A Simulation Investigation of Latent Variable Growth Models for Interaction Effects

by

Ian Clara

A Thesis submitted to the Faculty of Graduate Studies of

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Abstract

Latent growth curves are an effective tool for describing the change or growth of an attribute over time. Interactive effects between two latent variables on the rate of change of a latent outcome of interest are of great interest to researchers. Several models have been utilized to conceptualize the interaction in latent growth curves, but as yet there has been a limited amount of empirical research to assess each of these models. The current study used a Monte Carlo simulation approach to investigate three latent growth interaction models -- those by Wen (Wen et al., 2000), Duncan (Duncan et al., 1999), and a longitudinal extension of the model by Schumacker (2002), under varying conditions, with 5000 replications per condition. The factors of missing data mechanism (Complete, Missing Completely At Random, Missing Not At Random), correlation between latent intercept and slope factors (small, medium, large), sample size (250, 500, 1000), and the reliability of the observed variables (very low, low, average, high) were manipulated to determine their effects on overall model performance and model fit, bias of the estimates for the latent slope interaction effect, and rates of Type I error. Of the three models assessed, the Wen model showed the most reliable performance with respect to overall model fit, and the Duncan and Schumacker models showed the most reliable performance with respect to parameter estimation, and bias. The Schumacker model showed adequate Type I error control when the data was either Complete or Missing Completely at Random. When the missing data mechanism was Missing Not at Random none of the models performed well, however the Schumacker model showed the most promising behaviour with respect to bias and Type I error control. Recommendations for researchers utilizing these models are made, as well as considerations for their use.

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Dedication

To Keith – I couldn't have done this without you. You are my partner in love and in life, and your love and faith in me kept me going when nothing else would. You are a source of inspiration, strength, and courage. I did this for you and with you my love, and we finished it together, and I wouldn't want it any other way.

To my parents and family, who always believed that I would complete this project, even when they had no clue as to what it was about (and they're still not really sure).

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A Simulation Investigation of Latent Variable Growth Models for Interaction Effects

Overview

Longitudinal modeling has received increased interest in psychology and the social sciences. The ability to detect and investigate trends (or change) over time has great import for measuring developmental processes (Aiken & West, 1992), and for determining such phenomenon as the course of adult depression (Zuroff, Blatt, Sanislow, Bondi, & Pilkonis, 1999), the trajectory of cognition in the elderly (Raykov, 1993) and the effects of potential moderators on intervention practices in children (Dawson-McClure, Sandler, Wolchik, & Millsap, 2004a). An important area of research is the nature of interaction effects within the framework of latent variable growth modeling. Li, Duncan, Duncan, Yang-Wallentin, and Acock (2001) note that studying the impact of interactive relationships between growth factors (i.e., latent slopes) can be of substantive interest in those hypotheses that seek to determine how change in two latent attributes interact to produce a joint effect on the growth of an outcome.

Due to the relatively recent application of structural equation models to longitudinal data in terms of latent variable growth models, empirical research delving into the utility of these models needs to be carried out. More importantly, there has been a paucity of research to examine the utility of latent variable growth models to investigate interaction effects of rates of change in longitudinal designs (Curran & Hussong, 2003).

This dissertation begins with an overview of longitudinal designs and some of the associated conventional analytical methods in Chapter 1. Sections 2 through 4 outline some alternative approaches to longitudinal designs (e.g., mixed models), the use of structural equation models, and latent variable growth models. Sections 5 through 7 discuss the nature of interaction effects in latent variable growth models. Section 8 discusses the issue of missing data mechanisms in longitudinal designs, an important factor for researchers to consider in these types of designs. Section 9 outlines the procedural plan and the specific hypotheses addressed, and Section 10 gives information regarding the simulation conditions and the method of analysis. Results are presented in Section 11, and the final section, Section 12, discusses these findings and also presents recommendations for the researcher.

The current project investigated several basic issues in analyzing latent slope interactions using latent variable growth models with longitudinal data. Several methods have been proposed to represent such latent interaction effects in latent variable models, and few studies have compared them to each other. As a result, the conditions under which these methods have been studied has been limited. A Monte Carlo simulation was used to examine some of the empirical issues surrounding the estimation of latent slope interaction effects in latent variable growth models. Specifically, issues of overall model fit (including model convergence), estimation of the latent slope interaction effect (including bias in the estimates), and Type I error rates were compared for three model representations of a latent slope interaction effect, for three types of missing data mechanism. These issues were examined under the manipulation of the following factors: correlation between the latent intercept and slope for a factor, sample size, and the reliability of the observed indicators.

Chapter 1: Literature Review of Longitudinal Designs in Assessing Change Over Time

In cross-sectional studies individuals are measured at only a single time point. These types of studies allow the researcher to investigate the relationships between variables but are not optimal for the assessment of any causative influences on the observed outcomes of interest over time. In order for such causative influences to be determined, individuals (or groups) must be studied at consecutive time points, utilizing what is known as a longitudinal design. According to Curran and Hussong (2003) longitudinal designs "permit the systematic study of stability and change over time and thus can provide critically needed empirical evaluations of the course, causes, and consequences" of psychological phenomena (p. 526).

A particular type of longitudinal design is a repeated-measures design, which involves measurements of the same variables on the same subjects over a period of time. When using such a longitudinal perspective to modeling repeated-measures data the researcher is interested in how a response variable changes over time (Willett, Singer, & Martin, 1998) – be it a linear increase, a linear decrease, a non-linear type of change, or even no change at all. These types of models are also termed growth models. An obvious advantage of longitudinal designs over cross-sectional designs is the ability to assess the stability and growth behaviour of causal relationships between antecedent independent variables and the subsequent dependent variables which are the outcomes of interest.

The analysis of longitudinal data (including repeated-measures data) requires the use of techniques that take account of the correlations between successive measurement occasions (Raudenbush & Bryk, 2002; Rowe, 2002). Some analytic approaches, such as mixed models (Littell, Milliken, Stroup, & Wolfinger, 1996; Singer, 1998), hierarchical linear models (e.g., HLM; Raudenbush et al., 2002) and latent growth models (Meredith & Tisak, 1990) can be used for the proper analysis of longitudinal designs. One purpose of a longitudinal design is to relate the change in behaviour over time, represented as a growth parameter, to individual characteristics, background variables, and environmental factors. This approach makes it possible to detect systematic inter-individual differences in individual growth parameters (Stoel, van der Wittenboer, & Hox, 2004), and allows for the studying of predictors of growth or decline (Raykov & Marcoulides, 2000). The next sections describe several methods that can be used to analyze longitudinal data.

General Linear Modeling (GLM) Approaches to the Analysis of Longitudinal Data

A common approach to the analysis of longitudinal data is to utilize a general linear model approach, such as a repeated-measures analysis of variance (RM ANOVA) or a multivariate analysis of variance (MANOVA; Keppel, 1991). In an RM ANOVA the dependent variable is treated as a repeated-measures (within-subjects) factor (Games, 1990), and the interest is on mean differences among groups and whether any of those differences are likely to have occurred by chance. In a "pure" repeated-measures design, all subjects serve in all treatment conditions and, as a result, serve to control subject variability. The RM ANOVA has an assumption that the variances across the repeated measures all come from the same population, also known as the sphericity assumption (Keppel, 1991). The RM ANOVA can be used to investigate simple within-subjects designs, where time is assumed to be a categorical factor, with balanced data, and equal time spacing between assessment points. The RM ANOVA approach can also analyze a mixed within/between design, with interaction effects between time and other between-subjects factors. However, the RM NOVA can only incorporate time-invariant covariates (Kwok et al., 2008).

The MANOVA is a generalization of the RM ANOVA to the situation of there being several dependent variables (Tabachnick & Fidell, 1996). Like the RM ANOVA the focus is still on mean differences among groups. However, the sphericity assumption is no longer required when the multivariate approach is taken, an advantage of the MANOVA since this assumption is often violated in longitudinal designs.

Both of these approaches are based on the general linear model (GLM) as a framework, and many computer statistical packages offer a general linear modeling module. The GLM approach is limited in a number of ways. This approach requires complete data for all observed occasions of measurement, in that every subject must have complete data across all occasions of measurement, and any subjects with missing data are removed from the analysis. Some of the assumptions that are associated with the univariate GLM approach are often untenable, in particular the assumption concerning sphericity (McCall & Appelbaum, 1973; Raykov et al., 2000), and there are a limited number of variance/covariance structures that can be modeled. Finally, the GLM approach can only accommodate continuously distributed repeated measures, and its reliance on a fixed-effects approach means that systematic relations are evaluated by pooling across individuals, and the only source of variation is in the residual effect.

Hierarchical Modeling Approaches to Longitudinal Data

The data generated by a longitudinal design is sometimes referred to as hierarchical data, where occasions of measurement are considered to be nested within individual subjects (i.e., the data are nested with respect to time; for a review see Raudenbush & Bryk, 2002). With hierarchical models each of the levels in the nested data structure are formally represented by their own submodel. These submodels express relationships between the variables at a given level, as well as expressing how variables from different levels exert an influence on the variables at a given level. In a longitudinal design, the lowest level of measurement (also known as the Level 1 model) is the time or occasion of measurement. The general equation for the unconditional Level 1 model (i.e., no individual effects) can be represented as

$$Y_{it} = \beta_{0i} + \beta_{1i} X_{it} + \varepsilon_{it}, \qquad i = 1, 2, ..., N; t = 1, 2, ..., T, \qquad (1)$$

where N is the number of individuals, T is the number of measurement occasions (or time), Y_{it} is the response of person i at time t, X_{it} is the occasion of measurement (e.g., the age at time t for person i), and ε_{it} represents the residual. This general equation also represents the mean model or population average model. The X_{it} can indicate real time or the ordinal position of occasions (e.g., 1, 2, 3, etc.). This Level 1 model contains three random variables – the intercept (β_{0i}), the slope or growth trajectory parameter (β_{1i}), and the residual term (ε_{it}) which is assumed to have a constant variance, σ^2. Both a time-invariant covariate (X_i; e.g., gender) and a time-varying covariate (W_{it}; e.g., health status) can be introduced into these models if so desired.

The next level of measurement (also known as the Level 2 model) contains the individual difference variables or person-level characteristics (e.g., gender, socioeconomic status). These Level 2 model equations are proposed to explain the random variation present in the intercept (β_{0i}) and slope (β_{1i}) in the Level 1 model. In an unconditional Level 2 model, there are no individual difference variables included, and the model equations are given as:

$$\beta_{0i} = \beta_0 + u_{0i} \tag{2}$$

$$\beta_{1i} = \beta_1 + u_{1i}. \tag{3}$$

In this unconditional Level 2 model the β_0 and β_1 terms are fixed effects and are the grand means for the intercept and slope, respectively. The u_{0i} and u_{1i} terms are random variables representing the variation of individuals around these grand means. Subsequent model levels can be added as different types of variables are added (e.g., those measured at the group level are considered to be Level 3 variables). When these Level 2 (and any subsequent levels) model equations are inserted into the Level 1 model equation, we obtain a model of both fixed and random effects, given as:

$$\begin{aligned} Y_{it} &= \beta_0 + u_{0i} + (\beta_1 + u_{1i})\, X_{it} + \varepsilon_{it} \\ &= \beta_0 + \beta_1 X_{it} + u_{0i} + u_{1i}X_{it} + \varepsilon_{it} \end{aligned} \tag{4}$$

There have been several names for models that utilize longitudinal data in this fashion – multilevel linear models (Heck, 2001), mixed-effects models, random-effects models, and random-coefficient models (Hox, 2000; Littell et al., 1996; Raudenbush et al., 2002). These types of models all contain a random component for both the intercept (β_{0i}) and slope (β_{1i}) of the response variable. Raudenbush and Bryk (2002) use the term "hierarchical linear models" to encompass all of these types of models as it conveys information about the structure of the data that is common in many of these applications. The assumptions of the hierarchical model are that the random effects and the residuals are independent and multivariate normally distributed (Curran, 2003).

Hierarchical models are more flexible than the conventional MANOVA approach to longitudinal designs (Hox, 2000). Two drawbacks to the use of a MANOVA model are (1) the removal of any cases that have missing data at any of the observed time points and (2) the assumption that there is an unconstrained error covariance matrix. The hierarchical model approach does not suffer from these limitations -- it does not require all cases to have complete data, so that cases with incomplete data are not excluded from the analysis. The hierarchical model approach can allow for more complicated error structures (e.g., autocorrelational), can allow the residuals to be modeled as a function of time, or can allow for relaxation of the assumptions on the variance structure of the errors (due to the complicated error structure that can arise from longitudinal data).

SAS has introduced a similar procedure, termed mixed models, in their PROC MIXED (Littell et al., 1996) procedure that is deemed more appropriate for handling longitudinal designs than its general linear model approach (i.e., PROC GLM). Mixed models are a broad class of models that include hierarchical linear models (HLM), growth curve models, and random coefficient models. They have their basis in time series analysis (Elston & Grizzle, 1962), mixed and variance components models (Cochran & Cox, 1957), random effects models (Laird & Ware, 1982), and empirical Bayes models (Lindley & Smith, 1972), as well as both the nonlinear mixed model (NLMIXED) and the general linear mixed model (GLMMIX).

The mixed model approach can be used with simple within-subjects designs, where time is a categorical factor (allowing for contrasts of time effects) and data is balanced with equal time spacing between assessment points. If the interest is on the effects of between-subject factors on individual growth trajectories, mixed within/between designs which incorporate interactions with time and other between-subjects factors can be used. Mixed models can include both time-invariant and time-varying covariates, do not require complete data (i.e., the same number of assessments on all respondents), and allow researchers to specify a variety of covariance structures to account for specific patterns of correlations. Finally, mixed models allow certain assumptions associated with other methods used for analyzing longitudinal data to be relaxed. An example is the sphericity assumption associated with the RM ANOVA.

Chapter 2: Growth Modeling Approaches to Longitudinal Data

While the previous Chapter has outlined several methods for analyzing longitudinal data that are often adequate, at times it is the rate or patterns of change that are of interest to the researcher. As several authors have noted (Curran et al., 2003; Kline, 1998; Rogosa, 1993), approaches such as the GLM or mixed model analyze change only in group means, and any differences among individuals in their growth trajectories is treated as error variance.

Growth models allow for a flexible modeling of longitudinal data that can encompass many of the models described earlier. Growth models can accommodate data structures from a variety of modeling approaches, such as GLM, repeated-measures models and mixed models (MacCallum, Kim, Malarkey, & Kiecolt-Glaser, 1997). They are also capable of modeling individual patterns of change and are not restricted by the drawbacks that plague some of the other approaches described previously (Wolfinger & Chang, 1995). Fan and Fan (2005) have further noted that traditional approaches are often inferior to latent growth approaches for detecting linear growth under small sample sizes and low effect sizes.

The Linear Effect Growth Model

Similar to the hierarchical models presented earlier (see page 22), the linear effect growth model has two levels that are modeled (Singer, 1998; Willett & Sayer, 1994). The first level deals with the occasions of measurement and models the trajectory (growth) for each individual. This level represents the actual repeated measurements themselves, and is sometimes termed the “within-person” model since observations are taken “within” individuals. The second level represents the random variables from the first level (the intercept and slope) as outcomes that may depend on person-level characteristics (e.g., gender, socioeconomic status). The basic equations are the same as those presented earlier (Equations 2-4) and are reproduced here.

The Level 1 model (which is the same as the mean or population average model) is expressed as

$$Y_{it} = \beta_{0i} + \beta_{1i} X_{it} + \varepsilon_{it}, \tag{5}$$

with i and t defined as in *Section 1.2.* Y_{it} is the measure of the response variable for person i at time t, and X_{it} is the measure of time for individual i at time t (i.e., the occasion of measurement). The intercept (β_{0i}) in this Level 1 model represents the true level of the response variable for individual i at the first occasion of measurement. The slope (β_{1i}) represents the true rate of change on the response over time. These intercept and slope terms are random variables that vary across individuals and represent the individual-specific effects. This random variation is expressed as a set of Level 2 equations

$$\beta_{0i} = \beta_0 + u_{0i} \tag{6}$$

$$\beta_{1i} = \beta_1 + u_{1i}. \tag{7}$$

The complete model, joining the Level-1 and Level-2 models, is

$$Y_{it} = \beta_0 + u_{0i} + (\beta_1 + u_{1i}) X_{it} + \varepsilon_{it}$$

$$= \beta_0 + \beta_1 X_{it} + u_{0i} + u_{1i}X_{it} + \varepsilon_{it}. \tag{8}$$

Non-Linear Effects in Growth Models

Not all models of growth are purely linear in effect, i.e., following a monotonic rate of change with constant slope over time. For example, a growth model that has a quadratic effect is used when there is a smooth curved increase or decrease in scores over time (Willett et al., 1998). This is depicted in Figure 1.

Insert Figure 1 about here

This type of model, termed a non-linear-effect growth model, is useful for studying outcomes that change rapidly over time. An advantage of non-linear effect growth models is that, with only three occasions of measurement, the researcher is allowed more flexibility in testing possible models to explain the observed growth trajectory.

Continuing with the example of a quadratic-effects growth model, the Level 1 model equations presented earlier (Equations 6 to 8) change as there is now a squared polynomial term to represent the non-linear (quadratic in the current example) effect of growth over the occasions of measurement. The complete Level 1 model now becomes

$$Y_{it} = \beta_{0i} + \beta_{1i}\,(X_{it}) + \beta_{2i}\,(X_{it})^2 + \varepsilon_{it.} \qquad (9)$$

The addition of the squared polynomial term (X_{it}^2) permits the growth rate to differ smoothly and systematically as a function of the occasions of measurement, representing the non-linear quadratic component of the growth model.

The Level 2 model equations change slightly from those for the linear effect growth model as now there is a third random variable, β_{2i}, for the acceleration in each growth trajectory (how fast the trajectory changes over time) and to represent the non-linear effect. The system of Level 2 equations now becomes

$$\beta_{0i} = \beta_{00} + u_{0i}, \tag{10}$$

$$\beta_{1i} = \beta_{10} + u_{1i}, \tag{11}$$

$$\beta_{2i} = \beta_{20} + u_{2i}. \tag{12}$$

The intercept, β_{0i}, is the status of person i at time t, β_{1i} is the growth rate of person i, and β_{2i} is the acceleration in each growth trajectory (how quickly the trajectories change). The combined model is:

$$Y_{it} = \beta_{00} + \beta_{10}X_{it} + \beta_{20}\,X_{it}^{2} + u_{0i} + u_{1i}\,X_{it} + u_{2i}\,X_{it}^{2} + \beta_{ti} \tag{13}$$

Chapter 3: Structural Equation Modeling (SEM)

Background to SEM

Structural equation modeling (also called covariance structure modeling, causal modeling, or LISREL modeling) is a data analytic technique that can be used to test if a proposed causal structure is consistent with the covariances and variances for a given set of data (Breckler, 1990). There are two general purposes of structural equation modeling – the assessment of model fit, and the estimation of model parameters (Fan & Wang, 1998). The assessment of model fit gives information about the general pattern of the relationships among the variables, and the estimation of model parameters gives information about the direction and strength of those relationships. Model fit indices are meant to describe the fit of the proposed model to the data rather than to test the fit statistically.

The typical structural equation model has two components: the *measurement model*, which relates a series of observable indicators (often referred to as manifest variables) to a series of one or more latent (or unobserved) constructs (η), and the *structural model*, which defines the relations among the latent constructs (Muthen, 2002). The measurement component of the model is defined in terms of the p-dimensional outcome vector $\mathbf{y}$,

$$\mathbf{y} = \boldsymbol{\nu}_p + \boldsymbol{\Lambda}_p\, \boldsymbol{\eta}_m + \boldsymbol{\varepsilon}, \qquad (14)$$

With p representing the number of manifest variables, or indicators, and with m representing the number of latent variables, the elements of this model can be defined as follows: $\boldsymbol{\nu}$ is a p-dimensional parameter vector of measurement intercepts, $\boldsymbol{\Lambda}$ is a p-by-m matrix of measurement slopes (often referred to as model parameters or factor loadings), $\boldsymbol{\eta}$ is an m-dimensional vector of latent variables, and $\boldsymbol{\varepsilon}$ is a p-dimensional vector of residuals which are uncorrelated with other variables.

The equation that represents the structural portion of the model is:

$$\boldsymbol{\eta}_m = \boldsymbol{\alpha} + \mathbf{B}\boldsymbol{\eta}_m + \boldsymbol{\zeta}. \tag{15}$$

In this equation, $\boldsymbol{\alpha}$ is an m-dimensional parameter vector and $\mathbf{B}$ is an m-by-m matrix of latent slope parameter estimates, indicating the relationships among the latent variables. Lastly, $\boldsymbol{\zeta}$ is an m-dimensional vector of errors in prediction for the m dependent latent variable equations. The covariance matrix of $\boldsymbol{\zeta}$ is denoted by $\boldsymbol{\psi}$ and has dimensions m-by-m.

Estimation Methods in SEM

Model estimation involves the determination of a value for the unknown parameters in the proposed model. For each parameter, an estimate of the unstandardized coefficient and the standard error of the estimate are generated, with the goal being to minimize the difference between the observed and estimated population covariance matrices (Tabachnick & Fidell, 2001). The function that is minimized is

$$\mathrm{Q} = (\mathbf{s} - \boldsymbol{\sigma}(\boldsymbol{\theta}))'\mathbf{W}\,(\mathbf{s} - \boldsymbol{\sigma}(\boldsymbol{\theta})), \tag{16}$$

where $\mathbf{s}$ is the vector of the data, $\boldsymbol{\sigma}$ is the vector of the estimated population covariance matrix, $\boldsymbol{\theta}$ indicates that $\boldsymbol{\sigma}$ is derived from the parameters of the model, and ' is the transpose operator (indicating a reflection of a matrix along its main diagonal). $\mathbf{W}$ is a weight matrix that weights the squared differences between the sample and estimated population covariance matrix.

Different estimation procedures vary with respect to their choice of **W**. Estimation procedures include maximum likelihood (ML), generalized least squares (GLS), and unweighted least squares (ULS), among others (see Tabachnick & Fidell, 2001, for a review). Some methods are better than others with respect to correcting for the bias introduced by violation of statistical assumptions (Weston, Gore, Chan, & Catalano, 2008).

ML estimation is one of the most commonly used estimation methods and is robust to moderate violations of normality in the data. The ML estimation procedure is a normal distribution theory estimation procedure, and the ML estimator itself is the vector of arguments that minimizes the following ML fitting function

$$F_{ML} = \log [\, \mathbf{\Sigma}(\theta) \,] + \mathrm{tr}[\mathbf{S}\mathbf{\Sigma}^{-1}(\theta)] - \log [\mathbf{S}] - p, \qquad (17)$$

where **S** is the sample covariance matrix of the observed variables, $\mathbf{\Sigma}(\theta)$ is the covariance matrix implied by the model that is a function of θ, the model parameters, p is the number of observed variables, and tr is the trace function (the sum of the elements on the main diagonal for a square matrix).

The GLS estimator yields results that are asymptotically equivalent to those obtained from the use of the ML estimator, and uses the following fitting function:

$$F_{GLS} = 0.5 \;\; \mathrm{tr}[[\mathbf{S} - \mathbf{\Sigma}^{-1}(\theta)]\, \mathbf{W}^{-1}]^2. \qquad (18)$$

The ULS estimator is defined as (Siemsen & Bollen, 2007)

$$F_{ULS} = 0.5 \;\; \mathrm{tr}[\mathbf{S} - \mathbf{\Sigma}(\theta)]^2. \qquad (19)$$

SEM in Longitudinal Designs

SEM can be used when there are repeated observations on a set of individuals. This approach is applicable when the research question is framed as either a longitudinal model or as a latent variable growth curve model (Hoyle & Smith, 1994; Kline, 1998; MacCallum et al., 1997; Raykov et al., 2000). SEM offers advantages over the general linear model by its ability to allow for theoretical models of change to be specified (Raykov, 1992), and by providing an assessment of overall model fit which is not a large focus in hierarchical multilevel models (MacCallum et al., 1997). Further, SEM is useful for studying the influence of latent (error-free) constructs that are measured with fallible multiple indicators (the observed variables), and can yield accurate estimates of causal influences and relationships. SEM approaches are also able to take into account the effects of correlated errors of measurement in both the independent and dependent variables (Hox, 2000; McArdle & Hamagami, 1992), which are present in any longitudinal design. If desired, multiple-group analyses can be used to model between-subjects effects in longitudinal models (Hoyle et al., 1994) and can examine any time-by-group interactions (where there are different effects across time for different groups) as can be done in hierarchical and mixed-effects models.

Chapter 4: Latent Growth Models

A growth model can be formulated as a structural equation model (Meredith et al., 1990; Muthen, 2002), and is then called a latent growth model or a latent growth curve model. Here, the repeated measurements for each individual are modeled by a latent variable for the intercept of the growth curve and a second latent variable for the slope of the growth curve (Meredith et al., 1990; Muthen, 2002; Willett et al., 1994). The interest with a latent growth curve is to model individual change as a function of time (McArdle & Epstein, 1987), as the use of repeated measurements over time means that the latent factors represent common factors which are indicative of individual differences over time (Curran et al., 2003; Duncan, Duncan, Li, & Alpert, 1999; Hox, 2000).

With respect to the linear-effect growth model presented earlier (in *Section 2.1*), the random terms from the growth model (the intercept β_{0i} and the slope β_{1i}) are now represented as latent variables that vary across individuals. Also, the *p* manifest variables now represent the observed variables at each of the t time points.

History and Development of the Latent Growth Model

The latent growth model is based on the premise that a set of observed repeated assessments taken on a given individual over time can be used to estimate an unobserved trajectory that gives rise to the observed repeated measures – the focus is not on the set of observed measures, but instead on the underlying unobserved latent constructs that explain the relations between the observed measures (Burchinal, Nelson, & Poe, 2006). Several authors have proposed the idea of the analysis of individual trajectories (Gompertz, 1825; Palmer, Kawakami, & Reed, 1937; Wishart, 1938), and this was in an effort to capitalize on the rich and detailed information contained in continuous multi-wave data and also to address research questions regarding systematic interindividual differences in change (see Bryk & Raudenbush, 1987; Rogosa & Willett, 1985). With this approach, an individual growth model is derived that represents the change that each person experiences with time (also known as a within-person model). All members of a population are assumed to have trajectories of the same functional form, but different members can have different values of the individual growth parameters. If predictors of change are included in the modeling process, then these get linked to the individual growth parameters in a between-person model.

There have been a variety of methods proposed to estimate the parameters of these within and between models (also called Level 1 and Level 2 models, respectively). One avenue of estimation has been the use of the methods of covariance structure analysis, or structural equation modeling. Meredith and Tisak (1990) provide a technical framework for this, building on the earlier work of Tucker (1958) and Rao (1958), as well as drawing together other approaches to the analysis of longitudinal data (e.g., repeated-measures ANOVA, MANOVA). The approach of Meredith and Tisak (1990) is broad in that it allows the evaluation of the general shape of the individual growth trajectories (i.e., the individual growth parameters), but also provides estimates of the between-level means, variances, and covariances across all members of the population. Wilett and Sayer (1994) note that the integration of individual growth modeling and covariance structure analysis capitalizes on the mathematical equivalence of these two alternative methodologies of representing the same data structure. The formulation of the Level 1 and Level 2 models for individual change and for systematic interindividual change, respectively, is equivalent to proposing a specific structure for the matrix of population covariances among the repeated waves of observed data. By using a structural equation modeling approach with mean structures to articulate this covariance structure and to fit it to the matrix of sample covariances, estimates of the between-person parameters that were specified under the original growth modeling formulation can be obtained.

McArdle and Hamagami (2001) fostered the development of techniques for handling missing data, which aided in allowing the analysis of longitudinal data within a covariance structure modeling framework. Muthén (1991) and Muthén and Satorra (1989) have provided the technical basis and examples of the modeling of multilevel data using covariance structure methods, and have shown that the parameters of a linear growth model can vary across individuals in ways that are systematically related to selected time-invariance predictors of change. Curran and Hussong (2003) present extensions of the latent growth modeling framework to include mediation, moderation, and multivariate models.

Fundamentals of Latent Growth Models

The simplest latent growth curve has a single variable that is measured at two time points (i.e., $T = 2$), and is also known as a linear latent growth model. This linear growth model assumes that a one-unit change in time is associated with a β unit change in the outcome, and that the magnitude of this relation is constant over all points in time (Curran et al., 2003). While this is too basic of a model to represent alternative shapes of change over time (three or more observed time points are needed for this), it is useful for explaining the fundamentals of latent growth curve models. In matrix notation, the measurement model (which relates the repeated measurements to the latent growth variable, η) has the following form (Curran, Bauer, & Willoughby, 2004; Li, Duncan, Duncan, Yang-Wallentin, & Acock, 2001):

$$\mathbf{y} = \boldsymbol{\tau}_y + \boldsymbol{\Lambda}_y \boldsymbol{\eta} + \boldsymbol{\varepsilon}, \tag{20}$$

Here $\mathbf{y}$ is a T_i-by-1 vector which contains the observed values of Y (the outcome of interest) across time for each individual, $\boldsymbol{\tau}_y$ is a T_i-by-1 vector of observed variable intercepts (similar to β_0 given previously in Section 1.2), $\boldsymbol{\Lambda}_y$ is a T_i-by-m matrix of loadings (known as basis coefficients, with m being the number of latent factors in the model) that reflect the hypothesized growth pattern underlying Y (Meredith et al., 1990), $\boldsymbol{\eta}$ is an m-by-1 vector of latent growth factors that capture the facets of growth that are being modeled (in this case an intercept and a slope), and $\boldsymbol{\varepsilon}$ is a T_i-by-1 vector of measurement residuals which are assumed to be constant across time and uncorrelated with each other. In LISREL matrix form, the elements of this equation for a linear latent growth curve model are:

$$\begin{vmatrix} Y_{i1} \\ Y_{i2} \\ . \\ . \\ Y_{iTi} \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ 1 & 1 \\ . & . \\ . & . \\ 1 & T_i - 1 \end{vmatrix} \begin{vmatrix} \boldsymbol{\eta}_{\alpha i} \\ \boldsymbol{\eta}_{\beta i} \end{vmatrix} + \begin{vmatrix} \varepsilon_{i1} \\ \varepsilon_{i2} \\ . \\ . \\ \varepsilon_{iTi} \end{vmatrix} \tag{21}$$

In this linear latent growth model, both the intercepts and the measurement of time are parameterized by the factor loading matrix $\boldsymbol{\Lambda}$, which relates the repeated measurements to the latent factors (Curran et al., 2004; Rovine & Molenaar, 1998).

The structural model is represented by the following equation,

$$\eta = \alpha + \mathbf{B}\,\eta + \zeta, \qquad (22)$$

where α represents an m-by-1 vector of latent intercepts, **B** is an m-by-m matrix containing structural coefficients, and ζ is an m-by-1 latent residual vector, with ψ representing the covariance structure among the latent factors. A typical linear latent growth curve model of this type using two time points is given in Figure 2.

Insert Figure 2 about here

In this figure, observing from left to right, the first latent factor is the intercept (η_0; analogous to the β_{0i} in Section 1.2), and it is a constant for any given individual across time. As a result, all of the factor loadings from this latent factor to the observed variables are fixed to 1. This intercept factor also gives information about the mean (M_{int}, analogous to β_0 in Section 1.2) and variance (D_{int}, analogous to u_{0i} in Section 1.2) of the collection of intercepts that characterize the growth curve for each individual, with the mean representing the estimate of the common intercept across all individuals. The second latent factor is the slope (η_1; analogous to β_{1i} in Section 1.2), and is the slope of each individual trajectory (which in this example is a monotonically increasing trajectory). The slope factor also has a mean (M_{slp}, or β_1) and a variance (D_{slp}, or u_{1i}). The mean of the slope factor represents the common slope across all individuals, and the two latent factors of the intercept and the slope are allowed to covary (the double-headed arrow that links the two latent variables). Individual deviations from these common intercepts and slopes are modeled by their respective variances.

In order for this particular model to be identified (.e., to have a unique set of parameter estimates which can be calculated for the parameters in the model; Kline, 1998), some of the factor loadings for the paths from the observed variables to the two latent factors must be fixed to two different values. Fixing the factor loading from the latent intercept factor (η_0) to the first observation point (Y_1) at 0 and that from the latent slope factor (η_1) to the second observation point (Y_2) at 1 (see Figure 2) has the effect of locating the intercept at the first observation point, Y_1. With these choices of factor loadings, the latent intercept factor (η_0) now represents initial status and the latent slope factor (η_1) represents the difference in scores at the two observation points (Y_2 - Y_1), and the model can be identified.

In Figure 2, the observed measurements of Y_1 and Y_2 can be expressed as linear functions of the latent factor scores (the intercept and the slope), the factor loadings, and the latent factor means (the M_{int} and M_{slp}). We can then write the equations that represent the model as:

$$Y_1 = \eta_0 + \lambda_1 \eta_1 + e_1 \tag{23}$$

$$Y_2 = \eta_0 + \lambda_2 \eta_1 + e_2 \tag{24}$$

$$\eta_0 = M_{int} + D_{int} \tag{25}$$

$$\eta_1 = M_{slp} + D_{slp}. \tag{26}$$

Here, the λs represent the factor loadings (from the matrix **B** of structural coefficients) that relate the latent factors to the observed variables, η_0 and η_1 are the latent intercept and slope respectively, and e_1 and e_2 are individual measurement errors.

Assumptions of Latent Growth Models

There are common assumptions associated with latent growth models. These are: (1) that the trajectories of all individuals have the same functional growth form (e.g., all linear), (2) that the longitudinal data can be fully summarized by their means and covariances (which implies that the repeated measures are multinormally distributed), and (3) that the effects of the observed indicators are constant over the range of the trajectory parameter values (Curran & Bauer, 2003).

Latent Growth Models with More Than Two Repeated Observations

The basic linear effect latent growth curve presented earlier (p. 28) can easily be extended to incorporate more than two repeated observations per individual. Figure 3 gives an example of a basic linear effect latent growth model that utilizes three repeated observations per individual.

Insert Figure 3 about here

When there are three (or more) repeated observations for each individual there is the opportunity to test for more complex trajectories, such as quadratic or cubic effects. This can easily be accomplished with a latent growth model by adding another latent factor to represent each non-linear effect and fixing the latent factor loadings accordingly (McArdle et al., 1987). The model presented in Figure 4 is a hypothetical growth model with a quadratic-effect latent factor (denoted as η_2).

Insert Figure 4 about here

The factor loadings from the observed indicators to the latent variables can be fixed for particular shapes or effects (e.g., linear, quadratic) or they can be freely estimated if the shape of the trajectory is unknown.

Comparison of Latent Growth Models and RM ANOVA

As presented earlier, the RM ANOVA uses each of the repeated measures variables as a within-subjects factor which distinguishes measurements made on the same individual, rather than between different individuals (see *Section 1.1).* Further, with a traditional GLM-based approach only the factor means are of interest, and might not be the optimal choice of statistical procedure for researchers interested in the trajectory of change over time. Latent variable growth models offer several advantages over RM ANOVA approaches (Fan & Fan, 2005): they are more powerful for detecting growth with small effects with small to moderate sample sizes, can easily accommodate nonlinear growth patterns, and can handle facets of stability (correlations between latent variables adjacent in time), level (means) and inter-individual differences (variances) simultaneously (Rudinger & Rietz, 1998).

A latent growth model can be made comparable to the RM ANOVA by placing restrictions on the parameters that correspond to the assumptions from the RM ANOVA. First, an orthogonal polynomial transformation matrix needs to be generated, the entries of which become the factor loadings for each of the variable-factor relationships. Further, the RM ANOVA assumes that each of the observed variables are measured without error, requiring the error terms for each observed variable to be set equal to zero. Duncan et al. (1999) provide an analytic example that compares the two procedures.

Comparison of Latent Variable Growth Modeling and Multilevel Models

Curran (2003) had made direct comparisons between structural equation models and particular examples of multilevel models (i.e., hierarchical models), showing how the SEM and multilevel approaches to latent growth curve analyses have a high degree of isomorphism, and that they provide analytically identical solutions to two-level growth models (see also Rovine & Moelenaar, 2000). Latent growth curve models can be applied to the same range of longitudinal data structures as with multilevel approaches, can allow for both missing data and for individuals to be measured at different occasions and different numbers of occasions. Further, measurement error distributions within latent growth models can be either homoscedastic or heteroscedastic, as latent growth models can approximate random changes in measurement error. By allowing specific patterns of indicator-to-factor loadings, latent growth models can also test the adequacy of specific growth forms (e.g., linear, quadratic), and the interpretation of the intercept and shape factors for these growth forms are straightforward (Duncan et al., 1999).

As an example, consider the graphical representation of a latent variable growth model given in Figure 7. In this sample model a group of individuals are measured on a particular variable at four consecutive time points (Y_1 to Y_4). Some potential explanatory variables are also measured: the Z variable is a time-invariant covariate (i.e., a variable which does not change over time, e.g., gender), and the W_2 and W_3 variables are time-varying covariates (i.e,. a variable that can change over time) measured at the second and third assessment periods, respectively.

As can be seen from this example, an SEM approach to longitudinal models can easily incorporate all of the aspects of a hierarchical model when using longitudinal data.

Insert Figure 7 about here

The fitting of hierarchical data with mixed and multilevel models, while being powerful analytical tools in their own right and being able to explain variance in the parameters, are unable to provide information about the magnitude of direct, indirect, and total effects among the latent variables (Rowe, 2002). Further, since multilevel models are extensions of the general linear mixed model, which specifies the response vector to be a linear sum of the effects of the independent variables, there is no allowance made for examining the structure of the covariance matrix among the independent and dependent variables. As a result, it is not possible to jointly investigate the structural relationships (i.e., direct, indirect, and total effects) that exist among the independent and dependent variables. Such effects can be studied using the SEM framework (Rowe, 2002).

In summary, as a relatively newer analytical method, latent growth models can be applied to the same data that could be analyzed with traditional statistical models (e.g., RM ANOVA), which makes this method a viable one as a research strategy for existing data. The assessment of growth trajectories can be easily assessed with an LGM approach – in LGM the interest is about the underlying unobserved latent constructs that explain the relations among the observed measures, and not expressly in the characteristics of the set of observed measures (as can be the focus with other approaches such as the mixed model). The LGM approach can accommodate fixed effects, random effects, or a mixture of both, and is more flexible with incorporating interaction effects than the RM ANOVA or mixed model.

Chapter 5: Methods Used to Represent Latent Growth Models

Several methods have been proposed to represent latent growth models. These are: the LISREL univariate approach utilized by both Duncan et al. (1999) and MacCallum et al. (1997), the Raykov (1992) T_1-Congenericism model, and the Muthén (1994) Two-Level Disaggregated Model approach.

LISREL Univariate Approach

The LISREL univariate approach to longitudinal modeling is probably the most widely-used approach (Duncan et al., 1999; MacCallum et al., 1997), and it uses a covariance matrix of order t-by-t, where t is the number of measures of the latent outcome variable. Since these represent repeated measurements, t can also be used to represent the occasions of measurement (similar to that presented in previous sections). This covariance matrix provides the variances, covariances, and means of the t measures. The scores for each person i on a single response variable y, measured at t occasions, are organized into a vector **y**, where $\mathbf{y}' = (y_{i1}, y_{i2}, ..., y_{it})$ and ' is the transpose operator (indicating a transposition of rows of a matrix into columns). This LISREL model has the following form:

$$\mathbf{y} = \mathbf{\Lambda z} + \mathbf{e}, \qquad (27)$$

where Λ is a t-by-m matrix, with m representing the number of latent factors in the model. The columns of Λ represent specific trajectories of change, represented as the factor loadings from the observed indicators to the latent variables (and are sometimes referred to as "basis functions"). The vector **z** contains the scores on the m latent factors for a given person, and is analogous to the latent vector η given in previous sections. The diagrammatic representation of this LISREL model (as given in MacCallum et al., 1997) is given in Figure 5.

Insert Figure 5 about here

In this model, a dummy variable (represented as a diamond in the upper portion of Figure 5) represents the specification of non-zero means of the factors. The factor loadings from this dummy variable to the two latent variables for the intercept and slope (β_0 and β_1) are the factor means of these two latent variables. This model also estimates all of the parameters from the latent slope factor to the observed variables. Duncan and colleagues (1999) also use this approach but omit the dummy variable.

Raykov (1992) T_1-Congenericism Model

Raykov (1992) proposes a T_1-congenericism model for representing a latent growth model. A diagram of this model is given in Figure 6.

Insert Figure 6 about here

In this model the parameters of change are the factor loadings from the latent variable to each of the observed variables (e.g., Y_1 to Y_3). By setting the loading from the latent variable to Y_1 (i.e., b_1 in Figure 6) equal to 1, the given structure will model change over time in terms of true initial status at the time of the study. Furthermore, the scores of all assessments are now linear functions of the score at the first assessment.

In order for this model to be realized, a model needs to be fit to the cross-product moment matrix or the covariance-mean matrix (not the covariance or correlation matrix as is typically used in SEM procedures; Kline, 1998). The reason is that the cross-product moment matrix contains information about the means and their change over time, which is not incorporated into the covariance or correlation matrix. A dummy variable is also used in this model, represented by the diamond shape at the top of the model diagram in Figure 6, and it is a constant (usually set to the value of 1). The estimated value of the path from the latent variable to this dummy variable is interpretable as the mean of the individual scores at first assessment, as in the LISREL model approach.

Muthén's (1994) Two-Level Disaggregated Model Approach

Muthén (1994) provides a methodology for reducing a hierarchical longitudinal model to a model that can be represented using traditional structural equation modeling software. The outcome y_{it} for each individual is reformulated into a T-by-1 vector (y^*), where the elements are the observed outcome at each time point t, up to the final time point T (Muthén, 1994). This approach then divides the model into two sections: one section that expresses the mean as a function of initial status and the mean of the growth rate (representing the fixed parameter portion of the linear growth model), and a second section that expresses the within-group variation and error, both of which are random effects. Muthén (1994) proposes that this approach can be easily implemented in current SEM packages.

Chapter 6: Moderators / Interaction Effects

There is often the need to determine if the effect of one (or more) independent variable(s) on the dependent measure depends on the level of a second independent variable. Due to the potential theoretical importance of these interaction effects (also called moderator effects), their accurate detection is a crucial part of social science research (Wen, Marsh, & Hau, 2002). The assessment of interaction effects in regression and path analyses have been well documented by other authors (e.g., Aiken & West, 1992; Baron & Kenny, 1986; Jaccard et al., 1990; McClelland & Judd, 1993).

Definition and Purpose of Moderators

A formal definition of a moderator variable was introduced by Saunders (1956), who proposed that a moderator was any continuous type of variable which influenced the predictive effectiveness of other variables in a regression model. Zedeck (1971) furthered this definition by noting that a moderator variable could affect the nature and/or degree of association between an independent and a dependent variable. The current definition of a moderator variable is that put forward by Baron and Kenny (1986), which states that a moderator "is a qualitative or quantitative variable that affects the direction and/or strength of the relation between an independent or predictor variable and a dependent or criterion variable" (p. 1174). Other terms used to indicate moderator variables have been modifier, buffer, or vulnerability factor (Cleary & Kessler, 1982). In most analysis settings this type of moderator variable is called an interaction effect (Jaccard, Turrisi, & Wan, 1990).

A moderator can serve different functions in a given analysis. A moderator allows for the predictors in a model to be differentially valid for different groups (Zedeck, 1971), acting much like an interaction effect in an ANOVA setting. Banas (in Zedeck, 1971) reported that a moderator improves the usefulness of a predictor by isolating subgroups of individuals for which a particular predictor is most appropriate. Cleary and Kessler (1982) called a moderator a conditional effect, where the relationship between a risk factor and a dependent variable depends on the presence/absence or level of the potential modifier.

Two methods of identifying potential moderators are to use a theoretical approach and an empirical approach. With the theoretical approach the researcher uses intuition or theory based on previous research (e.g., hypothesis-formation) to discern the potential moderators. The empirical approach utilizes a statistical procedure (e.g., correlating variables within groupings) to determine the existence of a potential moderator. Kline (1998) has noted that if the R-square for a linear model is low then there could be a need for moderator effects due to the presence of possible complex relationships.

Regression Approach to Testing Moderator Effects

Saunders (1956) outlined a method for addressing moderated regression with continuous variables using product terms, which was elaborated upon by Zedeck (1971). This method used ordinary least-squares (OLS) regression, and was also advocated by Baron and Kenny (1986) for assessing moderated effects. The basic regression equation for a moderator effect is given as

$$Y = \alpha + \beta_1 X + \beta_2 M + \beta_3 (XM) + \varepsilon, \qquad (28)$$

where Y is the dependent variable of interest, α is the intercept, X is the predictor variable, M is the potential moderator variable, and XM is the product of these two variables.

Formal procedures are given in both Aiken and West (1992) and Jaccard et al. (1990) for testing moderator effects with both continuous and nominal variables using the Baron and Kenny (1986) approach. If the moderator is a continuous variable, and the product term in the regression is significant, then probing of the interaction effect involves plotting separate regression lines for different levels of the moderator. Aiken and West (1992) propose plotting at moderator values of the mean ± one standard deviation to probe the interaction effect, and these authors also provide a methodology for testing the slope parameters (pp. 14-16). This method has been used by some authors (e.g., Hewitt, Flett, & Ediger, 1996) to investigate the effects of psychological variables on mental health.

Cronbach (1987) found that the variance of the product term in this type of model increases as the individual predictor means differ from zero, leading to biased tests of the moderator effect. This increase in variance may be the reason why some authors have reported that moderator effects typically account for small portions of the variance in regression models in psychology studies (McClelland & Judd, 1993). Cronbach (1987) suggested that centering the individual predictors can minimize this variance and reduce multicollinearity, and Allison (1997) has noted that this centering does not bias the test of the moderator effect when raw (i.e., unstandardized) regression coefficients are used.

If the proposed moderator is categorical an alternative regression approach is to perform separate regressions for each subsample based on the level of the categorical moderator variable. This subsample-based analysis does not utilize the entire data set and results in a loss of statistical power (Morrish, Sherman, & Mansfield, 1986). Baron and Kenny (1986) noted further deficiencies with this subsample-analysis strategy. It assumes that the independent variable has equal variance at each level of the moderator and, if the amount of measurement error in the dependent variable varies as a function of the proposed moderator, then there will be spurious differences introduced into the correlations. Regression coefficients based on the entire sample are not affected by differences in measurement error of the independent variable, and so are more desirable in this case. Aiken and West (1992) propose substituting the different values of the categorical moderator variable into the regression equation and then plotting each regression line individually.

This regression-based approach to moderator effects can be represented as in Figure 8.

Insert Figure 8 about here

Chapter 7: Interaction Effects in Latent Variable Models

With respect to interaction effects in latent variable models, there are two general approaches that can be used depending on the nature of the individual interaction variables. If one of the variables involved in the interaction is a dichotomous or categorical variable, the common approach is to assess the structural model at each level of this categorical variable (Baron & Kenny, 1986). In this event, the researcher is seeking to test either (a) models that fit for some groups and not others, or (b) differential path coefficients for particular levels of the categorical interaction variable (i.e., test for differences in the path coefficients between levels of the interaction variable).

The second approach is used when all of the variables in the model are continuous in nature. The diagram in Figure 9 shows a latent variable model with an interaction term representing the interaction effect of two continuous latent variables (Kenny & Judd, 1984). As with regression and path analysis approaches (see Aiken & West, 1992), the latent interaction variable is constructed by creating product terms from all of the observed indicators for the two individual latent variables that are proposed to interact, and is termed a multiple indicator model.

Insert Figure 9 about here

The use of structural equation modeling to test for interaction effects allows for the correction of the estimates for measurement error (which introduces bias into the regression coefficients), thus increasing the power of the statistical test to detect interactions in comparison to regression and path analytic methods (Aiken et al., 1992; Li et al., 1998). The original latent variable interaction model posited by Kenny and Judd (1984) is

$$Y = \gamma_1\xi_1 + \gamma_2\xi_2 + \gamma_3\xi_1\xi_2 + \zeta, \tag{29}$$

where Y is the latent dependent variable, ξ_1, ξ_2 and $\xi_1\xi_2$ represent the latent factors for the independent variables, potential interaction variable, and the interaction product, respectively, and ζ is the residual term. This approach requires all of the observed variables to be in mean deviation form and to have normal distributions.

The Kenny and Judd (1984) model has led to several methods of specifying the interaction effect with latent variable models. These are (a) Bollen's (1996) 2-stage least squares (TSLS) method, (b) the Ping (1996) 2-step maximum likelihood (ML) method, (c) the Jaccard and Wan (1995) ML method, (d) a 2-step ML procedure by Joreskog and Yang (1996), and (e) a revised Joreskog-Yang model (Algina & Moulder, 2001).

Bollen (1996; Bollen & Paxton, 1998) suggested the TSLS approach for evaluating the latent interaction term as the Kenny and Judd (1984) approach produces non-normal indicators for the latent interaction term (even if the indicators for ξ_1 and ξ_2 are normally distributed). This non-normality can result in biased standard errors and fit statistics when ML estimation is used (Boomsma, 1983, cited in Moulder & Algina, 2002). The TSLS approach estimates the measurement model equations and the latent variable equations separately, so nonnormality in the indicators for $\xi_1\xi_2$ may not affect the standard errors of the interaction effect.

The procedure by Ping (1996) is also a two-step procedure. In this procedure a latent variable measurement model for defining the latent variables ξ_1 and ξ_2 is estimated in the first step. Parameter estimates from this step are treated as known parameters in a second step in which the original Kenny and Judd (1984) model is used.

The Jaccard and Wan (1995) procedure is a slight variation on the original Kenny and Judd (1984) model, where these authors added an extra main effect latent variable (ξ_3) to the model.

$$Y = \gamma_1\xi_1 + \gamma_2\xi_2 + \gamma_3\xi_3 + \gamma_4\xi_1\xi_2 + \zeta. \quad (30)$$

Using a multiple regression approach (with OLS estimation), these authors found a bias in detecting the interaction effect but almost no bias when an ML or asymptotic distribution free (ADF) estimation procedure was used, regardless of the effect size for γ_4. This approach requires the observed variables to be multivariate normal and to be in mean deviation form prior to estimation.

The Joreskog and Yang (1996) procedure is based upon what the authors call a misspecification in the original equation proposed by Kenny and Judd (1986), namely the assumption that the intercept (α) of the latent variable regression model is zero (even if all variables are in mean deviation form). A second difference in their model is that this procedure is sometimes called a single-indicator method since only a single observed indicator is used for the latent interaction term (versus the original model depicted in Figure 9 that has four indicators for the interaction term), and is presented in Figure 10.

Insert Figure 10 about here

Using a variety of estimation methods (ML, weighted least squares, weighted least squares applied to the augmented moment matrix) these authors fit the following latent variable regression model:

$$Y = \alpha + \gamma_1\xi_1 + \gamma_2\xi_2 + \gamma_3\xi_1\xi_2 + \zeta. \tag{31}$$

Joreskog and Yang (1996) set the intercept (α) equal to zero in their original study, but a subsequent simulation study by Algina and Moulder (2001) specified the intercept to be nonzero. Their results showed that the ML estimation procedure frequently did not converge when the intercept was nonzero. Algina and Moulder revised the model so that the measurement model intercepts for the indicators of the primary latent variables were zero but that the intercepts for the indicators of the interaction term were not. It was shown that this model was better able to converge under ML estimation than the original Joreskog and Yang (1996) model.

A subsequent simulation study by Moulder and Algina (2002) compared all five of these methods for testing the interaction effect, manipulating factors such as the effect size of the interaction, the squared multiple correlation for the full latent variable interaction regression model, the correlation between the primary latent factors, reliability of the indicators, sample size and the observed means of the indicators. Bollen's (1996) TSLS procedure showed a higher amount of bias in the estimation of parameters and a lower power than the other four methods. The original model by Joreskog and Yang (1996) under ML estimation showed Type I error rates that were very conservative when robust estimators (e.g., Satorra-Bentler chi-square; Satorra & Bentler, 1994) were used and very little power for detecting the interaction effect. The method by Ping (1996) showed conservative Type I error rates and little bias, but the bias did not decline as sample size increased so this procedure was discouraged from use. The Jaccard and Wan

(1995) method and the revised Joreskog-Yang model from the Algina and Moulder (2001) study showed adequate control of Type I errors and provided good power to detect interaction effects.

Li et al. (1998) compared the Joreskog and Yang (1996), the Ping (1996) and the Jaccard and Wan (1995) methods on a model for the interactive effects of perceptions of exercise competence. They found the Joreskog and Yang (1996) method to suffer from multicollinearity as it does not use mean centering for the indicators in the latent variable model, while the other two methods (which do utilize mean centering) are less susceptible to this. These authors also found the Joreskog and Yang (1996) and Jaccard and Wan (1995) methods to be more robust to the effects of non-normality introduced by use of the product indicator method, and simpler programming of the model specification in conventional SEM packages. Li et al. (1998) recommend the Joreskog and Yang (1996) and the Jaccard and Wan (1995) procedure for modeling latent interaction effects, in concordance with Algina and Moulder (2001).

These methods for modeling latent interactions described above, which all involve the use of the products of indicators, can be grouped into the general category of constrained methods, called such because non-linear constraints must be placed on the factor loadings and variances that are associated with the latent interaction term. These constraints are necessary because of the assumption of normality of the latent variables (Wen et al., 2002).

A related approach is the Generalized Appended Product Indicator (GAPI) approach (Wall & Amemiya, 2001). This approach is similar to the constrained approach but does not constrain the covariance matrix of the latent variables. It has been shown to be effective for when the latent variables are not normally distributed (i.e., when used with non-normal data; Wen et al., 2002). A third approach is the Unconstrained approach, based on the work of Algina and Moulder (2001). With this approach no nonlinear constraints are imposed on the relationships between the product indicators and the latent interaction factor. This method appears to be robust to violations of multivariate normality (Wen et al., 2002).

Schumacker (2002) outlined a latent variable score approach to interaction effects, in contrast to the product-indicator methods described previously, which does not involve the multiplication of the observed indicators to construct the latent interaction term. The procedure by Schumacker creates the latent interaction term by a multiplication of the latent scores of the individual latent factors, and is presented graphically in Figure 11.

Insert Figure 11 about here

Schumacker (2002) found that both procedures (traditional product-indicator and latent variable score) produced similar parameter estimates for the interaction effect but differed in the estimation of their standard errors. He concluded that the latent variable score approach was easier to implement yet called for more research into the computation of the standard errors. Wall and Amemiya (2003) proposed a technique for interaction effects based on factor score estimates, called the two-stage method of moments, which is a method similar to that put forward by Schumacker (2002). This method, through the use of factor scores, has no measurement error in the indicators.

Klein and Moosbrugger (2000) proposed an alternative method for analyzing interaction effects using LISREL that is similar to that of Schumacker (2002). Their method, Latent Moderated Structural Equations (LMS), takes the nonlinearity of the latent interaction term explicitly into account, which has caused some concern when linear methods are used to model the relationships (Joreskog & Yang, 1996). This approach utilizes a new method of ML estimation that is specifically designed to take the nonlinearity of the latent interaction term into account. These authors showed that the LMS method was efficient and unbiased with regards to computing standard errors and parameter estimation (see also Schermelleh-Engel, Klein, & Moosbrugger, 1998). This method has not yet been studied extensively, but does show some benefits over conventional approaches such as LISREL models and Bollen's (1995) two-stage least squares method. More specifically, the LMS method, when used with ML estimation, shows no bias in the parameter estimates and is capable of incorporating a non-normal distribution for the interaction term (Moosbrugger, Schermelleh-Engel, & Klein, 1998). This approach has now become the Quasi-Maximum Likelihood (QML; Klein & Muthen, 2002) approach. The QML approach uses all of the first-order factor indicators to estimate the latent interaction effect, and does not require the forming of any new indicators for the interaction term. The QML approach assumes that the first-order factors are normally distributed.

Chapter 8: Interaction Effects in Latent Variable Growth Models

Li et al. (1998) expressed a concern with traditional multiple regression approaches for investigating interaction effects. The measurement errors for the indicator variables have traditionally introduced bias in the regression coefficients and also decrease the power to detect nonlinear effects such as interactions in the regression approach (see also Jaccard & Wan, 1995). These authors and several others (e.g., Duncan et al., 1999; Li et al., 2001) have proposed that SEM techniques are useful for investigating interaction effects among change scores, especially when attempting to determine how a change in two latent attributes interact to produce a joint effect on the growth of an outcome attribute.

Duncan and colleagues (Duncan et al., 1999; Li et al., 1998; Li, Duncan, & Acock, 2000) have utilized an approach to interactions in latent growth modeling that is based upon the constrained product-indicator method. In this approach with cross-sectional data the product indicators are formed from all possible combinations of indicators for the two latent factors involved in the interaction. With a longitudinal design, it is reasonable to form the product indicators based on measures corresponding to the same time point. For example, for Latent Factor A and Latent Factor B measured at three time points, the interaction terms would be formed by the product of observations at Time 1 for Latent Factors A and B, the observations at Time 2 for Latent Factors A and B, and the observations at Time 3 for Latent Factors A and B. The first two parameters for the latent interaction slope factor are constrained (to 0 and 1, respectively) in order to identify the model, and the remaining parameters are freely estimated to approximate any potential curvilinear trajectories. A further result of this fixing of parameters is that the indicators for the first observed time point are not used to form a product indicator, since their respective loadings are zero. This conceptualization of the latent growth model with an interaction is given in Figure 12.

Insert Figure 12 about here

The interaction effect on the rate of change of the outcome can be observed in the following structural regression equation:

$$\eta_1 = \alpha_1 + \gamma_{22}\xi_2 + \gamma_{24}\xi_4 + \gamma_{25}\xi_2\xi_4 + \zeta_2$$

$$= (\gamma_{22} + \gamma_{25}\xi_4)\,\xi_2 + (\alpha_1 + \gamma_{24}\xi_4) + \zeta_2, \tag{32}$$

where the first part of the right-hand-side of equation 32, $(\gamma_{22} + \gamma_{25}\xi_4)\,\xi_2$, represents the simple slope of the regression of η_1 on ξ_2 for a given value of ξ_4. This characterization of the simple slope is similar to that given to the interaction effect in regression approaches (Aiken et al., 1992). Li et al. (2000) give the full equations and LISREL matrices for the complete latent growth interaction model.

