

**Viewing Learning as Complex Participation  
in a Community of Practice  
Characterized by Mathematical Inquiry**

by

Candy Skyhar

A Thesis submitted to the Faculty of Graduate Studies of  
The University of Manitoba  
in partial fulfilment of the requirements of the degree of

MASTER OF EDUCATION

Department of Curriculum, Teaching and Learning  
University of Manitoba  
Winnipeg, Manitoba

Dr. Thomas Falkenberg (advisor)  
Dr. Ralph Mason  
Dr. Paul Betts

Copyright © 2009 by Candy Skyhar

## **Abstract**

Using elements of design experiment research and autoethnography, this action research project investigated how viewing learning as complex participation in a community of practice characterized by mathematical inquiry impacted my teaching practice in a grade 10 Applied Mathematics class in a rural Manitoba high school. This report of the research project describes and analyzes both my attempts to change my teaching practice by drawing on theories of learning mathematics as complex participation in a community of practice and the changes that resulted from these attempts. The analysis focuses on the characteristics of a community of practice characterized by mathematical inquiry, how I attempted to foster such a community, what challenges I faced when I changed my teaching practice in this way, and how insights from this practitioner research project can inform the teaching of mathematics as well as theorizing about the learning of mathematics.

## Acknowledgements

First and foremost, I would like to thank my advisor, Dr. Thomas Falkenberg for his thoughtful advice through countless phonecalls, meetings, and conversations. His calm demeanor, thought provoking comments, and positive attitude inspired me to rise to every challenge before me.

I would also like to thank Dr. Ralph Mason for his support and encouragement during this research study and my entire Master's Program. He always had time to listen to a care or concern, and saw value in the work in which I was engaged. I cannot imagine this experience without him.

Special thanks also to Dr. Paul Betts of the University of Winnipeg for his advice as a member of my advisory committee, and in particular for making me pay attention to the connections between educational theory and teaching practice.

Finally, I would like to thank my colleagues – the fellow graduate students with whom I have had so many conversations about learning, teaching, and education. I couldn't have asked for better comrades in the journey that has led to this point. Thank you, and may you continue to discuss, learn and grow as both students and educators.

## Dedication

To the three pillars in my life: my husband, my boys, and my parents. Without you, none of this would be possible.

Wayne, you have encouraged me to strive for my dreams. Your love and patience made all of this possible. Thank you. I love you more now than ever.

Dawson and Zachary, I hope you can forgive the times Mom had to work on her papers, and I pray that you will value education as much as I do. You bring the sunshine into my life. I love you both so very much.

Mom and Dad, you made me who I am. You taught me that I could do anything I set my mind to, and be anything that I wanted to be. For this, and all of your love and support, I am truly grateful. I love you both.

## Table of Contents

<b>ABSTRACT .....</b>	<b>I</b>
<b>ACKNOWLEDGEMENTS .....</b>	<b>II</b>
<b>DEDICATION .....</b>	<b>III</b>
<b>TABLE OF CONTENTS .....</b>	<b>IV</b>
<b>LIST OF TABLES .....</b>	<b>VII</b>
<b>LIST OF FIGURES .....</b>	<b>VIII</b>
<b>CHAPTER 1 .....</b>	<b>1</b>
<b>INTRODUCTION .....</b>	<b>1</b>
<i>Introduction and Motivation for Research.....</i>	<i>1</i>
<i>The Research Setting .....</i>	<i>4</i>
<i>The Structure of the Thesis .....</i>	<i>5</i>
<b>CHAPTER 2 .....</b>	<b>9</b>
<b>LITERATURE REVIEW .....</b>	<b>9</b>
<i>Learning as a Social Activity.....</i>	<i>10</i>
<i>Learning as Participation in a Community of Practice .....</i>	<i>11</i>
<i>The Importance of Sociocultural Norms.....</i>	<i>13</i>
<i>The Role of the Teacher in a Community of Learners .....</i>	<i>17</i>
<i>Conditions Necessary for Learning to Take Place in a Community of Learners.....</i>	<i>19</i>
<b>CHAPTER 3 .....</b>	<b>26</b>
<b>THEORETICAL FRAMEWORK.....</b>	<b>26</b>
<b>CHAPTER 4 .....</b>	<b>36</b>
<b>THE PURPOSE AND METHODOLOGY OF THE RESEARCH STUDY.....</b>	<b>36</b>
<i>Purpose of the Study.....</i>	<i>36</i>
<i>Research Questions.....</i>	<i>37</i>
<i>The Role of Design Experiment Research in this Study.....</i>	<i>40</i>
<i>The Role of Autoethnography in this Study.....</i>	<i>48</i>
<b>CHAPTER 5 .....</b>	<b>55</b>
<b>MY STORY OF PLANNING AND TEACHING.....</b>	<b>55</b>
<i>Planning to Begin .....</i>	<i>56</i>
<i>Beginning the Year.....</i>	<i>65</i>
<i>Reverting Back to Old and Familiar Ways .....</i>	<i>69</i>
<i>Using Problem Solving Partners to Foster Independence and Collaboration .....</i>	<i>70</i>
<i>Struggling with Reporting.....</i>	<i>73</i>
<i>Discovering the Importance of my Role as Prompter.....</i>	<i>74</i>
<i>A Return to Problem Solving Partners .....</i>	<i>76</i>
<i>Time, Energy and Logistics Get in the Way.....</i>	<i>77</i>
<i>Christmas Break – A Chance to Refocus .....</i>	<i>78</i>
<i>A Taste of Success – The Student Lounge Project .....</i>	<i>80</i>
<i>Getting Students to Talk Strategy .....</i>	<i>82</i>
<i>Reverting Back to Old and Familiar Ways Again.....</i>	<i>86</i>
<i>Spring Break – Another Chance to Refocus.....</i>	<i>87</i>
<i>Using Old Activities Differently for Different Purposes.....</i>	<i>87</i>
<i>Adding More Open-Endedness to Learning Tasks.....</i>	<i>89</i>
<i>Moving Towards Performance Tasks for Assessment.....</i>	<i>91</i>
<i>Concluding and Reminiscing at the Year’s End .....</i>	<i>93</i>

<b>CHAPTER 6</b> .....	<b>96</b>
MY STORY OF RESEARCHING .....	96
<i>Data Collection</i> .....	96
<b>Planning and Observation Journal</b> .....	97
<b>Interactive Journals</b> .....	98
<b>Student Work</b> .....	99
<i>Data Analysis and Interpretation</i> .....	99
<b>CHAPTER 7</b> .....	<b>103</b>
CHARACTERIZING CHANGE IN MY TEACHING PRACTICE .....	103
<i>Parallel Planning</i> .....	104
<i>Mathematically and Communally Rich Learning Activities</i> .....	109
<i>Taking on the Role of Prompter</i> .....	115
<i>A Move Towards Performance Tasks for Evaluation</i> .....	118
<b>CHAPTER 8</b> .....	<b>124</b>
CHARACTERIZING A COMMUNITY OF PRACTICE CHARACTERIZED BY MATHEMATICAL INQUIRY .....	124
<i>Characteristics of a Community of Practice Characterized by Mathematical Inquiry</i> .....	124
<b>Discussion, Reflection, and Evaluation of Mathematical Ideas and Strategies</b> .....	127
<b>Application of Mathematical Concepts to Real World Contexts</b> .....	132
<b>Curiosity about Mathematics</b> .....	135
<b>Sense of Community Amongst Members</b> .....	137
<b>Increasing Sense of Individual and Collective Ownership of Learning</b> .....	140
<i>A Portrait of a Community of Practice Characterized by Mathematical Inquiry</i> .....	145
<b>CHAPTER 9</b> .....	<b>148</b>
CHALLENGES OF FOSTERING SUCH A COMMUNITY OF PRACTICE.....	148
<b>CHAPTER 10</b> .....	<b>154</b>
INFORMING PRACTICE AND THEORY .....	154
<i>Allowing Practice to Inform Broader Educational Practice</i> .....	155
<i>Allowing Practice to Inform Educational Theory</i> .....	160
<b>REFERENCES</b> .....	<b>169</b>
<b>APPENDIX A</b> .....	<b>172</b>
<i>Thinking Outside the Box</i> .....	172
<b>APPENDIX B</b> .....	<b>174</b>
<i>Measurement Debate</i> .....	174
<b>APPENDIX C</b> .....	<b>176</b>
<i>3D Geometry Research Assignment</i> .....	176
<b>APPENDIX D</b> .....	<b>178</b>
<i>Grandpa's Tool Shed</i> .....	178
<b>APPENDIX E</b> .....	<b>179</b>
<i>Mystifying Measurement Markings</i> .....	179
<b>APPENDIX F</b> .....	<b>180</b>
<i>Student Lounge Project</i> .....	180
<b>APPENDIX G</b> .....	<b>183</b>
<i>Trigonometry Challenge</i> .....	183

**APPENDIX H .....185**  
*Ball Bounce Activity.....185*

**APPENDIX I.....188**  
*DaVinci Project .....188*

**APPENDIX J .....189**  
*Linear Models and Patterns Performance Task .....189*

**APPENDIX K .....190**  
*Table A1: Planning Prior to the Study.....190*

**APPENDIX L.....191**  
*Table A2: Planning at the Beginning of the Study .....191*

**APPENDIX M .....192**  
*Table A3: Planning at the End of the Study.....192*

**APPENDIX N .....195**  
*Highlighting Key and Analysis Notes .....195*

**APPENDIX O .....196**  
*Journal Summary April 29.....196*

**APPENDIX P .....199**  
*Performance Task Feedback Form .....199*

**APPENDIX Q .....202**  
*Table A4: Interactive Journal Prompts .....202*

**APPENDIX R .....204**  
*Consent Form .....204*

## List of Tables

Table 5.1	
<i>Timeline for Activities</i> .....	95
Table 6.1	
<i>Interactive Journal Prompts</i> .....	98
Table 7.1	
<i>Characteristics of Mathematically and Communally Rich Learning Activities</i> .....	110
Table 8.1	
<i>Characteristics of a Community of Practice Characterized by Mathematical Inquiry</i> ..	125
Table 8.2	
<i>Summary of Characteristics of Community Identified by Students</i> .....	138
Table A1	
<i>Planning Prior to the Study</i> .....	190
Table A2	
<i>Planning at the Beginning of the Study</i> .....	191
Table A3	
<i>Planning at the End of the Study</i> .....	192
Table A4	
<i>List of Interactive Journal Prompts</i> .....	202

## List of Figures

Figure 8.1	
<i>First Sample Student Lounge</i> .....	144
Figure 8.2	
<i>Second Sample Student Lounge</i> .....	144

## **Chapter 1**

### ***Introduction***

The introduction to this document has three purposes: to introduce myself and describe my motivation for conducting this research study, to give a brief description of the setting under which the research was conducted, and to provide an overview of the structure of the thesis document itself.

#### **Introduction and Motivation for Research**

In 2006, I enrolled in my Master of Education program at the University of Manitoba. After fifteen years of teaching, I recognized that I needed to grow more as a professional, and I decided to embark on a journey towards improving myself and my practice, although I was not entirely sure what it was that I was unhappy with at the time. I knew only that I had become complacent, and was out of touch with educational theory and changes in mathematics education in general. I had always thought that I would return to university, and at last my life allowed me to be in a situation where returning for a Master's degree was possible. With naïve hopefulness, I entered the faculty open to change and growth, completely unaware of what the journey would bring.

Prior to entering the faculty, I taught as I suspected many mathematics teachers taught. I presented lessons each day at the front of the room and assigned practice for students to do based on the lesson. In hindsight, I suppose I viewed learning as an act of acquisition, whereby students would absorb information as it was presented to them.

Repetition was used to reinforce processes on assignments, and students contributed minimally to the lessons each day. The classes were led by me almost entirely, except in a few instances where I attempted to incorporate a project in order to help students apply what they knew to what could have been a real world situation. Most of the things I did in my classroom were inspired not by theories about how students learn mathematics, but by the resources that were available to me as an educator.

While I generally had good relationships with students, and students seemed to be able to succeed in achieving satisfactory marks in my classes, I was increasingly disturbed by several things. First of all, students had difficulty completing any problems that were not exactly like those I had demonstrated on the board at the front of the classroom. They were not able to extend their thinking to slightly novel situations, and relied heavily on me to tell them what to do or how to approach problems. This was even more evident when I engaged students in projects that required application of mathematical ideas and strategies to new situations or scenarios. Secondly, students at times made reference to the fact that they would never use the math I was teaching them in the real world. This troubled me greatly. Students obviously did not see connections between what they did in the classroom and what they might do outside of it. As a teacher of Applied Mathematics courses, I felt very strongly that I was failing in this capacity. After all, my students, if any, should have been able to see connections between the mathematics they learned in school and the real world. Finally, I was troubled by the lack of interest and enthusiasm exhibited by students in my classroom. While I considered myself to have good rapport with students, I found that students just did not find the mathematics interesting or thought-provoking. They did not ask questions. They did not

comment on the relationships they saw before them. This, I thought, should have been something all students would experience in a mathematics class. I wondered why I saw very little of this in my own classroom.

As I worked on the course work components of my Master's program, I was introduced to the notion of learning as a social activity through the work of Lev. S. Vygotsky (1978) and others. I began reading about learning as both participation and acquisition (Cobb, 1994, Sfard, 1998). I learned about communities of practice (Lave & Wenger, 1991), and about communities of mathematical inquiry (Goos, 2004). I read about sociomathematical norms (Cobb, 2000) and the importance of developing norms of practice within a classroom community (Lampert, 1990). I read about complexity theory and its contributions to the view of classrooms as complex learning systems (Davis & Simmt, 2003). Being introduced to a variety of educational theories at this point in my career proved to be a catalyst for change for me as an educator.

After completing the course work for my Master of Education degree, I found myself in an unsettling position as I tried to reconcile my own teaching practice with the new educational theories to which I had been exposed in my graduate program. As I began to formulate my own opinions about learning and teaching, I recognized a significant tension between my own teaching practices and what I had begun to view as effective educational practice based on educational theory. I knew that my teaching practices would have to change because of my changed beliefs about learning, and I found myself wanting to look at how the educational theories to which I had been exposed would impact my teaching practice. As I began to put into words what I believed to be true about learning from those educational theories, I recognized that I had begun to

view learning as *complex participation in a community of practice characterized by mathematical inquiry*, a phrase which I will explicate later in this thesis. I also began to realize that, as such, I needed to find ways to foster the development of such a community of practice within my classroom. The research study reported on in this thesis describes the research I undertook as I inquired into how viewing learning as complex participation in a community of practice characterized by mathematical inquiry would change my teaching practice, how such a community of practice could be fostered, and what such a community of practice would look like.

### **The Research Setting**

The research study described in this document took place in a small rural Manitoba high school of approximately two hundred thirty students. It was conducted in my own grade 10 Applied Mathematics class, which included eighteen students, all of whom participated in the research project (see Appendix R for consent form), allowing their journals and work to be used anonymously as research data. Because the research was focused on identifying and characterizing the changes in my own teaching practice by fostering the development of a community of practice characterized by mathematical inquiry, I used autoethnography as a research method in the self study portion of my research. I also employed the use of design experiment research, as I engaged in the process of designing learning activities that I expected would foster the emergence of a community of practice characterized by mathematical inquiry. These activities were designed in a cyclical manner, allowing my observations as students engaged with such

learning activities to inform further development of activities, as well as strategies for fostering the emergence of a community of practice characterized by mathematical inquiry. The study took place over the course of one school year, September to June, after which the data was analyzed and the thesis written.

### **The Structure of the Thesis**

This paper is divided up into ten chapters, followed by several appendices containing materials pertinent to the study. This chapter, chapter 1, contains the introduction to the thesis, including the motivation for the research, a description of the research setting, and an overview of the structure of the thesis.

Chapter 2 is comprised of a literature review which looks at the literature on learning as a social activity, learning as participation in a community of practice, the importance of sociocultural norms, the role of the teacher in a community of learners, and the conditions necessary for learning to take place in a community of learners.

Chapter 3 outlines the theoretical framework from which I attempted to change my teaching practice: *viewing learning as complex participation in a community of practice characterized by mathematical inquiry*. The chapter explains each part of this framework, relating it to the literature discussed in chapter 2.

Chapter 4 contains the research questions as well as a description of the research methods I used to conduct the study. In this chapter, elements of design experiment research are discussed, including both how the study utilized design experiment research, as well as how the design experiment research methodology was modified to suit the

purposes of the study. In addition to this, autoethnography as a research method is discussed, including the characteristics of autoethnography that I found helpful in conducting my research and answering my research questions.

Chapter 5, is an narrative, chronological story of the school year during which the research study was conducted. It provides an overview of the general experiences I had, as well as a timeline from which the reader can understand how smaller parts of the puzzle fit within the overall process. In this chapter, I describe what decisions I made in planning activities for students, as well as how my plans unfolded in the classroom, further informing my teaching. It highlights pivotal moments in my planning and thinking, as well as key moments in my teaching and interaction with students that provided insight into what a community of practice characterized by mathematical inquiry might look like. Since the chapter is written as a narrative, it allows me to describe my own experiences as an educator struggling to conceptualize what it means to plan and teach with this view of learning in mind. It is through this narrative, that other educators may recognize elements of their own educational practice, allowing them to consider how my experiences might apply to their situations.

Whereas chapter 5 tells the story of my planning and teaching, chapter 6 relates my story of researching in this study. The chapter outlines how I collected data through my own planning and observation journal, the interactive journals used within my classroom, and the collection of student work during the research study. It also outlines how the data was analyzed at the end of the study in order to draw conclusions from it. This chapter, like chapter 5, is written as a narrative to emphasize the importance of my journey as both an educator and a researcher during this study. Having the dual roles of

educator and researcher allowed me to characterize not only how my educational practice changed as a result of viewing learning as complex participation in a community of practice characterized by mathematical inquiry, but also what such a community looks like, and how it might be fostered.

Chapter 7 characterizes the way in which my teaching practice changed as a result of viewing learning as complex participation in a community of practice characterized by mathematical inquiry. In this chapter, I discuss four major changes in my teaching practice: the use of parallel planning, the creation of mathematically and communally rich learning activities, taking on the role of prompter, and my own movement towards the use of performance tasks for assessing student understanding. This characterization of change not only answers one of my research questions, but also provides valuable insight for other teachers who view learning as I do. In describing the changes in my own practice, other educators might see where their own practices could be changed to better match the ideas put forth in the educational theories that are described in chapter 2.

Chapter 8 identifies five characteristics of a community of practice characterized by mathematical inquiry, and describes how each of these characteristics emerged within the classroom community that was the subject of the research study. Following the discussion of the five characteristics, a portrait is painted through words that describes what such a classroom community looks like in practice.

Chapter 9 looks at the challenges and greater complexities I faced as I attempted to foster the emergence of a community of practice characterized by mathematical inquiry. Understanding these challenges and greater complexities is important if one is to

understand the strengths and weaknesses of viewing learning in this way, and of allowing this view of learning to inform one's teaching practice.

The final chapter, chapter 10, looks at how my own experiences can inform the practice of other teachers who view learning as complex participation in a community of practice characterized by mathematical inquiry. It also looks at how my experiences in practice can, in turn, inform educational theory. Because I felt that my own experiences strengthened Brent Davis and Elaine Simmt's (2003) view of mathematics classrooms as "adaptive and self-organizing complex systems" (p. 138), and the characteristics of such systems as identified by these two authors, most of the final chapter discusses the characteristics of complex systems and how these characteristics emerged within my own classroom community.

At the end of this document, there are several appendices that include slides outlining the mathematically and communally rich learning activities that students engaged in (Appendices A-J), samples of my own planning charts (Appendices K-M), and data summarized from student feedback forms and journals (Appendices N and O).

## Chapter 2

### *Literature Review*

The emergence of sociocultural theories about learning and development has impacted the field of mathematics education and educational theory significantly in the past thirty years. The predominant view of learning as an act of internalization or acquisition of knowledge has given way, gradually, to the view of learning as an emergent phenomenon or byproduct of interaction. The complex nature of the process of learning has been analyzed by educational theorists, and the importance of interaction and the situated nature of educational communities have come to the forefront of educational discussions in the past three decades. The classroom has come to be seen as a complex system in which participants evolve and mutually adapt to each other through the complex choreography we term 'learning'. In order to view the impact of sociocultural theories of learning on mathematics education, one must first look at the beliefs on which such theories are premised. Sociocultural theorists believe that learning is inherently a social activity, an idea that stems from the work of Lev. S. Vygotsky. Many sociocultural theorists also identify with the metaphor of *participation* (Sfard, 1998; Lave & Wenger, 1991) when speaking about learning. The concept of participation in a community of practice or in a community of learners has received significant attention of late in the field, as have the role of sociocultural norms and the teacher in such communities. If one is to look at possible ways to put sociocultural theories into practice in mathematics education, the conditions under which mathematical learning can be seen as complex

participation in a community of practice must be examined. That is the purpose of this literature review.

### **Learning as a Social Activity**

In the early 1900s, Lev. S. Vygotsky as well as other Soviet psychologists, struggled with the first sociocultural views of education and learning. Vygotsky suggested that “human learning presupposes a specific social nature and a process by which children grow into the intellectual life of those around them” (1978, p. 88). Vygotsky saw learning as a process that was necessarily social in nature and recognized the impact of others on the process for the individual. He believed that the social and cultural context of education necessarily shaped the learning process, and that learning, in fact, was dependent upon the interaction of people within the educational environment. Vygotsky proposed that:

an essential feature of learning is that it creates the zone of proximal development; that is, learning awakens a variety of internal developmental processes that are able to operate only when the child is interacting with people in his environment and in cooperation with his peers. Once these processes are internalized, they become part of the child’s independent developmental achievement (1978, p. 90).

Others have followed in Vygotsky’s direction of thought, suggesting that the social interactions between students in a learning environment can help develop the zone of proximal development, and thus enable learning. Merrilyn Goos (2004) further suggests that “working in collaborative peer groups, students have an opportunity to own the ideas they are constructing and to experience themselves and their partners as active participants in creating mathematical insights” (p. 263). Clearly the social nature of the learning environment is of paramount importance in the process of learning. Brent Davis,

Dennis Sumara, and Rebecca Luce-Kapler (2000) similarly argue that “collectives of persons are capable of actions and understandings that transcend the capabilities of the individuals on their own” (p. 68). Learning is a socially situated activity that individuals participate in and experience not only as individuals, but also as part of a collective.

### **Learning as Participation in a Community of Practice**

The opposing metaphors of *acquisition* and *participation* (Sfard, 1998) have come to the forefront of educational debate in the past decade. The acquisition metaphor refers largely to the conception that students acquire, or internalize knowledge in the process of learning. The participation metaphor refers, instead, to the idea that learning occurs as students participate in -- and become a member of -- a community. Anna Sfard (1998) advocates the inclusion of both of these metaphors when considering the process of learning. Similarly, Paul Cobb (1994) argues that “mathematical learning should be viewed as both a process of active individual construction and a process of enculturation into the mathematical practices of wider society” (p. 13). These viewpoints, which do not differ significantly from the ideas of Vygotsky, are based on the foundation that the social context of learning takes place within the community of learners that make up the classroom, including the teacher. While individuals most certainly learn for themselves, they learn more than content; they also learn what it means to be a member of a collective community of learners. “When classroom culture is taken into consideration, it becomes clear that teaching is not only about teaching what is conventionally called *content*. It is

also teaching students what a lesson is and how to participate in it” (Lampert, 1990, p. 34).

The concept of participating in a community of practice is rooted largely in the work of Jean Lave and Etienne Wenger (1991), who oppose the conventional view of learning as the process of internalization. Lave and Wenger propose instead that “learners inevitably participate in communities of practitioners and that the mastery of knowledge and skill requires newcomers to move toward full participation in the sociocultural practices of a community” (p. 29). Lave and Wenger suggest that learning is the process through which students participate in a community of practice, gradually becoming members themselves – a process which they term *legitimate peripheral participation*. According to them, “there is no activity that is not situated” (p. 33), meaning not only that learning takes place in a context, but also that the community is “an integral part of generative social practice of the lived-in world” (p. 35). According to Lave and Wenger, as learners interact within their communities, the communities themselves change in addition to the individuals. This process creates a constantly evolving community as individuals and collectives mutually adapt and change.

Paul Cobb (1994), like Lave and Wenger, argues that classroom culture and the individual student’s mathematical activity are mutually adaptive. He proposes that the individual construction of meaning on the part of the learner is constrained by their participation in the activities and by the interaction of others in the community. In this way, individual construction of meaning is a byproduct of both the individual’s interaction with mathematics and their interaction with others in the community. Learning “is not seen as a ‘taking in’ or a ‘theorizing about’ a reality that is external to

and separate from the learner. Rather, learning is coming to be understood as a participation in the world, a co-evolution of knower and known that transforms both” (Davis, Luce-Kapler, & Sumara, 2000, p. 64).

### **The Importance of Sociocultural Norms**

Part of what makes a group of learners a collective or community is the set of negotiated sociocultural norms that exist within the culture of the classroom. Students constantly learn what it means to participate in the practices of the community as they encounter activities within their community and interact with its members. These social norms are of tremendous importance to the learning process. The norms and behaviors that are established within the classroom community govern what sort of learning takes place. Even when teachers do not consider their classrooms as a community, but rather as a series of individual learners acquiring information, social norms are established within the classroom that help students understand what it means to do mathematics and how they should behave when learning mathematics. In traditional classrooms, students might, for example, become accustomed to quietly watching a teacher present information on a board at the front of the room followed by a series of similar questions from a textbook that they are expected to complete. Even if the teacher does not perceive that there is a community within the classroom, they are establishing norms, beliefs, and values about mathematics and students are “learning” what it means to participate in doing mathematics.

If one views learning mathematics as the act of participation in a community of practice, one must recognize that the “sociomathematical norms” (Cobb, 2000, p. 8) of a community are continually negotiated and regenerated through the constant interaction of its members (Lampert, 1990; Cobb, 2000; Boaler 1999; Davis & Simmt, 2003; Davis, Luce-Kapler & Sumara, 2000). Cobb (2000) refers to these norms as including things like what constitutes a good mathematical solution, who can validate a solution within the classroom and what is considered validation. He notes that such norms can be structured in such a way as to foster the development of “intellectual autonomy” (p. 8) within a classroom. Cobb recognizes the concept of autonomy as “synonymous with the gradual movement from relatively peripheral participation in classroom activities to more substantial participation, in which students increasingly rely on their own judgments rather than on those of the teacher” (p. 8), a concept very closely related to Lave and Wenger’s view of learning as legitimate peripheral participation. Cobb, like many others, discusses the importance of the sociocultural norms established in the learning community to the process of learning. The generation of such norms, he views as an “emergent” (p. 31) process, whereby the norms and practices of the community gradually emerge and evolve as a result of the interaction of community members as they become full participants in the community of practice.

Magdalene Lampert (1990) further examines the importance of community and the sociocultural norms established in the classroom indicating that:

from the activities the teacher sets for them, students learn what counts as knowledge and what kind of activities constitute legitimate academic tasks (Cazden, 1988; Doyle, 1985,1986; Leinhardt & Putnam, 1987; Lemke, 1982; Palinscar & Brown, 1984). Face-to-face interaction between students and their teacher follows context-specific rules, and cues within these contexts signal how what anyone says is to be understood in relation

to the task everyone is assembled to accomplish (Cazden, 1988; Mehan, 1979). The teacher has more power over how acts and utterances get interpreted, being in a position of social and intellectual authority, but these interpretations are finally the result of negotiation with students about how the activity is regarded (p. 34-35).

Lampert contends that the activities that teachers utilize as well as the interaction between members of the community provide context cues about what mathematics is and how mathematics is learned. These norms may not be openly negotiated, but are communicated nonetheless through the nature of the things students are required to do in the mathematics classroom.

In an analysis of two opposing methods of teaching mathematics at two schools named Amber Hill and Phoenix Park, Jo Boaler (1999) illustrates the importance of classroom culture and norms in the quality of learning and transfer of knowledge. In her discussion of Amber Hill, a traditional classroom culture, Boaler describes the norms that had been established. The students were separated into ability groups and taught in a “traditional” manner. The teacher demonstrated concepts on the chalk board and the students then practiced the procedures through the use of textbook questions. Boaler identifies the norms and practices that were established in such a setting. The students expected to use the method demonstrated by the teacher in their textbook practice, becoming confused by questions that did not follow the methods demonstrated by the teacher. They also expected to use all of the information given in a question, rarely had to make choices about which procedure or information to use, and relied on the teacher’s help to approach questions that confused them. The students in Amber Hill also saw very little connection between what they considered to be doing mathematics in the classroom, and what mathematics they would need to use in the ‘real world’. Phoenix Park students,

on the other hand, were divided up into mixed-ability groups and were given open projects that they worked on themselves at whatever rate they chose. They were expected to produce extended pieces of work with no guidance from the teacher about how to approach them. Students tended to ask the teacher only if what they were doing was going in an interesting direction, as opposed to how to do them. Quality rather than quantity of work was emphasized, and the tasks had many of the attributes of real-world problem solving: they were complex, students had to decide which information was relevant as well as what procedures would be helpful, students' beliefs and values were involved, and they had an opportunity to engage collaboratively in interpersonal activities. Boaler finds in her paper that the students at Phoenix Park had much more transferability of mathematical knowledge in that they saw what they did in the classroom as applicable to real life. Despite the fact that both sets of students came from similar background experiences, the Phoenix Park students out-performed the Amber Hill students on an end of year assessment, particularly on the conceptual understanding portions of the test. This occurred in spite of the fact that students were not explicitly taught procedures or content.

Boaler's account of these two opposing views of teaching is interesting for two reasons. First of all, she allows her readers to understand the concept of norms of practice by describing the structure of classroom life for the students in Amber Hill and Phoenix Park. In doing so, the reader is able to understand what behaviours are encouraged, enabled, and expected in each of the settings, something that many studies are not able to do. The complexity of classroom interactions, organization, and teaching are difficult to capture in a research study. Boaler's paper illustrates how these things combine and

manifest themselves in established norms and practices. Secondly, Boaler not only looks at behaviors, but also at performance on a end of year assessment in her research study. Not only did the Phoenix Park students exhibit desired behaviors, but they also outperformed their counterparts on the assessment.

Clearly, the norms that are established within a community of learners are of utmost importance and have a phenomenal impact on the type of learning that takes place within the community. Davis and Simmt (2003) suggest that:

the teacher's main attentions should perhaps be focused on the establishment of a classroom collective – that is, on ensuring that conditions are met for the possibility of a mathematical community. Such an emphasis is not meant to displace concern for individual understanding. The suggestion, rather, is that the individual learner's mathematical understandings might be better supported – not compromised – if the teacher pays more attention to the grander learning system (p. 164).

Perhaps by ensuring that the sociomathematical norms established in the classroom are conducive to the idea that the learning of mathematics can be seen as participation in a community of practice, students will develop a deeper understanding of mathematics individually as well as collectively.

### **The Role of the Teacher in a Community of Learners**

The role of the teacher in a community of learners is first and foremost to be a member of the community. The teacher walks a delicate line between the leader and a participant in the learning process, weaving in and out of each role as the situation warrants. He can lead a discussion or make suggestions to promote further thought, but must ultimately give up being a validator of truth. The teacher cannot be the source of information for

learners since it is they who must participate in order to learn. “So understood, the most critical aspect of the teacher’s role is not provision of information, but participation with learners in the development of strategies to interpret that information” (Davis, Luce-Kapler & Sumara, 2000, p. 131). Because the teacher is “the more experienced knower in the discipline” (Goos, 2004, p. 263) it is his responsibility to aid students in selecting which ideas are worth further examination and which ones are not. It is his responsibility to help students “notice what they haven’t noticed” (Davis, Luce-Kapler & Sumara, 2000, p. 26).

Merrilyn Goos (2004) highlights the “pivotal position of the teacher in structuring learning activities and social interactions to facilitate students’ increasing participation in a culture of mathematical inquiry” (p. 264). Not only must a teacher help negotiate the sociomathematical norms of the community, he must structure the activities and interactions that are necessary to engage students in the process of learning about mathematics as well as about what it means to be a member of the community. Davis and Simmt (2003) argue that while emergent events cannot be caused, they might be occasioned, suggesting that “decisions around planning are more about setting boundaries and conditions for activity than about predetermining outcomes and means – proscription rather than prescription” (p. 147). They refer to the term *liberating constraints* when speaking about the boundaries a teacher must set in order to achieve an environment conducive to learning. In another article (Davis & Towers, 2002), Davis further develops the idea of *Structuring Occasions*, discussing the role of planning and teaching in developing a classroom collective or community in mathematics education. Davis and Towers point out that:

planning might be more fruitfully understood as an exercise in anticipating how one might support many students simultaneously with a single intervention or prompt, or how one might respond to one student's formalized understanding with a prompt that is meaningful to a student whose understanding centres on images of the particular" (p. 338).

The role of the teacher, then, becomes one of anticipation. He plans activities and interactions that might become experiences in which students can participate and from which students might learn. Keeping in mind the need to negotiate the norms established by the community as well as the desired skills and outcomes, he must predict what might happen in the classroom. Only in the act of teaching/participating, however, is the teacher able to notice learning and adapt to the needs of the community in real time. Towers and Davis (2002) thus refer to teaching as "complex participation" (p. 338). It requires the teacher to plan for what might happen and react to what does – a difficult task. As such, "learning is *dependent on*, but cannot be *determined by* teaching" (Davis, Luce-Kapler & Sumara, 2000, p. 64). A teacher cannot determine what learning will take place in the classroom. He can only create mathematically and communally rich occasions and conditions in which learning can take place.

