THE PSYCHOPHYSICAL LAW AND THE MEASUREMENT
of
SENSATION.

By

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INTRODUCTION

THE MEASUREMENT OF SENSATION

The tremendous advance in the applications of the sciences of telephony and illuminating engineering in recent years has emphasized the need of a method of accurately measuring sensation, and the determination of sensation unit has become a legitimate subject for physical as well as psychological investigation. In this paper certain psycho-physical phenomena are reviewed, re-examined and co-ordinated in an attempt to determine a measure of sensation.

The treatment is made in four parts: the first part describes a group of psycho-physical phenomena, and a dimensional analysis of their empirical equations; the second, a second group of psycho-physical phenomena and an experimental determination of their empirical equations; in the third part the two groups of phenomena are related experimentally, and in the fourth section the two groups of equations are related analytically, the conclusion being drawn that on making one assumption, there are two distinct methods of accurately measuring sensation.

PART ONE

THE PHENOMENA OF INTERMITTENT SENSORY STIMULI

PHENOMENA OF INTERMITTENT SENSORY STIMULI

If a disc containing alternate equi-angular, closed and open segments be rotated between a source of illumination and the eye, so that the retina is receiving equal periods of stimulation and darkness, it is found, above a critical duration of the flashes of light, that the resulting sensation is discontinuous. Durations below this critical duration yield a continuous sensation. The value of the duration varies with the intensity of the light, and if D represent the critical duration of flicker, I the intensity of the light, and K and C are constants, a relation, known as the Ferry-Porter Law, connects these quantities.

$$\frac{1}{D} = K \text{ Log } I + C \qquad (1)$$

This is an empirical relation obtained by plotting graphically the reciprocal of the critical duration against the logarithm of the intensity of the stimulus and obtaining a straight line.

It has been shown by McDougal[1] that the least time during which a light must act on the retina in order to excite the most intense sensation of which it is capable is the same for lights of different wave length but of the same intensity. This "action time" should be closely allied to the critical duration of flicker and suggests that some function of the critical duration might be made to serve as a measure of sensation.

Frank Allen, extending the flicker phenomenon to the other senses, has used intermittent, adequate stimuli in the auditory[2] and tactile[3] senses, and found the critical duration to be related to the intensity of the stimulus by the equations,-

$$D = K \text{ Log } I + C \text{ ; } \quad \text{Auditory} \quad (2)$$

$$D = -K \text{ Log } I + C \text{ ; } \quad \text{Tactile} \quad (3)$$

By an electrical method of stimulation, he has also succeeded in obtaining intermittent gustatory stimuli which conform to the equation,-

$$\frac{1}{D} = -K \text{ Log } I + C \text{ ; } \quad \text{Taste} \quad (4)$$

The graphical representation of all these equations he has called Porter graphs. The author[4] has unsuccessfully attempted to obtain intermittent sensations of heat, cold, and pain, but has shown that these senses under certain conditions of electrical stimulation react in the same manner as touch, and has drawn the conclusion that the behaviour of all the cutaneous senses could be represented by the same law.

There are thus four known equations relating the critical duration of flicker to the intensity of stimulation. Their symmetry would suggest their being examined in two groups,-

$$\left.\begin{aligned} \frac{1}{D} &= K \log I + C \\ \frac{1}{D} &= -K \log I + C \end{aligned}\right) \quad \begin{aligned} &\text{Visual} \quad (5) \\ &\text{Gustatory} \quad (5) \end{aligned}$$

$$\left.\begin{aligned} D &= K \log I + C \\ D &= -K \log I + C \end{aligned}\right) \quad \begin{aligned} &\text{Auditory} \quad (6) \\ &\text{Cutaneous} \quad (6) \end{aligned}$$

The values of the intensity I, used in vision are only relative and we now define unit intensity in all the senses, as that necessary to excite the threshold sensation. This is to simplify a theoretical discussion only, and all the data given in this paper are given in whatever units happen to be the simplest to determine experimentally.

Letting Log I = 0, i.e. at unit intensity the visual equation reduces to

$$\frac{1}{D_0} = C$$

when D_0 is the duration at the threshold value.

The general visual equation then becomes

$$\frac{1}{D} - \frac{1}{D_o} = K \text{ Log } I \qquad (7)$$

Denoting

$$\frac{1}{D} - \frac{1}{d_o} \text{ by } \frac{1}{\delta T} \quad \text{gives}$$

$$\frac{1}{\delta T} = K \text{ Log } I \quad ; \qquad \text{Vision} \qquad (8)$$

Similarly the equation for taste becomes

$$\frac{1}{\delta t} = -K \text{ Log } I \quad ; \qquad \text{Taste} \qquad (9)$$

The fact that I occurs in the numerator in vision, in the denominator in taste, and that the quantity under a logarithm sign should have zero dimensions suggests that there may be two reactions present in each case, one varying as the logarithm of the intensity, the other as the logarithm of the reciprocal of the intensity, the relative magnitude of these reactions controlling the sign in front of the logarithmic sign, thus,-

$$\frac{1}{\delta t} = K^2 \text{ Log } \frac{I}{I_2} \text{ — Vision and Taste} \qquad (10)$$

$\frac{I_1}{I_2}$ is then a pure numeric, denoting this by N, these equations become

$$\frac{1}{\delta t} = K^2 \text{ Log } N \qquad (11)$$

It is difficult to discuss the physiological import of introducing the quantities I_1 and I_2, there being so little known about sensory reactions, though Allen has put forward an hypothesis of sensory reflex action, which coordinates all the phenomena of sensory induction by introducing two (imaginary) sensory principles, "......a stimulus applied to any sensory receptor arouses a sensation in the corresponding central organs, and in addition evokes simultaneously two sets of efferent impulses, one of which enhances, and the other depresses the sensitiveness of the whole sensory system. the degree of enhancement or depression being a measure of the excess of one process over the other."

Should exception be taken to introducing two physiological reactions, the quantity I_2 may be regarded as the threshold intensity, this having the dimensions of I and the numerical value unity.

The equations for the cutaneous and auditory senses may be treated in a similar way.

$$D = K^2 \operatorname{Log} N - C$$

at $N = 1$ $\quad C = D_0$

therefore

$$D - D_0 = K^2 \operatorname{Log} N$$

Denoting $(D - D_0)$ by δT this becomes

$$\delta T = K^2 \text{ Log } N \qquad (12)$$

In attempting to determine why the time function appears in the denominator in (11) and in the numerator in (12), it is to be remarked that in both the cutaneous and auditory senses the receptors receive direct mechanical stimulation, while in vision and taste, although the exact reactions involved are not known, the receptors probably receive stimulation as the result of some chemical reaction. It is then quite within reason to assume tentatively that the difference in the position of the time function is due, not to any difference in psychological reactions after stimulation, since in all cases the psychological phenomenon of a just continuous sensation is the same, but to some peculiarities in the method of stimulation of the receptors.

Writing the symmetry of (11) and (12) in terms of the time function, these become

$$\frac{\varphi'}{\delta t} = \text{Log } N \quad \text{Senses having receptors stimulated by chemical means} \qquad (13)$$

$\varphi' = \frac{1}{K^2}$

$$\frac{K^2}{T} = \frac{1}{\text{Log } N} \quad \text{Senses receiving direct mechanical stimulation} \qquad (14)$$

While the quantity on the right side of the above equations is a pure number, it is the controlling factor in a physiological reaction, and it is thus reasonable to suppose it represents the magnitude of the flow of energy transmitted by the nerve fibre. That is, the relation existing between the magnitude of the externally applied energy and the magnitude of the energy released by the receptor is not a linear, but a logarithmic function. The existence of some principle modifying the nervous action due to a large external stimulus is only to be expected as has been pointed out by Helmholtz[5]."...... Die inneren Veränderung im Nerven, welche den Eindruct des Reizes auf das Gehrin übertragen müssen können eben eine bestimmte Grösze nicht überschreiten, ohne das Organ zu zerstören , und jeder Wirkung des Reizes is dahr eine obere Grenze gesetzt....."

Consideration of recent work by Adrian[6] confirms this view. He has made a study of the action currents occurring in the sensory nerve fibres of cats. The results obtained show the frequency of the nerve currents to be proportional to the logarithm of the stimulating intensity. It was found, moreover, that the amplitude of the currents was constant, which would mean that the energy was proportional to the frequency and thus to the logarithm of the stimulating intensity.

From these considerations it is concluded that the logarithmic function in equations (13) and (14) is a measure of energy and this fact is incorporated in the equations by multiplying the logarithmic function by a unit quantity of energy, denoted by e

$$\frac{\varphi}{\delta t} = e \text{ Log } N \qquad (15)$$

$$\frac{K^2}{\delta t} = \frac{e}{\text{Log } N} \qquad (16)$$

The energy varying as Log N in (15) and as the reciprocal of Log N in (16).

The dimensions of φ and K^2 in these equations are both those of the product of an energy with a time

$$[\varphi] = [K^2] = [M]\,[L]\,[T^{-2}]\,[L]\,[T]$$

$$= [M]\,[L^2]\,[T^{-1}]$$ or of the dimensions of action.

The occurrence of action in these equations immediately suggests the possibility of a quantum of nerve action, which might have as its basis the physiological all or none principle. Quoting from a paper[6b] on the transmission of nervous energy in relation to this principle, "Changes of activity produced by local physical or chemical alteration in irritable cells have also a marked tendency to spread and to involve the entire cell, so that it reacts as a whole; and there is undoubtedly a common basis for the transmission in all of these cases, although normally stimulation, unless carried to excess, has no evident destructive effects. A muscle-cell stimulated at one end undergoes contraction as a whole: it is impossible in a normal cell to localize the contractile activity, which is instantly transmitted from one end to the other. Hence such a cell exhibits the peculiar type of response called "all or none", responding either completely or not at all - a form of behaviour highly characteristic of ittitable living elements.

The results obtained by Adrian giving the amplitude of the action currents as always constant shows the cells of the sensory nerve fibres to be controlled by this "all or none" principle.

Following Planck's treatment of the harmonic oscillator in the old quantum mechanics, let us consider the wave energy transmitted by the nerve fibre to be due to an harmonic oscillator whose energy is given by,-

Kinetic energy $= \frac{1}{2} M V^2$; Potential energy $= \frac{a^2 q^2}{2}$

"q" being a general space co-ordinate and "a" a constant.

Thus the total energy, E, is

$$E = \frac{1}{2} M V^2 + \frac{a^2 q^2}{2}$$

denoting MV by p, this may be written

$$E = \frac{p^2}{2M} + \frac{q^2}{\frac{2}{a^2}}$$

or

$$\frac{p^2}{2ME} + \frac{q^2}{\frac{2E}{a^2}} = 1$$

The area of this ellipse will be given by

$$2\pi \sqrt{\frac{M}{a^2}}\, E$$

Now if the energy of the nervous system is liberated in discrete quantities, the area of the ellipse may exist only as integral multiples of the smallest area due to the energy contained in a single nerve cell, since M and a^2 are constants.

The area of the ellipse is also given by

$$\oint p\,dq$$

Thus this integral may only have integral values.

The dimensions of pdq are however

$$[M][L][T^{-1}][L]$$

$$= [M][L^2][T^{-1}]$$

or action.

