MULTIDISCIPLINARY DESIGN OPTIMIZATION WITH COLLABORATION PURSUING AND DOMAIN DECOMPOSITION: APPLICATION TO AIRCRAFT DESIGN

by

Dapeng Wang

A THESIS SUBMITTED TO THE FACULTY OF GRADUATE STUDIES OF THE UNIVERSITY OF MANITOBA IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE OF

DOCTOR OF PHILOSOPHY

IN

MECHANICAL ENGINEERING

The University of Manitoba
Department of Mechanical and Manufacturing Engineering
Winnipeg, Manitoba, Canada

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"Multidisciplinary Design Optimization with Collaboration Pursuing and Domain Decomposition: Application to Aircraft Design"

BY

Dapeng Wang

A Thesis/Practicum submitted to the Faculty of Graduate Studies of The University of Manitoba in partial fulfillment of the requirement of the degree

Of

DOCTOR OF PHILOSOPHY

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To my wife and our parents
ABSTRACT

Multidisciplinary Design Optimization (MDO) has emerged as a new technology dealing with the design of complex systems. This thesis focuses on development of an effective and efficient collaboration mechanism for coordinating coupled disciplines, as couplings are dominant issues in MDO. In this thesis, an innovative collaboration model is developed to handle these couplings. Based on this newly proposed collaboration model, two novel methods are developed for solving MDO problems. The first method is named the Boundary Search and Simplex Decomposition Method (BSSDM). It proposes a new methodology to geometrically depict the coupling information. Difficulties encountered with the couplings are alleviated through the obtained geometric information by the BSSDM. The second method is named the Collaboration Pursuing Method (CPM). It is a sampling-based MDO method, which aims to deal with relatively large-scale MDO problems. Both new methods are successfully tested. A conceptual aircraft design is implemented with the CPM and the results show that the CPM is competitively efficient as compared with other MDO methods. Limitations of the newly proposed MDO methods are discussed, along with suggestions for future work.
ACKNOWLEDGEMENTS

The author would like to thank his co-supervisors, Dr. Greg F. Naterer and Dr. G. Gary Wang for their encouragement, advice and friendship, as well as financial support from the Natural Sciences and Engineering Research Council of Canada (NSERC). Moreover, I would like to thank the Department of Mechanical and Manufacturing Engineering for providing teaching assistantships, as well as a sessional instructorship.

The author also thanks his committee members, Dr. Robert A. Canfield, Dr. Robert W. Derksen, and Dr. Xikui Wang for their respective inputs to this dissertation.

I would like further acknowledge Dr. Evin J. Cramer for reminding me to apply a conceptual aircraft design in my thesis, and Dr. S. Balakrishnan, Dr. N. Popplewell, Dr. Myron G. Britton, Mr. John M. Symonds and my colleagues, Mr. Darryl K. Stoyko, Mr. Songqing Shan, Mr. Olusola Adeyinka, Mr. Mike Fagundes, Mr. Peter S. Glockner, Mr. Lee Ming Wong, Mr. Fang Li, Mr. Maikel Sianturi, Mr. Emmanuel Ogedengbe, and Mr. Qin Sun for their help and friendship.

I am extremely grateful to my wife, Lin, who shares the excitement and hard work of my research with me the most. Without her love and support, all of this would be impossible. The support and love from my parents are also essential.

Dapeng Wang
Winnipeg, April 2005
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NOMENCLATURE

AR    Aspect ratio
$C_f$  Skin friction coefficient
$c_p$ Specific heat
D     Drag
ESF   Engine scale factor
$f$    System objective function, e.g., the range of an aircraft design
$g$    Vector of constraints
$G$    Vector of constraint functions associated with $g$
$g_c$ Vector of constraints for subsystems
$G_c$ Vector of constraint functions associated with $g_c$
$g_s$ Vector of system constraints
$G_s$ Vector of constraint functions associated with $g_s$
GF    Guidance function
$h$    Convection coefficient in aircraft component design / Altitude in conceptual aircraft design
$I$    Integer value to specify the size of a local region around the current best solution for Adaptive Sampling in the CPM
$ip$   Integration point
$k$    Thermal conductivity
K      Kelvin units of temperature
L      Lift
Nomenclature

L/D  Lift to drag ratio

$M$  Number of specified strips in the Strip-measure Strategy

$M$  Mach number

$n$  Number of state parameters

$N$  Interpolation (shape) function

$ns$  Number of discretized values of a design variable

$na$  Number of reactivated design variables for Active Design Variable Control

$ng$  Number of constraints

$nm$  Nautical mile

$N_Z$  Maximum load factor

$P$  Number of samples

$P_q$  Number of samples in quadrant $q$ in the Explosion Strategy

$Q$  Number of quadrants in the Explosion Strategy

$r$  Fraction of uniced surface area

$R$  Range

$s, t$  Local coordinates within elements

$S_{REF}$  Wing surface area

$SFC$  Specific fuel consumption

$SP$  Speed factor of Guidance Functions

$T$  Temperature

$T$  Throttle setting

$t/c$  Thickness / chord ratio

$u, v$  Velocity components

$V$  Velocity
Nomenclature

- $W_{BE}$: Baseline engine weight
- $W_E$: Engine weight
- $W_F$: Fuel weight
- $W_{FO}$: Miscellaneous fuel weight
- $W_O$: Miscellaneous weight
- $W_T$: Total weight
- $x$: Wingbox x-sectional area
- $x, y$: Cartesian coordinates
- $x_i$: Vector of disciplinary / local design variables and interdisciplinary design variables of $y_i$; $x_i \cap x_j (i \neq j)$ does not have to be empty
- $x_{Li}$: Vector of local design variables in subsystem $i$; $x_{Li} \cap x_{Lj} = \emptyset (i \neq j)$
- $x_{csi}$: Vector of system-subsystem design variables in subsystem $i$; $x_{csi} \cap x_{cSJ} (i \neq j)$ does not have to be empty
- $x$: Union of disciplinary design variables and system-subsystem design variables, $\{x_1, \ldots, x_i, \ldots, x_n, x_{cs}\}$
- $x_{cs}$: Union of system-subsystem design variables, $\{x_{csi}, \ldots, x_{cSII}, \ldots, x_{cSn}\}$
- $x_s$: Vector of system design variables; $x_s \cap x = \emptyset$
- $y$: Union of state parameters, $\{y_1, \ldots, y_i, \ldots, y_n\}$
- $y_{ci}$: Sub-set of $y$ in subsystem $i$ (excluding $y_i$)
- $y_i$: State parameter given by subsystem $i$, e.g., lift output from aerodynamic analysis
- $Y_i$: State parameter function associated with $y_i$
- $Z$: Vector of interdisciplinary design variables in the BLISS
- $\Delta_1$: Percentage value to specify the size of a local region around the current best solution for Adaptive Sampling in the CPM
### Nomenclature

#### Greek

- $\beta$: Regression coefficient
- $\rho$: Density
- $\phi$: Velocity potential
- $\lambda$: Wing taper ratio
- $\Lambda$: Wing sweep

#### Superscripts

- $^*$: Optimum value

#### Subscripts

- $a$: Air
- $c$: Contributing analysis or subsystem
- $ce$: Geometric center of the feasible state parameter region
- $d$: Discrete variables
- $f$: Fluid
- $g$: Gas
- $L$: Local
- $Lb$: Lower bound of design variables
- $q$: Sample index / ranking number
- $s$: System
- $Ub$: Upper bound of design variables
- $w$: Wall
Nomenclature

Overheads

- Implicit approximation

= Explicit approximation
# LIST OF ABBREVIATIONS

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<td>Adaptive Response Surface Method</td>
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<td>BLISS</td>
<td>Bi-level Integrated System Synthesis</td>
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<td>BS</td>
<td>Boundary Search</td>
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CHAPTER 1

INTRODUCTION TO MULTIDISCIPLINARY DESIGN OPTIMIZATION

1.1 BACKGROUND

According to the website of the AIAA Multidisciplinary Design Optimization Technical Committee [1], Multidisciplinary Design Optimization (MDO), (or Multidisciplinary Design Synthesis), is "a methodology for the design of complex engineering systems and subsystems that coherently exploits the synergism of mutually interacting phenomena". In other words, MDO seeks the "optimal design of complex engineering systems which requires analysis that accounts for interactions amongst the disciplines (or parts of the system) and which seeks to synergistically exploit these interactions" [1].
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MDO is an important design strategy in industry today. In the increasingly competitive global marketplace, its motivation and importance arise through requirements from a design process. A design process generally consists of conceptual design, preliminary design, detailed design, production and support. As a design procedure evolves over time and covers more information, designers lose certain design freedom due to sequentially casting decisions “in concrete”. As a result, a sub-optimal design occurs at the end. Moreover, due to the sequential characteristics of a conventional design method, certain changes may require designers to re-work the entire design analysis. MDO allows the design process to shift from the conventional means of sequentially handling a design procedure, by only focusing on the performance, to concurrently considering all aspects of design over the product life cycle from the Concurrent Engineering (CE) point of view [2]. Some typical elements of design aspects include performance, maintainability, life-cycle cost, reliability, vulnerability and so forth. MDO is a broad area that includes design synthesis, sensitivity analysis, approximation concepts and optimization methods and strategies. Also, it involves artificial intelligence, rule-based design, physical programming, parallel computing, variable-fidelity models, human interfaces, and so forth. Hence MDO is a highly complex subject involving many subsystems.  

An engineering system often entails several complex and coupled physical disciplines. For example, a helicopter air-intake scoop design involves couplings amongst de-icing, aerodynamic performance and engine performance [3], [4] (see Appendix I). These disciplines usually rely on computationally intensive processes, e.g., Finite Element

---

1 A subsystem usually designates a physical discipline, and is interchangeable with “contributing analysis” in this thesis.
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Analysis (FEA) and Computational Fluid Dynamics (CFD). Optimizing such a design problem is difficult due to interactions amongst coupled disciplines. MDO aims to solve such complex problems.

A classical example to illustrate the practical application of MDO is known as the aeroelastic problem. Considering the wing as a flexible beam, the aeroelasticity problem is constrained by couplings between a structural analysis and an aerodynamic analysis. The structural analysis needs the input of a pressure distribution from the aerodynamic analysis. Similarly, the aerodynamic analysis requires the deformed wing shape from the structural analysis. Traditionally, an aircraft design is initiated with aerodynamic analysis, treating the structure as rigid. Structural analysis is commenced once air loads are available. This practice takes many design iterations and could lead to a sub-optimal solution of the whole design system, since the system performance should be simultaneously determined by all contributing disciplines / subsystems.

The mathematical formulation of a general MDO problem is defined by

\[
\begin{align*}
\min_{x, y} & \quad f(x, y) \\
\text{subject to:} & \quad y_i = Y_i \left( x_i, x_{\text{ref}}, y_{\text{ref}} \right), \quad i = 1, \ldots, n \\
& \quad g = G(x, y) \leq 0
\end{align*}
\]

(1.1)

where:

\( f \) is the system objective function,
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\( y_i \) denotes a state parameter / variable\(^2\) given by its corresponding subsystem or discipline, e.g., Lift given by aerodynamic analysis,

\( Y_i \) denotes the function associated with \( y_i \),

\( n \) denotes the number of state parameters,

\( x_i \) denotes a vector of disciplinary / local design variables and interdisciplinary design variables of \( y_i \); \(^3\) \((x_i \cap x_j (i \neq j) \text{ does not have to be } \emptyset)\),

\( x_{csi} \) denotes a vector of system-subsystem design variables shared by \( y_i \) and \( f \); \((x_{csi} \cap x_{cj} (i \neq j) \text{ does not have to be } \emptyset)\),

\( x = \{x_1, \ldots, x_i, \ldots, x_n, x_{cs}\} \), a union of the disciplinary / local design variables, interdisciplinary design variables, and system-subsystem design variables,

\( x_s \) denotes a vector of system design variables of \( f \) \((x_s \cap x = \emptyset)\),

\( x_{cs} = \{x_{cis1}, \ldots, x_{cis}, \ldots, x_{csn}\} \), a union of the system-subsystem design variables,

\( y = \{y_1, \ldots, y_i, \ldots, y_n\} \), a union of state parameters,

\( y_{ci} = \{y_j\}, j \neq i \), a vector of state parameters output from other subsystems to subsystem \( i \),

\( g \) denotes a vector of inequality constraints,

\( G \) denotes a vector of inequality constraint functions associated with \( g \).

For illustration convenience, a simplified conceptual aircraft design problem is defined below to facilitate the interpretation of equation (1.1).

---

\(^2\) Hereafter, \( y \) is called state parameter.

\(^3\) Interdisciplinary design variables are only shared by subsystems, rather than the system objective function, and are different from local design variables, \( x_{Li} \), which only appear in subsystem \( i \).
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\[
\begin{align*}
\text{max } R(M, h, W_T, W_F, L/D, \text{SFC}) \\
\text{subject to: } W_T &= Y_1(\lambda, x, t/c, \text{AR}, \Lambda, S_{\text{ref}}, W_F, L, W_E) \\
D &= Y_2(h, M, C_f, t/c, \text{AR}, x, \Lambda, S_{\text{ref}}, W_T, \Theta, \text{ESF}) \\
\Theta &= Y_4(\lambda, x, \text{AR}, S_{\text{ref}}, L) \\
L &= Y_5(W_T) \\
W_F &= Y_6(t/c, \text{AR}, S_{\text{ref}}) \\
L/D &= Y_7(h, M, C_f, t/c, \text{AR}, \Lambda, S_{\text{ref}}, W_T, \Theta, \text{ESF}) \\
W_E &= Y_8(\text{ESF}) \\
\text{SFC} &= Y_9(h, M, T) \\
\Theta &= G_1(\lambda, x, \text{AR}, S_{\text{ref}}, L) \\
\sigma &= G_2(t/c, x, S_{\text{ref}}, \text{AR}, \lambda, L)
\end{align*}
\]

where

- \( R \) - Range
- \( \text{AR} \) - aspect ratio
- \( C_f \) - skin friction coefficient
- \( D \) - drag
- \( \text{ESF} \) - engine scale factor
- \( h \) - altitude
- \( L \) - lift
- \( M \) - Mach number
- \( \text{SFC} \) - specific fuel consumption
- \( S_{\text{REF}} \) - wing surface area
- \( t/c \) - thickness/chord
- \( W_E \) - engine weight
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\( W_F \) - fuel weight
\( W_T \) - total weight
\( x \) – wingbox cross section
\( \Lambda \) -wing sweep
\( \lambda \) - taper ratio
\( L/D \) - lift to drag ratio
\( \Theta \) - wing twist
\( \sigma \) - stress

According to the general MDO formulation defined in equation (1.1), \( x \) consists of 
\( h, M, \lambda, x, t/c, AR, \Lambda, S_{ref}, T, \) and \( C_f \) and is the vector of independent design variables.
\( x_{cs1} \) consists of \( M \) and \( h \). \( x_{c1} \) is empty. Equation (1.2) is dominated by couplings amongst 
\( W_T, \Theta, D, \) and ESF. Given values of \( x, W_T, \Theta, D, \) and ESF, the other state parameters, 
such as \( L, W_F, L/D, W_E, \) and SFC, can be directly calculated. In the expression of \( W_T, \)
\( x_{cs1} \) is empty, \( x_1 \) is composed of \( \lambda, x, t/c, AR, \Lambda, \) and \( S_{ref}, \) and \( y_{c1} \) includes 
\( W_F, L, \) and \( W_E \). In the function of \( D, x_{cs2} \) includes \( M \) and \( h \), \( x_2 \) is composed of 
\( C_f, t/c, AR, x, \Lambda, \) and \( S_{ref}, \) and \( y_{c2} \) includes \( W_T, \Theta, \) and ESF. In the expression of ESF,
\( x_{cs3} \) is empty, \( x_3 \) is \( T, \) and \( y_{c3} \) is \( D \). More detailed information of the conceptual aircraft 
design is introduced in Chapter 6.

Physically, a discipline could output more than one state parameter. Disciplinary
Analysis (DA) is a process to calculate all state parameters in a discipline based on inputs.
In equation (1.1), \( y \) is governed by

\[ y = f(x) \]
Chapter 1 – Introduction to Multidisciplinary Design Optimization

\[
\begin{align*}
Y_1 &= Y_1(x_{c1}, x_i, y_{ci}) \\
& \vdots \\
Y_i &= Y_i(x_{ci}, x_i, y_{ci}) \\
& \vdots \\
Y_n &= Y_n(x_{cn}, x_i, y_{ci})
\end{align*}
\]

Equation (1.3) describes the System Analysis (SA) or the Multidisciplinary Analysis (MDA). The solution of equation (1.3) is calculated by an iterative procedure (such as the Gauss iterative method), when given a set of \( x \), the initial guess of \( y \), convergence criterion determined by a specified accuracy tolerance, and maximum number of iterations. The standard MDO formulation in equation (1.1) is also called the Multidisciplinary Feasible (MDF) method [5] [6] in the nonlinear programming community, or the All-in-One method in the engineering community [7]. It can be solved by conventional optimization algorithms, such as gradient-based methods. The main difficulty of applying the All-in-One method in practice is that the computational cost could be prohibitive since the SA/MDA is called at each iteration during the optimization process. Therefore, reducing the number of calls to the SA/MDA can improve the performance on an MDO algorithm.

1.2 OBJECTIVES

Traditional optimization methods, such as the All-in-One method, need frequently call the SA/MDA for any increment of design variables, \( x \), during the optimization process. It is generally cost-prohibitive to apply traditional optimization methodologies for pursuing the global optimal solution of MDO problems in practice if disciplinary analyses involve
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computationally intensive processes, such as CFD. Reducing the number of calls to the SA/MDA is a major challenge to develop new MDO methods. Also, since MDO focuses on the system design, another challenge is successfully and efficiently handling a large-scale design problem, such as an aircraft design involving many design variables, constraints and subsystems. In general, the MDO methodology is dedicated to reducing the total turn-around time for pursuing an optimal system solution by efficiently collaborating coupled disciplines.

The basic idea of sampling-based MDO methods is to have a large number of samples, from which some desirable samples are selected for optimization. In doing so, an obvious question is how the feasibility of samples with subject to the SA/MDA can be determined and maintained. In this regard, the first objective of this thesis is to develop a new collaboration model for selecting feasible samples.

Since MDO is dominated by the SA/MDA, intuitively, it is believed that more understanding of the SA/MDA can benefit the process of solving MDO problems. Based on the collaboration model, the second objective of this thesis is focused on exploring the essence of the SA/MDA in a different approach, which allows designers to capture the relation between coupled state parameters explicitly. Given the relation between coupled state parameters, optimizing the system objective function can be implemented easily. New boundary search and domain decomposition strategies will be developed for geometrically depicting the relation between coupled state parameters. Eventually, a new MDO methodology will be developed and tested.
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The third objective of this thesis is to develop a sampling-based MDO methodology, which is capable of solving general and relatively large-scale MDO problems. The collaboration model described previously will be applied for selecting the feasible sample candidates from sampling processes. For achieving the global optimum of an MDO problem, a sampling-based optimization method, named the Mode-pursuing Sampling (MPS) method [8], will be applied as a global optimizer in the new MDO method. Also, strategies for solving large-scale MDO problems will be developed. At the end, some benchmark test cases and a conceptual aircraft design problem will be solved for validation of this new MDO method.

The objectives of this thesis can be summarized as follows:

1. To be able to efficiently coordinate coupled subsystems of MDO problems in a sampling process, a new collaboration model will be developed.
2. To describe the couplings explicitly when optimizing MDO problems, a new MDO methodology will be developed based on the collaboration model, boundary search strategies, and decomposition methods.
3. For optimizing general and relatively large-scale MDO problems, a new sampling-based MDO methodology will be developed based on the collaboration model.

1.3 THESIS OUTLINE

Chapter 2 gives an extensive literature review of MDO.
Chapter 1 – Introduction to Multidisciplinary Design Optimization

Chapter 3 introduces a newly proposed collaboration model for effectively coordinating coupled disciplines in MDO problems.

Chapter 4 elaborates a newly proposed MDO method, named the Boundary Search and Simplex Decomposition Method (BSSDM).

Chapter 5 presents a newly proposed MDO method, called the Collaboration Pursuing Method (CPM), as well as results of benchmark MDO problems solved by the CPM.

Chapter 6 shows the application of the Collaboration Pursuing Method to a conceptual aircraft design.

Chapter 7 concludes the thesis and makes recommendations for future research and development of the proposed MDO methodologies.
2.1 INTRODUCTION

In 1991, AIAA (American Institute of Aeronautics and Astronautics) established a Technical Committee for Multidisciplinary Design Optimization (TC-MDO) to advance MDO technology [1]. The National Aeronautics and Space Administration (NASA) Langley Research Center has established a leading role in this area. References [9]-[12] provide extensive reviews on the current development of MDO. Many currently available MDO methods have been systematically tested and compared in references [13] and [14].

Formulations of MDO can be categorized into decomposition-based and non-decomposition methods. Under the decomposition-based category, the methods include Optimization by Linear Decomposition (OLD) and Collaborative Optimization (CO) methods. The MDF method (mentioned in Chapter 1), All-at-Once method, and
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Individual Discipline Feasible method belong to the non-decomposition category. The Response Surface Method (RSM) works as a nested technique in almost all of the currently available MDO methods.

2.2 OPTIMIZATION BY LINEAR DECOMPOSITION

The sensitivity analysis is a basic tool for Optimization by Linear Decomposition (OLD). OLD methods were motivated by personnel organization in the workplace, e.g., design teams consisting of different mission departments and concurrently dealing with all subsystems. The OLD methods decompose a large design into subsystems (or disciplines) with effective collaboration, and they drive all subsystems to concurrently contribute the system objective. In this way, the turn-around time is expected to be shortened. Three representative decomposition infrastructures are the Hierarchic Decomposition, Non-hierarchic Decomposition, and Bi-level Integrated System Synthesis (BLISS).

2.2.1 Sensitivity Analysis

In the area of Operations Research or Linear Programming, a sensitivity analysis, which is also called the post-optimality analysis, is used to explore “how a Linear Programming’s optimal solution depends on its parameters” [15]. For the standard MDO problem in equation (1.1), a difficulty is the costly SA/MDA involving interdisciplinary couplings amongst disciplines. Intuitively, if there is a way to relax or decouple the
couplings to achieve certain disciplinary autonomy, the process of repeatedly evaluating the SA/MDA could be avoided during the optimization. The sensitivity analysis was applied into engineering applications by Sobieszczanski-Sobieski and Riley in 1981 [16], [17]. According to references [16] and [17], an “optimum sensitivity analysis is a technique which permits investigation of the sensitivity of an optimization problem’s solution to variations of the problem’s parameters”. Details regarding the optimum sensitivity analysis can be found in references [16]-[19].

A straightforward method to perform the sensitivity analysis is the Finite Difference Method (FDM) [20]. But the FDM is costly since it needs to repeat the SA/MDA for every perturbed design variable. It was reported that the sensitivity analysis accounts for more than 90% of the total computational cost in the OLD methods [21]. Some work has been done for investigating other efficient ways of performing the sensitivity analysis, such as the direct method, adjoint method, and linear approximate method [19], [20], [22]. Analytical methods are always preferred due to high efficiency. The optimum sensitivity analysis is usually applied to link two adjacent levels of the decomposed system.

Due to its nature, the sensitivity analysis can be used for extrapolation [16], [17], [23]. The increment of a parameter can vary up to 20%, or at least 10%, and still satisfy engineering accuracy requirements, provided the change of the parameter will not influence the set of binding constraints. The other potential application of the sensitivity analysis is to determine weighting factors in a composite objective function in the multiple-objective optimization [16], [17].
2.2.2 Hierarchic Decomposition

In 1982, the NASA Langley Research Center published a formal generic decomposition method called the Hierarchic Decomposition [24]. Literally, the Hierarchic Decomposition method decomposes a design problem into subsystems corresponding to ‘parent’ and ‘daughter’ levels. There is no interaction between parents or daughters at the same level, as shown in Figure 2-1. In other words, a daughter problem only talks to its own parent problem, while one parent could have more than one daughter. Also, the whole system has only one parent at the top, which is in charge of the entire system optimization.

The Hierarchic Decomposition method has been successfully applied in structural analysis with a large number of design variables and degrees of freedom [25], [26]. In hierarchic decomposition, all subsystems can not concurrently pursue the system objective during the optimization procedure. Also, due to its nature of no connection between subsystems, the Hierarchic Decomposition method has difficulties to organize more coupled disciplines.

![Hierarchic Decomposition Diagram](image_url)
2.2.3 Non-Hierarchic Decomposition

Non-hierarchic Decomposition methods were motivated from the observation that most design problems are non-hierarchical in nature. Solving an MDO problem is like answering "what if" questions during a design procedure. However, for a large-scale, coupled system, it is difficult to answer the "what if" questions because everything influences everything else in an extreme situation. The sensitivity analysis herein is implemented with the Global Sensitivity Equations (GSE), which linearize nonlinear couplings amongst subsystems and serve an important role in the Non-hierarchic Decomposition methods [27].

A non-hierarchic system, as an example, involving structural, aerodynamic, and control subsystems in an aircraft design is depicted in Figure 2-2. The couplings are governed by the SA/MDA in equation (2.1).

\[
\begin{align*}
\alpha\left((x, y_\beta, y_\gamma), y_x\right) &= 0 \\
\beta\left((x, y_\alpha, y_\gamma), y_\beta\right) &= 0 \\
\gamma\left((x, y_\alpha, y_\beta), y_\gamma\right) &= 0
\end{align*}
\] (2.1)

where \(\alpha, \beta, \) and \(\gamma\) represent structural, aerodynamic, and control analyses, respectively. \(x\) is the design variable vector. \(y_\alpha, y_\beta,\) and \(y_\gamma\) are three behavior variables or state parameters output from their respective subsystem, and are coupled to each other by input-output relations.
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The GSE is the sensitivity derivative of state parameters, \( y \), with respect to design variables \( x \), i.e., \( \begin{bmatrix} \frac{\partial y}{\partial x} \end{bmatrix} \). It reflects the influence of the change from local design variables on state parameters. Two different ways of deriving the GSE are introduced in reference [27]. Two representative GSE-based MDO methods are briefly reviewed in Sections 2.2.3 and 2.2.4.

2.2.4 Concurrent Subspace Optimization

The first representative algorithm of Non-hierarchic Decomposition methods is the Concurrent Subspace Optimization (CSSO) [28]. The CSSO needs to uniquely allocate all independent design variables into separate subsystems / Contributing Analyses (CA). In other words, one design variable only appears in one subsystem. The SA/MDA of an MDO problem involving three coupled state parameters is formulated in the CSSO fashion by
The optimization process of the CSSO includes two steps. First, during the subspace optimization, the system objective function is optimized concurrently in each subsystem with respect to local (disciplinary) design variables. The GSE is applied to approximate interdisciplinary relations between the local design variables in one subsystem and state parameters output from other subsystems, e.g., \[
\frac{\partial y_{\beta}}{\partial x_a} \]
in subsystem \(a\) in equation (2.2).

Cumulative constraints are carried out with the Kreisselmeier-Steinhauser (KS) function, which is similar to the quadratic exterior penalty function [29]. Non-local constraints are approximated using a first-order Taylor series.

Then, in the system Coordination Procedure (COP), constraint responsibility and tradeoff factors are used to coordinate the subspace optimization [28]. It was reported from reference [28] that a linear approximation based on the GSE can be used to replace the factors for coordinating the subspace optimization. Similarly, first-order and second-order accumulated approximations are used to coordinate the subspace optimization [30], [31]. Neural network approximations are also applied in this regard [32].
2.2.5 Bi-level Integrated System Synthesis

The Bi-level Integrated System Synthesis (BLISS) is a two-level decomposition infrastructure consisting of the Black Box (BB) level and the System Optimization (SO) level [33], [34]. The BB corresponds to a CA or subsystem, while the system objective value is an output value from one of the subsystems. The BLISS focuses on MDO problems that have a large number of local design variables, $x_L$, but a small number of interdisciplinary design variables, $Z$, existing in at least two subsystems. All constraints are preferred to be local constraints, which are only functions of the local design variables, $x_L$, and state parameters, $y$. If constraints are functions of $Z$ and $y$, they can be handled in the System Optimization. Rewrite the original formulation of the MDO problem in the BLISS fashion by

$$\begin{align*}
\min_{x_L, Z, y} f(x_L, Z, y) \\
\text{subject to: } g = G(x_L, Z, y) \leq 0
\end{align*} \quad (2.3)$$

In the level of the BB optimization, given a feasible design, $f_0$, the BLISS linearizes the system objective function with respect to local design variables, $x_L$, based on the GSE by freezing the state parameters and interdisciplinary design variables, as shown in the following equation

$$\bar{f} = (f)_0 + \left( \frac{\partial f}{\partial x_L} \right)^T \Delta x_L + \ldots + \left( \frac{\partial f}{\partial x_{Ln}} \right)^T \Delta x_{Ln} \quad (2.4)$$
Chapter 2 – Literature Review

where \( \left( \frac{\partial f}{\partial x_{L_i}} \right)^T \) is evaluated by the GSE. Thus, all BBs can concurrently and explicitly contribute to the system objective by optimizing its own objective as

\[
\min_{x_u} \phi_i = \left( \frac{\partial f}{\partial x_{L_i}} \right)^T \Delta x_{L_i}
\]

subject to: \( g_i = G(x_{L_i}) \leq 0 \), \( i = 1, ..., n \)  \( (2.5) \)

In the level of the SO, the interdisciplinary design variables, \( Z \), become active to further improve the entire system objective. The system level optimization can be formulated by

\[
\min_{Z} f = f^0 + \left( \frac{\partial f}{\partial Z} \right)^T \Delta Z
\]

subject to: \( Z_{Lb} \leq Z + \Delta Z \leq Z_{Ub} \), \( \Delta Z_{Lb} \leq \Delta Z \leq \Delta Z_{Ub} \)  \( (2.6) \)

Two different ways of calculating \( \frac{\partial f}{\partial Z} \) in equation (2.6) are introduced in references [33] and [34]. Compared with the CSSO, each BB (or subsystem) concurrently works on its own contributing part to optimize the system objective rather than the whole system objective.

