Excel for Control Charts with Applications to Short Runs

By

Jose W.P. But

A Practicum
Submitted to the Faculty of Graduate Studies
in Partial Fulfillment of the Requirements
for the Degree of

Master of Science

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University of Manitoba
Winnipeg, Manitoba

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THE UNIVERSITY OF MANITOBA

FACULTY OF GRADUATE STUDIES
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Abstract

This practicum provides step-by-step instructions for constructing short run control charts in Microsoft Excel. Several widely used methods of short runs are reviewed and the applications of those short run charts are provided. The scenarios of choosing the appropriate short run charts are also discussed. In addition, we construct an Excel program to test the special causes for control charts.

ACKNOWLEDGMENTS

The author would like to thank my advisor Dr. L. Chan of the University of Manitoba for guiding me the area of short runs; and, also his encouragement, advice and support. In addition, I would like to thank Dr. S. Samanta and Dr. Q. Ye of the University of Manitoba for examining my practicum; and also would like to thank Dr. B. Macpherson, Ms. J. Mojica and Mr. K. Ng of University of Manitoba for valuable discussion and information. Finally, I would like to thank my mum for her financial support and encouragement through out of my education.

Chapter 1

Excel Programs For Short Run Control Charts

1. Introduction

Recently, there have been increasing uses of Microsoft Excel for statistical analysis. The main reasons are listed as below:(1) Excel is available in almost all computers, (2) Many computer users know how to use Excel, (3) Frequently data for analysis are already kept in an Excel spreadsheet, (4) standard statistical packages such as SAS, Minitab and SPSS are less accessible due to licensing cost and limitation on number of users per license. Thus statistics books on" doing statistics by Excel " are now available, e.g. Zabowski,G. (1999), Zimmerman, S.M. & Icenogle, M.L. (1999) and Middleton, M.R. (1999) .

Some of these books include a section on the use of Excel for control charts in statistical process control. However, they are for the conventional Shewhart X-bar and R charts. Furthermore, when testing for special causes, they consider only one test, i.e., one point beyond the control limits.

In this chapter, we first show how to use Excel to create control charts. Then we demonstrate how to incorporate them to the creation of control charts for short runs through three examples of short run control charts for center and for variation: (1) Individual-X and Moving range charts, (2) Individual-X and Moving Range charts for non-normal Data. (3) Short run X-bar and Range charts.

In additional to the above three types of control charts, several other types of short run control charts are also considered. But we will not propose their Excel programs, as they are similar to the above three types. The formulas of all the short run charts are given in Chapter 2, together with examples.

In Chapter 3, we show how to use Excel to perform the well known eight tests for special causes in the X-bar chart suggested by Nelson (1984,85).

2. Steps to Create Control Charts

Here we are giving the steps for creating a general control chart such that the data for the plotted points, centerline and control limits are already entered to the Excel spread sheet. In section 3, 4 and 5, we will demonstrate how to enter data and to type formulas for manipulation of the same data set.

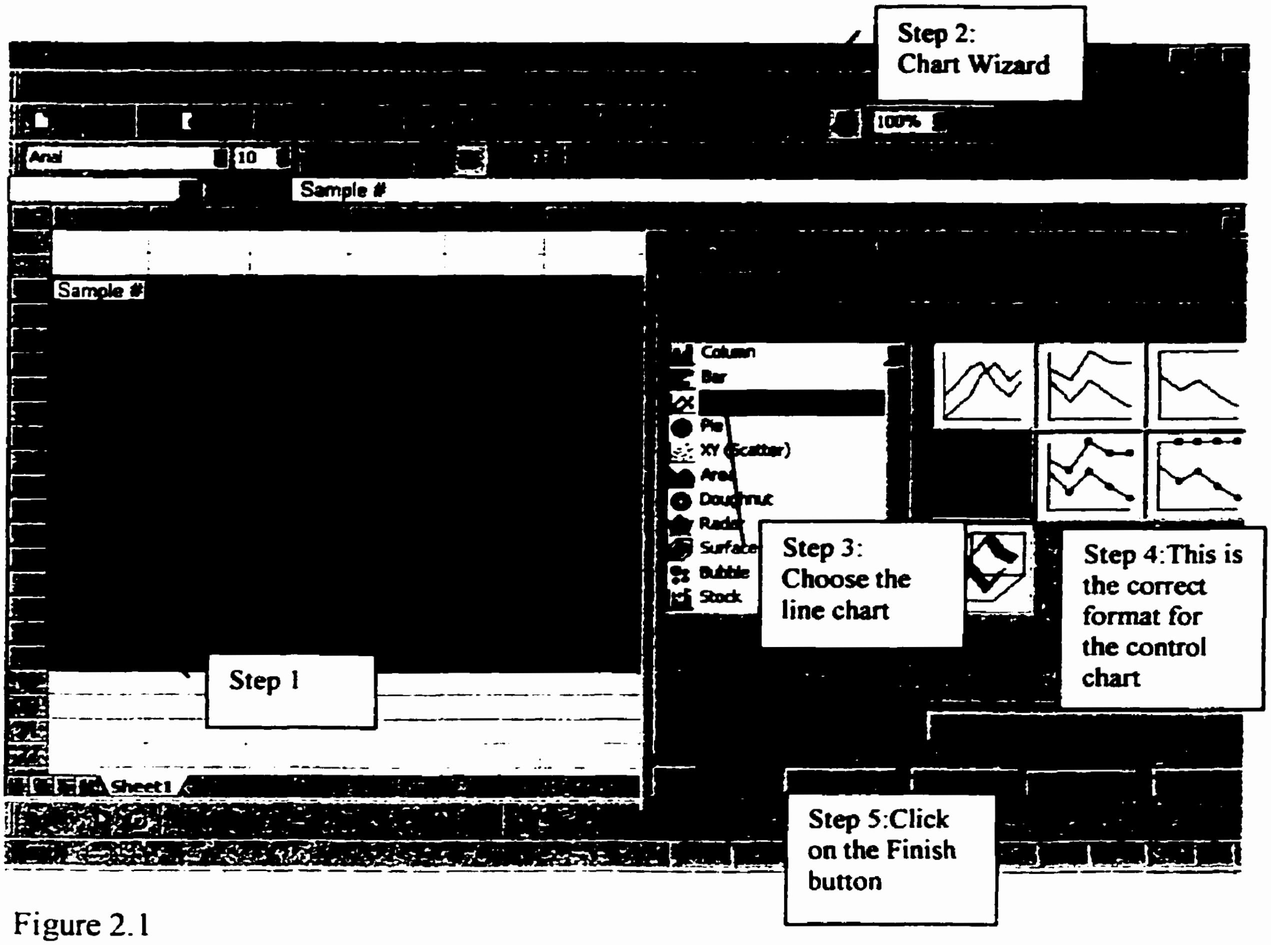


Figure 2.1

Step 1. Enter the data that we want to chart (Including Sample #, Part #, Plotted points (Di), Centerline (Di-bar), Upper Control Limit (UCL) and Lower Control Limit (LCL)) and choose the range A3 to F18 in this example.

Step 2. Click on the **Chart Wizard tool** on the tool bar.

Step 3. Choose the **line chart**.

Step 4. Choose the correct format for the control chart.

Step 5. Click on the **Finish** button.

After clicking on the Finish button, we could see the control chart as shown in below.

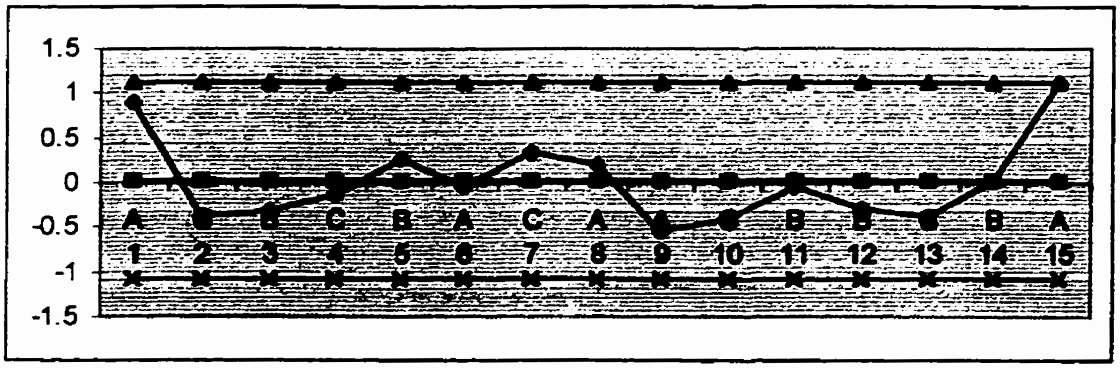


Figure 2.2

3. Steps for Formatting the Chart and Part Number Axis

3.1 Steps for formatting the chart

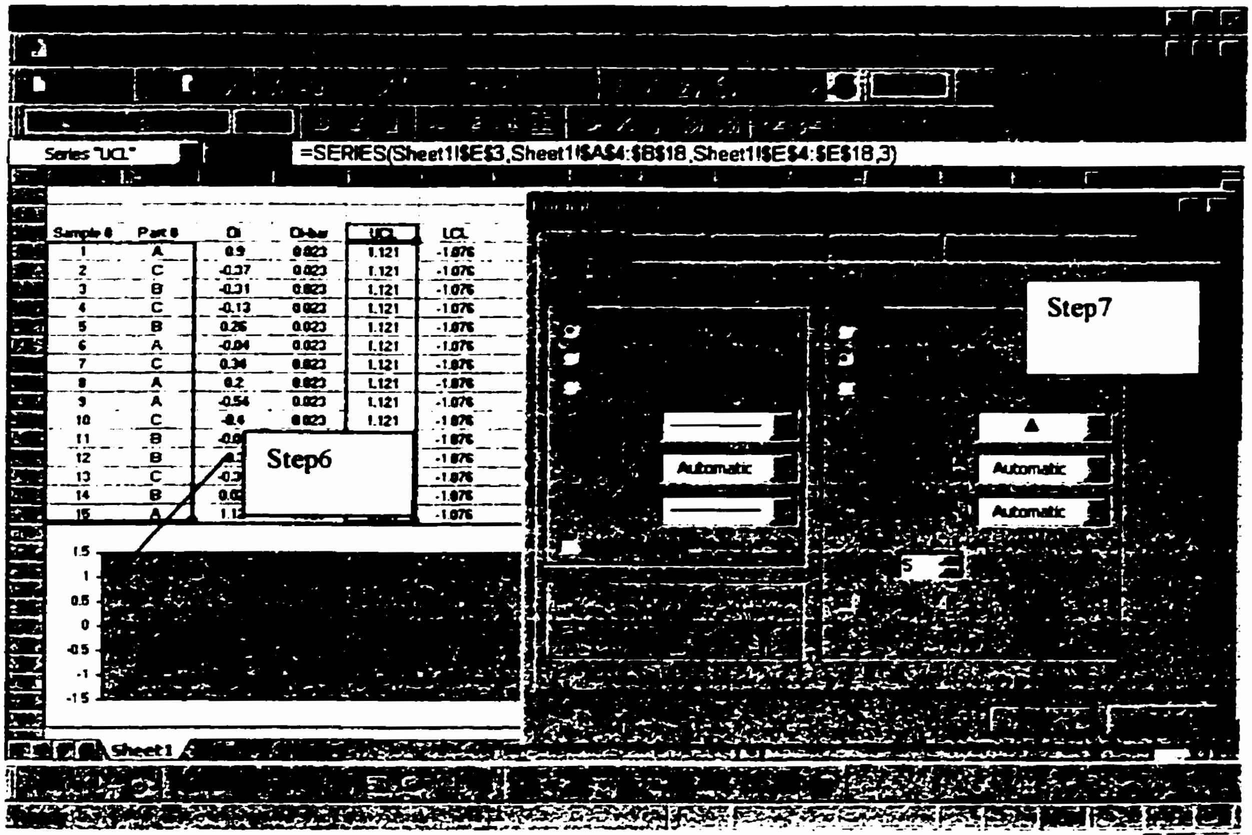


Figure 3.1

Step 6. Double click on the Upper Control Limit line in the graph (see Figure 3.1).

Step 7. Look for the Format Data Series dialog box. Choose **None** below the Marker heading.

Notice: We can format the centerline and lower control limit by simply repeating Step 6 (Double click on the line that we want to format) and Step 7 (see Figure 3.1).

3.2 Steps for formatting the part number Axis

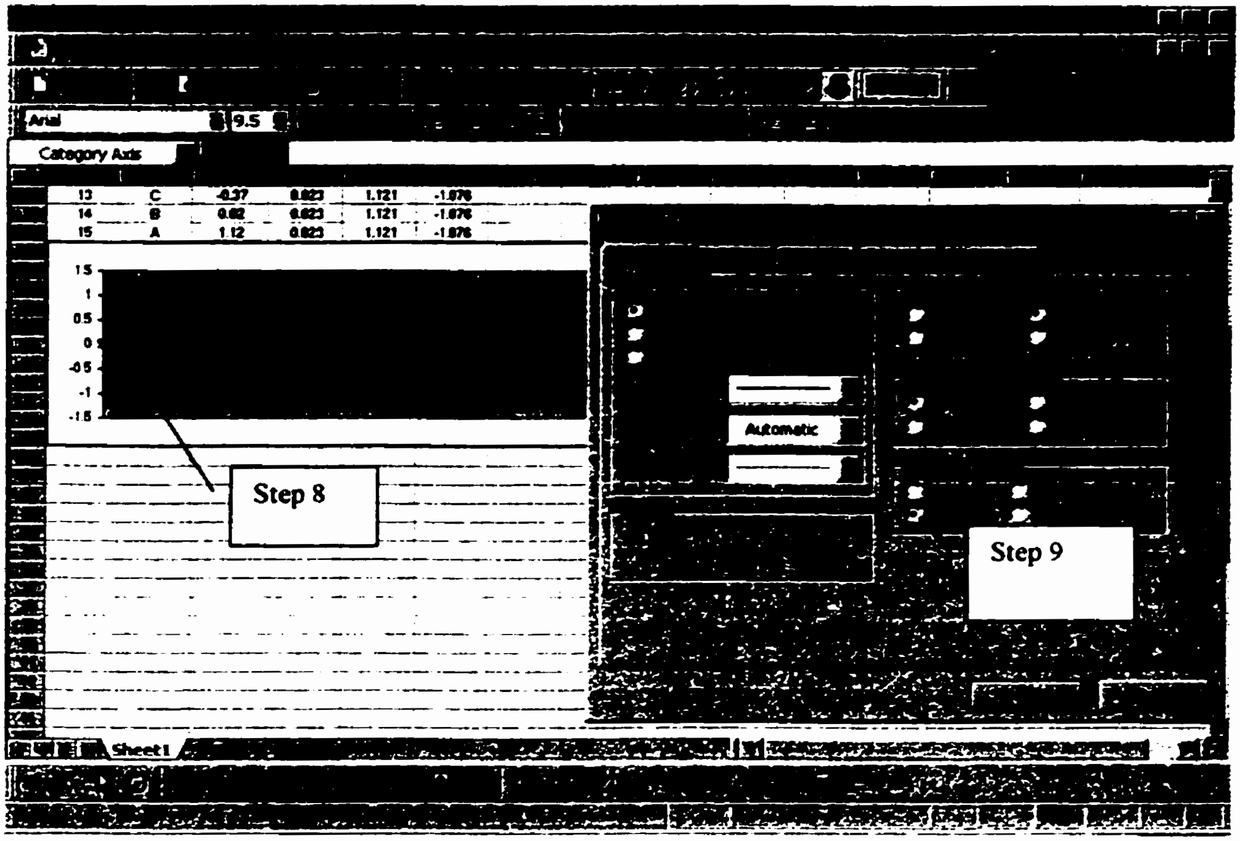


Figure 3.2

Step 8. Double click on the part number in the graph (see Figure 3.2).

Step 9. Look for the Format Axis dialog box. Choose **Low** below the Tick mark labels.

The following is the final control chart that we can get.

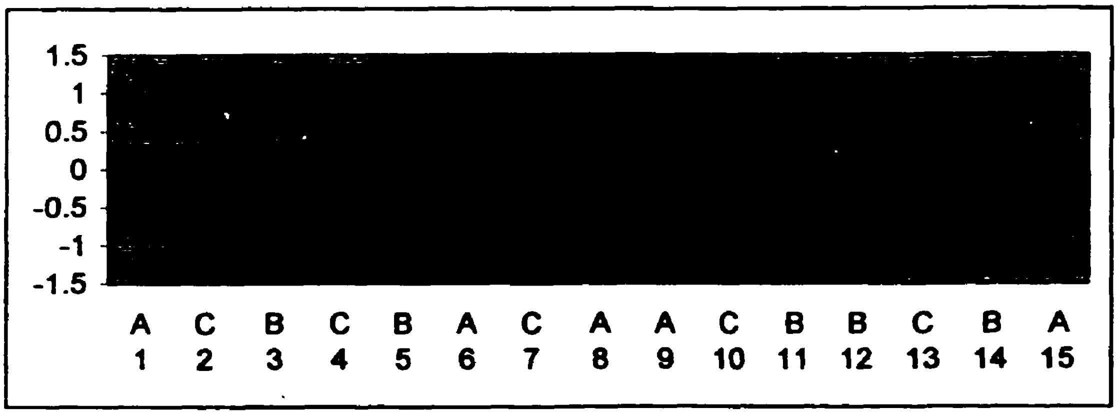


Figure 3.3

4. Creating the Short Run Target Individual X and Moving Range Charts

Here we consider the Short Run Target Individual X and Moving Range Charts by using the example in Section 5.1 of Chapter 2, in which the formulas for the plotted points, center lines and control limits are given.

SUBGROUP	1	2	3	4	5	6	7	8	9
PART #	A	C	B	C	B	A	C	A	A
Obs	15.9	39.6	24.7	39.9	25.3	14.9	40.3	15.2	15.5
Tar x-bar	15	40	25	40	25	15	40	15	15

SUBGROUP	10	11	12	13	14	15
PART #	C	B	B	C	B	A
Obs	39.6	24.9	24.7	39.7	25.1	16.1
Tar x-bar	40	25	25	40	25	15

4.1 Entering the texts into the worksheet

Table 1.

CELL	TEXT	DESCRIPTION
B2	Constant for IX-Chart	TEXT
D2	A2	TEXT
G2	Constant for R-Chart	TEXT
I2	D4	TEXT
J2	D3	TEXT
D3	2.66	A2 FOR SUBGROUP SIZE 1
I3	3.268	D4 FOR SUBGROUP SIZE 1
J3	0	D3 FOR SUBGROUP SIZE 1
B4	IX-Chart	TEXT
C4	C	CENTERLINE FOR IX-CHART
D4	UCL	UPPER CONTROL LIMIT OF IX-CHART
E4	LCL	LOWER CONTROL LIMIT OF IX-CHART
G4	R-Chart	TEXT
H4	R-C	CENTRE LINE FOR RANGE CHART
I4	R-UCL	UPPER CONTROL LIMIT OF RANGE CHART
J4	R-LCL	LOWER CONTROL LIMIT OF RANGE CHART
C5	=G8	AVERAGE OF Dj
D5	=C5+D3*H5	UPPER CONTROL LIMIT OF IX-CHART
E5	=C5-D3*H5	LOWER CONTROL LIMIT OF IX-CHART
H5	=H8	CENTRE LINE FOR RANGE CHART
I5	=I3*H5	UPPER CONTROL LIMIT OF RANGE CHART

J5	=J3*H5	LOWER CONTROL LIMIT OF RANGE CHART
A7	Sample #	SAMPLE NUMBERS
B7	Part #	PARTS NUMBER
C7	Xj	OBSERVATIONS
D7	TARGETj	TARGET VALUE OF Xj
E7	Dj	PLOT POINTS FOR IX-CHART
F7	DMRj	PLOT POINTS FOR RANGE CHART
H7	Dj-bar	AVERAGE OF Dj
I7	DMRj-bar	AVERAGE OF MOVING RANGE

The following display will appear:

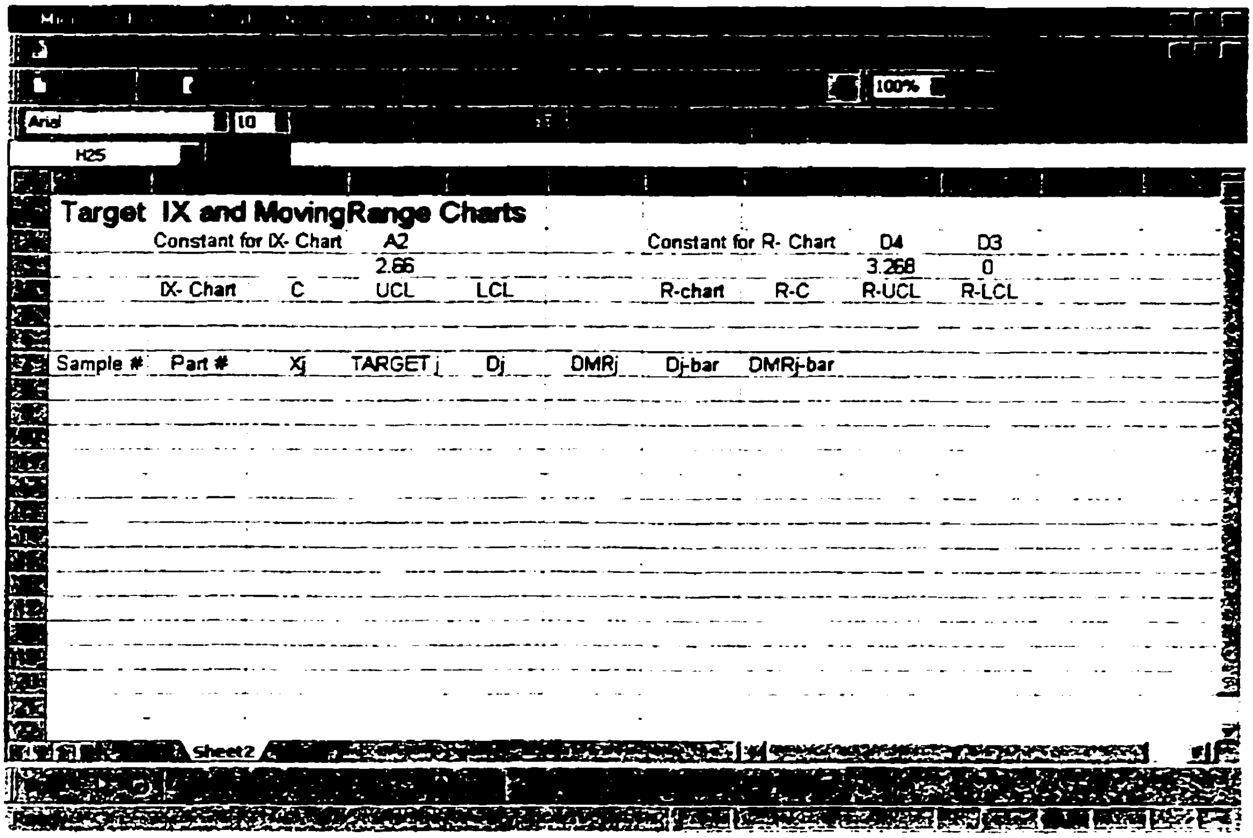


Figure 4 (a)

4.2 Steps for entering the data and typing the formula required for calculating the plotted points, center lines and control limits

Step 1. Enter the sample numbers

1.1 Enter 1 in cell A8 and press **enter**.

1.2 In cell A9 type in the formula: " **=1+A8** " and press **enter**.

1.3 Click on cell A9 and click on **copy** button.

1.4 Hold down the left mouse button, select the range A9: A22 and press **enter**.

Step 2. Enter the corresponding part numbers in column B (B8: B22).

Step 3. Enter all the observed values in column C (C8: C22).

Step 4. Enter the corresponding target values in column D (D8: D22).

Step 5. Calculate the plotted points for the IX-Chart in column E

5.1 Click on the cell E8 and type the formula :" = **C8-D8** " .

5.2 Press **enter**.

5.3 Click on cell E8 and choose the **copy** button.

5.4 Hold down the mouse button, select the range E8:E22 and press **enter**.

The plotted points of IX-Chart would be calculated and displayed in column E from E8 to E22.

Step 6. Calculate the Moving Range plotted points

6.1 Click on to the cell F9 and type the formula: " **=ABS(E8-E9)** "

6.2 Press **enter**.

6.3 Click on cell F9 and choose the **copy** button.

6.4 Hold down the mouse button, select the range F9: F22 and press **enter**.

The moving range would be calculated and displayed in column F from F9 to F22.

