THE EFFECT OF DISLOCATIONS INTRODUCED BY PLASTIC BENDING ON THE ENERGY RESOLUTION OF THALLIUM ACTIVATED SODIUM IODIDE SCINTILLATING PHOSPHORS

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ABSTRACT

It was found that when a thallium activated sodium iodide

phosphor was stressed the scintillation energy resolution increased

by a small amount. In one case, when the number of dislocations

was increased by an estimated 2 x 10^6 per square centimeter the

resolution was increased by 0.8%.

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I INTRODUCTION

The scintillation counter is now widely used for the detection of X- and gamma- radiation, and of energetic charged particles, such as alpha and beta particles. The scintillation counter consists of a phosphor, which emits a pulse of light when it absorbs energy from some form of radiation, and of a photomultiplier tube, which converts the light pulse to a pulse of electrons by means of the photoelectric effect and amplifies these pulses of electrons several times by means of the process of secondary electron emission, together with any associated electronic equipment used to further amplify the pulses, or to determine the pulse height distribution.

When a stream of monoenergetic radiation is incident on the phosphor and the energy of a quantum of electromagnetic radiation, or of a particle, is completely absorbed in the phosphor, the resulting pulse contributes to the photopeak which occurs in the pulse height distribution curve. The position of the photopeak is dependent on the energy of the incident particles or quanta. With gamma-rays, the pulse height corresponding to the centre of photopeak is very nearly proportional to the energy of the gamma-ray quanta.

It has been realized for some time that the detection of monoenergetic particles or quanta by a scintillation counter gives rise to voltage pulses in the photopeak which differ somewhat in size. The statistical nature of the processes involved contributes to this variance in pulse height as do imperfections in the component parts of the counter. Several papers have appeared which treat only the variance that arises out of one or other of the detection processes. Both Breitenberger ⁽¹⁾ and Wright ⁽²⁾ have published more comprehensive articles.

Defining photon transfer as a cascade of four steps, the escape of photons from the phosphor, the transmission of photons to the photocathode (possibly through a light guide), the emission of photoelectrons, and the collection of photoelectrons at the first dynode, Breitenberger discussed in detail the statistical nature of the processes of photon transfer, light collection, light "piping", and secondary electron emission, and touched on the statistical importance of the emission of after-electrons and satellite pulses, photocathode inhomogeneities, phosphor inhomogeneities, edge effects in the phosphor, the possibility of non-normal scintillation variance, amplifier and power supply instabilites, high counting rates, and the value of the collecting time constant at the anode of the photomultiplier tube.

Assuming a Gaussian distribution of pulses in the photopeak, the analysis of Wright leads to the result:

 $\frac{R^2}{5.56 \times 10^4} = \frac{\bar{p}^2(s_n^2 - \bar{n})}{(\bar{n}\bar{p} + \bar{m})^2} + \frac{s_p^2(s_n^2 + \bar{n}^2)}{(\bar{n}\bar{p} + \bar{m})^2} + \frac{1}{(\bar{n}\bar{p} + \bar{m})^$

where R = the resolution expressed as a percent (fractional full-width-athalf-maximum),

- s = mean gain of the second and ensuing dynode stages in the photomultiplier tube,
- \overline{m} = mean number of thermal electrons emitted from the photocathode during a scintillation pulse.

K = number of scintillations counted,

 $n_i = number of photons in the ith scintillation,$

P_{ij} = probability that the jth photon of the ith scintillation produces a photoelectron which reaches the first dynode,

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$$\bar{p}_{i} = \sum_{i=i}^{m_{i}} p_{ij}/n_{i}$$

$$\bar{p} = \sum_{i=i}^{k} \bar{p}_{i}/K$$

$$s_{p}^{2} = \sum_{i=i}^{k} \frac{(\bar{p}_{i} - \bar{p})^{2}}{K} ,$$

$$\bar{n} = \sum_{i=i}^{k} \frac{n_{i}}{K} ,$$

$$s_{n}^{2} = \sum_{i=i}^{k} \frac{(\bar{n}-n_{i})^{2}}{K} ,$$

r.

r =

= the number of secondary electrons produced at the first dynode by the ith incident photoelectron. Hence ri is a function of the point of incidence of the photoelectron. (The first dynode is assumed to be the only one that is non-uniform.) Also,

$$\vec{r} = \sum_{i=1}^{K} \frac{r_i}{K}$$
, and
 $S_r^2 = \sum_{i=1}^{K} \frac{(r_i - \bar{r})^2}{K}$. Wright has, in this expression, made the

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assumption that the variance of the tube gain is normal, an assumption defended by himself and questioned by Breitenberger.

It is often assumed that the scintillation variance, S_n^2 , is normal, i.e. that $S_n^2 = n$, in which case the first term vanishes. Seitz⁽³⁾ has shown that two thallium ions are involved in the emission of one quantum of light from NaI(T1). Breitenberger (1) has pointed out that if a similar interaction in the process of excitation occurs, then excitation of one

luminescence centre may affect the excitation of a neighbouring centre in which case the scintillation variance may be non-normal.

The transfer variance, S_p^2 , and hence the second term, vanishes if the optical geometry is the same for each scintillation, that is if $\bar{p} = p_i$, all i.

The third term, which often predominates, is due largely to the statistical nature of secondary electron emission. In the case where $S_r^2 = 0$, $\bar{r} = \bar{s}$, and \bar{m} is much smaller than $(\bar{n} \ \bar{p})$, this term becomes simply:

$$\frac{1}{\overline{n} \overline{p}} \left(\frac{\overline{s}}{\overline{s}-1} \right) \cdot$$

When \bar{m} is negligible, and S_n^2 is much less than \bar{n}^2 , we have

$$\frac{R^2}{5.56 \times 10^{44}} = \frac{S_n^2 - \bar{n}}{\bar{n}^2} + \frac{S_p^2}{\bar{p}^2} + \frac{1}{\bar{n}\bar{p}} \left(1 + \frac{\bar{s}}{\bar{r}(\bar{s}-1)} + \frac{S_r^2}{\bar{r}^2}\right) .$$

The dependence of R^2 on \bar{n} and \bar{p} is usually investigated by plotting R^2 versus $(\bar{n} \ \bar{p})^{-1}$ or, equivalently, versus the reciprocal of the pulse height, since the mean photopeak pulse height is proportional to $(\bar{n} \ \bar{p})$. In such experiments \bar{p} might be altered by the insertion of neutral density filters in front of the photocathode of the photomultiplier tube, and \bar{n} is varied by changing the energy of the radiation incident on the phosphor. In the case of gamma-ray scintillation spectroscopy, \bar{n} is very nearly proportional to the gamma-ray energy as mentioned above.