### **Conditions Necessary for Learning to Take Place in a Community of Learners**

Merrilyn Goos (2004) examines the actions that a teacher might take to create a *culture of inquiry* in a secondary mathematics classroom. She describes a *community of mathematical inquiry* as a classroom in which discussion and collaboration are deemed important and in which students are expected to "propose and defend mathematical ideas and conjectures and to respond thoughtfully to the mathematical arguments of their

peers” (p. 259). In her article outlining her research as part of her doctoral dissertation in Queensland, Goos outlines nine categories expressed as action statements in her data analysis:

1. The teacher models mathematical thinking.
2. The teacher asks students to clarify, elaborate, and justify their responses and strategies.
3. The teacher emphasizes sense-making.
4. The teacher makes explicit reference to mathematical conventions and symbolism.
5. The teacher encourages reflection, self-monitoring, and self-checking.
6. The teacher uses the students’ ideas as starting points for discussion.
7. The teacher structures students’ thinking.
8. The teacher encourages exploratory discussion.
9. The teacher structures students’ social interactions. (p. 267).

These actions are all things teachers do to create conditions that are ripe for learning to take place. They make no reference to content or presentation of information. What they do refer to are specific things that a teacher might do to create a culture of inquiry within a classroom. They focus on the establishment of norms and practices that are conducive to student participation in a community of practice.

Brent Davis and Elaine Simmt (2003) discuss the concept of mathematics classrooms as “adaptive and self-organizing complex systems” (p. 138). It is their contention that as a complex system, the classroom is adaptive in that it changes its own structure and emergent because it is composed of individual agents that together can be seen as a cognizing agent on the collective level. Davis and Simmt outline five conditions that must be met for learning to emerge in the complex classroom community: “(a) internal diversity, (b) redundancy, (c) decentralized control, (d) organized randomness,

and (e) neighbor interactions” (p. 147). These five conditions are not unlike the nine actions of a teacher listed by Marilyn Goos.

The first condition proposed by Davis and Simmt is internal diversity. It is necessary for complex systems, including classroom communities, to have diversity. It is the diversity of the members of the community that allow for social interaction and collective knowledge to emerge. Because each of the participants in a community of learners brings with them their own sets of background experiences, beliefs, and values, they each represent a different viewpoint and their diverse perspectives increase the potential of the collective. The complex system’s ability to adapt and learn is inherently determined by the internal diversity of the system. Davis and Simmt concede in their writing that by their nature, all classroom communities have internal diversity, which causes the participants to self-organize, establishing norms of practices. They suggest that creating internal diversity is not the concern, however. Rather the concern “becomes how the mathematics teacher might occasion the emergence of a complex collective whose interactions and products are mathematical” (p. 149). Jo Boaler’s (1999) account of Phoenix Park speaks to this concern. Students in Phoenix Park were placed in mixed-ability groups for project work and were encouraged to share ideas and respond to the ideas of others. Being in groups does not necessarily mean that students will develop collective understandings or create mathematical byproducts. Teachers must strive to create situations where complex interactions between students can occur in such a way that mathematical ideas, expressions, and thinking are the byproduct.

The second condition of redundancy proposed by Davis and Simmt refers largely to the need for members of a community to be somewhat similar. The members of a

classroom community often have similar backgrounds, education, age, and purpose, which enables them to form a collective identity in some manner. Redundancy within a community allows for the interaction among its agents. Qualities such as shared vocabularies, symbols, experiences, expectations, and purpose allow for a certain amount of stability within the community so that shared understandings can emerge through participation. While traditional classrooms that require students to listen and complete assignments in the same way promote redundancy, more student-centred approaches promote diversity. Davis and Simmt argue that a balance between internal diversity and redundancy must be met to optimize learning and for the development of collective knowledge to emerge. Jo Boaler's (1999) account of Phoenix Park students' interactions might provide some insight into what this balance might look like. While students worked at their own pace, the process was not entirely student-centred. Students were encouraged to interact and share ideas. Discussion and explanation were expected norms of conduct in the classroom, and as such, mathematics became a byproduct of the classroom community.

Davis and Simmt list the decentralization of control as the third condition which must be met in order for learning to emerge. They describe the need for control to be dispersed amongst the agents in a complex system – the students and the teacher – so that the system or community, itself, is able to decide what is appropriate or correct. Davis and Simmt question both the teacher-centred and student-centred approaches to learning because they are both based on the premise that learning is an individual enterprise. While teacher-centred approaches require the teacher to teach to the ideal of the “normal” student (an idea based on redundancy), student-centred approaches see individuals as so

diverse that they must be treated independently. Davis and Simmt advocate, instead, viewing learning as a discourse or shared action in which the center of the collective is not the teacher or the student, but rather the “collective phenomenon of a shared insight” (p. 153).

Organized randomness, Davis and Simmt’s fourth condition for the emergence of learning, entails creating an environment that is structured enough to generate activity and learning, but open enough to allow potential ideas to emerge. Davis and Simmt use the term “liberating constraints” (p. 155) when referring to this environment which does not mean that structure is abandoned allowing anything to happen, but rather that teachers must “maintain a delicate balance between sufficient organization to orient agents’ actions and sufficient randomness to allow for flexible and varied response” (Davis & Simmt, 2003, p. 155). Davis and Simmt further note that this environment must be negotiated in the act of teaching. Jo Boaler’s (1999) description of Phoenix Park illustrates this principal beautifully. Students were given open-ended projects that they worked on at their own pace. Students were expected to produce extended pieces of work for each project, but were never told how to approach them. These activities are indicative of the concept of organized randomness. They were structured enough to generate activity and learning, but yet open enough to allow potential ideas to emerge. Students were allowed to spend as much time as they desired developing their ideas, creating greater potentials for ideas to come into fruition. Not only was the control dispersed within the community about what and how much learning took place at any given time, but the whole process was enabling – the activities were catalysts for learning.

The final condition that must be met for learning to emerge as cited by Davis and Simmt is neighbor interactions. A conscious effort must be made by teachers to provide opportunities for members of the classroom community to interact with each other about mathematics. Davis and Simmt note that “group work, pod seating, and class projects may be no more effective at occasioning complex interactivity than traditional straight rows – if the focus is not on the display and interpretation of diverse, emergent ideas” (p. 156). This notion is similar to what many of the other authors cited in this paper have expressed. In fact, most of Goos’ nine action statements refer to this very concept: the teacher models mathematical thinking, asks students to clarify, elaborate, and justify their responses and strategies, emphasizes sense-making, uses the students’ ideas as starting points for discussion, structures students’ thinking, encourages exploratory discussion, and structures students’ social interactions. If the students at Phoenix Park (Boaler, 1999) were to only work individually on projects, and were not expected to interact with others, discuss and clarify ideas, and work collaboratively to solve problems, the environment would not have been as productive as it was. The potential of the collective is clearly greater than the potential of a group of individuals.

Mathematics education must strive to harness the power of the interactivity of complex learning environments. Mathematics teachers need to help students participate in the community of learners by negotiating the norms and practices of the learning community with its members such that complex interactions arise whose products are mathematical. They need to provide rich, open tasks that are structured enough to engage students in participation and learning, yet open enough to allow for potential thoughts to

emerge. Only then will mathematical learning be seen as complex participation in a community of practice.

## Chapter 3

### *Theoretical Framework*

As mentioned in the introduction in chapter 1, as I became aware of the educational theories described in chapter 2, I began to formulate my own view of what it means to *learn* mathematics. I came to the conclusion that mathematical learning should be viewed as *complex participation in a community of practice characterized by mathematical inquiry*. This view of learning, which I will refer to as a “theoretical framework”, became a lens for me as an educator, through which I viewed and reflected on my own teaching practice. It also became the basis for this research study as I looked at how my practice would need to change if I was to view learning in this way, and bring about change in my practice based on the educational theories to which I had been exposed. Before describing the study itself, it is important to consider what this view of mathematical learning entails. Each part of the the phrase *complex participation in a community of practice characterized by mathematical inquiry* was chosen from the writing of theorists who pointed out in their work key concepts about mathematical learning that I found to be particularly poignant. Together, the parts of the phrase connect the ideas of those theorists who have contributed to my view of what it means to learn mathematics. This chapter will outline the key parts of the theoretical framework I have and will continue to refer to, and will make connections between the framework and the theoretical foundations on which it is built.

By introducing the theoretical framework with the statement that learning should be viewed as *complex participation*, two very important theories about what

mathematical learning is surface. The term *complex participation* encapsulates, simultaneously, both the notions of complexity theory and the concept of learning as participation. Just as Anna Sfard (1998) recognizes the notion of participation as synonymous with both “taking part” and “being part”, both of which she considers to “signalize that learning should be viewed as a process of becoming a part of a greater whole” (p. 6), I too view learning as a process of “taking part in” and “being part of” the cultural practices of a community. Unlike Sfard, who refers to both of these phrases as indicative of becoming part of a greater whole, I believe that these statements represent two somewhat different parts of *complex participation* in that “taking part in” signifies an acceptance of the existing norms and practices of a community and working within them as an individual, while “being part of” indicates more a sense of being whereby learners make up the community, forming it, and changing it, by their very membership. Both of these aspects of participation are extremely important to me as an educator. Taking part in the activities of a community of learners is important to me because of the knowledge that is required to do so. In order to take part in, or participate in, the activities within a mathematics classroom, a student must understand the norms of conduct, conventions, language, and symbols of the classroom community. They must, as Lampert (1990) suggests, learn what a mathematics lesson is and how to participate in it. Being part of a community of learners is important to me as an educator because of the importance of the contributions of all members of the classroom community to the learning process. The comments of one member of the community can change the direction of a conversation and completely change the outcome of a lesson. Being part of a mathematics lesson or classroom community requires not only a sense of belonging, but also a sense of creation.

Members of a mathematics class form, by their very presence, the community itself, including all parts of the community that they touch through their participation and presence within it. This concept is reminiscent of Lave and Wenger's (1991) *legitimate peripheral participation*, which promotes a view of learning as peripheral participation in a community of practice in which learners gradually become members of that community. In the process of being transformed into members of the community, the participants become part of the evolution of the community – itself – as it continuously and dynamically evolves.

This view of participation and the community as constantly transforming the individual and being transformed *by* the individual parallels the basic notions of complexity theory, and thus *complex participation* is an apt phrase to embody the particular notion of participation I have come to view as part of learning. In using the term *complex* in front of participation, many ideas surface. The view of learning as an emerging byproduct of the interactions between agents in a system is of key importance. *Complex participation*, then, encompasses all of the interactions of the individuals within a community as well as the context that surrounds those interactions. It includes all of the interrelated, complex variables that come together to allow learning to emerge as agents within the community relate to one another. If learning truly emerges as a product of a community, as complexity theory would suggest, then a teacher must focus on creating a community within their classrooms whose byproducts are mathematics and mathematical learning. I believe how a teacher does this is of key importance. How can teachers facilitate the development of such a community in their mathematics classroom? What sorts of activities should they engage their students in? What norms and practices must

first be established so as to create an environment in which students are able to interact as individual agents do in a complex system, in addition to mathematical learning being the byproduct of those interactions? These are some of the questions that helped guide me towards this study as I thought about learning in this way.

According to Davis and Simmt (2003), there are two qualities that identify complex phenomena: adaptive and emergent. These two adjectives describe the classroom community. The mathematics classroom community adapts continually as it performs different tasks from discussion to problem solving, from working individually to working collaboratively, from performing experiments to explaining solutions to problems. The community is able to continually renegotiate what behaviors are expected and what constitutes “doing mathematics”. What one individual offers as a perspective on a concept can completely change the trajectory of learning in the room. All members of the community participate in a journey led by the interactions of the members of the community rather than by a leader. Essentially, the community must continuously adapt. For this reason, the mathematics community in a classroom is also emergent. The community itself is “composed of and arises in the co-implicated activities of individual agents” (Davis & Simmt, 2003, p. 138). The community changes and evolves through the interactions of its members, emerging as learning does and evolving continuously.

In the theoretical framework described as viewing learning as complex participation in a community of practice characterized by mathematical inquiry, the term *community of practice* is also of key importance. While Lave and Wenger’s (1991) view of learning as legitimate peripheral participation has traditionally been equated with forms of master-apprentice relations, they note that they “might equally have turned to

studies of socialization; children are, after all, quintessentially legitimate peripheral participants in adult social worlds” (p. 32). Lave and Wenger acknowledged that while they chose to look at learning as legitimate peripheral participation in communities of practice from the point of view of traditional apprenticeships, this view of learning was applicable to other circumstances, not just master-apprentice situations. What, then, does the term *community of practice* mean when looking at mathematics learning? Is the community of practice the larger community of society? Is it the community of mathematicians? Is it simply the community of learners that find themselves in the room for a given course at a given time? My personal belief is closest to the latter view. In the theoretical framework of viewing mathematical learning as complex participation in a community of practice characterized by mathematical inquiry, the term *community of practice*, for me, refers to the group of people, students and teacher, which is assembled in a given classroom at a given time with the common purpose of students learning mathematics. Most students will not become mathematicians, so it is not to the community of mathematicians to which they strive to become members. While many of the students will use mathematics in their lives as members of society in the future, the communities to which they will belong will have little to do with their mathematics classes. Therefore, the point that Lave and Wenger (1991) raise – that learning is participation in a community of practice as learners gradually become members of the community – does not apply in a formal educational institution. Rather, the classroom and school must be seen as smaller communities nested within the larger societal community that teaches and moulds individuals to become participants as adults. If one is to use Lave and Wenger’s (1991) concept of legitimate peripheral participation, one must

think of high school mathematics class as one episode of learning in the life of an apprentice. It is but one thing that moulds and shapes an individual, and yet, it is a microcosm unto itself. The classroom, on a micro-level functions as a complex system, with its own conventions, rules, symbols, and norms of conduct, all of which are continually negotiated amongst its members. Simultaneously, this classroom community “takes part in” and “is part of” the school community, the local community, the country, and the world. In looking at mathematics learning as complex participation in a community of practice, then, the term *community of practice* is intended to refer to the immediate classroom community as well as the things that the community does in the context of the mathematics classroom, keeping in mind that students are part of and take part in increasingly larger communities outside of the classroom. This is important to me as an educator, because by thinking of my mathematics classroom as a community of practice, I acknowledge not only that learning occurs through participation in the community, but also that the community itself is dynamic and can be changed to foster the emergence of various byproducts, including mathematics, mathematical thinking, and even inquiry. Viewing learning this way requires that I pay attention to the norms and practices that are established within the community as well as the complex choreography that makes up a mathematics lesson and a community of learners. This leads me to consider not only what mathematical content I wish to teach in my classroom, but also how students will interact with the content, how students view mathematics and mathematical learning, what sorts of opportunities for discussion and engagement I will structure, and how I can shape the trajectories of ideas and discussions in my classroom

such that the byproduct of the interactions within the community result in learning mathematics.

Viewing learning in this way also leads me to think about how I can foster the emergence of a community whose norms and practices are conducive to mathematical thinking and noticing. Paying attention to the characteristics of the community that emerges within my mathematics classroom becomes a critical enterprise when I consider these things.

The final part of the theoretical framework, which states that learning should be viewed as complex participation in a community of practice *characterized by mathematical inquiry*, adds yet another dimension to this view of mathematical learning.

This phrase comes from the work of Merrilyn Goos (2004), who suggests that:

*all classrooms are communities of practice – but classroom communities differ in the kinds of learning practices that become codified and accepted as appropriate by teachers and students (Jo Boaler, 1999). For example, in mathematics classrooms using a traditional, textbook-dominated approach, effective participation involves students in listening to and watching the teacher demonstrate mathematical procedures, and then practicing what was demonstrated by completing textbook exercises. Teaching methods that foster learning mathematics by memorization and reproduction of procedures can be contrasted with the more open approaches in reform-oriented mathematics classrooms, where quite different learning practices such as discussion and collaboration are valued in building a climate of intellectual challenge. Rather than rely on the teacher as an unquestioned authority, students in these classrooms are expected to propose and defend mathematical ideas and conjectures and to respond thoughtfully to the mathematical arguments of their peers. Thus, the practices and beliefs developed within reform classrooms frame learning as participation in a community of practice characterized by *inquiry mathematics* (p. 259).*

Goos' description of a community of practice characterized by inquiry mathematics matches my own views about what type of learning practices I wish to foster in my own classroom community. The addition of *characterized by mathematical inquiry* adds a

qualification about what sort of community I feel is necessary for me to foster if I am to alleviate some of my concerns about my educational practice. As I indicated in the introduction, the things I was most discontented with in my own classroom community were: (1) the inability of students to approach novel situations or problems that were slightly different than those they had seen demonstrated, (2) the inability of students to see connections between real life and the mathematics they engaged in within the classroom, and (3) the lack of curiosity and interest exhibited by students in my classroom. Goos's concept of a community of practice characterized by inquiry mathematics struck a chord with me, largely because it seemed to be a possible solution to the problems I had experienced in my own classroom. By making Goos' concept of inquiry mathematics, or as I termed it, *mathematical inquiry*, the characteristic or byproduct I was looking for from my classroom community, the very things that I wished to remedy were addressed. Therefore, this phrase was used to distinguish the sort of community I felt was needed to promote mathematical learning.

Having identified the nature of the mathematical community of practice I wished to foster in my own classroom, I was forced to consider the question of how such a community of practice could be fostered by teachers. According to Goos's (2004) study of one Australian school, teachers wishing to foster such a community could incorporate the nine teacher actions discussed in the literature review of this paper (see p. 20). These nine action statements are a good starting point for thinking about how one might go about attempting to develop such a community of practice. Goos's (2004) study of one teacher in Australia characterizes the process through which a teacher has established such a community of practice. In formulating the nine teacher actions and discussing both

a mature culture of mathematical inquiry and one that is beginning to form, Goos provides both a snapshot of the *product* and a description of the *process* of achieving that product to teachers who wish to foster such communities in their classrooms. She discusses how using broad problems to open discussions about mathematics can be beneficial, focusing largely on the actions of the teachers that promote the development of this classroom culture. How I could foster the emergence of such a community of practice became an important question for me as an educator who was looking to change her practice. It also became an important focus for this research study. As a result, I found it necessary to consider more specifically what I thought would be the characteristics of a community of practice characterized by mathematical inquiry. Using Goos's nine categories as a starting point, I developed a list of eleven characteristics I felt a community of practice characterized by mathematical inquiry would exhibit:

1. mathematical thinking and noticing
2. discussion of mathematical ideas
3. the proposing, clarifying, defending, and refuting of mathematical strategies
4. curiosity/asking questions about mathematics
5. individual and collective ownership of learning
6. application of mathematics to real world contexts
7. decreasing reliance on the teacher as validator of mathematical ideas and increasing reliance on peers and self as validators of mathematical ideas
8. reflection on mathematical ideas
9. metacognitive awareness
10. understanding of the norms and practices of the community
11. recognition of common purposes amongst community members

With these characteristics in mind, I conceptualized the study, thinking about how viewing learning as complex participation in a community of practice characterized by mathematical inquiry would inform change in my teaching practice. The study was designed to not only look at how the emergence of such a community could be fostered,

but also at the challenges I would face and the characteristics that would actually emerge within my classroom.

## Chapter 4

### *The Purpose and Methodology of the Research Study*

#### **Purpose of the Study**

This research study had essentially two purposes. Its primary purpose was to document the way in which theory would inform my teaching practice. More specifically, I wished to look at how viewing learning as complex participation in a community of practice characterized by mathematical inquiry would inform my teaching. I was curious about what decisions I would make with this view of learning in mind. I wondered what my classroom would look like, and if the areas of discontent I have already described would be remedied. At the heart of this purpose was the belief that in order for educational reform to occur, educational theories must first be enacted in the messiness and reality of the classroom. It is when educators allow educational theories to come to life in their classrooms that the usefulness of such theories is determined, and those theories that help teachers achieve their goals are the one that truly inspire educational reform. While I knew that every classroom and teacher were different, I felt that by changing my own practice to reflect the view of learning as complex participation in a community of practice characterized by mathematical inquiry, I would be able to provide an example for others who may have experienced the same discontent, or who might have a similar view of learning. I hoped that through my characterization of the changes my teaching underwent, I would offer insight into what this view of learning, or the educational theories from which it was built, might mean for classroom teachers.

The secondary purpose of this research study was to improve my own teaching practice. As I indicated earlier, prior to my Master's degree and this research study, I was unhappy with the inability of students to approach new problems and situations, the inability of students to see connections between mathematics class and real life, and the lack of interest and curiosity exhibited by students. Although this list is not exhaustive, it represents the three main areas that I felt I needed to give attention to in my classroom. I wanted to address these problems through improving my own teaching practice, and this became a major focus of the study. I suspected that improving my teaching practice in these areas would require me to develop activities, procedures, and practices that would foster the emergence of a community of practice characterized by mathematical inquiry in my classroom, which is what I set out to do in this study.

### **Research Questions**

As I thought about the two purposes of this study, two sets of research questions developed:

1. How does viewing mathematics learning as complex participation in a community of practice characterized by mathematical inquiry inform my teaching practice?
2. How can I foster the emergence of a community of practice characterized by mathematical inquiry in my classroom? What challenges will I face? Which characteristics of a community of practice will emerge?

These two questions guided me throughout the research study as well as throughout the analysis and interpretation of data. The thesis, too, has been structured to answer these questions. In chapter 5, I relate my story of planning and teaching. This chapter presents a narrative of my experiences throughout the school year, including both the decisions I made to change my teaching practice, as well as my experiences as those changes were enacted within the classroom. Chapter 5 begins the process of answering the first research question. By telling the story of my planning and teaching, I am able to describe to the reader in a narrative form how my practice changed as a result of viewing learning as complex participation in a community of practice characterized by mathematical inquiry. Following the narrative in chapter 5, in chapter 7 I explicate the ways in which my teaching practice changed. I elaborate on four specific changes to my teaching practice: the use of parallel planning, the creation of mathematically and communally rich learning activities, taking on the role of prompter, and my own movement towards the use of performance tasks for assessing student understanding.

The second research question is divided up into three parts. The first and third parts of the question – *How can I foster the emergence of a community of practice characterized by mathematical inquiry in my classroom?* and *Which characteristics of a community of practice will emerge?* – are also addressed through both the narrative describing my experiences attempting to foster the emergence of such a community of practice in chapter 5, as well as in chapter 8, in which I describe five characteristics of a community of practice characterized by mathematical inquiry and how they emerged within my classroom. The second part of the second research question – *What challenges will I face?* – is addressed in two ways in the thesis as well. In chapter 5, my own

personal experiences throughout the school year are described, including some of the challenges I faced along the way. In chapter 9 these challenges are more explicitly discussed as I both characterize the challenges I faced, as well as interpret what these challenges meant for me as an educator attempting to integrate theory and practice.

While at first glance the research questions appear to be personal, and perhaps even self-indulgent in nature, they make valuable contributions for other educators and educational theorists. For other educators who may be interested in this particular view of learning, or who are looking to change their teaching practice in similar ways, the study provides an example from which they may develop significant understandings for their own teaching practice. Through the rich, descriptive narrative account found in chapter 5 and the accompanying analysis contained in chapters 6 through 9, other educators may be able to draw parallels between my experiences and their own, finding ways they can use some of the ideas expressed in this thesis to affect change in their own practice. For other educational theorists and those who write about the theory that is behind the theoretical framework described in chapter 3, there is significant benefit in hearing about the lived experiences of a teacher who has looked at what such theories mean when she attempts to enact them in the practical setting of the classroom. While the research questions are about my own practice and experiences, others can also benefit from the answers I provide to the questions.

### **The Role of Design Experiment Research in this Study**

In order to achieve the second purpose of the study – to improve my own practice – I employed the use of design experiment research methods. Design experiment research provided me with a framework which allowed me to make changes to my teaching practice that I felt would foster the emergence of a community of practice characterized by mathematical inquiry. In using this methodology, I was able to make the changes to my teaching practice that were necessary to answer all of my research questions. In order for the questions to be answered, I first had to go about making changes in my practice based on the educational theories described in chapter 2, and design experiment research provided a method for making such changes.

Design experiments enable researchers to conceptualize learning theories and put them into practice. They allow researchers and practitioners to test theories of learning through essentially two steps. The first step of this method is to anticipate the hypothetical learning trajectories in a classroom and develop support and materials to facilitate the desired learning. Once this is done, the process requires the researcher to implement the measures decided upon in order to test the hypothesis or theory about learning. It is a largely “iterative design” (Cobb et al., 2003, p. 10), that evolves as researchers and practitioners work together, or as a researcher/practitioner works alone, to hypothesize about and test theories and processes in order to determine their effectiveness in the particular classroom. Researchers continually modify approaches to learning as the experiment goes on, creating two alternating phases: “prospective and reflective” (Cobb et al., 2003, p. 10). The experiment, itself, constantly evolves as

research practitioners modify instruction and materials based on their experiences and observations. As a result, an approach tested through design experiment research is grounded in theory and validated in practice. While the results of design experiment research are not generalizable, much can be learned by others by looking at situations in which design experiments were conducted and how the iterated findings lead to changed practices.

While the design of my research study did not fit perfectly with the traditions of design experiment research, it contained enough of the characteristics of design experiment research to be classified as such. Whereas design experiment research has traditionally been conducted such that the cycles constitute a re-inventive process about similar educational ideas and practices, my own research did not really use cycles to improve a specific educational practice. For example, whereas a teacher might incorporate the use of problem solving groups within their classrooms, going through cycles of using the groups, making changes to the design of the groups, and beginning again in an attempt to improve the use of problem solving groups as an educational strategy, I found myself attempting to cultivate many educational practices, often different ones in different cycles, all in order to foster the emergence of a community of practice characterized by mathematical inquiry. In one cycle, I might have used problem solving partners to have students discuss and engage with measurement problems, while in the next cycle, I may not have used problem solving partners at all, focusing instead on the development of whole class discussions and the expression of mathematical ideas and strategies. This departure from traditional design experiment research does not, however, lessen the concept of continuing cycles occurring in an attempt to improve educational

practice. The complex nature of fostering a community of practice characterized by mathematical inquiry required multiple educational strategies to be used at various times during the course. The focus remained, however, on the development of the community of practice, and thus all cycles were designed to achieve this purpose.

According to Cobb et al. (2003), “design experiments ideally result in greater understanding of a *learning ecology* – a complex, interacting system involving multiple elements of different types and levels – by designing its elements and by anticipating how these elements function together to support learning” (p. 9). The purposes of my research study – to characterize how theory informs practice for me as an educator and to improve my teaching practice – both required me to look at the complex system that existed within my classroom. Because I desired to know how to foster a community of practice characterized by mathematical inquiry, I necessarily had to look at how the different elements of the classroom community functioned together, and how I could use this knowledge to improve my practice and the learning that occurred within the classroom community. Cobb et al. (2003) further note that:

elements of a learning ecology typically include the tasks or problems that students are asked to solve, the kinds of discourse that are encouraged, the norms of participation that are established, the tools and related material means provided, and the practical means by which classroom teachers can orchestrate relations among these elements (p. 9).

The focus of my research study was completely devoted to developing a learning ecology or classroom community that fostered the emergence of mathematical inquiry. The tasks students were asked to solve, the discussion that occurred in the classroom, and the norms and practices that were established were all part of the changes in my teaching practice that emerged as I grappled with improving my teaching practice as a result of viewing

learning as complex participation in a community of practice characterized by mathematical inquiry. Design experiment research was a natural fit for this process, and allowed me to pay attention to not only the changes in my practice, but also the complex choreography that existed within the community itself.

Cobb et al. (2003) identify five features of design experiments in their article *Design experiments in educational research*. Following, I argue that my research study displays all of these features, despite a modification to the fourth feature which was made because of the nature of the study itself.

The first feature identified by Cobb et al. (2003) is that “the purpose of design experimentation is to develop a class of theories about both the process of learning and the means that are designed to support that learning” (p. 9-10). The research conducted as part of this study was based on the theoretical framework that views learning as complex participation in a community of practice characterized by mathematical inquiry. Such a framework suggests that learning occurs through the act of participation, and the byproducts of the interactions between students constitute mathematical learning. As such, I had to consider what sorts of activities would provide opportunities for students to participate and engage in communal practices whose byproducts were mathematical inquiry. I also had to consider how the classroom norms and practices could be negotiated such that a community of practice characterized by mathematical inquiry would emerge. Using the eleven characteristics of a community of practice that I developed (see p. 34) as a guide, I planned activities that I thought would promote the emergence of these characteristics. In this way, the research study both developed the theory of viewing learning as complex participation in a community of practice

characterized by mathematical inquiry, and the means through which one can foster the emergence of such a community.

The second feature of design experiment research as identified by Cobb et al. (2003) is the “highly interventionist nature of the methodology” (p. 10). The goal of design experiment research must be the improvement of education, which is accomplished by generating and testing new theories about learning. Cobb et al. (2003) further note that “there is frequently a significant discontinuity between typical forms of education . . . and those that are the focus of a design experiment” (p. 10). This was certainly true in this research project. The entire study was based on a desire to change my educational practice to remedy the significant discontinuity between my beliefs about learning and my teaching practice. Prior to the study, teaching for me was largely an act of presenting content. The process was extremely teacher-centred, as I stood at the front of the classroom, working out a few examples on slides or on the white board. Students sat quietly in rows, watching my instruction, until they were given a textbook assignment or work sheet on which they were to repeat the processes I had just demonstrated at the front of the classroom. At times, I asked for students to provide the answers to simple calculations, and occasionally I asked students for ideas about how to approach a problem. The students knew, however, that I would eventually show them how to do the problem anyway, which became the expectation. Previously, in the Applied Mathematics course that the research was conducted in, I had, in the past, had students do some projects, although the projects themselves were quite prescriptive. Not much was left open for students, although students generally seemed to enjoy doing them nonetheless. As I began to view learning as complex participation in a community of practice

characterized by mathematical inquiry, it became increasingly clear to me that my teaching practice would need to change dramatically if I was going to foster the learning community I had described. The research itself caused significant interventions in my teaching practice, more so than would have occurred had I not engaged in the research. By committing to the design experiments, I committed also to significant changes in my teaching practice.

The third feature identified by Cobb et al. (2003) is that “design experiments create the conditions for developing theories yet must place these theories in harm’s way. Thus, design experiments always have two faces: prospective and reflective” (p. 10). The prospective part of design experiments require researchers to identify *hypothetical learning trajectories* (Cobb, 2000, p. 31) in order to develop processes and practices that will allow such learning to emerge. In addition to this, during the implementation of such strategies, flexibility is key as the plan unfolds. The reflective element of design experiments follows the prospective part as several levels of analysis test the theory and design conjectures, and new or modified conjectures are developed for further study. Throughout the entire school year, I alternated between the prospective and the reflective phases of the design experiment cycles. I planned activities and teaching strategies for fostering the community of learners I wished to create, carried out those plans to varying extents, and then reflected on their effectiveness. Observations and reflections were then used to inform the planning of the next activity or class, restarting the design experiment cycle again. While the theoretical framework, itself, of viewing learning as complex participation in a community of practice characterized by mathematical inquiry was not placed in jeopardy during a research cycle, the characteristics I was hoping to foster may

or may not have emerged. Had they not emerged, the characteristics of such a community would have been called into question, or perhaps even the methods I used to attempt to foster them

This leads to Cobb et al.'s (2003) fourth feature of design experiment research – its “iterative design” (p. 10). As the researcher goes through the prospective generation of theory and process, and then through reflection and revision, they complete cycles of research that inform further cycles, allowing the researcher to test, modify, and retest theories of learning and pedagogy. As I have already indicated, the research conducted in this study followed an iterative pattern. While the characteristics of a community of practice characterized by mathematical inquiry I wished to foster were at times different in different cycles, each cycle was focused on improving my teaching practice through the development of the community of practice I have described. This research study is different from many traditional forms of design experiment research because each cycle of the design did not seek to improve on exactly the same element tested in the previous cycle. Because students could not be continuously exposed to the same activities, and because development of the community was the primary focus, cycles of planning were not based on the same activities or even the same methods of engaging students. Rather, cycles of planning and implementation were based on planned activities, strategies for developing certain characteristics within the community, and the subsequent observations, analysis, and reorganizing of strategies for fostering the emergence of the community of practice I have described. In one cycle, my focus could have been on decreasing reliance on the teacher as validator of ideas, while in another, the focus might have been on discussion of mathematical ideas and strategies. All eleven characteristics

(see p. 34) were not the focus of all cycles, and because of this, design experiment research in the traditional sense was modified to suit the purposes of this study.

The final feature of design experiment research as identified by Cobb et al. (2003) is that “theories developed during the process of experiment are humble not merely in the sense that they are concerned with domain-specific learning processes, but also because they are accountable to the activity of design” (p. 10). The practical and context-specific nature of design experiment research is both its strength and its weakness. While design experiments are developed and implemented in a specific context, “they also speak directly to the types of problems that practitioners address in the course of their work” (Cobb et al., 2003, p. 11). The results of design experiments may not be generalizable to other contexts, but they are specific and practical in their approach, offering valuable information for educators to apply in their own contexts and situations. While the results of my study may or may not be typical for other educators, they cannot be used to make generalizations about mathematics classrooms. My findings, however, do speak to some of the challenges educators face in attempting to change their teaching practice to better match the changes that have occurred in educational theory. My goal in the research study was never to come to generalizable conclusions about how mathematics should be taught. Rather, it was to characterize my journey of attempting to allow theory to inform my teaching practice so that others could read my story and look for areas that resonate with them in their particular situations.

### **The Role of Autoethnography in this Study**

While design experiment research provided a method for making changes to my teaching practice based on the theoretical framework I had developed, I also needed to undertake the self analysis portion of my study that would enable me to analyze and characterize what was accomplished through the design experiments. The research questions were not answered by the design experiments; rather, the design experiments only went as far as the point of planning and carrying out change in my teaching practices. The data collected during the process still had to be analyzed in order to characterize how exactly my teaching practice had changed, how I had attempted to foster the emergence of a community of practice characterized by mathematical inquiry, which characteristics of the community of practice actually emerged during the study, and what challenges I faced along the way. In order to conduct this portion of the research, I required a different research method – one which would allow me to study both myself as an educator and the culture which emerged in my mathematics classroom. For this process, I chose autoethnography.