Step 7. Calculate the average of Dj in Column G

7.1 Click on the cell G8.

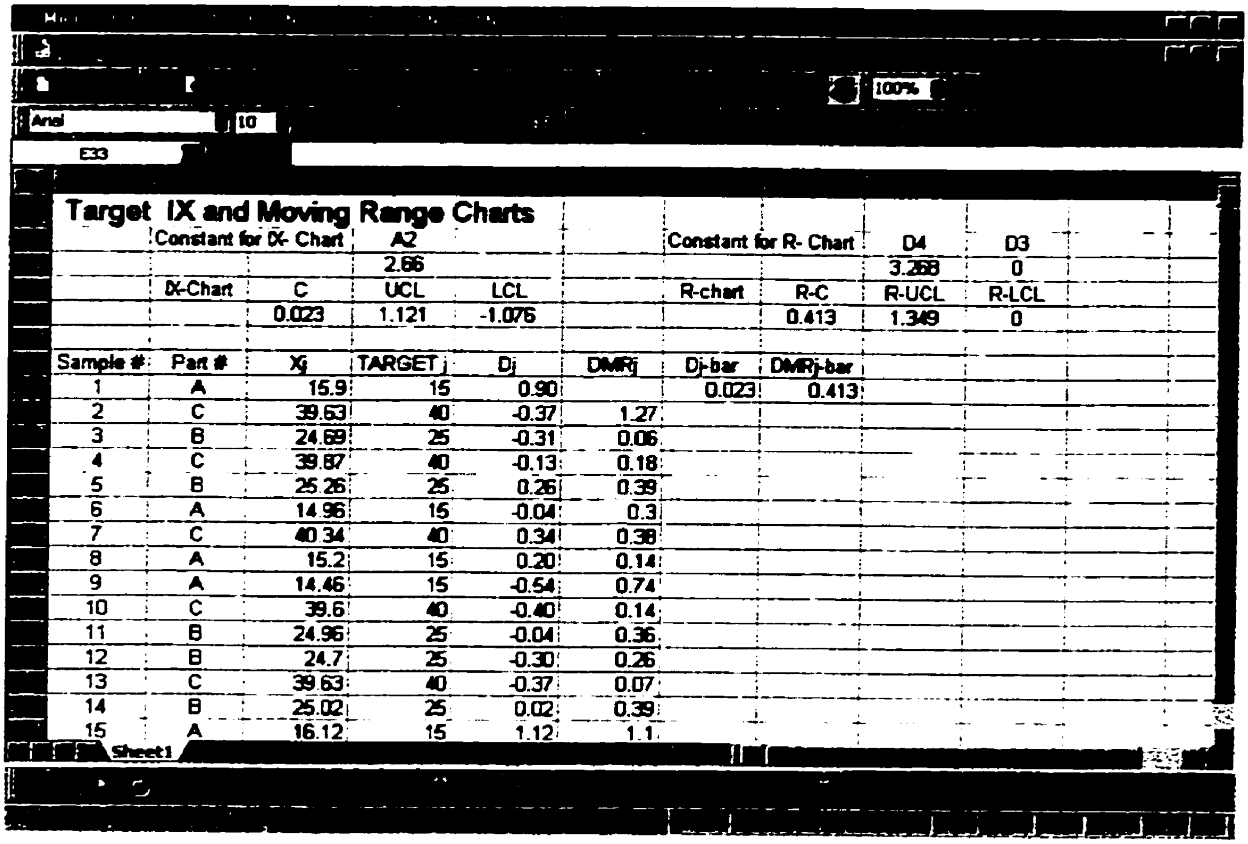
7.2 Type the formula: " = **AVERAGE(E8:E22)** " and press **enter**.

Step 8. Calculate the average of DMRj in column H

8.1 Click on the cell H8.

8.2 Type the formula: " = **AVERAGE(F9:F22)** " and press **enter**.

The following display will appear:

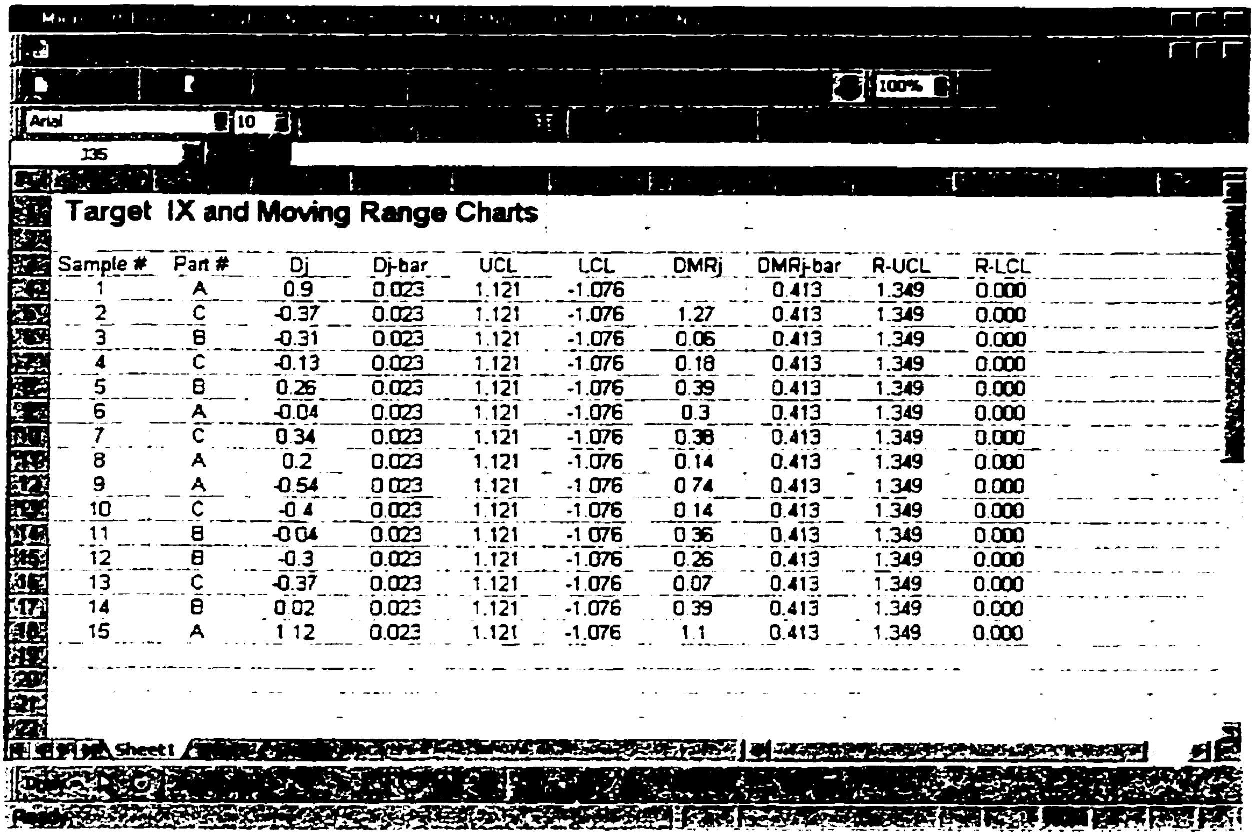


Target IX and Moving Range Charts									
	Constant for IX- Chart		A2			Constant for R- Chart		D4	D3
			2.66					3.268	0
	IX-Chart	C	UCL	LCL		R-chart	R-C	R-UCL	R-LCL
		0.023	1.121	-1.076			0.413	1.349	0
Sample #	Part #	Xj	TARGETj	Dj	DMRj	Dj-bar	DMRj-bar		
1	A	15.9	15	0.90		0.023	0.413		
2	C	39.63	40	-0.37	1.27				
3	B	24.69	25	-0.31	0.06				
4	C	39.87	40	-0.13	0.18				
5	B	25.26	25	0.26	0.39				
6	A	14.96	15	-0.04	0.3				
7	C	40.34	40	0.34	0.38				
8	A	15.2	15	0.20	0.14				
9	A	14.46	15	-0.54	0.74				
10	C	39.6	40	-0.40	0.14				
11	B	24.96	25	-0.04	0.36				
12	B	24.7	25	-0.30	0.26				
13	C	39.63	40	-0.37	0.07				
14	B	25.02	25	0.02	0.39				
15	A	16.12	15	1.12	1.1				

Figure 4(b)

4.3 Converting the Plotted Points and Control Limits into a New Excel Worksheet

After we calculate all the potted points and control limits for Target IX and Moving Range Chart, we can use **cut and paste** function to paste the "Sample number, Part number, Plotted points of Target IX, Control Limits of Target IX, Plotted Points of Moving Range, and Control Limits of Moving Range" into a new Excel Worksheet. These have to be done because it would then be easier to create a suitable control chart in Excel.


Target IX and Moving Range Charts

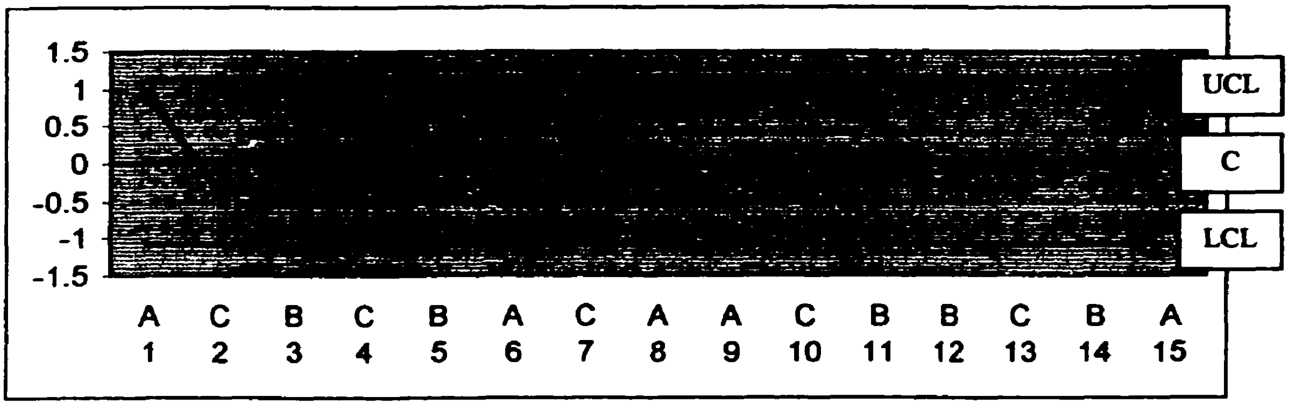
Sample #	Part #	Dj	Dj-bar	UCL	LCL	DMRj	DMRj-bar	R-UCL	R-LCL
1	A	0.9	0.023	1.121	-1.076		0.413	1.349	0.000
2	C	-0.37	0.023	1.121	-1.076	1.27	0.413	1.349	0.000
3	B	-0.31	0.023	1.121	-1.076	0.06	0.413	1.349	0.000
4	C	-0.13	0.023	1.121	-1.076	0.18	0.413	1.349	0.000
5	B	0.26	0.023	1.121	-1.076	0.39	0.413	1.349	0.000
6	A	-0.04	0.023	1.121	-1.076	0.3	0.413	1.349	0.000
7	C	0.34	0.023	1.121	-1.076	0.38	0.413	1.349	0.000
8	A	0.2	0.023	1.121	-1.076	0.14	0.413	1.349	0.000
9	A	-0.54	0.023	1.121	-1.076	0.74	0.413	1.349	0.000
10	C	-0.4	0.023	1.121	-1.076	0.14	0.413	1.349	0.000
11	B	-0.04	0.023	1.121	-1.076	0.36	0.413	1.349	0.000
12	B	-0.3	0.023	1.121	-1.076	0.26	0.413	1.349	0.000
13	C	-0.37	0.023	1.121	-1.076	0.07	0.413	1.349	0.000
14	B	0.02	0.023	1.121	-1.076	0.39	0.413	1.349	0.000
15	A	1.12	0.023	1.121	-1.076	1.1	0.413	1.349	0.000

Figure 4(c)

4.4 Creating the Control Charts

Follow the steps in Section 2 of this chapter to create the suitable control charts.

Target Individual X Control Chart



Moving Range Control Charts

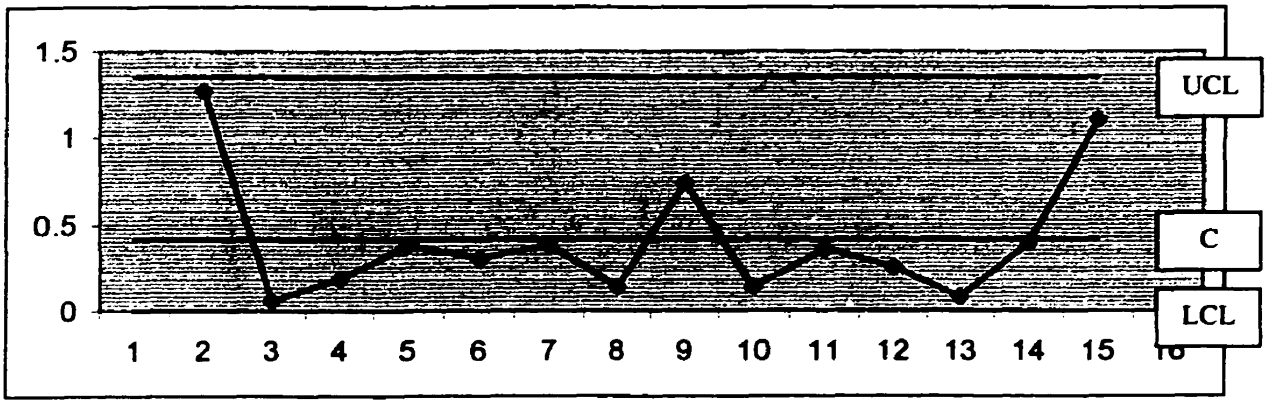


Figure 4 (d)

5. Creating the Short Run Target Individual X and Moving Range Charts (Non-normal data)

Here we consider the Short Run Individual X and Moving Range Charts (Non-Normal data) by using the example in Section 5.2 of Chapter 2, in which the formulas for the plotted points, center lines and control limits are given.

Example: The data is getting form Bothe's manual. (Bothe 1999, p28)

SUBGROUP	1	2	3	4	5	6	7	8	9
PART #	A	A	A	D	D	D	B	B	B
Obs	7.9	7.2	7.2	5.9	6.3	6.1	8.9	8.3	8.3
Tar x-bar	7.5	7.5	7.5	6.1	6.1	6.1	8.7	8.7	8.7

SUBGROUP	10	11	12	13	14	15
PART #	B	F	F	E	E	E
Obs	8.7	9.8	9.4	6.8	6.8	7.5
Tar x-bar	8.7	9.5	9.5	7.0	7.0	7.0

5.1 Entering the texts into the worksheet

Table 2.

CELL	TEXT	DESCRIPTION
B2	Constant for IX-Chart	TEXT
D2	A2	TEXT
H2	Constant for R-Chart	TEXT
J2	D4	TEXT
K2	D3	TEXT
D3	1.88	A2 FOR SUBGROUP SIZE 1
J3	3.268	D4 FOR SUBGROUP SIZE 1
K3	0	D3 FOR SUBGROUP SIZE 1
B4	IX-Chart	TEXT
C4	C	CENTERLINE FOR IX-CHART
D4	UCL	UPPER CONTROL LIMIT OF IX-CHART
E4	LCL	LOWER CONTROL LIMIT OF IX-CHART
H4	R-Chart	TEXT
I4	R-C	CENTRE LINE FOR RANGE CHART
J4	R-UCL	UPPER CONTROL LIMIT OF RANGE CHART
K4	R-LCL	LOWER CONTROL LIMIT OF RANGE CHART
C5	=H8	AVERAGE OF Dj
D5	=C5+D3*I5	UPPER CONTROL LIMIT OF IX-CHART

E5	=C5-D3*I5	LOWER CONTROL LIMIT OF IX-CHART
I5	=I8	CENTRE LINE FOR RANGE CHART
J5	=J3*I5	UPPER CONTROL LIMIT OF RANGE CHART
K5	=K3*I5	LOWER CONTROL LIMIT OF RANGE CHART
A7	Sample #	SAMPLE NUMBERS
B7	Part #	PARTS NUMBER
C7	Xj	OBSERVATIONS
D7	TARGETj	TARGET VALUE OF Xj
E7	Dj	TEXT
F7	DMRi	PLOT POINTS FOR RANGE CHART
G7	DMAj	PLOT POINTS FOR IX-CHART
H7	Dj-bar	AVERAGE OF Dj
I7	DMRj-bar	AVERAGE OF MOVING RANGE

The following display will appear:

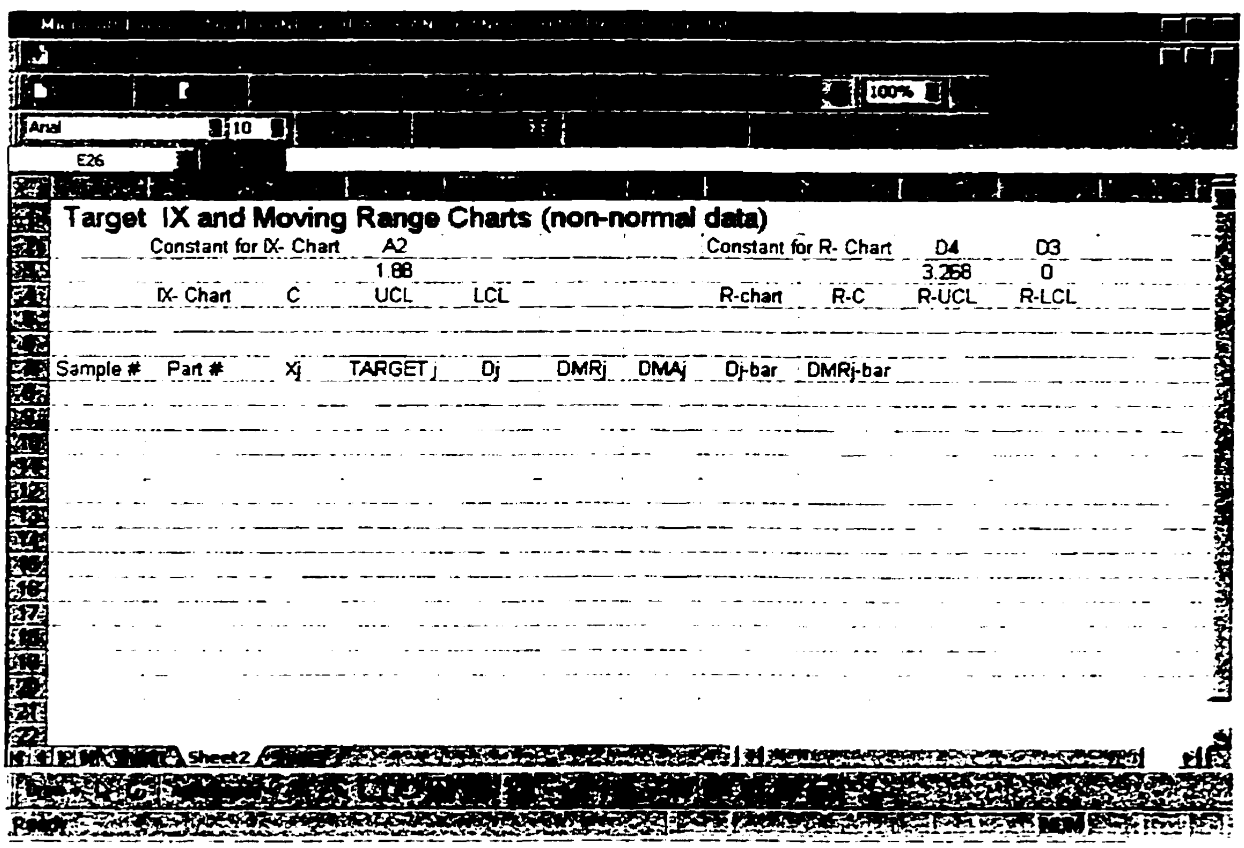


Figure 5(a)

5.2 Steps for entering the data and typing the formulas required for calculating the plotted points, center lines and control limits

Step 1. Enter the sample numbers

1.1 Enter 1 in cell A8 and press **enter**.

1.2 In cell A9 type in the formula: " **=1+A8** " and press **enter**.

1.3 Click on cell A9 and click on **copy** button.

1.4 Hold down the left mouse button, select the range A9: A22 and press **enter**.

Step 2. Enter the corresponding parts number in column B (B8: B22).

Step 3. Enter all the observed values in column C (C8: C22).

Step 4. Enter the corresponding target values in column D (D8: D22).

Step 5. Calculate the Dj=Xj-Targetj in column E

5.1 Click on the cell E8 and type the formula:" = **C8-D8** " .

5.2 Press **enter**.

5.3 Click on cell E8 and choose the **copy** button.

5.4 Hold down the mouse button, select the range E8:E22 and press **enter**.

The values of Dj would be calculated and displayed in column E from E8 to E22.

Step 6. Calculate the range plotted points

6.1, Click on to the cell F9 and type the formula: " **=ABS(E8-E9)** "

6.2, Press **enter**.

6.3, Click on cell F9 and choose the **copy** button.

6.4, Hold down the mouse button, select the range F9: F22 and press **enter**.

The moving range would be calculated and displayed in column F from F9 to F22.

Step 7. Calculate the plotted points for the IX-Chart in column G

7.1, Click on the cell G9 and type the formula: " = **(E9+E8)/2** " .

7.2, Press **enter**.

7.3, Click on cell G9 and choose the **copy** button.

7.4, Hold down the mouse button, select the range G9:G22 and press **enter**.

The plotted points of IX-Chart would be calculated and displayed in column G from G8 to G22.

Step 8. Calculate the average of Dj in Column H

8.1, Click on the cell H8.

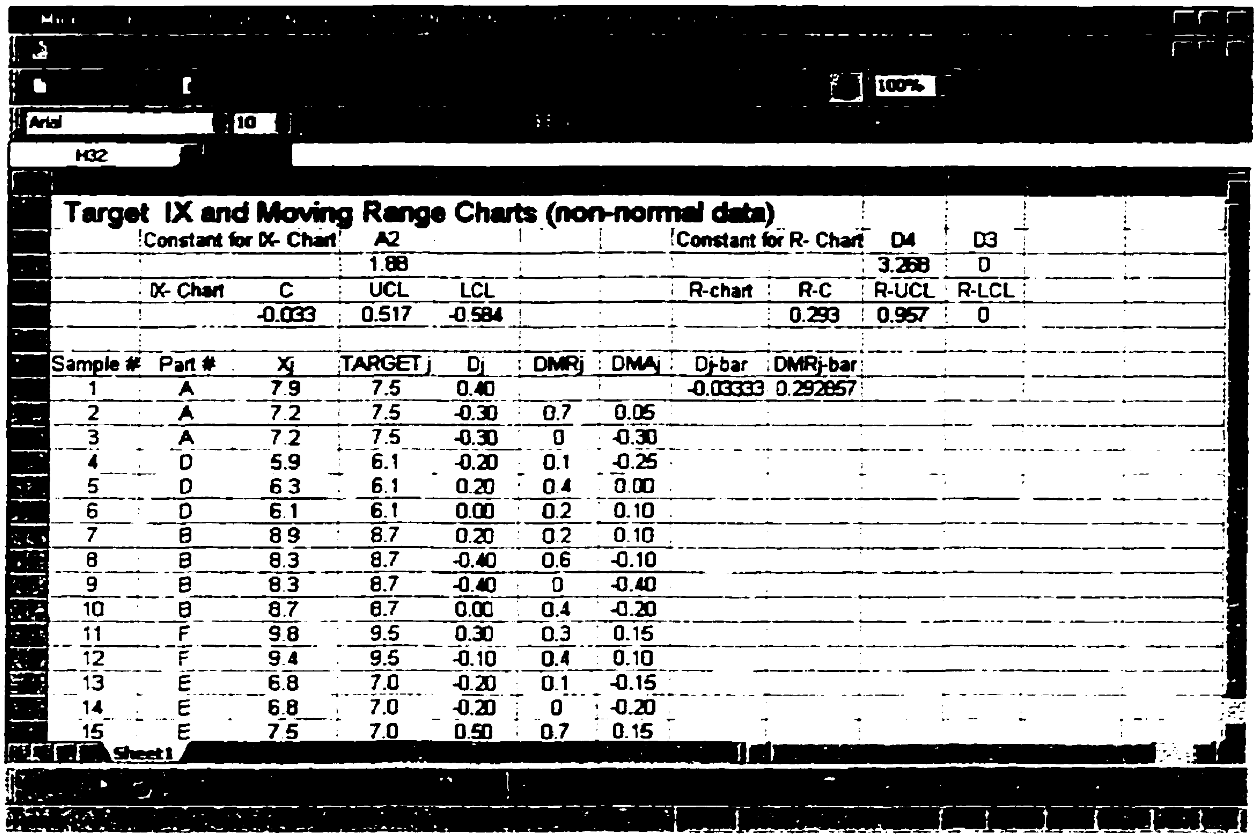
8.2, Type the formula: " = **AVERAGE(E8:E22)** " and press **enter**.

Step 9. Calculate the average of DMRj in column I

9.1, Click on the cell I8.

9.2, Type the formula: " = **AVERAGE(F9:F22)** " and press **enter**.

