The Challenges of Divergent Thinking in Curriculum

By

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Abstract

This study deals with curricular and problem solving issues as they relate to convergent and divergent thought. The value of the latter is emphasized, particularly as an aid to problem solving. Differences between the two modes of thought are discussed and reasons suggested as to why convergent thought is promoted in schools to a much greater degree than divergent thinking.

An emphasis on convergent thought and a single correct answer mentality can lead to rote memorization and a transmission-type of teaching and ultimately, an erosion of self-confidence on the part of the learner. This type of thinking has become the norm due in part to the mind-set engendered by the traditional form of curricula that are frequently a reflection of the fragmented and disjointed nature of the subject matter presented. Numerous topics and skill-based outcomes in curriculum can obscure vital connections between the disciplines while, at the same time, encouraging memorization.

The study also found that, in order to implement divergent thinking, a major paradigm shift is needed, one that requires teachers to also view themselves as learners. This study suggests ways in which classrooms and teacher attitudes may reflect a more divergent approach.

The study suggests that divergent thinking is an important part of problem solving and constructivism.

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Dedications

To my late father Glen Dempsey who was the best divergent thinker I have ever known and my wife, Lindor Reynolds, and family whose love gave me the courage to complete this.

Chapter 1

Problem Statement and Context

General Statement of the Problem

To a large extent, the value of any discipline lies in how that discipline is utilized. The primary focus of school-based mathematics, for example, is problem solving. This serves to remove it from the realm of the esoteric, and place it firmly within a context of everyday usefulness. However, problem solving, regardless of the discipline in which it is used, remains an area of great difficulty for students. In fact, some research indicates that students show literally no growth in this area as they progress through school.

This inability to problem solve appears to run deep and involve a lack of understanding of what is being asked as well as difficulty in seeing a problem from multiple perspectives. With this in mind, it may be time to consider that the remedy lies in teaching students a different mode of thought. Divergent thinking (vide Chapter 2), much different than the convergent thinking traditionally taught by schools, may allow students to view problems in a broader and more successful context.

This study seeks to answer this question: Given the current nature of the curriculum, what are the challenges and obstacles to promoting divergent thought in schools? To address this question, this study will examine the historical basis of teaching and learning and the reasons for the prevalence of convergent thought as well as what the literature says about divergent thought. As a representation of curricular thought, the Manitoba Grade 7 mathematics curriculum will be examined in terms of the degree to which it promotes divergent thought.

Methodology

Since this study is a review and synthesis of the literature, an integrative inquiry approach has been used. The purpose of such an approach is that it yields, "...the type of knowledge that brings together what is known from various, perhaps disparate studies, that may be relevant to the particular needs of practice" (Marsh, 1991, p. 271). Hence, the goal of this study is to summarize the accumulated knowledge and perhaps even more importantly, according to Taveggia (1974), to highlight important issues left unresolved by past research.

As with any integrative review conceptualized as a research project, a major concern centers on determining which potentially relevant studies to examine. This is a point raised by Strike and Posner (1983) when they discuss the difficulties inherent in aligning various existing frameworks and assumptions. The writer has chosen to include those works which have proven applicable to curricular knowledge, teaching, and learning. Following the recommendations of Cooper (1982), it is hoped that narrow, superficial concepts have been avoided in order that this study may maintain a sense of robustness and ultimately, practicality.

Immediate Problem Context

It is the purpose of schools, among other things, to convey content to students. This is done through an array of methods and techniques far too broad for the scope of this study. However, at the core of it all are the modes of thinking promoted by schools and internalized by the learners. The purpose of this study is to discuss these modes of curricular thought and determine whether the emphasis on particular modalities should be re-examined in terms of the teaching of problem solving. Although problem solving forms an integral part of many disciplines, it tends to appear most often and in a more formalized manner in the area of mathematics. It is for this reason that mathematics will be used as a representative example throughout this paper. It will become clear to the reader that the issues presented are by no means unique to mathematics.

This chapter will attempt to outline the evidence that illustrates the difficulties with problem solving and the ramifications of these difficulties.

Three questions present themselves at this point, all of which are valid and vital to further discussion: What constitutes a problem; what is problem solving; and why is problem solving emphasized as a skill for learning?

According to the National Council of Teachers of Mathematics, "(P)roblem solving should be the major focus of all mathematical instruction" (1980, p. xiv). The document *An Agenda for Action* (NCTM, 1980) echoes that: "...problem solving should be the focus of school mathematics" (p. 2). To Polya (1945), problem solving was a major theme of doing mathematics and "teaching students how to think" was of paramount importance (p. 4). Hersh (1997) concurs with this when he says, "(I)t is the questions that drive mathematics. Solving problems and making up new ones is the essence of mathematical life" (p. 18). The views held by some researchers are even more global. According to Bailin (1988), problem solving "...pervades all aspects of our lives" (p. 24). Rowe (1985) posits that the ability to solve problems is vital to human survival.

From an educational point of view, problem solving generally has a much narrower focus and commonly refers to specific types of tasks. Throughout the literature, there exists a consensus that problem solving is an integral part of mathematics learning. In fact, it is considered one of the primary goals. In addition, most curriculum guides stress that students should be able to solve problems that arise both in mathematics as well as other arenas. The National Council of Teachers of Mathematics states that, "(I)n everyday life and in the workplace, being able to solve problems can lead to great advantages."

Despite this widespread agreement regarding the value of problem solving, expectations for students are not being met. It has been suggested that current efforts to teach problem solving skills are falling short of the mark and that students leaving high school are no better in that capacity than when they began (Yager, 2000). Lester (1994) states the issue even more emphatically when he says, "(T)he situation in schools with respect to student performance in mathematical problem solving is desperate!" (p. 660)

This feeling is a common theme among researchers. The National Assessment of Educational Progress in the United States notes "...on extended constructed response tasks which required students to solve problems requiring a greater depth of understanding then explanation at some length, of specific features of their solutions, the average percentage of students producing satisfactory or better responses was 16% at grade 4, 8% at grade 8, and 9% at grade 12." Referencing this, Verschaffel and DeCorte (1997) point out that in recent studies in which 10 to 13 year olds were presented with word problems, "(T)he analysis of pupils' reactions to the problematic items yielded an alarmingly small number of realistic responses or comments. Only 17% of all reactions…to a set of problematic items could be considered realistic" (p. 578). The National Commission on Excellence in Education (1983) concurred with this in their findings in which only one-third of high school students were able to solve problems requiring several steps.

This is a reflection of the thinking processes utilized by students. It has been shown that a significant fraction of high school graduates are at best, ill prepared for the kind of thinking that their college experience will require of them (Carpenter, 1980; Karplus, 1974; Renner and Lawson, 1973; Tomlinson-Keasey, 1972). Equally distressing are the conclusions that students frequently get through mathematics and science courses with at best a superficial grasp of concepts and the inability to apply those concepts effectively to real-world problems (Carpenter, Corbitt, Kepner, Linquist, & Reys, 1980; Clement, 1982; McClosky, Carmazza, & Green, 1980; Trowbridge & McDermott, 1980). As Hyde and Bizar (1989) point out, "(M)athematics (and life) does not fall neatly into a linear progression as follows: First learn the math facts; then learn the concepts; then apply this knowledge in the real world through problems" (p. 95).

However, it is precisely this linear progression which is inculcated into students and teachers. Children are adept at performing the required algorithms but in many cases are unable to apply any thought to the calculations they are performing. Max Wertheimer illustrated this in his classic monograph, *Productive Thinking* (1945). He asked students to perform arithmetic operations such as this:

$$\frac{274 + 274 + 274 + 274 + 274}{5}$$

While some children recognized that the repeated addition in the numerator (equivalent to multiplying by 5) was nullified by division by 5 in the denominator, a great many lacked this insight. However, they were capable of performing the algorithm, tedious as it was.

This is not an isolated example. Wertheimer discusses a similar case in which a teacher exhibits a proof regarding the area of parallelograms. Although the lesson has been thorough, Wertheimer asks the students a question regarding a parallelogram viewed from a different angle. They are completely baffled and Wertheimer points out that, rather than learning properties of parallelograms, the students had memorized the steps of the solution. Although they had mastered the relevant procedures, they had, in a very critical way, failed to understand the underlying ideas. To him, such devotion to rote procedures was "ugly" and "foolish" (p. 57). As Schoenfeld (1989) puts it, "(I)f students can only employ a procedure blindly or can only use a technique in circumstances precisely like those in which they have been taught, then schooling has in large part failed them" (p. 85).

Simply put, "...a substantial number of students are not effective thinkers" (French & Rhoder, 1992, p. 5). According to one researcher,

> It is quite certain that only a negligible portion of children...could possibly justify the methods used by them. In my experience, even the very bright children in school simply do not have any idea on this point, and if they are asked to explain, they simply say that they were taught to do it that way. What a travesty of mathematics education! (Dienes, 1969, p. 71).

Declining test scores on certain widely used instruments, notably the National Assessment of Educational Progress, seem to support this position (Benderson, 1984; Halpern, 1989; Jones, 1986; McTighe, Cutlip, & Schollenberger, 1985).

There is no lack of consensus regarding the importance of problem solving. In spite of agreement, success in this area has eluded educators. The traditional methods of teaching and framing problem solving appear to be inadequate. Concerns in this area prompted an interest in the development of problem solving skills and critical thinking skills (National Council of Teachers of Mathematics, 1980, 1987; Romberg, 1984). Generally, there is an emphasis on teaching students to identify vital aspects of problem situations (Polya, 1945; Sowder, Threadgill-Sowder, Moyer & Moyer, 1986). This is a positive step in that it elevates learning above rote but unfortunately reinforces the view that mathematics is an objective body of knowledge that can simply be placed in the minds of the students.

It should be established what defines a problem. Davis (1973) identifies a problem as "...a stimulus situation for which an organism does not have a ready response" (p. 13). This is echoed by Krulik and Rudnick (1987) when they assert, "(A) problem is a situation...that confronts an individual...in which the individual sees no apparent or obvious means or path to obtaining a solution." Mills and Dean (1960) state that "...a genuine problem...exists when something, no matter how slight or commonplace in character, puzzles or perplexes him; when something appears to him as unexpected, strange, or disconcerting" (p. 3) and that "(A) problem exists when straight-line action is no longer possible" (p. 10). To Guilford (1977), a problem is simply the "...need for new intellectual activity" (p. 159). Zeitz (1999) is somewhat lyrical in his description: "(A) good problem is mysterious and interesting. It is mysterious, because at first, you don't know how to solve it. If it is not interesting, you won't think about it much" (p. 4).

The NCTM website lists three criteria for defining a mathematics problem: 1) The person confronting it wants or needs to find a solution, 2) The person has no readily available procedure for finding a solution, and 3) The person must make an attempt to find a solution. Although these criteria are framed around obtaining a solution, the quality of a solution is important. Students are frequently satisfied with their results simply if there is a reduction of the initial difficulty in the problem. This leads to low-level solutions or single solutions or one that is not the most satisfactory. To consider a problem solved, one's solution should meet certain standards. It should "...be internally consistent, should promote useful actions or attitudes, and should be acceptable for a reasonable length of time" (Mills & Dean, 1960, p. 4). Ultimately, all these definitions see a problem as an obstacle to action. Logically then, the perception of what constitutes a problem varies from individual to individual. Certainly, the student must first recognize that a problem has presented itself: "(I)f people do not realize the existence of a problem, one cannot expect them to look for a solution" (Bransford, Sherwood, Reiser & Vye, 1986, p. 22). Further to this, Mills and Dean (1960) suggest, "...a student who feels he knows the answer to a problem is not going to recognize the problem" (p. 11). This is echoed by Wheatley (1991) when he points out, "...what is a problem for a person may not be for another" (p. 12). Also, as Wilson, Fernandez & Hadaway (2000) wryly note, "...persons not enthralled with mathematics may describe *any* mathematics activity as problem solving" (p. 7).

There is a general consensus that problem solving requires skills in several areas. According to Treffinger and Huber (1975), an individual should be able to identify various aspects of the problem situation. A number of checklists have been developed for this purpose (Davis, 1973; de Bono, 1970, 1976; Noller, Parnes and Biondi, 1976; Parnes, Noller & Biondi, 1977). Basically, when confronted with a puzzling situation, the person should be able to recognize the "real" problem. It may not be immediately obvious but through practice, the solver will be able to redefine or broaden the issue as presented. This may result in the identification of more manageable sub-problems. This is sometimes referred to as a means-end analysis and may allow the solver to deal with the problem in smaller pieces (Woolfolk, 1990, p. 273). The feeling is that such skills are not directly correlated to intelligence nor does the ability to think divergently depend on intelligence (de Bono, 1975; Torrance, 1979). This stands in direct contrast to earlier work done by Getzels and Jackson (1962) in which they make a clear distinction between "high IQ" and "high creative" children, indicating little or no overlap between the two.

Problems present themselves in a context not necessarily theoretical. In fact, by their very nature, problems frequently arise in a practical context. As Meacham and Emont (1989) put it, "...the essence of a problem is in the having of it" (p. 9). In other words, if a solution is immediately obvious, the situation is not a problem. Contrast this with the usual type of situation presented as a problem in mathematics: "(I)n 1991, the average Canadian spent 23 hours weekly watching television. About 64% was American programming. About how many hours was Canadian programming?" (Elchuk et al, 1996, p. 164) While the answer might not be immediately apparent, the path to it generally is and the student simply needs to apply algorithms of one type or another in order to determine a solution. Thus, it becomes not so much a problem as an exercise "...designed to reinforce mathematical skills" (Schrock, 2000). This is echoed by other researchers (Kaplan, Yamamoto, & Ginsburg, 1989). The definition of problem solving according to Woolfolk (1990) implies that if one employs an algorithm, one is not problem solving: "(P)roblem solving is what happens when routine or automatic responses do not fit the current situation" (p. 267).

Frederikson (1984) points out that clearly defined, well-structured problems are generally the norm in problem solving and refers to such as "...the kind of problem which is clearly presented with all the information needed at hand and with an appropriate algorithm available that generates a correct answer, such as long-division, areas of triangles, Ohm's law and linear equations" (p.303). Zeitz (1999) concurs when he makes a distinction between a problem and an exercise: "(A)n exercise is a question that you know how to resolve immediately. Whether or not you get it right depends on how expertly you apply specific techniques but you don't need to puzzle out what techniques to use" (p. 3). Some researchers are more blunt: "(I)n math class, if the student already knows how to do the assignment, the student has exercises to do, not problems to solve" (Woodward, 2000, p. 2).

What does it mean to solve a problem? Davis (1973) suggests a problem solution is a creative idea or a new combination of existing ideas. Attaining this state is difficult because there are obstacles to the creation of new meanings. These are referred to in experimental psychology literature as rigidity, fixation, mental sets, and prior sets. A well-known example of rigidity as a barrier to problem solving was illustrated by Maier's (1931) research. In this experiment, individuals were asked to tie together the ends of two strings suspended from the ceiling. The problem was that the two strings were hanging some distance apart and both could not be grasped at once. The individuals could use any of a number of items present in the room to accomplish this task. Less than half of those tested were able to solve the problem by tying a weight (a pair of scissors was available) to one string and start it swinging, thereby allowing the person to grasp it while holding the other string. Most people viewed the scissors as an instrument for cutting, not as a pendulum. This was termed "functional fixedness" by Duncker (1945). In another experiment, a group of subjects were taught how to use tools in the conventional manner and then given problems that could only be solved by using the tools in unconventional ways. Their success rate was 11% compared to a group of people who scored 97% with no instruction (Michalko, 2001). Such preconceived notions impose severe limitations on the problem solver in that most of the problems that students will face in real-life are of the "fuzzy" and "ill-structured" variety (Simon, 1973). Crovitz (1970) puts it succinctly when he notes that in problem solving, "(T)he natural tendency is to keep trying the same old thing when illumination requires more flexibility than that" (p. 80). This tendency is the epitome of what is generally referred to as vertical or convergent thinking. This is the traditional type of thought process encouraged by schools whereby one selects the immediately obvious and applies sequential steps to formulate a solution. The result is that, "(S)chool education...will produce men and women who are all of one pattern, as if turned in a lathe..." (Sumner, 1959). One researcher has suggested that this narrow range of thought, "...reflects tradition and convenience more than necessity" (Perkins, 1992, p. 24). Further, as far as problem-solving skills are concerned, apart from working backwards, schools are "narrow and convergent" (Perkins, 1992, p. 21).

In the realm of problem solving, a number of things present themselves as difficulties. A major one, of course, is that of dealing with problems in unfamiliar domains. An example of this is the work done by Green, McCloskey, and Caramazza (1985) in which they found that most of the difficulties encountered in physics problem solving were the result of inaccurate and incomplete prior knowledge in the content area. Deficiencies in this regard lead to difficulties when it comes to representing the problem both in memory and graphically (Gick, 1986; Nickerson, Perkins & Smith, 1985). Representation is held to be the first step in the problem solving process (Resnick & Ford, 1981). This is the stage at which the solver notes the features of the problem that can be encoded in a familiar way. Not surprisingly, there appears to be a correlation between representation issues and incomplete prior knowledge. While someone accomplished at solving problems in a particular domain is able to represent it accurately in terms of a schema containing both content and process, one who is less adept is likely to produce what Larkin (1985) refers to as a "naïve representation". She found that inexperienced problem solvers in physics were greatly hampered by their lack of scientific knowledge as well as the lack of procedural knowledge.

The same problem may be represented at differing levels of complexity based on the expertise of the solver. It follows that if this is done incompletely or at a level not commensurate with a particular problem, the result is ineffective solving (Chi, Feltovich, & Glaser, 1981; Halpern, 1989; Siegler, 1985). According to Polson and Jeffries (1985), effective and efficient representations must be coherent, reflect the prior knowledge of the solver, and accurately describe the task.

Research further indicates that the act of problem solving is affected by the problem itself. These include such things as the ease of identifying the most useful strategy, the nature of the skills required, the complexity of the task at hand, and the relevance of the problem to the solver (Nickerson, Perkins, & Smith, 1985; Wearne & Hiebert, 1984).

According to research by Sowder (1989), middle school students tend to respond to word problems with one of four basic strategies. The lowest level would be that of a coping strategy in which the student simply guesses at the operation to be used. In this case, the numbers in the problem are dealt with in a manner most comfortable to the student or what the most recent work in class has been.

The next level is that of a computation-driven strategy. The student derives clues as to the operation to use from the numbers themselves. In other words, if the numbers are relatively close in size, such as 81 and 63, the student is most likely to choose addition or multiplication whereas if they are far apart (i.e. 81 and 4), the student will select division as the most viable option. In some instances, the student will simply try all four operations and choose the answer that appears most reasonable.

Another frequently used method by students is to look for "key" words or phrases as a clue to the correct strategy; "all together" for example, would be interpreted as addition. This is a strategy that is promoted in textbooks despite claims that it is of no value and possibly damaging to the problem solving mentality of students. Van de Walle and Lovin (2006) propose a number of arguments against the use of the key word strategy. Frequently, key words suggest an operation which is not the correct one. As well, many problems lack key words. Most importantly,

> The key word strategy sends a terribly wrong message about doing mathematics. The most important approach to solving any contextual problem is to analyze its structure – to make sense of it. The key word approach encourages students to ignore the meaning and structure of the problem and look for an easy way out. Mathematics is about reasoning and making sense of situations. A sense-making strategy will *always* work (Van de Walle & Lovin, 2006, p. 70).

Sowder believes that the most mature and desirable strategy is that of selecting the operation which best fits the meaning of the story.

Loftus and Suppes (1972) use the term "structural variables" to refer to problem characteristics which determine complexity. Certain of these variables were found to be significant in terms of their effect on increasing difficulty of solving. These included the number of different arithmetic operations required, if a problem was solvable by the same operations in the same order as previous problems, the length of the problem, the grammatical complexity of the problem, and if conversions of measurement was necessary.

This research builds on work done several decades ago. Researchers found that difficulty experienced by students when solving word problems were the result of several factors. A number of studies were done in which children were presented with word problems dealing with identical operations and identical numbers (Brownell & Stretch, 1931; Hydle & Clapp, 1927; Kramer, 1933). The variables appeared to lay with the wording and context. Certain factors seemed to significantly decrease the success rate of students. These included such things as the difficulty of the vocabulary, the amount of non-essential information, and whether the story problem was inherently interesting. As a result, Hydle and Clapp (1927) suggested that word problems should involve situations familiar to children. Other researchers such as Brownell and Stretch (1931) took the opposite view; they believed that exposure to unfamiliar situations was desirable in that it provided children with an appreciation of the applicability of operations to a wide array of circumstances.

