Analysis of a Cyclic Polling System with More Than One Priority for Design of a Data Acquisition System

By

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ANALYSIS OF A CYCLIC POLLING SYSTEM WITH MORE THAN ONE PRIORITY FOR DESIGN OF A DATA ACQUISITION SYSTEM

BY

IMED FRIGUI

A Thesis submitted to the Faculty of Graduate Studies of the University of Manitoba in partial fulfillment of the requirements of the degree of

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Dedicated with love to my mother Mamia and my father Mustapha

Abstract

We model a data acquisition system as a cyclic polling system with two priority levels. Two types of service disciplines are studied, exhaustive and time limited, with application to automatic meter reading. In the case of the exhaustive service discipline, stations are served exhaustively for both priorities, with high priority messages served first on a non-preemptive basis. In the case of the time limited service discipline, each station is served for not more than τ_{max} but not less than τ_{min} with the high priority messages served first.

Approximate results are obtained for the high and low priority mean waiting time for cyclic polling systems with the exhaustive service discipline, constant switchover times, N stations, general service times distributions, and independent Poisson arrival for each priority level. The performance of the exhaustive service discipline is studied under different service distributions, traffic intensities, connect time, and load combinations.

Also, the upper bound of the mean waiting time for symmetric cyclic polling systems with time limited service discipline is obtained. The performance of the time limited polling algorithm is studied under different arrival rates, τ_{min} parameter, τ_{max} parameter, mean service time, connect time, and number of stations.

The results are then applied to study the performance of a data acquisition system, System 2020, under the exhaustive and the time limited service disciplines.

Keywords: Polling Systems, Mean Waiting Time, Non-preemptive Exhaustive Service Discipline, Time Limited, Vacation Models, System 2020.

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The assistance of Richard Stone during this study is greatly appreciated.

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Nomenclature

Latin Letters

A	mean vacation period
$A^{(2)}$	second moment of the vacation period
C	mean cycle time
N	number of stations
T	mean service period
W	mean waiting time
$var(A)$	variance of the random variable A
h	mean service time
$h^{(2)}$	second moment of service time
r	mean connect time
$r^{(2)}$	second moment of connect time
i, j	indices

Greek Letters

λ	arrival rate
ρ	utilization
τ	time

CHAPTER 1

Introduction

1.1 General

A polling model is a system of multiqueues accessed in cyclic or random order by a single server. In a polling system the central controller (server) interrogates each queue to find whether it has customers to serve. The addressed queue receives services, and the server then interrogates the next queue. Polling schemes have been used in the area of computer communication networks, distributed systems, Local Area Networks (token ring, token bus), data acquisition systems, patrolling machine repair person, vehicle actuated traffic signal control (alternating queues), and moving customers on circular or back-and-forth routes (elevators). The widespread use of polling in any field of engineering is because polling is a good way of ensuring fair and guaranteed access to the server. Previous works have focused on polling systems with one priority at each station. However, in real life applications, more than one priority at each station often exists. For example, in the area of distributed system where fault recovery and/or routing traffic update need to be transmitted with priority over regular messages. Another application is in the area of data acquisition system, such as System 2020, where alarm messages need to be transmitted with priority over regular messages.

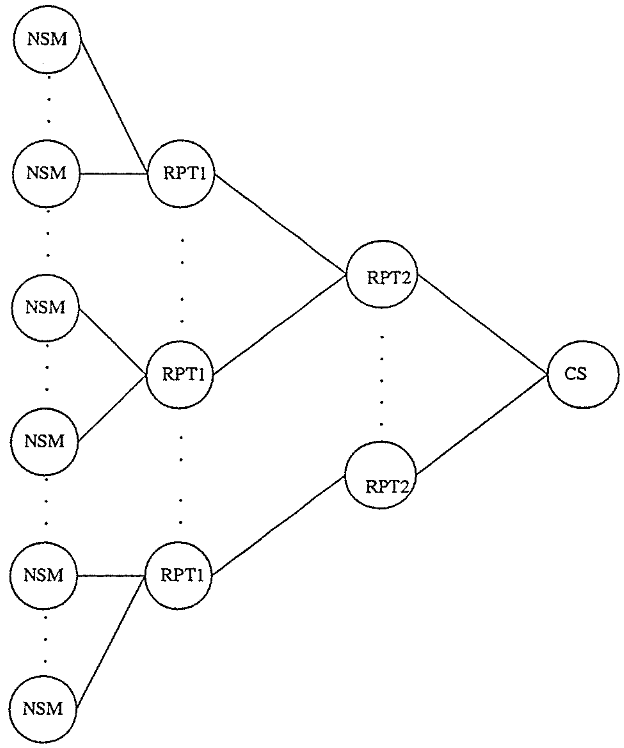

Figure 1.1: System 2020

1.2 System 2020

System 2020 is a data acquisition system designed by IRIS Systems Inc. to support automatic meter reading and distribution automation with application to gas, water, and electric utilities. In addition to the communication links, the system consists of Network Service Modules (NSMs), 1st Level Repeaters (RPT1s), 2nd Level Repeaters (RPT2s), and a Central Server (CS). NSMs are installed in the electric meters to carry out the reading. In the case of a power outage or a meter-cover removal the NSM enters an alarm mode. The NSMs transmit meter readings and alarm messages to the RPT1s. The RPT2s poll the RPT1s for these messages, store them, and then forward them after being polled to the CS. A schematic diagram of System 2020 is shown in Fig. 1.1. Our interest is in the last stage of the network i.e.

the RPT2s to CS portion.

1.3 Scope and Objective

The objective of this thesis is to model the data acquisition System 2020 as a polling system where each station has two queues: one for low priority messages and one for high priority messages. Each station holds messages and waits for the server to start transmission. Once the station seizes the server it transmits messages in accordance with the service discipline. The performance of the polling system will be studied using an approximate approach. The approximation is based on the results of the $M/G/1$ queueing system with vacation (intervisit) periods and occupation (service) periods. Two models are considered, an exhaustive service discipline and a time limited service discipline. The proposed models will approximate the mean waiting time of the exhaustive service discipline and the upper bound of the mean waiting time of the time limited service discipline. The effect of system parameters on the network performance under the exhaustive and the time limited service disciplines are studied.

1.4 Thesis Outline

Chapter 2 is a literature review of polling systems and the $M/G/1$ queueing system. Chapter 3 is a brief description of System 2020, its components, and the polling algorithms for the exhaustive and the time limited service disciplines. The analysis and effect of system parameters on the network performance for the exhaustive service discipline is presented in Chapter 4 and that of the time limited service discipline in

Chapter 5. In Chapter 6, a comparative study of the two polling algorithms with application to Manitoba Hydro is performed. Chapter 7 concludes this thesis and suggests some work for the future.

CHAPTER 2

Literature Review

2.1 Introduction

There is extensive literature in the area of polling systems. Work in this area can be classified into two categories, exact analysis and approximate analysis. The exact analysis is based on the moment generating function and Laplace Steiljtes transform (LST) of the busy period and the number of messages present at a station at polling instant. The approximate analysis is based on considering each station as a one node single server queue with occupation and vacation periods. In this chapter we review polling systems and the class of the $M/G/1$ queueing system, which is frequently used to obtain approximate results for polling systems.

Fig. 2.1 shows a multiqueue polling system. Each station holds messages and waits for the server to start transmission. In general, there is an overhead time R_i associated with each station i and is equal to the time it takes the server to switch from station i to station $i+1$. This overhead time is known as the walk time, connect time, or switchover time. The overhead time is usually modeled as a random variable R_i with mean r_i and second moment $r_i^{(2)}$.

The polling system shown in Fig. 2.1 consists of N stations $(0 < N < \infty)$ and n priority levels. Hence, each station has n queues where the lowest priority is n and the highest priority is 1. Arrival of customers type j at station i $(i = 1, 2, ...N; j = 1, 2, ...n)$ are independent Poisson processes with parameters $\lambda_{i,j}$; service times (trans-

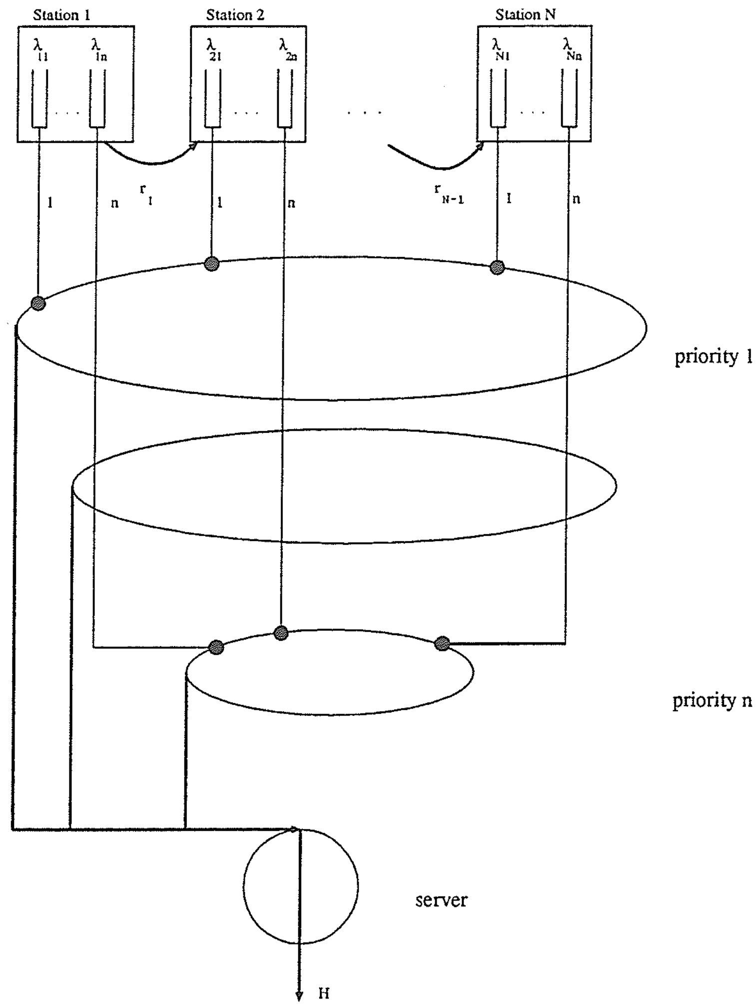

Figure 2.1: Multiple Priorities Polling System

mission times) are generally distributed random variables $H_{i,j}$ with mean $h_{i,j}$ and second moment $h_{i,j}^{(2)}$. Thus, $\rho_{i,j} = \lambda_{i,j} h_{i,j}$ represents the utilization at station i for priority j; $\rho_i = \sum_j \rho_{i,j}$ is the station utilization, and $\rho = \sum_i \rho_i$ is the system utilization. To ensure stability, the system utilization ρ must be less than one.

Several service disciplines exist. The most frequently used are the exhaustive, gated, and limited.

Exhaustive: For the exhaustive service policy, when station i is polled for service, the service continues until the station is empty. Hence, all messages found in the station, and those which arrived during the service period, are transmitted during the current service period.

Gated: For the gated service policy, when a station is polled for service only those messages present at the beginning of the service period are transmitted. Thus, during a service period, the arriving messages are transmitted during the next service period.

Limited: For the limited service policy, the server polls a station and transmits at most k ($0 < k < \infty$) messages, given that at least one message is waiting.

The service time, for queue i, is the period beginning when the server becomes available for queue i in cycle j and ending when the server leaves queue i in cycle j.

The intervisit time, for queue i, is the period beginning when the server leaves queue i in cycle j and ending when queue i is polled in cycle $j+1$.

The cycle time, for queue i, is the period beginning when queue i is polled in cycle j and ending when queue i is polled in cycle $j+1$. A schematic diagram of the relation between the cycle time, intervisit time, and service time is shown in Fig. 2.2.

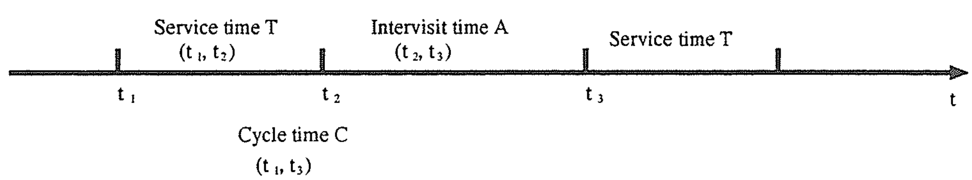

Figure 2.2: Cycle, Service, and Intervisit time

2.2 Polling Systems

Most of the work done in the area of polling systems has been limited to the case of single priority at each station with cyclic or random polling discipline. Few authors have attempted to solve the multi-priorities polling systems. A typical single priority polling system, as shown in Fig. 2.3, consists of N stations and a single server. The arrival at station i is a Poisson process with parameter λ_i (note that the second subscript is dropped because each station has only one priority level); service times are arbitrarily distributed random variables H_i with mean h_i and second moment $h_i^{(2)}$. Hence, the station utilization is $\rho_i = \lambda_i h_i$ and the system utilization is $\rho = \sum_i \rho_i$. The overhead time is modeled by a random variable R_i. In general, the random variables $R_1, R_2, ...R_N$ are assumed to be independent random variables with mean r_i and second moment $r_i^{(2)}$ $i = 1, 2, ..., N$.