The following display will appear:



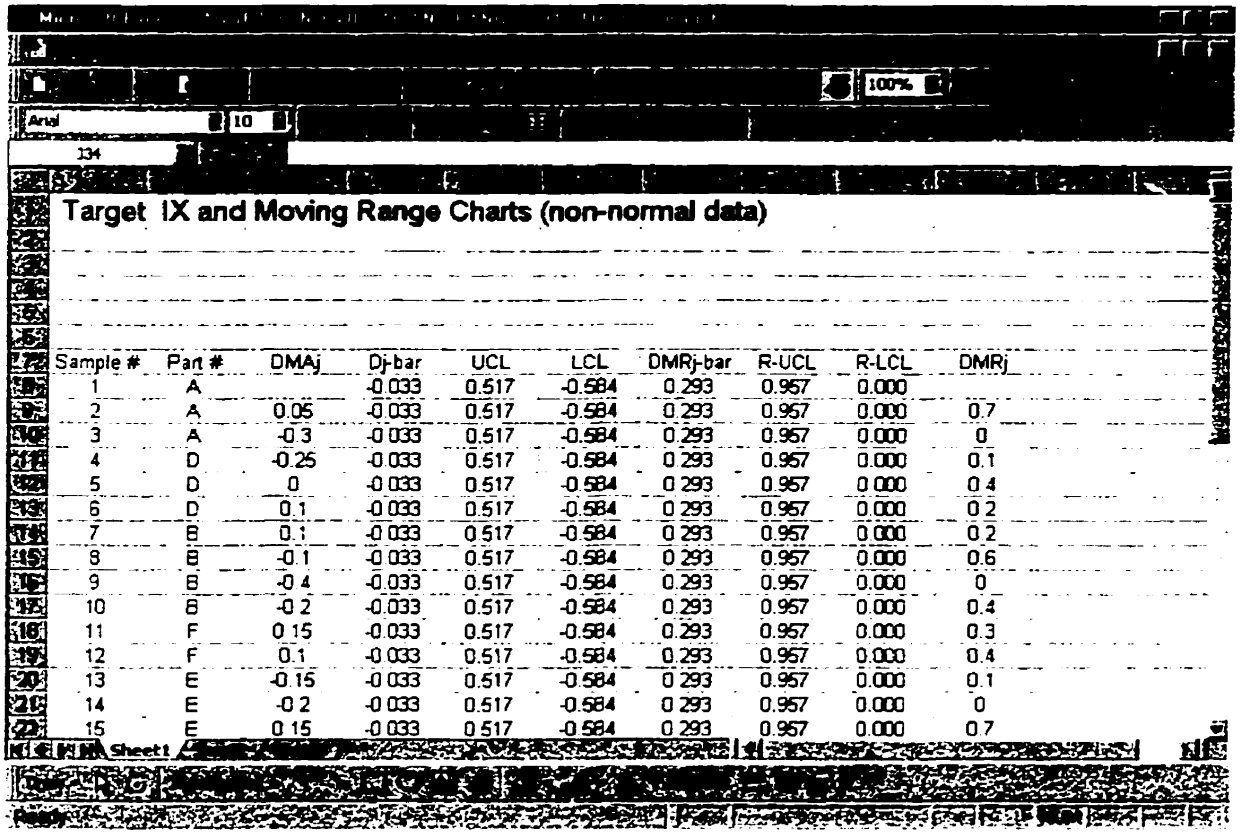
Target IX and Moving Range Charts (non-normal data)

	Constant for IX- Chart		A2				Constant for R- Chart		D4	D3
			1.88						3.268	0
	IX- Chart	C	UCL	LCL			R-chart	R-C	R-UCL	R-LCL
		-0.033	0.517	-0.584				0.293	0.957	0
Sample #	Part #	Xj	TARGET j	Dj	DMRj	DMAj	Dj-bar	DMRj-bar		
1	A	7.9	7.5	0.40			-0.03333	0.292857		
2	A	7.2	7.5	-0.30	0.7	0.05				
3	A	7.2	7.5	-0.30	0	-0.30				
4	D	5.9	6.1	-0.20	0.1	-0.25				
5	D	6.3	6.1	0.20	0.4	0.00				
6	D	6.1	6.1	0.00	0.2	0.10				
7	B	8.9	8.7	0.20	0.2	0.10				
8	B	8.3	8.7	-0.40	0.6	-0.10				
9	B	8.3	8.7	-0.40	0	-0.40				
10	B	8.7	8.7	0.00	0.4	-0.20				
11	F	9.8	9.5	0.30	0.3	0.15				
12	F	9.4	9.5	-0.10	0.4	0.10				
13	E	6.8	7.0	-0.20	0.1	-0.15				
14	E	6.8	7.0	-0.20	0	-0.20				
15	E	7.5	7.0	0.50	0.7	0.15				

Figure 5(b)

5.3 Converting the Plotted Points and Control Limits into a New Excel Worksheet

After we calculate all the potted points and control limits for Target IX and Moving Range Chart, we can use **cut and paste** function to paste the "Sample number, Part number, Plotted points of Target IX (DMAj), Control Limits of Target IX, Plotted Points of Moving Range (DMRj), and Control Limits of Moving Range" into a new Excel Worksheet. These have to be done because it would then be easier to create a suitable control chart in Excel.



Target IX and Moving Range Charts (non-normal data)

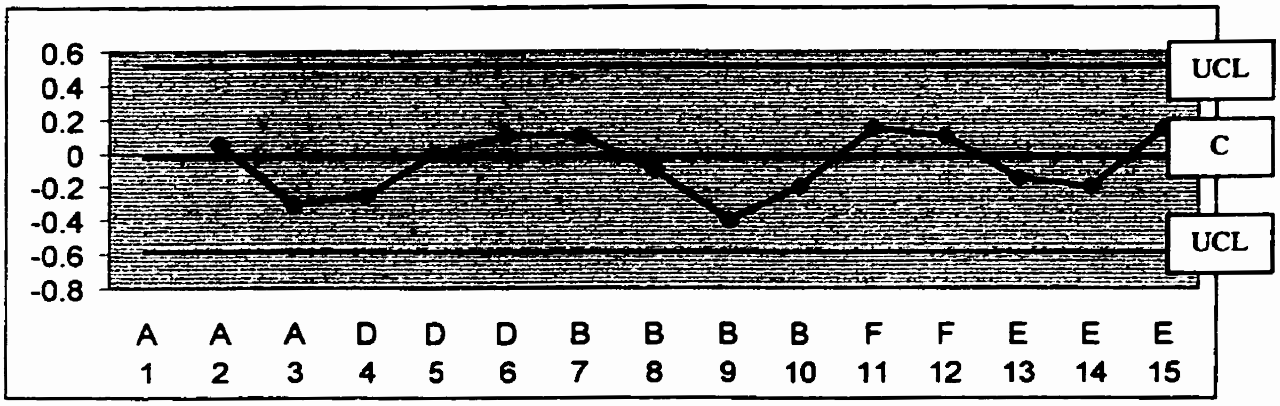
Sample #	Part #	DMAj	Dj-bar	UCL	LCL	DMRj-bar	R-UCL	R-LCL	DMRj
1	A		-0.033	0.517	-0.584	0.293	0.957	0.000	
2	A	0.05	-0.033	0.517	-0.584	0.293	0.957	0.000	0.7
3	A	-0.3	-0.033	0.517	-0.584	0.293	0.957	0.000	0
4	D	-0.25	-0.033	0.517	-0.584	0.293	0.957	0.000	0.1
5	D	0	-0.033	0.517	-0.584	0.293	0.957	0.000	0.4
6	D	0.1	-0.033	0.517	-0.584	0.293	0.957	0.000	0.2
7	B	0.1	-0.033	0.517	-0.584	0.293	0.957	0.000	0.2
8	B	-0.1	-0.033	0.517	-0.584	0.293	0.957	0.000	0.6
9	B	-0.4	-0.033	0.517	-0.584	0.293	0.957	0.000	0
10	B	-0.2	-0.033	0.517	-0.584	0.293	0.957	0.000	0.4
11	F	0.15	-0.033	0.517	-0.584	0.293	0.957	0.000	0.3
12	F	0.1	-0.033	0.517	-0.584	0.293	0.957	0.000	0.4
13	E	-0.15	-0.033	0.517	-0.584	0.293	0.957	0.000	0.1
14	E	-0.2	-0.033	0.517	-0.584	0.293	0.957	0.000	0
15	E	0.15	-0.033	0.517	-0.584	0.293	0.957	0.000	0.7

Figure 5(c)

5.4 Creating the Control Charts

Follow the steps in Section 2 of this chapter to create the suitable control charts.

Target Individual X Control Chart (non-normal data)



Moving Range Control Charts

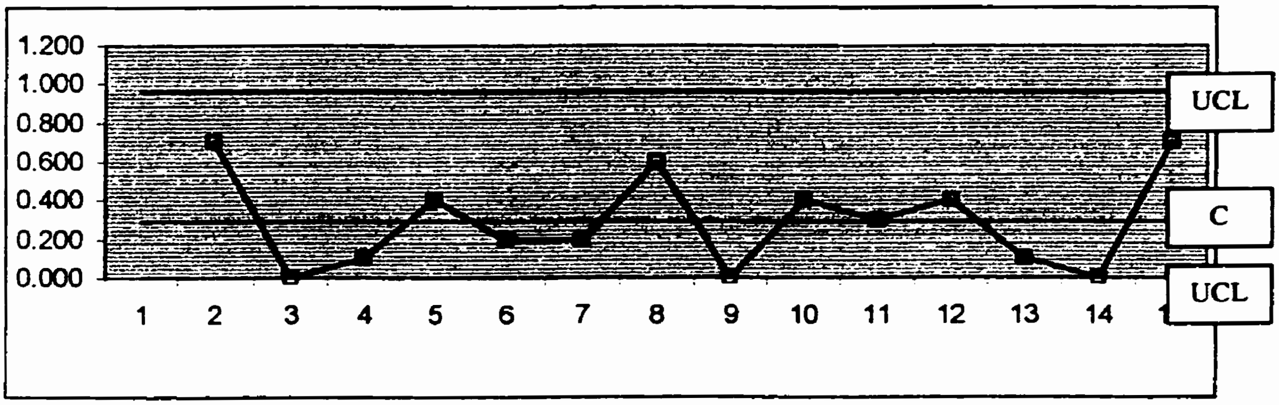


Figure 5(d)

6. Creating the Short Run X-bar and Range Charts

Here we consider the Short Run X-bar and Range Charts that will be considered in section 8.1 of Chapter 2, in which the formulas for the plotted points, center lines and control limits are given.

Notice: The data set of the example is not the same as the data set in section 8.1 of Chapter 2.

Example: The data is getting from Bothe's manual. (Bothe 1991, p.12)

SUBGROUP	1	2	3	4	5	6	7	8	9
PART #	C	C	C	A	A	A	A	B	B
Obs 1	6	14	8	48	38	35	35	98	109
Obs 2	11	12	6	55	55	13	57	101	108
Obs 3	10	13	10	53	27	24	67	98	92
Tar x-bar	10	10	10	50	50	50	50	100	100
Tar r-bar	4	4	4	20	20	20	20	9	9

6.1 Entering the texts into the worksheet

CELL	TEXT	DESCRIPTION
A3	x-bar chart	
B3	C	CENTER LINE FOR X-BAR CHART
C3	UCL	UPPER CONTROL LIMIT OF X-BAR CHART
D3	LCL	LOWER CONTROL LIMIT OF X-BAR CHART
F3	R-CHART	
G3	R-C	CENTRE LINE FOR RANGE CHART
H3	R-UCL	UPPER CONTROL LIMIT OF RANGE CHART
I3	R-LCL	LOWER CONTROL LIMIT OF RANGE CHART
B4	0	CENTER LINE FOR X-BAR CHART
C4	1.02	A2 FOR SUBGROUP SIZE 3
D4	-1.02	-A2 FOR SUBGROUP SIZE 3
G4	1	CENTERLINE FOR RANGE CHART
H4	2.57	D4 FOR RANGE SUBGROUP SIZE 3
I4	0	D3 FOR RANGE SUBGROUP SIZE 3
A6	Sample #	SAMPLE NUMBERS
B6	Part #	PARTS NUMBER
C6	Obs 1	OBSERVATIONS 1
D6	Obs 2	OBSERVATIONS 2
E6	Obs 3	OBSERVATIONS 3
F6	Ave x-bar	AVERAGE OF THREE SUBGROUPS
G6	Tar x-bar	TARGET VALUE OF X-BAR

H6	Tar r-bar	TARGET VALUE OF RANGE-BAR
I6	Plot pts	PLOT POINTS FOR X-BAR CHART
J6	Range-r	RANGE OF SUBGROUP
K6	R-plot pts	PLOT POINTS FOR RANGE CHART

The following display will appear:

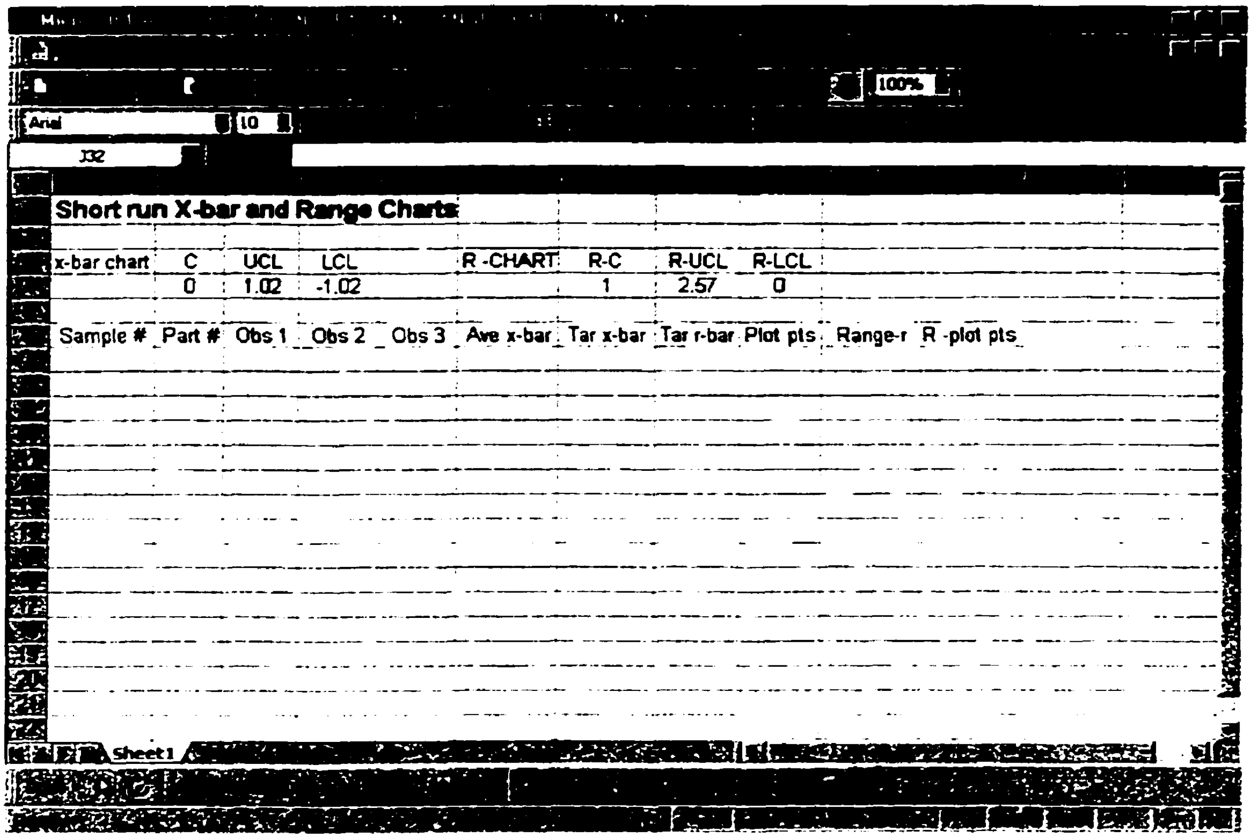


Figure 6(a)

6.2 Steps for entering the data and typing the formulas required for calculating the plotted points, center lines and control limits

Step 1. Enter the sample numbers in column A at the range A7:A15.

Step 2. Enter the corresponding part numbers in column B.

Step 3. Enter the first observed values of the samples to column C.

Step 4. Enter the second observed values of the samples to column D.

Step 5. Type the third observed values of the samples to column E.

Step 6. Entering the formula to calculate the average of different subgroups in column F
6.1 Click on the cell F7 and type the formula:" **=AVERAGE(C7:E7)**" .
6.2 press **enter**.
6.3 Click on cell F7 and click on the **copy** button.
6.3 Hold down the mouse button, select the range F8:F15 and press **enter**.
The average of subgroup would be calculated and appeared in column F from F7:F15.

Step 7. Enter the corresponding target x-bar in column G.

Step 8. Enter the corresponding target r-bar in column H.

Step 9. Calculate the plotted points for the x-bar chart in column I
9.1 Click on to the cell I7 and type the formula:" **=(F7-G7)/H7** " .
9.2 Press **enter**.
9.3 Click on cell I7 and choose the **copy** button.
9.4 Hold down the mouse button, select the range I8:I15 and press **enter**.
The plotted points of short run X-bar chart would be calculated and displayed in column I from I7 to I15.

Step 10. Calculate the range of range-chart in column J
10.1 Click on to the cell J7 and type the formula:
" **=ABS(MAX(C7:E7)-MIN(C7:E7))** " .
10.2 Press **enter.**
10.3 Click on cell J7 and choose the **copy** button.
10.4 Holding down the mouse button, select the range J8:J15 and press **enter**.
The subgroup range would be calculated and displayed in column J from J7 to J15.

Step 11. Calculate the range plotted points

11.1 Click on to the cell K7 and type the formula:" **=J7/H7** ".

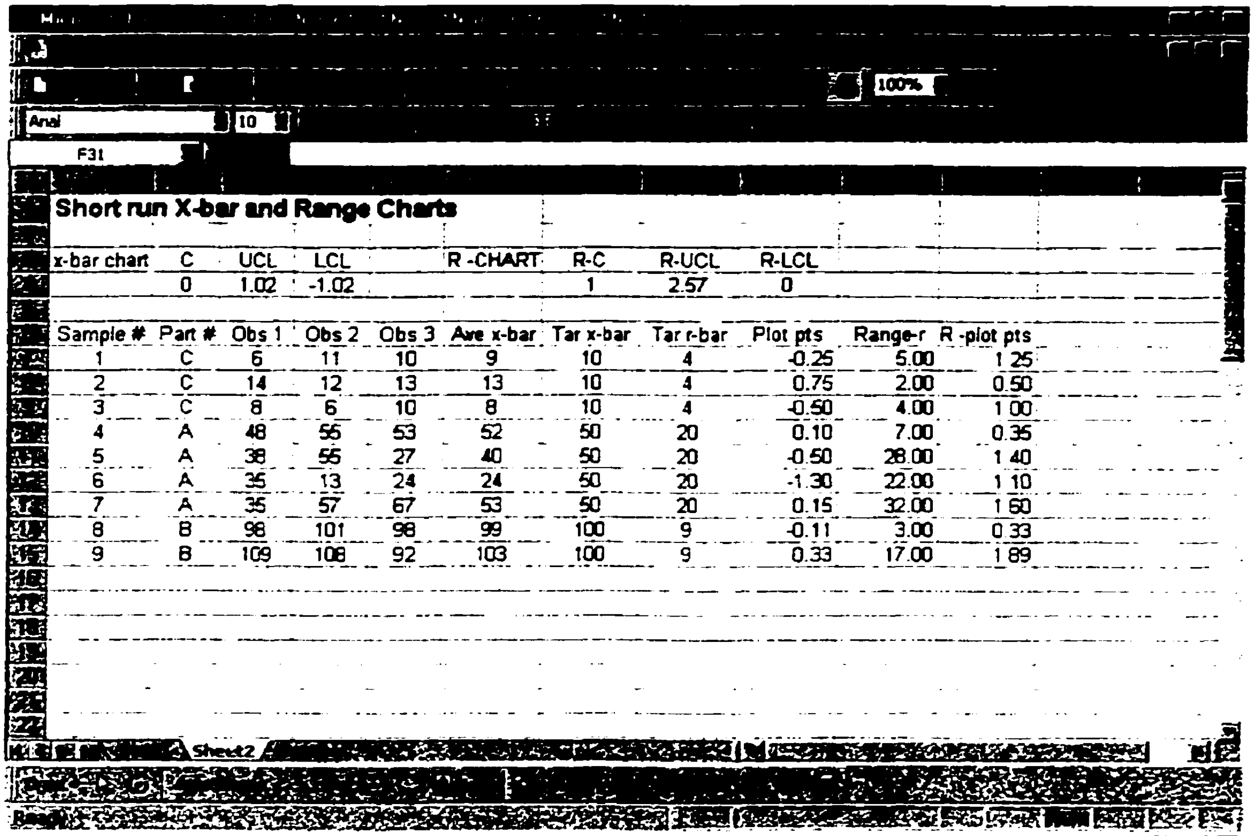
11.2 Press **enter**.

11.3 Click on cell K7 and choose the **copy** button.

11.4 Hold down the mouse button, select the range K8:K15 and press **enter**.

The plotted points of range chart would be calculated and appeared in column K.

The following display will appear:



Short run X-bar and Range Charts

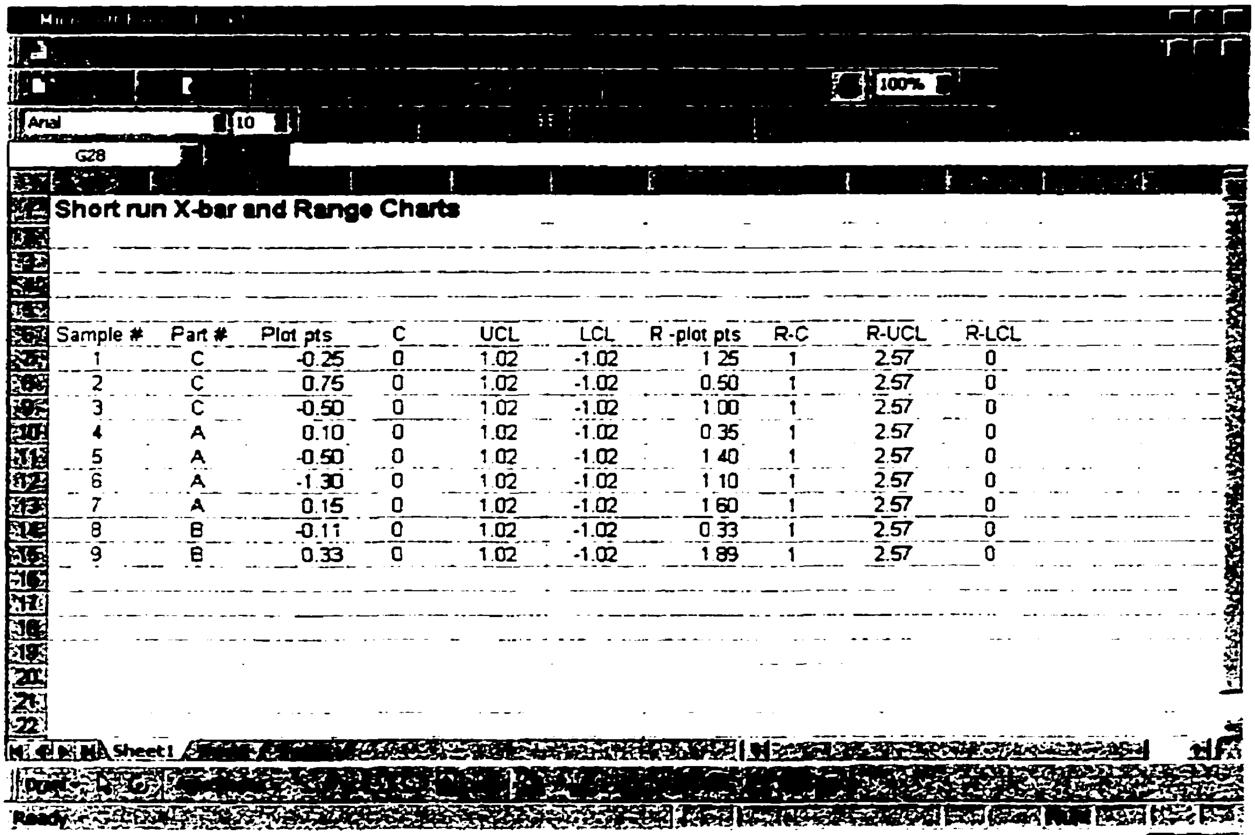
x-bar chart	C	UCL	LCL		R -CHART	R-C	R-UCL	R-LCL
	0	1.02	-1.02			1	2.57	0

Sample #	Part #	Obs 1	Obs 2	Obs 3	Ave x-bar	Tar x-bar	Tar r-bar	Plot pts	Range-r	R -plot pts
1	C	6	11	10	9	10	4	-0.25	5.00	1.25
2	C	14	12	13	13	10	4	0.75	2.00	0.50
3	C	8	6	10	8	10	4	-0.50	4.00	1.00
4	A	48	56	53	52	50	20	0.10	7.00	0.35
5	A	38	56	27	40	50	20	-0.50	28.00	1.40
6	A	35	13	24	24	50	20	-1.30	22.00	1.10
7	A	35	57	67	53	50	20	0.15	32.00	1.60
8	B	98	101	98	99	100	9	-0.11	3.00	0.33
9	B	109	108	92	103	100	9	0.33	17.00	1.89

Figure 6(b)

6.3 Converting the Plotted Points and Control Limits into a New Excel Worksheet

After we calculate all the potted points and control limits for Short Run X-bar and Moving Range Chart, we can use **cut and paste** function to paste the "Sample number, Part number, Plotted points of Short Run X-bar, Control Limits of Short Run X-bar, Plotted Points of Moving Range, and Control Limits of Moving Range" into a new Excel Worksheet. These have to be done because it would then be easier to create a suitable control chart in Excel.



