DESIGN AND REALIZATION OF FIR AND BIRECIPROCAL WAVE DIGITAL FILTERS

by

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A Thesis

Presented to the Faculty of Graduate Studies

in Partial Fulfillment of the Requirements

for the Degree of

MASTER OF SCIENCE

Department of Electrical and Computer Engineering

University of Manitoba

Winnipeg, Manitoba

Canada

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April, 2000

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A Thesis/Practicum submitted to the Faculty of Graduate Studies of The University

of Manitoba in partial fulfillment of the requirements of the degree

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Acknowledgments

I would like to express my deep appreciation and gratitude to my advisor, Professor G. O. Martens for his encouragement, patience, and ever-helpful guidance throughout the course of my study. I have greatly benefited from his expertise and constant help and advice, which will continue to have positive influence on my life. I also extend my thanks to the members of my committee, Dr. Shwedyk and Dr. Jarmasz for their valuable inputs.

Special thanks go to my husband and my parents for their love, understanding and encouragement at all times.

DESIGN AND REALIZATION OF FIR AND BIRECIPROCAL WAVE DIGITAL FILTERS

Yuhong Zhang

Abstract

This thesis concentrates on two subjects related to wave digital filter design and realization. The first one considers the cascade synthesis of lossless two-port networks, which is based on the factorization of the transfer matrix or the scattering matrix. Another subject of this thesis is the design and realization of bireciprocal filters.

Jarmasz provided an efficient cascade synthesis algorithm of lossless two-port networks by extracting elementary sections step by step. Following this approach, based on the factorization of the transfer matrix, necessary and sufficient conditions for cascade synthesis of lossless two-port networks from a given canonic set of scattering polynomials is presented. An algorithm to realize a digital filter with ladder structure based on the cascade decomposition and an illustrative example are also provided.

Fettweis proposed another approach to the cascade decomposition of lossless two-port networks based on the factorization of the scattering matrix. A proof that this approach can be applied to FIR filters is provided and at the same time a realization structure and an algorithm in a very general form is developed. Several other realization structures and algorithms for FIR filters are derived directly from this general form, including

Fettweis' two structures. Two example are included to demonstrate the efficiency of the algorithms and to compare the implementation structures.

An analytical formula method and an optimization method for the design of bireciprocal filters are presented. The analytical formula method is simple, direct and uses simple calculations. It is obtained by reducing the design of bireciprocal filters to a Chebyshev approximation problem and making use of a formula due to Cauer. The optimization method for the design of bireciprocal filters is developed by applying a minimax algorithm proposed by Dutta and Vidyasagar, and is an alternative to Wegener's solution. A lattice implementation structure is derived which clearly shows the advantages of bireciprocal filters which exhibit a saving in hardware of nearly one-half compared to nonbireciprocal filters.

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Chapter 1

Introduction

Wave digital filters introduced by Fettweis[1] are modeled on classical analog filters and therefore preserve some of the good properties of passive lossless analog filters, including low round-off noise, large dynamic range, low sensitivity, and stability. There is a detailed discussion of digital filters and their advantages in the review paper by Fettweis[1].

There are many different realization structures for wave digital filters. Ladder and lattice structures play important roles in them. The ladder structure is built on a decomposed analog circuit[10], which is one reason why network cascade synthesis is of interest. Fettweis[3] gave a detailed discussion of the cascade synthesis of lossless two-port networks by the transfer matrix factorization. Jarmasz[2] presented an efficient synthesis algorithm for lossless two-port networks which is also based on the factorization of the transfer matrix. For the given filter specifications, the canonic polynomial set $\{f(\psi), g(\psi), h(\psi)\}$ can be obtained by using a classical filter design method. Then by using Jarmasz' synthesis algorithm, an analog and a wave digital network can be derived at the same time. Fettweis also proposed another approach[1] that the decomposition of lossless two-port networks be based on the factorization of the scattering matrix instead of the transfer matrix. This approach is suitable for application to FIR filters which have a very simple form for g, where g is the canonic polynomial of Belevitch's representation.

Lattice wave digital filters are one of the most attractive ones among the different structures of IIR digital filters[1][19]. Especially, the lattice wave digital filters with bireciprocal characteristic function form an important subclass of the lattice wave digital filters. These kinds of filters, called bireciprocal filters lead to a significant saving in the number of multipliers and adders since only less than half the number of adaptors is required if they are implemented with lattice wave digital structures[14]. One of the popular design methods [24] uses the aid of optimization, e.g. nonlinear optimization as in the Fletcher-Powell algorithm or a Remez-type optimization.Wegener offered a general realization structure for bireciprocal filters in [14].

The goal and motivation of this thesis are as follows:

1) To provide a complete proof to the theory of the realizability of cascade decomposition of lossless networks, which is not found in [2].

2) To develop an efficient decomposition algorithm for FIR filters and to prove that by applying the approach in [1] that the cascade synthesis of lossless two-port networks based on the factorization of scattering matrix applied to FIR filters is successful. Fettwis suggested two realization structures for FIR filters in [1], but no algorithm and no proof of the realizability are given.

3) To give an analytical formula method and an alternative optimization method to design bireciprocal filters.

In order to realize a digital filter using a ladder structure, a decomposed analog network is necessary. Chapter 2 follows the decomposition scheme in [2], extracts elementary sections step by step based on the factorization of the transfer matrix. In particular, a necessary and sufficient condition for cascade synthesis of lossless two-port networks from a given canonical set of scattering polynomials is proven. Therefore, it shows in theory that Jarmasz' decomposition approach of lossless two-port networks is realizable. An algorithm and an example that realize a ladder structure based on the decomposed network are presented at the end of this chapter.

Chapter 3 applies Fettweis' suggestion that cascade decomposition can be based on the factorization of the scattering matrix of FIR filters, and it develops a realization structure and algorithm in a very general form. Using this approach several realization structures and algorithms for FIR filters are derived directly, including two structures proposed by Fettweis. The fact that linear phase FIR filters have symmetric or antisymmetric (antimetric) structures is proven, which means that only half the number of multipliers needs to be calculated for linear phase FIR filters. Two examples which are used to demonstrate the efficiency of the algorithm and to compare between the implementation structures are included.

Chapter 4 discusses the design and realization of bireciprocal filters. An analytical formula method and an optimization method are presented. From the definition of bireciprocal filters, some useful properties are derived. Based on the definition and properties, the design of bireciprocal filters reduces to a Chebyshev approximation problem, which can be solved by Cauer's formula[30]. The analytical formula method proposed in this thesis is simple and direct. Optimization is an important tool for the design of bireciprocal filters[21][24]. In Chapter 4, the optimization method for the design of bireciprocal filters is achieved by applying a minimax algorithm proposed by Dutta and Vidyasagar and is an alternative solution to that of [14] for bireciprocal filters. A lattice implementation structure is derived which shows the savings in the number of required multipliers and adders.

Finally, conclusions are included in Chapter 5.

Chapter 2

Cascade Synthesis of Lossless Two-port Networks Using the Transfer Matrices

Jarmasz[2] provided an efficient synthesis algorithm for lossless two-port networks (Fig. 2.1) based on a simplified characterization of elementary sections. However, a complete proof of the theory of decomposition is not found in [2]. In this Chapter, following the approach in [2] that uses the transfer matrix as a tool to complete the decomposition of lossless two-port networks, a proof of the realizability of the decomposition procedure is given. Finally, a wave digital realization algorithm based on the decomposed structure and an illustrative example are presented.

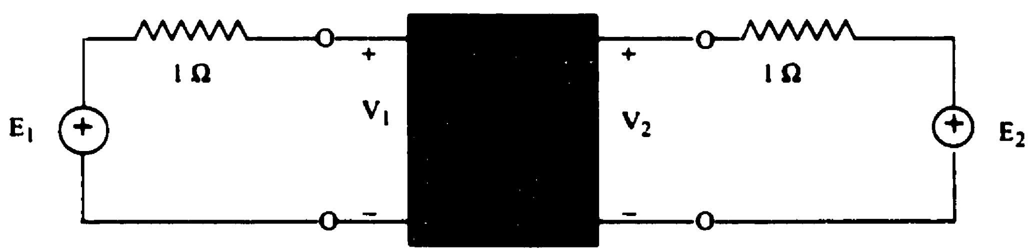


Fig. 2.1 A lossless two-port network

2.1 Belevitch's Representation for Two-Port Networks and the Definition of the Canonic Parameters

A lossless, passive, two-port network N as shown in Fig. 2.1 can be represented by canonic polynomials in the form of a scattering matrix S, or a transfer matrix T[1],

$$S = \frac{1}{g}\begin{bmatrix} h & \sigma f_* \\ f & -\sigma h_* \end{bmatrix}, \qquad T = \frac{1}{f}\begin{bmatrix} \sigma g_* & h \\ \sigma h_* & g \end{bmatrix} \tag{2.1a,b}$$

where the subscript asterisk denotes para conjugation, i.e., for a real polynomials $f_*(\psi) = f(-\psi)$ and the polynomials f, g and h satisfy the following necessary and sufficient conditions:

1. Polynomials f, g and h are real in some complex frequency variable, say ψ.
2. f, g and h are related by the Feldtkeller equation

$$gg_* = ff_* + hh_* \tag{2.2}$$

3. $g(\psi)$ is a Hurwitz polynomial with all its zeros strictly in the left-hand plane.
4. σ is either +1 or -1. For reciprocal two-ports,

$$\sigma = f/f_* \tag{2.3}$$

That the above four conditions are necessary and sufficient means that for any lossless, passive, two-port network, there are three polynomials which fulfil the four conditions and correspond to the network via the scattering matrix S. Vice-versa, if the three polynomials f, g, and h, satisfy the four conditions, there must be a two-port network which has a scattering matrix S as in (2.1a).

Three further parameters, transmission zero ψ_0, reflectance ρ, and delay d play important roles in the synthesis of lossless two-ports[2]. They have the following definitions:

$$\psi_0 : f(\psi)\big|_{\psi = \psi_0} = 0 \tag{2.4}$$

$$\rho(\psi_0) = \left.\frac{h(\psi)}{g(\psi)}\right|_{\psi = \psi_0} \tag{2.5}$$

$$d(\psi_0) = \left.\left(\frac{g'}{g} - \frac{h'}{h}\right)\right|_{\psi = \psi_0} \tag{2.6}$$

where the prime denotes differentiation. The special case of $\psi_0 = \infty$, called a transmission zero at infinity, definitions (2.4′)-(2.6′) are applied instead of (2.4)-(2.6).

$$\psi_0 = \infty,\ f(\infty) = 0,\ \text{if } \hat{f}(\bar{\psi})\big|_{\bar{\psi} = 0} = 0 \tag{2.4′}$$

$$\rho(\infty) = \left.\frac{\hat{h}(\bar{\psi})}{\hat{g}(\bar{\psi})}\right|_{\bar{\psi} = 0} \tag{2.5′}$$

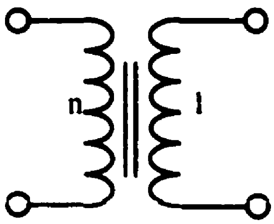
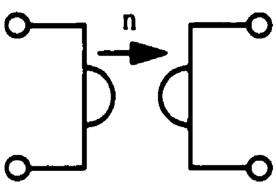
$$d(\infty) = \left.\left(\frac{\hat{g}'}{\hat{g}} - \frac{\hat{h}'}{\hat{h}}\right)\right|_{\bar{\psi} = 0} \tag{2.6′}$$

with $\hat{f}(\bar{\psi}) = \bar{\psi}^n f(1/\bar{\psi})$, $\hat{h}(\bar{\psi}) = \bar{\psi}^n h(1/\bar{\psi})$ and $\hat{g}(\bar{\psi}) = \bar{\psi}^n g(1/\bar{\psi})$, where n is the degree of g and $\bar{\psi} = 1/\psi$.

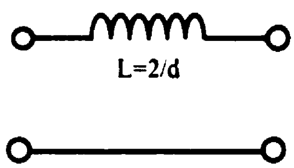
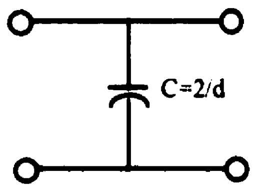
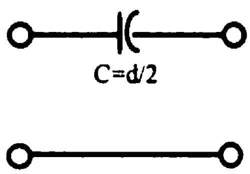
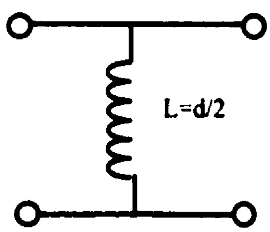
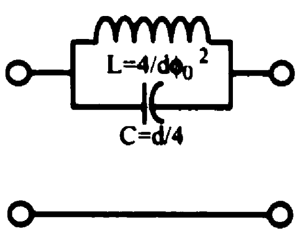
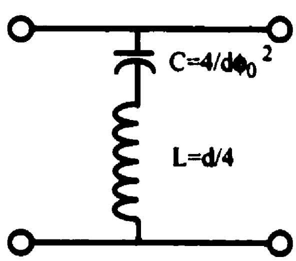
2.2 Elementary Sections

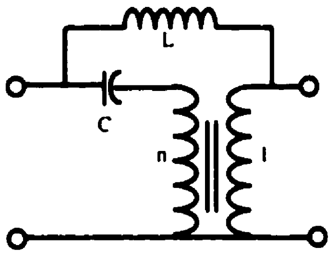
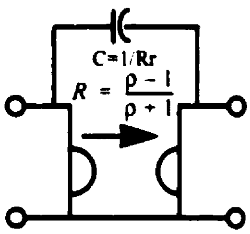
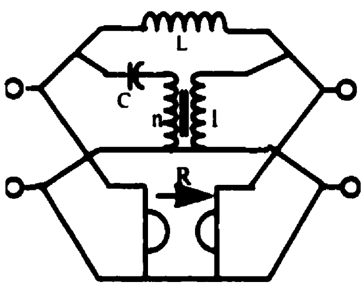
In this part, the elementary sections, i.e., zero-, first- and second-order polynomials f, g, h and their corresponding two-port networks, are presented as tables which are included in Jarmasz' thesis[2]. For convenient reference, they are shown below.

Zero order sections

Ideal Transformer		$S = \begin{bmatrix} \dfrac{n^2-1}{n^2+1} & \dfrac{2n}{n^2+1} \\ \dfrac{2n}{n^2+1} & \dfrac{1-n^2}{n^2+1} \end{bmatrix}$ $\sigma = 1, \quad n = \dfrac{g+h}{f}$
Gyrator		$S = \begin{bmatrix} \dfrac{1-n^2}{n^2+1} & \dfrac{-2n}{n^2+1} \\ \dfrac{2n}{n^2+1} & \dfrac{1-n^2}{n^2+1} \end{bmatrix}$ $\sigma = -1, \quad n = \dfrac{g-h}{f}$

First and second order sections

Sec#	Circuit components	Polynomials	Realizability Conditions
1st Section 1		$\sigma = 1$ $f = d$ $g = \psi + d$ $h = \psi$	$\rho(\infty) = 1$ $d > 0$
1st Section 2		$\sigma = 1$ $f = d$ $g = \psi + d$ $h = -\psi$	$\rho(\infty) = -1$ $d > 0$
1st Section 3		$\sigma = -1$ $f = d\psi$ $g = d\psi + 1$ $h = 1$	$\rho(0) = 1$ d > 0
1st Section 4		$\sigma = -1$ $f = d\psi$ $g = d\psi + 1$ $h = -1$	$\rho(0) = -1$ $d > 0$
2nd Section 5		$\sigma = 1$ $f = \psi^2 + \phi_0^2$ $g = \psi^2 + 2/d\ \psi + \phi_0^2$ $h = 2/d\ \psi$	$\rho(j\phi_0) = 1$ $d > 0$
2nd Section 6		$\sigma = 1$ $f = \psi^2 + \phi_0^2$ $g = \psi^2 + 2/d\ \psi + \phi_0^2$ $h = -2/d\ \psi$	$\rho(j\phi_0) = -1$ d > 0

Sec #	Circuit components	Polynomials	Realizability Conditions
2nd reciprocal (Brune) Section 7	 $n = \frac{1-\cos\gamma}{1+\cos\gamma}$, $L = \frac{n-1}{n\phi_0 \tan\frac{\alpha}{2}}$ $C = \frac{\tan\frac{\alpha}{2}}{\phi_0(n-1)}$	$\sigma = 1$ $f = \psi^2 + \phi_0^2$ $g = \frac{\cos\gamma^2 + 1}{\sin\gamma^2}\phi^2 + \frac{2(1-\cos\alpha\cos\gamma)}{d\sin\gamma^2}\phi + \phi_0^2$ $h = \frac{2\cos\gamma}{\sin\gamma^2}\phi^2 + \frac{2(\cos\alpha - \cos\gamma)}{d\sin\gamma^2}\phi$ $\frac{h}{g}(j\phi_0) = \rho = e^{j\alpha}$ $d = \frac{g'}{g}(j\phi_0) - \frac{h'}{h}(j\phi_0)$	$\cos\gamma = \frac{\sin\alpha}{d\phi_0}$ $d > \left\|\frac{\sin\alpha}{\phi_0}\right\|$ $d > 0$
1st nonreciprocal Section 8		$\sigma = 1, f = \psi + r$ $g = \psi + \frac{r(\rho^2+1)}{\rho^2-1}$ $h = \frac{2r\rho}{\rho^2-1}$, $\frac{h}{g}(-r) = \rho$	$r > 0, \leftrightarrow \rho^2 > 1$ $r < 0, \leftrightarrow \rho^2 < 1$
2nd nonreciprocal section 9	 $K = \frac{2r\cos\phi_0(\beta^2 + 2\beta\cos\alpha + 1)}{1-\beta^2}$ $n = \cot\frac{\gamma}{2}^2$, $C = \frac{1-\cos\gamma}{K}$ $L = \frac{K}{r^2(1+\cos\gamma)}$ $R = \frac{K}{-2r(1+\cos\gamma)\cos\phi_0}$	$\sigma = 1$ $f = \psi^2 - 2r(\cos\phi_0)\psi + r^2$ $g = \frac{\cos\gamma^2+1}{\sin\gamma^2}\psi^2 + 2r\left(\frac{(1-\beta^2)\cos\phi_0}{(1-\beta^2)\sin\gamma^2} - \frac{\sin\phi_0}{\tan\alpha\tan\gamma^2}\right)\psi + r^2$ $h = \frac{2\cos\gamma}{\sin\gamma^2}\left(\psi^2 + r\left(\frac{\sin\phi_0}{\tan\alpha} - \frac{(1-\beta^2)\cos\phi_0}{1+\beta^2}\right)\psi\right)$ $\rho = \frac{h}{g}(\psi_0) = \beta e^{j\alpha}$ ψ_0 is a zero of f	$\cos\phi_0 > 0, \leftrightarrow \beta^2 > 1$ $\cos\phi_0 < 0, \leftrightarrow \beta^2 < 1$ $\cos\gamma = \frac{2\beta\sin\alpha}{(1-\beta^2)\tan\phi_0}$ $\left\|\frac{2\beta\sin\alpha}{(1-\beta^2)\tan\phi_0}\right\| < 1$

2.3 Cascade Decomposition from the Scattering Polynomials

Suppose that the three polynomials f, g, h and unimodular number σ, which satisfy the four conditions, are given. Our objective is to synthesize the two-port network which corresponds to the given polynomials as a cascade of elementary sections.

One effective synthesis method consists of decomposing the transfer matrix T of a lossless two-port network (Fig. 2.1) into a product form[2]:

$$T = T_a T_b = T = \frac{1}{f}\begin{bmatrix}\sigma g_* & h \\ \sigma h_* & g\end{bmatrix} \tag{2.7}$$

where T_a and T_b are both transfer matrices of lossless two-ports, in particular T_a corresponds to one of the elementary sections. The canonic polynomials corresponding to T_a shall be designated by f_a, g_a, and h_a, and those corresponding to T_b by f_b, g_b, and h_b. From (2.7),

$$\sigma = \sigma_a \sigma_b, \quad f = f_a f_b \tag{2.8}$$

$$g = g_a g_b + \sigma_a h_{a*} h_b \tag{2.9}$$

$$h = h_a g_b + \sigma_a g_{a*} h_b \tag{2.10}$$

The equation $T = T_a T_b$ is equivalent to

$$T_b = T_a^{-1} T = \frac{1}{f_b}\begin{bmatrix}\sigma_b g_{b*} & h_b \\ \sigma_b h_{b*} & g_b\end{bmatrix} \tag{2.11}$$

where

$$\sigma_b = \sigma/\sigma_a, \quad f_b = \frac{f}{f_a} \tag{2.12}$$

$$g_b = \frac{g_{a*}g - h_{a*}h}{f_a f_{a*}},\ h_b = \frac{g_a h - h_a g}{\sigma_a f_a f_{a*}} \tag{2.13a,b}$$

***Lemma* 2.1**: The reflectance $\rho(\psi_0)$ and delay $d(\psi_0)$ defined by (2.5) and (2.6) or (2.5′) and (2.6′) have the following properties:

1) $\rho(\psi_0)\rho_*(\psi_0) = 1$ (2.14)

2) $|\rho(\psi_0)| = 1$, if ψ_0 is on the $j\omega$ axis

3) $d(\psi_0) = d_*(\psi_0)$, if $f_*(\psi_0) = f(\psi_0) = 0$ (2.15)

4) $d(\psi_0)$ is real if ψ_0 is on the $j\omega$ axis

5) $d(\psi_0)$ is positive if ψ_0 is on the $j\omega$ axis

where ψ_0 is a transmission zero, i.e.

$$f(\psi_0) = 0 \tag{2.16}$$

and

$$\rho_*(\psi_0) = \frac{h_*(\psi_0)}{g_*(\psi_0)},\quad d_*(\psi_0) = \frac{g'(\psi_0)_*}{g(\psi_0)_*} - \frac{h'(\psi_0)_*}{h(\psi_0)_*} \tag{2.17a,b}$$

Proof: 1) The Feldtkeller equation (2.2) and equation (2.16) imply $g(\psi_0)g_*(\psi_0) = h(\psi_0)h_*(\psi_0)$ or $(h(\psi_0))/(g(\psi_0)) = (g_*(\psi_0))/(h_*(\psi_0))$, i.e. $\rho(\psi_0) = 1/(\rho_*(\psi_0))$, which yields equation (2.14).

2) If ψ_0 is on the $j\omega$ axis, say $\psi_0 = j\phi_0$, substitute ψ_0 into (2.14), then

$$\rho(j\phi_0)\rho_*(j\phi_0) = 1$$

Since

$$\rho(j\phi_0)\rho_*(j\phi_0) = \rho(j\phi_0)\rho(-j\phi_0) = \rho(j\phi_0)\rho((j\phi_0)^*) = \rho(j\phi_0)(\rho(j\phi_0))^* = |\rho(j\phi_0)|^2$$

property 2) holds.

3) Differentiating the Feldtkeller equation (2.2), and evaluating at ψ_0, yields

$$(g'g_* - gg'_*)|_{\psi=\psi_0} = (h'h_* - hh'_*)|_{\psi=\psi_0} + (f'f_* - ff'_*)|_{\psi=\psi_0} \tag{2.18}$$

Since $f_*(\psi_0) = f(\psi_0) = 0$, (2.18) becomes

$$(g'g_* - gg'_* - h'h_* + hh'_*)|_{\psi=\psi_0} = 0$$

and

$$hh_*\left(\frac{g'g_*}{hh_*} - \frac{gg'_*}{hh_*} - \frac{h'}{h} + \frac{g'}{g}\right)\Bigg|_{\psi=\psi_0} = 0 \tag{2.19}$$

Substituting $\left(\frac{g_*}{h_*}\right)\Big|_{\psi=\psi_0} = \left(\frac{h}{g}\right)\Big|_{\psi=\psi_0}$ and $\left(\frac{g}{h}\right)\Big|_{\psi=\psi_0} = \left(\frac{h_*}{g_*}\right)\Big|_{\psi=\psi_0}$ into (2.19), (2.19) becomes $hh_*\left(\frac{g'}{g} - \frac{g'_*}{g_*} - \frac{h'}{h} + \frac{h'_*}{h_*}\right)\Big|_{\psi=\psi_0} = 0$. Taking the definitions of $d(\psi_0)$ and $d_*(\psi_0)$ into account, gives $d(\psi_0) = d_*(\psi_0)$, i.e. (2.15) holds.