II A CONTRIBUTION TO RESOLUTION INTRINSIC TO THE SCINTILLATING PHOSPHOR

Wright and Garlick ⁽⁴⁾ investigated extensively the variance in the pulse heights from scintillation counters. Replacing the scintillating phosphor with a light flasher and using neutral density filters to vary \bar{p} , it was found that R^2 was in fact proportional (1/ \bar{p}) as expected if S_p^2 is assumed to be zero in this case.

Next, using various phosphors irradiated by 5 Mev. alpha particles, \bar{p} was again varied by means of neutral density filters. In the case of each crystal used the dependence of R on \bar{p} could be fitted to $R^2 = R_0^2 + a/\bar{p}$, where R_0 and are constants for a given crystal and R_0 is regarded as a contribution to resolution intrinsic to the phosphor.

For blocks of thallium-activated potassium iodide, diamond, and calcium tungstate the value of R_0 ranged from 12% to 26% while for a lamina of anthracene it was zero. The slope of the plot of R^2 versus $(1/\bar{p})$ was different from that with the light flasher for all phosphors but anthracene.

The dependence of R^2 on \bar{n} was investigated in crystals of calcium fluoride with ground and with freshly cleaved surfaces by measuring the resolution corresponding to different alpha particle energies. The usual plot of R^2 versus $(1/\bar{n})$ yielded a straight line through the origin for the freshly cleaved crystal and a **straight** line with a positive R^2 -intercept and a greater slope for the other crystal.

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Wright and Garlick also determined the dependence of R_0 on energy in anthracene and in diamond. It was found that in anthracene R_0 was zero at high energies while at low energies R_0 was proportional to (1/n). In the diamond it was found that R_0^2 was proportional to (1/n).

Suspecting that the high density of ionization of alpha particles might have a peculiar effect on the resolution, the pulse height variance of fluorescent pulses induced by ultra-violet light was then investigated. Light pulses from an argon lamp in a sawtooth oscillator circuit were filtered by a piece of Wood's glass which passed light between the wave lengths 3500 to 4000 Angstrom units. These light pulses caused fluorescence pulses in the phosphor which passed to the photomultiplier tube through a filter which removed ultra-violet light. Using anthracene in this arrangement R_0 was zero, while in the diamond the value of R_0 was reduced. Wright and Garlick predicted that anthracene irradiated by beta particles would have R_0 equal to zero.

Wright and Garlick also investigated, in samples of both anthracene and diamond, the effect on R_0 of a prolonged flux of alpha particles. While no effect was observed in diamond it was found in anthracene that the luminescence efficiency decreased and that the dependence of R_0 on $(1/\bar{n})_{Was}$ similar to that already described except that the energy at which R_0 departed from zero became lower and lower as the radiation damage increased.

G. G. Kelly, P. R. Bell, R. C. Davis, and N. H. Lazar⁽⁵⁾ investigated the resolution of a scintillation counter using thallium-activated sodium iodide crystals to detect gamma rays. This group used a pulsed cathode ray

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tube (RCA type C73687A) to simulate a phosphor. Assuming that the resolution obtained with the flasher represented the contribution of the photomultiplier tube to the total variance, it was possible to calculate the additional variance caused by the use of the scintillating phosphor. It was assumed that processes in the crystal and phosphor were gaussian so that $R^2 = R_c^2 + R_t^2$, where R_t is the resolution obtained with the light flasher, and R_c is the contribution to the resolution due to the phosphor.

It was found that when the same crystal was used with different photomultiplier tubes the calculated values of R_c were the same.

When the light guide was altered between the crystal or light flasher, and the photomultiplier tube, it was found that again the value of R_c was essentially unchanged.

The value of R_c , the "intrinsic resolution", was unchanged when the light from the crystal was semicollimated or diffused onto the photocathode, when the crystal and light flasher were used simultaneously, when the gamma ray source was collimated, and when the crystal was broken into successively smaller pieces.

The dependence of R_c on the gamma ray energy was also investigated by this group for energies from 61 kev. to 1850 kev. It was found that R_c was approximately inversely proportional to the fourth root of the energy.

Bernstein, as reported by Cooper and Morton⁽⁶⁾, investigated the dependence of resolution on the energy of gamma rays incident on anthracene and NaI(T1) phosphors. Plotting R^2 versus the reciprocal of the energy,

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Bernstein obtained a straight line at low energies intersecting another of greater slope at high energies. For the crystals used the R^2 intercept of the low energy line indicates an "intrinsic resolution" in thallium activated sodium iodide of from 4 to 5 per cent and in anthracene of 10 per cent.

Breitenberger⁽¹⁾has analyzed some data by Hopkins⁽⁷⁾ regarding the energy dependence of the resolution of a scintillation counter with an anthracene phosphor irradiated by monoenergetic electrons. A plot of \mathbb{R}^2 versus the reciprocal of the energy results in a straight line with an \mathbb{R}^2 -intercept corresponding to a resolution of 9.1 per cent.

Wright and Garlick suggest that in the diamond the intrinsic resolution was due to the fact that the diamond was composed of domains some of which were luminescent and some of which were not. This situation would affect the scintillation variance, S_n^2 , and, therefore, the resolution.

For the anthracene phosphor Wright and Garlick suggested a different explanation. Near the end of its path in anthracene the slowed alpha particle produces knock-on protons, the luminescence efficiency of which differs from that of alpha particles. The result is an increase in the scintillation variance. This explanation gives no reason for the observed effect of radiation damage on the intrinsic resolution of anthracene.

No explanation is offered by Wright and Garlick for the existence of an intrinsic resolution in other phosphors.

Breitenberger⁽¹⁾ has mentioned that the value of the relative transfer variance, (S_p^2/p^2) , may not be negligible and that the optical geometry may

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be responsible for a large fraction of the intrinsic resolution. For a transparent, optically perfect phosphor which is optically matched to a homogeneous photocathode, the relative transfer variance is in part due to the relative variance of the probability that a photon emitted in the phosphor will escape from the phosphor. If the sides and front of the phosphor are regarded as mirrors of isotropic reflectivity g, and the relative variance in the mean number of reflections necessary for the escape of photons emitted throughout the phosphor is V_n , then, according to Breitenberger, the relative variance in the escape probability is $(1 - g)^2 V_r$. For cubes and short cylinders V_r is approximately 0.15 according to Breitenberger. In this case, then, the contribution to the resolution is about 91(1 - g) per cent. The above-mentioned 9.1 per cent intrinsic resolution at infinite energy found by Breitenberger in the data of Hopkins can be explained as being due to the variance in the probability of excape of photons from a phosphor whose sides and front have a reflectivity of 0.90. It is to be noticed that the existence of an intrinsic resolution with an anthracene crystal irradiated by electrons is not in agreement with the prediction of Wright and Garlick $^{(4)}$.