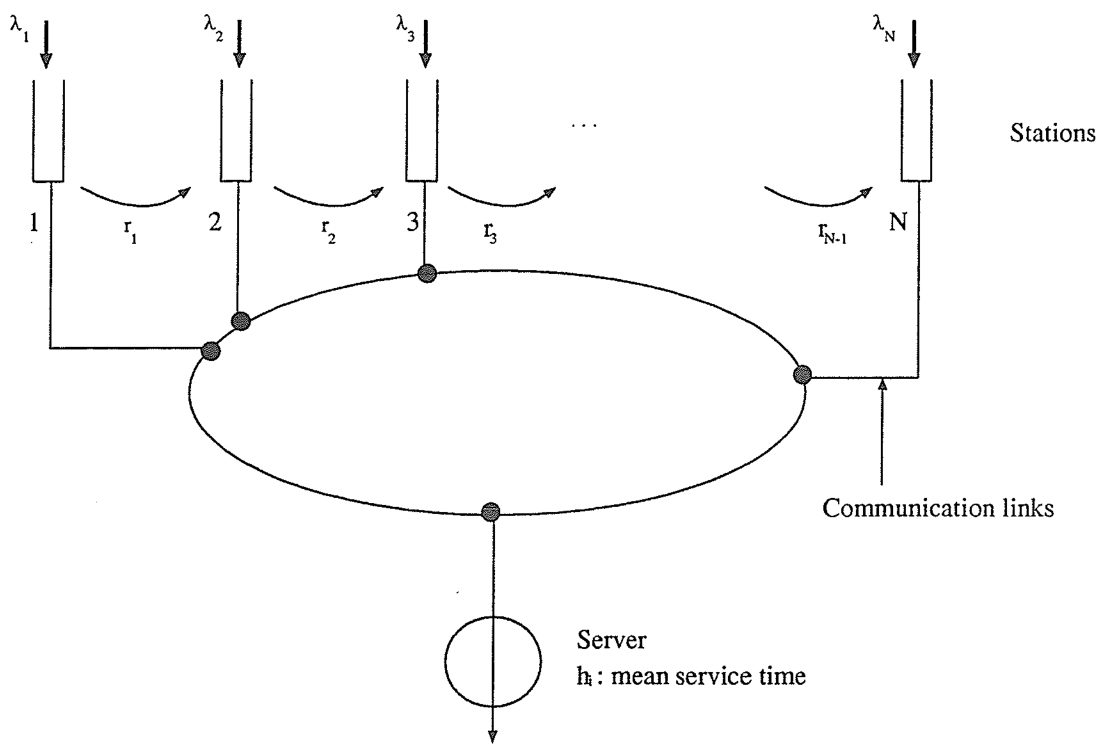

Figure 2.3: Single Priority Polling System

Polling systems are studied in continuous and discrete time. A continuous time polling model is a process in which transitions between states may take place at any instant in time. A discrete time polling model is a process in which transitions between states may take place only at discrete points in time. In the case of discrete time models, time is slotted with the slot size equal to the service of a customer, and all time units are normalized to this slot size.

2.2.1 Continuous time models

Among the first attempts to solve multiqueue system is the work of Eisenberg [1]. He considered a continuous time polling system similar to Fig. 2.3 with exhaustive service discipline. He obtained the (LST) of the intervisit time and the LST of the waiting time at queue i. He considered embedded Markov chains at the instant of service beginning, service completion, beginning of queue visit, and the end of queue visit. His solution relies on the relationship between the probabilities of the embedded Markov chains mentioned above. For instance, the beginning of a queue visit must coincide with a service beginning.

Takagi [2] considered a cyclic polling system as shown in Fig. 2.3 with infinite buffers and exhaustive or gated service disciplines. His solution is based on defining the joint marginal generating function F_i of the number of messages at station i at polling instants. He then related F_i to F_{i+1} and obtained analytical expressions for the first and second moments of the queue length. For symmetric systems (arrival rate, connect time and service time are independent of the station's number) a closed form solution was obtained for the first and second moments of the queue length. For asymmetric systems (arrival rate, connect time, and service time depend on the station's number) the second moments of the queues' length are obtained by solving numerically a set of $O(N^3)$ equations. Takagi [2] obtained the LST of queue length distribution by defining regeneration points as the points when station one is polled and all the queues are empty. The LST of the waiting time distribution was obtained from the relationship between the LST of the queue length and busy period distributions. For the limited service policy, Takagi [2] considered a symmetric cyclic polling system. Following the same procedure as for the exhaustive and gated service disciplines, he obtained the mean number of messages at station i and the mean waiting time. For symmetric cyclic polling systems (continuous time)

$$E(W)|_{exhaustive} \leq E(W)|_{gated} \leq E(W)|_{limited}$$

This relationship states that for symmetric cyclic polling systems the exhaustive service discipline has the least mean waiting time.

Leung [3] analyzed an asymmetric polling system with a probabilistically limited service policy. In this service policy, the maximum number of customers at a queue served during a server visit is determined by a probability. The queue length distribution is obtained via the discrete Fourier transforms. From the mean queue length, Leung [3] obtained the mean waiting time using Little's law. Since the solution is based on a numerical approach, the memory and CPU time are exponential functions of the number of queues. Hence, under heavy loads only relatively small systems can be solved.

Kuehn [4] considered a cyclic polling system with batch Poisson arrivals. He obtained the LST of the waiting time distribution using the embedded Markov chain approach.

Manfield [5] considered a polling system with two way data traffic. In this polling system, priority is given to messages going from the central controller (server) to the stations. The system is analyzed by considering $(N+1)$ stations, where N stations are dedicated to the incoming messages (messages going from the station to the server), and the $(N+1)$st station is dedicated for the outgoing messages (messages going from the server to the stations). The mean delay for the outgoing messages is exact and for the incoming messages is an approximation.

Gianini and Manfield [6] considered a polling system where each station in the system has two priority levels. They considered the case of exhaustive and gated (at the priority level) service disciplines. In these service disciplines, a station is polled at low priority level only if there are no high priority messages anywhere in the system. Their method of solution is based on defining a low priority poll busy period, a high priority poll busy period, and the moment generating function of the queue length at polling instants. They derived the first and second moment of the queue length and the waiting time for the high and low priority messages. However, their numerical results for the mean waiting time of the high priority messages do not seem to match their equation.

Tsai and Rubin [7] obtained exact results for a polling system with two priority levels with exhaustive or limited service disciplines. Their system is different from that of Gianini and Manfield [6] since they considered the case where each station has a single buffer high priority queue and an infinite buffer low priority queue. A station can seize the server at low priority only if all high priority buffers are empty. During a low priority poll with exhaustive service policy, the server continues to transmit messages until both queues are empty. Thus, in a low priority poll, all messages found in the queue and those that arrive (high or low) during the service period are transmitted in the current cycle. If a station seizes the server at a high priority level, then only the high priority message is transmitted. For the limited service policy, during a high (low) priority poll the server transmits one high (low) priority message.

Several attempts have been made to approximate the performance of cyclic polling systems. Pseudoconservation laws have been used by Boxma and Meister [8] to approximate the mean waiting time of non-exhaustive cyclic polling systems. Chang and Sandhu [9] used the pseudoconservation laws to approximate the mean waiting time for limited service policy (the server will switch from queue i to queue $i+1$

if the queue becomes empty or k_i messages are transmitted, whichever comes first). Everitt [10] summarized the pseudoconservation laws for cyclic service systems with exhaustive, gated, and limited service disciplines. He also derived a new result for the exhaustive limited service policy.

For asymmetric polling systems with an exhaustive service policy, to get around the numerical complexity in solving a system of $O(N^3)$ equations, Bux and Truong [11] considered the system as a $M/G/1$ queue with occupation and vacation periods. The mean waiting time of a $M/G/1$ queue with vacation periods depends on the mean and variance of the vacation period. The mean of the vacation period was obtained from the mean of the cycle time and service period. The variance of the vacation period was obtained by using a heuristic extrapolation from the case of $N = 2$.

2.2.2 Discrete time models

Takagi [2] considered a cyclic polling system with infinite buffers and exhaustive, gated, or limited service disciplines. In the case of the exhaustive service policy, the number of messages at arbitrary times and the waiting times were obtained using the same technique as in the case of continuous time models. His results show that, for the same total utilization, the mean waiting time at station one, in the case where all utilization is concentrated at station one, is smaller than the mean waiting time in the symmetric polling system.

For the gated service policy, the number of messages and waiting time were obtained using the same technique as in the case of continuous time models. Also, Takagi [2] proved that for the same total utilization, the mean waiting time at station one, in the case where all utilization is concentrated at station one, is larger than the mean waiting time in the symmetric polling system.

For the limited service policy, only the symmetric limited (discrete time) model is considered and it is shown that the condition for stability is $N\lambda(r_i + h_i) < 1$. The number of messages and the waiting time were obtained. As in the case of continuous time, for symmetric cyclic polling systems the exhaustive service discipline has the least mean waiting time and the limited service policy has the largest mean waiting time.

$$E(W)|_{exhaustive} \leq E(W)|_{gated} \leq E(W)|_{limited}$$

Similar results for the exhaustive, gated, or limited service disciplines were obtained by Kleinrok and Levy [12] for the case of random polling systems using the same analysis as Takagi [2]. For symmetric random polling systems the exhaustive service discipline has the least mean waiting time and the limited service discipline has the largest mean waiting time.

$$E(W)|_{exhaustive} \leq E(W)|_{gated} \leq E(W)|_{limited}$$

Kleinrock and Levy [12] showed that for the same system parameters, cyclic polling systems (under any service discipline) have a lower mean waiting time than random polling systems.

2.3 Variation of $M/G/1$ Model

Each station in a polling system can be represented by a single server queue with vacation periods and occupation periods which correspond to the intervisit period and the service period in a polling system, respectively. Many authors have used the results of the $M/G/1$ queueing system to approximate the performance of polling systems. In this section we review the $M/G/1$ queueing model.

Several authors have studied the $M/G/1$ queue with occupation and vacation periods. Doshi [13] studied the stochastic decomposition of the $GI/G/1$ queue with vacation periods. Fuhrmann and Cooper [14] showed that the stationary number of customers present in the system is distributed as the sum of two or more random variables, one of which is the stationary number of customers present in the standard $M/G/1$ queue. They showed that this property holds for a general class of $M/G/1$ queueing system such as the N-policy, in which the server waits until N customers are waiting to start service, and the limited service queueing model, in which the number of customers served during an occupation period is limited to k. Doshi [15] studied the stochastic decomposition of the $M/G/1$ queue with single and multiple vacation periods. He showed that the results obtained by Fuhrmann and Cooper can be proved by three arguments. 1) Embedded Markov chain in which transition occurs at service completion or vacation termination. 2) Level crossing arguments in which the rate at which the functions of the process downcross a level $x > 0$ should equal the rate at which it jumps from below x to above x. 3) Sample path arguments in which the vacation period is treated as additional work and that the work in the system seen by an arrival is its waiting time for the FIFO discipline.

Levy and Yechiali [16] considered a $M/G/1$ queue with single priority with single and multiple vacations. They used an embedded Markov chain with transition occurring at service completion or vacation termination. They obtained the LST of the queue length and the LST of the waiting time for single and multiple vacations models. Shanthikumar [17] studied the multiple priorities non-preemptive $M/G/1$ queue with single and multiple vacations models. He used the method of level crossing to obtain the LST and the waiting time for priority k. Kella and Yechiali [18] extended the work of Shanthikumar [17] to the case of a multiple priorities preemptive $M/G/1$ queue with single and multiple vacations models.

For a $M/G/1$ queue with linear holding cost and fixed charge for activating the server, Heyman [19] showed that the optimal N policy (the server vacation terminates when there are N customers in the queue) is better than the optimal T policy (the server will take a vacation of length T).

Leung and Eisenberg [20] and [21] studied the gated and non-gated time limited $M/G/1$ queue. For each model they derived a functional equation which characterizes the amount of work at polling instant. These equations were then solved using a numerical technique based on Laguerre functions. Using the stochastic decomposition and because Poisson arrival see time average (PASTA), the average waiting time is related to the amount of work found by an arrival.

Lee [22] analyzed the $M/G/1$ queue with vacation and finite capacity queue using embedded Markov chain. His results were used to study the performance of a cyclic polling system with an exhaustive service policy, where each station has a finite capacity. He also considered a $M/G/1$ queue with finite capacity and exhaustive limited service discipline. For this service policy, the server will take a vacation of random length if the queue is empty or M customers are served. The LST of the busy period and cycle time were obtained using an embedded Markov chain. The waiting time distribution, blocking probability, and queue length were obtained by the method of supplementary variables and sample biasing techniques.

Lee and Sengupta [23] considered a polling system with limited service and reservation. For this service policy, each station makes a reservation for the number of services required for cycle $j+1$ after receiving service in cycle j. However, the minimum number of services must be at least one and at most M. Their solution is based on the concept of a single queue with occupation and vacation periods. Their iterative procedure assumes that the vacation period of queue 1 in iteration $(k+1)$ is given by the sum of the following two terms: $\sum_{i=2}^{N} S_i^{(k)}$ with probability $(1-P)$, where $S_i^{(k)}$ is the service period for queue i in iteration k, $S_i^{(k)}$ are independently identically random variables (sum of independent service periods), and $(N-1)S^{(k)}$ with probability P, where $S^{(k)}$ is a generic service period (sum of dependent service periods). The results obtained consist of the queue length and sojourn-time distributions.

CHAPTER 3

RPT2-CS Network

3.1 System 2020

The main components of System 2020 are Network Service Modules (NSMs), 1st Level Repeaters (RPT1s), 2nd Level Repeaters (RPT2s), a Central Server (CS), and communication links.

