FAULT ISOLATION IN ACTIVE NETWORKS
BY TRANSFER FUNCTION ANALYSIS

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ABSTRACT

This thesis describes a method of isolating a single fault in an active network where only the input and output terminals are available for test measurements.

The method relies on the measurement and analysis of the coefficients of the network transfer function.

The significance of this solution is that it allows direct computation of the fault instead of having to choose from a set of precomputed faults. The computations per se avoid the pitfalls of symbolic transfer functions and pole-zero calculations.

TABLE OF CONTENTS

CHAPTER		PAGE
I.	INTRODUCTION	1
	Nature of Problem	1
	Definition of the Problem	2
	Review of Previous Work	2
	Proposed Procedure	3
II.	COEFFICIENT-SIGNATURE METHOD	5
	Formal Development	5
	Coefficient properties	5
	Identification procedure	9
	Pragmatic Solution	14
	Modification of rules of elimination	14
	Error reduction	16
	Example	17
III.	DISCUSSION AND CONCLUSION	21
BIBLIOGRAPHY		26

LIST OF FIGURES

FIGURES PAGE

1. Two-port Network 5
2. Mathematical Model of Problem 10
3. Example Network 12
4. Example Network 17
5. Flow Diagram of the Frequency-Signature Procedure 22
6. Flow Diagram of the Coefficient-Signature Procedure 22

CHAPTER I

INTRODUCTION

I. NATURE OF PROBLEM

The maintenance of electronic systems has long been the domain of technicians whose procedure relies upon accumulated experience with a particular type of equipment. Unfortunately, a consequence of the present trend of accelerated obsolescence is that experience becomes more difficult to acquire.

The time honored technique of probing a network for signals at various points is rapidly losing favor because of high packaging density. It is at best tedious to probe a printed circuit and virtually impossible to probe an integrated circuit.

This investigation is particularly relevant to the problem of diagnosing a fault in mass produced integrated circuits [1] where a large portion of the network is inaccessible. Unlike in discrete networks, the detection of a fault in an integrated circuit would mean rejection; however, the information regarding the element at fault would serve as the basis for redesigning the network and/or the assembly line.

The use of extra test terminals is undesirable because stray capacitance would be introduced, which would be objectionable in fast response networks.

It is therefore necessary to devise new fault isolation techniques which do not demand access to very many interior points in a network.

II. DEFINITION OF THE PROBLEM

Consider a linear, lumped, time-invariant network for which only the input and output terminals are available for test measurements. Given the network graph, the nominal value of every element, and the occurrence of at most a single fault, the problem is to identify the faulty element and its approximate value.

In order to simplify the presentation, the general network will be considered to be a two-port network. Catastrophic faults (i.e. short-circuits and open-circuits) which prevent measurement at either port, will be ignored.

III. REVIEW OF PREVIOUS WORK

Seshu and Waxman [2] have given a solution to the problem of analyzing the variations in magnitude of the network transfer function. The basis of their procedure is that a change in one of the network elements must change the position of the break frequencies and/or the value of the constant multiplier (i.e. in the sense of the Bode plot).

The network is described by measuring the magnitude of the transfer function between and around each of the nominal break frequencies. The measurements are quantized (i.e. the measurements are truncated to an integral number of dB.) and the ordered set of such measurements is called a frequency signature in this thesis.

A fault dictionary is precomputed by computing the frequency signature for each quantum variation of every element in the network. By having a computer match a measured frequency signature to a precomputed frequency signature, a fault may be identified.

Preliminary computation in this method involves symbolic transfer function and pole-zero computations which restrict the size of networks that can be considered. The motivation for another solution was to circumvent the above difficult computations in order to consider larger networks.

IV. PROPOSED PROCEDURE

In this thesis, a novel approach was taken by associating the variation of a network element with the variation of the coefficients of the network transfer function. The ordered set of values of these coefficients will be called a coefficient-signature vector.

In the following chapter a method is developed which uses the coefficient-signature vector of a network to identify a fault to within a proper subset of the set of all network elements. An example is cited to demonstrate that the subset of elements which are suspected of being at fault can be reduced to one element in some cases.

The problem may be reduced to two parts: the measurement and the analysis of the coefficient-signature vector. The first problem involves estimating the coefficients of the transfer function where the type and order of the transfer function are known, a priori. Signal levels are assumed to be large enough so that noise causes no difficulty, but estimates are still hampered by measurement error and numerical error in computation. This problem is beyond the scope of this thesis and will not be considered here; however, J. E. Valstar [3] has done a very thorough survey of this measurement problem and should be consulted for the details.

The body of this thesis considers the second problem which is the analysis of the coefficient-signature vector in order to identify a network fault.