It has already been shown that K^2 and φ^2 are of dimensions action.

Now φ^2 is the reciprocal of the slope of the Porter graphs in Vision and Taste. Thus the reciprocals of these slopes should all be constant multiples of a fundamental constant. Similarly the slopes of the Porter graphs in the cutaneous and auditory senses should exhibit an integral relation.

EXPERIMENTAL CONFIRMATION:

A number of Porter graphs for vision were picked at random, the slopes measured from the graphs, and the reciprocals examined for an integral relation. The results are given below. Since there are as many as four branches to some of the graphs, the different branches are designated by α, β, γ etc., where α denotes the branch at lowest intensity, β the adjacent branch, etc.

TABLE SHOWING QUANTUM CONDITION IN VISION

	Branch:	Reciprocal of slope x 10^2:	Factor:	Remainder:	Units of Action:
410		388	4		
410		968	8	0	121
420		112	8	0	14
420		504	8	0	63
550		615	8	-1	77
550		128	8	0	16
550		345	8	1	43
590		581	8	-3	73
590		162	8	2	20
590		478	8	-2	60
630		655	8	-1	82
630		177	8	1	22
630		935	8	-1	117
700		190	8	-2	24
700		410	8	2	51

The slopes are not experimentally determined correct to the third decimal, thus the agreement is within the limit of experimental error. It will be seen that the last column is in many cases divisable by 7, making 56 a possible factor, with more accurate determination of the Porter graphs.

The integral relation existing between the slopes as required in the tactile sense has already been found by Blair[7].

The data for taste are given below. The factors 7 and 9 have been chosen to show the existence of the quantum condition, though a study of the data would indicate larger factors if the slopes were more accurately determined.

TABLE SHOWING QUANTUM CONDITION IN TASTE.

Curve:	Branch:	Reciprocal of Slope:	Factor:	Remainder:
A		280.9	9	1
Sweet Sense		122.5	9	1
		63.1	9	1
		104.5	8	2
B		72.5	8	2
Salt Sense		52.1	8	1
		82.0	7	1
C		67.1	7	-1
Sour Sense		44.6	7	-2
		59.5	7	0
D		49.5	7	1
Bitter Sense		39.2	7	0

PART 2

THE WEBER-FECHNER PRINCIPLE IN THE VISUAL, AUDITORY AND TACTILE SENSES.

HISTORICAL OUTLINE OF THE WEBER-FECHNER PRINCIPLE

It has long been known that the physical strength of a sensory stimulus may be increased by an amount so small that the increase does not cause a corresponding increase in the sensation evoked by the stimulus. Denoting the physical strength of a stimulus by I, and the small increase by δI, experiment has shown that δI may be increased to a critical value such that a just perceptible increase in the sensation results. This value of δI is known as the differential threshold and is usually denoted by ΔI.

The behaviour of the magnitude of ΔI in conjunction with different values of I was first seriously investigated by Weber, who concluded that a simple relation existed between I and ΔI, the relation being expressed by the equation,-

$$\frac{\Delta I}{I} = C \qquad (1)$$

where C is constant.

An attempt to obtain a connection between a physical stimulus and the corresponding mental sensation was made by Fechner, using as a basis Weber's relation. Fechner argued that since a differential threshold always existed, sensation must be an atomic phenomenon, and that ΔI would always liberate the same whole number of sensation units, the C in Weber's relation being the measure of this number of sensation units. Denoting the number by ds in place of C, Weber's relation became

$$K \frac{\Delta I}{I} = ds \tag{2}$$

K being constant. Then assuming that ΔI was sufficiently small to be treated as an infinitesimal, integration of (2) yielded the law,-

$$S = K \operatorname{Log} I + C \tag{3}$$

This law has been subjected to many experimental tests, possibly the best work being done by König, using the sensory receptors of the eye. The original work of observers in senses other than vision is not discussed in detail by authors, though they conclude that the law is only an approximation for any sense. As there has been a large amount of work done on the eye, and as König's reputation would preclude any errors in his observations, only his work will be considered in this treatment and taken as typical of the best experimental work and procedure.

KÖNIG'S EXPERIMENTAL WORK:

König[8], keeping his eye in darkness adaptation, has used as a sensory stimulus a patch of light viewed through a suitable ocular. The upper half of the patch maintained at a definite intensity, the intensity of the lower half was varied by Nicol prisms until it was just perceptibly brighter than the upper. The difference between the physical intensities of the two patches was then determined by the relative rotations of the Nicols, and taken as the value of the differential threshold ΔI. The intensity I was then altered by the numerical value of ΔI, to give a new I, and in this manner the readings repeated throughout the whole intensity range of the eye. Data were obtained for various monochromatic radiations and for white light - some of these are given in Table 1. It will be seen that there is no evidence of a linear relation in these data.

THE PRESENT STATUS OF THE WEBER-FECHNER LAW:

In a recent paper by Hecht[9] (1924), data from various sources dealing with the Weber-Fechner Law in vision have been discussed, and he concludes:

"There is presented a series of data, assembled from various sources, which proves that in the visual discrimination of intensity the threshold "differences ΔI bears no constant relation to the intensity I. The evidence shows unequivocally that as the intensity rises, the ratio $\frac{\Delta I}{I}$ first decreases and then increases."

Parsons[10] (1924) also states:

"Weber's Law does not hold good for very low or very high intensities of stimuli, and is only approximate at the best."

DISCUSSION OF KÖNIG'S WORK:

In considering the interpretation to be put on experimental results it should be borne in mind that the essential condition of the scientific method of experimentation is to reduce to a minimum the number of variables observed at any one time. While König's readings are beyond criticism, it is believed that interpreting them as a complete experimental examination of the Weber Law is not sound for the following reasons:

(1) In examining the patch of light, one-half being at an intensity I, and the other at a greater intensity $I + \Delta I$, two sensations are evoked, while the Weber-Fechner Law deals with but a single sensation at any one time. Further, the retina is being unequally stimulated at different

portions of its surface, thus using different groups of receptors, which condition arouses a series of complex reactions for it is known that under similar conditions there is an inductive process started which tends to modify the intensity of the receptors in such a manner that equal sensation is evoked from all points on the retina.

It has also been shown by Allen[11] that in keeping one eye in darkness adaptation and exposing the other to visible radiation further modification of the strength of sensory impressions results.

(II) One of the outstanding features of the visual mechanism is its ability to respond accurately to stimuli varying over a wide range of intensity. It might be expected in order that this reaction should occur in the most efficient manner there would be certain modifications in the reaction process at different ranges of intensity. König, however, does not seem to have taken precautions against concealing any such possible modifications. Consider, for example, his curve obtained with a radiation of wave length 670 $\mu\mu$. There are only sixteen points covering a range of intensities from the least to the greatest perceptible. Points so widely separated would hardly show any modification in the visual process, and thus should not be interpreted as being satisfactory experimental examination of Weber's Law.

(III) The most extensive work done in vision on the Weber Law has been carried out with white light. Since white light is composed of all wave lengths in the visible spectrum and in varying amounts depending on the nature of the source, it would contain practically an infinite number of physical variables and would arouse a corresponding complexity of sensations.

CONCLUSIONS DRAWN FROM KÖNIG'S WORK

(i) Owing to the nature of the apparatus two distinct sensations of different intensity are obtained at the same time. This is not the condition required by Fechner's interpretation of Weber's ratio.

(ii) The normal condition of the sensitivity of the eye is not being examined due to inductive processes.

(iii) The method of adding the value of the differential threshold to the intensity in order to determine the next intensity at which to examine Weber's ratio leaves too great a gap between two successive readings for an exacting examination of the law.

(iv) The use of white light in examining simple visual laws introduces an unnecessary complexity.

