An Analysis of Grades and Placement in the Transition from Pre-Calculus to Calculus at the University of Manitoba

by

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A Thesis submitted to the Faculty of Graduate Studies of

The University of Manitoba

in partial fulfillment of the requirements of the degree of

DOCTOR OF PHILOSOPHY

Faculty of Education

University of Manitoba

Winnipeg

Declaration

I, Darja Barr, declare this document to be my own unaided work, and where published sources are used, they are acknowledged.

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Abstract

Recently, first-year mathematics instructors at universities across North America and around the globe have been noticing a decline in the mathematics skills and preparation of the incoming student body, and these students have been failing out of first-year mathematics courses at alarming rates (Crowther, Thompson & Cullingford, 2007; Hourigan & O'Donaghue, 2007; Kajander & Lovric, 2005; Rylands & Coady, 2009). Though some universities have implemented placement or diagnostic tests to measure the preparedness of their incoming students, many still use high school grades as the only indicator of students' readiness for university mathematics. However, researchers have found mixed results in terms of the effectiveness of high school grades at predicting success in university mathematics courses (Finnie et. al, 2010; Geiser, 2007; Salim & Al-Zarooni, 2009) due to factors such as the miss-alignment of teaching methods, students' entering knowledge, and skills, the curricula in high school and university, and high school grade inflation.

This dissertation includes two studies. The first study analyzed the relationship between grade 12 Pre-Calculus grades and first-year university Calculus grades at a large Canadian university over the period from 2001-2015. The results show that the disconnect between high school grades in mathematics and university grades in mathematics has been growing over time, and is significant for students who are not performing well in university Calculus. Recommendations include the implementation of placement examination, and increased communication and collaboration between K-12 and university mathematics educators and administrators.

The second study looked at the effectiveness of the ALEKS placement test at predicting students' readiness for first-year Calculus. Results show that including this mathematics placement test along with high school grades and some demographic variables can help predict the students' success in Calculus more validly. Recommendations include the implementation of the ALEKS placement test for entry into university Calculus courses, as well as some additional measures that could aid in placing underrepresented minority students more appropriately. Furthermore, the sharing of information between university and high school mathematics teachers is recommended.

Acknowledgements

I would like to acknowledge the support of my Advisor, Dr. Robert Renaud, and committee members, Dr. Xikui Wang and Dr. Rodney Clifton. Their time, effort, and encouragement were critical to the completion of this work.

I would also like to thank my colleagues Dr. Anna Stokke, Dr. Robert Craigen, and once again Dr. Rodney Clifton, who stood by my side through the difficult times in this doctoral process. Without their support, this work would never have come to fruition.

In addition, I thank the teachers and administrators of the Winnipeg School Division, who not only listened to these research results, but also because they have been working for the past two years to implement the recommendations based on this research. This has truly been one of the most valuable parts of the doctoral process.

Finally, I thank the never wavering support and encouragement of my parents and friends. Most of all my number one teammate, Myles Barr. Thank you for the many hours of help with reading, writing, and emotional support through the past years!

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Chapter 1: Introduction

In the past decade or so, certain researchers have described a trend at the post-secondary level as a move from "elite" to "mass" university education (Hong et al., 2009, p. 2). They refer to the fact that what was once thought of as an exclusive ivory tower for a few privileged and gifted students, has now become much more accessible to an increasing multitude of young people who wish to receive university education. This trend is evident in the number of students attending universities in Canada: While the population of Canada increased by 6-fold over the past century, university enrollment increased by 145-fold, from about 7,000 students in 1900 to over 1 million in 2005 (Canadian Association of University Teachers, 2005; Cote & Allahar, 2007; Statistics Canada, 2005). In fact, today, more students earn PhDs than graduated from high school at the end of the 19th century (Schoenfeld, 2004).

According to the Organization for Economic Co-operation and Development (OECD), around 20% of the Canadian workforce holds at least one university degree, which has increased from around 5% forty years ago. While only around 10% of the baby boom generation attended university, over 40% of their children did so (Cote & Allahar, 2007). In Canada, the population went from having an average of 7.7 years of schooling 75 years ago—in 1941—to nearly double that number 55 years later—in 1996, meaning that those who in the past would not have considered themselves eligible or qualified to attend university are now attending in droves.

In Canada, the number of students in high school increased from 788,800 in 1960 to 1.6 million in 1998, and in that same timeframe, university enrollment increased from 113,900 to 580,400—a five-fold increase (Clifton, 2000), then grew to 978,000 by 2007 and to about 1.3 million by the 2013/14 academic year (Statistics Canada, 2015). Concurrently, the number of full-time university teachers in Canada went from nearly 8,000 in 1960 to just over 40,000 in 2007 (Statistics Canada, 2012), meaning that the ratio of university students to teachers increased from about 14:1 in 1960 to roughly 25:1 in 2007, and this ratio has continued to rise. The given proliferation of students has, in turn, led to the strain on university resources, as enrollments grow at a faster rate than university teachers and policies and processes can keep up with (Cote & Allahar, 2007).

At the University of Manitoba, undergraduate enrollment has increased from about 18,000 in 1996 to just over 25,000 in 2014, almost a 40% increase, while the population of Manitoba increased by only about 18%, from around 1.1 million to nearly 1.3 million (Statistics Canada, 2011). This indicates that students are entering the university education system at greater rates than at any time in the past—whether successfully or not—and it is clear that more and more of the responsibility for educating the future generations is being placed on the shoulders of university instructors. University education has become a crucial part of a well-established life and successful career in a way that it was not in the past. But, as increasing proportions of high school graduates are attending universities, some unintended consequences have begun to emerge.

In the two studies in this dissertation, one of these consequences is explored in great detail. Specifically, as more students enroll in universities with increasingly diverse backgrounds, there is an increasing variability in their incoming knowledge and skills. The two studies focus on this trend, which has been documented by mathematics instructors across the globe: An increasing proportion of students coming into first-year university mathematics courses are seriously underprepared. The result of this lack of preparation for these first-year students has led to high, and continuously growing, failure and withdrawal rates. The possible causes of this issue were explored in this dissertation and recommendations for improving this situation were made.

This chapter focuses on the rising challenges of university education and will examine the context of university mathematics courses. Subsequent to that will be a detailed discussion of the two studies, the dissertations' significant contributions to the field, the limitations of the studies, and an overview of the dissertation will conclude the chapter.

Challenges in University Education

The explosion of students attending universities has created a host of critical problems that are becoming more and more evident. Some of these problems include large class sizes leading to higher student-instructor ratios, and more administrative duties for both administrative and teaching personnel. For example, today, on average, professors face three times as many students as they did a generation ago (Cote & Allahar, 2007), and the students themselves are, as noted above, different from those of the past decades. Similarly, American statistics show that, over 90% of American high school students expect to attend college or university (Arum & Roksa, 2011). In other words, students today are coming to post-secondary education in both the U.S. and Canada with a 'college for all' mentality (Arum & Roksa, 2011).

One of the unintended consequences of the unbridled growth in the university student population is that universities are now charged with many of the teaching and administrative tasks that once belonged to the high schools. Whereas in previous generations high schools did the sorting and advising about students' potential future education and career paths, given their interests and academic capabilities, this is now largely left to university staff. Some authors have claimed that currently, in a high school setting, students are getting little information about the future options that may be best suited to their aptitudes and skills, or what levels of achievement are needed to reach their goals (Cote & Allahar, 2007; Rosenbaum, 2001). Hence, the responsibilities of "weeding"—reducing the numbers of students who will reach university levels, "sorting"—determining who has the potential to go where, and "cooling out"—dealing with the "over-promised" students who have been promoted "beyond the limits of their abilities and motivations"—are left to colleges and universities—the institutions with fewer resources and more students than ever before (Cote & Allahar, 2007, pp. 44-45 & 55). Further, as a result of students' "soft-sorting"—giving them the greatest opportunities possible in their educational careers—other serious issues emerge.

Cote and Allahar (2007), for example, suggest that with the pressures to educate an increasing percentage of the university-aged cohort in the population, high schools have lowered their standards to ensure that most students graduate. In fact, they claim that high schools often allow students to move from one grade to the next and eventually to graduate irrespective of their performances. This results in grade inflation, when, increasingly, students are receiving higher grades for less knowledge, less skill, and less effort. From the 1970s to the 2000s, for example, the number of A's given out in U.S. high schools went from about 20% to nearly 50%. Moreover, according to the U.S. Census Bureau, in 2007 about 95% of first-year university and college students reported having earned mainly As and Bs in their last year of high school (as cited in Cole &

Allahar, 2007). Not surprisingly, Canadian students are not far behind, with data showing that in Ontario, for example, in the 1980s, the average high school grade of first-year university students was a B, which subsequently rose to an A in the early 2000s (Cote & Allahar, 2007).

One may wonder whether this implies that today students are achieving at higher levels then past generations of students, and if they are, whether they are doing better. However, the lack of comparable data on the increase, or in some cases *decrease*, in standardized test scores, such as the American College Test (ACT) and the Scholastic Achievement Test (SAT), suggests differently—students are, in fact, doing worse than they were in the past (Cote & Allahar, 2007).

In Alberta, a recent article outlines students' experience of grade inflation and the effects of this phenomenon on their university performance. It points at the difference in students' class grades compared to their grades on the Provincial Examinations as the evidence of grade inflation being real and tangible in Alberta, at least, and perhaps in most of Canada (French, 2017).

These data show a trend that Cote and Allahar (2007) call a grade "compression," or grade "conflation," predominantly happening at a high school level, that makes it increasingly difficult for university educators and administrators to distinguish between incoming students who are outstanding, good, mediocre, or poor (p. 45). This, in turn, contributes to the growing gulf separating high schools from post-secondary institutions, and within this sector, colleges from universities, by not clarifying the appropriate sets of expectations required in the various institutions. Specifically, it seems that at a high school level, students become used to moving ahead from grade to grade without acquiring the necessary knowledge and skills and without devoting much effort to their studies, which leaves them ill-prepared for the demands of college and university. Subsequently, students' high school grades alone do not accurately reflect their probable success in university courses. This is the key issue explored in the two studies included in this dissertation.

Thus, increasingly more students with inflated high school grades are coming into college and university classrooms. Many of these students are not used to putting in the required work that previous generations did, and they are not appropriately prepared for the rigors of university-level coursework. Cote and Allahar (2007) describe this scenario as being at the heart of the crisis that universities are currently experiencing. They believe that due to these factors, the high participation rates in Canadian universities may not accurately reflect the true educational prowess of our country. Simply put, today's students may have graduated from high school, college, or university, but they may not have the knowledge and skills that previous generations of graduates had.

Though certain studies suggest that high school grades are the best predictor of success in university and the successful completion of a university degree (Adelman, 2006; Geiser, 2007), it is actually first- and second-year university grades that are stronger predictors of graduation than high school grades (Finnie & Qui, 2008). In fact, Finnie and Qui (2008) analyzed data from a large sample of Canadian post-secondary students and found that first-year grades are strongly related to persistence and eventual graduation. Thus, what happens to students in their first couple of years at university has a large effect on whether or not they stay in university and eventually complete their degrees. Unfortunately, data about student achievement in those first years paints a rather bleak picture of the students' future academic success.

As Finnie, Frenette, Mueller, and Sweetman (2010) showed, on average, GPA drops about 10% between high school and first-year university. In other words, students who graduated from high school with an average grade of an A will likely receive an average grade of a B in their first-year university courses. Surprisingly, the grade drop is most pronounced for students who enter with the highest high school GPAs. For Manitoba students, specifically, the grade drop from their last year of high school to first year of university is almost 10% (Finnie et al., 2010), and of all of the fields of study, this drop is most pronounced for students entering the sciences and mathematics at the university level. Finnie and colleagues (2010) found that about 75% of students in Manitoba enter university with a GPA of 80% or higher, but in first year, that number drops substantially so that 75% of first-year students report grades of 79% or lower.

The consequences of this "grade shock" are numerous, ranging from students experiencing extreme stress, to the loss of scholarships, to even dropping out of university. All of these consequences have been leading administrators and teachers to recommend a stronger alignment between high school and university course content, expectations, and grading scales (Finnie et al., 2010). As Finnie and his colleagues put it, grades provide critical information for students, and being able to properly predict their performances in university based, at least partially on their high school grades provides a more accurate way to make decisions about what disciplines to study, and what to expect when they get into specific programs of study. This is most critical in the sciences and mathematics because of the "vertical" way these subjects are structured requiring students to have specific knowledge and to have mastered particular skills before they can be successful in subsequent topics and courses. For example, to be able to analyze the limits of functions, which is a foundation for first-year Calculus, students must be able to confidently and accurately perform the algebraic manipulations that are covered in the high school Pre-Calculus course. Unfortunately, not all students are coming to university with the Pre-Calculus knowledge and skills that should be part of their acumen.

Mathematics Achievement in University

A number of writers from the United States claim that in an ever-more technological society, it is increasingly critical for people to be mathematically proficient to ensure that the society continues to be competitive in the global economy (Madison & Hart, 1990; National Mathematics Advisory Panel, 2008; National Research Council, 1991; White, 1987). According to the 2006 Leitch Review of Skills (an independent review commissioned in 2004 by the Chairman of the National Employment Panel of the UK to identify current and optimal skill levels with a goal to maximize economic growth and productivity), the society is condemned to a bleaker future through a diminished economic growth and a decline in international competitiveness if citizens do not have basic mathematical literacy while many employers demand advanced mathematical knowledge and skills. Not only is mathematics at the heart of many other disciplines, such as physics and engineering which benefit our society directly, but it also plays a central role in the economic growth of a country (White, 1987). Thus, having an adequate pool of “trained talent” and “intellectual capital” in mathematics means that the U.S. and Canada can “harness science and technology for economic growth” (White, 1987, pp. 22).

Unfortunately, there are currently very high failure and withdrawal rates of students in first-year mathematics courses occurring at universities around the globe, in Canada, and, specifically, at universities in Manitoba (Kalajdzievska, 2011). The lack of success of students in first-year mathematics courses is not helping these young people to reach a level that can help Canada become competitive in the global economy (Mattern & Packman, 2009; Parker, 2005). Moreover, if students are not successful in first-year mathematics courses, then the future of their education may hang in the balance, and their employment prospects may be limited, meaning that they will also have limited opportunities to get jobs in other provinces or other countries. It is imperative that incoming students are placed accurately into first-year mathematics courses that are appropriate for their skill level, so that they are less likely to experience failure that could lead them to dropping out.

There are various reasons why students experience low success rates in first-year mathematics courses. Although research stretches back to the 1970's (Cornelius, 1972), as university students' performances declined, there has been a surge in interest in researching this topic. University instructors are increasingly concerned that more and more first-year students lack the appropriate mathematical skills coming out of high school. Over the past decade, faculty members teaching mathematics have become ever more frustrated by the lack of basic skills, understanding, and knowledge of first-year students (Hourigan & O'Donaghue, 2007; Kajander & Lovric, 2005; Rylands & Coady, 2009). Surprisingly, little is known about these incoming students' education in mathematics, except they have graduated from high school and passed the required high school mathematics courses. This creates a host of problems with placing students in appropriate mathematics courses and teaching courses that will address the various levels the students are working at. It is these problems that this dissertation is addressing.

The Two Studies

Some research has shown that high school grades, particularly grades in mathematics, are the best predictor of success in first-year university courses in mathematics (Adamuti-Trache, Bluman, & Tiedje, 2013; Geiser, 2007; Islam & Al-Ghassani, 2015). Thus, a hypothesis for the two studies in this dissertation is that high school grades in the Pre-Calculus course taught in Manitoba high schools is a strong predictor of the grades that these students receive in a first-year university Calculus course. However, in this province, the grades students earn in high school Pre-Calculus are not standardized; rather, these grades are determined, to a large extent, by individual teachers, or perhaps by departments in large high schools. The grades probably differ across schools based on factors such as the way the teachers decide to assign marks, the amount of material covered, and the rigor of the course. Students take a "standardized" examination at the end of the grade 12 mathematics course, but this exam only accounts for 20% of their final course grade while the remaining 80% is determined by the teachers using whichever assessments and evaluations methods they consider appropriate. For this reason, there may be considerable variation in grades that students with the same designated skill level will receive in the Pre-Calculus high school courses. More specifically, an A may not mean the same thing in different schools, and students with the same grade may have significant variance in their preparation for the first-year Calculus course in university. This makes it very difficult to accurately place incoming Manitoba students into appropriate university mathematics courses.

This dissertation contains two studies, which focus on the transition of students from Manitoba high school Pre-Calculus to the first-year Calculus course at the University of Manitoba.

Study 1: An Analysis of the Relationship Between Pre-Calculus and Calculus Grades

The objective of Study 1 is to analyze the relationship between the Pre-Calculus grades that students obtained in high school and the Calculus grades they obtained in the first-year Calculus course that they took at the University of Manitoba. The purpose of the study is to see how well this measure has been predicting students' first-year Calculus grades. In this analysis, a number of demographic variables, such as gender, school, and achievement level, will be controlled. Specifically, the analysis will determine what proportion of students first-year Calculus grades are out of the range predicted by their incoming grade 12 Pre-Calculus grades. Previous analyses suggest that nearly all university students perform at a lower level in university compared to high school (Finnie et al., 2010), but high failure and withdrawal rates at the University of Manitoba suggest that these students may be performing even lower than expected.

This study uses institutional data gathered over the last 15 years from about 12,000 students who have graduated from public and private high schools in Manitoba. The relationship across various demographic variables, such as gender, school, school division, and achievement levels in the university Calculus course will be examined to identify students who are better prepared for first-year Calculus at the University of Manitoba.

Study 2: Analyzing the Effectiveness of the ALEKS Mathematics Placement Test

Many universities employ standardized entrance examinations, such as the SAT or the ACT, to compensate for the lack of information or misinformation about incoming high school students' skill levels. Some universities use discipline-specific *placement tests* to direct students to appropriate courses based on their competencies. In Study 2, a review of the technical literature on placement tests and testing will be used to examine the efficacy of one of the most widely used placement tests—the Assessment and Learning in Knowledge Spaces (ALEKS) test— used along with students' high school Pre-Calculus grades as well as some demographic variables in predicting students' first-year Calculus achievement. This study will be used to recommend a placement strategy that the University of Manitoba, or any university experiencing high failure and withdrawal rates, could use for incoming first-year students enrolling in Calculus. The intent of this strategy would be to have more incoming students successfully complete the Calculus course.

The data for Study 2 was collected in the Fall of 2016 from approximately 700 students enrolling in Calculus 1500 to determine the correlation between the ALEKS placement score and the final grade in Calculus, including analyses based on demographic variables, such as gender, school location, age, and English as a first language. This study aims to test the placement accuracy of the ALEKS placement test as compared with incoming high school Pre-Calculus grade, and to provide recommendations for the implementation of an effective placement strategy for Calculus.

Significance of the Studies

Contributions to the Field

Currently, the mathematics achievement of students is a central issue in educational debates at many universities in North America (Ansari, 2016; Wright, 2012). The focus of this research, specifically, is on students coming to the University of Manitoba. Although issues such as low mathematical test results of Manitoba students or grade inflation at a high school level are popular topics in the media, no analysis of historical grade data has been undertaken in Canada, or even in the United states, that reports the correlation between high school Pre-Calculus course grades and first-year university Calculus course grades, especially when a number of important demographic variables are controlled. While some authors speculate about the relationship between students' high school mathematics achievement and their university mathematics achievement (see Geiser, 2007, for example), surprisingly, no correlation has yet been reported in the research literature. Nevertheless, a number of studies have shown that *overall*, high school grades are the most significant predictor of university grades, while only a few studies specifically examined the relationship between high school mathematics course grades and university mathematics course grades in a large group of students (see Adamuti-Trache,Tiedje, 2013; Islam & Al-Ghassani, 2015). As stated previously, the examination of this issue is especially meaningful given the role that Calculus courses play for other courses in a multitude of programs in science, arts, and professional degrees.

Study 1 contributes to the body of research by including the data that represents the entire population of students coming from Manitoba high schools and taking first-year Calculus course at the University of Manitoba over a number of years. This is a large data set that covers a significant period of time. Thus, generalizations can be made to the entire institution, the University of Manitoba, and many other Canadian and North American universities.

With regards to the significance of Study 2, there is very limited evidence regarding the effectiveness of commercial placement tests, such as ALEKS, at Canadian universities. Some admission tests, the ACT (American College Test) and the SAT (Scholastic Aptitude Test), are used in the United States, and tests such as the LSAT (Law School Admission Test) used in a number of professional programs in Canada, have been studied previously:

> College admission tests such as the SAT or the ACT measure students' general readiness for college, whereas placement tests seek to measure students' knowledge and skills that are prerequisite for specific entry-level college mathematics courses. Nationally administered tests such as SAT and ACT measure a broad range of quantitative skills, and this measure is often too general to distinguish between readiness for entry-level mathematics courses such as college algebra, trigonometry, pre-calculus, and calculus. (Mathematical Association of America, 2010, p. iii)

Thus, there is a gap in the Canadian literature about the use of placement tests in mathematics.