The Response Surface Method (RSM) is also applied to the BLISS for approximating \( f \) and \( g \) in \( Z \) space [35], [36]. One way of building up the RSM is based on data from the SA/MDA, and the other way is to obtaining data from the BB optimization.
2.3 COLLABORATIVE OPTIMIZATION

The Collaborative Optimization (CO) is also a bi-level decomposition algorithm [37]-[39]. In the CO, the interdisciplinary consistency is pursued by the match between the targets issued from the system level and the feedback returned from subsystems. For a two state-parameter MDO problem including a system-subsystem variable, $x_{cs}$, shared by both subsystems, the SA/MDA is formulated by

$$y_1 = Y_1(x_{cs}, x_{L1}, y_2)$$
$$y_2 = Y_2(x_{cs}, x_{L2}, y_1)$$

(2.7)

where $x_{L1}$ and $x_{L2}$ are local design variable vectors in subsystems. Then the two state-parameter MDO problem can be re-formulated in the CO fashion by

System-level optimization:

$$\min_{x_{cs}, y_1, y_2} f(x_{cs}, y_1, y_2)$$
subject to: $c_1(x_{cs}, y_1, y_2) = \frac{1}{2} \left[ \|\bar{\sigma}_1 - x_{cs}\|^2 + \|Y_1(\bar{\sigma}_1, \bar{x}_{L1}, y_2) - y_1\|^2 \right] = 0$ (2.8)
$$c_2(x_{cs}, y_1, y_2) = \frac{1}{2} \left[ \|\bar{\sigma}_2 - x_{cs}\|^2 + \|Y_2(\bar{\sigma}_2, \bar{x}_{L2}, y_1) - y_2\|^2 \right] = 0$$

where $\bar{\sigma}_1$ and $\bar{\sigma}_2$ indicate the copies of $x_{cs}$ in subsystems. Also, $\bar{\sigma}_1, \bar{x}_{L1}, \bar{\sigma}_2$ and $\bar{x}_{L2}$ are constants values returned from solving the subsystem optimization, given target variables of $(x_{cs}^*, y_1^*, y_2^*)$ from the system level.
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Subsystem-level problem:

Subsystem 1: \[
\begin{align*}
\text{min}_{\sigma_1, x_{L1}} & \quad \frac{1}{2} \left( \| \sigma_1 - x_{cs}^* \|^2 + \| Y_1 \left( \sigma_1, x_{L1}, y_2^* \right) - y_1^* \|^2 \right) \\
\text{subject to:} & \quad g_{L1} \left( \sigma_1, x_{L1}, Y_1 \left( \sigma_1, x_{L1}, y_2^* \right) \right) \leq 0
\end{align*}
\] (2.9)

where \( y_2^*, y_1^* \), and \( x_{cs}^* \) are constants given from the system level and \( \sigma_1 \) is the duplicate of \( x_{cs} \) in Subsystem 1. By the end of the subsystem optimization, \( \bar{\sigma}_1 \) and \( \bar{x}_{L1} \) are outputs from Subsystem 1.

Subsystem 2: \[
\begin{align*}
\text{min}_{\sigma_2, x_{L2}} & \quad \frac{1}{2} \left( \| \sigma_2 - x_{cs}^* \|^2 + \| Y_2 \left( \sigma_2, x_{L2}, y_1^* \right) - y_2^* \|^2 \right) \\
\text{subject to:} & \quad g_{L2} \left( \sigma_2, x_{L2}, Y_2 \left( \sigma_2, x_{L2}, y_1^* \right) \right) \leq 0
\end{align*}
\] (2.10)

where, \( y_2^*, y_1^* \), and \( x_{cs}^* \) are constants given from the system level and \( \sigma_2 \) is the duplicate of \( x_{cs} \) in Subsystem 2. By the end of the subsystem optimization, \( \bar{\sigma}_2 \) and \( \bar{x}_{L2} \) are outputs from Subsystem 2.

According to the above CO formulation, the system level and the subsystem level are linked by introducing extra variables, like \( \sigma_1 \) and \( \sigma_2 \) as local copies of the system design variable, \( x_{cs} \), in subsystems. The couplings amongst subsystems are relaxed by issuing target values of \( (x_{cs}^*, y_1^*, y_2^*) \) from the system level to the subsystem level. Therefore, each subsystem has its own autonomy and works independently of each other. The interdisciplinary consistency is maintained by pursuing the match between the issued
targets and the targets' copy in the subsystem level. Feedback from the subsystem level is used to adjust the value of the issued targets in the system level. Iteratively, the system-level optimization minimizes the objective function and reduces the interdisciplinary inconsistencies. The basic structure of the CO is shown in Figure 2-3.

![Collaborative Optimization Architecture](image)

Figure 2-3  The basic Collaborative Optimization architecture [38]

The basic idea of the CO fits well in industry settings. The chief designer assigns design requirements (targets) into each disciplinary analysis team to ask if they can implement it. Then, the design is re-adjusted based on the feedback from all design teams until all teams agree. However, it was reported that the CO has some serious computational difficulties [40]-[42], i.e.,
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(1) the CO leads to system level optimization problems that fail to satisfy the standard Karush-Kuhn-Tucker condition, and

(2) the CO leads to system level optimization problems that are more nonlinear than the original problem.

The Response Surface Method has been applied in the CO to approximate either the system objective function or subsystem objective function [43]. Nonetheless, the RSM cannot completely alleviate the problem raised from the CO’s nature [41].

2.4 ALL-AT-ONCE METHOD

If state parameters are considered as variables for optimization, and the SA/MDA is viewed as equality constraints, the formulation of the All-at-Once method (without considering the system design variables, \( x_r \)) of a two state-parameter MDO problem is given by (references [7], [9], and [10])

\[
\begin{align*}
\min_{x,y} & \quad f(x_{r1}, y_1, y_2) \\
\text{subject to:} & \quad y_1 - Y_1(x_{r1}, x_1, y_2) = 0 \\
& \quad y_2 - Y_2(x_{r2}, x_2, y_1) = 0 \\
& \quad g(x, y) \leq 0 
\end{align*}
\]  

At each optimization iteration, there is no feasibility maintained. The optimality is achieved together with feasibility only at solutions.
2.5 INDIVIDUAL DISCIPLINE FEASIBLE METHOD

The Individual Discipline Feasible (IDF) method (or called “in-between” approach) uses auxiliary variables and consistency constraints in MDO problems so that the SA/MDA can be decoupled [5], [7]. A two state-parameter MDO problem (without considering the system design variables, $x$) can be formulated in the IDF fashion by

$$
\begin{equation}
\begin{align*}
\min_{x,t} & \quad f(x, t_1, t_2) \\
\text{subject to:} & \quad t_1 = y_1 \\
& \quad t_2 = y_2 \\
& \quad g(x, t) \leq 0
\end{align*}
\end{equation}
$$

(2.12)

where $t_1$ and $t_2$ are the auxiliary variables of $y_1$ and $y_2$, respectively, and $t = \{t_1, t_2\}$. When given $x$ and $t$, $y_1$ and $y_2$ are evaluated by

$$
\begin{align*}
y_1 &= Y_1(x_{a1}, x, t_2) \\
y_2 &= Y_2(x_{a2}, x, t_1)
\end{align*}
$$

(2.13)

The individual discipline feasibility is always maintained during the optimization process, while the multidisciplinary feasibility is only guaranteed at optimization convergence.

2.6 OVERVIEW

Based on the GSE, the OLD methods linearize the coupled state parameters in the
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neighborhood of a feasible design, and search for the optimum solution along a piecewise path in the design variable space. Therefore, the global optimum solution is not assured. The efficiency is expected to be high, since they are gradient-based methods, while confined by the move limit. The infrastructure of the CO fits well in the framework of an engineering organization. However, the convergence is not guaranteed and it has difficulties when incorporating conventional optimization techniques due to its nature. The non-decomposition methods are not as efficient as the decomposition-based methods, while working well with conventional optimization techniques.

Like most currently available MDO methods, new MDO methods should be capable of effectively coordinating couplings among subsystems. It would be advantageous if new methods were capable of searching for the global optimum solution of an MDO problem with a short turn-around time. Since the SA/MDA is the biggest hurdle in complex MDO problems, a better design could result from more information about the SA/MDA.
CHAPTER 3

COLLABORATION MODEL

3.1 INTRODUCTION

MDO problems are dominated by couplings amongst state parameters. Effective and efficient collaboration between subsystems is always desirable when solving MDO problems. This chapter introduces a new Collaboration Model (CM) 4 for selecting feasible samples (generated in the design variable space) subject to the SA/MDA in a sampling process. A feasible sample subject to the SA/MDA means that the sample satisfies the simultaneous equations defined in equation (1.3). Aforementioned in Chapter 1, equation (1.3) is usually solved by an iterative procedure (such as the Gauss iterative method), when given a set of $x$, the initial guess of $y$, convergence criterion determined by a specified accuracy tolerance, and maximum number of iterations. The Collaboration

4 Hereafter, 'Collaboration Model' refers to the proposed collaboration model.
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Model is constructed by two mutually dependent approximations, which reflect the physical characteristics and mathematical dependency of the SA/MDA. A feasibility distribution for samples subject to the SA/MDA is plotted in terms of the interdisciplinary discrepancy / consistency value of samples based on the Collaboration Model. Given the feasibility distribution, samples are distinguished and feasible samples are chosen accordingly. The Collaboration Model serves a key role for sampling-based MDO methods. A Radial-basis Function, briefly reviewed in Section 3.3, is applied to implement the Collaboration Model.

3.2 COLLABORATION MODEL

Collaboration amongst subsystems (i.e., interdisciplinary consistency) in Linear Decomposition methods is maintained by the GSE. In the CO, coupled subsystems are relaxed by issuing some slack-variables from the system level to subsystems. Then the interdisciplinary consistency is pursued in the system level by extra equality constraints, which are the match between the targets issued from the system level and their corresponding values returned from subsystems.

In a sampling process, it is not certain if any set of design variables, \( x \), is feasible in terms of values of its state parameters governed by equation (1.3). The new Collaboration Model, which is expected to maintain the interdisciplinary consistency between coupled state parameters, is built for selecting feasible candidates from a sample pool, so as to optimize the system objective effectively. These selected samples are expected to produce feasible solutions of state parameters subject to the SA/MDA. So, the Collaboration
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Model can be viewed as a filter for a sampling process. The new Collaboration Model is formulated incorporating equations (1.1) and (1.3), and elaborated using a general two-state-parameter SA/MDA problem defined in equation (3.1) according to equation (1.1).

\[ y_1 = Y_1(x_1, x_{ct1}, y_2) \]
\[ y_2 = Y_2(x_2, x_{ct2}, y_1) \]  \hspace{1cm} (3.1)

In equation (3.1), \( y_1 \) is explicitly a function of \( x_{ct1} \), \( x_1 \), and \( y_2 \). The explicit expression reflects the physical relation between \( y_1 \) and its variables, i.e., \( x_{ct1}, x_1 \), and \( y_2 \). Meanwhile, \( y_1 \) is implicitly a function of \( x \), which is the union of \( x_1, x_2, x_{ct1} \) and \( x_{ct2} \). The situation is similar for \( y_2 \). Intrinsically, all state parameters, \( y \), are only implicitly affected by their associated design variables. The implicit function of state parameters uncovers mathematical dependencies between state parameters, \( y \), and design variables, \( x_{ct1}, x_1, x_{ct2} \), and \( x_2 \).

Recall equation (1.3). The proposed Collaboration Model is built based on two dependent approximations of coupled state parameters. One is the approximation of the implicit function and the other is for the explicit function. A Radial-basis Function (RBF) is applied to modeling the implicit and explicit approximations as follows:

\[ \bar{y}_i = \bar{Y}_i(\bar{x}_i), i = 1, ..., n \]  \hspace{1cm} (3.2)
\[ \bar{y}_i = \bar{Y}_i(x_i, x_{cti}, \bar{y}_{ct}), i = 1, ..., n \]  \hspace{1cm} (3.3)

where, \( \bar{x}_i \) includes all associated design variables of the state parameter \( y_i \) evaluated by
the approximate implicit function. In MDO problems where all state parameters are coupled with each other (in other words, values of \( y \) are evaluated simultaneously with a SA/MDA), \( \tilde{x}_i \) is same as \( x \) defined in equation (1.1). For example, \( \tilde{x}_i \) is the union of \( x_1 \), \( x_2 \), \( x_{cs1} \), and \( x_{cs2} \) in equation (3.1). In other situations where, at least, one state parameter is not coupled with any of other state parameters (e.g., \( y_{cl} \) is empty in the \( y_i \) function in equation (1.1)), and some or all of its design variables do not exist in other coupled state parameters’ function as well, \( \tilde{x}_i \) is a sub-set of \( x \) (Suppose that the formulation of MDO problems in equation (1.1) only has a set of nonlinear couplings, i.e., equation (1.3)). In general, the dependency analysis of state parameters is necessary to determine which state parameter should be in equation (1.3) and which design variable should be considered in the Collaboration Model. The value of \( y_i \) calculated by the approximate implicit function, i.e., equation (3.2), is represented by \( \bar{y}_i \), and sequentially, the approximate value of \( y_i \) marked as \( \bar{y}_i \) can be explicitly evaluated in equation (3.3) given \( x_i \), \( x_{cs} \) and \( \bar{y}_{cl} \). For a set of design variables, \( x \), the interdisciplinary consistency / discrepancy of state parameters can be determined by

\[
D = \sum_{i=1}^{n} |\bar{y}_i - \bar{y}_i| \quad (3.4)
\]

The Collaboration Model defined in equations (3.2)-(3.4) gives a distribution of the interdisciplinary discrepancy / consistency to a group of samples subject to the SA/MDA. It means that samples with a smaller value of \( D \) are more likely feasible subject to the SA/MDA than those with a larger value of \( D \). In other words, samples with a smaller
value of the interdisciplinary discrepancy more accurately maintain the nonlinear couplings amongst state parameters than those with a larger value of the interdisciplinary discrepancy.

The effectiveness of the Collaboration Model is shown by applying the Collaboration Model to a problem, which is the SA/MDA of Test Case 1 (in Section 4.6.1) defined by

\[
\begin{align*}
    y_1 &= x_1 + x_2 - 2 + (y_2 / 1.5)^4 \\
    y_2 &= x_3 + x_4 - 2 + (y_1 / 1.8)^4 \\
    \text{subject to: } &1 \leq x_i, x_j, x_k, x_l \leq 1.9
\end{align*}
\] (3.5)

In total, 100 samples generated through a random sampling in the design variable space, \((x_1, x_2, x_3, x_4)\), were applied for studying the effectiveness of the proposed Collaboration Model. Five experimental points, listed in Table 3-1, were randomly generated and used for the RBF approximations defined in equations (3.2) and (3.3). After applying the Collaboration Model for calculating the interdisciplinary discrepancy / consistency value, \(D\), of the 100 random samples, the distribution of \(D\) over the 100 samples is depicted in an ascending order in terms of the value of \(D\) in Figure 3-1. The real feasibility of the 100 samples subject to the SA/MDA in Figure 3-1 was determined based on evaluations of \(y\) of the 100 samples by calling the SA/MDA. Samples marked with ‘red dot’ signs are feasible, and samples plotted with “blue plus” signs are infeasible. As expected, the samples with a small value of \(D\) in Figure 3-1 are more possibly feasible subject to the SA/MDA than other samples with a large value of \(D\). The same study was also applied to cases with 500 and 1000 random samples, as shown in Figure 3-2, and Figure 3-3.
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Table 3-1 Experimental points for the RBF approximation

<table>
<thead>
<tr>
<th>Experiment</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_3$</th>
<th>$x_4$</th>
<th>$y_1$</th>
<th>$y_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.306581</td>
<td>1.398711</td>
<td>1.007338</td>
<td>1.048180</td>
<td>0.705300</td>
<td>0.079091</td>
</tr>
<tr>
<td>2</td>
<td>1.430430</td>
<td>1.504102</td>
<td>1.106891</td>
<td>1.172904</td>
<td>0.937613</td>
<td>0.353416</td>
</tr>
<tr>
<td>3</td>
<td>1.472354</td>
<td>1.571369</td>
<td>1.160585</td>
<td>1.242747</td>
<td>1.058492</td>
<td>0.522912</td>
</tr>
<tr>
<td>4</td>
<td>1.047014</td>
<td>1.668556</td>
<td>1.249554</td>
<td>1.318132</td>
<td>0.740550</td>
<td>0.596337</td>
</tr>
<tr>
<td>5</td>
<td>1.108217</td>
<td>1.734028</td>
<td>1.302309</td>
<td>1.383587</td>
<td>0.904644</td>
<td>0.749696</td>
</tr>
</tbody>
</table>

Figure 3-1 Distribution of $D$ - interdisciplinary consistency / discrepancy of 100 random samples
Figure 3-2  Distribution of $D$ - interdisciplinary consistency / discrepancy of 500 random samples
Chapter 3 – Collaboration Model

Figure 3-3  Distribution of $D$ - interdisciplinary consistency / discrepancy of 1000 random samples

Considering that the number of random samples is relatively large in practice, such as $10^4$, there could still be many feasible samples with a small $D$ to select for optimization based on the Collaboration Model. The effectiveness of the Collaboration Model was also intensively studied in reference [44]. It shows that the prediction of feasible samples subject to the SA/MDA based on the $D$ distribution given by the Collaboration Model becomes more accurate with more experimental points.
3.3 REVIEW OF RADIAL-BASIS FUNCTION

A Radial-basis Function can implement an \( R^m \rightarrow R^1 \) nonlinear mapping from a design variable space, \( x \), to a state parameter \( y_i \), where \( m \) is the dimensionality of design variables, \( x \).\(^5\) Radial-basis Functions were initially introduced in the solution of the real multivariate interpolation problem. From the perspective of neural networks, it involves three layers, which are the input layer corresponding to design variables, \( x \), the hidden layer executing a nonlinear mapping, and the linear output layer corresponding to state parameters \( y \). The nonlinear mapping becomes more accurate with a higher dimension of the hidden space [45]. In general, given a set of \( E \) different design variables \( \{ x^{(e)} \in R^m, e = 1, 2, ..., E \} \) and a corresponding set of \( E \) real numbers \( \{ y^{(e)}_i \in R^1 | e = 1, 2, ..., E \} \), the RBF can be expressed in the following form

\[
\tilde{y}_i(x) = \sum_{e=1}^{E} \alpha_e \varphi \left( \|x - x^{(e)}\| \right) \tag{3.6}
\]

where \( \tilde{y}_i(x^{(e)}) = y^{(e)}_i \), \( \{ \varphi (\|x - x^{(e)}\|), e = 1, 2, ..., E \} \) is a set of \( E \) functions, known as radial-basis functions, \( \|\cdot\| \) denotes a norm, which is usually Euclidean, and \( \alpha_i \) are unknown coefficients (weights) calculated by a set of simultaneous linear equations. The input data points, \( x^{(e)} \in R^m, e = 1, 2, ..., E \), are used as centers of the radial-basis functions.

According to Micchelli’s Theorem [45], as long as \( \{ x^{(e)} \}_{e=1}^{E} \) is a set of distinct points,

\(^5\) \( x \) represents variables of a state parameter, such as \( y_i \).
equation (3.6) is not singular in the solution for $\alpha_i$. There is a large class of radial-basis functions including the multiquadrics, inverse multiquadrics, Gaussian functions, thin-plate spline, biharmonic spline, etc. For simplicity, the biharmonic spline Radial-basis Function is chosen in this thesis as follows:

$$\tilde{y}_i(x) = \sum_{e=1}^{E} \alpha_i \|x - x^{(e)}\|$$  \hspace{1cm} (3.7)

The reason to choose the RBF is because the RBF approximation passes through all experimental points and gives a good accuracy around the experimental points. Also, it is easy to implement. The biharmonic spline Radial-basis Function is applied to equations (3.2) and (3.3) for implementing the Collaboration Model.

### 3.4 SUMMARY

The Collaboration Model constructed by two mutually dependent approximations reflects both physical and mathematical characteristics of nonlinear couplings. It gives a reasonable distribution (interdisciplinary discrepancy / consistency distribution - $D$) over samples in terms of their feasibility subject to the SA/MDA. This feature distinguishes samples from the point of view of the SA/MDA. The Collaboration Model works effectively to filter out infeasible samples subject to the SA/MDA, and in turn, efficiently coordinate coupled disciplines. Given the $D$ distribution, feasible samples subject to the SA/MDA can be selected. For example, a sampling-based global optimizer, called the
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Mode-pursuing Sampling (MPS) method [8], is used to select feasible samples based on the $D$ distribution for optimization. More details about using the MPS in new MDO methodology developments are introduced in Chapter 5. In general, the Collaboration Model serves as a key role for checking the feasibility of samples subject to the SA/MDA in a sampling-based optimization process when solving MDO problems. Its application is elaborated in the following newly developed MDO methods.
CHAPTER 4

BOUNDARY SEARCH AND SIMPLEX DECOMPOSITION METHOD

4.1 INTRODUCTION

This chapter presents a new methodology to MDO problems based on the Collaboration Model proposed in Chapter 3. The author’s proposed method, named the Boundary Search and Simplex Decomposition Method (BSSDM), geometrically depicts the relation between coupled state parameters by exploring the region formed by state parameters, which is called the feasible state parameter region in this thesis [46]. The state parameter region can be further decomposed into simplexes. Given the decomposed state parameter region, the relation between coupled state parameters becomes explicit when optimizing the system objective function. Geometrically, the feasible state parameter region can be classified into a convex region or a non-convex region. Three
boundary search strategies were developed to search for boundary points of the feasible state parameter region. Given the boundary points, a robust simplex decomposition algorithm for convex regions is applied and was tested by a numerical case. A decomposition algorithm for convex-like regions was also proposed and tested by a numerical case. All results were validated with exhaustive enumerations and show promising capabilities of the BSSDM for solving MDO problems. The BSSDM was implemented using MATLAB® 6.0 (reference [47]). Section 4.2 addresses the motivation of the BSSDM. Sections 4.3 - 4.5 explain the boundary search strategies, decomposition methods, and the framework of the BSSDM, respectively. Test cases and discussion are reported in Section 4.6 at the end of Chapter 4.

4.2 MOTIVATION

The Boundary Search and Simplex Decomposition Method (BSSDM) was motivated by the nature of MDO problems, which is the dominant couplings between state parameters. The relation between coupled state parameters is not a one-to-one mapping. Re-write the formulation of the two-state-parameter SA/MDA problem defined in equation (3.1) as follows:

\[ y_1 = Y_1(x_1, x_{a1}, y_2) \]
\[ y_2 = Y_2(x_2, x_{a2}, y_1) \]  

(4.1)

Given the value of \( y_1 \) in equation (4.1), the value of \( y_2 \) can vary within a certain range due to many feasible combinations of \( x_2 \) and \( x_{a2} \). This reality indicates that one state
Chapter 4 – Boundary Search and Simplex Decomposition Method

parameter cannot be approximated with a simple fitting function (such as quadratic fitting) of its own variables. In other words, such a fitting function only partly captures the whole relation between state parameters. The relation between \( n \) coupled state parameters can be described with an \( n \)-D feasible state parameter region. Geometrically, the feasible state parameter region can be further classified into convex and non-convex types. Correspondingly, the SA/MDA can be divided into convex and non-convex problems.

The SA/MDA can be also viewed as possessing difficult constraints. The optimal solution is usually located at the constraint boundary for constrained optimization problems. Therefore, it would be beneficial to know the feasible boundary of state parameters for solving MDO problems.

The above analyses would provide useful new insights, particularly in regards to a better understanding of the solution of the SA/MDA represented by a feasible state parameter region. In the BSSDM, the feasible state parameter region is approximated based on boundary points, which are iteratively explored by a boundary search strategy through a sampling process based on the Collaboration Model. The approximated feasible state parameter region can be further decomposed into simplexes to show the relation between state parameters explicitly and geometrically. The BSSDM gives a different approach of depicting and obtaining the solution of the SA/MDA.

The BSSDM focuses on certain MDO problems, in which the system objective function, \( f \), is a function of only state parameters, \( y \), and its own system design variables, \( x \). The MDO formulation required by the BSSDM is defined by
Chapter 4 – Boundary Search and Simplex Decomposition Method

\[
\begin{align*}
\min_{x,y} f(x, y) \\
\text{subject to: } y_i = Y_i(x_i, y_i), i = 1, ..., n \\
g_c = G_c(x, y) \leq 0 \\
g_s = G_s(x, y) \leq 0
\end{align*}
\] (4.2)

where \(g_c\) is a vector of disciplinary constraints; \(g_s\) is a vector of system constraints; \(G_c\) is a vector of disciplinary constraint functions associated with \(g_c\); \(G_s\) is a vector of system constraint functions associated with \(g_s\); and \(x\) is the union of \(x_i\) \((i = 1, ..., n)\). The only difference between equation (4.2) and equation (1.1) is that the MDO formulation defined in equation (4.2) does not involve the system-subsystem variables, \(x_{cs}\). The specified formulation defined in equation (4.2) might not work with some MDO problems. Thus, it is a limitation of the BSSDM in terms of the applicability to MDO problems. When the relation between coupled state parameters is explicitly available, optimizing the system objective function does not need to call disciplinary analyses based on the formulation defined in equation (4.2). This formulation sub-divides an MDO problem into three levels, which are the system optimization level, the state parameter level, and the disciplinary analysis level. In equation (4.2), having understood the relation between state parameters can eliminate the burden of calling the SA/MDA in the system optimization level. Also, the problem given by equation (4.2) is assumed to have a convex or convex-like state parameter region in the current development stage of the BSSDM. In general, it is difficult to define a convex-like region in a mathematical way. Some features of a convex-like region is discussed in Section 4.4. In this thesis, the term, ‘convex region’, is equivalent to the term, ‘convex set.’ Boundary search strategies are introduced in the next section.
Chapter 4 – Boundary Search and Simplex Decomposition Method

4.3 BOUNDARY SEARCH STRATEGY

The boundary search strategy is to find some boundary points of a feasible state parameter region, based on which the real state parameter region can be approximated. Three boundary search strategies, which are the Stair-climbing Strategy, Strip-measure Strategy, and Explosion Strategy, were developed and discussed. After comparing their efficiency and applicability, the Explosion Strategy is applied in the framework of the BSSDM. For illustration convenience, the SA/MDA in equation (4.2) can be expressed by

\[
\begin{align*}
    y_1 &= Y_1(x_1, y_{cl}) \\
    \vdots \\
    y_i &= Y_i(x_1, y_{cl}) \\
    \vdots \\
    y_n &= Y_n(x_n, y_{cl})
\end{align*}
\]

Equation (4.3) is used for elaborating the proposed search strategies.