4) If, as in property 2), ψ_0 is on the $j\omega$ axis, i.e. $\psi_0 = j\phi_0$; then

$$d_*(\psi_0) = d(-j\omega_0) = d(\psi_0^*) = (d(\psi_0))^*$$

This means that the value of the delay at the transmission zero on the $j\omega$ axis is real, i.e. property 4) is proven.

5) From the definitions of the admittance and the impedance

$$Y(\psi) = \frac{g(\psi) - h(\psi)}{g(\psi) + h(\psi)},\ Z(\psi) = \frac{g(\psi) + h(\psi)}{g(\psi) - h(\psi)}$$

which are positive real [33], it follows that

$$Y'(\psi) = \frac{2gh}{(g+h)^2}\left(\frac{g'}{g} - \frac{h'}{h}\right), \text{ if } \frac{h(\psi)}{g(\psi)} \neq -1 \text{ and } Z'(\psi) = \frac{-2gh}{(g-h)^2}\left(\frac{g'}{g} - \frac{h'}{h}\right), \text{ if } \frac{h(\psi)}{g(\psi)} \neq 1$$

Furthermore,

$$d(\psi) = \frac{1}{2}Y'(\psi)\left(1 + \frac{h(\psi)}{g(\psi)}\right)\left(1 + \frac{g(\psi)}{h(\psi)}\right), \text{ if } \frac{h(\psi)}{g(\psi)} \neq -1$$

$$d(\psi) = \frac{1}{2}Z'(\psi)\left(1 - \frac{h(\psi)}{g(\psi)}\right)\left(1 - \frac{g(\psi)}{h(\psi)}\right), \text{ if } \frac{h(\psi)}{g(\psi)} \neq 1$$

Substituting $\psi = \psi_0 = j\phi_0$ into the above two equations, yields

$$d(\psi_0) = \frac{1}{2}Y'(j\phi_0)(1 + \rho(j\phi_0))\left(1 + \frac{1}{\rho(j\phi_0)}\right), \text{ if } \rho(j\phi_0) \neq -1$$

$$d(\psi_0) = \frac{1}{2}Z'(j\phi_0)(1-\rho(j\phi_0))\left(1 - \frac{1}{\rho(j\phi_0)}\right), \text{ if } \rho(j\phi_0) \neq 1,$$

By property (2), it can be assumed that $\rho(j\phi_0) = e^{j\alpha}$, $-\pi < \alpha \leq \pi$, then the above two equations become

$$d(\psi_0) = Y'(j\phi_0)(1 + \cos\alpha), \ \alpha \neq \pi$$

$$d(\psi_0) = Z'(j\phi_0)(1 - \cos\alpha), \ \alpha \neq 0$$

Finally, the property 5) can be obtained by referring to a property of positive real functions, namely that $Y'(j\phi_0) > 0$ and $Z'(j\phi_0) > 0$ [32].

***Lemma* 2.2**[2]: The value of the reflectance and delay functions of a lossless two-port network evaluated at a transmission zero of the first member of a cascade are equal to the corresponding value of that member, i.e.

$$\rho_a(\psi_0) = \rho(\psi_0) \tag{2.20}$$

$$d_a(\psi_0) = d(\psi_0) \tag{2.21}$$

where $f_a(\psi_0) = 0$.

Proof: Starting from the definition of reflectance and substituting (2.9)-(2.10) for g and h yields

$$\rho = \frac{h}{g} = \frac{h_a g_b + \sigma_a g_{a*} h_b}{g_a g_b + \sigma_a h_{a*} h_b} = \frac{h_a}{g_a} \frac{g_b + \sigma_a \frac{g_{a*}}{h_a} h_b}{g_b + \sigma_a \frac{h_{a*}}{g_a} h_b} \tag{2.22}$$

$g_a g_{a*} = f_a f_{a*} + h_a h_{a*}$ and $f_a(\psi_0) = 0$ imply $\frac{g_{a*}}{h_a} = \frac{h_{a*}}{g_a}$ at $\psi = \psi_0$. Substituting this result into (2.22) yields the desired (2.20).

From the definition of delay and again substituting (2.9)-(2.10) for g and h yields

$$d = \left(\ln\frac{g}{h}\right)' = \left(\ln\left(\frac{g_a}{h_a}\frac{1 + \sigma_a \frac{h_{a*}}{g_a}\frac{h_b}{g_b}}{1 + \sigma_a \frac{g_{a*}}{h_a}\frac{h_b}{g_b}}\right)\right)' = d_a + \left(\ln\left(\frac{1 + \sigma_a \frac{h_{a*}}{g_a}\frac{h_b}{g_b}}{1 + \sigma_a \frac{g_{a*}}{h_a}\frac{h_b}{g_b}}\right)\right)' \tag{2.23}$$

Making use of $\frac{g_{a*}}{h_a} = \frac{h_{a*}}{g_a}$ and $d_a = (d_a)_*$ at $\psi = \psi_0$, after some calculations, the value of the second term on the right hand side of (2.23) is zero which implies (2.21) holds.

***Theorem* 2.1**: Suppose that number σ and three polynomials g, h and $f = f_a f_b$ satisfy conditions 1, 2, 3, 4 (see 2.1), then so do σ_a and $\{f_a, g_a, h_a\}$ which correspond to one of the elementary sections, then the number σ_b and $\{f_b, g_b, h_b\}$ determined by (2.12)-(2.13) also fulfil the four conditions (rewritten here for easy reference)

1. Polynomials f_b, g_b and h_b are real in some complex frequency variable, say ψ,

2. f_b, g_b and h_b are related by the Feldtkeller equation

$$g_b g_{b*} = f_b f_{b*} + h_b h_{b*} \tag{2.24}$$

3. $g_b(\psi)$ is a Hurwitz polynomial with all its zeros strictly in the left-hand plane.

4. σ_b is either +1 or -1. For reciprocal two-ports,

$$\sigma_b = f_b / f_{b*} \tag{2.25}$$

Proof: 1. Since f_a is a factor of f, so taking (2.12) into account it is obvious that f_b is a real polynomial. From (2.13), it is known that, in order to prove g_b and h_b are both real polynomials, it must be shown that $f_a f_{a*}$ divides the numerators of (2.13). In other words, the numerators should contain the zeros of $f_a f_{a*}$.

The numerators of $g_b(\psi)$ and $h_b(\psi)$ will be called $p(\psi)$ and $q(\psi)$, respectively, i.e.

$$p(\psi) = g(\psi)g_{a*}(\psi) - h(\psi)h_{a*}(\psi) \tag{2.26}$$

$$q(\psi) = g_a(\psi)h(\psi) - h_a(\psi)g(\psi) \tag{2.27}$$

Assume ψ_0 is a zero of $f_a(\psi)f_{a*}(\psi)$, then there are two situations:

1) ψ_0 is not on the $j\omega$ axis

2) ψ_0 is on the $j\omega$ axis

Situation1) corresponds to the elementary sections 8-9 in which ψ_0 is a single zero of $f_a(\psi)f_{a*}(\psi)$. Situation 2) corresponds to the elementary sections 1-7. In this case, if ψ_0 is a zero of $f_a(\psi)$, it must also be a zero of $f_{a*}(\psi)$, and vise versa, i.e. $f_a(\psi)f_{a*}(\psi)$

has a factor $(\psi-\psi_0)^2$.

Therefore, for situation 1), it is only necessary to prove that

$$p(\psi_0) = 0,\ q(\psi_0) = 0 \tag{2.28a,b}$$

From (2.26),

$$p(\psi_0) = g(\psi_0)g_{a*}(\psi_0)\left(1-\frac{h(\psi_0)h_{a*}(\psi_0)}{g(\psi_0)g_{a*}(\psi_0)}\right) = g(\psi_0)g_{a*}(\psi_0)(1-\rho(\psi_0)\rho_{a*}(\psi_0))$$

the equation (2.28a) follows from ***Lemmas* 2.1**, **2.2**, namely,

$$\rho_{a*}(\psi_0) = 1/\rho_a(\psi_0) = 1/\rho(\psi_0)$$

From (2.27) and ***Lemma* 2.2**

$$q(\psi_0) = g(\psi_0)g_a(\psi_0)\left(\frac{h(\psi_0)}{g(\psi_0)}-\frac{h_a(\psi_0)}{g_a(\psi_0)}\right) = g(\psi_0)g_{a*}(\psi_0)(\rho(\psi_0)-\rho_a(\psi_0)) = 0$$

For situation 2), rewrite equations (2.26) and (2.27) in a Taylor expansion form:

$$p(\psi) = p(\psi_0)+p'(\psi_0)(\psi-\psi_0)+O(\psi-\psi_0)^2 \tag{2.29}$$

and

$$q(\psi) = q(\psi_0)+p'(\psi_0)(\psi-\psi_0)+O(\psi-\psi_0)^2 \tag{2.30}$$

Since $p(\psi_0) = 0$ and $q(\psi_0) = 0$, it is required to show that

$$p'(\psi_0) = 0,\quad q'(\psi_0) = 0 \tag{2.31}$$

In fact,

$$p'(\psi_0) = (g'g_{a*}-g(g_a{}')_*+h(h_a{}')_*-h'h_{a*})\big|_{\psi=\psi_0}$$

$$= hh_{a*}\left(\frac{g'g_{a*}}{hh_{a*}}-\frac{g(g_a{}')_*}{hh_{a*}}+\left(\frac{h_a{}'}{h_a}\right)_*-\frac{h'}{h}\right)\Bigg|_{\psi=\psi_0}$$

By ***Lemmas*** **2.1, 2.2,** $\frac{g_{a*}}{h_{a*}} = \rho_a = \rho = \frac{h}{g}$ and $\frac{g}{h} = \frac{1}{\rho} = \frac{h_{a*}}{g_{a*}}$ at $\psi = \psi_0$. Thus, from the definitions of $d(\psi_0)$, $d_*(\psi_0)$ and ***Lemmas*** **2.1, 2.2,**

$$p'(\psi_0) = hh_{a*}\left(\frac{g'}{g} - \left(\frac{g_a'}{g_a}\right)_* + \left(\frac{h_a'}{h_a}\right)_* - \frac{h'}{h}\right)\Bigg|_{\psi = \psi_0} = hh_{a*}(d - d_{a*}) = 0$$

Differentiating equation (2.27) and using ***Lemmas*** **2.1, 2.2,** yields

$$q'(\psi_0) = (g_a'h + g_ah' - h_a'g - h_ag')|_{\psi = \psi_0}$$

$$= gg_a\left(\frac{g_a'h}{g_ag} + \frac{h'}{g} - \frac{h_a'}{g_a} - \frac{h_ag'}{g_ag}\right)\Bigg|_{\psi = \psi_0}$$

$$= gg_a\left(\frac{g_a'}{g_a}\rho - \frac{h_a'}{h_a}\rho - \frac{g'}{g}\rho + \frac{h'}{h}\rho\right)\Bigg|_{\psi = \psi_0}$$

$$= gg_a\rho(d_a - d) = 0$$

Therefore, $p(\psi) = O(\psi - \psi_0)^2$ and $p(\psi) = O(\psi - \psi_0)^2$ hold, which means the numerators of g_b and h_b do include the factor f_af_{a*}, and the fact that g_b and h_b are real polynomials is proved.

2. Substituting (2.13) into (2.24) and by simple calculation,

$$g_bg_{b*} - h_bh_{b*} = \frac{(g_{a*}g - h_{a*}h)(g_ag_* - h_ah_*)}{f_af_{a*}f_af_{a*}} - \frac{(g_ah - h_ag)(g_{a*}h_* - h_{a*}g_*)}{f_af_{a*}f_af_{a*}}$$

$$= \frac{(gg_* - hh_*)(g_ag_{a*} - h_ah_{a*})}{f_af_{a*}f_af_{a*}} = \frac{ff_*f_af_{a*}}{f_af_{a*}f_af_{a*}} = f_bf_{b*}$$

which means the Feldtkeller equation holds.

3. From (2.2) (with subscript a) and (2.24), it follows that

$$\left|\frac{h_a}{g_a}\right| \le 1,\ \left|\frac{h_b}{g_b}\right| \le 1,\ \psi = j\phi\ (\infty < \phi < \infty)$$

From (2.9), $g = g_a g_b + \sigma_a h_{a*} h_b \neq 0$, at $\psi = j\phi$, because $g(\psi)$ is a Hurwitz polynomial and $\left|\frac{\sigma h_{a*} h_b}{g_a g_b}\right| = \left|\frac{\sigma h_{a*}}{g_a}\right|\left|\frac{h_b}{g_b}\right| = \left|\frac{h_a}{g_a}\right|\left|\frac{h_b}{g_b}\right| \le 1$, i.e., $|\sigma h_{a*} h_b| \le |g_a g_b|$. So, it follows by the extended Rouche's theorem[4] that g and $g_a g_b$ have the same number of zeros in the right-hand plane. Hence the fact that g is Hurwitz, i.e., there are no zeros in the right-hand plane implies g_a, g_b are Hurwitz polynomials.

4. It is obvious that $\sigma_b = \frac{\sigma}{\sigma_a} = \pm 1$, and for a reciprocal two-port and a reciprocal section a:

$$\sigma_b = \frac{\sigma}{\sigma_a} = \frac{f/f_*}{f_a/f_{a*}} = f_b/f_{b*}$$

Therefore, ***Theorem* 2.1** is proved.

***Theorem* 2.2**. Assume that polynomials g, g_a and g_b are the same as in ***Theorem* 2.1**. Let n denote the degree of g, and n_a and n_b are the degrees of g_a and g_b respectively, then the equation

$$n = n_a + n_b \tag{2.32}$$

holds.

Proof: First, from (2.9), it is easy to see that

$$n \le n_a + n_b \tag{2.33}$$

Therefore, it is required to prove that

$$n \ge n_a + n_b \tag{2.34}$$

From the Feldtkeller equation

$$g_a g_{a*} = f_a f_{a*} + h_a h_{a*} \tag{2.35}$$

it is known that the degree of f_a and the degree of h_a are less than or equal to n_a and either the degree of f_a or the degree of h_a equals n_a. Hence only the following two situations are in consideration.

1) degree of $f_a = n_a$

equation (2.13) yields $n_b \le n_a + n - 2n_a = n - n_a$, i.e., $n \ge n_a + n_b$.

2) degree of $f_a < n_a$

From the table of elementary sections, it is known that this case includes only two sections:

Section 1: $f_a = d_a$, $g_a = \psi + d_a$, $h_a = \psi$, $\sigma_a = 1$, $\rho_a(\infty) = 1$, $d_a > 0$,

Section 2: $f_a = d_a$, $g_a = \psi + d_a$, $h_a = -\psi$, $\sigma_a = 1$, $\rho_a(\infty) = -1$, $d_a > 0$,

In this case, based on $f = f_a f_b$ and (2.2), it follows directly that the degree of $f < n$ and the degree of $h = n$. Therefore it is can be assumed that

$$g(\psi) = g_0\psi^n + g_1\psi^{n-1} + \dots + g_n,\ h(\psi) = kg_0\psi^n + h_1\psi^{n-1} + \dots + h_n \tag{2.36}$$

where $k = \begin{cases} 1, if\ \ \rho_a(\infty) = 1 \\ -1, if\ \ \rho_a(\infty) = -1 \end{cases}$. Substituting $f_a = d_a$, $g_a = \psi + d_a$, $h_a = k\psi$ into

(2.13a),

$$g_b(\psi) = \frac{(-\psi + d_a)(g_0\psi^n + g_1\psi^{n-1} + \dots + g_n) + k\psi(kg_0\psi^n + h_1\psi^{n-1} + \dots + h_n)}{dd}$$

On the right-hand side of the above equation the coefficient of ψ^{n+1} is

$-g_0 + k^2 g_0 = 0$ and the coefficient of ψ^n, say c, is given by

$$c = d_a g_0 - g_1 + kh_1 . \tag{2.37}$$

On the other hand, the delay at infinity is computed by using (2.6′) which yields

$d = \frac{g_1}{g_0} - \frac{h_1}{kg_0}$. Now take into account that $d = d_a$, it follows that

$$d_a g_0 = g_1 - \frac{h_1}{k} = g_1 - kh_1$$

which yields $c = 0$.

Thus the inequality $n_b \le n - 1 = n - n_a$ holds and therefore the inequality (2.34) holds.

Thus if a set of canonic polynomials $\{f, g, h\}$ with f in factored form is given, then based on ***Theorem* 1** and ***Theorem* 2**, a synthesis algorithm which realizes the circuits can be given as follows:

1). Select a transmission zero $\{\psi_a : f_a(\psi_a) = 0\}$, compute the reflectance ρ_a according (2.5) or (2.5′) and the delay d_a for a reciprocal section according to (2.6) or (2.6′).

2). Referring to the elementary section tables, obtain $\{\sigma_a, f_a, g_a, h_a\}$ and compute $\{\sigma_b, f_b, g_b, h_b\}$ according to formulae (2.12)-(2.13).

3). Drop subscript b and return to step 1) until all the transmission zeros are extracted.

4). Extract a zero-order section.

After all the factors of f have been exhausted, the remainder polynomials correspond to a zero-order section, i.e., an ideal transformer or a gyrator must be extracted to complete the synthesis, as shown in Fig. 2.2 (a), (b), depending on whether σ is +1 or -1, respectively.

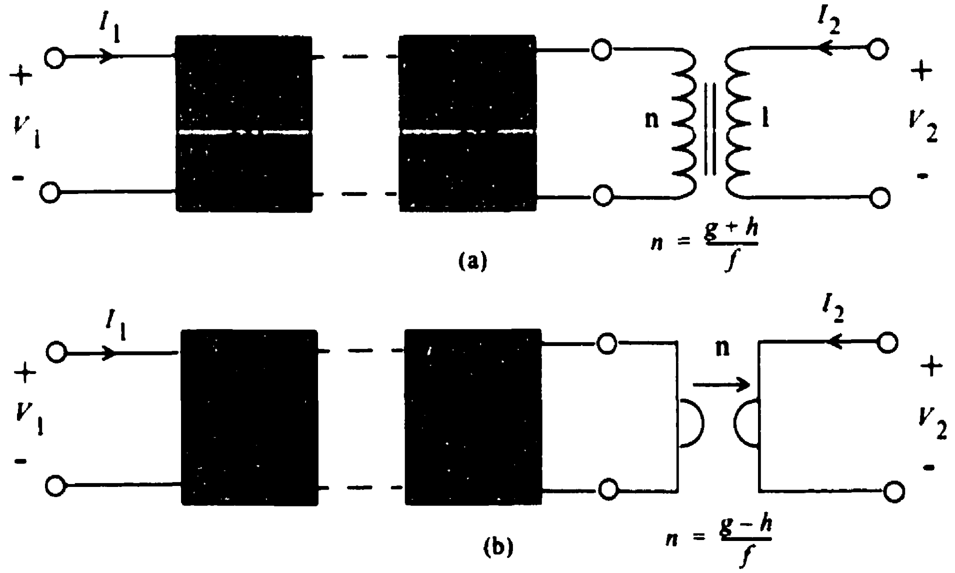


Fig. 2.2

Note: when considering numerical computation, more attention must be paid to step 3 where g_b and h_b are calculated by using (2.13). The polynomials in (2.13) are in product representation which is preferred to coefficient representation, since the frequency responses of narrow-band filters are very sensitive to coefficients. g_b and h_b can be obtained[29] in the same form by using the Newton-Maehly algorithm[31]. If this algorithm does not converge for a particular starting value, Muller's method followed by the Secant method[31] can be used to obtained an improved starting value. The combination of these algorithms has proven to be successful for a large number of circuits which have been decomposed (see Appendix I for more details).

2.3 Wave Digital Filter Realization

After the decomposition of a lossless two-port network is finished, i.e, a network like Fig. 2.2 (a) or (b) is obtained, there are two efficient methods to transform it into a wave digital filter realization structure.

The first one is to refer to the tables in [2], where for every analog elementary section there is a wave digital elementary section corresponding to it. So it is easy to obtain the wave digital equivalents for the analog networks presented in Fig. 2.2.

Another method is using the ladder wave digital structure. The details about how to map an analog network into its ladder wave digital equivalent are presented in Antoniou's book[10]: Digital filters.

Therefore, for a given digital specification, the wave digital filter which satisfies the specification can be obtained by the following steps.

Step 1. Pre-warp the frequency axis of the frequency specification using

$\phi = \tan\left(\frac{\omega T}{2}\right)$, where ω and ϕ are digital and analog frequencies respectively, T is the sampling frequency.

Step 2. Use this pre-warped frequency to design the analog filter, for example, a Butter worth, a Chebyshev, or a Cauer filter and obtain the Belevitch's polynomial set $\{f(\psi), g(\psi), h(\psi)\}$.

Step 3. Follow the algorithm proposed in section **2.2** and derive a decomposed analog network, Fig. 2.2 (a) or (b).

Step 4. Transfer the analog network to its ladder wave digital equivalent.

An option at Step 2, is to switch to the following Step 3′.

Step 3′. Follow the algorithm proposed in section **2.2** and refer to the tables in [2], then derive the wave digital structure directly.

2.4 Illustrative Algorithm Example

To illustrate the algorithm proposed in the previous section, a simple example is shown here.

Example: specifications $A_p = 0.4\text{dB}$, $A_s = 40\text{dB}$, $\omega_p = 25$, $\omega_s = 41$, $T = \frac{\pi}{50}$

Step 1. Pre-warp the frequency axis and obtain $\phi_p = 1$, $\phi_s = 3.4$.

Step 2. Three canonic polynomials and the number σ are obtained as follow:

$$f = \psi^2 + 16,\ \sigma = 1$$

$$g = \frac{7}{9}\psi^4 + \frac{1363}{100}\psi^3 + \frac{2353}{100}\psi^2 + \frac{6268}{225}\psi + \frac{724}{45}$$

$$h = -\frac{7}{9}\psi^4 + \frac{1237}{100}\psi^3 - \frac{247}{100}\psi^2 + \frac{1732}{225}\psi - \frac{76}{45}$$

Step 3: Follow the algorithm proposed in section **2.2** and derive a decomposed analog network:

1) Calculate the reflectance and the delay at the first transmission zero:

$$\psi_1 = 4j,\ \rho_1 = 1,\ d_1 = \frac{1}{4}$$

2) From the elementary section tables, select $\{\sigma_1, f_1, g_1, h_1\}$ and compute

$\{\sigma_b, f_b, g_b, h_b\}$, according to (2.13):

$$\sigma_1 = 1,\ f_1 = \psi^2 + 16,\ g_1 = \psi^2 + 8\psi + 16,\ h_1 = 8\psi$$

$$\sigma_b = 1,\ f_b = 1,\ g_b = \frac{7}{9}\psi^2 + \frac{1067}{900}\psi^2 + \frac{181}{180},\ h_b = -\frac{7}{9}\psi^2 - \frac{67}{900}\psi - \frac{19}{180}$$

3) Drop subscript b and return to 1), the values of the reflectance and the delay at transmission zero $\psi_2 = \infty$ are $\rho_2 = -1$, $d_2 = 10/7$, and also

$$\sigma_2 = 1, f_2 = \frac{10}{7}, g_2 = \psi + \frac{10}{7}, h_2 = -\psi$$

$$\sigma_b = 1, f_b = \frac{7}{10}, g_b = \frac{7}{18}\psi + \frac{1267}{1800}, h_b = \frac{7}{18}\psi - \frac{133}{1800}$$

Return to 1) again, compute the values of the reflectance and the delay at transmission zero $\psi_3 = \infty$ as $\rho_3 = 1$, $d_3 = 2$, and select

$$\sigma_3 = 1, f_3 = 2, g_3 = \psi + 2, h_3 = \psi$$

4) Extract the ideal transformer.

$$f_4 = \frac{7}{20}, g_4 = \frac{1267}{3600}, h_4 = -\frac{133}{3600}, n = \frac{g_4 + h_4}{f_4} = \frac{9}{10}$$

The final realized circuit is shown in Fig. 2.3.