Contributions to University Instructors and Students

Reporting on the effectiveness of high school Pre-Calculus grades in predicting university Calculus achievement, and using multiple measures to more effectively predict the performance of students in Introductory Calculus and to place these students accurately into appropriate first-year mathematics courses can have an enormous benefit to students, Calculus instructors, and administrators. From the students' perspective, benefits will include the ability to make better decisions about the courses they could or should take and the remediation they may need. This can lead to fewer students failing or withdrawing from courses, leading to higher retention and graduation rates.

As well, Calculus instructors will be able to better tailor their course-work and assessments if they have a better understanding of what their students already know and what they are capable of doing. If students were assigned to teachers in a more homogenous way, this would potentially reduce the need for in-course remediation, which takes time away from teaching course content, and would allow more time for applications and enrichment problems and opportunities.

Finally, for administrators, having students directed appropriately into courses will result in fewer cases of failure and repetition, which, until now, has made it difficult for departmental administrators to predict course enrollment. This could improve the reputation of the University of Manitoba and the Department of Mathematics in the eyes of the instructors, students, parents, taxpayers, and provincial governments.

For these reasons, the studies presented in this dissertation will add to the literature in mathematics education and hopefully to the success of more students in the future. Nevertheless, some limitations of each study are to be noted.

Limitations of the Studies

The two studies included in this dissertation have limitations coming from the fact that both of them focus on the teaching and learning of Calculus in one large Canadian university, the University of Manitoba. Study 1 is a large-scale analysis of data on students graduating from Manitoba high schools and thus does not include international students or students from other provinces who are enrolled at the University of Manitoba. The variation in high school courses and content that these non-Manitoba students completed in preparation for the first-year Calculus course has not been examined. Hence, this study is limited to examining the transition from Pre-Calculus to Calculus in a population of students who graduated from public and private schools in Manitoba. The general structure and content of the Pre-Calculus course and the structure of the Calculus course are quite well known, but it is not known if the high school mathematics teachers covered all the topics necessary for students to succeed in the university-level Calculus course. In addition, the University of Manitoba is a very accessible university with lower entrance requirements compared to other Canadian universities with more rigorous entrance requirements and less diverse students, which may be linked to smaller gaps between Pre-Calculus and Calculus.

Although many students voluntarily withdraw (VW) from the Calculus course, often due to poor performance, others withdraw for reasons unrelated to their academic achievement, such as emotional or family reasons. Thus, the question becomes whether it is necessary to include the data for students who received a VW. Including them would over-represent students who did not succeed by including students who dropped the course for reasons other than poor performances. Consequently, eliminating the VW data from the analyses ensures that the measures are more conservative and represent the true success of students from Manitoba high schools in the University of Manitoba Calculus course.

In Study 2, there are a few additional limitations associated with the placement testing procedure. First, the purpose of a placement test is to assess students' knowledge and skill levels before they begin the Calculus course. In this study, the students taking the placement test have already begun taking the Calculus course. This means that these students had little or no stake in doing well on the placement test. As a result, the students might not have put the same effort into the test as they normally would have if they needed a good score to enroll in the Calculus course. This means that the results of the placement test might be skewed and may misrepresent the relationship between students' scores on the placement test and in the Calculus course.

Second, the fact that students' high school grades were self-reported might have contributed to some inaccuracy in the data. This, however, should not pose a significant problem because self-reported grades in mathematics have previously been found to be very accurate (Sanchez & Budin, 2015).

Third, as the results are specific to the students attending the University of Manitoba, they may not be readily generalizable to students attending other universities. Further, since Canadian universities tend not to offer remedial Pre-Calculus courses, as many universities in the United States do, high school grades may be less relevant to the U.S. universities than these grades are to Canadian universities. Finally, because high school curricula vary across schools and because the mathematics curriculum changes over time, the results of these studies, from this particular time period, may not reflect what happened in the distant past or what will possibly happen in the future.

Even in light of these limitations, the two studies provide an original and generalizable set of results for the first-year Calculus course at the University of Manitoba. The recommendations drawn from the studies could help students, instructors, and administrators function more effectively at this institution. The results may also be useful for other Canadian universities.

Overview of the Dissertation

The research presented in this dissertation focuses on the predictors of success of first-year students in the Introductory Calculus courses at the University of Manitoba. Chapter 1, the Introduction, presents historical data on the enrollment of students in Canadian universities and in the University of Manitoba specifically, and outlines the challenges faced. Correlations between high school and university mathematics grades are computed, specifically, the relationship between the grades in Pre-Calculus taken by Manitoba high school students and the grades in Introductory Calculus, the course these students take at the University of Manitoba. Also introduced is the question of effective placement and placement accuracy, specifically into Introductory Calculus, using the ALEKS placement test.

Chapter 2 will review the literature on the predictive power of a variety of factors on students' success in university and, particularly, their success in the university-level mathematics courses. A description of the data set and the analyses of the relationship between students' performances in the high school Pre-Calculus courses and their performances in the Calculus course at the University of Manitoba will follow. The

Chapter will then focus on the specific relationship between Pre-Calculus high school grades and first-year university Calculus grades among selected groups of students, based on their gender, the schools they graduated from, and the year they took high school and university-level courses. This Chapter will conclude with recommendations for 'bridging the gap' between Pre-Calculus and Calculus at the University of Manitoba.

Chapter 3 will examine the effectiveness of the specific placement examination—the ALEKS placement test. The Chapter will begin with a review of the research literature on placement testing in mathematics. Next, the chapter will discuss the population of students and the methodology used in the study. The Chapter will then report the analyses of the data from a cohort of Introductory Calculus students and will correlate their results on the ALEKS test with their success in the University of Manitoba Calculus course. This relationship will be compared with the relationship between success in the Calculus course with the students' self-reported grades from the Pre-Calculus course they completed in high school. A multiple-measure, called the P-Score, is used to provide an accurate placement prediction for the students' Introductory Calculus grades. Based on the results of this study, a number of recommendations are made for assessing, placing, and teaching first-year students in the Calculus courses at the University of Manitoba.

Finally, Chapter 4, the Conclusion, summarizes the two studies, along with the generalizations that have been derived from the studies. The Chapter will outline directions for future research that can aid students and instructors in their quest to improve mathematics education of university students and also help students in their transition from high school to first-year university mathematics.

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Connecting Section

In Chapter 1, the stage was set for the exploration of students' transition issues from high school to first-year university mathematics. This Chapter outlined historical data about increasing university enrollments and the consequences that follow. Specifically, Chapter 1 described the challenges faced by university students, instructors, and administrators in first-year university mathematics courses. High failure and withdrawal rates in first-year mathematics courses at the University of Manitoba suggest that research should focus on the accuracy of high school Pre-Calculus grades in prediction achievement in first-year university Calculus, and on the effective use of placement procedures to direct students appropriately into these first-year mathematics courses.

Chapter 1 provided an overview of the dissertation, with details about the two studies included. It also laid out the significant contributions to the field of mathematics education, and the limitations of the two studies to follow.

Chapter 2 builds on the research examined in Chapter 1 by exploring the issue of students' unpreparedness for first-year mathematics courses. This Chapter analyzes data on the relationship between Manitoba students' incoming Pre-Calculus grades, and their related first-year Calculus grades. It aims to provide empirical evidence that these Pre-Calculus grades may not be a sufficient measure of readiness for university Calculus.

In addition, Chapter 2 then sets forth specific recommendations based on the data analyses that can aid future generations of university stakeholders (administrators, instructors, and students) in bridging the gap between high school and university mathematics courses.

Chapter 2: An Analysis of the Relationship Between Pre-Calculus and Calculus Grades

A version of this chapter was submitted to the "International Journal of Mathematical Education in Science and Technology".

Barr, D., Clifton, R., Renaud, R., & Wang, X. (2018). An Analysis of the relationship between high school pre-calculus and university calculus grades. International Journal of Mathematical Education in Science and Technology (under review).

The first author was responsible for the conception and design of the work, the data collection, data analysis and the drafting of the journal paper. The second, third, and fourth authors supervised the work and critically revised the paper.

Around the globe, mathematics professors have been lamenting the lack of basic skills of their incoming undergraduate students (Hourigan & O'Donaghue, 2007), and those students have been failing their first-year mathematics courses at alarming rates. While this concern stretches back to the 1970's (Cornelius, 1972), it appears that over the last decade, many more professors are becoming increasingly frustrated by the inadequacies of the mathematical prowess exhibited by their first-year calculus students (Hourigan & O'Donaghue, 2007; Kajander & Lovric, 2005; Rylands & Coady, 2009). Furthermore, the number of students who have struggled to pass introductory calculus courses has also increased, at some universities from around 20% to around 50% (Kalajdzievska, 2011). This highlights the main concern regarding mathematics achievement in first-year university mathematics courses; the gap between students' achievement in high school Pre-Calculus and their achievement in university Calculus courses is widening.

In order to adequately prepare first-year university students for subsequent entry and success into science, technology, engineering, and mathematics (STEM) programs and to minimize the number of students who withdraw from first-year mathematics courses, this gap must be reduced. Specifically, universities must find effective ways to reverse the escalating failure and withdrawal rates in first-year university mathematics courses.

Researchers and mathematics educators have outlined a number of factors that contribute to the disconnect between high school and university mathematics. Some point to environmental factors that can cause a culture shock for virtually all students entering first-year university (Clark & Lovric, 2009). Others focus on specific aspects that differ between high school and university mathematics, such as course content (O'Shea, 2003; Thomas et. al, 2010), the lack of coordination between high school and university mathematics teachers (Dossey, 1998; Hull & Seeley, 2010; Morgan, 2012; Thomas et. al., 2010), and differences in the teaching style and emphasis between high schools and universities (Hull & Seeley, 2010; Thomas et. al., 2010). In sum, this direction of research suggests that the misalignment between high school and university mathematics has many possible causes, but nevertheless it has a detrimental effect on student achievement, specifically in first-year mathematics courses.

The purpose of the study reported in this chapter is to explore the relationship between students' grades in a high school Pre-Calculus course and their subsequent grades in a first-year university Calculus course, and the selected conditions under which this relationship varies. These results will provide a better understanding of the link between high school Pre-Calculus and Calculus courses, which will help inform the development and use of mathematics placement tests, which is Study 2 presented in the next chapter. The following sections outline the background research which relates to predicting university achievement for specific demographic groups, and the negative consequences of grade inflation in the specific context of mathematics. Then the Pre-Calculus to Calculus transition in the University of Manitoba context will be described. Next, the method and results of the study will be explained, and the results will be discussed and a few recommendations will conclude the Chapter.

Predictors of University Achievement

Universities across North America, and, in fact, internationally, typically use a specific high school grade as a cut-off for the admission of students to first-year university courses, but it is not clear how often these grade cut-offs are justified with empirical evidence. When institutions believe that secondary school success is not a strong predictor of success in first-year university courses, they will often use entrance examinations to determine students' readiness for university coursework (Bressoud & Rasmussen, 2015; Hawkes & Savage, 2000; Mattern & Packman, 2009; NiFhloin, Bhaird & Nolan, 2014).

In looking specifically at the relationship between high school grades and university success, Salim and Al-Zarooni (2009) found that the correlation between students' high school GPAs and their GPAs in an engineering program was 0.67 after two years in the program, 0.62 after 4 years, and then 0.57 for the graduating students. Finnie et. al. (2010) looked at the connection between high school GPAs and university

GPAs in Canada, and found that GPA dropped about 10% as students made the transition from high school to first-year university. In other words, they found that students who graduated from high school with an A average, generally received a B average in their first-year university courses. In addition, Finnie and his colleagues (2010) found that this drop is most pronounced for students entering university science programs, which, of course, includes students enrolled in mathematics courses.

Geiser (2007) compared the predictive power of high school grades with the predictive power of university entrance examinations. He found that High School Grade Point Average (HSGPA) was a stronger predictor than the students' cumulative college GPAs or their standardized entrance examination results, such as the American College Test (ACT) or the Scholastic Achievement Test (SAT). In a meta-analysis of studies conducted from 1980 to the mid-1990s, Burton and Ramist (2001) found that the correlation between SAT verbal and math standardized scores and cumulative undergraduate GPAs was 0.36, the correlation between high school grades and cumulative undergraduate GPAs was 0.42, and the multiple correlation between the combination of these two measures and cumulative undergraduate GPAs was 0.52. However, neither Geiser (2007) nor the authors included in Burton and Ramist's (2001) meta-analysis looked into the predictive power of certain parts of the entrance examinations on certain courses, such as the predictive power of the mathematics sections of the SAT test with the success of first-year students in mathematics courses. Thus, an examination of more discipline-specific data is needed to provide more information on the transition from Pre-Calculus in high school to Calculus in first-year university.

Grade Inflation in High School

When examining high school grades, it should be relatively straightforward to distinguish between high performing students (i.e., those who earned As or A+s) and low performing students (i.e., those who earned Cs or lower). In theory, this information should be a good predictor of how well the students perform in the related, first-year university courses they enroll in. However, this is not always the case. One factor that appears to have a significant effect on the relationship between high school grades and university grades is *grade inflation* (for example, see Clark & Lovric, 2009; Geiser, 2007). Eislzer (2002, p. 489) defines grade inflation as "student attainment of higher grades independent of increased levels of academic attainment", and the ACT (2005) define it as "an increase in students' grades without an accompanying increase in their academic achievement".

There is much talk in a variety of public venues about grade inflation, but not much scholarly research. In fact, a search of the term "grade inflation" in the ERIC search engine reveals around 150 studies in peer-reviewed journals over the past 20 years, almost all of which focus on the post-secondary level. In comparison, a Google search of the same term showed over 800,000 news articles on grade inflation.

Finefter-Rosenbluh and Levinson (2015) explored the ethics of grade inflation and reviewed studies that identified three types of grade inflation – *longitudinal ,* *compressed* (narrowing the range of marks awarded), and *comparative* (among different institutions) – that each affect different stakeholders – *individuals, institutions,* and *society*. They concluded that grade inflation violates the principle of meritocracy. Since students are awarded grades that they have not earned, institutions of higher education are prevented from effectively sorting incoming students to make more valid admissions decisions. Oleinik (2009) suggests that the increased pervasiveness of grade inflation is the result of academic institutions reacting to market, industrial, and political forces that prioritize outcomes rather than the pursuit of truth.

While some claim that there is insufficient evidence to support the conclusion that grade inflation is increasing (e.g., Pattison, Grodsky & Muller, 2013), the majority of scholarly studies conclude that grade inflation is pervasive, harmful, and getting worse (Bachan, 2017; Kostal, Kuncel & Sackett, 2016; O'Halloran & Gordon, 2014; Wongsurawat, 2009). In 2005, the organization that manages the American College Test (ACT), a standardized admission test similar to the Scholastic Achievement Test (SAT), examined the degree of grade inflation by comparing high school GPAs and ACT composite scores from 1991 to 2003. Among students who obtained a score of 24 on the ACT, high school grade point average (HSGPA) increased over that period from 3.15 to approximately 3.45 (ACT, 2007). This trend was most significant for students in the low to middle range on the ACT composite scores. However, a follow up report published by the ACT in 2013 comparing high school GPAs and ACT composite scores from 2004 to 2011 found that after 2003, there was comparatively little change in HSGPA scores for the selected ACT composite scores. During this time period, students who obtained a score of 24 on the ACT had a consistent HSGPA of approximately 3.50 (ACT, 2013). Researchers also made claims that grade inflation is present in the grades of many incoming university students, and that as more students enter university with increasingly higher grades, the informational value of those grades, in fact, decreases (Camara et. al., 2003). Since these two organizations make considerable money developing and administrating their examinations, they may be inclined to criticize the validity of high school grades (Geiser, 2007). With that being said, the evidence of grade inflation is difficult to dismiss.

In summary, it appears that the increasing presence of grade inflation will make it more difficult for universities to predict their students' achievement, and within specific subject areas, based on high school grades alone. While there is a significant body of research examining the statistical predictive power of high school grades, entrance examinations, and other demographic variables on university achievement, the research is limited with regards to discipline-specific predictors of achievement in subjects such as mathematics.

The "Mathematics Problem"

Research on the mathematics performance of first-year university students shows an increasing gap between their success in high school mathematics courses and their success in university mathematics courses. Anecdotally, the lack of success of students in mathematics courses at university is perceived by many mathematics professors as a serious problem that has been growing more serious since the mid-1990's (Crowther, Thompson & Cullingford 2007; Kajander & Lovric, 2005). Hawkes and Savage (2000) have coined the term "*The Mathematics Problem*" to describe the challenge faced by students entering university with poor mathematical skills and knowledge.

Moreover, this problem has been noted by mathematics professors around the globe, for example, in Sweden (Brandell, Hemmi & Thunberg, 2008), South Africa (Engelbrecht & Harding, 2008), the United Kingdom (Hawkes & Savage, 2000), New Zealand (Hong et. al, 2009), Ireland (Hourigan & O'Donaghue, 2007), Canada (Kajander

& Lovric, 2005), the United States (Morgan, 2012), and Australia (Wilson & MacGillivray, 2007). In many of these articles, researchers describe how the decline in students' preparedness for calculus, and their declining achievement in the course, is not reflected in their incoming pre-calculus (or equivalent) course grades. More specifically, it appears that while students' high school mathematics grades have either remained relatively constant or, in many cases, has increased over the years, their performance in first-year university mathematics courses has declined.

In order to substantiate these claims, Hawkes and Savage (2000) analyzed the results of a 50-item multiple-choice diagnostic test given to the incoming A-level cohort (high school graduates who had completed course work that would admit them to university) at Coventry University in the United Kingdom during their "induction week". The researchers found that the incoming A-Level students' scores on the diagnostic test declined from 35.2 in 1991 to 30.4 in 1998, a decrease of around 10%, which was not reflected in a decline in their A-level grades in high school. Unfortunately, the authors did not ask whether or not successive cohorts of students also had a drop in their mathematics achievement at university. Nonetheless, these results highlight the decline in the level of mathematics preparedness of incoming students at this particular university over a period of time.

Hourigan and O'Donaghue (2007), note the "mismatch" between students' mathematical experiences in high school and their experiences in university in their qualitative study focusing on the experiences of mathematics students at two Irish secondary schools. They found that students achieving 'A' grades in mathematics in high school were often unable to use mathematics concepts and procedures properly except in the simplest and most practiced way. Furthermore, their research showed large gaps in the students' knowledge that would traditionally be associated with performing poorly in high school. About one-third of the incoming students who had achieved high grades in high school mathematics were diagnosed as being 'at risk' of failing their university mathematics courses. These students lacked a fundamental knowledge of mathematics, yet they passed the required mathematics courses for high school graduation, but even those with high school grades that indicate that they were 'high attaining' students also had considerable trouble in mathematics (see Hong et. al, 2009; Hull & Seely, 2010).

Results from the Educational Longitudinal Study in the USA found that of the 26.8% of students who completed 'high-level coursework' (at least one year of coursework beyond Algebra II), needed to complete a developmental course in mathematics in first-year university to be successful in higher level mathematics courses (National Centre for Education Statistics, 2010). Similarly, the ACT reported that of high school graduates who completed an extra year of mathematics in addition to the core high school mathematics course, only 62% were ready for college-level mathematics courses (ACT, 2007). This study reinforce the findings of Crowther, Thompson, and Cullingford (2007) who claim that generally, students' university mathematics achievement is somewhat unrelated to their high school mathematics achievement.

More detailed research on the gap between students' success in high school mathematics and their subsequent success in university mathematics emphasizes several factors. Some factors may be influenced more directly by high school and university math teachers (e.g., teaching style), others are broader (e.g., culture shock) and therefore, may be difficult for individual teachers or professors to change.

Clark and Lovric (2009) put forward a notion that a “rite of passage” (p. 756), in which, among other things, certain cognitive and pedagogical problems that students encounter influence their difficulties in making a transition from high school mathematics courses to university mathematics courses. For example, they describe the transition from more informal language and reasoning in high school to more formalized language and reasoning in university as one of the more common cognitive and pedagogical disconnects that students generally face. The need to progress from surface learning to deeper mathematical understanding can be one of the factors causing students to feel ‘culture shock’ when they enter university mathematics courses. Of course, this can make their transition difficult.

Another factor that contributes to the gap in students’ achievement between high school mathematics and university mathematics is the misalignment of various aspects of high school and university mathematics courses. Course content is one of these aspects (Thomas et. al, 2010). Whereas in the past, the content of high school mathematics courses was much more closely aligned with the skills and knowledge required in first-year mathematics course in university, this is not generally true now. Over the years, while first-year university calculus course content has remained largely unchanged, frequent reforms in high school course content has resulted in this course moving farther away from the requisite skills and knowledge required in university calculus courses (O’Shea, 2003). For example, topics such as conic sections, which appear in most first-year calculus courses, were removed from the Manitoba Pre-Calculus course in 2013 (Manitoba Education and Advanced Learning, 2014).