4.3.1 Stair-climbing Strategy

Assume that subsystems do not share common design variables, i.e., \( x_i \cap x_j = \emptyset \) (\( i \neq j \)); and do not have system-subsystem design variables, \( x_{cs} \), either. Given a feasible design of state parameters, \( y^d \), by calling the SA/MDA, boundary points of a convex state parameter region can be obtained along each state parameter direction by fixing values of the remaining state parameters at the feasible design point, \( y^d \). A two-state-parameter case is
Chapter 4 – Boundary Search and Simplex Decomposition Method

shown in Figure 4-1. In each subsystem defined in equation (4.3), one state parameter, $y_i$, can be maximized and minimized when given the other state parameters by

$$\max_{x_j} y_{ij} = Y_j(x_j, y_{ij}^{d}) \quad \text{and} \quad \min_{x_j} y_{ij} = Y_j(x_j, y_{ij}^{d})$$

subject to: $g_{cj} = G_{cj}(x_j, y_{ij}, y_{ij}^{d}) \leq 0$  \hspace{1cm} (4.4)

where $y_{ij}$ means the optimum value of $y_i$ output from subsystem $j$, $y_{ij}^{d}$ is a sub-set of the union of $y_{cj}$ and $y_j$ (excluding $y_i$) at the feasible design point, $y^{d}$, and $g_{cj}$ is a sub-set of $g_c$. The feasible range of $y_i$ is determined by the intersection of the ranges of $y_i$ evaluated from all subsystems as follows:

$$\max(y_i) = \min(\max(y_{i1}), ..., \max(y_{in}))$$
$$\min(y_i) = \max(\min(y_{i1}), ..., \min(y_{in}))$$ \hspace{1cm} (4.5)

Besides the limitation of the design variable allocation, i.e., $x_1 \cap x_2 = \emptyset$, the Stair-climbing Strategy also requires that each subsystem should be formulated explicitly. This situation rarely exists in engineering problems. Solving equation (4.4) for all state parameters could be very time-consuming when MDO problems involve computationally expensive processes. Each subsystem has to run $2n_i$ optimization processes, where $n_i$ is the number of state parameters involved in subsystem $i$. For high dimensional problems, the search process becomes very complicated. Therefore, the Stair-climbing Strategy is not suited in practice.
4.3.2 Strip-measure Strategy

Imagine that a feasible convex state parameter region can be enclosed by a hyper-rectangle based on a rough guess of the range of state parameters. Then the shape of the feasible convex state parameter region can be determined by measuring the length of strips (analogous to scales), which are positioned along one state parameter direction, $y_i$, and specified with a preset width of $y_j$ ($j = 1, ..., n, j \neq i$) on the hyper-rectangle. The search process for a two-state-parameter case is shown in Figure 4-2. Using approximation techniques, such as the RBF, the value of the state parameters of a large group of samples (generated by a random sampling) can be approximated. Then, the samples are allocated into positioned strips based on approximated state parameters’ value. The length of a positioned strip can be determined in equation (4.6) by choosing samples (in the strip) having the maximum and minimum values of $y_i$. 

Figure 4-1  Stair-climbing Strategy implemented in a 2-D state parameter region
Chapter 4 – Boundary Search and Simplex Decomposition Method

\[
\begin{align*}
\max y_{im} \text{ and } \min y_{m} \\
\text{subject to: } y_{jLm} &\leq y_j \leq y_{jUm}, \ j = 1, \ldots, n, \ f \neq i \\
m & = 1, \ldots, M
\end{align*}
\]  

(4.6)

where M is the number of positioned strips, \( y_{jLm} \) and \( y_{jUm} \) are the lower and upper bounds of \( y_j \), respectively.

In the Strip-measure Strategy, the range of state parameters could be difficult to guess before starting. Strips could be positioned outside the feasible region. The number of applied strips, M, is not certain and could be problem-dependent. Also, this strategy is difficult to organize for high dimensional cases.

Figure 4-2  Strip-measure Strategy applied in a 2-D state parameter region
4.3.3 Explosion Strategy

Literally, the Explosion Strategy is analogous to an explosion process. The search procedure is to explore the furthest point with respect to the geometric center along different directions based on some initial feasible experimental points in the state parameter space.

Given $K$ feasible state parameter points in the state parameter space, e.g., points marked with '■' signs in a two state-parameter case in Figure 4-3, the geometric center $ce$ is defined by

$$y_{ice} = \frac{\max(y_{ik}, k = 1, \ldots, K) + \min(y_{ik}, k = 1, \ldots, K)}{2}, i = 1, \ldots, n$$ (4.7)

First, the state parameter space can be divided into quadrants with respect to the geometric center, $ce$. For example, four quadrants, in a 2-D space, can be defined by

Quadrant 1: $y_1 \geq y_{1ce}$ and $y_2 \geq y_{2ce}$
Quadrant 2: $y_1 \leq y_{1ce}$ and $y_2 \geq y_{2ce}$
Quadrant 3: $y_1 \leq y_{1ce}$ and $y_2 \leq y_{2ce}$
Quadrant 4: $y_1 \geq y_{1ce}$ and $y_2 \leq y_{2ce}$ (4.8)

The number of quadrants, $Q$, is equal to $2^n$. Samples (generated in the design variable space based on a random sampling) can be allocated into their corresponding quadrants based on their approximated state parameters' value by equation (3.2). Then, in each
quadrant, the feasible boundary is pursued by searching for a sample, from samples in the quadrant, with the longest Euclidean distance with respect to the geometric center, \( ce \). Therefore, the objective function implemented with the RBF approximation is defined by

\[
\max \left( L_1, ..., L_s, ..., L_{P_q} \right) \tag{4.9}
\]

where \( P_q \) is the number of samples in quadrant \( q \), and \( L_s \) denotes the Euclidean distance between sample \( s \) and \( ce \), and is defined by

\[
L_s = \sqrt{\sum_{i=1}^{n} (\bar{y}_i - \bar{y}_{ice})^2} \tag{4.10}
\]

where \( \bar{y}_i \) is the approximate value of \( y_i \) calculated by equation (3.2). Meanwhile, the center \( ce \) is dynamically refreshed in the next iteration with more feasible state parameter points. Finally, the furthest vertex in each quadrant is found, e.g. points marked with '●' signs in Figure 4-3. These vertices and some other feasible design points are considered to be boundary point candidates and connected by facets to build up a convex hull for domain decomposition. Compared with the other two strategies, the Explosion Strategy is more robust and applicable for high dimensional problems. Also, it is easy to implement.

Having boundary points given by the Explosion Strategy, the domain decomposition process is carried out in the next section.
4.4 SIMPLEX DECOMPOSITION METHOD

Suppose that equation (4.2) has a feasible convex state parameter region. Given boundary points of the feasible convex state parameter region, the boundary of the feasible region can be tessellated into facets. Each facet consists of $n$ state parameter points. In a 2-D state parameter space ($n = 2$ in equation (4.2)), a facet is a straight line segment, while in a 3-D space, it is a triangle. A facet and an interior point in the feasible region construct a simplex, which is a triangle in the 2-D space and a tetrahedron in the 3-D space. In this way, a convex region can be uniquely decomposed into simplexes centered about a common interior point in the feasible region. Geometrically, each simplex is confined by its facets. In an $n$-D space, a facet can be described by
where \( a_i \) \((i = 1, \ldots, n+1)\) can be uniquely determined by \( n \) state parameter points. For example, in Figure 4-4, facet \( AB \) can be defined by state parameter points \( A \) and \( B \). For defining a simplex, the equal sign in equation (4.11) is replaced by either \( \leq \) or \( \geq \) when substituting one remaining vertex in the simplex into a facet, which does not pass through the remaining vertex, e.g., substitution of Point \( C \) into facet \( AB \) in Figure 4-4. Eventually, in the \( n \)-D state parameter space, a simplex can be defined with \( n+1 \) linear constraints by

\[
a_{i,1}y_1 + a_{i,2}y_2 + \ldots + a_{i,n}y_n \leq \text{or} \geq a_{i,n+1}
\]

\[
\ldots
\]

\[
a_{i,n+1}y_1 + a_{i,n+2}y_2 + \ldots + a_{i,n+\ldots,n+1}y_n \leq \text{or} \geq a_{i,n+1,n+1}
\]

The boundary points can be found by the Quickhull algorithm from all real state parameter points prepared by the Explosion Strategy. The Quickhull algorithm also connects all boundary points to generate facets enclosing the smallest convex region that contains all real state parameter points [48]. This process was implemented using a function named \textit{convhulln} in MATLAB\textsuperscript{®} 6.0 (reference [47]). The \textit{convhulln} function returns the indices of the boundary points in a data set that comprises of the facets of the convex hull for the set.
Due to the complexity of a general non-convex region, this thesis has no intention to handle a general non-convex region. For certain situations, namely convex-like regions, a convex-like decomposition algorithm was proposed and studied in Test Case 2 in Section 4.6.2. A general convex-like region should have the following features:

1) There are no other enclosed sub-regions within the feasible region.
2) Most boundary segments should be convex with respect to the center of the feasible region defined in equation (4.7).
3) The center of the convex-like region should be located inside the feasible region.

Other sufficient conditions need to be developed to rigorously define a convex-like region in the future. A convex-like region case is shown in Figure 4-5. The pseudo-code of the proposed convex-like decomposition algorithm is elaborated incorporating Figure 4-5 as follows:
Chapter 4 – Boundary Search and Simplex Decomposition Method

1) Given all current available boundary points, e.g., Points A, B, C, D, E and F, apply the Quickhull algorithm to construct a convex hull and decompose the region into simplexes, such as $S_1$.

2) Find a boundary point not listed on any facet given by step 1), e.g., Point C. Locate this point in one of the current simplexes based on equation (4.12), i.e., $S_1$. Break up this simplex and reconstruct it with new facets consisting of any $n-1$ vertices of $S_1$ (excluding the interior vertex), i.e., Point B or D, and this boundary point, i.e., Point C. This process can be easily implemented by the Quickhull algorithm when given Points B, C, and D and the interior point. Update all simplexes and repeat step 2) until all boundary points are listed on one of the facets.

By now, with all required techniques in the BSSDM, the architecture of the BSSDM is introduced in the next section.

Figure 4-5 Illustration of the convex-like decomposition algorithm for a 2-D state parameter case
4.5 **BOUNDARY SEARCH AND SIMPLEX DECOMPOSITION METHOD**

As shown in Figure 4-6, the BSSDM consists of 4 parts, which are the Boundary Search Process (BSP), Domain Decomposition, System Optimization and Boundary Refining.

The BSSDM starts with the Boundary Search Process (BSP). In the BSP, first, an initialization procedure creates some initial experimental points through a random sampling process, based on which the RBF approximation can be implemented in each discipline. Then a larger number of samples, e.g., \( p = 10^4 \), are randomly generated in the design variable space, \([x_{lb}, x_{ub}]\). The RBF is applied to fit samples in both explicit and implicit ways based on the Collaboration Model defined in equations (3.2) and (3.3) as follows:

\[
\bar{Y}_{q,i} = \bar{Y}_i(\bar{x}_{q,i}), \quad i = 1, ..., n, \quad q = 1, ..., p
\]  
(4.13)

\[
\bar{Y}_{q,i} = \bar{Y}_i(x_{q,i}, \bar{y}_{q,i}), \quad i = 1, ..., n, \quad q = 1, ..., p
\]  
(4.14)

The interdisciplinary discrepancy / consistency for each sample is evaluated by

\[
D_q = \sum_{i=1}^n |\bar{y}_{q,i} - \bar{y}_{q,i}|, \quad q = 1, ..., p
\]  
(4.15)

Based on the RBF approximation models, the constraint check is implemented by
Chapter 4 – Boundary Search and Simplex Decomposition Method

\[
\begin{align*}
\bar{g}_{c,q} &= \bar{G}(x_q, \bar{y}_q), q = 1, ..., p \\
\bar{g}_{c,q} &= \bar{G}(x_q, \bar{y}_q), q = 1, ..., p
\end{align*}
\] (4.16) (4.17)

Samples violating constraints based on equations (4.16) and (4.17) are identified by adding a big penalty value, such as $10^5$, to their $D$ values so that they can be judged as infeasible points, regardless if they are infeasible subject to the SA/MDA. This is a conservative way to ensure that selected samples are feasible. Consequently, the discrepancy check takes place to filter out ‘bad’ samples according to a preset threshold of the interdisciplinary discrepancy, $D$, e.g., 1000. The remaining samples, e.g., $p_1$ samples, are allocated into each quadrant defined by equations (4.7) and (4.8). Finally, a sample with the biggest Euclidean distance defined in equation (4.9) amongst samples allocated in each quadrant is chosen as one of experimental points for the next Boundary Search (BS) iteration, as shown in Figure 4-6. These chosen points are evaluated by calling the SA/MDA to verify their feasibility (therefore, one experimental point refers to one call to the SA/MDA), and saved, as state parameter points, in the database of experiments (if feasible subject to the SA/MDA). Given all currently available feasible state parameter points (or experimental points), the Boundary Search Process stops if the position of the geometric center of the explored state parameter region stays stationary according to a movement criterion over a certain number of consecutive BS iterations, e.g., 2. The movement criterion of $ce$ is defined by

---

6 The value of a threshold depends on the range of $D$ of solved problems. In general, a small threshold results in a conservative filtering process; a large one could have more infeasible samples selected.
Chapter 4 – Boundary Search and Simplex Decomposition Method

\[
\sqrt{\sum_{i=1}^{n} (y_{ice}^{(t)} - y_{ice}^{(t+1)})^2} \leq 1e^{-3}
\]  

(4.18)

or

\[
|ce^{(t)} - ce^{(t+1)}| \leq 1e^{-3}
\]  

(4.19)

where \( t \) indicates the \( t \)th BS iteration.

Given the vertices representing the longest Euclidean distance in each quadrant and all other real experimental points over one BSP, a convex hull can be constructed by applying the Quickhull method in the Domain Decomposition module. Also a non-convex region check follows. As a result, geometrically, the feasible state parameter region is decomposed into simplexes confined by linear constraints defined by equation (4.12).

Since the relation between the state parameters can be explicitly described by the simplexes, the optimization of the system objective function, \( f \), within each simplex, can be easily implemented in the System Optimization module by

\[
\min_{x,y} f(x,y)
\]

subject to:

\[
a_{1,1}y_1 + a_{1,2}y_2 + \ldots + a_{n,1}y_n \leq \text{or} \geq a_{n+1,1} \\
\]

\[
\ldots \\
\]

\[
a_{1,n+1}y_1 + a_{2,n+1}y_2 + \ldots + a_{n,n+1}y_n \leq \text{or} \geq a_{n+1,n+1} \\
g_s = G_s(x,y) \leq 0
\]

(4.20)
Chapter 4 – Boundary Search and Simplex Decomposition Method

As a result, the System Optimization module outputs optimum values, i.e., \( y^*, f^* \), and \( x_i^* \).

The best value of the system objective function can be improved by continually exploring unknown regions between current facets and the real boundary. The Boundary Refining module geometrically defines the unknown regions based on all current facets so as to allocate random samples into the unknown regions in terms of their approximated state parameters in the next BSP. In the BSSDM, it is suggested that the unknown region, as shown by the cross-hatched area in Figure 4-4, enclosed by a facet belonging to the simplex owing the current best value of \( f \) and its corresponding real boundary segment, is worthy of further investigation. This is because that the real optimum solution might be in the small neighborhood of the current best solution in the state parameter space for some continuum problems. Generally, all unknown regions should be further explored by applying the Boundary Refining module to run another BSP, as shown in Figure 4-6, until a satisfactory solution is found.

Given \( y^* \) and \( f^* \), the optimum value of disciplinary design variables, \( x^* \), can be traced back by running an optimization process in the Disciplinary Optimization module as follows:

\[
\min_{x} S = \sum_{i=1}^{n} (y_i^* - y_i)^2
\]

subject to: \( y_i = Y_i(x_i, y_i^*), i = 1, ..., n \)

\[
g_c = G_c(x, y^*) \leq 0
\]

(4.21)
Eventually, the optimum solution is given as \( y^*, x^*, x_s^*, \) and \( f^* \). The flow chart of the BSSDM is shown in Figure 4-7.
Figure 4-7  Flow chart of the Boundary Search and Simplex Decomposition Method
4.6 TEST PROBLEMS AND DISCUSSION

The BSSDM was tested by two cases. The author formulated the first numerical test case by placing two coupled state parameters, as variables, into the well-known six-hump camel-back problem. Two BSPs were implemented in Test Case 1. Test Case 2 is a problem of the SA/MDA cited from reference [49] to demonstrate the ability of the BSSDM in dealing with a convex-like state parameter region. One BSP was implemented in this problem. Both cases were validated by exhaustive enumerations. The convergence tolerance applied in a function named ‘fsolve’ in MATLAB® 6.0 (reference [47]) for solving the SA/MDA is a default value set to $10^{-4}$.

4.6.1 Test Case 1

$$\min f = 4y_1^2 - 2.1y_1^4 + \frac{y_1^6}{3} + y_1y_2 - 4y_2^2 + 4y_2^4$$

subject to: $y_1 = x_1 + x_2 - 2 + (y_2/1.5)^4$

$y_2 = x_3 + x_4 - 2 + (y_1/1.8)^4$

$y_1 \leq 1.0$

$y_2 \leq 1.0$

$1 \leq x_1, x_2, x_3, x_4 \leq 1.9$  

(4.22)

The BSSDM started with 5 initial feasible experimental points through a random sampling process by calling the SA/MDA. In total, 10 runs were carried out for Test Case 1 to show the robustness of the BSSDM. Results of Test Case 1 given by the BSSDM are shown in Table 4-1. According to the convergence criterion of the BSSDM, Runs 1-5, 6-7,
and 8-10 were stopped over 2, 3, and 4 consecutive BS iterations, respectively, without further movement of the geometric center, $ce$, during the 2nd BSP. All runs applied 2 consecutive BS iterations as the convergence criterion during the 1st BSP. In Table 4-1, each BSP output one solution based on the best experiment; one intermediate solution was calculated by optimizing $f$ in simplexes at the end of the 1st BSP; and the final optimum solution was evaluated by a trace-back process given $y^*$ after 2 BSPs. For Run 1, the complete process information of solving Test Case 1 is depicted in Figure 4-8, which shows all experimental points for exploring the boundary, the real state parameter region given from an exhaustive enumeration, the boundary of the feasible state parameter region approximated by the BSSDM, and the contour of $f$. Also, the cumulative number of the SA/MDA and the convergence curve of each BSP are shown in Figure 4-9 and Figure 4-10, respectively. The optimal value given by the BSSDM is very close to the real optimum according to the contour of $f$. Besides the optimum solution of $f$, the BSSDM almost gives a full geometric description of the relation between coupled state parameters rather than some experimental points. Understanding the relation between state parameters can help robust design.
## Table 4-1  Results of Test Case 1

<table>
<thead>
<tr>
<th>No. of Run</th>
<th># of SA/MDA Initial</th>
<th>Infeasible After Int.</th>
<th>1st Boundary Search Process</th>
<th>Solutions</th>
<th>2nd Boundary Search Process</th>
<th>Optimum Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>55</td>
<td>10</td>
<td>Best Experiment</td>
<td>Intermediate</td>
<td>Best Experiment</td>
<td>Optimun Solution</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$y_1 = 0.194433$, $y_2 = 0.934829$</td>
<td>$y_1 = 0.145198$, $y_2 = 0.678303$</td>
<td>$y_1 = 0.069245$, $y_2 = 0.678303$</td>
<td>$x_1 = 1.025010$, $x_2 = 1.436627$, $x_3 = 1.498066$</td>
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<td></td>
<td></td>
<td></td>
<td>$f = -0.110784$</td>
<td>$f = -0.927530$</td>
<td>$f = -0.927530$</td>
<td>$x_4 = 1.667369$, $x_5 = 1.339150$</td>
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<td>42</td>
<td>8</td>
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<td>Intermediate</td>
<td>Best Experiment</td>
<td>Optimun Solution</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
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<td>$y_1 = 0.153910$, $y_2 = 0.739779$</td>
<td>$y_1 = 0.069443$, $y_2 = 0.685478$</td>
<td>$x_1 = 1.028258$, $x_2 = 1.407857$, $x_3 = 1.510113$</td>
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<td></td>
<td></td>
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<td>$f = -0.227642$</td>
<td>$f = -0.905514$</td>
<td>$f = -0.929529$</td>
<td>$x_4 = 1.336352$, $x_5 = 1.342738$</td>
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<td>16</td>
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<td>Intermediate</td>
<td>Best Experiment</td>
<td>Optimun Solution</td>
</tr>
<tr>
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<td>$f = -0.852553$</td>
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<td>Intermediate</td>
<td>Best Experiment</td>
<td>Optimun Solution</td>
</tr>
<tr>
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<td></td>
<td></td>
<td>$y_1 = 0.160730$, $y_2 = 0.921624$</td>
<td>$y_1 = 0.139206$, $y_2 = 0.673956$</td>
<td>$y_1 = 0.095530$, $y_2 = 0.675328$</td>
<td>$x_1 = 1.004504$, $x_2 = 1.687412$, $x_4 = 1.336688$</td>
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<td>$f = -0.891423$</td>
<td>$f = -0.891423$</td>
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<td>47</td>
<td>15</td>
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<td>Intermediate</td>
<td>Best Experiment</td>
<td>Optimun Solution</td>
</tr>
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<td></td>
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<td>17</td>
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<td>Intermediate</td>
<td>Best Experiment</td>
<td>Optimun Solution</td>
</tr>
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<td></td>
<td></td>
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<td>$y_1 = 0.139213$, $y_2 = 0.072176$</td>
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<td>48</td>
<td>4</td>
<td>Best Experiment</td>
<td>Intermediate</td>
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<td>Optimun Solution</td>
</tr>
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<td></td>
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<td>82</td>
<td>6</td>
<td>Best Experiment</td>
<td>Intermediate</td>
<td>Best Experiment</td>
<td>Optimun Solution</td>
</tr>
<tr>
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<td>Intermediate</td>
<td>Best Experiment</td>
<td>Optimun Solution</td>
</tr>
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<td>65</td>
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<td>Intermediate</td>
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<td>Optimun Solution</td>
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<td>$f = -0.892977$</td>
<td>$f = -0.892977$</td>
<td>$x_4 = 1.335858$</td>
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</tbody>
</table>

7 F represents `Feasible`
Figure 4-8  Complete information of the results of Run No. 1 in Test Case 1 given by the BSSDM
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Figure 4-9  Cumulative number of the SA/MDA over 16 BS iterations of Run No. 1 in Test Case 1
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Figure 4-10  History of the convergence criterion value over 16 BS iterations of Run No. 1 in Test Case 1

4.6.2 Test Case 2

\[ y_1 = x_1^2 + x_2 + x_3 - 4 - 0.2y_2 \]
\[ y_2 = x_1 + x_3 - 2 + \sqrt{y_1} \]

subject to: \[ 0 \leq x_1 \leq 7 \]
\[ 2 \leq x_2 \leq 7 \]
\[ 2 \leq x_3 \leq 7 \]
\[ y_1 / 8 - 1 \geq 0 \]
\[ 1 - y_2 / 10 \geq 0 \]
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Test Case 2 is part of a problem solved in reference [49]. The complete solution of $y$, which is given by an exhaustive enumeration and depicted by a feasible convex-like state parameter region, is shown in Figure 4-11. The solution of Test Case 2 given by the BSSDM, depicted in Figure 4-12 and Figure 4-13, takes 46 feasible experimental points and 21 infeasible experimental points including 11 infeasible experimental points for initialization. The number of initial feasible experimental points is 5. The BSSDM stopped after 2 consecutive BS iterations without further movement of the geometric center, $ce$. Only one BSP was implemented. The cumulative number of the SA/MDA and the convergence curve are shown in Figure 4-14 and Figure 4-15, respectively. The result showed a good approximation of the feasible convex-like state parameter region.
Figure 4-11. Full solution of $y$ given by an exhaustive enumeration.
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Figure 4-12 Complete information of the results of Test Case 2 given by the BSSDM
Figure 4-13  Zoomed-in figure of the complete information of the results of Test Case 2

given by the BSSDM
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![Graph]

Figure 4-14  Cumulative number of the SA/MDA over 14 BS iterations of Test Case 2
4.6.3 Discussion

The BSSDM can robustly solve MDO problems with a feasible convex state parameter region. When the feasible state parameter region is known, the rest of the work is inexpensive. The optimization process can be readily implemented by conventional optimization techniques, such as a gradient-based algorithm. Also, the feasible state parameter region shows a 'global' picture of the relation between state parameters. This selectable and preserved information gives designers the freedom to choose a robust
design. However, it currently can not deal with a generic non-convex region. The proposed convex-like decomposition shows possibilities that a generic non-convex region can be further decomposed into local convex sub-regions. Yet this still relies on future research.

It was observed that the Collaboration Model was effective for coordinating coupled subsystems. The infrastructure of the BSSDM is inheritable since all experimental points are used to improve the accuracy of the Collaboration Model implemented with the RBF. Each call to the SA/MDA regardless of its feasibility subject to constraints benefits the RBF approximation.

For a pure convex state parameter region problem, a global optimization method is not necessary to explore the boundary in terms of the Euclidean distance. On the other hand, the Boundary Search Process can quickly converge. This reality could compensate the total computational cost for boundary exploration, which exponentially increases when the dimensionality of state parameters increases. However, compared with the number of disciplinary design variables, $x$, the number of state parameters, $n$, could be relatively small.

As reliability assessment also involves identification of complex integration regions [50]-[52], the BSSDM has potential to be applied to reliability assessment problems to depict a safe region corresponding to a specified performance function.
4.7 OVERVIEW

This chapter has shown new developments of the concept of a feasible state parameter region (further classified into the convex and non-convex regions), which geometrically depicts the complex couplings between subsystems (state parameters). Having the information of the feasible state parameter region benefit the design process. The BSSDM developed based on the Collaboration Model shows a different approach to analyze MDO problems, particularly for the SA/MDA. To the best of the author’s knowledge, it is the first time that MDO problems have been classified and solved in the context of the convex and non-convex state parameter regions in the MDO community. Due to its region exploration, the BSSDM has potential to be applied in other areas, such as reliability engineering. At the current stage, the BSSDM requires a specified MDO formation and is constrained by solving MDO problems with a convex or convex-like state parameter region. The newly developed BSSDM is considered to provide a solid basis for further research dealing with more generic MDO problems. In the next chapter, a new MDO method named the Collaboration Pursuing Method for solving a general MDO problem is introduced.
CHAPTER 5

COLLABORATION PURSUING METHOD

5.1 INTRODUCTION

This chapter introduces a new sampling-based MDO method called the Collaboration Pursuing Method (CPM) (reference [53]) to solve a general MDO problem defined in equation (1.1). The Collaboration Model developed in Chapter 3 is used in the CPM to coordinate coupled disciplines. Motivated by the Mode-pursuing Sampling (MPS) method [8], the CPM treats an MDO problem as a black-box function, instead of decoupling the SA/MDA explicitly. The MPS is used as a global optimizer in the CPM. An adaptive sampling method was developed and applied in the CPM to speed up the optimization process and enhance the solution accuracy for local optimum solutions. A

\footnote{\[x_i\] is not considered in the framework of the CPM since all test cases and applications do not involve \(x_i\). However, \(x_i\) can be included easily in practice because it is independent of \(x\) and not shared by any subsystem.}

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strategy was developed for controlling the number of active design variables when solving large-scale MDO problems. A discrete sampling method was developed for allowing the CPM to solve MDO problems with continuous design variables, discrete design variables, or hybrid design variables. The proposed CPM was implemented using MATLAB® 6.0 (reference [47]) and successfully applied to solve four test problems, including an engineering design application. Section 5.2 first reviews the MPS method; then the CPM is elaborated in Section 5.3; finally, Section 5.4 shows the result of test cases and discussions.

5.2 REVIEW OF THE MODE-PURSUEING SAMPLING METHOD

In the Collaboration Pursuing Method (CPM), the Mode-pursuing Sampling (MPS) method is applied as a global optimizer module. The MPS method was developed as a method to search for the global optimum of a black-box function. It is a discriminative sampling method that generates more sample points around the current minimum than other areas in the design space, while statistically covering the entire design space [8]. The main procedure of the MPS can be elaborated when solving equation (5.1) (the general formulation of an optimization problem) as follows:

\[
\begin{align*}
\min_{x} & \quad f(x) \\
\text{subject to:} & \quad g(x) \leq 0
\end{align*}
\]  

(5.1)

1. Generate \(k_i\) initial experimental points through a random sampling process in the
design variable space: $x^{(1)}, x^{(2)}, \ldots, x^{(ki)}$, and evaluate their corresponding objective function values of $f(x^{(1)}), f(x^{(2)}), \ldots, f(x^{(ki)})$. The objective function, $f$, in equation (5.1) is approximated with a Radial-basis Function in equation (5.2) based on all currently available function values, i.e., $f(x^{(1)}), f(x^{(2)}), \ldots, f(x^{(ki)})$.

$$\hat{f}(\mathbf{x}) = \sum_{e=1}^{ki} \alpha_e \| \mathbf{x} - \mathbf{x}^{(e)} \|$$

(5.2)

such that $\hat{f}(\mathbf{x}^{(e)}) = f(\mathbf{x}^{(e)}), e = 1, 2, \ldots, ki$.

2. Randomly create a large number of samples, $p$, e.g., $p = 10^4$, in the design variable space, $[\mathbf{x}_{lb}, \mathbf{x}_{ub}]$. All samples’ approximated objective function values, $\hat{f}_q(\mathbf{x})$ ($q = 1, \ldots, p$), are evaluated in equation (5.2). A distribution, $GF$, as shown in Figure 5-1-(a), is defined by ranking $p$ samples in an ascending order in terms of the value of $\hat{f}(\mathbf{x})$. $^9$

3. Define a Guidance Function based on the $GF$ distribution generated in Step 2, $\tilde{GF}(\mathbf{x}_q) = c_0 - \hat{f}(\mathbf{x}_q), q = 1, \ldots, p$, where $c_0$ is a constant such that $c_0 \geq \hat{f}(\mathbf{x}_q), q = 1, \ldots, p$, as shown in Figure 5-1-(b).