In the experiments of Wright and Garlick on ground and freshly cleaved crystals of calcium fluoride it may well be that an alteration of the variance of the photon escape probability produced the observed effects. Further, the unique results obtained with anthracene when various phosphors were irradiated by 5 Mev. alpha particles and the light was attenuated by neutral density filters, may have been due to the fact that the anthracene phosphor, alone, was in the shape of a lamina resulting in a reduced, or negligible, value of the variance of the photon escape probability.

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This contribution to the photon transfer variance is probably included in the intrinsic resolution of NaI(Tl) noticed in the experiments of Kelly et al.

In impurity activated phosphors such as NaI(T1) any inhomogeneity in the concentration of the activating ions would result in a resolution intrinsic to the phosphor. Gross inhomogeneity of the thallium concentration would have been detected in the experiments of Kelly et al when the crystal was repeatedly reduced in size until the sample was about the size of a matchhead.

If the gamma ray scintillation response of NaI(Tl) were nonlinear then the amount of light emitted following an interaction of one event could differ from the amount emitted following an interaction of several events. In other words, nonlinear response would result in an increased value of the scintillation variance. This explanation of the intrinsic resolution of NaI(Tl) is not supported by the above experiment since it is unlikely an interaction of several events would occur in the smallest piece of crystal used.

In spite of this result and the fact that previously there had been some controversy as to whether or not NaI(T1) was nonlinear, Zerby, Meyer, and Murray⁽⁸⁾ have recently given a paper in which a nonlinearity of NaI(T1) was assumed and by means of a Monte Carlo calculation have calculated a reasonable value of the intrinsic resolution.

Breitenberger⁽¹⁾ has reported an interesting result relating to crystals of anthracene grown from solution and from the melt of the same extremely pure

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material. The crystals grown from the solution were found to have a luminescence efficiency thirty per cent greater than that of the crystals grown from the melt. Breitenberger assumed that defects in the crystals grown from the melt acted as quenching traps thus reducing the light output. The degree of perfection of a crystal can vary throughout its volume causing an increase in the scintillation variance. The damage to an anthracene crystal caused by a flux of alpha particles could therefore cause a change in the intrinsic resolution of the crystal as observed by Wright and Garlick.

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FIG.I BLOCK DIAGRAM OF SCINTILLATION COUNTER



FIG.2 ARRANGEMENT FOR STRESSING THE CRYSTAL

III THE PROJECT

It was supposed that perhaps some of the intrinsic resolution of thallium activated sodium iodide was due to lattice defects and an experiment was undertaken which would detect a change in resolution of a scintillation counter if any occurred when the phosphor was stressed in such a way as to produce dislocations within the crystal.

The apparatus used consisted of a scintillation counter with provision for stressing the crystal. A cesium- 137 source of 662 kev. gamma rays was placed near the 1.5 centimeter square Harshaw crystal under investigation. The thickness of the crystals varied from one to four millimeters. Light from the crystals was detected by a Dumont 6292 photomultiplier tube whose dynode potentials were maintained by a Northeast model RE-10010 high voltage power supply. The pulse output from the photomultiplier tube was fed to a cathode follower whose plate supply was obtained from a Lambda model C281M power supply. The output from the cathode follower was amplified by a Nuclear Enterprise model NE5202 pulse amplifier whose output was connected by a terminated coaxial cable to Computing Devices of Canada model AEP2230 hundredchannel pulse height analyser in another laboratory.

The apparatus used for stressing the crystal is sketched in figure 2. The pitch of the screw depressing the so-called "knife-edge" was one-fortieth of an inch and the top of the screw was marked at twenty regularly spaced intervals so that it was possible to move the screw downward by an amount as small as 3.18×10^{-3} centimeters. The radius of curvature of the 1.5 centimeter square crystals used was approximately ($\frac{0.283}{a}$), where a is the depression of the centre of the crystal in centimeters relative to the edges. It is suspected

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that during stressing the edges of the crystal were also forced downward so that the depression, a, may have been less than the downward displacement of the screw.

Dekker⁽⁹⁾ has shown that the dislocation density, q, introduced when a crystal is plastically bent into a radius of curvature of r is $q = \left(\frac{1}{r v}\right)$, where v is the magnitude of the Burger's vestor. In this experiment, then, $q = \frac{a}{0.283 v}$.

The resolution in each trial was determined by the method outlined in Appendix A using a computer. The computer programme is given in Appendix B.

IV EXPERIMENTAL RESULTS

The earliest measurements showed that any change in energy resolution with the dislocations introduced by plastic bending was small. It was therefore necessary that the electronic apparatus be of high stability. Where a choice of amplifiers and power supplies was possible the units used were the ones having the higher or highest stability. Instability caused by rather sudden and large temperature changes of the laboratory was quite troublesome in the earlier experiments.

One of the first experiments showing any effect at all was that labelled Run A the results of which are presented in Table A and in Graph A. At each value of the strain the resolution was measured several times resulting in a mean value of the resolution, \overline{R} , and in a mean deviation from the mean of ΔR for that strain. The variation in \overline{R} is, in the case of Run A, not much larger than ΔR and is not monotonic as the strain increases.

Run B (see Table B and Graph B) consists of resolution measurements made at two values of strain. The strain was increased rapidly in this experiment in order to reduce the effect of instabilities. The results of this run were encouraging. The crystal broke when the strain was increased further.

Run C (see Table C and Graph C) was done over a period of several days. The strain was only increased once a day so that a relatively large number of determinations of the resolution at **each** strain was made. The temperature changed throughout some of these days and also from day to day during this run. In the case where for two consecutive days the strain was the same, the agreement was not good.

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Through the kindness of Dr. F. M. Kelly and Mr. E. Tomchuk it was possible to move most of the apparatus into a laboratory in which the temperature was controlled to within a fraction of a Centigrade degree. The only equipment not moved into this laboratory were the pulse height analyser and its A.C. voltage regulator. Runs D and E were done in this laboratory.

Before Run D was done the stability of the apparatus was checked and it was found that with constant crystal strain the value of ΔR was 0.06% on one day and 0.05% on another. The results of Run D given in Table D and in Graph D indicate an increase in resolution with phosphor strain.

A few days later the same phosphor was further strained in Run E (see Table E and Graph E) and, fortunately, it was possible to strain this phosphor sample much further than previous samples without breaking it. The curve in Graph E indicates a definite increase in resolution with increasing crystal strain. A stability test preceded this run yielding a value of ΔR of 0.07% for ten determinations of the resolution at constant strain. It is this value of ΔR that is indicated in Graph E since in this run the resolution was only measured once for each value of strain.