Step 4. Transfer the analog two-port network in Fig. 2.3 to its ladder wave digital equivalent which is shown in Fig. 2.4.

Step 5 Calculate all the parameters according to the approach in [1], [11], [10].

$$\gamma_1 = \frac{1}{17}, \gamma_2 = \frac{85}{316}, \beta_1 = \frac{17}{33}, \beta_2 = \frac{1102}{2461}, \beta_3 = \frac{1975}{2303}$$

After 8 bits of quantization, the above parameters become

$$\gamma_1 = \frac{15}{256}, \gamma_2 = \frac{69}{256}, \beta_1 = \frac{33}{64}, \beta_2 = \frac{115}{256}, \beta_3 = \frac{55}{64}, n = \frac{115}{128}$$

The frequency response of the above ladder wave digital filter is presented in Fig. 2.5 which shows that the specifications are satisfied.

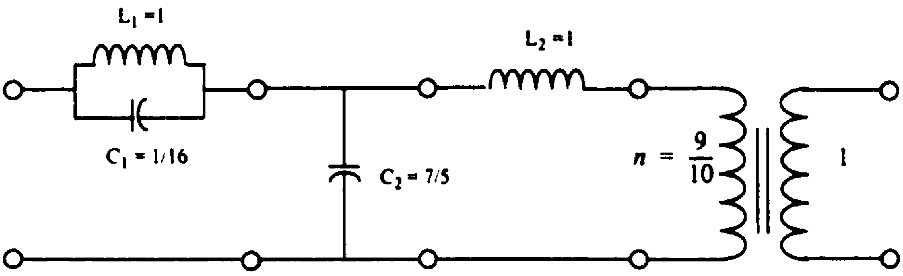


Fig. 2.3 Analog two-port network

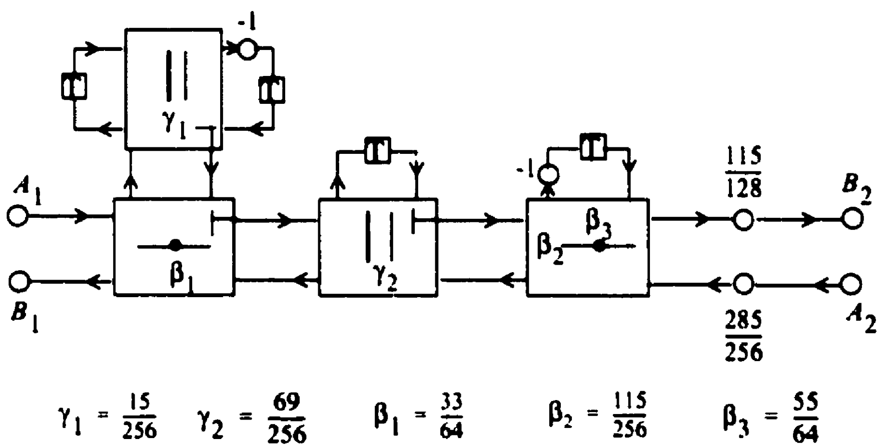
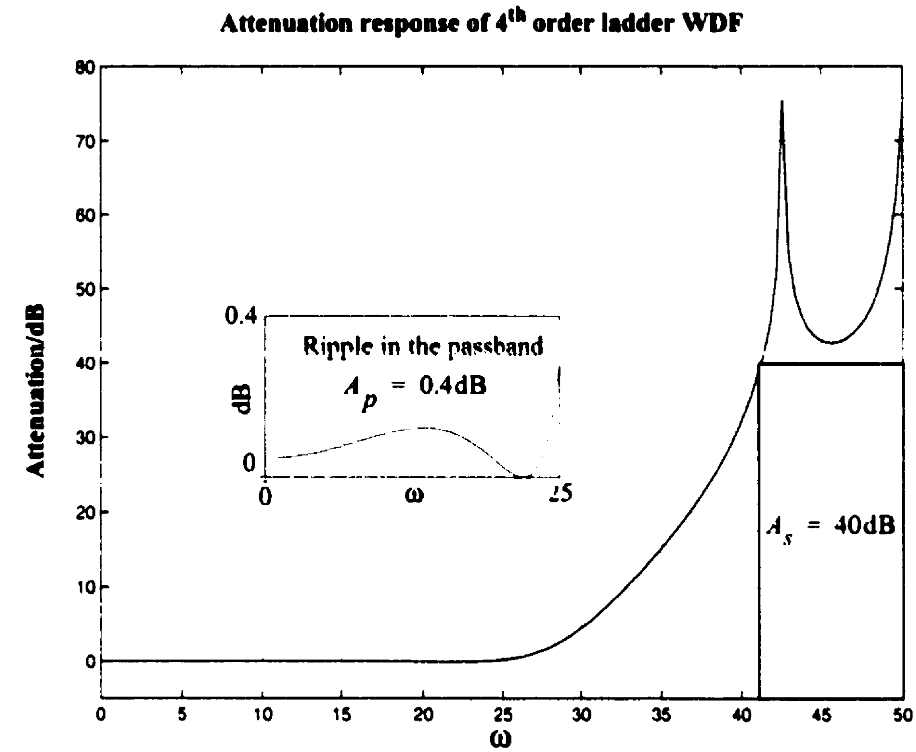


Fig. 2.4 Ladder wave digital equvalent


Attenuation response of 4th order ladder WDF
Attenuation/dB
Ripple in the passband
A_p = 0.4dB
dB
ω
A_s = 40dB
ω

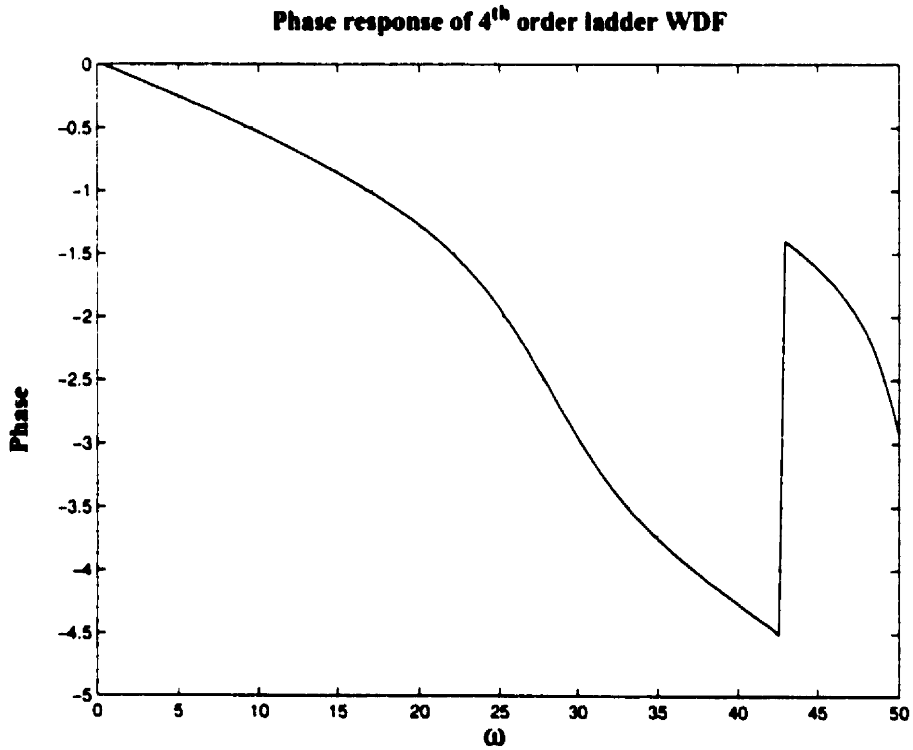

Phase response of 4th order ladder WDF
Phase
ω

Fig. 2.5

Chapter 3
Realization of FIR Wave Digital Filters by Factorization of the Scattering Matrix

In the previous chapter, the cascade decomposition with the transfer matrix of a lossless two-port network is discussed. In fact, a lossless two-port network N can also be synthesized by factorization of the scattering matrix [1]. Fettweis presented the resulting structure and its corresponding wave flow diagram as follows.

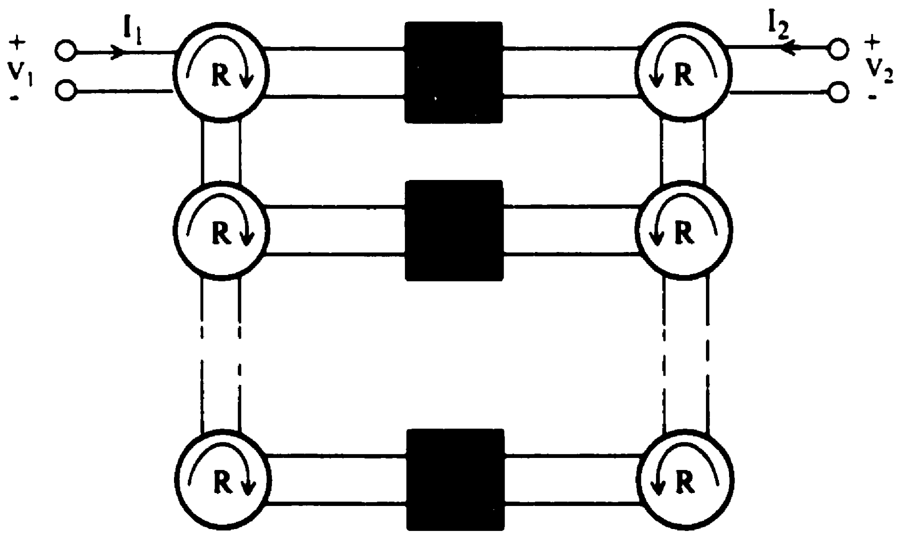


Fig. 3. 1. Synthesis of a classical two-port by scattering matrix factorization

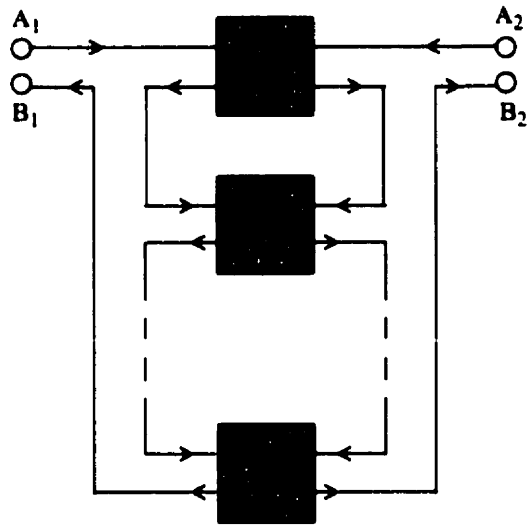


Fig. 3.2. Wave Flow Diagram

In Figs 3.1, 3.2, N_l to N_n are lossless two-ports of lower degree than that of N. Therefore, the question is how to decompose the network N into a series sub-networks N_i (i = 1, 2,...., n), which appear to offer advantages over those obtained directly. In the next section, an application of this approach to an FIR filter is discussed.

3.1 Some Basic Characteristics of FIR Filters [10]

FIR filters have a finite number of terms in the impulse response, therefore, the output can be written as a finite convolution sum

$$y(n) = \sum_{m=0}^{N} h(m)x(n-m) \tag{3.1}$$

where $x(n)$ is the input and $h(n)$ is the length-N impulse response. The transfer function of an FIR filter is given by the z transform of $h(n)$ as

$$H(z) = \sum_{n=0}^{N} h(n)z^{-n} \tag{3.2}$$

For the FIR filter to have linear phase, the impulse response $h(n)$ must be either symmetric ($h(n) = h(N-n)$) or anti-symmetric ($h(n) = -h(N-n)$.

There are several design methods[10], [12]. The basic design procedure is to construct an ideal lowpass filter in the frequency domain and use the inverse z transform to derive $h_{ideal}(n)$. The impulse response $h_{ideal}(n)$ is then truncated by a window function. However, the truncated filter is typically not causal, i.e $h(n) \neq 0$, $n < 0$. To make the truncated filter causal, multiply by z^{-k}, where k is selected as the minimum positive integer such that the truncated FIR filter is causal.

The classical implementation of an FIR filter is to use the transversal structure as follows:

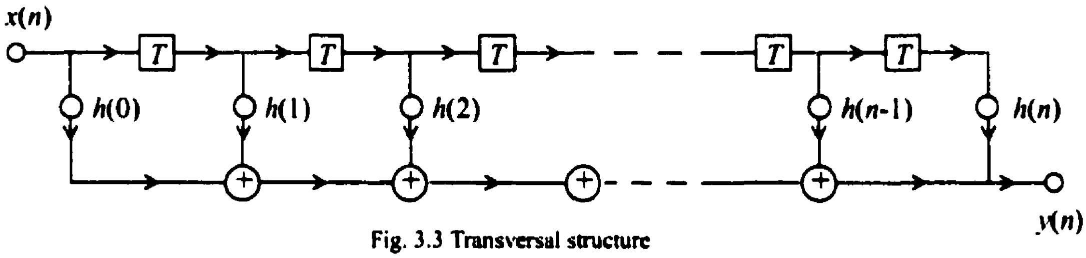


Fig. 3.3 Transversal structure

3.2 Belevitch's Representation for an FIR filter

Consider the given FIR filter (3.2), for convenience, rewrite it as

$$H(z) = f_0 + f_1 z^{-1} + f_2 z^{-2} + \ldots + f_n z^{-n},\ f_n, f_0 \neq 0 \tag{3.3}$$

and assume $H(z)$ is scaled so that $|H(z)| \le 1$ on the unit circle in the z^{-1}-domain. Also assume the polynomial set $\{f(z), g(z), h(z)\}$ is the Belevitch's representation in the z^{-1}-domain. Then

$$f(z) = f_0 + f_1 z^{-1} + f_2 z^{-2} + \dots + f_n z^{-n}$$

and its para conjugate in the z^{-1}-domain is

$$f_*(z) = z^{-n} f(z^{-1}) = f_n + f_{n-1} z^{-1} + f_{n-2} z^{-2} + \dots + f_0 z^{-n}$$

Further $g(z) = 1$, $g_*(z) = z^{-n}$, where $g_*(z)$ is the para conjugate of $g(z)$. The equation

$$h(z)h_*(z) = g(z)g_*(z) - f(z)f_*(z)$$

follows from the Feldtkeller equation $gg_* = ff_* + hh_*$. Next rewrite the right-hand side in product form as

$$h(z)h_*(z) = c\prod_{i=1}^{2n}(z^{-1} - a_i) \tag{3.4}$$

where c is a constant and a_i, $i = 1, 2, \dots, n$, are the zeros of $h(z)h_*(z)$. $h(z)$ can be separated from (3.4) by using the property that every zero of $h(z)$ must be a reciprocal of a zero of $h_*(z)$ [29]. If a_i is a zero of $h(z)$, there must exist an a_j satisfying $a_j = 1/a_i$ in the set of zeros of $h(z)h_*(z)$, which can be allotted as a zero to $h_*(z)$. After all elements in the set of zeros of $h(z)h_*(z)$ have been exhausted, all the zeros of $h(z)$ are obtained.

The constant factor of $h(z)$, namely k can be determined by $k = \pm\sqrt{c/\left(\prod_{i=1}^{n}(-b_i)\right)}$, where b_i, $i = 1, 2, \ldots, n$, are the zeros of $h(z)$ and the ratio $c/\left(\prod_{i=1}^{n}(-b_i)\right)$ must be positive (see Appendix II). Based on this idea, using a program written in MATLAB, $h(z)$ which has the same degree as $f(z)$ can be obtained, say,

$$h(z) = h_0 + h_1 z^{-1} + h_2 z^{-2} + \ldots + h_n z^{-n}, \quad h_n, h_0 \neq 0$$

and by the definition: $h_*(z) = z^{-n}h(z^{-1})$, its para conjugate is

$$h_*(z) = h_n + h_{n-1}z^{-1} + h_{n-2}z^{-2} + \ldots + h_0 z^{-n}$$

Thus, the scattering matrix $S = \frac{1}{g(z)}\begin{bmatrix} h(z) & \sigma f_*(z) \\ f(z) & -\sigma h_*(z) \end{bmatrix}$ can be written as

$$S = \begin{bmatrix} h_0 + h_1 z^{-1} + \ldots + h_n z^{-n} & \sigma(f_n + f_{n-1}z^{-1} + \ldots + f_0 z^{-n}) \\ f_0 + f_1 z^{-1} + \ldots + f_n z^{-n} & -\sigma(h_n + h_{n-1}z^{-1} + \ldots + h_0 z^{-n}) \end{bmatrix} \tag{3.5}$$

Additionally, by the Feldtkeller equation, the elements of the matrix should satisfy

$$\sum_{i=0}^{n}\sum_{j=0}^{n} f_i f_{n-j} z^{-(i+j)} + \sum_{i=0}^{n}\sum_{j=0}^{n} h_i h_{n-j} z^{-(i+j)} = z^{-n}$$

Consider the term z^{-2n} and obtain an important equation

$$f_n f_0 + h_n h_0 = 0 \tag{3.6}$$

which will be useful later.

3.3 A New Implementation Structure and Algorithm for FIR Filters

In [1], Fettweis proposed two implementation structures which are shown in Figs. 3.7-3.8. In this section, applying the approach of Figs. 3.1, 3.2 to FIR filter (3.3), a general implementation structure and algorithm are presented. Starting from this general implementation structure and algorithm, a series of implementation structures and algorithms, which include Fettweis' structures, are derived.

Assume S and S_i, $i = 1, 2, \ldots, n$, are the scattering matrices corresponding to networks N and N_i ($i = 1, 2, \ldots, n$) (Figs. 3.1, 3.2) respectively,

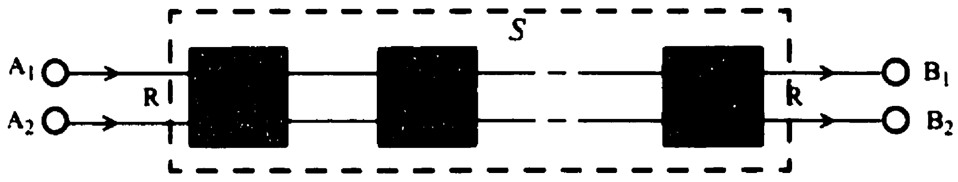


Fig. 3. 4.

Then from Fig. 3.4, it is easy to derive that

$$S = S_n S_{n-1} \ldots S_1 \tag{3.7}$$

Therefore, the question is how to decide the form of S_i ($i = 1, 2, \ldots, n$) such that network N is decomposed into a series of realizable sub-networks N_i ($i = 1, 2, \ldots, n$). This is discussed next.

Suppose that a lossless two-port network N for the FIR filter can be decomposed into two lossless two-port sub-networks N_a, N_b, (see Fig. 3.4), where N_a is called an elementary section,

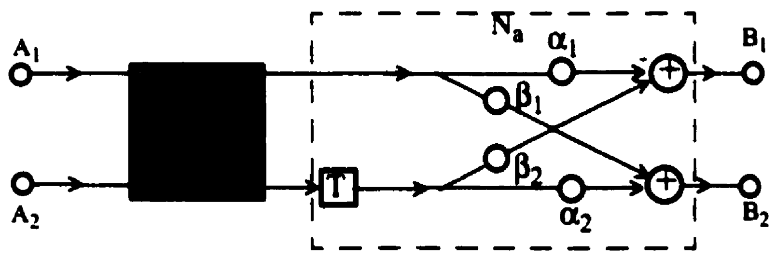


Fig. 3.5

and S, S_a and S_b are the scattering matrices corresponding to Networks N, N_a, and N_b respectively.

Then $S = S_a S_b$ and $S_a = S_{\alpha\beta} S_z = \begin{bmatrix} -\alpha_1 & \beta_1 \\ \beta_2 & \alpha_2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & z^{-1} \end{bmatrix}$, i.e. $S = S_{\alpha\beta} S_z S_b$, which implies

$S_z S_b = S_{\alpha\beta}^{-1} S$. Note that $S_{\alpha\beta}^{-1} = \dfrac{1}{\alpha_1\alpha_2 + \beta_1\beta_2} \begin{bmatrix} -\alpha_2 & \beta_1 \\ \beta_2 & \alpha_1 \end{bmatrix}$ and consider (3.5). This yields

$$S_z S_b = \frac{1}{\alpha_1\alpha_2 + \beta_1\beta_2} \begin{bmatrix} -\alpha_2 & \beta_1 \\ \beta_2 & \alpha_1 \end{bmatrix} \begin{bmatrix} \sum\limits_{i=0}^{n} h_i z^{-i} & \sigma \sum\limits_{i=0}^{n} f_{n-i} z^{-i} \\ \sum\limits_{i=0}^{n} f_i z^{-i} & -\sigma \sum\limits_{i=0}^{n} h_{n-i} z^{-i} \end{bmatrix}$$

$$= \frac{1}{\alpha_1\alpha_2 + \beta_1\beta_2} \begin{bmatrix} \sum\limits_{i=0}^{n} (f_i\beta_1 - h_i\alpha_2) z^{-i} & -\sigma \sum\limits_{i=0}^{n} (h_{n-i}\beta_1 + f_{n-i}\alpha_2) z^{-i} \\ \sum\limits_{i=0}^{n} (h_i\beta_2 + f_i\alpha_1) z^{-i} & \sigma \sum\limits_{i=0}^{n} (f_{n-i}\beta_2 - h_{n-i}\alpha_1) z^{-i} \end{bmatrix} \tag{3.8}$$

The desired degree of S_b is n-1 which can be reached if and only if

$$\begin{cases} f_n\beta_1 - h_n\alpha_2 = 0 & (a) \\ h_0\beta_2 + f_0\alpha_1 = 0 & (b) \\ h_0\beta_1 + f_0\alpha_2 = 0 & (c) \\ f_n\beta_2 - h_n\alpha_1 = 0 & (d) \end{cases} \tag{3.9}$$

Making use of (3.6), i.e., $\dfrac{h_n}{f_n} = -\dfrac{f_0}{h_0}$, then from (3.9), $\dfrac{\beta_2}{\alpha_1} = \dfrac{\beta_1}{\alpha_2} = \dfrac{h_n}{f_n}$. Let $\alpha_1 = \alpha$, $\beta_2 = \beta$ and

$\alpha_2 = \kappa\alpha$, $\beta_1 = \kappa\beta$, $\kappa \neq 0$. Then the general form for $S_{\alpha\beta}$ is

$$S_{\alpha\beta} = \begin{bmatrix} -\alpha & \kappa\beta \\ \beta & \kappa\alpha \end{bmatrix}, \quad \frac{\beta}{\alpha} = \frac{h_n}{f_n} \tag{3.10a,b}$$

If $S_{\alpha\beta}$ is required to be normalized, i.e, $S_{\alpha\beta}^T S_{\alpha\beta} = I$, where I is the identity matrix, then κ, α and β should satisfy $\kappa^2 = 1$ and $\alpha^2 + \beta^2 = 1$. Therefore, let $\kappa = 1$, $\alpha = \cos\theta$ and $\beta = \sin\theta$. Then $S_{\alpha\beta}$ can be rewritten as

$$S_\theta = \begin{bmatrix} -\cos\theta & \sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \tag{3.11}$$

where $\tan\theta = \dfrac{h_n}{f_n}$ and (3.8) becomes

$$S_z S_b = \begin{bmatrix} 1 & 0 \\ 0 & z^{-1} \end{bmatrix} \begin{bmatrix} \sum_{i=0}^{n-1} (f_i \sin\theta - h_i \cos\theta) z^{-i} & -\sigma \sum_{i=0}^{n-1} (h_{n-i} \sin\theta + f_{n-i} \cos\theta) z^{-i} \\ \sum_{i=0}^{n-1} (h_{i+1} \sin\theta + f_{i+1} \cos\theta) z^{-i} & \sigma \sum_{i=0}^{n-1} (f_{n-1-i} \sin\theta - h_{n-1-i} \cos\theta) z^{-i} \end{bmatrix}$$

and

$$S_b = \begin{bmatrix} \sum_{i=0}^{n-1} (f_i \sin\theta - h_i \cos\theta) z^{-i} & \sigma_b \sum_{i=0}^{n-1} (h_{n-i} \sin\theta + f_{n-i} \cos\theta) z^{-i} \\ \sum_{i=0}^{n-1} (h_{i+1} \sin\theta + f_{i+1} \cos\theta) z^{-i} & -\sigma_b \sum_{i=0}^{n-1} (f_{n-1-i} \sin\theta - h_{n-1-i} \cos\theta) z^{-i} \end{bmatrix} \tag{3.12}$$

where $\sigma_b = -\sigma$. It is obvious that S_b has the same form as S, except that it is one degree lower than S. If its Belevitch representation in the z^{-1}-domain is $\{f_b(z), g_b(z), h_b(z)\}$, then

$$f_b(z) = \sum_{i=0}^{n-1} (h_{i+1} \sin\theta + f_{i+1} \cos\theta) z^{-i}, \quad h_b(z) = \sum_{i=0}^{n-1} (f_i \sin\theta - h_i \cos\theta) z^{-i},$$

$g_b(z) = 1$, and $g_{b*}(z) = z^{-(n-1)}$.