$^9$ Figure 5-1 was plotted with the CPM for solving Test Case 1 in Section 5.4.1, and is used, as an example, for illustrating the general process of the MPS when optimizing equation (5.1).
4. Cumulatively sum up $\tilde{G}F(x)$ over $p$ samples to build up a new function $CG(x)$ by

$$CG(x_q) = \frac{\sum_{i=1}^{q} \tilde{G}F(x_i)}{\sum_{i=1}^{p} \tilde{G}F(x_i)}, \quad q = 1, \ldots, p \quad (5.3)$$

$CG(x)$ is plotted in Figure 5-1-(c). This new function reflects a certain ‘bias’ to a random selection from the set of $\hat{f}(x_q) \ (q = 1, \ldots, p)$ due to its upper convex shape. In other words, the possibility of being selected for each sample in the space of $\hat{f}(x_q) \ (q = 1, \ldots, p)$ is not equal. Instead, samples with a small value of $CG(x)$ have a higher possibility to be selected than other design samples with a large value of $CG(x)$.

5. Furthermore, applying a speed factor $SP$, the intensity of preference of $CG(x)$ to samples whose value of $\hat{f}$ is small can be increased by

$$CG(x) = \left(CG(x_q)\right)^{SP}, \quad 0 < SP \leq 1, \quad q = 1, \ldots, p \quad (5.4)$$

As a result, a certain number of samples can be selected randomly by a one-to-one mapping between a series of random values generated within $[0, 1]$ and samples’

---

10 The value of $SP$ is up to designers. In general, a small $SP$ could result in a local optimum, and a large $SP$ costs more computational efforts in searching for the global optimum.
Chapter 5 – Collaboration Pursuing Method

ranking number based on the distribution of $\tilde{CG}(x)$, as shown in Figure 5-1-(d). For example, a random value is given as 0.4899. Its corresponding sample ranking number is 600 in Figure 5-1-(d). Then the sample with the ranking number of 600 in $\tilde{CG}(x)$ is chosen as one of the new experiments. In this way, more samples with low ranking number values are selected than those with high ranking number values for the next sampling iteration. The total number of chosen samples is usually between three and five.\footnote{The number of new experiments depends on designers. The more experiments chosen, the more computational cost needed.}

6. Use all experiments including the initial experiments to form equation (5.2). Then, repeat steps 2–6 until a certain convergence criterion is satisfied.

In reference [8], constraints, $g$, are considered to be inexpensive functions of design variables, $x$. Therefore, samples violating $g$ were discarded in the random sampling process in step 2 before approximating $\hat{f}$. This situation rarely happens in MDO problems since $g$ involves state parameters, which are usually evaluated by computationally expensive processes. It was noted that the MPS gradually refines the RBF approximation of solved problems as the number of experiments increases. At each iteration, most new experiments are selected from samples where a small value of $\hat{f}$ exists. Meanwhile, samples are also chosen from other potential areas, even though these samples have a large value of $\hat{f}$. The MPS is thus, in essence, a discriminative sampling method.

Compared with other optimization methods applying approximation techniques, the MPS
Chapter 5 – Collaboration Pursuing Method

retains the possibility to pursue the optimum solution, not only along the direction with a high gradient value, but also statistically in other potential directions. The way that MPS works is reasonable since the accuracy of approximations is uncertain and depends on many factors. It can be proved that the MPS converges to the global optimum of \( f(x) \), as long as \( f(x) \) is continuous in the neighborhood of the global optimum [8]. Based on the idea of building a Guidance Function, the MPS can be customized and applied to other problems, such as multi-objective optimization [54].

![Graphs](image)

**Figure 5-1** Construction of the Sampling Guidance Function
Chapter 5 – Collaboration Pursuing Method

5.3 COLLABORATION PURSUING METHOD

The Collaboration Pursuing Method (CPM) is a sampling-based method. The idea of the CPM is to select some sample candidates from a sample pool. These selected samples are preferred to be feasible subject to the SA/MDA and constraints. Having the selected samples, the CPM should be able to do the following things:

1) achieve the global optimum iteratively,
2) and solve relatively large-scale design problems \(^{12}\) effectively and efficiently.

The architecture of the CPM is shown in Figure 5-2. The feasibility of selected samples subject to the SA/MDA is ensured by the Collaboration Model introduced in Chapter 3. The global optimum solution is achieved through the MPS method. Since the efficiency and effectiveness of sampling-based methods are related to the number of design variables as well as the range of design variables, an Adaptive Sampling process is applied within a small local area around the current best solution, \(x^*\). The small local area, which is similar to a trust region [55], makes a limited number of samples effective within it. Similar work has been done in reference [56], while the Adaptive Sampling in this thesis is much simpler. Also, the number of active design variables is controlled by an Active Design Variable Control process based on the sensitivity information of the intermediate best solution with respect to design variables, i.e., \(\frac{\partial f^*}{\partial x}\). The Adaptive Sampling and Active Design Variable Control can effectively help the CPM pursue local

\(^{12}\) In this thesis, a relatively large-scale problem has about 10 design variables.
optima and speed up the optimization process as well. The sampling process is capable of doing continuous sampling or discrete sampling. Discretizing continuous variables also can partly alleviate difficulties caused by a large number of design variables since the number of possible solutions becomes finite. On the other hand, only certain accuracy of design variables is meaningful in a real situation.

Figure 5-2 Architecture of the Collaboration Pursuing Method
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Figure 5-3 Flowchart of the Collaboration Pursuing Method

The detailed process of the CPM is elaborated incorporating Figure 5-3 as follows:
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(1) **Discrete Sampling**\(^{13}\) – step 1: For continuous design variables, a random sampling procedure is employed in the original range of design variables, \([x_{Lb}, x_{Ub}]\), where \(x_{Lb}\) and \(x_{Ub}\) are the lower and upper bound of design variables, respectively. To make samples meaningful to engineering applications as well as effectively deal with large-scale design problems, a discrete sampling is implemented by a one-to-one mapping process between continuous samples and discretized design variables. For example, given the range of \(x_1\), e.g., \(2 \leq x_1 \leq 3\), and the meaningful accuracy of \(x_1\), e.g., \(a_d = 0.1\), \(x_1\) can be discretized by

\[
x_{1d} = \{2.0 2.1 2.2 2.3 2.4 2.5 2.6 2.7 2.8 2.9 3.0\}
\]  

(5.5)

For a continuous random sample of \(x_1\), e.g., 2.435677, its corresponding discrete value is determined by

\[
x_i = x_{1d, \{for \ \forall d = 1, \ldots, ns, \ |x_i - x_{1d}| \leq |x_i - x_{1d} |\}}
\]  

(5.6)

where \(ns\) is the number of discrete values of \(x_1\). According to \(x_{1d}\) defined in equation (5.5), the discrete value of 2.435677 should be 2.4. The discrete sampling helps the optimization process reduce the number of possible combinations of design variables. Therefore the efficiency and capability of the CPM can be improved with a limited number of random samples. The discrete sampling extends the applicability of the CPM to MDO problems with hybrid design variables.

\(^{13}\) Step numbers correspond to box numbers in Figure 5-3.
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(2) *Initialization* – steps 2 and 3: The CPM starts with several initial experiments, \(^{14}\) e.g., 4, by calling the SA/MDA. This procedure is based on a random sampling process in the design variable space. In this process, experiments are saved in the database of experiments regardless of their feasibilities subject to constraints, while the experiments which do not satisfy the SA/MDA are discarded and marked to avoid future repeating. In the following CPM iterations, the initial infeasible experiments will be replaced by feasible experiments.

(3) From the 1\(^{st}\) CPM iteration on, three sampling processes take place in the beginning of each CPM iteration. All sampling processes work with both discrete and continuous design variables according to the Discrete Sampling explained above.

a) *Global Sampling* – step 4: A random sampling process employed in the original design space, \([x_{lb}, x_{ub}]\), is called the Global Sampling. The Global Sampling generates \(p\) random samples, \(^{15}\) e.g., \(10^4\), which are processed by the MPS to search for the global optimum solution in Step 9.

b) *Adaptive Sampling* – step 5: A random sampling process for generating \(p_3\) random samples in the neighborhood of the current best solution, \(x^*\), is called the Adaptive Sampling. The idea of the Adaptive Sampling is to have more samples in a small local region around the current best solution. If the Adaptive Sampling

\(^{14}\) The number of initial experiments is not fixed. It’s up to designers’ choice. It could be from 4 to 6.

\(^{15}\) The number of random samples depends on the computational capacity. Generally, more samples (if affordable), better result and efficiency.
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still cannot improve the objective function value with a certain number of random samples, the size of the small region should be decreased to increase the effectiveness of the Adaptive Sampling. As expected, the best sample in the small local region is more likely covered by a limited number of samples. The center of the small region is being continually updated until the optimization process reaches a local optimum, as shown in Figure 5-4. For continuous design variables, the range of design variables for Adaptive Sampling is determined by

$$
\left[ (x^* - \Delta_1 (x_{ub} - x_{lb})), (x^* + \Delta_1 (x_{ub} - x_{lb})) \right]
$$

(5.7)

where \(\Delta_1\) is a preset ratio between 15% and 30%.\(^{16}\) For discretized variables, the range of variables for Adaptive Sampling can be assigned by

$$
\left[ (x^* - Ia_d), (x^* + Ia_d) \right]
$$

(5.8)

where \(a_d\) is the accuracy or interval used to discretize continuous variables, such as \(a_d = 0.1\) in equation (5.5), and \(I\) is an integer,\(^ {17}\) such as 4.

---

\(^{16}\) The value of \(\Delta_1\) depends on the number of random samples, \(p_3\), and the number of design variables. Generally, more samples can cover a wider range of design variables. For different problems, more design variables, more samples are needed to cover the same range of design variables effectively.

\(^{17}\) Similar to \(\Delta_1\), the value of \(I\) depends on the value of \(p_3\) and the number of design variables.
c) **Active Design Variable Control**\(^{18}\) - step 6: The number of active design variables is controlled to create a limited number of random samples that can effectively cover desired samples where a better objective value exists. The value of a design variable of all samples (duplicated from the Adaptive Sampling,\(^ {19}\) i.e., \(p_3\) random samples) is fixed at the current best solution, if this design variable value of the current best solution does not further change after two consecutive CPM iterations. In doing so, each variable has a register to record the CPM iteration number when the \(f\) value does not further change. Then all design variables can be sorted in an ascending order in terms of the value of their registers. As all design variables are frozen at the current best solution, a certain

---

\(^{18}\) The Active Design Variable Control process can be applied either in the original design variable space, \(x\), or in the neighborhood around the current best solution, \(x^*\). In this work, it is only used in the latter situation, as shown in Figure 5-2.

\(^{19}\) As mentioned in Footnote 18, the Active Design Variable Control process is currently applied in the local region defined by the Active Sampling process. So, \(p_3\) samples from the Adaptive Sampling are simply duplicated for the Active Design Variable Control process.
number of design variables, which have a larger value of the register than the remaining design variables, will be reactivated. The number of reactivated design variables is represented by $na$ in the CPM.\footnote{The number of reactivated design variables depends on the random sample size, i.e., $p_3$, and the sampling region for Active Design Variable Control defined in equation (5.7) or (5.8). In general, a large number of random samples (large value of $p_3$) and a small sampling region (defined by $\Delta_1$ or $I$) allow more active design variables. In practice, the value of $na$ should be assigned by users manually, e.g., $na$ specified in a conceptual aircraft design problem on Pages 130 and 142 in Chapter 6. Feedback from previous CPM iterations in the optimization process also can be used for specifying the value of $na$. For example, if the $f$ value does not improve over a certain number of consecutive CPM iterations (other parameters, e.g., $\Delta_1$ and $I$, kept fixed), the value of $na$ should be reduced to make the Active Design Variable Control effective.} The rational of reactivating design variables is that those reactivated design variables could be pre-mature when frozen. As a result, the Active Design Variable Control process reduces the dimensionality of the original problem for sampling. Since the CPM is fundamentally built on experiments, all sensitivity information is given by two adjacent intermediate best solutions over the past optimization process. Utilizing the given sensitivity information can determine the importance of the influence of design variables to the objective function in a local area. Intrinsically, the mechanism of the Active Design Variable Control reflects the sensitivity information, i.e., $\partial f^* / \partial \alpha$.

The accuracy and capability of the CPM in searching for a local optimum is significantly enhanced by the Active Design Variable Control and the Adaptive Sampling. This advantage of applying the Adaptive Sampling is shown in Test Case 4 in this chapter.
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(4) Collaboration Model and Feasibility Check – steps 7 and 8: Like the BSSDM, \( p \) samples from the Global Sampling, \( p_3 \) samples from the Adaptive Sampling, and \( p_3 \) samples from the Active Design Variable Control process are separately approximated with the RBF approximations based on the database of experimental points by

\[
\bar{y}_{q,i} = \bar{Y}_i \left( \bar{x}_{q,i} \right), i = 1, ..., n, (q = 1, ..., p \text{ or } p_3 \text{ or } p_3) \tag{5.9}
\]

\[
\bar{y}_{q,i} = \bar{Y}_i \left( x_{q,i}, x_{q,ci}, \bar{y}_{q,ci} \right), i = 1, ..., n, (q = 1, ..., p \text{ or } p_3 \text{ or } p_3) \tag{5.10}
\]

Then the interdisciplinary discrepancy / consistency of each sample is calculated by

\[
D_q = \sum_{i=1}^{n} |y_{q,i} - \bar{y}_{q,i}|, q = 1, ..., p \text{ or } p_3 \text{ or } p_3 \tag{5.11}
\]

The Collaboration Model shows the distribution of the interdisciplinary discrepancy / consistency subject to the SA/MDA over all samples by equation (5.11). Based on the RBF approximation models, the constraint check is implemented by

\[
\bar{g}_q = \bar{G} \left( x_q, \bar{y}_q \right), q = 1, ..., p \text{ or } p_3 \text{ or } p_3 \tag{5.12}
\]

\[
\bar{g}_q = \bar{G} \left( x_q, \bar{y}_q \right), q = 1, ..., p \text{ or } p_3 \text{ or } p_3 \tag{5.13}
\]

Similar to the BSSDM, infeasible samples subject to the constraint check are identified by adding a big positive value, such as \( 10^5 \), to their discrepancy values, and are in turn discarded. This is a conservative way to make sure that selected samples are feasible.
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The numbers of remaining samples are $p_1$, $p_4$, and $p_5$ inherited from the Global Sampling, the Adaptive Sampling, and the Active Design Variable Control, respectively.

(5) In steps 10 – 13, from the remaining samples, two samples are chosen as new experiments or seeds for the next CPM iteration. For a minimization problem, a sample with the smallest value of $\hat{f}$ (approximate $f$) selected by the Adaptive Sampling from $p_4$ samples is called a Local Seed, and a sample with the smallest value of $\hat{f}$ chosen by the Active Design Variable Control from $p_5$ samples is called an Optimal Seed.

(6) *Global Optimizer – MPS –* step 9: Global Seeds chosen from $p_1$ samples are determined by the MPS method. Two guidance functions are built for this process.

a) In steps 9.1 and 9.2, $p_1$ samples are sorted in an ascending order in terms of the value of the interdisciplinary discrepancy / consistency given by equation (5.11).

Guidance Function I is formed by

$$ GD(x_q) = \sum_{i=1}^{n} |\tilde{y}_{q,i} - \bar{y}_{q,i}|, \; q = 1, \ldots, p_1 $$  \hspace{2cm} (5.14) 

Then the equation (5.14) is cumulated over $p_1$ samples to have
Chapter 5 – Collaboration Pursuing Method

\[
CD(x_q) = \frac{\sum_{j=1}^{q} GD(x_j)}{\sum_{i=1}^{n} GD(x_i)}, \; q = 1, ..., p_i
\] (5.15)

Finally, \(p_2\) samples are statistically selected from a modified Guidance Function I given by

\[
\tilde{CD}(x_q) = \left( CD(x_q) \right)^{\text{RI}}, \; q = 1, ..., p_i
\] (5.16)

The purpose of the Guidance Function I is to select \(p_2\) feasible samples subject to the SA/MDA and constraints.

b) In steps 9.3 - 9.5, the approximate objective value, \(\tilde{f}(x)\), of all \(p_2\) samples is calculated. \(p_2\) samples are sorted in an ascending order in terms of \(\tilde{f}(x)\). Guidance Function II is constructed by

\[
GF(x_q) = \max \left( \tilde{f}(x_{i=1,...,p_2}) \right) - \tilde{f}(x_q), \; q = 1, ..., p_2
\] (5.17)

Then equation (5.17) is cumulated over \(p_2\) samples by

---

21 The value of \(p_2\) is set to 200 in the thesis with respect to \(10^4\) random samples for each sampling. It could vary according to problems and be adjusted by users.
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\[
CF(x_q) = \frac{\sum_{j=1}^{q} GF(x_j)}{\sum_{i=1}^{p} GF(x_i)}, \quad q = 1, \ldots, p_2
\]  

Finally, \( k \) samples, such as 4, are selected as the new experiments (Global Seeds) randomly from a modified guidance function defined by

\[
\tilde{CF}(x_q) = \left( CF(x_q) \right)^{s_{R2}}, \quad q = 1, \ldots, p_2
\]  

The sample with the smallest value of \( \tilde{f}(x) \) in \( p_2 \) samples should be included in \( k \) selected samples since this one is very promising.

(7) All selected samples are passed to the SA/MDA to calculate the state parameters, \( y \), as well as the objective function, \( f \), in step 3. Samples which satisfy the SA/MDA are saved into the database of experiments to improve the quality of the RBF approximation. The CPM will be stopped if there is no further improvement of \( f \) after a certain number of consecutive CPM iterations, such as 6 iterations.

In summary, the Collaboration Model allows the CPM to extract useful information in compliance with the SA/MDA. Based on the Collaboration Model, the CPM selects desired samples to tune the RBF approximation, and consequently the RBF approximation model expands itself towards the optimum solution of an MDO problem. The discrete sampling, Adaptive Sampling, and Active Design Variable Control make
samples effective in dealing with large-scale design problems. The MPS retains the possibility to pursue the global optimum solution of MDO problems.

5.4 TEST CASES AND DISCUSSION

Three numerical test cases and one engineering application were solved with the CPM. Test Case 1 was also solved with the BSSDM in Chapter 4. Test Cases 2 and 3 were obtained from reference [49]. Finally, a power converter problem was solved as a benchmark MDO problem [57], [58]. According to Figure 5-2 and Figure 5-3, the CPM started with initial feasible experiments for solving all test cases, used a random sampling for continuous design variables, and did not apply the Active Design Variable Control module. Only in Test Case 2 was the Adaptive Sampling module applied. To observe the robustness and consistency of the CPM, 10 independent runs have been carried out for each test case. The convergence tolerance applied in a function named ‘fsolve’ in MATLAB® 6.0 (reference [47]) for solving the SA/MDA is set to $10^{-4}$ in Test Cases 1, 2, and 4, and set to $10^{-7}$ in Test Case 3. For each test case, one of the 10 independent runs is randomly chosen for plotting the convergence history and the cumulative number of the SA/MDA evaluations.
5.4.1 Test Case 1

\[
\begin{align*}
\text{minimize } f &= 4y_1^2 - 2.1y_1^4 + \frac{y_1^6}{3} - y_1y_2 - 4y_2^2 + 4y_2^4 \\
\text{subject to: } & y_1 = x_1 + x_2 - 2 + (y_2 / 1.5)^4 \\
& y_2 = x_3 + x_4 - 2 + (y_1 / 1.8)^4 \\
& y_1 \leq 1.2 \\
& y_2 \leq 1.0 \\
& 1 \leq x_1, x_2, x_3, x_4 \leq 1.9
\end{align*}
\]  

(5.20)

Based on an exhaustive enumeration by calling the SA/MDA, a feasible region, subject to the SA/MDA and constraints in the design space of the six-camel hump-back problem, is shown in Figure 5-5. Also, Figure 5-5 shows the optimization process of the CPM by experimental points marked with different signs. The optimum solution of Test Case 1 should be located in the feasible region in compliance with its SA/MDA and constraints, and it should be as close as possible to \( f = -0.1 \) according to the contour of \( f \). Speed factors of the first and the second Guidance Functions were fixed as 0.51 and 0.07, respectively. At the end of each CPM iteration, 4 samples (Global Seeds) were selected. Results of Test Case 1 are shown in Table 5-1. All runs started with 5 feasible experiments. Runs from 1 to 9 were stopped if the value of \( f \) is less than \(-0.9\). In particular, the optimization process of Run No. 10 was terminated when no further improvement of \( f \) occurred after 10 consecutive CPM iterations. Run No. 10 is also selected for generating the convergence history plot, as shown in Figure 5-6. Figure 5-7 shows the cumulative number of the SA/MDA at each CMP iteration step. According to the objective function contour of the six-camel hump-back problem in Figure 5-5, the
global optimum was reached successfully.

Figure 5-5  Optimization process of Test Case 1 solved with the CPM
### Table 5-1 Results of Test Case 1

<table>
<thead>
<tr>
<th>No.</th>
<th># of SA/MDA of Run</th>
<th>Feasible</th>
<th>Infeasible</th>
<th>Optimum Value of ( f )</th>
<th>Optimum Value of ( x )</th>
<th>Optimum Value of ( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>55</td>
<td>2</td>
<td>0</td>
<td>(-9.070575e^{-1})</td>
<td>1.001897</td>
<td>1.035303</td>
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<td>1.012684</td>
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<td>(-9.093511e^{-1})</td>
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<td>(-9.343211e^{-1})</td>
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<td>1.009576</td>
</tr>
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<td>1.005740</td>
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### Figure 5-6 Intermediate best objective function value over 32 CPM iterations of Run

No. 10 in Test Case 1

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Chapter 5 – Collaboration Pursuing Method

5.4.2 Test Case 2

\[
\begin{align*}
\min f &= y_1 + (x_2 - 2)^2 + x_3 - 2 + e^{-y_2} \\
\text{subject to:} & \\
y_1 &= x_1^2 + x_2 + x_3 - 4 - 0.2y_2 \\
y_2 &= x_1 + x_3 - 2 + \sqrt{y_1} \\
0 &\leq x_1 \leq 7 \\
2 &\leq x_2 \leq 7 \\
2 &\leq x_3 \leq 7 \\
y_1 / 8 - 1 &\geq 0 \\
1 - y_2 / 10 &\geq 0
\end{align*}
\] (5.21)
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Speed factors of the first and the second Guidance Functions were fixed as 1 and 0.41, respectively. At the end of each CPM iteration, 3 samples (1 Local Seed and 2 Global Seeds) were selected. Results are listed in Table 5-2. Runs from 1 to 3 were stopped if the value of $f$ is less than 8.08; Runs from 4 to 6 were stopped if the value of $f$ is less than 8.04; and Runs from 7 to 9 were terminated after 6 CPM iterations with no further improvement of the value of $f$. All runs started with 5 random feasible experiments. The Adaptive Sampling module is applied in Runs 1-9 by setting $\Delta_1 = 5\%$. It was observed that the Adaptive Sampling can efficiently solve MDO problems in which the system objective function $f$ is sensitive to the variation of its design variables. Comparisons are given between Runs 1-9 and Run No. 10, which does not apply the Adaptive Sampling and was stopped if the value of $f$ is less than 8.08. Apparently, Run No. 10 has the worst optimum value with the highest number of calls to the SA/MDA. For Run No. 9, the intermediate best objective function and the cumulative number of the SA/MDA at each CPM iteration step are shown in Figure 5-8 and Figure 5-9, respectively. The value of $f$ equals 8.0337 at the 16$^{th}$ CPM iteration with a total of 64 calls to the SA/MDA, in which 14 calls were spent on generating 5 initial feasible experimental points.
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Table 5-2 Results of Test Case 2

<table>
<thead>
<tr>
<th>No. of Run</th>
<th># of SA/MDA</th>
<th>Infeasible</th>
<th>Feasible</th>
<th>Initial</th>
<th>After Initialization</th>
<th>Optimum Value of (x)</th>
<th>Optimum Value of (y)</th>
</tr>
</thead>
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<tr>
<td>1</td>
<td>21</td>
<td>25</td>
<td>8</td>
<td>8.061338</td>
<td>3.016562</td>
<td>2.042900</td>
<td>2.046421</td>
</tr>
<tr>
<td>2</td>
<td>23</td>
<td>20</td>
<td>9</td>
<td>8.066726</td>
<td>3.004791</td>
<td>2.157272</td>
<td>2.011495</td>
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<td>17</td>
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<td>34</td>
<td>10</td>
<td>8.021729</td>
<td>3.020842</td>
<td>2.048775</td>
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<td>3.013692</td>
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<td>2.005488</td>
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<td>6</td>
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<td>36</td>
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<td>3.022309</td>
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<td>10</td>
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<td>21</td>
<td>7</td>
<td>8.075582</td>
<td>3.011512</td>
<td>2.118991</td>
<td>2.022045</td>
</tr>
</tbody>
</table>

Results from Ref. [49] 8.003 3.025 2.000 2.000

Figure 5-8 Intermediate best objective function value over 37 CPM iterations of Run

No. 9 in Test Case 2
5.4.3 Test Case 3

minimize \( f = \frac{y_1^2}{x_1} + y_2^2 - x_2 \)

subject to:

\[ y_1 = \frac{x_1 y_2 y_3^2}{x_2 y_4} \]

\[ y_2 = \sqrt{\frac{x_1 y_3}{x_2}} \]

\[ y_3 = \frac{y_1 y_4}{2x_1 y_2} \]

\[ y_4 = \frac{x_1 y_3^2}{x_2^2} \]

\[ h_1 = y_4 - y_3 - 2 = 0 \]

\[ g_1 = y_2^2 + 1 - x_1 \geq 0 \]

\[ 0 < x_i \leq 1, \; i = 1, 2 \]
Chapter 5 – Collaboration Pursuing Method

Speed factors of the first and the second Guidance Functions were set to 0.51 and 0.81, respectively. At the end of each CPM iteration, 5 samples (Global Seeds) were selected. It is very difficult to maintain the feasibility of samples subject to the SA/MDA, since the SA/MDA is over constrained by an equality constraint in Test Case 3. Upper bounds of the design variables were assigned by the author. The SA/MDA solver within the CPM was applied to evaluate the optimal solution given by reference [49] for verifying its accuracy. For the given design variable set, the optimal objective function value is same as that in reference [49], which is 2.984 as shown in Table 5-3. All runs were terminated after 6 consecutive CPM iterations without further improvement of $f$. All of the 10 runs started with 3 random feasible experiments. Results by running the CPM are shown in Table 5-4. The optimal function values obtained are in the neighborhood of 1.0, which is much better than 2.984, given by the CSSO in reference [49]. The difficulty of achieving feasible samples, raised by the equality constraint, resulted in a large amount of computational effort for initialization. For Run No. 8, the intermediate best objective function value and the cumulative number of the SA/MDA at each CMP iteration step are shown in Figure 5-10 and Figure 5-11, respectively.

Table 5-3   Accuracy comparison between the CPM and the CSSO applied in Ref. [49]

<table>
<thead>
<tr>
<th>Method</th>
<th>Optimum Value of $f$</th>
<th>Optimum Value of $x$ from Ref. [49]</th>
<th>Optimum Value of $y$</th>
</tr>
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<td>CPM</td>
<td>2.984</td>
<td>$x_1 = 0.998$</td>
<td>$y_1 = 1.4071, y_2 = 1.4128$</td>
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<td>CSSO used in Ref. [49]</td>
<td>2.984</td>
<td>$x_2 = 0.996$</td>
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### Chapter 5 – Collaboration Pursuing Method

#### Table 5-4 Results of Test Case 3

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<th># of SA/MDA</th>
<th>Feasible</th>
<th>Infeasible</th>
<th>Optimum Value of $f$</th>
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<td>Initial</td>
<td>After</td>
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</table>

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Figure 5-10  Intermediate best objective function value over 12 CPM iterations of Run No. 8 in Test Case 3

Figure 5-11  Cumulative number of the SA/MDA over 12 CPM iterations of Run No. 8 in Test Case 3
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5.4.4 Test Case 4

The power converter problem comprises a coupling between an electrical subsystem and a loss subsystem. An optimal power stage design is essential to the development of a quality power converter. The power stage design dominates the overall efficiency, size, and weight of the power converter. The objective of the power converter problem is to minimize the weight. The problem consists of six design variables and twelve state variables, of which four define constraints. All constant values given from reference [58] are listed in Appendix II. A schematic of the power converter problem is shown in Figure 5-12, and the geometry of the transformer core is shown in Figure 5-13.