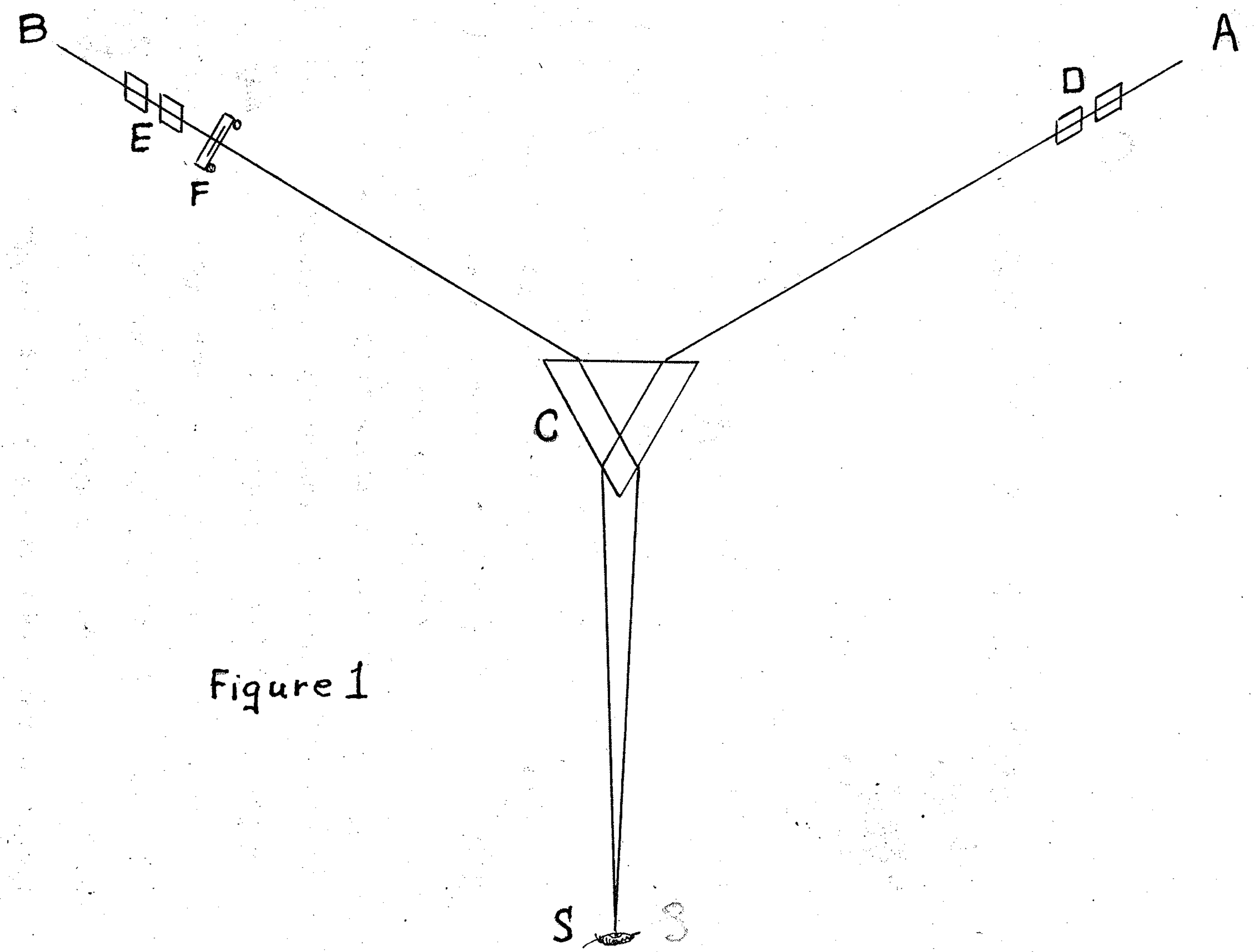


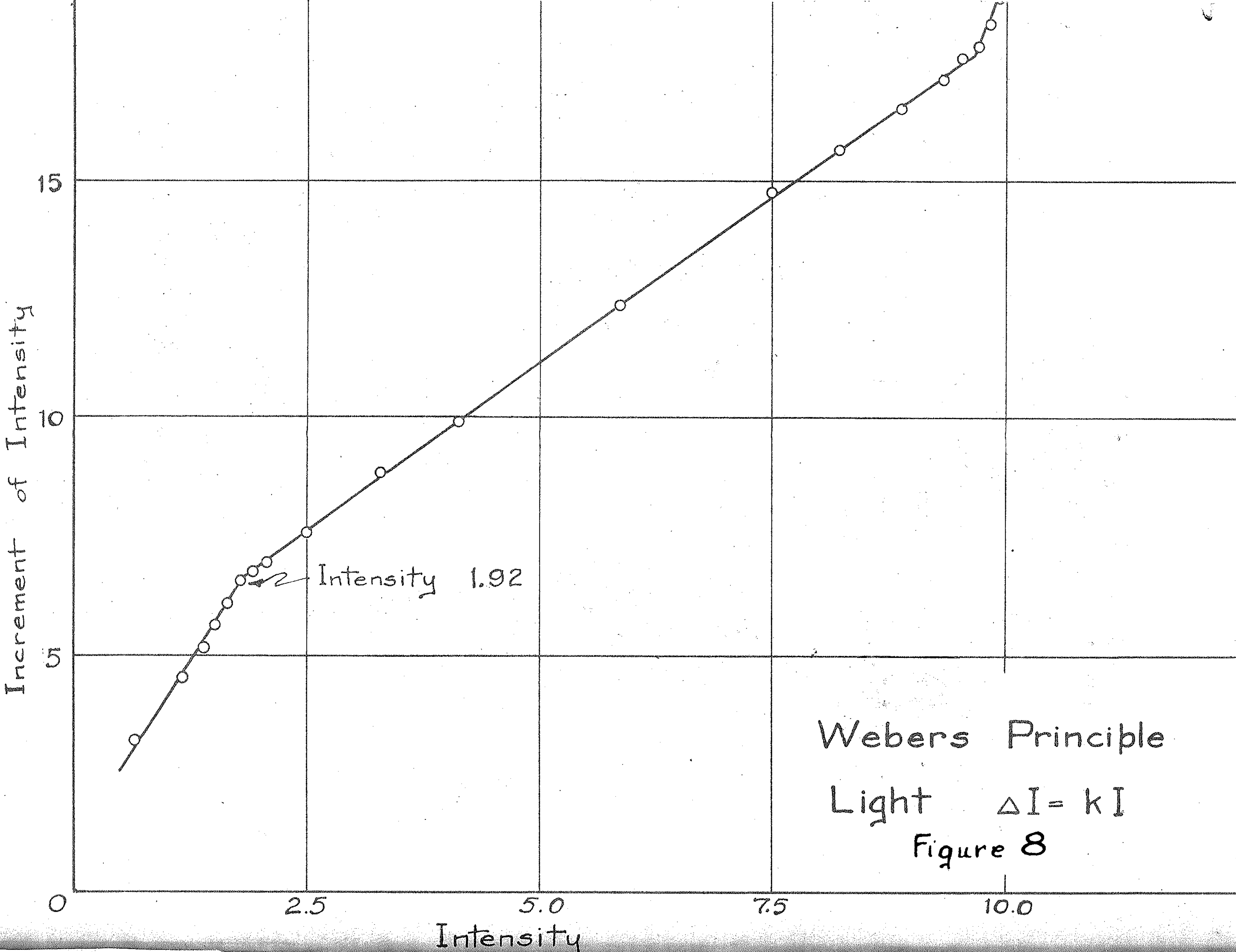
Figure 1

NEW EXPERIMENTAL WORK ON THE VISUAL SENSE

Apparatus:

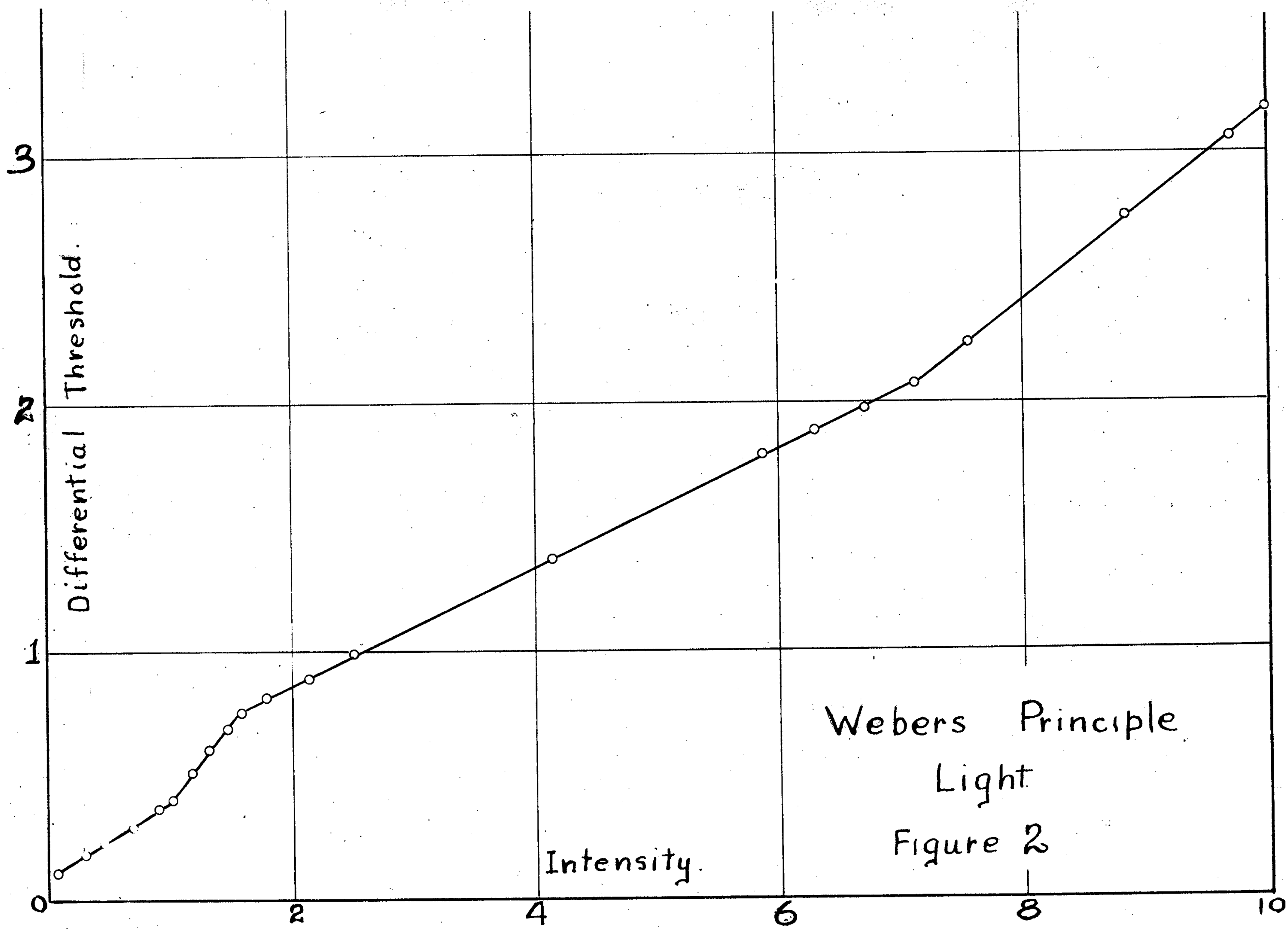
In an attempt to examine Weber's Law and overcome the criticism made of König's work, apparatus was set up similar in principle to Diagram (1).

Two incandescent lamps, A and B, run from the 110 Volt alternating current main, were used as sources of radiation, which was passed through a prism C, in such a manner that radiations of the same wave length from each source were superimposed on passing through an open slit S, which was viewed by the eye through a suitable ocular. The actual instrument used was a tri-colour spectrometer, designed by Professor Frank Allen.[12] The intensity of the radiation from each source was controlled by sets of nicol prisms D and E, placed between the sources and the collimator slits of the spectrometer arms. Between the source B, and the corresponding nicol prism E, was placed in addition a camera shutter F, which was normally kept closed.



Increment of Intensity
15
10
5
0
2.5
5.0
7.5
10.0
Intensity
Intensity 1.92
Webers Principle
Light $\Delta I = kI$

Figure 8



Differential Threshold.
Intensity.
3
2
1
0
2
4
6
8
10
Webers Principle
Light
Figure 2

Experimental Procedure:

The apparatus was used in a room well lighted by indirect sunlight, the eyes thus being adapted to the state in which they are ordinarily used. A definite intensity, I, of stimulation was obtained by rotating the polarizer of the nicol, D, This stimulus was received on the right eye for approximately two seconds, the camera shutter being then released. This allowed an increase of radiation of intensity, ΔI, coming from the source B, to be superimposed on the intensity I, for a period of 1/5 second. A wait of three minutes was then observed to allow the eye to return to its normal condition, and the process repeated, the value of the intensity ΔI, being either lowered or raised, depending on whether it had been noticeable or not at the previous opening of the shutter.

When the point had been reached at which the increment was just perceptible, its magnitude was determined by measuring the angle between the nicols, E, the magnitude of the intensity, I, being determined in the same manner from the nicols D. There was no attempt to obtain the absolute numerical value of the ratio.

The data obtained are given in Tables (2) and (6) and plotted graphically in Figures (2) and (8).