Now it is required to prove that the polynomials $f_b(z)$, $g_b(z)$ and $h_b(z)$ also satisfy the Feldtkeller equation

$$g_b(z)g_{b*}(z) = f_b(z)f_{b*}(z) + h_b(z)h_{b*}(z) \tag{3.13}$$

Since $S = S_\theta S_z S_b$ and $\det(S) = \det(S_\theta)\det(S_z)\det(S_b)$, i.e.,

$$-\sigma(f(z)f_*(z) + h(z)h_*(z)) = -z^{-1}\sigma(f_b(z)f_{b*}(z) + h_b(z)h_{b*}(z))$$

equation $f_b(z)f_{b*}(z) + h_b(z)h_{b*}(z) = (f(z)f_*(z) + h(z)h_*(z))z = z^{-(n-1)}$ holds, which yields equation (3.13).

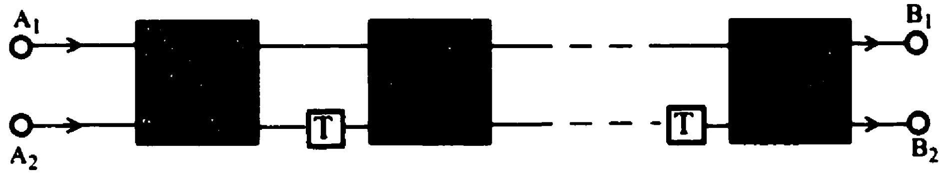
The above conclusion that S_b has the same properties as S except a lower degree shows that by replacing S with S_b, the decomposed procedure can be repeated until all elementary sections are extracted. Based on the above derivation, the realization structure and an algorithm to solve for all parameters θ_i, $i = 0, 1, 2, \ldots, n$, is given below.

When n is even, the realization structure is as presented in Fig. 3.6 (a), and the scattering matrix S is factorized as

$$S = \begin{bmatrix} -\cos\theta_n & \sin\theta_n \\ \sin\theta_n & \cos\theta_n \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & z^{-1} \end{bmatrix} \begin{bmatrix} -\cos\theta_{n-1} & \sin\theta_{n-1} \\ \sin\theta_{n-1} & \cos\theta_{n-1} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & z^{-1} \end{bmatrix} \cdots \begin{bmatrix} 1 & 0 \\ 0 & z^{-1} \end{bmatrix} \begin{bmatrix} -\cos\theta_0 & \sin\theta_0 \\ \sin\theta_0 & \cos\theta_0 \end{bmatrix} \tag{3.14}$$

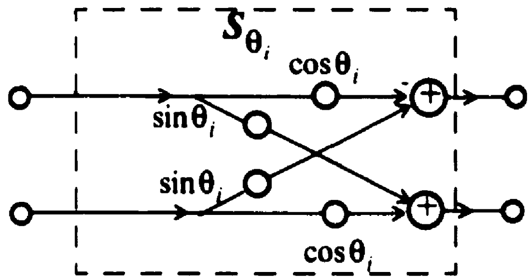
When n is odd, there is a small change in the realization structure and the factorization of the scattering matrix (see Fig. 3.6 (b) and equation (3.15)).

$$S = \begin{bmatrix} -\cos\theta_n & \sin\theta_n \\ \sin\theta_n & \cos\theta_n \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & z^{-1} \end{bmatrix} \begin{bmatrix} -\cos\theta_{n-1} & \sin\theta_{n-1} \\ \sin\theta_{n-1} & \cos\theta_{n-1} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & z^{-1} \end{bmatrix} \cdots \begin{bmatrix} 1 & 0 \\ 0 & z^{-1} \end{bmatrix} \begin{bmatrix} -\cos\theta_0 & \sin\theta_0 \\ \sin\theta_0 & \cos\theta_0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \tag{3.15}$$



(a) n is even

(b) n is odd



(c) Normalized two-port

Fig. 3.6 Wave digital realization of an FIR filter

Now, the algorithm is presented as follows:

Step 1: $j = n$,

$$h_{k,j} = h_k, f_{k,j} = f_k, k = n-2, n-1, \mathrm{n}$$

$$\tan\theta_n = \frac{h_n}{f_n},\ \cos\theta_n = \frac{f_n}{\sqrt{h_n^2 + f_n^2}},\ \sin\theta_n = \frac{h_n}{\sqrt{h_n^2 + f_n^2}}$$

Step 2: $j = j-1$

$$h_{k,j} = f_{k,j+1}\sin\theta_{j+1} - h_{k,j+1}\cos\theta_{j+1}$$

$$f_{k,j} = h_{k+1,j+1}\sin\theta_{j+1} + f_{k+1,j+1}\cos\theta_{j+1},\ 0 \le k = j-2, j-1, j$$

Step 3: if $j \ge 1$, $\tan\theta_j = \frac{h_{j,j}}{f_{j,j}}$, $\cos\theta_j = \frac{f_{j,j}}{\sqrt{h_{j,j}^2 + f_{j,j}^2}}$, $\sin\theta_j = \frac{h_{j,j}}{\sqrt{h_{j,j}^2 + f_{j,j}^2}}$

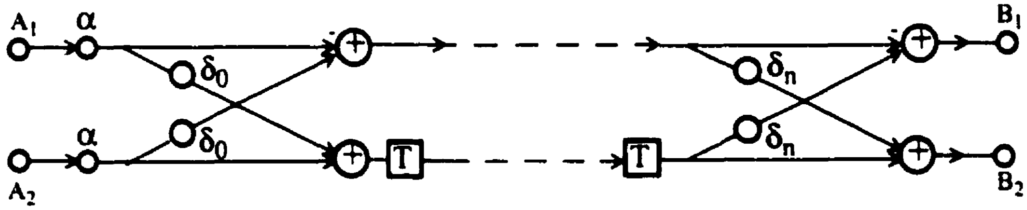
go to Step 2, otherwise go to Step 4.

Step 4: $\tan\theta_0 = -\frac{f_{0,0}}{h_{0,0}}$, $\cos\theta_0 = -h_{0,0}$, $\sin\theta_0 = f_{0,0}$

3.4 Other Implementation Structures

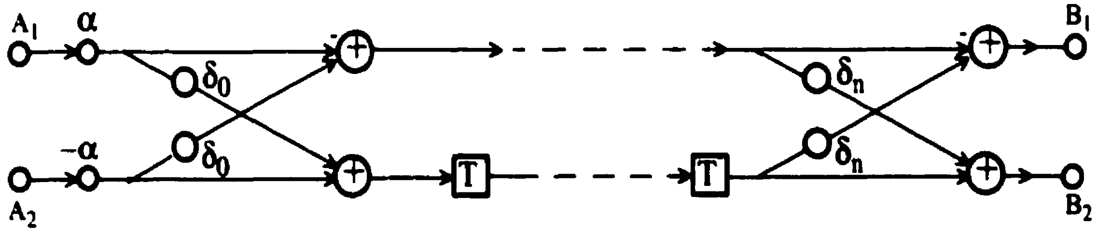
In (3.10a), let $\kappa = 1$, $\alpha = 1$ and set $\beta = \delta$. Then $S_\delta = \begin{bmatrix} -1 & \delta \\ \delta & 1 \end{bmatrix}$ which yields the

Fettweis implementation structure:



(a) n is even

or



(b) n is odd

Fig. 3.7 S_δ – structure

where the corresponding scattering matrices S are factorized as

$$S = \alpha\begin{bmatrix}-1 & \delta_n \\ \delta_n & 1\end{bmatrix}\begin{bmatrix}1 & 0 \\ 0 & z^{-1}\end{bmatrix}\begin{bmatrix}-1 & \delta_{n-1} \\ \delta_{n-1} & 1\end{bmatrix}\begin{bmatrix}1 & 0 \\ 0 & z^{-1}\end{bmatrix}\cdots\begin{bmatrix}1 & 0 \\ 0 & z^{-1}\end{bmatrix}\begin{bmatrix}-1 & \delta_0 \\ \delta_0 & 1\end{bmatrix},$$

and

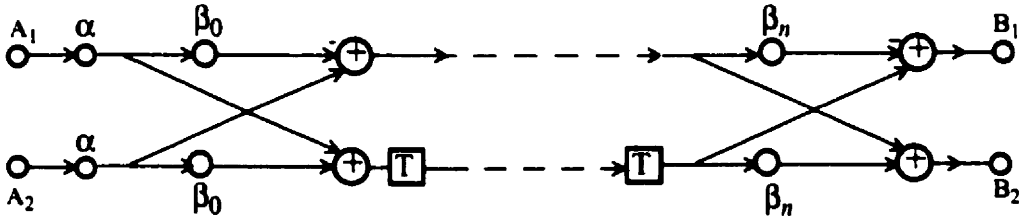
$$S = \begin{bmatrix}-1 & \delta_n \\ \delta_n & 1\end{bmatrix}\begin{bmatrix}1 & 0 \\ 0 & z^{-1}\end{bmatrix}\begin{bmatrix}-1 & \delta_{n-1} \\ \delta_{n-1} & 1\end{bmatrix}\begin{bmatrix}1 & 0 \\ 0 & z^{-1}\end{bmatrix}\cdots\begin{bmatrix}1 & 0 \\ 0 & -z^{-1}\end{bmatrix}\begin{bmatrix}-1 & \delta_0 \\ \delta_0 & 1\end{bmatrix}\begin{bmatrix}\alpha & 0 \\ 0 & -\alpha\end{bmatrix}$$

Compared the algorithm proposed in last section, here,

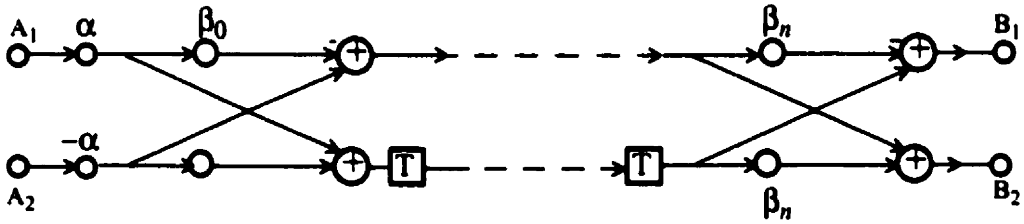
$$\delta_i = \tan\theta_i,\ i = 0, 1, \ldots, n,\ \alpha = \prod_{i=0}^{n}\cos\theta_i \tag{3.16}$$

Reconsider (3.10a), but this time let $\kappa = 1$, $\beta = 1$ and set $\alpha = \beta$. Then

$S_\beta = \begin{bmatrix}-\beta & 1 \\ 1 & \beta\end{bmatrix}$ and another Fettweis realization structure[1] is obtained as



(a) n is even



(b) n is odd

Fig. 3.8 S_β – structure

where

$$\beta_i = \cot\theta_i,\ i = 0, 1, \ldots, n,\ \alpha = \prod_{i=0}^{n} \sin\theta_i \tag{3.17}$$

It is noticed that in the above two structures, both multipliers $\delta_i = \tan\theta_i$ and $\beta_i = \cot\theta_i$ may be greater than one. But in the realization of a digital filter, it is required to quantize the multipliers for implementation in hardware and it is desirable to have all the multipliers less than one. This can be achieved as follows: Consider

$S_{\theta_i} = \begin{bmatrix} -\cos\theta_i & \sin\theta_i \\ \sin\theta_i & \cos\theta_i \end{bmatrix}$. If $|\cos\theta_i| \geq |\sin\theta_i|$, extract $\cos\theta_i$, and rewrite S_{θ_i} as

$$S_{\theta_i} = \cos\theta_i \begin{bmatrix} -1 & \tan\theta_i \\ \tan\theta_i & 1 \end{bmatrix} = \cos\theta_i \begin{bmatrix} -1 & \delta_i \\ \delta_i & 1 \end{bmatrix},$$

where

$$\delta_i = \tan\theta_i, \text{ for } |\tan\theta_i| \leq 1; \tag{3.18}$$

otherwise $|\sin\theta_i| > |\cos\theta_i|$, then extract $\sin\theta_i$. S_{θ_i} becomes

$$S_{\theta_i} = \sin\theta_i \begin{bmatrix} -\cot\theta_i & 1 \\ 1 & \cot\theta_i \end{bmatrix} = \sin\theta_i \begin{bmatrix} -\beta_i & 1 \\ 1 & \beta_i \end{bmatrix},$$

where

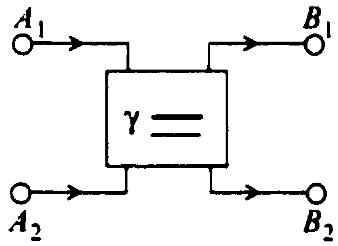
$$\beta_i = \cot\theta_i, \text{ for } |\cot\theta_i| < 1 \tag{3.19}$$

So, after the above manipulation, the scattering matrix S can be factored as a mixed product of S_δ and S_β, and a coefficient

$$\alpha = \prod_{|\cos\theta_i| \geq |\sin\theta_i|} \cos\theta_i \prod_{|\sin\theta_i| > |\cos\theta_i|} \sin\theta_i \tag{3.20}$$

Thus a mixed structure which has the desired properties can be obtained.

Next, by considering two other equivalent forms of $S_\theta = \begin{bmatrix} -\cos\theta & \sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, another two realization structures are obtained. One of the equivalent forms of S_θ is a 2-port adaptor



$$B_1 = -\gamma A_1 + (1+\gamma)A_2$$
$$B_2 = (1-\gamma)A_1 + \gamma A_2$$

Fig. 3. 9 2-port adaptor

cascaded with a pair of inverse multipliers [2], i.e., $S_\theta = P^{-1} S_\gamma P$, where $P = \begin{bmatrix} 1 & 0 \\ 0 & k \end{bmatrix}$, $k = \tan\frac{\theta}{2}$, and the corresponding scattering matrix $S_\gamma = \begin{bmatrix} -\gamma & 1+\gamma \\ 1-\gamma & \gamma \end{bmatrix}$ [1], $\gamma = \cos\theta$.

Thus, another factored form of the scattering matrix S is obtained as

$$S = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{k_n} \end{bmatrix} \begin{bmatrix} -\gamma_n & 1+\gamma_n \\ 1-\gamma_n & \gamma_n \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & \frac{k_n}{k_{n-1}} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & z^{-1} \end{bmatrix} \cdots \begin{bmatrix} 1 & 0 \\ 0 & z^{-1} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & \frac{k_1}{k_0} \end{bmatrix} \begin{bmatrix} -1 & 1+\gamma_0 \\ 1-\gamma_0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & \pm k_0 \end{bmatrix},$$

where

$$\gamma_i = \cos\theta_i,\ k_i = \tan\frac{\theta_i}{2},\ i = 0, 1, 2, \ldots, n \tag{3.21}$$

and the sign before k_0 depends on whether the degree n is even or odd. If n is even, $+k_0$ is preferred, otherwise, $-k_0$, and its realization structure is presented in Fig. 3.10.

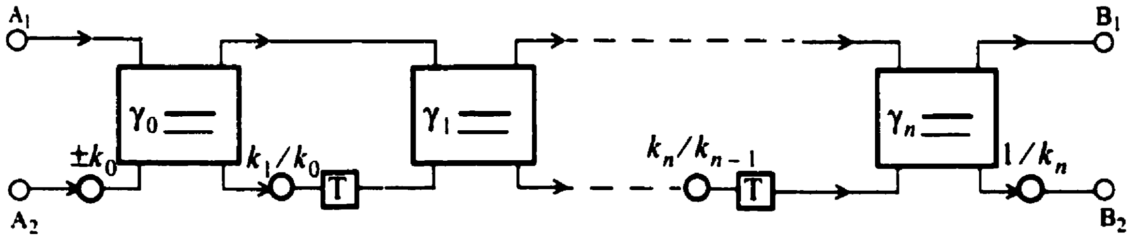
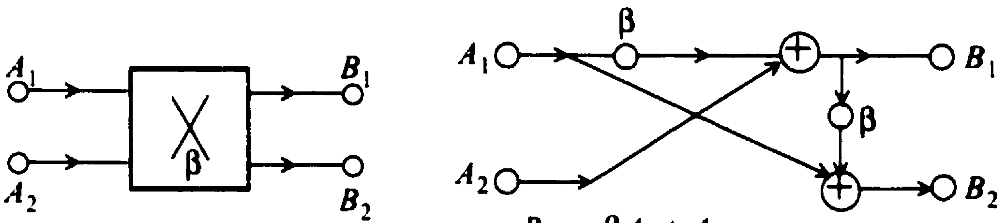


Fig. 3.10 Two-port adaptor structure

Another equivalent form of S_θ is a cross adaptor



(a) Cross adaptor

$B_1 = \beta A_1 + A_2$

$B_2 = A_1 - \beta B_1 = (1-\beta^2)A_1 - \beta A_2$

(b) Signal-flow diagram

Fig. 3.11

cascaded with a pair of inverse multipliers[18][13], $P = \begin{bmatrix} 1 & 0 \\ 0 & k \end{bmatrix}$, where $k = \sin\theta$, i.e.,

$S_\theta = P^{-1}S_\beta P$ with $S_\beta = \begin{bmatrix} \beta & 1 \\ 1-\beta^2 & -\beta \end{bmatrix}$, $\beta = -\cos\theta$. Thus the scattering matrix S can also

be factored as

$$S = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{k_n} \end{bmatrix} \begin{bmatrix} \beta & 1 \\ 1-\beta^2 & -\beta \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & \frac{k_n}{k_{n-1}} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & z^{-1} \end{bmatrix} \cdots \begin{bmatrix} 1 & 0 \\ 0 & z^{-1} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & \frac{k_1}{k_0} \end{bmatrix} \begin{bmatrix} \beta_0 & 1 \\ 1-\beta_0 & -\beta \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & \pm k_0 \end{bmatrix},$$

where

$$\beta_i = -\cos\theta_i,\ k_i = \sin\theta_i,\ i = 0, 1, 2, \ldots, n \tag{3.22}$$

Therefore, it is easy to give its realization structure as

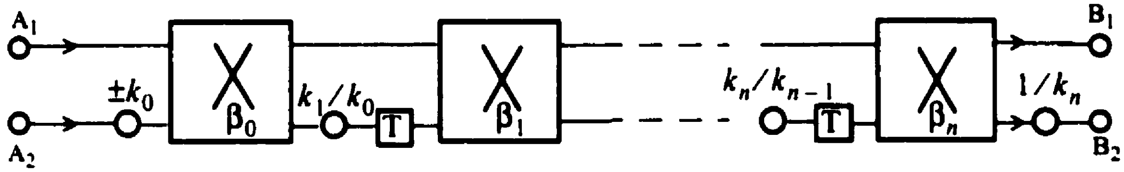


Fig. 3.12. Cross adaptor structure

So far, six different realization structures have been derived. The original one, presented in Fig. 3.6 (a), (b), is named the S_θ - structure. The one in Fig. 3.7 is named the S_δ - structure and the one in Fig. 3.8 the S_β - structure. The mixed S_δ and S_β structure is called the S_δ - S_β -structure and the last two are the 2-port-adaptor-structure and the cross-adaptor-structure. For the S_θ - structure, the algorithm is already given in **3.3**. In fact, it is easy to obtain different algorithms for the different structures by making small modifications on the known algorithm according to the definitions of the different structures.

3.5 Illustrative Algorithm Example

In order to demonstrate the algorithm, a simple FIR filter

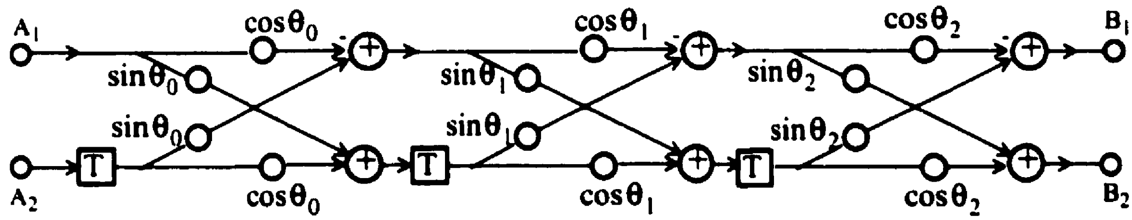
$$H(z) = \frac{1}{4} + \frac{1}{2}z^{-1} + \frac{1}{4}z^{-2},$$

with its scattering matrix

$$S = \begin{bmatrix} \frac{1+\sqrt{2}}{4} - \frac{1}{2}z^{-1} + \frac{1-\sqrt{2}}{4}z^{-2} & \frac{1}{4} + \frac{1}{2}z^{-1} + \frac{1}{4}z^{-2} \\ \frac{1}{4} + \frac{1}{2}z^{-1} + \frac{1}{4}z^{-2} & -\left(\frac{1-\sqrt{2}}{4} - \frac{1}{2}z^{-1} + \frac{1+\sqrt{2}}{4}z^{-2}\right) \end{bmatrix}.$$

will be considered.

