EXPEDITIONARY LEARNING FOR INCLUSIVE MATHEMATICS:
Five case studies in the field

by

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ABSTRACT

The dynamic nature of our knowledge of the world, ever-changing and expanding, requires the adoption of a new pedagogical stance for learning and living mathematics. Mathematics education should excite all students about the world around them and inspire them to investigate problems and propose solutions. Expeditionary Learning (MacLeod, 2016) seeks to inspire a spirit of wonder and curiosity that stays with each learner for life.

Expeditionary Learning is the result of a search to create a highly inclusive approach to teaching and learning through which mathematics is learned by being in the field, where mathematicians do their learning and work, and by telling the stories of our adventures. Initially, pieces of the constructivist approach (Bruner, 1961, Dewey, 1938), universal design for learning (Katz, 2013; Metcalf, 2011), inquiry learning (Alvarado & Herr, 2003; McQueen, 2003), problem-based learning (Evensen & Hmelo, 2000; Savery & Duffy, 2001), place-based learning (Brunswick, 1943; Washor & Mojkowski, 2013), the Reggio Emilia approach (Edwards, Gandini & Forman, 2012; Moss, 2004), Markerspace (Martinez & Stager, 2013; Papert, 1972) and traditional direct instruction (Kirschner, Sweller & Clark, 2006) that allowed all learners to be mathematicians were combined. Further refinement and experimentation steered Expeditionary Learning towards engineering, its philosophies and its actions. Engineers are models of learners for life. Their example invites all learners to approach problems from their own starting points,

carve their own paths, and present multiple solutions. In other words, it allows students to develop flexible responses to problems and creates learning spaces and activities that allow for the seamless inclusion of all learners. In this study, Expeditionary Learning is framed by engineering, inclusion, and adventure.

This is a study of the actions, thinking and learning of students and teachers engaged in mathematical learning expeditions. The goal of this research was to explore the potential of Expeditionary Learning as an inclusive, student-led pedagogy for learning mathematics. Grounded in interpretive phenomenology, this qualitative research project involved gathering stories and interpreting the meanings teachers and learners attached to their experiences. Analysis and synthesis of the individual stories will contribute to the development of a more global picture of what Expeditionary Learning has to offer to teaching and learning mathematics in inclusive settings.

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Introduction

A Culture of Curiosity and Adventure

The dynamic nature of our knowledge of the world, ever-changing and expanding, requires the adoption of a new pedagogical stance for learning and living mathematics. Mathematics education should excite all students about the world around them and inspire them to investigate problems and propose solutions. To take on the challenges of the 21st century (Wagner & Dintersmith, 2015), all people will need to be more than just literate and numerate; we will need to be passionately curious, fearless in exploration, and relentless life-long learners. Expeditionary learning (MacLeod, 2016) seeks to inspire a spirit of wonder and curiosity that stays with each learner for life.

In this introductory chapter, I provide an overview of the need for a pedagogical shift from learning as the transmission of facts, verified by testing; to learning as a focused experience driven by curiosity, imagination and a commitment to ongoing growth, confirmed through authentic assessment measures. The literature review presented in Chapter Two traces the theoretical background of Expeditionary Learning and its roots in engineering and inclusive education practices. A detailed picture of the research methodology used for this study is presented in chapter three. The results of the study are presented in Chapter Four and discussed in Chapter Five. Chapter Six concludes this study with a summary of the findings, limitations, and recommendations.

My Stance as a Mathematics Educator. With the turn of the millennium and the promise of change in education (Hayes Jacobs, 2010, Gardner, 2008, Dufour & Dufour, 2010), I anticipated that classes filled with disengaged students, working silently and compliantly at individual desks, on the same textbook question, to find the same answer, already known to the teacher, and of little meaning to anyone, would come to an end. I assumed that by the second decade of the new century, classes would be filled with learners actively leading the way in student driven communities; where each learner's unique strengths would be valued, and where teachers would focus on a students' learning trajectories and growth, and not on a final exam. I imagined that student curiosity would take over, and that students would become passionate explorers, intent on creating positive changes in the world through innovation and invention. I hoped that, as result of working collaboratively with others in creativity-intense environments, students would develop rich mathematical minds along with the confidence to seek out and ask authentic questions, to investigate unknowns, and to embrace uncertainty as an endless potential for learning. I dreamt of exploring local communities in search of meaningful problems that would require my students and I to draw upon a variety of flexible, non-standard problem-solving strategies. I imagined that schools would now "place in students' hands the exhilarating power to follow trails of interest, to make connections, to reformulate ideas, and to reach unique conclusions" (Brooks & Brooks, 1999, p.22). I saw every space as a learning place, and all people and all things as teachers.

However, bringing this inspired vision of learning to life has been a challenge. Along the way, I have come to know with certainty that shared experience brings shared understanding, that knowledge is not a commodity but a continuous event, that learning is best when flexible and that content is richest when it is unbounded. Living these beliefs has allowed for the creation of the Expeditionary Learning model for teaching mathematics (MacLeod, 2016). Initially, I had looked to teaching strategies from the constructivist approach (Bruner, 1961, Dewey, 1938), universal design for learning (Katz, 2013; Metcalf, 2011), inquiry learning (Alvarado & Herr, 2003; McQueen, 2003), problem-based learning (Evensen & Hmelo, 2000; Savery & Duffy, 2001), place-based learning (Brunswick, 1943; Washor & Mojkowski, 2013), the Reggio Emilia approach (Edwards, Gandini & Forman, 2012; Moss, 2005,), Markerspace (Martinez & Stager, 2013; Papert, 1972) and traditional direct instruction (Kirschner, Sweller & Clark, 2006) to create a philosophy of learning in which mathematics is learned by being in the field, where mathematicians do their learning and work, and by retelling the stories of their adventures. Although great ideas and practices exist in each of these aspects of education, none fully captured what I was looking for. Each aspect of education felt too tight, too restrictive, too prescribed. I wanted to define myself as a responsive learner, adjusting to the possibilities, and open to new ideas, while ensuring that my students were becoming powerful mathematicians. To say Expeditionary Learning is most like one would feel like giving up. Instead, I gathered the pieces that fit best with my vision and continued exploring. Eventually, this journey led me to engineering and inclusion.

Expeditionary Learning. In Expeditionary Learning, students adopt a mathematical point of view, seeing the world as mathematicians do. They engage in authentic inquiry and original research within their communities. They develop strengths as learners, so that no matter what the context, they will have access to their own strategies to deepen their understandings, create solutions or develop revolutionary inventions. As teachers strive to develop the skills, attitudes and abilities of mathematicians in their students, they need to create opportunities for them to explore their world as mathematicians do. The importance of curiosity, imagination, critical thinking, synthesis, invention, reasoning, reflection and communication must be at the forefront of every Expeditionary Learning experience teachers orchestrate.

The new criteria for intelligence is the ability to recognize a problem, define its variables, build its potential and create a solution that will be valuable to society (Wagner & Dintersmith, 2015). Given that new problems arise with countless unknowns, "learning to learn" (Wagner & Dintersmith, 2015) is the greatest skill that educators can teach their students. Experiences in Expeditionary Learning create opportunities for students to be engineers, architects, environmental researchers, physicists, astronomers, historians, geographers, artists and explorers - to name only a few possibilities. A learning expedition invites all learners to adopt a learners' point of view, to be curious and to engage in authentic inquires through direct encounters with the content being studied.

An expedition begins when students explore their community. When learning reaches from the classroom into the community, students build connections between theory and practice, concept and process, and content and context. In sharp contrast to a field trip, a learning expedition requires students to become a part of an uncertain, attention-grabbing and problematic situation. This experience of "learning at the edge means challenging old boundaries between school and community, between academic subjects and real-world problems, between theory and action, between thinking and doing" (Boss, 2012, p.123). Expeditionary Learning is about learning to think independently and as part of a community. It is about adapting to uncertainty and learning to be relentless in the pursuit of understanding.

Through these lived expeditionary experiences, learners shape their own ways of knowing and thinking about their world, their own approaches to solving problems, and their own learning processes. To be actively learning means to be fully engaged in an ongoing cycle of wondering, exploring, researching, developing, creating, constructing, connecting, communicating, reasoning and reflecting. These skills are best developed out in our communities, as deep thinking “cannot be removed from the world in which we live” (Roberts, 2012, p. 51).

The flexibility of the Expeditionary Learning path affords each student the opportunity to imagine and develop new strategies, applications and thinking. The unpredictable nature of an expedition and the learning that goes along with it create an environment where learners can try out their own ideas, pursue their own questions, test the limits, experiment, hear from others, share their thinking and draw conclusions based on experiences. Expeditionary Learning is not meant to fit nicely into a formal, sequential, lock-step diagram. The sequence of the learning strategies is never known at the beginning; so it demands that teachers and students learn to think in the moment, as in the unpredictable and uncertain real world.

Teachers using the Expeditionary Learning model re-imagine what a classroom is and what it could become. Expeditionary Learning lets teachers and students change their view of schools to one that is boundary free; where classrooms are “less a place and more a range of opportunities” (Zull, 2010, p. 44). By extending our learning in all directions, all learners are welcomed with accessible content, a range of actions and endless possible paths. Learning for all flourishes when educators engage their communities in the development of curious, self-directive and reflective learners. It is only when lived personal experience and theoretical content are tied together that a true understanding of the world begins to take shape.

Pedagogy. It is a critical time for educational institutions. Will Canadian schools emerge as centers for innovation, creativity and learning or remain mired in the past? The current system is outdated, exclusive and tired (Wagner & Dintersmith, 2015). Schedules and subject areas were created for a different world (Wagner & Dintersmith, 2015). Today’s students will graduate into a world of incredible complexity and infinite possibility. Traditional mathematics teaching “that emphasizes computation, rules, and procedures, at the expense of depth of understanding, is disadvantageous to students, primarily because it encourages learning that is inflexible, school bound, and of limited use" (Boaler, p.60). It is not reasonable to believe that the same pedagogical practices that have failed so many students in the past will succeed at offering all learners the chance to thrive in today's classrooms. Just as the world is constantly changing, so must educational practices.

The traditional approach to mathematical problem solving, where pre-written problems in a textbook is solved through routine processes, is far too restrictive (Ridlon, 2009). The variables found in these controlled questions are already defined, taking opportunities for research and development away from learners. After all, problem-solving and critical thinking skills are built through experimentation, failure and reflection (Bruner, 1961). Through an expedition, students learn mathematics by being mathematicians in the very environments where mathematicians learn and work. This repositioning moves educators beyond their role as purveyors of content and expands their role to focus on the facilitation of learning. Schoenfeld (1992) agrees that mathematics should be lived beyond the classroom, stating that a "fundamental component of thinking mathematically is having a mathematical point of view - seeing the world in ways like mathematicians do" (p.19).

With the growing push in education to develop 21st century skills (Hayes Jacobs, 2010; Gardner, 2008; Dufour & Dufour, 2010) and to encourage Science, Technology, Engineering and Mathematics (STEM) (Honey, Pearson & Schweingruber, 2014), educators may be experiencing a shift in paradigm. A movement from a teacher-centered to a student-centered model of education is emerging. The Expeditionary Learning model, with its dynamic balance of fieldwork, mini-lessons and workshops is student-centered and student led. This is because it is the original questions and problems, identified by the students, which guide the learning experience. This is consistent with the demands of citizenship in the 21st century. Those demands require students to develop skills in critical thinking, problem solving, communicating, and collaborating across disciplines and within the context of global perspectives (Hayes Jacobs, 2010; Gardner, 2008; Dufour & Dufour, 2010). The focus of Expeditionary Learning is on the development of learning behaviours that will empower learners to carry their skills and knowledge into every corner of their world.

Dewey suggested that “a system of education based on lived experiences” (Dewey, 1938, p.39) might create powerful learners who could carry their knowledge into new scenarios and use it to build new understandings. By experience, I mean the personal encounter: the active, engaged participant living the experience.

There is an ongoing increase in diversity in the classroom. To encourage inclusion in all aspects of society, education must play a lead role (Katz, 2013). Student needs vary considerably - socially, emotionally and academically. Addressing these needs is a challenge for classroom teachers. Research and scholarship calling for changes to instructional design (Katz, 2013) must give way to practices that teachers can use today with their students. There is an ongoing need to build confidence in classroom teachers and resource teachers in inclusionary, student driven learning practices that welcome all learners, including students with physical, sensory and intellectual disabilities. Expeditionary Learning is a model in which all learners are included from the initial question, where the balance of field work, mini-lessons and workshops provides multiple entry points, multiple levels of support, and multiple extensions beyond the original outcome. In Expeditionary Learning, open ended, authentic problems welcome a multitude of innovative solutions at various levels of difficulty. The focus on reflective story telling allows for learners to develop an understanding of themselves, including their strengths and needs as learners. Successful inclusive school communities increase the possibility of extending the values of inclusion beyond the school into an inclusive society.

Engineering. Engineers in the field are models of learners for life. They have blended specific mathematical content knowledge, skills and competencies into an intuitive engineering sense. Engineers work collaboratively with others, knowing that the best possible solutions will be negotiated between perspectives, boundaries and possibilities. They are flexible in their approach to new tasks, they learn from failures and they welcome uncertainty. Engineers are able to assess a situation, identify a component or process that could be improved, gather data, develop ideas, and create something, such as an item, process or service, that is valued. Engineering offers an intriguing blueprint for education. As stated by the National Academy of Engineering (2008) “no profession unleashes the spirit of innovation like engineering. From research to real-world applications, engineers constantly discover how to improve our lives by creating bold new solutions” (p.15). Engineers exhibit the very attitudes and competencies that educators seek to instil in their learners. What might education learn from engineering?

According to engineering educator Billy Koen (1985), the work of an engineer is to employ “the strategy for causing the best change in a poorly understood or uncertain situation within available resources” (p.5). With this statement, Koen reveals his engineering philosophy. Insights into the beliefs that shape the work of engineers may be gleaned through a discussion of the four main components of Koen’s approach: (a) employing strategies, (b) causing the best change, (c) being flexible with poorly understood or uncertain situations, and (d) working within available resources.

The strategies an engineer relies upon are varied, flexible and adapted to the situation at hand. Koen (1985) refers to these as heuristics; "anything that provides a plausible aid or direction in the solution of a problem" (p.16). These heuristics are often highly personal, not necessarily universally applicable, and dependent upon the variables of the situation and the expected solution. Selecting and implementing a specific strategy requires an engineers' professional intuition. Intuition is a feeling, an internal sense, a personal compass that points the engineer towards a possible path, alerts the engineer of possible danger or inaccuracy and reassures the engineer that a decision is right. This intuition is unique to each engineer, developed through experience, and emerges from a history of successes and failures.

The best change is dependent upon the availability of resources, time, current industry standards, our immediate level of understanding, and results in a positive impact for ourselves and others. Best, refers to one of any number of possibilities not simply the right calculation. This optimum outcome for the given situation will make the most positive impact possible for as many of the people and places involved. Pursuing the best outcome may require a reconfiguration, an adjustment or a completely new invention. It is also dependent upon public understanding and demands. Positive change comes when people with a variety of perspectives collaborate in an environment that supports the generation of new and unique ideas. Creative combinations of ideas from a number of sources are common. Engineers welcome the expertise of others, knowing that a mechanical eye, an electrical eye and a civil eye are better than only one eye. In other words, the best solution may be a negotiation, a trade off, between experts. By inviting individuals to share in the learning adventure together, educators may more easily see how to build the scaffolding and competencies students will need to be contributing teammates, leaning partners and valued citizens.

Purposeful, authentic problems create spaces for real thinking, negotiation and reasoning (Boaler, 2015). A commitment to working towards positive change in the world through innovation and invention is realized when students have opportunities to ask their own questions. Teamwork and collaborative skills are best developed through peer interactions, peers supporting peers, and through a collective responsibility to the shared learning of the group (Katz, 2013). It is likely that the solutions to tomorrow's problems will require rich reservoirs of imagination and innovation. Consequently, schools must become creativity hubs where the buzz of innovation calls to all learners. A passion to create positive change in the world through innovation and invention, as a result of working collaboratively with others, is best achieved by tackling questions for which the answers are not yet known; but for which the development of new insights will contribute to causes greater than our own. By inviting individuals to share in the learning adventure together, educators may more easily see how to build the scaffolding and competencies students will need to be contributing teammates, leaning partners and valued citizens. Establishing schools as creativity-intense spaces lets brilliant designs take shape.

Problems may be poorly understood and filled with uncertainty when first uncovered. Most are complex and ill-defined. Many layers exist below the surface of every initial picture. To better understand, engineers peel back the layers, questioning, interpreting, researching and analyzing every facet of a problematic situation from multiple perspectives. Dealing with the unknown is an integral part of engineering design. Curiosity is essential. Failures are probable. At times, the design may be faulty, one component may fail, or an entire system may crash. Each of these unpredictable events carries with it new understandings to be explored and new processes to discover. Engineering is accompanied by risk. However, over time, experiences with success and failure bring comfort with risk-taking. What at first seems ambiguous may just be the first step to invention and innovation.

The availability of resources can enhance or constrain possibilities. An engineer must consider all constraints such as budgets, time, the availability of materials, safety and industry standards. At any moment in time, the current level of understanding of an engineer or the engineering community also may act as a limiting factor. Personal strengths and experiences carried into new situations are significant resources; as are the community, its places, its history, and its people.

This philosophy of engineering and the process of learning are remarkably similar. Engineers share several incredibly valuable characteristics of strong, independent learners. Learning is seeking the best change for ourselves - the creation of new connections, new synapses and new competencies. When looking to learn something new, a learner is faced with uncertainty, and his or her understanding of a situation is poor. Learning can flourish when rich experiences and resources are readily available. Of particular interest to educators are the strategies (heuristics) that engineers use to build solutions, develop their intuition and comfort with ambiguity, and strengthen their ability to work creatively within restrictions. Drawing parallels between engineering and education allows us to rewrite Koen's description of engineering as a statement about learning. To learn is to employ strategies for creating new understandings, new competencies and new actions derived from wondering about and reflecting on experiences that are both rich and uncertain. Learning and engineering are flexible, fluid processes of creating positive change from unknowns.

My understanding of Expeditionary Learning has evolved through this philosophy of engineering. For educators, cultivating an intuitive sense in each learner (i.e., a disposition to naturally draw upon a variety of flexible, non-standard, personal strategies in response to any problematic situation) is best accomplished by: (a) offering learners opportunities to define their own questions, (b) encouraging an open, rather than prescriptive, approach to solving problem, (c) welcoming novel ideas and invented strategies, and (d) supporting learners in developing their own paths.

The confidence to explore what is unknown and to pose authentic questions, to investigate additional factors that will influence a solution, to turn ambiguity into determination and failure into invention, are realized though new experiences and adventures where learners are met with challenges far greater than those they have seen before. As engineers continue to demonstrate, the confidence and ability to go on learning is the most valuable resource available to students.

Inclusion. Each of us has much to offer. Education must help us to find our strengths and to build the skills needed to find fulfilment in our lives, to seek opportunities to help others, and to ensure that all people can and do make meaningful contributions to their communities. All students contribute to the shared learning of the group in an inclusive classroom community. The learning activities are designed so that each member can share their strengths and find the challenge they need to keep growing. To be known as a contributor, each learner would add something of value to the team. The contribution may be a measure that others need to start a calculation, organizing pieces of data to prepare for an analysis or building a model for a given scale.

The right to an education and appropriate educational programming is a key element of the Canadian charter of Rights and Freedoms (Canadian Charter of Rights and Freedoms, Section 15, 1985) and the Manitoba Provincial School Act (Province of Manitoba, Section 41.1).

Inclusion, as defined by The Public Schools Amendment Act (Appropriate Educational Programming, S.M. 2004, c.9, proclaimed on October 28, 2005), “is a way of thinking and acting that allows every individual to feel accepted, valued and safe. An inclusive community consciously evolves to meet the changing needs of its members”. Furthermore, it states “we embrace inclusion as a means of enhancing the well-being of every member of the community. By working together, we strengthen our capacity to provide the foundation for a richer future for all of us” (Appropriate Educational Programming, S.M. 2004, c.9, proclaimed on October 28, 2005).

An inclusive school is defined by Ofsted (2017) as “one in which the teaching and learning, achievements, attitudes and well-being of every young person matter” (p. 7). Evans highlights four key dimensions of inclusive practice.

1. The quality of each individual’s experiences in school, in terms of learning certainly, but also in terms of being respected for who they are.
2. Recognising different types of gifts and abilities and providing opportunities for everyone to succeed (everyone is good at something).
3. Identifying individual learning needs and providing for them.
4. The creation of a learning environment where barriers to learning are avoided wherever possible. (Evans, 2007, p.5)

Inclusion is not about treating all students the same. It is not about having students with different learning, physical, social and emotional needs seated within a classroom, but isolated to a back corner to work with an educational assistant. Inclusion is not forgoing the high standards of the curriculum for activities that are easy. Rather, inclusion is about belonging. Inclusion is about growth for both the individual student and the school community. Learners belong when they are welcomed as a teammate and have access to the learning resources they need to thrive, succeed, share and contribute.

Students who have been working towards individual learning goals as written into their Individual Education Plans are sometimes assigned Life Skills goals. Life Skills goals are not part of the provincial curriculum and are assigned on an individual basis. These goals, intended to offer students a path towards independence, often include learning to brush your teeth, learning to load a dishwasher, learning to fold a swim towel or learning to take turns in a conversation. Why are such limits placed on students in a place intended for growth? Students do not come school to learn to brush their teeth. Students come to school to be mathematicians, artists, scientists, explorers, athletes, writers, dancers, readers and teammates. They come to make discoveries, to create something new, to share in the stories of a community and to build connections. All learners come to school to flourish. Expeditionary Learning attempts to fulfil that promise by engaging students in a rich, open ended question and providing the unique scaffolds and supports for each learner to be constantly growing in terms of their mathematical content knowledge, their problem solving and critical thinking skills and their sense of social and emotional belonging.

The inclusion of all learners must be seamless. Originally, the concept of Universal Design (Pisha & Coyne, 2001) was used to describe a philosophy for architecture in which buildings and structures were designed to offer open access to all. When the concept was transferred to the field of education as Universal Design for Learning (UDL), it became a way to design, from the beginning, lessons and programs to include the widest spectrum of learners. UDL lessons have a built-in flexibility in terms of "setting goals, selecting materials and methods to support students in reaching their goals, and designing accurate ongoing assessments" (Rose & Meyer, 2006). The Center for Applied Special Technology (CAST) offers three guidelines for UDL upon which all learning experiences should stand. Students should be provided with "multiple means of representation", "multiple means of action and expression", and "multiple means of engagement" (2011).

From the very beginning, a learning experience must offer "multiple means of representation". In other words, a lesson must offer a wide variety of entry points. Students receive and interpret information in unique ways (Pisha & Coyne, 2001). A single representation, such as a written explanation read from a textbook is not fully recognized by some students. Presenting information or creating a situation where students can access information in a number of ways allows a maximum number of students to access the content. Students are then able to enter into the lesson through a representation that meets their learning needs building upon their prior knowledge and skills. For example, during an expeditionary experience where students are studying human impacts on a forest by measuring tree growth patterns, students would have a direct encounter with the trees in the forest. Additionally, students would have access to computers for internet searches, watching videos, a presentation from or an interview with an arborist, as well as access to print materials. In each of these representations, scaffolds and supports such as organizational frames, sentence starters, calculators, cameras, and electronic measuring tools should be available. At all times, the teacher is carefully monitoring student learning and is prepared to work with individuals or groups on a mini-lesson through which students can have direct instructional guidance. In this example (i.e., on the human impacts on a forest, measuring tree growth patterns), a mini-lesson might focus on using a tape measure, multiplicative thinking, determining a ratio or creating and using a T chart.

Each of us has our own distinct learning method. In a UDL experience, students with exceptionalities and students who require adaptations within the regular classroom can pursue their learning with their peers through “multiple means of action and expression” (CAST, 2011). This allows all students to plan actions, select strategies and processes that best fit their needs and the situation. In Expeditionary Learning, the initial question comes from the learner; so that a path to a solution is not known, and the teacher does not have a predetermined single, right answer. For example, multiple actions may arise when students are investigating how a sundial is constructed. Some students may begin with a large circle, knowing that a clock and a watch are circular. Then, they may mark the location of their shadow on the edge of the circle over a period of time. Other students may begin with a half circle thinking that their shadows will only fall in one direction. Another student may track the distance their shadow travels over a certain time. Multiple strategies arise when students do not receive initial direct instruction or a step-by-step process to follow. Within each of these active responses, students demonstrate their understandings of circles, angles, measurement and operations.

The larger context offered in expeditionary learning provides “multiple means of engagement” and the “latitude for learner choice and offer[s] a range of instructional approaches, supports, and scaffolds” (Pisha & Coyne, 2001). When engaging in a larger context such as a debate over the ethics of zoos, students can choose how to defend their opinions. Some student may decide to compare the size of the enclosure with the natural range of the animal, requiring them to measure and calculate the area. Other students may investigate how being in an enclosure affects the animals. They may choose to compare the distance that a kangaroo can hop within their enclosure to their natural hop length, or to compare the average age of animals in the wild against the average age of animals in captivity. The student responses can be captured through a variety of expressions including photos, videos, recorded interviews, letters to the zookeepers and models of alternative enclosures. Scaffolds such as data collection guides, organizational frames, sentence starters, and mini-lessons focusing on measurement, area and perimeter calculations, ratios and rates can be embedded within each student's choice of expression.

UDL includes the use of essential understandings. Essential understandings are more holistic learning targets that encompass knowledge, comprehension and application, with respect to a topic. These understandings remain relevant beyond the scope of the lesson and are applicable to a number of situations. Several specific learning outcomes often are combined with learning behaviours to create a target for which many entry points and many successful outcomes exist. For example, when working towards a goal of understanding metric measurements, one essential understanding may be to develop an understanding of the importance of ten in our number system. This allows students to enter at an appropriate starting point and grow in their understanding. Some students may be establishing their understanding of tens and ones. Other students may be working towards regrouping hundreds, tens and ones in three-digit addition and subtraction. Another student may be investigating the infinite number of decimal numbers that exist between one-tenth and one-hundredth.

Expeditionary Math. Expeditionary Learning is the result of a search to create a highly inclusive approach to teaching and learning through which mathematics is learned by being in the field, where mathematicians do their learning and work, and by telling the stories of our adventures. Initially, pieces of the constructivist approach (Bruner, 1961, Dewey, 1938), universal design for learning (Katz, 2013; Metcalf, 2011), inquiry learning (Alvarado & Herr, 2003; McQueen, 2003), problem-based learning (Evensen & Hmelo, 2000; Savery & Duffy, 2001), place-based learning (Brunswick, 1943; Washor & Mojkowski, 2013), the Reggio Emilia approach (Edwards, Gandini & Forman, 2012; Moss, 2004), Markerspace (Martinez & Stager, 2013; Papert, 1972) and traditional direct instruction (Kirschner, Sweller & Clark, 2006) that allowed all learners to be mathematicians were combined. Further refinement and experimentation steered Expeditionary Learning towards engineering, its philosophies and its actions. Engineers are models of learners for life. Their example invites all learners to approach problems from their own starting points, carve their own paths and present multiple solutions. In other words, it allows students to develop flexible responses to problems and creates learning spaces and activities that allow for the seamless inclusion of all learners. In this study, Expeditionary Learning is framed by engineering, inclusion, and adventure.

Inquiry learning starts with the wondering of the learner. It is his or her personal interests, passions and questions that guide the content and contexts to be explored. Students engage in original research, pursuing questions and ideas that arise from their experiences. In a problem-based approach, students are asked to identify what they already know, what they need to know, and how they will access new information that may lead them to a solution. In problem-based learning, it is assumed the content is best learned when it is anchored in a larger context. Place-based learning brings learners to the actual environments where the topics they are studying exist. This “on location” learning encourages participation in local and global events beyond the school day, as learners become deeply involved in the exploration of their connection with their own community. The Reggio approach is a holistic, constructivist model to early childhood education in which play, active participation and positive self-image are paramount. In a Reggio environment, children explore ideas and questions. Makerspace refers to a creative production that moves the learners from consumers of knowledge to become knowledge makers. In a Makerspace, learners are given an assortment of components with which to freely build or a specific challenge to overcome. A traditional lesson is characterized by a flow of information from the teacher to the students; while in a constructivist setting, students build their own understandings of the world through interactions. Each of these pedagogies has much to offer. By taking the most inclusive elements of each pedagogical approach, educators can create unlimited opportunities for differentiated instruction and personalized learning.

Research Questions

Over time, the research and scholarship that seeks to explain how we learn and what programming schools should offer has grown exponentially. Within this body of research, there are many contested theories, ideas, methods and tools, and considerable controversy. Often times, educators attend professional development sessions promoting one approach at the expense of another. Strategies and processes are labelled and categorized according to subject area and their theoretical underpinnings. This has resulted in a fragmented and static view of learning.

Through my attempts to personalize learning, I have known students to require direct instruction, inquiry, moments of quiet reflection, experiments, individual practice, times of free play, access to high and low technologies, opportunities to collaborate with peers, uncertainty, risk, success, failure and strong connections to places beyond the school.

This inquiry involves an analysis of students' and teachers' reflections about an expeditionary math experience. Learning Stories (i.e., a retelling of the teachers' and students' Expeditionary Learning experiences) will provide the lens through which the teachers' and students' actions and thinking can be analyzed. Specifically, within each Learning Story, I hope that the depth of knowledge of students' mathematical concepts will be revealed; as will the processes through which each learner made sense of problematic situations. The inclusiveness of the expedition also will be examined by exploring the meaning of the social interactions between students and their understanding of the academic, mathematical goals they pursued.

Importance of the study. Mathematics touches all aspects of our lives and all life on earth. Continuous advancement in mathematics is essential for continued growth in many diverse fields of human endeavour. Graduates who are ready to pursue careers in medicine, technology, engineering, genetics, accounting, management, architecture, biomechanics, environmental sustainability and countless other fields are invaluable for our future. Many factors impact a student's success in mathematics. Difficulties may be a result of early misconceptions or a lack of fluency with number. Inconsistent math instruction can contribute to a student's own belief that they "can't do math". Experiences with teachers and parents who were not confident in math or had a weak understanding of the relevance of math to practical life, also can hinder progress. Students with learning disabilities have struggled to acquire the skills and knowledge needed to become successful mathematicians (Herner and Lee, 2005). Suggested interventions for struggling math learners exist, but a gap remains between the research and the lived curriculum. A goal of the proposed study is to outline the characteristics of a culture that supports the development of learning behaviours for all students, including students with learning and intellectual disabilities, as well as reluctant and resistant learners.

Today's rapidly changing society leaves no question as to the value of a solid mathematics education. The National Council of Teachers of Mathematics (NCTM) reports that "those who understand and can do mathematics will have significantly enhanced opportunities and options for shaping their futures" (2012). A lack of mathematical skills and knowledge places limitations on one's economic and social potential. The Programme for International

Student Assessment (PISA) defines mathematical literacy as “the capacity to identify, to understand and to engage in mathematics and make well-founded judgements about the role that mathematics plays” (PISA, 2009). Data from the 2012 study of student performance in mathematics revealed that Manitoba’s students were performing significantly below the Canadian average. If Manitoba’s students are not able to access the resources and supports they need to be successful math learners, improvements are required. New pedagogies for engaging students in authentic, purposeful and deep mathematical thinking, such as those offered through Expeditionary Learning, may be essential to moving towards improved results.

Initial Research Findings. Initial research into the impact of Expeditionary Learning at five schools by Nicholas-Barrer and Hamison (2013) found “a positive and statistically significant impact on student achievement in reading and math” (pp.ix). Although these results are extremely promising, but further research is needed to determine the affects of Expeditionary Learning on student learning and inclusion.

A goal of this study was to outline the characteristics of a pedagogical culture that supports the development of learning behaviours for all students. If Expeditionary Learning succeeds at strengthening student learning behaviours, all learners stand to benefit. The scaffolded nature of the workshop and fieldwork, which include options for any number of mini-lessons, ensures that all students can find an entry point into the expedition. In addition, Expeditionary Learning supports the development of skills in collaboration, creativity, respect and supports for others. All of which are considered essential for success in the 21st Century (Dede, 2010, p.52).

An educator’s own experiences, confidence and competence impact his or her students and the learning culture. Teaching mathematics through Expeditionary Learning supports teachers as they continue to develop their own learning behaviours, mathematical content knowledge and connections to the community.

For teachers, Expeditionary Learning highlights the importance of learning behaviours and the development of skills, qualities and attributes that have been associated with successful learning opportunities for all. It also establishes opportunities for teachers to welcome learners at all levels into the same experience, providing solid examples of inclusion in mathematics education. This study contributes to our understanding and ability to implement quality programming that welcomes diversity in math learning.

Diversity in the classroom and in society is a valuable asset. An increase in diversity is vital to our understanding of the learning and teaching of mathematics. It is hoped that the results of the study will open further conversations between researchers and educators interested in examining the relationships between Inclusive Education and Mathematics Education. It is hoped that the results of this research will contribute to the literature that inspires teaching innovations in mathematics that value inclusion, creativity, diversity of talent and collaboration.

Outline of Succeeding Chapters. In this introduction, I have examined the need for a movement towards Expeditionary Learning in mathematics education, outlined the study and presented the statement of purpose. In Chapter two, I trace the development of Expeditionary Learning from its roots in constructivism, the Reggio Emilia approach Universal Design for Learning (UDL), 21st Century Learning, problem-based learning, inquiry, Makerspace, place-based learning, and traditional direct instruction to engineering. Connections between the opportunities of Expeditionary Learning and the goals of inclusive mathematics education are highlighted. A review of current research and scholarship in Expeditionary Learning is presented. In Chapter three, the problem statement, the three research questions, and the context of where and how the study was conducted are presented. I also describe the research design of this qualitative study, including the general theoretical stance of the researcher, the methodology and its rational, as well as the means of data collection and analysis. A description of the participants is shared in Chapter four followed by a description of the data gathered from the initial interviews, the teacher field notes, student Leaning Stories, and the focus group discussion. In Chapter five, two new themes that emerged from the study; taking risks to re-define mathematics, and inclusion by opportunity and choice are discussed. Finally, in Chapter six, responses to the three original research questions are explored followed by a discussion of the potential benefits of this study for classroom teachers, middle-years learners, and for the communities within which they are situated. The delimitations and limitations of the study are presented along with recommendations for future research and practices with respect to expeditionary math.

Chapter 2

Literature Review

In this chapter, I introduce Expeditionary Learning as a pedagogical approach to inclusive, student-driven mathematics learning. First, I examine the need for a change in pedagogy by reviewing policies, scholarship and research. Second, Expeditionary Learning has grown from experimenting with a number of different approaches to teaching and learning. I have explored aspects of education such as; constructivism (Bruner, 1961; Dewey, 1938), Universal Design for Learning (Katz, 2013; Metcalf, 2011), inquiry (Alvarado & Herr, 2003; McQueen, 2003), problem-based learning (Evensen & Hmelo, 2000; Savery & Duffy, 2001), place-based learning (Brunswick, 1943; Washor & Mojkowski, 2013), the Reggio Emilia (Edwards, Gandini & Forman, 2012; Moss, 2005) approach, Makerspace (Martinez & Stager, 2013; Papert, 1972) and traditional direct instruction (Kirschner, Sweller & Clark, 2006) looking for philosophies, beliefs, and practices to guide my thinking and to shape the inclusive, authentic learning experiences I wanted to create for my students. Third, to establish inclusive goals for high quality mathematics experiences for all, I turn to the processes of engineering and engineering philosophy as outlined by Koen (1985). Then, these goals for mathematics education are situated in engineering and inclusion and explored as a foundation for Expeditionary Learning. Fourth, the inclusive goals are further supported with educational research and scholarship on Expeditionary Learning. Finally, the limitations of Expeditionary Learning are explored.

A Change in Pedagogy

As mentioned in Chapter 1, a change in pedagogy is required. The ever-changing nature of our world demands a set of skills and abilities far different than those for which the current system of mathematics education was created (Dede, 2010, Hayes Jacobs, 2010). It is not reasonable to expect that the same pedagogy that has historically excluded so many students will now be able to welcome the full range of student learners in contemporary Canadian classrooms. That is, learners who excel, learners for whom English is a new language, learners with intellectual disabilities, and learners with unique sensory, physiological and social-emotional needs, as well as learners who require adaptations due to learning disabilities or gaps in their school experiences, and learners whose potential has yet to be uncovered. The time is right for a switch. Policies support the inclusion of all students in the regular classroom, technologists have created devises, materials and practices that welcome adaptations into the regular classroom, and 21st Century pedagogy calls for students to take on active roles in owning their learning and engaging in their communities.

For years, mathematicians, mathematics educators, parents and students have debated the value of traditional mathematics instruction and constructivist pedagogy. These debates, known as the “Math Wars” (Marshal, 2003; Schoenfeld, 2004), have consumed the energy of many math educators and researchers and taken valuable time away from purposeful discussions directed at improving student learning. Both the traditional and the constructivist philosophical stances have their own strengths and limitations. Rather than choosing one teaching model, Expeditionary Learning marries teaching strategies from a number of teaching and learning approaches, with ideas from engineering and inclusive education, in an original, flexible and highly personalized manner for each learner. In the moment, teachers will find a balance, drawing the best strategies from many pedagogical models to match the needs of the learners and to propel them towards great mathematical thinking.

Schools, as buildings, cannot encompass all learning. Traditional education, in which students are recipients of information from a teacher, has been shown to develop young people who lack independence and creativity, as well as skills in collaboration, problems solving and critical thinking (Dede, 2010, Hayes Jacobs, 2010). Yet, today's rapidly changing world leaves no doubt that these skills are essential for the 21st century (Christensen, Horn & Johnson, 2008). Establishing a culture of curiosity that invites students to bring their questions with them to school opens the door to an endless array of subjects and resources. Content can be learned through a variety of contexts in a number of locations so long as the spirit of adventure remains focused on learning. As teachers, we must look for ways to let others teach. Rather than always commanding a spot at the front of the classroom, we can learn alongside students as guides, coaches and pathfinders. Independence and self-direction are cultivated when moments of risk and uncertainty lead to new connections and knowledge. In 2010, The Partnership for 21st Century Skills outlined the knowledge and skills essential for a successful future. Their report "stressed that the traditional school culture was not designed to deliver those outcomes" (p.77). Dede adds that traditional programming continues to ignore "students ability to transfer their understanding" (p.55) and their ability to contribute to the success of a team (Dede, 2010).

Mathematics education also requires a shift towards more inclusive experiences. An inclusive educational experience provides "all students, including those with significant disabilities, equitable opportunities to receive effective educational services, with the needed supplementary aids and support services, in age appropriate classrooms in their neighbourhood schools, in order to prepare students for productive lives as full members of society" (Lipsky & Gartner, 1997, p.99). In inclusive classrooms, teachers create a culture where the norms, practices and actions of all who learn there contribute to the shared learning of the community.

Schools where educators follow scripted routines, based on predetermined curricula, resources and assessments, result in uniformity and exclusion (Dede, 2010). Such an approach does not allow for variations in thinking and actions, nor does it celebrate the uniqueness of each student. Asking students to share their discoveries and the processes they have used to learn lets them become learning resources for their classmates. Consequently, students develop confidence as leaders when they take over the responsibility for their own learning and the collective learning of the group.

Research and philosophy must give way to practical strategies that educators can count on to support learners every day. Each of the pedagogical approaches briefly described in the following sections are incorporated into Expeditionary Learning. When combined, Expeditionary Learning is able to draw on the best practices of many approaches and coalesce them into an original, highly inclusive, student driven pedagogy.

Constructivism

Constructivists such as Jean Piaget (1973), John Dewey (1938) and Maria Montessori (Renzuli, 1982) have developed a vision of teaching and learning in which meaning is built through experience. Knowledge is built through interactions with others and with the environment. For the constructivist teacher “to understanding is to discover” (Piaget, 1973, p.20). In a classroom, this ‘learning by doing’ takes place in the form of problem solving situations where students draw on their own experiences and prior knowledge to make sense of a new problem, to interact with others and their ideas, to select tools, to develop models and to justify their solutions. The focus of each learning experience is on student thinking, questioning and reasoning.

Supporters of the constructivist model, such as Bruner, point out that “practice in discovering for oneself teaches one to acquire information in a way that makes the information more readily viable in problem-solving” (Bruner, 1961). Those in favour of a constructivist approach believe that students develop higher order thinking skills through problem solving. They encourage the use of invented algorithms and personal strategies. A constructivist lesson may begin with a question such as, “Can two rectangles have the same area but different perimeters? To reach a solution, students may have access to manipulatives, paper for sketching and reference resources. They are encouraged to chat about their ideas and possible solutions. The teacher poses questions about the length and height of the rectangle and its resulting area, about past experiences with area and perimeter, and about how the student team is moving forward together. As the relationships between length, height, perimeter and area are established, students are challenged to make a generalized statement. They are encouraged to test their statement to see if it holds true in different situations perhaps with squares and triangles. Opportunities for all students to contribute to the shared learning of the group are endless. To be successful, a team member needs to identify rectangles in the environment, draw rectangles, measure the side lengths of rectangles, and create a comparison chart to analyze relationships, discuss their observations, and share conclusions. Within these roles there is room for students, with varying degrees of knowledge and understanding, to make meaningful contributions to both the required calculations and the concept discussion.

Key elements of the constructivist model are embedded into the Expeditionary Learning model. First, Expeditionary Learning contains opportunities for students to create meaning for themselves by finding themselves stuck, in just over their heads, so that new discoveries are within reach but not without struggle. Second, Expeditionary Learning supports the development of higher order thinking skills, beyond standard curricular outcomes, in order to develop habits of thinking deeply. Third, Expeditionary Learning allows for individual learning trajectories through which each learner finds his or her own personal path.

A study by Robertson, Meyer, and Wilkerson (2012) reported positive results for a constructivist approach to teaching and learning mathematics. In this study, 85 high school students participated in the “Gaining Early Awareness and Readiness for Undergraduate Programs (GEAR UP)” (p.32) program over a 6-week period. The program “was designed to create positive associations and experiences in the field of mathematics for at-risk students or those who might be academically marginalized” (p.32). For this study, students visited a skateboard park. Their own inquiry questions led them to new understandings about angular motion, rate of incline versus speed, acceleration and velocity. These concepts were further developed in a workshop setting where students continued to experiment and construct their knowledge. Data was collected from student self-assessments in the form of a survey and student recording sheets and calculations. From the analysis Robertson, Meyer, and Wilkerson (2012) concluded that throughout the constructivist activity “students were immersed in mathematical content and participated at a higher cognitive level, while having fun” (p.36).

Kirschner et al. (2006) have described constructivism as unguided and highlight the possibility of confusion and misunderstanding that can result from a lack of direct instruction.

Expeditionary Learning embodies the constructivist vision. Elements of the constructivist pedagogy are embedded in the Reggio Emilia approach, Universal Design for Learning, 21st Century learning, Problem-based learning, Place-based learning, Inquiry models and Makerspaces. In the sections that follow, each of these approaches is analyzed and then illustrated through the lens of Expeditionary Learning. The strengths that each approach brings to inclusion and to mathematics learning also are discussed.

Reggio Emilia

Referred to as the “gold standard” of early childhood education, the Reggio approach is a holistic, constructivist model of early childhood education in which play, active participation and positive self-image are paramount. Constructivist Loris Malguzzi, who believed that children constructed their own knowing and understanding through life experiences, first imagined the Reggio approach. He was interested in “how imagination was cultivated there, reinforcing at the same time the children’s sense of the possible” (Bruner, 2004).

In a Reggio environment, children explore ideas and questions. Learners communicate in “one hundred languages” including the “expressive, communicative, symbolic, cognitive, ethical, metaphorical, logical, imaginative, and relational” (Edwards, Gandini & Forman, 2012, p.7). Through student-centered activities, students develop an emotional connection to the content being learned.

In the Reggio environment, the role of the teacher is to document “what children are and the process of what they do” (Edwards, Gandini & Forman, 2012, p.7). The idea is for the teacher to “learn how to watch children for signs of thinking, and to point that child in the direction of deeper learning” (Martinez & Stager, 2013, p.43). Teachers attempt to learn what strategies students are trying, what questions they are asking themselves, and what frustration look like in each learner. Each learner has his or her own strategies and rhythm of learning. Careful assessment lets teachers see the “100 languages of children” so that they can come to know the students as people and as learners. Equipped with this understanding of each child, the teacher can orchestrate environments and situations where every child can flourish. Reggio is built upon a learner's freedom to explore and wonder. The Reggio curriculum is negotiated so that a "balance between providing structure and encouraging children's free exploration" (Tarini & White, 1998, p.379) is evident. The environment takes on a critical role as the third teacher. By taking learning to the environment, realness and relevance are embraced more fully than in traditional classroom-based instruction.

A study conducted by DeGroot (2012) found promising results when math was taught to young children using a Reggio approach. DeGroot created *Math Play* a "set of activities and environments that address mathematics growth and development that can be modified to include a classrooms emergent themes and interests" (p.38). These activities were designed to be developmentally appropriate for young math learners. While students engaged in these activities, the researchers gathered observational data and informal interview notes. Data was also collected through formative assessment portfolios, teacher journals, daily activity templates, teacher planning journals and curriculum checklists. Through a detailed analysis, DeGroot concluded that *Math Play* was an accessible approach to learning math for young children and that the use of "authentic formative portfolios provided an effective way to discuss individual child mathematic growth and development" (p.95). Similarly, the learning portfolios created during a math expedition create an avenue for young learners to share their discoveries and conclusions.

Children with special needs are known as "children with special rights" (Edwards, Gandini & Forman, 2012, p.189) in Reggio Emilia schools. Children whose learning needs are different are to be embraced by their teachers. It is believed that these children have much to teach us all. Loris Malaguzzi, Reggio expert and founder of the Loris Malaguzzi Center, "strongly believed that having the children with special rights in the school could stimulate us, as teachers, to think in terms of a much broader pedagogical approach for all children, to broaden our horizons for all the children" (Edwards, Gandini & Forman, 2012, p.189). This belief in inclusion is evident in the Reggio practices of connecting with the community services before a student enters the school and partnering with families. In schools, the classroom is organized so that scaffolds and supports are built in. The Reggio approach further supports the inclusion of all students by asking teachers to "learn to help each child exactly where the child finds difficulties" (Edwards, Gandini & Forman, 2012, p.207).

Potential challenges of the Reggio model include the time required to form such deep relationships with each and every learner, access to materials and resources that are consistent with the natural environment, and the demands of accurately documenting the learning of each child.

Expeditionary learning incorporates the Reggio belief that learning stems from the freedom to explore and wonder. Through negotiating the curriculum with students, teachers employing Expeditionary Learning seek a "balance between providing structure and encouraging children's free exploration" (Tarini & White, 1998, p.379). The opportunities afforded through the dynamic mix of fieldwork, mini-lessons and workshops is intended to put students in the lead, deciding with the teacher which activities will offer the most rewarding learning. The learning outcomes are reflected through student-authored learning stories, created throughout the learning experience, that are open to many forms of communication. Expeditionary Learning teachers encourage their students to reflect on and share their experiences in "100 languages".

Universal Design for Learning

As stated in the introduction, the Centre for Applied Special Technology (CAST) has outlines three guidelines for UDL. Lessons that are truly inclusive will have "multiple means of representation", "multiple means of engagement", and "multiple means of expression".

Expeditionary Learning illustrates the guidelines of UDL by placing students in the problematic situations, where the problem can be interpreted through the eyes of each learner. Multiple representations, action and expression are evident in the workshop and in the field, where students direct their own plans and determine their own methods for sharing what they have learned.

UDL is a model for inclusive practice (Katz, 2013). When teachers offer multiple means of representation, action, expression and engagement, they accommodate a range of student abilities. For example, a middle year's math teacher may adopt the principles of UDL for a lesson about area. The curricular outcome indicates that students will determine the area of a regular rectangle. This outcome leads to essential understandings of area as a measure of the surface of an object in square units. The teacher could pose an open problem such as "what is the largest enclosure that we can make using 24m of fencing?" To reach a solution, all students might have access to string cut in lengths of 24cm, square tiles, centimeter sticks, graph paper, and rulers: thereby encouraging multiple means of action and expression. Additional resources such as textbooks, reference charts, content word walls, visual dictionaries, digital textbooks and graphic organizers might be available to all students allowing for multiple means of representation. Scaffolds such as visual cue cards and graphic organizers might also be available. Students would work in teams and be encouraged to act as educational resources for each other. Responses to the problem could be shared using pictures, models, numbers, calculations and words. The open flexibility of UDL provides access points into the lesson for English as an Additional Language Learners, student with learning disabilities and students with intellectual disabilities. The essential understanding of area as a measure of the surface of an object in square units can be deconstructed into the smaller goals of counting, counting in groups, multiplication using arrays and other multiplication strategies. The essential understanding also can be extended to develop the relationship between area and perimeter, the area of a triangle, and the derivation of formulas.

A study by Katz (2013) examined the social and academic benefits of a UDL approach. The Three Block Model for UDL developed by Katz "provides teachers with a method for creating inclusive environments and improving student engagement" (p.6). The study involved 631 students, in grades 1 through 12, at 10 schools, and 58 educators. Data was collected from classroom observations and educator surveys. Katz (2013) reported "improved academic and social engagement" (p.163). Although research evidence does suggest that increased engagement does lead to increased academic performance Katz's study did not identify improvement to specific curricular outcomes.

21st Century Learning

What lies ahead is not known to us now. Preparing students for success in the future is a daunting task. Successful 21st Century learners are innovative thinkers (Delvin, 2012) who are able to gather and analyze information from a number of sources. They are creative thinkers, looking for solutions that are not written in the last pages of a textbook. They are flexible thinkers, able to adjust according to the demands of the situation. They are divergent thinkers, pulling ideas from a variety of sources and experiences to offer innovative pathways to solutions. They are collaborative and skilled communicators, who are self-reliant, interpersonal and intrapersonal. As such, teaching must move "in ways that help students learn without being taught" (Loong, 2004). As recommended by the Partnership for 21st Century Skills (p21.org), curriculum ought to focus on the development of the four c's (www.p21.org). The four c's are critical thinking and problem solving, communication, collaboration and creativity.

Critical thinking. Duran and Sendag (2012) defined critical thinking as a necessary skill for the 21st century. Specifically, critical thinking "begins with a physical or psychological inconvenience stemming from lacking the solution for a problem" (p. 241). It is "based on relating and drawing conclusions on notions and events and involves a variety of different cognitive processes such as implicating, problem solving, examining, reflecting and criticizing" (p.241). To test the development of critical thinking skills a science team, tech team, engineering team and a mathematics team collaborated to deliver a FI^3T (Fostering Interest in IT) project for student participants. The FI^3T project was implemented in two phases. The first phase allowed students to develop their technology skills and content knowledge in the science, technology, engineering and mathematics (STEM) fields through short lessons and hands-on workshops. The second phase employed the Community of Designers (Mishra, Koehler, and Zhao, 2006) approach. A community of designers "is an environment in which groups of individuals work collaboratively to design and develop solutions to authentic problems" (p. 243). A highlight of this approach was the "sustained inquiry and revisions of ideas" (p. 243). This is similar to the processes of Expeditionary Learning in that questions stem from student wonderings, that one question leads to another, and that learning is contextualized. Each question demonstrates the deepening of student understanding and critical thinking. Duran and Sendag (2012) investigated four specific questions. First, the researchers investigated the critical thinking skills profiles of the participants their study. Second, they asked how the profiles differed after completing the FI^3T program. Third, they considered if the scores of those students who had completed the FI^3T program had increased. Finally, they asked if there was a "significant increase in the subscale (analysis, inference, evaluation, inductive reasoning, deductive reasoning) test scores" (p. 243) of participating students. A total of 47 high school students participated in the study. The Test of

Everyday Reasoning (TER) is a multiple-choice test consisting of 35 items that must be completed in 50 minutes. Analysis of the results TER suggested that students who completed the program showed significant improvement in their critical thinking skills. The results of this study are consistent with other studies concluding that "focusing on authentic real-life problems within the context of a content area or focusing on problem solving process through problem-based and project-based learning improves students' critical thinking skills" (Ernst & Monroe, 2006; Hove, 2011).

Communication. The National Council of Teachers of Mathematics (NCTM) has also identified communication as a key component of mathematics education (2000, p. 60). Communication is the ability to express ideas with math language, symbols, sketches, models and equations. Valuing discussion in the mathematics classroom, McCrone (2005) studied "specific ways in which discussion can contribute to mathematical learning" (p. 248). Video tapes from 20, 60 to 90-minute discussions were analysed to identify specific dialogues in which students appeared to be learning. McCrone (2005) noted that in discussions students were active, that their arguments led to reasoning at a higher cognitive level, that students were more likely to challenge each other's thinking, and that questions were more thoroughly examined. McCrone concluded that discussions "allow students to test their ideas, to hear and incorporate their thinking by putting their ideas into words, and hence, to build a deeper understanding of key concepts" (p.11). Furthermore, a study of teacher perspectives conducted by Bennett (2013) found that middle years mathematics teachers who believed that communication was valuable noted greater student participation in lessons. Through a detailed analysis of questionnaires, and semi-structured interviews, Bennett (2013) was able to establish that the role of the teacher in learning discussions is three-fold. The teacher is "(1) probing student thinking, (2) shifting whole-class discourse responsibilities to students, and (3) being aware of a supporting students' affective development" (p. 482). Bennett (2013) also noted that discussions moved the focus from the teacher to the learner allowing student ideas to pop up and be explored and defended.

Collaboration. Cesar and Santos (2006) conducted an action research study of peer collaboration. The purpose of their project was to establish "the contributions of collaborative work to the promotion of more inclusive learning settings" (p. 335). This study followed a class of 9th grade mathematics students for one full school year. A member of the class had special educational needs. Throughout the course, students worked on problems in heterogeneous groupings. Data was collected through observations, analysis of video and audio recordings, photos, questionnaires, interviews, analysis of student work samples and academic documents. Cesar and Santos (2005) concluded that "social interactions that take place within lessons can play a decisive role in the promotion of more inclusive settings" (p.342). An additional finding related to the students' own realization that diversity benefits the development of all students.

