THE UNIVERSITY OF MANITOBA

ANALYSIS OF SHEAR WALL-FRAME STRUCTURES SUBJECTED TO LATERAL LOADS

BY

G. VISWANATHAN

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES

IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF SCIENCE IN CIVIL ENGINEERING

WINNIPEG, MANITOBA

OCTOBER 1972

ACKNOWLEDGEMENTS

The author gratefully acknowledges the financial support of the National Research Council of Canada.

The author sincerely expresses his thanks to his adviser Dr. G. A. Morris for his excellent guidance and assistance rendered during the preparation of this thesis.

Also, the author greatly appreciates the help given by Prof. R. B. Pinkney. His criticism and guidance in the computer program were invaluable.

Finally, thanks are extended to Mr. R. M. Godse, Chief Structural Engineer with a Firm of Consulting Engineers in Winnipeg, for reading the thesis and his valuable criticism and to the Computer Centre, University of Manitoba for allotting computer time for the preparation of the computer program.

TABLE OF CONTENTS

CHAPTER I

INTRODUCTION

1.1 INTRODUCTION

In recent years, construction of tall buildings has increased enormously. The trend in modern tall buildings is to have rigidly connected frames, providing spacious interior areas, and stiff interior shear walls for the elevator core and stair wells. In addition, shear walls are sometimes provided on shorter exterior faces of buildings. These walls, in conjunction with frames, provide additional stiffness to resist lateral movements caused by wind or earthquakes, hence, serving structural and non-structural functions. The increasing number of shear wall-frame structures has produced a need for a better knowledge of the interaction forces between the wall and the frame in order to permit more economical designs.

The shear wall is usually considered to be the principal lateral load-resisting element in tall shear wall-frame structures. At one time it was common practice to consider the shear wall to be a vertical cantilever beam resisting all of the lateral load. This resulted in very uneconomical designs. It has since been recognized that the frame substantially enhances the lateral stiffness in two ways. Firstly, the beams framing into the shear wall provide resisting moments which tend to reduce the shear wall deflection. Secondly, toward the top of the structure the frame rather than the shear wall tends to resist the major portion of the lateral shear. If the shear wall alone resisted the lateral loads, it would deflect as a free cantilever as shown in Fig. 1.1(b), while if the frame alone were to resist the lateral load, it would deflect in a shear mode as shown in Fig. 1.1(c). Since the shear wall and frame are interconnected at each floor level, interaction forces are developed between them. Thus, the combined action results in the deflected shape shown in Fig. 1.1(d).

1.2 REVIEW OF THE PREVIOUS WORK

Many investigations have been carried out on shear wall-frame structures in the elastic range. They can be classified as frame analogy methods and finite element methods. Work carried out on these two methods is briefly reviewed here.

1.2(i) FRAME ANALOGY METHODS

In this method the wall is idealized as a series of bending elements which approximate the structural characteristics of the wall. Fig. 1.2 shows a typical frame analogy for a structure.

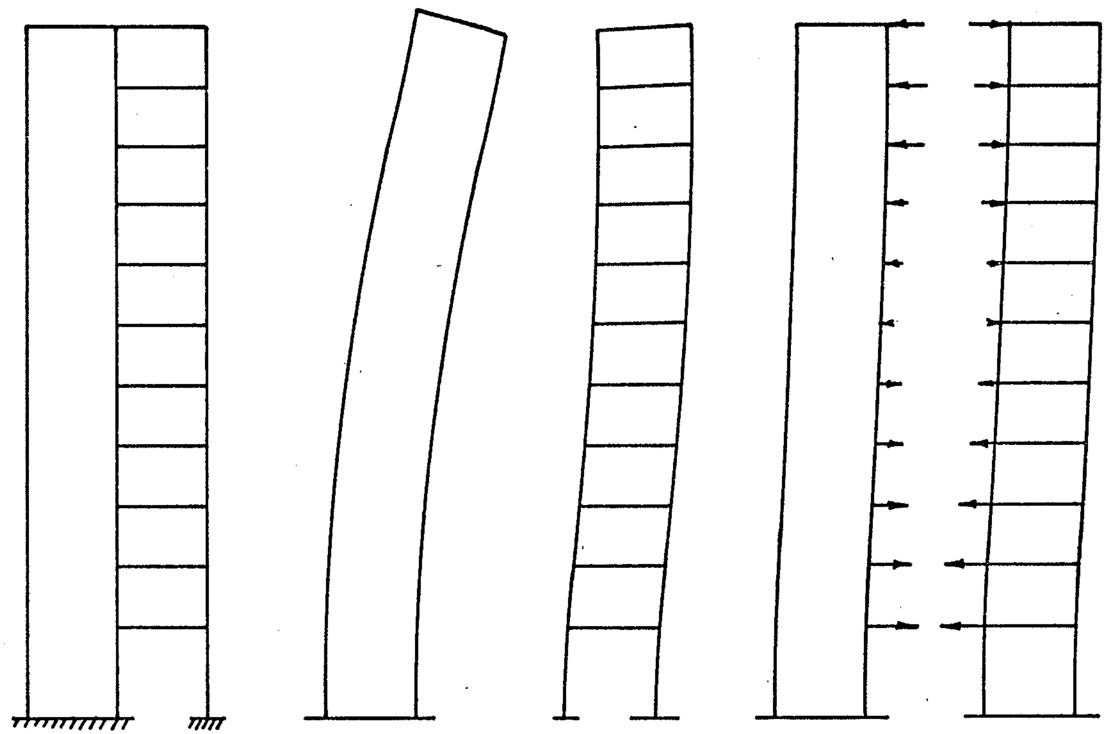


FIG. 1.1 DEFLECTED SHAPE OF STRUCTURE

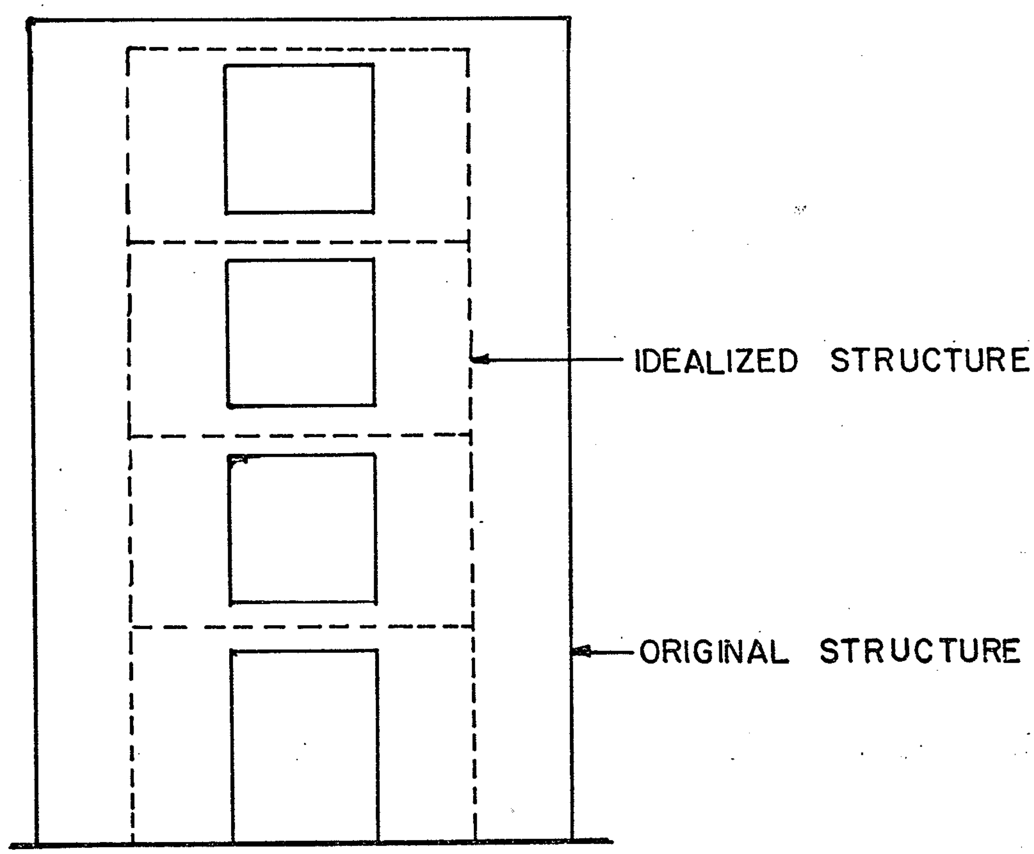


FIG. 1.2 FRAME ANALOGY

Rosenblueth and Holtz (1) presented an approximate method of analysis for a rigidly jointed frame which includes a single shear wall. They assumed that the drift of a frame in any storey was proportional to the shear acting on the given frame in that storey and the moments and shears in girders supported by a shear wall were proportional to the flexural slopes of the deflected wall. In their study the wall was idealized as a beam on an elastic foundation, in which the reactions were proportional to the wall rotations rather than to its displacements. The problem could then be treated following the procedure of successive approximations described by Newmark (21) for beams on elastic foundations. Two methods were given for a suitable first configuration which could then be improved by successive approximations. Column shortening and foundation deformations were neglected in the analysis.

Cardon (2) developed expressions for the lateral stiffness of various types of beam and column arrangements. He expressed the angular deflection at all points with a second degree differential equation, taking into account the effect of bending and shear. The main assumptions in this approach were that the properties of the wall were constant throughout and that the forces acting on the wall were continuously distributed over the height of the building.

Gould (3) replaced the frame with rotational and translational springs, linked to the centre line of the shear wall with rigid bars concentrated at frame levels. He used the finite difference technique in solving the problem. The interaction moments at each storey level were included in the beam equation by replacing them by statically equivalent equal and opposite forces at floors above and below. The assumptions were the same as those in Cardon's (2) paper.

Shear wall-frame structures have also been analysed by replacing the beams with a continuous lamina of equivalent stiffness.

Hubert Beck (4) proceeded on this basis and presented an approximate method which furnished simple formulae for the determination of statically redundant values. All redundant values were combined to form only one single unknown function. Hence, only one differential equation had to be solved in calculating the unknown function, instead of having a system of linear functions. The assumptions made were that all connecting beams had the same distance from each other and that the stiffness of all beams, except the end one, were the same. The end beam had one-half the cross section and one-half the moment of inertia of a normal connecting beam. The wind load was assumed distributed uniformly throughout the height of the building.

Bandel (5) introduced an energy method in which he replaced the shear wall by an equivalent truss, as shown in Fig. 1.3. He used a power series to represent the applied

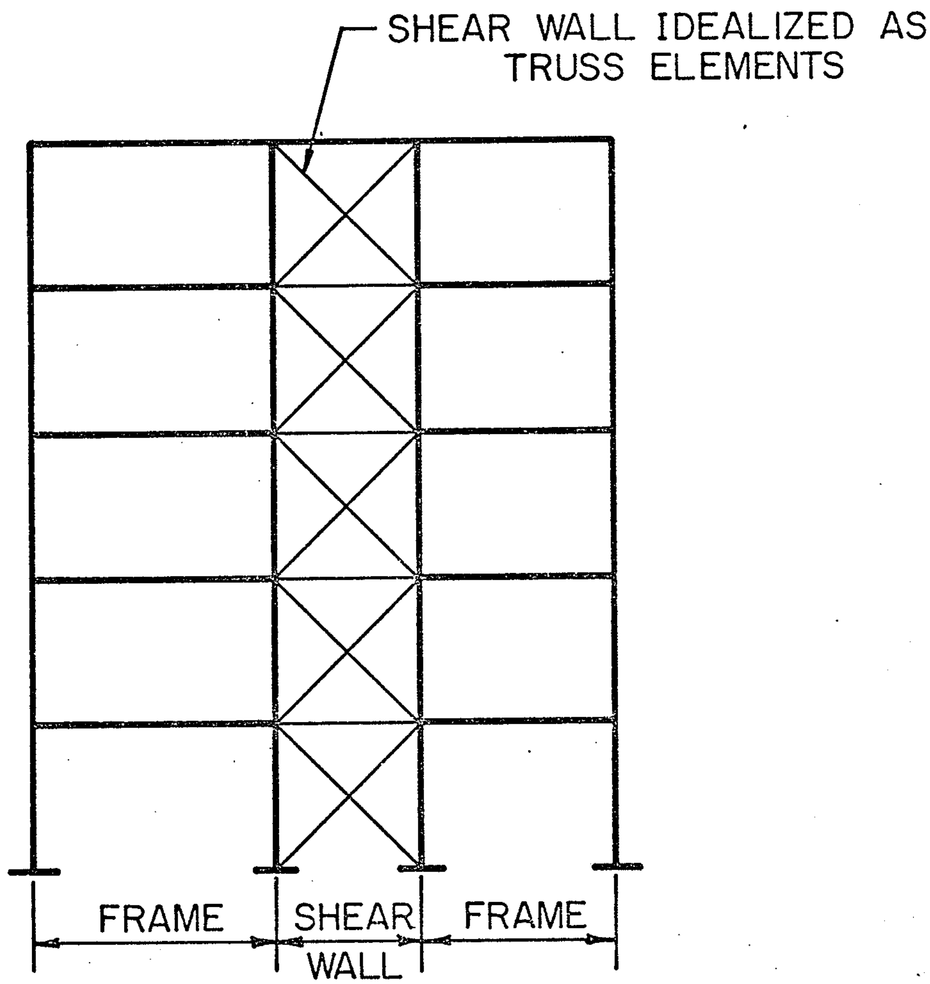


FIG. 1.3 BANDEL'S MODEL

loading and the minimization of potential energy yielded a set of linear simultaneous equations. The axial deformations were neglected and only rigid foundations were considered.

Riko Rosman (6) used the same continuous system method as that used by Hubert Beck. The integral shear forces in the continuous connections of individual piers were chosen as the redundant functions. He dealt with only a single concentrated lateral load at the top of the wall. The assumptions were exactly the same as Bandel's as far as the properties of the beams and the stiffnesses were concerned.

A. L. Parme (9) published a paper on shear wall-frame structures where the procedure consisted of relating the total load at each floor level to the displacement of that floor and the two floors above and below. By writing an equation in terms of the relative stiffnesses of the different elements at any level, a set of simultaneous equations was obtained which could then be solved easily. The assumption was made that the lateral displacement at each level was the same for the shear wall and all columns. The axial deformations of the columns and the elongation and contraction of the outer fibres of the shear wall were neglected.

An iterative analysis procedure was presented by Khan and Sbarounis (8). The shear wall-frame structure was analysed as an analytical model consisting of a shear wall system and a frame system. An initial deflection of the shear wall alone was computed either directly or with a set of curves that was presented. By forcing the link beams between the frame and shear wall system to be constrained to follow the distorted form of the wall, as shown in Fig. 1.4 the bending moments throughout the frame could be determined, either by moment distribution or by an iterative, modified slope-deflection procedure. Hence the interaction forces on the wall for over all equilibrium at each floor level were obtained. An iteration technique employing initial and final deflections and rotations at the end of any cycle in conjunction with free wall deflections, as a starting point for the next cycle of iteration, was used to determine the final mode of the deformation of the combined system. Consideration was given to the effects of foundation deformations, local yielding of the wall, axial and shear deformations, torsion of the frame, shear walls which do not run the full height of the building, and to the problem of the determination of the effective widths of floor slabs which can be taken to act as beams. For design purposes, a set of influence curves was given, which showed the distribution of storey shears between shear wall and frame members for a range of wall-column and column-beam stiffness ratios and for several different forms of applied lateral loading.

Clough, King and Wilson (7) developed a computer

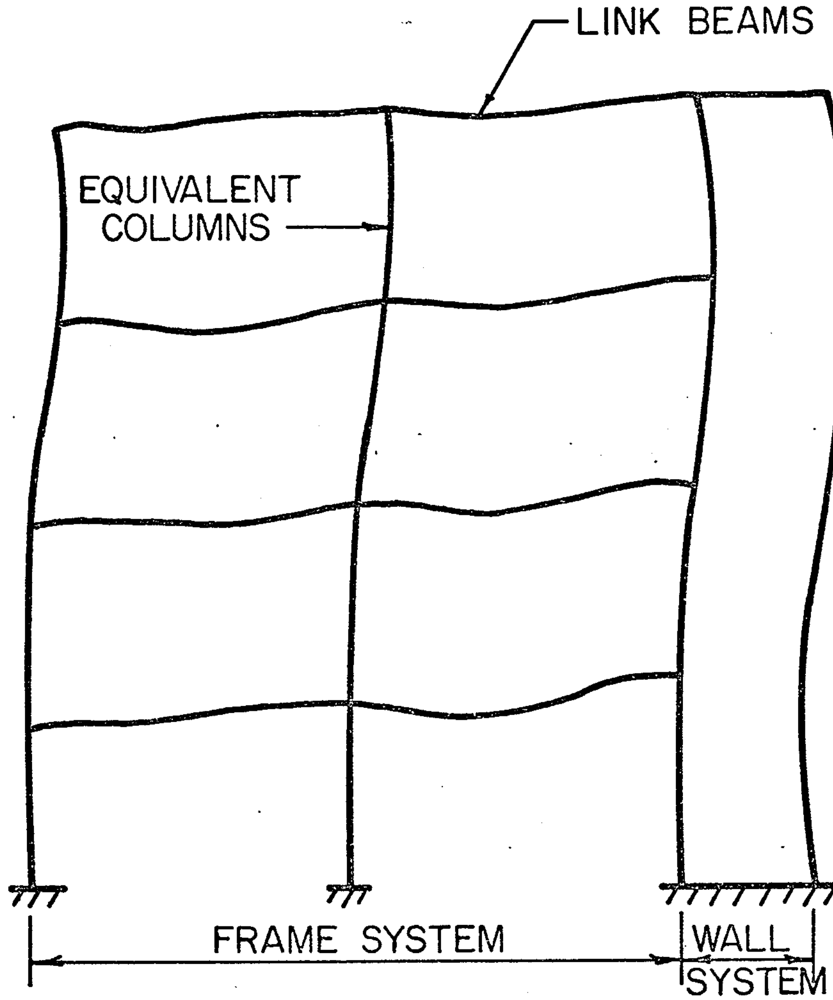


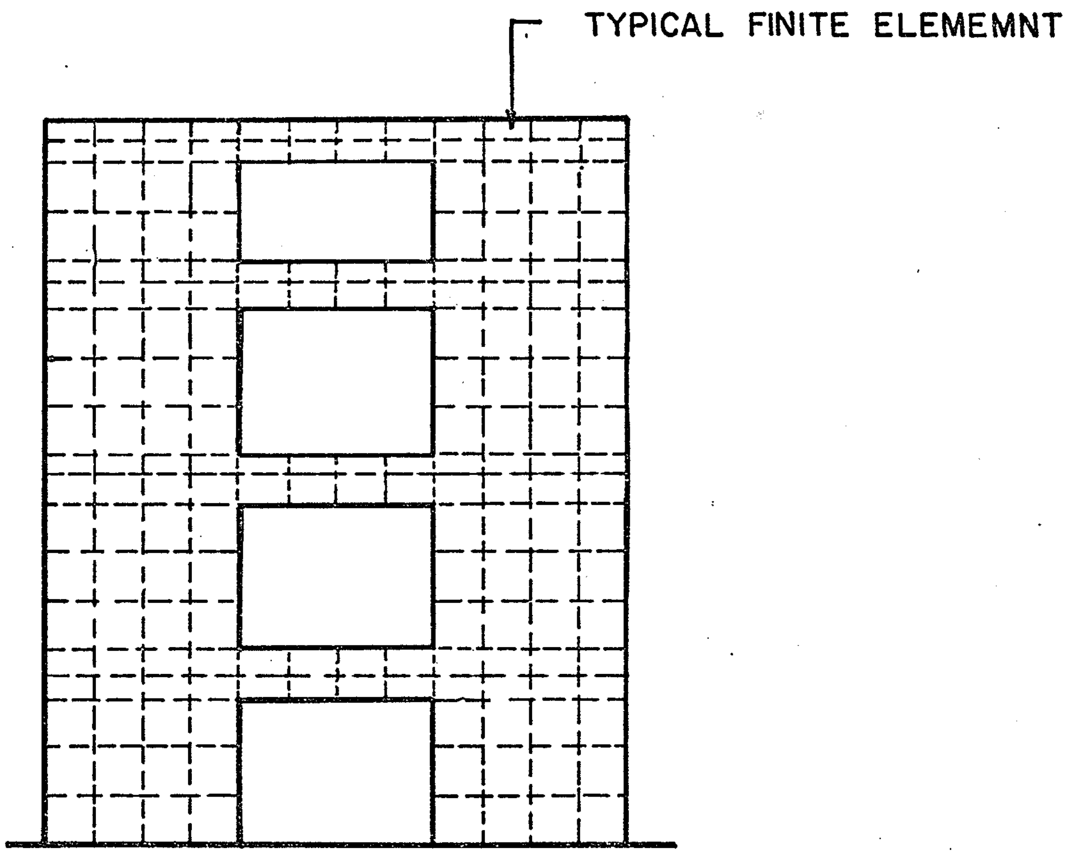
FIG. 1.4 KHAN'S MODEL

program for shear walls acting in conjunction with a frame. The stiffness method employed included flexural, shear and axial distortions of the members. However, the floor slab was considered to be rigid in its own plane, so that axial deformations of beams were neglected. The assumption was made that the building was laid out in a regular grid pattern with each floor level constrained to translate but not to rotate under the action of lateral loads. It was furthur assumed that the shear walls were of uniform width throughout the entire height -although variations in stiffness were allowed.

1.2(ii) FINITE ELEMENT METHODS

In these methods the wall is idealized as a system of elements, the behavior of which is similar to that of the real, continuous structure. The structure is then analysed by evaluating the properties of these interaconnecting elements. Fig. 1.5 shows a typical shear wall structure divided into finite elements.

C. V. Girijavallabhan (14) dealt with a shear wall with openings, by finite element method. He discussed the stress distribution in the shear wall with openings and predicted more accurate values for the bending moments, shear and axial forces which act upon the lintel beams when the shear walls are subjected to lateral loads. He also discussed the effects of changes in material properties such as Poisson's



DOTTED LINE INDICATES
THE STRUCTURE DIVIDED INTO
FINITE ELEMENTS

FIG. 1.5 FINITE ELEMENT IDEALIZATION

ratio upon the stresses in shear walls and bending moments and axial forces in the connecting lintel beams. He developed a program which employed the conventional procedures of matrix structural analysis. He observed that the stress distribution in the medium depends upon: (1) external load distribution, (2) relative stiffnesses of lintel beams and shear wall, and (3) the value of Poisson's ratio of the material used.

The linearly elastic plane stress analysis of a shear wall may be carried out using finite element idealizations. However, when the connecting beams are slender, existing types of elements are not suitable since they can not be connected with line elements in bending. I. A. Macleod (15) developed a new rectangular element which has a rotational degree of freedom at each node. This element overcomes the difficulty of combining line elements in bending with plane stress elements. He discussed the types of displacement functions used and the derivation of the element stiffness matrix. He developed a program which can include these new elements together with line elements in bending. The program is written in Algol and uses the direct stiffness method of solution. Some guidance in the proportioning of finite elements for shear wall analysis was given and comparison of finite element solutions with a frame solution was made for a wall with two different patterns of openings. This technique shows promise of being most useful in the analysis of shear walls with openings.

Recently, R. G. Oakberg and William Weaver (13) analysed a shear wall-frame structure by finite element technique. The finite element model included rectangular openings in the shear wall, and edge pilasters. They employed the method of substructures in the matrix stiffness analysis. The method took account of the effects of shearing and local distortions. The authors developed special elements which were used to combine with elements in bending. A program was written in Algol and the method of substructures allowed the solution of a fairly large problem in a reasonable amount of computer time. Using the substructure analysis, only the displacements at the connection points and the corresponding actions were retained and the lateral loads were applied only at the floor levels. The base was assumed to be fixed. The results were compared with those obtained using the deep column method and the discrepancies in the calculated rotations at the connection points were considerable. For larger shear wall height to width ratios, the discrepancies between the two sets of rotations were reduced. Therefore, this method would be suitable for lower ratios of height to width and the deep column method would be more economical for higher ratios.

1.3 OBJECTIVE OF STUDY

Computer methods are indispensable for the analysis of large structures, particularly in dealing with the finite element technique and most of the methods, as discussed in the previous section, lean heavily on a digital computer. The choice of a method depends on whether the most accurate method is sought regardless of the arithmatic labour involved or whether a practical method with a minimum amount of calculation or input data is desired.

In many instances, a series of openings are provided either for architectural or environmental reasons, thereby reducing the efficiency of the shear wall and altering its deformation characteristics. Other factors affecting the shear wall stiffness are thickness, material properties, height to width ratio, stiffness of beams framing into shear wall, width of openings and position of openings relative to the edges of the wall.

A completely rigorous analysis of a shear wall-frame structure generally requires extensive computation. In view of the approximate nature of the design wind, earthquake or blast loading assumed in the analysis and because the material properties can usually be estimated only approximately it is often felt that a completely rigorous solution is not warranted.

Several authors (11,13,14,15) have indicated that the finite element method gives better results than other methods. Unfortunately this method often gives rise to storage problems in the computer and becomes very uneconomical for large structures. However, if certain kinematic assumptions and other approximations are used, this problem can be overcome. It will be shown here that fairly large structures can be treated on finite element idealizations making the best use of the available computer storage.

Hence, the object of this dissertation is to develop a method of analysis of shear wall-frame structures subjected to lateral loads with an ultimate aim of developing a fully automatic solution technique with a minimum computational effort. Shear walls with or without openings are considered and the study is limited to linear elastic analysis only.

The analysis presented herein uses the matrix stiffness method. While treating the wall on finite element idealizations, a special kind of element with a rotational degree of freedom at each node is required to combine the shear wall stiffness with those of beams framing into shear wall. Such an element is given by R. G. Oakberg and William Weaver (13) and also by I.A. Macleod (15). The element presented by the former authors has been used in this analysis. The matrix reduction process has been intensively used here, utilizing the fact that the loads are applied only at the framing levels or at joints. A systematic procedure is adopted in the analysis and finally, a computer program is presented wherein the input consists of the joint coordinates, material properties, coordinates of the element divisions in the shear wall, beam and column properties and other general information.

Certain kinematic assumptions are made which will simplify the problem without much affecting the accuracy of the results. However, due consideration is given to the following points.

(i) When the thickness of the shear wall of the lower level differs from that at the top in a real shear wall problem, the element stiffness properties at the lower levels are changed linearly in relation to the thickness.

(ii) If the value of the elastic modulus is different at different levels it is possible to easily incorporate the different moduli into the computer program.

(iii) The stiffness matrix being symmetrical, only one-half of the banded matrix is stored in order to save storage space in the computer.

(iv) As much advantage as possible is taken from symmetry, and identity of substructures in particular, in generating elements, nodal coordinates and the stiffness matrices.

1.3 ASSUMPTIONS AND LIMITATIONS

The assumptions made in the analysis are briefly outlined below. Certain kinematic assumptions reduce the number of generalized coordinates and in a few cases reduce the band width of the stiffness matrix. Wherever necessary these assumptions are explained in detail with mathematical derivations in subsequent chapters.

(i) The shear wall is assumed to run the full height of the building and the floor slab at each framing level divides the wall into segments. Hence, the number of segments in a shear wall is equal to the number of storeys. The middle surface of the slab is assumed to coincide exactly with the framing level.

(ii) Each floor slab is assumed to be infinitely rigid in its own plane. Hence, there is only one transverse degree of freedom at each floor level. Consequently, the axial deformations of the beams are neglected.

(iii) It is assumed that plane transverse sections through the shear wall at all floor levels remain plane.

(iv) The force-displacement relationship at any level of a shear wall is related to those at adjacent levels only.

(v) The shear wall and the columns of finite elements comprising it are of uniform width throughout the height of the structure. However, the thickness can be varied.

(vi) The frame has the same number of bays at each framing level.

(vii) The structure is assumed to have either fixed or pinned bases and all connections are assumed to be rigid.

(viii) External forces are assumed to be applied only at the corner nodes of a shear wall segment, or at joints.

(ix) The coordinate system used in the analysis is shown in Fig. 1.6.

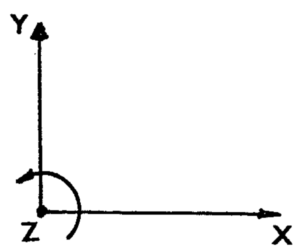


FIG. 1.6 POSITIVE COORDINATE DIRECTIONS

The directions shown are assumed positive for both forces and displacements.

(x) The structure is assumed to be linearly elastic.

(xi) Only rectangular panels are considered and no inclined members can be treated in the program.

(xii) Only rectangular finite elements are considered in the shear wall analysis.

CHAPTER II

GENERAL ANALYSIS OF STRUCTURE

2.1 INTRODUCTION

The use of "Numerical Methods" is inevitable in analysing complex structures which require reasonably accurate results in a short time. The numerical methods can be divided into two types, (i) numerical solutions of differential equations for displacements or stresses and (ii) matrix methods based on discrete-element idealizations. In the former case, for any particular structural configuration, the equations of elasticity are solved either by finite difference technique or by direct numerical integration. In this approach the analysis is based on mathematical approximation of differential equations and applications of these methods are restricted, due to practical limitations, to simple structures. In the second case, however, the structure is first idealized into an assembly of discrete structural elements with assumed forms of displacement or stress distribution and then the complete solution is obtained by combination of these distributions in a manner which satisfies equilibrium and compatibility at different joints. Methods based on this approach have proven to be most suitable for the analysis of complex structures.

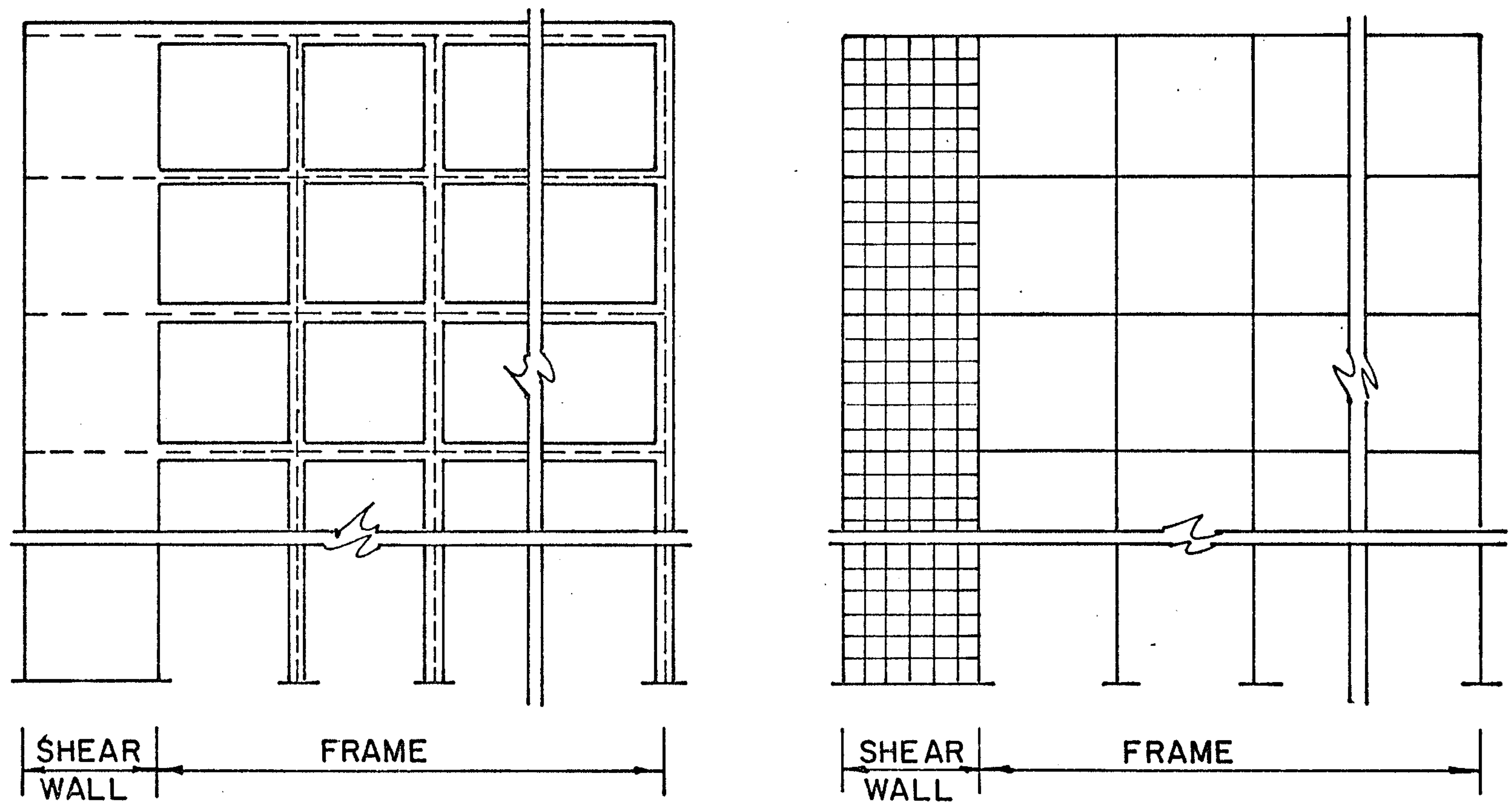
The two possible approaches in the matrix analysis are (i) the stiffness method (or displacement method) and (ii) the flexibility method (or force method). In both cases the conditions of equilibrium and compatibility are satisfied. The flexibility method involves fewer equations to be solved than stiffness method. For large structures, however, the difference is insignificant. The main advantage in stiffness method is its systematic approach which is well suited to programming. The flexibility method requires the exercising of judgement in the selection of suitable "redundant" force components and this choice has a significant bearing on the accuracy of the results obtained from a computer analysis. Hence, the flexibilty method is best suited to hand calculations. Present day matrix structural analysis using the computer is based mainly on the stiffness method and this method has been adopted in this study.

2.2 ANALYTICAL MODEL (STRUCTURAL IDEALIZATION)

The first step in matrix structural analysis is the formulation of a discrete element mathematical model which is equivalent to the actual continuous structure. The model is necessary since it establishes the finite number of degrees of freedom upon which matrix algebra operations can be performed. This is accomplished by equating energies of the continuous and discrete element systems. A typical analytical model for a shear wall frame structure is shown in Fig. 2.1. The model essentially consists of rectangular panels with shear walls and columns connected by means of beams. The shear walls may have openings in them or there may be shear walls connected by lintel beams only.

The frame portion of the structure is composed of columns and beams which present no difficulty in the formulation of their discrete models. With this discrete system, the energy equivalence leads to exact representation of the frame system. However, for the shear wall it is necessary to use approximations. The shear wall is subdivided into a number of smaller elements with fictitious boundaries and with assumed displacement distributions within the elements. As the number of elements is increased the solutions for the structural displacements and stress resultants should tend to the exact values for the continuous system. The nodal points are considered to be the segment corners, intersections of the centre lines of beams and columns and intersections of the centre lines of beams and edges of shear walls. The shear wall is assumed to be divided into segments with boundaries at the various levels. This is shown by the dotted lines in Fig. 2.1(a).

Although, the beam depths may vary and centre lines of adjacent beams and the centre lines of floor slab at any level may not coincide, this discripancy is considerd to be



(a) GENERAL STRUCTURE

(b) IDEALIZED STRUCTURE

FIG· 2·1 ANALYTICAL MODEL FOR A SHEAR WALL - FRAME STRUCTURE minor. Fig. 2.1(b) shows the idealized structure.

In the stiffness method it is necessary first to ascertain the degrees of freedom at each joint; i.e. the generalized coordinate system. A sample shear wall frame structure is shown in Fig. 2.2. All degrees of freedom are referred to a global system of axes as represented in Fig. 2.2(b). It is evident that each node has three degrees of freedom. But with some useful assumptions the total number of degrees of freedom for the whole structure can be reduced considerably. Since the floor slabs are assumed to be infinitely rigid in their own planes, a single horizontal degree of freedom at each level can be considered. This means that all nodal points at any particular level undergo the same displacement in the horizontal direction. Each node has a degree of freedom in the vertical direction. If the assumption from classical elastic theory, that plane sections remain plane before and after bending, can be applied to the shear wall, the left hand and right hand node points of a shear wall at any particular level undergo the same rotation. Further, this can be geometrically related to the corresponding vertical displacements of these nodes. Therefore the rotational degree of freedom can be suppressed at these nodes and the stiffness properties accordingly modified. This also leads to the fact that any beam end framing into the shear wall has only a vertical degree of freedom at the junction node. In the subsequent chapters

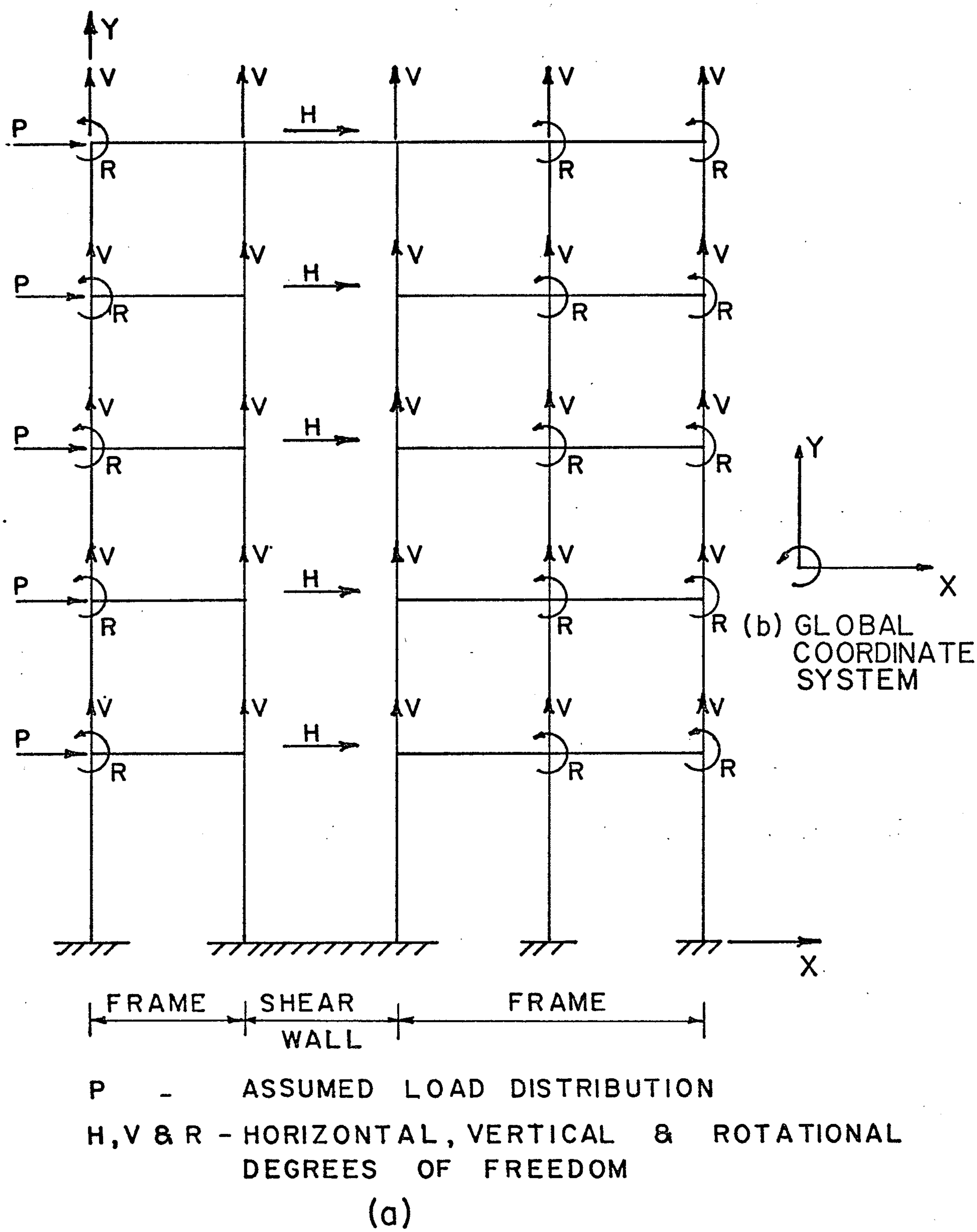


FIG. 2.2 SAMPLE OF SHEAR WALL - FRAME WITH GENERALIZED COORDINATE SYSTEM

these modifications are presented in mathematical terms. All other nodes namely those at the beam column junctions, have a rotational degree of freedom. These are represented in Fig. 2.2 as H, V and R. In the sample frame considered, there are 30 nodes and if there are 3-degrees of freedom at each node, the total number of degrees of freedom is 90. With the assumption described above, the number is reduced to 54. This will be the number of equations to be solved in the stiffness analysis. The lateral loads are assumed to be applied as a series of concentrated loads, P, at the floor levels.

2.3 STIFFNESS METHOD OF ANALYSIS

Having idealized the structure as a system of discrete elements and ascertained the degrees of freedom at the different nodes, the next step in the analysis is to determine the stiffness characterstics of individual structural elements. For this purpose, the structure is subdivided into a shear wall system and a frame system.

In the shear wall system, the shear wall is subdivided into a number of finite elements. The stiffness of each element is computed first and the element stiffness matrices are superimposed to give the overall stiffness matrix for the shear wall. In this process of developing the stiffness matrix for the shear wall, certain internal nodes can be suppressed as will be explained in article 2.4 of this chapter. A further reduction is done, as will be explained in chapter III, to give the shear wall force-displacement relationships relating forces and displacements at the shear wall segment corners only. However, in the frame system, the frame is already divided into discrete elements consisting of beams and columns connected by rigid joints. The stiffness properties of these members are evaluated individually and then they are superimposed at the common joints. Furhur, the stiffnesses at the nodes common to the shear wall and frame are superimposed to give the overall stiffness matrix of the structure.

The functional relationship between the nodal forces P (forces acting at the nodes) and their corresponding displacements D (nodal displacements) forms the basis of the stiffness approach. In its generalized form this can be represented as,

$$\begin{Bmatrix} P_1 \\ P_2 \\ P_3 \\ -- \\ -- \\ P_i \\ -- \\ P_j \end{Bmatrix} = \begin{bmatrix} K_{11} & K_{12} & K_{13} \cdots & K_{1i} \cdots & K_{1j} \\ K_{21} & K_{22} & K_{23} \cdots & K_{2i} \cdots & K_{2j} \\ K_{31} & K_{32} & K_{33} \cdots & K_{3i} \cdots & K_{3j} \\ \cdots & & & & \\ \cdots & & & & \\ K_{i1} & K_{i2} & K_{i3} \cdots & K_{ii} \cdots & K_{ij} \\ \cdots & & & & \\ K_{j1} & K_{j2} & k_{j3} \cdots & K_{ji} \cdots & K_{jj} \end{bmatrix} \begin{Bmatrix} D_1 \\ D_2 \\ D_3 \\ --- \\ --- \\ D_i \\ --- \\ D_j \end{Bmatrix} \qquad (2.1)$$

Or, in symbolic form this can be written as,

$$\{P\}=[K]\{D\} \tag{2.1a}$$

where P is the nodal force vector and D is the nodal displacement vector These are related by the structure stiffness matrix K. P, K and D are expressed in generalized coordinates. The order of [K], as explained earlier,is dictated by the total number of degrees of freedom for the structure. The main diagonal stiffness coefficients K_{ii} are always positive. The subscripts 'i' and 'j' represent generalized coordinates and 'K_{ij}' is the stiffness coefficient which represents the force in generalized coordinate direction i, due to a unit displacement in generalized coordinate direction j, with all other displacements zero. Therefore, the off diagonal elements in the stiffness matrix namely K_{ij} for $i \neq j$, by Maxwell's Reciprocal theorem, are symmetrical ($K_{ij} = K_{ji}$ for $i \neq j$).

Since by definition, the stiffness coefficient is the force developed in generalized coordinate direction i due to a unit displacement in generalized coordinate direction j, the problem of deriving the element stiffness matrix is handled systematically by giving the jth coordinate a unit displacement, holding all other coordinates at zero displacements. The resulting forces at all other coordinates due to this unit displacement form the coefficients of the jth column of the stiffness matrix $\underline{K}$. The formulation of the complete stiffness matrix can be achieved by giving each coordinate a unit displacement (treating one at a time) holding all others zero and evaluating the resulting forces at all coordinates.

The purpose of dividing the structure into discrete elements can now be more clearly understood. Because of the complexity of the structure, the amount of work involved in deriving directly the overall stiffness matrix becomes too involved, if not impossible. Therefore it is necessary to have individual stiffness matrices and then by superposing these matrices, the over all stiffness matrix for the structure can be obtained. As a demonstration the beam in Fig. 2.3, which has two degrees of freedom at each end, is considered. The force-displacement relationship for this beam can be written as,

$$\begin{Bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{Bmatrix} = \begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} \\ K_{21} & K_{22} & K_{23} & K_{24} \\ K_{31} & K_{32} & K_{33} & K_{34} \\ K_{41} & K_{42} & K_{43} & K_{44} \end{bmatrix} \begin{Bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \end{Bmatrix} \tag{2.2}$$

Now considering the two-beam structure assembly Fig. 2.4(a), connected at node 2, it is desired to arrive at the overall structure stiffness matrix for nodes 1,2 and 3.

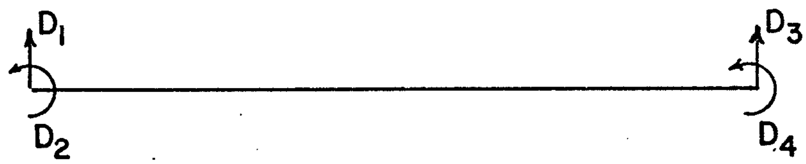
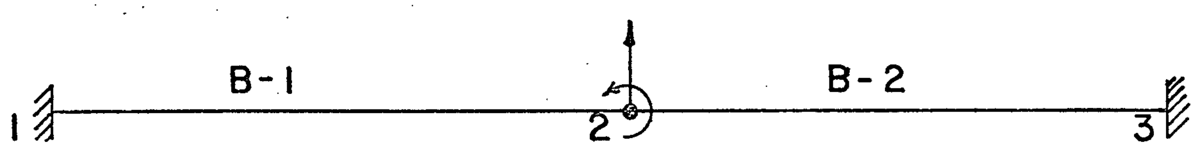
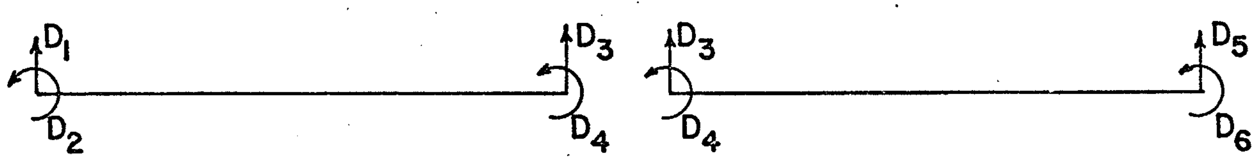


FIG. 2.3 BEAM WITH FOUR DEGREES OF FREEDOM



(a) IDEALIZED TWO BEAM STRUCTURAL ASSEMBLY



(b) DISASSEMBLED STRUCTURE

FIG. 2.4

The equilibrium equations for the two can now be written as ,

$$\begin{Bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{Bmatrix} = \begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} \\ K_{21} & K_{22} & K_{23} & K_{24} \\ K_{31} & K_{32} & K_{33} & K_{34} \\ K_{41} & K_{42} & K_{43} & K_{44} \end{bmatrix} \begin{Bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \end{Bmatrix}, \quad \begin{Bmatrix} P_3 \\ P_4 \\ P_5 \\ P_6 \end{Bmatrix} = \begin{bmatrix} K_{33} & K_{34} & K_{35} & K_{36} \\ K_{43} & K_{44} & K_{45} & K_{46} \\ K_{53} & K_{54} & K_{55} & K_{56} \\ K_{63} & K_{64} & K_{65} & K_{66} \end{bmatrix} \begin{Bmatrix} D_3 \\ D_4 \\ D_5 \\ D_6 \end{Bmatrix}$$

Beam - B1 Beam - B2

Superposition of these two equations yields the structure stiffness matrix,

$$\begin{Bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \end{Bmatrix} = \begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} & K_{15} & K_{16} \\ K_{21} & K_{22} & K_{23} & K_{24} & K_{25} & K_{26} \\ K_{31} & K_{32} & (K^{1}_{33} + K^{2}_{33}) & K_{34} & K_{35} & K_{36} \\ K_{41} & K_{42} & K_{43} & (K^{1}_{44} + K^{2}_{44}) & K_{45} & K_{46} \\ K_{51} & K_{52} & K_{53} & K_{54} & K_{55} & K_{56} \\ K_{61} & K_{62} & K_{63} & K_{64} & K_{65} & K_{66} \end{bmatrix} \begin{Bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \end{Bmatrix} \tag{2.3}$$

Because of the common node for the two coordinates, the order of the matrix is only 6X6. This structure stiffness can be symbolically put as 2.1(a)

$$\{P\} = [K]\{D\}$$

where K is a square matrix. By introduction of joint constraints(support conditions), the stiffness matrix can be made non-singular. Then, by confirming to the law of matrices, the joint displacements can be found by solving Eq. (2.1a) i.e. $$\{D\} = [K]^{-1}\{P\} \tag{2.4}$$

To satisfy compatibility, all member ends framing into any particular joint undergo the same displacement as does the joint. Therefore, once the displacement vector D has been determined from Eq. 2.4, the member end displacements (which are the same as the corresponding joint displacements) can be incorporated into the individual member force-displacement relationship to calculate the member end forces.

i.e. $$\{P\}^{I} = [K]^{I}\{D\}^{I} \tag{2.5}$$

where I represents any member.

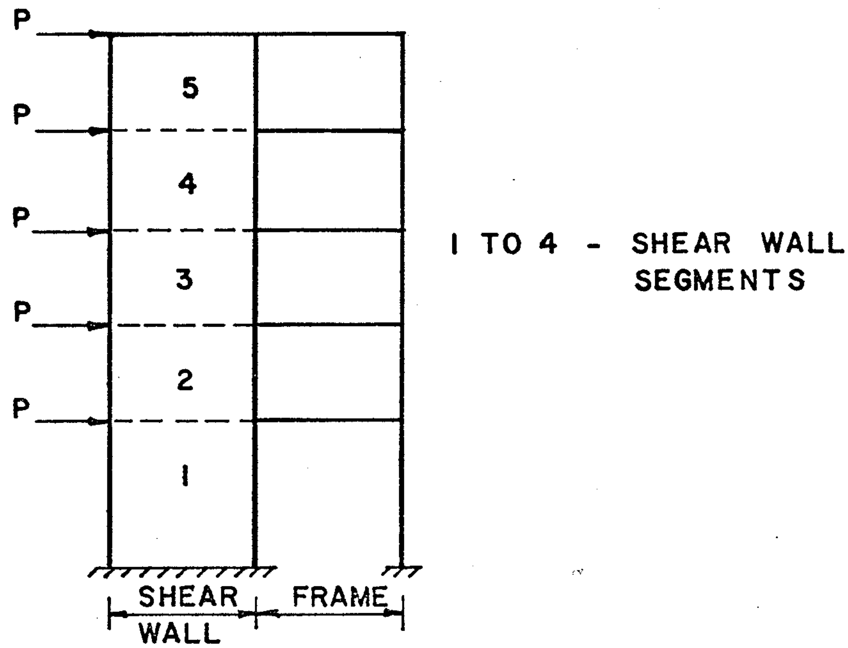
In this study all element stiffness matrices are expressed directly in the global system to avoid transformation of stiffness matrices and force and displacement vectors. This is convenient because the study is limited to structures with rectangular frames and shear walls which can be subdivided into rectangular segments. Three kinds of element stiffness matrices are required for the analysis of this type of structure: (i) finite element stiffness matrix, (ii) beam stiffness matrix and (iii) column stiffness matrix. The finite element stiffness matrix is used for the shear wall and beam and column stiffness matrices are employed for the frame system. They are treated separately in Chapters III and IV respectively.

2.4 CONDENSATION PROCESS

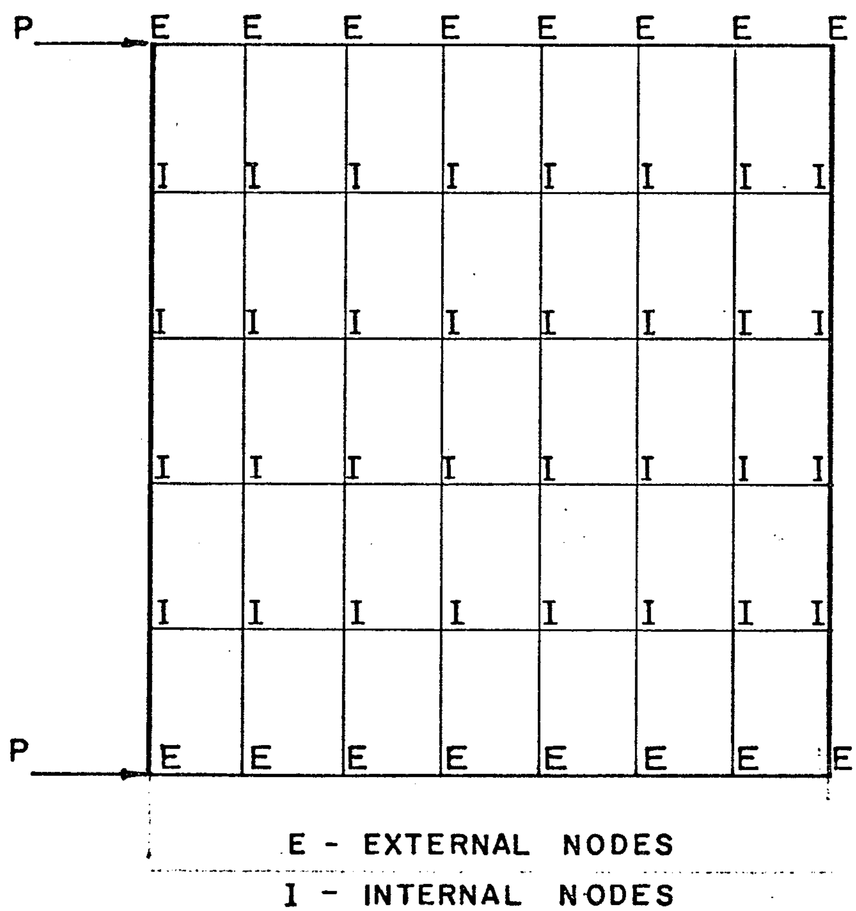
The lateral loads due to wind or earthquake are assumed to act only at the joints or at segment corner nodes. Also, since the shear wall is primarily designed to resist lateral loads it is furthur safe to assume that no external load is ever applied to any interior point of a shear wall. Since the shear wall in the present analysis is treated by finite element idealizations, there is always a problem of storage and additional computational work. The purpose herein is to demonstrate how the storage problem can be avoided making use of the assumption that internal nodes are not loaded externally.

Consider a segment of the shear wall, as shown in Fig. 2.5(b). Assume, for the purpose of analysis that the segment is subdivided into a number of finite elements having 'n' external nodes and 'm' internal nodes. The nodes marked 'E' are external nodes and those marked 'I' are internal nodes. In order that continuity between the segments should not be broken, the nodes on the top and bottom edges of each segment are considered as external nodes and all other nodes, where no external loads are applied, are considered as internal nodes. The equation of equilibrium can now be written as

$$P = [K]\ D$$



(a) SHEAR WALL ASSUMED TO BE LOADED EXTERNALLY AT SEGMENT CORNERS



(b) TYPICAL SEGMENT OF A SHEAR WALL

FIG·2·5

i.e
$$\begin{Bmatrix} P_E \\ P_I \end{Bmatrix} = \begin{bmatrix} K_{EE} & K_{EI} \\ K_{IE} & K_{II} \end{bmatrix} \begin{Bmatrix} D_E \\ D_I \end{Bmatrix} \tag{2.6}$$

where P denotes the external force,

K denotes the stiffness matrix

and D denotes the displacement vector.

The subscripts E and I denote the external and internal nodal actions respectively. For the sake of simplicity, let each node have 'C' degrees of freedom, the stiffness matrix 'K' will then be of the order (n+m)c X (n+m)c.

Then since the internal nodes are not loaded, P_I is zero, and

$$\begin{Bmatrix} P_E \\ 0 \end{Bmatrix} = \begin{bmatrix} K_{EE} & K_{EI} \\ K_{IE} & K_{II} \end{bmatrix} \begin{Bmatrix} D_E \\ D_I \end{Bmatrix}$$

Therefore without violating the law of matrices, the above expression can be expressed by the two matrix equations

$$P_E = K_{EE}\, D_E + K_{EI}\, D_I \tag{2.7a}$$

$$0 = K_{IE}\, D_E + K_{II}\, D_I \tag{2.7b}$$

From the second equation,

$$D_I = -K_{II}^{-1}\, K_{IE}\, D_E$$

Substituting for D_I in equation (2.7a),

$$P_E = K_{EE}\, D_E + K_{EI}(-K_{II}^{-1}\, K_{IE})\, D_E$$

or

$$P_E = (K_{EE} - K_{EI} K_{II}^{-1}\, K_{IE})\, D_E$$

which relates the external nodal forces to external nodal displacements only. This reduces the resultant stiffness matrix to the order of (nc X nc).

:Therefore,

$$\underline{K_{reduced} = K_{EE} - K_{EI} K_{II}^{-1} K_{IE}}$$

Thus the internal nodes are suppressed with a subsequent saving in storage.

2.5 METHOD OF ELIMINATION

The elimination phase and the subsequent expression of the stiffness matrix in terms of the external nodal forces and external nodal displacements follows "The Method of Aitken"(17) which is reproduced in Appendix A. Referring to this appendix the following correspondence of symbols will be used.

$$\begin{bmatrix} A & B \\ C & 0 \end{bmatrix} \equiv \begin{bmatrix} K_{II} & K_{IE} \\ K_{EI} & 0 \end{bmatrix} \tag{2.8}$$

and

$$\underline{-CA^{-1} B = -K_{EI} K_{II}^{-1} K_{IE}}$$

With 'n' external nodes and 'm' internal nodes , the over all stiffness matrix can be written as,

$$\left[\begin{array}{ccc|cc} K_{11} \; K_{12} & & K_{1n} & K_{1(n+1)} & K_{1(n+m)} \\ & K_{EE} & & K_{EI} & \\ K_{n1} & & K_{nn} & & K_{n(n+m)} \\ \hline K_{(n+1)1} & & & K_{(n+1)(n+1)} & K_{(n+1)(n+m)} \\ & K_{IE} & & K_{II} & \\ K_{(n+m)1} & & & & K_{(n+m)(n+m)} \end{array}\right] \tag{2.9}$$

Rearranging the terms as per equation (2.8), we have

$$\left[\begin{array}{c|c} K_{(n+1)(n+1)} & K_{(n+1)1} \\ K_{II} & K_{IE} \\ \hdashline K_{(n+m)(n+m)} & K_{(n+m)n} \\ \hdashline K_{1(n+1)} & 0 \\ K_{EI} & \text{null matrix} \\ K_{n(n+m)} & 0 \end{array}\right] \qquad (2.10)$$

1st pivotal row and $K_{(n+m)(n+m)}$ is first pivotal element

The aim is to reduce the K_{EI} matrix in the expression (2.10) to a null matrix. The process as we see in Aitken's method, produces the required matrix on the bottom right corner to replace the existing null matrix. To achieve this, the ordinary Gaussian elimination technique is used. To facilitate easy programming, the backward decomposition is adopted. Let the final matrix on the bottom right corner of expression (2.10), be represented, after reduction by [Q]. In the backward decomposition, the first pivotal row is the last row of K_{II}, as indicated in expression (2.10). The first pivotal element is the last element of K_{II}, i.e $K_{(n+m)(n+m)}$ in the above expression and the column below this is to be made zero. In general this process can be written in a simplified form as follws:

$$\boxed{K_{ij} = K_{ij} - \frac{K_{1j}}{K_{11}} \times K_{i1}}$$

where,

'l' is the pivotal row and K_{ll} is the pivotal element.

'i' and 'j' are any typical row and column, respectively, that undergo subsequent changes.

The matrix [Q], which replaces the null matrix, represents the expression $-K_{EI} K_{II}^{-1} K_{IE}$. Since we are interested in the expression $(K_{EE} - Q)$, the null matrix in equation (2.10) can be replaced by K_{EE} which gives directly the required expression. The flow chart of the above process is presented in Fig. 2.6

For the purpose of comparison it is interesting to note here that the direct procedure to obtain the expression,

$K_{EE} - K_{EI} K_{II}^{-1} K_{IE}$

involves an inverse of a matrix of very high order, two multplications and a subtraction. This is very uneconomical from the computer point of view. On the other hand, the reduction process explained in this chapter involves only a single operation which takes care of everything relating to matrix operations in the above expression.

2.6 SUMMARY

The general analysis procedure is summarised in the following steps:

(i) The shear wall is subdivided into a number of finite elements and individual element stiffness matrices are evaluated from the element properties.