However, Wen et al. (2002) contend that the model (and the matrices) used by both Duncan et al. (1999) and Li et al. (2000) are incorrect. They considered the constraints on the exogenous latent mean vector and on the covariance matrix to be inappropriate, as was the variance-covariance matrix for the errors of measurement. Wen et al. (2002) instead proposed a full interaction model that had more appropriate constraints, and a graphical representation of their full interaction model is given in Figure 13.

Insert Figure 13 about here

These authors showed that their model yielded more accurate estimates than the approach utilized by Duncan et al. (1999) and Li et al. (2000). No further empirical studies have been reported that compare these two models.

As noted previously, the two approaches by Duncan et al. (1999) and Wen et al. (2002) are both product-indicator approaches to interaction effects in latent growth models. However, also noted previously, the model proposed by Schumacker (2002) is a latent interaction model that does not require the use of product indicators. However, the Schumacker (2002) model for latent interaction effects has not been extended to applications with longitudinal data. The original development of the Schumacker model proceeded in three steps. In the first step, factor scores for the main effect latent factors are created. In the second step these factor scores are multiplied together to create a latent interaction factor score. In the third step, the latent factor scores (from both the main effects and the interaction) are used in an OLS regression model with the latent score for the outcome variable as the dependent variable.

The extension of the Schumacker (2002) model to longitudinal data is straightforward. In the first step, a latent growth model using the main effects only is used to create the latent factor scores for the main effects. In the second step, the latent scores from this growth model are used to create the latent interaction scores. In the third step, there are two latent dependent variables, one for the latent intercept and one for the latent slope. As a result, the model in the third step is estimated as a path analytic regression model, containing all the latent main and interaction effects. A graphical depiction of this process is given in Figure 14.

Insert Figure 14 about here

One difference between the two product-indicator method models (i.e., the Wen and the Duncan) and the Schumacker model are the function of the indicator variables. In all three of the models, with respect to the latent main effects, the main indicator variables are playing the same roles, as they are indicators of the same main effect latent slope and intercept factors. However, the characterization of the latent interaction terms is different in the three models. For the Wen and Duncan models, the indicators actually play two roles – they are indicators of both a latent main effect and a latent interaction effect, essentially appearing twice in the latent growth interaction model. The indicators for the latent interaction effect are created by forming products of the main-effect indicators, and the resulting product indicators for the latent interaction effect that are formed introduce non-normality into the model. Further, to incorporate these product-indicators into the model they need to be specified as a function of the latent variable that has variances and covariances that reflect the multiplicative relationship. This is achieved through the imposition of nonlinear constraints on several of the matrices involved in the estimation of the latent model. For the Schumacker model, the indicators only appear once, in the formation of the latent factor scores, after which they are not required further. As a result, there are no product-indicators in the model, and the imposition of nonlinear constraints is not necessary as there is no concern of multicollinearity among the indicators for the latent factors.

Chapter 9. Attrition in Longitudinal Studies (Missing Data Mechanisms)

Missing data occurs quite often in both cross-sectional and longitudinal research, and should be an important consideration for researchers pursuing longitudinal studies (Figueredo, McKnight, McKnight, & Sidani, 2000). The concept of missing data has also been referred to as coarsened data, aggregated data, rounded data, truncated data, or censored data (Schafer & Graham, 2002). The reasons for missing data are varied. In longitudinal studies a missing data point can occur from events that preclude measurement, such as attrition or dropout, or a respondent may not be available for one or more data collection instances (Schafer & Olsen, 1998). This is sometimes referred to as wave nonresponse. A respondent may simply choose to not respond to a particular survey questionnaire, or they may even miss a single item from an entire questionnaire. Some studies have planned missing values incorporated into the study design (Graham, Taylor, & Cumsille, 2001). When the missing values are a result of the data collection procedure (as is seen in most of survey research) this is termed unit nonresponse (Schafer et al., 2002).

Missing data can contain important information for the researcher and their hypotheses of interest, and not addressing the missing data properly can lead to biased results, especially in longitudinal designs (Cox, Rutter, Yule, & Quinton, 1977). When using statistical methods that assume responding on all variables of interest, missing data can greatly reduce the effective sample size which results in a loss of statistical power, making the detection of significant effects difficult (Delucchi & Bostrom, 1999). A further consequence is that statistical results may evidence some bias if the missing data contains influential information.

In a longitudinal study missing data can take on a variety of patterns (Little, 1992). One pattern is termed univariate missing data where the missing values are confined to a single variable at a single time point, and another is a monotone missing data pattern where once an observation is missed all future observations are also missed. This pattern is usually seen in longitudinal studies when there is participant attrition. A third type of pattern is when there is no special pattern to the missing data. Graphical representations of these three patterns are given in Figure 15.

Insert Figure 15 about here

Missing Data Mechanisms

The rate and pattern of missing data within a dataset has been described as a probabilistic phenomenon by Rubin (1976) called missingness. Missingness is used to capture the relationship between the pattern of the missing data and the values of the missing items. Rubin (1976) introduced the concept of a missing-data indicator matrix to quantify this relationship and then formalized the notion of a missing-data mechanism in terms of a conditional distribution. The three missing data mechanisms are described below.

Missing At Random (MAR)

MAR occurs when the distribution of missingness does not depend on the missing values of the response variable – the probability of missingness depends on the observed data in the covariates. In other words, once the covariates are taken into account there is no residual relationship between the missingness and the response variable. This type of mechanism is also known as an ignorable response. This type of mechanism usually holds when the missingness is planned (Schafer et al., 2002). An example of this would be if an individual in a longitudinal study missed an item on a survey at a particular assessment point. There is no definitive way to test for the presence of the MAR mechanism, but even making an erroneous assumption of an MAR mechanism does not severely impact estimates and standard errors (Collins, Schafer, & Kam, 2001).

Missing Completely At Random (MCAR)

The MCAR mechanism is similar to the MAR mechanism but here the missing response occurs by chance (Sinharay, Stern, & Russel, 2001), usually by some features of the study itself rather than the observed behaviour of the participants (Little, 1995). In other words, the missing response is not related to or is independent of the observed variables of interest. Compared to MAR, Figueredo et al. (2000) give an additional condition for MCAR, namely that the missing cases are a random subsample of the total sample. The presence of this mechanism can be assessed by testing across patterns of the missing data (e.g., complete cases versus missing cases) using t-tests for location (Bingham, Stemmler, Petersen, & Graber, 1998). Little (1995) has also called this mechanism covariate-dependent dropout, and notes that analysis of complete cases in this instance will not yield biased estimates but will be inefficient.

Missing Not At Random (MNAR)

This is known as nonignorable missing data, and the missing data is related to the observed values of the response variable (Sinharay et al., 2001). Sometimes MNAR is referred to as informative dropout. An example of this in a longitudinal design is if a participant drops out because their observed score on a previous measurement was over (or under) a particular value. With an MNAR mechanism there is residual dependence between the missingness function and the response variable after the covariates are accounted for. Figueredo et al. (2000) note that the MNAR condition can be detected by locating significant differences between means on subgroups of the data with complete cases versus those with incomplete cases. With an MNAR mechanism the researcher needs to specify a model for the missingness that is approximately correct otherwise bias will be present.

A specific kind of missing data pattern, specific to longitudinal designs, is attrition. Attrition, or dropout, occurs when a participant leaves the study and does not return, and is a special case of participant non-response. However, because later responses are not available, it is possible that scores on the missing covariates are the cause of the missingness. When attrition is present in a longitudinal study, the mechanism of missingness (MAR, MCAR, MNAR) can have some implications with respect to the dropout of the data. MCAR requires attrition to be independent of responses at every occasion, MAR allows attrition to depend on responses at any or all occasions prior to the dropout occasion so that missingness may be related to other variables, and MNAR means that the attrition depends on the unseen responses after the participant dropped out (Schafer et al., 2002).

Effects of Missing Data on Estimates

Determining the severity of missing data on the estimates of model parameters can be difficult and depends on many factors including sample size, proportion of missing data, the pattern (if any) of missing data, the type of analysis, and the number of variables being analyzed

(Rovine & Delaney, 1990). It is recognized that a failure to incorporate the missing-data mechanism into an analysis will result in biased parameter estimates (Figueredo et al., 2000; Rubin, 1976).

Missing data can represent a significant difficulty in longitudinal designs, since the assumptions of data being MCAR or MAR may not be tenable. Under MCAR, the use of maximum likelihood can give estimates that are efficient and which have a lower sampling variability (Duncan & Duncan, 1994; Rovine et al., 1990). Under MAR, maximum likelihood yields unbiased parameter estimates in structural equation models (Muthen, Kaplan, & Hollis, 1987). Under MNAR, the implications regarding bias are unclear, and researchers may find that a latent growth model may not fit the data and standard errors inflated, but parameter estimates are accurate (McArdle et al., 1992).

Methods of Addressing Missing Data

There are several methods within the SEM framework that are currently available to researchers for addressing missing data. Conventional procedures for handling missing data include listwise and pairwise deletion. In listwise deletion (or complete case analysis) only those observations that have complete data on all variables are included in the analysis (Little, 1992). With respect to the accuracy of parameter estimation this is usually a valid procedure under an MCAR mechanism, but rarely for MAR. In a multivariate setting this approach can result in discarding an unacceptably high proportion of participants (Schafer, 2001). A second conventional method is pairwise deletion (or available case analysis), where different sets of data are used to estimate the different parameters depending on if they have the necessary data or not.

A drawback to this method is that the parameters are generally estimated from different units within the sample so it is difficult to compute standard errors or other measures of uncertainty.

These procedures have the advantage of being simple to implement (Little & Rubin, 1987). However, both of these case deletion methods can result in biased estimates if the data are not MCAR due to the resulting complete cases not being representative of the full population (Hedecker & Gibbons, 1997; Schafer & Olsen, 1998) and most likely resulting in non-normal data (Figueredo et al., 2000). Yet even if the MCAR mechanism is present the results will be inefficient due to the discarded information contained in the missing data (Figueredo et al., 2000; Little & Rubin, 1987). Little (1992) and Bingham et al. (1998) have noted that these methods can result in an estimated covariance matrix of the observed predictor variables that is not always positive definite, which can lead to indeterminate slope estimates. This is usually seen when the predictors are highly correlated. Bingham et al. (1998) further note that there can be a loss of power (which will increase the rate of Type II errors; Figueredo et al., 2000), and recommends that in longitudinal designs these case deletion methods should not be used.

Tomarken and Waller (2005) outline four methods for addressing missing data that are considered specific to structural equation modeling procedures. The first is multiple imputation (Rubin, 1987; Schafer, 1997). This method first creates multiple samples in which all missing data values are estimated (i.e., imputed), then estimates the model of interest separately for each sample, and finally generates aggregate estimates of the parameters, standard errors, and model fit by taking into account variability both within and between samples.

The remaining three methods are based on maximum likelihood procedures (Enders, 2001). The first is multisample analysis (Allison, 1987; Muthen et al., 1987), where a sample is divided into G subgroups, such that each subgroup has the same pattern of missing data (all of the members in the same subgroup are missing/present on the same set of variables). Likelihood estimates are computed for each group, and then are accumulated across the entire sample and maximized. An advantage of this approach is that parameters and standard errors are estimated directly from the data. A second advantage is that this approach yields the usual measures of model fit in SEM approaches, although the degrees of freedom may be inaccurate (this can be remedied by subtracting an appropriate value from the degrees-of-freedom term). A limitation of this multisample approach is that the specification of multiple-group analyses is difficult, especially if there are many patterns of missing data that result in a high number of groups.

The second maximum likelihood approach is full-information maximum likelihood (FIML; Finkbeiner, 1979), which generates maximum likelihood estimates of the parameters of a specified model based on all of the available data per participant. This approach is similar to the multiple-group approach, except that the FIML approach uses the sum of all the casewise likelihood values, whereas the multiple-group approach uses the sum of all the G groupwise likelihood values. The FIML approach is flexible, being applicable to a variety of analyses including the estimation of covariance matrices, multiple regression, and SEM. Like the multiple-group approach, parameter estimates and standard errors are estimated directly from the available data. With respect to SEM, FIML yields a chi-square test of model fit and several model fit indices.

The third maximum likelihood approach to missing data is the expectation-maximization (EM) algorithm (Dempster, Laird, & Rubin, 1977). This uses a two-step iterative procedure where the missing observations are imputed and then unknown parameters are estimated. At the imputation stage (E step), missing values are replaced with the conditional expectation of the missing data given the observed data. Then, maximum likelihood estimates of the mean vector and covariance matrix are obtained (M step) as if there was no missing data using the statistics calculated in the previous step. The resulting covariance matrix and estimates are used to derive new estimates of the missing values at the next E step, and the process repeats until the difference between covariance matrices in subsequent M steps falls below some specified convergence criterion. The EM procedure cannot be used to directly obtain parameter estimates and standard errors, however the covariance matrix that is generated can be used as input to regression and SEM analyses. A drawback to the EM algorithm is that the imputed values lack the residual variability that is present in the hypothetically complete data set, since the imputed values fall directly on a regression line and are imputed without a random error component. As a result the standard errors from the EM approach will be negatively biased, and bootstrap procedures must be employed to obtain correct estimates of the standard errors.

Missing Data and Estimation Methods

If the data is MCAR or MAR, the maximum likelihood approaches will generally yield unbiased estimates of population parameters, more accurate coverage probabilities for confidence intervals, and more efficient estimates (i.e., smaller standard errors) than other traditional methods (e.g., listwise deletion; Sinharay et al., 2001) There is limited evidence that these methods can produce adequate results when the data are MNAR (Schafer et al., 2002; Sinharay et al., 2001). Arbuckle (1996) found that, under both MCAR and MAR, full-information maximum likelihood estimation (FIML) yielded unbiased estimates in SEM models when compared to pairwise and listwise deletion. Using multivariate normal data and an MCAR mechanism, Graham, Hofer, and MacKinnon (1996) found that estimated variances and covariances were unbiased under FIML. Enders and Bandalos (2001) further found that FIML showed equivalent or better performance with respect to convergence failure, bias, and efficiency of parameter estimates compared to listwise and deletion methods using normal data. This was the case regardless of factor loading magnitude, sample size, missing data rate, and missing data mechanism (MCAR or MAR). Enders and Bandalos (2001) further noted that the efficiency of the FIML approach improved as the percentage of missing data increased, and concluded that the FIML approach is appropriate when researchers do not know the appropriate missing data mechanism.

The previous studies have all included data that was normal in distribution. Enders (Enders, 2001) used a Monte Carlo study to investigate the impact of non-normality in the presence of missing data with the FIML approach, noting that a potential drawback to the use of FIML is that it was developed under the assumption of MAR and multivariate normality, and may not show the same efficiency under multivariate non-normality. Using both MAR and MCAR mechanisms, and comparing FIML against conventional missing data techniques such as listwise and pairwise deletion, Enders found that FIML was affected most by non-normality of the data, and that the presence of missing data (either MAR or MCAR) did not have a noticeable effect over and above the impact of non-normality.

Missing Data in Latent Growth Models

With latent growth models, model estimation in the face of missing data can utilize an

FIML approach, as outlined previously with research on structural equation models. Users of the multisample approach run the risk of there being many distinct patterns of missingness, thus resulting in some models with small samples sizes that cannot be estimated with SEM (Duncan, Oman, & Duncan, 1994). If all individuals are missing the same observation (i.e., nobody was assessed at that particular time point), then a simple solution has been to represent the missing data point as a latent variable (Ferrer, Hamagami, & McArdle, 2004), however in most naturalistic longitudinal studies this scenario would be rare.

It has been shown by previous authors (e.g., Enders & Bandalos, 2001; Graham, Hofer, & MacKinnon, 1996) that FIML estimation will exhibit no large-sample bias if the assumptions of MCAR or MAR are tenable, especially against conventional methods such as listwise deletion and pairwise deletion (Wothke, 2000). A study by Duncan, Duncan, and Li (1998) showed that, under MCAR, maximum likelihood methods performed as well or better than conventional methods (e.g., listwise deletion), but multiple imputation procedures performed poorly when missing data was due to both attrition and design (in their study a cohort-sequential design was used). McArdle and Hamagami (1992) assumed an MNAR mechanism for their missing data, and found that this did not affect the estimates of parameters using maximum likelihood procedures, although standard errors were inflated.

Much of the empirical research involving missing data in latent growth modeling has utilized a multi-sample procedure (e.g., Duncan et al., 1994; McArdle et al., 1992). There is very little research on the performance of likelihood-based methods with respect to overall model fit and parameter estimation and bias.

Chapter 10: Proposed Project

General Research Objectives and Hypotheses

The primary objective of the proposed research was to examine the performance of three approaches to latent variable modeling of an interaction effect between latent growth slope factors. These approaches were: (1) the full interaction model ("Wen") proposed by Wen et al. (2002), (2) the reduced interaction model ("Duncan") proposed by Duncan et al. (1999), and (3) the modified latent interaction model based on the model by Schumacker (2002). There has only been one published study comparing the Wen full interaction model against the Duncan reduced interaction model, and this comparison was based on an empirical data set with no manipulation of factors (e.g,. sample size, reliability) to evaluate the robustness of either of the models. Wen et al. (2002) contend that their model is superior to that of Duncan and colleagues with respect to estimating model parameters, but there is a paucity of empirical evidence besides their single study to support this contention. Further, the Wen et al. study only concerned itself with a comparison of parameter estimates, and did not investigate issues of overall model fit or bias.

The Schumacker (2002) model is a latent interaction model based on a cross-sectional design, and which does not use the cross-product of indicators method to characterize the latent interaction as do the Wen and Duncan models. It has not previously been extended into the latent growth modeling perspective. With its lack of product terms of the indicators, it does not require any complex constraints to be placed on the model parameters in order to take into account the non-normal nature of these product terms (see Chapter 8, p. 64), and as a result it shows promise as a viable approach to modeling latent slope interaction effects. However, as it is not known how the extension of the Schumacker model will perform, this model may not perform as well as either the Wen or Duncan models.

There are several issues that were investigated to assess the performance of the three models. The primary focus was on the model performance with respect to overall model fit statistics, as well as an examination of convergence rates, in the presence of several missing data conditions. The examination of overall model fit was carried out in a qualitative fashion, as we expected to observe particular trends in the overall model fit indices for each of the models. Specifically, it was expected that the Wen would show superior performance on the overall model fit indices compared to both the Duncan and Schumacker models. As there were no specific expectations about the performance of the Schumacker model in relation to the other two models, we expected that it would perform as good or worse than the Duncan model. This was based on its similarity to the Duncan model (i.e., only having a latent interaction term for the slopes, and not for any of the other latent factors as represented in the Wen model).

A secondary focus was on the estimation of the latent slope interaction parameter for the three models across the three missing data conditions. Specific aspects that were examined were the amount of bias in the parameter estimate and the rate of Type I error. Bias of a parameter estimate is the difference between the observed value of the estimate and the true value of the parameter being estimated (Mood, Graybill, & Boes, 1974). Type I error is considered to be an error of inference (Keppel, 1991), and occurs when we have rejected the null hypothesis in favour of the alternative when in fact the null hypothesis is true. A Type I error can occur in a latent growth curve interaction model when there is actually no presence of an interaction effect, yet the model estimates a parameter for this interaction effect that is significantly different from zero. A measure of robustness to Type I error is Bradley's (1978) liberal criterion of robustness, where a model is considered robust if its empirical rate of Type I error $\hat{\alpha}$ is contained in the interval $0.5\alpha \leq \hat{\alpha} \leq 1.5\alpha$. For an $\alpha = 0.05$ level of significance, a test is considered robust in a particular condition if its empirical rate of Type I error falls within the interval (0.25, 0.75) with values less than 0.25 being considered conservative and values higher than 0.75 being considered liberal.

There were two hypotheses generated with respect to bias and Type I error, and each hypothesis was evaluated separately for each missing data condition:

Hypothesis 1:

Null: All three models will show similar bias in the estimation of the unstandardized latent slope interaction parameter, as evidenced by similar average mean squared error (MSE) values and standardized bias values, across all study conditions.

Alternative: The Wen model will show a lesser degree of bias than both the Duncan and Schumacker models, and the Duncan model will show an equal or lesser degree of bias than the Schumacker model, across all study conditions. Specifically, the ordering of values for the MSE and standardized bias will be Wen < Duncan ≤ Schumacker.

Hypothesis 2:

Null: All three models will be similarly effective at controlling the rate of Type I error at the nominal level of significance of $\alpha = 0.05$ (using Bradley's criterion), across all study conditions.

Alternative: The Wen model will be provide adequate control of Type I error (i.e., maintaining a Type I error rate close to the nominal level of significance of $\alpha = 0.05$ using

Bradley's criterion) than both the Duncan and Schumacker models, across all study conditions. In other words, the Wen model will falsely detect the presence of the latent slope interaction effect at a rate that is closer to a nominal level of $\alpha = 0.05$ than both the Duncan and Schumacker models. Further, the Duncan model will provide adequate control of Type I error in as many or more study conditions than the Schumacker model.

Procedural Plan

A Monte Carlo simulation study was well suited to this type of project, as previous studies have utilized Monte Carlo simulations to investigate issues related to Type I error rates in latent variable models (e.g., Moulder & Algina, 2002). The following procedural plan was used to conduct the Monte Carlo simulation and analysis. The three latent growth interaction models – Wen (Wen et al., 2002), Duncan (Duncan et al., 1999; Li et al., 2000), and Schumacker (2002) were implemented in the SAS software using the PROC CALIS procedure. Data were generated according to a longitudinal repeated-measures design with four assessment points (with no specification of the time period between assessment points), with the Duncan model acting as the population model. Four factors were manipulated in the simulation: (1) the correlation between the latent intercept and slope for each factor, (2) the sample size, (3) the reliability of the observed indicators for each latent factor, and (4) the type of missing data mechanism.

To assess the main objective of the current study, the unstandardized value of the latent slope interaction parameter (which represents the effect of the latent interaction of the slopes of the independent factors on the latent slope of the outcome factor) was set equal to 2.0. The following overall model fit indices were extracted from the CALIS procedure for each model:

the comparative fit index, the normed fit index, the goodness of fit index, and the root mean square error of approximation.

To assess the secondary objective of the current project, and the two hypotheses, two approaches were used. Based on the original data used to assess overall model fit in the first objective, an evaluation of the estimation of the latent interaction slope parameter was also carried out for each model, examining the issues of estimation of the unstandardized latent slope interaction parameter, and the amount of bias in the estimation of this parameter. Bias was assessed through two indices, the mean square error (MSE) and the standardized bias. To assess the rates of Type I error for each of the three latent growth models in estimating the latent slope interaction parameter, the data were re-generated with the population value of the unstandardized latent slope interaction parameter being set equal to zero. The proportion of cases that showed inadequate control of Type I error rates (i.e., by being too liberal or too conservative) were reported, with values less than 0.25 being considered conservative and values higher than 0.75 being considered liberal.

Chapter 11: Method

Data Simulation Conditions

Four factors were manipulated in the simulation of data for this study: the correlation between the latent intercept and slope factors for the independent variables (3 levels), the sample size (3 levels), the reliability of the observed indicator variables for the independent variables (4 levels), and the mechanism of attrition (3 levels).

For the correlation between the latent intercepts and slopes, three values corresponding to low (0.20), medium (0.50) and high (0.80) were chosen. Sivo, Fan, and Witta (2005) have noted that the correlation between the latent intercept and slope is an important question (similar to correlated errors, which can introduce bias into the estimates). Correlations between latent constructs that approach 0.90 are considered to be indicative of significant overlap and are suggestive of collapsing the two latent constructs into a single construct. We chose three values of correlation that reflected a low, medium, and high correlation, but that were not high enough as to be reflective of too much overlap, but also not low enough to be considered trivial. A previous study by Hertzog and colleagues (Hertzog, Lindenberger, Ghisletta, & von Oertzen, 2006) used slope correlations of 0.25, 0.50, and 0.75 in their simulation using latent growth curves.

Three sample sizes were used in the study: n = 250, 500, and 1000. We chose a lower bound of 250 as a result of most basic articles on structural equation modeling reporting a minimum sample size of 200 being required for any SEM analysis (Weston & Gore, 2006), and some of the empirical literature has utilized samples close to this size (e.g., Dawson-McClure, Sandler, Wolchik, & Millsap, 2004b; Gilliom & Shaw, 2004). Further, we desired to take into account the loss in sample size that would occur through the process of generating missing data which would reduce the absolute sample size, and we wanted to be confident that we had enough of a sample size to meet what are considered general requirements for analyzing structural equation models in the missing data conditions. The middle sample size of 500 was chosen as many empirical studies have samples that are close to this size, ranging between 450 and 600 (e.g., Curran et al., 2003; DeShon, Kozlowksi, Schmidt, Milner, & Wiechmann, 2004). The highest sample size, 1000, was chosen as an upper limit as few longitudinal studies approached this number of participants (e.g., Duncan, Duncan, & Strycker, 2001), had a sample size of 770), and very few longitudinal studies have sample sizes above 1000.

A third factor that was manipulated was the reliability of the observed indicators, as this can influence the amount of measurement error. Since SEM models are aimed towards distinguishing true score from measurement error, the degree of reliability of the indicators is important in determining the best procedure for modeling an interaction. Following the procedure of Moulder and Algina (2002), four levels of reliability of the observed indicators for the latent variables were chosen, 0.30, 0.50, 0.70, and 0.90, corresponding to a range of poor, good, and exceptional reliabilities for the indicator variables. The reliability of each indicator is a function of the error variance for that indicator, given by the following formula (Allen & Yen, 1979):

$$\text{Error variance} = \sqrt{(\text{observed score variance} * (1\text{-reliability}))}. \qquad (33)$$

The observed score variance was set to increase non-linearly as the number of assessment points increased, with an observed score variance of 1 at the first assessment point, 3 at the second, 6 and the third, and 9 at the fourth. This pattern was chosen as other authors (e.g., Rudinger & Rietz, 1998) have noted that the relationship between time and the variance of scores in a linear latent growth model follows a quadratic relationship.

A final factor was the mechanism of missing data, or attrition. All cases had complete data for the first assessment point. In the Complete data condition, there was no attrition, and all cases had complete data for all assessment points. Two mechanisms of missing data were chosen: Missing Completely at Random (MCAR) and Missing Not at Random (MNAR). The full details for the generation of these attrition-based datasets is given in Step 3 of the Data Generation section below.

With 5000 replications for each condition, this resulted in 1,620,000 total replications being generated (3 latent growth interaction models X 3 latent intercept-slope correlations X 3 sample sizes X 4 reliabilities X 3 missing data mechanisms X 5000 replications).

Data Generation

Step 1: Specification of Population Model

Data were generated according to the latent growth model given in Figure 16, which is based on the model proposed by Duncan et al. (1999). This model has two latent slope factors, Factor 2 (ξ_2) and Factor 4 (ξ_4), that are proposed to have a direct effect on the latent slope of the outcome variable (η_1). These two slope factors are also proposed to have an interaction effect ($\xi_2\xi_4$) on the latent slope of the outcome. The symbol δ represents an error term, similar to that of ε. The choice of the population value of 2.0 for the latent slope interaction parameter was guided by the research of Li et al. (2000) and Wen et al. (2002). Li et al. used empirical data and found that the latent slope interaction parameter was -0.006, and Wen et al. used a value of 1.2 in their simulation study. In our study we strove to utilize a latent slope interaction parameter value that would be clearly identifiable as an interaction effect.

Insert Figure 16 about here

The LISREL specification of the measurement portion of the latent growth interaction model is:

$$\begin{vmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ X_2Z_2 \\ X_3Z_3 \\ X_4Z_4 \end{vmatrix} = \begin{vmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \\ \tau_4 \\ \tau_5 \\ \tau_6 \\ \tau_7 \\ \tau_8 \\ \tau_2\,\tau_6 \\ \tau_3\,\tau_7 \\ \tau_4\,\tau_8 \end{vmatrix} + \begin{vmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & \lambda_{32} & 0 & 0 & 0 \\ 1 & \lambda_{42} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & \lambda_{74} & 0 \\ 0 & 0 & 1 & \lambda_{84} & 0 \\ \tau_6 & \tau_6 & 0 & \tau_2 & 1 \\ \tau_7 & \tau_7\,\lambda_{32} & 0 & \tau_3\,\lambda_{74} & \lambda_{32}\,\lambda_{74} \\ \tau_8 & \tau_8\,\lambda_{42} & 0 & \tau_4\,\lambda_{84} & \lambda_{42}\lambda_{84} \end{vmatrix} \begin{vmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \\ \xi_4 \\ \xi_2\,\xi_4 \end{vmatrix} + \begin{vmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \\ \delta_5 \\ \delta_6 \\ \delta_7 \\ \delta_8 \\ \delta_9 \\ \delta_{10} \\ \delta_{11} \end{vmatrix} \quad (34)$$

For Factor 1, all observed variables (X_1-X_4) were specified to have an intercept ($\tau_1 - \tau_4$) equal to 1. For Factor 2, all observed variables (Z_1-Z_4) were specified to have an intercept ($\tau_5 - \tau_8$) equal to 1. From Figure 16, $\lambda_{32} = 2.0$; $\lambda_{42} = 3.0$; $\lambda_{74} = 2.0$; $\lambda_{84} = 4.0$.

Following Wen et al. (2002) the latent exogenous mean vector (K) is specified as

$$\begin{vmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \varphi_{31} \\ \varphi_{41} \\ \varphi_{32} \\ \varphi_{42} \end{vmatrix} \qquad (35)$$

and the variance-covariance matrix (Φ) for the latent variables is specified as

$$\begin{vmatrix} \varphi_{11} & & & & & & & \\ \varphi_{21} & \varphi_{22} & & & & & & \\ \varphi_{31} & \varphi_{32} & \varphi_{33} & & & & & \\ \varphi_{41} & \varphi_{42} & \varphi_{43} & \varphi_{44} & & & & \\ 0 & 0 & 0 & 0 & \varphi_{55} & & & \\ 0 & 0 & 0 & 0 & \varphi_{65} & \varphi_{66} & & \\ 0 & 0 & 0 & 0 & \varphi_{75} & \varphi_{76} & \varphi_{77} & \\ 0 & 0 & 0 & 0 & \varphi_{85} & \varphi_{86} & \varphi_{87} & \varphi_{88} \end{vmatrix} \qquad (36)$$

Where

$\varphi_{55} = \varphi_{11}\,\varphi_{33} + \varphi_{31}^{2}$; $\varphi_{66} = \varphi_{11}\,\varphi_{44} + \varphi_{41}^{2}$; $\varphi_{77} = \varphi_{22}\,\varphi_{33} + \varphi_{32}^{2}$; $\varphi_{88} = \varphi_{22}\,\varphi_{44} + \varphi_{42}^{2}$; $\varphi_{65} = \varphi_{11}\,\varphi_{43}$

$+ \varphi_{31}\,\varphi_{41}$; $\varphi_{75} = \varphi_{21}\,\varphi_{33} + \varphi_{31}\,\varphi_{32}$; $\varphi_{85} = \varphi_{21}\,\varphi_{43} + \varphi_{32}\,\varphi_{41}$; $\varphi_{76} = \varphi_{21}\,\varphi_{43} + \varphi_{31}\,\varphi_{42}$; $\varphi_{86} = \varphi_{21}\,\varphi_{44} + \varphi_{41}\,\varphi_{42}$; $\varphi_{87} = \varphi_{22}\,\varphi_{43} + \varphi_{32}\,\varphi_{42}$.

The variances of the latent intercept and slopes (φ_{11} to φ_{44}) were set equal to 1, the expectation (mean) of each of the latent variables was set equal to 0, and the covariances of the latent intercepts (φ_{31}) were set equal to 0.

The variance-covariance matrix of error, Θ_δ, is defined as

$$\begin{vmatrix}
\theta_1 & & & & & & & & & & \\
0 & \theta_2 & & & & & & & & & \\
0 & 0 & \Theta_3 & & & & & & & & \\
0 & 0 & 0 & \theta_4 & & & & & & & \\
0 & 0 & 0 & 0 & \theta_5 & & & & & & \\
0 & 0 & 0 & 0 & 0 & \theta_6 & & & & & \\
0 & 0 & 0 & 0 & 0 & 0 & \theta_7 & & & & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \theta_8 & & & \\
0 & \theta_{92} & 0 & 0 & 0 & \theta_{96} & 0 & 0 & \theta_9 & & \\
0 & 0 & \theta_{10,3} & 0 & 0 & 0 & \theta_{10,7} & 0 & 0 & \theta_{10} & \\
0 & 0 & 0 & \theta_{11,4} & 0 & 0 & 0 & \theta_{11,8} & 0 & 0 & \theta_{11}
\end{vmatrix} \quad (37)$$

Where

$\theta_9 = \tau_6^2\,\theta_2 + \varphi_{33}\theta_2 + \varphi_{44}\theta_2 + \tau_6^2\,\theta_6 + \varphi_{11}\theta_6 + \varphi_{22}\theta_6 + \theta_2\theta_6$

$\theta_{10} = \tau_7^2\,\theta_3 + \varphi_{33}\theta_3 + \lambda_{74}^2\varphi_{44}\theta_3 + \tau_3^2\,\theta_7 + \varphi_{11}\theta_7 + \lambda_{32}^2\varphi_{22}\theta_7 + \theta_3\theta_7$

$\theta_{11} = \tau_8^2\,\theta_4 + \varphi_{33}\theta_4 + \lambda_{84}^2\varphi_{44}\theta_4 + \tau_4^2\,\theta_8 + \varphi_{11}\theta_8 + \lambda_{42}^2\varphi_{22}\theta_8 + \theta_4\theta_8$

$\theta_{92} = \tau_6\,\theta_2$, $\theta_{96} = \tau_2\,\theta_6$, $\theta_{10,3} = \tau_7\,\theta_3$, $\theta_{10,7} = \tau_3\,\theta_7$, $\theta_{11,4} = \tau_8\,\theta_4$, $\theta_{11,8} = \tau_4\,\theta_8$.

Step 2: Generating Model Parameters

The SAS code used to generate the data for the longitudinal models is given in Appendix A. The SAS RANNOR procedure was used to generate the raw data for the current study. The RANNOR procedure is based on the RANUNI procedure, and uses the Box-Muller transformation of RANUNI uniform variates, and which has a period of 2,147,483,646 (SAS Institute Inc., 2009). When using the SAS random number functions or subroutines, one should specify a SEED in the range of 1 to 2**31-2 to initialize the starting point of the pseudorandom number stream, or use a nonpositive integer (0 or negative) to create the initial seed from the system clock. If a nonpositive number is used, SAS reads the system time and computes the initial seed value using an algorithm equivalent to SEED = 1e3 * mod(round(1e3 * datetime()), 1e6) + 1.

For all simulations, a negative seed number was used, which resulted in the initial seed being derived from the system clock. Bang, Schumacker, and Schlieve (1998) have shown that the sample size is important factor when generating normal random numbers, and advocated for sample sizes larger than 1000 in order to ensure small departures from the expected mean = 0 and standard deviation = 1. A random sample of 1000 cases from three separate simulation conditions showed that the observed variables all had means close to 0 (ranging from 0 to 0.02) with small standard deviations (ranging from 0.08 to 0.34), with skewness and kurtosis values less than 1 in absolute value. Further, the observed variables showed an increasing amount of variance acorss the assessment points – that is, the variance of the observations at the second assessment point was larger than the variance at the first assessment point, and the variance of the observations at the third assessment point was larger than the variance at the second assessment point, and so on through to the fourth assessment point.

Step 3: Generating Missing Data from the Complete Population Data

To generate the missing values under the MCAR and MNAR mechanisms, the following attrition process was utilized (e.g., Jamshidian & Bentler, 1999; Newman, 2003). MCAR missing samples were simulated by randomly removing a percentage of scores from each of the last three repeated observations in the Complete data set. To accomplish this, the raw data for the second, third, and fourth assessment points were paired with corresponding random values (denoted as RV_T2, RV_T3, RV_T4) from a normal distribution, and these random values were used to select scores for deletion. The deletion process was performed as follows: (1) starting with the second assessment point, all those cases with an RV_T2 value greater than a z-score of 1.30 (corresponding to the 90^{th} percentile) were removed and, further, those cases had their third and fourth observations eliminated; (2) from the remaining undeleted scores at the third assessment point, those cases with an RV_T3 value greater than 1.30 were selected to be removed, along with those cases' fourth observation; (3) finally, from the remaining undeleted scores at the fourth assessment point, those cases with an RV_T4 value greater than 1.30 were selected to be deleted. Through this attrition process, a monotone pattern of MCAR samples was obtained. An inspection of a random selection of 1000 datasets from each of the three sample size conditions (250, 500, 1000) showed average dropout rates of 10% for the second assessment point, 8.2% for the third assessment point, and 8.0% for the fourth assessment point, resulting in a cumulative attrition rate of 26%.

For MNAR samples, the dropout process was different from the MCAR process in that missingness depended on the observed scores of the outcome variable. The MNAR data was generated from the Complete data in the following manner. (1) The scores for the Y-variable at the second observation point were standardized into a new variable (STDY2) which had a mean of 0 and a standard deviation of 1. Those STDY2 scores that were above 1.30 (corresponding to the 90^{th} percentile) were removed, as were their corresponding observations for the remaining assessment points; (2) The remaining observations for the third observation point were standardized into a new variable (STDY3) which had a mean of 0 and a standard deviation of 1. Those STDY3 scores that were above a value of 1.30 were removed, as were their corresponding observations at the fourth assessment point; (3) Of the remaining observations, their fourth observation point scores were standardized into a new variable (STDY4) which had a mean of 0 and a standard deviation of 1, and those STDY4 scores which were above 1.30 were removed. An inspection of a random selection of 1000 datasets from each of the three sample size conditions (250, 500, 1000) showed average dropout rates of 10% for the second assessment point, 4% for the third assessment point, and 1% for the fourth assessment point, resulting in a cumulate attrition rate of 15%.

As the models are independent of the data, there is not modification of the raw data by any of the models, beyond the generation of the new variables for the interaction terms. The process of the missing data mechanisms does modify the raw data to some extent, but these modifications are natural outcomes of the missing data mechanisms. As a result, we can confirm that the generalization of the results will be valid for all of that data that is generated by the mechanisms described above.

Assessment of Overall Model Fit

The analysis of the results began with a descriptive examination of the rates of convergence for the three latent growth interaction models in the three data conditions. Convergence occurs when the estimation function has reached a minimum and the model parameters are estimated (Tabachnick & Fidell, 2001). This was followed by an examination of the overall model fit of the three latent growth interaction models. To assess overall model fit for the three models, the following model fit indices were used: the comparative fit index (CFI; Bentler, 1988), the Bentler and Bonnett normed fit index (NFI; Bentler & Bonnett, 1980), the goodness-of-fit index (GFI; Joreskog & Sorbom, 1986), and the root mean square error of approximation (RMSEA; Steiger & Lind, 1980).

CFI

The CFI (Bentler, 1988) is part of a class of comparative fit indexes, where models are conceptualized as being nested within one another. At one extreme is a model that is saturated (full or perfect) with zero degrees of freedom. At the other extreme is a model that corresponds to completely unrelated variables, called an independence model. The CFI assesses fit of the specified model relative to the independence model. The CFI uses a noncentral χ^2 distribution with noncentrality parameter, τ_i. The larger the value of τ_i, the greater the model misspecification. That is, if the estimated model is perfect, $\tau_i = 0$ and the CFI = 1. The CFI is defined as

$$CFI = 1 - (\tau_{\text{estimated model}} / \tau_{\text{independence model}}). \quad (38)$$

The τ value for a model can be estimated by

$$\tau_{\text{independence model}} = \chi^2_{\text{independence model}} - df_{\text{independence model}}, \text{ and} \quad (39)$$

$$\tau_{\text{estimated model}} = \chi^2_{\text{estimated model}} - df_{\text{estimated model}}. \quad (40)$$

NFI

The Bentler-Bonnett (Bentler & Bonnett, 1980) NFI is also part of the class of comparative fit indexes, where models are conceptualized as being nested within one another. The NFI evaluates the estimated model by comparing the χ^2 value of the model to the χ^2 value of the independence model,

$$NFI = (\chi^2_{\text{independence}} - \chi^2_{\text{model}}) / \chi^2_{\text{independence}}. \quad (41)$$

This yields a descriptive fit index that lies in the 0 to 1 range, with higher values indicative of a good-fitting model.

GFI

The GFI is analogous to the R^2 in multiple regression (Hoyle et al., 1994), and assesses the degree to which the reproduced covariance matrix based on the specified model has accounted for the original sample covariance matrix (Tanaka & Huba, 1985). The GFI calculates a weighted proportion of variance in the sample covariance accounted for by the estimated population covariance matrix (Tabachnick & Fidell, 2001). The GFI can be defined by

$$\mathrm{GFI} = \mathrm{tr}(\sigma'W\sigma) / \mathrm{tr}(s'Ws), \qquad (42)$$

where the numerator is the sum of the weighted variances from the estimated model covariance matrix (σ) and the denominator is the sum of the squared weighted variances from the sample covariance (s). W is the weight matrix that is selected by the choice of estimation method. The GFI ranges in value from 0 to 1, with values close to 0 indicating poor model fit and values close to 1 indicating good model fit.

RMSEA

The RMSEA was proposed by Steiger and Lind (1980), and was developed further by Browne and Cudeck (1993). The RMSEA is a test of exact model fit, comparing the proposed model to a perfect (saturated) model, with values less than 0.05 being indicative of good model fit. The equation for the estimated RMSEA is given by

$$\text{estimated RMSEA} = \sqrt{(F_o / df_{model})}, \qquad (43)$$

where

$$F_o = (\chi^2_{model} - df_{model}) / N \qquad (44)$$

or

$$F_o = 0,$$

whichever is smaller but positive. When the model is perfect, $F_o = 0$. The greater the model misspecification, the larger F_o.

Data Analysis Procedure

The following frequently used criteria were used to evaluate the adequacy of the models: CFI > 0.90, NFI > 0.90, GFI > 0.90, and RMSEA < .05 (Fan et al., 1998; Tabachnick & Fidell, 2001). The analysis of the results began with a descriptive examination of the rates of convergence for the three latent growth interaction models in the three data conditions. Convergence occurs when the estimation function has reached a minimum and the model parameters are estimated (Tabachnick & Fidell, 2001). Only those models that successfully converged and had CFI, NFI, or GFI values greater than 0 were analyzed (as these models were considered to be no different from an independence model).

The first objective was to examine the performance of the three latent growth interaction models in order to assess the strengths and weaknesses of the models in the presence of missing data. The majority of the analyses for this objective were focused on the evaluation of overall model fit for the three latent growth interaction models to the simulated data, including an evaluation of convergence rates for each of the three models. For each condition in the study, across type of missing data, the number of models that converged and the mean number of iterations were tabulated. Only those models that converged successfully were subsequently analyzed. Means, standard deviations and 95% confidence intervals for each of the model fit indices were calculated. To identify those factors that affected overall model fit statistics, analysis of variance (ANOVA) and partial omega squared were evaluated. For all of the ANOVA models, the dependent variables were the overall model fit statistics (i.e., CFI, NFI, GFI, RMSEA), and the independent variables were the factors of: type of latent interaction growth model, latent intercept-slope correlation, sample size, reliability of the observed indicators, and all of their interactions. Separate ANOVA were carried out for each type of missing data mechanism, and effects were assessed at an $\alpha = 0.05$ level of significance. To assess the magnitude of the main and interaction effects, partial omega squared was used to estimate the effect size for each effect. The partial omega squared estimates equaled the ratio of the variance due to an effect to the sum of (a) the variance due to the effect and (b) the error variance. According to Cohen (1988, cited in Olejnik & Algina, 2000), suggested values of 0.01, 0.06, and 0.14 correspond to small, medium, and large associations. Pairwise multiple comparisons of the means in a given condition were conducted using a Ryan-Einot-Gabriel-Welch (REGWQ) procedure to control familywise experiment error rate (Westfall & Young, 1993).

The second objective was to evaluate these models with respect to parameter estimation of the latent slope interaction, bias, and Type I error rates, in accordance with the two proposed hypotheses. For the first hypothesis, three measures of bias were calculated: the difference of the estimate of the latent slope interaction parameter from the population parameter, the mean square error (MSE), and the standardized bias. For this assessment the population value of the latent slope interaction parameter was set to 2.0.

To evaluate the bias in the parameter estimate for the latent slope interaction effect three aspects of bias were examined. The first was to calculate the mean bias in the estimate itself, by computing the difference between the population value and the estimated model value. In the case of the current study, mean bias was the difference between the estimated model value and the population value of 2.0 (see Figure 16). The second aspect was to evaluate the mean square error (MSE; Degroot, 1980) of the estimated latent slope interaction parameter. The MSE quantifies the expected (average) squared deviation of an estimator from a population parameter (Olejnik & Porter, 1981), and provides information on the spread of the parameter estimates around the true estimate. The value of the MSE is given by the following equation:

$$\text{MSE} = (\text{population value} - \text{model estimate})^2 + \text{variance of model estimate}. \qquad (45)$$

The third aspect of bias assessed was to utilize a standardized estimate of bias, which is the average deviation of the sample estimate from the population parameter estimate, divided by the standard error of the estimate. Values of this standardized estimate which are close to 1.0 indicate no bias in the estimate. Values which are above 1.0 are indicative of an overestimating bias, while values less than 1.0 are indicative of an underestimating bias.

A one-way analysis of variance was used to examine differences between the means for the MSE and standardized bias across the levels of manipulated factors, separately for each missing data mechanism. Pairwise multiple comparisons of the means in a given condition were conducted using a Ryan-Einot-Gabriel-Welch (REGWQ) procedure to control familywise experiment error rate.

For the second hypothesis, which addressed Type I error rates, the entire simulation was re-done, specifying a population value of 0.0 for the latent slope interaction parameter (i.e., $\gamma_{28} = 0$). The proportion of models that generated a significant effect for the latent slope interaction effect were tabulated. Bradley's (1978) liberal criterion of robustness was used to evaluate each of the models: a model is considered robust if its empirical rate of Type I error ($\hat{\alpha}$) is contained in the interval $0.5\alpha \leq \hat{\alpha} \leq 1.5\alpha$. A model was considered robust if its empirical Type I error rate for the latent interaction slope parameter was contained within the interval $0.5\alpha \leq \hat{\alpha} \leq 1.5\alpha$. For the five percent level of significance used in this study, a model was considered robust in a particular condition if its empirical rate of Type I error fell within the interval $0.25 \leq \hat{\alpha} \leq 0.75$. Correspondingly, a model was considered to be nonrobust if, for a particular condition, its Type I error rate was not contained in this interval. Models that showed Type I error rates that were above 0.75 were considered to be liberal, and those that showed Type I error rates that were below 0.25 were considered to be conservative.

All models were estimated using the maximum likelihood estimation method in SAS. The SAS code for evaluating each of the models are given in Appendix B (for the Wen model), Appendix C (for the Duncan model), and Appendix D (for the Schumacker model). Once the models were run for each simulation condition, the resulting parameter estimates, standard errors, and model fit indices were read into the SAS statistical package and analyzed.

Chapter 12: Results

Each of the approaches to modeling the interaction effect in the latent growth models were assessed for their overall model fit to the simulated data (as indicated by the selected model fit indices). This included descriptive information such as means and standard deviations, as well as 95% confidence intervals for the model fit indices. Further to this, evaluations of the accuracy of estimation of the latent slope interaction parameter (accuracy, MSE, standardized bias), and assessment of Type I error rates in testing the latent slope interaction effect were carried out. These were done separately for each type of missing data (Complete, MCAR, MNAR). Prior to the analyses, the rates of convergence for all models across all conditions were examined, and only those models that converged successfully and had CFI, NFI, or GFI values that were above 0.0 were considered afterwards.

The main set of analyses were a detailed examination of the overall model fit indices for the three latent interaction growth models across the three missing data conditions. This included descriptive information such as the mean and standard error for each model fit index for each model in each simulation condition, as well as the calculation of 95% confidence intervals for each of these mean estimates. This was followed by ANOVA models, with an examination of the main and interaction effects for all fixed effects in the models (i.e., the simulation conditions), and mean comparisons. The last section presents an examination of the bias in the estimation of the latent slope interaction parameter (γ_{28}), as well as the empirical Type I error rate for each of the three latent growth interaction models when the population latent slope interaction parameter was set to zero (i.e., $\gamma_{28} = 0$).

Assessment of Convergence Rates

Table 2 shows the convergence rates for all three latent growth interaction models (Wen, Duncan, and Schumacker) across the four manipulated factors (latent intercept-slope correlation, sample size, reliability of observed indicators, and missing data mechanism) where the population value of the latent slope interaction was set to 2.0 for each missing data type.

Insert Table 2 about here

In the Complete data condition, only the Wen latent interaction model showed convergence problems, failing to converge in all 5000 replications when the reliability was lowest (i.e., 0.30 or 0.50) across all three levels of sample size and latent intercept-slope correlation. In those conditions without full convergence (i.e., not converging for all 5000 replications within a condition), the Wen model converged in over 95% of the replications. The Duncan and Schumacker models showed convergence in all 5000 replications across all factors in the Complete data condition. Similar findings were seen for the MCAR data, with convergence rates being 93% or higher for the Wen model in the individual conditions.

In the MNAR data, the Wen model showed incomplete convergence in many of the simulation conditions, with rates ranging from 29% to 100%. For the Wen model the highest rates of convergence were observed at the highest level of reliability across the levels of sample size and latent intercept-slope correlation. The Duncan model showed a convergence problem in only a single condition, when the latent intercept-slope correlation was lowest (0.20), the sample size was smallest (250), and the reliability of the observed indicators was lowest (0.30), however there was convergence in 4999 out of 5000 simulations in this condition. The Schumacker model showed complete convergence across all conditions. Table 3 has the mean number of iterations for convergence to be achieved in only those models that converged successfully and had CFI, GFI, or NFI values that were above 0.0.

Insert Table 3 about here

Assessment of Overall Model Fit

The following sections outline the average values, 95% confidence intervals, multiple comparisons, and ANOVAs for the overall model fit indices (CFI, NFI, GFI, RMSEA) within those simulations for models that converged successfully, and which had CFI, NFI, or GFI values that were non-zero. The following frequently used criteria were used to evaluate the adequacy of the models: CFI > 0.90, NFI > 0.90, GFI > 0.90, and RMSEA < 0.05 (Fan et al., 1998; Tabachnick & Fidell, 2001).

CFI

Table 4 presents the average CFI values for all models across the three levels of missing data, with 95% confidence intervals for these average values given in Table 5. Table 6 contains the results of an ANOVA model, using the CFI model fit index as the dependent variable, with latent interaction model type (3 levels), correlation of the latent intercept and slope (3 levels), sample size (3 levels) and reliability of the observed indicators (4 levels) as independent factors. All interactions between these factors were included in the model. These analyses were carried out separately for each data type condition (Complete data, MCAR data, MNAR data).

Insert Tables 4, 5, and 6 about here

Complete Data Analyses

The average CFI value for the Wen model was 0.92 (range 0.91 – 0.93), for the Duncan model 0.90 (range 0.89 – 0.91), and for the Schumacker model 0.76 (range 0.66 – 0.85). The 95% confidence intervals in Table 5 for the Complete and MCAR data show the Wen and Duncan model having a large proportion of conditions where the lower and upper bounds of the confidence intervals are at or above 0.90. This indicates that, for these models, it can be expected that the average CFI values will be above 0.90 in over 95% of the cases. It was also seen that the proportion of conditions with 95% confidence intervals above 0.90 increased as sample size increased for the Wen model. A similar effect was seen for the Duncan model as reliability of the observed indicators increased. Both of these models showed an increased proportion of confidence intervals above 0.90 as the correlation between the latent intercept and slope increased. The Schumacker model did not have any 95% confidence intervals whose lower bound was above 0.90. Multiple comparisons between the means (using a REGWQ comparison procedure to control familywise experiment error rate) showed that only a single pair of means were not significantly different from each other, and they are presented in bold text in Table 5. This single pair was for the Duncan and Schumacker models in the MNAR condition for the correlation-sample size-reliability condition of (0.70, 250, 0.30). All other pairs of means were significantly different from each other.

The ANOVA in Table 6 for the Complete data condition showed significant effects ($p < 0.05$) for all main effects and their interactions. The overall model effect was significant ($F_{(107, 531703)} = 13599.94$, $p < 0.01$, partial eta$^2 = 0.73$). The largest effect sizes were seen for the two-way interactions of latent interaction model with reliability of the observed indicators (0.13) and latent interaction model with correlation between latent intercepts and slopes (0.10), and for the main effects of reliability (0.11) and latent interaction model (0.70), according to Olejnik and Algina (2000).

The four-way interaction of latent interaction model type, latent intercept-slope correlation, sample size, and reliability of the observed indicators was significant ($F_{(24, 531703)} = 2.15$, $p < 0.01$) but with a small effect size (partial eta$^2 < 0.01$). Normally such a significant interaction would be broken down into simple effects that would help to interpret the interaction between the factors. However, the large error degrees of freedom for the ANOVA model results in even trivial mean differences emerging as significant. To avoid this we have chosen to focus on only those ANOVA effects that produced effect sizes of at least a medium effect (i.e., 0.06 or greater for the partial eta-squared). The ANOVA effects that met this criterion were for the two-way interactions of latent interaction model with reliability of the observed indicators (0.13) and latent interaction model with correlation between latent intercepts and slopes (0.10), and for the main effects of reliability (0.11) and latent interaction model (0.70), according to Olejnik and Algina (2000).

To examine the significant two-way interactions, simple plots of the average CFI values with the reliability of the observed indicators were produced for each latent growth interaction model, and are given in Figure 17.

Insert Figure 17 about here

Seen clearly in the simple plots is the difference in pattern of average values of the CFI for the Schumacker model from both the Wen and the Duncan models. Also of note is the differing pattern of CFI values. When examining the simple plot for the interaction of latent growth interaction model with reliability, across the levels of reliability both the Wen and the Duncan model produced stable values, while the Schumacker model showed a trend of increasing CFI values as reliability increased. However, when examining the two-way interaction involving latent model type and correlation between the latent intercept and slope, the Wen and the Duncan models again showed a stable pattern while the Schumacker model showed a decreasing trend of CFI values as the correlation increased.

MCAR and MNAR Data Condition Analyses

In the MCAR data condition the average CFI value for the Wen model was 0.92 (range 0.91-0.93), for the Duncan model 0.90 (range 0.89-0.91), and for the Schumacker model 0.75 (range 0.69-0.81). The results for the analyses of the average CFI model fit statistic were identical in pattern to those for the Complete data condition. The largest effect sizes were seen for the two-way interactions of latent interaction model with both reliability of the observed indicators (0.12) and latent intercept-slope correlation (0.09), and for the main effects of reliability (0.10) and latent interaction model (0.69). The description of these results, including the average CFI value for each condition and the figures of the simple effects plots, are given in Appendix E.

In the MNAR data condition the average CFI value for the Wen model was 0.76 (range 0.69-0.84), for the Duncan model 0.70 (range 0.69-0.74), and for the Schumacker model 0.76 (range 0.69-0.81). None of the models in this data condition showed average CFI values that were above the cutoff of 0.90 for good model fit. These models are not discussed here, but are presented in Appendix E.

Summary

Across the three missing data conditions, the Wen model showed the most consistent and optimal performance with respect to the CFI model fit index. In the Complete data condition the Wen model yielded average CFI values that were above the cutoff of 0.90 in all 36 conditions, while the Duncan model produced values that were above this cutoff in 30 conditions. The Schumacker model did not provide any average CFI values that were above the cutoff. A similar pattern was seen in the 95% confidence intervals for the average CFI values. Namely, the Wen model showed confidence intervals where the lower bound was at or above 0.90 in almost all of the conditions for the Complete and MCAR data, with the Duncan model showing such behaviour only at higher levels of latent intercept-slope correlation and reliability. The Schumacker model did not have any confidence intervals whose lower bound was above 0.90. These findings were paralleled in the MCAR data condition. In the MNAR data condition none of the models had average CFI values that were above the cutoff for good model fit.

The Wen model consistently produced average CFI values that were higher than those of the Duncan model, the only exceptions being in the MNAR data condition when the reliability was at its highest level. The Schumacker model provided average CFI values that were superior to the Wen model in the MNAR data condition only, occurring only when the reliability of the observed indicators was at its highest values (i.e., 0.70 or 0.90), and this was seen across all three sample sizes and levels of latent intercept-slope covariance. Further, the average CFI values for the Schumacker model decreased as the latent intercept-slope correlation increased.

The ANOVAs for the CFI statistic showed a consistent two-way interaction of latent growth interaction model type with the reliability of the observed indicators in all three data conditions. In both the Complete and MCAR data conditions the pattern of the Wen and Duncan models showing stable trends of average values around the cutoff of 0.90 as reliability increased was seen, while the Schumacker model showed a pattern of decreasing average CFI values as reliability increased. In the MNAR data condition the pattern was more variable, with the Wen model showing a decreasing trend while the Duncan and Schumacker models showed increasing trends. An additional interaction was seen in the Complete data, namely that of latent growth interaction model type with latent intercept-slope correlation. For this interaction, the simple plot showed that both the Wen and Duncan models showed stable patterns of average CFI values as the correlation increased, while the Schumacker model showed a pattern of decreasing average CFI values.

NFI

The NFI (Bentler & Bonnett, 1980) evaluates the estimated model by comparing the χ^2 value of the model to the χ^2 value of the independence model, yielding an index that lies on a continuum from 0 (indicating a poor fit to the data) to 1 (indicating a perfect fit to the data).

Table 7 presents the average NFI values for all of the latent interaction models, across the three conditions of missing data, and Table 8 presents the 95% confidence intervals for the average NFI. Table 9 contains the results of three ANOVA models, using the NFI statistic as the dependent variable, with latent interaction model type (3 levels), correlation of the latent intercept and slope (3 levels), sample size (3 levels) and reliability of the observed indicators (4 levels), and all of their interactions, as independent factors. These analyses were carried out separately for each missing data condition (Complete, MCAR, MNAR).