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DATE	DEPRESSION (cm.)	NUMBER OF TRIALS	RESOLUTION (%)	MEAN DEVIATION (%)
August 15	1.59×10^{-3}	2	15.35	0,30
	3.18 x 10 ⁻³	3	15.72	0.03
·	4.76 x 10-3	5	15.81	0.09
	6.36 x 10-3	3	15.70	0.01
	7.94 x 10 ⁻³	2	15.94	0.03
•		TABLE B		
August 16	0.00×10^{-3}	5	14.82	0.07
	4.76×10^{-3}	3	15.61	0.15
		TABLE C		
August 21	0.00×10^{-3}	13	14.41	0.07
August 22	0.00×10^{-3}	16	14.75	0.06
August 23	2.38 x 10 ⁻³	18	15.02	0.07
August 24	4.76 x 10 ⁻³	14	15.17	0.07
August 25	6.36 x 10 ⁻³	14	15.51	0.13
August 26	7.94 x 10 ⁻³	11	15.35	0.08
August 27	7.94 x 10-3	12	15.56	0.11
August 28	9.54 x 10 ⁻³	10	15.62	0.09

TABLE A

	TABLE D		
DEPRESSION (cm.)	NUMBER OF TRIALS	RESOLUTION (%)	MEAN DEVIATION (%)
0.00×10^{-3}	3	14.95	0.06
1.59 x 10-3	3	14.91	0.08
2.38 x 10 ⁻³	3	15.12	0.02
3.18 x 10 ⁻³	3	15.04	0.05
4.76 x 10 ⁻³	3	15.00	0.07
4.76×10^{-3}	4	15.16	0.12
6.36 x 10 ⁻³	3	15.08	0.07
7.94×10^{-3}	3	15.04	0.06
9.54 x 10 ⁻³	2	15.15	0.02
	TABLE E		
9.54 x 10 ⁻³	2	15.65	0.05
11.1 x 10 ⁻³	1	15.64	
12.7 x 10-3	1	15.75	
14.3 x 10 ⁻³	l	15.58	
15.9×10^{-3}	1	15.88	
17.5×10^{-3}	l	15.69	
19.1×10^{-3}	l	15.68	
22.3 x 10 ⁻³	1	15.78	
25.4 x 10 ⁻³	l	15.88	*** ===
28.7×10^{-3}	l	16.04.	
31.8 x 10 ⁻³	l	16.28	
38.2×10^{-3}	1	16.23	
44.5×10^{-3}	1	16.43	
	DEPRESSION (cm.) 0.00×10^{-3} 1.59×10^{-3} 2.38×10^{-3} 3.18×10^{-3} 4.76×10^{-3} 4.76×10^{-3} 6.36×10^{-3} 7.94×10^{-3} 9.54×10^{-3} 12.7×10^{-3} 12.7×10^{-3} 14.3×10^{-3} 15.9×10^{-3} 15.9×10^{-3} 17.5×10^{-3} 17.5×10^{-3} 19.1×10^{-3} 22.3×10^{-3} 25.4×10^{-3} 25.4×10^{-3} 31.8×10^{-3} 31.8×10^{-3} 31.8×10^{-3}	TABLE DDEPRESSION (cm.)NUMBER OF TRIALS 0.00×10^{-3} 3 1.59×10^{-3} 3 2.38×10^{-3} 3 2.38×10^{-3} 3 3.18×10^{-3} 3 4.76×10^{-3} 4 6.36×10^{-3} 3 7.94×10^{-3} 3 9.54×10^{-3} 1 12.77×10^{-3} 1 14.3×10^{-3} 1 15.9×10^{-3} 1 17.5×10^{-3} 1 19.1×10^{-3} 1 22.3×10^{-3} 1 28.7×10^{-3} 1 31.8×10^{-3} 1 31.8×10^{-3} 1 38.2×10^{-3} 1 44.5×10^{-3} 1	TABLE DDEPRESSION (cm.)NUMBER OF TRIALSRESOLUTION ($\%$)0.00 x 10 ⁻³ 314.951.59 x 10 ⁻³ 314.912.38 x 10 ⁻³ 315.123.18 x 10 ⁻³ 315.044.76 x 10 ⁻³ 315.004.76 x 10 ⁻³ 415.166.36 x 10 ⁻³ 315.087.94 x 10 ⁻³ 315.04TABLE E9.54 x 10 ⁻³ 21.1 x 10 ⁻³ 11.5.7514.3 x 10 ⁻³ 11.5.8817.5 x 10 ⁻³ 115.9 x 10 ⁻³ 115.6817.5 x 10 ⁻³ 119.1 x 10 ⁻³ 115.6815.7822.3 x 10 ⁻³ 116.6422.3 x 10 ⁻³ 116.0431.8 x 10 ⁻³ 116.0431.8 x 10 ⁻³ 116.0431.8 x 10 ⁻³ 116.2838.2 x 10 ⁻³ 116.43

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V DISCUSSION OF RESULTS

Instability of the apparatus has been present to a varying degree throughout the experiments. The results do, however, indicate that the introduction of dislocations by the manner employed does in fact cause an increase in the energy resolution of thallium activated sodium iodide phosphors.

Various questions now arise. Are all solid phosphors similarly affected? Is the effect caused by the presence, or instead by the non-uniform presence, of dislocations? Is the resolution "intrinsic to the phosphor" caused partly, mostly, or wholly, by the presence of dislocations?

The work of Zerby et al.⁽⁸⁾ does give an explanation of the intrinsic resolution which yields reasonable values for NaI(T1). The stressing of the crystal in the last run resulted in an increase in resolution of only 0.8%. If the lattice constant of NaI is taken to be 6.5 Angstrom units and is assumed equal to the Burger's vector, then the density of dislocations introduced in this run is about 2×10^6 cm⁻² for a depression of 4.45 x 10^{-2} centimeters. Johnston⁽¹⁰⁾ has estimated that carefully handled crystals of NaI(T1) which are annealed at the melting point and

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then slowly cooled would have a dislocation density of from 10^5 to 10^6 per square centimeter. It may well be that the affect of dislocations on the resolution is far more pronounced at lower dislocation densities and that the effect of adding 2 x 10^6 per square centimeter is appreciably reduced when there are already 10^5 or 10^6 dislocations per square centimeter. If this were true then the contribution of dislocations to the intrinsic resolution of this phosphor could be appreciable.

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APPENDIX A

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Best Fit to a Gaussian

Since the determination of the energy resolution of a scintillation spectrometer was an important part of the project it was necessary to devise a more accurate method than the simple evaluation of the fullwidth-at-half-maximum from a graph of a scintillation spectrum. An outline of the method used follows:

1. The photopeak is "Gaussian" over some region which is determined by a graphical method outlined by Onno (See Appendix C).

