log\frac{1}{af.bf.cf}\right)Idl \quad (5)$$

$$Va1 = j\,\frac{0.00466\,f}{3}\left(\log\frac{1}{ad.be.cf} + a^2\log\frac{1}{ae.bf.cd} + a\log\frac{1}{af.bd.ce}\right)Idl \quad (6)$$

$$Va2 = j\,\frac{0.00466\,f}{3}\left(\log\frac{1}{ad.bf.ce} + a^2\log\frac{1}{ae.bd.cf} + a\log\frac{1}{af.be.cd}\right)Idl \quad (7)$$

If a is written $-0.5 + j\,0.866$ and a^2 is written $-0.5 - j\,0.866$ and the real and imaginary terms collected with all terms divided by 3 the equations become

$$Vao = j\,0.00466\,f\,\Bigg(\log\frac{\sqrt[6]{ae.be.ce.af.bf.cf}}{\sqrt[3]{ad.bd.cd}} +$$

$$j\,0.866\,\log\sqrt[3]{\frac{ae.be.ce}{af.bf.cf}}\Bigg)Idl \quad (8)$$

$$Va1 = j\,0.00466\,f\,\Bigg(\log\frac{\sqrt[6]{ae.bf.cd.af.bd.ce}}{\sqrt[3]{ad.be.cf}} +$$

$$j\,0.866\,\log\sqrt[3]{\frac{ae.bf.cd}{af.bd.ce}}\Bigg)Idl \quad (9)$$

$$Va2 = j\,0.00466\,f\,\Bigg(\log\frac{\sqrt[6]{ae.bd.cf.af.be.cd}}{\sqrt[3]{ad.bf.ce}} +$$

$$j\,0.866\,\log\sqrt[3]{\frac{ae.bd.cf}{af.be.cd}}\Bigg)Idl \quad (10)$$

All the above equations are in vector volts per mile.

The positive-sequence mutual impedance between two parallel circuits is defined as the ratio of the positive-sequence voltage induced in one circuit to the positive-sequence current flowing in the other circuit which produces it.

The positive-sequence mutual impedance $Zadl = \frac{Val}{Idl}$

and

$$Zadl = j\,0.00466\,f\left(\log\frac{\sqrt[6]{ae.bf.cd.af.bd.ce}}{\sqrt[3]{ae.be.cf}} + j\,0.866\log\sqrt[3]{\frac{ae.bf.cd}{af.bd.ce}}\right) \text{ vector ohms per mile} \qquad (11)$$

likewise the negative-sequence mutual impedance between two parallel circuits is defined as the ratio of the negative-sequence voltage induced in one circuit to the negative-sequence-current flowing in the other circuit which produces it. The negative-and the zero-sequence mutual impedances will be considered in detail later.

The same positive-negative and zero-sequence mutual impedance can be obtained for circuit 2 with respect to circuit 1 by considering the flow of positive-sequence currents flowing in circuit a, b, c, (P1). The fall of potential per mile in the three conductors of circuit d, e, f, (P2) may be written down by inspection from equations 2, 3, and 4. This is effectively the same as changing the letters a, b, c for d, e, f, respectively.

The resulting equations for zero-, positive-, and negative-sequence components of the mutually induced potentials in line d due to positive-sequence components of current in the a, b, c, circuit are:

$$Vdo = j\,0.00466\,f\left(\log \frac{\sqrt[6]{db.eb.fb.dc.ec.fc}}{\sqrt[3]{da.ea.fa}} + j\,0.866 \log \sqrt[3]{\frac{db.eb.fb}{dc.ec.fc}}\right) Ia1 \tag{12}$$

$$Vd1 = j\,0.00466\,f\left(\log \frac{\sqrt[6]{db.ec.af.dc.ae.bf}}{\sqrt[3]{da.eb.fc}} + j\,0.866 \log \sqrt[3]{\frac{db.ec.af}{dc.ae.fb}}\right) Ia1 \tag{13}$$

$$Vd2 = j\,0.00466\,f\left(\log \frac{\sqrt[6]{db.ea.fc.dc.eb.fa}}{\sqrt[3]{da.ec.fb}} + j\,0.866 \log \sqrt[3]{\frac{db.ea.fc}{dc.eb.fa}}\right) Ia1 \tag{14}$$

All expressed in vector volts per mile.

With the spacings listed previously for these conductors, the actual values of zero sequence mutual impedance per mile for the first untransposed section for each of the lines when positive-sequence currents flow in the other line, are:

$$\frac{Vdo}{Ia1} = -0.0338 + j\,0.0419 \text{ ohms per mile}$$

$$\frac{Vao}{Id1} = +0.0193 + j\,0.0504 \text{ ohms per mile}$$

The ratio of Vdo/Ia1 does not equal Vao/Id1

This finding agrees with the statement that: [1]

"In a static network, the mutual impedances between two conductors are reciprocal; in the sequence networks, the mutual impedances are not, in general, reciprocal".

Lyon[2] also agrees with this in his book.

This is particularly true of the lines under consideration because of their disposition on the tower. More will be said of this later.

$$\frac{Val}{Idl} = +0.00875 + j\ 0.0461 \text{ ohms per mile}$$

$$\frac{Vdl}{Ial} = -0.00875 + j\ 0.0461 \text{ ohms per mile}$$

The ratio Val/Idl is the negative of the conjugate of the ratio Vdl/Ial.

[1] Edith Clarke - Circuit Analysis of A.C. Power Systems. Vol. I p.420. Loc. cit.

[2] W.V. Lyon - Applications of the Method of Symmetrical Components. p.141. Loc. cit.

From an inspection of equations 10 and 14 it can be seen that the ratio Va2/Id1 equals the ratio Vd2/Ia1. This value will not be calculated for the untransposed section of the line, however its numerical value will be determined for the complete line, see Equation 24, Chapter III.

Dealing with the same section of line, the zero,-positive-and negative-sequence components of potential in the circuit a, b, c (P-1) can be determined for the flow of negative-sequence currents in circuit d, e, f (P-2). The equations for Va, Vb and Vc due to negative-sequence current Id2 are the same as equations 2, 3, and 4 except that a^2 will replace a and vice-versa, likewise Id2 will take the place of Id1. Following the same procedure that was used in developing equations 8, 9, and 10 we can write down

$$Vao = j\,0.00466\,f\left(\log \frac{\sqrt[6]{ae \cdot be \cdot ce \cdot af \cdot bf \cdot cf}}{\sqrt[3]{ad \cdot bd \cdot cd}} - j\,0.866 \log \sqrt[3]{\frac{ae \cdot be \cdot ce}{af \cdot bf \cdot cf}}\right) Id2 \qquad (15)$$

$$Va1 = j\,0.00466\,f\left(\log \frac{\sqrt[6]{ae \cdot bd \cdot cf \cdot af \cdot be \cdot cd}}{\sqrt[3]{ad \cdot bf \cdot ce}} - j\,0.866 \log \sqrt[3]{\frac{ae \cdot bd \cdot cf}{af \cdot be \cdot cd}}\right) Id2 \qquad (16)$$

$$Va2 = j\,0.00466\,f\left(\log \frac{\sqrt[6]{ae \cdot bf \cdot cd \cdot af \cdot bd \cdot ce}}{\sqrt[3]{ad \cdot be \cdot cf}} - j\,0.866 \log \sqrt[3]{\frac{ae \cdot bf \cdot cd}{af \cdot bd \cdot ce}}\right) Id2 \qquad (17)$$

all in vector volts per mile.

These expressions for the zero-, positive-and negative-sequence voltage give impedance ratios Vao/Id2, Va1/Id2, and Va2/ Id2 respectively.

The same positive-, negative-, and zero-sequence mutual impedances can be obtained for circuit 2 with respect to circuit 1 by considering the flow of negative-sequence currents in circuit a, b, c (P1). The fall of potential per mile in the three conductors of circuit d, e, f (P2) may be written down by inspection from equations 2, 3, and 4, when a^2 replaces a and Id2 replaces Id1. This is accomplished by changing the letters a, b, c for d, e, f, respectively.

The resulting equations for zero-, positive-, and negative-sequence components of the mutually induced potentials in line d due to negative-sequence components of current in the a, b, c, circuit are -

$$V_{do} = j\,0.00466\,f\left(\log\frac{\sqrt[6]{db.be.fb.dc.ec.cf}}{\sqrt[3]{ad.ea.af}} - j\,0.866\log\sqrt[3]{\frac{db.be.bf}{dc.ec.cf}}\right) I_{a2} \qquad (18)$$

$$V_{d1} = j\,0.00466\,f\left(\log\frac{\sqrt[6]{db.ea.cf.dc.be.af}}{\sqrt[3]{ad.ec.bf}} - j\,0.866\log\sqrt[3]{\frac{bd.ae.cf}{dc.be.fa}}\right) I_{a2} \qquad (19)$$

$$V_{d2} = j\,0.00466\,f\left(\log\frac{\sqrt[6]{db.ec.af.cd.ea.bf}}{\sqrt[3]{ad.be.cf}} - j\,0.866\log\sqrt[3]{\frac{bd.ec.fa}{dc.ea.bf}}\right) I_{a2} \qquad (20)$$

Equations 8, 9, 10, 12, 13, 14, 15, 16, 17, 18, 19, and 20 can be compared and the following generalities written down.

The impedance ratio $Va1/Id2$ (16) is the negative of the conjugate of $Va2/Id1$ (10).

The impedance ratio $Va2/Id2$ (17) is the negative of the conjugate of $Va1/Id1$ (9).

The impedance ratio $Va1/Id2$ (16) equals $Vd1/Ia2$ (19).

The impedance ratio $Va2/Id2$ (17) is the negative of the conjugate $Vd2/Ia2$ (20).

The impedance ratio $\frac{Vdo}{Ia1}$ (12) equals the impedance ratio $\frac{Vdo}{Ia2}$ (18)

The impedance ratio $\frac{Vao}{Id2}$ (15) equals the impedance ratio $\frac{Vao}{Id1}$ (8)

And hence these relationships follow:

$$Va1/Id1 = Vd2/Ia2 = -\text{conj}\,(Va2/Id2)$$

$$= -\text{conj}\,(Vd1/Ia1)$$

$$Va2/Id1 = Vd2/Ia1 = -\text{conj}\,(Va1/Id2)$$

$$= -\text{conj}\,(Vd1/Ia2)$$

$$\frac{Vdo}{Ia1} = \frac{Vdo}{Ia2} \quad \text{and} \quad \frac{Vao}{Id2} = \frac{Vao}{Id1}$$

From these general relations Lyon states "that it is necessary to compute only the positive-and negative sequence components of the potential induced in the first circuit by positive-sequence current in the second circuit."[1]

[1] Lyon p.142 Loc. cit.

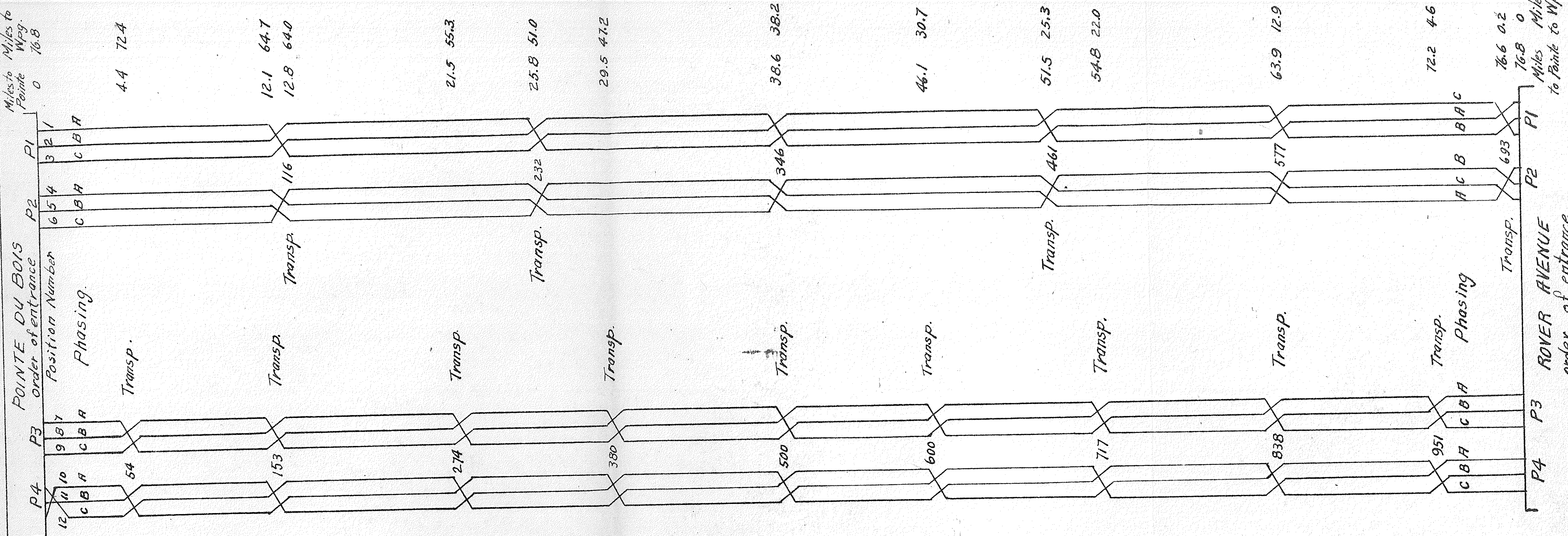
It is of interest to note that the ratio of the positive-sequence voltage drop to the negative-sequence current producing it is not the same as the ratio of the negative-sequence voltage drop to the positive-sequence current producing it. This is the negative of the conjugate in this case. The other cross-sequence mutual impedances except in special cases as further explored will be unequal in the general case.