CHAPTER II

COEFFICIENT-SIGNATURE METHOD

I. FORMAL DEVELOPMENT

Coefficient properties

Consider the voltage transfer function of the two-port network in Figure 1 to be written:

$$\frac{V_2}{V_1} = \frac{\sum_{j=0}^{n} a_{m+1+j} \quad s^{n-j}}{\sum_{i=0}^{m} a_i \quad s^{m-i}} \qquad (1)$$

Since the network transfer function is unique with regard to derivation, assume equation (1) to be derived from the topological admittance formula[1].

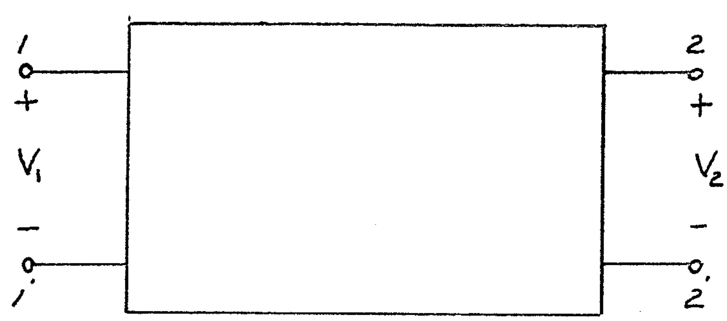


Figure 1. Two-port network

[1]Topological notations and definitions were adopted from [4].

This derivation depends on the existence of the admittance matrix. The assumption that the admittance matrix exists thus prohibits all dependent voltage generators (i.e. unless they are converted into current generators by the use of Norton's theorem); even current generators that depend on currents can be admitted only if the governing current is an element with a finite admittance. An example of a prohibited network would be a network containing an ideal transformer.

As a consequence of the assumption that at most a single fault has occurred, the coefficients of equation (1) are linearly related to each network element where the network contains no mutual inductance. This may be seen more clearly if the topological formula for the transfer function is examined:

$$\frac{V_2}{V_1} = \frac{\Delta_{12} - \Delta_{12}}{\Delta_{11}} \tag{2}$$

where $\Delta_{ij} = (-1)^{i+j} \sum_{\text{all complete 2-trees}} \epsilon_t \left(\text{complete 2-tree} \begin{bmatrix} i,r \\ j,r \end{bmatrix} \text{product}\right)$

ϵ_t = sign of complete 2-tree product,

and r = reference node

First observe that if equation (1) is identified with equation (2), then every term in equation (1) is a sum of complete 2-tree products with a common power of s. The next point to note is that by definition of a complete 2-tree product, every branch admittance in that product is a factor of multiplicity one. Consequently every network element in a complete 2-tree product is also a factor of multiplicity one. Thus the relation between a coefficient a and some particular element e may be written:

$$a = \alpha' e + \beta \qquad (3)$$

where α' and β are real constants and e is a real variable with the dimension of conductance, reciprocal inductance, or capacitance.

Equation (3) may be normalized by expressing the variable e as:

$$e = e_o x \quad 0 \leq x \qquad (4)$$

where e_o is the nominal value of e and x is a positive real dimensionless variable. The normalized form of equation (3) may be written as:

$$a = \alpha x + \beta \qquad (5)$$

Equation (1) is now written in standard form and as a result the relationship between a coefficient a and a normalized element x is the bilinear form:

$$a = \frac{\alpha x + \beta}{\gamma x + \delta} \qquad (6)$$

where α, β, γ and δ are all real constants.

The bilinear relationship of equation (6) requires only three constants to describe it and for computational purposes will be expressed as:

$$a = \frac{c_1 x + 1}{c_2 x + c_3} \qquad (7)$$

In order to determine the three constants c_1, c_2 and c_3, three equations may be derived from equation (6) by setting x equal to zero, one and infinity (i.e. the element e is open-circuited, set to nominal value and short-circuited).

To recapitulate the assumption that at most a single fault has occurred implies a general relationship between the coefficients of the standard voltage transfer function and a particular normalized network element. The array of m+n+1 coefficients is ordered as seen in equation (1) and is called the coefficient-signature vector, A. The k^{th} coefficient function, $F_k(x)$, relates the coefficient-signature vector to a scalar variable x which is the k^{th} normalized element. The above relationship may be expressed as:

$$A = F_k(x) \tag{8}$$

or

$$\left[a_i\right] = \left[f_{j,k}(x)\right]$$

$$\left[f_{j,k}(x)\right] = \left[\frac{c_{j,1}\,x + 1}{c_{j,2}\,x + c_{j,3}}\right]$$

There are p network elements and therefore p relationships of the type in equation (8).