Despite differing orientations, there is agreement as to the importance of context. As Resnick and Ford (1981) put it, "(T)he importance of contextually-based arithmetic is widely recognized in educational practice, as witnessed by the predominance of 'word' or 'story' problems in the curriculum" (p. 89).

The value of this has been acknowledged for some time. Students involved in the Eight Year Study, which will be discussed later in this paper, write of their experience:

> In our mathematics work, we have studied business mathematics, elementary algebra, and plane geometry. However, we studied these subjects for a different reason and in a different way. Our work was not done to cover certain pages in a textbook but to give us a good understanding of the entire field of mathematics. We brought up our own problems; ones with which each of us had come in contact. They were much more valuable to us since we ourselves had felt the need of solving them (University High School, 1938, p. 199).

This is an important point for the way in which problem solving is approached. Instead of separate teaching of algorithms which are then embedded in word problems, the data suggest that the two must be taught simultaneously. Resnick and Ford (1981) concur when they say, "...algorithms should be learned in the context of the structures underlying them" (p.138). Failure to learn an algorithm without a context results in students who, while skillful with a particular procedure "...are very reluctant to attach meanings to it after the fact" (Van de Walle & Lovin, 2006, p. 8). This is echoed by other researchers including Wearne and Hiebert (1988) who state, "(I)t is more difficult for students to acquire conceptual understanding once they have learned rote procedures. Thus, it is essential to focus initial instruction on building conceptual understanding" (p. 23). Also, learning algorithms without context increases the risk that they will be used haphazardly (Woolfolk, 1990). Other researchers (Kamii, 1994; Kamii & Dominick, 1998) extend the idea even further and suggest that algorithms, with or without context, should not be taught until grade 4. Doing so earlier appears to have a detrimental effect on the development of number sense in that algorithms encourages the student to abandon his or her operational thinking.

According to Chapin, O'Connor and Anderson (2003):

> If taught in isolation, problem-solving strategies...may appear to be merely tricks or shortcuts rather than powerful tools for mathematical thinking. Therefore, rather than focus on discussing particular

> problem-solving strategies in isolation. ...it's most effective to incorporate problem-solving strategies in the context of the content of the mathematics curriculum (p. 67).

In a study by Roman (1975), fourth grade children were taught mathematics on a contextual basis. Not only were their problem solving skills superior, they had significant gains over-all in their mathematics achievement. Learning within a context and focusing on concepts as opposed to solely on procedures provides the student with the opportunity to form important connections between topics. As Hiebert and Lefevre (1986) state "(C)onceptual knowledge is characterized most clearly as knowledge that is rich in relationships" (p. 6).

Piaget makes this point when he states:

> It is not by knowing the Pythagorean Theorem that free exercise of personal reasoning power is assured. It is in having rediscovered its existence and its usage. The goal of intellectual education is not to know how to repeat or retain ready-made truths. On becomes educated by learning to master the truth by oneself (Piaget, 1973, p. 106).

This is echoed by Alfie Kohn (1999) when he says learning happens ..."when children aren't handed rulers but in effect asked to invent them, when they recreate the marvelously consistent relation among the three sides of a right triangle" (p. 177).

This is a point raised by Bruner (1960) in his proposal of a spiral curriculum. He suggested presenting incomplete structures to learners in ways that would promote intuitive understanding of the interrelationships of the different parts, with later learning completing the structures. In other words, topics would be taken up repeatedly, with each treatment being more formalized than the previous encounter and demonstrating relationships in a wider and wider set of mathematical concepts. According to Bruner, the learning of structure should take precedence over the mastery of facts in order to provide an overview to the learner of the relationships between concepts encountered at different times. Simply, education is the process of acquiring knowledge:

> To instruct someone…is not a matter of getting him to commit results to mind. Rather, it is to teach him to participate in the process that makes possible the establishment of knowledge. We teach a subject not to produce little living libraries on that subject, but rather to get a student to think mathematically for himself, to consider matters as an historian does, to take part in the process of knowledge-getting. Knowledge is a process, not a product (Bruner, 1966, p. 72).

Not all researchers are in agreement as to the value of a spiral curriculum. McKnight (1987) believes that teaching in this manner puts students at a distinct disadvantage in that it promotes a fragmentation of "computationally-oriented content" (p.97.) Further, this fragmentation results in an emphasis of topic breadth over depth. Schmidt, Houang, and Wolfe (1999) concur that a highly repetitive focus results in compromising depth. The scenario caused by this is what Cogan, Houang, and Wang (2004) term a curriculum that appears to be "...a laundry list of topics" with little relationship between content (p. 3).

Although much thought has gone into curricula in terms of what concepts and topics to include, not enough attention has been directed towards providing meaningful contexts. As Hyde and Bizar (1989) put it: "...an essential ingredient for motivation and understanding is the real world and its situations, problems, and phenomena" (p. 90). Unfortunately, research does not show this to be the case. Curricula and textbooks continue to deal with topics on a disconnected and cursory level, with inadequate links between between topics (Valverde & Schmidt, 1997; Schmidt, Jakwerth & McKnight, 1998).

The manner in which concepts are presented to students determines the degree to which they are able to construct their own meaning. A recent study indicated that in North America, one-fifth of lessons were structured in such a way as to allow students to develop concepts. In the remaining four-fifths, the concepts were simply stated by the teacher (Stigler & Hiebert, 1999). It is interesting to note that this is the opposite of what occurs in countries such as Germany and Japan and that "(T)hese data add more weight to the impression that students in Japan and Germany have richer opportunities to learn the meanings behind the formulas and procedures they are acquiring" (Stigler & Hiebert, 1999, p. 60).

This process of constructing meaning through contextual investigations is referred to as "mathematizing" by Cathy Fosnot (2002, p. 9). She describes it as follows: "(C)hildren are finding ways to explore situations mathematically, they are noticing and exploring relationships, putting forth explanations and conjectures, and trying to convince one another of their thinking – all processes that beg a verb form" (Fosnot, 2002, p. 9).

As students construct meaning, it must be borne in mind that when they do so, it is from their "lived reality". This is well illustrated by the results on a question from the 1996 NAEP (National Assessment of Educational Progress):

> Julie wants to fence in an area in her yard for her dog. After paying for the materials to build her doghouse, she can only afford to buy 36 feet of fencing. She is considering different shapes for the enclosed area.

> However, she wants all of her shapes to have 4 sides that are whole number lengths and contain 4 right angles. All 4 sides are to have fencing. What is the largest area that Julie can enclose with 36 feet of fencing?

Of the 1615 grade eight students in the sample, less than 1% provided a satisfactory response, 29% gave a partial response, 4% gave a minimal response, nearly 40% gave an incorrect response and approximately one-third omitted the question (Kouba, Champagne, & Roy-Campbell, 2000). This question makes the assumption that students can envision or have experience with a yard large enough to house a dog. In fact, about 3% of the students gave divergent answers to this problem and suggested that possibly it could not be solved because the yard was too small. This response was congruent with the "lived reality" of these students.

Research suggests that children tend to view the context as reality which may then lead to an inability to respond (Kouba et al, 2000). As opposed to adults who tend to see the context as a vehicle for understanding the mathematics, children may not be able to extend beyond contextual details. In the previously noted example of building a fence to enclose the largest possible area for a dog to run, many students became very concerned about a gate, where it would have to be placed, the amount of lumber it would take, and so on (Kouba et al, 2000).

What this demonstrates is that there is likely no way to make a context free of alternative interpretations, nor should there be a desire to do so. Instead, the task of the educator becomes one of encouraging students to come up with multiple solutions (that is, if a gate is considered, the answer is x; if a gate is not considered, the answer is y).

If a teacher is to encourage students to construct their own meaning, then it becomes important to allow problem posing. By drawing upon their own experience, they are able to find problems which they believe require solutions. Researchers suggest that this exposes students to new realms of meaning as they construct knowledge (Driver, Asoko, Leach, Mortimer & Scott, 1994; Hill, 1996). It must be remembered however that since knowledge is a function of personal history and social constructivism what students ultimately construct will possibly be different from that which teachers intend (Osborne, 1996).

This chapter has attempted to show that, while problem solving is deemed to be of much importance, more is needed to be done to facilitate students experience success in this area. There are many difficulties inherent in problem solving. These include working in domains unfamiliar to the student and incomplete prior knowledge, as well as variables within the problem such as wording, context, and relevance. Despite a measurable and persistent lack of success in teaching problem solving, schools continue to promote convergent, linear modes of thought. The ramification of this is that students leave school with both a superficial grasp of concepts and a distinct inability to apply what they have learned to real-life problems. Reliance solely on convergent thinking is not adequate. Students must be exposed to divergent, lateral thinking as well. This is a difficult proposition at this point in that most schools promote convergent thinking.

The remainder of this study will seek to examine through the literature, how schools continue to emphasize linear thought, often to the exclusion of divergent thought and the reasons for this. The value of divergent thinking will be examined, including a review of the latter in the literature. As well, and as a representative example, there will be a review of the Manitoba grade 7 mathematics curriculum and suggestions as to how it may be approached in a more divergent manner.

The next chapter will review the literature on divergent thinking as well as a clarification of terms and a comparison and contrast of divergent and convergent thought.

Chapter 2

A Review of the Literature on Divergent Thinking

This chapter will introduce and compare convergent and divergent thinking, and discuss the widespread use of the latter and the value of the former. As well, there will be a review of the various terms used, including lateral, linear, divergent, and convergent thinking.

For the purposes of this study, linear and convergent thought are taken to mean the same thing: a pattern of thinking which proceeds in a step-by-step fashion to a single, correct answer. As there is a constant awareness of the logical, right nature of this approach, any avenues of thought that deviate from this are dismissed.

Divergent/lateral thinking is a much more generative type of thinking. All manner of options are considered, with no consideration of correctness. While the alternatives are ultimately evaluated, one of the hallmarks of this type of thinking is that proposed solutions, while possibly not practical for the matter at hand, may prove useful in other arenas or may even result in a change in the way the original problem is viewed.

Convergent, also known as vertical, logical, or linear thinking is much different than its divergent, or lateral, counterpart. While the former is sequential and is characterized by thinking within the frame of reference, the less constrained lateral thinking "...tends to restructure the problem space" (Nickerson et al, 1985, p. 214). As de Bono (1970) suggests, "(L)ateral thinking generates the ideas and vertical thinking develops them" (p. 12). Further, de Bono believes that the strength of logic lies not so much in reaching a conclusion as in does in testing the soundness of such conclusions. Polya repeatedly made this point when he discussed mathematical reasoning. To him, the thought processes that ultimately led to a theorem were unconstrained and non-linear.

At first glance, it may appear that the obvious, and indeed, natural course of action would be to look for alternative ways to solve a problem. While this is the case, it must be remembered that it is a matter of degree. The goal of divergent thinking is to produce as many alternatives as possible. According to de Bono (1970), "(O)ne is not looking for the *best* approach but for as many *different* approaches as possible" (p. 63).

As a concept, divergent thinking has been with us for some time. Granted, it has undergone an evolution of sorts, and has been refined into that which we know today. In his seminal 1950 paper, Guilford makes a distinction between convergent and divergent thinking. The former was characterized as recognition of the familiar. Either the answer to a question could be recalled from stored information or it could be worked out using conventional decision-making strategies. The latter, divergent thought, involved producing multiple answers.

A review of the literature suggests references to the concepts of divergent and convergent thought in the work of Getzels and Jackson (1962). They make a distinction between two types of children, the 'High IQ' (intelligence quotient) and the 'High Creative', based on the results of two contrasted types of mental tests. The former, as the name suggests, is particularly good at intelligence tests while the 'High Creative' performs better on tests of creativity. One could make a very good argument that the Getzels and Jackson tests were weak at assessing either. A typical intelligence question might be something such as: *Brick is to house as plank is to…orange, grass, egg, boat, ostrich.* The questions also include logical relations expressed in terms of patterns, and include a wide range of material, aimed to assess vocabulary, general knowledge, and immediate recall. The common factor in all the questions is that each has only one right answer. Compare this to a typical question from a creativity test: *How many uses can you think of for a brick?* According to Hudson (1966) in his review of Getzels and Jackson, such questions invite "…the individual to diverge, to think fluently and tangentially, without examining any one line of reasoning in detail" (p.37).

Individuals who were able to think in a divergent manner had no difficulty in producing a lengthy list of uses, including such suggestions as "(T)o break windows for robbery, to determine the depth of a well, use as a pendulum, prop up a wobbly table, keep a door open, use to make a path, demonstrate Archimedes' Principle" (Hudson, 1966, p. 38). Those deemed as 'High IQ' tended to list fewer, more obvious, suggestions such as "(B)uilding material".

Hudson (1966) proceeded further with his work on divergent thinking and developed a test that he referred to as "(U)ses of Objects" (p.41). This test asked for as many different uses as possible of everyday objects. Responses were scored based not only on number but also on their novelty and statistical rarity. Some students were unable to think of any but the most obvious responses. These individuals were designated as "convergers" while the multitude of uses produced by others indicated they could be called "divergers". Hudson illustrates the size of the gap between two boys, one mathematically inclined (a "converger"), the other an arts specialist (a "diverger"). He lists the responses given by each when asked to think of as many uses as possible for a barrel:

> Converger – Keeping wine in, playing football.
>
> Diverger – For storing old clothes, shoes, tools, paper, etc. For pickling onions in. For growing a yew-tree in. For inverting and sitting on. As a

table. As firewood chopped up. As a drain or sump for rainwater. As a sand pit. At a party for games. For making cider or beer in. As a play-pen for a small child. As a rabbit hutch, inverted with a door cut out of the side. On top of a pole as a dove-cote. Let into a wall as a night exit for a dog or cat. As the base for a large lamp. As a vase for golden rod and Michaelmas daisies, as an ornament, especially if it is a small one. With holes cut in the top and sides, either for growing wall-flowers and strawberries in, or for stacking pots, and kitchen utensils. As a proper garbage can or wastepaper basket. As a ladder to reach the top shelves of a high bookcase. As a casing for a homemade bomb. Sawn in half, as a doll's crib. As a drum. As a large bird's nest (Hudson, 1966, pp. 90-91).

Based on his sample, Hudson drew some questionable conclusions. He believed that the majority of convergent thinkers were math/science-oriented while the divergers were mainly specialized in the arts. He further suggested that these interests, with their attendant cognitive abilities, resulted from child-rearing practices. This in turn led him to the conclusion that convergent thinkers were emotionally inhibited and that this inability to express emotion stemmed from being raised by cool, over-demanding mothers. These conclusions are likely over-simplified. This is a view also held by Morrison (1973).

Hudson also found that certain subjects, notably history, literature, and modern language attracted more divergent thinkers while subjects such as mathematics, physics, and chemistry appealed to more convergent thinkers (Hudson, 1966, p. 180). One wonders if the students were inherently "convergers" or "divergers" or if it was the manner in which these subjects were taught that encouraged or discouraged creative thought. It is telling that convergent thinkers flourished in mathematics.

Entwhistle and Ramsden (1983) claim that "(U)ses of Objects" is a weak test because it accepts both plausible and implausible ideas. Raaheim (1976) concurs with this view and has developed a test which demands realistic alternatives. However, the authors in each case are attempting to apply linear ideas and frameworks to that which is lateral thinking. The essence of lateral thinking is the generation, without review or censure, of as many alternatives as possible.

As noted previously, there is a consensus among researchers that schools emphasize convergent thinking (deBono, 1970; Polya, 1945; Wilson et al, 1999; Ziv, 1983). This is not inherently undesirable. Throughout the education process, students are taught to proceed in a logical manner. Thinking is a matter of right and wrong. Ideas of the latter variety are discarded while those with promise are pursued further. Botkin, Elmandjra and Malitza (1979) refer to the two types of thinking as maintenance and innovative. The former is "...the acquisition of fixed outlooks, methods, and rules for dealing with known and recurrent situations" (p. 10). While this type of thinking has been adequate in the past, the authors contend that its usefulness is diminishing. Innovative learning, on the other hand, questions long-held assumptions and seeks new perspectives.

Other researchers have referenced two contrasting strategies as well. Bartlett (1932) made a distinction between "open" and "closed" thinking while Rothenberg (1988) coined the term "janusian" thinking. This is named for the two-faced Roman god Janus who was able to look in multiple directions simultaneously.

One of the fundamental challenges of education is to prepare students to shape the future as opposed to accommodating to it and the need to teach new thinking skills. If this is not done, students tend to rely more on memory and application and are not encouraged to take risks and explore a variety of methods. The result is that few students are able to solve real problems in real-world contexts. Physics Nobel Prize winner Richard Feynman recalls the time spent as a lecturer at a university in Brazil and notes "...I finally figured out that the students had memorized everything, but they didn't know what anything meant" (1985, p. 212). It is this reality that Howard Gardner refers to as a "correct answer compromise" (Brandt, 1993, p. 4). As Gardner states, "...students read a text, they take a test, and everybody agrees that if they say a certain thing, it'll be counted as understanding" (Brandt, 1993, p. 5).

One must question if learning has taken place. Being able to demonstrate the skills is only a part of the learning process. Students must be able to transfer this knowledge to new situations. As Yager (2000) points out, "(E)veryone must use information and skills in new contexts before there is any evidence that learning has occurred." This is echoed by Woodward (2000) when he suggests that since problems are constantly changing, students must be able to apply a general heuristic or problem solving scheme:

> The information we teach to students often remains true only for a period of time. The problems we solve for students benefit them only if they face the same problem at some point in the future and they can remember our solution. When the problem or the context changes, only the general process of problem solving will be of benefit to students. We all have many opportunities to solve problems but we do not get better through practice alone. Practice does not make perfect. Practice makes permanent (Woodward, 2000, p. 4).

Gardner expresses a similar sentiment when he states:

> In mathematics, the problem is that kids learn formulas by rote, and they learn to plug numbers into those formulas. As long as the problem is presented with the items in the right order, so to speak, everything is all right. But as soon as the problem is given another way requiring the students to understand what the formula refers to, to be able to use it flexibly, then the students fail (Brandt, 1993, p. 4).

Anderson et al (2000) agree when they call for an increase in children's capabilities in their "present and non-present school lives" (p. 12).

While this is a worthy goal, it may be somewhat more difficult to achieve than it appears. There is research which suggests that claims involving the transferability of problem solving skills are somewhat exaggerated (Howe, 1996; McCormick, 1993, 1997). As Lewis et al (1998) state: "(T)here is little evidence to suggest that solving one type of problem at work informs problems of the home or school" (p. 5). This could be a comment more on the traditional problem solving checklist than the type of thought behind it. This brings to mind the work of E.L. Thorndike in which he drew the conclusion that the transfer of learning occurred only in those situations where the experience and application shared identical elements (1906, p. 246).

There is now general acceptance that transfer of learning can occur to at least some degree in all situations. However, according to Krug (1957), the similarity between situations plays an important role:

> Learning to play a violin then will not only transfer more readily to other musical learnings than it will to other fields of endeavor, but it will also transfer more readily to playing a cello or viola than it will to the playing of a piano or horn. The study of physics will have more transfer value for the study of chemistry than it will for history (p. 39).

Researchers have documented a number of cases in which transfer of learning occurs when the student recognizes a relationship between a learned experience and a new one, without the benefit of identical elements (Nisbett, Fong, Lehman & Cheng, 1988). These findings are congruent with other studies in which researchers have made a distinction between two types of transfer of learning, namely lateral and vertical. The former refers to a transfer of skills across situations while vertical transfer denotes a transfer between lower-level and higher-level concepts. Frequently, transfer of learning is a combination of both types (Gagne, 1968).

These findings support the position taken by Charles Judd (1927) when he held that the degree of transfer was dependent upon the extent to which the principles involved were applicable to a wide variety of situations:

> Mental development consists not in storing the mind with knowledge

> nor in training the nervous system to perform with readiness particular habitual acts but rather in equipping the individual with the power to think abstractly and to form general ideas.
>
> When the ends thus described are attained, transfer of training, or formal discipline, has taken place because it is the very nature of generalization and abstraction that they extend beyond the particular experience in which they originate (p. 441).

Age may possibly be a factor in the degree of transference. In a study by Lesh, Landau, and Hamilton (1983), it was found that while young students produced incorrect answers for written calculations and the correct answers using concrete materials, more than half kept the incorrect answers when asked to show their written work again. The conclusion was that connections between formal and informal mathematics were unable to be made.

According to constructivist theory, Ben-Hur (1999) states, "...meaningful learning always transfers. If transfer is not evident, one must examine the state of the learning process and consider it incomplete" (p. 54).

Some studies have shown that the use of prescribed methods can become a ritual and reflect a classroom culture more than actual problem solving (McCormick et al, 1996). According to Lewis et al (1998), it then becomes "...just another tool among a landscape of cultural artifacts to be used, ignored, or reconditioned in the everyday problems of life and work" (p. 5).