Before progressing with this section of the line, the effects of transpositions and the Earth will be considered and then these general equations applied to the entire line.

<u>Transpositions</u> <u>on</u> <u>P1</u> <u>and</u> <u>P2</u> <u>lines</u>. The transpositions occur on P1 and P2 as shown in the following tabulation.

	0 edfedf 5				0 bcabca 2		
0 fedfed 6		P2	0 dfedfe 4	0 cabcab 3		P1	0 abcabc 1

The first letters indicate the original position of the conductor, the second letters indicate the conductor position in the second transposition section and so on. These transpositions may be seen on the Plan of Transpositions, Fig. 31.



Note:-
See Fig. 31 Page 18 for typical cross section of Pointe Transmission Lines looking toward Pointe du Bois.

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Figure 31
Plan of Transpositions

April 1948

L. A. Bateman.

Positive-Sequence Mutual Impedance (P1 and P2 Lines). The transpositions are symmetrical and the line is symmetrically transposed for its length. The mutual impedances for the line should be the average of the three values for the first three sections. This average value can be obtained from equation 9 expanded to include the three sections (a complete transposition) as follows.

$$Val = j\,0.00466\,f \left(\log \frac{\sqrt[18]{\begin{matrix} ae \cdot bf \cdot cd \cdot af \cdot bd \cdot ce \\ cf \cdot ad \cdot be \cdot cd \cdot ae \cdot bf \\ bd \cdot ce \cdot af \cdot be \cdot cf \cdot ad \end{matrix}}}{\sqrt[9]{\begin{matrix} ad \cdot be \cdot cf \\ ce \cdot af \cdot bd \\ bf \cdot cd \cdot ae \end{matrix}}} + j\,0.866 \log \sqrt[9]{\frac{\begin{matrix} ae \cdot bf \cdot cd \\ cf \cdot ad \cdot be \\ bd \cdot ce \cdot af \end{matrix}}{\begin{matrix} af \cdot bd \cdot ce \\ cd \cdot ae \cdot bf \\ be \cdot cf \cdot ad \end{matrix}}} \right) Idl \qquad (21)$$

Using values from page 19 Chapter III

$$\frac{Val}{Idl} = j\,0.2794 \left(\log \frac{\sqrt[18]{26.10 \times 10^{18}}}{\sqrt[9]{5.1 \times 10^{9}}} + j\,0.866 \log \sqrt[9]{\frac{1}{1}} \right)$$

Zadl = 0.00 ohms per mile.

Thus the positive-sequence mutual impedance between Pointe #1 and Pointe #2 lines is zero.

The positive-sequence impedance of two transmission lines connected in parallel and fed from the same source at each end has an impedance

$$Z = \frac{ZaaZdd - Z^2ad}{Zaa + Zdd - 2Zad} \qquad (22)$$

Proof of this can be obtained by referring to p.32 Chapter I of E. Clarke's book referred to earlier. Since in this case P1 = P2 in design and physical characteristics, equation 22 reduces to

$$Z = 1/2\ (Zaa + Zad) \qquad (23)$$

The arrangement of the conductors can be changed to alter the value of Zad.

If the mutual coupling between the two circuits for positive-and negative-sequence is neglected and using the average mutual impedance given by equation 21, then 23 will be a minimum when Zad has its maximum negative value. If P1 line is arranged so that phases a, b, and c occupy positions 1, 2, and 3, then P2 line can be in positions 4, 5, and 6, in several different arrangements. From equation 21, it is evident that the distances ad, be, and cf between conductors of the same phases should be large relative to distances between conductors of different phases to give Zad its maximum negative value. There is no easy way of arriving at this conductor configuration. The "cut-and-try"[1] method should be resorted to.

Lyon does show conductor arrangements for a pair of lines with vertical configuration[2] in which the mutual reactance is found to be negative if the conductors of the two lines roll counter-clockwise, whereas if the a,b, c and d, e, f circuits occupy the same relative position on the tower then the mutual reactance is positive.

[1] E. Clarke. Vol. I./p.421 Loc. cit.

[2] Page 145. W.V. Lyon. Loc. cit.

<u>Cross-Sequence Cross-Circuit Mutual Impedance</u> ($Z21$). The negative sequence potential induced in the first circuit by positive-sequence current in the second circuit can be found from equation 10 expanded to include three transpositions of line.

$$V_{a2} = j\,0.00466\,f \left(\log \frac{\sqrt[18]{\begin{matrix} ae \cdot bd \cdot cf \cdot af \cdot be \cdot cd \\ cf \cdot ae \cdot bd \cdot cd \cdot af \cdot be \\ bd \cdot cf \cdot ae \cdot be \cdot cd \cdot af \end{matrix}}}{\sqrt[9]{\begin{matrix} ad \cdot bf \cdot ce \\ ce \cdot ad \cdot bf \\ bf \cdot ce \cdot ad \end{matrix}}} + j\,0.866 \log \sqrt[9]{\frac{\begin{matrix} ae \cdot bd \cdot cf \\ cf \cdot ae \cdot bd \\ bd \cdot cf \cdot ae \end{matrix}}{\begin{matrix} af \cdot be \cdot cd \\ cd \cdot af \cdot be \\ be \cdot cd \cdot af \end{matrix}}} \right) I_{d1} \qquad (24)$$

Using values from page 19 Chapter III

$$\frac{V_{a2}}{I_{d1}} = -\,0.01492 - j\,0.005525 \text{ ohms per mile.}$$

$$\frac{V_{a2}}{I_{d1}} = Z21$$

This value is relatively small. It is brought into the discussion here to show the manner in which a positive-sequence current flowing in an unsymmetrical circuit and meeting a positive-sequence impedance Z_{a1} in the circuit will produce a negative-sequence voltage drop in the negative-sequence network. The positive-sequence network is therefore coupled with the negative-sequence network. This can be taken into account in short circuit studies using a network analyzer but the complexity of the networks required to do a rigorous solution by network reduction methods makes the refinement impractical.

Cross-Sequence Cross-Circuit Mutual Impedances (Z01 & Z02).

The zero-sequence voltage induced in the first circuit by positive-sequence current in the second circuit can be found from equation (8) expanded to include three transposition sections of the line.

$$\frac{Vao}{Id1} = j\,0.2794\left(\log \sqrt[18]{\frac{\begin{matrix} ae \cdot be \cdot ce \cdot af \cdot bf \cdot cf \\ cf \cdot af \cdot bf \cdot cd \cdot ad \cdot bd \\ bd \cdot cd \cdot ad \cdot be \cdot ce \cdot ae \end{matrix}}{\sqrt[9]{\begin{matrix} ad \cdot bd \cdot cd \\ ce \cdot ae \cdot be \\ bf \cdot cf \cdot af \end{matrix}}}} + j\,0.866 \log \sqrt[9]{\frac{\begin{matrix} ae \cdot be \cdot ce \\ cf \cdot af \cdot bf \\ bd \cdot cd \cdot ad \end{matrix}}{\begin{matrix} af \cdot bf \cdot cf \\ cd \cdot ad \cdot bd \\ be \cdot ce \cdot ae \end{matrix}}}\right) \tag{25}$$

Using values from page 19 Chapter III

$$= j\,0.2794\left(\log \frac{1}{1} + j\,0.866 \log \frac{1}{1}\right)$$

$$\frac{Vao}{Id1} = 0$$

From equation (15) it can be seen that $\frac{Vao}{Id2}$ = equation (25) when written for a complete transposition on the line.

The zero-sequence voltage induced in the second circuit by positive-sequence current in the first circuit can be found from equation (12) expanded to include a complete transposition on the line (3 sections).

$$\frac{Vdo}{Ia1} = j\,0.2794\left(\log \sqrt[18]{\frac{\begin{matrix} db \cdot eb \cdot fb \cdot dc \cdot ec \cdot fc \\ ae \cdot af \cdot ad \cdot be \cdot bf \cdot db \\ cf \cdot cd \cdot ce \cdot af \cdot ad \cdot ae \end{matrix}}{\sqrt[9]{\begin{matrix} da \cdot ea \cdot fa \\ ec \cdot cf \cdot ed \\ bf \cdot bd \cdot be \end{matrix}}}} + j\,0.866 \log \sqrt[9]{\frac{\begin{matrix} db \cdot eb \cdot fb \\ ae \cdot af \cdot ad \\ cf \cdot cd \cdot ce \end{matrix}}{\begin{matrix} de \cdot ec \cdot fc \\ be \cdot bf \cdot db \\ af \cdot ad \cdot ae \end{matrix}}}\right) \tag{26}$$

Using values from page 19 Chapter III

$$\frac{Vdo}{Ia1} = 0$$

From equation (18) it can be seen that $\frac{Vdo}{Ia2}$ = equation (26) when written for a complete transposition on the line.

<u>Effect of Ground Wires.</u> "For a line that is properly transposed, the positive-and negative-sequence components of line drop are not affected by the presence of ground wires."[1] This reference justifies the omission of ground wires from our consideration of the positive-and negative-sequence mutual impedances. The P3 and P4 steel tower line has a light steel ground wire for approximately half its length and two light steel ground wires for the remaining length. The effect of the light steel ground wires or wire on the P3 and P4 steel towers has not been taken into the calculation of the mutual cross-circuit impedances or the cross-sequence cross-circuit mutual impedances of P1 and P2 lines. The ground wire will not be considered in calculating the zero-sequence mutual impedance of the P1 and P2 lines. It will be considered when the P3 and P4 lines are calculated. "Light steel ground wires affect zero-sequence self-and mutual reactances but slightly."[2] This refers to light steel ground wires on the same towers, while the ground wires in question are some 40 feet distant from P1 and P2 lines.

[1] Lyon. Page 135. Loc. cit.

[2] E. Clarke. p.411. Loc. cit.

__Effect of Earth.__ "There can be no conductor currents in the earth due to the positive-and negative-sequence component currents in the conductors since the sum of each of these components in the three conductors is zero."[1]

The earth is a conducting body and will carry induced currents due to the action of both positive-and negative-sequence currents. This has the effect of increasing the apparent resistance of the conductors. There is also a decreasing effect on the reactance of the conductors due to the magnetic field surrounding the induced currents in the earth.

Lyon[2] finds that for a typical line the difference between the impedance to positive and negative sequence currents when calculated by the two methods, namely, considering the effect of the earth and neglecting it, is not large enough to justify taking the earth into account. However, for zero-sequence currents which flow by conduction in the earth, its effect must be taken into account.

The resistivity of the earth varies from 500 to approximately 1,000,000 ohms per centimeter cube.[3] An average value of 2500 ohms per centimeter cube is still one billion times that of copper. Therefore the eddy currents induced in the earth by the action of the currents in the transmission line conductors are of minor importance.

[1] E. Clarke Vol. I. p.419. Loc. cit.

[2] Lyon p.129. Loc. cit.

[3] Clem, J.E. Reactance of Transmission Lines with Ground Return, American Institute of Electrical Engineers, Vol. 50 p.901.

Zero-Sequence Quantities. The theory outlining the zero-sequence quantities has been developed very concisely in Vol. I of E. Clarke's book Circuit Analysis of A.C. Power Systems. The work that follows in calculating the zero-sequence self-and mutual impedances of the lines will conform to the method contained in the book mentioned. The outline of the theory here required is not contained in Chapter II of this thesis as no credit for the theory is claimed herein. Rather the method of using the equations to obtain the desired mutual impedances and their effect on the calculation of short-circuit currents in transmission networks is sought. Full use will be made of the charts and tables contained in the aforementioned book to arrive at values of zero-sequence impedance for the lines discussed.