FLOW CHART FOR CONDENSATION OF INTERNAL NODES

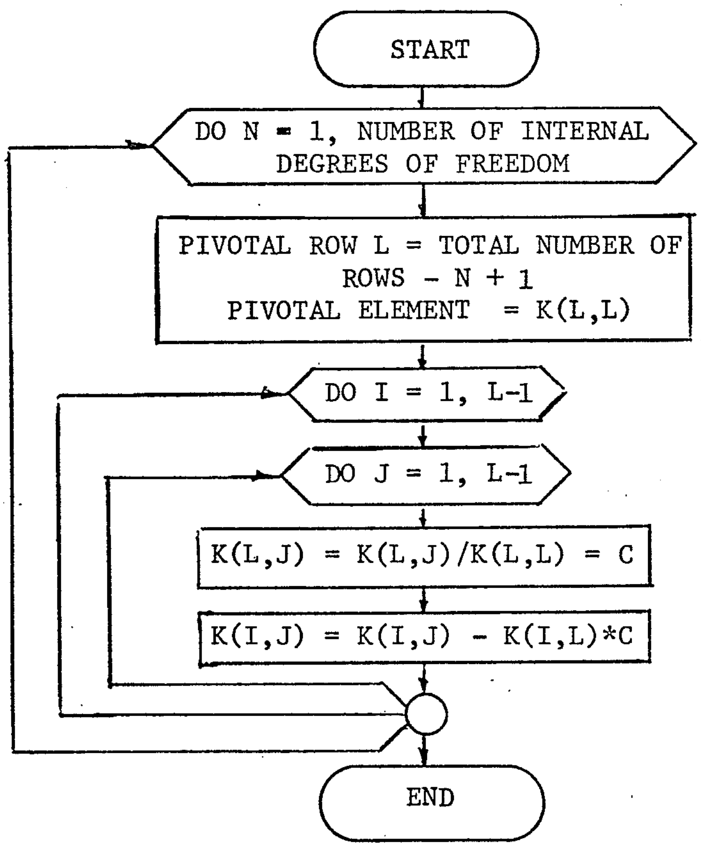


FIG. 2.6

(ii) The stiffness matrices for the elements in a given shear wall segment are superimposed to obtain the stiffness matrix for the segment. The internal nodes for the segment are then condensed off, using force and kinematic assumptions. This gives the force-displacement relationship for the shear wall segment corners.

(iii) Beam and column stiffness matrices are evaluated individually from their respective properties and they are superposed at their common joints to give the frame stiffness matrix.

(iv) The frame stiffness and the shear wall stiffness are combined (the stiffnesses of the two systems are superimposed at the connection points between them) to give the over all stiffness matrix.

(v) The support condition is incorporated in the stiffness matrix formulated in step (iv).

(vi) With known applied loads at the node points, the equations of equilibrium for the whole structure are solved for the unknown displacements at the joints.

(vii) Knowing the segment corner displacements found from step (vi), the forces in the shear wall are calculated from direct multiplication of individual shear wall stiffness matrices and their corresponding segment corner displacements.

(viii) The beam end forces and column end forces are calculated in a similar manner, knowing their end displacements and the individual stiffness matrices.

(ix) Finally, the reactions are calculated.

CHAPTER III

SHEAR WALL ANALYSIS

3.1 INTRODUCTION

In most practical cases, the shear wall properties do not change for any particular storey, that is, between adjacent framing levels. Therefore, it is most convenient to calculate the stiffness matrix for each shear wall segment separately and then to combine the matrices to obtain the stiffness matrix for the overall shear wall. In the finite element analysis of the shear wall, it is obvious that conventional plane stress elements, which have two translational degrees of freedom at each node, are unsuitable to take into account the interaction between the shear wall and frame, or to combine with line elements in bending. Therefore, a third degree of freedom, namely the rotation at each node, becomes absolutely necessary. The question still remains as to whether all the interior nodes should also have the three degrees of freedom. The third degree of freedom, namely the rotational degree of freedom at the internal nodes, can be suppressed, or simple plane stress elements for the internal ones can be considered. This type of approach obviously tends to reduce the accuracy of results. However, the three degrees of freedom at each node involve increased computational work and increased storage requirements. A compromise can be struck between the two approaches. A number of papers, the most recent of which is by R. G. Oakberg and William Weaver (13), point out the above facts. Oakberg and Weaver separated the edge elements and interior elements and introduced a new element called the transition element. The stiffness matrices for these elements are presented later in this chapter.

C. V. Girijavallabhan (14) has done considerable work on element shape. He subdivided a model shear wall into discrete elements. Triangular elements were used for discretization, and later, rectangular elements were adopted. To obtain accurate results, he subdivided the structure into 1,568 triangular elements with 918 nodal points. Employing the direct stiffness method, the nodal displacement vector for the given boundary forces was determined for the complete assemblage of finite elements. The same model problem was again solved using 264 discrete rectangular elements with 334 nodal points, without altering the other properties of the wall. The nodal displacements obtained by the two analyses were compared and it was observed that the overall displacement patterns were the same. Strains and stresses in each element were computed from nodal displacements in both cases and the results agreed very well. The solution obtained was sufficiently accurate for design purposes when 264 rectangular elements instead of 1,568 triangular elements, were used with about one-third the total number of nodal points. Consequently, further analysis of the shear wall was made with rectangular elements only.

Additional advantages of using rectangular elements are:

(i) ease of forming the element mesh and generating nodal coordinates. Only coordinates of one horizontal row and one vertical row need be included among the input data. In addition, the divisions in the X-direction are constant throughout the height of the wall.

(ii) increased flexibility and decreased number of elements when coarser elements are used.

(iii) since, in the shear wall problem, more degrees have to be considered at some of the nodal points, the smaller the number of elements, the smaller the amount of computational work and storage requirements while forming the nodal equilibrium equation.

(iv) ease of systematic generation of element stiffness matrices in the global axes system for the structure. Hence, no transformation of the matrices is required.

If the existing stiffness matrix for any element has a coordinate system other than the global coordinate system then it is taken as the local system for the element which then is transformed to global system using the transformation matrix.

3.2 ELEMENT MESH AND ELEMENT TYPES

Following the method adopted by Oakberg and Weaver, the element mesh for a typical panel is shown in Fig. 3.1. Since, on either edge of the panel, there may be beams framing into the shear wall, the two extreme vertical rows of elements are considered as edge elements to combine with beams in bending. These edge elements have three degrees of freedom at each of their nodes; two translational and one rotation. The interior nodes need not have the rotational degree of freedom since they are not directly connected to bending elements. But to satisfy compatibility at the nodes between edge elements and interior elements, a new element called the transition element is introduced between the two element types. That is, in the two vertical rows of elements adjacent to edge elements. They are further classified as left transition elements and right transition elements. A typical action of the combination of the three types of elements is shown in Fig. 3.2. It is clear from this figure that the two nodes of the transition element which are common to edge elements as well have three degrees of freedom, whereas the nodes that are common with interior elements need have only two degrees of freedom. Therefore, for a transition element, there are altogether 10 degrees of

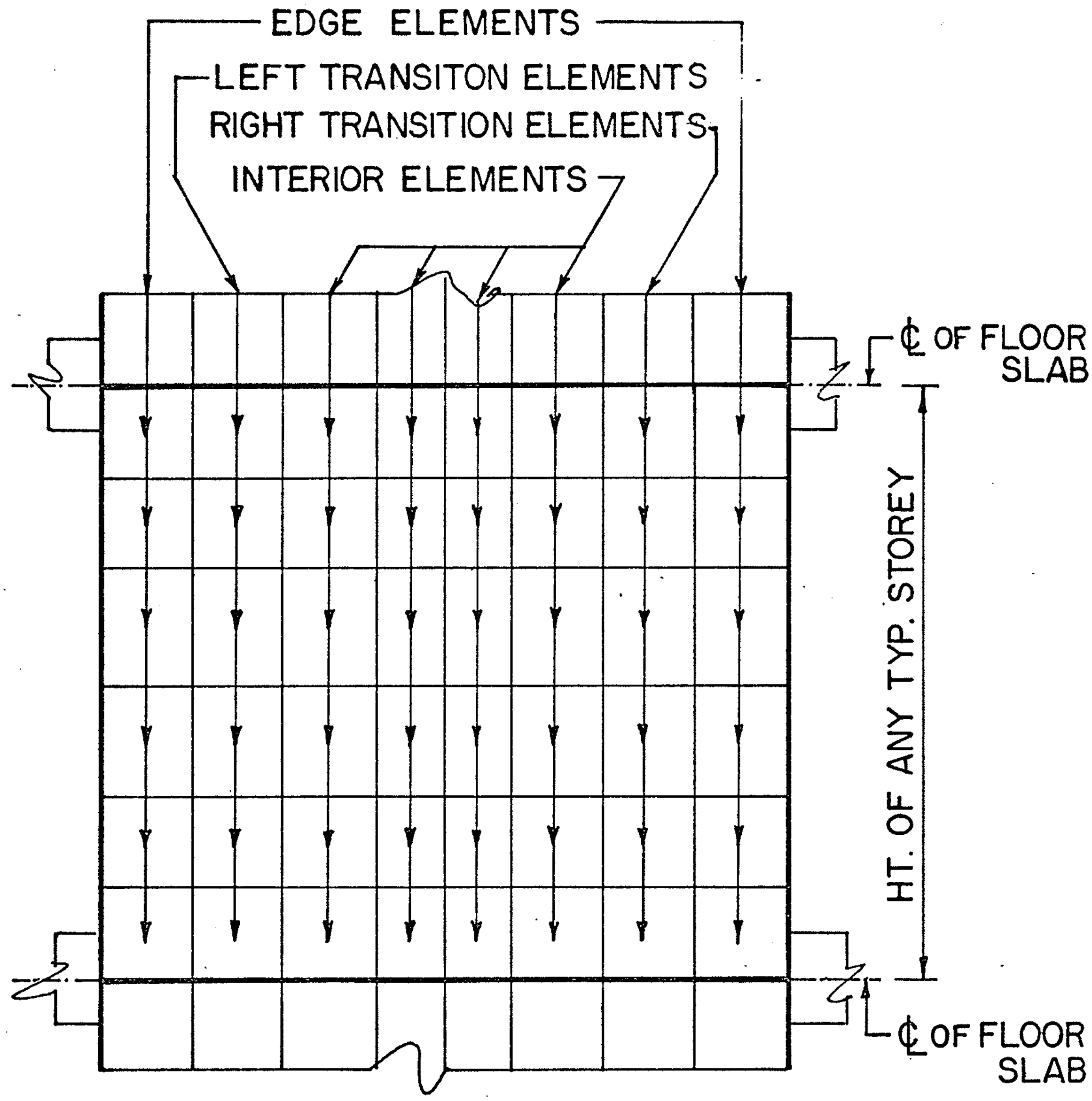


FIG. 3.1 ELEMENT MESH FOR ANY TYPICAL SEGMENT

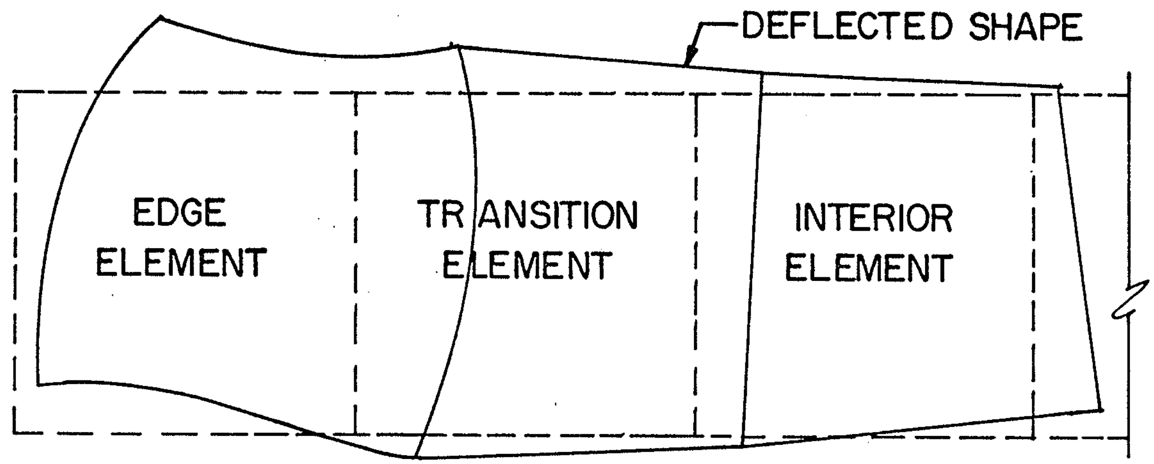


FIG. 3.2 COMBINATION OF EDGE, TRANSITION AND INTERIOR ELEMENTS.

freedom. The stiffness matrix as given by Oakberg and Weaver (13) for these two types of elements are employed here.

3.3 ELEMENT STIFFNESS MATRICES

(i) EDGE ELEMENT

An edge element has three degrees of freedom at each of its nodes; two translations and one rotation. It thus has twelve generalized displacements, as shown in Fig. 3.3. The symbols 'a' and 'b' denote the width and the depth of the element. The displacement functions for this element are the products of linear and cubic polynomials in the dimensionless variables $\xi = x/a$ and $\eta = y/b$ as follows:

$$
\begin{aligned}
u_1 (\xi,\eta,\underset{\sim}{d}) = & -(1-\xi)\ (2\eta^3 - 3\eta^2)d_1 - b\ (1-\xi)\ (\eta^3 - \eta^2)d_3 \\
& -\xi(2\eta^3 - 3\eta^2)d_4 - b\xi(\eta^3 - \eta^2)d_6 \\
& + (1-\xi)\ (2\eta^3 - 3\eta^2 + 1)d_7 - b(1-\xi)(\eta^3 - 2\eta^2 + \eta)d_9 \\
& + \xi(2\eta^3 - 3\eta^2 + 1)d_{10} - b\xi\ (\eta^3 - 2\eta^2 + \eta)d_{12} \qquad (3.1.a.)
\end{aligned}
$$

$$
\begin{aligned}
u_2 (\xi,\eta,\underset{\sim}{d}) = & \ (2\xi^3 - 3\xi^2 + 1)d_2 + a\ \eta(\xi^3 - 2\xi^2 + \xi)d_3 \\
& - \eta(2\xi^3 - 3\xi^2)\ d_4 - a\ \eta(\xi^3 - \xi^2)\ d_6 \\
& + (1-\eta)(2\xi^3 - 3\xi^2 + 1)d_8 + a(1-\eta)\ (\xi^3 - 2\xi^2 + \xi)d_9 \\
& - (1-\eta)(2\xi^3 - 3\xi^2)d_{11} - a(1-\eta)\ (\xi^3 - \xi^2)d_{12} \qquad (3.1.b.)
\end{aligned}
$$

where d is the element displacement vector and u_1 and u_2 are the two displacement functions.

These functions provide identical rotation for adjacent edges at each corner of the element. Since the adjacent element edges experience the same rotation at each corner,

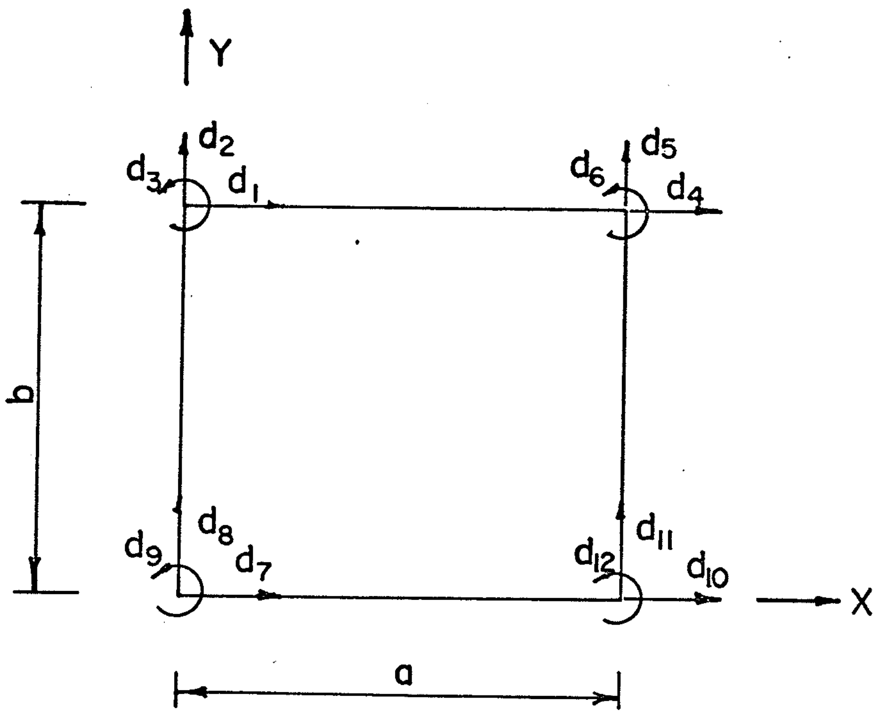


FIG. 3.3 EDGE ELEMENT

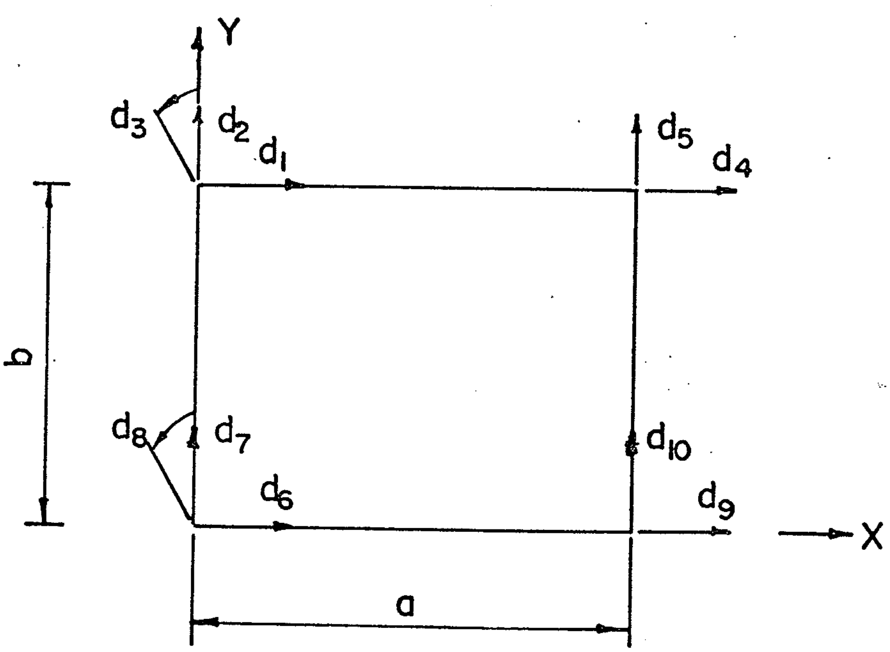


FIG. 3.4 LEFT TRANSITION ELEMENT

TABLE 3.1

STIFFNESS MATRIX FOR EDGE ELEMENT

$$
\frac{Et}{(1-\mu^2)}
\begin{bmatrix}
\rho_1 & -\rho_2 & -\rho_3 & \rho_4 & -\rho_5 & -\rho_6 & \rho_7 & \rho_5 & -\rho_9 & \rho_{10} & \rho_2 & -\rho_{12} \\
-\rho_2 & \beta_2 & \beta_3 & \rho_5 & \beta_5 & \beta_6 & -\rho_5 & \beta_8 & \beta_9 & \rho_2 & \beta_{11} & \beta_{12} \\
-\rho_3 & \beta_3 & \gamma_3 & -\rho_6 & -\beta_6 & \gamma_6 & \rho_9 & \beta_9 & \gamma_9 & \rho_{12} & -\beta_{12} & \gamma_{12} \\
\rho_4 & \rho_5 & -\rho_6 & \rho_1 & \rho_2 & -\rho_3 & \rho_{10} & -\rho_2 & -\rho_{12} & \rho_7 & -\rho_5 & -\rho_9 \\
-\rho_5 & \beta_5 & -\beta_6 & \rho_2 & \beta_2 & -\beta_3 & -\rho_2 & \beta_{11} & -\beta_{12} & \rho_5 & \beta_8 & -\beta_9 \\
-\rho_6 & \beta_6 & \gamma_6 & -\rho_3 & -\beta_3 & \gamma_3 & \rho_{12} & \beta_{12} & \gamma_{12} & \rho_9 & -\beta_9 & \gamma_9 \\
\rho_7 & -\rho_5 & \rho_9 & \rho_{10} & -\rho_2 & \rho_{12} & \rho_1 & \rho_2 & \rho_3 & \rho_4 & \rho_5 & \rho_6 \\
\rho_5 & \beta_8 & \beta_9 & -\rho_2 & \beta_{11} & \beta_{12} & \rho_2 & \beta_2 & \beta_3 & -\rho_5 & \beta_5 & \beta_6 \\
-\rho_9 & \beta_9 & \gamma_9 & -\rho_{12} & -\beta_{12} & \gamma_{12} & \rho_3 & \beta_3 & \gamma_3 & \rho_6 & -\beta_6 & \gamma_6 \\
\rho_{10} & \rho_2 & \rho_{12} & \rho_7 & \rho_5 & \rho_9 & \rho_4 & -\rho_5 & \rho_6 & \rho_1 & -\rho_2 & \rho_3 \\
\rho_2 & \beta_{11} & -\beta_{12} & -\rho_5 & \beta_8 & -\beta_9 & \rho_5 & \beta_5 & -\beta_6 & -\rho_2 & \beta_2 & -\beta_3 \\
-\rho_{12} & \beta_{12} & \gamma_{12} & -\rho_9 & -\beta_9 & \gamma_9 & \rho_6 & \beta_6 & \gamma_6 & \rho_3 & -\beta_3 & \gamma_3
\end{bmatrix}
$$

where,

$$\rho_1 = \frac{13b}{35a} + \frac{2\lambda a}{5b} \qquad \rho_2 = \frac{(\lambda+\mu)}{4}$$

$$\rho_3 = \frac{-11b^2}{210a} + \frac{\mu a}{24} - \frac{3\lambda a}{40} \qquad \rho_4 = \frac{-13b}{35a} + \frac{\lambda a}{5b}$$

$$\rho_5 = \frac{(\mu-\lambda)}{4} \qquad \rho_6 = \frac{11b^2}{210a} - \frac{\mu a}{24} + \frac{\lambda a}{40}$$

$$\rho_7 = \frac{9b}{70a} - \frac{2\lambda a}{5b} \qquad \rho_9 = \frac{13b^2}{420a} - \frac{\mu a}{24} - \frac{3\lambda a}{40}$$

$$\rho_{10} = \frac{-9b}{70a} - \frac{\lambda a}{5b} \qquad \rho_{12} = \frac{-13b^2}{420a} + \frac{\mu a}{24} + \frac{\lambda a}{40}$$

$$\beta_2 = \frac{13a}{35b} + \frac{2\lambda b}{5a} \qquad \beta_3 = \frac{11a^2}{210b} - \frac{\mu b}{24} + \frac{3\lambda b}{40}$$

$$\beta_5 = \frac{9a}{70b} - \frac{2\lambda b}{5a} \qquad \beta_6 = \frac{-13a^2}{420b} + \frac{\mu b}{24} + \frac{3\lambda b}{40}$$

$$\beta_8 = \frac{-13a}{35b} + \frac{\lambda b}{5a} \qquad \beta_9 = \frac{-11a^2}{210b} + \frac{\mu b}{24} - \frac{\lambda b}{40}$$

$$\beta_{11} = \frac{-9a}{70b} - \frac{\lambda b}{5a} \qquad \beta_{12} = \frac{13a^2}{420b} - \frac{\mu b}{24} - \frac{\lambda b}{40}$$

$$\gamma_3 = \frac{b^3}{105a} + \frac{a^3}{105b} - \frac{\mu ab}{72} + \frac{3\lambda ab}{40}$$

$$\gamma_6 = \frac{-b^3}{105a} - \frac{a^3}{140b} + \frac{\mu ab}{72} + \frac{\lambda ab}{40}$$

$$\gamma_9 = \frac{-b^3}{140a} - \frac{a^3}{105b} + \frac{\mu ab}{72} + \frac{\lambda ab}{40}$$

$$\gamma_{12} = \frac{b^3}{140a} + \frac{a^3}{140b} - \frac{\mu ab}{72} - \frac{\lambda ab}{40}$$

and

$$\lambda = \frac{1 - \mu}{2}$$

the shear strain is zero at four points on the element. The stiffness matrix for an edge element is presented in Table 3.1. In the stiffness matrix, E is the modulus of elasticity, 'μ' is the Poisson's ratio, and 't' is the thickness of the element.

(ii) LEFT TRANSITION ELEMENT

Oakberg (18) introduced a transition element between an edge element and an interior element to assure displacement continuity. There are a total of ten generalized displacements for this element as shown in Fig. 3.4. The displacement functions for this element are as follows:

$$u_1(\xi,\eta,\underset{\sim}{d}) = -(1-\xi)\ (2\eta^3 - 3\eta^2)\ d_1 - b(1-\xi)\ (\eta^3 - \eta^2)d_3 + \xi\eta d_4 + (1-\xi)\ (2\eta^3 - 3\eta^2 + 1)d_6 - b(1-\xi)\ (\eta^3 - 2\eta^2 + \eta)d_8 + \xi(1-\eta)d_9 \tag{3.2.a}$$

$$u_2(\xi,\eta,\underset{\sim}{d}) = \eta(1-\xi)d_2 + \xi\eta d_5 + (1-\xi)\ (1-\eta)d_7 + \xi(1-\eta)d_{10} \tag{3.2.b}$$

The symbols are the same as for the edge element and the stiffness matrix for this element is presented in Table 3.2.

TABLE 3.2

STIFFNESS MATRIX FOR LEFT HAND TRANSITION ELEMENT

$$
\frac{Et}{(1-\mu^2)}
\begin{bmatrix}
\theta_1 & -\theta_2 & \theta_3 & -\theta_4 & \theta_5 & \theta_6 & -\theta_5 & -\theta_8 & -\theta_9 & \theta_2 \\
-\theta_2 & \phi_2 & \phi_3 & -\theta_5 & \phi_5 & \theta_5 & -\phi_7 & -\phi_3 & \theta_2 & -\phi_{10} \\
\theta_3 & \phi_3 & \alpha_3 & -\alpha_4 & -\alpha_5 & \theta_8 & -\phi_3 & -\alpha_8 & \alpha_9 & \alpha_5 \\
-\theta_4 & -\theta_5 & -\alpha_4 & \sigma_4 & \theta_2 & -\theta_9 & -\theta_2 & \alpha_9 & \sigma_9 & \theta_5 \\
\theta_5 & \phi_5 & -\alpha_5 & \theta_2 & \phi_2 & -\theta_2 & -\phi_{10} & \alpha_5 & -\theta_5 & -\phi_7 \\
\theta_6 & \theta_5 & \theta_8 & -\theta_9 & -\theta_2 & \theta_1 & \theta_2 & -\theta_3 & -\theta_4 & -\theta_5 \\
-\theta_5 & -\phi_7 & -\phi_3 & -\theta_2 & -\phi_{10} & \theta_2 & \phi_2 & \phi_3 & \theta_5 & \phi_5 \\
-\theta_8 & -\phi_3 & -\alpha_8 & \alpha_9 & \alpha_5 & -\theta_3 & \phi_3 & \alpha_3 & \alpha_4 & -\alpha_5 \\
-\theta_9 & \theta_2 & \alpha_9 & \sigma_9 & -\theta_5 & -\theta_4 & \theta_5 & \alpha_4 & \sigma_4 & -\theta_2 \\
\theta_2 & -\theta_{10} & \alpha_5 & \theta_5 & -\phi_7 & -\theta_5 & \phi_5 & -\alpha_5 & -\theta_2 & \phi_2
\end{bmatrix}
$$

where,

$$\theta_1 = \frac{13b}{35a} + \frac{2\lambda a}{5b} \qquad \theta_2 = \frac{\lambda+\mu}{4}$$

$$\theta_3 = \frac{11b^2}{210a} + \frac{\lambda a}{30} \qquad \theta_4 = \frac{7b}{20a} - \frac{\lambda a}{6b}$$

$$\theta_5 = \frac{\lambda-\mu}{4} \qquad \theta_6 = \frac{9b}{70a} - \frac{2\lambda a}{5b}$$

$$\theta_8 = \frac{13b^2}{420a} - \frac{\lambda a}{30} \qquad \theta_9 = \frac{3b}{20a} + \frac{\lambda a}{6b}$$

$$\phi_2 = \frac{a}{3b} + \frac{\lambda b}{3a} \qquad \phi_3 = \frac{(\lambda-\mu)b}{24}$$

$$\phi_5 = \frac{a}{6b} - \frac{\lambda b}{3a} \qquad \phi_7 = \frac{a}{3b} - \frac{\lambda b}{6a}$$

$$\phi_{10} = \frac{a}{6b} + \frac{\lambda b}{6a}$$

$$\alpha_3 = \frac{b^3}{105a} + \frac{2\lambda ab}{45} \qquad \alpha_4 = \frac{b^2}{20a}$$

$$\alpha_5 = \frac{(\lambda+\mu)b}{24} \qquad \alpha_8 = \frac{b^3}{140a} + \frac{\lambda ab}{90}$$

$$\alpha_9 = -\frac{b^2}{30a}$$

$$\sigma_4 = \frac{b}{3a} + \frac{\lambda a}{3b} \qquad \sigma_9 = \frac{b}{6a} - \frac{\lambda a}{3b}$$

and

$$\lambda = \frac{(1-\mu)}{2}$$

(iii) RIGHT TRANSITION ELEMENT

The right transition element is actually the mirror image of the left hand element with the axes rotated through 180 degrees. Therefore, the stiffness matrix for this element can be derived from that for the left transition element by rearranging the terms and using the transformation matrix. First, the left transition element is rotated through 180 degrees about the Z-axis, as shown in Fig. 3.5(a) where the element displacement vector has components ordered from 1 to 10. Considering this as the local system for the right transition element, the stiffness matrix is the same as for the left transition element. Since the stiffness matrix in the global system is desired components 1 to 10 of the element displacement vector are first rearranged as shown in Fig 3.5(b). Accordingly, the stiffness coefficients in the local system are rearranged to give the revised stiffness matrix for the right transition element, again in the local system. This matrix appears in Table 3.3. To identify the two coordinate systems used, local system has primes in them.

The transformation to the global system is through 180 degrees about the Z-axis of the translational displacement vectors only. This transformation matrix is presented in Table 3.4. There is no sign change for the rotation transformation. Finally, the following matrix operation produces the required stiffness matrix for the right

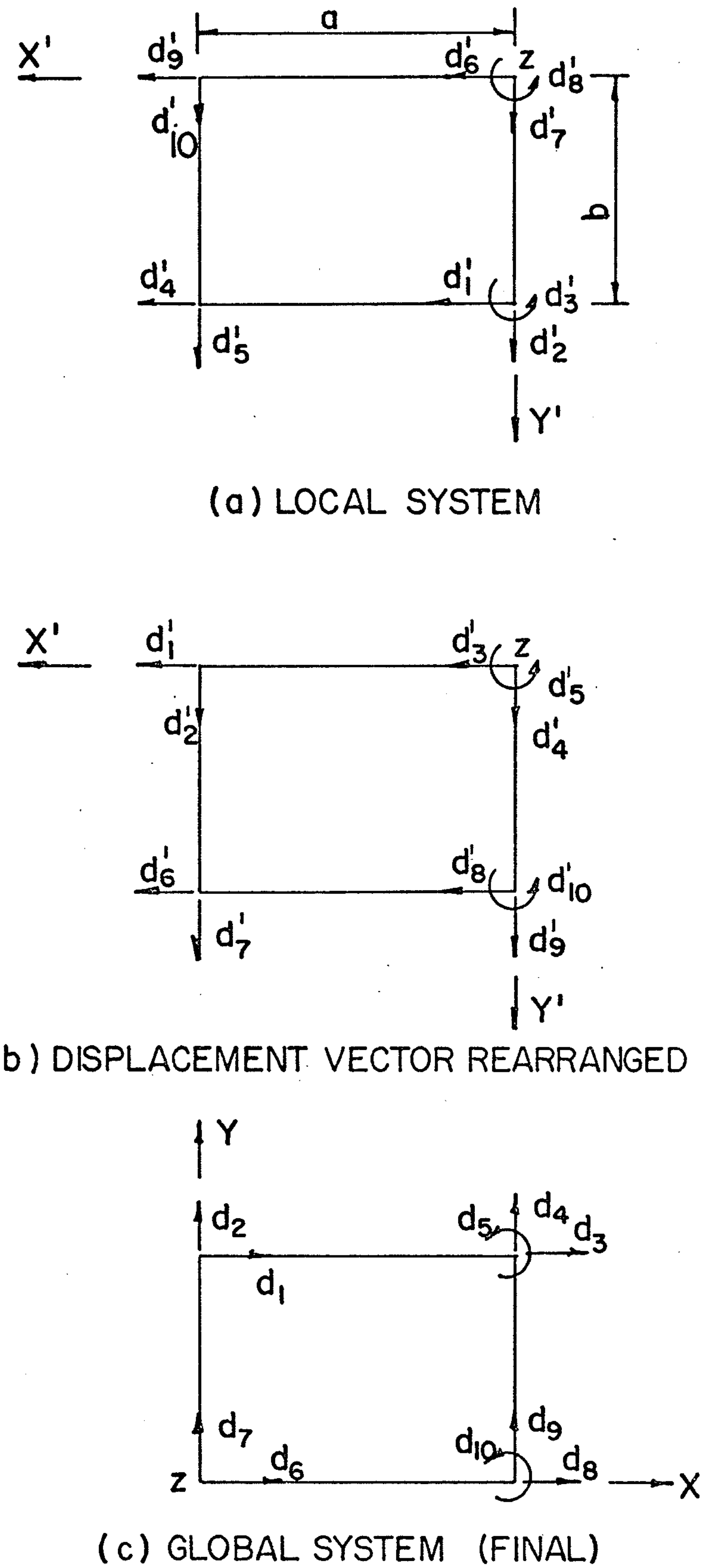


FIG. 3.5 RIGHT TRANSITION ELEMENT

TABLE 3.3

STIFFNESS MATRIX OF RIGHT TRANSITION ELEMENT IN THE LOCAL SYSTEM 'S_r'

$$\frac{Et}{(1-\mu^2)}\begin{bmatrix}
\sigma_4 & -\theta_2 & -\theta_4 & \theta_5 & \alpha_4 & \sigma_9 & -\theta_5 & -\theta_9 & \theta_2 & \alpha_9 \\
-\theta_2 & \phi_2 & -\theta_5 & \phi_5 & -\alpha_5 & \theta_5 & -\phi_7 & \theta_2 & -\phi_{10} & \alpha_5 \\
-\theta_4 & -\theta_5 & \theta_1 & \theta_2 & -\theta_3 & -\theta_9 & -\theta_2 & \theta_6 & \theta_5 & \theta_8 \\
\theta_5 & \phi_5 & \theta_2 & \phi_2 & \phi_3 & -\theta_2 & -\phi_{10} & -\theta_5 & -\phi_7 & -\phi_3 \\
\alpha_4 & -\alpha_5 & -\theta_3 & \phi_3 & \alpha_3 & \alpha_9 & \alpha_5 & -\theta_8 & -\phi_3 & -\alpha_8 \\
\sigma_9 & \theta_5 & -\theta_9 & -\theta_2 & \alpha_9 & \sigma_4 & \alpha_2 & -\theta_4 & -\theta_5 & -\alpha_4 \\
-\theta_5 & -\phi_7 & -\theta_2 & -\phi_{10} & \alpha_5 & \alpha_2 & \phi_2 & \theta_5 & \phi_5 & -\alpha_5 \\
-\theta_9 & \theta_2 & \theta_6 & -\theta_5 & -\theta_8 & -\theta_4 & \theta_5 & \theta_1 & -\theta_2 & \theta_3 \\
\theta_2 & -\phi_{10} & \theta_5 & -\phi_7 & -\phi_3 & -\theta_5 & \phi_5 & -\theta_2 & \phi_2 & \phi_3 \\
\alpha_9 & \alpha_5 & \theta_8 & -\phi_3 & -\alpha_8 & -\alpha_4 & -\alpha_5 & \theta_3 & \phi_3 & \alpha_3
\end{bmatrix}$$

TABLE 3.4

TRANSFORMATION MATRIX 'T'

$$\begin{bmatrix}
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}$$

transition element.

$$S_r = T\, S'_r\, T^T$$

where S_r is the stiffness matrix in the global system, S'_r is the stiffness matrix in the local system, and T is the transformation matrix.

The final stiffness matrix for right transition element appears in Table 3.5.

(iv) INTERIOR ELEMENT

An interior element has two translational degrees of freedom at each of its nodes; giving eight generalized displacements. The stiffness matrix for this element was presented by R. J. Melosh (16). The element, and the assumed generalized coordinates are shown in Fig. 3.6(a). The origin is chosen at the centre of gravity of the element. The displacement functions (obtained from Lagrange interpolation formulae in two dimensions) are as follows:

$$u_1\,(ab\underset{\sim}{d}) = (x-a/2)(y-b/2)d'_1 - (x-a/2)(y+b/2)d'_3 + (x+a/2)(y+b/2)d'_5 - (x+a/2)(y-b/2)d'_7 \qquad (3.4a)$$

$$u_2\,(ab\underset{\sim}{d}) = (x-a/2)(y-b/2)d'_2 - (x-a/2)(y+b/2)d'_4 + (x+a/2)(y+b/2)d'_6 - (x+a/2)(y-b/2)d'_8 \qquad (3.4b)$$

where u_1 and u_2 are displacement functions and $\underset{\sim}{d}$ is the element displacement vector.

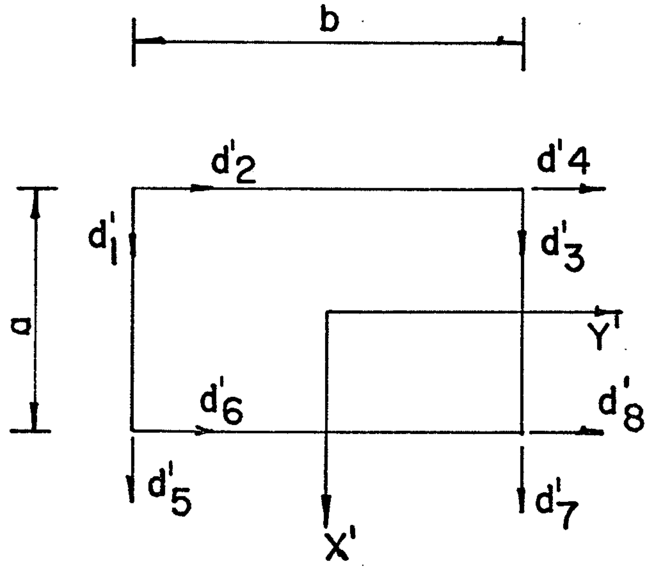
The sides of the element are parallel to the x and y axes and are of length a and b, respectively.

TABLE 3.5

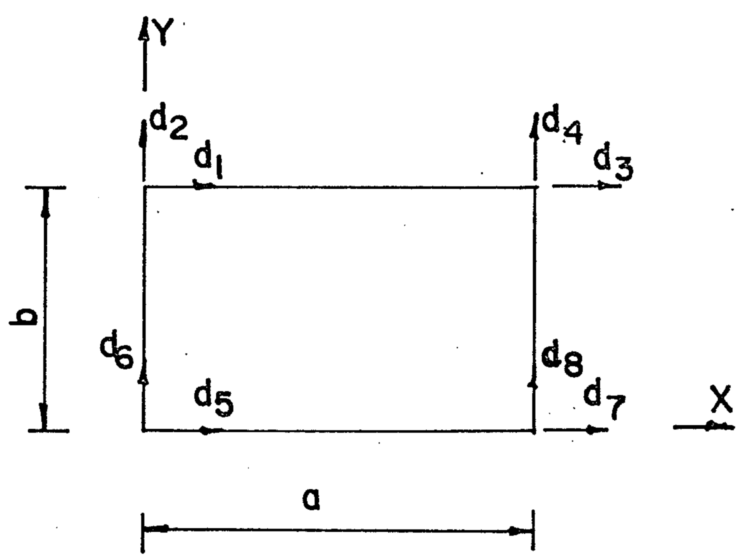
STIFFNESS MATRIX FOR RIGHT TRANSITION
ELEMENT IN THE GLOBAL SYSTEM

$$\frac{Et}{(1-\mu^2)}\begin{bmatrix}
\sigma_4 & -\theta_2 & -\theta_4 & \theta_5 & -\alpha_4 & \sigma_9 & -\theta_5 & -\theta_9 & \theta_2 & -\alpha_9 \\
-\theta_2 & \phi_2 & -\theta_5 & \phi_5 & \alpha_5 & \theta_5 & -\phi_7 & \theta_2 & -\phi_{10} & -\alpha_5 \\
-\theta_4 & -\theta_5 & \theta_1 & \theta_2 & \theta_3 & -\theta_9 & -\theta_2 & \theta_6 & \theta_5 & -\theta_8 \\
\theta_5 & \phi_5 & \theta_2 & \phi_2 & -\phi_3 & -\theta_2 & -\phi_{10} & -\theta_5 & -\phi_7 & \phi_3 \\
-\alpha_4 & \alpha_5 & \theta_3 & -\phi_3 & \alpha_3 & -\alpha_9 & -\alpha_5 & \theta_8 & \phi_3 & -\alpha_8 \\
\sigma_9 & \theta_5 & -\theta_9 & -\theta_2 & -\alpha_9 & \sigma_4 & \theta_2 & -\theta_4 & -\theta_5 & \alpha_4 \\
-\theta_5 & -\phi_7 & -\theta_2 & -\phi_{10} & -\alpha_5 & \theta_2 & \phi_2 & \theta_5 & \phi_5 & \alpha_5 \\
-\theta_9 & \theta_2 & \theta_6 & -\theta_5 & \theta_8 & -\theta_4 & \theta_5 & \theta_1 & -\theta_2 & -\theta_3 \\
\theta_2 & -\phi_{10} & \theta_5 & -\phi_7 & \phi_3 & -\theta_5 & \phi_5 & -\theta_2 & \phi_2 & -\phi_3 \\
-\alpha_9 & -\alpha_5 & -\theta_8 & \phi_3 & -\alpha_8 & \alpha_4 & \alpha_5 & -\theta_3 & -\phi_3 & \alpha_3
\end{bmatrix}$$

$$\boxed{S_r = T\,S'_r\,T^t}$$



(a) ELEMENT DISPLACEMENTS IN LOCAL SYSTEM



(b) ELEMENT DISPLACEMENTS IN GLOBAL SYSTEM

FIG. 3.6 INTERIOR ELEMENT

The stiffness matrix as given by R. J. Melosh (16) appears below.

$$K = \begin{bmatrix} K_{11} & K_{21}^{T} \\ K_{21} & K_{22} \end{bmatrix}$$

where,

$$K_{11} = \begin{bmatrix} 2a_{11} + 2\bar{a}_{44} & & & \\ a_{11} - 2\bar{a}_{44} & 2a_{11} + 2\bar{a}_{44} & \text{SYMMETRIC} & \\ -a_{11} - \bar{a}_{44} & -2a_{11} + \bar{a}_{44} & 2a_{11} + 2\bar{a}_{44} & \\ -2a_{11} + \bar{a}_{44} & -a_{11} - \bar{a}_{44} & a_{11} - 2\bar{a}_{44} & 2a_{11} + 2\bar{a}_{44} \end{bmatrix}$$

$$K_{21} = \begin{bmatrix} a_{12} + a_{44} & a_{12} - a_{44} & -a_{12} - a_{44} & -a_{12} + a_{44} \\ -a_{12} - a_{44} & -a_{12} - a_{44} & a_{12} - a_{44} & a_{12} + a_{44} \\ -a_{12} - a_{44} & -a_{12} + a_{44} & a_{12} + a_{44} & a_{12} - a_{44} \\ a_{12} - a_{44} & a_{12} + a_{44} & -a_{12} + a_{44} & -a_{12} - a_{44} \end{bmatrix}$$

and

$$K_{22} = \begin{bmatrix} 2a_{22} + 2\dot{a}_{44} & & \text{SYMMETRIC} & \\ -2a_{22} + \dot{a}_{44} & 2a_{22} + 2\dot{a}_{44} & & \\ -a_{22} - \dot{a}_{44} & a_{22} - 2\dot{a}_{44} & 2a_{22} + 2\dot{a}_{44} & \\ a_{22} - 2\dot{a}_{44} & -a_{22} - \dot{a}_{44} & -2a_{22} + \dot{a}_{44} & 2a_{22} + 2\dot{a}_{44} \end{bmatrix}$$

in which

$$a_{11} = \frac{Et}{(1-\mu^2)} \times \frac{b}{6a} \qquad a_{22} = \frac{Et}{(1-\mu^2)} \times \frac{a}{6b}$$

$$a_{12} = \frac{Et}{(1-\mu^2)} \times \frac{1}{4} \qquad \bar{a}_{44} = \frac{Et}{(1-\mu^2)} \times \frac{(1-\mu)a}{12b}$$

$$\dot{a}_{44} = \frac{Et}{1-\mu^2} \times \frac{(1-\mu)b}{12a} \quad \text{and} \quad a_{44} = \frac{Et}{(1-\mu^2)} \times \frac{(1-\mu)}{8}$$

'E' and 'μ' are the modulus of elasticity and Poisson's ratio, respectively, and 't' is the thickness of the element.

Substituting these values in the stiffness matrix, the matrix expressed in the local system as represented in Table 3.6, is obtained.

TABLE 3.6

STIFFNESS MATRIX FOR INTERIOR ELEMENT IN THE LOCAL SYSTEM

$$
\frac{Et}{(1-\mu^2)}
\begin{bmatrix}
\frac{b}{3a} + \frac{(1-\mu)a}{6b} & & & & & & & \\
\frac{b}{6a} - \frac{(1-\mu)a}{6b} & \frac{b}{3a} + \frac{(1-\mu)a}{6b} & & & & & & \\
\frac{-b}{6a} - \frac{(1-\mu)a}{12b} & \frac{-b}{3a} + \frac{(1-\mu)a}{12b} & \frac{b}{3a} + \frac{(1-\mu)a}{6b} & & & & & \\
\frac{-b}{3a} + \frac{(1-\mu)a}{12b} & \frac{-b}{6a} - \frac{(1-\mu)a}{12b} & \frac{b}{6a} - \frac{(1-\mu)a}{6b} & \frac{b}{3a} + \frac{(1-\mu)a}{6b} & & & \text{SYMMETRIC} & \\
\frac{\mu}{4} + \frac{1-\mu}{8} & \frac{\mu}{4} - \frac{1-\mu}{8} & \frac{-\mu}{4} - \frac{1-\mu}{8} & \frac{-\mu}{4} + \frac{1-\mu}{8} & \frac{a}{3b} + \frac{(1-\mu)b}{6a} & & & \\
\frac{-\mu}{4} + \frac{1-\mu}{8} & \frac{-\mu}{4} - \frac{1-\mu}{8} & \frac{\mu}{4} - \frac{1-\mu}{8} & \frac{\mu}{4} + \frac{1-\mu}{8} & \frac{-a}{3b} + \frac{(1-\mu)b}{12a} & \frac{a}{3b} + \frac{(1-\mu)b}{6a} & & \\
\frac{-\mu}{4} - \frac{1-\mu}{8} & \frac{-\mu}{4} + \frac{1-\mu}{8} & \frac{\mu}{4} + \frac{1-\mu}{8} & \frac{\mu}{4} - \frac{1-\mu}{8} & \frac{-a}{6b} - \frac{(1-\mu)b}{12a} & \frac{a}{6b} - \frac{(1-\mu)b}{6a} & \frac{a}{3b} + \frac{(1-\mu)b}{6a} & \\
\frac{\mu}{4} - \frac{1-\mu}{8} & \frac{\mu}{4} + \frac{1-\mu}{8} & \frac{-\mu}{4} + \frac{1-\mu}{8} & \frac{-\mu}{4} - \frac{1-\mu}{8} & \frac{a}{6b} - \frac{(1-\mu)b}{6a} & \frac{-a}{6b} - \frac{(1-\mu)b}{12a} & \frac{-a}{3b} + \frac{(1-\mu)b}{12a} & \frac{a}{3b} + \frac{(1-\mu)b}{6a}
\end{bmatrix}
$$

In the expanded form, the equilibrium equations for the interior element can be written as follows:

$$\begin{Bmatrix} P_1 \\ P_3 \\ P_5 \\ P_7 \\ P_2 \\ P_4 \\ P_6 \\ P_8 \end{Bmatrix} = \begin{bmatrix} K_{11} & K_{13} & K_{15} & K_{17} & K_{12} & K_{14} & K_{16} & K_{18} \\ - & - & - & - & - & - & - & - \\ - & - & - & - & - & - & - & - \\ - & - & - & - & - & - & - & - \\ K_{21} & K_{23} & K_{25} & K_{27} & K_{22} & K_{24} & K_{26} & K_{28} \\ - & - & - & - & - & - & - & - \\ - & - & - & - & - & - & - & - \\ K_{81} & K_{83} & K_{85} & K_{87} & K_{82} & K_{84} & K_{86} & K_{88} \end{bmatrix} \begin{Bmatrix} d'_1 \\ d'_3 \\ d'_5 \\ d'_7 \\ d'_2 \\ d'_4 \\ d'_6 \\ d'_8 \end{Bmatrix}$$

The terms are rearranged according to Fig. 3.6(b) to give the stiffness matrix for the interior element in the global system. This appears in Table 3.7.

3.4 SEGMENT STIFFNESS MATRIX

As mentioned earlier, the shear wall stiffness matrix is generated segment-wise and then the combination of the segmental matrices yields the required shear wall stiffness matrix. After the segment is subdivided into a number of rectangular finite elements, it is desired to develop the stiffness matrix that will relate the nodal displacements and the corresponding actions of the nodes that lie along the top and bottom edges of the segments. As was discussed in article 2.4, the nodes that lie along the edge are termed external nodes, while the remaining nodes in the segment are designated internal nodes.

TABLE 3.7

STIFFNESS MATRIX FOR INTERIOR ELEMENT IN GLOBAL SYSTEM

$$\frac{Et}{(1-\mu^2)}\begin{bmatrix} \alpha_1 & -\alpha_2 & \alpha_3 & \alpha_4 & \alpha_5 & -\alpha_4 & -\alpha_7 & \alpha_2 \\ -\alpha_2 & \beta_2 & -\alpha_4 & \beta_4 & \alpha_4 & \beta_6 & \alpha_2 & -\beta_8 \\ \alpha_3 & -\alpha_4 & \alpha_1 & \alpha_2 & -\alpha_7 & -\alpha_2 & \alpha_5 & \alpha_4 \\ \alpha_4 & \beta_4 & \alpha_2 & \beta_2 & -\alpha_2 & -\beta_8 & -\alpha_4 & \beta_6 \\ \alpha_5 & \alpha_4 & -\alpha_7 & -\alpha_2 & \alpha_1 & \alpha_2 & \alpha_3 & -\alpha_4 \\ -\alpha_4 & \beta_6 & -\alpha_2 & -\beta_8 & \alpha_2 & \beta_2 & \alpha_4 & \beta_4 \\ -\alpha_7 & \alpha_2 & \alpha_5 & -\alpha_4 & \alpha_3 & \alpha_4 & \alpha_1 & -\alpha_2 \\ \alpha_2 & -\beta_8 & \alpha_4 & \beta_6 & -\alpha_4 & \beta_4 & -\alpha_2 & \beta_2 \end{bmatrix}$$

where,

$$\alpha_1 = \frac{b}{3a} + \frac{\lambda a}{3b} \qquad \alpha_2 = \frac{\lambda + \mu}{4}$$

$$\alpha_3 = \frac{-b}{3a} + \frac{\lambda a}{6b} \qquad \alpha_4 = \frac{\lambda - \mu}{4}$$

$$\alpha_5 = \frac{b}{6a} - \frac{\lambda a}{3b} \qquad \alpha_7 = \frac{b}{6a} + \frac{\lambda a}{6b}$$

$$\beta_2 = \frac{a}{3b} + \frac{\lambda b}{3a} \qquad \beta_4 = \frac{a}{6b} - \frac{\lambda b}{3a}$$

$$\beta_6 = \frac{-a}{3b} + \frac{\lambda b}{6a} \qquad \beta_8 = \frac{a}{6b} + \frac{\lambda b}{6a}$$

and

$$\lambda = \frac{1-\mu}{2}$$

:Generation of the stiffness matrix starts from the topmost segment and the topmost row of elements. If all the elements in the segment are to be considered in one step to form the segment stiffness matrix, there is a problem of storage. To avoid this, the elements are considered row-wise starting from the top most row.

The top most row of elements, along with their nodal actions are shown in Fig. 3.7. Considering each element in this row in turn, the corresponding element stiffness matrix is generated. The stiffness matrix for the whole row is formed by superimposing the stiffnesses at common nodes. Since the top row of elements is pictured in Fig. 3.7, there is only one lateral displacement along line 1, which coincides with the centre line of the floor slab and is considered as the framing level. The matrix for this top row can be written as

$$\begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}^{\text{row -1}}$$

where subscripts 1 and 2 denote the nodes on lines 1 and 2, respectively.

As a second step, the next row of elements is considered and the stiffness matrix for this row is generated in a similar manner. Once again, the stiffnesses at the nodes along line 2 are obtained by superimposing the values for the elements in the adjacent two rows. The procedure is illustrated in Fig 3.8 and the corresponding stiffness matrix is as follows:

$$\begin{bmatrix} K_{11} & K_{12} & 0 \\ K_{21} & (K_{22}^{row1} + K_{22}^{row2}) & K_{23} \\ 0 & K_{32} & K_{33} \end{bmatrix}$$

Considering the nodes on line 2 as internal nodes, 'I', and the rest as external nodes, 'E', the previous stiffness matrix can be rewritten as

$$\begin{bmatrix} K^{(1)}_{EE} & K^{(1)}_{EI} & 0 \\ K^{(1)}_{IE} & (K^{(1)}_{II} + K^{(2)}_{II}) & K^{(2)}_{IE} \\ 0 & K^{(2)}_{EI} & K^{(2)}_{EE} \end{bmatrix}$$

where the superscript denotes the rows.

The terms can then be rearranged and partitioned as follows.

$$\left[\begin{array}{cc|c} K^{(1)}_{EE} & 0 & K^{(1)}_{EI} \\ 0 & K^{(2)}_{EE} & K^{(2)}_{EI} \\ \hline K^{(1)}_{IE} & K^{(2)}_{IE} & K^{(1+2)}_{II} \end{array}\right]$$

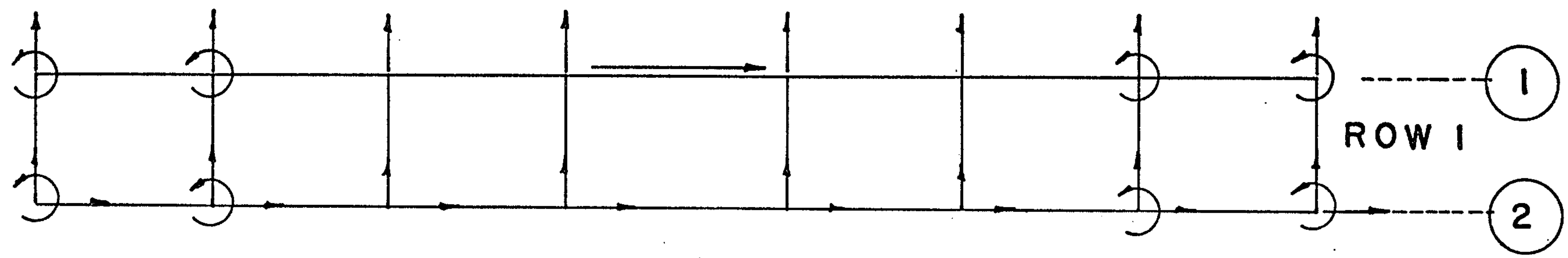


FIG. 3.7 ACTIONS ON TOP MOST ROW OF ELEMENTS

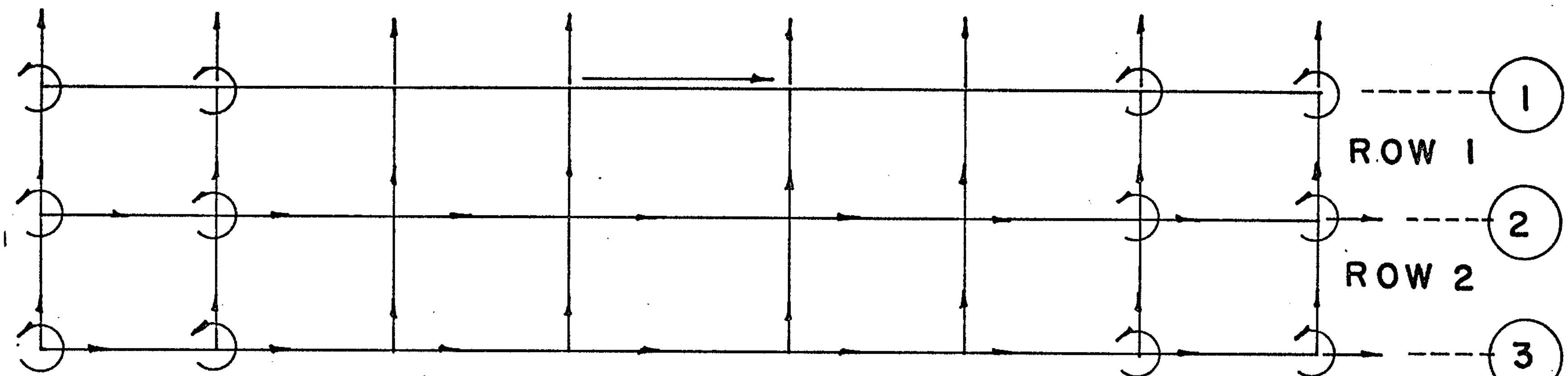


FIG. 3.8 ACTIONS ON TOP TWO ROWS OF ELEMENTS

The previous matrix has the form:

$$\begin{bmatrix} K_{EE} & K_{EI} \\ K_{IE} & K_{II} \end{bmatrix}$$

from which the matrix can be reduced to the form:

$$(K_{EE} - K_{EI} \ K_{II}^{-1} \ K_{IE})$$

as explained in article 2.4. This gives the reduced matrix:

$$\begin{bmatrix} K'_{11} & K'_{13} \\ K'_{31} & K'_{33} \end{bmatrix}$$

which relates the displacements and forces at the nodes on lines 1 and 3, only, the effects at the internal nodes being suppressed. This appears in Fig. 3.9.

Next, the third row of elements is considered, and after the stiffness matrix is generated for the row, it is combined with the previous stiffness matrices as illustrated in Fig. 3.10. The resultant matrix has the form:

$$\begin{bmatrix} K'_{11} & K'_{13} & 0 \\ K'_{31} & K'_{33} + K^{3}_{33} & K^{3}_{34} \\ 0 & K^{3}_{43} & K^{3}_{44} \end{bmatrix}$$

As explained in the previous paragraph, the terms of the above matrix can be rearranged and the reduction process repeated for this new matrix. the resulting matrix has the

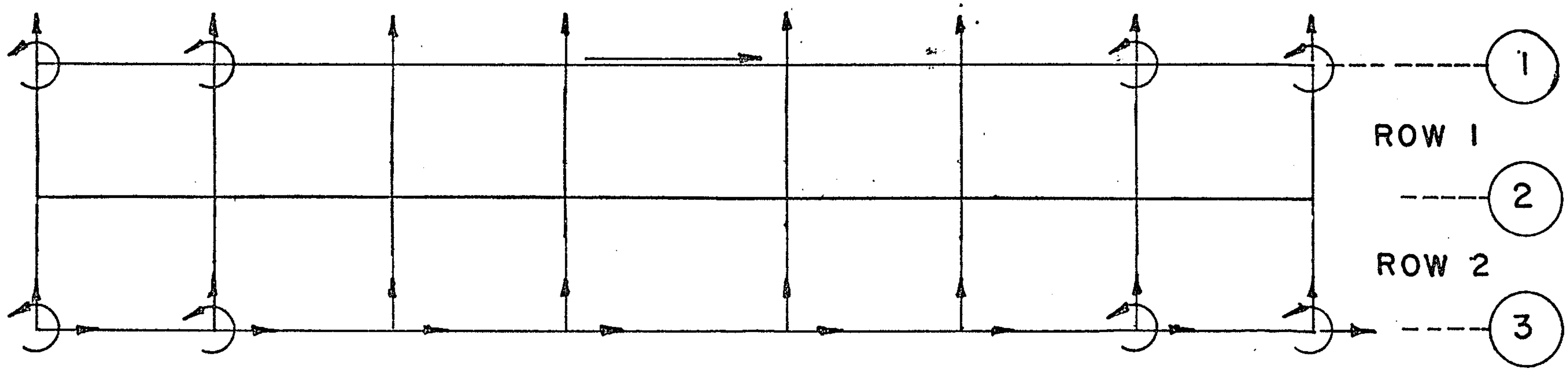


FIG. 3.9 ACTIONS ON LINE 2 ELIMINATED

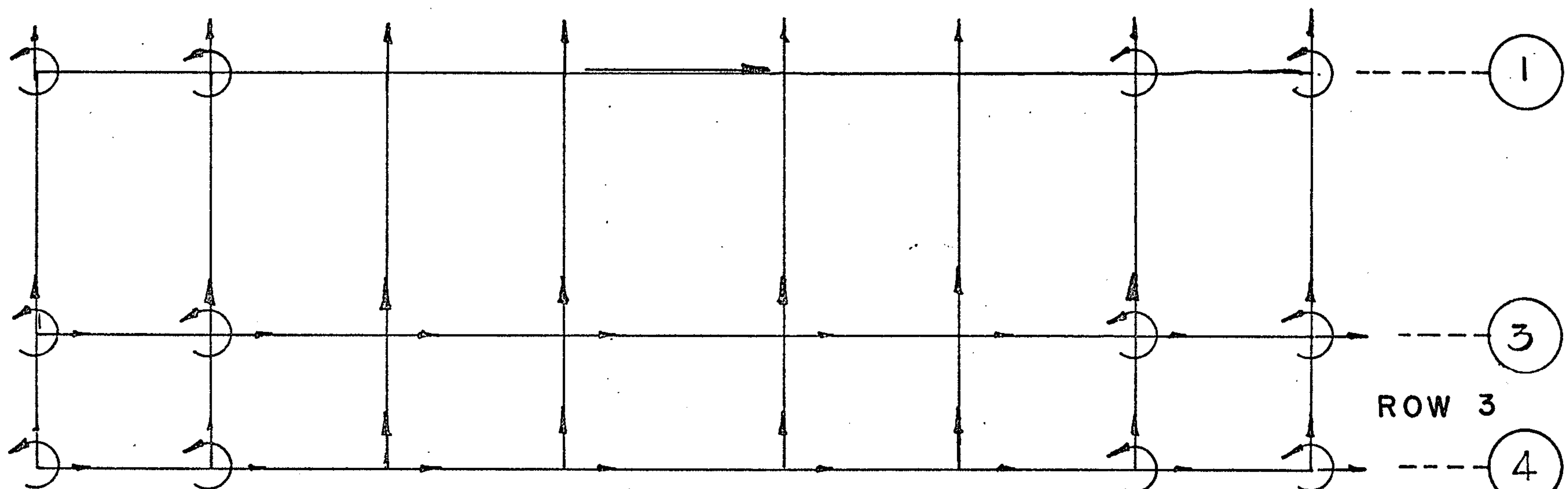


FIG. 3.10 THIRD ROW OF ELEMENTS SUPERIMPOSED

form:

$$\begin{bmatrix} K''_{11} & K''_{14} \\ K''_{41} & K''_{44} \end{bmatrix}$$

This process is repeated for all the rows in the segment, treating one row at a time until the bottom edge of the segment is reached. Fig. 3.11 shows the nodal actions for a segment, developed in this manner. The final stiffness matrix at this step is the required segment stiffness matrix.

