MINIMUM AVERAGE COST SAMPLING TABLES

A Thesis

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CHAPTER I

THE PROBLEM AND DEFINITIONS OF SYMBOLS USED

Statement of the problem It was the purpose of this study (a) to devise labour saving methods for the construction of sampling inspection tables; and (b) with these methods to develop a set of tables of single sampling plans providing lot quality protection at a minimum average cost of inspection per lot.

Importance of the study A number of sampling inspection tables have been published, notably those developed by Dodge and Romig (4) who based their calculations on certain approximate formulae, thus effecting a great saving in computative effort. In the present study the same approximate formulae were used; however they were evaluated here with *Tables of the Incomplete Beta-Function* (6) and *Tables of the Incomplete Gamma-Function* (7) to further lessen the work of computing.

The Dodge and Romig tables give sampling plans that minimize the average amount of inspection per lot, but they do not state what that amount is; hence they fail to provide an estimate of the inspection costs involved. Furthermore, the sampling plans they list will minimize the average cost of inspection only when the cost of inspection per piece in the remainder of those lots that fail to be accepted by sample is the same as the cost of inspection per piece in the sample. Since this is not always the case in practice, the tables in this study were compiled for a number of different ratios of the two costs. Not only was the appropriate sample plan for each ratio tabulated, but also the relative average amount of the inspection costs involved in that plan.

Definitions of symbols used

N = number of pieces in lot,

n = number of pieces in sample,

p_t = lot tolerance fraction defective,

$\bar{p}$ = process average (expected) fraction defective,

M = $p_t N$ = number of defective pieces in lot of tolerance quality,

m = number of defects found in sample,

c = acceptance number, the maximum allowable number of defective pieces in sample,

P_C = Consumer's Risk, the probability of accepting a submitted lot of tolerance (p_t) quality,

P_P = Producer's Risk, the probability of rejecting a submitted lot drawn from a product of process average ($\bar{p}$) quality,

$\binom{N}{n} = \frac{N!}{(N-n)!\, n!}$ = number of combinations of N things taken n at a time,

b = cost (in a monetary unit) of inspection per piece in the sample,

B = cost (in the same monetary unit as b) of inspection per piece in the remainder of those lots that fail to be accepted by sample,

C = b/B = inspection cost ratio,

Φ_C = average cost (in the same monetary unit as b) of inspection per lot for product of process average ($\bar{p}$) quality,

$\varphi_C = \Phi_C/B$ = relative average cost of inspection per lot for product of process average quality,

p = fraction defective in general.

Prime notation was used in numbering probability formulae in this study as follows:

Exact formulae - no primes;

Formulae based on the binomial approximation - single primes;

Formulae based on the Poisson approximation - double primes.

Organization of the remainder of the thesis The remainder of the thesis was organized in the following manner. The general considerations that influence the choice of a particular method of sampling inspection are discussed in Chapter II. This is followed in Chapter III by an outline of the basic principles of sampling inspection upon which the tables in this study are based. The mathematical development appears in Chapter IV under two distinct headings, (a) sampling from a finite universe and (b) sampling from an infinite universe. Both parts begin with a statement of the applicable exact formulae, from which the approximate formulae are then derived. Finally, this chapter indicates how these approximate formulae may be evaluated exactly with the use of Tables of the Incomplete Beta-Function and Tables of the Incomplete Gamma-Function. Chapter V outlines the computing procedure followed in the construction of the Minimum Average Cost Sampling Tables (Table V), with numerical examples. The uses of these tables are described in Chapter VI, while Chapter VII comprises a study of the nature and magnitude of the errors resulting from the use of approximate formulae. Finally, a general discussion of the problems encountered in the construction of these tables is presented in Chapter VIII, along with a few observations concerning the limitations of the methods developed here.

All tabulations are given in the Addenda, Tables I-V. Tables I-IV illustrate the different steps in the construction of the Minimum Average Cost Sampling Tables, which are presented in Table V.

CHAPTER II

THE CHOICE OF A PARTICULAR METHOD OF SAMPLING INSPECTION

Sampling inspection, as against total inspection, implies that the product that is to be inspected will be considered acceptable even though a certain small percentage of the pieces do not conform to specifications. The choice of a particular method of sampling inspection will depend on certain general considerations such as those described by Dodge and Romig (4, pp. 1-10).

Lot Quality or Average Quality Protection   The choice of a method may start with the fixing of a specific value for the allowable per cent defective, and then choosing either one of two kinds of consumer protection:

(1) Lot Quality Protection, in which this value applies to a finite lot.

(2) Average Quality Protection, in which it applies to the general output of a product.

Tables based on Lot Quality Protection are most useful where each lot is considered as a distinct unit as, for example, where the product goes out to a large number of consumers, each making only intermittent purchases. On the other hand, when constant purchases of large shipments are being made the individual lots tend to lose their identity, and the concept of Average Quality Protection will be more useful.

Single sampling or double sampling   A choice must also be made as to inspection procedure. Either single sampling, double sampling, or multiple sampling may be employed with either type of protection. Double sampling generally requires less sampling than single sampling, especially for large lot sizes and a high grade product. Single sampling however has the advantage of being simpler, both in the use and the development of the tables.

Tables of sample plans giving Lot Quality Protection and Average Quality Protection with a minimum amount of inspection have been developed and published (4) for both single sampling and double sampling procedure.

CHAPTER III

BASIC PRINCIPLES EMPLOYED

The tables developed in this study are for single sampling and are based on lot quality protection with a minimum average cost of inspection. They are drawn up for stated values of the inspection cost ratio, which is the ratio of b, the cost of inspection per piece in the sample, to B, the cost of inspection per piece in the remainder of those lots that fail to be accepted by sample.

The general conditions under which these tables are applicable, and the principles used in their development, are the same as those described by Dodge and Romig (4, pp. 10-14, 25-31), except for the inspection cost ratio which is introduced here. They have therefore been reviewed only briefly here. The basic requirements for the method are:

(a) The specified degree of consumer protection shall be provided for, and

(b) The average cost of inspection per lot shall be a minimum for a product of process (expected) quality.

<u>Inspection procedure</u> The inspection procedure is assumed to be as follows:

(a) Inspect a sample of n pieces.

(b) If the number of defects found in the sample does not exceed c, the allowable defect number, accept the lot.

(c) If the number of defects found in the sample exceeds c, inspect all the pieces in the remainder of the lot.

(d) Correct or replace all defective pieces found.

The protection aspect Consumer protection is defined numerically by specifying values of:

(a) Lot tolerance fraction defective, the allowable fraction defective in a lot.

(b) Consumer's Risk, the probability of accepting a submitted lot having exactly lot tolerance fraction defective.

In this study a Consumer's Risk of 0.10 was used throughout. Hence if a lot of worse than tolerance quality is submitted, the probability of accepting it will be less than one tenth. The sample size n corresponding to each value of c will be uniquely determined by specifying the degree of protection desired.

The economy aspect For each such sample plan (paired values of n and c) there will be an average cost of inspection per lot for a submitted product of process average quality. This cost will consist of two parts:

(a) The cost of inspecting the sample. This is nb, where n is the number of pieces in the sample, and b is the cost of inspection per piece in the sample.

(b) The average (expected) cost of inspecting the remainder of those lots that fail to be accepted by sample. This is $(N-n)\ P_p\ B$, where N is the number of pieces in the lot, P_p is the probability of rejecting a lot of process average quality, known as the Producer's Risk, and B is the cost of inspection per piece in the remainder portions of the rejected lots.

Clearly the first cost factor will be minimized by taking the smallest sample that will give the desired protection, that is, the value of n corresponding to $c = 0$. The second cost factor on the other hand will be minimized by inspecting the whole lot, when it will obviously be zero. The problem was to find the sample plan that would strike a balance between these two cost factors so as to make their sum a minimum for stated values of the inspection cost ratio, b/B.

CHAPTER IV

MATHEMATICAL BACKGROUND

Mathematical probability formulae used in sampling work are either one or the other of two types, depending on whether they involve

(a) Sampling from a finite universe, or

(b) Sampling from an infinite universe.

In determining the sample size, which involves the Consumer's Risk, the sample is considered as being drawn from a finite lot, and probabilities are therefore based on (a). In determining the Producer's Risk the sample is considered as being drawn from the general output of product, a source of supply, and probabilities are therefore based on (b).

I. SAMPLING FROM A FINITE UNIVERSE

The probability of finding m defects in a random sample of n pieces drawn from a finite universe (lot) of N pieces in which the number of defective pieces is $M = p\,N$, is given exactly by

$$P(m,n,N,M) = \frac{\binom{M}{m}\binom{N-M}{n-m}}{\binom{N}{n}} \ . \tag{1}$$

<u>The binomial approximation</u> When $p < 0.10$ a good approximation to equation (1) is given by the $m + 1$st term of the expansion of the binomial, $\left[(1 - \frac{n}{N}) + \frac{n}{N}\right]^M$, that is

$$P(m,n,N,M) \doteq P(m,\tfrac{n}{N},M)$$

where $$P(m,\tfrac{n}{N},M) = \binom{M}{m}\left(1-\frac{n}{N}\right)^{M-m}\left(\frac{n}{N}\right)^{m}.$$

The approximation is arrived at as follows:

$$P(m,n,N,M) = \frac{\binom{M}{m}\binom{N-M}{n-m}}{\binom{N}{n}}$$

$$= \frac{\binom{M}{m}\dfrac{(N-M)!}{(n-m)!\,(N-M-n+m)!}}{\dfrac{N!}{n!\,(N-n)!}}$$

$$= \frac{\binom{M}{m}\dfrac{n!}{(n-m)!}\,\dfrac{(N-n)!}{(N-M-n+m)!}}{\dfrac{N!}{(N-M)!}}$$

$$= \frac{\binom{M}{m}n^{m}\,(N-n)^{M-m}}{N^{M}}\left[\frac{\left\{1\,(1-\frac{1}{n})\ldots.(1-\frac{m-1}{n})\right\}\left\{1\,(1-\frac{1}{N-n})\ldots.(1-\frac{M-m-1}{N-n})\right\}}{\left\{1\,(1-\frac{1}{N})\ldots.(1-\frac{M-1}{N})\right\}}\right]$$

$$= \binom{M}{m}\left(1-\frac{n}{N}\right)^{M-m}\left(\frac{n}{N}\right)^{m} F(N,n,M,m),$$

where $F(N,n,M,m)$ is the expression in the large brackets.

Since $\log\,(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3}\ldots$ for $|x| < 1$, it follows that

$$\log F(N,n,M,m) = \sum_{y=0}^{M-1}\sum_{r=1}^{\infty}\frac{1}{r}\left(\frac{y}{N}\right)^{r} - \sum_{y=0}^{m-1}\sum_{r=1}^{\infty}\frac{1}{r}\left(\frac{y}{n}\right)^{r} - \sum_{y=0}^{M-m-1}\sum_{r=1}^{\infty}\frac{1}{r}\left(\frac{y}{N-n}\right)^{r}.$$

Neglecting terms of the order $1/N^2$, $1/n^2$, and $1/(N-n)^2$, and replacing m by its expected value $(\frac{n}{N})M$ gives the approximation

$$\log F(N,n,M,m) \doteq e^{M/2N} = e^{p/2} \text{ for the maximum term where } m = \left(\frac{n}{N}\right)M,$$

so that

$$P(m,n,N,M) \doteq \binom{M}{m}\left(1-\frac{n}{N}\right)^{M-m}\left(\frac{n}{N}\right)^{m} e^{p/2}.$$

Finally, setting $F(N,n,M,m) = 1$ gives the approximation (See Chapter VII for errors involved)

$$P(m,n,N,M) \doteq P(m,\tfrac{n}{N},M) \tag{1'}$$

where $P(m,\frac{n}{N},M) = \binom{M}{m}(1-\frac{n}{N})^{M-m}(\frac{n}{N})^{m}$.