Creativity. A study by Eisnmann, Novotna, Pribyl and Brenhovsky (2015) found that student creativity is enhanced through inquiry based, problem solving in mathematics. The researchers define creativity as "fluency", "originality", "flexibility" and "elaboration" (p. 551). Eisnmann, Novotna, Pribyl and Brenhovsky (2015) created a tool to describe a student's ability to solve mathematical problems. The tool was used to produce a profile of intelligence, reading comprehension, creativity and the ability to draw on prior mathematical knowledge. In this study, 62 students, aged 12 to 18 years, completed the pre and post evaluations 16 months apart. During the 16 months interval teachers taught heuristic strategies to students in a "culture of problem solving" (p. 541). Eisnmann, Novotna, Pribyl and Brenhovsky (2015) define a "culture of problem solving" (p. 541) as a "culture of inquiry" (p. 541). Students had numerous opportunities to construct their understanding of problematic situations. Comparing the pre and post evaluations, the researchers noted that there was a "significant increase in the creativity index, which doubled" (p. 557) or in some classes "more than tripled" (p. 557).

Proponents of Expeditionary Learning incorporate social responsibility and cultural and global awareness of 21st Century topics by pursuing solutions to authentic, meaningful problems. While in the field, students interact with members of the community and live the diversity of Canadian neighbourhoods first hand. Throughout this process, consideration is given to social, cultural and global factors that may contribute to, or limit, a possible solution to a problem. As students lead their own research, they gather information, judge the validity of sources, manage and organize information from a number of sources, and summarize information so that only the most relevant pieces are considered. Technology is a tool for researching, gathering data, and sharing questions, ideas and solutions. Technology may extend learning even further into the local and global community. Such a complex perspective on curricula is non-linear and built over time as teachers and students seek to incorporate 21st Century Learning with core content and skills (McTighe & Seif, 2010).

Problem-based Learning

A problem-based approach seeks to empower the learner to conduct research, integrate theory and apply new knowledge to a defined problem. Problem-based learning has its roots in medical education (Evenson & Hmelo, 2000). It was developed in response to concerns over the passivity of students and the vast amount of irrelevant information presented to students to memorize. Medical schools adopted a model where students worked alongside practicing doctors, assisting in diagnosis and treatment.

When given a problem, students identify what they already know, what they need to know, and how they will access new information that may lead them to the solution. The role of the teacher is to facilitate learning, pose questions, support and monitor the learning process. Teachers in a problem-based setting should "challenge the learner's thinking - not attempt to proceduralize their thinking" (Savery & Duffy, 2001, p. 5). The problems students tackle should be tough. For learning to occur, students should find that they are able to enter into a problem and make some progress towards its solution. They should also find themselves stuck, at an impasse or on the brink of failure. By working through these tricky times, students come to depend upon themselves as learners. In problem-based learning, students learn to make a decision as to what to try next requires students to reflect on their thinking processes, and the choices they have made up to that point. By revisiting each step, they learn to evaluate their progress and to base decisions on evidence. Failure becomes productive when it is used to strengthen the reasons that support their next best possible solution.

Savery and Duffy (2001) outline key instructional principles for problem-based learning that are relevant to Expeditionary Learning. They recommend that teachers "anchor all learning activities to a larger task or problem" (p.5). This larger context provides a reference point to tie various learning activities together. Savery and Duffy (2001) encourage teachers to design tasks reflecting the "complexity of the environment" (p.5) in which students will need to draw upon their skills. They recommend that teachers challenge students' thinking and provide opportunities for students to reflect upon what they have learned and how they learned it.

The problem-based approach is not intended to test a learner's understanding or ability to demonstrate learning by solving problems for which the answer and process are already known. All learning is to take place during the problem-solving process. Problems should be relevant and meaningful to the students, their communities and their futures. A problem does not need to be limited to a single subject area. Most problems encountered in the world are not. The critical thinking, and analytical and innovative problem-solving skills developed in problem-based learning are universal - applicable in a number of contexts.

A study of inclusion and problem-based learning by Belland, Glazewski and Ertmer (2015) found that group members felt that they had all contributed to the response, all group members agreed that they had benefited from working in a collaborative problem-solving group, and "each group member served a specific role that counterbalanced the shortcomings of their group members" (p.14). To further explore the benefits of open-ended, problem-based mathematics education, Boaler conducted a three-year case study of two schools in London. In her study, Boaler used quantitative assessment, observations, interviews and questionnaires to hear directly from students and teachers. She reported that "students who learned mathematics in an open, project-based environment developed a conceptual understanding that provided them with advantages in a range of assessment situations" (Boaler, 2015, p.41). Boaler concluded that the problem-based approach supported students in developing a "system of thinking and using mathematics that helped them in both school and non-school settings" (p.41). By incorporating problem-based learning in expeditionary math students benefit by deepening their collaborative team skills and their conceptual understanding beyond classroom mathematics.

Inquiry

All people are surrounded by mathematics, by science, history, geography, language and technology. These exist in all that we do. To develop the mathematical point of view, as suggested by Shoenfeld (1992), young mathematicians need to explore, to wonder about, and to question the spaces, events, patterns and relationships that surround them. Through the inquiry process, students “explore mathematical ideas from multiple perspectives” (NCTM, 2000, p.60-61). Inquiry learning starts with the wondering of the learner. It is their own questions that set the direction for the content to be learned and the contexts to be used. From there, students “are challenged and engaged in their own quest for understanding and knowledge, which prepares them for learning throughout their lives” (Kuhltau, Maniotes & Capari, 2007, p.16). Kolb notes that “if the education process begins by bringing out the learner’s beliefs and theories, examining and testing them, and then integrating the new, more refined ideas into the persons belief system, the learning process will be facilitated” (Kolb,1984, p.28). Kuhlthau, Maniotes and Caspari (2007) outline six principles for inquiry learning that are adopted in Expeditionary Learning. They recommend that students “learn by being actively engaged in and reflection on an experience” (p.25) and that students be offered explicit supports when needed. Kuhlthau, Maniotes and Caspari suggest that students need “opportunities to develop higher-order thinking through guidance at critical points in the learning process” (p.25). They add that children learn through social interactions with others. They also note that learning will be most meaningful when instruction and experiences are designed according to a student’s cognitive development.

Inquiry learning welcomes inclusive practices as they are flexible and can be incorporated into an open approach to teaching. An inquiry activity offers multiple entry points stemming from each student’s or student team’s own questions. This variety of starting points is engaging for reluctant learners as they can determine the focus of their learning context. It is equally engaging for students with intellectual disabilities, as they too can find a meaningful learning goal within the same context. For example, a teacher may target an understanding of volume using an inquiry activity. To begin, the teacher might invite students to explore a variety of boxes. The boxes are rectangular prisms, right triangular prisms and cubes. As the students explore the boxes, questions about size are encouraged. Drawing the class together, the teacher zeros in on determining which box is the biggest. The variety of outcomes is now endless. Students will build upon their understanding of comparing, ordering, estimating, measuring, experimenting and calculating. A student with intellectual disabilities working on an Individual Education Plan could work towards a goal of identifying objects that are taller and smaller than a given item, a fine motor goal of matching the 0 marker on a ruler to the end of one of the edges of the box or reading the numbers on the ruler and comparing the measured lengths. A student looking for a challenge could define big as occupying the greatest amount of space. By comparing the number of $1cm^2$ cubes required to build each box, the student could be challenged to generalize a formula for determining the volume of the boxes. Next, the student could adjust his or her formula to determine the volumes of the right triangular prisms. This inquiry can also be made accessible for students who are learning English as an additional language (EAL) by having a word bank, complete with photos, and a “4 point personal dictionary” (Freeze, 2002). The opportunity to work collaboratively with peers, practicing new words and expressions, is an additional benefit of inquiry learning for EAL students.

A study of inclusive inquiry learning by Jimenez, Browder, Spooner and Dibiase (2012) reported positive results for both students with intellectual disabilities and students enrolled in regular programming. In that study, five students with moderate intellectual disabilities were included in an inquiry-based science unit. Through observations and interviews Jimeneze et al. (2012) were able to report positive peer interactions and successful academic learning around several key scientific topics for all students.

Makerspace

Seymour Papert (1972) grounded his theory of constructionism in the belief that bringing minds and hands together creates engaging learning opportunities for all (Papert, 1972). Makerspace refers to a creative production that moves learners from consumers of knowledge to knowledge makers. In a Makerspace, learners are given an assortment of components with which to freely build or a specific challenge to overcome. Mark Hatch (2014), CEO and cofounder of TechShop, proposed a Maker Movement Manifesto that described maker activities and mind-sets "organized around nine key ideas: make, share, give, learn, tool up (i.e., secure access to necessary tools), play, participate, support, and change" (p. 496). Kurti, Kurti and Fleming (2014) reported that the Makerspace culture is "very conducive to collaboration and teamwork, and makers with complementary skill sets and viewpoints can work together to find solutions that would not have occurred to any of them alone" (Kurti, Kurti, & Fleming, 2014, p.8). A Makerspace gives the entrepreneurial and inventive spirit a place in schools.

In their study of Makerspaces in libraries, Slatter and Howard (2013) found that Makerspace was an accessible learning experience for all where "enhanced community engagement" (p. 276) allowed all participants a place to make and learn together. To that end, Expeditionary Learning looks to make available spaces and materials that spark ideas to initiate adventures. In a study of creativity in Makerspaces by Saorin, Melian-Diaz, Bonnet, Carrera, Meier, and De La Torre-Canteroa (2016), forty-four engineering students took part in 3d printing course. Saorin et al. (2016) used the Abreaction Test of Creativity to assess student creativity before and after the course. Individual interviews were also used to assess the students' "perception of their creativity" (Saorin et al., 2016, p.188). In the post test, student creativity competence scores increased 24.0 points while students reported that the 3d printer activities increased their creative abilities.

A Makerspace overflows with productive failure. The cycle of imagining, experimenting, failing and adjusting is ongoing. This creates a place where persistence, critical thinking and self-determination are naturally developed. These are key learning goals for all students. In particular, students with intellectual disabilities, physical disabilities and EAL learners rely upon their ability to find creative ways to accomplish tasks that others take for granted. Students with disabilities can find multiple ways to engage with materials in a Makerspace. Specifically, students can develop their abilities in counting, grouping objects, measuring, comparing, ordering, adding, subtracting, multiplying and dividing materials. Reluctant students may find the self-direction of the Makerspace inviting. While ELL students now have an opportunity to develop language beyond the language of the formal academic classroom. A Makerspace experience presents the perfect opportunity to develop those skills and to share their creative thinking with others.

Access to materials and time are two limiting factors that affect the success of a Makerspace.

Place-based Learning

Place-based learning builds upon the idea that current models of education are decontextualized, far removed from authentic environments, and lacking in ecological validity (Brunswick, 1943). In the artificial settings of classrooms, too much is controlled. Students struggle to connect the managed, fabricated experiences in school with their own lived experiences outside of school. Place-based learning brings learners to the actual environments where the topics they are studying live. This “on location” learning encourages participation in local and global events beyond the school day as students become deeply involved in the exploration of their connections with their own communities. The “constructivist approach common in these environments learning programs also contributes to deeper cognitive and critical thinking skills in students" (Kenney, Militana, and Horrocks-Donahue, 2003) while maintaining an emphasis on active, outdoor learning, teamwork and character building (Black, 2005). By tying learning to place, Gruenewald (2003) suggested that education "might have some direct bearing on the well-being of the social and ecological places people actually inhabit" (p.4) so that students will make responsible and well-informed decisions about their communities and their environmental impacts.

For example, in place-based learning, students might study forest diversity by conducting scientific studies in a forest. They might measure the impact of pollution on pond life by gathering data from a polluted pond. In essence, they learn scientific principles and skills by conducting research as a scientist would, where scientists do. When in these spaces, learners' skills in observation, defining relationships and making connections are enhanced. In addition, sharing the adventure with a collaborative team of peers facilitates the development of social skills, cooperation and communication. Learning occurs in every space and continues in every direction.

Place-based learning provides a live context for learning math. For example, decimals, ratios and percentages are lived in a grocery store where prices, best buys and discounts can be analyzed. As a second example, topics such as personal finances, interest rates and loans can be experienced at a car dealership. Trigonometry and slope are essential for creating a topographic map or planning for the construction of a fence on a hill.

Place-based pedagogy supports inclusion in that it forces us all to consider the physical accessibility of the spaces people inhabit. Challenging students to think about how a classmate with a physical disability or a classmate who is a wheelchair user will be able to move around a public place creates an awareness that all students will carry forward. This awareness leads to the mathematical thinking involved in estimating and organizing a space where there is enough room to turn a wheelchair or a walker and where items are easily accessible.

Seeking to identify the goals, strengths and challenges of place-based learning, Powers (2004) compared four programs within the Place-Based Education Evaluation Collaborative (PEEC). Powers (2004) considered two specific research questions. First, Powers (2004) wanted to know "which aspects of the program models are most consistent and effective" (p.20). Secondly, Powers (2004) studied "whether and how teachers change their practices as a result of the program" (p.20). Each of the four place-based programs studied had its own goals and local focus. However, four common themes linked the four programs. First, all four programs supported community and school connections. Second, each enhanced connections to place. Third, all four programs contributed to an "increased understanding of ecological concepts, enhanced stewardship behaviour, and increased academic performance in students" (p.21). Finally, all four programs focused on improving the environment and opening opportunities for civic participation. From this starting point, Powers (2004) used a mixed methods research design consisting of structured interviews with teachers, school administrators, students and community members, as well as partner focus groups and observational data (p.21). Following the analysis, Powers (2004) identified four areas of strength in Place-based learning. First, Powers (2004) noted that connecting with community members allowed students to consider diverse viewpoints. This connection also created opportunities for students to learn through real-world problems and add value to their communities. Second, teachers had developed strengths in their curricular knowledge, process facilitation as well as specific skills related to the context of the problems their classes worked on. Third, the program offered professional development for teachers over the summer. These sessions proved to be invaluable in deepening teachers' pedagogical skills and content knowledge. Finally, relationships between teachers and community members, teachers and students, and between students and community members were strengthened. This produced a boost in confidence and belonging. Two key challenges were identified. First, pursuing Place-based learning opportunities is time consuming. There is a steep learning curve in terms of adopting this new pedagogy as well as for making sense of the spaces and places being investigated. Similar challenges exist in Expeditionary Learning. Successful teachers require in-depth knowledge of curriculum and of the neighbourhoods that surround their schools. Being able to respond to students' unpredictable learning paths with on-demand lessons requires a deep understanding of mathematics. Teachers also require time to build connections with their communities. An emergent finding reported by Powers (2004) was the improved participation and achievement of students with special needs. Throughout the interviews teachers and students reported that students with special needs worked with greater independence, enthusiastically engaging with community members and began to thrive academically. I have noticed similar results during my own Expeditionary Learning experiences with students. Students with special needs were able to actively engage with a team of peers while learning at their own unique levels. Students with special needs also developed social skills and teamwork by becoming contributing members of their teams.

Howley, Showalter, Howley, Howley, Klein and Johnson (2011) investigated the challenges faced by place-based mathematics educators in seven school communities. Data collection included: (a) interviews with teachers, parents and school administrators, (b) observations from mathematics lessons, and (c) field notes and other relevant community documents (Howley et al., 2011). Their findings were presented under three specific themes.

First, the place-based learning of mathematics provided a relevance and realness to the content. Consequently, student motivation and engagement subsequently increased. Second, place-based mathematics lessons were taught more frequently in lower level mathematics classes while university prep classes preferred traditional methods. These opposing views created a tension and reinforced the belief that not all people are intended to excel at the higher levels of math (Howley et al., 2011). Finally, Howley et al. (2011) reported that "the strength of the champion educator and allied network" (p.199) and the shared belief in the future of the community contributed to the success of place-based mathematics.

Promising results were also reported by Endrery (2009) and Muthersbaugh (2012). Endrery (2009) conducted a study with thirty-three fifth grade students in an urban school setting. The fifth graders conducted a place-based inquiry about a local watershed. The data sources included concepts maps completed before and after the project, student science notebooks from the field, and interviews. Endrery (2009) reported that 77% were able to describe each of the four major curricular outcomes as a result of this inquiry. Muthersbaugh (2012) worked with twenty-five elementary students while investigating students' understandings of environmental science concepts. Murthersbaugh, Kern and Charvoz (2014) collected data from student journals, student interviews, and teacher interviews as well as through detailed analysis of artifacts created by the participants. Reported findings from this study indicated that the students had an increased confidence in their ability to lead their own learning and there was an increase in the number of questions originating from the students (Muthersbaugh, Kern and Charvoz, 2014).

Critics of place-based education note that there can be serious risks involved in taking groups of young people out of the school such as being involved in an accident due to unsafe conditions or becoming lost or separated from the group. In addition, learning off the school site requires planning and does have costs associated for the school and the students involved.

Traditional Mathematics Instruction

A traditionalist defines knowledge as the set of "facts and procedures one can reliably and correctly use" (Byers, 2010, p.25). A traditional lesson is characterized by "direct instructional guidance" (Kirschner, Sweller, & Clark, 2006, p.1) from the teacher to the students. The teacher's role is well defined as the keeper of knowledge to be transmitted to students through lecture, notes, examples and practice. Often, course materials are predetermined as lesson plans, texts, assignments and evaluations are similar from year to year, and from class to class.

A report from the National Commission on Mathematics and Science Teaching for the 21st Century characterized instruction in traditional classrooms as a "consistent pattern, which consists of:

1. a review of previous material and homework,
2. a problem illustrated by the teacher,
3. drill on low level procedures that imitate those demonstrated by the teacher,
4. supervised seat work by the students, often in isolation,
5. checking of seat work problems, and
6. assignment of homework" (2000, p.20).

This routine ensures that the content is systematically presented to students in isolated, step-by-step chunks. The immediate practice of new skills using worksheets and textbooks is supported by the findings of Nadolski, Kirschner and van Merriërch (2005) as an effective way of ensuring retention. Their research found that students who had immediate access to practice worksheets outperformed learners who did not. This is consistent with the findings of Kirschner, Sweller and Clark (2006) who found that expert student written word problem solvers benefit from guided instruction and practice as it enabled them to store and access knowledge in their long-term memories, while students who do not practice can only hold information in their working memories for the short term.

The traditionalists support the teaching and learning of formal algorithms and specific processes. An algorithm offers a specific method or set of steps to be used to reach the correct answer. A specific process provides a set of discrete steps to follow, in order, from start to finish. Both can offer efficient paths to solutions. However, they can also be extremely limiting in scope, as there is little opportunity to include unique details and variations. The traditionalist curriculum follows a specific sequence so that students are ready to add more complex concepts to their knowledge of basic skills.

Elements of the traditional classroom are incorporated into Expeditionary Learning as mini-lessons and as a part of the learning trajectory. Mini-lessons offer guided interventions as needed, while learning trajectories provide teachers with a planning tool for anticipating the concepts upon which students may stumble. By incorporating mini-lessons, rather than full class one size fits all lessons, teachers have the flexibility to focus on small group instruction targeting specific outcomes. For example, a teacher may move between student groups as they investigate prices and discounts at a local grocery store. Differentiated goals for student teams might include reading decimal numbers, ordering decimal numbers on a number line, rounding decimal numbers to the nearest whole number, adding, subtracting, multiplying and dividing decimal numbers, calculating taxes and using a ratio table to determine unit prices. Tools, scaffolds and supports that best fit the learning needs of each student are used during their specific mini-lesson. This wide range of goals, supported by brief direct instruction, provides a maximum number of students access to meaningful learning within their zone of proximal development.

In *Visible Learning*, Hattie (2003) identifies the most promising factors for student learning. Four of the top five most effective strategies are feedback (effect size 1.13), instructional quality (effect size 1.0), direct instruction (effect size 0.82) and remediation (effect size 0.65). Each of these strategies can be successfully incorporated into expeditionary math as part of a workshop or mini-lesson. The nature of the mini-lesson allows teachers to provide in the moment, on-demand, teaching and learning for students.

The inclusion of these strategies is also supported by the findings of Kortering, Debettencourt and Braziel (2005). In this study, Kortering, Debettencourt and Braziel (2005) surveyed 410 general education students and forty-six of their peers with diagnosed learning disabilities. The surveys were used to collect information about successful interventions and adaptations used in algebra classes. Among the most effective strategies identified were group and paired activities (80.4%), instruction in learning strategies (69,6%), additional encouragement from the teacher (65.2%), and more individual and small group help (67.4%). Again, the flexibility of the workshop and mini-lesson approach allows for these strategies to be easily included in Expeditionary Learning.

In comparing traditional mathematics instruction with constructivist approaches such as problem-based and inquiry, Ojose (2010) identifies “fuzzification” (p.105) as a major concern. Fuzzification is defined as an ambiguous or incomplete understanding of math concepts. Ojose (2010) states that “according to the traditionalists, what is at the heart of such fuzzification is the deliberate attempt to ask questions so vague that students feel comfortable in tendering partial answers" (p. 105). When examining the problem-based approach Ojose found that traditionalists identified two specific concerns; the uncertainty in interpreting data, which then lead to a multiplicity of possible solutions, is often confused with an intrinsic indeterminacy in mathematics; and an overemphasise on real-world applications robs mathematics of its coherence and internal structure" (p.105).

To become mathematicians, students and teachers need to live mathematics within and beyond the classroom. By combining the most inclusive and constructivist elements of several teaching approaches, Expeditionary Learning offers access to high quality math education for all learners. In the next section, I look to examples from engineering for guidance in mathematics education.

Engineering for Inclusive Mathematics

As noted in the introduction, engineering educator Billy Koen (1985) defines the work of an engineer as "the strategy for causing the best change in a poorly understood or uncertain situation within available resources" (p.5). Interpretations of each of the four main components (strategy, best change, poorly understood or uncertain situation, and within available resources) offer insight into the beliefs that shape the processes of engineering and provide a frame for inclusive math education.

Drawing parallels between engineering and education allows us to rewrite Koen's description of engineering as goals for inclusive mathematics learning.

1. **Strategy**: Cultivate in each learner an intuitive sense and disposition to naturally draw upon a variety of flexible, non-standard, personal strategies in response to any problematic situation.
2. **Best change**: Unleash in each learner a passion to create positive change in the world through innovation and invention as a result of working collaboratively with others in creativity-intense environments.
3. **Poorly understood or uncertain situation**: Develop in each learner the confidence to let curiosity take over. That is, to explore, to pose authentic questions, to investigate additional factors that will influence the solution, to turn ambiguity into determination and failure into invention.
4. **Available Resources**: Inspire in each learner a belief that where there are limits, there are possibilities. In other words, to create an atmosphere of optimistic enthusiasm where all spaces are learning places, where each of us and all things are teachers.

These goals come to life through Expeditionary Learning.

In the next section I discuss each of these four goals, their roots in engineering philosophy, and their ties to aspects of education. However, a thorough understanding of pedagogy and content will not suffice. Equally important is the relationship with the learner. A commitment to inclusion requires that teachers focus on offering meaningful, personalized academic, behavioural and social experiences throughout the day. I present Expeditionary Learning as a pedagogy that embraces these goals through inclusive, active, experiences that redefine the mathematics classroom.

Strategy

As noted above, strategy means to cultivate in each learner an intuitive sense, a disposition to naturally draw upon a variety of flexible, non-standard, personal strategies in response to any problematic situation. Within the goal of cultivating strategies, I include the learning strategies of explore, research, plan, create and construct, reason, connect, communicate and reflect used in Expeditionary Learning. To do this, I draw on scholarship from the constructivists, the Reggio Emilia model and UDL.

Strategy in Engineering. In the moment, engineers are called upon to make judgments and decisions that may result in the most unprecedented advancement for the betterment of humankind or in catastrophic failures. These “in the moment decisions” require confidence in oneself and the ability to synthesize prior experiences, evaluate innovative alternatives, reason through consequences and determine the most fitting choice at that time. To do so, one might suggest that engineers require a solid foundation of knowledge. Rather, this internal ability is so much more than the recall facts and theorems. To Bucciarelli (2003) this engineering intuition means to:

> Have some sense of the range of phenomena which might be so explained and how one constructs an explanation, develops ones open narrative and mathematical analyses when confronted with new phenomena, a new structural form, a new design. To know the meaning means to speak the language, to join the game, to know the rules and how and when they apply. (p.71)

It is a knowing how elements work together, what pieces will fit, which pieces will not fit, what alternative solutions to try and when to regroup and start fresh.

Variables in any problematic situation differ from those previously experienced by an engineer. Although some situations appear to be similar, where lessons learned in past experiences may prove to be incredibly valuable, the engineering process is not perfectly repeatable. How an engineer will make their way from problem to solution is not a set path. As more and more layers of a situation are exposed, more and more complex issues come to the surface. These contribute to the unique path developed by each engineer through his or her personal design process. An engineer is then defined by the set of personal strategies “including the heuristics he has learned in school, developed by experiences, and gleaned from the physical world around him” (Koen, 1985, p.49). From this perspective, Koen (1985) concluded, “no two engineers are alike” (p.49).

Uncharted experimentation, that begins with a thorough investigation of the current situation and a hypothesis of what is possible, more closely describes the process than a labelled diagram with arrows and numbered steps to follow. Ferguson (1994) adds that:

> Engineering design is always a contingent process, subject to unforeseen complications and influences as the design develops. The precise outcome of the process cannot be deduced from its initial goal. Design is not, as some textbooks would have us believe, a formal, sequential process that can be summarized in a block diagram. (p.37)

For Ferguson (1994), a pre-set sequence of steps to follow implies a “division of design into discrete segments, each of which can be processed before one turns to the next” (p.37) rather than “the usual chaotic growth of a design” (Ferguson, 1994, p.37). The open-endedness of an engineering design, the wealth of strategies, materials and processes that can be pursued allows for “many, many different paths, some of which are better than others but none of which is in all respects the one best way" (Ferguson, 1994, p.23). Every situation is unique therefore every approach is an original.

In an engineer, this intuition stirs a need to wonder, to question and to remain curious about the world around us. Engineers are looking for new problems to solve, opportunities to make improvements and reasons for innovation. At any moment, individuals should intuitively be able to assemble a number of possible approaches to problem solving and design, evaluate the potential of each method, select a response best suited to the situation at hand, and engage in action with confidence. It is only when the lived personal experience, the theoretical content and personal reflections are tied together that a true understanding of the world begins to take shape.

Strategy in Learning. In an educational setting, students should be given opportunities to develop a wide variety of personal learning strategies. When students find themselves in the middle of an ill-defined, problematic situation, they will need to select an appropriate next step from their kit of strategies. An ill-defined or ill-structured problem refers to realistic, authentic problems "that are so complex, messy and intriguing that they do not lend themselves to a right or wrong answer approach" (Barell, 2010, p.178). Consequently, students need to self-analyze their current level of understanding, assess what information they require, and implement a strategy to support the uncovering of that information. The ability to invent a new strategy, when common or standard processes have failed, is an equally important skill. A linear learning path is therefore difficult to draw as true learning paths are always in motion. A learner's path, from initial wonderings to a solution, is a story composed by the learner rather than a recipe to follow given by the teacher. Learning values serendipity over conformity.

Creating opportunities for students to explore problematic situations is key for the development of intuition. By orchestrating experiences where students "experiment, take risks, and play with their own ideas, we give them permission to trust themselves" (Martinez & Stager, 2013, p.36). The classroom community should be a place where students can try a hunch, experiment with possibilities and reaffirm ideas. Educators wishing to develop an intuition for learning in students, similar to the intuition of an engineer, can look to Vannevar Bush, American engineer and inventor, who recommended that students learn "how to tackle a comprehensive engineering problem in its entirety, drawing his [or her] tools from various sources" (Zachary, 1999, p.68) and to Israel Sheffler, professor of education who noted that:

> The aims of education must encompass also the formation of habits of judgment and the development of character, the elevation of standards, the facilitation of understanding, the development of taste and discrimination, the stimulation of curiosity and wonder, the fostering of style and sense of beauty, the growth of a thirst for new ideas and vision of the yet unknown. (Buccarelli, 2003, p.86)

Bush and Sheffler both encouraged educators to move beyond simple transmission and repetition and into problem solving challenges that teach learners to think. Their focus is on developing a personal thinking process rather than the recall of de-contextualized facts. Constructivism and the Reggio Emilia approach to education encourage learners to do just that.

The focus on problem solving is not new to mathematics learning. The National Council of Teachers of Mathematics (NCTM) Principles and Standards for School Mathematics has stated that "solving problems is not only a goal of learning mathematics but also a means of doing so" (p.4). NCTM recommends that learners have time to "grapple with", "apply and adapt" the strategies they develop over time and through a variety of different problem-solving experiences. As a result of "solving mathematic problems, students acquire ways of thinking, habits of persistence and curiosity, and confidence in unfamiliar situations" (p.4) that are similar to the heuristics described by Koen. Furthermore, in *Everybody Counts* (National Research Council) it is noted that “to understand what they learn, [students] must enact for themselves verbs that permeate the mathematics curriculum, examine, represent, transform, solve, apply, prove, communicate (1989, p.58).

Strategy in Expeditionary Learning. Drawing upon the constructivist and Reggio approaches, students on an expedition must develop their intuitive sense and their own set of heuristics to carve out their learning paths. As an engineer develops a strategy for designing solutions and inventions, each student must develop a strategy for learning. This unique, individually defined schema is echoed by Kolb (1984) who stated that “the learning process is not identical for all human beings. Rather, it appears that the physiological structures that govern learning allow for the emergence of unique adaptive processes” (p.62). A student’s approach to learning should be personally defined over time through experiences in a variety of learning environments. Students should come to recognize which strategies are best suited for themselves as learners and best suited to the task at hand. Just as Koen (1985) noted the uniqueness of each engineer, the same is true for learners, no two are exactly alike.

By marrying ideas from engineering and education, the Expeditionary Learning strategies of explore, research, create and construct, connect, reason, communicate and reflect show that the choice of strategy, the path to take, is in the hands of the learner. These strategies are linked (as no one strategy is enough on its own) but not prescribed. What is learned through one strategy becomes the next step into another. Believing that each learner is wonderfully unique, no two series of steps taken will ever be the same.

These strategies, although presented here in order, are not intended to be used in sequence. At times, students will find themselves using just one, while at other times they may feel they are using many at once. With practice, students will become familiar with the strengths of each strategy and naturally move between them, developing their own intuitive sense for learning. Their sequencing of the strategies is dependent upon their varied purposes.

Explore. Mathematicians wonder. They observe their surroundings: the interactions, the changes, and the processes that are taking place. Their observations form the basis of their questions. Due to the complex nature of interactions, mathematicians examine problems from multiple perspectives. They hypothesize, making reasonable predictions based on initial evidence, as a way to guide their thinking. Curiosity drives exploration, which in turn, leads to more questions, experimentation and new theories. In Expeditionary Learning, exploring signals the start of a new adventure. As students explore, they wonder about the dynamics of a situation. They observe. They estimate, using known referents, facts, benchmarks and by building onto insights from past experiences. They identify curious situations and pose questions. These initial questions deconstruct the situation, pulling pieces apart so that all factors, parameters, limitations, boundaries and possibilities are exposed. Students gather initial data, noting what is known to them now and what they will need to learn. This exploration leads to an understanding of the need for a solution and allows for the creation of solution criteria. Students are asked to make connections to past experiences and problems that are similar as well as to those that are not similar. This exploring phase is emphasized by Zull (2001) who notes that "all members of the community can serve as teachers and all buildings as schools" (p.44) and by Martinez and Stager (2013) who add "students engaged in direct experience with materials, unforeseen obstacles, and serendipitous discoveries may result in understanding never anticipated by the teacher" (p.52).

Research. Research in Expeditionary Learning consists of identifying the measurements, data, observations and information needed to define a solution. Students may need to research information about the context, the physical geography of the space, the scientific processes present, the mathematical content, contributing factors or possible similar solutions. Information is gathered from a variety of sources including reference materials, technology, experts in the field and other classmates. Not all of the information available is significant to the problem at hand. Students need to decide when and why to include or dismiss information. Students enter into the problem to gather information through measuring, observing and analysing. They evaluate the information and make decisions about its relevance to the solution and the reliability of the source.

To be successful at this stage, a student must maintain a balance of innovative personal strategies, known strategies, and formal processes and algorithms. Considering the successes and failures they have experienced in the past will support students in justifying their selection of a strategy, algorithm, process or operation. The learners also identify information that might be helpful in their process, but that is not yet known to them. For example, a student may realize that the desired solution requires him or her to find the area of a triangle. The student is aware of the concept of area and is able to apply the formula for finding the area of a rectangle. The student is able to develop a strategy for determining the area of the triangle by counting but knows that a formula must exist and that the formula will provide him or her with the most efficient path to the solution. The student then sets out to find a source and learn to use the formula to calculate the area of a triangle. Sources might include classmates, textbooks, internet resources or the teacher.

Deciding what information is needed to solve a problem is a challenge. Often the first picture of a problem does not reveal all of its complexities. Learning about a situation brings new questions to the surface that can alter perceptions and objectives. In a traditional word problem found in a textbook or on a standardized test, the research stage is absent. The possibilities and boundaries have already been set. This eliminates the need for students to identify the variables in a problem. It also removes the possibility that outside factors will influence the solution. Managing the impact of environmental, health, economic, social, and cultural factors expands a student's awareness of the world around them. Time is given to fully develop the question.

Expeditionary Learning affords students the opportunity to judge the effectiveness of a research approach by using as many of those methods as possible while generating data through experimentation and observation in the field.

As noted earlier, the desired skills of an engineer and those of a learner are transferable. Authentic questions and problems often cross the boundaries between school disciplines. For example, a question related to carbon dioxide emissions and fuel efficiency connects with science topics around environmental impacts and to social studies topics connected to populations, natural resources, markets and transportation. As another example, a question about the cost of food connects with science and social studies topics related to the production of food, and to health and physical education topics related to healthy living and fitness. A math problem about fractions in music connects to the arts curriculum, to science topics related to sound and to social studies topics related to the role of music in culture. The openness of the questions student pose in Expeditionary Learning create a platform for interdisciplinary learning.

Develop. This planning stage invites learners to suggest multiple strategies and processes without judgement. Innovative and creative strategies uniquely designed for the specific problem are welcomed as are strategies and processes that have been successfully used in the past for questions that have similar features. Each proposed strategy and process is evaluated in terms of its potential success. A plan for deepening understanding and acquiring new knowledge is made. Appropriate tools are assessed and chosen.

This plan includes a detailed list of the data to be collected such as measurements of length, time, amount, as well as the formulas to try and the steps to follow. The plan includes a list of tools, such as measuring tapes, clipboards, paper and a pencil as well as a short description of how information will be gathered. Collaborative student teams plan for the active involvement of all members by sharing their own strengths and goals. Teams also sketch a timeline, identifying benchmarks to complete and likely due dates. The final plan is checked against the initial solution criteria to ensure that it will lead to the solution.

Choices in the planning stage are intended to empower students. Setting the direction for one's learning cultivates confidence and persistence. Plans remain 'in development' throughout the process and are constantly re-evaluated and adjusted. Adjustments are not considered mistakes or errors. Instead, they are seen as incredibly useful in zeroing in on the best possible solution. Getting stuck, being puzzled and doubling back are all useful trials that will, at some point, offer greater clarity and confidence. Developing the plan is a difficult stage as it is complex, with multiple viewpoints coming together, and an extensive list of possibilities to consider. This stage is also important because an incomplete plan can leave the team at a standstill without key information needed to reach their solution.

The plan also includes a description of how each member of the team will contribute to the learning of the group as well as work toward his or her personal learning goals. Strong communication skills and a welcome team environment allow the team to arrive at a mutual understanding of what each member has in mind. The positives and negatives of each plan are discussed until the team agrees. A team may settle on one plan or agree to try several approaches and compare the results.

Consider the following example. A set of stands had recently been built at our local football field. As a class, we walked over to the field to explore this new addition. While there, the students wondered how many fans could sit in the stands. In teams, students turned their initial wondering into preliminary plans. The first team organized themselves as follows:

Student 1 directed the gathering of information and recorded the information for the team.

Student 2 counted the total number of benches and shared the information with Student 1, so that it was recorded.

Student 3 measured the length of 5 benches to verify that they were the same length and shared the measurements with Student 1 to be recorded.

Student 2 then sat on the bench so that Student 3 could measure the length of bench needed for one person to sit.

Student 3 then sat on the bench so that Student 2 could measure the length of the bench he or she occupied.

In this case, Student 2 was working on an Individualized Program and had a math goal of counting by 1's to 100. A Hundred Chart was offered to assist the student in keeping track of the benches. Finding the total number of benches was essential information for this problem. Student 2 was making a meaningful contribution to the success of the team while working towards the IEP goal.

Another team chose to implement a different plan, as follows:

Student 1 directed the gathering and recording the information.

Students 2 and 3 sat side by side starting at the edge of one bench.

Student 2 then moved to the other side of Student 3, who then moves to the other side of Student 2 and so on.

This way, they counted the number of people who can comfortably fit on a bench. The process was then repeated on 4 other benches to be sure that the benches would hold the same number of people. The total number of benches was then counted so that the number of fans per bench could be multiplied by the total number of benches.

The ideal plan is flexible enough to provide a starting point and flexible enough to be adjusted in the moment to maximize learning.

Create and Construct. At this stage, learners are encouraged to experiment with new strategies and processes. The plan is adjusted as needed and includes creative and innovative ideas that arise spontaneously during the process. Consequently, careful attention is given to the development of persistence during this stage. While errors can be frustrating, time consuming and overwhelming, it is important create a habit of dealing with errors as opportunities. The realization that a path or strategy pursued is not a correct one, further supports the students' justification for selecting a more efficient and more effective path. A mistake is an invitation to examine why, retracing steps until the last truth is found and then move forward again with more confidence. Mistakes, like failures, promote success only if students take the time to review, analyze and adjust (Lang, 2012, p.108). To welcome and anticipate errors, the teacher continuously monitors the level of engagement and satisfaction in the students and adjusts the length and depth of the problem as needed. Discoveries and innovations made by wiggling the plan are celebrated because learning is “not about arriving somewhere expected, but about deliberately moving outside of what is known” (Zull,2011, p.67).

The create and construct stage is ideal for individual, small group and full class mini-lessons. While observing students working through calculations, misunderstandings and misconceptions come to light. It is at this time that students make their understanding of, and ability with, mathematical concepts visible. Selecting the right moment to intervene, while still giving enough time for productive struggle, means that teachers need to know their learners well. Meaningful practice is incorporated to emphasize thinking and connections over memorized steps or routine calculations. A high level of support is given to students as they learn to persist and stick with a problem until a solution is reached.

Connect. Understanding can be imagined as a complex web of ties between ideas. Highlighting these connections with learners allows them to see that what we study in school does not come neatly packaged into subject area binders. A key element in understanding content is one’s ability to connect it to experiences. Strong learners demonstrate multidisciplinary thinking, incorporating ideas from mathematics, the humanities, language and science, and the ability to transfer knowledge from one situation to another. Mason and Spence (1999) argue that traditional school lessons compartmentalize the subjects creating breaks between concepts rather than highlighting the links (p.141). It is therefore vital that what is learned is not tied to a single, specific area of content. Making connections and transferring knowledge involves adopting new perspectives and seeing situations through new eyes. Feinstein (2010) finds that “people selectively integrate scientific ideas with other sources of meaning, connecting those ideas with their lived experience to draw conclusions and make decisions that are personally and socially meaningful" (p.180). Expeditionary Learning encourages these connections.

Reason. Reasoning involves synthesizing new and old ideas, refining thinking, deciding what is important and what is not, putting all the ideas together in a way that makes sense to the learner and in a way that can be shared with others as proof of new discovery. The ability to reason, to develop a "line of thought adopted to produce assertions and reach conclusions" (Boesen, Lithner, & Palm, 2010, p.92) develops through appropriate experiences. Exposure to positive arguments and productive conflict is energizing. As students learn to reason, they examine patterns in solutions. They construct relationships between representations and the lived problem scenario. Students accept or reject the solution. As mathematicians, historians or astronomers, students "reflect on the context in which the problem arose, to decide if something unexpected has arisen, to raise further questions, or in some other way to enrich or extend knowledge" (Watson, 2008, p.3). They construct an argument based on past experiences and new learning. They search for counter-examples. Support for discussions and sharing while reasoning, in workshops, mini-lessons and field work, is provided by Johnson and Johnson (1989) who found that "more frequent discovery and development of high-quality cognitive reasoning strategies occurs in cooperative environments than in competitive or individualistic situations" (p. 848).

In addition, a fundamental aspect of thinking like a learner occurs when students engage in the practice of explaining to themselves and others why a discovery is true or how widely a new theory applies. In order for a teacher to understand a student's thinking, when assessing his or her reasoning, the teacher needs assessment tasks that reveal not only what factual knowledge the student knows, but also how the student connects that information to other ideas, and when and how he or she uses it (Schoenfeld, 1992). Schoenfeld goes on to explain that when assessing a student's decision making and reasoning, the teacher needs to know what options the student had available. That is, did a student fail to pursue particular options because he or she overlooked using them, or was the student completely unaware those solutions existed? He argues that this type of assessment of students' reasoning is important for understanding their misconceptions and misunderstandings.

Questioning by the teacher and peers should clarify for the learner that imitation is not the same as reason. A student who imitates a process illustrated by the teacher or copied from a peer hasn't necessarily shown that his or her decisions are the best choices for the given situation. A student's ability to justify his or her selection of a strategy or a formula, defend his or her choice of variables, respond to arguments against his or her thinking, draw on examples and counter examples as proofs, and explain what he or she learned when pursuing a wrong path is of great importance.

Communicate. Each of us comes with our own lens through which we view the world. Our interpretation of events is shaped by our past. Communicating the experience to others is vital to build a common understanding and a shared solution. Learning is a process of social construction and so requires that, as learners, we share what we have learned so that we can all learn together.

A focus on communication skills serves a number of purposes. First, effective communication is required as students interact and collaborate with peers during the learning process. In the planning stage, students generate ideas for the active involvement of all members of their group. In doing so, they commit to sharing their strengths and drawing out the strengths of others. Each student is responsible for making meaningful contributions to discussions aimed at shared understandings. They learn to present new ideas and to respond to new ideas presented by their classmates. All students make an effort to understand each other's strategies and solutions. They learn to respectfully express opinions and to consider the opinions of others. Students demonstrate an understanding of their responsibility to contribute to the shared learning of the group. A learning community that values the diverse contributions of all its members fosters confidence.

Second, ideas, insights and experiences are strongest when they are shared. Students need opportunities to communicate solutions to the group and to audiences beyond their classroom. The audience adds an extra challenge as students need to select appropriate language, symbols, models and presentation formats that will convey their solution without sacrificing the integrity of the math. A mathematical solution includes justifications, reasoning, correct units, labelled diagrams, tables, charts and other representational tools. Formal mathematical terminology and standard units help all audiences come to a common understanding.

Reflect. During Expeditionary Learning, students and teachers are constantly encouraged to evaluate their learning, processes and strategies. During an expedition or while in a workshop or mini-lesson setting, students are encouraged to make adjustments to their plans as new discoveries lead them to better understandings. Growth is evident when ineffective strategies are replaced with more efficient and accurate ones. Reasons for doing so should be explored. Through conversations, presentations, products and actions students can share what they have learned and how they did so. Portfolios and learning stories are also used to promote self-reflection and goal setting. The purpose of reflection is to commit learners to ongoing growth, to honour their achievements and to share their learning with their communities. A learning story is a retelling of how a learner has made sense of the world and his or her experience of problem solving (Egan, 1998). A learning story is a collection of photos and examples of student learning from inquiry question to successful solution. Snapshots of students while learning are extremely valuable when teaching students to reflect. Each picture serves as a visual reminder of the experience and the personal feelings they evoke. The stories are talking props that students can revisit when looking for ideas for solve a new problem and when asked to talk about their learning.

A learning story serves many purposes. First, it is a student's own re-telling of events through which he or she has learned new techniques, skills or content. The process is highlighted so that steps can be revisited and analyzed in relation to the next activity. The story belongs to the student and so represents his or her autonomy and ownership of his or her learning. The learning story is the student's own perspective on his or her learning, what helped him or her to learn, what did not help, what actions he or she took and why. Consequently, students and teachers are able to look back and revisit mistakes with the confidence of knowing that they now have a successful solution. In other words, they can identify missteps, misunderstandings and misconceptions and discuss why they were unsuccessful. Perhaps most importantly it is a visual reminder of commitment to their learning community. No one has gone about their field work, mini lessons and workshops alone. All learners have contributed to the shared knowledge of the group, just as all have all enjoyed personal successes. Fieldwork, mini lessons and workshop create an interwoven fabric of mathematical knowledge, skill, competencies and strategies. It is difficult to separate these experiences into strands and individual outcomes. A cyclic curriculum in which outcomes are revisited and strengthened throughout the year makes it challenging to assess stand-alone outcomes. Yet the students' successful achievement of these outcomes is important for reporting, planning for learning and for the students themselves. To start the year, students spend time previewing the curricular learning goals in their outcome portfolios.

Outcome portfolios are tailored for some students as needed. The portfolios are made accessible to students for the entire year and students can access them at all times. At regular intervals, collected snapshots are passed on to the students to be included in their portfolios along with a caption describing what they are doing, what they are learning and how they are feeling about the process. These pictures and captions are very telling as to the comfort level learners have developed around content. Regular review of the portfolios offers key insights for planning and targeted mini lessons.

No two stories will be the same. The unique starting points, initial questions, plans developed, and actions taken will illustrate the interconnectedness of students' learning, the variability in their starting points, their growth, and their meaningful contributions to the shared learning of the group. A class set of Learning Stories written after a class visit to a local music store will start with the students' investigating a set of plastic bottles filled to various levels with water. Students include their reflections of how they interpreted and compared these bottles using fractions. Some stories will have pictures of groups of students measuring the liquid and rearranging the bottles. Other stories will have groups of students dividing the fullest bottles into smaller, equal parts to match the others. Other stories will have individual students comparing the bottles using terms such as more than, less than and equal. The next photos and captions show student groups naming the bottles using fractional values. Some stories will show students naming the fractions as more than or less than one half, others will show students naming the fractions using a common denominator while still other stories will show students naming the fraction by comparing decimal values. The stories will go on to show how each learner developed further questions related to math and music such as how does the size of a drum affect the sound it makes? Where are the notes on a trombone? How do the notes change when the time signature changes? As each story continues it focuses on the question posed by the learner and their understanding of how fractions are important to music. Because the initial question was open each story branches out towards meaningful learning for each student. Each personal story contributes to the collective story of the class. The story provides a platform on with teachers and students can discuss the growth, from each learner's individual starting point. Mistakes are included as loops and detours when learners pursued a path that did not take them where they had originally planned. Scaffolds, such as organization frames, planning templates, and checklists are included in the story. The strategies and processes of the learner, such as using an area model to compare fractions, finding a common denominator or converting to a decimal value, are to be retold with as much detail as possible. These strategies and processes serve as evidence of reasoning and thinking that will support the teacher in assessment and support the learner next time that they find themselves challenged by a new problem. Throughout the learning experience all students would have been part of a mini-lesson. Snap shots and work samples from the mini-lessons are also to be included in the Learning Story. These paths tell how learners deepened their understanding of why another path was needed. Photos and work samples are included in the story as reference points. This individual reflection will allow us to identify the differentiated content and processes used by each individual student. The depth to which each story conveys that the learner has found meaning and purpose in the experience will let us judge the inclusive nature of the expedition.

Kimber and Wyatt-Smith (2009) found that storytelling offers a platform for problem solving and critical thinking. This focus on the process of learning is meant to emphasize the belief that the "the most important attitude that can be formed is that of the desire to go on learning" (Dewey, 1938, p.48).

The intuition of engineers describes a disposition, an attitude and a way of living that is personal and dependent upon the places they live and the interactions they have with their environments. To adopt the engineers' method of heuristics, educators need to be able to read learning situations and respond accordingly. It is not enough to have a recipe to follow. In expeditionary learning, students are encouraged to develop their own sense of which strategy will be most successful depending upon where they find themselves in their inquiry process. The competencies described in Expeditionary Learning as explore, research, develop, create, construct, connect, reason, share and reflect are universal and transferable to other learning situation.

Coming to know oneself as a learner is to develop an intuitive sense. Knowing which strategy to choose and where to go next requires a confidence that grows through experience. Teachers can support the development of intuition and of knowing how to act in the moment by offering choices in how students might approach a problem. This requires opportunities for students to create meaning by finding themselves stuck, in just over their heads, so that new discoveries are within reach but not without struggle. Such an approach is necessary for students to develop the habit of thinking deeply and for each student to exploit the individual learning trajectory through which he or she finds his or her own personal path as a learner.

Best Change

Best change is intended to unleash in each learner a passion to create positive change in the world through innovation and invention as a result of working collaboratively with others. Within the goal of best change, I include the use of authentic problems. I outline how creativity, collaboration and belonging can be grown through workshops. To do that, I build upon research and scholarship from problem-based learning, 21st century learning, UDL and place-based learning.

Best Change in Engineering. The work of an engineer is to make the world a better place. Through design, engineers envision structures, places, machines, systems and processes that will be valued in society. Engineers are responsible for great inventions and innovations as well as the best fit solution in an emergency. To meet these demands, engineers work collaboratively with a variety of experts immersed in “an atmosphere of optimistic enthusiasm” (Ferguson, 1994, p.173).

Engineers specialize in a variety of fields (e.g., electrical, chemical, civil, mechanical, etc.). They bring different sets of experiences and strengths that shape their views and frame their thinking. As different as they are, the process of engineering is common to them all.

The shared objective of the best change and the shared methodology of engineering combine to provide a common ground for working together. Developing the best possible outcome involves a “process which engages different individuals, each with different ways of seeing the object of design… individuals who in collaboration, one with another, must work together” (Bucciarelli, 2003, p.9). In collaboration, individuals add to the collective picture by sharing their own distinctive views. Each view is defined by the beliefs and experiences of a unique individual. On the surface, these differences may appear to be a deficit, inhibiting cooperation and understanding among teammates. However, when viewed as wealth, this multitude of perspectives offers the best picture of reality. To arrive at the best possible outcome, engineers expect a give and take, a negotiation of the best each perspective has to offer.

Design team members must appreciate how they know each other. Knowing together is strongest when all team members’ voices are heard. What is known by an individual is stagnant.

It is unstable in that reality changes so quickly that knowledge is soon out of date. It is context-dependent in that what is true in one location, with specific elements, may not be true in the next. Knowing "is a construct that emerges between individuals and binds them together with ideas and places" (Mason & Spence, 1999, p.141). It is "not a property of a person, but of a person in a situation" (Mason & Spence, 1999, p.141) with others. Petroski (2012), like Buccarelli (2003), notes the unique perspective of each member of the design team. They view knowledge as social and dynamic, overflowing from each member into a collective web of knowing that connects the team. The "knower, knowledge, and the phenomenon known can't be separated" (Davis, Sumara, & Luce-Kapler, 2008, p.8) from one another. What is known exists in the in-between.

Best Change in Learning. Problems and topics that students examine in Canadian schools should be original, meaningful and authentic. Solving problems that only the teacher poses, for which the one right answer is already known, does little to stimulate an interest in contributing to the betterment of the world. As described in problem-based learning, the context provides the story to share, a common reference point. The need for a solution stems not only from interest but also from the investment in the context and the communities that it involves. Learners "should be helped to see the purpose in what they are asked to learn" (McTighe & Seif, 2010) and its transferability to other applications.

Examples of problems that facilitate a deeper understanding of our worlds and a commitment to looking out for each other include designing a universally accessible entrance to a second floor dance studio, evaluating the cost of food and the need for food banks in our city, monitoring water use and creating a water management plan for the city, or evaluating global trade patterns and the need for fair trade.

Best Change in Expeditionary Learning. Within any Expeditionary Learning context, the specific curricular outcomes are identified and become the focus of mini-lessons and workshops. Connections are highlighted between the content and the current context, between past experiences in different contexts, and new contexts yet to be explored. A study conducted by Hmelo and Lin (2000) resulted in five major findings associated with problem-based learning and its place as a 21st century, student centered, inclusive pedagogy. First, Hmelo and Lin found that "students took much of the responsibility for their own learning" (p. 229). Second, students responded to problems that were just beyond their ability by recognizing their "own personally relevant learning needs" and by planning for new learning. Third, learners "develop the skills and goals orientation that they need to be mindful, self-directed learners" (p. 230). Fourth, as learners use and develop their "information seeking skills" they are able to "construct conceptual knowledge as well as procedures for solving problems" (p.230). Finally, Hmelo and Lin found that learners learned to "critically evaluate the resources they have used" (p.230)

Each question posed by a student is full of potential. These initial wonderings do not come with all the variables neatly defined. The students themselves must enter into the problematic situation to fill in the blanks. In a traditional mathematics word problem, the variables are defined, and higher-level thinking is not required to identify them. Pursuing questions posed by students illustrates the interconnectedness of mathematics. Oftentimes, students are familiar with a unit model in which concepts are taught in isolation: fractions in September, geometry in December, and graphing in June. This disconnected approach encourages the belief that math is a collection of facts to be memorized and discourages a meaningful connection between school math and the math we live. As an expedition unfolds, students do not restrict themselves to a topic that fits into one unit binder. Instead, they search for answers from all strands of the math curriculum and other related curricula. Consequently, it becomes evident that mathematical concepts are tied together in a complex, yet beautiful web strung across the landscape of the world.

At this time, there are many problems in the world that students could investigate as part of their required curriculum. In Transforming our World: The 2030 Agenda for Sustainable Development (United Nations, 2015), the United Nations suggests engaging students in questions relating to eliminating poverty, ending hunger and food insecurity worldwide, creating and implementing a global health care standard, managing sustainable waste systems, increasing access to reliable, clean energy for all, creating opportunities for meaningful, productive employment for all, eliminating inequality, designing safe and sustainable cities, and conserving oceans, forests and global biodiversity (Kelly, 2016, p.xiii). Local questions such as developing a community garden for a food bank or designing an accessible entrance to a second-floor classroom are equally valuable. Each of these questions opens an infinite number of responses. Each topic will need to be considered from a variety of viewpoints. Any solution will be a negotiated give and take between learners.

Given that these are authentic problems, any solution demands an innovative approach. Students will need to combine pieces of past experiences and successes with novel ideas to create something new. Problems that students take on should be relevant to their lives and have authentic implication for their communities. As recommended in problem-based learning, Expeditionary Learning empowers students to actively seek out problems.

Collaboration. All learners are unique in their experiences and view the world from their own perspective. Collaborating with teammates whose experiences are different and who have expertise in other areas to share is essential for success in the 21st century (Dede, 2010; Wagner & Dintersmith, 2015). Individuals are empowered when personal intelligence is pooled and steered towards a common goal. The resultant collective intelligence maximizes the probability of success.

