A COMPUTER STUDY OF SOME ASPECTS OF POWER SYSTEM STABILITY

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ABSTRACT

The purpose of this thesis is to study transient stability, to devise a method of solving the swing equation that will make full use of the potentials of the IBM 1620 (20K) Digital Computer and to investigate three methods of improving transient stability, that is, the effects of exciter response, governor response and rapid reclosing on a two machine system.

PREFACE

This thesis is a computer study of some of the aspects of transient stability in power systems. Stability became a problem when machines were first paralleled and today power systems are more complex and expensive and the results of stability studies continue to be an important consideration in power system design.

Although a computer program has been written to handle up to thirteen machines (30 buses), a two machine system was used here. The swing equation is solved by the Kutta-Runge method which provides an accurate way of solving a large number of first order differential equations. In succeeding chapters the effects of exciter response, governor response and rapid reclosing are considered. The final chapter takes into account the combined effect on the two machine system when all improvement methods are used at once.

I wish to acknowledge my indebtedness to Professor G. W. Swift for his never failing guidance and assistance, to R. W. Haywood of Manitoba Hydro for reviewing the thesis and making many suggestions for its improvement, and to my wife for all that she has done.

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CHAPTER I

TRANSIENT STABILITY

The purpose of this thesis is to study transient stability, to devise a method of solving the swing equation that will make the maximum use of the potentials of the IBM 1620 (20K) Digital Computer, and to investigate three methods of improving transient stability.

Transient stability studies are by no means rare, in fact, the opposite is true. Every power company must investigate the transient stability characteristics of its system each time a major generator addition is proposed. This thesis will not attempt to carry out a detailed study of a particular power system but it will however, endeavor to investigate the general over-all effect of governor response, exciter response and rapid reclosing on a system. The point is again made that the purpose is to observe the general effects of these improvement methods in order to gain insight into the benefits available from using them.

1.1 Background

Stability, when referring to a power system, may be defined as that condition wherein all the machines in that system remain in synchronism with each other. Conversely then, instability denotes that condition when one or more machines within the system lose synchronism with the others, or, in other words, fall out of step.

Instability became a problem when machines were first paralleled. These original machines were operated by directly coupling the generator to a steam engine, which delivered a pulsating torque causing hunting of the generator. Hunting, however, was reduced by the introduction of a damper winding in the generator, which in turn, developed a damping torque due to the rate of change of the rotor position with respect to the armature m.m.f. It is noted that today most generators are run by coupling them to a waterwheel or to steam turbines, both of which develop a relatively non-pulsating torque. [1]

Before the advent of automatic regulating systems, such as governors and exciters, all systems had to be designed with a good inherent voltage regulation, that is, circuits, machines and transformers that had a low reactance. This was possible due to the fact that the lines between the load and the power sources were comparatively short and the voltage comparatively low. Stability, or for that matter, instability, became a problem when it was necessary to go farther and farther away to reach sources of power. The development of automatic voltage regulating devices made it possible to increase the reactance in order to obtain a more economical design and to limit short circuit currents.

[1]. See Bibliography, Ref. #1. Superscripts, henceforth, will refer to numbered references in Bibliography.

Today, the problem has not vanished. The engineer, because of economic considerations, cannot arbitrarily apply methods such as rapid reclosing; it is necessary that he realize the effect of what each method can do for his specific problem. This thesis intends to show generally the ramifications of each of the three methods studied-- with respect to transient stability for the particular system studied. The system used was a two machine system, i.e. a finite machine connected to an infinite bus. (See Figure 2.3)

With the era of long lines and rising costs, the engineer has tried to transmit more and more power through the line, approaching the theoretical power limit of that line. With these increased loadings, transient stability has thus become the focal point of many investigations.

1.2 Procedure

In order to have a common base on which to compare the various improvement methods, a sample power system was selected. It was also decided that a three phase fault lasting a specified time would be applied for each series of tests. (See Chapter II)

The first problem encountered in transient stability studies is how to solve the swing equation. Because a small computer was being used and because governor and exciter response were to be considered, it was felt that a method other than the classic step-by-step solution had to be found. Gills' variation of the Kutta-Runge process was employed because of the accuracy and flexibility it offered in solving first order differential equations[2].

This thesis is grouped into five sections. The first will deal with the solving of the swing equation and its application to the "raw" or uncompensated system[a]. The second will consider governor response and its abilities to compensate the "raw" system. The third and fourth sections will consider exciter response and rapid reclosing respectively and their effect on improving stability. The last section will deal with the combined effects of governor response, exciter response and reclosing on the system.

In order to supply the transient stability program with all the data needed it was necessary to prepare several programs. For each loading condition, the IBM Load Flow program was used in order to obtain the output power,

[a] By "raw" system it is meant a system based on the following assumptions:

1. Synchronous-machine transient reactances in the direct and quadrature axes are assumed to be alike.
2. Voltages behind transient reactances of the synchronous machines are assumed to remain constant.
3. Damping torques are neglected.
4. The influence of saturation may be either entirely neglected, or taken into account in an approximate manner by modifying the value of transient reactance.
5. Constant shaft torques are assumed for all of the machine groups, and governor-action and load-speed characteristics are neglected.

[2] See Bibliography, Ref. #19.

voltage and angle at the generator internal buses. Secondly, it was necessary to obtain the values of the admittance (self and mutual) between each generator and every other generator. Two programs were prepared and for each loading condition, and a set of admittances was procured[b].

The transient stability program was written so as to accept data directly from the previous program. The first program solved the swing equation by the Kutta-Runge method and considered only reclosing. This program has the capacity of handling up to thirteen machines (30 buses)[c]. The second program written again solved the swing equation by the Kutta-Runge method and considered reclosing. However, this time logic for both governor response and exciter response was added. This program handles five machines (30 buses)[d].

The transient stability programs were then used to compute the resulting swing curve in the test runs[e]. The results of each test will be given in the appropriate chapters.

[b] New England Electric ("Transient Stability" package).

[c] See Appendix I.

[d] See Appendix II.

[e] See Appendix III for typical outputs of various programs.

CHAPTER II

SOLUTION OF THE SWING EQUATION

The purpose of this chapter is to introduce the method used to solve the swing equation and to present the sample system studied, along with the criteria applied to the faulting and fault clearing of the system. Furthermore, the results of a three phase fault applied under various loading conditions to the sample system will also be given.

2.1 Swing Equation

Consider the swing equation:

$$\frac{d^2\delta}{dt^2} = \frac{P_m - P_e}{M} \qquad 2.1$$

where: δ = the displacement angle of the rotor of a synchronous machine, with respect to a reference axis rotating at normal speed in radians. In this thesis a two machine system was studied, with one of the machines used as reference.

M = the intertia constant of a machine in per unit megawatts per radian per second squared.

$M = \frac{GH}{B\pi f}$ where: G = the machine rating in Megavolt-amperes.

H = the stored energy in magawatts.

B = the system base (MVA).

f = the frequency in cycles per second.

p_m = the shaft power input corrected for rotational losses in per unit magawatts.

p_e = the electrical power output corrected for electrical losses in per unit magawatts. Note that the difference between p_m and p_e is the accelerating power of the machine.

By examining the swing equation we see that the acceleration of the machine, given by the second derivative of delta, varies directly with the accelerating power and, inversely with the inertia constant.

2.2 Classical Solution

The classical approach in solving the swing equation has been a formal solution, which proves in fact, to be impractible. As an example, consider a three machine system: by examining the equations given below we see that the output of a machine and therefore its accelerating power, depends on its angular position and angular speed with respect to every other machine in the system.

$$M_1 \frac{d^2\delta_1}{dt^2} = P_{m_1} - P_{e_1}\left(\delta_1, \delta_2, \delta_3, \frac{d\delta_1}{dt}, \frac{d\delta_2}{dt}, \frac{d\delta_3}{dt}\right) \quad 2.2$$

$$M_2 \frac{d^2\delta_2}{dt^2} = P_{m_2} - P_{e_2}\left(\delta_1, \delta_2, \delta_3, \frac{d\delta_1}{dt}, \frac{d\delta_2}{dt}, \frac{d\delta_3}{dt}\right) \quad 2.3$$

$$M_3 \frac{d^2\delta_3}{dt^2} = P_{m_3} - P_{e_3}\left(\delta_1, \delta_2, \delta_3, \frac{d\delta_1}{dt}, \frac{d\delta_2}{dt}, \frac{d\delta_3}{dt}\right) \quad 2.4$$

The simplest system of one finite machine connected through a reactance to an infinite bus, with damping neglected, yields the equation

$$M\frac{d^2\delta}{dt^2} = P_m - P_e \sin\delta \qquad 2.5$$

for which the formal method (with input power equal to zero) gives a solution involving elliptic integrals.

Another method used to solve the problem of stability of a very simple system is the equal area criterion. A solution of a swing equation, with the usual assumptions of constant input power, and constant voltage behind the transient reactance, shows that δ oscillates about some equilibrium point with a constant amplitude, providing the system is stable. The method used to indicate stability without solving the swing equation is called the equal area criterion of stability. This criterion applies only to a two machine system, but because exciter and governor response are to be considered this method proves to be inadequate for the purpose of this thesis.

However, the method which is usually used to solve the swing equation is the step-by-step solution. In this solution one or more of the variables are assumed to be constant and another is varied according to the assumed laws over a small interval Δt. It is usual practice to assume accelerating power and hence the acceleration to be constant over the time interval Δt. The mechanical input power is also assumed constant. Good accuracy can be obtained by the step-by-step method and the computations are fairly simple.

The step-by-step method of solving the swing equation is familiar to most persons in the field of power systems. The author felt that in view of the fact that regulators and exciters were to be taken into account in this study, another method more conducive to digital computation had to be used. The method selected was Gills' variation of the Kutta-Runge solution for simple first order differential equations[2].

2.3 Kutta-Runge Method

The Kutta-Runge method has been used recently in studies involving large systems using large computers[3]. Most methods used in computing the step-by-step integration of differential equations have one essential in common with each other; that is, at each step of the integration one must use values previously calculated in order to proceed.

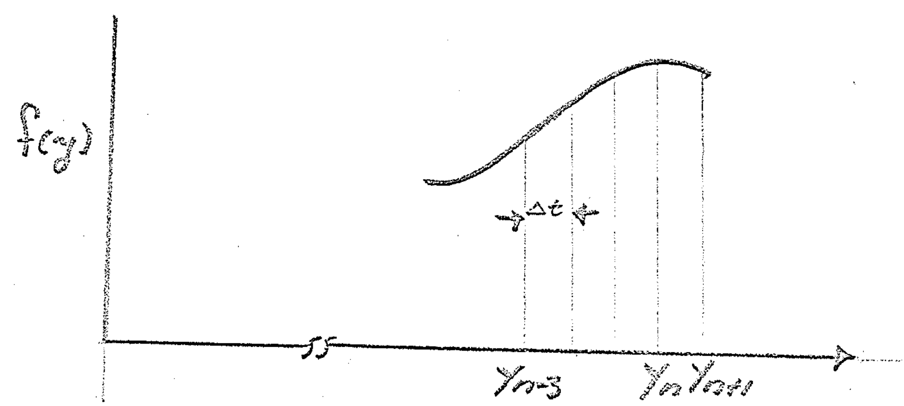


Figure 2.1

In other words, if one has arrived at y_n and wishes to calculate y_{n+1} (see figure 2.1) one must have a knowledge of y_{n-1}, y_{n-2}etc.--the number of the previous values required depending on the accuracy needed and the method used. The previous values are indications of how the function will act in the region of y_n to y_{n+1} and it would indeed be wasteful to disregard these values. In hand calculations most formulae are of the difference type, are simple and are easily remembered. This is a definite advantage, wherein mental labour is to be kept at a minimum. There are however, disadvantages with this method. The first is that it cannot be used at the beginning of the integration because two or more values of y_n are needed and therefore some auxiliary method must be used to begin the calculations. The second disadvantage is that it is difficult to change the size of the time interval in the middle of a run. Halving or doubling the size is relatively simple but changing by other factors proves to be combersome.

When using digital computers as opposed to hand calculations, these considerations assume different proportions. It is a serious drawback now to supply the computer with special methods and instructions to begin the calculation. A simple operation when solving the swing equation by hand, that of changing the variable y_{ni} in order to carry on to the next interval, can assume serious proportions in time and instructions in a digital computer.

As an example, having calculated y_{n+1} and wishing to calculate y_{n+2}, one must first write the program to take y_n, shift it to y_{n+1} (used to calculate y_n) and shift this in the memory to position y_n, whereas for hand calculation the operator would merely shift his eyes down the page.

There is another consideration when using computers, and this is storage space. Storage becomes critical as the number of machines in a study increases. Therefore, one is led to look for processes which do not make use of preceeding values. A large general classification of such processes that do not require previous values is given by Kutta[4, 5]. Kutta investigated a large number of these processes dealing with various orders of accuracy. A most attractive method is given by the fourth order Kutta-Runge process. By fourth order it is meant that the error in each step is of the order of h^5, where h is the interval size between calculations.

The Kutta-Runge process was used in this thesis in the form of Gills' variation[2]. This method consists of four approximations (see figure 2.2).

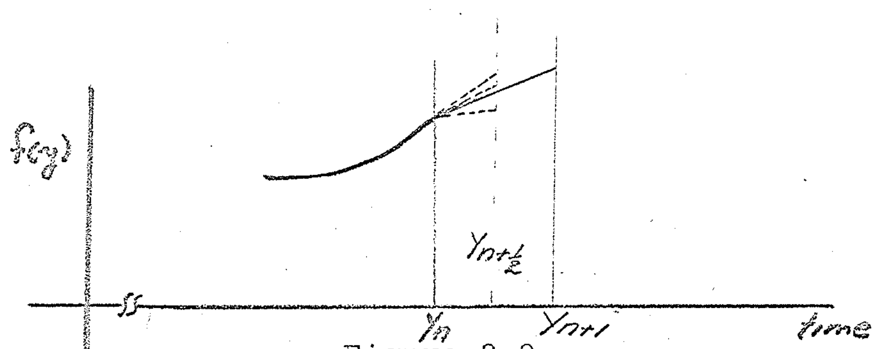


Figure 2.2

The first approximation calculates the slope of the function at half the interval length between y_n and y_{n+1}, the second approximation again calculates the slope at the half-way point, this time using the value of the first approximation as a weighted term; the third approximation uses the first and second approximations as weighted values and calculates another value of slope at half the distance between this interval; the fourth approximation then calculates the value of the slope at the end of the interval using the weighted values of the first, second and third approximations.

The weighting of the previous value of the slope has been derived by Kutta, extended by Runge, and amplified by Gill. Kutta suggested five special cases or solutions which Runge soon developed into the simplest particular solution. Briefly, these weighted terms allow us to obtain a high degree of accuracy without any knowledge of previous terms.

2.4 Application of the Kutta-Runge Method

The swing equation is a second order differential equation. In order to use the Kutta-Runge process the swing equation is simplified into the following two first order differential equations.

$$\frac{d\omega}{dt} = \frac{P_a}{M} \qquad 2.6$$

$$\frac{d\delta}{dt} = \omega \qquad 2.7$$

where: w = the speed in radians per second.

p_a = the accelerating power (i.e. $p_m - p_e$).

M = the inertia constant.

One can now begin to understand the reason for the choice of the Kutta-Runge process. The method is capable of handling as many single order differential equations as the memory will allow. It handles each differential equation separately, thus a concise subroutine within the computer program can be written and used for each differential equation in turn. Since part of the purpose of this thesis is to consider exciter and governor response, both of which can be approximated by first and second order differential equations, the simplicity of the method becomes apparent. The exact formulae used to consider regulation will be dealt with in separate chapters.

The accuracy of the Kutta-Runge process as compared to the step-by-step method of solution has been investigated and shown to be superior[15]. In order to check the accuracy of the Kutta-Runge process two problems with known solutions were solved and both times results obtained proved more accurate than those obtained by the step-by-step solution.

2.5 Power Equation

Consider the equation 2.6; the accelerating power, is calculated from the following equation

$$P_a = P_o - E_i \sum_{j=1}^{n} \left(E_j \left(\cos[\delta_i - \delta_j] G_{ij} + \sin[\delta_i - \delta_j] B_{ij} \right) \right) \quad 2.8^*$$

where: P_o = the initial electrical generator power.

E_i, E_j = the internal voltages (magnitude only) behind the transient reactance of the machines.

δ_i, δ_j = the generator internal voltage angle with respect to a synchronously rotating reference vector (infinite bus).

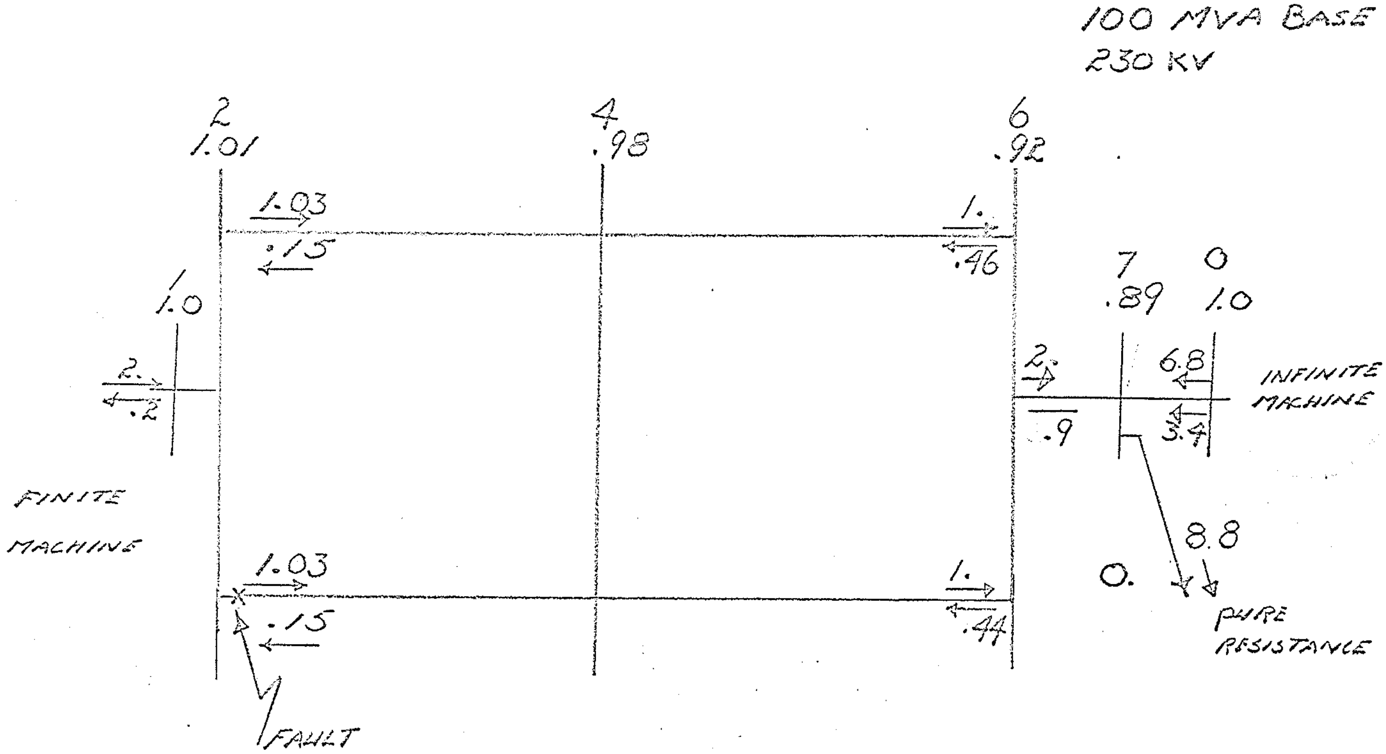
$B_{i,j}$ = the imaginary part of the driving point and transfer admittances between generators. Similar to $G_{i,j}$, the admittances are calculated from behind the transient reactance of the machines.

$G_{i,j}$ = the real part of the admittance between generators i, j. Admittance is calculated from behind the transient reactance of one machine to behind the transient reactance of the other.

2.6 Sample System

Due to the small size of the computers' 20K memory, separate programs, along with the 1620 load flow program, had to be used to solve for the mutual and self admittances in order to obtain all the various parameters needed to do a

* For derivation see Appendix IV.



