THE EFFECT OF DISIOCATIONS INTRODUCED BY PLASTIC BENDING ON THE ENERGY RESOLUTION OF THAIIIUM ACTIVATED SODIUM IODIDE SCINTILLATING PHOSPHORS


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## ABSTRACT

It was found that when a thallium activated sodium iodide
phosphor was stressed the scintillation energy resolution increased
by a small amount. In one case, when the number of dislocations
was increased by an estimated $2 \times 10^{6}$ per square centimeter the
resolution was increased by $0.8 \%$.

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The scintillation counter is now widely used for the detection of X- and garma- radiation, and of energetic charged particles, such as alpha and beta particles. The scintillation counter consists of a phosphor, which emits a pulse of light when it absorbs energy from some form of radiation, and of a photomultiplier tube, which converts the light pulse to a pulse of electrons by means of the photoelectric effect and amplifies these pulses of electrons several times by means of the process of secondary electron emission, together with any associated electronic equipment used to further amplify the pulses, or to determine the pulse height distribution.

When a stream of monoenergetic radiation is incident on the phosphor and the energy of a quantum of electromagnetic radiation, or of a particle, is completely absorbed in the phosphor, the resulting pulse contributes to the photopeak which occurs in the pulse height distribution curve. The position of the photopeak is dependent on the energy of the incident particles or quanta. With gamma-rays, the pulse height corresponding to the centre of photopeak is very nearly proportional to the energy of the ganma-ray quanta.

It has been realized for some time that the detection of monoenergetic particles or quanta by a scintillation counter gives rise to voltage pulses in the photopeak which differ somewhat in size. The statistical nature of the processes involved contributes to this variance in pulse height as do imperfections in the component parts of the counter. Several papers have
appeared which treat only the variance that arises out of one or other of the detection processes. Both Breitenberger (1) and Wright (2) have published more comprehensive articles.

Defining photon transfer as a cascade of four steps, the escape of photons from the phosphor, the transmission of photons to the photocathode (possibly through a light guide), the emission of photoelectrons, and the collection of photoelectrons at the first dynode, Breitenberger discussed in detail the statistical nature of the processes of photon transfer, light collection, light "piping", and secondary electron emission, and touched on the statistical importance of the emission of after-electrons and satellite pulses, photocathode inhomogeneities, phosphor inhomogeneities, edge effects in the phosphor, the possibility of non-normal scintillation variance, amplifier and power supply instabilites, high counting rates, and the value of the collecting time constant at the anode of the photomultiplier tube.

Assuming a Gaussian distribution of pulses in the photopeak, the analysis of Wright leads to the result:
$\frac{\mathrm{R}^{2}}{5.56 \times 10^{4}}=\frac{\bar{p}^{2}\left(S_{n}^{2}-\bar{n}\right)}{(\bar{n} \bar{p}+\bar{m})^{2}}+\frac{S_{p}^{2}\left(S_{n}^{2}+\bar{n}^{2}\right)}{(\bar{n} \bar{p}+\bar{m})^{2}}+\left(\frac{1}{\bar{n} \bar{p}+\bar{m}}\right)\left(1+\frac{\bar{s}}{\bar{r}(\bar{s}-1)}+\frac{S_{r}^{2}}{\bar{r}^{2}}\right)$
where $R=$ the resolution expressed as a percent ( fractional full-width-at-half-maximura),
$\bar{s}=$ mean gain of the second and ensuing dynode stages in the photomultiplier tube,
$\bar{m}=$ mean number of thermal electrons emitted from the photocathode during a scintillation pulse,
$K=$ number of scintillations counted,
$n_{i}=$ number of photons in the ith scintillation,
$p_{i j}=$ probability that the $j$ th photon of the ith scintillation produces a photoelectron which reaches the first dynode,
$\bar{p}_{i}=\sum_{j=i}^{m_{i}} p_{i j} / n_{i}$
$\bar{p}=\quad \sum_{i=1}^{K} \bar{p}_{i} / K$
$S_{p}^{2}=\sum_{i=1}^{k} \frac{\left(\overline{p_{1}}-\bar{p}\right)^{2}}{K}$,
$\bar{n}=\quad \sum_{i=1}^{k} \frac{n_{i}}{K}$,
$S_{n}^{2}=\sum_{i=1}^{k} \frac{\left(\bar{n}-n_{i}\right)^{2}}{K}$,
$r_{i}=$ the number of secondary electrons produced at the first dynode by the ith incident photoelectron. Hence $r_{i}$ is a function of the point of incidence of the photoelectron. (The first dynode is assumed to be the only one that is non-uniform* Also,
$\bar{r}=\sum_{i=1}^{K} \frac{r_{i}}{K}$, and
$S_{r}{ }^{2}=\sum_{i=1}^{k} \frac{\left(r_{i}-\bar{r}^{2}\right.}{K}$. Wright has, in this expression, made the assumption that the variance of the tube gain is normal, an assumption defended by himself and questioned by Breitenberger.

It is often assumed that the scintillation variance, $S_{n}{ }^{2}$, is normal, i.e. that $S_{n}{ }^{2}=\bar{n}$, in which case the first term vanishes. Seitz ${ }^{(3)}$ has shown that two thallium ions are involved in the emission of one quantum of light from NaI(TI). Breitenberger ( 1 ) has pointed out that if a similar interaction in the process of excitation occurs, then excitation of one
luminescence centre may affect the excitation of a neighbouring centre in which case the scintillation variance may be non-normal.

The transfer variance, $\mathrm{S}_{\mathrm{p}}{ }^{2}$, and hence the second term, vanishes if the optical geometry is the same for each scintillation, that is if $\bar{p}=p_{i}$, all i.

The third term, which often predominates, is due largely to the statistical nature of secondary electron emission. In the case where $S_{r}{ }^{2}=0, \bar{r}=\bar{s}$, and $\bar{m}$ is much smaller than $(\bar{n} \bar{p})$, this term becomes simply:


When $\bar{m}$ is negligible, and $S_{n}^{2}$ is much less than $\bar{n}^{2}$, we have


The dependence of $R^{2}$ on $\bar{n}$ and $\bar{p}$ is usually investigated by plotting $R^{2}$ versus ( $\left.\bar{n} \bar{p}\right)^{-1}$ or, equivalently, versus the reciprocal of the pulse height, since the mean photopeak pulse height is proportional to ( $\bar{n} \bar{p}$ ). In such experiments $\overline{\mathrm{p}}$ might be altered by the insertion of neutral density filters in front of the photocathode of the photomultiplier tube, and $\bar{n}$ is varied by changing the energy of the radiation incident on the phosphor. In the case of gamma-ray scintillation spectroscopy, $\bar{n}$ is very nearly proportional to the gamma-ray energy as mentioned above.

## II A CONTRTBUTION TO RESOLUTION INTRINSIC TO THE SCINTILIATTNG PHOSPHOR

Wright and Garlick
(4) investigated extensively the variance in the pulse heights from scintillation counters. Replacing the scintillating phosphor with a light flasher and using neutral density filters to vary $\bar{p}$, it was found that $R^{2}$ was in fact proportional ( $1 / \bar{p}$ ) as expected if $S_{p}{ }^{2}$ is assumed to be zero in this case.