The Poisson approximation Dodge and Romig (4, p. 43) state that when $p < 0.10$ and when $\frac{n}{N} < 0.10$, a good approximation to equation (1) is given by the $m + 1$st term of the Poisson exponential distribution, that is

$$P(m,n,N,M) \doteq P(m,pn), \quad \text{where} \quad P(m,pn) = \frac{e^{-pn}(pn)^m}{m!}.$$

This approximation is arrived at as follows:

From equation (1')

$$P(m,\tfrac{n}{N},M) = \binom{M}{m}(1-\tfrac{n}{N})^{M-m}(\tfrac{n}{N})^{m}$$

$$= \frac{M(M-1)\ldots.(M-\overline{m-1})}{m!}(1-\tfrac{n}{N})^{M}(1-\tfrac{n}{N})^{-m}(\tfrac{n}{N})^{m},$$

which, when $\frac{n}{N} \to 0$ and $M \to \infty$ in such a way that $\frac{Mn}{N} = pn$ remains constant, yields

$$P(m,\tfrac{n}{N},M) = \frac{1(1-\frac{1}{M})\ldots.(1-\frac{m-1}{M})}{m!}(1-\tfrac{pn}{M})^{M}(1-\tfrac{n}{N})^{-m}(pn)^{m}.$$

Hence in the limit

$$P(m,\tfrac{n}{N},M) \doteq \frac{e^{-pn}(pn)^m}{m!}.$$

From equation (1') we have also

$$P(m,n,N,M) \doteq P(m,\tfrac{n}{N},M),$$

thus giving the approximation (See Chapter VII for errors involved)

$$P(m,n,N,M) \doteq P(m,pn), \qquad \text{where} \quad P(m,pn) = \frac{e^{-pn}(pn)^m}{m!}. \tag{1''}$$

<u>Application of the foregoing approximations to the problem</u> Equations (1') and (1'') are general equations applicable for any fraction defective, p, but were used in this study only for the specific case where $p = p_t$, the lot tolerance fraction defective, and where in turn $M = p_t N$.

The Consumer's Risk, P_C, is the probability of meeting the acceptance criterion, c, in samples drawn from a lot of N pieces containing exactly the tolerance number of defects, $M = p_t N$, so that

$$P_C = \sum_{m=0}^{c} P(m,n,N,M), \quad \text{when } p = p_t, \tag{2}$$

or, using approximations (1') and (1'') with $p = p_t$,

$$P_C = \sum_{m=0}^{c} \binom{M}{m} \left(1-\frac{n}{N}\right)^{M-m} \left(\frac{n}{N}\right)^{m} \tag{2'}$$

and $$P_C = \sum_{m=0}^{c} \frac{e^{-p_t n}(p_t n)^m}{m!}. \tag{2''}$$

Equations (2') and (2'') may be evaluated exactly with <u>Tables of the Incomplete Beta-Function</u> (6) and <u>Tables of the Incomplete Gamma-Function</u> (7) respectively, as shown by Deming (3, pp. 18-20).

From equation (2')

$$P_C = \sum_{m=0}^{c} \binom{M}{m} (1-\frac{n}{N})^{M-m} (\frac{n}{N})^m, \quad \text{when } p = p_t,$$

$$= 1 - \frac{\int_0^{n/N} x^c (1-x)^{M-c-1} dx}{\int_0^1 x^c (1-x)^{M-c-1} dx}$$

$$= 1 - I_x(p,q), \quad \text{where } x = \frac{n}{N} \qquad (3'_a)$$

$$p = c+1$$

$$q = M-c,$$

$$= I_x(p,q), \quad \text{where } x = 1-\frac{n}{N} \qquad (3'_b)$$

$$p = M-c$$

$$q = c+1,$$

where the notation $I_x(p,q)$ for the incomplete Beta-function is that of Pearson (6, p.vi).

From equation (2'')

$$P_C = \sum_{m=0}^{c} \frac{e^{-p_t n} (p_t n)^m}{m!}$$

$$= 1 - \frac{\int_0^{p_t n} x^c e^{-x} dx}{\int_0^{\infty} x^c e^{-x} dx}$$

$$= 1 - I(u,p), \quad \text{where } u = \frac{p_t n}{\sqrt{(c+1)}} \qquad (3'')$$

$$p = c,$$

where the notation I(u,p) for the incomplete Gamma-function is that of

Pearson (7, p. vii).

II. SAMPLING FROM AN INFINITE UNIVERSE

The probability of finding m defects in a random sample of n pieces drawn from an infinite universe (general output of a uniform product) in which the fraction defective is p, is given exactly by the m + 1st term of the expansion of the binomial, $\left[(1-p) + p\right]^n$, that is

$$P(m,n,p) = \binom{n}{m} (1-p)^{n-m} p^m. \tag{4}$$

The Poisson approximation Dodge and Romig (4, p. 44) state that when $p < 0.10$, a good approximation to equation (4) is given by the m +1st term of the Poisson exponential distribution, that is

$$P(m,n,p) \doteq P(m,pn), \quad \text{where} \quad P(m,pn) = \frac{e^{-pn}(pn)^m}{m!}.$$

This approximation is arrived at as follows:

From equation (4)

$$P(m,n,p) = \binom{n}{m} (1-p)^{n-m} p^m$$

$$= \frac{n(n-1)\ldots.(n-\overline{m-1})}{m!} (1-p)^n (1-p)^{-m} p^m$$

$$= \frac{pn(pn-p)\ldots.(pn-p\,\overline{m-1})}{m!} \left(1-\frac{pn}{n}\right)^n (1-p)^{-m},$$

which, when $p \rightarrow 0$ and $n \rightarrow \infty$ in such a way that pn remains constant, gives the approximation (See Chapter VII for errors involved)

$$P(m,n,p) \doteq P(m,pn), \quad \text{where} \quad P(m,pn) = \frac{e^{-pn}(pn)^m}{m!}. \tag{4''}$$

Application of the foregoing approximation to the problem The Producer's Risk, P_p, is the probability of failing to meet the acceptance criterion, c, in samples drawn from a product of process average ($\bar{p}$) quality, so that

$$P_P = 1 - \sum_{m=0}^{c} P(m,n,p), \quad \text{when } p = \bar{p}, \tag{5}$$

or, using approximation (4'') with $p = \bar{p}$,

$$P_P = 1 - \sum_{m=0}^{c} \frac{e^{-\bar{p}n}(\bar{p}n)^m}{m!}, \tag{5''}$$

which, like equation (2''), may be evaluated exactly with Tables of the Incomplete Gamma-Function, that is

$$P_P = 1 - \sum_{m=0}^{c} \frac{e^{-\bar{p}n}(\bar{p}n)^m}{m!}$$

$$= \frac{\int_0^{\bar{p}n} x^c e^{-x}\,dx}{\int_0^{\infty} x^c e^{-x}\,dx}$$

$$= I(u,p), \quad \text{where} \quad u = \frac{\bar{p}n}{\sqrt{(c+1)}} \tag{6''}$$

$$p = c.$$

CHAPTER V

CONSTRUCTION OF MINIMUM AVERAGE COST SAMPLING TABLES

The construction of minimum average cost sampling tables giving lot quality protection consists of the solving of three distinct problems:

I. Determining sample size;

II. Determining Producer's Risk;

III. Determining the minimum average cost of inspection per lot.

The methods employed in this study are therefore discussed under these three headings.

I. DETERMINING SAMPLE SIZE

Given: Lot size (N), lot tolerance fraction defective (p_t), Consumer's Risk ($P_C = 0.10$).

To find: Sample size (n) corresponding to allowable number of defects (c = 0, 1, 2...) to give the specified protection to the consumer.

Method I - using Tables of the Incomplete Beta-Function (6) These tables could only be used where $M = p_t N \leq 50$. Since they are so tabulated that $q \leq p$, it was necessary to use equation ($3_b'$) where $c \leq \frac{M-1}{2}$, and equation ($3_a'$) where $c > \frac{M-1}{2}$.

For $c \leq \frac{M-1}{2}$ values of n were found from equation ($3_b'$) by setting $P_C = 0.10$ giving

$$I_x(p,q) = 0.10, \quad \text{where} \quad p = M-c \qquad (7_a')$$

$$q = c+1$$

$$n = N(1-x).$$

This equation was solved for x, using linear interpolation in the tables. For example, given $N = 1000$, $p_t = 0.05$ and $c = 8$, it follows that

$$M = p_t N = 50$$

$$p = M-c = 42$$

$$q = c+1 = 9.$$

From <u>Tables of the Incomplete Beta-Function</u> then

$$I_x(42,9) = .09160 \quad \text{for } x = .75$$

and $$I_x(42,9) = .12062 \quad \text{for } x = .76,$$

which by linear interpolation gives

$$I_x(42,9) = .10000 \text{ for } x = .7529,$$

so that $$n = N(1-x) = 1000(1-.7529) = 247$$ to the nearest integer.

For $c > \frac{M-1}{2}$ values of n were found from equation ($3_a'$) by setting $P_C = 0.10$ giving

$$I_x(p,q) = 0.90, \quad \text{where} \quad p = c+1 \qquad (7_b')$$
$$q = M-c$$
$$n = Nx,$$

where again x was found from the tables by linear interpolation. For example, given $N = 500$, $p_t = 0.05$, $c = 13$, it follows that

$$M = p_t N = 25$$

$$p = c+1 = 14$$

$$q = M-c = 12.$$

<u>Tables of the Incomplete Beta-Function</u> give

$$I_x(14,12) = .89559 \quad \text{for } x = .66$$

and $$I_x(14,12) = .91411 \quad \text{for } x = .67.$$

By linear interpolation then

$$I_x(14,12) = .90000 \quad \text{for } x = .6624,$$

so that $$n = Nx = 500(.6624) = 331$$ to the nearest integer.

The method is illustrated in tabular form in Table I.

Method II - using Tables of the Incomplete Gamma-Function (7)

For those parts of the sampling tables where $M = p_t N > 50$, values of n were found by setting $P_C = 0.10$ in equation (3''), giving

$$I(u,p) = 0.90, \quad \text{where} \quad p = c \tag{8''}$$

$$n = \frac{u\sqrt{(c+1)}}{p_t} .$$

This equation was solved for u, using linear interpolation in the tables. (Note: Since the lot size is considered infinite in deriving equation (8''), the value of n does not depend on N in Method II) For example, for $p_t = 0.02$ and $c = 5$, Tables of the Incomplete Gamma-Function give

$I(u,5) = .8881$ for $u = 3.7$

and $I(u,5) = .9018$ for $u = 3.8$. By linear interpolation then

$I(u,5) = .9000$ for $u = 3.787$,

so that $n = \frac{u\sqrt{(c+1)}}{p_t} = \frac{3.787\sqrt{6}}{.02} = 464$ to the nearest integer.