Figure 5-12  A schematic of the power stage of the power converter [58]
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Figure 5-13  The geometry of the transformer core [58]

The formulation of the power converter problem is defined by

\[
\text{minimize } y_1 = f(x) = W_c + W_w + W_{cap} + W_{hs}
\]

subject to:  \textit{Electrical Design State Analysis}

\[
\text{duty cycle: } y_3 = \frac{EO}{\frac{y_2 EI}{2(XN)}}
\]

\[
\text{minimum duty cycle: } y_4 = \frac{EO}{\frac{y_2 EIMAX}{2XN}}
\]
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inductor resistance: \( y_5 = \frac{XMLT x_2 (RO)}{x_3} \)

where \( XMLT = 2x_1 (1 + K_1) FC \)

core cross-sectional area: \( y_6 = K_1 |x_1| x_1 \)

magnetic path length: \( y_7 = \frac{\pi}{2} x_1 \)

inductor value: \( y_8 = \frac{(EO + VD)(1 - y_3)}{y_6 x_2 (FR)} \)

Loss Design State Analysis

circuit efficiency: \( y_2 = \frac{PO}{PS + PQ + PD + PO F + PX FR} \)

Fill Window Constraint

\( g_1 = WA - \frac{x_3 x_2}{FW} + (-WBOB)x_6 K_2 \geq 0 \), where \( WA = K_2 x_6 |x_6| \)

Ripple Specification

\( g_2 = \frac{VR - DELI (ESR)}{EO} \geq 0 \), where \( DELI = \frac{(EO + VD)(1 - y_3)}{x_4 FR} \)

Core Saturation

\( g_3 = BSP - \frac{x_4 (XIMAX)}{y_6 x_2} \geq 0 \),

where \( XIMAX = \frac{PO}{EO} + \frac{(EO + VD)(1 - y_4)}{2x_4 FR} \)

Minimum Inductor Size for Continuous Mode at EIMAX and POMIN

\( g_4 = x_4 - XLCRIT \geq 0 \),

where \( XLCRIT = \frac{(EO + VD)(1 - y_4) EO}{2(POMIN)FR} \)

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where \( W_e = \sum_1 D_l y_6 (ZP_1 + \gamma_1) \), \( ZP_1 = 2(1 + K_z) x_6 \), \( W_w = \sum_1 (XMLT) (DC) x_3 x_2 \), \( XMLT = 2x_1(1 + K_1)FC \), \( W_{cap} = \sum_1 (DK_1) x_5 \), and \( W_{as} = \frac{PO}{KH} (\frac{1}{y_2} - 1) \).

The power converter problem has 6 design variables, as shown in Table 5-5. Relatively large upper bounds of all design variables and the lower bound of \( x_4 \) were assigned by the author, by referring to the optimum solution and the lower bounds of design variables from references [57] and [58]. The problem is mainly dominated by the couplings amongst \( y_2, y_3 \) and \( y_8 \), and the explicit dependency matrix is shown in Table 5-6. Speed factors of the first and the second Guidance Functions were fixed as 1 and 0.00011, respectively. At the end of each CPM iteration, 2 samples (Global Seeds) were selected. Similar to Test Case 3, the SA/MDA solver within the CPM was applied to evaluate the optimum solution in reference [57] for verifying its accuracy. As shown in Table 5-7, the CPM has the same accuracy as the CSSO in reference [57]. 10 independent runs solved with the CPM, as shown in Table 5-8, were stopped without further improvement of \( f \) after 6 consecutive CPM iterations. All runs started with 4 random feasible experiments. For Run No. 4 (with the highest number of calls to the SA/MDA in Table 5-8), the intermediate best objective function value and the cumulative number of the SA/MDA at each CMP iteration step are shown in Figure 5-14 and Figure 5-15, respectively.

According to reference [57], the optimum solution \( (f = 1.48) \) was given by the CSSO with 54 ‘Design Point System Iterations’, and a ‘Design Point System Iteration’ refers to a call to the SA/MDA. Based on the results in Table 5-7 and Table 5-8, the CPM is more
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efficient than the CSSO for solving the power converter problem.

Table 5-5  Design variables of the power converter problem

<table>
<thead>
<tr>
<th>Variable</th>
<th>Name</th>
<th>Description</th>
<th>Range</th>
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<td></td>
<td></td>
<td></td>
<td>Lower Bound</td>
</tr>
<tr>
<td>x₁</td>
<td>( C_w )</td>
<td>Core center leg width</td>
<td>0.001</td>
</tr>
<tr>
<td>x₂</td>
<td>Turns</td>
<td>Inductor turns</td>
<td>1.0</td>
</tr>
<tr>
<td>x₃</td>
<td>( A_{cp} )</td>
<td>Copper size</td>
<td>7.29e⁸</td>
</tr>
<tr>
<td>x₄</td>
<td>( L_f/PINDUC )</td>
<td>Inductance</td>
<td>1.0e⁶</td>
</tr>
<tr>
<td>x₅</td>
<td>( C_f )</td>
<td>Capacitance</td>
<td>1.0e⁻⁵</td>
</tr>
<tr>
<td>x₆</td>
<td>( W_w )</td>
<td>Core window width</td>
<td>0.001</td>
</tr>
</tbody>
</table>

Table 5-6  Explicit dependency matrix of the power converter problem

Table 5-7  Accuracy comparison between the CPM and the CSSO applied in Ref. [57]

<table>
<thead>
<tr>
<th>Method</th>
<th>Optimum Value of ( f )</th>
<th>Optimum Design of ( x ) from Ref. [57]</th>
<th>Optimum Design of ( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>CPM</td>
<td>1.4856</td>
<td>( x_1 = 0.0191 ); ( x_2 = 4.91 )</td>
<td>( y_2 = 0.8302 ); ( y_3 = 0.5929 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( x_3 = 0.00000677 )</td>
<td>( y_4 = 0.4535 ); ( y_5 = 0.0018 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( x_4 = 0.00000524 )</td>
<td>( y_6 = 0.0003648 ); ( y_7 = 0.0300 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( x_5 = 0.0026 )</td>
<td>( y_8 = 0.0128 )</td>
</tr>
<tr>
<td>CSSO used in Ref. [57]</td>
<td>1.48</td>
<td>( x_6 = 0.00759 )</td>
<td>( y_2 = 0.830 ); ( y_3 = 0.593 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>( y_4 = 0.453 ); ( y_5 = 0.00182 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>( y_6 = 0.000367 ); ( y_7 = 0.0301 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>( y_8 = 0.0128 )</td>
</tr>
</tbody>
</table>
## Table 5-8  Results of Test Case 4 given by the CPM

<table>
<thead>
<tr>
<th>No. of Run</th>
<th># of SA/MDA Feasible Initial</th>
<th>Infeasible After Initialization</th>
<th>Optimum Value of ( f )</th>
<th>Optimum Value of ( x )</th>
<th>Optimum Value of ( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>35</td>
<td>1</td>
<td>3</td>
<td>1.40026</td>
<td>( x_1 = 0.011965 ) ( x_3 = 5.137405 ) ( x_5 = 0.008793 ) ( x_6 = 0.008656 ) ( y_2 = 0.841751 ) ( y_3 = 0.584688 ) ( y_6 = 0.018794 )</td>
</tr>
<tr>
<td>2</td>
<td>30</td>
<td>3</td>
<td>1</td>
<td>1.46960</td>
<td>( x_1 = 0.018990 ) ( x_3 = 3.261643 ) ( x_5 = 0.006239 ) ( x_6 = 0.007036 ) ( y_2 = 0.840183 ) ( y_3 = 0.585882 ) ( y_6 = 0.029829 )</td>
</tr>
<tr>
<td>3</td>
<td>20</td>
<td>0</td>
<td>7</td>
<td>1.46435</td>
<td>( x_1 = 0.015288 ) ( x_3 = 4.819028 ) ( x_5 = 0.007675 ) ( x_6 = 0.007685 ) ( y_2 = 0.836584 ) ( y_3 = 0.588444 ) ( y_8 = 0.024015 )</td>
</tr>
<tr>
<td>4</td>
<td>33</td>
<td>6</td>
<td>2</td>
<td>1.48278</td>
<td>( x_1 = 0.019377 ) ( x_3 = 2.581968 ) ( x_5 = 0.007388 ) ( x_6 = 0.008073 ) ( y_2 = 0.845515 ) ( y_3 = 0.582134 ) ( y_8 = 0.030437 )</td>
</tr>
<tr>
<td>5</td>
<td>22</td>
<td>31</td>
<td>0</td>
<td>1.44344</td>
<td>( x_1 = 0.017046 ) ( x_3 = 3.373527 ) ( x_5 = 0.007254 ) ( x_6 = 0.008487 ) ( y_2 = 0.843580 ) ( y_3 = 0.583488 ) ( y_8 = 0.026776 )</td>
</tr>
<tr>
<td>6</td>
<td>25</td>
<td>0</td>
<td>4</td>
<td>1.49415</td>
<td>( x_1 = 0.020723 ) ( x_3 = 3.178963 ) ( x_5 = 0.004352 ) ( x_6 = 0.005940 ) ( y_2 = 0.833817 ) ( y_3 = 0.590396 ) ( y_8 = 0.032552 )</td>
</tr>
<tr>
<td>7</td>
<td>29</td>
<td>2</td>
<td>3</td>
<td>1.44674</td>
<td>( x_1 = 0.016082 ) ( x_3 = 5.453818 ) ( x_5 = 0.004478 ) ( x_6 = 0.007960 ) ( y_2 = 0.832099 ) ( y_3 = 0.591687 ) ( y_8 = 0.025262 )</td>
</tr>
<tr>
<td>8</td>
<td>26</td>
<td>9</td>
<td>3</td>
<td>1.39168</td>
<td>( x_1 = 0.012810 ) ( x_3 = 5.567491 ) ( x_5 = 0.006761 ) ( x_6 = 0.009895 ) ( y_2 = 0.840247 ) ( y_3 = 0.585794 ) ( y_8 = 0.020122 )</td>
</tr>
<tr>
<td>9</td>
<td>27</td>
<td>5</td>
<td>0</td>
<td>1.43167</td>
<td>( x_1 = 0.012973 ) ( x_3 = 7.524132 ) ( x_5 = 0.005366 ) ( x_6 = 0.009920 ) ( y_2 = 0.832640 ) ( y_3 = 0.591244 ) ( y_8 = 0.020378 )</td>
</tr>
<tr>
<td>10</td>
<td>29</td>
<td>1</td>
<td>0</td>
<td>1.396361</td>
<td>( x_1 = 0.015761 ) ( x_3 = 3.054849 ) ( x_5 = 0.007356 ) ( x_6 = 0.007650 ) ( y_2 = 0.843796 ) ( y_3 = 0.583243 ) ( y_8 = 0.024757 )</td>
</tr>
</tbody>
</table>
Chapter 5 – Collaboration Pursuing Method

Figure 5-14 Intermediate best objective function value over 18 CPM iterations of Run No. 4 in Test Case 4

Figure 5-15 Cumulative number of the SA/MDA over 18 CPM iterations of Run No. 4 in Test Case 4
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5.4.5 Discussion

It was observed that the Collaboration Model was effective. For the implementation of the MPS, if the Guidance Function I is sped up intensively by applying a small value of $SP_1$, e.g., $SP_1 = 0.01$, new samples given from the current CPM iteration to the next one are most likely feasible. The speed factor value influences the efficiency of the MPS. An aggressive speed factor could lead to a local optimum and a large speed factor value could slow down the optimization procedure. Based on the feedback from the last CPM iteration, the speed factor could be dynamically adjusted by designers’ intervention. For example, the value of the speed factor should be reduced if the value of $f$ is not improved after several consecutive CPM iterations and the number of feasible experiments is larger than that of infeasible experiments at each CPM iteration. In this thesis for fair comparison, the speed factors are fixed throughout one optimization process and fixed for a number of independent runs. The efficiency of the CPM can be potentially further improved, however, by dynamically tuning the speed factors.

A common problem from all results was that the CPM spends a lot of efforts to generate initial feasible experiments. As the number of state parameters and design variables increase, the initialization procedure could become a serious problem. Intuitively, a good initialization procedure can be helpful. In this thesis, a strategy to generate feasible experiments was tested by giving a small offset to the design variables of the last feasible experimental point. However, this strategy does not guarantee that feasible experiments are generated. As mentioned previously, the CPM can start with either feasible or infeasible experiments as long as these experimental points are feasible.
subject to the SA/MDA. More feasible experiments are expected to be created after the initialization step to replace the infeasible experiments in the database of experiments. This is much better than randomly generating a certain number of initial feasible experiments subject to both the SA/MDA and constraints. The advantage of implementing the initialization with infeasible experiments, subject to constraints, is shown in Chapter 6 in a conceptual aircraft design problem. Also, for engineering problems, experimental data and other related experience can partly alleviate the difficulty for initialization.

The efficiency and capability of sampling-based optimization methods are always related to the range and number of design variables. As the range and number of design variables become large, the CPM encounters difficulties caused by a limited number of samples, which can not effectively cover the design space. The Adaptive Sampling showed its advantage in this regard when solving Test Case 2. For large-scale design problems, the key to the sampling-based optimization methods is to effectively reduce the number and range of the design variables, while maintaining the optimization progress. Due to the nature of sampling, the capability and efficiency of the CPM can be improved by applying parallel computing for generating a very large number of samples.

5.5 OVERVIEW

This chapter introduced the Collaboration Pursuing Method (CPM). The fundamental idea of the CPM is to handle an MDO problem as a black-box function, based on the
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Collaboration Model developed in Chapter 3, rather than explicitly depicting the couplings amongst subsystems. In the CPM, the Collaboration Model effectively coordinates coupled subsystems; the MPS method aims to search for the global optimum; the discrete sampling approach makes the CPM flexible in dealing with hybrid design variables; and the Adaptive Sampling and Active Design Variable Control modules enhance the CPM’s capability to solve large-scale MDO problems. Four test cases, including a power converter design problem, have been solved with the CPM successfully and efficiently. In the next chapter, a conceptual aircraft design problem is applied for further testing the CPM in solving a relatively large-scale MDO problem.
CHAPTER 6

CONCEPTUAL AIRCRAFT DESIGN

6.1 INTRODUCTION

In this chapter, a conceptual aircraft design problem from reference [33] was solved with the CPM. This design problem consists of 10 design variables, 3 nonlinear coupled disciplines (structures, aerodynamics, and propulsion), and 12 constraints. The goal is to maximize the range of the conceptual aircraft. Section 6.2 introduces the conceptual aircraft design problem. Section 6.3 presents solutions given by the CPM as well as discussions.

22 This problem also can be found in references [34] and [14].
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6.2 PROBLEM FORMULATION

The data dependencies of the conceptual aircraft design problem are shown in Figure 6-1. The aircraft design problem is to maximize the range computed through the Breguet range equation, as shown in Table 6-1. The disciplines are coupled by the output-to-input data transfers (design structure matrix) depicted in Figure 6-1. Note that the module calculating Range does not output data to other disciplines. All design variables are listed in Table 6-2.

![Diagram showing data dependencies](image)

**Figure 6-1** Data dependencies for Range optimization [33]
## Chapter 6 – Conceptual Aircraft Design

### Table 6-1  Formulation of the conceptual aircraft design [33]

<table>
<thead>
<tr>
<th>Sub-problem</th>
<th>Input</th>
<th>Internal</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Structures</strong></td>
<td>AR, ( \Lambda, \frac{U_c}{c} ), ( S_{REF} ), ( \Theta ), ( W_0 ), ( W_E ), ( L, N_Z ), ( \lambda, x )</td>
<td>[ W_w = 0.0051 \left( \frac{W_T N_Z}{S_{REF}} \right)^{0.57} \left( \frac{t}{c} \right)^{0.4} \left( 1 + \lambda \right)^{0.1} \left( \frac{0.1875 S_{REF}}{\cos(\Lambda)} \right) ]</td>
<td>[ W_T = W_w + W_F + W_E; \sigma_1 \rightarrow \sigma_5 = pf \left( \frac{t}{c}, L, x, \frac{b}{2}, R_1 \right) ]</td>
</tr>
<tr>
<td>Constraints</td>
<td>( \sigma_1 \rightarrow \sigma_5 \leq 1.09; 0.96 \leq \Theta \leq 1.04 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Aerodynamics</strong></td>
<td>( M, h, \Lambda, \frac{U_c}{c} ), AR, ( S_{REF} ), W(_T), ( \Theta ), ESF, C(<em>D</em>{min})</td>
<td>[ CL = \frac{W_T}{0.5 \rho V^2 S_{REF}}; Fo2 = pf(ESF, C_t) ]</td>
<td>L, D, SFC, W_E, ESF</td>
</tr>
<tr>
<td>( \rho = \left( 2.377e^{-3} \right) \left( 1 - \left( 6.875e^{-4} h \right)^{0.2561} \right) )</td>
<td>[ C_{D_{min} M \rightarrow \theta} + 3.05 \left( \frac{t}{c} \right)^{0.5} \cos(\Lambda)^{0.5} ]</td>
<td>[ k = 1/(\pi 0.8 AR) ]</td>
<td></td>
</tr>
<tr>
<td>( L = W_T; D = C_D 0.5 \rho V^2 S_{REF} )</td>
<td>dp/dx = pf \left( \frac{t}{c} \right) ]</td>
<td>dp/dx \leq 1.04 ]</td>
<td></td>
</tr>
<tr>
<td>Constraints</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Propulsion</strong></td>
<td>( M, h, D, W_{BE} ), ( T )</td>
<td>[ \bar{T} = 16168.6T; \text{Temp} = pf(M, h, T); ESF = \left( \frac{D}{3} \right) \bar{T}; ]</td>
<td>SFC, W_E, ESF</td>
</tr>
<tr>
<td>( \text{SFC} = 1.1324 + 1.5344M - \left( 3.2956e^{-5} \right) \bar{T} M - (8.574e^{11}) h^2 )</td>
<td>[ + \left( 3.8042e^9 \right) \bar{T} h + (1.06e^{-9}) \bar{T}^2; W_E = 3 W_{BE} ESF^{1.05}; ]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( T_{UA} = 11484 + 10856M - 0.50802h + 3200.2M^2 )</td>
<td>[ - 0.29326M h + (6.8572e^9) h^2 ]</td>
<td>Constraints</td>
<td>( 0.5 \leq ESF \leq 1.5; \bar{T} \leq T_{UA}; \text{Temp} \leq 1.02 )</td>
</tr>
</tbody>
</table>
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<table>
<thead>
<tr>
<th>Range</th>
<th>Design variable</th>
<th>Constraints</th>
<th>Objective function</th>
</tr>
</thead>
<tbody>
<tr>
<td>M, h, L/D, W_f, W_o, SFC</td>
<td>if h &lt; 36089 ft, ( \theta = 1 - 6.875 e^{4} h ); if h &gt; 36089 ft, ( \theta = 0.7519 );</td>
<td>maximize ( R = \frac{M(L/D)661/\theta}{SFC} \ln \left( \frac{W_f}{W_o - W_p} \right) )</td>
<td>( R )</td>
</tr>
</tbody>
</table>

| Constants | \( W_f = 2000 \text{ lb}; W_o = 25000 \text{ lb}; N_Z = 6g; W_{BE} = 4360 \text{ lb}; C_{D_{min,M<1}} = 0.01375 \) |

**Table 6-2** Design variables of the conceptual aircraft design

<table>
<thead>
<tr>
<th>Variables</th>
<th>Description</th>
<th>Unit</th>
<th>LB</th>
<th>UB</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 ( \lambda )</td>
<td>Wing taper ratio</td>
<td>N/A</td>
<td>0.1</td>
<td>0.4</td>
</tr>
<tr>
<td>2 ( x )</td>
<td>Wingbox x-sectional area as polynomial function</td>
<td>p.f.</td>
<td>0.75</td>
<td>1.25</td>
</tr>
<tr>
<td>3 ( C_f )</td>
<td>Skin friction coefficient as polynomial function</td>
<td>p.f.</td>
<td>0.75</td>
<td>1.25</td>
</tr>
<tr>
<td>4 ( T )</td>
<td>Throttle setting</td>
<td>N/A</td>
<td>0.1</td>
<td>1</td>
</tr>
<tr>
<td>5 ( t/c )</td>
<td>Thickness / chord ratio</td>
<td>N/A</td>
<td>0.01</td>
<td>0.09</td>
</tr>
<tr>
<td>6 ( h )</td>
<td>Altitude</td>
<td>ft</td>
<td>30000</td>
<td>60000</td>
</tr>
<tr>
<td>7 ( M )</td>
<td>Mach Number</td>
<td>N/A</td>
<td>1.4</td>
<td>1.8</td>
</tr>
<tr>
<td>8 ( AR )</td>
<td>Aspect Ratio</td>
<td>N/A</td>
<td>2.5</td>
<td>8.5</td>
</tr>
<tr>
<td>9 ( \Lambda )</td>
<td>Wing sweep</td>
<td>degree</td>
<td>40</td>
<td>70</td>
</tr>
<tr>
<td>10 ( S_{REF} )</td>
<td>Wing surface area</td>
<td>( \text{ft}^2 )</td>
<td>500</td>
<td>1500</td>
</tr>
</tbody>
</table>

The detailed description of design variables and some parameters are elaborated as follows [59]:

1. Wing taper ratio, \( \lambda \), is the ratio between the tip chord and the centerline root chord, i.e., \( c_{\text{tip}}/c_{\text{root}} \), as shown in Figure 6-2.

2. Wingbox x-sectional area, \( x \), determines if the fuel and other internal components will fit within the wing. It can be easily developed once airfoils are chosen.
3. Skin friction coefficient, $C_f$, is used to calculate the drag that is not related to lift. Such drag, called the "zero-lift", or "parasite" drag, is directly proportional to the total surface area of the aircraft exposed (wetted) to the air.

4. Throttle setting, $T$, is to control the mass flow ratio through the inlet.

5. Thickness / chord ratio, $t/c$, refers to the maximum thickness of the airfoil divided by its chord, as shown in Figure 6-3.
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6. Mach number, \( M \), is the ratio between the aircraft speed and the sound speed.

7. Aspect ratio, \( AR \), is defined as the square of the wing span, \( (b/2)^2 \), divided by the wing reference area, \( S_{REF}/2 \), as shown in Figure 6-2.

8. Wing sweep, \( \Lambda \), is used primarily to reduce the adverse effects of transonic and supersonic flow, as shown in Figure 6-2.

9. Specific Fuel Consumption (SFC) is the rate of fuel consumption divided by the resulting thrust.

10. Wing twist, \( \Theta \), is used to prevent tip stall and to revise the lift distribution to approximate an ellipse.

11. Engine Scale Factor (ESF) is the ratio between the required thrust and the actual thrust of the nominal engine. The engine typical geometries, such as length, height and diameter, vary with ESF.

12. Maximum Load Factor, \( N_z \), is the acceleration during a turn divided by \( g \), gravitation at constant.

The dependencies of state parameters are listed in Table 6-3. The coupled subsystems are defined by their input and output state parameters, and by the functions that link these state parameters in Table 6-1. Also, constraints and constants are listed in Table 6-1.
According to reference [33], some functional relationships are supplied to reflect some commonly known relationships involved in design. For example, stress is expected to fall as the skin thickness in a wing box increases. Such relationships are represented by polynomial functions. In Table 6-1, ‘pf(.)’ denotes a polynomial function of independent variables in the parentheses. Each polynomial function is of the form

$$pf(S) = A_0 + A_i S^T + (1/2) S A_{ij} S^T$$

(6.1)

where $S$ is the vector of independent variables, and $A_0$, $A_i$, and $A_{ij}$ are coefficient terms.

In calculating the polynomial functions defined in equation (6.1), terms in the $S$ vectors are in the same order as they appear in pf(.) in Table 6-1. The off diagonal terms of $A_{ij}$ were generated randomly between 0 and 1. For this model, they are

---

23 S, A, P and R represent Structures, Aerodynamics, Propulsion, and Range, respectively. Hereafter, the same fashion is applied in the thesis.
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\[ A_{ij} = \begin{bmatrix} -------- & 0.3970 & 0.8152 & 0.9230 & 0.1108 \\ 0.4252 & -------- & 0.6357 & 0.7435 & 0.1138 \\ 0.0329 & 0.8856 & -------- & 0.3657 & 0.0019 \\ 0.0878 & 0.7248 & 0.1978 & -------- & 0.0169 \\ 0.8955 & 0.4568 & 0.8075 & 0.9239 & -------- \end{bmatrix} \] (6.2)

The remaining coefficients are listed in Table 6-4.

Table 6-4 Coefficients for the polynomial functions

<table>
<thead>
<tr>
<th></th>
<th>( A_0 )</th>
<th>( A_i )</th>
<th>( A_{ni} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Theta )</td>
<td>[1.0]</td>
<td>[0.3 -0.3 -0.3 -0.2]</td>
<td>[0.4 -0.4 -0.4 0]</td>
</tr>
<tr>
<td>Fo1</td>
<td>[1.0]</td>
<td>[6.25]</td>
<td>[0]</td>
</tr>
<tr>
<td>( \sigma_1 )</td>
<td>[1.0]</td>
<td>[-0.75 0.5 -0.75 0.5 0.5]</td>
<td>[-2.5 0 -2.5 0 0]</td>
</tr>
<tr>
<td>( \sigma_2 )</td>
<td>[1.0]</td>
<td>[-0.5 0.333 -0.5 0.333 0.333]</td>
<td>[-1.111 0 -1.111 0 0]</td>
</tr>
<tr>
<td>( \sigma_3 )</td>
<td>[1.0]</td>
<td>[-0.375 0.25 -0.375 0.25 0.25]</td>
<td>[-0.625 0 -0.625 0 0]</td>
</tr>
<tr>
<td>( \sigma_4 )</td>
<td>[1.0]</td>
<td>[-0.3 0.2 -0.3 0.2 0.2]</td>
<td>[-0.4 0 -0.4 0 0]</td>
</tr>
<tr>
<td>( \sigma_5 )</td>
<td>[1.0]</td>
<td>[-0.25 0.1667 -0.25 0.1667 0.1667]</td>
<td>[-0.2778 0 -0.2778 0 0]</td>
</tr>
<tr>
<td>Fo2</td>
<td>[1.0]</td>
<td>[0.2 0.2]</td>
<td>[0 0]</td>
</tr>
<tr>
<td>Fo3</td>
<td>[1.0]</td>
<td>[0]</td>
<td>[0.04]</td>
</tr>
<tr>
<td>dp/dx</td>
<td>[1.0]</td>
<td>[0.2]</td>
<td>[0]</td>
</tr>
<tr>
<td>Temp</td>
<td>[1.0]</td>
<td>[0.3 -0.3 0.3]</td>
<td>[0.4 -0.4 0.4]</td>
</tr>
</tbody>
</table>

6.3 APPLICATION OF THE CPM TO CONCEPTUAL AIRCRAFT DESIGN

According to the framework of the CPM, as shown in Figure 5-2, the CPM used the Hybrid (both continuous and discrete) Sampling, Adaptive Sampling, and Active Design Variable Control modules to maximize the range of the conceptual aircraft design defined in Section 6.2. \( \lambda \) and \( T \) are kept as continuous variables and the remaining variables listed in Table 6-5 are discretized.
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Table 6-5  Discretized design variables

<table>
<thead>
<tr>
<th>Variables</th>
<th>Discretized Values</th>
<th>Accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>0.75 0.8 0.85 0.9 0.95 1.0 1.05 1.1 1.15 1.2 1.25</td>
<td>0.05</td>
</tr>
<tr>
<td>C_f</td>
<td>0.75 0.8 0.85 0.9 0.95 1.0 1.05 1.1 1.15 1.2 1.25</td>
<td>0.05</td>
</tr>
<tr>
<td>t/c</td>
<td>0.01 0.02 0.03 0.04 0.05 0.06 0.07 0.08 0.09</td>
<td>0.01</td>
</tr>
<tr>
<td>h (ft)</td>
<td>30000 31000 32000 33000 34000 35000 36000 37000 38000 39000 40000</td>
<td>1000</td>
</tr>
<tr>
<td>M</td>
<td>1.4 1.5 1.6 1.7 1.8</td>
<td>0.1</td>
</tr>
<tr>
<td>AR</td>
<td>2.5 2.6 2.7 2.8 2.9 3.0 3.1 3.2 3.3 3.4 3.5 3.6 3.7 3.8 3.9 4.0 4.1 4.2 4.3 4.4 4.5</td>
<td>0.1</td>
</tr>
<tr>
<td>A (°)</td>
<td>4.6 4.7 4.8 4.9 5.0 5.1 5.2 5.3 5.4 5.5 5.6 5.7 5.8 5.9 6.0 6.1 6.2 6.3 6.4 6.5 6.6</td>
<td>0.1</td>
</tr>
<tr>
<td>S_REF (ft^2)</td>
<td>6.7 6.8 6.9 7.0 7.1 7.2 7.3 7.4 7.5 7.6 7.7 7.8 7.9 8.0 8.1 8.2 8.3 8.4 8.5</td>
<td>0.1</td>
</tr>
</tbody>
</table>

The MPS is not utilized in the CPM for solving the conceptual aircraft design. Instead, a sample (Global Seed) with the smallest approximate value of Range from the Global Sampling is selected. For the internal control of the CPM, the Global Seed will be eliminated if its value of Range is smaller than that of the Optimal Seed or Local Seed. To use the Collaboration Model in the CPM, the explicit relations between state parameters and their corresponding design variables are shown in Table 6-6 according to the

24 In Section 6.3.1, it shows that $3 \times 10^4$ random samples are not enough to effectively cover the whole design variable space. Consequently, Global Seeds selected by the MPS from samples given by the Global Sampling can not effectively improve the optimization process. Therefore, the MPS is not used for solving the conceptual aircraft design problem, while it could be used in a sub-design variable region.
formulation in Table 6-1. For example, $W_T$ is an explicit function of $W_F$, $L$, $W_E$, $\lambda$, $x$, $t/c$, AR, $\Lambda$ and $S_{REF}$ in the structure subsystem. Also, it is observed that $\lambda$, $x$, and $C_f$ are local variables in structures, aerodynamics and propulsion, respectively, and the couplings amongst $W_T$, $D$, and ESF dominate the whole system. The implicit relations between the state parameters, $W_T$, $D$, and ESF, and their associated design variables are listed in Table 6-7, where ‘o’ signs indicate dependencies added by the implicit relations.