Mutual Impedance to Zero-Sequence. By definition, zero-sequence line currents and voltages to ground in the three phases at any point in the system are equal in magnitude and phase. Zero-sequence mutual impedances are therefore independent of the arrangements of the phases of the two circuits in their tower positions.

Considering P1 and P2 lines (Fig. 30). The zero-sequence self-impedance of either of the two circuits is its self-impedance with the other circuit open. Equation 47 page 388 of E. Clarke's book Circuit Analysis of A.C. Power Systems - Vol. I expresses the zero-sequence in ohms per mile as follows:

$Z_{aao} = r_{oo} + j\, X_{00}$

$= Z_{\overline{aa} - g} + 2Z_{\overline{ab} - g}$

$$Z_{aao} = (r_c + R_{\overline{aa} - g} + 2R_{\overline{ab} - g}) + j(X_{\overline{aa} - g} + 2X_{\overline{ab} - g} + X_i) \quad (27)$$

Where bars over the subscripts indicate average values.

d = diameter of Conductor = .605" refer to Fig. 30 page 17

r_c = resistance of " = 0.306 ohms per mile.

X_i = internal reactance in ohms per mile

$$X_i = 2\pi f 10^{-3}\left(0.7411 \log_{10} \frac{r}{gmr}\right) \quad (28)$$

where r = radius in feet

gmr = equivalent self geometric mean radius of the conductor in feet.

$X_i = 0.0549$ ohms per mile

$GMD = \sqrt[3]{6 \times 6 \times 6} = 6$ feet

h = height above ground of conductors

The line spans are 600 feet with a sag of 23 feet at 60°F. The effect of the sag in the transmission line conductors may be accounted for by choosing the height h, as the minimum clearance above the conducting mass of the earth plus one-third of the sag. Conductors are 50 to 55 feet above the ground and an average value of h = 40 feet will be reasonable.

H = horizontal spacing between two conductors in feet.

H = 0 for self impedance

$= 1/3(6 + 3 + 3) = 4$ feet for mutual impedance of one line.

From Figure 32 with earth resistivity in ohms per meter cube =

ρ = 100 being an average value.[1]

[1] J.E. Clem - Reactances of Transmission Lines with Ground Return. Vol. 50 AIEE Transactions p.901

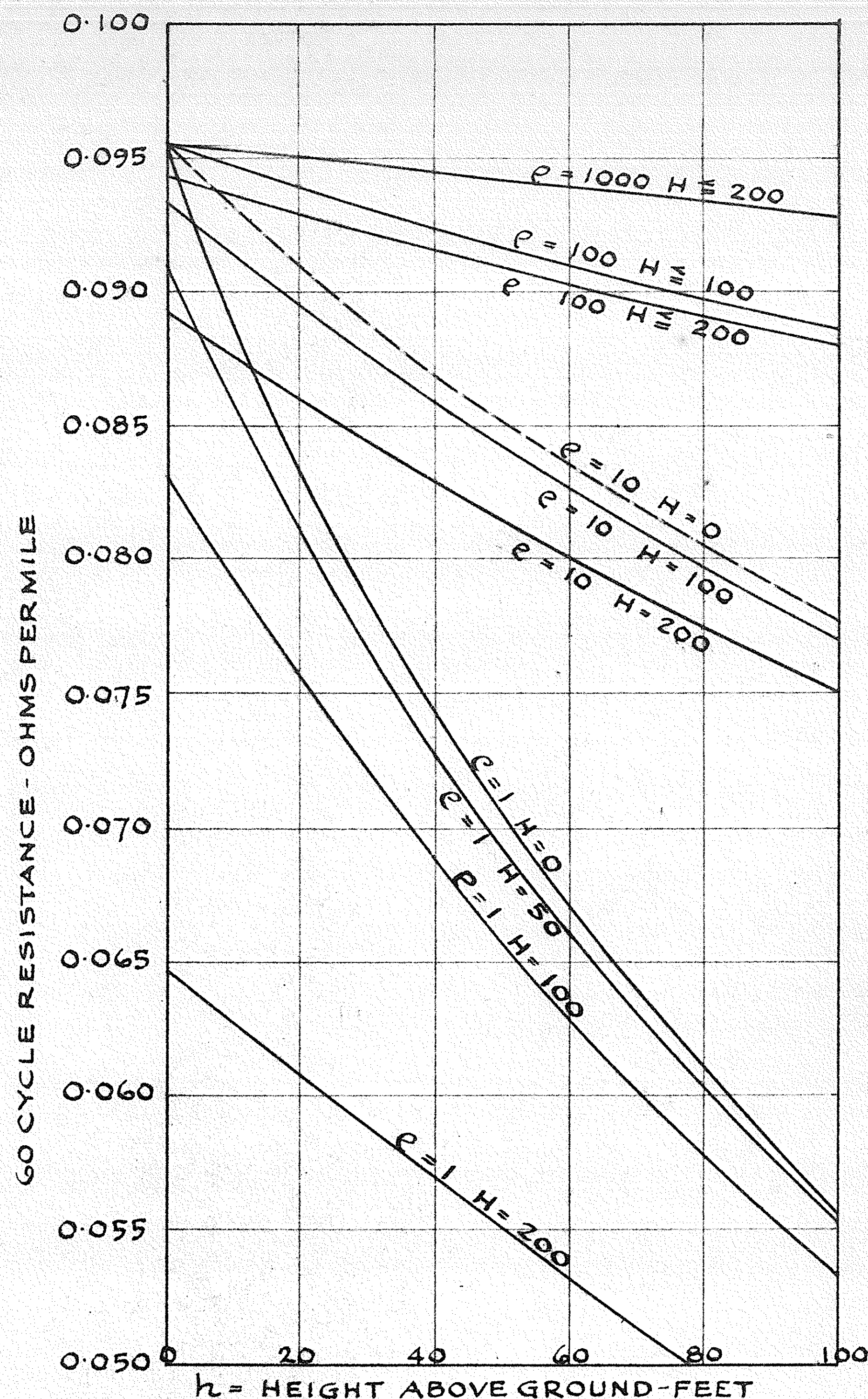


FIG. 32 60 cycle self and mutual resistances of the earth.

From E. Clarke Vol. 1 Circuit Analysis of A.C. Power Systems.

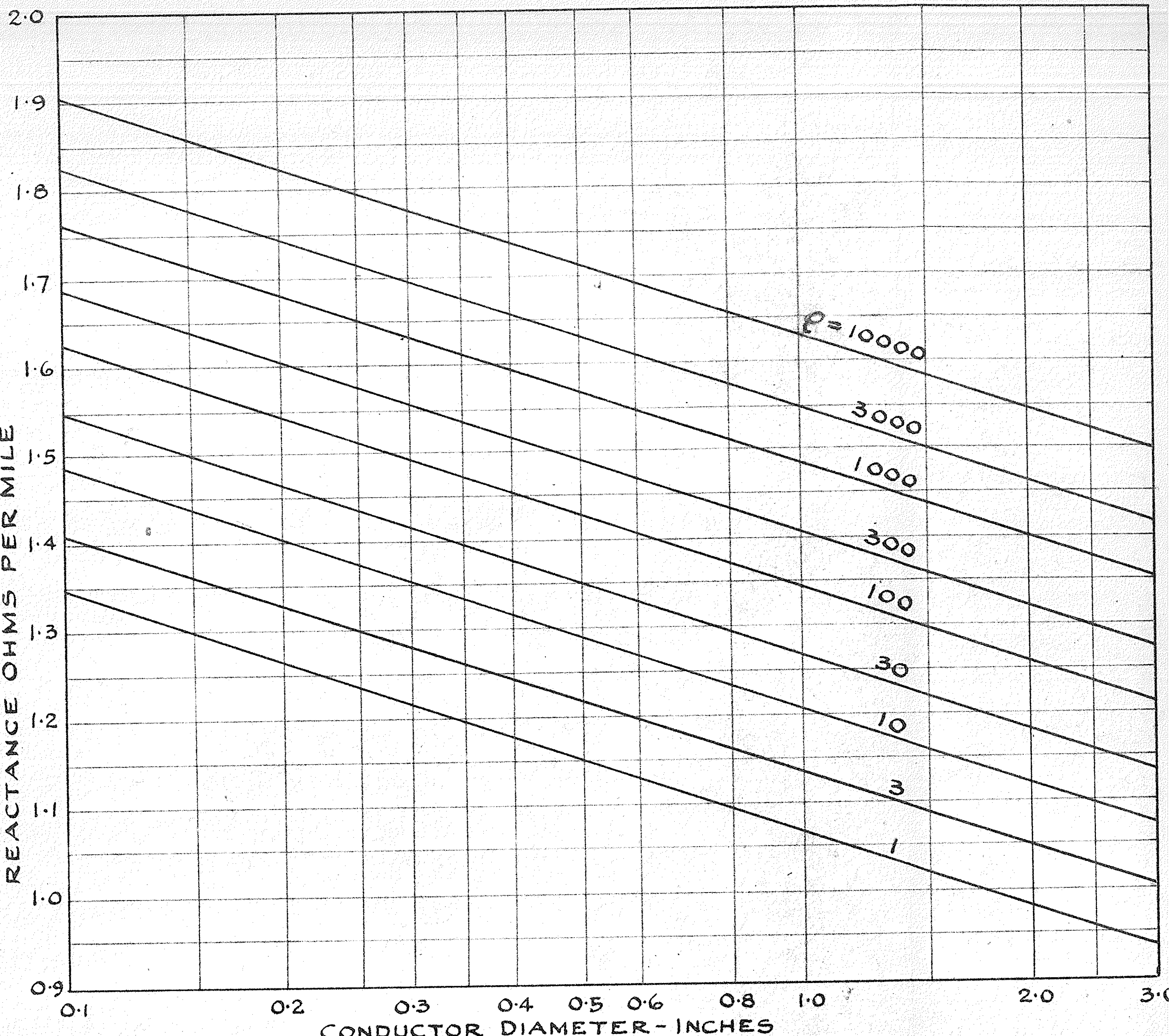


FIG. 33 60 cycle self reactance external to the conductor. Height above ground neglected.
From E. Clarke Vol. 1 Circuit Analysis of A.C. Power Systems.

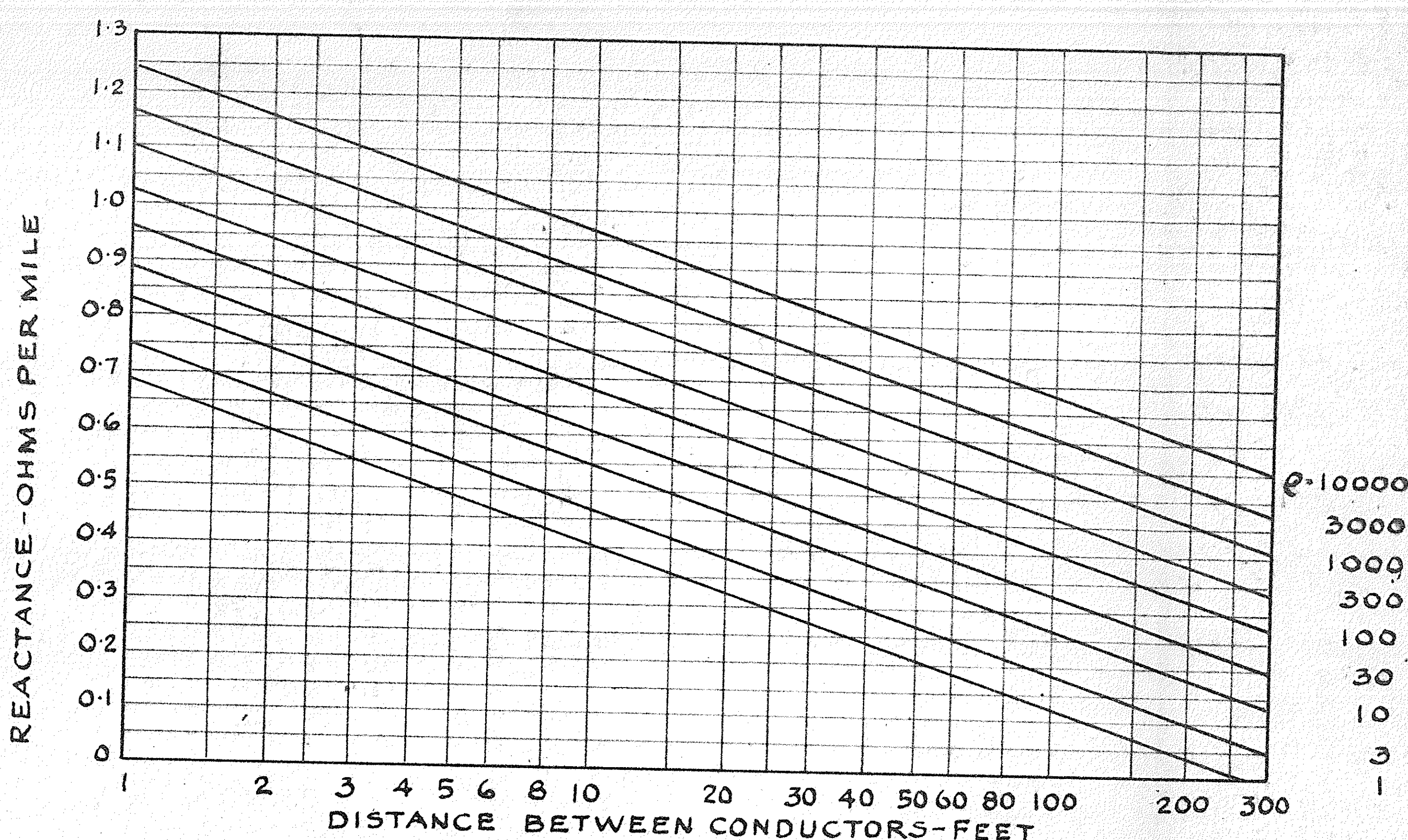


FIG. 34 60 cycle mutual reactance between two conductors with earth return. Height above ground of conductors neglected.