The shear wall stiffness matrix could be formulated by developing the stiffness matrix for each segment in turn and superimposing them along the common edges. However, there is a serious drawback in this, from the computational point of view. To illustrate, consider two segments, as shown in Fig. 3.12. For convenience, the actions at the segment corners are differentiated from those at the internal nodes on the common segment boundaries. The stiffness matrix for the two segments can be written as:

$$\begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} & 0 & 0 \\ K_{21} & K_{22} & K_{23} & K_{24} & 0 & 0 \\ K_{31} & K_{32} & K_{33}^{1+2} & K_{34}^{1+2} & K_{35} & K_{36} \\ K_{41} & K_{42} & K_{43}^{1+2} & K_{44}^{1+2} & K_{45} & K_{46} \\ 0 & 0 & K_{53} & K_{54} & K_{55} & K_{56} \\ 0 & 0 & K_{63} & K_{64} & K_{65} & K_{66} \end{bmatrix}$$

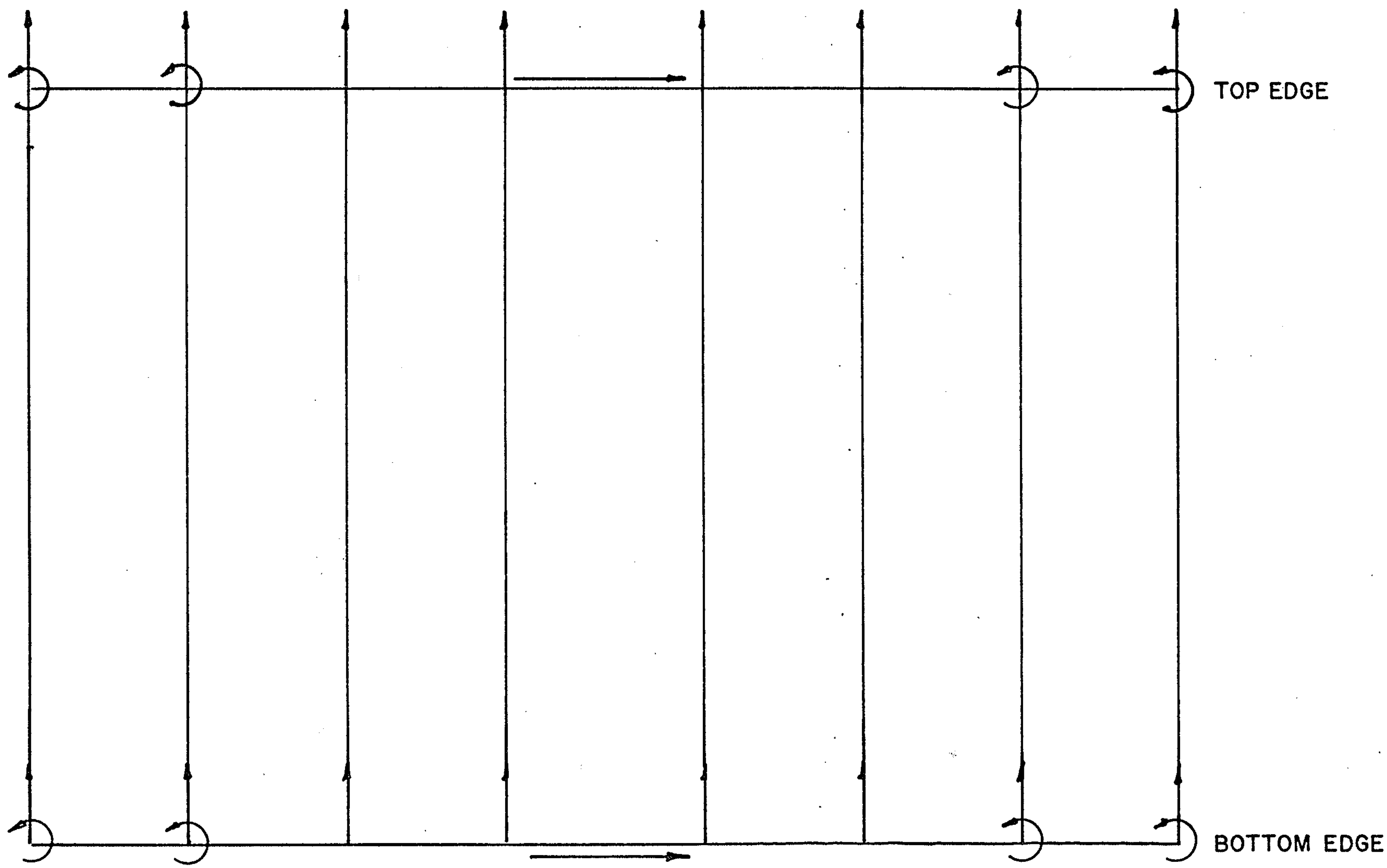


FIG. 3.11 TYPICAL ACTIONS ON SEGMENT EDGES

(AFTER INTERNAL NODES ARE SUPPRESSED)

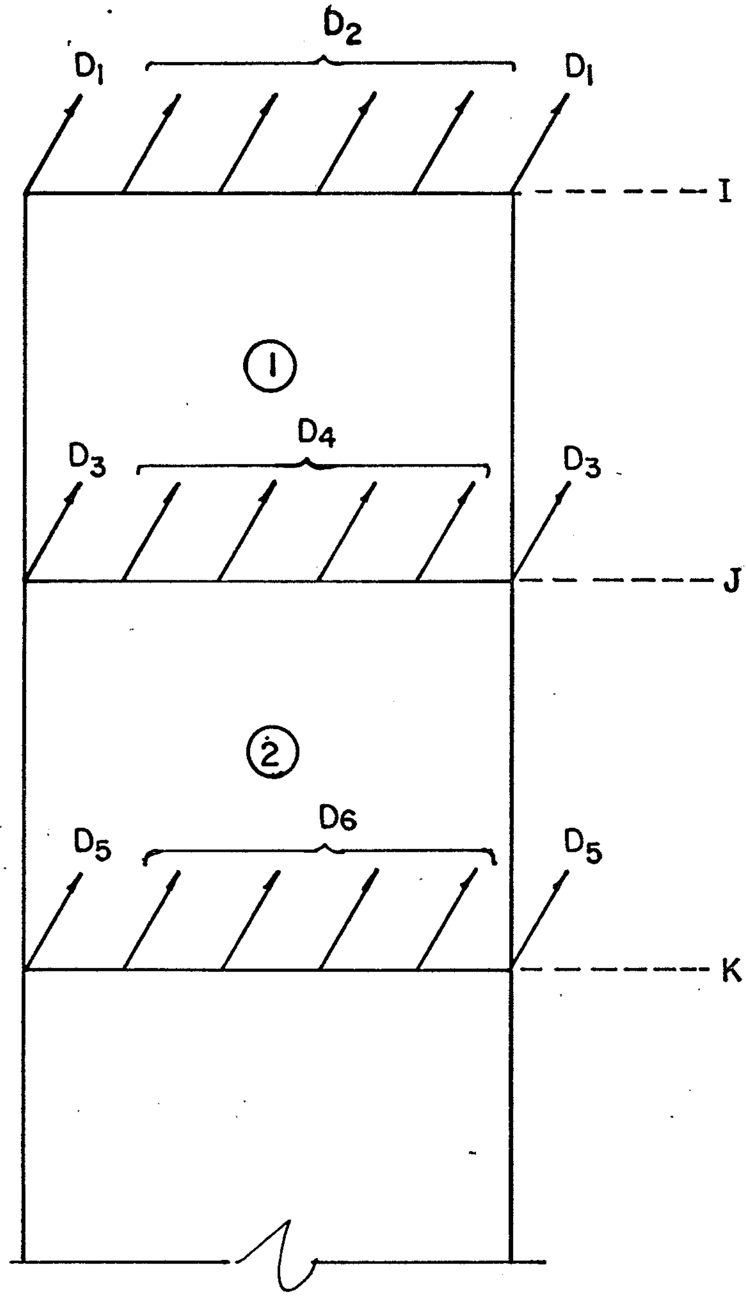


FIG. 3.12 COMBINATION OF TWO SEGMENTS

It is obtained by the combination of the two matrices for segments 1 and 2. By rearranging the terms in the above matrix and treating nodes 1, 3, 5 and 6 as external and nodes 2 and 4 as internal, the reduction process can be applied to condense off nodes 2 and 4. If this is done, the zero terms in the above matrix are replaced by non-zero terms, due to condensation. Consequently, the shear wall stiffness matrix becomes full, and unless further simplifying assumptions are made, a great deal of computational effort and large storage requirements result.

To avoid these problems, kinematic assumptions, described in the next section, are made at the shear wall segment boundaries, in order to produce a banded shear wall stiffness matrix.

3.5 LINEAR INTERPOLATION OF INTERNAL DISPLACEMENTS AND CORRESPONDING MODIFICATION TO STIFFNESS MATRIX

From classical elasticity theory, it is reasonable to assume that the plane sections of a long slender flexural member remain plane during bending. The assumption is therefore made here that the plane transverse sections through the shear wall at all floor levels remain plane as the structure is subjected to lateral loads, as illustrated in Fig. 3.13(a).

This assumption has two useful consequences:

(i) There is linear variation of vertical and rotational displacements along the boundary between any two segments.

(ii) The rotation of the left end of the boundary is equal to that at the right end.

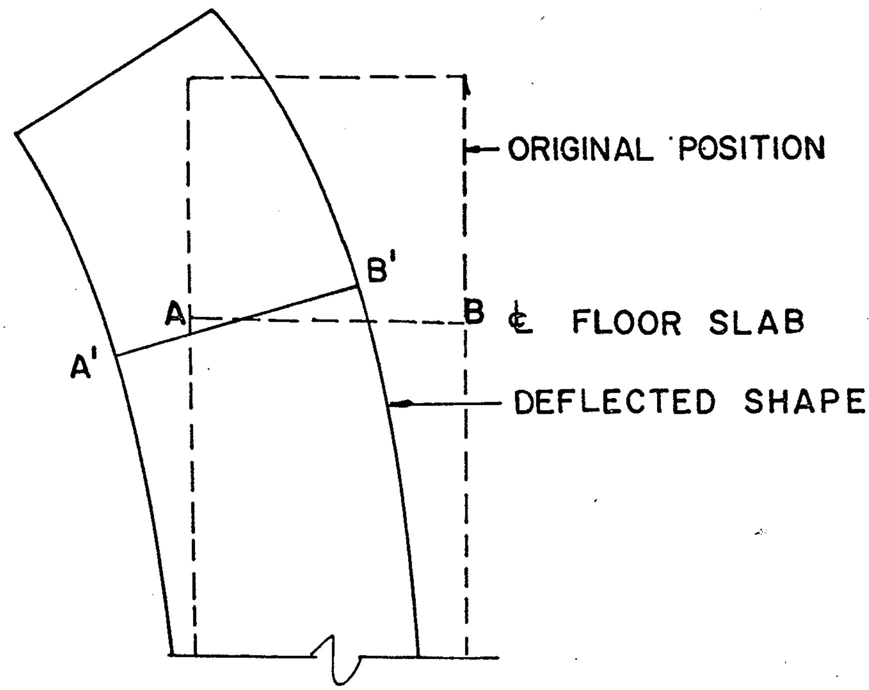
3.5(i) LINEAR INTERPOLATION OF DISPLACEMENTS

In Fig. 3.13(b), D_A and D_B are the displacements at ends A and B, respectively, of a segment boundary. The displacement D_C at any point C, between A and B, can be represented as

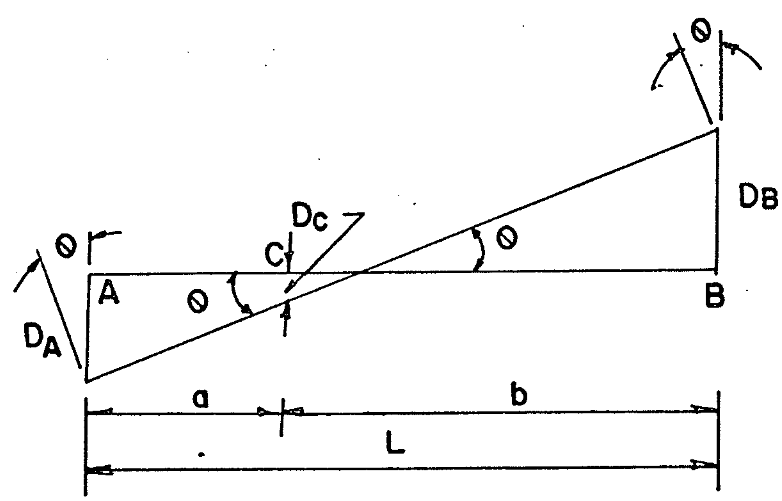
$$D_C = \frac{b}{L} D_A + \frac{a}{L} D_B \tag{3.5}$$

For the two shear wall segments shown in Fig. 3.14, it is assumed that the stiffness matrices for segments 1 and 2 are formed separately and the stiffness coefficients along boundary J are superimposed to give the following equilibrium equations for this portion of the structure:

$$\begin{Bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \end{Bmatrix} = \begin{bmatrix} K_{11} & K_{12} & K_{13} & 0 & K_{15} \\ K_{21} & K_{22} & K_{23} & 0 & K_{25} \\ K_{31} & K_{32} & K_{33} & K_{34} & K_{35} \\ 0 & 0 & K_{43} & K_{44} & 0 \\ K_{51} & K_{52} & K_{53} & 0 & K_{55} \end{bmatrix} \begin{Bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \end{Bmatrix} \tag{3.6}$$

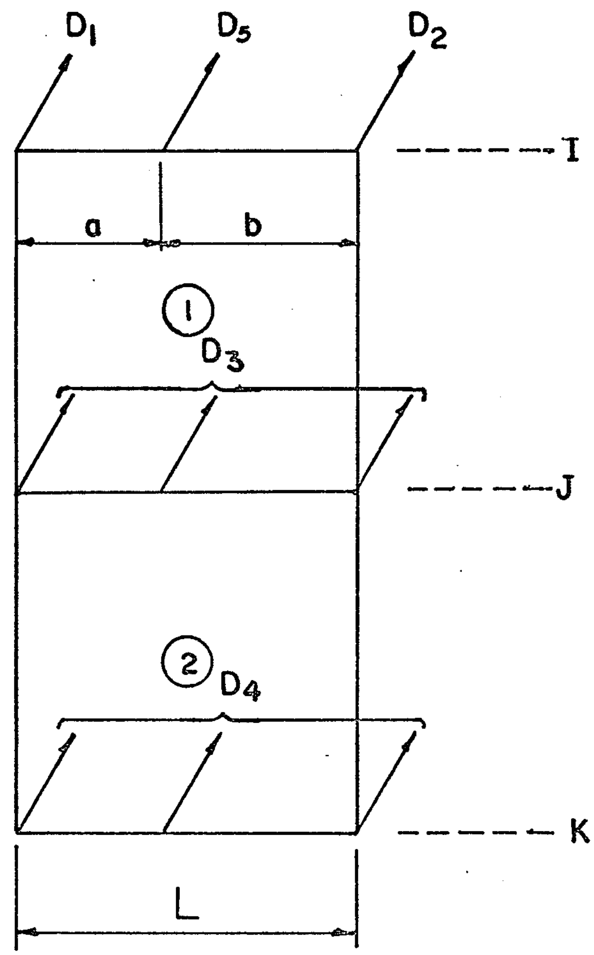


(a) DEFLECTED SHAPE OF SHEAR WALL



(b) GEOMETRY OF DISPLACEMENT ALONG EDGE

FIG. 3.13



FIG·3·14 INTERNAL AND CORNER DISPLACEMENT VECTOR ALONG THE SEGMENT EDGES

The aim here is to relate the displacement D_5 to displacements D_1 and D_3 in order to reduce the order of the matrix in (3.6). For simplicity, the displacements along boundaries J and are represented by symbolic vectors D3 and D_4, respectively.

Let C_1=b/L and C_2=a/L where a, b and L are as denoted in Fig. 3.14. Then from equation 3.5:

$$D_5 = C_1 D_1 + C_2 D_2 \tag{3.5a}$$

Substituting Eq. 3.5a into Eq. 3.6,

$$\begin{Bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \end{Bmatrix} = \begin{bmatrix} K_{11} & K_{12} & K_{13} & 0 & K_{15} \\ K_{21} & K_{22} & K_{23} & 0 & K_{25} \\ K_{31} & K_{32} & K_{33} & K_{34} & K_{35} \\ 0 & 0 & K_{43} & K_{44} & 0 \\ K_{51} & K_{52} & K_{53} & 0 & K_{55} \end{bmatrix} \begin{Bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ C_1 D_1 + C_2 D_2 \end{Bmatrix} \tag{3.7}$$

or

$$\underset{5 \times 1}{\begin{Bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \end{Bmatrix}} = \underset{5 \times 4}{\begin{bmatrix} (K_{11} + C_1 K_{15}) & (K_{12} + C_2 K_{15}) & K_{13} & 0 \\ (K_{21} + C_1 K_{25}) & (K_{22} + C_2 K_{25}) & K_{23} & 0 \\ (K_{31} + C_1 K_{35}) & (K_{32} + C_2 K_{35}) & K_{33} & K_{34} \\ 0 & 0 & K_{43} & K_{44} \\ (K_{51} + C_1 K_{55}) & (K_{52} + C_2 K_{55}) & K_{53} & 0 \end{bmatrix}} \underset{4 \times 1}{\begin{Bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \end{Bmatrix}} \tag{3.7a}$$

The number of unknowns is thus reduced to 4. However, there are five equations, the last of which is redundant. Also, the stiffness matrix in the above equation is not -symmetrical. To achieve symmetry, the fifth equation above is expanded as follows:

$$P_5 = (K_{51} + C_1K_{55})\,D_1 + (K_{52} + C_2K_{55})\,D_2 + K_{53}\,D_3 \qquad (3.8)$$

Multiplying both sides of Eq. 3.8 by C_1,

$$C_1P_5 = C_1(K_{51} + C_1K_{55})\,D_1 + C_1\,(K_{52} + C_2K_{55})\,D_2 + C_1K_{53}D_3 \qquad (3.9)$$

Eq. 3.8 can be multiplied by C_2 to give:

$$C_2P_5 = C_2\,(K_{51} + C_1K_{55})\,D_1 + C_2\,(K_{52} + C_2K_{55})\,D_2 + C_2K_{53}D_3 \qquad (3.10)$$

Expanding the first two of Eqs. (3.7a)

$$P_1 = (K_{11} + C_1K_{15})\,D_1 + (K_{12} + C_2K_{15})\,D_2 + K_{13}D_3 \qquad (3.11)$$

$$P_2 = (K_{21} + C_1K_{25})\,D_1 + (K_{22} + C_2K_{25})\,D_2 + K_{23}D_3 \qquad (3.12)$$

Adding Eqs. 3.11 and 3.9,

$$P_1 + C_1P_5 = \{(K_{11} + C_1K_{15}) + C_1\,(K_{51} + C_1K_{55})\}\,D_1 +$$

$$\{(K_{12} + C_2K_{15}) + C_1\,(K_{52} + C_2K_{55})\}\,D_2 + (K_{13} + C_1K_{53})\,D_3 \qquad (3.13)$$

Since, from symmetry, $K_{15} = K_{51}$, the Eq. (3.13) can be rewritten as

$$P_1 + C_1P_5 = (K_{11} + 2C_1K_{15}+C_1^2K_{55})\ D_1 + (K_{12} + C_2K_{15} + C_1K_{52} + C_1C_2K_{55})\ D_2$$
$$+ (K_{13} + C_1K_{53})\ D_3 \qquad (3.13a)$$

Similarly, addition of Eqs. 3.12 and 3.10 yields

$$P_2 + C_2P_5 = \{(K_{21} + C_1K_{25}) + C_2\ (K_{51} + C_1K_{55})\}\ D_1 + \{(K_{22} + C_2K_{25}) + C_2$$
$$(K_{52} + C_2K_{55})\}\ D_2 + (K_{23} + C_2K_{53})\ D_3 \qquad (3.14)$$

or, since $K_{52} = K_{25}$,

$$P_2 + C_2P_5 = (K_{21} + C_1K_{25} + C_2K_{51} + C_1C_2K_{55})\ D_1 + (K_{22} + 2C_2K_{25} + C_2^2K_{55})\ D_2$$
$$+ (K_{23} + C_2K_{53})\ D_3$$

'P_5' is the external force vector at C, a point between A and B. Since it is assumed that the external forces are applied only at the corner nodes, P_5 can be taken as zero. Also, the fifth of equations (3.7a) is redundant and can be omitted. The final matrix after reduction is thus:

$$\begin{Bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{Bmatrix} = \begin{array}{c} \\ I_1 \\ I_2 \\ \\ \\ \end{array} \begin{array}{c} \begin{array}{cccc} J1 & J2 & & \end{array} \\ \begin{bmatrix} (K_{11} + 2C_1K_{15} + C_1^2K_{55}) & (K_{12} + C_2K_{15} + C_1K_{52} + C_1C_2K_{55}) & (K_{13} + C_1K_{53}) & 0 \\ (K_{21} + C_1K_{25} + C_2K_{51} + C_1C_2K_{55}) & (K_{22} + 2C_2K_{25} + C_2^2K_{55}) & (K_{23} + C_2K_{53}) & 0 \\ (K_{31} + C_1K_{35}) & (K_{32} + C_2K_{35}) & K_{33} & K_{34} \\ 0 & 0 & K_{43} & K_{44} \end{bmatrix} \end{array} \begin{Bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \end{Bmatrix} \tag{3.15}$$

The stiffness matrix in Eq. (3.15) is symmetrical and it can be noted that K_{14}, K_{24}, K_{41} and K_{42} remain zero after the reduction. Also, K_{33}, K_{34}, K_{43} and K_{44} remain unaltered. In other words, only the rows and columns corresponding to the terms which are related to the displacement that is being reduced are altered. These are represented by rows I1 and I2 and columns J1 and J2 in Eq. (3.15).

In general, if 'K' is the displacement being reduced and 'I' and 'J' are the displacements to which 'K' is related by an expression of the form

$$D_k = C_1 D_I + C_2 D_J$$

the terms in the Ith row and Jth row are altered as follows:

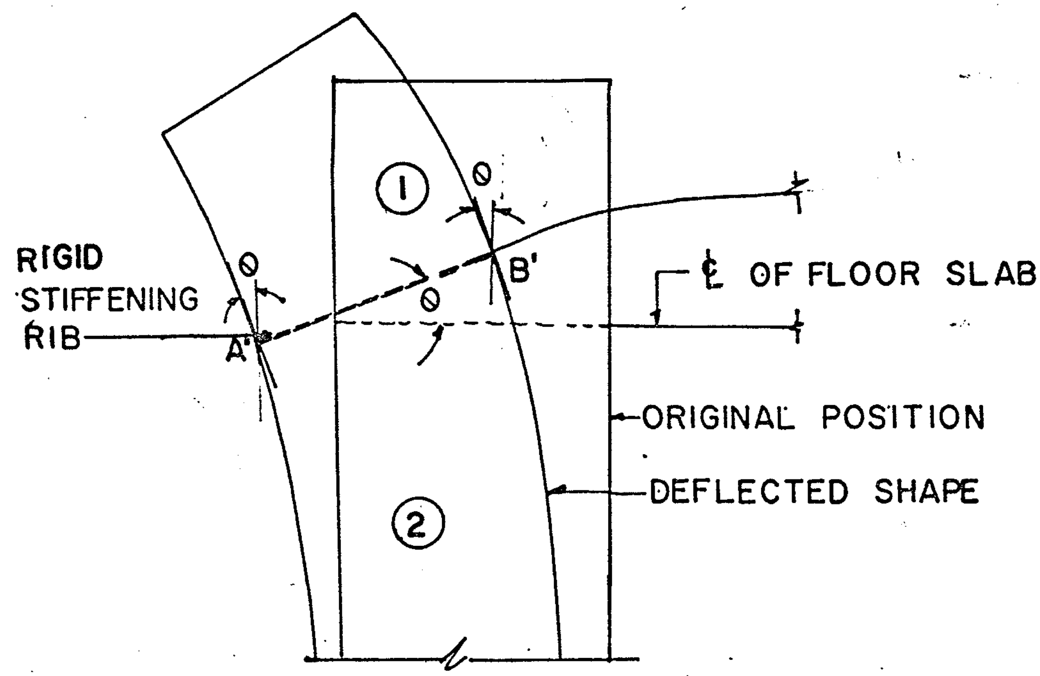
$$
\begin{array}{c}
\\
\text{Ith row} \\
\text{Jth row} \\
\\
\text{Lth row} \\
\\
\\
\end{array}
\left[
\begin{array}{l}
- - - - - - - - - - - - - - - - - - \\
\cdots\; (K_{II} + 2C_1K_{Ik} + C_1^2K_{kk}) \;\cdots\; (K_{IJ}+C_2K_{Ik}+C_1K_{JK}+C_1C_2K_{kk}) \;\cdots\; (K_{IL}+C_1K_{KL}) \\
\cdots\; (K_{JI} + C_1K_{JK} + C_2K_{Ik} + C_1C_2K_{kk}) \;\cdots\; (K_{JJ}+2C_2K_{JK}+C_2^2K_{kk}) \;\cdots \\
- - - - - - - - - - - - - - - - - - \\
\cdots\; K_{LI} + C_1K_{Ik} - - - - - - - - - - - - - \\
- - - - - - - - - - - - - - - - - - \\
- - - - - - - - - - - - - - - - - - \\
\end{array}
\right]
$$

where L is any other term, related to I, J or K, that -undergoes subsequent changes. Therefore, the actions that are not directly related to I, J or K do not undergo any change during the modification.

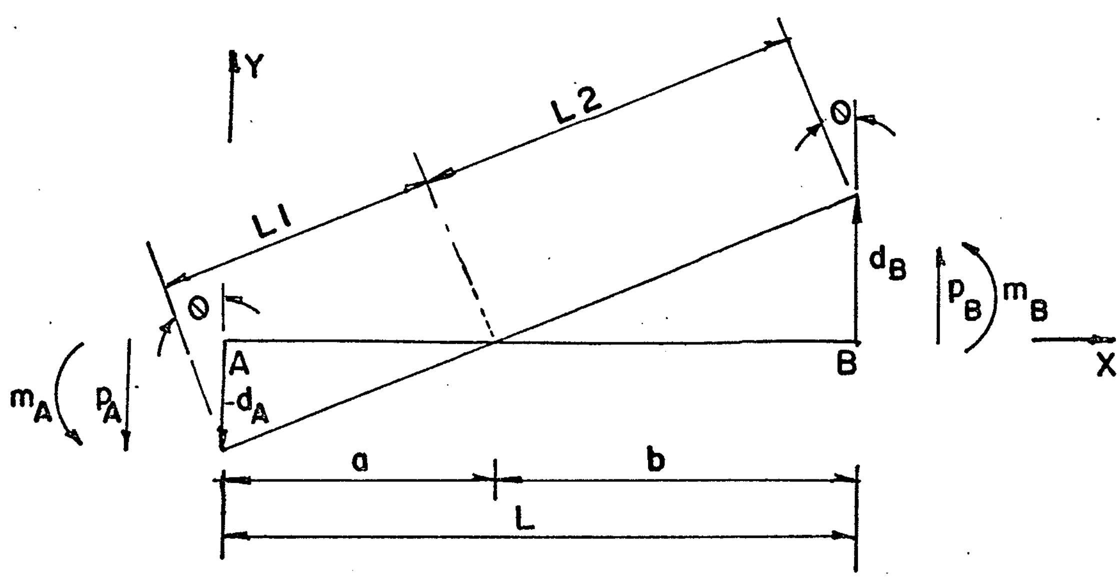
Any number of internal nodal displacement vectors on a boundary between two segments can be related to the end displacements in turn and the size of stiffness matrix can be correspondingly reduced.

3.5(ii) REDUCTION OF END ROTATIONS OF AN EDGE

When a beam with a large bending stiffness frames into -a shear wall at any level, there is a considerable amount of local deformation at the junction. This results in artificially large values of rotational displacement, since the moment applied to the shear wall by the beam is applied as a concentrated couple at a point, rather than being distributed over the finite beam depth. The error is reduced for comparatively slender beams. To avoid this problem, the rotations at the two ends of a shear wall segment boundary are assumed equal. This assumption is consistent with that made earlier, that the plane transverse sections through the shear wall remain plane during bending. The assumption is also equivalent to assuming a rigid horizontal stiffening rib attached to the shear wall at each floor level. Consequently, the rotation at the ends of the imaginary rib can be related to the vertical displacement at its ends. This approach not only avoids the problem of local



(a) ROTATION OF SHEAR WALL



(b) GEOMETRY OF END ROTATION

FIG· 3·15

deformation, it also reduces the number of generalized coordinates by two at each segment boundary and, hence, reduces the number of equations to be solved. The corresponding modification to the shear wall stiffness is outlined here. The reduction of the beam end rotation at the junction of a beam and a shear wall is explained in the next chapter.

Fig. 3.15(a) shows an imaginary stiffening rib AB on the boundary between two shear wall segments 1 and 2. The displacements of the rib during loading are shown in Fig. 3.15(b).

It can be seen from the figure that

$$\text{Sin } \theta = d_B/L_2 = -d_A/L_1 = \theta$$

(since, for small angles, $\sin\theta \simeq \theta$).

Also,

$$L_1 = L - L_2 = L - \frac{d_B}{\theta} = -\frac{d_A}{\theta}$$

Thus,

$$\frac{1}{\theta}(d_B - d_A) = L \qquad \text{or}$$

$$\theta = (d_B - d_A)/L \tag{3.16}$$

where θ is the shear wall rotation at each end of the stiffening rib. Thus, the end rotations can be related to the vertical deflections at the ends of the rib. Similarly, the force components (rotational) at the two ends can be written as.

$$(m_B + m_A)/L = -p_A = p_B$$

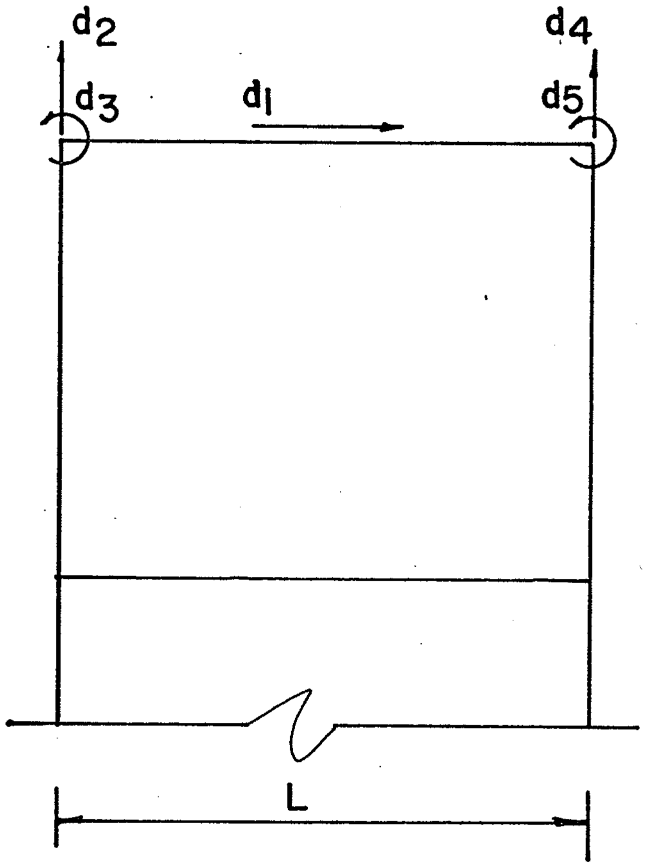


FIG. 3.16 DISPLACEMENT VECTOR FOR REDUCTION OF END ROTATIONS

The stiffness matrix for each shear wall segment can thus be modified using relationships of the form of Eqs. (3.16) and (3.17) to reflect the reduced number of degrees of freedom for the segment.

Fig. (3.16) shows the displacement vector for a typical segment. From Eqs. (3.16) and (3.17),

$$d_3 = d_5 = \frac{d_4 - d_2}{L} \tag{3.16a}$$

and

$$-p_2 = p_4 = \frac{p_3 + p_5}{L} \tag{3.17a}$$

The equilibrium equations relating displacement components d_1 to d_5 and force components p_1 to p_5, with Eq. (3.16a) employed to eliminate d_3 and d_5, are:

$$\begin{Bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \end{Bmatrix} = \begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} & K_{15} \\ K_{21} & K_{22} & K_{23} & K_{24} & K_{25} \\ K_{31} & K_{32} & K_{33} & K_{34} & K_{35} \\ K_{41} & K_{42} & K_{43} & K_{44} & K_{45} \\ K_{51} & K_{52} & K_{53} & K_{54} & K_{55} \end{bmatrix} \begin{Bmatrix} d_1 \\ d_2 \\ \dfrac{d_4 - d_2}{L} \\ d_4 \\ \dfrac{d_4 - d_2}{L} \end{Bmatrix} \tag{3.18}$$

Expanding each of the Eqs. 3.18,

$$\left.\begin{aligned}
p_1 &= K_{11}\, d_1 + \left(K_{12} - \frac{K_{13}}{L} - \frac{K_{15}}{L}\right) d_2 + \left(K_{14} \pm \frac{K_{13}}{L} + \frac{K_{15}}{L}\right) d_4 \\
p_2 &= K_{21}\, d_1 + \left(K_{22} - \frac{K_{23}}{L} - \frac{K_{25}}{L}\right) d_2 + \left(K_{24} + \frac{K_{23}}{L} + \frac{K_{25}}{L}\right) d_4 \\
p_3 &= K_{31}\, d_1 + \left(K_{32} - \frac{K_{33}}{L} - \frac{K_{35}}{L}\right) d_2 + \left(K_{34} + \frac{K_{33}}{L} + \frac{K_{35}}{L}\right) d_4 \\
p_4 &= K_{41}\, d_1 + \left(K_{42} - \frac{K_{43}}{L} - \frac{K_{45}}{L}\right) d_2 + \left(K_{44} + \frac{K_{43}}{L} + \frac{K_{45}}{L}\right) d_4 \\
p_5 &= K_{51}\, d_1 + \left(K_{52} - \frac{K_{53}}{L} - \frac{K_{55}}{L}\right) d_2 + \left(K_{54} + \frac{K_{53}}{L} + \frac{K_{55}}{L}\right) d_4
\end{aligned}\right\} \qquad (3.18a)$$

From these equations,

$$\frac{p_3}{L} = K_{31}\, d_1 + \frac{1}{L}\left(K_{32} - \frac{K_{33}}{L} - \frac{K_{35}}{L}\right) d_2 + \frac{1}{L}\left(K_{34} + \frac{K_{33}}{L} + \frac{K_{35}}{L}\right) d_4$$

$$\frac{p_5}{L} = K_{51}\, d_1 + \frac{1}{L}\left(K_{52} - \frac{K_{53}}{L} - \frac{K_{55}}{L}\right) d_2 + \frac{1}{L}\left(K_{54} + \frac{K_{53}}{L} + \frac{K_{55}}{L}\right) d_4$$

$$\begin{aligned}
p_2 - \left(\frac{p_3}{L} + \frac{p_5}{L}\right) = {} & \left(K_{21} - \frac{K_{31}}{L} - \frac{K_{51}}{L}\right) d_1 + \Big\{ \left(K_{22} - \frac{K_{23}}{L} - \frac{K_{25}}{L}\right) \\
& - \frac{1}{L}\left(K_{32} - \frac{K_{33}}{L} - \frac{K_{35}}{L}\right) - \frac{1}{L}\left(K_{52} - \frac{K_{53}}{L} - \frac{K_{55}}{L}\right) \Big\}\, d_2 \\
& + \Big\{\left(K_{24} + \frac{K_{23}}{L} + \frac{K_{25}}{L}\right) - \frac{1}{L}\left(K_{34} + \frac{K_{33}}{L} + \frac{K_{35}}{L}\right) - \frac{1}{L}\left(K_{54} + \frac{K_{53}}{L} + \frac{K_{55}}{L}\right)\Big\}\, d_4
\end{aligned}$$

$$\begin{aligned}
p_4 + \frac{p_3}{L} + \frac{p_5}{L} = {} & \left(K_{41} + \frac{K_{31}}{L} + \frac{K_{51}}{L}\right) d_1 + \Big\{ \left(K_{42} - \frac{K_{43}}{L} - \frac{K_{45}}{L}\right) \\
& + \frac{1}{L}\left(K_{32} - \frac{K_{33}}{L} - \frac{K_{35}}{L}\right) + \frac{1}{L}\left(K_{52} - \frac{K_{53}}{L} - \frac{K_{55}}{L}\right) \Big\}\, d_2 \\
& + \Big\{\left(K_{44} + \frac{K_{43}}{L} + \frac{K_{45}}{L}\right) + \frac{1}{L}\left(K_{34} - \frac{K_{33}}{L} + \frac{K_{35}}{L}\right) + \frac{1}{L}\left(K_{54} + \frac{K_{53}}{L} + \frac{K_{55}}{L}\right)\Big\}\, d_4
\end{aligned}$$

p_3 and p_5 are the external moments corresponding to displacement components d_3 and d_5. Since the structure is assumed to be loaded by horizontal external forces only, moments p_3 and p_5 are assumed to be zero. In any event, the corresponding equilibrium equations are redundant, and the equations relating forces and displacements at the top boundary of the segment reduce to the Eq. (3.19) shown in the next page.

$$\begin{Bmatrix} P_1 \\ P_2 \\ P_4 \end{Bmatrix} = \begin{bmatrix} K_{11} & (K_{12} - \frac{K_{13}}{L} - \frac{K_{15}}{L}) & (K_{14} + \frac{K_{13}}{L} + \frac{K_{15}}{L}) \\ & (K_{22} - 2\frac{K_{23}}{L} - 2\frac{K_{25}}{L} + \frac{K_{33}}{L^2} + 2\frac{K_{53}}{L^2} + \frac{K_{55}}{L^2}) & (K_{24} + \frac{K_{23}}{L} + \frac{K_{25}}{L} - \frac{K_{34}}{L} - \frac{K_{33}}{L^2} - \frac{K_{55}}{L^2} - \frac{K_{54}}{L} - \frac{K_{53}}{L^2} - \frac{K_{55}}{L^2}) \\ \text{Symmetrical} & & (K_{44} + 2\frac{K_{34}}{L} + 2\frac{K_{45}}{L} + \frac{K_{33}}{L^2} + 2\frac{K_{35}}{L^2} + \frac{K_{55}}{L^2}) \end{bmatrix} \begin{Bmatrix} d_1 \\ d_2 \\ d_4 \end{Bmatrix} \qquad (3.19)$$

3.6 SUMMARY

In summary, the shear wall stiffness matrix is generated by subdividing the wall into segments, each segment representing the portion of the shear wall between two consecutive floors.

Then proceeding from the top segment on any wall and working toward the base of the wall, the following operations are performed for each segment:

(a) The segment is subdivided into a rectangular finite element array.

(b) Using the procedure described in Section 3.4, the interior nodes for the segment (all nodes except on the top and bottom boundaries of the segment) are condensed out of the segment stiffness matrix. That is, the matrix is modified to relate forces and displacements along the top and bottom boundaries of the segment.

(c) The stiffness matrix for the segment is superimposed on that portion of the shear wall above it. Then employing the assumptions discussed in Section 3.5, all nodes along the upper boundary of the segment, except the end nodes, are condensed off.

(d) Finally, using the assumption that plane transverse sections of the shear wall remain plane during bending, the rows and columns of the stiffness matrix corresponding to the rotational degrees of freedom at the ends of the upper boundary of the segment, are eliminated.

When all segments have been considered, the resulting stiffness matrix relates vertical and horizontal forces applied to the shear wall at each floor level to the resulting displacements.

After the complete structure has been analyzed, the relationships described in Section 3.5 can be used to calculate the moments and rotations at the edges of the shear wall at each floor level.

CHAPTER IV

ANALYSIS OF FRAME

4.1 INTRODUCTION

The generation of the stiffness matrix for the frame is easier than that for the shear wall, since the frame is already idealized as consisting of discrete elements, the elements being beams and columns. The individual stiffness matrices for the beams and columns can be calculated exactly, in closed form, using the principle of virtual work or classical theories. However, to be consistent, the stiffness matrix for the frame should be calculated with an accuracy comparable to that of shear wall stiffness matrix. For this reason, only effective lengths of beams and columns are considered for the purpose of analysis. By effective length is meant the clear length between support faces. However, the beams and columns have finite depths which result in discrepancies between the member end displacements and the corresponding joint displacements. The stiffness matrices for the various members can, however, be developed in terms of clear lengths and then modified to relate forces and displacements at the joint. The joints are considered to be rigid and as shown in Fig. 4.1, any point in the

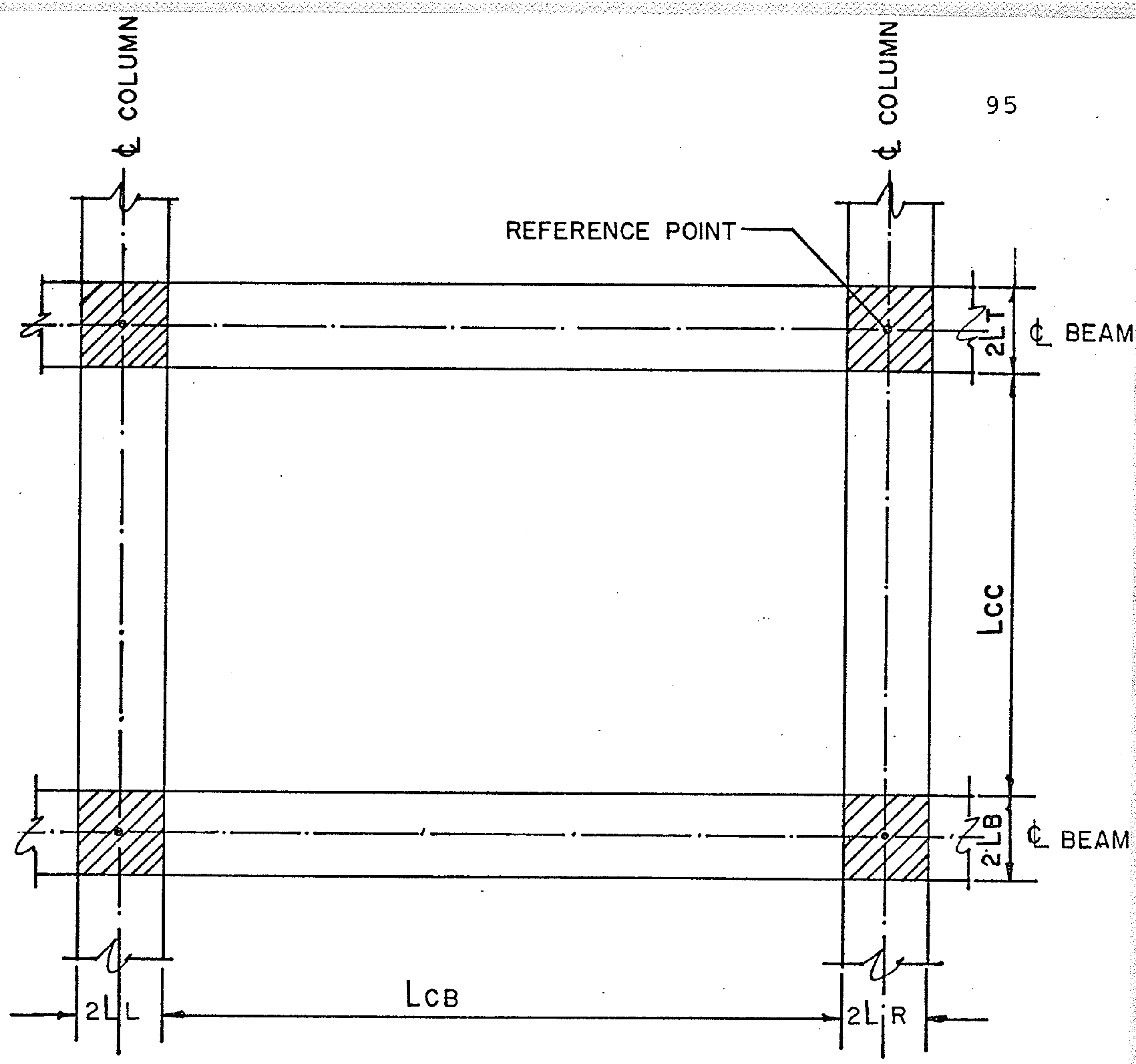


FIG. 4.1 INTERSECTION OF COLUMNS & BEAMS

LCB – CLEAR LENGTH OF BEAM

LCC – CLEAR LENGTH OF COLUMN

LL – LEFT RIGID LENGTH OF BEAM

LR – RIGHT RIGID LENGTH OF BEAM

LT – TOP RIGID LENGTH OF COLUMN

LB – BOTTOM RIGID LENGTH OF COLUMN

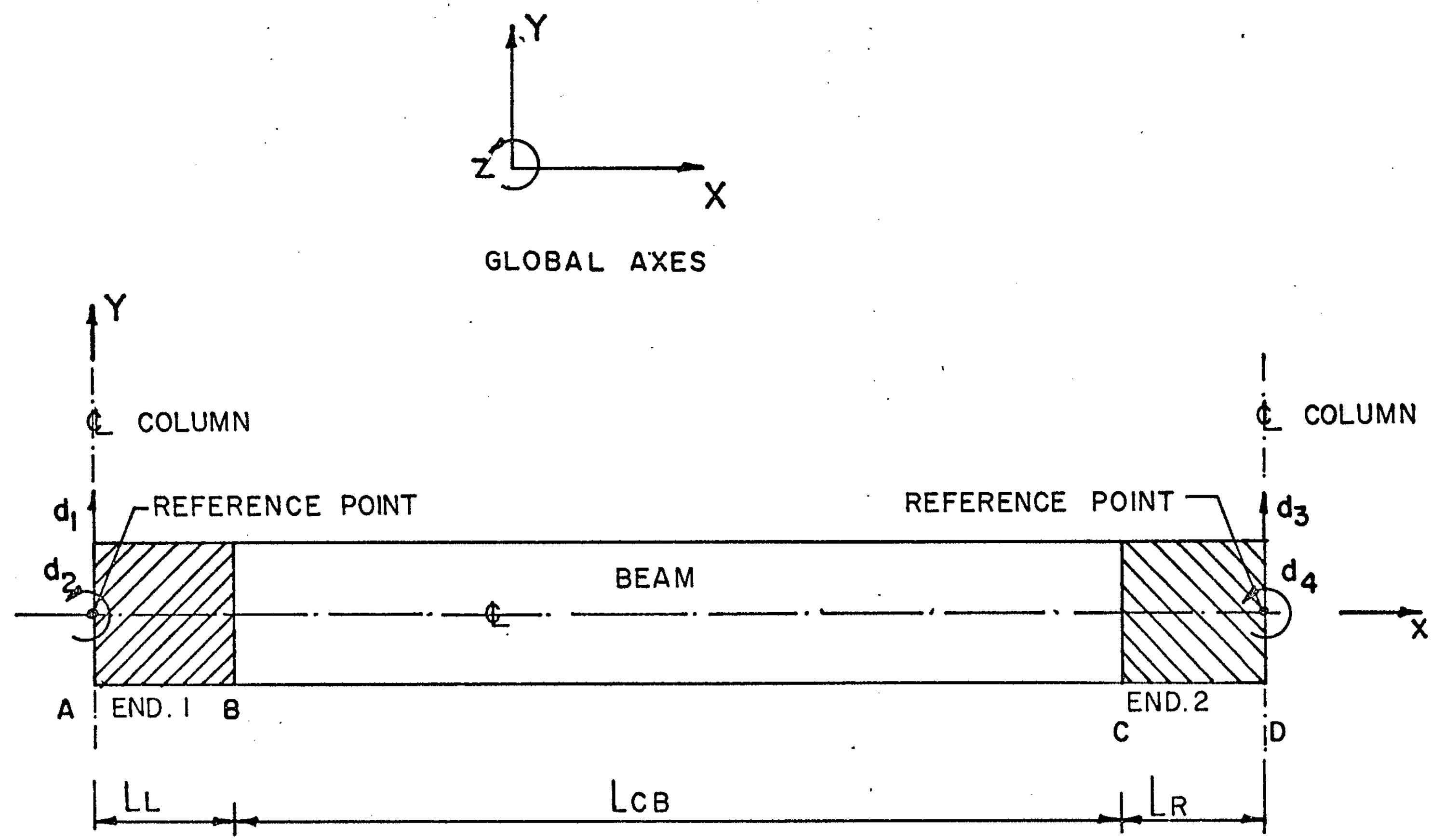
hatched portion undergoes the same displacement. The reference point is taken as the intersection of the centre lines of columns and beams for any particular joint.

The member local coordinate system is assumed as follows:

The X-axis lies along the axis of the member and axes y and Z are parallel to the principal axes of the member cross section. They are assumed to be positive according to a right hand coordinate system. Wherever necessary, the local system is identified with primes.

4.2 STIFFNESS MATRIX FOR BEAM

Since it is assumed that all floor slabs are infinitely rigid in their own planes, there is a rigid body translation of the beam in the horizontal direction. In other words, the axial deformation of the beam is considered negligible. Hence, there are only four displacement components to be considered in developing the stiffness matrix for the beam. They are shown in Fig. 4.2. It can be noted that the local coordinate system for the beam coincides with the global system for the structure. Hence, no rotation transformation of the beam stiffness matrix is required. With the four generalized coordinates, shown in Fig. 4.2, and ignoring shearing deformations, the stiffness matrix for the beam can



L_L LEFT RIGID LENGTH

L_R RIGHT RIGID LENGTH

L_{CB} CLEAR LENGTH

FIG. 4.2 BEAM WITH END RIGID LENGTHS

be written as follows:

$$\left[\begin{array}{cc|cc} \beta & \frac{\beta L}{2} & -\beta & \frac{\beta L}{2} \\ & K_{BB} & & K_{BC} \\ \frac{\beta L}{2} & \frac{\beta L^2}{3} & -\frac{\beta L}{2} & \frac{\beta L^2}{6} \\ \hline -\beta & -\frac{\beta L}{2} & \beta & -\frac{\beta L}{2} \\ & K_{CB} & & K_{CC} \\ \frac{\beta L}{2} & \frac{\beta L^2}{6} & -\frac{\beta L}{2} & \frac{\beta L^2}{3} \end{array}\right] \qquad (4.1)$$

where $\beta = 12EI/L^3$

E and I are the modulus of elasticity and moment of inertia of the beam, respectively.

The transformation of the stiffness matrix to the end reference points is achieved by the following procedure.

4.2(i) STIFFNESS MATRIX FOR END 1

Member BC is connected to rigid bodies AB and CD. Let the displacements and forces at A be D_A and P_A, respectively, where

$$D_A = \begin{Bmatrix} dA_1 \\ dA_2 \end{Bmatrix} \qquad P_A = \begin{Bmatrix} pA_1 \\ pA_2 \end{Bmatrix}$$

dA_1, dA_2 and pA_1, pA_2 are the displacement and force components respectively at A.

Similarly, the displacements and forces at B can be represented by the corresponding vectors, D_B and P_B, respectively.

The force transformation from B to A is achieved by the following transformation:

$$P_A = H_{AB}\ P_B \tag{4.2}$$

where

$$P_A = \begin{Bmatrix} pA_1 \\ pA_2 \end{Bmatrix}, \quad H_{AB} = \begin{bmatrix} 1 & 0 \\ L_L & 1 \end{bmatrix} \quad \text{and} \quad P_B = \begin{Bmatrix} pB_1 \\ pB_2 \end{Bmatrix}$$

For the rigid body translation, the displacements at B can be expressed in terms of the displacement at A using the displacement transformation:

$$\begin{Bmatrix} dB_1 \\ dB_2 \end{Bmatrix} = \begin{bmatrix} 1 & L_L \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} dA_1 \\ dA_2 \end{Bmatrix}$$

It can be seen from equation (4.2) that

$$D_B = (H_{AB})^T\ D_A \tag{4.3}$$

The displacement transformation from D to C can be written as

$$\begin{Bmatrix} dC_1 \\ dC_2 \end{Bmatrix} = \begin{bmatrix} 1 & -L_R \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} dD_1 \\ dD_2 \end{Bmatrix}$$

In symbolic form,

$$D_C = (H_{CD}^{-1})^T\ D_D \tag{4.4}$$

where

$$H_{CD} = \begin{bmatrix} 1 & 0 \\ L_R & 1 \end{bmatrix}$$

H_{CD} is the force transformation matrix from D to C.

Considering member BC in Figure 4.2, the force at B can be written as,

$$P_B = K_{BB} D_B + K_{BC} D_C \tag{4.5}$$

where K_{BB} relates force at B due to displacements at B, K_{BC} relates force at B due to displacements at C, D_B is the displacement vector at B and D_C is the displacement vector at C.

Substituting for D_B and D_C from equations (4.3) and (4.4),

$$P_B = K_{BB} H_{AB}^T D_A + K_{BC} (H_{CD}^{-1}) D_D \tag{4.6}$$

But, from equation (4.2),

$$P_A = H_{AB} P_B$$

Therefore, substituting for P_B from (4.6),

$$P_A = (H_{AB} K_{BB} H_{AB}^T) D_A + (H_{AB} K_{BC} (H_{CD}^{-1})^T) D_D \tag{4.7}$$

It is clear from this equation that

$$P_A = K_{AA} D_A + K_{AD} D_D$$

where

$K_{AA} = (H_{AB} K_{BB} H_{AB}^T)$ is the force vector at A due to unit displacement at A, with $D_B = 0$,

and

$K_{AD} = (H_{AB} \; K_{BC} \; (H_{CD}^{-1})^T)$ is the force vector at A due to unit displacement at D, with $D_A = 0$.

It should be noted here that the stiffness matrix K_{BB} reflects deformations of member BC only. From equation (4.1),

$$K_{BB} = \begin{bmatrix} \beta & \beta L/2 \\ \beta L/2 & \beta L^2/3 \end{bmatrix}, \quad K_{BC} = \begin{bmatrix} -\beta & \beta L/2 \\ -\beta L/2 & \beta L^2/6 \end{bmatrix}$$

Therefore,

$K_{AA} = H_{AB} \; K_{BB} \; H_{AB}^T$

$$= \begin{bmatrix} 1 & 0 \\ L_L & 0 \end{bmatrix} \begin{bmatrix} \beta & \frac{\beta L_{CB}}{2} \\ \frac{\beta L_{CB}}{2} & \frac{\beta L^2_{CB}}{3} \end{bmatrix} \begin{bmatrix} 1 & L_L \\ 0 & 1 \end{bmatrix}$$

or,

$$K_{AA} = \begin{bmatrix} \beta & \beta(L_L + \frac{L_{CB}}{2}) \\ \beta(L_L + \frac{L_{CB}}{2}) & \beta L_L(L_L + L_{CB}) + \frac{\beta L_{CB}^2}{3} \end{bmatrix} \qquad (4.8)$$

Similarly,

$$K_{AD} = H_{AB}\, K_{BC}\, (H_{CD}^{-1})^{T}$$

$$= \begin{bmatrix} 1 & 0 \\ L_L & 0 \end{bmatrix} \begin{bmatrix} -\beta & \beta \dfrac{L_{CB}}{2} \\ -\beta \dfrac{L_{CB}}{2} & \beta \dfrac{L_{CB}^2}{6} \end{bmatrix} \begin{bmatrix} 1 & -L_R \\ 0 & 1 \end{bmatrix}$$

which reduces to

$$K_{AD} = \begin{bmatrix} -\beta & \beta\left(L_R + \dfrac{L_{CB}}{2}\right) \\ -\beta\left(L_L + \dfrac{L_{CB}}{2}\right) & \beta\left(L_L L_R + \dfrac{L_{CB}}{2}(L_L + L_R) + \dfrac{L_{CB}^2}{3}\right) \end{bmatrix} \qquad (4.9)$$

4.2(ii) STIFFNESS MATRIX FOR END 2

The force transformation matrix from C to D is obtained using the following transformation.

$$\begin{Bmatrix} pD_1 \\ pD_2 \end{Bmatrix} = \begin{bmatrix} 1 & 0 \\ -L_R & 1 \end{bmatrix} \begin{Bmatrix} pC_1 \\ pC_2 \end{Bmatrix}$$

pC_1, pC_2 and pD_1, pD_2 are the force components at C & D respectively.

or,

$$P_D = H_{CD}^{-1}\, P_C \qquad (4.10)$$

The force at C is

$$P_C = K_{CC}\, D_C + K_{CB}\, D_B \qquad (4.11)$$

where K_{CC} is the force vector at C due to a unit displacement vector at C, and K_{CB} is the force vector at C due to a unit displacement vector at B. D_C and D_{B-} are the displacement vectors at C and B, respectively.

Substituting for D_B and D_C from Eqs. (4.3) and (4.4),

$$P_C = K_{CC}(H_{CD}^{-1})^T D_D + K_{CB}\ H_{AB}^T D_A \qquad (4.12)$$

But from equation (4.10),

$$P_D = H_{CD}^{-1} \cdot P_C$$

Therefore,

$$P_D = (H_{CD}^{-1}\ K_{CC}\ (H_{CD}^{-1})^T)\ D_D + (H_{CD}^{-1}\ K_{CB}\ H_{AB}^T)\ D_A \qquad (4.13)$$

It is clear from the above equation that

$K_{DD} = H_{CD}^{-1}\ K_{CC}\ (H_{CD}^{-1})^T$ and

$K_{DA} = H_{CD}^{-1}\ K_{CB}\ H_{AB}^T$

Again, K_{CC} and K_{CB} reflect the deformations of member BC only.

From Eq. (4.1),

$$K_{CC} = \begin{bmatrix} \beta & -\beta \dfrac{L_{CB}}{2} \\ -\beta \dfrac{L_{CB}}{2} & \beta \dfrac{L_{CB}^2}{3} \end{bmatrix} \quad \text{and } K_{CB} = \begin{bmatrix} -\beta & -\dfrac{\beta L_{CB}}{2} \\ \beta \dfrac{L_{CB}}{2} & \beta \dfrac{L_{CB}^2}{6} \end{bmatrix}$$

Therefore,

$$K_{DD} = H_{CD}^{-1} \cdot K_{CC}\ (H_{CD}^{-1})^T$$

$$= \begin{bmatrix} 1 & 0 \\ -L_R & 0 \end{bmatrix} \begin{bmatrix} \beta & -\dfrac{\beta L_{CB}}{2} \\ -\dfrac{\beta L_{CB}}{2} & \dfrac{\beta L_{CB}^2}{3} \end{bmatrix} \begin{bmatrix} 1 & -L_R \\ 0 & 1 \end{bmatrix}$$

Or

$$K_{DD} = \begin{bmatrix} \beta & -\beta\left(L_R + \frac{L_{CB}}{2}\right) \\ -\beta\left(L_R + \frac{L_{CB}}{2}\right) & \beta L_R\,(L_R + L_{CB}) + \beta \frac{L_{CB}^{2}}{3} \end{bmatrix} \tag{4.14}$$

And

$$K_{DA} = H_{CD}^{-1}\, K_{CB}\, H_{AB}^{T}$$

$$= \begin{bmatrix} 1 & 0 \\ -L_R & 1 \end{bmatrix} \begin{bmatrix} -\beta & -\beta \frac{L_{CB}}{2} \\ \beta \frac{L_{CB}}{2} & \beta \frac{L_{CB}^{2}}{6} \end{bmatrix} \begin{bmatrix} 1 & L_L \\ 0 & 1 \end{bmatrix}$$

Or

$$K_{DA} = \begin{bmatrix} -\beta & -\beta\left(L_L + \frac{L_{CB}}{2}\right) \\ \beta\left(L_R + \frac{L_{CB}}{2}\right) & \beta\left(L_L L_R + \frac{L_{CB}}{2}(L_L + L_R) + \frac{L_{CB}^{2}}{6}\right) \end{bmatrix} \tag{4.15}$$

Note that $K_{DA} = K_{AD}^{T}$, in accordance with the Maxwell-Betti reciprocal theorem.