The term *autoethnography* refers to “writing about the personal and its relationship to culture” (Ellis, 2004, p. 37). Deborah Reed-Danahay (1997) defines autoethnography as:

a form of self-narrative that places the self within a social context. It is both a method and a text, as is in the case of ethnography. Autoethnography can be done by either an anthropologist who is doing “home” or “native” ethnography or by a non-anthropologist/ethnographer. It can also be done by an autobiographer who places the story of his or her life within a story of the social context in which it occurs. (p. 9)

In my research study I was interested in studying both myself in terms of my own educational practices, and the social context and culture of the classroom community in which the research was conducted. For this reason, I felt that autoethnography was an appropriate research method for achieving my purposes.

Much of the literature on autoethnography discusses three characteristics, which I identified as useful for my research purposes in this study. Autoethnography (1) connects the self to the cultural, (2) requires the construction of rich, descriptive narrative that is written for the “other” despite its personal nature, and (3) enables both the researcher/author and the reader to take a more active role in the improvement of the culture being studied. These prominent characteristics of autoethnography were the basis on which I made my decision about research methods, as well as the characteristics which allowed me to answer my research questions.

The primary characteristic that identifies autoethnography from other research methods and forms of writing is that it connects the self to the cultural. Reed-Danahay (1997) further distinguishes this relationship between self and culture in two ways, noting that the term autoethnography refers: “either to the ethnography of one’s own group or to autobiographical writing that has ethnographic interest” (p. 2). For my purposes in this research study, I would characterize my own use of autoethnography as the latter of the two – autobiographical writing that has ethnographic interest. While I do not write about the culture of mathematics education per se, what I do do in this study is write about my experiences as a math educator struggling to change her practice and my experiences as I attempted to foster the development of a community of practice characterized by mathematical inquiry. This autobiographical writing has ethnographic qualities in two

ways. First, it looks at my own experiences as a math teacher, both in terms of my views about education and in terms of my experiences. Secondly, it describes the micro-culture which emerged in my mathematics classroom community as a result of changes in my practice. Both ways include writing about my personal experiences within an established culture – that of mathematics education or the classroom culture itself.

The second characteristic that I identified about autoethnography as useful for my purposes was that it requires the construction of rich, descriptive narrative that is written for “the other” despite its personal nature. In characterizing how my teaching practice changed as a result of viewing learning as complex participation in a community of practice characterized by mathematical inquiry I wanted to create a rich, descriptive narrative that would relate my experiences to readers.

Written narrative accounts have the capacity to illuminate the often complex and deeply problematic nature of people’s lived experience. In contrast to psychological case studies that interpret individual behavior from within a framework of disciplinary theory (personality, behaviorism, etc.), biographies and ethnographies provide the means to understand people’s lives from their own perspective. (Stringer, 2004, p. 130)

It was my hope in writing about my experiences that others would see things in my narrative writing that resonated with their own lived experiences, allowing them to draw conclusions from the narrative account that were pertinent to their own lives. By “others” I mean those who might have similar views about mathematics and learning, those who also want to change their practice, or even those who can simply identify with or benefit from my situation and experiences. According to W.-M. Roth (2005):

Any auto/biography or auto/ethnography is therefore never quite owned by the person who signs through assuming authorship, and who is the principal figure of the account. Rather, because any meaning of the text arises from the interaction of text and reader, the reader also owns it.  
(p. 11)

Despite the fact that this study was conducted to change my own practice, it was also conducted to characterize that change in order for others to learn from my experiences. Autoethnography provided a meaningful way in which to both analyze my experiences and use rich, descriptive narrative to share my experiences with the reader.

The third characteristic that I identified about autoethnography as useful for my purposes was that it enables both the researcher/author and the reader to take a more active role in the improvement of the culture being studied. It is for this reason that:

those who conduct auto/biographical research have a responsibility to readers to make insights (if any were gained, from the re-telling of their lived experiences) part of the narrative. In this way, richer contextual data can be made available for readers to move beyond empathy or superficial affirmation of their own lived experiences as these resonate with the author's. Hence, critical engagement in the telling and listening of auto/biographies could enable us to take a more active role in the improvement – e.g., of equity in science education.” (Rodriguez, p. 123)

While Rodriguez wrote specifically about autobiography in this passage, the same could be said for autoethnography. It is my hope that through this study I will be able to contribute to the improvement of mathematics education and teaching. For this reason, chapter 7 discusses the changes I made to my own teaching practice, chapter 8 discusses the characteristics of a community of practice characterized by mathematical inquiry, and chapter 9 discusses the challenges I faced. By analyzing my own insights into these things, I am able to contribute to the body of knowledge in mathematics education in order to convey to others how mathematics education might be improved. In addition to this, chapter 10 goes beyond my research questions by looking at what my experiences can offer theories of mathematics education.

Using autoethnography as one of my research methods during the course of the year, I collected three forms of data: writing from my own planning and observation journal and from student interactive journals, and different student work (see chapter 6). My own planning and observation journal was the primary source of data for the writing of chapter 5, a narrative account of my experiences throughout the year, although some of the student work and journals were also used to reconstruct my experiences. It was this narrative in chapter 5 that comprised the first phase of the autoethnographic portion of the study. Whereas design experiment research was used to actually make changes to my teaching practice, once the experience of this change was complete, the self study portion of the research project formally began.

The purpose of writing the narrative contained in chapter 5 was to take the lived experiences of my year and characterize them in a meaningful way (1) to relate the experiences through thick, rich description to my readers, and (2) to objectify the lived experiences in order to interpret them in terms of my research questions.

Without lived experience, there is no primary understanding that the person can reflect upon. Only after having exposed themselves to the irremediably unfolding events in the classroom, from which there is no time out to reflect, do teachers have a ground on and through which reflection on teaching can be developed. However, this experience, to become object of reflection, has to be objectified – raw experience in the making cannot serve as object because it has not yet been completed. Auto/biography and auto/ethnography both constituted forms of inquiry and writing that produce these primary objectifications, which then, in a second step, become the object of critical interpretation (reflection). (Roth, 2005, p. 177)

The writing of this narrative allowed me to look at my own experience as an object in order to analyze the experience and answer my research questions.

The analysis of my experiences, particularly through the narrative description included as chapter 5, made up the second phase of the autoethnographic method used in this study. During this phase, I looked at the narrative story I had written, in addition to the other data collected for the study, in order to answer each of the research questions. I first analyzed how my teaching practice had changed as a result of viewing learning as complex participation in a community of practice characterized by mathematical inquiry. The results of this analysis are described in chapter 7, where I discuss four major changes made to my practice. Next, I looked at all of the data collected in order to identify the characteristics of a community of practice characterized by mathematical inquiry, how it might be fostered, and what it might look like to an outsider looking in. The results of this analysis and interpretation are described in chapter 8. Finally, I considered my own journal and narrative story in order to identify the challenges I faced as I allowed theory to inform changes to my teaching practice. This analysis is reported on in chapter 9.

The third and final phase of the autoethnographic method used in this study included the process of looking outward at what my experiences could mean for other educators or educational theorists. Autoethnography does not just entail writing about the self. It pertains not only to what one experiences but also to what those experiences can offer the larger culture or society in which the research is conducted. In this case, while the research questions were about my practice and about myself as an educator, the questions also had potential to inform others who read about my experience. This is the nature of autoethnography. By extrapolating beyond my own experiences in chapter 10, making suggestions about how these experiences may be relevant to other educators, and discussing how my experiences could be seen to strengthen Davis and Simmt's (2003)

view of mathematics classrooms as complex learning systems I fulfil the potential of the autoethnographic method in an unconventional context – as an individual teacher studying change in her own practice.

## Chapter 5

### *My Story of Planning and Teaching*

This chapter includes a narrative overview of the school year in which the study was conducted. It provides a first person account of my lived experiences as a teacher struggling to affect change in her classroom. The first goal of the research project was to characterize how viewing learning as complex participation in a community of practice characterized by mathematical inquiry informed my teaching practice. By using narrative writing in chapter 5, I was able to present the data I collected in my planning and observation journal in a meaningful way to the reader. Using narrative writing and telling my story of the year also allowed me to provide a timeline for events that occurred within the classroom. It is important for the reader to understand the journey I have been on in order to understand the conclusions that follow in subsequent chapters. By writing the narrative in this chapter, I was able to objectify my experience both for the reader to understand and for my own purposes of analysis and answering the research questions. The act of writing this narrative allowed me to begin to characterize how my practice had changed and enabled me to look at the entire year analytically. This allowed me to identify strategies I implemented to foster the emergence of a community of practice characterized by mathematical inquiry, as well as the characteristics that emerged in the process. Through the writing of the narrative, challenges I faced in the process of changing my teaching practice came to the forefront, allowing me to look at deeper issues beneath such difficulties. It is through my story that I hope other educators will recognize

similarities or even differences between my experiences and their own, allowing them to decide if this research has relevance for them.

### **Planning to Begin**

I began planning for the new school year in July. I struggled with where to start my planning, and so I began where I always had in the past, with an overview of the months and units, attempting to put together some sort of sequence of units to fit within the time allotted for the course. For the first week or so, I struggled with finding a starting point. In the past, I had always begun by making a year plan or timeline. This time, I found myself going through the motions of making a year plan, knowing that I probably would not be able to stick to it. Still, I arranged the units in order, dividing up the year into relatively equally spaced intervals for each unit. This would be a recurring theme for me as the year continued – falling back into old and familiar ways of doing things. Perhaps this was evidence as to why I have failed to effect significant change in my educational practice in the past. It is difficult to seek out a new way of doing things. One is inevitably drawn to the way they have done things before, the path of least resistance.

It took me several days to decide on a starting point for the course. I did not know the students very well that I would be teaching, although I had taught a few of them the previous year in a computer course. I knew only that there would be approximately nineteen students in the class, and that they were the same students I would teach in both grade 10 Applied Math and grade 10 Precalculus Math. Recognizing that there would be a time issue as far as covering content was concerned, I found myself looking at both

curricula for ways to eliminate overlap as well as places where hands-on, experiential activities might be best suited. In the end, I decided to begin the Applied course with the measurement unit for several reasons. First of all, I recognized that a lot of the outcomes of the Measurement unit were based on prior knowledge students would likely have from previous grades. Students would have already been exposed to perimeter and area problems, measurement with metric rulers and other measurement instruments, and most of them would have some knowledge about the Metric and Imperial measurement systems. I felt this would allow me to get students doing things right away without having to do a lot of direct teaching. I wanted to begin the year in a very hands-on manner, establishing the norms and practices that would guide the classroom community. I felt that measurement would afford me this opportunity and I set my sights on conceptualizing the sorts of activities I wanted students to participate in that would foster the development of a community of practice characterized by mathematical inquiry.

In the beginning stages of planning, I found myself discouraged by the variety of outcomes contained in the curriculum document. The outcomes did not fit together in nice conceptual packages. They were quite diverse and there was no way to effectively draw all of them together. At one point, I considered having students develop a landscaping plan for the school yard. I felt that this would be hands-on, requiring students to use measurement instruments, it would have real world applications, it would require students to convert measures, it would require students to work collaboratively, and it would be somewhat open for students to express creativity and individuality. I was discouraged, however by the notion that this activity did not cover all of the outcomes, and began thinking how I could both give students a taste of what the course would be

like and yet ease them into it without throwing a very large project at them. I was surprised by how difficult it was for me to think about content and developing a community of practice characterized by mathematical inquiry simultaneously. The old and familiar ways enticed me to think about content, and reconciling this way of thinking with the educational theory I wished to incorporate into my teaching practice proved to be difficult.

A little over one week into the process, I first created a t-chart (see Appendix L) with *Content* as a heading in the left-hand column and *Dev. A Community of Math Inquiry* as a heading in the right-hand column. On the left, I listed in point form ideas that dealt with the content outcomes in the curriculum documents. On the right, I began by simply listing the eleven characteristics I had identified of a community of practice characterized by mathematical inquiry (see p. 34). Then, I began to brainstorm ideas about what kinds of activities could be used to get at the content in the left column, and I started to think about what made the activities good in terms of developing some of those eleven characteristics on the right. This was really the beginning of what I later began to refer to as *parallel planning*, where I attempted to look at content and development of the community of practice simultaneously in my planning. From here I began to conceptualize some activities as I thought about both content and the characteristics of the community of practice I hoped would emerge.

The first activity I planned was entitled Thinking Outside the Box (see Appendix A). The first thing I wanted students to do in the course was to be active, and I wanted the activity to embody all of the things I felt were important in an activity. I knew that in order to foster the eleven characteristics I was looking for, the activities that I planned

needed to be hands-on in nature. They needed to foster both independence on the part of the individual learner as well as social interdependence and collaboration. They needed to apply math to real world situations, and they needed to afford students the opportunity to determine mathematical strategies for solving problems as well as discuss and evaluate the effectiveness of such strategies. They needed to promote curiosity and inquiry about mathematics and its relation to the world. Finally, the activities needed to be open in nature, perhaps even somewhat ill-defined in that they needed to require students to think mathematically and allow them to explore some of their own mathematical ideas.

Thinking Outside the Box was designed to meet many of these qualities. It served as an activity which would show students what *doing math* was going to look like in this course. The metaphor of thinking outside the box provided an apt phrase for students to start challenging their perceptions about what a math class should look like and what it meant to do math. Through the activity, I hoped students would be able to see what the norms and expectations for the class would be as well as perhaps get excited about a more hands-on, active approach to learning mathematics. Although the activity required some work with measuring and area, its primary purpose was introducing students to the community of practice I hoped to establish.

The Thinking Outside the Box activity itself required students to take a box that was given to them and calculate the total area of cardboard used to make the box, the dimensions of the smallest piece from which the box could be cut, the amount of waste resulting from the cutting pattern, and the total volume of the box. They then had to create a box that could be cut from the same size piece of cardboard, but that had a better design, and present their design to the class. Finally, students were required to research

one industry concerned with boxes and packaging and suggest how something like box design might relate in a real world context. for example, how it might relate to profitability or marketing.

What I particularly liked about the design of Thinking Outside the Box was first and foremost that the students would be working collaboratively in a hands-on manner to approach a problem. This would require students to collectively come up with a strategy for finding the area of the box as well as the dimensions of the cardboard from which it would be cut. They would have to conceptualize and discuss a better design for the box and present it to the class, requiring them to think mathematically, to discuss ideas, and to propose and defend mathematical strategies, which were three of the eleven characteristics I hoped to see emerge. In addition to these characteristics, students would also have to make connections to real world industry and perhaps even ask a question or two about how math relates to the real world in the process of engaging in the research portion of the activity. As I began to think about what I would be doing as students worked at the task in groups, I liked the idea that I could choose not to take on the role of validator when students asked questions. I could use this opportunity to let students know that I would not tell them *how* to approach activities or problems; I would be expecting them to rely on their own ideas and the ideas of their peers. I could also use the opportunity to encourage students to take some risks, and let them know that sometimes thinking things through and deciding there might have been a better or faster way to accomplish something is one way in which we learn.

As I planned Thinking Outside the Box, I started with the hands-on nature of the activity, whereby students would take a box and find the area of cardboard used to make

it. I decided that this would lead nicely into the content of the course as students were expected to calculate volume and surface area of rectangular prisms. I added some depth to the problem by having students also calculate the smallest piece of cardboard from which the box could be cut, how much waste there would be, and the volume of the box. These things satisfied some content objectives, but I also wanted to look at how I could foster the emergence of several of the eleven characteristics I hoped to see emerge in my classroom. As I looked through the eleven characteristics, I recognized there was room to add in some discussion about what strategies they used as well as for a design element that might encourage students to work more closely and think more deeply about the design of a better box. This as well as a research component which asked students to make real world connections were added to the original activity. In the past, I have engaged students in the process of calculating the amount of cardboard used to make a box. The first part of the activity, or the content portion, is where I have traditionally stopped, however. Thinking about fostering the emergence of a community of practice characterized by mathematical inquiry simultaneously, or parallel planning, enabled me to extend the activity to include things like discussing mathematical ideas, proposing and defending strategies, and making real world connections.

After planning Thinking Outside the Box, I decided that coming up with a few activities like this might be worthwhile, even though I did want my experiences with students to guide future planning. One of these activities was entitled Mystifying Measurement Markings (see Appendix E). The idea for the activity came from my own curiosity about some of the measurement markings I see in the world around me. For example, I was always curious about the numbers on a car tire as well as what the

numbers meant on electrical wire. I decided that there are many things like this in the real world that are measurements. So I collected several items with such markings on them and asked students to figure out what the markings meant in an activity resembling a scavenger hunt. I hoped that by modelling curiosity I could inspire students to notice mathematics in the world around them and begin to ask some of their own questions. I anticipated that students might talk about some of the markings they had seen at home or in other places and that the activity might spark some interesting conversation. As far as content was concerned, this activity required students to use rulers, calipers, micrometers, and other measurement devices to see if they could solve the problem of figuring out what the numbers meant.

As far as developing a community of practice characterized by mathematical inquiry was concerned, however, the activity provided much more depth. Students would, naturally, work collaboratively in a hands-on manner. They would be busy physically moving around the room and discussing with their partners their ideas and what strategies they should try next. They would be drawn to ask questions about the nature of the numbers they saw and would be using real world items to learn with. I hoped that the competitive time frame might encourage students to work as a team with their partners and I would be able again to refuse to be a validator, further reinforcing the expectation that students did not come to me to find out if they were right. *Mystifying Measurement Markings*, although a simple one-day activity, was one that achieved many purposes in terms of both content and community development.

Another activity that I planned for the measurement unit was an activity entitled *Grandpa's Tool Shed* (see Appendix D). For this activity students would be given a

measurement tool, asked to learn how it worked, and then teach the class how to use it. Tools such as a carpenter's square, a spark plug gapper, and a center finder would be given to partners for them to learn and teach the rest of the class how to use. Students would then be asked to find a measurement tool or gadget at home to bring and show to the class. This activity, like *Mystifying Measurement Markings*, was designed to try to inspire curiosity and a sense of inquiry in students. I hoped that students would look at their worlds and find places that measurement and measurement tools are used. Aside from the obvious content goal of having students work with measurement instruments, this activity would require students to work collaboratively in a hands-on manner and discuss mathematical ideas and strategies. They would have to take ownership for their task of teaching the rest of the class about how to use the instrument, as well as of thinking about and explaining the usefulness of the instrument in the real world.

Another activity that I planned prior to beginning the course was the 3D Geometry Research Assignment (see Appendix C). I struggled considerably, as I thought about how I would get to looking at the surface area and volume of cylinders, prisms, pyramids, cones, and spheres in meaningful and hands-on way. I thought about having students derive formulas through activities and worried about the amount of time that would be required to undergo such a process. There was a large number of formulas that would need to be derived, and I could not imagine how I would do this in a way that was not teacher led. I wondered if I could just derive some formulas and then give others to students, an idea which made me considerably uncomfortable, since I did not want to be giving any information to students. Finally, I decided to create an activity which required students to find the information that I would have given to them in the past on their own.

I would assign each of the five 3D objects to a group and ask them to find the information and present it in poster form. Groups would have to create a net and three dimensional model of the shape that matched. They would have to find any pertinent formulas for the shape as well as interesting facts about the shape that related to the real world. Finally, each group would have to design a problem that required the calculation of volume and surface area of the figure. Aside from content, my goals in this activity were numerous. First of all, students had to work collaboratively to create the final product, a poster. They had to design questions which required discussion and consensus. They had to rely on themselves and their peers for information rather than me. They also had to take responsibility for their shape, because their information was the information that the rest of the class was going to depend on for that shape. I particularly liked the last point, since it strongly supported the interconnectedness of community members and the importance of recognizing a common goal amongst community members.

The final activity that I planned prior to the start of the course was the Measurement Debate (see Appendix B). This activity required students to work in much larger groups to debate whether or not Canada should revert back to the Imperial measurement system. I liked several things about this activity. First of all, it required students to work as one small part of a much larger group on a larger task. Students were each assigned roles for which they had individual responsibility. In addition to this, they had to anticipate the arguments of the other side, and work collaboratively to neutralize these arguments. Rather than presenting the information to students, they had to find it themselves. I knew from past experience that students often would question why Canada does not use the same system as the United States, as the two systems came up in

measurement. In planning this activity, I hoped to spark some interest in something I had already found students to be curious about. This activity, while familiarizing students with both measurement systems and forcing them to compare them, perhaps even needing to look at some conversions, would require students to propose, clarify, and especially defend and refute mathematical ideas. It would naturally lend itself to real world application and discussion as students would be forced to look at practical strengths and weaknesses of one system over the other in the real world. I suspected students would draw on their own experiences at home or in agricultural settings to make their arguments. I liked the opportunity for students to connect measurement with their lives.

### **Beginning the Year**

As I met the class for the first time and we began working on Thinking Outside the Box, I began to see the planning unfold before my eyes. While some of the things I thought might happen did, many more things I had not thought of also happened. As I had anticipated, students discussed how they would approach the tasks and came to a consensus about which strategy to use. They compared their boxes and strategies with those they saw and heard as other groups worked openly and naturally. Many students asked me if their strategy was right and I was able to tell each group that I would not answer that question. It worked out very nicely that students immediately began to repeat this as others asked the question, reiterating that I would not tell them *how* to do it; they had to figure it out for themselves. In addition to the things that I had expected to see, two

very important things emerged that I had not expected to see. First of all, there was an energy about the room that I was not used to seeing. Students spoke with relatively loud voices and seemed excited when they came up with a strategy that would work. Many of the students seemed very engaged in the conversation and process of participating in the activity. However, I noticed one group in which one member was engaged in the task but where three other students were off task. This was the beginning of yet another issue for me that became a recurring theme: How do you get students to remain on task amidst the messiness of collaborative, hands-on tasks?

Some discomfort with the number of questions about how to do things and with a couple of students not really engaging in the task caused me to vary from what I had intended and planned. While students did find the area and volume of the box and the smallest size of piece from which the box could be cut, they struggled a lot with the prospect of designing a new box with a better design. When students got to this part of the assignment, it was pretty clear that this was stretching their ideas about what should happen in the math classroom. They started to ask a lot of questions and to complain about not knowing what to do for the assignment. I found myself getting rather frustrated and ended up cutting the activity short. I told students to plan the new box and that they would describe the changes and basic measurements the next class. When the next class came, we all went through the designs and discussed what was better about the design, but presentations were not really formal. I lowered my expectations and we really just ended up having a discussion. I also completely left out the research part of the assignment.

I was disturbed after the fact by these decisions for a few reasons. It bothered me that the students were able to push me off my intended plan first of all. I suppose that this is one way that students and teachers must negotiate the norms and practices of the community together. I was obviously pushing them beyond their own tolerance levels for open-endedness and creativity. They, in turn, pushed beyond my own tolerance levels for off task behaviour as well as neediness and questioning behaviour. In the end, the initial activity was good at the beginning, but the higher level thinking part was a bit too much too soon.

Another thing I did as I started the course and the Thinking Outside the Box activity was to have students journal about their prior experiences in math and what it meant to them to “do math”. We discussed how their experiences in this course might be somewhat different in that they would be doing a lot more hands-on and collaborative work than they were likely used to. I was able to explicitly address my expectations for behaviour in the classroom and brought up expectations like wanting them to try to figure out things for themselves and not just ask me how to do things. I explained that a lot of what I was looking for was in the conversations they had with me and with other students, rather than on paper. I explained that Applied Math is a course that would require them to truly *apply* mathematics to the real world, and that I hoped to see them asking questions and being curious about mathematical things they noticed in the world around them. I believe that this explicit discussion about what “doing math” would be in the course and what it would look like was essential to establishing a community of practice characterized by mathematical inquiry, even though the initial activity was challenging on both sides. It was obvious that we still had a distance to travel in order to

negotiate the norms of our classroom community. For the first time, I started to understand the practical meaning of the word negotiate. My expectations and the norms established were not the same thing. It was something that I would need to work on.

After Thinking Outside the Box and some introductory discussions about the nature of the course and the experiences they would be having in it, we began to look at the Imperial and Metric measurement systems. As I introduced briefly what each one was, students began to argue about which one was better. This is what I had anticipated. I was thankful that I had already planned the Measurement Debate and the result was that the task appeared to arise from their input and discussion, although I had planned the activity in advance. For a few days, I showed students how to use Imperial rulers, calipers and micrometers in small installments. They practiced with the instruments and showed me that they knew how to use them. At the same time, they worked for part of each period on preparing for the Measurement Debate.

As it turned out, I was moderately disappointed with the Measurement Debate as it unfolded in the classroom. I found it very difficult to get students to actually prepare for the debate. They seemed content to research, but very little was done to put it together as an argument. I kept having to remind students that they needed to have an eight minute speech and that they needed to come up with the best argument they could during that time. Students came across some interesting facts and information, but it really did not translate into a good argument. On the day of the actual debate, many of the arguments were weak and short. In addition to this, the attackers and closers did not really listen and respond to the arguments of the opposing group. After the debate, I thought about what went wrong with the activity. A few things did go right, though: students worked

somewhat collaboratively to contribute to a larger cause and students did encounter some very good information about both measurement systems. However, the norms and practices that would be required for students to really take responsibility and ownership for their part were not yet established. The activity was too much, too soon, and required previously established norms that just were not yet there. Perhaps later on in the year students would have been more focused and perhaps they would have felt a greater sense of responsibility for the learning of the others in the community.

### **Reverting Back to Old and Familiar Ways**

During the time that preparation for the debate took place, I found myself wanting to revert back to old ways of doing things. I did a little direct teaching for reading calipers and micrometers, as well as for looking at measurement conversions. Both of these procedural tasks were in the content to be covered in the curriculum, but neither one of them could really be taught without at least some direct instruction. It was also at this time that I was away for inservicing for the first time. I struggled with what to leave for the substitute teacher, since students were still finishing their preparation for the debate. I knew that leaving students the entire class to work on their debate materials was too long and elected to have them spend some time doing a traditional worksheet on conversion problems. When I returned, I was not prepared for what I encountered. Students were upset, angry, and frustrated by the worksheet. They didn't know how to do the questions and most of them shut down and quit while I was gone. This disturbed me for a few reasons. First of all, in my absence, students completely stopped relying on themselves

and their peers to attack problems. It was fine when I was in class refusing to tell them the answers, but in my absence, they completely shut down. I felt a certain amount of guilt turning to the worksheet in the first place. This was amplified by the students' reactions to the task. I felt as though I had gone against the norms and expectations they were used to in giving them the assignment. The result was toxic to the classroom community and students began exhibiting the very qualities I wanted to eliminate: refusal to think about a problem, blaming the teacher, looking for answers from the teacher, and loss of independence. While the assignment was not important to me as far as content was concerned, I became determined to really emphasize students becoming less reliant on me and more reliant on themselves and their peers. I especially wanted to use my own time away from the classroom to work on further solidifying the characteristics of independence and collaboration within the community of learners.

### **Using Problem Solving Partners to Foster Independence and Collaboration**

In response to the weaknesses I saw emerging in the community developing, I modified an old assignment that consisted of measurement problems to help students understand what independence and collaboration looked like for problems. The old assignment was one I would have traditionally used as a worksheet. In the past, I would have shown students a couple of examples and then asked them to do the sheet. Whatever students would not complete in class would be homework, and then we would have gone over the problems the next class. One of the reasons I wanted to change my teaching practice was due to the fact that this model for teaching was not working well for me. On the

assignment I just described, only a handful of students would really engage in the problems. Many of the weaker students would shut down and simply not do the assignment, or they would copy the answers off of someone else never really learning to approach problems and learn from mistakes. Most students were unable to do such problems at home since they did not have good problem solving skills to begin with. What I decided to do instead, was to have students work in what I called *problem solving partners*. These partners would do the assignment collaboratively. It was clearly stated up front that they were not able to look to me or to other groups for answers. I told them that I wanted to see some effort on the page; I wanted to see that they had decided to approach the problem with some strategy, right or wrong. They were not allowed to put down nothing. We, in fact, spent three classes on the assignment that I previously would have only spent one class on. At the end, students handed in problems to me and I commented on the thinking processes I saw on the paper. The feedback was constructive and reinforced the things I had asked to see. While the process was lengthy, I think the outcome was very good, both in terms of content and in terms of community development. The structure of problem solving partners and the slower pace helped students really attempt several problems and discuss things with someone else in the community. My impression was that much more mathematical thinking went into attempting the problems than I had experienced in the past when I approached these problems as a worksheet that was handed out and assigned for homework. The problems were the same, but the outcome was significantly different.

Following the problem solving assignment, I asked the same problem solving partners to create a measurement problem of their own. This activity ended up being

quite interesting. Its strongest advantage was that students of varying ability levels could all contribute in a variety of ways. Students with weaker conceptual understanding were able to model a problem after one they had already seen in the previous assignment. Stronger students, on the other hand, were encouraged to relate the concepts to something else in the real world that they could think of where conversions could be used. Students naturally were curious about the problems other partners were making and they talked openly about them. Students in other groups often offered advice to others as to how they could make their questions better. I also noticed another benefit. I was able to help a couple of really strong students think a little harder and go a little further by posing a few questions as I looked at their questions. For example, one student created a question that required a calculation to see if a person had enough cloth to make a set of curtains a certain size, given the dimensions of the piece of cloth. He brought the question to me and suggested that he felt it was too easy and that he wanted to go a little further. I suggested that perhaps using all centimetres and metres might be easier than combining units and requiring conversions across two different systems. He agreed. I also suggested that fabric is often sold in square yards. I asked him what else a person making curtains might want to do with the fabric. He indicated perhaps they would want to make tiebacks for the curtains. I agreed, and he went off to adjust and add to his problem. This question was a small assignment, but creating a question different from any he had seen before, required good conceptual understanding of area and conversions for the student. I ended up using this particular activity as an evaluative piece of work. It gave me fairly good feedback about where different students were in terms of understanding the concepts.

## **Struggling with Reporting**

At the end of October, with report cards looming, I elected to give the students a traditional test. I struggled considerably with how I would come up with a percentage grade for the students, especially with the reporting period coming to a close. Compared with other years teaching the course, this time I felt very prepared to discuss with parents their child's progress in the course. However, when it came to giving a percentage mark for the course, I was very worried. I only had a few assignments that actually had marks attached, and so I reverted back to the old and familiar test with a great deal of apprehension. Students, interestingly enough, did not seem phased at all by this, as they had regularly encountered tests throughout their school careers. Math had always included tests for them, and they did not show any indication that this would not be appropriate for the material and method of learning they had been experiencing. When the tests were marked, the marks looked similar to what I might have expected from my observations. A couple of students surprised me a little in that their test marks were quite a bit lower than I thought they would be. I found myself wondering if all of the collaboration propped up some students' marks and quality of work or thought because students were capable of more when they collaborated, or if it was a matter of them sitting back while others completed the work. Of course this could also have been due to poor test-writing skills. I decided to keep an eye on these students and to look at how the weaker students participated a little more closely as the year went on. In the end, the traditional test did give me a number that I could compare with other smaller assignments and left me feeling a little more confident in the grades I was recording on report cards,

despite the sense of discomfort it gave me for not building the sense of community I hoped for.

### **Discovering the Importance of my Role as Prompter**

The next activity that students engaged in after the problem solving partners and the traditional test was the 3D Geometry research assignment I had planned in the summer. Several things came out of this activity. The end products, posters, were really mediocre in terms of quality. I think this was due to several things. During the middle of November, being sick and run down, took its toll on me as a teacher. I found it very difficult to be “on” each day and I was not able to perform my role as teacher as well as I would have planned or as well as I would have liked. As a result, I did not go around asking students questions and prompting them to dive a little deeper into their work. This confirmed for me the importance of the role of the teacher in the classroom. The teacher needs to be a catalyst for students, encouraging them to explore ideas and put forth that extra little bit of effort on things. When the teacher is unable to play the role of prompter, the learning suffers. Students still learned about 3D solids, and they still created the final project; however, the quality of thinking and of the final products was not as high as I feel it might otherwise have been if I had been able to be more active as a prompter.

As I looked at the final posters and graded them, I thought a lot about my role as a teacher and how poor health had severely hampered my performance. I thought about what was lacking in the assignment and was thinking about how the students were not making the connections between ideas on their own. As I thought about this, I thought

about one connection in particular. I wondered if students would be able to see that a cylinder is really just a prism whose base has an infinite number of sides. I wondered if they would be able to understand that a cone is just a pyramid whose base has an infinite number of sides. As I thought about this, I decided to go back and strengthen what students had already done. I decided to ask them the questions: Is a cylinder a prism? Is a cone a pyramid? In essence, I refocused on my role as prompter and looked at a prompt or question that would help me help students to make some of the connections between concepts that I knew they could make with some prompting. As I refocused on my role, I had an interesting conversation with a student I will call Damian that I recounted in my own observation notes as follows:

One student's comments took me by surprise. He was sort of thinking out loud and looking for input from me about whether he is on the right track. I could only think of Jo Boaler's account of Phoenix Park when he was asking and I really wanted to only tell him it was an interesting line of thought. He said, by definition, a cylinder has congruent, parallel polygon ends with sides that are parallelograms. His first reaction was that therefore a cylinder was not a prism since the ends were circles not polygons. Hmmm, I said. Are you sure a circle is not a polygon? About 10 minutes later, he said to me that a polygon has lines for sides and therefore a circle is not a polygon (once he looked it up). I said to him – how many sides does a polygon have? He said poly means many, an impressive answer for a grade ten student I thought. So I said in response, what does a 5-sided polygon look like? He showed me with his hands. I asked what does a 6-sided polygon look like? Again he showed me. A seven-sided polygon – he thought but did not show me with his hands. An eight-sided one? He thought quietly. Finally, I said what about a hundred-sided polygon or a thousand-sided polygon. He thought. Suddenly he sat up in his chair, smiled and said . . . “like a circle!” I told him well that was something to think about and I left. (Planning and Observation Journal, 2008, November 26)

Even though I was disappointed with the quality of the posters, the chance to refocus students in a timely manner and help draw some connections between ideas was invaluable. Even from the planning stage, I found it difficult to approach this topic in an

experiential way. After this act of refocusing and prompting, I came to the realization that perhaps it isn't only the activity that one engages students in, but also, even more importantly, the participation and role of the teacher that determines the quality of learning and depth of understanding that is developed within a community of learners.