From E. Clarke Vol. 1 Circuit Analysis of A.C. Power Systems.

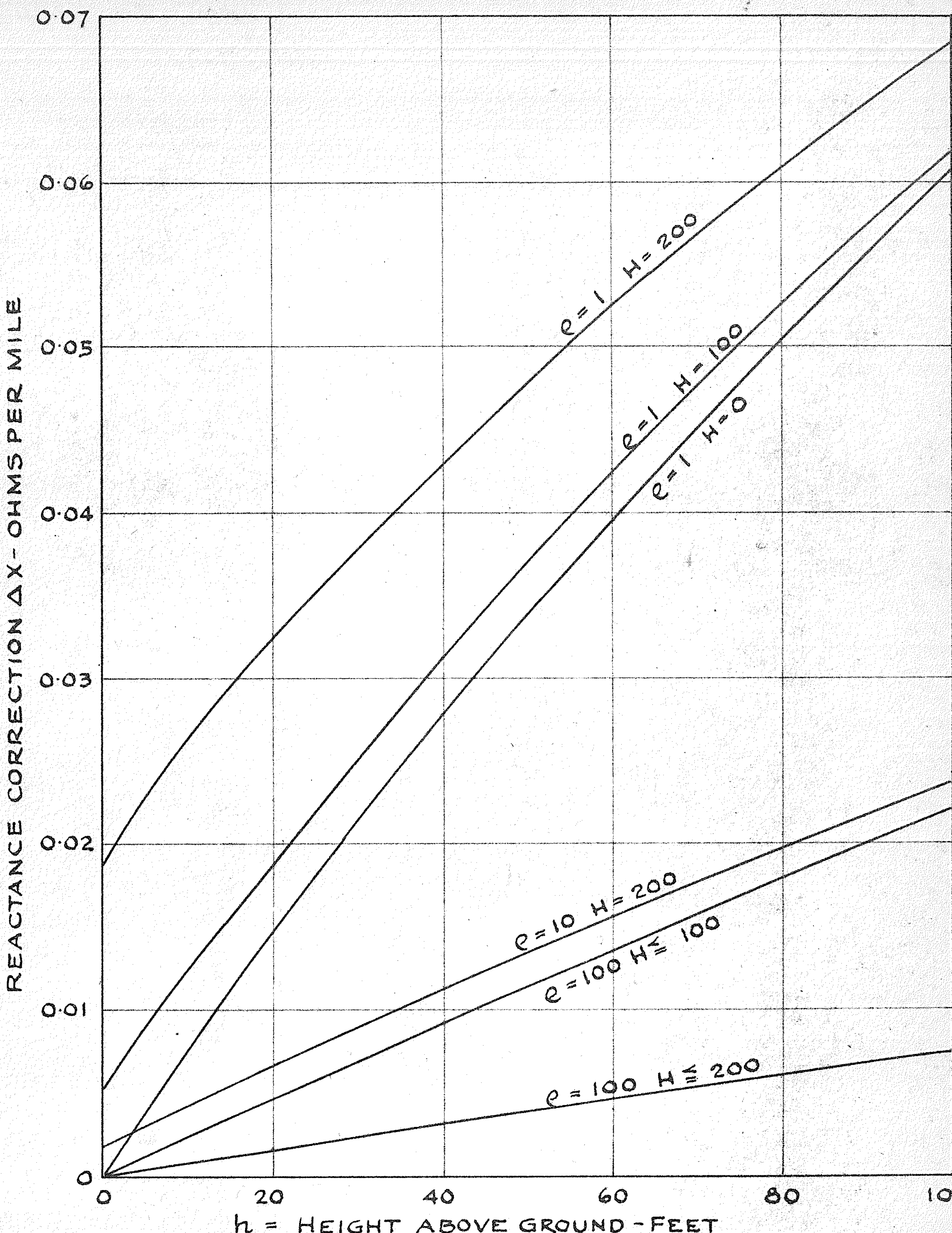


FIG. 35 Corrections to 60 cycle self and mutual reactances to account for height of conductors above ground.

From E. Clarke Vol. 1 Circuit Analysis of A.C. Power Systems.

$R_{aa} - g = 0.0925$ ohms per mile

$R_{ab} - g = 0.0925$ ohms per mile

From Fig. 33, with $\varphi = 100$ and $d = .605''$

$X_{\overline{\overline{aa}}} - g$ (conductor height neglected) $= 1.41$ ohms per mile

From Fig. 34 with $\varphi = 100$ and $s = 6$ feet

$X_{\overline{\overline{ab}}} - g$ (conductor height neglected) $= 0.745$ ohms per mile

The corrections to allow for conductor height above ground from Fig. 35 with $\varphi = 100$, $h = 40$, and $H = 0$ and 4 feet for self-and mutual reactances respectively

delta $X_{\overline{\overline{aa}}} - g = 0.003$ ohms per mile

delta $X_{\overline{\overline{ab}}} - g = 0.003$ ohms per mile

adding these correction factors

$X_{\overline{\overline{aa}}} - g = 1.413$ ohms per mile

$X_{\overline{\overline{ab}}} - g = 0.748$ ohms per mile

and substituting these values in equation (27)

$$Z_{aao} = 0.306 + 3(0.0925) + j(1.413 + 2(0.748) + 0.0549)$$

$$Z_{aao} = 0.58 + j\,2.96 \text{ ohms per mile}$$

Only the zero-sequence components of voltage induced by zero-sequence currents flowing in an adjacent line will be large enough to warrant consideration at this time. The zero-sequence mutual impedance between two parallel transmission circuits will be defined as the ratio of the zero-sequence voltage induced in one circuit to the zero-sequence current per phase flowing in the other circuit which induces it.

The mutual impedance for two parallel circuits with no ground wires.[1]

$$Z_{ado} = 3(R_{ad-g} + j\, X_{\overline{ad}-g}) \qquad (29)$$

From Fig. 30

Conductor average height above ground 40 feet.

Average horizontal spacing between conductors of P1 circuit and those of P2 circuit is 12 feet.

The geometric mean spacing between conductors of P1 circuit and those of P2 circuit is

$$S_{\overline{ad}} = \sqrt[9]{12 \times 15.9 \times 18 \times 10.4 \times 12 \times 15.9 \times 6 \times 10.4 \times 12}$$

$$S_{\overline{ad}} = 12.0$$

From Fig. 32 $R_{\overline{ad}-g} = 0.0925$ ohms per mile

From Fig. 34 $X_{\overline{ad}-g} = 0.67$ ohms per mile

$$Z_{ado} = 3(0.0925 + j\, 0.67)$$

$$Z_{ado} = 0.277 + j\, 2.01 \text{ ohms per mile}$$

<u>Summary of Impedances for P1 and P2 Lines</u>. The self-and mutual impedances to all sequences for P1 and P2 lines may be conveniently summarized as follows:

[1] Equation 76 Page 404 E. Clarke Vol. I Loc. cit.

Sequence	Equation	Self Impedance Ohms per Mile	Mutual Impedance Full Line Unless Noted. Ohms per Mile	On Line Length of 76.8 Miles Self Ohms	On Line Length of 76.8 Miles Mutual Ohms
Zaa1 = Zdd1	1	0.306 + j 0.717		23.5 + j 55.1	
Zaa2 = Zdd2	1	0.306 + j 0.717		23.5 + j 55.1	
Zdoa1 = $\frac{Vdo}{Ia1}$	12		-0.0338 - j0.0419	1st Section no transpositions	
Zaod1 = $\frac{Vao}{Id1}$	8		+0.0193 + j0.0504	1st " " "	
Za1d1 = $\frac{Va1}{Id1}$	9		+0.00875 + j0.0461	1st " " "	
Zd2a2 = $\frac{Vd2}{Ia2}$	20		+0.00875 + j0.0461	1st " " "	
Zd1a1 = $\frac{Vd1}{Ia1}$	13		-0.00875 + j0.0461	1st " " "	
Za2d2 = $\frac{Va2}{Id2}$	17		-0.00875 + j0.0461	1st " " "	
Za1d1 = $\frac{Va1}{Id1}$	9 & 21		0.0 + j0.0		0.0 + j0.0
Zd2a2 = $\frac{Vd2}{Ia2}$	20		0.0 + j0.0		0.0 + j0.0
Zd1a1 = $\frac{Vd1}{Ia1}$	13		0.0 + j0.0		0.0 + j0.0
Za2d2 = $\frac{Va2}{Id2}$	17		0.0 / j0.0		0.0 + j0.0

Sequence	Equation	Self Impedance Ohms per Mile	Mutual Impedance Full Line Unless Noted. Ohms per Mile	On Line Length of 76.8 Miles Self Ohms	On Line Length of 76.8 Miles Mutual Ohms
$Z21 = \frac{Va2}{Id1}$	10 & 24	−0.01492−j0.005525			−1.146 − j0.4245
$Z21 = \frac{Vd2}{Ia1}$	14	−0.01492−j0.005525			−1.146 − j0.4245
$Z12 = \frac{Va1}{Id2}$	16	+0.01492−j0.005525			+1.146 − j0.4245
$Z12 = \frac{Vd1}{Ia2}$	19	+ 0.01492−j0.005525			+1.146 − j0.4245
$Z01 = \frac{Vao}{Id1}$	25	0.0 + j0.0			0.0 + j0.0
$Z02 = \frac{Vao}{Id2}$	15 & 25	0.0 + j0.0			0.0 + j0.0
$Z01 = \frac{Vdo}{Ia1}$	26	0.0 + j0.0			0.0 + j0.0
$Z02 = \frac{Vdo}{Ia2}$	18 & 26	0.0 + j0.0			0.0 + j0.0
$Zaao = \frac{Vao}{Iao}$	27	0.58 + j2.96		44.56 + j227.4	
$Zddo = \frac{Vdo}{Ido}$	27	0.58 + j2.96		44.56 + j227.4	
$Zado = \frac{Vao}{Ido}$	29		0.277 + j2.01		21.28 + j154.4

Chapter IV

Calculations of Relative Effects of Mutual Impedance

CHAPTER IV

CALCULATIONS OF RELATIVE EFFECTS OF MUTUAL IMPEDANCE

Section of Network Considered. The effect of mutual impedances in P1 and P2 transmission lines for a short-circuit at Rover Avenue 60 kv tie bus will be considered.

The connections assumed for this study are shown in Fig. 41. The P1 and P2 lines are paralleled at both ends by means of the tie bus located in each station. There are eight generators available to feed the two lines with 33,750 kva rated capacity. There are also two 15,000 kva transformer banks, which for the purpose of this study we shall assume are connected delta-wye to provide a path for zero-sequence currents into the transmission lines under consideration. The wye is solidly grounded. The connections external to this network will not be considered.

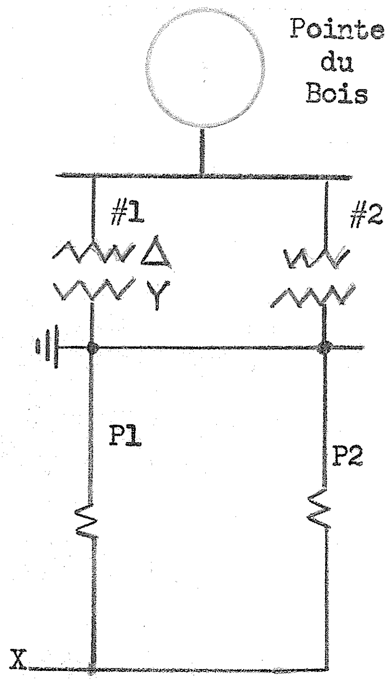
Assumptions. The following standard assumptions will be made in reducing this portion of the network.