Darling-Hammond (1997) observes that rote learning can actually result in students losing confidence. She points out that, "...children eventually become unable to reason things through, to estimate whether their answer is plausible, or to tackle a problem not already set up for them" (p. 54). In fact, research indicates that children may understand less and less as they proceed through the curriculum simply because they are applying meaningless algorithms to artificial situations (Gardner, 1991; Gardner & Winner, 1982; Strauss, 1982). Costa and Liebmann link the increasing lack of understanding as students proceed through school to the organization of the curriculum. They refer to the disconnected nature of the subjects when they state:

> The disciplines, presented as organized bodies of content, may deceive students into thinking they are incapable of constructing meaning. Students have frequently been indirectly taught that they lack the means to create, construct, connect, and classify knowledge. All they can hope for is to acquire other people's meanings and answers to questions that someone else deems important (1997, p. 26).

Costa and Liebmann urge educators to help students recognize that "...what they desire to know can come from within them" (1997, p. 26).

To this end, lateral thinking is a most efficacious medium. Lateral thinking is very distinct from its vertical counterpart. It has been described as a combination of "...liberation from old ideas and the stimulation of new ones" (de Bono, 1970, p. 12). As de Bono says, "(Y)ou cannot dig a hole in a different place by digging the same hole deeper. Lateral thinking is used to dig a hole in a different place" (1970, p. 13). Keith Devlin (2000) puts it this way: "(D)oing mathematics does not require new mental abilities, but rather a novel use of some existing capacities" (p. 180).

Different as they may be, it is widely held that proficiency in lateral thinking improves the efficiency of vertical thinking. According to de Bono, "…there is no antagonism between the two sorts of thinking. Both are necessary" (1970, p. 8). Research would seem to indicate that divergent or lateral thinking is the end result of making connections and that individuals can improve their creativity by seeking such connections between the new and the familiar. (ASA Critical Issues Reports, 1970, p. 35). William Gordon, a major proponent of this approach to lateral thinking, uses the following example: "(T)he Schick Injector Razor was invented by an army man who perceived an analogy between loading a repeating rifle and loading a razor" (ASA Critical Issues Reports, 1970, p. 35). Edward de Bono (1970) concurs when he describes a student's design for a vehicle to travel over rough ground (p. 118). The design involves laying down a carpet of "smooth stuff" which is held in a reservoir at the front of the vehicle. After it is driven over, a vacuum at the rear of the car sucks it up and returns it to the reservoir for immediate re-use. Impractical as the design is, it could provide the basis for other, related concepts such as tracked vehicles. The attitude embraced here is one acknowledging that, although an idea may not work, it can provide insight into others that will. According to Osborn (1963), it is an example of "...reaching out for targets by grasping problems, and how one target can create another" (p. 88). It is this connection-making that is the hallmark of divergent thinking. As Polya puts it: "(M)any a guess has turned out to be wrong but nevertheless useful in leading to a better one" (p. 99). Multiple solutions are encouraged with a view that there is not necessarily a right answer so much as a best answer. Polya points out, "(N)o idea is really bad unless we are uncritical. What is really bad is to have no idea at all" (Polya, 1945, p. 99).

Unfortunately, a great deal of mathematics as it is taught in schools focuses on a single correct answer arrived at via a single "best" manner. While a single correct answer may be the reality of the particular problem, the determination of this end result should not necessarily come about through a prescribed manner. Traditional models of problem solving generally all exhibit similar defects in that problem solving is depicted as a linear process involving a series of steps. Wilson et al (1999) point out that "(L)inear models of problem solving found in textbooks are inconsistent with genuine problem solving" (p. 4). The implication is that solving problems is basically a procedure to be memorized. Sizer (1984) says, "(T)hinking…is rarely as neat as this…analyzing paradigm suggests. Nor does all useful thinking flow in a step-by-step sequence" (p. 105).

A reliance on textbooks tends to contribute to this linear mind-set. Apart from the fact that, by their nature, textbooks promote a "cookbook" view of mathematics through lengthy lists of topics to be covered, they also stress algorithmic procedures. This is frequently done at the expense of understanding. As Darling-Hammond (1997) says, this results in "…only the most trivial aspects of the underlying knowledge sought" (p. 51). Tyson-Bernstein (1988) has dubbed this the "mentioning" problem: brief references to hundreds of ideas but insightful analysis of none. Darling-Hammond (1997) sums it up as follows:

> In mathematics, texts require coverage of dozens of topics each year. Students whisk through rows of problems and march quickly through chapter after chapter, applying algorithms rather than delving into concepts. Because they do not deeply understand much of what they have covered, they must be taught the same topics year after year, and

> many graduate with very little ability to use mathematics beyond simple operations (p. 51).

In addition, there is evidence that over the past number of years, textbooks have declined markedly in rigor (Chall & Conrad, 1991). Kirst (1982) believes textbooks have dropped by at least two grade levels over the last 10 to 15 years.

It falls to the classroom teacher to utilize a textbook, not as the driving force behind day-to-day classroom work but as an additional layer to already rich conversations. Traditional textbook problems can be re-framed in order to reflect divergent thinking. An example is the problem mentioned in chapter 1: "(I)n 1991, the average Canadian spent 23 hours weekly watching television. About 64% was American programming. About how many hours was Canadian programming?" (Elchuk et al, 1996, p. 164)

The divergency inherent in this problem is the discussion that could ensue as a result of the wording. Since the question asks "About how many hours?" the students are not faced with a single, correct answer and multiple responses can be justified. Arguments could be made for 8 or 9 hours or even a range on either side, depending on the interpretation of "about 64%". As well, discussions could result concerning 23 hours a week, how it is arrived at and so forth.

Virtually any textbook problem can be altered to be more divergent. Take the following traditional example: "(A) rectangular aquarium is 12 inches wide by 14 inches long by 12 inches high. What is the volume of water needed to fill the aquarium?" (Houghton Mifflin, 2002, p. 475) When the problem is stated in a manner to invite multiple responses, it could look something like this:

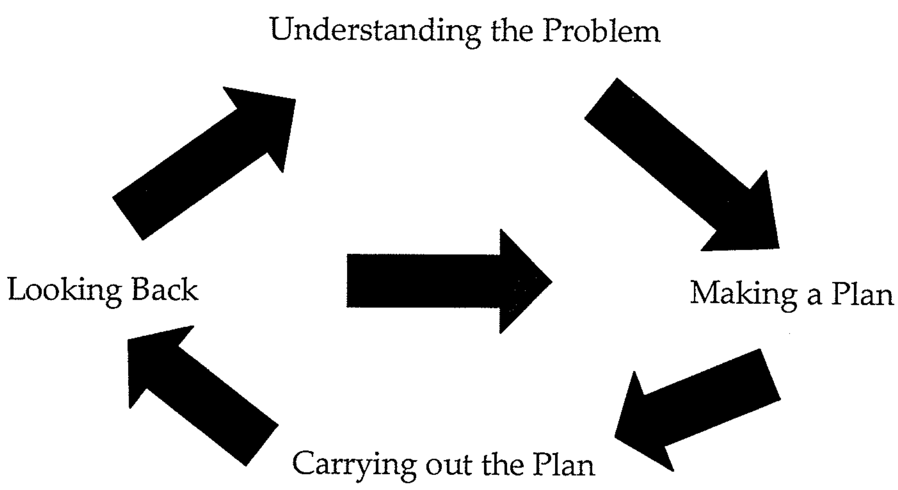
> You have been asked to design an aquarium in the shape of a rectangular prism for the school visitor's lounge. Because of the type of fish being purchased, the pet store recommends that the aquarium should hold 24 cubic feet of water. Find as many different dimensions for the aquarium as possible. Then decide which aquarium you would recommend for the lounge and explain why you made that choice (Mathematics Teaching in the Middle School, 2003).

Jerome Bruner, in comparing linear and lateral thinking, points out that in contrast to the former, lateral thought "...does not advance in careful, well-defined steps" (1960, p. 58). He further asserts that, "(U)nfortunately, the formulation of school learning has somehow devolved" this mode of thought Bruner, 1960, p. 58).

Above all, the emphasis is on obtaining the correct answer. According to Dienes (1969), teachers are in "...the business of providing children with the most effective methods of finding the correct answers" (p. 71). The result is determined in terms of student thought. "(S)chools that always insist on the right answers, with no concern as to how a student reaches it, smother the student's efforts to become an effective intuitive thinker" (Sizer, 1984, p. 105).

As an alternative to the traditional step-by-step linear model, the University of Georgia has developed a framework which appears to recognize the dynamic and cyclical nature of problem solving and begins with understanding the problem:

Figure 1: University of Georgia heuristic



This is applicable to divergent thinking in that the answer is assessed in terms of its applicability to the problem. If it is not appropriate, the problem is revisited.

Certain other heuristics, including the following one by Cyert (1980), paraphrased by Fredericksen (1984), reflect many of the notions held by divergent thinking:

1. Get the total picture; don't get lost in detail.
2. Withhold judgment; don't commit yourself too early.
3. Create models to simplify the problem.
4. Try changing the representation of the problem.
5. Vary the form of the question.
6. Be flexible; question the flexibility of your premise.
7. Try working backwards.
8. Proceed in a way that permits you to return to your partial solution.
9. Use analogies and metaphors.
10. Talk about the problem.

In order to develop thinking skills and more importantly, not reject out-of-hand ideas that are initially unpromising, a shift in thinking patterns is needed. Throughout the education process, students are taught to proceed in a logical manner. Thinking is a matter of right and wrong. Ideas of the latter variety are discarded while those with promise are pursued further.

A major component of divergent thinking is the recognition of the assumptions that are being made and being able to challenge those assumptions. As de Bono (1970) says, "(I)t is historical continuity that maintains most assumptions – not a repeated assessment of their validity" (p. 91). Problem solving, particularly as taught in mathematics, always assumes certain boundaries. Indeed, students are encouraged to establish parameters for the problem at hand. However, not only are such limits self-imposed, they are generally established on the basis of convenience. Clearly, ill-considered boundaries can provide a large, possibly insurmountable barrier to success in reaching a solution. Guilford (1977) refers to such limits as "hardening of the categories" (p. 167). In other words, once an object or idea is placed in a certain class or viewed in a particular manner, that is where it tends to remain.

It has been suggested that the ability to fulfill the criteria of the problem is a function of metacognition (Wallach, 1970). This is echoed by Suddendorf and Fletcher-Finn (2004) when they point out "(D)ivergent thinking, by its very definition, appears to require the individual to search his/her own knowledge base beyond the currently activated domain of mental content" (p. 2). This view is supported by Sternberg and Lubart (1991) when they point out new creative insights occur when one disengages from a current paradigm and "invests" in disregarded areas.

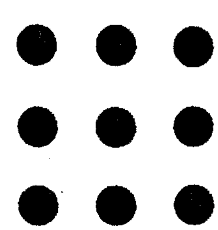
It is imperative that students are made aware of such self-imposed barriers. This is an attitude which does not necessarily change with age. As

Raudsepp (1982) states in reference to corporations: "(R)esistance to change is a major impediment to creative problem solving…" (p. 112).

In essence, divergent thinking is a challenge to the necessity of limits as it attempts to restructure patterns. According to de. Bono (1970), "…assumptions are patterns which usually escape the restructuring process" (p. 94). Consider the following problem: A landscape gardener is given instructions to plant four special trees so that each one is exactly the same distance from each of the others. How can this be done? The assumption is that all the trees are planted on a level piece of ground. If this assumption is not challenged, the problem cannot be solved.

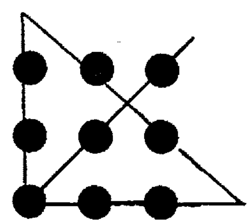
This is similar to the problem in which the individual is presented with the following picture and the instructions to connect the dots using no more than four lines and without lifting the writing instrument off the paper.

Figure 2: Three-Dot Problem



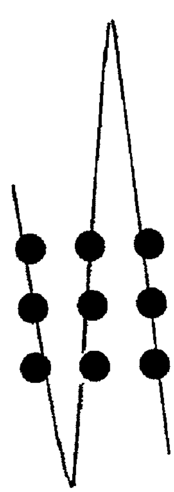
If the solver does not look beyond the perimeter delineated by the dots, the problem cannot be done. There are, however, a number of solutions. If one is not constrained by the boundaries of the dots, it could be solved as follows:

Figure 3: Three-Dot Problem solution



Interestingly, most people assume any lines must be drawn through the center of the dots, even though this is not mentioned in the instructions. This yields the following three-line solution:

Figure 4: Three-Dot Problem, alternative solution



There are a number of other solutions as well. They include:

- Cutting out the dots and arranging them in a row. A single line joins them all.
- Rolling the paper into a cylinder and using a single, continuous line.
- Using a very wide writing instrument and a single line.

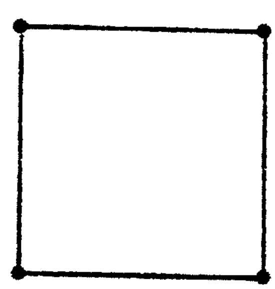
The writer has noted that the difficulties which people experience with this puzzle all stem, without exception, from preconceived notions which they hold.

Torrance (1979) believes this problem to be representative of first- and second-order changes. The former denotes attempts at solving which do not look beyond the information presented. The solution is "...a second-order change which involves leaving the 'field'" (p. 179). Second-order change then, is that which examines other possibilities. As Torrance says, "(T)he analogy between this and many real life...situations is obvious" (1979, p. 179). In fact, there is a large amount of research that supports this. Johntz (1967), using second-order changes, had great success teaching algebra and geometry to children with learning disabilities. These successes have been replicated in other studies (Boehm, 1970).

In their work on second-order changes and the relationship to successful problem solving, Watzlawick, Weakland, and Fisch (1974) noted that the key was looking outside the problem situation or at least reframing the problem. By redefining or rewording problems, the solver attempts to find the question to which a successful answer would activate second-order changes. As well, they found that solutions found in this manner tended to move away from what they termed "the more the better recipe" (p. 78). At first glance, such solutions can appear unexpected and perhaps even against common sense. However, further examination frequently shows the solution to be so simple and obvious that it is overlooked. Jerome Bruner (1973) believed that going beyond the information at hand was an important aspect of thinking generally and not limited to episodes of creativity.

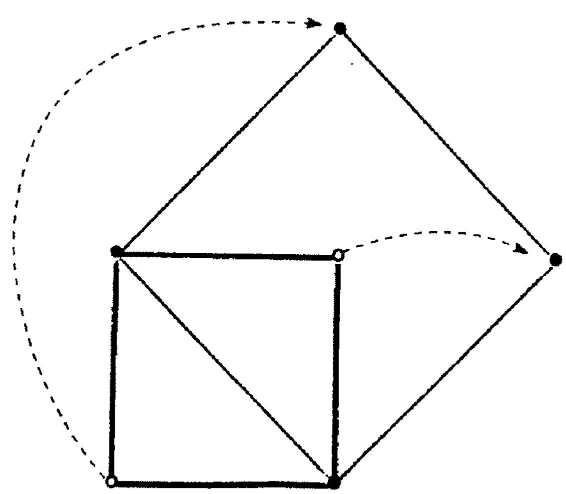
Even in cases where the options are truly limited, it can be very difficult to get past the initial response to the problem. The writer has presented the following picture and scenario to a number of people: Using the square you see in this illustration, how can you move two dots and make a bigger square? (Figure 5)

Figure 5



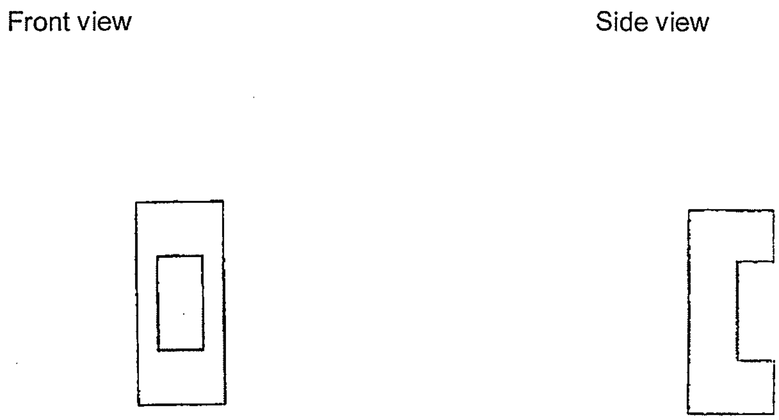
The assumption here is that the dots to be moved must be on the same side of the square. While it becomes an impossible situation to move two dots in this fashion and still maintain a square, people cannot see past this to alternative solutions. In cases where the audience has been high school mathematics teachers, the response has been no different. The answer truly provides an "ah ha" moment as it becomes clear that dots diagonally across from each other provide the only means of fashioning longer sides. (Figure 6)

Figure 6



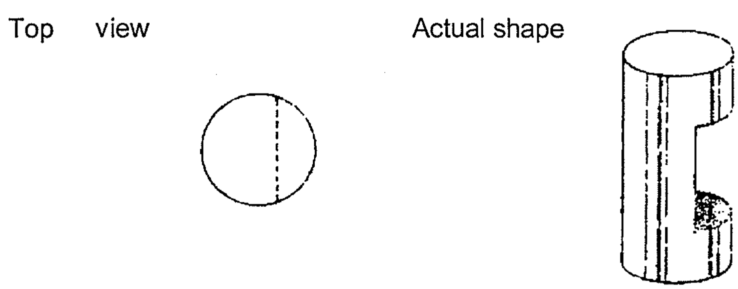
The following picture is an example of a seemingly impossible situation. The front and side views of the object seem incompatible with one another and the mind is unable to reconcile how the hole can be seen from both perspectives.

Figure 7: Incompatible Views



The assumption is that the figure is rectangular. Once that barrier is recognized and broken down, the solution becomes clear (figure 8).

Figure 8: Incompatible Views Reconciled

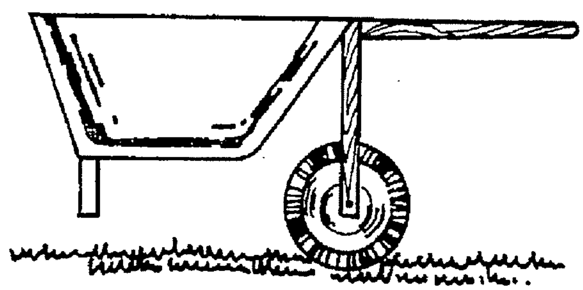


The key to developing divergent thinking patterns in students and ultimately increasing their ability to solve problems lies in enabling them to recognize barriers in their assessment of situations. The writer has found that students appear to go through a numbers of stages as they evolve towards this state. Initially, there is complete unwillingness to accept the situation as anything other than the way it is perceived. A great many students express the belief that it cannot be resolved or that information vital to the process is being withheld. This is followed by an acceptance that the problem may be viewed from different perspectives and finally, by a realization that they are able to do this with increasing frequency. The various reasons given for not doing well at problem solving are strikingly similar to those cited by Zeitz (1999). According to him, the three main reasons are: 1) an inability to know how to begin, 2) initial progress is made but the student cannot proceed further, and 3) the first few attempts produce no tangible result and so the student gives up (p. 4).

Students reach a point where they are able to reflect on their own thinking and determine the point at which their thought patterns are resulting in a barrier to further possibilities. Although it takes some time, the writer has found that a vast majority of students are able to reach this stage.

Habit of thought, or rigid patterns resulting from repetition can be extremely difficult to break. According to Koestler (1964), once a skill is learned or a pattern is found to be useful, the hierarchical nature of the brain relegates it to a lower center where it becomes a mechanical task. While this sequence is valuable from the perspective of organizing information, it can become a barrier to new thought. A number of cognitive researchers, notably Boden (1990), Perkins (1981), and Sternberg (1985), concur with this. McCormack (1977) believes adults have greater difficulty breaking down such barriers than do children. For example, when shown the following picture (Figure 9)

Figure 9



adult comments tend to focus on the negative such as "The wheel has been put on wrong" whereas younger children are more positive with such comments as "It would be easier to push around corners", "It would be useful for emptying a load over a low wall" and so on. Some researchers believe that this should not be taken as a sign that children are necessarily more creative than their adult counterparts. Wolf and Larson (1981) hold that what appears to be creativity may be more accidental than deliberate and stems from an inability to incorporate all of the facts.

It is of interest to note that some research suggests that there exists a strong connection between divergent thinking in children and the understanding of false beliefs (Suddendorf & Fletcher-Finn, 1999). False belief refers to the knowledge that people's perceptions of reality may differ. For example, a young child is told the following story:

> Bill has a bar of chocolate, which he puts in the green cupboard. He goes out to play, and, while he is out, his mother moves the chocolate to the blue cupboard. Then Bill comes in, and he wants to eat some chocolate. Where will he look for the chocolate?