2. A Gaussian may be converted into a parabola and the photopeak is converted in this way.

3. Of the family of curves that constitute a parabola, namely $w = -bx^2 + dx + c$, one is chosen such that the sum of the squares of the vertical distances from the parabola to each of the points of the photopeak, converted in the above way, is a minimum. This requires that the partial derivatives of the above sum, with respect to b, c, and d, are zero. That is, we have three linear equations in b, c, and d, which are solved by the usual method of determinants for b, c, and d. The resolution and peak position may then be found,

 μ_{\bullet} Weighting: The standard error in y counts is \sqrt{y}_{\bullet} Weighting was accomplished by assuming that a point on the graph of a scintillation spectrum representing y counts was actually \sqrt{y} coincident points.

Theory

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Gaussian:
$$y = \frac{T}{S\sqrt{2\pi}} \exp \frac{-(x-\bar{x})^2}{2S^2}$$

where

y is the number of pulses in the voltage range x to (x + dx), S is the standard deviation, T is the total number of pulses in a Gaussian distribution,

Set w = ln y = ln $\frac{T}{S\sqrt{2\pi}} = \frac{(x-\bar{x})^2}{2S^2} = a-b(x-\bar{x})^2 = -bx^2 + 2b\bar{x}x + a-b\bar{x}^2$,

where

and

$$a = ln \frac{T}{S \sqrt{2\pi}},$$

x is the position of the peak.

$$b = \frac{1}{2 s^2} \cdot$$

i.e. $w = -bx^2 + dx + c$, where $c = a - bx^{2}$, d = 2bx.

Best Fit to Parabola:

The photopeak consists of points (x_i, y_i) which are converted points (x_i, w_i) to which we must fit a parabola.

Set S = $\Sigma (w_i - w(x_i))^2 = \Sigma (w_i + bx_i^2 - dx_i - c)^2$.

As outlined above we set the three partial derivatives of S equal to $zero_{\bullet}$

Ignoring the subscript "i",

$$(\Sigma x^{4}) b - (\Sigma x^{3}) d - (\Sigma x^{2}) c = -(\Sigma w x^{2}),$$

 $(\Sigma x^{3}) b - (\Sigma x^{2}) d - (\Sigma x) c = -(\Sigma w x),$
 $(\Sigma x^{2}) b - (\Sigma x) d - Nc = - (\Sigma w),$

where N is the number of points considered.

Set $A = \Sigma x$, $J = \Sigma w$, $B = \Sigma x^2$, $K = \Sigma wx$, $C = \Sigma x^3$, $L = \Sigma wx^2$. $D = \Sigma x^{l_1}$,

Further, set

$$F = \begin{vmatrix} D & C & B \\ C & B & A \\ B & A & N \end{vmatrix},$$
$$G = \begin{vmatrix} D & L & B \\ C & K & A \\ B & J & N \end{vmatrix},$$

and

Then

b = H, and d = G.

 $H = - \begin{vmatrix} L & C & B \\ K & B & A \\ J & A & N \end{vmatrix}$

i.e.

$$S^{2} = 1 = F$$
, $x = d = G$,
 $2b = 2H$, $2b = 2H$

Weighting

Let z_i be $\overline{y_i}$, then z_i are the weighting factors.

A becomes Σ_{xz} , B becomes Σ_{x^2z} , C becomes Σ_{x^3z} - 27 -

D becomes $\Sigma x^{l_{1}}z$, J becomes Σwz , K becomes Σwxz , L becomes $\Sigma wx^{2}z$,

Σz.

N becomes

and

Full-Width-At-Half-Maximum, W

From the equation of a Gaussian given above we see that when

$$x = \bar{x}, y = \underline{T}$$

 $s\sqrt{2\pi}$

i.e. The half-maximum value of y is \underline{T} 2 S $\sqrt{2\pi}$

Let x! be the values of x at these points.

then
$$\frac{1}{2} = e \frac{-(x^{i} - \bar{x})^{2}}{2S^{2}}$$

 $\ln(\frac{1}{2}) = \frac{-(x^{i} - \bar{x})^{2}}{2S^{2}}$
 $i_{\bullet}e_{\bullet} (x^{i} - \bar{x})^{2} = 2S^{2} \ln 2.$
 $W = 2 |x^{i} - \bar{x}| = 2S \sqrt{2 \ln 2} = 2.3550 S.$

A program for a Bendix G -15 computer has been prepared to do the above analysis of scintillation spectra (See Appendix B).

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APPENDIX B

Programme for a Weighted Best Fit to a Gaussian

Data was obtained from a 100 channel pulse height analyzer.

Each of the hundred channels were of width U (U = 0.3, 0.15, 0.06, or 0.03 volts) and the voltage corresponding to a pulse falling in the first channel was determined by the setting of the bias control, V (V = 1,2,3,....10).

The height, h, of a pulse falling in channel Q when the channel width and bias controls were respectively set at U and V is:

 $h = 0.3 + 3(V-1) + UQ + \frac{1}{2}U + \frac{1}{2}U$ volts.

For the purpose of programming the pulse height scale is in channels and the pulse height origin is the least value of x, x_0 , for which the photopeak is Gaussian. In this coordinate system pulse heights are denoted by x¹, i.e. $x = x_0 + x^1$. Also, A,B,C,D,F,G,H,K, and L, become A¹, B¹, C¹, D¹, F¹, G¹, H¹, K¹, and L¹ respectively.

The full-width-at-half-maximum is found from W = 2.355 S = 2.355 $\frac{F!}{2H!}$ and the resolution R is found from R = UW, where h is the height of a pulse in volts for x = \overline{x} . (it is assumed that all pulses in a channel have a height corresponding to the midpoint of that channel.) Partial Expansion of Determinants

$$F = \begin{vmatrix} D & C & B \\ C & B & A \\ B & A & N \end{vmatrix} = D(NB - A^2) + C(-NC + AB) - B(-AC + B^2)$$

$$G = \begin{vmatrix} D & L & B \\ C & K & A \\ B & J & N \end{vmatrix} = D(NK - AJ) + L(-NC + AB) + BCJ - B^2K$$

$$H = -\begin{vmatrix} L & C & B \\ K & B & A \\ J & A & N \end{vmatrix} = -L(NB - A^2) + C(NK - AJ) - ABK + B^2J$$

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PROGRAMME

		"LOC" gives the location of th	e command KOPADI	DR
LOC	K OP ADDR	CONTENTS OF ACCUMULATOR	NOTES	
	First, we	set a word base according to	the number of po	oints
consi	dered:			
1)†00	1 72 OOWL		$WI_{1} = 2(N-1),$	(WL is set according to the number of points considered)
	Storage o	f some useful numbers:		······
1401	0 41 2100	° 0		
	0 49 0900		0 at 0900	
	0 42 0994	xo		• • • • • • • • • • • • • • • • • • •
	0 48 2100	$x_0/x_0=1$		
1405	0 49 0902		l at 0902	
	3 70 0000		WB3 =0	
	3 71 0002		WD ₃ =2	
	3 72 0016		WI3 =16	
	3 42 0902	1,2,,9		
1410	0 43 0902	2,3,,10		
	3 49 0904		2,3,,10 at	0904,,0920
	3 76 1409		loop to comman	d 11109
	3 70 0000		WB3 =0	
	0 42 0904	2		
1415	0 49 0970		2 at 0970	
	0 42 0906	3		
	0 48 0920	0 . 3		
	0 1.3 0970	0.2		