Table 6-6  Explicit dependency matrix between state parameters and their variables

<table>
<thead>
<tr>
<th>State Parameters</th>
<th>Structures</th>
<th>Aerodynamics</th>
<th>Propulsion</th>
<th>Local Variables</th>
<th>Interdisciplinary Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$W_F$</td>
<td>$W_T$</td>
<td>$\Theta$</td>
<td>$D$</td>
<td>$L/D$</td>
</tr>
<tr>
<td>$W_F$</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>$W_T$</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>$\Theta$</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>$\sigma_1$</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>$\sigma_2$</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>$\sigma_3$</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>$\sigma_4$</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>$\sigma_5$</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>$L$</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>$D$</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>$L/D$</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>$dp/dx$</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>$W_E$</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>$SFC$</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>$ESF$</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>$T$</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Temp</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>R</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
</tbody>
</table>
Table 6-7  Implicit dependency matrix between state parameters and their variables

<table>
<thead>
<tr>
<th>State Parameters</th>
<th>Local Variables</th>
<th>Interdisciplinary Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>Structures</td>
<td>W_F</td>
<td>W_T</td>
</tr>
<tr>
<td>Aerodynamics</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Propulsion</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S</td>
<td></td>
<td></td>
</tr>
<tr>
<td>W_F</td>
<td>x</td>
<td></td>
</tr>
<tr>
<td>W_T</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>Θ</td>
<td></td>
<td></td>
</tr>
<tr>
<td>σ1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>σ2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>σ3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>σ4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>σ5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>L</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D</td>
<td></td>
<td></td>
</tr>
<tr>
<td>L/D</td>
<td></td>
<td></td>
</tr>
<tr>
<td>dp/dx</td>
<td></td>
<td></td>
</tr>
<tr>
<td>P</td>
<td></td>
<td></td>
</tr>
<tr>
<td>W_E</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SFC</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ESF</td>
<td></td>
<td></td>
</tr>
<tr>
<td>T</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Temp</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For optimizing the conceptual aircraft design with the CPM, the subroutine codes of the SA/MDA, disciplinary analyses (structures, aerodynamics, and propulsion) and Range calculation were directly applied from reference [33]. To be able to compare the results given by the CPM and the results from reference [33], some baseline cases were tested based on intermediate and optimum solutions from the literature, as shown in Table 6-8. Results in CPM column in Table 6-8 were given by running the SA/MDA embedded in the CPM rather than the whole CPM optimization process. Since the values of Range calculated with the CPM are very close to the values from reference [33], we can conclude that the CPM has a comparable accuracy with the BLISS in reference [33] in terms of the SA/MDA. However, one of the constraints, Θ, of the optimum solution from reference [33] is violated. Therefore, in the optimization process of the conceptual aircraft
design with the CPM, two types of $\Theta$ are applied, i.e., $0.96 \leq \Theta \leq 1.04$ defined in Table 6-1, and $0.9049 \leq \Theta \leq 1.04$ based on the optimum solution in reference [33].

<table>
<thead>
<tr>
<th>Design Variables</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
<th>Case 5&lt;sup&gt;25&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$</td>
<td>0.25</td>
<td>0.14951</td>
<td>0.17476</td>
<td>0.25775</td>
<td>0.38757</td>
</tr>
<tr>
<td>$x$</td>
<td>1</td>
<td>0.75</td>
<td>0.75</td>
<td>0.75</td>
<td>0.75</td>
</tr>
<tr>
<td>$C_r$</td>
<td>1</td>
<td>0.75</td>
<td>0.75</td>
<td>0.75</td>
<td>0.75</td>
</tr>
<tr>
<td>$T$</td>
<td>1</td>
<td>0.1676</td>
<td>0.20703</td>
<td>0.15624</td>
<td>0.15624</td>
</tr>
<tr>
<td>t/c</td>
<td>0.05</td>
<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
</tr>
<tr>
<td>h (ft)</td>
<td>45000</td>
<td>54000</td>
<td>60000</td>
<td>60000</td>
<td>60000</td>
</tr>
<tr>
<td>M</td>
<td>1.6</td>
<td>1.4</td>
<td>1.4</td>
<td>1.4</td>
<td>1.4</td>
</tr>
<tr>
<td>AR</td>
<td>5.5</td>
<td>4.4</td>
<td>3.3</td>
<td>2.5</td>
<td>2.5</td>
</tr>
<tr>
<td>$\Lambda(\lambda)$</td>
<td>55</td>
<td>66</td>
<td>70</td>
<td>70</td>
<td>70</td>
</tr>
<tr>
<td>$S_{eff} (ft^2)$</td>
<td>1000</td>
<td>1200</td>
<td>1400</td>
<td>1500</td>
<td>1500</td>
</tr>
</tbody>
</table>

6.3.1 Numerical Studies of the Conceptual Aircraft Design Problem

The complexity of the conceptual aircraft design problem is uncovered by exhaustive enumerations through the SA/MDA subject to the original constraints and modified constraints, while the number of random samples is fixed. $10^4$ random samples were implemented 4 times subject to the modified constraints ($0.9049 \leq \Theta \leq 1.04$), as shown in Table 6-9. The distributions of feasible and infeasible samples of Run No. 2 in Table 6-9

---

<sup>25</sup> Case 5 is the optimum solution given by the BLISS in reference [33].
are depicted in Figure 6-4 and Figure 6-5. Random samples of $10^4$, $2 \times 10^4$, and $3 \times 10^4$ were executed 4 times subject to the original constraints ($0.96 \leq \Theta \leq 1.04$), as shown in Table 6-10. All Range values are much less than the optimum solution from the literature. A clear conclusion based on the above studies is that $3 \times 10^4$ random samples are not enough to effectively cover the design variable space of the conceptual aircraft design problem. This fact exactly reflects the real challenge of sampling-based optimization methods when faced with large-scale design problems. The optimization ability of the sampling-based optimization methods is constrained by the sample size, which is related to the number and range of design variables. Also, based on the difference between the results of $10^4$ random samples in Table 6-9 and the results of $10^4$ random samples in Table 6-10, the feasible region, represented by a feasible-to-infeasible ratio, is narrowed down and the feasible optimum solution decreases due to the tightened constraint of $\Theta$.

Table 6-9  Results subject to the modified constraints given by exhaustive enumerations

<table>
<thead>
<tr>
<th># of Random Samples (0.9049 $\leq \Theta \leq$ 1.04)</th>
<th>Run</th>
<th># of Feasible Samples</th>
<th># of Infeasible Samples</th>
<th>Maximum Range (nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1 \times 10^4$</td>
<td>1</td>
<td>258</td>
<td>9742</td>
<td>1564</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>279</td>
<td>9721</td>
<td>2248</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>278</td>
<td>9722</td>
<td>2147</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>279</td>
<td>9721</td>
<td>1698</td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td>273.5</td>
<td>9726.5</td>
<td>1914.25</td>
</tr>
<tr>
<td>Average Feasible-to-infeasible Ratio</td>
<td></td>
<td></td>
<td></td>
<td>0.02812</td>
</tr>
</tbody>
</table>
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Table 6-10  Results subject to the original constraints given by exhaustive enumerations

<table>
<thead>
<tr>
<th># of Random Samples (0.96≤θ≤1.04)</th>
<th>Run</th>
<th># of Feasible Samples</th>
<th># of Infeasible Samples</th>
<th>Maximum Range (nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1x10^4</td>
<td>1</td>
<td>234</td>
<td>9766</td>
<td>1545</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>226</td>
<td>9774</td>
<td>1426</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>224</td>
<td>9776</td>
<td>1647</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>198</td>
<td>9802</td>
<td>1442</td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td>220.5</td>
<td>9779.5</td>
<td>1515</td>
</tr>
<tr>
<td>Average Feasible-to-infeasible Ratio</td>
<td></td>
<td></td>
<td></td>
<td>0.02255</td>
</tr>
<tr>
<td>2x10^4</td>
<td>5</td>
<td>427</td>
<td>19573</td>
<td>1473</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>418</td>
<td>19582</td>
<td>2014</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>462</td>
<td>19538</td>
<td>2704</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>456</td>
<td>19544</td>
<td>1645</td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td>440.75</td>
<td>19559.25</td>
<td>1959</td>
</tr>
<tr>
<td>Average Feasible-to-infeasible Ratio</td>
<td></td>
<td></td>
<td></td>
<td>0.02253</td>
</tr>
<tr>
<td>3x10^4</td>
<td>9</td>
<td>653</td>
<td>29347</td>
<td>1637</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>689</td>
<td>29311</td>
<td>2221</td>
</tr>
<tr>
<td></td>
<td>11</td>
<td>662</td>
<td>29338</td>
<td>1772</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>659</td>
<td>29341</td>
<td>2310</td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td>665.75</td>
<td>29334.25</td>
<td>1985</td>
</tr>
<tr>
<td>Average Feasible-to-infeasible Ratio</td>
<td></td>
<td></td>
<td></td>
<td>0.02270</td>
</tr>
</tbody>
</table>

Figure 6-4  Feasible samples of Run No. 2 in Table 6-9
Subject to the original constraints, the effectiveness of applying the Adaptive Sampling in the CPM is shown by 10 runs with random initial experiments\textsuperscript{26} over 35 CPM iterations, as shown in Table 6-11.\textsuperscript{27} The size of random samples is $10^4$, $\Delta_1 = 0.2$, and $I = 4$. In each run, only a Local Seed and a Global Seed were selected. As mentioned before, the Global Seed will be eliminated if its Range value is less than the Range value of the Local Seed over 2 consecutive CPM iterations. The distribution of experiments, the trend of Range, and the cumulative # of the SA/MDA (or experimental points) of Run No. 1 are depicted in Figure 6-6, Figure 6-7, and Figure 6-8, respectively. In comparison with the results in Table 6-10, the Adaptive Sampling effectively lifted up the value of Range

\textsuperscript{26} Initial experiments could be either feasible or infeasible to the constraints. This feature will be discussed in Section 6.3.3.

\textsuperscript{27} In this study, the Active Design Variable Control module is not applied.
from about 1500 to 3500. Based on the author's observation, the value of Range given by new experiments is repeated between 2500 and 3500 over many CPM iterations when applying the Adaptive Sampling in the CPM.

Table 6-11 Effectiveness of applying the Adaptive Sampling in the CPM

<table>
<thead>
<tr>
<th>Run</th>
<th>Maximum Range</th>
<th>Index Number of the CPM Iteration</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3480.071</td>
<td>15</td>
</tr>
<tr>
<td>2</td>
<td>3480.071</td>
<td>15</td>
</tr>
<tr>
<td>3</td>
<td>3480.071</td>
<td>15</td>
</tr>
<tr>
<td>4</td>
<td>3480.071</td>
<td>15</td>
</tr>
<tr>
<td>5</td>
<td>3156.942</td>
<td>9</td>
</tr>
<tr>
<td>6</td>
<td>3480.071</td>
<td>15</td>
</tr>
<tr>
<td>7</td>
<td>3156.942</td>
<td>9</td>
</tr>
<tr>
<td>8</td>
<td>3480.071</td>
<td>15</td>
</tr>
<tr>
<td>9</td>
<td>3480.071</td>
<td>15</td>
</tr>
<tr>
<td>10</td>
<td>3156.942</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>Average</td>
<td>3383.132</td>
</tr>
</tbody>
</table>

Figure 6-6 Distribution of experimental points over 35 CPM iterations with $10^4$ random samples

The index number of the CPM iteration indicates when the optimum solution occurred over a total of 35 CPM iterations.
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Figure 6-7  Trend of Range over 35 CPM iterations with $10^4$ random samples

Figure 6-8  Cumulative # of the SA/MDA over 35 CPM iterations with $10^4$ random samples
6.3.2 Parameter Studies in the CPM

In this conceptual aircraft design problem, the CPM involves 3 parameters, which are \( \Delta_1 \), \( I \), and \( na \) introduced in Section 5.3 - (3). \( \Delta_1 \) and \( I \) determine the size of a local area around the current best solution, and \( na \) is the number of active design variables. Based on the same initial experiments, studies of how these parameters influence the CPM’s performance were implemented. The number of random samples is still \( 10^4 \). Four Initial experiments listed in Table 6-12 are infeasible subject to the modified constraints. Also, the maximum number of the CPM iteration is set to 35.

Table 6-12 Initial infeasible experimental points for the parameter studies in the CPM

<table>
<thead>
<tr>
<th>Variables</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda )</td>
<td>0.312689</td>
<td>0.346180</td>
<td>0.192750</td>
<td>0.378284</td>
</tr>
<tr>
<td>( x )</td>
<td>1.15</td>
<td>1.05</td>
<td>1.05</td>
<td>1.10</td>
</tr>
<tr>
<td>( C_f )</td>
<td>1.05</td>
<td>1.2</td>
<td>1.0</td>
<td>0.95</td>
</tr>
<tr>
<td>( T )</td>
<td>0.602455</td>
<td>0.805926</td>
<td>0.611939</td>
<td>0.478591</td>
</tr>
<tr>
<td>( t/c )</td>
<td>0.05</td>
<td>0.07</td>
<td>0.03</td>
<td>0.09</td>
</tr>
<tr>
<td>( h (ft) )</td>
<td>52000</td>
<td>46000</td>
<td>31000</td>
<td>57000</td>
</tr>
<tr>
<td>( M )</td>
<td>1.4</td>
<td>1.6</td>
<td>1.7</td>
<td>1.5</td>
</tr>
<tr>
<td>( AR )</td>
<td>6.1</td>
<td>8.0</td>
<td>6.6</td>
<td>3.4</td>
</tr>
<tr>
<td>( \Lambda^* )</td>
<td>45</td>
<td>53</td>
<td>56</td>
<td>61</td>
</tr>
<tr>
<td>( S_{\text{REF}}(\text{ft}^2) )</td>
<td>1010</td>
<td>660</td>
<td>1110</td>
<td>1290</td>
</tr>
<tr>
<td>Range (nm)</td>
<td>640.061194</td>
<td>337.649591</td>
<td>254.079841</td>
<td>1634.393249</td>
</tr>
</tbody>
</table>

(1) Study of \( na \): This study was implemented by changing the value of \( na \) with respect to two settings of \( \Delta_1 \) and \( I \). In the first setting, \( \Delta_1 = 0.4 \) and \( I = 5 \). In the second setting, \( \Delta_1 = 0.3 \) and \( I = 5 \). According to results listed in Table 6-13, we can roughly conclude that a small \( na \), such as \( na = 1 \), results in a good accuracy of the optimum solution, and a relatively large \( na \), e.g., \( na = 2 \) or \( 3 \), gives a good efficiency to converge to the optimum solution. As expected, the Active Design Variable
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Control is less effective (the optimum solution is low) when na is very large, such as 4. As a suggestion in this problem, the value of na is between 1 and 3 subject to $10^4$ random samples for the conceptual aircraft design problem.

Table 6-13  Results of the study of na

<table>
<thead>
<tr>
<th>Case</th>
<th>1 (na = 1)</th>
<th>2 (na = 2)</th>
<th>3 (na = 3)</th>
<th>4 (na = 4)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Maximum</td>
<td>Index</td>
<td>Maximum</td>
<td>Index</td>
</tr>
<tr>
<td></td>
<td>Range</td>
<td>Number of</td>
<td>Range</td>
<td>Number of</td>
</tr>
<tr>
<td></td>
<td></td>
<td>CPM</td>
<td></td>
<td>CPM</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Iteration</td>
<td></td>
<td>Iteration</td>
</tr>
<tr>
<td>Δ1 = 0.4</td>
<td>3960.907</td>
<td>26</td>
<td>3961.077</td>
<td>21</td>
</tr>
<tr>
<td>1 = 5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Δ1 = 0.3</td>
<td>3958.993</td>
<td>26</td>
<td>3834.720</td>
<td>22</td>
</tr>
<tr>
<td>1 = 5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(2) Study of Δ1 and I:  Similarly, the value of na is fixed to be 2 and the study of the influence of Δ1 and I on the CPM’s performance was implemented, as shown in Table 6-14. Large values of Δ1 and I correspond to a large local area around the current best solution, and vice versa. Consequently, a small local area results in achieving a local optimum solution early, such as case 1, and a large local area causes the Adaptive Sampling to be less effective, such as case 5. In this problem, the value of Δ1 is suggested between 0.2 and 0.4, and the value of I is recommended between 3 and 4 subject to $10^4$ random samples.
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Table 6-14 Results of the study of $\Delta_1$ and $I$

<table>
<thead>
<tr>
<th>Case</th>
<th>$\Delta_1 = 0.1$</th>
<th>$\Delta_1 = 0.2$</th>
<th>$\Delta_1 = 0.3$</th>
<th>$\Delta_1 = 0.4$</th>
<th>$\Delta_1 = 0.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Range of CPM</td>
<td>Iteration</td>
<td>Range of CPM</td>
<td>Iteration</td>
<td>Range of CPM</td>
</tr>
<tr>
<td>$na$</td>
<td>2</td>
<td>3559.96</td>
<td>19</td>
<td>3528.280</td>
<td>23</td>
</tr>
</tbody>
</table>

6.3.3 Optimization Using the CPM subject to the Modified Constraints

This conceptual aircraft design problem was solved with the All-in-One and BLISS methods in reference [14], and the BLISS method with Response Surface in references [35] and [36]. The results reported in references [14], [35], and [36] are shown in Table 6-15. Based on the value of the Number of Subsystem Analyses in the All-in-One, it seems that each SA takes 4 iterations and one iteration costs 3 subsystem analyses, i.e., structural, aerodynamic and propulsion analyses. Since the subroutine codes of the SA and subsystem analyses applied in the CPM are exactly the same as the codes used in references [33] and [14], the way of calculating the number of subsystem analyses in the reference is applied in the CPM for solving the conceptual aircraft design problem.

According to Table 6-8, the constraint of $\Theta$ is modified to be $0.9049 \leq \Theta \leq 1.04$ based on the optimum solution from reference [33]. The remaining constraints are still same as the original. Based on the parameter studies of $\Delta_1$, $I$, and $na$, the values of these parameters are specified as follows:
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\[
\Delta_I = 0.3 \\
i = 4 \\
na = 2
\]

(6.3)

Table 6-15  Results of the conceptual aircraft design from references [14] and [35]

<table>
<thead>
<tr>
<th>Method</th>
<th>Initial Objective</th>
<th>Initial Max. Constraint Value</th>
<th>Final Objective</th>
<th>Final Max. Constraint Value</th>
<th>Computational Effort</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Number of SA/MDA</td>
</tr>
<tr>
<td>All-in-One</td>
<td>535.79</td>
<td>-0.162</td>
<td>3964.19</td>
<td>1.0e(^{-6})</td>
<td>119</td>
</tr>
<tr>
<td>All-in-One/RS(^{29})</td>
<td>535.79</td>
<td>-0.162</td>
<td>3974.84</td>
<td>0.0013</td>
<td>72</td>
</tr>
<tr>
<td>BLISS</td>
<td>535.79</td>
<td>-0.162</td>
<td>3964.07</td>
<td>192e(^{-5})</td>
<td>7</td>
</tr>
<tr>
<td>BLISS/RS(^{30})</td>
<td>535.79</td>
<td>-0.162</td>
<td>3961.50</td>
<td>0.0</td>
<td>17</td>
</tr>
<tr>
<td>BLISS/RS(^{31})</td>
<td>535.79</td>
<td>-0.162</td>
<td>3964.12</td>
<td>0.0</td>
<td>12</td>
</tr>
</tbody>
</table>

The optimization process of the CPM was implemented based on 5 cases, each of which started with 4 different initial infeasible experiments, as shown in Table 6-16. Constraint values at the optimum solutions of 5 cases are listed in Table 6-17. Each case in Table 6-16 was executed 6 times with same initial experiments (listed in Table 6-16), and the results are shown in Table 6-18. Due to the statistic feature of the random sampling, the CPM could have different converged solutions with the same initial experiments. Based on the average computational cost of each case in Table 6-18 and the costs listed in Table 6-15, the CPM is more efficient than the All-in-One method and All-

\(^{29}\) The All-in-One/RS is a sequential approximation-based All-in-One optimization strategy that involves the use of the response surface model for approximation evaluations of the design objective and constraint functions [35].

\(^{30}\) The BLISS/RS1 builds up a response surface to approximate the objective function and constraints in \(Z\) space based on the data from the SA/MDA.

\(^{31}\) The BLISS/RS2 builds up a response surface to approximate the objective function and constraints in \(Z\) space based on the data from the BB optimization.
in-One/RS in solving this problem. The CPM is also competitively efficient against the BLISS, BLISS/RS1, and BLISS/RS2 in solving the conceptual aircraft design problem.

The distributions of experiments of Cases 1 – 5 in Table 6-16 are plotted in Figure 6-9, Figure 6-12, Figure 6-15, Figure 6-18, and Figure 6-21, respectively. From these figures, we can see that all Cases started with 4 initial infeasible experiments, as shown in ‘V’ signs, with a very poor Range. At the very beginning of the optimization process, the Global Seeds, marked with ‘*’ signs, led the optimization process. Then the Local Seeds, marked by ‘o’ signs, took over the leading role, while they are under about 3500 nm. Finally, the Optimal Seeds, marked in ‘0’ signs, led the optimization process towards the optimum solution. Based on the above observations and the results listed in Table 6-11, the Active Design Variable Control process worked effectively in the CPM to improve the CPM’s ability in solving the conceptual aircraft design problem. The trends of Range of Cases 1 – 5 are depicted in Figure 6-10, Figure 6-13, Figure 6-16, Figure 6-19, and Figure 6-22, respectively, and the computational costs, represented by the cumulative # of the SA/MDA (experimental points), of Cases 1 – 5 are shown Figure 6-11, Figure 6-14, Figure 6-17, Figure 6-20, and Figure 6-23, respectively.
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Table 6-16  Results given by the CPM subject to the modified constraints

<table>
<thead>
<tr>
<th>Case</th>
<th>Design Variables</th>
<th># of the SA/MDA When Range Occurred</th>
<th># of CPM Iterations When Range Occurred</th>
<th># of Subsystem Analyses</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \lambda )</td>
<td>x</td>
<td>( C_f )</td>
<td>T</td>
</tr>
<tr>
<td>1</td>
<td>Initial Design Variables</td>
<td>( 0.385039 )</td>
<td>0.85</td>
<td>1.05</td>
</tr>
<tr>
<td></td>
<td>2346003</td>
<td>1.15</td>
<td>200</td>
<td>0.764387</td>
</tr>
<tr>
<td></td>
<td>0.117367</td>
<td>0.95</td>
<td>1.15</td>
<td>0.108875</td>
</tr>
<tr>
<td></td>
<td>0.104582</td>
<td>1.10</td>
<td>0.95</td>
<td>0.936633</td>
</tr>
<tr>
<td>Optimum</td>
<td>( 0.370513 )</td>
<td>0.75</td>
<td>0.75</td>
<td>0.156244</td>
</tr>
<tr>
<td>2</td>
<td>Initial Design Variables</td>
<td>( 0.241243 )</td>
<td>1.20</td>
<td>1.10</td>
</tr>
<tr>
<td></td>
<td>0.14925</td>
<td>1.10</td>
<td>1.00</td>
<td>0.106188</td>
</tr>
<tr>
<td></td>
<td>0.389819</td>
<td>0.80</td>
<td>0.85</td>
<td>0.453128</td>
</tr>
<tr>
<td></td>
<td>0.196969</td>
<td>0.80</td>
<td>1.00</td>
<td>0.719632</td>
</tr>
<tr>
<td>Optimum</td>
<td>( 0.128208 )</td>
<td>0.75</td>
<td>0.75</td>
<td>0.156210</td>
</tr>
<tr>
<td>3</td>
<td>Initial Design Variables</td>
<td>( 0.106628 )</td>
<td>0.90</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>0.14264</td>
<td>0.85</td>
<td>1.15</td>
<td>0.982815</td>
</tr>
<tr>
<td></td>
<td>0.300568</td>
<td>1.15</td>
<td>1.00</td>
<td>0.932244</td>
</tr>
<tr>
<td></td>
<td>0.259309</td>
<td>1.15</td>
<td>0.80</td>
<td>0.687146</td>
</tr>
<tr>
<td>Optimum</td>
<td>( 0.270599 )</td>
<td>0.75</td>
<td>0.75</td>
<td>0.156240</td>
</tr>
<tr>
<td>4</td>
<td>Initial Design Variables</td>
<td>( 0.199809 )</td>
<td>0.80</td>
<td>1.10</td>
</tr>
<tr>
<td></td>
<td>0.36446</td>
<td>1.00</td>
<td>0.80</td>
<td>0.113999</td>
</tr>
<tr>
<td></td>
<td>0.174475</td>
<td>1.20</td>
<td>1.20</td>
<td>0.194790</td>
</tr>
<tr>
<td></td>
<td>0.353402</td>
<td>1.10</td>
<td>0.95</td>
<td>0.705562</td>
</tr>
<tr>
<td>Optimum</td>
<td>( 0.325759 )</td>
<td>0.75</td>
<td>0.75</td>
<td>0.156231</td>
</tr>
<tr>
<td>5</td>
<td>Initial Design Variables</td>
<td>( 0.241510 )</td>
<td>1.00</td>
<td>1.20</td>
</tr>
<tr>
<td></td>
<td>0.276891</td>
<td>1.20</td>
<td>1.25</td>
<td>0.959758</td>
</tr>
<tr>
<td></td>
<td>0.171966</td>
<td>0.95</td>
<td>1.15</td>
<td>0.995664</td>
</tr>
<tr>
<td></td>
<td>0.233039</td>
<td>0.80</td>
<td>0.80</td>
<td>0.814310</td>
</tr>
<tr>
<td>Optimum</td>
<td>( 0.351410 )</td>
<td>0.75</td>
<td>0.75</td>
<td>0.156213</td>
</tr>
</tbody>
</table>

Table 6-17  Constraint values of optimum solutions of the 5 cases in Table 6-16

<table>
<thead>
<tr>
<th>Case</th>
<th>( \sigma_1 )</th>
<th>( \sigma_2 )</th>
<th>( \sigma_3 )</th>
<th>( \sigma_4 )</th>
<th>( \sigma_5 )</th>
<th>( \Theta )</th>
<th>dp/dx</th>
<th>ESF</th>
<th>Temp</th>
<th>( T-T_{UA} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \leq 1.09 )</td>
<td>( \leq 1.09 )</td>
<td>( \leq 1.09 )</td>
<td>( \leq 1.09 )</td>
<td>( \leq 1.09 )</td>
<td>(0.9049 &lt; \Theta \leq 1.04)</td>
<td>(0.5 \leq \text{ESF} \leq 1.02)</td>
<td>(0.5 \leq \text{ESF} \leq 1.02)</td>
<td>(0.5 \leq \text{ESF} \leq 1.02)</td>
<td>(0.5 \leq \text{ESF} \leq 1.02)</td>
</tr>
<tr>
<td>1</td>
<td>1.0655041</td>
<td>0.0520671</td>
<td>0.0421991</td>
<td>0.0352711</td>
<td>0.030232</td>
<td>0.9089</td>
<td>1.04</td>
<td>0.729716</td>
<td>0.836745</td>
<td>-0.000004</td>
</tr>
<tr>
<td>2</td>
<td>1.0080251</td>
<td>0.123171</td>
<td>0.0118511</td>
<td>0.0107351</td>
<td>0.006642</td>
<td>0.956374</td>
<td>1.04</td>
<td>0.732871</td>
<td>0.836745</td>
<td>-0.000225</td>
</tr>
<tr>
<td>3</td>
<td>1.0449251</td>
<td>0.0379151</td>
<td>0.0341231</td>
<td>0.0265721</td>
<td>0.022940</td>
<td>0.926447</td>
<td>1.04</td>
<td>0.732774</td>
<td>0.836745</td>
<td>-0.000028</td>
</tr>
<tr>
<td>4</td>
<td>1.0570901</td>
<td>0.0463531</td>
<td>0.0378751</td>
<td>0.0317931</td>
<td>0.027323</td>
<td>0.919580</td>
<td>1.04</td>
<td>0.732842</td>
<td>0.836745</td>
<td>-0.000089</td>
</tr>
<tr>
<td>5</td>
<td>1.0624081</td>
<td>0.0500431</td>
<td>0.0406961</td>
<td>0.0340751</td>
<td>0.029240</td>
<td>0.911311</td>
<td>1.04</td>
<td>0.732936</td>
<td>0.836745</td>
<td>-0.000202</td>
</tr>
</tbody>
</table>

\[32\] All initial experiments are infeasible subject to constraints, and Range* denotes the optimum solution.
### Table 6-18  Results of multiple runs of the 5 cases in Table 6-16