Short run X-bar and Range Charts

Sample #	Part #	Plot pts	C	UCL	LCL	R -plot pts	R-C	R-UCL	R-LCL
1	C	-0.25	0	1.02	-1.02	1.25	1	2.57	0
2	C	0.75	0	1.02	-1.02	0.50	1	2.57	0
3	C	-0.50	0	1.02	-1.02	1.00	1	2.57	0
4	A	0.10	0	1.02	-1.02	0.35	1	2.57	0
5	A	-0.50	0	1.02	-1.02	1.40	1	2.57	0
6	A	-1.30	0	1.02	-1.02	1.10	1	2.57	0
7	A	0.15	0	1.02	-1.02	1.60	1	2.57	0
8	B	-0.11	0	1.02	-1.02	0.33	1	2.57	0
9	B	0.33	0	1.02	-1.02	1.89	1	2.57	0

Figure 6(c)

6.4 Creating the Control Charts

Follow the steps in Section 2 of this chapter to create control charts we need:

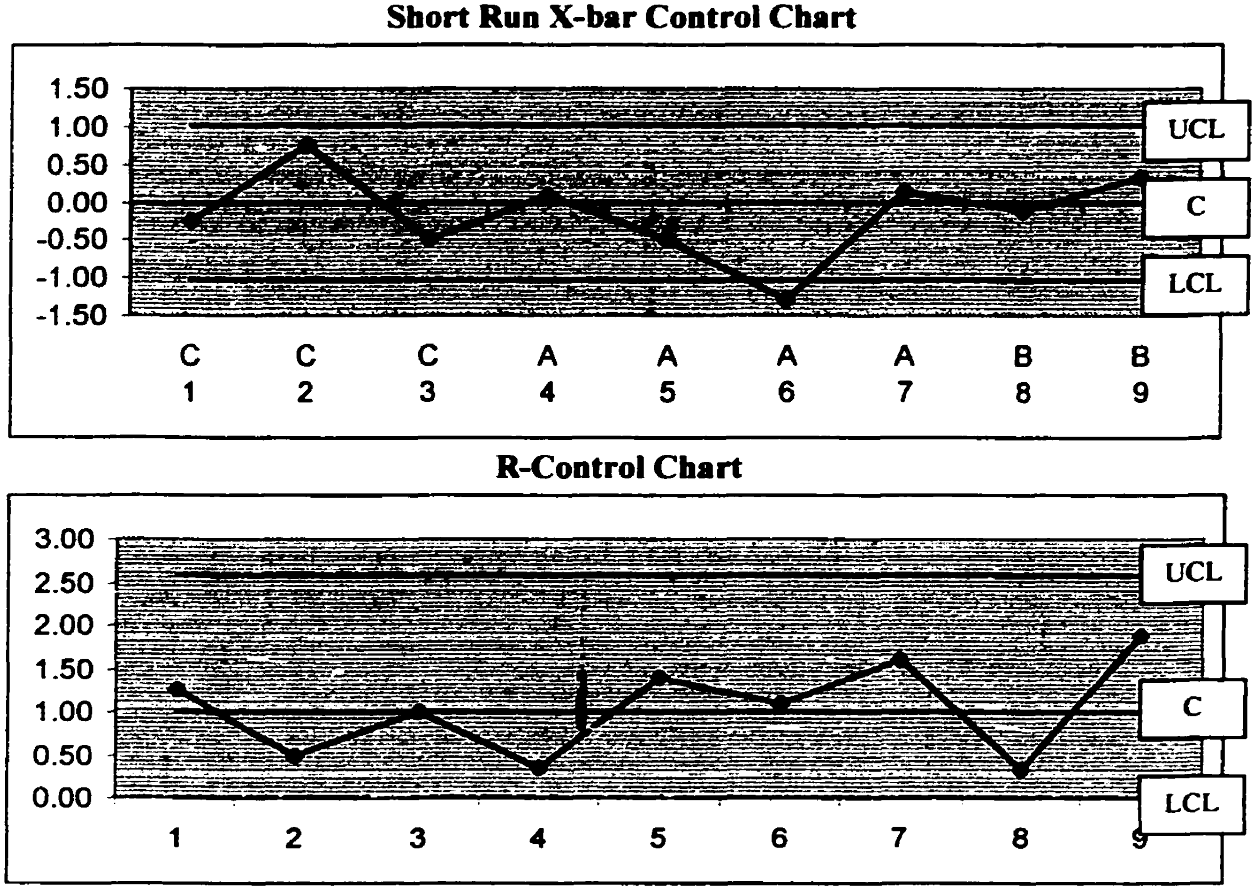


Figure 6(d)

Chapter 2

Control Charts for Short Runs

1. Introduction

Statistical Process Control (SPC) techniques are used for preventing defects and monitoring processes. The philosophy of SPC is to bring the process under control. When a production process is in control, it is stable and predictable. However, traditional techniques such as Shewhart type control charts were originally intended for applications in high volume manufacturing. In today's competitive world, these traditional SPC techniques are not suitable in many situations. For example, a machine can produce similar parts but not identical. Therefore, new techniques have to be developed in order to solve these kinds of problems. The short run control charts have been developed and they could be used in different process streams. Short-run or low production run refers to the process in which only a small amount of items are produced at a time. This chapter summarizes several techniques of short run control charts and the programs in Chapter 1 are used. In addition, different scenarios are provided to choose appropriate short run charts.

2. Why Use Short Run Control Charts

Shewhart's X-bar and R Charts have greatly evolved since the first application in the 1920's. However, these traditional control charts were developed for large production of parts with the same characteristic. In today's manufacturing environment, short production runs have become more and more common. Short run control charts have to be used due to the fact of the application limitation of the traditional control charts. For them, long production runs such as 25 subgroups (i.e., samples) or more would be required and one chart can just be used to keep track on one characteristic (e.g., one part type). If there are 5 quality characteristics, each will need an X-bar and a range chart, Thus 10 different charts would be required. It is tedious and takes a lot of effort.

3. Features of Short Run Control Charts

Short run control charts are developed to allow all parts with different characteristics to be plotted on the same chart. This is made possible by using different transformations of the actual measurements. " The data transformation standardizes all measurement data, this allows a chart to follow the material for a particular lot through the plant, with the data from all operations plotted on it" (Bothe 1989 p.265). This special transformation changes the scales of data so that different part numbers convert to one common distribution.

4. Consideration of the Short Run Charts

There are different types of short run control charts, which are used to deal with different situations. In this Chapter, we only considered several widely used or referred short run control charts. Their formulas and charting bear similarities to the traditional Shewhart control charts, and hence the charts are familiar to practitioners. However, we like to point out that these charts frequently are ad hoc in nature and their applications and validity might be in question in practice. Furthermore, few theoretical analytical works have been done to study their properties. Thus one should use them with caution.

5. Target Individual-X and Moving Range Charts

Nominal or Target Individual-X and Moving Range Charts could handle characteristics dealing with similar variation, shape and material. They can be used to detect changes of individual measurements of one characteristic at a time. They allow similar characteristics to be plotted on the same chart. This can be done by appropriate transformation. Different characteristics in the chart have different nominal or target values. The plotted point is calculated by subtracting the nominal value from the individual measurement. This method works well, when the R-bar values for the different parts are similar. Nominal Individual-X charts determine how far away the actual measurements fall from the nominal values. In this manner, we expect the measurement will be similar to the nominal values in order to get the process in control. There are several things that we have to consider in Individual-X charts. The measurements should be independent of each other. Constant subgroup size of one would be used, and the chart monitors similar characteristics with different dimensions from the process. The similar R-bar values would be expected among the different dimensions of the process. The control limits of the chart assume the whole process to follow the normal distribution. Bothe (1991) points out that if the variation of each different product in a process is more than 30%, this chart should not be used. In addition, there are several things that must be adapted in order to use the Target Individual-X and Moving Range control charts. They are: 1. The same machine must be used, 2. The same processing method is used, 3. The same materials are used.

5.1 Coded values and control limits of Target Individual-X and Moving Range Charts (normal data)

Sources: (Bothe 1991), (Thompson 1989), (Wise and Fair 1998) and (Wheeler 1991)

(a) PLOTTED POINTS:

For Target IX:

$D_j = IX_j - Target_j$ j=1,2……. g^{th} subgroup

where $Target_j$ is the target value for the IX_j

For Moving Range :

DMR_j = Absolute difference between 2 consecutive D_j's $= |D_j - D_{j-1}|$.

(b) CENTER LINE AND CONTROL LIMITS:

For Target IX:

$$CL = \overline{D} = \frac{\sum_{j=1}^{g} D_j}{g}$$

$$UCL = \overline{D} + A_2\,\overline{DMR} = \overline{D} + 2.66\,\overline{DMR}$$

$$LCL = \overline{D} - A_2\,\overline{DMR} = \overline{D} - 2.66\,\overline{DMR}$$

For Moving Range:

$$CL = \overline{DMR} = \frac{\sum_{j=2}^{g} DMR_j}{g-1}$$

$$UCL = D_4\,\overline{DMR} = 3.268\,\overline{DMR}$$

$$LCL = D_3\,\overline{DMR} = 0$$

Example 1: A cutting machine is used to cut rods of varying thickness. The rate of cutting rods would be different due to the thickness of the rod. There are three kinds of rods (A,B, and C) that have to be cut. The materials are the same for all different sizes of rods. The standard deviations are expected to be similar. 15 samples are selected randomly and the rods are measured. The measurement might vary because of the failure of machine parts. A target IX chart would be used to monitor the process.

The following data was generated by computer simulation:

SAMPLE	1	2	3	4	5	6	7	8	9
PART #	A	C	B	C	B	A	C	A	A
Obs	15.9	39.6	24.7	39.9	25.3	14.9	40.3	15.2	15.5

SAMPLE	10	11	12	13	14	15
PART #	C	B	B	C	B	A
Obs	39.6	24.9	24.7	39.7	25.1	16.1

The target values for part A, part B and part C are 15,25 and 40 respectively.

(a) Calculation of plotted points

For Target IX:

$D_j = IX_j - Target_j$ j=1,2·······.15th subgroup

where $Target_j$ is the target value for the IX_j

$D_1 = IX_1 - Target_1 =$ 15.90- 15.00 = 0.9.

$D_2 = IX_2 - Target_2 =$ 39.63- 40.00 = -0.37.

For Moving Range :

DMR_j = Absolute difference between 2 consecutive D_j's

$DMR_2 = |D_2 - D_1|$ = |-0.37-0.9| = 1.27.

$DMR_3 = |D_3 - D_2|$ = |-0.31+0.37| = 0.06.

(b) Calculation of Center Lines and Control Limits

For Target IX:

$$\text{Center line : CL} = \overline{D} = \frac{\sum_{j=1}^{g} D_j}{g} = \frac{0.33}{15} = 0.022187.$$

Upper control limit: UCL = $\overline{D} + 2.66\ \overline{DMR}$ $= 0.022187 + 2.66*0.412781$
$= 0.022187+1.097999$
$=1.120186.$

Lower control limit: LCL = $\overline{D} - 2.66\ \overline{DMR}$ $= 0.022187 - 2.66*0.412781$
$= 0.022187-1.097999$
$=-1.07581.$

<u>For Moving Range</u> :

Center line: CL = $\overline{DMR} = \dfrac{\sum_{j=2}^{g} DMR_j}{g-1} = \dfrac{5.78}{14} = 0.412781.$

Upper control limit: UCL = $D_4\ \overline{DMR} =$ 3.268 *0.412781= 1.34897.

Lower control limit: LCL = $D_3\ \overline{DMR} =$ 0 *0.412781=0.

The data are shown in Table 1.

TABLE 1.

Sample #	Product #	IX_j	$Target_j$	D_j	DMR_j	$\overline{D}$	$\overline{DMR}$
1	A	15.90	15.00	0.90		0.022187	0.412781
2	C	39.63	40.00	-0.37	1.27		
3	B	24.69	25.00	-0.31	0.06		
4	C	39.87	40.00	-0.13	0.19	R-UCL	R-LCL
5	B	25.26	25.00	0.26	0.38	1.34897	0
6	A	14.96	15.00	-0.04	0.30		
7	C	40.34	40.00	0.34	0.38	IX-UCL	IX-LCL
8	A	15.20	15.00	0.20	0.14	1.120186	-1.07581
9	A	14.46	15.00	-0.54	0.74		
10	C	39.60	40.00	-0.40	0.14		
11	B	24.96	25.00	-0.04	0.36		
12	B	24.70	25.00	-0.30	0.25		
13	C	39.63	40.00	-0.37	0.08		
14	B	25.02	25.00	0.02	0.39		
15	A	16.12	15.00	1.12	1.10		

Target Individual-X Control Chart

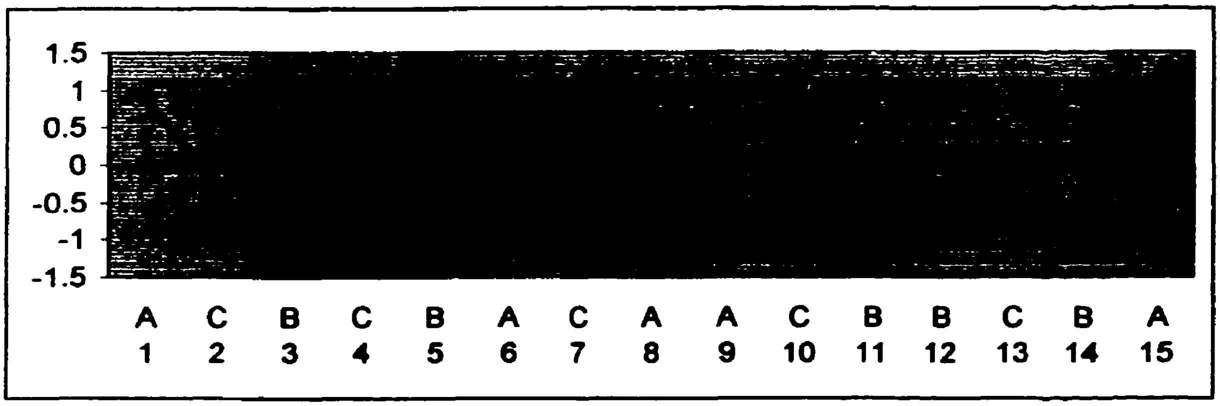


Figure 1(a)

Moving Range Chart

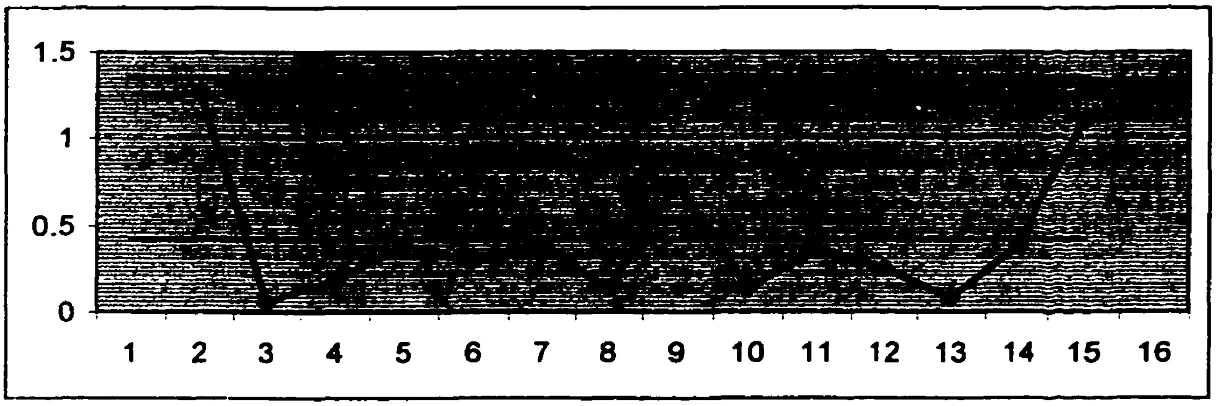


Figure 1(b)

The moving ranges in Figure 1(b) show that no point exceeds the upper control limit. There is no apparent indication of probable shift in the production process. However, the moving ranges of only 3 out of 14 fall above the center line. One should pay more attention to assignable cause and give appropriate adjustments. One should disregard the IX-chart when the moving range chart shows a pattern or points outside the control limits.

5.2 Coded values and control limits of Target Individual-X and Moving Range Charts (non-normal data)

For non-normal data we have to use Moving average as potted points instead of using just coded values of D_j's.

Sources: (Bothe 1991)

(a) PLOTTED POINTS:

For Target MA:

$$DMA_j = \frac{D_j + D_{j-1}}{2} \qquad j = 1,2,\cdots\cdots..15 \text{ subgroup}$$

where $D_j = IX_j - Target_j$ and $Target_j$ is the target value for the IX_j

For Target MR:

DMR_j = Absolute difference between 2 consecutive D_j's = $|D_j - D_{j-1}|$.

(b) CENTER LINES AND CONTROL LIMITS:

For Target MA:

$$\text{CL} = \overline{D} = \frac{\sum_{j=1}^{g} D_j}{g}$$

$$\text{UCL} = \overline{D} + A_2 \overline{DMR} = \overline{D} + 1.88 \overline{DMR}$$

$$\text{LCL} = \overline{D} - A_2 \overline{DMR} = \overline{D} - 1.88 \overline{DMR}$$

where A_2 depends on the subgroup size n

For Target MR:

$$\text{CL} = \overline{DMR} = \frac{\sum_{j=2}^{g} DMR_j}{g-1}$$

$$\text{UCL} = D_4 \overline{DMR} = 3.27 \overline{DMR}$$

$$\text{LCL} = D_3 \overline{DMR} = 0$$

where D_4 and D_3 depend on the subgroup size n

Example 2 : (Bothe 1991, p 28).

SUBGROUP	1	2	3	4	5	6	7	8	9
PART #	A	A	A	D	D	D	B	B	B
Obs	7.9	7.2	7.2	5.9	6.3	6.1	8.9	8.3	8.3
Tar x-bar	7.5	7.5	7.5	6.1	6.1	6.1	8.7	8.7	8.7

SUBGROUP	10	11	12	13	14	15
PART #	B	F	F	E	E	E
Obs	8.7	9.8	9.4	6.8	6.8	7.5
Tar x-bar	8.7	9.5	9.5	7.0	7.0	7.0

(a) Calculation of plotted points

For Target MA :

$DMA_j = \dfrac{D_{j+1} + D_j}{2}$ j= 1,2,⋯⋯..15 subgroup

where $D_j = IX_j - Target_j$ and $Target_j$ is the target value for the IX_j

$D_1 = IX_1 - Target_1$= 7.9-7.5= 0.4 . $D_2 = IX_2 - Target_2$= 7.2-7.5= -0.3 .

$D_3 = IX_3 - Target_3$= 7.2-7.5= -0.3 . $D_4 = IX_4 - Target_4$= 5.9-6.1= -0.2 .

$$DMA_2 = \frac{D_2 + D_1}{2} = \frac{-0.3 + 0.4}{2} = 0.05.$$

$$DMA_3 = \frac{D_3 + D_2}{2} = \frac{-0.3 - 0.3}{2} = -0.3.$$

For Moving Range:

DMR_j = Absolute difference between 2 consecutive D_j's. $= |D_j - D_{j-1}|$.

$DMR_2 = |D_2 - D_1| = |-0.3-0.4| = 0.7.$

$DMR_3 = |D_3 - D_2| = |-0.3+0.3| = 0.$

(b) Calculation of Center Line and Control Limits

<u>For Target MA:</u>

$$\text{Center line:} \quad CL = \overline{D} = \frac{\sum_{j=1}^{g} D_j}{g} = \frac{-0.95}{15} = -0.033.$$

$$\text{Upper control limit: } UCL = \overline{D} + 1.88\ \overline{DMR} = -0.033 + 1.88*0.293$$
$$= -0.033+0.55084$$
$$=0.518.$$

$$\text{Lower control limit: } LCL = \overline{D} - 1.88\ \overline{DMR} = 0.022187 - 1.88*0.293$$
$$= -0.033-0.55084$$
$$=-0.584.$$

<u>For Moving Range:</u>

$$\text{Center line:} \quad CL = \overline{DMR} = \frac{\sum_{j=2}^{g} DMR_j}{g-1} = \frac{4.1}{14} = 0.293.$$

$$\text{Upper control limit: } UCL = D_4\ \overline{DMR} = 3.268 *0.293 = 0.957.$$

$$\text{Lower control limit: } LCL = D_3\ \overline{DMR} = 0 *0.293=0.$$

The data are shown in Table 2.

Table 2.

Sample #	Product #	IX_j	$Target_j$	D_j	DMR_j	DMA_j	$\overline{D}$	$\overline{DMR}$
1	A	7.9	7.5	0.40			-0.03333	0.292857
2	A	7.2	7.5	-0.30	0.7	0.05		
3	A	7.2	7.5	-0.30	0	-0.30	R-UCL	R-LCL
4	D	5.9	6.1	-0.20	0.1	-0.25	0.957057	0
5	D	6.3	6.1	0.20	0.4	0.00		
6	D	6.1	6.1	0.00	0.2	0.10	IX-UCL	IX-LCL
7	B	8.9	8.7	0.20	0.2	0.10	0.517238	-0.5839
8	B	8.3	8.7	-0.40	0.6	-0.10		
9	B	8.3	8.7	-0.40	0	-0.40		
10	B	8.7	8.7	0.00	0.4	-0.20		
11	F	9.8	9.5	0.30	0.3	0.15		
12	F	9.4	9.5	-0.10	0.4	0.10		
13	E	6.8	7.0	-0.20	0.1	-0.15		
14	E	6.8	7.0	-0.20	0	-0.20		
15	E	7.5	7.0	0.50	0.7	0.15		

Target Individual-X Control Chart

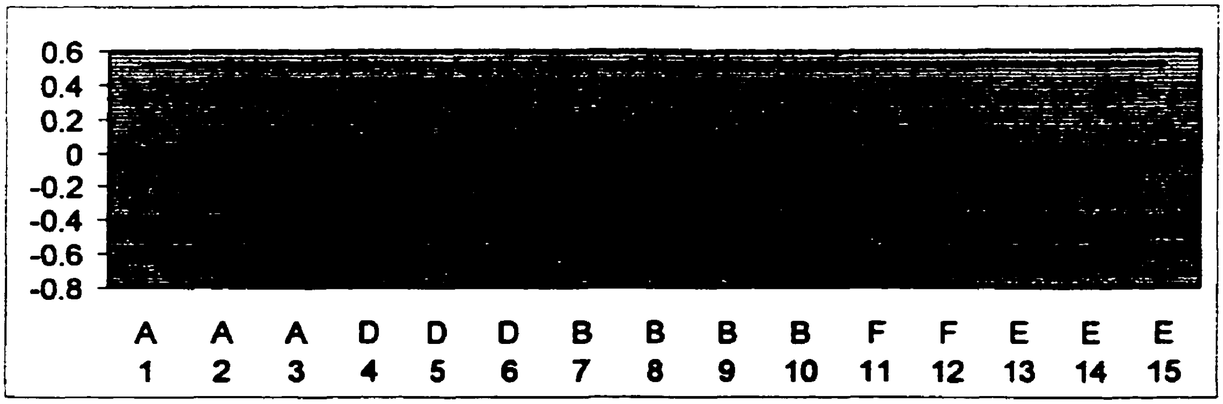


Figure 2(a)

Moving Range Chart

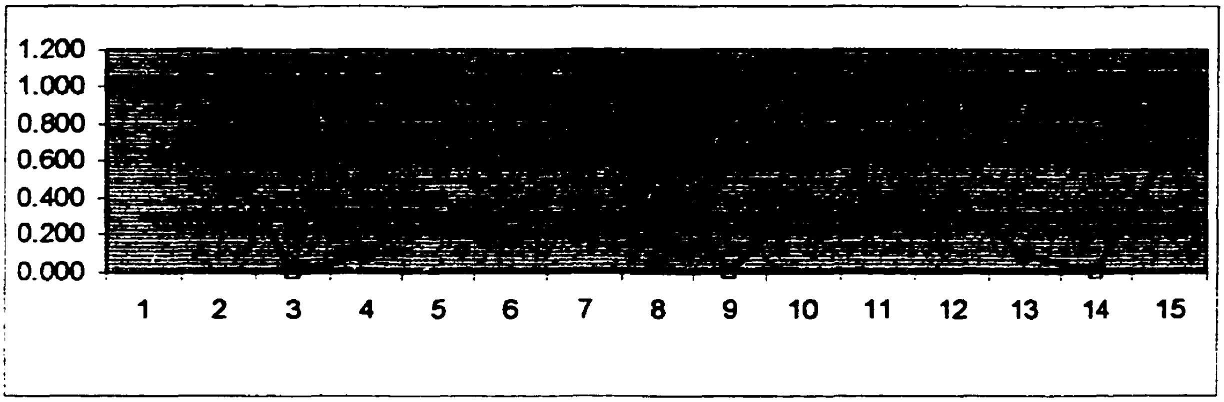


Figure 2(b)

The moving ranges in Figure 2(b) show no point exceeding the upper control limit. There is also no apparent indication to show probable shift in the production process. Most of the moving range plotted points appear to be random. There is no apparent indication of assignable cause.

When the moving range chart do not show any patterns or points out of control limits, we can try to interpret the Individual-X chart. The Individual-X chart shows no plotted points exceeding the control limits. The plotted points of part D (sample # 4, 5 and 6) show an upward trend. But there are only three points. This is not considered to be due to an assignable cause because it requires six or more points in a row steadily increasing or decreasing.

6. Target X-bar and Range Charts (subgroup size $\neq$ 1)

Target X-bar and Range Charts work almost the same as Target Individual-X and Moving Range Charts. The difference is just that the subgroup size is bigger than one. Target X-bar and Range charts are used to detect changes in the average of a single type of measured characteristic. It takes the average of measurements within a subgroup. The subgroup size is between 2 to 5, because of the shorter running time associated with short runs. The plotted point is calculated by subtracting the nominal or target values from the average of subgroup measurements. If the average measurement is equal to the nominal value, it means the plotted point would fall directly on the zero point of the control chart. When using the Target X-bar and Range Charts, there are several things with which we have to be concerned. The measurements should be independent of each other. Measurements within each subgroup have the same characteristic. The subgroup size also should be constant in order to maintain constant control limits. The process variation is expected to be about the same for all parts. In addition, there are several things that must be adapted in order to use the Target X-bar and Range Control Charts. They are: 1. the same machine must be used, 2. The same processing method is used, 3. The same materials are used. For a given process, all the part numbers could be plotted in the same chart.

6.1 Coded values and control limits of Target X-bar and Range charts

Sources: (Bothe 1991), (Farnum 1992), (Griffith 1989), (Montgomery 1996), (Pyzdek 1993), (Wise and Fair 1998) and (Wheeler 1991)

(a) PLOTTED POINTS:

For Target $\overline{X}$:

$\overline{D_j} = \overline{X_j} - Target_j$ i=1,2,..n[th] observation and j=1,2,⋯g[th] subgroup

where $\overline{X_j} = \dfrac{\sum_{i=1}^{n} X_{ij}}{n}$ and $Target_j$ is the target value for the $\overline{X_j}$.

For Range:

R_j = Maximum X_{ij} - Minimum X_{ij}

where maximum and minimum are over i.