1. The load currents will be neglected.
2. Generated internal voltages of all synchronous machines feeding these lines will be considered equal in magnitude and in phase with each other.
3. From statement 2 above it follows that all points in the positive-sequence network, where positive-sequence internal

voltages appear, may be connected together.

4. Lumped line constants will be used.
5. Effects of capacitance and transformer magnetizing currents will be neglected.
6. Assume positive direction of current flow the same in each of the sequence networks.
7. Phase a will be used as reference phase as heretofore.
8. All voltages unless otherwise specified are phase-to-neutral.
9. Resistance will be neglected in machines and transformers but will be taken into account in transmission lines.
10. The short-circuit will be taken on the Rover Avenue end of P1 line.
11. All impedances will be in ohms on the H.T. line base of a nominal 66 kv.

Positive-Negative and Zero-Sequence Diagrams.



Rover Ave.

Generators #1 - #8, 33750 kva total

$X'd = 24.5$ ohms
$X2 = 46.2$ ohms

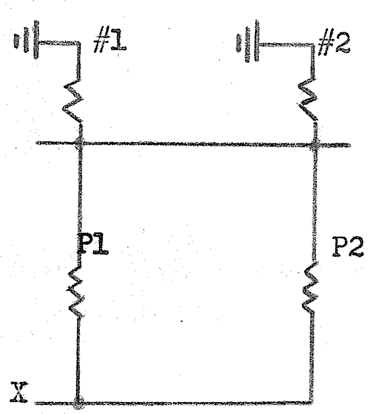
Banks #1 & #2, 15000 kva each

Ratio 6600-72000 = 10.92

Reactance = j19.07 ohms

Lines P1 & P2 Line Voltage 72000 volts

Zaa1 = Zdd1 = 23.5 + j55.1 ohms



Banks

Reactance = + j19.07 ohms

Lines

Zaao = Zddo = 44.56 + j227.4 ohms

Zado = Zdao = 21.28 + j154.4 ohms

Fig. 41 Impedance Diagrams. All values on a nominal 66 kv Base.

Positive- Sequence Network. Two lines in parallel. The positive-sequence network will readily be seen from Fig. 41. The reduction of this network produces a positive-sequence equivalent impedance of 11.75 ≠ j61.58 ohms.

Negative-Sequence Network. Two lines in parallel. The negative-sequence network is identical with the positive-sequence network in all but the generator reactance. The reduction of the negative-sequence network produces an equivalent impedance of 11.75 ≠ j83.28 ohms.

Zero-Sequence Network. Two lines in parallel. The equivalent circuit for two parallel and equal transmission lines with mutual impedance between them produces an equivalent impedance [1] of 1/2(Zaao ≠ Zado). Using the equivalent circuit for two parallel transmission lines with mutual impedance between them the zero-sequence network (as shown in Figure 41) reduces to 32.92 ≠ j200.43 ohms.

When mutual impedance to zero-sequence currents is disregarded, the zero-sequence network impedance is 22.28 ≠ j123.23 ohms.

Fault Currents - Short-Circuit on P1 at Rover Avenue. Using a voltage of 72,000 volts, the short-circuit current at the Rover Avenue end of P1 line for the case considering mutual impedance is given by $i = \frac{72,000}{\sqrt{3}(56.42 \neq j345.29)}$

Here i = 119.5 amps. as against 153.0 amps. when mutual impedance is disregarded. This represents an increase of 28.0%, a value which cannot be disregarded in short-circuit calculations. The effect of transmission line resistance in the positive-, negative,- and zero-sequence network is considered.

[1] E. Clarke. Vol. I. p.32. Loc. cit.

Positive-Sequence Network. P2 Line open at Rover Avenue.

The resulting value of positive-sequence impedance is 23.5 + j39.13 ohms.

Negative-Sequence Network. P2 Line open at Rover Avenue.

The resulting value of negative-sequence impedance is 23.5 + j110.73 ohms.

Zero-Sequence Network P2 Line Open at Rover Avenue. If the circuit fault is the same, namely P1 line involved in a line-to-ground fault at Rover Avenue, then the resulting equivalent circuit will be a three terminal circuit [1], for the case where mutual impedance is considered. The equivalent circuit reduces to a value of 44.56 + j236.93 ohms. If the mutual impedance to zero sequence currents is neglected the resulting equivalent impedance for the zero-sequence network is 44.56 + j236.93 ohms.

Fault Currents - Short on P1 Line at Rover Avenue P2 Line Open at Rover Avenue.

The fault current is the same whether the adjacent line is open or closed. The magnitude of this current is 173.5 amperes. A further point to observe is the magnitude of the induced voltage in the open line. This value approaches $27000 \underline{/82.3^{\circ}}$ vector volts which when added to normal line voltage is a value to contend with.

[1] E. Clarke. Vol. I p.35. Loc. cit.

Further Effects of Z21. With reference to equation (24) Chapter III, the mutual impedance between the positive-and negative-sequence network is relatively small.

$Va2/Id1 = Z21 = -0.01492 - j0.005525$ ohms per mile, on full line length $Z21 = -1.146 - j0.4245$ ohms.

This mutual coupling can be used by taking advantage of the equivalent circuits [1] developed for mutual coupling between a two terminal network. This is best demonstrated on an A.C. network analyzer.

The effect of the impedance Z21 can be seen when the negative-sequence voltage drop in line a, b, c (P1) is calculated for positive-sequence current flowing in line d, e, f (P2). If the fault current is 119.5 amperes as determined for the case considering mutual impedance to zero-sequence. Then

$Va2 = -146. \underline{/200^{\circ}.25}$ which is only 0.203% of normal line voltage of 72000 volts.

[1] E. Clarke. Vol. I. p.32 & p.185 Loc. cit.

Chapter V

Conclusions

CHAPTER V

CONCLUSIONS

In reviewing the preceding work the interesting features are grouped under the following four main headings.

Mutual Impedances affected by Transpositions. It is to be noted from equations 8 and 12 Chapter III that an untransposed transmission line may have non-reciprocal zero-sequence voltage drops due to positive-sequence currents. These are the impedances Z01 in line with the definitions set forth on page 12. By completely transposing these lines the mutual impedance between the positive-zero-and negative-zero-sequence networks (Equations 25 and 26 Chapter III) can be made zero. Thus we may conclude that the impedances Z01 and Z02 for the pair of lines under consideration are zero. Generally speaking we could say that these impedance values are reciprocal for the case considered. However, in an unsymmetrical circuit these impedances are non-reciprocal for the general case.

The positive-sequence mutual impedance for the P1 and P2 transmission lines, as previously pointed out in Chapter III Page 31 can have any reactance value from a negative quantity to a positive quantity depending upon whether the numerator or the denominator of the fraction of the logarithm of the reactance term in equation 21 is the greater. The resistance or real term in this equation is usually zero.

The order of magnitude of the positive-sequence mutual impedance is "usually not more than $\pm$ 5% of the positive-sequence self-impedance of either circuit".[1]

As far as my observations of the available literature on this subject have gone, I believe it has not previously been pointed out that the positive-sequence mutual impedance (Zad1) for two parallel lines, each with an equilateral triangle conductor configuration, is zero for completely transposed lines.

An investigation of the conductor configuration of these lines that would produce a negative value of positive-sequence mutual impedance would be of importance to determine if the regulation of these lines, when operating in parallel could be improved. The positive-sequence mutual impedance can be made wholly reactive if the two circuits are arranged symmetrically on opposite sides of an imaginary plane between circuits in such a manner that conductors of the same phase appear as images with respect to this plane. The physical difficulties of such a scheme for the lines under consideration are many. It would require rearrangement of the end connections to the line and the problem hence resolves itself into an economic study. What would be the saving in kilowatt hours over a 10-year period? This would have to be balanced against the cost of rearranging the end connections. The full effect of any such rearrangement would have to be considered for the other cross-circuit cross-sequence impedances before its value could be properly assessed.

[1] E. Clarke Vol. I p.420. Loc. cit.

There would be no effect on the zero-sequence mutual impedance.

The mutual impedance between the positive-and negative-sequence networks for these lines can be used to determine the negative-sequence voltage drop in the one circuit when positive-sequence current is flowing in the other circuit. The value of 0.20% of normal line voltage found in Chapter IV makes this mutual impedance insignificant relative to the self impedance of the line. The mutual impedance will depend upon the arrangements of the phases of the two circuits in their tower positions.

<u>Short-Circuit Studies.</u> In the usual transmission line short-circuit study, the mutual impedances between parallel lines are neglected unless a high degree of precision is required. This however is not true of the zero-sequence mutual impedance found to exist between P1 and P2 lines, in the discussion of zero-sequence mutual impedance. The summary of mutual and self impedances in Chapter III, pages 43 and 44, lists the zero-sequence self impedance as $44.56 + j227.4$ ohms per line. The mutual impedance to zero-sequence between lines is $21.28 + j154.4$ ohms, or 67.5% of the zero-sequence self impedance. The effect on the short-circuit current for the simple case discussed in Chapter IV is of considerable importance and must be considered in calculating fault currents.

In short-circuit studies for circuit breaker application or for preliminary relay settings on a high-tension system an error of 5% in the calculated values of short-circuit current would be negligible. However the short-circuit current values would be in error by 28.0% if zero-sequence mutual impedance were disregarded in the circuit considered in Chapter IV. This would be entirely unreasonable. We may therefore conclude that the mutual impedance between adjacent circuits to zero-sequence currents should be considered in all unbalanced short-circuit studies.

<u>New Line Design</u>. In new line extensions involving one or more circuits, the engineer must arrive at a design that is economical as well as practical. This requires a comprehensive investigation in order that voltage drops, energy losses, corona loss, stability and general performance under normal as well as fault conditions can be predicted accurately.

If a new line were built so that each conductor was equidistant from all other conductors carrying current and also equidistant from the ground, then the circuit would be completely symmetrical. In this case the positive-negative-and zero-sequence inductance would be the same for each phase. However such a line arrangement is impossible, and the practical alternative to this, to obtain average values of positive-negative-and zero-sequence reactance is to have each conductor occupy each tower position for one-third the total distance.

If a new line or double circuit line were not transposed, then the unsymmetrical voltage drops would have to be calculated for each of the sequences by the method used in Chapter III. The lack of symmetry between the three phases would result in the introduction of negative- and zero-sequence induced voltage drops in the other conductors. These in turn would produce their own sequence voltage drop as well as voltage drops to each of the other sequences. The reduction or total elimination of the majority of these mutual impedances can be accomplished by properly transposing the line. This is demonstrated for positive-sequence mutual impedance by equations 13 and 21. Equation 13 represents a definite value of positive-sequence mutual impedance for the first untransposed section of the line whereas equation 21 for the completely transposed line produces a zero- value of Zadl.

<u>Summary</u>. The foregoing investigation into the relative effects of mutual impedance was carried out for a pair of lines of the City of Winnipeg Hydro Electric System's high tension network. The need for taking the line configuration and transpositions into account for the positive-and negative-sequence mutual impedances and the cross-sequence mutual impedances was demonstrated. While transpositions have no effect on the zero-sequence mutual impedance between parallel lines, this mutual impedance is the most important of those encountered.

All parallel lines in the high tension network will have mutual impedances for all sequences to varying degrees, depending upon conductor configuration, proximity of ground wires and adjacent circuits.

These could readily be calculated for all lines by a similar procedure to that used in Chapter III taking full advantage of the simplifications developed therein.

The relative effect on the values of short-circuit currents and induced potentials by using the significant values from these calculations could be shown for a short-circuit fed by the entire network. However, it has been demonstrated in Chapter III that the zero-sequence mutual impedance is the only value of significance for short-circuit studies on this particular pair of lines. Therefore I believe that the relative effects of the mutual impedance in the zero-sequence network during a line-to-ground short-circuit involving this pair of parallel lines only, have been adequately demonstrated in the preceding investigation.

BIBLIOGRAPHY

E. Clarke. Circuit Analysis of A.C. Power Systems Vol. I Symmetrical and Related Components. New York John Wiley and Sons 1943.

W.V. Lyon Applications of the Method of Symmetrical Components New York McGraw-Hill 1937.

C.F. Wagner & R.D. Evans Symmetrical Components as applied to the analysis of unbalanced electrical circuits New York McGraw-Hill 1933.

Westinghouse Electric & Manufacturing Co. Electrical Transmission & Distribution Reference Book. Chicago Lakeside Press. 1942.