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IOC	K OP ADDR	CONTENTS OF ACCUMULATOR	NOTES
	0 49 0970		2.3 at 0970
1)420	0 42 0910	5	
	0 48 0920	0•5	•
	0 48 0920	0.05	
	0 43 0970	2•35	
	0 49 0970	· · · · · · · · · · · · · · · · · · ·	2.35 at 0970
1425	0 42 0910	5	
	0 48 0920	0.5	
	0 48 0920	0.05	
	0 48 0920	0.005	
	0 43 0970	2.355	
1/130	0 49 0922		2.355 at 0922
	Calculat	tion of the z _i :	
1431	1 70 0000		WB ₁ = 0
•	1 71 0002		$WD_1 = 2$
	1 42 1000	y _i	
	0 08 1297	$\sqrt{y_i} = z_i$	
1435	1 49 1700		z_{i} at channel 17
	1 76 1433		loop to command 1433
	1 70 0000		$WB_1 = 0$
	Calculat	ion of x _i ':	
11,38	0 41 2100	0	
	0 49 1800		0 at 1800
ոյիւօ	1 42 1800	0,1,,	
	0 113 0902	1.2 i.e. x.!	

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LOC K OP ADDR CONTENTS OF ACCUMULATOR 0 49 1802

1 76 1440

1 70 0000

Calculation of A' :

1145	1 42 1800	x,'
	1441700	x _i 'z _i
	1 49 1300	
	1 76 1445	
	1 70 0000	
1450	0 41 2100	0
	1 43 1300	x _i ^t z _i
	1 76 1451	
	1 70 0000	
	0 49 0924	
1455	0 31 0001	
	0 34 2100	
	Calculat	tion of Bt
1457	1 42 1300	x,'z
	1 44 1800	x _i ² z _i
	1 49 1300	-
1460	1 76 1457	
	1 70 0000	
`	0 41 2100	0

:

- 1 43 1300 $x_i^2 z_i$

NOTES

x, 1 at channel 18, since 0 already at 1800 (see command 1439) loop to command 11440 $WB_1 = 0$

- x, 'z, at channel 13 loop to command 1445 $WB_1 = 0$
- loop to command 1451 getting $\sum_{i} x_{i}' z_{i} = A'$ $WB_1 = 0$ A! at 0924 type 1 type A!

x_i²z_i at channel 13 loop to command 1457 $WB_7 = 0$

LOC	K OP ADDR	CONTENTS OF ACCUMULATOR	N
	1 76 1463		l
1465	1 70 0000		W
	0 49 0926		в
<u>14</u> 67	0 31 0002		t
	0 34 2100		t
	Calculat	tion of C' :	
1469	1 42 1300	x _i ² z _i	
1470	1 44 1800	x _i ³ z _i	
	1 49 1300	-	x
	1 76 1469	· · · · · · · · · · · · · · · · · · ·	l
	1 70 0000		W
	0 41 2100	0	
1475	1 43 1300	x, ¹³ z,	
	1 76 1475		l
			Σ
	1 70 0000		W
	0 49 0928		C
	0 31 0003		'tı
1480	0 34 2100		t
	Calculat	tion of D':	
1481	1 42 1300	x. ^{1³z.}	
	1 44 1800	x _i ^{t4} z _i	
	1 49 1300	-	x
	1 76 1481		l
1485	1 70 0000		WI

NOTES

loop to command ll463 getting $\sum x_i^2 z_i = B^i$ WB₁ = 0 B' at 0926 type 2 type B'

 $x_i'^3 z_i$ at channel 13 loop to command 1469 WB₁ = 0

loop to command 1475 getting $\sum_{i} x_{i}^{3} z_{i} = C!$ WB₁ = 0 C! at 0928 type 3 type C!

 $x_{i}^{\mu}z_{i}$ at channel 13 loop to command 1481 WB₁ = 0

LOC	K OP ADDR	CONTENTS OF ACCUMULATOR	NOTES
	0 41 2100	0	
	1 43 1300	x _i ^{t^lz} i	
	1 76 1487		loop to command 1487 getting
			$\sum_{i} x_{i} t^{i} z_{i} = Dt$
1489	70 0000		$WB_1 = 0$
1490	0 49 0930		D' at 0930
	0 31 0004		type 4
	0 34 2100		type D'
	Calcula	tion of N :	; · · ·
149 <u>-</u>	3 0 41 2100	0	
	1 43 1700	zi	
אַנג 149	5 1 76 1494		loop to command 1494 getting
			$\sum_{i} z_{i} = N$
	1 70 0000		$WB_1 = 0$
	0 49 0992		N at 0992
	0 31 0005		type 5
	0 34 2100		type N
	Calcula	tion of J :	
150	1 42 1000	У ₁	
	0 08 1117	ln y _i = w _i	
	1 44 1700	^w i ^z i	
•	1 49 1300		w.z. at channel 13
	1 76 1500		loop to command 1500
150	5 1 70 0000		$WB_1 = 0$
	0 41 2100	0	
	1 43 1300	Wi ² i	
	1 76 1507		loop to command 1507 getting

 $\sum_{i} w_{i} z_{i} = J$

		·	
TOC	K OP ADDR	CONTENTS OF ACCUMULATOR	NOTES
	1 70 0000		$WB_1 = 0$
1510	0 49 0932		J at 0932
1511	0 31 0006		type 6
	0 34 2100		type J
	Calculat	ion of K':	
1513	1 42 1300	W, Z i i	×
	1 44 1800	w x ¹ z i i i	
15 1 5	1 49 1300		w.x'z. at i i i
	1 76 1513		loop to co
	1 70 0 000		WB _l = 0
	0 41 2100	0	
	1 43 1300		
1520	1 76 1519		loop to co
			∑ w _i x _i 'z _i
	1 70 0000		WB ₁ = 0
	0 49 0934		K! at 0934
	0 31 0007		type 7
	0 34 2100		type K!
	Calculat	ion of L':	
1525	1 42 1300	w _i x _i [†] z _i	
	1 44 1800	wixi' ² zi	
	1 49 1300		w _i x _i ' ² z _i a
	1 76 1525		loop to co
	1 70 0000		WB = O
1530	0 41 2100	0	alia
	1 43 1300		

1532 1 76 1531

w.x.z. at channel 13 loop to command 1513 WB. = 0

loop to command 1519 getting $\sum_{i} w_{i}x_{i} \cdot z_{i} = K^{i}$ WB₁ = 0 K' at 0934 type 7 type K'