(b) CENTER LINES AND CONTROL LIMITS:

For Target $\overline{X}$:

$$\text{CL} = \overline{\overline{D}} = \frac{\sum_{j=1}^{g} \overline{D_j}}{g}$$

$$\text{UCL} = \overline{\overline{D}} + A_2 \overline{R}$$

$$\text{LCL} = \overline{\overline{D}} - A_2 \overline{R}$$

where A_2 depends on the subgroup size n

**Target Line = 0

For Range:

$$\text{CL} = \overline{R} = \frac{\sum_{j=1}^{g} R_j}{g}$$

$\text{UCL} = D_4 \ \overline{R}$

$\text{LCL} = D_3 \ \overline{R}$

where D_4 and D_3 depends on the subgroup size n

Example 3: (Wheeler 1991, p24)
A machine can produce two different products. The material of these two products is considered as to be the same but the sizes of the two products are different. The standard deviations of these two products are approximately the same. 20 samples of size 3 are selected and the product sizes are measured. The target values of Part A and Part B are 6 and 9, respectively.

Table 3.

Sample #	Part #	Obs 1	Obs 2	Obs 3	$\overline{X_j}$	$Target_j$	$\overline{D_j}$	Rj	$\overline{\overline{D_j}}$	$\overline{R}$
1	A	8	8	7	7.67	6	1.67	1.00	0.15	3.05
2	B	12	10	7	9.67	9	0.67	5.00		
3	B	9	8	6	7.67	9	-1.33	3.00		
4	B	8	11	8	9.00	9	0.00	3.00		
5	B	8	11	8	9.00	9	0.00	3.00		
6	B	11	7	7	8.33	9	-0.67	4.00		
7	B	9	7	8	8.00	9	-1.00	2.00		
8	B	10	8	6	8.00	9	-1.00	4.00		
9	B	9	6	8	7.67	9	-1.33	3.00		
10	A	5	7	7	6.33	6	0.33	2.00		
11	A	10	5	7	7.33	6	1.33	5.00		
12	A	5	4	6	5.00	6	-1.00	2.00		
13	A	7	10	7	8.00	6	2.00	3.00		
14	A	6	2	8	5.33	6	-0.67	6.00		
15	A	6	6	8	6.67	6	0.67	2.00		
16	A	6	9	6	7.00	6	1.00	3.00		
17	B	7	13	9	9.67	9	0.67	6.00		
18	B	10	10	9	9.67	9	0.67	1.00		
19	A	7	7	5	6.33	6	0.33	2.00		
20	A	7	6	7	6.67	6	0.67	1.00		

(a) Calculation of one-subgroup average

$$\overline{X_1} = \frac{\sum_{i=1}^{3} X_{i1}}{3} = \frac{8+8+7}{3} = 7.67.$$

$$\overline{X_2} = \frac{\sum_{i=1}^{3} X_{i2}}{3} = \frac{12+10+7}{3} = 9.67.$$

(b) Calculation of plotted points

For Target $\overline{X}$:

$\overline{D_j} = \overline{X_j} - Target_j$ j=1,2⋯⋯.20 subgroup

where $Target_j$ is the target value for the $\overline{X_j}$

$\overline{D_1} = \overline{X_1}$ - $Target_1 = 7.67 - 6.0 = 1.67.$

$\overline{D_2} = \overline{X_2}$ - $Target_2 = 9.67 - 9.0 = 0.67.$

For Range:

R_j = Maximum X_{ij} - Minimum X_{ij}

where maximum and minimum are over i.

R_1 = Maximum X_{11} - Minimum X_{31} = 8-7 = 1.

R_2 = Maximum X_{12} - Minimum X_{32} = 12-7 = 5.

(c) Calculation of Center Lines and Control Limits

For Target $\overline{X}$:

$$\text{Center line: CL} = \overline{\overline{D}} = \frac{\sum_{j=1}^{20} \overline{D_j}}{20} = \frac{3}{20} = 0.15.$$

$$\text{Upper control limit: UCL} = \overline{\overline{D}} + A_2 \overline{R} = 0.15 + 1.023*3.15 = 3.12.$$

$$\text{Lower control limit: LCL} = \overline{\overline{D}} - A_2 \overline{R} = 0.15 - 1.023*3.15 = -3.12.$$

**Target Line = 0

For Range:

$$\text{Center line: CL} = \overline{R} = \frac{\sum_{j=1}^{20} R_j}{20} = \frac{61}{20} = 3.05.$$

Upper control limit: UCL = $D_4 \overline{DMR}$ = 2.574 *3.05= 7.8507.

Lower control limit: LCL = $D_3 \overline{DMR}$ = 0 *3.05=0.

Target X-bar Control Chart

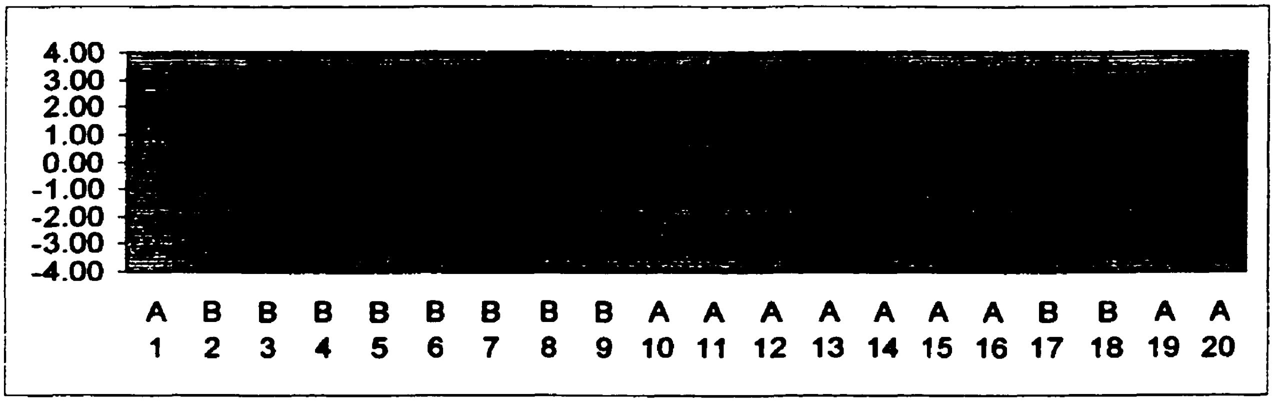


Figure 3(a)

Range Control Chart

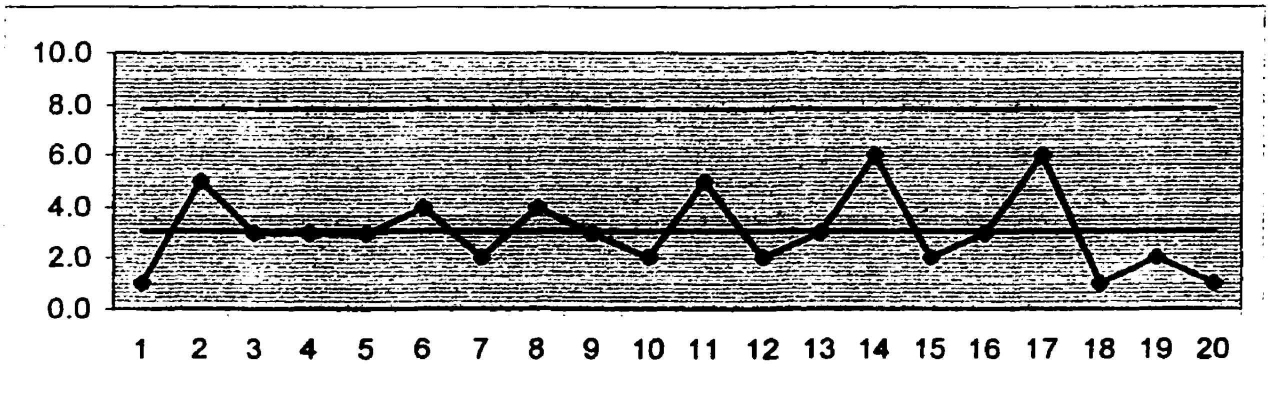


Figure 3(b)

The moving ranges in Figure3 (b) show no point exceeding the upper control limit. There is also no apparent indication to show probable shift in the production process. Most of the moving range plotted points appear to be random. There is no apparent indication of assignable cause.

By looking at the Target X-bar chart, the plotted points of samples 15 to 20 are all above center line. There may be the possibility of upward shift in the process.

7. Short Run IX and MR Charts (subgroup size =1)

The Short Run IX-MR charts allow dissimilar multiple quality characteristics with different standard deviations to be monitored on the same control chart. This can be done because the plotted points are standardized and the control limits remain constant. The control chart can be used for small amounts of data and the plotted points are coded into unitless ratio. Standardized transformations are taken by subtracting the corresponding target value (or grand average) from each individual measurement and the result is divided by R-bar. There are several things that we have to consider in Short Run IX and MR charts. The measurements should be independent of each other. Constant subgroup size of one would be used and dissimilar characteristics from different parts (such as different materials, volumes or thicknesses) of the process. Dissimilar R-bar values would be expected among the different dimensions. The methodology of the plotted points and control limits are shown in the following.

STATISTICAL ANALYSIS OF METHODOLOGY:

The SHORT RUN IX Chart was based on the traditional IX Chart except the plotted points are coded.

From the traditional RANGE Chart, the control limits are defined as:

$$\text{UCL} = D_4 \overline{R} \ , \qquad \text{LCL} = D_3 \overline{R}$$

A point (i.e. sample range R) falls between the UCL and LCL is said to be in control and could be expressed as:

$$\text{LCL} < R < \text{UCL}$$

$$\text{i.e., } D_3\,\overline{R} < R < D_4\,\overline{R}$$

Dividing all the terms by $\overline{R}$ result in a new plot point with

$$D_3 < \frac{R}{\overline{R}} < D_4$$

If $\overline{R}$ is considered as the target range, the range plotted point then becomes the unitless

$$W^{*} = \frac{R}{Target\,\overline{R}}$$

For the traditional IX chart, the control limits are defined as:

$$\text{UCL} = \overline{IX} + A_2\,\overline{MR} \quad , \qquad \text{LCL} = \overline{IX} - A_2\,\overline{MR}$$

Where $\overline{MR}$ is average moving range of a particular part.

A point (i.e. sample average) falling between the UCL and LCL is said to be in control and could be expressed as :

$$\text{LCL} < IX < \text{UCL}$$

$$\text{i.e., } \overline{IX} - A_2\,\overline{MR} < IX < \overline{IX} + A_2\,\overline{MR}$$

Subtracting $\overline{IX}$ and dividing by $\overline{MR}$ result in a new standardized plot point with

$$-A_2 < \frac{IX - \overline{IX}}{\overline{MR}} < A_2$$

If $\overline{IX}$ and $\overline{MR}$ are considered as the target $\overline{IX}$ and target $\overline{MR}$, respectively, then the IX plotted point becomes the unitless

$$Z^{*} = \frac{IX - Target\,\overline{IX}}{Target\,\overline{MR}}$$

There are two methods (Sections 7.1 and 7.2) that can both work in this particular situation. The difference between of these two methods is the estimation of $Target\,\overline{MR}$ and the control limits that are not the same.

7.1 Coded values and control limits of Short Run IX and MR charts (method 1)

Sources: (Bothe 1991) and (Wise and Fair 1998)

(a) PLOT POINTS:

For Short Run IX :

$$Z_j^{*} = \frac{IX_j - Target_j}{Target\ \overline{MR_j}}$$

where $Target_j$ is the target value for the IX_j and $Target\ \overline{MR_j}$ is the corresponding standard deviation for the IX_j

For Short Run Moving Range:

MR_j^{*} = Absolute difference between 2 consecutive Z_j^{*} value = $|Z_j^{*} - Z_{j-1}^{*}|$.

(b) CENTER LINE AND CONTROL LIMITS:

For Short Run IX:

CL = 0

UCL = A_2 = 2.66

LCL = - A_2 = -2.66

where A_2 depends on the subgroup size n

For Short Run Moving Range:

CL = 1

UCL = D_4 = 3.27

LCL = D_3 = 0

where D_4 and D_3 depend on the subgroup size n

Example 4 : (Bothe 1991, P24)

Table 4.

Sample #	Part #	IX_j	$Target_j$	$Target\ \overline{MR}_j$	z_j^{*}	MR_j^{*}
1	C	10.10	10.40	0.34	-0.88	
2	C	10.80	10.40	0.34	1.18	2.06
3	B	9.10	9.50	0.41	-0.98	2.15
4	B	9.40	9.50	0.41	-0.24	0.73
5	F	8.90	8.70	0.26	0.77	1.01
6	F	8.70	8.70	0.26	0.00	0.77
7	F	8.20	8.70	0.26	-1.92	1.92
8	D	12.20	10.90	0.32	4.06	5.99
9	A	10.40	10.20	0.28	0.71	3.35
10	A	10.60	10.20	0.28	1.43	0.71
11	A	9.90	10.20	0.28	-1.07	2.50

(a) Calculation of plotted points

For Short Run IX:

$$Z_j^{*} = \frac{IX_j - Target_j}{Target\ \overline{MR}_j} \qquad \text{j=1,2}\cdots\cdots\text{.11 subgroup}$$

where $Target_j$ is the target value for the IX_j and $Target\ \overline{MR}_j$ is the target value for the IX_j

$$Z_1^{*} = \frac{IX_1 - Target_1}{Target\ \overline{MR}_1} = \frac{10.1 - 10.4}{0.34} = -0.88.$$

$$Z_3^{*} = \frac{IX_3 - Target_3}{Target\ \overline{MR}_3} = \frac{9.1 - 9.5}{0.41} = -0.98.$$

For Short Run Moving Range:

MR_j^{*} = Absolute difference between 2 consecutive Z_j^{*} values = $|Z_j^{*} - Z_{j-1}^{*}|$.

$MR_2^{*} = |Z_2^{*} - Z_1^{*}| = |1.18+0.88| = 2.06.$

$MR_4^{*} = |Z_4^{*} - Z_3^{*}| = |-0.24+0.98| = 0.73.$

(b) Calculation of Center Line and Control Limits

<u>For Short Run IX</u>:

Center line: CL $=0.$

Upper control limit: UCL = A_2 $=2.66$

Lower control limit: LCL = $-A_2=-2.66$

<u>For Short Run Moving Range</u>:

Center line: CL = 1

Upper control limit: UCL = D_4 $=3.27$

Lower control limit: LCL = $D_3=0.$

Short run IX Control Chart

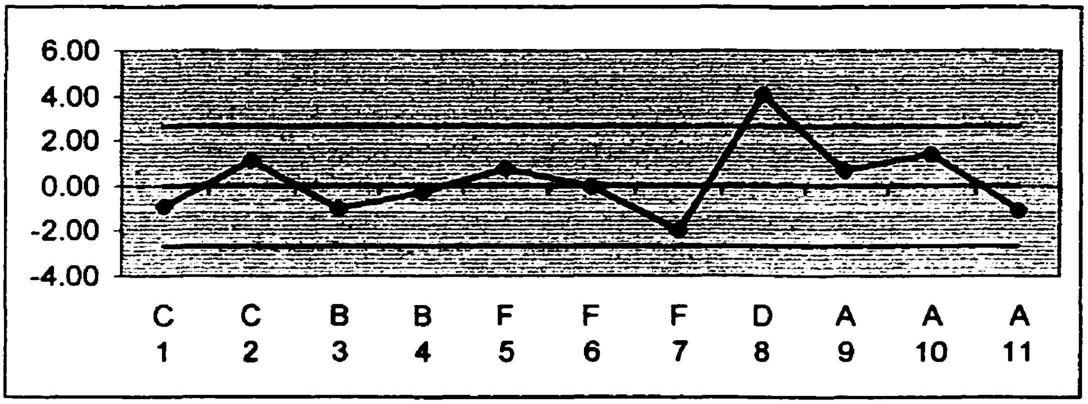


Figure 4(a)

Moving Range Control Chart

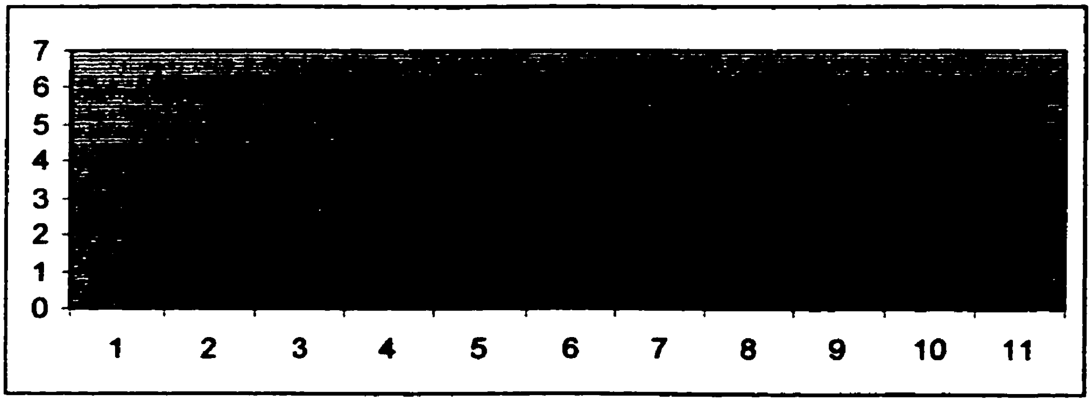


Figure 4(b)

The moving ranges in Figure 4(b) show two points (8 and 9) that exceed the upper control limit. There is apparent indication of probable shift in the production process. One should check for possible assignable cause and give an appropriate adjustment. Do not interpret the IX-chart when the moving range chart shows out of control limits signals.

7.2 Coded values and control limits of Zed for Individual X and Moving Range charts (method 2)

Source: (Wheeler 1991)
(a) PLOTTED POINTS:

For Zed IX:

$$Z_j = \frac{IX_j - No\min al_j}{Sigma(X_j)} \qquad j=1,2\cdots\cdots.,g^{th} \text{ subgroup}$$

where $No\min al_j$ is the target value of the IX_j and $Sigma(X_j)$ is the corresponding standard deviation of the IX_j

$$Sigma(X\ ) = \frac{\overline{MR}}{d_2}, \ d_2 = 1.128 \text{ for } n=2,$$

For Moving Range:

MR_j = Absolute difference between 2 consecutive Z_j values = $|Z_{j+1} - Z_j|$.

(b) CENTER LINES AND CONTROL LIMITS:

For Zed IX:

CL = 0

UCL = 3

LCL = -3

For Moving Range:

CL = d_2

UCL = $d_2\ D_4\ = d_2\ +\ 3\ d_3$

LCL = $d_2\ \ D_3\ = d_2\ -\ 3\ d_3$

Example 5: An automatic machine is used to fill cans with different fruits. There are two kinds of canned fruits. 20 samples are randomly selected and the nominal weights of canned fruit A and canned fruit B are 21.3 and 70 respectively. The standard deviations of the weights of these two kinds of fruit are not the same since the fruits are different. $Sigma(X)$ of part A and part B are 6.35 and 2.04. The data was generated by computer simulation.

Table 5.

Sample #	Part #	IX_j	$Nonimal_j$	$Sigma(X_j)$	Z_j	MR_j
1	A	29.02	21.3	6.35	1.22	
2	A	19.60	21.3	6.35	-0.27	1.48
3	A	21.98	21.3	6.35	0.11	0.37
4	B	68.74	70.0	2.04	-0.62	0.72
5	B	68.73	70.0	2.04	-0.62	0.00
6	A	14.58	21.3	6.35	-1.06	0.44
7	A	31.19	21.3	6.35	1.56	2.62
8	B	69.32	70.0	2.04	-0.33	1.89
9	A	16.88	21.3	6.35	-0.70	0.36
10	B	69.85	70.0	2.04	-0.07	0.62

11	B	71.93	70.0	2.04	0.95	1.02
12	B	67.57	70.0	2.04	-1.19	2.14
13	B	72.37	70.0	2.04	1.16	2.35
14	A	22.57	21.3	6.35	0.20	0.96
15	A	19.57	21.3	6.35	-0.27	0.47
16	A	17.03	21.3	6.35	-0.67	0.40
17	A	20.16	21.3	6.35	-0.18	0.49
18	B	69.31	70.0	2.04	-0.34	0.16
19	B	72.18	70.0	2.04	1.07	1.41
20	B	69.73	70.0	2.04	-0.13	1.20

(a) Calculation of $Sigma(X)$

Calculate the $Sigma(X)$ for part A.

$$Sigma(X_A) = \frac{\overline{MR_A}}{d_2}, \quad d_2 = 1.128 \text{ for n=2.}$$

$$= \frac{7.165}{1.128} = 6.35.$$

Notice: $\overline{MR_A}$ is estimated by Part A average moving range.

Calculate the $Sigma(X)$ for part B.

$$Sigma(X_B) = \frac{\overline{MR_B}}{d_2}, \quad d_2 = 1.128 \text{ for n=2.}$$

$$= \frac{2.305}{1.128} = 2.04.$$

Notice: $\overline{MR_B}$ is estimated by Part B average moving range.

(b) Calculation of plotted points

For Zed IX :

$$Z_j = \frac{IX_j - Nominal_j}{Sigma(X_j)} \quad \text{j=1,2}\cdots\cdots\text{.20 subgroup}$$

where $Nominal_j$ is the Nominal value for the IX_j and $Sigma(X_j)$ is the corresponding standard deviation for the IX_j.

$$Z_1 = \frac{IX_1 - Nominal_1}{Sigma(X_1)} = \frac{29.02 - 21.3}{6.35} = 1.22.$$

$$Z_4 = \frac{IX_4 - Nominal_4}{Sigma(X_4)} = \frac{68.74 - 70}{2.04} = -0.62.$$

For Moving Range:

MR_j = **Absolute difference between 2 consecutive Z_j values**

$= |Z_j - Z_{j-1}|$

$MR_2 = |Z_2 - Z_1| = |-0.27-1.22| = 1.48.$

$MR_4 = |Z_4 - Z_3| = |-0.62-0.11| = 0.72.$

(b) Calculation of Center Line and Control Limits

For Zed IX :

Center line : CL = 0.

Upper control limit: UCL = 3

Lower control limit: LCL = -3

For Moving Range:

Center line: CL = d_2 = 1.128

Upper control limit: UCL = $d_2 + 3d_3$ =1.128+3*0.8525=3.686.

Lower control limit: LCL = $d_2 - 3d_3$ = 0.

Zed Individual X Control Chart

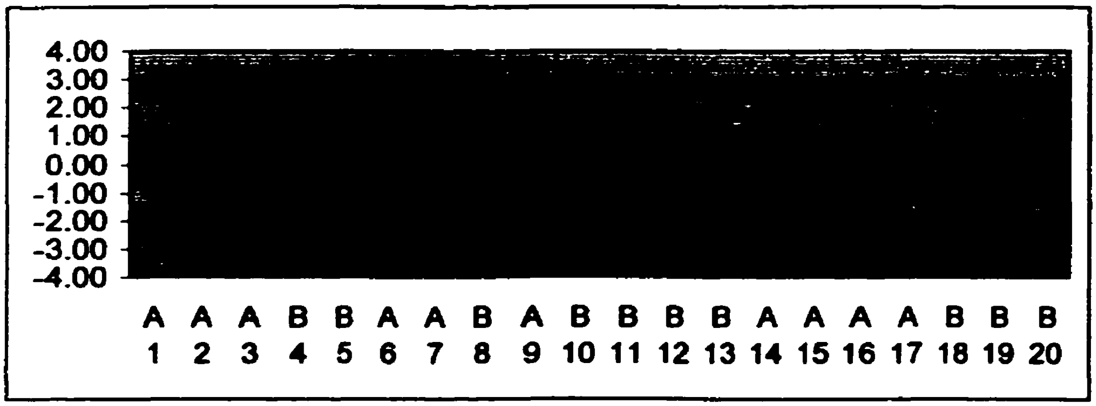


Figure 5(a)

Moving Range chart

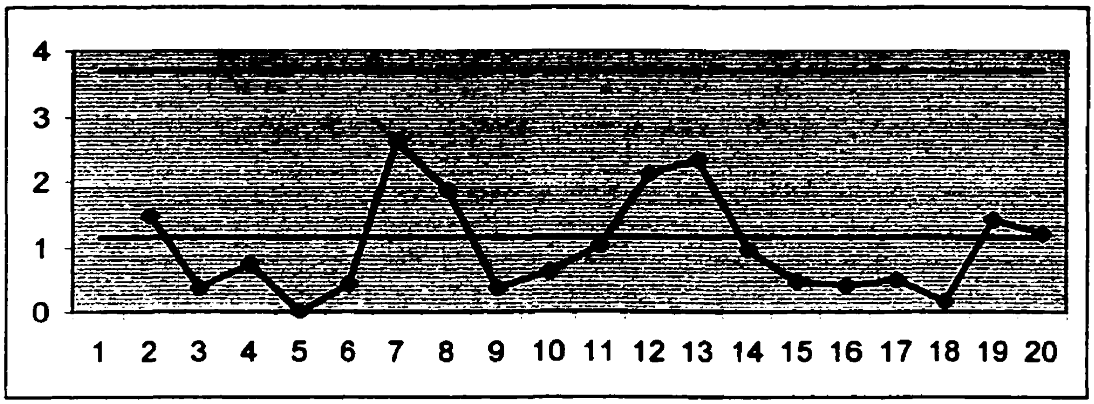


Figure 5(b)

The moving ranges in Figure 5(b) show no point exceeding the upper control limit. There is also no apparent indication of shift in the production process. Most of the moving range plotted points appear to be random. There is no apparent indication of assignable cause.

By looking at the Zed IX chart, there are neither apparent trends nor plotted points outside the control limits. The plotted points appear to be random about the center line.