Next, using various phosphors irradiated by 5 Mev . alpha particles, $\bar{p}$ was again varied by means of neutral density filters. In the case of each crystal used the dependence of $R$ on $\bar{p}$ could be fitted to $R^{2}=R_{0}^{2}+a / p$, where $R_{0}$ and are constants for a given crystal and $R_{0}$ is regarded as a contribution to resolution intrinsic to the phosphor.

For blocks of thallium-activated potassium iodide, diamond, and calcium tungstate the value of $\mathrm{R}_{0}$ ranged from $12 \%$ to $26 \%$ while for a lamina of anthracene it was zero. The slope of the plot of $R^{2}$ versus ( $1 / \bar{p}$ )was different from that with the light flasher for all phosphors but anthracene.

The dependence of $R^{2}$ on $\bar{n}$ was investigated in crystals of calcium fluoride with ground and with freshly cleaved surfaces by measuring the resolution corresponding to different alpha particle energies. The usual plot of $R^{2}$ versus ( $1 / \sqrt{n}$ ) yielded a straight line through the origin for the freshly cleaved crystal and a straight line with a positive $R^{2}$-intercept and a greater slope for the other crystal.

Wright and Garlick also determined the dependence of $\mathrm{R}_{0}$ on energy in anthracene and in diamond. It was found that in anthracene $R_{0}$ was zero at high energies while at low energies $R_{0}$ was proportional to ( $1 / \bar{n}$ ). In the diamond it was found that $R_{0}{ }^{2}$ was proportional to ( $1 / \bar{n}$ ).

Suspecting that the high density of ionization of alpha particles might have a peculiar effect on the resolution, the pulse height variance of fluorescent pulses induced by ultra-violet light was then investigated. Iight pulses from an argon lamp in a sawtooth oscillator circuit were filtered by a piece of Wood's glass which passed light between the wave lengths 3500 to 4000 Angstrom units. These light pulses caused fluorescence pulses in the phosphor which passed to the photomultiplier tube through a filter which removed ultra-violet light. Using anthracene in this arrangement $R_{0}$ was zero, while in the diamond the value of $R_{0}$ was reduced. Wright and Garlick predicted that anthracene irradiated by beta particles would have $R_{0}$ equal to zero.

Wright and Garlick also investigated, in samples of both anthracene and diamond, the effect on $R_{0}$ of a prolonged flux of alpha particles. While no effect was observed in diamond it was found in anthracene that the Iuminescence efficiency decreased and that the dependence of $R_{0}$ on $(I / \bar{n})_{\text {was }}$ similar to that already described except that the energy at which $R_{0}$ departed from zero became lower and lower as the radiation damage increased.
G. G. Kelly, P. R. Bell, R. C. Davis, and N. H. Lazar ${ }^{(5)}$ investigated the resolution of a scintillation counter using thallium-activated sodium iodide crystals to detect garma rays. This group used a pulsed cathode ray
tube (RCA type C73687A) to simulate a phosphor. Assuming that the resolution obtained with the flasher represented the contribution of the photomaltiplier tube to the total variance, it was possible to calculate the additional variance caused by the use of the scintillating phosphor. It was assumed that processes in the crystal and phosphor were gaussian so that $R^{2}=R_{c}^{2}+R_{t}^{2}$, where $R_{t}$ is the resolution obtained with the light flasher, and $R_{c}$ is the contribution to the resolution due to the phosphor.

It was found that when the same crystal was used with different photomultiplier tubes the calculated values of $R_{c}$ were the same.

When the light guide was altered between the crystal or light flasher, and the photomultiplier tube, it was found that again the value of $R_{c}$ was essentially unchanged.

The value of $R_{c}$, the "intrinsic resolution", was unchanged when the light from the crystal was semicollimated or diffused onto the photocathode, when the crystal and light flasher were used simultaneously, when the gamma ray source was collimated, and when the crystal was broken into successively smaller pieces.

The dependence of $\mathrm{R}_{\mathrm{c}}$ on the gamma ray energy was also investigated by this group for energies from 61 kev . to 1850 kev . It was found that $R_{c}$ was approximately inversely proportional to the fourth root of the energy.

Bernstein, as reported by Cooper and Morton ${ }^{(6)}$, investigated the dependence of resolution on the energy of gamma rays incident on anthracene and NaI(Tl) phosphors. Plotting $R^{2}$ versus the reciprocal of the energy,

Bernstein obtained a straight line at low energies intersecting another of greater slope at high energies. For the crystals used the $R^{2}$ intercept of the low energy line indicates an "intrinsic resolution" in thallium activated sodium iodide of from 4 to 5 per cent and in anthracene of 10 per cent.

Breitenberger ${ }^{(1)}$ has analyzed some data by Hopkins ${ }^{(7)}$ regarding the energy dependence of the resolution of a scintillation counter with an anthracene phosphor irradiated by monoenergetic electrons. A plot of $R^{2}$ versus the reciprocal of the energy results in a straight line with an $R^{2}$ intercept corresponding to a resolution of 9.1 per cent.

Wright and Garlick suggest that in the diamond the intrinsic resolution was due to the fact that the diamond was composed of domains some of which were luminescent and some of which were not. This situation would affect the scintillation variance, $S_{n}^{2}$, and, therefore, the resolution.

For the anthracene phosphor Wright and Garlick suggested a different explanation. Near the end of its path in anthracene the slowed alpha particle produces knock-on protons, the Iuminescence efficiency of which differs from that of alpha particles. The result is an increase in the scintillation variance. This explanation gives no reason for the observed effect of radiation damage on the intrinsic resolution of anthracene.

No explanation is offered by Wright and Garlick for the existence of an intrinsic resolution in other phosphors.

Breitenberger ${ }^{(1)}$ has mentioned that the value of the relative transfer variance, $\left(S_{p}^{2} / \bar{p}^{2}\right)$, may not be negligible and that the optical geometry may
be responsible for a large fraction of the intrinsic resolution For a transparent, optically perfect phosphor which is optically matched to a homogeneous photocathode, the relative transfer variance is in part due to the relative variance of the probability that a photon emitted in the phosphor will escape from the phosphor. If the sides and front of the phosphor are regarded as mirrors of isotropic reflectivity g, and the relative variance in the mean number of reflections necessary for the escape of photons emitted throughout the phosphor is $V_{r}$, then, according to Breitenberger, the relative variance in the escape probability is $(I-g)^{2} V_{r}$. For cubes and short cylinders $V_{r}$ is approximately 0.15 according to Breitenberger. In this case, then, the contribution to the resolution is about $91(1-g)$ per cent. The abovementioned 9.1 per cent intrinsic resolution at infinite energy found by Breitenberger in the data of Hopkins can be explained as being due to the variance in the probability of excape of photons from a phosphor whose sides and front have a reflectivity of 0.90 . It is to be noticed that the existence of an intrinsic resolution with an anthracene crystal irradiated by electrons is not in agreement with the prediction of Wright and Garlick ${ }^{(4)}$.