The need to develop skills in communication and collaboration is at an all-time high. In *21st Century Skills*, Dede (2010) notes that "the degree of importance for collaborative capacity is growing now that work in knowledge-based economies is increasingly accomplished by teams with complementary expertise and roles" (p. 52) rather than by individuals working on their own. In educational settings, communication is developed through collaborative learning groups, discussions, writing and reading in all content areas, producing videos, managing websites and learning blogs as well as through the arts. For the 21st century learner, communication is not limited to the spaces and people within their classroom walls. Authentic audiences provide a purpose and a vision for why their work is important. Through technology, students may find themselves collaborating with peers from all over the world.

Creativity. Creativity, as a current, should flow beyond the arts and into all subject areas. In Deep Creativity (2018a) Shamas defines creativity as both freshness and transcendence. This definition describes creativity not only as a product, but as actions and processes as well. Freshness "is a subjective determination made by the artist based on the intensity of what is being experienced at this moment." (p.18). A learner can consider their work as creative if they are exploring something new, making new and novel connections, or identifying a possible solution that they had not previously considered. Transcendence is defined as "moving beyond your own limitations" (p. 19). Shamas highlights that creativity occurs throughout the learning process

> "As you engage in the creative process, you may get a sense of expanding your knowledge base, breaking through false or restrictive assumptions, uncovering new thought processes,

> restructuring your worldview, solving a mystery, or discovering new abilities. The creative process tends to stretch you in some way, pushing you in terms of your ability to respond to circumstances that are far beyond the ordinary. (p.19)

With these definitions in mind, learning can be considered a creative process. For the learner, making discoveries and connections for themselves is creating knowledge and "yields the same qualities of freshness and transcendence associated with creativity" (Shamas, 2018b, p.132).

Wagner recommends organizing the learning culture of a school around "thoughtful risk-taking, trial and error, creating, intrinsic motivation: play, passion, and purpose" (Wagner, 2012, p.200). With the goal of creating rather than consuming knowledge, schools must offer rich learning environments that promote a constructivist approach to teaching. Constructivists Rousseau (Dent, 2005), Pestalozzi (Anderson, 1970), Montessori (Renzulli, 1982), and Dewey (Dewey, 1938), all argued that through play in environments that are varied, rich in color, textures and sounds, learners will construct authentic understandings of their world for themselves. They encouraged first hand activities such as exploring, building, taking apart, and designing. The role of the educator in these environments is to prepare situations where questions sprout from the experiences of students and then meet them at the edge of their current state of understanding (Vygotsky, 1978). Students share what has peeked their curiosity or has them puzzled and then, together with teachers, set out to investigate. In an innovative culture, students enjoy a "serendipitous climate, which makes room for creativity, coincidence, and playfulness" (Boss, 2012, p.83).

Schools as creativity-intense environments, for both teachers and students, will spark the development of fresh thinking, original ideas and positive change.

The UDL model includes the use of essential questions and essential understandings (CAST, 2011). Essential understandings are more holistic learning targets that encompass knowing, understanding and doing in relation to a topic. These understandings remain relevant beyond the scope of the lesson and are applicable to a number of situations. Essential questions welcome responses rather than answers. Responses are open to an infinite range of possibilities. Several specific outcomes often are combined with learning behaviours to create a target for which many entry points and many successful outcomes exist. A common outcome in early mathematics is counting. Written as an outcome it might read “The student can count forwards to 20.” As an essential understanding, counting may be written as “Number symbols represent specific amounts” and “Patterns in our number system are based on 10”. Both these understandings will carry the learner forward providing a base for learning about larger numbers, decimals and fractions. When written as an essential question “How is ten an important number in our number system?” Students may jump in at a number of levels and carry their responses in any number of directions. Essential understandings and essential questions can be embedded into authentic questions posed by students. For example, when my students wondered about the amount of carbon dioxide being emitted by cars on a busy street near our school, we had to deepen our understanding and skills related to exponents. The volume of carbon dioxide is expressed as cubed units (x^3) and the total amount is so large that it would need to be expressed using the short hand system of scientific notation. An essential understanding would be that exponents are used to represent very large and very small numbers. While the essential question might be “how can very large and very small numbers be represented?”

Workshops. A workshop is a lively exchange of ideas. The purpose of the workshop varies according to the immediate needs of the students. Workshops may be discussions, planning sessions, or times to calculate, build models, experiment or share results. Students may work in small collaborative groups or individually. It is not intended that all students take part in the same activity at the same time. Workshops that follow fieldwork are spirited debates where cooperation and argument mingle. Students communicate their learning and solutions using technical language, symbols and models. These workshops encourage exchanges between learners where new discoveries can be verified, and new understandings shared. When a team makes a new discovery, such as a strategy, connection or conclusion that would be beneficial for others to know, educators send other students over to consult or bring everyone together for a few minutes to learn from that team. Through sharing what they have tried and learned, students become learning resources for each other.

For the teacher, the workshop is a time for observation and documentation. Students will struggle with misconceptions and errors. Productive struggle can be beneficial, but frustration is discouraging. Ideally teachers balance having students make their own way with targeted interventions as needed. Carefully monitoring the levels of engagement and student success will indicate the best times to conference with a learner or group of learners.

Workshops provide extensive opportunities for social inclusion. As defined by Katz (2013), social inclusion means that “all students have opportunities to be part of the school community and to learn alongside their peers” (p.11). Furthermore, to be socially included means to have a “sense of belonging and connectedness” (Katz, 2013, p.11). Because workshops are fluid and grow from the inquiry questions and plans of so many students, they allow for students to branch out in many different ways. According to Tomlinson and Imbeau (2010), being part of a community “meets a fundamental human need for acceptance, belonging, affinity, respect, and caring” (p.84). The workshop is significantly different from a traditional classroom in that student interactions are constant. Students work together, teams meet with other teams, students consult with the teacher, experts are welcomed into the class, all in an effort to maximize positive interactions. This model more closely resembles that of a community, where each person has much to offer and much to learn from others. The workshop is a busy place. Students who are making use of additional supports such as an educational assistant, a visual dictionary or speech to text writing technology blend in. These supports are available to be accessed by all, not just by the few for whom they were originally designed.

When planning for a workshop, teachers must consider the needs of all learners from all their potential starting points. A concept is mapped using backward design (Wiggins & McTighe, 2005) isolating pieces of the puzzle that students should know or be able to do in order to make sense of the new material. It is of great importance to remember that learning is not linear. By creating a hypothetical learning trajectory, teachers can anticipate areas where students will get stuck and need support. Thus, the scaffolds, materials, mini lessons and feedback can be pre-planned to some extent.

During the workshop the teacher is observing and interacting with students as needed. Moving between students and student groups creates a space for immediate assessment and feedback. Short conferences between students and teachers are used to clarify the learner's thinking for the learner, clarify the learner's thinking for the teacher, reveal where a misconception or misunderstanding is rooted, reveal where solid understandings have been made, draw out the learner's reasoning to better understand their process, highlight connections for the learner to prior knowledge, and to determine the next steps for learning together. Conferencing with small groups and individual students in the moment, when the learning is happening, enables the teacher to personalize learning through differentiated materials, topics and supports. These interventions are seamlessly interwoven into the flow of the workshop. Through focused observations, workshops present the ideal conditions to uncover how students are thinking. Aspiring to create the best change in the understanding and knowledge of the students fits the definition by Whitin and Cox (2003), that “mathematicians are inventors. They invent tools to solve problems and to make life easier” (p.55).

Uncertainty

One goal of Expeditionary Learning is to develop in each learner the confidence to let curiosity take over, to explore, to pose authentic questions, to investigate additional factors that will influence the solution, and to turn ambiguity into determination and failure into invention.

Uncertainty in Engineering. Koen (1985) captures the poorly understood and uncertain nature of engineering in writing that “engineering has no hint of the absolute, the deterministic, the guaranteed, the true. Instead it fairly reeks of the uncertain, the provisional and the doubtful” (p. 23). Each problem encountered by an engineer has its own set of context dependent characteristics and outside factors. Each of these elements can impact the process and solution in any number of ways. They may derail the perfect plan with additional industry requirements, reroute a process as a result of time or entirely roadblock a project due to budget restraints. The engineering process is creative, inventive and fluid so that the engineer can respond to the unexpected. Each step is determined by carefully considering the results of the last step, the current conditions and demands.

Confidence and comfort with not knowing is cultivated through experiences where uncertainty exists. Both successful and unsuccessful experiences deepen understanding. Failures, both small scale and catastrophic, are instructive. For learning to come from a failure, engineers engage in an in-depth study of what was involved. The uncertainty remains, but the thrill of discovery is so great it fuels the need to carry on.

Uncertainty in Learning. Much of a child's day at school is predictable. The routine of classes and schedules provides a structured, similar pattern to every day. The traditional classroom role of the teacher as a keeper of knowledge and student as the obedient recipient of information are well defined and understood by even the youngest children before beginning school. Often, course materials are predetermined as lessons plans, texts, assignments and evaluations are similar from year to year, class to class. Problem sets given by the teacher may not afford students a wonderfully complex question to analyze. Often these repetitive assignments ask that students repeat a process illustrated by the teacher to arrive at the same answers as their classmates. The single right answer to these assignments has already been printed in the back of the book. Schools have succeeded at creating a very consistent, reliable model. They have also succeeded in creating students who are compliant memorizers. Why teach students to solve problems for which the answer is already known?

The certainty experienced in schools does not match the unpredictable nature of the world beyond our classrooms. When students leave the school, they are met with ill-defined and messy situations. The skills needed to be successful here are not at the same as those of the artificial, traditional classroom. In the real world, students need to be able to recognize a problematic situation, explore, wonder and question. They need to be able to develop a plan for learning more, noting what they know already and what they need to know. They need to create possible solutions, run tests, experiment, adjust and recreate models, build prototypes and analyze procedures. Each decision and action need to be justified as the best possible choice at that time. To do so, students need to feel comfortable with uncertainty. They need confidence in carving their own path towards solutions. This confidence and comfort comes from experience. This zone is described by Vygotsky (1978) as the zone of proximal development, "the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance, or in collaboration with more capable peers" (Vygotsky, 1978, p. 86). Learning is most likely when students are met at the edge with a challenge that will allow them to discover something new, something foreign, something exciting.

As noted in the literature review, an inquiry approach welcomes uncertainty into the classroom. Whitin and Cox use inquiry to create opportunities for students to experience "some of the principle activities of mathematicians: keep records, pose their own problems, assume a skeptical stance, go beyond the data, seek patterns, and develop theories" (p.5). However, knowing how to look around yourself, observe your world, ask meaningful questions, and pursue new understandings requires that we become comfortable with uncertainty, beyond the classroom, in the field.

Uncertainty in Expeditionary Learning. At the heart of Expeditionary Learning is fieldwork.

Fieldwork. It is during these expeditions into the community that students have the opportunity to consciously assume the role of mathematician. Where do mathematicians do their work? In the hardware store, at the zoo, in the grocery store, at the park, in a sports arena, at a construction site and in all places where questions and problems arise. While on site, students become active investigators using the research tools and techniques used by professionals in the field. Fieldwork is inspiring for learners as it places them directly into a problem. They are no longer outsiders reading about a problem or spectators watching a lesson, but the main characters in a learning adventure. Students interact with and respond to the elements of the situation. Reacting in the field is consistent with Mason and Spence's (1999) view of knowing as knowing how to act in the moment, when the information is immediately required. This knowing how to act is the culmination of knowing why, knowing how, knowing the technique and knowing the process. This knowing is found within interactions between individuals and the elements, it is alive and evolving. As an essential component of Expeditionary Learning, fieldwork is intended to be risky and rewarding. Students and teachers find themselves in unfamiliar environments that "push people out of their comfort zones and force them to see themselves differently start thinking differently about what they are capable of doing" (Lang, 2012, p.75). This risk provides the uncertainty that cannot be matched in the classroom. Just as professionals dealing with unknowns develop an intuitive sense, fieldwork develops intuition in young learners so that they know how to respond when faced with a problematic situation. The lived experience in the field lets students construct for themselves a more complete understanding of the world around them. Examples of fieldwork might include:

- launching student made rockets to test how the angle of elevation at the launch affects the distance travelled,
- launching student made rockets to test how the area of the wings affects flight time,
- interviewing the planetarium guides to learn about representing large numbers such as those numbers used to communicate distances in space and scientific notation,
- gathering measurements at a skateboard park to test how the steepness of a slope affects the speed of the skateboard, and to build a scale model of our favourite features such as "the bowl" or "the wave",

- using building plans to determine the amount of materials and cost to build a storage shed at the hardware store,
- playing a round of mini golf to determine how an understanding of angles can make you a better golfer,
- measuring enclosures at the zoo to compare these habitats with the natural range of animals to support an argument for or against zoos,
- measuring drum sets at the music store to find out how the volume of a drum affects the sound it produces,
- building to scale models of statues at an art gallery to investigate fractions, ratios and proportions,
- comparing prices and amounts at the grocery store to determine the best buys,
- measuring the speed (rate) of the skaters and the speed of slap shot at the hockey rink,
- investigating changes in populations and population density at the forest,
- measuring rate of growth at the community garden,
- using similar triangles to measure height at the football field,
- counting cars to determine the rate of CO_2 emissions,
- touring the neighbourhood looking for ramps, measuring rise and run, to determine how steepness affects safety, or
- studying populations of animals and plants in a local forested area to prepare arguments for and against a new housing development.

Developing intuition to navigate uncertainty requires educators to create authentic learning opportunities beyond their classrooms. Fieldwork offers students the chance to take on the role of the engineer in the kinds of environments where engineers do their learning and work.

While on site, students are active investigators, applying the research tools, techniques of inquiry and standards of presentation used by professionals in the field. Being in the field pushes both the teacher and the learner to try new things that they may not have had the opportunity to try elsewhere. Roberts (2012) insists that "children must actively sense and feel their immediate surroundings and it is the educator's job to provide the freedom necessary for the child to follow his or her natural inclinations" (p.33).

As mentioned earlier, fieldwork is intended to be risky and rewarding. A key element "of being a mathematical inquirer is traveling paths that are not well marked" (Whitin & Cox, 2003, p.37). Students and teachers find themselves in unfamiliar environments that "push people out of their comfort zones and force them to see themselves differently" (Lang, 2012, p.75).

As recommended in place-based learning, fieldwork is a unique and successful experience that fosters community, belonging, and the social-emotional well-being of the group. Sharing an adventure beyond the classroom requires trust: trust in the teacher, trust in the group and trust in oneself. When risks are taken in unfamiliar places, risks become common in known places like the classroom. Sharing the adventure creates a community of learners by learning together. Through experiences out in the communities that surround our schools, students and teachers build stories to share that tell of their belonging. This on location learning lets us know that what is sketched on the board or seen in a book can be experienced instead as a part of a living system. An expeditionary math classroom has no walls, no boundaries and no limits: all spaces are learning places.

Productive Failure. The ability to navigate ambiguity and uncertainty requires a balance of content knowledge and the capacity to develop creative and innovative ideas. As students learn to construct their own problem strategies, they will inevitably encounter "desirable difficulties" (Schmidt & Bjork, 1992, p.207). These difficulties, misunderstandings, misconceptions and errors, generate an even greater platform for learning. Van Lehn, Siler, Murray, Yamauchi and Baggett (2003) name this approach "impasse-driven learning" in that students are presented with an ill-defined problem, begin to work towards a solution, and then find themselves at an impasse. By confronting difficulty students must examine their process by justifying and verifying each decision. This revision can offer valuable insight into thinking and must be encouraged. Once a student has worked through a difficulty or an impasse, his or her new understanding is used to further support his or her reasoning towards a more viable solution.

With uncertainty comes failure. Applying Petroski's (2012) statement that "scientific understanding didn't progress by looking for truth; it did so by looking for mistakes" (p.87) to education suggests that failures, and their examination, should play a prominent role in developing learning behaviours. Productive failure, as described by Kapur and Bielaczyc (2011), sees students wrestle with challenging problems for which they do not yet have the skill level to complete. Consequently, they reach a point at which scaffolding, facilitation and instruction can be introduced to guide them further. Thus, productive failure "engages students in solving problems requiring concepts they have yet to learn, followed by consolidation and instruction on the targeted concept" (Kapur, 2016, p. 289). Through this combination, learners benefit from a balance of self-directed problem solving and instruction, as needed, to clarify any misconceptions. Schmidt and Bjork (1992) and Van Lehn, Siler, Murray, Yamauchi and Baggett (2003) have studied the positive effect of "desirable difficulties" (p.207) and "impasse driven learning" (p. 209). In both studies, the researchers found that learning was enhanced when students first found themselves unsure and struggling with a problem and then received guidance or coaching from a teacher. The moment of intervention on the part of the teacher in these situations was critical. Intervening too soon inhibited the students' ability to comb through all their own ideas while intervening too late lead to frustration and disengagement. Kapur and Toh (2013) recommend delaying the intervention as long as possible until the learner "makes a demonstrable error or is stuck" (p. 344) as it is much more conducive to learning if that scaffolding is delayed "until the students reaches an impasse – a form of failure – and is subsequently unable to generate an adequate way forward" (p. 344). Selecting the initial problem to present to students is also of great importance. Problems should be just beyond the learner's current ability level so that they find themselves in just over their heads – balancing a genuine challenge with rich enthusiasm for new learning.

Kapur and Toh (2015) propose a system of three layers to designing productive failure learning experiences. First, educators should "create problem solving contexts that involve working on complex problems that challenge but do not frustrate" (Kapur & Toh, 2015, p. 346). In their responses, students should be encouraged to present several different strategies and representations. Second, educators should "provide opportunities for explanation and elaboration" (Kapur & Toh, 2015, p. 346) that encourage learners to extend their ideas with sketches, descriptive language, examples and models. Third, once students have come to an appropriate solution, educators should "provide opportunities to compare and contrast the affordances and constraints of failed suboptimal" (Kapur & Toh, 2015, p. 346) responses with an appropriate response. This reflection allows students and teachers to identify where their thinking was inaccurate and how they went about making new meaning. It is critical that failures are examined so that misconceptions are revealed. This new understanding should be used as evidence in support of the more appropriate responses. The importance of this reflection stage is echoed by Petroski (2012), who stated "success comes from understanding failure and acting upon this knowledge" (p.87). In Expeditionary Learning, students reflect upon these impasses as they tell their Learning Stories. Including photos in the stories lets learners see how they were stuck and how they worked through it. The fluid nature of the mini-lessons affords teachers the opportunity to let students struggle while providing the comfort that if they need support, it will be there.

Defining Variables and Unknowns. In an educational setting, educators can create a culture of wonder by encouraging students to pose their own questions. Open-ended questions, where no single right answer exists, serve as powerful vehicles for active learning and engaging students in ways not possible through traditional instruction. Questions such as, "Can two triangles have the same area but different perimeters?" or "Can we construct a new kind of packaging that eliminates excess materials?" invite "the student to deal with anomalies, to address incongruities, to conceive of all sorts of possibilities, to improvise the unheard-of" (Bucciarelli, 2003, p.94) and to witness a number of right answers.

The best adventures come from students. Students have wonderful ideas: it is up to us, the teachers, to listen. Once students come to believe that their ideas will be honoured, they will share more ideas and take new risks. Whitin and Cox (2003) point out that mathematicians are skeptical. They "question results, challenge assumptions, and wonder if conclusions are valid. Skepticism is a sign of good mathematical health because it means that learners are thinking in critical ways" (p.37).

Inquiry supports the development of analytical thinking by encouraging students to define their own problems. Refining and adjusting the original question shows that students are examining their thinking, reflecting on what is already known and what processes they will need to reach the best possible solution. Inquiry experiences aim to create students who are "fearlessly curious and hooked on learning" (Lang, 2012, p.231). Learners thrive in a culture of curiosity and adventure.

The inquiry experience lived in the field lets students construct a more complete understanding of the world around them. For example, students might be asked to use the Pythagorean Theorem to find the length of a bridge given two points on the shore. Students would need to recall the formula, assign the known variables and solve for the missing side length. However, if students were asked to consider a design for a new bridge in their city, the problem-solving process expands exponentially. Not only would they need to develop the theorem using points on shore and a right triangle, students would need to first select an appropriate site by investigating the need, traffic patterns and accessibility. Once on site, the list of variables grows to include: pedestrian traffic, cyclists, length of the bridge, materials, weight, snowfall, freeze-up of the river, ice-melt, flooding, soil stability, erosion, and aesthetics, disruptions to businesses and homes, and safety. An on-site interview with an engineer may also bring costs, construction time, design limitations, and industry standards into question. Not only do these questions provide strong links to mathematics, they also connect to science, geography, history and art curricular outcomes.

As a second example, students may be asked to complete a textbook question where they determine the slope of a line. In a traditional classroom, the students would be responsible for selecting the appropriate formula and correctly substituting the variables. However, if students were asked to design a wheelchair ramp that would make the second floor dance studio accessible for all, they would need to explore: the height of the dance studio, the slope and design of other ramps in the neighbourhood, the safety guideline for the steepness of slope used for wheelchair ramps, the angle of elevation that would result in the safest slope, how to physically measure the angle of elevation with a clinometer, how to calculate the angle when two other variables can be measured, the distance from the base of the building from which the ramp extends and whether or not this is an acceptable distance or if the ramp needs to include turns, the area of the arc needed to turn the wheelchair, how best to communicate their plans through language, calculations, scale diagrams and models, and the cost of materials and managing the budget. A much richer experience results.

The cyclic curriculum of Expeditionary Learning, in which the same content is explored through several different lived contexts, supports UDL guideline of multiple representations. For example, students might revisit variables related to measurement in the kitchen while measuring ingredients for baking, at the track while measuring distances jumped by athletes, at the hardware store while investigating building materials, at a local nature area while investigating population density and at the art gallery while considering scale. Each different representation broadens the students understanding, provides them with opportunities make adjustments in their thinking, and enhances their comprehension of the interconnectedness of school concepts and life.

Available Resources

Within the goal of available resources, I group fieldwork, mini-lessons and conferences along with a growth mindset (Dweck, 2006) and belonging. I draw on theoretical reasoning from place-based education, traditionalists, and UDL.

Available Resources in Engineering. When faced with a problem to solve or a design challenge to overcome, an engineer must evaluate a multitude of factors. Consideration is given to political interests, social and economic constraints, ethical issues, manufacturing, environmental sustainability and aesthetics. For example, initial discussions related to the construction of a new shopping mall would include questions about: location, geographic features of the area, geology, transportation needs, foot, bike and vehicle traffic patterns, accessibility, safety, time constraints, construction materials, duration of the project, costs, weather and climate, architecture of other structures close by, aesthetics, the clients vision, purpose and the political impacts of the project. In each of these areas, an engineer's thinking is divergent, branching outwards to gather as much information as possible about the factors that will influence their project. Consideration is also given to how the project might impact others. Working within the parameters is key to successful engineering. What might at first look like a series of "constraints can be a springboard for new ways of thinking and problem solving" (Boss, 2012 p.19).

Along with these visible parameters are the available personnel resources. Each member of a design team, as well as the consultants and external collaborators, brings a strength to share. Expertise in a subject area, technical understanding, and past experiences also are considered available resources. For the engineer, knowing his or her own strengths is key. Of equal importance is knowing one's limitations. Being able to identify an area for growth, and together devising a path towards knowing, further strengthens each team member.

Available Resources in Learning. In a traditional education model, students are given tasks for which all the necessary information is known to them. There are no variables to define, no measurements to gather, no decisions to be made. Students are therefore denied the opportunity to develop the analytic thinking skills needed to generate a list of limitations and possibilities as well as the critical thinking skills required to scrutinize each factor on that list to assess its potential impact. Most often "these problems are contrived, matching a procedure rather than engaging students on relevant, important, or conceptual challenges" (Wagner & Dintersmith, 2015, p.89). These types of problems create the conditions for students to believe that every problem has a readymade solution, that following the step by step procedure laid out by the teacher is what learning is all about and that if you can arrive at that one correct answer quickly, you must be really smart. For example; a problem that asks students to determine the area of a rectangle with side lengths of 3m and 5m has taken the need to define the variables away from the learner. The problem has provided all the necessary information; we are looking for area, the terms are 3m and 5m.

In More Good Questions (2010), Marian Small recommends using open questions as a strategy to meet the needs of all learners in the classroom (p.7). According to Small, a "question is open when it is framed in such a way that a variety of responses or approaches are possible" (p.7). For example;

> Draw a graph of y = 3x2 – 12x + 17. Tel What you notice. (Small, 2010, p.7)

Small (2010) recommends that when teachers are creating open questions they consider:

- "turning around a question,
- asking for similarities and differences,
- replacing a number, shape, measurement unit, and so forth with a blank,
- asking for a number sentence" (p.8)

Open questions create an opportunity for students to be the ones who define the variables and frame the problem.

Available Resources in Expeditionary Learning. A wealth of available resources can be found in the communities beyond the doors of our schools. Places, spaces, people and activities can serve as learning resources. To access these resources, students need to explore their communities. When learning reaches from the classroom into neighbourhoods, it builds a connection between theory and practice, between concept and process, and between content and context.

Developing the ability to define the constraints acting upon a problem and mesh those with possibilities takes place in the field. Roberts (2012) highlights the importance of living the learning when he notes that “thinking cannot be removed from the world in which we live” (p.51) and that problem solving and critical thinking skills “are generated from experimentation in unique times and places” (p.52). Education lived solely in the fabricated contexts of a well-managed classroom will not suffice. In *Experience and Education* Dewey (1938) recommends a system of education based on lived experiences. For convenience and control, the term experience has come to mean, to some educators, the experience of copying off the board and filling in the blanks while seated quietly at a desk with a pencil. This style of education offers students the opportunity to develop a very narrow set of skills favouring theoretical knowledge and conformity. These lessons often make use of “fake world” (Meyers, 2014) problems. Such problems claim to have connections to the real world. But, in fact, they fall far short, in that they are well defined, decontextualized, compartmentalized, incomplete and predictable. Authentic problem-solving situations are lived through Expeditionary Learning by engaging learners in authentic, non-routine inquiries in a direct encounter with the content being studied. Learners become the main characters in these problems interacting with resources, places, people and activities.

Mini-lessons and Conferences. Mathematicians are efficient. Algorithms, formal processes, and structures, once fully developed by the individual and deeply understood, can allow for more efficient work. This research stage can be used to thoroughly investigate and solidify the relationships students have discovered. As students become more skilled at posing questions, the distances from problematic situations to solutions grow. Drawing on prior knowledge shortens this route and reduces the extent of loops and detours. Connecting back to a past assignment or problem can assist students in formalizing their knowledge and prepare them to build new ideas based on what they already understand. Maintaining a Learning Story and encouraging students to return to it can be a helpful reminder.

Structuring the research with students will help them to learn to manage time and resources. Gathering essential information during fieldwork presents a different challenge; if students run short of time they may not be able to develop a solution. While researching, students identify which measurements need to be taken, what data and information is needed to define the solution, what is already known about the context, as well as what they already know about the content.

For example, a traditional written problem might ask students to find the unit price for a can of soup when 6 cans of soup are sold together for $8.25. Here the students are given the two variables, price and amount. They need to divide the total cost by the number of cans and record their answer.

As participants in an expeditionary problem, students would have to decide what they wanted to buy and what information they need to be able to determine if it is a good price. Students will first decide that they wanted to buy the soup, how much soup they want, the cost of the soup, compare the price for the 6 cans with the price for one can, identify a strategy for calculations and determine which is the better buy. Then, students will justify that the information gathered is accurate and has been collected in the best way. Students may have gathered information from a number of sources including reference materials, technology, experts in the field, and other classmates. In this soup example, students may also want to consider customer loyalty rewards or discounts, storage space, a brand preference and the purpose of the soup. In this example, the topics and concepts to be learned are 'on demand' according to the students' own questions rather than according to the next chapter in the text.

As the teacher observes students, he or she notes what aspects of the content each student is able to successfully master and which pieces are a challenge. Through questioning or in conversation with their teacher or peers, students may gain clarification; while, at other times, the correction may need to be more direct. A mini-lesson involves a small group of students for whom similar learning goals are a challenge. The lesson is targeted and short. It may include direct instruction from the teacher, the use of manipulatives or models, or it may be a demonstration problem that the teacher and students solve together. Tomlinson and McTighe (2006) note that students are better supported in their learning if teachers have identified specific targets and prepared appropriate tasks. A mini lesson is followed by practice. Practice can take the form of continued work with the manipulatives, a similar set of problems to solve (e.g., where the context stays the same, but the variables are given as different amounts), or as individual practice. This practice reviews the methods used to teach the concept, offers guided practice with supports and then challenges the learner with independent questions. The type and amount of practice is variable and is decided with the student.

A mini-lesson is taught to individual students, student teams or to the whole class as needed. Mini-lessons are intended to build upon the knowledge base by solidifying ideas students have constructed for themselves in the field and during workshops with their classmates. Mini-lessons and conferences are consistent with the UDL principle of multiple representations

(CAST, 2011). Topics for theses targeted small group or individual lessons are carefully selected to support learners "where they are at" in their thinking, allowing for the most variation in intent.

Expeditionary Learning succeeds at providing many entry points. As learners explore a problematic situation they do so through their own lenses. The observations that they make and the questions that they pose provide the teacher with information about their unique starting points. A variety of supports can be added for the learner such as guiding questions, peer support, mathematical tools, information gathering frames, vocabulary references, and explicit connections to past experiences. By selecting locations that appeal to the different senses more students can access the information. Outdoor environments and environments that are rich in sounds, visuals, touch and actions should be considered to maximize the opportunities for students to find materials and information in a format that works well for them. The questions that students pose for themselves are of interest and represent an accessible starting point.

During a mini-lesson or conference, the teacher first explores what the students have uncovered. By asking guiding questions or by presenting new information, the teacher supports students in clarifying, making connections, and moving forward efficiently. This gives the teachers and the students an opportunity to transform invented strategies into more formal processes and efficient algorithms. Support for the practice of embedding mini-lessons and conferences into workshops and fieldwork comes from Boaler (2016) in stating that "it is true that while ideal mathematics discussion are those in which the students use mathematical methods and ideas to solve problems, there are times when teachers need to introduce students to new methods and ideas" (p.66). The teacher may bring manipulates, concept frames or text materials to support the learning. For example, students learning about fractions may be offered a mini lesson using manipulatives, a contextual investigation, an action problem, a length model, an area model or a set model, symbols and calculations, or a combination of several different representations.

Topics for mini-lessons and conferences are purposefully selected based on ongoing formative assessment data. Intervening at appropriate times once the students have had ample time to wrestle with a concept is key. Intervening too soon may limit learning from productive failure; while intervening too late may result in misconceptions that are tricky to resolve. A study conducted by Schwartz and Bransford (1998) compared three methods of teaching mathematics. The first model asked the teacher to teach new concepts, followed by students solving problems by following the process illustrated by the teacher. In the second model, students were given problems to wrestle with and make sense of through discovery. In the third model, students were first given problems to wrestle with: but then the teacher was able to intervene when students were stuck and offer explanations and instructions. The third group proved to be the most successful. The researchers observed that "when students were given problems to solve, and they did not know methods to solve them, but were given opportunity to explore the problems, they became curious, and their brains were primed to learn new methods, so that when the teachers taught the methods, students paid greater attention to them and were more motivated to learn" (Boaler, 2016, p.66). Through a well-timed mini-lesson or conference, students are given immediate, purposeful feedback, modeling, guided practice and individual practice, resulting in a co-created, individualized learning path.

Growth Mindset. In Mathematical Mindsets, Jo Boaler (2016) illustrates how a growth mindset can change students' beliefs about their abilities as mathematicians. The growth mindset is the mindset of a learner who believes that with experience, effort and practice, anything is possible. Learners who embody the mathematical mindset know that "math is a subject of growth and their role is to learn and think about new ideas" (Boaler, 2016, p.34). Expeditionary Learning encourages learners to see math as "a broad landscape of unexplored puzzles in which they can wander around, asking questions and thinking about relationships, they understand that their role is thinking, sense making, and growing" (Boaler, 2016, p.34). Adopting a growth mindset and an intense sense of wonder may be the most valuable resources available to today's learners. Regardless of the academic, social or emotional needs of a learner, developing an intuitive sense for learning is critical. All students, whether they are enrolled in regular, modified or individualized programming, have the right to develop a learner's perspective and adopt a growth mindset.

Belonging. The merging of ideas or "collective intelligence can be defined as empowerment through the development and pooling of intelligence to attain common goals or resolve common problems. It is inspired by a spirit of cooperation" (Brown & Lauder, 2000, p.234). To collaborate at this intensity, a common language among contributors is required.

In educational experiences, other learners are our greatest resource. Our own views and expertise, added to a design, create a more fluid and inclusive definition of knowledge. To say that "knowledge becomes, not a packaged commodity, but an event" (Petroski, 2012, p.41) is to adopt a growth mindset, a genuine learner's perspective. When reviewing the experiences of the Wright brothers in comparison to what is known in the field of aerodynamics today, the brothers knew very little. However, they did build and fly a plane. The successful flight of the Wright brothers became a starting point for others. Engineers studied the brothers' work, made adjustments and improvements and continue to do so to the planes used today. As part of a collaborative team, "collective intelligence can be defined as empowerment through the development and pooling of intelligence to attain common goals or resolve common problems. It is inspired by a spirit of cooperation" (Brown & Lauder, 2000, p.234).

The results of a study conducted by Boaler (2016) supported inclusive, collaborative groups in finding that "reasoning had a particular role to play in the promotion of equity, as it helped to reduce the gap between students who understood and students who were struggling" (p.86). The study concluded that conversations with peers "opened up mathematical pathways and allowed students who had not understood to both gain understanding and ask questions, adding to the understanding of the original student" (p.86) which illustrates the benefits of inclusion for all students. Inclusion "is about embracing everyone and making a commitment to provide each student in the community with the inalienable right to belong" (Falvey & Givner, 2005, p.3). Expeditionary Learning "assumes that living and learning together benefits everyone, not just children who are labelled as having a difference" (Falvey & Givner, 2005, p.3). Furthermore, Expeditionary Learning creates stories to share in which each of us has a meaningful role to play.

When combined, these four goals of inclusive, mathematics education become the central principles of Expeditionary Learning. First, we create unity by learning together in a community to which all students feel that they belong. These shared experiences create stories to tell, memories to hold on to and knowledge to build on. Second, field work brings content to life with sound, smell, touch and emotion. We live the math. Third, these experiences are naturally tailored so that each learner is working on appropriate content, with available scaffolding, and embedded extensions. Knowledge is not a commodity to be collected, but an event to be shared. Confidence and persistence are cultivated by having students define their goals and their own learning paths. Fourth, learning is strongest when fieldwork, workshops and mini-lessons are interconnected, linking personal experience, theory and practice. No single learning strategy is effective for all learners at all times. The built-in variety of fieldwork, workshops, and mini-lessons opens a range of learning options and extensions. Teaming with classmates, who bring different expertise and experiences to the group, develops in each of us, an appreciation of our own strengths and the strengths of others. Mathematics is learned best when the interconnectedness of topics and skills is highlighted. Knowing what to do in a problematic situation is a combination of knowing what, knowing how, and knowing why. And finally, learning has no boundaries and no finish line. Every space is a learning place, each of us and all things are teachers.

A Critic of Expeditionary Math

At this time, the research base to support expeditionary math is limited. There are several factors that can limit the success of this approach. To successfully implement an expeditionary program, teachers will need a wide and solid base of content knowledge. Being able to anticipate student stumbling points and to prepare scaffolds and supports requires experience and expertise. The enthusiasm of the teacher and the support of colleagues and school leaders is essential.

Criticisms of constructivist learning are easy to find. Klein and Milgram (2000) note the danger of untested personal algorithms as being that “such algorithms may only be valid for some subclass of problems” and that students can become easily confused as to when to apply which algorithm. These strategies may be “inefficient and cumbersome methods for solving arithmetic problems” (Stokke, 2014) resulting in frustration and confusion for both students and teachers. Kirschner, Sweller and Clark, (2006) point out that at the time that Bruner supported discovery constructivist learning, little was known about the function of long term and working memory. Taking into account what is known today, constructivist pedagogy is out of date and inconsistent with current research findings (p.77). Critics also point out that, without formal instruction, students may develop fuzzy interpretations of concepts and, in fact, expand the achievement gap.

In this study, I hope to discover if Expeditionary Learning holds the promise imagined by the proponents of the constructivists approaches I have discussed, the failings predicted by detractors, or other unanticipated characteristics.

Conclusion

Expeditionary Learning is not meant to fit nicely into a formal, sequential, lock step diagram. The sequence of the learning strategies is never known at the beginning, and demands that students learn to think in the moment. When learners embark on an expedition they explore, research, develop, create and construct, connect, reason, communicate and reflect. These experiences are intended to be fluid, dynamic and flexible; guided by the ever-evolving learning path of each student. Expeditionary Learning changes traditional view of schools to one that is boundary free. Where classrooms are “less a place and more a range of opportunities” (Zull, 2010, p. 44). By extending learning in all directions teachers welcome all learners with accessible content, a range of actions and endless possible paths. The dynamic mix of teaching and learning strategies from workshop to mini lesson to fieldwork invites all learners into an inclusive environment where multiple opportunities exist for learning at multiple levels. The workshop, field study, mini lesson model requires that student’s work together sharing data and questions. Field studies offer special moments where memories and stories are made and strong feelings of belonging are cultivated. These experiences, rich with diversity, lead to understanding, acceptance and collaboration.

Education should inspire in us all a need “to create, a passion to help humankind, a strategy to affect change, and the desire to impact the world” (Berger, 2009, p.29). Schools should “place in students’ hands the exhilarating power to follow trails of interest, to make connections, to reformulate ideas, and to reach unique conclusions” (Brooks & Brooks, 1999, p.22). Expeditionary Learning has evolved through these guiding principles. For educators, cultivating in each learner an intuitive sense, a disposition to naturally draw upon a variety of flexible, non-standard, personal strategies in response to any problematic situation is best accomplished by offering learners opportunities to define their own questions, by encouraging an open approach to solving problems rather than a prescriptive one, by welcoming novel ideas and invented strategies and supporting learners in developing their own paths.

Purposeful authentic problems create space for real thinking, negotiation and reasoning. A commitment to working towards positive change in the world through innovation and invention is realized when students have opportunities to ask their own questions. Teamwork and collaborative skills are best developed through interaction with others, supporting peers and through a responsibility to the shared learning of the group. The solutions to today’s problems will require a rich imagination as such schools must become creativity hubs where the buzz of innovation calls to all learners. Unleashing, in each learner, a passion to create positive change in the world through innovation and invention, as a result of working collaboratively with others, is best achieved by tackling questions for which the answer is not yet known, but for which any development of ideas will contribute to a cause greater than ourselves. Establishing schools as creativity-intense spaces lets brilliant designs take shape.

Developing in each learner the confidence to explore what is unknown and to pose authentic questions, to investigate additional factors that will influence the solution, to turn ambiguity into determination and failure into invention is realized though new experiences and adventures where learners are met with challenges far greater than they have seen before. As in engineering, the starting point for learning is a poorly understood or uncertain situation. To be fully prepared for any situation is unlikely but the ability to see possibility in uncertainty is a valued characteristic. Learners must know that as educators, we value their contributions and that we are willing to follow their lead. Uncertainty uncovers potential for courageous actions and perseverance atypical of a traditional schools' rectangular classrooms.

Inspiring in each learner a belief that where there are limits, there are possibilities is built first by analyzing the context of each problem, its core and periphery, so as to determine its boundaries. Then examining these boundaries, we see where we can go further, what new ideas can push forward, and which new skills will need to be developed. As engineers continue to demonstrate, the ability to go on learning is the most valuable resource available to us.

Chapter 3

Research Methodology

This was a study of the actions, thinking and learning of students and teachers engaged in mathematical learning expeditions. The goal of this research was to explore the potential of expeditionary math as an inclusive, student-led pedagogy. This chapter outlines the research methods used to complete this study as well as the methods of data collection, analysis and interpretation. Grounded in interpretive phenomenology, this qualitative research project involved gathering stories and interpreting the meanings teachers and learners attached to their experiences. Interpretive phenomenology, as defined by Creswell (1998) "describes the meaning of the lived experiences for several individuals" (p.51). Moustakas (1994) further developed the goals of phenomenology as determining "what an experience means for the persons who have had the experience and are able to provide a comprehensive description of it" (p.13). I hope that the analysis and synthesis of the individual stories contributes to the development of a more global picture of what Expeditionary Learning has to offer to teaching and learning mathematics in inclusive settings.

Research Questions

To be become mathematicians, students need to experience mathematics first hand. They need opportunities to explore their neighbourhoods, engaging in problematic situations, making use of the tools of mathematics, developing solutions, and communicating with a wider audience. Operating from this premise, the overall issue addressed in this study was the development of rich, inclusive, active, mathematics learning experiences through Expeditionary Learning. To assess the potential of Expeditionary Learning, I was looking for evidence that all students are able to learn new, increasingly complex, mathematical content while developing more and more sophisticated problem-solving strategies.

This study was both descriptive and explanatory. I sought to describe the experiences of learners and teachers in Expeditionary Learning and to explain how the components of the model shape those experiences. Specifically, I investigated the following research questions:

1. How does the Expeditionary Learning approach support the meaningful inclusion of all learners in mathematics education?
2. What specific mathematical content knowledge is learned, omitted, or transformed through the Expeditionary Learning model?
3. What problem solving strategies are developed, omitted or transformed through a learning expedition?

Qualitative Research

Qualitative research offers rich descriptions of individual experiences and perceptions**.** Denzin and Lincoln (2011) define qualitative inquiry:

> Qualitative research is a situated activity that locates the observer in the world. Qualitative research consists of a set of interpretive, material practices that make the world visible. These practices transform the world. They turn the world into a series of representation including field note, interviews, conversations, photographs, recordings, and memos to the self. At this level, qualitative research involves an interpretive, naturalistic approach to the world. This mean that qualitative researchers study things in their natural settings, attempting to make sense of, or interpret, phenomena in terms of the meaning people bring to them (Denzin & Lincoln, 2011, p.3).

According to Creswell (2013) qualitative researchers: a) often collect data in natural settings, b) collect data themselves and may use an instrument designed by the researcher using open-ended questions, c) typically gather data from interviews, observations, and documents, d) use inductive-deductive logic process that uses complex reasoning skills throughout the research, e) keep a focus on the participants' perspectives, f) may have changes or shifts after the data collection process is begun, and g) convey reflexivity of the researchers (i.e., their background).

Qualitative research is a method to "empower individuals to share their stories, hear their voices, and minimize the power relationships that often exist between a researcher and the participants in a study (Creswell, 2013, p.48).

Narrative Inquiry

The research methodology to be used in this study is narrative inquiry (Clandinin & Connelly, 2000). Expeditionary Learning is a 'choose your own adventure' story. Each experience has characters, with unique traits, settings with which to interact, problems to resolve and a conclusion to defend. A story that tells the experience of learning will reveal to the learner and the researcher details about the mathematics involved, the problem-solving strategies used, and the community of learners with whom the story was lived. Clandinin and Connelly (2000) define narrative inquiry as "a way of understanding experience" (p.20) through our "stories lived and told" (p.20). Narrative inquiry "is aimed at understanding and making meaning of experience" (Clandinin & Connelly, 2000, p.80). In this study, both teachers and students shared their experiences as Learning Stories. The teachers added depth to our understanding of these stories during two semi-structured interviews.

Story telling is a powerful learning tool. It allows both the author and the audience to reflect upon the events that occurred, and the thinking involved. As described by Riessman and Speedy (2007), a narrative shares "how the facts got assembled that way" (p.248). In other words, a narrative describing a learning experience shows how a student's actions and thinking assemble into new knowledge and understandings. I chose to follow Clandinin and Connelly's (1990) method of narrative inquiry because,

> ….humans are storytelling organisms who, individually and socially, lead storied lives. Thus, the study of narrative is the study of the ways humans experience the world. This general notion translates into the view that education is the construction and reconstruction of personal and social stories; teachers and learners are storytellers and characters in their own and other's stories. (Connelly & Clandinin, 1990, p.2)

Our stories reveal how we interpret our world and how we think about ourselves in it (Egan, 1988). Stories where math plays an important role cast a light on how students understand mathematical concepts, how they build connections between ideas, and how they see the relevance of mathematical topics to their own lives (Gaeta, Loia, Mangione, Orciuoli, Ritrovato, and Salerno, 2014). Stories where learning plays an important role invite students into a metacognitive reflection. This leads them to examine their thinking processes, the decisions they have made, the prior knowledge they relied on, and the new discoveries they have made. A story has room for a setting and a plot, descriptive language, characters, and emotions and therefore carries the potential to reveal so much more about a learner than a traditional math test. Clandinin and Connelly (2000) highlight the metacognitive value of narrative in learning when they write that "narrative inquiry is aimed at understanding and making meaning of experience" (p.80).

Learning Stories

In this study, each student's Learning Story was constructed by the learner for the learner. Each story is a collection of photos, work samples and written reflections created by the learner, that tells the steps involved as they moved from initial wonderings to proposed solution. Creating the Learning Story was a metacognitive process through which learners might examine their thinking and come to know themselves, their needs and their strengths as learners. The Learning Story provided a space for learners to share their understanding and knowledge as well as the missteps they took along the way. In expeditionary math, the story begins with the exploring stage. From here students followed their own paths through the research, develop, create and construct, connect, communicate, reason and reflect stages. Each story is exceptionally unique. The stages and sequence are not consistent from one story to the next. Because these stories retell a problem-solving process, they include the characters encountered, the classmates and community members, the setting, details of the problematic situation and the solution. In contrast to a simple set of calculations, these Learning Stories provide a medium for learners to include their emotions.

Learning Stories were piloted by Karlsdottir and Garoarsdottir (2010) as a collaborative way for teachers to assess pre-school aged children's' knowledge, well-being and learning dispositions. The researchers were looking to create a tool to "enable both researchers and practitioners to identify young children's learning processes, well-being and learning dispositions within the sociocultural context" (p. 255). In their study, Karlsdottir and Garoarsdottir (2010) found that "learning stories gave a better picture of the children's strengths and capabilities" (p.260) and that the stories were "an effective way to make children strengths and self-image visible" (p.264).

A Learning Story is not a straight line. It may resemble the net-like shape of a concept map. A concept map provides visual data of understanding. It illustrates the connections that learners are making, the vocabulary and schema that they are using, and their depth of knowledge in a chosen topic area. Ruiz-Primo (2004) agreed that concept maps can be used for assessment because they invite the learner to share evidence of their knowledge and understanding. Assessing the displayed connections or systems of interconnecting concepts can reveal the extent to which meaningful learning has taken place leading to further reflection and discourse (Afamasaga-Fuata'I, 2009, McPhan, 2008). Concept maps clarify the links between concepts in the same way that a Learning Story serves to clarify the connections between each problem-solving strategy.

As both a researcher and a teacher, I am intensely interested in student thinking. How students make sense of an experience, the choices they make during problem-solving and how they reason their responses offers great insight into their learning.

Research Plan

The data collection process of this study was complex consisting of interviews, field notes, individual student reflective Learning Stories and a focus group discussion. This variety of data sources allowed for several perspectives to be considered, the identification of themes across methods, and a fuller understanding of the themes in each context.

Triangulation. Since the focus of this study was to assess the potential of Expeditionary Learning to engage all learners in the development of mathematical knowledge, problem solving thinking and the meaningful inclusion of all students, cross referencing and triangulation between data sources were used. Similarities and differences were highlighted between the interview transcripts, the teacher field notes, the Learning Stories, and the focus group discussions, in terms of the mathematical content that was referenced, the problem-solving thinking that was identified, and the inclusive, active involvement of all learners that was observed. The consistencies and inconsistencies of the teachers' interpretations in the field, in their reflections, and in discussion further reinforces the credibility of these themes.

Credibility. To ensure credibility, thick, rich and exhaustive descriptions of the expeditionary experiences were given by the participating teachers. These descriptions included a description of the school communities, the participating student groups, the teachers, and the events leading up to the expeditions. Descriptive details about the expeditions were gathered from the teaches' field notes and the students' Learning Stories. The final focus group discussion allowed for additional information to be added to the descriptions.

Member Checking and Trustworthiness. During the focus group discussion, teachers had the opportunity to re-examine their field notes and student Learning Stories. This allowed teams to correct misinterpretations, seek further clarification, add additional details, and summarize thinking after a period of reflection.

Researcher Reflexivity and Bracketing-out. The intent of this study was to tell the stories of the teachers and students. My own beliefs did not influence the results. Throughout the process, I used "memoing" to record my reactions to the process, methods and observations. These memos are a record of my own thoughts. They were used to cross reference my analyses and to ensure that what I described was true to the experiences of the participants.

Transferability. This was a very small study. To determine the extent to which these results are transferable will require data from further studies with similar goals in the future. To assist any future studies, I have kept a detailed audit trail of my process. A careful, thorough description of my research methodology will allow others to assess how relevant my findings are to their situations and also allow future researchers to replicate my study.

Data Collection Tools

Interviews. According to Bogdan and Biklen (2011) interviews are "used to gather descriptive data in the subjects' own words so that the researcher can develop insights on how subjects interpret some piece of the world" (p.103). A semi-structured interview offered the flexibility to follow the stories of the participants while maintaining a structured plan. An interview can "provide rich data filled with words that reveal the respondents' perspectives" (Bogdan & Biklen, 2011, p.104). Data gathered through interviews can be incredibly valuable. However, Clandinin and Connelly (2000) caution that interview data can be misleading. They note that the interviewer must be aware of his or her actions and responses, as these can influence the experience of the subjects.

Creswell (1998) recommends a series of steps be followed to ensure the most successful interview experience. First, the researcher should purposefully identify the subjects to be interviewed. Second, the researcher should determine the type of interview best suited to reveal the desired information. Next, the researcher should design the interview protocol which will include a structure to follow, norms of behavior and questions to be asked. Finally, the researcher should arrange the recording equipment and select a location. Rubin and Rubin (2012) caution that an interview does not follow the same norms of a social conversation. They suggest that the researcher ask only one question at a time, gives the participants time to answer without interrupting, nod to show understanding, ask questions as needed for clarification, thank the participants and be sure to indicate the next steps before the end of the interview (Rubin & Rubin, 2012).

Field Notes. Clandinin and Connelly (2001) describe field notes as being "full of details and moments of our inquiry lives in the field" (p.105). They are the "text from which we can tell stories of our story of experience" (p.105). Field notes are written during the experience and contain immediate thinking and action details. They complement reflective journals. Clandinin and Connelly (2001) caution that field notes may carry different information if the author sees themselves as a participant or as an observer. Creswell (1998) cautioned that while in the field the author of the field notes will be distracted from the action while recording and may miss some of the action. To accommodate, Clandinin and Connelly (2001) recommend using photographs to support the written notes.

Learning Stories. As previously discussed, Learning Stories are a type of personalized, event specific journal. The stories are developed with photos, sketches and work samples that provide a firsthand illustration of the experience. Photos are taken during the experience by the teacher and attempt to capture moments of learning, failure and success. Work samples are produced by the individual student or student group. They are the planning notes, research findings, sketches and calculations. When complied the photos, work samples and student reflections build the story of their learning. Learning Stories share several common features of journals and photo-elicitation research methods. Journals are written and drawn accounts of an experience. Clandinin and Connelly (2001) describe journals as a very personal account of events. When several journals are compared themes related to the individual as well as themes that are shared between many are revealed. Young students, English language learners, and students with specific learning needs may find it difficult to use words to describe their learning.

Therefore, sketches in Learning Stories are encouraged for all students. Findings from a study conducted by Nedelcu (2013) indicate that student drawings are an efficient way to increase the level of involvement of young students in research. Nedelcu analyzed 146 individual student drawings of "*My Classroom*" by placing the drawings side by side and identifying themes in compositional elements. The same method can be used to compare elements of Learning Stories. Photos are also included in Learning Stories to capture even more of the learning moments. Photo-elicitation "involves having individuals take or select photographs that are representative of their reality and discuss the photographs during interview sessions" (Kronk, Weideman, Cunningham & Resick, 2015, p.99). In a study conducted by Kronk, Weideman, Cunningham and Resick (2015) eight individuals took photos during a service learning trip. Four weeks later the participants were able to provide details describing the deep personal transformations they experienced while referring to the photos. The photos "enabled a deeper, more poignant exploration of the impact" (Kronk, Weideman, Cunningham & Resick, 2015, p.102) of the experience. By combining journal writing, photos, sketches and work samples, a Learning Story becomes a deeper understanding of the lived experience.

Focus Group. Bogdan and Biklien (2011) describe focus group discussions as "group interviews that are structured to foster talk among the participants about particular issues" (p.109). In this study, teachers were trying Expeditionary Learning in their own unique environments and encountered a variety of challenges and successes. The purpose of the focus group discussion was so "that the researchers can learn what the range of views are" (p.109) and to encourage talk that may not have surfaced during an individual interview. Bogdan and Biklien (2011) caution that individuals may not feel as comfortable sharing their stories in a group setting. Seal, Bogart and Ehrhardt (1998) conducted a study to determine the strengths of focus groups as a research method. To do so, they engaged forty-four men in individual interviews and focus groups discussing sensitive and controversial topics. Their findings indicate that a "group setting may be better used to generate thought and discussion around shared and unshared attitudes and beliefs" (p.262). However, they also reported that "the individual setting may be better used to identify the range and depth of individual values and experiences" (p.262).

To encourage the greatest depth and variety of themes the group was asked to agree to a set of norms before our first discussion began. Participants also agreed to follow a positive and polite discussion format. We agreed to not interrupt the speaker, to raise a hand to make a comment, and to engage in discussion that is focussed on student and teacher learning. Should the discussion go off topic, participants were made aware that I would redirect them.

I reminded the participants that the purpose of the discussion was to support our learning. I reminded participants that I was not able to guarantee that the information shared will be kept confidential by all participants. However, I assured them that the audio recordings of the discussion would be stored safely and kept confidential. Participants were asked to omit student names during the discussion. Any actual name that was used has been omitted.

By compiling data from these sources, I will have a composite, balanced picture of the Expeditionary Learning experience from both the students' and teachers' perspectives.

Participants

Educators were invited to participate in the proposed study because of their desire to increase the active engagement of all learners in math education. A range of grade level participants, grade 6 to grade 8, was selected to ensure that the successes and challenges of Expeditionary Learning at each grade level were addressed. A key piece of the Expeditionary

Learning theory is that learning to learn is a critical educational outcome at all levels. These behaviours may become more and more refined with experience but will remain essential learning outcomes. We will never find ourselves at the finish line of learning.