Single line diagram of the "raw" system for the critically stable system.

Figure 2.3

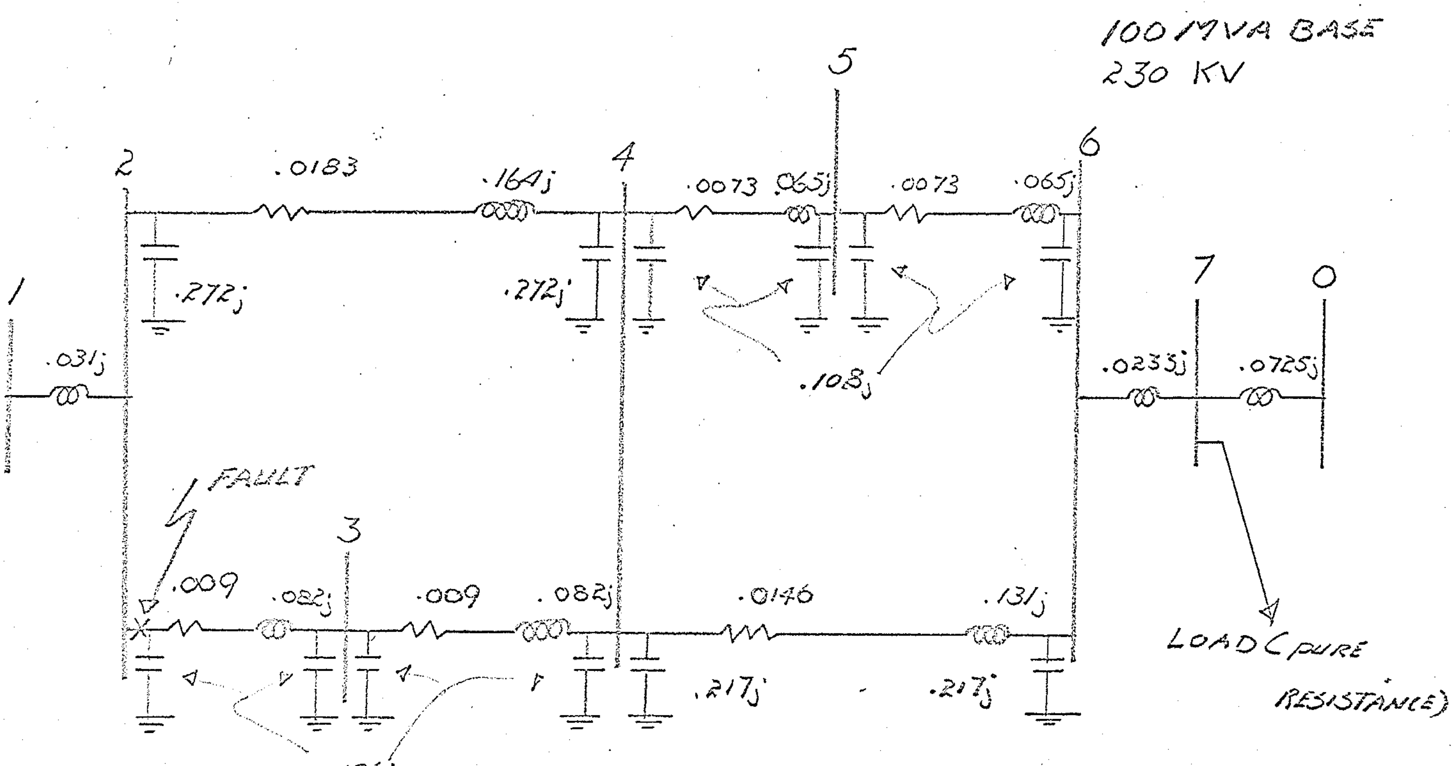
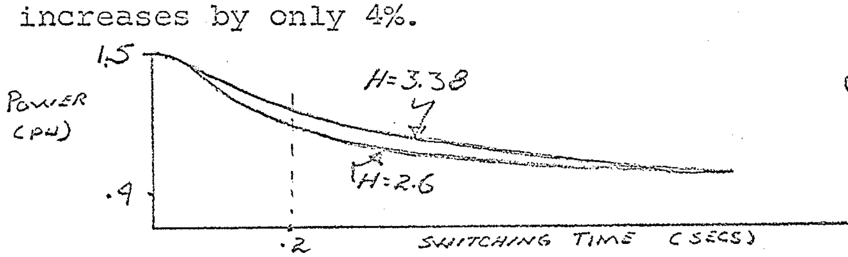


Figure 2.4

transient stability study[a]. All parameters were obtained for various loading conditions. Referring to figure 2.3, the single line diagram for the system studied, one sees that the power out for the infinite machine is 6.8 p.u. on a 100 MVA, 230 KV base for a load on bus 7 of 8.8 p.u. (unity power factor). For every loading condition from 7 p.u. to 9.8 p.u. the power flow out of the infinite bus was kept constant at 6.8 p.u. and the power difference required by the load was supplied by the finite machine.

Referring to figure 2.4, the circuit diagram of the system studied is shown. This system is very similar to the Grand Rapids-Winnipeg section of the Manitoba Hydro system. The inertia constant (H) of the finite machine was selected to be 27 on a 100 MVA base while the inertia constant for the second machine was infinity. The selection of H is important but not critical as can be seen by referring to figure 4.36 of reference 10 reproduced below as figure 2.5 where the effect of raising the inertia constant (H) by 30% on a system that is similar to the one studied in this thesis is that the output power at a switching time of .2 seconds increases by only 4%.



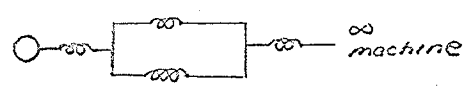


Figure 2.5

[a] By "20K" it is meant that there are 20,000 positions in the magnetic core of the computer in which to store data.

In this chapter, the system was studied as a "raw" system. That is, the finite machine was allowed to swing with no governor action and with air gap flux remaining constant. In order to simplify the study a number of conditions were applied. As the need for each condition arises throughout the thesis, it will be dealt with separately at that time.

For the entire study the type of fault and the clearing time of the fault were standardized. A three phase fault was chosen because it is the most severe type that may be applied to a power system. The fault clearing time (T_{fc}) was set at .1 seconds.

Over 90 per cent of the faults on a system are lightning faults and the arc created is extinguished relatively quickly. In many power companies, such as Manitoba Hydro, the circuit breakers are set to clear the fault in about five cycles or .0833 seconds. A clearing time of .1 seconds as opposed to .0833 seconds was selected because of its lending itself to ease in time incrementation.

Throughout the entire thesis a three phase fault was applied close to bus #2 on line 2--3. Summarizing then, a three phase fault was applied on line 2--3 and cleared in .1 seconds for every case studied in this thesis.

2.7 <u>Results of a Three Phase Fault on the "Raw" System</u>

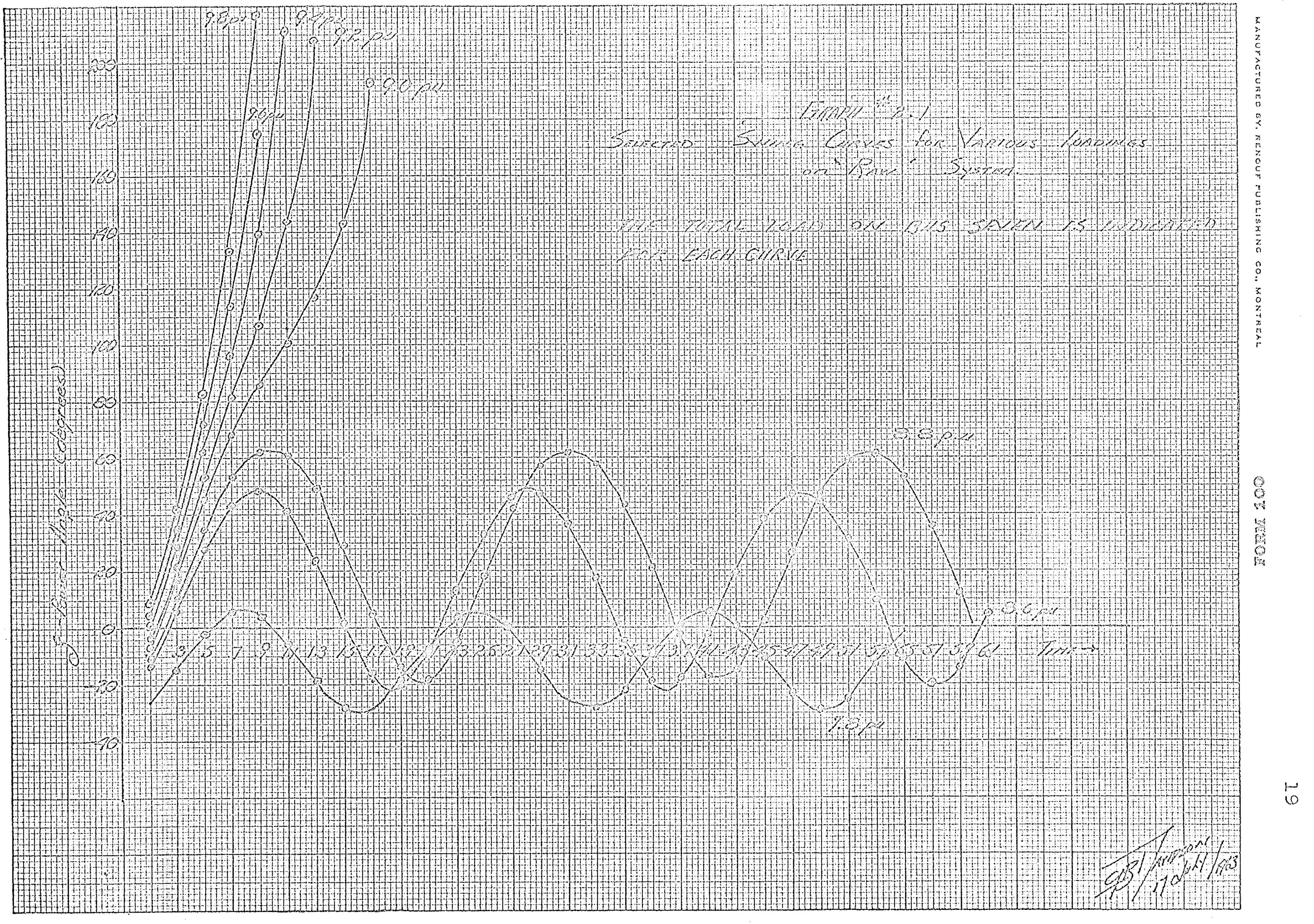
In order to investigate the reaction of the sample system under the various conditions of control, separate tests were carried out. To begin, a "raw" or simple system in which neither governor nor exciter systems were used was faulted while supplying various loads. The load bus (#7) demanded loads varying from 7.8 p.u. to 9.8 p.u. incremented in steps of .2 p.u.

Referring to graph 2.1, which gives the swing curves for selected loadings, it can be seen that the system remains stable after a three phase fault is applied on line 2--3 and cleared in .1 seconds, for loads of 7.8 p.u. to 8.6 p.u. The swing curve is plotted out to six seconds and it can be seen that each successive swing decreases in magnitude, thus indicating stability.

For a resistive load of 8.8 p.u. the swing has increased noticeably from 8.6 p.u. yet does not go unstable. At this loading, the system appears to be critically stable. For a load of 900 MW or 9. p.u. the system is definitely unstable. As one would expect as the load is increased up to 9.2, 9.4, 9.6 and 9.8 p.u. the system becomes unstable more quickly for each upward increment in power.

The foundation of the problem has now been set. The system ranges from being an inherently stable system to one that is critically stable (8.8 p.u.) oscillating in an undamped curve, to a system that is unstable when the load is grater than 8.8 p.u.

For this series of tests the voltage behind the transient reactances and the input power was assumed to be


GRAPH # 8.1
SELECTED SWING CURVES FOR VARIOUS LOADINGS
ON 'RAW' SYSTEM.
THE TOTAL LOAD ON BUS SEVEN IS INDICATED
FOR EACH CURVE
9.8 p.u
9.7 p.u
9.2 p.u
9.0 p.u
9.6 p.u
8.8 p.u
8.6 p.u
7.8 p.u
δ - Power Angle (degrees)
Time →
200
180
160
140
120
100
80
60
40
20
0
-20
-40

constant and neither governor nor rapid exciter control was applied. The next logical step is to determine the effect of each of the methods to improve transient stability when separately applied to the system and finally to apply all methods of control to the system to gain the over-all effect.

CHAPTER III

GOVERNOR RESPONSE

The swing equation alone gives a good indication of whether or not a machine or a system is unstable, but the purpose of a transient stability study is not just to take note of where a system goes unstable, but to find how to make the system more stable so that one may transmit more power along a particular line. This chapter will be devoted to one aspect of improving the transient stability: that of governor response.

3.1 Governor Response System

The purpose of a governor in this thesis is to sense a speed change, due to a sudden loading or unloading of the generator and try to regulate the speed so as to have it remain constant.

Referring to figure 3.1, a block diagram of a governor response system is given.

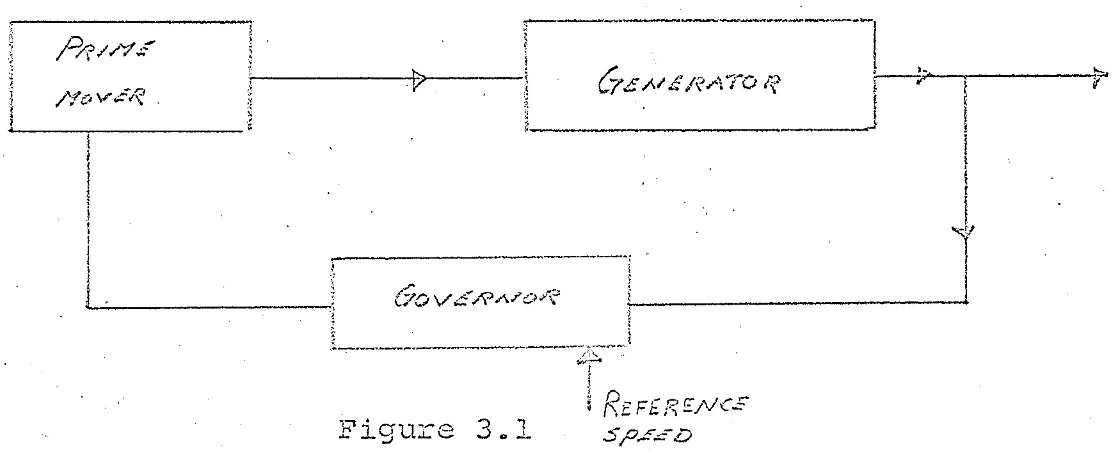


Figure 3.1

The block diagram may be redrawn as shown in figure 3.2:

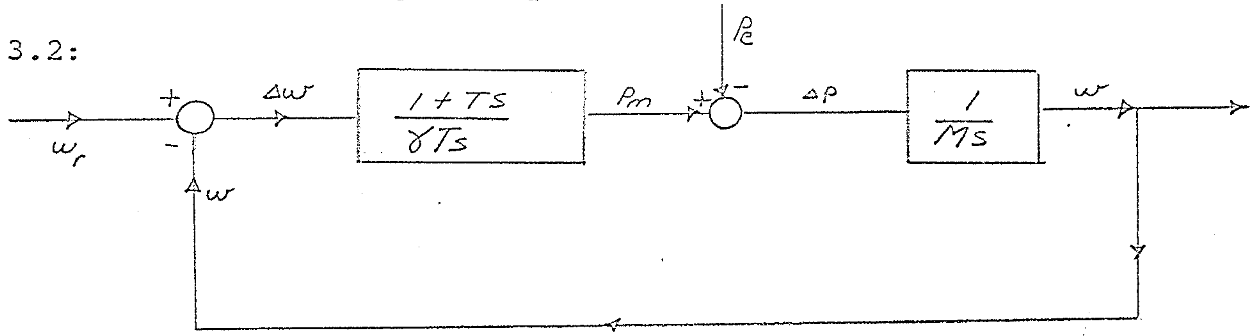


Figure 3.2

where: w_r*= 0. The reference speed in radians per second. The reference speed corresponding to the synchronous speed is set to zero.

w = the deviation from the reference speed in radians per second. Because the reference speed is zero, "Δw" represents the deviation from the normal speed. This deviation is defined as the error in speed that the governor acts on.

s = Laplacian operator.

T = the over-all time constant of the governing and hydraulic system in seconds.

γ = the temporary droop in per unit. The permanent droop of the machine is ignored. Conceptually, temporary droop can be thought of as that value of droop that is required by the machine in question to share in proportion to its rating the load demanded when a sudden change of load

* Lower case letters refer to the time domain while the upper case letters for the corresponding variables refer to the Laplacian domain.

is placed on the system.

P_e = a small step load change in electrical power. For the purpose of this analysis a value of .05 u (t) was chosen[a].

3.2 Development of Governor Response Equation

Considering figure 3.2, the transfer function of the system is

$$TF = \frac{W(s)}{P_e(s)} \qquad 3.1$$

also

$$M s W(s) = P_m - P_e(s) \qquad 3.2$$

and

$$-\gamma T s P_m = T s W(s) + W(s) \qquad 3.3$$

Now solving for p_m in equation 3.3, and substituting into equation 3.2, we eliminate p_m:

$$P_e(s) + M s W(s) = \frac{-T s W(s) - W(s)}{\gamma T s}$$

$$= \frac{-W(s)}{\gamma} - \frac{W(s)}{\gamma T s}$$

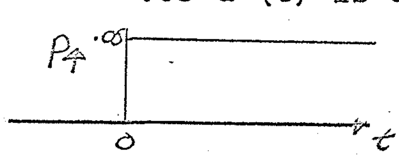
thus

$$-P_e(s) = \left(M(s) + \frac{1}{\gamma} + \frac{1}{\gamma T s} \right) W(s)$$

$$-\gamma T s P_e(s) = (1 + T s + \gamma T M s^2) W(s)$$

$$\frac{W(s)}{P_e(s)} = \frac{-\gamma T s}{(1 + T s + \gamma T M s^2)} \qquad 3.4$$

[a] .05 u (t) is a step function, i.e.



Consider, for example, a step disturbance of $p_e = .05\ u\ (t)$ when placed on the system. Solving for W(s) equation 3.5 is obtained.

$$W(s) = \frac{-.05\ \gamma T s}{1 + Ts + \gamma T M s^2} \qquad 3.5$$

The equation is now in a standard form that is, the Laplace transform for a second order differential equation when a step input is applied. Ignoring the negative sign, the solution of this equation in the time domain is given in the following form:

$$\omega(t) = \frac{.05 T \omega_n e^{-\gamma \omega_n t} \sin(\omega_n \sqrt{1-\gamma^2}\, t + \psi)}{\sqrt{1-\gamma^2}} \qquad 3.6$$

where:

$$\omega_n = \sqrt{\frac{1}{\gamma T M}} \qquad 3.7$$

and

$$T = \frac{2\gamma}{\omega_n} \qquad 3.8$$

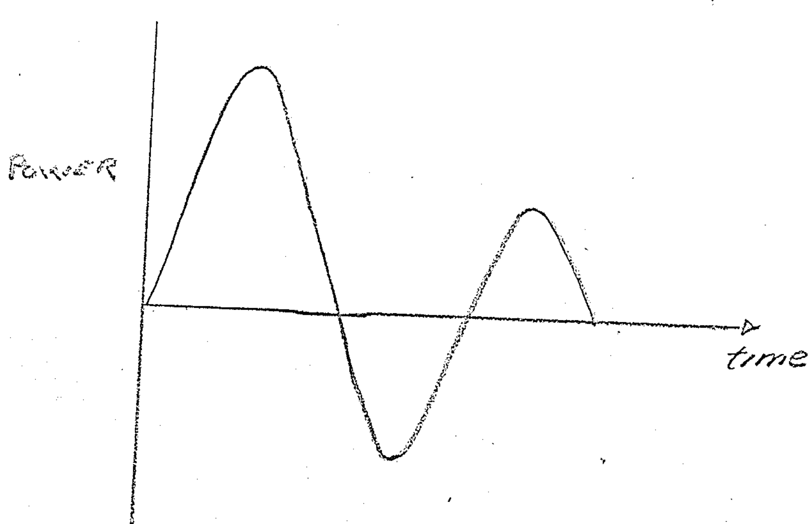


Figure 3.3

In order to select values of w_n and γ that will provide practical parameters for the governor response equation, it is necessary to refer to references 6 and 7 where a theoretical optimum response transient for the governor has been plotted. The plotted curve (see figure 4 reference 7) can be approximated by equation 3.6. Referring again to figure 3.3 it is seen that this curve can be made to approximate closely figure 4 in reference 7 by setting $w_n = .4$ and $\gamma = .5$.