Insert Tables 7, 8, and 9 about here

Complete Data Condition Analyses

In the Complete data condition the average NFI value for the Wen model was 0.91 (range 0.84-0.94), for the Duncan model 0.89 (range 0.82-0.93), and for the Schumacker model 0.76 (range 0.19-0.94). Only the Wen and Duncan models produced 95% confidence intervals for the mean NFI values that had lower bounds above the value of 0.90. For the Wen model these were consistently produced when the reliability was at its highest value (0.90), and occurred more often as both sample size increased and as the correlation between the latent intercept and slope increased. The Duncan model showed such confidence intervals only at the highest levels of reliability, with the highest proportion in the conditions where the correlation between the latent intercept and slope was highest (0.70) and the sample size was highest (1000). The Schumacker model did not produce any 95% confidence intervals whose lower bound was at or above 0.90. Multiple comparisons between the means (using a REGWQ comparison procedure to control familywise experiment error rate) showed that every pair of means were significantly different from each other.

The ANOVA in Table 9 for the Complete data shows significant effects ($p < 0.05$) for all main effects and their interactions. The overall model effect was significant ($F_{(107, 531703)}$ = 12496.27, $p < 0.01$), and had a large effect size (partial eta^2 = 0.72). The four-way interaction of latent interaction model type, latent intercept-slope correlation, sample size, and reliability of the observed indicators was significant ($F_{(24, 531703)} = 2.06$, $p < 0.01$) with a small effect size (partial eta$^2 < 0.01$). However, as noted earlier with the CFI, the large error degrees of freedom for the ANOVA model results in even trivial mean differences emerging as significant, and effects that produced effect sizes of at least a medium effect (i.e., 0.06 or greater for the partial eta-squared) were examined.

The ANOVA effects that met this criteria were for the two-way interactions of latent interaction model with both reliability (0.12) and correlation of latent intercept and slope (0.10), and the main effects of latent interaction model (0.68) and reliability (0.13). To examine the significant two-way interactions, separate simple plots of the average NFI values with the reliability of the observed indicators and the latent interaction-slope correlation were produced for each latent growth interaction model, and are given in Figure 18.

Insert Figure 18 about here

Seen clearly in the simple plots is the difference in pattern of average values of the NFI for the Schumacker model from both the Wen and the Duncan models. When examining the simple plot for the interaction of latent growth interaction model with reliability, across the levels of reliability both the Wen and the Duncan model produced stable values around 0.90, while the Schumacker model showed a trend of increasing NFI values as reliability increased. However, when examining the two-way interaction involving latent model type and correlation between the latent intercept and slope, the Wen and the Duncan models again showed a stable pattern while the Schumacker model showed a decreasing trend of NFI values as the correlation increased.

For the MCAR data condition, the means, ranges, 95% confidence intervals, and

ANOVA results were similar in pattern to those for the Complete data condition and are not presented here. They are reported in Appendix F.

MNAR Data Condition Analyses

None of the models for this data condition produced average NFI values that were above the cutoff of 0.90 for good model fit. These results are not discussed here and are presented in Appendix F.

Summary

The Wen model was consistently able to produce average NFI values that were above the cutoff of 0.90 for good model fit, and this was seen in the Complete and MCAR data only (for 32 of the 36 conditions in both data conditions). The Duncan model was only able to produce acceptable average NFI values in some of the conditions, doing so in only 20 conditions in the Complete data and 17 in the MCAR data. In a parallel manner, the Wen model was able to produce 95% confidence intervals whose lower bound was at or above 0.90 in almost half of the conditions in the Complete and MCAR data (16 and 14, respectively), while the Duncan model produced less than 10. The average NFI values of the Duncan model were generally lower than those of the Wen model. For both of these models in these two data conditions the average NFI values were stable across all levels of latent intercept-slope correlation, sample size, and reliability. The Schumacker model did not produce average NFI values above the cutoff for good model fit in any of the study conditions, regardless of the type of data.

GFI

The goodness of fit index (GFI) is analogous to the R^2 in regression, and assesses the degree to which the reproduced covariance matrix based on the specific model has accounted for the original sample covariance matrix (Tanaka & Huba, 1985). Table 10 presents the average GFI values across the three data conditions.

Insert Table 10 about here

Complete and MCAR Data Condition Analyses

The average GFI value for the Wen model was 0.87 (range 0.74 – 0.90), for the Duncan model 0.86 (range 0.74 – 0.90), and for the Schumacker model 0.77 (range 0.54 – 0.93) in the Complete data, and the results for the MCAR data were virtually identical. None of the average GFI values were above the cutoff of 0.90 for good model fit in either of these types of data. The 95% confidence intervals and ANOVA results are presented in Appendix G and are not discussed here.

MNAR Data Condition Analyses

In the MNAR data condition the average GFI value for the Wen model was 0.73 (range 0.52-0.83), for the Duncan model 0.81 (range 0.68-0.87), and for the Schumacker model 0.76 (range 0.06-0.96). Only the Schumacker model produced 95% confidence intervals that had a lower bound at or above the value of 0.90, and this occurred in only two of the study conditions for this data. As a result of the poor performance on this model fit index, these findings are presented in Appendix G and are not discussed here.

Summary

Generally, all three of the latent growth interaction models performed poorly with respect to the GFI. The Wen and the Duncan models produced similar average GFI values in both the Complete and MCAR data conditions, with the Duncan model having higher average GFI values in the MNAR data condition. The Schumacker model produced the lowest average GFI values in the Complete and MCAR data conditions, but produced the highest average values in the MNAR data condition. In all three data conditions neither the Wen nor the Duncan models produced average GFI values that were above the cutoff for good model fit of 0.90 in any of the study conditions, and similarly none of the 95% confidence intervals for the mean GFI value for these models had a lower bound that exceeded 0.90. The Schumacker model performed poorly in both the Complete and MCAR data conditions, but did show a trend of increasing average GFI values as both reliability and sample size increased. The Schumacker model was the only model to produce average GFI values that were above the cutoff of 0.90 in the MNAR data condition, doing so in 12 of the 36 conditions, and produced 95% confidence intervals whose lower bound was above 0.90 in two of the study conditions for this type of data. Adequate model fit only occurred at the highest levels of reliability across all of the levels of sample size and latent intercept-slope correlation.

RMSEA

The RMSEA is a test of exact fit, with the cut-off value of 0.05 (Browne & Cudeck, 1993). The average RMSEA values for all of the models are given in Table 11. None of the models showed average RMSEA values that were equal to or below the cut-off value of 0.05.

Across the three missing data conditions, the average RMSEA values for the Wen model ranged from 0.09 to 0.26, those for the Duncan model ranged from 0.12 to 0.24, and those for the Schumacker model ranged from 0.22 to 0.70. Due to none of the models passing the criterion for good fit, all of these results are presented in Appendix H and are not reported here.

Insert Table 11 about here

While the model fit indices presented previously are informative, it is difficult to use them to directly compare the models with each other. However, for models that are nested within each other, a chi-square different test can be used to determine the more parsimonious model (Tabachnick & Fidell, 2001). Table 12 presents the difference in model chi-square between the Wen and Duncan models. The chi-square for the Schumacker model could not be compared in this was as it is not considered to be a nested model of either the Wen or the Duncan models. The Wen model has 91 degrees of freedom and the Duncan model has 92 degrees of freedom, and the difference was taken as (model chi-square for the Duncan – model chi-square for the Wen). All difference values were evaluated against a chi-square distribution with degrees of freedom of 1.

Insert Table 12 about here

For the Complete and MCAR data, across all conditions, the Wen model showed a significant reduction in model chi-square over that of the Duncan model. These reductions showed a non-linear trend, with decreasing differences between the two models as reliability increased, the exception being that at the highest level of reliability there were large differences. This was consistent across the levels of sample size and latent intercept-slope correlation. In the

MNAR data, a similar trend occurred, again with the exception being at the highest level of reliability across the levels of sample size and latent intercept-slope correlation. In this case, however, the Duncan model showed a better fit to the data.

While not reported, an examination of the chi-square value for each model was undertaken. Specifically, the ratio of the model chi-square to the degrees of freedom was computed, with ratios less than 2.5 considered to be indicative of a parsimonious model that was fitting the observed data adequately (Kline, 1998). None of the models produced ratios of this nature in any of the missing data conditions. An examination of the chi-square values for the individual models in each simulation condition, using the G*Power software (Faul, Erdfelder, Lang, & Buchner, 2007), showed that all models had power equal to 1, across all three missing data conditions.

Estimation of the Latent Slope Interaction Parameter (γ_{28}) when the Population Value of the Parameter was Set to 2.0.

Table 13 shows the average of the unstandardized parameter estimates (with standard deviations in parentheses) for the latent slope interaction, which was set to a value of 2.0 in the population model (see the parameter γ_{28} in Figure 16).

Insert Table 13 about here

Complete Data Condition Analyses

In the Complete data condition, the Wen model provided unstandardized estimates that ranged in average value from a minimum of 1.90 to a maximum of 33.58. This model showed poor estimation at lower reliability values (i.e., 0.30, 0.50) compared to the higher reliability values. However, as both sample size and the latent intercept-slope correlation increased the estimated values for this parameter at lower reliabilities became increasingly accurate, reducing in value from 33.58 at the lowest reliability/sample size/correlation condition to 2.27 in the lowest reliability/highest sample size/correlation condition. Further, as both reliability and latent intercept-slope correlation increased, the Wen model showed a tendency to underestimate the latent slope interaction parameter. The Duncan model showed poor estimation of the latent slope interaction parameter, with values ranging from 2.90 to 0.84. This model produced average values that predominantly underestimated the latent slope interaction parameter, doing so in 35 of the 36 conditions. The estimates for the Duncan model decreased in value towards zero as the factors of reliability, sample size, and latent intercept-slope correlation increased, and had a smaller range of values than the Wen model. The Schumacker model underestimated the population parameter across all conditions, with mean parameter estimates ranging from 1.37 to 1.87. For this model, the estimates improved and approached the population value as the reliability of the observed indicators increased, but there was no influence on the estimates by increases in either sample size or latent intercept-slope correlation. This model had the smallest range of all three models.

An alternative method to assess the accuracy of estimation is to tabulate those estimates that fell within a particular distance from the population value (e.g., +/- 2 standard deviations). To remain consistent to our earlier investigation of the model fit indices, we constructed 95% confidence intervals around each of the mean estimates, and tabulated the number of intervals that contained the population value of 2.0. For the Wen model, this occurred only 5 conditions, for the Duncan model 3, and for the Schumacker model only 4 of the conditions, and these conditions are presented in bold text in Table 14.

Insert Table 14 about here

MCAR Data Condition Analyses

The estimation of the latent slope interaction parameter in this data condition closely followed that of the Complete data condition, especially as the reliability of the observed indicators increased. The average values for the Wen model ranged from a low of 1.90 to a high of 45.54, those for the Duncan model from 0.84 to 4.63, and those for the Schumacker model from 1.38 to 1.87. The 95% confidence intervals presented in Table 14 show that the population value of the parameter was contained in 8 of the study conditions for the Wen model, 4 for the Duncan model, and 3 for the Schumacker model.

MNAR Data Condition Analyses

All three of the latent growth interaction models showed severe underestimation of the latent slope interaction parameter across all conditions. The Wen model had average estimates that ranged from 0.003 to 0.87; for the Duncan model they ranged from 0.11 to 0.15, and those for the Schumacker model ranged from 0.003 to 0.21. None of these models produced 95% confidence intervals of the estimate that included the population value of 2.0.

Summary

There were two distinct patterns of estimation of the latent slope interaction parameter.

Both the Duncan and Wen models showed a trend of increasing underestimation of the latent slope interaction parameter as the reliability of the observed indicators increased. Conversely, the Schumacker model showed increasing values of the latent intercept slope parameter as reliability increased, with a pattern of decreasing underestimation. Overall, in the Complete and MCAR data conditions, both the Wen and the Schumacker models showed better performance than the Duncan model, with the Schumacker model providing more consistent parameter estimates across all of the study conditions. With Complete data the Wen model produced outlier estimates when reliability was low, and the number of outliers rose as the correlation between the latent slope and intercept increased. The Duncan model also produced some outliers, doing so predominantly at the very lowest level of reliability, with the number of outliers decreasing as both sample size and latent intercept-slope correlation increased. The Schumacker model produced very few outliers. A similar pattern was also seen in the MCAR data. In the MNAR data condition none of the models performed in a satisfactory manner, with all of the models underestimating the latent slope interaction parameter.

Average Mean Square Error (MSE) and Average Standardized Bias in the Latent Slope Interaction Parameter Estimates when the Population Value of the Parameter was 2.0.

The average MSE values and the average standardized bias values for the estimate of the latent interaction slope parameter (γ_{28}) are given in Tables 15 and 16, respectively. For the MSE, lower values are indicative of less bias; for the standardized bias, values close to 1.0 are desirable, with values greater than 1.0 indicating overestimation and values lower than 1.0 indicating underestimation.

Insert Tables 15 and 16 about here

Average MSE Bias

Complete Data Condition Analyses

In the Complete Data condition, the average MSE values for the Wen model ranged from 5.90 to 78267.01; those for the Duncan model from 2.52 to 2096.34, and those for the Schumacker model from 2.43 to 82.13. The Wen model showed a general trend of decreasing MSE values as reliability, sample size, and latent intercept-slope correlation increased, with lower bias values at the highest levels of reliability across the factors of sample size and latent intercept-slope correlation. The only exception to this trend was at the smallest level of all three factors, where the average MSE value for the reliability level of 0.70 was higher than that for the level of 0.50. The Duncan model showed a similar pattern to that of the Wen model with respect to average MSE bias values, namely a decreasing trend as reliability, sample size, and latent intercept-slope correlation increased, with the largest MSE values occurring at the lowest level of reliability. However, the Duncan model consistently produced lower average MSE values than the Wen model across all conditions. The Schumacker model also showed a decreasing trend in average MSE bias values across the levels of reliability, sample size, and latent intercept-slope correlation. The Schumacker model produced average MSE values that were similar to those of the Duncan model at all but the lowest level of reliability, where its MSE values were lower than both the Wen and Duncan models.

The results for the average MSE bias values in the MCAR data condition (reported in Table 14) followed the same pattern as seen in the Complete data condition and, when compared to the average MSE values for the Complete data condition, at higher reliabilities the three models produced average MSE values that were very similar.

MNAR Data Condition Analyses

The average MSE values for all three latent interaction models in this data condition are given in Table 14. The average MSE values for the Wen model ranged from 3.21 to 29.06, those for the Duncan model from 3.49 to 3.61, and those for the Schumacker model ranged from 3.89 to 105.36. As none of the models performed poorly with respect to overall model fit and estimation of the latent slope interaction parameter, the MSE results are not discussed further for this data condition.

Summary

A comparison among the three models showed that the Schumacker model had the best performance with respect to values of MSE – this model produced values that were routinely lower than or similar to the other models when the data was Complete or MCAR, and in the MNAR condition the MSE values for this model were similar across all conditions, showing a high degree of stability. The Duncan model was comparable to the Schumacker model at moderate to high levels of reliability in each of the three data conditions, the exceptions being at low sample sizes and low reliabilities. The Wen model generally performed poorly compared to the other two models in the Complete and MCAR data conditions, especially across all conditions.

Average Standardized Bias Estimates

The average standardized bias estimates for each of the models across all of the conditions are given in Table 15. For this measure of bias, values close to 1 are desirable, and values that are less than 1 are indicative of underestimation and values greater than 1 are indicative of overestimation.

Complete Data Condition Analyses

The Wen model showed average bias values that generally decreased as the factors of reliability, sample size, and latent intercept-slope correlation increased, with bias values that ranged from -0.06 to over 1000. Exceptions to this decreasing pattern occurred at the highest level of the latent intercept-slope correlation, where the trend became one of lower bias values at lower reliabilities and higher bias values at higher reliabilities as sample size increased. The Duncan model showed a decreasing pattern of standardized bias values, indicating an increasing underestimation of the latent slope interaction parameter in almost all of the study conditions, with values ranging from -1.01 to 30.01. The only exception to this pattern was in the condition of latent intercept-slope correlation = 0.20, sample size = 1000, reliability = 0.90, where the value of the bias estimate was 0.75. The degree of underestimating bias increased as reliability increased, and bias also increased as there were increases in sample size.

The Schumacker model showed a pattern of increasing bias values, with values that indicated an increasingly lesser degree of underestimation as the study factors of latent intercept-slope correlation, sample size, and reliability increased. The values of bias ranged from -0.35 to 0.20. The exception to this pattern was in a single condition (latent intercept-slope correlation =

0.20, sample size = 500), where the bias value for a reliability of 0.70 was positive whereas the bias for the other reliabilities at this condition were all negative. The bias values were similar across the level of sample size, especially at higher reliabilities.

MCAR Data Condition Analyses

The overall pattern of results for the MCAR data followed those of the Complete data closely, with some exceptions. The Wen model showed average standardized bias values that ranged from -0.88 to over 1000, and showed a similar decreasing trend in bias values (indicative of biased estimates that ranged from overestimation to underestimation). The exception to this pattern again was at the highest level of latent intercept-slope correlation, where the pattern at the lowest sample size (n=250) was for increasing bias values (moving from severe underestimation to almost no underestimation), at the middle sample size (n=500) the pattern was non-linear (moved from moderate underestimation to severe overestimation to moderate underestimation), and to a flat pattern of moderate underestimation at the largest sample size (n=1000). The Duncan model had standardized bias values that ranged from -1.01 to over 1000. This model showed a pattern of decreasing standardized bias values as reliability increased, and this was seen across the levels of sample size and latent intercept-slope correlation. More specifically, the bias values indicated that at low reliabilities the Duncan model would overestimate the latent slope interaction parameter (i.e., show a positive bias), and that as reliability increased this model would underestimate the parameter (i.e., show a negative bias). The Schumacker model produced average standardized bias values that were similar to those for Complete data, with values ranging from -0.36 to 0.19. The pattern of bias was also similar to that seen with

Complete data, whereby as reliability increased the estimates moved towards a lower degree of bias. The exception to the pattern, whereby the bias increased at the highest level of reliability, was seen at the largest sample size (n=1000) and the lowest latent intercept-slope correlation (0.20), where the bias value for the reliability level of 0.70 was positive (0.19), whereas at other reliabilities the bias value was negative.

MNAR Data Condition Analyses

As noted previously, due to the poor performance of all three of the models in this condition, the results of the standardized bias values are not discussed in detail. In brief, the Wen model produced standardized bias values that ranged from less than -1000 to greater than 1000, and there was no consistent pattern that emerged across all of the study conditions. The Duncan model produced average standardized bias values that ranged from -20.26 to -5.68, and showed a similar trend to that seen in the Complete and MCAR data conditions, with steadily increasing values of underestimating bias as reliability increased. The Schumacker model had average standardized bias values that ranged from -33.64 to -3.87, and the pattern was predominantly one of decreasing bias as reliability increased, across the factors of sample size and latent intercept-slope correlation.

Summary

All three of the latent growth interaction models tended to show an underestimating bias in the estimate of the latent slope interaction parameter according to this index of bias. The Wen model showed a smaller degree of bias in the estimation of the latent slope interaction parameter compared to the Duncan and Schumacker models, across both the Complete and MCAR data conditions, but this only occurred at the highest levels of reliability. At other levels of reliability the Duncan and Schumacker models showed modest amounts of underestimating bias, whereas the Wen model showed large amounts of bias. In the MNAR data condition the Duncan model showed the smallest degree of bias at lower reliabilities, and the Schumacker model showed smaller degrees of bias at higher reliabilities. The Schumacker model was the only model to show a decrease in bias as the factors of reliability, sample size, and latent intercept-slope correlation increased, and this was seen across all three data conditions.

Type I Error Rates

For this analysis, all of the models were re-run with the population value of the latent slope interaction parameter set equal to zero. Table 17 provides a tabulation of the proportion of cases (with frequencies in parentheses) where the absolute value of the estimate of the latent slope interaction parameter was significantly different from zero. Bradley's (1978) liberal criterion of robustness was used to evaluate the Type I error rate for each of the models. Using a 5% level of significance, Type I error rates outside of the range of (0.25, 0.75) are considered to be conservative or liberal, respectively. Type I error rates were examined separately for each missing data condition. In Table 16 those conditions with a liberal Type I error rate are presented in bold text, and those conditions with a conservative Type I error rate are presented in underlined text.

Insert Table 17 about here

Complete Data Condition Analyses

Out of the 36 study conditions, the Wen model showed Type I error rates that were liberal (over 7.5%) in 26 of the conditions, with adequate Type I error control in only 7 of the conditions. The liberal error rates were consistently associated with all but the highest level of reliability (0.90). As the sample size increased the amount of liberal rates increased, but as the latent intercept-slope correlation increased the rates decreased (i.e., the number of liberal rates for a sample size of 250 were higher at a correlation value of 0.20 than at the correlation value of 0.50). The Duncan model was liberal in a single condition, showed conservative Type I error rates (i.e., under 2.5%) in 22 conditions, and adequate Type I error control in 13 conditions. The conservative rates occurred at the highest levels of reliability (0.70, 0.90) across the factors of sample size and latent intercept-slope correlation. The rates at the combination of lower latent intercept-slope correlation and reliability were the highest among all conditions for this model. The Schumacker model was conservative in 18 conditions, and showed adequate control in the remaining 18 conditions. There was no consistent pattern to the presence of conservative conditions.

MCAR Data Condition Analyses

The pattern of Type I error rates for the Wen model were similar to that seen in the Complete Data condition. Type I error rates for this model were liberal in 25 of the conditions and adequately controlled in the remaining 11 conditions. As in the Complete data condition, the majority of the liberal rates occurred at the lowest reliabilities, and adequate Type I error control was seen only at the highest level of reliability across the levels of sample size and latent intercept-slope correlation. The rates of Type I error were higher at lower levels of the latent intercept-slope correlation, and also increased with sample size.

For the Duncan model, Type I error rates were liberal in 2 of the conditions, conservative in 16 of the conditions, and adequately controlled in the remaining 18 conditions. A similar pattern to that seen in the Complete data condition emerged, with almost all of the conservative error rates occurring when the reliability ranged from 0.70 to 0.90, across all of the latent intercept-slope correlation and sample size conditions. The rates at the combination of lower latent intercept-slope correlation and reliability were the highest among all conditions for this model.

The Schumacker model showed Type I error rates that were conservative in 17 of the conditions and adequately controlled in the remaining 19 conditions. There was no consistent pattern to the rates of Type I error.

MNAR Data Condition Analyses

The Wen model showed Type I error rates that were liberal in 7 conditions, conservative in 23, and only adequately controlled in 6 of the conditions. The presence of liberal Type I error rates was associated with the lowest level of reliability across the factors of sample size and latent intercept-slope correlation for this model. As sample size increased there was a strong tendency towards conservative Type I error rates.

The Duncan model was conservative in 33 of the conditions, showing adequate control in only 3 of the study conditions. Those conditions that were well-controlled were at the middle level of reliability (0.50) at the lowest level of latent intercept-slope correlation.

The Schumacker model was liberal in 11 conditions, and showed adequate Type I error rate control in the remaining 25 conditions. The liberal rates were associated with the largest sample size condition (n=1000), and for all of the reliabilities except the lowest (0.30).

Summary

The Schumacker showed the best control of Type I error rates of the three models, showing a rough split of adequate and conservative control in both the Complete and MCAR data conditions, and showing adequate control in the majority of conditions with MNAR data. The Wen model was largely liberal with respect to Type I error control when the data were Complete or MCAR, but was conservative in almost two-thirds of the conditions when the data was MNAR. The Duncan model had a more variable pattern, with a slightly higher rate of conservative control in the Complete data condition, a more even split between conservative and adequate control in the MCAR data condition, and almost entirely conservative in the MNAR data condition.

Chapter 13. Discussion

There were two objectives to the current project. The first objective was to investigate the performance of three latent interaction growth models by Wen et al. (2002), Duncan et al., (1999), and an extension of the Schumacker (2002) cross-sectional model, under several varying conditions: the correlation between the latent intercept and slope for each latent factor, the sample size, and the reliability of the observed indicators for the latent factors, with respect to three types of missing data. The performance of these three models were evaluated with respect to convergence rates over 5000 replications and overall model fit on several commonly-used model fit indices (CFI, NFI, GFI, RMSEA).

The second objective was to examine the performance of each of the three latent interaction growth models with respect to bias in the estimation of the latent slope interaction parameter (γ_{28} in Figure 15), and included an examination of the amount of bias in estimating the latent slope interaction parameter (i.e., absolute bias, mean square error, and an estimate of standardized bias), and an evaluation of each of the three latent interaction growth models with respect to the rates of Type I error for the latent interaction slope parameter.

A particular challenge for these models is that the presence of an interaction term introduces non-normality into the data, either in the form of cross-products of the observed indicators as in the case of the Wen and Duncan models, or in the form of a multiplication of factor scores as in the case of the Schumacker model. This non-normality can introduce bias into the model fit indices and the parameter estimates.

Overall Model Performance

Model Convergence

Convergence occurs when the estimation function has reached a minimum and the model parameters are estimated (Tabachnick & Fidell, 2001). The Wen model showed minor problems of convergence when the data was either Complete or MCAR, occurring primarily when the reliability of the observed indicators were at their lowest levels (i.e., 0.30 or 0.50), and this was regardless of the value of the latent intercept-slope correlation or sample size. However, in these instances, the Wen model still converged in 93% of the 5000 replications. In the MNAR data condition the Wen model showed poor convergence across many conditions, sometimes converging in less than 30% of the replications for a given condition, the only exceptions being when the reliability of the observed indicators were at their highest level (0.90). Both the Duncan and the Schumacker models converged for nearly all 5000 replications in the 36 conditions across all three of the missing data conditions. The only exception was for the Duncan model in the specific MNAR data condition where the latent intercept-slope correlation was at its lowest value (0.20), the sample size was smallest (250), and when the reliability of the observed indicators was at its lowest value (0.30).

Nonconvergence can indicate that the model does not fit the data, but it does not provide information regarding the rejection or non-rejection of any hypotheses concerning specific model parameters, and as such provides limited diagnostic information. Convergence of a model to a proper solution typically fails if the sample size is small, and failure to converge occurs more often with maximum likelihood estimation than with generalized least squares estimation (Anderson & Gerbing, 1984; Fan et al., 1998) in the case of complete data. Newman (2003) found that the use of maximum likelihood estimation in structural models did not result in convergence problems with MCAR or MNAR data if there was less than 50% missingness. Convergence rates do increase if there are more time points assessed in a longitudinal design (Hamilton, Gagne, & Hancock, 2003). When this was investigated by Fan and Fan (2005), they found that with only three repeated assessment points non-convergence occurred frequently, especially if there is little growth over those three assessment points.

In the current study, convergence problems occurred consistently for one model in particular, the Wen model. Nonconvergence for this model occurred primarily when the reliability of the observed indicators was low in both the Complete and MCAR data conditions, which is consistent with previous research. The high rate of nonconvergence with the MNAR data for this model goes against the findings of Newman (2003). Since sample sizes were quite high in some conditions, there were more than three repeated assessment points, and the rate of missing data was not greater than 50%, other factors may be contributing to the poor convergence rates seen with the Wen model compared to the other two models. Some of these issues may involve the presence of the interaction effects, which introduce non-normality into the observed data. A second potential explanation may lie within the differences between the Duncan and the Wen models. The Duncan model was the population model for generating the data, and the Wen model represents a misspecification of the model as it contains paths that are not present in the Duncan model (see Figures 12 and 13). A novel finding from this study is that convergence rates for the Duncan and Schumacker models using ML estimation were high and appeared to not be affected by the value of the latent intercept-slope correlation, sample size, or by the reliability of the observed indicators, a finding which has not been reported elsewhere.

Further to this, neither the Duncan nor the Schumacker model had the poor rates of convergence in the MNAR data condition as the Wen model, which leads to two avenues of exploration: what characteristics of these two models makes them have stable convergence rates in the presence of MNAR data, and what about the Wen model makes it susceptible to convergence problems, especially in the presence of MNAR data.

In summary, if longitudinal data are Complete or MCAR, convergence of any of the three latent growth models assessed in the current project is not a troublesome issue across conditions of latent intercept-slope correlation, sample size, or reliability of the observed indicators, with convergence rates above 93% for all of the models under these conditions. Further, the Duncan and Schumacker models showed better convergence rates than the Wen model, converging in nearly 100% of the simulations. If the data are MNAR, either the Duncan or the Schumacker model are the most reliable with respect to converging to a solution, while the Wen model is problematic. It should be cautioned that convergence of a model does not automatically imply that the overall model fit indices and parameter estimates are correct, only that estimates of these values have been achieved.

CFI Values

The CFI (Bentler, 1988) ranges in value from 0 to 1.0 and assesses fit relative to a null model, with values between 0.90 and 1.0 being indicative of models that have a better fit to the data. The Wen and Duncan models showed similar performance across all factors in both the Complete and MCAR data conditions, with both models producing CFI values that were above the cutoff of 0.90 for good model fit in many of the conditions. At high sample sizes and moderate to high correlations between latent intercepts and slopes the Schumacker model showed rising average CFI values as reliability of the observed indicators increased, however the value of the CFI in these conditions was lower than that seen when the latent intercept-slope correlation was at its lowest value. For all three of the models, the trend in both the Complete and the MCAR data conditions was that as reliability of the observed indicators increased there were increases in the mean CFI values, especially for the Schumacker model; the increases seen in the Wen and Duncan models were incremental.

These findings are further bolstered by the evidence of the 95% confidence intervals for these mean CFI values. Both the Wen and Duncan models produced confidence intervals whose lower bound was at or above the model cut-off of 0.90, with the Wen model producing a higher proportion of such intervals across both the Complete and MCAR data conditions. The implication of this, when considered in conjunction with the preponderance of mean CFI values that were above the cut-off for good model fit, is that when a model such as the Wen full interaction model produces a CFI value that indicates good fit to the data, the researcher can have a high degree of confidence that such a result is not a spurious finding.

The presence of meaningful (i.e., having at least a medium effect size) two-way interactions of (a) latent growth interaction model with reliability and (b) latent growth interaction model with correlation between latent intercept and slope was noted in both the Complete and MCAR data conditions. For both types of interactions, the Wen and Ducnan models showed stable patterns of CFI values across the levels of reliability and correlation. For (a) the Schumacker model showed a trend of increasing CFI values as reliability increased, and for (b) this model showed a trend of decreasing CFI vales as the correlation increased. These findings imply that the largest influences on a model fit index like the CFI are the type of model, the reliability of the observed indicators, and the correlation between the latent intercept and slope, and a factor such as sample size has very limited influence. In the MNAR data condition, none of the three latent interaction growth models produced average CFI values that were above the cutoff of 0.90 for good model fit.

Previous authors have noted that the CFI is susceptible to particular facets of study design. The CFI is susceptible to the type of estimation method used and shows a downward bias when the model is incorrectly specified (Sugawara & MacCallum, 1993), and this was confirmed by Fan, Thompson, and Wang (1999) with both ML and GLS estimation. The CFI is not affected by nonnormality of the data, nor by sample size (Fan et al., 1998), although these authors only studied minor degrees of non-normality in the observed variables (i.e., skewness of ± 1.5, kurtosis between +3 and +4). Hu and Bentler (1998) examined more severe non-normality (e.g., kurtosis values of -1, 2, 5, and elliptical distributions of the data), and found that this accounted for a small proportion of variance in model fit with ML estimation.

These findings from the current study contradict what has been reported in previous research, as the CFI did not show a downward bias for the Wen model, which is a misspecification of the population model used in this study. As the Schumacker model was not comparable in this manner to either the Wen or Duncan models, it is difficult to determine if the CFI model fit index was performing as expected for this model. However, the CFI did show an increase in value as sample size increased for the Schumacker model, which is similar to that seen in previous research.

NFI Values

The NFI compares the specified model against an independence model and, similar to the CFI, ranges in value from 0 to 1.0 and uses the same cutoff of 0.90 for indicating good fit to the data by the model. For the three latent growth interaction models studied in the current project, the results for the NFI were very similar to those of the CFI. The Wen model produced average NFI values that were above the cutoff for good model fit in a higher proportion of conditions than the Duncan and Schumacker models in both the Complete and MCAR data conditions. The Duncan model produced average NFI values that were consistently lower than those of the Wen model. The Schumacker model produced average NFI values that were lower than those of the other two models at low levels of reliability, and which never approached the cutoff for good model fit. For the MNAR data condition, none of the models produced average NFI values that were above the cutoff for good model fit.

Similar to the findings with the CFI, the NFI values showed slight increases as sample size increased, which is consistent with previous research. The performance of the NFI with respect to measurement error (i.e., reliability of the observed indicators) and missing data has not been studied previously. While there was a significant effect associated with the measurement error factor, this was most likely due to the variation in average values between the three models across the three data conditions. The Wen and Duncan models had stable average NFI values across all manipulated conditions in the Complete and MCAR data, while the Schumacker model showed an increasing trend in these two data conditions. In the presence of MNAR data each of the models showed a different pattern: the Wen model showed a decreasing pattern of values, the Duncan model an increasing pattern, and the Schumacker model showed a stable pattern. The similarity of performance for the NFI with respect to missing data type also represents a new finding, with the NFI performing in the MCAR data similarly to that seen in the Complete data condition, and the poor performance in the MNAR data condition. This result confirms and extends the findings from previous studies on other fit indices to that of the NFI.

These findings are further bolstered by the supporting evidence of the 95% confidence intervals for the mean NFI values. Both the Wen and Duncan models produced confidence intervals whose lower bound was at or above the model cut-off of 0.90, with the Wen model producing a higher proportion of such intervals across both the Complete and MCAR data conditions (22 vs 15 and 19 vs 9, respectively). The implication of this, when considered in conjunction with the preponderance of mean NFI values that were above the cut-off for good model fit, is that when a model such as the Wen full interaction model produces an NFI value that indicates good fit to the data, the researcher can have a high degree of confidence that such a result is not a spurious finding.

Similar to the findings with the CFI, the largest effect sizes were seen with the two-way interactions of (a) latent interaction model and reliability of the observed indicators, and (b) latent interaction model with correlation between the latent intercept and slope. These large effect sizes were seen with the same two-way interactions as with the CFI, and the pattern of NFI values in the corresponding simple plots were similar as well. This implies that the largest influences on a model fit index like the NFI are the type of model, the reliability of the observed indicators, and the size of the latent intercept-slope correlation, and a factor such as sample size has very limited influence.

The NFI is sensitive to the type of model that is being analyzed, with both nested confirmatory factor analysis models and structural models with misspecified paths leading to unreliable NFI values for comparison across models (Fan & Sivo, 2007). The NFI statistic was designed to be free of the influences of sample size as well as to provide accurate model fit assessment across several estimation methods (e.g., ML, GLS; Bentler et al., 1980). However, the NFI has been shown to be affected by sample size (Bearden, Sharma, & Teel, 1982; McDonald & Marsh, 1990), with larger values as sample size increases and smaller values as sample size decreases when used with ML estimation (Fan et al., 1998; La Du & Tanaka, 1989). Marsh et al. (1988) found that the NFI is not affected by non-normality of the data. La Du and Tanaka (1989) found that this model fit index was insensitive to model misspecification when a ULS estimation method was used.

GFI Values

The GFI is analogous to the R^2 in regression, and assesses the degree to which the reproduced covariance matrix based on the specific model has accounted for the original sample covariance matrix (Tanaka et al., 1985). Values of the GFI which are closer to 1.0 are desirable, with a value of 0.90 being recommended as a cutoff value for good model fit. The GFI model fit index is sensitive to sample size when used with ML estimation (Anderson et al., 1984; Marsh, Balla, & McDonald, 1988), with increasing GFI values as sample size increases (La Du et al., 1989), and a downward bias as sample size decreases. Fan and Wang (1998) have found that the GFI is not affected by non-normality of the data.

In the current study, only the Schumacker model showed average GFI values that were above the cutoff of 0.90 for good model fit, and this was only in one-third of the cases in the MNAR data condition. In the Complete and MCAR data conditions, even though none of the models showed average GFI values that were above the cutoff, the Wen and Duncan models showed stability in their values across all of the manipulated factors, while the Schumacker model showed a trend of rising average values as reliability increased. In the MNAR data condition, the Wen model continued to show a stable pattern of average GFI values, generally between 0.70 and 0.80. The Duncan model showed a decreasing pattern, with values decreasing from 0.85 to 0.74 across all of the conditions. As noted earlier, only the Schumacker model produced average GFI values above the cutoff in this condition, and this same model showed an increasing pattern of average values as the reliability of the observed indicators increased. The positive results of the Schumacker model in the MNAR data should be tempered with the evidence from the 95% confidence intervals for the GFI values. There were only two conditions in the MNAR data where the lower bound of the confidence interval was at or above the cutoff for good model fit. The implication of this is that a GFI value produced by the Schumacker model that indicates adequate fit to the data may not be the typical GFI value obtained, and it is likely that a GFI value that indicates poor fit to the data would also have been obtained.

The findings reported for the current study with respect to the GFI show some consistency with previous research with respect to the factor of sample size. To this author's current knowledge, there have been few studies that have looked at influences of either the latent intercept-slope correlation or the reliability of the observed indicators on the GFI. This study presents novel findings with respect to these factors and their influence on the GFI statistic. For both the Wen and the Duncan models the value of the latent intercept-slope correlation has little impact on the GFI, and this was seen across both the Complete and MCAR data conditions. For the Schumacker model there were slight decreases in the average GFI values as the latent intercept-slope value increased across both the Complete and MCAR data conditions. As noted earlier, a potential reason for this decrease in GFI values may be due to the presence of multicollinearity introduced by the increasing value of the correlation. In the MNAR data condition, there were no differences at the two lowest levels of correlation (0.20, 0.50), but lower values were seen at the highest level (0.70), for the Wen and Schumacker models only. These differences were slight.

With respect to the reliability of the observed indicators, this factor was also seen to have little impact on the average GFI values produced by the Wen and Duncan models in both the Complete and MCAR data conditions. In the MNAR data condition, this same pattern held for the Wen model only, while the Duncan model showed a decreasing pattern of average GFI values as reliability increased. For the Schumacker model, across all three data conditions, the reliability of the observed indicators had the strongest influence, and the influence of the latent intercept-slope correlation and sample size were minimal.

If the Duncan model can be considered the properly specified "correct model", the average GFI indices for this model in particular should have been well above the cutoff required for good model fit to the data. That this did not occur is evidence that more research on the utility of this statistic is needed, especially in the context of latent growth modeling. The GFI also utilizes the non-central chi-square distribution as part of its computation (Curran et al., 2002), and some of the concerns noted previously with respect to the RMSEA may also be applicable to the GFI.

RMSEA Values

The RMSEA was proposed by Steiger and Lind (1980), developed further by Browne and Cudeck (1993), and is a test of exact fit. Values of the RMSEA that are lower than 0.05 are considered to represent a model that fits the data well, with values up to 0.08 also being indicative of a well-fitting model. None of the three latent growth interaction models showed average RMSEA values that were below the minimum criteria of 0.05 (nor of 0.08) used in this study for adequate model fit across any of the three data conditions. Despite this lack of positive findings, some conclusions regarding the performance of the models can be presented. The Wen and Duncan models showed a trend of increasing average RMSEA values (indicating an increasing degree of poor fit to the data) as there were increases in the latent intercept-slope correlation, sample size, and reliability of the observed indicators, in all three of the data conditions. Conversely, the Schumacker model showed a decreasing pattern of average RMSEA values (indicating an increasing degree of good fit to the data) across all of the manipulated factors for all of the missing data conditions.

The RMSEA is sensitive to model misspecification (Fan et al., 2007), which is actually an advantage since an index that identifies misspecified models is desirable. The RMSEA has shown itself to be insensitive to sample size, and is not affected by the nonnormality of the data (Fan et al., 1998). However, Nevitt and Hancock (2004) found that there was an interaction between sample size and non-normality of the data. At large sample sizes there was no influence of non-normality on the RMSEA, and at more moderate sample sizes (e.g., n=500) the RMSEA was affected by non-normality, yielding reduced power. In general, Nevitt and Hancock (2000) found that, if the model was misspecified, the RMSEA did not seem to be affected by changes in sample size, but increasing non-normality led to increases in the RMSEA values, which can be exacerbated by decreasing sample size.

These previously reported findings were largely corroborated in the current study for the Wen and Duncan models, as these models showed average RMSEA values that consistently increased across the rising levels of reliability, sample size, and latent intercept-slope correlation. What is surprising is that the Duncan model, which can be considered a correctly specified model, showed performance that was similar to that of the Wen model (a misspecified model) and that none of the average RMSEA values for the Duncan model approached the criterion for good model fit.

There are several possibilities for why this unexpected relationship occurred. Previous authors (e.g., Fan et al., 1999) have shown that the RMSEA can be overestimated at small sample sizes even if the model is properly specified, and also that the value of the RMSEA can be inflated if there is multivariate non-normality present in the data. Due to the aforementioned inclusion of interaction terms in the models, which introduce non-normality into the data, this could be contributing to the poor performance of the RMSEA in the current study. Curran, Bollen, Paxton, Kirby, and Chen (2002) have noted that, in the presence of uncorrelated variables in structural equation models the distribution of the model chi-square, part of the formula for the RMSEA (see page 79), may not follow a non-central chi-square distribution. This is problematic for fit indices such as the RMSEA, which are assessed based on the assumption that this model chi-square follows a non-central chi-square distribution. If this assumption is not met then the RMSEA as an statistic of overall model fit may not be suitable. In the current study, the two latent independent factors were specified as being uncorrelated, and this may have led to the situation of the model chi-square not following the proper distribution.

The Wen and the Duncan models use the product-indicator method for the interaction effect, which contains a procedure for mean centering that reduces the multicollinearity that is introduced by the creation of the product terms from the individual observed indicators. However, since the nature of the interaction terms used in both the Wen and Duncan models produces non-normal data, the high values of the RMSEA may be a result of the presence of these interaction terms (i.e., even though multicollinearity is reduced, the influence of non-normal data may not be entirely controlled by centering). The Schumacker model had substantially higher RMSEA values than either of these models across all of the study conditions. The Schumacker model does not explicitly outline a method for reducing multicollinearity, and this may have further contributed to the poor performance of the Schumacker model with respect to non-normal data and mean values of the RMSEA. However, this finding is mitigated to some degree by the fact that the Schumacker model showed increasingly good fit to the data as the study factors increased. The finding that the correlation between the latent intercept and slope did not adversely affect the average RMSEA values for any of the models is a novel result.

Bias in the Unstandardized Parameter Estimate for the Latent Slope Interaction Parameter (γ_{28}) with a Population Value of 2.0

A secondary objective was to evaluate the accuracy of each of the three latent growth interaction models in estimating the unstandardized structural parameter for the effect of the latent slope interaction on the latent slope of the outcome variable (i.e., γ_{28} in Figure 16). Accuracy in parameter estimation is one of the important goals of structural equation modeling (Fan & Wang, 1998). When measurement error is present, bias is introduced into the parameter estimates, power to detect differences is lowered, and inference becomes problematic (DeShon, Ployhart, & Sacco, 1998).

The first method used to assess the accuracy in parameter estimation was to examine the difference in the estimated parameter from the population value of 2.0. In the Complete data condition, the Wen model showed parameter estimates that closely approximated the population value as reliability increased, with values above the population value at low to moderate levels of reliability (i.e., 0.30, 0.50, 0.70), and values below the population value at the highest level of reliability (0.90). Further, the estimate approached the population value as both the latent intercept-slope correlation and sample size increased, however this effect was more apparent at estimates associated with lower levels of reliability. The Duncan model consistently produced estimates that were below the population value across almost all study conditions, and these estimates became more distant as reliability increased. Similar to the Wen model, as the latent intercept-slope correlation and sample size increased, the deviation from the population value decreased, especially at lower levels of reliability. The Schumacker model was consistent in its estimation of the population value of the latent slope interaction parameter, with estimates increasing towards the population value as the correlation between the latent intercept and slope increased and as the reliability of the observed indicators increased. An examination of 95% confidence intervals around the estimates showed that very few of them included the population value of zero, indicating that the models were not estimating the population parameter very accurately.

Sample size appeared to have very little influence on the estimates provided by the Schumacker model. In the MCAR condition findings were similar to those seen in the Complete data condition, with the exception that at lower reliabilities the inaccuracy in estimation was greater, especially for the Wen and Duncan models. In the MNAR condition all of the models provided inaccurate estimates, consistently estimating the latent slope interaction parameter to be close to zero.

The second facet of parameter estimation examined was the amount of bias in the parameter estimates. The degree of bias in the estimate of the latent slope interaction parameter was assessed by two methods, the mean square error (MSE) and the standardized bias. The MSE quantifies the expected (average) squared deviation of an estimator from a population parameter (Olejnik & Porter, 1981), and provides information on the spread of the parameter estimates around the true estimate. For all of the models, across all three data conditions, the values of the MSE consistently decreased as sample size and reliability increased, which is a desirable property. In other words, as reliability and sample size increased, the amount of deviation of the estimates from the population value decreased. The Schumacker model consistently had the lowest MSE values except in the MNAR data condition, indicating less bias in the estimated latent slope interaction parameter when data was Complete or MCAR (this was also reflected in the unstandardized parameter estimates for the Schumacker model). The Duncan model produced MSE values that were slightly higher than those for the Schumacker model at the highest value of the latent intercept-slope correlation. The Wen model was consistent in producing the largest MSE values across the Complete and MCAR data conditions, but showed similar MSE values to the other models as reliability increased in the MNAR data condition.

For all three of the latent growth interaction models the average MSE values were largest at the lowest levels of the reliability of the observed indicators, which is not surprising since the degree of reliability has an effect on the amount of error variance of the parameter estimates, with lower reliabilities yielding larger error variances (see Equation 33). In general, the combination of a low intercept-slope correlation, small sample size, and low reliability produced inaccurate parameter estimates and a substantial amount of bias for all three models in the Complete and MCAR data conditions. All of the models performed similarly in the MNAR condition, producing low MSE values in almost all conditions, however as noted earlier all of the models were poor in estimating the latent slope interaction parameter in this data condition.

The last measure of bias examined was the standardized estimate of bias, which is the average deviation of the sample estimate from the population parameter estimate, divided by the empirical standard error of the estimate. Values of this standardized estimate which are close to 1.0 indicate no bias in the estimate, those above 1.0 are indicative of an overestimating bias, and those less than 1.0 are indicative of an underestimating bias. The standardized bias for the latent slope interaction parameter estimates showed a pattern very similar to that seen with the actual estimates of the latent slope interaction parameter itself, with the Wen model showing consistent overestimation in the Complete and MCAR data conditions, the Duncan model showing consistent underestimation, and the Schumacker model showing slight underestimation in some conditions but providing more accurate estimates in other conditions. The degree of standardized bias in the MNAR data was large for the Wen and Duncan models, both of which showed an increasing underestimating bias as the study factors increased. The Schumacker model showed a large degree of underestimating bias, with a trend of decreasing amounts of bias as the reliability of the observed indicators increased – in other words, the degree of bias lessened for this model as the reliability of the observed indicators increased. Increases in bias were seen as both sample size and the correlation between latent intercepts and slopes increased.

Overall, the Schumacker model was consistent in the bias values that it provided, especially at higher reliabilities. The Wen model showed a large amount of variability in its bias values, to such a degree as to make the trustworthiness of its estimates questionable. The Duncan model, while providing biased estimates, was not as biased as the Wen model and at times provided bias estimates that were similar to those of the Schumacker model. However, all of the models were biased to some degree.

Type I Error Rates

The last facet of the parameter estimate that was assessed was Type I error rates in detecting the latent slope interaction parameter. For this facet, all of the simulations were re-run with the population parameter of the latent slope interaction set to zero. For each model, the p-value for the test of the latent slope interaction parameter was evaluated to determine if it was within the acceptable range (i.e., 0.025 to 0.075), or if it was liberal (i.e., greater than 0.075) or conservative (i.e., less than 0.025). All of the models were poor in their control of Type I error rate, with the Schumacker model showing adequate control in only 14 of 36 conditions when the data was Complete (15 when MCAR), and the Wen and the Duncan models being predominantly liberal and conservative, respectively.

Assessment of Objectives and Hypotheses

The first objective of this study was to evaluate the overall performance of the models in each of the three missing data conditions with respect to model fit. It was our expectation that particular trends in the overall model fit indices would emerge. Specifically, it was expected that the Wen would show superior performance on the overall model fit indices compared to both the Duncan and Schumacker models. Further, it was expected that the Schumacker model would perform as good or worse than the Duncan model. This was based on its similarity to the Duncan model (i.e., only having a latent interaction term for the slopes, and not for any of the other latent factors as represented in the Wen model).

Relative to the other two models, the Wen model showed the best performance with respect to overall model fit, providing fit values that were indicative of a good-fitting model for both the CFI and NFI when the data was either Complete or MCAR. However, none of the models were satisfactory in their performance on the other two indices of model fit used (the GFI and RMSEA) in these two data conditions. Further, when the data was MNAR, all of the models performed poorly.

The secondary objective was an evaluation of each of the models for their ability to estimate the latent slope interaction parameter across the three missing data conditions. Specific aspects that were examined were the amount of bias in the parameter estimate and the rate of

Type I error. To recap, there were two hypotheses generated with respect to bias and Type I error:

Hypothesis 1:

Null: All three models will show similar bias in the estimation of the unstandardized latent slope interaction parameter, as evidenced by similar average mean squared error (MSE) values and standardized bias values, across all study conditions.

Alternative: The Wen model will show a lesser degree of bias than both the Duncan and Schumacker models, and the Duncan model will show an equal or lesser degree of bias than the Schumacker model, across all study conditions. Specifically, the ordering of values for the MSE and standardized bias will be Wen < Duncan ≤ Schumacker.

Hypothesis 2:

Null: All three models will be similarly effective at controlling the rate of Type I error at the nominal level of significance of $\alpha = 0.05$ (using Bradley's criterion), across all study conditions.

Alternative: The Wen model will be provide adequate control of Type I error (i.e., maintaining a Type I error rate close to the nominal level of significance of $\alpha = 0.05$ using Bradley's criterion) than both the Duncan and Schumacker models, across all study conditions. In other words, the Wen model will falsely detect the presence of the latent slope interaction

Hypothesis 1 was marginally supported. The Wen model did not produce the lowest bias values of the three models, as evidenced by lower average MSE values and standardized bias values close to 1.0, than the Duncan model, across all study conditions. With respect to the MSE, the values for the Duncan model were significantly smaller than those for the Wen model in all but two conditions for the Complete data. When the data was MCAR there were a majority of conditions where the Wen model showed larger MSE values than the Duncan model, and multiple comparisons showed more conditions of non-significant differences between these two models (however, the variability in the estimates of the bias were quite large). In the MNAR data condition the Duncan model showed lower average MSE values than the Wen model at all but the highest levels of reliability. However, it should be noted that neither of these models performed particularly well in providing an accurate estimate of the population parameter.

The second half of the first hypothesis, which proposed that the Schumacker model would show a level of bias that was similar or worse than the Duncan model, was not supported when the data were Complete or MCAR, and only partially supported when the data was MNAR. With Complete and MCAR data the Schumacker model had average MSE values that were lower than the Wen model in all conditions, and which were similar or lower than those of the Duncan model. In these two data conditions, the standardized bias values for the Schumacker model indicated that it was more accurately estimating the population estimate than either the Wen or Duncan models. When the data was MNAR the Schumacker model showed average MSE values that were similar to both the Wen and the Duncan model. Similar to the MSE values in the Complete and MCAR data, the standardized bias values for the Schumacker model in MNAR data indicated that it was more accurately estimating the latent slope interaction parameter than either the Wen or Duncan models. Further, it was the only model to show a trend of lesser bias as the reliability of the observed indicators increased.

The second hypothesis was not supported. Both the Wen and the Duncan model had similar rates of adequate Type I error control when the data was Complete, and the Duncan model showed a higher rate of error control when the data was MCAR. In both of these data conditions the Wen model was largely biased towards a more liberal rate of Type I error control while the Duncan model was biased towards a conservative rate of error control. In the MNAR data condition both of these models showed conservative Type I error rates in the majority of conditions studied. The Wen model showed control of Type I error rate in fewer than 1/3 of the study conditions across the three types of missing data, and the Duncan model showed Type I error control in 1/3 of the conditions in the Complete data and ½ of the conditions in the MCAR data. In the MNAR data both of these models showed Type I error control in fewer than 1/3 of the 36 study conditions.

The second part of this hypothesis proposed that the Schumacker model would perform similar to or worse than the Duncan model with respect to controlling Type I error. This was not seen in the current results, as the Schumacker model was better than both the Wen and the Duncan models in controlling the rate of Type I error across the three data conditions. The Schumacker model showed Type I error control that was split between being adequately controlled or being conservative when the data was either Complete or MCAR, and controlled Type I error rates in 25 of the 36 conditions when the data was MNAR. However, in the MNAR data all of the parameter estimates for the latent slope interaction were close to zero, so the impact of this finding is lessened.

A question that can be addressed with the results of these two hypotheses is "Does one of the models provide a more accurate estimate of the latent slope interaction parameter than the others?" The Schumacker model appears to be the model that provides the most accurate estimate of this parameter, with estimated values that were qualitatively close to the population value, and had a small amount of variability compared to the Wen and Duncan models, and this was seen in the Complete and MCAR data conditions only. The Wen and Duncan models showed large amounts of variability when reliability of the observed indicators was low, but this was reduced to levels comparable to those of the Schumacker at higher levels of reliability. The Duncan model outperformed the Wen model with respect to MSE. When the data was MNAR, none of the models showed adequate estimation of this parameter, although MSE values were low for all models in this data condition.

It is not uncommon to have missing data in longitudinal studies, and statistical inferences are improved if the effect of missing data has been taken into account (Song & Lee, 2003). The attrition from multivariate (e.g., longitudinal) studies can lead to large standard errors in parameter estimates due to non-response being compounded across the waves of data collection to produce small longitudinal data sizes (Newman, 2003). If the missing data is due to a non-random mechanism, this can further bias model parameters and lead to both misspecification and misestimation of the model (Chan, 1998; Muthen et al., 1987). As the percentage of missing data approaches 15-20%, the choice of estimation used with missing data can have implications for the parameter estimates (Roth, 1994). Taken together, the finding that all of the models performed poorly with respect to both the overall model fit indices and the latent slope interaction estimate when the missing data mechanism was MNAR is not surprising.

Bias in the parameter estimates can also come from other sources. Curran (2003) has noted that parameter estimates in structural equation models are biased if the assumption of independence of observations is violated. Given that the framework used in the current project is that of a longitudinal design, this assumption is violated with latent growth curves and some bias in the parameter estimates is expected. The current study provides evidence that bias in the parameter estimates of the latent growth structural model, in particular that of the latent interaction of two slope factors, can be affected by the value of the correlation of the latent intercept and slope, sample size, and reliability, especially in those models that use the product-indicator approach to form interaction effects. The Schumacker model, which does not use the product-indicator method to form the latent slope interaction effect, showed consistent estimation across all conditions regardless of whether the data was Complete, MCAR, or MNAR, which would indicate that this model is preferable to the other two models in obtaining accurate parameter estimates in latent growth models that involve interaction effects. Further, the Schumacker model showed a decreasing amount of bias as the study factor increased.

The reliability of the observed indicators for a model can have an influence on the robustness and quality of the estimates. Indicators that are more reliable, and which contain more information about the latent constructs being assessed, may be able to compensate for the biasing effects of small sample sizes to some degree (Boomsma & Hoogland, 2001). Other authors (e.g., Schmidt & Hunter, 1996) have noted that increased measurement error (i.e., low reliability) can produce a downward bias in the correlations (or covariances) between variables. As the accurate estimation of the variance-covariance matrix is the goal of latent modeling, an increased amount of measurement error will lead to a higher rate of erroneous models, and can also produce a downward influence on the estimates (Schwartz & Coull, 2003). Cronbach's alpha is a commonly used measure testing the extent to which multiple indicators for a latent variable belong together (Green & Yang, 2009). It varies from 0 to 1.0. A common rule of thumb is that the indicators should have a Cronbach's alpha of 0.7 to judge the set of items reliable (Moulder & Algina, 2002). Alpha may be low because of lack of homogeneity of variances among items, for instance, and it is also lower when there are few items in the scale/factor. Another method to assess reliability is to use Raykov's reliability rho, also called reliability rho or composite reliability, which tests the assumption that a single common factor underlies a set of variables (Raykov, 1998). Raykov (1998) has demonstrated that Cronbach's alpha may over- or under-estimate scale reliability, with underestimation being more common. For this reason, rho is now preferred and may lead to higher estimates of true reliability. Raykov's reliability rho is not to be confused with Spearman's median rho, an ordinal alternative to Cronbach's alpha. The acceptable cutoff for rho would be the same as the researcher sets for Cronbach's alpha since both attempt to measure true reliability.

The correlation between the latent factors can also have an influence on the ML estimates of the factor loadings (Cudeck & O'Dell, 1994). Further, the standard errors of the estimates show an upwards bias as the amount of missing data increases (Newman, 2003). This same phenomenon was seen in the MCAR data condition for the latent interaction slope parameter estimates (see Table 13) where the standard deviation for the estimates were larger in the MCAR data condition than in the Complete data condition. However, the same influence was not seen in the MNAR data condition, where the estimates of the latent slope interaction parameter were close to zero in all conditions for all models, which most likely contributed to the lack of variance in the estimates for this data condition. The reasons for the poor performance of all three of these models in the MNAR condition may be due to a combination of the MNAR mechanism and the violation of the assumption of independence of observations.

In the current study, an advantage of the Schumacker model may be attributed to the use of factor scores from a confirmatory factor analysis as a preliminary step in forming the latent growth interaction model (see Figure 14). Rowe (2002) has noted that using factor scores resulting from a confirmatory approach minimizes the measurement error that is present in the indicator variables themselves. This is a potential explanation as to why the Schumacker model showed performance that was consistently better than either the Wen or the Duncan models in terms of estimating the latent slope interaction parameter. This may also account for the low amount of bias seen across the levels of the reliability factor for both the latent slope interaction parameter estimates themselves and for the estimates of MSE and standardized bias (see Tables 13 through 16).

The mean values of the model fit indices did not appear to be affected when the data condition was either Complete or MCAR, as the values for each individual model were similar across these two data conditions. The effect of missing data was largest for the MNAR condition, and showed an influence on the model fit indices by severely lowering their values. The impact of data that is MNAR is especially pernicious, as even the Duncan model (which is correctly specified as the population model) showed poor fit to the data in this condition. The nature of this impact by MNAR data needs further exploration.