W_ix_i²z_i at channel 13 loop to command 1525 WB = 0

loop to command 1531 getting $\sum_{i} w_{i} x_{i} r^{2} z_{i} = L r$

		*	
LOC	K OP ADDR	CONTENTS OF ACCUMULATOR	NOTES
	1 70 0000		WB ₁ = 0
	0 49 0936		L' at 0936
1535	8000 18 0		type 8
	0 34 2100		type Lt
	Calcula	tion of three sub-determinant	s:
1537	0 42 0992	N	
	0 44 0926	NB	
	0 49 0954		NB! at 0954
1540	0 40 0924	-A 1	
	0 44 0924	-A ^{t²}	
	0 43 0954	$(NB^{\dagger} - A^{\dagger^2})$	
	0 49 0954		$(NB^{\dagger} - A^{\dagger^2})$ at 0954
	0 40 0992	-N	en e
1545	0 44 0928	-NC [†]	
	0 49 0956		(-NC1) at 0956
	0 42 0924	A *	
	0 44 0926	A ^t B ^t	
	0 43 0956	$(-NC^{\dagger} + A^{\dagger}B^{\dagger})$	
1 550	0 49 0956	• • • • •	(-NC! + A'B!) at 0956
	0 40 0924	- <u>A</u> t	
	0 44 0928	-A 1C 1	
	0 49 0958		(-A ^t C ¹) at 0958
1553	0 42 0926	Bı	
	0 44 0926	B ¹²	
1555	0 113 0958	$(-A^{1}C^{1} + B^{1})$	

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LOC	K OP ADDR	CONTENTS OF ACCUMULATOR	NOTES
	0 49 0958		$(-A^{\dagger}C^{\dagger} + B^{\dagger}^{2})$ at 0958
	0 42 0992	N	
	0 44 0934	NKI	
	0 49 0960		(NK1) at 0960
1560	0 40 0924	-41	
	0 44 0932	-A' J	
	0 43 0960	$(NK^{\dagger} - A^{\dagger}J)$	(see command 1559)
	0 49 0960		(NK! - A!J) at 0960
	Calculat	ion of F [†] :	
1564	0 42 0930	Dı	
1565	0 44 0954	$D! (NB! - A!^2)$	
	0 49 0970		$D!(NB! - A!^2)$ at 0970
	0 42 0928	01	n an
	0 44 0956	$C^{\dagger}(-NC^{\dagger} + A^{\dagger}B^{\dagger})$	
1570	0 43 0970	$D^{\dagger}(NB^{\dagger}-A^{\dagger}^{2}) + C^{\dagger}(-NC^{\dagger}+A^{\dagger}B^{\dagger})$	
	0 49 0970	n, n , n, , , , , , , , , , , , , , , , , ,	$D!(NB!-A!^2) + C!(-NC!+A!B!)$ at 0970
	0 40 0926	-Bt	и и и
	0 44 0958	$-B^{\dagger}(-A^{\dagger}C^{\dagger}+B^{\dagger})$	
	0 43 0970	Ft	
1575	0 49 0938		F' at 0938
	0 31 0009		type 9
	0 34 2100		type F'
	Calculat	tion of G ¹ :	
1578	0 42 0932	J	
	0 11 0028	CLI	

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LOC	K OP ADDR	CONTENTS OF ACCUMULATOR	NOTES
1580	0 44 0926	BICIJ	
	0 49 0970		B'C'J at 0970
	0 40 0934	-K'	
	0 44 0926	⇔B i K i	
	0 44 0926	-B ^{*2} K*	
1585	0 43 0970	$(-B^{\dagger 2}K^{\dagger} + B^{\dagger}C^{\dagger}J)$	
	0 49 0970		$(-B^2K^{\dagger} + B^{\dagger}C^{\dagger}J)$ at 0970
	0 42 0936	L1	
	0 44 0956	$L^{\dagger}(-NC^{\dagger} + A^{\dagger}B^{\dagger})$	
	0 43 0970	$L^{\dagger}(-NC^{\dagger} + A^{\dagger}B^{\dagger}J - B^{\dagger}K^{\dagger}K^{\dagger}$	
1590	0 49 0970		$L^{\dagger}(-NC^{\dagger} + A^{\dagger}B^{\dagger}) + B^{\dagger}C^{\dagger}J = B^{\dagger}K^{\dagger}$ at 0970
	0 42 0930	Dt	
	0 44 0960	$D^{\dagger}(NK^{\dagger} - A^{\dagger}J)$	
	0 43 0970	G 1	
	0 49 0940		G! at 0940
1595	0 31 0010		type 10
	0 34 2100		type G'
	Calculat	tion of H [‡] :	
1597	0 42 0932	J	
	0 44 0926	BIJ	
	0 44 0926	B ¹² J	
1600	0 49 0970	•	B ² J at 0970
	0 40 0924	-A I	-
	0 44 0926	-A'B'	
1603	0 111 0931	-AIBIKI	

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LOC	K OP ADDR	CONTENTS OF ACCUMULATOR	NOTES
	0 43 0970	$-B_i \ \mathbf{f} - \mathbf{y}_i B_i \mathbf{K}_i$	
1605	0 49 0970		$-B^{2}J - A^{B}K^{T}$ at 0970
	0 42 0928	<u>O</u> t	
	0 44 0960	C'(NK'-A'J)	
	0 43 0970	$C^{\dagger}(NK^{\dagger}-A^{\dagger}J)-A^{\dagger}B^{\dagger}K^{\dagger}+B^{\dagger}^{2}J$	
	0 49 0970	<u>.</u>	$C^{1}(NK^{1}-A^{1}J)-A^{1}B^{1}K^{1}+B^{1}J$ at 0970
1610	0 40 0936]t	· · · · · · · · · · · · · · · · · · ·
	0 44 0954	$-L^{\dagger}(NB^{\dagger}-A^{\dagger}^{2})$	
	0 43 0970	Ht.	
	0 49 0942		H! at 0942
	0 31 0011		type 11
1615	0 34 2100		type H ¹
	Calculation	of x':	
1616	0 42 0940	Gı	
	0 48 0942	G1/H1	
	0 48 0904	(G'/2H') = x'	
	0 49 0944		x' at 0944
1620	0 42 0910	5	
	0 48 0920	0•2	
	0 43 0944	$x^{\dagger} + 0_{\bullet}5$	
	0 43 0994	$x_0 + \bar{x} + 0.5 = \bar{x}$	
	0 49 0944		x at 0944
1625	0 31 0012		type 12
	0 34 2100		type x