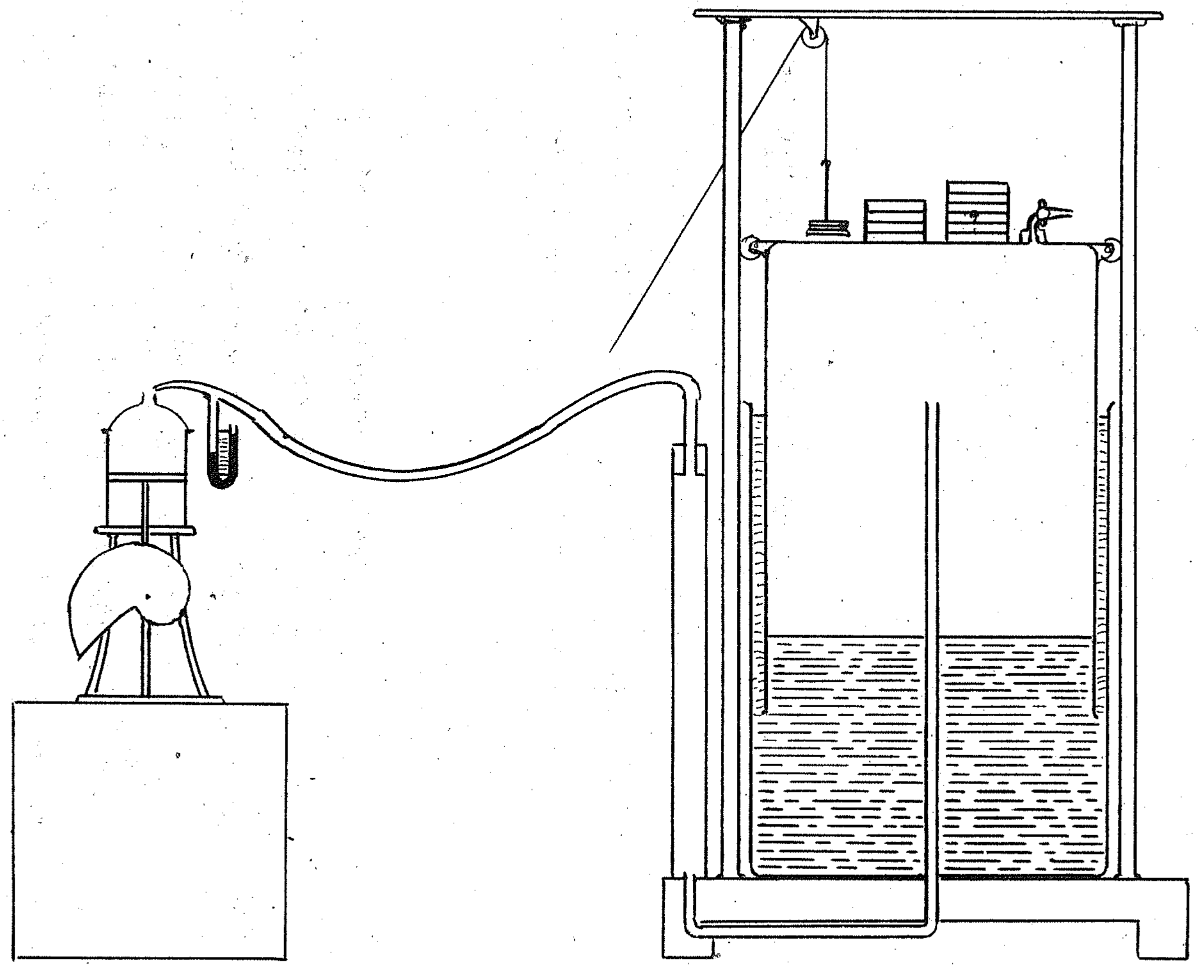


Figure 3

CONCLUSIONS

It is seen from the curve presented in this paper that Weber's Law holds exactly over all ranges of intensity, there being however at definite values of the intensity sudden changes in the value of the ratio of the differential threshold to the stimulating intensity.

THE WEBER-FECHNER PRINCIPLE IN AUDITION

In subjecting Weber's Law to an examination for the sense of audition the same fundamental principle and precautions observed in the visual examination were employed.

Apparatus:

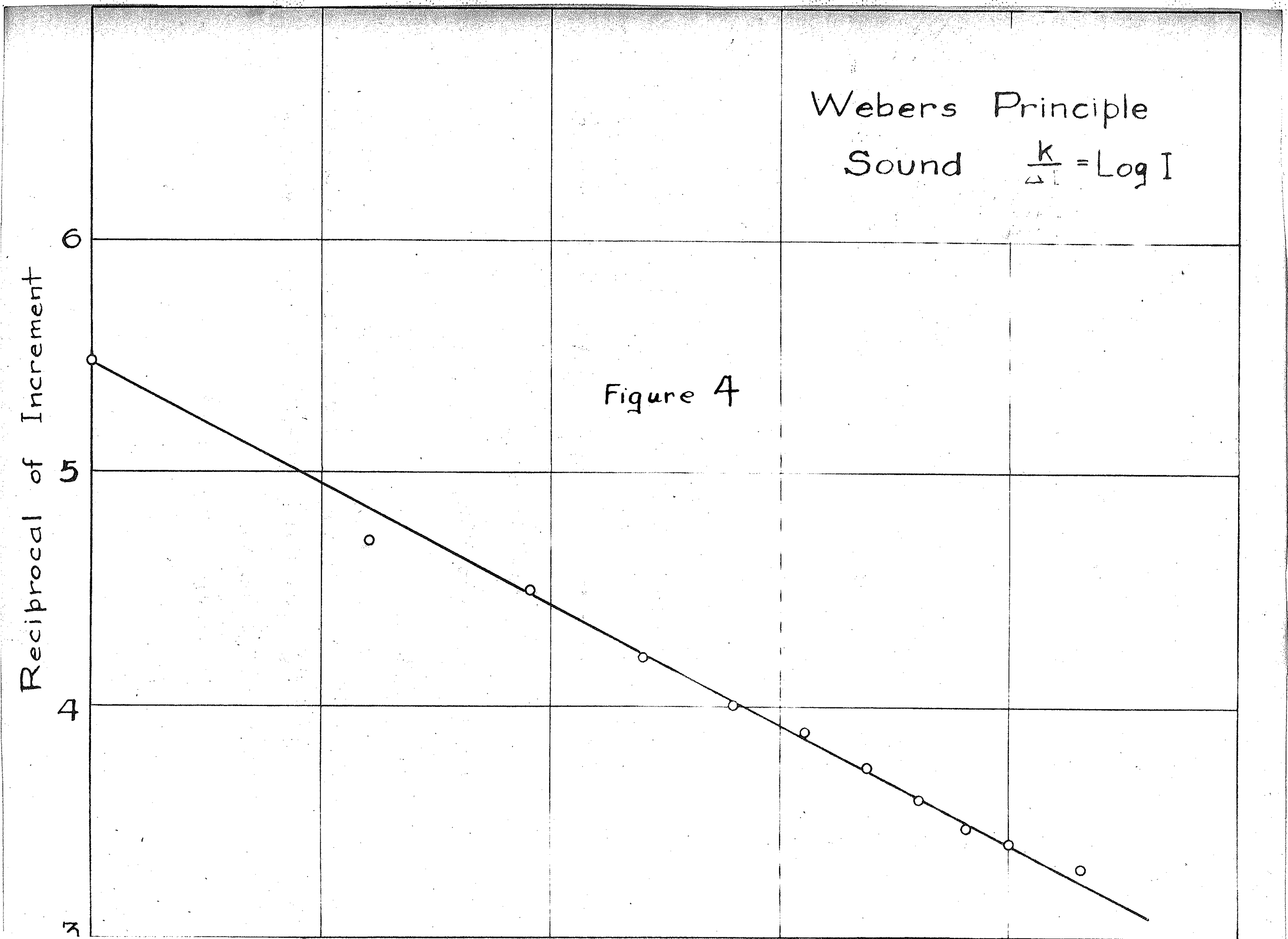
To obtain a source of sound of constant intensity and frequency a Stern Tonvariator was employed being sounded by a stream of air previously collected over water in a constant pressure tank (Fig. 3). Since with different blowing pressures both the intensity and frequency of the sound alter, it was necessary before using the instrument to calibrate both these variables as functions of the blowing pressure. The two operations were carried out simultaneously using a Rayleigh disc.

The Rayleigh disc being set approximately in resonance with the tonvariator, by mechanical adjustment of the length of the resonator tube containing the disc, the room was vacated and further adjustments made on the tonvariator from the adjacent room. A definite blowing pressure was obtained by placing weights on the top of the tank and the frequency of the tonvariator altered until the deflection of the Rayleigh disc was a maximum, Under this condition the tonvariator would be in exact resonance with the Rayleigh disc and the deflection of the disc would be a measure of the intensity of the sound, the deflection being read on a tangent scale and being in no case greater than ten degrees.

Data obtained in this manner showed that the intensity of the sound at constant frequency was, over a small range, directly proportional to the weight on the tank. In this range then, the addition of any weight to the top of the tank would result in a corresponding increase in the intensity of the sound.

EXPERIMENTAL PROCEDURE:

A definite weight was placed on the top of the air tank, the tonvariator adjusted for frequency and the sound stopped, in order to allow the ear to be restored


Webers Principle
Sound $\frac{k}{\Delta I}$ = Log I
Figure 4
Reciprocal of Increment
6
5
4
3

to a state of normal sensitivity. The observer then placed his head against a rest and allowing the tonvariator to sound for approximately two seconds, lowered by a cord a small weight to the top of the air tank. Observing a five minute rest period between successive trials, the small weight was altered in value until it caused a just perceptible increase in the intensity of the resulting sound. The small weight was then taken as a measure of the magnitude of the differential threshold and the large weight as the stimulating intensity of the sound. The data obtained are given in Table 3. These data when plotted in the form $\frac{1}{\Delta I}$ against Log I showed the linear relation of Figure 4. Thus the relation between the differential threshold and the intensity is given by the equation,-

$$\frac{1}{\Delta I} = K \operatorname{Log} I + C \qquad (1)$$

K and C being constants.

CONCLUSION:

It is thus seen that Weber's Law does not hold for audition. There does exist however an intimate relation between the differential threshold and the total stimulus given by the equation,

$$\frac{1}{\Delta I} = K \operatorname{Log} I + C \qquad (1)$$

THE WEBER-FECHNER PRINCIPLE IN THE TACTILE

Sense:

In order to examine the behaviour of the tactile sense, air was collected over water in a constant pressure metal tank, and after passing through a rubber tube, was delivered through a suitable nozzle, to the surface of the lower lip, where it acted as an adequate tactile stimulus. The lip was kept at a constant distance from the nozzle by a protecting plate.

The pressure of the air, read by a water manometer placed in the air circuit immediately behind the nozzle, was taken as the strength of the stimulus, this pressure being varied by placing weights on the top of the constant pressure tank. Since an increase in weight resulted in a corresponding increase in air pressure, it was possible to obtain the increment of stimulation, by adding small weights to the top of the tank. The value of these small weights was taken as a direct measure of the magnitude of the increment as the actual change in the numerical value of the pressure in the case of the increment was too small to be measured accurately by the manometer.