This value of n was then used wherever $p_t = 0.02$ and $c = 5$ regardless of the value of N except for $N \leq 2000$ where it was possible to determine n by Method I which is more accurate. Method II is illustrated in tabular form in Table II.

II. DETERMINING PRODUCER'S RISK

Given: Sample size (n), acceptance number (c), process average fraction defective ($\bar{p}$).

To find: Producer's Risk (P_p), the probability of rejecting a submitted lot drawn from a product of process average ($\bar{p}$) quality.

Method: Values of P_p were found from equation (6''),

$$P_p = I(u,p), \quad \text{where} \quad u = \frac{\bar{p}n}{\sqrt{(c+1)}}$$

$$p = c,$$

using Tables of the Incomplete Gamma-Function with linear interpolation. For example, given that $p_t = 0.04$, $n = 97$, $c = 1$ and $\bar{p} = 0.02$, it follows

that $u = \frac{\bar{p}n}{\sqrt{(c+1)}} = \frac{.02(97)}{\sqrt{2}} = 1.3718$

and $p = c = 1.$

Tables of the Incomplete Gamma-Function give

$I(u,1) = .5485$ for $u = 1.3$

and $I(u,1) = .5885$ for $u = 1.4$. By linear interpolation then

$I(u,1) = .577$ for $u = 1.3718$,

so that $P_p = .577$.

The tables presented in this study were drawn up with three values of $\bar{p}$ for each value of p_t and hence each pair of n and c values yielded three values of P_p, one for each value of $\bar{p}$. A saving in time was therefore effected by tabulating the work as shown in Table III.

III. DETERMINING THE MINIMUM AVERAGE COST OF INSPECTION PER LOT

Given: Lot size (N), sample plans (paired values of n and c), Producer's Risk (P_p), inspection cost ratio ($C = b/B$, where b is the cost of inspection per piece in the sample and B is the cost of inspection per piece in the remainder of those lots that fail to be accepted by sample).

To find: (a) The sample plan that will minimize Φ_C, the average cost of inspection per lot for a product of process average ($\bar{p}$) quality, and (b) φ_C, the relative amount of that cost.

Method: Φ_C is composed of two parts:

(1) Cost of inspecting the sample. This is always nb.

(2) Expected or average cost per lot of inspecting the remainder of those lots that fail to be accepted by sample. This is $(N-n)\ P_P\ B$. It follows that

$$\Phi_C = nb + (N-n)\ P_P\ B, \tag{9}$$

where P_P is as defined in equation (6''). To find the sample plan to minimize Φ_C it was only necessary to find the plan that would minimize the relative average cost of inspection, φ_C, given by

$$\varphi_C = \frac{\Phi_C}{B} = nC + (N-n)\ P_P, \tag{10}$$

where $C = b/B$, the inspection cost ratio. From equation (10) φ_C was calculated for each inspection cost ratio, C, a number of sample plans being tried until a minimum φ_C was found. The computing was simplified by tabulating the work as shown in Table IV.

The product $(N-n)\ P_P$ was first punched in the machine and then the different multiples of n, viz., 10n, 9n, 8n..., added to it, the results being tabulated in rows. After a few minima had been found, they were underlined in red. Succeeding minima were then found with only a few trials by observing the trend. Finally, these minimum values of φ_C, were rounded to three significant figures and tabulated along with the corresponding sample plans in Minimum Average Cost Sampling Tables (Table V).

CHAPTER VI

USES OF THE TABLES

These tables give the combinations of the sample size and allowable defect number that will provide the specified consumer protection at a minimum average cost of inspection per lot, and they also give the relative amount of that cost. From this relative cost figure may be obtained the average cost in dollars simply by multiplying the relative cost by B, the cost of inspection per piece in the remainder of those lots that fail to be accepted by sample. These cost figures, besides being useful as an estimate of the actual inspection costs, serve as a basis of comparison of different plans on a cost basis.

They will show, for example, to what extent inspection costs could be reduced by a certain decrease in the process average. This saving might then be compared with the decrease in costs that would result from the use of a higher lot tolerance per cent defective. Still another form of savings in inspection costs could be effected by the use of larger lot sizes. Inspection costs relative to the lot size will always be lower for larger lot sizes and, since these tables make this difference measurable, this difference might now be considered as one of the factors in setting the price differential between small and large quantity purchases. Numerical examples are given below to illustrate the method.

How to find the appropriate sample plan and the average cost of inspection per lot

The problem: Suppose the ABC Co. is manufacturing a product that has been observed over a period of a year to have an average fraction defective of 0.02, with only minor fluctuations in the quality from day to day. They wish to set up a sampling inspection plan such that they will be able to guarantee their customers that if any lot should have a fraction defective of 0.04 then the chances that it will pass inspection will not be more than one in ten. For lots having more than 0.04 fraction defective the chances of passing inspection will then of course be less than one in ten. The product is sold in lots of 500 pieces each, and the cost of inspecting a piece in the sample is about 16 cents whereas the cost of inspecting a piece in the remainder of those lots that fail to be accepted by sample is about 20 cents. What sampling plan will insure the desired protection to the consumer at a minimum average cost of inspection per lot?

<u>The solution</u>: Lot tolerance per cent defective is 4 per cent, and process average is 2 per cent, lot size is 500, and the inspection cost ratio is .16/.20 or 0.8. For these values the tables give n = 208, c = 5, and relative average cost = 237. This means that a sample of 208 pieces should be drawn at random from each lot and inspected. The lot should be accepted if the sample has five or less defective pieces, and rejected if it has more than five defective pieces. The average cost of this inspection plan will be 237($0.20) = $47.40 per lot of 500 pieces.

<u>How to compare alternative means of reducing inspection costs</u>

<u>The problem</u>: Suppose the company wishes to know how they could reduce this inspection cost, and to what extent.

<u>The solution</u>: They might investigate the possibility of improving their manufacturing process to get a process average of 1 per cent, say, instead of 2 per cent. Other factors being unchanged, they would then require a sample size of only 152 with an allowable defect number of 3. The average cost of the plan would then be 146($0.20) = $29.20 per lot of 500 pieces.

A second alternative would be to guarantee the consumer less protection by using a lot tolerance per cent defective of 5 per cent instead of 4 per cent. Other factors being unchanged, they would then require a sample of size 170 with an allowable defect number of 5. The average cost of inspection would then be 179($0.20) = $35.80 per lot of 500 pieces.

A third alternative would be to sell their product in larger lots, say of size 1000. With other factors the same, the required sample size would then be 224 with an allowable defect number of 7. Average cost of the plan would then be 245($0.20) = $49.00 per lot of 1000 or $24.50 per 500 pieces.

All of the three alternatives effect a saving over the method given in the preceding section, and although cost is only one of the many factors to be considered in setting up a sampling scheme, a comparison of the amounts saved by the respective schemes should serve as a useful guide in making the choice.

CHAPTER VII

NATURE AND MAGNITUDE OF ERRORS

The use of the binomial approximation, equation (2'), and the Poisson approximation, equation (2''), in determining the values of n resulted in errors in the value of the Consumer's Risk. Errors in the Producer's Risk resulted from the use of the Poisson approximation, equation (5''). The following is a brief study of the magnitude of the errors arising from these three sources.

Errors in the Consumer's Risk due to the binomial approximation

Equation (2') is based on equation (1'), which involved two approximations in its derivation, viz.:

(a) Neglecting terms of the order $1/N^2$, $1/n^2$ and $1/(N-n)^2$ in the expansion of log F(N,n,M,m), and

(b) Replacing m by (n/N) M, and then setting $e^{p/2} = 1$.

Although an upper bound to the error caused by (a) can be found in the form of a function of the variables involved, this function was found to be too unwieldy to determine its maximum numerical value over the range of the tables. For the error resulting from (b), however, a numerical upper bound was easily established as follows:

Assuming that terms of the order $1/N^2$, $1/n^2$ and $1/(N-n)^2$ can be neglected, as they in fact were, it follows that

$$\log F(N,n,M,m) \doteq A_m ,$$

where $$A_m = \sum_{y=0}^{M-1} (y/N) - \sum_{y=0}^{m-1} (y/n) - \sum_{y=0}^{M-m-1} (y/(N-n))$$

$$= \frac{1}{2}\left[\frac{M^2-M}{N} - \frac{m^2-m}{n} - \frac{M^2-2Mm+m^2-M+m}{N-n}\right] \quad \text{for all } m.$$

For the maximum term where $m = (n/N)M$ this general expression for A_m reduces to $M/2N = p/2$. Consider now the terms where $m \neq (n/N)M$, in particular those terms where $m < (n/N)M$, since these are the only values of m for which equation (1') was used in this study.

From the definition of A_m it follows that

$$A_{m-1} = A_m + \left[\frac{m-1}{n} - \frac{M-m}{N-n}\right] \quad \text{so that} \quad A_{m-1} \leq A_m$$

if $\frac{m-1}{n} \leq \frac{M-m}{N-n}$ or if $Nm-N \leq Mn-n$.

This last inequality is certainly true for $m \leq (n/N)M$ since n is always $\leq N$. Hence it follows that $A_{m-1} \leq A_m$ for $m \leq (n/N)M$. Similarly $A_{m-2} \leq A_{m-1}$, etc. Hence $A_m \leq p/2$ for all $m \leq (n/N)M$, and $F(N,n,M,m) \leq e^{p/2}$ for all calculations that were based on equation (1') (with the assumption that terms of the order $1/N^2$, $1/n^2$, and $1/(N-n)^2$ are equal to zero) so that

$$P(m,n,N,M) \leq e^{p/2} P(m,\tfrac{n}{N},M)$$

$$= 1.0513\ P(m,\tfrac{n}{N},M) \quad \text{for } p = 0.10.$$

Hence when n is determined from equation (2'), which is based on equation (1'), and where $p = p_t \leq 0.10$, the Consumer's Risk, P_C, by exact mehhods should not exceed the stated value of 0.10 by more than about five per cent of 0.10.

Since $A_{m-1} \leq A_m$ it follows that log F(N,n,M,m) may be negative for small m and thus result in a Consumer's Risk that will be less than the stated value by more than five per cent; however an error in this direction will result in more protection to the consumer than is specified, never less.

Exploratory checks over that part of the tables where n was found from the binomial approximation gave a Consumer's Risk by exact methods as low as 0.0836 but never greater than the stated value, 0.10. In general these checks showed that the error in the Consumer's Risk was greater for larger values of p_t and smaller values of n.

Errors in the Consumer's Risk due to the Poisson approximation

A method of checking the accuracy of the Poisson exponential approximation to the binomial $\left[(1-p) + p\right]^n$ is given by Fig. 5 of Campbell's paper (1, p. 100). However it is not applicable here since the Poisson used in equation (2'') is derived from the binomial $\left[(1-\frac{n}{N}) + \frac{n}{N}\right]^M$.

Exploratory checks were made over that part of the table where n was found from the Poisson approximation. These gave a Consumer's Risk by exact methods as low as 0.0450 but never greater than the stated value, 0.10. The checks showed that the error in the Consumer's Risk was greater for larger values of p_t and n/N.