The final stiffness matrix for the beam can be written as,

$$K = \begin{bmatrix} K_{AA} & K_{AD} \\ K_{DA} & K_{DD} \end{bmatrix}$$

Substituting for K_{AA}, K_{AD}, K_{DA} and K_{DD} from equations (4.8), (4.9), (4.14) and (4.15), we have the final matrix for the beam as shown in Table 4.1.

4.3 STIFFNESS MATRIX FOR COLUMN

Each column has three degrees of freedom at each of its ends, giving rise to six generalized coordinates as illustrated in Fig. 4.3. In general, the stiffness matrix for a planar member is given by,

$$K' = \begin{bmatrix} \frac{AE}{L} & 0 & 0 & -\frac{AE}{L} & 0 & 0 \\ 0 & \beta & \frac{\beta L}{2} & 0 & \beta & \frac{\beta L}{2} \\ 0 & \frac{\beta L}{2} & \frac{\beta L^2}{3} & 0 & -\frac{\beta L}{2} & \frac{\beta L^2}{6} \\ -\frac{AE}{L} & 0 & 0 & \frac{AE}{L} & 0 & 0 \\ 0 & \beta & -\frac{\beta L}{2} & 0 & \beta & -\frac{\beta L}{2} \\ 0 & \frac{\beta L}{2} & \frac{\beta L^2}{6} & 0 & -\frac{\beta L}{2} & \frac{\beta L^2}{3} \end{bmatrix}$$

where $\beta = 12EI/L^3$ and E and I are modulus of elasticity and moment of inertia, respectively. Shearing deformations are again considered negligible. The above stiffness matrix is expressed in terms of the local coordinate system for the column, as shown in Fig. 4.3, where the primed axes represent the local system. It must therefore be given a rotation transformation to transform it to the global system before the translation is performed to refer the stiffness coefficients to the joints.

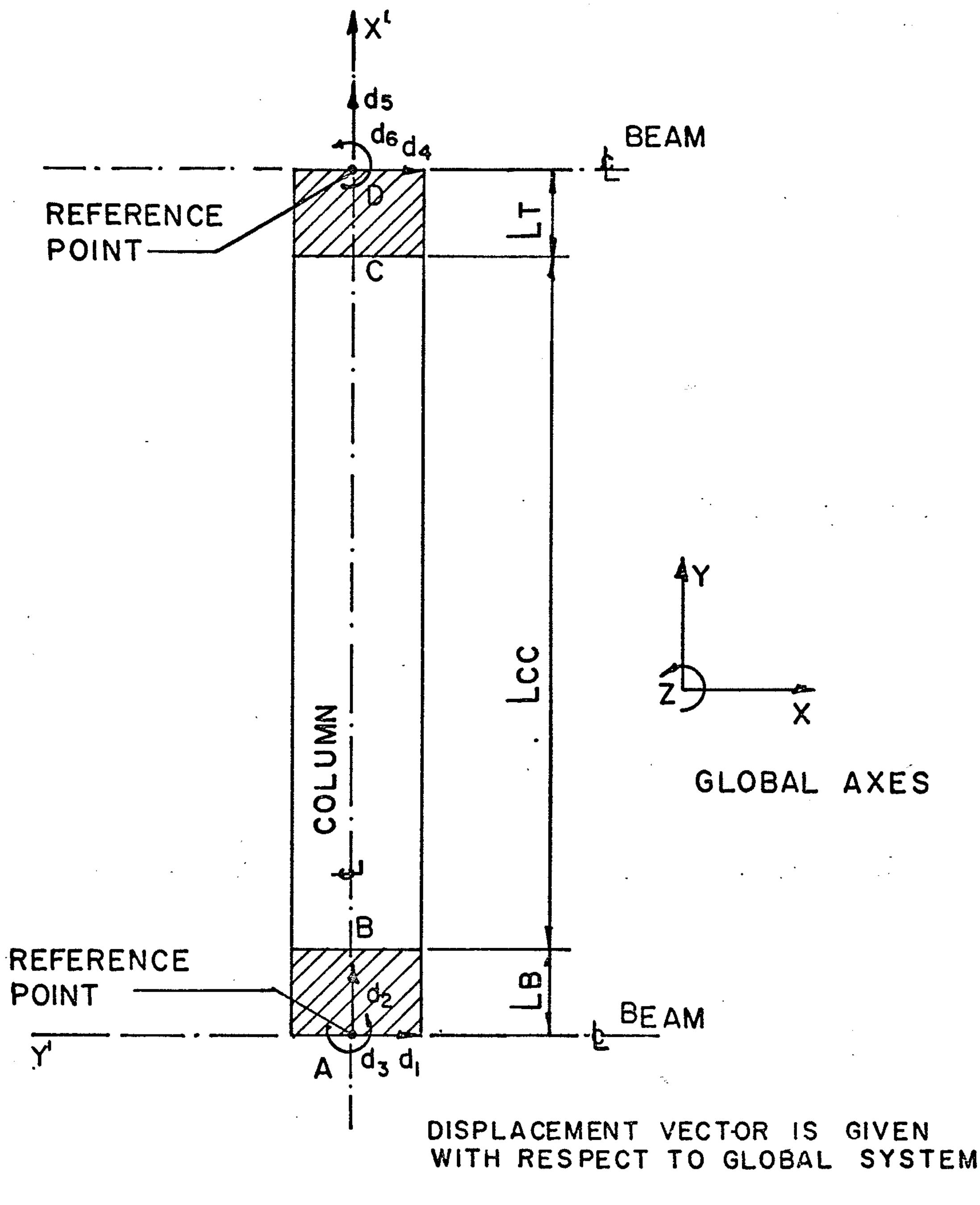
TABLE 4.1

STIFFNESS MATRIX FOR BEAM

$$
\left[\begin{array}{c|c|c|c}
\beta & \beta\left(L_L + \frac{L_{CB}}{2}\right) & -\beta & \beta\left(L_R + \frac{L_{CB}}{2}\right) \\
\hline
 & \beta L_L(L_L + L_{CB}) + \frac{\beta L_{CB}^{2}}{3} & -\beta\left(L_L + \frac{L_{CB}}{2}\right) & L_L L_R \beta + \beta \frac{L_{CB}}{2}(L_L + L_R) + \frac{\beta L^{2}_{B}}{6} \\
\hline
 & \text{Symmetric} & \beta & -\beta\left(L_R + \frac{L_{CB}}{2}\right) \\
\hline
 & & & \beta L_R(L_R + L_{CB}) + \frac{\beta L_{CB}^{2}}{3}
\end{array}\right]
$$

where

$\beta = 12EI/L_{CB}^{3}$



Lcc CLEAR LENGTH
LT TOP RIGID LENGTH
LB BOTTOM RIGID LENGTH

FIG. 4.3 COLUMN WITH END RIGID LENGTHS

The rotation transformation matrix is given by

$$R = \begin{bmatrix} 0 & -1 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & -1 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

The stiffness matrix in the global system can be obtained using $K = RK' R^T$ (where the prime represents the local coordinate system). This matrix operation is shown on the next page.

4.3(i) STIFFNESS MATRIX FOR END 1

The force transformation matrix from B to A and D to C, both in the global system, are given by

$$H_{AB} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -L_B & 0 & 1 \end{bmatrix} \quad \text{and} \quad H_{CD} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -L_T & 0 & 1 \end{bmatrix}$$

As shown for the beam,

$$P_A = H_{AB} P_B \qquad (4.18)$$

and

$$D_B = H_{AB}^T D_A \qquad (4.19)$$

where P_A and P_B represent forces at A and B, respectively, and D_A and D_B represent the corresponding displacements.

$$
K = \underset{R}{\begin{bmatrix} 0 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}}
\underset{K'}{\begin{bmatrix} AE/L & 0 & 0 & -AE/L & 0 & 0 \\ 0 & \beta & \beta L/2 & 0 & \beta & \beta L/2 \\ 0 & \beta L/2 & \beta L^2/3 & 0 & -\beta L/2 & \beta L^2/6 \\ -AE/L & 0 & 0 & AE/L & 0 & 0 \\ 0 & \beta & -\beta L/2 & 0 & \beta & -\beta L/2 \\ 0 & \beta L/2 & \beta L^2/6 & 0 & -\beta L/2 & \beta L^2/3 \end{bmatrix}}
\underset{R^T}{\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}}
$$

$$
\text{or } K = \begin{array}{c} \begin{array}{cc} K_{BB} \qquad\qquad & \qquad\qquad K_{BC} \end{array} \\ \left[\begin{array}{ccc|ccc} \beta & 0 & -\beta L/2 & -\beta & 0 & -\beta L/2 \\ 0 & AE/L & 0 & 0 & -AE/L & 0 \\ -\beta L/2 & 0 & \beta L^2/3 & \beta L/2 & 0 & \beta L^2/6 \\ \hline -\beta & 0 & \beta L/2 & \beta & 0 & \beta L/2 \\ 0 & -AE/L & 0 & 0 & AE/L & 0 \\ -\beta L/2 & 0 & \beta L^2/6 & \beta L/2 & 0 & \beta L^2/3 \end{array}\right] \\ \begin{array}{cc} K_{CB} \qquad\qquad & \qquad\qquad K_{CC} \end{array} \end{array} \qquad (4.17)
$$

As before,

$$P_A = (H_{AB}\ K_{BB}\ H_{AB}^T)\ D_A + (H_{AB}\ K_{BC}\ (H_{CD}^{-1})^T)\ D_D$$

$$= K_{AA}\ D_A + K_{AD}\ D_D \qquad (4.20)$$

K_{BB} and K_{BC} are taken from equation (4.17). Therefore,

$$K_{AA} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -L_B & 0 & 1 \end{bmatrix} \begin{bmatrix} \beta & 0 & -\beta L/2 \\ 0 & AE/L & 0 \\ -\beta L/2 & 0 & \beta L^2/3 \end{bmatrix} \begin{bmatrix} 1 & 0 & -L_B \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

or,

$$K_{AA} = \begin{bmatrix} \beta & 0 & -\beta(L_B + L_{CC}/2) \\ 0 & AE/L & 0 \\ -\beta(L_B + L_{CC}/2) & 0 & \beta L_B (L_B + L_{CC}) + \beta \frac{L_{CC}^2}{3} \end{bmatrix} \qquad (4.21)$$

Similarly, $K_{AD} = H_{AB} K_{BC} (H_{CD}^{-1})^T$. Thus,

$$K_{AD} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -L_B & 0 & 1 \end{bmatrix} \begin{bmatrix} -\beta & 0 & -\beta L_{CC}/2 \\ 0 & AE/L & 0 \\ \beta L/2 & 0 & \beta L^2/6 \end{bmatrix} \begin{bmatrix} 1 & 0 & L_T \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

from which

$$K_{AD} = \begin{bmatrix} -\beta & 0 & -\beta\left(L_T + \frac{L_{CC}}{2}\right) \\ 0 & -AE/L & 0 \\ \beta\left(L_B + \frac{L_{CC}}{2}\right) & 0 & \beta\left(L_T L_B + \frac{L_B L_{CC}}{2} + \frac{L_T L_{CC}}{2} + \frac{L_{CC}^2}{6}\right) \end{bmatrix} \qquad (4.22)$$

4.3 (ii) STIFFNESS MATRIX FOR END 2

Proceeding on a similar basis to that above,

$$P_D = H_{CD}^{-1} P_C$$

$$P_C = K_{CC} D_C + K_{CB} D_B$$

$$= K_{CC} (H_{CD}^{-1})^T D_D + K_{CB} H_{AB}^T D_A$$

$$\text{or } P_D = H_{CD}^{-1} K_{CC} (H_{CD}^{-1})^T D_C + H_{CD}^{-1} K_{CB} H_{AB}^T D_A$$

Therefore,

$$K_{DD} = H_{CD}^{-1} K_{CC} (H_{CD}^{-1})^T \qquad (4.23)$$

and

$$K_{DA} = H_{CD}^{-1} K_{CB} H_{AB}^T \qquad (4.24)$$

Thus,

$$K_{DD} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ L_T & 0 & 1 \end{bmatrix} \begin{bmatrix} \beta & 0 & \beta L_{CC}/2 \\ 0 & AE/L & 0 \\ \beta \frac{L_{CC}}{2} & 0 & \beta \frac{L_{CC}^2}{3} \end{bmatrix} \begin{bmatrix} 1 & 0 & L_T \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

or,

$$K_{DD} = \begin{bmatrix} \beta & 0 & (L_T + \frac{L_{CC}}{2})\beta \\ 0 & AE/L & 0 \\ \beta(L_T + \frac{L_{CC}}{2}) & 0 & L_T \beta (L_T + L_{CC}) + \beta \frac{L_{CC}^2}{3} \end{bmatrix} \qquad (4.25)$$

By symmetry, $K_{DA} = K_{AD}^T$. Therefore, from equation (4.22),

$$K_{DA} = \begin{bmatrix} -\beta & 0 & \beta\left(L_B + \frac{L_{CC}}{2}\right) \\ 0 & -AE/L & 0 \\ -\left(L_T + \frac{L_{CC}}{2}\right) & 0 & \beta\left(L_B L_T + L_T \frac{L_{CC}}{2} + L_B \frac{L_{CC}}{2}\right) + \beta \frac{L_{CC}^2}{6} \end{bmatrix} \tag{4.26}$$

Combining equations (4.21), (4.22), (4.25) and (4.26), we have the stiffness matrix for the column as

$$K = \begin{bmatrix} K_{AA} & K_{AD} \\ K_{DA} & K_{DD} \end{bmatrix}$$

This appears in Table 4.2.

4.4 MODIFICATION TO BEAM STIFFNESS FOR BEAMS FRAMING INTO SHEAR WALL

It was pointed out in Section 3.5 (ii) that large local distortions tend to occur in a shear wall where a beam, represented as a line element, frames into it. A procedure for eliminating the rotational degree of freedom for the shear wall at the point was discussed. Since the beam has a rotational degree of freedom at its end, this cannot be directly combined with the shear wall stiffness at the wall beam junction. This section deals with the modification to the beam stiffness, if either end or both ends frame into the shear wall.

TABLE 4.2

COLUMN STIFFNESS MATRIX

$$\begin{bmatrix}
\beta & 0 & -\beta(L_B + \frac{L_{CC}}{2}) & -\beta & 0 & -\beta(L_T + \frac{L_{CC}}{2}) \\
0 & \frac{AE}{L} & 0 & 0 & \frac{-AE}{L} & 0 \\
-\beta(L_B + \frac{L_{CC})}{2} & 0 & \beta L_B(L_B + L_{CC}) + \frac{\beta L_{CC}^2}{3} & \beta(L_B + \frac{L_{CC}}{2}) & 0 & \beta(L_T L_B + L_B\frac{L_{CC}}{2} + L_T\frac{L_{CC}}{2} + \frac{L_{CC}^2}{6}) \\
-\beta & 0 & \beta(L_B + \frac{L_{CC}}{2}) & \beta & 0 & \beta(L_T + \frac{L_{CC}}{2}) \\
0 & \frac{-AE}{L} & 0 & 0 & \frac{AE}{L} & 0 \\
-\beta(L_T + \frac{L_{CC}}{2}) & 0 & \beta(L_T L_B + L_B\frac{L_{CC}}{2} + L_T\frac{L_{CC}}{2} + \frac{L_{CC}^2}{6}) & \beta(L_T + \frac{L_{CC}}{2}) & 0 & \beta L_T(L_T + L_{CC}) + \frac{\beta L_{CC}^2}{3}
\end{bmatrix}$$

where

$\beta = 12EI/L_{CC}^3$

4.4 (i) LEFT END OF BEAM FRAMING INTO SHEAR WALL

Fig. 4.4 shows the left end of a beam framing into a shear wall. The nodes are designated by A, B and C. In the overall structure, the displacement components at A, B and C are d_1 to d_4 as represented in the figure. But when the beam is isolated, it has four degrees of freedom, represented by the displacement vectors d_I, d_J, d_K and d_L in the figure. As can be seen from the figure, the stiffness factor corresponding to d_J cannot be directly combined with the shear wall stiffness. Therefore, by geometry, the rotation of the beam where it meets the shear wall can be expressed in terms of the vertical shear wall displacements as follows:

$$d_J = (d_2 - d_1)/L \qquad (4.27)$$

Similarly, the beam end moment at the shear wall-beam junction can be expressed as

$$p_J/L = -p_1 = p_2 \qquad (4.28)$$

where L = shear wall width and p stands for the force vector. With the above relations, d_J has to be eliminated and there is subsequent modification to stiffness at nodes A, B and C.

The stiffness matrix relating forces and displacements at A, B and C can be written as:

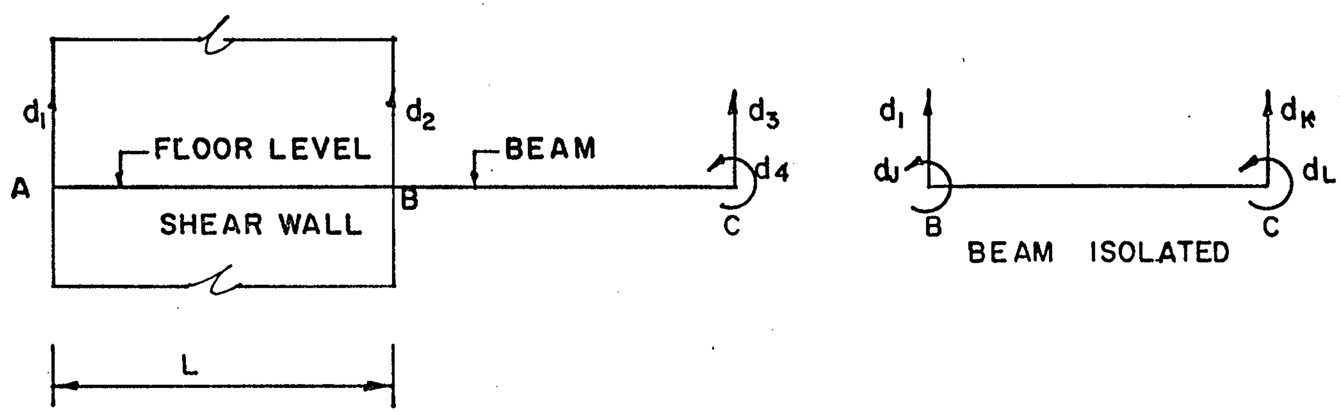


FIG. 4.4 LEFT END OF BEAM FRAMING INTO SHEAR WALL

$$\begin{Bmatrix} p_1 \\ p_2 \\ p_J \\ p_3 \\ p_4 \end{Bmatrix} = \begin{bmatrix} K_{11} & K_{12} & 0 & 0 & 0 \\ K_{21} & (K_{22}+K_{II}) & K_{IJ} & K_{IK} & K_{IL} \\ 0 & K_{JI} & K_{JJ} & K_{JK} & K_{JL} \\ 0 & K_{KI} & K_{KJ} & K_{KK} & K_{KL} \\ 0 & K_{LI} & K_{LJ} & K_{LK} & K_{LL} \end{bmatrix} \begin{Bmatrix} d_1 \\ d_2 \\ \dfrac{d_2 - d_1}{L} \\ d_3 \\ d_4 \end{Bmatrix} \tag{4.29}$$

Substituting for p_1, p_2 and p_J,

$$p_1 = K_{11}\, d_1 + K_{12}\, d_2 \tag{4.29a}$$

$$p_2 = (K_{21} - \frac{K_{IJ}}{L})\, d_1 + (K_{22} + K_{II} + \frac{K_{IJ}}{L})\, d_2 + K_{IK}\, d_3 + K_{IL}\, d_4 \tag{4.29b}$$

$$p_J = -\frac{K_{JJ}}{L}\, d_1 + (K_{JI} + \frac{K_{JJ}}{L})\, d_2 + K_{JK}\, d_3 + K_{JL}\, d_4 \tag{4.29c}$$

or

$$\frac{p_J}{L} = -\frac{K_{JJ}}{L^2}\, d_1 + \frac{1}{L}(K_{JI} + \frac{K_{JJ}}{L})\, d_2 + \frac{K_{JK}}{L}\, d_3 + \frac{K_{JL}}{L}\, d_4$$

Combining equations (4.29.a) and (4.29.c),

$$p_1 - \frac{p_J}{L} = (K_{11} + \frac{K_{JJ}}{L^2})\, d_1 + \left\{ K_{12} - \frac{1}{L}(K_{JI} + \frac{K_{JJ}}{L}) \right\} d_2 - \frac{K_{JK}}{L}\, d_3 - \frac{K_{JL}}{L}\, d_4 \tag{4.30}$$

and combining equations (4.29.b) and (4.29.c),

$$p_2 + \frac{p_J}{L} = \left\{ \left(K_{21} - \frac{K_{IJ}}{L}\right) - \frac{K_{JJ}}{L^2} \right\} d_1 + \left\{ (K_{22} + K_{II}) + \frac{K_{IJ}}{L} + \frac{1}{L}\left(K_{JI} + \frac{K_{JJ}}{L}\right) \right\} d_2$$

$$+ \left(K_{IK} + \frac{K_{JK}}{L}\right) d_3 + \left(K_{IL} + \frac{K_{JL}}{L}\right) d_4 \qquad (4.31)$$

From equation (4.29),

$$p_3 = -\frac{K_{KJ}}{L} d_1 + \left(K_{KI} + \frac{K_{KJ}}{L}\right) d_2 + K_{kk}\, d_3 + K_{KL}\, d_4 \qquad (4.32)$$

and

$$p_4 = -\frac{K_{LJ}}{L} d_1 + \left(K_{LI} + \frac{K_{LJ}}{L}\right) d_2 + K_{IK}\, d_3 + K_{LL}\, d_4 \qquad (4.33)$$

As before, p_J is the external moment applied at J and in our analysis, this is zero. Therefore, equations (4.30) to (4.33) can be combined to give the following matrix.

$$\begin{Bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{Bmatrix} = \begin{bmatrix} \left(K_{11} + \frac{K_{JJ}}{L^2}\right) & \left(K_{12} - \frac{1}{L}\left(K_{JI} + \frac{K_{JJ}}{L}\right)\right) & -\frac{K_{JK}}{L} & -\frac{K_{JL}}{L} \\ \left(K_{21} - \frac{K_{IJ}}{L} - \frac{K_{JJ}}{L^2}\right) & \left(K_{22} + K_{II} + 2\frac{K_{IJ}}{L} + \frac{K_{JJ}}{L^2}\right) & K_{IK} + \frac{K_{JK}}{L} & K_{IL} + \frac{K_{JL}}{L} \\ -\frac{K_{KJ}}{L} & K_{KI} + \frac{K_{KJ}}{L} & K_{KK} & K_{KL} \\ -\frac{K_{LJ}}{L} & K_{LI} + \frac{K_{LJ}}{L} & K_{LK} & K_{LL} \end{bmatrix} \begin{Bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \end{Bmatrix}$$

(4.34)

Since there are only four unknowns, the equation for p_J in equations (4.29) is redundant and hence omitted. Thus, the rotation d_J at the left end of the beam is transferred to d_1, d_2, d_3 and d_4, without affecting the symmetry of the stiffness matrix.

4.4(ii) RIGHT END OF BEAM FRAMING INTO SHEAR WALL

This is illustrated in Fig. 4.5 and the nomenclature is the same as for the previous case.

$$d_L = (d_4 - d_3)/L \qquad (4.35)$$

(displacement relationship)

and,

$$p_L/L = -p_3 = p_4 \qquad (4.36)$$

(force relationship)

Proceeding on a similar basis as for the previous case, the stiffness matrix for nodes A, B and C can be written. p_L, being the external moment, is taken as zero for our analysis and writing out the expression for

$p_3 - p_L/L$ and $p_4 + p_3/L$,

the final stiffness matrix can be obtained as follows:

$$\begin{Bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{Bmatrix} = \begin{bmatrix} K_{II} & K_{IJ} & (K_{IK}-\frac{K_{IL}}{L}) & \frac{K_{IL}}{L} \\ K_{JI} & K_{JJ} & (K_{JK}-\frac{K_{JL}}{L}) & \frac{K_{JL}}{L} \\ (K_{KI}-\frac{K_{LI}}{L}) & (K_{KJ}-\frac{K_{LJ}}{L}) & K_{33}+K_{KK}-\frac{2K_{KL}}{L}+\frac{K_{LL}}{L^2} & (K_{34}+\frac{K_{KL}}{L}-\frac{K_{LL}}{L^2}) \\ \frac{K_{LI}}{L} & \frac{K_{LJ}}{L} & K_{43}+\frac{1}{L}(K_{LK}-\frac{K_{LL}}{L}) & K_{44}+\frac{K_{LL}}{L^2} \end{bmatrix} \begin{Bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \end{Bmatrix} \qquad (4.37)$$

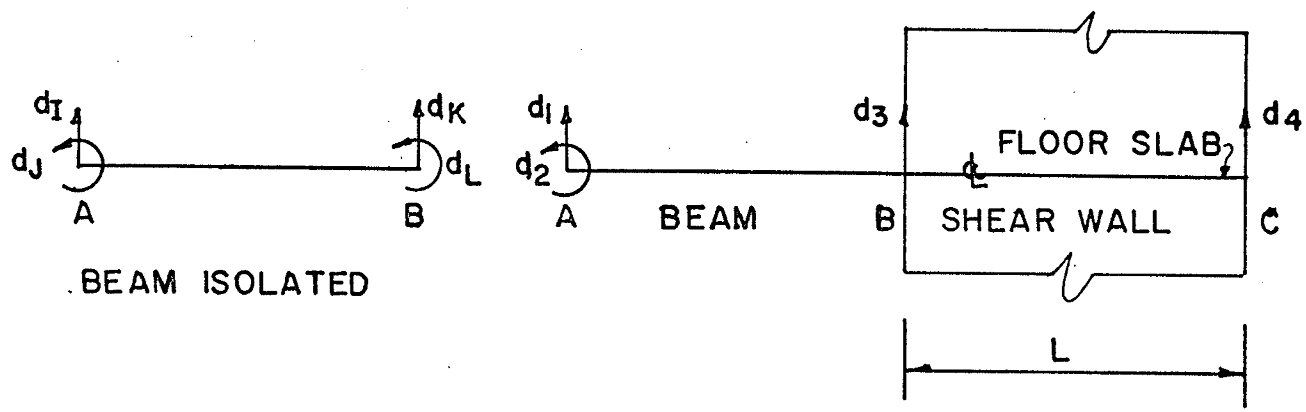


FIG. 4.5 RIGHT END OF BEAM FRAMING INTO SHEAR WALL

4.4(iii) BOTH ENDS OF BEAM FRAMING INTO SHEAR WALL

Fig. 4.6 shows an arrangement of both ends of beam framing into shear wall. The two rotational degrees of freedom, namely d_J and d_L, have to be modified. These end rotations can be expressed as follows:

$$d_J = (d_2 - d_1)/L1 \tag{4.38}$$

$$d_L = (d_4 - d_3)/L2 \tag{4.39}$$

where L1 and L2 are widths of shear walls.

Writing equilibrium equation for this set-up, we have, after substituting for d_J and d_L from (4.38) and (4.39),

$$\begin{Bmatrix} p_1 \\ p_2 \\ p_J \\ p_3 \\ p_L \\ p_4 \end{Bmatrix} = \begin{bmatrix} K_{11} & K_{12} & 0 & 0 & 0 & 0 \\ K_{21} & (K_{22}+K_{II}) & K_{2J} & K_{23} & K_{2L} & 0 \\ 0 & K_{J2} & K_{JJ} & K_{J3} & K_{JL} & 0 \\ 0 & K_{32} & K_{3J} & (K_{33}+K_{KK}) & K_{3L} & K_{34} \\ 0 & K_{L2} & K_{LJ} & K_{L3} & K_{LL} & 0 \\ 0 & 0 & 0 & K_{43} & 0 & K_{44} \end{bmatrix} \begin{Bmatrix} d_1 \\ d_2 \\ \dfrac{d_2 - d_1}{L1} \\ d_3 \\ \dfrac{d_4 - d_3}{L2} \\ d_4 \end{Bmatrix} \tag{4.40}$$

There are four unknowns and six equations. Equations for p_J and p_L may be treated as redundant. Accordingly, the stiffness matrix has to be modified.

Substituting for p_J,

$$p_J = -\frac{K_{JJ}}{L1} d_1 + \left(K_{J2}+\frac{K_{JJ}}{L1}\right) d_2 + \left(K_{J3}-\frac{K_{JL}}{L2}\right) d_3 + \frac{K_{JL}}{L2} d_4 \quad -$$

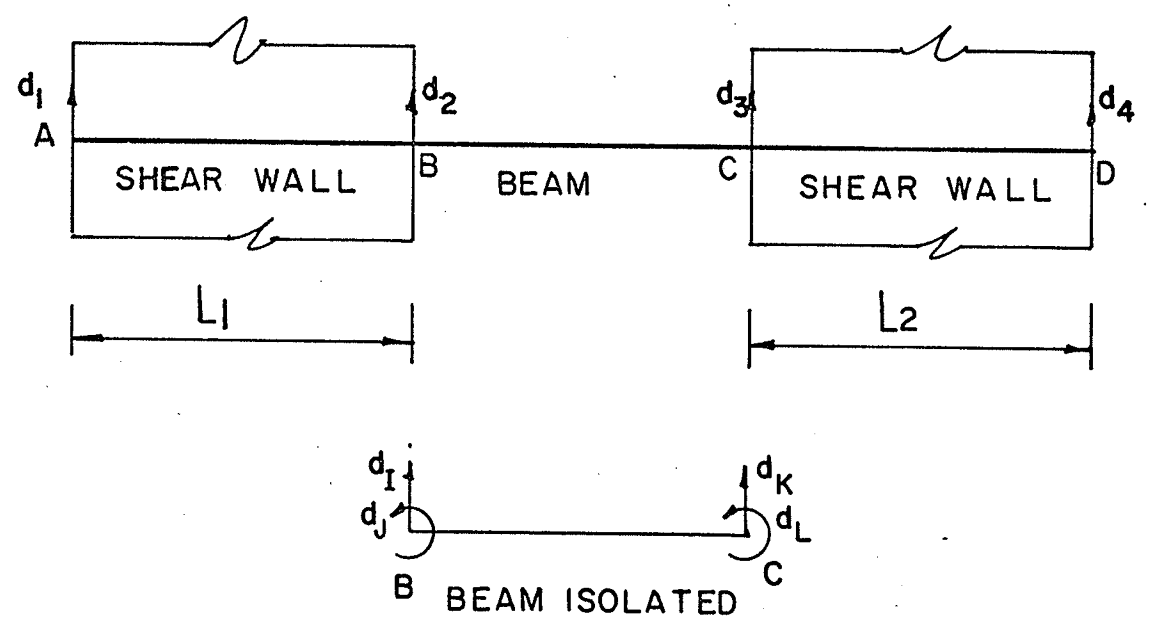


FIG 4·6 BOTH ENDS OF BEAM FRAMING INTO SHEAR WALL or dividing throughout by L1,

$$\frac{p_J}{L1} = -\frac{K_{JJ}}{L_1^2} d_1 + \left(\frac{K_{J2}}{L1} + \frac{K_{JJ}}{L_1^2}\right) d_2 + \left(\frac{K_{J3}}{L_1} - \frac{K_{JL}}{L_1L_2}\right) d_3 + \frac{K_{JL}}{L_1L_2} d_4 \qquad (4.41)$$

Similarly, substituting for p_L,

$$p_L = -\frac{K_{LJ}}{L1} d_1 + \left(K_{L2} + \frac{K_{LJ}}{L_1}\right) d_2 + \left(K_{L3} - \frac{K_{LL}}{L_2}\right) d_3 + \frac{K_{LL}}{L_2} d_4$$

Dividing throughout by L2, we have

$$\frac{p_L}{L_2} = -\frac{K_{LJ}}{L_1L_2} d_1 + \left(\frac{K_{L2}}{L_2} + \frac{K_{LJ}}{L_1L_2}\right) d_2 + \left(\frac{K_{L3}}{L_2} - \frac{K_{LL}}{L_2^2}\right) d_3 + \frac{K_{LL}}{L_2^2} d_4 \qquad (4.42)$$

Since by geometry the beam end moments can be expressed as

$$p_J/L1 = -p_1 = p_2 \quad \text{and}$$

$$p_L/L2 = -p_3 = p_4$$

Combining equations (4.41) and the first of (4.40),

$$p_1 - \frac{p_J}{L1} = \left(K_{11} + \frac{K_{JJ}}{L_1^2}\right) d_1 + \left(K_{12} - \frac{K_{J2}}{L1} - \frac{K_{JJ}}{L_1^2}\right) d_2 - \left(\frac{K_{J3}}{L1} - \frac{K_{JL}}{L_1L_2}\right) d_3 - \frac{K_{JL}}{L_1L_2} d_4 \qquad (4.43)$$

and combining equations (4.41) and the second of (4.40),

$$\frac{p_2+p_J}{L1} = \left(K_{21} - \frac{K_{2J}}{L1} - \frac{K_{JJ}}{L_1^2}\right) d_1 + \left(K_{22} + K_{II} + \frac{K_{2J}}{L1} + \frac{K_{J2}}{L1} + \frac{K_{JJ}}{L_1^2}\right) d_2 + \left(K_{23} - \frac{K_{2L}}{L_2} + \frac{K_{J3}}{L1} - \frac{K_{JL}}{L_1L_2}\right) d_3 + \left(\frac{K_{2L}}{L_2} + \frac{K_{JL}}{L_1L_2}\right) d_4 \qquad (4.44)$$

Similarly, working on p_L, p_3 and p_4,

$$\frac{p_3-p_L}{L_2} = \left(-\frac{K_{3J}}{L_1}+\frac{K_{LJ}}{L_1L_2}\right) d_1 + \left(K_{32}+\frac{K_{3J}}{L_1}-\frac{K_{L2}}{L_2}-\frac{K_{LJ}}{L_1L_2}\right) d_2 + \left(K_{33}+K_{KK}-\frac{K_{3L}}{L_2}-\frac{K_{L3}}{L_2}-\frac{K_{LL}}{L_2^2}\right) d_3 + \left(\frac{K_{3L}}{L_2}+K_{34}-\frac{K_{LL}}{L_2^2}\right) d_4 \tag{4.45}$$

and

$$\frac{p_4+p_L}{L_2} = -\frac{K_{LJ}}{L_1L_2} d_1 + \left(\frac{K_{L2}}{L_2}+\frac{K_{LJ}}{L_1L_2}\right) d_2 + \left(K_{43}+\frac{K_{L3}}{L_2}-\frac{K_{LL}}{L_2^2}\right) d_3 + \left(K_{44}+\frac{K_{LL}}{L_2^2}\right) d_4 \tag{4.46}$$

Combining (4.43) to (4.46) and treating p_L and p_J as zero, we have the final matrix as shown below.

$$\begin{Bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{Bmatrix} = \begin{bmatrix} \left(K_{11}+\frac{K_{JJ}}{L_1^2}\right) & \left(K_{12}-\frac{K_{L2}}{L_1}-\frac{K_{JJ}}{L_1^2}\right) & -\frac{K_{J3}}{L_1}+\frac{K_{JL}}{L_1L_2} & -\frac{K_{JL}}{L_1L_2} \\ \left(K_{21}-\frac{K_{2J}}{L_1}-\frac{K_{JJ}}{L_1^2}\right) & \left(K_{22}+K_{II}+\frac{2K_{2J}}{L_1}+\frac{K_{JJ}}{L_1^2}\right) & \left(K_{23}-\frac{K_{2L}}{L_2}+\frac{K_{J3}}{L_1}-\frac{K_{JL}}{L_1L_2}\right) & \left(\frac{K_{2L}}{L_2}+\frac{K_{JL}}{L_1L_2}\right) \\ \left(-\frac{K_{3J}}{L_1}+\frac{K_{LJ}}{L_1L_2}\right) & \left(K_{32}+\frac{K_{3J}}{L_1}-\frac{K_{L2}}{L_2}-\frac{K_{LJ}}{L_1L_2}\right) & \left(K_{33}+K_{KK}-\frac{2K_{L3}}{L_2}-\frac{K_{LL}}{L_2^2}\right) & \left(\frac{K_{3L}}{L_2}+K_{34}-\frac{K_{LL}}{L_2^2}\right) \\ -\frac{K_{LJ}}{L_1L_2} & \left(\frac{K_{L2}}{L_2}+\frac{K_{LJ}}{L_1L_2}\right) & \left(K_{43}+\frac{K_{L3}}{L_2}-\frac{K_{LL}}{L_2^2}\right) & \left(K_{44}+\frac{K_{LL}}{L_2^2}\right) \end{bmatrix} \begin{Bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \end{Bmatrix} \tag{4.47}$$

CHAPTER V

EXAMPLES

Four examples are presented in this chapter to demonstrate the analysis procedure. The results of the first three examples are compared with those obtained in previous work and the percentage deviations are presented in tables for each case. Before proceeding with different examples it is necessary to predict approximately the number of elements required to produce a realistic stiffness for a shear wall. Therefore, this has been studied in the first example and the results are presented in graphs. Due to the unavailability of relevant experimental results for the problem, the results are compared with the work carried out by other classical methods.

5.1 EXAMPLE 1 - FOUR STOREY SHEAR WALL-FRAME WITHOUT OPENINGS IN SHEAR WALL

The structure shown in Fig. 5.1 is analysed for the loading shown in the figure. The structure and the loading are identical to those considered by R.G. Oakberg (18) who solved the problem by stiffness method using sub-structure analysis and also by a deep column method, in which the shear wall is represented as a deep column.

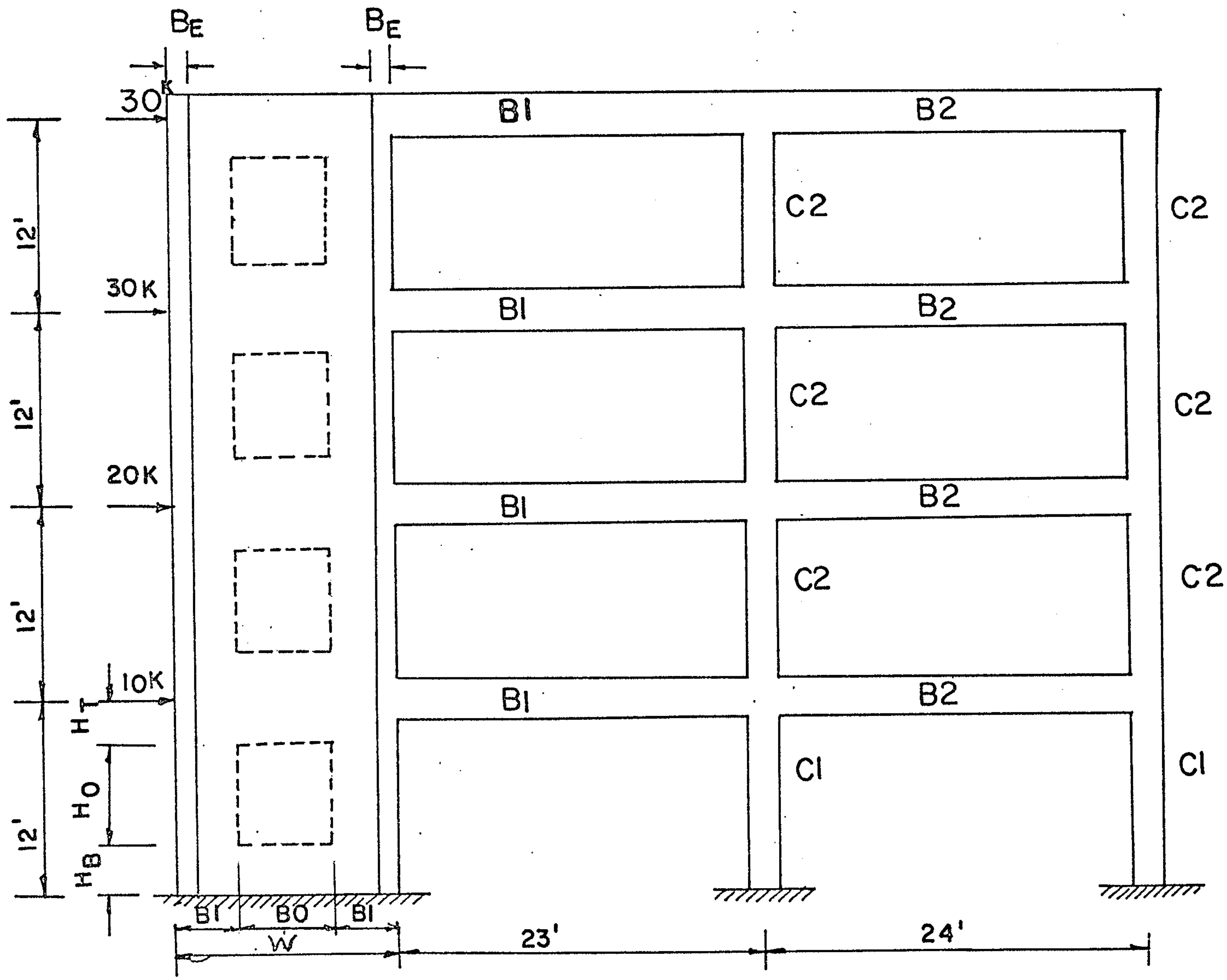


FIG. 5.1 FOUR STOREY SHEAR WALL-FRAME

TABLE 5.1

PROPERTIES OF SHEAR WALL

Story	Height	Width W	H_o	B_o	B_1	H_B	H_T	B_E	Thickness Edge Element	Thickness Remainder
1	12'0"	18'0"	6'6"	6'0"	4'0"	3'0"	2'6"	24"	16"	8"
2	12'0"	18'0"	6'6"	6'0"	4'0"	3'0"	2'6"	24"	16"	8"
3	12'0"	18'0"	6'6"	6'0"	4'0"	3'0"	2'6"	24"	16"	8"
4	12'0"	18'0"	6'6"	6'0"	4'0"	3'0"	2'6"	24"	16"	8"

Modulus of Elasticity For All Storeys = 3,000 ksi

Poisson's Ratio For All Storeys = 0.25

TABLE 5.2

PROPERTIES OF BEAMS

Beam No.	Clear Length	Left Rigid Length	Right Rigid Length	Modulus of Elasticity	Moment of Inertia in^4	Cross Sectional Area in^2
B1	22'0"	0	12"	3000 ksi	69675	656
B2	22'0"	12"	12"	3000 ksi	69675	656

TABLE 5.3

PROPERTIES OF COLUMNS

Column No.	Clear Length	Top Rigid Length	Bottom Rigid Length	Modulus of Elasticity ksi	Moment of Inertia in^4	Cross Sectional Area in^2
C1	10'9"	15"	0	3000	26112	384
C2	9'6"	15"	15"	3000	26112	384

The sub-structure analysis is based on the stiffness method and uses finite elements for generating the shear wall stiffness matrix. The deep column method is also based on the stiffness analysis but idealizes the shear wall as a deep column. The properties of the shear wall, beams and columns are given in Tables 5.1, 5.2 and 5.3, respectively. No openings in the shear wall are considered for this example.

The structure was analysed using 20, 48, 80, 120 and 180 elements per shear wall segment and the results were compared with those obtained by the sub-structure method (also with varying numbers of elements) and with those obtained by the deep column method.

Figs. 5.2, 5.3 and 5.4 show plots of lateral displacements at the top floor level for the three types of analysis. The variation in displacement with number of elements is more pronounced for the sub-structure analysis than for the analysis procedure described in this study. The rotational component in the sub-structure analysis was very inconsistent due to local deformation at the wall beam connection points. In the present analysis all the displacement components converge from a lower value to a higher value indicating that the model becomes more flexible as the number of elements is increased. Comparison of the plots for the three cases shows that the model employed in this study and that used in the sub-structure analysis which

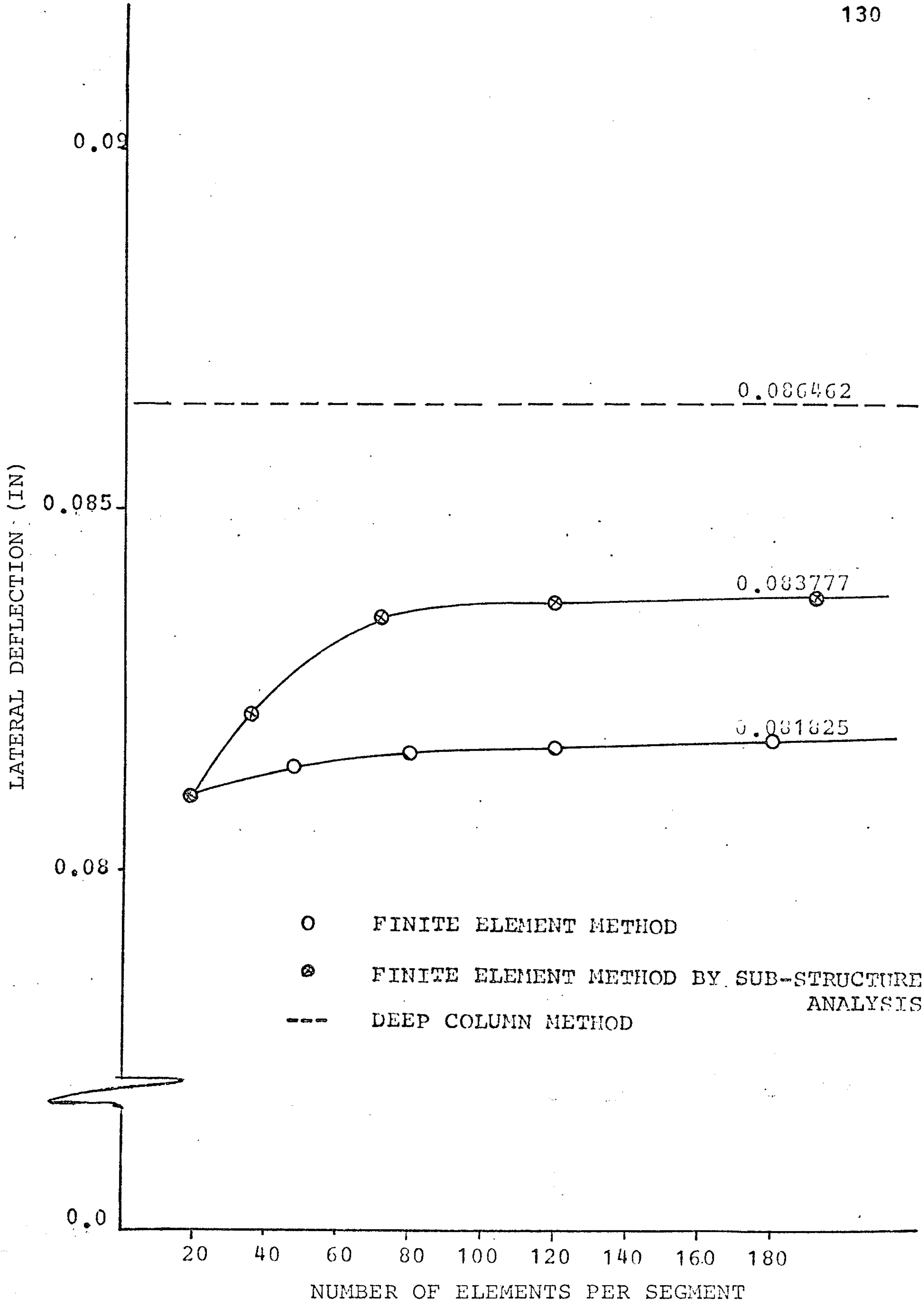


FIG. 5.2 CONVERGENCE OF LATERAL DEFLECTION AT TOP FLOOR LEVEL, EXAMPLE - 1

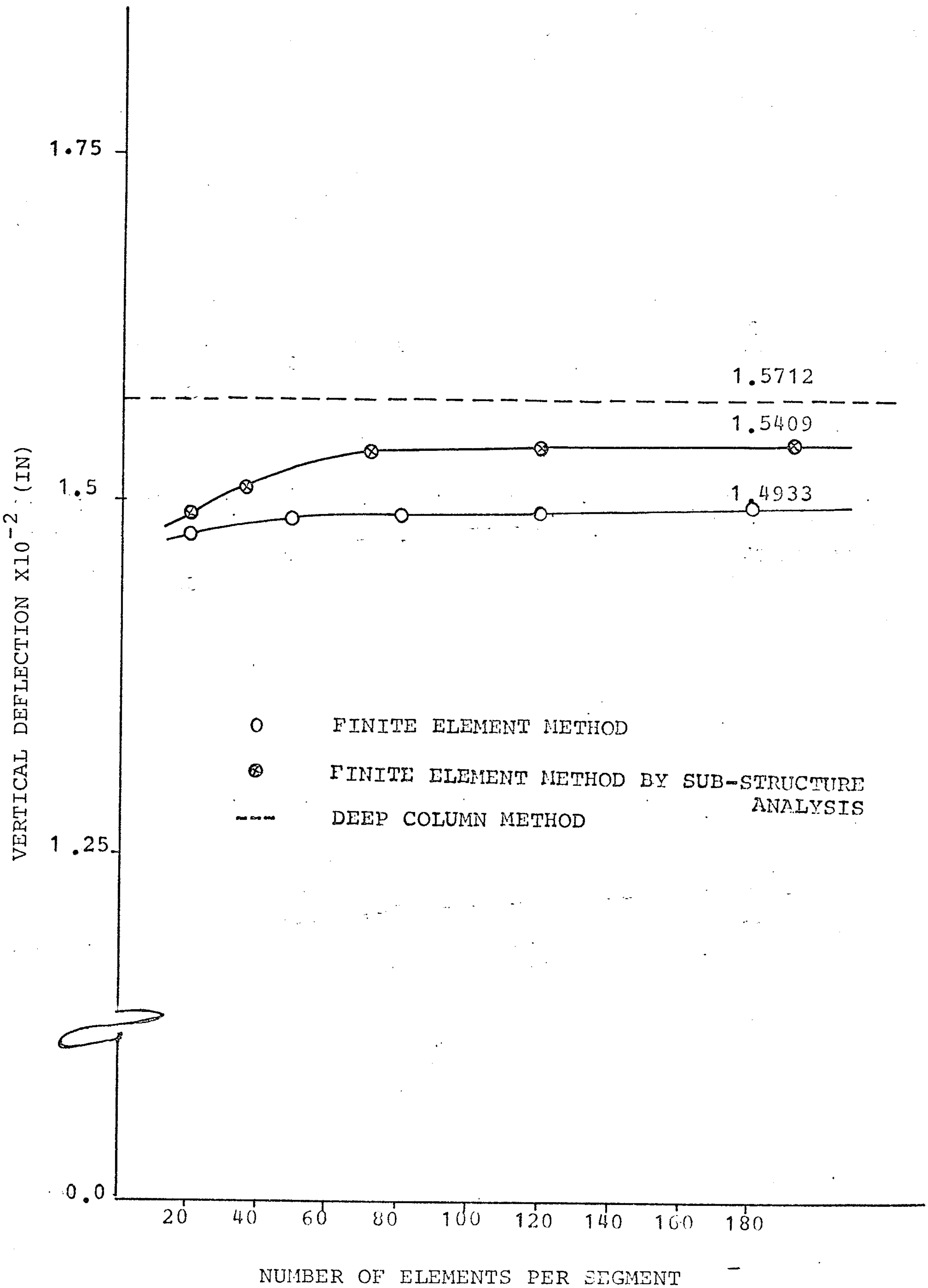


FIG. 5.3 CONVERGENCE OF VERTICAL DEFLECTION AT WALL - BEAM CONNECTION POINT AT FIRST FLOOR LEVEL

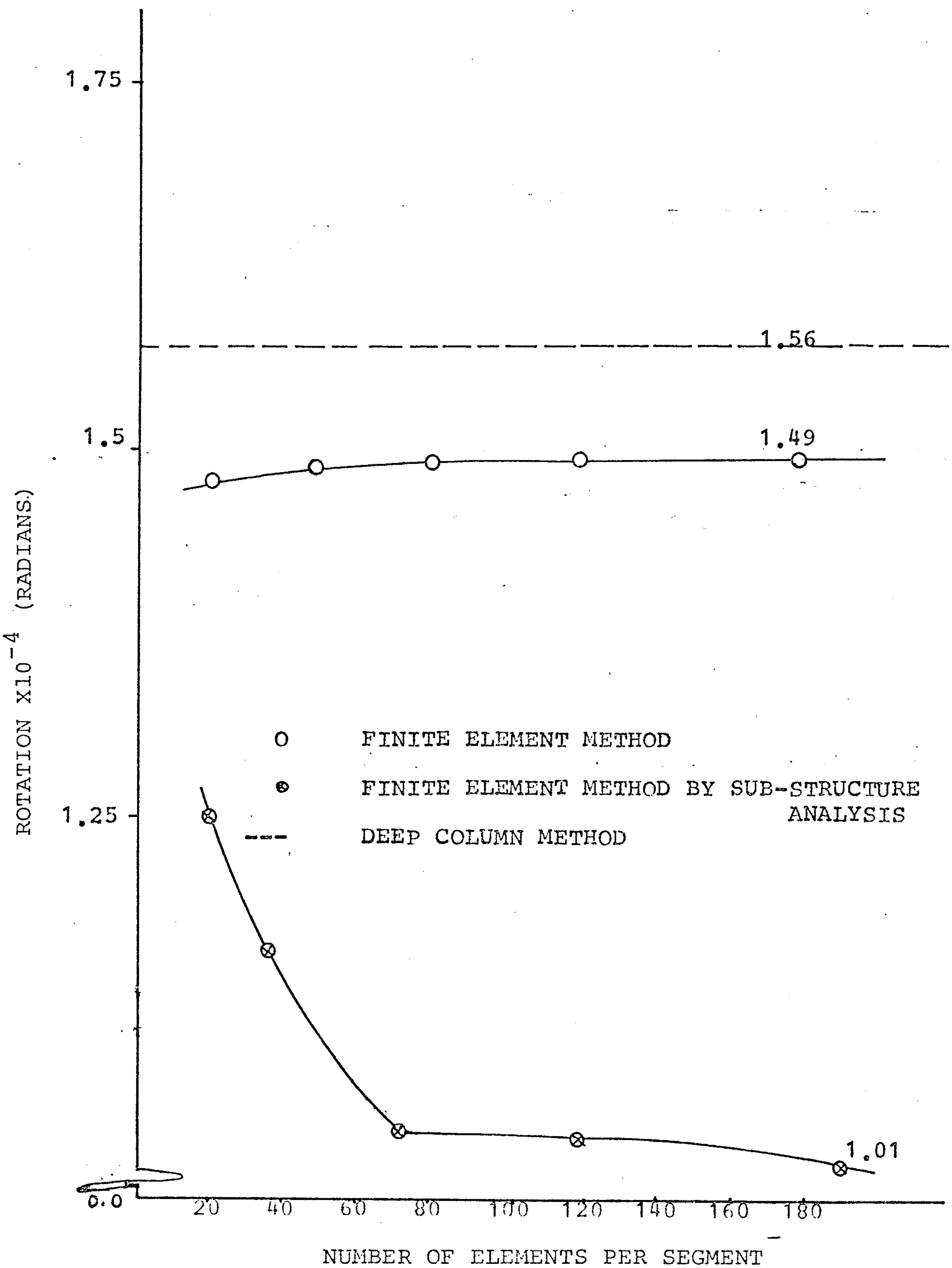


FIG. 5.4 CONVERGENCE OF ROTATION AT WALL-BEAM CONNECTION POINT AT TOP FLOOR LEVEL EXAMPLE - 1

also employs a finite element model, are more rigid than that used in the deep column method. It should be expected that the finite element model deveoped in this study will be stiffer than that described by Oakberg. For the latter model, the unrealistically large local distortions at the shear wall beam junctions (these local distortions have been noted in other investigations (10,14)), will significantly increase the flexibility. On the other hand the kinematic constraint applied at the floor levels will cause the former model to be slightly stiffer than it should be.

It is difficult to determine whether or not the shear wall model used in this study should be stiffer than the deep column model. While in general, any finite element model over estimates the stiffness of the structure, the deep column model assumes plane transverse sections at all shear wall cross-sections. Hence, it too over estimates the shear wall stiffness.

Figs. 5.5, 5.6 and 5.7 show plots of force components obtained by the three methods. The rates of convergence with increasing numbers of elements per shear wall segment, are approximately the same for the two methods employing finite element idealizations. The results of the present analysis are in close agreement with those of the deep column method rather than with those of the sub-structure analysis. This suggests that local shear wall deformations have a significant bearing on the final results.