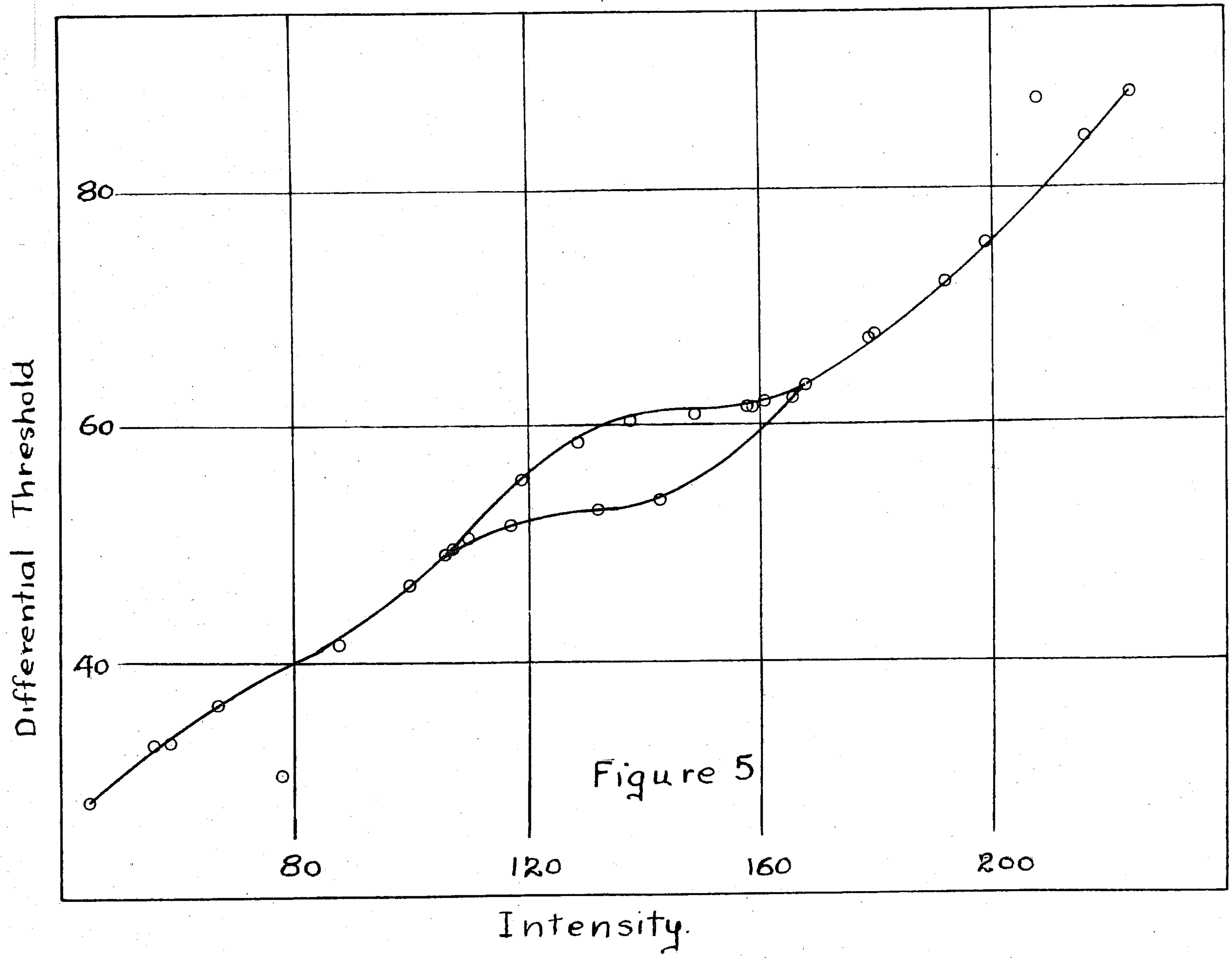
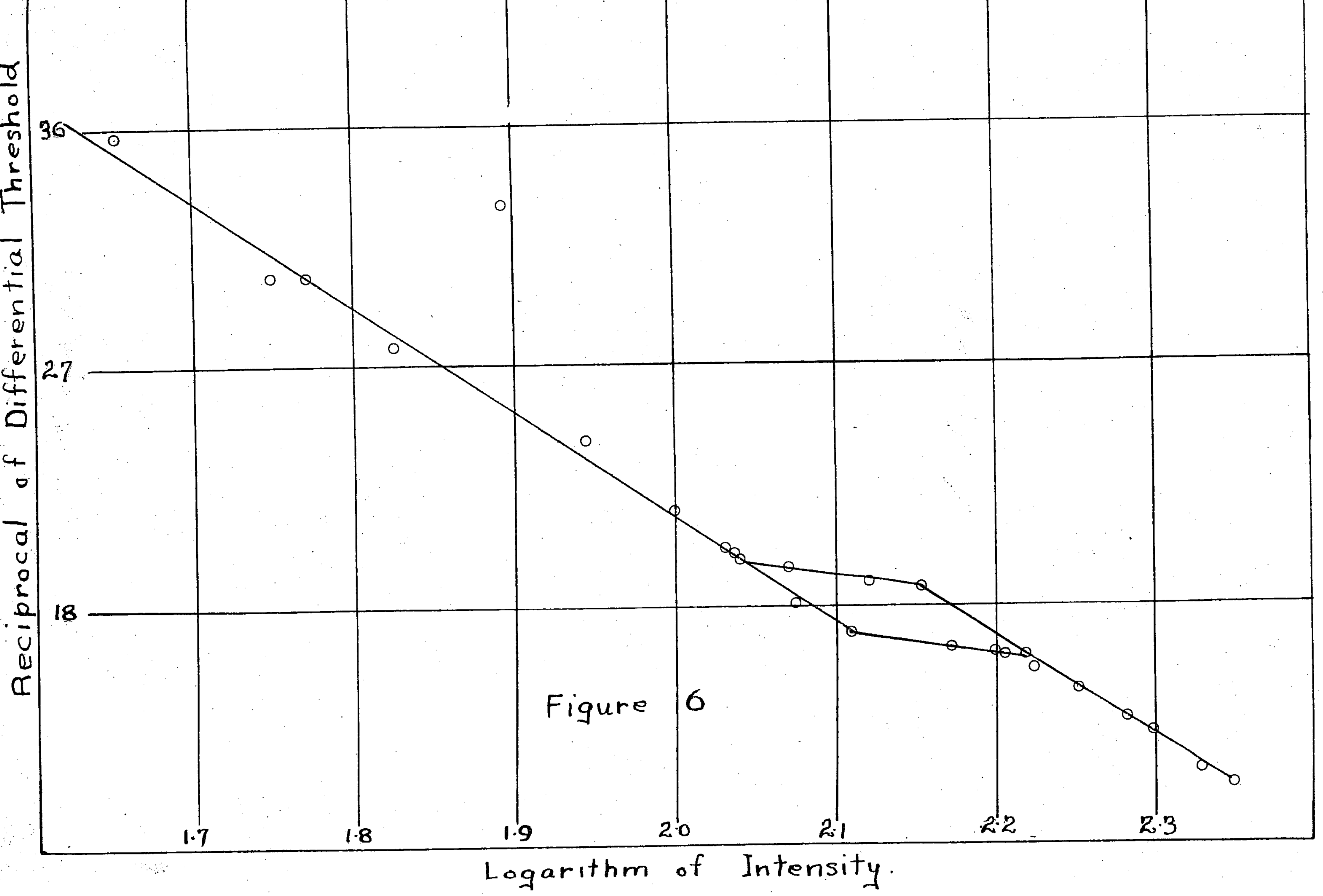


Figure 5



Reciprocal of Differential Threshold
36
27
18
1·7
1·8
1·9
2·0
2·1
2·2
2·3
Figure 6
Logarithm of Intensity.

Experimental Procedure:

Air at a measured pressure was allowed to flow through the nozzle directly upon the lip which was previously coated with a thin film of vaseline to prevent drying. After a period of approximately one second a small weight was added to the top of the tank by a lever, and a change in sensation strength looked for. A rest period of ten minutes was observed, following which another trial was made, with a decrease or increase in the magnitude of the small weight, depending on whether or not the previous increment had been perceptible. Considerable experience is required to determine, in this manner, the differential threshold with a high degree of accuracy. The data obtained for a number of pressures are given in Table 4, and shown graphically in Figure 5. In Figure 6, the same readings are plotted $\frac{1}{\Delta I}$ against $\text{Log}\frac{1}{I}$. The resulting linear relation shows the differential threshold to be related to the intensity of the stimulus by the equation,-

$$\frac{1}{\Delta I} = K \text{Log}\left(\frac{1}{I}\right) + C \qquad (2)$$

there being abrupt changes in the value of the constant at definite values of the intensity.

CONCLUSION:

The Weber-Fechner Law does not hold for the tactile sense, the law relating the increment to the intensity being,

$$\frac{1}{\Delta I} = K^2 \quad \text{Log} \left(\frac{1}{I}\right) + C \qquad (2)$$

RECAPITULATION

A detailed examination of the magnitude of the differential threshold at different intensities of stimulation ranging from the threshold value to intensity greater than those usually met with in every day life has been made. The results of this examination show that the differential threshold is related to the total intensity of stimulation by different laws for different senses, these laws being,-

$$\text{Vision} = \Delta I = K^2 I + C$$

$$\text{Sound} = \frac{1}{\Delta I} = K^2 \quad \text{Log } I + C$$

$$\text{Touch} = \frac{1}{\Delta I} = -K^2 \quad \text{Log } I + C$$

PART 3

EXPERIMENTAL CO-ORDINATION OF FERRY-PORTER AND WEBER-FECHNER LAWS

It has been pointed out that the critical duration of flicker is probably related to sensation and as Fechner's interpretation of Weber's Law also deals with sensation, similarities in the experimental curves might be expected. With this in view the Ferry-Porter and Weber-Fechner laws were examined with the same apparatus under the same conditions and covering the same range of intensity.

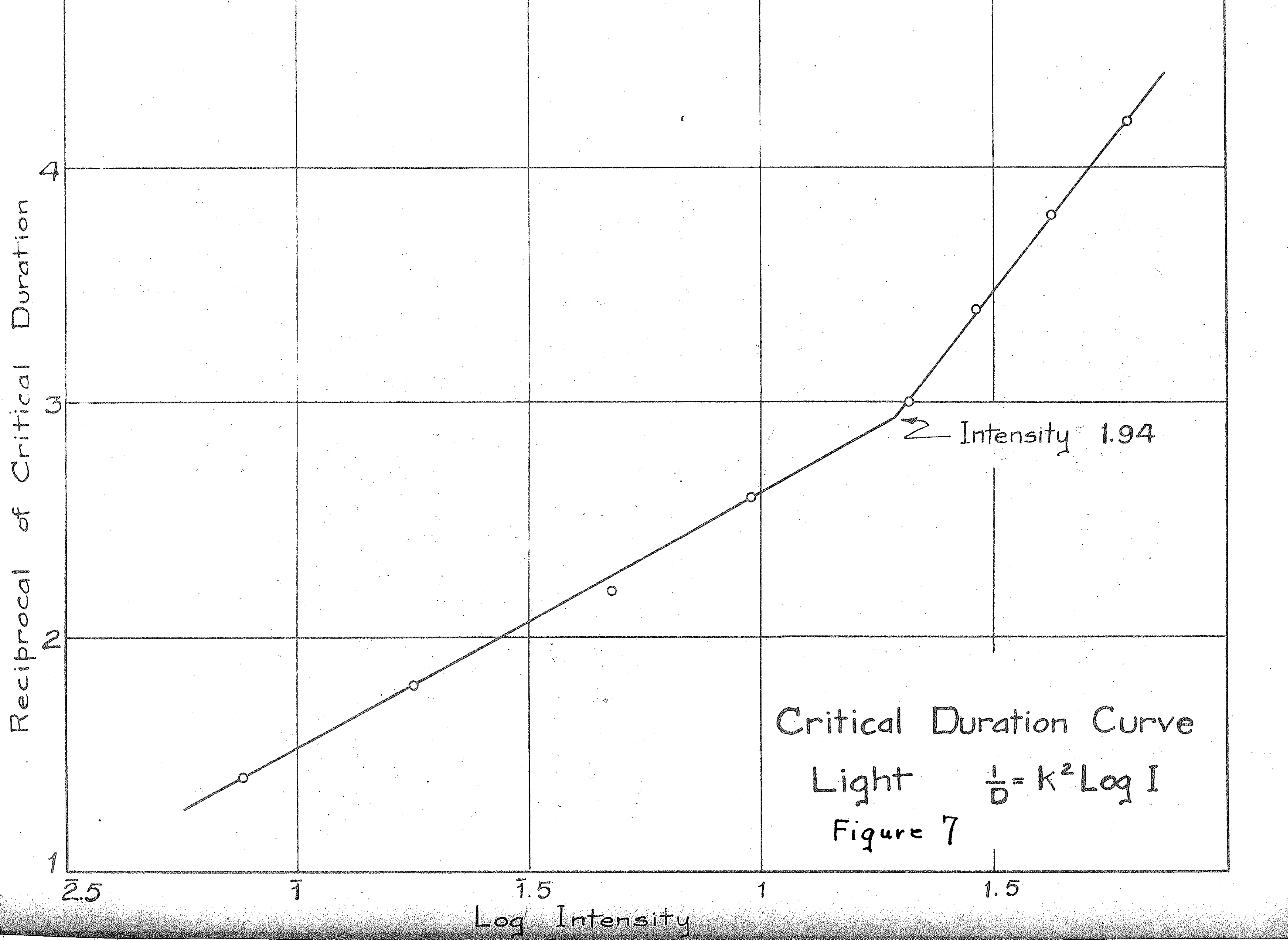
VISUAL EXPERIMENTS

Apparatus:

Without altering the apparatus used in the examination of Weber's Law, a sectored metallic disc was inserted between the source A, and the nicol prisms D. This disc was rotated by an electric motor, and its velocity determined by chronograph.