The written products of students as they grappled with the question were also slightly disappointing. Even Damian, the student mentioned above, had difficulty putting into words what he had discovered. Students were able to discuss the connections between the shapes much more easily than they were able to write about them. This was likely due, in part, to lack of practice. This was really the first time I asked them to create a written argument to support their points, which they had difficulty with even on a second attempt. If writing to support their ideas was something they had more experience with, I think the results might have been better.

### **A Return to Problem Solving Partners**

The problems that were generated during the poster assignment created an interesting bit of practice for students to engage in with a problem solving partner. Again, the problem solving partners successfully looked at many problems, and students were guided to attempt every question using guidance from within their partnership. They were not allowed to ask me or others for help, and I wanted to see evidence of them grappling with the problems. When we looked at the problems afterward as a whole class, we discussed what made them good problems or not so good problems, and students seemed much more connected to the problems in the past due to the fact that they were made by

members of their classroom community. Having students create questions and solve questions made by other students in their class was, in my opinion, a successful strategy and one well worth the time it took to complete.

### **Time, Energy and Logistics Get in the Way**

Another activity that students were engaged in before the Christmas break, was Grandpa's Tool Shed (see Appendix D). This activity was a bit rough around the edges due to two main factors. The tools chosen had varying levels of complexity, which made it a bit awkward. One of the brightest students in the class chose, unknowingly, one of the easiest instruments to use, while several of the weaker students chose more difficult instruments. In hindsight, the activity would have been much better if I had assigned the tools so that the difficulty levels matched student ability a bit better. Students presented their instruments to the class and all students managed to explain how to use their tools satisfactorily. I was forced to abandon the part where students were to bring in a tool from home due to our focus on the prism and cylinder questions and the fast approaching Christmas Break. This was the second major omission from a planned activity that occurred for me during the term. Logistics got in the way of good teaching and learning, somewhat. Again, the extension part of the assignment was what got left out. I chose to leave out the part that asked them to make applications to the real world. I found myself wondering why it is that I make that decision over and over. I suppose this is the hardest part to do. It is the most open and ill-defined for students and for me and subconsciously I have tended to shy away from such things given the chance. This tendency would prove

to be yet another emerging theme for me as a teacher struggling to integrate theory and practice.

At the end of the term leading up to the break, I elected to give students a formal test once again. This time, due to time constraints, I decided to give them a take-home test so that they would be able to think about the problems more and so that they would not be rushed. Intuitively, I knew that I needed to gather marks for the fast-approaching report cards coming up, but I also knew that it did not match my feeling about how I should be coming up with grades for students. Time did not permit for me to create a hands-on assessment form and my own energy levels were depleted. Students performed well on the assessment. They may have helped each other outside of class. I knew that the assessment really may not be that valid, but I also knew that it was not really any more invalid than working on problems in class. Grading, again, became a burden and a troublesome task, despite the fact that I felt very confident that I knew how students were progressing in anecdotal terms. Assigning the percentage proved to be the most difficult part.

### **Christmas Break – A Chance to Refocus**

Over the Christmas Break, I revisited the idea of having students create a plan to landscape the school yard. I revamped the idea - due to weather constraints - into the Student Lounge Project (see appendix F). In this project, students were asked to prepare a proposal for taking their existing classroom and turning it into a student lounge for the students in their school, given a \$10,000 budget. They had to include a scale drawing of

the empty room after measuring it, as well as a final scale drawing with all attributes of the new lounge included. They had to build a three dimensional scale model of their design, as well as create a cost analysis outlining all costs for creating the lounge. The sketches, model, cost analysis and other information was to be presented formally to the class as well as the principal and other invited guests. While the project took over a month to complete, it required a good understanding of scale factor and conversions. What I liked about the idea was its real world connections, as renovations are likely something most students will come across in their lives. As far as creating a community of practice characterized by mathematical inquiry, I felt that working with partners to create and present a design fostered both individual and collective ownership of their learning. Students would need to collaborate on the task and rely on themselves as well as their partner to complete it. Because the students were required to do two scale drawings as well as a scale model, I anticipated that students would need to choose appropriate scales as well as use them and that discussions about this would ensue between partners. I felt that students may learn the importance of choosing a scale wisely as they figured out how to fit their drawing on one page. I also hoped that in trying to come to a consensus about the design of the room, cost and measurements relating to costs might cause students to discuss mathematical strategies and ideas. Finally, I really liked the idea of having students present their plan to the principal and our classroom community at the end of the project. I felt this would allow the class to discuss the strengths and weaknesses of different proposals as well as build on the students' sense of community as they shared in the achievements of other groups.

### **A Taste of Success – The Student Lounge Project**

As students worked on this project, I noticed several things. Students approached me very seldom. Most of the conversations were between partners as they discussed their plans. They were physically and mentally engaged in the process and interest was very high. Students truly seemed to enjoy the control and ownership they had over the project, and loved to share their ideas and designs with others. At one point, I watched as students were thoroughly engaged in creating their models. I was so amazed at what I saw that I grabbed a camera and took pictures. Even the students who had been difficult to keep on task were focused on the task and were working hard. For me, this was an inspiring moment. I recall thinking: “So, this is what doing math looks like!” The presentations at the end of the project were equally inspiring for me as a teacher. The students did an excellent job of the entire project and really went that extra step to put forth excellent presentations. They truly took ownership of their work, and there was an air of celebration as the final presentations were viewed. We discussed the things we liked about each design and students enjoyed looking at the final models. I even overheard students asking each other what scale they used and why. Presenting to the principal in the end proved to be a motivating factor for students. He was impressed by the effort and mathematical thinking that had gone into the projects, as was I. I was even more impressed by the accuracy of the three dimensional models. In the past I had done projects like this one, but I usually found one or two pairs that would have a model that was completely not to scale. In this class, none of the pairs were in this predicament. All

of the models were largely done to scale, and students applied scale correctly to most of the items in their model.

During the time students were working on the project, I found myself taking on the role of the prompter or catalyst again. I walked around asking different pairs questions about what they were doing, what scale they chose and why, and what was difficult or easy about using that scale. I recall one discussion in particular that I had with a student I will call Richard that was interesting:

I asked him what his scale was for his 3D model. He said it was 1 inch is 2 feet or 1:24. This intrigued me. He needed no help converting the scale to a ratio, which is often difficult for students. I decided to probe a little further to explore his understanding of scale. I asked him if he found this scale difficult to work with. He said "Yeah, a bit when you get decimals of an inch." I asked him what was difficult about it and he explained that at the moment, he was trying to draw in a door using the actual classroom door dimensions. I asked him how he decided how big it should be on the model. He described more or less this way: (paraphrased) "The door was 81 inches so I divided it by 24 to find out how many inches it should be on the model, which was 3.375. Then I knew that there were 3 and 375 thousandths of an inch which I needed to convert to sixteenths. I multiplied .375 by 16 and figured out that it was 6 sixteenths, which didn't seem right to me." When I asked him why it didn't seem right, he said that 6/16 seemed too large and then said "but I guess 8/16 is .5 so maybe 6/16 being .375 seems reasonable." When I asked him how he would use what he just described to create the door on his scale model, he described how he would then go to 3 inches and the 6<sup>th</sup> sixteenth line and mark that off as the height. I believe his ability to describe this fairly complex mathematical process of applying scale indicates excellent communication skills. I think that I was definitely witnessing the emergence of inquiry as he described his mathematical process. Another group listened as he did this and nodded. This was a valuable moment for me as a teacher to recognize understanding as well as a moment where knowledge was shared with other members of the community. (Planning and Observation Journal, 2009, January 20)

The end of this entry indicates one of the key things that came out of this assignment for me as an educator. I saw before me how the establishment of a community of practice characterized by mathematical inquiry can lead to situations where students not only

inquire about math themselves, but where they witness others who are doing the same things as them. They are able to see questions being asked and solved all around them. It becomes the norm and students' understanding of mathematics evolves. Had this student not engaged in this hands-on assignment and chosen a scale for himself, had I not asked questions of him that required him to think through the process of using scale and converting measures, had he not worked through this in his head, this student would not have created the depth of understanding he did in dealing with fractional and decimal measurements and scales at this point in time. Perhaps more importantly, the students who were watching and learning from his explanation would also not have had the experience they did. To me, this was an illuminative moment.

### **Getting Students to Talk Strategy**

Following the Student Lounge Project, I next prepared to engage students with trigonometry. As I began planning for the project, I wondered how I would get students working in a hands-on manner to investigate and learn about trigonometry. After considerable thought, I developed a Trigonometry Challenge (see Appendix G) to introduce the new topic to students and get them actively involved. The Challenge had two parts. The first part required students to find the height of the school flag pole and school gymnasium using only a metre stick, a clinometer, a piece of string, and a tape measure. Students were given no direction and were told that they would have to share the strategy they used later. As far as content was concerned, I was interested in developing the students' understanding of basic trigonometry as well as the notion of the

trigonometry functions as ratios. In developing a community of practice characterized by mathematical inquiry, I hoped to focus on the proposing, clarifying, defending, and refuting of mathematical strategies. Although this activity required groups of about four students to collaborate, come up with a strategy and then use it to solve a problem, it was the ability of students to describe their strategies and evaluate their effectiveness that I wanted to emphasize. I also wanted to work on students validating the ideas and strategies of other students in the process, as well as developing metacognitive awareness in individual students.

Once students had approached the problem and come up with a solution, I had the students write out as clearly as they could what they did on the board. We had a whole class discussion about what each group did, and I asked the groups to clarify any unclear parts of their work. Once we had looked at all the work, we talked about the similarities and differences between strategies as well as the strengths and weaknesses of the different strategies. At the end of the assignment, students responded to journal questions asking them individually to identify and evaluate their group's work on the challenge (see Appendix G).

When we looked at the strategies, we noticed some similarities as well as some differences. Three of the groups measured the angle between the line of sight of an observer to the top of the pole and horizontal. They then measured the observer's distance from the flag pole and drew a right triangle from the observer's eye, to the flagpole at eye level, and to the top of the flagpole. This allowed them to use the trigonometric ratios they learned in grade nine to solve the problem. Recognizing the similar strategy in the work of the three groups was not easy, however. Each group took

different measurements and drew their right triangle differently, and students really needed to look hard to recognize that a similar strategy was in fact used. The discussion surrounding this was intriguing. Some students explained to others that what they did was similar to what another group did, but that they just didn't draw in the observer, or that they drew it the other way around. Also, even though the strategy lent itself to using tangent to solve the problem, one group used cosine first to find the hypotenuse between the top of the flagpole and the observer's eye. The work looked different, but they really ended up using a similar strategy – trigonometric ratios. I was very pleased with the fact that there was no correct answer, and that students compared their answers to the answers of others to see how close they were. This, for me, helped establish the notion of peers validating each other's work.

What made this activity even more interesting than the discussion about how three groups had used a similar strategy was the contrast of the one group that did it differently. The fourth group used a scale drawing to calculate the pole height. They took the measurements and drew the triangle, using scale to solve for the height of the pole. Even though they still used a right triangle and their drawing looked similar, the group was able to measure the sides of their scale drawing and estimate the height of the flagpole using their scale. As the group explained their strategy, the other groups were surprised and interested. Several of the other students nodded and expressed, physically and verbally, their approval of this as a valid method. I thought this was a powerful moment. The similarities and differences in strategies made this a great activity to work on not only right angle trigonometry but also on proposing, clarifying, defending, and refuting mathematical strategies.

The second part of the Trigonometry Challenge required students to perform the same task, this time without the clinometer. This activity hit on a Friday afternoon, last period. At first I thought the activity was a bust. Students were off task and difficult to focus. They goofed around and generally did not appear to be doing what they were supposed to. Most of the groups managed to pull something together towards the end of class, although I was certain that Tuesday's discussion would be disastrous. In fact, the discussion on Tuesday turned out to be just fine. Again we looked at each group's work on the board and discussed the strategies used. All of the groups used a small triangle using either a person or a meter stick as the vertical side, nested within the larger right triangle that had the flag pole as its vertical side. Some groups eyeballed the diagonal hypotenuse. One or two used string to help focus a line of sight to the top of the flagpole. It was nice that one group decided to create a triangle with a meter stick so that the "viewer" would not have to be on the ground. They were the only group to create a smaller triangle without using a person's height. As we discussed the similar, yet slightly different methods of finding a solution, it became clear to me once again that it was the discussion about mathematical strategy that made this such a great activity. Students had to look hard and understand trigonometric concepts to see the similarities between strategies. One group in particular had the rest of the class fooled. They, rather than using tangent to solve the problem, used the Pythagorean Theorem to find the missing side of the triangle and then set up cosine to find the angle. It appeared to be a very different approach, but yet the strategy was similar in how the problem was set up. The group just used the trigonometric ratios differently to get to the same place. Because the group did not show their work clearly, it took even more thought before a couple of students

identified what work had been done. These students were not in the group, but they recognized what the strategy was and explained it. A couple of other students verified the work on their calculators and indicated that this was in fact what the group did. This happened spontaneously without much being said by me at all, which I felt was a testament to norms and practices being more solidly established about students discussing and validating each other's work. Students worked together in the discussion to analyze the work and evaluate its appropriateness. There was a common purpose that was understood, and the characteristics of a community of practice characterized by mathematical inquiry emerged.

### **Reverting Back to Old and Familiar Ways Again**

After the initial trigonometry challenges, I went through another dark time as I found myself again exhausted, out of planned material, and unsure as to how I was going to get to the sine and cosine laws without direct teaching. I attempted to have students use right angle trigonometry to solve sine law situations, but only a few students were able to get to the level of solving those situations on their own. I ended up having to show students and then tried to drag them through the steps using letters instead of real numbers, coming up with the sine law formula. This process was painful and students were not very engaged. The remainder of the trigonometry unit also saw me reverting back to the old and familiar ways of direct teaching and problem sheets. However, I did use problem solving partners as we looked at some of the problems and asked students to continue to take responsibility for their own learning and that of their partners. At the end of the unit,

I used a traditional review and test and felt very guilty for getting dragged back into the old and familiar.

### **Spring Break – Another Chance to Refocus**

Spring Break allowed me to refocus my efforts on developing a community of practice characterized by mathematical inquiry, and I returned with activities and ideas as I approached both 20S Applied units: Linear Models and Patterns and Relations and Functions simultaneously. When planning the unit over the break, I again made a chart depicting the activities as well as my goals for fostering the emergence of a community of practice characterized by mathematical inquiry and my content goals. The chart looked somewhat different, however (see Appendix M). The focus of my planning was clearly on the goals for creating a community of practice characterized by mathematical inquiry.

### **Using Old Activities Differently for Different Purposes**

During this last part of the school year, students were engaged in a variety of activities requiring them to collect data, graph it, and use technology to find the equation of a line of best fit. I was able to use several activities I had used in the past, because they were already hands-on in nature. The activities at the beginning of the unit were very hands-on but not open in nature. This was necessary, because students needed to first practice using technology to find a line of best fit before I could ask them to apply this strategy to a novel situation. For example, one of these activities, entitled The Ball Bounce Activity

(see Appendix H) required students to collect data by dropping a ball from different heights and recording its bounce height. During this activity, students became familiar with some of the terminology of linear functions such as interpolation, extrapolation, discrete, and continuous, as well as determining a line of best fit. Following the activity, we discussed how bouncy each group's ball was and how we could use the slope to determine this.

Following the Ball Bounce Activity, the students participated in a few more activities that required them to use their calculators to find a line of best fit. These structured activities were hands-on, but guided for the most part. My goals in these activities were quite content-based due to the necessary skills that the students needed to develop. However, something that changed for me dramatically in planning for this portion of the year, was that I added in a few other things to directly get at developing the community of practice that I desired. For example, during the Cricket Chirps Activity, I asked students to be responsible for their partner's understanding of the calculator work and to advocate for themselves. This focused students on the community aspect of learning *while* we looked at the technical parts of using the graphing calculator. After The Wave activity, I asked students to create a study sheet outlining the steps to finding a linear regression equation on a graphing calculator and then share it with a friend. The job of the friend was to evaluate the list of steps and help verify that nothing was incorrect or was missed. When I had to be away for a class, I left a practice worksheet for students, but focused them on being reliant on their peers in my absence. Before I left, I instructed students to ask a classmate when they were having difficulty with a question and that I would be asking them to tell me about a time they had to do this when I

returned. I emphasized the notion of community and of their responsibilities to help others in the room when they had difficulties. I encouraged them to be validators for each other and asked them about how this went in their journals once I returned. After reading their journal responses, I noted in my own Planning and Observation Journal:

While I was away, I asked students to help each other out on a practice sheet. They were practicing entering data into their calculators and finding the linear regression equation on the calculator. Then they had to use a line of best fit to interpolate/extrapolate. This is quite technical since there are a lot of steps. Since I knew I would be away, I decided the practice was needed and it would also give me an opportunity to work on the students' sense of independence. When they returned, I didn't have a chance due to interruptions etc. but today I asked them to comment in their journals about how they worked and helped each other in my absence. I asked them if they were more confident relying on themselves and their peers than they were at the beginning of the course. Only one person felt they were already good at that so they didn't think they had gotten better. Several other students felt:

- They were definitely more confident
- They asked me for help less and depended on themselves more and their peers
- A couple of students noted that the capabilities of all those in the room were great and that someone always seemed to be able to help

I believe that I saw in their responses a portrait of what the community that has been established looks like from a student perspective. I was amazed at their honesty and candor in the journals. Responding was a great joy to me. This is the first time I truly feel as though I can SEE the community emerge and I can SEE their understanding of the norms and practices of that community. Students know that they are expected to help each other. They know that I will not just tell them answers. They know that they need to try before asking. These have now become established norms. (Planning and Observation Journal, 2009, May 13)

### **Adding More Open-Endedness to Learning Tasks**

After a few activities that allowed students to practice skills with creating a line of best fit and finding its equation, I engaged students in a much more open task entitled DaVinci's Proportions (see Appendix I). In this activity, students were asked to research Leonardo

DaVinci's proportions based on The Vitruvian Man and to prove or disprove one of them using linear models. Although students were given the general topic, how they chose to approach the problem was up to them. More importantly, they were forced to collect and interpret their own data for their own purpose. Students worked in partners on this assignment and created PowerPoint presentations to present as they argued to prove or disprove the proportion they were investigating.

This more open-ended task once again excited students. They seemed to enjoy choosing a proportion as well as the fact that the activity involved taking measurements. In general, students began to refer to "their proportion", indicating ownership of the proportion they chose and of their work. Students struggled somewhat as they tried to interpret the data they took, but it was in these moments of struggling that I saw inquiry and mathematical thinking emerge. Students often spoke to a member of another group, asking them if they thought the line of best fit was close enough to the data or not. They bounced ideas off of each other and tried to help each other make sense of the data. I also had the opportunity to help students think things through by asking questions, as is evident in one of my entries in my planning and observation journal about a conversation I had with a student I will call Elizabeth:

One conversation sticks out in my mind during their work on this project. One student had graphed the relationship between height and face length. The property was that the length of the head, chin to top, should be one seven and a halfth the their height, including the head – that is that the height including the head should be seven and a half times the length of the head. She had created a linear model and we were discussing the equation. She was getting a big number and couldn't figure out why or what that meant because it was 2 point something and because it didn't appear to make sense. I asked her what was in  $L_1$  and  $L_2$  and asked her then what the slope might represent. She was able to tell me that the slope would be the change in height over the change in head length. I asked her if I was an inch taller, how much might she expect my face to be taller?

She said she wasn't sure. As we spoke we started referring to that change as one seven and a halfth of an inch. I asked her if my face was an inch longer than hers, how much taller did she think I would be. She said seven and a half inches taller. I said well, if your model is giving you the change in height over change in face length as 2 point something, what does that tell you. She said it isn't close to either number ( $1/7.5$  or  $7.5$ ). I agreed. I said what should be the difference in height for every inch or centimeter difference in face length? She said 7.5. I said "Okay, now go prove or disprove the theory using your data." What a great conversation that truly helped a student understand slope as a rate of change. I was very happy. Again my role as a prompter came out and her learning emerged through our interaction. (Planning and Observation Journal, 2009, May 13)

The DaVinci Project, in the same way as the Student Lounge Project, provided a powerful moment at the end as students presented their arguments to the class and we discussed them together. Students seemed genuinely interested in the work of other groups and proud of their own work as the presentations occurred. The air of celebration returned once again, and students openly discussed and critiqued arguments, looking at how they and others could make them stronger. I was very pleased with how the activity turned out. It ended up being a nice fusion of measurement, proof and logic, technology use, linear models, collaboration, and building a community of practice.

### **Moving Towards Performance Tasks for Assessment**

The final activity in the combined Linear Models and Patterns/Relations and Functions unit was a performance task (see Appendix J). I decided that since students had done so much work with linear models in a hands-on manner, that I would try something different than a traditional test, which I had been frustrated with all year. I knew that the evaluation and grading I was using did not match the norms and practices established within the classroom community, and I managed to put together a performance task that would

allow students to *show* me what they knew about linear models. The performance task required students to find something in the real world that had a linear relationship and model it using linear models. They were asked to show how the model could be used to make predictions and were given complete freedom to choose their own topic and method of presenting their information. As I assigned the performance task, I worried about whether or not students would be able to handle the open nature of the assignment as well as what kind of quality their work might demonstrate, given that they had never been evaluated in this way before.

As it turned out, my fears were unfounded. The students knew that the performance task was essentially their “test” and the word “test” still resonated with importance for students. They put forth excellent effort on the task and I was pleasantly surprised at the products that were presented. As with the DaVinci project, students were very curious about the work of others and openly discussed the things they were looking at. This task was completed individually, because it was an evaluative task, which was different for students since they had worked in pairs and groups all year to complete most activities. Students looked at a variety of relationships. Some represented linear data and some did not, although students were not able to tell at the beginning that the data would come out as non-linear. I modified my expectations a little, allowing for this problem in the marking. Some students simply chose something and went with it but were unlucky enough to have picked non-linear data.

At the end of the performance task, I had students go through self and peer evaluations of the final product before I evaluated it (see forms in Appendix P). The students were relatively good judges of the quality of the work, and many students gave

good feedback in their evaluations. I wanted to include this portion of the task, because I thought that it might encourage students to think critically about their own work and the work of others. I also liked the notion of asking for this critical feedback from peers so that I was not seen as the only validator in the activity. Although the peer and self evaluation could have been set up better and students might have been a bit better at it if they had had more practice, I was pleased with at least a few of the self and peer evaluations. This sort of activity takes a little more practice for students to be good at it, and doing it for the first time at the end of the course did not really allow for this.

### **Concluding and Reminiscing at the Year's End**

As the course was coming to a close, I concluded my formal data collection with summative questions in students' interactive journals on June 9, 2009. Table 5.1 (see p. 95) is a summary of the order in which activities were used with students.

The remainder of the year, which was only about four classes, was spent looking at isolated content pieces that did not fit in with the rest of the Linear Models and Patterns/Relations and Functions unit. As students responded to these last few questions, I noticed a sense of closure and celebration in the room once again. As the students were writing and responding to questions like "What was your favorite activity", they reminisced about some of the things they had done over the year. This emphasized the lasting nature of concrete, hands-on experiences for students. The things that they remembered were things like the Measurement Debate, the DaVinci project, the Trigonometry Challenge, the Student Lounge Project, and Grandpa's Tool Shed. The

active, hands-on activities were the ones that resonated in their minds, more than worksheets, reviews, tests, or even problem solving partners. As we talked about the activities they had participated in, I also asked them to journal about what it meant to do math in our classroom and what that looked like. I asked them if their views about what it meant to do math had changed since the beginning of the year, and students gave fairly thoughtful responses to all of my questions. Something I had not expected, when I began the project, was for such a strong bond to develop between me and the students in the class. The activities, collaboration and shared experiences helped form a sense of community within the classroom that was clearly demonstrated in moments where students were able to celebrate their accomplishments. This was a surprising but rewarding outcome for me.

Table 5.1

*Timeline for Activities*

Timeline	20S Applied Topics	Activities
September – January	Measurement 2D/3D Geometry	Thinking Outside the Box (see Appendix A) Measurement Debate (see Appendix B) Problem Solving Partners 3D Geometry Research Assignment (see Appendix C) Grandpa’s Tool Shed (see Appendix D) Mystifying Measurement Markings (see Appendix E) Student Lounge (see Appendix F)
February – March	Trigonometry	Trig Challenges (see Appendix G)
April – June	Linear Models and Patterns Relations and Functions	Ball Bounce Activity (see Appendix H) Cricket Chirps The Wave Height/Armspan Activity DaVinci Proportions (see Appendix I) Performance Task (see Appendix J)

## Chapter 6

### *My Story of Researching*

Although it comes after *My Story of Planning and Teaching* in this thesis, research began at the same time as my planning and teaching did in July. I placed *My Story of Researching* in chapter 6 after *My Story of Planning and Teaching* because of the nature of the autoethnographic methods described in chapter 4. I felt it was important to look at the narrative in chapter 5 before I discussed how my data was analyzed because *My Story of Planning and Teaching* served two purposes: sharing my lived experiences with the reader of this thesis and objectifying that lived experience for analysis. In order to describe how my teaching practice changed, or how I fostered the emergence of a community of practice characterized by mathematical inquiry, I first had to mould my lived experiences into an object that could be examined. Writing *My Story of Planning and Teaching* prior to beginning *My Story of Researching* allowed this to happen.

#### **Data Collection**

Data collection began in July as I began the planning stage of this research project. The data took on three main forms: entries in my own planning and observation journal, entries in interactive student journals, and student work. All three forms of data were used to inform future teaching practice, and as such, were analyzed during the research process in addition to being analyzed after the process.

### *Planning and Observation Journal*

Beginning in July, and continuing until the end of the research process in June, I maintained a planning and observation journal that allowed me to record my thoughts as I planned as well as my observations as my plans unfolded within the classroom. My original idea was to separate these two sets of notes; however, soon into the research project, I realized that because of the strong influence of my observations on future planning, it was more beneficial to keep notes about my observations as well as future planning considerations in the same journal. In the journal, I recorded my struggles as I grappled with conceptualizing activities that would engage students in learning about mathematics as well as my reasons for making decisions about what would happen in the classroom. As I observed my plans unfolding in the classroom, I recorded my observations and thoughts about what went well, what did not, and what I would do next time. I also recorded things I noticed about student behavior and the development of a classroom community characterized by mathematical inquiry. The journal allowed me to record my own thoughts and, perhaps just as important, my own memories of conversations and key moments that occurred within the classroom from my perspective. This journal provided the basis for the narrative writing in the previous chapter, allowing me to characterize my journey as an educator attempting to integrate educational theory about communities of practice into her teaching. This narrative account, in addition to the planning and observation journal, was also used as a form of data for analysis at the end of the study.

### *Interactive Journals*

Throughout the school year, I used interactive journals for a variety of purposes, including data collection. These journals each began with a prompt, which directed students towards discussion of a particular question or topic. Students then responded in writing and handed the journals in to me. In turn, I read their responses and commented in the journals, sometimes asking for further clarification, commenting on their entries, or asking more questions to help extend their thinking. This process continued as the year went on, allowing for ongoing discussion to take place on a more personal level with students. At the end of the school year, these journals were kept as a form of data for this research project. Table 6.1 includes the first five journal entries used during the study (for a complete list of all journal entries, see Appendix Q):

Table 6.1

#### *Interactive Journal Prompts*

Date	Interactive Journal Prompts
Sept. 5, 2008	<ul style="list-style-type: none"> <li>• What is mathematics to you?</li> <li>• What does “doing mathematics” look like at school?</li> <li>• What does “doing mathematics” look like outside of school?</li> <li>• Can you think of a time that you used math outside of school?</li> </ul>
Sept. 30, 2008	<ul style="list-style-type: none"> <li>• List 5 points made by each of the sides (Pro and Con) in the measurement debate.</li> <li>• Who do you think won the debate?</li> <li>• What do you think Canada should do? Explain.</li> <li>• What did you learn during this activity?</li> </ul>
Oct. 16, 2008	<ul style="list-style-type: none"> <li>• Where do you think you might need to convert measurements in real life?</li> <li>• What strategy (eg. Convert, online, unit ratios) would you use the most? Why?</li> </ul>
Oct. 24, 2008	<ul style="list-style-type: none"> <li>• What part of the questions/problems was “doing math”?</li> <li>• What part wasn’t?</li> <li>• Did you like working with a partner? Why or why not?</li> <li>• What do you think you learned?</li> </ul>
Nov. 27, 2008	<ul style="list-style-type: none"> <li>• I asked verbally, now that we have discussed this again, would you change anything about your previous answer? What would you change and why?</li> </ul>

### ***Student Work***

In addition to collecting the interactive journals from students at the end of the course, I also collected pieces of work students had completed throughout the year. Some of these pieces of work were final products of larger activities such as the Student Lounge Project and the DaVinci Project. Other pieces included reflective writing pieces such as the one in which I asked students to argue, in writing, whether a cylinder is a prism and whether a cone is a pyramid. I also collected some rough work as students worked in problem solving partners as well as the original piece of paper on which they created their measurement conversion problems. Finally, all of the performance tasks as well as student reflections about the task were collected at the end of the course. These artifacts were collected, because they contained some evidence of the characteristics of a community of practice I hoped to see emerge. They provided tangible evidence of students inquiring about mathematics as well as the development of a community of practice within our classroom.

### **Data Analysis and Interpretation**

The data collected for this research project was analyzed in several ways. I highlighted and colour coded my planning and observation journal as I attempted to identify common themes as well as evidence of the emergence of a community of practice characterized by mathematical inquiry. This allowed me to later look through the journal and draw together common threads of evidence to formulate an overall analysis of emergent concepts and themes. In addition to this, I noted any particularly illuminative moments or

experiences that I felt were important ideas for inclusion in my analysis. For example, I made note of interesting conversations I had with students which I felt I could later draw on to illustrate the emergence of a particular characteristic within the classroom community. An example of a highlighting key and point form notes about journal entries is included in Appendix N in order to demonstrate the this preliminary stage of analysis. In Appendix N, a key is shown illustrating how several themes were highlighted and color coded. Several point form notes appear in Appendix N as well, demonstrating further how illuminative moments were marked for further analysis at a later date and for use as examples in the writing of this thesis.

In addition to identifying illuminative moments and emerging themes in my journal, I also constructed a narrative account or outline of the year's events, which is included in chapter 5. This narrative was used, in conjunction with the other forms of data, to construct themes that emerged from the data in order to answer my research questions.

Student journals were analyzed primarily in two ways: first, by flagging particularly inciteful examples of student thinking or the emergence of characteristics of a community of practice characterized by mathematical inquiry, and, second, by summarizing student responses to some of the prompts. Flagging inciteful examples allowed me to maintain the integrity of the text and yet still identify student writing that illustrated the emergence of mathematical thinking, for example. These flags were later used to identify student writing that supported the statements made in the thesis. Summaries of student responses were used to identify overall trends in student responses as well as to note entries of particular interest. Appendix O has been included to provide

an example of how the summaries were done. Overall comments were sometimes made about the entries in addition to direct quotations from student journals indicating points of interest pertaining to the development of a community of practice characterized by mathematical inquiry. These summaries as well as the flagged or copied direct journal entries were used to identify characteristics of the community, to establish the opinions and expectations of students, and to evaluate the effectiveness of the research design.

The first and last interactive journal entries students engaged in (see Appendix Q) were designed to investigate student beliefs about what it means to do mathematics as well as to get feedback on student perceptions of the community of which they were part. The final entry in June was analyzed much more in depth than many of the others because of this. As the last part of this entry, students were asked to identify five characteristics of the classroom community they had participated in. The characteristics identified by students were listed, categorized, and a tally was created in order to talk about the frequency of students bringing up particular characteristics of the community. The results of this tally are included in Table 8.2 (in chapter 8).

Student work was analyzed by flagging particular examples of work that demonstrated the emergence of the characteristics of a community of practice characterized by mathematical inquiry. Not much student work was included in this document, with the exception of two pictures of Student Lounge Projects in chapter 8. The work was used, however, to describe student interaction and engagement in the learning activities that were constructed as part of this study. One group of data, however, included under the heading of student work that was drawn from quite extensively was the self/peer/teacher evaluation sheet and accompanying feedback questions after the

performance task. Appendix P includes a copy of the blank form students filled in. This feedback form provided valuable information about student perceptions regarding completing performance tasks instead of traditional tests. While the questions could have been asked through their Interactive Journals, I felt it was best to have students provide feedback at the same time they were doing self and peer evaluations of the project. This form, as a result was completed on separate paper and included in the folders of student work for each student, along with their performance task work. The feedback for this particular task was tallied for student opinions about whether they liked performance tasks or traditional tests better, and illuminative examples of responses were flagged to be used in the thesis.

Interpreting the data and writing the thesis required triangulation of data from all sources. As I looked at my own journal and story of planning and teaching, the interactive journals collected, and the pieces of student work, I began to interpret meaning with respect to my research questions. The remainder of this thesis is devoted to the presentation of those interpretations, including answers to the research questions posed. Chapter 7 uses the data to characterize how my teaching practice changed as a result of viewing learning as complex participation in a community of practice characterized by mathematical inquiry. Chapter 8 discusses the characteristics of a community of practice characterized by mathematical inquiry, how these characteristics were fostered, and what such a community of practice might look like from the outside looking in. Chapter 9 identifies and discusses the challenges I faced as I attempted to change my teaching practice, and chapter 10 looks at how my own experiences are relevant to both other educators and educational theory.