Automatic meter reading is carried out by the NSMs which can be installed in the electric meters. Messages, which can range from meter reading to alarms, are transmitted at random intervals by the NSMs, through the RPT1s to the RPT2s. When a RPT2 receives a message, it stores it, and waits to be polled by the CS.

Manitoba Hydro and IRIS Systems Inc. have divided the messages into five priority levels, PL 1 through PL 5, with PL 1 being the highest priority:

(a) Load Survey Mode (PL 5): Load survey messages are transmitted at random intervals.

(b) Normal Mode (PL 4): Meter readings are generated by NSMs and are transmitted at random intervals.

(c) Status Messages (PL 3): These messages are transmitted directly to the RPT2s.

(d) Test Messages (PL 2): These are test messages and are generated only upon installation of the network.

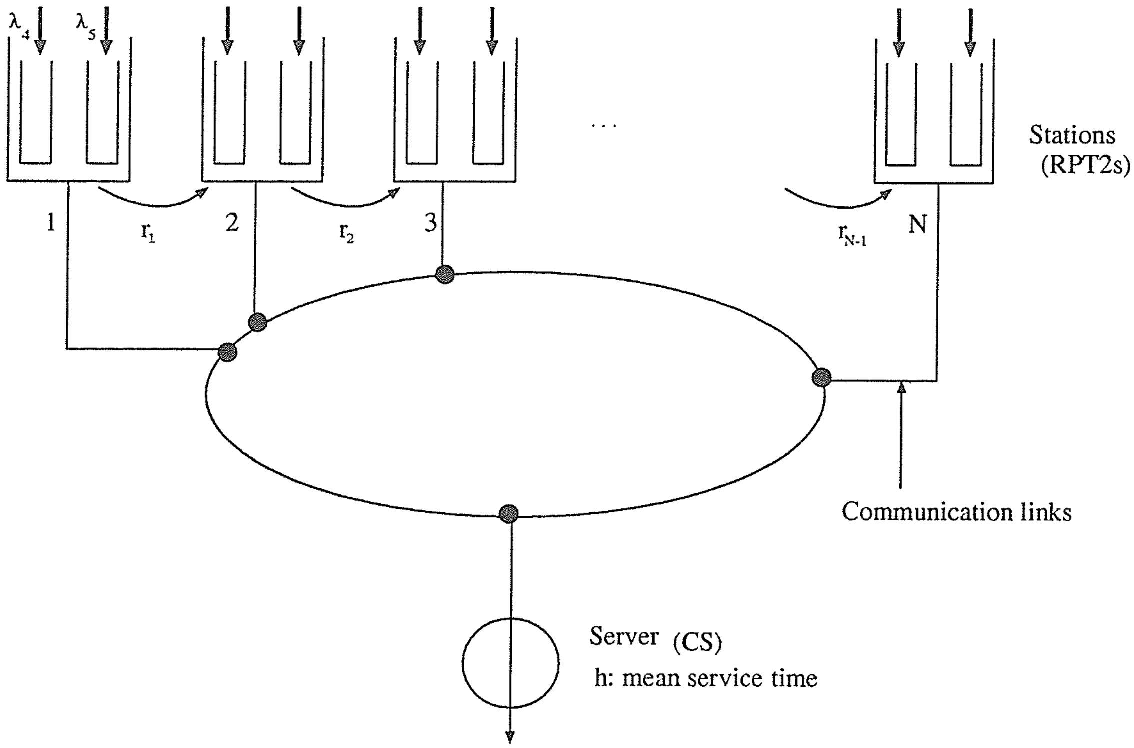

Figure 3.1: RPT2-CS Network Model

(e) Alarm Mode (PL 1): Messages that could result from, among other things, meter cover removals or power outages.

3.2 RPT2-CS Model

The RPT2s-CS network will generally consist of N ($1 \leq N < \infty$) stations and one server. Each station will represent a RPT2 with five priority queues, one for each priority level. In this thesis we focus on two priorities only, priority level four and priority level five. Fig. 3.1 shows the RPT2s-CS network with the CS polling the RPT2s in a cyclic order, according to the polling algorithm, for PL 5 and PL 4 messages.

Two different types of polling disciplines will be considered: Time limited service discipline and exhaustive service discipline. Several key parameters of the RPT2s-CS network will now be explained, including the two polling algorithms.

3.2.1 RPT2 switchover time

For the RPT2s-CS network we assume that the connect time between two consecutive RPT2s is deterministic and denoted by R. We also assume that the connect time distribution between $(i, i+1)$ and $(i+1, i+2)$ is the same. These assumptions are based on the fact that Manitoba Hydro intends to use telecommunication links between the CS and RPT2s, which yield a connect time between two stations of approximately 20 seconds. Also, it is assumed that Manitoba Hydro uses the same kind of modems throughout the network.

3.2.2 Message arrival process

Messages arriving at each RPT2 are either of priority level PL 5 (low priority) or PL 4 (high priority). Since the statistical distributions of the arrival processes of PL 5 and PL 4 messages are not available, we assume that the arrivals of PL 5 and PL 4 follow a Poisson distribution (this assumption will ease the mathematical derivation of the waiting time distribution) and have arrival rates of λ_5 and λ_4, respectively.

3.2.3 CS service rate

In general, the service time depends on the message length. However, according to Iris Systems Inc., for the RTP2s-CS polling system the message length is constant. Therefore, the CS will transmit messages with a deterministic service times distribution and we denote the transmission time random variable by H.

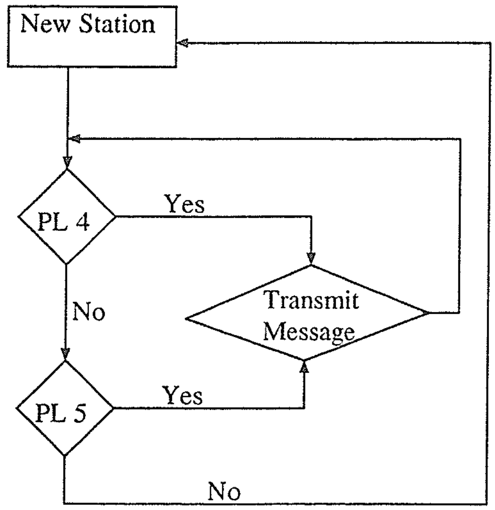

Figure 3.2: Polling Algorithm: Exhaustive Service Discipline

3.2.4 RPT2 buffers

In real life the buffer size is finite. However, for the RPT2s-CS network each priority queue has a large capacity. Therefore, for ease of analysis the buffer size is assumed to be infinite, and arriving messages cannot balk or switch to another RPT2.

3.2.5 CS polling disciplines

(a) Exhaustive Service Discipline: The CS will poll the RPT2s sequentially and service will continue at the polled station until all messages of both priority levels are transmitted. Fig. 3.2 is the algorithm for this polling discipline. Service continues until all enqueued messages of both priority levels have been transmitted. The length of an occupation period, T_i, is the time it takes to transmit all enqueued messages of both priority levels.

(b) Time Limited Service Discipline: The algorithm for the time limited polling discipline is shown in Fig. 3.3 and is described as follows:

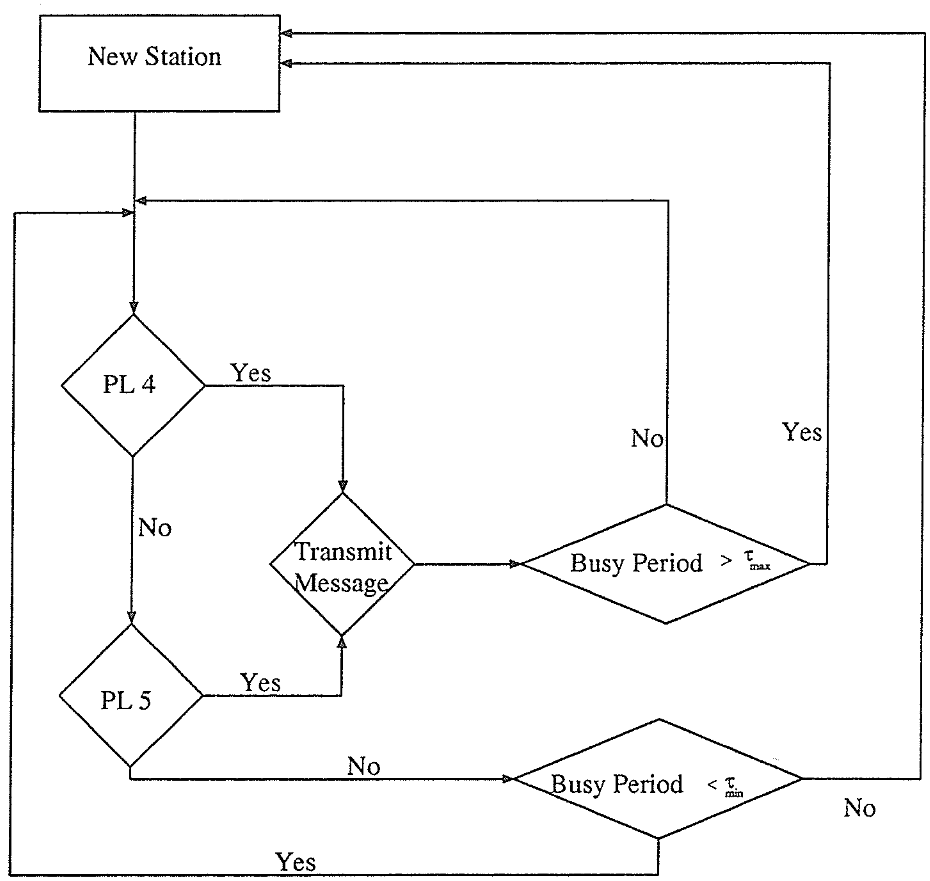

Figure 3.3: Polling Algorithm: Time Limited Service Discipline

RPT2s are polled sequentially by the CS. Each time a station is polled it may send a high priority message, a low priority message, or have no message to transmit.

During a poll, the server tends to a queue until one of the following two conditions is violated:

1. The station occupation period T_i (i stands for the RPT2 number) reaches its maximum length τ_{max}, which is the time set for one service period. In this case the CS will switch to the next RPT2 and the remaining messages will be transmitted during a subsequent cycle.
2. Both the PL 4 and PL 5 queues are empty, and the occupation period has exceeded the minimum occupation period required, τ_{min}.

The exhaustive and time limited polling disciplines are chosen because the connect time is relatively high compared to the mean service time (an initial estimate of the mean service time is 1/12 sec. and that of the connect time is 20 sec.). Also, it is shown in Section 2.2.1 that the exhaustive service discipline gives the least mean waiting time for symmetric polling systems. The time limited service discipline is chosen to give Manitoba Hydro the option of synchronizing the polling time.

CHAPTER 4

Exhaustive Service Discipline

4.1 Introduction

In this chapter the polling algorithm of the exhaustive service discipline shown in Fig. 3.2 is analyzed using an approximate approach. Each station is considered to be a $M/G/1$ queue with two priorities, an occupation or service period, and a vacation period, which is known as the intervisit period in polling systems terminology. After the mean waiting time analysis, several examples are considered in order to study the effect of high priority arrival rate, low priority arrival rate, number of stations, connect time, mean service time, service time distribution, high priority percentage, and asymmetric system on the mean waiting time of high and low priority messages.

4.2 Mean Waiting Time Analysis

The polling system shown in Fig. 3.1 consists of N $(0 < N < \infty)$ stations. Each station in the system has two priority levels, a high priority level and a low priority level. Because only two priorities are considered, the high priority is labeled priority 1 and the low priority is labeled priority 2. The arrivals at station i, priority j $(i = 1, 2, ..., N; j = 1, 2)$ are independent Poisson processes with parameters $\lambda_{i,j}$, let $\lambda_i = \lambda_{i,1} + \lambda_{i,2}$. The message transmission times (service times) are arbitrarily distributed random variables $H_{i,j}$ with mean $h_{i,j}$ and second moment $h_{i,j}^{(2)}$ $(i = 1, 2, ...N; j = 1, 2)$. Consider an arbitrary message at node i, let H_i be the weighted service times of this message at station i for $i = 1, 2, ...N$, H_i is given by

$$H_i = \frac{\lambda_{i,1}}{\lambda_i} H_{i,1} + \frac{\lambda_{i,2}}{\lambda_i} H_{i,2} \tag{4.1}$$

Let the utilization at station $i = 1, 2, ..., N$ for priority $j = 1, 2$ be $\rho_{i,j} = \lambda_{i,j} h_{i,j}$.
The station total utilization $\rho_i = \rho_{i,1} + \rho_{i,2}$, $i = 1, 2, ...N$.
The system total utilization $\rho = \sum_i \rho_i$
The first moment (h_i) and the second moment $(h_i^{(2)})$ of the service times of this message are

$$h_i = \frac{\lambda_{i,1}}{\lambda_i} h_{i,1} + \frac{\lambda_{i,2}}{\lambda_i} h_{i,2} \ , \tag{4.2}$$

and

$$h_i^{(2)} = (\frac{\lambda_{i,1}}{\lambda_i})^2 h_{i,1}^{(2)} + (\frac{\lambda_{i,2}}{\lambda_i})^2 h_{i,2}^{(2)} + 2 \frac{\lambda_{i,1} \lambda_{i,2}}{\lambda_i^2} h_{i,1} h_{i,2} \ , \tag{4.3}$$

respectively.

This aggregation procedure is key to our approximation because then we are able to use Kella and Yechiali's [18] results for the vacation model.

Let r_i be the overhead time associated with station i. The overhead time is the time taken by the server to disconnect from station i and connect to station $i + 1$. Let R be the sum of all overhead times $R = \sum_i r_i$.