Therefore:

$$T = \frac{2 \times .5}{.4} = 2.5 \text{ seconds}$$

Now, referring to equation 3.3, equation 3.9 is obtained[b]

$$-\frac{\gamma T \, dp_m}{dt} = \frac{T d\omega}{dt} + \omega \qquad 3.9$$

where: T = an overall time constant, relating the governor and all the various components such as water acceleration and inertia parameters, to the machine. Generally, this can be called the

[b] It is interesting to note the similarity between the approximation for the governor response equation developed here and the more accurate development done in reference 6.

Rewrite equation 3 of reference 6 by substituting w for n, where w is the p.u. speed of the machine in radians per second and n is the p.u. speed of the machine.

then
$$-\gamma T_r \frac{dg}{dt} = T_r \frac{d\omega}{dt} + \omega$$

where: T_r = the dashpot time constant in seconds.
p = the differential d/dt.
g = the per unit wicket gate opening.

governor system time constant, in that it relates the speed of response of the governor to the load change.

w = the change from base speed in p.u. radians per second.

$\frac{dp_m}{dt}$ = the change of the input power with respect to time.

γ = the temporary droop.

By noting that $dw/dt = d^2\delta/dt^2 = (p_m - p_e)/M$, where all values are the same as previously defined, one may, by substituting into 3.9, obtain:

$$\frac{dp_m}{dt} = \frac{-1(p_m - p_e)}{M\gamma} - \frac{w}{\gamma T} \qquad 3.10$$

3.3 Application of Governor Response Equation

At this time it is necessary to refer back to the Kutta-Runge subroutine that was used to solve the swing equation and to note the simple means offered for solving equation 3.10. Note that dw/dt has already been solved in the program (see equation 2.6), and that w will also be readily available. Very little extra logic was required to add equation 3.10 to the program. Examining equations 3.9 and 3.10 it is seen that the governor is responding to a change of speed and also to a rate of change of speed. Being a theoretical governor, deadband effects have been ignored, giving the governor instantaneous action.

As shown in figure 3.4, the total response of the governor is the combined effect of dw/dt and w. A straight line characteristic passing through the origin is quite applicable for the purposes of this thesis because now the governor has optimum response and its effect on stability will be indicated more clearly.

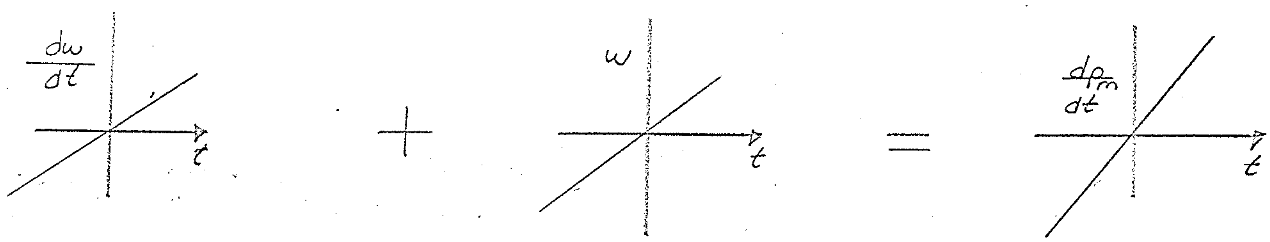


Figure 3.4

The governor used in this thesis, besides having no deadband effects, has no upper or lower cutoff. The author appreciated that it is easy to sit at a computer and change parameters but it was felt that it is, at this time, more important to gain insight into the effects of governor response addition, than to maintain a rigid hold onto reality.

3.4 Results

Referring to figure 3.5, one notes the results of the method used to obtain the optimum temporary droop. The system used was the same as the system described in section 2.6. Again, a three phase fault was applied for .1 seconds. It was noted that for every loading condition above 8.8 p.u.

a value for the temporary droop (δ) that just provided stability was .5 p.u. This value is well within practical limits. It is noted that in reference 7, the values given for the temporary droop range from .27 p.u. to .75 p.u.

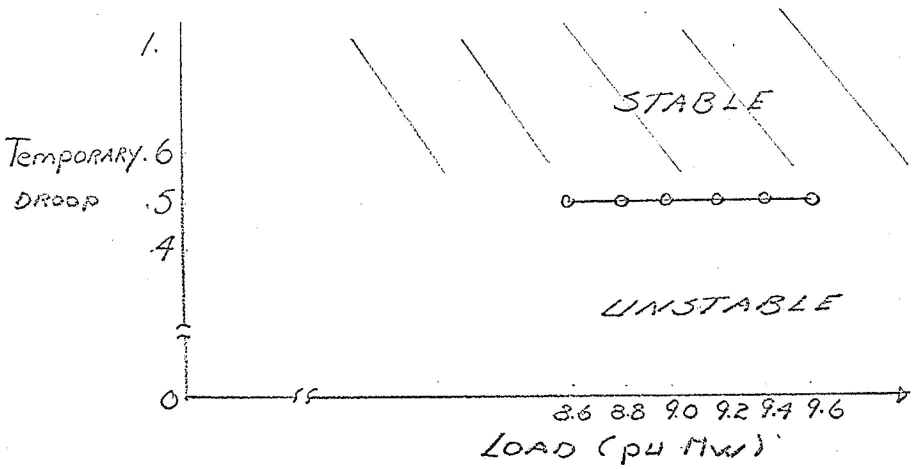


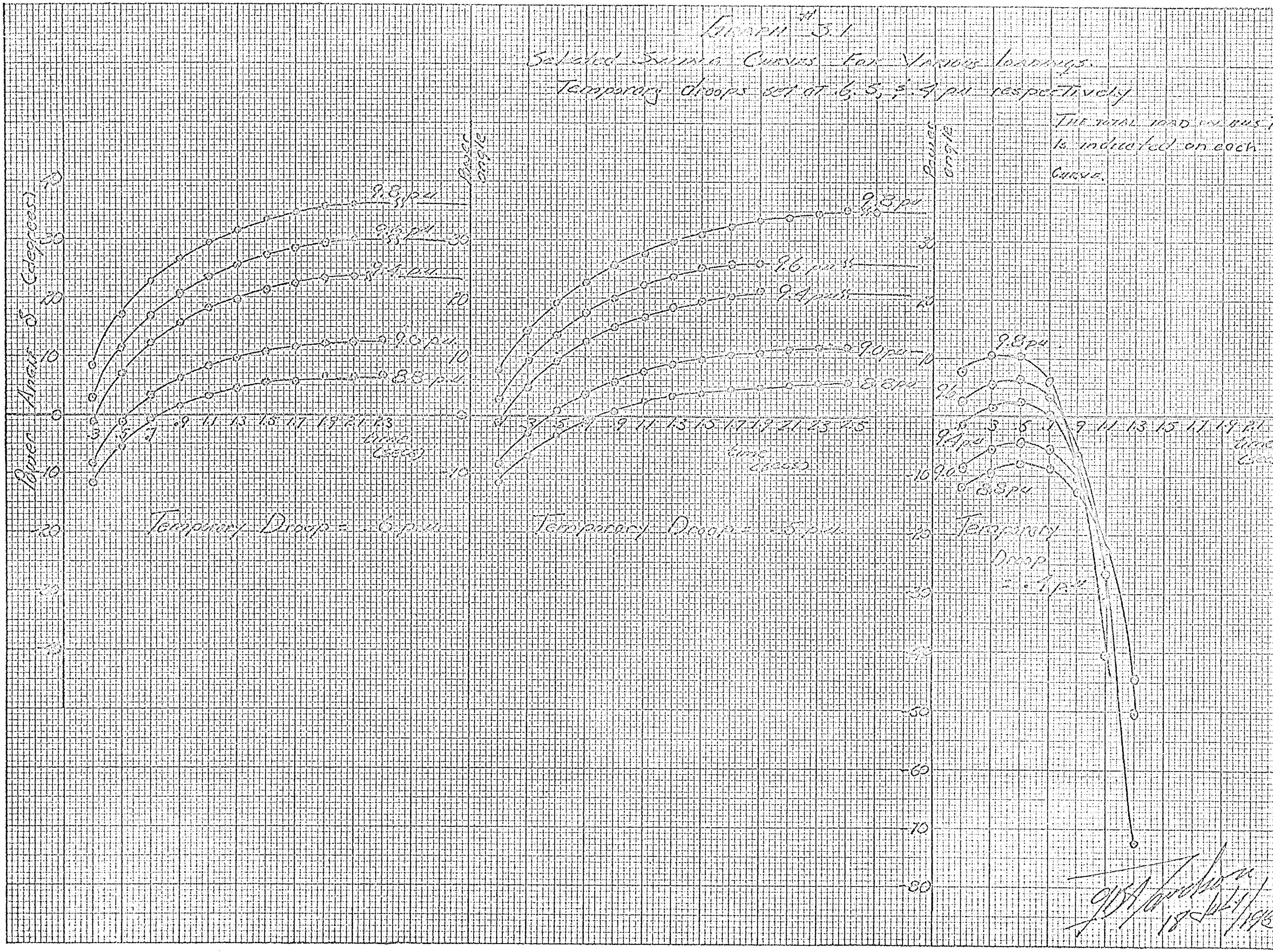
Figure 3.5

The method used to find the correct value of temporary droop was: the system was faulted with T set to 2.5 seconds and various values of droop applied. Referring to graph 3.1, it appeared that as the droop was increased above .5 p.u. the system became more and more stable, that is the curve flattened out. For values below .5 p.u., the system (no matter the loading) became unstable.

Referring to graph 3.2, note that for this case of a loading of 9.2 p.u. on the system, that as the droop was increased, the response (although faster) still levelled off at much the same angle.

Furthermore, it is noted again that equation 3.10 represents a theoretical governor; one having no time lags of any kind and where all the parameters were optimized to give the fastest response. Although this may not sound too practical it is reiterated that the purpose here was to note general effects rather than absolute magnitude of effects.

Further consideration will be given in the final chapter to the validity of the assumptions made with respect to governor response because of the resulting over damped swing curve whenever the governor mathematical model was used in the system.


Graph #3.1
Selected Swing Curves for Various loadings.
Temporary droops set at .6, .5, & .4 pu respectively
The total load on bus 7 is indicated on each curve.
Power Angle δ (degrees)
Power angle
time (secs)
9.8 pu
9.6 pu
9.4 pu
9.0 pu
8.8 pu
Temporary Droop = .6 p.u.
Temporary Droop = .5 p.u.
Temporary Droop = .4 p.u.

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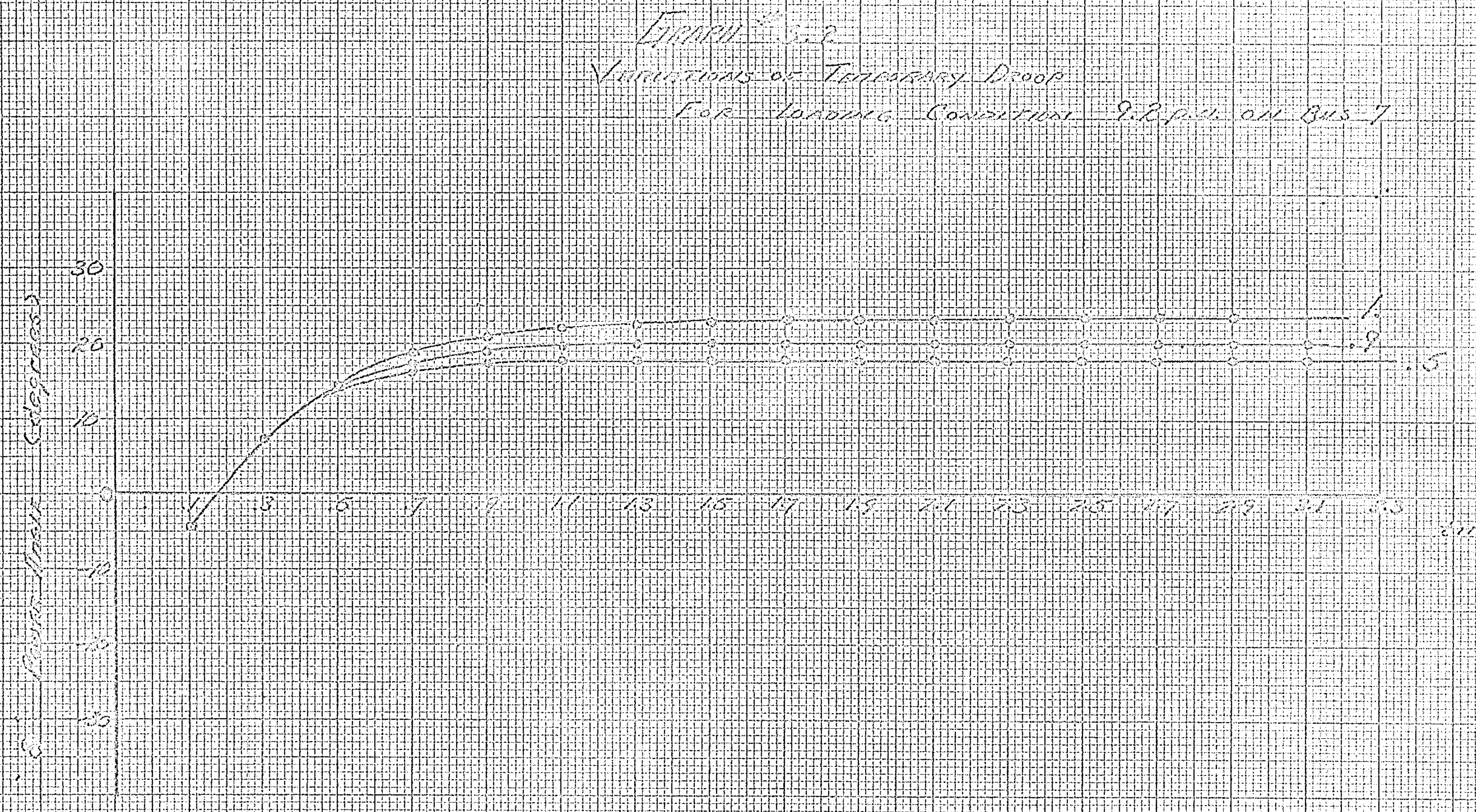

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CHAPTER IV

EXCITATION RESPONSE

This chapter will deal with the problem of considering exciter response as a method of improving the transient stability of a power system. As pointed out previously, exciter response proves to be a complex problem of taking into account the exact effect of the excitation system on the machine. Exciter response in itself is worthy of being considered as a separate thesis topic.

However, with the advent of computing devices the assumption in the calculation of accelerating power (see chapter 2.5) of constant voltage behind the transient reactance can now be replaced by a mathematical expression to enable adjustment of the voltage and thus improve stability.

4.1 General Considerations

Exciter system response may be defined, for the purpose of this thesis, as the rate of increase or decrease of the main exciter voltage when a fault is applied to or removed from the machine.

The equation to be developed will represent a system which senses a change in terminal voltage and attempts, by changing the voltage behind transient reactance, to return the terminal voltage to its normal value.

Exciter response was one of the first methods used to improve the stability characteristics of a system. The power that may be transmitted between machines varies directly as the voltage behind the transient reactance. By raising the internal voltages thereby increasing the flux linkages during a transient condition the prospect of a machine remaining stable is enhanced.

For a short fault duration, the over-all decrease of flux linkages is small, thus the effect of the excitation system is also small. However, even with the use of high-speed clearing, exciter response remains important because now the demagnetizing effect of armature current is smaller and it is possible for the exciter to maintain the flux linkages and perhaps even increase the number of linkages.

Consider figure 4.1, the vector diagram of a salient pole synchronous machine, for the transient state[9].

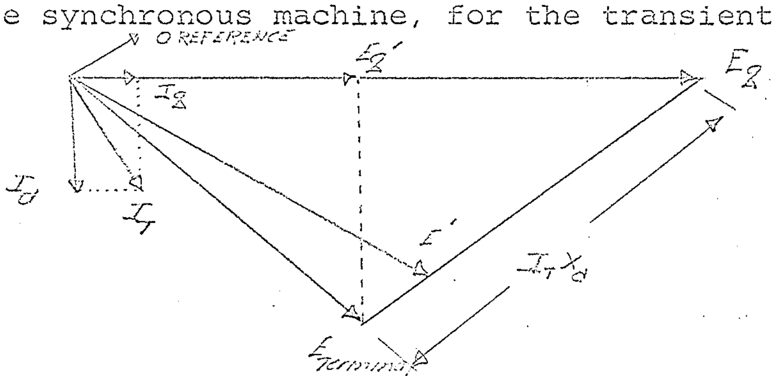


Figure 4.1*

* Throughout this thesis, capital "E's" represent phasor voltages which vary in magnitude and phase with respect to time but vary slowly with respect to power frequency (60 cps.).

If the armature current changes very slowly when compared to the transient decrement, the field current and the fictitious voltage behind the transient reactance remains constant. However, consider now a faster change in the armature current, such as would occur when a fault was applied or removed, or if the machine was swinging with respect to a reference. During the change, the flux linkages of the field remain constant. A new fictitious armature voltage will be defined proportional to the field flux linkages.

$$E_q' = \frac{w M_f \psi_f}{L_{ff}} \tag{4.1}$$

where: E_q' = voltage proportional to field flux linkages.

w = radians per second.

M_f = the mutual inductance between the field and the armature phase.

ψ_f = flux linkages with the field winding.

L_{ff} = self inductance of the field (henries).

Similarly, for the quadrature axis rotor circuit, when a fast change of armature current appears, the flux linkages remain constant. E_d', a voltage proportional to the quadrature axis flux linkages can also be defined.

$$E_d' = \frac{w M_q \psi_q}{L_{qq}} \tag{4.2}$$

where: $E_d{}'$ = voltage proportional to the linkages with the quadrature axis rotor circuit.

M_q = mutual inductance between the quadrature axis rotor circuit and the field.

L_{qq} = self inductance of the quadrature axis.

w = as per equation 4.1

ψ_q = flux linkages with the quadrature axis.

Combining these two voltages, we are now able to define the voltage behind the transient reactance as being

$$E' = E_d{}' + jE_q{}' \qquad 4.3$$

Now, if we restrict our study to salient pole machines, this equation may be simplified because of the fact that a salient pole machine has no quadrature axis field circuit. Thus:

$$E_d{}' = 0 \qquad 4.4$$

Therefore:

$$|E'| = |E_q{}'| \qquad 4.5$$

Therefore it can be said that the voltage behind the transient reactance varies as the flux linkages with the field.

4.2 Exciter Response System

Due to the complexity of analysing exciter response rigorously, and realizing the shortage of storage space remaining, it was necessary to develop an equivalent exciter system analysis that would react similar to a more exact analysis, and yet demand little storage space.