Teachers were selected from different school communities within a suburban school division in a western Canadian city. An essential component of Expeditionary Learning is that the initial question comes from the learners. Having teachers and students from across the division allowed for a variety of local questions and perspectives to be considered. Mathematical content knowledge plays an important role in Expeditionary Learning. Students are co-creating their learning paths with their teachers. To do so, teachers are highlighting mathematical connections and drawing out the most relevant mathematical concepts at the very best moments. To encourage further discussion and to begin to examine transferability, teachers with varying years of experience and with varying levels of confidence were selected to participate in the study.

All learners in each of the classes were invited to participate. Parents and families received information letters about the study via school email and access to the expeditionary math website (www.expeditionarymath.com). They were offered the opportunity to ask questions and share concerns with the classroom math teachers and with myself. Including all learners was critical. Every classroom, in every school, has a rich mix of learners; each with his or her own strengths and talents, and unique in his or her challenges. Expeditionary Learning seeks to engage each learner in authentic, meaningful learning that will welcome his or her individual learning style and challenge him or her to expand his or her set of learning strategies. In this study, teachers and numeracy coaches planned for the inclusion of students working towards outcomes, outcomes with adaptations, modified outcomes, and individual programming goals.

Process

The proposed study took place between September 2017 and June 2018. After approval from my advisor and my committee, and approval by the University of Manitoba Education and Nursing Research Ethics Board (ENREB), I invited teams of two teachers to take part in the study. A copy of the ENREB approval is found in Appendix A. To recruit these teams, I first contacted the superintendent of the school division by email to ask permission. I adhered to the research guidelines of the school division. A copy of the email and the attached letter are found in Appendix B. Upon approval from the superintendent, school principals were made aware of the study and invited to share the invitation with their teachers. At this point it was suggested that I work with the divisional numeracy coaches and classroom teachers rather than school-based resource teachers and classroom teachers. In this school division, the role of a numeracy coach is similar to that of a resource teacher in that they work side-by-side with the classroom teacher to plan for an implement inclusive mathematics programming. This switch was approved by the University of Manitoba ENREB committee. An initial meeting was planned to further explain the study to the numeracy coaches and to begin planning for our work with the teachers.

The numeracy coaches had previously been supporting math teachers who were interested in developing problem-solving and inclusive practices in their classrooms. The coaches presented the opportunity to take part in the study to these teachers. Interested teachers then contacted one of the numeracy coached by email. Through email, the numeracy coaches, participating teachers, and I were able to arrange a date to meet for a brief information session about the study.

Participation in the study was optional. School administration teams and divisional superintendents were not told which teachers chose to participate in the study. All participants had the right to withdraw from the study for any reason, at any time.

Five teams of two teachers, each consisting of one classroom teacher and one numeracy coach, indicated that they were interested in participating in this research project. Each classroom teacher was provided with information letters for their students and their families (see Appendix H), parental consent letters, and letters of assent for each student (see Appendix J). The letters asked that parents and students indicate whether or not they wished to participate in the study, so that all students will return a letter and not only those who will participate. Families and students were asked to return their letters of consent and letters of assent to the classroom teacher in a sealed envelope within seven days. These letters were stored in a locked cabinet within the classroom.

Participation in this study was voluntary. Parents, teachers and students had the right to withdraw from any part of my study at any time with no consequences. Any student who did not wish to participate in the study was still included in the Expeditionary Learning project; however, they were not included in any photos, nor were their Learning Stories collected for analysis. In all documentation, teachers are referred to as T1, T2, T3, T4 and T5. The numeracy coaches are referred to as NC1, NC2, and NC3. Students are named according to their teachers' assigned letter and a randomly selected number between 1 and 30. For example; students in teacher team 1s' class are known as student T1S1, student T1S2 and so on.

Each teacher completed an introductory interview. The interviews were recorded and later transcribed. Transcribed notes were kept in a locked cabinet in a locked office. The interviews were conducted over the phone at a time that was suggested by the participant. The interviews lasted approximately thirty minutes. The purpose of these interviews was to gain an understanding of the teacher’s beliefs, goals and experiences with Expeditionary Learning and inclusion. The semi-structured interview style allowed for general questions relating to these themes to be asked while affording the teachers the room to share their personal contexts and experiences. Appendix K contains a list of interview questions. Transcribed interviews are found in Appendix L. During these semi structured interviews, teachers were asked to share a description of their classes and to describe the level of inclusion in their classroom activities and the problem-solving abilities of their students. Their responses to these questions served as a base line for comparison with the expedition. Demographic information related to the age, the culture of their school, and the nature of their school community also were collected through these initial interviews.

An initial Professional Development session followed these interviews (see Appendix M for an agenda). The session included an overview of the theoretical background, examples of expeditions and a description of the learning strategies.

Following this initial session, the numeracy coaches, the teachers and myself worked together to plan an expedition for each class. Specific attention was given to the creation of the individual students' Learning Stories and teachers' field notes.

It is important to note that each expedition was unique, as it was driven by the interests and needs of each teacher's class. Participants were not directed to a specific topic or curricular outcome, as that restriction would have contradicted the student-led inquiry focus of Expeditionary Learning. Instead, teachers were supported in developing their expeditions once their students had set the focus. Ongoing support through emails, phone conversations, and meetings between myself and the teacher teams also took place as needed.

Each teacher and numeracy coach team returned to their school to further develop their Expeditionary Learning experiences with their students. The teachers and numeracy coaches then created an initial introduction or exploration. Then, the teachers and numeracy coaches tried to anticipate the potential questions and points of struggle their students might encounter. For each expedition, they planned mini-lessons, scaffolds and supports. The supports included research frames and additional measurement tools that would be made available to their learners if and when they needed them.

Throughout the process, the teachers collected photos of student involvement in problem-solving situations and included them in the students' Learning Stories and their own field notes.

At the conclusion of the expedition, the students created their Learning Stories. Since I could not anticipate the exact moments in the experience that would be the most meaningful to each learner, I could not prepare a set of questions for the teachers to ask (Trahar, 2009). Rather, I invited the students and teachers to freely capture the elements of their experiences that most resonated with them. To do so, I suggested that students retrace their steps by creating a timeline of their process from their initial wonderings to their final solutions. The following questions were suggested to help them as they worked.

1. What did you do? What made you decide to try that?
2. What questions were you and your team asking at each step along the way?
3. How did you answer those questions?
4. What math were you using and learning along the way?
5. What strategies; explore, research, develop, create and construct, communicate, reason, connect and reflect, were you using along the way?

The classroom teacher compiled the stories and their notes and brought them to the final focus group session.

Next, the five teams and the researcher met for a focus group discussion. This discussion lasted for approximately two hours and took place at a time and location determined by the teachers. The focus group had two central goals. First, two components of Expeditionary Learning are communication and collaboration; so, it was fitting that the focus group explored those elements. Second, the focus group offered participants a chance to reflect on their experiences, and to revisit their field notes and Learning Stories. Additional information offered by the teachers, such as the clarification of my interpretation of a photo or extra details about a conversation with a learner, contributed to the credibility of the data. I aimed to create credible local data from the story of each participant. When participating in a team discussion, educators “can stimulate each other to articulate their views or even to realize what their own views are” (Bogdan & Biklen, 2007, p.109). The open conversation style of the focus group offered the teachers the opportunity to share their experiences, to reflect on common goals, and to discuss the potential of Expeditionary Learning as an inclusive teaching method for developing mathematical content knowledge and problem-solving strategies. The field notes and Learning Stories acted as a third point from which the educators could share evidence. Specifically, this session focused on the teachers' observations of the inclusive nature of the expedition, the mathematical content students accessed through the expedition, and the problem-solving strategies they witnessed students using.

The role of the researcher in the focus group was to redirect and refocus the conversation. Probes, used to guide the conversation, included: What did you find to be some of the strengths of Expeditionary Learning? What pieces of the experience did you find difficult to manage?

What evidence of inclusion did you notice? Describe your role during the experience. The interviews were recorded and later transcribed. The transcriptions were kept in a locked desk within a locked office.

An analysis of these data sources offered an indication of the mathematical content explored, the learning strategies used, and the inclusivity of the experience. The analysis therefore required five specific phases.

1. The mathematical content developed through the expedition was identified in all data sources. Mathematical concepts evident in the Learning Stories and focus groups transcripts was coded and cross referenced to the Manitoba curriculum. Specific topics were decomposed into grade level expectations. Each story was first analysed separately to identify content themes. All stories were then analysed as a conglomerate to identify mathematical content themes that emerged as well as those that did not. Themes were tabulated in a chart. This data was used to respond to the first research question "How does the Expeditionary Learning approach support the meaningful inclusion of all learners in mathematics education?" and the second research question "What specific mathematical content knowledge is learned, omitted, or transformed through the Expeditionary Learning model?" by identifying the actual outcomes students targeted.

2. The extent to which mathematical concepts are described in the Learning Stories and the focus groups transcripts was analysed using the Manitoba Provincial Report Card Grade Scale Mathematics Achievement Profiles: Knowledge and Understanding and Mental Math and Estimation (see Appendix Q). This data further supports the responses to the first two research questions. Not only does an expedition provide

exposure to mathematical content it should also deepen understanding. Vocabulary, calculations and explanations were cross-referenced with the relevant curricular outcomes to assess accuracy and depth of understanding.

3. The problem-solving strategies were identified in the Learning Stories and focus group transcripts. The actions and descriptions were matched to the descriptors of the Expeditionary Learning strategies of explore, research, develop, create and construct, reason, connect, communicate and reflect (see Appendix Q). Each of the actions and descriptions was further analyzed to identify common themes associated with each of the Expeditionary Learning strategies. This data supports the response to question three "What problem-solving strategies are developed, omitted or transformed through a learning expedition?" Each theme was explored as it related to content and action. Photos gave further insight into the different paths students may have tried and their reasons for selecting a final path.

4. The inclusivity of the experience was analyzed by identifying themes that are consistent with the descriptions of the three UDL Guidelines: Multiple Means of Representation, Multiple Means of Action and Expression and Multiple Means of Engagement (CAST, 2011). Each Learning Story and the focus group transcript contained examples of inclusivity and exclusivity. By categorizing these examples into the UDL Guidelines, a richer understanding of which UDL practices were most supportive of inclusive mathematics emerged. This data was used to support the response to the first research question "How does the Expeditionary Learning approach support the meaningful inclusion of all learners in mathematics education?" Each photo and comment also spoke to the inclusive nature of the experience.

5. Additional themes, that emerged throughout the experience, also were examined.

Once the transcripts and Learning Stories were analysed, common threads and themes were identified and explained. As outlined by Creswell (1998), this process began by reviewing the interview and focus group transcripts searching for “statements of how individuals are experiencing the topic” (p.147) so that a list of descriptions could be developed. Next, these statements were grouped into “meaning units” (Creswell, 1998, p.148) to capture the action of the Expeditionary Learning experience. A “description of how the phenomenon was experienced” was complied (p.150) and finally, a “description of the meaning” (Creswell, 1998, p.150) or the essence of the experience was written.

To complete this analysis, I read through the interview transcripts, notes and Learning Stories. I coded emergent themes, common ideas, examples, counter-examples and outliers. I matched interview quotations, field note samples and evidence from the Learning Stories to each theme. Then I analyzed each theme in terms of its significance to the research questions, implications for teaching and learning mathematics and for future research.

The analysis of these varied data sources offers a well-rounded picture of the Expeditionary Learning experience. Pursuing this mix of data sources was key to advancing theoretical knowledge about Expeditionary Learning, its practical applications in inclusive mathematics learning, and how to further development and refinement the method.

Delimitations and Limitations

The choice of a qualitative research methodology limited the applicability of the findings. However, learning is highly personalized. Part of the strength of Expeditionary Learning lies in "telling our stories". Each story is shaped by the personal lenses through which individuals view the world.

The following delimitations and limitations applied to this study:

1. This research was delimited to the study of four middle years classrooms and one high school classroom in which the teachers responsible for teaching the math classes had indicated an interest in adopting an expeditionary model.
2. In each case, the classroom math teacher took primary responsibility for leading the learning expeditions. The study was therefore delimited to the perceptions, commitments and actions of those teachers.
3. Atkinson and Delamont (2006) note that "a narrative of a personal experience is not a clear route into the truth" (p.166). In this study, where students and teachers were retelling the experiences of their personal learning, more factors are at play than those reported. Photos of their experiences and interviews with the teachers allowed more factors to be considered. However, an unknown number of factors, that either contributed to or limited learning, may not have been uncovered.
4. The individual stories were analyzed to reveal a collection of common themes and recommendation for educators looking to increase the levels of inclusion and mathematical problem solving. However, the grouping of stories into categories may have overshadowed the individual voices of the students; putting the richness of the participants unique, individual voices at odds with the needs of the collective (Andrews, 2007, p.491).

Chapter 4

Results

This chapters begins with a summary of the background information about the schools, teachers and the students who participated in the study. Credibility is established by identifying similarities and differences between the interview notes, the focus group discussion and the students' Learning Stories in terms of the mathematical content that was referenced, the problem-solving thinking that was identified, and the inclusive, active involvement of all learners. To ensure credibility, descriptions of the school communities, the teachers, and the events leading up to the expedition are presented. Descriptive details about the expeditions from the teaches' field notes, the focus group discussion, and the students' Learning Stories were examined. The focus group discussion added another layer of trustworthiness by giving teachers and numeracy coaches and opportunity to re-examine their field notes and the student Learning Stories. This opportunity allowed misinterpretations to be corrected, clarifications to be made, details to be added, and thinking to be summarized after a period of reflection

Following that, the major themes that emerged from the initial interviews, the focus group conversation, and the student Learning Stories are presented. Throughout the chapter the following abbreviations are used to name participants.

Table 1.

Abbreviations for Naming Participants

Abbreviation	Meaning
T1	Teacher 1
T2	Teacger 2
T3	Teacher 3
T4	Teacher 4
T5	Teacher 5
NC1	Numeracy Coach 1
NC2	Numeracy Coach 2
NC3	Numeracy Coach 3
T1S1, T1S2, T1S3…	Teacher 1 Student 1, Teacher 1 Student2, Teacher 1 Student 3…
T2S1, T2S2, T2S3…	Teacher 2 Student 1, Teacher 2 Student 2, Teacher 2 Student3…
T3S1, T3S2, T3S3…	Teacher 3 Student 1, Teacher 3 Student2, Teacher 3 Student3…
T4S1, T4S2, T4S3…	Teacher 4 Student 1, Teacher 4 Student 2, Teacher 4 Student3…
T5S1, T5S2, T5S3…	Teacher 5 Student 1, Teacher 5 Student2, Teacher 5 Student3…

Background Information

All 5 of the teachers had been teaching at their current schools for a minimum of three years. All had previous experience teaching mathematics at their current grade level. One of the teachers (T2) had not taught mathematics for five years and was teaching only one class of mathematics during the study. This teacher (T2) shared that she preferred to teach technology. All teachers were involved in a divisional professional development opportunity with the divisional math coaches that focused on problem solving. Each classroom teacher had previously worked with his or her numeracy coach.

Two of the numeracy coaches (NC1 and NC2) had been in their role for a minimum of two years. One of the math coaches (NC3) was new to the role and midway through the first year. The role of the numeracy coach, as defined by the St. James-Assiniboia School Division, is "to enable teachers to implement effective research based instructional practices that respond to the literacy and numeracy needs of all students" (St. James-Assiniboia School Division Policy #1035792 Appendix 2 CCA).

On Thursday, February 1, 2018, the three numeracy coaches had a preplanned Professional Development afternoon session with Middle Year's teachers within the school division. There were five classroom teachers in attendance representing five different middle schools. An overview of the research study and its goals was presented. The presentation lasted approximate twenty minutes. The teachers in attendance received a project synopsis (Appendix N) outlining the goals of the study and offering a summary of Expeditionary Math. The numeracy coaches responded by email two weeks later, indicating that five teachers had agreed to participate in the study. Their email addresses were shared, and a plan was made to meet for a professional development and planning afternoon.

Teacher 1 (T1). School 1 is in a suburban setting next to a large hospital and close to small grocery and convenience stores. The principal of school 1 described the school community as follows:

> Over the years, [redacted] Middle School has built a "Tradition of Excellence". This is due in part to the expectations placed on the students by the parents and teachers. The foundation of this tradition, however, rests with our outstanding staff of experienced and dedicated teachers and educational assistants. The academic performance of [redacted] students is exceptional. Year after year, we encourage students to strive for their personal best and to reach for personal excellence. We are careful to consider the needs of each individual and challenge each child to raise their personal bar of achievement. The rewards for the athletic and academic effort are evident both in our trophy cases and in the success achieved by [redacted] students when they proceed to high school. The "Tradition of Excellence" continues to be one of the main focuses of [redacted] Middle School.

T1 is a relatively new teacher with approximately ten years of experience. T1 is currently teaching grade 6 mathematics to three classes. T1 was supported by NC3.

Teacher 2 (T2). School 2 is located in a suburban setting. The school is in its final year of a transition from a dual track, offering French Immersion and English programming, to a single-track English school. The school website displays the schools' vision as "collaborating with parents and community in a bilingual setting, we empower students to develop their personal road to success while fostering accountability, integrity, and self-esteem" ([redacted] Middle School website). T2 is currently teaching one class of mathematics to grade 7 students. T2 shared that she has not taught mathematics for five years. T2 was supported by NC3.

Teacher 3 (T3). School 3 is located in a suburban setting adjacent to a major city route. Several businesses are located across the street and right next door to the school. The school is a school within a school as it is attached to a large high school. The school website displays the schools' mission statement as "[redacted] Middle School is devoted to academic excellence, respect for individual rights and differences as well as to the development of individual strengths" ([redacted] Middle School website, 2018). T3 is a new teacher having taught for five years. It is important to note that T3 completed one of her student teaching blocks with T4 at school 4. Prior to beginning her initial interview T3 stated that she had no idea what this project was about. T3 was supported by NC1.

Teacher 4 (T4). School 4 is located in a suburban setting. It is adjacent to an elementary school and shared an athletic field with a high school. The school is across the street from a shopping center. The school website displays the schools' mission statement as "[redacted] School is committed to providing students with opportunities to develop personally, socially and academically in a safe environment and to become lifelong learners" ([redacted] Middle School 2018). T4 has been teaching mathematics at this school for more than 18 years. She is currently teaching grade 8 math. T4 was supported by NC1.

Teacher 5 (T5). School 5 is located in a suburban setting on a major route. It is adjacent to the local recreation center. School 5 is in it's final year of transitioning from an English track school to a French Immersion milieu school. The school website displays the schools' mission statement as "The mission of [redacted] Middle School is to provide a safe, nurturing and cooperative environment which will guide and encourage each individual to develop mind and body to full potential, to foster respect for self, school, home, and community, to value learning, to develop responsibility, to enjoy and take pride in belonging, achieving, and caring" (2018). Teacher 5 has been teaching mathematics for many years and is currently teaching grade 7 French Immersion math. T5 was supported by NC2.

Initial Interviews

Following the presentation of the study and offer to participate interested classroom teachers connected with their numeracy coaches and sent an email to indicate when was a good time for the initial interview. The interviews lasted approximately thirty minutes and were done over the phone. Each participant received a copy of the initial interview questions (Appendix K.). To begin each interview, the purpose of the study was explained, the protocols for consent, assent and confidentiality were reviewed and questions that the participants had at that time were answered. At the end of the interview I thanked the participant and explained the next steps. The interviews were recorded using an iPhone and then transcribed (Appendix L).

Initial Interview Emerging Themes

Teacher Hesitation. During the initial interviews, teachers communicated a hesitation or lack of confidence around teaching mathematics. When asked to describe a typical class T2 stated that "I haven't taught math in five years" and "I haven't taught math for a while" (T2, l9 –

10) later adding “I don’t use technology as much as I should because I am all over the place” (T2, l32).

Teacher 3 stated that she modeled her math class after a cooperating teacher she worked with as a student teacher. She also commented “I’m only in my fifth year so I just do that” (T3, 15 – 16).

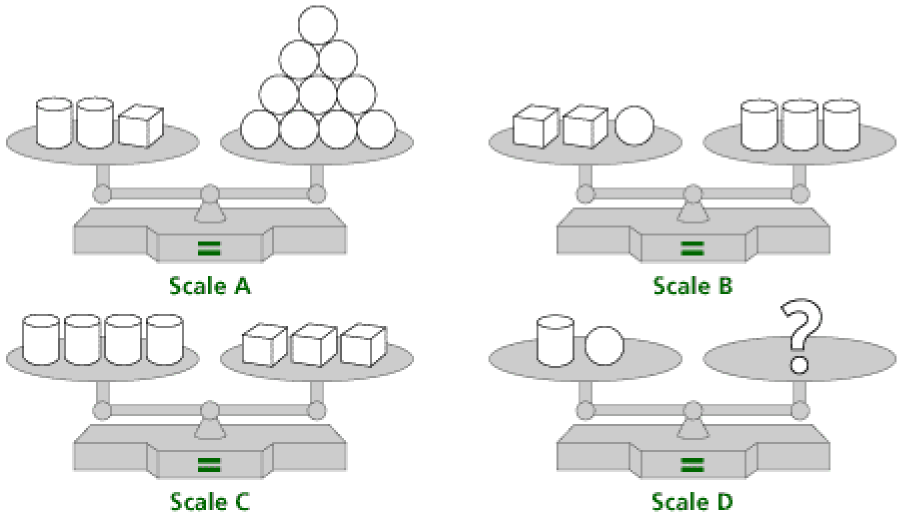
Written, word problems as Mathematical Problems. When asked to share an example of a problem that students had recently tried in class, all five teachers shared examples of written word problems.

T1 shared an ice cream problem.

> In shops with lots of ice cream flavors there are many different flavor combinations, even with only a 2-scoop cone. With 1 ice-cream flavor there is 1 kind of 2-scoop ice cream, but with 2 flavours there are 3 possible combinations (eg. vanilla/vanilla, chocolate/chocolate, and vanilla/chocolate)
>
> How many kinds of 2-scoop cones are there with 3 flavours?
>
> How many kinds of 2-scoop cones are there with 6 flavours?
>
> How many kinds of 2-scoop cones are there with 10 flavours?
>
> What about “n” flavours? Create a poster that represents your group’s thinking.

T2 shared an equilibrium problem.

“There’s three pictures of balance scales with the rectangles, the circles and the square and each one weighs different, so you try to find out what each one weighs.” (T2, L41-42)



http://www.learner.org/courses/learningmath/algebra/images/session6/6a6_shapes.gif

T3 shared a written problem

> Paublo spent the entire day at an amusement park on the roller-coaster. On the way out, he saw a stuffed dinosaur at the ring toss. He said “I have to win that dinosaur for my little sister Arian. Paublo had five rings to toss that could land on 5, 10, 15, 20 and 25 points. To win the dinosaur Paublo needed a total of 40 points. How many different combinations of points are possible?

T4 shared that she had given a Magic Square as a problem to solve.

-7	**-14**	
	-12	
		-13

T5 shared an open calculation problem and a percentage problem.

> The sum of two numbers is equal to 15. The difference between the two numbers is -7. What are the two numbers?

> One student got 18/20 on a test and one student got 87%. Who got the higher mark and why?

Inclusion. Through the answers to the question "how all students are included in your class and lessons" it became evident that inclusion is understood in a variety of ways. Teachers shared that they incorporated random groupings into the daily structure of the class. For example, T2 stated that "They are all random groupings. Maybe on an odd day I use a strategic grouping" (T1, 56 - 57). T3 noted that "we do random groupings so that they are always working with other people" (T3, 74). T2 noted that there is "one student who has an IEP and does the programming from St. Amant. It's one to one with an EA" (T2, 61) in her classroom.

Teachers described a parallel task that would have students included in the physical space of the classroom but not necessarily engaged in contributing to the learning community. For example: T2, referring to the student with an IEP, noted that if she had "something that I think that he would be able to do, then I include him. Like patterning, if we are working on patterns then he can work on his patterning goals at the same time but that's not too often" (T2, 63 - 65). T4 stated that she tries to "include them in the vertical learning spaces even if they are just recorders" (T4, 47).

T3 indicated that to include students she reduced the number of questions assigned "I have some kids, that you know, are below level, so I take off a couple of questions" (T3, 60 – 61). T3 also shared that she offered puzzles for students who were able to "whip through" and "be done in like five minutes" (T3, 67)

T4 stated that she struggled with inclusion. She noted that “inclusion is hard” (T4, 41). She explained that “If we are talking best practices, so we’re learning about operations with fractions and the IEP students are working on fractions does that count? Is that inclusion? (T4, 45 – 47). T4 also shared a desire to learn more and to increase her ability to authentically include all students “I haven’t found a way to truly have it the way I think that they should be included. I want them to be moving forward at the level they are at, so I struggle” (T4, 52-53).

T5 noted that he uses manipulatives “so that they can see the math, not just do it” (T5, 79) as a means of inclusion. T4 was the only teacher to indicate that she wanted to learn more about inclusion and saw a need for improvement.

Professional Development Session

The Professional Development session took place on Friday, February 9, at the school division professional development center. All teacher teams were able to attend. Throughout the session, teachers were invited to ask questions and make comments. They were encouraged to think about their students, what they were interested in, and which mathematics concepts they would benefit from exploring.

Initial Plans. Two teachers planned for in-school adventures. T1 and T3 wanted to ease into Expeditionary Learning by first taking their classes to the gym. Once in the gym, they would invite their students to wonder by asking “what do you notice?” and “what do you wonder?” Both teachers indicated that they had previously used these phrases to initiate an in-class number talk using photos. The teachers were confident that the students would be able to ask questions about what they saw in the gym. Once they had visited the gym and found solutions to the student questions they would continue with an expedition outside of the school.

T1 planned to follow the visit to the school gym with an investigation of yards and gardens. He planned to have students map out their own backyards and then develop a plan for landscaping his yard. The students would plot features of their yards on a Cartesian Plane, include measurements for perimeter and area, determine the number of trees, scrubs and plants required and consider the volume of soil. He envisioned using a section of the school field as his hypothetical backyard.

T2 had considered several curricular outcomes to try but was not able to select a location where students would find a problem that would lead them to those outcomes. T2 shared a strong desire to have the questions come from the students and so agreed to share the idea of Expeditionary Learning with her class and then to see what ideas her students had.

T3 had planned to follow the visit to the school gym with a visit to a local, independent, grocery store. T3 had hoped to have students investigate operations with decimals.

T4 had earlier decided that her class would go to Canadian Tire. The Canadian Tire store is located less than a five-minute walk from the school. T4 had previously discussed the project with the numeracy coach and had created a plan to have students determine the 'best storage shed' for her and her family. She had already visited the Canadian Tire store and had spoken with the store manager. The store manager, who she referred to by name, had suggested a date in late May as the sample storage sheds would be on display in front of the store. T4 had wanted her students to explore geometry concepts such as measurement, perimeter, area and volume.

T5 had planned to visit a local coffee shop and grocery store. He anticipated that the students would ask questions relating to costs and amount. Their questions would be connected to curricular outcomes relating to decimals, taxes, percentages, volume and weight.

Focus Group Discussion

The final focus group discussion took place on Tuesday, June 5 from 1:00pm to 3:30pm. All participants were present. The session took place at the divisional professional learning center. The discussion was recorded using an iPhone and then transcribed (Appendix O).

All five teacher teams shared that they had tried an expedition. T5 had completed two expeditions, one within the school to the school gym and one to the local grocery store. T2 shared that given that she had left the expedition until so late in the year they had not let the school and instead used the school cafeteria for an expedition.

All the teachers stated that they had had great difficulty collecting the parent consent forms as students had misplaced them or forgotten to return them. As a result, they had brought only a small representation of the student's experience.

The full group had not met since the last professional development session and so we began by hearing a summary from each teacher team. As teachers shared their experiences, several themes emerged; teachers as risk takers, the changing role of the classroom teacher, responsive teaching, what is learned through Expeditionary Learning, student engagement, inclusion, students as leaders of their own learning, and teacher supports.

Teachers as Risk Takers. Expeditionary Learning was a new approach for teaching to all participants. No teacher mentioned that they had any experience taking students outside of the school to learn mathematics.

Three teachers had concerns about the openness of the expeditions. T1 stated that he had avoided some risk by not leaving the school building but that he begun the project with an open mind stating "I went in thinking I am going to try this and it could completely flop and if it does oh well. It's scary" (T1, 582 – 583). T2 noted that her decision to stay in the school was based on her feeling that she "wasn't ready to jump into going somewhere far" (T2, 173). T3 shared her concerns about the open-endedness of expeditions stating "It was scary, like not knowing, it being so open and letting the project go kind of wherever it went and not knowing exactly what we are going to do today. I'm not used to that." (T3, 580 – 581). The numeracy coach that supported T3 echoed this feeling stating, "I was a little bit worried because she has a busy class and I was like oh, how is this going to be?" (NC1, 326).

T4 described her journey towards a more student-led experience:

> "for me I found that I was going into this still needing to have a lot of control and I found myself being able to give up more and more the more I saw the project developing and I think I am a lot more comfortable now having gone through it. I am a lot more comfortable now trying something that I have less control over" (T4, 591 – 594).

The numeracy coach who worked with T1 welcomed the uncertainty. She noted the need to be open to possibilities stating, "you have to be open with it" (NC3, 261) and that "you just have to run with it" (NC3, 263).

The Changing Role of the Classroom Teacher: Towards responsive teaching. Responsive teaching requires "teachers to become keen observers of how learners construct knowledge and then use that information to build on learners' strengths" (Dozier, Garnett and Tabatabai, 2011). This approach is a significant departure from traditional teaching methods where the teacher has preplanned the lessons according to a preset curriculum.

Four of the teachers and two of the numeracy coaches commented on the change towards responsive teaching. T1 noted that having the numeracy coach as a partner was helpful because they "were able to debrief quickly and say ok next class is coming in so what are we going to change" (T1, 30 – 31). He shared his comparison of preplanned and open lessons, realizing that lessons "where I am just going to try to figure out where the kids are going to go with this and those lessons and those activities are so much more rich" (T1, 609 – 611). T2 agreed that it was a different type of preparation and the her and the numeracy coach did "a lot of figuring out as we go" (T2, 176). She shared that she "did a lot of planning ahead of time" and had "ended up not even using half of it" (T2, 617). She added that "there was a lot of making decisions on the fly and we had to plan right after" (T2, 619). She stressed that "you cannot preplan this, you cannot plan out the whole thing. You have that original, tiny place to start and then you've got to plan every day and you have to be ok with that and it be normal" (T2, 621 – 622). T3 added that "we kinda kept morphing what we were going to do" (T3, 289) and "We didn't know exactly what it was going to look like so we kept switching it" (T3, 291 – 292).

T4 described the shift she experienced from teacher-centered to student-led. She explained that "at the end of each class that they needed to tell me what they needed from me for the next class" (T4, 623 – 624). She continued saying that "sometimes I had to go take more pictures and sometimes it was you have to go measure how high your roof is" (T4, 626 – 627). At one-point students had said to her "we need to know how to find volume" (T4, 627). She concluded that she had begun "letting the kids dictate what they need and if [she] knew that there was something that they needed, and they just weren't asking [she] might slip it in in some ways but most of the time they figured it out" (T4, 629 – 631).

The numeracy coach (NC3) summarized the conversation saying "the difference that I hear is that you need to know the difference between teaching curriculum and teaching kids (NC3, 842 – 843). Adding that "when you are teaching kids you will take risks (NC, 845). NC2 also noted the difference between traditional and expeditionary teaching and learning saying that "it wasn't you dictating, yeah, do this next, do this thing" (NC2, 600) "they were the kids ideas" (NC2, 589).

What More Can Be Learned Through Expeditionary Mathematics? The teachers initially had identified specific curricular outcomes that they had envisioned students exploring though their expeditions. During the focus group discussions, the teachers and numeracy coaches noted that there was more being learned than the mathematical content.

All of the teachers shared examples of how the problems they had investigated with their students involved more than just factual math concepts. They identified that their students had also developed broader skills in problem solving, communication, teamwork, reasoning through positive argument, identifying variables and productive struggle.

T1 noted that his students explored the tools that mathematicians use and learned to select appropriate tools for the task at hand. He stated that the students had originally asked for a ruler to measure the gym but that through conversation and experimentation they realized that a meter stick or tape measure would be more efficient.

T2 and T4 noted that students had engaged in positive arguments when making decisions in problem solving. This positive argument required that they reason for or against ideas and conjectures. T2 noted that this type of argument was present, but that students needed to learn how to do it (T2, 764) and T4 noted that she had "heard them arguing with each other all the time and positively" (T4, 772). She went on to say that "the discussion was just so rich" (T4, 776).

The numeracy coach noted that the experience was about "trying to get them thinking beyond what they are seeing" (NC3, 46 – 47). She added that the experience was similar to teaching through problem solving "because there is a whole lot more thought that has to go into what they are doing rather than us just giving them a question and telling them here use this and go measure that. It's the more openness of it" (NC3, 735 – 737). NC3 also noted that Expeditionary Math is about teaching thinking that will transfer from one situation to the next. That is:

> "teaching kids to think rather than teaching them 'here's some content'. They have to think for themselves and make sense of things if we are doing that most of the time in our classrooms then when they are presented with a dry problem, then it will look different to them but if they are used to thinking for themselves then they will try to work it out. Well if they are attacking something like ok, I know some things I can figure it I know I can do it rather than here's a question on a page and I don't know how to do percent it's just a different mindset" (NC3. 745 – 752).

T2 added that in addition to mathematical content and problem-solving skills, Expeditionary Learning is "the struggle" (T2, 738) and T3 noted that students developed "perseverance." (T3, 753).

Inclusion. During the first introduction to the study, it was mentioned that one of the specific research questions was related to the inclusion of all students in meaningful math activities. A solid, deep, common understanding of inclusion was not developed with this group.

Four of the classroom teachers and all three numeracy coaches identified examples of inclusion during the expedition.

NC2 shared a story from the expedition to the gym with T1. She recalled that they had gone into the gym and asked the students 'what do you notice?' and 'what do you wonder?'. One student wondered about the total baskets he could make if he shot "three baskets on Monday and two baskets on Tuesday". (NC2, 851) NC2 added that the question was celebrated because "that's where he was at" (NC2, 852).

T2 described a student finding his own measurement tool and exploring a question using a self-developed strategy. In this example, the students were trying to determine the total number of people that could eat lunch at a set of picnic tables. The students were trying to determine how many people could sit along one picnic bench. She stated that the student "actually grabbed a chair, cause he's like, you can sit comfortably, and he used the size of the chair as his measurement tool and he literally took the chair and put it on the bench and sat on it, and [redacted] [NC3] and I were kinda looking at each other like what? And this is one of my kids that is not very verbal, and he dives into every question. He sees things differently than us, or than most of us. And it was, that was his measurement tool, but he got the exact same result as just four people sitting there knowing that we are not just sitting bum to bum so it was the same result, but it was interesting that his measurement tool was a chair, knowing that that was the size. He never thought about a ruler or using a meter stick and not knowing what do to with that measurement" (T2, 713 – 720).

T3 recalled that the two students in her class that have individualized learning goals “both got involved and even if they weren’t doing the same math as the other kids they were able to help decide what food they were picking out” (T3, 853 – 855). She added that all the students had “something to contribute to the group” (T3, 859).

T4 stated that “the engagement was through the roof, like through the roof, I had kids who are weak and math phobic and they were the ones digging into it more than anyone. It was amazing, um, the inclusion was amazing as well, like I was able to include a lot of kids” (T4, 63- 65). She shared that the students who had individualized goals “felt like they were a part of it” (T4, 67) and the “Some of my weakest kids who would normally give up at any site of a challenge really were into it and, and. And with ideas and interesting enough had a lot of higher end ideas” (T4,)

T5 shared that when his class had prepared their lists of items to buy at the grocery store they had considered classmates with allergies. One of the students in the class needed to maintain a gluten free diet. Once at the grocery store students noticed that there was a price difference between the cost of gluten free cupcakes and regular cupcakes. This turned into “an interesting conversation back in class as well because those gluten free cupcakes that came in a pack of six” “were very expensive and some kids were like ‘nine dollars for cupcakes’ that’s crazy, so it was a good opportunity to say, well yes, but if we are including everyone in this party we can’t just not provide a cupcake” (T5, 445 – 448). In this case the inclusion represented social inclusion. T5 also shared that there were opportunities for students to approach the problem in different ways. He recalled, while referring to students working in a group of three to find the circumference of the basketball hoop, that “there are two that are so called strong and one that is so called not, and she had the brilliant idea to try one of the four square balls that they play with outside at lunch and that's a lot bigger and then see how well that fits through" (T5,). This further highlights the opportunities that students will create for themselves when the path is opened to them.

NC1 noted that because there were so many different pieces to the expedition, the experience built empathy and collaboration skills as students could say "I can help you here, but can you help me there" (NC1, 526).

Students as Leaders of Their Own Learning. To direct their own learning, students identify a learning goal, plan the steps needed to reach that goal, and then reflect upon their success in achieving that goal. Within each expedition, students did have opportunities to direct their own learning by posing their own questions, selecting their own strategies, choosing their own tools, presenting their own findings, and sharing their own stories.

T4 had captured a key piece of handing the learning over to the students when she told her students that they were to identify what they needed for the next class. She stated that it was a mixture of needs saying that "sometimes I had to go take more pictures and sometimes it was you have to go measure how high your roof is" (T4, 626 – 627) or a lesson on "how to find volume" (T4, 626 – 627). Handing over this aspect of learning to the students was empowering for both the students and the teacher. The teachers and numeracy coaches all commented that aspects of what the students had done had surprised them.

The movement towards student leadership in learning is a challenge. T2 identified a concern around student leadership. She shared that she had “a very compliant class” (T2, 275) and as a result did not see the student ownership like others described. She further explained that sometimes her students were confused noting that “some groups wanted to measure things even though they didn’t really know what they were measuring and even though they didn’t realize that they could just count” (T2, 205 – 207). T2 did indicate that she believed in the value of having students find their own way saying that “having the patience to let them figure out what they needed, like you know in the back of your mind that they need this, like they will need this measurement at some point, but to give them that freedom to not tell them” (T2,). Further concerns were raised about students making decisions about the tools to use to gather data. T2 stated that “you give them tools and they are like, oh, let’s use this tool even though we don’t know why” (T2, 217). This was supported by T5 who added that “that speaks to how well we have engrained the traditional way into them. They don’t know what equipment they need” (T5, 670 – 671). NC3 also supported this idea stating that “they are used to us handing it over. We are going to do this and here’s what you need” (NC3, 673). These two comments support the need for learning models like Expeditionary Math. For students to learn to take the lead, they will need to be presented with opportunities where they can develop those skills. When we see that learners are struggling with a skill, especially when that skill is affecting their learning, as teachers we need to create opportunities for learners to develop those very skills.

Teacher Support Group. All five classroom teachers and the numeracy coaches agreed that having a partner for a first try at Expeditionary Math was essential. T1 stated that "it was scary, but it was really nice having someone else there, being able to bounce ideas off each other. I think you probably all had similar experiences where you're like I don't know what to do here and we're both sitting here going 'I don't know what to do here but we'll talk it through" (T1, 582 – 586). NC1 added that "just having the opportunity to have the conversation and talk about successes and talk about where you hit road blocks" (NC1, 821 – 822). T2 stressed that she did not "think that you can do this alone. You need somebody whether it is a coach or someone in your building" (T2, 824-825).

Challenging our Perception of What Math Is. Two teachers noted that their students had been busy 'doing math' but that what they were doing was not contributing to developing a solution for the problem. T2 stated that she had observed her students "doing some kind of math, measuring, calculating, but the big picture wasn't there" (T2, 729 – 730). T5 recalled observing a student measuring in the corner of the gym. He noted that the student was "using a tool, measuring something" (T5, 724) and that the student believed that he was accomplishing the task, however, "his measurement was serving no purpose, but it certainly looked like math probably to him" (T5, 724-728).

Student Learning Stories and Portfolio Pages

A Learning Story is a collection of photos, work samples and written reflections created by the learner, that tell the steps involved as they move from initial wonderings to proposed solution.

A total of fifty-five Learning Stories were collected. Each teacher adjusted the instructions of the Learning Story to meet the needs of their students. This is a positive outcome in that teachers took advantage of making the experience unique and is negative in that is has created a lack of consistency between the expectations of the stories and portfolio pages. T1's Learning Stories consisted of students' writing, sketches, and calculations. These Learning Stories were completed individually. T2's Learning Stories consisted of student photos, student writing, sketches and calculations. These Learning Stories were completed in groups. T3's Learning Stories were a mix of the portfolio pages and Learning Story. They consisted of student calculations around a specific outcome with some students writing. T4's Learning Stories and portfolio pages are mixed and include student photos as well as calculations and some writing around a specific curricular outcome. T5's Learning Stories consist of student photos, student writing, calculation and sketches.

All teachers had their students create a Learning Story. However, several of the Learning Stories were not shared because students had not returned their parent consent forms. The Learning Stories are pictured in Appendix R. Student writing, sketches and calculations have been matched with the specific curricular learning outcomes that they represent.

T1 Expedition to The Gym. Seventeen Learning Stories were collected from T1's expedition to the gym. Within these Learning Stories students had given evidence indicating that the following curricular outcomes were being explored.

- 5.N.2 Students will apply estimation strategies, including front end rounding, compensation, compatible numbers in problem solving contexts.

- 5.N.3 Student will apply mental math strategies to determine multiplication and related division facts to 81 (9 x 9).

- 5.N.5 Students will demonstrate an understanding of multiplication (1 – and 2 – digit multipliers and up to 4 – digit multiplicands) concretely, pictorially, and symbolically, by using personal strategies, using the standard algorithm, estimating products to solve problems.

- 5.N.10 Students will compare and order decimals (tenths, hundredths, thousandths) by using benchmarks, place value, equivalent decimals.

- 5.N.11. Students will demonstrate an understanding of addition and subtraction of decimals (to thousandths), concretely, pictorially, and symbolically, by using personal strategies, using the standard algorithms, using estimation, solving problems.

- 5.SS.2 Student will demonstrate an understanding of measuring length by selecting and justifying referent for the unit ~~(mm), modeling and describing the relationship between mm and cm units and between mm and m units.~~

- 6.N.5 Students will demonstrate an understanding of ratio, concretely, pictorially, and symbolically.

- 6.N.8 Students will demonstrate an understanding of multiplication and division of decimals (involving 1- digit whole number multipliers, 1- digit natural number divisors, and multipliers and divisors that are multiples of 10), concretely, pictorially, and symbolically, by using personal strategies, using the standard algorithms, using estimation, solving problems.

- 6.SS.3 Students will develop and apply a formula for determining the perimeter of polygons, area of rectangles, ~~volume of right rectangular prisms~~.

- 6.PR.1 Students will demonstrate an understanding of the relationships within a table of values to solve problems.

- 6.PR.2 Students will represent and describe patterns and relationships using graphs and tables.

Portions of some multi-component outcomes have a strikethrough to indicate that those components of the outcome were not addressed.

T1's student Learning Stories also contained evidence of the strategies of Expeditionary Learning.

Explore. Each Learning Story begins with a question that required some form of mathematical thinking to solve. The depth of the questions varies. Each question is a closed question, in that each does have the potential to have one correct solution. Students had wondered about many things.

- How many chairs would you need to stack to reach the top of the gym?
- Which color line has the greatest total length and by how much?
- How much Velcro was needed to hang the mats on the walls?
- How many benches could fit around the gym?
- How many people could fit comfortably on the bleachers?
- Is the gym floor made of morel light or dark wood?
- If we were to recover the green mats in the gym, how much fabric would you need?
- If the banners were all lined up in a single row around the top of the gym, how many could we fit before needing to drop down to a new row?
- How many buckets of paint would be needed to repaint the gym?
- How many laps would it take to run 10k?
- How many people enter the gym on an average school cycle?
- How much area would each volleyball player be responsible for on a volleyball court?

Research. Each Learning Story showed evidence of research to identify and gather the data and information that was needed to determine a solution. Stories that required measurements also showed evidence of the students' abilities to select and use tools for gathering data. Although a formula could have been used to support the process of finding a solution for the perimeter of the gym, the area of the light and dark wood in the floor, the area of the volleyball court and the surface area of the green mats, students did not specifically state the required formula. Their stories only showed their calculations. Each Learning Story had a sketch. Forms of research evident in the Learning Stories were:

- determining the height of the gym ceiling,
- determining the height of ten stacked chairs,
- measuring the length of the floor lines,
- measuring the length and width of the gym and using that data to determine the perimeter,
- measuring the length of a bench,
- counting the number of rows of bleachers,
- measuring the dimensions of the green mats and using that data to determine surface area,
- measuring the width of a banner,
- measuring the length and height of each gym wall and using that data to find area,
- counting the number of students in each class and the number of times each class comes to the gym, and
- measuring the length and width of the volleyball court.

The Learning Stories did not show that students had identified their own strengths or shared them with others.

Develop. Each of the student Learning Stories showed evidence of a plan. However, much of the evidence was collected under the headings Important Information, Other *Things We Need to Know and What We Need* as requested by the classroom teacher. As the Learning Stories were created as a reflection piece following the expedition it is not possible to say if the plan was made before or if it was a retelling. One of the Learning Stories was incomplete and missing a plan that could be followed. The Learning Stories do show that students considered what information they need and how they could gather it. They do not show that multiple strategies were considered, or that plans had been rejected or reworked. The stories do not show evidence of collaboration.

Create and Construct. The Learning Stories did show evidence of creating solutions. Students included the necessary units of measurement and all relevant data. There was evidence that they had correctly selected and used the necessary operations. Evidence of creating and constructing in the Learning Stories included the following:

- doubling the height of ten chairs (measured) until the height of the gym ceiling (measured) was reached,
- finding the difference is length between the two longest lines,
- dividing the perimeter of the gym by the length of a bench,
- multiplying to find the total number of seats from the number per row,
- calculating surface area,
- doubling the length of each measured line (measured half the gym),
- dividing the total perimeter of the gym by the width of a banner,
- dividing the total area of the gym walls by the area covered by one can of paint,
- dividing 10k by the perimeter of the gym,

- adding to find the total number of students in each gym class, and the total per cycle, and
- diving the total area of the volleyball court by twelve (players).

Reason. Some evidence was found in the Learning Stories to show that the students were reasoning. Evidence from the stories that showed reasoning included the following.

- Rather than counting and measuring by one, the student reasoned that using the height of ten chairs would be more efficient.
- Noting that each matt required two pieces of Velcro to be hung on the wall, the student reasoned that the perimeter of the gym could be multiplied by two to find the total length of Velcro required.
- A student reasoned that five people could sit on one row of the bleachers (but did not provided evidence as to why).

Students did not include in their reasoning any notes to indicate that they recognized that the perimeter could not be used to determine the number of benches or banners if the length of the benches or banners was not a factor of the perimeter. There was no reasoning to indicate that the student had considered that to run a lap of the gym is not the same distance as the perimeter of the gym. There was no reasoning to indicate that the area of the gym walls that the students had considered to calculate the total amount of paint required should not include painting over windows, doors and bulletin boards. Students did not identify and correct all errors in their stories. The student counting the ten chairs directly compared centimeters to meters and has indicated that a stack of forty chairs would reach the height of the gym at 732m. The student determining the amount of floor space that each player on the volleyball court is required to cover divided the floor evenly amongst the players and did not reason that different positions have different responsibilities and would therefore be responsible for different areas.

Communicate. Students used words, sketches and calculations to communicate their thinking through their Learning Stories. They communicated some ideas under the headings *Things We Need to Know* and *What We Need*. However, they were communicating a solution and not a process. Solutions communicated included the following.

- "You would need 40 chairs to reach the top of the gym." (T1S1)
- "The white lines are greater than the rest by 326.6m." (T1S2)
- "You will need 1768cm of Velcro to use to put all the mats up." (T1S3)
- "You will need 30 benches to go around the perimeter of the gym." (T1S4)
- "29 benches would fit around the perimeter of the gym." (T1S8)
- "64 banners would fit around a whole gym before dropping down to a new row." (T1 S9)
- "it takes 43.7636 paint buckets to repaint the whole gym." (T1S10)
- "1 chair is 77.5cm. It takes 52 chairs to reach the top of the gym (6.6m)." (T1S11)
- "30 benches could fit around the perimeter of the gym." (T1S12)
- "You would have to go around the gyn 141 times to reach 10k." (T1S13)
- "An average of about 1250 students enter the gym every school cycle." (T1S14)
- "Each player is responsible for 13.5m of the volleyball court." (T1S15)

Two students communicated part of their process along with their solutions:

> "5 people can sit on one row, and there are 16 rows on one bleacher. There was two bleachers so we had to multiply the bleachers by two. 160 people can sit on bot of the bleachers comfortably." (T1S5)

"First you find the line that looks the longest, times that by 2, the longest one is the long black one on the sides is 23m and times that by two and equals 46." (unclear/units) (T1S7)

Furthermore, there is no evidence to suggest that the Learning Stories were shared or that students had shared their learning within the group.

Connect. There is little evidence to show that students had made connections to other experiences of math concepts.

Reflect. The Learning Story is a tool for reflection, however, these pieces did not show evidence of reflection. No story contained any suggestions for improvement of further learning. No story spoke of students' strengths.

T2 Expedition to the cafeteria. Four Learning Stories were collected from T2's expedition to the cafeteria. Within these Learning Stories students had given evidence indicating that the following curricular outcomes were being explored.

- 5.N.2 Students will apply estimation strategies, including front end rounding, compensation, compatible numbers in problem solving contexts.
- 5.N.3 Student will apply mental math strategies to determine multiplication and related division facts to 81 (9 x 9).
- 5.SS.2 Student will demonstrate an understanding of measuring length by selecting and justifying referent for the unit ~~(mm), modeling and describing the relationship between mm and cm units and between mm and m units.~~

Portions of some multi-component outcomes have a strikethrough to indicate that those components of the outcome were not addressed.

T2's student Learning Stories also contained evidence of the strategies of Expeditionary Learning.

Explore. Each of the Learning Story began with the same question, "Will we be able to have all students comfortably eat in the cafeteria next year?" There was no evidence to suggest that students explored beyond that question.

Research. All four Learning Stories showed evidence of students gathering data to be used in creating their solutions. Each story had a photo of the students measuring. Three of the stories included a sketch with dimensions or drawn seats.

Develop. Although students did take steps to come to a solution, there was no evidence that a plan was created before they began. One of the Learning Stories did retell the process used using *What We Did* and *What We Learned* headings.

Create and Construct. Each Learning Story showed evidence of students creating a solution to the problem. Evidence of creating and constructing in the Learning Stories included the following:

- determining the total number of students how stay for lunch,
- adding to find the total number of seats at one table,
- multiplying the number of students who can comfortably sit at one table by the total number of tables in the cafeteria, and
- finding the difference between the total number of students who stay for lunch and the total number of seats available.

Absent from the Learning Stories was any evidence to suggest that the students needed to be flexible. Three of the four Learning Stories were completed which suggests that these students did persist with the problem until a solution was reached.

Reason. There was some evidence of reasoning in two of the Learning Stories. In T2S1's Learning Story the student identified that they had gathered unnecessary data stating "We measured the length of the table. But after we relisd that we din't have to" (T2S1). In this case the student has identified an error but did not give an explanation for the correction. In T2S2's story there is a photo of a student using a chair to measure the amount of space that a student would need while sitting on the bench. They later wrote that "not everybody is the same sizes and maybe only 3 can fit in a section" (T2S2). The photo and note show that they were making sense of the amount of space that a body needs versus the amount of space that a student would like to have to feel comfortable.

Communicate. All four Learning Stories communicated a story. For example, T2S1's retold their process.

> We measured the length of the table. But after we realized that we didn't have to. After calculating the data and copying the data on the poster we determined that only 288 people can fit in the caff. We learned that we can fit all student in the caff comfortably" (T2S1).

Their story did communicate some of the steps involved in working towards a solution but did not clearly communicate how they arrived at the solution.

T2S2 told their story as follows.

> We used a chair to measure. We could fit 4 chairs in each section. We wrote down all the information that we found. We learnd how many kids are going to school here next year. We presented the information that we found and answered yes to the question. Not everybody is the same sizes and maybe only 3 can fit in a section. Most of the kids going into gr.8 want to eat in the hallway." (T2S2).

The student has retold their actions but not how they arrived at a yes answer.

T2S3's story was incomplete but did begin to retell their process.

> "What we did: 1st we measured the seat, because the tables has different lengths. During lunch we found out that 4 – 5 people can fit at one seat. And all the lunch tables are relatively the same length, each table has 5 seats and there

> are 18 table in the cafeteria. 250-300 people can sit in the cafeteria. We have found out that the biggest class will have 30 students but it's very rare to have 30 students and the lowest amount is 18 students in one class. There are 3 grade six academics, and both grade 7 & 8 have 4 academics" (T2S3).

T2S4's story contained only writing, with no sketches or calculations to support the group's thinking.

> "We had to measure the table benches to estimate how many people could sit at a table. We learned that two people equaled 72cm. So we just estimated that 4 people could sit at a table. We were talking and seeing how many people can fit comfortably at one table. We thought back to the morning and four people fit comfortable. We learned that four people can sit at one bench. So we did some calculating and 288 people can fit in the cafeteria" (T2S4).

Although all four of the stories shared a retelling of what the students did, they did not show evidence of mathematical language, collaboration or contribution.

Connect. The four Learning Stories did not show evidence of making connections.

Reflect. The Learning Stories had been completed after the learning activity and so provide some evidence of reflection. However, no story made suggestions for improvement or evaluated participation in the group. One of the stories showed that an inefficient strategy had been replaced with a more effective strategies by stating "we measured the length of the table. But after we relisd that we din't have to" (T2S1).

T3 Expedition to the Grocery Store. Thirteen Learning Stories were collected from T3's expedition to the grocery store. T3's Learning Stories were similar to T5's mixed version that included calculations and photos and some additional student writing. T3 and her students were able to visit the grocery store twice. Following their first visit they wrote postcards, one side of the card being photos of the students and the other side a space to write what they were doing and what they were learning. The Learning Story and portfolio page mixed documents were created by the students following the second trip to the grocery store.

The thirteen Learning Stories and portfolio pages showed that students were exploring the following two curricular outcomes.

- 7.N.2 Students will demonstrate an understanding of the addition, subtraction, multiplication and division of decimal to solve problems for more that 1-digit divisors or two-digit multipliers, technology could be used.
- 7.N.3 Student will demonstrate an understanding of solving problems involving percent.

The teacher created the reflection pages with the curricular outcomes already identified for the students.

T3's Learning Stories also contained some evidence of the strategies of Expeditionary Math.

Explore. Each story contained a photo of the student at the grocery store. Most of the photos do contain more than one product and more than one price that the students looked at.