## Chapter 7

### *Characterizing Change in my Teaching Practice*

Whereas many theorists write about communities of mathematical inquiry, very little has been published in the area of what it means to be an educator, struggling to incite change in her practice as she attempts to reconcile the tensions between the theory with which she agrees and the practice in which she engages. In order for educational reform to occur, much more work must be done by teachers to examine educational theory and search for ways that such theories can improve their teaching practice within their own specific contexts. While educational theory is the birthplace of educational reform, it is through the teacher, in the messiness of the classroom environment, that educational reform is enacted. Characterizing how one teacher attempts to reconcile theory and practice in a context-specific situation may provide insight for educators and theorists alike into the complexities of educational reform. The perspective of the educator, and perhaps more important, the perspective of the teacher-researcher is valuable and important to characterize as the complexities and messiness of any classroom provide their own challenges to teaching and learning. If theory is to inform practice, it must first be tested in practice so that teachers, researchers, and theorists can learn from each other. With this in mind, in this chapter I characterize how my own practice has changed as a result of viewing learning as complex participation in a community of practice characterized by mathematical inquiry. While some of the more subtle changes that came as I attempted to negotiate the norms and practices of our community of learners will be discussed in the next chapter, this chapter will be primarily concerned with the areas that I felt were the most significant changes in my teaching practice: parallel planning, using

mathematically and communally rich learning activities, taking on the role of the prompter, and moving towards evaluating with performance tasks. All of the activities referred to in this chapter can be found in the Appendices A-J. I will not refer to these as I mention each activity in this chapter.

### **Parallel Planning**

One of the most significant changes to my practice concerned the form of my planning. In past years, as I planned for upcoming classes, I always started with the content in the curriculum guide and worked with the time frame I had allotted for a particular unit to come up with a day by day plan that ensured that all of the outcomes were met within the time I had allowed. The chart in Appendix K illustrates how I engaged in this process. I fit pieces of work (textbook assignments, exercises, worksheets, projects, activities, and assessment pieces) together like a jigsaw puzzle, trying to organize the time I had into the right combination of these work pieces to cover all of the curricular outcomes. Planning occurred under such categories as: day, outcome, method, materials, and assessment. Classroom culture, behaviour, and discussion topics were not part of my planning. Occasionally, I would create a new activity or worksheet to fill in the holes where I thought the outcome might not be covered very well, but planning was, for the most part, an act of organizing those pieces of work.

As I began looking at the year in July for this research project, I found myself wanting to reject textbook work and worksheets for the most part in favor of more hands-on activities that fostered the characteristics of a community of practice characterized by

mathematical inquiry I had previously established. In general terms, I wanted the activities to be active and hands-on. I wanted them to require discussion and interaction amongst community members. I wanted the activities to be as open as possible so that multiple strategies and solutions to problems were possible and could be discussed. I wanted to stop trying to present or give content to students and instead wanted to give them opportunities to experience mathematics and own it in the classroom. In essence, I began planning by setting aside the traditional math work I was used to piecing together and looking at what else we might *do* in the classroom to both cover the content of the course *and* establish a community of practice characterized by mathematical inquiry. I began to develop a concept of what I will later discuss as *mathematically and communally rich learning activities*, which were essentially activities that would simultaneously involve students in the process of engaging with mathematical content and provide opportunities for the development of the classroom community I wished to foster.

*Parallel planning* was a concept that naturally evolved from this line of thinking as I attempted to move away from what resources were available and to move towards planning activities that achieved these two purposes. In the beginning, I created a t-chart (see Appendix L) in which the left side represented content and the right side represented fostering a community of practice characterized by mathematical inquiry. The term *parallel planning*, seemed appropriate for this dual focus, which in the beginning was represented equally in the chart, with content being slightly more important to me as is evidenced by it lying on the left side of the chart. Traditionally, the left hand column of a chart contains the organizing or key element when a chart is used for planning. The fact

that I put content in this column indicates not only that content was important to me, but also that it was what I used to organize and categorize my lessons. However, content was not the only focus of the chart. The chart contained columns for both content and developing a community of practice, which was a dramatic change for me as an educator. Whereas time, content, and materials were previously my foci in the process of planning, I found myself thinking about learning with parallel foci – content and community.

As the year progressed, I continued to think about both content and community in planning activities for my classroom; however, I noticed an important change in my perception of the significance of each. As is evident in an organizational table created at the end of the year (see Appendix M), I clearly began to think about developing a community of practice characterized by mathematical inquiry as more significant than content; that is, I began to plan in terms of *community* and content. The middle column entitled “Goals for fostering the emergence of a community of practice characterized by mathematical inquiry” encompassed half of the three column chart with a column on its left titled “Activity/Description” and one on the right titled “Content Goals”. A clear shift can be seen as I tried to consider each activity first in terms of community development and second in terms of its contribution to content goals. This is consistent with Davis and Simmt’s (2003) suggestion that:

the teacher’s main attentions should perhaps be focused on the establishment of a classroom collective – that is, on ensuring that conditions are met for the possibility of a mathematical community. Such an emphasis is not meant to displace concern for individual understanding. The suggestion, rather, is that the individual learner’s mathematical understandings might be better supported – not compromised – if the teacher pays more attention to the grander learning system (p. 164).

While content is important, and by extension individual understanding of mathematical concepts, perhaps focusing on the establishment of a classroom community is the best way to facilitate the development of individual understanding. Without the development of a classroom community in which mathematical ideas are discussed, compared, and evaluated, students do not have the same depth of experiences using these skills on a regular basis. Without the development of a classroom community in which students are encouraged to take ownership of their learning and make connections between mathematics and their world, individual students may not see how math from the classroom relates to their lives outside of school. Without the development of a classroom community in which students feel safe to express ideas, ask questions, and respond to their peers, valuable discussions and learning opportunities may be lost. Perhaps one of the most important things a teacher does is to attend to norms and practices of the classroom community of which they are part. Perhaps the best efforts in planning come from creating an environment in which the characteristics of a community of practice characterized by mathematical inquiry are encouraged and celebrated.

I believe that parallel planning provides an important key in this process. Prior to this research project, I focused on content in my planning. I never thought about what things I could do to foster a learning environment or community that would strengthen student interactions and discussions. Planning in this way actually changed and strengthened the activities that I engaged students in. After planning the Linear Models unit and creating the chart in Appendix M, I noted in my journal:

Something I noticed when I was creating this unit plan was that it actually changed what I was going to teach and how I was going to teach it. For example, I took out a formal test (which is what I would have traditionally done) and put in a performance task. In addition to this, I did some

different things with activities I have used before. Because I knew that I would be away for professional development one day, I decided to use the time to give students an old practice sheet on graphing linear functions with a calculator, but to pair them up in partners and focus them on working together to complete the task. I hoped not only to see them master the content objectives, but also to strengthen their confidence in themselves and in their peers. (Planning and Observation Journal, 2009, April 26)

While I was planning the unit using the dual focus expressed in the chart, I thought about how I could get students to practice graphing on the calculator, while still achieving the development of a community of practice characterized by mathematical inquiry. This is when I decided to use my time away from students to encourage them to be more independent as well as to rely on the expertise of their peers for help when needed. During this planning phase, I also came up with the idea of a performance task. I wondered how I could evaluate students in a summative way, without giving a traditional test. Giving traditional tests created tension for me as it was inconsistent with the mathematically and communally rich learning activities students were used to experiencing in the classroom community. As I planned, I distinctly remember coming up with the idea for the performance task and being pleased with its match to my own sense of community.

Both the changes to the use of the practice sheet while I was away and to my evaluation techniques occurred as a result of not only viewing learning as complex participation in a community of practice characterized by mathematical inquiry, but also because of the use of parallel planning. This way of planning managed to keep me focused on community in addition to content, and I believe it helped strengthen the community as well as the activities in which they participated.

### **Mathematically and Communally Rich Learning Activities**

The learning activities that I chose to engage my students in could be characterized as *mathematically and communally rich learning activities* due to the fact that they were chosen precisely because they provided experiences that were rich in both mathematical content and in opportunity for the development of the community of learners I was looking for. Initially, I referred to the things I planned for my students to do in the mathematics classroom simply as “activities”. As time went on, however, I felt the need to characterize these activities in some way. I thought about the notion of rich learning activities, but found myself asking: “Rich in what way?” I recognized through the process of parallel planning that the things I engaged students in had a dual focus: content and community. As a result, I put together the phrase *mathematically and communally rich learning activities*, which I felt encapsulated the nature and goal of these activities.

Not all of the activities students engaged in during this study contained the same characteristics, but a general list of characteristics could be compiled using the breadth of activities in which students engaged. Table 7.1 summarizes these characteristics.

The mathematically and communally rich learning activities in which students engaged were usually hands-on in nature, requiring students to be involved physically as well as mentally. For example, during the Trigonometry Challenge, students worked with measuring equipment to determine the height of the flag pole at the school. Similarly, during the DaVinci Project, students took measurements of at least fifty people in order to prove or disprove one of DaVinci’s proportions.

Table 7.1

*Characteristics of Mathematically and Communally Rich Learning Activities*

Mathematically and communally rich learning activities should:
<ul style="list-style-type: none"> <li>• be hands-on in nature</li> <li>• be aimed at arousing curiosity or a sense of inquiry</li> <li>• be open or ill-defined</li> <li>• be applicable to real-world situations</li> <li>• foster independence as well as interdependence</li> <li>• promote discussion about mathematics and mathematical strategies</li> </ul>

Another characteristic of these activities was that some of them were aimed at arousing curiosity and a sense of inquiry in the community. For example, Grandpa's Tool Shed and Mystifying Measurement Markings were both activities conceptualized from my own curiosity about things I saw in the real world and was curious about. I hoped that through the activities I would see students asking questions about the things they saw around them.

The activities were sometimes open or ill-defined, requiring students to make decisions about how to approach the problem, or even what topic they might investigate. The Measurement Debate, for example, was very open, requiring students to formulate their own arguments and respond to the arguments of their peers, as was the Trigonometry Challenge, where students had to come up with their own strategies for solving the problem.

The learning activities often had real world application, as was evident in the Student Lounge Project as students prepared a cost analysis and presented their designs in front of the principal of the school.

The activities fostered both independence and social interdependence as students had to engage in the learning process as opposed to sitting back and attempting to take in material presented by a teacher. Students were required to participate, individually, and were responsible to their peers as part of the larger community of learners. The activities necessarily had to require this on the part of the students, as was evident in the Measurement Debate where students had a role to play in the unfolding debate that marked the culmination of the activity.

Finally, the mathematically and communally rich learning activities students engaged in promoted discussion of mathematics, and in particular, of mathematical strategies. The Trigonometry Challenge was perhaps the best example of an activity that had this characteristic. Students were required to come up with a strategy for finding the height of a flag pole using mathematics and simple tools. The entire activity was designed to have students generate a strategy for solving a problem and relate the strategy to the rest of the classroom community. The most important purpose of the activity was to promote discussion about mathematical strategies.

Looking further at the phrase *viewing learning as complex participation in a community of practice characterized by mathematical inquiry*, several of the characteristics of the learning activities I have described are evident in the phrase itself. The notion of complex participation requires that students engage in a hands-on, interactive manner. The notion of a community of practice suggests not only that the

activities should foster interdependence and collaboration but also a sense of authenticity or application to real world contexts as Lave and Wenger (1991) intimated in their concept of communities of practice. Finally, the notion mathematical inquiry suggests both an element of inquiry or curiosity as well as an open, thoughtful approach to looking at mathematics.

Planning all of the activities required a careful look at what each could contribute to individual and collective understandings of mathematics, as well as how each one could help foster the emergence of a community of practice characterized by mathematical inquiry. The process of planning essentially was what Cobb (1999) refers to as an *anticipatory thought experiment*, requiring me to think about what the activity offered, how students might respond to the activity, and what *hypothetical learning trajectories* (Cobb, 1999) might emerge as the plan came into being. For example, in the Trigonometry Challenge, I anticipated that students would look for a triangle and that they might use the string to create a line of sight to the top of the flagpole from the ground. I anticipated that students may use the tangent function to solve the problem once they created this triangle. I also thought that one of the groups may do it differently, using more than one triangle, or measuring from eye level. As I thought about this, I decided to discuss all of the strategies used by the groups as a whole class, strengthening students' ability to express their own strategies, compare them to the strategies of others, and evaluate the appropriateness and accuracy of the strategies used to solve the problem. In this case, I anticipated what students might come up with and planned for differences in strategies to be a direction we could go in. Although I could not predict what strategies

students would use, I could anticipate that the strategies would look different and that it might be a good time to work on that aspect of building community.

As the Trigonometry Challenge was enacted in the classroom, what I had anticipated came to life. While most of the groups did some sort of variation on the strategy I had suspected they would use, each of the solutions presented by the groups looked different. Students had to unpack each solution in order to recognize that it was similar to their own group's solution, which made them consider not only their own strategy, but also the strategies of the others, noticing similarities between them. In addition to this, one group solved the problem by creating a scale drawing of the triangle they had measured and did not use trigonometric ratios at all. This was something I had not anticipated. It was a surprising but effective strategy for solving the problem and led to a great discussion about using multiple strategies to solve the same problem. All of the groups, even with the variances in strategies, came up with similar numbers, which allowed them to feel confident in their strategies and to be proud of their work.

Attempting to anticipate the learning trajectories in the classroom allowed me to prepare to discuss and have students write about strategies for solving problems. Many of the students provided great responses to the journal prompts after the activity, which would not have been possible had I not conducted the thought experiment and considered what might happen. The following excerpt from the journal of a student I will call Sarah shows the depth of thinking about strategy that occurred as a result of planning this way:

*What strategy did your group employ to find the two heights? Our group used a scale drawing to figure out the height of the flag pole and the gym. We measured the angle from my eye-level looking at the top of the pole or gym to the ground. We then subtracted the angle from  $90^\circ$  and we had the three angles of the triangle. We stood 26 feet away from the object we were trying to find one height of. Our scale was  $0.5 \text{ cm} = 1 \text{ foot}$ . We drew*

the triangle and made all the angles the same as we measured and drew the rest of the measurements to scale as well. We then measured the side we needed.

*Why did you decide to use this strategy?* We decided to use this strategy because it was the first idea that was suggested and it sounded like it would work. Also we just finished doing our project on a student lounge and it used a lot of scale that probably helped us make the decision.

*Was the strategy effective? Is there anything that would have improved it?* I think that the strategy was effective because most of the other groups got the same answers or close to as us even though they used different strategies. I think that if there could have been a way so that our measurements were more exact it would have improved our results. There was snow in the way sometimes and the thickness of the string took up a whole degree on the clinometer [sic].

*Describe in your own words what you learned about trigonometry during this activity.* I didn't learn anything new about trigonometry but I did learn that trig isn't the only way to solve problems with triangles.

(Sarah's Interactive Journal, 2009, March 3; italics added)

Had my only concerns been content and time when I was planning the trigonometry unit, and had I not considered the development of the characteristics of the community I wished to foster, I likely would not have engaged students in the very active, hands-on activity of the Trigonometry Challenge. Had I not anticipated the emergence of multiple different strategies as well as the opportunity for discussion about strategies, Sarah's experience would have been very different in the classroom. The type of thinking and learning that she expressed in her journal would not have been possible.

The development of mathematically and communally rich activities, then, is a critical change in my teaching practice and in particular in my planning. In considering both content and community in my planning, and in anticipating the way that these activities will play out, I have been able to improve my teaching practice as well as the quality of learning that takes place within my classroom community.

### **Taking on the Role of Prompter**

Another important change in my teaching practice was taking on the role of *prompter* in the classroom. All three of the quotations from my observation notes in chapter 5 provide evidence of the importance of this role for me as an educator. Taking on the role of prompter was not planned as the research study started; it naturally evolved as I began teaching and students began participating in the activities I had planned. It became very clear to me that my role in the classroom was essentially to walk around and ask questions, which was somewhat opposite to what I had always done. In previous years, I would have found myself walking around the room *answering* questions students had about what they were working on. By changing the norms of the classroom, I was able to get students to stop asking me to tell them how to do something and start discussing with me interesting things about what they were doing. One of my students I will call Trevor, when asked to describe the course at the end of the year to a student thinking of taking it next year, stated: “The teacher is very willing to help you if you have a question, but won’t answer all of them” (Trevor’s Interactive Journal, 2009, June 9). Students became used to me talking to them but not giving them the answers. They knew that I would not feed them a series of steps to do to solve a problem, I would ask them questions. They also knew that I would be walking around asking them questions even if they were not having difficulties. This became the norm, and I found it very helpful in two ways: I was able to help students who were stuck get past the difficulty they were encountering, and I was also able to help students extend their thinking, pushing them a little farther than they might otherwise have gone. This is evident in the conversation I had with Elizabeth (see

chapter 5, p. 90/91). She was stuck when she knew her linear model didn't match what she thought she should see, but she was not able to figure out exactly how to use this information to prove or disprove the proportion. By asking her some questions, I was able to help her solidify her understanding of slope as a rate of change. She started to discuss the relationship of  $1/7.5$  or  $7.5/1$  as a rate of change and began to see that her model did not reflect the same rate. I did not tell her if the proportion was true or false. I did not tell her how to go about proving things; I simply asked her enough questions to get past the point of being stuck and move on. This was much more effective than telling her what to do. Similarly, in the conversation I had with Damian (see chapter 5, p. 75), I was able to ask some questions that focused him and enabled him to get past the superficial answer he was stuck with. At first, he thought that there was a simple "no" answer to whether a cylinder was a prism. I asked him first what the definition of a prism was and then for the definition of a polygon, and helped him then make the connection between a polygon and a circle. Because of this, Damian began to think more deeply about the problem and recognized both sides of the argument. In his own journal he later wrote "The most challenging thing to do was to try to think of how to word my argument and prove it with evidence" (Damian's Interactive Journal, 2009, January 5), which indicated to me that after he managed to understand conceptually how a cylinder could be considered a prism, and he made up his mind that it in fact was not, formulating an argument that had sufficient proof was his biggest concern. It was clear to me that his original quick answer of "no" required some intervention and prompting from me, the teacher, to help him make some connections and take his thinking to a deeper level, which he was clearly capable of with some prompting.

Some other important benefits of prompting are evident in my journal entry about the conversation I had with Richard (see chapter 5, p. 81). In this conversation, as I asked Richard about his scale and how he used it. Several things happened in the classroom community that were beneficial for Richard, his classmates, and for me as the teacher. Richard, in the process of explaining how he used his scale and the difficulties he had using it, went through a metacognitive exercise of thinking about his own thinking. He was able to recount the steps he followed and even explain why he used the strategies he did. This is difficult for many students to do, and in asking students to respond to questions about their thinking, teachers can make students become stronger at metacognition as well as thinking about strategies for solving problems. This conversation also helped Richard's classmates, as many of the students sitting nearby listened not only to the questions I asked, but also to Richard's responses. They nodded as he explained, and they also learned from the interaction. Finally, I was able as a teacher to gain valuable information about Richard's thinking and understanding of scale. Richard clearly had a very strong conceptual understanding of scale factor, proportional thinking, and measurement. This information gave me a concrete experience from which I could comment to Richard and his parents on his ability and progress in the course, much moreso than if I had assessed him through a test or piece of written work. This was largely due to the fact that I was able to witness Richard's thinking process as he relayed it to me, something that one only guesses when looking at an answer on a test.

Although I had used prompting in my classroom in the past, it was not a focus for me when I thought about my role in the classroom. Thinking about my role in the classroom as a prompter changed how I interacted with students, what the classroom

environment looked like, my understanding of student learning, and the quality of discussion and mathematical thinking that occurred in my classroom. For me, this was a critical change in my teaching practice, both in how I thought about my role, and in how I interacted with students.

### **A Move Towards Performance Tasks for Evaluation**

Finally, I found it necessary to change my methods of evaluation as I began to plan and teach with both community and content in mind. As I began the course, I was unsure how I would evaluate students. I thought I might be able to make a rubric for observation and mark students as they worked on activities; however, in the first year at least, there was not sufficient time to think about organizing students, prompting, and evaluating all at the same time. I spent much of my time getting students started and then walking around asking questions and promoting discussions within the classroom. Evaluating all eighteen students at the same time was nearly impossible.

When the first set of report cards came up, I found myself struggling with how to assign the required grade for each student. I recorded this dilemma in my journal:

After having an extremely successful experience in problem solving partners, reality has hit for me. Report cards are coming due and I have to assign a mark on them for students after really only about 20 classes. When I looked at my grade book yesterday, I realized that while I know a lot about the students anecdotally, I have very few marks in for them. I decided to do a traditional paper and pencil test, as a result, because I do not feel that I am able to adequately assign a grade on the reports without some sort of summative evaluation.

When I began this research, I wasn't sure how exactly I would assess students. I thought about doing much more in class observations and rubrics/checklists. What I have found out is that in theory this is a good idea, but in practice, not so good. I am busy. I have a lot of

administrative burdens – attendance, percentages, reports, phonecalls, school activities, etc. that affect my ability to teach the way I envision. It is somewhat discouraging to think that the constraints of my job, the environment, my course load, and my own human limitations are making my ability to teach how I believe is best nearly impossible. I have gone through a series of guilt-ridden decisions and decided that in order to give a percentage, I need to get more concrete data in the form of a test. This goes against what I envisioned but I am finding it necessary in this situation. I wonder if I will ever be able to continually assess student progress. Turning these “observational” assessments into marks is problematic. How do you observe say that a student is very reluctant to attack a problem on their own and translate that into a number? I struggle with this idea. I have tried to assess quality of journal entries and I assessed the problems that students created both on a 4 level rubric, but these numbers are often at odds with my concept of a student’s grade range. For instance, one student who is very bright and who works very hard turned in a simple problem basically modeled after one we did in class. On a test, this student would normally score mid eighties and up. I gave him 2/4 and that was generous. How do I reconcile how he normally does on tests where he knows what is coming with a mark like this? How do I justify that? (Planning and Observation Journal, 2008, October 28)

This journal entry encapsulates the difficulty I had with assessment throughout the year.

In theory, I felt that I should have been able to grade on scales in class, and I really felt that I knew much more about students and their abilities than I had ever known in the past. The problem was converting that knowledge into a percentage. I was clearly more upset about my decision to return to a formal test than students were. They did not seem to mind, likely because the norms of the community were not fully established yet. At the end of the course, however, several students noted that their least favorite part of the course was the tests.

By putting a formal test into the list of things we did in the math classroom, I felt as though I had destroyed the community I wished to create. I wanted students to truly experience something different than that which they were used to, and by giving a formal test, it seemed disjointed and out of place. The assessment did not really match what their

experiences had been in the first twenty classes, and this bothered me considerably, even though the class went on, grades were assigned, and none of the students seemed disturbed at all.

This scenario repeated itself twice more during the year as I found myself turning to a test to assess students in a summative manner. During activities, I found that I was able to give a couple of smaller marks for daily things, and a group mark for larger group tasks, but I was quite uncomfortable without having individual assessments when it came time to put numbers on report cards. As a result, I turned to formal tests to alleviate this concern.

At the end of the course, I found myself again in the situation of having to assess students for a final grade, but I decided that I wanted the assessment to match their experiences more closely. This is when I created what I called a *performance task* (see Appendix J), which required students to choose data with a linear relationship and create a linear model from which they could make some conclusions and predictions about the relationship. This was a summative, individual assessment task which enabled me to give them a mark based on their own performance on the task, as opposed to other tasks in which students worked collaboratively. That was not to say that students did not still discuss their data with others and elicit the opinions of their peers as they completed the task, it just meant that the product that they created was theirs and the interpretation was their own despite the fact that it might have been influenced by input from their peers. I was much more comfortable with this form of assessment as it had all of the same characteristics as the activities they had been participating in. At the same time, the task

was open to students to pursue a topic of their own choice and they could really take ownership of their work and their learning.

The performance task worked out even better than I had anticipated. Students chose interesting topics such as: height and circumference of trees, fuel economy and weight of vehicles, a country's GDP and its population, the size of a country and the size of its ecological footprint, length of prison terms and prior convictions, seed required for planting a crop and acres to be planted, number of restaurants and population, leg length and vertical jumping ability, MLB salaries and years played, SAT scores and hours studied, a country's birth rate and its population, age and shoe size, and fuel economy and speed travelled. I noticed that students, in general chose a topic that they were capable of but that was not too easy. Gregory, one of my students who was not always consistent about completing assignments, chose to compare Major League Baseball salaries with number of years played in the league, which proved to be a difficult task. The amount of effort he put into his presentation was phenomenal, and his understanding of linear models was quite strong. The data turned out to have a relationship, but there were many outliers on his graph. Gregory was able to describe these outliers and how they skewed the data, indicating significant understanding of linear models. This understanding, together with the interest he displayed through his tremendous effort was encouraging for both Gregory and myself. Clearly the openness of the activity was engaging for Gregory, which he indicated at the end of the course when he stated: "My favorite activities have been . . .the final assignment because I got to do a project on baseball, and baseball players" (Gregory's Interactive Journal, 2009, June 9). Gregory's increased effort and interest in a self-generated topic were typical of the performance task as many of the

students enjoyed being able to research something they were interested in. The activity proved to be motivating for students and all of them were able to participate, regardless of having chosen age and shoe size to compare, MLB salaries and years playing, or a country's GDP and its population.

After the performance task, students completed a self and peer evaluation, in addition to answering some feedback questions on an evaluation form (see Appendix P). On the feedback form I asked students their opinions about the experience. Thirteen out of eighteen said they liked the performance task better than a traditional test, while only four preferred a traditional test. One said they liked both. Those who liked the task better cited several similar things that they appreciated about the performance task. In general, students enjoyed the change, the ability to be creative, the freedom to choose their own topic, being able to discuss data with others, having more time to think and work, and many of them expressed that the performance task was much less stressful for them than a traditional test would have been.

The four students who said they would have preferred a traditional test were of two distinct types. Two of them were very high achievers and felt their mark would have been the same or better on a test without all the time required to do the performance task. The other two were two of the weaker students in the class, both having some difficulty with organization and staying on task on larger assignments. One of the latter two, whom I will call Philip, wrote: "I did worse than if I did a test because this project required several days of work, which I get sidetracked from, but a test is hard to get distracted from. I prefer tests because I find it easier" (Philip's feedback form, 2009, June 5).

When I compared the marks students had on previous tests with the marks they received on the performance task, I noticed that they did not vary significantly from the marks students would have scored on tests. None of the students failed the performance task. I was very relieved to find out that the marks on the performance task were consistent with other grades in the course for most students, since this was one of my fears about the task. As a teacher, I found this form of assessment viable and reliable, and I plan on incorporating this form of assessment in my classes in the future.

In addition to maintaining fairly consistent marks, performance tasks enabled me to continue to develop the community of practice characterized by mathematical inquiry that I sought. I was able to have students do self and peer evaluations in addition to my own evaluation, which required students to look at and respond to the work of others in a more formal way. Students in general seemed to appreciate sharing their work, and there was a celebratory air about the classroom when these tasks were completed and shared. Whereas I felt that traditional tests destroyed the classroom community that I was trying to establish earlier in the year, performance tasks had the potential to build the community of practice characterized by mathematical inquiry while still achieving my purpose of acquiring summative grades for individual students on their understanding of mathematical concepts.

## Chapter 8

### *Characterizing a Community of Practice Characterized by Mathematical Inquiry*

#### **Characteristics of a Community of Practice Characterized by Mathematical Inquiry**

In this chapter, I will identify five characteristics of a community of practice characterized by mathematical inquiry. I will describe each of these characteristics and how I saw each characteristic emerge within the classroom community that was the subject of this research study. I will also discuss the strategies I used for fostering the emergence of each characteristic within the classroom community. At the end of the chapter, I will describe what the classroom community might have looked like from the outside looking in, including what an observer might have been able to see as well as what she might not have noticed.

In order to look at developing both community and content through my planning and teaching, I first had to identify what a community of practice characterized by mathematical inquiry looked like. As I conceptualized the study, I identified eleven characteristics that I felt such a community of practice would have. These characteristics were influenced significantly by the work of Merrilyn Goos (2004), as I described earlier in chapter 3. However, I found these characteristics were too numerous to keep track of during the planning process, and I decided to consolidate the list into five more concise statements about the community I hoped to foster. Table 8.1 (p. 125) summarizes the characteristics as I identified them at the beginning and at the end of the study. While I did not want to eliminate any of the eleven characteristics I had identified prior to the study, I found that these needed to be broken down into smaller chunks in order to use

them for planning. As I made these changes, I envisioned the concept of parallel planning becoming easier as I was able to use an abbreviation like DACCO to summarize the characteristics and keep them at the forefront of my planning. DACCO (**d**iscussing, **a**pplication, **c**uriosity, **c**ommunity, and **o**wnership) would enable me to characterize the characteristics in one word, as opposed to the long list of eleven I began with. Even though the characteristics were cut down, I felt the spirit of the original eleven was maintained in the new five characteristics.

Table 8.1

*Characteristics of a Community of Practice Characterized by Mathematical Inquiry*

Eleven Characteristics at the Beginning of the Study	Five Characteristics at the End of the Study
<ol style="list-style-type: none"> <li>1. Mathematical thinking and noticing</li> <li>2. Discussion of mathematical ideas</li> <li>3. The proposing, clarifying, defending, and refuting of mathematical strategies</li> <li>4. Curiosity/asking questions about mathematics</li> <li>5. Individual and collective ownership of learning</li> <li>6. Application of mathematics to real world contexts</li> <li>7. Decreasing reliance on the teacher as validator of mathematical ideas and increasing reliance on peers and self as validators of mathematical ideas</li> <li>8. Reflection on mathematical ideas</li> <li>9. Metacognitive awareness</li> <li>10. Understanding of the norms and practices of the community</li> <li>11. Recognition of common purposes amongst community members</li> </ol>	<ol style="list-style-type: none"> <li>1. Discussion, reflection, and evaluation of mathematical ideas and strategies</li> <li>2. Application of mathematical concepts to real world contexts</li> <li>3. Curiosity about mathematics</li> <li>4. Sense of community amongst members</li> <li>5. Increasing sense of individual and collective ownership of learning</li> </ol>

The first of these five characteristics - discussion, reflection, and evaluation of mathematical ideas and strategies - combined the original characteristics one, two, three, and eight as all of these centered around thinking about and discussing mathematical ideas and strategies. In addition to this, I felt that it might also include the metacognitive aspect, or characteristic nine, if students were discussing their own ideas. The second and third new characteristics - application of mathematical concepts to real world contexts and curiosity about mathematics - I believed were fundamental and important enough to stand on their own as characteristics. The fourth characteristic - sense of community amongst members - replaced the tenth and eleventh old characteristics - understanding of the norms and practices of the community and recognition of common purposes amongst community members - to make the notion of community more concise. Finally, the idea of ownership in the old characteristic five and the idea of students looking to themselves and their peers as validators in the old number seven were combined into a new fifth characteristic: increasing sense of individual and collective ownership of learning.

Although the eleven original characteristics were used as I planned activities and made observations, the five new characteristics of a community of practice characterized by mathematical inquiry are what I now use to discuss the things I observed during this research study. These characteristics both summarize what I attempted to foster in my classroom as well as what I observed as a result, and they provide a framework for characterizing a community of practice characterized by mathematical inquiry.

***Discussion, Reflection, and Evaluation of Mathematical Ideas and Strategies.***

Perhaps the most important characteristic of the five I identified was the notion of discussion, reflection, and evaluation of mathematical ideas and strategies. This characteristic stemmed from the work of Marilyn Goos (2004), who suggested that one of the characteristics of a community of mathematical inquiry was that “the teacher asks students to clarify, elaborate, and justify their responses and strategies” (p. 267).

Similarly, Magdalene Lampert (1990) argued that: “Generating a strategy and arguing for its legitimacy indicates what the student knows about mathematics” (p. 40). Nadia Stoyanova Kennedy (2009) also suggests that “ideal mathematical inquiry proceeds through a form of argumentation” (p. 73). All of these dialogical models focus on the student’s ability to discuss, reflect, and evaluate mathematical ideas, an important characteristic to foster if one wishes to develop a community of practice characterized by mathematical inquiry.

In order to promote discussion, reflection, and evaluation of mathematical ideas and strategies, students needed primarily to engage in activities that required them to come up with and discuss their own strategies on a regular basis. During the course of the research study, the mathematically and communally rich learning activities provided such opportunities for students. For example, during the Trigonometry Challenge, students were given the problem of finding the heights of the flag pole and the gymnasium but were asked to come up with their own strategies with no help from me or other groups. Afterwards, we looked at these strategies, compared them, and discussed how they were similar, appropriate, and effective. Students also described and evaluated their group’s strategy in their journals as individuals. Sarah’s journal entry (p. 113/114) was one

example of a student's comments about her group's strategy as well as her evaluation of it. Sarah was able to clearly indicate the strategy used by her group and showed some insight into the group members' thinking process when she acknowledged that having just finished the Student Lounge Project probably influenced their decision to use a scale drawing to solve the problem. Even though Sarah's group came up with a very different strategy than the other groups, she was still confident that her strategy was good, due in part to the similarity between her group's final height and that of the other groups. What Sarah did not say in her journal was that she also knew that it was an appropriate strategy because it was validated in the classroom discussion as her group presented it to the class. The fact that Sarah's group came up with a strategy completely different than the strategies of the other groups provided a valuable learning opportunity for the entire class. By having students explain their strategies within the classroom community and by having a discussion about each group's work, students gained valuable experience discussing and evaluating mathematical strategies.