7.3 Coded values and control limits of Short run MA and MR charts (non-normal data)

Source: (Bothe 1991)

(a) PLOTTED POINTS:

For Short Run MA:

$$MA_j = \frac{Z_j + Z_{j-1}}{2}$$

where $Z_j = \dfrac{IX_j - Target_j}{Target\ \overline{MR_j}}$,

$Target_j$ = the target value for the IX_j and

$Target\ \overline{MR_j}$ = the corresponding target moving range for the IX_j

For Short Run MR:

MR_j = Absolute difference between 2 consecutive Z_j' s values $= |Z_j - Z_{j-1}|$.

(b) CENTER LINES AND CONTROL LIMITS:

For Short Run MA:

CL = 0

UCL = A_2 = 1.88

LCL = - A_2 = -1.88

For Short Run MR :

CL = 0

UCL = D_4 = 3.27

LCL = D_3 = 0

Example 6: (Bothe 1991, P24) The data is the same as example 4.

Table 6.

Sample #	Part #	IX_j	$Target_j$	Target $\overline{MR}_j$	Z_j	MR_j	MA_j
1	C	10.10	10.40	0.34	-0.88		
2	C	10.80	10.40	0.34	1.18	2.06	0.15
3	B	9.10	9.50	0.41	-0.98	2.15	0.10
4	B	9.40	9.50	0.41	-0.24	0.73	-0.61
5	F	8.90	8.70	0.26	0.77	1.01	0.26
6	F	8.70	8.70	0.26	0.00	0.77	0.38
7	F	8.20	8.70	0.26	-1.92	1.92	-0.96
8	D	12.20	10.90	0.32	4.06	5.99	1.07
9	A	10.40	10.20	0.28	0.71	3.35	2.39
10	A	10.60	10.20	0.28	1.43	0.71	1.07
11	A	9.90	10.20	0.28	-1.07	2.50	0.18

(a) Calculation of plotted points

For Short run MA :

$$MA_j = \frac{Z_j + Z_{j-1}}{2}$$

where $Z_j = \dfrac{IX_j - Target_j}{Target\ \overline{MR}_j}$

$Target_j$ = Target value for the IX_j.

$Target\ \overline{MR}_j$ = Corresponding target moving range for the IX_j.

$Z_1 = \dfrac{X_1 - Target_1}{Target\ \overline{MR}_1} = \dfrac{10.1 - 10.4}{0.34} = -0.88$. $Z_2 = \dfrac{X_2 - Target_2}{Target\ \overline{MR}_2} = \dfrac{10.8 - 10.4}{0.34} = 1.18$.

$Z_3 = \dfrac{X_3 - Target_3}{Target\ \overline{MR}_3} = \dfrac{9.1 - 9.5}{0.41} = -0.98$. $Z_4 = \dfrac{X_4 - Target_4}{Target\ \overline{MR}_4} = \dfrac{9.4 - 9.5}{0.41} = -0.24$.

For Short run MA:

$$MA_2 = \frac{Z_2 + Z_1}{2} = \frac{1.18 - 0.88}{2} = 0.15.$$

$$MA_4 = \frac{Z_4 + Z_3}{2} = \frac{-0.24 - 0.98}{2} = -0.61.$$

<u>For Short Run Moving Range</u> :

MR_j = Absolute difference between 2 consecutive Z_j values = $|Z_j - Z_{j-1}|$.

$MR_2 = |Z_2 - Z_1| = |1.18+0.88| = 2.06.$

$MR_4 = |Z_4 - Z_3| = |-0.24+0.98| = 0.73.$

(b) Calculation of Center Line and Control Limits

<u>For Short run MA</u> :

Center line: CL = 0.

Upper control limit: UCL = A_2 = 1.88

Lower control limit: LCL = $-A_2$ = -1.88

<u>For Short Run Moving Range</u> :

Center line : CL = 1

Upper control limit: UCL = D_4 =3.27

Lower control limit: LCL = D_3 = 0.

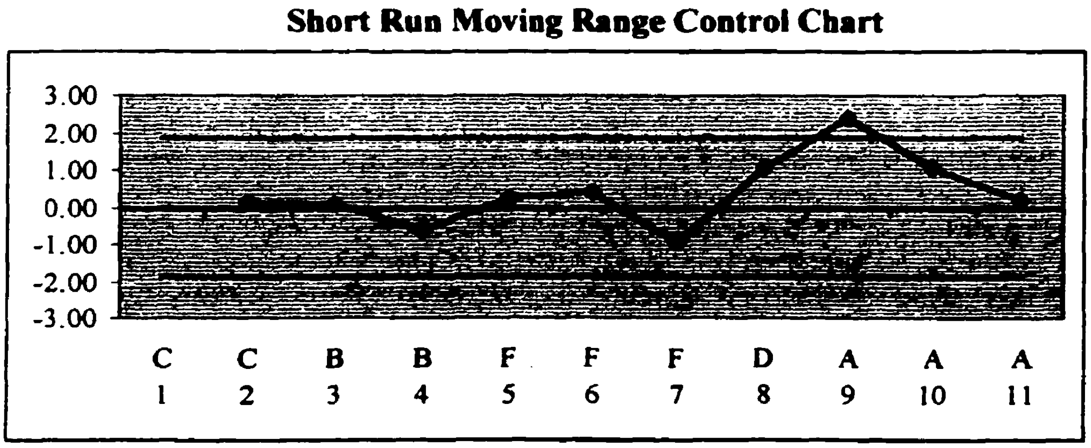


Figure 6(a).

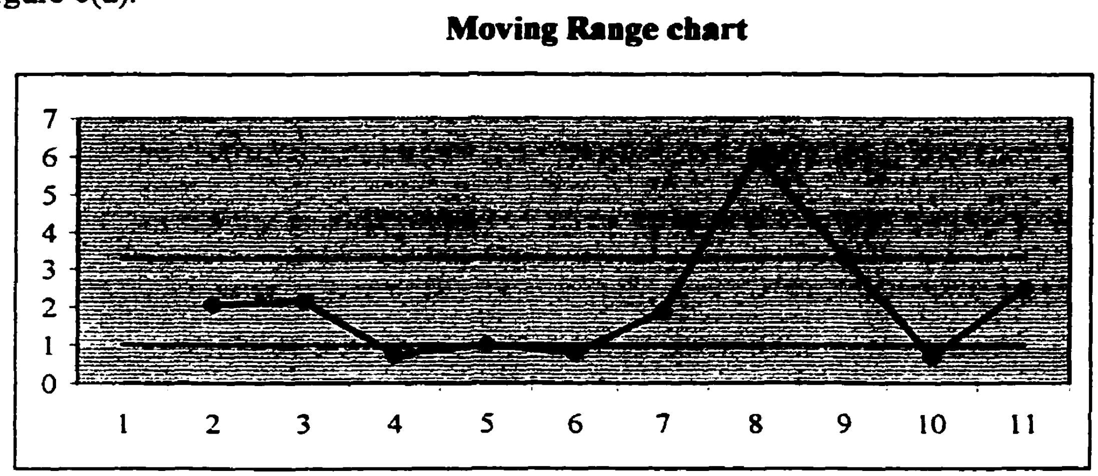


Figure 6(b).

The moving ranges in Figure 6(b) shows two points exceeding the upper control limit. There is apparent indication of probable shift in the production process. One should check for possible assignable cause and give an appropriate adjustment. Do not interpret the IX-chart when the moving range chart shows points that are out of control limits.

8. Short Run Target X-bar and Range Chart (subgroup size ≠ one)

Short run Target X-bar and Range charts work almost the same as Short Run IX and MR charts. The difference is just that the subgroup size is greater than one. The control charts could be used for small amounts of data and the plotted points are coded into unitless ratios (Farnum 1992). Different characteristics should not be in the same subgroup. Doing this would result in wrong calculations of plotted points and control limits. The subgroup size should also be constant in order to maintain constant control limits. There are several things that we have to consider in Short Run Target X-bar and Range charts. The measurements should be independent of each other. Constant subgroup size of 2 to 5 would be used and the charts monitor dissimilar characteristics (different target values, different units of measurement and different standard deviations) with different dimensions from the process. The variety of R-bar values would be expected among the different dimensions of the process. The methodology of the plotted points and control limits are shown in below.

STATISTICAL ANALYSIS OF METHODOLOGY:

The SHORT RUN TARGET $\overline{X}$ AND RANGE ™Charts are based on the traditional $\overline{X}$ AND RANGE Charts except the plotted points are coded.

From the traditional RANGE Chart, the control limits are defined as:

$$\text{UCL} = D_4 \overline{R} \text{ , LCL} = D_3 \overline{R}$$

A point (i.e. sample range R) falling between the UCL and LCL is said to be in control and could be expressed as :

$$\text{LCL} < R < \text{UCL}$$

$$\text{i.e., } D_3 \overline{R} < R < D_4 \overline{R}$$

Dividing all the terms by $\overline{R}$ result to a new plot point with

$$D_3 < \frac{R}{\overline{\overline{R}}} < D_4$$

If $\overline{\text{R}}$ is considered as the target range, the range plotted point then becomes the unitless

$$W^{*} = \frac{R}{Target\overline{R}}$$

From the traditional $\overline{X}$ Chart, the control limits are defined as:

$$UCL = \overline{\overline{X}} + A_2\overline{R}, \qquad LCL = \overline{\overline{X}} - A_2\overline{R}$$

A point (i.e. sample average) falling between the UCL and LCL is said to be in control and could be expressed as :

$$LCL < \overline{X} < UCL$$

$$\text{i.e., } \overline{\overline{X}} - A_2\overline{R} < \overline{X} < \overline{\overline{X}} + A_2\overline{R}$$

Subtracting all the terms by $\overline{\overline{X}}$ and dividing by $\overline{R}$ result to a new standardized plotted point with

$$-A_2 < \frac{\overline{X} - \overline{\overline{X}}}{\overline{R}} < A_2$$

If $\overline{\overline{X}}$ is considered as the target average and $\overline{R}$ as the target range, the $\overline{X}$ plot point then becomes the unitless

$$\overline{Z^{*}} = \frac{\overline{X} - Target\overline{\overline{X}}}{Target\overline{R}}$$

There are two methods(section 8.1 and section 8.2) that can both work in this particular situation. The difference between of these two methods is that the estimation of $Target\ \overline{R}$ and the control limits is not the same.

8.1 Coded values and control limits of Short Run Target X-bar and Range Chart (method 1)

Sources: (Bothe 1991), (Farnum 1992, 1994), (Montgomery 1996), (Pyzdek 1993), (Wise and Fair 1998) and (Wheeler 1991)

(a) PLOTTED POINTS:

For Short Run Target $\overline{\overline{X}}$:

$$\overline{z_j^*} = \frac{\overline{X_j} - Target\ \overline{\overline{X}}_j}{Target\ \overline{R_j}}$$

where $Target\ \overline{\overline{X}}_j$ is the target value for the $\overline{X}_j$ and $Target\ \overline{R_j}$ is the corresponding target range for the $\overline{X}_j$.

For Short Run Range:

R_j = Maximum X_{ij} - Minimum X_{ij}

where maximum and minimum are over i.

$$W_j^* = \frac{R_j}{Target\ \overline{R_j}}$$

(b) CENTER LINES AND CONTROL LIMITS:

For Short Run Target $\overline{\overline{X}}$:

CL = 0

UCL = A_2

LCL = - A_2

where A_2 depends on the subgroup size n

For Short Run Range :

CL = 1

UCL = D_4

LCL = D_3

where D_4 and D_3 **depend on the subgroup size n**

Example 7: (Wise and Fair 1998, p156-157)

Table 7.

Sample #	Part #	Obs1	Obs2	Obs3	Obs4	Obs5	$\overline{X}_j$	Target $\overline{\overline{X}}_j$	Target $\overline{R}_j$	$\overline{Z_j^*}$	R_j	W_j^*
1	A	3.95	3.99	4.14	4.02	4.08	4.04	4.00	0.23	0.16	0.19	0.83
2	A	3.96	4.08	3.73	4.15	4.07	4.00	4.00	0.23	-0.01	0.42	1.83
3	A	3.91	4.12	4.02	3.76	3.98	3.96	4.00	0.23	-0.18	0.36	1.57
4	B	8.38	8.36	8.49	8.35	8.46	8.41	8.20	0.35	0.59	0.14	0.40
5	B	8.48	8.52	8.37	8.48	8.16	8.40	8.20	0.35	0.58	0.36	1.03
6	B	8.33	8.44	8.53	8.34	8.31	8.39	8.20	0.35	0.54	0.22	0.63
7	B	8.23	8.40	8.26	8.30	8.19	8.28	8.20	0.35	0.22	0.21	0.60
8	B	8.70	8.76	8.14	7.87	8.27	8.35	8.20	0.35	0.42	0.89	2.54
9	B	8.31	8.30	8.33	8.53	8.32	8.36	8.20	0.35	0.45	0.23	0.66
10	C	1.19	1.17	1.12	1.17	1.19	1.17	1.30	0.19	-0.69	0.07	0.37
11	C	1.25	1.21	1.21	1.22	1.21	1.22	1.30	0.19	-0.42	0.04	0.21
12	B	8.27	7.87	8.35	8.50	8.74	8.35	8.20	0.35	0.42	0.87	2.49
13	B	8.07	8.70	8.60	8.28	8.27	8.38	8.20	0.35	0.53	0.63	1.80
14	B	8.60	8.66	8.65	8.57	8.51	8.60	8.20	0.35	1.14	0.15	0.43
15	B	8.53	8.74	8.62	8.64	7.98	8.50	8.20	0.35	0.86	0.76	2.17
16	B	8.38	8.36	8.56	8.29	8.50	8.42	8.20	0.35	0.62	0.27	0.77
17	A	3.79	4.01	3.81	4.11	4.00	3.94	4.00	0.23	-0.24	0.32	1.39
18	A	3.83	4.08	3.80	3.99	3.89	3.92	4.00	0.23	-0.36	0.28	1.22
19	A	3.96	4.08	4.04	3.95	4.04	4.01	4.00	0.23	0.06	0.13	0.57
20	C	1.24	1.20	1.20	1.24	1.19	1.21	1.30	0.19	-0.45	0.05	0.26

(a) Calculation of plotted points

For Short Run $\overline{X}$:

$$\overline{Z_j^*} = \frac{\overline{X_j} - Target\ \overline{\overline{X}}_j}{Target\ \overline{R_j}}$$

where $Target\ \overline{\overline{X}}_j$ is the target value for the $\overline{X}_j$ and $Target\ \overline{R_j}$ is the corresponding standard deviation for the $\overline{X}_j$.

$$\overline{Z_1^*} = \frac{\overline{X_1} - Target\ \overline{\overline{X}}_1}{Target\ \overline{R_1}} = \frac{4.04 - 4.0}{0.23} = 0.16.$$

$$\overline{Z_4^*} = \frac{\overline{X_4} - Target\ \overline{\overline{X_4}}}{Target\ \overline{R_4}} = \frac{8.41 - 8.2}{0.35} = 0.59.$$

For Short Run Moving Range :

R_j = Maximum X_{ij} - Minimum X_{ij}
where maximum and minimum are over i.

R_1 = Maximum X_{31} - Minimum X_{11} = 4.14 -3.95 = 0.19.

R_4 = Maximum X_{34} - Minimum X_{44} = 8.49 - 8.35 = 0.14.

$$W_1^* = \frac{R_1}{Target\ \overline{R_1}} = \frac{0.19}{0.23} = 0.83.$$

$$W_4^* = \frac{R_4}{Target\ \overline{R_4}} = \frac{0.14}{0.35} = 0.4.$$

(b) Calculation of Center Lines and Control Limits

For Short Run $\overline{X}$:

Center line: CL = 0.

Upper control limit: UCL = A_2 = 0.577

Lower control limit: LCL = $-A_2$ = -0.577

For Short Run Moving Range :

Center line: CL = 1.

Upper control limit: UCL = D_4 = 2.114.

Lower control limit: LCL = D_3 = 0.

Short run Target X-bar Control Chart

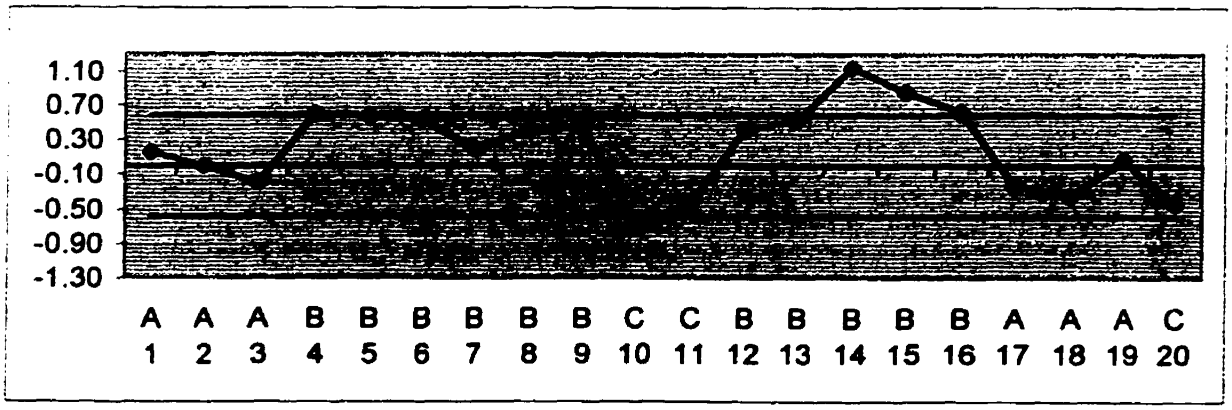


Figure 7(a).

Range Control Chart

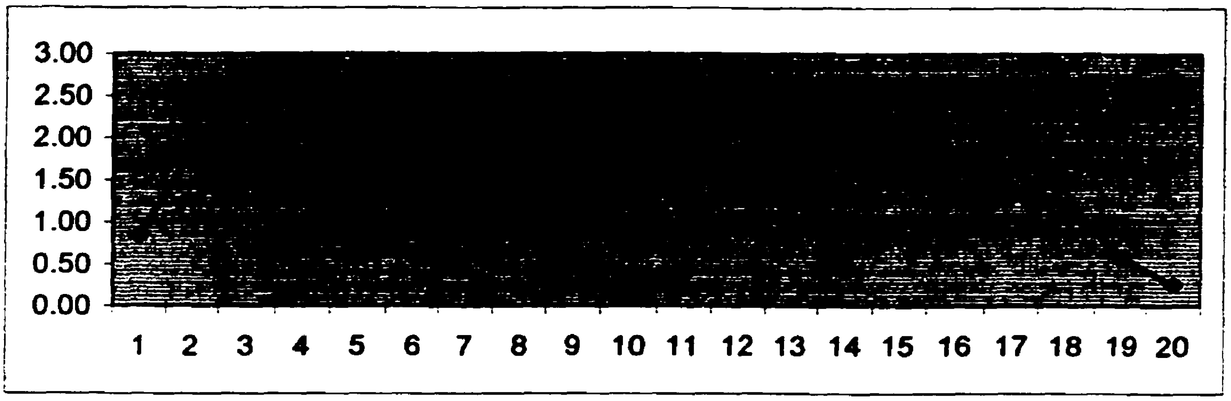


Figure 7(b).

The moving ranges in Figure 7(b) show three points exceed the upper control limit. There is apparent indication of probable shift in the production process. One should check for more assignable cause and give an appropriate adjustment. Do not interpret the IX-chart when the moving range chart shows points that are out of control limits.

8.2 Coded values and control limits of Zed-bar and Range Charts (method 2)

Source: (Wheeler 1991)

(a) PLOTTED POINTS:

For Zed-bar:

$$\overline{Z}_j = \frac{\overline{X}_j - Target\ \overline{\overline{X}}_j}{Sigma\ (\overline{X}_j)}$$

where $Target\ \overline{\overline{X}}_j$ is the target value for the $\overline{X}_j$ and $Sigma(\overline{X}_j)$ is the corresponding standard deviation for the $\overline{X}_j$.

For Range:

R_j = Maximum X_{ij} - Minimum X_{ij} i=1,2,..n[th] observation and j=1,2,...g[th] subgroup

where maximum and minimum are over i.

$$W_j = \frac{R_j}{Sigma\ (X_j)}$$

Note: $Sigma(X)$ and $Sigma(\overline{X})$ are computed separately for each characteristic k, k=1,2,...m,

$$Sigma(X) = \frac{\overline{R}}{d_2}$$

$$Sigma(\overline{X}) = \frac{\overline{R}}{d_2\sqrt{n}}$$

where d_2 depends on subgroup size

(b) CENTER LINES AND CONTROL LIMITS:

For Zed-bar:

CL = 0

UCL = 3

LCL = -3

For Range :

CL $= d_2$

UCL $= d_2 + 3\ d_3$

LCL $= d_2 - 3\ d_3$

Example 8: (Wheeler 1991 p.29)

Table 8.

Sample #	Part #	Obs 1	Obs 2	Obs 3	Obs 4	Obs 5	$\overline{X}_j$	$Target\ \overline{X}_j$	$Sigma(\overline{X})$	$\overline{Z}_j$	Rj	$Sigma(X_j)$	W_j
1	A	16.0	11.0	13.0	7.0	10.0	11.4	9.5	2.02	0.9	9.0	4.5	2.0
2	A	14.0	15.0	15.0	10.0	16.0	14.0	9.5	2.02	2.2	6.0	4.5	1.3
3	B	4.0	0.0	6.0	4.0	4.0	3.6	4.5	0.79	-1.1	6.0	1.8	3.4
4	B	2.0	4.0	5.0	8.0	5.0	4.8	4.5	0.79	0.4	6.0	1.8	3.4
5	B	5.0	8.0	3.0	5.0	3.0	4.8	4.5	0.79	0.4	5.0	1.8	2.8
6	B	6.0	6.0	5.0	3.0	5.0	5.0	4.5	0.79	0.6	3.0	1.8	1.7
7	B	4.0	5.0	4.0	2.0	8.0	4.6	4.5	0.79	0.1	6.0	1.8	3.4
8	A	5.0	7.0	7.0	8.0	15.0	8.4	9.5	2.02	-0.5	10.0	4.5	2.2
9	C	0.0	9.0	9.0	7.0	0.0	5.0	8.5	1.52	-2.3	9.0	3.4	2.6
10	C	5.0	7.0	7.0	12.0	8.0	7.8	8.5	1.52	-0.5	7.0	3.4	2.1
11	B	4.0	2.0	7.0	2.0	2.0	3.4	4.5	0.79	-1.4	5.0	1.8	2.8
12	B	3.0	6.0	4.0	2.0	6.0	4.2	4.5	0.79	-0.4	4.0	1.8	2.3
13	A	3.0	10.0	16.0	6.0	10.0	9.0	9.5	2.02	-0.2	13.0	4.5	2.9
14	C	9.0	7.0	8.0	8.0	13.0	9.0	8.5	1.52	0.3	6.0	3.4	1.8
15	A	5.0	15.0	9.0	5.0	10.0	8.8	9.5	2.02	-0.3	10.0	1.8	2.2
16	C	8.0	8.0	8.0	10.0	10.0	8.8	8.5	1.52	0.2	2.0	3.4	0.6
17	A	11.0	15.0	12.0	8.0	14.0	12.0	9.5	2.02	1.2	7.0	4.5	1.6
18	B	3.0	2.0	5.0	4.0	4.0	3.6	4.5	0.79	-1.1	3.0	1.8	1.7
19	B	4.0	5.0	5.0	4.0	8.0	5.2	4.5	0.79	0.9	4.0	1.8	2.3
20	B	2.0	3.0	1.0	4.0	1.0	2.2	4.5	0.79	-2.9	3.0	1.8	1.7

(a) Calculation of sigma(X)

$\overline{R}_A$ is estimated by historical average Range of subgroups of product A. The average range of product A is 10.5.

$$Sigma(X)_A = \frac{\overline{R_A}}{d_2} = \frac{10.5}{2.326} = 4.51.$$

$$Sigma(\overline{X})_A = \frac{\overline{R_A}}{d_2\sqrt{n}} = \frac{10.5}{2.326 * \sqrt{5}} = 2.02.$$

$\overline{R}_B$ is estimated by historical average Range of subgroups of product B. The average range of product B is 4.1.

$$Sigma(X)_B = \frac{\overline{R_B}}{d_2} = \frac{4.1}{2.326} = 1.8.$$

$$Sigma(\overline{X})_B = \frac{\overline{R_B}}{d_2\sqrt{n}} = \frac{4.1}{2.326 * \sqrt{5}} = 0.79.$$

$\overline{R}_C$ is estimated by historical average Range of subgroups of product C. The average range of product C is 7.9.

$$Sigma(X)_C = \frac{\overline{R_C}}{d_2} = \frac{7.9}{2.326} = 3.4.$$

$$Sigma(\overline{X})_C = \frac{\overline{R_C}}{d_2\sqrt{n}} = \frac{7.9}{2.326 * \sqrt{5}} = 1.52.$$

The target values for part A, part B and part C are 9.5,4.5 and 8.5 respectively.

(b) Calculation of plotted points

For Short Run $\overline{X}$:

$$\overline{Z_j} = \frac{\overline{X_j} - Target\ \overline{\overline{X}}}{Sigma\ (\overline{X})}$$

where $Target\ \overline{\overline{X}}$ is the target value for the $\overline{X}_j$ and $Sigma(\overline{X})$ is the corresponding standard deviation for the $\overline{X}_j$.

$$\overline{z}_1 = \frac{\overline{X_1} - Target\ \overline{\overline{X_1}}}{Sigma\ (\overline{X_1})} = \frac{11.4 - 9.5}{2.02} = 0.9.$$

$$\bar{z}_3 = \frac{\overline{X_3} - Target\ \overline{\overline{X_3}}}{Sigma\ (\overline{X_3})} = \frac{3.6-4.5}{0.79} = -1.1.$$

<u>For Short Run Moving Range</u> :

R_j = Maximum X_{ij} - Minimum X_{ij}
where maximum and minimum are over i.

R_1 = Maximum X_{11} - Minimum X_{41} = 16 - 7 = 9.