The service discipline at each station is non-preemptive exhaustive. When a station is polled, the server will continue to transmit until both queues become empty. Thus, all customers of both priority levels found in the station at polling instant, and those which arrived during the service period, are transmitted during the current station visit. Let A_i, the time interval from the server's departure from station i until it returns to the same station i, denote the vacation period or the intervisit time for station i. This leads to the mean waiting time for the high and low priority in station i (see Kella and Yechiali [18] or Shanthikumar [17]).

$$W_{i,1} = \frac{\lambda_i h_i^{(2)} + (1-\rho_i)\frac{A_i^{(2)}}{A_i}}{2(1-\rho_{i,1})} \tag{4.4}$$

$$W_{i,2} = \frac{\lambda_i h_i^{(2)} + (1-\rho_i)\frac{A_i^{(2)}}{A_i}}{2(1-\rho_{i,1})(1-\rho_i)} \tag{4.5}$$

The only unknowns in equations Eq. 4 and Eq. 5 of the mean waiting times are the first and second moment of the vacation period for station i. If we assume that the arrival rate for station i is λ_i and the service time distribution is H_i with mean h_i and second moment $h_i^{(2)}$, then the first moment and the variance of the vacation period are given in Bux and Truong [11] as

$$A_i = \frac{1-\rho_i}{1-\rho} R \tag{4.6}$$

$$var(A_i) = \frac{R}{(1-\rho)^2} \sum_{\substack{k=1 \\ k \neq i}}^{N} \frac{\lambda_k h_k^{(2)}(1-\rho_i)^2 + \lambda_i h_i^{(2)} \rho_k^2}{1-\rho_i-\rho_k+2\rho_i\rho_k} \tag{4.7}$$

The second moment of the vacation period can be obtained by

$$A_i^{(2)} = var(A_i) + A_i^2 \tag{4.8}$$

For $i = 1, 2, ..., N$.

4.3 Effect of System Parameters on the Mean Waiting Time

In this section, we present some numerical examples for the exhaustive service discipline. The accuracy of this approximation is discussed in a paper by Frigui, Stone and Alfa [24].

4.3.1 Effect of high priority arrival rate

This section discusses the effect of changing the arrival rate of the high priority messages, for a symmetric cyclic polling system with a deterministic service times distribution having a mean of ten units of time for both priority levels, a connect time of one unit of time, an arrival rate for the low priority messages $\lambda_2 = 0.003$ messages per unit of time, and ten stations.

Fig. 4.1 shows that as the arrival rate of the high priority messages increases, the mean waiting times of the low and high priority messages increase exponentially. This is because as the arrival rate of the high priority messages increases the vacation period increases, hence the increase in the mean waiting times. Also, note that the difference between the mean waiting times of the low and high priority messages increases as the arrival rate of the high priority messages increases. This is because as more high priority messages arrive to the system, the server spends more time serving these messages and hence the increase in the difference between the mean waiting times of the low and high priority messages. However, the relative difference, Rel. Diff. $= \frac{W_2 - W_1}{W_2}$, is not affected by the change of the arrival rate of the high priority messages (Table A.1).

4.3.2 Effect of low priority arrival rate

This section discusses the effect of changing the arrival rate of the low priority messages, for a symmetric cyclic polling system with a deterministic service times distribution having a mean of ten units of time for both priority levels, a connect time of one unit of time, an arrival rate for the high priority $\lambda_1 = 0.003$ messages per unit of time, and ten stations.

Fig. 4.2 shows that as the arrival rate of the low priority messages increases, the

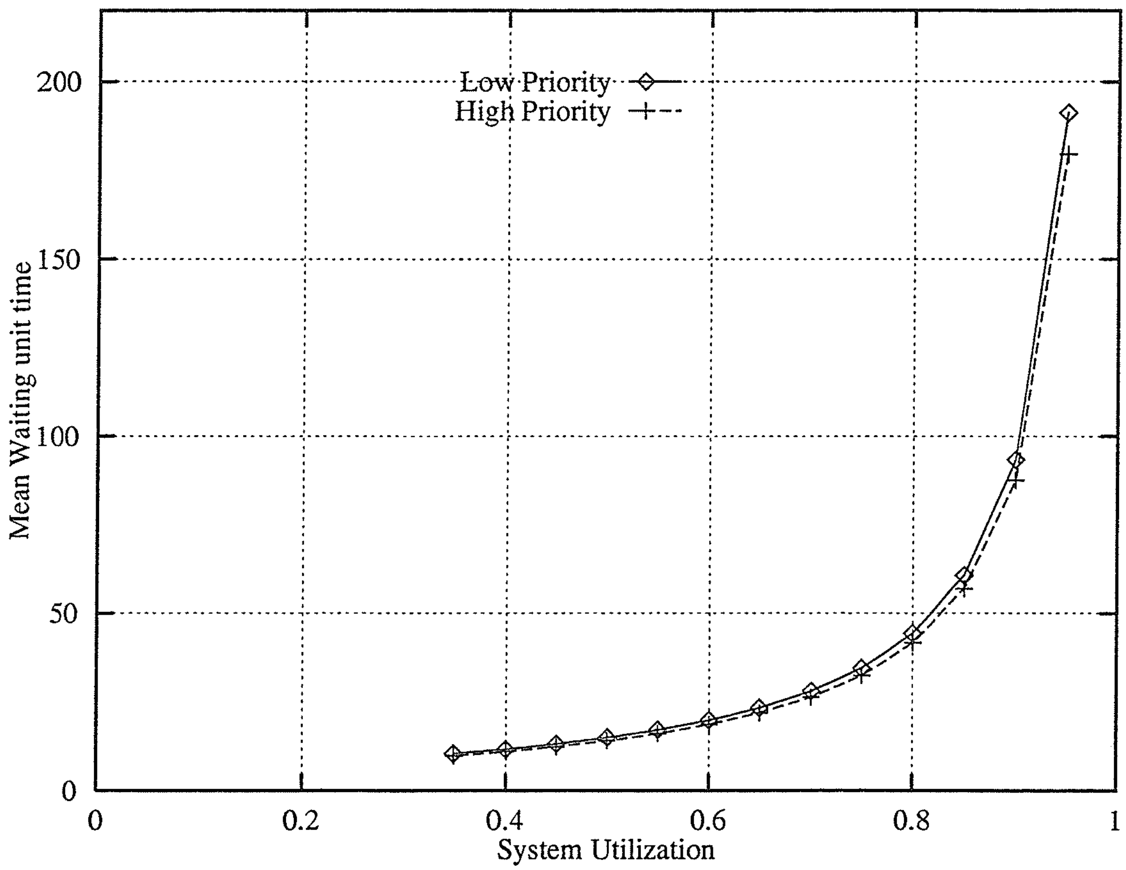

Figure 4.1: Effect of High Priority Arrival Rate

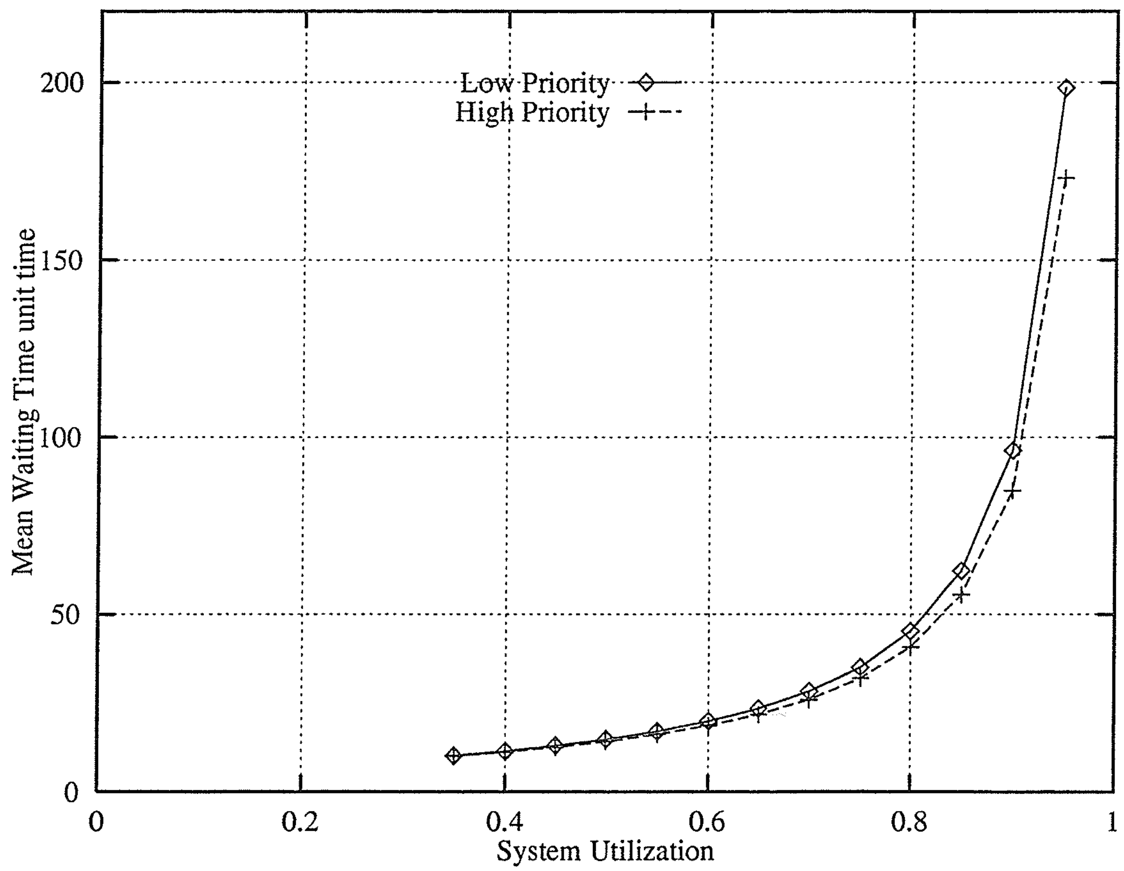

Figure 4.2: Effect of Low Priority Arrival Rate

mean waiting times of the low and high priority messages increase exponentially. This is because as the arrival rate of the low priority increases, the vacation period increases and hence the increase in the mean waiting times. Note that in this case (Table A.2), the difference between the mean waiting times of the low and high priority messages increases faster than that of the previous case (Table A.1). This is because as more low priority messages arrive at each station, the vacation period increases. This leads to an increase of the number of both the high and low priority messages, but the increase of the low priority messages is faster than that of the high priority messages. Therefore, the increase in the mean waiting time of the high priority messages is smaller than that of the low priority. Hence, the increase in the relative difference between the mean waiting times.

4.3.3 Effect of number of stations

Fig. 4.3 is for a symmetric cyclic polling system with a deterministic service times distribution having a mean of ten units of time for both priority levels, an arrival rate for the low priority $\lambda_2 = 0.002$ messages per unit of time, an arrival rate for the high priority $\lambda_1 = 0.0001$ messages per unit of time, and a connect time of one unit of time. Fig. 4.3 shows that as the system utilization increases from increasing the number of stations the mean waiting times of the low and high priority messages increase exponentially. However, the relative difference stays constant as the number of stations increases. This fact can be verified from equations Eq. 4.4 and Eq. 4.5.

4.3.4 Effect of connect time

For a symmetric cyclic polling system with a deterministic service times distribution having a mean of ten units for both priority levels, an arrival rate for the high priority $\lambda_1 = 0.0001$ messages per unit of time, an arrival rate for the low priority $\lambda_2 = 0.002$

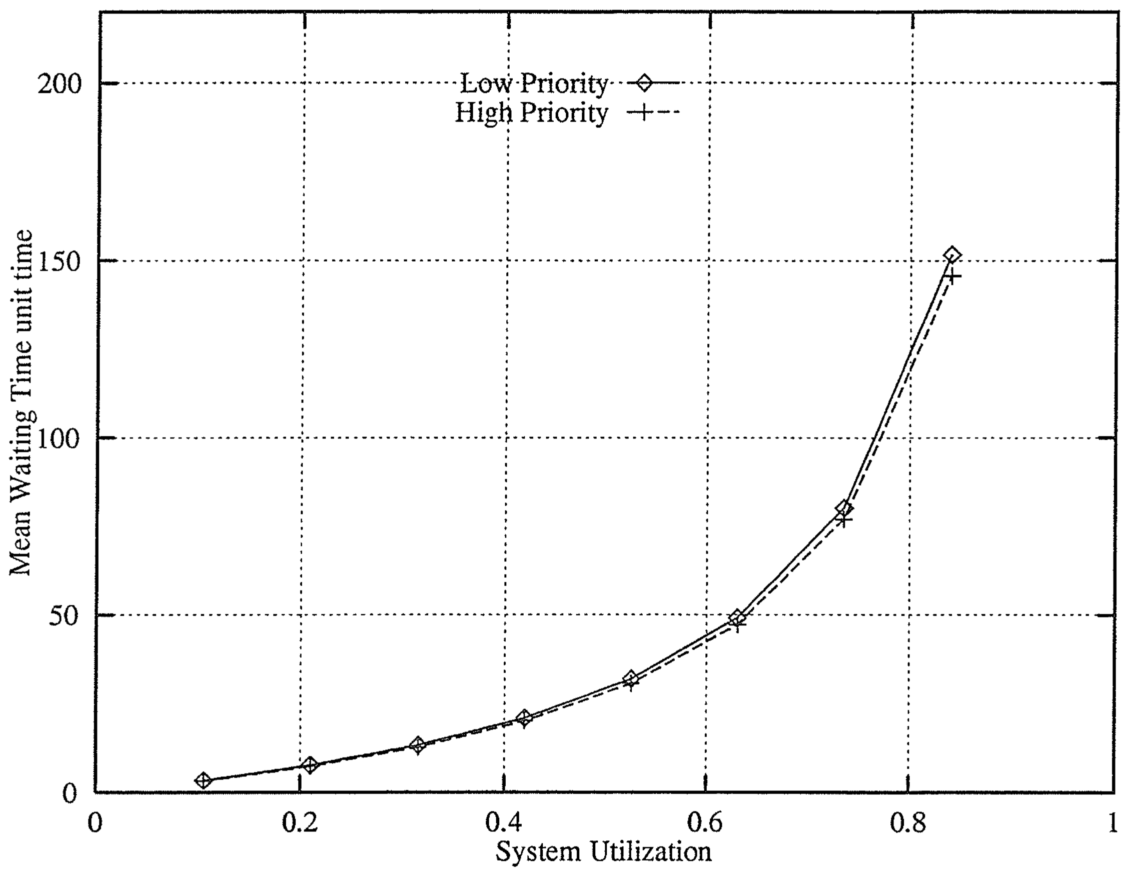

Figure 4.3: Effect of Number of Stations

messages per unit of time, and ten stations. Table A.4 shows that as the connect time increases the mean waiting times of the low and high priority messages increase. Fig. 4.4 shows that for both the low and high priority messages, the mean waiting time is a linear function of the connect time.