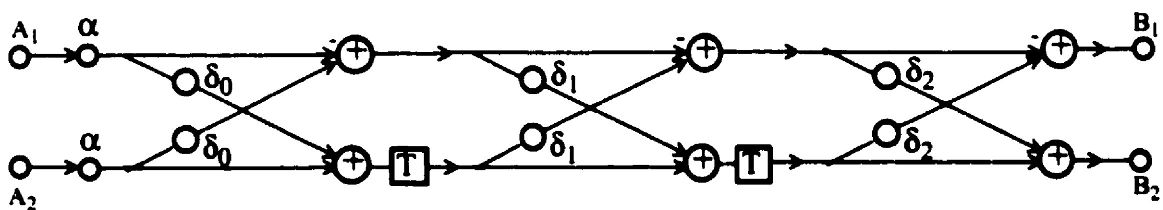
Based on the algorithm proposed in section 3.3, the factorization result is obtained in Fig. 3.13. If it is realized according to Fig. 3.7, by the formula (3.16), $\alpha = -\frac{1+\sqrt{2}}{4}$, $\delta_0 = 1-\sqrt{2}$, $\delta_1 = 1$ and $\delta_2 = 1-\sqrt{2}$. The implementation structure is shown is Fig. 3.14. Based on Fig. 3.8 and equation (3.17), the S_β structure is obtained and shown in Fig. 3.15. For this example, because all $|\cos\theta_i| \geq 1$, the S_δ-S_β-structure is the same as the S_δ-structure. Similarly, by Fig. 3.10 and equation (3.21), Fig. 3.12 and equation (3.22), the 2-port-adaptor-structure and cross-adaptor-structure are obtained, respectively (see Figs. 3.16, 3.17).



$$\cos\theta_2 = \frac{\sqrt{2+\sqrt{2}}}{2} \quad \sin\theta_2 = \frac{\sqrt{2-\sqrt{2}}}{2} \quad \cos\theta_1 = \frac{\sqrt{2}}{2} \quad \sin\theta_1 = \frac{\sqrt{2}}{2}$$

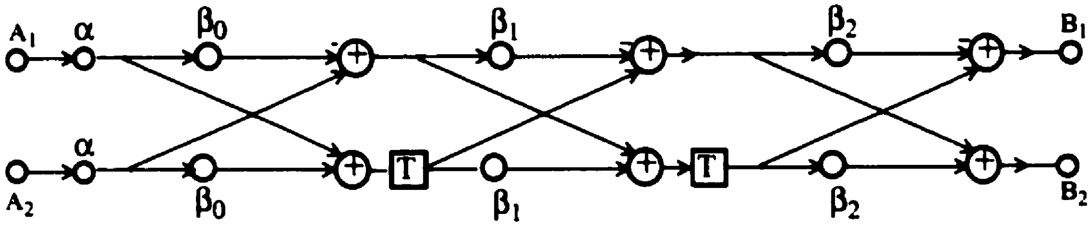
$$\cos\theta_0 = \frac{\sqrt{2+\sqrt{2}}}{2} \quad \sin\theta_0 = \frac{\sqrt{2-\sqrt{2}}}{2}$$

Fig. 3.13 S_θ - structure for an example



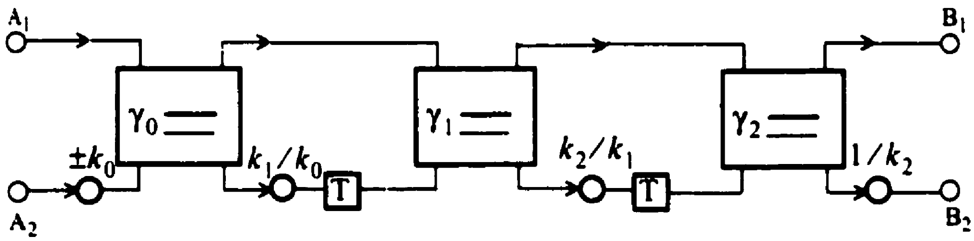
$$\alpha = -\frac{1+\sqrt{2}}{4} \qquad \delta_0 = 1-\sqrt{2} \qquad \delta_1 = 1 \qquad \delta_2 = 1-\sqrt{2}$$

Fig. 3.14 S_δ - structure for an example



$$\alpha = \frac{\sqrt{2}-1}{4} \qquad \beta_0 = -(1+\sqrt{2}) \qquad \beta_1 = 1 \qquad \beta_2 = -(1+\sqrt{2})$$

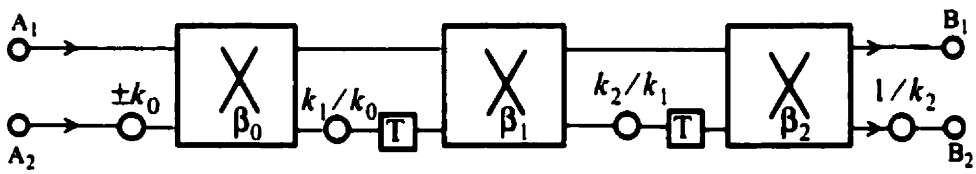
Fig.3.15. S_β -structure for an example



$$\gamma_0 = \frac{\sqrt{2+\sqrt{2}}}{2} \qquad \gamma_1 = \frac{\sqrt{2}}{2} \qquad \gamma_2 = \frac{\sqrt{2+\sqrt{2}}}{2}$$

$$k_0 = \sqrt{2(2+\sqrt{2})}-\sqrt{2}-1 \qquad k_1 = \sqrt{2}-1 \quad k_2 = \sqrt{2(2+\sqrt{2})}-\sqrt{2}-1$$

Fig. 3.16. Two-port adaptor structure for an example



$$\beta_0 = -\frac{\sqrt{2+\sqrt{2}}}{2} \qquad \beta_1 = -\frac{\sqrt{2}}{2} \qquad \beta_2 = -\frac{\sqrt{2+\sqrt{2}}}{2}$$

$$k_0 = \frac{\sqrt{2-\sqrt{2}}}{2} \qquad k_1 = \frac{\sqrt{2}}{2} \qquad k_2 = \frac{\sqrt{2-\sqrt{2}}}{2}$$

Fig. 3.17. Cross adaptor structure for an example

3.6 Linear Phase FIR Filters

FIR filters have the important characteristic that they can achieve exactly linear phase and cannot be unstable[12]. For convenience, rewrite (3.3) here,

$$H(z) = f_0 + f_1 z^{-1} + f_2 z^{-2} + \dots + f_n z^{-n}, f_n, f_0 \neq 0$$

There are four possible types of FIR filters leading to a linear phase.

Type 1. n is odd and $f_i = f_{n-i}$, $i = 0, \dots, \frac{n-1}{2}$;

Type 2. n is even and $f_i = f_{n-i}$, $i = 0, \dots, \frac{n-2}{2}$;

Type 3. n is odd and $f_i = -f_{n-i}$, $i = 0, \dots, \frac{n-1}{2}$;

Type 4. n is even and $f_i = -f_{n-i}$, $i = 0, \dots, \frac{n-2}{2}$ and $f_{\frac{n}{2}} = 0$.

For the four Types of FIR filter, let us consider the scattering matrix

$$S = \begin{bmatrix} h_0 + h_1 z^{-1} + \dots + h_n z^{-n} & \sigma(f_n + f_{n-1} z^{-1} + \dots + f_0 z^{-n}) \\ f_0 + f_1 z^{-1} + \dots + f_n z^{-n} & -\sigma(h_n + h_{n-1} z^{-1} + \dots + h_0 z^{-n}) \end{bmatrix}$$

For Type 1 and 2, let $\sigma = 1$, then $S = S^T$, where S^T means the transpose of S, for Type 3 and 4, let $\sigma = -1$, $S = S^T$ also holds. It is known that the decomposition proposed in 3.3 does not require any change no matter if $\sigma = 1$ or -1. Therefore the factorization structure will be discussed under the situation $S = S^T$. The interesting point here is to see if there is any special relationship between S_{θ_i} and $S_{\theta_{n-i}}$, for $i = 0, \dots, \frac{n-2}{2}$, if n is even; $i = 0, \dots, \frac{n-1}{2}$, if n is odd.

If the degree n is even, according to the discussion in 3.3, the matrix S can be factorized as

$$S = S_{\theta_n} S_z \ldots S_{\theta_{\frac{n}{2}}} \ldots S_z S_{\theta_0}. \tag{3.23}$$

Its central factor is $S_{\theta_{\frac{n}{2}}}$. Where the definitions of S_z and S_{θ_i}, $i = 0, 1, \ldots, n$, are the same as before. Note that all these matrices are symmetric, i.e., $S_z = S_z^T$ and $S_{\theta_i} = S_{\theta_i}^T$. Hence from $S = S^T$, another factorization of S

$$S = S_{\theta_0} S_z \ldots S_{\theta_{\frac{n}{2}}} \ldots S_z S_{\theta_n} \tag{3.24}$$

is obtained.

If n is odd, from 3.3, the scattering matrix can be factorized as

$$S = -S_{\theta_n} S_z \ldots S_z \ldots S_z S_{\theta_0}, \tag{3.25}$$

where the central factor is S_z. Similar to the situation where n is even, here S also has another factorization

$$S = -S_{\theta_0} S_z \ldots S_z \ldots S_z S_{\theta_n} \tag{3.26}$$

Rewrite (3.23) and (3.24) as

$$S = S_{\theta_n} S_z S_{b_1}, \quad S = S_{\theta_0} S_z S_{b_2} \tag{3.27a,b}$$

where $S_{b_1} = S_{\theta_{n-1}} \ldots S_{\theta_{\frac{n}{2}}} \ldots S_z S_{\theta_0}$ and $S_{b_2} = S_{\theta_1} \ldots S_{\theta_{\frac{n}{2}}} \ldots S_z S_{\theta_n}$. From the decomposition and the algorithm in 3.3, $\tan\theta_n = \frac{h_n}{f_n} = \tan\theta_0$ which implies $S_{\theta_n} = \pm S_{\theta_0}$ and $S_{b_1} = \pm S_{b_2}$. Replacing S with S_{b_1} or S_{b_2}, yields $\tan\theta_{n-1} = \tan\theta_1$ and $S_{\theta_{n-1}} = \pm S_{\theta_1}$.

This procedure can be repeated $\frac{n-2}{2}$ times. Therefore the relationship between S_{θ_i} and $S_{\theta_{n-i}}$ can be described as

$$S_{\theta_{n-i}} = \pm S_{\theta_i},\ i = 0, \ldots, \frac{n-2}{2}. \tag{3.28}$$

For n odd, equation (3.28) also holds for $i = 0, \ldots, \frac{n-1}{2}$.

The above conclusions show that for a linear phase FIR filter, it is only required to extract half of the elementary sections and obtain the other half according to (3.28). In particular, if n is even, it is required to calculate S_{θ_n}, $S_{\theta_{n-1}}$, $\ldots$, $S_{\theta_{\frac{n}{2}}}$; if n is odd, S_{θ_n}, $S_{\theta_{n-1}}$, $\ldots$, $S_{\theta_{\frac{n-1}{2}}}$. Therefore, half the computions for the decomposition are required.

3.7 The Design Procedure and Examples

In this section, the design procedure for an FIR filter according to the factored structure will be presented first, then two examples to illustrate the procedure follow.

3.7.1 Procedure

The general design procedure can be carried out as follows: for a given specification in terms of attenuation as illustrated in Fig. 3.18

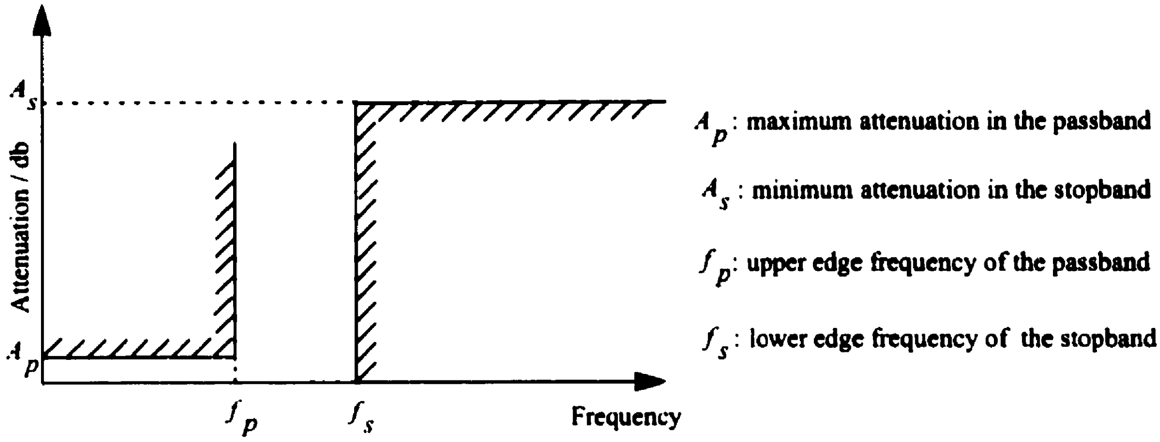


Fig. 3.18 Design specification

A MATLAB function REMEZ is used to create the FIR frequency response function, say,

$$H(z) = f_0 + f_1 z^{-1} + f_2 z^{-2} + \ldots + f_n z^{-n},$$

which (after scaling so that $|H(z)| \leq 1$ on the unit circle) can also be seen as $f(z)$ of the Belevitch representation $\{f(z), g(z), h(z)\}$. As described in **3.2**, $g(z) = 1$ and $h(z)$ can be obtained by MATLAB programming, i.e. the scattering matrix

$S = \dfrac{1}{g(z)} \begin{bmatrix} h(z) & \sigma f_*(z) \\ f(z) & -\sigma h_*(z) \end{bmatrix}$ is then known. Next by applying the algorithm stated in **3.3**,

the basic factored structure, namely the S_θ - structure (see Fig. 3.19)

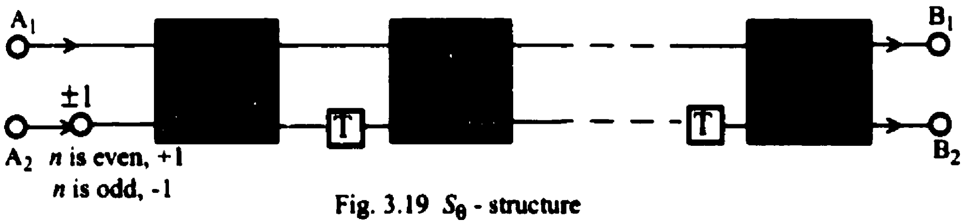


Fig. 3.19 S_θ - structure

and all multipliers $\cos\theta_i$, $\sin\theta_i$, $i = 0, 1, \ldots, n$ are determined. Based on this basic structure, it is easy to derive other structures, such as the S_δ-structure, S_β-structure, S_δ-

S_β-structure, S_δ-S_β-structure, 2-port-adaptor-structure and cross-adaptor-structure. The formulae to determine all the structures are listed below.

For the S_δ - structure (see Fig. 3.7),

$$\delta_i = \tan\theta_i,\ i = 0, 1, \ldots, n,\ \alpha = \prod_{i=0}^{n} \cos\theta_i \tag{3.29}$$

For the S_β - structure (see Fig. 3.8),

$$\beta_i = \cot\theta_i,\ i = 0, 1, \ldots, n,\ \alpha = \prod_{i=0}^{n} \sin\theta_i \tag{3.30}$$

For the S_δ-S_β-structure,

$$\delta_i = \tan\theta_i, \text{ for } |\tan\theta_i| \le 1,\ \beta_i = \cot\theta_i, \text{ for } |\cot\theta_i| < 1 \tag{3.31a}$$

$$\alpha = \prod_{|\cos\theta_i| \ge |\sin\theta_i|} \cos\theta_i \prod_{|\sin\theta_i| > |\cos\theta_i|} \sin\theta_i \tag{3.31b}$$

For the 2-port-adaptor-structure (see Fig. 10)

$$\gamma_i = \cos\theta_i,\ k_i = \tan\frac{\theta_i}{2},\ i = 0, 1, 2, \ldots, n \tag{3.32}$$

For the Cross-adaptor-structure (see Fig. 3. 12),

$$\beta_i = -\cos\theta_i,\ k_i = \sin\theta_i,\ i = 0, 1, 2, \ldots, n \tag{3.33}$$

After all the multipliers for a selected structure are determined, they must be quantized to a limited number of bits for implementation in hardware. The last step is to implement the selected structure with the quantized multipliers.

Several structures have been established so far. The question then arises as to which one is the preferred one. The S_δ and S_β mixed structure, namely, the S_δ-S_β-structure is preferred. There are two reasons for this: the first one is that the S_δ-S_β-structure has a relative ly low sensitivity to changes in the multipliers; the second reason is that the S_δ-S_β-structure has only n+1 multipliers, nearly half of that in the 2-port-adaptor-structure and the Cross-adaptor-structure, both of which have $2n$ multipliers.

3.7.2 Examples

Next, two examples are presented according to the above design procedure.

***Example* 1**: Specification: $A_p = 0.7\,\text{dB}$, $A_s = 27\,\text{dB}$, $f_p/F = 0.19$, $f_s/F = 0.31$

For the specification, an 18th order linear phase FIR filter

$$H(z) = f_0 + f_1 z^{-1} + f_2 z^{-2} + \ldots + f_{18} z^{-18}$$

is obtained by using the MATLAB function REMEZ. Its coefficients are listed in Table 1.

Table 1: The Coefficients of 18th FIR Filter

f_0 to f_4	f_5 to f_9	f_{10} to f_{14}	f_{15} to f_{18}
0.00852976116030	0.00006700756621	0.30237336760195	-0.00011210088575
-0.00027229773499	- 0.09144000481338	- 0.00167956359552	-0.00207147488452
-0.00207147488452	- 0.00167956359552	- 0.09144000481338	-0.00027229773499
-0.00011210088575	0.30237336760195	0.00006700756621	0.00852976116030
0.04325196780017	0.48270667557107	0.04325196780017	

From the Feldtkeller equation $gg_* = ff_* + hh_*$,

$$h(z)h_*(z) = g(z)g_*(z) - f(z)f_*(z)$$

Using a program written in MATLAB, the coefficients of polynomial

$$h(z) = h_0 + h_1 z^{-1} + h_2 z^{-2} + \ldots + h_{18} z^{-18}$$

are obtained and are listed in Table 2

Table 2: The Coefficients of $h(z)$

h_0 to h_4	h_5 to h_9	h_{10} to h_{14}	h_{15} to h_{18}
0.00014898602099	0.00146461024170	0.02025633839697	-0.15902734981553
0.00014092406863	-0.00248467048961	-0.00046542490305	-0.16789904854537
0.00001886135183	-0.00344262073546	0.04776333715866	0.49310040472701
-0.00007952875273	0.01228141317667	0.13623545506986	-0.48834665809613
0.00141074774734	0.03162019272969	0.07730405960422	

Next, by the algorithm proposed in 3.3, all the multipliers in the S_θ - structure, $\cos\theta_i$, $\sin\theta_i$, i = 0, 1, ..., 18 are calculated and presented in Table 3

Table 3: The Multipliers of S_θ - Structure

$\cos\theta_0$ to $\cos\theta_9$	$\sin\theta_0$ to $\sin\theta_9$	$\cos\theta_{10}$ to $\cos\theta_{18}$	$\sin\theta_{10}$ to $\sin\theta_{18}$
-0.01746394733352	0.99984749364267	0.91437521742956	0.40486783244738
0.99985427389608	-0.01707134944094	0.98587254086448	0.16749726316991
0.99985427389608	0.00698883712781	0.99324153334487	-0.11606574188251
0.99998883744629	0.00472493204264	0.99633918901237	-0.08548813039352
0.99692940283443	0.07830559216426	0.99692940283443	0.07830559216425
0.99633918901236	-0.08548813039354	0.99998883744630	0.00472493204262
0.99324153334487	-0.11606574188250	0.99997557777958	0.00698883712784
0.98587254086448	0.16749726316992	0.99324153334487	-0.01707134944096
0.91437521742957	0.40486783244735	0.01746394733353	-0.99984749364267
0.61783822787511	-0.78630523600962		

Substituting the value of $\cos\theta_i$, $\sin\theta_i$, i = 0, 1, ..., 18, into (3.31a), (3.31b), the multipliers of the S_δ-S_β-structure are obtained and listed in Table 4 after 8 bit quantization.

Table 4: The Multipliers of S_δ-S_β Structure

$\alpha = 0.62109375$	$\delta_4 = 0.0781250$	$\beta_9 = -0.78515625$	$\delta_{14} = 0.0781250$
$\beta_0 = -0.015625$	$\delta_5 = -0.0859375$	$\delta_{10} = 0.44140625$	$\delta_{15} = 0.00390625$
$\delta_1 = -0.015625$	$\delta_6 = -0.1171875$	$\delta_{11} = 0.16796875$	$\delta_{16} = 0.0078125$
$\delta_2 = 0.0078125$	$\delta_7 = 0.16796875$	$\delta_{12} = -0.1171875$	$\delta_{17} = -0.015625$
$\delta_3 = 0.00390625$	$\delta_8 = 0.44140625$	$\delta_{13} = -0.0859375$	$\beta_{18} = -0.015625$

Similarly, from (3.32) and (3.33), the coefficients of the 2-port-adaptor-structure and cross-adaptor-structure can be solved for respectively. The results after 8 bit quantization are presented in Table 5 and 6. Here it is noticed that the relationship between the κ_i in Table 5 and 6 and k_i in equation (3.32) or (3.33) is

$$\kappa_0 = k_0,\ \kappa_i = k_i/k_{i-1},\ i = 1, 2, \ldots, 18,\ \kappa_{19} = 1/k_{19}$$

Table 5: The Multipliers of 2-port Adaptor Structure

γ_0 to γ_9	γ_{10} to γ_{18}	κ_1 to κ_{10}	κ_{11} to κ_{19}
-0.015625	0.9140625	-0.0078125	0.3984375
1.0000000	0.9843750	-0.41015625	0.69140625
1.0000000	0.9921875	0.67578125	0.734375
1.0000000	0.99609375	16.59765625	-0.9140625
0.99609375	0.99609375	-1.0937500	0.05859375
0.99609375	1.0000000	1.3593750	1.48046875
0.9921875	1.0000000	-1.44921875	-2.44140625
0.9843750	1.0000000	2.5078125	115.1171875
0.9140625	0.015625	-2.296875	-1.01953125
0.6171875		-0.43359375	

Table 6: The Multipliers of Cross Adaptor Structure

β_0 to β_9	β_{10} to β_{18}	κ_1 to κ_{10}	κ_{11} to κ_{19}
-0.015625	0.91406250	-0.0156250	0.4140625
1.0000000	0.98437500	-0.41015625	-0.69140625
1.0000000	0.99218750	0.67578125	0.73828125
1.0000000	0.99609375	16.57421875	-0.91406250
0.99609375	0.99609375	-1.08984375	0.05859375
0.99609375	1.0000000	1.35937500	1.48046875
0.99218750	1.0000000	-1.44140625	-2.44140625
0.98437500	1.0000000	2.41796875	58.5703125
0.91406250	0.015625	-1.94140625	-1.00000000
0.61718750		-0.51562500	

The frequency responses for the three structures, S_δ-S_β-structure, 2-port-adaptor-structure and cross-adaptor-structure are presented in Figs 3. 20, 3.21. Fig. 3.20 shows all the three structures satisfy the specification given in (3.34), and Fig. 3.21 shows that they have an exact linear phase in the passband except for jumps of π in the stopband.

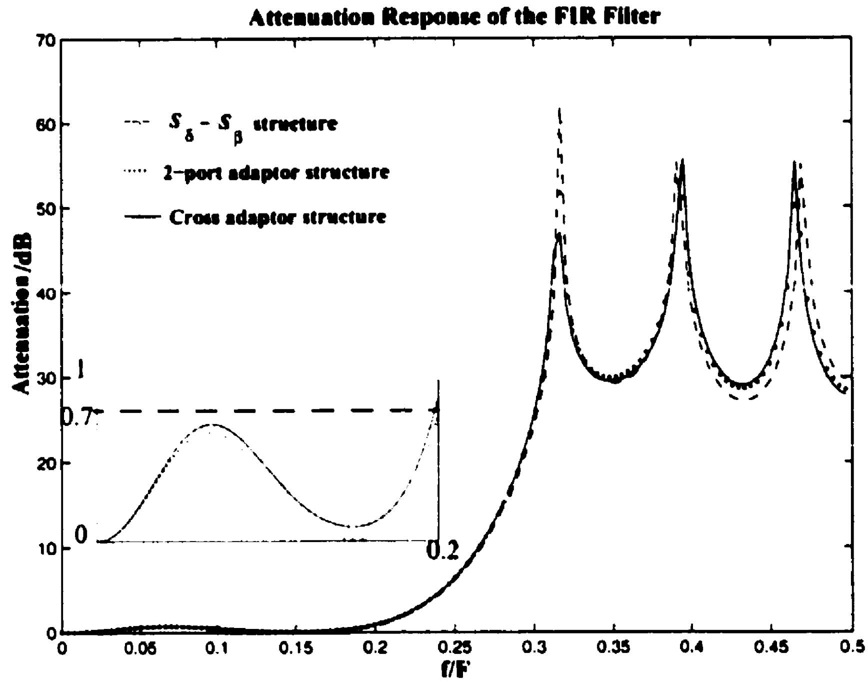


Fig. 3.20

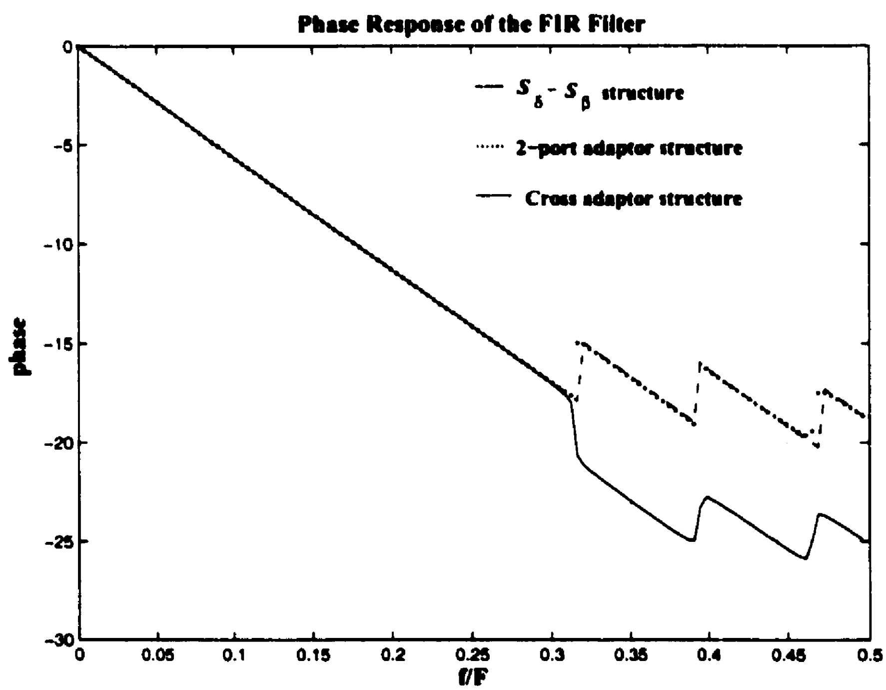


Fig. 3.21

Example 2. Specification: $A_p = 0.1\,\text{dB}$, $A_s = 35\,\text{dB}$, $f_p/F = 0.05$, $f_s/F = 0.2$

A 29th order linear phase FIR filter, whose coefficients are listed in Table 7, satisfies this specification. The objective of this example is to make a comparison between the S_δ-S_β-structure and the traditional direct form also called the transversal structure (Fig. 3.3). The plots of frequency responses of the two structures are showed in Figs. 3.22-3.24. Where all multipliers are quantized to 8 bit numbers. Fig. 3.23 shows that the frequency response of the S_δ-S_β-structure in the passband is much better than that of the direct form structure. This means the S_δ-S_β-structure has lower sensitivity than the direct form structure does.