If that child has not yet developed an understanding of false beliefs, he will assume that his knowledge, that of the chocolate's location, is shared by others. Research indicates that children who pass false beliefs test score significantly higher on measures of divergent thinking due to "...the ability to disengage from immediate perception and close associations in order to form more novel ideas" (Suddendorf & Fletcher-Finn, 1999, p. 116). This also suggests that a shift to a more divergent point of view is developmental, at least to a degree.

De Bono (1972) has compiled interesting work on divergent responses from children to various situations. One of the questions posed, for example, was how can you stop a cat and a dog from fighting? In a larger sense, this represents a basic political problem in that it can be applied to people. The responses included ways to keep them separate, allowing them time to get to know each other, and smearing each with the food of the other. In human dynamics, these could translate to national boundaries, assimilation, and the principle of self-interest. As de Bono notes, what is most striking about the attempts of children to solve this political problem is the variety of responses. The number of approaches suggested far exceeds that of adults, not because "...children have a special ability to look at things in a different way, but simply that adults have almost completely lost this ability" (1972, p. 43). Adults tend to seek the best and most sensible way which is that which most closely aligns to their experience. Since children lack this experience, they are more likely to try new ideas.

Jerome Bruner (1964) held that, on the basis of experience, the brain would group in categories events that repeatedly occurred together. This process of assigning events to categories is referred to as "coding" (Bruner, 1973, p. 222). Davis (1973) refers to this as "(f)orming an equivalence-class" (p. 38). This allows one to generalize beyond the concrete event. For example, if the unknown something is a part of the equivalence-class *fruit*, a considerable amount will already be known about it. In other words, if a new event is judged to match the properties of a particular category, the event is encoded into that category and the event is then assumed to have all the characteristics of the category. Once the brain has coded an event or object, it becomes very difficult to see it as anything other than a part of that group. This is not unlike the functional fixation reported by Maier decades earlier. However, he tended to apply the term to items with a specific function such as a hammer as a device for driving nails as opposed to a weight, a hook, and so on. Cropley (2001) suggests that encoding anything, while giving a sense of familiarity and predictability, shuts out alternative meanings. He cites as an example

> ...the difficulty experienced in eating kangaroo by many Australians despite the fact that kangaroo is a plentiful, tasty, healthy and cheap source of protein. This animal is coded as 'lovable and cuddly' or 'symbol of our country', categories that are incompatible with eating them. Eating kangaroo is something like eating one's national pride or dignity, an obvious impossibility. Sheep, on the other hand, have the misfortune to be coded into the category 'food' and therefore readily eaten, despite the fact that they are woolly and much more cuddly, while they also contribute far more to Australia's economic well-being than kangaroo (p. 37).

When dealing with a problem, the coding needs to go beyond the obvious and have the flexibility to be recoded according to the situation at hand. This is well illustrated by the urban legend sometimes attributed to Niels Bohr, Nobel Prize winner in 1922, which showed that alternative codings of an object could produce a number of divergent possibilities to solve a problem. On a physics examination in 1905, Bohr was asked a question as to how a barometer could be used to measure the height of a building. The usual coding of a barometer is "instrument for measuring air pressure" and hence, the answer was expected to reflect that: the height of the building calculated from the difference in readings of air pressure taken at ground level and the top of the building. Bohr's response is allegedly as follows:

> "You tie a long piece of string to the neck of the barometer, and then lower the barometer from the roof of the skyscraper to the ground. The length of the string plus the length of the barometer will equal the height of the building." *(coding of barometer as "measuring device")*
>
> This highly original answer so incensed the examiner that he failed Bohr who immediately appealed on the grounds that his answer was indisputably correct.
>
> The university appointed an independent arbiter to decide the case.
>
> The arbiter ruled that the answer was indeed correct, but did not display any noticeable knowledge of physics. It was decided to call Bohr in and allow him six minutes in which to provide a verbal answer

which showed at least a minimal familiarity with the basic principles of physics.

For five minutes, Bohr sat in silence, forehead creased in thought. The arbiter reminded him that time was running out, to which he replied that he had several extremely relevant answers, but couldn't make up his mind which to use.

On being advised to hurry up, Bohr replied: "First, you could take the barometer up to the roof of the skyscraper, drop it over the edge, and measure the time it takes to reach the ground. The height of the building can then be worked out from this formula I have worked out for you on my test paper here." *(coding of barometer as "object with mass")*

Then Bohr added, "But, Sir, I wouldn't recommend it. Bad luck on the barometer."

"Another alternative", offered Bohr, "is this: If the sun is shining you could measure the height of the barometer, then set it on end and measure the length of its shadow. Then you measure the length of the skyscraper's shadow, and thereafter it is a simple matter of proportional geometry to work out the height of the skyscraper. On the paper is the formula for that as well." *(coding of barometer as "object with fixed length")*

"But, Sir, if you wanted to be highly scientific about it, you could tie a short piece of string to the barometer and swing it like a pendulum, first at ground level and then on the roof of the skyscraper. The height is worked out by the difference in a gravitational formula, which I have determined here this time on a long sheet of paper with a very long and complicated calculation." *(coding of barometer as "object with weight")*

"Or, Sir, here's another way, and not a bad one at all. If the skyscraper has an outside emergency staircase, it would be easier to walk up it and mark off the height of the skyscraper in barometer lengths, then add them up. *(coding of barometer as "measuring device")*

But if you merely wanted to be very boring and very orthodox about the answer you seem to seek, of course, you could use the barometer to measure the air pressure on the roof, and on the ground, and then convert the difference in millibars into feet to give the height of the building.

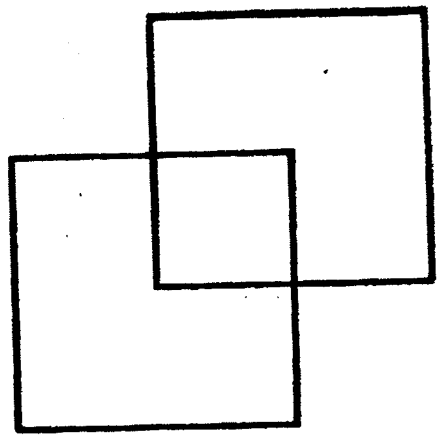
But since we are constantly being exhorted to exercise independence of mind and apply scientific methods, undoubtedly the best way would be to knock on the janitor's door and say to him 'If you would like a nice new barometer, I will give you this one if you tell me the height of this skyscraper'." *(coding of barometer as "object with monetary value")* (Chicago Tribune, September 1988).

The responses in the foregoing quote are excellent examples of divergent thinking. Although there is ultimately a single, correct answer, that of the building's height, there exist multiple ways to determine it.

Sometimes the wording of the problem influences the solver in ways that preclude him from seeing the solution clearly. Consider the following: Patches of water lilies double in area every twenty-four hours. On the first day of summer, there is one water lily on the lake. Sixty days later, the lake is completely covered with water lilies. On which day is the lake half covered? The words "double", "twenty four", "one", "on which day", and "sixty" influence many people to divide sixty days by two and arrive at thirty as the solution. Since the lilies increase geometrically, the answer is incorrect and the pond is half-covered on the second last day.

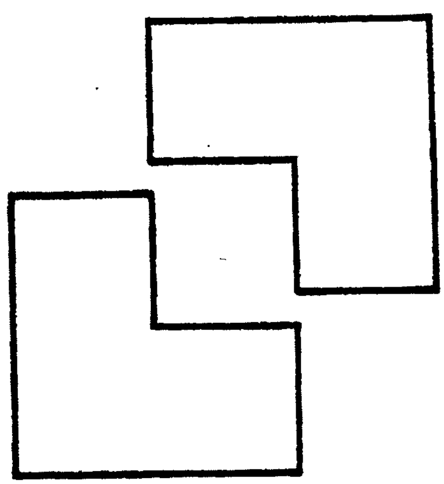
With practice, students can begin to view problems from a different perspective. In the following straightforward example, students are asked to identify the shapes in figure 10:

Figure 10



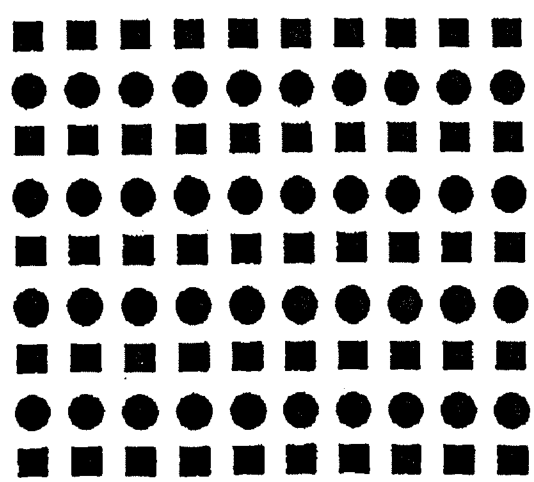
The majority of responses indicate that the picture is that of two overlapping squares. However, it could be seen as two L-shaped figures which do not overlap at all (figure 11).

Figure 11



The following picture (figure 12) is another example of perception and how one's immediate reaction determines how it is seen. When asked to describe this collection of squares and dots, the vast majority of people see it as rows of squares alternating with rows of dots. Very few describe in a columnar fashion. This is not to imply that one way is superior or holds some inherent advantage over the other but that perceptions are common. Commonly held perceptions sometimes foster the mistaken belief that other views are not possible or do not exist.

Figure 12



The salient differences between convergent and divergent thinking can be summed up as follows in figure 13:

Figure 13: Comparison of Convergent and Divergent Thinking

Convergent Thinking	Divergent Thinking
Looks for a single, right approach	Looks for as many approaches as possible
Rightness	Richness or multiple interpretations
Proceeds if there is any obvious direction	Proceeds to generate a direction
Is sequential	Does not require sequential development
The solver must be successful at every step	The solver does not have to be successful at every step
Uses negatives to block off certain pathways	There are no negatives
Excludes what is irrelevant	Welcomes the irrelevant in order to establish connections
Focuses on the end-point of a single correct answer	Focuses on possibilities which may satisfy the question
Tends to view problems from a narrow perspective	Views problems in a much wider context
Tends to use lower levels of Bloom's Taxonomy	Tends to use higher levels of Bloom's Taxonomy
Assumes there is a single correct answer which can be accessed by logical reasoning	Recognizes that many possibilities for a solution may exist but they cannot necessarily be recognized with linear thinking

The implications of each type of thinking for curriculum development and learning are profound. Convergent thought, by its nature, does not view knowledge as on-going conversation with rich inter-connectedness between topics. Instead, it presents "knowledge-out-of-context" (Applebee, 1996). This reinforces the belief that learning need not be integrated since it represents a vast accumulation of unrelated facts. It is hence divorced from the individual learner and is seen as a body of immutable knowledge, simply transferred from one individual to another without modification. Rote memorization becomes not only acceptable but expected. Despite the fact that such "...methods violate the nature of the child and lead to boredom, apathy, hostility, stupidity, and other negative effects", this continues to be the norm in schools (Graubard, 1972, p. 17). To Emile Durkheim (1956), this was not surprising since he believed this to be in the best interests of society as a whole: "(E)ducation, far from having as its unique or principal object the individual and his interests, is above all the means by which society perpetually recreates the conditions of its very existence" (p. 71).

Schools have traditionally promoted convergent thinking. While a sole reliance on this type of thought is undesirable, convergency has value when used in conjunction with divergent thinking. Although many solutions may be proposed, ultimately one must be selected as having the "best" attributes at a given time.

There is an emphasis on thought largely as a linear process and answers as being right or wrong. For students to mature as problem solvers, there must be a consideration of divergent thought as a means to generate possibilities. As Edward de Bono (1970) asserts:

The purpose of thinking is not to be right but to be effective. Being

> effective does eventually involve being right but there is a very important difference between the two. Being right means being right all the time. Being effective means being right only at the end (p.11).

This chapter has sought to review the literature on divergent thought and compared the attributes of convergent and divergent thinking. While both possibly arriving at similar conclusions, the two modes of thought take vastly different approaches to that end. Students can benefit from both divergent and convergent thinking in that former generates ideas and latter evaluates them. However, it would appear that convergent thought is promoted by schools to the near-exclusion of divergent thinking.

The next chapter will discuss the historical basis of teaching and learning and how the emphasis on convergent thought in schools has been maintained. It will also discuss approaches that have strived to enhance the personal experience of the individual and how such approaches have been influenced by, as well as influence, the social context in which they existed.

Chapter 3

A Historical Perspective of Learning and Thinking

Although it is apparent that the convergent mode of thought as traditionally taught by schools is inadequate for problem solving, it continues to be the norm. This chapter will attempt to focus on the reasons behind this historical continuity.

To provide guidance for the reader, a timeline of certain important events in the progression of divergent thought has been included (figure 14):

Figure 14: A Timeline of Events in the Progression of Divergent Thought

Dates	Event/Individual	Focus
Early 20th Century	Progressive Education	An educational paradigm, the aim of which was to make schools more effective agents of a democratic society. Students should be independent and creative thinkers.
1920s –1930s	John Dewey	Education as preparation for life. Knowledge emerges only from situations in which the learner has to draw them out of meaningful experiences.
1920s – 1930s	Jean Piaget	The basis of learning is discovery. Knowledge not acquired external of individual but constructed from within.

1920s – 1930s	Lev Vygotsky	Investigated child development and the effect of culture and interpersonal development. Construction of knowledge driven by learned habits of culture.
1932 – 1940	The Eight Year Study	A major research undertaking which demonstrated that traditional curricula/delivery were not necessarily the sole routes to success.
1950	J.P. Guilford	Provided seminal research into varied modes of thinking.
1968	Edward deBono	Coined the term "lateral thinking" to describe unconventional ways of viewing a problem and reaching a solution.
Can be traced back centuries in various forms. Has become an important educational approach in the 20th century	Constructivism	Knowledge is not about the world but constitutive of the world. It is not fixed but constructed by each individual.

The ideal of schools as informed and focused on thoughtful learning is not a new concept. In fact, it has long figured in the history of education, particularly in the past century (Power, 1982, p. 287).

Historically, education has attempted to achieve cultural homogeneity and produce dutiful citizens. Progressive education arose at the turn of the century in response to this and with it, a view that diversity and critical thinking could help make schools more effective agencies of a democratic society. The movement and its proponents held that participation by all citizens in social, economic, and political decisions was vital to democracy. The major elements of progressive education have come to be known as reconstructionism and child-centered approaches.

John Dewey (1933) encouraged thoughtful learning. The progressive thinking of the time that Dewey promoted believed that "...information is an undigested burden unless it is understood...and understanding, comprehension, means that the various parts of the information acquired are grasped in their relations to one another – a result that is attained only when acquisition is accompanied by constant reflection upon the meaning of what is studied" (p. 21.) To Dewey, the nurturing of reflective individuals capable of reflective thought was a major educational objective (Dewey, 1933). In the face of a growing avenue of thought which advocated academic education for the few and vocational training for the majority, Dewey believed that schools should reflect the needs of society. For example, he suggested that it was the responsibility of the education system to introduce immigrants to the new culture. As well, a number of curricular changes were proposed, many of which are still visible today, including for example, allowing students to work in groups and theme-based teaching.

Dewey's philosophy was one of education as dependent on action. Learners are presented with situations which produce knowledge and experience. These situations must occur in a social context (such as a classroom) thereby creating a community of learners building knowledge together.

This position is supported by Dewey's ideas of continuity and interaction. Continuity refers to the notion that learning begets further learning, and each experience is formed by those that have preceded it. All experiences, both positive and negative, historical and contemporary, provide a learning opportunity. It is the accumulation of these experiences that influence the nature of future learning.

Interaction builds upon the notion of continuity and refers to the idea that acquired knowledge may have to be reviewed and adapted as a response to new learning in social context. In other words, the interaction of past experience and the present situation serve to create the present experience. The important implication for educators is that the same situation can, and likely will, be viewed in very different ways by a group of learners.

The ideas of continuity and interaction are the essence of constructivism. The student constructs meaning and revises understanding based on new information. Prior structures, which result from past experiences, are modified by the learner as these experiences come in contact with the present situation.

Exposure to different ideas frees individuals from dogmatic thinking and suggests new lines of inquiry. Subsequent discussion and debate improves the quality of association with others and the teacher becomes a student as he or she is exposed to the new perspectives of the class.

In 1937, the National Education Association's Educational Policies Commission listed ten imperatives, which included this statement: "(A)ll youth need to grow in their ability to think rationally, to express their thoughts clearly, and to read and listen with understanding" (Educational Policies Commission, 1937).

However, educational practice of the time did not generally reflect this view. By most measures, schools were inadequately preparing students for post-school life (Chamberlin, Chamberlin, Drought, and Scott, 1942; Smith and Tyler, 1942). As Aikin (1942) put it: "(I)t was easy for him to 'get his lessons', pass his courses. The result was that many...developed habits of laziness, carelessness, superficiality. These habits, becoming firmly established during adolescence, prevented the full development of powers" (p. 5).

By contrast, many of the schools initiated by progressive educators were very successful. This was shown to be the case by the famous Eight-Year Study (1932-1940). At the time, the majority of students entering high school either did not finish or did not continue on to college. However, the high school curriculum was almost entirely focused on college preparatory courses and hence, not serving the majority of secondary students. The leading educators of the time persuaded over 300 colleges and universities throughout the United States to accept students from 30 pre-selected schools based not on academic achievement but on the recommendation of the school administrator. Freed from the burden of teaching with a view to college acceptance, the schools were allowed to abandon their curricula and proceed in a much more student-oriented fashion.

Upon arrival at college, 1 475 students from these schools were closely matched on a number of factors with students from traditional high schools and monitored throughout their time at college. The students from the progressive schools were ultimately shown to be "...more academically successful, practically resourceful, and socially responsible" (Darling-Hammond, 1997, p. 10). In fact, the students who showed the greatest gains were from schools that differed the most from mainstream practice (Aikin, 1942). Despite the proof in favour of this approach to education, it failed to have a lasting impact on the educational business of the day. It was published in 1942 when the collective American mind was focused elsewhere. As Harold B. Alberty (1953) states, "(C)onsequently, it did not receive the attention it deserved. The impact upon the rank and file of secondary schools was very slight indeed. Teachers, by and large, went on assigning daily lessons from textbooks" (p. 287).

Progressive education virtually disappeared during the war years with the effect that, by 1950, even schools that had been studied and found to be successful had reverted to "fundamentals" (Redefer, 1950, p. 35). As one principal said," (T)he strong breeze of the Eight Year Study has passed and now we are getting back to fundamentals. Our students write fewer articles in English and social science but they are better spellers" (Redefer, 1950, p. 35). Wilford Aikin, the chairman and director of the study, upon reflecting on it some years later said:

> After the Eight-Year Study ended in 1942, each school was entirely on its own. It was easy to lose the spirit of cooperative adventure and to slip back into old, easy ways. Not many schools have the courage or strength to be different and stand alone (Akin, 1953, p. 12).

When given free rein and allowed to learn in a more divergent manner, students responded with a much greater involvement with the material. Student interest and engagement were notably higher than in traditional schools. This reflects the findings by contemporary researchers when they note that in schools which stress active learning methods, the students demonstrate "…significantly higher achievement as measured by the National Assessment of Educational Progress" (Rettig & Canady, 1996, p. 2). Further, Darling-Hammond and Falk (1997) observe that

> Teachers in these [successful] schools offer students challenging, interesting activities and rich materials for learning that foster thinking, creativity, and production. They make available a variety of pathways to learning that accommodate different intelligences and learning styles, they allow students to make choices and contribute to some of their learning experiences, and they use methods that engage students in hands-on learning. Their instruction focuses on reasoning and problem solving (p. 193).

Adherents of progressivism advocated a close relation between process and content. However, the pendulum has since swung back and forth between intellectual quality and "life adjustment education" which was viewed as practical preparation for real life (Perkins, 1992, p.9). The present contemporary effort to reform educational practice towards thoughtful learning follows on the heels of the back-to-basics movement of the 1970s. This movement did not succeed in raising student performances. In fact, "(Y)oungsters did not know what it seemed they should. Youngsters did not understand what they were learning. They could not solve problems with the knowledge they had gained" (Perkins, 1992, p. 10).

Some writers contend that current problems with schools stem from the fact that they have embraced progressive practices and have abandoned rote learning and memorization (Hirsch, 1996). In fact, studies have shown that a very large percentage of North American schools emphasize rote learning, drill, and memorization (Darling-Hammond, 1997). This brings to mind the quote from Alfie Kohn: "Back to basics? When did we ever leave?" (Kohn, 2004, p, 12).