Experimental Procedure:

The sectored disc being rotated at a constant velocity, the intensity of the light was altered by the nicols until the intensity at which the critical flicker occurred was obtained. The critical duration was then



Critical Duration Curve

Light $\frac{1}{D} = K^2 \text{Log } I$

Figure 7

computed from the velocity of the motor and the number of segments in the disc, and the intensity at which the critical duration occurred deduced from the angle between the nicol prisms. Three settings of the nicols were made to obtain a reading. At the first setting only the approximate intensity was obtained. Allowing a rest period for the eye to be restored to normal daylight adaptation, the nicols were then set within 15 or 20 minutes of arc of the true value, another rest period observed, and the third adjustment of the nicols made. The time required to expose the eye to the radiation in making the final determination was approximately 2 seconds. The third reading could generally be repeated within five minutes of arc.

By using different velocities of the motor, readings of the critical duration were obtained over the same range of intensity at which the Weber ratio had been examined.

Results:

The data obtained are given in Table 5, and plotted in semi-logarithmic form in Figure 7. The value of the intensity at which the change of slope occurred in this curve was measured from the graph and is given below in comparison with the change of slope in the Weber Law.

It will be seen that the change occurs at the same value of stimulating intensity:

Ferry-Porter Law - I = 1.94

Weber-Fechner Law - I = 1.92

Conclusion:

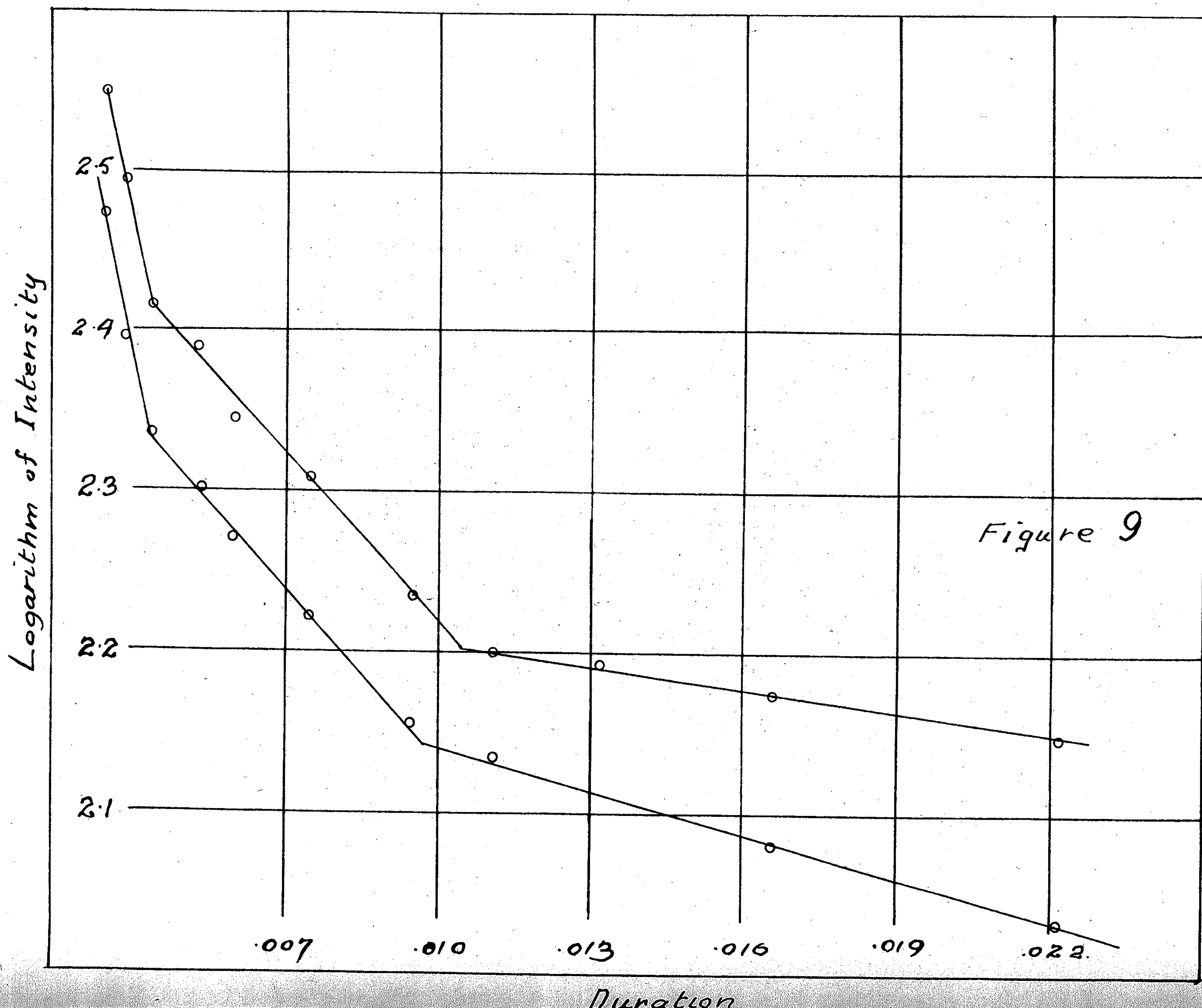
There exists an intimate relation between the Weber-Fechner and the Ferry-Porter Laws, from which it may be concluded that the critical duration of flicker is closely allied to sensation.

TACTILE EXPERIMENTS

To ascertain if the pressure at which abrupt changes in slope occurred in the increment corresponded with the pressure at which they occurred in the graph for critical duration of stimulation, an attempt was made to obtain a critical duration curve with the same apparatus covering the same range of pressures as that used in obtaining the graphs in Figures 5 and 6.

Apparatus:

In order to interrupt periodically the air playing on the lip, a cardboard disc containing an equal number of open and closed equiangular segments was rotated



2.5
2.4
2.3
2.2
2.1
Logarithm of Intensity
.007
.010
.013
.016
.019
.022
Duration

Figure 9

between the protecting disc and the air nozzle by a low velocity synchronous motor. The duration of a pulse of air was computed from the velocity of the motor and the number of segments cut from the disc. Different values of the duration could be obtained by substituting discs containing different numbers of segments. To obtain a point on the critical duration curve, a particular disc was rotated and the pressure of the air altered until it was found to evoke a just continuous sensation. A rest period of ten minutes was allowed, a new disc substituted and the corresponding alteration in air pressure determined.

Considerable difficulty was experienced in obtaining consistent readings with the apparatus, and after careful examination this was attributed to the size of the air nozzle, which was too small. A new nozzle of larger bore was substituted, whereupon accurate readings were obtained without difficulty, and the value of the intensities at which changes in the constant occurred, where found by plotting the results in a semi-logarithmic form as shown in Figure 9. The data rare given in Table 7.