R_3 = Maximum X_{53} - Minimum X_{43} = 16 - 10 = 6.

$$W_1 = \frac{R_1}{Sigma\ (X_1)} = \frac{9}{4.5} = 2.$$

$$W_3^* = \frac{R_3}{Sigma\ (X_3)} = \frac{6}{1.8} = 3.4.$$

(c) Calculation of Center Line and Control Limits

<u>For Short Run $\bar{X}$</u> :

Center line: CL = 0.

Upper control limit: UCL = 3.

Lower control limit: LCL = -3.

<u>For Short Run Moving Range</u> :

Center line: CL = d_2 =2.326.

Upper control limit: UCL = $d_2 + 3d_3$=2.326+3*0.8641=4.9183.

Lower control limit: LCL = $d_2 - 3\ d_3$ = 0.

Notice: The lower control limit of range chart will be equal to zero if the value is negative.

Zed-bar Control Chart

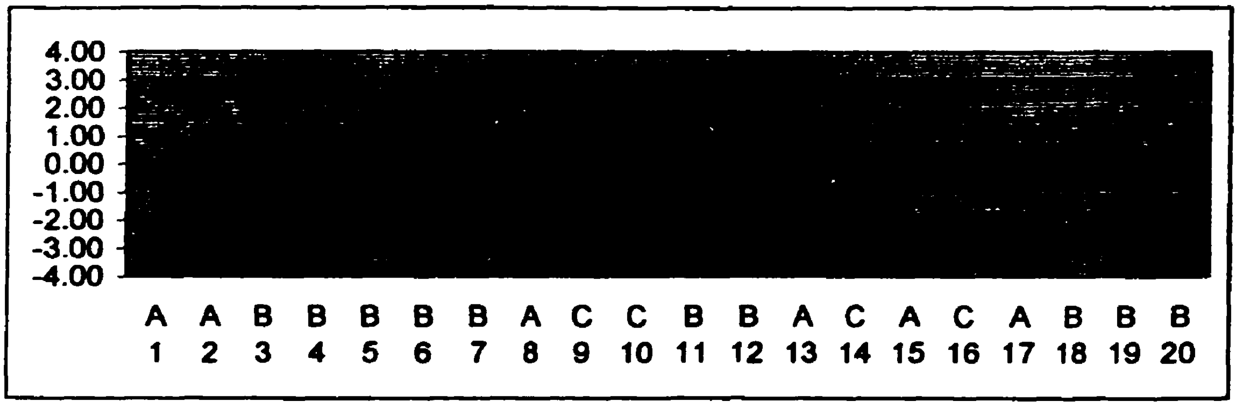


Figure 8(a)

Range Chart

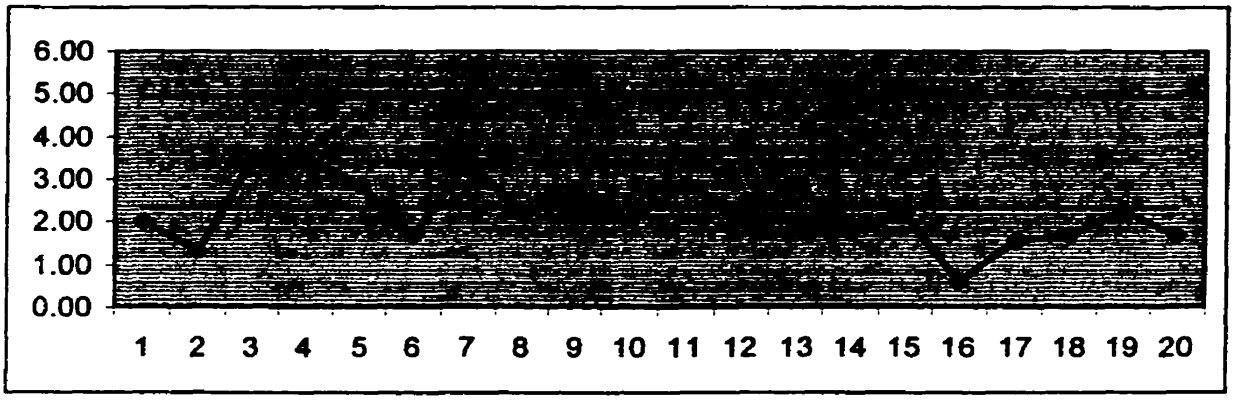


Figure 8(b)

The moving ranges in Figure 8(b) show no point is exceeding the upper control limit. The plotted points of moving ranges also appear to be random. There is also no apparent indication of probable shift in the production process.

By looking at the Zed bar chart, there are no apparent trends or plotted points outside the process control limits. The plotted points appear to be random about the center line. Therefore no specific process action has to be considered at this point.

9. The Importance's of Short Production Run Problem

When charting short production runs, the basic principal is to quickly recognize two critical conditions: 1. Being on target but out of control. 2. Being in control but at the wrong target level. Both scenarios cause problems and must be recognized and action must be taken at a given process. "Maintaining a process in control at the wrong target level (even no false-alarm rates) is not a relevant issue in short run situations"(Quality Progress Feb 1998). To maintain a certain target level is usually more important than simply just establishing a state of control at some level. With short runs, the process must not only be in control, but also in control at a specific targeted level. Thus we have to put in a sufficient amount of time to determine the appropriate target values for a given process.

10. Method of Estimating Target Values

10.1 With previous data

1. If traditional control charts of each part number are available and the process was in control, simply use the center line of the traditional control chart as the target value.
2. Use historically similar characteristics, parts or process parameters to estimate the initial target values.

10.2 Without previous data

1. If the data (should be at least 10 subgroups) represents normal production output, the average of the data can be used as the target value. Remember to remove all the outliers before calculating the average of the data. (Bothe 1991, p48)

2. Use engineering nominal values (the midpoint between the upper and lower control limits) as target values. (Bothe 1991, p48)

11. Method of estimating the Target $\overline{R}$

11.1 With previous data

1. If traditional moving range or range control charts of each part number are available and the process was in control, simply use the center line of the traditional range control chart as the target $\overline{R}$.
2. Use historical similar characteristics, parts or process parameters to estimate the initial target $\overline{R}$.

11.2 Without previous data

1. Use the existing data of at least ten subgroups of each part number to calculate its range-bar and then use that range-bar to estimate their own Target $\overline{R} = \frac{\overline{R}}{d_2}$. (Wheeler 1991, p 12-13)

   The constant of d_2 is depending on different sample size.

2. For $\hat{\sigma} = \frac{\overline{R}}{d_2}$ and $\hat{\sigma} = \frac{\overline{s}}{c_4}$ (Bothe 1991, p48)

$$\frac{\overline{R}}{d_2} = \hat{\sigma} = \frac{\overline{s}}{c_4}$$

$$\therefore \text{Target } \overline{R} = \frac{d_2}{c_4}\overline{s}.$$

The $\overline{s}$ can be estimated by the average sample standard deviation if the number of measurements of historical data were at least 10.

The constants of d_2 and c_4 are depending on different sample size.

3. For $\hat{\sigma} = \frac{\overline{R}}{d_2}$ and $\hat{\sigma} = \frac{\overline{s}}{c_4}$ (Wise 1998, p 129-130 and p 276-277)

$$\frac{\overline{R}}{d_2} = \hat{\sigma} = \frac{\overline{s}}{c_4}$$

$\therefore$ Target $\overline{R} = \frac{d_2}{c_4}\overline{s}$.

Notice: The $\overline{s}$ can be estimated by engineering tolerance $\overline{s}$ = (tolerance*2)/6 .

4. For two side specifications. (Wise 1998, p 129-130 and p 276-277)

When the Cp= Cpk = 1, then

$$\hat{\sigma} = \frac{1}{6}(USL - LSL) \quad \text{and} \quad \hat{\sigma} = \frac{\overline{R}}{d_2}$$

$$\hat{\sigma} = \frac{\overline{R}}{d_2} = \frac{1}{6}(USL - LSL)$$

$\therefore$ Target $\overline{R} = \frac{d_2}{6}(USL - LSL)$.

The constant of d_2 is depending on different sample size.

5. For one side specification. (Wise 1998, p 129-130 and p 276-277)

When Cpk = 1 then

$$\hat{\sigma} = \frac{1}{3}(spec\ \text{limit} - \overline{\overline{x}}) \quad \text{and} \quad \hat{\sigma} = \frac{\overline{R}}{d_2}$$

$$\hat{\sigma} = \frac{\overline{R}}{d_2} = \frac{1}{3}|\,\text{spec limit} - \overline{\overline{x}}\,|$$

$\therefore$ Target $\overline{R} = \frac{d_2}{3}|\,\text{spec limit} - \overline{\overline{x}}\,|$

The constant of d_2 is depending on different sample size.

Notice: In the above cases 3,4 and 5, the target $\overline{R}$ has no connection to the actual data; it is based on the engineering tolerance. The target $\overline{R}$ should be updated for any new information.

12. Appropriate Control Limits for Small Amount of Data

Hillier (1969) proposed a technique which can be used to calculate the appropriate control limits due to lack of data. It is because conventional control limits cannot be established until a large amount of subgroup is available. There are two stages for setting the desired control limits. In the first stage, control limits are set when the subgroups are being drawn. If all the average x-bar and range R fall inside the first stage control limits, these control limits can be used. Otherwise one has to remove all the out of control points and then the first stage control limits have to be recalculated according to existing subgroup data. After all the subgroups fall inside the control limits of the first stage, then we can calculate the control limits of the second stage for statistical control for the future.

Sources: (Hiller 1964, 1969)

(a) PLOTTED POINTS:

For $\overline{X}$:

$$\overline{X}_j = \frac{\sum_{i=1}^{n} X_{ij}}{n} \qquad i=1,2,..n^{th} \text{ observation and } j=1,2,\cdots g^{th} \text{ subgroup}$$

For Range:

R_j = Maximum X_{ij} - Minimum X_{ij}

where maximum and minimum are over i.

12.1 Two stages for setting the control limits:

1. First Stage (Restrospective Testing): Establishing a statistical control process using a small number of initial subgroups (denoted by m).

   i. The choice of m initial subgroups depends on the "extent of the need of the early control" and "the power of the control limits in detecting an out-of-control process" Hillier (1969). The preferable size is m=5.

   ii. The procedure requires choosing of the appropriate probabilities of a Type I error, namely, α_2, α_3 and α_4 for each initial subgroup; where α_2 corresponds to the probability that $\bar{X}$ would fall outside the $\bar{X}$ Chart control limits, α_3 corresponds to the probability that R would fall below the R Chart lower control limit and α_4 corresponds to the probability that R would fall above the R Chart upper control limit. The choice of values for α_2, α_3 and α_4 requires a balance between the "smallness of the probability of a Type I error and the sensitivity of the control chart as well as the costs involved" Hillier (1969). This then suggests different values for α_2, α_3 and α_4. The values of α_2=0.0027, α_3=0 and α_4=0.005 perform closely to the conventional with large m.

   iii. Using the constants m, α_2, α_3 and α_4, the control limits are calculated using the following formulas:

   For $\bar{X}$:

   $$UCL = \bar{\bar{X}} + A_2^{**}\,\bar{R}$$

   $$CL = \bar{\bar{X}}$$

   $$LCL = \bar{\bar{X}} - A_2^{**}\,\bar{R}$$

   For R:

   $$UCL = D_4^{**}\,\bar{R}$$

   $$CL = \bar{R}$$

   $$LCL = D_3^{**}\,\bar{R}$$

   The values of A_2^{**}, D_3^{**} and D_4^{**} given the values of m, α_2, α_3 and α_4 are shown in Tables B,C and D respectively for the case where the subgroups are each of size 5.

   iv. For each of the initial subgroups, its $\bar{X}$ and R will be checked if falling inside the first stage control limits for the $\bar{X}$-Chart and R Chart respectively. If not, then this subgroup will be discarded and the values of $\bar{\bar{X}}$, $\bar{R}$ and the first stage control limits will be recomputed based on the new value of m. The process will be repeated until all the remaining subgroups are within the current control limits.

2. Second Stage: Testing whether the process remains in statistical control when new subgroups are drawn in the future.

i. The procedure again requires the choice of α_2, α_3 and α_4 values (and define similarly as in the first stage).

ii. The control limits are calculated with the values of $\overline{\overline{X}}$ and $\overline{R}$ based from the initial "in-control" subgroups of the first stage but the constant factors are changed.

For $\overline{X}$:

$$UCL = \overline{\overline{X}} + A_2^* \overline{R}$$

$$CL = \overline{\overline{X}}$$

$$LCL = \overline{\overline{X}} - A_2^* \overline{R}$$

For R:

$$UCL = D_4^* \overline{R}$$

$$CL = \overline{R}$$

$$LCL = D_3^* \overline{R}$$

The values of A_2^*, D_3^* and D_4^* given the values of m, α_2, α_3 and α_4 are shown in Tables E,F and G respectively for the case where the subgroups are each of size 5

iii. After setting the control limits, the subgroups are inspected periodically in order to maintain a statistical control process.

iv. As m increases, new control limits need to be computed but this would be computationally cumbersome, thus a compromise is to compute new control limits for values of m=5,10, 25 and 100 or when a point falls very close to the recent control limits.

Subgroup(s) with values of $\overline{X}$ and R lying outside the current control limits will be excluded from the calculation of $\overline{\overline{X}}$ and $\overline{R}$. Thus, the value of m would not refer to the number of subgroups observed but to the number of "in-control" subgroups included in the calculation of $\overline{\overline{X}}$ and $\overline{R}$.

Example 9: The data is from the manual. (Chrysler etc 1991)

Table 9.

Sample #	Obs 1	Obs 2	Obs 3	Obs 4	Obs 5	$\overline{X}_j$	R_j	$\overline{\overline{X}}$	$\overline{R}$
1	0.65	0.70	0.65	0.65	0.85	0.70	0.20	0.732	0.17
2	0.75	0.85	0.75	0.85	0.65	0.77	0.20	0.732	0.17

3	0.75	0.80	0.80	0.70	0.75	0.76	0.10	0.732	0.17
4	0.60	0.70	0.70	0.75	0.65	0.68	0.15	0.732	0.17
5	0.70	0.75	0.65	0.85	0.80	0.75	0.20	0.732	0.17

(a) Calculation of plotted points of first stage

For $\overline{X}$:

$$\overline{X}_j = \frac{\sum_{i=1}^{n} X_{ij}}{n}$$ i=1,2,..5th observation and j=1,2,⋯15th subgroup

$$\overline{X}_1 = \frac{\sum_{i=1}^{n} X_{i1}}{n} = \frac{(X_{11}+X_{21}+X_{31}+X_{41}+X_{51})}{5} = \frac{0.65+0.70+0.65+0.65+0.85}{5} = 0.7.$$

$$\overline{X}_2 = \frac{\sum_{i=1}^{n} X_{i2}}{n} = \frac{(X_{12}+X_{22}+X_{32}+X_{42}+X_{52})}{5} = \frac{0.75+0.85+0.75+0.85+0.65}{5} = 0.77.$$

For Range:

R_j = Maximum X_{ij} - Minimum X_{ij}
where maximum and minimum are over i.

R_1 = Maximum X_{i1} - Minimum X_{i1} = $X_{51} - X_{11}$ = 0.85-0.65 =0.2.

Notice that: X_{11}, X_{31} and X_{41} has the same value of 0.65.

(b) Calculation of center lines and control limits of first stage

The constants for number of subgroup size 5 with probabilities α_2=0.027, α_3=0 and α_4=0.005 are A_2^{**}=0.588, D_3^{**}=0 and D_4^{**}=1.96.

For $\overline{X}$:

$$CL = \overline{\overline{X}} = \frac{\sum_{j=1}^{5} \overline{X_j}}{5} = \frac{3.66}{5} = 0.732. \quad \text{** (Same of the first stage)}$$

$$UCL = \overline{\overline{X}} + A_2^{**} \overline{R} = 0.732 + 0.588*0.17 = 0.83.$$

$$LCL = \overline{\overline{X}} - A_2^{**} \overline{R} = 0.732 - 0.588*0.17 = 0.63.$$

<u>For Range</u>:

$$CL = \overline{R} = \frac{\sum_{j=1}^{5} R_j}{5} = \frac{.85}{5} = 0.17. \quad \text{** (Same of the first stage)}$$

$$UCL = D_4^{**} \overline{R} = 1.96*0.17 = 0.33.$$

$$LCL = D_3^{**} \overline{R} = 0*0.17 = 0.$$

X-bar Chart

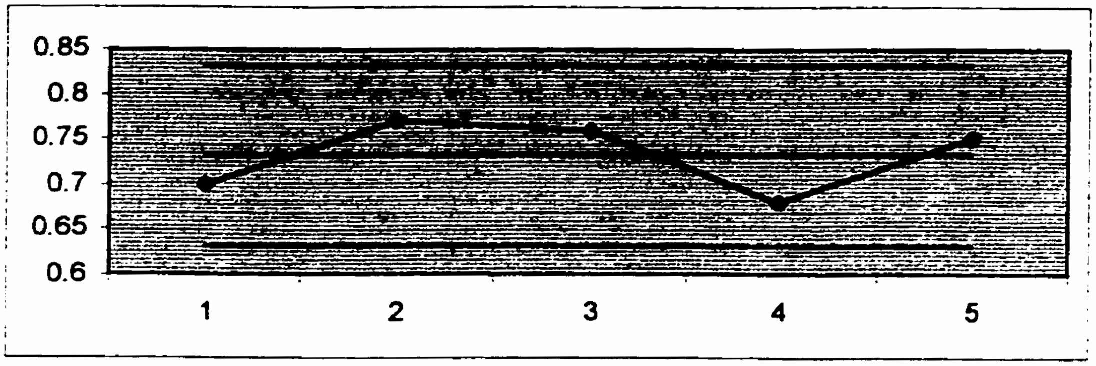


Figure 9(a)

Range Chart

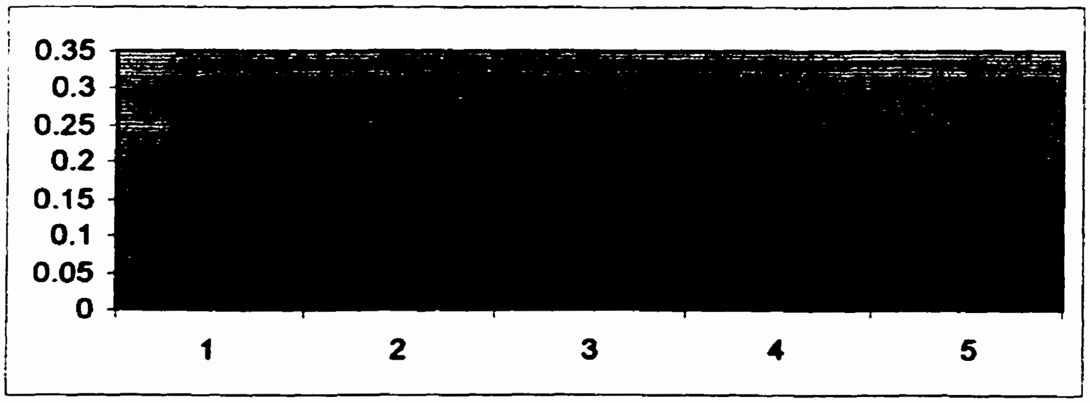


Figure 9(b)

All the plotted points of $\overline{X}$ and R fall inside the first stage control limits for the $\overline{X}$-Chart and R-Chart respectively. The initial subgroups have been identified to be in control. Finally we can proceed to the second stage for testing the future runs.

Second stage also requires the choice of α_2, α_3 and α_4 values. The control limits are calculated by using past values of $\overline{\overline{X}}$ and $\overline{R}$ based from the initial "in-control" subgroups of the first stage but the constant factors have to be changed.

(c) **Calculation of control limits of second stage**

The constants for number of subgroup size 5 with probabilities α_2=0.027, α_3=0 and α_4=0.005 are $A_2^{\bullet}$=0.720, $D_3^{\bullet}$=0 and $D_4^{\bullet}$=2.47.

<u>For $\overline{X}$:</u>

$$CL = \overline{\overline{X}} = \frac{\sum_{j=1}^{5} \overline{X_j}}{5} = \frac{3.66}{5} = 0.732.$$

$$UCL = \overline{\overline{X}} + A_2^{\bullet}\,\overline{R} = 0.732 + 0.720*0.17 = 0.8544.$$

$$LCL = \overline{\overline{X}} - A_2^{\bullet}\,\overline{R} = 0.732 - 0.720*0.17 = 0.6096.$$

For Range:

$$\text{CL} = \overline{R} = \frac{\sum_{j=1}^{5} R_j}{5} = \frac{.85}{5} = 0.17.$$

$$\text{UCL} = D_4^{**}\,\overline{R} = 2.47*0.18 = 0.4199.$$

$$\text{LCL} = D_3^{**}\,\overline{R} = 0*0.18 = 0.$$

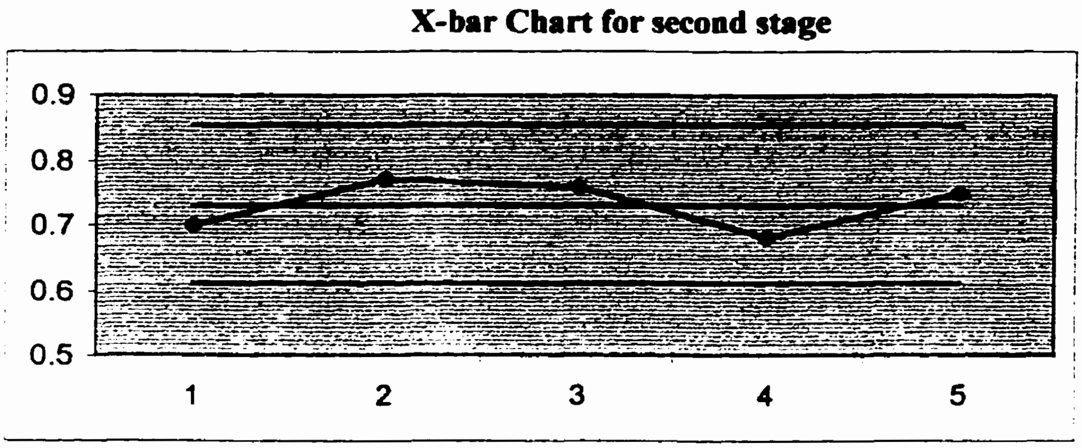


Figure 9(c)

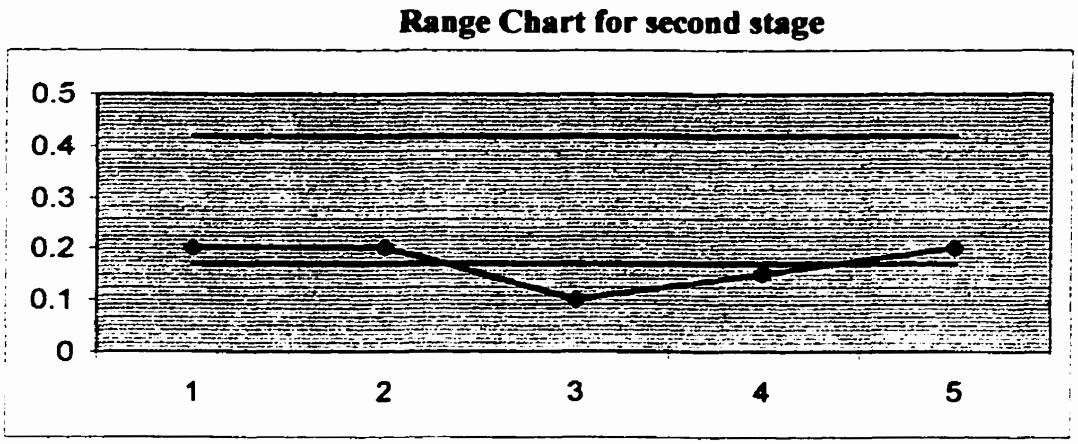


Figure 9(d)

After calculating the control limits of second stage, these control limits are used to monitor the process for the future.

Now consider the future runs of the process.

Table 10.