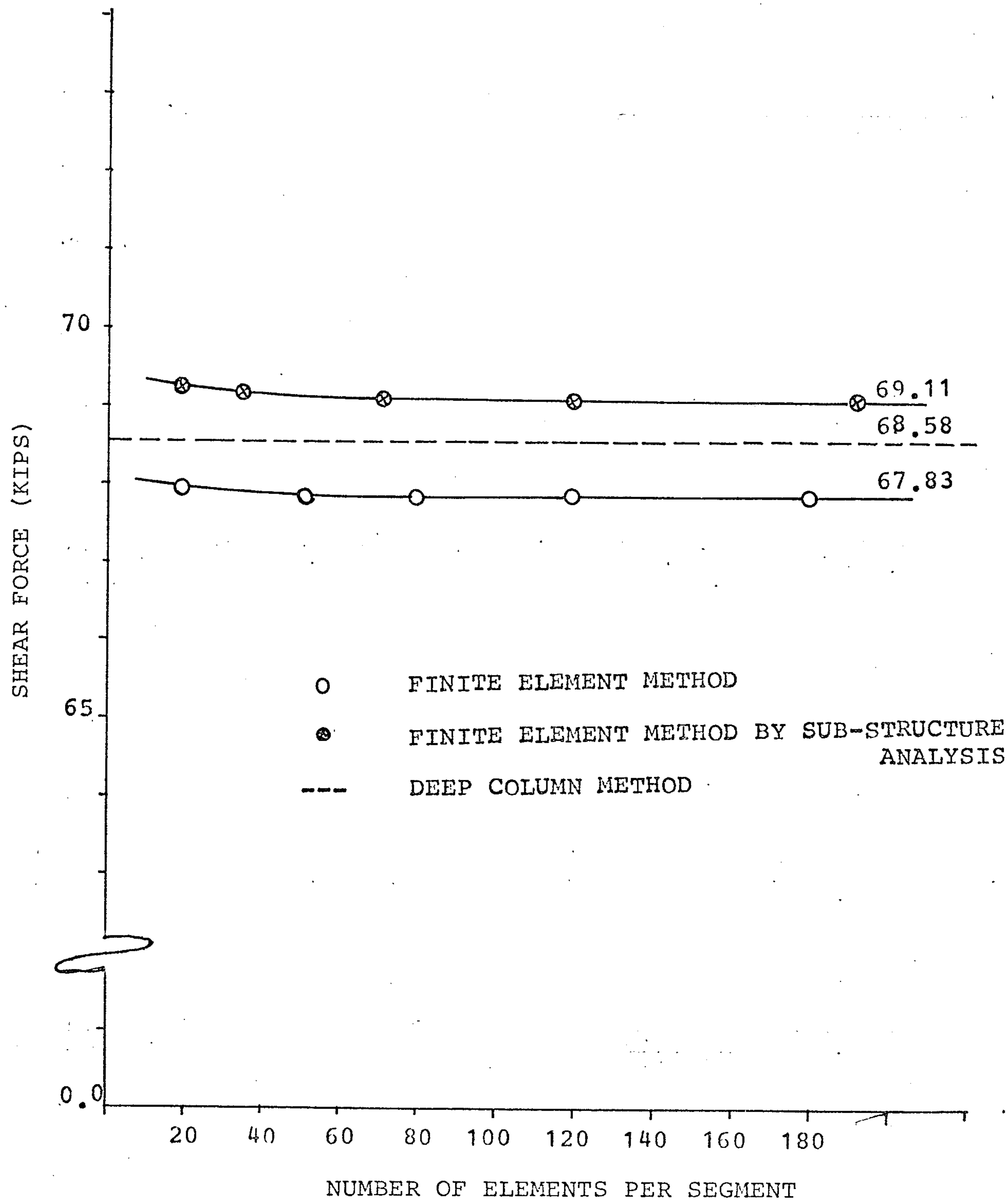


FIG. 5.5 CONVERGENCE OF SHEAR FORCE IN SHEAR WALL AT FIRST FLOOR LEVEL, EXAMPLE - 1

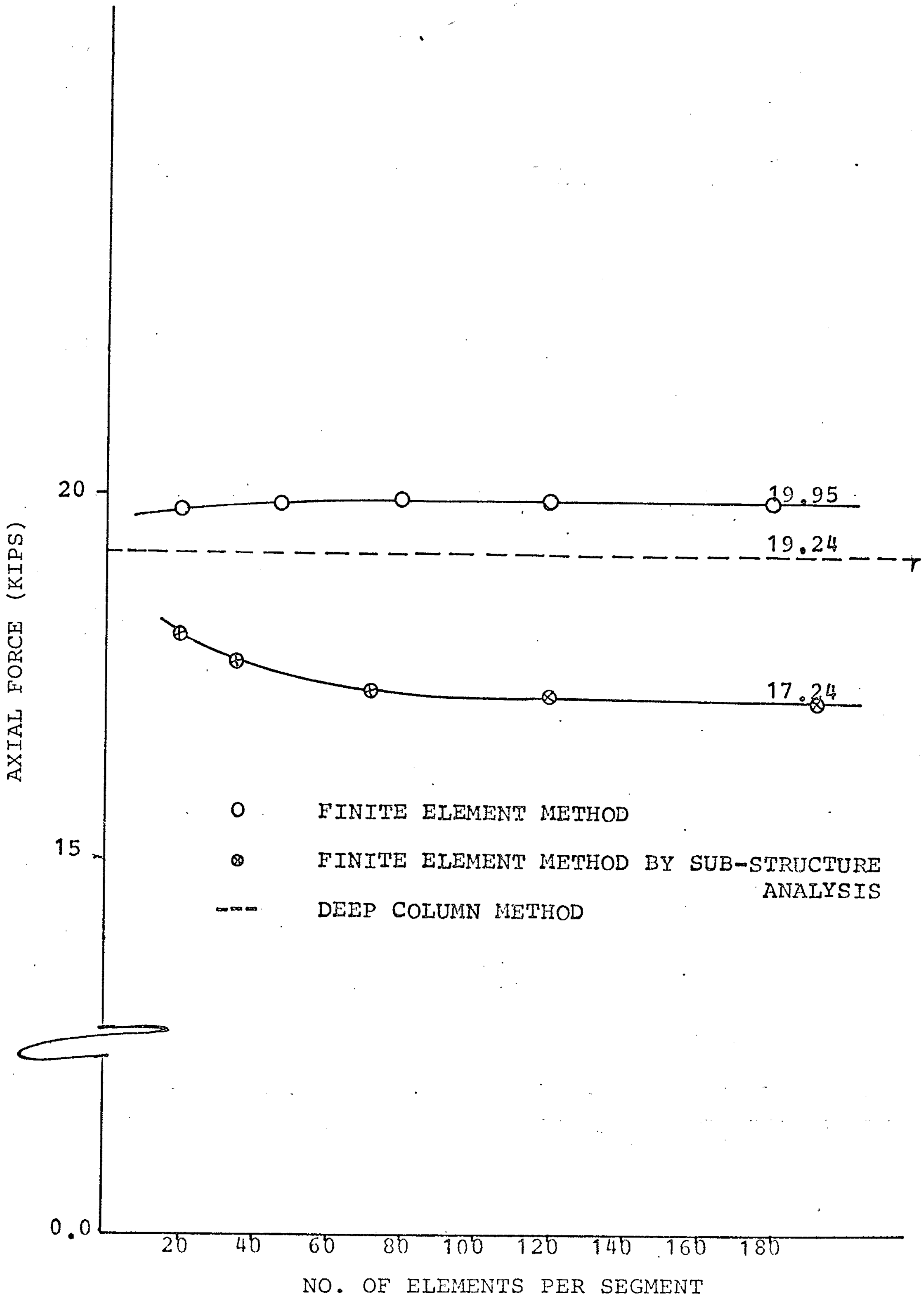


FIG. 5.6 CONVERGENCE OF AXIAL FORCE IN SHEAR WALL AT BASE , EXAMPLE - 1

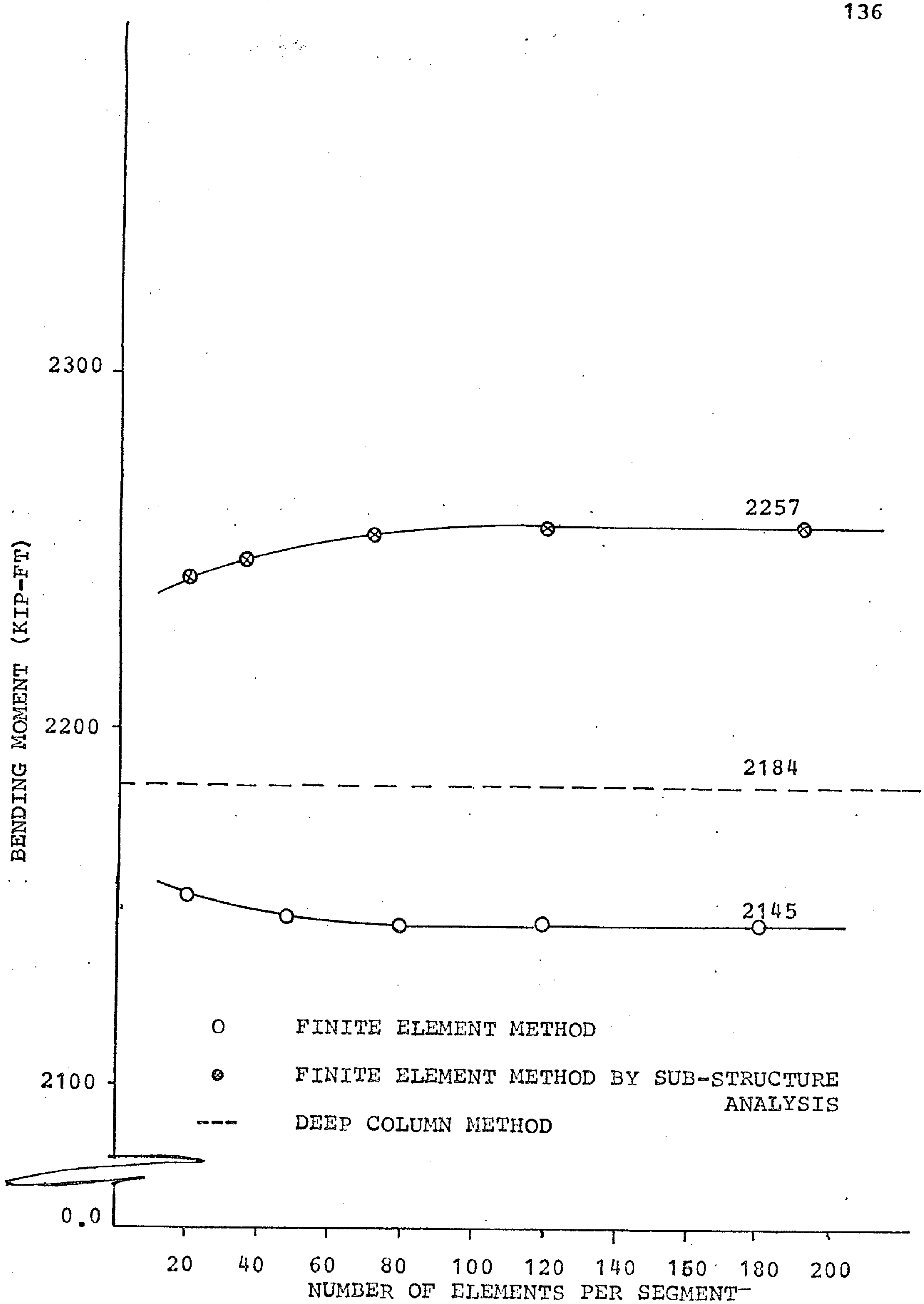


FIG. 5.7 CONVERGENCE OF BENDING MOMENT IN SHEAR WALL AT BASE, EXAMPLE - 1

TABLE 5.4

LATERAL DEFLECTIONS (IN)

Framing Level Top To Ground	Finite Element Method	Sub-Structure Method	Difference
5	0.682	0.084	2.4%
4	0.0592	0.0601	1.52%
3	0.0349	0.0351	0.575%
2	0.0132	0.0131	0.76%

TABLE 5.5

CONNECTION POINTS VERTICAL DEFLECTIONS (IN)

Framing Level Top To Ground	Finite Element Method	Sub-Structure Method	Difference
5	-0.0149	-0.0154	3.35%
4	-0.0151	-0.0156	3.3%
3	-0.0139	-0.0141	1.44%
2	-0.0094	-0.0095	1.06%

TABLE 5.6

CONNECTION POINTS ROTATIONS (RADIANS x 10^{-3})

Framing Level Top To Ground	Finite Element Method	Sub-Structure Method	Difference
5	-0.149	-0.101	32.2%
4	-0.149	-0.117	21.5%
3	-0.136	-0.114	16.2%
2	-0.091	-0.086	5.5%

TABLE 5.7

CONNECTION POINTS FORCES

Framing Level Top To Ground	Shear Force (Kips)				Bending Moment (Kip-Ft.)		
	Finite Element Method	Sub-Structure Method	Difference		Finite Element Method	Sub-Structure Method	Difference
5	4.66	3.86	17%		60.24	47.6	21%
4	5.98	5.13	14.2%		70.59	59.3	15.8%
3	5.49	4.8	12.6%		64.82	55.9	14.0%
2	3.82	3.45	9.7%		44.59	40.5	9.10%

TABLE 5.8

ACTIONS IN SHEAR WALL

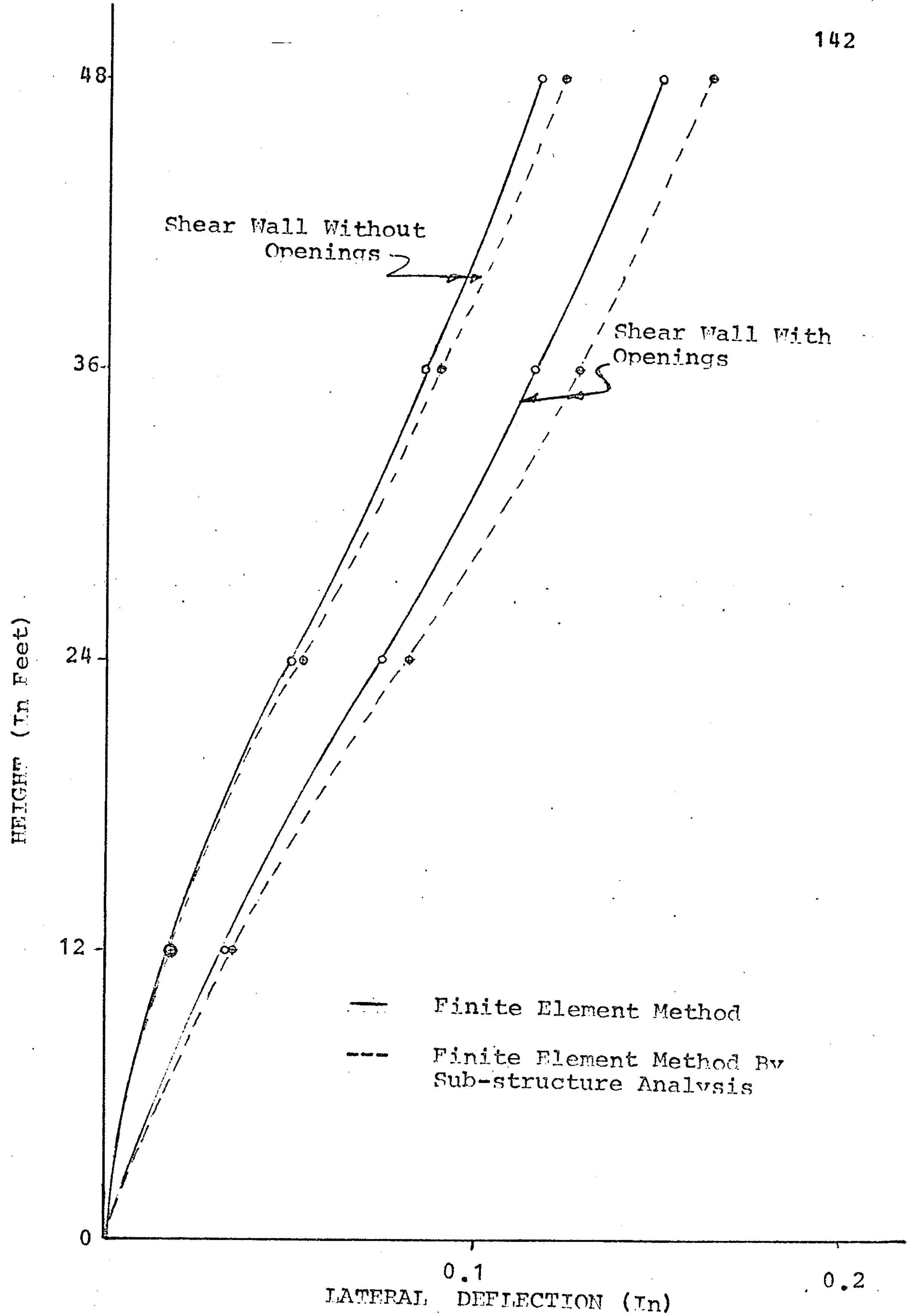
Framing Level Top To Ground	Shear Force (Kips)			Axial Force (Kips)			Bending Moment (K-Ft.)		
	Finite Element Method	Sub-Structure Method	Difference	Finite Element Method	Sub-Structure Method	Difference	Finite Element Method	Sub-Structure Method	Difference
5	15.27	16.73	9.6%	4.66	3.86	17.0%	102.2	82.4	21.0%
4	47.12	48.29	2.46%	10.64	8.99	15.3%	43.4	-12.86	----
3	67.83	69.1	1.88%	16.13	13.79	14.5%	-407.8	-493.2	21.0%
2	83.58	83.91	0.4%	19.95	17.24	13.7%	-1142.8	-1250.9	9.4%
Ground	83.58	83.91	0.4%	19.95	17.24	13.7%	-2145.8	-2257.7	5.25%

Since the sub-structure analysis was made on finite element idealizations the final results are compared with this method and they are presented in Tables 5.4, 5.5 and 5.6. The lateral and vertical deflections are in excellent agreement, the maximum deviation being only 3.3%. The discrepancies in connection point rotations, however, ranged up to 32.2%. The high percentage variation is primarily due to the local distortions at the connection points. The effect of this can be well understood from the connection point forces where the deviation ranged up to 21%. The values of shear force in shear wall, calculated by the two methods are in good agreement.

5.2 <u>EXAMPLE - 2, EFFECT OF OPENINGS IN SHEAR WALL</u>

The structure shown in Fig. 5.1, with a shear wall width, W, of 14'-0", was analysed firstly with a solid shear wall, and then with a shear wall with the openings, indicated dotted in the figure. Other properties of the structure were as listed in Table 5.1.

The results obtained were compared with corresponding values reported by Oakberg and Weaver (13). The plots of the lateral deflections of the shear wall with and without openings are shown in Fig. 5.8. The shear wall lateral deflection and bending moments at various levels are presented in Table 5.9. It can be seen that the openings in the shear wall tend to make the model more flexible, as they



EXAMPLE - 2

FIG. 5.8

TABLE 5.9

COMPARISON OF RESULTS FOR EXAMPLE - 2

Framing Level Top to Base	Lateral Deflection (in)						Bending Moment (K-Ft)					
	Without openings In Shear Wall			With Openings In Shear Wall			Without Openings In Shear Wall			With Openings In Shear Wall		
	F.E Method	Sub-Structure Method	Difference	F.E Method	Sub-Structure Method	Difference	F.E Method	Sub-Structure Method	Difference	F.E Method	Sub-Structure Method	Difference
5	0.118	0.123	4.2%	0.15	0.165	10.0%	122.6	102.0	17.2%	117.7	95.7	18.8%
4	0.086	0.089	3.5%	0.117	0.129	9.3%	156.7	102.0	34.0%	141.8	90.0	36.0%
3	0.051	0.052	2.0%	0.076	0.083	9.1%	-197.4	-285.4	45.4%	-174.5	-283.7	62.2%
2	0.019	0.019	Nil	0.034	0.035	3.0%	-838.4	-957.1	15.5%	-757.9	-809.4	6.9%
Base	---	---	---	---	---	---	-1809.0	-1933.1	6.6%	-1635.0	-1681.7	2.8%

increase the deflections by approximately 30%. The comparison of the results by the two methods shows that the percentage deviation increases if there are openings in the shear wall. This is predominantly due to the constraint that was provided at shear wall segment boundaries. As the depth of the lintel beams between the openings becomes smaller, the assumption that plane transverse sections through the shear wall at the floor levels remain plane after bending, will no more be valid. Therefore, for small openings in the shear wall and when the depths of lintel beams between the openings are large, the method outlined in this study may be employed. For large openings, the shear wall may be treated as two walls connected by lintel beams as illustrated in the next example.

5.3 EXAMPLE 3 - SIX STOREY SHEAR WALL

As a third example, shear walls connected by link beams only, is analysed and a study on the effect Poisson's ratio on the final results is made. Fig. 5.9 shows the structure and loading which are identical to those analysed by German Gurfinkel (12) and the final results are compared with his solution which was based on the cantilever moment distribution method. The properties of the shear walls and beams are presented in Tables 5.10 and 5.11.

The plots of lateral deflection versus height for both the methods is shown in Fig. 5.10. Table 5.12 shows the lateral deflections obtained by both the methods and the percentage deviation. The difference in results ranged upto 15.2% at the first floor level. The discrepancy reduced with increasing height. As can be seen from the plot of lateral deflection, the model analysed by the present method is more flexible than the cantilever moment distribution method. A study was performed to see the variation in deflection and force components with different values of Poisson's ratio. It is observed that deflection and force components varied from higher value to lower value with increasing values of Poisson's ratio. The values of Poisson's ratio used for this purpose were 0.2, 0.25, 0.3, 0.35, 0.4 and 0.45. The difference for the two extreme cases was never more than 5%.

The forces calculated by the two methods are in good agreement. The maximum percentage deviation for axial force

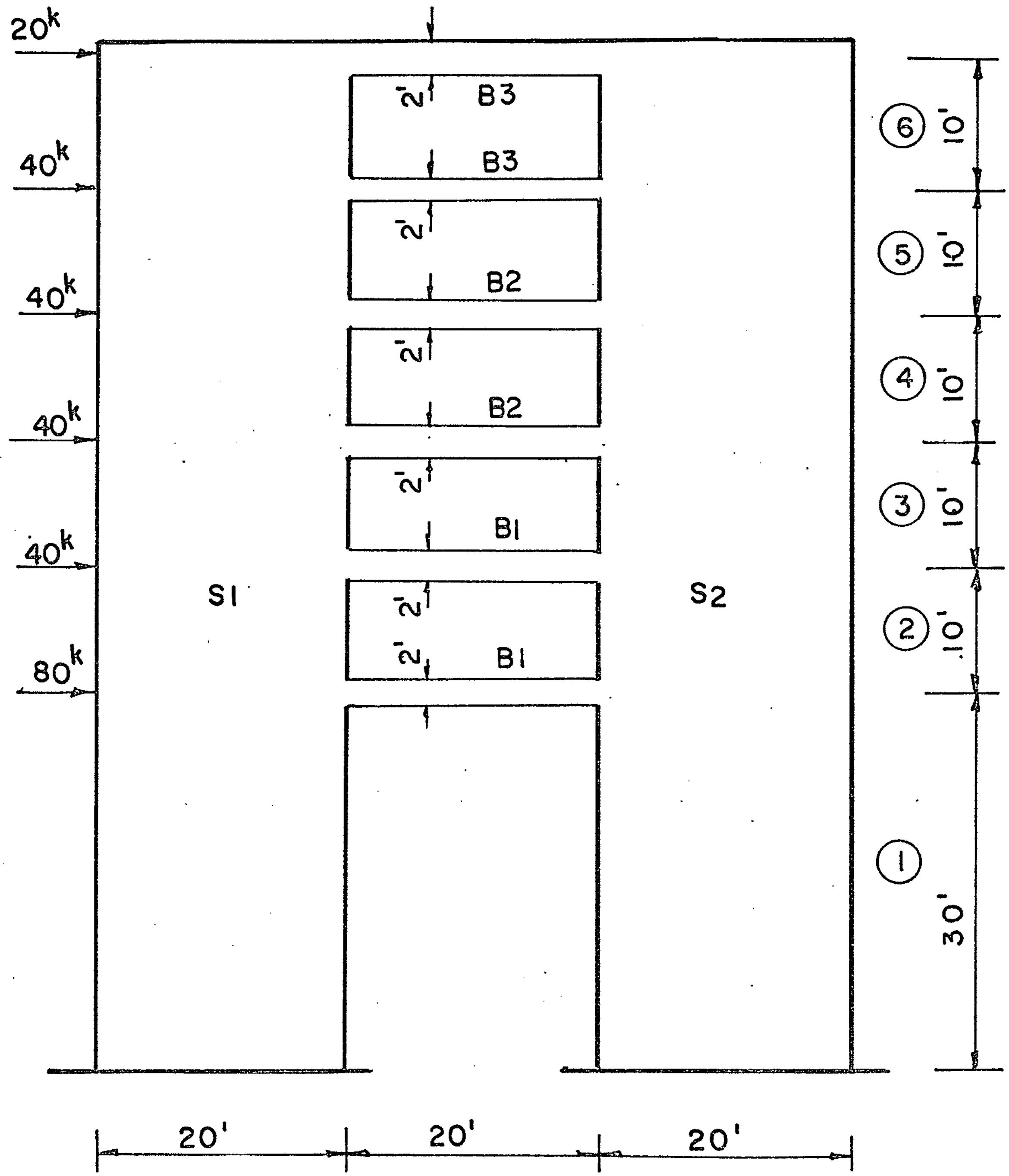


FIG. 5.9 SIX STOREY FRAME , EXAMPLE-3

TABLE 5.10

PROPERTIES OF BOTH SHEAR WALLS

Storey	Height	Width	Modulus of Elasticity (KS1)	Poisson's Ratio	Thickness
1	30'0"	20'0"	5000	0.3	12"
2	10'0"	20'0"	5000	0.3	12"
3	10'0"	20'0"	4000	0.3	12"
4	10'0"	20'0"	4000	0.3	12"
5	10'0"	20'0"	3000	0.3	12"
6	10'0"	20'0"	3000	0.3	12"

TABLE 5.11

PROPERTIES OF BEAMS

Beam No.	Clear Length	Modulus of Elasticity (KS1)	Moment of Inertia (in^4)
B1	20'0"	5000	13824
B2	20'0"	4000	13824
B3	20'0"	3000	13824

is 1.44 and for bending moment is 5.9. The variation of results with respect to shear force were nil, however. There is a good agreement on the bending moment of the lintel beams as well. For the purpose of comparison the results of the force components by both methods are presented in Tables 5.13 to 5.16. From symmetry considerations, the results are presented only for shear wall S1. Forces on shear wall S2 are exactly the same as that for S1 except that the signs are reversed for axial force alone. Fig. 5.11 shows the shear force, axial force and bending moment for left hand shear wall and Fig. 5.12 shows the deflected shape of the whole structure. As can be seen from this figure, the shortening and the elongation of the shear wall edges caused vertical deflections at the connection points between shear wall and beams which in turn caused bending moments on the beam as the load was transformed from left to right. This gives a clear picture of the interaction of shear wall and beams. These bending moments and shear forces on beam ends in turn act on the surfaces of wall due to which there is local deformation at the connection point between wall and beam. This is another important reason for transferring the connection point rotational stiffness to two ends of the wall at any particular level, by relating them to vertical displacements, therby avoiding any local deformation. It is the moment, rather than the shear at the end of the beam, that causes the local distortion of the wall.

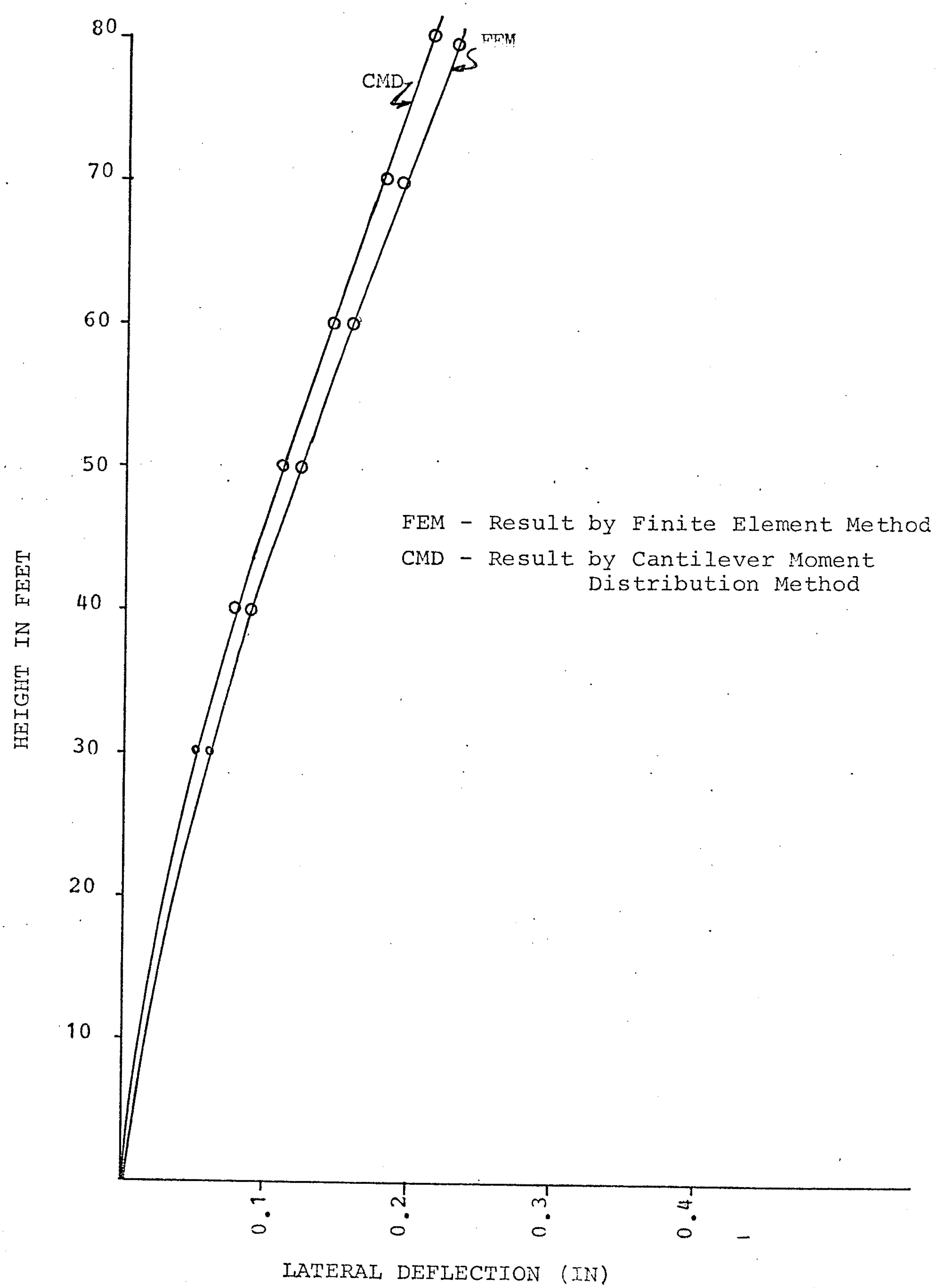

80
70
60
50
40
30
20
10
HEIGHT IN FEET
CMD
FEM
FEM - Result by Finite Element Method
CMD - Result by Cantilever Moment Distribution Method
0.1
0.2
0.3
0.4
LATERAL DEFLECTION (IN)

FIG. 5.10

TABLE 5.12

COMPARISON OF LATERAL DEFLECTION

Framing Level	Lateral Deflection (in)		Difference
	Finite Element Method	Cantilever Moment Dist'n	
7	0.230	0.213	+7.4%
6	0.195	0.179	+8.21%
5	0.159	0.145	+8.8%
4	0.124	0.111	+10.5%
3	0.089	0.078	+12.4%
2	0.058	0.049	+15.2%
Ground	0.000	0.000	--

TABLE 5.13

COMPARISON OF SHEAR FORCE ON LEFT HAND SHEAR WALL

Framing Level	Shear Force (KIPS)		Difference
	Finite Element Method	Cantilever Moment Dist'n	
7	10	10	Nil
6	30	30	Nil
5	50	50	Nil
4	70	70	Nil
3	90	90	Nil
2	130	130	Nil
Ground	130	130	Nil

TABLE 5.14

COMPARISON OF AXIAL FORCE ON LEFT HAND SHEAR WALL

Framing Level	Axial Force (KIPS)		Difference
	Finite Element Method	Cantilever Moment Dist'n	
7	4.84	4.91	-1.44%
6	9.72	9.85	-1.34%
5	16.19	16.41	-1.36%
4	22.48	22.80	-1.42%
3	29.79	30.21	-1.41%
2	36.28	36.78	-1.375%
Ground	36.28	36.78	-1.375%

TABLE 5.15

COMPARISON OF BENDING MOMENT ON LEFT HAND SHEAR WALL

Framing Level	Bending Moment (Kip. Ft)		Difference
	Finite Element Method	Cantilever Moment Dist'n	
7	96.85	98.17	-1.36%
6	94.34	97.02	-2.73%
5	-76.29	-71.77	-5.9%
4	-450.42	-444.09	-1.41%
3	-1004.22	-995.84	-1.76%
2	-1774.42	-1764.33	-0.562%
Ground	-5674.42	-5664.33	-0.176%

TABLE 5.16

COMPARISON OF BEAM END MOMENTS

Beam At Framing Level	Bending Moment (Kip. Ft.)				Difference
	Left End		Right End		
	Fem	Cmd	Fem	Cmd	
7	-48.43	-49.09	-48.43	-49.09	1.36%
6	-48.75	-49.43	-48.75	-49.43	1.40%
5	-64.68	-65.61	-64.68	-65.61	1.44%
4	-62.93	-63.84	-62.93	-63.84	1.44%
3	-73.10	-74.13	-73.10	-74.13	1.40%
2	-64.90	-65.76	-64.90	-65.76	1.33%

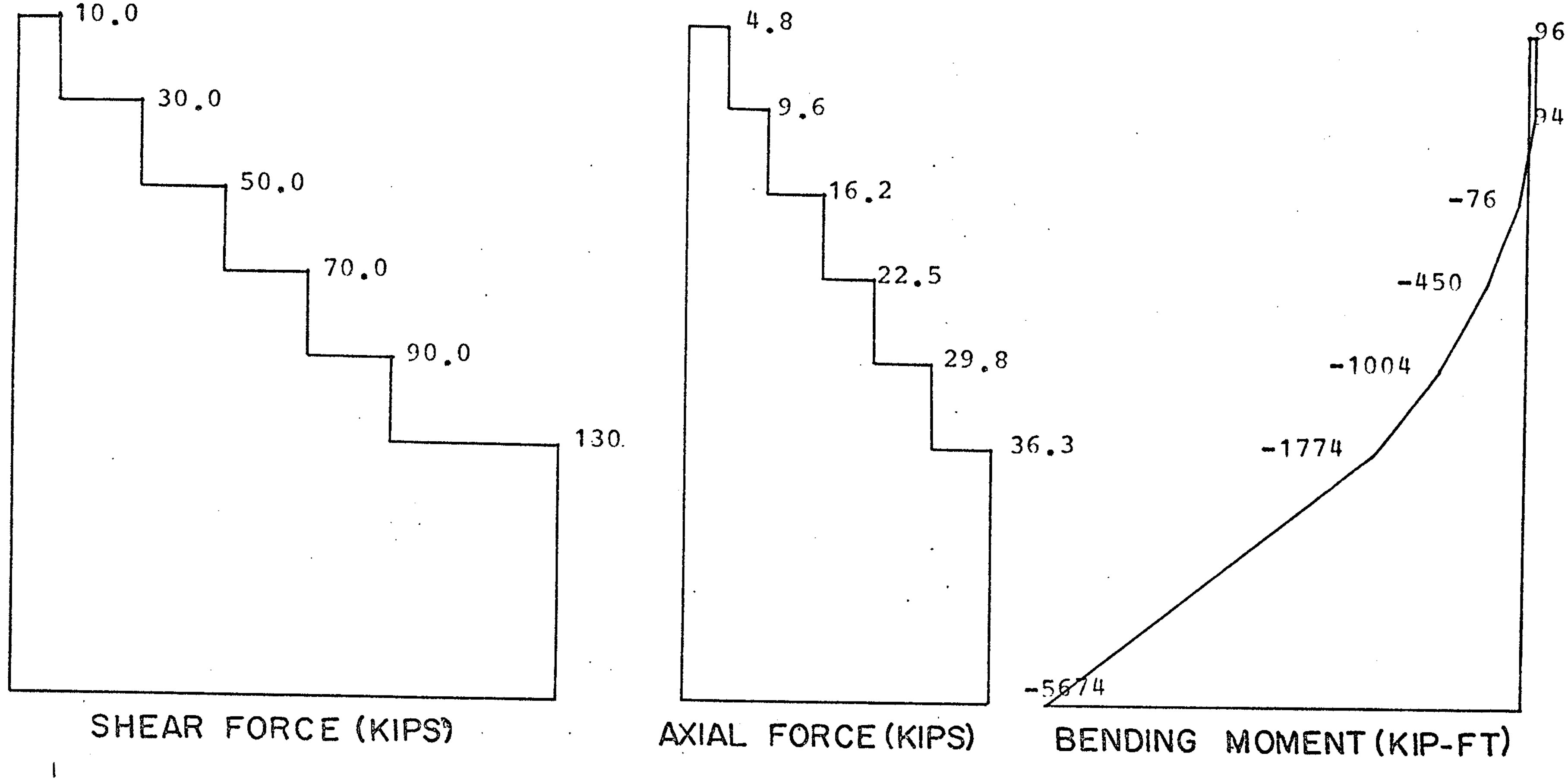


FIG 5.11 FINAL FORCES ON LEFT HAND SHEAR WALL ABOUT ITS ℄

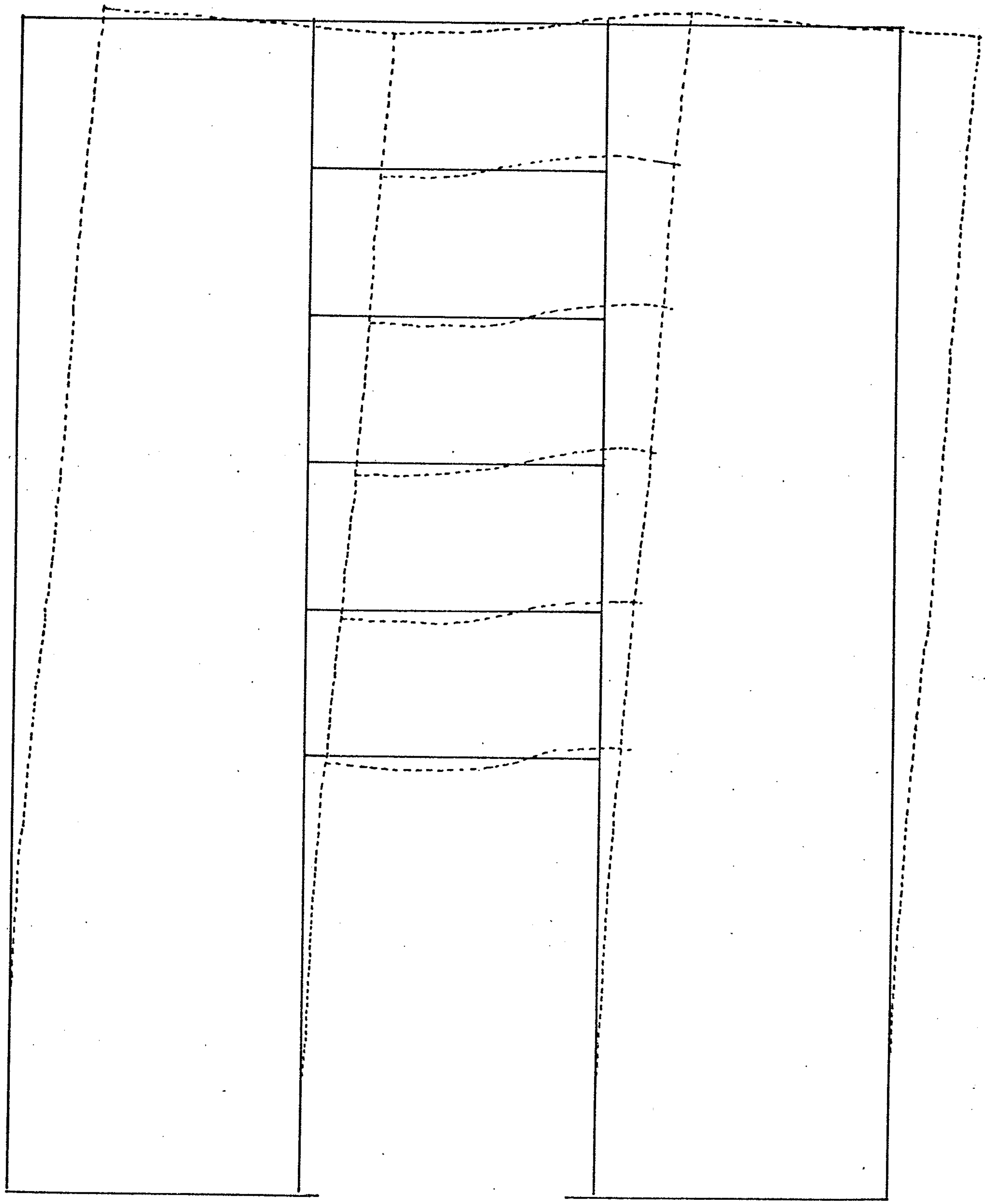


FIG. 5.12 FINAL DEFLECTED SHAPE OF STRUCTURE

5.4 EXAMPLE 4 - THIRTEEN STOREY SHEAR WALL-FRAME

A practical example is given to illustrate the efficiency of the program in analysing large structures. A typical floor plan of the building considered is shown in Fig. 5.13. For the purpose of illustration, only one lateral section, that on column line (N), is chosen and other similar sections can be analysed in the same manner. Since this is only an illustration of the analysis procedure, the wind pressure and other material properties are assumed to be constant throughout the height of the building. Fig. 5.14 shows the transverse section of the building on column line (N).

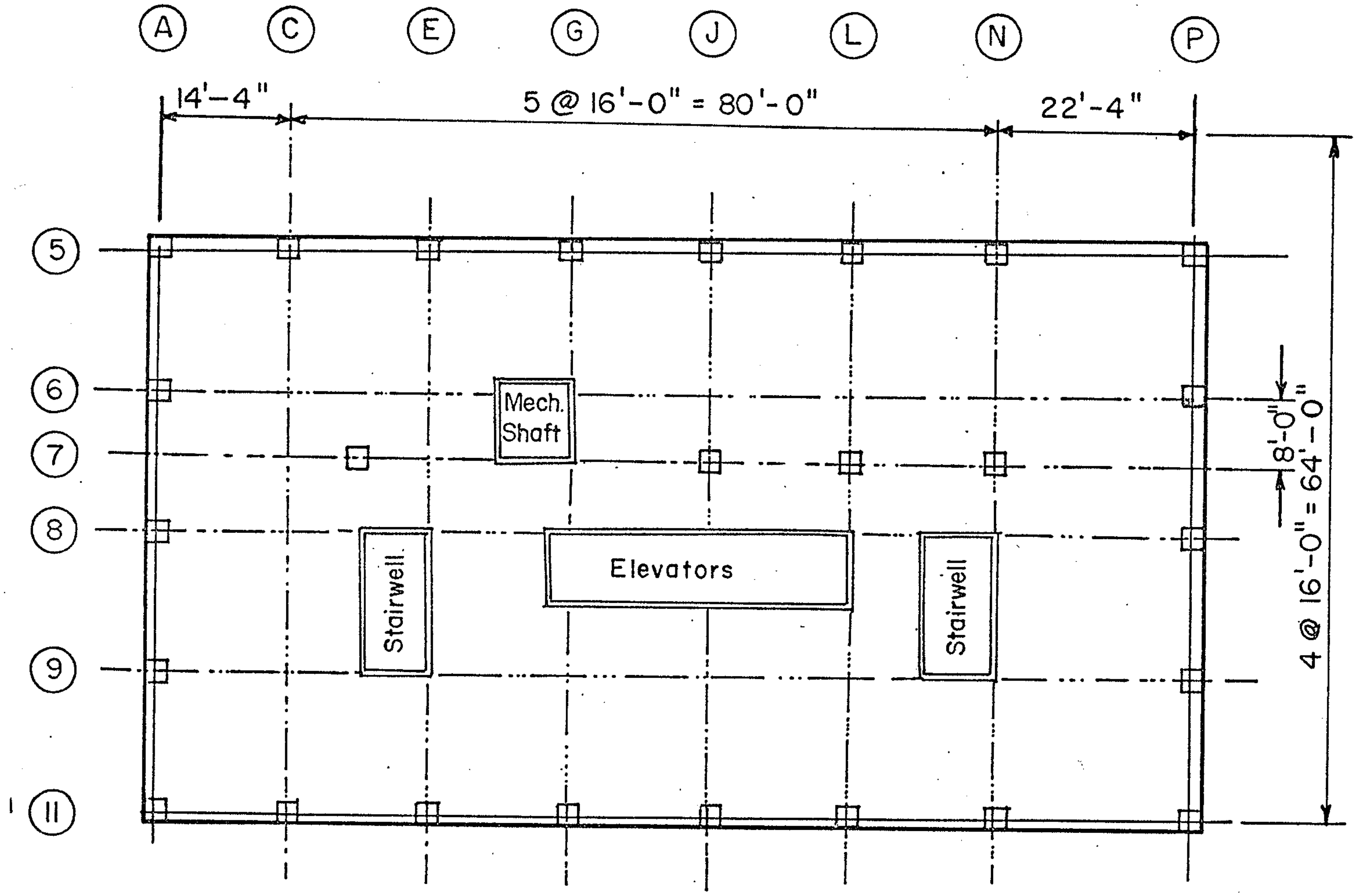
The building has a flat plate floor system and hence, portions of the slab are treated as beam elements in the transverse frame. The equivalent width of slab that can be treated as a beam, calculated using a procedure outlined by Khan and Sbarounis (8), is 10 feet. Other properties of the equivalent link beams and columns are given in Tables 5.17, 5.18 and 5.19.

The basic wind pressure assumed is 26psf.

Therefore,

Wind force at each floor level upto 13th =26X(8+11.17)X11.17

=5500 lbs =5.5K



TYPICAL FLOOR PLAN FOR FLAT SLAB BUILDING FIG. 5.13 EXAMPLE 4

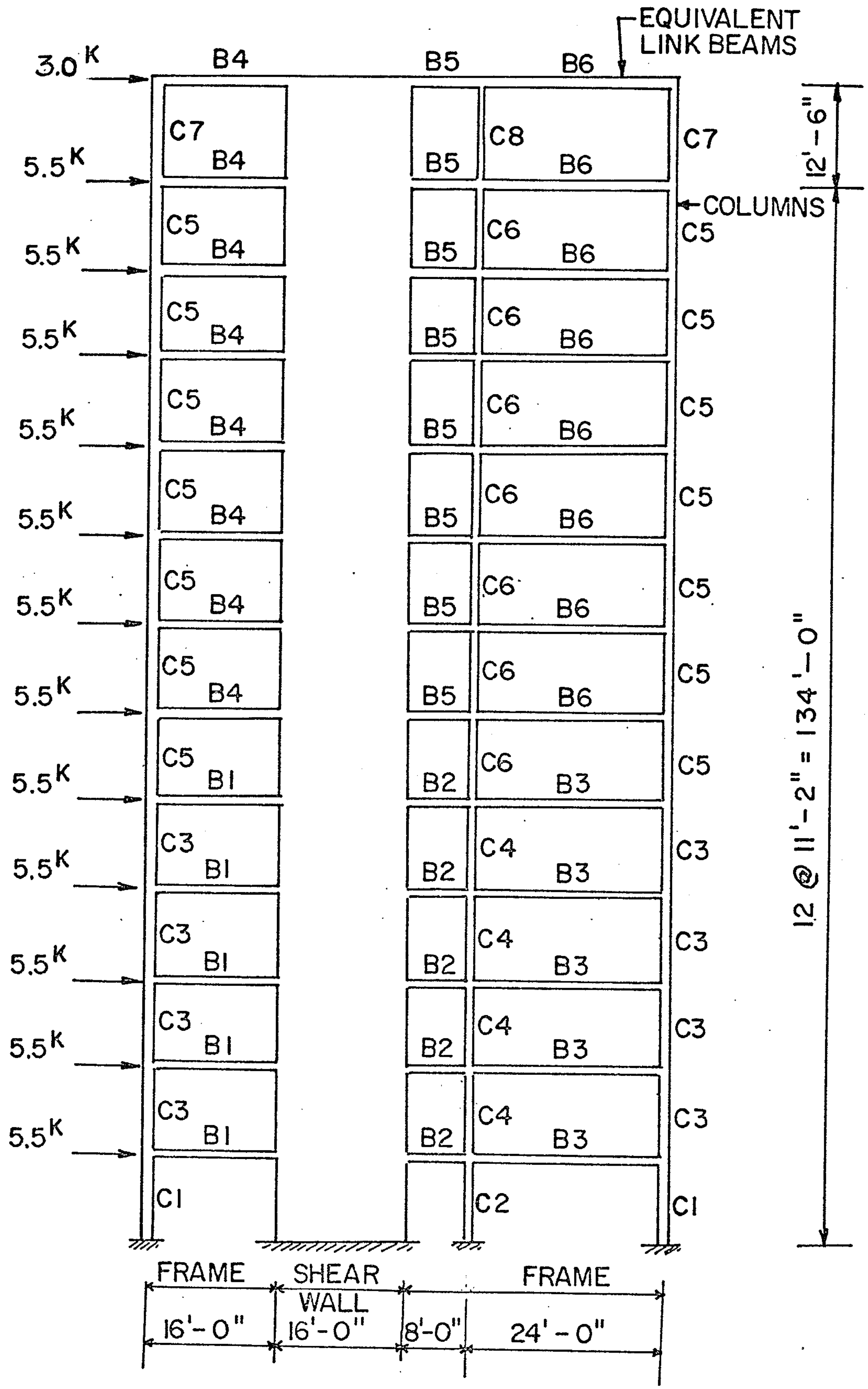


FIG. 5.14 SECTION ON COLUMN LINE (N)

TABLE 5.17

PROPERTIES OF SHEAR WALL

Storey	Height (Ft)	Width (Ft)	Modulus Of Elasticity (Ksi)	Poisson's Ratio	Thickness
1-5	11.167	16.0	4000	0.3	8.0"
5-12	11.167	16.0	3500	0.3	8.0"
13	12.500	16.0	3500	0.3	8.0"

TABLE 5.18

PROPERTIES OF BEAMS

Beam Type	Clear Length	Left Rigid Length	Right Rigid Length	Modulus Of Elasticity (Ksi)	I (in^4)
B1	15'3"	18"	0"	4000	5200
B2	7'3"	0"	18"	4000	5200
B3	22'6"	18"	18"	4000	5200
B4	15'3"	18"	0"	3500	5200
B5	7'3"	0"	18"	3500	5200
B6	22'6"	18"	18"	3500	5200

TABLE 5.19

PROPERTIES OF COLUMNS

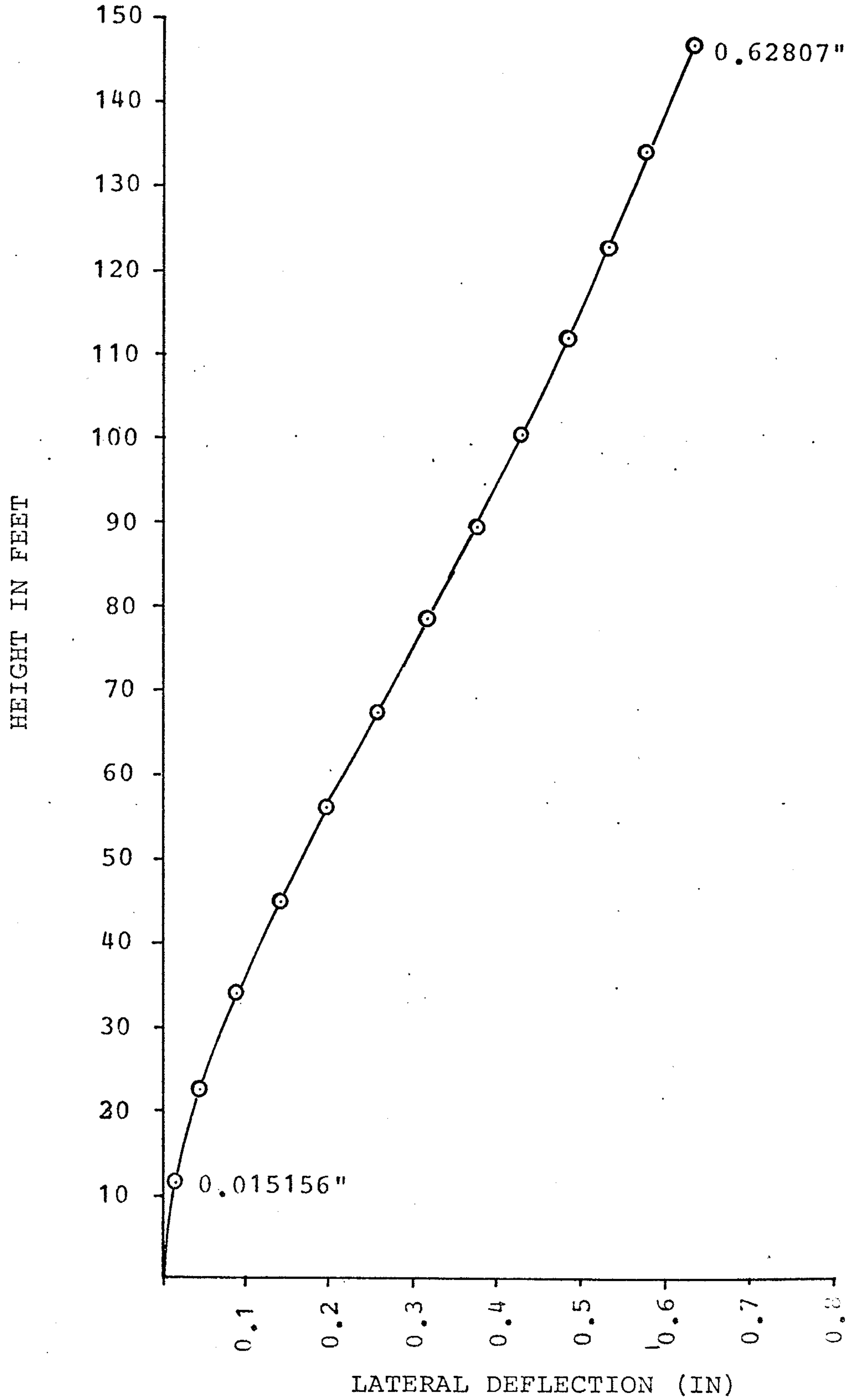
Column Type	Clear Length	Top Rigid Length	Bottom Rigid Length	Modulus of Elasticity (Ksi)	Moment of Inertia (in^4)	C. S. Area (in^2)
C1	10'8"	12"	0"	4000	10,000	346
C2	10'10"	8"	0"	4000	10,000	340
C3	10'2"	12"	12"	4000	10,000	340
C4	10'6"	8"	8"	4000	10,000	340
C5	10'2"	12"	12"	3500	10,000	340
C6	10'6"	8"	8"	3500	10,000	340
C7	11'8"	12"	12"	3500	10,000	340
C8	11'10"	8"	8"	3500	10,000	340

For the top most floor level, wind force =26X(8+11.17)X6.25

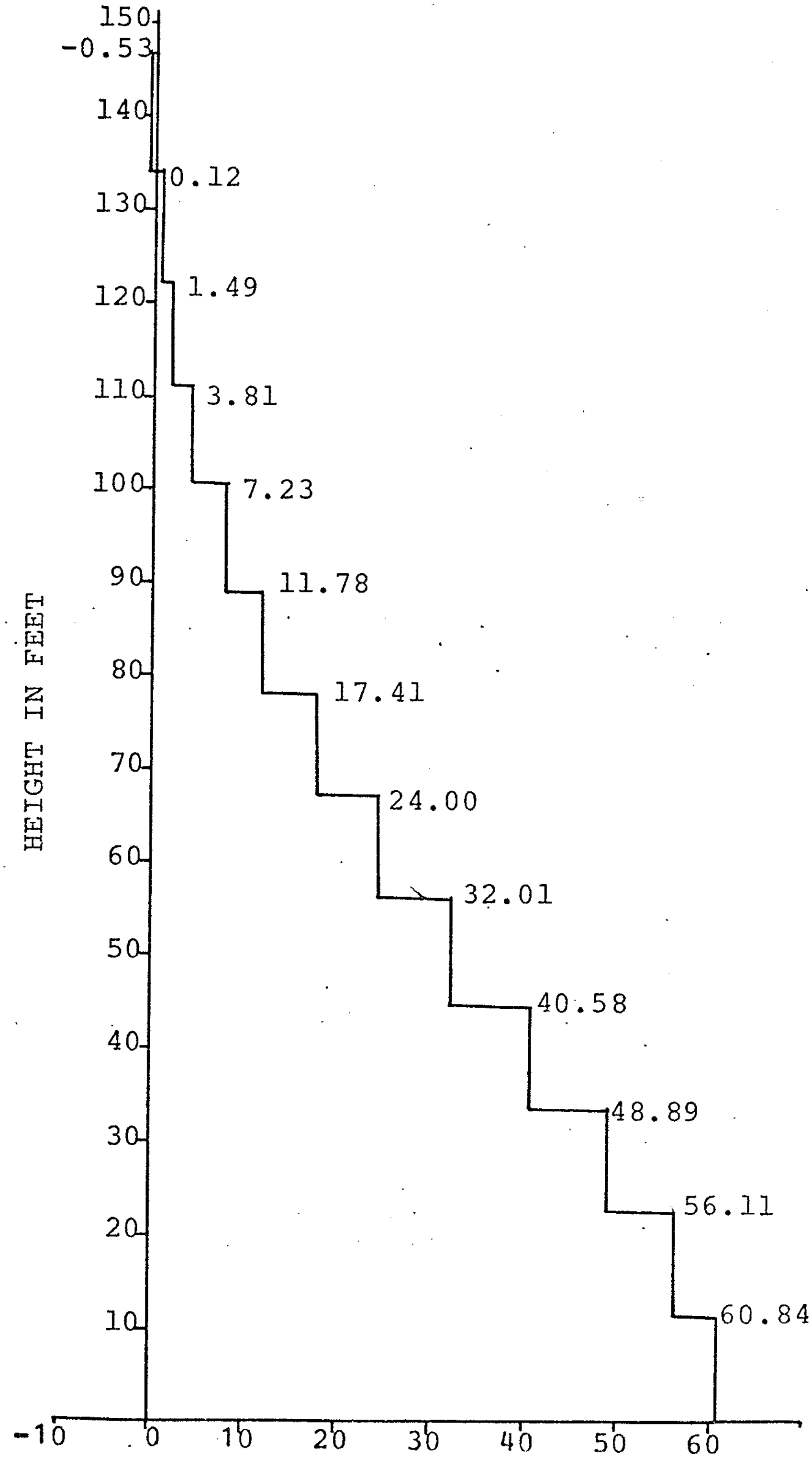
=3000 lbs =3.0 K

The values of lateral deflection at each floor level are shown in Fig. 5.15. The force components in the shear wall at different levels are given in Figs. 5.16, 5.17 and 5.18.

The maximum half band width for this structure was 18 and the maximum number of equations to be solved was 126. The storage requirement was approximately 176K and the example was run on double precision. The execution time was observed to be 0.53M. In commercial terms the cost of running the program was approximately $ 15.

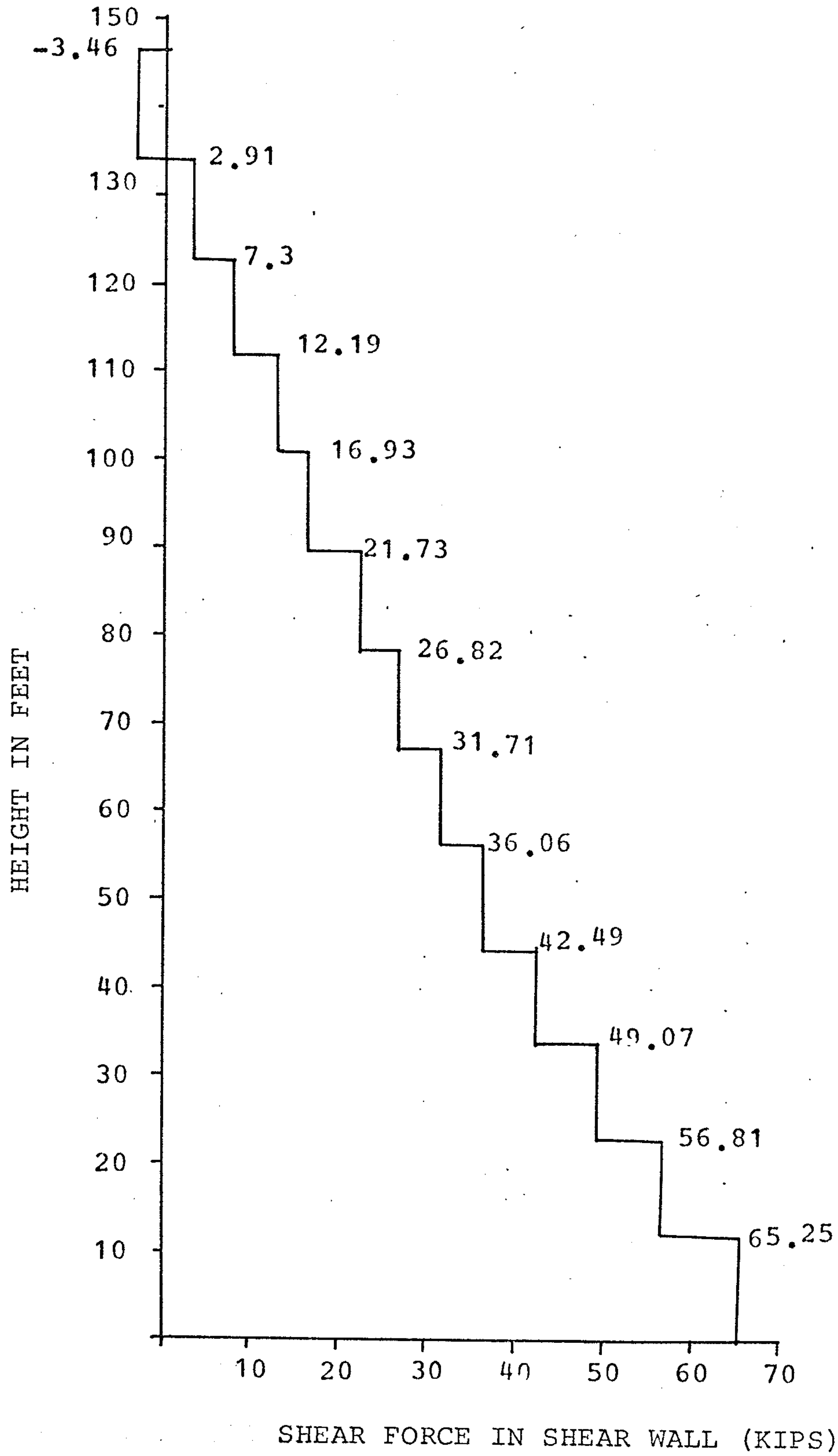


EXAMPLE - 4
FIG. 5.15



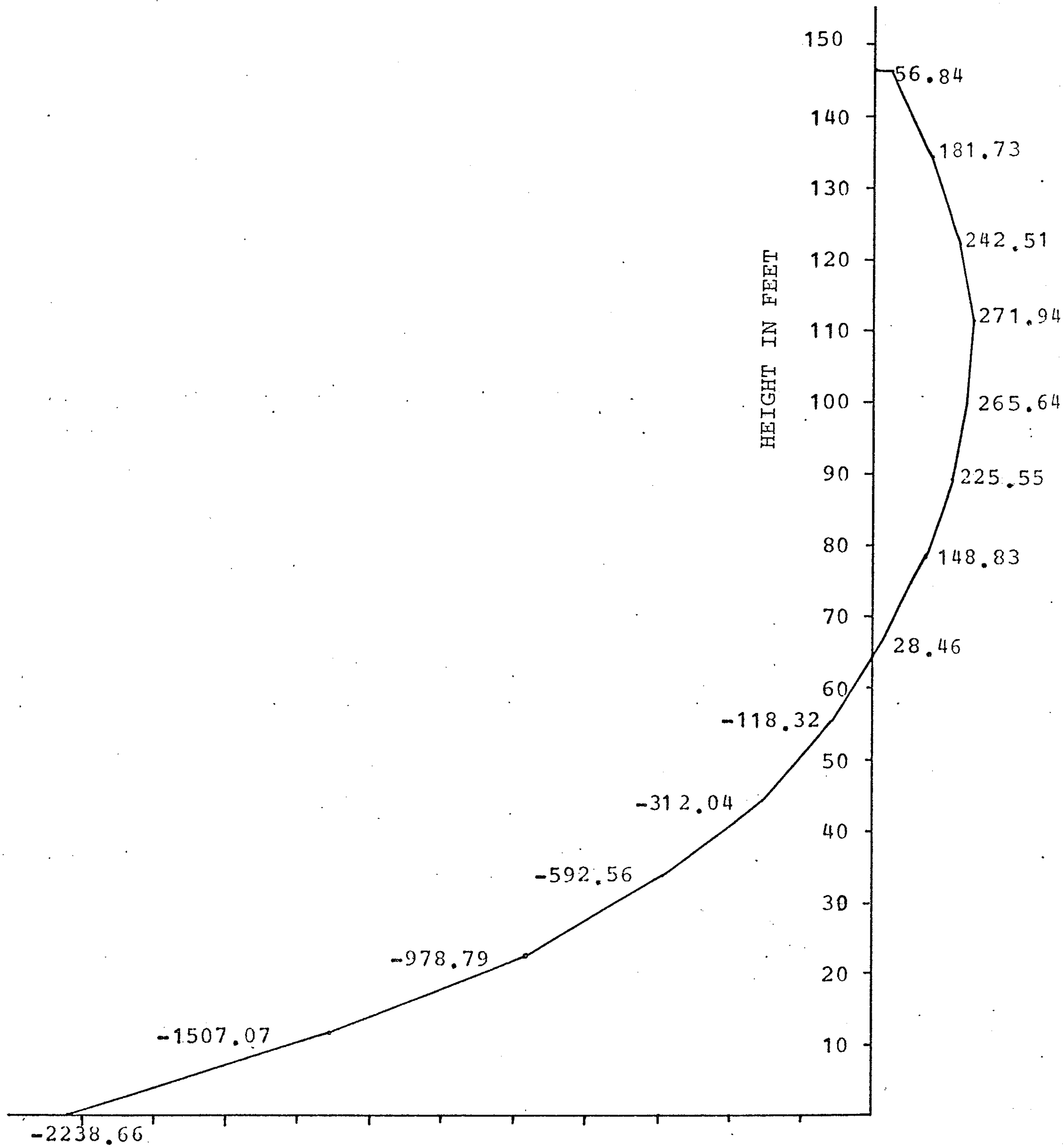
AXIAL FORCE IN SHEAR WALL (KIPS)

FIG. 5.16



EXAMPLE - 4

FIG. 5.17



BENDING MOMENT IN SHEAR WALL (KIP-FT)

EXAMPLE - 4

FIG. 5.18

5.5 COMPARISON OF THE COMPUTER REQUIREMENT FOR THE THREE EXAMPLES

As an illustration, Table 5.20 is presented to show the different requirements from the computer point of view for the three examples solved in this Chapter.