In the experiments of Wright and Garlick on ground and freshly cleaved crystals of calcium fluoride it may well be that an alteration of the variance of the photon escape probability produced the observed effects. Further, the unique results obtained with anthracene when various phosphors were irradiated by 5 Mev . alpha particles and the light was attenuated by neutral density filters, may have been due to the fact that the anthracene phosphor, alone, was in the shape of a lamina resulting in a reduced, or negligible, value of the variance of the photon escape probability.

This contribution to the photon transfer variance is probably included in the intrinsic resolution of $\mathrm{NaI}(\mathrm{Tl})$ noticed in the experiments of Kelly et al.

In impurity activated phosphors such as $\mathrm{NaI}(\mathrm{Tl})$ any inhomogeneity in the concentration of the activating ions would result in a resolution intrinsic to the phosphor. Gross inhomogeneity of the thallium concentration would have been detected in the experiments of Kelly et al when the crystal was repeatedly reduced in size until the sample was about the size of a matchhead.

If the gamma ray scintillation response of $\mathrm{NaI}(\mathrm{Tl})$ were nonlinear then the amount of light emitted following an interaction of one event could differ from the amount emitted following an interaction of several events. In other words, nonlinear response would result in an increased value of the scintillation variance. This explangtion of the intrinsic resolution of $\mathrm{NaI}(\mathrm{Tl})$ is not supported by the above experiment since it is unlikely an interaction of several events would occur in the smallest piece of crystal used.

In spite of this result and the fact that previously there had been some controversy as to whether or not $\mathrm{NaI}(\mathrm{Tl}$ ) was nonlinear, Zerby, Meyer, and Murray ${ }^{(8)}$ have recently given a paper in which a nonlinearity of $\mathrm{NaI}(\mathrm{TI})$ was assumed and by means of a Monte Carlo calculation have calculated a reasonable value of the intrinsic resolution.

Breitenberger ${ }^{(1)}$ has reported an interesting result relating to crystals of anthracene grown from solution and from the melt of the same extremely pure
material. The crystals grown from the solution were found to have a Iuminescence efficiency thirty per cent greater than that of the crystals grown from the melt. Breitenberger assumed that defects in the crystals grown from the melt acted as quenching traps thus reducing the light output. The degree of perfection of a crystal can vary throughout its volume causing an increase in the scintillation variance. The damage to an anthracene crystal caused by a fiux of alpha particles could therefore cause a change in the intrinsic resolution of the crystal as observed by Wright and Garlick.


Fig. 1 Block Diagram Of Scintillation Counter

# NOT TO <br> SCALE 



Fig. 2 Arrangement for Stressing The Crystal

It was supposed that perhaps some of the intrinsic resolution of thallium activated sodium iodide was due to lattice defects and an experiment was undertaken which would detect a change in resolution of a scintillation counter if any occurred when the phosphor was stressed in such a way as to produce dislocations within the crystal.

The apparatus used consisted of a scintillation counter with provision for stressing the crystal. A cesium- 137 source of 662 kev . gamma rays was placed near the 1.5 centimeter square Harshaw crystal under investigation. The thickness of the crystals varied from one to four millimeters. Light from the crystals was detected by a Dumont 6292 photomultiplier tube whose dynode potentials were maintained by a Northeast model RE-10010 high voltage power supply. The pulse output from the photomultiplier tube was fed to a cathode follower whose plate supply was obtained from a Lambda model C281M power supply. The output from the cathode follower was amplified by a Nuclear Enterprise model NE5202 pulse amplifier whose output was connected by a terminated coaxial cable to Computing Devices of Canada model AEP2230 hundredchannel pulse height analyser in another laboratory.

The apparatus used for stressing the crystal is sketched in figure 2. The pitch of the screw depressing the so-called "knife-edge" was one-fortieth of an inch and the top of the screw was marked at twenty regularly spaced intervals so that it was possible to move the screw downward by an amount as small as $3.18 \times 10^{-3}$ centimeters. The radius of curvature of the 1.5 centimeter square crystals used was approximately $\left(\frac{0.283}{a}\right)$, where $a$ is the depression of the centre of the crystal in centimeters relative to the edges. It is suspected
that during stressing the edges of the crystal were also forced downward so that the depression, a, may have been less than the downward displacenent of the screw.

Dekker ${ }^{(9)}$ has shown that the dislocation density, $q$, introduced when a crystal is plastically bent into a radius of curvature of $r$ is $q=\left(\frac{I}{r v}\right)$, where $v$ is the magnitude of the Burger's vestor. In this experiment, then, $q=\frac{a}{0.283 \mathrm{v}}$.

The resolution in each trial was determined by the method outlined in Appendix A using a computer. The computer programme is given in Appendix B.

The earliest measurements showed that any change in energy resolution with the dislocations introduced by plastic bending was small. It was therefore necessary that the electronic apparatus be of high stability。 Where a choice of amplifiers and power supplies was possible the units used were the ones having the higher or highest stability. Instability caused by rather sudden and large temperature changes of the laboratory was quite troublesome in the earlier experiments.

One of the first experiments showing any effect at all was that labelled Run A the results of which are presented in Table A and in Graph A. At each value of the strain the resolution was measured several times resulting in a mean value of the resolution, $\vec{R}$, and in a mean deviation from the mean of $\Delta R$ for that strain. The variation in $\bar{R}$ is, in the case of Run $A$, not much larger than $\Delta R$ and is not monotonic as the strain increases.

Run B (see Table B and Graph B) consists of resolution measurements made at two values of strain. The strain was increased rapidly in this experiment in order to reduce the effect of instabilities. The results of this run were encouraging. The crystal broke when the strain was increased further.

Run C (see Table C and Graph C) was done over a period of several days. The strain was only increased once a day so that a relatively large number of determinations of the resolution at each strain was made. The temperature changed throughout some of these days and also from day to day during this run. In the case where for two consecutive days the strain was the same, the agreement was not good.

Through the kindness of Dr. F. M. Kelly and Mr. E. Tomchuk it was possible to move most of the apparatus into a laboratory in which the temperature was controlled to within a fraction of a Centigrade degree. The only equipment not moved into this laboratory were the pulse height analyser and its A.C. voltage regulator. Runs $D$ and $E$ were done in this laboratory.

Before Run D was done the stability of the apparatus was checked and it was found that with constant crystal strain the value of $\Delta R$ was $0.06 \%$ on one day and $0.05 \%$ on another. The results of Run D given in Table $D$ and in Graph D indicate an increase in resolution with phosphor strain.

A few days later the same phosphor was further strained in Run $E$ (see Table E and Graph E) and, fortunately, it was possible to strain this phosphor sample much further than previous samples without breaking it. The curve in Graph E indicates a definite increase in resolution with increasing crystal strain. A stability test preceded this run yielding a value of $\Delta R$ of $0.07 \%$ for ten determinations of the resolution at constant strain. It is this value of $\Delta R$ that is indicated in Graph E since in this run the resolution was only measured once for each value of strain.