As the connect time increases, there is an increase in the number of messages arriving at each station. Because the high priority messages are served first, their mean waiting time increases less than that of the low priority messages. Hence, the increase in the difference between the mean waiting times of the low and high priority messages, as shown in Table A.4. However, the relative difference between the mean waiting time of the high and low priority messages is not affected by the change in the connect time.

4.3.5 Effect of mean service time

This section discusses the effect of changing the mean service time (both priorities have the same mean service time) for a symmetric cyclic polling system with a connect time of one unit of time, an arrival rate for the low priority $\lambda_2 = 0.002$ messages per unit of time, an arrival rate for the high priority $\lambda_1 = 0.0001$ messages per unit of time, and ten stations.

Fig. 4.5 shows that as the mean service time increases, system utilization approaches one, the mean waiting times of the low and high priority messages increase exponentially. This is an expected behavior of many queueing systems, since it is well known that as the mean service time increases, the system utilization approaches one which is the saturation point.

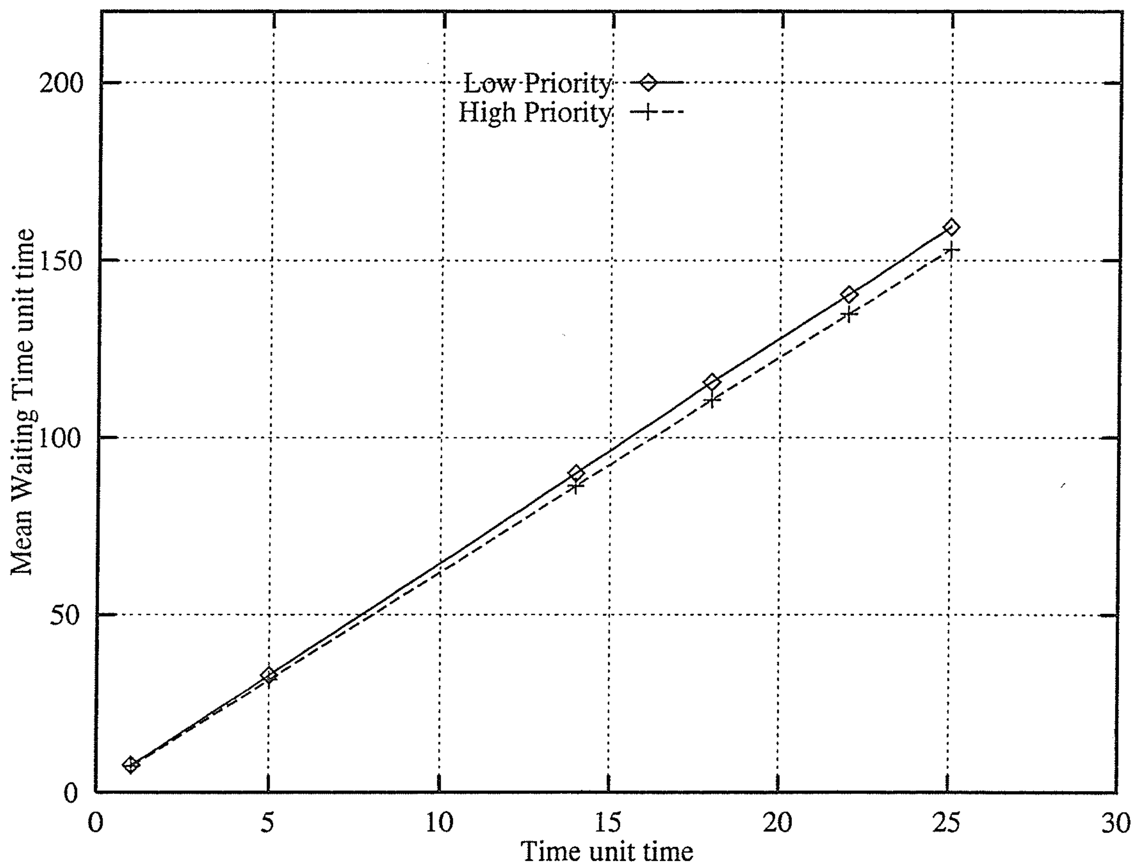

Figure 4.4: Effect of Connect Time

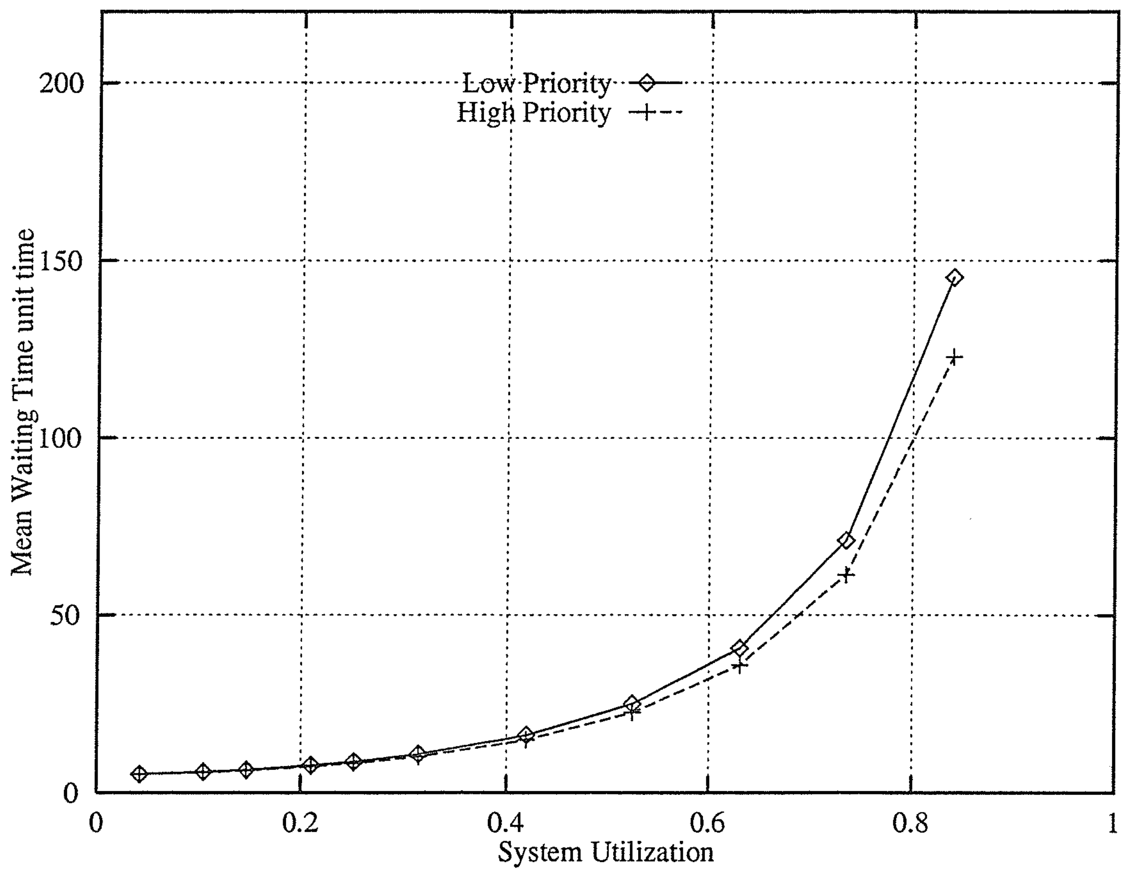

Figure 4.5: Effect of Mean Service Time (Deterministic Dist.)

4.3.6 Effect of service times distribution

This section discusses the effect of the service times distribution for a symmetric polling system having a mean of ten units of time for both priority levels, a connect time of one unit of time, 10% high priority messages, 90% low priority messages, and five stations. Table A.6 shows the effect of the service times distribution on the mean waiting times for different utilizations. For any utilization the deterministic service times distribution yields the smallest mean waiting times. The difference can be attributed to the fact that the variance of the deterministic service times distribution is zero.

4.3.7 Effect of high priority percentage

This section discusses the effect of the high priority messages percentage for a symmetric system with the same deterministic service times distribution having a mean of ten units of time for both priority levels, a connect time of one unit of time, a traffic intensity $\rho = 0.5$, and five stations. It can be seen from Table A.7 that the weighted mean waiting time is constant. Note that as the percentage of the high priority messages increases the mean waiting time of the low priority messages converges toward the mean waiting time of the high priority messages.

4.3.8 Asymmetric polling system

This set of data is for an asymmetric cyclic polling system with the same deterministic service times distribution having a mean of ten units of time for both priority levels, a connect time of one unit of time, five stations in the network where station one utilization $\rho_1 = 0.3$, and stations two through five utilization is $\rho_i = 0.1$ $(i = 2, 3, 4, 5)$, 10% high priority messages, and 90% low priority messages. Table A.8 shows that the station with the heaviest traffic has the least mean waiting time. This can be explained by the fact that messages arriving to station one have a better chance of finding the server available. When we reverse the case (i.e. station one is lightly loaded and stations $2, 3, 4, 5$ are heavily loaded, Table A.9), we confirm our reasoning and see that messages arriving at station one experience the longest mean waiting time.

CHAPTER 5

Time Limited Discipline

5.1 Introduction

In this chapter the polling algorithm of the time limited service discipline shown in Fig. 3.3 is studied. In this polling algorithm each time a station is polled, the server will serve the queue for a period of not less than τ_{min} and not greater than τ_{max}. The analysis yields an upper bound for the mean waiting time. Each station is considered to be a $M/G/1$ queue with an exhaustive limited service policy, an occupation or service period, and a vacation period which is known as the intervisit period in polling systems terminology. After the mean waiting time analysis, several examples are considered in order to study the effect of message arrival rate, parameter τ_{min}, parameter τ_{max}, mean service time, connect time, and number of stations on the upper bound of the mean waiting time.

5.2 Delay Analysis

Let C denote the cycle time which is the time between subsequent visits to a queue, T the service period which is the time the server spends to serve a queue, and A the intervisit period which is the time from the server's departure from a queue until it returns to the same queue.

The stability condition for cyclic multiqueue systems with time limited service policy is given as $\rho = \sum_{i=1}^{N} h\lambda_i < 1$ and $\lambda_i hC < T$ for $i = 1, 2, ...N$, where ρ is the system utilization, h is the mean service time, λ_i is the arrival rate for station i, and N is the number of stations. This inequality states that the total service time of customers that arrive during one cycle must be less than one service period ($\lambda_i C$ is the average number of customers that arrive during one cycle to queue i). Hence, for this queueing system to be stable, the total service time of customer arriving during one cycle must be less than τ_{max}. The stability conditions for the time limited service discipline are given by $\rho < 1$ and $\lambda_i hC < \tau_{max}$. The first condition, $\rho < 1$, ensures that the whole queueing system is stable. The second condition, $\lambda_i hC < \tau_{max}$ for all i, ensures that each individual queue is stable. To use the server effectively the time spent at each queue must be greater than τ_{min}, therefore $\tau_{min} < \lambda_i hC$ for $i = 1, 2, ...N$. These two conditions yield a maximum arrival rate $\lambda_{i,max}$, above which the polling system becomes unstable, and a minimum arrival rate $\lambda_{i,min}$, below which some allocated resources are wasted. If the number of customers that arrive at queue i during each cycle is larger than the number of customers that can be serviced during τ_{max}, a queue buildup will result. This is because at each cycle some messages are not transmitted, therefore the queue will grow boundlessly . When the number of customers that arrive at queue i during one cycle is less than the number that can be serviced during τ_{min}, the server will stay idle while some other messages at another queue are waiting for transmission. This is an unacceptable situation and therefore $\lambda_{i,min}$ must be smaller than the arrival rate.