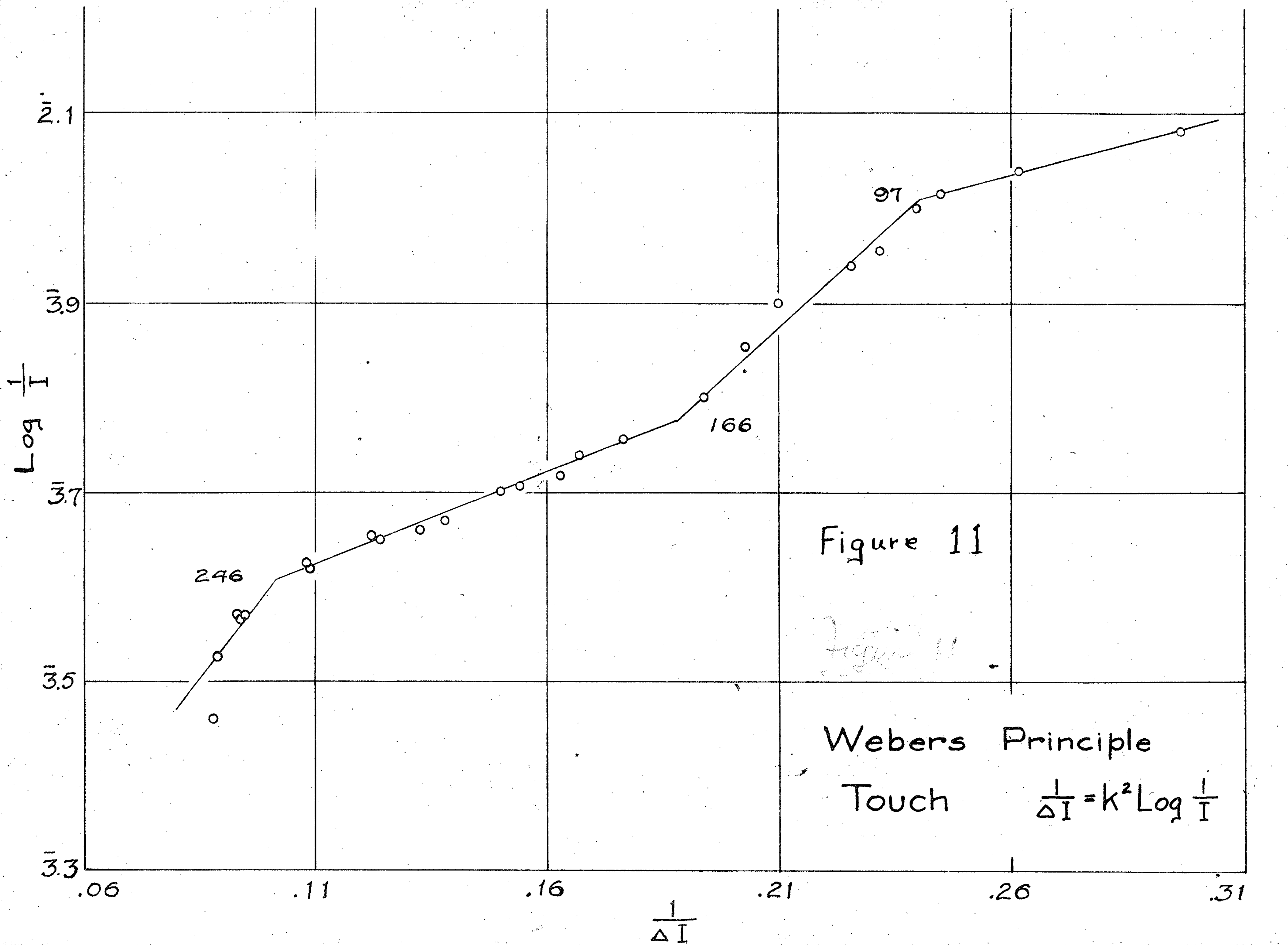
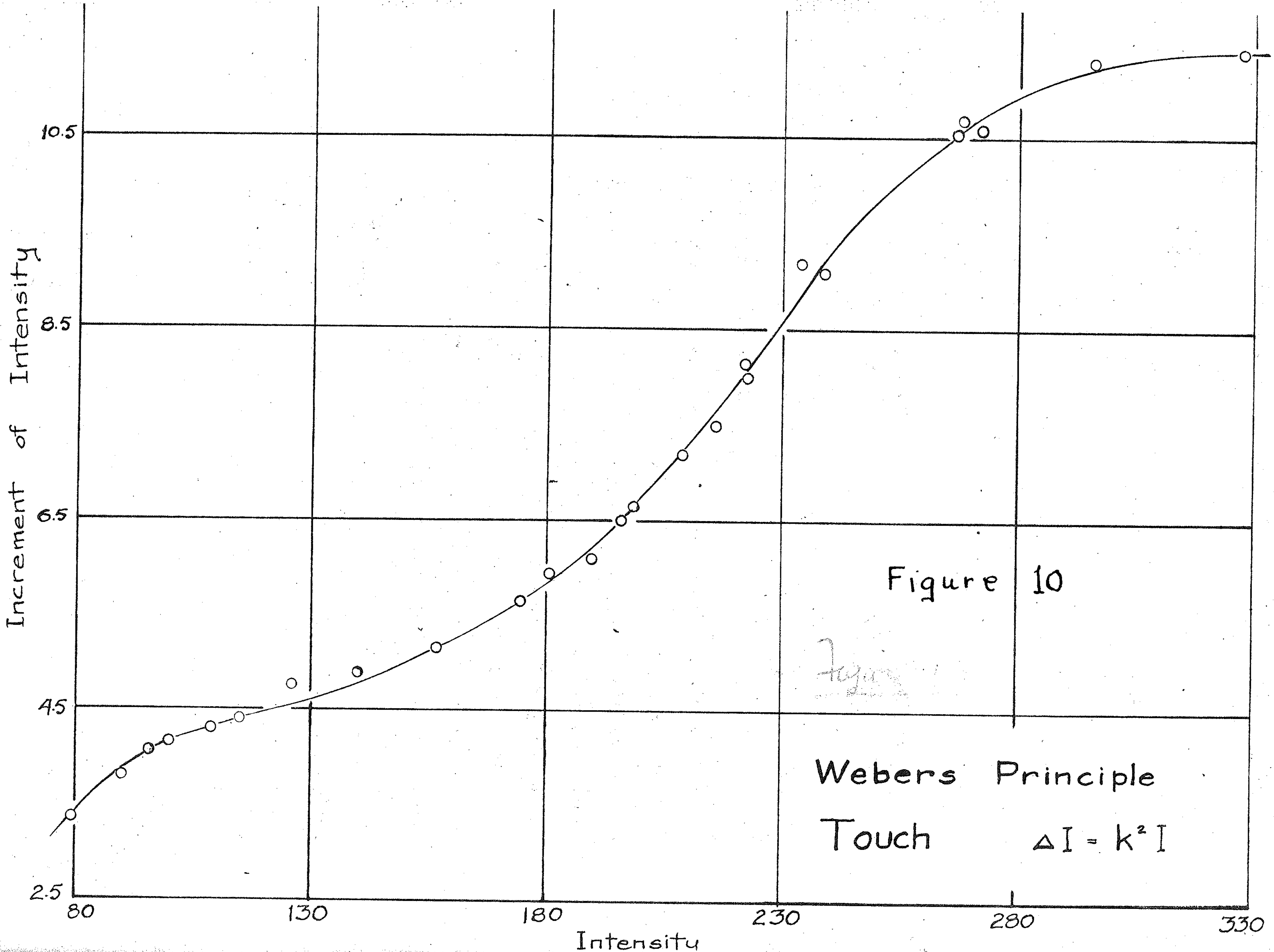


Figure 11

Webers Principle

Touch $\frac{1}{\Delta I} = k^2 \operatorname{Log} \frac{1}{I}$



Increment of Intensity
Intensity
10.5
8.5
6.5
4.5
2.5
80
130
180
230
280
330
Figure 10
Webers Principle
Touch
$\Delta I = k^2 I$

It will be seen that two curves are obtained. The upper one of these is probably due to the superficial tactile sense, which would be the sense employed in determining the increment curve, the lower curve being due to deep tactile sense.

The change in the size of the nozzle necessitated repetition of the increment curve. Difficulty was here experienced by finding that the nozzle was too large for accurate increment readings. New data, given in Table 8, which are shown in Figures 10 and 11, were sufficiently accurate to determine the intensity at which the changes in slope occur. A comparison of the position of these inflection points in this increment curve with those in the corresponding Porter graph is given below. It will be seen that agreement is obtained within the limits of experimental error:

Changes of slope - Ferry-Porter Law (I = 245
(I = 160

Changes og slope - Weber-Fechner Law (I = 246
(I = 166

CONCLUSION:

, There is an intimate relation between the critical duration and the differential threshold as shown by the fact that the inflexion points occur at the same places in the Ferry-Porter and the Weber-Fechner graphs.

THE MEASUREMENT OF SENSATION.

(i) Vision:

The Weber-Fechner equation is

$$\Delta I = K_1 I + C \qquad (1)$$

at $I = 0$, $\Delta I =$ threshold value, a small quantity which we shall neglect. Thus equation 1, becomes

$$\frac{\Delta I}{I} = K_1 \qquad (2)$$

Suppose that K_1 is a unit of sensation, denoting this by dS

$$\frac{\Delta I}{I} = dS \qquad (3)$$

If ΔI is sufficiently small to be treated as an infinitesimal

$$\frac{dI}{I} = ds \quad \text{or} \quad s = \text{Log } I + C \qquad (4)$$

At $I =$ unity, C is given as the value of the sensation at the threshold, denoting this by S_0

$$S = \text{Log } I + S_0 \qquad (5)$$

The Ferry Porter equation is

$$\frac{I}{D} = K_2 \text{ Log } I + \frac{1}{D_0} \qquad (6)$$

Equating 5 and 6

$$\left(\frac{1}{D} - \frac{1}{D_0}\right)\frac{1}{K_2} = S - S_0 \qquad (7)$$

$$\frac{1}{D} = K_2S - \left(K_2S_o - \frac{1}{D_o}\right) \qquad (8)$$

Neglecting the small quantity $\left(K_2S_o - \frac{1}{D_o}\right)$

$$\frac{1}{D} = KS \qquad (9)$$

That is $\frac{1}{D}$ is a measure of sensation provided that:

(i) $\triangle I$ always liberates one sensation unit,

(ii) $\triangle I = dI$

It may be shown that (ii) is legitimate.

Suppose $S = \text{Log } I$

Then $S + \triangle S = \text{Log }(I + \triangle I)$

Or $S = \text{Log }\left(1 + \frac{\triangle I}{I}\right)$

Expanding $S = \frac{\triangle I}{I} - \frac{1}{2}\left(\frac{\triangle I}{I}\right)^2 + \frac{1}{3}\left(\frac{\triangle I}{I}\right)^3 \ldots\ldots.$

Assuming $\frac{\triangle I}{I}$ to be at the most $\frac{1}{150}$, a value determined by Helmholtz, only the first term in the expansion need be retained, so that

$$\frac{\triangle I}{I} = \triangle S$$

APPLICATION TO THE LUMINOSITY CURVE OF THE SPECTRUM:

The sensation of light has a physical basis, apparently being caused by the action of ether waves on physiological receptors. For such an action there would be a transference of energy from the wave to the receptor. The transference of energy in this manner implies the presence of oscillators. If the oscillator has the same period as the wave a transfer of energy takes place. If, however, the oscillator has a different period an energy change will not take place over a period of time. The criterion that a change of energy takes place is that the oscillator and the wave have the same frequency, consequently in an attempt to relate wave motion and sensation, such as is made in determining the luminosity curve, a step liable to reveal simplification in the law would be to use frequency in place of wave length throughout the spectrum.

Now since $\frac{1}{D}$ is a measure of sensation its value for different frequencies when plotted against these frequencies should yield the luminosity curve. Such curves were plotted and had the same general shape as curves obtained by other methods, when they were plotted against frequency. Plotting the logarithm of $\frac{1}{D}$ however, against the frequency yielded a series of straight lines, thus revealing the law of the luminosity curve.