Table 7: The Coefficients of FIR Filter

f_0 to f_7	f_8 to f_{15}	f_{16} to f_{23}	f_{24} to f_{29}
-0.00042630463719	-0.02453618953949	0.19075151308210	0.00830962149705
-0.00102730266266	-0.03551867388150	0.10741363204064	0.00506311841444
-0.00096202663252	-0.02168898167948	0.02807531585830	0.00104626691673
0.00104626691673	0.02807531585830	-0.02168898167948	-0.00096202663252
0.00506311841444	0.10741363204064	-0.03551867388150	-0.00102730266266
0.00830962149705	0.19075151308210	-0.02453618953949	-0.00042630463719
0.00565576915057	0.24430988827920	-0.00646614620569	
-0.00646614620569	0.24430988827920	0.00565576915057	

Table 8: The Coefficients of $h(z)$

h_0 to h_7	h_8 to h_{15}	h_{16} to h_{23}	h_{24} to h_{29}
0.00000034276696	-0.00009420872339	-0.01021909665958	-0.12425888037911
0.00000206763716	0.00021469741907	-0.00392449346170	-0.15486851688892
0.00000615487718	0.00076408490255	0.01161776668001	-0.07901344306601
0.00001012223697	0.00120744895716	0.03296766382364	0.17033614952607
0.00000339277886	0.00079832454232	0.04967801129910	0.64293557240964
-0.00003101506972	-0.00128293958644	0.04680430158976	-0.53020175423781
-0.00009794928814	-0.00514140783996	0.01231422944750	
-0.00015553255273	-0.00925678376415	-0.05252852187793	

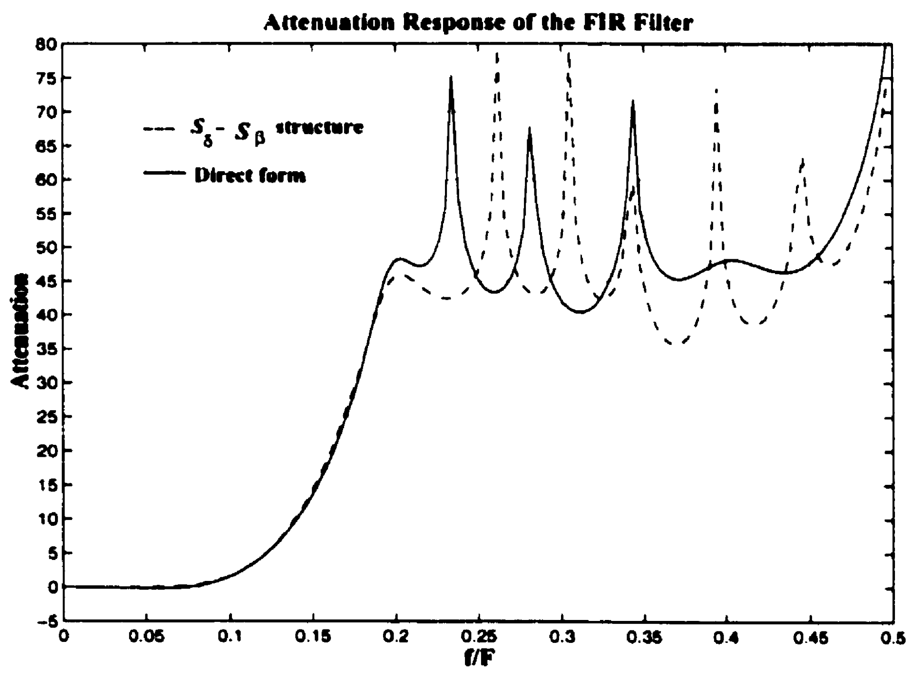


Fig. 3.22

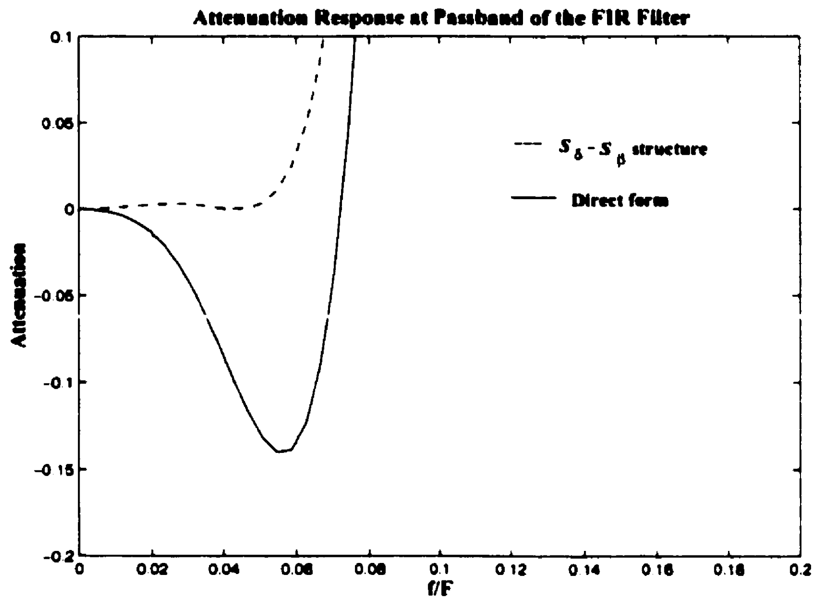


Fig. 3.23

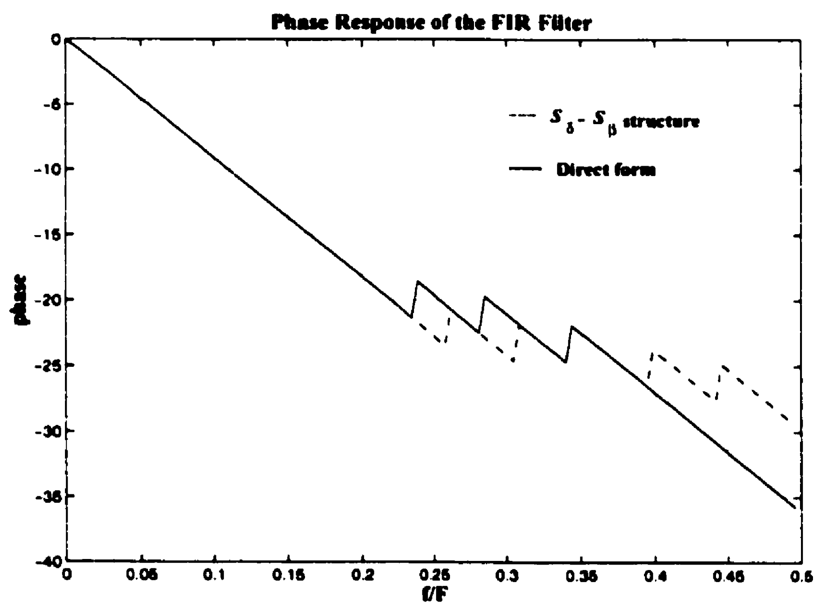


Fig. 3.24

Chapter 4

Design and Realization of Bireciprocal Filters

Bireciprocal IIR filters have big advantages in many communication applications of digital filters in that the lattice structure with bireciprocal characteristic function leads to enormous savings in hardware [14], [19], [24]. In this chapter, an analytical formula for the design of bireciprocal filters is presented first, followed by an optimization design method, and finally by a lattice realization structure. Some examples which demonstrate the realization structures are shown in sections **4.4, 4.5**.

4.1 The Definition and Some Properties of Bireciprocal Filters

Here three canonic polynomials $f(\psi)$, $g(\psi)$ and $h(\psi)$ are used to represent an IIR filter. The frequency ψ is defined[8] by

$$\psi = \tanh\left(\frac{sT}{2}\right) = \frac{z-1}{z+1},\ z = e^{sT} \tag{4.1}$$

where s is the actual complex frequency and T the sampling period. Furthermore, $g(\psi)$ is a Hurwitz polynomials of degree n.

If the characteristic function[8]

$$k(\psi) = \frac{h(\psi)}{f(\psi)} \tag{4.2}$$

satisfies

$$k\left(\frac{1}{\psi}\right) = \frac{1}{k(\psi)} \tag{4.3}$$

then the characteristic function is called a bireciprocal or a mirror-image function [1] and the corresponding filter $H(\psi) = \dfrac{f(\psi)}{g(\psi)}$ is a bireciprocal filter. Based on the definition of a bireciprocal filter defined by (4.2), some useful properties of $f(\psi)$, $g(\psi)$ and $h(\psi)$ can be derived, i.e.

$$h(\psi) = \pm\psi^n f\left(\frac{1}{\psi}\right), f(\psi) = \pm\psi^n h\left(\frac{1}{\psi}\right) \tag{4.4a,b}$$

$$g(\psi) = \pm\psi^n g\left(\frac{1}{\psi}\right) \tag{4.5}$$

where n is the degree of $g(\psi)$.

Proof[29]: substituting (4.2) into (4.3) and multiplying the left-hand ratio by ψ^n, yields

$$\frac{\psi^n h(1/\psi)}{\psi^n f(1/\psi)} = \frac{f(\psi)}{h(\psi)} \tag{4.6}$$

Let $\hat{h}(\psi) = \psi^n h(1/\psi)$ and $\hat{f}(\psi) = \psi^n f(1/\psi)$, then (4.6) becomes

$$\hat{h}(\psi)h(\psi) = \hat{f}(\psi)f(\psi) \tag{4.7}$$

Assume polynomial $h(\psi)$ and $f(\psi)$ are relatively prime. Then $\hat{h}(\psi)$ and $\hat{f}(\psi)$ are also relatively prime. This can be proven as follows: Suppose $\hat{h}(\psi)$ and $\hat{f}(\psi)$ are not relatively prime, then there exist a polynomial $p(\psi)$ such that $p(\psi)|\hat{h}(\psi)$ and $p(\psi)|\hat{h}(\psi)$ and $deg\ p = n_0 > 0$. Let $\hat{h}(\psi) = p(\psi)\hat{h}_1(\psi)$, then $deg\ \hat{h}_1 = deg\ \hat{h} - n_0$, and let $\hat{f}(\psi) = p(\psi)\hat{f}_1(\psi)$, then $deg\ \hat{f}_1 = deg\ \hat{f} - n_0$. Thus

$$h(\psi) = \hat{h}\left(\frac{1}{\psi}\right)\psi^n = p\left(\frac{1}{\psi}\right)\psi^{n_0}\hat{h}_1\left(\frac{1}{\psi}\right)\psi^{deg\ \hat{h} - n_0}$$

Let $\hat{p}(\psi) = p(1/\psi)\psi^{n_0}$, then $\hat{p}(\psi)|h(\psi)$. Similarly $\hat{p}(\psi)|f(\psi)$. That is $\hat{p}(\psi)|h(\psi)$, $\hat{p}(\psi)|f(\psi)$ and $deg\ \hat{p} = n_0 > 0$, which is a contradiction.

Now from (4.7), $f(\psi)|\hat{h}(\psi)$ and $h(\psi)|\hat{f}(\psi)$. Also $\hat{f}(\psi)|h(\psi)$ and $\hat{h}(\psi)|f(\psi)$ hold. $f(\psi)|\hat{h}(\psi)$ and $\hat{h}(\psi)|f(\psi)$ imply $deg\ \hat{h} = deg\ f$. Then

$$f(\psi) = k\hat{h}(\psi), \tag{4.8}$$

where k is a constant. Substituting (4.8) into (4.7),

$$h(\psi) = k\hat{f}(\psi) \tag{4.9}$$

then $\hat{h}(\psi) = \psi^n h(1/\psi) = k\psi^n\hat{f}(1/\psi) = kf(\psi)$, which implies

$$\hat{h}(\psi) = kf(\psi) \tag{4.10}$$

From (4.8)-(4.10), $k^2 = 1$ which means (4.4a,b) are proven.

Next, the proof of (4.5) is considered. From the Feldkeller equation and (4.8), it follows that

$$g(\psi)g_*(\psi) = h(\psi)h_*(\psi) + \hat{h}(\psi)\hat{h}_*(\psi)$$

and $$\psi^{2n} g\left(\frac{1}{\psi}\right) g_*\left(\frac{1}{\psi}\right) = \psi^{2n} h\left(\frac{1}{\psi}\right) h_*\left(\frac{1}{\psi}\right) + \psi^{2n} h\left(\frac{1}{\psi}\right) \hat{h}_*\left(\frac{1}{\psi}\right)$$

i.e.

$$\hat{g}(\psi)\hat{g}_*(\psi) = \hat{h}(\psi)\hat{h}_*(\psi) + h(\psi)h_*(\psi) = g(\psi)g_*(\psi) \tag{4.11}$$

where $\hat{g}(\psi) = \psi^n g(1/\psi)$. Since $g(\psi)$ is Hurwitz, $g(\psi)$ and $g_*(\psi)$ are relatively prime and consequently so are $\hat{g}(\psi)$ and $\hat{g}_*(\psi)$. Additionally, $g(\psi)$ is Hurwitz also implies $\hat{g}(\psi)$ is Hurwitz, which implies $g(\psi)$ and $\hat{g}_*(\psi)$ and $\hat{g}(\psi)$ and $g_*(\psi)$ are relatively prime. So $g(\psi)g_*(\psi) = \hat{g}(\psi)\hat{g}_*(\psi)$ implies $g(\psi)|\hat{g}(\psi)$ and $\hat{g}(\psi)|g(\psi)$ and finally $g(\psi) = \pm\hat{g}(\psi)$, that is (4.5) is proven.

4.2 Analytical Method for Design of Bireciprocal Filters

Generally, in filter design, a specification in terms of attenuation as illustrated in Fig. 3.18 is given first. Then based on this specification, different kinds of filters can be designed. In this section, an analytical method to design bireciprocal filters by building a bireciprocal characteristic function $k(\psi)$ which satisfies a specification (see Fig. 4.1) derived from Fig. 3.18 is presented.

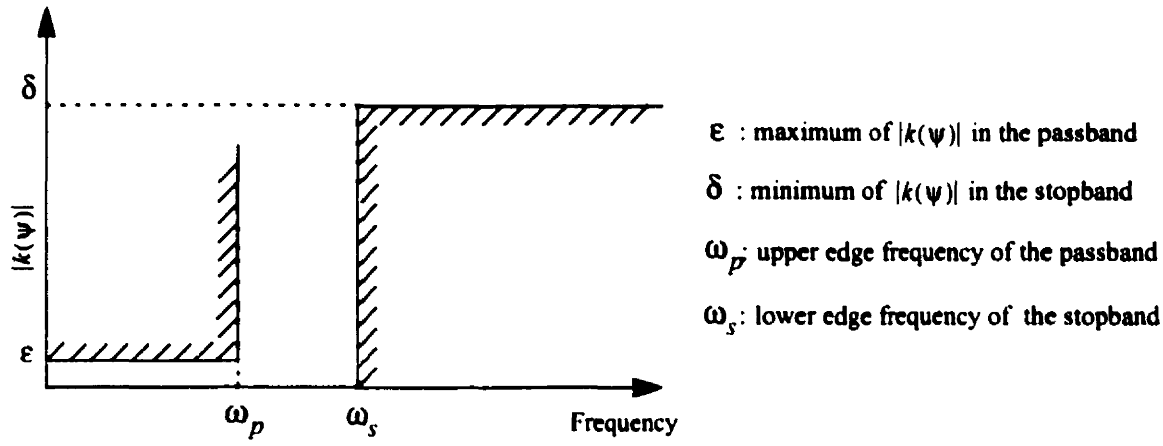


Fig. 4.1 Design specification for $k(\psi)$

If the maximum attenuation in the passband is A_p dB and the minimum attenuation in the stopband is A_s dB, then

$$\varepsilon = \sqrt{1 - 10^{A_p/10}}, \quad \delta = \sqrt{1 - 10^{A_s/10}} \tag{4.12}$$

because the attenuation α can be expressed in terms of the characteristic function $k(\psi)$ as

$$\alpha = 10\log(1 + |k(\psi)|^2) = 10\log\left(1 + \left|\frac{h(\psi)}{f(\psi)}\right|^2\right), \text{ where } \psi = j\phi = j\tan\left(\frac{\omega T}{2}\right).$$ Hence,

if only the attenuation is of interest, all zeros and poles of the characteristic function are usually distributed on the imaginary axis. Furthermore, taking into account the definition of a bireciprocal filter (4.3) and the properties (4.4a,b), the characteristic function $k(\psi)$ can be assumed to have the form

$$k(\psi) = \begin{cases} \displaystyle\prod_{i=1}^{n/2} \frac{\psi^2 + p_i^2}{p_i^2 \psi^2 + 1}, & \text{if } n \text{ is even} \\ \displaystyle\psi \prod_{i=1}^{(n-1)/2} \frac{\psi^2 + p_i^2}{p_i^2 \psi^2 + 1}, & \text{if } n \text{ is odd} \end{cases} \tag{4.13}$$

Next, solve for the p_i, $i = 1, 2, \ldots, [n/2]$ (where [ ] denotes integer part) such that $k(\psi)$ satisfies the specification as illustrated in Fig. 4.1. That is, in the passband, the maximum of $|k(\psi)|$ is as small as possible and in the stopband, its minimum is as great as possible. This is a Chebyshev approximation problem and it has a unique solution [30]:

$$p_i = \hat{\omega}_p \mathrm{sn}\left(\frac{n-2i+1}{n}K\right), \quad i = 1, 2, \ldots, [n/2] \tag{4.14}$$

where sn is the Jacobi elliptic function, and K is the complete elliptic integral of the first kind with modulus $\hat{\omega}_p^2$, that is,

$$K = \int_0^1 \frac{1}{\sqrt{(1-t^2)(1-\hat{\omega}_p^4 t^2)}}dt, \qquad \hat{\omega}_p = \frac{\omega_p}{\omega_0}, \quad \omega_0 = \sqrt{\omega_p \omega_s} \tag{4.15}$$

The maximum of $|k(\psi)|$ in the passband can be calculated by[30]

$$\varepsilon = \hat{\omega}_p^n \prod_{i=1}^{[n/2]} \mathrm{sn}\left(\frac{2i-1}{n}K\right) \tag{4.16}$$

The characteristic function $k(\psi)$ defined by (4.13) has the property[30]: if its maximum in the interval $[0, \hat{\omega}_p]$ is ε, then its minimum in $[\hat{\omega}_s, \infty]$ must be $1/\varepsilon$. This property means that if the specification about ε and δ illustrated in Fig. 4.1 does not satisfy $\varepsilon\delta \le 1$, but $\varepsilon\delta > 1$, a small adjustment to ε or δ is required: Let the new

$$\varepsilon = \frac{1}{\delta} \text{ or } \delta = \frac{1}{\varepsilon} \tag{4.17}$$

which satisfies $\varepsilon\delta \le 1$.

The design procedure for bireciprocal filters using the analytical method is summarized as follows:

- for an attenuation specification A_p, A_s, ω_p and ω_s, calculate ε and δ according formulae (4.12). If $\varepsilon\delta > 1$, compute a new ε or a new δ according (4.17).
- using (4.15) calculate $\hat{\omega}_p$ first, then K. Next determine the degree n using (4.16).
- use (4.14) to calculate p_i, $i = 1, 2, \ldots, [n/2]$, which yields $k(\psi)$.
- from $k(\psi)$, $h(\psi)$ and $f(\psi)$ immediately obtain

$$h(\psi) = \begin{cases} \prod_{i=1}^{\frac{n}{2}} (\psi^2 + p_i^2) \text{ , if } n \text{ is even} \\ \psi \prod_{i=1}^{\frac{(n-1)}{2}} (\psi^2 + p_i^2) \text{ , if } n \text{ is odd} \end{cases}, \quad f(\psi) = \prod_{i=1}^{\left[\frac{n}{2}\right]} (p_i^2 \psi^2 + 1) \qquad (4.18)$$

- write the right-hand side of the Feldtkeller equation $gg_* = ff_* + hh_*$ in product

form: $g(\psi)g_*(\psi) = \sum_{i=1}^{2n} (\psi - \psi_i)$. Since $g(\psi)$ is a Hurwitz polynomial, all zeros located in the left-half of the ψ-plane can be allotted to $g(\psi)$, the reminder to $g_*(\psi)$.

In order to demonstrate the above design procedure, an example is shown below.

Example: Specification: $A_p = 0.5\text{dB}$, $A_s = 53\text{dB}$, $\omega_p = 1$, $\omega_s = 2$

Step 1: calculate ε, δ according to (4.13) and obtain

$\varepsilon = 0.3493$, $\delta = 446.6825$ and $\varepsilon\delta = 156.0313 > 1$.

From (4.17) obtain a new $\varepsilon = 0.0022387$.

Step 2: From (4.15), (4.16),

$\hat{\omega}_p = 0.7071$, $K = 1.6858$, $n = 7$.