Research. Each story included a photo of students at the grocery store gathering data. They began their calculations with information about the prices and quantities of items on their list.

Develop. The Learning Stories were dominated by calculations. Some of the stories did show evidence of organizing the data (prices and amounts) that students were working with but did not indicate if the plan for organizing was developed prior to, during or after gathering the data.

Create and Construct. Each of the Learning Stories shows calculations. T3S4 has indicated that they have rounded all decimal numbers to whole numbers before beginning their calculations. T3S9 has used a compatible number strategy for adding decimal numbers. All other Learning Stories show that the students have used a standard, vertical algorithm for addition of decimals. T3S3 has included vertical lines to show that the decimal place is lined up. All Learning Stories show that the students have used an equivalent decimal to represent the taxes and multiplied by their total. No student has shown how the percentage was converted to a decimal.

Reason. There is evidence to show that students have reasoned. T3S1 has included a note saying "I am over budget" however, the note does not indicate how they have determined that they are over budget or by how much. T3S2 has included a written note that says, "we have enough money for the party" and "I have enough money to buy another drink" however, T3S2 has not included a reason why or demonstrated that the totals they have found are less than the original budget.

Communicate. The Learning Stories are dominated by calculations. The students have included the name of the items for which they have found the prices. The photos of the students at the grocery store do communicate that this project was done in teams.

Connect. There is no evidence that connections to other contexts of concepts have been made.

Reflect. The Learning Stories were completed after the second expedition to the grocery store. They are reflection pieces however, they do not contain any suggestions for improvement or further learning for the student. No story speaks to the student's strengths or contributions.

T4 Expedition to Canadian Tire. Eight Learning Stories were collected from T4's expedition to Canadian Tire. Two of the Learning Stories were left blank. Within these Learning Stories students had given evidence indicating that the following curricular outcomes were being explored:

- 8.SS.1Students will develop and apply the Pythagorean theorem to solve problems.

- 8.SS.3 Students will determine the surface area of right rectangular prisms, right triangular prisms and right cylinders.

- 8.SS.4 Students will develop and apply formulas for determining the volume of right prisms and right cylinders.

The Learning Stories have been combined with the portfolio pages. Each Learning Story tells how the students decided which storage shed their teacher needed to purchase to store the items in her yard.

Explore. Four of the eight Learning Stories have photos of students at Canadian Tire. In each photo students are exploring the store and investigating the necessary items. Students are exploring how to measure each item.

Research. In the photos students can be seen measuring items at Canadian Tire. The students have written captions for these photos:

- “In this picture we were measuring the length width and height for the lawn mower. These measurements helped us find the total volume of the lawn mower” (T4S1),
- “We measured width of the bike and also the length and height to know the surface area” (T4S2),
- “We measured the height of the lawn mower because we needed to find the volume so we would know know if how much space it would take up” (T4S3), and
- “This is a picture of us measuring height this is related because without height we can’t find volume.” (T4S4).

Six of the eight Learning Stories show calculations for area and volume with dimensions gathered at Canadian Tire.

Develop. The Learning Stories do not show evidence of developing or following a plan.

Create and Construct. Six of the Learning Stories do show calculations. However, the calculations are in response to the question given on the side of the portfolio page and not as part of the student response to the shed question.

Reason. Four of the Learning Stories offered a reason for collecting the measurements at Canadian Tire.

- “These measurements helped us find the total volume of the lawn mower” (T4S1).
- “We measured width of the bike and also the length and height to know the surface area” (T4S2).
- “We measured the height of the lawn mower because we needed to find the volume so we would know if how much space it would take up” (T4S3).
- “This is a picture of us measuring height this is related because without height we can’t find volume” (T4S4).

Communicate. Four of the Learning Stories had photos, calculations and student writing. However, the writing was limited to describing what was happening in the photo and not a process of solution finding.

Connect. These Learning Stories, which are linked to the portfolio page, do connect between the lived experience at Canadian Tire and a traditional word problem. The connection is visible through the student written comment and the calculations relating to the word problem. For example, the students have written that they are gathering data to calculate volume and surface area. The word problems shown on the portfolio page are surface area and volume calculations.

Reflect. The portfolio pages and Learning Stories were intended to offer students an opportunity to reflect on their learning. These pages do show that students are capable of doing the required calculations, and that a link has been made between the calculations and a lived experience, however, the students do not offer suggestions for improvement, correct errors, replace inefficient strategies with more efficient ones, or comment on their strengths as learners

T5 Expeditions to the Gym and Grocery Store. T5 completed two expeditions; one to the school gym and one to a local grocery store. A total of thirteen Learning Stories were collected. Each Learning Story has a photo of students conducting field work, sketches, calculations and written notes. The Learning Stories that were collected following the expedition to the gym show evidence of the following curricular outcomes:

- 5.SS.2 Student will demonstrate an understanding of measuring length by selecting and justifying referent for the unit ~~(mm), modeling and describing the relationship between mm and cm units and between mm and m units.~~

- 6.SS3 Students will develop and apply a formula for determining the perimeter of polygons, area of rectangles, ~~volume of right rectangular prisms~~.
- 7.SP.1 Students will demonstrate and understanding of central tendency and range by determining the measures of central tendency (mean, median, mode) and range, and by determining the most appropriate measures of central tendency to report findings.
- 7.N.2 Students will demonstrate an understanding of the addition, subtraction, multiplication and division of decimals to solve problems.

Portions of some multi-component outcomes have a strikethrough to indicate that those components of the outcome were not addressed.

The Learning Stories that were collected following the expedition to the grocery store show evidence of the following curricular outcomes.

- 7.N.2 Students will demonstrate an understanding of the addition, subtraction, multiplication and division of decimals to solve problems.

- 7.N.7 Students will compare and order fractions, decimals (to thousandths), and integers by using benchmarks, place value and equivalent fractions and/or decimals.

Each Learning Story also showed evidence of the strategies of Expeditionary Learning.

Explore. Each story has a photo of student during their field work in the gym and at the grocery store. The photos show the students engaged in exploring the math in these locations. The Learning Stories are based on a student's journey towards a solution to a question that they posed.

- "How many bags of chips will we need for our party?" (T5S1)
- "How much will pop drinks cost?" (T5S2)
- "How much will pop drinks cost?" (T5S3)
- "How much money do we need to buy the candy we want?" (T5S4)
- "How many packs of Twizzlers do we need for 21 students?" (T5S5)
- "Is it better to buy a fruit tray or individual fruits? Which is more expensive?" (T5S6)
- "What is the area of the gym?" (T5S7) (T5S9)
- "How many people can comfortably sit in the bleachers?" (T5S8)
- "Which line on the floor is the longest? By how much?" (T5S10) (T5S11)
- "How many green mats would we need to cover the whole gym floor?" (T5S12)

Research. Each Learning Story included data gathered in the field. Student have measured in the gym or located information such as prices or quantities at the grocery store. T5S7 and T5S9 have included a list of materials required for gathering data.

Develop. These Learning Stories are a reflection of their adventure it is difficult to know if the students are following a plan or finding their way as they go.

Create and Construct. The stories do show evidence of calculations and of creating a solution. There is evidence that the students have correctly selected and used the necessary operations. This includes:

- measuring the length and width of the gym to calculate the area,
- finding the total number of students that can sit comfortably along one bleacher and then multiplying by the total number of bleachers,
- measuring the length of the colored lines and then subtracting the second longest length from the longest length,
- measuring the length and width of a green mat to calculate the area, measuring the length and width of the gym to calculate the area, then dividing the area of the gym by the area of a mat to find the total number of mats needed to cover the gym floor,
- adding to find the total cost of the chips,
- multiplying to find the total cost of the soft drinks,
- multiplying to find the total cost of each type of candy, adding to find the total cost of all the candy, and

- adding to find the total cost of buying individual fruits, comparing this total with the cost of a prepared fruit tray.

Reason. The students have included a description of how they arrived at their solutions. The solutions are a step by step retelling. The reasons for decisions that students have made are not clear. T5S6 has included a written description of comparing the cost of a fruit tray and the cost of buying individual fruits. In their description they explain that they had first selected the prepared fruit tray that cost $24.99. Then, they noticed that the individual fruit may be cheaper. They selected strawberries, oranges, apples and grapes, as these were the most popular fruits in their class and found the total cost for these fruits. They found that the total was less than that of the prepared fruit tray. They explain that the reason that they choose the individual fruits is because they are cheaper than the prepared fruit tray.

Communicate. The students have included written notes, sketches, calculations and photos in their stories. The written notes are a retelling of the steps they took towards their solution and do not fully describe their thinking or reasoning. *

Connect. The stories do not show evidence of students making connections to context or content.

Reflection. The stories are a reflection piece however, they do not show evidence of students making suggestions for improvement or for further learning. No student identifies a strength in their story.

Throughout the Learning Stories and portfolio pages reason, connect and reflect are not as well developed as the other strategies. Reason is described as an Expeditionary Math strategy in chapter 2 as i) judge the appropriateness of an answer using known facts and experiences, ii) accept and reject solutions using known facts and experiences, iii) draw conclusions based on current results and past experiences, iv) identify patterns and non patterns in solutions, v) derive formulas from experience, vi) identify errors and give reasons, vii) construct mathematical arguments using calculations, formulas, and experiences to support statements, and viii) organize, thinking, data, calculations and results. T3, T4 and T5 all commented that students had been engaged in production arguments, reasoning for and against different ideas. The details of those arguments were not captured in the student Learning Stories of portfolios.

Connect is described as i) identify and explain mathematical relationships, ii) apply newly learned mathematical skills and knowledge to new experiences, iii) make connections between new and old, iv) identify more questions for research, and v) extend learning to new situations. It was noted earlier that connections to other concepts and contexts were possible but not pursued in the expeditions of this study. Connections can be anticipated and emphasized by the teachers to teach students to do the same.

Reflect was earlier described as i) asses the effectiveness of a strategy of process, ii) suggest improvements to known strategies and processes, iii) replace inefficient and ineffective strategies with more efficient ones, iv) evaluate ones' participation and contribution to the learning community, v) persist, stick with a problem until a solution is reached, vi) recommend improvements to the way we work together, and vii) evaluate individual strengths and identify next steps.

Chapter 5

Synthesis and Analysis

Initially this study set out to answer three specific research questions:

1. How does the Expeditionary Learning approach support the meaningful inclusion of all learners in mathematics education?
2. What specific mathematical content knowledge is learned, omitted, or transformed through the Expeditionary Learning model?
3. What problem solving strategies are developed, omitted or transformed through a learning expedition?

When examining the interview notes, the focus group discussion and the students' work samples, common themes emerged; teacher hesitation, written word problems as mathematical problems, inclusion, teachers as risk takers, the changing role of the classroom teacher: towards responsive teaching, what more can be learned through expeditionary mathematics, students as leaders of their own learning, challenging our perception of what math is, and teacher support groups. Similarities between the themes were analysed. Two new combined themes emerged. This chapter explores those new themes; i) taking risks to re-define mathematics and ii) inclusion by opportunity and choice. The original three research questions are included in these themes.

Taking Risks to Re-Define Mathematics

Webster's dictionary defines mathematics as "the science of numbers and their operations interrelations, combinations, generalizations, and abstractions and of space configurations and their structure, measurement, transformations, and generalizations. Algebra, arithmetic, calculus, geometry, and trigonometry are branches of mathematics" (Merriam Webster). From this perspective, it is understandable that schools, teachers, and students have developed a narrow view of learning mathematics. The mathematics of Expeditionary Math is to be living, rich, and connected.

Expeditionary Mathematics is intended to be an adventure. At the beginning of an expedition, much of what is ahead is unknown. There is risk involved. T2's responses to the initial interview question relating to a typical math class, conveyed a lack of confidence and an uncertainty around teaching math. In recent years there has been much debate about how best to teach math (Marshal, 2003; Schoenfeld, 2004). Teachers have been surrounded by the arguments of traditional mathematics instruction and the more innovative, 21st century, student-centered approaches. Chernoff and Stone (2014) describe this feeling of uneasiness as math anxiety. They found that "teachers who were math anxious retreated from current best practice guidelines and instead used more traditional teaching styles involving individual seat work and focusing on skills instead of concepts" (Chernoff & Stone, 30, 2014). To take on the challenge of Expeditionary Math teachers must also be risk takers. The other teacher participants described a typical lesson as a very typical, traditional lesson. They used words such as "structured" (T4, 6), "routine" (T4, 6), and "direct teaching" (T1, 7). These words describe certainty and knowing,

It was noted in Koen's description of engineering, "the strategy for causing the best change in a poorly understood or uncertain situation within available resources" (Koen, 1985, p.5), that a problem is a poorly understood or uncertain situation. Risk is required to explore what is not understood or uncertain. In the earlier chapters, it was suggested that students be ready to take on the challenge of jumping into these poorly understood and uncertain situations as this is where learning happens. This challenge must also be passed on to teachers. Risk in Expeditionary Mathematics is essential and threefold. First, there is risk involved in changing what teachers think of as their classrooms. Venturing out of our rectangular, four walled boxes should be welcomed as an adventure full of possibilities. Secondly, exploring new, open, undefined, messy, mathematics for which the teacher does not yet have an answer key, should be exhilarating. Finally, creating an opportunity for students to lead the way, to make decisions, to own the learning, the challenges and the failures should be encouraged. This is how students will develop independence as learners, leaders and citizens.

Changing What We Think of as Our Classrooms. Expeditionary Mathematics was a new idea presented to the participants of this study. Participation in the study was optional and, initially, all the teachers were able to identify a goal for their expedition, indicating that they were willing to give it a try. Once back in their schools preparing for the expeditions, teachers shared that they did feel hesitant. Two of the teachers avoided leaving their school altogether to minimize the risk involved. The purpose of exploring the community beyond the school is to build ties between the people who live and work in the community and the students. It is to support the development of a sense of place in students and a sense of respect for those who take care of and use these areas. Ardoin found that “a sense of belonging to a specific, localized community is one of the most profound sources of human identity” (Ardoin, 2006). Investigating the spaces that surround the school increases the potential for equity of experience for of all learners. Many of our students have not lived the same lives as we have and have perhaps not visited all the spaces and places our communities have to offer. Following the expeditions teachers used terms such as; “awesome” (T4, 110), and “hugely beneficial” (T2, 248) to describe their adventures. Both teachers who remained at school for their expeditions did comment that they would like to explore their communities and expressed an interest in looking for locations beyond the school to try.

An expedition beyond the school is an avenue to share the work of developing citizens with our communities. In “What Kind of Citizen?” Westheimer (2015) notes that “students who acquire knowledge in conjunction with community experiences gain an understanding of knowledge as fluid, collaborative, and context-driven” (p.91). He added that “to develop the kinds of relationships, attitudes, dispositions, and skills that are necessary for them to engage in democratic and community life” (p.91) students need to know more that formulas. T3 also commented on the benefits of sharing this experience with the community. She said, “it was just so good seeing them in Family Foods and the staff there was really pumped that we were there and we got to see kids like in the produce section, they were taking stuff and weighing it and finding prices that way” (T3, 309-312). The expeditions to Canadian Tire and the grocery stores allowed a community store to engage with student learning, to have a fresh look at what school can become, and to gain experience teaming with youth.

Exploring New, Open, Undefined, Messy Mathematics. During the initial interviews teachers gave examples of traditional, written word problems when asked to share a problem that they had recently tried with their classes. These problems were straightforward, contained the necessary variables, indicated a possible solution, and left little room for wondering. Four of the expeditions were based on a teacher question, while two expedition, both to the gym, gave students the opportunity to start with their own questions. An analysis of the questions posed by students during their expeditions revealed that these questions were also closed, “math questions”. By “math questions” I mean a question that would involve a calculation or would be solved by working with numbers. The questions did require data to be gathered to answer but, in each case, there would be only one correct answer. The question posed for the expedition to the cafeteria, “Will we be able to have all the students eat comfortably in the cafeteria next year?”

was created by the teacher. Although it had the potential to be open, the pre-existing understanding that the same number picnic tables would be used each day and arranged in the same fashion closed the question. The question did allow students to experiment with multiple strategies. The expeditions to the grocery store were open to student questions that fell under the main question of planning for a class party. As T3'S and T5's expeditions unfolded, they created space for leaving to diverge from the main question. Both T3 and T5 commented that some students had been wondering about taxes and that that pursuing those questions had resulted in new learning for the students and for themselves. Table 2 compares student questions and strategies. In the end, only one total number of students eating comfortably in the cafeteria was possible. The expedition to Canadian Tire supported students' investigation to find the most suitable storage shed for their teacher. Initially this question had the potential to be closed as the teacher had defined the specific items that needed to be stored in the shed and specified the location in her yard where the shed would be placed. As the expedition unfolded, the teacher opened the question. Students challenged the teacher's need to have all the items in the shed and she responded by asking them to prepare a proposal, with reasoning, indicating which items may not need to be included in the shed. This allowed the students some flexibility to create multiple arrangements of the items within the shed. The students challenged the teacher on where the shed needed to be purchased. Initially the teacher had determined that the shed would be bought from Canadian Tire as that is where they were going to be visiting. The teacher had wanted the students to be able to see and walk into the sample sheds at the store. This opportunity would present the students with a lived experience of being inside a closed $4m^2$, $8m^2$ and $10m^2$ space. This representation would allow students to connect the sketches and measurements with an authentic space. Once they were back in the classroom and working on their solutions the students wanted to explore more options. Again, the teacher opened the question. She encouraged the students to look for storage sheds from other stores. Because visiting those stores and exploring those spaces was not possible, the teacher encouraged the students to map out the base of the shed on the classroom floor. This gave students another representation to consider. In this expedition the teacher started with some significant concerns relating to the 'how to' of an expedition and in the moment, she was able to repeatedly create opportunities for the problem to become messy and undefined.

Ostroff (2016) lists curiosity as one of the driving forces of learning. The ability ask questions is key to understanding our world. How do we encourage students to move from closed math questions to deeper questions that challenge their thinking and push them towards new growth? Questions that develop into expeditions are wider than a traditional word problem. The mathematics to be developed isn't the focus. It is a woven into the context. For example, the question that sparked the adventure to the zoo contained no numbers. When students wondered "Are Zoos Ethical?" they were referring to the treatment of animals. The mathematics that they engaged with was in the details of their arguments. The area of the enclosure was used to argue about the space provided for each animal in the zoo in comparison to the natural range. The length of a jump of a kangaroo was used in a pattern to demonstrate that the length of a jump was compromised in the given enclosure. The volume of water in the penguin exhibit was used to compared to the top speed of a penguin swimming in a zoo versus the top speed of a penguin swimming in the wild. The question "Are Zoos Ethical?" is open, messy and undefined and therefore full of mathematical potential. Similarly, the question "Who Eats?" was not easily answered with a calculation. The question lead students to investigate the costs of food and the earnings of someone working a minimum wage job. Further investigations in the field allowed students to explore weight, volumes, rate, ratios and taxes. These questions are equally accompanied by risk and possibilities. Britton emphasizes the importance of questioning in *On Design* (2017) where he states, "you probably spend as much (or more) time trying to identify the problem that needs to be solved as you do solving it" (p.6).

When questions are broader the potential for learning broadens as well. In each of the expeditions studied, students had opportunities to develop skills and knowledge beyond the intended curricular outcome. NC3 noted the expedition was about "just trying to get them thinking beyond what they are seeing" (NC3. 45 – 46). T4 illustrated the idea of thinking beyond the question when she retold the story of her students asking for the roof height. The students were determining how best to fit the storage shed into the location that the teacher had specified. They were able to make the footprint of the shed fit in the space but then carried their investigation further, considering how snow and rain would fall from the roof of the house onto the shed, how the shed door would open next to the fence and how, if by turning the shed, they could increase the space afforded to it. If we compare this thinking to a more traditional word problem, students may have been asked to determine the area of a 2m by 3.5m storage shed. The mathematical calculations would be the same, but T4's students also sharpened their ability to consider contributing factors and experience a problem from multiple perspectives.

For students and teachers to experience the definition of learning mathematics through engineering and inclusion as described in Chapter 1, the following goals need to be considered.

1. **Strategy**: Cultivate in each learner an intuitive sense and disposition to naturally draw upon a variety of flexible, non-standard, personal strategies in response to any problematic situation.

2. **Best change**: Unleash in each learner a passion to create positive change in the world through innovation and invention as a result of working collaboratively with others in creativity-intense environments.

3. **Poorly understood or uncertain situation**: Develop in each learner the confidence to let curiosity take over. That is, to explore, to pose authentic questions, to investigate additional factors that will influence the solution, to turn ambiguity into determination and failure into invention.
4. **Available Resources**: Inspire in each learner a belief that where there are limits, there are possibilities. In other words, to create an atmosphere of optimistic enthusiasm where all spaces are learning places, where each of us and all things are teachers.

To realize these goals, students and teachers will need to embrace the possibilities of risk.

Students as Leaders of Their Own Learning. If students are to take the lead, then teachers will need to be ready to respond in ways that supports their learning paths. Responsive teaching is student-centered, connected to students' thinking, and follows students' ideas in the classroom (Robertson, Atkins, Levin, & Richards, 2015). To teach responsively, "teachers notice and interpret their students' thinking, making sense of their ideas and approaches, including to identify possible beginning of disciplinary practices and concepts. Then, teachers respond by adapting their objectives and plans based on their interpretations" (Watkins, McCormick, Bethke Wendell, Spencer, Milto, Portsmore, & Hammer, 2018).

Four of the teachers and two of the numeracy coaches commented on the changes they noticed towards responsive teaching. T1 noted that having the numeracy coach as a partner was helpful because they "were able to debrief quickly and say ok next class is coming in so what are we going to change" (T1, 30 – 31). He shared his comparison of preplanned and open lessons, realizing that lessons “where I am just going to try to figure out where the kids are going to go with this and those lessons and those activities are so much more rich” (T1, 609 – 611). T2 agreed that it was a different type of preparation and the her and the numeracy coach did “a lot of figuring out as we go” (T2, 176). She shared that she “did a lot of planning ahead of time” and had “ended up not even using half of it” (T2, 617). She added that “there was a lot of making decisions on the fly and we had to plan right after” (T2, 619). She stressed that “you cannot preplan this, you cannot plan out the whole thing. You have that original, tiny place to start and then you’ve got to plan every day and you have to be ok with that and it be normal” (T2, 621 – 622).

T4 described the shift she experienced from teacher-centered to student-led learning. She explained that “at the end of each class that they needed to tell me what they needed from me for the next class” (T4, 623 – 624). She continued saying that “sometimes I had to go take more pictures and sometimes it was you have to go measure how high your roof your roof is” (T4, 626 – 627). At one-point students had said to her “we need to know how to find volume” (T4, 627). She concluded that she had begun “letting the kids dictate what they need and if (she) knew that there was something that they needed, and they just weren’t asking (she) might slip it in in some ways but most of the time they figured it out” (T4, 629 – 631).

One of the numeracy coach (NC3) summarized the conversation saying “the difference that I hear is that you need to know the difference between teaching curriculum and teaching kids (NC3, 842 – 843). Adding that “when you are teaching kids you will take risks (NC3, 845). NC2 also noted the difference between traditional and expeditionary teaching and learning saying that “it wasn’t you dictating, yeah, do this next, do this thing” (NC2, 600) “they were the kids ideas” (NC2, 589).

To be responsive, teachers will need to be confident and knowledgeable about the curriculum, the necessary math concepts, and about each student's personal learning style. Responding to the students may look like directing students to a mini-lesson or conference for further investigation of a topic that they are just beginning to uncover, it may look like spending time working through similar questions or modeling with manipulatives in a workshop, or it may look like further time spent exploring in the field. Teacher planning and their role in the classroom will need to shift towards anticipating struggles and connections. For example, students in Group T1S1 had been engaged in measuring and recording. They had measured the height of one chair, then stacked another chair on top, measured, then stacked another chair on top, and measured again. This pattern went on until the height of ten chairs was measured. The students prepared a table to show their pattern. To the right of their table they calculated the total number of chairs needed by doubling the height of ten chairs until the height of the roof was reached. A big idea of patterning is that patterns repeat, and that once the core of the pattern is known, an equation can be developed and used to solve any term in the pattern. In this case, $y = 9x + 77$.

As another example, one of the portfolio pages from the Canadian Tire expedition showed students trying to determine the length of a lawn mower. The handle of the lawn mower was adjustable to the height of the person walking with the mower. Students struggled to get an accurate measurement because the handle sat at an angle. Students could have used the Pythagorean theorem to solve for the distance that the handle extends from the mower (height of the handle bar, a, length of the handle, c, unknown, b).

By taking advantage of the teachable moments that spontaneously happen with students, we can support students with ‘just in time’ rather than ‘just in case’ learning. ‘Just in time’ learning meets an authentic need for new knowledge in the immediate moment.

For teachers to begin to take risks and re-define their vision of mathematics we will need the support of colleagues. Earlier, having the responsibility to contribute to the shared learning of the group, was listed as a key element of inclusive schools. The same principle is true for teachers. As educators, we all have a responsibility to contribute to the shared body of knowledge about teaching and learning. Supporting each other by asking questions from a place of curiosity, sharing experiences and resources, and debriefing experiences will contribute to our collective learning and ultimately, our shared strength as teachers.

Inclusion by Opportunity and Choice

Expeditionary Learning attempts to create opportunities for all learners to explore, grow and celebrate their learning. It attempts to meet learners where they are, by having students set the starting point with their own questions, and by being open to any number of paths towards a solution. The balance of fieldwork, mini-lessons and conferences, and workshops is designed to provide multiple means of presentation, action and expression, and engagement (CAST, 2011) In Expeditionary Learning, students experience the same concepts through a variety of contexts; how the concept is lived in world during fieldwork, how it can be modelled with manipulatives during a mini-lesson, illustrated as a sketch, or described using symbols and how it can be explained with words during a workshop.

Creating Space for Opportunity. During the initial interviews it became evident that the participants did not share a common understanding of inclusion. When asked to describe how all students were included in their classes the responses varied considerably. T1 listed rich tasks with multiple entry points and flexible groupings as practices that promote inclusion. A rich task is defined by McDougall (2004) as: 1) encompassing a set of attributes that are grounded in a real-world problem, 2) allowing for multiple solutions, 3) providing opportunity for engagement with many solution strategies developed by students, 4) involving a variety of representations, 5) leading students towards making connections between mathematical ideas, 6) expecting students to communicate their reasoning process, and 7) assigning reflection as a continuous process. The problem that T1 shared as a recent problem that the class had worked on doesn't meet these criteria. T1 had given the ice cream problem, where students were to determine the total number of possible combinations of ice cream for a two scoop-cone. T1 led an expedition to the school gym. Once there, students had the freedom to wonder and develop their own questions. All students in T1's class participated in the adventure and were able to pose questions and work towards their solutions.

As further defined by T1, flexible groupings are a way to create small student groups that change each class. This allows all students to the opportunity to work closely with several different students during a project. T2 and T4 also shared that they use random groupings as an inclusive strategy.

T4 shared her personal struggle with inclusion. She provided an example of a parallel task, where some students may be working on identifying right triangles while others are developing the Pythagorean theorem as academic inclusion and an example of groupings with roles to socially include classmates. It is her struggle that is most interesting. She had identified her goal for inclusion as having the students “moving forward at the level that they are at” (T4, 52) but admitted that she had not found a way to have all students as fully included as she would have liked.

During the Focus Group Discussion, the teachers were asked to reflect on the inclusivity of their experiences. T4 indicated that “the inclusion was amazing” (T4, 65) and that she had been “able to include a lot of kids” (T4, 65). She added that the two students working towards individualized learning goals were “a part of it” and “able to participate” (T4, 67). Her excitement and confidence shown through as she happily shared her stories. T4 had prepared a portfolio page and Learning Story combination. In this case, the learning objective had been identified for the students. Six of the eight portfolio pages did show evidence of the students' ability to successfully respond to questions related to that outcome. Six of the responses showed that students selected the appropriate formula, substituted the variables and calculated the correct answer. Four of the pages included photos of the students on site at Canadian Tire. In each of the photos, all the students pictured are engaged in gathering the necessary data. Each student is fulfilling a role. It is evident that the task could not have been completed without the help of all the team members. Two of the Learning Stories from this expedition to Canadian Tire were blank. This is evidence that the project was not fully inclusive, as two students were not able to reflect on the task to share their learning in a way that was offered.

T4 also shared two stories that illustrated the inclusiveness of the expedition. She noted that, following the visit to Canadian Tire, the engagement of the students did not diminish. She said, “some of my weakest kids who would normally give up at any site of a challenge really were into it and, and with ideas and interesting enough, had a lot of higher end ideas” (T4 129 –

131). She also shared the story of T4S8 “who is math phobic and struggles, struggles, struggles, and she was the one who had the idea that if we turned the bikes on their sides, if we turned them up then they would reach that height (motioning with her hand), I did talk with them about that but she was the only one that went there and yeah, and for her, usually she is like, there this is good enough, and she felt like, like she has talked about it and said it was the nest math class and this was awesome” (T4, 136 – 140). These observations were echoed by the numeracy coach who supported this expedition “right from your intro she was like, oh this is awesome” (NC1, 142).

Multiple Means to Choose From. In Chapter 1, an inclusive school was defined as “one in which the teaching and learning, achievements, attitudes and well-being of every young person matter” (Ofsted, 2000). In addition, I noted that the Center for Applied Special Technology (CAST) offers three guidelines for UDL upon which all learning experiences should stand. According to CAST, students should be provided with “multiple means of representation”, “multiple means of action and expression”, and “multiple means of engagement” (2011). In Table 2, examples drawn from the expeditions have been sorted into a chart relating them to these three UDL Guidelines.

Table 2

Modified Universal Design for Learning Guidelines with Expeditionary Examples

Multiple Means of Representation	Multiple Means of Action and Expression	Multiple Means of Engagement

Options for Perception
- Visits to locations being investigated (cafeteria, gym, grocery store, Canadian Tire)
	Options for physical action
	- Movement to and from the locations (cafeteria, gym, grocery store, Canadian Tire)
	- Physically exploring:
	  - Picnic table and open space in the cafeteria
	  - Lines and equipment in the gym
	  - Items in the grocery store
	  - Locating and measuring items to be stored, exploring the sample sheds at Canadian Tire
	- Choose of tools for gathering data
		Options for recruiting interest
		- Choice of strategy and process to determine the number of seats in the cafeteria
		- Choice of what question to pose and solve in the gym
		- Choice of questions items to 'buy' at the grocery store
		- Choice of how to arrange items in the storage shed and which items to exclude at Canadian Tire
		- Relevance and authenticity of cafeteria problem to students determining where they will eat,
		- Relevance of planning a class party
		- Relevance and authenticity of the storage shed question because student truly believed they were helping their teacher

Options for language, mathematical expressions, and symbols - Photos of students at work in the cafeteria, the gym, the grocery store, and at Canadian Tire, - Catalogues and websites for items and shed at Canadian Tire	Options for expression and communication - Learning Stories and portfolio pages contained samples of student written explanations, calculations, sketches and photos.	Options for sustaining effort and persistence - Teacher adjusting (opening) the criteria as the project developed - Photos of students at work with teams of peers (cafeteria, gym, the grocery store, Canadian Tire)
Options for comprehension - Pre-expedition questioning activity before visiting the gym and the grocery store - Storage shed story from T4 to develop an understanding of the need for a shed - T5 mini-expedition to the gym prior to expedition to the grocery store	Options for executive function - Students used tables, charts and sketches to organize - "things we need to know" and "what we need" questions answered before visiting the gym	Options for self-regulation - Completed Learning Stories and portfolio pages

The analysis of the expeditions did reveal several examples of the three UDL guidelines. Visiting locations that were being studied allowed students "options for perception" that would not have been available to them in the classroom. Being in a place includes hearing, feeling, smelling and experiencing the vibe of the surroundings. The physical movement from the classroom to the location and back again provides students with an authentic and purposeful movement break. While on site students continue to have opportunities to move and experience space. Selecting and using tools to gather data requires that students move around, strengthening their special awareness in a 3-dimensional way. Teachers encouraged students to share their thinking with written explanations, sketches, diagrams and calculations providing each learner with multiple ways of sharing. All three expeditions included opportunities for choice and autonomy. Students were able to define their own process and path to a solution. They were presented with options for tools to use for gathering data and how to present their findings.

These expeditions did not provide students with opportunities to "highlight patterns, critical features, big ideas, and relationships" (CAST, 2011). However, in each expedition opportunities to highlight patterns and big ideas were missed.

One of the inclusive features of Expeditionary Learning is the fluid movement between fieldwork, mini-lessons and conferences, and workshops. The Learning Stories collected from the expedition to the gym show student misunderstandings. The stories also show inefficient strategies. For example, Learning Story T1S1 shows a table comparing the number of stacked chairs and their height. Once the student had reached ten chairs they doubled this number until they reached the height of the gym. It is not clear if doubling was a new strategy for this student and this calculation represented a new success or if this strategy was inefficient for this learner and that they could have benefited from a mini-lesson on deriving a formula from a pattern.

Learning Story T1S6 showed a student's calculation in which they used meters, decameters and centimeters in the same answer. The answer is intended to show an area. Again, it is not clear if this was a significant challenge for this student or if there was room for a mini-lesson. Could the students have benefited from a mini-lesson or conference exploring units of measure? Perhaps measuring a string that is one meter, one decameter and one centimeter in length so that the comparison could be visual? Could the student have benefited from a conversation around the relationships between length and area? Might some experience with some manipulatives have helped the student to make sense of the need for square units when describing and area?

T2 recalled a moment during a workshop where the students were preparing a list of materials that they would need. The students had listed only a camera (T2, 663). This could have become a perfect moment for a conference or a mini-lesson. Students could have been given a camera and invited to explore how the camera would be useful, what data could be gathered and what data might be missed. There could have been a connection to scale, where students could have determined the scale of a photo and sketch in comparison to the actual table and seats. Later in the discussion, T2 recalled that some students had wanted to use the meter sticks for measuring but hadn't been able to explain their reasoning. Again, this could have been a great moment for a mini-lesson. Do the students know how to measure? Do the students know what measurements they can find using a meter stick? For what kinds of measurements is a meter stick a good choice and for what kinds of measurements is a meter stick a poor choice? Had students been supported in spending a few moments exploring the potential of meter sticks, their understanding of measurement and data collection could have been deepened.

The Learning Stories and remarks from the teacher and numeracy coach indicated that different strategies were observed. The expedition to the gym started when students were invited to wonder about the gym. Students posed their own questions and developed their own solutions. No two questions or solutions were alike. Once the teacher had posed the original storage shed questions, student teams directed their own processes. They were able to choose how to gather the data, in which order to find items at Canadian Tire, how to arrange the items in the shed and ultimately which storage shed to choose. Two of the expeditions had relevance and authenticity. Determining the number of students who could eat comfortably in the cafeteria was a real question that the principal of the school needed to consider when planning for the next school year. The students involved in finding the solution will be affected by the solution. It was an authentic and relevant problem to them. Determining which shed was the best option for their teacher was an authentic problem for which the relevance was increased due to the strong relationship that was built between the students and their teacher. Table 3 illustrates how the choices students had and the opportunities that were available to them provided an inclusive experience. By choosing their own questions and strategies, students found opportunities to start at their own entry points and to create their own extension challenges.

Table 3.

Inclusion Through Opportunity and Choice

Teacher prompt	Student questions	Student Strategies
What do you notice? What do you wonder? (Gym) (T1)	How many chairs would you need to reach the top of the gym? (T1S1)	• Measure the height of one chair, two chairs, three chairs...ten chairs • Double the height of ten chairs until the height of the gym is reached
	How many chairs would you need to reach the top of the gym? (T1S11)	• Find the height of the gym roof – count the number of bricks to the top of one of the walls • Measure one brick • Multiply the height of one brick by the number of bricks (unclear) • Measure the height of two chairs • Double the height of the two chairs until the height of the roof is reached
	Which colour lone has the greatest total length and by how much? (T1S7)	• Measure the length of all the lines in the gym (using half the gym) • Double those lengths (full gym) (symmetry of the lines) • Compare the lengths
	How much Velcro was needed to hang the mats on the walls? (T1S3)	• Measure the length of Velcro on half the mat • Multiply the length of Velcro by two for both sides of the mat • Count the total number of mats • Multiply the length of Velcro needed for one mat by the total number of mats
	How many benches could fit around the gym? (T1S8)	• Measure the length of each of the walls of the gym • Add the lengths to find perimeter • Measure the length of a bench • Divide the perimeter by the length of a bench
	How many benches could fit around the gym? (T1S12)	• Measure the length of each of the walls of the gym • Add the lengths to find the perimeter (double length and width then add) • Measure the length of a bench • Divide the perimeter by the length of one bench
	How many benches could fit around the gym? (T1S17)	• Measure the length of each of the walls of the gym • Add the lengths to find perimeter • Measure the length of a bench
	How many people could sit comfortably on the bleachers? (T1S5)	• Count the number of rows • Count the number of seats per row • Multiply to find the total number of seats

Teacher prompt	Student questions	Student Strategies
	Is the gym floor made of more dark or light wood? (T1S6)	• Measure the length and width of the dark wood • Measure the length and width of the light wood • Subtract the area of the dark wood from the area of the light wood (unclear calculations)
	If we were to recover the green mats in the gym, how much fabric would we need? (T1S16)	• Count the total number of mats • Measure the perimeter of one of the sides of the mats • Calculate the area of one of the sides of the mat • Multiply the area of one side by 4 for the 4 sides • Multiply the surface of the fabric by 4o for the forty mats
	If all the banners were all lined up in a single row around the top of the gym, how many could we fit before needing to drop down to a new row? (T1S9)	• Measure the length of each gym wall • Add the lengths to find perimeter (standard algorithm) • Measure the length of one banner • Convert the perimeter from m to cm • Double the length of 6 banners, 12 banners, 24 banner, then add the length of one banner at a time until the total perimeter is reached
	How many buckets of paint would be needed to repaint the gym? (T1S10)	• Process is unclear (calculations are multiplication and division)
	How many laps would it take to run 10k? (T1S13)	• Measure the length and width of the gym • Add the lengths to find perimeter (double length and double width, then add) • Convert 10k to m • Divide the 10k by the perimeter of the gym
	How many people enter the gym on an average school cycle? (T1S14)	• Find the number of students in each class (unclear) • Find the number of students coming into the gym each day of the school cycle • Add the total number of students coming into the gym each day together (standard algorithm)
	How much area would each volleyball player be responsible for on a volleyball court (12 players)? (T1S15)	• Measure the length and width of the volley court • Multiply the length and the width to find area • Divide the area by 12 players
Will we be able to have all students comfortably eat in the cafeteria next year? (T2)	(T2S1)	• Measure the length of a table • Count the number of people who can sit on one bench • Multiply the number of benches by the number of people that can sit on one bench
	(T2S2)	• Place a chair on the bench to see how many chairs can fit on one bench • Find the number of students who will be at the school next year • Multiply the number of tables by the number of seats at one table • Subtract the number of seats from the number of students to find the number of seats left over

Teacher prompt	Student questions	Student Strategies
	(T2S3)	• Measure the length of a bench • Count the number of tables • Observe the tables at lunch time, count the number of students per bench • Multiply the number of tables by the number of students who can sit on one bench
	(T2S4)	• Measure the length of the bench • Measure the space needed for two people to sit comfortably • Calculations (unclear)
(T3) What can we buy for our party with a $50 budget?	(T3S1)	• Addition to find total cost (standard algorithm) • Multiplication to find total cost per item (standard algorithm)
	(T3S2)	• Addition to find total cost (standard algorithm) • Multiplication to find total cost per item (standard algorithm)
	(T3S3)	• Addition to find total cost (standard algorithm) • Multiplication to find total cost per item (standard algorithm)
	(T3S4)	• Rounded (all prices to whole numbers) • Adding to find total cost
	(T3S5)	• Addition to find total cost (standard algorithm) • Multiplication to find total cost per item (standard algorithm)
	(T3S6)	• Addition to find total cost (standard algorithm) • Multiplication to find total cost per item (standard algorithm)
	(T3S7)	• Addition to find total cost (standard algorithm) • Multiplication to find total cost per item (standard algorithm)
	(T3S8)	• Addition to find total cost (standard algorithm) • Multiplication to find total cost per item (standard algorithm)
	(T3S9)	• Addition to find total cost (standard algorithm) • Multiplication to find total cost per item (standard algorithm)
	(T3S10)	• Addition to find total cost (standard algorithm) • Multiplication to find total cost per item (standard algorithm)
	(T3S11)	• Addition to find total cost (standard algorithm) • Multiplication to find total cost per item (standard algorithm)
	(T3S12)	• Subtraction (standard algorithm)
	(T3S13)	• Addition to find total cost (standard algorithm) • Multiplication to find total cost per item (standard algorithm)
Which shed should I buy? (T4)	(T4S1)	• Measure the length, height and width of the lawnmower Calculate volume to know how much space it will take up in the shed
	(T4S2)	• Measure the length, height and width of a bicycle • Calculate surface area (?)

Teacher prompt	Student questions	Student Strategies
	(T4S3)	• Measure the length, height and width of the lawn chairs • Calculate volume to know how much space it will take up in the shed
	(T4S4)	• Measure the length, height and width of the lawn chairs • Calculate volume to know how much space it will take up in the shed
	(T4S5)	• Measure the length, height and width of a bicycle • Calculate surface area (?)
	(T4S6)	•
What do you notice? What do you wonder? (Gym) (T5)	What is the area of the gym? (T5S7)	• Measuring the length of each wall of the gym • Multiply
	What is the area of the gym? (T5S9)	• Measuring the length of each wall of the gym • Multiply
	How many people can comfortably sit in the bleachers? (T5S8)	• Sit side by side to count the number of people that can fit on one bleacher • Multiply the number of bleachers by the number of people that can fit comfortably on a bleacher
	Which line on the floor is the longest? By how much? (T5S10)	• Measure the length of each line • Compare decimal numbers
	Which line on the floor is the longest? By how much? (T5S11)	• Measure the length of each line
	How many green mats would we need to cover the whole gym floor? (T5S12)	• Measure the length and width of a green mat • Calculate area • Measure the length and width of the gym • Calculate area • Divide the area of the gym by the area of a mat to find the total number of mats needed
	How many green mats would we need to cover the whole gym floor? (T5S12)	• Measure the length and width of a green mat • Calculate area • Measure the length and width of the gym • Calculate area • Divide the area of the gym by the area of a mat to find the total number of mats needed
(T5) What can we buy for our party?	How many bags of chips will we need for our party? (T5S1)	• Comparing decimal numbers • Addition to find total (unclear)
	How much will pop drinks cost? (T5S2)	• Multiplying decimal numbers
	How much will pop drinks cost? (T5S3)	• Multiplying decimal numbers
	How much money do we need to buy the candy we want? (T5S4)	• Multiplying decimal numbers • Addition to find total (standard algorithm)
	How many packs of Twizzlers do we need for 21 students? (T5S5)	• Multiplying decimal numbers
	Is it better to buy a fruit tray or individual fruits? Which is more expensive? (T5S6)	• Multiplying decimal numbers • Addition to fin total (standard algorithm) • Comparing decimal numbers

Opportunities for inclusion, meaningful and purposeful learning for all, should be evident in all activities of a school. In *One Without the Other*, Moore offers a new definition of inclusion. She writes that "inclusive education is about providing opportunities with supports for all students to have access to, and contribute to, an education rich in content and experience with their peers" (2016, p.17). Throughout the stories of the expeditions that the teachers and numeracy coaches shared opportunities for inclusion were abundant. Students were socially included. There were accessible learning spaces and peer connections. Opportunities to contribute to the shared learning of the group were available. Learning at the edge of one's current strengths was developed. However, the built-in supports could have been strengthened. Moments that sprung up during field work and workshops could have been caught and exploited for the potential growth within them.

Chapter 6

Conclusion

In this concluding chapter, responses to the three research questions are explored; followed by a discussion of the limitations of this study and recommendations for future research and practices with respect to Expeditionary Learning.

Findings

As earlier stated, the new criteria for intelligence is the ability to recognize a problem, define its variables, build its potential and create a solution that will be valuable to society (Wagner & Dintersmith, 2015). Given that new problems arise with countless unknowns, "learning to learn" (Wagner & Dintersmith, 2015) is the greatest skill that educators can teach their students. To achieve these goals, schools, classes, teachers must be able to create a culture of curiosity and adventure. Expeditionary Learning is one way to build inclusive communities from big, rich and authentic curriculum-based questions.

This research study set out to investigate three questions:

1. **How does the expeditionary learning approach support the meaningful inclusion of all learners in mathematics education?**

Expeditionary Mathematics welcomes learners into adventures where students can pose their own questions, select their own strategies and present their learning in multiple ways. Expeditionary Learning provides multiple means of presentation by having students explore authentic places and problems, through multiple modalities. Expeditionary Learning creates engagement by pursuing questions that are interesting to the learner and by adjusting problems as students move through them. Expeditionary Learning creates spaces for action, allowing students to move about and investigate in their own ways. In addition, solutions can be presented in many different ways.

Expeditions are shared experiences where students can find a role for themselves and contribute to the shared learning of the group. Opportunities and choice emerged as two factors that contributed significantly to the inclusion of all learners in Expeditionary Mathematics. In this study, the students had many opportunities to choose: (a) topics of interest, (b) peers to team with, (c) the difficulty of the challenge, (d) strategies and processes, and (e) how to communicate their thinking and solutions. The flexibility and uncertainty of Expeditionary Learning allowed for each problem to become open, creating space for a wide range of approaches to learning.

2. **What specific mathematical content knowledge is learned, omitted, or transformed through the Expeditionary Learning model?**

In each of the expeditions studied, measurement was the most common curricular outcome developed. In some cases, the measurements provided a spring board to other outcomes such as addition and subtraction, multiplication and division, comparing and ordering decimals, operations with decimals, and calculations of perimeter, area, volume and rate. In many of the questions that the students explored, more connections could have been made to other curricular outcomes. For example, there were examples of patterning and algebraic equations and symmetry in the gym questions, there were links to rate, area, and statistics in the cafeteria, and there were connections to trigonometry and taxes at Canadian Tire. Reasons for not exploring these additional curricular outcomes is not known; but a lack or experience in authentic settings or a set by step view of the curriculum may have contributed to these overlooked opportunities.

Opportunities for learning beyond the curriculum also were highlighted. Students and teachers were able to develop skills as leaders, learners and citizens.

3. What problem solving strategies are developed, omitted or transformed through a learning expedition?

The expeditions studied revealed that exploring, researching, creating and constructing, and communicating were the strategies that were used most often by the students. Exploring was evident as students investigated new places, posed questions, and experimented with the tools of mathematicians. Researching was evident when students gathered data in the field. The variables in each question were not predefined for the students. Students needed to first research what data they would need and then gather that data. Creating and constructing were demonstrated as students worked through calculations to find solutions. Communicating was demonstrated through the written notes, sketches and symbols the students included in their Learning Stories and portfolios. However, the evidence gathered suggested that these strategies were only beginning to be developed. To explore fully, students would need to take the lead and identify curious and problematic situations. To research fully, students would need to look beyond the immediate numbers to fill a chosen formula and consider what other factors might be influencing their data. They would need to be comfortable and competent using a variety of tools efficiently. To create and construct more fully, students would need to develop new, innovative approaches to analyzing data. To communicate fully, students would need to incorporate more mathematical language as they describe their thinking.

Noticeably absent from the samples collected were reasoning, connecting and reflecting. There are two possible explanations for their absence. First, it is possible that these strategies are not required as part of developing a solution to a problem. Or, it is possible that these strategies are more difficult to express and to analyse. Comments from the teachers during the focus group discussion supported the second explanation. As described in Chapter 2, reflection is critical to the learning process (Bruner, 1961, Tarini & White, 1998, Durant & Sendag, 2012). Reasoning, connecting and reflecting are essential pieces of the expeditionary process. Each one plays a valuable role in learning. Their absence from the expeditions studies was most likely due to the teacher and students' inexperience in making these elements visible.

Limitations

1. This research is limited to the study of five middle years classrooms in which the teacher responsible for teaching the math classes had indicated an interest in problem solving. This is a very small study. The results of this study are limited to the specific experiences of these five teachers and their students.
2. In each case, the classroom math teacher took primary responsibility for leading the learning expeditions. The study will therefore be delimited to the perceptions, commitments and actions of those teachers.
3. This study is built on the stories of five 'first time' expeditions. The true value of Expeditionary Learning is developed over time, as teachers and students become more confident in their abilities to lead an expedition and more comfortable taking risks. The study may underestimate the potential of Expeditionary Learning when experienced by students and teachers over time.
4. In this study, students and teachers retold their experiences and it is likely that more factors were at play than those reported. An unknown number of factors that either contributed to or limited the students' learning may not have been uncovered in the stories.

5. The individual stories have been analysed to offer a collection of common themes and recommendations for educators looking to increase the levels of inclusion and mathematical problem solving in their programs. In the analysis, common themes were combined and so have put the richness of the unique, individual voices at odds with the needs of the collective (Andrews, 2007, p.491).

Recommendations

Teacher Learning. It is important to continue to support teachers learning to think of mathematics as a living subject. To be successful at Expeditionary Learning, teachers must anticipate the learning needs of their students. A solid understanding of the curriculum and the underlying mathematical relationships will support teachers as they reframe their definition of math to include developing the strategies of Expeditionary Learning. The unknowns that accompany Expeditionary Mathematics are key to it's success. Successfully managing a math adventure requires that teachers see beyond the immediate math topic. They will need to be able to uncover math in everyday situations, explicitly connect mathematical ideas, identify examples and counter-examples and expose their students to math in a variety of contexts on the fly. Preparation for mini-lessons and conferences is essential. A set of mini-lessons, that address the key components of a learning trajectory, can be developed ahead of time in anticipation.

As Expeditionary Learning emphasises living the math, the best preparation for teachers would be opportunities to explore their neighborhoods, asking own questions, and experiencing the processes for themselves. Teachers must be knowledgeable and confident in their abilities to highlight connections between what their students are uncovering through their explorations and relevant underlying mathematical concepts.

Support Risk Taking. It is also vital that teachers have opportunities to support each other as they take risks. Expeditions beyond the school require trust. Experiences that are led by students and their wonderings require trust. School-based administrators can support their teachers by asking questions, reassuring teachers that risk is a part of teaching and learning, and by creating safe classroom communities where risks, successes and failures are celebrated as part of the learning process. School-based administrators can also model risk by taking part in expeditions or by highlighting new experiences in which they have taken a risk.

Creativity and risk-taking occur in all subjects. For example, teachers take risks in science by having students conduct science experiments and in language arts classes when students perform plays. This same confidence and freedom should be transferred to and amplified throughout math teaching.

To support teachers in moving from a controlled math lesson to a riskier lesson, a teacher guide could be created. The guide could provide teachers with examples of a structured, teacher-led expedition, a teacher-guided or co-led expedition, and finally a student-led inquiry expedition. This guidance could be helpful for teachers and students as they try to become riskier.

School administrators can create a culture of belonging and innovation. By supporting teachers, filling their buckets, administrators can share a message that risk taking is safe, expected, and celebrated. To do so, administrators would regularly engage in conversation around teaching and learning, be aware of the wonderings the teachers have and the practices they are trying. Administrators can create time and space for teacher teams to meet, to visit each others' classrooms and to co-teach. Furthermore, administrators could accompany teachers on an expedition so that they can engage in rich dialogue, offering construction feedback, and encouragement.

Expeditionary Learning came to be as a response to my questions around inclusive practices and meaningful, authentic math learning. The need for innovation in teaching and learning was outlined in Chapter 1. We cannot expect the pedagogies of the past, that have failed to include so many, to now be inclusive by changing the name from mainstreamed to integrated to included. New approaches are required. Expeditionary Learning is one idea. The purpose of sharing my story of Expeditionary Mathematics is to offer a possibility. I encourage teachers to wonder, to imagine, to experiment, and to create their own approaches to inclusive mathematics.

Community Partnerships. The development of community partnerships, through which businesses and local experts can register as safe, inclusive places and people for learning outside of the school may be helpful in implementing expeditionary math. As teachers, we have all experienced somewhat similar educational experiences, pre-service preparation and in-service professional development. Taping into experts in the field who have expertise and experiences that are different from teachers will bring a different perspective to curricular math concepts.

As I look around my community and my city, I see areas for growth. Starting with a simple Math Trail, a walking tour of our neighborhood with our math lens, can inspire questions hat become springboards. If the questions we ask and the solutions we propose are indeed valuable, then sharing them with our communities could lead to improvements. To begin these partnerships schools can invite community members in to share their stories, their work, and the issues that they are facing. Students can be invited to tackle these issues.

Parents and community member may find this approach to learning mathematics somewhat different from their own experiences in school. Information brochures, presentation evenings, and technologies such as online student portfolios that are regularly shard with families can assist in creating clarity around our activities and purpose.

As citizens we all have a responsibility to each other. Cultivating relationships between schools and communities will deepen respect and understanding for one another. In addition, schools have been leaders in inclusive practices. Sharing what educators have learned and what they are able to do within school communities will support the spread of inclusive practices throughout the community.

Permission. One way to facilitate expeditionary mathematics is to develop a more efficient system for obtaining parental permission and school division approval for leaving the school grounds. So much learning is waiting for our students just beyond the schoolyard fence. However, teachers sometimes shy away from the paperwork required for a field trip. A smoother system of obtaining permission would be helpful.

Local Research. Strengthening the relationships between university researchers and local school divisions also may support the introduction of expeditionary mathematics. Local research is valuable. Teachers in classrooms would benefit from more research and recommendations that are directly related to the unique experience of going to school in Winnipeg. Finding research participants for this study was a challenge. Teachers were willing and interested but reaching them through the school division office was slow. There was a significant delay to the start of this study, as communications between the school division and myself was challenging. I recommend that university researchers and the school division superintendents responsible for research create a plan to share upcoming research participation opportunities in a more open and convenient way.

In summary, this small-scale study demonstrates that Expeditionary Learning has the potential to support teachers and students as they take risks to re-define mathematics and create inclusive learning experiences through opportunity and choice. To achieve a goal of exciting all students about the mathematics around them and inspiring them to investigate authentic, relevant problems; new methods of teaching and learning, such as Expeditionary Learning, will need to be explored further. The hope is that experiences in Expeditionary Learning will inspire a spirit of wonder and curiosity that stays with each learner for life; so that they can explore the world as passionately curious, learners for life.