TABLE A

| DATE | $\begin{aligned} & \text { DEPRESSION } \\ & (\mathrm{cm} .) \end{aligned}$ | NUMBER OF TRIAIS | $\frac{\text { RESOLUTION }}{(\%)}$ | NEAN DEVIATION (\%) |
| :---: | :---: | :---: | :---: | :---: |
| August 15 | $1.59 \times 10^{-3}$ | 2 | 15.35 | 0.30 |
|  | $3.18 \times 10^{-3}$ | 3 | 15.72 | 0.03 |
|  | $4.76 \times 10-3$ | 5 | 15.81 | 0.09 |
|  | $6.36 \times 10^{-3}$ | 3 | 15.70 | 0.01 |
|  | $7.94 \times 10^{-3}$ | 2 | 15.94 | 0.03 |

## TABLE B

| August 16 | $0.00 \times 10^{-3}$ | 5 | 14.82 | 0.07 |
| ---: | :--- | :--- | :--- | :--- |
|  | $4.76 \times 10^{-3}$ | 3 | 15.61 | 0.15 |

## TABLE C

| August 21 | $0.00 \times 10^{-3}$ | 13 | 14.41 | 0.07 |
| :--- | :--- | :--- | :--- | :--- |
| August 22 | $0.00 \times 10^{-3}$ | 16 | 14.75 | 0.06 |
| August 23 | $2.38 \times 10^{-3}$ | 18 | 15.02 | 0.07 |
| August 24 | $4.76 \times 10^{-3}$ | 14 | 15.17 | 0.07 |
| August 25 | $6.36 \times 10^{-3}$ | 14 | 15.51 | 0.13 |
| August 26 | $7.94 \times 10^{-3}$ | 11 | 15.35 | 0.08 |
| August 27 | $7.94 \times 10^{-3}$ | 12 | 15.56 | 0.11 |
| August 28 | $9.54 \times 10^{-3}$ | 10 | 15.62 | 0.09 |

TABLE D

September $8 \quad 0.00 \times 10^{-3}$
$1.59 \times 10^{-3}$
$2.38 \times 10^{-3}$
$3.18 \times 10^{-3}$
$4.76 \times 10^{-3}$
September $114076 \times 10^{-3}$
$6.36 \times 10^{-3}$
$7.94 \times 10^{-3}$
$9.54 \times 10^{-3}$
$\begin{array}{ccc}\text { NUMBER OF } & \text { RESOLUTION MEAN DEVIATION } \\ \text { TRIALS } & (\%) & (\%)\end{array}$
14.95
0.06
140.91
0.08
15.12
0.02
15.04
0.05
15.00
0.07
15.16
0.12
15.08
0.07
15.04
0.06
15.15
0.02

TABIFE E
September $16 \quad 9.54 \times 10^{-3}$ $11.1 \times 10^{-3}$
$12.7 \times 10^{-3}$
$14.3 \times 10^{-3}$
$15.9 \times 10^{-3}$
$17.5 \times 10^{-3}$ 19.1 $\times 10^{-3}$ $22.3 \times 10^{-3}$ $25.4 \times 10^{-3}$ $28.7 \times 10^{-3}$ $31.8 \times 10^{-3}$ $38.2 \times 10^{-3}$ $44.5 \times 10^{-3}$

2
15.65
0.05
15.64
15.75
$15 \cdot 58$
15.88
15.69
15.68
15.78
15.88
16.04 .
16.28
16.23
16.43






## V DISCUSSION OF RESULTS

Instability of the apparatus has been present to a varying degree throughout the experiments. The results do, however, indicate that the introduction of dislocations by the manner employed does in fact cause an increase in the energy resolution of thallium activated sodium iodide phosphors.

Various questions now arise. Are all solid phosphors similarly affected? Is the effect caused by the presence, or instead by the non-uniform presence, of dislocations? Is the resolution "intrinsic to the phosphor" caused partly, mostly, or wholly, by the presence of dislocations?

The work of Zerby et al. ${ }^{(8)}$ does give an explanation of the intrinsic resolution which yields reasonable values for $\mathrm{NaI}(T I)$. The stressing of the crystal in the last run resulted in an increase in resolution of only $0.8 \%$. If the lattice constant of NaI is taken to be 6.5 Angstrom units and is assumed equal to the Burger's vector, then the density of dislocations introduced in this run is about $2 \times 10^{6} \mathrm{~cm}^{-2}$ for a depression of $4.45 \times 10^{-2}$ centimeters. Johnston ${ }^{(10)}$ has estimated that carefully handled crystals of NaI (TI) which are annealed at the melting point and
then slowly cooled would have a dislocation density of from $10^{5}$ to $10^{6}$ per square centimeter. It may well be that the affect of dislocations on the resolution is far more pronounced at lower dislocation densities and that the effect of adding $2 \times 10^{6}$ per square centimeter is appreciably reduced when there are already $10^{5}$ or $10^{6}$ dislocations per square centimeter. If this were true then the contribution of dislocations to the intrinsic resolution of this phosphor could be appreciable.

## APPENDIX A

Best Fit to a Gaussian

Since the determination of the energy resolution of a scintillation spectrometer was an important part of the project it was necessary to devise a more accurate method than the simple evaluation of the full-width-at-half-maximum from a graph of a scintillation spectrum. An outline of the method used follows:

1. The photopeak is "Gaussian" over some region which is determined by a graphical method outlined by Onno (See Appendix C).
2. A Gaussian may be converted into a parabola and the photopeak is converted in this way.
3. Of the family of curves that constitute a parabola, namely $\mathrm{w}=-\mathrm{bx} \mathrm{x}^{2}+d x+c$, one is chosen such that the sum of the squares of the vertical distances from the parabola to each of the points of the photopeak, converted in the above way, is a minimum. This requires that the partial derivatives of the above sum, with respect to $b, c$, and $d$, are zero. That is, we have three linear equations in $b, c$, and $d$, which are solved by the usual method of determinants for $b$, $c$, and $d_{\text {' The resolution and peak }}$ position may then be found.
4. Weighting: The standard error in $y$ counts is $\sqrt{y}$. Weighting was accomplished by assuming that a point on the graph of a scintillation spectrum representing $y$ counts was actually $\sqrt{y}$ coincident points.

## Theory

## Gonversion of Gaussian to Parabola:

Gaussian: $\quad y=\frac{T}{s \sqrt{2 \pi}} \exp \frac{-(x-\bar{x})^{2}}{2 s^{2}}$
where
$y$ is the number of pulses in the voltage range $x$ to $(x+d x)$,
$S$ is the standard deviation,
I is the total number of pulses in a Gaussian distribution,
and $\overline{\mathrm{X}}$ is the position of the peak.
Set $w=\ln y=\ln \frac{T}{S \sqrt{2 \pi}}-\frac{(x-\bar{x})^{2}}{2 s^{2}}=a-b(x-\bar{x})^{2}=-b x^{2}+2 b \bar{x} x+a-b \bar{x}^{2}$,
where

$$
\begin{aligned}
& a=\ln \frac{T}{S \sqrt{2 \pi}}, \\
& b=\frac{1}{2 S^{2}}
\end{aligned}
$$

i.e. $w=-b x^{2}+d x+c$,
where $c=a-b x^{-2}$,
$d=2 b \bar{x}$.