In addition to the mathematically and communally rich learning activities that provided students with tasks requiring discussion, reflection, and evaluation of mathematical ideas and strategies, deliberate emphasis was put on students working collaboratively whenever possible. Students were organized in this way to facilitate discussion, and the implicit and explicit expectation was that students discussed and collaborated with their peers to solve mathematical problems. One of the key benefits of working collaboratively on tasks that require a group to strategize about how to solve a problem – such as the Trigonometry Challenge – is that the group must come to a consensus about how to approach a given problem. The very nature of this requires

students to discuss, propose and defend, or argue their strategies, allowing them to use their own ideas and the ideas of others to construct meaning. Challenges to their own thinking stretch and strengthen students' understanding, and they are able to consider more complex mathematical ideas than they might otherwise have been able to consider in the absence of collaboration. One strategy that I used to promote collaboration in the classroom community during this research study was problem solving partners. In one instance, I managed to use this structuring of partners in conjunction with explicitly stated expectations to net a very different result from a traditional problem solving sheet I had used in the past. Recognizing that the students needed some practice in order to master the topic of conversions, I decided to use an old measurement work sheet but in the process of attempting to develop the community of practice by pairing students up into problem solving partners. This strategy required students to help each other, and it created a sense of responsibility for the other person on the part of students. Students, in general, seemed to appreciate working with a peer, often acknowledging in true Vygotskian fashion that working with a partner allowed them to go beyond where they could on their own: "I liked working with a partner because if I or my partner don't understand we can figure it out together or discuss it" (Janine's Interactive Journal, 2008, October 24). In addition to these sorts of statements, at the end of the course when I asked students to describe five characteristics of our community of learners, or rephrased, to list five things about this class that describes it to someone who was thinking of taking it the next year, thirteen out of seventeen students indicated that they got to work in groups a lot in the course. Collaboration became a way of doing math in the community, and thus discussion of mathematical ideas and strategies became part of what we did also.

As discussed in chapter 7, taking on the role of the prompter was an important part of fostering a community in which discussion about mathematical ideas and strategies were a part. During group work, I was able to walk around and ask questions of students to help them think deeper, as was evident in my conversation with Damian about the cylinder being a prism (see Chapter 5, p. 75) and my conversation with Richard about the scale he was using on his Student Lounge model (see Chapter 5, p. 81). In addition to this, I was also able to prompt students during full class discussions and in their interactive journals in order to promote thinking, reflecting, and evaluation of mathematical strategies and ideas.

Modelling and celebrating examples of strong mathematical thinking and noticing were also essential in developing an environment in which students could discuss mathematical ideas. After the Trigonometry Challenge and subsequent class discussion, I noted in my observation notes: “Students are proud when they come up with a solution that is unique and different from others. Other students appreciate making the connections. Celebrate this!” (Planning and Observation Journal, 2009, March 4). When students were able to get excited about what they had done and were able to share their thinking with others, they truly engaged in the processes of discussion, reflection, and evaluation. This was evident in the Trigonometry Challenge as well as in the DaVinci Project. In both cases, students were asked to share their thinking with the class, resulting in a celebratory air in the classroom as students presented and watched the presentations of other members of the community.

By fostering discussion, reflection and evaluation of mathematical ideas and strategies, an educator can promote development of the thinking and reasoning abilities

of community members. All too often, students do not learn how to think mathematically, proposing and defending ideas in the context of the classroom community. These things need to be valued in a classroom community if students are to become better at reasoning and logical thinking. If they are to notice mathematics in the world around them, there must be an invitation for dialogue in mathematics education where such noticings are valued. If students are to come to a sophisticated understanding of what constitutes a good mathematical argument, they must engage in the process of argumentation (Kennedy, 2009). In the very utterings of students, through the process of their thoughts turning to words, students learn. What was once hidden becomes explicit; what was once intuitive, finds a voice. In the process of putting things into words, new thoughts are born, and students are capable of more than they were before they spoke. Others in the group, operating within the zone of proximal development (Vygotsky, 1978), can become capable of more once other community members speak; communication by community members scaffolds the learning of others. Students are capable of more than they were before as their ideas are enriched by listening to the ideas of other community members and by the thoughts they have when talking about their own ideas. Kennedy's (2009) concept of *distributed thinking*, whereby the ideas of each community member as they are expressed build on previous understandings, forming new ones, breathes life into the notion of collective understandings. None of these can happen without the educator paying particular attention to the development of a community of practice in which these things are practiced and valued.

### *Application of Mathematical Concepts to Real World Contexts*

In addition to the discussion, reflection, and evaluation of mathematical ideas and strategies, another characteristic I hoped to foster in my classroom community was the application of mathematical concepts to real world contexts. By fostering a community in which application of mathematics to real world problems and situations is practiced and valued, students learn that mathematics is not only applicable to their lives, it comes from their lives. In her article entitled “Real-world connections in secondary mathematics teaching”, Julie Gainsburg (2008, pp. 199-200) suggests:

The K-12 mathematics-education community is virtually united on the importance of connecting classroom mathematics to the real world (e.g., Boaler 1997; National Council of Teachers of Mathematics [NCTM] 2000; National Research Council [NRC] 1990; Steen 1997). Real-world connections are expected to have many benefits, such as enhancing students’ understanding of mathematical concepts (De Lange 1996; Steen and Forman 1995), motivating mathematics learning (National Academy of Sciences 2003), and helping students apply mathematics to real problems, particularly those arising in the workplace (NRC 1998). The mathematics education literature as a whole locates a range of practices under the umbrella of real-world connections, including:

- simple analogies (e.g., relating negative numbers to subzero temperatures)
- classic “word problems” (e.g., “Two trains leave the same station...”)
- the analysis of real data (e.g., finding the mean and median heights of classmates)
- discussions of mathematics in society (e.g., media misuses of statistics to sway public opinion)
- “hands-on” representations of mathematics concepts (e.g., models of regular solids, dice)
- mathematically modeling real phenomena (e.g., writing a formula to express temperature as an approximate function of the day of the year).

Many of the mathematically and communally rich learning activities the class engaged in during the study required students to apply mathematical concepts to real world contexts. For example, in *Thinking Outside the Box*, students were asked to expand what they had done in class to industry and look at how certain industries could

benefit from better designs in packaging. This involved discussing mathematics in society, as did the Measurement Debate, in which students had to take the measurement systems they were taught about in school and form an argument using real life information and situations in favor of or against Canada adopting the Imperial System. In Mystifying Measurement Markings and Grandpa's Tool Shed, items from outside school became the focus of investigation, many relating to careers in the working world. In the Student Lounge Project, students were given a scenario that was not real, but that could have potentially been real. The project required them to create a plan to create a student lounge in their math classroom on a budget and present their design, complete with scale drawings and a model to a committee, which included their class, the principal, and some other invited guests. The scenario, while not real, culminated in a very real presentation, and students could easily see the relationship between what they were doing and home design or bidding on projects. In the Trigonometry Challenge, a very hands-on activity, students went out to learn about how they could use typical measurement tools to estimate heights and discussion ensued about how surveyors use trigonometry in their careers. In the DaVinci Project, and the following performance task, students collected real data and analyzed it, fitting it to their understanding of linear models in mathematics. Students worked collaboratively to solve some classic word problems, and even designed some problems of their own that related to the real world. All of these activities helped students apply mathematics to real world contexts, making mathematics in the real world more accessible to students.

While Gainsburg (2008) suggests that there is a disconnect between what the mathematics education community values in terms of real world connections and what is

actually happening in mathematics classrooms, this need not be the case. Gainsburg suggests that: “Secondary mathematics teachers count a wide range of practices as real-world connections. Teachers make connections frequently, but most are brief and many appear to require no action or thinking on the students’ part” (p. 215). By fostering the emergence of a community of practice characterized by mathematical inquiry, application of mathematical ideas to real world contexts can be the norm rather than the exception. In such a context, the thinking is done by students as opposed to connections being made by teachers, and those connections are valued and celebrated within the community.

During the course of this research study, students were given ample opportunities to make connections by considering how some of the mathematical concepts they were using could be used in the real world. At the end of the course, when asked to identify the characteristics of their classroom community, five different students identified application of mathematics to the real world as one of the characteristics that described their mathematics classroom community (see Table 8.1, p. 125, for complete results). When asked what math is and what it looks like in school, one student, Damian, wrote at the beginning of the course in his journal:

I think mathematics is a series of numbers and symbols and letters that can be put together in many different orders to form equations. I think doing math in school looks like this: sitting in a classroom with a pencil and eraser in your hand trying to understand complicated equations that a lot of the time don’t even help you in the real world. (Damian’s Interactive Journal, 2008, September 5)

At the end of the course Damian responded to the same questions:

Mathematics is a way of solving problems in the real world based on formulas and theories. We learn about these different formulas and theories in school so that we can use them in the real world. Doing math at school can look very different depending on the type of math you take. For

example in Applied Math the type of work we do is problem solving, formulas for solving problems, experiments, etc. The Applied is so much more relevant to real life than the Precalculus is. (Damian's Interactive Journal, 2008, June 9)

Making connections to the real world became a characteristic of the classroom community as was evidenced by comments such as Damian's. Although Damian and a few other students associated the real world connections with their Applied Math course, this sort of application was due to the classroom expectations, norms, and practices that had developed over the course of the year, moreso than the content of the course itself. The comment about Precalculus Mathematics not being as relevant is possibly more directed at the algebraic nature of the content as well as the more traditional way in which it was taught. Their Precalculus Mathematics course was taught using notes and exercises for the most part, with very few activities or opportunities to relate the mathematics to real world contexts. Students were able to see the Applied Mathematics activities as useful and applicable to the real world, moreso than other math courses they had taken. It is clear through Damian's journal entry that such real world connections were valued, and for Damian, at least, mathematics had become more accessible and pertinent to him in his life.

### *Curiosity about Mathematics*

Curiosity about mathematics is something I believe that a community of practice characterized by mathematical inquiry must exhibit. The word inquiry itself necessitates a sense of curiosity or asking questions about mathematics or about mathematical things in the world around us. This characteristic was the least represented of the five in the data from this study. Students were not encouraged openly to ask questions about math,

except in the form of guided questions such as the Thinking Outside the Box activity, where they were asked to look at what industries used packaging like boxes as part of their business. Students also were guided to investigate measurement markings and tools through the Mystifying Measurement Markings and Grandpa's Tool Shed projects. While these activities were conceptualized from my own curiosity about the things I noticed around me, students did not share the same curiosity as I did. Several students seemed rather uninterested in these activities, and at the end of the course, six students also noted that Grandpa's Tool Shed was one of their least favorite activities. This was in part due to the lack of modelling on my part about curiosity in mathematics. I think with more emphasis on this, an environment that truly encourages the asking of questions and curiosity about mathematics and its relationship to the world could be fostered.

Aside from the lack of interest in a few of the activities, a couple of the mathematically and communally rich learning activities that students engaged in during the research project spawned curiosity on their own. The first one was the Student Lounge Project. Students asked a lot of questions about industry standards, how to build things, how large certain things were, how wide was wide enough to leave between pieces of furniture for a person to walk through, and much more. This activity required students to go beyond their comfort zones and into areas that they needed information about, thus creating some questioning and curiosity. A second activity that sparked some curiosity was the DaVinci Project. Quite a few students began to ask questions about DaVinci and his proportions as they worked on the project. One student, whom I will call Tracy, said: "During the DaVinci project, I always wondered who the Vituvian Man is!

Was it DaVinci, was it that Vitruvian architect [sic]? Plus, how did he decide this man was perfect? That always troubled me!” (Tracy’s Interactive Journal, 2009, May 15).

By valuing curiosity, asking questions, and noticing things about mathematics and the world, a teacher can foster a community of practice in which students truly inquire about mathematics. Moreover, when twenty students exhibit and share their own curiosity and noticing, a classroom community can have a different dynamic to it completely. Effort is required, however, on the part of teachers to model and notice curiosity on a daily basis if it is truly to become a characteristic of a classroom community. If one is able to foster the emergence of curiosity within a classroom community, however, the potential of student engagement, mathematical thinking, and dialogue is dramatically increased. If mathematical inquiry is the goal, then curiosity is the pathway to it.

### ***Sense of Community Amongst Members***

The development of a sense of community is an important aspect of fostering a community of practice characterized by mathematical inquiry. Not all communities of practice are characterized by mathematical inquiry. In order for a community of practice characterized by mathematical inquiry to emerge within a classroom, the community members must have a sense of what that community is and what their role is within it. According to Magdalene Lampert (1990), “when classroom culture is taken into consideration, it becomes clear that teaching is not only about teaching what is conventionally called *content*. It is also teaching students what a lesson is and how to participate in it” (p. 34). As a result, a community of students must learn what is

appropriate participation within the community in addition to mathematical content. They must learn that discussion of mathematical ideas and coming up with strategies to solve problems are valued. They must learn to rely on themselves and their peers to validate their ideas, and they must learn that application of mathematics to real world situations is what is done in their classroom community. Curiosity and individual and collective ownership over learning need to be valued within the classroom, and students need to develop not only the ability to participate in the community of practice but also have an awareness of what is valued and expected as well as what is not.

When students were asked at the end of the research study to identify five characteristics of our community of learners, their responses fell into five distinct categories. Seventeen students were present for the journal response, although not all seventeen gave five characteristics. Some students also repeated similar ideas. Table 8.2 (p. 138) summarizes the five categories that were evident in their responses. The labels for the categories are quotations from some of the students' journal entries on June 9.

Table 8.2

*Summary of Characteristics of Community Identified by Students*

Characteristic	Number of students
We don't use text book or work sheets very much	14
We do lots of hands-on projects	14
We do a lot of partner and group work	13
We solve problems by thinking for ourselves and we learn not to ask the teacher for every little thing. We learn to think for ourselves and ask others for help	6
We learn to apply math to the real world	5

Other, less common comments included four comments about the course being fun, three comments about liking the teacher, and two comments each about lack of homework and tests in the course, which is not unlike the first characteristic about textbooks and worksheets being used minimally.

The student responses to the request to identify characteristics of their classroom community were surprisingly similar, which demonstrates a common sense of community among students. Similarly, the use of the word “we” by many students suggests an implicit understanding of the collective nature of the community as well as ownership of it. The fact that five characteristics emerged so prominently in the descriptions of students as well as the dominance of the first three characteristics points to the establishing of social norms and practices within the community that were broadly accepted by its members. Students understood what it meant to participate in their community of learning and were able to articulate the characteristics of the community in a fairly cohesive way. A sense of community was evident amongst the members of the research community, demonstrating the presence of this important characteristic of a community of practice characterized by mathematical inquiry.

I fostered the development of such a sense of community in several ways. The primary way was talking openly about norms and expectations within the classroom. For example, students were told regularly that I would not tell them the answer and that they needed to rely on themselves and their peers to figure out how to solve problems. We also used verbs to describe math and talked openly about what it was that we *do* in this course, classroom, and community, often referring to “hands-on” activities, class discussions and coming up with strategies for solving problems. Another way of fostering

the development of a sense of community in the classroom was through the types of experiences I created for students. Students had repeated experiences of participating in hands-on activities, working collaboratively, and making connections to real world contexts. These experiences moulded their sense of community and helped them recognize the characteristics of their own classroom community in the process.

In order for the members of a community of learners to be able to identify the characteristics and purpose of that community, they must have developed a solid understanding of what the community is assembled for and what their own role is within it. They must have understood the social norms of the community and must have been able to operate within those norms to contribute to the community. Paying attention to the development of this sense of community in their classroom can help a teacher foster the emergence of a community of practice whose byproduct is mathematical inquiry. When students recognize what is valued within their community, they are able to participate and contribute in appropriate ways to further their own understandings as well as the collective understandings of the community itself.

***Increasing Sense of Individual and Collective Ownership of Learning.***

Perhaps the biggest difference in my classroom community from a teacher's point of view this year was that students genuinely attempted to solve problems themselves and stopped asking me to tell them how to do it. This was something I stressed from the first day of the course until the last, and students accepted it as the norm in our classroom. Six students referred to this very idea as a characteristic of our community in their journals at

the end of the course, which attests to how important students thought this particular behaviour was in our classroom community.

One of the things that had always bothered me prior to this project was that many students refused to really engage in problem solving and thinking about mathematics. They tended, instead, to ask for help as soon as more thinking was required and often complained about the problem being difficult. Magdalene Lampert (1990) describes this dilemma in her characterization of how mathematics is commonly viewed:

Commonly, mathematics is associated with certainty: knowing it, with being able to get the right answer, quickly (Ball, 1988; Schoenfeld, 1985a; Stodolosky, 1985). These cultural assumptions are shaped by school experience, in which *doing* mathematics means following the rules laid down by the teacher; *knowing* mathematics means remembering and applying the correct rule when the teacher asks a question; and mathematical *truth is determined* when the answer is ratified by the teacher. Beliefs about how to do mathematics and what it means to know it in school are acquired through years of watching, listening, and practicing. (p. 32)

By letting students know on the first day that this sort of behaviour would not be acceptable in our classroom, students immediately began to reconceptualize what it meant to do math in high school. As a result, their beliefs about how to do mathematics changed. It did not take very long for students to stop asking me how to solve a problem at all. Students began telling each other that I would not tell them how to solve the problem and, eventually, they quit asking altogether.

One example of this process in action was when I was concerned about how I could get students to practice converting measures without just giving them an individual worksheet on conversions to do for homework. As I thought about the process, I decided to use an old worksheet I had, but to focus simultaneously on having students take more

responsibility for their learning. In my planning and observation journal I recorded this thinking process as well as the result:

In the past, I used to give a single work sheet on application questions. I acknowledged in thinking about what I should do that in the past, many students were needy and asked how to do the question almost before they finished reading them. This was very frustrating. Students were reluctant to try them at all. Also, I would often just look at one or two in class and then assign the rest to be done in class or at home. I would spend the entire period running around like crazy, getting frustrated at having to tell students over and over how to do each question. Then the rest would be assigned for homework, and students would come back citing that it was too hard and could I help them. This was a defeating activity so I decided to change my approach in two ways. First of all, I made the students work in partners, telling them that they had to collaborate with their partner and that I would NOT tell them how to do the question. Secondly, I spent 3 classes doing the same problems that I usually touched on and then assigned in one class. All students got to consider each question and were forced to consider it with only their partner's help. This, I hoped would eliminate both frustrating things I had previously experienced – asking for help immediately and failing to attempt the questions. I was right in so many ways. The results were astounding. First of all, even the weakest students were able to try with the help of partners. The collaborative work was excellent. When a student asked me for help, I didn't even have to respond; the other students told the student immediately that I was not going to answer, and that they had to try the question with their partner first. Talk about an amazing transformation of classroom norms and expectations! Secondly, we were able to really discuss the problems at the end and students provided their own work and solutions for discussions. A few times, different sets of partners provided different but equally effective solutions to the same problem. Wow! What an amazing use of time. The three classes were at least 3 times as effective as the past methods were. (Planning and Observation Journal, 2008, October 26)

The use of problem solving partners was a key strategy for me in having students rely on themselves and their peers to solve problems. This was required if I was to foster both a sense of independence as well as interdependence within the classroom community and if I was going to see students decrease their reliance on the teacher for validation of mathematical ideas.

Another feature of a community of practice exhibiting an increasing sense of individual and collective ownership of learning was that students felt responsible for and took pride in their own learning as well as the products created in the process of that learning. This was something I thought about continually when planning activities for students. I often thought about how students could *make it their own*, not wanting the activities they were engaged in to all be about what the teacher was imposing on them. I looked for ways that I could give students open activities that would allow them to be creative and develop a sense of ownership and pride in their accomplishments.

Following the episode involving problem solving partners working on word problems described above, I engaged the partners in the process of creating their own measurement problems, in the hopes of helping students develop a sense of ownership of the problems they were looking at. Having students create their own problems proved to be a good exercise for several reasons, as I noted in my journal:

It was interesting to see that this really allowed me to differentiate instruction. Students who were weaker and had less conceptual understanding were able to model a problem after one they had already seen. Students with deeper understandings naturally tried to create something new and different, often relating it to their own lives. What a great outcome! All students were able to participate and grow. Students naturally were curious about the problems others were making and they talked openly about where conversions were needed. (Planning and Observation Journal, 2008, October 26)

When students were able to create their own problems, the level of engagement increased. Students were proud of their problems and talked about their problems with others. Both the solving of problems and the creating of problems in problem solving partners provided a vehicle for practicing skills and allowing students to rely more on themselves and their peers for validation of and feedback on their ideas. In addition to

this, creating their own problems gave students the sense of ownership that enabled them to be more engaged and interested in sharing their work as well as in the work of others.

Out of all of the activities students were engaged in during the course, ten students identified the Student Lounge Project as their favorite activity. Also, the DaVinci Project and the Performance Task after the DaVinci Project were identified as favorites by four and three students respectively. I think that this was largely due to the ownership students felt as they participated in these projects. The Student Lounge Project allowed students to be creative and make it their own. It allowed them to inject personality and be proud of their design, which students really enjoyed. One student, whom I will call Brittany, indicated: “My favorite activity was the student lounge because it gave me the most freedom. I also enjoyed decorating the room” (Brittany’s Interactive Journal, 2009, June 9). If I were to reflect on the activities, I also would choose the Student Lounge as my favorite activity precisely because of its open nature and opportunities for students to be creative. Students were very engaged in the project and they produced phenomenal work. Figures 9.2 and 9.3 below are photos taken of two of the models created by students. An extraordinary amount of effort was put into these as is evident in the photos.

Figure 8.1  
*First Sample Student Lounge*



Figure 8.2  
*Second Sample Student Lounge*



Not only do the student lounge models in the photos above indicate tremendous effort, they were also impeccably done to scale, down to the tiniest of details. Mathematically, the students' work was as intricate as it was aesthetically pleasing, which was phenomenal to see as a teacher. When students are able to have ownership both individually and collectively of their work and of their learning, the level of engagement and quality of their work increases tremendously.

### **A Portrait of a Community of Practice Characterized by Mathematical Inquiry**

If I were asked what my classroom looked like as I attempted to foster a community of practice characterized by mathematical inquiry, I would have to say that the answer was characterized by what the members of the community were *doing* as well as in what they were *saying*. In this research study, students came into their classroom asking what it was that we were going to be doing that day. They expected to engage in something hands-on, collaborative, or both when they arrived in the classroom. The activities that they engaged in within the classroom community were both mathematically and communally rich, although students really only recognized that they would be doing something that had to do with mathematics and that they would most likely be working in groups. While working on these activities, students generally recognized that they were expected to discuss ideas and come to a consensus within their groups about how to proceed. They were often told that they would need to share their work or strategies with the class following the activity and were used to discussing, comparing, and evaluating different strategies in classroom discussions. Sometimes students were required to create extended

pieces of work such as the DaVinci PowerPoint presentations, or the presentation of plans during the Student Lounge Project. Such presentations of work were generally celebrated as significant accomplishments, and students took great pleasure in sharing them.

Discussion was encouraged from all community members and curiosity and application to real world contexts were valued and promoted. Students were encouraged to think for themselves but also to listen to the ideas of others, add to them, and reformulate their own ideas about mathematical concepts. Respecting other members of the community and allowing them to participate in making decisions and in discussions was emphasized.

If a person was to walk into our classroom community, on most days they would have immediately noticed one of two things: Either they would have seen a classroom discussion going on as some students presented their work on a project or activity of some sort, or they would have seen groups of students, dispersed about the classroom, working together energetically on a task. What they may not have noticed, unless they watched and listened closely, was that the students would not have been asking the teacher how to solve a problem. Instead, the students would have been discussing with other students how they would solve a problem or comparing their own methods with the methods of others. What they likely would not have noticed was the increasing confidence students had in their own abilities to *do math* or the sense of community that was growing by the minute within the classroom. While collaboration could have been witnessed from the outside looking in, qualities such as confidence, ownership, curiosity, and connections could only have been witnessed from the inside out. My goal as a teacher was to create a culture within the classroom that would allow such inner qualities to emerge externally, through the actions of each individual within the community.

If, as Cobb (2000) suggests, student activity and the classroom culture are reflexively related, then attention must be paid to the development of a classroom microculture in which students can learn and grow. By attempting to foster the five characteristics described above as well as the picture of learning described thereafter, the teacher pays attention to the development of a classroom community that is conducive to the emergence of mathematical inquiry as a byproduct of the interactions of its members. It is only through careful attention being paid to the development of community that this can happen, making this perhaps the most important role of the teacher hoping to affect change in mathematics education.

## Chapter 9

### *Challenges of Fostering Such a Community of Practice*

While viewing learning as complex participation in a community of practice characterized by mathematical inquiry changed my teaching practice as well as the classroom community itself, attempting to integrate theory and practice was not without some challenges. Characterizing the community without acknowledging these complexities would not be a true depiction of the community of practice I saw emerge. During the course of the study, I noted several challenges as I attempted to change my teaching practice: my own human limitations made it difficult to have the energy required to be *always on* as a teacher, prompting, encouraging, and discussing things with students; the noise level and mobility of students working on the activities challenged my beliefs about what good classroom management and control were; evaluation and curriculum coverage became issues for me as I renegotiated my classroom community and the activities I engaged students in; difficulties with students disengaging and “riding the coat tails” of others posed some problems for me as an educator; and falling back into old ways and the subsequent feelings of guilt all emerged as challenges as I attempted to teach with this view of learning in mind. These challenges are an important part of characterizing both the changes in my teaching practice and the community of practice that emerged within my classroom. This chapter will discuss each of these five challenges, in turn, as I attempt to characterize not only the positive changes but also the challenges I faced as I attempted to allow theory to inform practice in my own classroom.

One of the most difficult parts of attempting to integrate theory and practice for me over the course of the research study was my own human limitations. I found it very difficult to change my own teaching practice so dramatically for such a length of time, and what I theorized about in the planning phases was not always what was enacted in the classroom due to a variety of human limitations. The research project required me to essentially rewrite my teaching curriculum unit by unit, piece by piece, while still continuing to teach six other courses at the same time. I attempted to let my experiences in the classroom inform further practice, and so, I was not able to plan the entire year ahead of time. While this was beneficial to me in that I was truly able to allow my practice to inform further practice, it put significant pressure on me to continually be developing new activities, while still trying to keep up with the demands of my teaching load and research responsibilities at the same time. In addition to this, I was at times tired and unable to put forth the energy required to constantly take on the role of the prompter in the classroom, engaging students at every turn. What I envisioned my own role to be did not always happen due to these factors. This problem definitely emerged as a theme in my own notes. I found it difficult to be *always on* and questioned the sustainability of teaching this way. In hindsight, however, I now think that once a teacher builds up a repertoire of mathematically and communally rich learning activities, much more of her time and energy can be spent on paying attention to the discussions and prompting students to consider mathematical ideas more deeply. A good part of the difficulty came for me from trying to develop the curriculum, conceptualize the community, and conduct research at the same time.

Another complexity that emerged as a result of this research and viewing learning in this way, was that I found that the hands-on nature of the activities caused a classroom environment that was, at times, at odds with my fairly traditional beliefs about classroom control and management. Subconsciously, at least, I had always thought of a good teacher as one who had control of her students. This meant that the students were quiet and were seated, working individually or with a group. I was forced to confront this view of classroom control when it did not match how my students engaged in the mathematically and communally rich learning activities I had planned. I was surprised by my discomfort when students were moving around the school completing tasks or when my administrator would come and ask what we were doing. In theory, I knew that what the students were engaged in was valid and that great learning opportunities were occurring; however, the physical mobility and volume that resulted contradicted my own beliefs about what it meant to have good classroom management. This caused some very mixed feelings on my part. I felt distinctly uncomfortable during the Student Lounge Project as students needed to move about to get materials for their models. I also felt uncomfortable during the Trigonometry Challenge when my administrator came out of his office to see what was going on as we were outside with meter sticks, string, and tape measures, trying to estimate the height of the flag pole. For change to truly occur, sometimes it has to challenge an educator's deep-seated beliefs. For this reason, I both acknowledged and embraced this necessary dissonance I saw emerge during the study. To deny its existence, however, would not be an accurate portrayal of change in my teaching practice or change in my classroom environment.

Evaluation also formed a significant challenge for me as I attempted to allow theory to inform my teaching practice. While changing and renegotiating the classroom community and the activities in which students were engaged, the need to assign a percentage on report cards was not something that I had license to change. As I described earlier, I struggled with evaluating students individually while they worked collaboratively. I worried about how valid or reliable their marks were, and I struggled intensely with the notion of testing. I originally thought that I would be able to evaluate individual students by observing and asking questions as they worked. In reality, this was much more difficult than I predicted. I had enough difficulty just prompting students and working at developing opportunities for discussion. Evaluating at the same time proved almost impossible. As a result, in the beginning I continued to use some tests as a form of summative evaluation. By the end of the course, however, I began to incorporate performance tasks (see Chapter 7 and Appendix J) to evaluate individual achievement. I think this trend will continue for me. Performance tasks offer a viable solution to the problem of grading and assessment of learning.

Another challenge that I faced during this research study was time constraints. The mathematically and communally rich learning activities that I engaged students in took time, and as a result, there was no way that I could expose students to the entire curriculum for the course. I was lucky in that I taught the same group of students Precalculus Mathematics as well, and so I was able to pick up some of the overlapping content in the Precalculus course so that students would not be missing as much of the content after this study. Curriculum coverage and time constraints are a significant challenge, however, and this challenge has to be addressed by any teacher attempting to

use such time consuming activities in their practice. Some of the content will inevitably have to be eliminated if one is to teach in this manner, which is a dilemma for all concerned.

One of the most obvious challenges to collaborative group work is students using it to disengage or “ride the coat tails” of others. This was a problem for a handful of students, and it was something that I believe could be addressed through more specific and careful establishment of norms and practices in the community. While I did not address this problem by doing much more than trying to encourage those students to participate, I feel that in the future I could explicitly call attention to this behaviour, hopefully deterring it. Just as students learned that they were not allowed to ask the teacher to validate ideas, they could also learn that they are not allowed to disengage and ride coat tails in this community. It is something that would require critical evaluation and noticing by the teacher, and in addition to explicit class discussions about such behaviour, a teacher would need to have more personal discussions with individuals prone to this behaviour so as to curb it.

Guilt was the final emergent challenge I noted during this research project. Perhaps more precisely, the recurring falling back into old ways of doing things and subsequent feelings of guilt were an emergent theme during the course of the study. It is difficult to attempt enacting theory in practice, and the complexities of attempting to do so make the challenge almost impossible. Theories are perfect by nature, and teachers and classroom communities are not. In order for change to occur, however, teachers must engage in attempting to allow theory to inform practice in the messiness of the classroom. It is here where educational reform is enacted, as imperfect as it might seem. During this

research study, I found myself falling back into old and familiar ways when I was under pressure to assign grades, or to cover certain elements of curricular content. When time frames mattered, or when grades were due, I tended to revert to using tests or giving the odd worksheet to students to try to speed up the process or get things done quickly. When I was tired and unable to come up with something new in a short period of time, I found myself adding in things I had used before, even if they did not necessarily fit with my view of learning in this way very well. This was almost an act of self-preservation on my part, as I attempted to alleviate the pressure put on me as an educator. It was during these times that I could see the tendency emerge to go back to old ways of teaching, and guilt for not teaching the way I had planned would ensue. Again, I noticed and embraced the feelings of guilt, and areas where I found myself falling back into old and familiar ways of teaching. These were the areas in which I found the greatest challenges: in times of great physical and personal stress, when time constraints loomed, and when I needed to come up with summative grades. It is in these same areas that other teachers would likely experience challenges with and fall back into old and familiar ways of teaching. It is also in these areas that the greatest amount of attention must be given if viewing learning as complex participation in a community of practice characterized by mathematical inquiry is to impact my practice as well as the practice of other educators who wish to view learning in this way.

## Chapter 10

### *Informing Practice and Theory*

One of the goals of this research study was to characterize how theory informs practice for me as an educator. This goal was based on the belief that in order for educational theory to change practice in a broader sense, that is for educational reform to occur as a result of theory, the ideas of educational theorists must first be enacted in practice. While my own experiences can never be the same as those of other educators, some parts of them may be similar, making sections of my research applicable to other teachers and their teaching practices. The first part of this chapter is devoted to allowing my own experiences and the changes I made to my teaching practice to inform other educators who view learning as complex participation in a community of practice characterized by mathematical inquiry. In the second part of this chapter, I will discuss how my own experiences through this research project can inform the educational theory from which my theoretical framework was drawn. In particular, I feel that my experiences support the view that mathematics classrooms can be seen as “adaptive and self-organizing complex systems” (Davis & Simmt, 2003, p. 138). By extending my experiences outward, I hope to inform not only other educators who view learning as I do, but also the educational theories that have contributed to this view of learning. Teaching, theory, and research, as such, may be seen as part of a complex system of their own, evolving simultaneously, influencing each other.

### **Allowing Practice to Inform Broader Educational Practice**

An important question as I consider the overall product of this research study is “What do I hold in high enough esteem still to pass on to others who would embark on a similar journey, who seek to change their teaching practice, and who view mathematical learning as I do?” The answer to this comes, for me, in the form of five recommendations. I feel that educators should consider the following five aspects of teaching practice:

- the characteristics they are fostering in their classroom communities,
- what sorts of activities they engage their students in,
- what their role is in the classroom,
- how they evaluate student understanding, and
- how educational research can be of benefit to them.

By considering these five aspects, educators who view learning as complex participation in a community of practice characterized by mathematical inquiry can begin to change their own teaching practice in order to foster a community of learners whose byproducts are mathematical inquiry. In doing so, I believe that a movement can be made towards broader educational reform, thus allowing this view of learning to emerge in the educational practices of mathematics educators everywhere.