Consider figure 4.2; the block diagram of the exciter response control system used in this thesis:

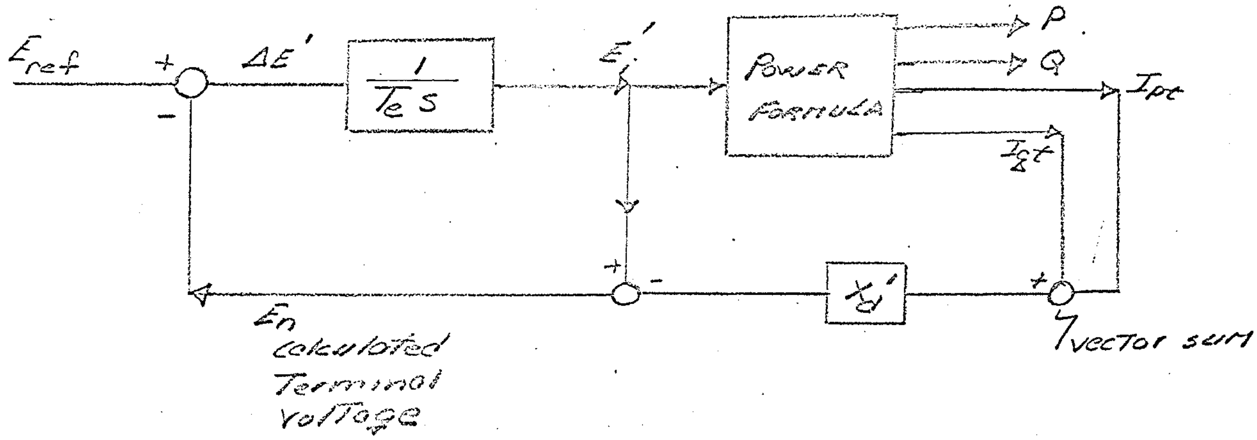


Figure 4.2

E_{ref}, the reference voltage was taken to be the terminal voltage of the machine (pre-vault). For each time increment, the real and imaginary components of power were calculated. Knowing E_i' (the voltage behind the transient reactance), I_{pt} and I_{qt} (the respective components of current), could then be calculated. Referring to figure 4.3, we see that the terminal voltage is readily available by applying Pythagoras' theorem.

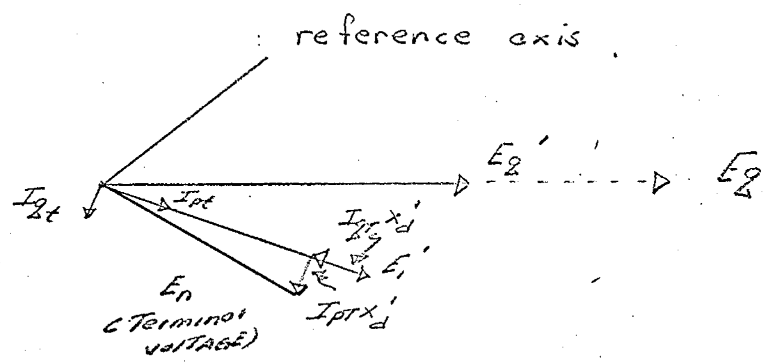


Figure 4.3

The terminal voltage can now be calculated at the end of each time increment. By subtracting the newly calculated terminal voltage from the reference voltage, an error voltage which our exciter can act upon is produced.

The exciter is assumed to have a straight line characteristic as shown in figure 4.4:

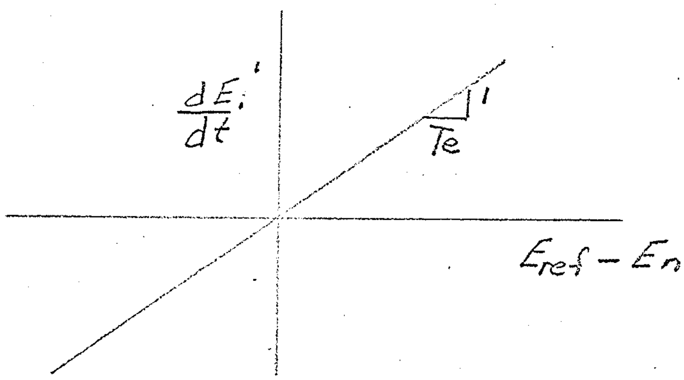


Figure 4.4

In other words, as seen in figure 4.2;

$$\frac{dE_i'}{dt} = \frac{E_{ref} - E_n}{T_e} \qquad 4.6.$$

where: E_{ref} = reference voltage: taken to be the voltage at the output terminals of the generator before the fault has been applied.

E_n = the calculated terminal voltage at the end of each time increment.

T_e = the equivalent exciter system constant (seconds). T_e is of the order of the open circuit time constant of an actual machine, that is, between one and 10 seconds. Much like the governor time constant this time constant must incorporate in an approximate way the variation of the open circuit time constant T_{do}' of the machine and the time constants of the exciter and its related components. It must also compensate for the assumption of linearity in the exciter and the fact that in solving for the output powers we allow E', the voltage behind the transient reactance, to vary with the field flux linkages as opposed to E_q'.

It is realized that these assumptions are by no means rigorous, however it is pointed out again that the exciter system developed here does react, though not exactly, to the changes in flux linkages as would a real exciter. It is felt that for the purposes of this thesis, the accuracy obtained is adequate. The exciter response as given by equation 4.6 indicates the type of response and effect to be expected from an actual system.

4.3 Solution of Exciter Equation by Kutta-Runge Process

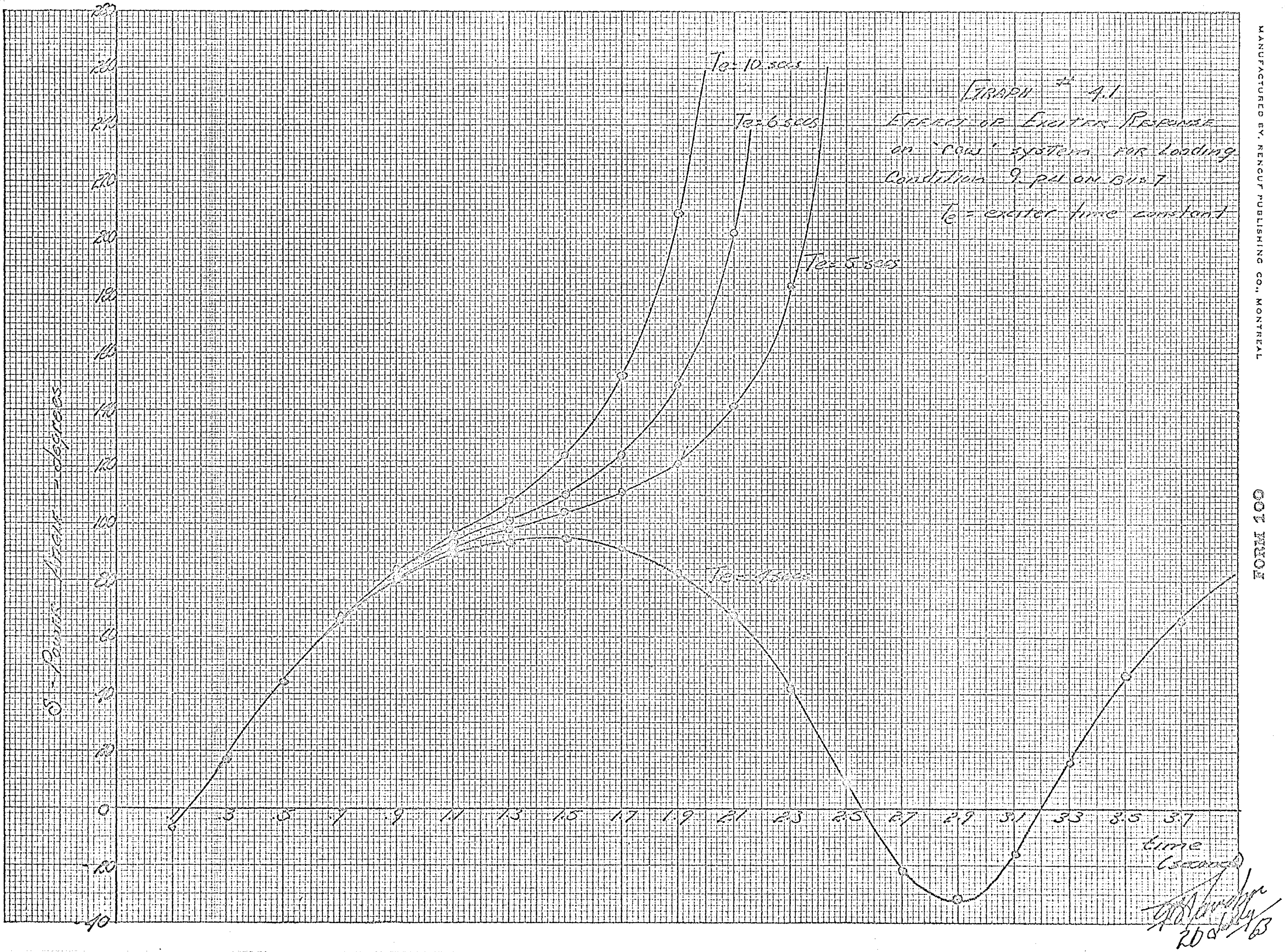
Equation 4.6 is a first order differential equation and is therfore easily solved by the Kutta-Runge subroutine. This ability of the Kutta-Runge process of handling a large number of first order differential equations without requiring a large amount of logic for each, is especially beneficial when using a small computer such as the IBM 1620 (20K).

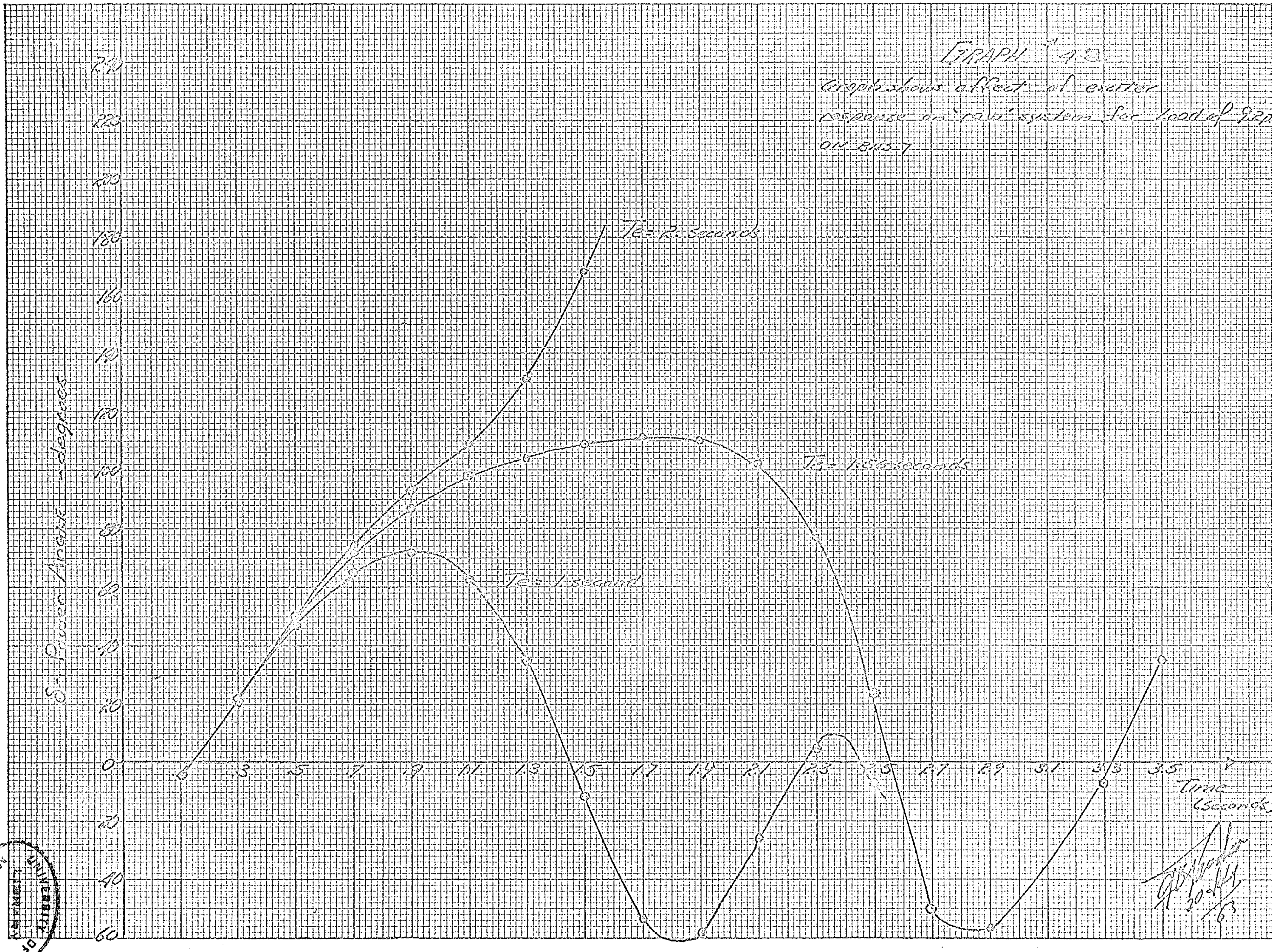
4.4 Results of Tests with Exciter Response Applied

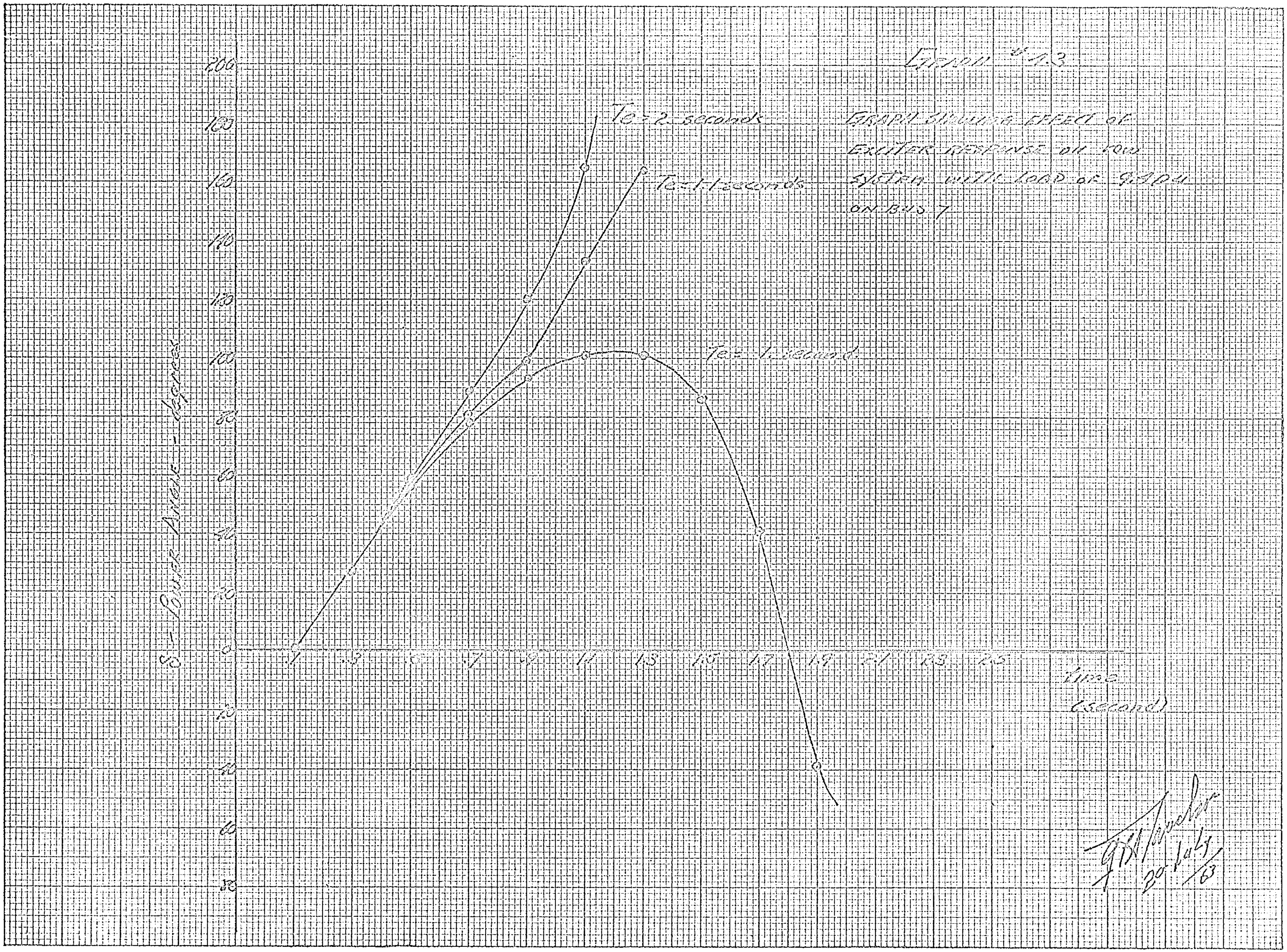
A series of tests were run on the sample system with exciter response considered. At each stage of loading (from 7.8 p.u. to 9.8 p.u.) various time constants (T_e) were fixed and the fault applied and cleared as previously stated. Refer to graphs 4.1, 4.2, 4.3, 4.4 for selected examples.

Note that in graph 4.1, where the loading on bus 7 is 9.0 p.u., the system remains stable for a time constant of 4 seconds. The machine remained unstable for a time constant of 5 seconds or greater. Recall that for the "raw" system, the loading of 9.0 p.u. definitely caused instability when the fault was applied. Therefore, by applying exciter response the allowable amount of power that may be transmitted has increased.

Referring to Graphs 4.2, 4.3 and 4.4, one can see that by applying the appropriate time constant it is possible to make the system stable after a fault even though the finite machine was transmitting power at a level that proved to be, in chapter II, definitely unstable. Refer to figure 4.5 for a table giving the load drawn (p.u.) and the maximum time constant (seconds) that stabilized the system. It is to be noted that the lowest value of T_e used was one second. It was felt that one second was the lowest practical time constant that could be used.