Another contributing factor to the misalignment between high school and university mathematics courses is the revised high school graduation requirements, which have led many students to be eligible to take mathematics at university, while university requirements have stayed the same. In the 1960's, most high school students did not study mathematics past grade 8 or 9 (O'Shea, 2003), whereas currently Manitoba students are required to complete a grade 12 level mathematics course in order to graduate from high school. This has led to a diverse population of first-year students entering mathematics courses. The variety of students with diverse skills and interest are two contributing factors that result in students having difficulty in completing university mathematics courses (Hawkes & Savage, 2000; Hong et. al., 2009; Kajander & Lovric, 2005; Rylands & Coady, 2009).

Insufficient communication between high school mathematics teachers and university mathematics instructors, and the different beliefs about student preparation, are another factor which causes difficulty for first-year mathematics students (Dossey, 1998; Hull & Seeley, 2010; Morgan, 2012; Thomas et. al., 2010). The disconnect between these two levels in mathematics has generated a gulf between high school mathematics teachers' and university mathematics instructors' perceptions of their students success. For example, Hong et. al (2009) found that 66% of the high school teachers thought that their mathematics students were well prepared to study mathematics at the university level, but only 26% of university mathematics lecturers felt the same way about the same incoming students. This evidence suggests that high school teachers tend to overestimate the level of readiness of their students in comparison with university lecturers. The disparity between teachers' and instructors' perceptions of students' level of readiness has led teachers to send under-prepared students to university believing that they have gained the necessary skills to succeed, especially in mathematics.

There also exists a misalignment between teaching style at the different levels. This misalignment may be caused by a lack of information shared between high school teachers and university mathematics instructors, or it may simply be that the insufficient communication has caused a lack of knowledge about instructional practices at the two levels. However, different expectations of students' capabilities (such as independent learning skills and reliance on calculators), the lower lecturer-student interaction, the increased pace of the course, the higher workloads at university, and larger classes in university mathematics courses all seem to contribute to the negative transition for some students (Hull & Seeley, 2010; Thomas et. al., 2010). In high school, there is typically a focus on minimal competency, surface learning, and symbolic manipulation in mathematics that results in graduates who are not adequately prepared for the more rigorous expectations and formal thinking processes in university mathematics courses (Hong et al, 2009; Hourigan & O'Donaghue, 2007; Kajander & Lovric, 2005; Morgan, 2012). This suggests that many students who have succeeded in high school mathematics courses often do not have adequate foundational knowledge and skills to do well in their university mathematics courses.

Thus, environment, content, teaching style, teachers' perceptions, and mathematical rigour differences between high school and first-year university mathematics are all factors that affect the correlation between high school Pre-Calculus grades and first-year university Calculus grades. This literature emphasizes the need to analyze the predictive power of high school mathematics grades on university mathematics grades at the University of Manitoba in more detail.

Predictors of University Mathematics Achievement

The results of studies that explored the relationship between grades in high school and grades in university mathematics courses are varied. Bengmark, Thunberg, and Winberg (2017) found that upper-level high school mathematics grades accounted for only about 17% of the variation in university mathematics grades, which increased to about 47% when student motivation, self-efficacy, study habits, and views on mathematics were included in the analyses. James, Montelle, and Williams (2008) found that the relation between students' high school and university mathematics grades may be a function of how many mathematics courses they completed in high school. They noted that students who completed more high school mathematics courses and obtained higher grades in those courses, were more likely to succeed in their subsequent first-year university mathematics courses.

When comparing the predictive power of high school mathematics grades to mathematics entrance examination scores, Brown, Halpin, and Halpin (2015) examined adjusted ACT math section scores and the grades in high school mathematics courses (such as calculus and algebra) as predictors of GPA in what they call 'quantitative courses' (defined as "a college course whose conceptual foundation is based in mathematics" (p. 4)) at university. They found that the adjusted ACT Math score had the strongest correlation with quantitative GPA ($r = 0.46$), followed closely by high school calculus grades ($r = 0.42$).

Rylands and Coady (2009) also compared the predictive power of high school mathematics grades and university mathematics entrance examination scores. They concluded that neither students' grades in their Advanced mathematics courses in high school (most closely resembling Pre-Calculus in the Manitoba curricula) nor their scores on the entrance examination used by their institution (called the University Admissions Index or UAI) were significantly associated with grades in either Discrete Mathematics or Basic Mathematics. However, they found that first-year Discrete Mathematics grades were predicted by high school Intermediate Mathematics course grades ($r = 0.51$), which resembles the Applied Mathematics course in Manitoba rather than the Pre-Calculus course.

These results of these studies lead to the question of why high school mathematics grades are not a better predictor of subsequent university mathematics grades in related coursework. One possible factor which has been analyzed is the practice of grade inflation at the high school level.

Grade Inflation in Mathematics Courses

Though many studies have examined grade inflation in high school, relatively few have focused on inflation in mathematics courses specifically. A search through Google Scholar, the University of Manitoba's library database, and ERIC, confirms this to be true. Using the search term 'grade inflation' in these engines reveals only around 150 studies about grade inflation published in peer-reviewed journals over the past 20 years, almost all of which focused on grade inflation at the post-secondary level. Less than 20 of these studies concentrate on grade inflation in high school, and only a handful concerned grade inflation in mathematics. Meanwhile, a Google search of the same term shows over

800,000 articles in the popular news media covering grade inflation. Adamanti-Trache, Bluman, and Tiedje (2013) substantiated the claim that the effects of grade inflation are being experienced by Canadian post-secondary institutions, specifically in mathematics-laden subjects, by discussing the effects that a lack of standardized provincial examinations in these subjects can have. Specifically, they mention that without such standardized examinations, school-to-school variability likely increases, and there are fewer checks and balances to ensure that teachers are following the curriculum. Though they do not provide direct evidence of grade inflation in high school grades, these researchers assert that standardized provincial examinations are a safeguard against grade inflation. In this context, the authors are saying that grade inflation can occur when students are assigned grades that do not reflect their capabilities based on what was intended to be covered by the curricula. Grades also vary from subject-to-subject, from teacher-to-teacher, and from school-to-school, and consequently grades may not accurately represent the students' skill level that would be assessed by standardized examinations.

Further evidence of grade inflation in mathematics was found in Alberta where increasing numbers of students who entered university with inflated grades were left "floundering"; the evidence was a rather large gap between their high school mathematics grades and their scores on the provincial standardized examinations (French, 2017). The difference between the grades in one high school mathematics course (Math 30-1) and scores on the provincial examination increased from 5.8% in 2013 to 14.8% in 2016. Though exact figures are not given, in 8 out of 12 Grade 12 courses (one of which is a mathematics course), the number of students earning 'excellent marks' (which appears to be defined as marks over 80 percent) has increased from 2008 to 2016, while the proportion achieving 80% or above on the provincial examination has stayed about the same (French, 2017).

The evidence showing the increasing gap between students' high school grades and their actual skills as reflected on standardized examinations is particularly interesting as it comes from a context that is rather similar to the context in Manitoba. While students' incoming average high school Pre-Calculus grades increased from an average of around 79% in 2001 to an average of 84% in 2015, their provincial examination grades in Pre-Calculus over the past three years have remained in the high 60's (Manitoba Education and Training, 2016). This finding highlights the increasing disparity between high school mathematics students' standardized achievement and their course grades. The effect of grade inflation makes it difficult for university instructors to distinguish between students who lack mathematical preparation from those who are well-prepared, weakening the predictive power of high school pre-Calculus grades, most significantly for students who achieve low grades in first-year university Calculus.

While the overall predictability of high school mathematics grades appears to be declining, there may be other factors that are associated with predictability of high school grades for university grades, such as gender (Adamuti-Trache, Bluman & Tiedje, 2013; Islam & Al-Ghassani, 2015), high school type and location (Adamuti-Trache, Bluman & Tiedje, 2013; Geiser, 2007), trends over time (Kalajdzievska, 2011) and mathematical achievement level (Islam & Al-Ghassani, 2015; Rylands & Coady, 2009).

Factors Associated with Predictability

Gender. Islam and Al-Ghassani (2015) compared the predictive relationship between HSGPA in mathematics and grades in university math courses between female and male students. Students in Sultan Qaboos University in Oman are accepted into courses and programs on the basis of their high school grades alone; in other words, there are no entrance examinations. These researchers found that overall high school grades and grades in a first-year calculus course showed a strong positive linear relationship ($r = 0.51$). Building a multiple regression model, high school math grades and gender added 5.4% and 1.2% respectively to the $r = 0.51$ variation accounted for in predicting first-year calculus grades. Though Islam and Al-Ghassani (2015) found that females significantly outperformed their male counterparts both at high school (overall average high school grade for females 93.0%, males 86.3% and average high school math grade 90.9% for females and 80.5% for males) as well as in university calculus (an average of 2.83 grade points for females (between a B- and B) and an average of 1.74 for males (between a C- and a C)), they do not compare the predictability of high school grades between the genders. These results are similar to the results reported by Adamuti-Trache, Bluman and Tiedje (2013), who used a hierarchical linear model to show that students' university grades were highly influenced by student-level variables such as gender. These authors also show that females outperformed males, but they do not compare the predictability of the model across genders. Although these two studies analyzed the predictive power of high school grades with gender as a factor, the difference in predictability between the genders is not clearly stated. Nevertheless, these results show that gender should be controlled as a possible confounding variable.

School type. Adamuti-Trache (2013) and her colleagues focused on the effect of high school type (private versus public) and the location of the schools (urban versus rural) on students' successful transitions in first-year university mathematics and physics courses at the University of British Columbia. This study is the only one that used the predictability of high school grades on university grades as a measure of success of high school instruction, and the only study that compared the predictability between schools and school types (private and public). Specifically, these researchers hypothesized that current measures of high school effectiveness, based on the results of standardized examinations, are much weaker indicators of the quality of the education that students received in high school than measures of successful secondary to post-secondary transition such as grades in subsequent university courses. In other words, these researchers claimed that it is the relationship between the performance of students in university subjects and their performance in the related high school subject that truly shows the effectiveness of schools in teaching their students that subject. Their results showed that students from different high schools entering first-year university with the same high school grades in mathematics and physics consistently received different grades in the related university courses. They also found that when the measure of success was the strength of the relationship between the students' high school course grades and their related university course grades rather than outcomes on standardized examinations, public schools ranked considerably higher than independent (private) schools. Based on their research, these academics claim that the ranking for public schools was 9.4, compared to 15.0 for independent schools (a lower mean rank value corresponds to a stronger relation between the high school math grades and university math grades). This study also highlighted differences between genders, school types, school locations, and socio-economic status on predictive power of high school marks in physics and the students' performances in university physics courses. In sum, these results indicate that the relation between high school and university math performances vary considerably as a function of gender and school type.

Trends over time. Data collected from the University of Manitoba suggest discouraging trends in average Calculus GPAs and failure rates in the first-year Calculus course. A Cox-Stuart Test for Trend found a statistically significant decrease in average GPAs and an increase in the rate of students who either received grades of Ds or Fs, or voluntarily withdrew from the course (DFW) in the years from 2001 to 2011. These DFW rates have now reached levels of approximately 50% of the students who enrolled in the first-year Calculus course at the University of Manitoba (Kalajdzievska, 2011). With enrollments in this course at about 2000 students per year, this represents an alarming trend for a large proportion of the students. These trends, in fact, mirror those described by Hourigan and O'Donaghue (2007), whose incoming A-Level cohort students have average scores that declined from 35.2 in 1991 to 30.4 in 1998. It is thus very important to look at the associated high school grades to see whether or not these trends are reflected in the University of Manitoba data. If not, this would highlight the increasing disconnect between high school mathematics grades and university mathematics grades.

Many universities, such as the University of Manitoba, require students to complete at least one mathematics course in order to graduate, and the students' success in first-year mathematics courses is obviously linked to persistence in science, technology, engineering and mathematics (STEM) programs and to retention in university as a whole (Parker, 2005; Rylands & Coady, 2009). Therefore, it is imperative that students entering first-year university need to be adequately prepared for their mathematics courses. As this research literature describes, using Pre-Calculus grades as the only criterion for accepting students into first-year mathematics courses may lack the necessary predictability to be effective, and the relationship may vary for different demographic groups.

Pre-Calculus and Calculus in the Manitoba Context

In Manitoba, mathematics students, instructors, and administrators face unique challenges. First, the final grade for high school Pre-Calculus 40S is computed so that only 20% comes from the standardized Provincial Examination, with the remaining 80% based on teachers' assessments. This means that the largest proportion of the students' Pre-Calculus grades is determined in a non-standardized way, by individual teachers, using their own criteria. This is troubling when considering the difference between term marks and provincial examination marks as illustrated in the precious section. Second, the provincial examination is not as 'standardized' as it could be because it is not graded in a central location as most standardized exams are graded in other jurisdictions, such as the SAT exams that are used in the U.S.A. Instead, the exams are marked locally within each school division and in some cases in the school by the teacher or teachers who taught the pre-calculus course. Of course, the teacher or teachers are expected to follow a marking guide provided by Manitoba Education and Training (https://www.edu.gov.mb.ca/k12/assess/docs/pol_proc/document.pdf), but there is no way of knowing if they do this or not. Without seeing some psychometric properties of the test grades, such as their reliability and validity, it is impossible to confirm how accurately the test scores reflect the students' mathematics understanding and skill.

The University of Manitoba's Introductory Calculus (MATH 1500) course is used as a prerequisite for many areas of study, especially those in the STEM programs, meaning that it is a gateway course for numerous programs in a number of faculties. Currently, the pre-requisite to be admitted into MATH 1500 is a 60% or higher in a high school Pre-Calculus (or an equivalent) course. Failing the Calculus course can prevent students from studying STEM subjects, and in some cases from obtaining a university degree (Mattern & Packman, 2009; Parker, 2005). Hillel (2001) notes that Calculus is often used as a gateway into STEM majors. If this is indeed the case, then current success rates at the University of Manitoba indicate that the mathematics program may need significant improvement. Using incoming Pre-Calculus grades alone to place students into the Calculus course deserves a much more critical analysis.

The Research Questions

To explore how well high school Pre-Calculus courses prepare students for introductory Calculus in first-year university, and some of the conditions that influence that relationship, the following research questions were examined:

1. What is the relationship between students' Pre-Calculus grades in high schools and their Calculus grades in university over the period from 2001-2015?
2. What proportion of students obtained a final grade that was significantly below their expected grade in university Calculus based on their grades in high school Pre-Calculus?

3. How much does the relationship between students' Pre-Calculus grades in high schools and their Calculus grades in university vary across gender (male/female), school type (public/private, English/French), school division, and yearly (from 2001 to 2015)? How much do these relationships vary among students who earned low Calculus grades?
4. What are the characteristics of students who obtained final grades that are significantly below their expected grades in Calculus based on their grades in high school Pre-Calculus?

Method

Data for this study was obtained from the University of Manitoba's Office of Institutional Analysis with all identifying information of individual students removed. In order to obtain this data, written permission was obtained from the Chair of the Mathematics Department, the Associate Dean of Undergraduate Studies, and the Dean of the Faculty of Science.

The Students

The students included in this study were 17,196 students who took Introductory Calculus (MATH 1500) at the University of Manitoba from 2001 to 2015. All of these students were enrolled in MATH 1500 for the first time and completed Pre-Calculus 40S in a Manitoba high school. In addition, students with a grade of VW (voluntary withdrawal) in MATH 1500 ($n = 4{,}988$) were not included in the data set. This decision was based on the fact that a VW can be due to a variety of factors (emotional, financial, etc.), and may not necessarily indicate lack of ability in Introductory Calculus. Approximately 29% of the students in the sample voluntarily withdrew from the course.

As a result, 12,208 students completed the math course on which the primary analyses were performed. The number of male and female students in each year are reported in Table 2.1

Number of Students by Gender and Year

Year	Gender		Total
	Male	Female	
2001	494	405	899
2002	485	422	907
2003	457	416	873
2004	500	386	886
2005	510	457	967
2006	454	427	881
2007	358	395	753
2008	387	348	735
2009	371	366	737
2010	382	339	721
2011	357	370	727
2012	358	322	680
2013	392	369	761
2014	482	349	831
2015	452	398	850
	Total = 6,439 (52.7%)	Total = 5,769 (47.4%)	Total = 12,208

Table 2.1 shows that the proportion of male and female students is approximately equal, and that the number of Manitoba students in the course has had some variation across the years from 680 to 967.

Table 2.2

Number of Students by School Division

Division Code	Total
A	155
B	1,637
C	539
D	1,751
E	516
F	4
G	1,229
H	14
I	1,744
J	1,759
Outside Winnipeg	2,860
Total	12,208

Table 2.2 shows that the number of students entering the Calculus courses varies across actual divisions, with the largest divisions sending the highest numbers of students from Pre-Calculus to Calculus.

The Courses

In 2015, approximately 2,500 students completed grade 12 Pre-Calculus and wrote the Provincial Pre-Calculus examination. The Provincial pass rate was 80 percent, with a mean grade of 67 percent (http://www.edu.gov.mb.ca/k12/assess/tests/results.html). The Manitoba Curriculum Framework of Outcomes describes the Grade 12 (40S) Pre-Calculus mathematics course as being "designed for students who intend to study Calculus and related mathematics as part of post-secondary education" (Manitoba Education & Advanced Learning, 2014). So the Pre-Calculus course is specifically designed with the objective of transitioning students into a university Calculus course.

At the University of Manitoba, approximately 2,000 students enroll in Introductory Calculus (MATH 1500) each year (this includes between 1,000 and 1,500 student coming from outside of Manitoba and between 325 and 400 students who withdrew from the course, both of whom were not included in this study). This course is, in fact, the largest first-year course taught in the Department of Mathematics, and it is one of the largest enrollment courses at the university.

During the past 10 year period, the course evaluation consists of a 10 percent lab component (worksheets or quizzes done in tutorial sections), 30 percent midterm exam, and 60 percent final exam. The success rate for this course, students who receive a final grade of C or better (D is rarely accepted as an acceptable grade for university programs that requires MATH 1500) has decreased steadily from about 65 and 70% 5 years ago to approximately 50% now (see, for example, Kalajdzievska, 2011).

Variables

Independent variables. The main independent variable in this study is the Pre-Calculus grade. Pre-Calculus grades are reported as both percentages and a grade-point (GP) scale ranging from 0 to 4.5, which is the same scale used in the Calculus course. This scale is used to calculate the difference factor in the results section. The Pre-Calculus grade was converted to a GP scale using the Department of Mathematics standard grade-GP conversion table in which 90-100% = 4.5 (A+ or Excellent), 80-89% = 4 (A or Very Good), 75-79% = 3.5 (B+ or Good), 70-74% = 3 (B or Good), 65-69% = 2.5 (C+ or Satisfactory), 60-64% = 2 (C or Satisfactory), 50-59% = 1 (D or Unsatisfactory), 0-49% = 0 (F or Fail).

Four other independent variables are used: Gender, School Type (public/private, French/English), School Division, and the Year the Calculus course was Taken*.* For the school divisions, I assigned the letter from A to J to help maintain anonymity.

Dependent variable. The dependent variable is the students' Calculus (MATH 1500) final grades.

Results

Research Question 1

> What is the relationship between students' Pre-Calculus grades in high schools and their Calculus grades in university over the period from 2001-2015?

This study begins with an analysis of the distributions of the Pre-Calculus and Calculus grades to show trends. The frequency distribution for Pre-Calculus and Calculus grades are reported in Table 2.3.

Table 2.3

Grade Distributions in Pre-Calculus and Calculus

Final Grade (GPA scale)	Number of Students	
	Pre-Calculus	Calculus
0.0	7 (0.06%)*	1175 (9.6%)
1.0	333 (2.7%)*	488 (4.0%)
2.0	790 (6.5%)	1347 (11.0%)
2.5	886 (7.3%)	1383 (11.3%)
3.0	1249 (10.2%)	1504 (12.3%)
3.5	1353 (11.1%)	1775 (14.5%)
4.0	4045 (33.1%)	2565 (21.0%)
4.5	3545 (29.0%)	1971 (16.1%)
Total	12,208 (100%)	12,208 (100%)

*Note: Though the pre-requisite Pre-Calculus grade to be allowed entry into Calculus is 60 percent, or a 2.0 GPA, some students (34) with lower grades are allowed in through special permission waivers.