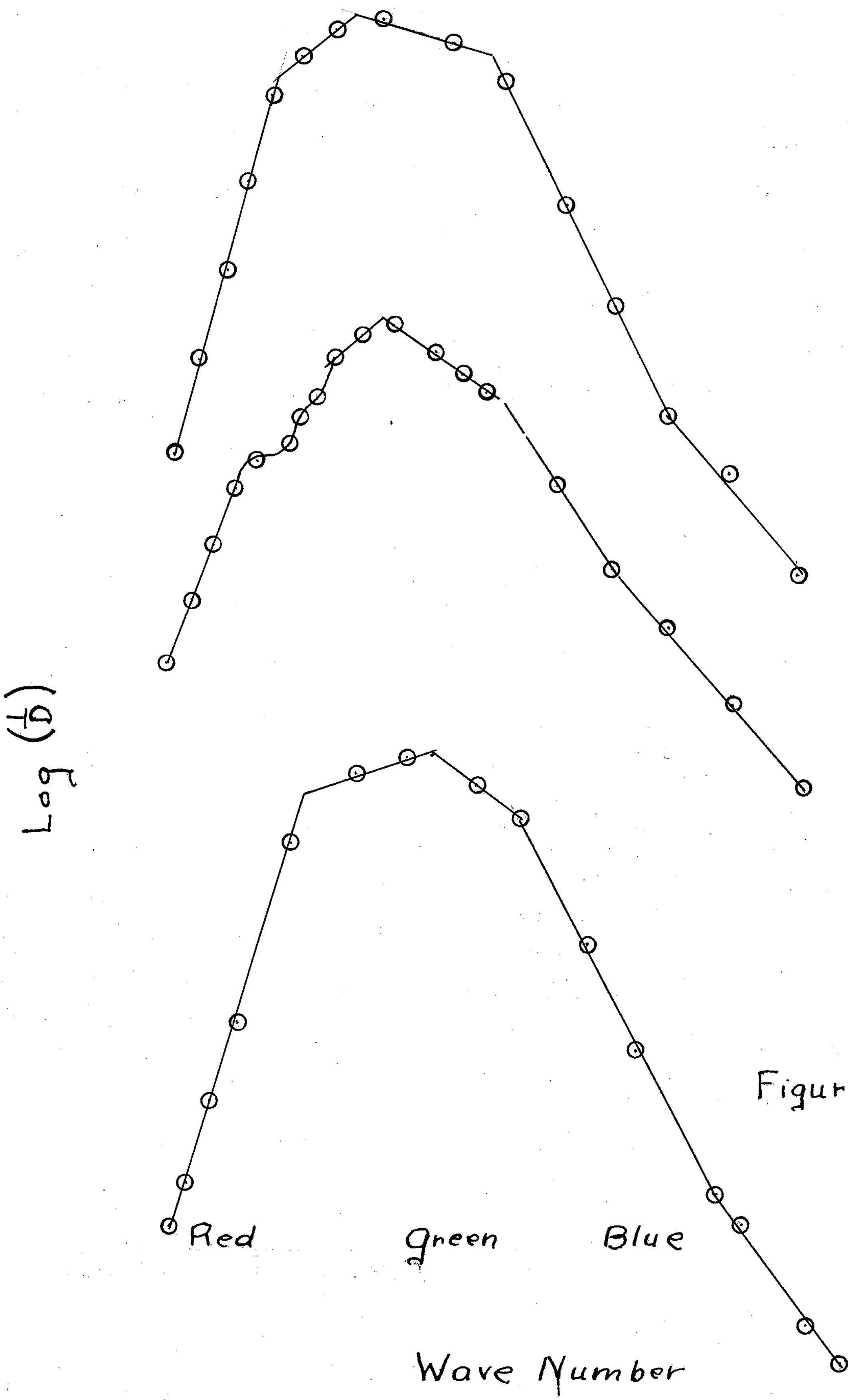


Figure 12.

Data to plot these curves were obtained from published and unpublished papers by Dr. Allen and his students dealing with studies in colour vision. The curves are shown in Figure 12, where it will be seen that the centre curve displays the linear relation at its best, there being more points available. The departure from the linear relation in the green is due to the fact that the observer was slightly colour blind.

$$\text{Log}\ \frac{1}{D} = \text{Log Sensation} = \text{Log}\ S$$

$$\text{Log}\ S = K\nu$$

where ν is the frequency of the wave.

Thus $$S = e^{K\nu}$$

(ii) TACTILE AND AUDITORY SENSES:

Weber-Fechner Equation is $= \frac{1}{\Delta I} = K, \text{Log}\ \frac{1}{I} + C$

Ferry-Porter Equation is $= D = K_2 \text{ Log}\ \frac{1}{I} + C_2$

Eliminating $\text{Log}\ \frac{1}{I}$ the result may be written

$$\Delta I = K\left(\frac{1}{D}\right) + C$$

If as before $I = S$, $\frac{1}{D}$ gives a method of examining the variations of sensation under different conditions.

Thus the critical duration and the differential threshold are closely related and the formula for sensation should be given by

$$S = \sum \Delta I \ \text{Log} \ \frac{1}{I} + C$$

Since a simple relation exists between the critical duration and the intensity and between the increment and the intensity the existence of such a complicated formula for sensation as deduced above seems artificial and not as all satisfactory especially since the numerical value of $\frac{\Delta I}{I}$ is approximately one in eight, preventing integration. This problem is being held over for further investigation.

<u>CONCLUSION</u>:

The critical duration or the differential threshold may be used to measure sensation in the visual sense. In the auditory and cutaneous senses these two quantities differ from each other by constant quantity and may be used to give an indication of the variation of the sensitivity of the receptors since they each measure an element of sensation.

I would like to take this opportunity to express my thanks to Dr. Frank Allen, not only for the facilities of his laboratory but for a great deal of constructive criticism during the progress of the work: to Dr. N. R. Wilson for suggesting the treatment showing under what conditions finite quantities may be integrated; to Mr. John Allen and Mr. M. Robertson for taking readings on the various senses. Mr. John Allen has also constructed and operated the Rayleigh Disc for the measurement of sound intensities.

TABLE 1

KÖNIG'S DATA

$\lambda = 670$		$\lambda = 505$	
I	$\frac{\Delta I}{I}$	I	$\frac{\Delta I}{I}$
48950	.0215	19610	.0197
19680	.0163	9819	.0184
9844	.0158	4920	.0163
4912	.0180	1965	.0179
1967	.0169	982	.0188
983	.0172	490	.0197
490	.0206	196	.0222
196	.0224	97.6	.0250
97.1	.0300	48.7	.0258
48.1	.0391	19.4	.0306
19.1	.0465	9.64	.0375
9.35	.0701	4.76	.0513
4.54	.101	1.87	.0701
1.66	.207	0.920	.0874
.742	.347	0.454	.100
.312	.603	0.178	.124
		0.0866	.154
		0.0408	.224
		0.0150	.336
		0.00729	.372
		0.00339	.475

TABLE 2

WEBER'S PRINCIPLE - LIGHT

I	ΔI
1.000	.3167
.970	.3059
.8830	.2742
.755	.224
.7114	.2073
.6712	.1979
.6292	.1887
.5867	.1796
.4134	.1372
.2500	.0997
.2132	.0896
.1786	.0814
.1590	.0751
.1464	.0691
.1313	.0606
.117	.0513
.1006	.0403
.0904	.0369
.0588	.0288
.0467	.0231
.0301	.0181
.0076	.0116

TABLE 3

WEBER'S PRINCIPLE - SOUND

I	ΔI
2.0	.212
4.0	.237
6.0	.257
8.0	.277
10.0	.292
12.0	.302
1.0	.162
3.0	.222
5.0	.247
7.0	.267
9.0	.287
11.0	.297
2.0	.207
3.0	.222
3.5	.2275
3.0	.2075
1.5	.1975
1.0	.1825
0.5	.1575

TABLE 4

DATA FOR WEBER'S RATIO - TOUCH

Intensity of Stimulus I	$\text{Log } \frac{1}{I}$	Differential Threshold ΔI	$\frac{1}{\Delta I}$
7	$\bar{2}.1523$	.760	1.31
14	$\bar{2}.8537$	1.400	.714
16	$\bar{2}.7958$	1.560	.641
45	$\bar{2}.3463$	3.110	.321
56	$\bar{2}.2504$	3.660	.273
59	$\bar{2}.2279$	3.660	.273
67	$\bar{2}.1732$	4.010	.249
78	$\bar{2}.1072$	3.360	.229
88	$\bar{2}.0531$	4.610	.217
100	$\bar{2}.0000$	5.160	.194
108	$\bar{3}.9661$	5.510	.181
109	$\bar{3}.9624$	5.560	.180
110	$\bar{3}.9585$	5.610	.178
117	$\bar{3}.9314$	5.710	.175
119	$\bar{3}.9243$	6.160	.162
129	$\bar{3}.8893$	6.510	.154
132	$\bar{3}.8791$	5.860	.171
138	$\bar{3}.8597$	6.710	.149
143	$\bar{3}.8445$	5.960	.168
149	$\bar{3}.8267$	6.760	.148
158	$\bar{3}.8014$	6.810	.147
159	$\bar{3}.7986$	6.810	.147
161	$\bar{3}.7931$	6.860	.146
166	$\bar{3}.7796$	6.910	.145
168	$\bar{3}.7745$	7.060	.142
179	$\bar{3}.7474$	7.460	.134
180	$\bar{3}.7443$	7.510	.133
192	$\bar{3}.7168$	8.010	.125
199	$\bar{3}.7007$	8.360	.119
208	$\bar{3}.6821$	9.710	.115
214	$\bar{3}.6693$	9.360	.107
224	$\bar{2}.6493$	9.860	.101

TABLE 5

FLICKER CURVE LIGHT

1/D	Log I
217.1	$\bar{3}.8806$
271.4	$\bar{2}.2504$
325.7	
379.9	$\bar{2}.9798$
434.4	$\bar{1}.3140$
488.5	$\bar{1}.4644$
542.9	$\bar{1}.6242$
597.0	$\bar{1}.7900$

TABLE 6

WEBER'S PRINCIPLE - LIGHT

I	ΔI
0.2500	.1513
0.4130	.1980
0.5867	.2462
0.9532	.3522
0.9808	.3662
0.7499	.2950
0.8215	.3136
0.8830	.3306
0.9325	.3437
0.9700	.3576
0.9824	.3772
1.0000	.3893
0.2061	.1391
0.1922	.1354
0.0668	.0634
0.1169	.0900
0.1402	.1033
0.1525	.1122
0.1653	.1215
0.1784	.1513

TABLE 7

CRITICAL DURATION DATA - TOUCH

Duration	Intensity of Stimulus		Log $\frac{1}{I}$	
	Curve A	Curve B	A	B
.0222	140	108	$\bar{3}.854$	$\bar{3}.966$
.0166	149	120	$\bar{3}.827$	$\bar{3}.921$
.0133	156		$\bar{3}.807$	
.0111	158	136	$\bar{3}.801$	$\bar{3}.866$
.0095	171	143	$\bar{3}.767$	$\bar{3}.841$
.0075	203	167	$\bar{3}.693$	$\bar{3}.777$
.0060	221	186	$\bar{3}.655$	$\bar{3}.731$
.0053	245	200	$\bar{3}.611$	$\bar{3}.699$
.0044	260	219	$\bar{3}.584$	$\bar{3}.660$
.0039	313	250	$\bar{3}.504$	$\bar{3}.602$
.0035	354	297	$\bar{3}.450$	$\bar{3}.528$

TABLE 8

DATA FOR WEBER'S RATIO - TOUCH

Intensity of Stimulus I	Log $\frac{1}{I}$	Differential Threshold $\triangle I$	$\frac{1}{\triangle I}$
79	$\bar{2}.079$	3.360	.297
90	$\bar{2}.041$	3.810	.262
96	$\bar{2}.017$	4.060	.246
100	$\bar{2}.000$	4.160	.240
109	$\bar{3}.954$	4.310	.232
115	$\bar{3}.939$	4.410	.226
126	$\bar{3}.897$	4.760	.210
140	$\bar{3}.854$	4.910	.203
157	$\bar{3}.799$	5.141	.194
175	$\bar{3}.755$	5.641	.177
181	$\bar{3}.740$	5.960	.167
190	$\bar{3}.716$	6.110	.163
196	$\bar{3}.707$	6.491	.154
199	$\bar{3}.702$	6.660	.150
209	$\bar{3}.672$	7.210	.138
216	$\bar{3}.662$	7.491	.133
222	$\bar{3}.653$	8.160	.122
223	$\bar{3}.651$	8.010	.124
234	$\bar{3}.623$	9.210	.108
239	$\bar{3}.620$	9.110	.109
267	$\bar{3}.572$	10.541	.095
268	$\bar{3}.570$	10.710	.093
272	$\bar{3}.566$	10.610	.094
296	$\bar{3}.529$	11.291	.089
328	$\bar{3}.484$	11.391	.088

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