Graph # 4.1
Effect of Exciter Response on 'CGW' system for Loading Condition 9 pu on Bus 7
Te = exciter time constant
Te = 10 secs
Te = 6 secs
Te = 5 secs
δ - Power Angle - degrees
time (seconds)


GRAPH # 42
Graph shows affect of exciter response on 'raw' system for load of 9.2pu. on Bus 7
Te = 2 seconds
Te = 1.55 seconds
Te = 1 second
δ - Power Angle - degrees
Time (seconds)
240
220
200
180
160
140
120
100
80
60
40
20
0
20
40
60
.3
.5
.7
.9
1.1
1.3
1.5
1.7
1.9
2.1
2.3
2.5
2.7
2.9
3.1
3.3
3.5


Graph #13
GRAPH SHOWING EFFECT OF
EXCITER RESPONSE ON
SYSTEM WITH LOAD OF
ON BUS 7
Te = 2 seconds
Te = 1.1 seconds
Te = 1 second
δ - Power Angle - degrees
Time (seconds)
200
180
160
140
120
100
80
60
40
20
0
20
40
60
80
.1 .3 .5 .7 .9 1.1 1.3 1.5 1.7 1.9 2.1 2.3 2.5
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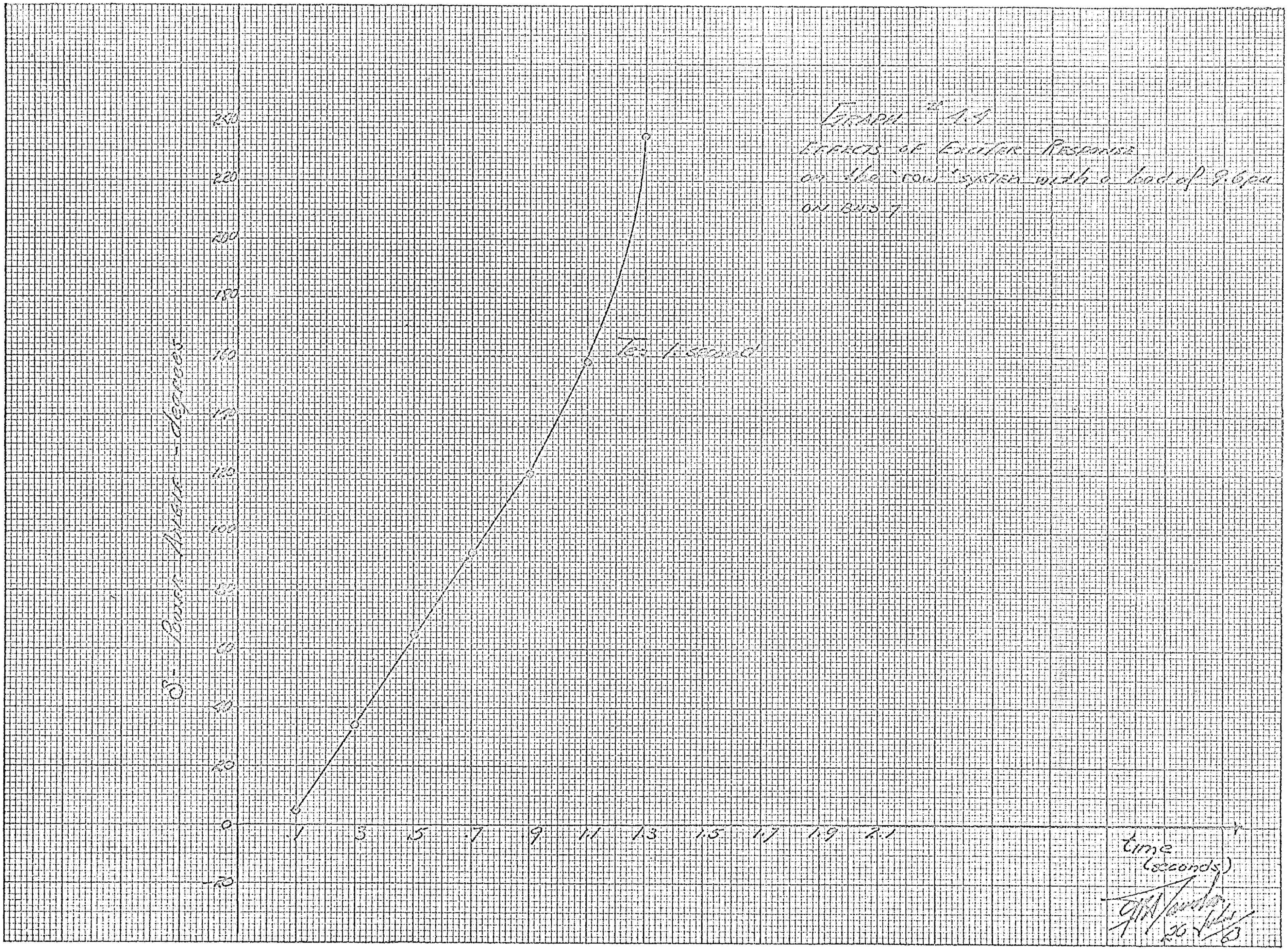

GRAPH #14
EFFECTS OF EXCITER RESPONSE
on the 'cow' system with a load of 9.6pu
ON BUS 7
Te = 1 second
δ - Power Angle - degrees
240
220
200
180
160
140
120
100
80
60
40
20
0
-20
.1
.3
.5
.7
.9
1.1
1.3
1.5
1.7
1.9
2.1
time (seconds)

Load Drawn (p.u.)	T_e (secs)
9.0	4.0
9.2	1.5
9.4	1.0
9.6	not obtained

Figure 4.5

4.5 Summary

In summarizing, recall equation 4.6, which gives the equivalent exciter response equation used in this thesis. The equation, which is an approximation, was used for two reasons. The first is that to consider exciter response rigorously at this time would defeat the purposes set out in chapter I. That is, the advantages and disadvantages of employing exciter response is the objective and not the exact magnitude of effects. The second and equally important is that the storage requirements demanded by a rigorous treatment far exceeded the capacity of the 1620 but it was shown that exciter response models are conducive to representations that are suitable for use in digital computers.

The exciter response model has shown that its effects on improving transient stability can not be denied. It was shown that by employing exciter response, the maximum allowable power transmittable was raised by 60 Megawatts in this two-machine problem.

A comparison of exciter response and the other methods used to improve stability will be dealt with in chapter VI.

CHAPTER V

RAPID RECLOSING

This chapter will deal with the problem of improving transient stability by means of rapid reclosing.

5.1 General Considerations

Practice has shown that high speed reclosing circuit-breakers are a definite advantage in maintaining the transient stability of a power system.

The fundamental problem is to find the maximum permissible time available after the fault has been cleared to reclose and also the minimum time permissible for the deionization of the fault arc.

When a three phase fault occurs on a section of a line and the line is removed by breakers (in this case line 2--3), the impedance between the generators is increased. This increase in impedance lowers the amount of available power that may be transmitted through the system, thus increasing the difference between input power and electrical power which increases the accelerating power imposed on the machine. The sooner the line can be replaced into the circuit so as to again lower the impedance between the generator and the load the better the chance the system has of remaining stable. For the purpose of this thesis three phase reclosing breakers were employed.

5.2 Program Considerations

Including rapid reclosing in the computer program was very easy. All that was required of the program was that it accept a new set of admittances (G_{ij}, B_{ij}) for the new circuit condition, that is, the admittances that were calculated for the circuit in chapter II with no three phase fault applied.

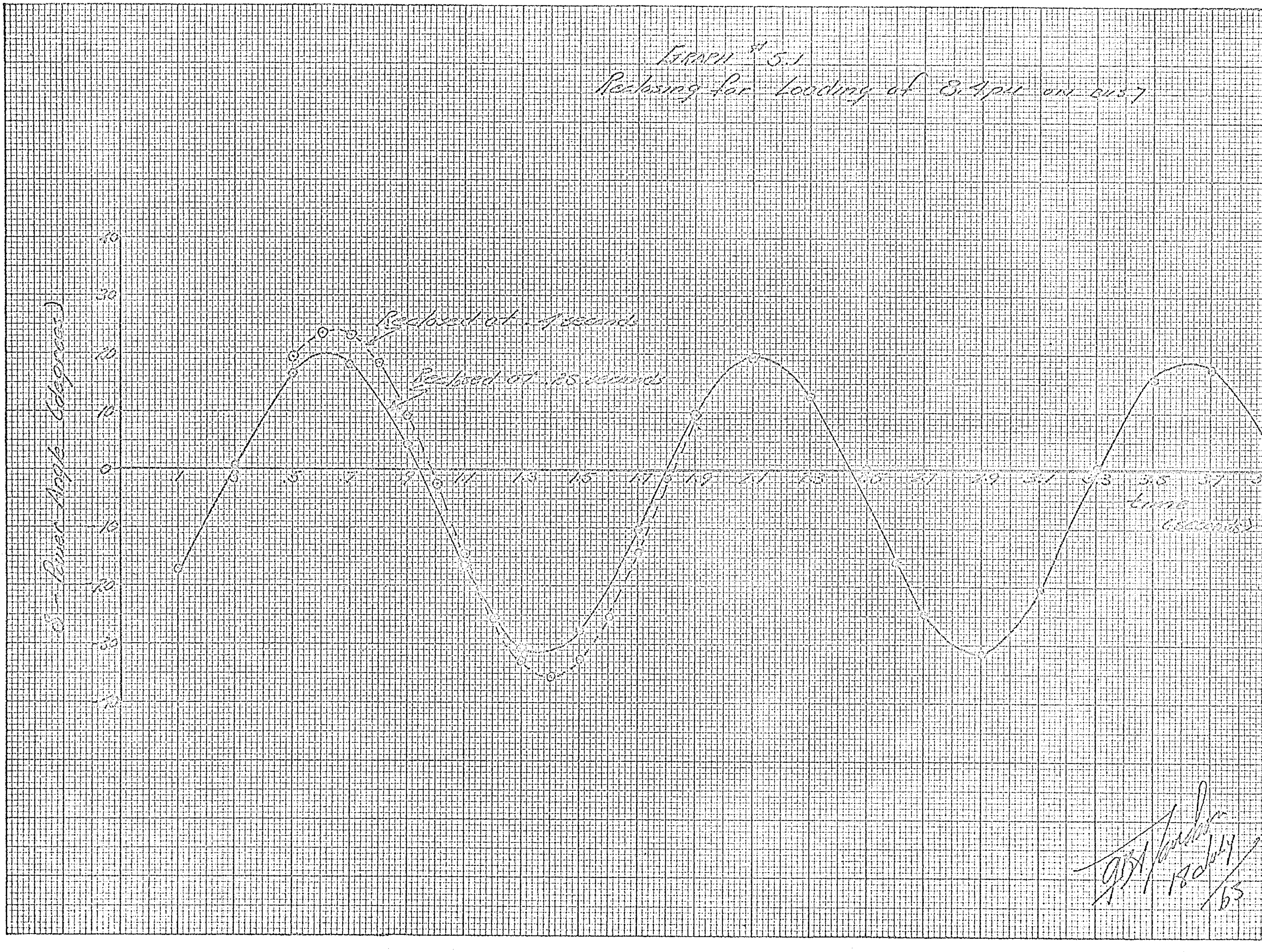
5.3 Results of Rapid Reclosing

For the next series of tests on the sample power system, a three phase fault, cleared at .1 seconds, was again applied to line 2--3. However, the line was reclosed at varying intervals for each power level from 8.4 to 9.8 p.u. Refer to graphs 5.1, 5.2, which give selected typical samples of tests carried out. It is seen that the system remained stable for loads of 8.4 and 8.8 p.u.

In chapter II, a load of 8.8 p.u. on bus 7 had proved to be the load wherein the system was critically stable. However, the system with rapid reclosing again proved to be critically stable.

For a loading of 9. p.u., we can see from graph 5.3, that now the system is stable for all values of reclosing time up to and including one second. At 1.1 seconds, the machine fell out of step, and became unstable as it did for the "raw" system.

As the load on the system was increased, the maximum time allowed before reclosing so that the system would remain


Graph # 5.1
Reclosing for Loading of 8.4 pu on Bus 7
Reclosed at .4 seconds
Reclosed at .25 seconds
δ – Power Angle (degrees)
Time (seconds)
40
30
20
10
0
-10
-20
-30
-40
.1
.3
.5
.7
.9
1.1
1.3
1.5
1.7
1.9
2.1
2.3
2.5
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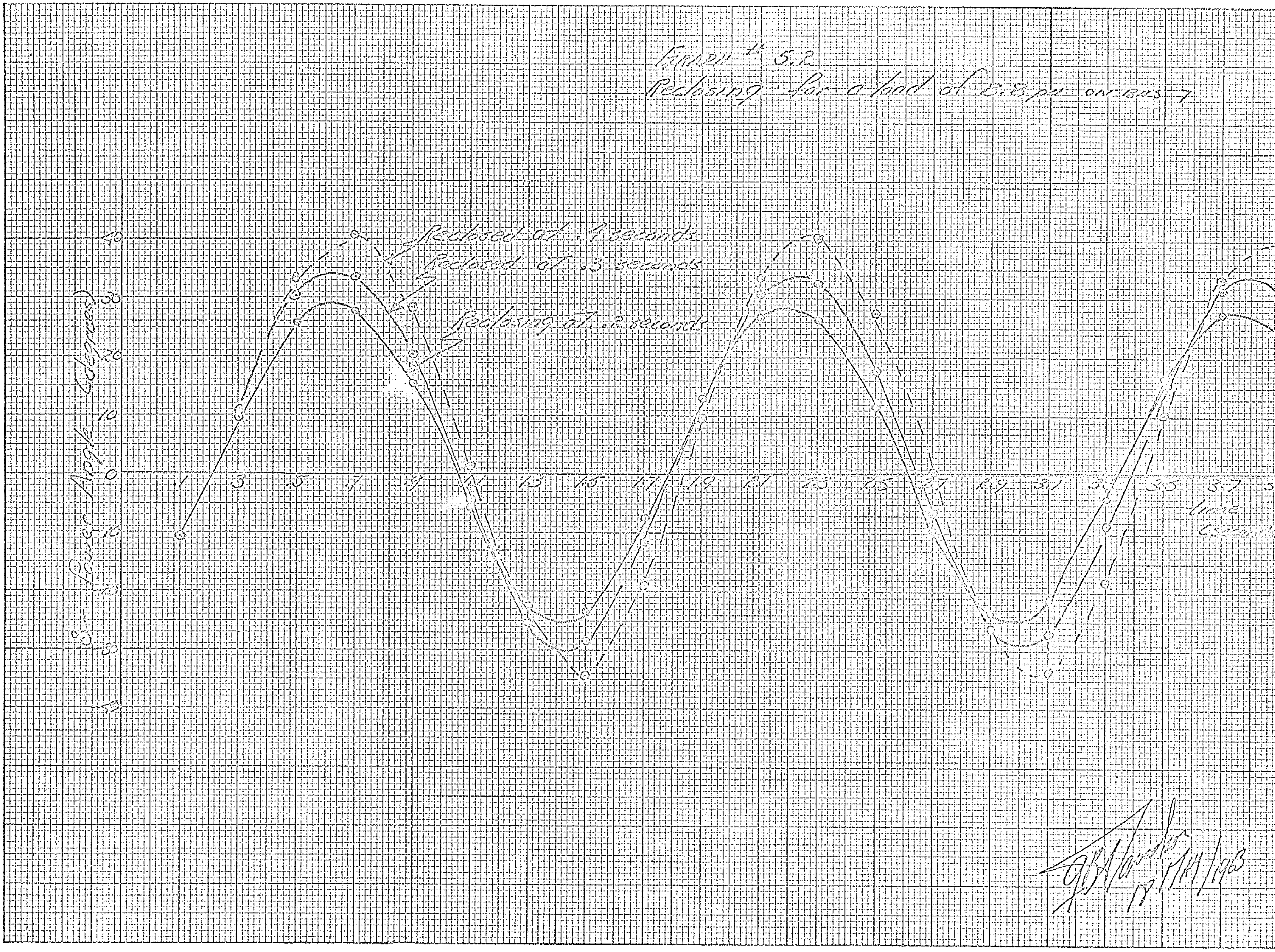
2.9
3.1
3.3
3.5
3.7
3.9


Graph # 5.2
Reclosing for a load of 8.8 pu on bus 7
Reclosed at .4 seconds
Reclosed at .3 seconds
Reclosing at .2 seconds
δ - Power Angle (degrees)
40
30
20
10
0
1
3
5
7
9
11
13
15
17
19
21
23
25
27
29
31
33
35
37
39
Time (Seconds)

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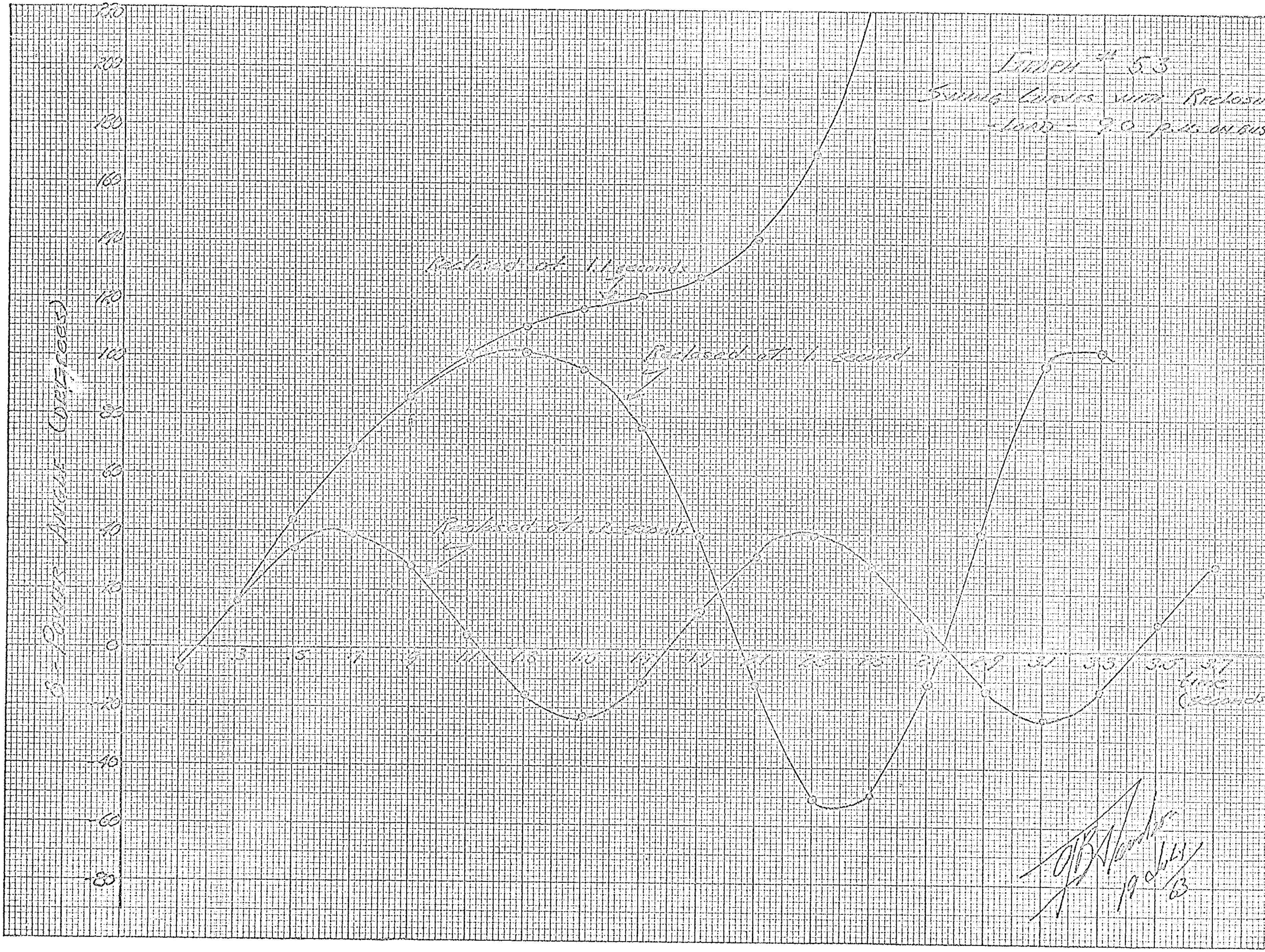

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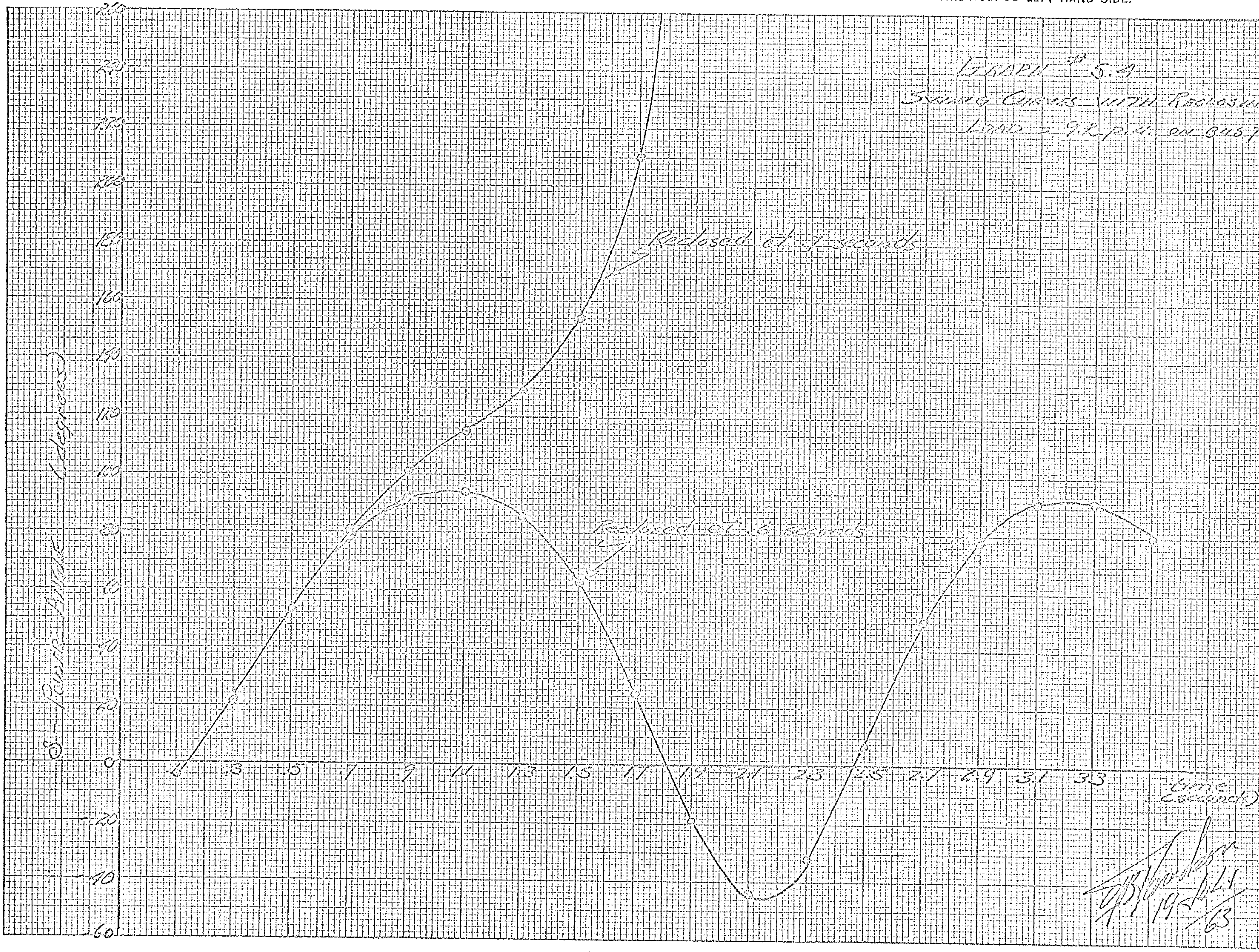
stable, decreased. Referring to graph 5.4, we see the time for reclosing had to be .6 seconds or less in order to maintain stability. It is pointed out that with reclosing we can now transmit more power. Not until a load of 9.8 p.u. (see graph 5.5) was placed on the system did reclosing prove to be futile. As pointed out previously, .2 seconds was the minimum time that could be used for reclosing and still stay within practical boundaries.