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Appendix A

ENREB Protocol Approval

UNIVERSITY OF MANITOBA | Research Ethics and Compliance

Human Ethics
208-194 Dafoe Road
Winnipeg, MB
Canada R3T 2N2
Phone +204-474-7122
Email: humanethics@umanitoba.ca

PROTOCOL APPROVAL

TO: **Glenys MacLeod** (Advisor: Rick Freeze)
Principal Investigator

FROM: **Todd Duhamel, Vice Chair** [redacted]
Education/Nursing Research Ethics Board (ENREB)

Re: **Protocol #E2017:063 (HS20971)**
"Expeditionary Mathematics for Inclusive Education: Three Case Studies in the Field"

Effective: July 17, 2017 **Expiry:** July 17, 2018

Education/Nursing Research Ethics Board (ENREB) has reviewed and approved the above research. ENREB is constituted and operates in accordance with the current *Tri-Council Policy Statement: Ethical Conduct for Research Involving Humans.*

This approval is subject to the following conditions:

1. Approval is granted only for the research and purposes described in the application.
2. Any modification to the research must be submitted to ENREB for approval before implementation.
3. Any deviations to the research or adverse events must be submitted to ENREB as soon as possible.
4. This approval is valid for one year only and a Renewal Request must be submitted and approved by the above expiry date.
5. A Study Closure form must be submitted to ENREB when the research is complete or terminated.
6. The University of Manitoba may request to review research documentation from this project to demonstrate compliance with this approved protocol and the University of Manitoba *Ethics of Research Involving Humans.*

Funded Protocols:
- **Please mail/e-mail a copy of this Approval, identifying the related UM Project Number, to the Research Grants Officer in ORS.**

Research Ethics and Compliance is a part of the Office of the Vice-President (Research and International)
umanitoba.ca/research

UNIVERSITY OF MANITOBA | Research Ethics and Compliance

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AMENDMENT APPROVAL

April 12, 2018

TO:	**Glenys MacLeod** Principal Investigator	**(Advisor: Rick Freeze)**
FROM:	**Zana Lutfiyya, Chair** Education/Nursing Research Ethics Board (ENREB)	[redacted]
Re:	**Protocol #E2017:063 (HS20971)** **Expeditionary Mathematics for Inclusive Education: Three Case Studies in the Field**	

Education/Nursing Research Ethics Board (ENREB) has reviewed and approved your Amendment Request received on **April 4, 2018** to the above-noted protocol. ENREB is constituted and operates in accordance with the current *Tri-Council Policy Statement: Ethical Conduct for Research Involving Humans.*

This approval is subject to the following conditions:

1. Approval is given for this amendment only. Any further changes to the protocol must be reported to the Human Ethics Coordinator in advance of implementation.
2. Any deviations to the research or adverse events must be submitted to ENREB as soon as possible.
3. Amendment Approvals do not change the protocol expiry date. Please refer to the original Protocol Approval or subsequent Renewal Approvals for the protocol expiry date.

Appendix B

Letter to Superintendent of School Division

[redacted]

Superintendent of Schools

[redacted]

Winnipeg, MB

October 22, 2016

[redacted]

I am a graduate student at the University of Manitoba in the Faculty of Education. I am interested in examining Expeditionary Learning as a potential teaching and learning approach that will engage all learners in the development of both mathematical content knowledge and problem-solving thinking in the Middle Years. This is the focus of my research for my doctoral thesis. Using qualitative research methods of interviews, focus groups, field notes and learning stories, my study will specifically:

a) Explore Expeditionary Learning as an approach to support the meaningful inclusion of all learners in mathematics education.
b) Identify the specific mathematical content knowledge learned through the Expeditionary Learning model.
c) Assess the problem-solving strategies are developed through a learning expedition.

In Expeditionary Learning, students adopt a mathematical point of view, seeing the world as mathematicians do. They engage in authentic inquiry and original research within their communities. They develop strengths as learners, so that no matter what the context, they will have access to their own strategies to deepen their understanding, create a solution or develop a revolutionary invention. As teachers strive to develop the skills, attitudes and abilities of mathematicians, their students need opportunities to explore their world as mathematicians do. The importance of curiosity, imagination, critical thinking, synthesis, invention, reasoning, reflection and communication must be at the forefront of every learning experience teachers orchestrate.

The ability to recognize a problem, define its variables, build its potential and create a solution that will be valuable to society is the new criteria for intelligence. Given that new problems arise with countless unknowns, learning to learn is the greatest skill we can teach our students. Experiences in Expeditionary Learning create opportunities for students to be engineers, architects, environmental researchers, physicists, astronomers, historians, geographers, artists and explorers; to name only a few possibilities. A Learning Expedition invites all learners to adopt a learners' point of view, to be curious and to engage in authentic inquires through direct encounters with the content being studied.

I am writing to you to request your permission to invite members of the St. James Assiniboia teaching staff and their students to take part in the study. With your permission, I will write to the principals of [redacted] School, [redacted] School and [redacted] School to share information about my study and invite them to share the opportunity with their staff. Interested teachers will contact me directly to make arrangements.

Potential participants for the study (i.e., Grade 5, 7, 8 and 9 Math and resource teachers) will be given a letter outlining the study and will contact me if they are interested. Participation in the study is optional and teachers will be able to end their participation at any time should they choose to. Teacher teams, consisting of a classroom teacher and a resource teacher, interested in participating in the study will be required to give written consent prior to the beginning of the study. Letters outlining the study and detailing how student work samples will be gathered, analyzed, stored and returned to students will be shared with the students and sent to parents. The students will be asked to return the letters in a sealed envelope that was provided to them within seven days. The letter will allow their parents to indicate whether or not they agree to have their child participate. All participation is voluntary, and participants can withdraw from the study at any time. If school administrators, teachers, students and parents would like to withdraw from the study, they ca inform me by email ([redacted] or by phone [redacted] or in person,

An initial interview with teacher teams will arranged at a convenient time and location. During these semi structured interviews teams will be asked to share a description of the mathematical content knowledge of their class, identifying a sample of three topics that their students find challenging. Teams will be asked to describe the level of inclusion in their classroom activities and the problem-solving abilities of their students. Although the identities of the teachers will be known to me this will be kept strictly confidential. An audio recording of the interviews will be password protected and will be kept in a locked office at all times. The recording will be transcribed and then destroyed at the conclusion of the study. All written documents will also be kept in a locked office to ensure confidentiality. Names of teachers, their students and their schools will not be included in any written documents.

Participating teachers will be invited to a Professional Development session held during the evening at the University of Manitoba. Teachers will be introduced to the Expeditionary Learning model and will take part in a learning expedition. From here teacher teams will be supported in creating their own expeditionary learning experience will their students. Throughout the experience teachers will be asked to keep field notes in which they record their observations relating to the inclusivity of the experiences, the specific mathematical content being learned and the problem-solving strategies their students are using and developing. Again, these field notes will be kept in a locked office and will not contain any identifying information about the students, teachers or the schools. At the conclusion of the experience teachers will join a focus group discussion where they will be able to dialogue about the potential of Expeditionary Learning as an inclusive approach to 21st Century, student-led, mathematics learning.

Again, participation in this study is completely voluntary. If any participant would like to withdraw they can do so by contacting me at [redacted] or at [redacted] or by contacting my thesis advisor, Dr. Richard Freeze, at [redacted] or at [redacted]

All participants in this study who would like to receive a summary of the results can do so by emailing me at [redacted] or by phoning me [redacted]

Thank you for considering my request.

Sincerely,

Glenys MacLeod

Appendix C

Consent Form for Superintendent of School Division

Dear;

This consent form, a copy of which is to be left with you for your records, is part of the process of informed consent to participate in this research project. This letter offers an outline of the research and details of your participation. If you would like more details, please ask.

I am a graduate student at the University of Manitoba in the Faculty of Education. I am interested in examining Expeditionary Learning as a potential teaching and learning approach that will engage all learners in the development of both mathematical content knowledge and problem-solving thinking in the Middle Years. This is the focus of my research for my doctoral thesis. Using qualitative research methods of interviews, focus groups, field notes and Learning Stories, my study will specifically:

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The ability to recognize a problem, define its variables, build its potential and create a solution that will be valuable to society is the new criteria for intelligence. Given that new problems arise with countless unknowns, learning to learn is the greatest skill we can teach our students. Experiences in Expeditionary Learning create opportunities for students to be engineers, architects, environmental researchers, physicists, astronomers, historians, geographers, artists and explorers; to name only a few possibilities. A Learning Expedition invites all learners to adopt a learners' point of view, to be curious and to engage in authentic inquires through direct encounters with the content being studied.

I am writing to you to request your permission to invite members the Stevenson-Britannica School teaching staff and their students to take part in this study. Interested teachers will contact me directly to make arrangements.

Potential participants for the study (i.e., Grade 5, 7, 8 and 9 Math and resource teachers) will be given a letter outlining the study and will contact me if they are interested. Participation in the study is optional and teachers will be able to end their participation at any time should they choose to. Teacher teams, consisting of a classroom teacher and a resource teacher, interested in participating in the study will be required to give written consent prior to the beginning of the study. Letters outlining the study and detailing how student work samples will be gathered, analyzed, stored and returned to students will be shared with the students and sent to parents. The students will be asked to return the letters in a sealed envelope that was provided to them within seven days. The letter will allow their parents to indicate whether or not they agree to have their child participate. All participation is voluntary and participants can withdraw from the study at any time. If school administrators, teachers, students and parents would like to withdraw from the study, they ca inform me by email ([redacted] or by phone [redacted] or in person,

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Participating teachers will be invited to a Professional Development session held during the evening at the University of Manitoba. Teachers will be introduced to the Expeditionary Learning model and will take part in a learning expedition. From here teacher teams will be supported in creating their own expeditionary learning experience will their students. Throughout the experience teachers will be asked to keep field notes in which they record their observations relating to the inclusivity of the experiences, the specific mathematical content being learned and the problem-solving strategies their students are using and developing. Again, these field notes will be kept in a locked office and will not contain any identifying information about the students, teachers or the schools. At the conclusion of the experience teachers will join a focus group discussion where they will be able to dialogue about the potential of Expeditionary Learning as an inclusive approach to 21st Century, student-led, mathematics learning.

I am asking that principals distribute a letter to math teachers and resources teachers inviting them to take part in the study.

Again, participation in this study is completely voluntary. If any participant would like to withdraw they can do so by contacting me at [redacted] or at [redacted] or by contacting my thesis advisor, Dr. Richard Freeze, at [redacted] or at [redacted]

All participants in this study who would like to receive a summary of the results can do so by emailing me at [redacted] or by phoning me at [redacted]

Thank you.

Sincerely,

Glenys MacLeod

_______________________________________ _____________________________

Participant's Signature Date

_______________________________________ _____________________________

Researcher's Signature Date

____ Please send me a written summary of the results of the study:

____ (email or mailing address)

Appendix D

Letter to School Administrator

October 22, 2016

Dear;

I am a graduate student at the University of Manitoba in the Faculty of Education. I am interested in examining Expeditionary Learning as a potential teaching and learning approach that will engage all learners in the development of both mathematical content knowledge and problem-solving thinking in the Middle Years. This is the focus of my research for my doctoral thesis. Using qualitative research methods of interviews, focus groups, field notes and learning stories, my study will specifically:

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All participants in this study who would like to receive a summary of the results can do so by emailing me at [redacted] or by phoning me at [redacted].

Thank you for considering my request.

Sincerely,

Glenys MacLeod

Appendix E

Consent Form for School Administrators

October 22, 2016

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d) Explore Expeditionary Learning as an approach to support the meaningful inclusion of all learners in mathematics education.
e) Identify the specific mathematical content knowledge learned through the Expeditionary Learning model.
f) Assess the problem-solving strategies are developed through a learning expedition.

In Expeditionary Learning, students adopt a mathematical point of view, seeing the world as mathematicians do. They engage in authentic inquiry and original research within their communities. They develop strengths as learners, so that no matter what the context, they will have access to their own strategies to deepen their understanding, create a solution or develop a revolutionary invention. As teachers strive to develop the skills, attitudes and abilities of mathematicians, their students need opportunities to explore their world as mathematicians do. The importance of curiosity, imagination, critical thinking, synthesis, invention, reasoning, reflection and communication must be at the forefront of every learning experience teachers orchestrate.

The ability to recognize a problem, define its variables, build its potential and create a solution that will be valuable to society is the new criteria for intelligence. Given that new problems arise with countless unknowns, learning to learn is the greatest skill we can teach our students. Experiences in Expeditionary Learning create opportunities for students to be engineers, architects, environmental researchers, physicists, astronomers, historians, geographers, artists and explorers; to name only a few possibilities. A Learning Expedition invites all learners to adopt a learners' point of view, to be curious and to engage in authentic inquires through direct encounters with the content being studied.

I am writing to you to request your permission to invite members the Stevenson-Britannica School teaching staff and their students to take part in this study. Interested teachers will contact me directly to make arrangements.

Potential participants for the study (i.e., Grade 5, 7, 8 and 9 Math and resource teachers) will be given a letter outlining the study and will contact me if they are interested. Participation in the study is optional and teachers will be able to end their participation at any time should they choose to. Teacher teams, consisting of a classroom teacher and a resource teacher, interested in participating in the study will be required to give written consent prior to the beginning of the study. Letters outlining the study and detailing how student work samples will be gathered, analyzed, stored and returned to students will be shared with the students and sent to parents. The students will be asked to return the letters in a sealed envelope that was provided to them within seven days. The letter will allow their parents to indicate whether or not they agree to have their child participate. All participation is voluntary and participants can withdraw from the study at any time. If school administrators, teachers, students and parents would like to withdraw from the study, they ca inform me by email [redacted]), or by phone ([redacted]), or in person,

An initial interview with teacher teams will arranged at a convenient time and location. During these semi structured interviews teams will be asked to share a description of the mathematical content knowledge of their class, identifying a sample of three topics that their students find challenging. Teams will be asked to describe the level of inclusion in their classroom activities and the problem-solving abilities of their students. Although the identities of the teachers will be known to me this will be kept strictly confidential. An audio recording of the interviews will be password protected and will be kept in a locked office at all times. The recording will be transcribed and then destroyed at the conclusion of the study. All written documents will also be kept in a locked office to ensure confidentiality. Names of teachers, their students and their schools will not be included in any written documents.

Participating teachers will be invited to a Professional Development session held during the evening at the University of Manitoba. Teachers will be introduced to the Expeditionary Learning model and will take part in a learning expedition. From here teacher teams will be supported in creating their own expeditionary learning experience will their students. Throughout the experience teachers will be asked to keep field notes in which they record their observations relating to the inclusivity of the experiences, the specific mathematical content being learned and the problem-solving strategies their students are using and developing. Again, these field notes will be kept in a locked office and will not contain any identifying information about the students, teachers or the schools. At the conclusion of the experience teachers will join a focus group discussion where they will be able to dialogue about the potential of Expeditionary Learning as an inclusive approach to 21st Century, student-led, mathematics learning.

I am asking that principals distribute a letter to math teachers and resources teachers inviting them to take part in the study.

Again, participation in this study is completely voluntary. If any participant would like to withdraw they can do so by contacting me at [redacted] or at [redacted] or by contacting my thesis advisor, Dr. Richard Freeze, at [redacted] or at [redacted].

All participants in this study who would like to receive a summary of the results can do so by emailing me at [redacted] or by phoning me at [redacted].

Thank you.

Sincerely,

Glenys MacLeod

_______________________________________ _____________________________

Participant's Signature Date

_______________________________________ _____________________________

Researcher's Signature Date

____ Please send me a written summary of the results of the study:

____ (email or mailing address)

Appendix F

Information Letter for Teacher Participants

October 22, 2016

Dear (Teacher's name);

I am a graduate student at the University of Manitoba in the Faculty of Education. I am interested in examining Expeditionary Learning as a potential teaching and learning approach that will engage all learners in the development of both mathematical content knowledge and problem-solving thinking in the Middle Years. This is the focus of my research for my doctoral thesis. Using qualitative research methods of interviews, focus groups, field notes and Learning Stories, my study will specifically:

a) explore Expeditionary Learning as an approach to support the meaningful inclusion of all learners in mathematics education.
b) identify the specific mathematical content knowledge learned through the Expeditionary Learning model.
c) assess the problem-solving strategies are developed through a learning expedition.

In Expeditionary Learning, students adopt a mathematical point of view, seeing the world as mathematicians do. They engage in authentic inquiry and original research within their communities. They develop strengths as learners, so that no matter what the context, they will have access to their own strategies to deepen their understanding, create a solution or develop a revolutionary invention. As teachers strive to develop the skills, attitudes and abilities of mathematicians, their students need opportunities to explore their world as mathematicians do. The importance of curiosity, imagination, critical thinking, synthesis, invention, reasoning, reflection and communication must be at the forefront of every learning experience teachers orchestrate.

The ability to recognize a problem, define its variables, build its potential and create a solution that will be valuable to society is the new criteria for intelligence. Given that new problems arise with countless unknowns, learning to learn is the greatest skill we can teach our students. Experiences in Expeditionary Learning create opportunities for students to be engineers, architects, environmental researchers, physicists, astronomers, historians, geographers, artists and explorers; to name only a few possibilities. A Learning Expedition invites all learners to adopt a learners' point of view, to be curious and to engage in authentic inquires through direct encounters with the content being studied.

I am writing to you to invite you to take part in this study. Participation in the study is optional and you will be able to end your participation at any time should you choose to. Teacher teams, consisting of a classroom teacher and a resource teacher, interested in participating in the study will be required to give written consent prior to the beginning of the study. Letters outlining the study and detailing how student work samples will be gathered, analyzed, stored and returned to students will be shared with the students and sent to parents. The students will be asked to return the letters in a sealed envelope that was provided to them within seven days. The letter will allow their parents to indicate whether or not they agree to have their child participate. All participation is voluntary and students can also withdraw from the study at any time. If school administrators, teachers, students and parents would like to withdraw from the study, they ca inform me by email [redacted]), or by phone ([redacted]), or in person,

An initial interview with teacher teams will arranged at a convenient time and location. During these semi structured interviews teams will be asked to share a description of the mathematical content knowledge of their class, identifying a sample of three topics that their students find challenging. Teams will be asked to describe the level of inclusion in their classroom activities and the problem-solving abilities of their students. Although the identities of the teachers will be known to me this will be kept strictly confidential. An audio recording of the interviews will be password protected and will be kept in a locked office at all times. The recording will be transcribed and then destroyed at the conclusion of the study. All written documents will also be kept in a locked office to ensure confidentiality. Names of teachers, their students and their schools will not be included in any written documents.

Participating teachers will be invited to a Professional Development session held during the evening at the University of Manitoba. Teachers will be introduced to the Expeditionary Learning model and will take part in a learning expedition. From here teacher teams will be supported in creating their own expeditionary learning experience will their students. Throughout the experience teachers will be asked to keep field notes in which they record their observations relating to the inclusivity of the experiences, the specific mathematical content being learned and the problem-solving strategies their students are using and developing. Again, these field notes will be kept in a locked office and will not contain any identifying information about the students, teachers or the schools. At the conclusion of the experience teachers will join a focus group discussion where they will be able to dialogue about the potential of Expeditionary Learning as an inclusive approach to 21st Century, student-led, mathematics learning.

Again, participation in this study is completely voluntary. If any participant would like to withdraw they can do so by contacting me at [redacted] or at [redacted] or by contacting my thesis advisor, Dr. Richard Freeze, at [redacted] or at [redacted].

All participants in this study who would like to receive a summary of the results can do so by emailing me at [redacted] or by phoning me at [redacted].

Thank you for considering my request.

Sincerely,

Glenys MacLeod

Appendix G

Consent Form for Teacher Participants

Date

Researcher: Glenys MacLeod

This consent form, a copy of which is to be left with you for your records, is part of the process of informed consent to participate in this research project. This letter offers an outline of the research and details of your participation. If you would like more details, please ask.

I am a graduate student at the University of Manitoba in the Faculty of Education. I am interested in examining Expeditionary Learning as a potential teaching and learning approach that will engage all learners in the development of both mathematical content knowledge and problem-solving thinking in the Middle Years. This is the focus of my research for my doctoral thesis. Using qualitative research methods of interviews, focus groups, field notes and Learning Stories, my study will specifically:

a) explore Expeditionary Learning as an approach to support the meaningful inclusion of all learners in mathematics education.
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The ability to recognize a problem, define its variables, build its potential and create a solution that will be valuable to society is the new criteria for intelligence. Given that new problems arise with countless unknowns, learning to learn is the greatest skill we can teach our students. Experiences in Expeditionary Learning create opportunities for students to be engineers, architects, environmental researchers, physicists, astronomers, historians, geographers, artists and explorers; to name only a few possibilities. A Learning Expedition invites all learners to adopt a learners' point of view, to be curious and to engage in authentic inquires through direct encounters with the content being studied.

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Participating teachers will be invited to a Professional Development session held during the evening at the University of Manitoba. Teachers will be introduced to the Expeditionary Learning model and will take part in a learning expedition. From here teacher teams will be supported in creating their own expeditionary learning experience will their students. Throughout the experience teachers will be asked to keep field notes in which they record their observations relating to the inclusivity of the experiences, the specific mathematical content being learned and the problem-solving strategies their students are using and developing. Again, these field notes will be kept in a locked office and will not contain any identifying information about the students, teachers or the schools. At the conclusion of the experience teachers will join a focus group discussion where they will be able to dialogue about the potential of Expeditionary Learning as an inclusive approach to 21st Century, student-led, mathematics learning.

Again, participation in this study is completely voluntary. If any participant would like to withdraw they can do so by contacting me at [redacted] or at [redacted] or by contacting my thesis advisor, Dr. Richard Freeze, at [redacted] or at [redacted]

All participants in this study who would like to receive a summary of the results can do so by emailing me at [redacted] or by phoning me at [redacted].

Thank you.

Sincerely,

Glenys MacLeod

_______________________________________ _____________________________

Participant's Signature Date

_______________________________________ _____________________________

Researcher's Signature Date

____ Please send me a written summary of the results of the study:

____ (email or mailing address)

Appendix H

Information Letter to Parents

UNIVERSITY OF MANITOBA
Faculty of Education

March, 2018

Dear parents and guardians,

I am a graduate student at the University of Manitoba in the Faculty of Education. I am interested in examining expeditionary learning as a potential teaching and learning approach that will engage all learners in the development of both mathematical content knowledge and problem-solving thinking in the Middle Years. This is the focus of my research for my doctoral thesis.

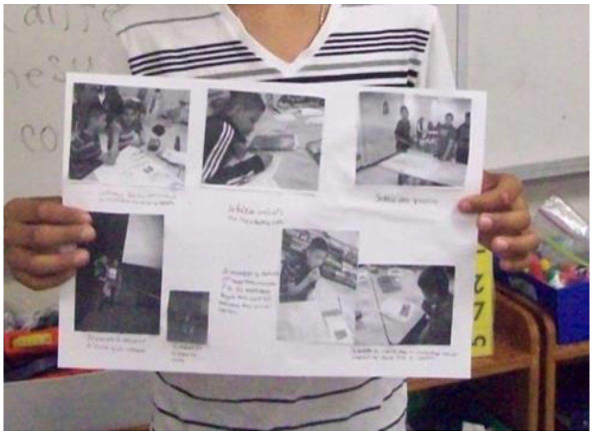
Expeditionary learning is a student centered, problem-based approach to teaching and learning mathematics that incorporates field work, mini lessons and conferences and workshops. In expeditionary learning, students adopt a mathematical point of view, seeing the world as mathematicians do. They engage in authentic inquiry and original research within their communities. They develop strengths as learners, so that no matter what the context, they will have access to their own strategies to deepen their understanding, create a solution or develop a revolutionary invention. Students need opportunities to explore their world as mathematicians do. The importance of curiosity, imagination, critical thinking, synthesis, invention, reasoning, reflection and communication must be at the forefront of every learning experience teachers orchestrate.

The ability to recognize a problem, define its variables, build its potential and create a solution that will be valuable to society is the new criteria for intelligence. Given that new problems arise with countless unknowns, learning to learn is the greatest skill we can teach our students. An expedition begins when students explore their community. When learning reaches from the classroom into the community, students build connections between theory and practice, concept and process, and content and context. In sharp contrast to a field trip, a learning expedition requires students to become a part of an uncertain, attention-grabbing and problematic situation. Expeditionary learning is about learning to think independently and as part of a community. It is about adapting to uncertainty and learning to be relentless in the pursuit of understanding.

Through these lived expeditionary experiences, learners shape their own ways of knowing and thinking about their world, their own approaches to solving problems, and their own learning processes. To be actively learning means to be fully engaged in an ongoing cycle of wondering, exploring, researching, developing, creating, constructing, connecting, communicating, reasoning and reflecting. A learning expedition invites all learners to adopt a learners' point of view, to be curious and to engage in authentic inquires through direct encounters with the content being studied. More information about expeditionary learning can be found on my website at www.expeditionarymath.com

I am writing to you to request that you consider agreeing to have your child participate in the study. Participation in the study is optional and students will be able to end their participation at any time should they choose to.

This research study will take place between April 2018 and June 2018. Students will be going on a mathematical field trip with their class and teacher. While there, students will be learning about specific mathematical outcomes. The classroom mathematics teacher will be writing some notes and taking photos. These notes will focus on the math, they will refer to the content being studied and the strategies that students are using. They will not refer to student achievement or behavior. These notes and photos will be kept confidential. Student names and identities will not be revealed. At the end of the field trip students will create a summary of their learning experience using photos, writing and calculations. These will be shared with the researcher and returned to the classroom teacher within three months. No copies will be made, and the student's names will not be included in the study. These reflections will also be kept in a locked home office. Below is a sample of a student reflection.



Students whose parents and guardians have signed the consent form, will also be asked to sign an Assent Form for Student Participation. The Assent Form gives students the opportunity to agree or to not agree to participate in the study. Both the Parent or Guardian Consent Form and the Assent Form for Student Participation must be signed for the student to participate. Assent, agreeing to participate in the study, will be explained to students at school by the classroom teacher. A copy of the Assent Form for Student Participation is attached to this letter for your information.

Participation in this study is completely voluntary. If any student would like to withdraw they can do so by contacting me at [redacted] or at [redacted] or by contacting my thesis advisor, Dr. Richard Freeze, at [redacted] or at [redacted].

The name of the school division, the school, school principals, teachers and students will not be identified or used in any publication. The names of the participants will be known to the researcher, but any other identity will not be shared. Field notes, interviews and student reflections will use a coding system so that no names appear on any documentation. The student reflections may include photos of the student taken during the experience. These photos will not be used in the study or any publication. All files will be stored in a password protected computer file, in a locked home office. Consent forms and contact information will be stored in a separate locked filling cabinet. Only the researcher will have access to the data. Participating teachers will be sharing their observations and reflections during the focus group discussion however, if it is necessary to identify a student they will use the established coding system. All documents will be destroyed by June 2018.

Students who do not have parental permission to participate in the study will still take part in the learning experience. Their reflections will not be included in the data and teachers will not include observations about the student in their field notes.

Participants in this study who would like to receive a summary of the results can do so by indicating on the informed consent document.

Appendix I

Consent Form for Parents

UNIVERSITY OF MANITOBA
Faculty of Education

March 2018

Research Project Title: Expeditionary Learning for Inclusive Mathematics: Three case studies from the field.

Principal Investigator and Contact Information: G. MacLeod [redacted]

Research Supervisor and Contact Information: Dr. Rick Freeze [redacted]

This consent form, a copy of which will be left with you for your records and reference, is only part of the process of informed consent. It should give you the basic idea of what the research is about and what your participation will involve. If you would like more detail about something mentioned here, or information not included here, you should feel free to ask. Please take the time to read this carefully and to understand any accompanying information.

Dear parents and guardians,

I am a graduate student at the University of Manitoba in the Faculty of Education. I am interested in examining expeditionary learning as a potential teaching and learning approach that will engage all learners in the development of both mathematical content knowledge and problem-solving thinking in the Middle Years. This is the focus of my research for my doctoral thesis.

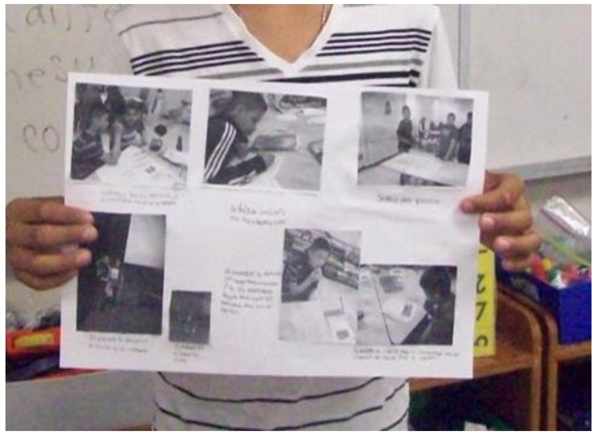
Expeditionary learning is a student centered, problem-based approach to teaching and learning mathematics that incorporates field work, mini lessons and conferences and workshops. In expeditionary learning, students adopt a mathematical point of view, seeing the world as mathematicians do. They engage in authentic inquiry and original research within their communities. They develop strengths as learners, so that no matter what the context, they will have access to their own strategies to deepen their understanding, create a solution or develop a revolutionary invention. Students need opportunities to explore their world as mathematicians do. The importance of curiosity, imagination, critical thinking, synthesis, invention, reasoning, reflection and communication must be at the forefront of every learning experience teachers orchestrate.

The ability to recognize a problem, define its variables, build its potential and create a solution that will be valuable to society is the new criteria for intelligence. Given that new problems arise with countless unknowns, learning to learn is the greatest skill we can teach our students. An expedition begins when students explore their community. When learning reaches from the classroom into the community, students build connections between theory and practice, concept and process, and content and context. In sharp contrast to a field trip, a learning expedition requires students to become a part of an uncertain, attention-grabbing and problematic situation. Expeditionary learning is about learning to think independently and as part of a community. It is about adapting to uncertainty and learning to be relentless in the pursuit of understanding.

Through these lived expeditionary experiences, learners shape their own ways of knowing and thinking about their world, their own approaches to solving problems, and their own learning processes. To be actively learning means to be fully engaged in an ongoing cycle of wondering, exploring, researching, developing, creating, constructing, connecting, communicating, reasoning and reflecting. A learning expedition invites all learners to adopt a learners' point of view, to be curious and to engage in authentic inquires through direct encounters with the content being studied. More information about expeditionary learning can be found on my website at www.expeditionarymath.com

I am writing to you to request that you consider agreeing to have your child participate in the study. Participation in the study is optional and students will be able to end their participation at any time should they choose to.

This research study will take place between April 2018 and June 2018. Students will be going on a mathematical field trip with their class and teacher. While there, students will be learning about specific mathematical outcomes. The classroom mathematics teacher will be writing some notes and taking photos. These notes will focus on the math, they will refer to the content being studied and the strategies that students are using. They will not refer to student achievement or behavior. These notes and photos will be kept confidential. Student names and identities will not be revealed. At the end of the field trip students will create a summary of their learning experience using photos, writing and calculations. These will be shared with the researcher and returned to the classroom mathematics teacher within three months. No copies will be made, and the student's names will not be included in the study. These reflections will also be kept in a locked home office. Below is a sample of a student reflection.



Students whose parents and guardians have signed the consent form, will also be asked to sign an Assent Form for Student Participation. The Assent Form gives students the opportunity to agree or to not agree to participate in the study. Both the Parent or Guardian Consent Form and the Assent Form for Student Participation must be signed for the student to participate. Assent, agreeing to participate in the study, will be explained to students at school by the school resource teacher. A copy of the Assent Form for Student Participation is attached to this letter for your information.

The name of the school division, the school, school principals, teachers and students will not be identified or used in any publication. The names of the participants will be known to the researcher, but any other identity will not be shared. Field notes, interviews and student reflection will use a coding system so that no names appear on any documentation. The student reflections may include photos of the student taken during the experience. These photos will not be used in the study or any publication. All files will be stored in a password protected computer file, in a locked home office. Consent forms and contact information will be stored in a separate locked filling cabinet. Only the researcher will have access to the data. Participating teachers will be sharing their observations and reflections during the focus group discussion however, if it is necessary to identify a student they will use the established coding system. All documents will be destroyed by June 2018.

Participation in this study is completely voluntary. If any student would like to withdraw they can do so by contacting me at [redacted] or at [redacted] or by contacting my thesis advisor, Dr. Richard Freeze, at [redacted] or at [redacted]

Students who do not have parental permission to participate in the study will still take part in the learning experience. Their reflections will not be included in the data and teachers will not include observations about the student in their field notes.

Participants in this study who would like to receive a summary of the results can do so by indicating on the informed consent document.

Your signature on this form indicates that you have understood to your satisfaction the information regarding participation in the research project and agree to participate as a subject. In no way does this waive your legal rights nor release the researchers, sponsors, or involved institutions from their legal and professional responsibilities. You are free to withdraw from the study at any time, and/or refrain from answering and questions you prefer to omit, without prejudice or consequence. Your continued participation should be as informed as your initial consent, so you should feel free to ask for clarification or new information throughout your participation.

The University of Manitoba may look at your research records to see that the research is being done in a safe and proper way.

This research has been approved by the Fort Garry Campus Research Ethics Board, Education/Nursing REB. If you have any concerns or complaints about this project you may contact any of the above-named persons or the Human Ethics Coordinator at 204-474-7122. A copy of this consent form has been given to you to keep for your records and reference.

Parent or guardians Signature ______________________ Date ______________________

Researcher's Signature _____________________ Date ______________________

If you would like a summary of the results of this project, please list your email address below and I will send a copy to you directly.

Email address ____

Appendix J

Assent Form for Student Participants

UNIVERSITY OF MANITOBA
Faculty of Education

March 2018

Research Project Title: Expeditionary Learning for Inclusive Mathematics: Three case studies from the field.

Principal Investigator and Contact Information: G. MacLeod [redacted]

Research Supervisor and Contact Information: Dr. Rick Freeze [redacted]

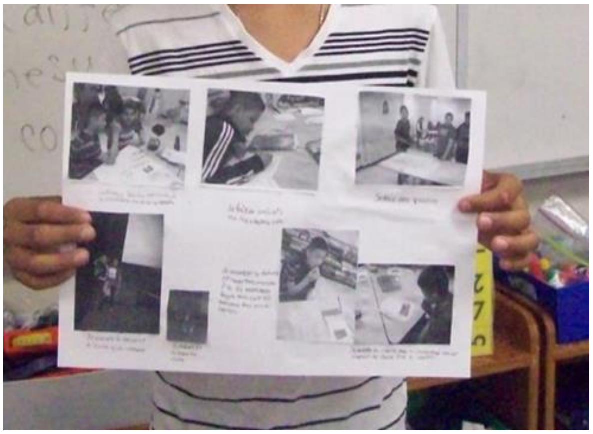
This consent form, a copy of which will be left with you for your records and reference, is only part of the process of informed consent. It should give you the basic idea of what the research is about and what your participation will involve. If you would like more detail about something mentioned here, or information not included here, you should feel free to ask. Please take the time to read this carefully and to understand any accompanying information.

I am a graduate student at the University of Manitoba in the Faculty of Education. I am interested in examining expeditionary learning as a potential teaching and learning approach that will engage all learners in the development of both mathematical content knowledge and problem-solving thinking in the Middle Years. This is the focus of my research for my doctoral thesis.

In expeditionary learning, students adopt a mathematical point of view, seeing the world as mathematicians do. They engage in authentic inquiry and original research within their communities. They develop strengths as learners, so that no matter what the context, they will have access to their own strategies to deepen their understanding, create a solution or develop a revolutionary invention. More information about expeditionary learning can be found on my website at www.expeditionarymath.com

I am writing to you to request that you consider agreeing to participate in the study. Participation in the study is optional and students will be able to end their participation at any time should they choose to.

This research study will take place between April 2018 and June 2018. You will be going on a mathematical field trip with your class and teacher. While there, you will be learning about specific mathematical outcomes. The classroom mathematics teacher, will be writing some notes and taking photos. These notes will focus on the math, they will refer to the content being studied and the strategies that you and your classmates are using. They will not refer to student achievement or behavior. These notes and photos will be kept confidential. Student names and identities will not be revealed. At the end of the field trip you will be asked to create a summary of your learning experience using photos, writing and calculations. These will be shared with the researcher and returned to your classroom mathematics teacher within three months. No copies will be made, and student names will not be included in the study. These reflections will also be kept in a locked home office. Below is a sample of a student reflection.



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may contact any of the above-named persons or the Human Ethics Coordinator at 204-474-7122. A copy of this consent form has been given to you to keep for your records and reference.

Participant's Signature ____________________ Date _____________________

Researcher's Signature ________________________ Date _____________________

If you would like a summary of the results of this project, please list your email address below and I will send a copy to you directly.

Email address _______________________________________

Appendix K

Initial Questions for the Semi-structured Interview

1. Please describe a typical math class with your students.

2. What resources do you incorporate into your lessons?

3. Please give an example of a problem that you have recently asked your students to solve.

4. Please describe how your students respond when given a problem to solve.

5. How are all students included in your class and lessons?

6. What mathematical concept or learning behavior would you like to most develop in your students? Why? How might you do that?

Appendix L

Initial Interview Transcripts

Teacher 3

To start the interview, I thanked [redacted] [T3] for being willing to give up some of her time to talk with me. I explained that the initial interview was simply to give us a starting point and to have some ideas about where she would like to go with this project.

G.M. Question 1. Please describe a typical math class with your students.

So, um, typical... that's hard, that's a tough one. I am totally all over the place. I haven't taught math in five years, I started teaching math, but I moved to technology, so I haven't taught math for a while. I only have one math class.

Ok so, I'd say that I am not routine like other math teachers. Recently I've been starting with number talks to work on communicating and reasoning and that takes like seven to ten minutes.

Then I flip into problem solving or questions or a new topic. So, for like an example this afternoon I'll have a lesson and questions and students will be working together. I don't do a ton of direct teaching. There's lots of talking and lots of exploring, sometimes with the document camera.

We are all over the place. I feel like we never finish anything, it's like a little bit here and then a little bit there and so yeah.

> *As she is speaking I can here a bit of nervousness. She almost sounds embarrassed that she doesn't have a specific routine to share.*

Okay, thank you.

G.M. Question 2. What resources do you incorporate into your lessons?

Ok we have this side shelf where there are like mini whiteboards, fraction pieces, rulers… its always accessible for them.

(pause)

We do almost all our work on white boards.

We use iPads because I'm in the room right next door to the computer room.

I have moveable tables.

(pause)

There are fourteen iPads that are just easier to get together than the computers because I'm not always planned ahead. I don't use technology as much as I should because I am all over the place.

Yeah, that's pretty much it.

Great, thank you.

G.M. Question 3. Please give an example of a problem that you have recently asked your students to solve.

Ok so one we did most recent was you know the balance scale questions where there's like those pictures.

Yes, I think I know which ones.

So there's three pictures of balance scales with the rectangles, the circle and the square and each one weighs different you try to find out what each one weighs?

We dissect that one in vertical spaces. We are working towards algebra.

G.M. Question 4. Please describe how your students respond when given a problem to solve.

Right now, they just dive in. we've been doing so much that they just jump in, so much group work. It's kind of the norm. they just go.

Was there a time when that was different?

They just do everything together. Its an established norm. At first it took them a while to get into it and they didn't have that persistence but now its March, so their perseverance is way up. Sometimes a kid wanders off, at the beginning a few kids wander – one kid goes over to friends and watches but then when he gets it he comes back.

Ok, great, thank you.

G.M. Question 5. How are all students included in your class and lessons?

They are all random groupings. Maybe on an odd day I use a strategic grouping. The teacher that teaches ELA and Social uses random groupings too, so they are used to it. They come in everyday and are "ok are in the same groups or different groups"

I have one student that needs t know, can't handle the random part he has to know if the group is going to be different, so I'll just go over and tell him.

I have one student who has an IEP and does the programming that comes from St. Amant. It's a one to one with an EA. I'd say 97% of the time he's not included in the same stuff that the rest of us are doing but he's in the class sometimes and sometimes he's not. If I have something that I think that he would be able to do, then I include him. Like patterning, if we are working on patterns then he can work on his patterning goals at the same time but that's not too often.

I should also mention that I have another student that is with an EA that works on communication. Working on oral communication. So, the kid totally understands the math but can can't tell me anything like I'll wait for a really long time and not get anything out of him but then he'll say something to the EA so we are working on oral communication.

They're included in everything, but the EA might take him elsewhere.

G.M. Ok, thank you. This last question is to help us think about what you would like to do in the project so it's a bit of a look ahead.

G.M. Question 6. What mathematical concept or learning behavior would you like to most develop in you students? Why? How might you do that?

Right now, I am really hoping to get kids to ask questions when they don't understand. I want them to put their hands up. We do a lot of listening.

I want them to be able to tell why they did something.

I would really like to see kids asking more questions. Communication is like a huge goal.

I don't know…

I thought that they would be able to ask questions by now, but they don’t.

G.M. Thank you very much.

Initial Interview

Teacher 5

Monday, March 5 8 – 8:30pm

To start the interview, I thanked Jon for being willing to give up some of her time to talk with me. I explained that the initial interview was simply to give us a starting point and to have some ideas about where she would like to go with this project.

G.M. Question 1. Please describe a typical math class with your students.

Well it’s all over the map because I don’t know what I want it to look like. We’ve worked a lot with John Hattie and Visible Learning. I’m in a divisional leaders group and we did a deep dive into John Hattie’s work last year. I never set out to be a math teacher or a middle year’s teacher. My major was geography so once I tried grade 5 and now grade 7 I started doing lots of research like through the BEF we did PRIME based on Marian Small. That changed my perspective to more process and conceptual understanding. At times it may appear old school with worksheets, but it has the background of being more process focused. So typical math class I would say has lots of discussion, we sit at table groups, the students are in groups of three and they change every day. That comes from Peter Liljedahl and the vertical non-permanent spaces – the division has had him in he talks about visible spaces. Each day when the students come in they take a card and that determines their group for the day. It’s hard because its lots of logistics but I have seen how it works and so it’s worth it – I’m quiet long winded you’ve noticed I’m sure

G.M. Not at all, I am enjoying hearing about your class and your thinking. Please keep going.

The other piece that I wonder about is that because I teach in French Immersion it's the vocabulary and the language. When you're in a group of three it's hard to hide.

So, a typical lesson I would say starts with mental math, a number talk, which is based on the work of, well I'm not sure but I think that you know what I mean, then we would be exploring conceptually, doing some problem solving we're all over the map but we start with mental math as an activator.

G.M. Okay, thank you.

G.M. Question 2. What resources do you incorporate into your lessons?

That depends what you mean by resources?

G.M. Great question. I would say anything that you bring in to support the concept being learned.

Well since I am learning about teaching math I am fairly well read – not only the textbook. In grade 7 for example we use lots of physical manipulatives like for fraction because its hard to move to that conceptual piece. And as a resource, but maybe more as an instructional resource that a math resource, we use white boards. We have white boards all over the place here. That's a vertical, non-permanent space strategy from Peter Liljedhail as well. He's pretty sure that non-permanent space means more ideas that come quicker, students aren't married to them, so they are more willing to get working and let's you be more committed to the process that n the product. We have cheneliere text books too but not a copy for each student. If you were to ask one of the students to get a copy of the textbook they wouldn't know what you were talking about its more that my partner and I use it. We get questions from it we use resources from other places too like Marian Small, university of Waterloo we find what's going to work and we use it. I should mention too the human resources as well. I get to be in my partner's class four times a cycle. She's new to teaching math so I get to go in and support her once a cycle with each class that she teaches. In the division we have the math coaches too. I know Lisa for four years now so she's really easy to reach out to and ask a question. I use the BEF a lot too you know Nicole Allain Fox she' great, she's been out a few times. I can email her, and she'll come out and we can work together.

G.M. Great, thank you. That is very interesting how you included the human factor in the resources that you use.

G.M. Question 3. Please give an example of a problem that you have recently asked your students to solve.

Well after we met I started talking with Tricia from George Waters and she shared some open problems with me, but I would have to say that the ones I try are usually routine, usually straight forward not too open. I blame French. I've been thinking about it, I just started this a couple of weeks ago and now I am asking more challenging questions in that they involve many steps or have some information that the students don't need. I worry about what's suitable for their level of vocabulary and it may just be my bias, but I hold myself back because I worry that they won't understand and then I wonder if I am sacrificing the content of the math for the language. There is a lack of richness in the problems.

Here’s an example, we’ve been working on les nombres enteirs. La somme de deux nombres est egale a +15 et la différence est de -7. Que sont ces nombres. Its not terribly open. I also did one students got 18/20 on a test and one student got 87% who got the higher mark and why.

G.M. Ok, those are great thank you. You bring up a really interesting point about the French language piece and where do we find that balance or the right blend of mathematical challenge and languages.

G.M. Question 4. Please describe how your students respond when given a problem to solve.

Most of my students this year, they persevere. They persevere, they might have gaps, but they persevere. The challenge is the communication. Some of them have the procedural piece but they struggle with the communication. So, they are ‘good’ in a typical sense. Those that are weaker don’t hold back anymore because they know they have access to the manipulatives that we have, they use the open number line, we have it out, they give it their best

G.M. Ok, great, thank you.

G.M. Question 5. How are all students included in your class and lessons?

Well I vary what I do, process and product. This is where the talk comes into play. Another student might be able to say it in a way that connects. So days I’ll say ok label yourselves as A, B, and C. A just came in late, B you need to explain where we are at and C you listen to make sure that they have it right and be ready to ask questions. Then we practice saying what we are learning. They all have a chance to talk and to fill each other in. They all contribute something.

We have the manipulatives too so that they can see the math nit just do it. We use then for integers and it works really well you now with those two-sided chips. Even if we are doing a worksheet or preparing for a quiz they can still use them, and they can see it. The goal is the same for all of them, to operate in the symbolic but it may take some of them longer to get there.

I also change the groups a lot so that they can hear one another and how they go about solving the problems.

This year I'd say that I have fairly strong students. I find the spread between the low ones and the high ones really large. That's been really hard this year. I would say that about 5 of them don't have the prerequisite knowledge that required for these topics. For example, in fractions I have kids that aren't familiar with the concept of a fraction. They don't know numerator and denominator so it's really hard for them to jump in. the higher we go I find that the spread gets bigger. I know we just have to keep working and moving forward but I haven't found that one miracle solution just yet, the gap is so large. This year I have a small group that comes in for extra help in the mornings. It's not ideal because I know that they should be able to be supported in class, but this just gives them a bit of extra time to hopefully fill in those gaps.

G.M. Question 6. What mathematical concept or learning behavior would you like to most develop in you students? Why? How might you do that?

I want them to know that math isn't just a subject. Especially this year in grade 7 everything just seems to come together. It's all new, it's based on fundamental understandings and it's easy to get lost. There's so much to do and some get lost. As for learning behaviors its understanding math and where we see it. There is no 80 watermelon questions in real life. I want them to see math all around them.

You know when you were talking about the zoo project when you came to share this project with us? And you talked about how they actually used the math to figure out the area of the cages that was great. I don't have a specific outcome in mind because as this is later in the year we should be able to find something all encompassing. They will have done so much by the time we get to this project, I want the students to decide what to do. I want them to recognize that math is everywhere not just in room 207.

G.M. That's a great question that I would love to ask them after the project – were they able to recognize that there is math out there.

Initial Interview

Teacher 3

To start the interview, I thanked [redacted] [T3] for being willing to give up some of her time to talk with me. I explained that the initial interview was simply to give us a starting point and to have some ideas about where she would like to go with this project. She shared that did not know anything about the project and had just been told a few days before that she would be taking part.

> *This was a surprise to me as I didn't realize that not all the teachers had volunteered. The numeracy consultant had included* [redacted] *[T3] in the study as they felt it would be a good learning experience for her. It sounded like she wasn't too keen on the idea.*

At this point I reviewed the outline of the project and shared some background about expeditionary learning. I let her know that the study is confidential and that her name, the school name, the school division and all the students would remain confidential and would not be used in the study.

G.M. Question 1. Please describe a typical math class with your students.

Hmm, I'm not too sure…Have you talked with [redacted] [T4]? She is a teacher at [redacted]. I think that she is going to be doing the project too? I student taught with her, so I model my math class after her. I'm only in my fifth year so I just do that.

Ok so I start each class with a warm up. It's usually problem solving or mental math. So, the students do that and then we correct it. We go through it together.

Then we get into the lesson for the day, um…

G.M. Can you give me an example of a lesson topic?

Oh, for sure, like decimals, so I would do some pre-teaching, probably on the smartboard, then we would have games, questions for them to work on and a worksheet.

Do you know Jerry Amis at the University of Winnipeg? I'm doing a post bac and I took a class with him, so I do those non-routine problems every week. So yeah, that's kinda what it looks like.

G.M. Okay, thank you.

G.M. Question 2.What resources do you incorporate into your lessons?

What do you mean by resources?

G.M. Any additional tools that you and your students use in the classroom.

Ok, so like I use the smartboard, we have games, there's iPADS, we use the computers, sometimes we have blocks and bingo chips it depends, and you know things like that.

G.M. Great, thank you.

G.M. Question 3. Please give an example of a problem that you have recently asked your students to solve.

Ok, so do you want like a real problem that we just did?

G.M. Sure, that would be great.

Ok so I have one right here. Should I just read it?

G.M. That would be great, thank you.

Pablo spent the entire day at an amusement park on the roller-coaster. On the way out, he saw a stuffed dinosaur at the ring toss. He said "I have to win that dinosaur for my little sister Arian. Pablo had five rings to toss that could land on 5, 10, 15, 20 and 25 points. To win the dinosaur Pablo needed a total of 40 points. How many different combinations of points are possible using all five rings?

G.M. Oh, that sounds like an interesting problem. How did your students respond?

Well I have lots of EAL students, so I set up a ring toss in the class so that they could see what it was. Most of them got it but not all of them got all seven.

G.M. Thank you. Having the actual ring toss must have been so helpful for them.

G.M. Question 4. Please describe how your students respond when given a problem to solve.

Well, ok, first I try to make it as clear as I can. So, we read it all together, so I know its clear. They actually love word problems I'd say way more than number crunching. They enjoy it.

It is curious to me that I had asked for a 'problem to solve' and she has interpreted that as a written word problem.

They will jump right in and not really know what they are looking for. And then they keep going and they still don't even know what they are looking for, so they did all this work and they aren't looking to what the problem asked them to find out.

G.M. Ok, great, thank you.

G.M. Question 5. How are all students included in your class and lessons?

Ok, so I have lots of EAL kids, so I have to clarify the language all the time.

I have some kids, that you know, are below level, so I take off a couple of questions like for the ring toss questions instead of doing five rings I would just say that they only have three rings.

> *It is curious to me that there is only one solution offered for students who are finding the content difficult. Doing less questions that the y students don't understand maybe won't help them to understand better.*

I have some kids that just whip through and then they'd be done in like five minutes. So, I have like puzzles for them. Oh yeah, I also took a course with Ralph Mason at the University of Manitoba and he gave me some books, so I have a few puzzle books and them when they finish early I can just give them one of the puzzles.

Again, I am not sure that this comment represents an inclusive perspective as it may be interpreted by students as a way to keep busy not as a challenge or an attempt to deepen their sense of belonging to the group.

I use groups and we do random groupings so that they are always working with other people.

The reasoning for using random groupings

G.M. Ok, thank you. This last question is to help us think about what you would like to do in the project so it's a bit of a look ahead.

G.M. Question 6. What mathematical concept or learning behavior would you like to most develop in you students? Why? How might you do that?

I guess, well I just heard about this project, so I don't really know. I guess that I want them to enjoy math and to learn math.

Some kids have a negative attitude and say things like I hate math.

I just wanna show them how much I enjoy it and hope that helps.

Teacher 4

Initial Interview (telephone)

G.M. Question 1. Please describe a typical math class with your students.

Ok, yeah, well, my class is structured. I start with a routine problem. I have to admit they are not the greatest if problems but that's how we start. Then we have a mental math section and follow that with a number talk. Number talks are the big thing right now as you know. Then that opens up into the lesson. I would say half of our class time is spent on that opening routine and half is spent of the lesson and the practice.

(honesty, strengths in beliefs but questioning those beliefs at the same time)

G.M. Question 2. What resources do you incorporate into your lessons?

That depends what you mean by resources? I mean I use manipulatives as much as I can, I have a smartboard, it is my lifeline if they take that things away I'm going to retire. I love the interactive manipulatives. I do as much hands on as I can. We are all into these vertical learning spaces right now so I have those in my room. I am using the number talks techniques as well.

G.M. Question 3. Please give an example of a problem that you have recently asked your students to solve.

I struggled with this question and I am unhappy with my answer. The more in servicing I have and the more PD I do the more it makes me unsure. I am struggling to find the non-routine and rich tasks. So, for grade 8 I just did a magic square. So, the students had five positive integers and four negative inters and the needed to make the product of the diagonals, up and downs equal to minus one. So that wasn't the greatest, but it was what we did. It wasn't what I wanted it to be.

(*reflection/concern/disappointment)*

G.M. Question 4. Please describe how your students respond when given a problem to solve.

Well that's something that we have been working on is perseverance, that stick with it and dig into it, dig into a problem. We've progressed, they are much more into it now. I see it in the vertical learning spaces there's stamina is not what it was its slowly but surely improving. Id say I have 25% of my class disengaged, about 50% super engaged like they just want to run with it, like they could just take over and then 25% trying – at least they are trying

(honesty)

G.M. Question 5. How are all students included in your class and lessons?

So, inclusion is hard. You know [redacted] it's a special needs school in a low socio economical area. Of my 120 students I have 20 on IEPs. So that's a heavy load. Philosophically I struggle with inclusion. If we are taking best practice so we're learning about operations with fractions and the IEP students are working with fractions does that count? Is that inclusion? I try to include them in the vertical learning spaces even if they are just the recorders. I'll say I am evolving – philosophically. I just went to this UDL thing and I asked you know tell me what to do and I'll do it. I said what do I do when I am teaching the Pythagorean theorem? Cause that's a unit you know, so if I have them identifying right triangles is that inclusion? I haven't found a way to make it work yet. So, is it similar content? I haven't found a way to truly have it the way I think they should be included. I want then to be moving forward at the level that they are at so I struggle.

G.M. Question 6. What mathematical concept or learning behavior would you like to most develop in your students? Why? How might you do that?

Well that's where I want the perseverance to come through, its stamina, persist with something, be uncomfortable I want that productive struggle. I want them to critically think – that's something that they work hard to avoid. I don't know can I tackle that in April of grade 8 its something that we should have started from the beginning, its systemic change. So what I am thinking is to show two pictures, because I have been trying those three act tasks – some I like some I don't but that's because I don't understand them – anyway I thought that I would show two pictures and have then go what do you notice, what do you wonder.