Errors in the Producer's Risk due to the Poisson approximation

The Poisson approximation, equation (5''), was used here in place of the binomial $\left[(1-\bar{p}) + \bar{p}\right]^n$ to find the Producer's Risk, so that Fig. 5 of Campbell's paper was useful in determining the error involved. Since

$P(c,n,a)$ in that paper denotes the probability of finding c or more defects in a sample of size n when the expected number is a, it follows that the Producer's Risk, P_P, is given exactly by

$$P_P = P(c+1,n,a) \quad \text{where} \quad a = \bar{p}n.$$

Campbell's Fig. 5 gives curves of A, the first coefficient in the expansion of the ratio of the increments in probability due to a decrease in n (from ∞) and to unit increase in c. Denoting this ratio by r_P we have, using Campbell's notation,

$$r_P = \frac{P(c+1,n,a) - P(c+1,\infty,a)}{P(c+2,\infty,a) - P(c+1,\infty,a)}$$

$$= A/n - (\text{terms of the order } 1/n^2),$$

where $A = \frac{1}{2}(c+1)(c-a)$ and $a = \bar{p}n$.

It can be seen from Fig. 5 that the coefficient A, and hence the error ratio r_P, is zero for $P_P \doteq 0.45$, positive for P_P less than this, and negative for P_P greater than this, except for the small c values, where the zero error will occur for smaller values of P_P than this. Since the denominator in the expression

$$r_P = \frac{P(c+1,n,a) - P(c+1,\infty,a)}{P(c+2,\infty,a) - P(c+1,\infty,a)}$$

is essentially negative it follows that the numerator will be of opposite sign to that of r_P. Hence the lower values of the Producer's Risk will tend to be overstated and the higher values will tend to be understated by the Poisson approximation.

An upper bound to the absolute value of the error ratio over the range of the Minimum Average Cost Sampling Tables can be established from an inspection of Fig. 5 as follows. In the tables

$$0.00001 < P_P < 0.700$$

and $$0 \leq c \leq 46.$$

If c is replaced by c+1 in Fig. 5, these boundaries of P_P and c will determine the region of Fig. 5 to be considered. The value of the coefficient A in this region ranges from about -100 to +600, so that the value of r_P ranges from about -100/n to +600/n. Hence for n = 23, which is the lowest value of n in the tables, the upper bound to $|r_P|$ would be established at about 25.

However, since the smaller n values correspond to the smaller c values, which in turn correspond to smaller values of $|A|$, it was possible to establish a much lower upper bound than this by calculating values of r_P from the approximation

$$r_P \doteq A/n, \quad \text{where} \quad A = \frac{1}{2}(c+1)(c-\bar{p}n),$$

for exploratory cases throughout the tables. The values of the error ratio found in this way ranged from -0.025 to +0.755. Hence the error in the Producer's Risk due to the use of the Poisson approximation in place of the binomial should not be more than about three quarters of the error in the Producer's Risk that would have been caused by the use of a value of c greater by one than the specified value.

Finally a number of values of the Producer's Risk were calculated by the exact method and compared with the approximate values that were used in the construction of the tables. These checks showed that the lower values of the Producer's Risk were overstated and the higher values were understated by the Poisson approximation, and thus confirmed the information given by Campbell's paper. The error in the Producer's Risk in absolute value was found to be greater for larger values of $\bar{p}$ and smaller values of n. The largest error observed occurred when the exact method gave a Producer's Risk of 0.693 as against 0.683 by the Poisson approximation. This error of -0.010 in the Producer's Risk resulted in an understatement of the average minimum cost of inspection of not more than $1\frac{1}{4}$ per cent.

CHAPTER VIII

DISCUSSION

The binomial approximation is a better one than the Poisson approximation for determining values of n. However the binomial was evaluated in this paper with Tables of the Incomplete Beta-Function (6) in which both p and q range from 0.5 to 50 and therefore the use of the binomial for finding n was restricted to the following parts of the sampling tables:

$p_t = 0.10, 0.07$: for N = 500,

$p_t = 0.05, 0.04, 0.03$: for N = 500, 1000,

$p_t = 0.02$: for N = 500, 1000, 2000.

Values of n for the balance of the tables were found using the Poisson approximation which, according to Dodge and Romig (4, p. 43) is good when $p_t < 0.10$ and when $n/N < 0.10$. Although in the tables presented here, values of p_t never exceeded 0.10, the values of n/N did exceed 0.10 over a considerable portion of the tables, running as high as 0.50 in a few extreme cases. These high values of n/N, which were necessitated by the use of low inspection cost ratios, account for the tables giving a Consumer's Risk as low as 0.045 when it should be 0.100. However as a result of these errors the tables will always give more than the specified protection, never less.

The method outlined in this paper can be extended to the construction of minimum average cost sampling tables using other values of N, p_t,

$\bar{p}$, C and P_C, with certain restrictions on their range. For example, since the error introduced by using the binomial approximation to determine n increases with p_t, there exists a practical upper limit to the value of p_t when Tables of the Incomplete Beta-Function are used to find n. Furthermore since the error resulting from the use of the Poisson approximation for finding n increases with n/N, this fraction as well as p_t must be kept reasonably low over any part of the sampling tables where Tables of the Incomplete Gamma-Function (7) are used to find n. This in turn implies that the inspection cost ratio cannot be too low over that part of the tables, since the lower the inspection cost ratio the greater the fraction of the lot that must be inspected to minimize the average cost of inspection. Also if n/N is to be kept low, then the ratio $\bar{p}/p_t$ must be considerably less than unity. It was found necessary in the present tables to keep $\bar{p}/p_t \leq 0.5$ in order to keep $c \leq 50$, since that is the greatest value of p given in Tables of the Incomplete Gamma-Function.

BIBLIOGRAPHY

1. Campbell, George A., 'Probability Curves Showing Poisson's Exponential Summation', The Bell System Technical Journal, 2:95-113, January, 1923.

2. Coggins, Paul P., 'Some General Results of Elementary Sampling Theory for Engineering Use', The Bell System Technical Journal, 7:26-69, January, 1928.

3. Deming, William Edwards, The Gamma and Beta Functions. Washington: Graduate School, Dept. of Agriculture, 1944. 37 pp.

4. Dodge, Harold F., and Harry G. Romig, Sampling Inspection Tables. New York: John Wiley and Sons, Inc., 1946. 106 pp.

5. Kavanagh, A. J., 'On the Selection of an Inspection Plan', Industrial Quality Control, 2:10-11, March, 1946.

6. Pearson, Karl, Tables of the Incomplete Beta-Function. London: The Biometrika Office, University College, 1934. 494 pp.

7. ______, Tables of the Incomplete Gamma-Function. London: The Biometrika Office, University College, 1946. 164 pp.

ADDENDA

TABLE I

DETERMINING SAMPLE SIZE n USING TABLES OF THE INCOMPLETE BETA-FUNCTION

(p_t=.02, N=500, M=10)

c	p	q	x	1-x	n: 500x Using eqn. (7_b)	n: 500(1-x) Using eqn. (7_a)
0	10	1	.7942	.2058		103
1	9	2	.6630	.3370		169
2	8	3	.5503	.4497		225
3	7	4	.4482	.5518		276
4	6	5	.3541	.6459		323
5	6	5	.7328		366	
6	7	4	.8125		406	
7	8	3	.8843		442	
8	9	2	.9456		473	
9	10	1	.9895		495	

TABLE II

DETERMINING SAMPLE SIZE n USING TABLES OF THE INCOMPLETE GAMMA-FUNCTION

Col. (1) $p=c$	Col. (2) u	Col. (3) $u\sqrt{c+1}$	p_t .02 n=50.00 x Col. (3)	.03 n=33.33 x Col. (3)	.04 n=25.00 x Col. (3)	.05 n=20.00 x Col. (3)	.07 n=14.29 x Col. (3)	.10 n=10.00 x Col. (3)
0	2.303	2.303	115	77	58	46	33	23
1	2.752	3.892	195	130	97	78	56	39
2	3.074	5.324	266	177	133	106	76	53
etc.								

Lot Tolerance Per Cent Defective = 3.0%

Process Average	Cost Ratio	N = 500			1,000			2,000			3,000			4,000			5,000			10,000		
0.5%	10	71	0	838	74	0	1030	77	0	1380	130	1	1700	130	1	1840	130	1	1980	177	2	2370
	9	71	0	767	74	0	952	77	0	1310	130	1	1570	130	1	1710	130	1	1850	177	2	2190
	8	71	0	696	74	0	878	77	0	1230	130	1	1440	130	1	1580	177	2	1710	177	2	2010
	7	71	0	625	74	0	804	77	0	1150	130	1	1310	130	1	1450	177	2	1530	223	3	1850
	6	71	0	554	74	0	730	130	1	1040	130	1	1180	177	2	1290	177	2	1360	223	3	1610
	5	71	0	483	74	0	656	130	1	910	130	1	1050	177	2	1120	177	2	1180	223	3	1360
	4	71	0	412	74	0	582	130	1	780	177	2	879	177	2	940	177	2	1000	223	3	1160
	3	71	0	341	124	1	485	177	2	642	177	2	702	177	2	763	223	3	800	267	4	917
	2	71	0	270	124	1	361	177	2	465	223	3	522	223	3	550	223	3	577	267	4	650
	1	118	1	163	168	2	213	223	3	272	223	3	299	267	4	311	267	4	323	309	5	361
	.9	118	1	152	168	2	196	223	3	250	267	4	273	267	4	285	267	4	297	309	5	330
	.8	118	1	140	168	2	179	223	3	227	267	4	246	267	4	258	267	4	270	309	5	299
	.7	159	2	128	168	2	162	223	3	205	267	4	219	267	4	231	309	5	241	309	5	268
	.6	159	2	112	209	3	145	267	4	181	267	4	193	267	4	205	309	5	210	351	6	234
	.5	159	2	95.7	209	3	122	267	4	154	267	4	166	309	5	174	309	5	180	351	6	198
	.4	159	2	79.8	209	3	101	267	4	127	309	5	138	309	5	143	309	5	149	351	6	163
	.3	159	2	63.9	209	3	80.3	267	4	101	309	5	107	309	5	112	351	6	116	393	7	128
	.2	196	3	44.7	249	4	56.9	309	5	70.9	309	5	76.2	351	6	78.8	351	6	81.2	393	7	88.2
	.1	232	4	25.1	288	5	31.5	351	6	39.0	351	6	41.4	393	7	42.9	393	7	43.9	433	8	47.6

Lot Tolerance Per Cent Defective = 3.0%.