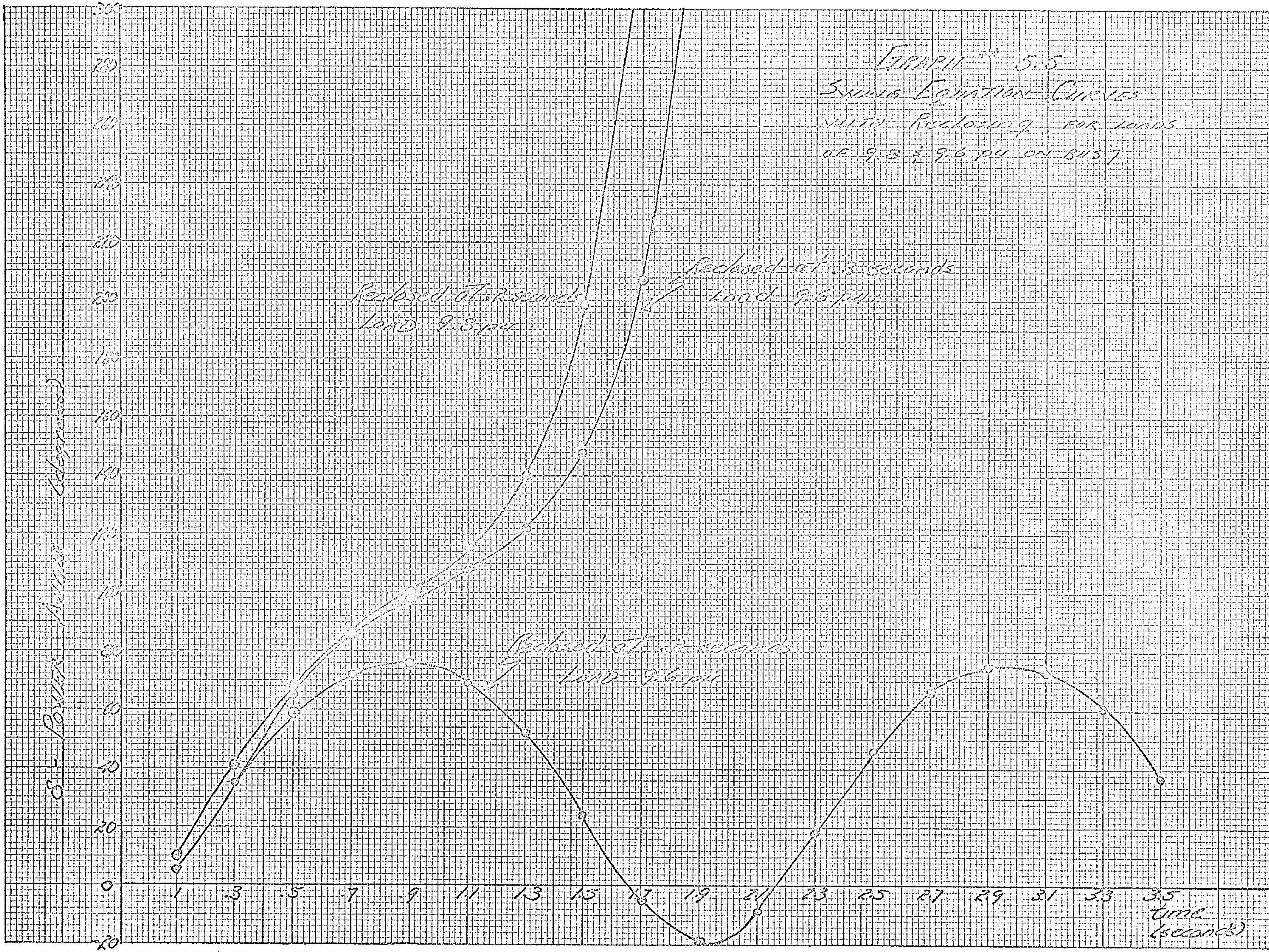
Refer to figure 5.1; where a table of the load vs. the maximum time allowable to maintain stability is given.

Load (p.u.)	Maximum Reclosing Time (seconds)
8.4	infinity
8.6	infinity
8.8	infinity
9.0	1.0
9.2	.6
9.4	.4
9.6	.2
9.8	unstable

Figure 5.1

For the sample system studied, a graph may be drawn indicating the maximum time allowed for reclosing at any power level in order to maintain stability. Referring to figure 5.2, it can be seen that for any load level, the faulted line must be reclosed at a time such that the plotted point of load vs. reclosing time falls beneath the curve drawn.


Graph # 5.4
Swing Curves with Reclosing
Load = 92 p.u. on 845.7
Reclosed at .7 seconds
Reclosed at .16 seconds
δ - Power Angle (degrees)
time (seconds)


Graph No 5.5
Swing Equation Curves
with Reclosing for loads
of 9.8 & 9.6 pu on Bus 7
Reclosed at .2 seconds
Load 9.8 pu
Reclosed at .3 seconds
Load 9.6 pu
Reclosed at .2 seconds
Load 9.6 pu
δ - Power Angle (degrees)
time (seconds)
300
280
260
240
220
200
180
160
140
120
100
80
60
40
20
0
-20
.1
.3
.5
.7
.9
1.1
1.3
1.5
1.7
1.9
2.1
2.3
2.5
2.7
2.9
3.1
3.3
3.5

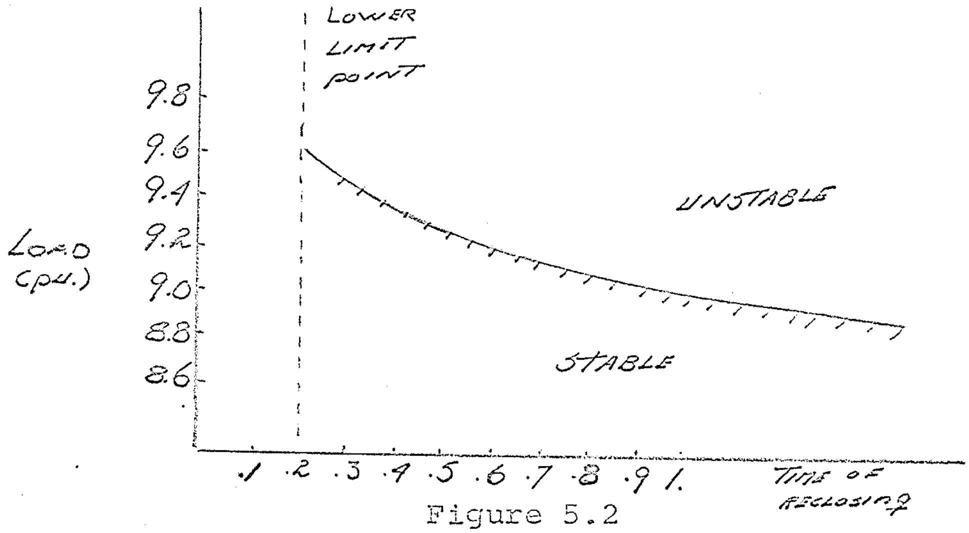


Figure 5.2

Furthermore, it is to be noted that as the load increases, the reclosing time interval to maintain stability on reclosing, shortens, until it gets to a point where no stability is obtainable.

Finally, it should be pointed out that the effect of reclosing alone can allow the system to transmit a greater amount of power and remain stable.

5.4 Summary

Rapid reclosing of a line after the fault is removed has been shown to be beneficial in improving the transient stability characteristics of the system studied.

CHAPTER VI

COMBINED EFFECT OF RAPID RECLOSING, EXCITER AND GOVERNOR RESPONSE.....CONCLUSION

Three methods of improving transient stability have been studied as separate entities. Now, it is the purpose of this final chapter to compare these methods with each other and to present the benefits gained by employing them.

6.1 Results

Only selected typical examples will be discussed due to the fact that the trends at one loading proved to be the same as the trends at another loading, varying only in magnitune. Therefore, for the sake of clarity and simplicity, specific examples will be presented.

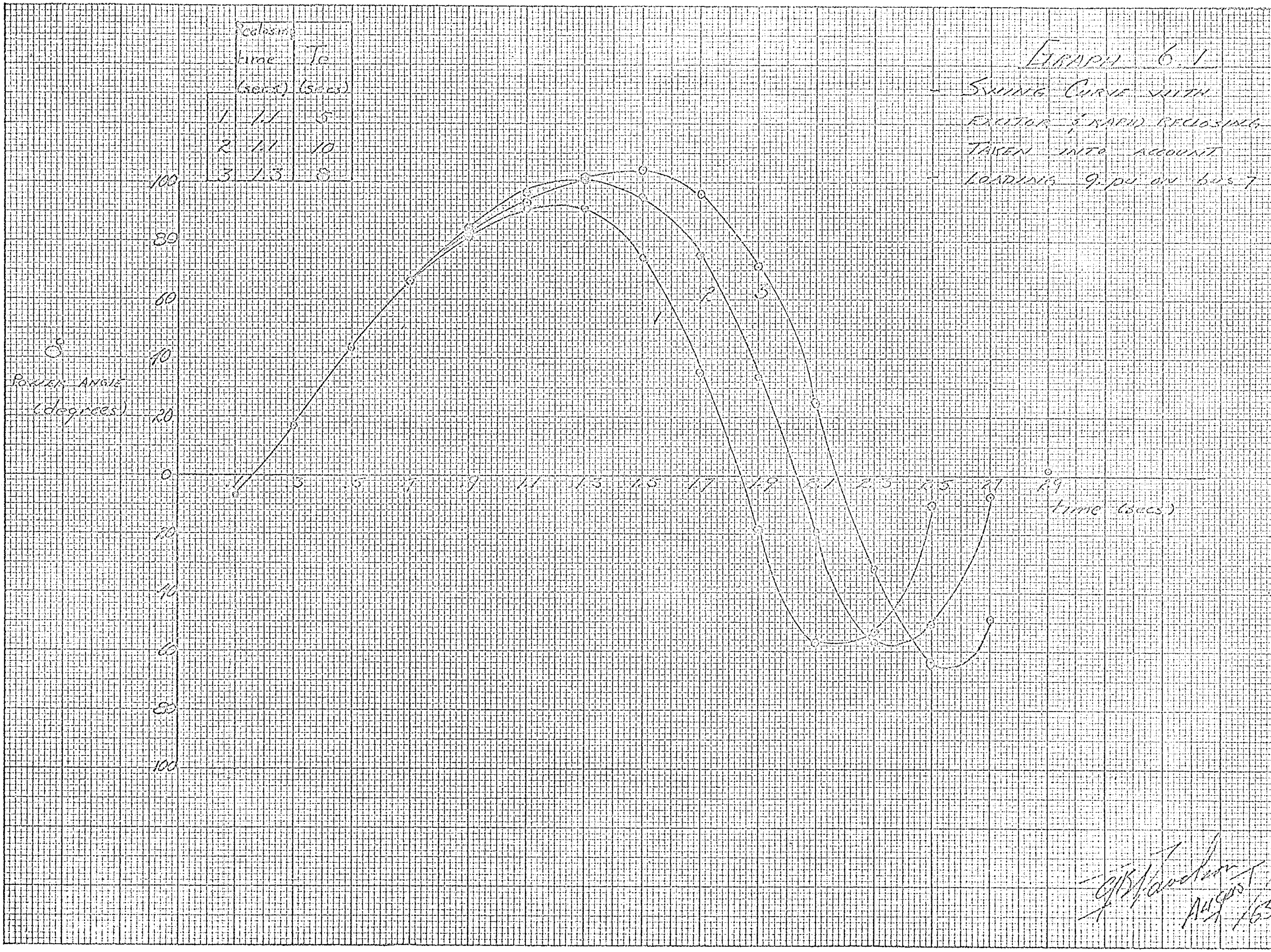
Recall that in chapter II, it was shown that a load on bus 7 of 9. p.u. for the "raw" system became unstable after a three phase fault was applied. Moreover, by employing reclosing, the system was stabilized if the line reclosing time was one second or less. The system could also be made stable, as shown in chapter IV, by applying exciter response having a time constant (T_e) of four seconds.

Now, by referring to graph 6.1, we see that for a loading of 9. p.u. and a reclosing time of 1.1 seconds and an exciter time constant (T_e) set to five seconds, that the system is stable. Note that the parameters used had all previously proven to be of little use alone in stabilizing the system. The fact that these values may be used because the overall effect is enough to stabilize the system has one advantage, that is in all cases the values are more easily obtainable in practice. Furthermore, to establish this point, refer again to graph 6.1 which is a selected typical example of the test series run and it is seen that the exciter time constant was raised to 10 seconds and still the system remained stable. Note also that in another case, the reclosing time was lengthened to 1.3 seconds with T_e=8 seconds, and the system again proved to be stable.

To compare the relative effect on the system of the various methods used to improve stability all parameters were held constant at a value that had proved to be of the correct order and one parameter was then varied and the effect noted.

Consider graph 6.2. We see the effect on the system with a loading of 9.4 p.u., of keeping the exciter and governor parameters constant and varying the reclosing time. The three phase fault was cleared at .1 seconds and line 2--3 was reclosed at .3, .5 and .7 seconds in three separate tests. The only effect of lengthening the reclosing time was that the overshoot was slightly higher. The final value in all three cases was reached before two seconds had elapsed. These results were indicative of the effect at each loading.

Consider now graph 6.3. In this case the reclosing and the governor parameters were held constant while the

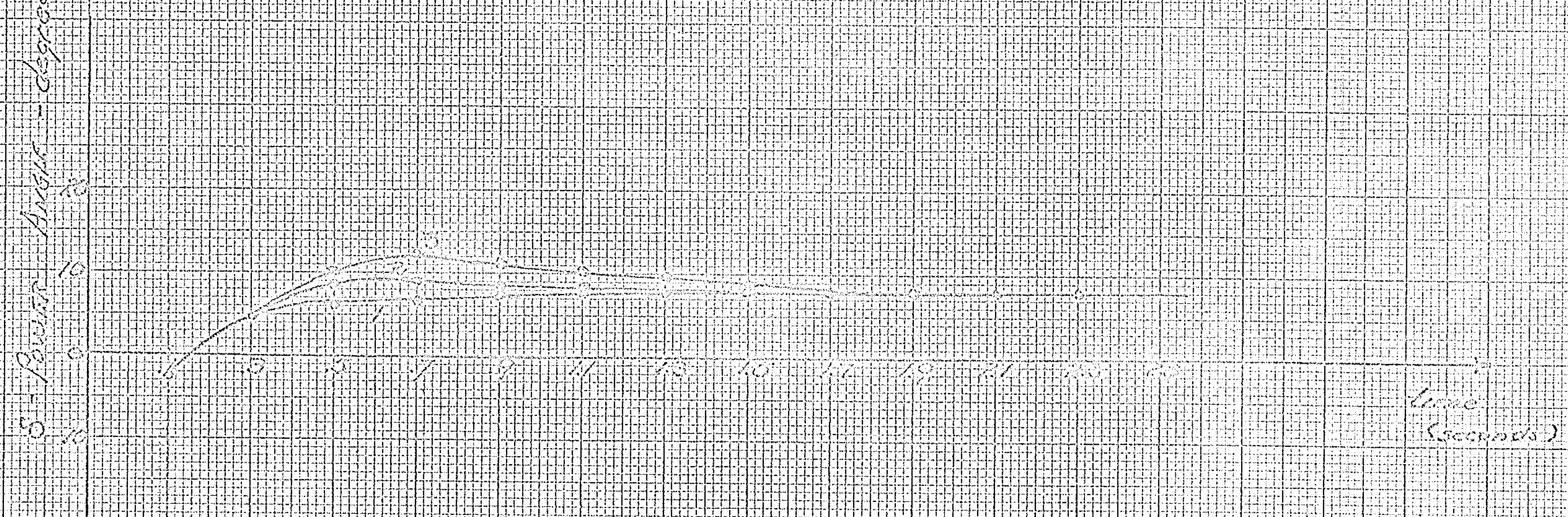

GRAPH 6.1
SWING CURVE WITH
EXCITOR & RAPID RECLOSING
TAKEN INTO ACCOUNT
LOADING 9 pu ON BUS 7
Reclosing time (secs)
Te (secs)
1 .11 5
2 .11 10
3 .13 8
δ
POWER ANGLE
(degrees)
100
80
60
40
20
0
20
40
60
80
100
.1
.3
.5
.7
.9
1.1
1.3
1.5
1.7
1.9
2.1
2.3
2.5
2.7
2.9
time (secs)
1
2
3
G.B. Goodwin
August 1/63

GRAPH 6.2

SWING CURVES SHOWING EFFECT OF RECLOSING WITH GOVERNOR & EXCITER RESPONSE INCLUDED

	Reclosing time (secs)	Te secs	Temp Droop pu
1	.3	.5	.5
2	.5	.5	.5
3	.7	.5	.5

Load 9.4 pu on bus 1

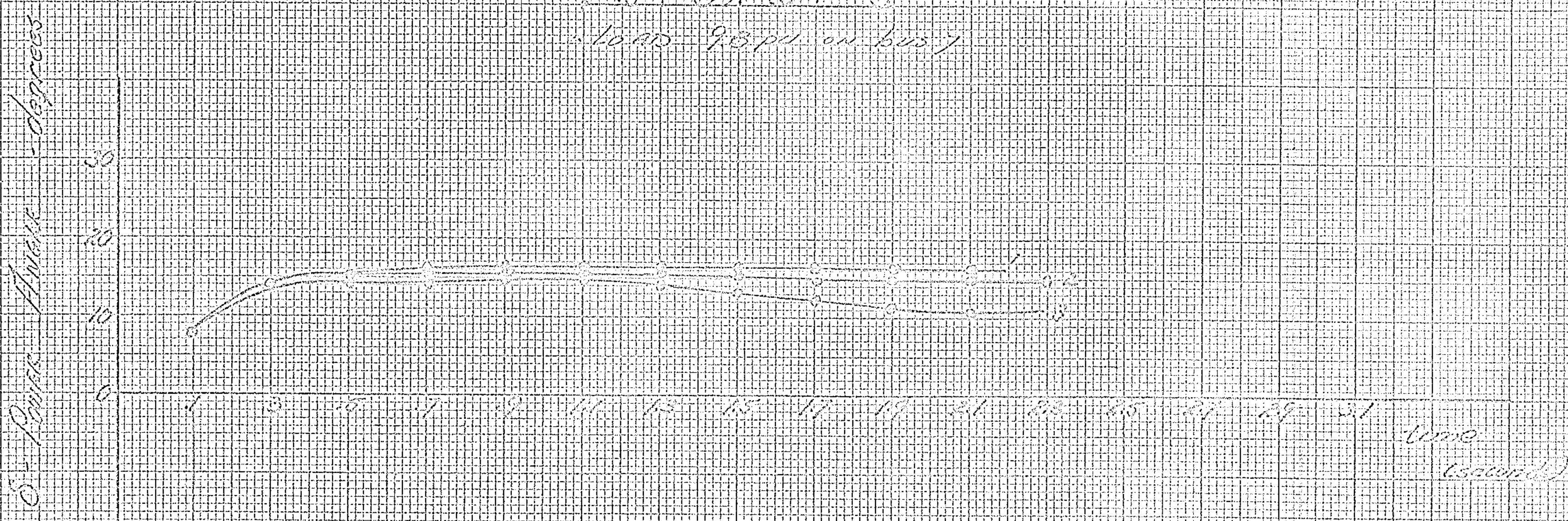


Graph # 6.3

Swing Curves showing effect of exciter response with governor & rapid reclosing included.