TABLE 5.20

COMPARISON OF COMPUTER REQUIREMENT

Example	No. of elements per segment of shear wall	storage requirement	Execution time	Cost Factor
1 (4 Storeys 3 Bays)	48	176K	0.24M	3.03
3 (6 Storeys 3 Bays)	48	176K	0.41M	3.91
4 (13 Store- ys 4 Bays)	54	176K	0.53M	5.00

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

6.1 CONCLUSION

A procedure for an efficient and economical analysis of shear wall-frame structures subjected to lateral load has been presented in this study. Four examples were given to demonstrate the analysis procedure and the efficiency and economy of the computer program. Despite the fact that the program does not use auxiliary storage, fairly large problems can be run economically with a reasonable amount of computer time.

Comparisons were made with another finite element method and two different methods which involved idealizing the shear wall as a deep column. These comparisons show that most of the displacements and forces calculated by the various methods agree within approximately 5%. Unfortunately, exact results are not available for comparison purposes.

The comparisons suggest that large local rotational deformations at shear wall-beam junctions occured when using a conventional finite element representation for the shear wall. The assumption employed in this study, that the plane transverse sections of the shear wall at the floor levels remain plane during loading, eliminates the large local deformations. Unfortunately, this assumption imposes a potentially significant constraint on the shear wall displacements, particularly for shear walls with openings. However, the comparisons suggest that the effect of the constraint on the calculated displacements and forces is minor, both for structures with solid shear walls and those with openings. Nevertheless, it does raise the question of the justification for the representation of the shear wall by a finite element model, when the constraint is subsequently imposed. As would be expected, the results further indicate that openings in the shear wall produced significantly increased horizontal deflections.

It was observed that changes in Poisson's ratio did not have a significant effect either on the deflection components or the force comnents.

The plots of deflection and force components with varying numbers of elements per shear wall segment indicate that any shear wall segment having 50 to 60 finite elements should produce a reasonably realistic stiffness for the wall.

6.2 RECOMMENDATIONS FOR FURTHER WORK

Because of the lack of an "exact" analytical procedure, it is recommended that an experimental study of shear wall frame structures be undertaken to corelate with the various approximate analytical procedures.

Since the assumption of a linear variation of displacements at the floor levels of the shear wall imposes a potentially significant constraint on the shear wall displacements, a study should be performed to find an alternate approach to the suppressing of excessive local deformations at the wall-beam connection points.

Instead of assuming a linear variation of displacements along the shear wall segment boundaries, a cubic variation in the displacement pattern, as shown in Fig. 6.1, should be tried.

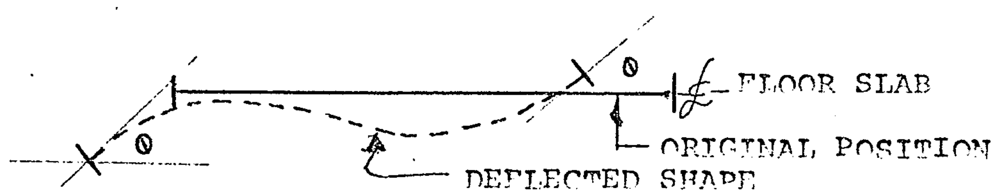


FIG. 6.1

This would definitely increase the accuracy of the results without much of an alteration to the storage requirement in the computer program. But this would not avoid the problem of local deformation. This approach would greatly increase the accuracy of the results for shear walls with openings. This kind of displacement pattern would require the modification of the RFDN subroutine in the computer program and the incorporation of the modified stiffness in the structure stiffness matrix of the MAIN program.

The rectangular finite element puts severe restriction on mesh refinement. As a further study, a quadrilateral element for shear walls should be tried. Recently, more refined rectangular elements have been developed (13,15). I.A. Macleod's element (15) is one of them. These elements could be incorporated into the model outlined in this study.

The program is limited to lateral load analysis. The same program could be modified to take account of more than one load vector. This will permit any suitable combination of loading for design purposes. In the present program, the rearranging of the equations after developing segment stiffness matrices can be avoided to save a considerable amount of execution time.

LIST OF REFERENCES

1.....Rosenblueth, E., and Holtz, J., "Elastic Analysis of Shear Walls in Tall Buildings", ACI Journal Proceedings, Vol. 56, June 1960.

2.....Cardon, B., "Concrete Shear Walls Combined with Rigid Frames in Multistory Buildings Subjected to Lateral Loads", ACI Journal Proceedings, Vol. 58, September 1961.

3.....Gould, P. L., "Interaction of shear wall frame systems in Multistory Buildings", ACI Journal Proceedings, Vol. 62, January 1962.

4.....Beck, Hubert, "Contribution to the Analysis of Coupled Shear Wall", ACI Journal Proceedings, Vol 59, August 1962.

5.....Bandel, H., "Frames Combined with Shear Frames under Lateral Loads", ASCE Proceedings, Vol. 88, 576, December 1962.

6.....Rosman, Riko, "Approximate Method of Analysis of Shear Walls Subjected to Lateral Loads", ACI Proceedings, Vol. 61, June 1964.

7.....Clough, King and Wilson, "Structural Analysis of Multistory Buildings", ASCE Proceedings, Vol. 90, ST3, June 1964.

8.....Khan and Sbarounis, "Interaction of Shear Walls and Frames", ASCE Proceedings, Vol. 90, No. ST3, June 1964.

9.....Parme, A. L., "Design of Combined Frames and Shear Walls", PCA, Adv. Engg. Bulletin, No. 14, 1965.

10....Coull and Smith, "Tall Buildings", Proceedings of a Symposium Held at the University of Southampton, April 1966.

11....Macleod, I. A., "Lateral Stiffness of Shear Wall with Openings", Tall Buildings - Proceedings of a Symposium held at the University of Southampton, April 1966.

12....German Gurfinkel, "Simple Method of Analysis of Vierendeel Structures", ASCE Proceedings, Vol. 93, No. ST3, June 1967, p. 273.

13....Oakberg, Robert G., William Weaver, Jr., "Analysis of Frames with Shear Walls by Finite Elements", ASCE Proceedings on "Application of Finite Element Methods in Civil Engineering", November 13 - 14, 1969.

14....Girijavallabhan Chiyyarath V., "Analysis of Shear Walls with Openings", ASCE Proceedings, Vol.95 , No. ST10, October 1969.

15....Macleod, I.A., "New Rectangular Finite Element for Shear Wall Analysis", ASCE Proceedings, Vol. 95, No. ST3, March 1969.

16....Melosh, Robert J., "Basis for Derivation of Matrices for the Direct Stiffness Method", AIAA Journal, Vol. 1, No. 7, July 1963.

17....Fox, L., "An Introduction to Numerical Linear Algebra", Oxford University Press, 1965.

18....Oakberg, R.G.," Analysis of Frames with Shear walls by Finite Elements," dissertation presented to Stanford University, at Stanford, California, in 1967

19....Majumdar S.N.G., MacGregar J.G., Adams P.F., "Experimental and Analytical study of the behavior of Steel Frame-Shear wall structures", Structural Engineering report No. 27, Department of Civil Engineering, University of Alberta, Edmonton, Canada

20....Przemieniecki J.S., "Theory of Matrix Structural Analysis ", MacGrew-Hill Book company, 1968

21....Newmark,N.M., " Numerical procedure for Computing Deflections, Moments and Buckling Loads ", Transactions A.S.C.E Vol. 108, 1943, pp. 1161 - 1234.

22....Michael, D., "The Effect of Local Wall Deformations on the Elastic Interaction of Cross Walls Coupled by Beams ", Proceedings of a Symposium on"Tall Buildings" held in the Department of Civil Engineering, University of Southampton April 1966.

APPENDIX A

AITKEN'S METHOD

In a structural analysis by the stiffness method it is often necessary to reduce the order of a structure stiffness matrix,

$$K_S = \left[\begin{array}{c|c} K_{EE} & K_{EI} \\ \hline K_{IE} & K_{II} \end{array}\right] \tag{A.1}$$

(where K_{EE} , K_{EI} , K_{II} and K_{IE} are sub-matrices and E and I represent the order of these matrices)

This can be accomplished by the relationship,

$$K = K_{EE} - K_{EI}\, K_{II}^{-1}\, K_{IE} \tag{A.2}$$

Normally, the matrix K_{II} is of very high order. A direct evaluation of matrix K_S involves an inverse of this high order matrix, two multiplications and a subtraction, which becomes uneconomical from the computer point of view. Aitken developed an expression ($Y = CA^{-1}B$) similar to that contained in Eq. (A.2), from a general matrix of the form,

$$\left[\begin{array}{c|c} A_{mm} & B_{mn} \\ \hline C_{pm} & O_{pn} \end{array}\right] \qquad (A.3)$$

by a simplified Gauss elimination technique in which O is a null matrix

This method can be used as a powerful tool in the static condensation. This needs, only rearranging of the terms in Eq. (A.1) as per Eq. (A.3). The null matrix can be replaced by K_{EE}.

By perfoming Gauss elimination of the matrix (A.3) till A becomes an upper triangular matrix, the null matrix O is replaced by a new matrix of the form $-CA^{-1}B$ which is the required expression for the stiffness analysis.

Proof

$$\text{Let } Y = C\,A^{-1}\,B \qquad (A.4)$$

where C, A and B are submatrices as in (A.3).

The Matrix (A.4) can be expressed by the two equations,

$$\left.\begin{array}{l} A\,X = B \\ C\,X = Y \end{array}\right\} \qquad (A.5)$$

Performing Gauss elimination for the first of Eqs. (A.5), matrix A is replaced by an upper triangular matrix (say 'U').

The equation can thus be rewritten as

$$U\,X = F \tag{A.6}$$

Now consider the array

$$\left[\begin{array}{c|c} U & F \\ \hline C & O \end{array}\right]$$

Performing Gauss elimination once again to make C a null matrix, O is replaced by another matrix Q. That is multiples of the first row of U are added to successive rows of C so that elements in the first column of C are replaced by zeros. Further addition of second row of U to the rows of C can produce zeros in the second column of C without affecting its first column since U is upper triangular. A similar operation with F added to rows of O is performed. This operation is equivalent to the following expression

$$\left[\begin{array}{c|c} I & O \\ \hline L & I \end{array}\right] \left[\begin{array}{c|c} U & F \\ \hline C & O \end{array}\right] = \left[\begin{array}{c|c} U & F \\ \hline M & Q \end{array}\right]$$

from which

$M = LU+C=0$ since M is a null matrix from the previous operation.

Therefore,

$$L = -CU^{-1}$$

and

$$Q = LF = -CU^{-1}F \tag{A.7}$$

From the first of equations (A.5) we have

$$X = A^{-1} B$$

and from (A.6)

$$X = U^{-1} F$$

Therefore, $U^{-1}F = A^{-1} B$

Equation (A.7) now becomes

$$Q = -CA^{-1} B$$

Hence the proof.

The reduction of A to U ,followed by that of C to O can be performed simultaneously, treating the rows of C for the purposes of elimination as if they belonged to A, pivots being chosen only from A. The relevent matrix operation is then,

$$\left[\begin{array}{c|c} J & O \\ \hline L & I \end{array}\right] \left[\begin{array}{c|c} A & B \\ \hline C & O \end{array}\right] = \left[\begin{array}{c|c} JA & JB \\ \hline LA+C & LB \end{array}\right]$$

where, 'J' is a unit lower triangular matrix which when multiplied by 'A' is nothing but 'U' and

$$LA+C = 0$$

Thus, $L = -CA^{-1}$

and $$[illegible]\ LB = -CA^{-1}B$$

which is the required expression.

APPENDIX B

COMPUTER ORGANIZATION

B.1 PURPOSE

The program performs a linear elastic analysis of shear wall-frame structures from the known material properties and over-all dimensions of the structure. The results consist of joint displacements, forces in the shear wall(s), beam and column end forces and reactions. The shear wall is analysed by finite element idealizations.

B.2 PROGRAMMING INFORMATION

The program is written in FOTRAN IV (version) for the IBM360 computer.

B.3 CAPACITY

The storage capacity is dependant on the following varibles:

NSH - Number of shear walls

NS - Number of storeys

NN - Number of joints

NBM - Number of beams

NC - Number of columns

MAXX - Maximum number of vertical rows of finite elements in any shear wall and

MAXY - Maximum number of horizontal rows of finite elements for a segment of any shear wall.

The dynamic storage allocation is used and the array area for this purpose can be determined from the known values of the above variables. By numbering the joints in a particular sequence the maximum half band width MBAND of the structure stiffness matrix is given by,

$$\text{MBAND} = (\text{N1*2+1-NSH*2})\text{*2} \qquad \text{(B.1)}$$

where N1 = NN/(NS+1)

The number of equations NEQN to be solved for the structure is given by,

$$\text{NEQN} = \text{2*NN+(NS+1)-2*(NS+1)*NSH} \qquad \text{(B.2)}$$

Denoting the storage pool as Z, the equation that decides the storage area is,

$$\begin{aligned} \text{Z} = {} & \text{6*NN+37MAXX +(NSH*NS+NSH+308)MAXX+(NSH*NS+NS)MAXY} \\ & \text{+NSH(31NS+21)+4NS+7(NBM+NC)+NEQN*(MBAND+1)+674} \qquad \text{(B.3)} \end{aligned}$$

As explained in Chapter V, any segment having 50 to 60 finite elements produces a reasonably accurate stiffness matrix for the shear wall. In this case, the values of MAXX and MAXY may be tentatively restricted to 8. With the above simplification and denoting by Z1 the storage required for equation solver and z2 the rest of the storage,

$$Z1 = NEQN*(MBAND+1) \tag{B.4}$$

$$Z2 = 6NN+47NSH*NS+29NSH+12NS+7(NBM+NC)+5506 \tag{B.5}$$

and

$$Z' = Z1+Z2$$

where MBAND and NEQN are as given by Eq. (B.1) and Eq. (B.2) respectively. Z' gives the storage required if MAXX and MAXY do not exceed 8. Otherwise the value of Z given by Eq. (B.3) should be used.

An indication of the size of the problem that can be handled is given in Table B.1.

TABLE B.1

SIZE OF PROBLEM FOR VARIOUS VALUES OF Z'

Z'	10000		20000		30000	40000
Number of Bays	2	10	2	10	10	10
Number of Storeys	30	4	98	17	30	42

Number of shear walls considered for 2 bays is one and that for 10 bays is 4.

Approximate core requirements are:

Z'	Single Precision	Double Precision
10000	134K	174K
20000	174K	254K
30000	214K	334K
40000	254K	414K

B.4 BASIC MESH UNITS FOR SHEAR WALL

The basic mesh unit for the shear wall is a rectangular element. The program generates the mesh automatically if the X coordinates of the nodal points of the elements in X-direction and Y-coordinates of the nodal points of the elements in Y-direction for a shear wall are given.

In Fig. B.1 if the values of X_1, X_2....X_M and Y_1, Y_2,... Y_N are given the mesh is automatically generated for the shear wall under consideration. m and n are the number of the element nodal points in the X and Y directions respectively.

B.5 PROGRAM STRUCTURE

The flow chart for the complete program is given at the end of this section.

MAIN

Reads the input required for allocating the storage area. It allocates storage for the various arrays and calls subroutine SWF.

SWF

Reads all input data, forms the structure stiffness matrix by calling subroutines SEGMT, REDN, STBEAM and STCOLM. It stores the stiffness matrix and the load vector in one dimensional arrays. If a joint is constrained to move in a particular direction (support condition - fixed

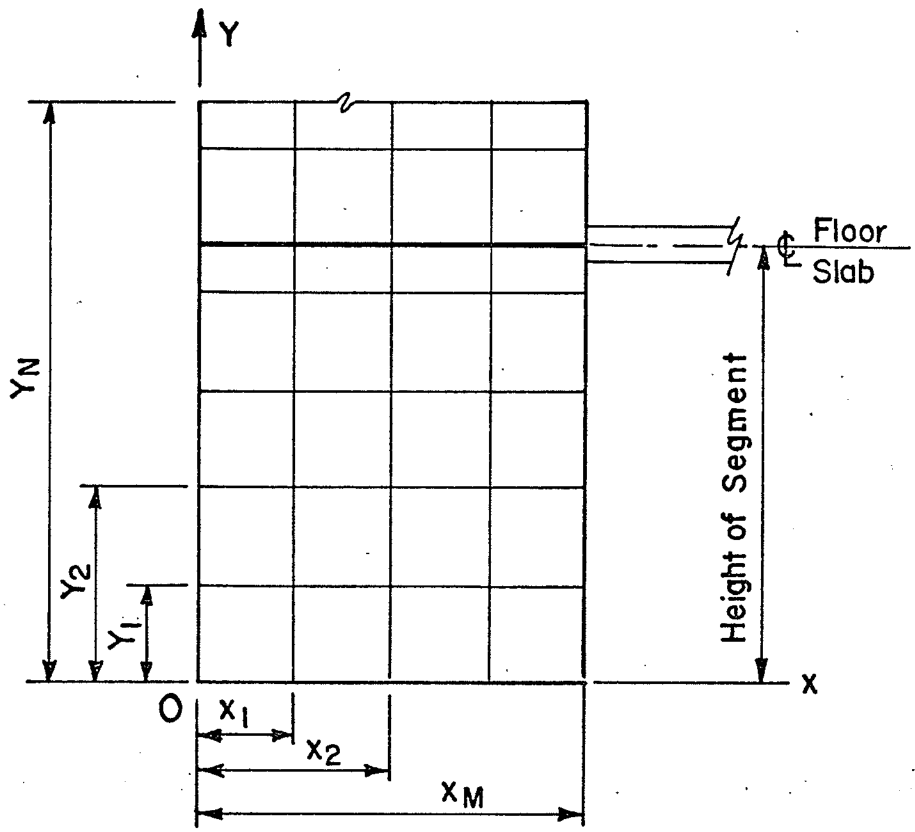


FIG. B.1 BASIC MESH UNIT FOR SHEAR WALL

or pinned), a very large number is inserted in the main diagonal of the corresponding row in the structure stiffness matrix and the corresponding external force component is set to zero. The equation is solved by calling equation solver INSOL and the joint displacements are printed. Then it calls the subroutine FORCE, which calculates the force in the shear walls and the reactions on the shear wall base. The subroutine MMULT is called to calculate and print the forces on the beam and column ends. Finally, the reactions at other support joints are calculated and printed.

SEGMT

Generates stiffness matrix for a segment and condenses off the internal nodes starting from top row to bottom row of elements using the condensation process explained in Article 2.4. The procedure is explained in Chapter III. This subroutine in turn calls one of the following subroutines STEDGE, STINT, STLTR and STRTR depending on the type of element under consideration while developing the stiffness matrix for any row. The matrix is returned as ST in two dimensional array.

REDN

Suppresses the internal displacement vector along any edge and the rotation at the two ends of the edge and returns the modified stiffness matrix as AK.

STBEAM

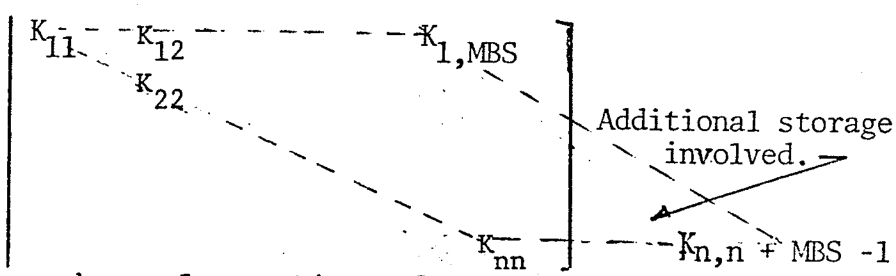
Generates stiffness matrix for a beam

STCOLM

Generates stiffness matrix for a column.

FORCE

Calculates forces in shear wall and prints them. The stiffness matrix for shear wall for this purpose is stored as STSH in a two dimensional array. The first subscript in the array denotes the shear wall number and the second denotes the position of the coefficient in the stiffness matrix for the shear wall. This is, incidentally, stored as a banded matrix in the form STSH (I,K) where I = 1, 2, 3...number of shear walls and K = K_{11}, K_{22}, K_{33}, K_{44}, K_{55}...K_{nn}, K_{12}, K_{23}, ...; $K_{n, n + MBS - 1}$; and K is given by



n is the number of equations for shear wall and MBS is the half band width for shear wall.

INSOL

Symmetric banded in core matrix equation solver used for the displacement solution. A is the stiffness matrix stored in one dimensional array stored in the same manner as explained for the second subscript of STSH. Here band width is the maximum half band width of the structure stiffness matrix and number of equations is the total number of equations for the structure. The load vector stored as B is used for back substitution and returns the displacement vector as B.

MMULT

Performs matrix multiplication and is made use of in calculating beam and column end forces.

STEDGE

Generates stiffness matrix for an edge element.

STLTR

Generates stiffness matrix for a left transition element.

STRTR

Generates stiffness matrix for a right transition element.

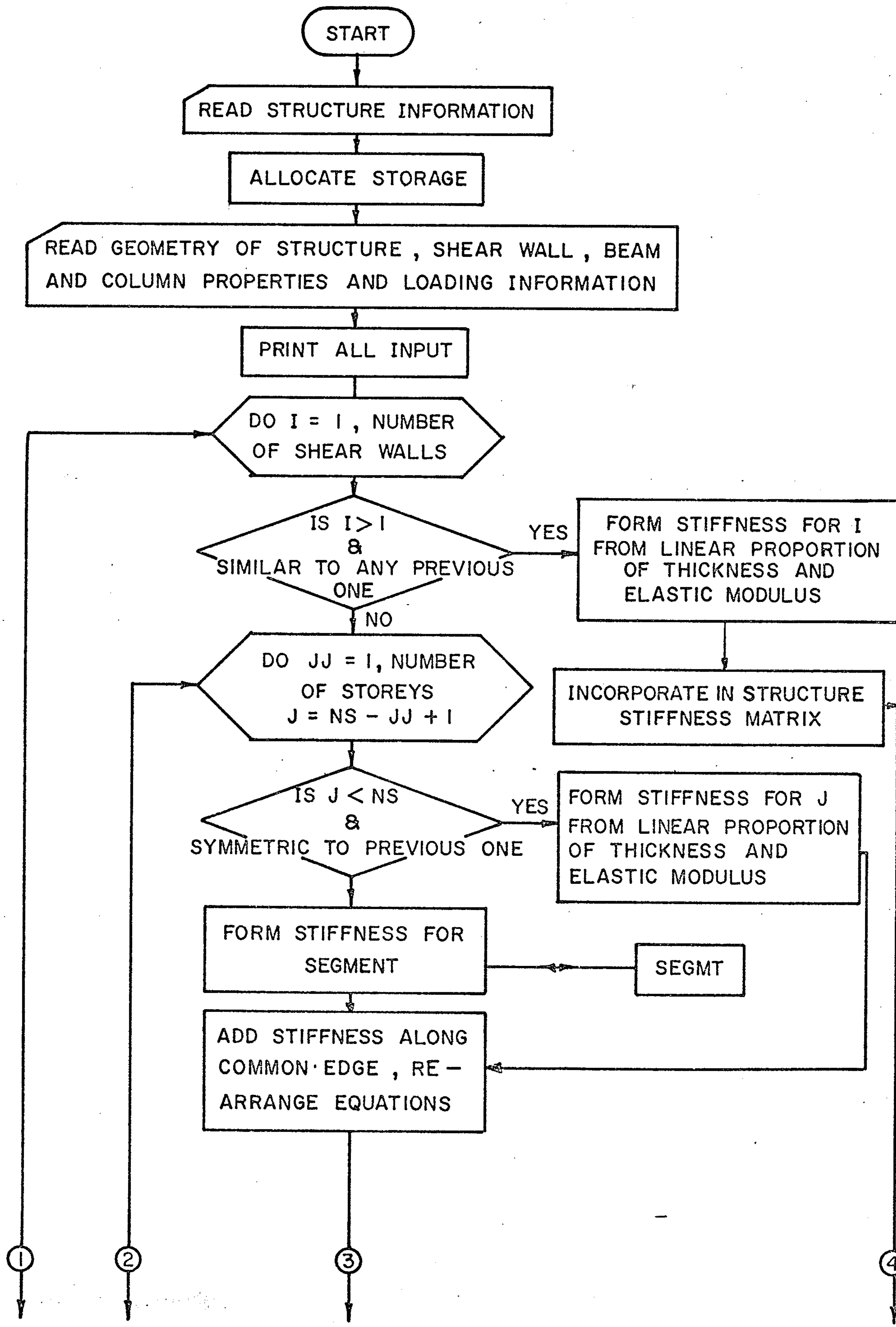
STINT

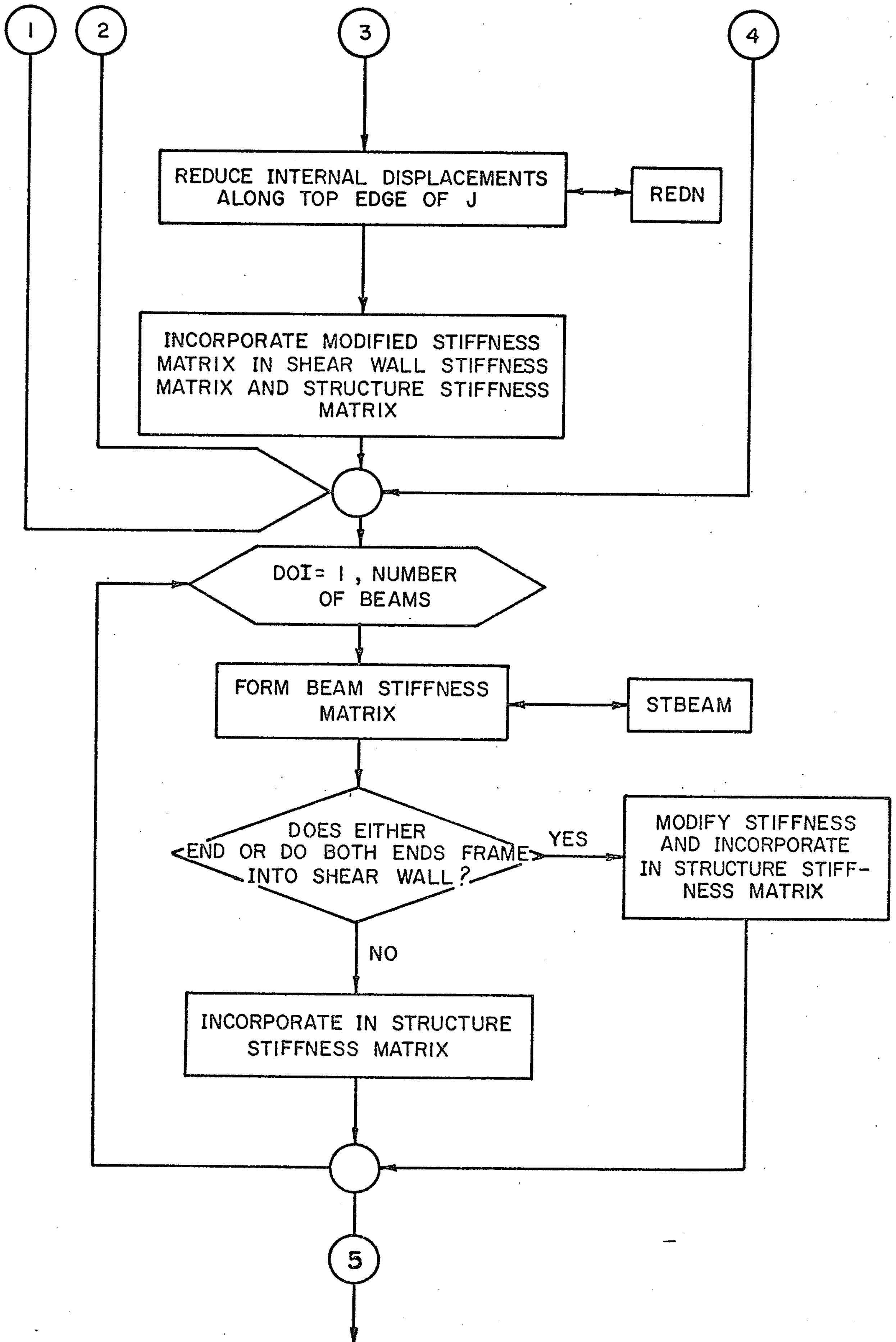
Generates stiffness matrix for an interior element.

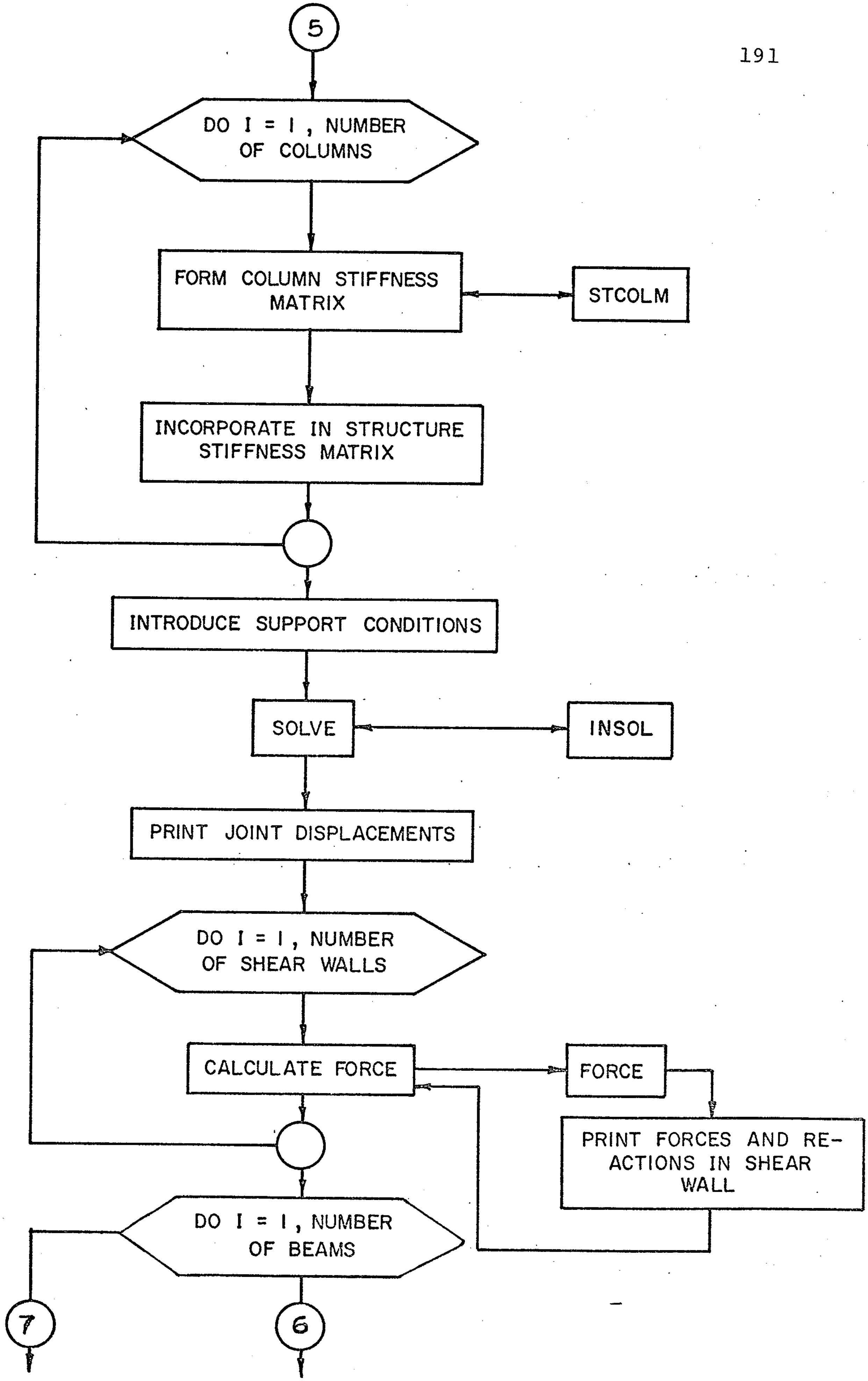
CNVRT

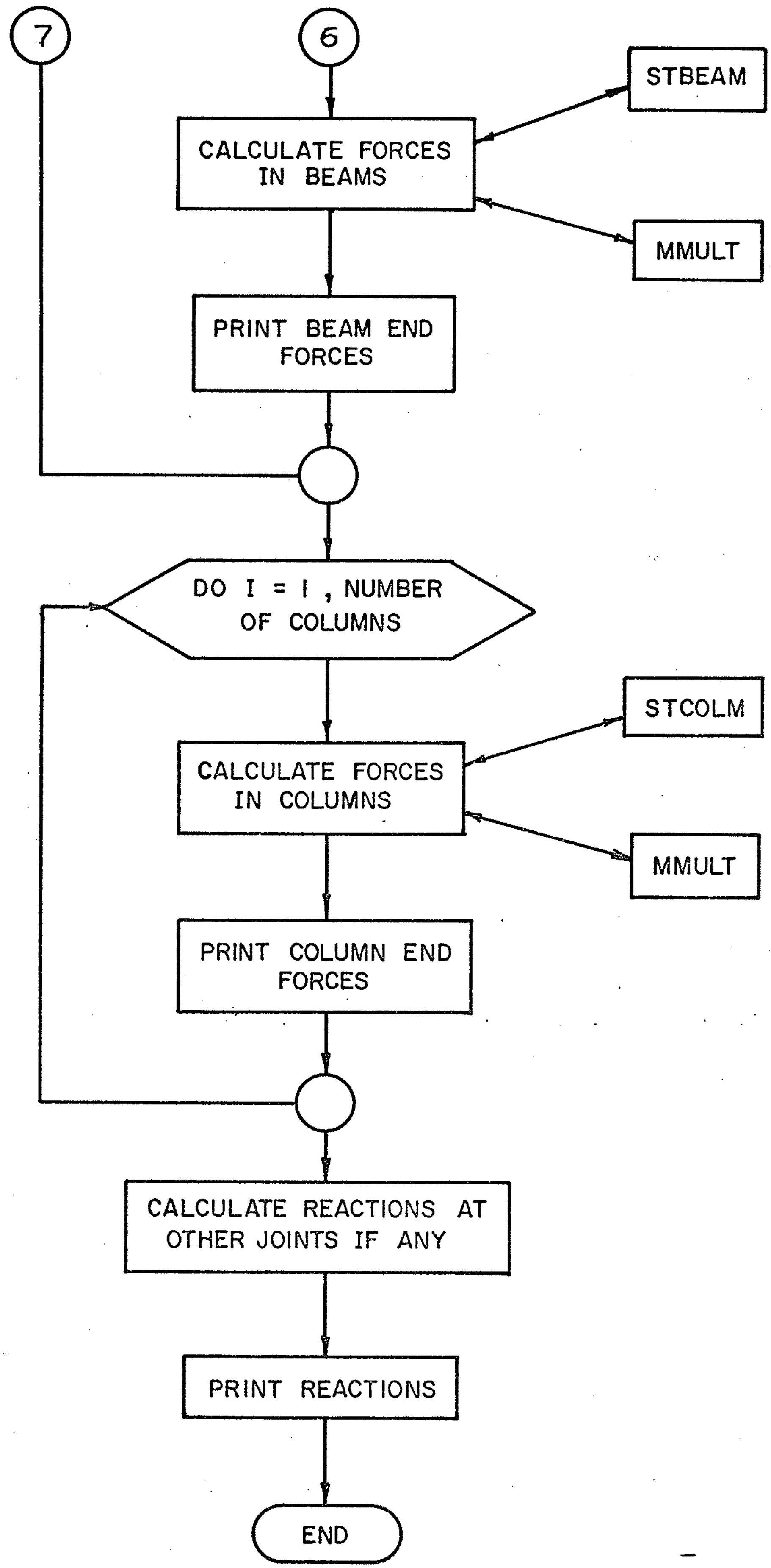
Converts input data to real numbers.

FLOW CHART FOR MAIN PROGRAM

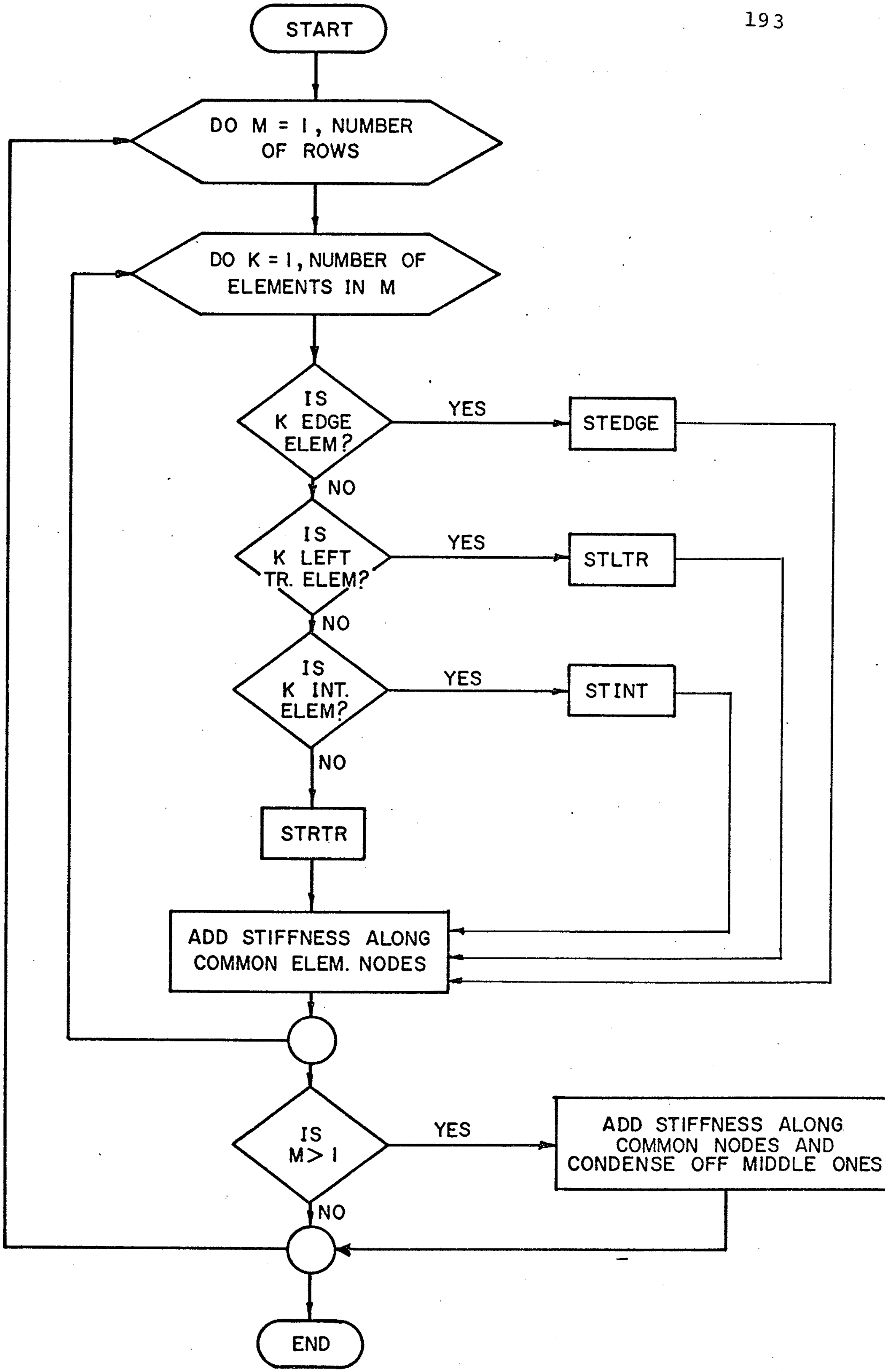

START
READ STRUCTURE INFORMATION
ALLOCATE STORAGE
READ GEOMETRY OF STRUCTURE , SHEAR WALL , BEAM AND COLUMN PROPERTIES AND LOADING INFORMATION
PRINT ALL INPUT
DO I = 1 , NUMBER OF SHEAR WALLS
IS I > 1 & SIMILAR TO ANY PREVIOUS ONE
YES
FORM STIFFNESS FOR I FROM LINEAR PROPORTION OF THICKNESS AND ELASTIC MODULUS
NO
INCORPORATE IN STRUCTURE STIFFNESS MATRIX
DO JJ = 1, NUMBER OF STOREYS J = NS - JJ + 1
IS J < NS & SYMMETRIC TO PREVIOUS ONE
YES
FORM STIFFNESS FOR J FROM LINEAR PROPORTION OF THICKNESS AND ELASTIC MODULUS
FORM STIFFNESS FOR SEGMENT
SEGMT
ADD STIFFNESS ALONG COMMON EDGE , RE - ARRANGE EQUATIONS
1
2
3
4


1
2
3
4
REDUCE INTERNAL DISPLACEMENTS ALONG TOP EDGE OF J
REDN
INCORPORATE MODIFIED STIFFNESS MATRIX IN SHEAR WALL STIFFNESS MATRIX AND STRUCTURE STIFFNESS MATRIX
DOI= 1 , NUMBER OF BEAMS
FORM BEAM STIFFNESS MATRIX
STBEAM
DOES EITHER END OR DO BOTH ENDS FRAME INTO SHEAR WALL ?
YES
MODIFY STIFFNESS AND INCORPORATE IN STRUCTURE STIFF-NESS MATRIX
NO
INCORPORATE IN STRUCTURE STIFFNESS MATRIX
5


5
DO I = 1 , NUMBER OF COLUMNS
FORM COLUMN STIFFNESS MATRIX
STCOLM
INCORPORATE IN STRUCTURE STIFFNESS MATRIX
INTRODUCE SUPPORT CONDITIONS
SOLVE
INSOL
PRINT JOINT DISPLACEMENTS
DO I = 1, NUMBER OF SHEAR WALLS
CALCULATE FORCE
FORCE
PRINT FORCES AND RE-ACTIONS IN SHEAR WALL
DO I = 1, NUMBER OF BEAMS
7
6


7
6
STBEAM
CALCULATE FORCES IN BEAMS
MMULT
PRINT BEAM END FORCES
DO I = 1 , NUMBER OF COLUMNS
STCOLM
CALCULATE FORCES IN COLUMNS
MMULT
PRINT COLUMN END FORCES
CALCULATE REACTIONS AT OTHER JOINTS IF ANY
PRINT REACTIONS
END

FLOW CHART FOR SUBROUTINE SEGMT


START
DO M = 1, NUMBER OF ROWS
DO K = 1, NUMBER OF ELEMENTS IN M
IS K EDGE ELEM?
YES
STEDGE
NO
IS K LEFT TR. ELEM?
YES
STLTR
NO
IS K INT. ELEM?
YES
STINT
NO
STRTR
ADD STIFFNESS ALONG COMMON ELEM. NODES
IS M > 1
YES
ADD STIFFNESS ALONG COMMON NODES AND CONDENSE OFF MIDDLE ONES
NO
END

APPENDIX C

USER'S MANUAL

C.1 PROGRAM IDENTIFICATION

SWALFRME - This program performs a linear elastic analysis of laterally loaded shear wall-frame structures.

C.2 DESCRIPTION OF STRUCTURE

In describing the structure all joints are numbered in sequence starting from the base and numbering them from left to right as shown in Fig. C.1. The shear walls, beams and columns are also numbered in sequence from left to right starting from the base. The shear wall is divided into segments with boundaries at the various levels as shown by dotted lines in Fig. C.1. The coordinates of the joints are expressed in a global coordinate system, the origin of which may lie anywhere.

Each shear wall segment is subdivided into rectangular finite elements. The vertical rows of elements should extend the full height of shear wall with constant width. If a segment has an opening, there should be atleast one horizontal row of elements in the segment, above and below the opening. The segments of each shear wall are numbered independantly from bottom to top.

A shear wall is similar to another one in the same structure only if all the properties and dimensions are identical, except that the thickness and modulus of elasticity can vary in the same proportion at corresponding points of the two shear walls.

A segment is similar to one above it only if all the properties and dimensions are identical, except that the thickness and modulus of elasticity can be proportional at corresponding points for the two segments.

C.3 INPUT

Except for descriptive heading cards, each data card is divided into 10 column fields unless otherwise mentioned.

DATA CARDS:

(a) Job description - one card - to contain a job description which is printed out as a title over the output.

(b) Structure information - one card

Field 1	number of shear walls
Field 2	number of storyes
Field 3	number of beams
Field 4	number of columns
Field 5	number of joints
Field 6	number of laterally loaded joints
Field 7	maximum number of vertical rows of elements encountered in any shear wall.
Field 8	maximum number of horizontal rows of elements encountered in a segment of any shear wall

(c) Joint Information - one card for each joint

Field 1	joint number
Field 2	joint x - coordinate (ft)
Field 3	joint y - coordinate (ft)
Field 4	support condition Blank - Non support joint F - Fixed support R - Rotation release (pinned support)

If field 1 is left blank on nth joint information card, joint number is taken as n.

(d) <u>Shear Wall Properties and Segment Incidence Table</u>

One set of cards for each shear wall

One card for number of vertical rows of elements in the shear wall followed by two cards for each segment of shear wall.

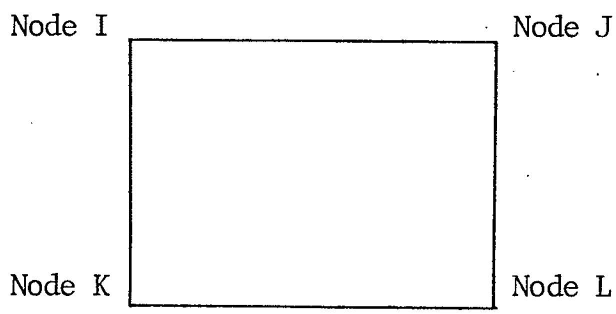
1st CARD	Field 1 -	shear wall number
	Field 2 -	number of vertical rows of elements
	Field 3 -	If the shear wall is similar to any previous one, the shear wall number to which it is similar should be given here.

2 cards for each segment of the shear wall.

Card 1	Field 1 -	segment number
	Field 2 -	number of horizontal rows of elements
	Field 3 -	modulus of elasticity. If left blank, it is taken as the value for the previous segment. Default value if first card is blank = 3000 ksi.
	Field 4 -	Poisson's ratio If left blank it is taken as the value for the previous segment. Default value = 0.3.
	Column 41	If the segment is similar to the one above, it may be entered here as T - otherwise blank.

Card 2	Field 1 -	joint number of node I
(Segment	Field 2 -	joint number of node J
incidence	Field 3 -	joint number of node K
table)	Field 4 -	joint number of node L

Joints I to L for a given segment occupy the postitions shown in Fig. C-2



FIG· C2

(Field 5 to 8 - are used only if there is an opening in the segment under consideration.)

Field 5 - bottom row line number of opening (The first row line of a segment is its bottom edge increasing with horizontal rows of elements.)

Field 6 - top row line number of opening.

Field 7 - left column line of the opening. (The first column line for a segment is its left vertical edge increasing with vertical rows of elements.)

Field 8 - right column line of the opening.

(e) Coordinates of Element Nodes in X and Y Directions

The input format for this 8F 10.3

Either case (i) or case (ii), which ever is applicable, should be used.

CASE (i) shear wall is similar to any previous one.

ONE CARD:

Field 1 thickness of first element of the first segment of the shear wall under consideration (FT)

CASE (ii) shear wall is not similar to any previous one.

3-sets of cards for each shear wall

SET 1 - Field 1 to n where n - number of vertical rows of elements in the shear wall +1	x-coordinates of boundaries of vertical rows of elements starting from left vertical edge of shear wall (FT)-use as many cards as necessary.
SET 2 - Field 1 to m where m = total number of horizontal rows of elements in the shear wall +1	Y-coordinates of boundaries of horizontal rows of elements, starting from base line proceeding toward top (FT)-use as many cards as necessary.

SET 3 - 1 sub set for each segment of shear wall

number of subsets = number of storeys

Field 1 to p where p = number of	Thickness of elements (FT)-use as many cards as necessary.

vertical rows of elements in each segment of the shear wall under considera- tion +1	(NOTE: thickness of any vertical row of elements for any segment is constant for that segment) Start with a fresh card for the new segment.

(f) Beam information - one card for each beam (proceed in sequence from 1st beam)

Field 1	beam number
Field 2	joint number at left end of beam
Field 3	joint number at right end of beam
Field 4	supporting column width at left end of beam (FT) If the beam left end into shear wall, width = 0
Field 5	supporting column width at right end of beam (FT) If beam right end into shear wall, width = 0
Field 6	Modulus of elasticity of beam (ksi) If this field left blank, value of E is taken same as preceding beam. Default value = 3000 ksi
Field 7	Moment of inertia (in^4) If this field left blank, value of I is taken same as for preceding beam. Default value = 1000 in^4

(g) Column Information - one card for each column (proceed in sequence from 1st column)

Field 1	column number
Field 2	joint number of the top of column
Field 3	joint number of the bottom of column
Field 4	top supported beam depth (FT)
Field 5	bottom supported beam depth (FT)
Field 6	Modulus of elasticity 'E' (ksi) If this field left blank E taken same as for preceding column. Default value = 3000 ksi
Field 7	Moment of inertia 'I' (in^4) If this field left blank, I is taken same as for preceding column. Default value = 1000 in^4
Field 8	Cross-sectional area 'A' (in^2) If field left blank, value is taken same as for preceding column. Default value = 100 in^2

(h) Loading Information - one card for each laterally loaded joint

Field 1	joint number
Field 2	horizontal load

C.4 Example Problem

The input data for the structure shown in Fig.C.1. are listed below. The two shear walls are identical and the finite element mesh for one of the walls is shown in Fig. C3.

10	20	30	40	50	60	70	80
EXAMPLE PROBLEM							
2	3	6	3	20	3	5	6
1	0	0	F				
2	10	0	F				
	20	0	F				
	30	0	F				
	40	0	F				
	0	12					
	10	12					
	20	12					
	30	12					
	40	12					
	0	24					
	10	24					
	20	24					
	30	24					
	40	24					
	0	36					
	10	36					
	20	36					
	30	36					
	40	36					

10	20	30	40/41	50	60	70	80
1	5						
1	6	3000	0.25	T			
7	8	2	3				
2	6						
12	13	7	8				
3	6	2800					
17	18	12	13	3	5	2	4
2	5	1					
1	6	3000	0.25	T			
9	10	4	5				
2	6	3000					
14	15	9	10				
3	6	2800		3	5	2	4
19	20	14	15				
10.0	12.	14.	16.	18.	20.		
0	2.	4.0	6.0	8.0	10.	12.	14.0
16.	18.	20.	22.	24.0	26.	28.	30.0
32.0	34.	36.					
1.5	1.5	1.5	1.5	1.5			
1.0	1.0	1.0	1.0	1.0			
1.0	1.0	1.0	1.0	1.0			
1.5							
1	6	7	1.0	0	3000	5000	
2	8	9	0	0			
	11	12	1	0			
	13	14	0	0		–	
	16	17	1.0	0	2800	4500	

10	20	30	40	50	60	70	80
	18	19	0	0			
1	6	1	1.0	0.0	3000	5000	
2	11	6	0.75	1.0	2800	4500	
3	16	11	0.75	0.75			
6	10						
11	10						
16	5						

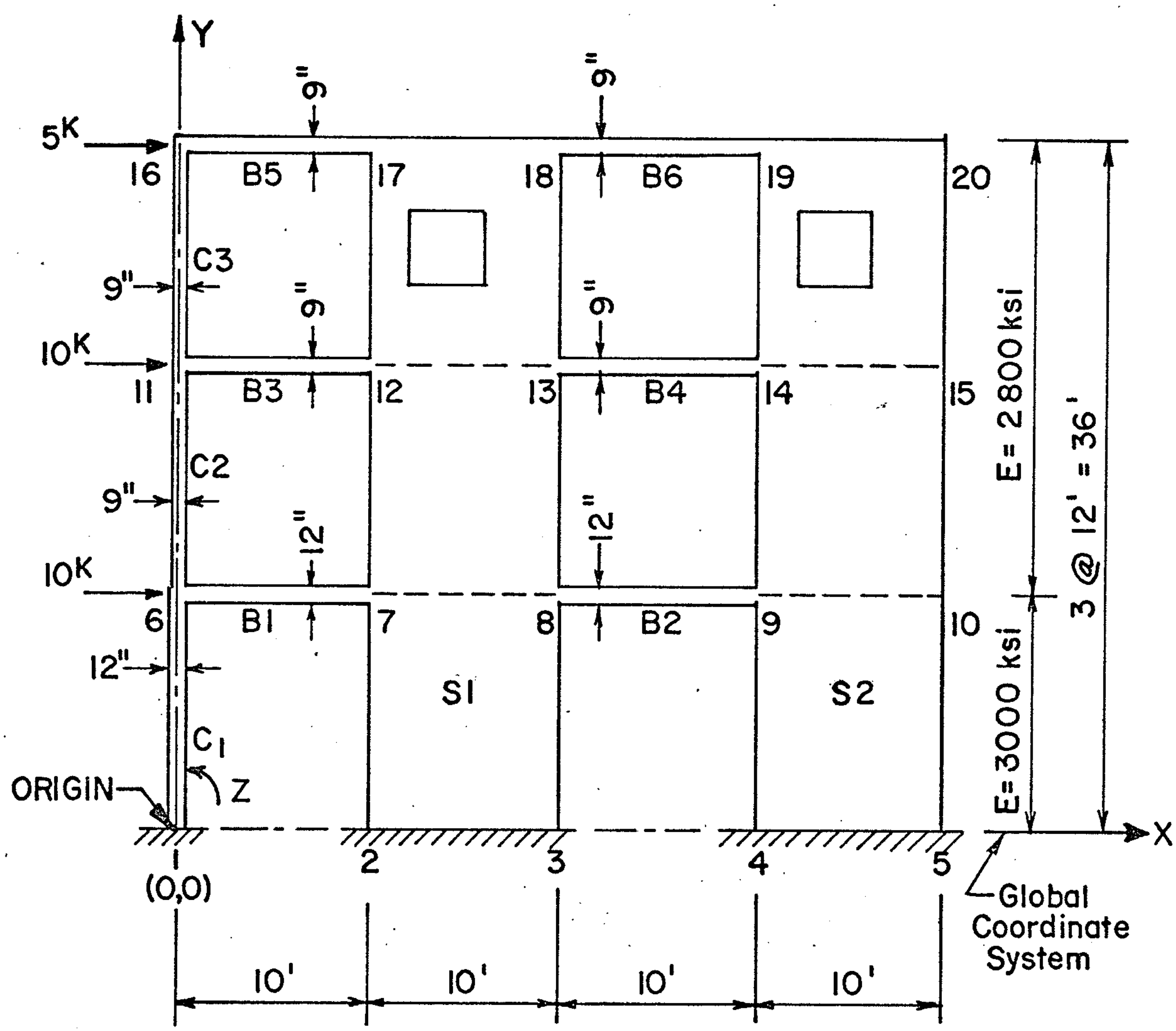
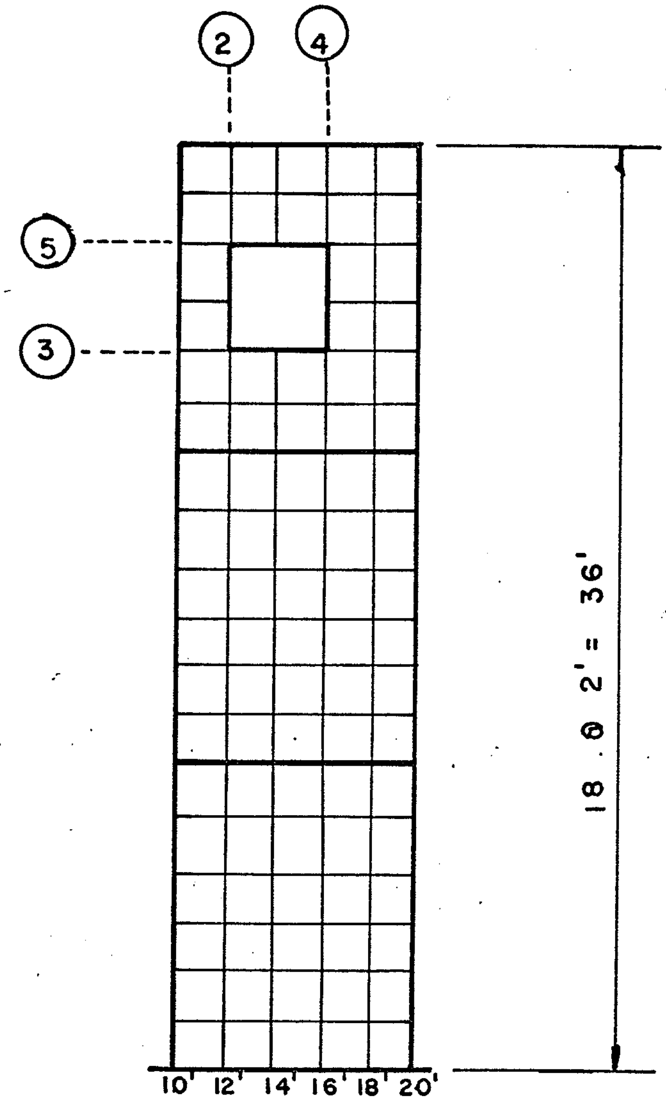


FIG. C.1 EXAMPLE PROBLEM



FIG· C-3 SHEAR WALL MKD S1 DIVIDED INTO FINITE ELEMENTS (TYPICAL FOR S2)