Process Average	Cost Ratio	N = 500			1,000			2,000			3,000			4,000			5,000			10,000		
1.0%	10	71	0	928	74	0	1220	77	0	1800	77	0	2340	130	1	2740	177	2	3030	267	4	3960
	9	71	0	857	74	0	1150	77	0	1720	130	1	2240	177	2	2590	177	2	2850	309	5	3690
	8	71	0	786	74	0	1080	77	0	1650	130	1	2110	177	2	2410	177	2	2680	309	5	3380
	7	71	0	715	74	0	1000	77	0	1570	177	2	1980	177	2	2240	223	3	2450	309	5	3070
	6	71	0	644	74	0	927	130	1	1480	177	2	1800	223	3	2040	223	3	2230	351	6	2750
	5	71	0	573	74	0	853	130	1	1350	177	2	1620	223	3	1820	267	4	1960	351	6	2400
	4	71	0	502	74	0	779	177	2	1180	223	3	1410	267	4	1560	309	5	1680	393	7	2020
	3	71	0	431	124	1	680	223	3	1000	267	4	1160	309	5	1270	351	6	1360	433	8	1620
	2	71	0	360	168	2	534	267	4	764	309	5	870	351	6	944	393	7	1000	474	9	1170
	1	159	2	232	249	4	330	351	6	460	393	7	516	433	8	552	474	9	580	553	11	662
	.9	159	2	216	249	4	305	351	6	425	433	8	475	433	8	508	474	9	532	553	11	606
	.8	196	3	198	249	4	280	351	6	390	433	8	432	474	9	462	474	9	485	553	11	551
	.7	196	3	178	288	5	253	393	7	351	433	8	389	474	9	414	514	10	435	593	12	493
	.6	196	3	159	288	5	225	393	7	312	474	9	344	514	10	367	553	11	383	632	13	432
	.5	232	4	139	325	6	195	433	8	269	474	9	296	514	10	315	553	11	328	632	13	369
	.4	232	4	116	325	6	162	474	9	225	514	10	247	553	11	261	553	11	272	632	13	306
	.3	266	5	92.4	361	7	128	474	9	178	553	11	194	553	11	206	593	12	214	671	14	239
	.2	266	5	65.8	397	8	91.9	553	11	127	593	12	138	632	13	146	632	13	151	749	16	168
	.1	329	7	36.3	432	9	50.9	593	12	70.9	671	14	76.5	710	15	80.3	710	15	83.1	825	18	91.1

Lot Tolerance Per Cent Defective = 3.0%.

Process Average	Cost Ratio	N = 500			1,000			2,000			3,000			4,000			5,000			10,000		
1.5%	10	71	0	991	74	0	1360	77	0	2090	77	0	2770	77	0	3460	130	1	4120	309	5	6190
	9	71	0	920	74	0	1290	77	0	2010	77	0	2700	77	0	3380	177	2	3980	351	6	5840
	8	71	0	849	74	0	1210	77	0	1930	77	0	2620	130	1	3280	177	2	3800	433	8	5460
	7	71	0	778	74	0	1140	77	0	1860	77	0	2540	177	2	3130	223	3	3610	433	8	5020
	6	71	0	707	74	0	1060	77	0	1780	130	1	2440	177	2	2950	309	5	3360	474	9	4570
	5	71	0	636	74	0	990	77	0	1700	177	2	2280	267	4	2720	309	5	3050	553	11	4040
	4	71	0	565	74	0	916	130	1	1600	223	3	2080	309	5	2420	433	8	2680	593	12	3470
	3	71	0	494	74	0	842	223	3	1430	309	5	1790	433	8	2040	474	9	2240	671	14	2820
	2	71	0	423	188	2	719	309	5	1160	433	8	1400	553	11	1570	593	12	1700	787	17	2090
	1	196	3	299	325	6	474	514	10	747	632	13	871	710	15	958	787	17	1020	977	22	1220
	.9	196	3	280	361	7	440	553	11	692	671	14	806	749	16	886	825	18	944	1015	23	1120
	.8	232	4	258	361	7	404	553	11	637	671	14	739	787	17	810	825	18	861	1015	23	1010
	.7	232	4	235	397	8	367	593	12	580	710	15	670	825	18	730	863	19	775	1015	23	913
	.6	266	5	210	397	8	328	632	13	517	787	17	596	863	19	647	863	19	689	1091	25	805
	.5	298	6	182	432	9	285	671	14	451	825	18	517	863	19	561	977	22	595	1165	27	691
	.4	298	6	152	466	10	239	749	16	381	863	19	434	940	21	471	1015	23	496	1165	27	575
	.3	329	7	120	500	11	190	787	17	304	940	21	346	1015	23	372	1091	25	393	1240	29	452
	.2	359	8	85.4	566	13	136	863	19	220	1015	23	248	1091	25	267	1165	27	280	1352	32	321
	.1	414	10	45.9	662	16	74.9	1015	23	124	1165	27	139	1240	29	149	1314	31	156	1499	36	176

Lot Tolerance Per Cent Defective = 4.0%

Process Average	Cost Ratio	N = 500			1,000			2,000			3,000			4,000			5,000			10,000		
0.5%	10	54	0	645	56	0	790	58	0	1070	97	1	1220	97	1	1310	97	1	1390	133	2	1630
	9	54	0	591	56	0	734	58	0	1010	97	1	1120	97	1	1210	97	1	1300	133	2	1500
	8	54	0	537	56	0	678	97	.	940	97	1	1030	97	1	1110	97	1	1200	133	2	1360
	7	54	0	483	56	0	622	97	1	843	97	1	930	97	1	1020	133	2	1080	133	2	1230
	6	54	0	429	56	0	566	97	1	746	97	1	833	133	2	916	133	2	946	133	2	1100
	5	54	0	375	56	0	510	97	1	649	97	1	736	133	2	783	133	2	813	167	3	941
	4	54	0	321	94	1	450	97	1	552	133	2	619	133	2	650	133	2	680	167	3	774
	3	54	0	267	94	1	356	97	1	455	133	2	486	133	2	517	133	2	547	167	3	607
	2	54	0	213	94	1	262	133	2	323	133	2	353	167	3	375	167	3	386	200	4	439
	1	91	1	123	128	2	152	167	3	187	167	3	198	167	3	208	200	4	219	200	4	239
	.9	91	1	114	128	2	140	167	3	170	167	3	181	167	3	192	200	4	199	200	4	219
	.8	91	1	104	128	2	127	167	3	153	167	3	164	167	3	175	200	4	179	200	4	199
	.7	91	1	95.4	128	2	114	167	3	137	167	3	148	200	4	155	200	4	159	232	5	176
	.6	123	2	83.4	128	2	101	167	3	120	167	3	131	200	4	135	200	4	139	232	5	152
	.5	123	2	71.1	160	3	87.6	167	3	103	200	4	111	200	4	115	200	4	119	232	5	129
	.4	123	2	58.8	160	3	71.6	167	3	86.6	200	4	91.1	200	4	95.0	200	4	99.0	232	5	106
	.3	123	2	46.5	160	3	55.6	200	4	67.1	200	4	71.1	232	5	74.7	232	5	76.1	232	5	82.9
	.2	152	3	33.2	160	3	39.6	200	4	47.1	232	5	50.2	232	5	51.5	232	5	52.9	263	6	56.9
	.1	152	3	18.0	190	4	21.6	232	5	25.6	232	5	27.0	263	6	27.9	263	6	28.4	263	6	30.6

Lot Tolerance Per Cent Defective = 4.0%.

Process Average	Cost Ratio	N ≤ 500			1,000			2,000			3,000			4,000			5,000			10,000		
1.0%	10	54	0	726	56	0	964	58	0	1430	97	1	1700	133	2	1910	133	2	2060	200	4	2520
	9	54	0	672	56	0	908	97	1	1360	97	1	1610	133	2	1780	133	2	1930	200	4	2320
	8	54	0	618	56	0	852	97	1	1260	133	2	1500	133	2	1640	167	3	1770	200	4	2120
	7	54	0	564	56	0	796	97	1	1160	133	2	1360	167	3	1511	167	3	1600	200	4	1920
	6	54	0	510	56	0	740	97	1	1060	133	2	1230	167	3	1340	167	3	1430	232	5	1700
	5	54	0	456	56	0	684	133	2	946	167	3	1090	167	3	1180	200	4	1250	232	5	1470
	4	54	0	402	94	1	596	133	2	813	167	3	921	200	4	1000	200	4	1050	263	6	1230
	3	54	0	348	94	1	502	167	3	665	200	4	748	200	4	801	232	5	846	263	6	966
	2	91	1	277	128	2	377	200	4	495	200	4	548	232	5	583	263	6	612	294	7	694
	1	123	2	171	190	4	226	232	5	288	263	6	313	263	6	331	294	7	345	325	8	387
	.9	123	2	159	190	4	207	232	5	264	263	6	287	263	6	305	294	7	316	325	8	354
	.8	152	3	146	190	4	188	232	5	241	263	6	260	294	7	276	294	7	287	355	9	320
	.7	152	3	130	190	4	169	263	6	216	263	6	234	294	7	246	294	7	257	355	9	285
	.6	152	3	115	190	4	150	263	6	189	294	7	206	294	7	217	325	8	225	355	9	249
	.5	152	3	99.9	220	5	129	263	6	163	294	7	176	325	8	186	325	8	192	385	10	214
	.4	180	4	83.7	220	5	107	294	7	136	325	8	147	325	8	154	355	9	160	385	10	175
	.3	180	4	65.7	249	6	85.4	294	7	107	325	8	115	355	9	120	355	9	124	415	11	136
	.2	208	5	47.5	249	6	60.5	325	8	75.7	355	9	80.9	355	9	84.7	385	10	87.2	415	11	95.0
	.1	234	6	26.2	277	7	33.3	355	9	41.7	385	10	44.3	415	11	46.0	415	11	47.2	474	13	51.6

Lot Tolerance Per Cent Defective = 4.0%

Process Average	Cost Ratio	N = 500			1,000			2,000			3,000			4,000			5,000			10,000		
2.0%	10	54	0	834	56	0	1200	58	0	1910	58	0	2600	97	1	3220	167	3	3740	294	7	5270
	9	54	0	780	56	0	1140	58	0	1850	58	0	2540	133	2	3120	167	3	3570	355	9	4930
	8	54	0	726	56	0	1080	58	0	1800	97	1	2450	167	3	2980	200	4	3380	385	10	4580
	7	54	0	672	56	0	1030	58	0	1740	133	2	2350	200	4	2810	263	6	3150	385	10	4190
	6	54	0	618	56	0	972	58	0	1680	167	3	2220	232	5	2600	263	6	2890	415	11	3780
	5	54	0	564	56	0	916	97	1	1580	200	4	2040	263	6	2350	294	7	2600	474	13	3330
	4	54	0	510	56	0	860	167	3	1450	263	6	1810	294	7	2060	355	9	2260	503	14	2840
	3	54	0	456	94	1	790	232	5	1260	294	7	1530	385	10	1720	415	11	1860	561	16	2290
	2	54	0	402	190	4	649	294	7	997	385	10	1180	474	13	1300	503	14	1400	648	19	1680
	1	180	4	274	277	7	419	445	12	628	532	15	717	590	17	780	648	19	829	761	23	970
	.9	180	4	256	305	8	388	474	13	561	561	16	662	619	18	721	676	20	763	818	25	891
	.8	208	5	237	332	9	356	503	14	532	561	16	606	648	19	658	676	20	696	818	25	809
	.7	234	6	215	332	9	323	503	14	482	590	17	548	676	20	592	705	21	626	874	27	725
	.6	234	6	192	359	10	287	532	15	429	619	18	487	676	20	523	761	23	552	874	27	637
	.5	259	7	166	386	11	250	561	16	374	676	20	421	733	22	453	761	23	476	930	29	548
	.4	284	8	140	412	12	210	619	18	314	705	21	353	761	23	378	818	25	398	930	29	454
	.3	308	9	111	463	14	166	676	20	250	761	23	279	818	25	299	874	27	313	1014	32	357
	.2	331	10	78.7	489	15	119	761	23	180	818	25	200	874	27	213	930	29	223	1069	34	252
	.1	376	12	43.0	563	18	65.9	874	27	101	930	29	112	1014	32	119	1069	34	124	1209	39	138

Lot Tolerance Per Cent Defective = 5.0%.