There are three conclusions that can be drawn from the preceding hypotheses. The first pertains to the question of "Which of the three models is uniformly better with respect to overall model fit?" The Wen model is the best model when the data are Complete or MCAR. Within these two data types, for a given condition, the multiple comparison tests favoured the Wen model as having the highest overall model fit indices, and that these values were significantly different from those of the other two models. When the data is MNAR, no model does well with respect to overall model fit, and so no real conclusion can be drawn other than that these models should not be used if the data is MNAR.

A second related conclusion pertains to the question "Does the same pattern of overall model fit indices hold across the different types of missing data?" In other words, is there a consistent ranking for the models that is replicated across the three data types? An examination of the means and the multiple comparisons showed that the order of models with respect to the CFI and NFI was Wen > Duncan > Schumacker, and for the RMSEA the ranking was Wen < Duncan < Schumacker when the data was Complete or MCAR. The GFI showed a slightly more variable pattern, with the Duncan model having slightly higher GFI values than the Wen model at lower reliabilities. The Wen model always produced GFI values that were higher than the Schumacker model. When the data was MNAR no clear ranking emerged, however some global patterns were observed as the study factors were manipulated. For the CFI and NFI, as reliability, sample size, and the latent intercept-slope correlation increased the Wen model consistently decreased, the Duncan increased, and the Schumacker showed stability. For the GFI the Wen and Duncan models decreased, and the Schumacker showed an increase. For the RMSEA the Wen and Duncan models increased, and the Schumacker model decreased. In other words, the pattern of model fit indices when the data is MNAR depends on which model and fit index is chosen.

A third conclusion relates to the finding in the previous paragraphs regarding the significant mean differences among the models. Given that almost all of these differences were significant, is this occurring because one model in particular is consistently passing (or failing) to meet the criterion for good model fit? An examination of the 95% confidence intervals for each of the models showed that the Wen model was passing the cutoff of 0.90 for the CFI (in 30 of the 36 conditions) and for the NFI (in 16 of the 36 conditions) when the data was Complete, and in slightly fewer conditions when the data was MCAR. For the GFI only the Schumacker model showed confidence intervals that were above 0.90, and this was only in two conditions. None of the models showed confidence intervals for the RMSEA that met the criterion for good fit. This finding shows that the Wen model is consistent in its performance on the indices for the CFI and NFI when the data is Complete or MCAR, and that the GFI and RMSEA are problematic fit indices for all models.

These findings with respect to overall model fit in each of the latent growth interaction models should be considered carefully in light of the fact that the models used in this study were longitudinal designs with a non-linear effect (i.e., the latent slope interaction), and the model design itself may be having an impact on the model fit indices. The model fit indices used in the current study are commonly used in articles that present structural equation models, and they are used mainly from a cross-sectional perspective. The performance of these fit indices in longitudinal research designs which utilize SEM is largely unexplored, and it is possible that some of the violations of the assumptions that are not seen with cross-sectional data (e.g., independence of observations,) may be impacting the proper estimation of these fit indices. An indication of this is with the poor performance of the Duncan model, which is the model that was used as a basis for the data generation. Of the latent interaction growth models examined in the current study, this model can be considered the "true" model, and the model fit indices should have been approaching perfect fit in the Complete data condition. That this did not occur may be indicative of the inadequacy of conventional model fit indices to evaluate latent growth models.

Addressing Missing Data in SEM

The current study indicates that, compared to models using complete data, when the missing data follows an MCAR mechanism the influence of the missing data may not be as severe and results are comparable to those found with complete data. When the missing data follow an MNAR mechanism the resulting poor model fit and bias in estimation becomes problematic.

A method to determine the type of missing data mechanism has been forwarded by Shin, Davison, and Long (2009). In their approach, missing data vectors are created that correspond to each assessment point after the initial assessment. Letting Y* represent the complete response variable that consists of both observed and missing values, and Y^*_{it} being a particular score for the i^{th} individual (i = 1, 2, …, N) at the t^{th} time point (t = 1, 2, …, T), at each time point a binary indicator variable, I_{iT}, can be created such that $I_{iT} = 1$ if Y^*_{it} was observed and $I_{it} = 0$ if Y^*_{it} was missing. Each I_{iT} can be regressed on the complete data for the T^{th} time point and all previous time points using the following regression formula,

$$I_{iT} = \beta_0 + \sum_{J=1}^{T} \beta_t Y^*_{it} + e_{it}, \tag{46}$$

for all time points T (e.g., one model for T=2, another for T=3, etc). For samples with an MCAR mechanism, the R^2 for all models should be 0 and all estimates of β_t to be nonsignificant. Under an MNAR mechanism, R^2 should be greater than 0 for all models and all β_t to be significant.

Other methods for assessing the mechanism of missing data have also been proposed. The presence of the MCAR mechanism can be assessed by testing across patterns of the missing data (e.g., complete cases versus missing cases) using t-tests for location (Bingham et al., 1998). Little (1995) has also called this mechanism covariate-dependent dropout, and notes that analysis of complete cases in this instance will not yield biased estimates but will be inefficient. Figueredo et al. (2000) note that the MNAR condition can be detected by locating significant differences between means on subgroups of the data with complete cases versus those with incomplete cases.

Several authors have proposed methods to addressing missing data using SEM techniques, and some of them have been discussed previously (see Section 8, p.60). While the implementation and comparison of the described methods was not a focus of this study, future research could investigate the use of these methods with longitudinal latent modeling.

Lee (1986) and Allison (1987) examined missing data in SEM, and also investigated the use of ML estimation with missing data. Both authors treated the problem of missing data as a multiple-group model. With this approach, the sample is split into groups based on the patterns of missing data, and equality constraints are then placed on the parameters of the groups. If the sample size in a group is small, the sample covariance matrices are singular and may not converge.

Jamshidian and Bentler (1999) proposed an EM algorithm for missing data which circumvented the problems of small sample sizes, however Dempster, Laird, and Rubin (1977) report that convergence with the EM algorithm is dependent to the proportion of missing information. Song and Lee (2002) proposed a Bayesian approach to SEM with data that was ignorably missing (MAR). In this approach, the observed data is augmented with missing quantities in a posterior analysis. A sequence of random observations are generated from a joint posterior distribution and the model has converged if the posterior distribution can be approximated adequately by the empirical distribution of the simulated observations. These authors performed their study with 100 replications for 2000 observations, and the procedure performed well under different patterns of missing data with small samples, as well as converging quickly.

Lee and Tang (2006) sought to develop a model that accounted for nonignorable missing data in nonlinear structural equation models, using a Bayesian approach as (1) it can provide more accurate estimates obtained when good prior information is available, (2) Bayesian methods do not rely on asymptotic theory and so may work well with small sample sizes, and (3) gives a more flexible statistic for model comparison / selection than the likelihood ratio test (i.e., the Bayes factor). These authors used a linear logistic regression model that produced assessable conditional distributions whose observations can be sampled in their method.

A procedure that has also been presented is to use a latent variable to represent the missing observations or assessment points, but this approach is only viable if there are *no observations* at that assessment point (Ferrer et al., 2004).

This section has highlighted some of the more recent approaches to handling missing data in latent variable modeling, and some of this research has focused on the MNAR mechanism. However, there is a paucity of research that examines these methods, and those described earlier, for their use with missing data in longitudinal designs. Future research should aim towards investigating the performance of these methods in longitudinal designs with latent variables.

Limitations

The findings of the current project, while being novel and extending the literature in several areas, should be considered within the light of several limitations. The first limitation is that only specific conditions were used in the current Monte Carlo study – three specific levels of latent intercept-slope correlation, three specific sample sizes, and four specific levels of reliability. The levels of latent intercept-slope correlation and reliability chosen covered a wide range of possible values. Only three sample sizes were chosen (250, 500, 1000) and longitudinal research with sample sizes approaching the largest sample size used may be both expensive and difficult to conduct in a practical manner. Smaller sample sizes should be examined in future studies. A related factor that could have been manipulated was the amount of missing data, with higher rates of missing data being incorporated into the study design, and missing data patterns other than a monotone attrition pattern (see Figure 15). Other potential factors that were not studied were the method of estimation for the models and the non-normality of the original data for the observed factors (not including the non-normality introduced by the formation of the interaction terms by the product-indicator method, nor by the product of latent factor scores).

A second limitation to the study is the modification of the Schumacker (2002) model to make it applicable to latent growth modeling. The original Schumacker model is based on a cross-sectional design, and the extension used in the current project, while straightforward, may not have taken into account some of the considerations of other latent interaction models. Namely, both the Wen and the Duncan models impose constraints on the model parameters that are meant to account for the non-linear nature of the interaction effect, but this is not a feature of the Schumacker model. A mitigating factor to this limitation are the findings of Algina and Moulder (2001) that a model with no cross-product indicators, such as the Schumacker model, is robust to violations of normality, and this may have contributed to the ability of this model to perform well in some respects compared to the Wen and Duncan models.

A third limitation is that the interaction effect studied in the current project may have been too weakly specified to be detected reliably. Interaction effects in cross-sectional studies generally account for a small amount of variance (Hewitt et al., 1996), and may need to be particularly strong in order to be detected reliably. Related to this limitation is the specification of the interaction effect in latent growth models. The approach advocated by both Wen and Duncan, namely the product-indicator approach, has also been utilized in standard regression approaches to interaction effects, and there is an additional assumption that the parameters which define the growth model are invariant across the levels of predictors (Curran & Hussong, 2003). The Wen and Duncan models place non-linear constraints on the parameters of the model which are difficult to program manually. Other product-indicator approaches are available (e.g., Marsh et al., 2004), some of which may be easier to implement than the one used by the Wen and Duncan models.

A fourth limitation is that the data generation scheme was based upon the model proposed by Duncan et al. (1999), and so this model should have demonstrated close to perfect fit in the Complete data condition. That this did not occur could indicate that (1) the model fit indices used to evaluate the overall performance of the latent growth interaction models may not be appropriate for these types of models, or (2) these latent growth interaction models have an underlying flaw that makes their performance sub-optimal, for example the parameterization of the models or the imposition of the nonlinear constraints on some of the model parameters. There is some support for this second option, as Rudinger and Rietz (1998) have noted that the coding of the time factor is an essential feature of a correctly-specified latent growth model. The models examined in the current study all represent the dimension of time as a function of the factor loadings, which Rudinger and Rietz contend is simply a confirmatory factor analysis model and not a true latent growth model.

A final limitation is with respect to the Type I error rates for the MNAR data condition. For the population model in this section of the study, the value of the latent slope interaction parameter was set to zero. However, even when the population value for this parameter is set to be non-zero, in the MNAR condition the estimated value was close to zero in almost all of the conditions (see Table 13). As a result, the Type I error rates for the MNAR data condition may be misleading, and future research using different population values are needed.

There are some further limitations that, while not specific to this study, are applicable to latent variable growth modeling in general. The latent growth model is based on the premise that a set of observed repeated measures taken on a given individual over time can be used to estimate an unobserved underlying trajectory that gives rise to the repeated measures (Curran & Hussong, 2003). The process of latent growth modeling imposes a very restrictive factor loading matrix, and so the usage of standard model fitting criteria may need to be loosened (e.g., relaxing the cutoff value of 0.90 for the CFI, GFI, and NFI). However, with strict parameter restrictions, passing the model fit indices using the existing cutoff values is desirable, since the particular growth trend is what researchers are aiming to test by imposing such restrictions.

The representation of latent growth models is varied, as can be seen in the models presented by authors such as Raykov (1993), Muthen (1994), and Wen et al. (2002) and Duncan et al. (1999). Further to this, the finding that latent growth models can be also be represented as a hierarchical model, which has a single representation (e.g., Curran, 2003), can leave a researcher with several options for modeling the basic latent growth curve. The various methods of representation for a latent growth model have not been extensively compared, and this could lead to confusion as to what representation to use, further complicated by an attempt to include interaction terms. The Duncan and Wen models have been the only models that have explicitly represented an interaction effect in a latent growth model, but the existence of other latent growth models does open up the possibility of other representations of the interaction effect.

Data preparation for these models is an important factor. As was noted in this study, all observed data was mean-centered prior to analysis in the latent growth models. Ideally the interaction term is uncorrelated with (i.e., is orthogonal to) its first-order effect terms. With continuous scores, transforming the raw-score variables to deviation-score variables (by subtracting the variable mean from all observations) results in a product term that is minimally correlated with the first-order variables, depending on their proximity to bivariate normality. However, an additional data-processing procedure can also be utilized prior to analysis – that of generating normalized data. The normalized data also serves to control sampling fluctuations in the generated data – by normalizing the data any outliers have their influence minimized, and skewness will be more controlled.

There are some practical limitations to the use of the latent growth modeling approach. At least three repeated measures are needed to overidentify a linear growth model. Currently the latent growth approach is largely restricted to multiple assessment points nested within individuals, and extensions into higher-order nesting (e.g., multiple individuals nested within a cluster) are difficult (and HLM approaches might be better suited for this). Further, the latent growth approach assumes that the same measure is used to assess the construct of interest over time (e.g., that the measure is invariant, which may not always be the case). Finally, the use of latent growth models requires careful screening and evaluation of the data prior to model fitting to remove outliers. Other limitations noted by other authors is that the function of time, or the role of time as a predictor, is not as clearly obvious as with other approaches (e.g., HLM; Curran et al., 2004).

Missing data is a difficult issue with latent growth modeling, as many latent modeling approaches utilize a complete-case method of analysis. However, recent improvements in handling missing data (e.g., FIML, multiple imputation) and in computational software (e.g. LISREL, Mplus) are making it increasingly easier to incorporate missing data into these models. Several authors have suggested various approaches for addressing missing data (see pp. 150-153) and their application in latent growth modeling needs to be explored further. Missing data of an attrition-based nature is of especial interest in this regard.

The following outlines some further factors that, while not limitations related directly to the factors manipulated in the current study, do represent factors that may have had an impact on the findings.

Estimation Method

The maximum likelihood (ML) estimation procedure was initially derived under the assumption that the observed variables were from a multivariate normal distribution, however Browne (1984) has shown that the ML estimator has desirable asymptotic properties under less restrictive conditions (e.g., having no excess multivariate kurtosis). Bollen (1996) and Siemsen and Bollen (2007) have shown that the ML estimator has several desirable properties – it is consistent, uses information from the model specification to obtain parameter estimates that have the highest likelihood of reproducing the data for a given model, and is asymptotically efficient. However, these desirable properties only hold if the model is correctly specified and if the sample size is sufficiently large. When these conditions are not met, the ML estimator is problematic (Anderson et al., 1984; Bollen, 1989), and can lead to large biases in the estimates of parameters when the underlying model is misspecified (Bollen, 1996).

The performance of the ML estimator with small sample sizes are not theoretically established, since only their asymptotic distributions have been derived, and consistent estimators can be biased in small samples. In using the ML estimator in structural equation models, the appropriate sample size required has varied from small (e.g., 100; Gorsuch, 1983) to moderate (e.g., 400; Cheung, 2004), while others have reported that the required sample size is a function of the number of estimated parameters contained within the model (e.g., Marsh et al., 1988). Further, in smaller samples the ML estimator may not converge successfully (Anderson et al., 1984), as a result of both the observed covariance estimates being farther away from their true population value and of the initial starting values used to start the parameter estimation. Finally, with small sample sizes ML estimation can have high rates of non-convergence and/or improper solutions, parameter standard errors that are attenuated, and Type I error rates which are inflated (non-normality of the data can further inflate the Type I error rates). In a related area, improper estimates can occur when sample sizes are small, leading to problems in the interpretation of the parameter estimates (Chen, Bollen, Paxton, Curran, & Kirby, 2001).

When used with missing data, ML estimation produces unbiased estimates when missing data is MAR (Jamshidian et al., 1999; McArdle et al., 1992; McArdle & Hamagami, 2001). ML estimation has been found to be a suitable estimation method when missing data due to attrition is "natural" (e.g., due to mortality; Feng, Silverstein, Giarrusso, McArdle, & Bengston, 2006). Newman (2003) found that ML estimation provided accurate standard errors when used with missing data, and that the ML estimation procedure was robust to data that was missing at random (MAR), but provided poor estimates for MNAR data. This was not a surprising finding, since non-random missingness leads to bias due to restriction of range. When data is MNAR, ML estimation gives larger errors for estimates and larger bias compared to MCAR (Jamshidian et al., 1999). With interaction models using cross-sectional data (Lee, Song, & Poon, 2004), ML estimation only produced satisfactory results in simple models with large sample sizes.

Data Normality

Mild degrees of data non-normality have little influence on the overall model fit indices (Fan et al., 1998). Non-normality in the data has also been shown to have little effect on the parameter estimates compared to when data is normal, and this non-normality has accounted for less than 1% of the variance in the estimates (Fan & Wang, 1998). While the individual variables for the individual latent factors were normally distributed, the use of interaction terms in the current study introduces non-normality into the data and a centering procedure (Aiken & West, 1992) was used to reduce the effects of multicollinearity between the interaction variables and their constituent variables. The findings of Fan and colleagues are supportive of the conclusion that the effects of non-normality on the model fit indices and parameter estimation in the current study are negligible.

Power

Most of the studies that have examined power within the context of latent growth models have done so in an attempt to detect group differences in latent growth trajectories. In an examination of power in latent growth models, Fan and Fan (2005) found that neither the magnitude of the effect nor the number of assessment points had an influence on the power of latent growth models to detect linear growth. These authors also found that the latent growth perspective was more powerful than traditional approaches (e.g., dependent t-tests, RM ANOVA) at smaller effect sizes (see Fan et al., 2005, for a review). In other words, the latent growth model is a powerful analytical tool, and is preferable when the effects being studied are small and may not be detected reliably by other methods.

Hertzog and colleagues (2006) found that the power to detect linear change and correlations between change is low unless the reliability of the observed indicators is quite high (i.e., above 0.90), sample size is substantial (i.e., greater than 500), and there is a large number of assessment points (i.e., greater than four). These authors used only a single indicator for each assessment point, as was done in the current study, and the authors proposed that their results could be improved if there were multiple indicators for each assessment point. A similar recommendation is made for the current study, and is an area of future research.

Coding of Time

Rudinger and Rietz (1998) propose that the coding or representation of time is crucial in a latent growth model. Typical SEM and LGM models lose the sequential nature of time, and these authors assert that one can change the ordering of the indicators in these models (which are generally individual assessment points) and still have an equivalent model. In other words, these models do not take the basis / factor loadings into account, instead treating all LGM models as pure CFA models (see p. 112 of their manuscript). In general, the variance of the true scores will have a quadratic relationship with time in a linear growth model, and the correlation (or covariance) between the level of a true score at a given time t and the growth rate is dependent on the time factor (Mehta & West, 2000), and this should be incorporated into the model design. The current project used the coding of time as outlined by Duncan and colleagues (Duncan et al., 1999), which codes time by fixing the parameter loadings for the observed variables on their respective latent variables (see Figure 15).

Chapter 14: Summary and Recommendations

There were several themes addressed by the current project of latent slope interaction effects in latent growth modeling. The first was the comparison of three alternative methods for constructing interaction effects in latent growth models – the methods of Duncan et al. (1999), Wen et al. (2002), and an extension of the cross-sectional model by Schumacker (2002). The findings of this project provide mixed support for the utility of these approaches in the modeling of a latent growth interaction effect. The Wen et al. (2002) model showed the best performance with respect to overall model fit, especially when the data is either Complete or MCAR, with the Duncan et al. (1999) model showing similar performance. The Schumacker model provided the most accurate estimation of the latent interaction slope parameter, with the Wen model showing accurate estimation at high levels of reliability and the Duncan model doing so at lower levels of reliability. The Duncan and Schumacker models performed similarly with respect to bias in the estimation of the latent interaction slope parameter across the three types of missing data, and the Wen model was biased in the majority of the conditions studied regardless of the type of missing data. However, Type I error was not well controlled by any of the models.

A second theme was the examination of the effects of missing data in the area of latent growth interactions. Previous studies that have examined the impact of missing data in longitudinal designs have done so with simple longitudinal models that have only included a single latent factor, and have not used more complex models that involve multiple factors. In the current study, it was seen that the impact of missing data that was the result of an MCAR mechanism does not result in performance that was qualitatively different from that of having Complete data. For all of the models, their performance under the MCAR mechanism was similar with respect to overall model fit, latent slope interaction parameter estimation, bias in the parameter estimation, and Type I error rates. The evaluation of these latent growth interaction models with MNAR data is a new contribution to the literature. Not surprisingly, none of the models performed particularly well when missing data was derived from an MNAR mechanism.

Each of the models studied in the current project had several strengths and weaknesses. The Duncan model was the population model (i.e., all data was generated from the framework of this model) and the Wen model is an extension of this model, so they are discussed jointly. The Wen model is a "full" model in the sense that it contains all estimated paths and effects, and one of its strengths is that it provided better model fit to the data and better estimation of the latent slope interaction parameter than the Duncan model, and this is impressive since the Wen model can be considered to be a misspecified model compared to the Duncan model from which the population data was derived. A strength of the Wen model was its accurate estimation of the latent slope interaction parameter, a conclusion supported by Wen et al. (2002) in their study. A third strength of the Wen model was its programming difficulty in relation to the Duncan model. Both the Duncan and the Wen model require substantial programming for imposing the constraints on the parameters in the model due to the non-linear nature of the interaction effects. While the Wen has some additional constraints that need to be imposed compared to the Duncan model, the addition of these constraints is not onerous, and the current findings would indicate that these extra constraints are worthwhile towards providing accurate model estimation.
A fourth strength is that the Wen model showed desirable characteristics in the context of the manipulated model factors (i.e., latent intercept-slope correlation, sample size, and reliability of the observed indicators), namely improved performance in overall model fit indices and

parameter estimation as all of these factors increased. Again, this is especially notable due to the fact that the Wen model was not the population model from which the data was derived. A final strength of the Wen model, and of the Duncan model but to a lesser extent, is the similarity of performance of the model when the data was either Complete or MCAR.

The Duncan model had several strengths. It showed a high rate of convergence across all of the conditions, regardless of the type of missing data, and converged faster than the Wen model at lower reliabilities. The values for the CFI and NFI model fit indices were very similar to those found for the Wen model in the Complete and MCAR data, and also showed slight increases as sample size increased. The estimate of the latent slope interaction parameter showed a lower degree of bias than the Wen model, and the bias also decreased as the study factors of reliability, sample size, and latent intercept-slope correlation increased. The programming for this model is simpler than that for the Wen model.

Weaknesses of both the Wen and the Duncan models include a poor control of Type I error rates, with the Wen model showing liberal control and the Duncan model showing conservative control when the data was either Complete or MCAR, and both of these models were conservative when the data was MNAR. A further weakness is that both of these models performed poorly when the data was MNAR, showing poor model fit, parameter estimation, and control of Type I error, across all of the manipulated study factors. The reasons behind the poor performance in the MNAR data condition are unknown at this time.

The Schumacker model had several strengths relative to the Wen and Duncan models. These include an ease of conceptualization, as forming the multiple cross-products of indicators was not needed. This model also provided qualitatively more accurate parameter estimates at all levels of the manipulated factors, and performed especially well when the reliability of the observed indicators were at their highest levels, surpassing the other two models studied. The Schumacker model showed reasonable control of Type I error rates when compared to either the Wen or the Duncan models, and showed better performance as both reliability and sample size increased.

Some weaknesses of the Schumacker model include the difficulty in programming, as there are several SEM models that are required to be constructed and evaluated, as well as the saving of the factor scores from one of the models as an intermediate step so that they can be manipulated to form the interaction terms in the final model. This can be a confusing and daunting task for researchers. A second weakness is that this model was extended from a cross-sectional perspective, and while it performed adequately in the current study, the model was not designed to investigate latent interaction effects in growth models. A third weakness is that the Schumacker model performed worse than the other two models at lower levels of reliability of the observed indicators, and did not show adequate model fit on any of the model fit indices when the data was Complete or MCAR.

Recommendations For the Researcher

There are several recommendations that can be made to researchers who are considering using one of the three latent growth interaction models examined in the current project.

Based on these findings, the following contributions and recommendations can be made. If the researcher believes that the factor of time is a fixed effect, then the Schumacker model should be used. If the researcher believes that time is a random effect, then the Schumacker model can provide accurate estimates but may not be wholly suitable for model fit, mainly due to the final step of the Schumacker model which is a purely fixed-effect model.

If the trend for growth is considered to be a random effect, then either the Wen or the Duncan model can be used. The Duncan model should be attempted first, as if it shows good model fit to the data the parameter estimates for the interaction effect will not show a large amount of bias compared to the Wen model. If the Duncan model does not provide adequate model fit, the Wen model can be used, with the careful consideration that the parameter estimates may be biased. Further to this, if the model is complex (e.g., the growth may not be purely linear) the Wen model may be preferred as it has a better chance to provide adequate model fit, especially when the data is Complete or MCAR.

A second recommendation is that researchers should examine their longitudinal data closely, paying particular attention to any missing data and the mechanism behind the missingness. If the missing data can be attributed to an MCAR mechanism the influence on model performance will be minimal. If the mechanism is determined to be MNAR then each of the models studied in the current project will show poor performance. The current findings indicate that model choice may depend on the type of data – if data is Complete or MCAR the Duncan and Wen models are acceptable.

A third recommendation is that researchers should make efforts to utilize observed measures that are highly reliable. Of all the factors in the current study the factor of reliability had the strongest influence, as seen in the large effect sizes associated with this factor in the ANOVA analyses. Reliable indicators will reduce the amount of measurement error, and can also mitigate the influence of other negative effects on latent variable modeling (e.g., small sample sizes; Boomsma & Hoogland, 2001).

Future Directions

This study can act as a starting point for an extensive body of research that examines interactions in latent growth modeling with respect to types of missing data (e.g., cohort-sequential designs, planned missingness designs), as well as their robustness to other factors not studied in the current project (e.g., non-normality of the observed indicators, estimation methods, ordinal data). Other methods of forming the interaction effect can also be explored. The traditional product-indicator approach used in the current study may not be the most suitable approach, given all of the constraints that need to be programmed. The latent interaction model by Schumacker, which was extended in this current project, shows some promise buy may also not be wholly suitable. There are other methods of characterizing the latent interaction term such as the GAPI approach (e.g., Marsh, Wen, & Hau, 2004) and the QML method of Klein and Muthén (2002) which should be investigated.

A promising area of future research is the comparison of the performance of latent growth models with other methods that can be used to analyze longitudinal data. As discussed briefly in the Introduction (page 38), mixed effects models are an alternative approach to modeling longitudinal repeated-measures data. Mixed effects models assume that individuals deviate randomly from the overall average response, and that the correlation between repeated observations on the same subject arises from the common random effects for this individual (i.e., a random intercept and slope). The mixed model possesses several advantages, namely that all available data on a case is used, is unaffected by randomly missing data, and can flexibly model time effects even with unequally spaced intervals. These models can also model the pattern of variability over time, which results in more accurate treatment effects and standard errors of estimates, and helps to control Type I error (Gueorguieva & Krystal, 2004). Mixed effects models can model the missingness mechanism along with the outcome variable, but it does yield biased estimates when missing data is MNAR, and if sample sizes are small (Gueorguieva et al., 2004). A further drawback to the mixed model approach is that it can only incorporate bivariate relationships with a single dependent variable (Gao, Thompson, Xiong, & Miller, 2006).

Mixed effects models and structural equation models for longitudinal data are analytically equivalent under a variety of conditions (see Raudenbush, 2001; Willett et al., 1994), despite differences in estimation procedure (Curran, 2003). SEM approaches longitudinal data through the use of multiple-indicator latent factors. However, due to the assumption of independence of observations, the analysis is still done on a single aggregate covariance matrix. The key facet of the SEM approach is to incorporate time as fixed values within the factor loading matrix ($\mathbf{\Lambda}$), whereas the mixed and hierarchical model approaches enter time as a predictor. An advantage of the SEM approach is the ability to separate the Level 1 and Level 2 effects in the aggregate covariance matrix. Curran (2003) gives formulas showing that the matrices for the fixed and random effects are identical for SEM and hierarchical models. He also gives methods for making them equivalent for a variety of models, achieved by ordering the data by groups (according to the level of the variables), and adding a latent variable for each second-level variable. This approach does not use repeated observations, but nested observations.

Some of the advantages of SEM are: (1) hierarchical / mixed models assume the predictors are error free, and with multiple indicators the SEM approach can model measurement error, (2) can have multiple-indicator dependent variables, (3) can decompose effects into direct, indirect, and specific indirect effects, and (4) can provide omnibus measures of model fit (the hierarchical model has no logical saturated model that it can compare to the fitted model).

Some of the limitations of SEM are: (1) unless there is something unique about the SEM approach, it is probably easier to use the hierarchical / mixed approach; (2) Curran (2003) outlines an approach with SEM that is more about data management, which can be tedious and difficult; (3) the interpretation of models done in SEM is non-standard, in that latent factor means represent regression coefficients, and that indirect effects are representative of cross-level interactions; and (4) the SEM technique discards data if it is incomplete (i.e., there are missing assessment points).

A final area of future research is to utilize an empirical data set with "real" missing data, and to examine how the three models from the current study perform with respect to overall model fit and parameter estimation. This could potentially be done with large nationally representative datasets that can be considered to be representative of the population (e.g., data from Statistics Canada). Findings from this approach may reduce the arbitrariness of the simulation and may improve the generalizations of the findings in the current study.

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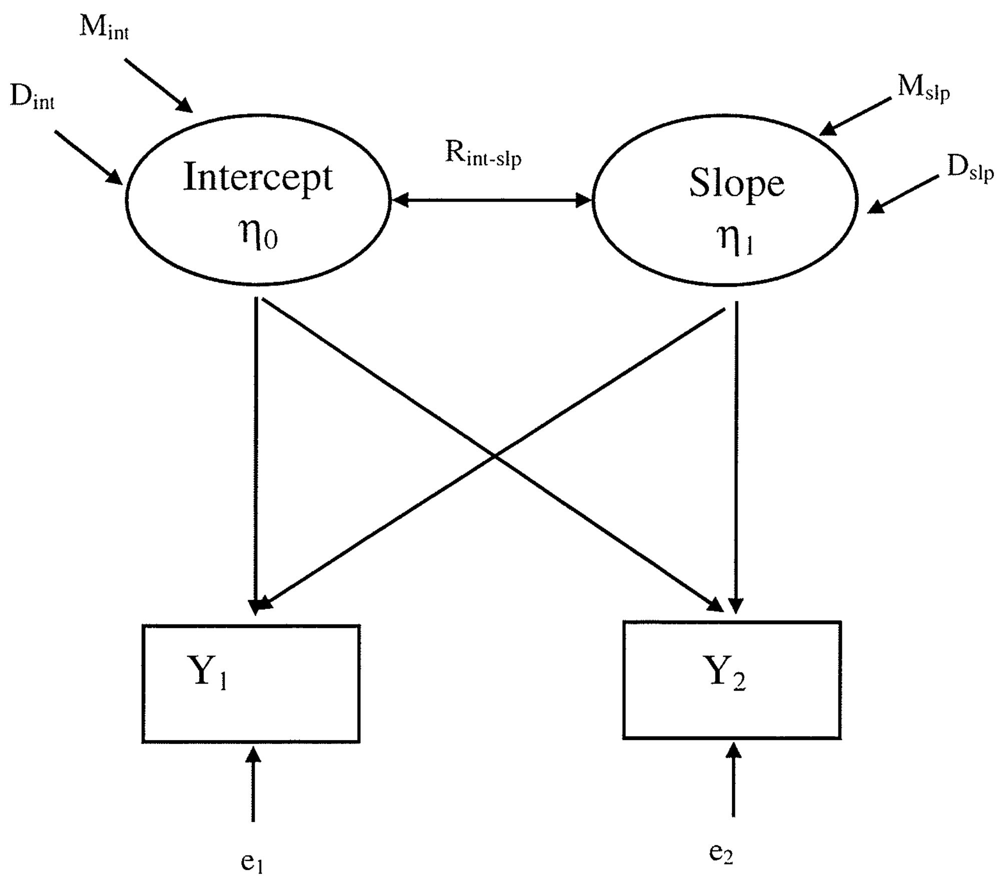
Figure 1

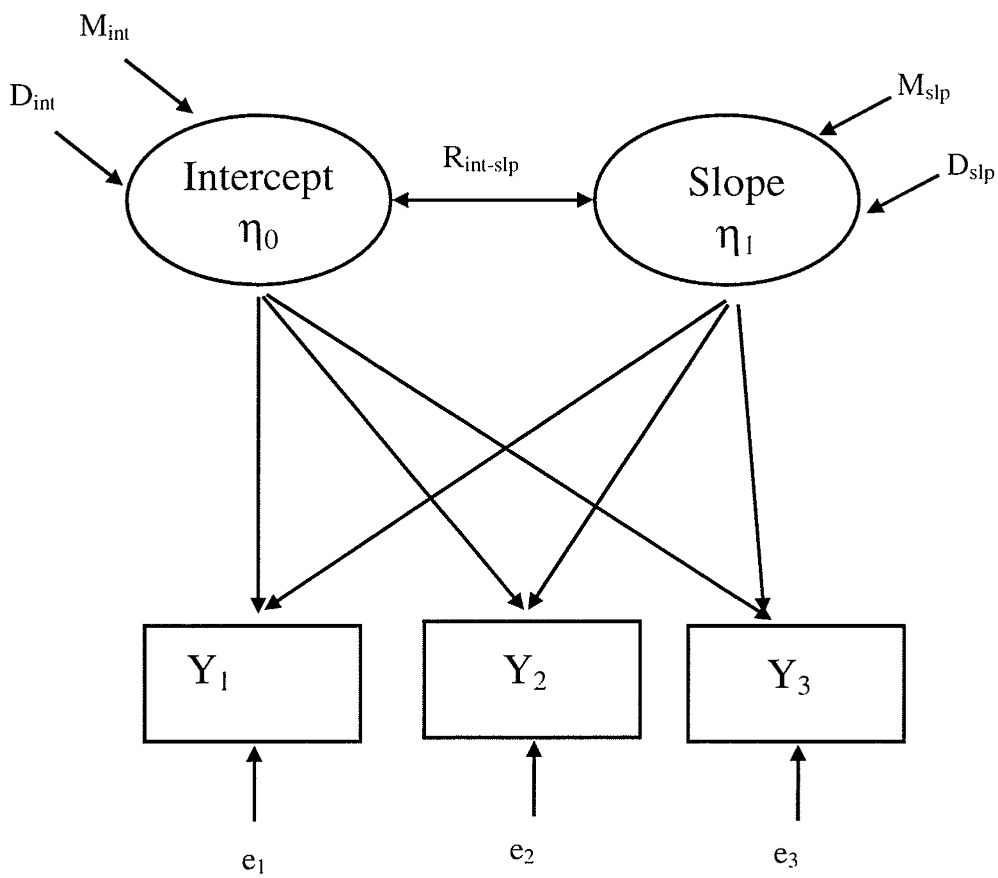
Example of a Non-Linear (Quadratic) Effects Growth Curve.

Figure 2
Basic Linear Latent Variable Growth Model.



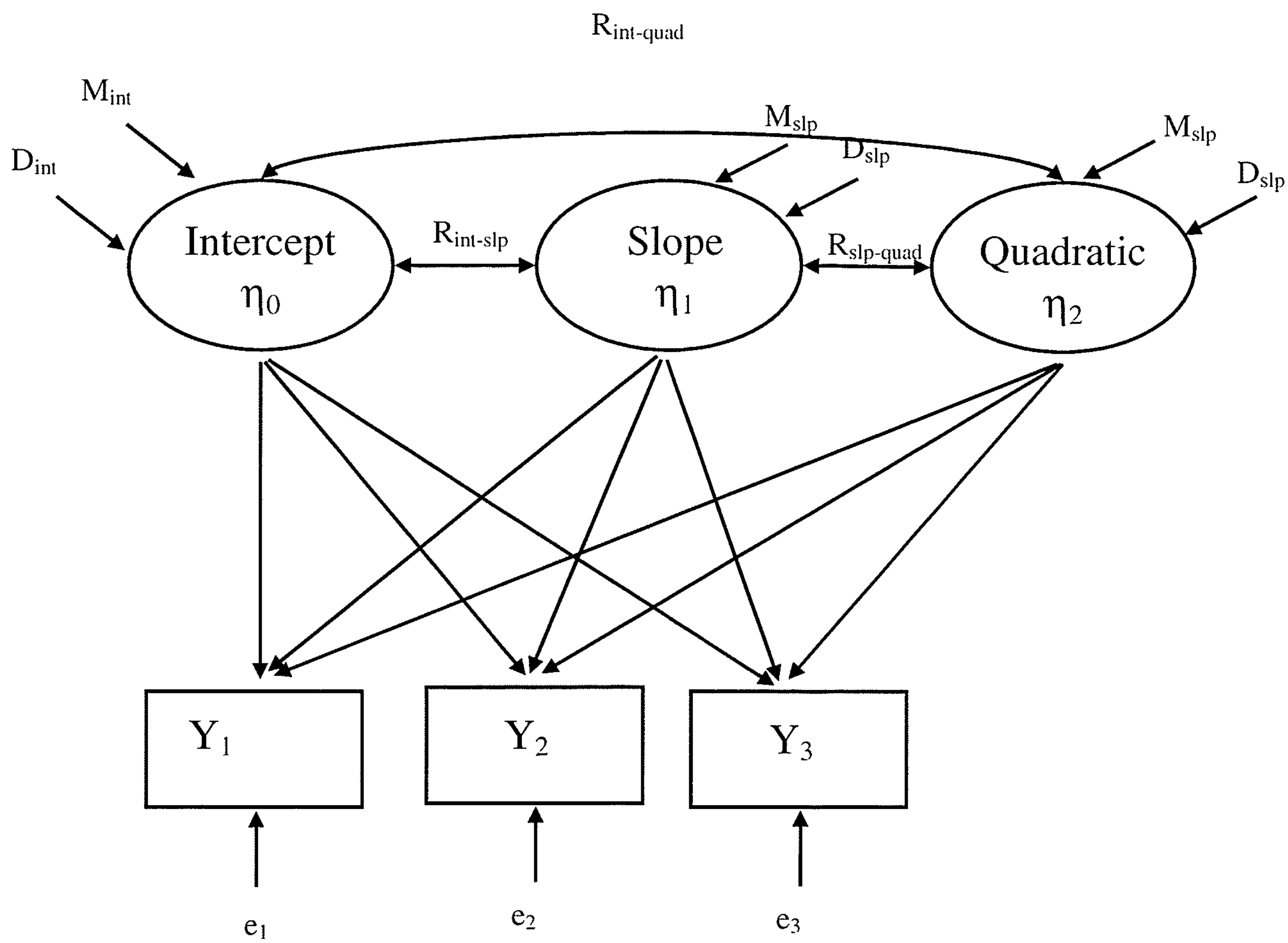
Note: M_{int} = Mean of the latent intercept; D_{int} = Variance of the latent intercept; M_{slp} = Mean of the latent slope; D_{slp} = Variance of the latent slope; $R_{int\text{-}slp}$ = correlation between the latent intercept and slope; e_1 and e_2 represent residual terms.

Figure 3
Basic Linear Latent Variable Growth Model With Three Repeated Observations.



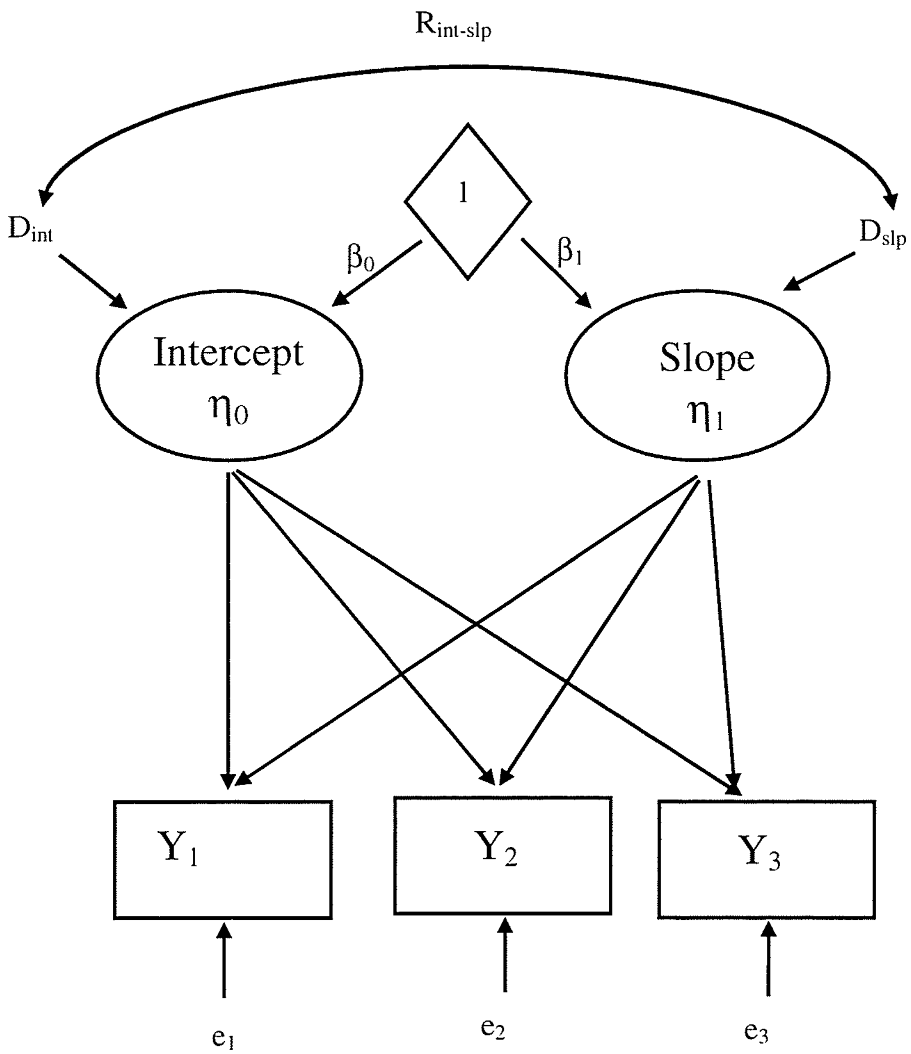
Note: M_{int} = Mean of the latent intercept; D_{int} = Variance of the latent intercept; M_{slp} = Mean of the latent slope; D_{slp} = Variance of the latent slope; $R_{int\text{-}slp}$ = correlation between the latent intercept and slope; e_1, e_2, and e_3 represent residual terms.

Figure 4
Quadratic Effect Latent Variable Growth Model with Three Repeated Observations.



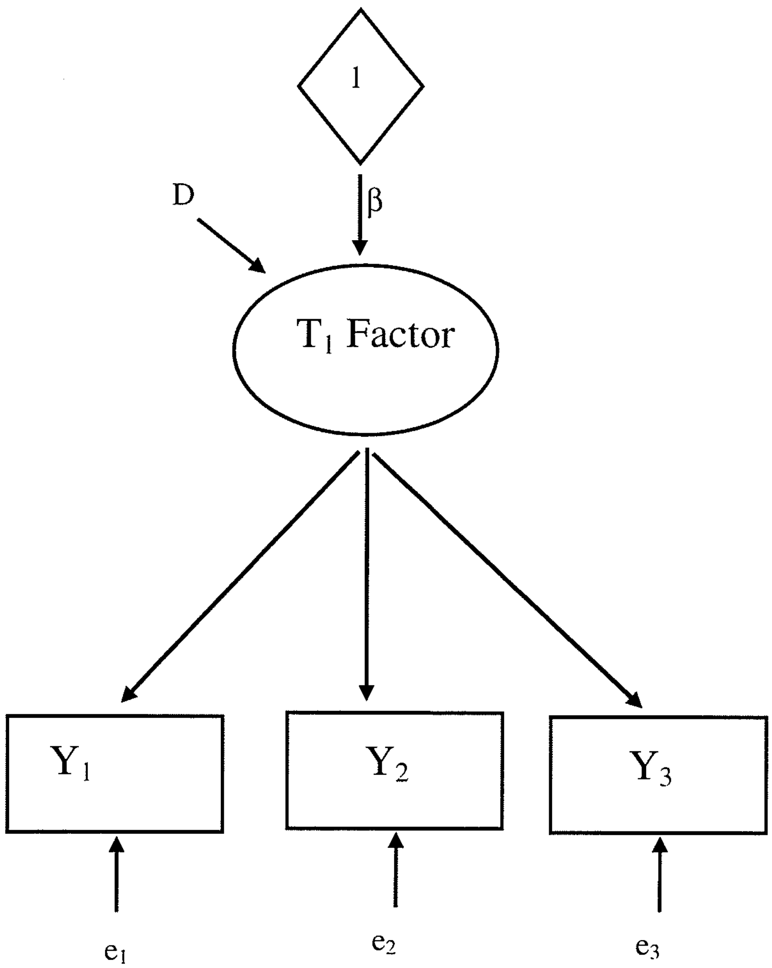
Note: M_{int} = Mean of the latent intercept; D_{int} = Variance of the latent intercept; M_{slp} = Mean of the latent slope; D_{slp} = Variance of the latent slope; M_{quad} = Mean of the quadratic slope; D_{quad} = Variance of the quadratic slope; $R_{int\text{-}slp}$ = correlation between the latent intercept and slope; $R_{int\text{-}quad}$ = correlation between the latent intercept and quadratic slope; $R_{slp\text{-}quad}$ = correlation between the latent slope and quadratic slope; e_1, e_2, and e_3 represent residual terms.

Figure 5
LISREL Univariate Latent Growth Curve Model.



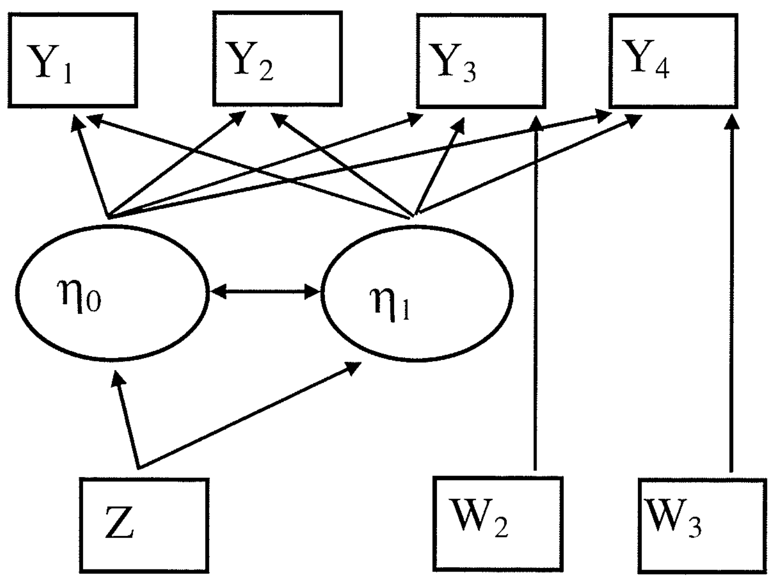
Note: D_{int} = Variance of the latent intercept; D_{slp} = Variance of the latent slope; $R_{int\text{-}slp}$ = correlation between the latent intercept and slope; the diamond shape represents the estimated non-zero means of the two latent factors, and β_0 and β_1 represent the factor means of the latent intercept and slope, respectively; e_1, e_2, and e_3 represent residual terms.

Figure 6
Raykov T_1-Congenericism Latent Growth Model.



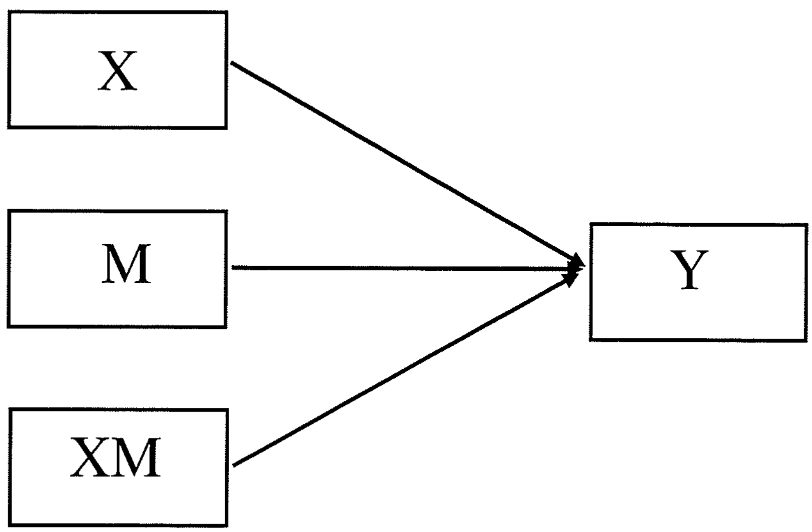
Note: D = Variance of the latent factor; the diamond shape represents the estimated non-zero mean of the latent factor, and β represents the factor mean of the latent factor; e_1, e_2, and e_3 represent residual terms.

Figure 7
Sample Structural Model for Longitudinal Analysis.



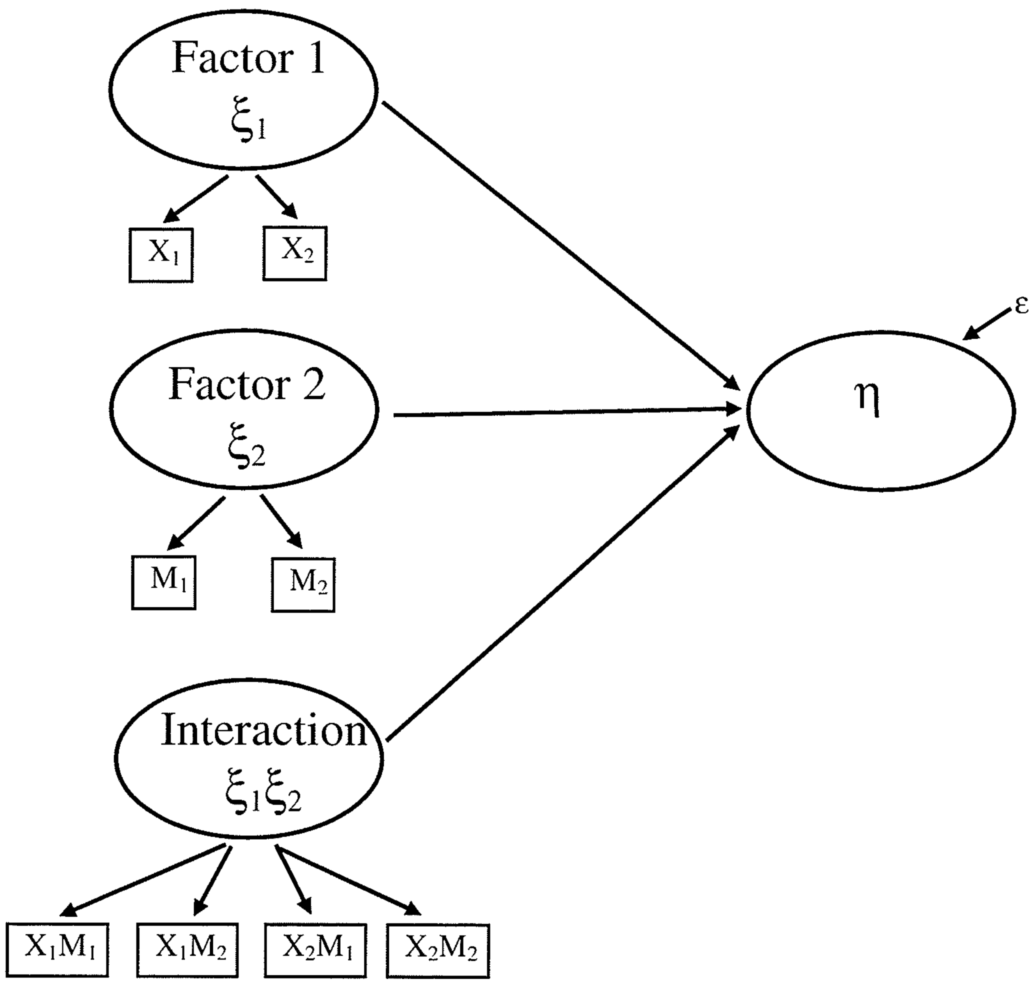
Note: The observed variables Y_1 to Y_4 represent repeated observations on individuals at four consecutive assessment points. The Z variable is a time-invariant covariate and the W_2 and W_3 variables are time-varying covariates measured at the second and third assessment point, respectively. The latent variables η_0 and η_1 represent a latent intercept and slope, respectively.

Figure 8
Graphical Depiction of an Interaction Effect



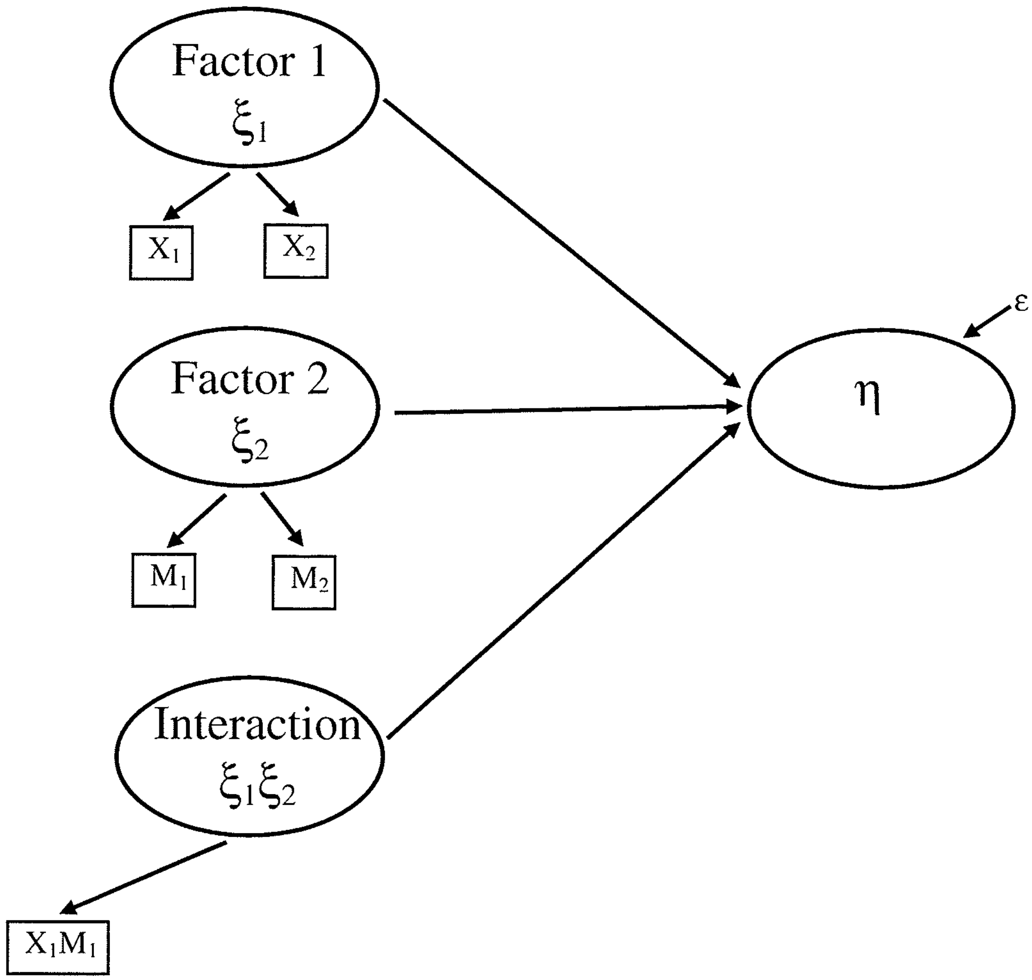
Note: All variables are observed variables, where Y is the dependent variable of interest, X is the predictor variable, M is the potential moderator variable, and XM is the product of these two variables.

Figure 9
The Multiple Indicator Regression Model by Kenny and Judd (1984).



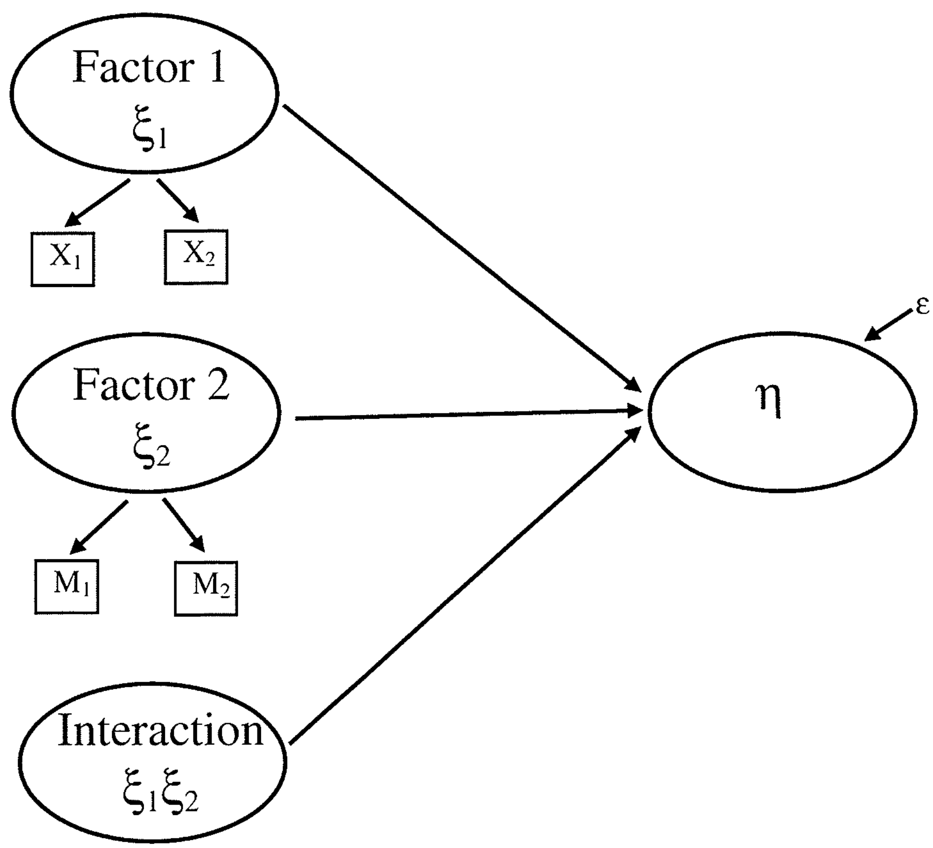
Note: ξ_1 represents a latent independent variable with X_1 and X_2 as observed indicators, ξ_2 represents the latent moderator variable with M_1 and M_2 as observed indicators, and $\xi_1\xi_2$ represents the interaction latent variable between the independent variable and the moderator; η represents the latent dependent variable, and ε represents the error term associated with the latent dependent variable.