For a symmetric polling system with N stations the cycle time is given by:

$$C = NT + R \tag{5.1}$$

where $R = Nr$ and r is the connect time between two stations. Using the stability condition $\lambda_i hC < \tau_{max}$ and Eq. 5.1 $\lambda_{i,max}$ is given by:

$$\lambda_{i,max} = \frac{\tau_{max}}{h(R + N\tau_{max})}. \tag{5.2}$$

Using the stability condition $\lambda_i hC > \tau_{min}$ and Eq. 5.1 $\lambda_{i,min}$ is given by:

$$\lambda_{i,min} = \frac{\tau_{min}}{h(R + N\tau_{min})}. \tag{5.3}$$

In order to determine the upper bound of the waiting time of messages in a given queue, it is convenient to consider each queue as a $M/G/1$ queue with server vacation times and an exhaustive limited service policy. In this service policy, during a queue visit the server will transmit messages until either the queue is empty or k messages are transmitted. The upper bound for the mean waiting time W of a symmetric cyclic polling system with an exhaustive limited service policy is given in Fuhrmann [25] as

$$W \leq \frac{1-\rho}{1-\rho-\lambda r/k}\left(\frac{(N-1)r}{2(1-\rho)} + \frac{r^{(2)}}{2r} + W_0\right), \tag{5.4}$$

where W_0 is the average waiting time for the standard $M/G/1$ queue given by:

$$W_0 = \frac{\lambda_i h^{(2)}}{2(1-\rho)} + h. \tag{5.5}$$

Therefore, the upper bound of the waiting time for each station can be known if the maximum expected number of messages, k, served during one service period can be found. The number of messages transmitted can be obtained from the length of the service period. We know that the following relation between the cycle time C, the service period T, and the vacation period A holds

$$C = T + A. \tag{5.6}$$

From equations Eq. 5.1 and Eq. 5.6, T is obtained as:

$$T = \frac{A - R}{N - 1}. \tag{5.7}$$

The maximum expected number of messages serviced during one queue visit is given by $k = T/h$. Hence k is given by

$$k = \frac{A - R}{h(N - 1)}. \tag{5.8}$$

The only unknown in Eq. 5.8 is the vacation period A.

To determine the length of the vacation period A we define A_{max} as the maximum vacation period. This maximum vacation period is obtained if, at a given cycle, the server spends exactly τ_{max} units of time at each queue. Therefore,

$$A_{max} = \tau_{max}(N - 1) + R. \tag{5.9}$$

Also, we define A_{min} as the minimum vacation period. The minimum vacation period is obtained if, at a given cycle, the server spends exactly τ_{min} units of time at each queue. Therefore,

$$A_{min} = \tau_{min}(N - 1) + R. \tag{5.10}$$

The vacation period can be obtained by a linear combination of the maximum vacation period, A_{max}, and the minimum vacation period, A_{min}, subject to the following two conditions: If the arrival rate is equal to $\lambda_{i,min}$ then the vacation period is equal to A_{min}. On the other hand, if the arrival rate is equal to $\lambda_{i,max}$ then the vacation period is equal to A_{max}. The following equation satisfies the above two conditions:

$$A = \frac{\rho_i}{\rho_{i,max}} \left(\frac{\rho_i - \rho_{i,min}}{\rho_{i,max} - \rho_{i,min}} \right) A_{max} + \frac{\rho_i}{\rho_{i,min}} \left(\frac{\rho_{i,max} - \rho_i}{\rho_{i,max} - \rho_{i,min}} \right) A_{min}. \tag{5.11}$$

Note that this equation satisfies the condition that the vacation period A must depend on the total connect time. Now that all the variables are known, one can use Eq. 5.4 to obtain the upper bound of the mean waiting time.

5.3 Effect of System Parameters on the Mean Waiting Time

In this section, we present some numerical results for the time limited service policy. The accuracy of this approximation procedure is discussed in a paper by Frigui, Stone and Alfa [24]. Because only the upper bound of the mean waiting time is obtained, the ratio of the high priority to the low priority messages is not important in this section.

5.3.1 Effect of messages arrival rate

The first set of data, Table B.1, is for a symmetric polling system with deterministic service times distribution having a mean of one unit of time, a connect time of one unit of time, 10 stations, $\tau_{min} = 0.5$ units of time, and $\tau_{max} = 45$ units of time. As expected, Fig. 5.1 shows that as the arrival rate increases the upper bound of the mean waiting time increases exponentially. Note that the system may become unstable before the utilization reaches one. This depends on whether $\rho < 1$ or $\lambda_i hC < \tau_{max}$ controls the stability of the polling system.

5.3.2 Effect of τ_{min} parameter

This section discusses the effect of changing τ_{min} on the mean waiting time. Table B.2 is for a polling system with deterministic service times having a mean of one unit of time, a connect time of one unit of time, 50 stations, a utilization of 60%, and τ_{max}

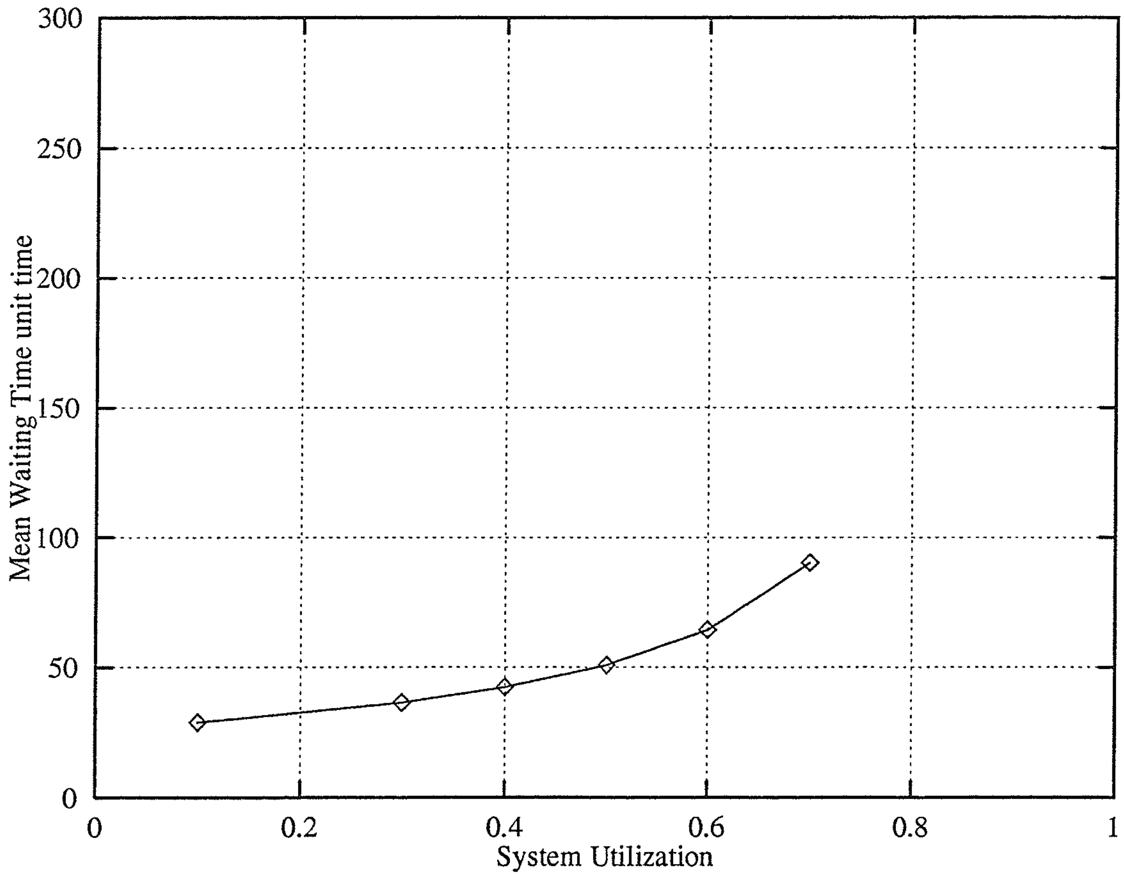

Figure 5.1: Effect of Messages Arrival Rate

equal to ten units of time. Note that for very small τ_{min} the system behaves almost like a k-limited exhaustive service policy. However, as τ_{min} increases the server is more likely to stay idle for a longer period at a queue. Because this idlen time is not accounted for in our approximation, as τ_{min} becomes large (a large τ_{min} is defined such that $\lambda_{i,min}1.3 > \lambda_i$), our approximate approach will underestimate the mean waiting time.

5.3.3 Effect of τ_{max} parameter

Table B.3 is for a symmetric polling system with deterministic service time having a mean of one unit of time, a connect time of one unit of time, 60% utilization, 50 stations, and $\tau_{min} = 0.5$ units of time. In this example we test the effect of τ_{max} on the approximation results. For small τ_{max} ($\lambda_{i,max} < 1.2\lambda_i$) the approximate method gives a good result for the low priority level but it fails to approximate the high priority mean waiting time (see Frigui, Stone, and Alfa [24] for a comparison between the simulation and the approximate approach). This is because our solution is based on the total average number of messages served rather than the individual average number of messages served in each cycle. However, as expected as τ_{max} increases the time limited service policy behaves more like an exhaustive k-limited service policy. Note that to obtain a good approximation one should pick τ_{max} such that $\lambda_{i,max} > 1.5\lambda_i$.

5.3.4 Effect of mean service time

This section discusses the effect of changing the mean service time (deterministic distribution) while keeping $\tau_{min} = 0.5$ units of time, $\tau_{max} = 45$ units of time, ten stations, connect time is 5 units of time, and an arrival rate of 0.05 messages per station per unit of time. Fig. 5.4 shows that as the mean service time increases, the