Process Average	Cost Ratio	N = 500			1,000			2,000			3,000			4,000			5,000			10,000		
0.5%	10	44	0	530	46	0	655	46	0	860	78	1	954	78	1	1010	78	1	1070	106	2	1230
	9	44	0	486	46	0	609	46	0	814	78	1	876	78	1	936	78	1	996	106	2	1120
	8	44	0	442	46	0	563	78	1	739	78	1	798	78	1	858	78	1	918	106	2	1020
	7	44	0	398	46	0	517	78	1	661	78	1	720	78	1	780	106	2	825	106	2	910
	6	44	0	354	46	0	471	78	1	583	78	1	642	78	1	702	106	2	719	106	2	804
	5	44	0	310	46	0	425	78	1	505	78	1	564	106	2	596	106	2	613	106	2	698
	4	44	0	266	75	1	352	78	1	427	106	2	473	106	2	490	106	2	507	134	3	589
	3	44	0	222	75	1	277	78	1	349	106	2	367	106	2	384	106	2	401	134	3	455
	2	74	1	171	75	1	202	106	2	244	106	2	261	106	2	278	134	3	294	134	3	321
	1	74	1	97.3	103	2	117	106	2	138	134	3	149	134	3	155	134	3	160	160	4	176
	.9	74	1	89.9	103	2	107	106	2	128	134	3	136	134	3	141	134	3	147	160	4	160
	.8	74	1	82.5	103	2	96.4	106	2	117	134	3	123	134	3	128	134	3	133	160	4	144
	.7	74	1	75.1	103	2	88.1	134	3	104	134	3	109	134	3	114	160	4	120	160	4	128
	.6	100	2	65.9	103	2	75.8	134	3	90.4	134	3	95.8	134	3	101	160	4	104	160	4	112
	.5	100	2	55.9	103	2	65.5	134	3	77.0	134	3	82.4	160	4	86.1	160	4	87.7	160	4	95.6
	.4	100	2	45.9	103	2	55.2	134	3	63.6	160	4	68.5	160	4	70.1	160	4	71.7	186	5	78.8
	.3	100	2	35.9	129	3	42.8	134	3	50.2	160	4	52.5	160	4	54.1	160	4	55.7	186	5	60.2
	.2	100	2	25.9	129	3	29.9	160	4	34.9	160	4	36.5	160	4	38.1	186	5	39.4	186	5	41.6
	.1	124	3	13.9	154	4	16.6	160	4	18.9	186	5	19.9	186	5	20.3	186	5	20.8	211	6	22.3

Lot Tolerance Per Cent Defective = 5.0%.

Process Average	Cost Ratio	N = 500			1,000			2,000			3,000			4,000			5,000			10,000		
1.0%	10	44	0	602	46	0	811	78	1	1130	78	1	1320	106	2	1420	106	2	1510	134	3	1810
	9	44	0	558	46	0	765	78	1	1060	106	2	1220	106	2	1310	106	2	1400	160	4	1680
	8	44	0	514	46	0	719	78	1	978	106	2	1110	106	2	1210	106	2	1300	160	4	1520
	7	44	0	470	46	0	673	78	1	900	106	2	1010	106	2	1100	134	3	1170	160	4	1360
	6	44	0	426	75	1	611	106	2	810	106	2	902	134	3	988	134	3	1040	160	4	1200
	5	44	0	382	75	1	536	106	2	704	106	2	796	134	3	854	134	3	902	160	4	1040
	4	44	0	338	75	1	461	106	2	598	134	3	673	134	3	720	160	4	756	186	5	865
	3	44	0	294	75	1	386	134	3	491	134	3	539	160	4	572	160	4	596	186	5	679
	2	74	1	220	103	2	283	134	3	357	160	4	388	160	4	412	186	5	432	211	6	481
	1	100	2	132	129	3	166	160	4	204	186	5	221	186	5	233	211	6	240	236	7	267
	.9	100	2	122	129	3	153	160	4	188	186	5	202	211	6	213	211	6	219	236	7	243
	.8	100	2	112	129	3	140	186	5	171	186	5	184	211	6	192	211	6	198	236	7	220
	.7	124	3	101	154	4	125	186	5	153	211	6	164	211	6	170	211	6	177	236	7	196
	.6	124	3	88.6	154	4	110	186	5	134	211	6	143	211	6	149	211	6	156	260	8	171
	.5	124	3	76.2	154	4	94.5	186	5	115	211	6	122	211	6	128	236	7	133	260	8	145
	.4	124	3	63.8	154	4	79.1	211	6	95.2	211	6	101	236	7	106	236	7	110	260	8	119
	.3	147	4	50.3	178	5	61.7	211	6	74.1	236	7	79.5	236	7	82.7	260	8	85.4	284	9	92.4
	.2	147	4	35.6	178	5	43.9	236	7	52.8	236	7	55.9	260	8	57.8	260	8	59.4	284	9	64.0
	.1	170	5	19.7	201	6	24.0	260	8	28.7	260	8	30.3	284	9	31.1	284	9	31.9	308	10	34.6

Lot Tolerance Per Cent Defective = 5.0%.

Process Average	Cost Ratio	N = 500			1,000			2,000			3,000			4,000			5,000			10,000		
2.0%	10	44	0	707	46	0	1030	46	0	1640	106	2	2090	134	3	2430	160	4	2660	236	7	3400
	9	44	0	663	46	0	988	46	0	1590	106	2	1980	160	4	2280	160	4	2500	260	8	3140
	8	44	0	619	46	0	942	78	1	1510	106	2	1880	160	4	2120	186	5	2320	260	8	2880
	7	44	0	575	46	0	896	106	2	1420	160	4	1740	186	5	1960	211	6	2120	284	9	2610
	6	44	0	531	46	0	850	106	2	1310	160	4	1580	186	5	1780	211	6	1910	284	9	2320
	5	44	0	487	75	1	784	134	3	1200	186	5	1420	211	6	1570	236	7	1690	308	10	2020
	4	44	0	443	75	1	709	160	4	1040	211	6	1220	236	7	1340	260	8	1430	332	11	1700
	3	44	0	399	129	3	613	186	5	872	236	7	1000	260	8	1090	284	9	1150	356	12	1360
	2	100	2	329	154	4	476	236	7	659	284	9	741	308	10	801	332	11	846	403	14	983
	1	147	4	209	224	7	290	308	10	393	332	11	436	379	13	464	379	13	488	472	17	553
	.9	147	4	194	224	7	268	308	10	362	356	12	401	379	13	426	403	14	448	472	17	506
	.8	170	5	179	224	7	245	332	11	330	379	13	365	379	13	388	426	15	406	495	18	459
	.7	170	5	162	247	8	222	332	11	297	379	13	327	403	14	348	426	15	364	495	18	409
	.6	192	6	144	269	9	196	356	12	264	379	13	289	426	15	307	449	16	320	518	19	359
	.5	192	6	125	269	9	169	379	13	228	403	14	249	449	16	264	472	17	275	518	19	307
	.4	213	7	105	291	10	142	379	13	190	426	15	207	449	16	219	472	17	227	541	20	254
	.3	234	8	85.4	313	11	112	426	15	150	449	16	163	495	18	172	518	19	178	564	21	198
	.2	254	9	59.4	335	12	80.4	449	16	107	495	18	116	518	19	121	541	20	126	609	23	139
	.1	293	11	32.9	377	14	44.5	518	19	59.4	564	21	63.8	586	22	66.7	609	23	68.8	677	26	75.3

Lot Tolerance Per Cent Defective = 7.0%.

Process Average	Cost Ratio	N = 500			1,000			2,000			3,000			4,000			5,000			10,000		
1.0%	10	32	0	448	33	0	601	56	1	772	56	1	881	76	2	927	76	2	970	95	3	1110
	9	32	0	416	33	0	568	56	1	716	76	2	809	76	2	851	76	2	894	95	3	1020
	8	32	0	384	33	0	535	56	1	660	76	2	733	76	2	775	76	2	818	95	3	923
	7	32	0	352	56	1	495	56	1	604	76	2	657	76	2	699	76	2	742	95	3	828
	6	32	0	320	56	1	439	76	2	538	76	2	581	76	2	623	95	3	651	95	3	733
	5	32	0	288	56	1	383	76	2	462	76	2	505	95	3	539	95	3	556	114	4	633
	4	32	0	256	56	1	327	76	2	386	95	3	428	95	3	444	95	3	461	114	4	519
	3	54	1	208	76	2	267	76	2	310	95	3	333	95	3	349	95	3	366	114	4	405
	2	54	1	154	76	2	191	95	3	221	95	3	238	114	4	253	114	4	259	114	4	291
	1	73	2	89.4	95	3	110	114	4	126	114	4	133	114	4	139	114	4	145	133	5	160
	.9	73	2	82.1	95	3	100	114	4	115	114	4	121	114	4	128	133	5	133	151	6	146
	.8	73	2	74.8	95	3	90.9	114	4	103	114	4	110	114	4	116	133	5	120	151	6	131
	.7	73	2	67.5	95	3	81.4	114	4	91.9	114	4	98.3	133	5	104	133	5	106	151	6	116
	.6	73	2	60.2	95	3	71.9	114	4	80.5	114	4	86.9	133	5	90.2	133	5	92.9	151	6	101
	.5	91	3	51.4	95	3	62.4	114	4	69.1	133	5	74.2	133	5	76.9	133	5	79.6	151	6	85.6
	.4	91	3	42.3	114	4	51.3	114	4	57.7	133	5	60.9	133	5	63.6	151	6	65.4	151	6	70.5
	.3	91	3	33.2	114	4	39.9	133	5	44.9	133	5	47.6	151	6	49.3	151	6	50.3	168	7	54.0
	.2	108	4	23.6	114	4	28.5	133	5	31.6	151	6	33.1	151	6	34.2	151	6	35.2	168	7	37.2
	.1	108	4	12.8	133	5	15.6	151	6	17.0	168	7	17.8	168	7	18.2	168	7	18.6	186	8	20.2

TABLE III

DETERMINING PRODUCER'S RISK, P_p, USING TABLES OF THE INCOMPLETE GAMMA-FUNCTION

($\bar{p}_t$ = .07)

	$\bar{p}$ = .01						$\bar{p}$ = .02				$\bar{p}$ = .03			
	N = 500			N = 1000 and over			N = 500		N = 1000 and over		N = 500		N = 1000 and over	
c	n	$u = \frac{.01n}{\sqrt{c+1}}$	$P_p = I(u,c)$	n	$u = \frac{.01n}{\sqrt{c+1}}$	$P_p = I(u,c)$	$u = \frac{.02n}{\sqrt{c+1}}$	$P_p = I(u,c)$	$u = \frac{.02n}{\sqrt{c+1}}$	$P_p = I(u,c)$	$u = \frac{.03n}{\sqrt{c+1}}$	$P_p = I(u,c)$	$u = \frac{.03n}{\sqrt{c+1}}$	$P_p = I(u,c)$
0	32	.3200	.2733	33	.3300	.2803	.6400	.4721	.6600	.4825	.9600	.6166	.9900	.6283
1	54	.3818	.1030	56	.3960	.1090	.7637	.2936	.7920	.3083	1.1455	.4812	1.1879	.5004
2	73	.4215	.0385	76	.4388	.0426	.8429	.1815	.8776	.1964	1.2644	.3745	1.3164	.3986
etc.														