Figure 10
Joreskog and Yang's (1996) Single-Indicator Interaction Model.



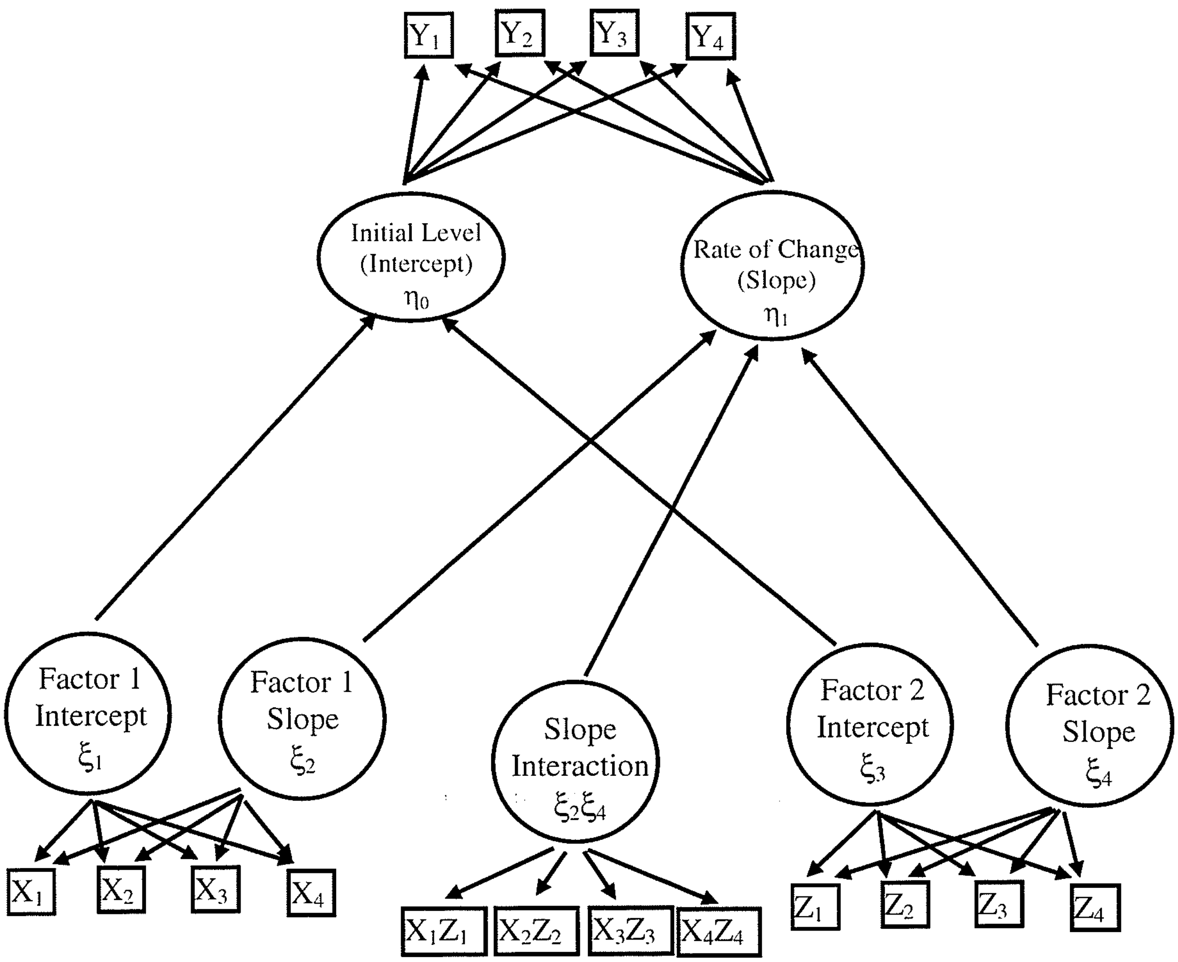
Note: ξ_1 represents a latent independent variable with X_1 and X_2 as observed indicators, ξ_2 represents the latent moderator variable with M_1 and M_2 as observed indicators, and $\xi_1\xi_2$ represents the interaction latent variable between the independent variable and the moderator; η represents the latent dependent variable, and ε represents the error term associated with the latent dependent variable.

Figure 11
Schumacker's (2002) Latent Interaction Model.



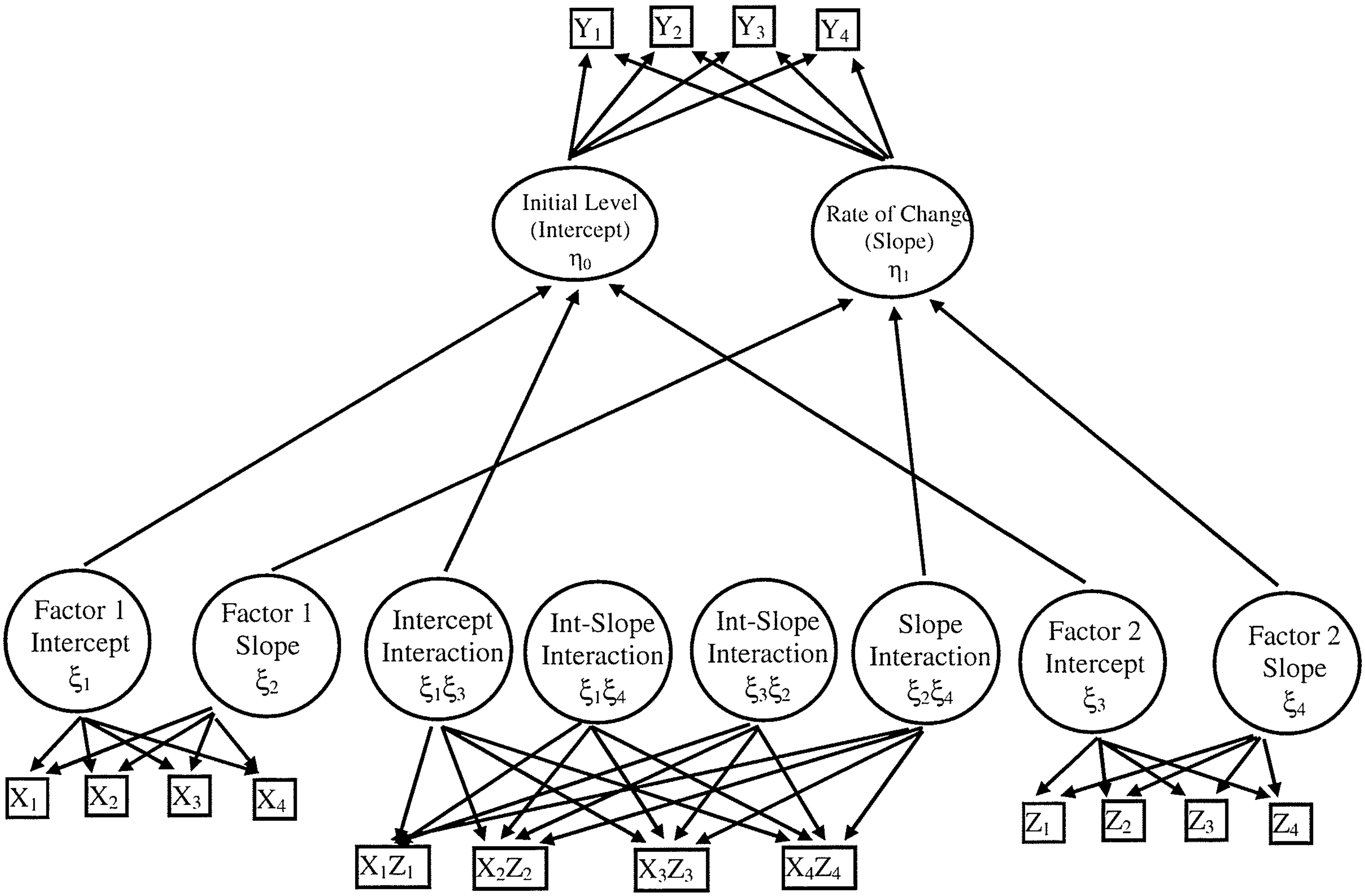
Note: ξ_1 represents a latent independent variable with X_1 and X_2 as observed indicators, ξ_2 represents the latent moderator variable with M_1 and M_2 as observed indicators, and $\xi_1\xi_2$ represents the interaction latent variable between the independent variable and the moderator; η represents the latent dependent variable, and ε represents the error term associated with the latent dependent variable.

Figure 12
Latent Interaction Growth Model (Based on Duncan et al., 1999) with Four Assessment Points.



Note: Error terms and covariances are omitted from the figure for clarity.

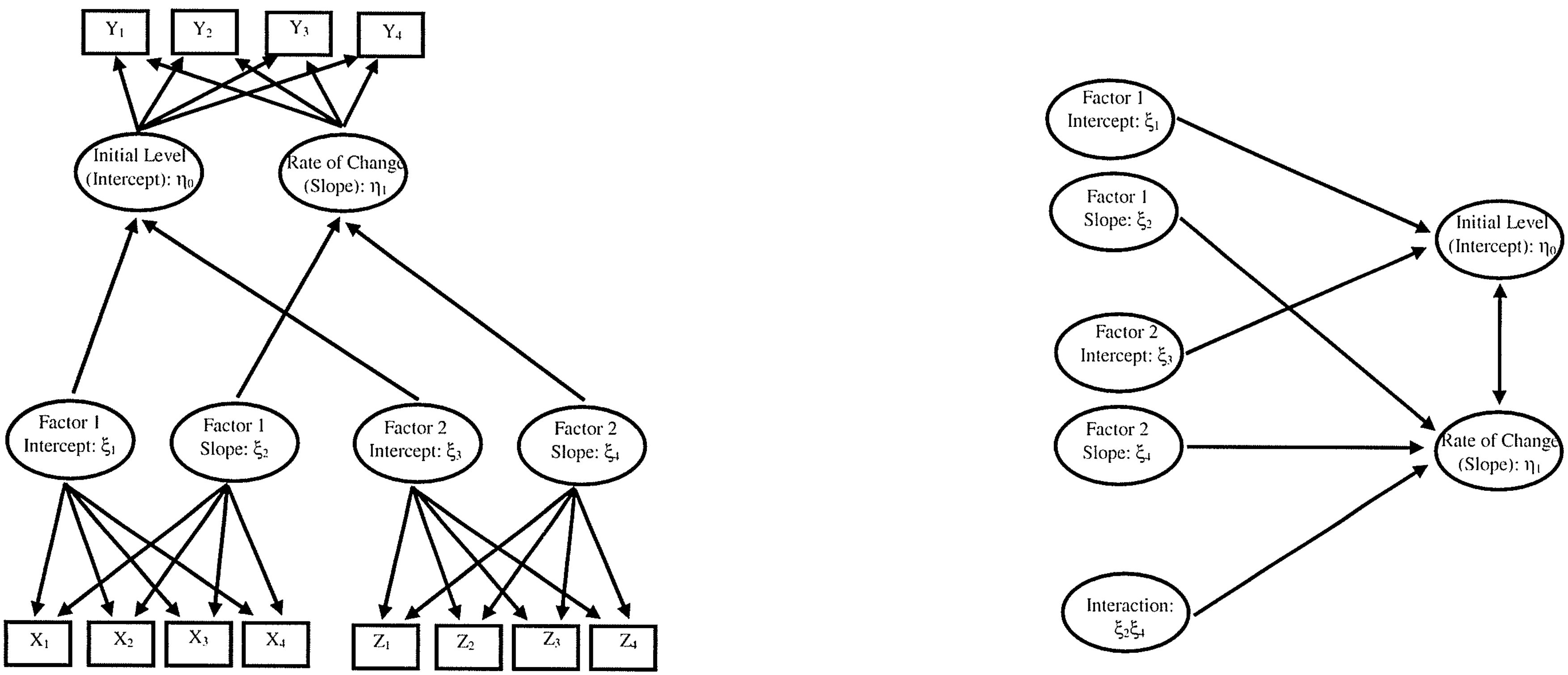
Figure 13
The Full Interaction Latent Growth Model (Wen et al., 2002) with Four Assessment Points.



Note: Error terms and covariances are omitted from the figure for clarity.

Figure 14

Graphical Depiction of the Steps Involved for the Schumacker Latent Growth Interaction Model.



Step 1: Model a latent growth model with no interaction terms.
Step 2: Save the latent factor scores from this model.

Step 3: Use those factor scores from Step 2 in the above path analytic model.

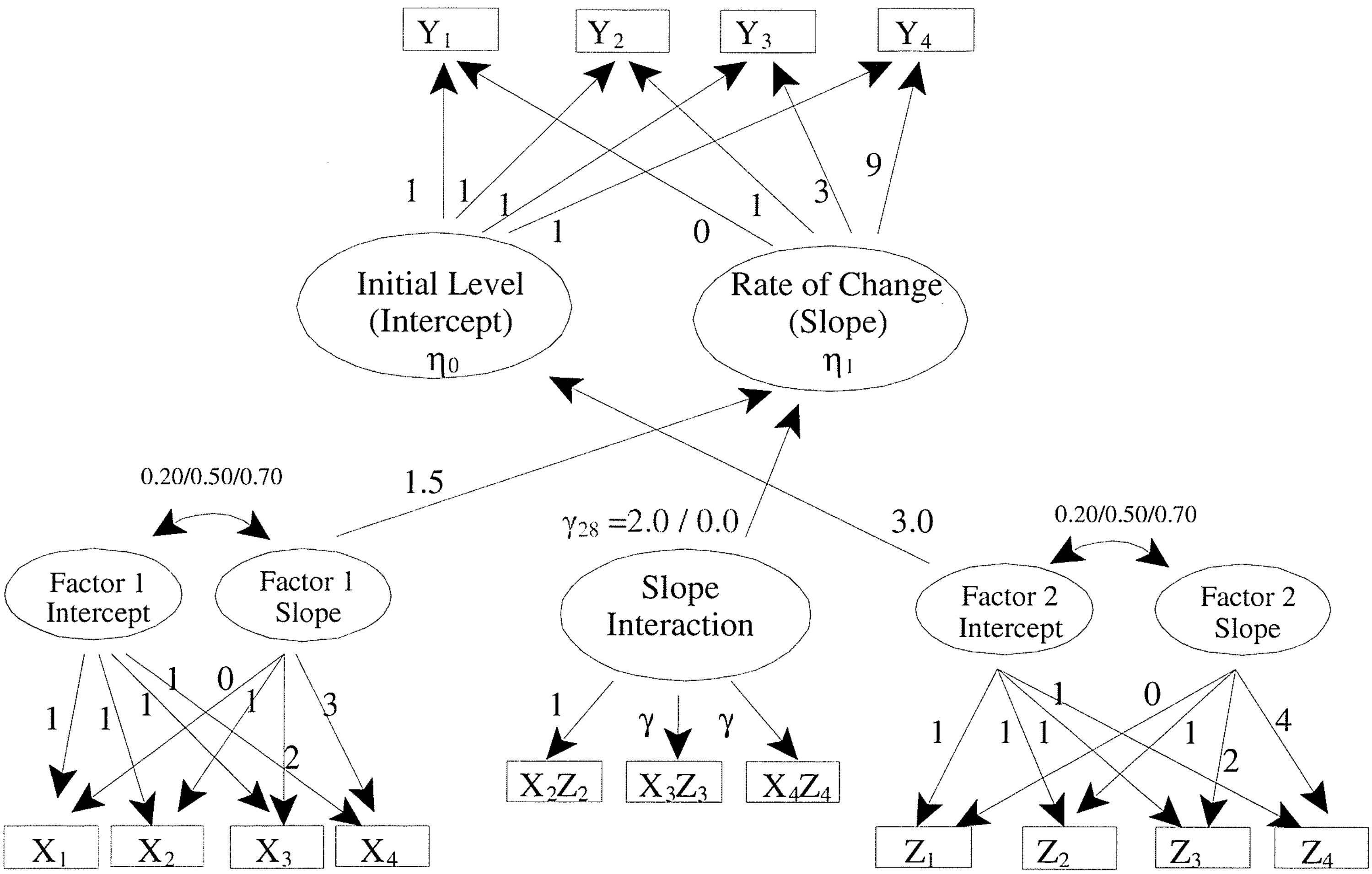
Note: Error terms and covariances are omitted from the figure for clarity.

Figure 15
Potential Patterns of Missing Data in a Longitudinal Study.

Subject	Univariate			Monotone			Intermittent		
	X_1	X_2	X_3	X_1	X_2	X_3	X_1	X_2	X_3
1							?		
2									?
3									
4		?						?	
5		?							
6		?			?	?			
7		?		?	?	?		?	?

Note: X_1, X_2, and X_3 represent repeated measurements on the same individuals; the "?" symbol represents those values that are missing for a particular subject at the repeated measurements.

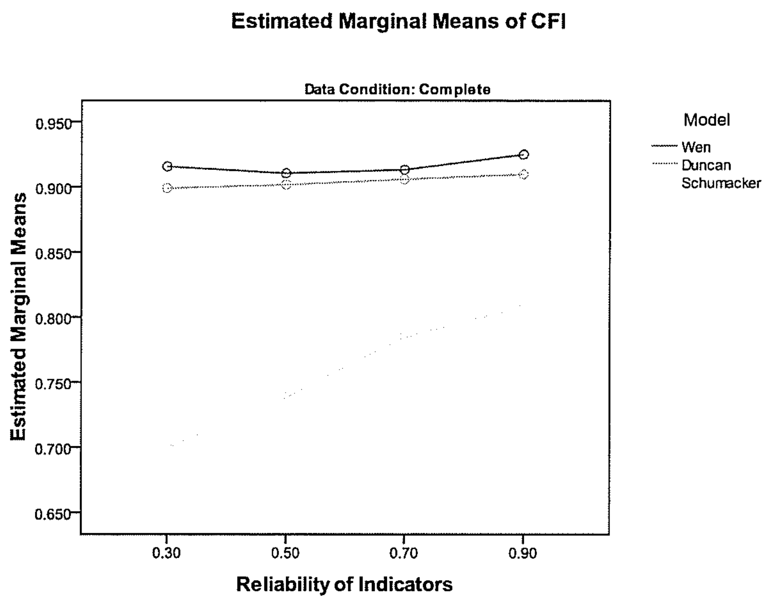
Figure 16
Latent Interaction Growth Model Used to Generate the Simulation Data.



Note: The correlation between the latent intercepts and slopes will be set at 0.20, 0.50, and 0.70. The reliability of the observed indicators (X_1-X_4, Z_1-Z_4, Y_1-Y_4) will be set at 0.30, 0.50, 0.70, and 0.90. The sample size will be set at 250, 500, and 1000. The type of missing data will be set at none (complete) data, missing completely at random data, and missing not at random data.

Figure 17

Plots of the Interaction Effect of Latent Interaction Model Type Factor with the Observed Indicator Reliability Factor (1) and Latent Intercept-Slope Correlation (2) on the Comparative Fit Index (CFI) in the Complete Data Condition in Those Models that Converged Successfully.



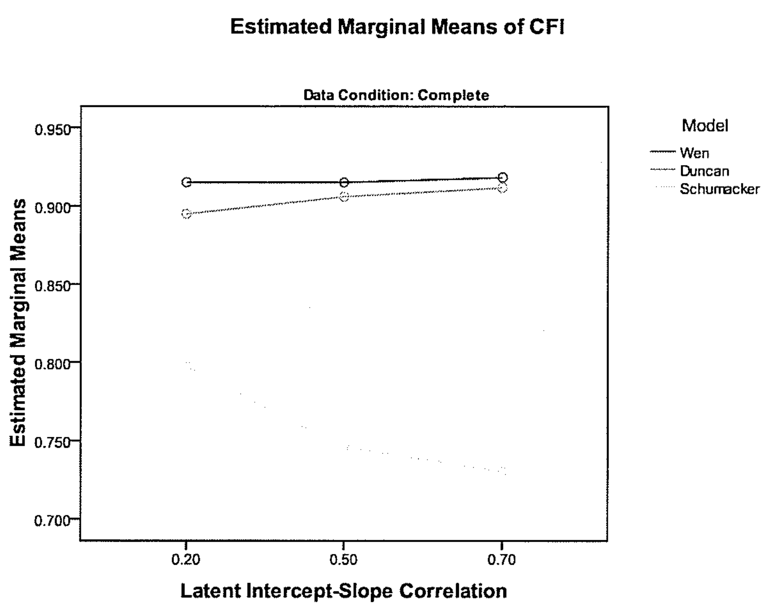
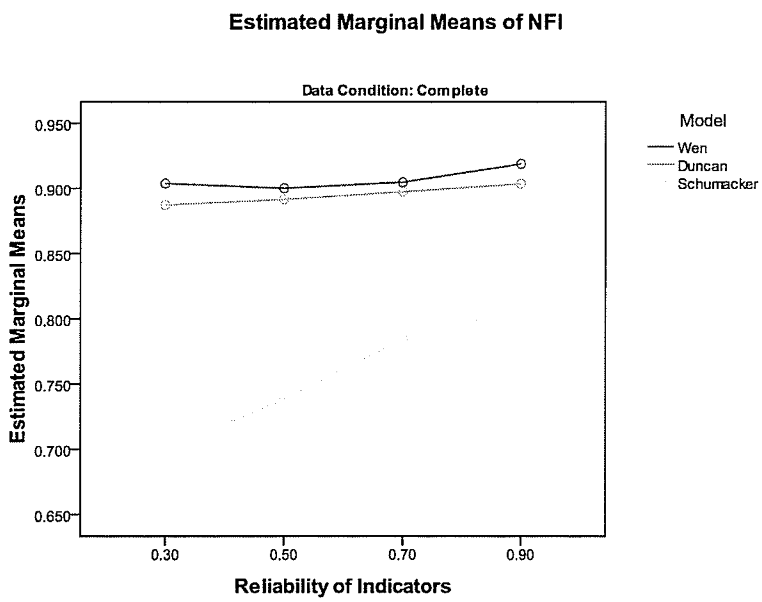


Figure 18

Plots of the Interaction Effect of Latent Interaction Model Type Factor with the Observed Indicator Reliability Factor (1) and Latent Intercept-Slope Correlation (2) on the Normed Fit Index (NFI) in the Complete Data Condition in Those Models that Converged Successfully.



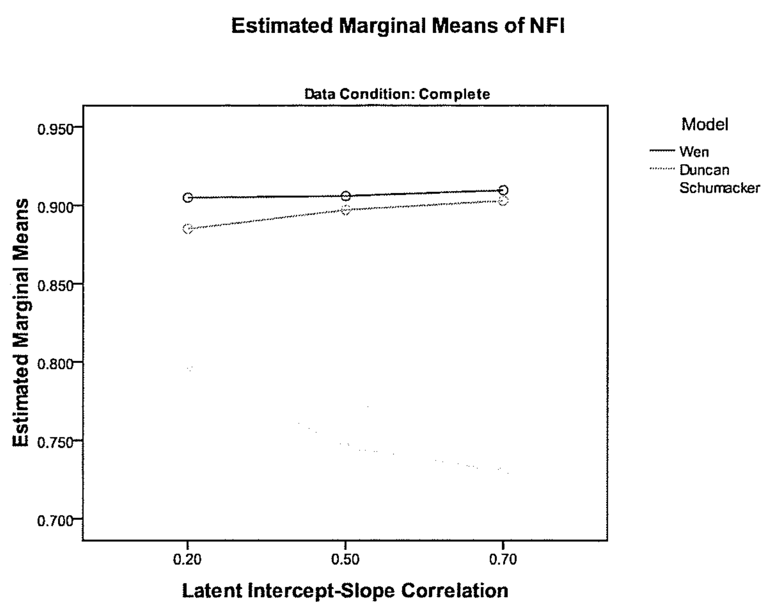
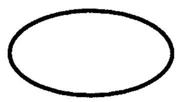


Table 1
Definition of Symbols Used in Structural Equation Models.

Symbol	Meaning
	Observed variable
	Latent variable / error variable
	Direct Path
	Correlation

Note: Notation for latent variables can be either with a η or an ξ symbol.

Table 2
Convergence Frequency for All Latent Growth Interaction Models (Population Value of Latent Interaction Parameter Equal to 2.0) For 5000 Simulations, For the Three Missing Data Conditions (Complete, MCAR, MNAR).

Corr	N	Rel	Complete Data			MCAR Data			MNAR Data		
			Wen	Duncan	Schumacker	Wen	Duncan	Schumacker	Wen	Duncan	Schumacker
0.20	250	0.30	4881	5000	5000	4854	5000	5000	3692	4999	5000
		0.50	4991	5000	5000	4975	5000	5000	3666	5000	5000
		0.70	5000	5000	5000	4999	5000	5000	4158	5000	5000
		0.90	5000	5000	5000	5000	5000	5000	4955	5000	5000
	500	0.30	4947	5000	5000	4943	5000	5000	3808	5000	5000
		0.50	4998	5000	5000	4996	5000	5000	3742	5000	5000
		0.70	5000	5000	5000	5000	5000	5000	4316	5000	5000
		0.90	5000	5000	5000	5000	5000	5000	4981	5000	5000
	1000	0.30	4979	5000	5000	4967	5000	5000	4026	5000	5000
		0.50	5000	5000	5000	5000	5000	5000	3884	5000	5000
		0.70	5000	5000	5000	5000	5000	5000	4498	5000	5000
		0.90	5000	5000	5000	5000	5000	5000	4996	5000	5000
.50	250	0.30	4809	5000	5000	4760	5000	5000	2670	5000	5000
		0.50	4934	5000	5000	4900	5000	5000	2410	5000	5000
		0.70	4999	5000	5000	5000	5000	5000	2290	5000	5000
		0.90	5000	5000	5000	5000	5000	5000	4581	5000	5000
	500	0.30	4932	5000	5000	4899	5000	5000	2617	5000	5000
		0.50	4985	5000	5000	4968	5000	5000	2313	5000	5000
		0.70	5000	5000	5000	5000	5000	5000	1902	5000	5000
		0.90	5000	5000	5000	5000	5000	5000	4755	5000	5000
	1000	0.30	4985	5000	5000	4971	5000	5000	2590	5000	5000
		0.50	5000	5000	5000	4996	5000	5000	2313	5000	5000
		0.70	5000	5000	5000	5000	5000	5000	1469	5000	5000
		0.90	5000	5000	5000	5000	5000	5000	4835	5000	5000
.70	250	0.30	4742	5000	5000	4639	5000	5000	2261	5000	5000
		0.50	4879	5000	5000	4838	5000	5000	2043	5000	5000
		0.70	4998	5000	5000	4991	5000	5000	2379	5000	5000
		0.90	5000	5000	5000	5000	5000	5000	4184	5000	5000
	500	0.30	4872	5000	5000	4822	5000	5000	1954	5000	5000
		0.50	4973	5000	5000	4933	5000	5000	1634	5000	5000
		0.70	5000	5000	5000	4998	5000	5000	2217	5000	5000
		0.90	5000	5000	5000	5000	5000	5000	4354	5000	5000
	1000	0.30	4960	5000	5000	4934	5000	5000	1854	5000	5000
		0.50	4996	5000	5000	4987	5000	5000	1467	5000	5000
		0.70	5000	5000	5000	5000	5000	5000	2252	5000	5000
		0.90	5000	5000	5000	5000	5000	5000	4539	5000	5000

Note: MCAR = Missing Completely At Random; MNAR = Missing Not At Random; Corr = Latent intercept-slope correlation; N = Sample size; Rel = Reliability of the observed indicator.

Table 3
Mean Number of Iterations (with Standard Deviation in parentheses) for All Latent Growth Interaction Models where the Population Value of Latent Interaction Parameter was Equal to 2.0, In Those Simulations That Converged For the Three Missing Data Conditions (Complete, MCAR, MNAR).

Corr	N	Rel	Complete Data			MCAR Data			MNAR Data		
			Wen	Duncan	Schumacker	Wen	Duncan	Schumacker	Wen	Duncan	Schumacker
0.20	250	0.30	509.83	230.51	5.16	525.00	241.91	5.65	1772.03	139.26	5.93
			(291.182)	(108.028)	(2.951)	(344.637)	(130.156)	(3.758)	(581.932)	(54.598)	(2.817)
		0.50	337.55	202.59	4.92	349.43	204.11	5.51	1658.82	132.68	5.77
			(177.743)	(22.211)	(2.551)	(205.007)	(29.143)	(4.233)	(632.107)	(18.636)	(2.903)
		0.70	183.85	181.66	4.56	192.01	183.91	5.02	1133.57	135.63	5.33
			(41.041)	(22.737)	(1.977)	(65.142)	(24.801)	(3.017)	(731.272)	(26.394)	(2.103)
		0.90	164.80	172.83	4.45	165.67	175.19	4.77	452.91	136.60	4.63
			(10.719)	(15.667)	(1.794)	(12.197)	(18.998)	(2.168)	(417.678)	(33.326)	(1.809)
	500	0.30	510.48	212.93	4.57	514.38	216.29	4.76	1745.22	132.37	5.74
			(233.485)	(43.199)	(1.194)	(275.198)	(59.484)	(1.919)	(584.006)	(11.907)	(1.246)
		0.50	311.98	201.25	4.30	319.62	201.95	4.50	1638.78	128.53	5.47
			(123.846)	(16.734)	(1.308)	(142.888)	(19.534)	(1.598)	(562.391)	(13.609)	(1.247)
		0.70	175.08	176.74	3.93	178.63	178.08	4.14	1050.72	133.90	5.04
			(16.861)	(17.322)	(1.176)	(31.368)	(19.145)	(1.359)	(690.658)	(24.065)	(1.348)
		0.90	164.51	169.48	3.96	164.92	170.47	4.16	374.28	133.18	3.99
			(8.100)	(8.750)	(1.362)	(9.086)	(10.658)	(1.537)	(329.442)	(29.425)	(1.328)
	1000	0.30	508.51	209.76	4.49	511.23	211.02	4.51	1699.82	129.62	5.85
			(168.585)	(20.127)	(0.988)	(202.820)	(34.844)	(1.054)	(576.568)	(8.624)	(0.890)
		0.50	306.17	199.45	4.13	309.31	200.71	4.19	1611.70	125.77	5.62
			(111.060)	(12.686)	(0.006)	(117.378)	(14.821)	(1.046)	(539.930)	(10.250)	(0.932)
		0.70	173.44	173.60	3.62	173.87	174.46	3.74	993.37	131.32	5.03
			(8.199)	(12.340)	(1.001)	(10.680)	(14.116)	(1.071)	(642.100)	(22.551)	(1.084)
		0.90	164.26	168.39	3.67	164.47	168.87	3.77	337.30	131.70	3.60
			(6.029)	(6.986)	(1.138)	(6.816)	(7.576)	(1.250)	(280.847)	(27.389)	(1.111)
.50	250	0.30	475.31	220.677	5.31	497.14	224.10	5.94	1899.96	144.11	5.18
			(387.922)	(40.016)	(3.750)	(409.886)	(58.098)	(4.881)	(578.413)	(30.013)	(3.482)
		0.50	300.43	197.81	5.16	322.71	199.68	5.76	1935.32	141.21	5.00
			(281.686)	(21.863)	(3.306)	(303.420)	(24.967)	(4.495)	(577.110)	(22.707)	(3.090)
		0.70	184.04	180.566	5.28	187.76	182.80	5.79	1859.73	142.76	4.87
			(14.596)	(12.504)	(2.851)	(48.837)	(15.023)	(3.826)	(676.576)	(27.470)	(2.692)
		0.90	171.28	196.05	6.13	172.04	197.27	6.33	537.66	145.00	5.38
			(11.997)	(12.201)	(2.365)	(13.646)	(14.336)	(3.074)	(600.108)	(30.722)	(2.542)
	500	0.30	424.02	216.75	4.29	452.08	218.04	4.67	1956.01	139.96	4.23
			(340.192)	(22.894)	(1.665)	(361.692)	(28.276)	(2.449)	(559.102)	(16.234)	(1.622)
		0.50	262.78	196.17	4.40	282.50	196.64	4.68	1986.29	138.11	4.08
			(224.428)	(17.606)	(1.710)	(261.391)	(19.591)	(2.336)	(588.549)	(20.185)	(1.390)
		0.70	182.38	177.54	4.73	182.89	178.84	4.88	1983.55	139.67	4.08
			(8.797)	(9.004)	(1.562)	(9.576)	(10.594)	(1.893)	(630.741)	(26.066)	(1.391)
		0.90	169.91	195.26	5.88	170.37	195.57	5.91	449.19	144.44	4.93
			(8.442)	(9.491)	(1.446)	(9.666)	(10.839)	(1.700)	(524.624)	(29.655)	(1.661)

	1000	0.30	369.94	216.88	3.80	391.66	217.03	3.98	2024.31	136.79	3.94
			(243.501)	(20.429)	(1.015)	(282.092)	(22.530)	(1.161)	(563.903)	(13.318)	(0.842)
		0.50	232.49	195.17	4.03	242.73	195.58	4.16	2053.01	136.07	3.63
			(140.593)	(14.715)	(1.096)	(172.962)	(15.821)	(1.269)	(563.394)	(18.784)	(0.889)
		0.70	181.78	175.74	4.59	181.94	176.49	4.61	2018.09	139.13	3.56
			(7.124)	(6.886)	(1.081)	(7.599)	(7.505)	(1.257)	(579.381)	(24.673)	(1.026)
		0.90	168.99	194.78	5.83	169.14	194.94	5.85	371.30	145.15	4.88
			(6.301)	(7.210)	(0.980)	(6.981)	(8.254)	(1.157)	(431.407)	(28.374)	(1.223)
.70	250	0.30	504.36	208.37	5.89	515.68	211.53	6.51	2039.36	150.25	5.46
			(480.770)	(29.493)	(4.060)	(486.118)	(33.969)	(5.052)	(573.146)	(23.359)	(4.043)
		0.50	336.32	194.86	5.93	359.06	196.28	6.37	2005.30	148.03	5.32
			(354.058)	(17.873)	(3.929)	(393.736)	(20.145)	(4.714)	(605.273)	(25.693)	(3.702)
		0.70	194.93	187.28	6.11	201.24	188.71	6.35	1799.88	148.44	5.43
			(71.971)	(11.625)	(3.385)	(109.403)	(13.809)	(3.933)	(622.313)	(28.667)	(3.133)
		0.90	185.36	208.75	7.07	186.55	210.06	7.22	671.38	151.26	6.48
			(17.227)	(13.493)	(2.479)	(19.209)	(15.945)	(3.180)	(730.176)	(30.869)	(2.820)
	500	0.30	442.32	203.30	4.96	474.78	204.45	5.31	2116.83	146.93	4.41
			(425.451)	(21.758)	(1.972)	(457.454)	(23.115)	(2.632)	(549.383)	(20.260)	(1.832)
		0.50	287.75	192.82	5.19	303.58	193.60	5.44	2089.26	144.81	4.39
			(283.163)	(14.074)	(1.937)	(312.653)	(15.492)	(2.769)	(578.033)	(23.874)	(1.550)
		0.70	190.31	185.28	5.70	190.78	186.17	5.88	1824.87	146.96	4.88
			(9.379)	(8.967)	(1.728)	(11.250)	(10.040)	(2.574)	(607.351)	(27.518)	(1.715)
		0.90	182.71	207.85	6.93	183.56	208.27	7.00	540.66	149.73	6.25
			(13.354)	(10.118)	(1.547)	(14.987)	(11.736)	(1.867)	(666.937)	(28.925)	(1.750)
	1000	0.30	383.59	201.71	4.71	410.79	202.65	4.81	2184.94	145.59	3.87
			(352.578)	(18.970)	(1.147)	(389.762)	(19.805)	(1.542)	(534.136)	(18.257)	(0.941)
		0.50	252.11	192.15	4.99	265.59	192.46	5.10	2101.92	143.37	4.03
			(182.462)	(12.509)	(1.113)	(228.990)	(13.158)	(1.383)	(580.371)	(22.614)	(1.044)
		0.70	189.98	184.01	5.55	190.07	184.58	5.57	1814.51	145.55	4.55
			(7.432)	(7.112)	(1.073)	(8.427)	(7.880)	(1.279)	(548.154)	(26.378)	(1.127)
		0.90	180.93	207.42	6.86	181.65	207.52	6.86	450.53	149.47	6.12
			(9.880)	(7.653)	(1.044)	(11.438)	(8.872)	(1.217)	(626.779)	(26.905)	(1.155)

Note: MCAR = Missing Completely At Random; MNAR = Missing Not At Random; Corr = Latent intercept-slope correlation; N = Sample size; Rel = Reliability of the observed indicator.

Table 4
Average Comparative Fit Index (CFI) Values (with Standard Deviations in Parentheses) for All Latent Growth Interaction Models (Population Value of Latent Interaction Parameter Equal to 2.0) For Those Simulations That Converged, For the Three Missing Data Conditions (Complete, MCAR, MNAR).

Corr	N	Rel	Complete Data			MCAR Data			MNAR Data		
			Wen	Duncan	Schumacker	Wen	Duncan	Schumacker	Wen	Duncan	Schumacker
0.20	250	0.30	0.92	0.89	0.72	0.92	0.89	0.71	0.80	0.64	0.71
			(0.012)	(0.013)	(0.123)	(0.014)	(0.016)	(0.127)	(0.023)	(0.018)	(0.136)
		0.50	0.91	0.89	0.75	0.91	0.89	0.74	0.77	0.66	0.74
			(0.011)	(0.009)	(0.122)	(0.013)	(0.011)	(0.119)	(0.033)	(0.016)	(0.120)
		0.70	0.91	0.90	0.79	0.91	0.90	0.78	0.72	0.69	0.76
			(0.008)	(0.008)	(0.097)	(0.009)	(0.009)	(0.108)	(0.043)	(0.014)	(0.097)
		0.90	0.92	0.90	0.84	0.92	0.90	0.84	0.69	0.71	0.74
			(0.005)	(0.006)	(0.035)	(0.006)	(0.007)	(0.046)	(0.021)	(0.011)	(0.076)
	500	0.30	0.92	0.89	0.74	0.92	0.89	0.73	0.79	0.64	0.77
			(0.009)	(0.008)	(0.120)	(0.010)	(0.009)	(0.122)	(0.016)	(0.013)	(0.085)
		0.50	0.91	0.89	0.78	0.91	0.89	0.77	0.77	0.66	0.79
			(0.009)	(0.006)	(0.103)	(0.009)	(0.007)	(0.111)	(0.019)	(0.011)	(0.073)
		0.70	0.91	0.90	0.82	0.91	0.90	0.81	0.72	0.69	0.79
			(0.005)	(0.005)	(0.068)	(0.006)	(0.006)	(0.083)	(0.040)	(0.010)	(0.061)
		0.90	0.92	0.90	0.85	0.92	0.90	0.85	0.69	0.71	0.75
			(0.004)	(0.004)	(0.020)	(0.004)	(0.005)	(0.025)	(0.018)	(0.008)	(0.052)
	1000	0.30	0.92	0.89	0.76	0.92	0.89	0.75	0.79	0.64	0.80
			(0.007)	(0.005)	(0.104)	(0.008)	(0.006)	(0.112)	(0.012)	(0.009)	(0.053)
		0.50	0.91	0.89	0.80	0.91	0.89	0.79	0.77	0.66	0.81
			(0.007)	(0.004)	(0.080)	(0.008)	(0.005)	(0.095)	(0.011)	(0.008)	(0.048)
		0.70	0.91	0.90	0.85	0.91	0.90	0.84	0.71	0.69	0.81
			(0.004)	(0.004)	(0.036)	(0.004)	(0.004)	(0.050)	(0.037)	(0.007)	(0.042)
		0.90	0.92	0.90	0.85	0.92	0.90	0.85	0.69	0.71	0.76
			(0.003)	(0.003)	(0.010)	(0.003)	(0.003)	(0.015)	(0.017)	(0.005)	(0.037)
	Average		0.92	0.89	0.80	0.92	0.90	0.79	0.74	0.68	0.77
			(0.101)	(0.010)	(0.010)	(0.011)	(0.011)	(0.104)	(0.051)	(0.029)	(0.085)
.50	250	0.30	0.91	0.90	0.67	0.91	0.90	0.67	0.82	0.68	0.71
			(0.010)	(0.009)	(0.120)	(0.011)	(0.011)	(0.122)	(0.021)	(0.016)	(0.134)
		0.50	0.91	0.90	0.70	0.91	0.90	0.69	0.81	0.69	0.75
			(0.008)	(0.008)	(0.111)	(0.010)	(0.010)	(0.117)	(0.022)	(0.014)	(0.104)
		0.70	0.91	0.91	0.75	0.91	0.91	0.73	0.79	0.71	0.76
			(0.007)	(0.007)	(0.090)	(0.008)	(0.009)	(0.100)	(0.029)	(0.013)	(0.077)
		0.90	0.93	0.91	0.79	0.92	0.91	0.79	0.71	0.73	0.78
			(0.005)	(0.006)	(0.024)	(0.006)	(0.007)	(0.034)	(0.031)	(0.010)	(0.057)
	500	0.30	0.91	0.90	0.68	0.91	0.90	0.68	0.82	0.68	0.77
			(0.007)	(0.006)	(0.115)	(0.008)	(0.008)	(0.117)	(0.017)	(0.011)	(0.084)
		0.50	0.91	0.90	0.73	0.91	0.90	0.72	0.81	0.69	0.78
			(0.006)	(0.006)	(0.098)	(0.007)	(0.007)	(0.103)	(0.019)	(0.010)	(0.063)
		0.70	0.91	0.91	0.78	0.91	0.91	0.76	0.79	0.71	0.79
			(0.005)	(0.005)	(0.061)	(0.006)	(0.006)	(0.076)	(0.022)	(0.009)	(0.049)
		0.90	0.93	0.91	0.80	0.93	0.91	0.80	0.70	0.73	0.79
			(0.003)	(0.004)	(0.016)	(0.004)	(0.005)	(0.019)	(0.025)	(0.007)	(0.038)

	1000	0.30	0.91	0.90	0.70	0.91	0.90	0.70	0.82	0.68	0.80
			(0.006)	(0.004)	(0.100)	(0.006)	(0.005)	(0.106)	(0.015)	(0.008)	(0.052)
		0.50	0.91	0.90	0.75	0.91	0.90	0.74	0.81	0.69	0.80
			(0.004)	(0.004)	(0.075)	(0.005)	(0.005)	(0.085)	(0.017)	(0.007)	(0.041)
		0.70	0.91	0.91	0.80	0.91	0.91	0.79	0.79	0.71	0.80
			(0.003)	(0.003)	(0.034)	(0.004)	(0.004)	(0.046)	(0.020)	(0.006)	(0.032)
		0.90	0.93	0.91	0.80	0.93	0.91	0.80	0.69	0.73	0.79
			(0.002)	(0.003)	(0.011)	(0.003)	(0.003)	(0.013)	(0.020)	(0.005)	(0.026)
	Average		0.92	0.91	0.75	0.92	0.91	0.74	0.77	0.70	0.78
			(0.009)	(0.007)	(0.093)	(0.009)	(0.008)	(0.098)	(0.059)	(0.023)	(0.075)
.70	250	0.30	0.92	0.91	0.66	0.92	0.91	0.65	0.84	**0.69**	**0.69**
			(0.009)	(0.009)	(0.116)	(0.011)	(0.011)	(0.120)	(0.021)	**(0.015)**	**(0.136)**
		0.50	0.91	0.91	0.69	0.91	0.91	0.68	0.83	0.71	0.71
			(0.008)	(0.009)	(0.109)	(0.010)	(0.009)	(0.116)	(0.023)	(0.014)	(0.110)
		0.70	0.92	0.91	0.73	0.92	0.91	0.72	0.81	0.72	0.74
			(0.007)	(0.007)	(0.089)	(0.008)	(0.008)	(0.096)	(0.022)	(0.012)	(0.077)
		0.90	0.93	0.91	0.77	0.93	0.91	0.77	0.72	0.74	0.75
			(0.005)	(0.005)	(0.029)	(0.006)	(0.006)	(0.040)	(0.037)	(0.010)	(0.059)
	500	0.30	0.91	0.91	0.67	0.91	0.91	0.66	0.84	0.69	0.74
			(0.007)	(0.006)	(0.108)	(0.008)	(0.007)	(0.116)	(0.017)	(0.010)	(0.092)
		0.50	0.91	0.91	0.71	0.91	0.91	0.70	0.83	0.70	0.75
			(0.006)	(0.005)	(0.096)	(0.006)	(0.006)	(0.101)	(0.019)	(0.010)	(0.070)
		0.70	0.92	0.91	0.76	0.92	0.91	0.75	0.82	0.72	0.76
			(0.005)	(0.005)	(0.058)	(0.005)	(0.006)	(0.069)	(0.018)	(0.008)	(0.050)
		0.90	0.93	0.91	0.78	0.93	0.91	0.78	0.71	0.74	0.76
			(0.003)	(0.004)	(0.016)	(0.004)	(0.004)	(0.019)	(0.032)	(0.007)	(0.039)
	1000	0.30	0.91	0.91	0.69	0.91	0.91	0.68	0.84	0.69	0.77
			(0.005)	(0.004)	(0.093)	(0.006)	(0.005)	(0.102)	(0.014)	(0.007)	(0.056)
		0.50	0.91	0.91	0.73	0.91	0.91	0.72	0.83	0.70	0.77
			(0.004)	(0.004)	(0.071)	(0.004)	(0.004)	(0.082)	(0.016)	(0.007)	(0.043)
		0.70	0.92	0.91	0.78	0.92	0.91	0.77	0.82	0.72	0.77
			(0.003)	(0.003)	(0.034)	(0.004)	(0.004)	(0.043)	(0.014)	(0.006)	(0.034)
		0.90	0.93	0.91	0.78	0.93	0.91	0.78	0.70	0.74	0.76
			(0.002)	(0.003)	(0.011)	(0.003)	(0.003)	(0.013)	(0.027)	(0.005)	(0.028)
	Average		0.90	0.91	0.73	0.92	0.91	0.072	0.78	0.71	0.75
			(0.008)	(0.006)	(0.088)	(0.009)	(0.007)	(0.095)	(0.062)	(0.020)	(0.086)

Note: MCAR = Missing Completely At Random; MNAR = Missing Not At Random; Corr = Latent intercept-slope correlation; N = Sample size; Rel = Reliability of the observed indicator.

Table 5

Confidence Intervals (95%) for the Average Comparative Fit Index (CFI) Values for All Latent Growth Interaction Models (Population Value of Latent Interaction Parameter Equal to 2.0) For the Three Missing Data Conditions (Complete, MCAR, MNAR).

				Complete			MCAR			MNAR		
Corr	N	Rel		Wen	Duncan	Schumacker	Wen	Duncan	Schumacker	Wen	Duncan	Schumacker
0.20	250	0.30	L	0.919	0.888	0.705	0.918	0.888	0.692	0.797	0.638	0.692
			U	0.921	0.892	0.735	0.922	0.892	0.728	0.803	0.642	0.728
		0.50	L	0.909	0.872	0.497	0.908	0.867	0.493	0.766	0.658	0.724
			U	0.911	0.908	1.003	0.912	0.913	0.987	0.774	0.662	0.756
		0.70	L	0.909	0.899	0.778	0.909	0.899	0.764	0.714	0.688	0.747
			U	0.911	0.901	0.802	0.911	0.901	0.796	0.726	0.692	0.773
		0.90	L	0.919	0.899	0.836	0.919	0.899	0.833	0.687	0.709	0.730
			U	0.921	0.901	0.844	0.921	0.901	0.847	0.693	0.711	0.750
	500	0.30	L	0.919	0.889	0.730	0.919	0.889	0.718	0.789	0.639	0.762
			U	0.921	0.891	0.751	0.921	0.891	0.742	0.792	0.641	0.778
		0.50	L	0.909	0.878	0.566	0.909	0.889	0.759	0.768	0.659	0.783
			U	0.911	0.902	0.994	0.911	0.891	0.781	0.772	0.661	0.797
		0.70	L	0.910	0.900	0.814	0.909	0.899	0.802	0.716	0.689	0.784
			U	0.910	0.900	0.826	0.911	0.901	0.818	0.724	0.691	0.796
		0.90	L	0.920	0.900	0.848	0.920	0.900	0.847	0.688	0.709	0.745
			U	0.920	0.900	0.852	0.920	0.901	0.853	0.692	0.711	0.755
	1000	0.30	L	0.920	0.890	0.754	0.919	0.890	0.742	0.789	0.639	0.796
			U	0.920	0.890	0.766	0.921	0.890	0.758	0.791	0.641	0.804
		0.50	L	0.910	0.882	0.634	0.909	0.890	0.783	0.769	0.660	0.807
			U	0.910	0.898	0.966	0.911	0.890	0.797	0.771	0.661	0.813
		0.70	L	0.910	0.900	0.848	0.910	0.900	0.836	0.708	0.690	0.807
			U	0.910	0.900	0.852	0.910	0.900	0.844	0.712	0.690	0.813
		0.90	L	0.920	0.900	0.849	0.920	0.900	0.849	0.689	0.710	0.758
			U	0.920	0.900	0.851	0.920	0.900	0.851	0.691	0.710	0.762
0.50	250	0.30	L	0.909	0.899	0.655	0.908	0.898	0.652	0.817	0.678	0.692
			U	0.911	0.901	0.685	0.912	0.902	0.688	0.823	0.682	0.728
		0.50	L	0.909	0.884	0.471	0.909	0.899	0.673	0.807	0.688	0.736
			U	0.911	0.916	0.929	0.911	0.901	0.707	0.813	0.692	0.764
		0.70	L	0.909	0.909	0.739	0.909	0.909	0.716	0.786	0.708	0.750
			U	0.911	0.911	0.761	0.911	0.911	0.744	0.794	0.712	0.770
		0.90	L	0.929	0.909	0.787	0.919	0.909	0.785	0.706	0.729	0.772
			U	0.931	0.911	0.793	0.921	0.911	0.795	0.714	0.731	0.788
	500	0.30	L	0.909	0.900	0.670	0.909	0.899	0.668	0.818	0.679	0.762
			U	0.911	0.901	0.690	0.911	0.901	0.692	0.822	0.681	0.778

		0.50	L	0.910	0.888	0.528	0.909	0.899	0.710	0.808	0.689	0.774
			U	0.911	0.912	0.932	0.911	0.901	0.731	0.812	0.691	0.786
		0.70	L	0.910	0.910	0.775	0.909	0.909	0.752	0.788	0.709	0.785
			U	0.910	0.910	0.785	0.911	0.911	0.768	0.792	0.711	0.795
		0.90	L	0.930	0.910	0.799	0.930	0.910	0.798	0.698	0.729	0.786
			U	0.930	0.910	0.801	0.930	0.911	0.802	0.702	0.731	0.794
	1000	0.30	L	0.910	0.900	0.694	0.910	0.900	0.692	0.819	0.680	0.797
			U	0.910	0.900	0.706	0.910	0.900	0.708	0.821	0.681	0.803
		0.50	L	0.910	0.892	0.595	0.910	0.900	0.734	0.809	0.690	0.797
			U	0.910	0.908	0.905	0.910	0.900	0.746	0.811	0.690	0.803
		0.70	L	0.910	0.910	0.798	0.910	0.910	0.787	0.789	0.710	0.798
			U	0.910	0.910	0.802	0.910	0.910	0.793	0.791	0.710	0.802
		0.90	L	0.930	0.910	0.799	0.930	0.910	0.799	0.689	0.730	0.788
			U	0.930	0.910	0.801	0.930	0.910	0.801	0.691	0.730	0.792
0.70	250	0.30	L	0.919	0.909	0.646	0.918	0.908	0.633	0.837	0.688	0.672
			U	0.921	0.911	0.674	0.922	0.912	0.667	0.843	0.692	0.708
		0.50	L	0.909	0.892	0.466	0.909	0.909	0.663	0.827	0.708	0.695
			U	0.911	0.928	0.914	0.911	0.911	0.697	0.833	0.712	0.725
		0.70	L	0.919	0.909	0.719	0.919	0.909	0.706	0.807	0.718	0.730
			U	0.921	0.911	0.741	0.921	0.911	0.734	0.813	0.722	0.750
		0.90	L	0.929	0.909	0.766	0.929	0.909	0.764	0.715	0.739	0.742
			U	0.931	0.911	0.774	0.931	0.911	0.776	0.725	0.741	0.758
	500	0.30	L	0.909	0.910	0.661	0.909	0.909	0.648	0.838	0.689	0.731
			U	0.911	0.911	0.679	0.911	0.911	0.672	0.842	0.691	0.749
		0.50	L	0.910	0.900	0.513	0.909	0.909	0.690	0.828	0.699	0.743
			U	0.911	0.920	0.907	0.911	0.911	0.710	0.832	0.701	0.757
		0.70	L	0.920	0.910	0.755	0.920	0.909	0.743	0.818	0.719	0.755
			U	0.920	0.910	0.765	0.921	0.911	0.757	0.822	0.721	0.765
		0.90	L	0.930	0.910	0.779	0.930	0.910	0.778	0.707	0.739	0.756
			U	0.930	0.910	0.781	0.930	0.910	0.782	0.713	0.741	0.764
	1000	0.30	L	0.910	0.910	0.684	0.910	0.910	0.673	0.839	0.690	0.766
			U	0.910	0.910	0.696	0.910	0.910	0.687	0.841	0.690	0.774
		0.50	L	0.910	0.902	0.584	0.910	0.910	0.714	0.829	0.700	0.767
			U	0.910	0.918	0.876	0.910	0.910	0.726	0.831	0.700	0.773
		0.70	L	0.920	0.910	0.778	0.920	0.910	0.767	0.819	0.720	0.768
			U	0.920	0.910	0.782	0.920	0.910	0.773	0.821	0.720	0.772
		0.90	L	0.930	0.910	0.779	0.930	0.910	0.779	0.698	0.740	0.758
			U	0.930	0.910	0.781	0.930	0.910	0.781	0.702	0.740	0.762

Note: MCAR = Missing Completely At Random; MNAR = Missing Not At Random; Corr = Latent intercept-slope correlation; N = Sample size; Rel = Reliability of the observed indicator; L = Lower 95% limit; U = Upper 95% limit.

Table 6

Analysis of Variance Results with Comparative Fit Index (CFI) Values as the Dependent Variable, with Latent Model Type, Latent Intercept-Slope Correlation, Sample Size, and Reliability of Observed Indicators as Between-Subjects Factors, For the Three Missing Data Conditions (Complete, MCAR, MNAR).

Model Effect	df	Complete Data		MCAR Data		MNAR Data	
		F-value	Partial eta^2	F-value	Partial eta^2	F-value	Partial eta^2
Corrected Model	107	13599.94	.732	12805.37	.722	4456.85	.501
Latent Interaction Model	2	625752.50	.702	597603.27	.693	120586.55	.337
Correlation (Corr)	2	6000.76	.022	5003.64	.019	13111.61	.052
Sample Size (N)	2	2934.51	.011	2923.65	.011	3984.76	.017
Reliability (Rel)	3	21817.36	.110	19681.74	.100	5431.93	.033
Latent Model X Corr	4	14691.94	.100	12317.34	.085	9606.90	.075
Latent Model X N	4	2940.86	.022	2881.55	.021	5535.97	.045
Latent Model X Rel	6	13231.16	.130	12102.99	.121	25305.06	.242
Corr X N	4	7.95	.000	7.54	.000	16.88	.000
Corr X Rel	6	57.55	.001	45.56	.001	250.26	.003
N X Rel	6	218.60	.002	192.60	.002	596.95	.007
Latent Model X Corr X N	8	8.66	.000	8.79	.000	31.64	.001
Latent Model X Corr X Rel	12	75.23	.002	61.59	.001	1128.74	.028
Latent Model X N X Rel	12	226.78	.005	187.52	.004	441.97	.011
Corr X N X Rel	12	1.58 ns	.000	2.34	.000	7.65	.000
Latent Model X Corr X N X Rel	24	2.15	.000	2.22	.000	9.49	.000
Error	531703						

Note: MCAR = Missing Completely At Random; MNAR = Missing Not At Random; Corr = Latent intercept-slope correlation; N = Sample size; Rel = Reliability of the observed indicator. The symbol “X” represents an interaction between two factors.
All effects are significant at the $p < 0.01$ level.

Table 7
Average Normed Fit Index (NFI) Values (with Standard Deviation in parentheses) for All Latent Growth Interaction Models (Population Value of Latent Interaction Parameter Equal to 2.0) For Those Simulations That Converged, For the Three Missing Data Conditions (Complete, MCAR, MNAR).