Best Fit to Parabola:
The photopeak consists of points ( $X_{i}, y_{i}$ ) which are converted points $\left(x_{i}, w_{i}\right)$ to which we must fit a parabola.

Set $S=\Sigma\left(w_{i}-w\left(x_{i}\right)\right)^{2}=\Sigma\left(w_{i}+b x_{i}^{2}-d x_{i}-c\right)^{2}$.
As outlined above we set the three partial derivatives of $s$ equal to zero.

Ignoring the subscript "i",

$$
\begin{aligned}
& \left(\Sigma x^{4}\right) b-\left(\Sigma x^{3}\right) d-\left(\Sigma x^{2}\right) c=-\left(\Sigma w x^{2}\right), \\
& \left(\Sigma x^{3}\right) b-\left(\Sigma x^{2}\right) d-(\Sigma x) c=-(\Sigma w x), \\
& \left(\Sigma x^{2}\right) b-(\Sigma x) d-N c=-(\Sigma w)
\end{aligned}
$$

where $N$ is the number of points considered.
Set

$$
\begin{array}{ll}
A=\Sigma x, & J=\Sigma w, \\
B=\Sigma x^{2}, & K=\Sigma w x, \\
C=\Sigma x^{3}, & L=\Sigma w x^{2} . \\
D=\Sigma X^{4}, &
\end{array}
$$

Further, set

$$
F=\left|\begin{array}{lll}
\mathrm{D} & \mathrm{C} & \mathrm{~B} \\
\mathrm{C} & \mathrm{~B} & \mathrm{~A} \\
\mathrm{~B} & \mathrm{~A} & \mathrm{~N}
\end{array}\right|,
$$

$$
G=\left|\begin{array}{lll}
D & L & B \\
C & K & A \\
B & J & N
\end{array}\right|,
$$

and

$$
H=-\left|\begin{array}{lll}
\mathrm{L} & \mathrm{C} & \mathrm{~B} \\
\mathrm{~K} & \mathrm{~B} & \mathrm{~A} \\
\mathrm{~J} & \mathrm{~A} & \mathrm{~N}
\end{array}\right|
$$

Then

$$
\mathrm{b}=\frac{\mathrm{H}}{\mathrm{~F}}, \text { and } \mathrm{d}=\frac{\mathrm{G}}{\mathrm{~F}} .
$$

ie.

$$
s^{2}=\frac{1}{2 b}=\frac{F}{2 H}, \quad \bar{x}=\frac{d}{2 b}=\frac{G}{2 H} .
$$

## Weighting

Let $z_{i}$ be $\sqrt{y_{i}}$, then $z_{i}$ are the weighting factors.
A becomes $\boldsymbol{\Sigma} \mathbf{X z}$,
$B$ becomes $\Sigma x^{2} z$,
$C$ becomes $\Sigma x^{3} z$
D becomes $\Sigma x^{4} z$,
J becomes $\Sigma w z$,
K becomes $\Sigma w x z$,
L becomes $\Sigma w x^{2} z$,
$N$ becomes $\Sigma z$.

## Full-Width-At-Half-Maximum, W

From the equation of a Gaussian given above we see that when $x=\bar{x}, y=\frac{T}{s \sqrt{2 \pi}}$.
i.e. The half-maximum value of $y$ is $T$

$$
\frac{1}{2 s \sqrt{2 \pi}}
$$

Let $x^{\prime}$ be the values of $x$ at these points.
then $\frac{1}{2}=e \frac{-\left(x^{1}-\bar{x}\right)^{2}}{2 s^{2}}$
$\ln \left(\frac{1}{2}\right)=\frac{-\left(x^{\prime}-\bar{x}\right)^{2}}{2 s^{2}}$
i.e. $\left(x^{1}-\bar{x}\right)^{2}=2 S^{2} \quad \ln 2$.
$W=2\left|x^{\prime}-\bar{x}\right|=2 s \sqrt{2 \ln 2}=2.3550 \mathrm{~s}$.
A program for a Bendix G -15 computer has been prepared to do the above analysis of scintillation spectra (See Appendix B).

## APPENDTX B

## Programme for a Weighted Best Fit to a Gaussian

Data was obtained from a 100 channel pulse height analyzer.

Each of the hundred channels were of width $U(U=0.3,0.15,0.06$, or 0.03 volts) and the voltage corresponding to a pulse falling in the first channel was determined by the setting of the bias control, V ( $V=1,2,3, \ldots . .10$ )

The height, $h$, of a pulse falling in channel $Q$ when the channel width and bias controls were respectively set at $U$ and $V$ is:

$$
h=0.3+3(V-1)+U Q+\frac{1}{2} U \pm \frac{1}{2} U \text { volts. }
$$

For the purpose of programming the pulse height scale is in channels and the pulse height origin is the least value of $x, x_{0}$, for which the photopeak is Gaussian. In this coordinate system pulse heights are denoted by $x^{\prime}$, i.e. $x=x_{0}+x^{\prime}$. Also, $A, B, C, D, F, G, H, K$, and $L$, become $A^{\prime}, B^{\prime}, C^{\prime}, D^{\prime}, F^{\prime}, G^{\prime}, H^{\prime}, K^{\prime}$, and $L^{\prime}$ respectively.

The full-width-at-half-maximum is found from $W=2.355^{\circ} \mathrm{S}=2.355^{\mathrm{Ft}} \frac{\mathrm{Ft}}{2 \mathrm{H}}$ and the resolution $R$ is found from $R=\frac{W W}{h}$, where $h$ is the height of a pulse in volts for $\mathrm{x}=\overline{\mathrm{x}}$. (it is assumed that all pulses in a channel have a height corresponding to the midpoint of that channel.)

Partial Expansion of Determinants

$$
\begin{aligned}
& F=\left|\begin{array}{lll}
D & C & B \\
C & B & A \\
B & A & N
\end{array}\right|=D\left(N B-A^{2}\right)+C(-N C+A B)-B\left(-A C+B^{2}\right) \\
& G=\left|\begin{array}{lll}
D & L & B \\
C & K & A \\
B & J & N
\end{array}\right|=D(N K-A J)+L(-N C+A B)+B C J-B^{2} K \\
& H=-\left|\begin{array}{lll}
I & C & B \\
K & B & A \\
J & A & N
\end{array}\right|=-I\left(N B-A^{2}\right)+C(N K-A J)-A B K+B^{2} J
\end{aligned}
$$

## PROGRAMME

"IOC" gives the location of the command KOPADDR
IOC
K OP ADDR
CONTENIS OF ACCUMULATOR
NOTES
First, we set a word base according to the number of points considered:

I400 172 OOWL

Storage of some useful numbers:
140104121000
$0490900 \quad 0$ at 0900
0420994
$0420994 \quad x_{0}$
$0482100 \quad x_{0} / x_{0}=1$
14050490902
3700000
3710002
3720016
$W I_{3}=16$
3420902 1,2,...,9
14100430902 2,3, ...,10
3490904
3761409
3700000
0420904
2

14150490970
04209063
$0480920 \quad 0.3$
$0430970 \quad 2.3$
$0182100 x_{0}$
$W_{3}=0$
$W_{1}=2$
$W I_{1}=2(\mathrm{~N}-1)$, (WL is set according to the number of points considered)