Strategies that emphasize rote learning imply that:

> Learning for most students should be passive – teachers transmit knowledge to students who receive it and remember it mostly in the form in which it was transmitted. In the light of this, it is hardly surprising that the achievement test items on which…students most often showed relatively greater growth were those most suited to performance of rote procedures (McKnight et al, 1987, p. 81).

Transmission teaching in this manner is simpler than the alternative. As Darling-Hammond says, "(T)here is a sense of certainty and accomplishment when a lecture has been given, a list of facts covered, or a chapter finished, even if the result is little learning for student" (p.13). That this type of teaching is the norm is not surprising. Most teachers have not been prepared by their training to engineer situations in which students can evaluate their own learning. They teach as they have been taught: in a convergent, linear manner. According to Costa and Liebmann (1997):

> Teachers tend to carry forth to their teaching those strategies and content that they were taught. The result is a maintenance of old and familiar models: Because I dissected a frog in my college biology class, I therefore have my biology students dissect frogs. Because I learned to conjugate verbs in my French class, I therefore have my students conjugate verbs (p. 29).

Traditionally, conventional teaching has been driven by associative and behaviourist psychologies. First, learning must be a constructivist process, not knowledge absorption. Constructivism is based on the premise that learning is a search for meaning and that this meaning requires the understanding of the parts constituting the whole as well as the whole itself. The emphasis therefore is on concepts as opposed to isolated facts. In the latter instance, the "right" answer becomes someone else's meaning.

In a constructivist classroom, the role of the teacher becomes one of facilitator and to establish an environment that gives students the opportunity to pose problems and test hypotheses.

Second, it follows that learning is knowledge-dependent. Students use knowledge to construct new knowledge. This is a multi-layered concept and has been defined as having four main principles.

1. *Knowledge consists of past constructions.* We can only know the world through our logical framework, which transforms, organizes, and interprets our perceptions. In essence, cognitive development comes about through the same processes as biological development – through self-regulation or adaptation.
2. *Constructions come about through assimilation and accommodation.* Assimilation simply refers to the logical framework or scheme we use to interpret or organize information. When this assimilatory scheme is contradicted or found to be insufficient, we accommodate; that is, we develop a higher-level theory or logic to encompass the information.
3. *Learning is an organic process of invention, rather than a mechanical process of accumulation.* A constructivist takes the position that the learner must

have experiences with hypothesizing and predicting, manipulating objects, posing questions, researching answers, imagining and inventing in order for new constructions to be developed.

4. *Meaningful learning occurs through reflection and resolution of cognitive conflict and thus serves to negate earlier, incomplete levels of understanding.* Both contradiction and cognitive conflict are constructions of the learner (Fosnot, 1989, p. 19).

Third, learning is sensitive to the situation in which it takes place. In other words, learning occurs not by recording information, but by interpreting it. As Lauren Resnick (1989) says, "(W)e need instructional theories that place the learner's constructive mental activity at the heart of any instructional exchange that treats instruction as an intervention in an ongoing knowledge construction process" (p. 4). Of course, this is not meant to imply that the students are left to discover everything for themselves. Teachers guide and provide information to fuel the construction process and through so doing, must strive to understand not only the students' mental models but the assumptions they are making to support these models. Robert Glaser (1984) provided strong evidence that both reasoning and learning are knowledge-driven and that those who are knowledge-rich reason more profoundly. Hence, students must be encouraged to interpret and analyze information.

It appears possible that a conscious effort to change one's mode of thinking may result in physiological changes in the brain. Practice may enable individuals to become better divergent thinkers while effecting permanent changes in their brain make-up. This is suggested by Wolfe and Brandt (1998) when they state, "(T)he brain changes physiologically as a result of experience. The environment in which a brain operates determines to a large degree the functioning ability of that brain" (p. 61). A number of researchers concur (Diamond and Hopson, 1998; Fitzpatrick, 1995) and believe the brain constantly undergoes changes in response to its environment, a concept termed "neural plasticity". Kotulak (1996) compares the process to a banquet:

> The brain gobbles up the external environment through its sensory system and then reassembles the digested world in the form of trillions of connections which are constantly growing or dying, becoming stronger or weaker depending on the richness of the banquet (p. 4).

The implication for educators is that the classroom is far from a neutral place. However, the extent to which it contributes to brain development is in the hands of the teacher. With this in mind, a constructivist approach in which the student makes meaning through connections with new material and that which is already known provides an environment in which the brain can flourish. Divergent thought, by its very nature, is constructivist in that new connections are constantly being sought.

Traditional instructional theory assumes that knowledge and skill can be analyzed into component parts that function in the same way regardless of where they are used. Complexity is avoided in favour of teaching separate components that can be combined later. However, it is now widely held that this is not effective. First, human memory for isolated facts is very limited. Knowledge is retained only when embedded in some organizing structure. Alfie Kohn (1999) describes this when he states:

> Consider, then, a teacher who tells her students what a "ratio" is, expecting them to remember the definition. Now imagine a teacher who has first-graders figure out how many plastic links placed on one side of a balance are equivalent to one metal washer on the other side. Then, after discovering that the same number of links must be added again to balance an additional washer, the children come to make sense of the concept of "ratio" for themselves. Which approach do you suppose will lead to a deeper understanding? (p. 176)

Second, skills and knowledge are not independent of the contexts in which they are used. Contextualized practice of skills is required. Most of our thinking about education stems from an implicit assumption that skill and knowledge exist independently of the contexts in which they are acquired, that once a person learns something, he knows it no matter where he is. On this assumption, failure to use a particular piece of knowledge or skill is attributed to the individual's not recognizing it's relevance to the situation or not being motivated to apply it. This has been termed the "bewitchment of intelligence" (Wittgenstein, 1991, p. 109).

Lev Vygotsky, (1978) in the proposal of his theory of teaching and learning, believed that there existed an objective body of mathematical knowledge to be learned by students. Although children possess their own mathematical beliefs, it becomes the responsibility of the adult to influence the learner to move beyond his level of competence. He termed this the zone of proximal development which represented the gap between the known and unknown. He believed that higher-order thinking was achieved through the opportunity to solve problems. This speaks directly to the value of divergent thought.

In school and in most other instructional settings, people are expected to learn and perform individually. This stands in sharp contrast to most work-place settings as well as personal life in which mental activity is done in the context of some shared task. Also, most real-world mental activity involves the use of tools that expand people's mental power. These could include anything from calculators to computer programs. Contrast this with schools where people are generally expected to perform without any props or crutches. In other words, schools tend to promote thought independent of tools, a situation Resnick says "...seems to derive from a belief that mental capabilities are encapsulated within individual minds." These are all linked to schools' aspirations to teach competencies that are general rather than situation specific.

The end result of this type of education is that students have not been taught to evaluate their thinking and worse, to understand. According to the National Assessment of Educational Progress in 1992:

- Only 43 percent of seventeen year–old high school students could read and understand material such as that typically presented at the high school level, and only 7 percent could synthesize and learn from specialized reading materials.
- Fewer than half could evaluate the results of a scientific study, and just 10 percent could draw conclusions using detailed scientific knowledge.
- Only 36 percent could write well enough to communicate their ideas, and just 2 percent were able to write in an in-depth fashion.
- Only 7 percent were able to use basic algebra or solve math problems requiring more than one step (National Center for Education Statistics,

1994).

Convergent thinking is the norm in schools (de Bono, 1970; Polya, 1945; Wilson, Fernandez, & Hadaway, 1999; Ziv, 1983). Despite the fact that research shows that thoughtful learning must focus on understanding and must be learned contextually, the implementation eludes educators. Rote learning, by its very nature, cannot stress anything other than convergent thinking. Knowledge must be embedded in an organizing structure for it is in the interpretation of information that learning occurs. As Lloyd (1997) states, "(M)athematics is no longer a set of isolated skills to be accumulated until someday when you know enough to use them but rather a tool for describing and making meaning from the world around us everyday" (p. 96).

As this chapter has described, such concepts as reflective thought and contextually embedded learning have been widely embraced although perhaps not widely implemented. Despite the fact that the research strongly suggests that students are more engaged and derive more value from a constructivist approach in which they draw their own meaning, many schools continue to emphasize rote learning and a presentation of mathematics as content, a fragmented set of isolated skills. It is the teacher who by virtue of the classroom atmosphere, the willingness to embrace "teachable moments", and the ability to ask the open-ended questions which continue the curricular conversations, can seek out and highlight the necessary divergent thought.

The next chapter will provide current perspectives on learning in a school situation and the relationship to divergent thinking.

Chapter 4

Current Perspectives on Divergent Thought

It has been suggested that there exist two major shortfalls in educational achievement, that of fragile knowledge and poor thinking (Perkins, 1992). The former refers to the fact that students do not remember or understand much of what they have learned whereas the latter suggests that students do not reason well with what they do know. This is due, in part, to what has been termed a "Trivial Pursuit theory of learning" which emphasizes the accumulation of facts and routines (Perkins, 1992, p. 20). These views are shared by a number of other researchers (Boyer, 1983; Goodlad, 1984). Kurfman and Cassidy (1977) propose that

> ...learning only easily testable fact-finding skills will prove increasingly inadequate for life in the modern world. Much more than fact-finding skills – that is, higher level thought processes, useful knowledge, and clear values – are needed for students to function effectively (p. 112).

It is clear that both knowledge and thinking are the primary issues to be addressed. Bloom, Englehart, Furst, Hill, and Krathwohl proposed a Taxonomy of Educational Objectives in 1956 with the belief that information could be organized in a hierarchal manner from basic factual recall to higher order thinking. Different levels of learning exist in each domain, with higher levels considered closer to mastery of the subject matter. The highest levels are those of analysis, synthesis, and evaluation. These involve making inferences and generalizations, proposing alternative solutions, and making judgments.

Following is a chart (figure 15) showing objectives, defined and illustrated, for the cognitive domain (Lefrancois, 1994, p.359):

Figure 15: Cognitive Domain Objectives

Class of Objectives	Example
Knowledge	Who wrote *A Midsummer Night's Dream*?
Comprehension	What was the author trying to say?
Application	Given what you know about the authenticity of the first quarto and about weather conditions in England in the summer of 1594, when do you think the play was written?
Analysis	Find the most basic metaphors in Act I and explain their meaning.
Synthesis	Identify the four themes in *A Midsummer Night's Dream* and discuss how they contribute to the central action.
Evaluation	Do you agree with the statement that *A Midsummer Night's Dream* is Shakespeare's first undisputed masterpiece? Explain your answer.

As can be seen from this list, the objectives progress from lowest (the recall of factual information to highest (the formulation of value judgments). The evaluation of students tends to be based largely on the former, rather than "...on the basis of how much they understand, how cleverly they generalize and extrapolate, or how elegantly they formulate hypotheses and generate new concepts" (Lefrancois, 1994, p.359).

A chart of cognitive objectives would certainly have a place in mathematics. Following (figure 16) is an example of what this might look like:

Figure 16: Cognitive Domain Objectives as applied to Mathematics

Class of Objectives	Example
Knowledge	What is the answer to this equation? $2x + 5 = 11$
Comprehension	Explain what this equation means.
Application	Name two situations in which a knowledge of algebra could be used.
Analysis	Explain at least two other ways in which this equation could be solved.
Synthesis	What are some connections between this and other things you have learned in mathematics this year?
Evaluation	Explain how you would teach this concept to someone

Lauren Resnick of the University of Pittsburgh Learning, Research and Development Center emphatically believes that higher-order thinking is largely non-existent (Perkins, 1992, p.30). According to Resnick, if students are not taught to think as they are acquiring knowledge, there is little point in knowledge acquisition in the first place. Lipman (1991) concurs and suggests that the higher order skills of analysis, synthesis, and evaluation could be renamed critical thinking, creative thinking, and judgment. He also points out that these skills should not be viewed as an add-on to solid knowledge skills although they frequently are, nor are they necessarily hierarchical and context-free, as suggested by Bloom et al (1956 p. 49).

In schools, mathematics tends to be viewed as a body of computational rules and procedures, with proficiency as a major goal of instruction (Stigler & Hiebert). In fact, computation forms the majority of students' mathematical experiences throughout school (Resnick & Ford, 1981). As Potter (2006) says, "(F)or most of my time at primary school, doing mathematics was the same as doing sums" (p. 27).

Lefrancois (1994) has noted that:

> … schools tend to evaluate students on the basis of how many textbook- and teacher-presented facts they remember rather than on the basis of how much they understand, how cleverly they generalize and extrapolate, or how elegantly they formulate hypotheses and generate new concepts (p. 359).

Research supports this viewpoint. Fleming and Chambers (1983), in their analysis of nearly 9 000 questions from high school tests, found that approximately 80% dealt with only knowledge of facts.

It is through this narrow focus that an emphasis on linear thinking is maintained. It is widely held that mathematics, more so than any other subject, has the greatest applicability outside of school (Evans, 1998). However, it also remains the subject most shaped by time spent in the classroom. The task then becomes rethinking that which takes place in the classroom to provide a different context for learning and problem solving. Wirtz (1985) suggests that, in the traditional mathematics curriculum, understanding "...is telling about an idea often enough so that all children will eventually understand it" (p. 97). The most common form of teaching in schools has been termed "recitation" (Hoetker & Ahlbrand, 1969; Tharp & Gallimore, 1988). This involves the teacher leading

> ...the class of students through the lesson material by asking question that can be answered with brief responses, often one word. The teacher acknowledges and evaluates each response, usually as right or wrong, and asks the next question (National Research Council, 2001, p. 48.).

It is widely held that attitudes and success in mathematics are closely linked. The Ontario Curriculum Grades 1 – 8: Mathematics (1997) states, "...students' attitudes have a significant effect on how they approach problem solving and how well they succeed in mathematics" (p. 73).

Traditional methods of teaching mathematics can shape the attitudes which learners carry with them into later life. This can be particularly profound in cases where the students eventually become teachers themselves. In a study of 53 preservice teachers, Harper and Daane (1998) found a high degree of math anxiety, fostered in most cases by negative experiences in schools. Without exception, they tended to "...view mathematics as rule-bound procedures and an arbitrary collection of facts" (p. 29).

It has been suggested that the attitudes leading to mathematics avoidance begin in the elementary school classroom (Hadfield & Lillibridge, 1991; Hilton, 1980). It is here that students recount the sources of their anxiety including the lack of mastery on the part of the teacher resulting in an authoritarian teaching style, as well as the prevalence of rote work, emphasis on memorization, and unrealistic, unengaging problems. There are a number of other studies which support these findings and point to an emphasis on drill and practice, getting a single, correct answer using one approach, and memorizing formulas as major factors in increasing mathematics anxiety (Frank, 1990; Reyes, 1984; Tobias & Weissbrod, 1980; Widmer & Chavez, 1982). It is worth noting that these factors are, by their nature, convergent. Potter (2006) puts it very well when he says, "(J)ust like any environment, the conditions of the mathematics classroom affect its inhabitants. The fear of getting an answer wrong means that for most, the best chance of survival is silence " (p. 12). Simply, most students are "...just too thankful to have an answer, any answer, to even dare to investigate further" (Hart, 1981, p. 12).

As Sternberg (1999) states:

> Although being successful often involves making mistakes along the way, schools are often unforgiving of mistakes. Errors on schoolwork are often marked with a large and pronounced X. When a student responds to a question with an incorrect answer, some teachers pounce on the student for not having read or understood the material, which results in classmates snickering. In hundreds of ways and thousands of instances over the course of a school career, children learn that it is not all right to make mistakes. The result is that they become afraid to risk the independent and the sometimes flawed thinking that leads to creativity (p. 102).

The ideal situation, acoording to Dweck (as quoted in Saphier, 2005, p. 89) would be one in which "(S)tudents do not interpret errors and difficulty as confirmations of their ineptitude".

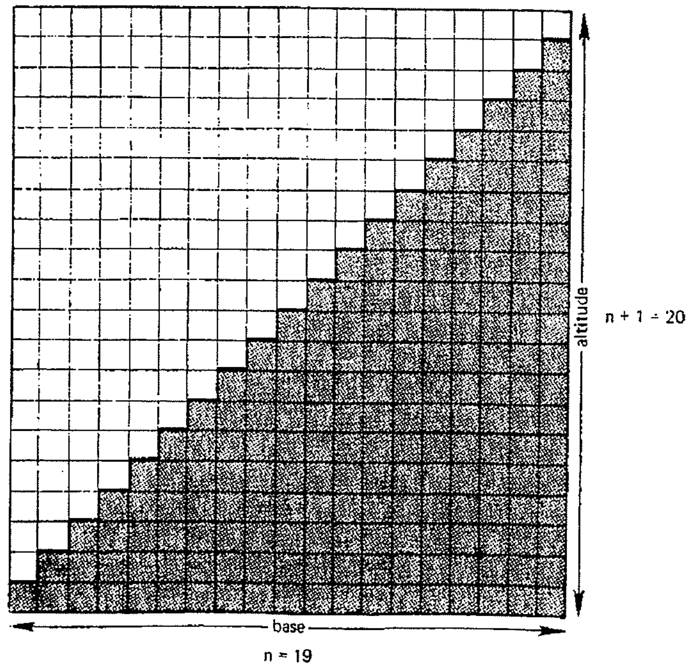
Linked to changing attitudes is a significant amount of evidence that suggests developmental declines in academic achievement occur through grades 6 to 8 (Eccles, Midgley, & Adler, 1984; Eccles et al, 1993). A number of factors have been cited as possible causes of this trend, including characteristics of the classroom environment, decrease in student choice, and teachers' beliefs about their personal effectiveness (Eccles et al, 1984; Midgley, Feldlaufer, & Eccles, 1989).

The gestalt approach that function and the interrelatedness of its parts are defined by the structure of the whole seems particularly appropriate for problem solving. This is well illustrated by Wertheimer (1945) in his Carpenter's Apprentice question:

> A staircase is being built along the wall of a new house. It has 19 steps. The side away from the wall is to be faced with square panels of the size of the ends of the steps. The carpenter tells his apprentice to fetch them from the shop. The apprentice asks, 'How many shall I bring?' 'Find out for yourself', rejoins the carpenter. The apprentice starts counting: 1 + 2 = 3; + 3 = 6; + 4 = 10; + 5 =... The carpenter laughs. 'Why don't you think? Must you count them out, one by one? What if the staircase was not along the wall and required the same number of wooden panels on both sides? Would it help if I suggested thinking of the patterns of the two sides cut out of paper?' (p. 108-116)

What he suggests is a reframing of the perspective of the problem to look as follows: (figure 17)

Figure 17: Visualization of the Carpenter's Apprentice Question



Such a solution implies a much deeper understanding of the problem and is reminiscent of Gauss's sum of series centuries earlier. This intuitive approach of Wertheimer's suggests that algorithms must be learned in the context of the structures underlying them. As Resnick (1981) says, "...when the reasons behind algorithms are clearly understood, then the thinker or problem solver is in a better position to choose the particular algorithm that is most appropriate to the problem at hand" (p. 138).

While the ability of students to think in a divergent manner is affected by a number of factors, it is the concept of the open-ended question that appears to hold the most promise. Naumann (1980) believes that ultimately, optimal cognition can only be accessed through such questions. To do so, the teacher must abandon the "…know-the-answer-beforehand questions and ask questions for which there are no 'right' answers" (Cecil, 1995, p. 139). The result is a broad range of responses. As Saphier (2005) states: "(Q)uestions are important; they regulate the level of thinking" (p. 91).

Certainly, the research demonstrates a very consistent and disturbing pattern with regards to the cognitive level of questions being asked in classrooms. Suydam (1985) found that 80 percent of questions asked by mathematics teachers were at a very low level. Other research has shown that there are approximately five times as many interactions at low cognitive levels than at high levels (Fennema & Peterson, 1986; Hart, 1989).

Torrance (1971) suggested teachers ask provocative questions. Generally speaking, stated problems are not open-ended and educators need to "…stress the importance of asking…questions that require discussion instead of yes-or-no answers" (Meisner, 1999). Other researchers concur with this view. Chapin, O'Connor and Anderson (2003) point out that in most cases, questioning by the teacher is quizzing to get at answers that are already known.

Teachers, textbooks, and in the bigger picture, curricula, must include more questions of an open-ended nature. The students are then encouraged to consider things from new and different perspectives. An important aspect of this is assisting students to become aware of their individual biases, or promoting "accurate observation" as Mills and Dean refer to it (1960, p. 11). The student is much more apt to think in a lateral fashion if he or she is not constrained in the way in which the problem is viewed. In fact, it is probably this feature more than any other that separates the convergent from the divergent thinker. With practice, students become able to identify the point at which their thinking is being guided by their preconceived notions. This proves to be an excellent starting point for proposing alternative solutions.