Corr	N	Rel	Complete Data			MCAR Data			MNAR Data		
			Wen	Duncan	Schumacker	Wen	Duncan	Schumacker	Wen	Duncan	Schumacker
0.20	250	0.30	0.90	0.87	0.72	0.89	0.86	0.71	0.77	0.62	0.71
			(0.012)	(0.013)	(0.122)	(0.013)	(0.015)	(0.125)	(0.022)	(0.018)	(0.131)
		0.50	0.89	0.87	0.75	0.89	0.86	0.74	0.75	0.65	0.74
			(0.011)	(0.009)	(0.121)	(0.012)	(0.011)	(0.117)	(0.032)	(0.016)	(0.115)
		0.70	0.89	0.88	0.79	0.89	0.88	0.78	0.71	0.67	0.76
			(0.008)	(0.008)	(0.096)	(0.009)	(0.009)	(0.106)	(0.042)	(0.014)	(0.091)
		0.90	0.91	0.89	0.84	0.91	0.89	0.83	0.68	0.70	0.74
			(0.005)	(0.006)	(0.035)	(0.006)	(0.007)	(0.045)	(0.021)	(0.011)	(0.070)
	500	0.30	0.91	0.88	0.74	0.91	0.87	0.73	0.78	0.63	0.77
			(0.009)	(0.008)	(0.119)	(0.010)	(0.009)	(0.121)	(0.016)	(0.013)	(0.083)
		0.50	0.90	0.88	0.79	0.90	0.88	0.77	0.76	0.65	0.79
			(0.009)	(0.006)	(0.103)	(0.009)	(0.007)	(0.110)	(0.019)	(0.011)	(0.071)
		0.70	0.90	0.89	0.82	0.90	0.89	0.81	0.71	0.68	0.79
			(0.005)	(0.005)	(0.067)	(0.006)	(0.006)	(0.082)	(0.040)	(0.010)	(0.059)
		0.90	0.92	0.90	0.85	0.92	0.90	0.85	0.68	0.71	0.75
			(0.004)	(0.005)	(0.020)	(0.004)	(0.005)	(0.025)	(0.018)	(0.008)	(0.050)
	1000	0.30	0.92	0.88	0.76	0.91	0.88	0.75	0.79	0.64	0.80
			(0.007)	(0.005)	(0.104)	(0.008)	(0.006)	(0.111)	(0.012)	(0.009)	(0.053)
		0.50	0.90	0.87	0.80	0.90	0.88	0.79	0.77	0.66	0.81
			(0.007)	(0.004)	(0.079)	(0.008)	(0.005)	(0.095)	(0.011)	(0.008)	(0.048)
		0.70	0.90	0.89	0.85	0.90	0.89	0.84	0.71	0.68	0.80
			(0.004)	(0.004)	(0.036)	(0.004)	(0.004)	(0.049)	(0.037)	(0.007)	(0.042)
		0.90	0.92	0.90	0.85	0.92	0.90	0.85	0.68	0.71	0.76
			(0.003)	(0.003)	(0.010)	(0.003)	(0.003)	(0.014)	(0.017)	(0.006)	(0.036)
	Average		0.90	0.88	0.80	0.90	0.88	0.79	0.73	0.67	0.77
			(0.012)	(0.013)	(0.097)	(0.013)	(0.015)	(0.103)	(0.048)	(0.031)	(0.082)
.50	250	0.30	0.89	0.88	0.67	0.89	0.87	0.67	0.80	0.66	0.71
			(0.010)	(0.009)	(0.119)	(0.011)	(0.011)	(0.120)	(0.020)	(0.016)	(0.131)
		0.50	0.89	0.89	0.70	0.89	0.88	0.69	0.79	0.68	0.74
			(0.008)	(0.008)	(0.110)	(0.010)	(0.010)	(0.116)	(0.021)	(0.014)	(0.101)
		0.70	0.90	0.89	0.75	0.89	0.89	0.73	0.78	0.70	0.76
			(0.007)	(0.007)	(0.090)	(0.008)	(0.008)	(0.099)	(0.028)	(0.012)	(0.074)
		0.90	0.91	0.90	0.79	0.91	0.90	0.79	0.70	0.72	0.78
			(0.005)	(0.006)	(0.024)	(0.006)	(0.007)	(0.034)	(0.030)	(0.010)	(0.054)
	500	0.30	0.90	0.89	0.68	0.90	0.89	0.68	0.81	0.67	0.77
			(0.007)	(0.006)	(0.114)	(0.008)	(0.008)	(0.116)	(0.017)	(0.011)	(0.083)
		0.50	0.90	0.90	0.73	0.90	0.89	0.72	0.80	0.69	0.78
			(0.006)	(0.006)	(0.098)	(0.007)	(0.007)	(0.103)	(0.019)	(0.010)	(0.062)
		0.70	0.91	0.90	0.78	0.90	0.90	0.76	0.79	0.71	0.79
			(0.005)	(0.005)	(0.060)	(0.006)	(0.006)	(0.076)	(0.022)	(0.009)	(0.048)

		0.90	0.92 (0.003)	0.91 (0.004)	0.80 (0.016)	0.92 (0.004)	0.90 (0.005)	0.80 (0.019)	0.70 (0.025)	0.73 (0.007)	0.79 (0.037)
	1000	0.30	0.90 (0.006)	0.90 (0.004)	0.70 (0.100)	0.90 (0.006)	0.89 (0.005)	0.70 (0.106)	0.82 (0.015)	0.67 (0.008)	0.80 (0.051)
		0.50	0.90 (0.004)	0.90 (0.004)	0.75 (0.075)	0.90 (0.005)	0.90 (0.005)	0.74 (0.085)	0.80 (0.017)	0.69 (0.007)	0.80 (0.040)
		0.70	0.91 (0.003)	0.90 (0.003)	0.80 (0.034)	0.91 (0.004)	0.90 (0.004)	0.79 (0.046)	0.79 (0.020)	0.71 (0.006)	0.80 (0.032)
		0.90	0.92 (0.002)	0.91 (0.003)	0.80 (0.011)	0.92 (0.003)	0.91 (0.003)	0.80 (0.013)	0.69 (0.020)	0.73 (0.005)	0.79 (0.026)
Average			0.91 (0.011)	0.90 (0.010)	0.75 (0.092)	0.90 (0.013)	0.89 (0.012)	0.74 (0.098)	0.76 (0.055)	0.70 (0.025)	0.78 (0.074)
.70	250	0.30	0.90 (0.009)	0.89 (0.009)	0.66 (0.115)	0.89 (0.011)	0.88 (0.010)	0.65 (0.119)	0.82 (0.020)	0.67 (0.015)	0.69 (0.133)
		0.50	0.90 (0.008)	0.89 (0.008)	0.69 (0.108)	0.89 (0.010)	0.89 (0.009)	0.68 (0.114)	0.81 (0.022)	0.69 (0.013)	0.71 (0.107)
		0.70	0.90 (0.007)	0.90 (0.007)	0.73 (0.089)	0.90 (0.008)	0.89 (0.008)	0.72 (0.094)	0.80 (0.021)	0.71 (0.012)	0.74 (0.075)
		0.90	0.92 (0.005)	0.90 (0.005)	0.77 (0.029)	0.91 (0.006)	0.90 (0.006)	0.77 (0.039)	0.71 (0.036)	0.73 (0.009)	0.75 (0.056)
	500	0.30	0.90 (0.007)	0.90 (0.006)	0.67 (0.107)	0.90 (0.008)	0.90 (0.007)	0.66 (0.115)	0.83 (0.016)	0.68 (0.010)	0.74 (0.091)
		0.50	0.90 (0.006)	0.90 (0.005)	0.71 (0.010)	0.90 (0.006)	0.90 (0.006)	0.70 (0.101)	0.82 (0.019)	0.70 (0.010)	0.75 (0.069)
		0.70	0.91 (0.005)	0.91 (0.005)	0.76 (0.058)	0.91 (0.005)	0.90 (0.006)	0.75 (0.069)	0.81 (0.018)	0.72 (0.008)	0.76 (0.049)
		0.90	0.92 (0.003)	0.91 (0.004)	0.78 (0.016)	0.92 (0.004)	0.91 (0.004)	0.78 (0.019)	0.71 (0.031)	0.73 (0.007)	0.76 (0.038)
	1000	0.30	0.91 (0.005)	0.90 (0.004)	0.69 (0.093)	0.91 (0.006)	0.90 (0.005)	0.68 (0.101)	0.83 (0.014)	0.68 (0.007)	0.77 (0.056)
		0.50	0.91 (0.004)	0.91 (0.004)	0.73 (0.071)	0.91 (0.004)	0.91 (0.004)	0.72 (0.082)	0.82 (0.016)	0.70 (0.007)	0.77 (0.043)
		0.70	0.91 (0.003)	0.91 (0.003)	0.78 (0.034)	0.91 (0.004)	0.91 (0.004)	0.77 (0.043)	0.81 (0.014)	0.72 (0.006)	0.77 (0.034)
		0.90	0.93 (0.002)	0.91 (0.003)	0.78 (0.011)	0.92 (0.003)	0.91 (0.003)	0.78 (0.013)	0.70 (0.026)	0.73 (0.005)	0.76 (0.027)
Average			0.91 (0.010)	0.90 (0.008)	0.73 (0.087)	0.91 (0.012)	0.90 (0.010)	0.72 (0.094)	0.77 (0.059)	0.71 (0.022)	0.75 (0.075)

Note: MCAR = Missing Completely At Random; MNAR = Missing Not At Random; Corr = Latent intercept-slope correlation; N = Sample size; Rel = Reliability of the observed indicator.

Table 8
Confidence Intervals (95%) for the Average Normed Fit Index (NFI) Values for All Latent Growth Interaction Models (Population Value of Latent Interaction Parameter Equal to 2.0) For the Three Missing Data Conditions (Complete, MCAR, MNAR).

				Complete Data			MCAR Data			MNAR Data		
Corr	N	Rel		Wen	Duncan	Schumacker	Wen	Duncan	Schumacker	Wen	Duncan	Schumacker
0.20	250	0.30	L	0.899	0.868	0.705	0.888	0.858	0.692	0.767	0.618	0.692
			U	0.901	0.872	0.735	0.892	0.862	0.728	0.773	0.622	0.728
		0.50	L	0.889	0.869	0.735	0.888	0.858	0.723	0.746	0.648	0.724
			U	0.891	0.871	0.765	0.892	0.862	0.757	0.754	0.652	0.756
		0.70	L	0.889	0.879	0.778	0.889	0.879	0.765	0.704	0.668	0.748
			U	0.891	0.881	0.802	0.891	0.881	0.795	0.716	0.672	0.772
		0.90	L	0.909	0.889	0.836	0.909	0.889	0.823	0.677	0.699	0.731
			U	0.911	0.891	0.844	0.911	0.891	0.837	0.683	0.701	0.749
	500	0.30	L	0.909	0.879	0.730	0.909	0.869	0.718	0.778	0.629	0.762
			U	0.911	0.881	0.750	0.911	0.871	0.742	0.782	0.631	0.778
		0.50	L	0.899	0.879	0.781	0.899	0.879	0.759	0.758	0.649	0.783
			U	0.901	0.881	0.799	0.901	0.881	0.781	0.762	0.651	0.797
		0.70	L	0.900	0.890	0.814	0.899	0.889	0.802	0.706	0.679	0.784
			U	0.900	0.890	0.826	0.901	0.891	0.818	0.714	0.681	0.796
		0.90	L	0.920	0.900	0.848	0.920	0.899	0.847	0.678	0.709	0.745
			U	0.920	0.900	0.852	0.920	0.901	0.853	0.682	0.711	0.755
	1000	0.30	L	0.920	0.880	0.754	0.909	0.880	0.742	0.789	0.639	0.796
			U	0.920	0.880	0.766	0.911	0.880	0.758	0.791	0.641	0.804
		0.50	L	0.900	0.870	0.795	0.899	0.880	0.783	0.769	0.659	0.807
			U	0.900	0.870	0.805	0.901	0.880	0.797	0.771	0.661	0.813
		0.70	L	0.900	0.890	0.848	0.900	0.890	0.836	0.708	0.680	0.797
			U	0.900	0.890	0.852	0.900	0.890	0.844	0.712	0.680	0.803
		0.90	L	0.920	0.900	0.849	0.920	0.900	0.849	0.679	0.710	0.758
			U	0.920	0.900	0.851	0.920	0.900	0.851	0.681	0.710	0.762
0.50	250	0.30	L	0.889	0.879	0.655	0.888	0.868	0.653	0.797	0.658	0.692
			U	0.891	0.881	0.685	0.892	0.872	0.687	0.803	0.662	0.728
		0.50	L	0.889	0.889	0.686	0.889	0.879	0.673	0.787	0.678	0.726
			U	0.891	0.891	0.714	0.891	0.881	0.707	0.793	0.682	0.754
		0.70	L	0.899	0.889	0.739	0.889	0.889	0.716	0.776	0.698	0.750
			U	0.901	0.891	0.761	0.891	0.891	0.744	0.784	0.702	0.770
		0.90	L	0.909	0.899	0.787	0.909	0.899	0.785	0.696	0.719	0.773
			U	0.911	0.901	0.793	0.911	0.901	0.795	0.704	0.721	0.787
	500	0.30	L	0.899	0.889	0.670	0.899	0.889	0.668	0.808	0.669	0.762
			U	0.901	0.891	0.690	0.901	0.891	0.692	0.812	0.671	0.778

		0.50	L	0.899	0.899	0.721	0.899	0.889	0.709	0.798	0.689	0.774
			U	0.901	0.901	0.739	0.901	0.891	0.731	0.802	0.691	0.786
		0.70	L	0.910	0.900	0.775	0.899	0.899	0.752	0.788	0.709	0.785
			U	0.910	0.900	0.785	0.901	0.901	0.768	0.792	0.711	0.795
		0.90	L	0.920	0.910	0.799	0.920	0.899	0.798	0.698	0.729	0.786
			U	0.920	0.910	0.801	0.920	0.901	0.802	0.702	0.731	0.794
	1000	0.30	L	0.900	0.900	0.694	0.900	0.890	0.692	0.819	0.669	0.797
			U	0.900	0.900	0.706	0.900	0.890	0.708	0.821	0.671	0.803
		0.50	L	0.900	0.900	0.745	0.900	0.900	0.734	0.799	0.690	0.797
			U	0.900	0.900	0.755	0.900	0.900	0.746	0.801	0.690	0.803
		0.70	L	0.910	0.900	0.798	0.910	0.900	0.787	0.789	0.710	0.798
			U	0.910	0.900	0.802	0.910	0.900	0.793	0.791	0.710	0.802
		0.90	L	0.920	0.910	0.799	0.920	0.910	0.799	0.689	0.730	0.788
			U	0.920	0.910	0.801	0.920	0.910	0.801	0.691	0.730	0.792
0.70	250	0.30	L	0.899	0.889	0.646	0.888	0.879	0.633	0.817	0.668	0.672
			U	0.901	0.891	0.674	0.892	0.881	0.667	0.823	0.672	0.708
		0.50	L	0.899	0.889	0.677	0.889	0.889	0.664	0.807	0.688	0.696
			U	0.901	0.891	0.703	0.891	0.891	0.696	0.813	0.692	0.724
		0.70	L	0.899	0.899	0.719	0.899	0.889	0.706	0.797	0.708	0.730
			U	0.901	0.901	0.741	0.901	0.891	0.734	0.803	0.712	0.750
		0.90	L	0.919	0.899	0.766	0.909	0.899	0.764	0.705	0.729	0.742
			U	0.921	0.901	0.774	0.911	0.901	0.776	0.715	0.731	0.758
	500	0.30	L	0.899	0.899	0.661	0.899	0.899	0.648	0.828	0.679	0.731
			U	0.901	0.901	0.679	0.901	0.901	0.672	0.832	0.681	0.749
		0.50	L	0.899	0.900	0.709	0.899	0.899	0.690	0.818	0.699	0.743
			U	0.901	0.900	0.711	0.901	0.901	0.710	0.822	0.701	0.757
		0.70	L	0.910	0.910	0.755	0.909	0.899	0.743	0.808	0.719	0.755
			U	0.910	0.910	0.765	0.911	0.901	0.757	0.812	0.721	0.765
		0.90	L	0.920	0.910	0.779	0.920	0.910	0.778	0.707	0.729	0.756
			U	0.920	0.910	0.781	0.920	0.910	0.782	0.713	0.731	0.764
	1000	0.30	L	0.910	0.900	0.684	0.910	0.900	0.673	0.829	0.680	0.766
			U	0.910	0.900	0.696	0.910	0.900	0.687	0.831	0.680	0.774
		0.50	L	0.910	0.910	0.726	0.910	0.910	0.714	0.819	0.700	0.767
			U	0.910	0.910	0.734	0.910	0.910	0.726	0.821	0.700	0.773
		0.70	L	0.910	0.910	0.778	0.910	0.910	0.767	0.809	0.720	0.768
			U	0.910	0.910	0.782	0.910	0.910	0.773	0.811	0.720	0.772
		0.90	L	0.930	0.910	0.779	0.920	0.910	0.779	0.698	0.730	0.758
			U	0.930	0.910	0.781	0.920	0.910	0.781	0.702	0.730	0.762

Complete = Complete Data; MCAR = Missing Complete at Random Data; MNAR = Missing Not at Random Data; Corr = Correlation between latent intercepts and slopes; N = Sample size; Rel = Reliability of the observed indicator.; L = Lower 95% limit; U = Upper 95% limit.

Table 9
Analysis of Variance Results with Normed Fit Index (NFI) Values as the Dependent Variable, with Latent Model Type, Latent Intercept-Slope Correlation, Sample Size, and Reliability of Observed Indicators as Between-Subjects Factors, For the Three Missing Data Conditions (Complete, MCAR, MNAR).

Model Effect		Complete Data		MCAR Data		MNAR Data	
	df	F-value	Partial eta^2	F-value	Partial eta^2	F-value	Partial eta^2
Corrected Model	107	12496.27	.715	11478.07	.699	4961.38	.528
Latent Interaction Model	2	561513.46	.679	519361.46	.663	141548.37	.374
Correlation (Corr)	2	5285.12	.019	4197.62	.016	14700.13	.058
Sample Size (N)	2	8322.38	.030	10067.81	.037	9748.44	.039
Reliability (Rel)	3	25774.02	.127	24467.69	.122	4365.73	.027
Latent Model X Corr	4	15188.22	.103	12885.76	.089	10153.64	.079
Latent Model X N	4	1381.96	.010	1042.62	.008	3563.60	.029
Latent Model X Rel	6	11858.39	.118	10447.35	.106	25221.59	.242
Corr X N	4	22.33	.000	25.92	.000	27.29	.000
Corr X Rel	6	59.16	.001	47.33	.001	262.64	.003
N X Rel	6	304.89	.003	265.02	.003	890.49	.011
Latent Model X Corr X N	8	6.18	.000	5.81	.000	28.74	.000
Latent Model X Corr X Rel	12	72.57	.002	58.80	.001	1159.34	.028
Latent Model X N X Rel	12	187.84	.004	162.12	.004	351.52	.009
Corr X N X Rel	12	1.92 (.03)	.000	2.86	.000	7.86	.000
Latent Model X Corr X N X Rel	24	2.06	.000	2.04	.000	10.39	.001
Error	531703						

Note: MCAR = Missing Completely At Random; MNAR = Missing Not At Random; Corr = Latent intercept-slope correlation; N = Sample size; Rel = Reliability of the observed indicator.
The symbol "X" represents an interaction between two factors.
All effects are significant at the $p < 0.01$ level.

Table 10

Average Goodness of Fit Index (GFI) Values (with Standard Deviation in parentheses) for All Latent Growth Interaction Models (Population Value of Latent Interaction Parameter Equal to 2.0) For Those Simulations That Converged, For the Three Missing Data Conditions (Complete, MCAR, MNAR).

Corr	N	Rel	Complete Data			MCAR Data			MNAR Data		
			Wen	Duncan	Schumacker	Wen	Duncan	Schumacker	Wen	Duncan	Schumacker
0.20	250	0.30	0.85	0.86	0.77	0.84	0.85	0.76	0.76	0.83	0.85
			(0.015)	(0.010)	(0.058)	(0.017)	(0.012)	(0.061)	(0.020)	(0.011)	(0.063)
		0.50	0.84	0.85	0.78	0.83	0.84	0.77	0.73	0.82	0.87
			(0.024)	(0.008)	(0.057)	(0.024)	(0.010)	(0.059)	(0.025)	(0.011)	(0.055)
		0.70	0.86	0.85	0.80	0.85	0.84	0.79	0.73	0.79	0.90
			(0.015)	(0.008)	(0.054)	(0.018)	(0.010)	(0.057)	(0.039)	(0.012)	(0.038)
		0.90	0.85	0.82	0.84	0.84	0.81	0.83	0.72	0.75	0.92
			(0.008)	(0.009)	(0.026)	(0.009)	0.011)	(0.032)	(0.024)	(0.012)	(0.022)
	500	0.30	0.86	0.88	0.77	0.85	0.87	0.77	0.77	0.84	0.87
			(0.012)	(0.007)	(0.052)	(0.013)	(0.008)	(0.055)	(0.016)	(0.007)	(0.042)
		0.50	0.85	0.87	0.79	0.85	0.87	0.79	0.74	0.83	0.90
			(0.024)	(0.005)	(0.052)	(0.024)	(0.007)	(0.054)	(0.019)	(0.008)	(0.032)
		0.70	0.87	0.86	0.81	0.87	0.86	0.81	0.74	0.81	0.92
			(0.008)	(0.006)	(0.045)	(0.011)	(0.007)	(0.049)	(0.039)	(0.009)	(0.022)
		0.90	0.87	0.83	0.85	0.86	0.83	0.85	0.74	0.77	0.93
			(0.005)	(0.006)	(0.017)	(0.006)	(0.007)	(0.021)	(0.019)	(0.008)	(0.012)
	1000	0.30	0.86	0.88	0.78	0.86	0.88	0.78	0.77	0.85	0.89
			(0.009)	(0.005)	(0.046)	(0.010)	(0.006)	(0.049)	(0.012)	(0.005)	(0.028)
		0.50	0.86	0.88	0.80	0.86	0.88	0.80	0.74	0.84	0.91
			(0.025)	(0.004)	(0.044)	(0.024)	(0.005)	(0.047)	(0.015)	(0.005)	(0.021)
		0.70	0.88	0.87	0.83	0.88	0.87	0.82	0.74	0.82	0.92
			(0.004)	(0.004)	(0.033)	(0.005)	(0.005)	(0.038)	(0.039)	(0.007)	(0.015)
		0.90	0.88	0.84	0.85	0.87	0.84	0.85	0.75	0.77	0.93
			(0.003)	(0.004)	(0.010)	(0.004)	(0.005)	(0.012)	(0.016)	(0.006)	(0.008)
	Average		0.86	0.86	0.81	0.86	0.85	0.80	0.74	0.81	0.90
			(0.019)	(0.021)	(0.053)	(0.021)	(0.023)	(0.055)	(0.029)	(0.032)	(0.042)
.50	250	0.30	0.84	0.86	0.73	0.83	0.85	0.73	0.76	0.83	0.83
			(0.021)	(0.009)	(0.056)	(0.021)	(0.011)	(0.059)	(0.023)	(0.012)	(0.059)
		0.50	0.86	0.86	0.74	**0.84**	**0.84**	0.73	0.73	0.81	0.85
			(0.022)	(0.008)	(0.054)	**(0.024)**	**(0.010)**	(0.057)	(0.022)	(0.012)	(0.049)
		0.70	0.86	0.85	0.76	0.85	0.83	0.75	0.69	0.79	0.87
			(0.008)	(0.009)	(0.050)	(0.012)	(0.011)	(0.054)	(0.027)	(0.013)	(0.036)
		0.90	0.86	0.82	0.80	0.85	0.81	0.79	0.71	0.75	0.91
			(0.008)	(0.009)	(0.027)	(0.009)	(0.011)	(0.033)	(0.033)	(0.013)	(0.020)
	500	0.30	0.86	0.88	0.73	0.85	0.87	0.73	0.77	0.85	0.85
			(0.021)	(0.006)	(0.049)	(0.021)	(0.007)	(0.053)	(0.020)	(0.008)	(0.043)
		0.50	0.88	0.87	0.75	0.87	0.87	0.74	0.73	0.83	0.87
			(0.014)	(0.006)	(0.047)	(0.018)	(0.007)	(0.050)	(0.022)	(0.009)	(0.034)
		0.70	0.88	0.86	0.77	0.87	0.86	0.76	0.69	0.81	0.89
			(0.005)	(0.006)	(0.040)	(0.007)	(0.007)	(0.046)	(0.023)	(0.010)	(0.024)
		0.90	0.87	0.83	0.80	0.87	0.83	0.80	0.72	0.76	0.91
			(0.005)	(0.006)	(0.017)	(0.006)	(0.008)	(0.021)	(0.026)	(0.009)	(0.015)

	1000	0.30	0.87	0.89	0.74	0.86	0.88	0.74	0.78	0.85	0.87
			(0.021)	(0.004)	(0.040)	(0.021)	(0.005)	(0.044)	(0.019)	(0.006)	(0.030)
		0.50	0.89	0.88	0.76	0.89	0.88	0.75	0.74	0.84	0.88
			(0.008)	(0.004)	(0.038)	(0.010)	(0.005)	(0.042)	(0.021)	(0.006)	(0.023)
		0.70	0.89	0.87	0.78	0.86	0.87	0.78	0.70	0.81	0.90
			(0.004)	(0.004)	(0.030)	(0.004)	(0.005)	(0.035)	(0.022)	(0.008)	(0.016)
		0.90	0.88	0.84	0.81	0.88	0.83	0.81	0.72	0.76	0.91
			(0.003)	(0.004)	(0.012)	(0.004)	(0.005)	(0.014)	(0.020)	(0.007)	(0.012)
	Average		0.87	0.86	0.76	0.86	0.85	0.76	0.73	0.81	0.88
			(0.019)	(0.022)	(0.049)	(0.022)	(0.023)	(0.052)	(0.035)	(0.035)	(0.042)
.70	250	0.30	0.85	0.86	0.71	0.84	0.85	0.71	0.76	0.83	0.81
			(0.020)	(0.009)	(0.055)	(0.021)	(0.011)	(0.059)	(0.022)	(0.012)	(0.059)
		0.50	0.86	0.86	0.72	0.85	0.85	0.72	0.73	0.81	0.82
			(0.018)	(0.009)	(0.053)	(0.020)	(0.011)	(0.056)	(0.023)	(0.013)	(0.052)
		0.70	0.86	0.85	0.74	0.85	0.84	0.73	0.69	0.79	0.85
			(0.009)	(0.009)	(0.050)	(0.012)	(0.011)	(0.054)	(0.023)	(0.013)	(0.039)
		0.90	0.86	0.82	0.77	0.85	0.81	0.77	0.70	0.74	0.88
			(0.008)	(0.009)	(0.029)	(0.010)	(0.011)	(0.035)	(0.040)	(0.014)	(0.026)
	500	0.30	0.87	0.88	0.72	0.86	0.87	0.71	0.77	0.84	0.83
			(0.018)	(0.006)	(0.047)	(0.018)	(0.007)	(0.052)	(0.018)	(0.008)	(0.047)
		0.50	0.88	0.87	0.73	0.87	0.87	0.72	0.74	0.83	0.84
			(0.012)	(0.006)	(0.046)	(0.014)	(0.007)	(0.049)	(0.020)	(0.009)	(0.038)
		0.70	0.88	0.86	0.75	0.88	0.86	0.74	0.70	0.80	0.86
			(0.006)	(0.006)	(0.040)	(0.007)	(0.007)	(0.044)	(0.018)	(0.010)	(0.028)
		0.90	0.88	0.83	0.78	0.87	0.82	0.78	0.71	0.75	0.88
			(0.005)	(0.006)	(0.019)	(0.006)	(0.008)	(0.023)	(0.035)	(0.010)	(0.018)
	1000	0.30	0.88	0.89	0.72	0.87	0.88	0.71	0.78	0.85	0.84
			(0.015)	(0.004)	(0.039)	(0.016)	(0.005)	(0.043)	(0.016)	(0.006)	(0.033)
		0.50	0.89	0.88	0.73	0.89	0.88	0.73	0.74	0.84	0.85
			(0.008)	(0.004)	(0.036)	(0.010)	(0.005)	(0.039)	(0.020)	(0.007)	(0.026)
		0.70	0.89	0.87	0.76	0.89	0.87	0.75	0.71	0.81	0.86
			(0.004)	(0.004)	(0.030)	(0.004)	(0.005)	(0.035)	(0.014)	(0.008)	(0.020)
		0.90	0.88	0.84	0.78	0.88	0.83	0.78	0.72	0.76	0.89
			(0.003)	(0.004)	(0.013)	(0.004)	(0.005)	(0.016)	(0.029)	(0.008)	(0.013)
	Average		0.87	0.86	0.74	0.87	0.85	0.74	0.72	0.80	0.85
			(0.017)	(0.022)	(0.047)	(0.012)	(0.024)	(0.050)	(0.038)	(0.038)	(0.043)

Note: MCAR = Missing Completely At Random; MNAR = Missing Not At Random; Corr = Latent intercept-slope correlation; N = Sample size; Rel = Reliability of the observed indicator.

Table 11

Average Root Mean Square Error of Approximation (RMSEA) Values (with Standard Deviation in parentheses) for All Latent Growth Interaction Models (Population Value of Latent Interaction Parameter Equal to 2.0) For the Three Missing Data Conditions (Complete, MCAR, MNAR).

Corr	N	Rel	Complete Data			MCAR Data			MNAR Data		
			Wen	Duncan	Schumacker	Wen	Duncan	Schumacker	Wen	Duncan	Schumacker
0.20	250	0.30	0.11	0.13	0.59	0.11	0.13	0.59	0.16	0.21	0.37
			(0.009)	(0.008)	(0.216)	(0.010)	(0.010)	(0.217)	(0.010)	(0.005)	(0.153)
		0.50	0.13	0.14	0.56	0.13	0.14	0.56	0.18	0.22	0.31
			(0.008)	(0.006)	(0.216)	(0.009)	(0.007)	(0.206)	(0.013)	(0.005)	(0.121)
		0.70	0.14	0.15	0.49	0.14	0.15	0.51	0.23	0.24	0.25
			(0.006)	(0.006)	(0.175)	(0.007)	(0.007)	(0.191)	(0.018)	(0.005)	(0.071)
		0.90	0.15	0.17	0.39	0.16	0.17	0.40	0.29	0.28	0.23
			(0.005)	(0.005)	(0.051)	(0.006)	(0.007)	(0.070)	(0.011)	(0.006)	(0.044)
	500	0.30	0.11	0.13	0.58	0.11	0.13	0.58	0.16	0.21	0.30
			(0.007)	(0.004)	(0.212)	(0.008)	(0.005)	(0.214)	(0.007)	(0.004)	(0.079)
		0.50	0.13	0.14	0.51	0.13	0.14	0.53	0.18	0.22	0.26
			(0.006)	(0.004)	(0.183)	(0.007)	(0.005)	(0.197)	(0.008)	(0.004)	(0.059)
		0.70	0.14	0.15	0.44	0.14	0.15	0.46	0.23	0.24	0.23
			(0.004)	(0.004)	(0.122)	(0.004)	(0.004)	(0.148)	(0.016)	(0.004)	(0.039)
		0.90	0.15	0.17	0.38	0.15	0.17	0.38	0.29	0.28	0.22
			(0.003)	(0.004)	(0.029)	(0.004)	(0.004)	(0.035)	(0.009)	(0.004)	(0.023)
	1000	0.30	0.11	0.13	0.55	0.11	0.13	0.56	0.16	0.21	0.28
			(0.005)	(0.003)	(0.184)	(0.006)	(0.003)	(0.197)	(0.005)	(0.002)	(0.045)
		0.50	0.13	0.14	0.48	0.13	0.14	0.50	0.18	0.22	0.25
			(0.005)	(0.003)	(0.142)	(0.006)	(0.003)	(0.170)	(0.005)	(0.002)	(0.035)
		0.70	0.14	0.15	0.40	0.14	0.15	0.42	0.23	0.24	0.22
			(0.002)	(0.003)	(0.067)	(0.003)	(0.003)	(0.091)	(0.015)	(0.003)	(0.026)
		0.90	0.15	0.17	0.38	0.15	0.17	0.38	0.29	0.28	0.22
			(0.002)	(0.003)	(0.015)	(0.003)	(0.003)	(0.020)	(0.009)	(0.003)	(0.016)
	Average		0.13	0.15	0.48	0.13	0.15	0.49	0.22	0.24	0.26
			(0.018)	(0.015)	(0.170)	(0.018)	(0.016)	(0.179)	(0.053)	(0.026)	(0.084)
.50	250	0.30	0.12	0.13	0.68	0.12	0.13	0.67	0.16	0.22	0.42
			(0.007)	(0.006)	(0.216)	(0.009)	(0.007)	(0.214)	(0.010)	(0.006)	(0.157)
		0.50	0.13	0.14	0.64	0.13	0.14	0.65	0.18	0.23	0.36
			(0.006)	(0.006)	(0.200)	(0.007)	(0.007)	(0.210)	(0.010)	(0.005)	(0.103)
		0.70	0.14	0.15	0.57	0.14	0.15	0.59	0.21	0.25	0.31
			(0.005)	(0.006)	(0.166)	(0.006)	(0.007)	(0.181)	(0.014)	(0.006)	(0.062)
		0.90	0.16	0.17	0.49	0.16	0.17	0.49	0.30	0.28	0.25
			(0.005)	(0.005)	(0.038)	(0.006)	(0.006)	(0.056)	(0.017)	(0.006)	(0.033)
	500	0.30	0.12	0.13	0.67	0.12	0.13	0.68	0.16	0.22	0.36
			(0.005)	(0.004)	(0.208)	(0.006)	(0.005)	(0.213)	(0.008)	(0.004)	(0.092)
		0.50	0.13	0.14	0.61	0.13	0.14	0.62	0.18	0.23	0.32
			(0.004)	(0.004)	(0.179)	(0.005)	(0.005)	(0.189)	(0.009)	(0.004)	(0.059)
		0.70	0.14	0.15	0.52	0.14	0.15	0.55	0.21	0.25	0.29
			(0.004)	(0.004)	(0.112)	(0.004)	(0.005)	(0.142)	(0.011)	(0.004)	(0.040)
		0.90	0.16	0.17	0.48	0.16	0.17	0.48	0.30	0.28	0.25
			(0.003)	(0.004)	(0.022)	(0.004)	(0.004)	(0.027)	(0.014)	(0.004)	(0.025)

	1000	0.30	0.12	0.13	0.65	0.12	0.13	0.66	0.16	0.22	0.33
			(0.004)	(0.003)	(0.182)	(0.004)	(0.003)	(0.194)	(0.007)	(0.003)	(0.055)
		0.50	0.13	0.14	0.57	0.13	0.14	0.59	0.18	0.23	0.30
			(0.003)	(0.003)	(0.138)	(0.003)	(0.003)	(0.157)	(0.008)	(0.003)	(0.038)
		0.70	0.14	0.15	0.50	0.14	0.15	0.51	0.21	0.25	0.28
			(0.002)	(0.003)	(0.063)	(0.003)	(0.003)	(0.085)	(0.010)	(0.003)	(0.026)
		0.90	0.16	0.17	0.47	0.16	0.17	0.48	0.30	0.28	0.25
			(0.002)	(0.003)	(0.015)	(0.003)	(0.003)	(0.017)	(0.011)	(0.003)	(0.021)
	Average		0.14	0.15	0.57	0.14	0.15	0.58	0.23	0.24	0.31
			(0.013)	(0.017)	(0.164)	(0.014)	(0.017)	(0.173)	(0.062)	(0.026)	(0.086)
.70	250	0.30	0.12	0.13	0.70	0.12	0.13	0.70	0.16	0.22	0.45
			(0.007)	(0.006)	(0.211)	(0.008)	(0.007)	(0.214)	(0.011)	(0.006)	(0.161)
		0.50	0.13	0.14	0.67	0.13	0.14	0.67	0.18	0.23	0.40
			(0.006)	(0.006)	(0.199)	(0.007)	(0.007)	(0.208)	(0.011)	(0.006)	(0.120)
		0.70	0.14	0.15	0.60	0.14	0.15	0.62	0.21	0.25	0.35
			(0.005)	(0.006)	(0.168)	(0.006)	(0.007)	(0.177)	(0.012)	(0.006)	(0.066)
		0.90	0.16	0.17	0.52	0.16	0.17	0.53	0.30	0.29	0.29
			(0.005)	(0.005)	(0.039)	(0.006)	(0.007)	(0.058)	(0.022)	(0.006)	(0.046)
	500	0.30	0.13	0.13	0.69	0.12	0.13	0.70	0.16	0.22	0.40
			(0.005)	(0.004)	(0.196)	(0.006)	(0.005)	(0.211)	(0.008)	(0.004)	(0.102)
		0.50	0.13	0.14	0.64	0.13	0.14	0.65	0.18	0.23	0.37
			(0.004)	(0.004)	(0.177)	(0.005)	(0.005)	(0.185)	(0.009)	(0.004)	(0.069)
		0.70	0.14	0.15	0.56	0.14	0.15	0.57	0.20	0.25	0.33
			(0.004)	(0.004)	(0.109)	(0.004)	(0.005)	(0.129)	(0.010)	(0.004)	(0.042)
		0.90	0.16	0.17	0.52	0.16	0.17	0.52	0.30	0.29	0.29
			(0.003)	(0.004)	(0.023)	(0.004)	(0.004)	(0.028)	(0.018)	(0.004)	(0.025)
	1000	0.30	0.13	0.13	0.68	0.13	0.13	0.69	0.16	0.22	0.37
			(0.003)	(0.003)	(0.171)	(0.004)	(0.003)	(0.185)	(0.007)	(0.003)	(0.059)
		0.50	0.13	0.14	0.61	0.13	0.14	0.62	0.18	0.23	0.35
			(0.003)	(0.003)	(0.130)	(0.003)	(0.003)	(0.150)	(0.008)	(0.003)	(0.041)
		0.70	0.14	0.15	0.53	0.14	0.15	0.54	0.20	0.25	0.33
			(0.002)	(0.003)	(0.063)	(0.003)	(0.003)	(0.080)	(0.007)	(0.003)	(0.029)
		0.90	0.16	0.17	0.51	0.16	0.17	0.51	0.31	0.29	0.28
			(0.002)	(0.003)	(0.015)	(0.003)	(0.003)	(0.018)	(0.015)	(0.003)	(0.019)
	Average		0.14	0.15	0.60	0.14	0.15	0.61	0.23	0.25	0.35
			(0.013)	(0.017)	(0.156)	(0.014)	(0.018)	(0.166)	(0.063)	(0.026)	(0.090)

Note: MCAR = Missing Completely At Random; MNAR = Missing Not At Random; Corr = Latent intercept-slope correlation; N = Sample size; Rel = Reliability of the observed indicator.

Table 12
Chi-Square Difference Values for the Wen and Duncan Latent Growth Interaction Models For the Three Missing Data Conditions (Complete, MCAR, MNAR).

Corr	N	Rel	Complete	MCAR	MNAR
0.20	250	0.30	119.85	87.59	366.14
		0.50	80.60	62.81	308.46
		0.70	52.69	39.57	132.29
		0.90	117.81	86.19	-121.83
	500	0.30	243.67	180.11	725.44
		0.50	142.88	109.71	625.40
		0.70	103.50	76.92	215.66
		0.90	237.59	174.16	-291.36
	1000	0.30	482.96	355.38	1451.34
		0.50	272.61	205.62	1253.65
		0.70	208.97	154.04	364.37
		0.90	477.29	350.93	-625.40
0.50	250	0.30	48.52	40.40	413.74
		0.50	25.90	22.23	382.54
		0.70	36.07	26.75	325.21
		0.90	112.65	82.97	-131.98
	500	0.30	76.67	63.95	820.90
		0.50	42.79	33.93	751.46
		0.70	73.12	53.92	671.45
		0.90	227.47	166.71	-362.34

	1000	0.30	127.19	102.04	1643.06
		0.50	80.98	60.63	1485.13
		0.70	145.32	107.44	1356.42
		0.90	455.69	335.77	-866.32
0.70	250	0.30	30.18	26.28	454.48
		0.50	17.30	14.99	427.51
		0.70	27.81	20.76	386.24
		0.90	109.93	80.74	-94.63
	500	0.30	39.79	35.29	914.05
		0.50	25.32	20.79	866.04
		0.70	55.51	41.10	798.54
		0.90	219.80	161.94	-315.75
	1000	0.30	55.08	48.00	1836.34
		0.50	43.43	34.23	1735.67
		0.70	111.17	82.22	1665.09
		0.90	439.96	323.49	-806.35

Note: MCAR = Missing Completely At Random; MNAR = Missing Not At Random; Corr = Latent intercept-slope correlation; N = Sample size; Rel = Reliability of the observed indicator. All differences are significant at $p < 0.05$.

Table 13
Average Unstandardized Latent Slope Interaction Parameter Estimate (γ_{28}) with Standard Deviation in Parentheses, for All Latent Growth Interaction Models (Population Value of Latent Interaction Parameter Equal to 2.0) For Those Simulations That Converged For the Three Missing Data Conditions (Complete, MCAR, MNAR).

Corr	N	Rel	Complete Data			MCAR Data			MNAR Data		
			Wen	Duncan	Schumacker	Wen	Duncan	Schumacker	Wen	Duncan	Schumacker
0.20	250	0.30	33.58	2.90	1.41	45.54	4.63	1.57	0.87	0.15	0.06
			(179.830)	(18.456)	(2.543)	(273.150)	(30.346)	(8.759)	(5.262)	(0.042)	(0.067)
		0.50	12.76	0.94	1.48	16.75	1.07	1.47	0.33	0.14	0.06
			(34.093)	(0.208)	(0.267)	(68.808)	(4.768)	(0.316)	(2.660)	(0.028)	(0.060)
		0.70	5.44	0.89	1.63	4.21	0.89	1.62	0.13	0.14	0.07
			(205.319)	(0.166)	(0.225)	(90.073)	(0.196)	(0.263)	(0.676)	(0.023)	(0.045)
		0.90	1.93	0.84	1.84	1.94	0.85	1.84	0.23	0.14	0.08
			(0.257)	(0.128)	(0.156)	(0.305)	(0.151)	(0.183)	(0.067)	(0.019)	(0.043)
	500	0.30	24.44	1.44	1.37	25.06	2.05	1.38	0.56	0.14	0.05
			(176.830)	(7.698)	(0.213)	(61.869)	(15.483)	(0.707)	(4.528)	(0.023)	(0.035)
		0.50	9.00	0.93	1.50	9.67	0.99	1.50	0.16	0.14	0.05
			(9.963)	(0.143)	(0.192)	(12.763)	(3.725)	(0.223)	(2.930)	(0.019)	(0.026)
		0.70	2.18	0.88	1.65	2.33	0.88	1.64	0.10	0.14	0.06
			(0.793)	(0.113)	(0.156)	(4.769)	(0.134)	(0.184)	(0.192)	(0.016)	(0.023)
		0.90	1.93	0.84	1.85	1.93	0.84	1.85	0.24	0.14	0.08
			(0.176)	(0.088)	(0.110)	(0.206)	(0.103)	(0.129)	(0.053)	(0.013)	(0.019)
	1000	0.30	18.08	1.09	1.38	18.18	1.21	1.38	0.25	0.14	0.05
			(31.931)	(1.901)	(0.150)	(23.463)	(4.537)	(0.175)	(1.887)	(0.016)	(0.018)
		0.50	8.12	0.93	1.50	8.26	0.93	1.51	0.08	0.14	0.06
			(7.513)	(0.100)	(0.133)	(8.377)	(0.119)	(0.156)	(0.816)	(0.013)	(0.015)
		0.70	2.13	0.88	1.66	2.13	0.88	1.66	0.10	0.14	0.07
			(0.188)	(0.079)	(0.112)	(0.345)	(0.093)	(0.131)	(0.171)	(0.012)	(0.014)
		0.90	1.93	0.84	1.85	1.93	0.84	1.85	0.25	0.14	0.08
			(0.125)	(0.063)	(0.075)	(0.147)	(0.074)	(0.089)	(0.047)	(0.010)	(0.014)
.50	250	0.30	8.63	1.59	1.47	10.15	3.15	1.46	0.04	0.12	0.01
			(73.580)	(16.196)	(4.790)	(48.388)	(104.172)	(4.760)	(0.659)	(0.025)	(0.019)
		0.50	3.77	0.91	1.52	3.56	0.91	1.51	0.02	0.12	0.01
			(60.868)	(0.154)	(0.248)	(15.878)	(0.184)	(0.298)	(0.118)	(0.021)	(0.021)
		0.70	1.99	0.88	1.66	2.01	0.88	1.65	0.04	0.12	0.02
			(0.276)	(0.126)	(0.206)	(0.877)	(0.150)	(0.239)	(0.067)	(0.017)	(0.019)
		0.90	1.91	0.86	1.85	1.91	0.86	1.84	0.17	0.12	0.03
			(0.203)	(0.099)	(0.147)	(0.241)	(0.116)	(0.172)	(0.069)	(0.014)	(0.018)
	500	0.30	4.85	0.95	1.41	5.77	1.00	1.40	0.01	0.12	0.01
			(6.235)	(0.190)	(0.196)	(18.936)	(3.014)	(0.230)	(0.112)	(0.016)	(0.014)
		0.50	2.22	0.91	1.53	2.38	0.91	1.53	0.01	0.12	0.01
			(1.794)	(0.108)	(0.175)	(2.668)	(0.126)	(0.207)	(0.03)	(0.014)	(0.013)
		0.70	1.97	0.87	1.67	1.98	0.88	1.67	0.03	0.12	0.02
			(0.192)	(0.088)	(0.146)	(0.225)	(0.103)	(0.170)	(0.063)	(0.012)	(0.013)
		0.90	1.91	0.86	1.86	1.91	0.86	1.85	0.18	0.12	0.03
			(0.141)	(0.069)	(0.103)	(0.166)	(0.082)	(0.121)	(0.053)	(0.010)	(0.018)

	1000	0.30	4.27	1.03	1.41	4.58	0.94	1.41	0.003	0.12	0.01
			(4.218)	(6.421)	(0.138)	(5.715)	(0.155)	(0.158)	(0.009)	(0.011)	(0.009)
		0.50	2.05	0.90	1.54	2.07	0.90	1.53	0.01	0.12	0.01
			(0.328)	(0.073)	(0.124)	(0.669)	(0.086)	(0.146)	(0.019)	(0.010)	(0.010)
		0.70	1.97	0.87	1.68	1.97	0.87	1.68	0.03	0.12	0.02
			(0.131)	(0.061)	(0.101)	(0.154)	(0.072)	(0.118)	(0.061)	(0.008)	(0.009)
		0.90	1.90	0.85	1.86	1.90	0.85	1.86	0.18	0.12	0.03
			(0.098)	(0.049)	(0.071)	(0.114)	(0.057)	(0.084)	(0.039)	(0.007)	(0.019)
.70	250	0.30	3.09	0.96	1.45	3.99	1.02	1.45	0.04	0.11	0.003
			(7.168)	(0.280)	(0.264)	(27.254)	(2.250)	(0.313)	(0.358)	(0.021)	(0.023)
		0.50	2.04	0.92	1.56	2.18	0.92	1.55	0.02	0.11	0.01
			(1.645)	(0.136)	(0.238)	(2.360)	(0.161)	(0.282)	(0.080)	(0.018)	(0.015)
		0.70	1.95	0.89	1.69	1.95	0.90	1.69	0.05	0.11	0.01
			(0.250)	(0.113)	(0.202)	(0.312)	(0.134)	(0.236)	(0.065)	(0.015)	(0.012)
		0.90	1.91	0.88	1.86	1.91	0.88	1.85	0.14	0.11	0.21
			(0.185)	(0.090)	(0.151)	(0.218)	(0.108)	(0.193)	(0.069)	(0.012)	(9.40)
	500	0.30	2.43	0.94	1.46	2.55	0.95	1.46	0.01	0.11	0.003
			(2.244)	(0.113)	(0.186)	(2.602)	(0.131)	(0.218)	(0.054)	(0.014)	(0.021)
		0.50	1.95	0.91	1.57	2.47	0.91	1.57	0.02	0.11	0.01
			(0.334)	(0.093)	(0.164)	(36.736)	(0.110)	(0.191)	(0.046)	(0.012)	(0.008)
		0.70	1.93	0.89	1.70	1.94	0.89	1.70	0.04	0.11	0.01
			(0.169)	(0.078)	(0.137)	(0.199)	(0.092)	(0.159)	(0.056)	(0.010)	(0.009)
		0.90	1.91	0.87	1.87	1.91	0.87	1.87	0.15	0.11	0.02
			(0.126)	(0.062)	(0.097)	(0.146)	(0.073)	(0.113)	(0.056)	(0.008)	(0.011)
	1000	0.30	2.27	0.94	1.46	2.33	0.94	1.46	0.01	0.11	0.003
			(1.305)	(0.078)	(0.130)	(1.491)	(0.091)	(0.152)	(0.020)	(0.009)	(0.006)
		0.50	1.95	0.91	1.57	1.95	0.91	1.57	0.01	0.11	0.01
			(0.202)	(0.063)	(0.113)	(0.249)	(0.075)	(0.135)	(0.029)	(0.008)	(0.006)
		0.70	1.93	0.89	1.70	1.94	0.89	1.70	0.03	0.11	0.01
			(0.117)	(0.054)	(0.097)	(0.137)	(0.064)	(0.114)	(0.042)	(0.007)	(0.006)
		0.90	1.90	0.87	1.87	1.90	0.87	1.87	0.16	0.11	0.02
			(0.087)	(0.043)	(0.068)	(0.102)	(0.051)	(0.080)	(0.045)	(0.006)	(0.012)

Note: MCAR = Missing Completely At Random; MNAR = Missing Not At Random; Corr = Latent intercept-slope correlation; N = Sample size; Rel = Reliability of the observed indicator.

Table 14

Confidence Intervals (95%) for the Unstandardized Parameter Estimate of the Latent Slope Interaction for All Latent Growth Interaction Models (Population Value of Latent Interaction Parameter Equal to 2.0) For the Three Missing Data Conditions (Complete, MCAR, MNAR).

				Complete Data			MCAR Data			MNAR Data		
Corr	N	Rel		Wen	Duncan	Schumacker	Wen	Duncan	Schumacker	Wen	Duncan	Schumacker
0.20	250	0.30	U	55.872	**5.188**	1.725	85.008	**9.015**	**2.836**	1.582	0.156	0.069
			L	11.288	**0.612**	1.095	6.072	**0.245**	**0.304**	0.158	0.144	0.051
		0.50	U	16.986	1.054	**2.020**	26.692	**3.353**	**2.069**	0.689	0.144	0.068
			L	8.534	0.826	**0.940**	6.808	**-1.213**	**0.871**	-0.029	0.136	0.052
		0.70	U	**30.892**	0.911	1.658	**17.225**	0.918	1.658	0.221	0.143	0.076
			L	**-20.012**	0.869	1.602	**-8.805**	0.862	1.582	0.039	0.137	0.064
		0.90	U	1.962	0.856	1.859	1.984	0.872	1.866	0.239	0.143	0.086
			L	1.898	0.824	1.821	1.896	0.828	1.814	0.221	0.137	0.074
	500	0.30	U	39.940	**2.115**	1.389	31.373	**3.630**	1.452	0.993	0.142	0.053
			L	8.940	**0.765**	1.351	18.747	**0.470**	1.308	0.127	0.138	0.047
		0.50	U	9.873	1.023	1.890	10.972	1.370	1.523	0.440	0.142	0.053
			L	8.127	0.837	1.110	8.368	0.610	1.477	-0.120	0.138	0.048
		0.70	U	2.250	0.890	1.664	**2.817**	0.894	1.659	0.118	0.142	0.062
			L	2.110	0.870	1.636	**1.843**	0.866	1.621	0.082	0.138	0.058
		0.90	U	1.945	0.848	1.860	1.951	0.851	1.863	0.245	0.141	0.082
			L	1.915	0.832	1.840	1.909	0.829	1.837	0.235	0.139	0.078
	1000	0.30	U	20.059	1.208	1.389	19.874	1.538	1.393	0.378	0.141	0.051
			L	16.101	0.972	1.371	16.486	0.882	1.367	0.122	0.139	0.049
		0.50	U	8.586	0.999	1.770	8.865	0.939	1.521	0.135	0.141	0.061
			L	7.654	0.861	1.230	7.655	0.921	1.499	0.025	0.139	0.059
		0.70	U	2.142	0.885	1.667	2.155	0.887	1.670	0.112	0.141	0.071
			L	2.118	0.875	1.653	2.105	0.873	1.651	0.088	0.139	0.069
		0.90	U	1.938	0.844	1.855	1.941	0.845	1.856	0.253	0.141	0.081
			L	1.922	0.836	1.845	1.919	0.835	1.844	0.247	0.139	0.079
0.50	250	0.30	U	**17.751**	**3.598**	**2.064**	17.142	**18.202**	**2.148**	0.129	0.123	0.013
			L	**-0.491**	**-0.418**	**0.876**	3.158	**-11.902**	**0.772**	-0.049	0.117	0.007
		0.50	U	**11.315**	1.066	**2.030**	**5.854**	0.937	1.553	0.036	0.123	0.013
			L	**-3.775**	0.755	**1.010**	**1.266**	0.883	1.467	0.004	0.117	0.007
		0.70	U	**2.024**	0.896	1.686	**2.137**	0.902	1.685	0.049	0.122	0.023
			L	**1.956**	0.864	1.634	**1.883**	0.858	1.615	0.031	0.118	0.017
		0.90	U	1.935	0.872	1.868	1.945	0.877	1.865	0.179	0.122	0.032
			L	1.885	0.848	1.832	1.875	0.843	1.815	0.161	0.118	0.028
	500	0.30	U	5.397	0.967	1.427	7.705	1.308	1.424	0.021	0.122	0.011
			L	4.303	0.933	1.393	3.835	0.692	1.377	-0.001	0.118	0.009

		0.50	U	2.377	1.052	1.890	2.652	0.923	1.551	0.013	0.121	0.011
			L	2.063	0.768	1.170	2.108	0.897	1.509	0.007	0.119	0.009
		0.70	U	1.987	0.878	1.683	**2.003**	0.891	1.687	0.036	0.121	0.021
			L	1.953	0.862	1.657	**1.957**	0.869	1.653	0.024	0.119	0.019
		0.90	U	1.922	0.866	1.869	1.927	0.868	1.862	0.185	0.121	0.032
			L	1.898	0.854	1.851	1.893	0.852	1.838	0.175	0.119	0.028
	1000	0.30	U	4.531	1.428	1.419	4.993	0.951	1.421	0.004	0.121	0.011
			L	4.009	0.632	1.401	4.167	0.929	1.399	0.002	0.119	0.009
		0.50	U	2.070	1.000	1.796	2.118	0.906	1.541	0.011	0.121	0.011
			L	2.030	0.800	1.284	2.022	0.894	1.519	0.009	0.119	0.009
		0.70	U	1.978	0.874	1.686	1.981	0.875	1.689	0.034	0.121	0.021
			L	1.962	0.866	1.674	1.959	0.865	1.671	0.026	0.119	0.019
		0.90	U	1.906	0.853	1.864	1.908	0.854	1.866	0.183	0.121	0.031
			L	1.894	0.847	1.856	1.892	0.846	1.854	0.177	0.120	0.029
0.70	250	0.30	U	3.979	0.995	1.483	**7.928**	1.345	1.495	0.088	0.113	0.006
			L	2.201	0.925	1.417	**0.052**	0.695	1.405	-0.008	0.107	0.000
		0.50	U	**2.244**	1.107	**2.046**	**2.521**	0.943	1.591	0.031	0.112	0.012
			L	**1.836**	0.733	**1.074**	**1.839**	0.897	1.509	0.009	0.108	0.008
		0.70	U	1.981	0.904	1.715	1.995	0.919	1.724	0.059	0.112	0.012
			L	1.919	0.876	1.665	1.905	0.881	1.656	0.041	0.108	0.008
		0.90	U	1.933	0.891	1.879	1.942	0.896	1.878	0.149	0.112	1.469
			L	1.887	0.869	1.841	1.879	0.864	1.822	0.131	0.108	-1.049
	500	0.30	U	2.627	0.950	1.476	2.816	0.963	1.482	0.015	0.111	0.005
			L	2.233	0.930	1.444	2.285	0.937	1.438	0.005	0.109	0.001
		0.50	U	1.979	1.041	1.907	**6.223**	0.921	1.590	0.024	0.111	0.011
			L	1.921	0.779	1.233	**-1.283**	0.899	1.550	0.016	0.109	0.009
		0.70	U	1.945	0.897	1.712	1.960	0.899	1.716	0.045	0.111	0.011
			L	1.915	0.883	1.688	1.920	0.881	1.684	0.035	0.109	0.009
		0.90	U	1.921	0.875	1.879	1.925	0.878	1.882	0.155	0.111	0.021
			L	1.899	0.865	1.861	1.895	0.863	1.858	0.145	0.109	0.019
	1000	0.30	U	2.351	0.945	1.468	2.438	0.947	1.471	0.011	0.111	0.003
			L	2.189	0.935	1.452	2.222	0.933	1.449	0.009	0.109	0.003
		0.50	U	1.963	0.998	1.802	1.968	0.915	1.580	0.012	0.111	0.010
			L	1.937	0.822	1.338	1.932	0.905	1.560	0.008	0.109	0.010
		0.70	U	1.937	0.893	1.706	1.950	0.895	1.708	0.033	0.111	0.010
			L	1.923	0.887	1.694	1.930	0.885	1.692	0.027	0.110	0.010
		0.90	U	1.905	0.873	1.874	1.907	0.874	1.876	0.163	0.110	0.021
			L	1.895	0.867	1.866	1.893	0.866	1.864	0.157	0.110	0.019

Complete = Complete Data; MCAR = Missing Complete at Random Data; MNAR = Missing Not at Random Data; Corr = Correlation between latent intercepts and slopes; N = Sample size; Rel = Reliability of the observed indicator.; L = Lower 95% limit; U = Upper 95% limit.

Table 15

Mean Square Error Bias of the Unstandardized Latent Slope Interaction Parameter Estimate (γ_{28}) for All Latent Growth Interaction Models (Population Value of Latent Interaction Parameter Equal to 2.0) For the Three Missing Data Conditions (Complete, MCAR, MNAR).