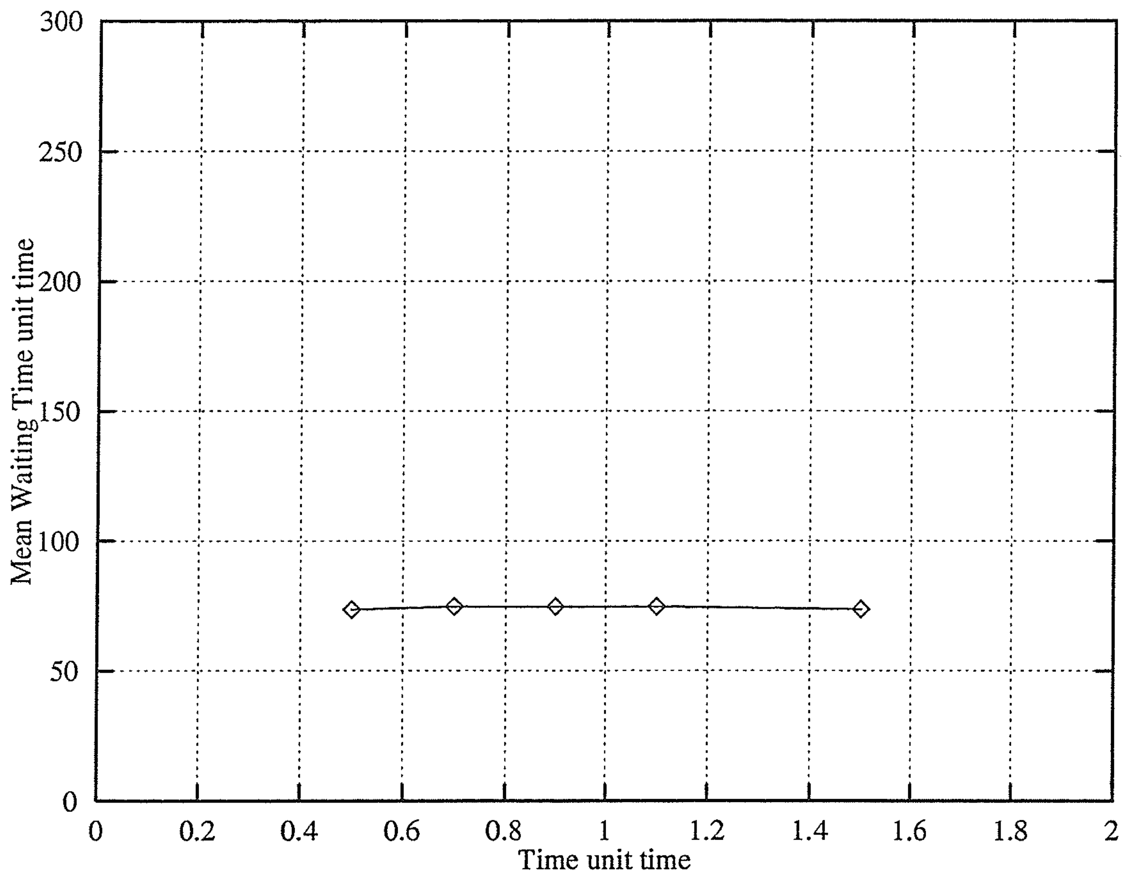

Figure 5.2: Effect of τ_{min} Parameter

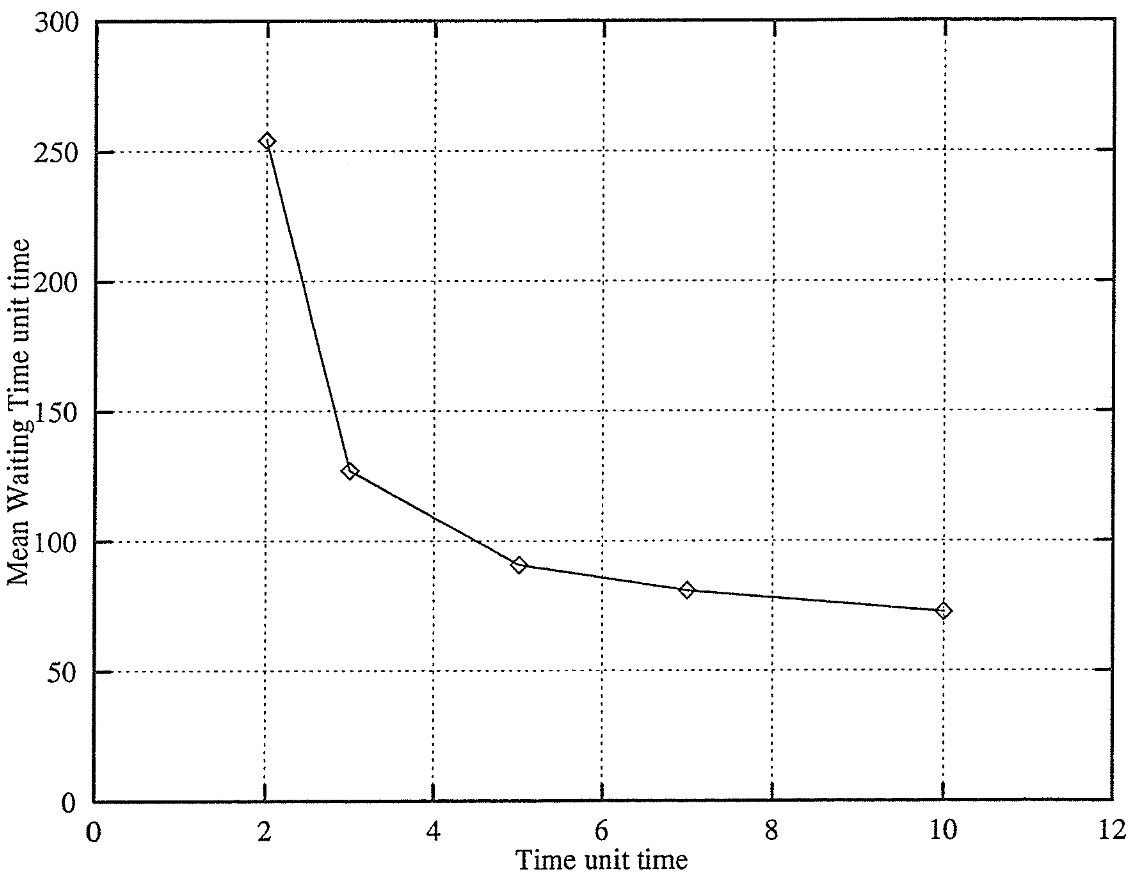

Figure 5.3: Effect of τ_{max} Parameter

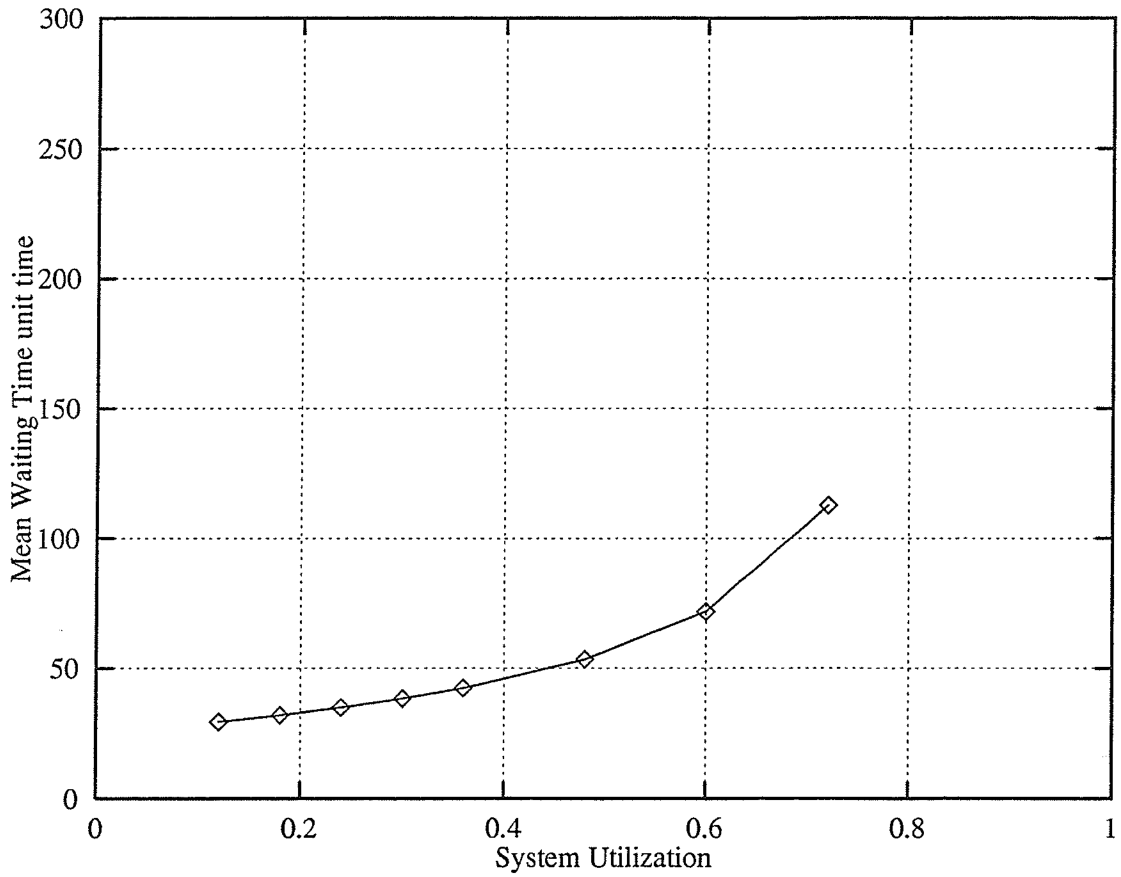

Figure 5.4: Effect of Mean Service Time

upper bound of the mean waiting time increases exponentially. Because τ_{max} is the upper bound for the time the server can spend at a station in a given cycle, the queue length will grow boundlessly as the number of messages in need of service becomes larger than k given by equation 5.8.

5.3.5 Effect of connect time

For a symmetric polling system with ten stations, $\tau_{min} = 0.5$ units of time, $\tau_{max} = 45$ units of time, a deterministic service time having a mean of one unit of time, and an arrival rate of 0.05 messages per station per unit of time, we vary the connect time between 4 and 25 units of time. Fig. 5.5 shows the linear relationship between the upper bound of the mean waiting time and the connect time. This is due to the linear relationship between the vacation period and the connect time (equations 5.9 and 5.10).

5.3.6 Effect of number of stations

The last set of data, Table B.6, is for a symmetric polling system with deterministic service times having a mean of one unit of time, a connect time of five units of time, $\tau_{min} = 0.5$ units of time, $\tau_{max} = 45$ units of time, and an arrival rate of 0.05 messages per station per unit of time. Fig. 5.6 shows that as the number of messages increases, the upper bound of the mean waiting time increases exponentially. Note that for a given τ_{max}, as the number of stations increases the maximum allowable arrival rate decreases (see equation 5.2).

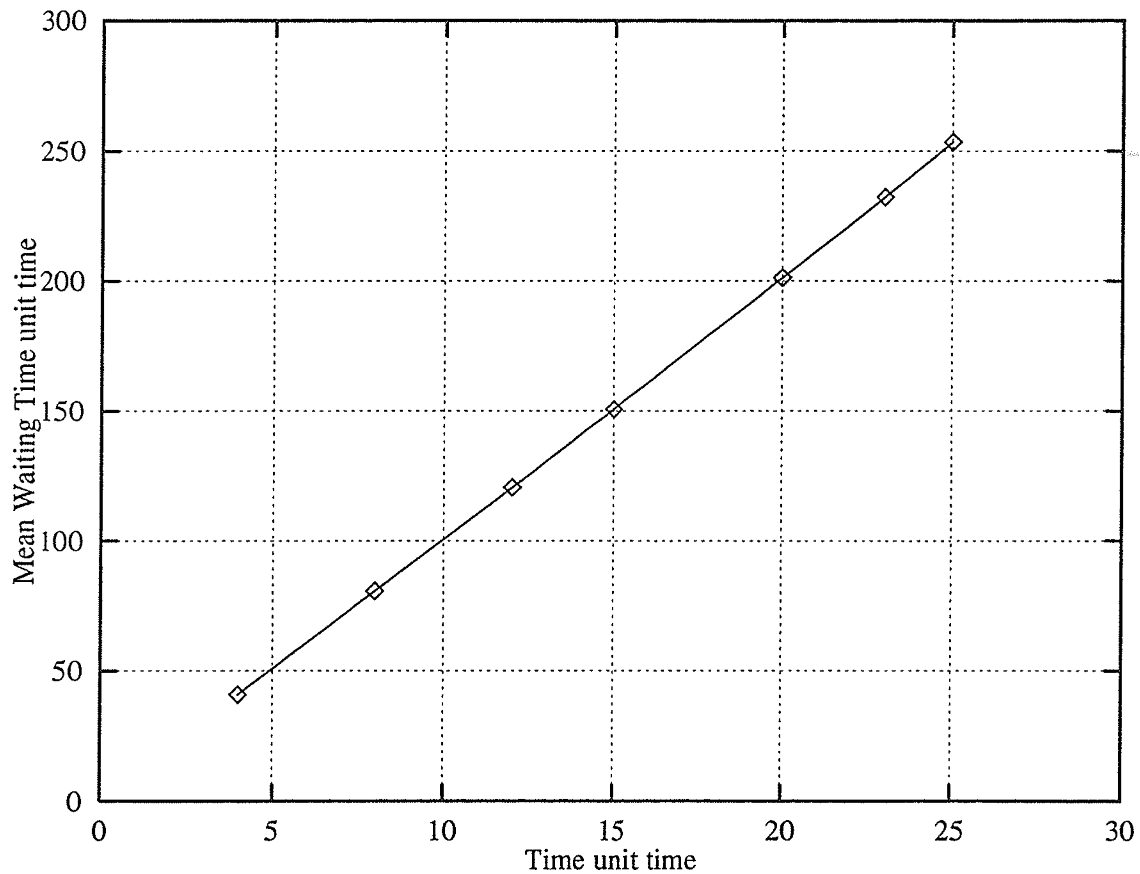

Figure 5.5: Effect of Connect Time

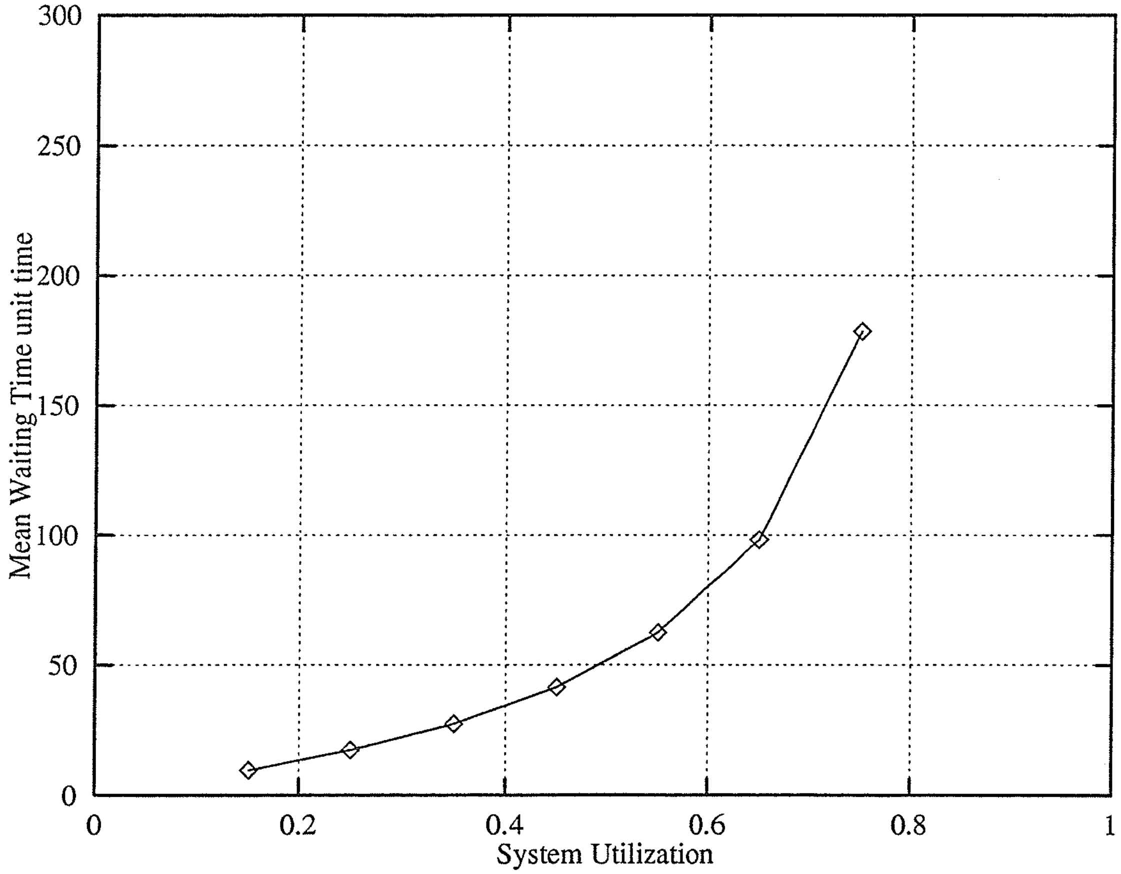

Figure 5.6: Effect of Number of Stations

CHAPTER 6

Application of Polling Algorithms to the RPT2s-CS Network

6.1 Introduction

This chapter is intended to discuss the performance of the RPT2s-CS network under the exhaustive and time limited service disciplines. The analyses are based on Manitoba Hydro implementation of the data acquisition System 2020 for the City of Winnipeg. The arrival rate per RPT2, mean service time, connect time, percentage of high priority messages, and number of stations are based on the data provided either by Manitoba Hydro or Iris Systems Inc..