D. Braymer Today's Network Calculators will Plan Tomorrow's Systems, Electrical World January 5, 1946. p.53

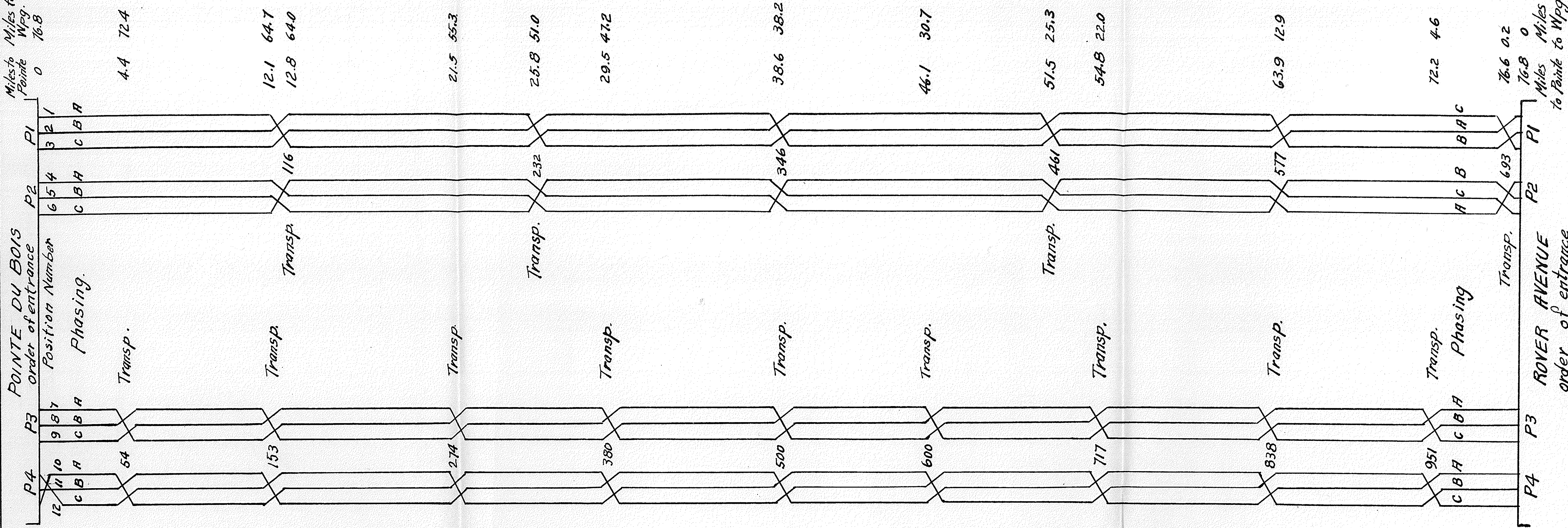
F.M. Starr Equivalent Circuits. Proc. A.I.E.E. Vol. 51, 1932 pp. 287-298.

W.M. Hanna Calculation of Short-Circuit Currents in A.C. Networks, (Series of Articles) General Electric Review, March, April, June and August, 1937, pp. 125-130, 189-196, 287-291, and 383-388 incl.

W.W. Lewis Single-Phase Short-Circuit Calculations, General Electric Review July, 1925.

John R. Carson Wave Propagation in Overhead Wires with Ground Return. Bell System Technical Journal Vol. 5, 1926. pp. 539-554.

J.E. Clem Reactance of Transmission Lines with Ground Return A.I.E.E. Transactions Vol. 50 September, 1931. pp. 901-918.



Note:-

See Fig. 31 Page 18 for typical cross section of Pointe Transmission Lines looking toward Pointe du Bois.

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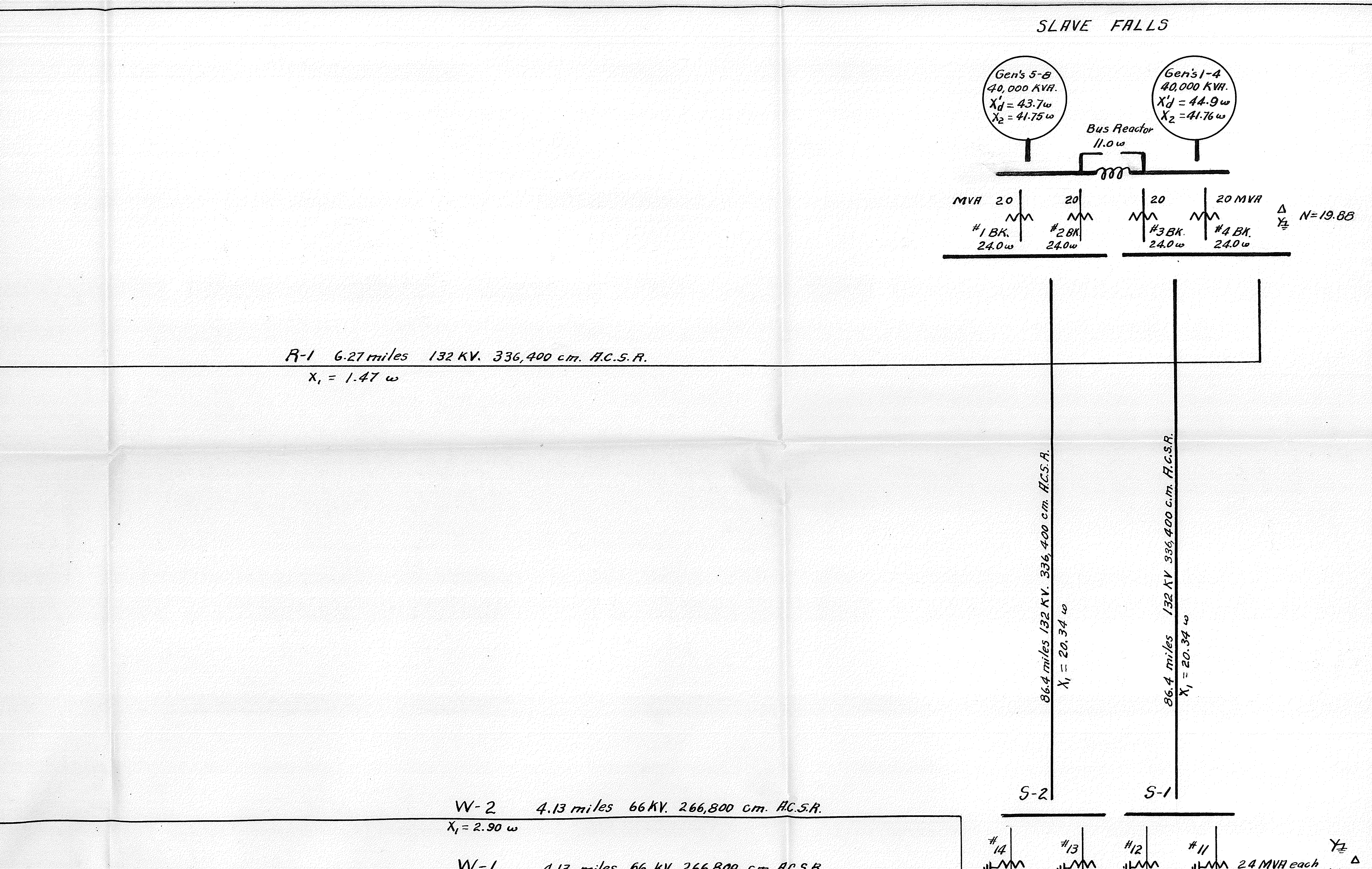
A mathematical investigation of the relative importance of mutual impedance in short circuit calculations for transmission networks

Figure 31

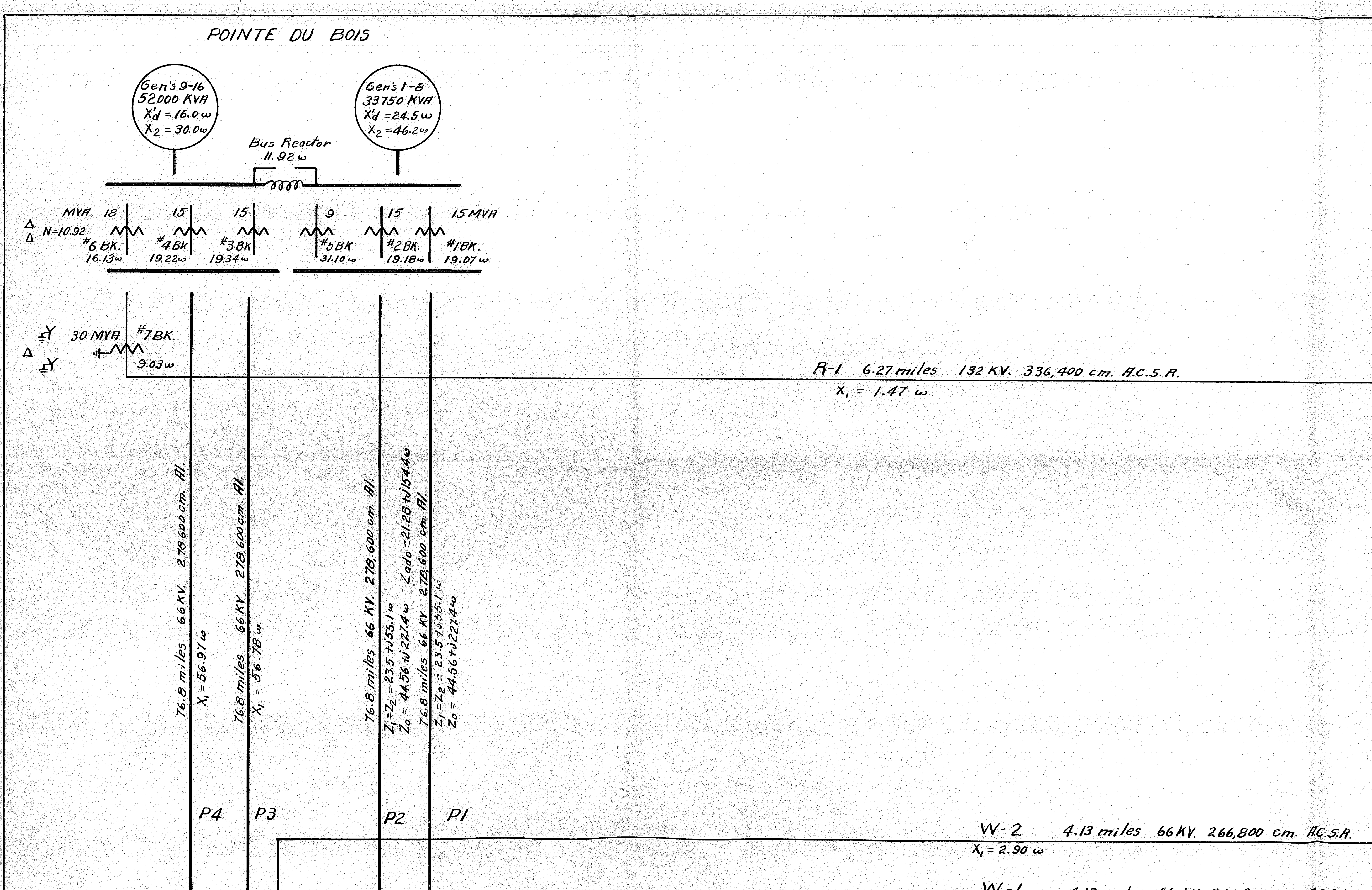
Plan of Transpositions

April 1948

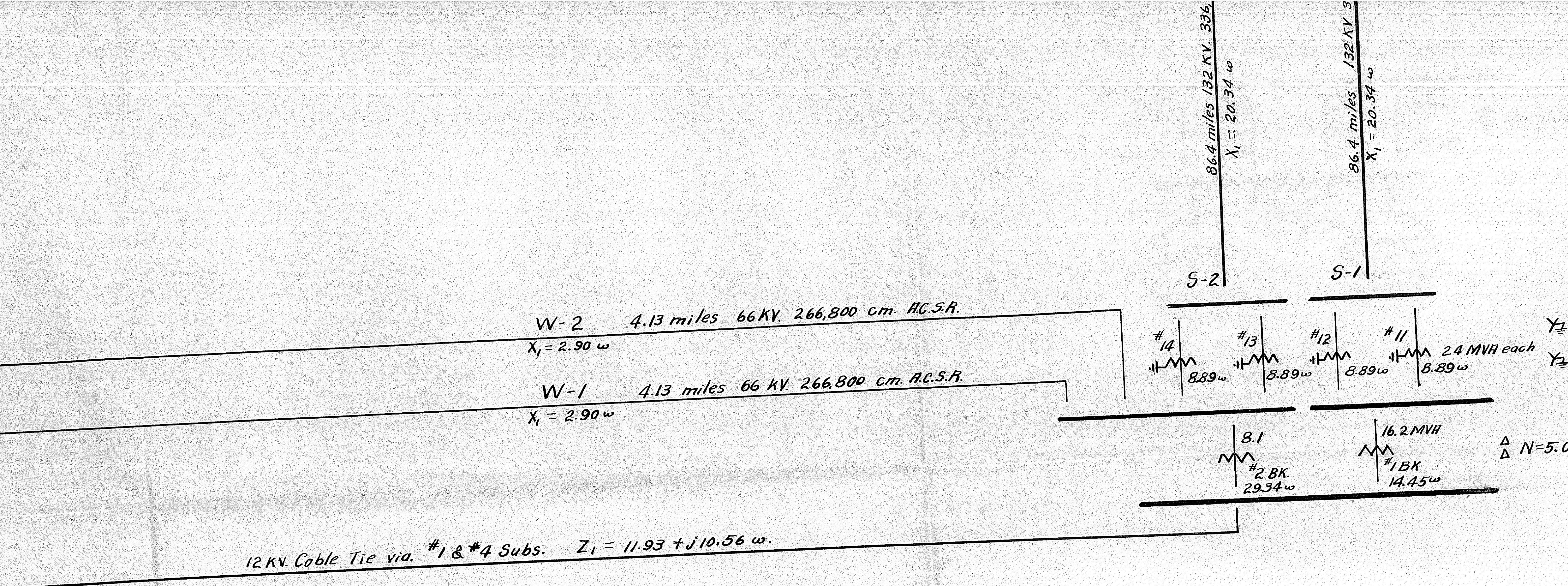
L. A. Bateman.