IOC K OP ADDR CONTENTS OF ACCUMULATOR NOTES
$0490970 \quad 2.3$ at 0970
$1420 \quad 0 \quad 420910 \quad 5$
$0480920 \quad 0.5$
$0480920 \quad 0.05$
$0430970 \quad 2.35$
$0490970 \quad 2.35$ at 0970
$14250420910 \quad 5$
$0480920 \quad 0.5$
$0480920 \quad 0.05$
$0480920 \quad 0.005$
$0430970 \quad 2.355$
II30 $0490922 \quad 2.355$ at 0922
Calculation of the $z_{i}$ :
14311700000
$W B_{2}=0$
1710002
$W D_{1}=2$
$1421000 \quad \mathrm{y}_{\mathrm{i}}$
$0081297 \sqrt{y_{i}}=z_{i}$
14351491700
1761433
$z_{i}$ at channel 17

1700000
Calculation of $x_{i}^{\prime}$ :
$11438 \quad 0412100 \quad 0$
0491800
14401421800 0,1,....,

0430902 I,2, ..., i, e. $x_{i}{ }^{\prime}$
$W_{1}=0$ 0 at 1800
loop to cormand 1433

IOC
K OP ADDR CONTENIS OF ACCUMULATOR
0491802

176 14 0
1700000
Calculation of $\mathrm{A}^{1}$ :
I145 $1421800 x_{i}{ }^{\prime}$
1 $41700 x_{1}{ }^{1} x_{i}$
$1491300 \quad x_{i}{ }^{\prime} \mathrm{z}_{1}$ at channel 13
1761445
1700000
$1450 \quad 0 \quad 412100 \quad 0$
$1431300 \quad x_{i}{ }^{\prime} x_{i}$
1761451

1700000
0490924
$1455 \quad 0310001$
0342100
Galculation of $B^{1}$ :
1457 I $421300 \quad x_{i}{ }^{\prime} z_{i}$
$1441800 \quad x_{i}{ }^{\prime}{ }^{2} z_{i}$
1491300
$\begin{array}{llll}1460 & 1 & 76 & 1457\end{array}$
1700000
$0422100 \quad 0$
$1431300 \quad x_{i} \prime^{2} z_{i}$
loop to cormand 1445
$W B_{1}=0$

100 p to command $1 \mathrm{l}_{5} 51$ getting
$\sum_{i} x_{i}{ }^{\prime} z_{i}=A^{\prime}$
$W B_{1}=0$
AI at 0924
type 1
type A1
$x_{i} \prime^{2} z_{i}$ at channel 13
loop to cormand $\mathrm{H}_{4} 57$
$W B_{1}=0$

NOTES
$\mathrm{x}_{\mathrm{i}}{ }^{\prime}$ at channel 18, since 0
already at 1800 (see command 1439)
loop to cormand 1440
$W_{1}=0$

INC K OP ADD CONTENTS OF ACCUMULATOR
I 76 1463
$1465 \quad 1700000$
0490926
14670310002
0342100
Calculation of $\mathrm{C}^{\prime}$ :
1469
1421300
$x_{i}{ }^{\prime 2} z_{i}$
1470 I $441800 x_{i} i^{3} z_{i}$
1491300
I 761469
1700000
$0422100 \quad 0$
$1475 \quad 1431300 \quad \mathrm{x}_{\mathrm{i}} \mathbf{\prime}^{3} \mathrm{z}_{\mathrm{i}}$
1761475

1. 700000

0490928
0310003
$1480 \quad 0 \quad 342100$
Calculation of $D^{\prime}$ :
$7481 \quad 1421300$
1441800 $x_{i}{ }^{3} z_{i}$

1491300
1761481
1485 1700000

NOTES
loop to command 1463 getting $\sum x_{i}{ }^{\prime}{ }^{2} z_{i}=B t$
$W_{1}=0$
$B^{\prime}$ at 0926
type 2
type $\mathrm{B}^{1}$
$x_{i}{ }^{3} z_{i}$ at channel 13
Lop to command 1469
$W B_{1}=0$
loop to command 1475 getting $\sum_{i} x_{i} i^{3} z_{i}=C^{\prime}$
$W B_{1}=0$
C1 at 0928
'type 3
type C1
$x_{i}{ }^{\prime 4} z_{i}$ at channel 13
loop to command 1481
$W B_{1}=0$

LDC K OP ADD CONTENTS OF ACCUMULATOR
NOTES
$0412100 \quad 0$
$1431300 \quad x_{i} \mathbf{t}^{4} z_{i}$
1761487
$1489 \quad 1700000$
loop to command 1487 getting
$\sum_{i} x_{i} \cdot{ }^{4} z_{i}=D^{\prime}$
$W B_{1}=0$
$1490 \quad 0490930$
Di at 0930

- 310004

0342100
Calculation of N :
114930 LI $2100 \quad 0$
$1431700 \quad z_{i}$
1495 I 761494

1700000
0490992
0310005
0342100
Calculation of J :
$1500 \quad 1421000 \quad y_{i}$
$0081117 \quad \ln y_{i}=w_{i}$
$141700 \quad w_{i} z_{i}$
1491300
I 761500
15051700000
$0412100 \quad 0$
$1431300 \quad w_{i}{ }_{i}$
1 761507

$$
\sum_{i} w_{i} z_{i}=J
$$

INC K OP ADDR CONTENTS OF ACCUMULATOR NOTES
I 700000
15100490932
15110310006
0342100
Calculation of $K^{\prime}$ :
1513
1421300 $w_{i} z_{i}$
$1441800 \quad w_{i} x_{i}^{1}{ }_{i}$
15151491300
1761513
1700000
$0472100 \quad 0$
1431300

$$
w_{i} x_{i}^{l} z_{i}
$$

15201761519

1 700000
0490934
0310007
0342100
Calculation of $L^{\prime}$ :
15251421300 $w_{i} x_{i}{ }^{\prime} z_{i}$
141800 $w_{i} x_{i} \prime^{2} z_{i}$
1491300
1761525
1700000
15300412100
$1431300 \quad w_{i} x_{i} \prime^{2} z_{i}$
15321761531
loop to command 1519 getting $\sum_{i} w_{i} x_{i}{ }^{\prime} z_{i}=K^{\prime}$
$W B_{I}=0$
$K^{\prime}$ at 0934
type 7
type $K^{\prime}$
$w_{i} x_{i}{ }^{2} z_{i}$ at channel 13
loop to command 1525
$W_{I}=0$