6.2 System Parameters

As discussed in Chapter 3, since Manitoba Hydro intends to use a telecommunication based system, the connect time is approximately 20 seconds. This will take into account the disconnect/dial/reconnect process from one RPT2 to the next in the cycle. The number of RPT2s required to cover the city of Winnipeg is approximately 25. This is based on the assumption that each RPT2 has an effective coverage area of 4 square miles (densely populated areas may require more than one RPT2 per 4 square miles).

Messages priority, type, and frequency of transmission by an NSM are given in Table 6.1. However, in this study the main concern is priority levels 4 and 5.

Table 6.1: System 2020 Message Types.

Priority	Type	Frequency	Distribution (% of meters)
PL 5	Load Survey	48/day/meter	0.5%
PL 4	Reading	3/day/meter	100%
PL 3	Status	1/day/meter	100%
PL 2	Test	Upon Installation	100%
PL 1	Alarm	Outages/Tampers	100%

Manitoba Hydro services approximately 150,000 customers. Based on the number of customers and due to message redundancy that exists at the RPT2 level of System 2020 (according to Iris Systems each message will be sent approximately 1.988 times) the arrival rates were calculated. The low priority messages, PL 5, arrival rate is given by:

$$150,000 \times \frac{48}{24} \times 1.988 \times 0.5\%,$$

and the high priority messages, PL 4, arrival rate is given by:

$$150,000 \times \frac{3}{24} \times 1.988 \times 100\%.$$

These arrival rates are summarized in Table 6.2

Iris Systems Inc. has designed the data acquisition system such that each message has a length of 200 bits. Because connect times will decrease the availability of the CS, a 9600 baud modem is recommended. Based on a message length of 200 bits and the 9600 baud modem, the mean service time is 1/48 of a seconds.

Table 6.2: Message Arrival Rates for Winnipeg.

Priority	Arrival Rate (Messages/hour)	% of Total
PL 5	2982	7.41%
PL 4	37280	92.59%

6.3 Exhaustive Service Discipline Performance

The results of Chapter 4 were used to evaluate the performance of the Winnipeg City operation of System 2020 under the exhaustive service discipline. The parameters used were those given in Section 6.2.

Table 6.3 shows the mean waiting time of the high and low priority messages. As expected, the high priority messages have a lower mean waiting time than the low priority messages. The difference between the mean waiting times is not that noticeable because the system is heavily loaded with the high priority messages. Note that under the conditions specified in Section 6.2 the mean waiting times for both priority levels are under 6 minutes.

Table 6.3: Messages Mean Waiting Time

Priority	Mean Waiting Time seconds
PL 4	322.63
PL 5	323.08

6.4 Time Limited Service Discipline Performance

The results of Chapter 5 were used to evaluate the performance of the Winnipeg City operation of System 2020 under the time limited service discipline. The parameters used were those given in Section 6.2.

An upper bound for the mean waiting time of 339.87 seconds was obtained by considering the case when $\tau_{min} = 1$ second and $\tau_{max} = 20$ seconds. Again the mean waiting time is under 6 minutes for the conditions specified in section 6.2.

Note that for both service disciplines the waiting time is mainly due to the connect time. On the average, the CS spends

$$20 \times \frac{25}{2} = 250 \quad \text{sec.}$$

on connect time to reach a station. This is because on the average the CS is $N/2$ stations away from a station requiring service in any given cycle.

CHAPTER 7

Summary and Conclusions

7.1 Summary

The overall objective of this research was to study the performance of a multi-priorities polling system under the exhaustive and the time limited service disciplines. The results were then used to study the performance of a data acquisition, System 2020. These objectives were accomplished by the following steps:

1) For the exhaustive service discipline, the mean waiting time was obtained by approximating each station by a single node queue with vacation periods and occupation period. By using an aggregation procedure, and based on the results of Kella and Yechiali, and those of Bux and Truong [11], the mean waiting times of the high and low priority messages were obtained in this thesis.

2) For the time limited service discipline, the upper bound of the mean waiting time was obtained by considering each station as a single queue with vacation periods and occupation (service) period. Using the existing relationship among the mean vacation period, the mean service period and the mean cycle time, the number of messages served during one service period was obtained. Based on the number of messages the upper bound of the mean waiting time was obtained using Fuhrmann's [25] upper bound for the mean waiting time of the k limited exhaustive service discipline.

3) For the exhaustive service discipline, the mean waiting times of the high and low priority messages were plotted against the high priority messages arrival rate, the low priority messages arrival rate, the number of stations, and the mean service time. The plots show the mean waiting times as a function of system utilization. Also, the mean waiting times of the high and low priority messages were plotted against the connect time. The plots show the mean waiting times as a function of time.

4) For the exhaustive service discipline and asymmetric polling system, the mean waiting times of the high and low priority messages were obtained for two cases. In the first case station one was heavily loaded and in the second case station one was lightly loaded (in comparison with the other stations in the network). Also, the effect of the service times distribution and the percentage of high priority messages were considered.

5) For the time limited service discipline, the upper bound of the mean waiting time was plotted against messages arrival rate, the mean service time, and the number of stations. The plots show the upper bound of the mean waiting time as a function of system utilization. Also, the upper bound of the mean waiting time was plotted against the connect time, parameter τ_{min}, and parameter τ_{max}. The plots show the upper bound of the mean waiting time as a function of time.

7.2 Conclusions

In this thesis, two service disciplines were considered an exhaustive and a time limited. A study of the effect of the system parameters on the mean waiting time was performed. The conclusions of this study are given in the following two sections.

7.2.1 Exhaustive service discipline

The study of the exhaustive service discipline yielded the following conclusions:

- The mean waiting times of the high priority and low priority messages increase exponentially as a function of any of the following parameters: high priority messages arrival rate, low priority messages arrival rate, number of stations, and mean service time.

- The mean waiting times of the high and low priority messages increase linearly as the connect time increases.

- The relative difference between the mean waiting times of the high and low priority messages is not affected by the high priority arrival rate, the number of stations, or the connect time. However, as the arrival rate of the low priority messages or the mean service time increases, the relative difference between the mean waiting times increases.

- For asymmetric polling systems, messages arriving to a heavily loaded station(s) have the lowest mean waiting times.

- The mean waiting times are lower when the service times distribution is of the deterministic type.

- The increase in the high priority percentage increases the mean waiting time of the high priority messages and decreases the mean waiting time of the low priority messages.

7.2.2 Time limited service discipline

The study of the time limited service discipline yielded the following conclusions:

- The upper bound of the mean waiting time increases exponentially as a function of any of the following parameters: messages arrival rate, mean service time, and number of stations.

- The upper bound of the mean waiting time increases linearly as a function of connect time.

- As the parameter τ_{max} increases the upper bound of the mean waiting time decreases exponentially. As τ_{max} goes to infinity the mean waiting time converges to that of the exhaustive service discipline.

- The paramater τ_{min} does not have a significant effect on the upper bound of the mean waiting time.

7.2.3 Limitations of this study

This study is based on an approximate approach and it has the following limitations:

- In real life the buffer capacity is finite. The assumption of infinite capacity would affect the results if the buffer capacity is very close to the mean queue length. This would result in messages being lost and not serviced. The buffer capacity should be set large enough to ensure that no messages get lost.

- This study considers only the mean waiting time as a performance measure. In many situations it is more appropriate to consider what the probability is of a customer having to wait longer than t. This problem is not solved for many advanced queueing systems.

- The analysis of the mean waiting time is based on the steady state conditions. The transient portion of the waiting time is not studied. The obtained results are intended for the steady state region only.

- For System 2020, the effect of messages redundancy is not considered. The arrival rates may have to be adjusted in order to consider messages redundancy. An internal mechanism must be set to avoid messages redundancy.

7.3 Recommendations for Further Research

Because of the limitations stated in Section 7.2.3, future work should attempt to:

- Extend this analysis to finite capacity buffer. Care should be taken to account for each priority buffer's capacity.
- Extend this analysis to consider the distribution of the waiting time instead of just the mean waiting time.
- Consider the waiting time in the transient region.
- Consider the case of gated service discipline.

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APPENDICES

APPENDIX A

Data for the Exhaustive Service Discipline

This appendix contains the data of the numerical examples of Chapter 4.

Table A.1: Effect of High Priority Arrival Rate

λ_1	ρ	W_1	W_2	Rel. Diff.
0.0005	0.35	9.81	10.43	5.9%
0.0010	0.40	10.99	11.68	5.9%
0.0015	0.45	12.38	13.17	6.0%
0.0020	0.50	14.06	14.95	6.0%
0.0025	0.55	16.10	17.12	6.0%
0.0030	0.60	18.65	19.84	6.0%
0.0035	0.65	21.94	23.34	6.0%
0.0040	0.70	26.32	28.00	6.0%
0.0045	0.75	32.45	34.54	6.1%
0.0050	0.80	41.64	44.44	6.3%
0.0055	0.85	56.97	60.65	6.1%
0.0060	0.90	87.62	93.30	6.1%
0.0065	0.95	179.55	191.24	6.1%

Table A.2: Effect of Low Priority Arrival Rate

λ_2	ρ	W_1	W_2	Rel. Diff.
0.0005	0.35	10.06	10.17	1.1%
0.0010	0.40	11.22	11.45	2.0%
0.0015	0.45	12.58	12.97	3.0%
0.0020	0.50	14.20	14.80	4.1%
0.0025	0.55	16.18	17.04	5.0%
0.0030	0.60	18.65	19.84	6.0%
0.0035	0.65	21.83	23.46	7.0%
0.0040	0.70	26.04	28.30	8.0%
0.0045	0.75	31.95	35.08	8.9%
0.0050	0.80	40.78	45.26	9.9%
0.0055	0.85	55.50	62.26	10.9%
0.0060	0.90	84.90	96.28	11.8%
0.0065	0.95	173.07	198.40	12.8%

Table A.3: Effect of Number of Stations

N	5	10	15	20	25	30	35	40
ρ	0.105	0.210	0.315	0.420	0.525	0.630	0.735	0.840
W_1	3.25	7.37	12.76	20.09	30.66	47.24	76.95	145.65
W_2	3.38	7.68	13.28	20.91	31.93	49.17	80.12	151.66
Rel. Diff.	3.9%	4.0%	3.9%	3.9%	4.0%	3.9%	4.0%	4.0%

Table A.4: Effect of Connect Time

R	1.0	5.0	14.0	18.0	22.0	25.0
W_1	7.37	31.66	86.31	110.60	134.89	153.10
W_2	7.68	32.97	89.87	115.16	140.45	159.42
Rel. Diff.	3.9%	4.0%	4.0%	4.0%	4.0%	4.0%

Table A.5: Effect of Mean Service Time

h	2	5	7	10	15	20	25	30	35
ρ	0.042	0.105	0.147	0.210	0.315	0.420	0.525	0.630	0.735
W_1	5.22	5.76	6.29	7.37	10.20	14.88	22.60	35.90	61.38
W_2	5.26	5.88	6.45	7.68	10.84	16.15	25.00	40.64	70.99
Rel. Diff.	0.8%	2.0%	2.5%	4.0%	5.9%	7.9%	9.6%	11.7%	13.5%

Table A.6: Effect of Service Times Distribution

Distribution	ρ	0.40	0.50	0.60	0.70	0.80
Exponential	W_1	9.18	12.36	17.05	24.77	40.04
Deterministic	W_1	6.65	8.64	11.58	16.43	26.04
Exponential	W_2	10.67	14.95	21.47	32.49	54.79
Deterministic	W_2	7.72	10.44	14.57	21.55	35.63

Table A.7: Effect of High Priority Percentage

$Per.of HP$	5%	20%	35%	50%	65%	80%	95%
W_1 analy.	8.59	8.72	8.86	9.00	9.14	9.29	9.44
W_2 analy.	10.50	10.33	10.16	10.00	9.84	9.69	9.55

Table A.8: Asymmetric Polling System

	Station	High priority waiting time	Low priority waiting time
Analy.	1	11.17	21.20
Analy.	2, 3, 4, 5	18.80	22.73

Table A.9: Asymmetric Polling System I

	Station	High priority waiting time	Low priority waiting time
Analy.	1	21.51	22.32
Analy.	2, 3, 4, 5	15.52	21.87

APPENDIX B

Data for the Time Limited Service Discipline

This appendix contains the data of the numerical examples of Chapter 5.

Table B.1: Effect of Messages Arrival Rate

λ	0.01	0.03	0.04	0.05	0.06	0.07
W	28.85	36.49	42.36	50.84	64.36	90.31

Table B.2: Effect of τ_{min} Parameter

τ_{min}	0.5	0.7	0.9	1.1	1.5
W	73.53	74.71	74.71	74.71	73.53

Table B.3: Effect of τ_{max} Parameter

τ_{max}	2	3	5	7	10
W	254.00	127.00	90.71	80.81	73.52

Table B.4: Effect of Mean Service Time (Deterministic Dist.)

h	1	1.5	2	2.5	3	4	5	6
W	29.5	32.0	34.9	38.4	42.4	53.4	71.8	112.7

Table B.5: Effect of Conncect Time

r	4	8	12	15	20	23	25
W	40.9	80.7	120.6	150.7	201.4	232.2	253.4

Table B.6: Effect of Number of Stations

N	3	5	7	9	11	13	15
W	9.6	17.3	27.4	41.5	62.5	98.2	178.5