APPENDIX D

PROGRAM LISTING

```
C      ****                          MAIN                             ****
C     ******************************************************************
C     *THIS PROGRAM PERFORMS THE LINEAR ELASTIC ANALYSIS OF MULTISTORY  *
C     *SHEAR WALL FRAME STRUCTURES FOR LATERAL LOADS. ONLY RECTANGULAR  *
C     *PANELS AND RECTANGULAR FINITE ELEMENTS FOR SHEAR WALL CONSIDERED *
C     ******************************************************************
      IMPLICIT REAL*8(A-H,O-Z),INTEGER(I-N)
C     Z IS A DATA STORAGE POOL
      DIMENSION Z(10000)
      INTEGER*2 INPT(80)
      COMMON JRD,JWT,FN(8),INPT,NSH,NS,MAXX,NBM,NC,NN,NNL,MAXY
      DIMENSION HDG(20)
      JRD = 5
      JWT = 6
C     READ JOB TITLE
C     ******************************************************************
C     *                             READ                               *
C     ******************************************************************
      READ (JRD,10,END=60) HDG
   10 FORMAT (20A4)
      WRITE (JWT,20) HDG
   20 FORMAT ('1'//20X20A4)
      WRITE (JWT,30)
   30 FORMAT (//31X'LINEAR ELASTIC ANALYSIS OF FRAME WITH SHEAR WALL'//
     &52X,'INPUT DATA'/)
C     READ NO. OF SHEAR WALLS,NO. OF STOREYS, NO. OF BEAMS, NO. OF COLS.
C     TOTAL NO. OF NODES, NO. OF NODES EXTERNALLY LOADED MAXIMUM X DIVNS
C     AND MAXIMUM Y DIVNS FOR FOR ANY PANEL    ALL INPUT PRINTED OUT FOR
C     CHECKING PURPOSES
C     ******************************************************************
C     *                             READ                               *
C     ******************************************************************
      READ (JRD,40,END=60) INPT
   40 FORMAT (80A1)
      CALL CNVRT(1,8)
      NSH = FN(1)
```

```
      NS = FN(2)                                                        MAIN0370
      NBM = FN(3)                                                       MAIN0380
      NC = FN(4)                                                        MAIN0390
      NN = FN(5)                                                        MAIN0400
      NNL = FN(6)                                                       MAIN0410
      MAXX = FN(7)                                                      MAIN0420
      MAXY = FN(8)                                                      MAIN0430
      WRITE (JWT,50) NSH,NS,NBM,NC,NN,NNL                               MAIN0440
   50 FORMAT (/34X'NO. OF SHEAR WALLS',16X'=',I4,//34X'NO. OF STOREYS', MAIN0450
     &20X'=',I4,//34X'NO. OF BEAMS',22X'=',I4,//34X'NO. OF COLUMNS',20X'MAIN0460
     &=',I4,//34X'NO. OF JOINTS',21X'=',I4,//34X'NO. OF HORIZONTAL JOINTMAIN0470
     & LOADS',5X'=',I4//)                                               MAIN0480
C     TO CALCULATE MAXIMUM HALF BAND WIDTH                              MAIN0490
      N1 = NN/(NS+1)                                                    MAIN0500
      NEQN = 2*NN+(NS+1)-2*(NS+1)*NSH                                   MAIN0510
      MBAND = (N1*2+1-NSH*2)*2                                          MAIN0520
      II2 = 5*MAXX+18                                                   MAIN0530
      II3 = MAX0((3*MAXX+12),2*MAXX+17)                                 MAIN0540
      II4 = MAXX+11                                                     MAIN0550
      II5 = MAXX+6                                                      MAIN0560
      NN1 = 1                                                           MAIN0570
      NN2 = NN1+2*NN                                                    MAIN0580
      N3 = NN2+NN                                                       MAIN0590
      N4 = N3+3*NN                                                      MAIN0600
      N5 = N4+MAX0(6,NS+1)                                              MAIN0610
      N6 = N5+(NS+1)*3                                                  MAIN0620
      N7 = N6+NSH                                                       MAIN0630
      N8 = N7+NSH*NS                                                    MAIN0640
      N9 = N8+NSH*(MAXX+1)                                              MAIN0650
      N10 = N9+MAXY*NS+1                                                MAIN0660
      N11 = N10+MAXX                                                    MAIN0670
      N12 = N11+NSH*NS*(MAXY+1)                                         MAIN0680
      N13 = N12+NSH*NS*4                                                MAIN0690
      N14 = N13+NSH*NS*4                                                MAIN0700
      N15 = N14+NSH*NS                                                  MAIN0710
      N16 = N15+NSH*NS                                                  MAIN0720
```

```
      N17 = N16+NSH*NS*MAXX                                             MAIN0730
      N18 = N17+MAXX                                                    MAIN0740
      N19 = N18+3*(MAXX+1)                                              MAIN0750
      N20 = N19+3*(MAXX+1)                                              MAIN0760
      N21 = N20+MAXX+1                                                  MAIN0770
      N22 = N21+MAXX+1                                                  MAIN0780
      N23 = N22+NSH*NS                                                  MAIN0790
      N24 = N23+NSH                                                     MAIN0800
      N25 = N24+II2**2                                                  MAIN0810
      N26 = N25+II3**2                                                  MAIN0820
      N27 = N26+II4**2                                                  MAIN0830
      N28 = N27+II5**2                                                  MAIN0840
      N29 = N28+II5**2                                                  MAIN0850
      N30 = N29+NSH*(NS+1)*18                                           MAIN0860
      N31 = N30+NBM*2                                                   MAIN0870
      N32 = N31+NBM                                                     MAIN0880
      N33 = N32+NBM                                                     MAIN0890
      N34 = N33+NBM                                                     MAIN0900
      N35 = N34+NBM                                                     MAIN0910
      N36 = N35+NBM                                                     MAIN0920
      N37 = N36+NC*2                                                    MAIN0930
      N38 = N37+NC                                                      MAIN0940
      N39 = N38+NC                                                      MAIN0950
      N40 = N39+NC                                                      MAIN0960
      N41 = N40+NC                                                      MAIN0970
      N42 = N41+NC                                                      MAIN0980
      N43 = N42+NEQN*MBAND                                              MAIN0990
      N44 = N43+NEQN                                                    MAIN1000
      CALL SWF(Z(NN1),Z(NN2),Z(N3),Z(N4),Z(N5),Z(N6),Z(N7),Z(N8),Z(N9),ZMAIN1010
     &(N10),Z(N11),Z(N12),Z(N13),Z(N14),Z(N15),Z(N16),Z(N17),Z(N18),Z( MAIN1020
     &N19),Z(N20),Z(N21),Z(N22),Z(N23),Z(N24),Z(N25),Z(N26),Z(N27),Z(N28MAIN1030
     &),Z(N29),Z(N30),Z(N31),Z(N32),Z(N33),Z(N34),Z(N35),Z(N36),Z(N37),ZMAIN1040
     &(N38),Z(N39),Z(N40),Z(N41),Z(N42),Z(N43),Z(N44),NEQN,MBAND,II2,II3MAIN1050
     &,II4,II5)                                                         MAIN1060
   60 STOP                                                              MAIN1070
      END                                                               MAIN1080
```

```
      SUBROUTINE SWF(CJ,ISR,R,P,DEF,NX,NY,CX,YY,TT,CY,IP,LO,ES,US,TS,KEL
     &,KK,ND,C1,C2,MTYP,KSH,ST,AK,BK,STB,STT,STSH,IB,BL,BP,EB,BMI,CAB,IC
     &,CT,CB,EC,CMI,CAC,A,B,STORE,NEQN,MBAND,II2,II3,II4,II5)
      IMPLICIT REAL*8(A-H,O-Z),INTEGER(I-N)
      INTEGER*2 INPT(80)
      COMMON JRD,JWT,FN(8),INPT,NSH,NS,MAXX,NBM,NC,NN,NNL,MAXY
      DIMENSION CJ(2,1),ISR(1),R(3,1),P(1),DEF(1),NX(1),NY(NSH,1),CX(NSH
     &,1),YY(1),CY(NSH,NS,1),TT(1),IP(NSH,NS,1),LO(NSH,NS,1),ES(NSH,1),
     &US(NSH,1),TS(NSH,NS,1),KEL(1),KK(3,1),ND(3,1),C1(1),C2(1),MTYP(NSH
     &,1),KSH(1),IB(2,1),BL(1),BR(1),EB(1),BMI(1),CAB(1),IC(2,1),CT(1),
     &CB(1),EC(1),CMI(1),CAC(1),A(1),B(1),STORE(1)
      DIMENSION ST(II2,1),AK(II3,1),BK(II4,1),STB(II5,1),STT(II5,1),STSH
     &(NSH,1)
      DIMENSION S(12,12),ID(12)
      INTEGER*2 BC(3)/' ','F','R'/
      INTEGER*2 INP(2)/' ','T'/
      WRITE (JWT,10)
   10 FORMAT (///48X'NODAL COORDINATES    (  FT  )',//20X'  NODE',13X'X C
     &OORD',13X'Y COORD',13X'SUPPORT CONDN.'//)
      DO 120 I = 1,NN
      II = I
C     READ COORDINATES OF ALL NODES AND BOUNDARY CONDITIONS.
C     *****************************************************************
C     *                             READ                              *
C     *****************************************************************
      READ (JRD,20,END=1900) INPT
   20 FORMAT (80A1)
      CALL CNVRT(1,4)
      IF (FN(1) .NE. 0) II = FN(1)
      IF (II .LE. NN .AND. II .GE. 0) GO TO 40
      WRITE (JWT,30) INPT
   30 FORMAT (//' ERROR IN JT. NO. ON JT. INFORMATION CARD'//80A1)
      GO TO 120
   40 CJ(1,II) = FN(2)
      CJ(2,II) = FN(3)
      ISR(II) = 0
```

```
      DO 100 K = 31,40
      IF (INPT(K) .EQ. BC(1)) GO TO 100
      IF (INPT(K) .NE. BC(2)) GO TO 60
      WRITE (JWT,50) II,CJ(1,II),CJ(2,II)
      ISR(II) = ISR(II)+1
   50 FORMAT (21XI4,14XF7.3,13XF7.3,13X'    FIXED')
      GO TO 120
   60 IF (INPT(K) .EQ. BC(3)) GO TO 80
      WRITE (JWT,70)
   70 FORMAT ('          INCORRECT BOUNDARY CONDITION')
      CALL EXIT
   80 WRITE (JWT,90) II,CJ(1,II),CJ(2,II)
      ISR(II) = ISR(II)+2
   90 FORMAT (21XI4,14XF7.3,13XF7.3,13X'ROT. RELEASE')
      GO TO 120
  100 CONTINUE
      WRITE (JWT,110) II,CJ(1,II),CJ(2,II)
  110 FORMAT (21XI4,14XF7.3,13XF7.3)
      CJ(1,II) = CJ(1,II)*12.
      CJ(2,II) = CJ(2,II)*12.
  120 CONTINUE
      WRITE (JWT,130)
  130 FORMAT ('1'//50X'SHEAR WALL DATA'//' SHEAR',5X'SEGMT',3X'NO. OF ',
     &5X'NO. OF',4X'JOINT 1',2X'JOINT 2',2X'JOINT 3',2X'JOINT 4',6X'OPEN
     &INGS',12X' E   ',4X'POISS.RATIO'/' WALL',5X'(TOP TO',2X'X-DIVNS.',
     &3X'Y-DIVNS.',39X'ROWS',6X'COLUMNS',6X'(KSI)'/11X'BASE)'/)
      DO 200 I = 1,NSH
      ISH = I
C     READ DATA FOR SHEAR WALL. READ PANEL INCIDENCE TABLE,YOUNGS MOD.
C     POISSONS RATIO, NO. OF DIVNS. IN X-DIRN., NO. OF DIVNS. IN Y DIRN.
C     ROW NO. AND COLUMN NO. OF OPENINGS IN SHEAR WALL
C     ******************************************************************
C     *                            READ                                *
C     ******************************************************************
      READ (JRD,20,END=1900) INPT
      CALL CNVRT(1,3)
```

```
      IF (FN(1) .NE. 0) ISH = FN(1)
      NX(ISH) = FN(2)
      IF (FN(2) .GT. MAXX) GO TO 140
      KSH(ISH) = FN(3)
      GO TO 160
  140 WRITE (JWT,150)
  150 FORMAT (///' NO. OF X DIVNS EXCEEDS MAXM. VALUE MENTIONED IN 2ND I
     &NPUT CARD         CHECK INPUT'//)
      CALL EXIT
  160 DO 200 J = 1,NS
      IS = J
C     ******************************************************************
C     *                            READ                                *
C     ******************************************************************
      READ (JRD,20,END=1900) INPT
      CALL CNVRT(1,5)
      IF (FN(1) .NE. 0) IS = FN(1)
      NY(ISH,IS) = FN(2)
      IF (FN(2) .GT. MAXY) GO TO 180
      ES(ISH,IS) = 3000.
      US(ISH,IS) = 0.3
      IF (FN(3) .NE. 0) ES(ISH,IS) = FN(3)
      IF (FN(3) .EQ. 0 .AND. IS .GT. 1) ES(ISH,IS) = ES(ISH,IS-1)
      IF (FN(4) .NE. 0) US(ISH,IS) = FN(4)
      IF (FN(4) .EQ. 0 .AND. IS .GT. 1) US(ISH,IS) = US(ISH,IS-1)
      IF (INPT(41) .EQ. INPT(1)) MTYP(ISH,IS) = 0
      IF (INPT(41) .EQ. INP(2)) MTYP(ISH,IS) = 1
C     ******************************************************************
C     *                            READ                                *
C     ******************************************************************
      READ (JRD,20,END=1900) INPT
      CALL CNVRT(1,8)
      L = 5
      DO 170 K = 1,4
      IP(ISH,IS,K) = FN(K)
      LO(ISH,IS,K) = FN(L)
```

```
170 L = L+1                                                              SWF0545
    GO TO 200                                                            SWF0550
180 WRITE (JWT,190)                                                      SWF0555
190 FORMAT (///' NO. OF Y DIVNS. EXCEEDS MAXM. VALUE MENTIONED IN 2ND    SWF0560
   &INPUT CARD       CHECK INPUT'//)                                     SWF0565
    CALL EXIT                                                            SWF0570
200 CONTINUE                                                             SWF0575
    DO 240 I = 1,NSH                                                     SWF0580
    DO 240 JJ = 1,NS                                                     SWF0585
    J = NS-JJ+1                                                          SWF0590
    K1 = LO(I,J,1)                                                       SWF0595
    K2 = LO(I,J,2)                                                       SWF0600
    K3 = LO(I,J,3)                                                       SWF0605
    K4 = LO(I,J,4)                                                       SWF0610
    IF (K1 .EQ. 0 .AND. K2 .EQ. 0 .AND. K3 .EQ. 0 .AND. K4 .EQ. 0)       SWF0615
   &GO TO 220                                                            SWF0620
    WRITE (JWT,210) I,J,NX(I),NY(I,J),(IP(I,J,K),K = 1,4),(LO(I,J,K),K   SWF0625
   & = 1,4),ES(I,J),US(I,J)                                              SWF0630
210 FORMAT (2XI3,6XI3,7XI3,8XI3,7XI3,6XI3,6XI3,6XI3,4XI2,' -',I2,5XI2,   SWF0635
   &' -',I2,6XF9.3,4XF7.3)                                               SWF0640
    GO TO 240                                                            SWF0645
220 WRITE (JWT,230) I,J,NX(I),NY(I,J),(IP(I,J,K),K = 1,4),ES(I,J),US(I   SWF0650
   &,J)                                                                  SWF0655
230 FORMAT (2XI3,6XI3,7XI3,8XI3,7XI3,6XI3,6XI3,6XI3,11X'NIL',12XF9.3,    SWF0660
   &4XF7.3)                                                              SWF0665
240 CONTINUE                                                             SWF0670
C   READ X AND Y COORDINATES OF INDIVIDUAL NODES ON EDGES OF EACH        SWF0675
C   PANEL OF SHEAR WALL                                                  SWF0680
    DO 420 I = 1,NSH                                                     SWF0685
    IF (KSH(I) .NE. 0) GO TO 390                                         SWF0690
    WRITE (JWT,250) I                                                    SWF0695
250 FORMAT (//50X'SHEAR WALL NO.   =',I3//)                              SWF0700
    K = NX(I)+1                                                          SWF0705
C   **************************************************************** SWF0710
C   *                          READ                                * SWF0715
C   **************************************************************** SWF0720
```

```
      READ (JRD,260,END=1900) (CX(I,J),J = 1,K)                         SWF0725
  260 FORMAT (8F10.3)                                                   SWF0730
      WRITE (JWT,270)                                                   SWF0735
  270 FORMAT (' X - COORDINATES OF SEGMT. SUBNODES IN X-DIRN.'/20X'NODE' SWF0740
     &,20X'X COORD. (FT)'/)                                             SWF0745
      DO 290 J = 1,K                                                    SWF0750
      WRITE (JWT,280) J,CX(I,J)                                         SWF0755
  280 FORMAT (20XI3,21XF10.3)                                           SWF0760
      CX(I,J) = CX(I,J)*12.                                             SWF0765
  290 CONTINUE                                                          SWF0770
      LNY = 0                                                           SWF0775
      DO 300 J = 1,NS                                                   SWF0780
  300 LNY = LNY+NY(I,J)                                                 SWF0785
      LNY = LNY+1                                                       SWF0790
C     ****************************************************************** SWF0795
C     *                             READ                               * SWF0800
C     ****************************************************************** SWF0805
      READ (JRD,260,END=1900) (YY(L),L = 1,LNY)                         SWF0810
      L = 1                                                             SWF0815
      WRITE (JWT,310)                                                   SWF0820
  310 FORMAT (//' Y - COORDINATES OF SUBNODES OF SEGMTS.'/20X'SEGMT',19X SWF0825
     &'NODE',21X'Y COORD.(FT)'/)                                        SWF0830
      DO 340 J = 1,NS                                                   SWF0835
      N = NY(I,J)+1                                                     SWF0840
      DO 330 K = 1,N                                                    SWF0845
      L = L+1                                                           SWF0850
      IF (K .EQ. 1) L = L-1                                             SWF0855
      CY(I,J,K) = YY(L)                                                 SWF0860
      WRITE (JWT,320) J,K,CY(I,J,K)                                     SWF0865
      CY(I,J,K) = CY(I,J,K)*12.                                         SWF0870
  320 FORMAT (20XI3,21XI3,21XF10.3)                                     SWF0875
  330 CONTINUE                                                          SWF0880
  340 CONTINUE                                                          SWF0885
      WRITE (JWT,350)                                                   SWF0890
  350 FORMAT (//' THICKNESS OF ELEMENTS IN SEGMTS.'/20X'SEGMT.',18X'ELEM SWF0895
     &ENT',19X'THICKNESS(FT)'/)                                         SWF0900
```

```
      DO 380 J = 1,NS
      L = NX(I)
C     ****************************************************************
C     *                          READ                                *
C     ****************************************************************
      READ (JRD,260,END=1900) (TT(M1),M1 = 1,L)
      DO 370 M2 = 1,L
      IF (TT(M2) .EQ. 0.) TT(M2) = 1.
      TS(I,J,M2) = TT(M2)
      WRITE (JWT,360) J,M2,TS(I,J,M2)
      TS(I,J,M2) = TS(I,J,M2)*12.
  360 FORMAT (20XI3,21XI3,21XF10.3)
  370 CONTINUE
  380 CONTINUE
      GO TO 420
  390 READ (JRD,400) TS(I,1,1)
  400 FORMAT (F10.3)
      K = KSH(I)
      TS(I,1,1) = TS(I,1,1)*12.
      PRN = TS(I,1,1)/TS(K,1,1)
      WRITE (JWT,410) I,K,PRN
  410 FORMAT (/' SHEAR WALL ',I3,' IS SIMILAR TO SHEAR WALL ',I3//'  THI
     &CKNESS PROPORTION IS ',F5.3//)
  420 CONTINUE
      IF (NBM .EQ. 0) GO TO 460
      WRITE (JWT,430)
  430 FORMAT ('1'/50X'BEAM DATA'//'  BEAM NO.   JOINT 1  JOINT 2',8X'SUP
     &PORTING COL. WIDTH (FT)',14X'E',11X'I',/34X'LEFT END',13X'RIGHT EN
     &D',10X'( KSI )',6X'(IN-4)'/)
      DO 450 I = 1,NBM
      II = I
C     READ PROPERTIES OF BEAM, BEAM INCIDENCE TABLE
C     ****************************************************************
C     *                          READ                                *
C     ****************************************************************
      READ (JRD,20,END=1900) INPT
```

```
      CALL CNVRT(1,7)
      IF (FN(1) .NE. 0) II = FN(1)
      IB(1,II) = FN(2)
      IB(2,II) = FN(3)
      EB(II) = 3000.
      BMI(II) = 1000.
      BL(II) = FN(4)
      BR(II) = FN(5)
      IF (FN(6) .NE. 0) EB(II) = FN(6)
      IF (FN(6) .EQ. 0 .AND. II .GT. 1) EB(II) = EB(II-1)
      IF (FN(7) .NE. 0) BMI(II) = FN(7)
      IF (FN(7) .EQ. 0 .AND. II .GT. 1) BMI(II) = BMI(II-1)
      CAB(II) = 0.
      WRITE (JWT,440) II,IB(1,II),IB(2,II),BL(II),BR(II),EB(II),BMI(II)
  440 FORMAT (4XI3,7XI3,6XI3,9XF6.3,15XF6.3,10XF9.3,4XF9.3)
      BL(II) = BL(II)*12.
      BR(II) = BR(II)*12.
  450 CONTINUE
  460 IF (NC .EQ. 0) GO TO 500
      WRITE (JWT,470)
  470 FORMAT ('1'/50X'COLUMN DATA'//' COLUMN NO.   JOINT 1  JOINT 2',6X'
     &DEPTH OF SUPPORTED BEAMS (FT)',14X'E',10X'I',12X'A'/39X'TOP',16X'B
     &OTTOM',13X'(KSI)',5X'(IN-4)',9X'(IN-2)'/)
      DO 490 I = 1,NC
      II = I
C     READ COLUMN PROPERTIES AND COLUMN INCIDENCE TABLE
C     ******************************************************************
C     *                              READ                              *
C     ******************************************************************
      READ (JRD,20,END=1900) INPT
      CALL CNVRT(1,8)
      IF (FN(1) .NE. 0) II = FN(1)
      EC(II) = 3000.
      CMI(II) = 1000.
      CAC(II) = 100.
      IC(1,II) = FN(2)
```

```
      IC(2,II) = FN(3)                                                  SWF1265
      CT(II) = FN(4)                                                    SWF1270
      CB(II) = FN(5)                                                    SWF1275
      IF (FN(6) .NE. 0) EC(II) = FN(6)                                  SWF1280
      IF (FN(6) .EQ. 0 .AND. II .GT. 1) EC(II) = EC(II-1)               SWF1285
      IF (FN(7) .NE. 0) CMI(II) = FN(7)                                 SWF1290
      IF (FN(7) .EQ. 0 .AND. II .GT. 1) CMI(II) = CMI(II-1)             SWF1295
      IF (FN(8) .NE. 0) CAC(II) = FN(8)                                 SWF1300
      IF (FN(8) .EQ. 0 .AND. II .GT. 1) CAC(II) = CAC(II-1)             SWF1305
      WRITE (JWT,480) II,IC(1,II),IC(2,II),CT(II),CB(II),EC(II),CMI(II),SWF1310
     &CAC(II)                                                           SWF1315
  480 FORMAT (4XI3,7XI3,6XI3,11XF6.3,15XF6.3,10XF9.3,3XF9.3,5XF8.3)     SWF1320
      CT(II) = CT(II)*12.                                               SWF1325
      CB(II) = CB(II)*12.                                               SWF1330
  490 CONTINUE                                                          SWF1335
  500 NO = MBAND*NEQN                                                   SWF1340
      NO = MBAND*NEQN                                                   SWF1345
      DO 510 J = 1,NO                                                   SWF1350
  510 A(J) = 0.                                                         SWF1355
      DO 520 J = 1,NEQN                                                 SWF1360
  520 B(J) = 0.                                                         SWF1365
      WRITE (JWT,530)                                                   SWF1370
  530 FORMAT ('1'/34X'JOINT',15X'HORIZONTAL LOAD (KIPS)'/)              SWF1375
      DO 550 J = 1,NNL                                                  SWF1380
C     READ LOAD VECTORS                                                 SWF1385
C     ****************************************************************** SWF1390
C     *                           READ                                 * SWF1395
C     ****************************************************************** SWF1400
      READ (JRD,20,END=1900) INPT                                       SWF1405
      CALL CNVRT(1,2)                                                   SWF1410
      I = FN(1)                                                         SWF1415
      P(1) = FN(2)                                                      SWF1420
      WRITE (JWT,540) I,P(1)                                            SWF1425
  540 FORMAT (35XI3,22XF8.3)                                            SWF1430
      N1 = NN/(NS+1)                                                    SWF1435
      II = (I-1)/N1                                                     SWF1440
```

```
      ID(1) = N1*II*2+II+1-2*NSH*II
      NOI = ID(1)
      B(NOI) = B(NOI)+P(1)
  550 CONTINUE
      LOOPB = 0
      LOOPC = 0
C     NEQS IS NUMBER EQUATIONS PER SHEAR WALL
C     MBS IS BAND WIDTH FOR SHEAR WALL
      MBS = 6
      NEQS = (NS+1)*3
      NES = NEQS*MBS
      DO 560 IK = 1,NSH
      DO 560 JK = 1,NES
  560 STSH(IK,JK) = 0.
C     INITIALIZE REACTION VECTORS
      DO 570 I = 1,NN
      DO 570 J = 1,3
  570 R(J,I) = 0.
      DO 1120 I = 1,NSH
      IF (KSH(I) .NE. 0) GO TO 1050
      LOOP = 0
      DO 1040 JJ = 1,NS
      J = NS-JJ+1
      IF (JJ .GT. 1 .AND. MTYP(I,J) .EQ. 1) GO TO 680
      E = ES(I,J)
      U = US(I,J)
      CALL SEGMT(I,J,E,U,CX,CY,NX,NY,TS,LO,AK,NEQ1,NEQ3,ST,II2,II3,KK,
     &KEL,ND)
      NEQ13 = NEQ1+NEQ3
C     REARRANGE ST MATRIX AS AK MATRIX AND STORE FINAL RESULTS (AFTER
C     CONDENSATION) AS BK MATRIX
      IF (JJ .NE. 1) GO TO 660
C     REARRANGE EQUATIONS FOR TOP MOST PANEL
      NEQ = NEQ1+NEQ3
      I1 = NEQ1-2
      II = 0
```

```
      DO 650 L = 1,NEQ                                              SWF1625
      MM = 0                                                        SWF1630
      J1 = NEQ1-2                                                   SWF1635
      IF (L .LT. 4 .OR. L .GT. 8) GO TO 580                         SWF1640
      I1 = I1+1                                                     SWF1645
      II = I1                                                       SWF1650
      GO TO 610                                                     SWF1655
  580 IF (L .EQ. 9) GO TO 590                                       SWF1660
      IF (L .EQ. 10) GO TO 600                                      SWF1665
      II = II+1                                                     SWF1670
      IF (II .EQ. NEQ+1) II = NEQ1+4                                SWF1675
      IF (II .EQ. NEQ-1) II = 4                                     SWF1680
      GO TO 610                                                     SWF1685
  590 II = NEQ-1                                                    SWF1690
      GO TO 610                                                     SWF1695
  600 II = NEQ                                                      SWF1700
  610 DO 650 M = 1,NEQ                                              SWF1705
      IF (M .LT. 4 .OR. M .GT. 8) GO TO 620                         SWF1710
      J1 = J1+1                                                     SWF1715
      MM = J1                                                       SWF1720
      GO TO 650                                                     SWF1725
  620 IF (M .EQ. 9) GO TO 630                                       SWF1730
      IF (M .EQ. 10) GO TO 640                                      SWF1735
      MM = MM+1                                                     SWF1740
      IF (MM .EQ. NEQ+1) MM = NEQ1+4                                SWF1745
      IF (MM .EQ. NEQ-1) MM = 4                                     SWF1750
      GO TO 650                                                     SWF1755
  630 MM = NEQ-1                                                    SWF1760
      GO TO 650                                                     SWF1765
  640 MM = NEQ                                                      SWF1770
  650 AK(L,M) = ST(II,MM)                                           SWF1775
      GO TO 890                                                     SWF1780
C     REARRANGE EQUATIONS FOR SUBSEQUENT PANELS                     SWF1785
C     ADD STIFFNESS ALONG COMMON EDGE                               SWF1790
  660 DO 670 II = 1,NEQ1                                            SWF1795
      DO 670 MM = 1,NEQ1                                            SWF1800
```

```
  670 ST(II,MM) = ST(II,MM)+STB(II,MM)                              SWF1805
      GO TO 730                                                     SWF1810
  680 IF (JJ .EQ. 2) GO TO 700                                      SWF1815
      K1 = NEQ1                                                     SWF1820
      DO 690 II = 1,NEQ1                                            SWF1825
      K1 = K1+1                                                     SWF1830
      K2 = NEQ1                                                     SWF1835
      DO 690 MM = 1,NEQ1                                            SWF1840
      ST(II,MM) = STT(II,MM)                                        SWF1845
  690 ST(K1,K2) = STB(II,MM)                                        SWF1850
  700 EO = ES(I,J+1)                                                SWF1855
      EN = ES(I,J)                                                  SWF1860
      TO = TS(I,J+1,1)                                              SWF1865
      TN = TS(I,J,1)                                                SWF1870
      DO 710 II = 1,NEQ13                                           SWF1875
      DO 710 MM = 1,NEQ13                                           SWF1880
  710 ST(II,MM) = ST(II,MM)*(EN/EO)*(TN/TO)                         SWF1885
      DO 720 I1 = 1,NEQ1                                            SWF1890
      DO 720 I2 = 1,NEQ1                                            SWF1895
  720 ST(I1,I2) = ST(I1,I2)+STB(I1,I2)                              SWF1900
  730 NEQ = NEQ1+NEQ3+5                                             SWF1905
C     KH3 IS THE HIGHEST EQUATION IN THE THIRD ROW                  SWF1910
C     KL2 IS THE LOWEST EQUATION TO BE CONDENSED OFF                SWF1915
      KH3 = 10+NEQ3                                                 SWF1920
      KL2 = KH3+1                                                   SWF1925
C     START REARRANGING EQUATIONS                                   SWF1930
      DO 740 II = 1,5                                               SWF1935
      DO 740 MM = II,10                                             SWF1940
      AK(II,MM) = BK(II,MM)                                         SWF1945
  740 AK(MM,II) = AK(II,MM)                                         SWF1950
      DO 750 II = 1,5                                               SWF1955
      K2 = 10                                                       SWF1960
      DO 750 MM = KL2,NEQ                                           SWF1965
      K2 = K2+1                                                     SWF1970
      AK(II,MM) = BK(II,K2)                                         SWF1975
  750 AK(MM,II) = AK(II,MM)                                         SWF1980
```

```
      DO 760 II = 1,5                                                  SWF1985
      DO 760 MM = 11,KH3                                               SWF1990
      AK(II,MM) = 0.                                                   SWF1995
  760 AK(MM,II) = 0.                                                   SWF2000
      K1 = 0                                                           SWF2005
      DO 770 II = 6,10                                                 SWF2010
      K1 = K1+1                                                        SWF2015
      IF (II .EQ. 9) K1 = NEQ1-1                                       SWF2020
      K2 = 0                                                           SWF2025
      DO 770 MM = 6,10                                                 SWF2030
      K2 = K2+1                                                        SWF2035
      IF (MM .EQ. 9) K2 = NEQ1-1                                       SWF2040
  770 AK(II,MM) = ST(K1,K2)                                            SWF2045
      K1 = 3                                                           SWF2050
      DO 780 II = KL2,NEQ                                              SWF2055
      K1 = K1+1                                                        SWF2060
      K2 = 3                                                           SWF2065
      DO 780 MM = KL2,NEQ                                              SWF2070
      K2 = K2+1                                                        SWF2075
  780 AK(II,MM) = ST(K1,K2)                                            SWF2080
      K1 = 0                                                           SWF2085
      DO 790 II = 6,10                                                 SWF2090
      K1 = K1+1                                                        SWF2095
      IF (II .EQ. 9) K1 = NEQ1-1                                       SWF2100
      K2 = 3                                                           SWF2105
      DO 790 MM = KL2,NEQ                                              SWF2110
      K2 = K2+1                                                        SWF2115
      AK(II,MM) = ST(K1,K2)                                            SWF2120
  790 AK(MM,II) = AK(II,MM)                                            SWF2125
      K1 = NEQ1                                                        SWF2130
      DO 800 II = 11,15                                                SWF2135
      K1 = K1+1                                                        SWF2140
      IF (II .EQ. 14) K1 = NEQ1+NEQ3-1                                 SWF2145
      K2 = NEQ1                                                        SWF2150
      DO 800 MM = 11,15                                                SWF2155
      K2 = K2+1                                                        SWF2160
```

```
      IF (MM .EQ. 14) K2 = NEQ1+NEQ3-1                                  SWF2165
  800 AK(II,MM) = ST(K1,K2)                                             SWF2170
      K1 = NEQ1+3                                                       SWF2175
      DO 810 II = 16,KH3                                                SWF2180
      K1 = K1+1                                                         SWF2185
      K2 = NEQ1+3                                                       SWF2190
      DO 810 MM = 16,KH3                                                SWF2195
      K2 = K2+1                                                         SWF2200
  810 AK(II,MM) = ST(K1,K2)                                             SWF2205
      K1 = NEQ1                                                         SWF2210
      DO 820 II = 11,15                                                 SWF2215
      K1 = K1+1                                                         SWF2220
      IF (II .EQ. 14) K1 = NEQ1+NEQ3-1                                  SWF2225
      K2 = NEQ1+3                                                       SWF2230
      DO 820 MM = 16,KH3                                                SWF2235
      K2 = K2+1                                                         SWF2240
      AK(II,MM) = ST(K1,K2)                                             SWF2245
  820 AK(MM,II) = AK(II,MM)                                             SWF2250
      K1 = 0                                                            SWF2255
      NEQ13 = NEQ1+NEQ3                                                 SWF2260
      DO 830 II = 6,10                                                  SWF2265
      K1 = K1+1                                                         SWF2270
      IF (II .EQ. 9) K1 = NEQ1-1                                        SWF2275
      K2 = NEQ1                                                         SWF2280
      DO 830 MM = 11,15                                                 SWF2285
      K2 = K2+1                                                         SWF2290
      IF (MM .EQ. 14) K2 = NEQ13-1                                      SWF2295
      AK(II,MM) = ST(K1,K2)                                             SWF2300
  830 AK(MM,II) = AK(II,MM)                                             SWF2305
      K1 = 3                                                            SWF2310
      DO 840 II = KL2,NEQ                                               SWF2315
      K1 = K1+1                                                         SWF2320
      K2 = NEQ1+3                                                       SWF2325
      DO 840 MM = 16,KH3                                                SWF2330
      K2 = K2+1                                                         SWF2335
      AK(II,MM) = ST(K1,K2)                                             SWF2340
```

```
840 AK(MM,II) = AK(II,MM)                                                SWF2345
    K1 = 3                                                               SWF2350
    DO 850 II = KL2,NEQ                                                  SWF2355
    K1 = K1+1                                                            SWF2360
    K2 = NEQ1                                                            SWF2365
    DO 850 MM = 11,15                                                    SWF2370
    K2 = K2+1                                                            SWF2375
    IF (MM .EQ. 14) K2 = NEQ13-1                                         SWF2380
    AK(II,MM) = ST(K1,K2)                                                SWF2385
850 AK(MM,II) = AK(II,MM)                                                SWF2390
    K1 = 0                                                               SWF2395
    DO 860 II = 6,10                                                     SWF2400
    K1 = K1+1                                                            SWF2405
    IF (II .EQ. 9) K1 = NEQ1-1                                           SWF2410
    K2 = NEQ1+3                                                          SWF2415
    DO 860 MM = 16,KH3                                                   SWF2420
    K2 = K2+1                                                            SWF2425
    AK(II,MM) = ST(K1,K2)                                                SWF2430
860 AK(MM,II) = ST(K1,K2)                                                SWF2435
    GO TO 890                                                            SWF2440
870 NEQ = NEQ3+5                                                         SWF2445
    DO 880 II = 1,NEQ                                                    SWF2450
    DO 880 MM = 1,NEQ                                                    SWF2455
880 AK(II,MM) = BK(II,MM)                                                SWF2460
890 CALL REDN(I,J,IP,CJ,NEQ,NEQ1,NEQ3,AK,NEQI,LOOP,II3,CX,C1,C2,NX)      SWF2465
    IF (J .EQ. NS) GO TO 910                                             SWF2470
    IF (J-1 .EQ. 0) GO TO 910                                            SWF2475
    IF (MTYP(I,J-1) .NE. 1) GO TO 910                                    SWF2480
    DO 900 II = 1,NEQ1                                                   SWF2485
    DO 900 MM = 1,NEQ1                                                   SWF2490
900 STT(II,MM) = ST(II,MM)-STB(II,MM)                                    SWF2495
910 K1 = NEQ1                                                            SWF2500
    DO 920 II = 1,NEQ3                                                   SWF2505
    K1 = K1+1                                                            SWF2510
    K2 = NEQ1                                                            SWF2515
    DO 920 MM = 1,NEQ3                                                   SWF2520
```

```
      K2 = K2+1                                              SWF2525
  920 STB(II,MM) = ST(K1,K2)                                 SWF2530
      NK = NEQ-NEQI                                          SWF2535
      K1 = 5                                                 SWF2540
      IF (J .EQ. NS) K1 = 0                                  SWF2545
      IF (J .NE. NS) NK = NK-5                               SWF2550
      DO 930 II = 1,NK                                       SWF2555
      K1 = K1+1                                              SWF2560
      K2 = 5                                                 SWF2565
      IF (J .EQ. NS) K2 = 0                                  SWF2570
      DO 930 MM = 1,NK                                       SWF2575
      K2 = K2+1                                              SWF2580
  930 BK(II,MM) = AK(K1,K2)                                  SWF2585
      N1 = NN/(NS+1)                                         SWF2590
      IF (J .NE. NS) GO TO 980                               SWF2595
      L1 = IP(I,J,1)/N1                                      SWF2600
      ID(1) = N1*L1*2+L1+1-2*NSH*L1                          SWF2605
      K = IP(I,J,1)-(N1*L1+1)                                SWF2610
      ID(2) = ID(1)+K*2-(I-1)*2+1                            SWF2615
      ID(4) = ID(2)+1                                        SWF2620
      DO 950 M = 1,4                                         SWF2625
      IF (M .EQ. 3) GO TO 950                                SWF2630
      DO 940 MM = 1,4                                        SWF2635
      IF (MM .EQ. 3) GO TO 940                               SWF2640
      IF (ID(M) .GT. ID(MM)) GO TO 940                       SWF2645
      NOK = (ID(MM)-ID(M))*NEQN+ID(M)                        SWF2650
      A(NOK) = A(NOK)+AK(M,MM)                               SWF2655
  940 CONTINUE                                               SWF2660
  950 CONTINUE                                               SWF2665
C     STORE SHEAR WALL STIFFNESS MATRIX AS STSH              SWF2670
      ID(1) = 3*(NS-J)+1                                     SWF2675
      ID(2) = ID(1)+1                                        SWF2680
      ID(4) = ID(2)+1                                        SWF2685
      DO 970 JK = 1,4                                        SWF2690
      IF (JK .EQ. 3) GO TO 970                               SWF2695
      DO 960 JL = 1,4                                        SWF2700
```

```
      IF (JL .EQ. 3) GO TO 960
      IF (ID(JK) .GT. ID(JL)) GO TO 960
      NOK = (ID(JL)-ID(JK))*NEQS+ID(JK)
      STSH(I,NOK) = STSH(I,NOK)+AK(JK,JL)
  960 CONTINUE
  970 CONTINUE
      GO TO 1030
  980 J1 = J+1
      IF (LOOP .EQ. 1) J1 = J
      L1 = IP(I,J1,1)/N1
      L3 = IP(I,J1,3)/N1
      ID(1) = N1*L1*2+L1+1-2*NSH*L1
      ID(6) = N1*L3*2+L3+1-2*NSH*L3
      K = IP(I,J1,1)-(N1*L1+1)
      ID(2) = ID(1)+K*2-(I-1)*2+1
      ID(4) = ID(2)+1
      K = IP(I,J1,3)-(N1*L3+1)
      ID(7) = ID(6)+K*2-(I-1)*2+1
      ID(9) = ID(7)+1
      DO 1000 M = 1,9
      IF (M .EQ. 3 .OR. M .EQ. 5 .OR. M .EQ. 8) GO TO 1000
      DO 990 MM = 1,9
      IF (MM .EQ. 3 .OR. MM .EQ. 5 .OR. MM .EQ. 8) GO TO 990
      IF (M .LE. 5 .AND. MM .LE. 5) GO TO 990
      IF (ID(M) .GT. ID(MM)) GO TO 990
      NOK = (ID(MM)-ID(M))*NEQN+ID(M)
      A(NOK) = A(NOK)+AK(M,MM)
  990 CONTINUE
 1000 CONTINUE
      ID(1) = 3*(NS-J1)+1
      ID(2) = ID(1)+1
      ID(4) = ID(2)+1
      ID(6) = 3*(NS-J1+1)+1
      ID(7) = ID(6)+1
      ID(9) = ID(7)+1
      DO 1020 JK = 1,9
```

```
      IF (JK .EQ. 3 .OR. JK .EQ. 5 .OR. JK .EQ. 8) GO TO 1020
      DO 1010 JL = 1,9
      IF (JL .EQ. 3 .OR. JL .EQ. 5 .OR. JL .EQ. 8) GO TO 1010
      IF (JK .LE. 5 .AND. JL .LE. 5) GO TO 1010
      IF (ID(JK) .GT. ID(JL)) GO TO 1010
      NOK = (ID(JL)-ID(JK))*NEQS+ID(JK)
      STSH(I,NOK) = STSH(I,NOK)+AK(JK,JL)
 1010 CONTINUE
 1020 CONTINUE
 1030 IF (JJ .EQ. NS) LOOP = LOOP+1
      IF (LOOP .EQ. 1) GO TO 870
 1040 CONTINUE
      GO TO 1120
 1050 K = KSH(I)
      EO = ES(K,1)
      EN = ES(I,1)
      TO = TS(K,1,1)
      TN = TS(I,1,1)
      DO 1060 J = 1,NES
 1060 STSH(I,J) = STSH(K,J)*(EN/EO)*(TN/TO)
      J = NS+1
      DO 1110 I1 = 1,NES,3
      J = J-1
      K1 = I1-1
      IF (J .EQ. 0) GO TO 1070
      L1 = IP(I,J,1)/N1
      L3 = IP(I,J,3)/N1
      ID(1) = N1*L1*2+L1+1-2*NSH*L1
      ID(4) = N1*L3*2+L3+1-2*NSH*L3
      K = IP(I,J,1)-(N1*L1+1)
      ID(2) = ID(1)+K*2-(I-1)*2+1
      ID(3) = ID(2)+1
      K = IP(I,J,3)-(N1*L3+1)
      ID(5) = ID(4)+K*2-(I-1)*2+1
      ID(6) = ID(5)+1
      GO TO 1080
```

```
1070 ID(1) = ID(4)                                                    SWF3065
     ID(2) = ID(5)                                                    SWF3070
     ID(3) = ID(6)                                                    SWF3075
1080 DO 1100 I2 = 1,6                                                 SWF3080
     K1 = K1+1                                                        SWF3085
     IF (K1 .GT. NEQS) GO TO 1100                                     SWF3090
     K2 = I1-1                                                        SWF3095
     DO 1090 I3 = 1,6                                                 SWF3100
     K2 = K2+1                                                        SWF3105
     IF (K2 .GT. NEQS) GO TO 1090                                     SWF3110
     IF (ID(I2) .GT. ID(I3)) GO TO 1090                               SWF3115
     IF (I2 .GT. 3 .AND. K2 .GE. K1) GO TO 1100                       SWF3120
     NOK = (ID(I3)-ID(I2))*NEQN+ID(I2)                                SWF3125
     NOK1 = (K2-K1)*NEQS+K1                                           SWF3130
     IF (K2 .LT. K1) NOK1 = (K1-K2)*NEQS+K2                           SWF3135
     A(NOK) = A(NOK)+STSH(I,NOK1)                                     SWF3140
1090 CONTINUE                                                         SWF3145
1100 CONTINUE                                                         SWF3150
1110 CONTINUE                                                         SWF3155
1120 CONTINUE                                                         SWF3160
     IF (NBM .EQ. 0) GO TO 1350                                       SWF3165
C    GENERATE BEAM STIFFNESS MATRIX AND INCORPORATE INTO EQ. EQNS.    SWF3170
1130 DO 1340 I = 1,NBM                                                SWF3175
     L1 = IB(1,I)                                                     SWF3180
     L2 = IB(2,I)                                                     SWF3185
     DLL = BL(I)                                                      SWF3190
     DLR = BR(I)                                                      SWF3195
     CL = CJ(1,L2)-CJ(1,L1)-DLL/2.-DLR/2.                             SWF3200
     E = EB(I)                                                        SWF3205
     CA = CAB(I)                                                      SWF3210
     BI = BMI(I)                                                      SWF3215
     DLL = DLL/2.                                                     SWF3220
     DLR = DLR/2.                                                     SWF3225
     CALL STBEAM(E,CA,BI,CL,DLL,DLR,S)                                SWF3230
     L1 = L1/N1                                                       SWF3235
     K = IB(1,I)-(N1*L1+1)                                            SWF3240
```

```
      ID(1) = N1*L1*2+L1+1-2*NSH*L1+K*2+1                               SWF3245
      ID(2) = ID(1)+1                                                   SWF3250
      ID(3) = ID(2)+1                                                   SWF3255
      ID(4) = ID(3)+1                                                   SWF3260
      J1 = 0                                                            SWF3265
      J2 = 0                                                            SWF3270
      DO 1200 II = 1,NSH                                                SWF3275
      J = L1                                                            SWF3280
      IF (J .EQ. 0) GO TO 1140                                          SWF3285
      K1 = IP(II,J,1)                                                   SWF3290
      GO TO 1150                                                        SWF3295
 1140 J = J+1                                                           SWF3300
      K1 = IP(II,J,3)                                                   SWF3305
 1150 IF (IB(1,I)-K1) 1180,1200,1160                                    SWF3310
 1160 ID(1) = ID(1)-1                                                   SWF3315
      IF ((IB(1,I)-K1) .GT. 1) ID(1) = ID(1)-1                          SWF3320
      IF ((IB(1,I)-K1) .EQ. 1) GO TO 1170                               SWF3325
      ID(2) = ID(1)+1                                                   SWF3330
      ID(3) = ID(2)+1                                                   SWF3335
      ID(4) = ID(3)+1                                                   SWF3340
      GO TO 1200                                                        SWF3345
 1170 ID(1) = ID(1)-1                                                   SWF3350
      ID(2) = ID(1)+1                                                   SWF3355
      ID(3) = ID(2)+1                                                   SWF3360
      ID(4) = ID(3)+1                                                   SWF3365
      J1 = 1                                                            SWF3370
      SL1 = CJ(1,K1+1)-CJ(1,K1)                                         SWF3375
      GO TO 1200                                                        SWF3380
 1180 IF ((IB(1,I)-K1) .EQ. -1) GO TO 1190                              SWF3385
      GO TO 1200                                                        SWF3390
 1190 J2 = 1                                                            SWF3395
      SL2 = CJ(1,K1+1)-CJ(1,K1)                                         SWF3400
 1200 CONTINUE                                                          SWF3405
 1210 DO 1310 L = 1,4                                                   SWF3410
      NOJ = ID(L)                                                       SWF3415
      IF (LOOPB .EQ. 0) GO TO 1260                                      SWF3420
```

```
      NOJ1 = ID(2)                                                      SWF3425
      NOJ2 = ID(3)                                                      SWF3430
      IF (J1-J2) 1250,1220,1240                                         SWF3435
 1220 IF (J1 .NE. 0 .AND. J2 .NE. 0) GO TO 1230                         SWF3440
      DEF(L) = B(NOJ)                                                   SWF3445
      GO TO 1310                                                        SWF3450
 1230 IF (L .EQ. 1) DEF(2) = (B(NOJ1)-B(NOJ))/SL1                       SWF3455
      IF (L .EQ. 2) DEF(1) = B(NOJ)                                     SWF3460
      IF (L .EQ. 3) DEF(3) = B(NOJ)                                     SWF3465
      IF (L .EQ. 4) DEF(4) = (B(NOJ)-B(NOJ2))/SL2                       SWF3470
      GO TO 1310                                                        SWF3475
 1240 IF (L .EQ. 1) DEF(2) = (B(NOJ1)-B(NOJ))/SL1                       SWF3480
      IF (L .EQ. 2) DEF(1) = B(NOJ)                                     SWF3485
      IF (L .EQ. 3) DEF(3) = B(NOJ)                                     SWF3490
      IF (L .EQ. 4) DEF(4) = B(NOJ)                                     SWF3495
      GO TO 1310                                                        SWF3500
 1250 DEF(L) = B(NOJ)                                                   SWF3505
      IF (L .EQ. 4) DEF(4) = DEF(4)-B(NOJ)+(B(NOJ)-B(NOJ2))/SL2         SWF3510
      GO TO 1310                                                        SWF3515
 1260 DO 1300 M = 1,4                                                   SWF3520
      IF (ID(L) .GT. ID(M)) GO TO 1300                                  SWF3525
      NOK = (ID(M)-ID(L))*NEQN+NOJ                                      SWF3530
      IF (J1 .NE. 0 .AND. J2 .NE. 0) GO TO 1270                         SWF3535
      IF (J1 .NE. 0 .AND. J2 .EQ. 0) GO TO 1280                         SWF3540
      IF (J1 .EQ. 0 .AND. J2 .NE. 0) GO TO 1290                         SWF3545
      A(NOK) = A(NOK)+S(L,M)                                            SWF3550
      GO TO 1300                                                        SWF3555
 1270 IF (L .EQ. 1 .AND. M .EQ. 1) A(NOK) = A(NOK)+S(2,2)/SL1**2        SWF3560
      IF (L .EQ. 2 .AND. M .EQ. 2) A(NOK) = A(NOK)+S(1,1)+2.*S(1,2)/SL1+SWF3565
     &S(2,2)/SL1**2                                                     SWF3570
      IF (L .EQ. 3 .AND. M .EQ. 3) A(NOK) = A(NOK)+S(3,3)-2.*S(3,4)/SL2+SWF3575
     &S(4,4)/SL2**2                                                     SWF3580
      IF (L .EQ. 4 .AND. M .EQ. 4) A(NOK) = A(NOK)+S(4,4)/SL2**2        SWF3585
      IF (L .EQ. 1 .AND. M .EQ. 2) A(NOK) = A(NOK)-(S(2,1)+S(2,2)/SL1)/ SWF3590
     &SL1                                                               SWF3595
      IF (L .EQ. 1 .AND. M .EQ. 3) A(NOK) = A(NOK)-(S(2,3)-S(2,4)/SL2)/ SWF3600
```

```
     &SL1                                                        SWF3605
      IF (L .EQ. 1 .AND. M .EQ. 4) A(NOK) = A(NOK)-S(2,4)/(SL1*SL2)     SWF3610
      IF (L .EQ. 2 .AND. M .EQ. 3) A(NOK) = A(NOK)+S(1,3)-S(1,4)/SL2+(S( SWF3615
     &2,3)-S(2,4)/SL2)/SL1                                       SWF3620
      IF (L .EQ. 2 .AND. M .EQ. 4) A(NOK) = A(NOK)+S(1,4)/SL2+S(2,4)/(  SWF3625
     &SL1*SL2)                                                   SWF3630
      IF (L .EQ. 3 .AND. M .EQ. 4) A(NOK) = A(NOK)+S(3,4)/SL2-S(4,4)/SL2 SWF3635
     &**2                                                        SWF3640
      GO TO 1300                                                 SWF3645
 1280 IF (L .EQ. 1 .AND. M .EQ. 1) A(NOK) = A(NOK)+S(2,2)/SL1**2        SWF3650
      IF (L .EQ. 2 .AND. M .EQ. 2) A(NOK) = A(NOK)+S(1,1)+2.*S(1,2)/SL1+ SWF3655
     &S(2,2)/SL1**2                                              SWF3660
      IF (L .EQ. 3 .AND. M .EQ. 3) A(NOK) = A(NOK)+S(3,3)               SWF3665
      IF (L .EQ. 4 .AND. M .EQ. 4) A(NOK) = A(NOK)+S(4,4)               SWF3670
      IF (L .EQ. 1 .AND. M .EQ. 2) A(NOK) = A(NOK)-(S(2,1)+S(2,2)/SL1)/ SWF3675
     &SL1                                                        SWF3680
      IF (L .EQ. 1 .AND. M .EQ. 3) A(NOK) = A(NOK)-S(2,3)/SL1           SWF3685
      IF (L .EQ. 1 .AND. M .EQ. 4) A(NOK) = A(NOK)-S(2,4)/SL1           SWF3690
      IF (L .EQ. 2 .AND. M .EQ. 3) A(NOK) = A(NOK)+S(2,3)/SL1+S(1,3)    SWF3695
      IF (L .EQ. 2 .AND. M .EQ. 4) A(NOK) = A(NOK)+S(2,4)/SL1+S(1,4)    SWF3700
      IF (L .EQ. 3 .AND. M .EQ. 4) A(NOK) = A(NOK)+S(3,4)               SWF3705
      GO TO 1300                                                 SWF3710
 1290 IF (L .EQ. 1 .AND. M .EQ. 1) A(NOK) = A(NOK)+S(1,1)               SWF3715
      IF (L .EQ. 2 .AND. M .EQ. 2) A(NOK) = A(NOK)+S(2,2)               SWF3720
      IF (L .EQ. 3 .AND. M .EQ. 3) A(NOK) = A(NOK)+S(3,3)+S(4,4)/SL2**2- SWF3725
     &2.*S(3,4)/SL2                                              SWF3730
      IF (L .EQ. 4 .AND. M .EQ. 4) A(NOK) = A(NOK)+S(4,4)/SL2**2        SWF3735
      IF (L .EQ. 1 .AND. M .EQ. 2) A(NOK) = A(NOK)+S(1,2)               SWF3740
      IF (L .EQ. 1 .AND. M .EQ. 3) A(NOK) = A(NOK)+S(1,3)-S(1,4)/SL2    SWF3745
      IF (L .EQ. 1 .AND. M .EQ. 4) A(NOK) = A(NOK)+S(1,4)/SL2           SWF3750
      IF (L .EQ. 2 .AND. M .EQ. 3) A(NOK) = A(NOK)+S(2,3)-S(2,4)/SL2    SWF3755
      IF (L .EQ. 2 .AND. M .EQ. 4) A(NOK) = A(NOK)+S(2,4)/SL2           SWF3760
      IF (L .EQ. 3 .AND. M .EQ. 4) A(NOK) = A(NOK)+S(3,4)/SL2-S(4,4)/SL2 SWF3765
     &**2                                                        SWF3770
 1300 CONTINUE                                                   SWF3775
 1310 CONTINUE                                                   SWF3780
```

```
      IF (LOOPB .EQ. 0) GO TO 1340                                      SWF3785
      M = 4                                                             SWF3790
      CALL MMULT(M,DEF,S,P)                                             SWF3795
      P(2) = P(2)/12.                                                   SWF3800
      P(4) = P(4)/12.                                                   SWF3805
      WRITE (JWT,1320) I,(P(K),K = 1,4)                                 SWF3810
 1320 FORMAT (15XI3,16XF8.2,2XF9.2,10XF8.2,2XF9.2)                      SWF3815
      L1 = IB(1,I)                                                      SWF3820
      L2 = IB(2,I)                                                      SWF3825
      IF (ISR(L1) .EQ. 0 .AND. ISR(L2) .EQ. 0) GO TO 1340               SWF3830
      IF (ISR(L1) .EQ. 0 .AND. ISR(L2) .NE. 0) GO TO 1330               SWF3835
      R(2,L1) = R(2,L1)+P(1)                                            SWF3840
      R(3,L1) = R(3,L1)+P(2)                                            SWF3845
      IF (ISR(L2) .EQ. 0) GO TO 1340                                    SWF3850
 1330 R(2,L2) = R(2,L2)+P(3)                                            SWF3855
      R(3,L2) = R(3,L2)+P(4)                                            SWF3860
 1340 CONTINUE                                                          SWF3865
      IF (LOOPB .EQ. 1) GO TO 1810                                      SWF3870
 1350 IF (NC .EQ. 0) GO TO 1520                                         SWF3875
C     GENERATE COLUMN STIFFNESS MATRIX AND INCORPORATE INTO EQ. EQNS.   SWF3880
 1360 DO 1510 I = 1,NC                                                  SWF3885
      L1 = IC(2,I)                                                      SWF3890
      L2 = IC(1,I)                                                      SWF3895
      DLT = CT(I)                                                       SWF3900
      DLB = CB(I)                                                       SWF3905
      CL = CJ(2,L2)-CJ(2,L1)-DLT/2.-DLB/2.                              SWF3910
      E = EC(I)                                                         SWF3915
      CA = CAC(I)                                                       SWF3920
      CI = CMI(I)                                                       SWF3925
      DLT = DLT/2.                                                      SWF3930
      DLB = DLB/2.                                                      SWF3935
      CALL STCOLM(E,CA,CI,CL,DLT,DLB,S)                                 SWF3940
      L1 = (L1-1)/N1                                                    SWF3945
      L2 = (L2-1)/N1                                                    SWF3950
      ID(1) = N1*L1*2+L1+1-2*NSH*L1                                     SWF3955
      ID(4) = N1*L2*2+L2+1-2*NSH*L2                                     SWF3960
```

```
      K = IC(2,I)-(N1*L1+1)                                          SWF3965
      J = L1                                                         SWF3970
      ID(2) = ID(1)+K*2+1                                            SWF3975
      DO 1400 II = 1,NSH                                             SWF3980
      IF (J .EQ. 0) GO TO 1370                                       SWF3985
      K1 = IP(II,J,1)                                                SWF3990
      GO TO 1380                                                     SWF3995
 1370 J = J+1                                                        SWF4000
      K1 = IP(II,J,3)                                                SWF4005
 1380 IF (IC(2,I)-K1) 1400,1400,1390                                 SWF4010
 1390 ID(2) = ID(2)-2                                                SWF4015
 1400 CONTINUE                                                       SWF4020
      ID(3) = ID(2)+1                                                SWF4025
      K = IC(1,I)-(N1*L2+1)                                          SWF4030
      ID(5) = ID(4)+K*2+1                                            SWF4035
      DO 1440 II = 1,NSH                                             SWF4040
      J = L2                                                         SWF4045
      IF (J .EQ. 0) GO TO 1410                                       SWF4050
      K1 = IP(II,J,1)                                                SWF4055
      GO TO 1420                                                     SWF4060
 1410 J = J+1                                                        SWF4065
      K1 = IP(II,J,3)                                                SWF4070
 1420 IF (IC(1,I)-K1) 1440,1440,1430                                 SWF4075
 1430 ID(5) = ID(5)-2                                                SWF4080
 1440 CONTINUE                                                       SWF4085
      ID(6) = ID(5)+1                                                SWF4090
      DO 1460 L = 1,6                                                SWF4095
      NOJ = ID(L)                                                    SWF4100
      IF (LOOPC .EQ. 1) DEF(L) = B(NOJ)                              SWF4105
      IF (LOOPC .EQ. 1) GO TO 1460                                   SWF4110
      DO 1450 M = 1,6                                                SWF4115
      IF (ID(L) .GT. ID(M)) GO TO 1450                               SWF4120
      NOK = (ID(M)-ID(L))*NEQN+ID(L)                                 SWF4125
      A(NOK) = A(NOK)+S(L,M)                                         SWF4130
 1450 CONTINUE                                                       SWF4135
 1460 CONTINUE                                                       SWF4140
```

```
      IF (LOOPC .EQ. 0) GO TO 1510                                      SWF4145
      M = 6                                                             SWF4150
      CALL MMULT(M,DEF,S,P)                                             SWF4155
      P(3) = P(3)/12.                                                   SWF4160
      P(6) = P(6)/12.                                                   SWF4165
      WRITE (JWT,1470) I,(P(K),K = 1,6)                                 SWF4170
 1470 FORMAT (16XI3,8XF8.2,2XF8.2,3XF9.2,11XF8.2,3XF8.2,2XF9.2)         SWF4175
      L1 = IC(2,I)                                                      SWF4180
      L2 = IC(1,I)                                                      SWF4185
      IF (ISR(L1) .EQ. 0 .AND. ISR(L2) .EQ. 0) GO TO 1510               SWF4190
      IF (ISR(L1) .EQ. 0 .AND. ISR(L2) .NE. 0) GO TO 1490               SWF4195
      DO 1480 J = 1,3                                                   SWF4200
 1480 R(J,L1) = R(J,L1)+P(J)                                            SWF4205
      IF (ISR(L2) .EQ. 0) GO TO 1510                                    SWF4210
 1490 K = 0                                                             SWF4215
      DO 1500 J = 4,6                                                   SWF4220
      K = K+1                                                           SWF4225
 1500 R(K,L2) = R(K,L2)+P(J)                                            SWF4230
 1510 CONTINUE                                                          SWF4235
      IF (LOOPC .EQ. 1) GO TO 1840                                      SWF4240
C     MODIFY EQUATION FOR ANY RELEASES AT JOINT I INSERT LARGE NO. ON   SWF4245
C     MAIN DIAGONAL AND MULTIPLY JOINT DISPLACEMENT BY SAME LARGE NO.   SWF4250
 1520 DO 1600 I = 1,NN                                                  SWF4255
      IF (ISR(I) .EQ. 0) GO TO 1600                                     SWF4260
      II = (I-1)/N1                                                     SWF4265
      ID(1) = N1*II*2+II+1-2*NSH*II                                     SWF4270