The distribution of Calculus grades and Pre-Calculus grades are both somewhat skewed distributions (skewness of -0.936 and -0.532 respectively). The Pre-Calculus grades are more skewed to the right with the majority of students near the top of the range (4.0 or higher). This is at least partially due to the fact that students with an incoming Pre-Calculus grade of 0 or 1 would not normally be permitted to enroll in the Calculus course. The mean Pre-Calculus grade was 81.6% (or a 4.0, an A) with a standard deviation of 11.2%. In fact, 62.1% of incoming university students have Pre-

Calculus grades of 4 or 4.5. In comparison, the mean Calculus grade was 2.99, or C+, with a standard deviation of 1.33. In addition, the Calculus distribution has only 37.1% of students earning a grade of 4 or 4.5 – only about half as many as for the Pre-Calculus. These results clearly reflect a disconnect between Pre-Calculus and Calculus achievement for the students at the University of Manitoba.

The relationship between the Pre-Calculus grades and Calculus grades is illustrated with the scatterplot in Figure 2.1. In this figure, markers with darker colouring indicate larger numbers of data points.

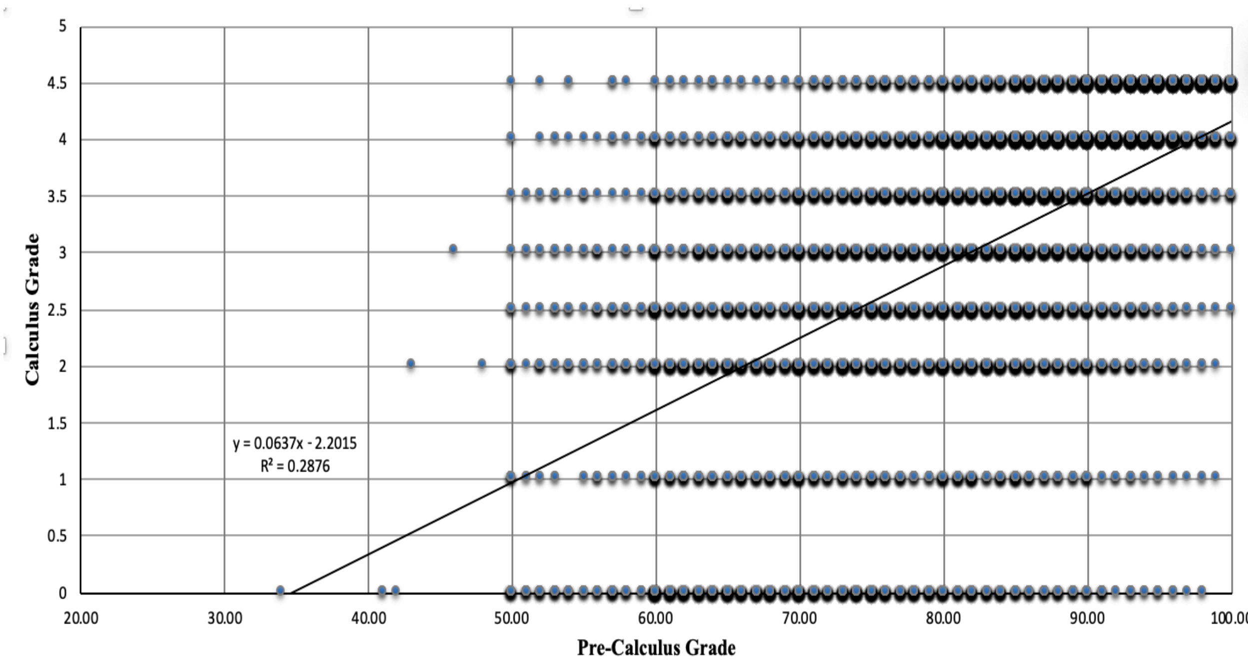

Figure 2.1: Scatterplot of Pre-Calculus Grades and Calculus Grades

Figure 2.1 indicates a positive linear relationship between the independent and dependent variables, and indicates the line of best fit for this data. The Pearson Product Moment Correlation shows a strong positive linear relationship between Pre-Calculus grades and Calculus grades ($r = .54$), indicating that almost 30% of the variation in students' Calculus grades can be explained by their Pre-Calculus grades. As expected, higher Calculus grades (4.5, or A+) correspond to a higher Pre-Calculus grades (4.0, or A, and 4.5, or A+) and low Calculus grades (0, or F) correspond to Pre-Calculus grades between a C+ and a B. In the mid-range, however, students achieving a Calculus grade higher than 2.5 but lower than 4.0 had a mean Pre-Calculus grade of at least an A. This pattern suggests heteroscedasticity in the data – that those with a lower Calculus grades may have a much greater variation in their incoming Pre-Calculus grade than students with higher Calculus grades. This trend is clearly illustrated in Figure 2.2.

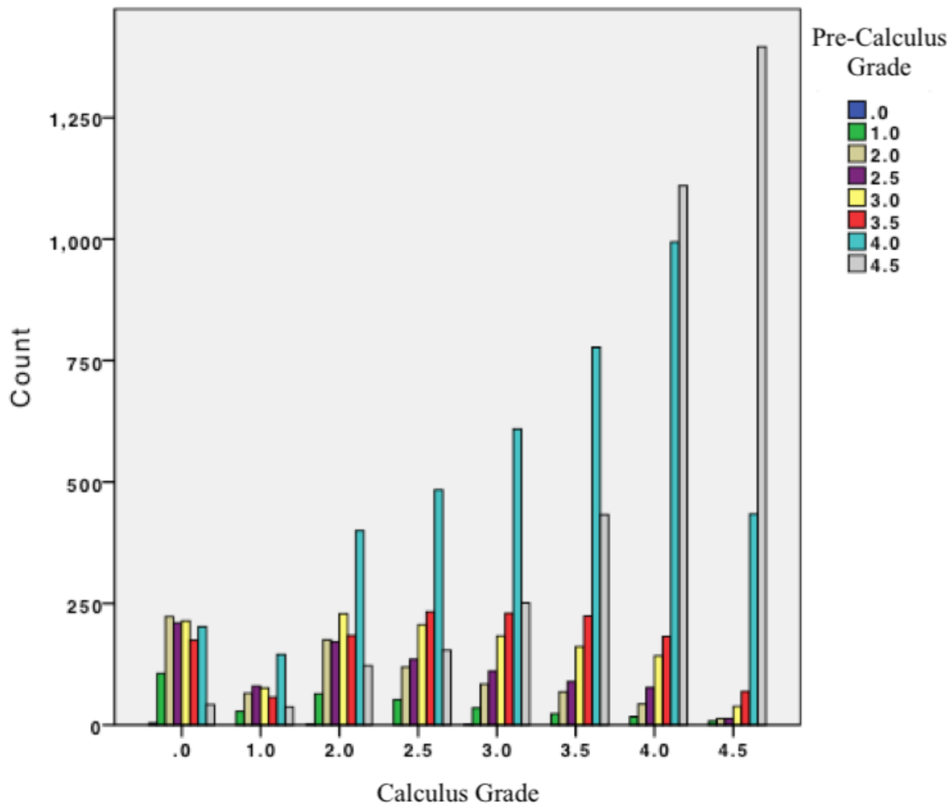

Figure 2.2: Distribution of Pre-Calculus Grades at each Calculus Grade Value

Figure 2.2 shows that the distribution of Calculus grades on the right hand side of the graph, the higher end (3.0 to 4.5) looks relatively reasonable, probably as expected, with greater numbers of students entering with higher Pre-Calculus grades. For example, students who received a 4.5 in Calculus most often came into the course with 4.5 in Pre-

Calculus. The other end of the scale, Calculus grades of 0.0 to 2.5 is, however, much different. Here, it is clear that a much similar number of students entered the Calculus course with a variety of grades in Pre-Calculus. The best example is for the students who failed the Calculus course (0.0) where students who enrolled with grades of 2.0 to 4.5 were almost equally likely to fail. Students scoring a D, C, or C+ (1, 2, or 2.5 grade points) in Calculus are much more likely to be entering with an A (4.0 grade points) in Pre-Calculus than any other grade, suggesting that at the lower Calculus grades, students' incoming Pre-Calculus grades are much more weakly related to their Calculus grades. Thus, I explored, in greater detail, the relationship between Pre-Calculus grades and Calculus grades for the students achieving a C+ (2.5 grade points) or less in Calculus.

While correlations are commonly used to show the relationship between scores, one limitation of correlations is that they show the patterns for two variables only as a straight line. Another way to illustrate the relationship between Pre-Calculus and Calculus achievement is to use a measure called the *"Difference Factor" (DF).* This procedural framework was used in the UBC Math Report and is a particularly useful measure of transitional success because "a smaller grade difference is a likely indicator that the high school is preparing students to enhance their potential for success at the next level" (Adamuti-Trache, Bluman & Tiedje, 2013, pp. 2907). Building on this framework, the DF value was calculated from the absolute difference between Pre-Calculus grade point and Calculus grade point with 0.5 representing a difference of 1 letter grade (e.g., B+ to A). Lower DF values reflect greater agreement between the students performances in the Pre-Calculus and the Calculus courses. The values are interpreted as follows: 0 (perfect agreement), 0.5 (very strong), 1.0 (moderately strong), 1.5/2.0 (medium), 2.5

(weak), 3.0/3.5 (very weak), and 4.0/4.5 (no relationship). For example, a student who scored an A in Pre-Calculus (4.0 grade points) and received a B in Calculus (3.0 grade points) would have a DF of $|4-3|=1$, indicating a difference of 1 full letter grade. As noted above, nearly all of the Pre-Calculus grade point values (97.2%) are above 2.0, which means that DF values 2.0 or greater are very unlikely to be discovered. In addition, 85% of the students in the sample had Calculus grades that were below their Pre-Calculus grades, so the calculated difference between these scores in almost all of the cases was negative. In the DF calculation, however, the absolute value was applied in order to maintain consistency of the values for analyses. Though this DF measure does not distinguish between differences where Pre-Calculus grade was better than Calculus or vice-versa, it remains valid as the absolute difference is of interest, and the proportion of students with Pre-Calculus grades better than Calculus grades is very small.

Not surprisingly, the mean DF scores reflect the three groups suggested in Figure 2.3. Students achieving a Pre-Calculus grade of 4.5 ($n = 3545$) have a DF mean of approximately 0.6, students with Pre-Calculus grades between 2.0 and 4.0 ($n = 8323$) have DF scores between 1.0 and 1.1, and those with a Pre-Calculus grade of 1.0 ($n = 333$) have a DF score of almost 1.4. The number of students coming in with a Pre-Calculus grade of 0 is negligible (less than 5 in total), so those results should not be analyzed as part of the overall trend. There is a negative correlation between Pre-Calculus grades and DF scores ($r = -0.22$) indicating that as Pre-Calculus grades increase, the DF scores decrease. The evidence shows that students performing extremely well in Pre-Calculus have a low DF score.

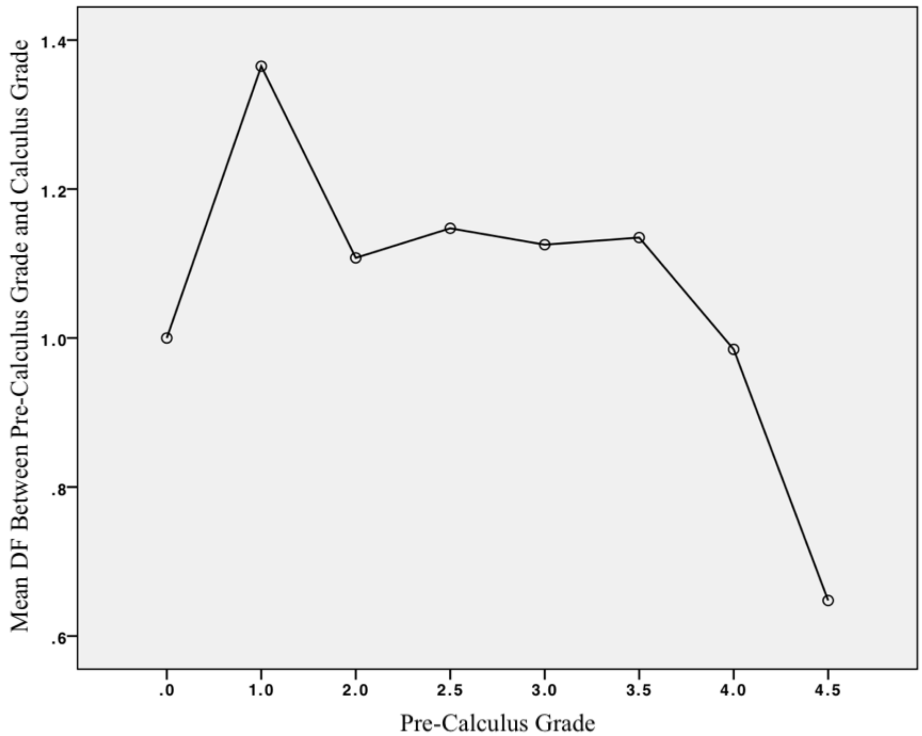

Figure 2.3: Mean Difference Factor (DF) at Pre-Calculus Grade Point Values

Figure 2.3 shows the trend in DF values for the Pre-Calculus grade point values. There is a steep drop in DF from Pre-Calculus grades of 1 to 2, then the DF value levels off for Pre-Calculus grades of 2 to 3.5, followed by a steep drop for Pre-Calculus grades of 3.5 to 4.

Examining the relationship between the DF and Calculus grades, the mean DF scores show a clear, almost linear trend in Figure 2.4 – as Calculus grade increases, the DF values decrease. Students achieving a Calculus grade of 4.5 (n = 1971) have a DF value of approximately 0.25, students with a Calculus grade of 2.0 (n = 1347) have a DF value of 1.3, and students with a Calculus grade of 0 or fail (n = 1175) have a DF value of almost 3. There is a very strong negative correlation between Calculus grades and DF scores (r = -.78) indicating that as Calculus grades increase, the DF decreases. This finding indicates that students who are less successful in Calculus most often have Pre-Calculus grades that are not closely related to their Calculus grades. In fact, for the great majority of these students, their Pre-Calculus grades are high.

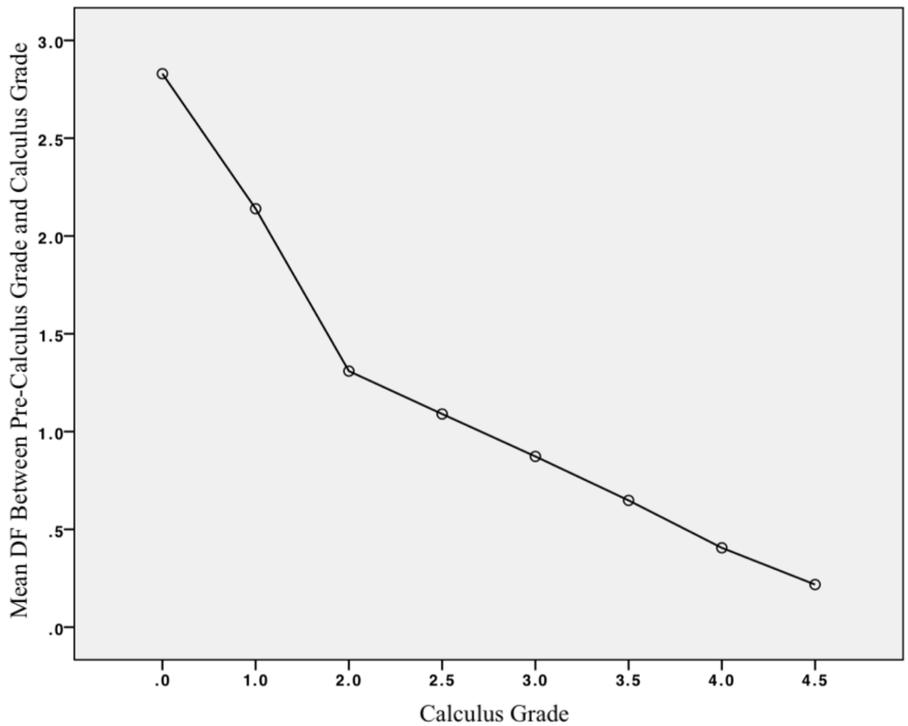

Figure 2.4: Mean Difference Factor (DF) for Calculus Grade Point Values

What Figures 2.3 and 2.4 show is that students coming from Pre-Calculus courses in Manitoba who have high grades in Calculus have most often earned similar grades in Pre-Calculus, but that those who did not do well in Calculus, generally have a large variation in their incoming grades. In other words, success in Pre-Calculus is a necessary but not a sufficient, condition for success in Calculus. These results suggest a specific need to examine, more carefully, the low achieving students in the Calculus course.

The literature, as well as the preliminary analyses reported above have suggested a more careful look into the differences between Pre-Calculus grades and Calculus grades. The Pearson Product Moment Correlation shows a moderate positive linear relationship between Pre-Calculus grades and Calculus grades (r = .23) for students with Calculus grades of 2.5 or lower (n = 4393), indicating that only approximately 5.3% of the variation in Calculus grades is explained by Pre-Calculus grades for these students. In other words, there is considerable variation that is potentially explained by other factors.

There is a significant negative correlation between Calculus grades and DFs (r = -.78). For lower Calculus grades, the DF values get larger (meaning that the relationship is weaker). Though those achieving a Calculus grade between 3.0 and 4.5 are generally within the 1 letter grade level suggested by Finnie et. al (2010), students achieving a Calculus grade of 2.5 or less show a higher grade decrease then would be otherwise predicted (Figure 2.4). In fact, students with a Calculus grade of F (0 grade points) show an average of almost 3 full grade points difference between their Calculus grades and their incoming Pre-Calculus grades. This means that those with low Calculus grades have grades that are very weakly related to their Pre-Calculus grades. With a steep decline in predictive power at the lower end, how are Calculus instructors and the incoming students to know how well students are prepared to succeed in the Calculus course? Should these students seek remediation? These results suggest examining the students performing poorly in Calculus, for example students with grades of 2.5 or lower (n = 4393).

Research Question 2

> What proportion of students have obtained final grades in their university Calculus that were significantly below expected based on their grades in high school Pre-Calculus?

Though some students may perform as expected in Calculus, based on their Pre-

Calculus grades, it is those who are performing well below what their Pre-Calculus grades would indicate who are of interest in terms of the high failure and withdrawal rates experienced in the Calculus course today. Students who obtained Calculus grades that were below expected (BE) are those for whom the DF is 1.5 or larger. For example, students receiving Excellent (A+)/Very Good (A) grades in Pre-Calculus and earning Satisfactory (C+ or C) grades in Calculus would have a DF score of about 1.5. Students whose DF score is 1.0 or less are considered to have obtained Calculus grades as expected (AE). For this analysis, we restrict the data to students coming from high schools in Winnipeg, due to the comparatively small number of students coming from rural high schools (n = 2860 over all 15 years) and due to the fact that students in rural high schools may experience more variation in their instruction and in the options for classes that they have.

Looking at the students coming from high schools in Winnipeg from 2001-2015 (n = 9348), Table 2.4 shows the number and proportion of students performing below expected based on their Pre-Calculus grades.

Table 2.4

Students Performing Below Expected in Calculus

	Pre-Calculus Grade			Total
	A+/A	B+/B	C+ or Lower	
Number of Students	5912 (63.2%)	1914 (20.5%)	1522 (16.3%)	9348
Number Performing Significantly Below Expected	1183 (20.0%)	360 (18.8%)	0 (0.0%)	1543
Number Performing As Expected	4729 (80.0%)	1554 (81.2%)	1522 (100%)	7805

In essence, about 20% of the high achieving students (those scoring a B or above in Pre-Calculus) earned Calculus grades that were below what their Pre-Calculus grades predicted. In other words, most students who do well in Calculus have also done well in Pre-Calculus, but there is a considerable proportion of those who do well in Pre-Calculus who do not do well in Calculus. For those students, we do not know why they have performed worse than expected. For this reason, we begin examining some demographic variables in order to identify potential causes.

Research Question 3

> How much does the relationship between students' Pre-Calculus grades in high schools and Calculus grades in university vary across gender (male/female), school type (public/private, English/French), school division, and yearly (from 2001 to 2015)? How much do these relationships vary among students who earned lower Calculus grades (of 2.5 or lower)?

Gender. The relationship between Pre-Calculus Grades and Calculus grades are very similar for male and female students (Table 2.5). The average DF for males and females were 0.98 and 0.91 respectively. As well, the correlations for males and females were 0.52 and 0.55 respectively.

Low achievers – gender. Among the lower-achieving students, however, these relationships differ (Table 2.5). For low achieving students, the DFs for males and females were 1.68 and 1.82 respectively, and the correlations for males and females were 0.21 and 0.26. This suggests that among those who have not done well in Calculus (grade of 2.5 or lower), females have slightly more variation than males. In addition, the correlation between Pre-Calculus grades and Calculus grades for males and females in the low achieving group is only about half as strong as compared to the population, and the DF values for both genders are higher for the lower achieving students than overall.