LOC	K OP ADDR	CONTENTS OF ACCUMULATOR	NOTES
	Calculation	of W :	
1627	0 42 0938	F t	
	0 48 0942	Ft/Ht	
	0 48 0904	Ft/2Ht	
1630	0 08 1297	$\sqrt{F^{\dagger}/2H^{\dagger}} = S$	
	0 44 0922	2.355S = W	
	0 31 0013		type 13
	0 34 2100		type W
	Calculation	of h :	
1634	0 44 0998	UW	
1635	0 49 0946		UW at 0946
	0 42 0996	V	
	0 41 0902	(V-1)	
	0 44 0906	3(V-1)	
	0 49 0970		3(V-1) at 0970
1640	0 42 0998	U	
	0 111 09111	Ux	
	0 43 0970	3(V-1) + Ux	
	0 49 0970		3(V-1) + Ux at 0970
	0 42 0906	3	i
1645	о 48 0920	0.3	
	0 43 0970	$0_{\bullet}3+3(V-1)+Ux = h$	UNIVERSITY OF
	0 31 001)4		type 14
	0 34 2100		type h

1654	0 67 0000		hald in manual mode
	0 34 2100		type R
,	0 31 0015		type 15
	0 144 0920	100 UW/h = R	
1650	0 44 0920	lo uw/h	
1649	0 47 0946	uw/h	
	Calculation	of R:	· · ·
LOC	K OP ADDR	CONTENTS OF ACCUMULATOR	NOTES

USE OF THE COMPUTER MEMORY IN THE PROGRAMME:

0900	0
0902	1
0904	2
0906	3
0908	4
0910	5
0912	6
091)4	7
0916	8
0918	9
0920	10
0922	2.355
0924	A.
0926	Bı
0928	C1
0930	Dı
0932	J
0934	K1
0936	Lŧ
0938	Ft
0940	Gt
0942	HI
09144	x' and x
0946	UW
0948	

50

 (NB^1) and (NB^1-A^{12}) 0956 $(-NG^1)$ and $(-NG^1+A^{1}B^1)$ 0958 $(-A^1G^1)$ and $(-A^1G^1+B^{12})$ 0960 (NK^1) and (NK^1-A^1J)

miscellaneous numbers

0990	
0992	N
0994	×o
0996	v
0998	U

channel 10	y _i
channel 11	In subroutine
channel 12	square root subroutine
channel 13	$\mathbf{x_i z_i, x_i}^2 \mathbf{z_i, x_i}^3 \mathbf{z_i, x_i}^4 \mathbf{z_i, w_i z_i, w_i x_i z_i, w_i x_i}^2 \mathbf{z_i}$
channel 1)4	programme
channel 15	programme
channel 16	programme
channel 17	
channel 18	x'i

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APPENDIX C

Determination of the Gaussian Region

The scintillation spectrum closely approximates a Gaussian only over a limited region near the photopeak. On the high energy side of the photopeak background counts become appreciable above a certain level and on the low energy side counts in the Compton peak become appreciable below another certain level. It was desired to determine the "Gaussian region" in order to fit a Gaussian only to those points that lay very nearly on a Gaussian.

Several graphical methods have been used in the analysis of scintillation spectra. Descriptions of three of these methods follow.

Ι

The equation of a Gaussian is:

$$y = \frac{T}{S \sqrt{2\pi}} \exp - \frac{(x-\overline{x})^2}{2S^2}$$
, as before.

Set
$$Y = \frac{T}{S\sqrt{2\pi}}$$
 and $x'' = x - \bar{x}$,

then
$$y/Y = \exp \frac{-x^{H^2}}{2S^2}$$
 and $\ln y/Y = \frac{-x^{H^2}}{2S^2}$.

Therefore a plot of $\ln(y/Y)$ versus x''^2 yields a straight line through the origin with slope $\frac{-1}{2S^2}$ or a plot of (y/Y) versus x''^2 on semilogarithmic

paper gives rise to a straight line over the Gaussian region.

This method assumes a knowledge of the values Y (the maximum value of y) and of \bar{x} (the position of the photopeak).

II Boekelheide Method.

Boekelheide⁽¹¹⁾ has outlined a method which makes use of "probability paper"⁽¹²⁾ on which the abscissa scale is linear and the ordinate scale is such that plotting

 $\frac{t}{T} = \frac{1}{S\sqrt{2 \pi}} \int \exp - \frac{(x-\bar{x})^2}{2S^2} dx \text{ versus x results in a straight line}$

of slope $-\frac{1}{S}$, where t is the area under that part of the Gaussian which lies above x, and T, the total area under the Gaussian, is determined by doubling the area above the peak position. (A Gaussian is symmetrical about the peak position. Asymmetry results in the photopeak from the Compton contribution.)

This method requires a knowledge of the peak position and a calculation of the fractions (t/T) by summing the contents of the channels of a multichannel pulse height analyzer from the top of the photopeak down to the channel in question and dividing the result by T.

III Onno Method

P. $Onno^{(13)}has$ produced graph paper⁽¹⁴⁾ ("Gaussian graph paper") with a linear abscissa scale and with such an ordinate scale that plotting $R = 100 \exp - \frac{(x-\bar{x})^2}{2S^2}$ versus x results in a straight line whose slope is proportional to (1/S) where R = 100 y.

This method requires a knowledge of Y and a calculation of R for each value of y, a calculation readily done on a slide rule. The method is much quicker than that of Boekelheide and is, as Onno has shown, far more shapesensitive to departures from a Gaussian distribution than the integral method of Boekelheide.

REFERENCES

(1)	Breitenberger: Progress in Nuclear Physics, 4, 56. London:Pergamon
	Press (1955).
(2)	G. T. Wright: Journal of Scientific Instruments, 31, 377, (1954).
(3)	F. Seitz: Journ. Chem. Phys. 6, 150, (1938).
(4)	G.F.J. Garlick and G.T. Wright: British Journal of Applied Physics,
	<u>5</u> , 13, (1954),
(5)	G.G. Kelly, P.R. Bell, R.C. Davis, and N.H. Lazar: Nucleonics,
	<u>14</u> , No.4, 53, (1956).
(6)	D.I. Cooper and G.A. Morton: Nucleonics, 14, No. 4, 57, (1956).
(7)	J.J. Hopkins: Review of Scientific Instruments, 22, 29, (1951).
(8)	C.D. Zerby, A. Meyer, R.B. Murray: Bulletin of the American Physical
	Society, Series II, 5, No.6, Abstract J5, 415,
	(November, 1960).
(9)	A.J. Dekker: Solid State Physics, page 96, Prentice-Hall (1959).
(10)	W.E. Johnston: private communication.
(11)	I.F. Boekelheide: Review of Scientific Instruments, 31, No.9, 1001, (1960).
(12)	Keuffel and Esser: No. 359 - 23.
(13)	P. Onno: Review of Scientific Instruments, 32, No. 11, 1253, (Nov., 1961).
(ı/ı)	Canadian Charts and Supplies. Itd Oakrille Ontario. No. 08-10055

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