A number of researchers have observed that educators tend to ask children short and simple easy-to-answer questions (Blank, 1975; Bruner, 1966; Leach, 1972; Schlicter, 1983). The result is that students become "responders rather than inventors" (Samples, 1975). As well, if teachers avoid yes/no questions in favour of ones that focus on how/why/if, the resultant delay in closure enables students to move towards divergent thinking and away from convergent thinking.

Research supports the notion that children must have the opportunity to pose problems in the classroom (Driver, Asoko, Leach, Mortimer, & Scott, 1994; Hill, 1996). Through so doing, they become active participants in the process. Paulo Freire (1970) made a sharp distinction between problem posing and traditional teacher-dominated education. While the former had a basis in "...creativity and could stimulate true reflection upon reality" (p. 71) and was based on probing, critical inquiry, the latter he termed "banking education". In Freire's view, this traditional approach served to separate the learner from both content and process. Under such a paradigm, the information is simply transmitted from teacher to pupil, an exchange that "...transforms students into receiving objects" (p. 76).

In distinguishing between banking education and problem posing, Freire (1970) stated:

> Whereas banking education...inhibits creative power, problem-posing education involves a constant unveiling of reality. Students, as they are increasingly posed with problems relating to themselves in the world and with the world, will feel increasingly challenged and obliged to respond to that challenge (p. 68).

To encourage problem posing, it falls to the teacher to create an atmosphere of freedom and risk-taking in the classroom, in other words, a climate of curiosity.

Research supports this as a major factor in the problem solving abilities of students. Appleton (1995) reports a high degree of correlation between social setting in the classroom and success in divergent problem solving. Unfortunately, it appears that such classroom climates are not the norm. According to Cecil (1995):

> The climate in many North American classrooms inhibits children from asking and answering questions. In such classrooms, silence and order are the most important features and strict adherence to teacher-imposed rules is dutifully enforced. The atmosphere can be described as chilly, and the brains of learners have great difficulty warming up (p. 19).

It would seem such classrooms do not reflect the essence of the comment that "...energy and curiosity make good deskmates" (Ashton-Warner, 1974, p. 117).

Dillon (1982) held the finding or posing of problems to be the quintessential creative endeavour. More than that however is the possibility that through the posing of problems, students become agents of social reconstruction. According to Lewis et al (1998)

> Whether the found problem is the presence of a hole in the ozone, or that people who smoke are prone to cancer, those keen enough to call our attention to them distinguish themselves by their astuteness. They help set community agendas that lead to discoveries and inventions that help make the world better (p. 9).

It has been suggested that problem posing denotes a high degree of creativity. According to Henle (1962), "...in particular cases, the important creative task may be precisely to pose a question rather than to answer one" (p.44). In fact, perhaps the creativity lies in the ability to reformulate a problem to allow it to be viewed in a new light. Echoing this, Csikszentmihalyi (1994) says, "(M)any creative individuals have pointed out...that the formulation of a problem is more important than its solution and that real advances occur...when new questions are asked or old problems are viewed from a different angle" (p.138). However, he goes on to point out that

> When measuring thinking processes, psychologists usually rely on problem solution, rather than problem formulation, as an index of creativity. They thus fail to deal with one of the most interesting

> characteristics of the creative process, namely the ability to define the nature of the problem (Csikszentmihalyi, 1994, p. 138).

Other research has indicated similar findings. When problem posing and exploration is encouraged, student creativity and learning are enriched (Hill, 1998).

Clearly, it is the teacher who determines the extent to which higher-level thinking is present in the classroom. Bellanca (1985) concurs when he states

> We hear much talk about students acquiring higher—level thinking skills. We know this occurs most successfully when a teacher uses higher-order teaching skills. For instance, asking students questions that demand complex responses – not just the simple recall of information – requires sophisticated teaching skills. Teachers must draw out and extend responses (p. 15).

Typically, classroom interactions tend not to support this behaviour. What does present itself is that which Dewey referred to as "collateral learning". Children quickly pick up a set of expectations and outcomes toward which they are expected to strive. These would include a view that teachers possess the correct answer which students are supposed to figure out. This has been termed the Quiz Show Model (Roby, 1981). As well, there is an expectation that students' responses should be short and as close to the right answer as possible (Barell, 1985). Stigler and Hiebert (1999) point out that "(T)he nature and tone of teachers' questions often give away the answer" (p. 45). It is not surprising that students experience difficulties with complex thinking.

Judith Sowder of the National Council of Teachers of Mathematics recounts the story of a colleague who is asked to look for an example of the "hidden" curriculum that she teaches without being aware of it. Upon reflection, she realizes that her students never ask "why" questions in mathematics class. She asks one of the better students why this is so and is told that there are no "why" questions to ask – it is just a matter of applying rules (NCTM, 2002 yearbook, p. 1).

Theodore Sizer (1984) described his impressions of teacher/student dialogue:

> The mode is a one-sentence or two-sentence exchange…Dialogue is strikingly absent, and as a result the opportunity of teachers to challenge students' ideas in a systematic and logical way is limited. One must infer that careful probing of students' thinking is not a high priority (p. 82).

There is evidence to suggest that cooperation among students, through working in small groups, aids divergent thinking and problem solving abilities. (Artzt & Armour-Thomas, 1992; Curcio & Artzt, 1992; Howe, 1996;

Stacey & Gooding, 1997) Students are given the opportunity to discuss problem-solving strategies and resolve misconceptions. The small group aspect appears to be less inhibiting for students to ask questions and explore their thoughts. Further, it has been suggested that the cognitive and metacognitive behaviours that occur among members of problem-solving groups closely mirror those of expert solvers working alone (Schoenfeld, 1987). The most recent evidence (Laughlin, 2006) suggests that people working in groups perform better than an equal number of individuals working alone, even in cases where the solo people are at the top of their class academically. However, despite the evidence which seems to indicate that students benefit from working in small groups, not everyone agrees. Von Glaserfeld (1995) holds that the essence of problem solving, that of seeing a problem as one's own, is in opposition to working in a group. As he says, "(T)o solve a problem intelligently…one must see it as an obstacle that obstructs one's progress towards a goal" (p. 14). Lewis et al (1998) are more direct: "(S)tudents must arrive at their own problem solving methods and strategies: they cannot rely on a communal strategy" (p.6).

Inhelder and Piaget (1958) believed that children were unable to think in a divergent manner until at least a junior high school level. Their correlation of ages and stages is well known although other research indicates that this may not be the case and that high-level questioning from teachers results in significant increases in scores on tests that include divergent thinking problems (Turner & Durrett, 1975). Clearly, teachers are an important aspect of the cultivation of divergent thinking. However, a very small percentage of teachers' questions require students to use this type of thinking (Cliatt, Shaw, & Sherwood, 1980). In fact, Boyer (1983) found that less than 1% of teacher questions encouraged students to respond in a deeper way. Others support these findings as well. Goodlad (1984) found that, on average, only about 5% of daily class time was spent on discussion. Not surprisingly, it would appear there is a correlation between these findings and the lack of higher order thinking.

Mills and Dean (1960) stress the importance of the classroom climate, in that the proper atmosphere can be very conducive to students' problem solving efforts. This is supported by other researchers as well:

> The development of a community of learners, especially in the mathematics classroom, allows students and teachers to work together towards developing understanding. In a community of learners, the teacher is no longer the sole source of expertise. With the establishment of a community, students are better able to engage in productive mathematical exploration and discovery (Ontario Ministry

of Education, 2004, p. 16).

As Alfie Kohn (1998) states, "(S)omeone who facilitates students' learning welcomes mistakes – first, because they are invaluable clues as to how the student is thinking, and second, because to do so creates a climate of safety that ultimately promotes more successful learning" (p. 213).

Such conditions result largely from the teacher and the manner in which problems are presented. To encourage problem solving, it falls to the teacher to create in the classroom an atmosphere of freedom and risk-taking, in other words, a climate of creativity. One researcher has termed such classrooms "high in challenge but low in threat" (Wolfe, 1999, p. 107). Other findings support this as a major factor in the solving abilities of students. Appleton (1995) reports a high degree of correlation between social setting in the classroom and the amount of divergent thinking that takes place. However, reaching this stage is not an easy matter for some teachers. Due to the relative positions of teacher and student and the fact that the teacher is managing a number of diverse personalities, there is a feeling that the environment is one in which control is a necessary factor. According to Lewis et al (1998), this very control may serve to neutralize creativity. The classroom environment has been cited by a number of researchers as a major determining factor in the motivation of students (Eccles et al, 1984; Midgley, Feldlaufer, & Eccles, 1989).

Interestingly, it has also been suggested that reinforcement from teacher to child should be kept to a minimum in that this practice encourages a focus on the reward as opposed to the task.

In addition to open-ended questions, several researchers have postulated ways by which divergent thinking can be encouraged in the classroom. Naumann (1980) advocates a warm atmosphere based on mutual trust and respect. Ziv (1983) found that a humorous atmosphere in the classroom significantly increased the divergent thinking ability of students. It has been suggested that since humour itself is basically creative, people exhibiting a sense of humour are able to facilitate further creativity (Torrance, 1979). This is supported by Moody (1978) who views humour as an important creative characteristic in that it can produce new and original remarks, stories, and so forth.

Dyer (1997) makes a direct link between divergent thought and humour when he states, "(T)he process of divergent thinking that is necessary to create humor frequently leads to imaginative, inventive, and original solutions or alternatives. Understanding humor is a cognitive process. It increases capacities for lateral thinking" (p. 214).

In *The Three Domains of Creativity* (1971), Arthur Koestler identifies artistic originality (the "ah" reaction), scientific discovery (the "aha" reaction), and comic inspiration (the "haha" reaction) and notes that all three are the combination of previously unrelated structures. Comic inspiration stems from the interaction of two mutually exclusive associative contexts. The result is humour which is "...an essential ingredient of healthy conceptualization" (Adams, 1986, p. 58). Guilford (1977) refers to such restructuring as transformation and believes this change in information, which can take a number of forms, results in new ways of viewing situations. An example of this as a semantic transformation would be puns such as "(C)ollege bred means a four-year loaf made from the old man's dough."

As noted earlier, according to Hudson (1966), certain subject matter may not lend itself to divergent thinking. For example, he found that there was a distinct lack of this type of thinking among science students. It has been suggested that their superior convergent thinking could be due to the fact that students with this inclination are attracted to study science (Rosenthal et al, p. 186). Also, and possibly more likely, science-teaching methods may not promote the use of more creative modes of thought. Certainly, the traditional emphasis in science teaching is one of conveying a large body of facts. The scientific method includes hypothesis-formulation and while it may remain important, ultimately it may be de-emphasized for the sake of teaching content. As Janus (1989) puts it, "...'knowing science' is sometimes thought to be accomplished when students can name, explain, and define terms, regurgitate scientific facts and concepts, and do a number of other convergent, or single-answer, mental activities" (p. 21).

On a practical note, there are certain indicators of improved student thinking. While competency might be demonstrated in a single test, what should be sought is effectiveness, or sustained performance in a variety of situations that employ the use of different problem solving strategies. Feuerstein (1980) noted ten characteristics of such growth:

1. Perseverance – Since students have a number of strategies available for use, they become more likely to reject an unproductive one in favour of an alternate one. In addition, studies indicate that, as time passes, the ideas being generated tend to become more useful and appropriate. Parnes (1962) found that, on average, the second half of ideas produced contained 78 percent more valuable ideas than the first half. Perseverance is a very important trait among people who are successful at mathematics. A recent study indicated that 55 out of 77 mathematicians interviewed reported struggle as an integral part of their work (Burton, 2004). According to one mathematician, "(T)he natural condition of doing mathematics research is to be stuck, most of the time, on most of the things you are doing" (Burton, 2004, p. 59).

2. Decreased Impulsiveness - Students tend to spend more time on assessing

the problem, determining the appropriate course of action, and reflecting on their answers.

3. Flexible Thinking – Students are able to consider alternate points of view and are more able to consider the merits of same.

4. Metacognition – This is a vital step in that students who are aware of their own thinking are much more likely to determine the point at which their thinking leads them into unproductive pathways.

5. Careful Review – Students become more concerned with clarity and checking to ensure that their answers are appropriate.

6. Problem Posing – This is one of the hallmarks of divergent thinking. Students are able to recognize discrepancies in their environment and re-frame the problem in a variety of ways.

7. Use of Past Knowledge and Experiences – Students are capable of extracting meaning from past experiences and applying it to new situations.

8. Transference Beyond the Learning Situation – The ultimate goal of divergent thinking is to enable the student to apply it in real-life situations and areas beyond which it was taught.

9. Precise Language – Students' speech becomes more descriptive and concise and as a result, they tend to voluntarily provide support for ideas.

10. Enjoyment of Problem Solving - Thinking is not perceived as hard work.

The writer has attempted to actively cultivate divergent thought in his practice with students. The topic is introduced over time with a number of lateral thinking problems. These are presented in concert with a warm, accepting classroom atmosphere in which all responses are valued. As students become more adept at accessing their divergent thought, they are gently guided to a metacognitive stage, wherein they are encouraged to examine the thought behind their decisions, or more importantly, their inability to reach solutions. Open-ended questions at this point might include the following: "What is it about the situation presented in the question that makes it difficult to move ahead?" "How would the situation have to be different in order for you to find a solution?" "Are any of these situations possible?" With guidance, students are able to discern the point at which their thinking is breaking down. This knowledge enables them to continue the conversation and not be hampered by self-imposed barriers.

The results have been striking. As their divergent thinking skills develop, students become more productive mathematicians. They are much more inclined to spend the time needed to solve traditional problems. As one child noted, "(I) used to think that information was missing and so I wouldn't be able to solve problems. This has shown me that all the information I need is there. I just need to look at it in a different way" (personal communication).

Equally important also is the manner in which students have been able to apply their divergent thinking skills to other curricular areas. This is a natural and welcome extension of this ability and is made easier in cases where different subject teachers share a similar vision.

While it is clear that changes in teacher practice would facilitate a more widespread use of divergent thinking in the classroom, this is only part of the solution. On the one hand, it is apparent that a deep grasp of content knowledge may provide the educational practitioner the latitude to teach in a divergent manner. As well, the environment in which the student finds himself plays a major role in fostering creativity. However, it is a truism that people teach the way they themselves were taught. As Martin Haberman (2002) points out, teaching is widely believed to consist of certain acts, so basic that not to engage in them would be considered deviant. The acts constituting the "core" of teaching would be as follows:

- giving information
- reviewing tests
- assigning homework
- asking questions
- monitoring seatwork
- marking papers
- reviewing assignments
- giving directions
- reviewing homework
- settling disputes
- marking assignments
- giving tests
- punishing non-compliance
- giving grades

(Haberman, 2002)

Note that none of these involve higher level thinking or indeed, very little thinking at all. They are reflective of the non-constructivist approach assumed by a great many teachers and reinforce the notion of teacher as dispenser of knowledge as opposed to teacher as learner. Further, according to Haberman (2002) these acts tie in with four syllogisms that underpin teaching. They are:

- Teaching is what teachers do. Learning is what students do. Therefore, students and teachers are engaged in different activities.
- Teachers are in charge and responsible. Students are those who still need to develop appropriate behaviour.
- Students represent a wide range of individual differences. Therefore, ranking of some sort is inevitable; some students will end up at the bottom of the class while others will finish at the top.
- Basic skills are a prerequisite for learning and living. Students are not necessarily interested in basic skills. Therefore, directive pedagogy must be used to ensure that youngsters are compelled to learn their basic skills (p. 3).

Since people teach in the manner they were taught, part of this mindset has evolved from their own school days. However, the teacher training which they have undergone is by no means blameless and in a sense, more culpable given the mission of universities and colleges. University programs tend to be bastions of convergency. The very nature of classes, that of lecturing by the teacher, results in a transmission-style of content delivery. Frequently, due to class size or the impersonal nature of the situation, questions and dialogue between teacher and student is not possible. In short, students are not placed in a position where they are able to construct their own meaning. Many lecturers would not see this as a drawback. The transmission style of teaching allows them to cover a greater amount of material, without concern for student engagement or learning. The situation is further exacerbated by the placement of pre-service teachers with established teachers who themselves teach this way. It is little wonder that the teacher who can enable the students to construct meaning and has the background to foster divergent thinking is the exception rather than the rule.

It is possible that a major impediment to a wide-scale implementation of divergent thinking in schools is that of the traditional method of assessment. Numerous studies have found that assigning letter or number grades to student work has the effect of decreasing interest in learning. As Kohn (2004) states "...the more people are rewarded for doing something, the more they tend to lose interest in whatever they had to do to get the reward" (p. 75). In other words, there appears to be an inverse relationship between an orientation directed towards grades and one concerned with learning (Beck, Rorrer-Woody, & Pierce, 1991; Milton, Pollio, & Eison, 1986).

Even more concerning are the findings that grades tend to lessen the likelihood that students will choose challenging tasks (Harter, 1978; Milton, Pollio, & Eison, 1986). This is a response to an educational culture that highly values correct answers and frequently sees marks as the only benchmark of learning. One must question if creative, divergent thought can flourish in such an environment.

Other researchers have drawn the conclusion that students' loss of interest in what they are learning reduces the quality and depth of their thinking (Butler, 1987; Butler & Nisan, 1986). In fact, numerical grades appear to result in less creative students and the greater the creativity required, the worse the performance of students who are evaluated with a grade (Butler, 1987). Consistently, highest achievement occurs when students are given formative comments instead of numerical, summative grades (Black & Wiliam, 1998; Butler & Nisan, 1986). Summative assessment provides right/wrong, yes/no feedback which is inconsistent with divergent thought. Formative assessment, on the other hand, is aligned with the concept of divergency in that it focuses more on the work done to that point, with suggestions for future guidance. As Wiliam (2006) states when referring to formative assessment, "...we found that a focus by teacher on assessment *for* learning, as opposed to assessment *of* learning, produced substantial increases in student achievement – typically doubling the rate of learning" (p. 4). For schools to truly embrace a divergent perspective, perhaps it is time to reconsider the manner by which marks are given.

The quest remains in mathematics, and other areas, to make students into better problem solvers. Most problems do not fit into a mold to which a linear step-by-step strategy can be applied. Success in this area using such a model has not been forthcoming. Further, the types of problems that will be faced by students in their lifetimes will likely not be of a sort that can be addressed with linear thinking. Global problems today, including such diverse situations as overpopulation, climate change, and energy issues, do not lend themselves to linear thinking. These demand a mode of thought which seeks alternatives and possibilities. Divergent thought is such a tool.

As this chapter has attempted to show, the mode of convergent thought is one that is perpetuated by schools through a number of factors including the classroom climate, the type of questions asked, and an emphasis on rote learning and the accumulation of facts. Since the raison d'etre of schools is the transmission of various curricula, it is reasonable to assume that such curricula do not reflect divergent thought to any appreciable degree. Were they to be divergent in their approach, one would expect that they would reflect what Csikszentmihalyi, Rathunde and Whalen term "optimal experiences" (1993, p. 14). Such powerful opportunities for new learning occur in response to "authentic questions" (Nystrand & Gamoran, 1991) and encourage students to extend beyond previous learning into new territory.

The next chapter will explore the principles of divergent thinking as they apply to curriculum.

Chapter 5

The Principles of Divergent Thinking in Curriculum

Curriculum has been defined as "...the collective story we tell our children about our past, our present, and our future" (Grumet, 1981, p. 115). This puts the focus on the learner and "...acknowledges the student's search for meaning as an interactive and reflective process undertaken in a social milieu" (Graham, 1992, p. 27).

Before any curriculum can be analyzed as an instructional program, some points of curriculum construction should be noted in order to provide the reader with a background lens through which it may be viewed.

One model that must be considered is that proposed by Ralph Tyler. It has been termed ..."the major influence on curriculum thought" (Posner, 1992, p. 13). Throughout his work, Basic Principles of Curriculum and Instruction, Tyler continually reinforced the notion that any evaluation is a cyclical process in that such programs demand constant reformulation and reappraisal. As a basis for so doing, the Tylerian model suggests four areas to be of paramount importance. These were articulated through four questions which defined the purpose, content, organization, and evaluation of a curriculum:

- What are the essential questions of the curriculum and the identifiable outcomes?
- What is the knowledge to be imparted to the learner and why is it important?
- Through what means will knowledge be imparted?
- What are the indications that learning has occurred? (Tyler, 1949)

In terms of purpose, it must be remembered that above all, any curriculum is simply a means to an end. As Tyler stated, "...if we are to study an educational program systematically and intelligently we must first be sure as to the educational objectives aimed at" (Tyler, 1949, p. 3). Identifying the objectives enables one to isolate and frame the essential questions and develop outcomes that reflect these.

Reconceptualism is a school of thought which proposes that the current curricula generally do a less than adequate job in terms of dealing with issues in a vital manner. Proponents argue that educators tightly control teaching and learning and lack critical reflection. As a result, problems may or may not be recognized but no solutions will be forthcoming.