Step 3: From (4.14), p_i, $i = 1, 2, 3$, can be calculated as

$$p_1 = 0.6917,\ p_2 = 0.5679,\ p_3 = 0.3248$$

Step 4: Substituting p_i, $i = 1, 2, 3$, into (4.18), the polynomials $h(\psi)$ and $f(\psi)$ are found immediately

$$h(\psi) = \psi(\psi^2 + 0.6917^2)(\psi^2 + 0.5679^2)(\psi^2 + 0.3248^2)$$

$$f(\psi) = (0.6917^2\psi^2 + 1)(0.5679^2\psi^2 + 1)(0.3248^2\psi^2 + 1)$$

Step 5: Using a program written in MATLAB, polynomial $g(\psi)$ is obtained as

$$g(\psi) = (\psi + 1)(\psi^2 + 0.2448\psi + 1)(\psi^2 + 1.5640\psi + 1)(\psi^2 + 0.4126\psi + 1)$$

Step 6: Finally, the bireciprocal filter which satisfies the specification has the transfer function

$$\frac{f(\psi)}{g(\psi)} = \frac{(0.6917^2\psi^2 + 1)(0.5679^2\psi^2 + 1)(0.3248^2\psi^2 + 1)}{(\psi + 1)(\psi^2 + 0.2448\psi + 1)(\psi^2 + 1.5640\psi + 1)(\psi^2 + 0.4126\psi + 1)}$$

The frequency response for this filter is shown in Fig. 4.2

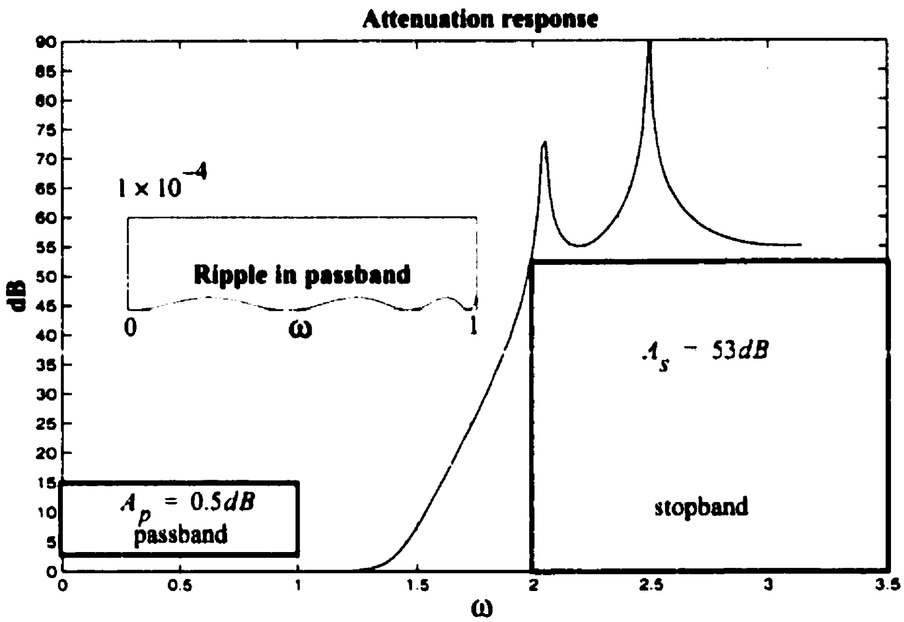
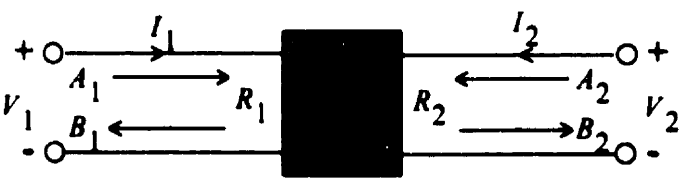


Fig.4.2

4.3 Structure of Wave Digital Lattice Filters[8]

Wave digital filters are derived from real lossless reference filters using voltage wave quantities. Consider a two-port network *N*:



where the relationship between the incident wave $A_i = V_i + I_i R_i$ and reflected wave $B_i = V_i - I_i R_i$, $(i = 1, 2)$ and the scattering matrix

$$S = \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix} \tag{4.19}$$

can be described as

$$B_1 = s_{11}A_1 + s_{12}A_2 \tag{4.20a}$$

$$B_2 = s_{21}A_1 + s_{22}A_2 \tag{4.20b}$$

It is assumed that the two-port network N is symmetric and reciprocal, i.e.

$$s_{11} = s_{22}, \quad s_{12} = s_{21} \tag{4.21}$$

Next, define reflections $S_1 = s_{11} - s_{12}$, $S_2 = s_{11} + s_{21}$ and take (4.21) into account.

Then equation (4.20a,b) can be written as

$$2B_1 = S_1(A_1 - A_2) + S_2(A_1 + A_2) \tag{4.22a}$$

$$2B_2 = -S_1(A_1 - A_2) + S_2(A_1 + A_2) \tag{4.22b}$$

These equations lead to the lattice realization of a wave digital filter as shown in Fig. 4.3(a). For $A_2 = 0$, the signal-flow diagram simplifies to Fig. 4.3(b).

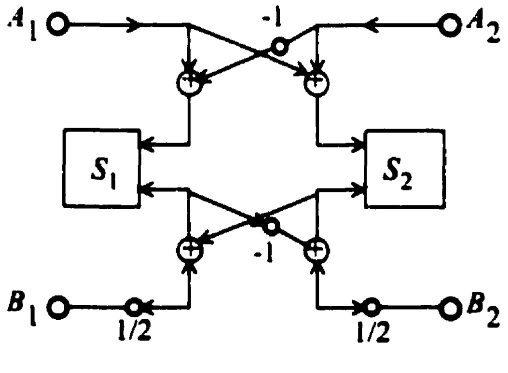


Fig. 4.3 (a) Signal-flow diagram of the Lattice structure

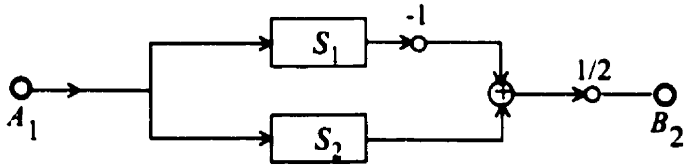


Fig. 4.3 (b) Simplified wave-flow diagram

Therefore the realization reduces to the realization of two reflectances: S_1, S_2. It is known that the scattering matrix (4.19) can also be described by canonic polynomials $f(\psi)$, $g(\psi)$ and $h(\psi)$, i.e. $S = \frac{1}{g(z)}\begin{bmatrix} h(\psi) & \sigma f_*(\psi) \\ f(\psi) & -\sigma h_*(\psi) \end{bmatrix}$. So in view of (4.20) and (4.21), it follows that

$$s_{11} = s_{22} = \frac{h(\psi)}{g(\psi)}, \quad s_{12} = s_{21} = \frac{h(\psi)}{g(\psi)} \tag{4.23a,b}$$

with

$$f_*(\psi) = \sigma f(\psi), \quad h_*(\psi) = -\sigma h(\psi) \tag{4.23c,d}$$

In addition, from the Feldtkeller equation $g(\psi)g_*(\psi) = f(\psi)f_*(\psi) + h(\psi)h_*(\psi)$ and considering 4.23a,b,c,d, it follows that

$$g(\psi)g_*(\psi) = \sigma(f(\psi) - h(\psi))(f(\psi) + h(\psi)) \tag{4.24}$$

The polynomial $g(\psi)$ can always be factored[1] as $g(\psi) = g_1(\psi)g_2(\psi)$, so that $f(\psi) + h(\psi) = g_1(\psi)g_{2*}(\psi)$ and $\sigma(f(\psi) - h(\psi)) = g_{1*}(\psi)g_2(\psi)$, where $g_1(\psi)$ and $g_2(\psi)$ are also Hurwitz polynomials. Next from the definition for S_1 and S_2,

$$S_1(\psi) = \frac{h(\psi) - f(\psi)}{g(\psi)} = (-\sigma)\frac{g_{1*}(\psi)}{g_1(\psi)}, \; S_2(\psi) = \frac{h(\psi) + f(\psi)}{g(\psi)} = \frac{g_{2*}(\psi)}{g_2(\psi)}$$

Thus both S_1 and S_2 are allpass functions. There are several methods[1] to realize allpass functions S_1 and S_2.

4.4 Optimization Method for the Design of Bireciprocal Filters

In the previous section, an analytical solution for the design of bireciprocal filters was provided. The main advantage of the analytical method is simplicity, direct and easy calculations since both the Jacobi elliptic function sn and the complete elliptic integral are available in MATLAB. However for some situations, in order to obtain a fit to the prescribed specification, overdesign is required. In this section, applying Yli-Kaakinen's optimization approach[15] to the design of bireciprocal filters, another efficient design method is obtained.

4.4.1 The statement of problem

One kind of optimization problem can be stated in the following form[15]: Find an adjustable parameter vector x to minimize

$$\rho(x) = \max_{1 \le i \le N_p + N_s} \{y_i(x)\} \tag{4.25}$$

subject to L inequality constraints

$$g_l(x) < 0, \; l = 1, 2, \ldots, L \tag{4.26}$$

Now, the question arises, how to reduce the design of bireciprocal filters to an optimization problem, or how to define the objective function $y_i(x)$. The magnitude function is preferred rather than the characteristic function or attenuation function because there is an ideal function

$$H(\omega) = \begin{cases} 1 & 0 \le \omega \le \omega_p \\ 0 & \omega \ge \omega_s \end{cases}$$

which the magnitude function can approximate. Here ω_p and ω_s are the passband and stopband edges, respectively. For any attenuation specification illustrated in Fig. 3.18, it is easy to obtain the corresponding specification for the magnitude:

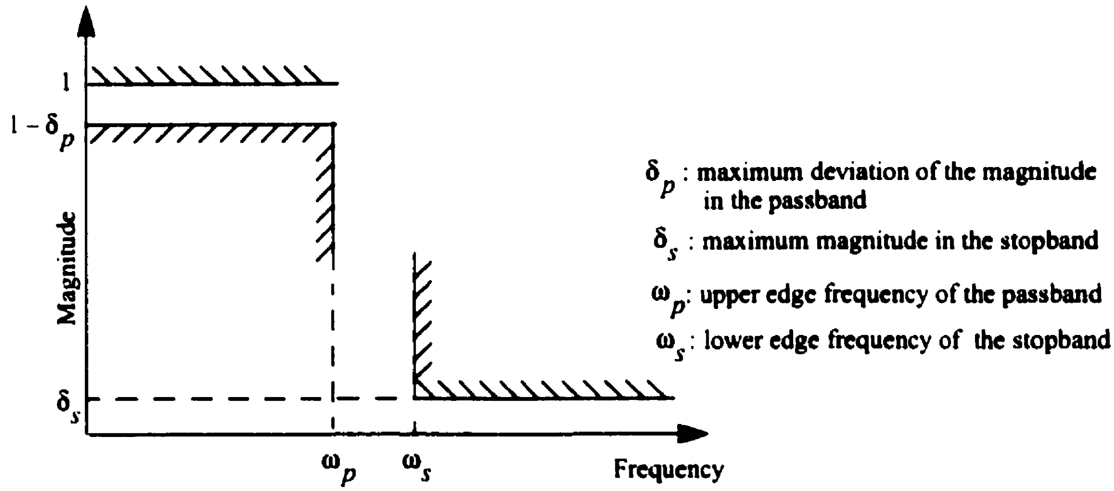


Fig. 4.4 Design specification for magnitude

where

$$\delta_p = 1 - \exp(-A_p \log 10/20),\ \delta_s = \exp(-A_s \log 10/20) \qquad (4.27a,b)$$

Generally, optimization methods are a good way to solve a problem which contains some unknown parameters. The adjustable parameters included in the bireciprocal filters which are designed are denoted as vector x. The magnitude function can be designated $H(x, \omega)$, where ω is the frequency. Thus the criteria illustrated in Fig. 4.4 can be stated mathematically[15] as

$$1-\delta_p \le |H(x,\omega)| \le 1 , \qquad \omega \in [0, \omega_p] \tag{4.28a}$$

$$|H(x,\omega)| \le \delta_s , \qquad \omega \in [\omega_s, C] \tag{4.28b}$$

For optimization purposes, or to find the appropriate $y_i(x)$ (cf 4.25) which have a uniform expression, it is beneficial to combine the passband and stopband criteria. Let

$$|E(x,\omega)| = W(\omega)|H(x,\omega)-D(\omega)| , \quad \omega \in [0,\omega_p] \cup [\omega_s, C] \tag{4.29}$$

with

$$D(\omega) = \begin{cases} 1, & \omega \in [0, \omega_p] \\ 0, & \omega \in [\omega_s, C] \end{cases} \text{ and } W(\omega) = \begin{cases} 1/\delta_p , & \omega \in [0, \omega_p] \\ 1/\delta_s, & \omega \in [\omega_s, C] \end{cases},$$

then

$$|E(x,\omega)| \le 1 , \quad \omega \in [0,\omega_p] \cup [\omega_s, C] \tag{4.30}$$

is equivalent to (4.28a,b). Function $E(x,\omega)$ is called the weighted error function. Next, focus on finding the adjustable parameter vector x to minimize

$$\max_{\omega \in [0,\omega_p] \cup [\omega_s, C]} |E(x,\omega)|$$

In order to make the above problem more suitable for optimization, the passband and the stopband regions are discretized into the frequency points $\omega_i \in [0, \omega_p]$, i=1,2, ... N_p and $\omega_i \in [\omega_s, C]$, i=$N_p+1$, N_p+2, ... , N_p+N_s. The resulting non-constrained discrete problem is to find x to minimize

$$\max_{1 \le i \le N_p+N_s} |E(x,\omega_i)| \tag{4.31}$$

i.e., here $|E(x,\omega_i)|$ is taken as the $y_i(x)$ in (4.25). In some situations where the parameter vector is required to satisfy additional conditions such as in (4.26), the problems is called a constrained minimax problem. It can be solved by applying a minimization algorithm proposed by Dutta and Vidyasagar[28], which is listed in the next section.

4.4.2 Minimax algorithm using the Dutta-Vidyasagar method

Problem:

In [28], Dutta and Vidyasagar proposed a method to solve a problem of minimax optimization under constraints. This problem can be stated as follows:

$$\text{minimize } F(x) = \max_{i \in I} f_i(x)$$

where $I = \{1, 2, \ldots, n\}$ is a finite set of integers, under the constraints

$$g_j(x) \le 0, \qquad j \in J$$

$$g_l(x) = 0 \qquad j \in L$$

Converting the above constrained problem to an unconstrained minimization of Least-squares type objective function yields

$$\text{minimize } P(x, \phi) = \sum_{i \in I(x)} (f_i(x) - \phi)^2 + \sum_{i \in J(x)} w_j g_i(x) + \sum_{l \in L} v_j v_l(x)$$

where ϕ is a prespecified constant, w_j, $j \in J$, and v_l, $l \in L$ are prespecified weights, and

$$I(x) = \{i \in I : f_i(x) > \phi\},\ J(x) = \{j \in J : g_j(x) > \phi\}$$

Algorithm:

Step 1. Set $B_1 < F(\bar{x})$, where $F(\bar{x})$ is the optimum and B_1 is a lower bound on $F(\bar{x})$.

Step 2. Set $\phi_1 \leftarrow B_1$, and $k \leftarrow 1$.

Step 3. Minimize $P(x, \phi_k)$, call the solution $\bar{x}_k$.

Step 4. Set B_u as an upper bound on $F(\bar{x})$. For an unconstrained problem one can set

$B_u \leftarrow \min\{F(x_0), F(\bar{x}_k)\}$, where x_0 is the initial guess of the parameter vector.

Step 5. Calculate

$$M^M \leftarrow \phi_k + \{P(\bar{x}_k, \phi_k)/n\}^{1/2}$$

$$M^T \leftarrow \phi_k + P(\bar{x}_k, \phi_k)/\sum(f_i(\bar{x}_k) - \phi_k), \quad i \in I(\bar{x}_k)$$

where $I(\bar{x}_k) = \{i : f_i(\bar{x}_k) > \phi_k\}$.

Step 6. If $M^T \le B_{L'}$, set $\phi_{k+1} \leftarrow M^T$, otherwise set $\phi_{k+1} \leftarrow M^M$. Also set

$\phi_0 \leftarrow \phi_{k+1} - \phi_k$.

Step 7. Set $B_l \leftarrow M^M$, and $S \leftarrow P(\bar{x}_k, \phi_k)$.

Step 8. Set $k \leftarrow k+1$.

Step 9. Minimize $P(x, \phi_k)$, call the solution $\bar{x}_k$.

Step 10. If $(B_u - B_l)$ or $\phi_0 \le \varepsilon$, STOP (ε is a small number).

Step 11. If $P(\bar{x}_k, \phi_k) > \delta$, go to Step 5, otherwise if $\bar{S}$ < SMALL, STOP (SMALL is a positive constant signifying the closeness of $P(\bar{x}_{k-1}, \phi_{k-1})$ to zero). If none is true, then set $B_u \leftarrow \phi_k$, $S \leftarrow 0$, and $\phi_k \leftarrow B_l$ and go to Step 9.

4.4.3 Constructing the magnitude function

From the discussion in section **4.4.1**, it is known that the first step of applying optimization for designing bireciprocal filters is to construct the magnitude function. In this section, two construction methods are presented. ***Method 1*** starts from the characteristic function $k(x, \psi)$, which includes the unknown parameter vector x. The magnitude function $H(x, \psi)$ can be easily be obtained from $k(x, \psi)$, as can the weighted error function $E(x, \psi)$. Parameter vector x is obtained after applying optimization to $E(x, \psi)$. Different from ***method 1***, ***method 2*** construct the magnitude function $H(x, \psi)$ from the lattice realization structure. The details of both methods are presented below.

Method 1. Based on the definition of the bireciprocal filter (cf (4.3)) and the property that all zeros and poles of the characteristic function are usually distributed on the imaginary axis, it can be assumed that the characteristic function which contains the adjustable parameter vector x has the following form:

$$k(x, \psi) = \psi \prod_{i=1}^{(n-1)/2} \frac{\psi^2 + x_i^2}{x_i^2 \psi^2 + 1} \tag{4.32}$$

Here $x = \left(x_1, x_2, \ldots, x_{\frac{n-1}{2}}\right)$, n is the degree of the filter which is always assumed to be odd in order to implement the lattice structure. Then the magnitude function is

$$|H(x, \phi)| = \left.\sqrt{\frac{1}{1 + |k(x, \psi)|^2}}\right|_{\psi = j\phi} \tag{4.33}$$

with $\phi = \tan\omega$. Consider (4.32), the magnitude function which contains the parameter vector x and frequency ω. It has the following form:

$$|H(x, \omega)| = \sqrt{1 \Big/ \left(1 - \tan\omega^2 \prod_{k=1}^{(n-1)/2} \frac{x_k^2 - \tan\omega^2}{1 - \tan\omega^2 x_k^2}\right)} \tag{4.34}$$

All that is needed next is to substitute (4.34) into (4.28a,b) and follow the procedure stated in section **4.4.1.** After applying the Dutta-Vidyasagar minimax algorithm, a solution for the parameter vector x will be obtained.

Method 2: The idea for this method comes from Wegener's paper[14], which derives the design and implementation of bireciprocal filters at the same time. Recall the lattice structure proposed in the previous section where the realization of a digital filter was reduced to the realization of two allpass filters $S_1(\psi)$ and $S_2(\psi)$:

$$S_1(\psi) = -\sigma\frac{g_{1*}(\psi)}{g_1(\psi)}, S_2(\psi) = \frac{g_{2*}(\psi)}{g_2(\psi)} \tag{4.35}$$

where $g(\psi) = g_1(\psi)g_2(\psi)$. Based on three properties:

1) $g(\psi)$ satisfies $g(\psi) = \pm\psi^n g\left(\frac{1}{\psi}\right)$ (cf 4.5).

2) $g(\psi)$ is Hurwitz polynomial.

3) the degree of $g(\psi)$ is odd.

$g(\psi)$ can be assumed to have the following product form:

$$g(\psi) = K(\psi + 1) \prod_{i=1}^{(n-1)/2} (\psi^2 + x_i\psi + 1) \tag{4.36}$$

From the lattice structure Fig. 4.3 and equations (4.35) and (4.36), it is can be assumed that a bireciprocal lowpass filter has the structure shown in Fig. 4.5, where

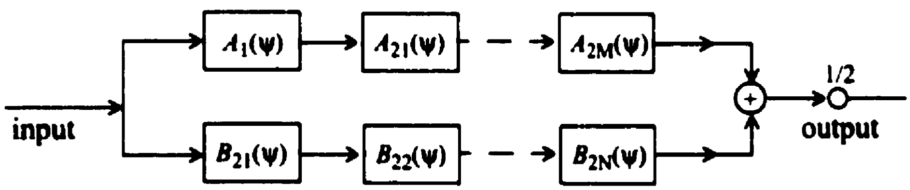


Fig. 4.5

$$A_1(\psi) = \frac{-\psi+1}{\psi+1} \tag{4.37}$$

$$A_{2i}(\psi) = \frac{\psi^2 - x_i\psi + 1}{\psi^2 + x_i\psi + 1}, \qquad x_i > 0, \quad i = 1, 2, \ldots, M \tag{4.38}$$

$$B_{2j}(\psi) = \frac{\psi^2 - x_{i+j}\psi + 1}{\psi^2 + x_{i+j}\psi + 1}, \qquad x_{i+j} > 0, \quad \mathrm{j} = 1, 2, \ldots, N \tag{4.39}$$

With $M = N \pm 1$ and $n = M + N + 1$, then

$$H(x, \psi) = \frac{1}{2}\left(A_1(\psi)\prod_{i=1}^{M} A_{2i}(x, \psi) + \prod_{j=1}^{N} B_{2j}(x, \psi)\right)$$

Let $\psi = j\phi$ and $\phi = \tan\omega$, and then obtain the magnitude function immediately.

$$|H(x, \omega)| = \frac{1}{2}\left|A_1(j\tan\omega)\prod_{i=1}^{M} A_{2i}(x_i, j\tan\omega) + \prod_{j=1}^{N} B_{2j}(x_{i+j}, j\tan\omega)\right| \tag{4.40}$$

where $x = (x_1, x_2, \ldots, x_M, x_{M+1}, \ldots, x_N)$ is the parameter vector. Similar to ***Method* 1**, by substituting (4.37) into (4.28a,b), following the procedure stated in section **4.4.1**, and then applying the Dutta and Vidyasagar minimax algorithm, a solution for parameter vector x is obtained.

Here it is appropriate to mention that this method can also be applied to any general lattice filter design. This can be carried out by changing equations (4.37)-(4.40) to

$$A_1(\psi) = \frac{-\psi+y_0}{\psi+y_0}, \qquad y_0 > 0$$

$$A_{2i}(\psi) = \frac{\psi^2 - x_i\psi + y_i}{\psi^2 + x_i\psi + y_i}, \qquad x_i, y_i > 0, \quad i = 1, 2, \ldots, M$$

$$B_{2j}(\psi) = \frac{\psi^2 - x_{i+j}\psi + y_{i+j}}{\psi^2 + x_{i+j}\psi + y_{i+j}}, \qquad x_{i+j}, y_{i+j} > 0, \quad j = 1, 2, \ldots, N$$

Now the adjustable parameter is $x = (x_1 \ldots x_{M+N} y_0 y_1 \ldots y_{M+N})$ whose dimension is $2(M+N)+1$ that is same as the degree of the filter.

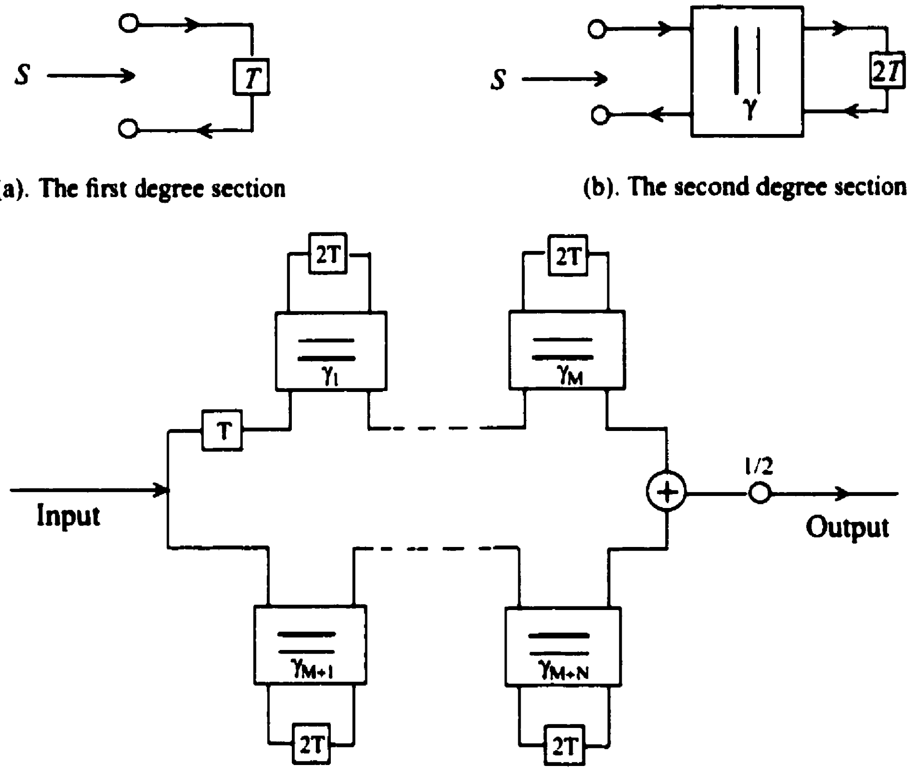
4.5 The realization of Bireciprocal Filters

The realization structure for design ***method 2*** has been given in the previous section (see Fig. 4.5). For ***method 1***, after solving for parameter vector x using the optimization method and obtaining characteristic function $k(x, \psi)$, it is easy to obtain $f(\psi)$, $h(\psi)$ and $g(\psi)$ (There is a statement about how to derive $g(\psi)$ from $f(\psi)$ and $h(\psi)$ in section **4.2.**). Then following the procedure stated in section **4.3**, $g_1(\psi)$ and $g_2(\psi)$ can be obtained, then $S_1(\psi)$ and $S_2(\psi)$, and finally a structure similar to Fig. 4.5 is obtained. This means that independently of whether the design for a bireciprocal filters uses ***method 1*** or ***method 2***, the realization structure can always be reduced to Fig. 4.5.