Table 2.5

Correlations and Average Difference Factor (DF) by Gender

	All Students		Low Achieving Students	
	Gender		Gender	
	Male	Female	Male	Female
Total Number	6439	5769	2506	1887
DF Value	0.98	0.91	1.68	1.82
Correlation between Pre-Calculus Grade and Calculus Grade	0.52	0.55	0.21	0.26

School type and division. Analysis in this section focuses on urban schools (n = 9348), and data from the two smallest divisions (H, n = 14; F, n = 4) were dropped due to their small sizes. This left n = 9330 for the analyses. The DF and correlation for the remaining school divisions are shown in Table 2.6. The correlations range from 0.38 to 0.59 and DFs range from 0.81 to 1.17. The DF values show that across these divisions, students are dropping approximately 1 grade in the transition from Pre-Calculus to Calculus. Comparing DF values between the division with French schools (Division A) (DF = 0.86), the division with private schools (Division J) (DF = 0.81) and the public school divisions (Divisions B through I) (mean DF = 0.98), we see that students coming from the French and private schools have a slightly stronger relationship than students coming from the public divisions. However, the correlation between Pre-Calculus and Calculus grades is somewhat lower for the French school division (r = 0.38) than for the other divisions (ranging from .47 to .59).

Table 2.6

Correlation and Average Difference Factor (DF) by School Division

Division	Number of Students	DF Value	Correlation between Pre-Calculus Grade and Calculus Grade
A	155	0.86	0.38
B	1637	0.86	0.56
C	539	1.17	0.47
D	1751	0.97	0.50
E	516	1.12	0.49
G	1229	1.00	0.53
I	1744	0.86	0.59
J	1759	0.81	0.59
	Total = 9330	Average = 0.92	

Focusing on low-achieving students, the DF and correlation for the school divisions are shown in Table 2.7. Not only are the DF values higher for the lower achieving students than for the population of students, but there is also considerable differences between the divisions. While some divisions show a grade drop of about 1.5 grade points (Divisions A and J), one division, Division C, showed a drop of 2 points in the transition from Pre-Calculus to Calculus. In this division, students achieving, say, a C in Calculus, would on average be coming in with a grade of A in Pre-Calculus.

In comparing school types, the private schools (J) and the French division (A) have moderately lower DF values than the other divisions, indicating slightly better predictive value of the students' Pre-calculus grades on their Calculus grades. The correlations between the students' Pre-Calculus and Calculus grades are only about half as strong for the low-achieving students as for the population of students. Interestingly, the correlation between Pre-Calculus grades and Calculus grades for the low achieving students from the French division (A) is very low (r = .06), but this may be caused by the relatively small number of students included in this analysis (n = 50).

Table 2.7

Correlation and Average Difference Factor (DF) by Division – Low Achievers

Division	Total Number	DF Value	Correlation between Pre-Calculus Grade and Calculus Grade
A	50	1.53	0.06
B	524	1.69	0.24
C	230	2.00	0.33
D	621	1.76	0.18
E	242	1.71	0.22
G	498	1.66	0.30
I	555	1.78	0.24
J	525	1.59	0.21
	Total = 3245	Average =1.72	

When comparing Pre-Calculus to Calculus grades over the years, important trends emerge. The correlation between Pre-Calculus and Calculus grades for each year range from 0.50 to 0.61 (see Table 2.8). When examining the mean Calculus grades year by year, they range from a high of 3.21 in 2012 to a low of 2.62 in 2015, which also has the highest standard deviation (1.50). So, while the mean incoming Pre-Calculus grade was relatively constant at around 79% until 2005, the mean grade has increased to almost 84% in 2015. A similar trend is not apparent for the mean Calculus grade, which have remained relatively constant at around 3.0 (representing a letter grade of B), with a low of 2.6 in 2015 (a grade of C+).

Table 2.8

Mean Pre-Calculus and Calculus Grades by Year

Year Calculus Taken	Total	Mean (SD) Pre-Calculus Grade*	Mean (SD) Calculus Grade	Correlation Between Pre-Calculus and Calculus Grades
2001	899	79.25 (11.41)	2.96 (1.44)	0.55
2002	907	79.07 (11.31)	2.92 (1.40)	0.53
2003	873	78.64 (11.44)	2.92 (1.35)	0.53
2004	886	78.28 (11.44)	2.81 (1.33)	0.51
2005	967	78.74 (11.30)	3.09 (1.27)	0.56
2006	881	80.55 (10.84)	3.01 (1.23)	0.53
2007	753	82.03 (10.85)	3.04 (1.29)	0.61
2008	735	82.88 (11.15)	3.05 (1.23)	0.59
2009	737	83.66 (10.87)	3.19 (1.28)	0.54
2010	721	83.75 (10.16)	2.95 (1.32)	0.50
2011	727	84.12 (10.57)	3.10 (1.23)	0.59

2012	680	84.05 (11.04)	3.21 (1.20)	0.52
2013	761	83.86 (10.83)	3.09 (1.28)	0.57
2014	831	83.51 (10.59)	3.00 (1.43)	0.50
2015	850	82.92 (10.65)	2.62 (1.50)	0.57
	Total =12,208	Average = 81.49 (11.21)	Average = 2.99 (1.33)	

*The Pre-Calculus grades are shown here as percentages in order to better illustrate the variation from year to year, because conversion to grade points puts the values into larger categories.

The DF values remained fairly constant, averaging around 0.9 during that period (see Figure 2.5). Nevertheless, the average DF value has increased sharply from 2012 to 2015, suggesting that the gap between Pre-Calculus achievement and Calculus achievement has become more pronounced.

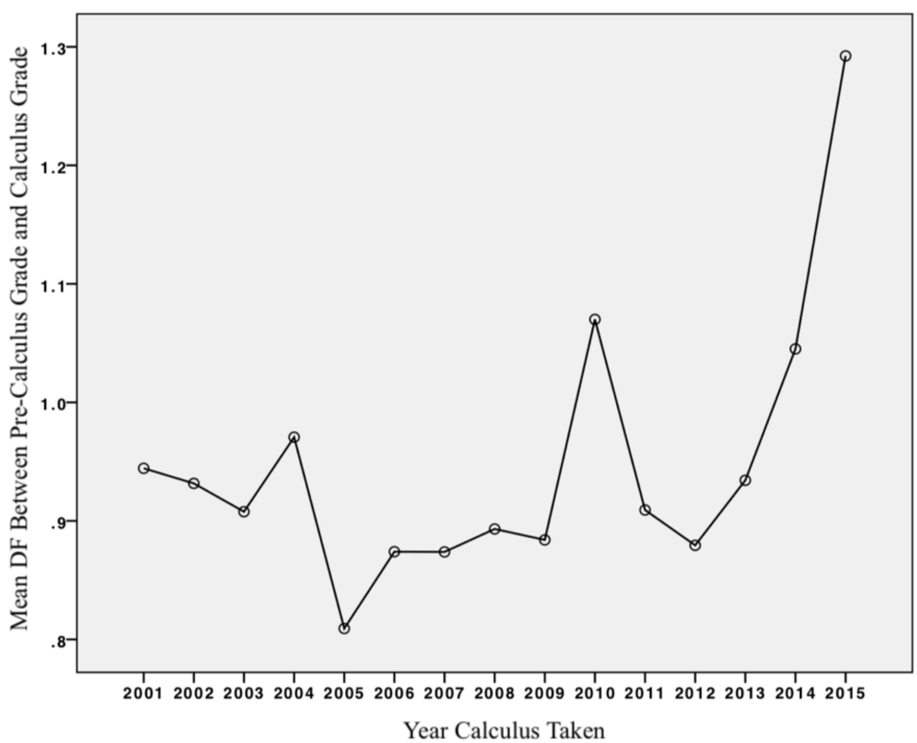

Figure 2.5: Average Difference Factor (DF) by Year

The proportion of students in the Low Achieving group (students scoring 2.5 or below in Calculus) varied from 25% in 2012 to 47% in 2015, and the correlation between Pre-Calculus grades and Calculus grades varied from r = .14 in 2006 to r = .37 in 2013 (Table 2.9). Again, we see that the strength of the relationship between Pre-Calculus and Calculus grades is much weaker for low achieving students (C+ or below) than for the higher achieving group.

Table 2.9

Number of Students, Proportion, and Correlation, by Year – Low Achievers

Year Calculus Taken	Number	Proportion of Total	Correlation between Pre-Calculus Grade and Calculus Grade
2001	317	35%	0.23
2002	351	39%	0.16
2003	356	41%	0.21
2004	363	41%	0.22
2005	316	33%	0.24
2006	327	37%	0.14
2007	257	34%	0.27
2008	248	34%	0.35
2009	225	31%	0.27
2010	265	36%	0.19
2011	236	33%	0.31
2012	169	25%	0.16
2013	257	34%	0.37

2014	305	37%	0.19
2015	401	47%	0.36
	Total =4,393	Average = 34%	

The mean DF values for the low-achieving students had considerable variation over the years, ranging from 1.4 to 2.2 (Figure 2.6), which were substantially larger compared to the population of students. In addition, the DF scores for the low achievers show a clearly worsening trend from 2005 (DF = 1.4) to 2015 (DF = 2.2). This means that in 2015, students obtained a D in Calculus, were likely to have obtained a B or higher in Pre-Calculus.

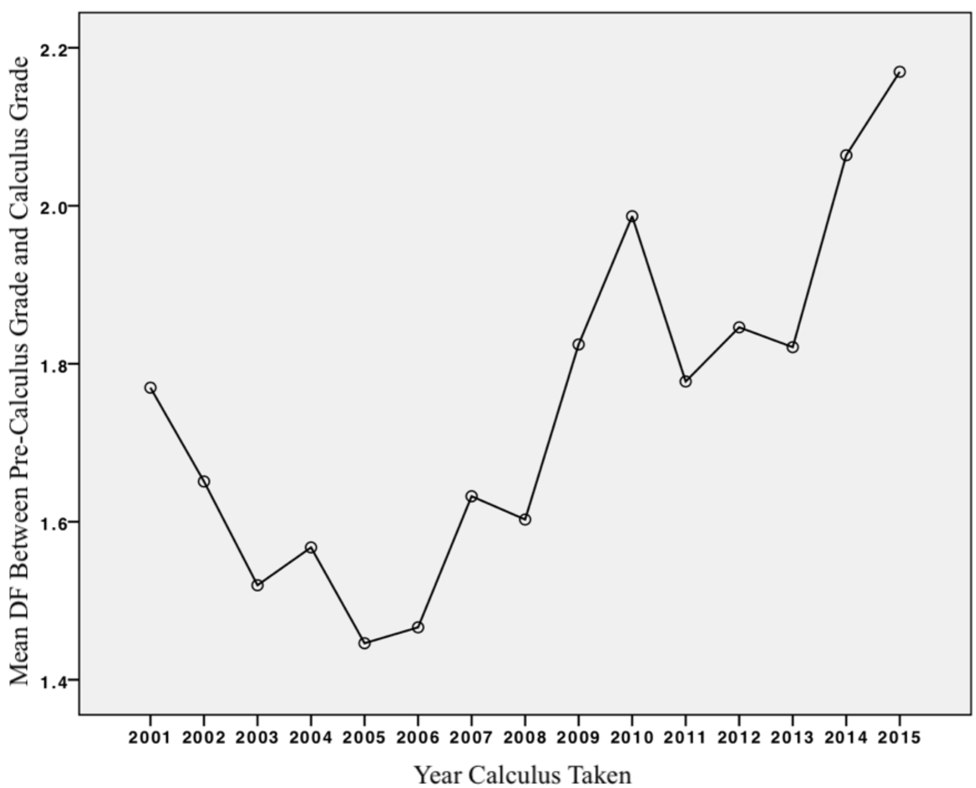

Figure 2.6: Average Difference Factor (DF) by Year – Low Achievers

Research Question 4

> What are the characteristics of students who obtained final grades that were significantly below their expected grades in university Calculus based on their grades in high school Pre-Calculus?

The numbers of students whose Calculus grades were significantly lower than expected are reported in Table 2.10. The gender proportions are quite similar (M=20.6% and F=18.8%). Interestingly, some meaningful differences were observed across divisions. While the French division (A) and private schools (J) have relatively few students performing below expectations (15.3% and 15.0% respectively), students in the other divisions have nearly double the proportion of students performing below expectations (30.7%).

Table 2.10

Students Performing Below Expected in Calculus

		Pre-Calculus Grade				Total	
		A+/A		B+/B			
		Total Number	Proportion of these Performing Significantly Below Expected	Total Number	Proportion of these Performing Significantly Below Expected	Total Number	Proportion of these Performing Significantly Below Expected
Gender	Male	2893	21.47%	1068	18.26%	3961	20.60%
	Female	3019	18.61%	846	19.27%	3865	18.76%
Division	A	85	18.82%	46	8.70%	131	15.27%
	B	1103	18.22%	310	16.77%	1413	17.91%
	C	332	31.93%	118	27.12%	450	30.67%
	D	1109	21.64%	367	18.80%	1476	20.93%

	E	292	31.16%	114	20.18%	406	28.08%
	G	674	22.85%	297	17.17%	971	21.11%
	I	1149	17.41%	319	21.63%	1468	18.32%
	J	1160	14.66%	338	16.27%	1498	15.02%
Year	2001	368	15.76%	144	19.44%	512	16.80%
	2002	349	19.77%	162	20.37%	511	19.96%
	2003	335	22.69%	145	13.79%	480	20.00%
	2004	353	19.55%	164	21.34%	517	20.12%
	2005	390	13.85%	177	10.73%	567	12.87%
	2006	383	16.45%	150	14.00%	533	15.76%
	2007	375	16.27%	118	16.95%	493	16.43%
	2008	367	20.16%	95	9.47%	462	17.97%
	2009	385	18.70%	102	16.67%	487	18.28%
	2010	393	23.66%	112	21.43%	505	23.17%
	2011	422	18.72%	114	22.81%	536	19.59%
	2012	385	13.25%	95	21.05%	480	14.79%
	2013	429	21.45%	108	17.59%	537	20.67%
	2014	480	23.33%	102	24.51%	582	23.54%
	2015	498	32.13%	126	34.92%	624	32.69%

If we look at the proportions over the time period, there appears to be an increasing trend as the proportion of students performing significantly below expected about doubled from about 17% in 2001 to about 33% in 2015. These results mirror the trends in the DF analyses, and suggest that the gap between high school Pre-Calculus achievement and university Calculus achievement has been increasing over this time period.

Discussion

The purpose of this study was to explore the relationship between students' grades in high school Pre-Calculus and their subsequent grades in the first-year university calculus course, and selected conditions under which the relationships vary. The findings showed that at the University of Manitoba, not only is there a 'gap' between Pre-Calculus and Calculus grades, but grades are increasing in the high school Pre-Calculus course and they are staying relatively stable in the university Calculus course.

A drop in grades is expected for students transitioning from high school to university, especially in mathematics (Finnie et. al, 2010), the results of this study show a very pronounced difference in the proportion of students entering with grades of 4 or 4.5 in Pre-Calculus (63%) and those receiving grades of 4 or 4.5 in Calculus (37%). The distributions of both Pre-Calculus and Calculus grades are heteroscedastic; the variation was larger for students at the lower end of Calculus grades than for the same students' grades in Pre-Calculus. This disconnect has been increasing steadily since 2005. Especially troubling is the correlation between Pre-Calculus grades and Calculus grades for students who receive less than a 2.5 grade in Calculus. The explained variation in Calculus grades based on Pre-Calculus grades is about 29% for the population and about 5% for the low-achieving students. This evidence suggests that students who are not doing well in Calculus are difficult to identify from their Pre-Calculus grades. Students who fail Calculus may come in with C's, A+'s or anything in between in Pre-Calculus.

It is interesting to look at this relationship between Calculus and Pre-Calculus grades. Students who perform poorly in Pre-Calculus also perform poorly in Calculus, because overall there are almost no students who performed poorly in Pre-Calculus and then do well in Calculus. Of course, the relationship between Pre-Calculus and Calculus works in only one way – those who did well in Calculus inevitably did well in Pre-Calculus, but those who did poorly in Calculus did not necessarily do poorly in Pre-Calculus.

Looking at the data, we see that about 85% of the differences for these students represent a *positive* difference between Pre-Calculus grade and Calculus grade; that is, the Pre-Calculus grade is *higher than* the Calculus grade. This indicates that students are probably receiving inflated grades in their high school Pre-Calculus courses, and this is especially significant for students who have grades of less than a 2.5 in Calculus. These results align with what the ACT reported about grade inflation (ACT, 2005), and suggest that there is a need for more effective placement procedures for students entering first-year mathematics courses at the University of Manitoba.

In terms of the relationship between students' Pre-Calculus grades and their Calculus grades, over the time period, a correlation of $r = .54$ is observed, which is considered large in the social sciences. However, considering that the stated aim of the Pre-Calculus course is that it is "designed for students who intend to study calculus and related mathematics as part of post-secondary education" (Manitoba Education and Advanced Learning, 2014, p. 17), one might expect a stronger relationship. Also, the Pre-Calculus grades explain only 29% of the variation in the Calculus grades, which is only slightly higher than the 20% variation explained by students' average high school grades as noted by Geiser (2007). One might expect that the correlation between high school and university grades in specific subjects, especially mathematics, would be higher, but we know that being well-prepared in a subject is only one of the many factors that affect the academic performance of students in university. One possible reason for the correlation between these two mathematics measures is the general transitional problems that students have when moving from the high school to university, where the environment, the students' motivation and expectations, and the size of university classes are significant factors that affect their academic performance (see, for example, Clark & Lovric, 2009). However, there are educational issues, such as a misalignment of curricula in high school and university, the differences in teaching styles, and different assessment methods (Dossey, 1998; Hull & Seeley, 2010; Morgan, 2012; Thomas et. al., 2010). The results of this study may be troubling to university instructors and administrators when they see that the University of Manitoba, like many other universities, only uses the students' Pre-Calculus grades for accepting them into the Calculus course.

This study also found that approximately 20% of the highly successful students in Pre-Calculus are earning grades in Calculus that are significantly below what they would probably expect, or what their Pre-Calculus grades would predict. This represents approximately 400 students each year. These results indicate that there is a significant disconnect between what students expect based on their incoming Pre-Calculus grades and what they actually achieve in Calculus. One possible factor contributing to this disconnect is grade inflation that seems to be taking place in the high school Pre-Calculus course. In fact, a number of researchers have identified grade inflation as a pervasive problem that seems to be getting worse (Bachan, 2017; Kostal, Kuncel &

Sackett, 2016; O'Halloran & Gordon, 2014; Wongsurawat, 2009). Another possible factor contributing to this disconnect is that high school students are generally being graded by their teachers using non-standardized measures which potentially brings considerable bias into their grading procedures (Adamanti-Trache, Bluman & Tiedje, 2013; Ahlgren-Reddy & Harper, 2013; Carlson, 2010). As such, high school students may be receiving grades that do not accurately reflect their understanding of the Pre-Calculus topics they are expected to have mastered by the time they enroll in the university Calculus course.

Another contributing factor may be that the Pre-Calculus courses are not adequately or appropriately covering important topics and skills that are required to be successful in Calculus. Students may have been quite successful in understanding the concepts when they were first introduced to them in Pre-Calculus, but they may not have assimilated the knowledge into their long-term memories and the knowledge and skills may have faded before they begin the university Calculus course. These results highlight the need for further measures to be taken to more accurately determine which incoming students are adequately prepared for Calculus at the University of Manitoba.

To explore possible factors contributing to the high failure rates and the large number of students who are performing well below their expected level in Calculus, further analyses are needed. While the relationship between Pre-Calculus achievement and Calculus achievement was similar for males and females, the relationship varied on other factors. Students from the French and private school divisions had a slightly stronger relationship between their Pre-Calculus and Calculus grades than students from the other divisions, and these groups had lower proportions of students who obtained final grades in Calculus that were significantly below those expected based on their grades in high school Pre-Calculus. Because these two groups are relatively small and they tend to be more selective in their students and teachers, it is possible that they are thus able to maintain higher standards in mathematics.

More significantly, the disconnect between high school mathematics grades and university mathematics grades has been growing as time goes on. While the average Calculus grade stayed relatively similar, at least over the past 15 years, the average Pre-Calculus grades have increased from around 79% to around 84%. In other words, the Difference Factor values have been increasing over the years. This holds for both the population of students, and even more so for the low achieving students, where the average Pre-Calculus grades show an increasing trend, from an average of around 70% to around 80%. These results also signal that grade inflation is happening at the Pre-Calculus level, particularly for low-performing students.

The time has certainly come to address this problem by putting appropriate corrective measures in place. Unfortunately, these results do not shed any light on possible remediation strategies. It appears that for the students who are not doing well, we have not identified their difficulties from their Pre-Calculus grades. We need to identify more accurately *who* needs remediation, and what can be done for these students. Nevertheless, these results suggest that high school and university math instructors could work more closely together to align Pre-Calculus course with the Calculus course.