Reconceptualists hold that the Tylerian model and others like it reinforce the notion that school problems can be dealt with in a "tool-box" manner. Once the problem is identified, one needs only to alter the curriculum in order to fix it. Student behaviours either support or refute whatever particular hypotheses have been put forth. The students do not participate in the planning or implementation of their education. In effect, they assume the role of the object of the study. However, this is considered to be a very narrow approach and one that fails to address the larger questions. The perspective needs to be broader and must include questions geared to understanding the discipline itself as well as links to other disciplines.

Acceptance of divergent thinking from a curricular perspective would require a major paradigm shift. Teaching and learning would have to be viewed in a post-modern point of view. As Slattery (1995) points out, the shift to a postmodern way of thinking involves the examination of "...some very sacred beliefs and structures that have been firmly entrenched in human consciousness for at least the past five hundred years" (p. 17). Further, it implies change without a return to a previous pre-modern existence. New developments must reflect the existing cultural context in which they have arisen. This point is made by Applebee (1996) when he addresses the issue of quality and how this is, by necessity, a function of the larger traditions in which it finds itself.

John Dewey was a strong advocate of educators being in touch with the needs of society. He believed that curricula should not be a collection of activities resulting in pre-determined ends (Dewey, 1934). Instead, Dewey felt it to be a continuous process of construction and reconstruction, involving an on-going reflection of one's experience.

This view of society-as-constantly-evolving leads to curricula that are both meaningful and valuable in the long term. Doll (1989) puts this in biological terms and indicates that continual adaptation and change are inevitable although meaningful is subjective in that the individual educator ultimately determines it. This is reminiscent of curriculum as mapping in that it becomes "...a journey involving increasingly wider ranges in modes of thinking" (MacNeill, 1999, p. 141). Along these lines, Elliot Eisner (1997) notes the importance of "...acknowledging the variety of ways through which our experience is coded" (p. 4).

Clearly, the teaching of divergent thinking as a means to enhance problem solving is very post-modernist in its approach. Slattery (1995) supports this when he characterizes postmodernism as a paradigm shift in which learners construct their own meaning from their relationships to others and knowledge. For this to be the case, material must be presented in a manner that promotes respect and encourages synthesis. This is termed Socratic dialogue. Currently, a great deal of mathematics instruction would have to be considered non-Socratic dialogue. Since the traditional emphasis has been on the correct answer with less regard for process, transmission of knowledge becomes what MacNeill (1999) has called "a closed map". A postmodern curriculum would be one characterized by student-centered learning. Doll (1989) believes that change results, not from the student being fed information but rather from the student being given the latitude to develop and organize his or her own program. This is reinforced by Usher and Edwards (1994): "(I)t is impossible to be a teacher without also being a learner, that in order to be a teacher, it is first necessary to abandon the position of the 'one who knows'" (p. 80).

Changes in perspective in terms of mathematics or any subject area would be in keeping with the idea of a responsive curriculum, one that is constantly evolving and adapting both to current research and demonstrated needs of the learner. Toepfer (1997) in his chronology of curriculum at a middle school level, points out that, by 1900, only 20% of students completed high school. One of the goals of the early curricular split was to retain students longer. This was taken a step further by the Committee on the Equal Division of the Twelve Years in the Public Schools when they recommended dividing the six-year high school tenure into two equal units. This was a beginning and a positive response to a problem at the time. However, the first three years or "junior high school" were seen as a diminutive version of high school.

Specific learning needs of students at that age were only beginning to be recognized through the efforts of educators such as G. Stanley Hall.

Several educators including J. Baker, the chairman of the Committee on the Equal Division, eventually addressed the gap that had been created between junior and senior high schools. It was apparent that something akin to a "bridge" was needed. These actions all illustrate a similar point: the curriculum was in a state of responsive flux and was constantly being modified to better serve the needs of the learners. Junior high school eventually received a unique curriculum and educators began to view the role of these grades as one of exploration (Briggs, 1920; Glass, 1923; Koos, 1927). Changes in the curriculum continue today as further research is done on students' learning needs.

The chronology shows the ever-changing nature of the curriculum. Above all, the curriculum should be a responsive tool, constantly being re-designed to best fit the needs of the learners. In this sense, it represents the best of teaching.

Teaching should constantly raise two questions: Are students learning? and What can be done to enhance that learning? As its development shows through the years, curricula have been designed and re-designed to reflect the current best response in the nature of learning and the knowledge of the learner. It is a relevant device that is responsive to change not necessarily curricular in nature.

It is possible that certain parts of any mathematics curriculum are, by their nature, more suited to the application of divergent thought. An example of this might be statistics and probability. Throughout this strand, students are asked to analyze data and make predictions. This presents an excellent opportunity for the teacher to elicit all manner of unusual, but plausible, responses. The probability of each can be examined with a discussion as to why certain ones are more likely than others.

Ultimately, regardless of the theory by which it is conceptualized, it could be argued that any curriculum by virtue of its stated goals, observable outcomes, and separation of disciplines and topics, is reductionist and consequently, non-divergent in nature. This leads to episodic and compartmentalized thinking on the part of students. The organization of knowledge into discrete components was originally a way to systematize the information at hand. However, the results neglect "...those who prefer interdependent, interpersonal, contextual, and synthetic thinking" (Costa & Liebmann, 1997, p. 23).

Further, "(T)he reductionist search seldom leads to a satisfying end. When we study the individual parts and try to understand the curriculum through the disciplines, we inevitably get lost in a meaningless world lacking quality, beauty, and interconnectedness" (Costa & Liebmann, 1997, p. 30). This sentiment is echoed by Costa (1999) when he says

> From an early age, employing a curriculum of fragmentation, competition, and reactiveness, we are trained to believe that deep learning means figuring out the truth rather than developing capacities for effective and thoughtful action. We are taught to value certainty rather than doubt, to give answers rather than to inquire, to know which choice is correct rather than to explore alternatives (p. 33).

If a curriculum is to be considered to embrace the principles of divergent thought, it clearly must be one in which the various disciplines are integrated. Ideally, the disciplines would not even be visible as distinct, separate entities. The concept of a thorough integration of subjects is not a new one. At the inception of the Eight Year Study, Wilford Aikin (1932) advocated "... a more coherent development of fields of study" (p. 432). He continues in a much more direct fashion:

> There should be less emphasis on subjects and more on continuous, unified sequence of subject matter. Continuous courses in the sciences and social sciences would take the place of ... fragments of subject

> matter. Mathematics ... would be reorganized in a manner to enable the pupil to get a "long" view (Aikin, 1932, p. 443).

With such a curriculum, students would investigate situations that would demand a holistic approach and would require them to make new connections and draw on knowledge from different areas. An example of this is a situation that occurred in the writer's classroom. The father of one of the students owned a construction company that specialized in salvage and re-use of usable equipment from demolished buildings. Through the course of his business, he had come into possession of a number of pieces of hospital equipment, including surgical room lights, triage lights, x-ray readers, curtains, bedpans, plus various other items. The challenge to the class, a multi-age grade 7/8 group, was to decide what to do with this equipment. The class assembled themselves into teams based on their initial ideas and proceeded to investigate various options. These ranged from donating it to a reserve in northern Manitoba to donating it to an African hospital. All of the possibilities required the students to make numerous connections between disciplines. Ultimately, the class collectively decided on the African hospital route and contacted the appropriate people in Ethiopia. Shipping the equipment posed a new set of challenges. Using scale models which they made, the students calculated the most efficient way to pack the items and computed shipping costs based on that.

Even in cases where the disciplines cannot be integrated to this extent, divergent thinking would dictate that the topics within the subject itself be treated in a non-fragmented manner. As this paper has indicated, this is not generally the case with the teaching of mathematics or any of the other disciplines. As a result, it is difficult for the student to make the important connections.

It has been suggested that perhaps there exists an irresolvable dichotomy between traditional school curricula and desired characteristics in students. Certainly, the two appear to be at odds in that the former is generally an amalgam of content and by its nature, cannot foster such characteristics as effective problem solver and complex thinker (Leibowitz, 2000). The implied connection between the two is that through the process of learning the content, "…students will become the kind of people we want them to be" (Seiger-Ehrenberg, 1991, p. 437).

To promote divergent thought, problems must be presented at the beginning for context as opposed to the end for application. As well, they must be grounded in real-life situations and present multiple ways to reach a solution. A curriculum based on the principles of divergent thought would be one in which skills would not be taught separately but rather embedded in context. As well, the use of algorithms, while not directly presented by the teacher, would evolve at the student's own hand and be refined as necessary by the individual.

A curriculum grounded in divergency would be constructivist in nature and would be structured to allow students multiple ways to problem solve while enabling them to make their own personal meaning through connections between the known and unknown.

The next chapter will examine the Manitoba Grade 7 mathematics curriculum and the type of thought it promotes.

Chapter 6

An Examination of the Manitoba Grade 7 Mathematics Curriculum and the Utilization of Divergent Thinking to Enhance Learning

The purpose of this chapter is to examine aspects of the Manitoba Grade 7 mathematics curriculum with a view to discussing the convergent and divergent approaches inherent in its design.

In the case of the Grade 7 mathematics curriculum, the outcomes are encapsulated within four major strands. These strands of number, patterns and relations, shape and space, and statistics and probability remain unchanged throughout kindergarten to grade 12. All are based on the seven processes of communication, problem solving, visualization, reasoning, mental mathematics/estimation, and technology.

In spite of the fact that curricula have traditionally been written in terms of outcomes, a number of researchers hold the belief that measurable outcomes actually represent the least significant aspect of the learning process (Kohn, 1998). This position is not unreasonable given that most goals emphasize skills as opposed to conceptual understanding. This is certainly the case with the Manitoba curriculum. The outcomes include such statements as "(D)emonstrates number sense for decimals, common fractions, and integers" (p. E-186), "(U)ses common multiples, common factors, lowest common multiples, greatest common factors, composites, primes, and prime factorization" (p. E-198), and "(D)istinguishes between rate and ratio, and uses them to solve problems" (p. E-274). In fact, one can select virtually any page in the Foundation for Implementation document and the prescribed outcome will be reflective of a particular skill set rather than calling on the student to demonstrate conceptual knowledge.

Stigler and Hiebert (1999) state that, while there is no single correct way to define learning goals, it must be borne in mind that "(W)hat is important, first, is that goals capture the kind of learning that we most value" (p. 141). Goals that stress recall of definitions, the use of algorithms, and demonstration of skills clearly do not value higher-level thinking. By its nature, the Manitoba grade 7 mathematics curriculum is promoting a convergent, low level of thought on the part of students.

The essential questions that give rise to the outcomes are very clear. They seek to have the learner develop a useful number sense, utilize patterns to describe the world and solve problems, use direct or indirect measurement to solve problems, and collect, analyze, and display data to solve problems. Despite the worthiness of the questions, the issue that arises is whether the learner truly has a useful number sense, for example, if he or she is assessed on the basis of a demonstration of a skill. For instance, one of the prescribed outcomes in the number strand is that the student will be able to read and write any number to any number of decimal places (p. E-192). However, only focusing on that which the student is able to do provides a very shallow picture. Routinely, students leave school with a knowledge of "face value" as opposed to the much deeper place value. The former refers to the case in which the student is able to recite the place value of a particular number and can read numbers successfully but has little or no grasp of the deeper underlying meaning of place value. This has been shown repeatedly in the results of the CAP (comprehensive assessment program) testing by the Winnipeg School Division. If their understanding is not examined on a deep level, students appear to meet the learning outcomes whereas testing shows that the knowledge exhibited is that of face, not place, value.

Once the purpose has been established, it then falls to determine what content will be presented to the learner to reflect these essential questions. In the case of the grade 7 mathematics curriculum, there is a developmental continuum. The content offered builds on concepts that arise in earlier grades. For example, fractions are first discussed in grade 1 with an introduction of the concept of half. This is followed by thirds and fourths in grade 2, fifths and tenths in grade 3, hundredths and a connection from fractions to decimals in grade 4, equivalent fractions in grade 5, and an introduction of mixed numbers and improper fractions in grade 6. Fractions, as a part of the bigger picture of rational numbers, which includes percents and decimals as well, is vital to any mathematics program. Research has shown that the proportionality aspect of rational number is a major underpinning of mathematical literacy. According to Post, Behr and Lesh (1988), "(T)he fact that many aspects of our world operate according to proportional rules makes proportional reasoning abilities extremely useful in the interpretation of real-world phenomena" (p. 79). However, despite the usefulness and wide applicability of rational number,

> The acquisition of proportional thinking skills in the population at large has been unsatisfactory. Not only do these skills emerge more slowly than suggested but there is evidence that a large segment of our society never acquires them at all (Hoffer, 1988, p. 285).

In the Manitoba curriculum, the concepts of fractions, percents, and decimals are addressed in an interconnected manner although, as will later be noted, this is one of the few areas where such connections are evident.

The content offered in other areas appears essential as well. The strand of patterns is very important and is in a sense, overriding in that in the final analysis, regardless of topic, mathematics is the study of patterns (Devlin, 1997, p. 9). In grade 7, the emphasis is on algebra and the use of linear equations to solve problems. This develops through the years, beginning with sorting attribute blocks in the early grades to making predictions based on addition and multiplication patterns to constructing tables to chart pattern growth.

Shape and space also deals with big questions, notably measurement and comparison. Both serve to link mathematics with the real world and by learning about length, area, and volume, "...students mentally structure and revise their construction of space, both large-scale and small-scale" (National Research Council, 2001, p. 281). This view is supported by earlier work that suggests understanding is closely linked to successive mental restructuring of space (Piaget & Inhelder, 1956).

The final strand of statistics and probability is treated less extensively than the other three. It largely deals with a few key processes, notably describing, organizing, representing, and analyzing data.

The Manitoba curriculum is somewhat sequential, in that knowledge is compiled in a particular order. As an example, one finds that patterns and integers are best dealt with before algebra. Beyond the sequential design, one could argue that it is structured in what Applebee (1996) terms an episodic manner. This is the case when there is a sense of an over-all topic layered on top of the sequential nature. In this sense, that could be deemed to be "mathematics as a useful tool". According to Applebee (1996), a curriculum such as this is "...easy to plan and teach, since each episode or segment flows logically from the previous ones yet remains self-contained" (p. 76). The drawback, and this would appear to be so in this case, is that while the various episodes cast light on the central topic, they tend not to illuminate one another and appear somewhat disconnected. In other words, conversations engendered by the last episode, for example, do not reflect, nor are necessarily related to, conversations surrounding the first. The result "...invites affirmation rather than...reassessment of earlier experiences" (Applebee, 1996, p. 76).

Convergent thinking tends to shut off curricular conversation and can lead, ultimately, to an atomistic perspective. Atomism stresses, "...segmentation and reduction of the curriculum into small, separate units" (Miller, 1988, p.13). A leading proponent of this approach was Franklin Bobbitt (1924) who advocated curricular design encompassing a great number of skills and behaviours, ultimately unconnected to one another. Such fragmentation results from, and in turn promotes, convergent thought.

There is a significant amount of disconnectedness between various topics in the Manitoba grade 7 mathematics curriculum. An examination of the number strand, for example, references several ideas. These include decimals, fractions, integers, expanded notation, multiples and factors, and divisibility rules among others. Apart from decimals and fractions, few connections are suggested. It falls to the classroom teacher to actively seek out and enable students to discover these links.

For truly in-depth curricular conversations, students must discover the interrelationships throughout the knowledge they are learning.

There is agreement on the need for mathematical literacy. Educators concur that students must be able to "…understand how mathematical concepts permeate daily life, business, industry, government, and our thinking about the environment. They must be able to use mathematics, not just in their work lives, but in their personal lives as citizens and consumers" (Manitoba Education, Citizenship and Youth website).

Curriculum documents everywhere contain similar statements. Mathematical literacy, as well as a comfort and facility in its use, is the overarching goal of educators involved with this discipline. There is likely tacit agreement that the mathematics referred to is not that of high-level specialized mathematics but rather the useful day-to-day number sense and problem solving ability which is frequently used in one's life.

A discussion of the Manitoba grade 7 curricular outcomes by strand suggests there is encouragement of creative and logical thinking as well as problem solving skills. However, an examination of the document does not immediately confirm this. While there appears to be an emphasis on logical thinking which is the hallmark of mathematics teaching, it is difficult to find references to, or opportunities for, divergent thought. As well, although the term "problem solving" is used frequently, the examples given are overwhelmingly practice for the algorithm most recently taught.

As discussed earlier, a curriculum cannot claim to foster creative thinking while at the same time listing prescribed learning outcomes. Such outcomes and competencies, that should be attained, point toward a very convergent model.

Consider the following paragraph which is presented to allow students to solve problems using patterns:

> Monthly, my uncle sends a package with an even number of hockey cards. My mother says I must give half of the cards to my younger sister. My sister thinks I have more, so I give her another card. I keep the rest, which ends up being one less than half. Every month, my uncle sends me 2 more cards than the prior month. At the end of 6 months, how many hockey cards will I have? How many cards will my

sister have? (A Foundation for Implementation: Grade 7 Mathematics Curricular Outcomes by Strand, p. B-54)

A number of issues present themselves with offerings such as this. A very strong case could be made in favour of this not being a problem at all, despite the label attached to it. As discussed earlier, this could simply be deemed practice. The student is aware that patterns are involved in that the question forms part of that particular unit. This knowledge alone serves to remove it from the realm of true problems. In addition, there are aspects of reading and decoding which could prove to be a problem for students. As well, one is forced to question the relevance of problems such as this to the student. Student engagement becomes difficult in that a situation is presented that is neither of interest nor likely to be encountered. The student is expected to arrive at the answer by putting the various pieces of information on a chart that will serve to reveal the pattern, thereby allowing predictions to be made.

From a divergent thinking point of view, the difficulties with this are manifold. As noted, the very fact that teachers are expected to follow a detailed curriculum, complete with outcomes and objectives, is in itself a very convergent approach to learning. If questioned, teachers would likely say without hesitation that they welcome alternative methods and that their primary function is to teach students how to learn. As shown, research does not support this. The reality is paradoxical. Despite being aware of constructivist techniques, the day-to-day regimen of listing course objectives, "covering" the curriculum, and asking questions such as the one above, with a single correct answer that is to be arrived at in a prescribed manner, teachers are forced into promoting convergence. The result is that Instead of teaching how to learn, teachers are inculcating students into a system that has historical continuity on its side and one in which success is measured by adherence to the narrow thought that is required.

The Manitoba curriculum is set up in such a way as to reflect a commonly held belief that mathematics is a collection of algorithms and methods. As Stigler and Hiebert (1999) point out, many teachers "...behave as if mathematics is a subject whose use for students, in the end, is as a set of procedures for solving problems" (p. 89). This is borne out by their recent study in which 61% of teachers interviewed cited skills as being the most important result of lessons (Stigler & Hiebert, 1999, p. 89).

Further, because of the emphasis on skills and the attendant practice, there is a decided lack of excitement in the mathematics curriculum. As a result, teachers frequently "...act as if student interest will be generated only by diversions outside of mathematics" (Stigler & Hiebert, 1999, p. 89).

The patterns and relations strand of the curriculum stresses logical thinking. The student is expected to describe patterns, make predictions, and extrapolate values from graphs. Although these all stress a step-by-step linear model of thought, "Students need to understand that there are often several equally valid ways of stating how a pattern continues" (A Foundation for Implementation, p. B-42). Teachers must be aware of the possibility of different answers and encourage same. Wherever possible, students must be prompted to explain their thinking which they are more apt to do in a classroom climate marked by creativity and acceptance, where different modes of thought are valued.

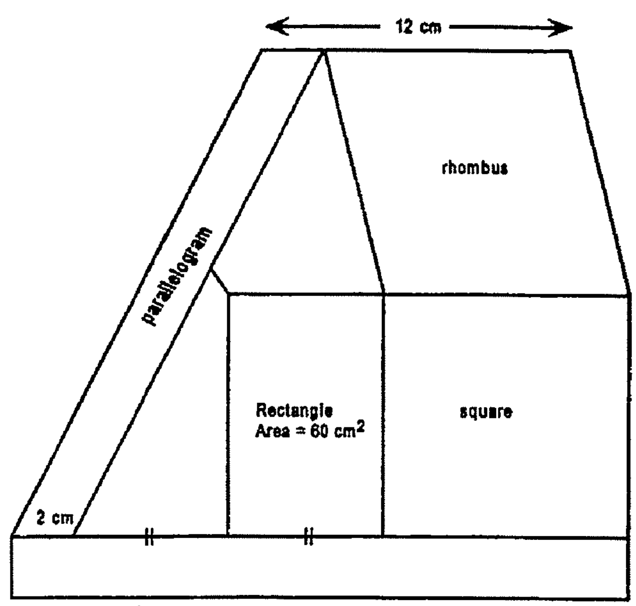
Throughout the patterns strand, one finds various suggestions for assessment. The following is an example:

> Measure the sides of each of the squares provided. Find the perimeter of each square. Label a graph so the horizontal axis represents the length of the sides and the vertical axis represents the perimeter. Plot the values. Describe the pattern. From the results of this graph, make a rule for finding the perimeter of a square (A Foundation for Implementation, p. B-49).