The values of n used for finding u are either one or the other of the values given in the second and fifth columns depending on whether N = 500 or N = 1000 and over.

TABLE IV

DETERMINING THE MINIMUM RELATIVE AVERAGE COST OF INSPECTION PER LOT

(p_t = .03, $\overline{p}$ = .01, N = 1000)

n	c	P_p	N-n	P_p(N-n)	C																		
					10	9	8	7	6	5	4	3	2	1	.9	.8	.7	.6	.5	.4	.3	.2	.1
74	0	.522	926	483	1223	1149	1075	1001	927	853	779	705											
124	1	.352	876	308						928	804	680	556	432									
168	2	.238	832	198							870	702	534	366									
209	3	.160	791	127									545	336	315	294							
249	4	.108	751	81									579	330	305	280	255	230					
288	5	.0727	712	51.8										340	311.0	282.2	253.4	224.6	195.8	167.0			
325	6	.0481	675	32.5												292.5	260.0	227.5	195.0	162.5	130.0	97.5	
361	7	.0315	639	20.1															200.6	164.5	128.4	92.3	56.2
397	8	.0207	603	12.5																171.3	131.6	91.9	52.2
432	9	.0135	568	7.7																		94.1	50.9
466	10	.00854	534	4.6																			51.2

TABLE V

MINIMUM AVERAGE COST SAMPLING TABLES

(For Single Sampling Lot Inspection-Based on stated values of "Lot Tolerance Per Cent Defective" and Consumer's Risk = 0.10)

Lot Tolerance Per Cent Defective = 2.0%.

Lot Size		500			1,000			2,000			3,000			4,000			5,000			10,000		
Process average	Inspection cost ratio	Sample plan		Relative average cost	Sample plan		Relative average cost	Sample plan		Relative average cost	Sample plan		Relative average cost	Sample plan		Relative average cost	Sample plan		Relative average cost	Sample plan		Relative average cost
		n	c		n	c		n	c		n	c		n	c		n	c		n	c	
0.1%	10	103	0	1070	109	0	1180	113	0	1330	115	0	1460	115	0	1570	115	0	1680	195	1	2130
	9	103	0	966	109	0	1070	113	0	1220	115	0	1350	115	0	1460	115	0	1560	195	1	1930
	8	103	0	863	109	0	964	113	0	1100	115	0	1230	115	0	1340	115	0	1450	195	1	1740
	7	103	0	760	109	0	855	113	0	991	115	0	1120	115	0	1220	115	0	1330	195	1	1540
	6	103	0	657	109	0	746	113	0	878	115	0	1000	115	0	1110	115	0	1220	195	1	1350
	5	103	0	554	109	0	637	113	0	765	115	0	887	115	0	995	195	1	1060	195	1	1150
	4	103	0	451	109	0	528	113	0	652	115	0	772	195	1	849	195	1	867	195	1	958
	3	103	0	348	109	0	419	113	0	539	195	1	636	195	1	654	195	1	672	195	1	763
	2	103	0	245	109	0	310	188	1	407	195	1	441	195	1	459	195	1	477	266	2	563
	1	103	0	142	181	1	194	188	1	219	195	1	246	195	1	264	266	2	281	266	2	297
	.9	103	0	132	181	1	176	188	1	200	195	1	227	195	1	245	266	2	254	266	2	270
	.8	103	0	121	181	1	158	188	1	181	195	1	207	266	2	225	266	2	228	266	2	244
	.7	103	0	111	181	1	140	188	1	162	195	1	188	266	2	198	266	2	201	266	2	217
	.6	103	0	101	181	1	122	188	1	144	195	1	168	266	2	172	266	2	175	266	2	191
	.5	169	1	89.1	181	1	104	188	1	125	266	2	142	266	2	145	266	2	148	266	2	164
	.4	169	1	72.2	181	1	85.4	188	1	106	266	2	115	266	2	118	266	2	122	266	2	138
	.3	169	1	55.3	181	1	67.3	256	2	82.0	266	2	88.5	266	2	91.7	266	2	94.9	334	3	105
	.2	169	1	38.4	181	1	49.2	256	2	56.4	266	2	61.9	266	2	65.1	266	2	68.3	334	3	71.6
	.1	169	1	21.5	245	2	26.5	256	2	30.8	334	3	34.7	334	3	35.2	334	3	35.7	334	3	38.2

Lot Tolerance Per Cent Defective = 2.0%.

Process Average	Cost Ratio	N = 500			1,000			2,000			3,000			4,000			5,000			10,000		
0.5%	10	103	0	1190	109	0	1460	113	0	1940	115	0	2410	115	0	2850	195	1	3180	266	2	4120
	9	103	0	1090	109	0	1350	113	0	1830	115	0	2300	195	1	2720	195	1	2980	266	2	3850
	8	103	0	984	109	0	1240	113	0	1720	115	0	2180	195	1	2530	195	1	2780	334	5	3540
	7	103	0	881	109	0	1140	113	0	1600	115	0	2070	195	1	2340	266	2	2570	334	3	3200
	6	103	0	778	109	0	1030	113	0	1490	195	1	1880	195	1	2140	266	2	2310	334	3	2870
	5	103	0	675	109	0	918	113	0	1380	195	1	1690	266	2	1890	266	2	2040	400	4	2510
	4	103	0	572	109	0	809	188	1	1190	266	2	1470	266	2	1620	334	3	1750	400	4	2110
	3	103	0	469	109	0	700	188	1	1000	266	2	1210	334	3	1330	334	3	1420	464	5	1690
	2	103	0	366	181	1	550	256	2	754	334	3	906	400	4	990	400	4	1040	527	6	1230
	1	169	1	238	245	2	340	319	3	450	400	4	537	464	5	575	464	5	607	589	7	693
	.9	169	1	221	245	2	316	380	4	414	400	4	497	464	5	529	527	6	557	589	7	634
	.8	169	1	204	245	2	291	380	4	376	464	5	451	464	5	482	527	6	504	589	7	575
	.7	225	2	186	304	3	261	380	4	338	464	5	405	527	6	433	527	6	451	650	8	515
	.6	225	2	164	304	3	230	380	4	300	464	5	358	527	6	380	527	6	398	650	8	450
	.5	225	2	141	304	3	200	439	5	258	527	6	309	527	6	327	589	7	343	650	8	385
	.4	225	2	119	361	4	168	439	5	214	527	6	256	589	7	273	589	7	284	710	9	319
	.3	276	3	94.4	361	4	132	439	5	170	589	7	203	589	7	214	650	8	223	710	9	248
	.2	276	3	66.8	415	5	94.6	497	6	121	589	7	144	650	8	151	650	8	158	771	10	175
	.1	323	4	36.7	467	6	52.2	554	7	66.7	710	9	79.6	710	9	83.4	771	10	86.6	830	11	94.5

Lot Tolerance Per Cent Defective = 2.0%.

Process Average	Cost Ratio	N = 500			1,000			2,000			3,000			4,000			5,000			10,000		
1.0%	10	103	0	1280	109	0	1680	113	0	2410	115	0	3120	115	0	3800	115	0	4490	334	3	7480
	9	103	0	1180	109	0	1570	113	0	2290	115	0	3000	115	0	3690	115	0	4370	334	3	7140
	8	103	0	1080	109	0	1460	113	0	2180	115	0	2890	115	0	3570	115	0	4260	400	4	6760
	7	103	0	976	109	0	1360	113	0	2070	115	0	2780	115	0	3460	115	0	4140	464	5	6310
	6	103	0	873	109	0	1250	113	0	1960	115	0	2660	115	0	3340	266	2	3940	527	6	5880
	5	103	0	770	109	0	1140	113	0	1840	115	0	2540	195	1	3180	334	3	3670	650	8	5200
	4	103	0	667	109	0	1030	113	0	1730	195	1	2410	334	3	2900	400	4	3310	710	9	4510
	3	103	0	564	109	0	919	188	1	1580	334	3	2140	464	5	2530	527	6	2820	830	11	3730
	2	103	0	461	181	1	804	380	4	1300	527	6	1740	650	8	2000	710	9	2190	1006	14	2790
	1	225	2	331	361	4	550	609	8	836	710	9	1120	889	12	1250	1006	14	1350	1295	19	1660
	.9	225	2	308	415	5	513	609	8	775	830	11	1040	948	13	1160	1006	14	1250	1295	19	1530
	.8	225	2	286	415	5	472	664	9	712	830	11	957	1006	14	1060	1065	15	1150	1409	21	1390
	.7	276	3	260	467	6	429	664	9	645	889	12	869	1006	14	964	1123	16	1040	1409	21	1250
	.6	276	3	233	467	6	382	718	10	574	948	13	778	1065	15	861	1181	17	923	1466	22	1110
	.5	323	4	201	518	7	333	771	11	499	1006	14	676	1181	17	749	1295	19	801	1523	23	955
	.4	366	5	168	568	8	280	823	12	418	1123	16	571	1238	18	629	1352	20	671	1636	25	795
	.3	366	5	132	615	9	221	875	13	333	1181	17	456	1352	20	500	1409	21	533	1748	27	627
	.2	406	6	92.2	662	10	157	977	15	238	1352	20	329	1466	22	361	1580	24	384	1860	29	446
	.1	473	8	48.7	751	12	85.9	1127	18	132	1523	23	186	1748	27	203	1804	28	215	2138	34	247

Lot Tolerance Per Cent Defective = 10.0%.

Process Average	Cost Ratio	n = 500			1,000			2,000			3,000			4,000			5,000			10,000		
5.0%	10	23	0	446	23	0	897	53	2	1490	105	6	1850	130	8	2110	142	9	2290	201	14	2860
	9	23	0	424	23	0	874	80	4	1430	105	6	1740	142	9	1970	154	10	2140	201	14	2650
	8	23	0	402	23	0	851	80	4	1350	105	6	1640	142	9	1830	166	11	1980	236	17	2430
	7	23	0	380	23	0	828	105	6	1260	142	9	1510	154	10	1680	178	12	1810	236	17	2200
	6	23	0	358	53	2	786	105	6	1150	142	9	1370	166	11	1510	201	14	1620	259	19	1960
	5	23	0	336	53	2	733	130	8	1040	166	11	1210	190	13	1340	201	14	1420	270	20	1690
	4	37	1	[illegible]	80	4	661	142	9	902	178	12	1040	201	14	1130	236	17	1210	270	20	1420
	3	64	3	[illegible]	105	6	561	166	11	745	201	14	844	236	17	918	259	19	973	293	22	1140
	2	89	5	207	142	9	438	201	14	557	236	17	626	270	20	672	270	20	707	338	26	815
	1	135	9	188	201	14	270	259	19	331	293	22	364	327	25	388	327	25	405	394	31	458
	.9	146	10	174	201	14	250	270	20	304	293	22	335	327	25	356	338	26	371	394	31	419
	.8	146	10	160	201	14	230	270	20	277	316	24	306	327	25	323	338	26	338	394	31	379
	.7	167	12	144	236	17	208	293	22	250	327	25	274	338	26	289	372	29	302	394	31	340
	.6	167	12	128	236	17	184	293	22	221	327	25	241	338	26	256	372	29	265	428	34	298
	.5	167	12	111	259	19	160	316	24	191	338	26	207	372	29	219	394	31	228	450	36	255
	.4	178	13	93.5	270	20	134	327	25	159	372	29	173	383	30	182	394	31	186	450	36	210
	.3	209	16	73.9	293	22	107	338	26	125	383	30	136	394	31	142	428	34	148	472	38	163
	.2	209	16	53.0	327	25	76.6	372	29	89.2	394	31	96.2	439	35	101	450	36	104	505	41	114
	.1	250	20	29.4	372	29	42.9	428	34	49.5	450	36	52.9	483	39	55.2	505	41	56.8	560	46	62.1

n = Size of sample.
c = Maximum allowable number of defective pieces per sample.
Inspection cost ratio = b/B = Cost of inspection per piece in the sample divided by cost of inspection per piece in the remainder of those lots that fail to be accepted by sample.
Relative average cost (rounded to three significant figures) = Relative average cost of inspection per lot for product of process average quality. The average cost of inspection per lot (in dollars) is obtained by multiplying this figure by B (expressed in dollars).