My first recommendation is that educators must consider the characteristics they are fostering in their classroom communities and ask themselves if these are the characteristics that will promote mathematical thinking and learning. All too often I thought primarily about content in the process of planning educational activities for my students, and if there was a secondary thought, it was usually about time or scheduling. It

is very important for teachers to think about what they want the characteristics of their classroom community to be and then plan to achieve them. For example, two important things that I wanted to change about my classroom community were the boring nature of textbook work and the disengagement I saw in students as they refused to even try to solve difficult problems. I saw in my own practice that I created such a culture by telling students how to do problems and by letting them quit rather than try. I fostered an environment in which students thought that math was about doing work sheets and textbook assignments rather than solving problems and applying mathematics to the real world. My suggestion for other educators is to look at the culture that is being cultivated in your own classrooms. What is good about it? What is bad? Every time that a lesson is planned or an activity is created, the thought that should be in an educator's mind is twofold: How will this activity help understand the content students are exploring, and how will this activity help to foster a culture or community that is more like what I want it to be? Even when an old activity is being dusted off for use again, the question should be: Now how can I use this not only to look at the content I have in the past, but also to improve the community of practice that exists within my classroom. This method of looking at both content and community in the planning process, which I have previously referred to as *parallel planning*, has a lot of potential to help educators pay attention to their classroom communities and improve the learning environments in which they participate.

In addition to paying attention to the characteristics they are fostering within their classroom communities, secondary educators in particular should also consider what sorts of activities they engage their students in. It has been my experience that secondary

teachers could take some advice from many elementary school mathematics teachers, because there seems to be a reluctance among many secondary teachers to engage students in hands-on, experiential learning. In saying this, I do not discount the very real pressures of class size, overladen curricula, departmental and provincial examinations, or the desire for consistency from one class to another or even one school to another. These are factors which, I suspect, have contributed to a general reluctance amongst secondary mathematics educators to engage students in activities such as the mathematically and communally rich learning activities I have proposed. What is more important to me is that students need to learn how to solve problems. They need to have experiences relating mathematics to the world around them and using mathematics in useful ways. If a picture is worth a thousand words, then an experience must be worth a million. More effort must be put into giving students experiences that allow them to make connections with mathematics. Only then will they see mathematics as something that is done, not something that is observed, or completed, or remembered.

Another suggestion I have for educators as a result of this research study is to consider what they think their role is in the classroom. I have reconsidered my own role in the mathematics classroom. At one time, I believed my role was to show, or present, students with the mathematical content. My perception of this has changed, however, leading me to consider my own role differently. I began to notice this during the research and noted it several times in my journal:

I am noticing that I am approaching teaching with new eyes. A description of teaching as participation in *Engaging Minds* comes to mind here:

A popular set of participatory-oriented synonyms for teaching has yet to emerge, although a number of suggestions have been put forward. The list includes *improvising, occasioning, conversing, caring, and*

*engaging minds*. To varying extents, these notions are intended to highlight the qualities of contingency, flexibility, emergence, and expansive possibility. Once again, the critical break with entrenched perspectives is in the realization that teaching is not about *telling* or *directing*, but *triggering* and *disturbing*.” (p. 171-2).

I notice that I am anticipating what will happen when I am planning, although I rarely know. I notice that I am adapting continuously to see if I can start a conversation, line of thought, or interesting discussion. I look for differences in student thought as well as ways to examine those differences and celebrate them. I relish in the moment when I see some of the characteristics I think represent a community of practice characterized by mathematical inquiry emerge such as the verifying of mathematical strategies and their appropriateness amongst community members. These are the things I strive to cause and that I consider success. Moreover, I know theoretically why I value these things. Really an epiphany for me. (Planning and Observation Journal, 2009, March 4)

As my concept of teaching changed, and as I created activities that both addressed content and community, my view of what it meant to teach necessarily changed as well. If I was not standing at the front of the room presenting or telling, then I had to consider what else I could be doing to help students understand content and help foster the community of practice I wanted to see emerge within my classroom. For me, this meant taking on the role of prompter, trying to look for ways I could start a line of thinking or a conversation with students that would help them make sense of the activity in which they were engaged. It is through the conversations with students amidst the noise and chaos of hands-on activities that sense is made of mathematical ideas and connections are made between our world and mathematics. It is also here that I learn what a student knows and does not know, what they might be capable of, and how I can best encourage them to think in new directions. My own role is much more complex than I originally thought, and yet much more satisfying.

Evaluation is also an area for consideration amongst educators today. If one is to engage students in the sorts of mathematically and communally rich activities that have been described in this research project, one will likely find that standard forms of evaluation such as practice sheets and tests no longer seem appropriate or adequate for evaluating student achievement. For me, I found significant dissonance between using formal tests as an evaluation tool and using the mathematically and communally rich learning activities I had created. Traditional tests did not match what students were used to experiencing and they did not account at all for the communal part of my parallel planning model. Instead, I found myself moving toward the use of anecdotal record keeping, questioning, and observation in formative assessment, and the inclusion of performance tasks in summative assessment. These performance tasks matched what students had been doing, and required students to show me what they knew about the mathematical topic at hand. For example, at the end of the linear models unit, students participated in a summative performance task that required them to choose a linear relationship and use it to show me what they knew about linear models. Not only did the performance task help me evaluate student understanding of linear models, it required students to engage in a very open problem for the purpose of evaluation. They had to formulate an argument to show that the relationship was linear and model the data using a linear model. They had to make predictions using that model, and they had to demonstrate to me in the process what they knew about linear models and patterns. This sort of evaluation tool matched what students had been doing in the activities they were engaged in, and it put the responsibility on students to demonstrate understanding. I

believe that educators must consider alternate forms of assessment of student learning and that the assessment must match what students do in their classroom communities.

My last suggestion for educators concerns the role of educational research for classroom teachers. While this research was done as part of a Master's program of study, I encourage all educators to participate in the process of formal or informal forms of practitioner research. The process of engaging in reading and discussions about educational theory and methods of putting that theory into practice is invaluable for educators in all areas. If it had not been for the formal research process I found myself engaged in, I am not sure that these changes to my teaching practice would have occurred. They certainly would not have occurred so quickly and on such a scale as they did. It is important for educators to treat their teaching practice as ongoing research and to engage in the process of making it better.

### **Allowing Practice to Inform Educational Theory**

Brent Davis and Elaine Simmt (2003) suggest that “mathematics classes are adaptive and self-organizing complex systems” (p. 138), looking beyond individual and social learning models, and towards the notion of the collective in a mathematics classroom. Davis and Simmt propose that complexity theory can be used pragmatically as educators look not only at the emergence of the collective as a cognizing agent, but also at how the emergence of collectives with “transcendent possibilities” (p. 145) might be caused. It is through looking at how such collectives might be caused that complexity theory offers practical advice for teachers. Educators who seek to foster the emergence of a collective

that is capable of more than the individual agents are on their own, can turn to the fundamental principles of complex systems and apply those principles to the complex dynamics that exist within a classroom community. Through comparing the elements of complex systems to those that exist within a concrete classroom community, more can be understood about how to cause the emergence of a collective whose byproducts are mathematical thinking and learning.

According to Davis and Simmt (2003), there are five conditions that must be met for complex systems to thrive and for learning to emerge: “(a) internal diversity, (b) redundancy, (c) decentralized control, (d) organized randomness, and (e) neighbor interactions” (p. 147). In order for a teacher to foster the emergence of a collective capable of more than its individual agents alone, she must ensure these five conditions are met within the classroom community. Only then can the complex system thrive, or as in the case of education, can the collective achieve its transcendent potential.

Although these five characteristics were discussed in the literature review in chapter 2 of this document, it is worthwhile to note how changes in my own teaching practice moved towards the establishment of these five conditions. While my purpose in changing my teaching practice was not to draw comparisons between complex systems and a mathematics classroom community, the changes in my teaching practice that resulted from this study further strengthen the notion of classroom communities as complex systems. As I worked towards the establishment of a collective whose byproducts were mathematical inquiry, I found the changes that I made strengthened rather than weakened the conditions identified by Davis and Simmt within my own classroom community. Internal diversity, redundancy, decentralized control, organized

randomness, and neighbor interactions were all conditions that not only existed within the community of practice established within my classroom, but were also strengthened as my teaching practice changed.

The condition of internal diversity is one that occurs fairly naturally within a classroom community. Each community of learners is inherently different due to the individuality of each agent within the community. However, this does not mean that simply being different is enough to create the condition of internal diversity. For example, if students are different and yet do not express their differences in the public domain of the classroom, not much is accomplished despite their diversity. However, during this research study, as I worked towards the goal of having students work collaboratively, taking responsibility for their own learning as well as the learning of their peers, the potential interactions and use of the existing diversity was heightened. By encouraging discussion of mathematical ideas and strategies, the interactions, and thus the variety of mathematical thoughts and learning trajectories expressed, were greatly increased. While my goals were not to increase the diversity of the students within the classroom, it was very much my goal to increase the diversity of ideas that were expressed within the community. This is a valuable point in terms of the connection between complexity theory and education for me. It is also where I find complexity theory has a lot to offer educators pragmatically. Perhaps the question really is how to optimize the condition of internal diversity in the mathematics classroom to optimize learning. For me, by encouraging and celebrating different viewpoints and methods of solving problems, I believe students were more willing to share diverse solutions and thought patterns. When differing strategies emerged and were discussed, the entire

collective benefitted from the expression of those ideas. For example, when Sarah's group (see p. 113/114) chose scale drawing as their strategy of finding the gym and flag pole heights instead of using trigonometry during the Trigonometry Challenge (see Appendix G), the entire class benefitted from the different perspective and strategy that was expressed. The classroom collective had a much stronger understanding of the problem and potentially of trigonometry as ratios due to the expression of different strategies during the activity. Internal diversity, while inherently present in any community of individuals, can be optimized by educators such that the potential learning of the collective is increased. This is, perhaps, where educators can place their focus.

The condition of redundancy proposed by Davis and Simmt refers to the fact that in order for individual agents within a complex system to interact, they must have some similarities, or redundancy. In order for members of a classroom community to interact, they must speak the same language, understand the same letters and symbols, have similar experiences and expectations, and share a common purpose. While redundancy was not my explicit goal in attempting to foster the emergence of a community of practice characterized by mathematical inquiry, it was implicit in the goal of developing a community of practice. In order for the fourth characteristic I identified to emerge, that is a sense of community amongst members, redundancy was essential. Students needed to have an understanding of what made them similar as well as what the expectations were within their classroom community. The mathematically and communally rich learning activities that were planned to engage students were by their very nature developing community. Students were expected to interact and collaborate. They were encouraged to discuss mathematical ideas and strategies, and the activities had not only mathematical

content as a focus, but also strengthening the community, which meant creating norms and practices that were established in the minds of all community members. This was essentially creating the condition of redundancy. Students began to get a much stronger picture of what participation in the community meant, and what it did not. Commonalities were established as were common expectations and norms. In addition to this, students engaged in activities that provided shared experiences, about which they were able to communicate. For example, even at the end of the course, students still spoke about the Student Lounge Project (see Appendix F) and using a scale factor in creating a model. The shared experiences provided a basis for discussion and enough redundancy for reflection to occur. Similarly, students were able during the Performance Task (see Appendix J) to compare their linear models with the models of others based on their common experiences. While the data was different, the shared experiences creating linear models in a variety of ways gave them individual and collective understanding of what linear models were and how they were represented. As a result, they were able to describe to others the relationships they were investigating. This is something I had underestimated in the course of my research. The more shared experiences that exist within a group, the greater the potential for interaction about them. Towards the end of the course, when students became more confident working with linear models, discussion about their data and their interpretations increased. Students became more confident, individually, in their ability to interpret the data using linear models, but they also became more confident in the ability of other members of the community to understand and listen to their interpretations of the data. This was interesting, and illustrates nicely the importance of redundancy within a classroom community.

Decentralization of control was a condition of complex systems that I worked more explicitly to foster within my classroom community throughout this research project. I agreed with Davis and Simmt in their recommendation that rather than taking a very teacher-centred or student-centred approach to learning, a teacher should pay attention to the collective, attempting to create “shared insight” (p. 153) within the community. As a result, one of the five characteristics I attempted to foster within the classroom community was an increasing sense of collective and individual ownership over learning. In addition to this, I wanted to foster a sense of community amongst members. Both of these characteristics were aimed at decentralizing the control within the classroom. I wanted students to be dependent on themselves and their peers to solve mathematical problems, not on their teacher. I wanted the conversations within the classroom to evolve and grow into something collectively, owned by all of the students collectively. While I designed and structured mathematically and communally rich learning activities that gave rise to such discussions, I wanted the bulk of what students did in the classroom to be centered around discussion of ideas and problems rather than on individual work. This is where the complex nature of the classroom community emerged. While in the past, I often was the center of control in the classroom, working examples on the board and showing students how to solve problems, I found myself expecting the students to determine how they would solve problems. My impression of the result was that the knowledge of the group, collectively, was much richer because of this change to my teaching practice. Multiple solutions to problems were discussed, and students gained confidence in their own ability to contribute to the knowledge that was held by the community. Perhaps teachers should pay attention to the development of

“shared insight” or collective knowledge generation. Perhaps one of the keys to creating a community of practice characterized by mathematical inquiry lies in carefully negotiating control within the classroom community such that students are focused on mathematical tasks but feel empowered as an individual to contribute to the collective understandings of the community. This delicate balance requires both a renegotiation of the role of the teacher and the expectations of what it means to be a mathematics student. I believe this research project provided a glimpse of what is possible as one teacher attempted to find this balance.

Organized Randomness, Davis and Simmt’s fourth characteristic of complex systems, applies pragmatically to mathematics education and teaching practices in that the environment of a classroom community must be structured enough to generate mathematical thinking, and yet still open enough to allow mathematical ideas to emerge that may add to the collective understanding of the members of the community. In the past, I often used problems and activities that were closed, having really only one solution. My own teaching practice changed significantly as a result of this research project, as I struggled with how to engage students in activities that were open enough to allow multiple solutions, strategies, and ideas to emerge within the context of the interactions occurring in the classroom. The multiple strategies expressed during the Trigonometry Challenge (see Appendix G) were an example of how the community benefitted from organized randomness. Students were focused on a mathematical task, which generated thinking and strategizing about a problem, and yet it was through the variety of solutions that rich collective conversations and understandings emerged. Davis and Towers (2002) refer to the notion of creating activities that are structured enough to

promote mathematical thinking and yet open enough to allow for the emergence of many ideas or strategies as *structuring occasions*, which is an apt term for this complicated process of developing activities that allow for organized randomness within the complex learning system. Developing such occasions, or rich learning activities, requires conducting anticipatory thought experiments (Cobb, 2000) in order to identify possible learning trajectories and careful attention must be paid to allow for other learning trajectories that might emerge during the activity itself. Through the changes to my teaching practices, I believe that the organized randomness within my classroom community was optimized, and by paying attention to creating an environment structured enough to generate mathematical thinking and open enough to allow other ideas to emerge, I believe a community of practice characterized by mathematical inquiry emerged.

In Chapter 3, I quoted Davis and Simmt (2003), who noted that “group work, pod seating, and class projects may be no more effective at occasioning complex interactivity than traditional straight rows – if the focus is not on the display and interpretation of diverse, emergent ideas” (p. 156). The changes to my teaching practice went far beyond simple group work or pod seating. The primary focus for me became discussion, reflection, and evaluation of mathematical ideas and strategies within my classroom community. As a result of these interactions, rich learning opportunities and complex mathematical understandings were generated. Davis and Simmt’s fifth characteristic also provides pragmatic information for teachers. By focusing on discussion of mathematical ideas and strategies, teachers can promote the neighbor interactions that result in the emergence of individual and collective mathematical inquiry. While the activities of a

complex system must be focused on mathematical thinking, they must also be open enough to allow for diverse ideas to emerge. By creating opportunities for discussion of the emergent ideas that surface within the classroom, teachers can promote neighbor interactions, thus optimizing the functions of the mathematical learning community as a complex system. Several times during the research study, ideas emerged within the context of activities and discussions that lead to deeper understandings. By paying attention to the learning system and the neighbor interactions within it teachers may be able to foster not only the emergence of a community of practice characterized by mathematical inquiry, but also the development of individual and collective knowledge that transcends the capability of the individual agents alone.

## References

- Boaler, J. (1999). Participation, knowledge and beliefs: A community perspective on mathematics learning. *Educational Studies in Mathematics*, 40, 259-281.
- Cobb, P. (1994). Where is the mind: Constructivist and sociocultural perspectives on mathematical development. *Educational Researcher*, 23(7), 13-20.
- Cobb, P. (2000). Individual and collective mathematical development: The case of statistical data analysis. *Mathematical Thinking and Learning*, 1(1), 5-43.
- Cobb, P., Confrey, J., diSessa, A., Lehrer, R., & Schauble, L. (2003). Design experiments in educational research. *Educational Researcher*, 32(1), 9-13.
- Davis, B., & Towers, J. (2002). Structuring occasions. *Educational Studies in Mathematics*, 49, 313-340.
- Davis, B., Luce-Kapler, R., & Sumara, D. (2000). *Engaging minds: Learning and teaching in a complex world*. Mahwah, NJ: Erlbaum.
- Davis, B., & Simmt, E. (2003). Understanding learning systems: Mathematics education and complexity science. *Journal for Research in Mathematics Education*, 34(2), 137-167.
- Ellis, C. (2004). *The ethnographic I: A methodological novel about autoethnography*. New York, NY: Altamira Press.
- Gainsburg, J. (2008). Real-world connections in secondary mathematics teaching. *Journal of Mathematics Teacher Education*, 11, 199-219.
- Goos, M. (2004). Learning mathematics in a classroom community of inquiry. *Journal for Research in Mathematics Education*, 35(4), 258-291.

- Griffiths, M.(1994). Autobiography, feminism and the practice of action research.  
*Educational Action Research*, 2(1), 71-82.
- Holt, N. (2003). Representation, legitimation, and autoethnography: An autoethnographic writing story. *International Journal of Qualitative Methods*, 2(1). Retrieved April 20, 2008 from [http://www.ualberta.ca/~iiqm/backissues/2\\_1final/html/holt.html](http://www.ualberta.ca/~iiqm/backissues/2_1final/html/holt.html)
- Kennedy, N. (2009). Towards a dialogical pedagogy: Some characteristics of a community of mathematical inquiry. *Eurasia Journal of Mathematics, Science & Technology Education*, 5(1), 71-78.
- Lampert, M. (1990). When the problem is not the question and the solution is not the answer: Mathematical knowing and teaching. *American Educational Research Journal*, 27(1), 29-63.
- Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. New York, NY: Cambridge University Press.
- Reed-Danahay, D. (1997). *Auto/ethnography: Rewriting the self and the social*. New York, NY: Oxford International Publishers.
- Roderiguez, A. J. (2005). Unraveling the allure of auto/biographies. In W.-M. Roth (Ed.), *Auto/biography and Auto/ethnography: Praxis of research method* (pp. 119-130). Rotterdam, UK: Sense Publishers.
- Roth, W. M. (Ed.).(1995). *Auto/biography and Auto/ethnography: Praxis of research method*. Rotterdam, UK: Sense Publishers.
- Sfard, A. (1998). On two metaphors for learning and the dangers of choosing just one. *Educational Researcher*, 27(2), 4-13.

Stringer, E. (2004). *Action research in education*. Upper Saddle River, NJ:

Pearson/Merrill Prentice Hall.

Vygotsky, L.S. (1978). *Mind in society*. Cambridge, MA: Harvard University Press.

## Appendix A

### Thinking Outside the Box

#### Thinking Outside the Box

*Connect all of the dots in the picture below using only 4 lines.*



#### Thinking Outside the Box



*According to Answers.com "thinking outside the box" means breaking away from traditional or conventional thought to develop a unique, superior solution to a difficult problem.*

## Thinking Outside the Box

### Directions:

You have been given a box. Your task is to do the following:

1. Calculate the total area of cardboard used to make the box.
2. Calculate the dimensions (length and width) of the piece of cardboard from which the box could be cut.
3. Calculate the amount of waste that would result from such a cutting pattern
4. Calculate the volume of the box



## Thinking Outside the Box

5. Using the same dimensions of cardboard calculated in #2, create a box that has a better design. What do you consider a better design? Include all dimensions, areas, and the volume of the box. Present your design to the class.
6. What industries are concerned with boxes and packaging? Choose one and research how packaging design impacts their industry. Explain in your presentation to the class how this activity relates to a real-world situation.



## Appendix B

### Measurement Debate

# DEBATE

Be it resolved that Canada should adopt the US/Imperial Measurement System as its official measurement system.

Pro	Con
-----	-----

Debate Procedure

**Staters**

Pro - states position (2 min.)  
 Con - states counterpoints/position (2 min)

**Provers**

Pro - prover brings on evidence (8 min.)  
 Con - prover brings on evidence (8 min.)

**Attackers**

Pro - attacks arguments (5 min.)  
 Con - responds in kind (5 min.)

\*\*\* 5 minute break \*\*\*

**Closers**

Pro - salvages undamaged arguments and summarizes (2 min.)  
 Con - salvages undamaged arguments and summarizes (2 min.)

**Questioners**

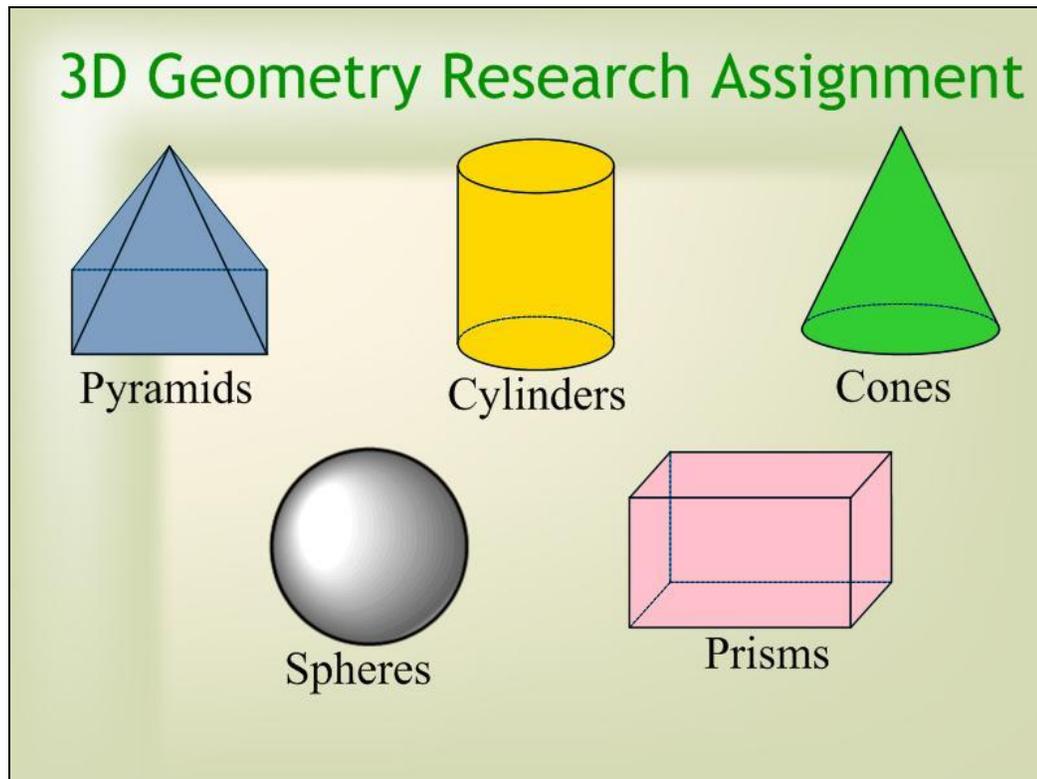
Pro - Poses one point of clarification or question  
 Con - responds  
 Con - Poses one point of clarification or question  
 Pro - responds

**Your team must prepare for the debate that will be held three classes from now. You will be graded on the following:**

- Use of supporting materials
- Grasp of the issue and important related points.
- Proper use of supporting evidence.
- Realization of points of agreement and points of disagreement.
- The ability to anticipate and counter opposing viewpoints.
- Use of supporting points not suggested by introductions.
- The ability to see and challenge flaws in the opposition's arguments and research as well as one's own flaws.
- Use of constructive criticism and rationales.
- The ability to anticipate questions.
- The ability to ask appropriate questions.

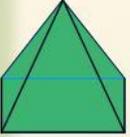
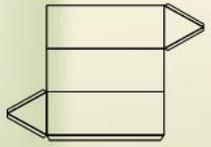
## Appendix C

### 3D Geometry Research Assignment



## The Assignment

Your group has been assigned a 3D shape. Research the shape and create a summative poster or other product to display in the classroom. Your poster/display must include:

$SA = 4\pi r^2$

- a 3D model made out of paper, cardboard, wood, or other substance
- a scale drawing of the net for your 3D model
- formulas of importance for SA, Vol. etc.

## The Assignment con't

- interesting facts about variations to the shape, uses, interesting relationships etc. (THIS IS THE IMPORTANT PART)

- Prisms can have any shape as a base - a square, a triangle, a rectangle, a pentagon as long as the sides are rectangular.
- Triangular prisms can be used to separate light

- A sample real life situation/problem where the volume of the 3D figure would need to be calculated
- A sample real life situation/problem where the surface area of the 3D figure would need to be calculated

## Appendix D

### Grandpa's Tool Shed

## Grandpa's Tool Shed

I have gathered some items I found in my Grandpa's tool shed. Your task is to figure out:

- What is the tool used for?
- What does it measure?
- How do you use it?
- Does it do more than one thing?

Present your tool and your findings to the class.

## Your own Tool Shed

Now go home and find a tool, kitchen gadget, or something else that is used for measurement. Figure out how it works and bring it to class to show the rest of us.

## Appendix E

### Mystifying Measurement Markings

# Mystifying Measurement Markings

There are measurement markings, scales, sizes and fractions all around us. All too often we use them without even thinking about what they mean. For example, Mr. Skyhar has an 800cc Polaris. What does 800cc mean? What does a 4/12 pitch on a roof mean?

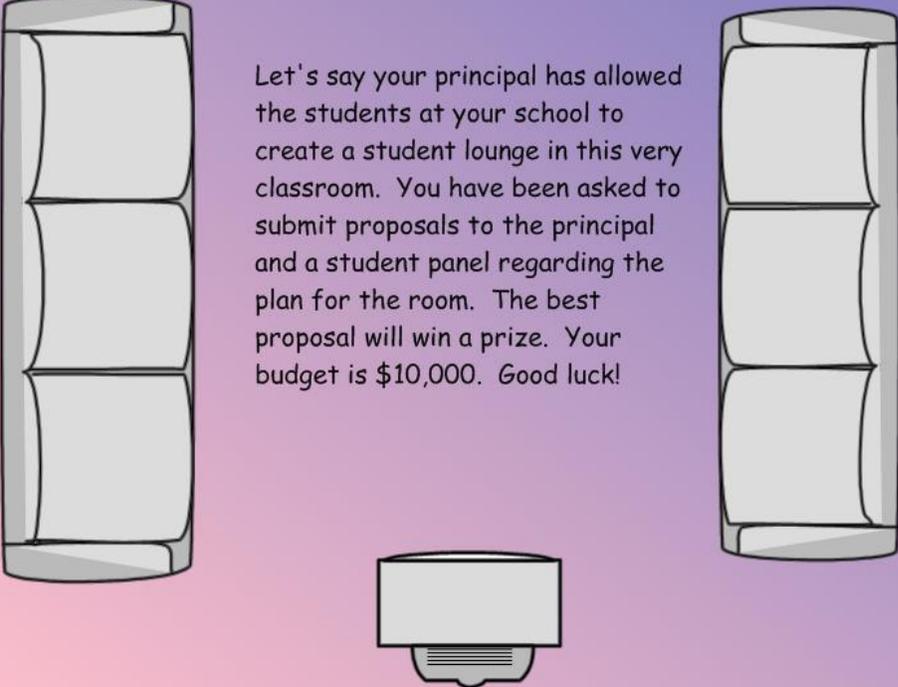


Your task today is to look at the items I have brought to class. Find the measurement markings or numbers on the item. Figure out what the numbers mean. Use any measurement instruments you like and/or the Internet. Turn in your group's sheet at the end of the class.

## Appendix F

### Student Lounge Project

**Student Lounge Project**



Let's say your principal has allowed the students at your school to create a student lounge in this very classroom. You have been asked to submit proposals to the principal and a student panel regarding the plan for the room. The best proposal will win a prize. Your budget is \$10,000. Good luck!

### Step 1: First Sketch

Your first step is to prepare a scale drawing of the classroom, empty. Take measurements of the classroom, windows, doors, etc. Then create a sketch of the room using an appropriate scale. Assume that everything inside the room (white boards, cupboards, shelves, etc.) will be removed. This should be four walls, one door and two windows. You should create drawings of the floor plan as well as drawings of each of the four walls. You will need these later.

What I will be looking for:

- An appropriate scale used consistently
- Drawings of 4 walls and floor plan (including windows and door)
- All dimensions labelled on drawings
- Area of windows and door calculated
- Area of walls calculated for paint
- Area of floor calculated for flooring

## Step 2: Second Sketch

Your next step is to redesign the space as a student lounge. You may not alter the size of the room or the location of the door or windows. Anything else goes. Walls, doors, etc. may be added. Furniture, flooring, paint, electronics, and decorative touches are a must. You will need to keep track of your purchases (price and dimensions) and work on the second sketch and cost analysis together. Pictures should be kept for the presentation to come.

What I will be looking for:

- neat, legible, ruler used, appropriate scale used
- Floor plan includes new walls, windows, doors etc. added
- Drawings and areas of new walls to be added are included
- Dimensions and area given for all types of flooring, furniture, and window coverings.
- All items are drawn in using appropriate scale

## Step 3: Cost Analysis

While you complete the second sketch and your plan, prepare a cost analysis outlining the cost of your paint, flooring, furniture, decoration, electronics, etc. Remember your budget is \$10,000 (including tax). Your cost analysis can be done on a spreadsheet or hand written. Be sure to total the final cost with taxes.

What I will be looking for:

- Organized and attractive in appearance
- Meets \$10,000 budget
- Paint and flooring calculated correctly, work shown
- Furniture, decorations, electronics included and itemized
- Total calculated with taxes

## Step 4: 3D Model



The fourth step is to create a 3 dimensional model of your plan for presentation to the class. You must scale the model, although you may want to choose a different scale than the one you used for the sketches. Every piece of the model should be to scale if I measure it. The floor, window, wall, door, and furniture dimensions will all be to scale. Use any materials that you have available to you. This model will be presented as part of your presentation and then checked by your teacher.

## Step 5: Presentation

The final part of this project will have you present your design to the class. We will judge the best design based on your presentation.

In your presentation, you should highlight the features of your design using the 3D model. Also, indicate the cost of the renovation.

Use good presentation skills as a good presentation will be reflected in your mark.

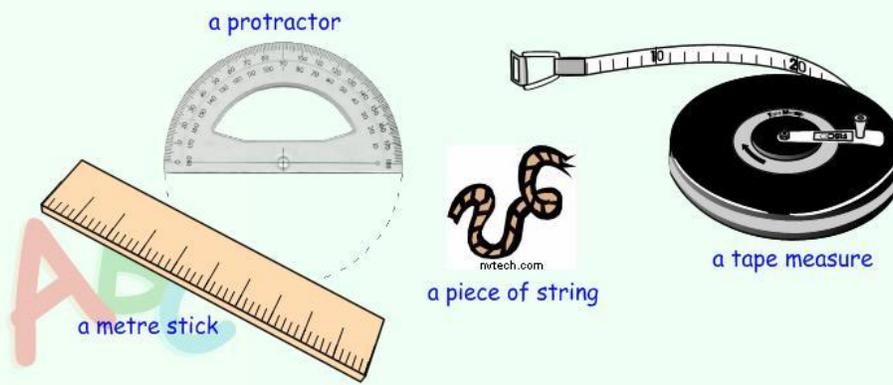
Good luck!

## Appendix G

### Trigonometry Challenge

#### Challenge

Your challenge today is to use trigonometry to estimate the height of the flag pole across the street at the Elementary School and our school gymnasium. You can NOT climb up top to measure either of these. You must use trigonometry. You can use the following things:



#### Interactive Journal

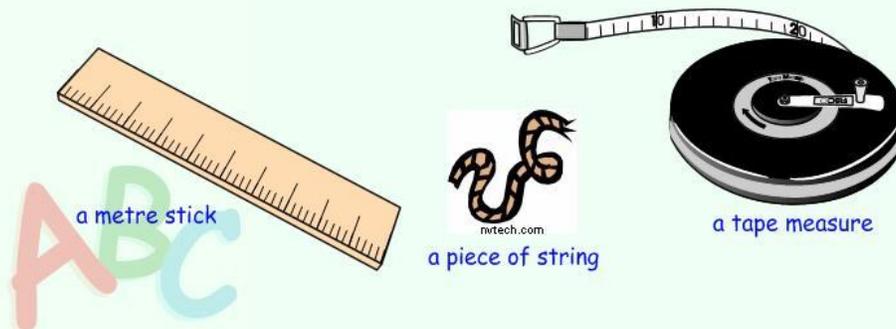
Now that you have completed the challenge, I would like you to respond in your journal to the following questions:

- What strategy did your group employ to find the two heights?
- Why did you decide to use this strategy?
- Was the strategy effective? Is there anything that would have improved it?
- Describe in your own words what you learned about trigonometry during this activity.



### Challenge #2

Your challenge today is to use trigonometry to estimate the height of the flag pole across the street at the Elementary School and our school gymnasium, but this time I am taking away your protractor. You can NOT climb up top to measure either of these. You must use trigonometry. You can use the following things:



### Interactive Journal

Now that you have completed the second challenge, I would like you to respond in your journal to the following questions:

- What strategy did your group employ to find the two heights?
- Why did you decide to use this strategy?
- Was the strategy effective? Is there anything that would have improved it?
- Describe in your own words what changed when I took away the protractor. How did this change the problem?