Limitations

There are several limitations of this study. Students who voluntarily withdrew from the course were not included in the analyses. These students make up about 30% of the students first enrolling in the Calculus course. A separate study of these students is needed to identify why they are withdrawing. In addition, the students in this study came only from high schools in Manitoba. This means that the results found here may not be generalizable to the population of Calculus students at the University of Manitoba because an increasing proportion of students come from other provinces and other nations. Finally, compared to other universities in Canada, the University of Manitoba is a highly accessible university with lower entrance requirements. Universities with more rigorous entrance requirements, or less diverse students, may not have similar gaps between Pre-Calculus and Calculus.

Conclusion

If students with different mathematical skill sets are directed into courses appropriate for their skills, then there should not be the high failure and withdrawal rates evident today (Kalajdzievska, 2011). However, mathematics instructors at universities around the globe are increasingly concerned about the preparation and capabilities of incoming students (Carlson et. al, 2010; Crowther et. al, 2007; Edwards, 1997; Hawkes & Savage, 2000; Hong et. al., 2009; Kajander & Lovric, 2005).

It seems that one of the issues is that students coming from high school to university mathematics are not, in fact, being placed properly based on their high school mathematics grades. Adamuti-Trache, Bluman and Tiedje (2013) claim that it is not so much demographic variables as it is “more accurate assessment in the high school subject [that] leads to better performance in related university courses” (p. 2922). As well, Carlson et. al (2010) point out that one of the reasons for high attrition in calculus is that “standards for what should be taught in courses like Pre-Calculus are unclear and inconsistent" (p. 2), and Ahlgren-Reddy and Harper (2013) note that "what constitutes Pre-Calculus at the various feeding institutions [schools] varies greatly, as do the grading procedures used by different high schools and university instructors" (p. 684). This variability means that the information that administrators and students are using to choose different courses may not be adequate for informed decision making. Thus, the next study, reported in Chapter 3, examines the use of an entrance examination for students enrolling in Calculus at the University of Manitoba.

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Connecting Section

Chapter 2 focused on quantifying the relationship between Manitoba students' Pre-Calculus grades and their subsequent Calculus grades in an effort to distinguish whether or not students' Pre-Calculus are an adequate measure of readiness for the Calculus course. What the study found is that the disparity between students' Pre-Calculus grades and their subsequent Calculus grades has been growing over time. In addition, results showed that though in general the predictive value of Pre-Calculus grades is not bad, the relationship is actually quite weak for students who are struggling in Calculus. This is of huge concern, given that these are the students who should be identified for remediation before they register in Calculus.

Building on these results, Chapter 3 aims to examine placement testing in mathematics as a stop-gap measure for predicting students' readiness for first-year mathematics courses, specifically, Calculus. The Chapter provides an overview of the most common mathematics placement tests used in North America, and delves into the issue of adding additional ways to assess students' readiness for Calculus to their incoming high school grades alone.

Chapter 3 outlines a large-scale analysis of the ALEKS placement test as a predictor of readiness for Calculus at the University of Manitoba. The Chapter concludes with recommendations for implementing a more effective procedure for placing incoming students into appropriate first-year mathematics courses, thus increasing their chances for success in mathematics and in university as a whole.

Chapter 3: Analyzing the Effectiveness of the ALEKS Mathematics Placement Test

At universities around the world, many mathematics instructors have become increasingly concerned about preparation and capabilities of their incoming students (Carlson, Madison, & West, 2010; Crowther, Thompson, & Cullingford, 2007; Edwards, 1997; Hawkes & Savage, 2000; Hong et al., 2009; Kajander & Lovric, 2005). Students' readiness for first-year university mathematics courses are assessed, in part, via their previous high school mathematics achievement. However, students' high school mathematics grades may not, in fact, be a reliable indicator of their true readiness for the successful completion of university-level courses. Due to the increasingly diverse incoming university student population and the ever-growing disconnect between high school and post-secondary mathematics (Barr, Clifton, Renaud & Wang, 2018), the rates of failing in many first-year university mathematics courses are alarmingly high, with some failure rates being as great as 60% (Hourigan & O'Donaghue, 2007; Kalajdzievska, 2011; Rylands & Coady, 2009). Failing in first-year mathematics courses, in turn, cause some students to change their intended majors or even to drop out of university (Mattern & Packman, 2009; Parker 2005).

Consequently, many universities are evaluating entering students' readiness for first-year mathematics courses using indicators other than their high school grades. Entrance examinations, or *placement tests,* often serve this purpose (Keller, Shreve, & Remmers, 1943). Currently, in the USA, approximately 90% of post-secondary institutions use placement tests to identify students who are not adequately prepared for first-year mathematics courses (Latterell & Regal, 2007). Placement tests offer students, instructors, and university administrators a standardized way of evaluating incoming students’ level of preparedness for successfully completing university mathematics courses.

Placement tests are most valuable when they are designed carefully and used properly, when they have acceptable levels of reliability and validity, and when the tests support students to become more successful in first-year mathematics courses. Of course, placement tests have some inherent limitations. They represent a snapshot of student learning rather than a comprehensive representation of their potential. Also, these tests often lack a clear connection to the context of local schools and universities, which may diminish their ability to measure students’ readiness for specific courses or programs.

In mathematics, placement testing is being used with an increasing frequency to measure how well students are prepared for university courses given the current high failure and withdrawal rates discussed above. This is especially true for university Calculus courses, which are often a gateway course to programs and degrees in the Science, Technology, Engineering and Mathematics (STEM) fields (Bressoud & Rasmussen, 2015). Across North America, to assess students’ readiness for enrolling in university-level Calculus courses, there are at least four commonly used and commercially available placement tests— ACCUPLACER, COMPASS, Maplesoft, and ALEKS.

The purpose of this study is to analyze the relationship between students’ performance on one of these placement tests, ALEKS, and their grades in the first-year Introduction to Calculus course (MATH 1500) at the University of Manitoba, which would then allow researchers and administrators to determine if this particular placement test could predict the students’ success in the course. In addition, the relationship between

ALEKS scores and Calculus grades will be examined with respect to various demographic variables (gender, high school location, first language, and age) to estimate the effect of these factors. The next section will describe various placement tests that are used at North American universities to evaluate students' readiness for the Calculus course. Following that, the effectiveness of the ALEKS placement test for a sample of University of Manitoba students will be discussed.

Why Use Mathematics Placement Testing?

With increased access to university for a wider variety of students, there is a greater diversity of backgrounds in the student population (Ahlgren-Reddy & Harper, 2013; Edwards, 1997; Hong et al., 2009; Kajander & Lovric, 2005; Lee & Robinson, 2004). Mathematics departments in many universities are addressing this issue by offering a wide range of first-year mathematics courses. Thirty to forty years ago, first-year courses were typically limited to Calculus and Linear Algebra, whereas today, numerous first-year mathematics courses have become available. At the University of Manitoba, for example, first-year courses include the classical Calculus course, Mathematical Reasoning for students in the Faculty of Education, and even Math in Art for Fine Arts students. Given these course options, if students with different mathematical skills and knowledge were directed into appropriate courses based on their interests and skill levels, they should have had little difficulty completing the requirements in their first-year university mathematics courses. However, the rates of failing in first-year mathematics courses remain disturbingly high (Barr, Clifton, Renaud & Wang, 2018; Hourigan & O'Donaghue, 2007; Kalajdzievska, 2011; Rylands & Coady, 2009). This means that the information that students are using to enroll in university mathematics courses may be misleading.

This mis-placement affects both students and faculty in several ways. Students who enter courses that are too difficult for them probably experience considerable frustration, whereas those who select courses that are not challenging enough are likely to become bored (Morgan, 2012). For those who are frustrated and cannot successfully complete their mathematics course, there are ramifications in terms of their future success in university (Mattern & Packman, 2009) as well as their competencies to pursue STEM careers (Parker, 2005). For faculty members teaching first-year courses, frustration can build when students lack the knowledge and skills to understand the material presented to them (Carlson et al., 2010). All of these consequences can be addressed, at least to some degree, by the proper use of placement testing prior to students entering their first-year mathematics courses.

According to a recent report on calculus programs in the United States, effective placement is an essential part of the most successful calculus programs (Bressoud & Rasmussen, 2015). Approximately 50% of the U.S. universities use mandatory placement examinations; an additional 22% do not require placement tests but nevertheless encourage students to take them. The Mathematical Association of America (MAA) places a special emphasis on placement testing recognizing the potential of such testing to reduce failure and withdrawal rates from 50% to about 10%, if used appropriately (Douglas, 1986).

In short, with the main objective of reducing failure and drop-out rates, placement tests are believed to be effective in determining students' mathematical knowledge and skill before they begin university mathematics courses, in identifying their strengths and weaknesses, and, ultimately, in foreseeing which students are likely to succeed and which students are likely to need remediation and support (Hawkes & Savage, 2000; NíFhloinn, Bhaird, & Nolan, 2013).

What is Mathematics Placement Testing?

Placement tests are used to indicate students' readiness for specific courses, to reveal the skill and knowledge levels of incoming students, and to identify those who are likely to succeed and those who need remediation. They are different from general admission tests such as the American College Test (ACT) and the Scholastic Achievement Test (SAT), which have different goals:

> College admission tests such as the SAT or ACT measure students' general readiness for college, whereas placement tests seek to measure students' knowledge and skills that are prerequisite for specific entry-level college mathematics courses. Nationally administered tests such as SAT and ACT measure a broad range of quantitative skills, and this measure is often too general to distinguish between readiness for entry-level mathematics courses such as college algebra, trigonometry, precalculus, and calculus. (Mathematical Association of America, 2010, p. iii)

Mathematics placement tests are typically administered to students before registering in first-year courses. These tests are often pencil and paper multiple-choice tests, but some include comprehensive, open-ended, online items (Morgan, 2012). Computer adaptive tests (CATs) typically use advanced computer algorithms which select test items based on students' responses to previous items. These tests contain large item banks so that each student gets a different set of items.

The most important characteristics of a "good" placement test are its psychometric properties—validity and reliability. The ACT and the College Board have defined *validity* as the degree to which a test actually measures what it is supposed to measure, the degree to which it can reliably distinguish between students likely or not to do well in specific courses, and the degree to which there is a predictive relationship between test scores and subsequent grades in the target courses (Scott-Clayton, 2012). Some authors call this *predictive validity*, which is the correlation between students' performance on a placement test and their subsequent performance in a target course (Mattern & Packman, 2009; Medhanie, Dupuis, LeBeau, Harwell, & Post, 2012; Morgan, 2012).

However, Ahlgren-Reddy and Harper (2013) pointed out that evaluating a placement mechanism on the basis of predictive validity carries at least two limitations. First, the relationship can be affected by factors unrelated to a placement test. For example, students may occasionally perform below their true ability due to unforeseen reasons. Second, Ahlgren-Reddy and Harper (2013) claim that placement tests are not designed to predict course grades, but rather to measure students' readiness for successfully completing the course. However, one would expect these two measures to be strongly related, given the nature and purpose of placement tests.

Another way to determine a placement tests' validity is by calculating its *placement accuracy rate* (Scott-Clayton, 2012). This is the test's ability to minimize placement errors. It is possible to predict these rates using the relationship between students who were predicted to succeed in a given course and those who actually succeeded. Placement accuracy is then measured as the sum of *observed true positives* (students who are placed into the course and actually succeed) and *predicted true negatives* (students who are predicted not to succeed and are placed correctly into remediation), which can be extrapolated from the data using linear regression (Scott-Clayton, 2012).

The second psychometric property of a test is reliability. *Reliability* is defined as the consistency of a test in assessing students' results on quizzes and exams in a target course (Morgan, 2012). For placement tests, reliability can be measured by administering a test multiple times (with no intervention between retakes) and then analyzing the similarity between test results. A test with higher reliability results in students' test scores being within some reasonable measure of error from each other.

Morgan (2012) presents one main criterion that generally leads to more reliable placement tests, this being longer tests with more items. However, Morgan also comments on the fact that a positive correlation between test length and reliability may be circumvented with the use of "smart" tests or CATs, since such tests have the ability to select specifically targeted test items based on student responses using computer algorithms.

Overall, both reliability and validity of mathematics placement tests can be increased through the use of CATs, by choosing questions that have been shown to exhibit high correlations with outcome measures in previous versions of the test, and by employing advanced methods of analyzing the tests (e.g., Ahlgren-Reddy & Harper, 2013; Bressoud & Rassmussen, 2015). The effectiveness of placement can also be enhanced by administering multiple measures rather than a single test.

In addition, placement procedures are only effective if they are closely linked with remediation (Edwards, 1997; Hawkes & Savage, 2000; NíFhloinn et al., 2013; Scott-Clayton, 2012). Hawkes and Savage (2000) state that "diagnostic testing should be seen as part of a two-stage process," and that "prompt and effective follow-up is essential to deal with both individual weaknesses and those of the whole cohort" (p. iii). As far back as 1943, mathematicians were aware that one of the main aims of such testing is to allow students to take advantage of remedial programs based on their needs as identified by a placement test (Keller et al., 1943).

The following are descriptions of the mathematics placement tests that are most commonly used in North America.

Institutional Tests

Institutional tests created by faculty at a university are the most common type of a mathematics placement test. As reported in Bressoud and Rasmussen's (2015) MAA study, they are used by 64% of the universities in their study. These are large American universities with graduate programs, similar to the University of Manitoba. Ranging in type, length, difficulty, and target courses, these tests are recognized as effective placement examinations (Carr et al., 2013; Edwards, 1997; Haek, Yeld, Conradie, Robertson, & Shall, 1997; Keller et al., 1943; Latterell & Regal, 2007; Lee & Robinson, 2004; Madison et al., 2015; Moffat, 1991; NíFhloinn et al., 2013). Such placement tests can be tailored according to a particular context of a university; however, assessing and improving the tests' reliability and validity requires time and resources (Hsu & Bressoud, 2015; Medhanie et al., 2012).

The most commonly used commercial placement tests for mathematics are discussed below. They include ACCUPLACER, COMPASS, Maplesoft, and ALEKS.

ACCUPLACER

This test is "an integrated system of computer-adaptive assessments designed to evaluate students' skills in reading, writing, and mathematics" (College Board, 2019) and was developed by the College Board in the USA. It is used by 3% of the institutions, as estimated in the MAA study by Bressoud and Rasmussen (2015). The "College Level Math" test consists of 20 questions that cover algebraic operations, equalities and inequalities, coordinate geometry, applications, and other algebra topics. In a predictive validity conducted done by the College Board, it was identified that when measuring success for students obtaining a C or higher in a target course, ACCUPLACER had a correlation of $r = .32$ with the course grades; this correlation was .36 when the success measure was to obtain a B or higher (Mattern & Packman, 2009). Mattern and Packman report this correlation as indicating a moderate-to-strong relationship between the placement test scores and the subsequent course grades. In terms of placement accuracy, they found that ACCUPLACER correctly placed 74.1% of college-level math students when success was measured as a C or higher, and 66.5% when success was a B or higher (Mattern & Packman, 2009).

COMPASS

Also used by 3% of the institutions in the MAA study, COMPASS is an "untimed computerized test" (ACT, 2019). This test is a computer-adaptive, multiple-choice, mathematics placement test with multiple modules in pre-algebra, algebra, geometry, and trigonometry. The test also includes a number of modules that can be customized by administrators. Predictive validity and placement accuracy of the COMPASS test was measured by Scott-Clayton (2012). The correlation between the COMPASS algebra placement test scores and the first college-level math course grades was r = .36. The placement accuracy of the test was between 69.5% and 52.8% depending on the measure (C or higher or B or higher), with a placement error rate of about 24%. Unfortunately, this test has since been discontinued, and is no longer available for use.

Maplesoft

Maplesoft is a test that was designed by the Mathematics Association of America as a "mechanism…to place…incoming students into the appropriate first-year mathematics course" (Mathematics Association of America, 2010). It is used by 5% of the American institutions and was the only test that was designed specifically for Calculus (Bressoud & Rasmussen, 2015). Its Calculus Readiness (CR) and Calculus Concepts Readiness (CCR) instruments consist of 25 multiple-choice questions each. These instruments were developed through cycles of refinement, which included interviews with students, in order to determine if every test item has achieved proper validity in order to be interpreted consistently, to assess knowledge intended by design, and to have optimal distractors that would reflect students' thinking as was determined in the interviews (Carlson et al., 2010).

While the Maplesoft tests are computerized and items for individual students are generated by an algorithm, subsequent items are not determined by students' responses to previous items. Carlson and colleagues (2010) found that the predictive validity of CCR test with course grades in Calculus was r = .51. They also found that a cut score of 17/25 on the CCR test would have resulted in 100% of students receiving a C or higher in the target course, and a cut score of 11/25 resulted in 66% of students receiving a C or higher. Based on these results, the placement accuracy for a grade of C or higher was 66.2%.

ALEKS

The ALEKS (Assessment and Learning in Knowledge Spaces) placement test is an open-response adaptive online system that employs artificial intelligence to analyze the responses of millions of students to continuously refine the testing algorithm for future test-takers. ALEKS is used by 10% of the institutions. The number and type of items vary depending on the students' responses to previous items. The test covers over 300 topics, from developmental mathematics to pre-calculus, and includes built-in remediation modules specifically designed for an individual student on 8 subcategories—real numbers, equations and inequalities, linear and quadratic functions, exponents and polynomials, rational expressions, exponentials and logarithms, geometry, and trigonometry.

ALEKS placement scores are not based not on the percentage of items answered correctly, but on the types of questions students have successfully completed. In a study that examined the relationship between this test's placement scores and subsequent performance in target courses, Ahlgren-Reddy and Harper (2013) found that, as ALEKS score increased, so did the proportion of students earning grades at or above the cut score. Over 300 topics are covered in 30 questions or less reflecting the test's excellent efficiency: A similar but non-adaptive test would likely include hundreds of questions in order to provide results with the same level of insight. A decade of data has also refined the system to become highly accurate in measuring student knowledge state.

Among the aforementioned tests, only ACCUPLACER and ALEKS use modern computer-adaptive technology to select items based on the students' responses to previous items. In fact, only two tests (ACCUPLACER and ALEKS) have remediation programs built-in. These are very important features of a mathematics placement test since the first contributes to strong reliability and validity and the test's ability to diagnose and help students do well in a selected math course, while the second provides the support that students need to achieve their goal of eventually entering into the course. However, of the two, ACCUPLACER is designed only to identify readiness for "College-level" mathematics broadly rather than Calculus, specifically, and it does not provide a possibility for open-ended responses. This may lead to inaccurate results that are associated with guessing and careless errors, which often occur in multiple-choice tests but can be circumvented in open-response tests (Ahlgren-Reddy & Harper, 2013; Doignon & Falmagne, 2011). Given the above argumentation, ALEKS appears to have the greatest capabilities in one test, as well as being the only one capable of analyzing open-ended responses.

For institutions considering the implementation of mathematics placement examination, it is important to select a test that best meets the needs of that institution. The choice of a placement test may vary from university to university.

Using a Mathematics Placement Test at the University of Manitoba

A Calculus placement test was developed and piloted in 2012 by members of the Department of Mathematics at the University of Manitoba. Up to that time, only students' high school Pre-Calculus grades were used for admitting students into the first-year Calculus course. This test consisted of 36 multiple-choice items covering algebra, equalities and inequalities, functions and graphing, geometry, exponents and logarithms, and trigonometry. Over a two-year period, the committee gathered data pertinent to students' results on the placement test and course grades with a goal to refine the test and the testing procedure. In 2014, the placement test was revised and included 25 items. Data from that year showed that the revised test had a predictive value of $r = 0.47$, and that its placement accuracy for the Calculus course at the 70% and 60% cutoffs was 45.7% and 55.4%, respectively. This means among students who scored 70% and above, 45.7% of them would have passed the course; and among students who scored 60% and above, 55.4% of them would have passed the course. A reliability coefficient for the test was not calculated.

Given that this test only accurately placed about half of the students even after three years of development, the committee discontinued the test development and began using a commercial test that provided more reliable and valid results. The Department also had concerns about the future feasibility of a paper-and-pencil test because copies of the test could eventually make its way to high school students and their teachers, which would diminish the test's validity.

Among the various commercially developed mathematics placement tests available, the ALEKS test was chosen for a trial period beginning in 2016, based on several criteria. First, ALEKS was a Canadian test. This was important to the committee's members in the Department of Mathematics given the need to have the test reflecting the national, local standards of testing.

Second, ALEKS was the only placement test on the market that used artificial intelligence in an adaptive online format. In addition, it was the only commercial test that contained open-ended response items hence allowing students to demonstrate a variety of skills (e.g., graphing) that are important in Calculus. As stated before, such format reduces some potential inaccuracies from careless guessing in multiple-choice test items (Doignon & Falmagne, 2011).