Next, focus on the realization of structure Fig. 4.5. It is known [14] that the realization of allpass sections, like $A_1(\psi)$ in Fig. 4.5, require no arithmetic operations, but only a delay T. The wave-flow diagram is shown in Fig. 4.6 (a). For allpass sections of degree two, like $A_{2i}(\psi)$ $(i = 1, 2, \ldots, M)$ and $B_{2j}(\psi)$ $(j = 1, 2, \ldots, N)$, the realization requires a two-port adaptor and two delays $2T$. The coefficients γ_i of the two-port adaptors are given by

$$\gamma_i = (x_i - 2)/(x_i + 2), \; i = 1, 2, \ldots, M, M+1, \ldots, M+N \tag{4.41}$$

The wave-flow diagram for degree two is shown in Fig. 4.6 (b), and the wave-flow diagram for realization structure Fig. 4.5 is presented in Fig. 4.6 (c).



(c) Signal-flow diagram for realization of a bireciprocal filter

Fig. 4.6 Realization of a bireciprocal filter

The above signal-flow diagram shows that it only requires $M+N = (n-1)/2$ (n is odd) two-port adaptors and n delays to implement an n^{th} order bireciprocal filter, which means savings of almost one half in hardware compared with other digital filters.

4.6 Example

The attenuation requirements are illustrated in Fig. 4.7[14] and the sampling frequency $F = 48Hz$.

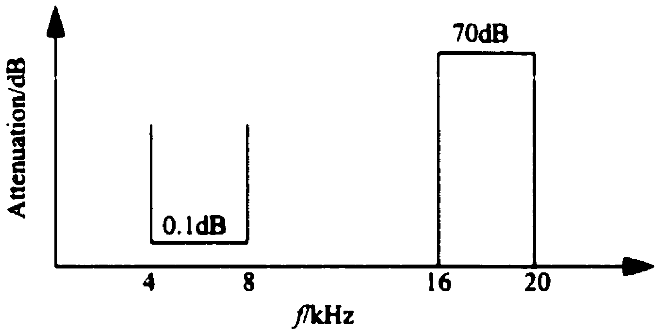


Fig. 4.7

With ***method*** **1**, starting from (4.32) and following the procedure described in section **4.4.1**, the adjustment parameter x can be solved for by applying the minimization algorithm proposed by Dutta and Vidyasagar. Here the parameter vector

$$x = (-0.565007, -0.312758, -0.469945)$$

is obtained and

$$k(x, \psi) = \psi\left(\frac{\psi^2 + (-0.565007)^2}{\psi^2(-0.565007)^2 + 1}\right)\left(\frac{\psi^2 + (-0.312758)^2}{\psi^2(-0.312758)^2 + 1}\right)\left(\frac{\psi^2 + (-0.469945)^2}{\psi^2(-0.469945)^2 + 1}\right)$$

then

$$f(\psi) = (\psi^2(-0.565007)^2 + 1)(\psi^2(-0.312758)^2 + 1)(\psi^2(-0.469945)^2 + 1)$$

$$h(\psi) = \psi(\psi^2 + (-0.565007)^2)(\psi^2 + (-0.312758)^2)(\psi^2 + (-0.469945)^2)$$

and $g(\psi)$ can be derived following the statement in section **4.2** as

$$g(\psi) = (\psi+1)(\psi^2+0.3032196\psi+1)(\psi^2+1.6476963\psi+1)(\psi^2+0.9578127\psi+1)$$

Next from section **4.2**, it follows that

$$g_1(\psi) = (\psi+1)(\psi^2+0.9578127\psi+1)$$

$$g_2(\psi) = (\psi^2+0.3032196\psi+1)(\psi^2+1.6476963\psi+1)$$

and from equation (4.41), the parameters of the three 2-port adaptors can be obtained as

$$\gamma_1 = -0.35235092,\ \gamma_2 = -0.09658259,\ \gamma_3 = -0.73669939$$

After 11 bit quantization, the three parameters become

$$\gamma_1 = -361/1024,\ \gamma_2 = -99/1024,\ \gamma_3 = -1509/2048 \tag{4.42}$$

With ***method*** **2**, as described in section **4.4**, starting from (4.40) and following the procedure stated in section **4.1**, the parameter x is obtained as

$$x = (0.95780467,\ 1.6476936,\ 0.3032167),$$

and the coefficients of three 2-port adaptors are obtained by (4.41) as

$$\gamma_1 = -0.35235435,\ \gamma_2 = -0.09658332,\ \gamma_3 = -0.73670154$$

After quantization, exactly the same results as in (4.42) are obtained which shows that the results of the two method are the same, i.e., for the same problem, no matter which method is applied, the same realization structure results. The advantage of ***method*** **2** is that the realization structure can be obtained directly after applying the optimization algorithm. However it requires the constrained minimax algorithm and uses more computer time than ***method*** **1**, which only requires the unconstraint minimax algorithm. Another advantage of ***method*** **1** is that it is easy to derive $f(\psi)$, $g(\psi)$ and $h(\psi)$ after applying the optimization algorithm.

The realization structure for the example is shown in Fig. 4.8.

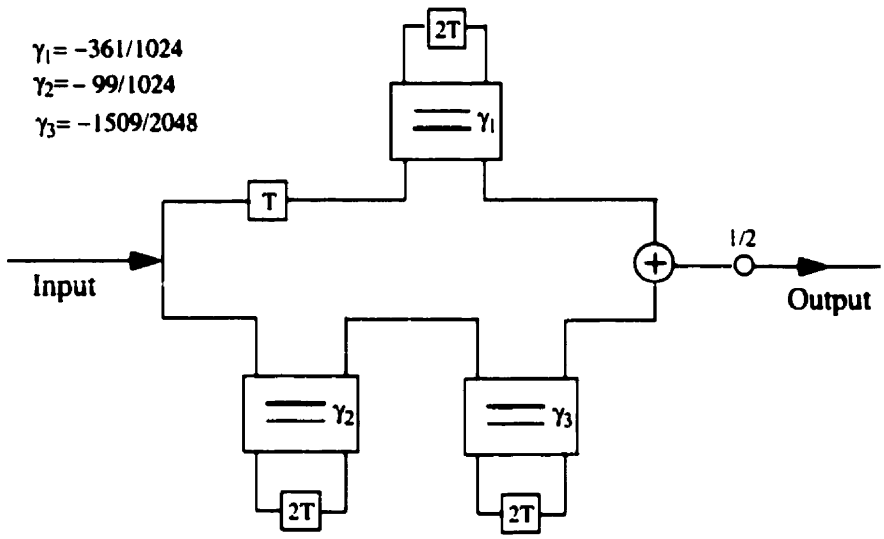


Fig. 4.8 Realization structure of the example

The frequency responses are presented in Fig 4.9 which shows that the specifications (see Fig. 4.7) are satisfied.

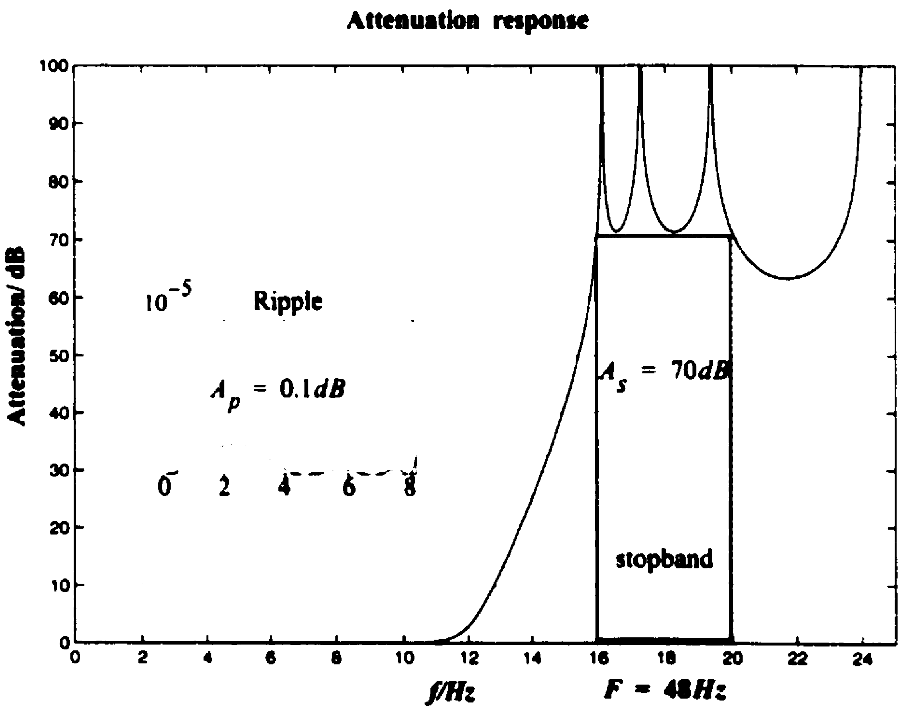


Fig. 4.9

Chapter 5

Conclusions

In this thesis the following has been proven:

- The cascade decomposition of lossless two-port networks by means of the factorization of the transfer matrix T is realizable. A proof of the necessary and sufficient conditions for the realizability is included.
- The cascade synthesis of two-port lossless networks by the factorization of the scattering matrix S applied to FIR filters is successful. An implementation structure and an algorithm in a very general form are derived. Several other implementation structures and related algorithms, including Fettweis' two structures, are obtained. Examples demonstrate that for broad-band FIR filters all proposed structures exhibit low sensitivity to multipliers, but for some narrow-band FIR filters, one of the derived structures, called the S_δ-S_β-structure, shows much lower sensitivity in the passband than the classical direct form. Therefore the S_δ-S_β-structure is the recommended structure for narrow-band FIR filters.

- Both the analytical formula method and the optimization method for designing bireciprocal filters are efficient. The analytic formula method for the design of bireciprocal filters appears to be new. A search of the literature of the past ten years has not produced any reference to it. The formula is simple, direct, and requires only simple calculations. The only disadvantage is that it requires overdesign in some situations. The optimization method developed in this thesis is very flexible. It is efficient not only for the design of bireciprocal filters but also for general lattice wave digital filters after minor modifications have been made. An example shows that the proposed optimization method is an alternate solution to Wegener's optimization method.

Future research directions might be the following:

- Apply the approach of cascade synthesis of two-port lossless networks by factorization of the transfer matrix to FIR filters and compare with the results obtained in chapter 3.
- Consider the design of half band filters using the same idea as in the design of bireciprocal filters.

Appendix I[29]

The Sum of Polynomials in Product Representation

To determine the product representation of the sum of two polynomials $p = a + b$, a and b both in product representation without converting to coefficient representation requires an iterative procedure. For this purpose, consider the first two terms of the Tayler series about a point s_k

$$p(s) \approx p(s_k) + p'(s_k)(s - s_k)$$

To determine an approximation s_{k+1} to a zero, set $p(s_{k+1}) = 0$ and solve for s_{k+1}:

$$s_{k+1} = s_k - \frac{p(s_k)}{p'(s_k)}$$

This is Newton's estimate[31] and can be used iteratively, $k = 0, 1, \ldots, k_m$, to improve the estimate of the zero.

After a zero has been found it is necessary to prevent finding the same zero repeatedly. This can be accomplished by zero suppression; i.e. by formally dividing out the found zero. For this purpose the found zeros are accumulated in a polynomial c in product representation. Next Newton's method is applied to

$$q(s) = \frac{p(s)}{c(s)}$$

giving

$$s_{k+1} = s_k - \frac{(p(s_k))/c(s_k)}{(p(s)/c(s))'|_{s = s_k}}$$

$$= s_k - \frac{1}{(p'(s_k))/(p(s_k)) - (c'(s_k))/(c(s_k))}$$

This is the Newton-Maehly estimate[31].

Note that $\frac{c'(s)}{c(s)} = \sum_{i=1}^{m} \frac{1}{s - c_i}$, where the c_i are the zeros of c (the found zeros) and

$$\frac{p'(s)}{p(s)} = \frac{a'(s)}{a(s)} \frac{a(s)}{a(s) + b(s)} + \frac{b'(s)}{b(s)} \frac{b(s)}{a(s) + b(s)}$$

Similarly to c above

$$\frac{a'(s)}{a(s)} = \sum_{i=1}^{m} \frac{1}{s - a_i}, \quad \frac{b'(s)}{b(s)} = \sum_{i=1}^{m} \frac{1}{s - b_i}$$

Thus the Newton-Maehly estimate for a zero is readily calculated from a, b and c. If the sequence of estimates $\{x_k\}$ does not converge for a given initial value (a starting value) x_0, an improved starting value can be obtained using Muller's method followed by the Secant method[31]. The sequence of estimates will be said not to converge if $|x_{k+1} - x_k|$ is not less than some $\varepsilon > 0$ for $k = k_{max}$.

A further consideration is the determination of the constant factor K of p. Let K_a and K_b be the constant factors of a and b. Then

$$K = \begin{cases} K_a & \text{if } \deg a > \deg b \\ K_b & \text{if } \deg b > \deg a \end{cases}$$

and if the $\deg a = \deg b$, $K = K_a + K_b$, unless $K_a + K_b = 0$ in which case degree reduction is said to occur; i.e. the degree of p is less than the degree of a and b. The degree of p and the constant factor K can be determined by converting to coefficient representation and determining the nonzero coefficient of the highest power of the sum of a and b.

In a practical implementation exact equality cannot be expected. Therefore a test like

$$\frac{|K_a + K_b|}{|K_a|} < \varepsilon$$

must be used to decide if $K_a + K_b$ should be considered to be equal to zero. A similar test is used for determining the highest nonzero coefficient of the sum if

$$\deg p \le \deg a - 1$$

It has also been found that $e(s_k) \equiv \left|\frac{p'(s_k)}{p(s_k)} - \frac{c'(s_k)}{c(s_k)}\right|$ may continue to increase and exceed the maximum number representable in the computer and in this case a test must be implemented and $1/e(s_k)$ set equal to zero, implying $s_{k+1} = s_k$.

The algorithm described above has been tested on a large number of filter examples including high order, narrowband filters which are critical because of the clustering of zeros. The described algorithm has been successfully used in the design of such filters.

Appendix II[29]

Solution of the Feldtkeller Equation for FIR Filters

Theorem 1: Given a polynomial

$$f(z) = f_n z^{-n} + f_{n-1} z^{-(n-1)} + \ldots + f_0, \quad f_n, f_0 \neq 0, n > 0$$

$|f(z)| \le 1$ for all ω with $z = e^{j\omega}$, and a polynomial $g(z) = 1$. Then there exists a polynomial

$$h(z) = h_n z^{-n} + h_{n-1} z^{-(n-1)} + \ldots + h_0, \quad h_n, h_0 \neq 0, \quad n > 0$$

such that

$$g(z)g_*(z) = f(z)f_*(z) + h(z)h_*(z) \tag{1}$$

where the para conjugate is defined by

$$f_*(z) = z^n f(1/z) = f_0 z^{-n} + f_1 z^{-(n-1)} + \ldots + f_n$$

Proof: From the given $f(z)$ and $g(z)$,

$$f(z)f_*(z) = f_0 f_n z^{-2n} + \ldots + f_0 f_n \text{ and } g(z)g_*(z) = z^{-n}$$

Let

$$a(z) = z^{-n} - f(z)f_*(z) \tag{2}$$

then

$$a_*(z) = z^{-2n}a(1/z) = z^{-n} - f_*(z)f(z) = a(z)$$

deg $a(z) = 2n$ and the constant term of $a(z)$, $f_0 f_n \neq 0$. Therefore $a(z)$ can be expressed in product representation in the form

$$a(z) = K\prod_{i=1}^{2n}(z^{-1} - a_i),\ a_i \neq 0,\ K = f_0 f_n \tag{3}$$

where the zeros are arranged so that $|a_i| \geq |a_j|$, $i < j$

Then

$$a_*(z) = K\prod_{i=1}^{2n}a_i\prod_{i=1}^{2n}(z^{-1} - 1/a_i)$$

But $a(z) = a_*(z)$ which implies that $\prod_{i=1}^{2n}a_i = 1$ and $a_i = \frac{1}{a_j}$ for some j, i.e. the zeros occur in pairs $\left(a_i, \frac{1}{a_j}\right)$ with $a_i = a_j$.

Let

$$\hat{h}(z) = \prod_{i=1}^{n}(z^{-1} - a_i) \tag{4}$$

Note that $|a_i| \geq 1$, $i = 1, 2, \ldots, n$, and $|a_i| \leq 1$, $i = n, n+1, \ldots, 2n$. Then

$$\hat{h}_*(z) = \prod_{i=1}^{n}(-a_i)\prod_{i=1}^{n}(z^{-1} - 1/a_i) = \prod_{i=1}^{n}(-a_i)\prod_{i=n+1}^{2n}(z^{-1} - a_i)$$

It follows that $\hat{h}(z)\hat{h}_*(z)$ and $a(z)$ have the same zeros, and, furthermore,

$$a(z) = \left(K/\prod_{i=1}^{n}(-a_i)\right)\hat{h}(z)\hat{h}_*(z) \tag{5}$$

From (2) and (5)

$$z^{-n} = \left(K/\prod_{i=1}^{n}(-a_i)\right)\hat{h}(z)\hat{h}_*(z) + f(z)f_*(z)$$

and then

$$1 = \left(K/\prod_{i=1}^{n}(-a_i)\right)\hat{h}(z)\hat{h}\left(\frac{1}{z}\right) + f(z)f\left(\frac{1}{z}\right)$$

and for $z = e^{j\omega}$

$$1 = \left(K/\prod_{i=1}^{n}(-a_i)\right)\left|\hat{h}(e^{j\omega})\right|^2 + \left|f(e^{j\omega})\right|^2 \tag{6}$$

Since $\left|f(e^{j\omega})\right|^2 \le 1$ for all ω and f is nonconstant, there exists an ω_0 such that

$\left|f(e^{j\omega_0})\right|^2 < 1$, which together with (6) implies $\left|\hat{h}(e^{j\omega_0})\right|^2 \ne 0$. Then from (6)

$$K/\prod_{i=1}^{n}(-a_i) = \frac{1-\left|f(e^{j\omega_0})\right|^2}{\left|\hat{h}(e^{j\omega_0})\right|^2} > 0$$

Now let

$$K_h = \sqrt{K/\prod_{i=1}^{n}(-a_i)} = \sqrt{f_0 f_n/\prod_{i=1}^{n}(-a_i)} \tag{7}$$

and let

$$h(z) = K_h\hat{h}(z) = K_h\prod_{i=1}^{n}(z^{-1}-a_i) \tag{8}$$

Then

$$h_*(z) = K_h \prod_{i=1}^{n} (-a_i) \prod_{i=1}^{n} \left(z^{-1} - \frac{1}{a_i}\right)$$

and

$$h(z)h_*(z) = K_h^2 \prod_{i=1}^{n} (-a_i) \prod_{i=1}^{n} (z^{-1} - a_i) \prod_{i=1}^{n} \left(z^{-1} - \frac{1}{a_i}\right) = K \prod_{i=1}^{2n} (z^{-1} - a_i) = a(z) \quad (9)$$

Finally from (2) and (9)

$$g(z)g_*(z) = z^{-n} = h(z)h_*(z) + f(z)f_*(z) \quad (10)$$

where $h(z)$ is defined by (7) and (8). From (10), $f_n f_0 + h_n h_0 = 0$ implying

$h_n h_0 = -f_n f_0 \neq 0$, and therefore $h_n, h_0 \neq 0$

Corollary 1: Given a polynomial

$$f(z) = f_n z^{-n} + f_{n-1} z^{-(n-1)} + \ldots + f_{n_0} z^{-n_0}, \quad f_n, f_{n_0} \neq 0, n > 0$$

$|f(z)| \leq 1$ for all ω with $z = e^{j\omega}$, and a polynomial $g(z) = 1$. Then there exists a polynomial $h(z) = z^{-n_1}\tilde{h}(z)$, where $\tilde{h}(z) = h_{n'} z^{-n'} + h_{n'-1} z^{-(n-1)} + \ldots + h_0$ with $n' = n - n_0$, $h_{n'}, h_0 \neq 0$ and $0 \leq n_1 \leq n_0$, such that

$$g(z)g_*(z) = f(z)f_*(z) + h(z)h_*(z)$$

Proof: Let $f(z) = z^{-n_0}\tilde{f}(z)$, where

$$\tilde{f}(z) = f_n z^{-(n-n_0)} + f_{n-1} z^{-(n-1-n_0)} + \ldots + f_{n_0}, \; f_{n_0} \neq 0$$

Then $|\tilde{f}(z)| \leq 1$ for all ω with $z = e^{j\omega}$. Let $n' = n - n_0$. By ***Theorem 1***, there exists

$\tilde{h}(z) = h_{n'}z^{-n'} + h_{n'-1}z^{-(n'-1)} + \ldots + h_0$, $h_{n'}, h_0 \neq 0$, such that

$$z^{-n'} = \tilde{h}(z)\tilde{h}_*(z) + \tilde{f}(z)\tilde{f}_*(z)$$

Note that $f_*(z) = f_n + f_{n-1}z + \ldots + f_{n_0}z^{-n'} = \tilde{f}_*(z)$. Now

$$z^{-n} = z^{-n_0}\tilde{h}(z)\tilde{h}_*(z) + z^{-n_0}\tilde{f}(z)\tilde{f}_*(z) = z^{-n_0}\tilde{h}(z)\tilde{h}_*(z) + f(z)f_*(z)$$

Let $h(z) = z^{-n_1}\tilde{h}(z)$ with $0 \leq n_1 \leq n_0$, then

$$h_*(z) = z^{-n}z^{n_1}\tilde{h}(1/z) = z^{-n}z^{n_1}z^{n'}\tilde{h}_*(z)$$

and $h(z)h_*(z) = z^{-n_0}\tilde{h}(z)\tilde{h}_*(z)$. And, finally,

$$g(z)g_*(z) = z^{-n} = h(z)h_*(z) + f(z)f_*(z)$$

Corollary 2: Given $f(z) = f_n z^{-n}$, $f_n \neq 0$, $|f_n| \leq 1$, and $g(z) = 1$, then there exists

$$h(z) = \sqrt{1 - f_n^2}\,z^{-n_1},\quad 0 \leq n_1 \leq n$$

which satisfies

$$g(z)g_*(z) = f(z)f_*(z) + h(z)h_*(z)$$

Proof: From the given $f(z)$ and $g(z)$, it is obvious that

$$f_*(z) = f_n,\ f(z)f_*(z) = f_n^2 z^{-n},\ g(z)g_*(z) = z^{-n}, \text{ and } h(z)h_*(z) = (1 - f_n^2)z^{-n}$$

If $f_n^2 = 1$, $h(z) \equiv 0$. Otherwise $f_n^2 < 1$. Let $h(z) = \sqrt{1 - f_n^2}\,z^{-n_1}$, $0 \leq n_1 \leq n$, then

$h_*(z) = \sqrt{1 - f_n^2}\,z^{-(n-n_1)}$ and $h(z)h_*(z) = (1 - f_n^2)z^{-n}$

Therefore $f(z)f_*(z) + h(z)h_*(z) = (1 - f_n^2)z^{-n} + f_n^2 z^{-n} = z^{-n} = g(z)g_*(z)$

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