SLAVE FALLS
Gen's 5-8
40,000 KVA.
X'd = 43.7ω
X2 = 41.75ω
Gen's 1-4
40,000 KVA.
X'd = 44.9ω
X2 = 41.76ω
Bus Reactor
11.0ω
MVA 20
20
20
20 MVA
#1 BK. 24.0ω
#2 BK. 24.0ω
#3 BK. 24.0ω
#4 BK. 24.0ω
N=19.88
R-1 6.27 miles 132 KV. 336,400 cm. A.C.S.R.
X1 = 1.47 ω
86.4 miles 132 KV. 336,400 cm. A.C.S.R.
X1 = 20.34 ω
86.4 miles 132 KV 336,400 c.m. A.C.S.R.
X1 = 20.34 ω
S-2
S-1
W-2 4.13 miles 66 KV. 266,800 cm. A.C.S.R.
X1 = 2.90 ω
#14
#13
#12
#11
24 MVA each



POINTE DU BOIS
Gen's 9-16
52000 KVA
X'd = 16.0 ω
X2 = 30.0 ω
Gen's 1-8
33750 KVA
X'd = 24.5 ω
X2 = 46.2 ω
Bus Reactor
11.92 ω
MVA 18
15
15
9
15
15 MVA
N=10.92
#6 BK.
16.13 ω
#4 BK
19.22 ω
#3 BK
19.34 ω
#5 BK
31.10 ω
#2 BK.
19.18 ω
#1 BK.
19.07 ω
30 MVA
#7 BK.
9.03 ω
R-1 6.27 miles 132 KV. 336,400 cm. A.C.S.R.
X1 = 1.47 ω
76.8 miles 66 KV. 278 600 cm. Al.
X1 = 56.97 ω
76.8 miles 66 KV 278,600 cm. Al.
X1 = 56.78 ω
76.8 miles 66 KV. 278,600 cm. Al.
Z1 = Z2 = 23.5 + j55.1 ω
Zo = 44.56 + j227.4 ω
Zado = 21.28 + j154.4 ω
76.8 miles 66 KV 278,600 cm. Al.
Z1 = Z2 = 23.5 + j55.1 ω
Zo = 44.56 + j227.4 ω
P4
P3
P2
P1
W-2 4.13 miles 66 KV. 266,800 cm. A.C.S.R.
X1 = 2.90 ω



SCOTLAND AVENUE

Notes:- All quantities shown in ohms as on 66 KV. System.

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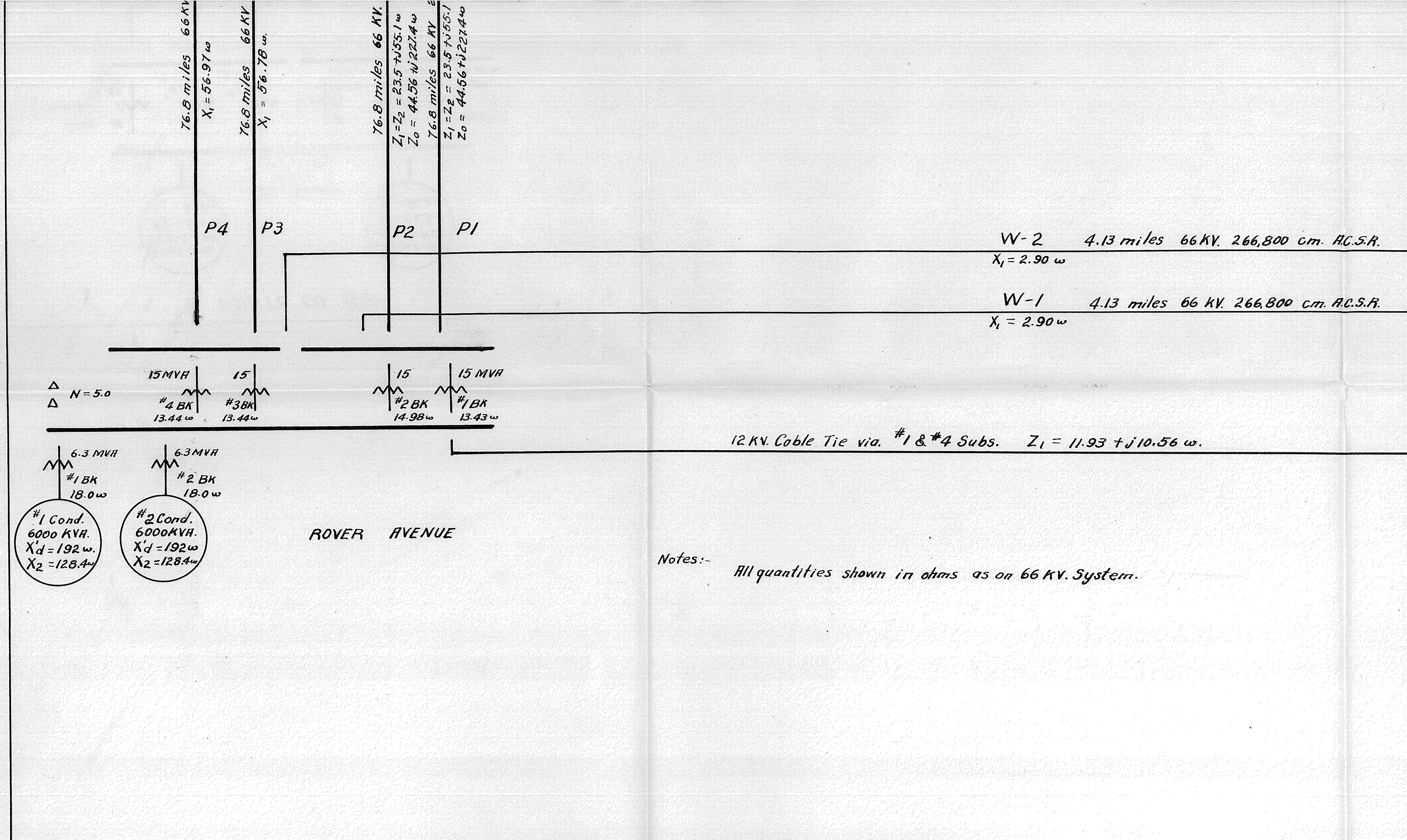
A mathematical investigation of the relative importance of mutual impedance in short circuit calculations for transmission networks

Figure No. 1

City of Winnipeg Hydro Electric System
High Tension Network

April 1948

L.A. Bateman.


76.8 miles 66 KV
$X_1 = 56.97\omega$
76.8 miles 66 KV
$X_1 = 56.78\omega$
76.8 miles 66 KV.
$Z_1 = Z_2 = 23.5 + j55.1\omega$
$Z_0 = 44.56 + j227.4\omega$
76.8 miles 66 KV
$Z_1 = Z_2 = 23.5 + j55.1$
$Z_0 = 44.56 + j227.4\omega$
P4
P3
P2
P1
W-2
4.13 miles 66 KV. 266,800 cm. A.C.S.R.
$X_1 = 2.90\omega$
W-1
4.13 miles 66 KV. 266,800 cm. A.C.S.R.
$X_1 = 2.90\omega$
15 MVA
#4 BK
13.44ω
15
#3 BK
13.44ω
15
#2 BK
14.98ω
15 MVA
#1 BK
13.43ω
Δ N = 5.0
Δ
12 KV. Cable Tie via. #1 & #4 Subs. $Z_1 = 11.93 + j10.56\omega$.
6.3 MVA
#1 BK
18.0ω
6.3 MVA
#2 BK
18.0ω
#1 Cond.
6000 KVA.
$X'_d = 192\omega$.
$X_2 = 128.4\omega$
#2 Cond.
6000 KVA.
$X'_d = 192\omega$
$X_2 = 128.4\omega$
ROVER AVENUE
Notes:-
All quantities shown in ohms as on 66 KV. System.

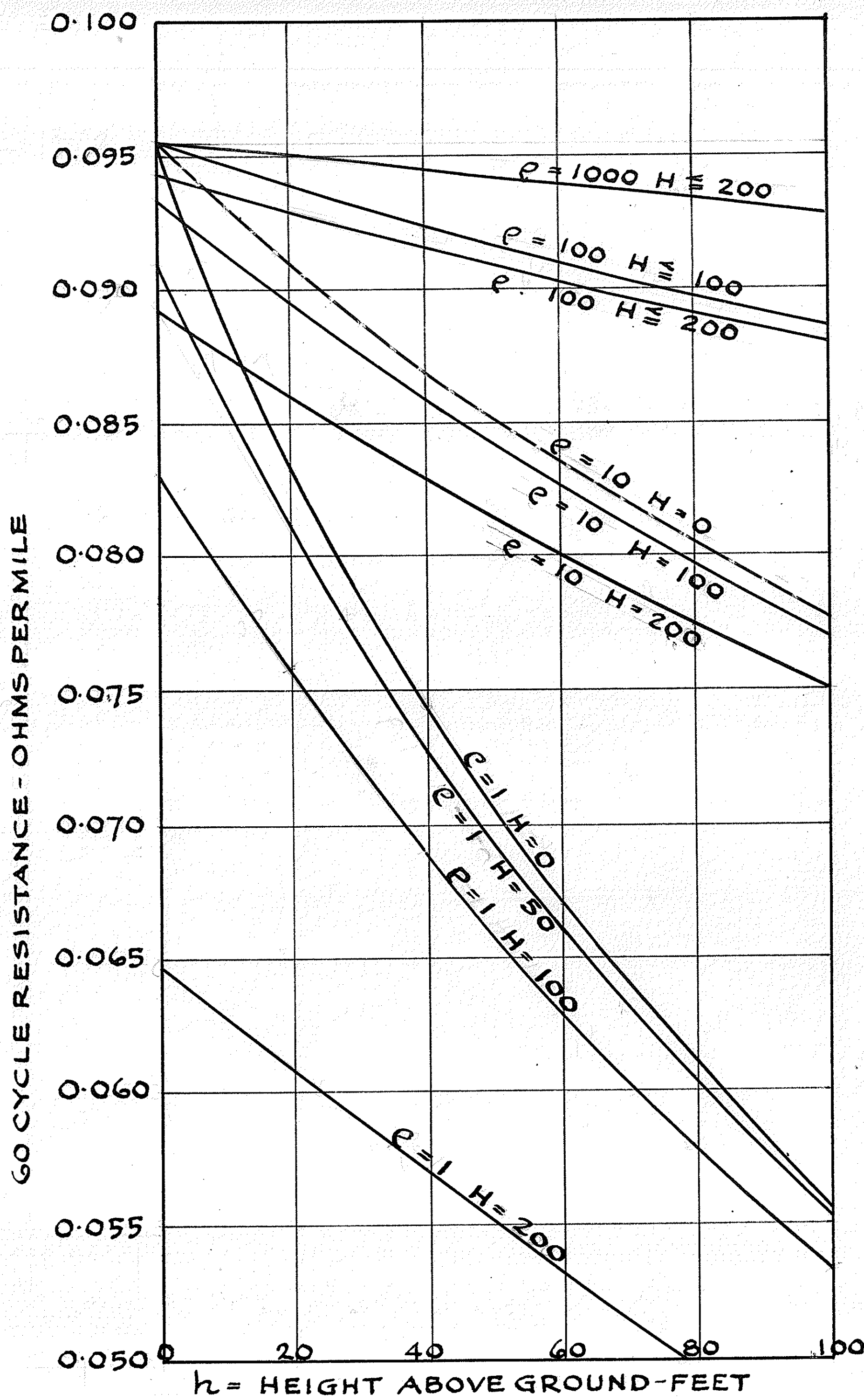


FIG. 32 60 cycle self and mutual resistances of the earth.