I'm a control freak I don't to have this project all planned out I want to keep it open. I don't want it to get too narrow.

Initial Interview

Teacher 1 (telephone)

G.M. Please describe a typical math class with your students.

A typical math class for my students will begin with an activity to prime students for what they are about to learn. This is mostly done through number talks or a series of questions students answer. Once we have completed that, students will be in small groups at whiteboards. I will give students an initial problem, or question to answer. I will then circulate the room helping small groups or providing addition questions. Sometimes, I will use direct teaching to help teach the steps to a new concept, or one that needs clarifying.

G.M. What resources do you incorporate into your lessons?

I use whiteboards a lot in my classroom, as students are mostly using them for math. I will also use games or activities to help students understand a concept in a different way. Use of base 10 blocks, pattern blocks, dice, counters are all use as well.

G.M. Please give an example of a problem that you have recently asked your students to solve.

Ok, here's one that I do every year. It's the Ice Cream Problem

In shops with lots of ice-cream flavors there are many different flavor combinations, even with only a 2-scoop cone.
With 1 ice-cream flavor there is 1 kind of 2-scoop ice cream, but with 2 flavors there are 3 possible combinations (eg vanilla/vanilla, chocolate/chocolate, and vanilla/chocolate).

How many kinds of 2-scoop cones are there with 3 flavors?

How many kinds of 2-scoop cones are there with 6 flavors?

How many kinds of 2-scoop cones are there with 10 flavors?

What about "n" flavors? Create a poster that represents your group's thinking.

G.M. Great, thank you. Please describe how your students respond when given a problem to solve.

Well, that depends on the student. I will see a variety of responses to a new problem. Some students will dive right in, utilizing a variety of strategies to solve the problem. Others may see the problem as too difficult and will shut down, thinking they are not able to solve the problem.

Students will sometimes hit a wall when solving, not sure of where to take the solution next. Again, some students will persevere through their “road block”, and others will see the problem as “impossible.

G.M. How are all students included in your class and lessons?

Well, I utilize rich tasks, multiple entry points, and flexible grouping, all students are given the opportunity to contribute to a lesson. I do my best to give students tasks they are capable of completing at their level, but also pushing them to do what they think is not possible.

G.M. What mathematical concept or learning behavior would you like to most develop in your students? Why? How might you do that?

I would like students to have a stronger number sense, perseverance, and ability to “play with numbers”. I find many students look for one answer and if it is not attainable, they will not look to another strategy to solve a problem. Many students struggle with their basic facts, and this hurts them when solving complex tasks.

Appendix M

Professional Development Session Agenda

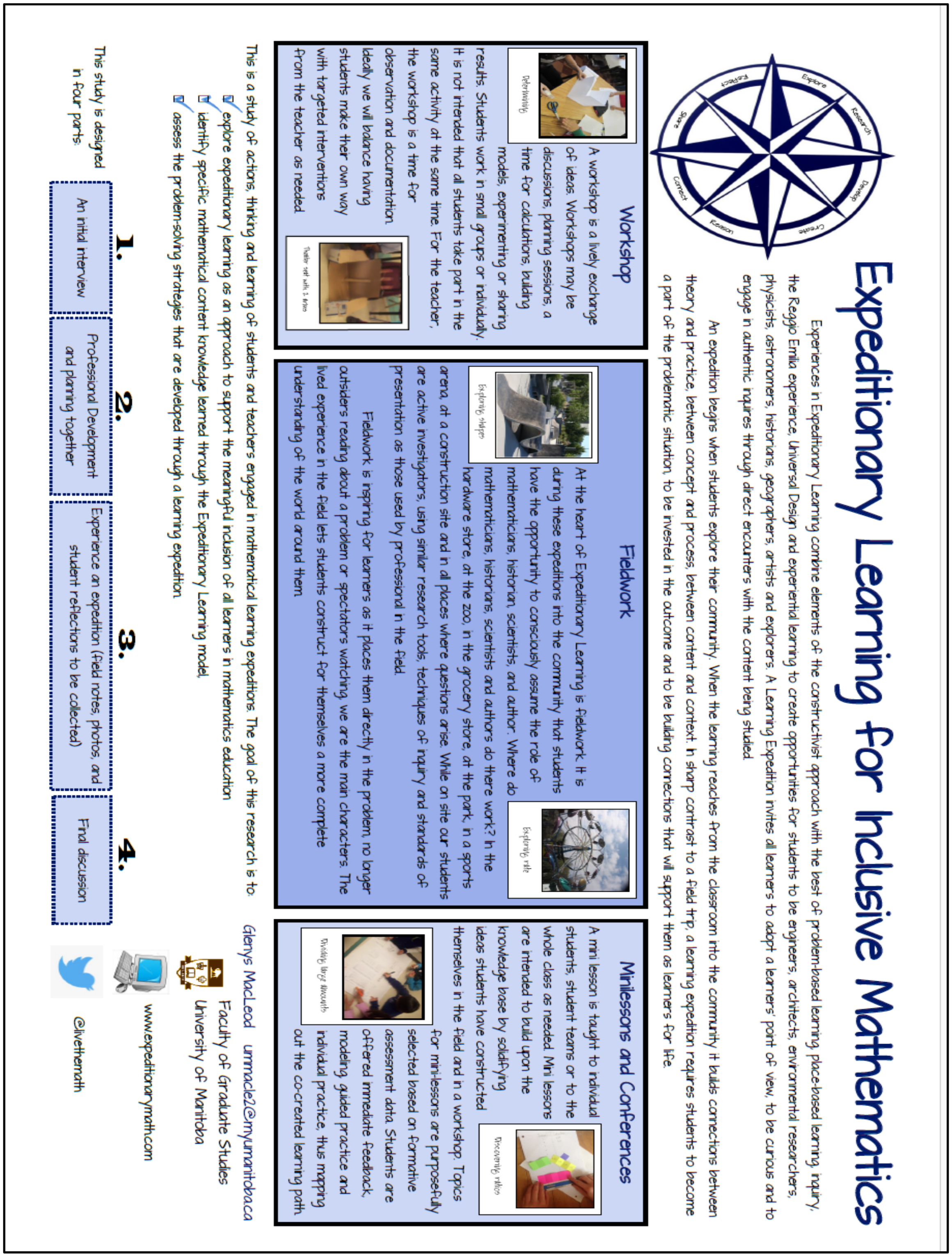
1. Welcome and introduction (5 minutes)
2. How I came to teach this way and the purpose of Expeditionary Learning (10minutes)
3. What is problem solving? What is learning? Chilean miner's activity (30minutes)
4. Learning Strategies to develop (30minutes)

Break (10 minutes)

5. Expeditionary Learning as Fieldwork, Workshops and Mini-lessons (30 minutes)
6. Planning for the study (30 minutes)

Appendix N

Study Placemat for Participants



Appendix O

Focus Group Discussion Transcript

Tuesday, June 5, 2018

G.M. The purpose of today's discussion is to reflect on the expeditionary learning experience. To start it would be great to hear a summary of your adventures.

So, how were your adventureS?

(laughing and nodding)

T1. Great

T4. they were really good, the student learning…um, I had [redacted] [NC1]in with me, and uh, for pretty much the majority of it, and the student learning was huge and you could see the student engagement. I had kids who are very non-attenders, actually engaged…uh, he didn't get to the part of actually completing a project, but, we were doing the gym expedition…

G.M. right

T1. and we were measuring different things in the gym, he was actually in there measuring, actually learning how to measure properly so I was glad that he was getting something out of this

G.M. right

T1. and the ones who were there from start to finish, their products were great, uh, they did a really good job and they really showed a different side of themselves.

G.M. ok, well a different side, that's so intriguing because even the difference of being at Canadian Tire to being in the classroom was like …wow.. they are like you could see it and feel it in their interactions. So, a different side, how lovely that a school can provide, or offer, chances for that kid to shine in different spots..

T1. yeah, yeah.. instead of just being here's the one way

G.M. can you tell us where you went and what you did, you mentioned the gym?

T1. yeah, we went into the gym...um…and we had kids… our focus was that we wanted to have kids working with area and perimeter or measurement of different lines so we just went into the school gym, I just took all three classes of mine and so it was a lot easier just going into the school gym I didn't have to worry about leaving the school building, or for how long or all that stuff, uh, it was also nice to keep the kids contained, so, if it wasn't working we had that option of, ok, let's go back to class. So that was nice, and we were able to, [redacted] [NC1] and I were able to debrief really quickly and say ok next class is coming in so what are we going to change, ok, we're going to photocopy stuff as they are coming in

(laughing)

T1. so that was nice, and I would like to take it further in the future, but we are, we are wrapping out heads around it and it's not the easiest thing at this time of year

G.M. ok, because we had sort of discussed the garden idea but the gym was

T1. the gym was great, they had come up With questions we didn't even think of, like they were saying "how many chairs would it take to to go to the ceiling?

G.M. oh wow,

T1. I have no idea, how would you figure that out, G.M. that is so interesting to hear you say that because that is one of the questions that you had for me in initial session was “how do you get the kids to ask the questions” and then there was a moment where we imagine a space and we all started throwing in questions and I said, this is what it’s like but until you have lived that experience with students and you have seen kids do it, it is an unknown, it is scary

NC3. well we laid a bit of the ground work, we uh, we did a number talk using pictures of the MTS Center and we threw out some off the wall questions like we said, there is no popcorn in this picture, but how much popcorn do you think that they prep for a game that is sold out? So just trying to get them thinking beyond what they are seeing

G.M. right

NC3. so that when we took them into the gym they knew what we were talking about when we said “think outside of your space” ([redacted]. nodding and agreeing) like what could you do in here that we could ask questions about so we tried to prime them a little bit for you know open their eyes and really have them take a look around

T1. that was probably really good, priming them and in the future if you were doing this more often then throughout the year those questions would come more..

NC3. …and they would get used to it then I don’t think that you would have to do it you know at this point in the year student would be able to see those questions… but being that this was the first time that we were taking them anywhere… we needed to do that.

T1. yup, yeah.

G.M. what a great connection you made between MTS and the gym because that's probably not the first connection that kids would have made, maybe they would have connected to other gyms but maybe not to the MTS Center so that's a great connection to highlight.

G.M. ok, thank you

T4. Ok, so we went to Canadian Tire and the engagement was through the roof, like through the roof, I had kids who are weak and math phobic and they were the ones digging into it more than anyone. It was amazing, um, the inclusion was amazing as well, like I was able to include a lot of kids, well not a lot of kids but a couple of kids who are on IEPs so they felt like they were a part of it and um, you know they were able to participate in the class which is fantastic. So, doing it again I think I would have done it with my grade seven's instead, it didn't get to my grade eight outcomes the way that I wanted it to get to my grade eight outcomes. So, I think it covers more of the grade 7 areas and outcomes more so to do it again, if I was to do that again I would do it with my grade seven's instead. But yeah, they had a lot of fun with it. They did not, no body picked a shed from Canadian Tire, they had to go online and find a shed somewhere else and unfortunately the shed that they picked was at Home Depot, but it was on Walmart site for two hundred dollars cheaper and then all of a sudden it disappeared from Walmart site so, I will have to pay the extra two hundred dollars

G.M. did you buy the shed already?

T4. no, not yet but I will be buying it. They did pitches as well and that was unbelievable. One of the groups made up a jingle for their shed and everything, it was pretty awesome, [redacted] [NC1] and I started marking them yesterday and just trying to wrap our heads around how we are going to do that and it was you know, the first question that we had was how they showed their understanding of the problem and I think, I think, every group, except that one group that didn't finish, I think every group had a really solid understanding of what they needed to do, so that was really cool. One group of boys kept on coming up with these sheds and then one would say that shed's too big and someone else would say no, no it isn't so I could see that they weren't really getting it but it was really neat to see them jump in and stick with it.

NC2. did you ask them to design a shed or just pick a shed

T4. no, just pick a shed, so I gave them…

NC2. the parameters

T4. I gave them what had to go into the shed and then we found that it was a pipe dream and not everything was going to fit into the shed and so then they asked me to prioritize what I wanted t put into the shed. I said the lawnmower had to be in the shed but the rocking chair maybe not. It was neat because they had to think in terms of volume as well because um, bicycles and stuff like that and I was hoping to get some of them to think of standing the bikes up, but nobody got there. They didn't think of it in terms of volume the way that I wanted them to but it was neat and, um, some of them had, well the criteria was, well I want to be able to put the equipment in the shed in a variety of different ways so I wanted more than one floor plan so not everybody was able to get to more than one floor plan, but some of them didn't get there and that's ok. Some of them you could tell right away didn't understand the floor plan because they made a scale and ok two squares equal one foot and then it didn't work out because there object was bigger than one foot so they tried but yeah it was neat and then looking at it, it was much easier to see what they got and what they were lacking and stuff like that. They had to do the cost, how much tax, was it on sale, one group even had how much the shipping was, and yeah, it was like, the engagement was, you can't match that engagement, it was amazing.

NC1. and then they had to measure each one of those items that you had wanted in your shed so that they could find how much space it would need in the shed.

T3. Did you bring the lawnmower in?

T4. I did not,

T3. oh, ok, I'm, I'm just wondering how the measured it

(group laughing)

T4. no, no the lovely people at Canadian Tire took a lawnmower off the rack for us and brought it down so that the kids could measure.

T1. that's great.

T4. It was awesome, it was awesome. Yeah, no, I said everything that everything that needs to go into my shed can be bought at Canadian Tire so that's why we were going there, and they were able to measure all the stuff there. There was an emergency, there was a type of chair, you know those old, white plastic chairs well they're not selling any of those this year so I had to, my son was at home so I texted him and he measured and sent me the measurements and (laughing) I gave them all the measurements and ah, yeah but it was neat yeah but to do it again I wish I would have done it with my grade sevens.

T4. And I am just wrapping my head around the final product and I am not sure what I want. Like, I've got the portfolio pages that I am not necessarily thrilled with, but I'm sort of not sure what to do with everything right now.

G.M. I just wanted to ask you one question because somewhere I had jotted down um, it was probably about the engagement that you both had mentioned so far and sometimes for myself I worry that engagement may be ‘fun and games ‘and the kids are… but at Canadian Tire when the group was moving between let’s say the chair and the lawnmower and the group was keen to get there and is that engagement or where they engaged when they were holding that tape measure getting the math piece out of it, were they engaged in the context and the content or just the context?

T4. no, no I was surprised that when they got back to the school and the days that we worked on it after, they were digging in, they were still really engaged and if you had seen, some of the students had laid out a floor plan of the shed on the floor and that continued for days. And there was a lot of arguments, like yeah, I called them geeks, I said you realize you are arguing about math right now, and they thought that that was funny but like yeah like I said some of my weakest kids who would normally give up at any site of a challenge really were into it and, and, and with ideas and interesting enough had a lot of higher end ideas.

G.M. Oh,

T4. yes!

G.M. can you give us an example?

T4. well ,yeas I am just thinking of my little [redacted] who is math phobic and struggles, struggles struggle, and she was the one who had the idea that if we turned the bikes on their sides, like if we turned them up then they would reach that height, I did talk with them about that but she was the only one that went there and yeah and for her, usually she is the one that is like, there this is good enough, and she felt like, like she has talked about it and said this was the best math class and this was awesome and I know that this is something that they will remember forever.

NC1. Right from your intro she was like, oh this is awesome, she was looking at the contents of your shed and thinking well here's my space and you know like that whole intro just hooked her.

T4. yeah, it was really neat with that whole intro and the pictures that I showed them with What do you notice, what do you wonder, what they came up with and then what they needed from me and some of them were like well we need to know how high your roof is, and I was like, well why do you need to know how high my roof is, and I was like ok, well I'll get that for you but I don't know why you would need that and so when we talked about that after they said well we thought that the peak may come into play with the peak of the shed but once they had the measurement and saw another picture that I took they were like oh no it so it won't come into play. So, they were thinking about things.

G.M. Its almost like here's the problem and then they are thinking around the problem.

T4. yeah, yeah..

G.M. Do you think kids would have thought of that if they weren't living the problem with you?

T4. oh no,

G.M. and had it been a text book question

T4. Oh no, no, no

G.M. well that's neat that their eyes are looking beyond the problem,

T4. oh and then some of them were well we could get you the wider shed if we could have the doors toward the fence and I said are you kidding me, how would I get into the shed. Well could you make a gate, and I said no, I am not going to make a gate and so they were trying, they were trying to get me the biggest shed they could but they had some interesting approaches to it.

G.M. is that investment in the problem?

T4. Um hum oh absolutely, like it meant something to them. But I am worried that if I don't go and buy the shed when they come back and visit in grade nine they'll be like did you buy the shed? Ok, I'll go buy the shed I promise. (laughing)

NC1. And then they'll want to know if it is their shed that you bought.

T4. and now they are like, well who won, so we found two different winners, the girls won because their pitch was the best and then the one we marked yesterday theirs blew us away, they had a couple of different floor plans and everything was covered so they won. But the girls jingle was so good.

T2. Ok, well we went to the cafeteria um we were struggling for spaces because, well the same idea as [redacted] [T1] I wasn't ready to jump into going somewhere far and doing the paperwork for leaving. I did just want to get their feet wet and figure things out. We weren't allowed to go to the gym because the Phys Ed teachers said no, so, the cafeteria is a space where everybody can go anytime because it is always open. So, we gave it a shot and I feel like our group, it was a lot of figuring it out as we go um, [redacted] [NC3] had mentioned that they did a lot of pre-priming stuff that we did not do with my group and I think that that was something that we learned the most afterwards was that we needed to prime them before. I went on the, my idea behind this was trying to keep things as open as possible and not give them a lot of direction and like [redacted] [T4] gave them a lot of parameters, I wanted mine to go into it with not really any so, um, but with that comes um a lot of really floundering, like what the heck are we doing? (laughing) so we did go and explore and come up with the questions and um I wasn't really excited about the the questions that they came up with and uh [redacted] [NC3] had gotten done with me and then [redacted] [T1] after and um and so the types of questions that we got weren't really that deep and they weren't really that exciting and they were just kinda, I felt like they were just doing it because they had to they weren't doing it because they wanted to like how [redacted] [T1'S] kids were super pumped like they were in the gym.

NC1. What grade did you do?

T2.. I did grade 7

T2. I don't know if it was the space, or that we didn't prime them or, we did the same MTS question for what do you notice, what do you wonder to practice the questioning piece but I don't know how much it translated into when we went to the cafeteria. I don't know if it was the space that wasn't that exciting, so we did end up having to pick the question for them. One of the things that jumped out for most of the groups was that they were interested in would the grade eights have to eat in the hallway again. So some background information, the grade 8s eating in the hallway right now and the six's and seven's eat in the cafeteria but our enrollment is going down with the French groups leaving so the grade seven's who will be grade eight's next year want to know if they have to go eat in the cafeteria next year with everybody else or will the get to go eat in the hallway which is something that they have been looking forward to doing. So, it was a real question that they actually wanted to know and so it was – so I just kinda picked that question.

G.M. It sounds relevant to them though.

T2. It was it was something that they were interested in knowing. Originally, I wanted to tackle um, multiplying and divining decimals, or even adding and subtracting decimals and that was one that I hadn't really touched on so I was hoping for. After we had gone through all the processes though the answer to the question was actually quite simple. It didn't involve really any decimals at all, um, I don't even know what else to say. It's been so long. Some of the groups wanted to measure things even though they didn't really know what they were measuring and even though they didn't realize that they could just count. It was a real simple question they could just see how many could sit at one table and then find out how many tables there were.

NC2. Some really interesting things did come up, like a does a grade 8 kid and a grade 6 kid need the same space or take up the same space? So then they started looking at if all the grade 8s sat together would they fit here and and do they mix.

T2. Right… like 6, 7s and 8s.

NC2. yeah, so they were thinking like do we take an average or some of them wanted to use measurements but it was almost like they didn't know..

T2. like they thought that they had to

NC2. Yeah,. Like the stuff was there so they thought that they had to.

T2. yeah, you give them tools and they are like oh let's use this tool even though we don't know why.

NC2. I think we messed up the last time, when we went back the last time, the tables were there.

T2. like the answer was there. The tables can only go in one way. So we needed to take the tables out or put them up or something so that they would have actually had to do measuring.

NC2. When we went the first time it was empty and then when we went back it was like oh no..

T2. yeah that was a big… on our part… once we realized that the answer was right there.

NC2. Like they could just sit kids at the tables. If they hadn't had that there they would have had to have measured to see how many could actually fit. Well they are all here laid out perfectly.

T2. So we were not happy about that piece so to uh wrap it up we did invite our principal in, two of the groups invited him in because they wanted him to know that yes they can fit everyone in the cafeteria to let him know and then he posed a number of really interesting questions like as an administrator there are more things to factor in that just if they can physically fit in there like do I have enough lunch supervisors and do you want to have the eights in there is it better to manage them if they are out in the hallways. Like, then he asked the kids, he just asked the class how many of you would like to be in the hallway if you had that choice? Pretty much all of them did because it is something that they were looking forward to year after year. So regardless of all this he's just like well I consider what you guys want too and so that experience was nice for them to be able to see that this was a real world question that you are talking to your principal about and he needs to make that decision for next year for real and now he has all this data and then yeah so it didn't work out the way that I thought it would but it was still beneficial in many ways.

G.M. and were all students in you class able to hop in and be a part of it?

T2. yeah, they were all able to I have one kid who has been the struggle all year who just struggles with participating in general. He was curled up in a ball on the coach today and wouldn't do anything so honestly it is hit and miss whether he wants to or not. There's times where [redacted] [T2] has come in and he's all hands up and giving all the answers and just like

100% and she was there today and he wouldn't do nothing. So we can't, we have no idea, no one can understand this kid and why he does what he does these things I thought that he would be all over this but no. he sat there and did nothing and so I had to talk to his parents and the parents didn't really care for this because it involved group work. I have said to the parent numerous times that this kid is capable of group work and can do really well at group work but there is lots of excuses from mom. I don't know if it has to do with what it was or if it's just he had checked out. But everybody else was engaged in it really well so. And these were mixed groupings, just random mixed groupings of kids. I think if I was to do it again I would prime a lot more. And I can see if we continue to do this or started at the beginning of the year and this was the norm it would be hugely beneficial and that I would get good at it and I would know what I am looking for better so I saw the potential even though It went crashing down at the end. But I can see the potential in it for sure.

G.M. it doesn't sound to me like it was a crash landing at the end. It sounds still like there had been some great conversation and…

NC2. There was, there was, and I think it is hard when you go in, like you wanted certain outcomes and that was, it's like our time is limited..

T2. Because I am running out of time…

NC2. Because it's hard..

NC3. It is hard to kinda steer it where you wanted to go, but be a little more open

T2. and give them that freedom too

NC3. So you have to be open with it

T2. What happens, happens

NC3. Exactly, you just have to run with it.

T2. I think that if you were to do it at the beginning of the year you have more of that freedom to know that if you don't hit any outcome, you know your year and you can go anywhere and I think that that would have been better than me trying to uh go that way without telling them that this is what we are doing so I uh struggle between that freedom and I don't know I'll have to try it. I will definitely want to do it more at the beginning of the year and have more of that freedom to go where it goes and hit whatever outcomes happen.

G.M. Ok because I remember when we first met and you were talking about how you wanted it to come from the kids..

T2. I wanted them to be engaged, I wanted it to be their question and be engaged about it but when they couldn't come up with something that exciting. Like even the question we did do isn't super pumping them like they weren't crazy about it. I have a very compliant class too, so they weren't. I didn't see that engagement.

T2. I don't know it could have just been the place we did or the time of year, it's a tough time of year for us. We did this in may.

T3. I'm next?

G.M. I think so.

T3. I had kinda the same issue. By the time I learned about this project we were well into the year. So, looking at the outcomes that we had left I couldn't really come up with anything so I went back and did an outcome that we had already covered at the start of the year. Um, so we did, we went to Family Foods and planned a class party. So, they, we started out just looking at flyers and I said you have a fifty-dollar budget and they went through in partners and picked out items. So that was kind of like our mini one there and they did really well, and they were super into it. From that we picked out some categories if what we needed to buy for the party so we came up with games, snack food, fruits and vegetables, drinks, and like cutlery, plates, napkins. And then we decided who wanted to go in a get what stuff. So we made groups so it was sort of random. That worked out well. And then we kinda kept morphing what we were going to do. We didn't have a clear idea. We knew it was going to be with adding decimals, subtracting decimals and multiplying to get tax. But we didn't know exactly what it was going to look like so we kept switching it. We went over in those groups and um we gave them a clip board and with each section we told them that they had a fifty dollar budget and each group ended up planning a party and then they did pitched just like your group and I would have given more time for the pitches if I was to do this again because um it was a little bit rushed. And so they each came up with, so that was our first tie going, to the store. We didn't actually buy anything they just went around the store and looked at what they wanted and the prices. And then they made their pitches and then we gave some requirements of what we would use to judge the best pitch and what the categories were and um what would be most appealing to the class. And then we went back the next day and picked up their items, brought them back. We held the party when we were done (laughing). So yeah, um, when did we do this, after that first time of just going to the store to find their items and make their pitches we had them, [redacted] [NC1] printed off some pictures like postcards and they wrote just what they had learned so far and we had some question that we gave them for it. We did, what did we do, where was the math in what we did and what was the purpose of our visit and we got some really good stuff and some really bad stuff in there. A real mix. And then we did the sheets we did demonstrate understanding of addition, subtraction, multiplication and division of decimals. And explain their thinking. I would explain this a little more clearly next time because some of them didn't know what we were doing with this part. And on the back it is solving problems involving percent's so that was the tax and it brought up a lot of good conversation around what is taxed in a grocery store, I learned some stuff. It was just so good seeing them in Family Foods and the staff there was really pumped that we were there and we got to see the kids like in the produce section they were taking stuff and weighing it and finding prices that way. Just trying to find the best prices and it was just really practical stuff that they were doing there.

And when we ended up having a um, the group that who's pitch we choice got to go through a ring up all the items and the others just waited outside at this point because it was getting a little crazy with that many kids up at the front. They said that when they first rang it up it came out to about sixty-five dollars and then they said ok we grabbed some of the wrong stuff and so they went back and switched some items and ended up spending forty-nine dollars and some cents. It worked out really well.

G.M. Ok, and while you were there, you mentioned that it was neat to see them in the produce and such, were you able to hear their conversations was there that thinking going on.

T3. Oh absolutely, I think we had four groups and four adults so we just floated around and listened and watched and took lots of pictures. So yeah it was really good. One thing that they noticed was that when they were looking at paper plates there was small ones and big ones, same brand, same amount in each but the smaller ones were more expensive, and they were shocked by that. And um in the produce comparing stuff that was priced per pound versus like a bag or oranges, making sure that they had enough of what they were buying for the whole class.

NC1. I was a little bit worried because she has a busy class and I was like o how is this going to be. And it was awesome, when we came back I was like oh you guys, like thank you they represented the school and they were great. It was a happy moment.

T3. they really enjoyed it too.

NC1. And then the little party that you had.

T5. Ok so we did two. The first one we went to the gym and used what [redacted] [NC3 and T1] had put together. So we picked a few questions from there for the kids to think about. They also came up with their own questions after we went down just to look around and to think about what kinds of things do you want to ask. Um, all of these expeditions were done in our visibly random groups, we switch groups everyday so I didn't organize them based on who might do well with group and who might not it is just by which card they drew when they came in and, um, coming up with questions went ok. Um, most of them were measurement based which is what we expected but we asked them to pick three of the questions that Monsior and Madame have given you and them pick three of the ones that you came up with – see, every kid came up with questions of their own so every group had a total of six questions that we did. That took one class. And then the next class we actually went to the gym and they took measurements and got data for each of their questions that they choose to answer. Six questions was fairly ambitious because measurement took like forever so most of them answered one or two if that really and then that was one class on its own. And as everyone has said so far everyone was fairly engaged it think both in context and content and I think that that was interesting. Certainly being in a new context lent a lot to it but they were also interested in what was going on and I noticed [redacted] [T2] you said that your kids were just measuring for the sake of measuring and we were like ‘what are you doing?’ and they were like we are measuring this corner (laughing from the group) and we were like ‘why?” so like their group was not doing what they were supposed to be doing so it was just a matter of redirecting or asking “so what data are you getting by what you are doing right now, or what question are you gathering information for” and they couldn’t answer so it was like well, maybe let’s try something else. We also saw some really innovative ways of exploring one question because they wanted to measure the circumference of the basketball hoop and they weren’t really sure how because it’s tall and they are not (laughing) one student had the really curious idea of getting the basketball and throwing it through and seeing how much room there would be and then they measured the diameter of the ball to see ok how much is off

NC1. That’s interesting!

T4. the other student that they were with, because in that group I would way that there are two that are so called strong and one that is so called not, and she had the brilliant idea to try one of the four square balls that they play outside with at lunch and that’s a lot bigger and then see how well that fits through and they found that it took up almost the whole space so they used that to measure instead so their estimation was actually fairly close to what it actually is, which is really cool (group agreeing) so after that they came back, they had photos, they picked a photo of their expedition and they wrote kind of a narrative of what they did and they showed their calculations. So that was the first one.

The second one that we did was much like what Sam had done, basically Lisa and I did what everyone else had done, (laughing from the group) but I don’t have a shed – that’s next.

(laughing). So, we went to Safeway and we are planning a class party. It came up with ok, let's plan a party, what questions do you have, what do we need to know. Um, so allergy, so there is every allergy in the book in this class. We uh, we listed, again visibly random groups, it wasn't prearranged or anything. Well, they came up with stuff like, how much money will we have? How much food will we need? What allergies are there and so on and so forth. There were some whole group things and some small group things and when we had a question for all of us I would call everyone back together and be like, ok so what's the budget, if everyone brings in five bucks how much is that, let's calculate that right now. We tried two dollars, three dollars and a number of them were like oh five dollars that's way too much. But it's funny because in the end that's what we found we would need. So that was step one, so the next day we thought ok, what are the categories of food we want. So some more questioning and because there was so many allergies we quickly discarded some of the food cause that may be too dangerous so why don't we just do this and this and this and they ranked it and prioritized it. So, we did some data analysis in some ways cause kids voted and we picked the most popular ones and uh. Then what we did was they had a sheet and we went to Safeway. And it was, there was very little front loading because we had already done a lot of decimal operations and those sorts of things. So, the purpose of our trip to Safeway on the most humid day of the year was just to write down information. So, you're responsible for accessories like plates, and all those things. Write down anything you want, anything you think that we might need, uh, write down the brand, write down the pack, write down the price just collect data just like we did when we went to the gym so I think of the one group who was responsible for the paper accessories there was one girl who really wanted those umbrellas for the drinks and I said, hey yeah put it on the list, you never know. Maybe we have enough money for it, this is you wish list. Some other groups that had the drinks found it hard to move past the big display in the front because they had each one for fifty cents.

NC2. yeah, each one was fifty cents, so I said, well, this is the sale price and will the sale still be on when we come back.

T5. and just because they saw the price they stopped there, I had to say, have you even gone to the drink isle? (laughing) And they hadn't yet and so when they did that was good to see them compare. And then with the fruit group (laughing) the group responsible for the fruit, they very quickly ran to the prepared which is always like $24.99. and I was like ok, could you just buy like a pineapple and then save. And it just so happened that the pineapples were right behind us and they were like $3 something so why wouldn't we buy that. So, it was interesting because then they left the one little area we were in and started like infiltrating the produce section it was interesting to see them shift from only wanting to buy the pre-prepared to wanting to get the best deal. They saw that they could even buy oranges and when we came back to the class they then decided what was absolutely essential and they also had to tally the sum of um what their group came up with and then we uh, each had to come up with what is our total as a class and then we, each group shared their total and then the whole class had to come up with what was our total as a class. So, they individually did those calculations to uh, use decimal addition. Once we had our number, we said to the kids that were done let's divide a hundred and whatever by twenty-one and let's see how that goes. So not all kids were able to do that because some didn't finish the addition of the decimal but a number of them actually got $4.99 which was the answer. So, it was really curious to see that their initial estimation of $5 was way too much, was actually right on the money. (laughing) yeah, so we haven't finished this we haven't done our portfolio page or our party yet simply because I haven't been there it seems for the past week or so so that's the next step and eventually I do plan to go back and get the food, I don't know that we will go as a group because prices will have changed drastically since then, but to actually purchase the stuff and have a party at the end of the year.

G.M. that's so convenient in both of your cases to be able to return more than once. Because it sounds like the first one, I should ask, was it just the excitement of being there and then the second time, was more like, ok, we've been here before so it ..

T3. Yeah, yeah they had to find everything the first time but by the second time they knew where to go. It was quick the second time.

G.M. This sounds just fabulous, and what incredible opportunities you were able to create for kids, so thank you again for sharing those with me. There are a couple of things that popped up for me. There was just when you were mentioning the umbrellas and when you were mentioning the different shed I think what struck me was the personalization of it because there is so much talk about personalized learning well when we have 32 kids in our classes where are those opportunities for personalization? And did you see that in your experiences?

T4. Well, in my case it was them that came up with "can we shop somewhere else". It started with "do you have a Costco membership?" and then it went "what about, could we look at Walmart?" so the idea was to stick with Canadian Tire but then they came up with, just out of necessity, that there just was not one that's going to work for you [redacted] [T4] so they searched all over and came up with great ideas and so that was them. It was really neat.

G.M. and then within groups it could even be different which one that they choose.

T4. yeah, it could have been but in the end they all came to the same conclusion.

G.M. and Safeway

T5. it was more about how to express yourself. One student, probably the top math student in the class wanted to find Frozen cups, and it was interesting to see him look for those and it was that expression of who you are. Like the chip group was like 'let's just get them all' and then their subtotal was like twenty-five dollars and the rest of the class was like 'what, for chips!" and they were beside themselves.

NC2. well at one point they have twenty dollars for candy. And I think you mentioned that we could take something off the list, like, do we need twenty dollars' worth of candy? It was interesting too I was with the group in the bakery for a little bit and their job was to find some cupcakes. I don't know if you had said something that they had to be vanilla and they found these little ones but they were too small and they kept coming back to but we have someone who is gluten free so we have to get gluten free and I finally asked them how many kids in your class have an allergy to gluten, they said one, do you think that they all need to be gluten free. So, they hadn't really thought about that outlier so I said well think about one gluten free cupcake.

T4. that led to an interesting conversation back in class as well because that gluten free cupcakes that came in a pack of six I believe were very expensive and some kids were like 'nine dollars for cupcakes' that's crazy, so it was a good opportunity to say, well yes, but if we are including everyone in this party we can't just not provide a cupcake so that was cool.

T3. Did you guys call Safeway?

T5. Yeah, yeah, we did, and he was all god with it, he was like, yup, sounds good.

NC2. It was a good time to go, it was a Tuesday morning, and the store was not busy.

T5. yeah, staff was good. Everyone was all oh cool.

T4. I like how you did a fifty-dollar budget and you were focused on the per student price, so it was interesting the two different takes.

T3. I was going to say, yeah, the fifty dollars ended up being the perfect amount of food for the class. We went through almost all of it, but like kids had enough.

T4. so, you're going to have lots!

T5. yeah and we are not done yet, we still have more to go through, I think on their list right now they still have like ten liters of cola, so (laughing)

T3. so, are they going to figure out how much like each kid would drink?

T5. yeah, yeah, per kid. I don't anticipate that we are going to spend a hundred dollars on this party but yeah, we still need to narrow it down to the essentials.

T2. so, you've like started this big and now you are narrowing it down.

T5. yeah because they have no sense of price really and uh, we did have the chance to talk about how a small can of cola or pop would be different than and two-liter bottle and why is that such a drastic change. And [redacted] [T3] we also had a discussion about tax, what would be taxed and what wouldn't it was fairly confusing for them and fair enough, it was confusing for me as well. It was really cool to do it at the end of the year in my view because we got to integrate so many different things that we had already done. So, you know going forward if I were to do this again, which I think I will, I kind of see it as those non-routine, big problem solving thing every now and then, and maybe one per term because there were a lot of kids who were super engaged with the context of course, and then just doing math in a different way and in a more realistic way.

T1. It's funny that you say that because I was just making notes to myself and I was like how many could I actually do (laughing) I actually came up with two per term because I figured, correct me if I'm wrong it took me about two weeks to do the first one and so ok, two weeks and then the second one may take two weeks and after that the kids should become a lot more familiar with it and the time frame is going to go down substantially and you are then looking at a week per expedition. I was just kind of making notes and I was like what could we do build a shed, class party, gym explore and then I also thought maybe ice-cream combinations you know we have that ice cream problem that we always do, and wouldn't it be great to actually take the kids to the ice-cream store and be ok, this is how many flavors they have.

G.M. and aren't you close to Dairy Queen?

T1. I am and I am close to Sargent Sundae.

NC3. You have to do the buffalo barbeque next year!

T1. I know!

NC3. If we had been planned it would have been just awesome. Because on day one the trucks are just all thereby the next day you could tell that they had planned it all out and everything was facing the right way, but nothing was set up yet, it wasn't until the next day that you saw things actually unfolding and bring set up. So, like if you could get someone to take you through and explain to the kids, this is how we do it, that would be awesome.

T1. It's a company, you could bring them in beforehand and even have them come into your classroom before.

NC1. I know [redacted] [T4] you were thinking that with grade 6 next year, because she has a bit of a grade change for next year, that you would even take the kids outside to look for angles. The possibilities are endless for what you can do with this. But, definitely, all of us going in, completely not knowing what to expect right, that was our whole 'oh, what are we going to do" I remember before [redacted] [T3's] I was thinking Oh no, what's this going to be like and oh what's going to happen? (laughing)

NC3. I think both times we wrestled with how much structure do we want? How much can we just sort of show up and to figure out how much do we do ahead of time.

T2. after listening to your experiences and that was something that you had said was to establish those norms at the beginning of the year and we are doing what you did at the beginning of the year now and so this would be normal for the beginning of the year type stuff and I think that most of can see that if you did integrate this throughout the whole year plus you know more strategic planning with the mini-lessons mixed in and if this was just the way you did it, right mix it in with your number talks here and there and then that kind of stuff then you could get through the whole curriculum which we tend to not ever do. But I could see it happening if this was something that you were doing all the time and you just got good at then you wouldn't teach any huge, formal lessons. And it was all just those mini-lessons throughout the year mixed in. If you just dedicate your year to seeing math this way I think that it would be huge for the kids. It would be hard for them maybe for the first few months depending on where they came from and the teacher that they had but I could definitely see how much stuff you can tackle, right, like you saw all the real-world applications that came out of this that I could see that if this was your norm, it'd be huge, it would work really well.

T3. I found that the kids that do really well in math with the worksheets they liked it the least.

NC1. That's true

T1. because it's out of their comfort zones

NC3. it was working for them the other way

T2. yeah, exactly!

T4. I find that with manipulatives as well, kids that want the rule hate the manipulatives, can't deal with the conceptual understanding of it and ah those ones that struggle really buy in when you get the manipulatives

G.M. How do you fell about that, the if I'm good at this I should just do this?

T3. I think that it is a balance, incorporate both into my teaching next year so that everyone has a chance to do that they like and those who aren't comfortable with one or the other get some experience with it. Because when they get into high school or university they will be doing worksheet kind of stuff.

NC1. Well it built empathy too right. You know, I can help you here, but can you help me there. So, it really shows another thing about working collaboratively. I know in rich tasks and problem solving having that independent time and then talking it out and going through the motions – like for the portfolio pages it was just show all your thinking and how do you communicate your thinking well some of them just wrote procedures and we had to say well what does this mean?

T3. we tried to tell them ‘can you explain that’ the really bright ones couldn’t and we said ‘you can’t write a sentence’ and they’re like no.

NC1. So, we said if we show this to people who did not come on the expedition with us, will they now what we did. So [redacted] [T1] I am wondering, if you are planning and expedition for next year for all you classes, 6, 7 and 8, then I am wondering, kids who have gone through that when they get to high school, what’s going to happen? Will it be like, you’ve got work to do.

T3. I think at some point it’s got t filter up.

T1. I think it will if you look at, I’m going to go to the business world, we are hearing that all these companies are wanting is communication, is collaboration is people being able to think outside the box, it’s all these things that schools and universities are going to have to produce, um, so we are going to have to do it eventually, it’s more fun to be at the front of it instead of the end.

G.M. Something that live worried about is the transfer, sure we can go to these places and live these experiences, but will they carry it to the next one. So if we are at the grocery store and I say oh, this is just like when we were at Canadian Tire, will they link those experiences and will they take the decimals that I used there and the decimals that they used here and be able to say, oh I learned something about decimals and you can put it on a paper for me and I’ll know how to do it. How will we encourage that transfer? How will kids know that that decimal on the page and the money in the store or the weight on the scale is the same decimal? How do we help kids to know? So, I would be super interested to know if there is an outcome that was targeted through your expedition and then shows up on the exam, how kids did on it. Did one support the other? Or did they see it as two separates.

T3. Yeah, because we were doing subtraction from the budget, I showed them the strategy of taking a penny away to make it $49.99 and that was easier to do subtraction that way and they had uh, I think they do have a question like that on their exam.

G.M. Lots of your stories mentioned decisions that students made in the moment. Do you have an example of a decision that a student made on the spot or a decision that the student could not make on the spot?

T4. For me it was what items are going to fit in and what items are not going to fit in. they were like. [redacted] [T4] we just can't get four bikes in this shed, you have to move on. So, they would what do you absolutely have to fit in this shed. So, they were making decisions that were ok well, the rocking chair you can really keep that in your basement and it wouldn't be a big deal and yeah, they were.

G.M. And so was this common in your math class or was this something different?

T4. Oh no, this was totally different

G.M. and is that a good thing?

T4. Oh yeah, it showed me that they were really thinking about the problem in its true form.

T3. I've got one. In the fruit section students were deciding between grapes, oranges and banana. Grapes were sold per pound. So, once they weighed them, they found that they were really expensive. The oranges we expensive but you got a lot of them and then the bananas were the cheapest and then they found the bulk oranges and they went with the bulk oranges and the bananas. And then they when they were adding up their prices on their sheet they found that they were a little bit over budget, so I told them to write I am over budget, don't erase it. And then they had to decide what to take out.

G.M. Do you see that as a math skill or a thinking skill that students should have opportunities to explore.

T3. oh, yeah, it shouldn't just be write down the numbers and here's your answer. There should be thinking. It's nice that there is actual thinking involved this way.

T4. and that's a real-life skill right. They're living paycheck to paycheck they won't have someone to put the extra dime in, so they will have to learn to prioritize so that's a valuable life skill.

G.M. And then yourselves, it sounded like you gave some extension opportunities or added to the question in the moment or on the fly. Is that true? Did you hear that in your stories too? What did that feel like?

T3. it was scary, like not knowing, it being so open and letting the project go kind of where ever it went and not knowing exactly what we are going to be doing today. I'm not used to that.

T1. I think that that's how I went into the whole thing. I went in thinking I am going to try this and it could completely flop and if it does oh well. It's scary but it was really nice having someone else there, being able to bounce ideas off each other. I think you probably all had similar experiences where like, I don't know what to do here and we're both sitting here going I don't know what to do here but we'll talk it through.

(laughter)

T3. the coaches were key because they talked and shared between the schools and then they would say like ’oh this didn’t work for [redacted] [T2] so we definitely don’t want to try that. (laughing) That sharing and the coaches are what was, was key to this.

T4. for me I found that I was going into this still needed to have a lot of control and I found myself being able to give up more and more the more I saw the project developing and I think I am a lot more comfortable now having gone through it, I am a lot more comfortable now tying something that I have less control over. Like I am thinking of things that I can do next year and ways that I can incorporate it. But just the confidence of sort of just um, as it developed, ok sure, why can’t we shop somewhere else, alright. And why can’t we do that, why can’t we lay it out on the floor and why can’t I say ok yeah, this doesn’t have to go in. So, yeah there were a lot of decisions that were made on the fly.

NC2. And it sounds like they were the kid’s ideas.

T4. yeah,

NC2. It wasn’t you dictating yeah do this next, do this thing.

T4. yeah, right and I think had I said that for example I had said nope, everything has to fit in the shed, I think that I would have lost half of them. Because they weren’t easily finding what they needed. Like, they knew what they needed, and it wasn’t working out, so I needed

T2. I would go the opposite, I would want to structure it more to start then I could gradually release freedom instead of going the opposite way and starting with a lot of freedom and trying to figure out a way to narrow it for them so that they could understand what was happening. I think some of them were like ‘what are we doing?’

T1. I think that that is almost ours to anticipate sometimes like I see something great and I am like ok let's just do it and then we go and it's just completely flat and then there are other times where I am just going to try to figure out where the kids are going to go with this and those lessons and those activities are so much more rich.

G.M. You are kind of making me wonder about, what does teacher planning look like for this and that anticipating where they are going to go with it?

T2. I did a lot of planning ahead of time, I had shared some sheets with [redacted] [NC2] and ended up not even using half of it. Um, I spent a lot of time thinking about like what kinds of questions are they going to come up with and what's this going to look like and then when we actually went to do it, it was totally not what I thought it was going to be like. And then there was a lot of making decisions on the fly and we had to plan right after like how are we going to get them where we want them to go. Um so it was a lot of planning on the spot. My preplanning was out the window pretty much right away so it's about being ready to plan as you go. Pretty much at the end of every class and every day you are planning for your next you cannot preplan this you cannot plan that whole thing you have that original tiny place to start and then you've got to plan every day and you have to be ok with that and it be normal.

T4. I told my kids that was part of it I told them at the end of the class that they needed to tell me what they needed from me for the next class.

T2. Oh, that's a good way to get them participating and planning with you.

T4. yeah, yeah, so sometimes I had to go take more pictures and sometimes it was you have to go measure how high is your roof and then you know ok we need to know how to find volume and stuff like that and so I just let and of course the more comfortable you are with the subject matter, I (cough) in my back pocket maybe a first year teacher that's going to be more difficult so yeah but letting the kids dictate what they need and if I knew that there was something that they needed and they just weren't asking I might slip it in in some ways but most of the time they figured it out.

T2. Having the patience to let them figure out what they needed, like you know in the back of your mind that they need this, like they will need this measurement at some point, but to give them that freedom to not tell them. Like not every teacher is able to do that.

T4. Like that's huge, you wait for it.

T2. And if it doesn't come it doesn't come, they don't get that information.

T3. the nice thing about that from the teacher perspective is that you are not doing the same boring lesson.

T4. yeah, totally.

T3. it will be different every year. Even different classes doing it at the same time they could go in different directions.

NC1. Oh, that'd be cool. Different directions and then they could share in like a celebration. Here's the problem here's where this class took it and here's where this class took it, you know. That's cool.

G.M. This what you are describing here is just a beautiful definition of that difference between just in case learning, like I am going to teach you all these things so that just in case at some point in your life you are going to need them versus here in the moment, you need it, this is where it fits and this is where the kid is going to go yeah, because they needed it in that moment to know what it is. The way you have just described that is the perfect picture of what that term means and not doing the same thing from year to year that's such a great idea too because it is so easy to come up with a binder and be ready and feel confident but the way you are talking about handing over the control and the trust to the kids I think a huge piece of it, you're saying I trust you to decide, you just said um, letting the kids dictate what they needed and they'll figure it out that's a huge concept as a teacher because we want to be helpers and we want to be ready and have everything set up but for them to be able to say I need to learn how to blah blah is huge.

S.F. Well I actually had the conversation with them. They said to me well you didn't teach us how to do this and I said well you didn't tell me that you needed to be taught that. I go them on that one. Because it was all up to them, so it was like what do you need me to teach you to be able to do this.

T3. It's interesting how your kids came to that. My kids really struggled to know what they needed like they assumed..

T4. Mine were eights

T2. oh, ok, like even one of the things that I gave them to help them with their planning was what questions do you have like we are not going to the cafeteria until you are prepared with what you need from the cafeteria. Like when I asked what tools do you need this one group put a camera, they didn't put anything else, nothing. It was like a whole forty-five minutes figuring out ok what do we need in the cafeteria, Still groups went there not knowing what they needed to figure out the question.

T1. That's similar to the sixes. Like we said that we were going into the gym and did the wondering thing and I was like ok, what tools do you need and a lot of kids were actually saying a ruler and I was like you're going to measure the gym with a ruler. And I pulled it out and I showed it to them and I was like this is what you want to use. No (laughter)

T1. I think that speaks to how well we have engrained the traditional way into them. They don't know what equipment they need to bring into the gym.

All yeah, yeah (laughter)

NC3. They are used to us handing it over. We are going to go do this and here's what you need.

T2. like we had meter sticks and we are measuring and we were asking what are you measuring, like what's that for? Just because you brought a meter stick.

T2. it was a big struggle for my kids to think about what it is that they actually needed to get to gather the data. They really just wanted to go there and then gather the data while they were there and figure out what they needed once we were there. I was trying to give them this idea of like, what if we were going somewhere like the MTS Center and we can't go back. You get one shot and you've got to be prepared. And that's what we were trying to get at and it didn't sink in right away.

T1. That's why I liked doing the expedition in our school to start. I think it was, for myself as well, to see ok, we are here what am I going to miss. I think that is really good at the start or the year or for a new teacher, do a couple in the school so, at least on the school grounds so that you become comfortable and the kids become comfortable with it. And then when you do go to an MTS Center or a Canadian Tire you do know. If you can't go back, then what do you do.

T2. Like you did that one to the movie theater and that's one that we thought would be super good for us to do but knowing that the kids only had one shot at it we thought the kids just weren't ready for that.

T1. Do you have any experience with kids going from, sorry, going from a range of grades so kids did expeditions in grade five and then grade six, just to see what those kids would be like at the end of this because we keep talking about getting those realistic life skills and because we see the benefits of these expeditions and if they do it one year with one teacher and then the next four years they are not doing it..

G.M. Right, where did it go.

T1. Exactly opposed to what if you had them do it in grade six, seven and eight, what is that kid going to look like what is there thought process like?

G.M. Something you had just mentioned here was what are you measuring for so I am just going to go back to like the traditional problem or exam, question that have that extra information in them that kids have to ignore like um, I have four apples, you have four apples, Shelly has ten apples well, How many apples do you and I have well Shelly's apples are not important to the problem. Is that of developing that skill of knowing what it is that you are looking for.

T2. you get that from doing these expeditions, but I don't think that I see that a lot in the other work and stuff that we do. The outlier stuff, like there might be the odd problem that, I know some of the ones that we have looked at that we got from you guys that are pretty wordy and stuff, but most of the stuff in there is essential to it. I don't think that there is anything as big as these types of things where there is a lot of outliers and things that the kids have to dissect.

G.M. and what about, how you had mentioned, that kids just wanted to use the tools, it was the novelty of using the tools. How do we get kids to know, I guess in their own minds, define that problem and know what the variables are that they are defining? In your case, are they picnic tables?

T2. similar and they fold up.

G.M. Yeah so you certainly are right, they could just sit side by side and count or measure it…

T2. I had a kid that actually grabbed a chair and, cause he's like, you can sit comfortably, and he used the size of the chair as his measurement tool and he literally took the chair and put it on the bench and sat on it, and Jones and I were kinda looking at each other like what? And this is one of my kids that is not very verbal, and he dives into every question. He sees things very different than us, or than most of use. And it was, that was his measurement tool but he got the exact same result as just four people sitting there knowing that we are not just sitting right bum to bum so it was the same result, but it was just interesting that his measurement tool was a chair know that that was the size. He never thought about a ruler or using a meter stick and not know what to do with that measurement. So that was one thing that kinda stood out for a tool.

T5. I keep thinking back to engagement with content and context and I keep thinking of the kid who was in the corner measuring nothing and I wonder if he was just in the corner doing math as he knows it. Right, he's using a tool, he's measuring something, it looks like he is accomplishing a task, we are just not upstairs we are in the gym. So them I am thinking ok if I look at what he produced from Safeway and think now was he really engaged or was he just doing what he thought math looked like, somewhere else with a bigger tool than say a rule because because it was clear, with reflection now, that his measurement was serving no purpose but it certainly looked like math probably to him.

T2. That sounds similar to what I had seen in my kids. They know that they are doing some kind of math, measuring, calculating but the big picture wasn't there.

T5. to an outsider, he was doing math.

T2. Oh yeah, if you walked into the cafeteria you would have been like oh what's going on here. A handful of groups were not engaged in what we were trying to get at.

NC3. Oh but that's half of teaching through problem solving. Having kids really wrap their heads around problem to begin with so I kind of look at this as a parallel to that because there is a whole lot more thought that has to go into what they are doing rather than us just giving them a question and telling them here use this to go and measure that. It's the more openness of it

T2. it's the struggling

NC3. Here I want five equations with y equaling twenty, but they all have to be different. Now there is a problem embedded in there and they have to make choices, so I think if we are doing more of that kind of thing and even how we teach the concept part that will help.

T1. I think the more we do math and approach it this way the less that student will see math as being divorced from a context, so you know I am hoping that I don't think that I teach them again next year but if I did I hope to see that maybe if we were to do something like that that that thinking has progressed.

NC3. well the research says that if you teach through problem solving then those kids do better than your average kids on exams at the end of the year. They make more that their year of progress. It's teaching kids to think rather than teaching them 'here's some content'. They have to think for themselves and makes sense of things if we are doing that most of the time in our classrooms then when they are presented with a dry problem, then it will look different to them but if they are used to thinking for themselves then they will try to work it out. Well if they are attacking something like ok, I know some things I can figure it I know I can do it rather than here's a question on a page and I don't know how to do percent it's just a different mindset.

T3. Perseverance.

T4. Absolutely

T1. We have to continually do this throughout the years. It can't just be ok we have this group working on expeditionary learning this year, ok check. (laughter) Same with number talks, same with rich tasks, problem-based curriculums. It has to be something that is constant and then like you said when kids are going to the high schools they go, no, this I how I learn, and then high schools are going to be going to middle schools going how are you teaching them. They don't know anything – no, I'm not saying that (laughing)

G.M. We kind of touched on this one too, the idea of argument. That kids will argue with each other. Was that evident in your expeditions? Was that positive, was that terrible…was it..

T4. Positive

T2. for the most part, once kids learned how to do that.

NC3. I think that that is something that develops over time because some groups have that one kid that takes over where there's like that one kid that explains it and then the others kind of check out. When we walk up and say what do you mean by that ..

T2. Everyone was like oh yeah so and so said that they were really quick to put it on that one person. Like I have learned that over the year it gets a little bit better and kids will own what they say but before it's like as soon as I question anything and the whole goal is for me to like question the group but they would be defensive right away and not just expeditionary but like just in any of the stuff we are doing.