Finally, ALEKS was superior because it assessed students' knowledge over a range of mathematics topics, from basic mathematics to pre-calculus topics, whereas some other placement tests would require a number of tests to do this. Given there were several different first-year courses offered in the Department of Mathematics at the University of Manitoba (e.g., Applied Finite Mathematics, Calculus, and Honours Calculus), ALEKS was the most convenient instrument for placing students into these various courses.

In light of these obvious advantages of the ALEKS test, there is still a need to evaluate its predictive validity. If ALEKS is found to be a stronger predictor of students' true achievement in university-level Calculus than high school Pre-Calculus grades, an argument can be made for the use of this test to help both students and instructors who are assigned to particular Calculus courses. As well, it is crucial to explore predictive validity of the ALEKS instrument across different groups of students to determine whether or not the placement accuracy is stronger for some groups of students.

The Research Questions

1. What is the relationship between students' scores on the ALEKS placement tests and their subsequent grades in the Calculus course at the University of Manitoba?

2. To what degree does the relationship vary across gender, high school location, first language, and age?

Method

Participants

The ALEKS test was taken by students enrolled in the Introductory Calculus course (MATH 1500) in the fall term of 2016 (N = 658). Of these students, 73% completed the course with a final grade making the final sample size 462. There were 264 male students and 213 female students. Most students completed their high school education in Winnipeg, spoke English as their first language, and were in their first year of university (see Table 3.1). Further**,** each student had the placement test score and the MATH 1500 final course grade, both of which were included in the analyses.

Table 3.1

Demographics of Study Sample

Characteristics				
Gender	Male 254(55%)		Female 208(45%)	
High School Location	In Winnipeg	Out of Winnipeg but in Manitoba	Out of Manitoba but in Canada	Out of Canada
	296(64%)	64(14%)	14(3%)	88(19%)
First Language	English		Non-English	

	347(75%)		115(25%)
Age	<20	20-21	>21
	328(71%)	69(15%)	65(14%)

Note. N = 462.

Variables

The independent variable for this study was the students' scores on the ALEKS placement test, which ranged from 0 to 100. This test covered the pre-requisite topics for MATH 1500 in 30 or fewer items depending on the students' performances. Artificial intelligence procedures were used to assign scores based on the number and the types of items answered correctly. Test items varied from student to student being contingent the students' responses to previous items. The mean ALEKS score for students completing the course was 68.0% with a standard deviation of 18.9%. The distribution of the ALEKS scores was close to a normal distribution with a slight negative skew (-0.55).

The dependent variable for this study was the students' grades in the Calculus course, which ranged from 0 (F) to 4.5 (A+). This course is typically taken by students in their first year at the university. Most of the students would have completed the grade 12 Pre-Calculus course with a grade of at least 60%.

The class size for Calculus at the University of Manitoba is around 120 students. A typical overall enrollment for the course per semester is 800-900 students, and an average pass rate is around 50% (Barr, Clifton, Renaud & Wang, 2018; Kalajdzievska, 2011). The mean Calculus GPA for the students in these analyses was 2.2 (i.e., a C) with a standard deviation of 1.5. The distribution of the Calculus grades was also close to normal, slightly skewed to the left (-0.15).

The Procedures

In Fall 2016, the students in all 8 sections of Calculus (MATH 1500) received 2% of their final course grade for completing the ALEKS placement test which could be taken during the first two weeks of the course. For the students who completed both the ALEKS test and the Calculus course, correlations between ALEKS score and final Calculus grade were calculated. Using a 60% cut-score, the placement accuracy score was derived by computing the proportion of students who scored over 60% on the placement test and who obtained a grade C or better in the Calculus course. Type I (over-placement) error was obtained by computing the proportion of students scoring above 60% on the placement test who did *not* complete the Calculus course successfully, and Type II (under-placement) error was obtained by computing the proportion of students scoring below 60% on the placement test but who also complete the Calculus course successfully.

In addition, a multi-level model was derived using a weighted combination of the ALEKS scores and the incoming high school math grades. This placement strategy is recommended in previous research (Denny, 2012; Geiser, 2007, Hsu & Bressoud, 2015; Scott-Clayton, 2012).

Results

Research Question 1

> What is the relationship between students' scores on the ALEKS placement tests and their grades in Introductory Calculus at the University of Manitoba?

The relationship between the ALEKS scores and the Calculus course grades is illustrated by the scatterplot in Figure 3.1. It indicates a positive linear relationship between the independent and dependent variables as least squares regression line of best fit.

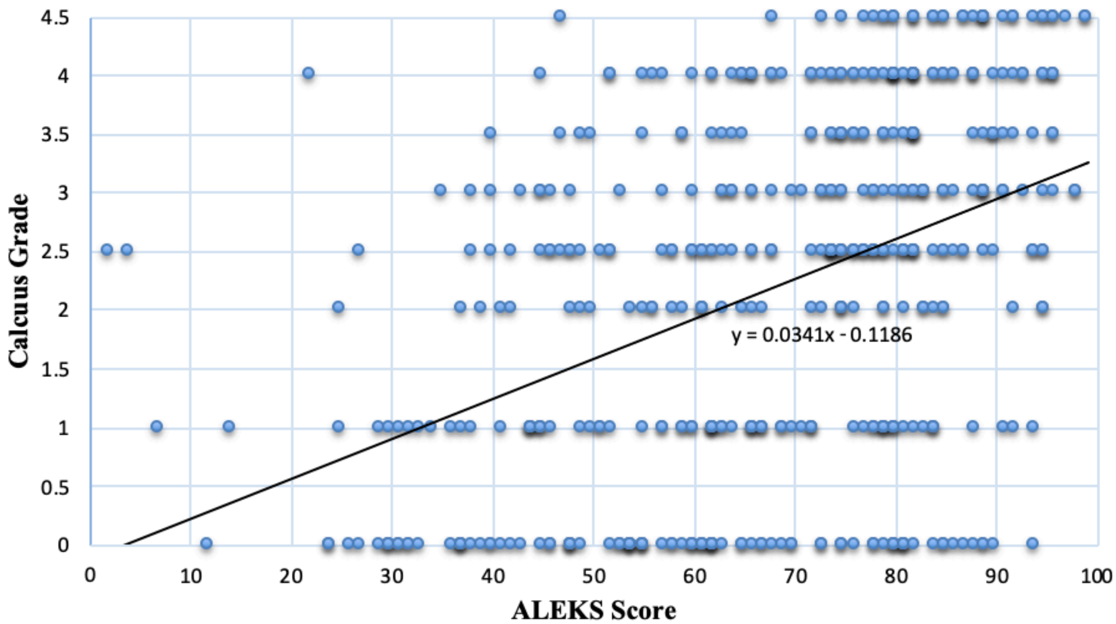

Figure 3.1. Scatterplot between ALEKS Scores and Calculus Grades.

Note: Markers that are shaded indicate a larger number of data points.

Table 3.2 reports the relationship between ALEKS placement score, self-reported high school mathematics grade, and a combined measure of ALEKS score and high school grade and their correlation with final Calculus grade. It also reports the placement accuracy as well as the Type I and Type II error proportions for each of these measures.

Table 3.2

ALEKS Fall 2016 Results

	Mean(SD)	Correlation with Calculus Grade	Placement Accuracy	Type I Error (Percent Overplaced)	Type II Error (Percent Underplaced)
High School Mathematics Grade	3.2(1.0)	.37*	66.5%	31.2%	2.4%
ALEKS	68.2(19.0)	.43*	72.9%	18.6%	11.7%
ALEKS and High School Math Grade Combined		.49*	71.9%	14.9%	13.2%

Note. *$p < 0.001$.

These results show that for Calculus grade, ALEKS has a slightly stronger correlation (r = 0.43) than the high school mathematics grades (r = .37). More importantly, a model including both the high school mathematics grades and ALEKS scores had the best predictive value for Calculus grades (r = .49). The placement accuracy is also greater when ALEKS alone or high school grades and ALEKS scores are used rather than when high school mathematics grades are used alone. Additionally, the Type 1 errors are reduced for ALEKS (18.6%) and the two-variable model (14.9%) compared to the high school grade (31.2%).

Research Question 2

To what degree does the relationship vary across gender, high school location, first language, and age?

Although ALEKS is a better predictor of the students' success in university

Calculus than their high school grades, the prediction value may vary for different demographic groups. Table 3.3 shows the correlation between ALEKS score and Calculus grade broken down by various demographic variables.

The table reveals that for nearly every demographic variable, the mean ALEKS score is similar to that reported in the previous analyses. Differences between the demographic groups exist, but they are small. For example, the mean ALEKS scores for females and males differed by less than half a percent (M = 68.2% for females, M = 67.9% for males), as did the scores for English First Language and English Language Learners (M = 68.1% vs. M = 67.9, respectively). High school location did not seem to be a factor in terms of the differences between the mean ALEKS scores, while age was a factor. The mean ALEKS score for the students over the age of 21 was lower compared to the younger students (56.5 versus 61.8 for those 20-21 and 71.1 for those under 20).

Table 3.3 also shows that for nearly every demographic variable, the correlation between ALEKS scores and Calculus grades are similar to the correlation reported in the previous analyses, except for two groups. The students who completed their high school mathematics courses outside of Manitoba but in Canada have a lower correlation between ALEKS score and final Calculus grade than the students overall (r = .16 vs. r = .53); and this correlation is not significant. For the students who were over the age of 21, the correlation between ALEKS score and final Calculus grade are much lower than expected (r = .22 for the group > 21, vs. *r* = .42 for the group < 21, and *r* = .53 for the group 21-22), and this correlation is also non-significant.

Table 3.3

ALEKS Fall 2016 Results—By Selected Variables

		Number	ALEKS Score Mean(SD)	Correlation Between ALEKS Score and Calculus Grade
Gender	Male	254	67.9(19.2)	.41**
	Female	208	68.2(18.6)	.45**
High School Location	In Winnipeg	296	69.6(18.4)	.45**
	Out of Winnipeg but in Manitoba	64	66.5(19.7)	.32*
	Out of Manitoba but in Canada	14	62.8(21.3)	.16
	Out of Canada	88	64.6(19.5)	.50**
English	First Language	347	68.1(19.1)	.41**
	Language Learner	115	67.9(18.5)	.48**
Age	<20	328	71.1(17.4)	.42**
	20-21	69	61.8(19.8)	.53**
	>21	65	56.5(21.7)	.22

Note. $N = 462$. $^{*}p<0.01$, $^{**}p<0.001$.

Discussion

The purpose of this study was to determine whether or not a mathematics placement test for Calculus, the ALEKS placement test, would be a good predictor of achievement of the first-year students in the University of Manitoba Calculus course (MATH 1500). This study had two research questions: First, what is the relationship between students' scores on the ALEKS placement test and their grades in Introductory Calculus at the University of Manitoba? Second, to what degree does the relationship vary across gender, high school location, first language, and age?

This study showed that the ALEKS test scores were a better predictor of students' success in Calculus than their high school mathematics grades. The ALEKS scores, in comparison with high school grades, improved the correlation with Calculus grades, from r = 0.37 to r = 0.43, and the placement accuracy from 66.5% to 72.9%. More importantly, using the information about students' readiness for Calculus based on their high school grades and their ALEKS score together showed an even greater improvement. The correlation with Calculus grades increased to r = 0.49, while the placement accuracy was similar to that of ALEKS alone, at 71.9%. This represents an increase in correlation by 0.12 and an increase in the placement accuracy of around 5% compared to the correlation and the placement accuracy achieved when using the high school grades alone.

Very few students who scored highly on ALEKS did not pass the Calculus course. Of the 462 students included in this study, 337 (72.9%) were placed accurately by ALEKS alone. Of those not placed accurately, 86 (18.6%) were over-placed (Type I error) and 52 (11.7%) were under-placed (Type II error). Scott-Clayton (2012) claims that under the passing criterion, a Type II error is much less problematic than a Type I error, since recommending remediation to students who could have otherwise passed the course is a less severe consequence then allowing them into the course only to fail. In this regard, ALEKS demonstrated good prediction outcomes, but ALEKS together with high school math grades was even better, with Type I errors reduced to 14.9%.

The predictive value of ALEKS for the different demographic groups of students were similar to the total group. The only exceptions to this were the students who took their high school mathematics course outside of Manitoba but in Canada, and those who were over the age of 21. There were only a few students (n = 14) in the first group (Non-Manitoba, Canadian students), which may have contributed to the non-significant correlation for this group. It is reasonable to think that students from outside of the province took high school mathematics courses that covered different topics than covered in the Manitoba Pre-Calculus course.

For the students over the age of 21 (n = 65), there was a noticeable difference in the correlation between their ALEKS scores and their Calculus grades from the total group. This may be due to the fact that this group had the lowest mean ALEKS scores (M = 56.5%) among all the demographic groups, differing markedly from the mean ALEKS grade (M = 68%). It seems to be that students who waited longer to take their first-year Calculus course were at a disadvantage writing the ALEKS test because their high school mathematics was not as fresh in their minds. In other words, their poor scores on ALEKS may have been a reflection of their time away from using mathematics. On the other hand, mature students have probably had more experience with self-directed learning, which hopefully makes them better able to meet the expectations of the Calculus course and which thus could have contributed to the lower correlation between ALEKS score and Calculus grade. They were able to overcome. Overall, greater maturity of these students appears to contribute to their being more successful in the Calculus course compared to younger students.

To conclude, the analyses point to age as important demographic variable to be included in a multiple-measure predictive model for the placement of students in the Calculus course. Thus, as university instructors and administrators, we can do a better job of informing ourselves and our students about students' readiness for university Calculus by using a placement strategy that takes into account high school mathematics grades and ALEKS scores, as well as age.

Limitations

There are several limitations in this study. First, students' high school grades were self-reported. This may have led to some inaccuracy in the correlation between their high school mathematics grades and their Calculus grades. However, as suggested by Sanchez and Buddin (2015), self-reported grades in mathematics courses tend to have the strongest correlations with the actual transcript grades, $r = 0.76$ to $r = 0.77$.

Second, students in this study had not been selected into the Calculus course on the basis of their ALEKS scores. This means that they may not have taken the test as seriously as they would have if higher stakes were attached to doing well on the test. Nevertheless, as revealed in this study, the ALEKS scores still better reflected the students' preparedness for Calculus than their high school grades. It is likely that when the ALEKS is used as a true placement test, students will be more motivated to do well, hence the test should become an even more accurate reflection of their preparedness for the Calculus course. Finally, students who withdrew from the course ($n = 181$) were excluded from the analyses, resulting in a reduction in the sample. These students did not complete the course and thus did not have a final grade with which to correlate their ALEKS score. While these students make up a relatively large proportion of the students studied, there are a variety of reasons why they may have withdrawn from the course. For this reason, assigning a numerical value to those students would have been arbitrary, and doing so could have led to less accuracy in the analyses.

Recommendations

The research on the validity of mathematics placement tests at university level programs has been the focus of this chapter. In this study, the ALEKS placement test was found to be a valid method for measuring students' readiness for Calculus at the University of Manitoba. Based on the results, the first recommendation is to begin using the ALEKS test as a placement test for the first-year Calculus course at the University of Manitoba. It is recommended that students have a score of 60 or higher in order to be admitted into the course.

Following the first year of implementation, correlation and the placement accuracy rate during that year should be analyzed to adjust the admission score to an optimal level for the accurate sorting of students into either the Calculus course or remedial courses. Of course, a model that takes into account students' ALEKS score or students' Pre-Calculus grades together with their ALEKS placement scores would have higher predictive power in sorting the students. This model could be improved further with the inclusion of variables such as students' age.

Students must enroll in courses that are best suited to their skills and knowledge or they must be provided with opportunities to improve their skills and knowledge if they are deemed as unprepared for the Calculus course. Those students who score below the required cut score but still wish to enter Calculus in the could use the ALEKS Prep & Learning Modules or they could enroll in a remediation course before attempting to re-take the placement test.

Another recommendation comes from the work of a group of mathematicians at Ohio State University. These faculty members collaborated with high school mathematics teachers in creation of the Early Mathematics Placement Test (EMPT). This test is intended for high school juniors to give an early alert to those students who may need remediation in mathematics *before* they entered university (Moffat, 1991). Although the

EMPT was a very basic, paper-and-pencil multiple-choice mathematics test, with no advanced features like those in ALEKS, this project produced successful results. In Ohio, about 25% of students graduating from non-EMPT schools need remedial math at university compared to only about 13% of those enrolled in the program. Further, since the year of the program implementation (1977) the percentage of freshmen at Ohio State University who need remedial mathematics courses has dropped from about 47% to an about 20% (Moffat, 1991).

This is a strategy that could help to identify weak students in Manitoba by opening lines of communication between Manitoba high school mathematics teachers and university mathematics educators. For a relatively low administration cost, students could be alerted to their weaknesses and given the opportunity to correct them before writing the ALEKS test for admission into university mathematics courses. By talking with high schools teachers, we can give them more information on the skills that university instructors consider most critical for being successful in first-year Calculus.

Conclusion

High failure rates in the first-year Calculus courses are a plague on the Mathematics Department at the University of Manitoba, and in North America in general. Many universities have attempted to address this problem by using mathematics placement tests. There are a variety of these tests available. What can be seen from the literature and from the analyses presented here is that placement testing has the potential to make a significant difference in the success of students in first-year mathematics courses. It gives students and instructors a better understanding of the students' current competencies in order to help them address the gaps in their knowledge before enrolling in a university mathematics course. Effective course placement can, in fact, maximize student achievement and increase retention of students in mathematics courses which are specifically important STEM programs. In addition, the use of a placement test can help to identify the knowledge and skills that are most important for success in Calculus. This information should be communicated to high school mathematics teachers, administrators, and curriculum developers. Maintaining stronger and more frequent communication between these groups is essential to ensure a seamless transition of students from high school to university mathematics courses.

At universities where students are struggling to succeed in their first-year calculus courses, the ALEKS placement test can assist universities to develop a placement mechanism that is more effective than the one currently used that relies solely on students' high school Pre-Calculus grades. Students should have an accurate picture of their incoming knowledge and skill level and how it matches the requirements. It is the responsibility of university instructors to ensure that they are doing all that they can to efficiently direct students to mathematics courses that are appropriate for them. As Morgan (2012) states:

> Until a seamless curriculum and pathway for students transitioning from high school into college or the skilled workforce can be devised and implemented, it will be necessary to continue placement testing of incoming students to ensure proper placement to allow the student the best opportunity for success…. Placement testing does not bridge the gap between the two parts of the system – secondary and

postsecondary – but it does allow for the identification of the students that are falling through the gap (pp. 381).

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Connecting Section

Chapter 3 focused on the positive impacts to the Pre-Calculus to Calculus transition of implementing an effective placement test prior to registration in Calculus. Results showed that including a placement test as a measure of readiness can improve administrators' and instructors' ability to advise students of their readiness for Calculus. It can also aid students in choosing first-year mathematics courses appropriately, and in seeking remediation, if necessary, rather than failing out of first-year mathematics.

Chapter 4 concludes the dissertation by once again describing the challenges faced by university educators today, especially those in the field of mathematics. It summarizes the methods and main results of the two studies presented above, and compares and contrasts these results with the related previous research.

The Conclusion Chapter also highlights directions for future research that could expand and extend the research presented here, and concludes with practical recommendations to address the Pre-Calculus to Calculus transition in Manitoba, and beyond.

Chapter 4: Conclusion

In many countries, especially in the Western world, there has been a substantial increase in the proportion of students enrolling in universities (Cote & Allahar, 2007; Hong et al., 2009; Schoenfeld, 2004; Statistics Canada, 2005; The Association of Universities and Colleges of Canada, 2011). In Canada, the number of undergraduate students has increased from about 190,000 in 1967 to over 600,000 in 1997, to close to 1 million in 2017, representing over a 5-fold increase over the 50-year period (Statistics Canada, 2012; Statistics Canada, 2018). Along with this unprecedented growth in the number of students has come greater difficulties faced by both students and universities. One of the most important difficulties—the focus of this research—has been a relatively large proportion of first-year undergraduate students who have not been successful in completing first-year courses, especially mathematics courses. This experience is often devastating for these students, and it is, of course, wasteful of university resources. Many instructors and administrators agree that something needs to be done to address this pressing problem.

This dissertation examined one aspect of the problem. Specifically, the two studies examined the relationship between the students' achievement in the Pre-Calculus math course in grade 12, which is the last year of high school, and their grades in the first-year Calculus course at the University of Manitoba. This study was reported in Chapter 2, followed by an evaluation of the use of the ALEKS placement test as the means of placing first-year university students into appropriate mathematics courses in Chapter 3.

The concluding chapter of this dissertation argues that the two studies make a significant contribution to research literature by extending the existing knowledge pertinent to first-year university students taking mathematics courses. Policy recommendations and suggestions for future research flow from this discussion.