IOC K OP ADDR CONIENIS OF ACCUMUTATOR
1700000
0490936
15350310008
0342100

NOTES
$W B_{I}=0$
L' at 0936
type 8
type $L^{\text {: }}$

Calculation of three sub-determinants:
15370420992 N
0 L4 0926 NB:
0490954 NB! at 0954
$15400400924-A^{\prime}$
$0440924-A i^{2}$
0430954 (NB1 $-A 1^{2}$ )
$0490954 \quad\left(\mathrm{NB}^{1}-\mathrm{A}^{2}\right)$ at 0954
$0400992-N$
15450440928 -NC
0490956 (-NC1) at 0956
0420924 A:
0440926 A'B'
0430956 ( $-\mathrm{NC}^{1}+\mathrm{A}^{1} \mathrm{~B}^{1}$ )
$1550 \quad 0490956$
$\left(-N C 1+A^{\prime} B^{\prime}\right)$ at 0956
0400924 -A
0440928 -ATC!
$\left.0490958 \quad\left(-A^{\prime \prime}\right)^{1}\right)$ at 0958
$15530420926 \quad B^{1}$
$0440926 \quad \mathrm{Bi}^{2}$
$15550430958 \quad\left(-A^{\prime} C^{\prime}+B^{2}\right)$

15750490938

Calculation of $G^{1}$ ：
15780420932 J
0 山 $40928 \quad \mathrm{CI}$

IOC K OP ADDR CONIENIS OF ACCUMULATOR
0490958
0420992 N
0440934 NK：
0490960
（NK＇）at 0960
$1560 \quad 0400924 \quad$－A＇
0 山 0932 －A＇J
0430960 （NK－ATJ）
0490960
Calculation of Ft ：
$1564 \quad 0420930 \quad$ D＇
$15650440954 \quad D^{\prime}\left(N B B^{\prime}-A^{2}\right)$
0490970
$D^{\prime}\left(N^{\prime}{ }^{\prime}-A^{\prime}\right)$ at 0970
0420928 C1
$0440956 \quad C^{1}\left(-\mathrm{NC}^{1}+\mathrm{A}^{1} \mathrm{~B}^{1}\right)$
$1570 \quad 0430970 \quad D^{\prime}\left(N B^{1}-A^{2}\right)+C i\left(-N C^{1}+A^{\prime} B^{\prime}\right)$
0490970
$D^{\prime}\left(N B^{1}-A^{t^{2}}\right)+C^{1}\left(-N^{1}+A^{\prime} B^{1}\right)$ at 0970
0400926 － $\mathrm{B}^{\prime}$
0 岖 $0958 \quad-B^{\prime}\left(-A^{\prime} C^{t}+B^{2}\right)$
0430970 F＇

0310009
0342100
NOTES
$\left(-A^{\prime} C^{\prime}+B^{2}\right)$ at 0958
（see command 1559）
（NK ${ }^{\prime}$ AIJ）at 0960

0
Fi at 0938
type 9
type Ft

NOTES
$0490970 \quad B^{\prime} C^{\prime J}$ at 0970
$0400934-K^{\prime}$
$0440926-B^{\prime} \mathrm{K}^{\prime}$
$0440926-\mathrm{Br}^{2} \mathrm{~K}^{1}$
$15850430970 \quad\left(-\mathrm{B}^{2} \mathrm{~K}^{1}+\mathrm{B}^{\prime} \mathrm{C}^{1} \mathrm{~J}\right)$
0490970
$\left(-B^{2} K^{1}+B^{1} C^{1} J\right)$ at 0970
0420936 Li
$0 \mathrm{~L}_{4} 0956 \mathrm{~L}^{1}\left(-\mathrm{NC}^{1}+\mathrm{A}^{1} \mathrm{~B}^{1}\right)$
0430970
$L^{\prime}\left(-N G^{\prime}+A^{\prime} B^{\prime} J-B^{2} K^{\prime}\right.$
1590
0490970
$\mathrm{I}^{\mathrm{I}}\left(-\mathrm{NC} I+\mathrm{A}^{\prime} \mathrm{B}^{\prime}\right)+\mathrm{B}^{\prime} \mathrm{CI} \mathrm{J}-\mathrm{Br}^{2} \mathrm{~K}^{\prime}$
at 0970

0420930 D
$0440960 \mathrm{D}^{\prime}\left(\mathrm{NK}^{\prime}-\mathrm{A}^{\mathbf{T}} \mathrm{J}\right)$
0430970 Gt
0490940 Gt at 0940
1595
0310010 type 10
0342100
Calculation of $\mathrm{H}^{\prime}$ :
1597
0420932 J
0440926 BiJ
$0440926 \quad B^{2} \mathrm{~J}$
1600
0490970
0400924 -A:
$0440926-A \cdot B 1$
$16030440934-A^{\prime} B^{\prime} K^{\prime}$
$B \mathbf{r}^{2} \mathrm{~J}$ at 0970

| IOC | K OP ADDR | CONTENTS OF AGCUMULATOR | NOTES |
| :---: | :---: | :---: | :---: |
|  | 0430970 | - $\mathrm{B}^{\prime} \mathrm{J}-\mathrm{A}^{\prime} \mathrm{B}^{\prime} \mathrm{K}^{\prime}$ |  |
| 1605 | 0490970 |  | - $\mathrm{B}^{2} \mathrm{~J}-\mathrm{A}^{\mathbf{r}} \mathrm{K}^{\prime}$ at 0970 |
|  | 0420928 | 61 |  |
|  | 0440960 | $C^{\prime}\left(N K^{\prime}-A^{\prime} \mathrm{J}\right)$ |  |
|  | 0430970 | $C \cdot\left(N K^{i}-A^{\prime} J\right)-A^{1} B^{\prime} K^{t}+B^{2} J$ |  |
|  | 0490970 |  | $C^{1}\left(N^{\prime}-A^{\prime} \mathrm{J}\right)-A^{\prime} \mathrm{B}^{\prime} \mathrm{K}^{\prime}+\mathrm{B}^{2} \mathrm{~J}$ at 0970 |
| 1610 | 0400936 | $-I^{\text {t }}$ |  |
|  | 0440954 | $-L^{\prime}\left(N B^{\prime}-A r^{2}\right)$ |  |
|  | 0430970 | H |  |
|  | 0490942 |  | $\mathrm{H}^{1}$ at 0942 |
|  | 0310011 |  | type 11 |
| 1615 | 0342100 |  | type H: |
|  | Calculation | of $\bar{x}^{\prime}$ : |  |
| 1616 | 0420940 | $G^{1}$ |  |
|  | 0.480942 | G $1 / \mathrm{H}^{1}$ |  |
|  | 04480904 | $\left(G^{1} / 2 H^{\prime}\right)=\bar{X}^{\prime}$ |  |
|  | 0490944 |  | $\overline{\mathrm{x}}$ at 0944 |
| 1620 | 0420910 | 5 |  |
|  | 0480920 | 0.5 |  |
|  | 0430944 | $\overline{x^{\prime}}+0.5$ |  |
|  | 0430994 | $\mathrm{x}_{0}+\overline{\mathrm{x}},+0.5=\overline{\mathrm{x}}$ |  |
|  | 0490944 |  | $\bar{x}$ at 0944 |
| 1625 | 0310012 |  | type 12 |
|  | 0342100 |  | type $\bar{x}$ |

Calculation of W :
1627 0420938 Ft

0480942 Fi/Et
$0480904 \quad \mathrm{Ft} / 2 \mathrm{H}^{t}$
$16300081297 \quad \sqrt{F^{1 / 2 H^{t}}}=S$
$040922 \quad 2.355 \mathrm{~S}=\mathrm{W}$
0310013
0342100
Calculation of h :
16340440998 UW
16350490946
0420996
V