Reclosing (secs)	Tc (secs)	Temp Drop pu	
.3	10.	.5	1
.3	5.	.5	2
.3	2.	.5	3

Load 9.8 pu on bus 7



23/11/63

exciter time constant was varied. The effect of increasing the exciter time constant (T_e) was to raise slightly the steady state value of the swing curve. Both with reclosing and exciter response the governor model had an over-powerful effect. (See General Conclusions, section 6.2.)

Referring to graph 6.4 which gives the swing curves for a loading of 9.6 p.u., on bus 7, it can be seen that by increasing the temporary droop to .8 p.u., there is a larger overshoot. If the droop is decreased below .5 p.u., the system becomes unstable. The governor time constant was fixed, as varying, as it proved to have little effect.

6.2 General Conclusions

It appears that the Kutta-Runge method offers an accurate and simple means of solving the swing equation when using digital computers. Its accuracy is definitely better than the classical point by point method previously used. It is not suggested by the author that the point by point method is not accurate enough for general purposes, and should no longer be used, but it is suggested that for digital computers the Kutta-Runge method does offer, besides greater accuracy, a very convenient method of calculating the effect of exciter and governor response; or, for that matter, any other effect that may be considered, and that may be described by single order differential equations.

	Relaxing time (secs)	Te secs	Temp Droop p.u.
1	.5	5	.8
2	.5	5	.15
3	.5	5	.4

Reviewing the results of governor response, it is seen that the mathematical model used gives questionable results for the first few seconds of the swing. The immediate effect of the governor on stabilizing the system is greater than one would expect from an actual governor. In order to explain, it is necessary to re-examine the assumptions of the original model.

Consider figure 6.1 which gives in block diagram form the mathematical representation used in reference #7 for the governor system.

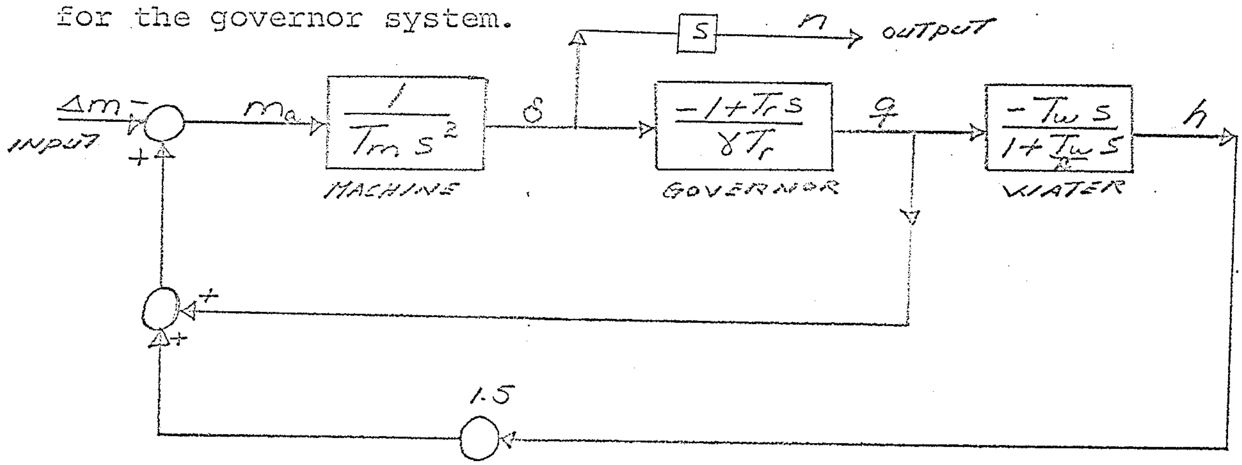


Figure 6.1

where: T_r = Dashpot time constant (secs.)

T_m = Mechanical starting time (secs.)

T_w = Water starting time (secs.)

γ = Temporary droop (p.u.)

For the purposes of this thesis a similar model was set up. However, because of storage space considerations, several approximations were used. Consider figure 6.2 which gives in block form the mathematical model used to simulate the governor system. Note that the dotted lines indicate that portion of figure 6.1 not included in figure 6.2.

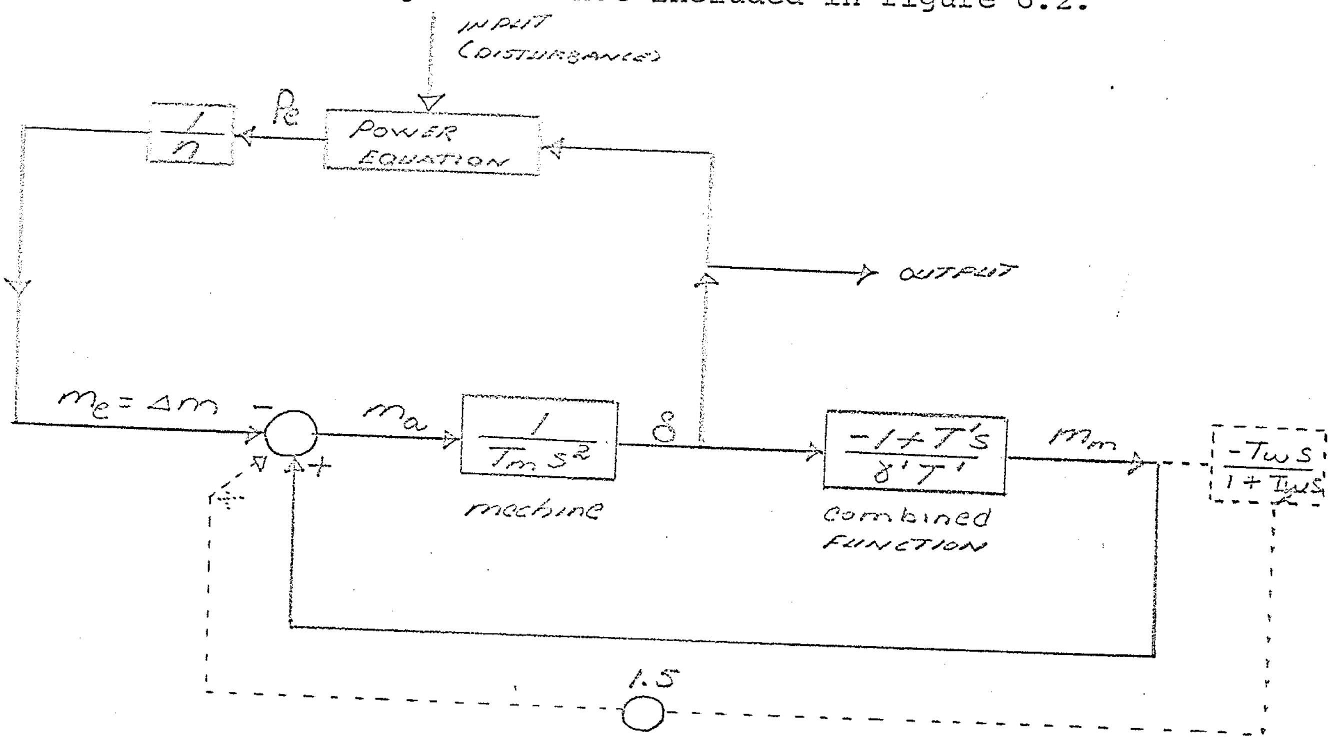


figure 6.2

where: δ' = Temporary droop (p.u.)

T' = Overall governor time constant (secs.) (See equation 3.9)

In figure 6.2 the overall time constant (T') and the temporary droop (δ') were selected to coincide with the "optimum transient response" curve shown in figure #4 of reference #7. A formula was developed to approximate this curve; the type of curve used is shown in figure 3.3.

The difference between the methods shown in figures 6.1 and 6.2 is that in the approximate method used for this thesis the full effect of T_W (the water starting time in seconds) was not fully taken into account by the overall time constant.

In order to develop this point further, consider figure 6.3 which shows the relative effect, in the first few seconds of the p.u. wicket gate opening (g) and the p.u. head (that is 1.5 x h) after the occurence of a load change.*

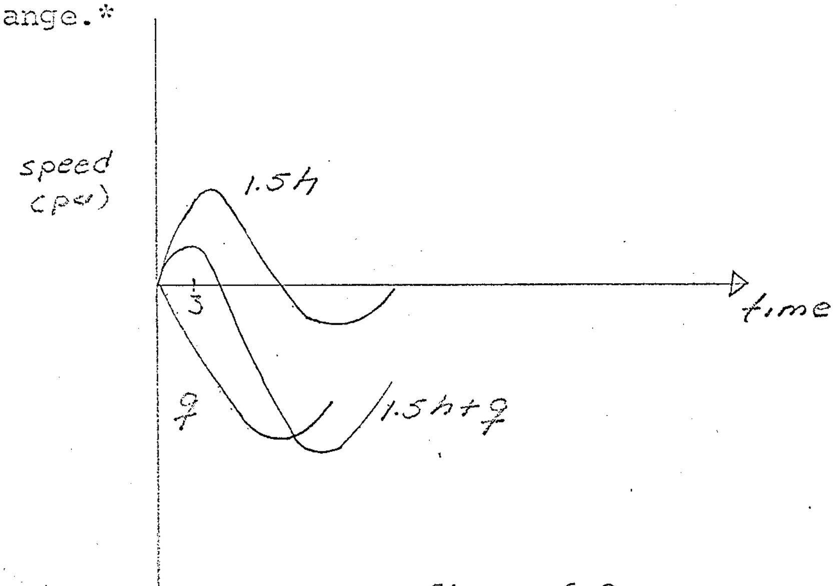


figure 6.3

Note that within the first three seconds the resultant effect of g and 1.5h is to accelerate the machine. In the mathematical model used in this thesis the full effect of water acceleration was not accounted for by the overall time constant (T); therefore causing the governor system in the first second to de-accelerate the machine, thus accounting for the damped curve when-ever the governor system model was employed.

* 1.5h is taken directly from the machine acceleration equation:

$$T_m \frac{dn}{dt} = g + 1.5h - \Delta m$$

However, for the purpose of this thesis it was shown that a mathematical model of a governor system does lend itself to analysis by digital computers and although the model used in this thesis was not accurate, the employing of digital computers in future studies of the behavior of governors during faults is suggested (see section 6.4).

The study of reclosing showed that its effect on improving transient stability was definitely worth considering. It proved capable of stabilizing the system up to a load at bus 7 of 9.6 p.u. with a reclosing time of .2 seconds. This thesis considered three phase reclosing but it may be pointed out that for longer lines with single phase reclosing, and for a non-three phase faults (say a line to ground fault), power may still be transmitted down the sound phases.

Exciter response has its greatest effect at the lower loads. The accurate consideration of exciter response so as to be cognizant with the program proved to be too large for the benefit gained. A reasonable approximation of exciter response was made. Several criteria had to be satisfied in this case. The first was that the equation used for exciters had only to be accurate enough to show the effect of the improvement on the system. The second was that no matter what approximation was used the logic needed to solve it had to be small, and preferably the equation decided on should be compatible with the Kutta-Runge method. By using the approximation derived in chapter

IV, all criteria were satisfied. It is suggested that for a more accurate analysis of exciter response, the effects of saturation and upper and lower cut-off be written into the program.

To summarize then, it has been shown that more power can be transmitted if improvement methods such as exciter or reclosing are used. It was also shown that by employing two or more methods, one could obtain a stable system. This time, however, the values of the parameters were less critical.

6.3 Conclusion

In conclusion, this thesis has covered transient stability of a typical power system from various angles. First the swing equation (usually solved by the point by point method) was solved by means of Gills Variation of the Kutta-Runge process. Secondly, three separate methods for improving transient stability were investigated and their effect on the stability when used singly was noted. The combined effect of the improvement methods used on the sample was studied. Advantages were noted, the major one being that appreciably more power can be transmitted.

6.4 Future Studies

This thesis could not hope to cover so broad a field as power systems stability in one study. At each section, problems that were out of the scope of this thesis presented themselves as possibilities for future studies.

The first suggestion for future work is that a separate study using a digital computer be carried out on governor behavior during a fault using the accurate model shown in figure 6.1. The study of governor behavior itself is worthy of a separate topic.

Secondly a separate study dealing with exciter response would be beneficial. Again, a separate study is suggested so that an exact accurate mathematical model of the exciter system can be used to study the effect of exciter response on a machine during a load change. To consider exciter response accurately involves a large amount of logic and storage space, much more than was allotted to the exciter portion of the program used in this thesis.

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06950 C     MAXIMUM OF 13 MACHINES
06950 C    G. B. DAVIDSON.....UNIVERSITY OF MANITOBA
06950 201FORMAT(F6.3,5H     ,F9.4,I5)
07000 202FORMAT(I3,F5.3,F5.3,F5.3)
07038 203FORMAT(F8.5,F8.5)
07066 204FORMAT(F6.4,F6.4,F6.4,F6.4,F6.4)
07108 205FORMAT(27HTIME         SWING   MACHINE)
07186 206FORMAT(19H(SEC)      (DEGREES))
07248 207FORMAT(19HNGEN  DT  FCT  TURC)
07310     DIMENSION RK(26),RQ(26),Y(26),E(13),PO(13)
07310     DIMENSION HM(13),XD(13),AR(13,13),AX(13,13)
07310     TYPE 207
07322 100ACCEPT 202,NGEN,DT,FCT,TURC
07382     TYPE 205
07394     TYPE 206
07406     M=NGEN*2
07442     DO 46 I=1,M
07454 46 ACCEPT TAPE 203,Y(I)
07538     DO 45 I= 1,NGEN
07550     ACCEPT TAPE 204,E(I),PO(I),Q,HM(I),XD(I)
07718     AMR=PO(I)/E(I)
07802     AMQ=Q/E(I)
07862     ER=E(I)+XD(I)*AMQ
07958     EQ=XD(I)*AMR
08018     E(I)=SQRT(ER**2+EQ**2)
08138 45 Y(I)=Y(I)+ATAN(EQ/ER)
08282 C    VOLTAGE AND DELTA BEHIND XD(I) HAS NOW BEEN CALCULATED
08282     T=DT
08306 101DO 40 I= 1,NGEN
08318     DO 40 J=1,NGEN
08330     ACCEPT TAPE 203,AR(I,J),AX(I,J)
08486     IF(J-I)39,40,39
08554 39 AX(I,J)=-AX(I,J)
08710     AR(I,J)=-AR(I,J)
08866 40 CONTINUE
08938 C    BEGIN GILLS VARIATION OF KUTTA RUNGE SUBROUTINE
08938 300 DO 309 JRK=1,4
08950     GO TO 301
08958 302DO 309 I=1,M
08970     GO TO (305,306,307,308),JRK
09054 305Y(I)=Y(I)+.5*RK(I)
09174     RQ(I)=RK(I)
09246     GO TO 309
09254 306Y(I)=Y(I)+.292893*(RK(I)-RQ(I))
09410     RQ(I)=.58579*RK(I)+.12132*RQ(I)
09566     GO TO 309
09574 307Y(I)=Y(I)+1.70711*(RK(I)-RQ(I))
09730     RQ(I)=3.4142*RK(I)-4.12132*RQ(I)
09898     GO TO 309
09906 308Y(I)=Y(I)+0.166667*RK(I)-.33333*RQ(I)
10110     IF(I-NGEN)303,303,309
10178 303DELTA=Y(I)-Y(1)
10238     DELTA=DELTA*57.29578
10274     TYPE 201,T,DELTA,I
10322 309CONTINUE
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T0394 C    CHECK FOR FAULT CLEARING TIME AND RECLOSING TIME
T0394      IF(SENSE SWITCH 1)317,315
T0414 317TYPE 207
T0426      ACCEPT 202,NGEN,DT,FCT,TURC
T0486 315T=T+DT
T0522      IF(FCT-T+DT)311,101,300
T0602 311IF(TURC-T+DT)300,101,300
T0682 C    BEGIN CALCULATION OF  RUNGE-KUTTA COEFFICIENTS..RK(I)
T0682 301DO 1 K=1,NGEN
T0694      NPK=NGEN+ K
T0730 1    RK(K)=DT*Y(NPK)
T0850      DO 2 I=1,NGEN
T0862      NPI=NGEN + I
T0898      IF(HM(I)-100.)14,14,12
T0990 14   PT=0.
T1014      DO 3 J=1,NGEN
T1026 4    PT=PT+E(J)*(COS(Y(I)-Y(J))*AR(I,J)+SIN(Y(I)-Y(J))*AX(I,J))
T1422 3    CONTINUE
T1458      RK(NPI)=(DT*(PO(I)-E(I)*PT))/HM(I)
T1638 2    CONTINUE
T1674      GO TO 302
T1682 12   RK(NPI)=0.
T1730      GO TO 2
T1738      END

T9999 SIN
T9989 SINF
T9979 COS
T9969 COSF
T9959 ATAN
T9949 ATANF
T9939 EXP
T9929 EXPF
T9919 LOG
T9909 LOGF
T9899 SQRT
T9889 SQRTF
T9879 SIGN
T9869 SIGNF
T9859 ABS
T9849 ABSF
T9839 0201
T9829 0202
T9819 0203
T9809 0204
T9799 0205
T9789 0206
T9779 0207
T9769 RK     T9519
T9509 RQ     T9259
T9249 Y      T8999
T8989 E      T8869
T8859 PO     T8739
T8729 HM     T8609
T8599 XD     T8479
T8469 AR     T6789
T6779 AX     T5099
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1̄5069 DT
1̄5059 FCT
1̄5049 TURC
1̄5039 M
1̄5029 0̄002̄
1̄5019 0̄00
1̄5009 0̄046
1̄4999 I
1̄4989 0̄045
1̄4979 Q
1̄4969 AMR
1̄4959 AMQ
1̄4949 ER
1̄4939 EQ
1̄4929 0̄01
1̄4919 T
1̄4909 0̄101
1̄4899 0̄040
1̄4889 J
1̄4879 0̄039
1̄4869 0̄300
1̄4859 0̄309
1̄4849 JRK
1̄4839 0̄301
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1̄4729 3̄4142000̄1
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1̄4679 DELTA
1̄4669 5̄7295780̄2
1̄4659 0̄317
1̄4649 0̄315
1̄4639 0̄311
1̄4629 0̄001
1̄4619 K
1̄4609 NPK
1̄4599 0̄002
1̄4589 NPI
1̄4579 1̄0000000̄3
1̄4569 0̄014
1̄4559 0̄012
1̄4549 PT
1̄4539 0̄0000000̄99̄
1̄4529 0̄003
1̄4519 0̄004
1̄4509 0̄02
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LOAD SUBROUTINES　　　　　　　　PROCESSING COMPLETE