Sample #	Obs 1	Obs 2	Obs 3	Obs 4	Obs 5	$\overline{X}_j$	R_j
6	0.60	0.75	0.75	0.85	0.70	0.73	0.25
7	0.75	0.80	0.65	0.75	0.70	0.73	0.15
8	0.60	0.70	0.80	0.75	0.75	0.72	0.20

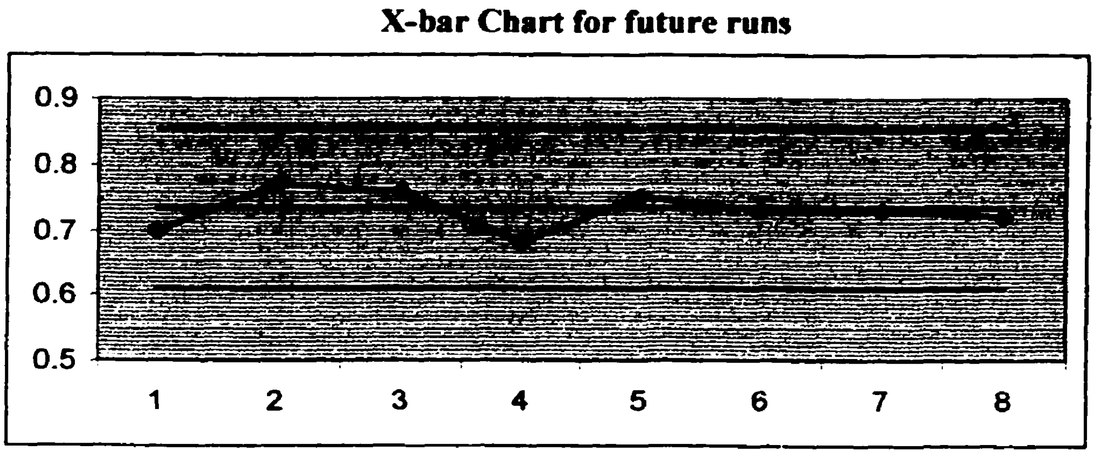


Figure 9(e)

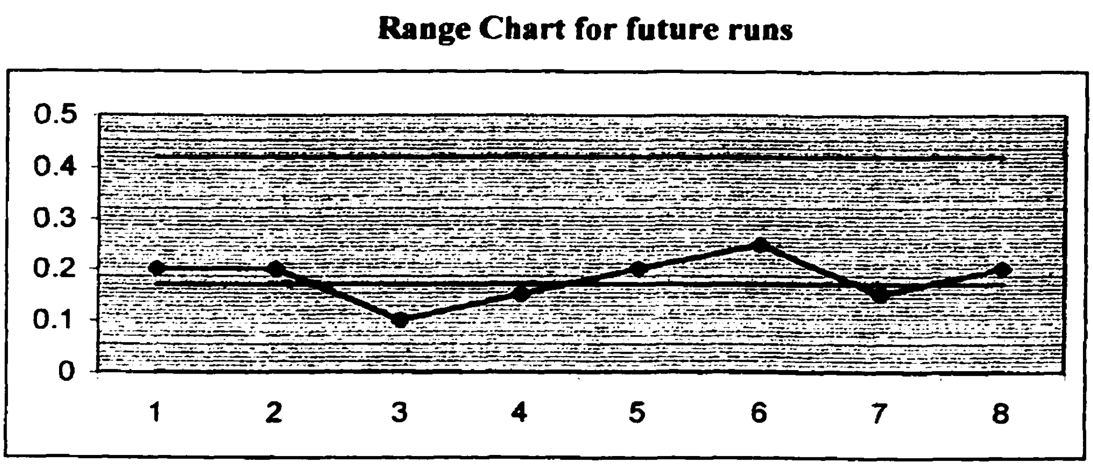


Figure 9(f)

Considering the future runs, no points are outside the second stage control limits. If there are $\overline{X}$ and R plotted points outside the control limits, one should take appropriate action to investigate the process. We should repeat the procedure of stage 1 and stage 2 to calculate the new control limits, if the new subgroups of m= 5,10,15..100, have been drawn.

Chapter 3

Excel for Tests for Special Causes in X-bar Chart

1. Introduction

Nelson (1984,1985) listed eight tests for special causes in a X-bar chart. These tests or a subset of them have become "standards" tests used for signaling that a process might be out-of-control in SPC manuals, books and computer packages. For example, Statistical Process Control Reference Manual (1991), Statistical Quality Control Handbooks (1982), Griffith (1990), Kiemele (1990) and Montgomery (1996).

Here we propose the use of Excel for all these eight tests. One example of the conventional X-bar chart for variables is used for illustration. However, as it will be seen, the formulas for the eight tests can be used for other types of control charts.

For easy reference to Nelson's paper (1984) , other SPC books, reference manuals and computer packages, we use the identical numbering sequence and descriptions of the eight tests as those by him.

2. The Excel Program for the Eight Tests

We illustrate the construction of the Excel program for the eight tests through the following example. The same program can then be kept and used for other examples by simply changing its data to the data of these examples.

Example 1: The data below is $\bar{x}$ for 23 samples (i.e. subgroups) of size n=5 taken from a manufacturing process every hour.

Table 1. (raw data of Example 1)

Sample #	$\bar{x}$	Sample #	$\bar{x}$	Sample #	$\bar{x}$
1	1092.7	9	1092.8	17	1111.4
2	1095.3	10	1103.4	18	1056.5
3	1095.3	11	1099.1	19	1099.2
4	1073.3	12	1100.0	20	1116.1
5	1152.1	13	1096.3	21	1106.4
6	1092.9	14	1075.1	22	1100.9
7	1129.8	15	1111.5	23	1114.4
8	1111.6	16	1113.0		

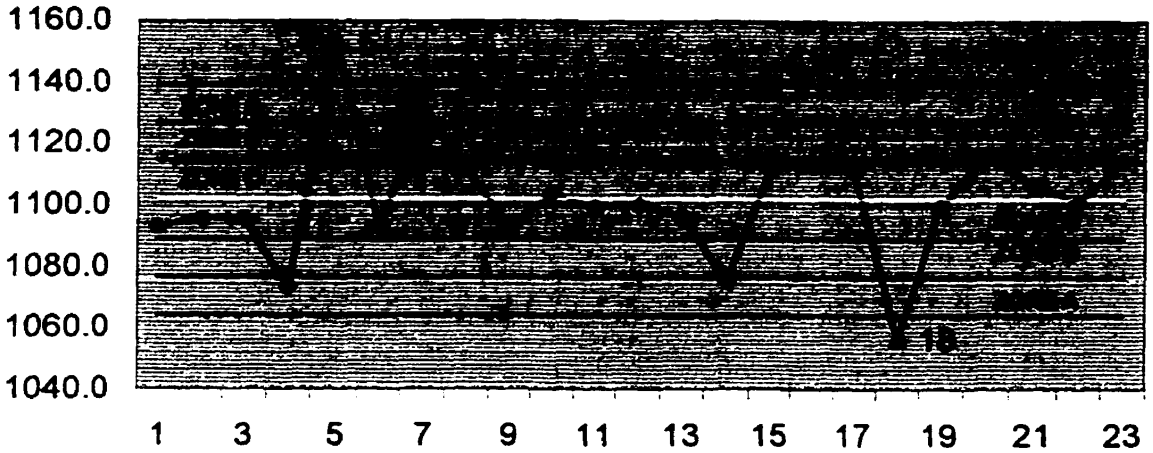


Figure 1 X-bar chart for the process.

The centerline is $\bar{\bar{x}}$ = 1101.7, the control limits are given by $\bar{x} \pm A_2 \bar{R}$ with UCL = 1139 and LCL = 1064, where $\bar{\bar{x}}$ and $\bar{R}$ are the usual grand average and average of ranges computed from the 23 samples. The control chart is shown in Figure 1. The charts between the lower and upper control limits are partitioned into six equal zones, labeled Zones A, B, C, C, B, A with Zones C closest to the center line.

2.1 Steps for entering the text in the Excel worksheet

Step1. Enter the text in the Excel worksheet as in Table 1. The printout is in Figure 2.

Table 1.

cell	Text	Description
A6	Sample #	Sample number
B1	Center Line	Text
B2	UCL	Upper control limit
B3	LCL	Lower control limit
B6	Data	Text
C2	U Zone A	Upper limit of Zone A
C3	L Zone A	Lower limit of Zone A
C6	Test 1	Text
D1	1101.7	The value of center line
D2	1139.0	The value of Upper control limit
D3	1064.1	The value of Lower control limit
D6	Test 2	Text
E6	Test 3	Text
F1	U ZONE B	Upper limit of Zone B
F2	U ZONE C	Upper limit of Zone C
F3	L ZONE C	Lower limit of Zone C
F4	L ZONE B	Lower limit of Zone B
F6	Test 4	Text
G1	=$D$1+(($D$1-$D$3)/3)*2	Formula for Upper limit of Zone B
G2	=$D$1+(($D$1-$D$3)/3)*1	Formula for Upper limit of Zone C
G3	=$D$1-(($D$1-$D$3)/3)*1	Formula for Lower limit of Zone C
G4	=$D$1-(($D$1-$D$3)/3)*2	Formula for Lower limit of Zone B
G6	Test 5	Text
H6	Test 6	Text
I6	Test 7	Text
J6	Test 8	Text

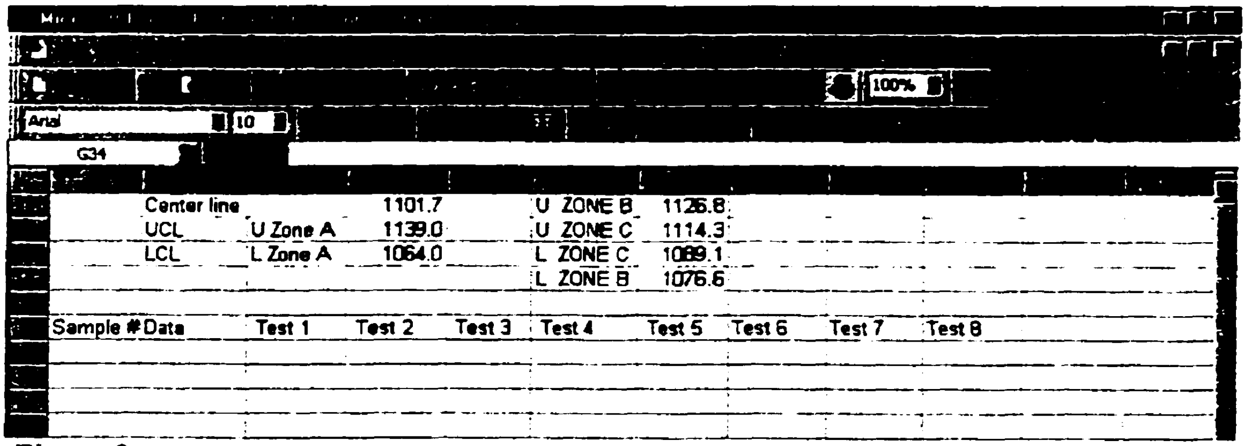


Figure 2

2.2 Steps for entering the data and formulas in Excel worksheet

Step 2. Enter the sample numbers 1 to 23 at cell A7 to cell A29 as shown in printout Figure 3.

Step 3. Enter the values of 23 sample averages at cell B7 to cell B29; then enter the text **"end"** at cell B30.

Step 4. Select cell C7 and enter the formula: = **IF(OR(B7>$D$2,B7<$D$3),"special cause","")**. Press **enter**.

Step 5. Select cell C7 again and choose **copy** button.

Step 6. Select the range from C8 to C29 and press **enter**.

The text "special cause" appearing in Test 1 at sample 5 and sample 18 indicates that the corresponding points in the control charts exceed the control limits, as can also be seen in Figure 1.

Note that in Step 3 the text **"end"** is used to identify the ending of the sample averages, so that a formula would not operate on that cell or beyond.

The formulas of the eight tests are given in Table 2. The numbering of the tests and their descriptions are taken directly from Nelson (1984) for easy reference.

For each of the remaining seven tests, follow the steps similar to Steps 4,5 and 6. For example, for Test 5, we have

Step 4. Select G7 and enter the formula: (as given in Table2). Press **enter**.

Step 5. Select cell G7 again and choose **copy** button.

Step 6. Select the range from G8 to G23. Press **enter**.

The text "special cause" indicates the beginning of the "special cause", i.e., the first point in the "Two out of three points in a row in Zone A or beyond". In Figure 3, under the title Test 5, the first "special cause" appearing at sample 5 indicates the two out of the three points corresponding to samples 5,6 and 7 in the control chart are in Zone A or beyond, as can be seen in Figure 1.

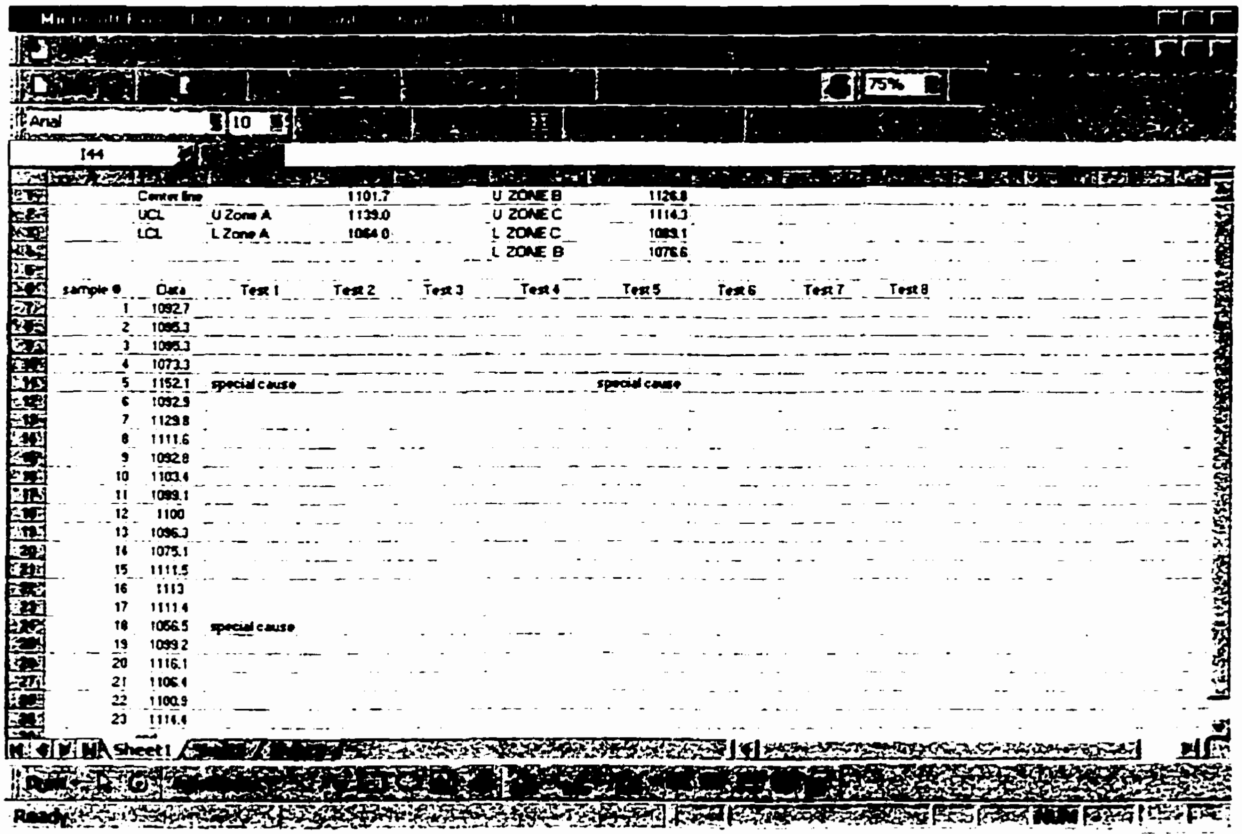


Figure 3

Formula worksheet of Nelson's (1984) eight tests for special causes.

Table 2.

Test	Description	Formula
Test 1	One point beyond zone A.	`=IF(OR(B7>$D$2,B7<$D$3),"special cause","")`
Test 2	Nine points in a row in zone C or beyond.	`=IF(OR(AND(B7<$D$1,B8<$D$1,B9<$D$1,B10<$D$1,B11<$D$1,B12<$D$1,B13<$D$1,B14<$D$1,B15<$D$1),AND(B7>$D$1,B8>$D$1,B9>$D$1,B10>$D$1,B11>$D$1,B12>$D$1,B13>$D$1,B14>$D$1,B15>$D$1)),"special cause","")`
Test 3	Six points in a row steadily increasing or decreasing.	`=IF(OR(AND(B7<B8,B8<B9,B9<B10,B10<B11,B11<B12),AND(B7>B8,B8>B9,B9>B10,B10>B11,B11>B12)),"special cause", "")`
Test 4	Fourteen points in a row alternating up and down.	`=IF(OR(AND(B7>B8,B8<B9,B9>B10,B10<B11,B11>B12,B12<B13,B13>B14,B14<B15,B15>B16,B16<B17,B17>B18,B18<B19,B19>B20),AND(B7<B8,B8>B9,B9<B10,B10>B11,B11<B12,B12>B13,B13<B14,B14>B15,B15<B16,B16>B17,B17<B18,B18>B19,B19<B20)), "special cause","")`
Test 5	Two out of three points in a row in zone A or beyond.	`=IF(OR(OR(AND(B7>$G$1, B8>$G$1),AND(B8>$G$1,B9>$G$1),AND(B7>$G$1,B9>$G$1)),OR(AND(B7<$G$4,B8<$G$4),AND(B8<$G$4,B9<$G$4),AND(B7<$G$4,B9<$G$4))),"special cause","")`
Test 6	Four out of five points in a row in zone B or beyond.	`=IF(OR(OR(AND(B7>$G$2,B8>$G$2,B9>$G$2,B10>$G$2),AND(B7>$G$2,B8>$G$2,B9>$G$2,B11>$G$2),AND(B8>$G$2,B9>$G$2,B10>$G$2,B11>$G$2),AND(B7>$G$2,B8>$G$2,B10>$G$2,B11>$G$2),AND(B7>$G$2,B9>$G$2,B10>$G$2,B11>$G$2)),OR(AND(B7<$G$3,B8<$G$3,B9<$G$3,B10<$G$3),AND(B7<$G$3,B8<$G$3,B9<$G$3,B11<$G$3),AND(B8<$G$3,B9<$G$3,B10<$G$3,B11<$G$3),AND(B7<$G$3,B8<$G$3,B10<$G$3,B11<$G$3),AND(B7<$G$3,B9<$G$3,B10<$G$3,B11<$G$3))),"special cause","")`

Table 2. (continued)

Test	Description	Formula
Test 7	Fifteen points in a row in zone C (above and below centerline)	=IF(AND(AND(B7>$G$3,B7<$G$2),AND(B8>$G$3,B8<$G$2),AND(B9>$G$3,B9<$G$2),AND(B10>$G$3,B10<$G$2),AND(B11>$G$3,B11<$G$2),AND(B12>$G$3,B12<$G$2),AND(B13>$G$3,B13<$G$2),AND(B14>$G$3,B14<$G$2),AND(B15>$G$3,B15<$G$2),AND(B16>$G$3,B16<$G$2),AND(B17>$G$3,B17<$G$2),AND(B18>$G$3,B18<$G$2),AND(B19>$G$3,B19<$G$2),AND(B20>$G$3,B20<$G$2),AND(B21>$G$3,B21<$G$2)),"special cause","")
Test 8	Eight points in a row on both sides of centerline with none in zones C.	=IF(AND(OR(B7>$G$2,B7<$G$3),OR(B8>$G$2,B8<$G$3),OR(B9>$G$2,B9<$G$3),OR(B10>$G$2,B10<$G$3),OR(B11>$G$2,B11<$G$3),OR(B12>$G$2,B12<$G$3),OR(B13>$G$2,B13<$G$3),OR(B14>$G$2,B14<$G$3)),"special cause","")

Conclusion

We described the methods of making and customizing the short run control charts in Microsoft Excel. These methods are flexible and can be modified to construct other control charts.

We also reviewed several widely used short run control charts under different scenarios. Further researches on those charts are required, because the theoretical validity of those charts has not yet been carefully studied. The reasons that we introduced these charts are that they are familiar to practitioners. The interpretation of those short run charts is similar to those widely used traditional control charts.

Finally, we provided the eight tests for special causes of x-bar charts in Excel. We can use the program for both the traditional and short run control charts. Therefore it is simpler for users to identify the special causes in control charts even if they do not have an expensive commercial software for control charts.

APPENDIX A.

Table A. Factors for Constructing Variable Control Charts

Observation in sample, n	A_2	D_3	D_4	d_2	d_3	c_4
1	2.660	0	3.267			
2	1.880	0	3.267	1.128	0.853	0.798
3	1.023	0	2.574	1.693	0.888	0.886
4	0.729	0	2.282	2.059	0.880	0.921
5	0.577	0	2.114	2.326	0.864	0.940
6	0.483	0	2.004	2.534	0.848	0.952
7	0.419	0.076	1.924	2.704	0.833	0.959
8	0.373	0.136	1.864	2.847	0.820	0.965
9	0.337	0.184	1.816	2.970	0.808	0.969
10	0.308	0.223	1.777	3.078	0.797	0.973
11	0.285	0.256	1.744	3.173	0.787	0.975
12	0.266	0.283	1.717	3.258	0.778	0.978

APPENDIX B.

Table B. A_2^{**} Factors for $\bar{X}$ Chart First-Stage Control Limits (Hiller (1969))

Number of Subgroup m	α_2				
	0.001	0.0027	0.01	0.025	0.05
2	0.684	0.575	0.449	0.366	0.306
3	0.679	0.590	0.476	0.397	0.337
4	0.669	0.589	0.483	0.408	0.349
5	0.662	0.588	0.487	0.414	0.356
6	0.658	0.587	0.489	0.418	0.360
7	0.655	0.586	0.490	0.420	0.362
8	0.652	0.585	0.491	0.421	0.363
9	0.650	0.584	0.492	0.423	0366
10	0.648	0.584	0.493	0.425	0.368
15	0.643	0.581	0.493	0.426	0.370
20	0.639	0.579	0.493	0.427	0.372
25	0.637	0.578	0.493	0.428	0.372
50	0.636	0.578	0.495	0.430	0.375
100	0.634	0.577	0.495	0.430	0.375
∞	0.633	0.577	0.495	0.431	0.377

Table C. D_3^{**} Factors for R Chart First-Stage Lower Control Limits (Hiller (1969))

Number of Subgroup m	α_3				
	0.001	0.005	0.01	0.025	0.05
2	0.236	0.339	0.394	0.487	0.560
3	0.202	0.296	0.349	0.434	0.513
4	0.189	0.279	0.330	0.414	0.493
5	0.182	0.270	0.320	0.403	0.482
6	0.177	0.264	0.314	0.396	0.475
7	0.174	0.259	0.309	0.391	0.470
8	0.172	0.257	0.307	0.388	0.466

9	0.170	0.255	0.304	0385	0.463
10	0.169	0.253	0.302	0.383	0.461
15	0.165	0.248	0.296	0.377	0.455
20	0.163	0.246	0.294	0.374	0.452
25	0.162	0.244	0.292	0.372	0.450
50	0.160	0.241	0.289	0.369	0.446
100	0.159	0.240	0.287	0.367	0.445
∞	0.158	0.239	0.286	0.365	0.443

Table D. D_4^{**} Factors for R Chart First-Stage Upper Control Limits (Hiller (1969))

Number of Subgroup m	α_4				
	0.001	0.005	0.01	0.025	0.05
2	1.78	1.67	1.62	1.53	1.44
3	2.02	1.84	1.76	1.63	1.52
4	2.12	1.92	1.82	1.68	1.56
5	2.18	1.96	1.85	1.71	1.58
6	2.21	1.98	1.88	1.72	1.60
7	2.23	2.00	1.89	1.73	1.60
8	2.25	2.01	1.90	1.74	1.61
9	2.26	2.02	1.91	1.75	1.62
10	2.27	2.03	1.92	1.76	1.62
15	2.30	2.06	1.94	1.77	1.63
20	2.32	2.07	1.95	1.78	1.64
25	2.32	2.07	1.96	1.79	1.64
50	2.34	2.09	1.97	1.80	1.65
100	2.35	2.09	1.97	1.80	1.65
∞	2.36	2.10	1.98	1.80	1.66

Table E. A_2^{*} Factors for $\bar{X}$ Chart Second-Stage Control Limits (Hiller (1969))

Number of Subgroup m	α_2				
	0.011	0.0027	0.01	0.025	0.05

1	2..27	1.74	1.21	0.911	0.720
2	1.19	1.00	0.781	0.637	0.532
3	0.960	0.834	0.673	0.562	0.477
4	0.864	0.760	0.624	0.527	0.451
5	0.811	0.720	0.596	0.507	0.436
6	0.779	0.695	0.579	0.495	0.426
7	0.756	0.677	0.564	0.485	0.418
8	0.738	0.662	0.556	0.477	0.412
9	0.729	0.655	0.551	0.474	0.410
10	0.719	0.647	0.545	0.470	0.407
15	0.667	0.621	0.527	0.455	0.396
20	0.672	0.609	0.518	0.449	0.391
25	0.663	0.602	0.513	0.445	0.387
50	0.649	0.590	0.505	0.439	0.383
100	0.640	0.583	0.500	0.434	0.379
∝	0.633	0.577	0.495	0.431	0.377

Table F. D_3^* Factors for R Chart Second-Stage Lower Control Limits (Hiller (1969))

Number of Subgroup m	α_3				
	0.001	0.005	0.01	0.025	0.05
1	0.134	0.204	0.245	0.317	0.389
2	0.144	0.219	0.263	0.338	0.413
3	0.148	0.225	0.270	0.346	0.422
4	0.151	0..228	0.274	0.351	0.437
5	0.152	0.230	0.276	0.353	0.430
6	0.153	0.231	0.278	0.355	0.432
7	0.154	0.232	0.279	0.357	0.433
8	0.154	0.233	0.280	0.358	0.434
9	0.155	0.234	0.280	0.359	0.435
10	0.155	0.234	0.281	0.359	0.436

15	0.156	0.236	0.282	0.361	0.438
20	0.156	0.236	0.283	0.362	0.439
25	0.157	0.237	0.284	0.363	0.440
50	0.157	0.238	0.285	0.364	0.441
100	0.158	0.238	0.285	0.365	0.442
∞	0.158	0.239	0.286	0.365	0.443

Table G. D_4^* Factors for R Chart Second-Stage Upper Control Limits (Hiller (1969))

Number of Subgroup m	α_4				
	0.001	0.005	0.01	0.025	0.05
1	7.94	5.10	4.20	3.22	2.60
2	4.14	3.19	2.83	2.39	2.07
3	3.39	2.76	2.50	2.17	1.92
4	3.08	2.57	2.36	2.07	1.85
5	2.91	2.47	2.27	2.01	1.81
6	2.81	2.40	2.22	1.98	1.78
7	2.74	2.35	2.18	1.95	1.77
8	2.69	2.32	2.16	1.93	1.75
9	2.65	2.30	2.14	1.92	1.74
10	2.62	2.27	2.12	1.91	1.73
15	2.53	2.21	2.07	1.87	1.71
20	2.48	2.18	2.05	1.85	1.70
25	2.46	2.17	2.03	1.84	1.69
50	2.41	2.13	2.01	1.82	1.67
100	2.38	2.12	1.99	1.81	1.67
∞	2.36	2.10	1.98	1.80	1.66

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