From E. Clarke Vol. 1 Circuit Analysis of A.C. Power Systems.

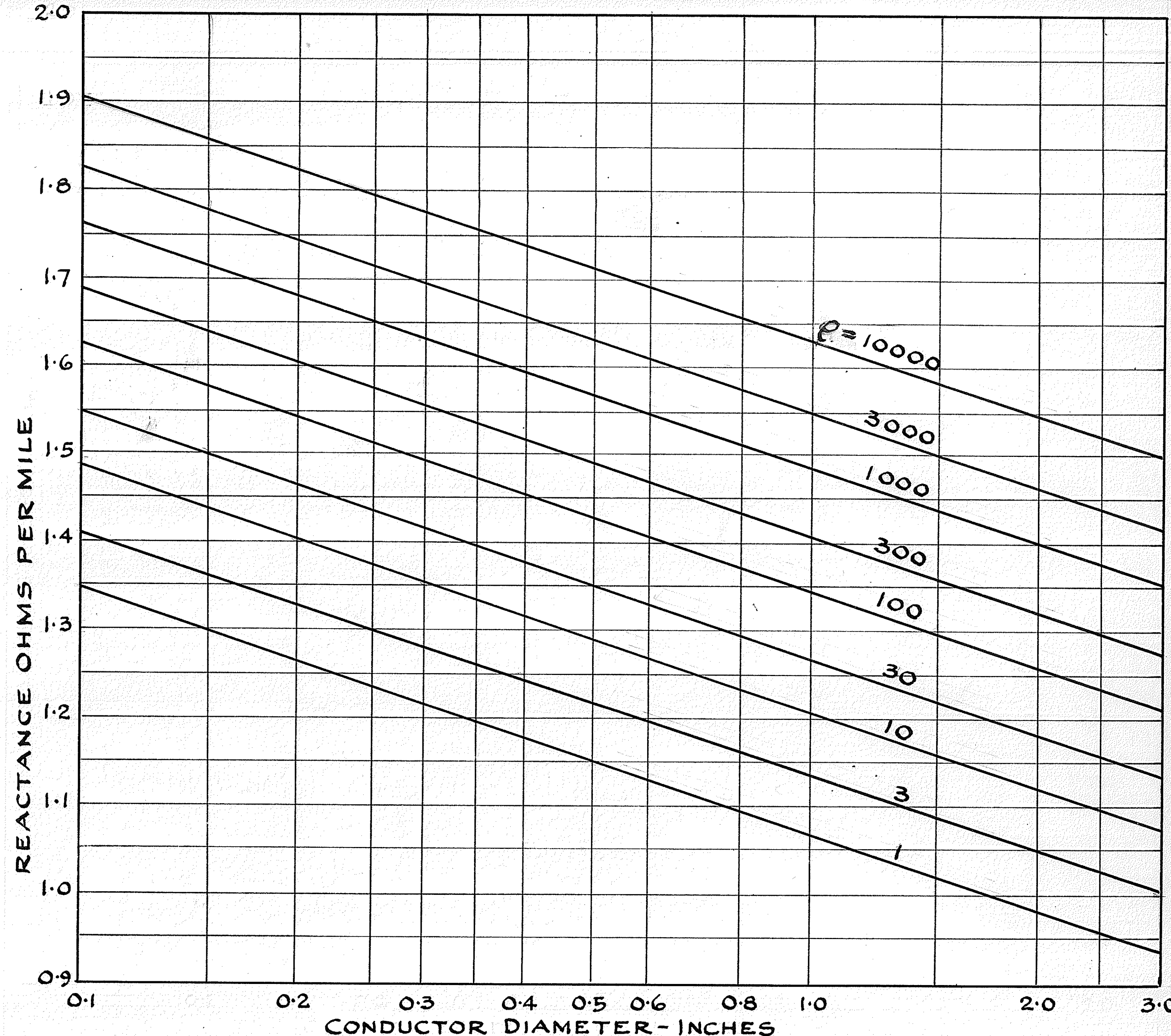


FIG. 33 60 cycle self reactance external to the conductor. Height above ground neglected.

From E. Clarke Vol. 1 Circuit Analysis of A.C. Power Systems.

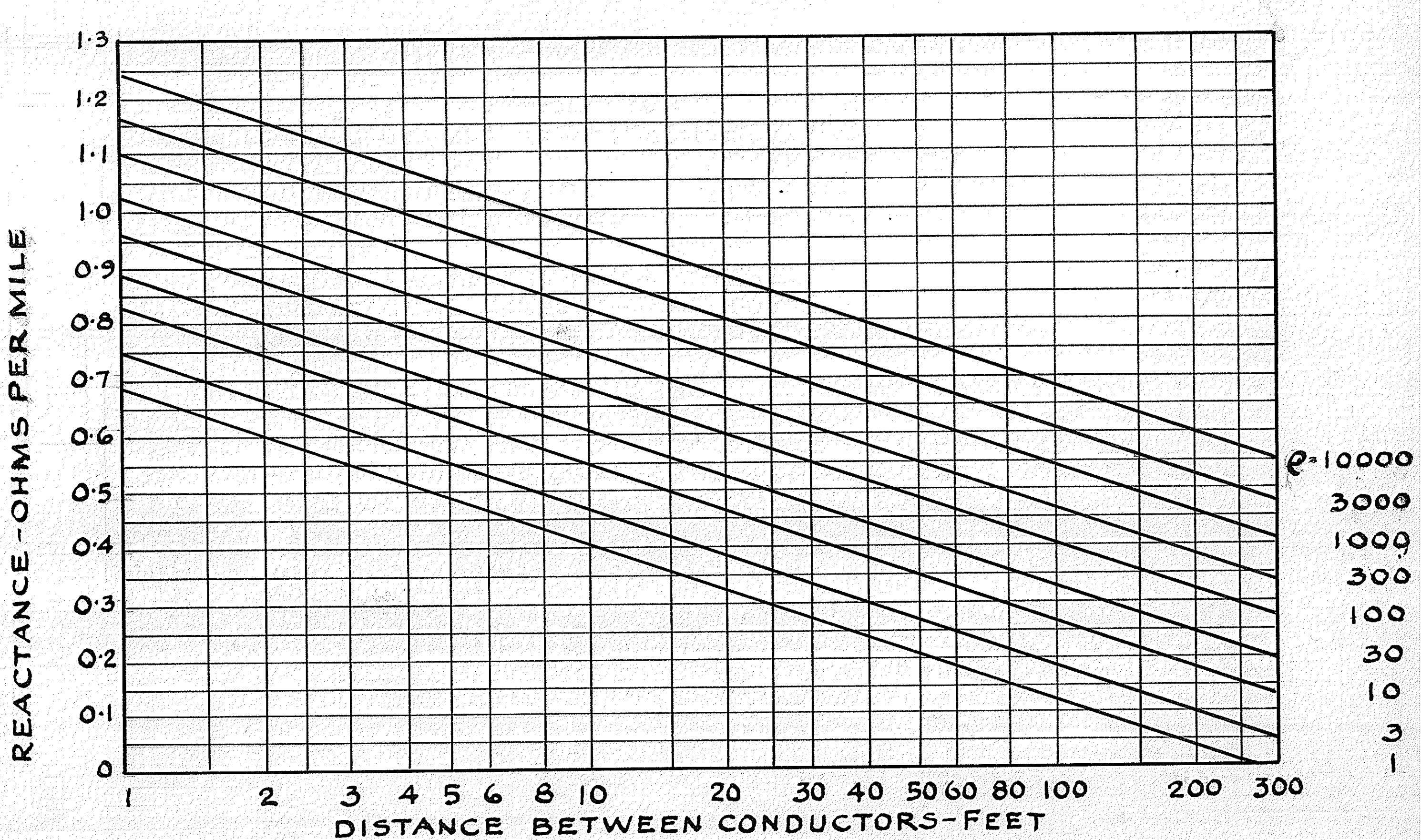


FIG. 34 60 cycle mutual reactance between two conductors with earth return. Height above ground of conductors neglected.

From E. Clarke Vol. 1 Circuit Analysis of A.C. Power Systems.

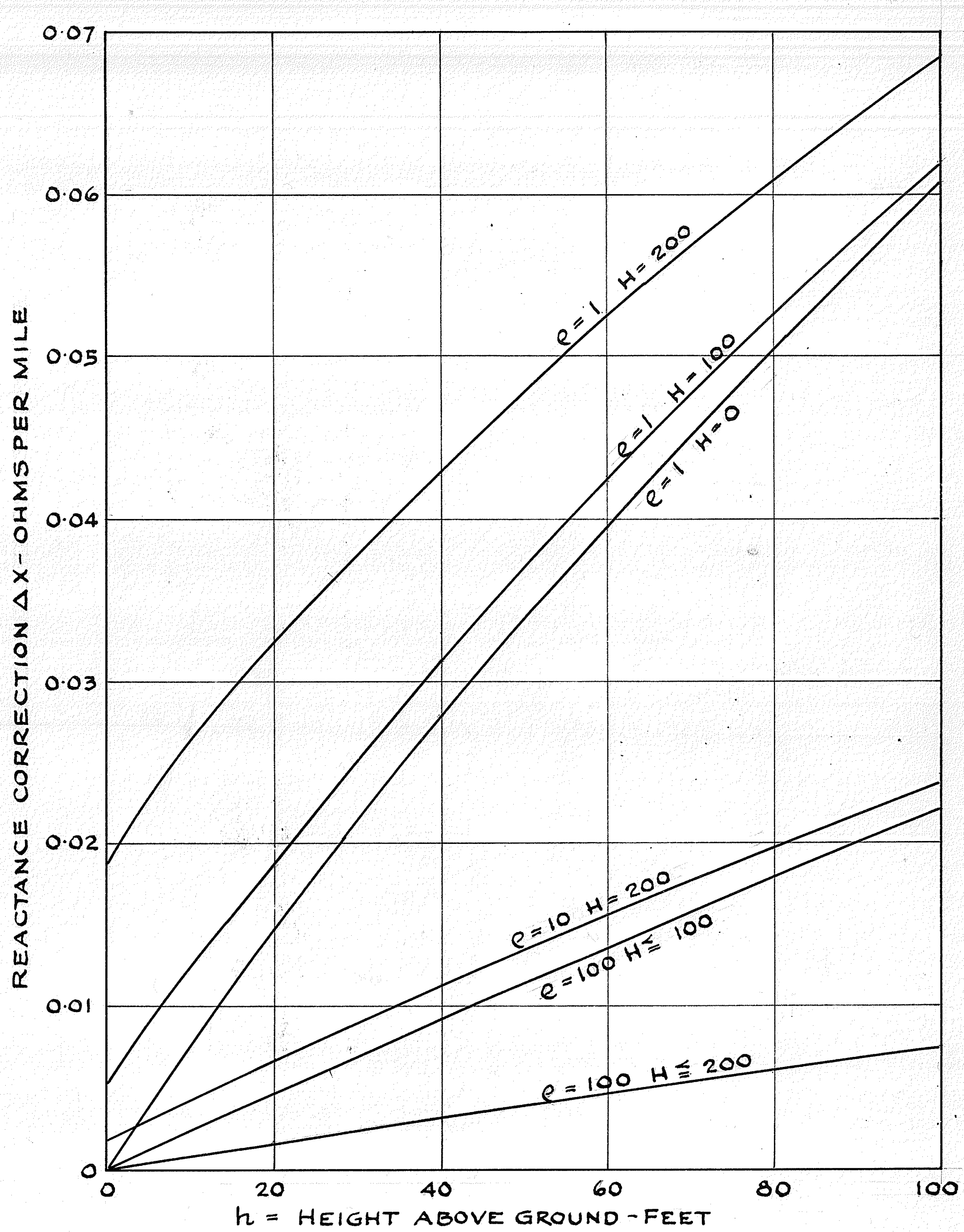


FIG. 35 Corrections to 60 cycle self and mutual reactances to account for height of conductors above ground.

From E. Clarke Vol. 1 Circuit Analysis of A.C. Power Systems.