<table>
<thead>
<tr>
<th>Run</th>
<th>Case1</th>
<th>Case2</th>
<th>Case3</th>
<th>Case4</th>
<th>Case5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Range</td>
<td>3946.9</td>
<td>3958.6</td>
<td>3961.3</td>
<td>3962.2</td>
</tr>
<tr>
<td></td>
<td># of SA/MDA</td>
<td>45</td>
<td>40</td>
<td>36</td>
<td>26</td>
</tr>
<tr>
<td></td>
<td>Number of Subsystem Analyses</td>
<td>540(45x4x3)</td>
<td>480(40x4x3)</td>
<td>432(36x4x3)</td>
<td>312(26x4x3)</td>
</tr>
<tr>
<td>2</td>
<td>Range</td>
<td>3946.9</td>
<td>3958.6</td>
<td>3914.4</td>
<td>3962.2</td>
</tr>
<tr>
<td></td>
<td># of SA/MDA</td>
<td>45</td>
<td>40</td>
<td>27</td>
<td>26</td>
</tr>
<tr>
<td></td>
<td>Number of Subsystem Analyses</td>
<td>540(45x4x3)</td>
<td>480(40x4x3)</td>
<td>324(27x4x3)</td>
<td>312(26x4x3)</td>
</tr>
<tr>
<td>3</td>
<td>Range</td>
<td>3946.9</td>
<td>3960.7</td>
<td>3863.5</td>
<td>3962.2</td>
</tr>
<tr>
<td></td>
<td># of SA/MDA</td>
<td>45</td>
<td>58</td>
<td>46</td>
<td>26</td>
</tr>
<tr>
<td></td>
<td>Number of Subsystem Analyses</td>
<td>540(45x4x3)</td>
<td>696(8x4x3)</td>
<td>552(46x4x3)</td>
<td>312(26x4x3)</td>
</tr>
<tr>
<td>4</td>
<td>Range</td>
<td>3946.9</td>
<td>3945.9</td>
<td>3898.8</td>
<td>3944.7</td>
</tr>
<tr>
<td></td>
<td># of SA/MDA</td>
<td>45</td>
<td>34</td>
<td>60</td>
<td>43</td>
</tr>
<tr>
<td></td>
<td>Number of Subsystem Analyses</td>
<td>540(45x4x3)</td>
<td>408(34x4x3)</td>
<td>720(60x4x3)</td>
<td>516(43x4x3)</td>
</tr>
<tr>
<td>5</td>
<td>Range</td>
<td>3946.9</td>
<td>3963.1</td>
<td>3930.5</td>
<td>3960.1</td>
</tr>
<tr>
<td></td>
<td># of SA/MDA</td>
<td>45</td>
<td>33</td>
<td>28</td>
<td>55</td>
</tr>
<tr>
<td></td>
<td>Number of Subsystem Analyses</td>
<td>540(45x4x3)</td>
<td>396(33x4x3)</td>
<td>336(28x4x3)</td>
<td>660(55x4x3)</td>
</tr>
<tr>
<td>6</td>
<td>Range</td>
<td>3959.9</td>
<td>3883.3</td>
<td>3639.8</td>
<td>3960.3</td>
</tr>
<tr>
<td></td>
<td># of SA/MDA</td>
<td>49</td>
<td>39</td>
<td>29</td>
<td>29</td>
</tr>
<tr>
<td></td>
<td>Number of Subsystem Analyses</td>
<td>588(49x4x3)</td>
<td>468(39x4x3)</td>
<td>348(29x4x3)</td>
<td>348(29x4x3)</td>
</tr>
</tbody>
</table>

**Legend**

- ▽: Infeasible Experiment
- △: Initial Feasible Experiment
- *: Global Seed/Experiment
- ○: Local Seed/Experiment
- ◊: Optimal Seed/Experiment

Figure 6-9  Distribution of experimental points over 35 CPM iterations of Case 1 in Table 6-16
Figure 6-10  Trend of Range over 35 CPM iterations of Case 1 in Table 6-16

Figure 6-11  Cumulative # of the SA/MDA over 35 CPM iterations of Case 1 in Table 6-16
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Figure 6-12  Distribution of experimental points over 35 CPM iterations of Case 2 in Table 6-16

Figure 6-13  Trend of Range over 35 CPM iterations of Case 2 in Table 6-16
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Figure 6-14  Cumulative # of the SA/MDA over 35 CPM iterations of Case 2 in Table 6-16

Figure 6-15  Distribution of experimental points over 35 CPM iterations of Case 3 in Table 6-16
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Figure 6-16  Trend of Range over 35 CPM iterations of Case 3 in Table 6-16

Figure 6-17  Cumulative # of the SA/MDA over 35 CPM iterations of Case 3 in Table 6-16
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Figure 6-18 Distribution of experimental points over 35 CPM iterations of Case 4 in Table 6-16

Figure 6-19 Trend of Range over 35 CPM iterations of Case 4 in Table 6-16
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Figure 6-20 Cumulative # of the SA/MDA over 35 CPM iterations of Case 4 in Table 6-16

Figure 6-21 Distribution of experimental points over 35 CPM iterations of Case 5 in Table 6-16

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Figure 6-22  Trend of Range over 35 CPM iterations of Case 5 in Table 6-16

Figure 6-23  Cumulative # of the SA/MDA over 35 CPM iterations of Case 5 in Table 6-16

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In Chapter 5, all test cases were optimized with initial feasible experiments subject to the SA/MDA and constraints. Apparently, it takes a long time to get initial feasible experiments required through a random sampling. In doing this in the conceptual aircraft design problem, the computational costs are listed in Table 6-19 based on 10 independent runs subject to the original constraints and the modified constraints, respectively. Based on the results in Table 6-19, initialization with feasible experiments subject to the constraints is too costly through random sampling. Aforementioned in Section 5.3, the CPM can start with infeasible experiments subject to constraints to generate feasible experiments over CPM iterations following the initialization process. By comparing the average computational cost in Table 6-18 with the costs in Table 6-19, the initialization strategy applied in the CPM is very efficient. The reason to initialize the CPM with infeasible experiments subject to the constraints is that these experiments still reflect the mathematical relation of the problem. In other words, they are mathematically valid, but not physically feasible. Thus, the infeasible experiments can be used for initialization to improve the RBF approximation model.

Table 6-19  Computational cost for generating initial feasible experimental points subject to constraints through a random sampling

<table>
<thead>
<tr>
<th>Constraint</th>
<th># of SA/MDA</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.9049 \leq \Theta \leq 1.04$</td>
<td></td>
<td>116</td>
<td>105</td>
<td>239</td>
<td>261</td>
<td>67</td>
<td>75</td>
<td>104</td>
<td>38</td>
<td>136</td>
<td>75</td>
<td>121.6</td>
</tr>
<tr>
<td>$0.96 \leq \Theta \leq 1.04$</td>
<td></td>
<td>431</td>
<td>331</td>
<td>108</td>
<td>192</td>
<td>101</td>
<td>203</td>
<td>101</td>
<td>266</td>
<td>117</td>
<td>288</td>
<td>213.8</td>
</tr>
</tbody>
</table>

Also, Case 4 in Table 6-16 was optimized with the CPM based on a continuous sampling, as shown in Table 6-20. The average value of Range is less than that of Case 4
listed in Table 6-18. This study shows that the discrete sampling effectively facilitates the optimization process in coping with the difficulties arising from a large-scale problem.

<table>
<thead>
<tr>
<th>Case</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range</td>
<td>3497</td>
<td>2966</td>
<td>3497</td>
<td>2966</td>
<td>3231.5</td>
</tr>
<tr>
<td># of experiments when Range occurred</td>
<td>54</td>
<td>38</td>
<td>54</td>
<td>38</td>
<td>46</td>
</tr>
<tr>
<td># of CPM iterations when Range occurred</td>
<td>25</td>
<td>17</td>
<td>25</td>
<td>17</td>
<td>21</td>
</tr>
</tbody>
</table>

6.3.4 **Optimization Using the CPM subject to the Original Constraints**

The parameters in the CPM are set by

\[
\begin{align*}
\Delta_i &= 0.2 \\
I &= 4 \\
na &= 4
\end{align*}
\]

(6.4)

The reason that \(na\) is increased is because more active design variables could increase the diversity of samples so that more feasible samples could survive subject to the tightened constraint of \(\Theta\). As a result, the chance to reach the real optimum is expected to be high. The constraint of \(\Theta\) is set to \(0.96 \leq \Theta \leq 1.04\). The optimization process was randomly executed 8 times with different initial infeasible experiments. The optimum solution and the computational cost of each Case are reported in Table 6-21. All original data are available in Appendix III. For Case 6, the distribution of experiments is depicted in Figure 6-24; the trend of Range is plotted in Figure 6-25; and the cumulative number of the SA/MDA is shown in Figure 6-26. By referring to the constraint value of the optimum
solution of each case, as shown in Appendix III, the conceptual aircraft design problem is a constrained optimization problem subject to $\Theta$. Clearly, the computational cost is increased in Table 6-21 compared with the cost required by the CPM subject to the modified constraints in Table 6-18. This is because the constraint of $\Theta$ is tightened and active at the optimum solution, and in turn the feasible region in the design space is narrowed down. Consequently, the number of feasible samples is less than that subject to the modified constraints. According to references [33], [34], and [14], optimal values of the design variables reflect many tradeoffs typical for aircraft design. For instance, optimal $t/c$ resulted, in part, from a trade-off between the wave drag and structural weight.

Table 6-21  Results given by the CPM subject to original constraints

<table>
<thead>
<tr>
<th>Case</th>
<th>Range Values of Initial Experiments</th>
<th>Range* (nm)</th>
<th># of the SA/MDA When Range* Occurred</th>
<th># of CPM Iterations When Range* Occurred</th>
<th># of Subsystem Analyses</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>373.8, 800.2, 380.9, 452.2</td>
<td>3910.6</td>
<td>66</td>
<td>33</td>
<td>792(66x4x3)</td>
</tr>
<tr>
<td>2</td>
<td>1005.6, 438.7, 186.3, 155.5</td>
<td>3830.3</td>
<td>33</td>
<td>16</td>
<td>396(33x4x3)</td>
</tr>
<tr>
<td>3</td>
<td>766.1, 223.5, 682.8, 262.7</td>
<td>3806.2</td>
<td>66</td>
<td>34</td>
<td>792(66x4x3)</td>
</tr>
<tr>
<td>4</td>
<td>530.6, 1015.9, 348.6, 445.1</td>
<td>3834.5</td>
<td>61</td>
<td>29</td>
<td>732(61x4x3)</td>
</tr>
<tr>
<td>5</td>
<td>196.5, 740.8, 383.6, 1060.5</td>
<td>3942.5</td>
<td>62</td>
<td>31</td>
<td>744(62x4x3)</td>
</tr>
<tr>
<td>6</td>
<td>397.9, 800.6, 374.7, 300.1</td>
<td>3924.4</td>
<td>39</td>
<td>18</td>
<td>468(39x4x3)</td>
</tr>
<tr>
<td>7</td>
<td>873.7, 874.0, 472.3, 571.9</td>
<td>3958.4</td>
<td>48</td>
<td>24</td>
<td>576(48x4x3)</td>
</tr>
<tr>
<td>8</td>
<td>290.6, 508.1, 405.7, 250.7</td>
<td>3792.2</td>
<td>65</td>
<td>31</td>
<td>780(65x4x3)</td>
</tr>
<tr>
<td>Average</td>
<td>3893.767</td>
<td>55</td>
<td>27</td>
<td>660(55x4x3)</td>
<td></td>
</tr>
</tbody>
</table>
Chapter 6 – Conceptual Aircraft Design

Figure 6-24  Distribution of experimental points over 35 CPM iterations of Case 6 in Table 6-21

Figure 6-25  Trend of Range over 35 CPM iterations of Case 6 in Table 6-21
6.4 OVERVIEW

The CPM successfully solved the conceptual aircraft design problem. The discrete sampling, Adaptive Sampling, initialization strategy, and Active Design Variable Control functioned effectively in the CPM in coping with difficulties arising from large-scale design problems. The initialization strategy increases the efficiency dramatically. The Adaptive Sampling ensures a local optimum solution. The discrete sampling allows the CPM to solve problems with hybrid design variables. The Active Design Variable Control effectively controls the number of active design variables so as to reduce the dimensionality of the solved problem. The idea behind the Active Design Variable
Control is to utilize the sensitivity information, which is a byproduct from the past optimization process. In solving the conceptual aircraft design problem, the CPM is competitively efficient and easily deals with constraints in comparison with the BLISS.

Due to its statistic characteristics, the CPM may have different converged solutions with the same initial experiments. As the number of design variable is large, more random samples are required to cover the design variable space effectively. High computational capacity, such as parallel computing, can facilitate the CPM for solving large-scale MDO problems.
CHAPTER 7

CLOSURE

7.1 CONCLUDING REMARKS

This thesis has developed new methodologies for coordination of coupled disciplines in problems of MDO (Multidisciplinary Design Optimization). These developments involve an innovative Collaboration Model, as well as new methods called BSSDM (Boundary Search and Simplex Decomposition Method) and CPM (Collaboration Pursuing Method). In Chapter 3, the newly proposed Collaboration Model (CM) was formulated based on two mutually dependent approximations from the perspective of physical and mathematical dependencies in couplings. The Collaboration Model plays a key role in selecting feasible sample candidates during the optimization process by giving an interdisciplinary discrepancy / consistency distribution of coupled state parameters subject to the SA/MDA (System Analysis / Multidisciplinary Analysis). The RBF (Radial-basis Function) was chosen, as approximation technique, for implementing the
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Collaboration Model.

In Chapter 4, based on the Collaboration Model, the BSSDM (Boundary Search and Simplex Decomposition Method) presented a different perspective and solution to MDO problems with both convex and non-convex state parameter regions. A novel and robust boundary search strategy was developed for exploring convex state parameter regions. A robust decomposition algorithm (called Quickhull) was applied for convex decomposition. Some features of a convex-like region were extracted and a convex-like decomposition algorithm was proposed. The BSSDM successfully depicted the feasible state parameter region for test cases with convex or convex-like state parameter regions, and yielded successful designs with rich information of the SA/MDA.

In Chapter 5, the CPM (Collaboration Pursuing Method) was developed based on the Collaboration Model and was shown to effectively and efficiently solve general MDO problems in comparison with the CSSO method. Features, as the discrete sampling, Adaptive Sampling, and Active Design Variable Control, effectively enhanced the CPM’s capability of solving relatively large-scale MDO problems. The discrete sampling also extended the CPM’s applicability to MDO problem with hybrid design variables. The adoption of the MPS (Mode-pursuing Sampling) ensured the global optimum solution.

In Chapter 6, a conceptual aircraft design solved with the CPM was demonstrated, as well as a comparison amongst the CPM, BLISS and All-in-One. Challenges of the conceptual aircraft design with sampling-based optimization methods were addressed and elucidated. It was shown that the CPM is efficient and capable of dealing with general
Chapter 7 – Closure

and relatively large-scale MDO problems, by solving various example problems and applications.

Contributions that distinguish this thesis from other researchers’ work are summarized as follows:

1. A novel Collaboration Model (CM) reflecting both the physical and mathematical characteristics of couplings was developed for effectively and efficiently coordinating coupled disciplines / subsystems in a sampling process.

2. New concepts of feasible convex and non-convex state parameter regions were introduced and applied to solve MDO problems.

3. A robust boundary search strategy for convex regions and a decomposition strategy for convex-like regions were developed.

4. A new MDO method named the Boundary Search and Simplex Decomposition Method (BSSDM) was developed and tested.

5. Strategies of discrete sampling, Adaptive Sampling, and Active Design Variable Control were developed for handling large-scale MDO problems.

6. Globally optimum solutions were sought in solving MDO problems by applying the Mode-pursuing Sampling (MPS) method to the MDO framework.

7. A new MDO method named the Collaboration Pursuing Method (CPM) was developed and tested.
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7.2 RECOMMENDATIONS FOR FUTURE RESEARCH

The BSSDM requires a specified MDO formulation, one without system-subsystem design variables. Introducing auxiliary variables may extend the BSSDM’s applicability to more general MDO problems. Also, the BSSDM is not sufficiently mature yet to solve MDO problems with non-convex or convex-like state parameter regions. The algorithm should be further developed. A suggestion for future research is improving the BSSDM’s applicability in handling non-convex state parameter regions, by decomposing a non-convex region into multiple local convex sub-regions.

Both the BSSDM and CPM rely on the Collaboration Model. The Collaboration Model fits state parameters implicitly and explicitly; it may have difficulties with a large number of disciplinary / local design variables. Also, similar difficulties arise in the sampling process (as observed in the conceptual aircraft design). Effectively dealing with large-scale MDO problems is a major challenge for the CPM. The Active Design Variable Control shows an advantage of utilizing the sensitivity information generated during the optimization process. Future integration between the sensitivity analysis and sampling is suggested to possibly alleviate difficulties with a large number of interdisciplinary and local design variables. On the other hand, the sampling itself has parallel characteristics, so future research could utilize parallel computing to overcome the large-scale issues.
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APPENDIX I

AIRCRAFT COMPONENT DESIGN

AI.1 INTRODUCTION

Appendix I presents a specific application problem regarding thermofluid optimization of a helicopter's air cooling intake scoop design [4]. This work was done by the author in his Ph.D. study to gain appreciation of the complexity of an MDO problem, which also involves computationally intensive processes, such as FEA (Finite Element Analysis). This work is thus included in the thesis to help readers for the same purpose.

In the design of external aircraft surfaces, shape is an important factor, but various trade-offs involving other factors must be considered. For example, the engine intake scoop of a helicopter’s cooling bay can be ice prone under certain atmospheric conditions. Its effective shape design involves a complicated trade-off between aerodynamic drag, ice protection and other factors (such as cost, manufacturing, and efficiency). In particular,
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shape optimization could be applied in a way to reduce droplet capturing and ice buildup on an aircraft surface. For example, various modifications of the surface profile could be used for passive shedding of runback water before it refreezes near the engine intake. In this way, an optimized geometrical configuration can lead to improvements in engine efficiency and aircraft controllability. Better ice-protected designs would improve the commercial viability, safety and endurance of flight in winter weather conditions.

This example represents a problem of optimizing the shape of a heated intake scoop under a certain range of air flow rates and ambient temperatures. It considers heat transfer inside the scoop and an external flow process around the scoop. A simplified objective function was motivated based on a curve-fitted cooling efficiency, as described by Hewitt et al. [60]. This objective function integrates the heat transfer process and the external flow process. Therefore, the interaction (coupling) between the two processes is simplified. However, the design problem itself is a multidisciplinary design problem. A heat conduction model and a potential flow model were selected for this problem. Then, a Control Volume-based Finite Element Method (CVFEM) [61], [62] was applied to simulate heat conduction and potential flow. A cubic B-spline curve was used to plot the shape of the scoop for mesh generation.

An Adaptive Response Surface Method (ARSM) [63], [64] was modified and used to optimize the air cooling intake scoop problem. The ARSM is a global optimization method. It fits a quadratic approximation model (called a response surface or surrogate) to the design objective function with a group of real experimental points by calling the simulations of heat transfer and fluid flow. The design variable space is gradually reduced
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according to a specified threshold of the objective function, and the surrogate is gradually refined with more experimental points until the optimum point converges within a desired small design space. Then the final optimal value of the objective function is calculated based on the design variable values of the optimum point of the surrogate by calling a CVFEM solver. The ARSM can find the global or close-to-global optimum solution. In this thesis, numerical formulations of heat conduction and potential flow are integrated with the ARSM. Section AI.2 introduces the CVFEM; Section AI.3 elaborates the ARSM; Section AI.4 shows the air cooling intake scoop design; and Section AI.5 discusses some related issues.

AI.2 NUMERICAL FORMULATION

Both heat conduction and potential flow problems can be described in Laplace’s equation as follows:

\[ \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 \]  

(I.1)

In equation (I.1), \( \phi \) represents either temperature, \( T \), for two-dimensional, steady, heat conduction without internal heat sources, or the velocity potential, \( \phi \), for external potential flow. In particular, the governing equation for two-dimensional heat conduction is given by

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and the governing equation for two-dimensional potential flow is given by

\[ k \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) = 0 \]  

(I.2)

where \( \phi = \phi(x, y) \). The velocity components are \( u = \frac{\partial \phi}{\partial x} \) and \( v = \frac{\partial \phi}{\partial y} \).

Although detailed flow predictions often require the full Navier-Stokes equations (i.e., see reference [65]), rather than the simplified potential flow equations, a low fidelity model can reduce the total computational cost, from the point of view of a Variable-Fidelity Model [66]. In other words, as long as the low fidelity model produces the same trends as a high fidelity model for optimization, the optimal solution given by the low fidelity model should be close to the real answer. In this problem, it is assumed that a potential flow model is reasonably adequate for predicting certain flow trends in the overall optimization problem. The solution of equation (I.1) is obtained using the CVFEM.

In the CVFEM, the two-dimensional solution domain is subdivided into an assembly of linear, quadrilateral, isoparametric finite elements and control volumes, as shown in Figure I-1. A control volume consists of four Sub-control Volumes (SCV) from elements
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surrounding a particular global node that uniquely identifies the control volume. Since equation (I.1) is discretized within each element, the discrete equations are formed locally and independently of the overall mesh configuration. Assembly rules are carried out by conventional finite element procedures [67].

![Schematic of finite element and control volume in the CVFEM](image)

Figure I-1  Schematic of finite element and control volume in the CVFEM

The transformation between local variables and global variables, as well as the interpolation of scalar values within an element, are performed through bilinear shape functions. These shape functions are denoted by \( N_i(s, t) \), where the subscript \( i \) refers to local nodes, and \( s \) and \( t \) refer to local coordinates, as shown in Figure I-1. The value of \( \phi \) and global coordinates corresponding to a local coordinate, \((s, t)\), can then be approximated by
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\[ \varphi(s,t) = \sum_{i=1}^{4} N_i(s,t) \varphi_i \]

\[ x(s,t) = \sum_{i=1}^{4} N_i(s,t) x_i \]  \hspace{1cm} (I.4)

\[ y(s,t) = \sum_{i=1}^{4} N_i(s,t) y_i \]

where

\[ N_1(s,t) = \frac{1}{4} (1+s)(1+t) \]

\[ N_2(s,t) = \frac{1}{4} (1-s)(1+t) \]  \hspace{1cm} (I.5)

\[ N_3(s,t) = \frac{1}{4} (1-s)(1-t) \]

\[ N_4(s,t) = \frac{1}{4} (1+s)(1-t) \]

The subscripts are \( i = 1, 2, 3 \) and 4 for the four local nodes within a linear, quadrilateral element.

Based on these definitions, the numerical solution can be obtained by spatial integration of equation (I.1) over a control volume yielding

\[ \sum_{e,j} Q_{e,j} = 0 \]  \hspace{1cm} (I.6)

where the summation refers to the elements surrounding the global nodes (i.e., \( e = 1, 2, 3, 4 \) for internal nodes). Also, the summation over \( j \) denotes the two surfaces of the Sub-
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control Volume within an element (note: not common edges of the Sub-control Volumes within a control volume shown in Figure I-1). The heat flow (or flow rate for a potential flow problem) across a surface is represented by

\[ Q = \int_s \vec{q} \cdot \vec{dn} \]  (I.7)

where \( \vec{dn} \) is the surface normal and the heat flux vector (or flow rate per unit area), \( \vec{q} \), is given by

\[ \vec{q} = -k \frac{\partial \phi}{\partial x} \hat{i} - k \frac{\partial \phi}{\partial y} \hat{j} \]  (I.8)

where \( k \) represents either the thermal conductivity for heat conduction, or the air density for potential flow. Boundary conditions are applied along the external boundaries of the problem domain. In particular, for the heat conduction problem, convective boundary conditions are applied as follows:

\[ q_w = -k \left. \frac{\partial T}{\partial n} \right|_w = h(T_f - T_w) \]  (I.9)

where the subscripts \( w \) and \( f \) refer to wall and freestream fluid, respectively. Each boundary element contains 2 global nodes along the external boundary, except corners with 3 global nodes on the boundary. Heat flows are supplied through the boundary.
surfaces to complete the heat balances in the boundary control volumes. For example, the convective heat flow supplied at the boundary, based on equation (I.9), is given by

\[ Q = \int h(T_f - T_w)\sqrt{\Delta x^2 + \Delta y^2} \]  

where the latter term (within the square root) represents the surface length of the boundary surface in that element. The values of \( \Delta x \) and \( \Delta y \) are determined from the shape functions, equations (I.4) and (I.5), and the local coordinates at each boundary node. The boundary heat flows from equation (I.10) are added to equation (I.6). In this way, the \( T_w \) portion becomes an active term on the left side of the global system of equations, whereas the \( T_f \) component is moved to the right side of the global system (i.e., constant; not multiplying any active temperature variables). In the following section, the implementation of an optimization method (ASRM) with the finite element solver is described.

**AI.3 OPTIMIZATION METHOD**

The Adaptive Response Surface Method (ARSM) is a global optimization method, as shown in Figure I-2. The ARSM uses a second-order polynomial function as the response surface model based on experimental designs given by a Latin Hypercube Sampling process [68]-[72]. Given the threshold of the surrogate, the design variable space is reduced by maximizing and minimizing a design variable’s bound, subject to other design variables. Two global optimization algorithms, the Simulated Annealing (SA) and the
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Boender-Timmer-Rinnoy-Kan (BTRK) clustering algorithm by Tibor Csendes [73], [74], are applied to search for the minimum of the surrogate and the reduced design variable space. The real value of the optimum point of the surrogate is evaluated by calling a computationally intensive process and is used as an experimental design. The ARSM updates new response according to the reduced design variable space, and converges until the design variable space can not reduced significantly.

![Flowchart of the Adaptive Response Surface Method](image-url)

**Figure I-2** Flowchart of the Adaptive Response Surface Method
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AI.4 AIR COOLING INTAKE SCOOP DESIGN

This section consists of the following four main parts: (1) problem description, (2) objective function, (3) geometry generation, and (4) physical analyses and boundary conditions for heat transfer and potential flow.

AI.4.1 Problem Description

In a helicopter, outside air is forced into the engine cooling bay through a separate intake (see Figure I-3). Under certain atmospheric conditions, the intake scoop is often ice prone, and so it can be heated to reduce or prevent ice buildup on the intake scoop. As the air flows into the intake and over various engine components, it experiences considerable drag forces (often up to 25 – 30% of the total drag) due to pressure, frictional and ice blockage effects. Also the cooling intake scoop should allow enough airflow to satisfy the engine performance requirements. As a result, it is desirable to obtain an optimized shape design of the air cooling intake scoop to reduce the drag and increase the engine efficiency, while preventing the ice buildup through surface heating.

Figure I-3  Schematic of engine intake flow
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In this thesis, the combined problem of icing and engine efficiency is considered. Since many complex parameters describe how engine systems actually operate, the intention here is to formulate the overall problem with simple model coefficients that identify certain overall trends (see Section AI.4.2). A closed form expression for the objective function is given and the ARSM can be applied for optimization. The coefficients in this objective function are not intended to represent a particular aircraft or engine, but rather, they are selected to yield certain expected physical trends. In this way, the suitability of the newly combined CVFEM-ARSM can be assessed.

In practical terms, ice on the helicopter intake scoop surface can be melted in two main ways. One way is to use bleed-air from a compressor to heat the scoop, while the other way uses electrical resistance heating. Generally, based on the material’s thermal limit, the efficiency of the engine depends on the temperature difference between engine burning gas and component \((\Delta T = T_g - T_w)\), where a higher \(\Delta T\) is considered to yield a better efficiency. This means that the temperature difference, \(\Delta T\), should be maximized.

From the point of view of thermodynamics, reducing the amount of compressed air to heat the air-intake scoop sufficiently is a way to improve the performance of the engine. Similarly, reducing the heat input of electrical resistance heating can also improve efficiency, since the power required in such heating must be generated locally. On the other hand, less heating may cause more ice buildup on the intake scoop surface, which may break off (damaging downstream hardware), reduce the air inflow, and/or increase friction drag. Thus, these processes involve certain trade-offs in the aircraft design, thereby requiring an effective optimization.
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AI.4.2 Objective Function

For aircraft, gas turbines need cooling. Practically the whole turbine flow passage is air cooled. The cooling should be intensive because the thermal resistance of metal alloys is limited. The general trend of research and development is to increase the turbine blade cooling efficiency which is determined by the depth of cooling, $\theta$, as follows:

$$
\theta = \frac{(T_g - T_w)}{(T_a - T_w)}
$$

(1.11)

where

$T_g$ is the temperature of the gas flowing over the blade.