As it stands, this is a very teacher-directed event and thus it is likely that student engagement would be quite low. An effort should be made in a case such as this to render it more open-ended and discovery-oriented. When attention is paid to a series of prescribed steps, mathematics becomes content driven and while not quite rote in this case, not far removed from it. The student must be given the latitude to experiment with squares and discover for himself the relationship that exists between side length and perimeter.

Divergent thinking does show up more in certain parts of the curriculum. The shape and space strand, for example, contains a number of questions which are somewhat open-ended and lend themselves to this type of thinking. Consider the following example: (figure 18)

Figure 18: Grade 7 Shape and Space Question



Even the wording of the question appears to be divergent: "(D)etermine as many measurements as possible and explain how you determined each" (A Foundation for Implementation, p. D-145). While there are a specific and finite number of measurements to be found, it is the possibility of explanation that sets this question apart. The teacher must be careful in cases such as this to encourage and accept alternative methods of determining measurements.

It is possible that certain parts of any mathematics curriculum are, by their nature, more suited to the application of divergent thought. An example of this might be statistics and probability. Throughout this strand, students are asked to analyze data and make predictions. This presents an excellent opportunity for the teacher to elicit all manner of unusual, but plausible, responses. The probability of each can be examined with a discussion as to why certain ones are more likely than others.

The Manitoba grade 7 mathematics curriculum, perhaps like any document listing prescribed outcomes and emphasizing that which students should be able to do, is convergent in its approach. This is in no way surprising given the nature of the educational system and the mind-set of the teachers who work in such settings. Were any one of curriculum, the system, or teachers to invoke wholesale changes and adopt a divergent, inquiry-based approach to learning, it would result in the others following suit.

The next chapter summarizes the major ideas raised in this study, draws conclusions from these data, and suggests recommendations based on this information.

Chapter 7

Summary, Conclusions, and Recommendations

Summary

As previously noted, although mathematics is used throughout this paper as a representative example, this study deals with curricular and problem solving issues raised in connection with convergent and divergent thought. Mathematics was selected as a suitable lens through which these may be viewed simply because of the importance of problem solving in the discipline.

The need for a thorough grounding in problem solving is more important today than ever. Society has become increasingly technological with the effect that "…people are much more exposed to numbers and quantitative ideas and so need to deal with mathematics on a higher level than they did just 20 years ago" (National Research Council, 2001, p. 407).

As this paper has shown, the problem solving abilities of students is an area of concern. The data indicate that skills in this regard remain at a fairly low level throughout the school years. Problem solving is a complex area and success can be affected by a number of factors, including the length and grammatical complexity of the problem, the context, the amount of non-essential information, and the level of engagement of the solver. At the core of the difficulties seems to be the linear progression of thought encouraged by schools. True problem solving depends on a re-combination of existing ideas, one of the obstacles to which is linear thinking. The solver must be encouraged to re-frame the problem and view it from a number of perspectives. By so doing, the brain is allowed to form new connections.

The disciplines are frequently viewed in a fragmented manner in which content is simply a mass of information to be deposited in the mind of the learner. Whereas rote learning made up a large part of mathematics in the past, whatever value it may have once had has diminished. Further, an emphasis on rote learning has been shown to erode confidence (vide chapter 2). The predominance of technology has resulted in a population that is less reliant on personal computation. Problem solving which has always played a central role in the transmission of knowledge from teacher to student is now recognized for what it is: a vehicle for practice of formulas and algorithms. While problem solving can and should be an important context to allow students to deepen their understanding of mathematical topics, it frequently emphasizes a single method to arrive at an answer. The result is that, instead of enhancing students' opportunities and allowing them to make connections among different strands, it has devolved to another way to reinforce rote learning. This results in "...a one-sided character to learning" (National Research Council, 2001, p. 421) whereas:

> Problem solving should be the site in which all of the strands of mathematics proficiency converge. It should provide opportunities for students to weave together the strands of proficiency and for teachers to assess performance on all of the strands (National Research Council, 2001, p. 421).

As discussed earlier, it is in the constructivist classroom that divergent thinking can thrive. Historically, teaching has been driven by associative and behaviourist psychologies. In turn, these give rise to a transmission style of teaching. This is an approach that does not value construction of knowledge by the learner and is easier to implement from the teacher's point of view.

The proponents of a constructivist theory of learning, including Dewey, Vygotsky, and Piaget believed that knowledge does not exist independent of the learner and only that which is constructed by the learner has validity. Constructivists would argue that unless each learner constructs knowledge independently, it does not represent learning in a meaningful way. Further, students reflect on previous knowledge, thereby progressively enriching their understanding and organizing their knowledge into increasingly complex structures.

As has been demonstrated throughout this paper, this line of thought is one not necessarily promoted by schools. Curricula are frequently an aggregate of facts, and success is based on assessments that measure how well these facts can be recalled. Ultimately, the question that must be addressed is whether students' understanding and perception of the world at large is affected by their procurement of such "knowledge". Learning is an active process. It cannot exist if there is a passive acceptance of knowledge and there is no engagement with the world. Important also is the recognition that people learn as they learn and that metacognition plays an important role in this process. As Hein (1991) puts it:

> We do not learn isolated facts and theories in some abstract ethereal land of the mind separate from the rest of our lives: we learn in relationship to what else we know, what we believe, our prejudices and our fears. We cannot divorce learning from our lives (p. 17).

While there may be general agreement with this point, one must accept that the reality of the current situation is closer to the observation made by Julyan and Duckworth (2005) when they speak of learning science: "(T)hese trademarks of a constructivist classroom may well be inconsistent with the view of science as a static body of facts" (p. 78).

As noted in chapter 4, the focus on linear, convergent thought is maintained through the types of questions that are asked. These tend not to invite conversation, and require a single, correct answer response. This situation, while not only repressing divergent thought, appears to foster mathematics anxiety.

Curricula (vide chapter 5), through the convergency which they engender, maintain this situation. To embrace a more divergent point of view, curricula should emphasize construction of meaning by the learner. The Manitoba Grade 7 mathematics curriculum is discussed and is shown to be somewhat disconnected and fragmented. As well, the skill-based outcomes promote convergent thinking.

Divergent thinking needs to be accorded a place of prominence in the curriculum as well as everyday teaching. As has been shown, a reliance on convergent thought leads to a situation in which rote memorization is construed as learning and the real business of problem solving as students will encounter it in real life goes unaddressed. One is reminded of the following statement:

> If our destination is excellence on a massive scale, not only must we change from the slow lane into the fast lane; we literally must change highways. Perhaps we need to abandon the highways altogether and

take flight, because the highest goals that we can imagine are well within reach for those who have the will to excellence (Hilliard, 1991, p. 5).

Conclusions

The study suggests that a number of conclusions can be drawn.

The first conclusion is that problem solving skills of students are at a very low level. Although it is taught throughout the grades, problem solving is usually presented either out of context or in a contrived and artificial manner. The strategies promoted by schools tend to be linear and not reflective of the cyclical and unclear nature of real problems. They do not contribute to understanding and tend to reinforce students' view of mathematics and the other subjects as something to be done in school, without real-world connections.

The second conclusion that can be drawn is that reliance on convergent, linear thought as a way to promote learning and facility with problem solving is, by itself, inadequate. Despite the effort expended to foster capable problem solvers, the data show that students do not improve appreciably in this area as they progress through school. Convergent thought does not allow the student to explore possibilities outside of those which define the problem.

Instead, the emphasis is on rote learning and the application of formulas that may or may not be understood. Although there exists, at least in the case of mathematics, a single "best" answer or one most closely aligned with the current situation presented as a problem, it is the various ways in which this may be arrived at that is the essence of divergent thinking.

The third conclusion is that the degree to which divergent thinking is promoted and encouraged in the classroom depends to a very large extent upon the teacher and the classroom climate that has been developed. Teachers must demonstrate a willingness to embrace non-traditional modes of thought through their actions, both in the application of divergent thought to problems and the encouragement offered to students.

The final conclusion is that the Manitoba grade 7 mathematics curriculum promotes convergency in that the majority of outcomes are skill-based in nature. When the emphasis is placed on how well students can perform specific tasks as opposed to being asked to explain their thinking or to de-code accepted algorithms and invent their own, divergent thinking becomes an afterthought, if it is considered at all.

Recommendations

It is clear that a reliance on convergent thought is not sufficient to promote success in curriculum, whether it be in mathematics or any other discipline or body of knowledge in education. Children must be exposed to a combination of divergent and convergent thinking, the former to generate a number of alternatives, the latter to evaluate the possibilities. The question remains: Given the convergent nature of schools and the education process, what steps can be taken to implement divergent thinking? A number of recommendations follow.

An initial recommendation is that problems must be seen not as an end in and of themselves but as a context for learning. There is no disagreement that problem solving is important. In fact, it is the mainstay of certain disciplines, notably mathematics and science. Researchers have pointed out that, "(S)tudies in almost every domain of mathematics have demonstrated that problem solving provides an important context in which students can learn about number and other mathematical topics" (National Research Council, 2001, p. 420).

However, if problems are to be viewed as a context for learning, this must be more than lip service. Problems need to be presented that reflect real life and more importantly, are engaging to students. If relevant problems are presented, by their nature these will be open-ended and somewhat fuzzy in their resolution. With this in mind, students will come to realize that frequently, it becomes a case not of a single, correct answer but rather one of an answer which is most appropriate in the circumstances.

Curricular areas must be treated as knowledge-in-action as opposed to knowledge-out-of-context (Applebee, 1996). As Lloyd (1997) says "Life is a series of problems to be solved. It is the process of problem solving that they will use every day of their lives if they are to survive" (p. 96). In other words, problem solving in mathematics, for example, is much more than simply the translation of a word problem into an equation that has a single "correct" answer.

Schools generally teach convergent thinking because the emphasis in that setting is on achieving the conventionally accepted answer. Simply, "(C)onvergent thinking is the sort of thinking usually required to answer examination questions and conventional intelligence tests" (Rosenthal et al, 1977, p. 186). As Harold Jaus (1989) puts it, convergent thinking is a "(S)ingle-answer mental activity" whereas divergent thinking represents an "(U)nlimited-answer endeavour" (p. 15).

A second recommendation is that authentic, contextual problems need to be presented at the beginning of lessons, prior to the introduction of formulae and algorithms. This allows students the opportunity to explore their thinking and construct a response that is unencumbered by rote procedure. To use mathematics as an example, this could result in a de-centering of the accepted way in which the subject is taught. Instead of learning concepts in isolation through drill and then applying them to contrived situations, the reverse would happen. The situations would be presented then de-coded and solved without the use of standard algorithms. These would be learned afterwards. Through co-operative learning, children would discover ways and processes they could apply. Divergent thought would be more readily accepted in that boundaries are less likely to have been erected through adherence to traditional mathematics.

A third recommendation is that, wherever possible, teaching should be based to a large degree on the examination of open-ended problems, and experience with situations that require divergent thinking. As discussed previously, the research on the value of open-ended questions is very compelling. As educators broaden their focus from the single correct answer attained through a single prescribed method, the result will be students who learn to wander and wonder, link and leap. In short, students will begin to think divergently. Teachers must make every effort to frame their questions in ways that promote, not hinder, conversation. They need to move away from being purveyors of knowledge-out-of-context and become instead creators of knowledge-in-action.

A fourth recommendation is that classrooms must support a process-oriented curriculum.. Learners in all disciplines would find their learning enhanced as a result. This is particularly true for mathematics. In fact, mathematics has been called "(T)he supreme example of the inability to separate content and process" (Lloyd, 1997, p. 96). However, one of the major difficulties in teaching problem solving is that the aspects of process and content are treated as two easily distinguished and ultimately not intimately connected strands. This is not an accurate reflection of any of the disciplines nor does it pay homage to their place in the world at large. Once the process is treated equally with content, divergent thought is more welcome in that it is very much process-oriented.

A fifth recommendation would be advocating a change in teacher practice. This is likely the most difficult area to address with success. As Vann (1995) says, "...as difficult as it is to change the *what* of curriculum, it is far more difficult to change the *how* of teaching" (p. 39). Despite the wealth of data attesting to the worth of teaching in an exploratory manner where divergent thinking is valued, schools continue to devise curricula and support pedagogy which make this difficult. Curricula remain a vast amount of information to be "covered", with assessment a narrow snapshot of learning that may or may not have taken place. The present situation has created a paradox: teachers who nominally honour constructivist techniques while being forced into a role framed from the outset to perpetuate convergent thinking. It falls to the classroom teacher to structure and present the material in a way that encourages a more divergent manner of thought. This includes, as mentioned in chapter 5, helping students to discover the relationships between concepts.

Coupled with this recommendation would be a re-examination of teacher education in light of the need to promote a constructivist way of teaching. If the agreed-upon premise among all parties is the improvement of student learning, it follows that teachers must be trained in ways that enable them to facilitate this.

To do so successfully requires an examination of what has been termed "perceptual orientations" (Caine & Caine, 1999, p. 17). These refer to personal characteristics and assumptions about learning and tend to support the maxim that teachers teach as they were taught. This truism is supported by research (Jones, 1975). Teachers' perceptions are shaped both by their own time in school and by the individuals by whom they were taught. As Fosnot (1989) points out:

> Teachers may be producers of passive learners, but they are products of the system as well. Not only did they sit for 12 years in elementary and secondary classrooms learning how to regurgitate what teachers wanted, but they refined these skills in college with approximately 3 more years of liberal arts education (p. 5).

As a result of this education, once teachers are in classrooms, they are encouraged to promote what has been termed "naïve realism" (Senge, 1999, p. 140). In other words, the interpretation of the teacher is not encouraged or valued. The teachers

> ...do not teach their opinions. They do not teach their interpretation of what happened. They teach what's actually fact. The kids learn that "this" is what happened in history, that history is just as the author said. They don't think that this is what this teacher (or author) has interpreted to have happened, but that things happened just as they were told (p. 14).

It is imperative to recognize is that construction in learning must apply to all learners, teachers and students alike. Long-term development is needed to encourage teachers to reflect on their practice, test their understandings, and build new ones. This must begin in teacher training in universities and colleges where learning activities are modeled and shown how these may be applied in classrooms (vide chapter 4). As Barth (1990) puts it

> To teacher and student alike there must be continuous evidence that it is occurring. For when teachers observe, examine, question, and reflect on their ideas and develop new practices that lead toward their ideals, students are alive. When teachers stop growing, so do their students (p. 50).

However, empowering teachers as thinkers is a difficult proposition, particularly at university where

> …teachers are viewed as passive recipients of knowledge to be imparted. Like sponges, they are expected to soak up a wealth of information from the liberal arts. They are responsible for all subjects but because they do not major in any of these fields and thus miss out on the higher-level seminar courses, they rarely do any intensive thinking in these areas (Fosnot, 1989, p. 7).

University courses need to focus more on integration of subject matter and must be designed from an experiential base, emphasizing exploration and meaning-making by the learner. Mathematics, for example, must be problem-based.

Brooks and Brooks (1999) point out that:

> So much of what aspiring and practicing teachers are taught is rooted in the behavioral soil of stimulus/response theory. But this soil has been used for too many years, and is becoming more widely seen as nutrient-deficient. It's time to replant our ideas about teaching and learning in richer fields (p. 121)

In addition to content, there must be a greater range in the manner by which this content is disseminated to students. According to Howey (1983), "(O)ne could logically expect that teachers of teachers would employ a wide variety of teaching technologies or methodologies. However, in general, instructional diversity is limited" (p. 13). At present, most courses tend to be taught in a lecture style. In this model, good teaching is equated with clear presentation, as opposed to the promotion of student-centered inquiry.

This situation does not change when pre-service teachers are placed in classrooms for practical experience. Student teachers are expected to emulate the style of the "master" teacher with whom they are placed and are positively reinforced for such things as efficient covering of the curriculum, well-managed classrooms and "...skilled imitation of clichéd ways of teaching" (Fosnot, 1989, p. 7). In short, thinking and meaning construction are not high priorities. Further, this emphasis on an agenda which does not focus on thinking is perpetrated by research on what is deemed to be effective teaching (Berliner, 1986; Brandt, 1986). The characteristics and behaviours of selected, competent teachers are used as models to be emulated, as opposed to determining what makes for creative teaching and enhanced student learning.

Once in classrooms of their own, many teachers continue to view teaching as something done in isolation and avoid or resent input from colleagues. As Barth (1990) points out:

> My experience suggests that the professional growth of teachers is closely related to relationships within schools, between teacher and principal, and between teacher and teacher. I am convinced that great untapped opportunities for the professional development of teachers reside within the school. Teachers favour principals who intrude on their classrooms least (p. 50).

With professional growth comes student growth. This is a major paradigm shift and involves a change of focus on the part of teachers and well as a view of themselves as learners. Tabor (1997) states that there must be an awareness of

> ...the link between the investment in [professional] development and the learning of students...[Professional] development must be continuous and focused on the improvement of practice which results

> in measurable advances in student learning...The key to student growth is educator growth. They happen together; each enhances the other (p. 58).

A seventh recommendation would be that teaching be done in an interdisciplinary manner. For example, as opposed to the traditional, fragmented approach generally adopted, mathematics should be presented in a manner that integrates it with other areas of study. Through this, the concepts are learned in ways where the important connections are revealed to the learner. The subject is then viewed, not as a separate entity, but rather as an inseparable component of other disciplines. Connection-making is a hallmark of divergent thinking.

The final recommendation would be that the classroom teacher make every effort to re-frame the curricular outcomes from ones that are skill-based to those which emphasize conceptual knowledge. In cases where the learning outcomes dictate that the student should be able to do a particular task, the teacher should encourage the student to show his or her knowledge not only through demonstration of the skill but also through an explanation and breakdown of the skill. Anything less only serves to condone and perpetuate rote learning and the notion that success in school is the ability to recite the single, correct answer.

As with any study, while certain questions are addressed, even more are raised. So it is with this study. A number of queries lend themselves to further study. These include the following:

- Although it appears that there exists a positive correlation between an improvement in divergent thinking and linear, convergent thought as manifested through problem solving, no hard data are available to corroborate this. This would be a very worthwhile undertaking to determine the relationship.
- The evidence would seem to suggest that traditional methods of teaching, with heavy emphasis on convergent thought can exacerbate or even result in math anxiety. Again, this is a study which would prove quite valuable.

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Appendix

Terms of Reference

There a number of terms used throughout this paper that require clarification as to their use within the scope of this study.

Convergent Thinking

Convergent thought is known by a number of names, including linear, logical, and vertical thinking. For the purposes of this study, all are taken to mean the same thing: a pattern of thinking which proceeds in a step-by-step fashion to a single, correct answer. As there is a constant awareness of the logical, right nature of this approach, any avenues of thought that deviate from this are dismissed. The orientation of convergent thinking is that of deriving an answer to a clearly articulated question.

Divergent Thinking

Divergent thinking, also known as lateral thinking, involves producing a number of answers based on the available information. The divergent thinker makes connections between ideas, often transforming the information in unexpected ways.

Divergent/lateral thinking is a much more generative type of thinking than its convergent counterpart. All manner of options are considered, with no consideration of correctness. While the alternatives are ultimately evaluated, one of the hallmarks of this type of thinking is that proposed solutions, while possibly not practical for the matter at hand, may prove useful in other arenas or may even result in a change in the way the original problem is viewed.

Constructivism

The constructivist theory of learning holds that students learn best when they are allowed to make their own meaning as opposed to absorbing ideas, facts, and skills passively. This is a discovery-oriented approach in that it focuses on cognitive development and deep understanding. It is very much aligned with the notion of divergent thought in that, rather than viewing learning as a linear process, it recognizes that the fundamental nature of learning is, in fact, non-linear.

Problem Solving

A problem is an obstacle to action and hence, problem solving is a means by which such barriers may be removed. It is a dynamic and on-going process. Although many of the examples in this paper concern mathematics, dealing with problems is not confined to a single discipline.

Heuristic

Heuristic refers to a general strategy, used without reference to any particular topic, which aids the solver in understanding the problem and assists in organizing the resources needed to find a solution.

Gestalt

Gestalt is the German word for pattern or configuration and refers to the tendency to organize sensory information into patterns in order to make sense of the world.