T4. And I found the opposite, I heard them arguing with each other all the time and like positively

When we want them to like it was again it was my week kids that were saying like what about this and did you think about it this way and this. And then they would say oh yeah. And they would see other groups doing it and they would say oh yeah well that's not right so why are they doing it that was and you know. So the discussion was just so rich and if I look at my class from five years ago that was never taking place. It's like you know

T2. I wonder if my sevens go to grade eight they will do that more, because now, towards the end of the year, you see [redacted] [students] arguing how many times now and Brayden was the kid that, when somethings hard he would just say 'I don't know this, I don't know this' he's like the fixed mindset kid all year and now when he's with [redacted] [student] they are just constantly...

T4. I think that it is the way that you set it up to like I had, I just tease them now, I say 'oh you're having a math argument, you're arguing about math'

G.M. I know some of you have to go right away so I will be quick. In your conversations you mentioned the support of the math coaches as being something and risk taking. So how much of an adventure was this for you? If you think of other risks you have taken in your career, on a scale of one to ten, was it a two or was it a ten in terms

T2. If I had had yearend expectations of testing or something, I would not have been comfortable doing this but because there was nothing that I had to like prepare for it was like I had the time. It was a little bit stressful not hitting all the outcomes because I know that this may not hit any of the outcomes which it didn't so it was two week so not hitting any outcomes but if I had that expectation that I would have had to prepare them for, yearend test or something, but I didn't have those expectations. But because I can go right up to report card writing teaching my outcomes then it wasn't a big deal.

T3. When [redacted] [NC1] first mentioned it to me I was like, yeah, I'm in and didn't like know what it even was (laughter) and I was like oh what did I get myself in to.

NC1. I said that you would be great (laughter)

T3. So it was probably, it is only my fifth-year teaching, so it is probably one of the more risky for me. Yeah and I want to do it again next year. I learned a lot.

G.M. What as a classroom teacher do you need to be able to be comfortable taking this kind of a risk in your teaching?

T2. Having the freedom to choose what you are doing in your classroom. Not having these expectations from admin that you have to be having certain things. Being able to teach however you want, have that support, for sure.

G.M. Sorry, and support would look like?

T2. From admin, from coaches, like this (pointing around the room to show the group support) People know that this is good.

T3. Financial too, I was able to have a budget for ours.

T1. I was going to say the admin support because this is a risky thing in that we've going to go off here and I don't know what the kids are going to learn. You need an administrator that is going to say, ok do it.

NC1. Yeah, take that chance.

T1. Ok see what happens. If you have an administrator going ok well, I want you to lay everything out form the start. And I have to say well I can't because I don't know where the kids are going. You have ideas but, that goes to the anticipating thing as well. Like I had my admin come into the gym when I had my kids all over the place and he said, looks great. He didn't say, what are you doing? So that was very comforting know that I had his support. And then having another teacher along with me being able to bounce ideas off each other it would be interesting to have someone on the same building doing the same one. It was nice having Heather there when I was doing it. She was in my classroom for those two weeks, but she was gone after that. It would be nice having someone in the same building doing it I might do these things more often.

NC1. Well even right now just having the opportunity to have the conversation and talk about successes and talk about where you hit roads blocks. I mean we had a few road blocks with uh our guest teacher coming in.

T2. I don't think that you can do this alone. You need somebody whether it is a coach or someone in your building.

T3. For sure at the beginning.

T2. Yeah, I'm sure after a while you are good to go but to start for sure.

T4. For me it was being able to pull the ideas, the real-life ideas out of math. I've been to a workshop that you have done before about inquiry in math and you were talking about music and I was like I don't know how to get the math out of that. Do you know what I mean? It's being able to find those real-life applications and lead them down that path.

G.M. What a neat point because in a team like this you might have someone who is a musician, and someone who is a home reno expert and someone who is a baker so we would all be able to say oh well I use cups and quarter cups and so on but, you're right, on your own you just have your own lens to look out. And the goal would be for students to have many lenses to look through.

T4. Absolutely, just to go back to the word control, so that if you do run into a road block you can say well let's think about it this way and then go from there.

T2. you need to have some familiarity with the curriculum (laughter). I haven't taught math that many years, but I know that I had to ask like I don't remember what the circle outcome is because I hadn't taught it for a couple years. You gotta know what your outcomes are for sure. A brand-new teacher would be tough.

NC3. But I think that the difference that I hear is that you need to know the difference between teaching the curriculum and teach kids. I still see lots of teachers that are still teaching the curriculum and so it's so easy to just stay in your room and have your whole year planned and that's all you need to do. When you are teaching kids, you will take a risk.

G.M. Are there any other comments around the inclusion of all kids. Were all kids able to access did all kids find an opening?

NC2. Well even in John's room, and I don't know this kid very well when we let them go in the gym and we said what math do you see and what do you wonder there was very little guidance there and most of them came up with the measurement questions that we saw. But one little boy, I guess, his question was if I can shoot three baskets on Monday and two baskets on Tuesday. So super traditional word problem but that's where he was at.

T3. I had two kids that were on IEPs that were in the group and they both got involved and even if they weren't doing the same math as the other kids they were able to help decide what foo they were picking out and the other kids were really good about making sure that they were included.

G.M. And so was it social emotional kind of inclusion a part of it as well.

T4. oh yeah, that was good.

G.M. Like you had said being a part of a group and being able to share a common experience.

T3. Got something to contribute to the group.

Appendix P

Emerging Themes from Focus Group Discussion

Teachers as risk takers

T1. I just took all three classes of mine and so it was a lot easier just going into the school gym I didn't have to worry about leaving the school building, or for how long or all that stuff, uh, it was also nice to keep the kids contained, so, if it wasn't working we had that option of, ok, let's go back to class.

T2. well we went to the cafeteria um we were struggling for spaces because, well the same idea as [redacted] I wasn't ready to jump into going somewhere far and doing the paperwork for leaving.

T3. it was scary, like not knowing, it being so open and letting the project go kind of where ever it went and not knowing exactly what we are going to be doing today. I'm not used to that.

T1. I think that that's how I went into the whole thing. I went in thinking I am going to try this and it could completely flop and if it does oh well. It's scary but it was really nice having someone else there, being able to bounce ideas off each other. I think you probably all had similar experiences where like, I don't know what to do here and we're both sitting here going I don't know what to do here but we'll talk it through.

T4. for me I found that I was going into this still needed to have a lot of control and I found myself being able to give up more and more the more I saw the project developing and I think I am a lot more comfortable now having gone through it, I am a lot more comfortable now trying something that I have less control over. Like I am thinking of things that I can do next year and ways that I can incorporate it. But just the confidence of sort of just um, as it developed, ok sure, why can't we shop somewhere else, alright. And why can't we do that, why can't we lay it out on the floor and why can't I say ok yeah, this doesn't have to go in. So, yeah there were a lot of decisions that were made on the fly. NC2. And it sounds like they were the kid's ideas. T4. yeah, NC2. It wasn't you dictating yeah do this next, do this thing. T4. yeah, right and I think had I said that for example I had said nope, everything has to fit in the shed, I think that I would have lost half of them.
Changing the role of the teacher/responsive teaching T1. [redacted] [NC3] and I were able to debrief really quickly and say ok next class is coming in so what are we going to change, ok, we're going to photocopy stuff as they were coming in T4. (Students telling the teacher what they need to find out) what they needed from me and some of them were like well we need to know how high your roof is, and I was like, well why do you need to know how high my roof is, and I was like ok, well I'll get that for you but I don't know why you would need that and so when we talked about that after they said well we thought that the peak may come into play with the peak of the shed and it may come into play

with the peak of the shed but once they had the measurement and saw another picture that I took they were like oh no it so it won't come into play. So, they were thinking about things.

G.M. It's almost like here's the problem and then they are thinking around the problem.

T2. it was a lot of figuring it out as we go

NC3. It is hard to kinda steer it where you wanted to go, but be a little more open

T2. and give them that freedom too

NC3. So, you have to be open with it

T2. What happens, happens

NC3. Exactly, you just have to run with it.

T3. And then we kinda kept morphing what we were going to do.

T3. we didn't know exactly what it was going to look like, so we kept switching it.

T4. for me I found that I was going into this still needed to have a lot of control and I found myself being able to give up more and more the more I saw the project developing and I think I am a lot more comfortable now having gone through it, I am a lot more comfortable now trying something that I have less control over. Like I am thinking of things that I can do next year and ways that I can incorporate it. But just the confidence of sort of just um, as it developed, ok sure, why can't we shop somewhere else, alright. And why can't we do that, why can't we lay it out on the floor and why can't I say ok yeah, this doesn't have to go in. So, yeah there were a lot of decisions that were made on the fly.

NC2. And it sounds like they were the kid's ideas.

T4. yeah,

NC2. It wasn't you dictating yeah do this next, do this thing.

T4. yeah, right and I think had I said that for example I had said nope, everything has to fit in the shed, I think that I would have lost half of them.

T1. I think that that is almost ours to anticipate sometimes like I see something great and I am like ok let's just do it and then we go and it's just completely flat and then there are other times where I am just going to try to figure out where the kids are going to go with this and those lessons and those activities are so much more rich

T2. I did a lot of planning ahead of time, I had shared some sheets with [redacted] [NC3] and ended up not even using half of it. Um, I spent a lot of time thinking about like what kinds of questions are they going to come up with and what's this going to look like and then when we actually went to do it, it was totally not what I thought it was going to be like. And then there was a lot of making decisions on the fly and we had to plan right after like how are we going to get them where we want them to go. Um so it was a lot of planning on the spot. My preplanning was out the window pretty much right away so it's about being ready to plan as you go. Pretty much at the end of every class and every day you are planning for your next you cannot preplan this you cannot plan that whole thing you have that original tiny place to start and then you've got to plan every day and you have to be ok with that and it be normal.

T4. I told my kids that was part of it I told them at the end of the class that they needed to tell me what they needed from me for the next class.

T2. Oh, that's a good way to get them participating and planning with you.

T4. yeah, yeah, so sometimes I had to go take more pictures and sometimes it was you have to go measure how high is your roof and then you know ok we need to know how to find volume and stuff like that and so I just let and of course the more comfortable you are with the subject matter, I (cough) in my back pocket maybe a first year teacher that's going to be more difficult so yeah but letting the kids dictate what they need and if I knew that there was something that they needed and they just weren't asking I might slip it in in some ways but most of the time they figured it out.

T4. For me it was being able to pull the ideas, the real-life ideas out of math. I've been to a workshop that you have done before about inquiry in math and you were talking about music and I was like I don't know how to get the math out of that. Do you know what I mean? It's being able to find those real-life applications and lead them down that path.

T4. Absolutely, just to go back to the word control, so that if you do run into a road block you can say well let's think about it this way and then go from there.

NC3. But I think that the difference that I hear is that you need to know the difference between teaching the curriculum and teach kids. I still see lots of teachers that are still teaching the curriculum and so it's so easy to just stay in your room and have your whole year planned and that's all you need to do. When you are teaching kids, you will take a risk.

What can be learned through expeditionary learning

NC3. trying to get them thinking beyond what they are seeing

T4. was how they showed their understanding of the problem and I think, I think, every group, except that one group that didn't finish, I think every group had a really solid understanding of what they needed to do, so that was really cool

T5. because they have no sense of price really and uh, we did have the chance to talk about how a small can of cola or pop would be different than and two-liter bottle and why is that such a drastic change.

T5. that led to an interesting conversation back in class as well because that gluten free cupcakes that came in a pack of six I believe were very expensive and some kids were like 'nine dollars for cupcakes' that's crazy, so it was a good opportunity to say, well yes, but if we are including everyone in this party we can't just not provide a cupcake so that was cool.

T4. I told my kids that was part of it I told them at the end of the class that they needed to tell me what they needed from me for the next class.

T2. Oh, that's a good way to get them participating and planning with you.

T4. yeah, yeah, so sometimes I had to go take more pictures and sometimes it was you have to go measure how high is your roof and then you know ok we need to know how to find volume and stuff like that and so I just let and of course the more comfortable you are with the subject matter, I (cough) in my back pocket maybe a first year teacher that's going to be more difficult so yeah but letting the kids dictate what they need and if I knew that there was something that they needed and they just weren't asking I might slip it in in some ways but most of the time they figured it out.

T1. Like we said that we were going into the gym and did the wondering thing and I was like ok, what tools do you need and a lot of kids were actually saying a ruler and I was like you're going to measure the gym with a ruler.

T4. And I found the opposite, I heard them arguing with each other all the time and like positively

When we want them to like it was again it was my week kids that were saying like what about this and did you think about it this way and this. And then they would say oh yeah. And they would see other groups doing it and they would say oh yeah well that's not right so why are they doing it that was and you know. So, the discussion was just so rich and if I look at my class from five years ago that was never taking place.

T4. Positive

T2. for the most part, once kids learned how to do that.

NC3. I think that that is something that develops over time because some groups have that one kid that takes over where there's like that one kid that explains it and then the others kind of check out. When we walk up and say what do you mean by that ..

T2. Everyone was like oh yeah so and so said that they were really quick to put it on that one person. Like I have learned that over the year it gets a little bit better and kids will own what they say but before it's like as soon as I question anything and the whole goal is for me to like question the group, but they would be defensive right away and not just expeditionary but like just in any of the stuff we are doing.

T4. And I found the opposite, I heard them arguing with each other all the time and like positively

When we want them to like it was again it was my week kids that were saying like what about this and did you think about it this way and this. And then they would say oh yeah. And they would see other groups doing it and they would say oh yeah well that's not right so why are they doing it that was, and you know. So, the discussion was just so rich and if I look at my class from five years ago that was never taking place. It's like you know

T2. I wonder if my sevens go to grade eight they will do that more, because now, towards the end of the year, you see [redacted] [students] arguing how many times now and Brayden was the kid that, when somethings hard he would just say 'I don't know this, I don't know this' he's like the fixed mindset kid all year and now when he's with [redacted] [student] they are just constantly...

T4. I think that it is the way that you set it up to like I had, I just tease them now, I say 'oh you're having a math argument, you're arguing about math'

NC3. Oh but that's half of teaching through problem solving. Having kids really wrap their heads around problem to begin with so I kind of look at this as a parallel to that because there is a whole lot more thought that has to go into what they are doing rather than us just giving them a question and telling them here use this to go and measure that. It's the more openness of it

T2. it's the struggling

Engagement (what do we mean – activity vs thinking engagement)

T4. you could see the student engagement.

T4. the engagement was through the roof

T4. no I was surprised that when they got back to the school and the days that we worked on it after, they were digging in, they were still really engaged and if you had seen, some of the students had laid out a floor plan of the shed on the floor and that continued for days. And there was a lot of arguments, like yeah, I called them geeks, I said you realize you are arguing about math right now, and they thought that that was funny but like yeah like I said some of my weakest kids who would normally give up at any site f a challenge really were into it and, and, and with ideas and interesting enough had a lot of higher end ideas.

T5. And as everyone has said so far everyone was fairly engaged it think both in context and content and I think that that was interesting. Certainly, being in a new context lent a lot to it but they were also interested in what was going on

T5. I keep thinking back to engagement with content and context and I keep thinking of the kid who was in the corner measuring nothing and I wonder if he was just in the corner doing math as he knows it. Right, he’s using a tool, he’s measuring something, it looks like he is accomplishing a task, we are just not upstairs we are in the gym. So them I am thinking ok if I look at what he produced from Safeway and think now was he really engaged or was he just doing what he thought math looked like, somewhere else with a bigger tool than say a rule because because it was clear, with reflection now, that his measurement was serving no purpose but it certainly looked like math probably to him.

T2. that sounds similar to what I had seen in my kids. They know that they are doing some kind of math, measuring, calculating but the big picture wasn't there.

Inclusion

T4. the inclusion was amazing as well, like I was able to include a lot of kids, well not a lot of kids but a couple of kids who are on IEPs so they felt like they were a part of it and um, you know they were able to participate in the class which is fantastic.

T1. that led to an interesting conversation back in class as well because that gluten free cupcakes that came in a pack of six I believe were very expensive and some kids were like 'nine dollars for cupcakes' that's crazy, so it was a good opportunity to say, well yes, but if we are including everyone in this party we can't just not provide a cupcake so that was cool.

NC1. Well it built empathy too right. You know, I can help you here, but can you help me there. So, it really shows another thing about working collaboratively.

T2. I had a kid that actually grabbed a chair and, cause he's like, you can sit comfortably, and he used the size of the chair as his measurement tool and he literally took the chair and put it on the bench and sat on it, and Jones and I were kinda looking at each other like what? And this is one of my kids that is not very verbal, and he dives into every question. He sees things very different than us, or than most of use. And it was, that was his measurement tool but he got the exact same result as just four people sitting there knowing that we are not just sitting right bum to bum so it was the same result, but it was just interesting that his measurement tool was a chair know that that was the size. He never thought about a ruler or using a meter stick and not know what to do with that measurement. So that was one thing that kinda stood out for a tool.

T4. And I found the opposite, I heard them arguing with each other all the time and like positively

When we want them to like it was again it was my week kids that were saying like what about this and did you think about it this way and this. And then they would say oh yeah. And they would see other groups doing it and they would say oh yeah well that's not right so why are they doing it that was, and you know. So, the discussion was just so rich and if I look at my class from five years ago that was never taking place.

NC2. Well even in John's room, and I don't know this kid very well when we let them go in the gym and we said what math do you see and what do you wonder there was very little guidance there and most of them came up with the measurement questions that we saw. But one little boy, I guess, his question was if I can shoot three baskets on Monday and two baskets on Tuesday. So super traditional word problem but that's where he was at.

S.M. I had two kids that were on IEPs that were in the group and they both got involved and even if they weren't doing the same math as the other kids they were able to help decide what

foo they were picking out and the other kids were really good about making sure that they were included. G.M. And so was it social emotional kind of inclusion a part of it as well. T4. oh yeah, that was good. G.M. Like you had said being a part of a group and being able to share a common experience. T3. Got something to contribute to the group.
Students knowing how to follow the teacher but not knowing how to lead themselves T2. Some of the groups wanted to measure things even though they didn't really know what they were measuring and even though they didn't realize that they could just count. It was a real simple question they could just see how many could sit at one table and then find out how many tables there were. T2. you give them tools and they are like oh let's use this tool even though we don't know why. T1. Like we said that we were going into the gym and did the wondering thing and I was like ok, what tools do you need and a lot of kids were actually saying a ruler and I was like you're going to measure the gym with a ruler. T5. I think that speaks to how well we have engrained the traditional way into them. They don't know what equipment they need

NC3. They are used to us handing it over. We are going to go do this and here's what you need.

T2. like we had meter sticks and we are measuring and we were asking what are you measuring, like what's that for?

Teacher Support Group

T1. The second one that we did was much like what [redacted] had done, basically [redacted] and I did what everyone else had done, (laughing from the group)

T5. The coaches were key because they talked and shared between the schools and then they would say like 'oh this didn't work for [redacted] [T2] so we definitely don't want to try that. That sharing and the coaches are what was, was key to this.

T1. Its scary but it was really nice having someone else there, being able to bounce ideas off each other. I think you probably all had similar experiences where like, I don't know what to do here and we're both sitting here going I don't know what to do here but we'll talk it through.

T1. I was going to say the admin support because this is a risky thing in that we've going to go off here and I don't know what the kids are going to learn. You need an administrator that is going to say, ok do it.

NC1. Yeah, take that chance.

T1. Ok see what happens. If you have an administrator going ok well I want you to lay everything out form the start. And I have to say well I can't because I don't know where the kids are going. You have ideas but, that goes to the anticipating thing as well. Like I had my admin come into the gym when I had my kids all over the place and he said, looks great. He didn't say, what are you doing? So that was very comforting know that I had his support. And then having another teacher along with me being able to bounce ideas off each other it would be interesting to have someone on the same building doing the same one. It was nice having Heather there when I was doing it. She was in my classroom for those two weeks but she was gone after that. It would be nice having someone in the same building doing it I might do these things more often. NC1 Well even right now just having the opportunity to have the conversation and talk about successes and talk about where you hit roads blocks. I mean we had a few road blocks with uh our guest teacher coming in. T2. I don't think that you can do this alone. You need somebody whether it is a coach or someone in your building.
Challenging the student perception of what math is T5. I keep thinking back to engagement with content and context and I keep thinking of the kid who was in the corner measuring nothing and I wonder if he was just in the corner doing math as he knows it. Right, he's using a tool, he's measuring something, it looks like he is accomplishing a task, we are just not upstairs we are in the gym. So them I am thinking ok if I look at what he produced from Safeway and think now was he really engaged or was he just

doing what he thought math looked like, somewhere else with a bigger tool than say a rule because it was clear, with reflection now, that his measurement was serving no purpose but it certainly looked like math probably to him.

T2. That sounds similar to what I had seen in my kids. They know that they are doing some kind of math, measuring, calculating but the big picture wasn't there.

T5. to an outsider, he was doing math.

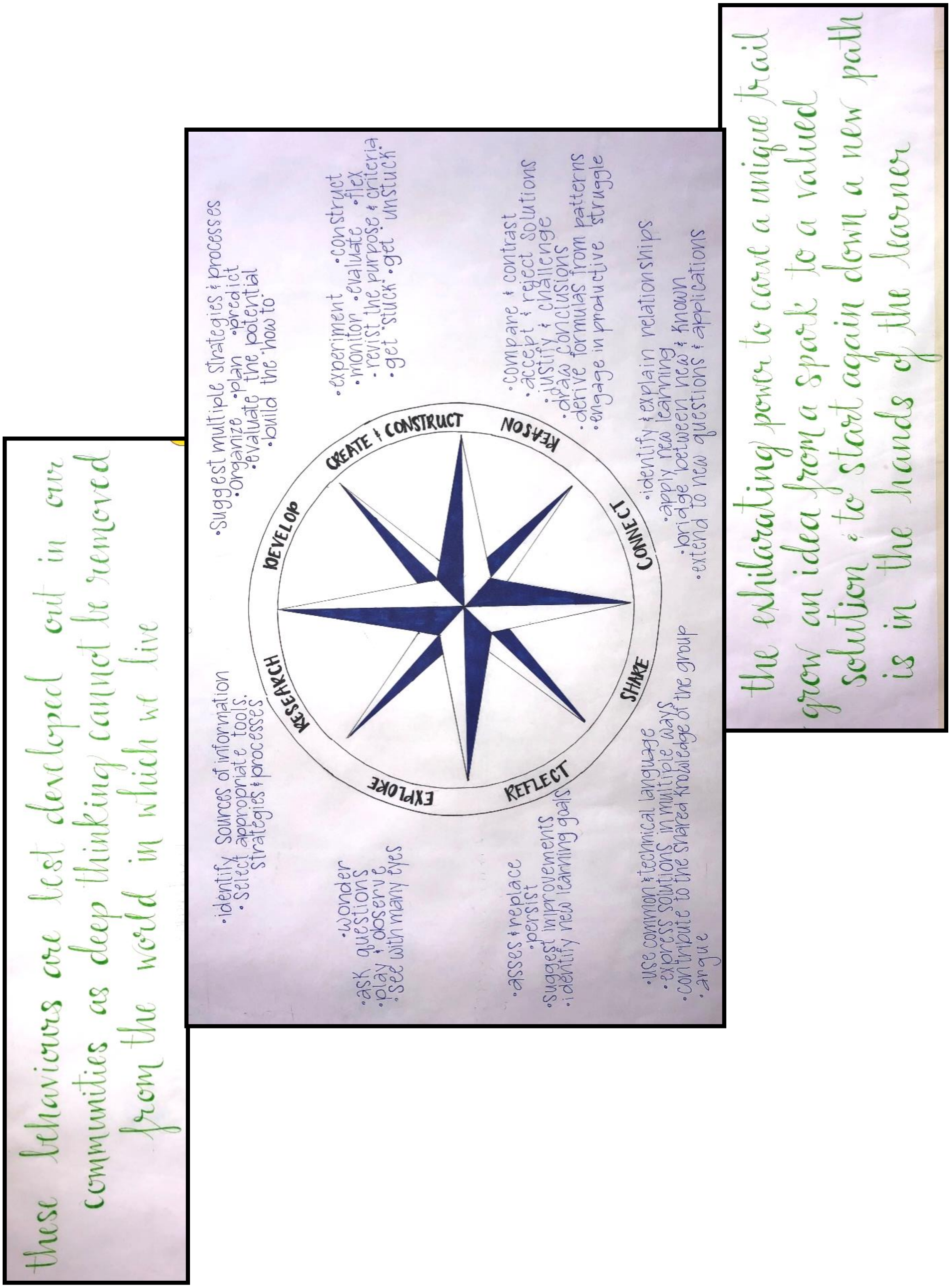
Appendix Q

Learning Story Analysis by Curricular Outcome and Strategy

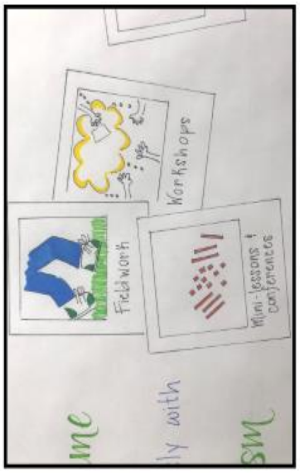
Expedition	Number of Learning Stories Available	Curricular Outcomes	Strategies
T1 Gym	17	5.N.2, 5.N.3, 5.N.5, 5.N.10, 5.N.11, 5.SS.2 6.N.5, 6.N.8 6.SS.3 6.SP.2, 6.PR.1, 6.PR.2	Explore, Develop, Create, Communicate, Reason, Connect
T2 Cafeteria	4	5.SS.2, 5.N.11 6.SP.2	Explore, Develop, Communicate, Create, Reason
T3 Grocery Store	13	7.N.2, 7.N.3	Explore, create, communicate
T4 Canadian Tire	8	8.SS.1, 8.SS.3, 8.SS.4,	Explore, Develop, Communicate, Create, Reason
T5 Gym	7	5.N.2, 5.SS.2 6.SS.3, 6.SP.2, 6.SP.3 7.N.2 7.SP.1	Explore, Develop, Communicate, Create, Reason
T5 Grocery Store	6	5.n.2 6.SP.2 7.N.2, 7.N.7	Explore, Develop, Communicate, Create, Reason
Total	55		

Appendix R

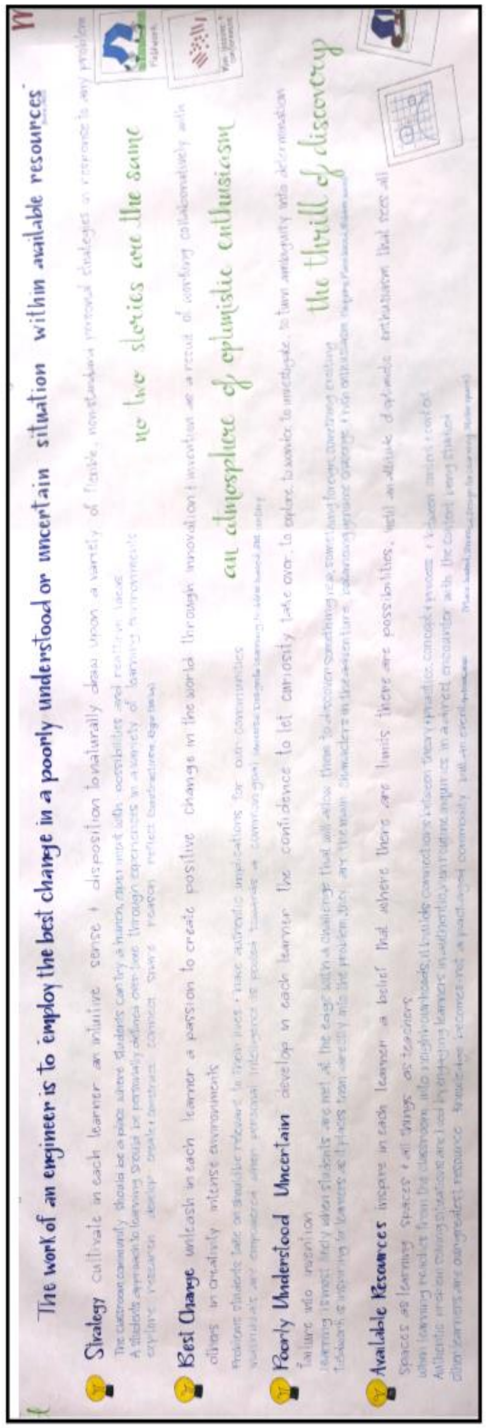
Dissertation Defense Handout


these behaviours are best developed out in our communities as deep thinking cannot be removed from the world in which we live
DEVELOP
CREATE & CONSTRUCT
REASON
CONNECT
SHARE
REFLECT
EXPLORE
RESEARCH
•suggest multiple strategies & processes •organize •plan •predict •evaluate the potential •build the "how to"
•experiment •construct •monitor •evaluate •flex •revisit the purpose & criteria •get "stuck" •get "unstuck"
•compare & contrast •accept & reject solutions •justify & challenge •draw conclusions •derive formulas from patterns •engage in productive struggle
•identify & explain relationships •apply new learning •bridge between new & known •extend to new questions & applications
•use common & technical language •express solutions in multiple ways •contribute to the shared knowledge of the group •argue
•asses & replace •persist •suggest improvements •identify new learning goals
•wonder •ask questions •play & observe •see with many eyes
•identify sources of information •select appropriate tools, strategies & processes
the exhilarating power to carve a unique trail grow an idea from a spark to a valued solution & to start again down a new path is in the hands of the learner

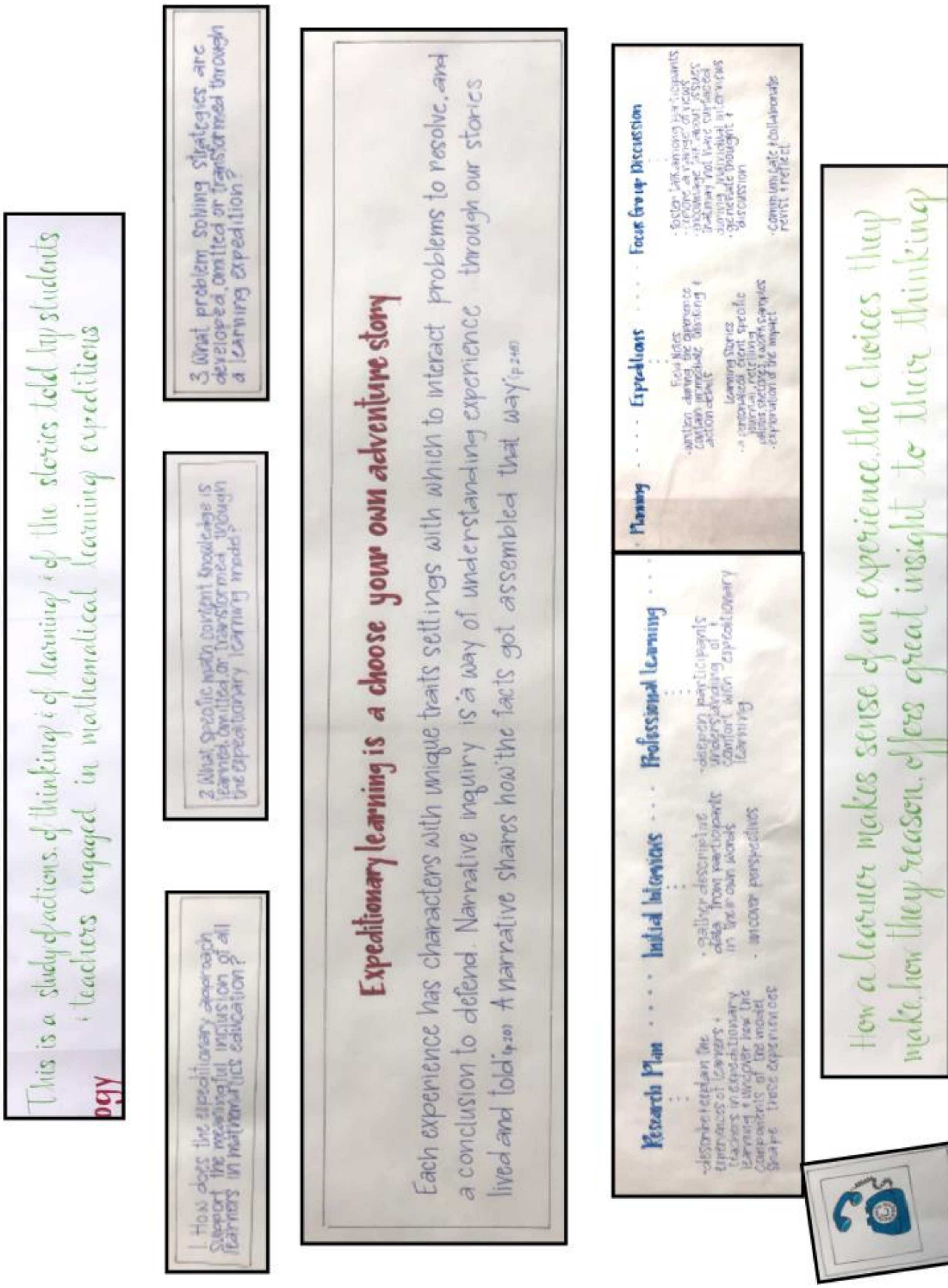

Engineers are models of learners for life


Fieldwork
Workshops
Mini-lessons & conferences

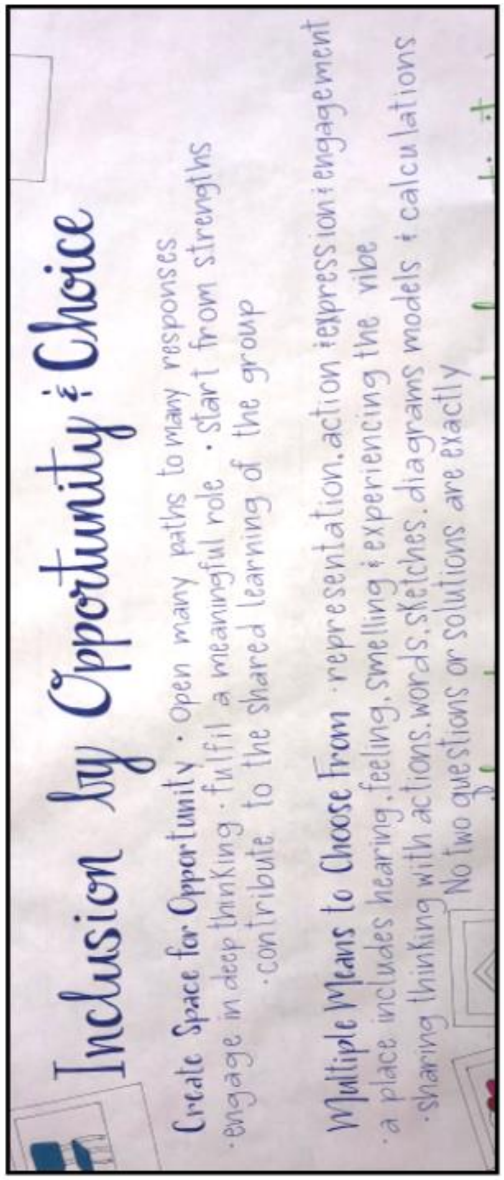

What might education learn from engineering?

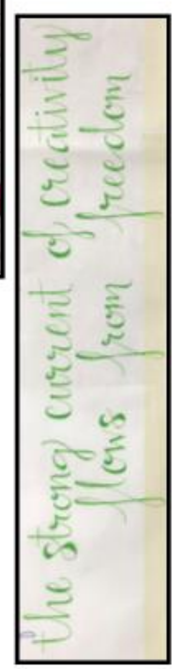

The work of an engineer is to employ the best change in a poorly understood or uncertain situation within available resources
Strategy
no two stories are the same
Best Change
an atmosphere of optimistic enthusiasm
Poorly Understood Uncertain
the thrill of discovery
Available Resources

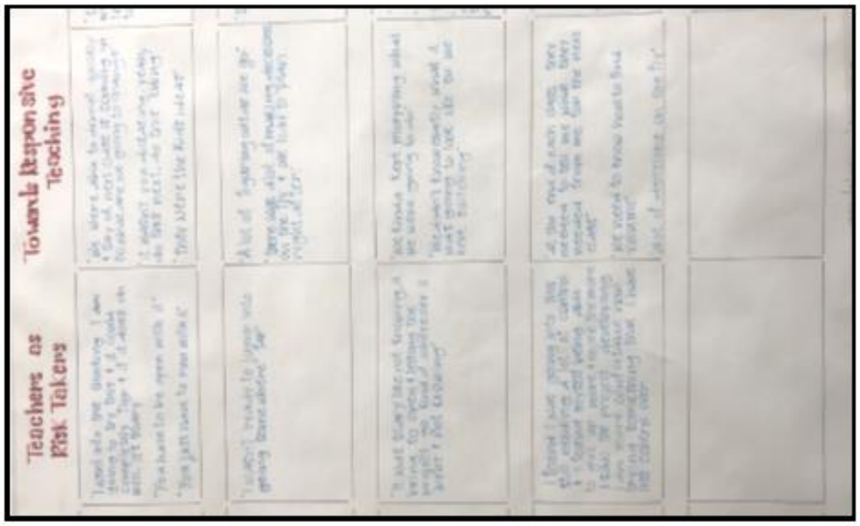

exists in the inbetweens

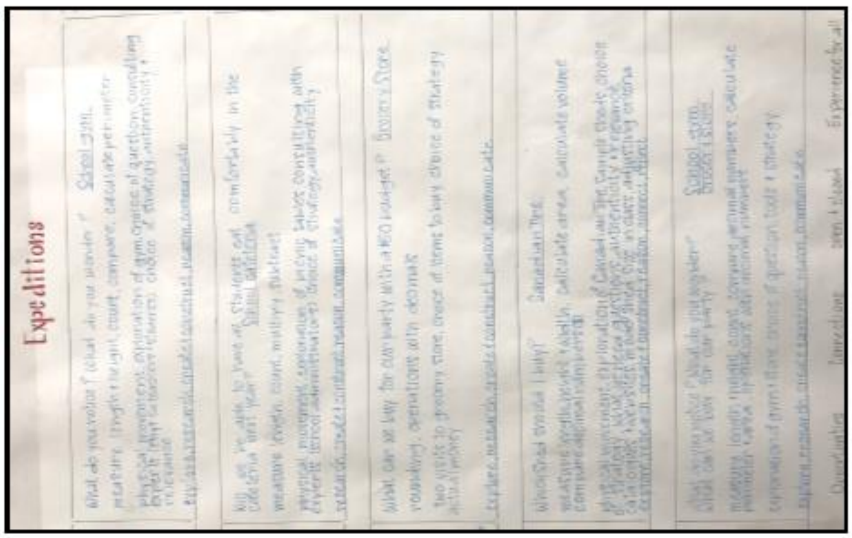

This is a study of actions of thinking (of learning) of the stories told by students & teachers engaged in mathematical learning expeditions
1. How does the expeditionary approach support the meaningful inclusion of all learners in mathematics education?
2. What specific math content knowledge is learned, omitted, or transformed through the expeditionary learning model?
3. What problem solving strategies are developed, omitted or transformed through a learning expedition?
Expeditionary learning is a choose your own adventure story
Each experience has characters with unique traits settings with which to interact problems to resolve, and a conclusion to defend. Narrative inquiry is a way of understanding experience through our stories lived and told (p.20) A narrative shares how the facts got assembled that way (p.240)
Research Plan
Initial Interviews
Professional Learning
Planning
Expeditions
Focus Group Discussion
How a learner makes sense of an experience, the choices they make, how they reason, offers great insight to their thinking

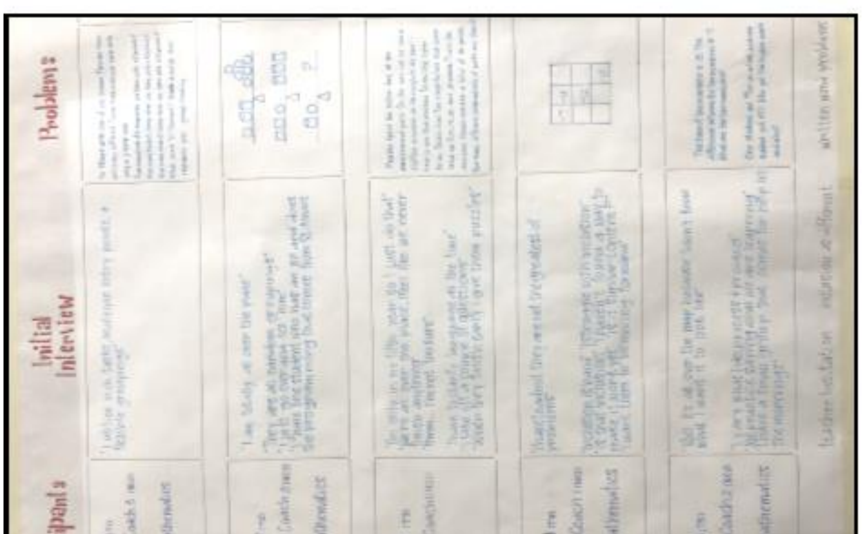

Synthesis · Analysis
Taking Risks to Redefine Mathematics
Take a Risk · math as an adventure that reaches far beyond the four walls
Explore new, open, undefined, messy math, without an answer key, creates opportunities
for students to lead the way, to make decisions, to fail, to succeed, & to develop
independence as learners, leaders, & citizens
Changing What we think class our Classroom · a Sense of place & respect for
those who take care of & use these places · relationships, attitudes &
skills to engage in community life
Access to shared experiences

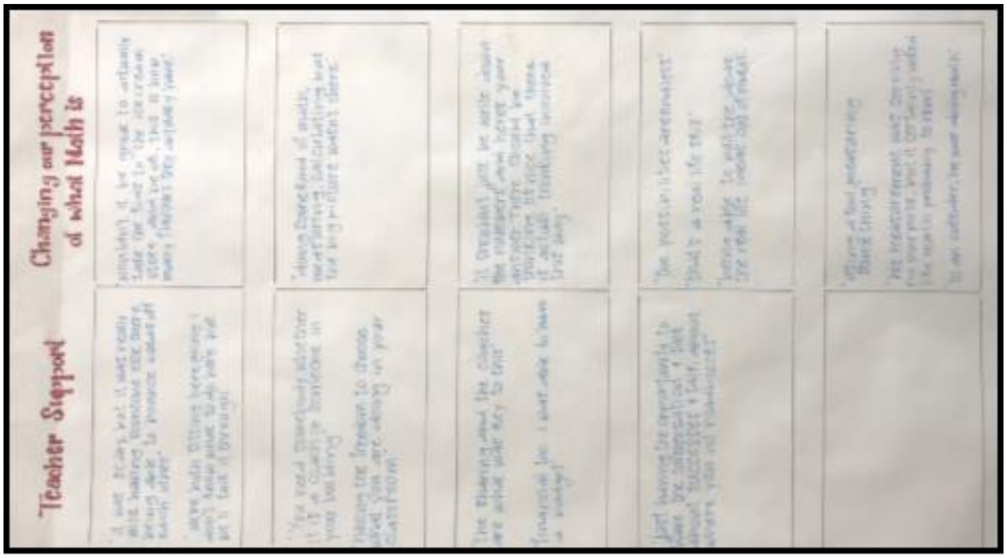

Inclusion by Opportunity & Choice
Create Space for Opportunity · Open many paths to many responses
·engage in deep thinking · fulfil a meaningful role · start from strengths
·contribute to the shared learning of the group
Multiple Means to Choose From · representation, action & expression & engagement
·a place includes hearing, feeling, smelling & experiencing the vibe
·sharing thinking with actions, words, sketches, diagrams, models & calculations
No two questions or solutions are exactly

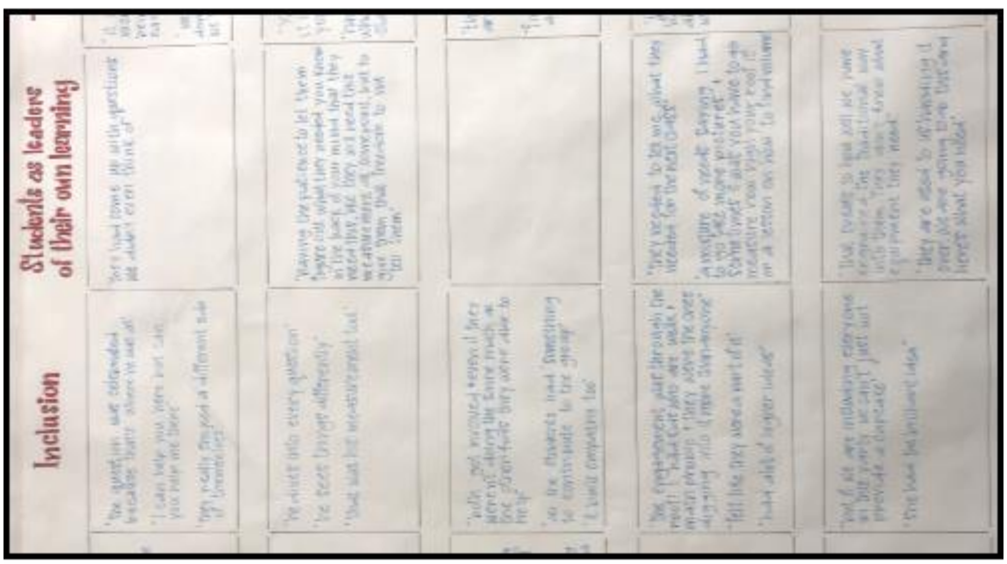

the strong current of creativity
flows from freedom


Teachers as Risk Takers
Towards Responsive Teaching

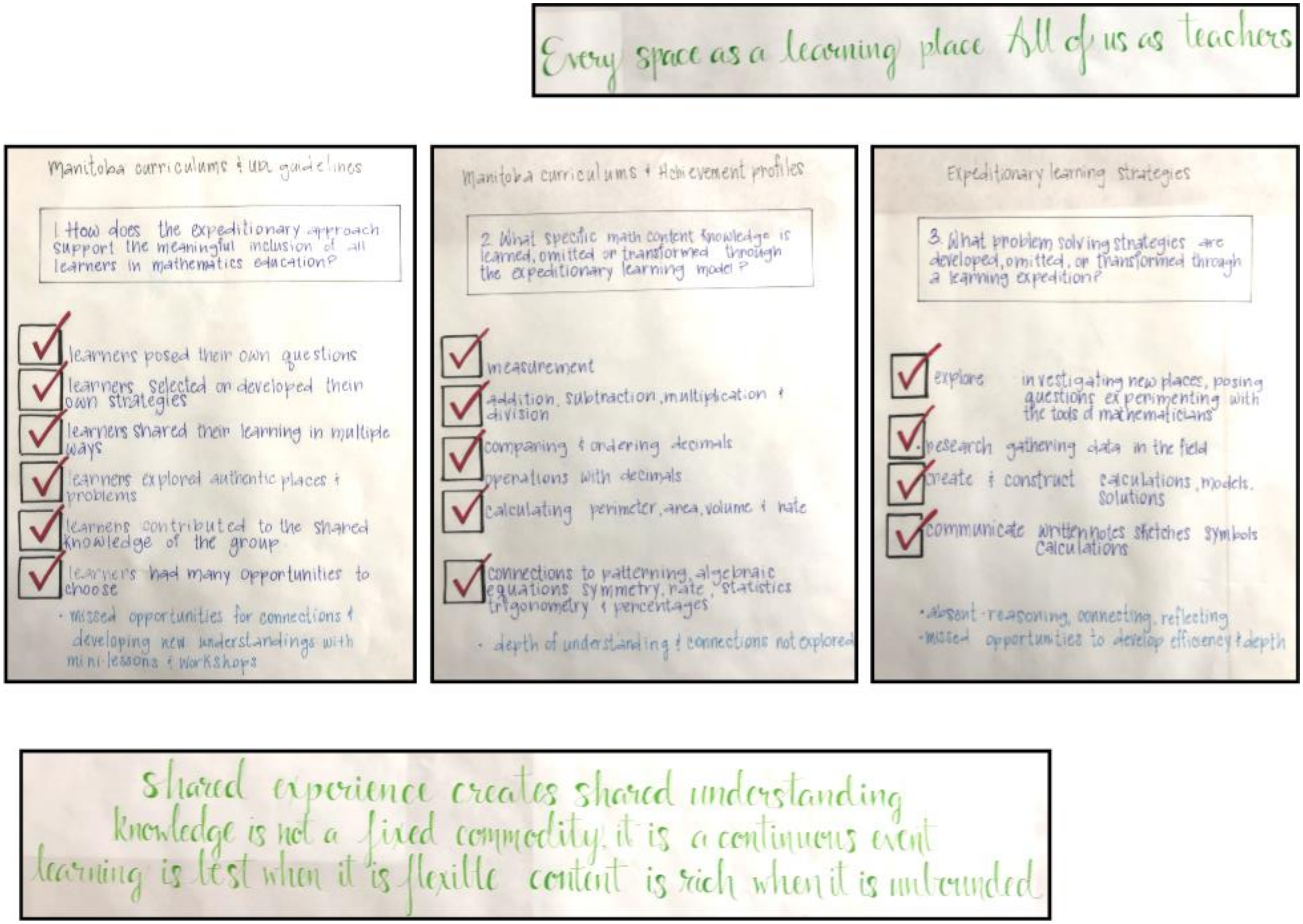

Expeditions

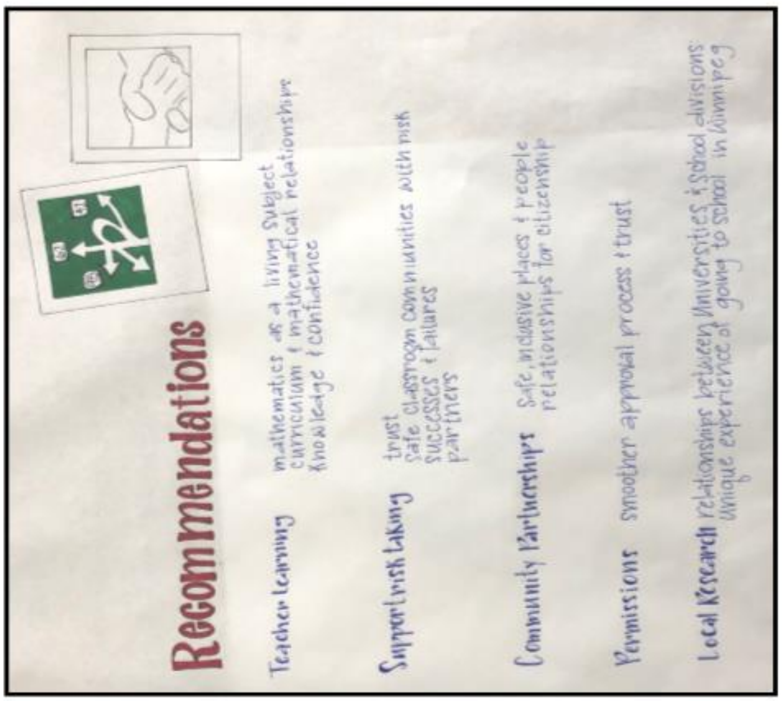

Participants
Initial Interview
Problems

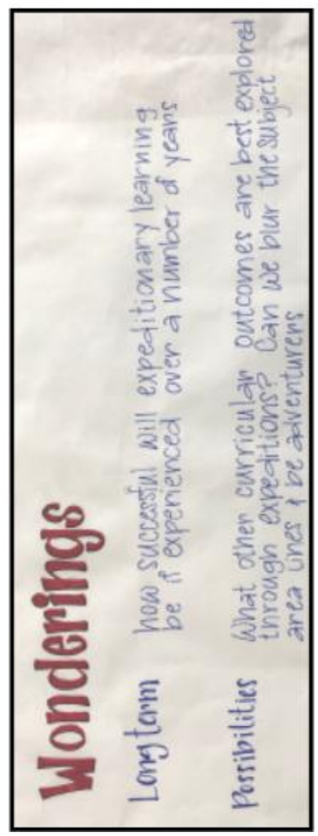

Teacher Support
Changing our perception of what Math is

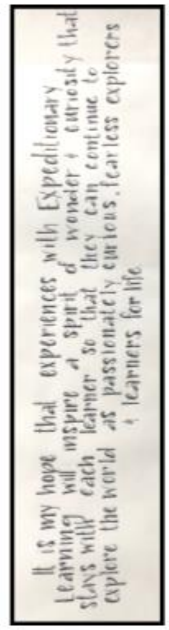

Inclusion
Students as leaders of their own learning


What more can be learned?


Every space as a learning place All of us as teachers
Manitoba curriculums & UDL guidelines
1. How does the expeditionary approach support the meaningful inclusion of all learners in mathematics education?
learners posed their own questions
learners selected on developed their own strategies
learners shared their learning in multiple ways
learners explored authentic places & problems
learners contributed to the shared knowledge of the group
learners had many opportunities to choose
· missed opportunities for connections & developing new understandings with mini-lessons & workshops
Manitoba curriculums & Achievement profiles
2. What specific math content knowledge is learned, omitted or transformed through the expeditionary learning model?
measurement
addition, subtraction, multiplication & division
comparing & ordering decimals
operations with decimals
calculating perimeter, area, volume & rate
connections to patterning, algebraic equations, symmetry, rate, statistics, trigonometry & percentages
· depth of understanding & connections not explored
Expeditionary learning strategies
3. What problem solving strategies are developed, omitted, or transformed through a learning expedition?
explore — investigating new places, posing questions, experimenting with the tools of mathematicians
research — gathering data in the field
create & construct — calculations, models, solutions
communicate — written notes, sketches, symbols, calculations
· absent - reasoning, connecting, reflecting
· missed opportunities to develop efficiency & depth
Shared experience creates shared understanding
Knowledge is not a fixed commodity, it is a continuous event
learning is best when it is flexible content is rich when it is unbounded


Recommendations
Teacher learning
mathematics as a living subject
curriculum & mathematical relationships
knowledge & confidence
Support risk taking
trust
safe classroom communities with risk
successes & failures
partners
Community Partnerships
safe, inclusive places & people
relationships for citizenship
Permissions
smoother approval process & trust
Local Research
relationships between Universities & School divisions
unique experience of going to school in Winnipeg


Wonderings
Long term
how successful will expeditionary learning be if experienced over a number of years
Possibilities
What other curricular outcomes are best explored through expeditions? Can we blur the subject area lines & be adventurers


It is my hope that experiences with Expeditionary Learning will inspire a spirit of wonder & curiosity that stays with each learner so that they can continue to explore the world as passionately curious, fearless explorers & learners for life