      K = I-(N1*II+1)                                                   SWF4275
      ID(2) = ID(1)+K*2+1                                               SWF4280
      ID(3) = ID(2)+1                                                   SWF4285
      K2 = 1                                                            SWF4290
      DO 1570 JJ = 1,NSH                                                SWF4295
      J = II                                                            SWF4300
      IF (J .EQ. 0) GO TO 1530                                          SWF4305
      K1 = IP(JJ,J,1)                                                   SWF4310
      GO TO 1540                                                        SWF4315
 1530 J = J+1                                                           SWF4320
```

```
      K1 = IP(JJ,J,3)
 1540 IF (I-K1) 1570,1560,1550
 1550 ID(2) = ID(2)-1
      IF (I-K1 .GT. 1) ID(2) = ID(2)-1
      IF (I-K1 .EQ. 1) K2 = 0
      ID(3) = ID(2)+1
      GO TO 1570
 1560 K2 = 0
 1570 CONTINUE
      DO 1590 L = 1,3
      IF (L .EQ. 3 .AND. K2 .EQ. 0) GO TO 1590
      NOI = ID(L)
      IF (ISR(I) .EQ. 2) GO TO 1580
      A(NOI) = 1.E60
      B(NOI) = 0.
      GO TO 1590
 1580 IF (L .EQ. 3) GO TO 1600
      A(NOI) = 1.E60
      B(NOI) = 0.
 1590 CONTINUE
 1600 CONTINUE
      WRITE (JWT,1610)
 1610 FORMAT ('1'///53X'****************'/53X'* R E S U L T S *'/53X'**
     &***************'/)
      CALL INSOL(A,B,NEQN,MBAND)
      WRITE (JWT,1620)
 1620 FORMAT (//54X'JOINT   DISPLACEMENTS',//20X'JOINT',20X'HORIZ DISP',
     &20X'VERT. DISP',18X'ROTATION'/47X'(IN)',26X'(IN)',21X'(RADIANS)'//
     &)
      N1 = NN/(NS+1)
      DO 1720 I = 1,NN
      L1 = (I-1)/N1
      ID(1) = N1*L1*2+L1+1-2*NSH*L1
      K = I-(N1*L1+1)
      ID(2) = ID(1)+K*2+1
      ID(3) = ID(2)+1
```

```
      ID(4) = 0
      ID(5) = 0
      DO 1670 II = 1,NSH
      J = L1
      IF (J .EQ. 0) GO TO 1630
      K1 = IP(II,J,1)
      GO TO 1640
 1630 J = J+1
      K1 = IP(II,J,3)
 1640 IF (I-K1) 1670,1660,1650
 1650 ID(2) = ID(2)-1
      IF (I-K1 .GT. 1) ID(2) = ID(2)-1
      IF (I-K1 .EQ. 1) ID(4) = ID(2)-1
      IF (I-K1 .EQ. 1) SL1 = CJ(1,K1+1)-CJ(1,K1)
      ID(3) = ID(2)+1
      GO TO 1670
 1660 ID(5) = ID(2)+1
      SL2 = CJ(1,K1+1)-CJ(1,K1)
 1670 CONTINUE
      DO 1700 J = 1,3
      K = J
      IF ((J .NE. 3) .OR. (ID(4) .EQ. 0 .AND. ID(5) .EQ. 0)) GO TO 1690
      IF (ID(4) .NE. 0) GO TO 1680
      NOJ2 = ID(5)
      K = 2
      GO TO 1690
 1680 NOJ1 = ID(4)
      K = 2
 1690 NOJ = ID(K)
      IF (J .EQ. 1) H = B(NOJ)
      IF (J .EQ. 2) V = B(NOJ)
      IF (J .EQ. 3 .AND. (ID(4) .EQ. 0 .AND. ID(5) .EQ. 0)) O = B(NOJ)
      IF (J .EQ. 3 .AND. (ID(4) .NE. 0 .AND. ID(5) .EQ. 0)) O = (B(NOJ)-
     &B(NOJ1))/SL1
      IF (J .EQ. 3 .AND. (ID(4) .EQ. 0 .AND. ID(5) .NE. 0)) O = (B(NOJ2)
     &-B(NOJ))/SL2
```

```
 1700 CONTINUE
      WRITE (JWT,1710) I,H,V,O
 1710 FORMAT (20XI3,21XF10.6,19XF10.6,18XF10.6)
 1720 CONTINUE
C     START CALCULATING FORCES IN SHEAR WALL
      WRITE (JWT,1730)
 1730 FORMAT ('1'/54X'FORCES IN SHEAR WALL'/)
      DO 1790 I = 1,NSH
      WRITE (JWT,1740) I
 1740 FORMAT (//54X'SHEAR WALL NO.=',I3///15X'FRAMING LEVEL',17X'AXIAL F
     &ORCE',18X'SHEAR FORCE',19X'MOMENT',/12X'(FROM TOP  TO  BASE)',15X'
     &(KIPS)',23X'(KIPS)',21X'(KIP-FT)'//)
      N1 = NN/(NS+1)
      K2 = 0
      NS1 = NS+1
      DO 1780 JJ = 1,NS1
      J = NS-JJ+1
      IF (J .EQ. 0) GO TO 1750
      K1 = IP(I,J,1)/N1
      K = IP(I,J,1)-(N1*K1+1)
      GO TO 1760
 1750 J = 1
      K1 = IP(I,J,3)/N1
      K = IP(I,J,3)-(N1*K1+1)
 1760 ID(1) = N1*K1*2+K1+1-2*NSH*K1
      ID(2) = ID(1)+K*2-(I-1)*2+1
      ID(3) = ID(2)+1
      DO 1770 K = 1,3
      K2 = K2+1
      NOJ = ID(K)
      DEF(K2) = B(NOJ)
 1770 CONTINUE
 1780 CONTINUE
      CALL FORCE(I,IP,CJ,NEQS,STSH,DEF,P)
 1790 CONTINUE
      IF (NBM .EQ. 0) GO TO 1820
```

```
      WRITE (JWT,1800)
 1800 FORMAT ('1'/50X'BEAM END FORCES',//15X'BEAM',20X'LEFT END',20X'RIG
     &HT END',//36X'SHEAR',5X'MOMENT',13X'SHEAR',5X'MOMENT'/36X'(KIPS)',
     &3X'(KIP-FT)',12X'(KIPS)',3X'(KIP-FT)'//)
      LOOPB = LOOPB+1
      IF (LOOPB .EQ. 1) GO TO 1130
 1810 LOOPC = LOOPC+1
 1820 IF (NC .EQ. 0) GO TO 1840
      WRITE (JWT,1830)
 1830 FORMAT ('1'/53X'COLUMN END FORCES',//15X'COLUMN',17X'BOTTOM END',
     &32X'TOP END'//30X'SHEAR',5X'AXIAL',5X'MOMENT',15X'SHEAR',5X'AXIAL'
     &,5X'MOMENT'/30X'(KIPS)',4X'(KIPS)',3X'(KIP-FT)',14X'(KIPS)',4X'(KI
     &PS)',3X'(KIP-FT)'//)
      IF (LOOPC .EQ. 1) GO TO 1360
 1840 L1 = 0
      DO 1880 I = 1,NN
      IF (ISR(I) .EQ. 0) GO TO 1880
      DO 1850 J = 1,NSH
      K = 1
      DO 1850 L = 3,4
      IF (IP(J,K,L) .EQ. I) GO TO 1880
 1850 CONTINUE
      L1 = L1+1
      IF (L1 .EQ. 1) WRITE (JWT,1860)
 1860 FORMAT (///52X'    COLUMN   REACTIONS    '//20X'JOINT',20X'HORIZONTAL
     &',20X'VERTICAL',20X'MOMENT'/47X'(KIPS)',23X'(KIPS)',20X'(KIP-FT)'/
     &)
      WRITE (JWT,1870) I,(R(J,I),J = 1,3)
 1870 FORMAT (20XI3,21XF9.3,21XF8.3,18XF9.3)
 1880 CONTINUE
 1890 CONTINUE
 1900 RETURN
      END
```

```
      SUBROUTINE CNVRT(I1, I2)
      IMPLICIT REAL*8(A-H, O-Z), INTEGER(I-N)
C     ******************************************************************
C     *     CONVERTS INPUT DATA INTO REAL NUMBERS                      *
C     ******************************************************************
      INTEGER*2 INPT(80)
      COMMON JRD, JWT, FN(8), INPT
      INTEGER*2 IN(5)/'0', '9', '.', ' ', '-'/
      J = 10*I2+1
      IFL = I2-I1+1
   10 FK = 0.
      K = 1
      L = 0
      DO 50 I = 1, 10
      J = J-1
      IF (INPT(J) .NE. IN(3)) GO TO 20
      FK = FK/10.**L
      L = 0
      GO TO 50
   20 IF (INPT(J) .LT. IN(1) .OR. INPT(J) .GT. IN(2)) GO TO 30
      IJ = INPT(J)/256+15
      L = L+1
      FK = FK+IJ*10.**L/10.
      GO TO 50
   30 IF (INPT(J) .NE. IN(5)) GO TO 40
      FK = -FK
      GO TO 50
   40 IF (INPT(J) .EQ. IN(4)) GO TO 50
      K = 0
   50 CONTINUE
   60 FN(IFL) = FK*K
      IFL = IFL-1
      IF (IFL .GT. 0) GO TO 10
      RETURN
      END
```

```
      SUBROUTINE SEGMT(I, J, E, U, CX, CY, NX, NY, TS, LO, AK, NEQ1,
     &NEQ3, ST, II2, II3, KK, KEL, ND)
      IMPLICIT REAL*8(A-H, O-Z), INTEGER(I-N)
C     ******************************************************************
C     *   GENERATES STIFFNESS MATRIX FOR A SEGMENT                     *
C     ******************************************************************
      INTEGER*2 INPT(80)
      COMMON JRD, JWT, FN(8), INPT, NSH, NS, MAXX
      DIMENSION CX(NSH, 1), CY(NSH, NS, 1), NX(1), NY(NSH, 1), TS(NSH,
     &NS, 1), LO(NSH, NS, 4), AK(II3, 1), ST(II2, 1)
      DIMENSION S(12, 12), KK(3, 1), KEL(1), ND(3, 1), IJ(12)
C     GENERATE NODAL NUMBERS FOR TOP MOST ROW OF PANEL J
      I1 = 0
      N1 = NX(I)+1
      DO 10 K = 1, 2
      DO 10 L = 1, N1
      I1 = I1+1
   10 KK(K, L) = I1
C     INITIALIZE ST MATRIX FOR TOP MOST ROW OF ELEMENTS
      IF (N1 .LE. 4) GO TO 20
      NEQ1 = N1+5
      NEQ2 = 4+2*N1
      NEQ = NEQ1+NEQ2
      GO TO 30
   20 NEQ = 5*N1+1
      NEQ1 = 2*(NX(I)+1)+1
   30 DO 40 K = 1, NEQ
      DO 40 L = 1, NEQ
   40 ST(K, L) = 0.
      NEL = NX(I)
      DO 60 K = 1, NEL
      IF (N1 .GT. 4) GO TO 50
      KEL(K) = 1
      GO TO 60
   50 IF (K .EQ. 1 .OR. K .EQ. NX(I)) KEL(K) = 1
      IF (K .EQ. 2) KEL(K) = 2
```

```
      IF (K .EQ. (NX(I)-1)) KEL(K) = 3                                  SEGM0370
      IF (K .GT. 2 .AND. K .LT. (NX(I)-1)) KEL(K) = 0                   SEGM0380
   60 CONTINUE                                                          SEGM0390
      I1 = 1                                                            SEGM0400
      I2 = NEQ1                                                         SEGM0410
      N = NY(I, J)+1                                                    SEGM0420
      Y = CY(I, J, N)-CY(I, J, N-1)                                     SEGM0430
      DO 260 K = 1, NEL                                                 SEGM0440
      X = CX(I, K+1)-CX(I, K)                                           SEGM0450
      T = TS(I, J, K)                                                   SEGM0460
      IF (KEL(K) .EQ. 1) GO TO 110                                      SEGM0470
      IF (KEL(K) .EQ. 2) GO TO 170                                      SEGM0480
      IF (KEL(K) .EQ. 0) GO TO 220                                      SEGM0490
      I1 = I1-1                                                         SEGM0500
      I2 = I2-2                                                         SEGM0510
      DO 70 L = 1, 3, 2                                                 SEGM0520
   70 IJ(L) = 1                                                         SEGM0530
      DO 80 L = 2, 5                                                    SEGM0540
      IF (L .EQ. 3) GO TO 80                                            SEGM0550
      I1 = I1+1                                                         SEGM0560
      IJ(L) = I1                                                        SEGM0570
   80 CONTINUE                                                          SEGM0580
      DO 90 L = 6, 10                                                   SEGM0590
      I2 = I2+1                                                         SEGM0600
   90 IJ(L) = I2                                                        SEGM0610
      CALL STRTR(X, Y, T, E, U, S)                                      SEGM0620
      DO 100 L = 1, 10                                                  SEGM0630
      L1 = IJ(L)                                                        SEGM0640
      DO 100 M = 1, 10                                                  SEGM0650
      L2 = IJ(M)                                                        SEGM0660
  100 ST(L1, L2) = ST(L1, L2)+S(L, M)                                   SEGM0670
      GO TO 260                                                         SEGM0680
  110 IF (K .NE. NEL) GO TO 120                                         SEGM0690
      I1 = I1-2                                                         SEGM0700
      I2 = I2-3                                                         SEGM0710
  120 DO 130 L = 1, 4, 3                                                SEGM0720
```

```
130 IJ(L) = 1                                          SEGM0730
    DO 140 L = 2, 6                                    SEGM0740
    IF (L .EQ. 4) GO TO 140                            SEGM0750
    I1 = I1+1                                          SEGM0760
    IJ(L) = I1                                         SEGM0770
140 CONTINUE                                           SEGM0780
    DO 150 L = 7, 12                                   SEGM0790
    I2 = I2+1                                          SEGM0800
150 IJ(L) = I2                                         SEGM0810
    CALL STEDGE(X, Y, T, E, U, S)                      SEGM0820
    DO 160 L = 1, 12                                   SEGM0830
    L1 = IJ(L)                                         SEGM0840
    DO 160 M = 1, 12                                   SEGM0850
    L2 = IJ(M)                                         SEGM0860
160 ST(L1, L2) = ST(L1, L2)+S(L, M)                    SEGM0870
    GO TO 260                                          SEGM0880
170 I1 = I1-2                                          SEGM0890
    I2 = I2-3                                          SEGM0900
    DO 180 L = 1, 4, 3                                 SEGM0910
180 IJ(L) = 1                                          SEGM0920
    DO 190 L = 2, 5                                    SEGM0930
    IF (L .EQ. 4) GO TO 190                            SEGM0940
    I1 = I1+1                                          SEGM0950
    IJ(L) = I1                                         SEGM0960
190 CONTINUE                                           SEGM0970
    DO 200 L = 6, 10                                   SEGM0980
    I2 = I2+1                                          SEGM0990
200 IJ(L) = I2                                         SEGM1000
    CALL STLTR(X, Y, T, E, U, S)                       SEGM1010
    DO 210 L = 1, 10                                   SEGM1020
    L1 = IJ(L)                                         SEGM1030
    DO 210 M = 1, 10                                   SEGM1040
    L2 = IJ(M)                                         SEGM1050
210 ST(L1, L2) = ST(L1, L2)+S(L, M)                    SEGM1060
    GO TO 260                                          SEGM1070
220 I1 = I1-1                                          SEGM1080
```

```
      I2 = I2-2                                                         SEGM1090
      DO 230 L = 1, 3, 2                                                SEGM1100
  230 IJ(L) = 1                                                         SEGM1110
      I1 = I1+1                                                         SEGM1120
      IJ(2) = I1                                                        SEGM1130
      I1 = I1+1                                                         SEGM1140
      IJ(4) = I1                                                        SEGM1150
      DO 240 L = 5, 8                                                   SEGM1160
      I2 = I2+1                                                         SEGM1170
  240 IJ(L) = I2                                                        SEGM1180
      CALL STINT(X, Y, T, E, U, S)                                      SEGM1190
      DO 250 L = 1, 8                                                   SEGM1200
      L1 = IJ(L)                                                        SEGM1210
      DO 250 M = 1, 8                                                   SEGM1220
      L2 = IJ(M)                                                        SEGM1230
  250 ST(L1, L2) = ST(L1, L2)+S(L, M)                                   SEGM1240
  260 CONTINUE                                                          SEGM1250
C     ADD STIFFNESS OF SUBSEQUENT ROWS OF ELEMENTS AND ELIMINATE THE    SEGM1260
C     INTERIOR ONES.                                                    SEGM1270
      N2 = NY(I, J)-1                                                   SEGM1280
      M1 = NY(I, J)+1                                                   SEGM1290
      DO 850 K2 = 1, N2                                                 SEGM1300
      K = (N2-K2)+1                                                     SEGM1310
      Y = CY(I, J, K+1)-CY(I, J, K)                                     SEGM1320
      I1 = NX(I)+1                                                      SEGM1330
      K1 = NEQ1                                                         SEGM1340
      DO 270 J1 = 2, 3                                                  SEGM1350
      DO 270 J2 = 1, N1                                                 SEGM1360
  270 KK(J1, J2) = 0                                                    SEGM1370
      M1 = M1-2                                                         SEGM1380
      DO 290 L = 1, N1                                                  SEGM1390
      IF (M1 .GE. LO(I, J, 2) .OR. M1 .LE. LO(I, J, 1)) GO TO 280       SEGM1400
      IF (L .GT. LO(I, J, 3) .AND. L .LT. LO(I, J, 4)) GO TO 290        SEGM1410
  280 I1 = I1+1                                                         SEGM1420
      KK(3, L) = I1                                                     SEGM1430
  290 CONTINUE                                                          SEGM1440
```

```
      I3 = I1-N1                                                        SEGM1450
      M1 = M1+1                                                         SEGM1460
      DO 310 L = 1, N1                                                  SEGM1470
      IF (M1 .GE. LO(I, J, 2) .OR. M1 .LE. LO(I, J, 1)) GO TO 300       SEGM1480
      IF (L .GT. LO(I, J, 3) .AND. L .LT. LO(I, J, 4)) GO TO 310        SEGM1490
  300 I1 = I1+1                                                         SEGM1500
      KK(2, L) = I1                                                     SEGM1510
  310 CONTINUE                                                          SEGM1520
      I2 = I1-I3-N1                                                     SEGM1530
      IF (N1 .LE. 4) GO TO 320                                          SEGM1540
      NEQ2 = 4+2*I2                                                     SEGM1550
      GO TO 330                                                         SEGM1560
  320 NEQ2 = 3*N1                                                       SEGM1570
  330 IF (M1-1 .EQ. 1) GO TO 340                                        SEGM1580
      NEQ3 = 4+2*I3                                                     SEGM1590
      IF (N1 .LE. 4) NEQ3 = 3*N1                                        SEGM1600
      GO TO 360                                                         SEGM1610
  340 IF (N1 .LE. 4) GO TO 350                                          SEGM1620
      NEQ3 = N1+5                                                       SEGM1630
      GO TO 360                                                         SEGM1640
  350 NEQ3 = 2*N1+1                                                     SEGM1650
  360 NEQ = NEQ1+NEQ2+NEQ3                                              SEGM1660
      NEQ4 = NEQ1+NEQ3+1                                                SEGM1670
      NEQ5 = NEQ1+NEQ2                                                  SEGM1680
      DO 370 L = 1, NEQ5                                                SEGM1690
      DO 370 M = 1, NEQ5                                                SEGM1700
  370 AK(L, M) = ST(L, M)                                               SEGM1710
      DO 380 L = 1, NEQ1                                                SEGM1720
      K1 = NEQ1                                                         SEGM1730
      DO 380 M = NEQ4, NEQ                                              SEGM1740
      K1 = K1+1                                                         SEGM1750
      ST(L, M) = AK(L, K1)                                              SEGM1760
  380 ST(M, L) = ST(L, M)                                               SEGM1770
      L1 = NEQ1                                                         SEGM1780
      DO 390 L = NEQ4, NEQ                                              SEGM1790
      L1 = L1+1                                                         SEGM1800
```

```
      K1 = NEQ1
      DO 390 M = NEQ4, NEQ
      K1 = K1+1
  390 ST(L, M) = AK(L1, K1)
C     INITIALIZE STIFFNESS MATRIX FOR NEW SET OF ELEMENTS
      K1 = NEQ1+1
      K3 = NEQ1+NEQ3
      DO 400 L = 1, NEQ1
      DO 400 M = K1, K3
      ST(L, M) = 0.
  400 ST(M, L) = 0.
      DO 410 L = K1, K3
      DO 410 M = K1, K3
  410 ST(L, M) = 0.
      DO 420 L = K1, K3
      DO 420 M = NEQ4, NEQ
      ST(L, M) = 0.
  420 ST(M, L) = 0.
      NEL1 = NX(I)
      N = 0
      J1 = 0
      M2 = 2
      DO 440 M = 1, NEL1
      N = N+1
      IF (KK(M2, N) .EQ. 0 .OR. KK(M2, N+1) .EQ. 0) GO TO 440
      IF (KK(M2+1, N) .EQ. 0 .OR. KK(M2+1, N+1) .EQ. 0) GO TO 440
      J1 = J1+1
      IF (N1 .LE. 4) GO TO 430
      IF (M .EQ. 1 .OR. M .EQ. NEL1) KEL(J1) = 1
      IF (M .EQ. 2) KEL(J1) = 2
      IF (M .EQ. NX(I)-1) KEL(J1) = 3
      IF (M .GT. 2 .AND. M .LT. NX(I)-1) KEL(J1) = 0
      GO TO 440
  430 KEL(J1) = 1
  440 CONTINUE
      NEL = J1
```

```
      DO 450 L = 2, 3
      DO 450 M = 1, N1
  450 ND(L, M) = 0
      DO 470 L = 2, 3
      DO 470 M = 1, N1
      IF (KK(L, M) .EQ. 0) GO TO 470
      IF (M .LE. 2 .OR. M .GE. NEL1) GO TO 460
      ND(L, M) = 2
      GO TO 470
  460 ND(L, M) = 3
  470 CONTINUE
      I1 = NEQ1
      I2 = NEQ1+NEQ3
      J1 = 0
      N = 0
      DO 820 L = 1, NEL1
      N = N+1
      IF (KK(M2, N) .EQ. 0 .OR. KK(M2, N+1) .EQ. 0) GO TO 480
      IF (KK(M2+1, N) .EQ. 0 .OR. KK(M2+1, N+1) .EQ. 0) GO TO 480
      J1 = J1+1
      X = CX(I, L+1)-CX(I, L)
      T = TS(I, J, L)
      GO TO 490
  480 I1 = I1+ND(3, L+1)
      I2 = I2+ND(2, L+1)
      GO TO 820
  490 IF (KEL(J1) .EQ. 1) GO TO 570
      IF (KEL(J1) .EQ. 2) GO TO 660
      IF (KEL(J1) .EQ. 0) GO TO 740
      I2 = I2-2
      DO 500 M = 1, 5
      I2 = I2+1
  500 IJ(M) = I2
      IF (M1-1 .EQ. 1) GO TO 520
      I1 = I1-2
      DO 510 M = 6, 10
```

```
      I1 = I1+1                                                    SEGM2530
  510 IJ(M) = I1                                                   SEGM2540
      GO TO 550                                                    SEGM2550
  520 I1 = I1-1                                                    SEGM2560
      DO 530 M = 6, 8, 2                                           SEGM2570
  530 IJ(M) = NEQ1+1                                               SEGM2580
      DO 540 M = 7, 10                                             SEGM2590
      IF (M .EQ. 8) GO TO 540                                      SEGM2600
      I1 = I1+1                                                    SEGM2610
      IJ(M) = I1                                                   SEGM2620
  540 CONTINUE                                                     SEGM2630
  550 CALL STRTR(X, Y, T, E, U, S)                                 SEGM2640
      DO 560 M = 1, 10                                             SEGM2650
      L1 = IJ(M)                                                   SEGM2660
      DO 560 MI = 1, 10                                            SEGM2670
      L2 = IJ(MI)                                                  SEGM2680
  560 ST(L1, L2) = ST(L1, L2)+S(M, MI)                             SEGM2690
      GO TO 820                                                    SEGM2700
  570 IF (J1 .NE. NEL) GO TO 580                                   SEGM2710
      I1 = I1-3                                                    SEGM2720
      I2 = I2-3                                                    SEGM2730
  580 DO 590 M = 1, 6                                              SEGM2740
      I2 = I2+1                                                    SEGM2750
  590 IJ(M) = I2                                                   SEGM2760
      IF (M1-1 .EQ. 1) GO TO 610                                   SEGM2770
      DO 600 M = 7, 12                                             SEGM2780
      I1 = I1+1                                                    SEGM2790
  600 IJ(M) = I1                                                   SEGM2800
      GO TO 640                                                    SEGM2810
  610 DO 620 M = 7, 10, 3                                          SEGM2820
  620 IJ(M) = NEQ1+1                                               SEGM2830
      I1 = I1+1                                                    SEGM2840
      DO 630 M = 8, 12                                             SEGM2850
      IF (M .EQ. 10) GO TO 630                                     SEGM2860
      I1 = I1+1                                                    SEGM2870
      IJ(M) = I1                                                   SEGM2880
```

```
630 CONTINUE                                                SEGM2890
640 CALL STEDGE(X, Y, T, E, U, S)                           SEGM2900
    DO 650 M = 1, 12                                        SEGM2910
    L1 = IJ(M)                                              SEGM2920
    DO 650 MI = 1, 12                                       SEGM2930
    L2 = IJ(MI)                                             SEGM2940
650 ST(L1, L2) = ST(L1, L2)+S(M, MI)                        SEGM2950
    GO TO 820                                               SEGM2960
660 I2 = I2-3                                               SEGM2970
    DO 670 M = 1, 5                                         SEGM2980
    I2 = I2+1                                               SEGM2990
670 IJ(M) = I2                                              SEGM3000
    IF ((M1-1) .EQ. 1) GO TO 690                            SEGM3010
    I1 = I1-3                                               SEGM3020
    DO 680 M = 6, 10                                        SEGM3030
    I1 = I1+1                                               SEGM3040
680 IJ(M) = I1                                              SEGM3050
    GO TO 720                                               SEGM3060
690 I1 = I1-2                                               SEGM3070
    DO 700 M = 6, 9, 3                                      SEGM3080
700 IJ(M) = NEQ1+1                                          SEGM3090
    DO 710 M = 7, 10                                        SEGM3100
    IF (M .EQ. 9) GO TO 710                                 SEGM3110
    I1 = I1+1                                               SEGM3120
    IJ(M) = I1                                              SEGM3130
710 CONTINUE                                                SEGM3140
720 CALL STLTR(X, Y, T, E, U, S)                            SEGM3150
    DO 730 M = 1, 10                                        SEGM3160
    L1 = IJ(M)                                              SEGM3170
    DO 730 MI = 1, 10                                       SEGM3180
    L2 = IJ(MI)                                             SEGM3190
730 ST(L1, L2) = ST(L1, L2)+S(M, MI)                        SEGM3200
    GO TO 820                                               SEGM3210
740 I2 = I2-2                                               SEGM3220
    DO 750 M = 1, 4                                         SEGM3230
    I2 = I2+1                                               SEGM3240
```

```
  750 IJ(M) = I2
      IF (M1-1 .EQ. 1) GO TO 770
      I1 = I1-2
      DO 760 M = 5, 8
      I1 = I1+1
  760 IJ(M) = I1
      GO TO 800
  770 I1 = I1-1
      DO 780 M = 5, 7, 2
  780 IJ(M) = NEQ1+1
      DO 790 M = 6, 8, 2
      I1 = I1+1
  790 IJ(M) = I1
  800 CALL STINT(X, Y, T, E, U, S)
      DO 810 M = 1, 8
      L1 = IJ(M)
      DO 810 MI = 1, 8
      L2 = IJ(MI)
  810 ST(L1, L2) = ST(L1, L2)+S(M, MI)
  820 CONTINUE
C     CONDENSATION OF INTERNAL NODES
  830 DO 840 J1 = 1, NEQ2
      J2 = NEQ-J1
      L = J2+1
      PIVOT = ST(L, L)
      DO 840 LL = 1, J2
      C = ST(L, LL)/PIVOT
      ST(L, LL) = C
      DO 840 MM = LL, J2
      ST(MM, LL) = ST(MM, LL)-C*ST(MM, L)
  840 ST(LL, MM) = ST(MM, LL)
  850 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE REDN(I, J, IP, CJ, NEQ, NEQ1, NEQ3, AK, NEQI, LOOP, II3
     &, CX, C1, C2, NX)
      IMPLICIT REAL*8(A-H, O-Z), INTEGER(I-N)
C     ******************************************************************
C     * REDUCTION OF INTERNAL DISPLACEMENTS AND END ROTATIONS OF AN EDGE
C     * OF A SEGMENT                                                   *
C     ******************************************************************
      INTEGER*2 INPT(80)
      COMMON JRD, JWT, FN(8), INPT, NSH, NS, MAXX
      DIMENSION CX(NSH, 1), NX(1), IP(NSH, NS, 1), CJ(2, 1), AK(II3, 1),
     &C1(1), C2(1)
C     CALCULATE LENGTH BETWEEN EXTREME NODES OF EDGE CONSIDERED
      L1 = IP(I, J, 1)
      L2 = IP(I, J, 2)
      IF (LOOP .EQ. 1) L1 = IP(I, J, 3)
      IF (LOOP .EQ. 1) L2 = IP(I, J, 4)
      CL = CJ(1, L2)-CJ(1, L1)
      I1 = NX(I)+1
      NEQI = NEQ1-5
      IF (LOOP .EQ. 1) NEQI = NEQ3-5
      DO 30 II = 1, NEQI
      K = NEQ+1-II
      I1 = I1-1
      IF (II .EQ. 2 .OR. II .EQ. NEQI) I1 = I1+1
      C1(II) = (CJ(1, L2)-CX(I, I1))/CL
      C2(II) = 1.-C1(II)
      PIV1 = 7
      PIV2 = 9
      IF (J .EQ. NS) PIV1 = 2
      IF (J .EQ. NS) PIV2 = 4
      IF (II .EQ. 1 .OR. II .EQ. NEQI-1) PIV1 = 8
      IF (II .EQ. 1 .OR. II .EQ. NEQI-1) PIV2 = 10
      IF ((II .EQ. 1 .OR. II .EQ. NEQI-1) .AND. J .EQ. NS) PIV1 = 3
      IF ((II .EQ. 1 .OR. II .EQ. NEQI-1) .AND. J .EQ. NS) PIV2 = 5
C     MODIFY STIFFNESS COEFFICIENTS
      K2 = K-1
```

```
      DO 10 K1 = 1, K2                                                  REDN0370
      AK(PIV1, K1) = AK(PIV1, K1)+C1(II)*AK(K, K1)                      REDN0380
      AK(PIV2, K1) = AK(PIV2, K1)+C2(II)*AK(K, K1)                      REDN0390
      IF (K1 .EQ. PIV1) AK(PIV1, K1) = AK(PIV1, K1)+C1(II)*AK(K, K)*C1( REDN0400
     &II)+C1(II)*AK(K1, K)                                              REDN0410
   10 IF (K1 .EQ. PIV2) AK(PIV2, K1) = AK(PIV2, K1)+C2(II)*AK(K, K)*C2( REDN0420
     &II)+C2(II)*AK(K1, K)                                              REDN0430
      AK(PIV1, PIV2) = AK(PIV1, PIV2)+C2(II)*AK(K, K)*C1(II)+C2(II)*AK( REDN0440
     &PIV1, K)                                                          REDN0450
      AK(PIV2, PIV1) = AK(PIV1, PIV2)                                   REDN0460
      DO 20 M = 1, K2                                                   REDN0470
      AK(M, PIV1) = AK(PIV1, M)                                         REDN0480
   20 AK(M, PIV2) = AK(PIV2, M)                                         REDN0490
   30 CONTINUE                                                          REDN0500
      NREQ = NEQ-NEQI                                                   REDN0510
      PIV1 = 7                                                          REDN0520
      PIV2 = 9                                                          REDN0530
      IF (J .EQ. NS) PIV1 = 2                                           REDN0540
      IF (J .EQ. NS) PIV2 = 4                                           REDN0550
      K1 = 8                                                            REDN0560
      K2 = 10                                                           REDN0570
      IF (J .EQ. NS) K1 = 3                                             REDN0580
      IF (J .EQ. NS) K2 = 5                                             REDN0590
      DO 40 K = 1, NREQ                                                 REDN0600
      IF (K .EQ. K1 .OR. K .EQ. K2) GO TO 40                            REDN0610
      AK(K, PIV1) = AK(K, PIV1)-AK(K, K1)/CL-AK(K, K2)/CL               REDN0620
      AK(K, PIV2) = AK(K, PIV2)+AK(K, K1)/CL+AK(K, K2)/CL               REDN0630
      IF (K .EQ. PIV1) AK(K, PIV1) = AK(K, PIV1)-(AK(K1, K)-AK(K1, K1)/ REDN0640
     &CL-AK(K1, K2)/CL)/CL-(AK(K2, K)-AK(K2, K1)/CL-AK(K2, K2)/CL)/CL   REDN0650
      IF (K .EQ. PIV2) AK(K, PIV2) = AK(K, PIV2)+(AK(K1, K)+AK(K1, K1)/ REDN0660
     &CL+AK(K1, K2)/CL)/CL+(AK(K2, K)+AK(K2, K1)/CL+AK(K2, K2)/CL)/CL   REDN0670
   40 CONTINUE                                                          REDN0680
      AK(PIV1, PIV2) = AK(PIV1, PIV2)-(AK(K1, PIV2)+AK(K1, K1)/CL+AK(K1,REDN0690
     &K2)/CL)/CL-(AK(K2, PIV2)+AK(K2, K1)/CL+AK(K2, K2)/CL)/CL          REDN0700
      AK(PIV2, PIV1) = AK(PIV1, PIV2)                                   REDN0710
      DO 50 K = 1, NREQ                                                 REDN0720
```

```
      AK(PIV1, K) = AK(K, PIV1)                                         REDN0730
   50 AK(PIV2, K) = AK(K, PIV2)                                         REDN0740
      RETURN                                                            REDN0750
      END                                                               REDN0760

      SUBROUTINE INSOL(A, B, NEQN, MBAND)                               INSO0010
      IMPLICIT REAL*8(A-H, O-Z), INTEGER(I-N)                           INSO0020
C     ******************************************************************INSO0030
C     *  IN CORE LINEAR EQUATION SOLVER FOR SYMMETRIC BANDED MATRIX    *INSO0040
C     ******************************************************************INSO0050
      DIMENSION A(1), B(1)                                              INSO0060
      NR = NEQN-1                                                       INSO0070
      NMA = NEQN*MBAND                                                  INSO0080
C     START REDUCTION                                                   INSO0090
      DO 30 N = 1, NR                                                   INSO0100
      J = N+1                                                           INSO0110
      PIVOT = A(N)                                                      INSO0120
      MAX = (NMA-NEQN)+N                                                INSO0130
      MR = N+NEQN                                                       INSO0140
      DO 20 L = MR, NMA, NEQN                                           INSO0150
      MAX = MAX-NR                                                      INSO0160
      C = A(L)/PIVOT                                                    INSO0170
      IF (C .EQ. 0.) GO TO 20                                           INSO0180
      M = L                                                             INSO0190
      DO 10 K = J, MAX, NEQN                                            INSO0200
      A(K) = A(K)-C*A(M)                                                INSO0210
   10 M = M+NEQN                                                        INSO0220
      B(J) = B(J)-C*B(N)                                                INSO0230
      A(L) = C                                                          INSO0240
   20 J = J+1                                                           INSO0250
      B(N) = B(N)/PIVOT                                                 INSO0260
   30 CONTINUE                                                          INSO0270
C     START BACK SUBSTITUTION                                           INSO0280
      B(NEQN) = B(NEQN)/A(NEQN)                                         INSO0290
      DO 40 K = 1, NR                                                   INSO0300
      N = NEQN-K                                                        INSO0310
      MR = MIN0(MBAND-1, K)                                             INSO0320
```

```
      J = N                                                             INSO0330
      L = N                                                             INSO0340
      DO 40 M = 1, MR                                                   INSO0350
      J = J+1                                                           INSO0360
      L = L+NEQN                                                        INSO0370
   40 B(N) = B(N)-A(L)*B(J)                                             INSO0380
      RETURN                                                            INSO0390
      END                                                               INSO0400

      SUBROUTINE FORCE(I, IP, CJ, NEQS, STSH, DEF, P)                   FORC0010
      IMPLICIT REAL*8(A-H, O-Z), INTEGER(I-N)                           FORC0020
C     ******************************************************************FORC0030
C     *         THIS SUBROUTINE CALCULATES FORCES IN SHEAR WALL        *FORC0040
C     ******************************************************************FORC0050
      INTEGER*2 INPT(80)                                                FORC0060
      COMMON JRD, JWT, FN(8), INPT, NSH, NS                             FORC0070
      DIMENSION IP(NSH, NS, 1), CJ(2, 1), DEF(1), STSH(NSH, 1), P(1), SMFORC0080
     &(4)                                                               FORC0090
      NS1 = NS+1                                                        FORC0100
      NMB= NEQS*6                                                       FORC0110
      INC = NEQS-1                                                      FORC0120
      K1 = 0                                                            FORC0130
      DO 10 II = 1, 4                                                   FORC0140
   10 SM(II) = 0.                                                       FORC0150
      DO 170 I1 = 1, NS1                                                FORC0160
      IK = NS1-I1+1                                                     FORC0170
      I2 = NS-I1+2                                                      FORC0180
      I3 = I2-1                                                         FORC0190
      IF (I2 .GT. NS) I2 = I3                                           FORC0200
      IF (I3 .EQ. 0) GO TO 20                                           FORC0210
      C1 = IP(I, I3, 1)                                                 FORC0220
      C2 = IP(I, I2, 1)                                                 FORC0230
      IF (I1 .GT. 1) GO TO 30                                           FORC0240
      C3 = IP(I, I3, 2)                                                 FORC0250
      GO TO 30                                                          FORC0260
   20 I3 = I2                                                           FORC0270
      C1 = IP(I, I3, 3)                                                 FORC0280
```

```
      C2 = IP(I, I3, 1)
   30 CL1 = CJ(2, C1)-CJ(2, C2)
      IF (I1 .GT. 1) GO TO 40
      CL2 = (CJ(1, C3)-CJ(1, C1))/2.
   40 DO 70 II = 1, 3
      K1 = K1+1
      MAX = (NMB-NEQS)+K1
      KK = K1
      DO 50 K = K1, MAX, NEQS
      IF (STSH(I, K) .EQ. 0) GO TO 50
      SM(II) = SM(II)+STSH(I, K)*DEF(KK)
   50 KK = KK+1
      IL = 1+(K1-1)*NEQS
      JJ = K1-1
      IF (JJ .EQ. 0) GO TO 70
      DO 60 K = 1, JJ
      IF (IL .GT. NMB) GO TO 60
      SM(II) = SM(II)+STSH(I, IL)*DEF(K)
   60 IL = IL-INC
   70 CONTINUE
      IF (I1 .EQ. 1) GO TO 90
      J1 = I1-1
      P(I1) = SM(1)
      DO 80 J = 1, J1
   80 P(I1) = P(I1)-P(J)
      GO TO 100
   90 P(I1) = SM(1)
  100 IF (I1 .EQ. 1 .OR. I1 .EQ. NS1) GO TO 120
      DO 110 J = 1, J1
  110 SM(4) = SM(4)+P(J)*CL1
  120 IF (I1 .EQ. NS1) GO TO 140
      SHEAR = SM(1)
      AXIAL = SM(2)+SM(3)
      AMOMT = (SM(4)+SM(2)*(-CL2)+SM(3)*CL2)/12.
      WRITE (JWT,130) IK, AXIAL, SHEAR, AMOMT
  130 FORMAT (20XI3, 22XF8.2, 22XF8.2, 18XF9.2)
```

```
      GO TO 170                                                          FORC0650
  140 SHEAR = SHEAR-SM(1)                                                FORC0660
      AXIAL = AXIAL-SM(2)-SM(3)                                          FORC0670
      DO 150 J = 1, J1                                                   FORC0680
  150 AMOMT = AMOMT+P(J)*CL1/12.                                         FORC0690
      WRITE (JWT,130) IK, AXIAL, SHEAR, AMOMT                            FORC0700
      SHEAR = -SHEAR                                                     FORC0710
      AXIAL = -AXIAL                                                     FORC0720
      AMOMT = -AMOMT                                                     FORC0730
      WRITE (JWT,160) AXIAL, SHEAR, AMOMT                                FORC0740
  160 FORMAT (/17X'REACTIONS', 19XF8.2, 22XF8.2, 18XF9.2)                FORC0750
  170 CONTINUE                                                           FORC0760
      RETURN                                                             FORC0770
      END                                                                FORC0780


      SUBROUTINE MMULT(M, DEF, S, P)                                     MMUL0010
      IMPLICIT REAL*8(A-H, O-Z), INTEGER(I-N)                            MMUL0020
C     ******************************************************************MMUL0030
C     *          THIS SUBROUTINE PERFORMS MATRIX MULTIPLICATION        *MMUL0040
C     ******************************************************************MMUL0050
      DIMENSION S(12, 12), DEF(1), P(1)                                  MMUL0060
      DO 20 I = 1, M                                                     MMUL0070
      SUM = 0.                                                           MMUL0080
      DO 10 K = 1, M                                                     MMUL0090
      IF (DABS(S(I, K)) .LT. 1.E-30 .AND. DABS(DEF(K)) .LT. 1.E-30)      MMUL0100
     &GO TO 10                                                           MMUL0110
      SUM = SUM+S(I, K)*DEF(K)                                           MMUL0120
   10 CONTINUE                                                           MMUL0130
   20 P(I) = SUM                                                         MMUL0140
      RETURN                                                             MMUL0150
      END                                                                MMUL0160
```

```
      SUBROUTINE STEDGE(X, Y, T, E, U, S)
      IMPLICIT REAL*8(A-H, O-Z), INTEGER(I-N)
C     ******************************************************************
C     * THIS SUBROUTINE GENERATES STIFFNESS MATRIX FOR AN EDGE ELEMENT *
C     ******************************************************************
      DIMENSION S(12, 12)
      RL = (1.-U)/2.
      R = (E*T)/(1.-U**2)
      DO 10 I = 1, 10, 3
   10 S(I, I) = (((13.*Y)/(35.*X))+((2.*RL*X)/(5.*Y)))*R
      S(1, 2) = -((RL+U)/4.)*R
      S(4, 8) = S(1, 2)
      S(5, 7) = S(1, 2)
      S(10, 11) = S(1, 2)
      S(1, 11) = -S(1, 2)
      S(2, 10) = S(1, 11)
      S(4, 5) = S(2, 10)
      S(7, 8) = S(4, 5)
      S(1, 3) = ((11.*Y**2)/(210.*X)-(U*X)/24.+(3.*RL*X)/40.)*R
      S(4, 6) = S(1, 3)
      S(7, 9) = -S(1, 3)
      S(10, 12) = S(7, 9)
      S(1, 4) = ((RL*X)/(5.*Y)-(13.*Y)/(35.*X))*R
      S(7, 10) = S(1, 4)
      S(1, 5) = ((RL-U)/4.)*R
      S(2, 7) = S(1, 5)
      S(4, 11) = S(1, 5)
      S(8, 10) = S(1, 5)
      S(1, 8) = -S(1, 5)
      S(2, 4) = S(1, 8)
      S(5, 10) = S(2, 4)
      S(7, 11) = S(5, 10)
      S(1, 6) = ((X*U)/24.-(11.*Y**2)/(210.*X)-(RL*X)/40.)*R
      S(3, 4) = S(1, 6)
      S(7, 12) = -S(1, 6)
      S(9, 10) = S(7, 12)
```

```
      S(1, 7) = ((9.*Y)/(70.*X)-(2.*RL*X)/(5.*Y))*R                     STED0370
      S(4, 10) = S(1, 7)                                                STED0380
      S(1, 9) = ((U*X)/24.-(13.*Y**2)/(420.*X)+(3.*RL*X)/40.)*R         STED0390
      S(4, 12) = S(1, 9)                                                STED0400
      S(3, 7) = -S(1, 9)                                                STED0410
      S(6, 10) = S(3, 7)                                                STED0420
      S(1, 10) = -((9.*Y)/(70.*X)+(RL*X)/(5.*Y))*R                      STED0430
      S(4, 7) = S(1, 10)                                                STED0440
      S(1, 12) = ((13.*Y**2)/(420.*X)-(U*X)/24.-RL*X/40.)*R             STED0450
      S(4, 9) = S(1, 12)                                                STED0460
      S(3, 10) = -S(1, 12)                                              STED0470
      S(6, 7) = -S(1, 12)                                               STED0480
      DO 20 I = 2, 11, 3                                                STED0490
   20 S(I, I) = ((13.*X)/(35.*Y)+(2.*RL*Y)/(5.*X))*R                    STED0500
      S(2, 3) = ((11.*X**2)/(210.*Y)-U*Y/24.+(3.*RL*Y)/40.)*R           STED0510
      S(8, 9) = S(2, 3)                                                 STED0520
      S(5, 6) = -S(2, 3)                                                STED0530
      S(11, 12) = -S(2, 3)                                              STED0540
      S(2, 5) = ((9.*X)/(70.*Y)-(2.*RL*Y)/(5.*X))*R                     STED0550
      S(8, 11) = S(2, 5)                                                STED0560
      S(2, 6) = ((Y*U)/24.-(13.*X**2)/(420.*Y)+(3.*RL*Y)/40.)*R         STED0570
      S(8, 12) = S(2, 6)                                                STED0580
      S(3, 5) = -S(2, 6)                                                STED0590
      S(9, 11) = -S(2, 6)                                               STED0600
      S(2, 8) = ((RL*Y)/(5.*X)-(13.*X)/(35.*Y))*R                       STED0610
      S(5, 11) = S(2, 8)                                                STED0620
      S(2, 9) = ((U*Y)/24.-(11.*X**2)/(210.*Y)-RL*Y/40.)*R              STED0630
      S(3, 8) = S(2, 9)                                                 STED0640
      S(5, 12) = -S(2, 9)                                               STED0650
      S(6, 11) = S(5, 12)                                               STED0660
      S(2, 11) = -((9.*X)/(70.*Y)+(RL*Y)/(5.*X))*R                      STED0670
      S(5, 8) = S(2, 11)                                                STED0680
      S(2, 12) = ((13.*X**2)/(420.*Y)-(U*Y)/24.-(RL*Y)/40.)*R           STED0690
      S(6, 8) = S(2, 12)                                                STED0700
      S(3, 11) = -S(2, 12)                                              STED0710
```

```
      S(5, 9) = -S(2, 12)                                               STED0720
      DO 30 I = 3, 12, 3                                                STED0730
   30 S(I, I) = (Y**3/(105.*X)+X**3/(105.*Y)-U*X*Y/72.+3.*RL*X*Y/40.)*R STED0740
      S(3, 6) = ((RL*X*Y)/40.+(U*X*Y)/72.-(X**3)/(140.*Y)-Y**3/(105.*X))STED0750
     &*R                                                                STED0760
      S(9, 12) = S(3, 6)                                                STED0770
      S(3, 9) = (RL*X*Y/40.+U*X*Y/72.-X**3/(105.*Y)-Y**3/(140.*X))*R    STED0780
      S(6, 12) = S(3, 9)                                                STED0790
      S(3, 12) = (Y**3/(140.*X)+X**3/(140.*Y)-U*X*Y/72.-RL*X*Y/40.)*R   STED0800
      S(6, 9) = S(3, 12)                                                STED0810
      DO 40 II = 1, 12                                                  STED0820
      LL = II                                                           STED0830
      DO 40 JJ = LL, 12                                                 STED0840
   40 S(JJ, II) = S(II, JJ)                                             STED0850
      RETURN                                                            STED0860
      END                                                               STED0870

      SUBROUTINE STRTR(X, Y, T, E, U, S)                                STRT0010
      IMPLICIT REAL*8(A-H, O-Z), INTEGER(I-N)                           STRT0020
C     ******************************************************************STRT0030
C     *    GENERATES STIFFNESS MATRIX FOR A RIGHT TRANSITION ELEMENT   *STRT0040
C     ******************************************************************STRT0050
      DIMENSION S(12, 12)                                               STRT0060
      RL = (1.-U)/2.                                                    STRT0070
      R = (E*T)/(1.-U**2)                                               STRT0080
      S(1, 1) = (Y/(3.*X)+RL*X/(3.*Y))*R                                STPT0090
      S(6, 6) = S(1, 1)                                                 STRT0100
      S(1, 2) = (-(RL+U)/4.)*R                                          STRT0110
      S(3, 7) = S(1, 2)                                                 STRT0120
      S(4, 6) = S(1, 2)                                                 STRT0130
      S(8, 9) = S(1, 2)                                                 STRT0140
      S(1, 9) = -S(1, 2)                                                STRT0150
      S(2, 8) = S(1, 9)                                                 STRT0160
      S(3, 4) = S(2, 8)                                                 STRT0170
      S(6, 7) = S(3, 4)                                                 STRT0180
      S(1, 3) = (RL*X/(6.*Y)-7.*Y/(20.*X))*R                            STRT0190
      S(6, 8) = S(1, 3)                                                 STRT0200
```

```
S(1, 4) = ((RL-U)/4.)*R                                  STRT0210
S(2, 6) = S(1, 4)                                        STRT0220
S(3, 9) = S(1, 4)                                        STRT0230
S(7, 8) = S(1, 4)                                        STRT0240
S(1, 7) = -S(1, 4)                                       STRT0250
S(2, 3) = S(1, 7)                                        STRT0260
S(4, 8) = S(2, 3)                                        STRT0270
S(6, 9) = S(4, 8)                                        STRT0280
S(1, 5) = (-(Y**2)/(20.*X))*R                            STRT0290
S(6, 10) = -S(1, 5)                                      STRT0300
S(1, 6) = (Y/(6.*X)-RL*X/(3.*Y))*R                       STRT0310
S(1, 8) = (-(3.*Y)/(20.*X)-(RL*X)/(6.*Y))*R              STRT0320
S(1, 10) = ((Y**2)/(30.*X))*R                            STRT0330
S(5, 6) = -S(1, 10)                                      STRT0340
S(2, 2) = (X/(3.*Y)+RL*Y/(3.*X))*R                       STRT0350
S(4, 4) = S(2, 2)                                        STRT0360
S(7, 7) = S(2, 2)                                        STRT0370
S(9, 9) = S(2, 2)                                        STRT0380
S(2, 4) = (X/(6.*Y)-RL*Y/(3.*X))*R                       STRT0390
S(7, 9) = S(2, 4)                                        STRT0400
S(2, 5) = (((RL+U)*Y)/24.)*R                             STRT0410
S(7, 10) = S(2, 5)                                       STRT0420
S(2, 10) = -S(2, 5)                                      STRT0430
S(5, 7) = S(2, 10)                                       STRT0440
S(2, 7) = (-(X/(3.*Y)-RL*Y/(6.*X)))*R                    STRT0450
S(4, 9) = S(2, 7)                                        STRT0460
S(2, 9) = (-(X/(6.*Y)+RL*Y/(6.*X)))*R                    STRT0470
S(4, 7) = S(2, 9)                                        STRT0480
S(3, 3) = (13.*Y/(35.*X)+2.*RL*X/(5.*Y))*R               STRT0490
S(8, 8) = S(3, 3)                                        STRT0500
S(3, 5) = (11.*(Y**2)/(210.*X)+RL*X/30.)*R               STRT0510
S(8, 10) = -S(3, 5)                                      STRT0520
S(3, 6) = (-3.*Y/(20.*X)-RL*X/(6.*Y))*R                  STRT0530
S(3, 8) = ((9.*Y)/(70.*X)-2.*RL*X/(5.*Y))*R              STRT0540
S(3, 10) = (-(13.*Y**2)/(420.*X)+RL*X/30.)*R             STRT0550
S(5, 8) = -S(3, 10)                                      STRT0560
```

```
      S(4, 5) = (Y*(U-RL)/24.)*R
      S(9, 10) = S(4, 5)
      S(4, 10) = -S(4, 5)
      S(5, 9) = S(4, 10)
      S(5, 5) = (Y**3/(105.*X)+2.*RL*X*Y/45.)*R
      S(10, 10) = S(5, 5)
      S(5, 10) = (-(Y**3/(140.*X)+RL*X*Y/90.))*R
      DO 10 II = 1, 10
      LL = II
      DO 10 JJ = LL, 10
   10 S(JJ, II) = S(II, JJ)
      RETURN
      END

      SUBROUTINE STLTR(X, Y, T, E, U, S)
      IMPLICIT REAL*8(A-H, O-Z), INTEGER(I-N)
C     ******************************************************************
C     *    GENERATES STIFFNESS MATRIX FOR A LEFT TRANSITION ELEMENT    *
C     ******************************************************************
      DIMENSION S(12, 12)
      RL = (1.-U)/2.
      R = (E*T)/(1.-U**2)
      S(1, 1) = ((13.*Y)/(35.*X)+(2.*RL*X)/(5.*Y))*R
      S(6, 6) = S(1, 1)
      S(1, 2) = -((RL+U)/4.)*R
      S(4, 7) = S(1, 2)
      S(1, 10) = -S(1, 2)
      S(2, 9) = S(1, 10)
      S(4, 5) = S(2, 9)
      S(6, 7) = S(4, 5)
      S(5, 6) = S(1, 2)
      S(9, 10) = S(1, 2)
      S(1, 3) = ((11.*Y**2)/(210.*X)+(RL*X)/30.)*R
      S(6, 8) = -S(1, 3)
      S(1, 4) = ((RL*X)/(6.*Y)-(7.*Y)/(20.*X))*R
      S(6, 9) = S(1, 4)
      S(1, 5) = ((RL-U)/4.)*R
```

```
      S(2, 6) = S(1, 5)                                                 STLT0240
      S(4, 10) = S(1, 5)                                                STLT0250
      S(7, 9) = S(1, 5)                                                 STLT0260
      S(1, 7) = -S(1, 5)                                                STLT0270
      S(2, 4) = S(1, 7)                                                 STLT0280
      S(5, 9) = S(2, 4)                                                 STLT0290
      S(6, 10) = S(5, 9)                                                STLT0300
      S(1, 6) = ((9.*Y)/(70.*X)-(2.*RL*X)/(5.*Y))*R                     STLT0310
      S(1, 8) = (-13.*Y**2/(420.*X)+(RL*X)/30.)*R                       STLT0320
      S(3, 6) = -S(1, 8)                                                STLT0330
      S(1, 9) = (-(RL*X/(6.*Y)+3.*Y/(20.*X)))*R                         STLT0340
      S(2, 2) = (X/(3.*Y)+RL*Y/(3.*X))*R                                STLT0350
      S(5, 5) = S(2, 2)                                                 STLT0360
      S(7, 7) = S(2, 2)                                                 STLT0370
      S(10, 10) = S(2, 2)                                               STLT0380
      S(2, 3) = ((RL-U)*Y/24.)*R                                        STLT0390
      S(7, 8) = S(2, 3)                                                 STLT0400
      S(2, 8) = -S(2, 3)                                                STLT0410
      S(3, 7) = S(2, 8)                                                 STLT0420
      S(2, 5) = (X/(6.*Y)-(RL*Y)/(3.*X))*R                              STLT0430
      S(7, 10) = S(2, 5)                                                STLT0440
      S(2, 7) = ((RL*Y)/(6.*X)-X/(3.*Y))*R                              STLT0450
      S(5, 10) = S(2, 7)                                                STLT0460
      S(2, 10) = (-X/(6.*Y)-(RL*Y)/(6.*X))*R                            STLT0470
      S(5, 7) = S(2, 10)                                                STLT0480
      S(3, 3) = (Y**3/(105.*X)+(2.*RL*X*Y)/45.)*R                       STLT0490
      S(8, 8) = S(3, 3)                                                 STLT0500
      S(3, 4) = (-Y**2/(20.*X))*R                                       STLT0510
      S(8, 9) = -S(3, 4)                                                STLT0520
      S(3, 5) = (-((RL+U)*Y/24.))*R                                     STLT0530
      S(8, 10) = S(3, 5)                                                STLT0540
      S(3, 10) = -S(3, 5)                                               STLT0550
      S(5, 8) = S(3, 10)                                                STLT0560
      S(3, 8) = (-(Y**3/(140.*X)+RL*X*Y/90.))*R                         STLT0570
      S(3, 9) = -(Y**2/(30.*X))*R                                       STLT0580
      S(4, 8) = -S(3, 9)                                                STLT0590
```

```
      S(4, 4) = (Y/(3.*X)+(RL*X)/(3.*Y))*R                              STLT0600
      S(9, 9) = S(4, 4)                                                 STLT0610
      S(4, 6) = (-(3.*Y/(20.*X)+RL*X/(6.*Y)))*R                         STLT0620
      S(4, 9) = (Y/(6.*X)-RL*X/(3.*Y))*R                                STLT0630
      DO 10 II = 1, 10                                                  STLT0640
      LL = II                                                           STLT0650
      DO 10 JJ = LL, 10                                                 STLT0660
   10 S(JJ, II) = S(II, JJ)                                             STLT0670
      RETURN                                                            STLT0680
      END                                                               STLT0690

      SUBROUTINE STINT(X, Y, T, E, U, S)                                STIN0010
      IMPLICIT REAL*8(A-H, O-Z), INTEGER(I-N)                           STIN0020
C     ******************************************************************STIN0030
C     *       GENERATES STIFFNESS MATRIX FOR AN INTERIOR ELEMENT       *STIN0040
C     ******************************************************************STIN0050
      DIMENSION S(12, 12)                                               STIN0060
      RL = (1.-U)/2.                                                    STIN0070
      R = E*T/(1.-U**2)                                                 STIN0080
      DO 10 I = 1, 7, 2                                                 STIN0090
   10 S(I, I) = (Y/(3.*X)+RL*X/(3.*Y))*R                                STIN0100
      S(1, 2) = ((-RL-U)/4.)*R                                          STIN0110
      S(3, 6) = S(1, 2)                                                 STIN0120
      S(4, 5) = S(1, 2)                                                 STIN0130
      S(7, 8) = S(1, 2)                                                 STIN0140
      S(1, 3) = (-Y/(3.*X)+RL*X/(6.*Y))*R                               STIN0150
      S(5, 7) = S(1, 3)                                                 STIN0160
      S(1, 8) = -S(1, 2)                                                STIN0170
      S(2, 7) = S(1, 8)                                                 STIN0180
      S(3, 4) = S(2, 7)                                                 STIN0190
      S(5, 6) = S(3, 4)                                                 STIN0200
      S(1, 4) = ((RL-U)/4.)*R                                           STIN0210
      S(2, 5) = S(1, 4)                                                 STIN0220
      S(3, 8) = S(1, 4)                                                 STIN0230
      S(6, 7) = S(1, 4)                                                 STIN0240
      S(1, 6) = -S(1, 4)                                                STIN0250
      S(2, 3) = S(1, 6)                                                 STIN0260
```

```
      S(4, 7) = S(2, 3)                                                 STIN0270
      S(5, 8) = S(4, 7)                                                 STIN0280
      S(1, 5) = (Y/(6.*X)-RL*X/(Y*3.))*R                                STIN0290
      S(3, 7) = S(1, 5)                                                 STIN0300
      S(1, 7) = (-(Y/(6.*X)+RL*X/(6.*Y)))*R                             STIN0310
      S(3, 5) = S(1, 7)                                                 STIN0320
      DO 20 I = 2, 8, 2                                                 STIN0330
   20 S(I, I) = (X/(3.*Y)+RL*Y/(3.*X))*R                                STIN0340
      S(2, 4) = (X/(6.*Y)-RL*Y/(3.*X))*R                                STIN0350
      S(6, 8) = S(2, 4)                                                 STIN0360
      S(2, 6) = (-X/(3.*Y)+RL*Y/(6.*X))*R                               STIN0370
      S(2, 8) = (-X/(6.*Y)-RL*Y/(6.*X))*R                               STIN0380
      S(4, 8) = S(2, 6)                                                 STIN0390
      S(4, 6) = S(2, 8)                                                 STIN0400
      DO 30 II = 1, 8                                                   STIN0410
      LL = II                                                           STIN0420
      DO 30 JJ = LL, 8                                                  STIN0430
   30 S(JJ, II) = S(II, JJ)                                             STIN0440
      RETURN                                                            STIN0450
      END                                                               STIN0460

      SUBROUTINE STBEAM(E, CA, BI, CL, DLL, DLR, S)                     STBE0010
      IMPLICIT REAL*8(A-H, O-Z), INTEGER(I-N)                           STBE0020
C     ******************************************************************STBE0030
C     *               GENERATES STIFFNESS MATRIX FOR A BEAM            *STBE0040
C     ******************************************************************STBE0050
      DIMENSION S(12, 12)                                               STBE0060
      BEETA = 12.*E*BI/(CL**3)                                          STBE0070
      S(1, 1) = BEETA                                                   STBE0080
      S(3, 3) = S(1, 1)                                                 STBE0090
      S(1, 2) = BEETA*(DLL+CL/2.)                                       STBE0100
      S(1, 3) = -S(1, 1)                                                STBE0110
      S(1, 4) = BEETA*(DLR+CL/2.)                                       STBE0120
      S(2, 2) = BEETA*DLL*(DLL+CL)+4.*E*BI/CL                           STBE0130
      S(2, 3) = -BEETA*(DLL+CL/2.)                                      STBE0140
      S(2, 4) = DLL*DLR*BEETA+BEETA*CL*(DLL+DLR)/2.+2.*E*BI/CL          STBE0150
      S(3, 4) = -BEETA*(DLR+CL/2.)                                      STBE0160
```

```
      S(4, 4) = BEETA*DLR*(DLR+CL)+4.*E*BI/CL                          STBE0170
      DO 10 I = 1, 4                                                   STBE0180
      L = I                                                            STBE0190
      DO 10 J = L, 4                                                   STBE0200
   10 S(J, I) = S(I, J)                                                STBE0210
      RETURN                                                           STBE0220
      END                                                              STBE0230

      SUBROUTINE STCOLM(E, CA, CI, CL, DLT, DLB, S)                    STCO0010
      IMPLICIT REAL*8(A-H, O-Z), INTEGER(I-N)                          STCO0020
C     *****************************************************************STCO0030
C     *            GENERATES STIFFNESS MATRIX FOR A COLUMN            *STCO0040
C     *****************************************************************STCO0050
      DIMENSION S(12, 12)                                              STCO0060
      BEETA = 12.*E*CI/CL**3                                           STCO0070
      S(1, 1) = BEETA                                                  STCO0080
      S(4, 4) = S(1, 1)                                                STCO0090
      S(1, 2) = 0.                                                     STCO0100
      S(1, 3) = -BEETA*(DLB+CL/2.)                                     STCO0110
      S(1, 4) = -S(1, 1)                                               STCO0120
      S(1, 5) = 0.                                                     STCO0130
      S(1, 6) = -BEETA*(DLT+CL/2.)                                     STCO0140
      S(2, 2) = (CA*E)/CL                                              STCO0150
      S(2, 3) = 0.                                                     STCO0160
      S(2, 4) = 0.                                                     STCO0170
      S(2, 5) = -S(2, 2)                                               STCO0180
      S(2, 6) = 0.                                                     STCO0190
      S(3, 3) = BEETA*DLB*(DLB+CL)+(4.*E*CI/CL)                        STCO0200
      S(3, 4) = BEETA*(DLB+CL/2.)                                      STCO0210
      S(3, 5) = 0.                                                     STCO0220
      S(3, 6) = BEETA*(DLT*DLB+DLB*CL/2.+DLT*CL/2.)+2.*E*CI/CL         STCO0230
      S(4, 5) = 0.                                                     STCO0240
      S(4, 6) = BEETA*(DLT+CL/2.)                                      STCO0250
      S(5, 5) = S(2, 2)                                                STCO0260
      S(5, 6) = 0.                                                     STCO0270
      S(6, 6) = BEETA*DLT*(DLT+CL)+4.*E*CI/CL                          STCO0280
      DO 10 I = 1, 6                                                   STCO0290
```

```
      L = I                                                             STCO0300
      DO 10 J = L, 6                                                    STCO0310
   10 S(J, I) = S(I, J)                                                 STCO0320
      RETURN                                                            STCO0330
      END                                                               STCO0340
```