Discussion

As access to university education continues to become easier, many important consequences arise, some of which have a direct effect on the educational dynamics in classrooms. Specifically, university instructors have noted that as the student body becomes increasingly larger and more diverse, classes that previously consisted of homogeneous groups of students with equally developed knowledge and skills now comprise students who vary considerably in both their background knowledge and skills. Such variability entails a great difficulty in predicting the level of readiness of these incoming students for university coursework. Consequently, it becomes more challenging for instructors to gauge what course material, and how it should be presented to students. Obviously, this can lead to frustration for both instructors and students (Cote & Allahar, 2007).

Another consequence of the growing proportions of university-bound students is that student "sorting" (i.e., directing students into appropriate courses based on ability and interest), which was previously done at a high school level, is now undertaken by university instructors and administrators (Cote & Allahar, 2007; Rosenbaum, 2001). Currently, many Canadian universities place incoming students into courses on the basis of their high school grades alone. However, the occurrence of "grade inflation," when students receive higher grades than they, in fact, deserve, has increasingly been documented in Canada and other countries (Clark & Lovric, 2009; Eislzer, 2002; Finefter-Rosenbluh & Levinson, 2015; Geiser, 2007). This trend, along with the existing disconnect between teaching practices, assessment methods, and class sizes, and students' cultural backgrounds and the curriculum of high school mathematics courses, has resulted in high school grades becoming less valid as measures of readiness for university math courses (Bressoud & Rasmussen, 2015; Hawkes & Savage, 2000; Mattern & Packman, 2009; NíFhloinn, Bhaird, & Nolan, 2014). Consequently, students with inflated grades enter universities having unrealistic expectations about their academic performances and hence they often do not seek out remediation when it is needed. As well, university instructors are less able to identify students who would benefit from remedial help. This all leads to a reduced chance for some first-year students to be successful in their university courses.

When it comes to courses in which students struggle, mathematics courses are often at the top of the list. This is, in part, due to the fact that mathematics is a vertically organized subject so that students' understanding of mathematical concepts at more advanced levels depends on their knowledge and skills in lower levels. When students fail to master certain skills or concepts, this limitation can cause serious problems in their learning for years to come. Hence, it appears especially alarming that mathematics competencies of first-year university students are becoming worse as time goes on (Hourigan & O'Donaghue, 2007; Kajander & Lovric, 2005; Rylands & Coady, 2009).

In essence, there seems to be a growing mismatch between students' high school mathematics achievement and their grades in first-year university mathematics courses. This trend contributes to both high failure and withdrawal rates in university mathematics courses (Kalajdzievska, 2011). Considering the significance of this subject for the pursuit of STEM careers (Madison & Hart, 1990; National Mathematics Advisory Panel, 2008; National Numeracy, 2015; National Research Council, 1991; White, 1987), this poses a considerable obstacle for technological progress of the province (Mattern & Packman, 2009; Parker, 2005).

In addressing this issue, universities started to seek ways of placing students into math courses that would match their level of knowledge so that appropriate skills would be learned. Increasingly, more universities are deciding to use placement tests to place students into courses and identify those who need remedial support. These tests are designed for specific university courses in mind, such as mathematics, and they assess students' preparedness for these courses (e.g., Mathematical Association of America, 2010). Particularly, placement tests have been shown to be particularly effective in predicting students' success in mathematics courses, identifying existing gaps in their relevant knowledge and skills.

The two studies in this dissertation examined the potential of one placement tests, the ALEKS placement test, to accurately predict the academic performance of the first-year students in Introductory Calculus at the University of Manitoba in comparison to the students' high school Pre-Calculus grades. In other words, these studies examine exactly how effective the ALEKS placement test is, versus students' high school grades, in providing accurate information for the sorting and placing of incoming students.

Study 1: An Analysis of the Relationship between Pre-Calculus and Calculus Grades

The first study was reported in Chapter 2 and investigated the relationship between Manitoba high school students' Pre-Calculus grades and their subsequent first-year university Introductory Calculus grades at the University of Manitoba. Specifically, this study examined the correlation between students' Pre-Calculus grades in high schools and their Calculus grades in university, and it also assessed variability in this correlation across a number of demographic variables, such as gender, school type, school division, and, academic year. The study also identified the proportion of students who obtained final grades that were significantly below their expected grades in the university Calculus course, which was calculated based on the students' grades in the high school Pre-Calculus course.

In order to conduct this study, data from 12,208 students enrolled in Introduction to Calculus (MATH 1500) who had also completed Pre-Calculus 40S in Manitoba high schools, from 2001 to 2015, were included. The number of students in each year varied from 680 to 967, with a nearly even division between females and males. The number of students from selected school divisions in Manitoba ranged from 4 to 1,759. The independent variable was Pre-Calculus grade reported as a percentage as well as a Grade Point Average (GPA) ranging from 0 to 4.5. The dependent variable was the students' university Calculus (MATH 1500) final grade reported on a GPA scale from 0 to 4.5.

This study showed that the difference between the Pre-Calculus grades and the Calculus grades was larger than the one grade-point drop previously estimated in the literature (Finnie, Frenette, Mueller, & Sweetman, 2010). Specifically, the study revealed that the correlation between the students' Pre-Calculus grades and Calculus grades was r = 0.54, which corresponds to 29% of the variation in the students' Calculus grade being explained by their Pre-Calculus grades. More significantly, approximately one fifth of the most successful students in Pre-Calculus (those with a grade of A or A+) earned grades in Calculus that were well below grades that would be predicted from their Pre-Calculus grades.

While the relationship between Pre-Calculus and Calculus grades was similar for male and female students, this relationship had a larger variation across other demographic variables. A notable example was the grade variability for students from the French division and private schools and for students from the other schools. Students in these two groups had an average difference between their Pre-Calculus and Calculus grades that was lower than the average difference for students from other schools. Specifically, the difference for students in the French division was 0.86 grade points, and the difference for the students from the private schools was 0.81 points, while the average difference for students in the other schools was 0.98 grade points.

The study also demonstrated that the difference between high school mathematics grades and university mathematics grades had noticeably increased over the 15-year period. While Calculus grades remained fairly constant over this time, Pre-Calculus grades increased from an average of around 79% to an average of around 84%. These results attested to the growing disconnect between students' grades in Pre-Calculus and their grades in Calculus and confirmed the presence of grade inflation in some high schools.

Also, quite noteworthy was the finding that for the students who received grades of 2.5 or lower in university Calculus, there was a significantly lower correlation between their Pre-Calculus grades and their Calculus grades ($r = 0.23$). In other words, the predictive value of the Pre-Calculus grades was only about 5% for this group of students, which is troubling given that these are exactly the students who are in need of remediation. Consequently, the university instructors and administrators are unable to identify those students based on their incoming grades, which until now remains the only means of pre-university academic assessment used at the University of Manitoba.

These results differed from the results of the study by Adamuti-Trache, Bluman, and Tiedje (2013). Those researchers found that the correlation between high school mathematics grades and university mathematics grades was stronger for students coming from public schools than for students coming from private schools. In this study, however, it was the students from private schools who had a closer link between their high school grades and their university mathematics grades. It is difficult to surmise why this discrepancy occurred. Perhaps it is based on the fact that in British Columbia, where the data in Adamuti-Trache et al.'s (2013) study was collected, there were no standardized examinations for grade 12 mathematics. There might have been pressure on private school teachers to inflate the mathematics grades to meet parental expectations, and in BC, the teachers might have had more freedom to do this because their students' grades would not be affected by their performance on a standardized examination. This could lead to a greater gap between private schools mathematics grades and university mathematics grades compared to the grade difference between public school and university.

There was also a difference between the results of this study and the work done by Finnie and colleagues (2010). While an average grade drop between high school and university was presented in Finnie et al.'s study and was one letter grade, this study showed that for Manitoba students, the average incoming Pre-Calculus grade was an A, and the average university Calculus grade was a C+, corresponding to a difference of two letter grades. One possible explanation for this difference is that Finnie and colleagues examined grades in schools and universities across Canada, whereas this study focused on grades in mathematics in Manitoba high schools and the University of Manitoba.

In Manitoba, there may be unique factors that place this province on the upper end of the grade drop spectrum in comparison to other provinces. For example, in some other universities, more rigorous entrance requirements are used, and some use alternate placement procedures prior to allowing students into university-level courses. This would mean that students in other universities who take Calculus courses may be better prepared than students entering the Calculus course at the University of Manitoba. Another possible explanation is that the transition between high school and university mathematics may be more difficult for students in mathematics than for those in other courses. This may result from the increasingly complex nature of mathematics from Kindergarten to university. Mathematics may be the course with the largest grade drop in comparison to other courses, a point not discussed by Finnie et al. (2010).

Results from this study, however, confirmed the work of Hawkes and Savage (2000) showing that the declining mathematics knowledge and skills of first-year university students was not reflected in their incoming mathematics grades. These results suggest grade inflation at the high school level, and reinforce the notion that high school grades alone are not effective indicators of readiness for university mathematics courses.

Study 2: The Effectiveness of the ALEKS Mathematics Placement Test at the University of Manitoba

The second study in this dissertation, presented in Chapter 3, explored the use of a mathematics placement test, ALEKS, at the University of Manitoba. This test has been used in assessing the skills and knowledge that students have in mathematics in a number of universities across North America. It is used to determine if students have the background knowledge and skills necessary to be successful in university-level mathematics courses. This study investigated the relationship between the students' ALEKS placement test results and their subsequent grades in Introductory Calculus and determined the extent to which this relationship varied across a few demographic variables (gender, high school location, first language, and age).

To conduct this study, data were collected in the Fall 2016 from 462 students who completed the ALEKS test at the beginning of the term and who also completed the Calculus course and received a final grade. Students who voluntarily dropped out of the course were not included. The sample was composed of approximately the same number of females as males, with two thirds of the students having graduated from high schools in Winnipeg and about three quarters of the students speaking English as their first language. As well, more than 71% of the sample was younger than 20 years of age. The independent variable was the students' scores on the ALEKS placement test, which ranged from 0 to 100. The dependent variable was the students' grades in the Calculus course (MATH 1500), with GPAs ranging from 0 (F) to 4.5 (A+).

This study showed that for this sample of University of Manitoba students, the correlation between the ALEKS placement test scores and the Calculus grades was $r = 0.43$, and the correlation between the ALEKS placement test and the Calculus grades when the Pre-Calculus grades were controlled was $r = 0.49$. Both of these correlations were stronger than the correlation between high school Pre-Calculus grades and Calculus grades alone ($r = 0.37$). In addition, ALEKS had a strong placement accuracy rate alone (68.8%), and even stronger in combination with the Pre-Calculus grades (71.9%) than the Pre-Calculus grades alone (66.5%).

When the correlation between ALEKS scores and Calculus grades were examined among different demographic groups (gender, high school location, English as a first language, and age), there were two groups of students that had substantially different results from the overall group. These two groups were students who took their high school mathematics course outside of Manitoba, but in Canada, and those who were over the age of 21 ($r = 0.16$ and $r = 0.22$, respectively). Both of these groups had much lower correlations that the overall group and, in fact, these correlations were not statistically significant. Results for the group of students coming from outside of Manitoba but in Canada should be interpreted with caution because the sample is very small ($n = 12$). For the mature student group (age over 21), these results could indicate that this group of students were able to overcome weaker incoming skills and thus more likely to achieve Calculus grades that were higher than the grades predicted by their ALEKS scores.

These results contradict those of Geiser (2007) who claimed that high school grades consistently provide a stronger predictor of university readiness than placement examinations. One possible explanation for this could be that the entrance examinations that Geiser used covered multiple subjects and were not geared to the university curriculum as closely as ALEKS is geared to university-level Calculus. In other words, the ALEKS test is designed to assess the students' knowledge and skills, those they need to succeed in Calculus. Hence, such placement tests as ALEKS should, in fact, be better predictors of students' achievement in Calculus than a more general examination like the one discussed in Geiser's (2007) study.

Finally, the results of this study corroborate the analyses of Ahlgren-Reddy and Harper (2013) who found that ALEKS was a highly effective predictor of university Calculus course grades at the University of Illinois. The study here is also supported by the findings of Bressoud and Rasmussen (2015) who demonstrated that multiple measures, such as high school grades, placement test score, and demographic variables, provide better predictions of students' preparedness for university Calculus than their high school grades alone.

Contributions to the Field

The two studies that are part of this dissertation make a contribution to research in mathematics education. They also provide practical suggestions for university instructors and administrators to address the problem of grade inflation and students' struggling in first-year university mathematics courses even though they had received good grades in mathematics in grade 12.

The study presented in Chapter 2 reflects an important conceptual point first put forth by Adamuti-Trache et al. (2013): It is the strength of the relationship between high school course grades and subsequent university course grades that show whether or not high schools are effective at preparing students for future success in similar university courses. Adamuti-Trache and colleagues were among the first researchers to evaluate the effectiveness of high school education not by students' grades alone, but by this relationship.

In the study in Chapter 2, a measure of the differences between high school grades in Pre-Calculus and university grades in Calculus, called a 'Difference Factor'(DF), was used. The DF, as a measure of students' preparation, is a viable way of illustrating the relationship between high school grades and university grades, confirming research by Adamuti-Trache et al. (2013). In most previous studies, this relationship is presented as a correlation coefficient between groups of students. The DF measure can, however, be used to assess academic preparation of individual students. This way of using this statistic gives both researchers and educators an important way to compare individual students in their preparation for specific university courses. While the focus of this study was, of course, on mathematics, there is no reason why similar procedures could not be used for other subjects.

Next, the study presented in Chapter 3 adds to the field by providing a snapshot of current placement practices in post-secondary mathematics education in North America. It also supports research that advocates for using placement tests as a measure of students' readiness for university mathematics, rather than just using high school grades (e.g., Bressoud & Rasmussen, 2015; Hawkes & Savage, 2000; Latterell & Regal, 2007; Mattern & Packman, 2009; NíFhloinn, Bhaird & Nolan, 2014). In addition, it adds to the body of literature on Canadian placement testing in mathematics—in that the amount of research in mathematics education, and specifically with respect to placement testing, in Canada is far less than in other countries, the members of the Organization for Economic

Co-operation and Development (OECD) or the United States. The Canadian educational context differs in several ways. First, in Canada, a large proportion of students remain home for post-secondary education, whereas in the US, many more students leave home for university. Second, education is more expensive south of the border. Both of these factors mean that Canadian universities are more accessible than some American universities thus having a more diverse incoming student population. The need for placement examination in the Canadian context is growing, and this study makes a valuable and timely contribution to this area of study.

Recommendations

Given the results presented in both of these studies, high school teachers and university instructors alike should begin regularly sharing information pertinent to students' performance in mathematics courses. If high school teachers know specifically which baseline skills and concepts are required for university Calculus—those that are tested in Calculus placement examinations—they could focus on these topics to better prepare their students for university. In turn, university instructors should be taking the time to learn where their students are coming from, what curriculum they had in high schools, and what instructional experiences they received. We, as university instructors, need to find ways to adjust our university teaching to the students' backgrounds and needs in order to be effective. Both high school teachers and university instructors should focus more time and resources on collecting and analyzing student data so that better placement mechanisms can be used and higher success rates in first-year mathematics courses can be achieved.

Among effective placement strategies is the use of the rigorously developed ALEKS test. Such tests should be used to sort students into first-year mathematics courses so they have a higher likelihood to succeed. For those students identified as being unprepared for university-level mathematics courses, there should be remediation programs in place, such as transition-to-university mathematics camps and online remedial modules.

The results of the studies presented in this dissertation strongly suggest that high school grades alone are not an effective means of predicting incoming students' readiness for first-year mathematics courses. Placement testing, such as the ALEKS placement test, can be a way to more accurately assess students' preparedness for university mathematics courses, especially when used in conjuncture with the students' high school grades and their demographic characteristics. If universities are serious about addressing the constantly high failure and withdrawal rates of students in first-year mathematics courses, sharing information with high schools and building a strong placement strategy can be a powerful first step.

Future Research

From the analyses, the directions for future research can be derived. In both studies, while the large samples allowed for confidence in the conclusions, the samples focused on one university with most of its students coming from nearby high schools. For universities with differing student demographics, programs, and selectivity, there may be additional factors to consider. In addition, future studies should explore placement accuracy in other first-year mathematics courses, such as Applied Finite Mathematics, Statistics, or Linear Algebra.

As an extension to the analyses in Study 1, future researchers should examine how interventions including an improved alignment between grade 12 and first-year university curricula, increased communication between high school teachers and university instructors and administrators, and modified teaching and assessment methods to provide a smoother transition could reduce the disconnect between high school Pre-Calculus and university Calculus. In order to achieve these goals, high school teachers and university mathematics instructors should work more closely together to create opportunities communication, and they should share more information. Over the last couple of years, members of various university mathematics departments and high school mathematics teachers from a large urban school division have been meeting regularly to discuss issues of curriculum, teaching, and assessment. These meetings have provided illumination to both groups, resulting in stronger relationships between the two groups. Other ways of reaching these goals can be online discussion forums, local and national conferences, and an increased emphasis in university outreach groups forging links with high school teachers and administrators for the purposes of sharing information. All this provides valuable stepping-off points for research in the transition students make from high school to university mathematics.

Building upon the placement test results in Study 2, further research should explore the predictive power of placement strategies that include multiple measures to determine student readiness for university mathematics courses. Placement tests can clearly improve the prediction of student achievement in university-level Calculus typically estimated by high school grades alone. Using placement tests as an additional predictor of mathematics achievement can be most beneficial when used with demographic variables, such as students' age. University instructors and administrators will benefit from more research that considers demographic variables other than those used in the studies presented here (e.g., chosen field of study) in order to increase the predictive power of the existing pre-university assessment measures. As well, to examine which types of placement tests are most effective in predicting students' success in different contexts will also be helpful. Finding a means to address the data from students who voluntarily withdraw may further shed light on how to better place students so that more of them are successful in their university mathematics courses.

In addition, modern technological advances offered by data science and machine learning provide new intriguing opportunities to extend the research in both Study 1 and Study 2. In both of these studies, the data was used to extract information and make inferences, but the algorithms developed to make these predictions are static. Machine learning represents a way to improve algorithms by self-learning as more data is included. While data science and machine learning are used extensively in industry, their implementation is very limited in educational settings (Ali, 2013; Kumar & Vijayalakshmi, 2013).

Recent publications describe the role of the emerging field of data science in education as an opportunity to improve student outcomes, teacher performance, and education metrics through data-driven evidence (Ali, 2013; Kaur, Singh, & Singh Josan, 2015; Kumar & Vijayalakshmi, 2013; Siemens, 2013; Singh, 2012). Some examples of the use of data science in education include building predictive models for student success and dropout rates, analyzing student performance in order to better adapt teaching practices, and providing transparent information to stakeholders such as prospective students and faculty members (Ali, 2013; Kaur et al., 2015; Kumar & Vijayalakshmi, 2013; Siemens, 2013; Singh, 2012; Sharma, 2017). Data analytics models that have been used for these purposes include Random Forest, Neural Network, Logistic Regression, Support Vector Machines, and Decision Trees (Ali, 2013; Kaur et al., 2015; Singh, 2012), and these methods have been shown to predict student performance with up to 75% accuracy (Kaur et al., 2015).

The use of such models to describe and predict student outcomes and the effect of teaching methods has been described as the new discipline of “Learning Analytics” (Kumar & Vijayalakshmi, 2013; Siemens, 2013; Singh, 2012). These models could be used in place of the simple linear regression models to improve placement accuracy. Future research should look at how advances in data science can aid in analyzing placement data gathered from high school grades, diagnostic testing, and demographic variables. By analyzing such large-scale data, universities can measure the effectiveness of their current policies, such as policies on course placement, in ways that are substantially more rigorous than they do currently.

Regarding machine learning, this advanced analytic method provides educators with an opportunity to use computers in a way that they can begin to “learn” and predict students’ behavior. This can be extremely useful in adapting practices and policies to suit students’ needs resulting in highly personalized educational experiences (Siemens, 2013). For example, by inputting student data into machine learning algorithms, mathematics teachers could more effectively predict which students need extra support and on which topics. This would be especially useful to university instructors who teach large classes where the opportunity to regularly interact with individual students one-on-one is limited.

Machine learning accurately predicted dropout rates and as a result, has many promising applications for analyzing educational data (Lykourentzou, Giannoukos, Nikololous, Mpardis, & Luomos, 2009). Machine learning can aid in building placement mechanisms online that are more flexible and reactionary than static placement tests, and it can be used as a tool for determining students' readiness for university mathematics courses in ways that have never been possible. Though these tools seem to have extremely promising applications in education, their use, along with research pertinent to these methods, has been extremely scant, especially at the post-secondary education level.

In conclusion, the two studies described in this dissertation add valuable quantitative information to the current body of research in education. They provide important practical recommendations that have the potential to significantly reduce failure and withdrawal rates in first-year mathematics courses and point to projects for future research to study.

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