$T_a$ is the coolant temperature.

$T_w$ is the temperature of the blade external surface.

The cooling efficiency can be evaluated based on Figure I-4.

![Figure I-4](image.png)

Figure I-4  Depth of cooling ($\theta$) as a function of relative cooling air flow, $m_r$ [60]
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In Figure I-4, the higher \( \theta \) is, the better is the cooling efficiency. \( m_r \) is the relative cooling air flow rate, which is determined as the ratio of the flow rate of the air used for cooling to the flow rate of the air at the compressor inlet. Curve 7 represents the cooling efficiency of the blade with a porous sheath; Curve 6 shows the cooling efficiency of blades with film cooling have several hundreds of small diameter holes (0.4 to 0.6 mm) in the thin-walled sheath; Curves 1 through 5 illustrate the cooling efficiency of blades cooled using different methods of convective cooling. According to reference [60], intensification of heat transfer in the internal passages of cooled blades and reduction of additional losses due to cooling may increase the efficiency of gas-turbine units with cooled turbines.

The following parameters are used in the helicopter’s air cooling intake scoop design.

\( \dot{m}_0 \): reference air mass flow rate, 5.6 kg/s (\( \dot{m}_0 = V_w W_{in} \rho \), where \( W_{in} \) indicates the width of the air-intake port),

\( \dot{m} \): air mass flow rate (as determined by the CVFEM simulation) through intake port (without considering ice prone on the intake surface) calculated by potential flow, with respect to different values of \( x_1, x_2, y_1 \) and \( y_2 \),

\( T_g \): gas temperature past engine component,

\( T_{in}, T_{ao} \): free stream air temperature (253 K),

\( T_c \): engine component temperature,

AR: aspect ratio (intake scoop height, \( H=y_1+y_2 \), divided by \( W = 0.1 \text{ m} \),

\( V_{ao} \): air velocity (40 m/s),

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$h$: air convection coefficient ($160 \text{ W/m}^2\text{K}$),

$r$: $r(x_1, y_1, x_2, y_2, q_w)$ is the fraction of the un-iced area (above $273 \text{ K}$) to the total area of the intake scoop exposed to air (as determined from the CVFEM simulation),

$T_{ad}$: $1900 \text{ K}$,

$\rho$: air density ($1.4 \text{ kg/m}^3$),

$k$: thermal conductivity ($177 \text{ W/mK}$),

$W$: width of intake scoop at the base ($0.1m$).

The general functional form of the objective function was motivated based on a curve-fitted cooling efficiency, as described by Hewitt et al. (1996) [60]. Other coefficients may be approximated to represent the physical trends and dependencies on the aspect ratio, AR, and un-iced ratio, $r$. Such values could include other factors, such as incoming turbulence level, or pressure gradient. However, as discussed earlier, the coefficients were selected to represent certain expected trends, rather than the actual and detailed processes of the internal engine dynamics. The objective function is derived next.

According to Figure I-4, the relation between $\theta$ and $m_r$ is defined as follows:

$$\theta = c_1 m_r^{c_2}$$  \hspace{1cm} (I.12)

where $c_1$ and $c_2$ are constant. Based on Curve 5, we have
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\[ \theta(0.02) = c_1(0.02)^2 = 0.42 \]
\[ \theta(0.06) = c_1(0.06)^2 = 0.61 \]  \hspace{1cm} (I.13)

By solving equation (I.13), we have

\[ \theta = 1.59 m_r^{0.34} \]  \hspace{1cm} (I.14)

Continually, we have

\[ (T_g - T_w) = (T_g - T_a) \theta = (T_g - T_a) 1.59 m_r^{0.34} \]  \hspace{1cm} (I.15)

Two parameters, Aspect Ratio, AR, and Un-iced Ratio, \( r \), are used to complete the formulation of the objective function. Deicing needs part of bleed air right after compressor or electrically generated heat. To increase the engine efficiency, the temperature of burning gas, that is \( T_g - T_w \), needs to be increased. The following physical analyses show interactions amongst engine performance, deicing, and aerodynamics.

1. When \( r \) decreases:
   - Intake port reduced; flow rate reduced; cooling efficiency worse; \( T_g \) reduced;
   - and a large \( r \) preferred.
   - Required heat for deicing small; and \( T_g \) increased.

2. When AR increases:
   - More heat for deicing required; \( T_g \) reduced; and aerodynamic drag increased.
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Air flow rate increased; better cooling efficiency; and $T_g$ increased.

Based on the above analyses, the objective function is formulated, subject to the spans of design variables (see Figure I-5), as follows:

$$\Delta T = 1.59 \left[ T_{ad} (1-r^4) e^{-AR/10} - T_e \right] \left( \frac{\dot{m}r}{\dot{m}_0} \right)^{0.34}$$  \hspace{1cm} (I.16)

Coefficient values of 4 and 10 in equation (I.16) were assigned by the author artificially to reflect the above physical analyses.

![Figure I-5](image)

Figure I-5  Intake scoop shape created with a cubic B-spline curve

The spans of design variables are specified as follows:

---

Unit: m
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\[ 5500 \, W/m^2 < q_w < 7500 \, W/m^2 \]
\[ 0.012 \, m < x_1 < 0.029 \, m \]
\[ 0.017 \, m < y_1 < 0.039 \, m \]
\[ 0.007 \, m < x_2 < 0.024 \, m \]
\[ 0.007 \, m < y_2 < 0.029 \, m \] \hspace{1cm} (I.17)

In these inequalities, \( x_1, y_1, x_2 \) and \( y_2 \) are the coordinate values of control point 1 (CP1) and control point 2 (CP2), respectively, as shown in Figure I-5, and \( q_w \) is the heat flux across the bottom of the cooling intake scoop, as shown in Figure I-6.

![Figure I-6 Schematic of heat transfer problem](image)

**AI.4.3 Geometry Generation**

The geometry of the cooling intake scoop was created using Pro/ENGINEER 2000i [75]. A cubic B-spline curve, as shown in Figure I-5, was used to plot the top surface of the intake scoop, since it has local properties. The whole B-spline curve is plotted based
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on seven control points (CP1, CP2 ..., CP7), in which only two points (CP1 and CP2) were chosen as design points for the optimization process. The geometric dimensions of CP1 and CP2 are set in relative coordinates. This simplifies geometric constraints for the optimization process, as shown in Figure I-5. The reason to choose CP1 and CP2 as design points is that both of them can change the height and width of the scoop. Also, the left segment of the scoop surface is more influential than other sections of the scoop surface on the incoming air through the intake port. Two explicit relationships are defined by

\[
\begin{align*}
y_3 &= y_4 = 1.1(y_1 + y_2) \\
y_5 &= 0.65(y_1 + y_2)
\end{align*}
\]  \hspace{1cm} (I.18)

where \(y_3, y_4\) and \(y_5\) are respective dimensions of CP3, CP4 and CP5 along the \(y\) coordinate direction. Equation (I.18) keeps the shape of the intake scoop consistent with variations of \(x_1, y_1, x_2\) and \(y_2\).

**AI.4.4 Physical Analyses and Boundary Conditions**

The air mass flow rate, \(\dot{m}_{in}\), depends significantly on the width and the height of the intake scoop, as shown in Figure I-7. A bigger width and a larger height can allow greater air mass inflow. The fraction of the un-iced scoop area to the total area, \(r\), is determined by both geometry of the intake scoop and the heat flux value, \(q_{w}\). A smaller size of scoop needs less heat to deliver the same value of \(r\). Several trends indicate that an optimum
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exists. For example, if \( y_1, y_2 \) become large and \( x_1, x_2 \) remain constant, then \( \dot{m}_m \) increases, and so \( \Delta T \) increases (per kg of fuel burned). On the other hand, the system will use more air (more heat) to melt the ice on the scoop surface. This is not desirable since excessive energy is extracted from the system for de-icing and \( \Delta T \) likely decreases. A similar situation occurs with different values of \( x_1 \) and \( x_2 \), with \( y_1 \) and \( y_2 \) held constant. It is shown that the maximum of the objective function occurs when \( r = 0.51 \). This result is given by an exhaustive numerical enumeration of equation (I.16), which output the value of \( \Delta T \) by changing \( r \), while fixing the values of \( AR \) and \( \dot{m} \). It shows that \( \Delta T \) has its biggest value, when \( r \) equals 0.51, for a set of \( AR \) and \( \dot{m} \) (regardless of the values of \( AR \) and \( \dot{m} \)). More details about this study are discussed in Section AI.5.

In order to find \( r \) and \( \dot{m} \) for optimizing equation (I.16), a heat transfer process and a potential flow simulation are respectively defined, as shown in Figure I-6 and Figure I-7. All meshes for the CVFEM simulations were created with ANSYS 6.0 [76] after importing a geometry file from Pro/ENGINEER in IGES format to ANSYS. For the heat transfer problem, the value of \( q_w \) is given as a uniform surface heat flux applied at the base of the intake scoop. The finite element solution of heat conduction with the CVFEM in the intake scoop evaluates the temperature distribution therein. Given the temperature distribution of the intake scoop, \( r \) can be calculated based on its definition. In this analysis, it is assumed that ice can accumulate on the intake surface when the surface temperature falls below zero degrees Celsius in the presence of incoming supercooled droplets. Convective boundary conditions are applied along all surfaces except the base of the intake scoop.
The potential flow solution with the CVFEM gives the velocity field around the intake scoop. In Figure I-7, the velocity of the incoming (freestream) air, $V_{\infty}$, which is parallel to the $x$ direction, is specified at the left inlet. It is known that $\phi = \phi(x, y)$. At the right outlet, $x$ is constant along the outlet boundaries. Thus, $\phi$ is only functionally dependent on $y$ there. Also, the cross-stream velocity component, $v$, is assumed to be zero at the right outlet, which means that $\phi$ is constant at that location. The velocity potential is a relative value in the potential flow field. Thus, $\phi$ is arbitrarily set as 100 at the right outlet. Consider the same value of $\phi$ at the bottom outlet, as the potential flow simulation only focuses on how the geometry of the intake scoop influences the air mass inflow rate. In other words, the influences from other factors, such as boundary conditions, should be
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eliminated. This hypothesis was tested by running the potential flow simulations on a fixed intake scoop geometry with different values of \( \phi \), while keeping the same value of \( \phi \) at both outlets for each simulation. As a result, the same air mass flow was obtained.

The boundary condition at the top of the computational domain is specified by

\[
\frac{\partial \phi}{\partial x} = u = V_w
\]  
(I.19)

or

\[
\frac{\partial \phi}{\partial y} = v = 0
\]  
(I.20)

It means that no air flow is perpendicular to the top boundary, provided that the height of the computational domain is sufficiently large. In this thesis, the boundary condition of \( \frac{\partial \phi}{\partial x} = u = V_w \) is called the Free Boundary Condition and the other condition, \( \frac{\partial \phi}{\partial y} = v = 0 \), is called the Zero Gradient Boundary Condition. Zero flux conditions are specified at the other boundaries, i.e., \( \frac{\partial \phi}{\partial n} = 0 \).

Under the case of a uniform \( \phi \) at both outlets, the air mass flow rate becomes independent of the length of both outlets, L1 and L2, provided that both lengths can enclose the intake scoop. The lengths of L1 and L2 are respectively set as 0.4m and 0.14m, as shown in Figure I-7.

For the potential flow, the streamlines are parallel to each other. Thus, the amount of
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air flowing through each outlet depends on where the stagnation point on the intake scoop is located. Physically, the position of the stagnation point should be located at the leading edge of the intake scoop. When the height of the computational domain exceeds a certain value with the Free Boundary Condition at the top, the position of the stagnation point only depends on the geometry of the intake scoop.

On the other hand, according to the specified boundary conditions of potential flow, we can interpret the external flow case as a duct or channel type flow. These flows (applied potential flow and duct flow) have an analogous type of boundary conditions, as the height of the computational domain is big enough, so that there is no air mass flux across the top boundary (or the $x$ component of air velocity is equal to the free-stream air component at the top of the computational domain). In turn, the role of the intake scoop resembles a valve in pipe flows, as the computational domain resembles a tee junction with one inlet and two outlets. It means that some geometry of the scoop (according to a certain position of the valve) can generate a high flow rate, regardless of the height of the domain (according to the diameter of the pipe). In this thesis, the purpose of the potential flow simulation is to find how the geometry of the scoop influences the air mass flow rates. The above analyses appear to be acceptable if all duct flow simulations maintain the same general trend as the potential flow. 33

To prove the above analyses, we can apply the Zero Gradient Boundary Condition,

---

33 The trend shows the sequence of different geometries of the intake scoop from the biggest air mass flow rate to the smallest rate.
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\[ \frac{\partial \phi}{\partial y} = v = 0, \text{ at the top of the computational domain to simulate the duct flow, and the Free Boundary Condition for the potential flow simulation.} \]

Two study cases were implemented with the duct and potential flow simulations based on different geometrical configurations, as shown in Figure I-8. The results in Figure I-8 show that the pipe flow with the Zero Gradient Boundary Condition has the same trend of air mass flow rate, with respect to the scoop geometry, as the potential flow with the Free Boundary Condition.

Eventually, a value of 0.2\(m\) was set as the height of the computational domain and the Zero Gradient Boundary Condition was applied during the optimization process, as shown in Figure I-7. In this way, the number of elements for the CVFEM simulation and computational time were reduced significantly during the optimization process. After finishing the optimization process, a big height of the computational domain (0.5\(m\) high) is used to calculate \(\dot{m}\) for the optimum case. This optimization strategy is summarized in Figure I-9.

Based on the optimization strategy of the ARSM, many experimental points\(^{34}\) are needed to fit the surrogate for reducing the design variable space during the optimization process. Thus, the above simplification with the use of the low fidelity external flow model significantly reduces the total cost of. The total computational time of the external flow simulations with a height of 0.5\(m\) and Free Top Boundary Condition, \(\frac{\partial \phi}{\partial x} = u = V_\infty\), is 9738 seconds, while the reduced computational domain with a height of 0.2\(m\) and the

\(^{34}\) Experimental points are given by doing computationally intensive simulations, i.e., CVFEM.
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Zero Gradient Boundary Condition, \( \frac{\partial \phi}{\partial y} = v = 0 \), requires 1628 seconds (Pentium III 550 MHz, 256 MB memory).

Case 1 with Zero Gradient Boundary:
\( \frac{\partial \phi}{\partial y} = 0 \)

Case 1 with Free Boundary:
\( \frac{\partial \phi}{\partial \alpha} = u = V_\infty \)

Optimum Case with Zero Gradient Boundary:
\( \frac{\partial \phi}{\partial y} = 0 \)

Optimum Case with Free Boundary:
\( \frac{\partial \phi}{\partial \alpha} = u = V_\infty \)

Figure I-8 Air mass flow rates with different domain heights and top boundary conditions

Height of computational domain (m) / approximate number of mesh elements

0.2/492 0.3/834 0.4/1079 0.5/1295

---

35 Case 1 and optimum case are based on different geometric configurations according to the values of \( x_1 \), \( x_2 \), \( y_1 \) and \( y_2 \).
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Potential Flow with the Free Boundary Condition,
\( \frac{\partial \phi}{\partial x} = u = V_0 \)
and a Big Height, \( H = 0.5m \), of the Computational Domain

Pipe / Duct Flow with the Zero Gradient Boundary Condition,
\( \frac{\partial \phi}{\partial y} = v = 0 \)
and a Small Height, \( H = 0.2m \), of the Computational Domain

End:
Calculate the Final Optimum Value of the Objective Function

Converge?

Start:
Initial Experimental Points

Adaptive Response Surface Method (ARSM)

Heat Transfer Simulation

Start:
Calculate the Final Optimum Value of the Objective Function

Figure I-9  Strategy of flow simulation for optimization\(^{36}\)

Actual aircraft icing processes involve viscous effects, phase change heat transfer, impinging droplets and other complex thermofluid processes [65], [77]. However, the main purpose of the cooling intake scoop design is to test the integration of the ARSM and the CVFEM. Despite the idealization of a potential flow solution, it is expected to provide reasonable trends for a main parameter of interest, namely the intake flow rate. Also, few or no previous studies have attempted to combine three parts of such problems

\(^{36}\) * indicates optimum values.
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(heat transfer, fluid flow and optimization with an ARSM) simultaneously. Once validated, the combined framework will serve as a solid basis, from which future extensions involving droplet dynamics, etc., can be incorporated.

AI.5 RESULTS AND DISCUSSION

During the optimization process, the ARSM called the CVFEM solver 37 times in 4 iteration steps. One call includes both the heat transfer and potential flow simulations. Each iteration step corresponds to one response surface constructed by the ARSM. An example plot of the spatial temperature distribution (subject to values of $x_1, y_1, x_2, y_2$ and $q_w$) is shown in Figure I-10. An example of the external potential flow field (subject to values of $x_1, y_1, x_2$ and $y_2$) is depicted in Figure I-11. Aforementioned, the heat transfer simulation yields the value of $r$, the flow simulation determines the value of $\dot{m}$, and AR is calculated based on $y_1$ and $y_2$.

The optimum value was obtained during the 22nd call to the CVFEM in the first iteration step of the whole optimization process, while retaining this same value in the rest of 3 iteration steps. The final optimum, $\Delta T^\ast$, is 1899.37K, when

$$x_1^\ast = 0.015m, \ y_1^\ast = 0.025m, \ x_2^\ast = 0.007m, \ y_2^\ast = 0.022m, \ q_w^\ast = 5701 W/m^2$$
$$AR^\ast = 0.47, \ r^\ast = 0.5002, \ \dot{m}^\ast = 6.399 kg/s$$

The optimal intake scoop is shown in Figure I-12 with a view scale of 2.0. The
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smallest and the biggest intake scoops are also shown in Figure I-13 and Figure I-14, respectively, with the same view scale of 2.0. As mentioned previously, $\dot{m}^*$ was eventually calculated based on a domain height of 0.5$m$ with the Free Boundary Condition.

The optimization results were validated by an exhaustive numerical enumeration of equation (I.16) with respect to AR, $r$ and $\dot{m}$. Based on the computational domain height of 0.2$m$ with the Zero Gradient Condition at the top, and the ranges of the geometric design variables, reasonable spans of these three parameters are given as follows:

$$0 < r < 1$$
$$5.0 \text{ kg/s} < \dot{m} < 5.5 \text{ kg/s}$$
$$0.24 < \text{AR} < 0.68$$

After finishing the above exhaust enumeration subject to the above spans of $r$, $\dot{m}$ and AR, it was found that the objective function reaches the maximum value when $r$ equals 0.51 for a set of AR and $\dot{m}$. In Figure I-15, the objective function, $\Delta T$, is proportional to the air mass flow rate, $\dot{m}$, which suggests a relatively large size of the intake scoop, thereby causing extra aerodynamic friction and heating. Furthermore, as the value of AR increases, the objective function value decreases at the same $\dot{m}$. Using $\Delta T = 1780K$ \textsuperscript{37} as a threshold with a given $r$ of 0.51, all solutions are depicted in Figure I-16, Figure I-17

\textsuperscript{37} This value is a moderate value chosen from all available experimental points over the optimization process, subject to the computational domain of 0.2$m$ with the Zero Gradient Boundary Condition at the top.
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and Figure I-18, based on all combinations of AR and $m$.

Figure I-10  Temperature distribution within the intake scoop

Figure I-11  Contours of velocity potential in the flow field
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Figure I-12  Optimal cooling intake scoop shape

Figure I-13  Smallest cooling intake scoop shape

Figure I-14  Biggest cooling intake scoop shape
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Figure I-15    Objective function versus Aspect Ratio, AR, and air mass flow rate, $\dot{m}$ \(^{38}\)

Figure I-16    Numerical solutions of the objective function over 1780K (view 1) \(^{39}\)

\(^{38}\) The figure is subject to $r = 0.51$.

\(^{39}\) The results are subject to $r = 0.51$.  

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Figure I-17  Numerical solutions of the objective function over 1780K (view 2)

Figure I-18  Numerical solutions of the objective function over 1780K (view 3)

In Figure I-16, Figure I-17 and Figure I-18, the largest objective function value arises
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at a low AR area with a high \( \dot{m} \). However, as expected in the real situation, the higher intake scoop and wider intake port lead to a high air mass flow rate, \( \dot{m} \). Thus, a low AR implies that both the size of the intake scoop and the air mass flow, \( \dot{m} \), are small. In other words, it suggests that this ideal maximum value of the objective function would not likely exist in practice. Based on the above analyses, in Figure I-16, Figure I-17 and Figure I-18, the corner with a minimum AR and maximum \( \dot{m} \) and the corner with maximum AR and minimum \( \dot{m} \) are impossible. Thus, the theoretical optimum value could be close to the range defined by the other two corners, which is roughly from 1782K to 1788K. During the optimization process of the ARSM, the maximum value of the objective function is 1810K with the computational domain height of 0.2m, subject to the Zero Gradient Boundary Condition. Since the theoretical optimum value (from 1782K to 1788K) is very close to the maximum value of 1810K from the optimization, the optimum solution given by the ARSM is viewed to be reasonably accurate and acceptable.

A1.6 SUMMARY

This chapter has demonstrated a detailed example with thermofluid optimization in aircraft component design (helicopter air cooling intake scoop design). The use of a variable-fidelity model for potential flow simulations has shown a significant improvement in computational efficiency. It is believed that the variable-fidelity model is an effective strategy for MDO problems [66]. Design of Experiment-based optimizers, such as the ARSM, can easily implement parallel computing. For large-scale design problems, parallel computing can speed up the design process.
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The design problem involves couplings between the external flow, de-icing, and engine efficiency. The framework based on the simplified formulation of the cooling intake scoop design provides a basis for future development by considering more realistic interactions between disciplines.
### APPENDIX II

#### CONSTANTS OF POWER CONVERTER PROBLEM

Constants of the Power Converter problem [58]

<table>
<thead>
<tr>
<th>Name</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>EI</td>
<td>Input Voltage (nominal)</td>
<td>$3.25 \times 10^2$</td>
</tr>
<tr>
<td>EIMIN</td>
<td>Input Voltage (minimum)</td>
<td>$2.25 \times 10^2$</td>
</tr>
<tr>
<td>EIMAX</td>
<td>Input Voltage (maximum)</td>
<td>$4.25 \times 10^2$</td>
</tr>
<tr>
<td>EO</td>
<td>Output Voltage</td>
<td>5.0</td>
</tr>
<tr>
<td>PO</td>
<td>Output Power</td>
<td>$5.0 \times 10^2$</td>
</tr>
<tr>
<td>POMIN</td>
<td>Output Power (minimum)</td>
<td>$0.5 \times 10^2$</td>
</tr>
<tr>
<td>VR</td>
<td>Output Ripple Spec.</td>
<td>$5.0 \times 10^{-2}$</td>
</tr>
<tr>
<td>K1</td>
<td>Aspect Ratio, center leg depth/width</td>
<td>1.0</td>
</tr>
<tr>
<td>K2</td>
<td>Aspect Ratio, window height/width</td>
<td>2.0</td>
</tr>
<tr>
<td>XN</td>
<td>Transformer Turns Ratio</td>
<td>16</td>
</tr>
<tr>
<td>PXFR</td>
<td>Transformer Related Losses</td>
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</tr>
<tr>
<td>FR</td>
<td>Switching Ripple Frequency</td>
<td>$0.1 \times 10^6$</td>
</tr>
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</table>
## Appendix II – Constants of Power Converter Problem

<table>
<thead>
<tr>
<th>FC</th>
<th>Winding Pitch Factor</th>
<th>1.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>FW</td>
<td>Window Fill Factor</td>
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</tr>
<tr>
<td>WBOB</td>
<td>Bobbin Thickness</td>
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</tr>
<tr>
<td>BSP</td>
<td>Maximum Flux Density</td>
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</tr>
<tr>
<td>DI</td>
<td>Core Density</td>
<td>0.78 ( \times 10^{4} )</td>
</tr>
<tr>
<td>DC</td>
<td>Copper Density</td>
<td>0.89 ( \times 10^{4} )</td>
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<tr>
<td>DK5</td>
<td>Capacitor Density</td>
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<tr>
<td>KH</td>
<td>Heat Sink Density</td>
<td>88.0</td>
</tr>
<tr>
<td>RO</td>
<td>Copper Resistivity</td>
<td>1.724 ( \times 10^{5} )</td>
</tr>
<tr>
<td>RCK</td>
<td>ESR Time Constant</td>
<td>0.3</td>
</tr>
<tr>
<td>CK</td>
<td>ESR Time Constant</td>
<td>0.1 ( \times 10^{-3} )</td>
</tr>
<tr>
<td>VD</td>
<td>Diode Conduction Drop</td>
<td>0.65</td>
</tr>
<tr>
<td>TND</td>
<td>Diode Turn-on Time</td>
<td>1.0 ( \times 10^{-7} )</td>
</tr>
<tr>
<td>TFD</td>
<td>Diode Turn-off Time</td>
<td>1.0 ( \times 10^{-7} )</td>
</tr>
<tr>
<td>TRE</td>
<td>Diode Reverse Recovery Time</td>
<td>0.5 ( \times 10^{-7} )</td>
</tr>
<tr>
<td>VST</td>
<td>Transistor Saturation Drop</td>
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<tr>
<td>VBE</td>
<td>Transistor Base-Emitter Drop</td>
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<td>Transistor Current Gain</td>
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<td>TSR</td>
<td>Transistor Turn-on Rise Time</td>
<td>1.0 ( \times 10^{-7} )</td>
</tr>
<tr>
<td>TSF</td>
<td>Transistor Turn-off Fall Time</td>
<td>1.0 ( \times 10^{-7} )</td>
</tr>
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<td>RDS</td>
<td>MOSFET On Resistance</td>
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<tr>
<td>CGS</td>
<td>MOSFET Gate-Source Capacitance</td>
<td>8.0 ( \times 10^{-9} )</td>
</tr>
<tr>
<td>COSS</td>
<td>MOSFET Output Capacitance</td>
<td>4.0 ( \times 10^{-10} )</td>
</tr>
<tr>
<td>VGS</td>
<td>MOSFET Gate-Source Voltage</td>
<td>10.0</td>
</tr>
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</table>
# APPENDIX III

## ORIGINAL DATA OF CONCEPTUAL AIRCRAFT DESIGN

Results of the conceptual aircraft optimization in Table 6-21

<table>
<thead>
<tr>
<th>Case</th>
<th>Initial Infeasible Experimental Points</th>
<th>Optimum Design Variables</th>
<th>Optimal State Parameters</th>
<th>Constraints at the Optimum Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>x1 = 0.234500 x2 = 0.355195 x3 = 0.254663 x4 = 0.155419</td>
<td>x1 = 0.120449 x2 = 0.750000 x3 = 0.868750 x4 = 0.800000</td>
<td>y1 = 0.750000 y2 = 0.750000 y3 = 0.750000 y4 = 0.900000</td>
<td>c1 = 44403.824060 c2 = 5442.048306 c3 = 18832.647098 c4 = 0.960814</td>
</tr>
</tbody>
</table>

40 All corresponding values of Range* are listed in Table 6-21.
41 In the column of 'Initial Infeasible Experimental Points', each column from x1 to x10 represents an initial experimental point.
### Appendix III – Original Data of Conceptual Aircraft Design

<table>
<thead>
<tr>
<th>x10 = 1500.</th>
<th>x10 = 1240.</th>
<th>x10 = 1330</th>
<th>x10 = 1140.00</th>
<th>x10 = 1470.00</th>
<th>x10 = -0.278e^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>f1 = 373.7561973</td>
<td>f2 = 800.160567</td>
<td>f3 = 380.8614004</td>
<td>f4 = 452.226646</td>
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<tr>
<td>x1 = 0.287451 x1 = 0.124758 x1 = 0.28205 x1 = 0.361815</td>
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<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>x2 = 1.050 x2 = 1.00000 x2 = 1.15000 x2 = 0.850000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>x3 = 1.050000 x3 = 1.000000 x3 = 0.850000 x3 = 0.800000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>x4 = 0.342438 x4 = 0.11952 x4 = 0.3629314 x4 = 0.14702</td>
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<td>x6 = 58000.00 x6 = 48000.00 x6 = 30000.00 x6 = 32000.00</td>
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<tr>
<td>x7 = 1.500000 x7 = 1.700000 x7 = 1.600000 x7 = 1.700000</td>
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<td>x8 = 5.300000 x8 = 4.000000 x8 = 8.200000 x8 = 7.300000</td>
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<td>x9 = 50.00000 x9 = 50.00000 x9 = 54.00000 x9 = 55.00000</td>
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| f3 = 348.636034 f4 = 445.106795 |

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## Appendix III – Original Data of Conceptual Aircraft Design

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