0410902
$0 \mu_{4} 0906 \quad 3(V-1)$
0490970
$1640 \quad 0420998$
U
0 U 40944
$04309703(\nabla-1)+U \mathbb{x}$
0490970
04209063
$16450480920 \quad 0.3$
$0430970 \quad 0.3+3(\nabla-1)+\mathbb{U}=h$
$031 \mathrm{OOH}_{4}$

- 342100
type 14
type h
type h
$3(\mathrm{~V}-1)+\mathbb{X}$ at 0970
"

3(V-1) at 0970
type W

UW at 0946

IOC K OP ADDR CONTE
Calculation of $R:$
$16490470946 \quad \mathrm{UW} / \mathrm{h}$
$16500440920 \quad 10 \mathrm{JW} / \mathrm{h}$
$0440920 \quad 100 \mathrm{UW} / \mathrm{h}=\mathrm{R}$
0310015 type 15
0342100
16540670000
type $R$
hald in manual mode

USE OF THE COMPUTER MEMORY IN THE PROGRAMME:

| 0900 | 0 |
| :---: | :---: |
| 0902 | 1 |
| 0904 | 2 |
| 0906 | 3 |
| 0908 | 4 |
| 0910 | 5 |
| 0912 | 6 |
| 0914 | 7 |
| 0916 | 8 |
| 0918 | 9 |
| 0920 | 10 |
| 0922 | 2.355 |
| 0924 | A' |
| 0926 | $B^{\prime \prime}$ |
| 0928 | $\mathrm{Cl}^{\text {P }}$ |
| 0930 | D' |
| 0932 | J |
| 0934 | K' |
| 0936 | L! |
| 0938 | Ft |
| 0940 | G! |
| 0942 | H! |
| 0944 | $\overline{\mathrm{X}}$ ' and $\overline{\mathrm{x}}$ |
| 0946 | UW |
| 0948 |  |

0950
0952
0954
0956
0958
0960
(NBI) and (NBImA ${ }^{2}$ )

0970
miscellaneous numbers

0980

0990
0992
0994
0996
0998

## N

$x_{0}$ V

U
channel 10
channel 11
channel 12
channel 13
channel $I_{4}$
channel 15
channel 16
channel 17
channel 18
$\Psi_{i}$
In subroutine square root subroutine $x_{i} z_{i}, x_{i}^{2} z_{i}, x_{i}{ }^{3} z_{i}, x_{i}{ }^{4} z_{i}, w_{i} z_{i}, w_{i} x_{i} z_{i}, w_{i} x_{i}{ }^{2} z_{i}$ programme
programme
progranme
$z_{i}$
$x_{i}$

## APPENDIX C

## Determination of the Gaussian Region

The scintillation spectrum closely approximates a Gaussian only over a limited region near the photopeak. On the high energy side of the photopeak background counts become appreciable above a certain level and on the low energy side counts in the Compton peak become appreciable below another certain level. It was desired to determine the "Gaussian region" in order to fit a Gaussian only to those points that lay very nearly on a Gaussian.

Several graphical methods have been used in the analysis of scintillation spectra. Descriptions of three of these methods follow. I

The equation of a Gaussian is:
$y=\frac{T}{S \sqrt{2 \pi}} \exp -\frac{(x-\bar{x})^{2}}{2 S^{2}}$, as before.
Set $Y=\frac{T}{S \sqrt{2 \pi}} \quad$ and $X^{\prime \prime}=x-\bar{x}$,
then $y / Y=\exp \frac{-x^{n^{2}}}{2 S^{2}} \quad$ and $\ln y / Y=\frac{-x^{\prime \prime}}{2 S^{2}}$.
Therefore a plot of $\ln (y / Y)$ versus $X^{\prime \prime}$ yields a straight line through the origin with slope $\frac{-1}{2 S^{2}}$ or a plot of $(y / Y)$ versus $x^{\prime \prime}$ on semilogarithmic paper gives rise to a straight line over the Gaussian region,

This method assumes a knowledge of the values $Y$ (the maximum value of $y$ ) and of $\bar{x}$ ( the position of the photopeak).

II BoekeIheide Method.
Boekelheide (11) has outlined a method which makes use of "probability paper" (12) on which the abscissa scale is linear and the ordinate scale is such that plotting
$\frac{t}{T}=\frac{1}{s \sqrt{2 \pi}} \int_{x}^{\infty} \exp -\frac{(x-\bar{x})^{2}}{2 s^{2}} d x$ versus $x$ results in a straight line of slope $\frac{-1}{S}$, where $t$ is the area under that part of the Gaussian which lies above $x$, and $T$, the total area under the Gaussian, is determined by doubling the area above the peak position. (A Gaussian is symmetrical about the peak position. Asymmetry results in the photopeak from the Compton contribution. )

This method requires a knowledge of the peak position and a calculation of the fractions ( $t / T$ ) by summing the contents of the channels of a multichannel pulse height analyzer from the top of the photopeak down to the channel in question and dividing the result by $T$.

## III Onno Method

P. Onno(13)has produced graph paper (14) ("Gaussian graph paper") with a linear abscissa scale and with such an ordinate scale that plotting $R=100 \exp -\frac{(x-\bar{x})^{2}}{2 S^{2}}$ versus $x$ results in a straight line whose slope is proportional to ( $1 / \mathrm{S}$ ) where $R=\frac{100 \mathrm{Y}}{Y}$.

This method requires a knowledge of $Y$ and a calculation of $R$ for each value of $y$, a calculation readily done on a slide rule. The method is much quicker than that of Boekelheide and is, as Onno has shown, far more shapesensitive to departures from a Gaussian distribution than the integral method of Boekelheide.
(1) Breitenberger: Progress in Nuclear Physics, 4, 56. London:Pergamon Press (1955).
(2) G. T. Wright: Journal of Scientific Instruments, 31, 377, (1954).
(3) F. Seitz: Joum. Chem. Phys. 6, 150, (1938).
(4) G.F.J. Garlick and G.T. Wright: British Journal of Applied Physics, 5, 13, (1954)
(5) G.G. Kelly, P.R. Bell, R.C. Davis, and N.H. Lazar: Nucleonics, 114, No.4, 53, (1956).
(6) D.I. Cooper and G.A. Morton: Nucleonics, I4, No. 4, 57, (1956).
(7) J.J. Hopkins: Review of Scientific Instruments, 22, 29, (1951).
(8) C.D. Zerby, A. Meyer, R.B. Nurray: Bulletin of the American Physical Society, Series II, 5, No.6, Abstract J5, 415, (November, 1960).
(9) A.J. Dekker: Solid State Physics, page 96, Prentice-Hall (1959).
(10) W.E. Johnston: private communication.
(11) I.F. Boekelheide: Review of Scientific Instruments, 31, No 99, 1001, (1960).
(12) Keuffel and Esser: No. 359 - 23.
(13) P. Onno: Review of Scientific Instruments, 32, No. 11, 1253, (Nov., 1961).
(14) Canadian Charts and Supplies, Itd, Oakville, Ontario: No. G8-10055.