Lot Tolerance Per Cent Defective = 10.0%.

Process Average	Cost Ratio	N = 500			1,000			2,000			3,000			4,000			5,000			10,000		
4.0%	10	23	0	517	23	0	817	67	3	1220	93	5	1430	105	6	1570	118	7	1700	154	10	2030
	9	23	0	494	23	0	794	80	4	1140	105	6	1330	105	6	1460	130	8	1570	166	11	1880
	8	23	0	471	39	1	756	80	4	1060	105	6	1220	118	7	1360	130	8	1440	166	11	1710
	7	23	0	448	53	2	708	93	5	981	105	6	1120	130	8	1230	142	9	1300	166	11	1540
	6	37	1	423	53	2	655	105	6	881	118	7	1010	142	9	1100	142	9	1160	178	12	1370
	5	37	1	386	67	3	598	105	6	776	130	8	886	142	9	955	154	10	1010	201	14	1180
	4	37	1	349	80	4	522	105	6	671	142	9	750	154	10	808	166	11	852	201	14	981
	3	64	3	303	105	6	434	130	8	544	154	10	604	166	11	647	178	12	681	213	15	779
	2	77	4	238	105	6	329	154	10	400	178	12	442	201	14	471	201	14	489	236	17	555
	1	112	7	145	154	10	196	190	13	233	201	14	252	213	15	267	236	17	277	270	20	308
	.9	112	7	134	154	10	181	201	14	214	213	15	232	236	17	244	236	17	253	270	20	281
	.8	112	7	123	166	11	165	201	14	193	213	15	210	236	17	221	236	17	229	270	20	254
	.7	112	7	112	166	11	149	201	14	173	225	16	189	236	17	197	259	19	206	282	21	227
	.6	135	9	98.8	178	12	132	201	14	153	236	17	165	236	17	174	259	19	180	282	21	199
	.5	135	9	85.3	178	12	114	213	15	132	236	17	142	259	19	149	270	20	154	293	22	169
	.4	146	10	71.4	201	14	94.9	236	17	109	259	19	118	270	20	123	270	20	126	305	23	140
	.3	157	11	56.5	201	14	74.8	236	17	85.8	270	20	91.7	270	20	95.6	282	21	98.9	327	25	108
	.2	167	12	40.0	225	16	53.7	259	19	60.7	282	21	64.7	293	22	67.3	305	23	69.6	338	26	75.3
	.1	199	15	22.5	259	19	29.7	293	22	33.3	316	24	35.4	327	25	36.7	338	26	37.5	361	28	40.8

Lot Tolerance Per Cent Defective = 7.0%.

Process Average	Cost Ratio	N = 500			1,000			2,000			3,000		
2.0%	10	32	0	541	33	0	797	76	2	1140	95	3	1320
	9	32	0	509	33	0	764	76	2	1060	95	3	1220
	8	32	0	477	33	0	731	76	2	986	95	3	1130
	7	32	0	445	56	1	683	95	3	905	95	3	1030
	6	32	0	413	56	1	627	95	3	810	114	4	920
	5	32	0	381	76	2	561	95	3	715	114	4	808
	4	54	1	347	76	2	485	114	4	610	133	5	686
	3	54	1	293	95	3	399	114	4	496	151	6	553
	2	73	2	224	114	4	300	133	5	366	168	7	398
	1	108	4	135	133	5	180	168	7	208	186	8	227
	.9	108	4	124	151	6	166	168	7	191	186	8	208
	.8	108	4	113	151	6	150	168	7	174	203	9	188
	.7	108	4	102	151	6	135	186	8	156	203	9	168
	.6	125	5	90.9	168	7	119	186	8	138	203	9	147
	.5	125	5	78.4	168	7	102	203	9	118	220	10	126
	.4	141	6	65.6	168	7	85.4	203	9	97.5	220	10	104
	.3	141	6	51.5	186	8	67.5	220	10	76.4	237	11	81.4
	.2	157	7	36.6	203	9	47.8	237	11	54.0	254	12	57.2
	.1	172	8	20.2	220	10	26.5	254	12	29.5	271	13	31.3

Process Average	Cost Ratio	4,000			5,000			10,000		
2.0%	10	95	3	1440	114	4	1540	151	6	1860
	9	114	4	1340	114	4	1420	151	6	1700
	8	114	4	1230	114	4	1310	151	6	1550
	7	114	4	1120	133	5	1190	168	7	1390
	6	114	4	1000	133	5	1060	168	7	1220
	5	133	5	873	151	6	925	168	7	1060
	4	151	6	739	151	6	774	186	8	885
	3	151	6	588	168	7	610	203	9	698
	2	168	7	420	186	8	441	203	9	495
	1	203	9	237	203	9	247	237	11	273
	.9	203	9	217	220	10	226	237	11	250
	.8	203	9	197	220	10	204	237	11	226
	.7	220	10	176	220	10	182	254	12	201
	.6	220	10	154	220	10	160	254	12	175
	.5	220	10	132	237	11	136	254	12	150
	.4	237	11	109	237	11	112	271	13	123
	.3	254	12	85.0	254	12	87.3	288	14	95.7
	.2	254	12	59.6	271	13	61.5	304	15	66.7
	.1	288	14	32.4	288	14	33.3	321	16	35.9

Lot Tolerance Per Cent Defective = 7.0%.

Process Average	Cost Ratio	N = 500			1,000			2,000			3,000			4,000			5,000			10,000		
3.0%	10	32	0	609	33	0	938	76	2	1530	95	3	1880	114	4	2150	168	7	2350	220	10	2910
	9	32	0	577	33	0	905	76	2	1450	114	4	1780	133	5	2020	168	7	2180	220	10	2690
	8	32	0	545	33	0	872	95	3	1370	114	4	1660	168	7	1870	168	7	2010	237	11	2470
	7	32	0	513	33	0	839	95	3	1270	133	5	1540	168	7	1700	168	7	1840	237	11	2230
	6	32	0	481	33	0	806	114	4	1170	168	7	1400	168	7	1540	203	9	1650	254	12	1990
	5	32	0	449	76	2	748	114	4	1060	168	7	1230	203	9	1360	220	10	1450	271	13	1730
	4	32	0	417	95	3	669	151	6	924	186	8	1060	220	10	1160	220	10	1230	288	14	1460
	3	54	1	377	114	4	572	168	7	756	220	10	862	237	11	933	254	12	987	304	15	1150
	2	91	3	302	151	6	449	203	9	569	237	11	637	271	13	685	288	14	723	337	17	828
	1	141	6	190	203	9	275	254	12	337	304	15	370	304	15	395	337	17	411	386	20	465
	.9	141	6	176	220	10	255	271	13	310	304	15	340	337	17	362	337	17	377	386	20	426
	.8	141	6	162	220	10	233	271	13	283	304	15	310	337	17	328	370	19	343	419	22	386
	.7	157	7	146	220	10	211	304	15	254	337	17	278	337	17	294	370	19	306	419	22	344
	.6	157	7	130	237	11	187	304	15	224	337	17	244	370	19	259	386	20	269	435	23	302
	.5	172	8	112	254	12	162	304	15	194	337	17	211	370	19	222	386	20	231	451	24	257
	.4	188	9	94.4	271	13	136	337	17	161	370	19	175	386	20	184	403	21	192	451	24	212
	.3	203	10	74.8	304	15	108	337	17	128	386	20	137	419	22	145	435	23	150	483	26	165
	.2	218	11	55.6	321	16	77.8	386	20	90.4	419	22	97.5	451	24	102	451	24	105	515	28	116
	.1	261	14	29.6	370	19	43.5	435	23	50.1	467	25	53.5	483	25	55.7	499	27	57.6	563	31	62.8

Lot Tolerance Per Cent Defective = 10.0%.

Process Average	Cost Ratio	N = 500			1,000			2,000			3,000			4,000			5,000			10,000		
3.0%	10	23	0	468	39	1	703	53	2	947	80	4	1080	80	4	1180	93	5	1250	105	6	1460
	9	23	0	445	39	1	664	53	2	894	80	4	1000	93	5	1090	105	6	1150	118	7	1350
	8	23	0	422	39	1	625	67	3	816	80	4	921	93	5	997	105	6	1040	130	8	1220
	7	23	0	39[illegible]	53	2	574	80	4	745	93	5	839	105	6	898	105	6	940	130	8	1090
	6	37	1	36[illegible]	53	2	521	80	4	665	93	5	746	105	6	793	105	6	840	130	8	963
	5	37	1	32[illegible]	53	2	468	80	4	585	105	6	646	105	6	688	105	6	730	142	9	832
	4	37	1	28[illegible]	67	3	403	93	5	495	105	6	541	105	6	585	130	8	610	142	9	690
	3	64	3	248	80	4	329	105	6	394	105	6	436	130	8	462	130	8	480	154	10	541
	2	64	3	184	93	5	245	105	6	289	130	8	313	130	8	332	142	9	544	166	11	385
	1	89	5	111	105	6	142	130	8	165	154	10	177	154	10	185	166	11	192	178	12	213
	.9	89	5	102	118	7	132	142	9	151	154	10	162	154	10	170	166	11	176	190	13	194
	.8	89	5	93.6	118	7	120	142	9	137	154	10	146	166	11	154	166	11	159	190	13	175
	.7	89	5	84.7	130	8	107	142	9	122	154	10	131	166	11	137	178	12	142	201	14	156
	.6	101	6	74.8	130	8	94.1	154	10	107	166	11	115	166	11	120	178	12	124	201	14	156
	.5	112	7	64.5	130	8	81.1	154	10	91.9	166	11	98.4	178	12	103	178	12	106	201	14	115
	.4	112	7	53.3	142	9	67.4	166	11	76.4	178	12	81.2	178	12	84.8	190	13	87.4	213	15	94.8
	.3	112	7	42.1	154	10	53.0	166	11	59.8	178	12	63.4	190	13	66.1	201	14	67.6	213	15	73.5
	.2	124	8	30.2	154	10	37.6	178	12	42.1	201	14	44.5	201	14	46.0	213	15	47.3	236	17	51.4
	.1	146	10	16.6	178	12	20.7	201	14	22.8	213	15	24.0	213	15	25.0	236	17	25.6	248	18	27.5