## Appendix H

### Ball Bounce Activity

### 20S Applied Mathematics Ball Bounce Activity

Tape a meter stick to the wall so that 0 is on the floor and the 100 cm mark is at the top. Using masking tape, mark off increments every 10 cm. These are going to be the points from which you will drop your ball.

Your task is to drop the ball from each height and record the bounce height of the ball. You will measure the drop and the bounce from the bottom of the ball. The person watching the bounce to take a reading will need to get down so that the ball is at eye level.

Drop the ball from each height five times and record the bounce height in cm. Then, discard the lowest and highest trials and average the remaining three. Do this for every 10 cm increment with 100 cm being the first drop height and 0 being the last.

<b>Observation Chart for a _____ Ball</b>							
<b>Drop Height (cm)</b>	<b>Bounce Height (cm) Trials</b>					<b>Calculations (____ + ____ + ____)/3</b>	<b>Average Bounce Height (cm)</b>
	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>		
100							
90							
80							
70							
60							
50							
40							
30							
20							
10							

## Questions

1. What is the linear regression equation that fits your data? Label each of the following in the equation:
  - a. the independent variable
  - b. the dependent variable
  - c. the slope of the equation
  - d. the y-intercept of the equation
  
2. Is this function a discrete or a continuous function? Remember **discrete** functions are those where the points in between data points do not have meaning and so they are not joined with a line. For example you can't buy  $\frac{1}{2}$  of a ticket. **Continuous** functions are those in which points between data points do have meaning and are often joined with a line or curve. For example, if you are travelling on the highway, you do not teleport down the highway only existing each hour. In between those hours, you are still travelling.
  
3. What are the domain and range of the function? Be careful – does it make sense to have a negative drop height or a bounce?

4. **Interpolation** means predicting values within a set of data. **Extrapolation** is the process of predicting values outside of a set of data. Make the following predictions and fill in the chart. Indicate whether each one is an example of interpolation or extrapolation.

<b>Drop height</b>	<b>Bounce Height</b>	<b>Interpolation or Extrapolation?</b>
30 cm		Interpolation
150 cm		Extrapolation
45 cm		
69 cm		
120 cm		
200 cm		
	50 cm	
	100 cm	
	200 cm	
	20 cm	

5. Use the data gathered from a different ball in a different group. Repeat #1-4 in the space below:

## Appendix I

## DaVinci Project

**Da Vinci's Proportions  
20S Applied Mathematics  
Inquiry Activity**

Leonardo Da Vinci was interested in the proportions of the human body. In his famous drawing *Vitruvian Man* (1487), Da Vinci drew the human body inscribed in a circle and a square. His drawing was based on the work of the Roman architect Vitruvius.

One of Da Vinci's ideal proportions was the hypothesis that a man's height is equal to his arm span. Let's see if that holds true for our class.

Your project for this unit is to research Da Vinci's beliefs about the human body's proportions and test one of them. You may use measuring instruments and what you have learned about linear regression models to aid you in this task.

You must present your findings to the class at the end of the project, giving evidence to support why or why not Da Vinci's belief about a certain proportion of the human body can be considered true. Your presentation must demonstrate logically as well as mathematically how you have arrived at your decision. Use PowerPoint to prepare slides to help you in the task of presenting this information.

If you have time, do a little extra research and tell us something more about Da Vinci or Vitruvius that is mathematically interesting.

You will have two full and one partial class to complete this project so use your time wisely. Presentations will take place on \_\_\_\_\_ . You may work with a partner if you so choose.

## Appendix J

### Linear Models and Patterns Performance Task

#### Linear Models and Patterns Final Assessment Performance Task

Instead of writing a formal test in this unit, you will be completing a performance task. This is an activity I will use to assess your knowledge about linear models and patterns. It is up to you to demonstrate your knowledge about linear models to me during this task.

For this unit, your task is to find something in your world that has a linear relationship and model it using linear models. This means you must come up with an idea, find data either by measuring or by finding data on the Internet, and creating a linear model to make some predictions about the data. The task is completely open. You get to decide on the topic as well as what your final product will look like.

You will be evaluated using a rubric. Look carefully at the rubric before you start working on the task.



Level	1	2	3
Topic Choice	Topic chosen does not relate to the world and/or does not model a linear relationship	Topic chosen does present a linear relationship, although it may be simple or have little application to the real world.	Topic chosen is linear in nature and applies in an interesting and relevant way to the real world.
Data Collection	Data collection techniques have many errors, are inappropriate or unreliable	There may be one feature of data collection which is susceptible to error, although methods are reasonably reliable	Data collection techniques are reliable and appropriate for data being sought. Care is taken to ensure few errors.
Understanding of Linear Models	There are several mistakes made in coming up with a linear model for the data or data doesn't lend itself to a linear model	One or two mistakes may be evident in work to come up with a linear model.	A linear model is correctly calculated to represent the data.
Making Predictions	Data is not used to make predictions of interest relating to the world or purpose	Data is used to make predictions, although the reason for making these predictions may not be clear.	Data is used to make predictions about the nature of the data in the real world. Why these predictions were made and their relevance is evident.
Connections to your world	How linear models connect to the real world is not outlined at all in final product.	Some indication of how linear models might fit into the real world is given, although it is not clearly explained.	A good understanding of how linear models fit into real life is evident.

## Appendix K

**Table A1: Planning Prior to the Study**

Table A1

*Planning Prior to the Study*

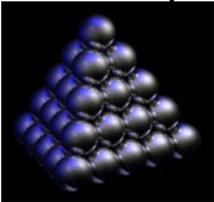
Day #	Outcomes	Methodology	Materials	Assessment
<b>Day 1</b> (1/2 class)	H-1	<ol style="list-style-type: none"> <li>1. Introduce Dreamroom Project (have students begin planning. Tonight take the measurements of the area you want to use – eg. Your spare room, your bedroom, the entire second floor, etc.) First criteria is to make a sketch of the empty space from which you will start.</li> </ol>	<ul style="list-style-type: none"> <li>▪ 2 Dreamroom sheets</li> <li>▪ Samples of Dreamroom Project</li> </ul>	
<b>Day 2</b>	H-1 H-2	<ol style="list-style-type: none"> <li>1. Mental Math #1</li> <li>2. Linear Measurement Systems Handout. – Go through the hand out together. Assign the assignment at the end.</li> <li>3. Assign Journal p. 8 (1.1)</li> </ol>	<ul style="list-style-type: none"> <li>▪ Linear Measurement Systems Handout</li> <li>▪ Small rulers with mm and inches on them. (Photocopy if necessary)</li> </ul>	<ul style="list-style-type: none"> <li>▪ Assign Lin. Meas. Systems Handout and Journal 1.1</li> </ul>
<b>Day 3</b>	H-1	<ol style="list-style-type: none"> <li>1. Technical Communication – Used Car Lot</li> <li>2. Correct last class’s assignment together for marks and record in mark book.</li> <li>3. CONVERT – Show how to link (Utility 4) Show how to load a program (Utility 5) Do Investigation 2 together as a class.</li> <li>4. Go through example 1 and 2 (p. 17/8) together.</li> <li>5. Discussion Questions – Do Orally together</li> <li>6. Assign p. 20 #1-6. Omit 6g. and Journal on page 21 (1.3)</li> </ol>	<ul style="list-style-type: none"> <li>▪ USED CAR LOT handout</li> <li>▪ Calculators (Graphing)</li> <li>▪ CONVERT linking cables</li> </ul>	<ul style="list-style-type: none"> <li>▪ Assign p. 20 #1-6. Omit 6g. and do Journal 1.3</li> </ul>
<b>Day 4</b>	H-1 H-2	<ol style="list-style-type: none"> <li>1. Mental Math #2</li> <li>2. Hand in p. 20 # 1-6</li> <li>3. <b>Quiz - Conversions</b></li> <li>4. Vernier Calipers and Micrometers Handout Go through the handout together and the examples.</li> <li>5. Do the Assignment in partners and go over together</li> <li>6. Pipe Investigation (p.5)</li> <li>7. Designing a Box (p. 6/7) – Finish for Homework</li> </ol>	<ul style="list-style-type: none"> <li>▪ Conversions Quiz</li> <li>▪ Vernier Calipers and Micrometers handout</li> <li>▪ Pieces of pipe, rulers, calipers (for pipe investigation)</li> <li>▪ Copy paper, micrometers, a copy box opened up as net, rulers (for box investigation)</li> </ul>	<ul style="list-style-type: none"> <li>▪ Quiz - Conversions</li> <li>▪ Assign – Designing a Box Presentation</li> </ul>
<b>Day 5</b>	H-1 H-2	<ol style="list-style-type: none"> <li>1. Mental Math # 3</li> <li>2. <b>Quiz – Reading Vernier Calipers and Micrometers</b></li> <li>3. Measurement Activity</li> <li>4. Do Journal on page 13 (1.2)</li> <li>5. Finish Box Investigation</li> <li>6. Rest of period to work on Dreamroom</li> </ol>	<ul style="list-style-type: none"> <li>▪ Quiz #2</li> <li>▪ Measurement Activity – activity sheets, pipe pieces, pencils, paperclips, staples</li> <li>▪ Dreamroom projects</li> </ul>	<ul style="list-style-type: none"> <li>▪ Quiz – Calipers and Micrometers</li> <li>▪ Assign Journal 1.2</li> <li>▪ Assign – Designing a Box Presentation</li> </ul>
<b>Day 6</b>	H-2	<ol style="list-style-type: none"> <li>1. Technical Communication – Media Clips (Ohio Speed Traps)</li> <li>2. Box Presentations</li> <li>3. Precision and Accuracy Notes</li> <li>4. Assign p. 25-26 #1-6 and Journal 1.4</li> </ol>	<ul style="list-style-type: none"> <li>▪ Ohio Speed Traps handout</li> <li>▪ Marking Checklist for Box Presentations</li> <li>▪ Precision and Accuracy Notes</li> </ul>	<ul style="list-style-type: none"> <li>▪ Assign p. 25-26 #1-6 and Journal 1.4</li> </ul>

### Appendix L

**Table A2: Planning at the Beginning of the Study**

Table A2

*Planning at the Beginning of the Study*

Content	Dev. A Community of Math Inquiry
<p>Measurement unit main ideas</p> <ul style="list-style-type: none"> <li>• Use metric and imperial systems</li> <li>• Concepts of precision and accuracy</li> <li>• Solve problems involving length, area, vol, time, mass and rates</li> <li>• Interpret scale drawings</li> </ul>	<ol style="list-style-type: none"> <li>1. mathematical thinking and noticing</li> <li>2. discussion of mathematical ideas</li> <li>3. the proposing, clarifying, defending, and refuting of mathematical strategies</li> <li>4. curiosity/asking questions about mathematics</li> <li>5. individual and collective ownership of learning</li> </ol>
<p>2D/3D Geometry main ideas</p> <ul style="list-style-type: none"> <li>• Volumes (prisms, pyramids, cones)</li> <li>• SA, Vol spheres</li> <li>• Scale factors (linear/area/vol)</li> <li>• Interpret scale drawings</li> </ul>	<ol style="list-style-type: none"> <li>6. application of mathematics to real world contexts</li> <li>7. decreasing reliance on the teacher as validator of mathematical ideas and increasing reliance on peers and self as validators of mathematical ideas</li> <li>8. reflection on mathematical ideas</li> <li>9. metacognitive awareness</li> <li>10. understanding of the norms and practices of the community</li> <li>11. recognition of common purposes amongst community members</li> </ol>
<p>Ideas for activities</p> <ol style="list-style-type: none"> <li>1. Give them a scale drawing (from a metal works company for example) – calculate the volume of material needed (ie. Metal) or better yet, the mass.</li> <li>2. Give them physical objects to measure and have them calculate the amount of material or mass based on known properties of the material.</li> <li>3. Give them a picture and find more theoretically the volume</li> </ol>  <ol style="list-style-type: none"> <li>4. Gap a spark plug with own instruments???</li> <li>5. Research metric and imperial systems? Debate use of one or other?</li> <li>6. Research about a sphere physically and/or on internet. The superior sphere/spectacular sphere!</li> <li>7. Derive formulas? Why bother?</li> <li>8. Create a plan for landscaping the school yard and prepare a cost analysis.</li> </ol>	<ol style="list-style-type: none"> <li>1. I like the idea in 1-3 of having students manipulate, discuss etc. I like the application idea and the idea of having them explain how they did it to others. I like the multiple approaches and discovering different paths to the same point idea. I think that this would foster interactivity and discussion, an important goal.</li> <li>4. I don't know if this is possible but it would certainly start some discussion.</li> <li>5. I like the importance of listening and reacting to the arguments of others while formulating an understanding of the two systems of measurement at the same time.</li> <li>6. I like discovery and the "inquiring" that would go on here. Students would have to inquire about spheres.</li> <li>8. I like the "project" approach because it requires several skills and creativity at the same time.</li> </ol>

## Appendix M

**Table A3: Planning at the End of the Study**

Table A3

*Planning at the End of the Study*

<b>Activity/Description</b>	<b>Goals for fostering the emergence of a community of practice characterized by mathematical inquiry</b>	<b>Content Goals</b>
<p>1. Ball Bounce Activity (Graph drawn by hand)</p> <p>Vocabulary</p> <ul style="list-style-type: none"> <li>• Independent</li> <li>• Dependent</li> <li>• Interpolation</li> <li>• Extrapolation</li> <li>• Line of best fit</li> <li>• Linear regression equation</li> <li>• Discrete</li> <li>• Continuous</li> </ul>	<p>Recognition of common vocabulary regarding linear functions.</p> <p>Students will work cooperatively to gather data, and examine it for trends.</p> <p>Students will make predictions requiring interpolation and extrapolation and students will defend their predictions with group members. (Propose and defend)</p> <p>Students will come to a consensus about the line that best models the data. (propose and defend)</p> <p>Establish the norms and practices of our community – how we will interact in the highly physical and interactive nature of our experiments</p>	<p>Review the notion of graphing data by hand and creating a line of best fit.</p> <p>Explore the ideas of independent and dependent variables and their relationships to linear equations <math>y = mx + b</math></p> <p>Also interpolation and extrapolation by hand</p>
<p>2. Chirps/Temperature activity</p> <p>Introduction to graphing a scatterplot and finding a line of best fit using a graphing calculator.</p>	<p>Although this is largely a technical class on how to find a line of best fit and make predictions with the calculator, I would also like to emphasize the use of technology in real world math as well as promote the collective nature of what we are doing in this unit and in this classroom. I hope to have students be responsible for their partners and in advocating for themselves. I will emphasize their own responsibility in helping the person next to them and in explaining the steps to each other. In this way, I will be working on establishing the norms and practices of the community and recognition of common purposes.</p>	<p>Intro to graphing data with a calculator and finding the linear regression equation.</p> <p>We will essentially cover most of the unit outcomes today in terms of technology and using linear models. Students will plot data using appropriate scales on the calculator. They will find the equation of a line of best fit and will use the equation to make predictions.</p>
<p>3. The Wave</p> <p>Make study sheet with steps to follow for finding the regression equation using a calculator. Then explain to a friend how you would go about this process. (Friend's job is to advise their peer of anything that may be missing in the description)</p>	<p>The Wave –</p> <ul style="list-style-type: none"> <li>• mathematical thinking and noticing</li> <li>• discussion of mathematical ideas</li> <li>• the proposing, clarifying, defending, and refuting of mathematical strategies</li> <li>• individual and collective ownership of learning</li> <li>• application of mathematics to real world contexts</li> <li>• understanding of the norms and practices of the community</li> <li>• recognition of common purposes amongst community members</li> </ul>	<p>Another experience graphing data with a calculator and finding the linear regression equation. Also making predictions based on the linear model, and finding intercepts, domain, and range.</p>
<p>4. Height Humerus Relationship</p>	<p>Similar to The Wave, these activities focus on the following characteristics:</p> <ul style="list-style-type: none"> <li>• mathematical thinking and noticing</li> </ul>	<p>Another 2 experiences graphing data with a calculator and finding the</p>

<p>Height, armspan activity</p>	<ul style="list-style-type: none"> <li>• discussion of mathematical ideas</li> <li>• the proposing, clarifying, defending, and refuting of mathematical strategies</li> <li>• individual and collective ownership of learning</li> <li>• application of mathematics to real world contexts</li> <li>• understanding of the norms and practices of the community</li> <li>• recognition of common purposes amongst community members</li> </ul>	<p>linear regression equation with two sets of data on same graph (As well as making predictions based on their linear model)</p> <p>Students will also get to graph two sets of data and find two regression equations at once on calculator.</p>
<p>5. AWAY Journal – What is a linear function to you? What does it mean to create a linear model? What good are linear models in the real world?</p> <p>Practice with these skills-sheet</p>	<p>Journal - discussion of mathematical ideas, reflection on mathematical ideas, metacognitive awareness</p> <p>Practice – generate <b>confidence</b> in their own individual and collective skills with using linear models to make predictions. Because I will be away, I plan on discussing with students their responsibilities and their collective roles in my absence. I am going to ask them to reflect on this next class.</p> <p>This is essential working on:</p> <ul style="list-style-type: none"> <li>• individual and collective ownership of learning</li> <li>• decreasing reliance on the teacher as validator of mathematical ideas and increasing reliance on peers and self as validators of mathematical ideas</li> <li>• recognition of common purposes amongst community members</li> <li>• understanding of the norms and practices of the community</li> </ul> <p>Perhaps the sign of good teaching here is using even the time away from students to increase self-reliance and to develop a sense of community.</p>	<p>Solidifying the skills with plotting linear data, creating linear models to describe the data and to make predictions, and extending these ideas to new situations.</p>
<p>6, 7, 8, 9. (LAB) DaVinci’s Proportions Assignment (1 class to research and choose one and start taking data. Take measurements and create lines of best fit in second class. Third class to make presentation)</p> <p>Fourth class to present</p>	<p>This project/activity focuses on application of the ideas already learned to a specific task. I will be looking for all of the following characteristics to emerge:</p> <ul style="list-style-type: none"> <li>• mathematical thinking and noticing (relationships between body proportions)</li> <li>• discussion of mathematical ideas (amongst group members and in presentation)</li> <li>• the proposing, clarifying, defending, and refuting of mathematical strategies (strong element of justification in the presentation)</li> <li>• curiosity/asking questions about mathematics (in choosing the proportion, there may develop a curiosity/questioning – this topic, however is not their own)</li> <li>• individual and collective ownership of learning (I will not in any way steer them how to approach the assignment)</li> <li>• application of mathematics to real world contexts</li> <li>• decreasing reliance on the teacher as validator of mathematical ideas and increasing reliance on peers and self as validators of mathematical ideas (Peer evaluation of presentations, verbal feedback,</li> </ul>	<p>All outcomes of the linear models and patterns unit.</p>

	<p>etc.)</p> <ul style="list-style-type: none"> <li>• reflection on mathematical ideas</li> </ul>	
<p>10 &amp; 11. Performance Task/Test (LAB)</p> <p>Students will have one task. To find something in their own worlds that has a linear relationship and model it using a linear equation. (Completely open)</p>	<ul style="list-style-type: none"> <li>• mathematical thinking and noticing</li> <li>• discussion of mathematical ideas</li> <li>• the proposing, clarifying, defending, and refuting of mathematical strategies</li> <li>• curiosity/asking questions about mathematics</li> <li>• individual and collective ownership of learning</li> <li>• application of mathematics to real world contexts</li> <li>• decreasing reliance on the teacher as validator of mathematical ideas and increasing reliance on peers and self as validators of mathematical ideas</li> <li>• reflection on mathematical ideas</li> <li>• metacognitive awareness</li> </ul>	<p>All outcomes of the linear models and patterns unit.</p>
<p>12. Reflection</p> <p>Self and peer evaluation of own performance task.</p> <p>Journal reflection about the community of learners of which they are part.</p>	<ul style="list-style-type: none"> <li>• mathematical thinking and noticing</li> <li>• discussion of mathematical ideas</li> <li>• the proposing, clarifying, defending, and refuting of mathematical strategies</li> <li>• curiosity/asking questions about mathematics</li> <li>• individual and collective ownership of learning</li> <li>• application of mathematics to real world contexts</li> <li>• decreasing reliance on the teacher as validator of mathematical ideas and increasing reliance on peers and self as validators of mathematical ideas</li> <li>• reflection on mathematical ideas</li> <li>• metacognitive awareness</li> <li>• understanding of the norms and practices of the community</li> <li>• recognition of common purposes amongst community members</li> </ul>	<p>None except a look back at what they have already learned about linear equations. I previously would never have included this element – perhaps this is evidence of growth or prior weakness</p> <p>This is the way it should be. A summative evaluation should include not only all of the mathematical content but evidence that the class is exhibiting the eleven characteristic I have identified ☺ An epiphany ☺</p>

## Appendix N

### Highlighting Key and Analysis Notes

#### Highlighting Key

Incompatibility of institutional/logistical requirements and effective teaching

Design Experiments – Info to be translated into summary charts

Classroom Management – Volume, off task behaviour, lack of engagement

My internal struggle and GUILT over not doing what I know is good teaching practice

Student Perceptions of what math is

Engagement/Energy and accessibility to all learners

SUCCESS – Inquiry, discourse, classroom culture emerges

Retention of what has been learned/Develop. Conceptual understanding vs. wrote memorization

Learned helplessness

Changes in norms and practices

Paying attention to developing the qualities of a community of practice characterized by inquiry as opposed to mathematical content

Noticing my own growth and change/ Sustainability of teaching this way

AHA moments – ideas emerging as I write

#### Illustrative Examples to use in Thesis

##### Measurement Debate

- I predicted where the natural curiosity would occur and designed an activity around it to engage students
- See DER chart – I think it helped develop many of the characteristics I wish to see emerge.

##### Abandoning my Plan (Oct. 2) to have students teach concepts to each other.

- Look at the reasons I identified for bailing out on plan

##### Problem Solving Partners (Oct. 6 & 26)

- Successful – stark contrast to my old ways of teaching
- Discussion with {name removed} – reaching varying levels of student ability
- Student responses in journals interesting (what it means to do math)

##### Morphing of 3D Research Assignment into Cylinder/Prism discussion (Nov. 7 – Dec. 2)

- Not happy with original idea – not very open
- Thought of discussion while I was journaling – a naturally occurring “inquiry” out of the community itself. Extensions. Proposing and defending ideas. Hard to come up with things that force students to do this.
- Interesting that I only asked two questions – the students did the work. Much more sustainable as a teacher. I often do all the work – the students watch.
- Discussion with {name removed}
- Promising oral discussions – disappointing written ones.
- Redo and recommunication of expectations net better written results?

*This is only a piece of the highlighted notes that provides a snapshot of what was done*

## Appendix O

### Journal Summary April 29

**Question:**

What is a linear model?

What might a linear model be used for in real life?

Does collecting data and graphing it together help you understand how to use linear models to make predictions? Explain

Several students tried to describe what a linear model was. (I was ecstatic when about half the class used the word “relationship” in their explanations. This, to me, indicates a deeper conceptual understanding of the ideas behind linear modelling) Lots weren’t too sure, I think due to our limited use of the terminology. Also, I think I wasn’t specific enough about them giving detailed descriptions about where linear models may be used in real life. I got a lot of token answers. I would like to see if this improves if I give the performance task at the end of the unit involving detailed research and descriptions of this. Perhaps I will get them to reflect on this journal entry as part of that process. What was very interesting was the number of students that commented on how collecting data and graphing it together was much better than “worksheets” or “writing notes”. Some of these interesting responses are noted below:

**Student 1**

“Ya, because I like the interaction rather than just writing notes.”

**Student 2**

“Collecting data for me is way better than do(ing) worksheet(‘s) in class because I tend to remember things that I do hands on better than worksheet(s). Altogether this whole program that we are doing is really good for me to understand.”

When I asked him what was better about hands-on activities than worksheets he replied . . . “I do think participating in activities is helping me because its fun and I tend to remember a little bit better things that I have done in an activity or fun action then just reading data off the board. And that is the same with worksheets. When I work with others and seeing the data form and seeing the objects develop into the questions. For example the height to arm examples I saw what we were doing and understood a lot more rather than it being a written up and having to see what happened and all that.”

**Student 3**

“Yes because when you collect data you can have fun and when you collect it yourself it helps you understand it more. When you graph it, it’s easier to understand when the teacher is showing an example because then you can see exactly what to do.” (fun, visual importance)

## Student 4

“Yes, collecting data helps me understand how to use linear models because collecting the data helps me point out who on the calculator was sitting where. For example {name removed} was easy to pick out because his arms are longer than his height.” (data relates to context and experience. It means something. Can pick out meaningful points on graph)

I asked her if she thought the data was more meaningful if she collected it and she replied: “Yes, the data and the graph has more meaning for me because I can visualize the problem so much better”

I asked her if picking one person out was easier and made more sense than just seeing data or outliers that didn't have a context in our classroom and she replied: “It was a lot easier to pick out {name removed} than just giving me a piece of work and not knowing anybody or the context. It (is) so much easier if you know one person saw the data and everythin(g)”

## Student 5

“Yes, collecting data helps to understand how to use linear models because its easy to remember things you've done. Like bouncing the ball and doing the wave. It was easy to see the relationship between the height of where the ball was being dropped and how high it bounced back up. Same with the number of people doing the wave and how long it took.” (visual aspect, relating visual experience to graph and a sense of remembering the activity)

After I asked her to give something specific these activities helped her understand . . .

“when I think about the experiments we did, it helps me know which one is the dependent and independent variables and such. It helps me to think about those experiments because it makes me picture what the data will look like on a graph and the line of best fit in my mind.”

## Student 5

“Yes, because it puts the data in perspective instead (of) it looking like a bunch of #s” (context is important to students) His reply to my question was great and visual so I copied it to put with his work in his folder.

## Student 6

“Collecting data as a class and graphing it together helped me to understand linear models a lot more because I can then easily see the relationship between the 2 different points. It helps when I can physicaly see the relationship between two things like the height a ball is dropped form and the height it bounces.” (visual terminology and reference to physical experience.)

## Student 7

“Yes, collect(ing) data and graphing it does help me because then I can tell what the line is representing, such as I could tell {name removed} was the top dot and {name removed} was a low dot. It also helps me understand slope such as our arm span is almost as long as our height.”

**Student 8**

“Yes it does. Some reasons why I think this is because when we collect the data together as a class we get to be actively involved in the collection. The data collection also helps me see what each point on the graph represents.”

**Student 9**

“Doing the work together does help you to understand because of you ever get stuck on a question you can always ask for help and someone your (you’re) working with usually knows the answer”

**Student 10**

“Collecting data helps me visualize the data. It is so much easier to understand when you collect data than being given data. If there is a random dot when graphing you know why that happened because you collected the data”

**Notes:**

- Brackets indicate thoughts or corrections added by me
- **Blue font** indicates secondary prompt responses

### Appendix P

#### Performance Task Feedback Form

Name: \_\_\_\_\_

Level	1	2	3
Topic Choice	Topic chosen does not relate to the world and/or does not model a linear relationship	Topic chosen does present a linear relationship, although it may be simple or have little application to the real world.	Topic chosen is linear in nature and applies in an interesting and relevant way to the real world.
Data Collection	Data collection techniques have many errors, are inappropriate or unreliable	There may be one feature of data collection which is susceptible to error, although methods are reasonably reliable	Data collection techniques are reliable and appropriate for data being sought. Care is taken to ensure few errors.
Understanding of Linear Models	There are several mistakes made in coming up with a linear model for the data or data doesn't lend itself to a linear model	One or two mistakes may be evident in work to come up with a linear model.	A linear model is correctly calculated to represent the data.
Making Predictions	Data is not used to make predictions of interest relating to the world or purpose	Data is used to make predictions, although the reason for making these predictions may not be clear.	Data is used to make predictions about the nature of the data in the real world. Why these predictions were made and their relevance is evident.
Connections to your world	How linear models connect to the real world is not outlined at all in final product.	Some indication of how linear models might fit into the real world is given, although it is not clearly explained.	A good understanding of how linear models fit into real life is evident.

**Self Evaluation**

Things you think you did well on this project:

Things you think you could have improved on:

Do you think you did better or worse on this task than you would have on a traditional test? Why?

What do you think are the strengths and weaknesses of using this method of evaluation?

Which do you prefer - performance tasks like this or traditional tests? Why?

**Peer Evaluation:** Done by \_\_\_\_\_

Things you think your partner did well on in their performance task:

Suggestions for improvement:

**Teacher Evaluation**

Things Mrs. Skyhar thinks you did well on in this performance task:

Suggestions for improvement:

Final Grade on task:
----------------------

## Appendix Q

**Table A4: Interactive Journal Prompts**

Table A4

*List of Interactive Journal Prompts*

Date	Interactive Journal Prompts
Sept. 5, 2008	<ul style="list-style-type: none"> <li>• What is mathematics to you?</li> <li>• What does “doing mathematics” look like at school?</li> <li>• What does “doing mathematics” look like outside of school?</li> <li>• Can you think of a time that you used math outside of school?</li> </ul>
Sept. 30, 2008	<ul style="list-style-type: none"> <li>• List 5 points made by each of the sides (Pro and Con) in the measurement debate.</li> <li>• Who do you think won the debate?</li> <li>• What do you think Canada should do? Explain.</li> <li>• What did you learn during this activity?</li> </ul>
Oct. 16, 2008	<ul style="list-style-type: none"> <li>• Where do you think you might need to convert measurements in real life?</li> <li>• What strategy (eg. Convert, online, unit ratios) would you use the most? Why?</li> </ul>
Oct. 24, 2008	<ul style="list-style-type: none"> <li>• What part of the questions/problems was “doing math”?</li> <li>• What part wasn’t?</li> <li>• Did you like working with a partner? Why or why not?</li> <li>• What do you think you learned?</li> </ul>
Nov. 27, 2008	<ul style="list-style-type: none"> <li>• I asked verbally, now that we have discussed this again, would you change anything about your previous answer? What would you change and why?</li> </ul>
Jan. 5, 2009	<ul style="list-style-type: none"> <li>• What is Trigonometry?</li> <li>• What careers use trigonometry?</li> </ul>
Feb. 23, 2009	<ul style="list-style-type: none"> <li>• What strategy did your group employ to find the two heights?</li> <li>• Why did you decide to use this strategy?</li> <li>• Was the strategy effective?</li> <li>• Is there anything that would have improved it?</li> <li>• Describe in your own words what you learned about trigonometry during this activity.</li> </ul>
Mar. 3, 2009	<ul style="list-style-type: none"> <li>• What strategy did your group employ to find the two heights?</li> <li>• Why did you decide to use this strategy?</li> <li>• Was the strategy effective?</li> <li>• Is there anything that would have improved it?</li> <li>• Describe in your own words what changed when I took away the protractor. How did this change the problem?</li> </ul>
Mar. 26, 2009	<ul style="list-style-type: none"> <li>• How do you decide which strategy to use when you look at a trig problem? Use an example in your explanation.</li> </ul>
Apr. 29, 2009	<ul style="list-style-type: none"> <li>• What is a linear model?</li> <li>• What can a linear model be used for in real life?</li> <li>• Does collecting data and graphing it together help you understand how to</li> </ul>

	use linear models to make predictions? Explain
May 13, 2009	<ul style="list-style-type: none"> <li>• While Mrs. Skyhar was away, you had to practice finding equations of lines of best fit for data on a work sheet. You also worked on your projects when she was away.</li> <li>• Did you struggle with any of the questions, or steps in finding a linear equation to model data? What did you struggle with?</li> <li>• How did you overcome these struggles? Who did you ask? What did you ask? Did they help you to get “unstuck”?</li> <li>• Do you feel more or less confident in your ability to solve problems and rely on yourself and your peers to figure things out than at the beginning of the course. Explain why you think that is.</li> </ul>
May 15, 2009	<ul style="list-style-type: none"> <li>• What did you learn during the DaVinci Project? (about linear models? About your understanding of them? About your ability to formulate an argument?)</li> </ul>
May 28, 2009	<ul style="list-style-type: none"> <li>• Do you think you did better or worse on the “performance task” than you would have on a traditional test?</li> <li>• Which do you prefer? Why? (Be specific)</li> </ul>
June 1, 2009	<ul style="list-style-type: none"> <li>• What is mathematics to you?</li> <li>• What does “doing mathematics” look like at school?</li> <li>• What does “doing mathematics” look like outside of school?</li> </ul> <p>REREAD YOUR SEPT. 5<sup>th</sup> ENTRY</p> <ul style="list-style-type: none"> <li>• How has your view of mathematics changed if at all?</li> <li>• How have your experiences in Applied Math this year been the same or different than previous years or other math courses? Can you give some examples to help illustrate this?</li> <li>• What were your most favorite activities/experiences in Applied math this year? Why?</li> <li>• What were your least favorite? Why?</li> <li>• What advice/feedback do you have for Mrs. Skyhar about this course?</li> <li>• If you had to write 5 things that described our “group” or “community” in Applied Math and what we do here, what would you write?</li> </ul>

## Appendix R

### Consent Form

### Consent Form

**Parent/Guardian** *(please check one and sign below)*

I  do

do not

give my consent for anonymous examples of my son/daughter's classroom work and interactive journals to be included in Candy Skyhar's Masters Thesis for the Department of Education: Curriculum, Teaching and Learning, at the University of Manitoba.

Name of Participant's Parent/Guardian	Signature	Date
---------------------------------------	-----------	------

~~~~~

**Student/Participant** *(please check one and sign below)*

I  do

do not

give my consent for anonymous examples of my classroom work and interactive journals to be included in Candy Skyhar's Masters Thesis for the Department of Education: Curriculum, Teaching and Learning, at the University of Manitoba.

|                     |           |      |
|---------------------|-----------|------|
| Name of Participant | Signature | Date |
|---------------------|-----------|------|

~~~~~

### Researcher

Name of Researcher	Signature	Date
--------------------	-----------	------