Corr	N	Rel	Complete Data			MCAR Data			MNAR Data		
			Wen	Duncan	Schumacker	Wen	Duncan	Schumacker	Wen	Duncan	Schumacker
0.20	250	0.30	78267.01	2096.34	28.93	129123.07	8286.00	405.28	29.06	3.56	3.90
		0.50	16390.26	6.12	4.34	23280.62	92.09	4.50	9.89	3.52	3.90
		0.70	42583.60	4.88	3.56	8664.67	4.99	3.67	4.02	3.50	3.90
		0.90	10.59	3.86	2.81	10.86	3.91	3.05	3.25	3.49	3.94
	500	0.30	66695.57	459.45	4.71	43694.34	901.81	6.58	22.59	3.55	3.92
		0.50	7428.15	5.89	4.13	10070.78	19.71	4.19	11.97	3.52	3.91
		0.70	30.66	4.78	3.45	89.89	4.82	3.50	3.69	3.50	3.90
		0.90	10.24	3.80	2.64	10.33	3.82	2.70	3.22	3.49	3.94
	1000	0.30	23682.06	58.32	4.61	27044.35	179.58	4.66	6.66	3.55	3.91
		0.50	3815.27	5.78	4.04	4959.39	5.82	4.08	4.37	3.52	3.90
		0.70	14.98	4.42	3.39	16.70	4.74	3.41	3.68	3.50	3.89
		0.90	10.08	3.77	2.54	10.15	3.78	2.60	3.21	3.49	3.93
.50	250	0.30	14624.92	290.77	82.13	14952.97	10913.87	79.89	4.28	3.60	4.00
		0.50	4871.29	4.17	4.15	2614.16	4.25	4.33	3.94	3.58	4.00
		0.70	10.61	3.49	3.48	23.76	3.54	3.59	3.87	3.57	3.99
		0.90	7.44	2.91	2.75	7.58	3.94	2.84	3.39	3.56	4.00
	500	0.30	2151.64	5.19	4.46	4691.79	15.34	4.54	3.98	3.59	4.00
		0.50	227.75	4.05	3.95	386.97	4.08	4.01	3.97	3.58	3.99
		0.70	10.11	3.42	3.36	10.29	3.45	3.41	3.88	3.57	3.98
		0.90	7.24	2.87	2.66	7.31	2.88	2.69	3.37	3.56	3.99
	1000	0.30	653.58	45.72	4.33	1431.05	4.96	4.36	3.99	3.59	3.99
		0.50	14.42	3.98	3.85	32.81	4.00	3.87	3.98	3.58	3.98
		0.70	9.91	3.40	3.29	10.00	3.41	3.32	3.89	3.57	3.98
		0.90	7.13	2.85	2.62	7.16	2.86	2.64	3.37	3.56	3.98
.70	250	0.30	1628.80	5.10	4.44	3170.42	1305.90	4.61	4.00	3.61	4.03
		0.50	174.89	3.55	3.90	398.69	3.60	4.03	3.92	3.61	4.00
		0.70	8.70	3.04	3.32	9.01	3.08	3.43	3.84	3.60	4.00
		0.90	6.15	2.57	2.95	6.25	2.58	3.69	3.49	3.59	1105.36
	500	0.30	585.74	4.06	4.18	395.42	4.10	4.25	3.96	3.61	4.02
		0.50	11.07	3.47	3.70	1361.86	3.50	3.76	3.95	3.60	4.00
		0.70	8.33	2.99	3.13	8.46	3.01	3.19	3.85	3.60	4.00
		0.90	5.99	2.54	2.48	6.04	2.55	2.52	3.46	3.59	4.00
	1000	0.30	45.27	3.98	4.04	60.73	4.00	4.08	3.97	3.61	4.00
		0.50	10.52	3.42	3.57	10.60	3.44	3.61	3.96	3.60	4.00
		0.70	8.16	2.96	3.06	8.22	2.97	3.09	3.88	3.60	3.99
		0.90	5.90	2.52	2.43	5.92	2.53	2.45	3.44	3.59	4.00

Note: Complete = Complete Data condition; MCAR = Missing Completely At Random Data condition; MNAR = Missing Not At Random Data condition; Corr = Latent intercept-slope correlation; N = Sample size; Rel = Reliability of the observed indicator.

Table 16

Standardized Bias Estimates of the Unstandardized Latent Slope Interaction Parameter Estimate (γ_{28}) for All Latent Growth Interaction Models (Population Value of Latent Interaction Parameter Equal to 2.0) For the Three Missing Data Conditions (Complete, MCAR, MNAR).

Corr	N	Rel	Complete Data			MCAR Data			MNAR Data		
			Wen	Duncan	Schumacker	Wen	Duncan	Schumacker	Wen	Duncan	Schumacker
0.20	250	0.30	> 1000	9.14	-0.35	> 1000	178.82	-0.36	> 1000	-8.67	-12.30
		0.50	> 1000	-0.51	-0.28	> 1000	-0.46	-0.29	> 1000	-7.21	-7.89
		0.70	> 1000	-0.61	-0.21	> 1000	-0.61	-0.22	> 1000	-8.95	-21.69
		0.90	-0.03	-0.75	-0.10	-0.03	-0.75	-0.11	-45.25	-11.17	-3.87
	500	0.30	> 1000	0.69	-0.32	> 1000	9.06	-0.32	> 1000	-5.68	-6.53
		0.50	0.18	-0.51	-0.27	1.30	-0.06	-0.27	> 1000	-7.18	-5.98
		0.70	0.03	-0.61	0.20	0.47	-0.61	-0.20	> 1000	-8.92	-5.13
		0.90	-0.03	-0.75	-0.10	-0.03	-0.75	-0.10	-12.65	-11.19	-3.89
	1000	0.30	> 1000	-0.41	-0.31	> 1000	-0.38	-0.31	> 1000	-5.69	-6.43
		0.50	0.15	-0.51	-0.26	0.14	-0.51	-0.26	> 1000	-7.16	-5.86
		0.70	0.03	-0.61	-0.19	0.03	-0.61	0.19	> 1000	-8.89	-5.11
		0.90	-0.02	0.75	-0.10	-0.03	-0.75	-0.10	-6.41	-11.21	-3.91
0.50	250	0.30	> 1000	30.01	-0.31	> 1000	> 1000	-0.32	> 1000	-13.00	-30.81
		0.50	> 1000	-0.67	-0.26	167.53	-0.67	-0.27	< -1000	-10.93	-13.40
		0.70	-0.02	-0.78	-0.20	-0.02	-0.78	-0.21	< -1000	-13.47	-9.48
		0.90	-0.04	-0.93	0.10	-0.04	-0.93	-0.10	< -1000	-16.70	-6.36
	500	0.30	0.05	-0.57	-0.31	124.00	-0.56	-0.31	< -1000	-8.99	-13.6
		0.50	-0.004	-0.67	-0.25	-0.01	-0.67	-0.26	< -1000	-10.75	-11.38
		0.70	-0.01	-0.78	-0.19	-0.01	-0.78	-0.19	< -1000	-13.46	-9.15
		0.90	-0.04	-0.93	-0.09	-0.04	-0.93	-0.09	-149.96	-16.75	-6.40
	1000	0.30	0.06	1.97	-0.30	0.06	-0.57	-0.30	< -1000	-8.97	-12.74
		0.50	0.004	-0.67	-0.25	0.001	-0.67	-0.25	< -1000	-10.800	-11.06
		0.70	-0.01	-0.78	-0.18	-0.01	-0.78	-0.19	< -1000	-13.42	-9.04
		0.90	-0.04	-0.93	-0.09	-0.04	-0.93	-0.09	-41.32	-16.73	-6.34
0.70	250	0.30	> 1000	-0.64	-0.29	< -1000	-0.63	-0.30	> 1000	-11.41	-33.64
		0.50	-0.06	-0.73	-0.24	-0.88	-0.74	-0.25	< -1000	-12.97	-19.89
		0.70	-0.03	-0.85	-0.18	-0.03	-0.85	-0.19	< -1000	-16.10	-12.83
		0.90	-0.04	-1.01	-0.09	-0.04	-1.01	-0.10	< -1000	-20.32	-8.24
	500	0.30	-0.04	-0.64	-0.28	-0.23	-0.64	-0.29	< -1000	-10.78	-19.36
		0.50	-0.04	-0.74	-0.24	> 1000	-0.74	-0.24	< -1000	-12.86	-15.91
		0.70	-0.03	-0.86	-0.18	-0.03	-0.85	-0.18	-510.54	-16.06	-12.25
		0.90	0.04	-1.01	-0.09	-0.04	-1.01	-0.09	< -1000	-20.26	-8.23
	1000	0.30	-0.02	-0.64	-0.28	-0.03	-0.64	-0.28	< -1000	-10.78	-17.79
		0.50	-0.02	-0.74	-0.24	-0.03	-0.74	-0.24	< -1000	-12.88	-15.02
		0.70	0.03	-0.86	-0.18	-0.03	-0.86	-0.18	-251.05	-16.03	-12.13
		0.90	0.04	1.01	-0.09	-0.04	-1.01	-0.09	< -1000	-20.26	-8.16

Note: Complete = Complete Data condition; MCAR = Missing Completely At Random Data condition; MNAR = Missing Not At Random Data condition; Corr = Latent intercept-slope correlation; N = Sample size; Rel = Reliability of the observed indicator.

Table 17
Type I Error Rates For Those Simulations (Frequencies in Parentheses) That Converged when the Latent Slope Interaction Parameter (γ_{28}) was set Equal to 0 For the Three Missing Data Conditions (Complete, MCAR, MNAR).

Corr	N	Rel	Complete Data			MCAR Data			MNAR Data		
			Wen	Duncan	Schumacker	Wen	Duncan	Schumacker	Wen	Duncan	Schumacker
0.20	250	0.30	**0.30**	**0.09**	0.03	**0.29**	**0.10**	0.03	**0.10**	0.003	0.04
			(868)	(427)	(133)	(824)	(466)	(142)	(320)	(17)	(218)
		0.50	**0.24**	0.05	0.02	**0.22**	0.06	0.03	0.03	0.004	0.05
			(1098)	(257)	(112)	(961)	(301)	(149)	(137)	(20)	(267)
		0.70	**0.14**	0.03	0.02	**0.15**	0.04	0.03	0.01	0.01	0.05
			(694)	(148)	(118)	(734)	(203)	(126)	(25)	(36)	(263)
		0.90	0.05	0.02	0.03	0.06	0.03	0.03	0.02	0.03	0.05
			(258)	(105)	(138)	(320)	(140)	(143)	(111)	(146)	(229)
	500	0.30	**0.37**	0.07	0.02	**0.31**	**0.08**	0.03	0.05	0.001	0.06
			(1060)	(360)	(110)	(885)	(405)	(137)	(155)	(5)	(277)
		0.50	**0.28**	0.04	0.02	**0.25**	0.05	0.03	0.01	0.001	0.07
			(1389)	(207)	(111)	(1154)	(246)	(134)	(29)	(4)	(330)
		0.70	**0.17**	0.02	0.02	**0.16**	0.03	0.02	0.001	0.004	0.07
			(832)	(109)	(112)	(783)	(170)	(116)	(7)	(19)	(371)
		0.90	0.04	0.01	0.03	0.04	0.02	0.03	0.01	0.04	0.06
			(204)	(68)	(129)	(219)	(80)	(140)	(53)	(177)	(315)
	1000	0.30	**0.43**	0.06	0.02	**0.37**	0.07	0.02	0.01	0.00	0.07
			(1248)	(296)	(111)	(1069)	(354)	(117)	(41)	(0)	(370)
		0.50	**0.33**	0.03	0.02	**0.28**	0.04	0.02	0.001	0.001	**0.09**
			(1653)	(141)	(113)	(1381)	(208)	(107)	(3)	(4)	(454)
		0.70	**0.18**	0.02	0.03	**0.16**	0.02	0.02	0.00	0.001	**0.10**
			(920)	(78)	(128)	(800)	(105)	(90)	(0)	(4)	(522)
		0.90	0.03	0.01	0.03	0.04	0.01	0.02	0.003	0.04	**0.09**
			(138)	(31)	(124)	(198)	(58)	(120)	(17)	(177)	(425)
.50	250	0.30	**0.15**	0.04	0.03	**0.15**	0.05	0.03	**0.15**	0.002	0.03
			(603)	(212)	(134)	(581)	(256)	(147)	(426)	(9)	(171)
		0.50	**0.12**	0.03	0.03	**0.12**	0.04	0.02	0.06	0.002	0.04
			(608)	(153)	(130)	(593)	(207)	(118)	(140)	(9)	(222)
		0.70	**0.09**	0.03	0.03	**0.09**	0.03	0.02	0.01	0.003	0.05
			(457)	(127)	(123)	(447)	(152)	(110)	(32)	(16)	(247)
		0.90	0.05	0.02	0.03	0.05	0.03	0.02	0.02	0.01	0.06
			(245)	(107)	(153)	(242)	(125)	(118)	(92)	(27)	(308)
	500	0.30	**0.16**	0.03	0.02	**0.13**	0.04	0.02	**0.11**	0.00	0.05
			(647)	(144)	(118)	(542)	(184)	(117)	(276)	(0)	(239)
		0.50	**0.15**	0.02	0.02	**0.14**	0.03	0.02	0.02	0.001	0.05
			(733)	(112)	(119)	(680)	(170)	(110)	(37)	(3)	(243)
		0.70	**0.10**	0.02	0.02	**0.09**	0.02	0.03	0.002	0.001	0.07
			(493)	(80)	(109)	(452)	(113)	(124)	(7)	(5)	(334)
		0.90	0.04	0.02	0.02	0.05	0.02	0.02	0.01	0.004	**0.09**
			(207)	(85)	(119)	(227)	(99)	(110)	(35)	(22)	(441)

	1000	0.30	**0.21** (870)	0.03 (139)	0.02 (116)	**0.16** (665)	0.03 (160)	0.03 (124)	0.07 (168)	0.00 (0)	0.07 (326)
		0.50	**0.17** (823)	0.02 (94)	0.03 (134)	**0.15** (755)	0.03 (137)	0.03 (132)	0.003 (7)	0.00 (0)	**0.08** (380)
		0.70	**0.11** (530)	0.01 (57)	0.03 (126)	**0.10** (514)	0.02 (98)	0.03 (135)	0.001 (3)	0.00 (0)	**0.09** (439)
		0.90	0.04 (188)	0.01 (53)	0.02 (118)	0.04 (217)	0.02 (83)	0.03 (139)	0.01 (23)	0.001 (7)	**0.12** (622)
.70	250	0.30	**0.08** (316)	0.03 (163)	0.02 (110)	**0.10** (399)	0.04 (221)	0.02 (110)	**0.20** (583)	0.001 (4)	0.05 (238)
		0.50	**0.09** (439)	0.03 (130)	0.02 (119)	**0.10** (448)	0.04 (173)	0.02 (82)	**0.08** (206)	0.001 (3)	0.04 (196)
		0.70	0.07 (372)	0.02 (102)	0.02 (108)	0.07 (358)	0.02 (118)	0.02 (110)	0.03 (84)	0.003 (14)	0.05 (253)
		0.90	0.04 (214)	0.02 (104)	0.03 (130)	0.04 (215)	0.02 (118)	0.02 (117)	0.01 (38)	0.003 (16)	0.06 (299)
	500	0.30	**0.08** (290)	0.02 (113)	0.02 (101)	0.07 (293)	0.03 (139)	0.02 (91)	**0.15** (389)	0.00 (0)	0.05 (238)
		0.50	**0.11** (524)	0.02 (97)	0.02 (113)	**0.10** (466)	0.03 (133)	0.02 (106)	0.04 (98)	0.00 (0)	0.07 (323)
		0.70	**0.08** (420)	0.01 (68)	0.02 (121)	**0.08** (391)	0.02 (111)	0.02 (100)	0.01 (38)	0.0002 (1)	0.06 (308)
		0.90	0.04 (213)	0.02 (82)	0.03 (135)	0.05 (247)	0.02 (118)	0.02 (112)	0.01 (40)	0.001 (4)	**0.09** (450)
	1000	0.30	**0.09** (295)	0.02 (83)	0.02 (120)	**0.08** (296)	0.03 (123)	0.05 (123)	**0.08** (190)	0.00 (0)	0.06 (310)
		0.50	**0.11** (567)	0.01 (58)	0.02 (120)	**0.10** (487)	0.02 (89)	0.02 (108)	0.01 (17)	0.00 (0)	**0.09** (424)
		0.70	**0.08** (418)	0.02 (73)	0.03 (125)	**0.08** (397)	0.01 (72)	0.02 (106)	0.01 (31)	0.00 (0)	**0.10** (497)
		0.90	0.04 (205)	0.01 (57)	0.02 (116)	0.04 (216)	0.02 (94)	0.03 (123)	0.002 (12)	0.00 (0)	**0.14** (713)

Note: MCAR = Missing Completely At Random; MNAR = Missing Not At Random; Corr = Latent intercept-slope correlation; N = Sample size; Rel = Reliability of the observed indicator. A liberal condition is where the Type I error rate is above 0.75, and is presented in **bold** text; a conservative condition is where the Type I error rate is below 0.25, and is presented in underlined text.

Appendix A

SAS Code for Generating the Latent Growth Data

```
libname fdata "C:\Ian\PhD Dissertation";
libname temp "C:\Ian\temp";
run;
OPTIONS mprint symbolgen;


%MACRO
COMPUTE (REPS, N, WAVES, M_INTx, V_INTx, M_SLPx, V_SLPx, PHIx, m_intz, v_intz,
m_slpz, v_slpz, phiz,
m_inty, v_inty, m_slpy, v_slpy, phiy, rel, PARM);

/*****************************************************************************
********
**********                    DATA GENERATION
**************************
*****************************************************************************
********/
%do j = 1 %to &reps;
data fdata.semt&j;
array errx errx1-errx&n; *arrays for error terms;
array errz errz1-errz&n;
array erry erry1-erry&n;

array alphx alphx1-alphx&n;  *arrays for latent intercepts and slopes;
array betax betax1-betax&n;
array alphz alphz1-alphz&n;
array betaz betaz1-betaz&n;
array alphy alphy1-alphy&n;
array betay betay1-betay&n;

array betaxz betaxz1-betaxz&n; *latent slope interaction array;

array var_x var_x1-var_x&waves;
array var_z var_z1-var_z&waves;
array var_y var_y1-var_y&waves;
array var_xz var_xz1-var_xz&waves;
seed1=-11;
seed2=-22;
seed3=-33;
seed4=-44;

* generating the data  ;
do i=1 to &n;
        errx(i) = rannor(seed1); * error term for X, root of variance X random;
        errz(i) = rannor(seed1); *error term for Z;
        erry(i) = rannor(seed1); * error term for Y;
```

```
        alphx(i) = &m_intx + rannor(seed2)*sqrt(&v_intx); *latent intercept X;
        betax(i) = &m_slpx + (sqrt(&corx) * alphx(i)) +
rannor(seed3)*sqrt(&v_slpx); *latent slope X;
        alphz(i) = &m_intz + rannor(seed2)*sqrt(&v_intz); *latent intercept Z;
        betaz(i) = &m_slpz + (sqrt(&corz) * alphz(i)) +
rannor(seed3)*sqrt(&v_slpz); *latent slope Z;
        betaxz(i) = betax(i) * betaz(i); * latent interaction of slopes;
        alphy(i) = &m_inty + rannor(seed2)*sqrt(&v_inty) + 1*alphx(i) +
3*alphz(i);
        betay(i) = &m_slpy + (sqrt(&cory) * alphy(i)) +
rannor(seed3)*sqrt(&v_slpy) + 1.5 *betax(i) + 1.4*betaz(i) + 2*betaxz(i);
        k=0;

        var_x(1) = alphx(i) + 0*betax(i) + rannor(seed4)*sqrt((1-&rel)*1);
        var_x(2) = alphx(i) + 1*betax(i) + rannor(seed4)*sqrt((1-&rel)*3);
        var_x(3) = alphx(i) + 2*betax(i) + rannor(seed4)*sqrt((1-&rel)*6);
        var_x(4) = alphx(i) + 3*betax(i) + rannor(seed4)*sqrt((1-&rel)*9);

        var_z(1) = alphz(i) + 0*betaz(i) + rannor(seed4)*sqrt((1-&rel)*1);
        var_z(2) = alphz(i) + 1*betaz(i) + rannor(seed4)*sqrt((1-&rel)*3);
        var_z(3) = alphz(i) + 2*betaz(i) + rannor(seed4)*sqrt((1-&rel)*6);
        var_z(4) = alphz(i) + 4*betaz(i) + rannor(seed4)*sqrt((1-&rel)*9);

        var_y(1) = alphy(i) + 0*betay(i) + rannor(seed4)*sqrt((1-&rel)*1);
        var_y(2) = alphy(i) + 1*betay(i) + rannor(seed4)*sqrt((1-&rel)*3);
        var_y(3) = alphy(i) + 3*betay(i) + rannor(seed4)*sqrt((1-&rel)*6);
        var_y(4) = alphy(i) + 9*betay(i) + rannor(seed4)*sqrt((1-&rel)*9);
        output;
end;

keep var_x1-var_x&waves var_z1-var_z&waves var_y1-var_y&waves;

%end;

*  generating the x and z means;
%do j = 1 %to &reps;
proc means data=fdata.semt&j;  var var_x1;  ods output summary=fdata.mxt1;
run;
data fdata.mxt1;  set fdata.mxt1; keep var_x1_mean; run;

proc means data=fdata.semt&j;  var var_x2;  ods output summary=fdata.mxt2;
run;
data fdata.mxt2;  set fdata.mxt2;  keep var_x2_mean; run;

proc means data=fdata.semt&j;  var var_x3;  ods output summary=fdata.mxt3;
run;
data fdata.mxt3;  set fdata.mxt3;  keep var_x3_mean;  run;

proc means data=fdata.semt&j;  var var_x4;  ods output summary=fdata.mxt4;
run;
data fdata.mxt4;  set fdata.mxt4;  keep var_x4_mean;  run;
```

```
proc means data=fdata.semt&j;  var var_z1;  ods output summary=fdata.mzt1;
run;
data fdata.mzt1;  set fdata.mzt1; keep var_z1_mean; run;

proc means data=fdata.semt&j;  var var_z2;  ods output summary=fdata.mzt2;
run;
data fdata.mzt2;  set fdata.mzt2;  keep var_z2_mean; run;

proc means data=fdata.semt&j;  var var_z3;  ods output summary=fdata.mzt3;
run;
data fdata.mzt3;  set fdata.mzt3;  keep var_z3_mean;  run;

proc means data=fdata.semt&j;  var var_z4;  ods output summary=fdata.mzt4;
run;
data fdata.mzt4;  set fdata.mzt4;  keep var_z4_mean;  run;


data fdata.tmeans&j;
merge fdata.mxt1 fdata.mxt2 fdata.mxt3 fdata.mxt4 fdata.mzt1 fdata.mzt2
fdata.mzt3 fdata.mzt4;
run;
%end;

* generating the x and z means for every observation;
%do j = 1 %to &reps;
data fdata.tmean&j;
set fdata.tmeans&j;
array mx  mx1 - mx&waves; *array for means;
array mz  mz1 - mz&waves; *array for means;
do i= 1 to &n;
                mx(1) = var_x1_mean;
                mx(2) = var_x2_mean;
                mx(3) = var_x3_mean;
                mx(4) = var_x4_mean;
                        mz(1) = var_z1_mean;
                        mz(2) = var_z2_mean;
                        mz(3) = var_z3_mean;
                        mz(4) = var_z4_mean;
            output;
end;
%end;


%do j = 1 %to &reps;
data fdata.semt&j;
merge fdata.tmean&j fdata.semt&j;
run;
%end;
```

```
%do j = 1 %to &reps;
data fdata.sem&j;
array phix  phix1-phix&n; *arrays for error terms;
array phiz phiz1-phiz&n;
array phiy phiy1-phiy&n;

array alphx alphx1-alphx&n;  *arrays for latent intercepts and slopes;
array betax betax1-betax&n;
array alphz alphz1-alphz&n;
array betaz betaz1-betaz&n;
array alphy alphy1-alphy&n;
array betay betay1-betay&n;

array betaxz betaxz1-betaxz&n; *latent slope interaction array;

array var_x var_x1-var_x&waves;
array var_z var_z1-var_z&waves;
array var_y var_y1-var_y&waves;
array var_xz var_xz1-var_xz&waves;
array mx  mx1 - mx&waves; *array for means;
array mz  mz1 - mz&waves; *array for means;

set fdata.semt&j;
        var_xz(1) = (var_x(1) - mx(1)) * (var_z(1) - mz(1) );
        var_xz(2) = (var_x(2) - mx(2)) * (var_z(2) - mz(2) );
        var_xz(3) = (var_x(3) - mx(3)) * (var_z(3) - mz(3));
        var_xz(4) = (var_x(4) - mx(4)) * (var_z(4) - mz(4));
output;

keep var_x1-var_x&waves var_z1-var_z&waves var_xz1-var_xz&waves var_y1-
var_y&waves;
%end;

/* MODIFYING THE COMPLETE DATA TO INCLUDE 10 PERCENT MISSING from T2, extra 10
perc from T3, and 10 perc from T4     */

%do j = 1 %to &reps;
data fdata.semmiss&j; set fdata.sem&j;
seed1=(123);
miss1 = abs(rannor (seed1));
IF (miss1 >= 1.30) then remove_x2 = .;
if (miss1 < 1.30) then remove_x2 = 1;

var_x2 = var_x2 * remove_x2;
var_x3 = var_x3 * remove_x2;
var_x4 = var_x4 * remove_x2;
run;
%end;

%do j = 1 %to &reps;
data fdata.semmiss&j; set fdata.semmiss&j;
seed2=(-11);
```

```
miss2 = abs(rannor (seed2));
IF ((remove_x2 = 1) & (miss2 >= 1.30)) then remove_x3 = .;
if ((remove_x2 = 1) & (miss2 < 1.30)) then remove_x3 = 1;

var_x3 = var_x3 * remove_x3;
var_x4 = var_x4 * remove_x3;
run;
%end;

%do j = 1 %to &reps;
data fdata.semmiss&j; set fdata.semmiss&j;
seed3 = (-55);
miss3 = abs(rannor (seed3));
IF ((remove_x2 = 1) & (remove_x3 = 1) & (miss2 >= 1.30)) then remove_x4 = .;
if ((remove_x2 = 1) & (remove_x3 = 1) & (miss2 < 1.30)) then remove_x4 = 1;

var_x4 = var_x4 * remove_x4;
run;
%end;

/* MODIFYING THE COMPLETE DATA TO INCLUDE 10 PERCENT MISSING NOT AT RANDOM*/
ods listing;
/* standardizing of Y1 scores to 0 mean and std of 1*/
%do j = 1 %to &reps;
proc standard data=fdata.sem&j m=0 std = 1 out=stan&j;
var var_y1; run;

data stan&j;
set stan&j;
if (var_y1 >= abs(1.3)) then remove_x2 = .;
if (var_y1 < abs(1.3)) then remove_x2 = 1;
var_y1 = var_y1 * remove_x2;
var_x2 = var_x2 * remove_x2;
var_x3 = var_x3 * remove_x2;
var_x4 = var_x4 * remove_x2;
run;

/* standardizing of remaining Y2 scores to 0 mean and std of 1*/
proc standard data=stan&j m=0 std = 1 out=stan&j;
var var_y2;  run;

data stan&j;     set stan&j;
if (var_y2 >= abs(1.3)) then remove_x3 = .;
if (var_y2 < abs(1.3)) then remove_x3 = 1;
var_y2 = var_y2 * remove_x3;
var_x3 = var_x3 * remove_x3;
var_x4 = var_x4 * remove_x3;
run;

/* standardizing of remaining Y3 scores to 0 mean and std of 1*/
proc standard data=stan&j m=0 std = 1 out=stan&j;
var var_y3; run;
```

```
data stan&j;     set stan&j;
if (var_y3 >= abs(1.3)) then remove_x4= .;
if (var_y3 < abs(1.3)) then remove_x4 = 1;
var_y3 = var_y3 * remove_x4;
var_x4 = var_x4 * remove_x4;
run;
data fdata.semmnar&j; set stan&j;
%end;
```

Appendix B

SAS Syntax for the Wen Latent Growth Interaction Model

```
/*****************************************************************************
*******/
proc calis ucov aug method=ml data=fdata.sem9 maxiter=5000 maxfunc=5000
outram=temp.wenfit;
title "CALIS output for Wen model sample ";
ods output iterstop = weniter; ods output convergencestatus = temp.wenconverge;
ods output stdlatenteq = temp.wenstdlatenteq;
  lineqs
      /* measurement model for y : assume the tau vector is a mistake in the
note */
      var_y1  =   Intercept + F_eta1                      +     e1,
      var_y2  =   Intercept + F_eta1 + 1 F_eta2           +     e2,
      var_y3  =   Intercept + F_eta1 + betay3 F_eta2 +     e3,
      var_y4  =   Intercept + F_eta1 + betay4 F_eta2 +     e4,

      /* measurement model for x and z */
      var_x1  =  tau1  Intercept +  1  F_xi1                                +
e5,
      var_x2  =  tau2  Intercept +  1  F_xi1 + 1   F_xi2                    +
e6,
      var_x3  =  tau3  Intercept +  1  F_xi1 + l32 F_xi2                    +
e7,
      var_x4  =  tau4  Intercept +  1  F_xi1 + l42 F_xi2                    +
e8,
      var_z1  =  tau5  Intercept +  1  F_xi3                                +
e9,
      var_z2  =  tau6  Intercept +  1  F_xi3 + 1   F_xi4                    +
e10,
      var_z3  =  tau7  Intercept +  1  F_xi3 + l74 F_xi4                    +
e11,
      var_z4  =  tau8  Intercept +  1  F_xi3 + l84 F_xi4                    +
e12,
      var_xz2 =  tau9  Intercept + tau6 F_xi1  + tau6  F_xi2   + tau2  F_xi3
+ tau2  F_xi4
                              +      F_xi13 +      F_xi14  +      F_xi23  +
F_xi24 + e13,
      var_xz3 =  tau10 Intercept + tau7 F_xi1  + l10_2 F_xi2   + tau3  F_xi3
+ l10_4 F_xi4
                              +      F_xi13 + l74   F_xi14  + l32   F_xi23  +
l10_8 F_xi24 + e14,
      var_xz4 =  tau11 Intercept + tau8 F_xi1  + l11_2 F_xi2   + tau4  F_xi3
+ l11_4 F_xi4
                              +      F_xi13 + l84   F_xi14  + l42   F_xi23  +
l11_8 F_xi24 + e15,

    /* structural model */
    F_eta1 = g11 F_xi1 + g13 F_xi3 + g15 F_xi13 + d1,
```

```
    F_eta2 = g22 F_xi2 + g24 F_xi4 + g28 F_xi24 + d2;

  std /*variance parameters */
     F_xi1 F_xi2 F_xi3 F_xi4 F_xi13 F_xi14 F_xi23 F_xi24 =
           phi11 phi22 phi33 phi44 phi55 phi66 phi77 phi88,
     e5-e15 = th1-th11,
     e1-e4 d1-d2 = any1-any6;  /* unconstrained parameters I added */

  cov  /* mean parameters */
      Intercept f_xi1 = 0,
      Intercept f_xi2 = 0,
      Intercept f_xi3 = 0,
      Intercept f_xi4 = 0,
      Intercept f_xi13 = phi31,
      Intercept f_xi14 = phi41,
      Intercept f_xi23 = phi32,
      Intercept f_xi24 = phi42,
      /* cov parameters */
      F_xi1-F_xi4 = phi21 phi31 phi32 phi41 phi42 phi43,
      F_xi13 F_xi14 F_xi23 F_xi24 = phi65 phi75 phi76 phi85 phi86 phi87,
      e13 e6  = th92,
      e14 e7  = th10_3,
      e15 e8  = th11_4,
      e13 e10 = th96,
      e14 e11 = th10_7,
      e15 e12 = th11_8;

/* programming statements for parameter constraints */
tau9  = tau2*tau6;
tau10 = tau3*tau7;
tau11 = tau4*tau8;
l10_2 = tau7*l32;
l10_4 = tau3*l74;
l10_8 = l32*l74;
l11_2 = tau8*l42;
l11_4 = tau4*l84;
l11_8 = l42*l84;
phi55 = phi11 * phi33 + phi31 * phi31;
phi66 = phi11 * phi44 + phi41 * phi41;
phi77 = phi22 * phi33 + phi32 * phi32;
phi88 = phi22 * phi44 + phi42 * phi42;
phi65 = phi11 * phi43 + phi31 * phi41;
phi75 = phi21 * phi33 + phi31 * phi32;
phi76 = phi21 * phi43 + phi31 * phi42;
phi85 = phi21 * phi43 + phi32 * phi41;
phi86 = phi21 * phi44 + phi41 * phi42;
phi87 = phi22 * phi43 + phi32 * phi42;
phi88 = phi22 * phi44 + phi42 * phi42;
th9 = tau6 * tau6 * th2 + phi33 * th2 + phi44*th2 + tau2*tau2*th6 + phi11*th6 +
phi22*th6 + th2*th6;
th10 = tau7*tau7*th3 + phi33*th3 + l74*l74*phi44*th3 + tau3*tau3*th7 +
phi11*th7 + l32*l32*phi22*th7 + th3*th7;
```

```
th11 = tau8*tau8*th4 + phi33*th4 + l84*l84*phi44*th4 + tau4*tau4*th8 +
phi11*th8 + l42*l42*phi22*th8 + th4*th8;
th92 = tau6 * th2;
th96 = tau2 * th6;
th10_3 = tau7 * th3;
th10_7 = tau3 * th7;
th11_4 = tau8 * th4;
th11_8 = tau4 * th8;

* outram statement for fit indices for WEN model;
data temp.wenfit2at; set temp.wenfit;
keep _name_ _estim_;

data temp.wenfit2bt; set temp.wenfit;
keep _name_ _stderr_;

proc transpose data=temp.wenfit2at out=temp.wenfit2a let;
proc transpose data=temp.wenfit2bt out=temp.wenfit2b let;

data temp.wenfit2b1;
set temp.wenfit2b;
l32_str = l32;
l42_str = l42;
l74_str = l74;
g11_str = g11;
g13_str = g13;
g15_str = g15;
g22_str = g22;
g24_str = g24;
g28_str = g28;  run;

data temp.wenfit2b2; set temp.wenfit2b1;
keep l32_str l42_str l74_str g11_str g13_str g15_str g22_str g24_str g28_str;
run;

/* merging of the two data rows to get estimates and std errors on one line for
each model */
data temp.wenfitx;
merge temp.wenfit2a temp.wenfit2b2;  run;

/* merging in ODS output of convergence*/
data temp.wenfits;
merge temp.wenfitx temp.wenconverge;
run;

/* pulling in standardized estimate from ODS file*/
data temp.std_temp;
set temp.wenstdlatenteq;
if parameter3 = 'F_xi24';
g28std = coefficient3;
run;
```

```
data temp.g28std;
set temp.std_temp;
keep g28std;
run;

data temp.wenfits;
merge temp.wenfits temp.g28std;
run;

/* adding of model index value */
data temp.wenfits;
set temp.wenfits;
model = 1;
run;

/* pulling in iterations from ODS file*/
data itr_temp;
set weniter;
if Label1 = 'Iterations';
iter = nvalue1;

data itr;
set itr_temp;
keep iter;

data temp.wenfits;
merge temp.wenfits itr;


data temp.wenfits; set temp.wenfits;
      keep status model n fit nparm df chisquar p_chisq gfi agfi rmseaest
compfiti bb_nonor bb_normd bol_rho1 bol_del2 centrali
      l32 l32_str l42 l42_str l74 l74_str l84 l84_str g11 g11_str g13 g13_str
g15 g15_str g22 g22_str g24 g24_str g28 g28_str
      g28std iter;
run;

/* merging all Wen replicates into one dataset*/
proc append base=fdata.zsemfits_wen force;
run;
```

Appendix C

SAS Syntax for the Duncan Latent Growth Interaction Model

```
/*****************************************************************************
**************/
/***************************           DUNCAN MODEL
*************************************/
/*****************************************************************************
**************/

%do j = 1 %to &reps;


proc calis ucov aug method=ml data=fdata.sem&j maxiter=5000 maxfunc=5000
outram=dunfit;
title "CALIS output for Duncan model sample &j";
ods output iterstop = dunciter;
ods output convergencestatus = dunconverge;
ods output stdlatenteq = dunstdlatenteq;
  lineqs
      /* measurement model for y : assume the tau vector is a mistake in the
note */
      var_y1  =   F_eta1                    +     e1,
      var_y2  =   F_eta1 + 1 F_eta2         +     e2,
      var_y3  =   F_eta1 + betay3  F_eta2 +     e3,
      var_y4  =   F_eta1 + betay4 F_eta2  +     e4,

      /* measurement model for x and z */
      var_x1  =  tau1  Intercept +  1  F_xi1                              +
e5,
      var_x2  =  tau2  Intercept +  1  F_xi1 + 1   F_xi2                  +
e6,
      var_x3  =  tau3  Intercept +  1  F_xi1 + l32 F_xi2                  +
e7,
      var_x4  =  tau4  Intercept +  1  F_xi1 + l42 F_xi2                  +
e8,
      var_z1  =  tau5  Intercept +  1  F_xi3                              +
e9,
      var_z2  =  tau6  Intercept +  1  F_xi3 + 1   F_xi4                  +
e10,
      var_z3  =  tau7  Intercept +  1  F_xi3 + l74 F_xi4                  +
e11,
      var_z4  =  tau8  Intercept +  1  F_xi3 + l84 F_xi4                  +
e12,

      var_xz2 =  tau9  Intercept + tau6 F_xi1  + tau6  F_xi2   + tau2  F_xi3
+ tau2  F_xi4
                                    + 1 F_xi24 + e13,
      var_xz3 =  tau10 Intercept + tau7 F_xi1  + l10_2 F_xi2   + tau3  F_xi3
+ l10_4 F_xi4
```

```
                                + l10_8 F_xi24 + e14,
      var_xz4 =  tau11 Intercept + tau8 F_xi1  + l11_2 F_xi2   + tau4  F_xi3
+ l11_4 F_xi4
                               + l11_8 F_xi24 + e15,

     /* structural model */
     F_eta1 = g11 F_xi1 + g13 F_xi3 + d1,
     F_eta2 = g22 F_xi2 + g24 F_xi4 + g28 F_xi24 + d2;

   std /*variance parameters */
      F_xi1 F_xi2 F_xi3 F_xi4 F_xi24 =
            phi11 phi22 phi33 phi44 phi55,
      e5-e15 = th1-th11,
      e1-e4 d1-d2 = any1-any6;  /* unconstrained parameters I added */

   cov  /* mean parameters */
       Intercept f_xi1 = 0,
       Intercept f_xi2 = 0,
       Intercept f_xi3 = 0,
       Intercept f_xi4 = 0,
       Intercept f_xi24 = phi42,  /*does this need to be there;
       /* cov parameters */
       F_xi1-F_xi4 = phi21 phi31 phi32 phi41 phi42 phi43,
       e13 e6  = th92,
       e14 e7  = th10_3,
       e15 e8  = th11_4,
       e13 e10 = th96,
       e14 e11 = th10_7,
       e15 e12 = th11_8;

/* programming statements for parameter constraints */
tau9  = tau2*tau6;
tau10 = tau3*tau7;
tau11 = tau4*tau8; *duncan ms has tau4Xtau7;

l10_2 = tau7*l32;
l10_4 = tau3*l74;
l10_8 = l32*l74;
l11_2 = tau8*l42;
l11_4 = tau4*l84;
l11_8 = l42*l84;

phi55 = phi22 * phi44 + phi42*phi42;

th9 = tau2*tau2*th6 + tau6*tau6*th2 + phi22*th6 + phi44*th2 + th2*th6;
th10 = tau3*tau3*th7 + tau7*tau7*th3 + l32*l32*phi22*th7 + l74*l74*phi44*th3 +
th3*th7;
th11 = tau4*tau4*th8 + tau8*tau8*th4 + l42*l42*phi22*th8 + l84*l84*phi44*th4 +
th4*th8;
th92 = tau6 * th2;
th96 = tau2 * th6;
th10_3 = tau7 * th3;
```

```
th10_7 = tau3 * th7;
th11_4 = tau8 * th4;
th11_8 = tau4 * th8;
*missing constraint on ka(5) of phi(4,2);

* outram statement for fit indices for Duncan model;
data dunfit2at; set dunfit;
keep _name_ _estim_;

data dunfit2bt; set dunfit;
keep _name_ _stderr_;

proc transpose data=dunfit2at out=dunfit2a let;
proc transpose data=dunfit2bt out=dunfit2b let;

data dunfit2b1;
set dunfit2b;
l32_str = l32;
l42_str = l42;
l74_str = l74;
g11_str = g11;
g13_str = g13;
g15_str = g15;
g22_str = g22;
g24_str = g24;
g28_str = g28;

data dunfit2b2; set dunfit2b1;
keep l32_str l42_str l74_str g11_str g13_str g15_str g22_str g24_str g28_str;

/* merging of the two data rows to get estimates and std errors on one line for
each model */
data dunfitx;
merge dunfit2a dunfit2b2;

/* merging in ODS output of convergence*/
data dunfits;
merge dunfitx dunconverge;

/* pulling in standardized estimate from ODS file*/
data std_temp;
set dunstdlatenteq;
if parameter3 = 'F_xi24';
g28std = coefficient3;

data g28std;
set std_temp;
keep g28std;

data dunfits;
merge dunfits g28std;
```

```
/* adding of model index value */
data dunfits;
set dunfits;
model = 2;

/* pulling in iterations from ODS file*/
data itr_temp;
set dunciter;
if Label1 = 'Iterations';
iter = nvalue1;

data itr;
set itr_temp;
keep iter;

data dunfits;
merge dunfits itr;

data dunfits; set dunfits;
      keep status model n fit nparm df chisquar p_chisq gfi agfi rmseaest
compfiti bb_nonor bb_normd bol_rho1 bol_del2 centrali
      l32 l32_str l42 l42_str l74 l74_str l84 l84_str g11 g11_str g13 g13_str
g15 g15_str g22 g22_str g24 g24_str g28 g28_str
      g28std iter;


/* merging of all Duncan replicates to one dataset */
proc append base=fdata.zsemfits_dun force;

%end;
```

Appendix D

SAS Syntax for the Schumacker Latent Growth Interaction Model

```
/*************************************************************************************
***************
/****************           SCHUMACKER FACTOR SCORE MODEL
***************************
*************************************************************************************
**************/
%do j = 1 %to &reps;

proc calis ucov aug method=ml data=fdata.sem&j maxiter=5000 maxfunc=5000
outram=temp.schfit_t&j
            noprint outstat=temp.schumout_t&j;
title "CALIS output for Schumacker model sample &j for creating factor scores";
  lineqs
      /* measurement model for y : assume the tau vector is a mistake in the
note */
      var_y1  =  F_eta1                    +     e1,
      var_y2  =  F_eta1 + 1. F_eta2        +     e2,
      var_y3  =  F_eta1 + betay3 F_eta2 +     e3,
      var_y4  =  F_eta1 +  betay4 F_eta2 +     e4,

      /* measurement model for x and z */
      var_x1  =  tau1  Intercept +  1  F_xi1                                  +
e5,
      var_x2  =  tau2  Intercept +  1  F_xi1 + 1   F_xi2                      +
e6,
      var_x3  =  tau3  Intercept +  1  F_xi1 + l32 F_xi2                      +
e7,
      var_x4  =  tau4  Intercept +  1  F_xi1 + l42 F_xi2                      +
e8,
      var_z1  =  tau5  Intercept +  1  F_xi3                                  +
e9,
      var_z2  =  tau6  Intercept +  1  F_xi3 + 1   F_xi4                      +
e10,
      var_z3  =  tau7  Intercept +  1  F_xi3 + l74 F_xi4                      +
e11,
      var_z4  =  tau8  Intercept +  1  F_xi3 + l84 F_xi4                      +
e12;

  std /*variance parameters */
    F_xi1 F_xi2 F_xi3 F_xi4 = phi11 phi22 phi33 phi44,
      F_eta1 = phiy1, F_eta2 = phiy2,
    e5-e12 = th1-th8,
    e1-e4 = any1-any4;  /* unconstrained parameters I added */

  cov  /* mean parameters */
     Intercept f_xi1 = 0,
     Intercept f_xi2 = 0,
```

```
        Intercept f_xi3 = 0,
        Intercept f_xi4 = 0,


        /* cov parameters */
        F_xi1-F_xi4 = phi21 phi31 phi32 phi41 phi42 phi43,
          F_eta1 F_eta2 = ceta12,
          F_eta1 F_xi1 = etaxy11,
          F_eta1 F_xi3 = etazy11,
          F_eta2 F_xi2 = etaxy22,
          F_eta2 F_xi4 = etazy22;

/*********    factor score creation for Schumacker model
************************/
proc score data=fdata.sem&j score=temp.schumout_t&j out=temp.schumxzfac&j;
var var_x1 var_x2 var_x3 var_x4 var_z1 var_z2 var_z3 var_z4 var_y1 var_y2
var_y3 var_y4;

%end;

/******* CREATING OF FACTOR SCORE INTERACTIONS FOR SCHUMACKER MODEL  *********/

%do j = 1 %to &reps;
data temp.schumf2&j;
      set temp.schumxzfac&j;
      F_xi13 = F_xi1 * F_xi3;
      F_xi24 = F_xi2 * F_xi4;

data temp.schumf2&j;
      set temp.schumf2&j;
      keep F_xi1 F_xi2 F_xi3 F_xi4 F_xi24 F_eta1 F_eta2;

%end;

/*************************************************************************
**
***********    CREATING OF SCHUMACKER INTERACTION MODEL USING CALIS
***********
**************************************************************************
**/

%do j = 1 %to &reps;

proc calis ucov aug method=ml data=temp.schumf2&j maxiter=5000 maxfunc=5000
outram=schfit
            outstat=schumout all nomod;

ods output iterstop = schiter;
ods output convergencestatus = schconverge;
ods output stdmanifesteq = schstdlatenteq;

title 'CALIS output for Schumacker model sample &j, actual model';
```

```
  lineqs
      F_eta1 = g11 F_xi1 + g13 F_xi3 + d1,
    F_eta2 = g22 F_xi2 + g24 F_xi4 + g28 F_xi24 + d2;

  std /*variance parameters */
     F_xi1 = 1,
       F_xi2 = 1,
     d1-d2 = any1 any2;  /* unconstrained parameters I added */

  cov  /* mean parameters */
      /* cov parameters */
      F_xi1 F_xi2 = phi12,
        F_xi3 F_xi4 = phi34;
run;

* outram statement for fit indices for SCHUMACKER model;
data schfit2at; set schfit;
keep _name_ _estim_;

data schfit2bt; set schfit;
keep _name_ _stderr_;

proc transpose data=schfit2at out=schfit2a let;
proc transpose data=schfit2bt out=schfit2b let;

data schfit2b1;
set schfit2b;
l32_str = l32;
l42_str = l42;
l74_str = l74;
g11_str = g11;
g13_str = g13;
g15_str = g15;
g22_str = g22;
g24_str = g24;
g28_str = g28;  run;
ods trace off;

data schfit2b2; set schfit2b1;
keep l32_str l42_str l74_str g11_str g13_str g15_str g22_str g24_str g28_str;
run;

/* merging of the two data rows to get estimates and std errors on one line for
each model */
data schfits;
merge schfit2a schfit2b2;  run;

/* merging in ODS output of convergence*/
data temp;
merge schfits schconverge;   run;

data schfits;
```

```
set temp;

/* pulling in standardized estimate from ODS file */
data std_temp;
set schstdlatenteq;
if parameter3 = 'F_xi24';
g28std = coefficient3;
run;

data g28std;
set std_temp;
keep g28std;
run;

data schfits;
merge schfits g28std;
run;

/* adding of model index value */
data schfits;
set schfits;
model = 3;

/* pulling in iterations from ODS file*/
data itr_temp;
set schiter;
if Label1 = 'Iterations';
iter = nvalue1;
run;

data itr;
set itr_temp;
keep iter;

data schfits;
merge schfits itr;  run;

/*  */
data schfits; set schfits;
      keep status model n fit nparm df chisquar p_chisq gfi agfi rmseaest
compfiti bb_nonor bb_normd bol_rho1 bol_del2 centrali
      l32 l32_str l42 l42_str l74 l74_str l84 l84_str g11 g11_str g13 g13_str
g15 g15_str g22 g22_str g24 g24_str g28 g28_str
      g28std iter;

/* merging of all Schumacker replicates into one dataset  */
proc append base=fdata.zsemfits_schum force;
run;

%end;
/* END OF SCHUMACKER MODEL*/
```

```
data fdata.ztotal;
set  fdata.zsemfits_dun fdata.zsemfits_schum;
run;
```

Appendix E

Results of the CFI Analyses for the MCAR and MNAR Data Conditions

MCAR Data Condition Analyses

The ANOVA in Table 5 for MCAR data shows significant effects ($p < 0.05$) for all main effects and their interactions. The overall model effect was significant ($F_{(107, 528871)} = 12805.37$, $p < 0.01$, partial $eta^2 = 0.72$). The four-way interaction of latent interaction model type, latent intercept-slope correlation, sample size, and reliability of the observed indicators, was significant ($F_{(24, 528871)} = 2.22$, $p < 0.01$, partial $eta^2 < 0.01$). However, as with the Complete data, only those effects with a partial eta-squared greater than 0.06 were considered strong enough to warrant further investigation. The ANOVA effects that met this criterion were for the two-way interactions of latent interaction model with reliability of the observed indicators (0.12) and the main effects of reliability (0.10) and latent interaction model (0.69), according to Olejnik and Algina (2000).

To examine these significant two-way interactions, simple plot of the average CFI values for each latent growth interaction model with both level of reliability of the observed indicators (0.30, 0.50, 0.70, 0.90) and latent intercept-slope correlation (0.20, 0.50, 0.70) on the X-axis. These simple plots are given in Figure ApxE-F1.

Insert Figure ApxE-F1 about here

Seen clearly in the simple plot is the difference in pattern of average values of the CFI for the Schumacker model from both the Wen and the Duncan models across the levels of indicator reliability. Both the Wen and the Duncan model produced stable values around the average value of 0.90, while the Schumacker model showed a trend of increasing CFI values as reliability increased, with values increasing from 0.69 to 0.81. However, when examining the two-way interaction involving latent model type and correlation between the latent intercept and slope, the Wen and the Duncan models again showed a stable pattern (with average values around clustered around 0.90) while the Schumacker model showed a decreasing trend of CFI values as the correlation increased.

MNAR Data Condition Analyses

The ANOVA in Table 6 for the MNAR data condition showed significant effects ($p < 0.05$) for all main effects and their interactions. The largest effect sizes were seen for the two-way interaction of latent interaction model with reliability of the observed indicators (0.24) and for the main effect of latent interaction model (0.34). The overall model effect was significant ($F_{(107, 474758)} = 4456.85$, $p < 0.01$, partial eta$^2 = 0.50$). The four-way interaction of latent interaction model type, latent intercept-slope correlation, sample size, and reliability of the observed indicators, was significant ($F_{(24, 474758)} = 9.49$, $p < 0.01$) and with a small effect size (partial eta$^2 < 0.01$). As with the Complete data, only those effects with a partial eta-squared greater than 0.06 were considered strong enough to warrant further investigation. The ANOVA effects that met this criterion were for the two-way interactions of latent interaction model with both reliability of the observed indicators (0.24) and latent intercept-slope correlation (0.08), and the main effect of latent interaction model (0.34), according to Olejnik and Algina (2000).

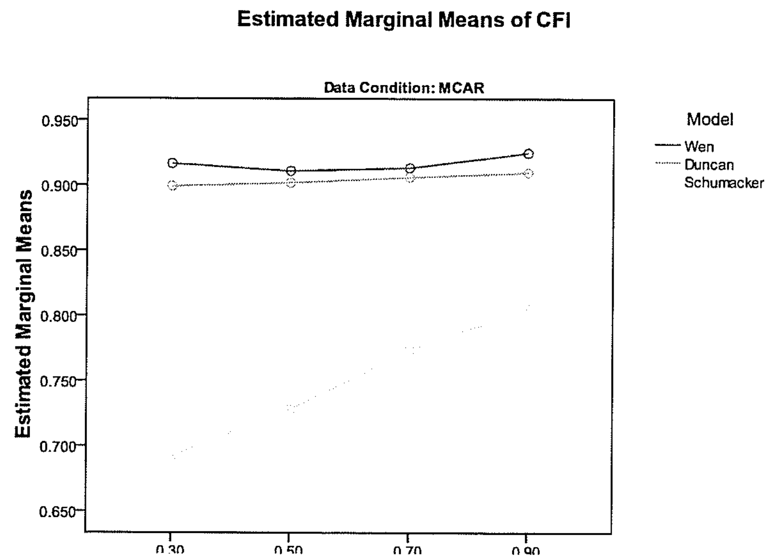
To examine the significant two-way interactions, a simple plot of the average CFI values for each latent growth interaction model with both level of reliability of the observed indicators and latent intercept-slope correlation on the X-axis. These simple plots are given in Figure ApxE-F2.

Insert Figure ApxE-F2 about here

Seen clearly in the simple plot is the difference in pattern of average values of the CFI for all three of the latent growth interaction models. The Wen model showed a pattern of decreasing average CFI values (from 0.81 to 0.70) as reliability increased. The Duncan and Schumacker models both showed patterns of increasing average CFI values (0.67 to 0.73 and 0.75 to 0.78, respectively), although the Schumacker model showed a slight decrease at the highest level of reliability. For the interaction of latent interaction model type with latent intercept-slope correlation, both the Wen and the Duncan models showed slightly increasing trends as the correlation increased, wihle the Schumacker model showed a decreasing trend.

Figure ApxE-F1

Plots of the Interaction Effect of Latent Interaction Model Type Factor with the Observed Indicator Reliability Factor (1) and Latent Intercept-Slope Correlation (2) on the Comparative Fit Index (CFI) in the Missing Completely At Random Data Condition in Those Models that Converged Successfully.



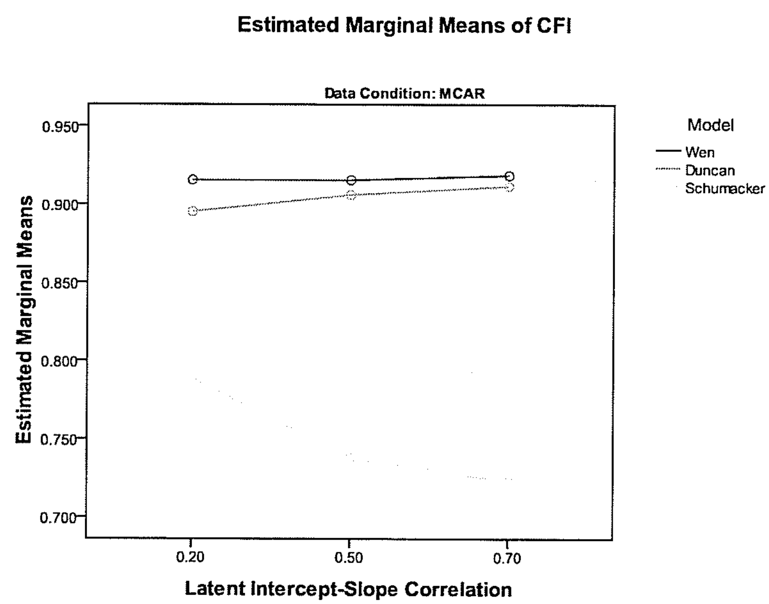
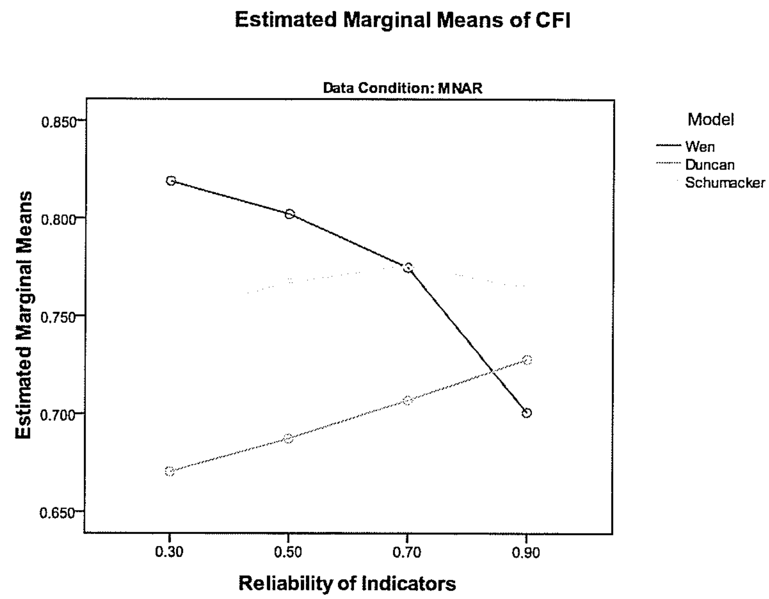
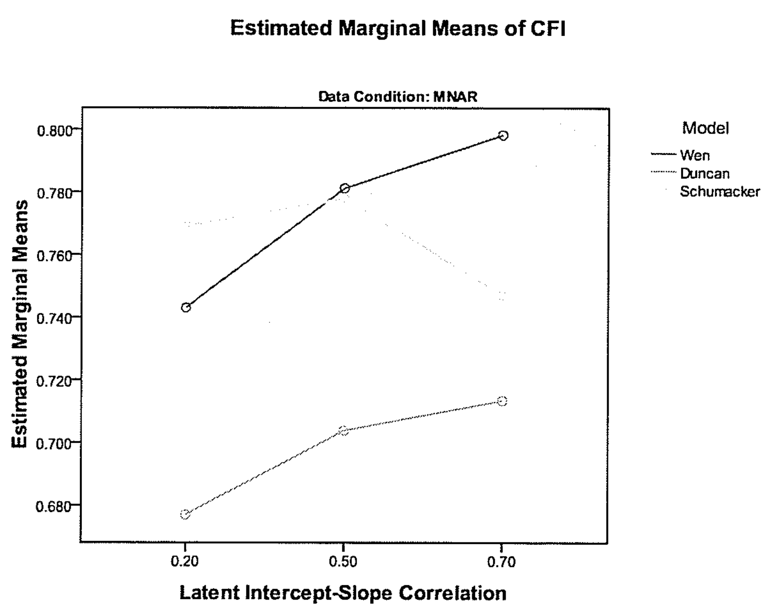


Figure ApxE-F2

Plots of the Interaction Effect of Latent Interaction Model Type Factor with the Observed Indicator Reliability Factor (1) and Latent Intercept-Slope Correlation (2) on the Comparative Fit Index (CFI) in the Missing Not At Random Data Condition in Those Models that Converged Successfully.





Appendix F

Results of the NFI Analyses for the MCAR and MNAR Data Conditions

MCAR Data Condition Analyses

In the MCAR data condition the average NFI value for the Wen model was 0.90 (range 0.83-0.94), for the Duncan model 0.89 (range 0.81-0.93), and for the Schumacker model 0.75 (range 0.18-0.95). Only the Wen and Duncan models produced 95% confidence intervals for the mean NFI values that had lower bounds above the value of 0.90. For the Wen model these were consistently produced when the reliability was at its highest value (0.90), and occurred more often as both sample size increased and as the correlation between the latent intercept and slope increased. The Duncan model showed such confidence intervals only at the highest levels of reliability, with the highest proportion in the conditions where the correlation between the latent intercept and slope was highest (0.70) and the sample size was highest (1000). The Schumacker model did not produce any 95% confidence intervals whose lower bound was at or above 0.90.