UTO FORTRAN SUBROUTINES. 1/63

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06950 C     THIS PROGRAM ADDS TO 14T04 EXCITATION RESPONSE AND
06950 C     GOVERNOR RESPONSE
06950 C     THIS ADDS TO 17T02 A  MORE EXACT METHOD OF CONSIDERING
06950 C     GOVERNOR RESPONSE...
06950 C      MAXIMUM OF 5 MACHINES
06950 C     TRANSIENT STABILITY PROGRAM
06950 C     G. B. DAVIDSON...UNIVERSITY OF MANITOBA
06950 201FORMAT(F5.3,5H     ,F9.4,I5)
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07038 203FORMAT(F8.5,F8.5)
07066 204FORMAT(F6.4,F6.4,F6.4,F6.4,F6.4)
07108 205FORMAT(27HTIME         SWING   MACHINE)
07186 206FORMAT(19H(SEC)      (DEGREES))
07248 207FORMAT(19HNGEN  DT  FCT  TURC)
07310 208FORMAT(18HEX-TC GOV-TC DROOP)
07370    DIMENSION RK(20),RQ(20),Y(20),EREF(5),TE(5)
07370    DIMENSION HM(5),XD(5),AR(5,5),AX(5,5)
07370    DIMENSION DRP(5),TC(5)
07370    TYPE 207
07382 100ACCEPT 202,NGEN,DT,FCT,TURC
07442    TYPE 208
07454    DO 49 I=1,NGEN
07466 49 ACCEPT 204,TE(I),TC(I),DRP(I)
07622    TYPE 205
07634    TYPE 206
07646    M=NGEN*2
07682    M3 = NGEN*3
07718    M4=NGEN*4
07754    DO 46 I=1,M
07766 46 ACCEPT TAPE 203,Y(I)
07850    DO 45 I= 1,NGEN
07862    ACCEPT TAPE 204,E,PO,Q,HM(I),XD(I)
07982    EREF(I)=E
08030    MPI=M3+I
08066    MI=M+I
08102    Y(MPI)=PO
08150    AMR=PO/E
08186    AMQ=Q/E
08222    ER=E+XD(I)*AMQ
08294    EQ=XD(I)*AMR
08354    Y(MI)=SQRT(ER**2+EQ**2)
08474 45 Y(I)=Y(I)+ATAN(EQ/ER)
08618 C   VOLTAGE AND DELTA BEHIND XD(I) HAS NOW BEEN CALCULATED
08618    T=DT
08642 101DO 40 I= 1,NGEN
08654    DO 40 J=1,NGEN
08666    ACCEPT TAPE 203,AR(I,J),AX(I,J)
08822    IF(J-I)39,40,39
08890 39 AX(I,J)=-AX(I,J)
09046    AR(I,J)=-AR(I,J)
09202 40 CONTINUE
```

```
09274 C   BEGIN GILLS VARIATION OF KUTTA RUNGE SUBROUTINE
09274 300 DO 309 JRK=1,4
09286     GO TO 301
09294 302DO 309 I=1,M4
09306     GO TO (305,306,307,308),JRK
09390 305Y(I)=Y(I)+.5*RK(I)
09510     RQ(I)=RK(I)
09582     GO TO 309
09590 306Y(I)=Y(I)+.292893*(RK(I)-RQ(I))
09746     RQ(I)=.58579*RK(I)+.12132*RQ(I)
09902     GO TO 309
09910 307Y(I)=Y(I)+1.70711*(RK(I)-RQ(I))
10066     RQ(I)=3.4142*RK(I)-4.12132*RQ(I)
10234     GO TO 309
10242 308Y(I)=Y(I)+0.166667*RK(I)-.33333*RQ(I)
10446     IF(I-NGEN)303,303,320
10514 303DELTA=Y(I)-Y(1)
10574     DELTA=DELTA*57.29578
10610     TYPE 201,T,DELTA,I
10658     GO TO 309
10666 320TYPE 201,T,Y(I),I
10738 309CONTINUE
10810 C   CHECK FAULT CLEARING TIME AND RECLOSE TIME
10810     IF(SENSE SWITCH 1)317,315
10830 317TYPE 207
10842     ACCEPT 202, NGEN, DT,FCT,TURC
10902 315T=T+DT
10938     IF(FCT-T+DT)311,101,300
11018 311IF(TURC-T+DT)300,101,300
11098 C   BEGIN CALCULATION OF THE KUTTA RUNGE COEFFICIENTS
11098 301DO 1 K=1,NGEN
11110     NPK= NGEN+ K
11146 1   RK(K)=DT*Y(NPK)
11266     DO 2 I=1,NGEN
11278     MI=M+I
11314     NPI=NGEN+I
11350     MPI=M3+I
11386     PT=0.
11410     QT=0.
11434     DO 3 J=1,NGEN
11446     MJ=M+J
11482     PT=PT+Y(MJ)*(COS(Y(I)-Y(J))*AR(I,J)+SIN(Y(I)-Y(J))*AX(I,J))
11878 3   QT=QT+Y(MJ)*(COS(Y(I)-Y(J))*AX(I,J)-SIN(Y(I)-Y(J))*AR(I,J))
12322     EN=SQRT((Y(MI)-(QT*XD(I)))**2+(PT*XD(I))**2)
12538     IF(HM(I)-99.)20,20,12
12630 12RK(MI)=0.
12678     RK(MPI)=0.
12726     RK(NPI)=0.
12774     GO TO 2
12782 20 RK(MI)=(EREF(I)-EN)*DT/TE(I)
12914     RK(NPI)=(DT*(Y(MPI)-Y(MI)*PT))/HM(I)
13094     RK(MPI)=-RK(NPI)/DRP(I)-Y(NPI)*DT/(DRP(I)*TC(I))
13382 2   CONTINUE
13418     GO TO 302
13426     END
```

```
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1̄9989 SINF
1̄9979 COS
1̄9969 COSF
1̄9959 ATAN
1̄9949 ATANF
1̄9939 EXP
1̄9929 EXPF
1̄9919 LOG
1̄9909 LOGF
1̄9899 SQRT
1̄9889 SQRTF
1̄9879 SIGN
1̄9869 SIGNF
1̄9859 ABS
1̄9849 ABSF
1̄9839 0̄201
1̄9829 0̄202
1̄9819 0̄203
1̄9809 0̄204
1̄9799 0̄205
1̄9789 0̄206
1̄9779 0̄207
1̄9769 0̄208
1̄9759 RK     1̄9569
1̄9559 RQ     1̄9369
1̄9359 Y      1̄9169
1̄9159 EREF   1̄9119
1̄9109 TE     1̄9069
1̄9059 HM     1̄9019
1̄9009 XD     1̄8969
1̄8959 AR     1̄8719
1̄8709 AX     1̄8469
1̄8459 DRP    1̄8419
1̄8409 TC     1̄8369
1̄8359 0̄100
1̄8349 NGEN
1̄8339 DT
1̄8329 FCT
1̄8319 TURC
1̄8309 0̄049
1̄8299 I
1̄8289 M
1̄8279 0̄002̄
1̄8269 0̄00
1̄8259 M3
1̄8249 0̄003̄
1̄8239 M4
1̄8229 0̄004̄
1̄8219 0̄046
1̄8209 0̄045
1̄8199 E
1̄8189 PO
1̄8179 Q
1̄8169 MPI
1̄8159 MI
1̄8149 AMR
1̄8139 AMQ
1̄8129 ER
```

```
18119 EQ
18109 001
18099 T
18089 0101
18079 0040
18069 J
18059 0039
18049 0300
18039 0309
18029 JRK
18019 0301
18009 0302
17999 0305
17989 0306
17979 0307
17969 0308
17959 5000000000
17949 2928930000
17939 5857900000
17929 1213200000
17919 1707110001
17909 3414200001
17899 4121320001
17889 1666670000
17879 3333300000
17869 0303
17859 0320
17849 DELTA
17839 5729578002
17829 0317
17819 0315
17809 0311
17799 0001
17789 K
17779 NPK
17769 0002
17759 NPI
17749 PT
17739 0000000099
17729 QT
17719 0003
17709 MJ
17699 002
17689 EN
17679 004
17669 9900000002
17659 0020
17649 0012
17639 003
```

LOAD SUBROUTINES
UTO FORTRAN SUBROUTINES. 1/63

APPENDIX III

TYPICAL OUTPUTS

Output of Load Flow Program for Load of 9.8 p.u.

Bus	Voltage (p.u.)		P	Q	
000	1.0000	-7.202	680.0	546.8	
		007	679.6	545.7	1.000
001	1.0000	.000	319.7	122.5	
		002	319.7	122.5	1.000
002	.9671	-5.881			
		003	160.7	43.8	
		004	159.2	43.4	
		001	-319.7	-86.2	
003	.9135	-14.124			
		004	158.3	43.4	
		002	-157.9	-42.4	
004	.8595	-23.269			
		005	155.2	36.7	
		006	153.2	36.5	
		002	-153.3	-36.2	
		003	-155.2	-37.2	
005	.8205	-31.225			
		006	153.3	30.0	
		004	-152.6	-29.0	
006	.7863	-39.867			
		007	298.9	39.0	1.000
		004	-148.0	-19.4	
		005	-150.6	-19.9	
007	.7798	-46.389	-980.0		
		000	-679.6	5.0	
		006	-298.9	-4.8	

END OF OUTPUT

APPENDIX III (continued) Output of Admittance Program

```
 1
 2
18
3.00959-5.37103          A 11
 .00000 -.00000          A 12
 2
 2
19
 .00152 -.00037          A 21
-.00000-.15723272E 02    A 22
 CONTROL FACTOR
 1÷
-1 ÷
 1
-2
24
3.72691-4.97273          A 11
-.96520-1.58785          A 12
 2
-2
28
-.95999-1.58992          A 21
 .54834-2.63290          A 22
 CONTROL FACTOR
 1÷
-1 ÷
 1
-2
20
3.39314-5.11480          A 11
-1.19052-2.07301         A 12
 2
-2
23
-1.17787-2.07069         A 21
 .79724-3.35950          A 22
```

Condition 1.
Fault On--breakers closed

Condition 2.
Fault On--breakers open

Condition 3.
Fault Off--line 2--3 reclosed

NGEN	DT	FCT	TURC
002	.10	.10	.70*

TIME (SEC)	SWING (DEGREES)	MACHINE
.100	.0000	1
.100	-3.6095	2
NGEN DT FCT TURC		
002 .20 .10 .70*		
.300	.0000	1
.300	22.3297	2
.500	.0000	1
.500	52.6761	2
.700	.0000	1
.700	80.6917	2
.900	.0000	1
.900	101.3077	2
1.100	.0000	1
1.100	114.8856	2
1.300	.0000	1
1.300	129.1520	2
1.500	.0000	1
1.500	154.1666	2
1.700	.0000	1
1.700	209.6471	2
1.900	.0000	1
1.900	324.8032	2
2.100	.0000	1
2.100	469.0837	2
2.300	.0000	1
2.300	622.5866	2
2.500	.0000	1
2.500	815.8204	2

NGEN	DT	FCT	~~TURC~~
002	.10	.10	.60*

TIME (SEC)	SWING (DEGREES)	MACHINE
.100	.0000	1
.100	-3.6095	2
.200	.0000	1
.200	8.1351	2
.300	.0000	1
.300	22.3819	2
.400	.0000	1
.400	37.6442	2
.500	.0000	1
.500	52.7775	2
.600	.0000	1
.600	67.1838	2
.700	.0000	1
.700	79.1258	2
NGEN DT FCT TURC		
002 .20 .10 .60*		
.900	.0000	1
.900	92.2801	2
1.100	.0000	1
1.100	94.1688	2
1.300	.0000	1
1.300	85.5117	2
1.500	.0000	1
1.500	63.4045	2
1.700	.0000	1
1.700	25.2729	2
1.900	.0000	1
1.900	-19.0955	2
2.100	.0000	1
2.100	-44.1345	2
2.300	.0000	1
2.300	-32.4629	2
2.500	.0000	1
2.500	7.4443	2
2.700	.0000	1
2.700	49.8516	2
2.900	.0000	1
2.900	77.7842	2
3.100	.0000	1
3.100	90.5115	2
3.300	.0000	1
3.300	91.2905	2
3.500	.0000	1
3.500	80.3919	2
3.700	.0000	1
3.700	54.7783	2
3.900	.0000	1
3.900	13.8772	2
4.100	.0000	1
4.100	-27.7707	2
4.300	.0000	1
4.300	-44.1759	2
4.500	.0000	1
4.500	-23.8772	2

*Machine 1 is the infinite generator and machine 2 is the finite generator.

APPENDIX III Output of Program 17T03

```
4906950
NGEN  DT   FCT   TURC
2  .1   .1   .3*
EX-TC GOV-TC DROOP
999. 999. 999.*
-10.  2.5   .5*
TIME          SWING    MACHINE
(SEC)        (DEGREES)
 .100           .0000      1
 .100          7.9731      2
 .100           .0000      3
 .100          1.1406      4
 .100          1.1687      5
 .100          1.0509      6
 .100          6.8000      7
 .100           .8537      8
NGEN   DT   FCT   TURC
2  .2   .1   .3*
 .300           .0000      1
 .300         14.2605      2
 .300           .0000      3
 .300           .8175      4
 .300          1.1687      5
 .300          1.0528      6
 .300          6.8000      7
 .300          1.4121      8
 .500           .0000      1
 .500         15.3666      2
 .500           .0000      3
 .500           .5133      4
 .500          1.1687      5
 .500          1.0551      6
 .500          6.8000      7
 .500          2.0051      8
 .700           .0000      1
 .700         15.9799      2
 .700           .0000      3
 .700           .3183      4
 .700          1.1687      5
 .700          1.0573      6
 .700          6.8000      7
 .700          2.3866      8
 .900           .0000      1
 .900         16.2872      2
 .900           .0000      3
 .900           .1941      4
 .900          1.1687      5
 .900          1.0597      6
 .900          6.8000      7
 .900          2.6306      8
 .11000000E 01              .0000      1
 .11000000E 01            16.4046      2
 .11000000E 01              .0000      3
 .11000000E 01              .1155      4
 .11000000E 01             1.1687      5
 .11000000E 01             1.0620      6
```

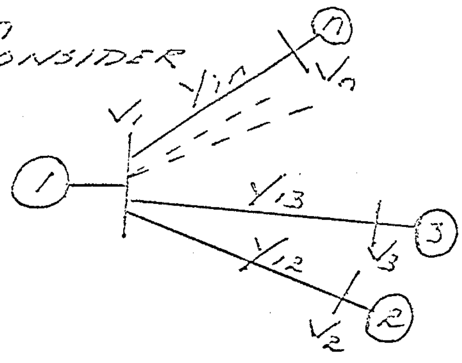
NOTE

- 1 & 2 denote: machine angles (degrees)
- 3 & 4 denote: per unit speed error
- 5 & 6 denote: voltage behind transient reactance
- 7 & 8 denote: P_O--output power

Appendix IV

The power equation (3.8) for an 'n' machine system is developed below.

Consider



note
P_1 = power machine (1) supplies to the system
Y_{in} = admittance between i & n
V_n = voltage @ Machine (behind x_d')

$$P_1 = \mathcal{R}e\,(\bar{V}_1^* \bar{I}_1) = \mathcal{R}e\, V_1 (V_2 - V_1) Y_{12} + \mathcal{R}e\, V_1 (V_3 - V_1) Y_{13} + \cdots\cdots + \mathcal{R}e\, V_1 (V_n - V_1) Y_{1n} \quad \ldots\ldots (1)$$

Expanding equation (1)

$$P_1 = \mathcal{R}e(V_1\cos\theta_1 - jV_1\sin\theta_1)(V_2\cos\theta_2 + jV_2\sin\theta_2 - V_1\cos\theta_1 - V_1 j\sin\theta_1)(-G_{12} + jB_{12})$$
$$+ \mathcal{R}e(V_1\cos\theta_1 - jV_1\sin\theta_1)(V_3\cos\theta_3 + jV_3\sin\theta_3 - V_1\cos\theta_1 - V_1 j\sin\theta_1)(-G_{13} + jB_{13})$$
$$+ \cdots\cdots$$
$$+ \mathcal{R}e(V_1\cos\theta_1 - jV_1\sin\theta_1)(V_n\cos\theta_n + jV_n\sin\theta_n - V_1\cos\theta_1 - V_1 j\sin\theta_1)(-G_{1n} + jB_{1n})$$

$$P_1 = -G_{12}[(V_1\cos\theta_1 (V_2\cos\theta_2 - V_1\cos\theta_1) - V_1\sin\theta_1(V_2\sin\theta_2 - V_1\sin\theta_1)]$$
$$+ B_{12}[(V_1\cos\theta_1(-V_2\sin\theta_2 + V_1\sin\theta_2) + V_1\sin\theta_1(V_2\cos\theta_2 - V_1\cos\theta_1)]$$
$$+ \cdots\cdots$$
$$+ -G_{1n}(V_1\cos\theta_1(V_n\cos\theta_n - V_1\cos\theta_1) - V_1\sin\theta_1(V_n\sin\theta_n - V_1\sin\theta_1))$$
$$+ B_{1n}(V_1\cos\theta_1(-V_n\sin\theta_n + V_1\sin\theta_n) + V_1\sin\theta_1(V_n\cos\theta_n - V_1\cos\theta_n))$$

Combining & cancelling

$$P_1 = -G_{12}(V_1\cos\theta_1 V_2\cos\theta_2 - V_1^2\cos^2\theta_1 + V_1\sin\theta_1 V_2\sin\theta_2$$
$$-V_1^2\sin^2\theta_1) + B_{12}(-V_1V_2\cos\theta_1\sin\theta_2 + V_1^2\sin\theta_1\cos\theta_1$$
$$+V_2V_1\sin\theta_1\cos\theta_2 - V_1^2\sin\theta_1\cos\theta_2) + \ldots\ldots$$
$$-G_{1n}(V_1\cos\theta_1 V_n\cos\theta_n - V_1^2\cos^2\theta_n + V_1\sin\theta_1 V_n\sin\theta_n$$
$$-V_1^2\sin^2\theta_1) + B_{1n}(-V_1V_n\cos\theta_1\sin\theta_n + V_1^2\sin\theta\cos\theta_1$$
$$+V_nV_1\sin\theta_1\cos\theta_n - V_1^2\sin\theta_1\cos\theta)$$

$$P_1 = -G_{12}(-V_1^2 + V_1V_2(\cos(\theta_1-\theta_2)))$$
$$+B_{12}(V_2V_1(\sin\theta_1\cos\theta_2 - \cos\theta_1\sin\theta_2)$$
$$+\ldots\ldots$$
$$+ -G_{1n}(-V_1^2 + V_1V_n(\cos(\theta_1-\theta_n)))$$
$$+B_{1n}(V_nV_1(\sin\theta_1\cos\theta_n - \cos\theta_1\sin\theta_n)$$

$$P_1 = G_{12}(V_1^2 - V_1V_2(\cos(\theta_1-\theta_2)) + B_{12}(V_2V_1\sin(\theta_1-\theta_2))$$
$$+\ldots\ldots$$
$$+G_{1n}(V_1^2 - V_nV_2(\cos(\theta_1-\theta_n) + B_{1n}(V_nV_1\sin(\theta_1-\theta_n)$$

$$P_1 = -G_{12}V_1^2 - G_{13}V_1^2 - \ldots - G_{1n}V_1^2$$
$$+V_1V_2\cos(\theta_1-\theta_2)G_{12} + V_1V_3\cos(\theta_1-\theta_3)G_{13} + \ldots V_1V_n\cos(\theta_1-\theta_n)G_{1n}$$
$$+V_1V_2\sin(\theta_1-\theta_2)B_{12} + \ldots\ldots V_1V_n\sin(\theta_1-\theta_n)B_{1n}$$

Thus for 'n' machines the power out

$$P_1 = -\sum_1^n V_1^2 G_{1n} + V_1\sum_1^n V_n(\cos(\theta_1-\theta_n)\cdot G_{1n} + \sin(\theta_1-\theta_n)B_{1n})$$

Substituting 'i' for '1' a more general equation may be written

$$P_i = -\sum_1^n V_1^2 G_{1n} + V_i \sum_1^n V_n[\cos(\theta_i - \theta_n) G_{in} + \sin(\theta_i - \theta_n) B_{in}]$$

& note that for $n = i$ $P_{i=j} = 0$ i.e. $j = n$

$$\therefore P_i = V_i \sum_{j=1}^n V_n (\cos(\theta_i - \theta_j) G_{ij} + \sin(\theta_i - \theta_j) B_{ij}) \Big/_{n \neq i}$$

WHERE

V_i & V_n = voltage behind the transient reactance.
θ_i & θ_j = power angle (degrees)
G_{in} = Real part of admittance
B_{in} = Imaginery part of admittance.