The ANOVA in Table 13 for MCAR data shows significant effects ($p < 0.05$) for all main effects and their interactions. The overall model effect was significant ($F_{(107, 528871)} = 11478.07$, $p < 0.01$), and had a large effect size (partial eta$^2 = 0.70$). The four-way interaction of latent interaction model type, latent intercept-slope correlation, sample size, and reliability of the observed indicators, was significant ($F_{(24, 528871)} = 2.04$, $p < 0.01$) with a small effect size (partial eta$^2 < 0.01$). However, as noted earlier with the CFI, the large error degrees of freedom for the ANOVA model results in even trivial mean differences emerging as significant, and effects that produced effect sizes of at least a medium effect (i.e., 0.06 or greater for the partial eta-squared) were examined.

The ANOVA effects that met this criteria were for the two-way interactions of latent interaction model with both reliability (0.11) and correlation of latent intercept and slope (0.09), and the main effects of latent interaction model (0.66) and reliability (0.12). To examine these significant two-way interactions, separate simple plots of the average NFI values with the reliability of the observed indicators and the latent interacept-slope correlation were produced for each latent growth interaction model, and are given in Figure ApxF-F1.

Insert Figure ApxF-F1 about here

Similar to that seen with the Complete data, the simple plot for the interaction of latent growth interaction model with reliability showed that both the Wen and Duncan models produced stable NFI values (around 0.90) across the levels of reliability, while the Schumacker model showed a trend of increasing NFI values as reliability increased. However, when examining the two-way interaction involving latent model type and correlation between the latent intercept and slope, the Wen and the Duncan models again showed a stable pattern (with average values around clustered around 0.90) while the Schumacker model showed a decreasing trend of NFI values as the correlation increased.

MNAR Data Condition Analyses

In the MNAR data condition the average NFI value for the Wen model was 0.75 (range 0.59-0.87), for the Duncan model 0.69 (range 0.56-0.80), and for the Schumacker model 0.76 (range 0.06-0.96).

The ANOVA in Table 13 for MNAR data shows significant effects ($p < 0.05$) for all main effects and their interactions. The overall model effect was significant ($F_{(107, 474758)} = 4961.38$, $p < 0.01$), and had a large effect size (partial eta$^2 = 0.53$). The four-way interaction of latent interaction model type, latent intercept-slope correlation, sample size, and reliability of the observed indicators, was significant ($F_{(24, 474758)} = 10.39$, $p < 0.01$) with a small effect size (partial eta$^2 < 0.01$). However, as noted earlier with the CFI, the large error degrees of freedom for the ANOVA model results in even trivial mean differences emerging as significant, and effects that produced effect sizes of at least a medium effect (i.e., 0.06 or greater for the partial eta-squared) were examined.

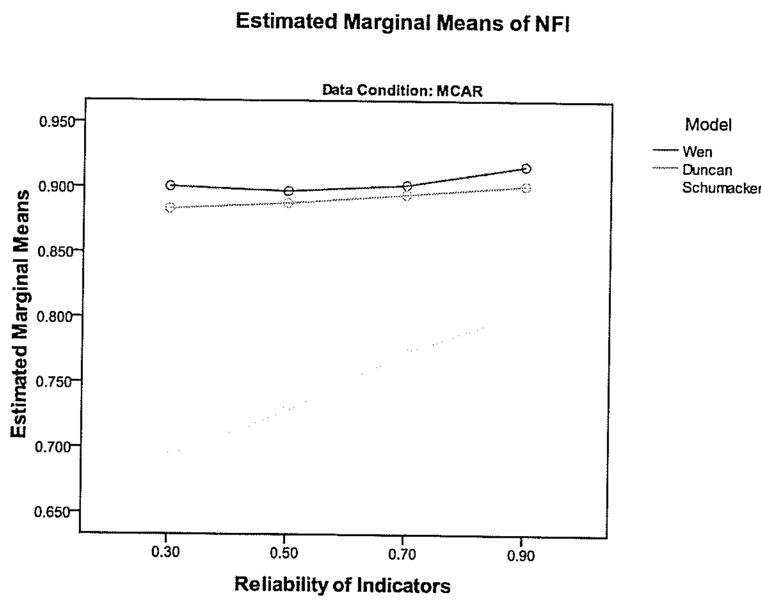
The ANOVA effects that met this criteria were the two-way interactions of latent interaction model with both reliability (0.24) and latent intercept-slope correlation (0.08), and the main effect of latent interaction model (0.37). To examine these significant two-way interactions, separate simple plots of the average NFI values with the reliability of the observed indicators and the latent interaction-slope correlation were produced for each latent growth interaction model, and are given in Figure ApxF-F2.

Insert Figure ApxF-F2 about here

Similar to that seen with the Complete data, the simple plot for the latent growth interaction model X reliability interaction showed that the Wen model showed a decreasing trend of average NFI values as reliability increased, and the Duncan and Schumacker models showed increasing trends, with the Schumacker model producing higher average values than the Duncan. With respect to the interaction of latent model type with correlation, both the Wen and the Duncan models showed slight increases as the correlation increased, and the Schumacker model showed a non-linear trend of first increasing average values and then decreasing average values as the correlation increased.

Figure ApxF-F1

Plots of the Interaction Effect of Latent Interaction Model Type Factor with the Observed Indicator Reliability Factor (1) and Latent Intercept-Slope Correlation (2) on the Normed Fit Index (NFI) in the Missing Completely At Random Data Condition in Those Models that Converged Successfully.



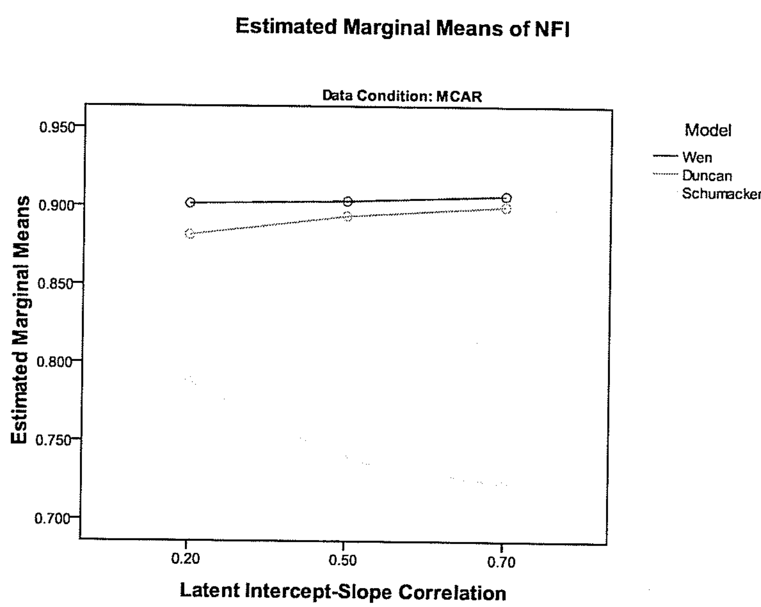
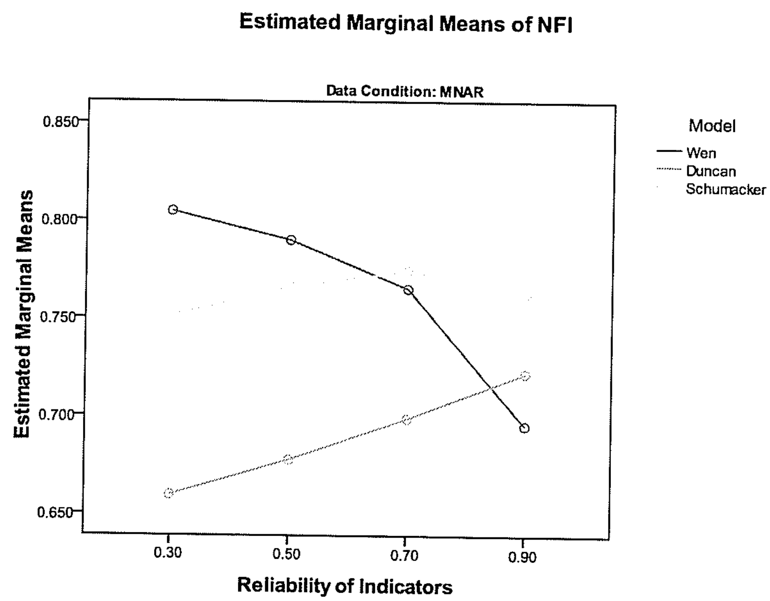
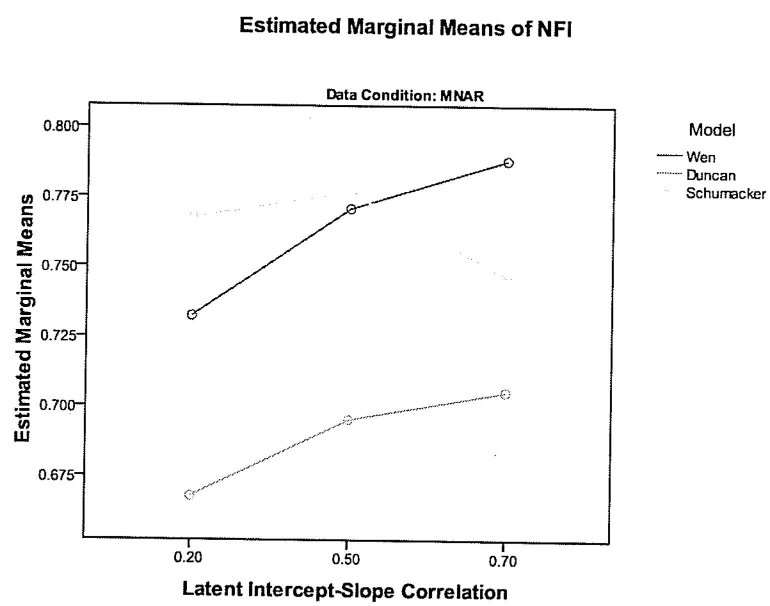


Figure ApxF-F2

Plots of the Interaction Effect of Latent Interaction Model Type Factor with the Observed Indicator Reliability Factor (1) and Latent Intercept-Slope Correlation (2) on the Normed Fit Index (NFI) in the Missing Not At Random Data Condition in Those Models that Converged Successfully.





Appendix G

Results of the GFI Analyses for the Complete, MCAR, and MNAR Data Conditions

Complete Data Condition Analyses

Table ApxG-T1 presents the 95% confidence intervals for the mean value. Table ApxG-T2 contains the results of three ANOVA models, using the GFI statistic as the dependent variable, with latent interaction model type (3 levels), correlation of the latent intercept and slope (3 levels), sample size (3 levels) and reliability of the observed indicators (4 levels), as well as their interactions, as independent factors. These analyses were carried out separately for each data condition (Complete data, MCAR data, MNAR data).

The ANOVA in Table ApxG-T2 for Complete data showed significant effects ($p < 0.05$) for all main effects and their interactions. The overall model showed a significant effect ($F_{(107, 531703)} = 19863.89$, $p < 0.01$), and had a large effect size (partial eta^2 = 0.44). The four-way interaction of latent interaction model type, latent intercept-slope correlation, sample size, and reliability of the observed indicators, was significant ($F_{(24, 531703)} = 18.02$, $p < 0.01$) with a small effect size (partial eta^2 < 0.01). However, the largest effects were seen for the two-way interactions of latent growth interaction model type with reliability of the observed indicators (0.34), latent growth interaction model with intercept-slope correlation (0.23), and for the main effects of latent growth interaction model (0.75), sample size (0.11) and correlation between latent intercept and slope (0.07).

To examine these significant two-way interactions, separate simple plots of the average GFI values with the reliability of the observed indicators and the latent interaction-slope correlation were produced for each latent growth interaction model, and are given in Figure ApfG-F1.

Insert Figure ApxG-F1 about here

The simple plot for the interaction of latent growth interaction model with reliability showed the Wen model having a stable pattern of GFI values as reliability increased, the Duncan model having a decreasing pattern of GFI values, and the Schumacker with a slight increasing pattern. With respect to the interaction of latent model type with correlation, both the Wen and the Duncan models showed stable patterns of average GFI values as the correlation increased, and the Schumacker model showed a decreasing trend as the correlation increased.

MCAR Data Condition Analyses

The ANOVA in Table ApxG-T2 for MCAR data shows significant effects ($p < 0.05$) for all main effects and their interactions. The overall model showed a significant effect ($F_{(107, 528871)} = 16169.74$, $p < 0.01$), and had a large effect size (partial eta^2 = 0.77). The four-way interaction of latent interaction model type, latent intercept-slope correlation, sample size, and reliability of the observed indicators, was significant ($F_{(24, 528871)} = 17.36$, $p < 0.01$) and a small effect size (partial eta^2 < 0.01).

However, the largest effects were seen for the two-way interactions of latent growth interaction model type with reliability of the observed indicators (0.30), latent growth interaction model with intercept-slope correlation (0.19), and for the main effects of latent growth interaction model (0.71), sample size (0.15) and correlation between latent intercept and slope (0.06).

To examine these significant two-way interactions, separate simple plots of the average GFI values with the reliability of the observed indicators and the latent interaction-slope correlation were produced for each latent growth interaction model, and are given in Figure ApfG-F2.

Insert Figure ApxG-F2 about here

The simple plot for the interaction of latent growth interaction model with reliability showed the Wen model having a stable pattern of GFI values as reliability increased, the Duncan model having a decreasing pattern of GFI values, and the Schumacker with a slight increasing pattern. With respect to the interaction of latent model type with correlation, both the Wen and the Duncan models showed stable patterns of average GFI values as the correlation increased, and the Schumacker model showed a decreasing trend as the correlation increased. For the main effect of sample size, the means show an increasing trend as sample size increased (0.82, 0.83, and 0.84, at initial sample sizes of 250, 500, and 1000, respectively).

MNAR Data condition Analyses

The ANOVA for the MCAR data (reported in ApxG-T2, and not reproduced here) showed significant effects ($p < 0.05$) for all main effects and their interactions. The overall model showed a significant effect ($F_{(107, 474758)} = 27877.66$, $p < 0.01$), and had a large effect size (partial eta^2 = 0.86). The four-way interaction of latent interaction model type, latent intercept-slope correlation, sample size, and reliability of the observed indicators, was significant ($F_{(24, 474758)} = 9.99$, $p < 0.01$) and produced a small effect size (partial eta^2 < 0.01). The largest effects were seen for the two-way interactions of: latent interaction model with reliability (0.49) and correlation of the latent intercept and slope (0.10), and the main effects of latent interaction model (0.82), correlation between latent intercept and slope (0.13), sample size (0.09), and reliability (0.12).

To examine these significant two-way interactions, separate simple plots of the average GFI values with the reliability of the observed indicators and the latent interaction-slope correlation were produced for each latent growth interaction model, and are given in Figure ApxG-F3.

Insert Figure ApxG-F3 about here

The simple plot for the interaction of latent growth interaction model with reliability showed both the Wen and Duncan models having a decreasing pattern of GFI values as reliability increased, with the Wen model being more affected at lower reliabilities and the Duncan model being affected at higher reliabilities. The Schumacker model showed a slight increasing pattern. With respect to the interaction of latent model type with correlation, both the Wen and the Duncan models showed stable patterns of average GFI values as the correlation increased, and the Schumacker model showed a decreasing trend as the correlation increased. For the main effect of sample size, the means show an increasing trend as sample size increased (0.80, 0.82, and 0.83, at initial sample sizes of 250, 500, and 1000, respectively).

ApxG-T1
Confidence Intervals (95%) for the Goodness of Fit Index (GFI) Values for All Latent Growth Interaction Models (Population Value of Latent Interaction Parameter Equal to 2.0) For the Three Missing Data Conditions (Complete Data, MCAR Data, MNAR Data).

Corr	N	Rel		Complete Data			MCAR Data			MNAR Data		
				Wen	Duncan	Schumacker	Wen	Duncan	Schumacker	Wen	Duncan	Schumacker
0.20	250	0.30	L	0.848	0.859	0.763	0.838	0.848	0.751	0.757	0.829	0.841
			U	0.852	0.861	0.777	0.842	0.852	0.769	0.763	0.831	0.859
		0.50	L	0.837	0.849	0.773	0.827	0.839	0.761	0.727	0.819	0.863
			U	0.843	0.851	0.787	0.833	0.841	0.779	0.733	0.821	0.877
		0.70	L	0.858	0.849	0.793	0.847	0.839	0.782	0.725	0.788	0.895
			U	0.862	0.851	0.807	0.853	0.841	0.798	0.735	0.792	0.905
		0.90	L	0.849	0.819	0.837	0.839	0.808	0.825	0.717	0.748	0.917
			U	0.851	0.821	0.843	0.841	0.812	0.835	0.723	0.752	0.923
	500	0.30	L	0.859	0.879	0.765	0.849	0.869	0.764	0.768	0.839	0.866
			U	0.861	0.881	0.775	0.851	0.871	0.776	0.772	0.841	0.874
		0.50	L	0.848	0.870	0.785	0.848	0.869	0.784	0.738	0.829	0.897
			U	0.852	0.870	0.795	0.852	0.871	0.796	0.742	0.831	0.903
		0.70	L	0.869	0.859	0.806	0.869	0.859	0.805	0.736	0.809	0.918
			U	0.871	0.861	0.814	0.871	0.861	0.815	0.744	0.811	0.922
		0.90	L	0.870	0.829	0.849	0.859	0.829	0.848	0.738	0.769	0.929
			U	0.870	0.831	0.851	0.861	0.831	0.852	0.742	0.771	0.931
	1000	0.30	L	0.859	0.880	0.777	0.859	0.880	0.776	0.769	0.850	0.888
			U	0.861	0.880	0.783	0.861	0.880	0.784	0.771	0.850	0.892
		0.50	L	0.858	0.880	0.797	0.858	0.880	0.797	0.739	0.840	0.909
			U	0.862	0.880	0.803	0.862	0.880	0.803	0.741	0.840	0.911
		0.70	L	0.880	0.870	0.828	0.880	0.870	0.817	0.737	0.820	0.919
			U	0.880	0.870	0.832	0.880	0.870	0.823	0.743	0.820	0.921
		0.90	L	0.880	0.840	0.849	0.870	0.840	0.849	0.749	0.770	0.929
			U	0.880	0.840	0.851	0.870	0.840	0.851	0.751	0.770	0.931
0.50	250	0.30	L	0.837	0.859	0.723	0.827	0.848	0.721	0.757	0.828	0.822
			U	0.843	0.861	0.737	0.833	0.852	0.739	0.763	0.832	0.838
		0.50	L	0.857	0.859	0.733	0.837	0.839	0.722	0.727	0.808	0.843
			U	0.863	0.861	0.747	0.843	0.841	0.738	0.733	0.812	0.857
		0.70	L	0.859	0.849	0.754	0.848	0.828	0.742	0.686	0.788	0.865
			U	0.861	0.851	0.766	0.852	0.832	0.758	0.694	0.792	0.875
		0.90	L	0.859	0.819	0.797	0.849	0.808	0.785	0.706	0.748	0.907
			U	0.861	0.821	0.803	0.851	0.812	0.795	0.714	0.752	0.913
	500	0.30	L	0.858	0.879	0.726	0.848	0.869	0.725	0.768	0.849	0.846
			U	0.862	0.881	0.734	0.852	0.871	0.735	0.772	0.851	0.854

		0.50	L	0.879	0.869	0.746	0.868	0.869	0.735	0.728	0.829	0.867
			U	0.881	0.871	0.754	0.872	0.871	0.745	0.732	0.831	0.873
		0.70	L	0.880	0.859	0.766	0.869	0.859	0.755	0.688	0.809	0.888
			U	0.880	0.861	0.774	0.871	0.861	0.765	0.692	0.811	0.892
		0.90	L	0.870	0.829	0.799	0.869	0.829	0.798	0.718	0.759	0.909
			U	0.870	0.831	0.801	0.871	0.831	0.802	0.722	0.761	0.911
	1000	0.30	L	0.869	0.890	0.738	0.858	0.880	0.737	0.779	0.850	0.868
			U	0.871	0.890	0.742	0.862	0.880	0.743	0.781	0.850	0.872
		0.50	L	0.890	0.880	0.758	0.889	0.880	0.747	0.739	0.840	0.878
			U	0.890	0.880	0.762	0.891	0.880	0.753	0.741	0.840	0.882
		0.70	L	0.890	0.870	0.778	0.860	0.870	0.777	0.699	0.809	0.899
			U	0.890	0.870	0.782	0.860	0.870	0.783	0.701	0.811	0.901
		0.90	L	0.880	0.840	0.809	0.880	0.830	0.809	0.719	0.760	0.909
			U	0.880	0.840	0.811	0.880	0.830	0.811	0.721	0.760	0.911
0.70	250	0.30	L	0.848	0.859	0.703	0.837	0.848	0.701	0.757	0.828	0.802
			U	0.852	0.861	0.717	0.843	0.852	0.719	0.763	0.832	0.818
		0.50	L	0.858	0.859	0.713	0.847	0.848	0.712	0.727	0.808	0.813
			U	0.862	0.861	0.727	0.853	0.852	0.728	0.733	0.812	0.827
		0.70	L	0.859	0.849	0.734	0.848	0.838	0.722	0.687	0.788	0.845
			U	0.861	0.851	0.746	0.852	0.842	0.738	0.693	0.792	0.855
		0.90	L	0.859	0.819	0.766	0.849	0.808	0.765	0.695	0.738	0.877
			U	0.861	0.821	0.774	0.851	0.812	0.775	0.705	0.742	0.883
	500	0.30	L	0.868	0.879	0.716	0.858	0.869	0.705	0.768	0.839	0.826
			U	0.872	0.881	0.724	0.862	0.871	0.715	0.772	0.841	0.834
		0.50	L	0.879	0.869	0.726	0.869	0.869	0.715	0.738	0.829	0.836
			U	0.881	0.871	0.734	0.871	0.871	0.725	0.742	0.831	0.844
		0.70	L	0.879	0.859	0.746	0.879	0.859	0.736	0.698	0.799	0.857
			U	0.881	0.861	0.754	0.881	0.861	0.744	0.702	0.801	0.863
		0.90	L	0.880	0.829	0.778	0.869	0.819	0.778	0.707	0.749	0.878
			U	0.880	0.831	0.782	0.871	0.821	0.782	0.713	0.751	0.882
	1000	0.30		0.879	0.890	0.718	0.869	0.880	0.707	0.779	0.850	0.838
				0.881	0.890	0.722	0.871	0.880	0.713	0.781	0.850	0.842
		0.50		0.890	0.880	0.728	0.889	0.880	0.727	0.739	0.840	0.848
				0.890	0.880	0.732	0.891	0.880	0.733	0.741	0.840	0.852
		0.70		0.890	0.870	0.758	0.890	0.870	0.747	0.709	0.809	0.859
				0.890	0.870	0.762	0.890	0.870	0.753	0.711	0.811	0.861
		0.90		0.880	0.840	0.779	0.880	0.830	0.779	0.718	0.759	0.889
				0.880	0.840	0.781	0.880	0.830	0.781	0.722	0.761	0.891

Note: MCAR = Missing Completely At Random; MNAR = Missing Not At Random; Corr = Latent intercept-slope correlation; N = Sample size; Rel = Reliability of the observed indicator; L = Lower 95% lilmit; U = Upper 95% limit.

ApxG-T2

Analysis of Variance Results with the Goodness of Fit Index (GFI) Values as the Dependent Variable, with Latent Model Type, Latent Intercept-Slope Correlation, Sample Size, and Reliability of Observed Indicators as Between-Subjects Factors, For the Three Missing Data Conditions (Complete, MCAR, MNAR).

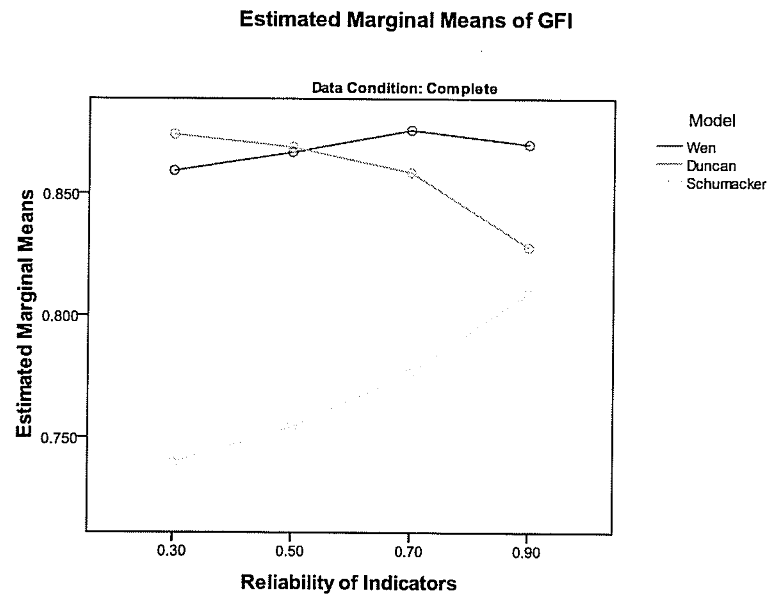
Corrected Model	df	Complete Data		MCAR Data		MNAR Data	
		F-value	Partial eta^2	F-value	Partial eta^2	F-value	Partial eta^2
Latent Interaction Model							
Covariance (Corr)	107	19863.89	.800	16169.74	.766	27877.66	.863
Sample Size (N)							
Reliability (Rel)	2	805126.83	.752	640801.51	.708	1042513.02	.815
Latent Model X Corr	2	20525.50	.072	17042.71	.061	34084.47	.126
Latent Model X N	2	31928.74	.107	44685.90	.145	24627.15	.094
Latent Model X Rel	3	7112.53	.039	5384.47	.030	21040.27	.117
Corr X N	4	39979.81	.231	31238.55	.191	13075.84	.099
Corr X Rel	4	598.83	.004	1347.12	.010	341.29	.003
N X Rel	6	45168.19	.338	37003.55	.296	76710.94	.492
Latent Model X Corr X N	4	19.42	.000	24.52	.000	25.02	.000
Latent Model X Corr X Rel	6	485.02	.005	336.68	.004	1577.68	.020
Latent Model X N X Rel	6	127.63	.001	133.60	.002	324.11	.004
Corr X N X Rel	8	108.35	.002	114.86	.002	57.57	.001
Latent Model X Corr X N X Rel	12	300.32	.007	215.68	.005	788.15	.020
Error	12	93.61	.002	113.25	.003	320.57	.008
Corrected Model	12	7.51	.000	8.75	.000	4.99	.000
	24	18.02	.001	17.36	.001	9.99	.001
Latent Interaction Model	531703						

Note: MCAR = Missing Completely At Random; MNAR = Missing Not At Random; Corr = Latent intercept-slope correlation; N = Sample size; Rel = Reliability of the observed indicator. The symbol "X" represents an interaction between two factors.

All effects are significant at the $p < 0.01$ level.

Figure ApxG-F1

Plots of the Interaction Effect of Latent Interaction Model Type Factor with the Observed Indicator Reliability Factor (1) and Latent Intercept-Slope Correlation (2) on the Goodness of Fit Index (GFI) in the Complete Data Condition in Those Models that Converged Successfully.



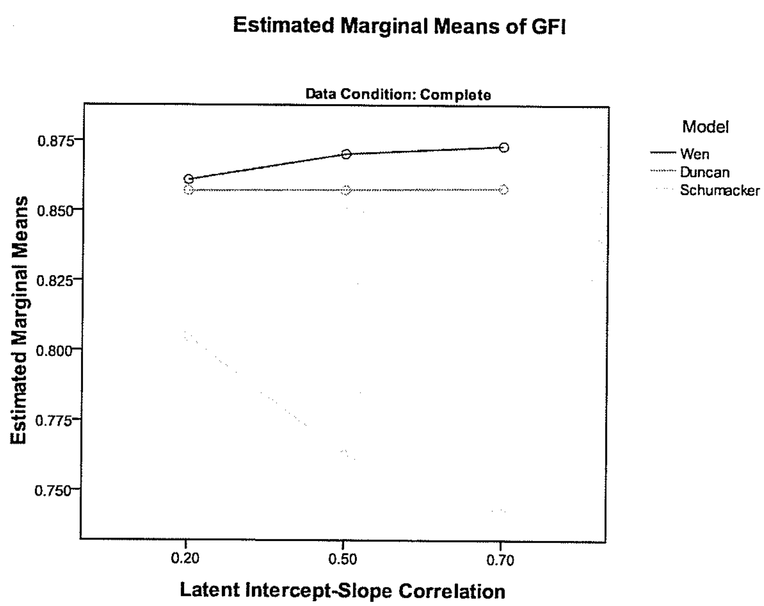
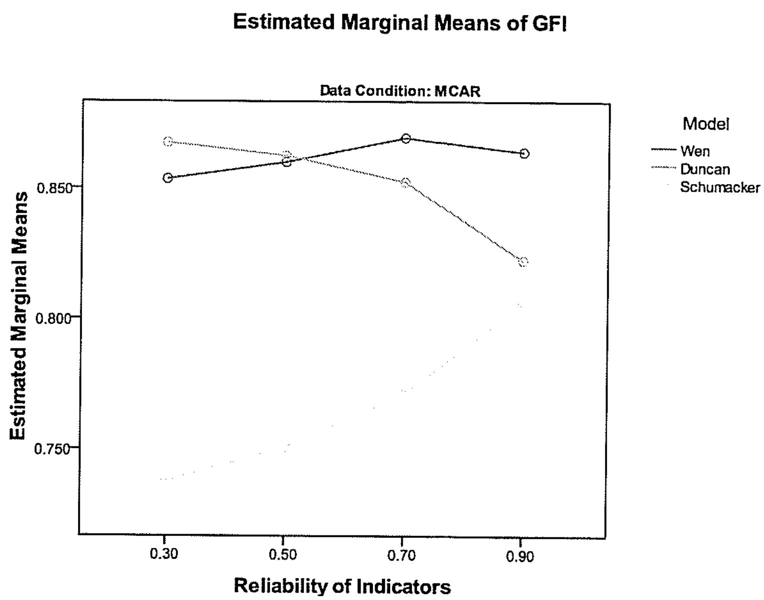


Figure ApxG-F2

Plots of the Interaction Effect of Latent Interaction Model Type Factor with the Observed Indicator Reliability Factor (1) and Latent Intercept-Slope Correlation (2) on the Goodness of Fit Index (GFI) in the Missing Completely At Random Data Condition in Those Models that Converged Successfully.



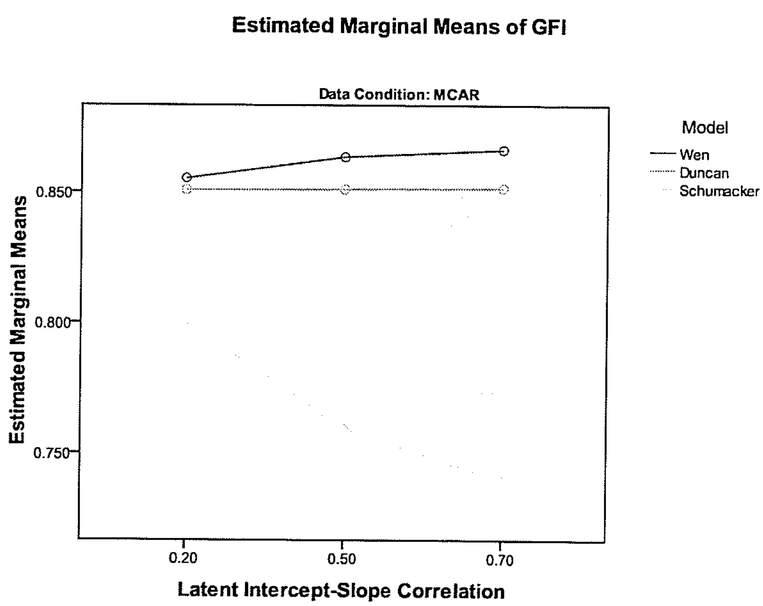
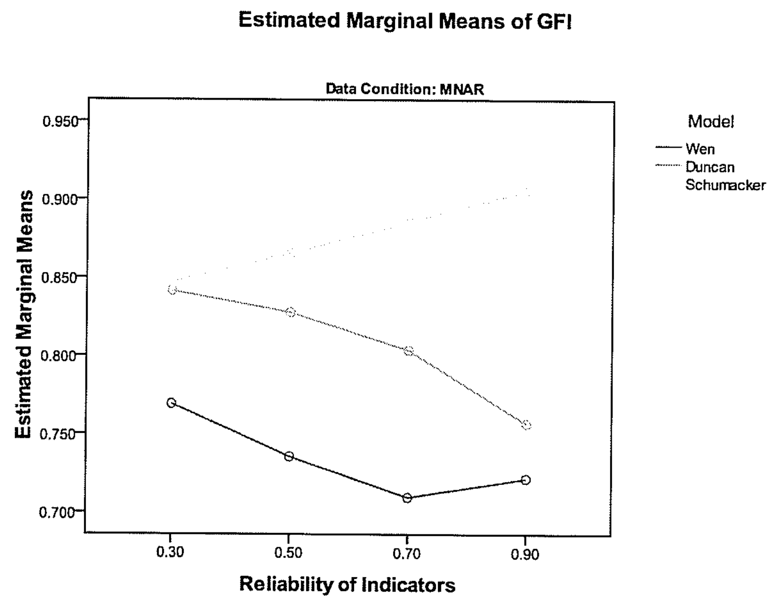
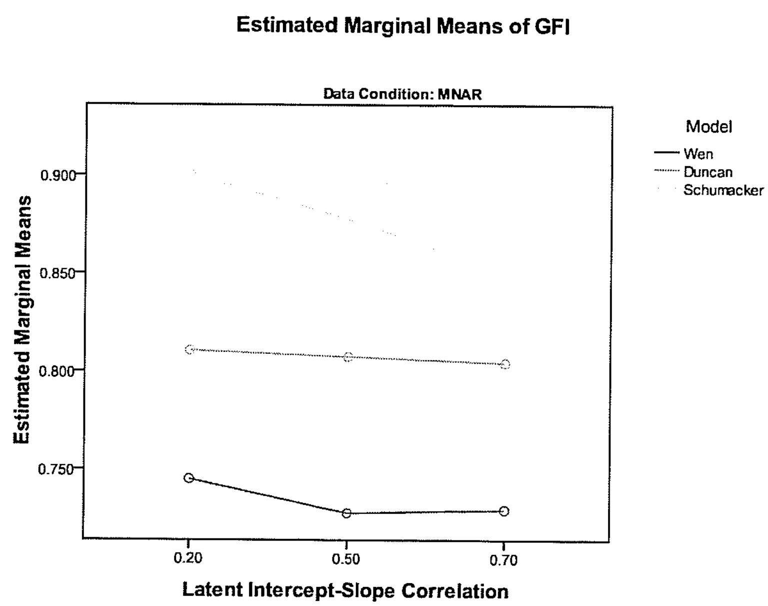


Figure ApxG-F3

Plots of the Interaction Effect of Latent Interaction Model Type Factor with the Observed Indicator Reliability Factor (1) and Latent Intercept-Slope Correlation (2) on the Goodness of Fit Index (GFI) in the Missing Not At Random Data Condition in Those Models that Converged Successfully.





Appendix H

Results of the RMSEA Analyses for All Data Conditions

Table ApxH-T1 contains the 95% confidence intervals for the mean RMSEA values for each of the three latent interaction growth models across all conditions, in each of the missing data conditions.

Analysis of Variance of RMSEA Values

Table ApxH-T2 contains the results of three ANOVA models, using the RMSEA statistic as the dependent variable, with latent interaction model type (3 levels), correlation of the latent intercept and slope (3 levels), sample size (3 levels) and reliability of the observed indicators (4 levels) as independent factors. These analyses were carried out separately for each data type condition (Complete data, MCAR data, MNAR data) using only those replications that successfully converged.

Complete Data Condition Analysis

The ANOVA in Table ApxH-T2 for Complete data shows significant effects ($p < 0.05$) for all main effects and their interactions. The overall model effect was significant ($F_{(107, 531703)} = 27467.60$, $p < 0.01$), with a large effect size (partial $eta^2 = 0.85$). The four-way interaction of latent interaction model type, latent intercept-slope correlation, sample size, and reliability of the observed indicators, was significant ($F_{(24, 531703)} = 1.63$, $p < 0.05$) and produced a small effect size (< 0.01). The largest effect sizes were seen for the two-way interactions of latent interaction model and reliability of the observed indicators (0.18) and the latent intercept-slope correlation (0.08) and for the main effect of latent interaction model (0.84).

To examine these significant two-way interactions, separate simple plots of the average RMSEA values with the reliability of the observed indicators and the latent intercept-slope correlation were produced for each latent growth interaction model, and are given in Figure ApxH-F1.

Insert Figure ApxH-F1 about here

The simple plot for the interaction of latent growth interaction model with reliability showed both the Wen and Duncan models having an increasing pattern of RMSEA values as reliability increased, with the Duncan model showing a sharper increase at higher levels of reliability. The Schumacker model showed a decreasing pattern of average RMSEA values as reliability increased. With respect to the interaction of latent model type with correlation, both the Wen and the Duncan models showed stable patterns of average RMSEA values as the correlation increased, and the Schumacker model showed an increasing trend as the correlation increased.

MCAR Data Condition Analysis

The ANOVA in Table ApxH-T2 for MCAR data shows significant effects ($p < 0.05$) for all main effects and their interactions. The overall model effect was significant ($F_{(107, 28871)} = 25003.44$, $p < 0.01$), with a large effect size (partial $eta^2 = 0.84$). The four-way interaction of latent interaction model type, latent intercept-slope correlation, sample size, and reliability of the observed indicators, was not significant ($F_{(24, 528871)} = 1.44$, ns). The largest effect sizes were seen with the two-way interactions of latent interaction model with both reliability of the observed indicators (0.17) and latent intercept-slope correlation (0.07), and the main effect of latent interaction model (0.83).

To examine these significant two-way interactions, separate simple plots of the average RMSEA values with the reliability of the observed indicators and the latent intercept-slope correlation were produced for each latent growth interaction model, and are given in Figure ApxH-F2.

Insert Figure ApxH-F2 about here

The results for the MCAR data closely resemble those of the Complete data. The simple plot for the interaction of latent growth interaction model with reliability showed both the Wen and Duncan models having an increasing pattern of RMSEA values as reliability increased, with the Duncan model showing a sharper increase at higher levels of reliability. The Schumacker model showed a decreasing pattern of average RMSEA values as reliability increased. With respect to the interaction of latent model type with correlation, both the Wen and the Duncan models showed stable patterns of average RMSEA values as the correlation increased, and the Schumacker model showed an increasing trend as the correlation increased.

MNAR Data Condition Analyses

The ANOVA in Table ApxH-T2 for MNAR data shows significant effects ($p < 0.05$) for all main effects and their interactions. The overall model effect was significant ($F_{(107, 474758)} = 7825.30$, $p < 0.01$), with a large effect size (partial eta$^2 = 0.64$). The four-way interaction of latent interaction model type, latent intercept-slope correlation, sample size, and reliability of the observed indicators was significant ($F_{(24, 474758)} = 6.02$, $p < 0.01$), with a small effect size (partial eta$^2 < 0.01$). However, the largest effect sizes were seen for the two-way interactions of latent interaction model with reliability (0.43) and correlation between latent intercepts and slopes (0.12), and the main effect of latent interaction model (0.41).

To examine these significant two-way interactions, separate simple plots of the average RMSEA values with the reliability of the observed indicators and the latent intercept-slope correlation were produced for each latent growth interaction model, and are given in Figure ApxH-F3.

Insert Figure ApxH-F3 about here

The simple plot for the interaction of latent growth interaction model with reliability showed both the Wen and Duncan models having an increasing pattern of RMSEA values as reliability increased, with the Wen model showing a sharper increase at higher levels of reliability (and having an average value that was higher than that of the Duncan model). The Schumacker model showed a decreasing pattern of average RMSEA values as reliability increased, with average RMSEA values that were comparable to those of the Wen and Duncan models at the highest level of reliability. With respect to the interaction of latent model type with correlation, both the Wen and the Duncan models showed stable patterns of average RMSEA values as the correlation increased, and the Schumacker model showed an increasing trend as the correlation increased.

Summary

None of the three latent growth interaction models showed average RMSEA values that were below the cutoff of 0.05 for adequate model fit. Even using a relaxed value of 0.08 as a cutoff for "acceptable" model fit (Fan et al., 1998), none of the latent growth interaction models had average RMSEA values that were lower than this value, although the Wen model did have average RMSEA values that were equal to 0.11 in several of the study conditions, most notably when the reliability of the observed indicators was at its lowest value (0.30). In the Complete and MCAR data conditions the average RMSEA values for the Wen and Duncan models were substantially lower than those of the

Schumacker model. In the MNAR data condition this same pattern held at lower reliabilities, but all three models produced similar average RMSEA values at higher reliabilities as sample size increased.

Under two of the three missing data conditions (Complete, MCAR) there was a trend for both the Wen and the Duncan models to show average RMSEA values that were stable as the latent intercept-slope correlation increased, and which increased as the reliability of the observed indicators increased. In these conditions the Schumacker model showed an increasing pattern of average RMSEA values as the latent intercept-slope correlation increased and a decreasing pattern of average RMSEA values as reliability increased.

Table ApxH-T1

Confidence Intervals (95%) for the Average Root Mean Square Error of Approximation (RMSEA) Values for All Latent Growth Interaction Models (Population Value of Latent Interaction Parameter Equal to 2.0) For the Three Missing Data Conditions (Complete, MCAR, MNAR).

Corr	N	Rel		Complete Data			MCAR Data			MNAR Data		
				Wen	Duncan	Schumacker	Wen	Duncan	Schumacker	Wen	Duncan	Schumacker
0.20	250	0.30	L	0.109	0.129	0.563	0.109	0.129	0.559	0.159	0.209	0.349
			U	0.111	0.131	0.617	0.111	0.131	0.621	0.161	0.211	0.391
		0.50	L	0.129	0.139	0.533	0.129	0.139	0.530	0.178	0.219	0.294
			U	0.131	0.141	0.587	0.131	0.141	0.590	0.182	0.221	0.326
		0.70	L	0.139	0.149	0.468	0.139	0.149	0.482	0.228	0.239	0.240
			U	0.141	0.151	0.512	0.141	0.151	0.538	0.232	0.241	0.260
		0.90	L	0.149	0.169	0.384	0.159	0.169	0.390	0.289	0.279	0.224
			U	0.151	0.171	0.396	0.161	0.171	0.410	0.291	0.281	0.236
	500	0.30	L	0.109	0.130	0.561	0.109	0.129	0.558	0.159	0.210	0.292
			U	0.111	0.130	0.599	0.111	0.131	0.602	0.161	0.210	0.308
		0.50	L	0.129	0.140	0.494	0.129	0.139	0.510	0.179	0.220	0.254
			U	0.131	0.140	0.526	0.131	0.141	0.550	0.181	0.220	0.266
		0.70	L	0.140	0.150	0.429	0.140	0.150	0.445	0.228	0.240	0.226
			U	0.140	0.150	0.451	0.140	0.150	0.475	0.232	0.240	0.234
		0.90	L	0.150	0.170	0.377	0.150	0.170	0.376	0.289	0.280	0.218
			U	0.150	0.170	0.383	0.150	0.170	0.384	0.291	0.280	0.222
	1000	0.30	L	0.110	0.130	0.539	0.110	0.130	0.546	0.160	0.210	0.277
			U	0.110	0.130	0.561	0.110	0.130	0.574	0.160	0.210	0.283
		0.50	L	0.130	0.140	0.471	0.130	0.140	0.488	0.180	0.220	0.248
			U	0.130	0.140	0.489	0.130	0.140	0.512	0.180	0.220	0.252
		0.70	L	0.140	0.150	0.396	0.140	0.150	0.413	0.229	0.240	0.218
			U	0.140	0.150	0.404	0.140	0.150	0.427	0.231	0.240	0.222
		0.90	L	0.150	0.170	0.379	0.150	0.170	0.379	0.289	0.280	0.219
			U	0.150	0.170	0.381	0.150	0.170	0.381	0.291	0.280	0.221
0.50	250	0.30	L	0.119	0.129	0.653	0.119	0.129	0.639	0.159	0.219	0.399
			U	0.121	0.131	0.707	0.121	0.131	0.701	0.161	0.221	0.441
		0.50	L	0.129	0.139	0.615	0.129	0.139	0.620	0.179	0.229	0.346
			U	0.131	0.141	0.665	0.131	0.141	0.680	0.181	0.231	0.374
		0.70	L	0.139	0.149	0.549	0.139	0.149	0.564	0.208	0.249	0.302
			U	0.141	0.151	0.591	0.141	0.151	0.616	0.212	0.251	0.318
		0.90	L	0.159	0.169	0.485	0.159	0.169	0.482	0.298	0.279	0.246
			U	0.161	0.171	0.495	0.161	0.171	0.498	0.302	0.281	0.254
	500	0.30	L	0.120	0.130	0.652	0.119	0.129	0.658	0.159	0.220	0.351
			U	0.120	0.130	0.688	0.121	0.131	0.702	0.161	0.220	0.369

		0.50	L	0.130	0.140	0.594	0.129	0.139	0.601	0.179	0.230	0.314
			U	0.130	0.140	0.626	0.131	0.141	0.639	0.181	0.230	0.326
		0.70	L	0.140	0.150	0.510	0.140	0.149	0.536	0.209	0.250	0.286
			U	0.140	0.150	0.530	0.140	0.151	0.564	0.211	0.250	0.294
		0.90	L	0.160	0.170	0.478	0.160	0.170	0.477	0.299	0.280	0.248
			U	0.160	0.170	0.482	0.160	0.170	0.483	0.301	0.280	0.252
	1000	0.30	L	0.120	0.130	0.639	0.120	0.130	0.646	0.160	0.220	0.326
			U	0.120	0.130	0.661	0.120	0.130	0.674	0.160	0.220	0.334
		0.50	L	0.130	0.140	0.561	0.130	0.140	0.579	0.179	0.230	0.297
			U	0.130	0.140	0.579	0.130	0.140	0.601	0.181	0.230	0.303
		0.70	L	0.140	0.150	0.496	0.140	0.150	0.504	0.209	0.250	0.278
			U	0.140	0.150	0.504	0.140	0.150	0.516	0.211	0.250	0.282
		0.90	L	0.160	0.170	0.469	0.160	0.170	0.479	0.299	0.280	0.249
			U	0.160	0.170	0.471	0.160	0.170	0.481	0.301	0.280	0.251
0.70	250	0.30	L	0.119	0.129	0.674	0.119	0.129	0.669	0.159	0.219	0.428
			U	0.121	0.131	0.726	0.121	0.131	0.731	0.161	0.221	0.472
		0.50	L	0.129	0.139	0.645	0.129	0.139	0.640	0.179	0.229	0.384
			U	0.131	0.141	0.695	0.131	0.141	0.700	0.181	0.231	0.416
		0.70	L	0.139	0.149	0.579	0.139	0.149	0.594	0.208	0.249	0.341
			U	0.141	0.151	0.621	0.141	0.151	0.646	0.212	0.251	0.359
		0.90	L	0.159	0.169	0.515	0.159	0.169	0.522	0.297	0.289	0.284
			U	0.161	0.171	0.525	0.161	0.171	0.538	0.303	0.291	0.296
	500	0.30	L	0.130	0.130	0.673	0.119	0.129	0.678	0.159	0.220	0.390
			U	0.130	0.130	0.707	0.121	0.131	0.722	0.161	0.220	0.410
		0.50	L	0.130	0.140	0.624	0.129	0.139	0.631	0.179	0.230	0.363
			U	0.130	0.140	0.656	0.131	0.141	0.669	0.181	0.230	0.377
		0.70	L	0.140	0.150	0.550	0.140	0.149	0.557	0.199	0.250	0.326
			U	0.140	0.150	0.570	0.140	0.151	0.583	0.201	0.250	0.334
		0.90	L	0.160	0.170	0.518	0.160	0.170	0.517	0.298	0.290	0.288
			U	0.160	0.170	0.522	0.160	0.170	0.523	0.302	0.290	0.292
	1000	0.30	L	0.130	0.130	0.669	0.130	0.130	0.677	0.160	0.220	0.366
			U	0.130	0.130	0.691	0.130	0.130	0.703	0.160	0.220	0.374
		0.50	L	0.130	0.140	0.602	0.130	0.140	0.609	0.179	0.230	0.347
			U	0.130	0.140	0.618	0.130	0.140	0.631	0.181	0.230	0.353
		0.70	L	0.140	0.150	0.526	0.140	0.150	0.534	0.200	0.250	0.328
			U	0.140	0.150	0.534	0.140	0.150	0.546	0.200	0.250	0.332
		0.90	L	0.160	0.170	0.509	0.160	0.170	0.509	0.309	0.290	0.279
			U	0.160	0.170	0.511	0.160	0.170	0.511	0.311	0.290	0.281

Note: MCAR = Missing Completely At Random; MNAR = Missing Not At Random; Corr = Latent intercept-slope correlation; N = Sample size; Rel = Reliability of the observed indicator; L = Lower 95% limit; U = Upper 95% limit.

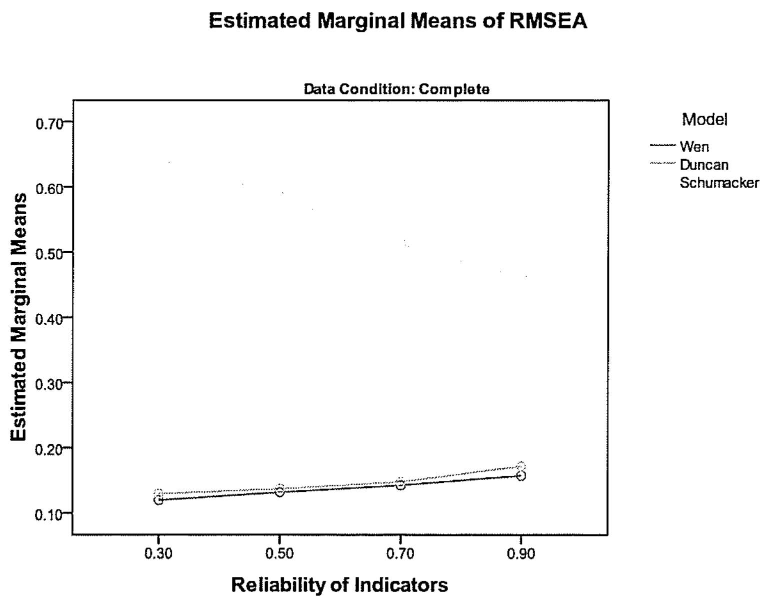
Table ApxH-T2
Analysis of Variance Results with Root Mean Square Error of Approximation (RMSEA) Values as the Dependent Variable, with Latent Model Type, Latent Intercept-Slope Correlation, Sample Size, and Reliability of Observed Indicators as Between-Subjects Factors, For the Three Missing Data Conditions (Complete, MCAR, MNAR).

Model Effect	df	Complete Data		MCAR Data		MNAR Data	
		F-value	Partial eta^2	F-value	Partial eta^2	F-value	Partial eta^2
Corrected Model	107	27467.60	.847	25003.44	.835	7825.30	.638
Latent Interaction Model	2	1382988.46	.839	1266019.26	.827	166830.89	.413
Covariance (Corr)	2	13551.28	.049	11584.31	.042	16514.99	.065
Sample Size (N)	2	1575.79	.006	1278.98	.005	3747.89	.016
Reliability (Rel)	3	4746.54	.026	4031.53	.022	15486.33	.089
Latent Model X Corr	4	10987.56	.076	9311.09	.066	16578.51	.123
Latent Model X N	4	1556.97	.012	1240.22	.009	4667.88	.038
Latent Model X Rel	6	19675.99	.182	17402.04	.165	60306.20	.433
Corr X N	4	7.96	.000	9.13	.000	20.20	.000
Corr X Rel	6	15.65	.000	12.29	.000	54.54	.001
N X Rel	6	170.00	.002	175.37	.002	706.96	.009
Latent Model X Corr X N	8	8.05	.000	9.47	.000	25.77	.000
Latent Model X Corr X Rel	12	19.03	.000	14.97	.000	446.79	.011
Latent Model X N X Rel	12	166.74	.004	162.78	.004	650.52	.016
Corr X N X Rel	12	1.47 ns	.000	1.61 ns	.000	4.72	.000
Latent Model X Corr X N X Rel	24	1.63 (.03)	.000	1.44 ns	.000	6.02	.000
Error	531703						

Note: MCAR = Missing Completely At Random; MNAR = Missing Not At Random; Corr = Latent intercept-slope correlation; N = Sample size; Rel = Reliability of the observed indicator.
All effects are significant at the $p < 0.01$ level.

Figure ApxH-F1

Plots of the Interaction Effect of Latent Interaction Model Type Factor with the Observed Indicator Reliability Factor (1) and Latent Intercept-Slope Correlation (2) on the Root Mean Square Error of Approximation (RMSEA) in the Complete Data Condition in Those Models that Converged Successfully.



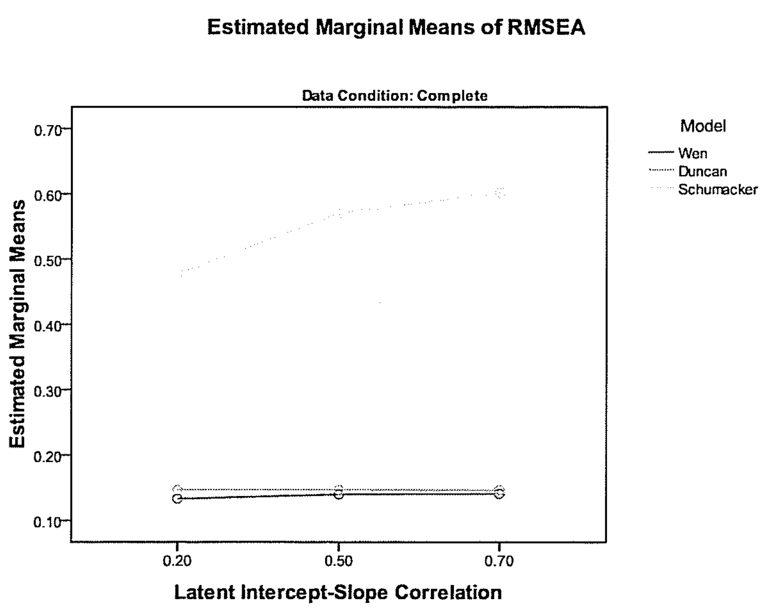
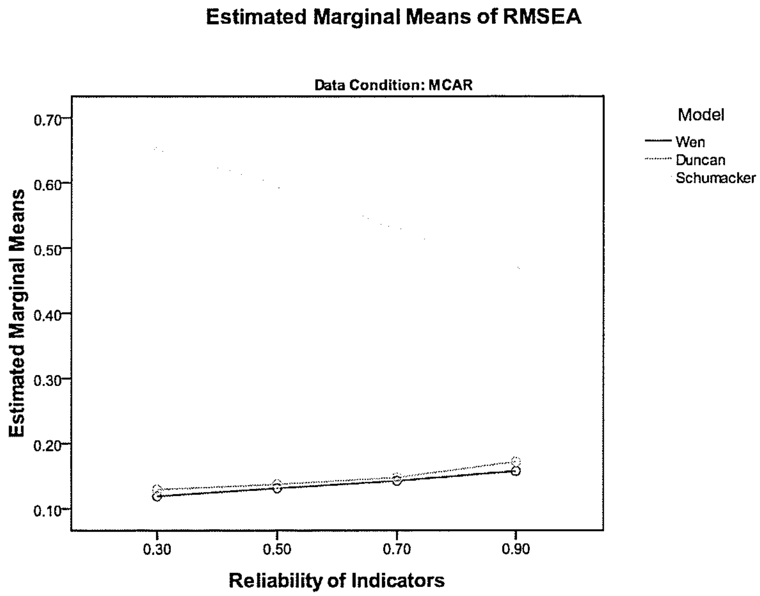


Figure ApxH-F2

Plots of the Interaction Effect of Latent Interaction Model Type Factor with the Observed Indicator Reliability Factor (1) and Latent Intercept-Slope Correlation (2) on the Root Mean Square Error of Approximation (RMSEA) in the Missing Completely At Random Data Condition in Those Models that Converged Successfully.



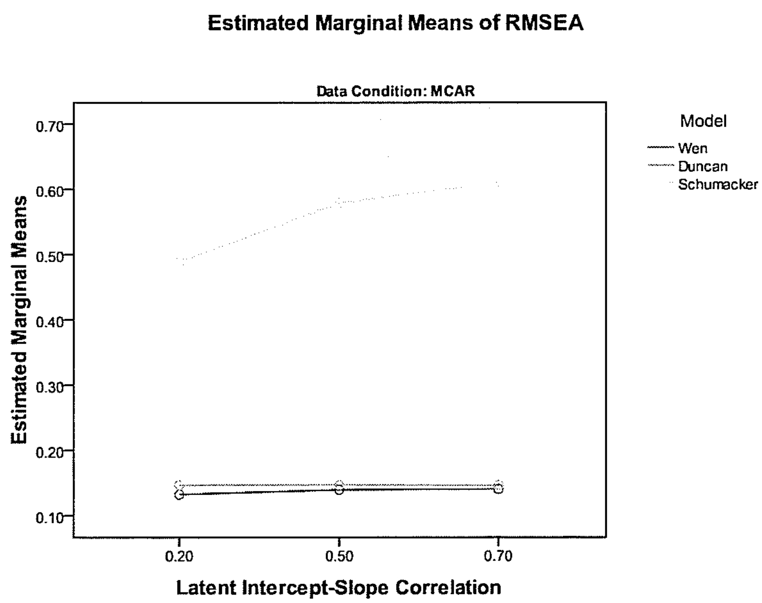


Figure ApxH-F3

Plots of the Interaction Effect of Latent Interaction Model Type Factor with the Observed Indicator Reliability Factor (1) and Latent Intercept-Slope Correlation (2) on the Root Mean Square Error of Approximation (RMSEA) in the Missing Not At Random Data Condition in Those Models that Converged Successfully.