It is stated without proof that the relationship of equation (8) is a function which maps the set of non-negative real numbers into a set of n-tuples in a one-to-one manner.

Identification procedure

In this section it is assumed that an exact coefficient-signature vector can be obtained from ideal measurements. Given an accurate coefficient-signature vector, an elimination procedure is initiated which eliminates those network elements whose variation could not possibly yield the given vector. The remaining network elements constitute a set of possible faults.

The mapping described may be written in set notation as:

$$F_k \; : R \longrightarrow S_k \tag{9}$$

where R = the set of non-negative real numbers

and $S_k = \left\{ A \mid A = F_k(x), \quad 0 \leq x \right\}$

At this point it seems that the complete mathematical model of the problem can be presented most lucidly by drawing attention to the pictorial representation in Figure 2.

A general characteristic of this model is that the intersection of all the proper subsets, S_1 to S_p, contain only one n-tuple - the nominal coefficient-signature vector. On the other hand, intersections between proper subsets occur as a result of such special cases as parallel reistors or other, possibly less trivial, cases; the variation of one element or another is indistinguishable in the coefficients of the transfer function.

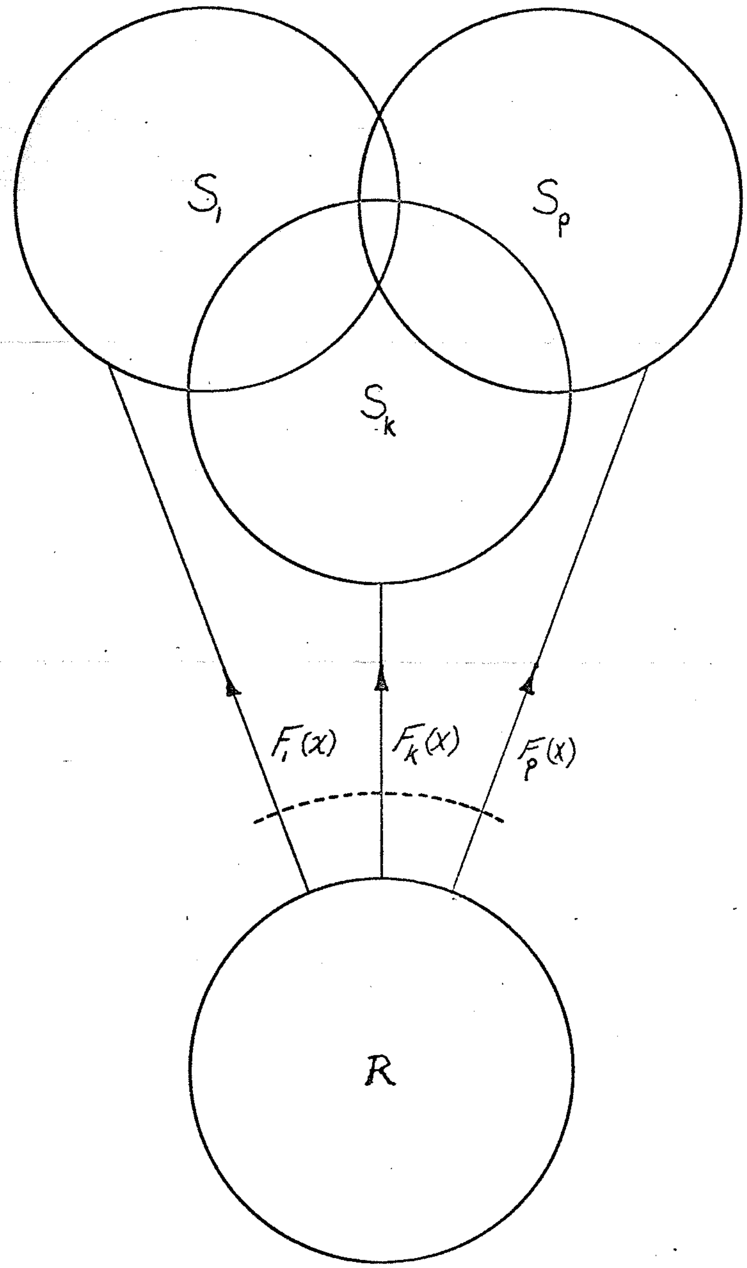


Figure 2. Mathematical model of problem

The principle of identification follows naturally from the above mathematical model: if a given coefficient-signature vector can be mapped into the set R, then the element e_k corresponding to that mapping is a possible fault. A possible fault becomes the one and only fault for the case of a unique coefficient-signature vector (i.e. the given coefficient-signature vector does not occur in any intersection).

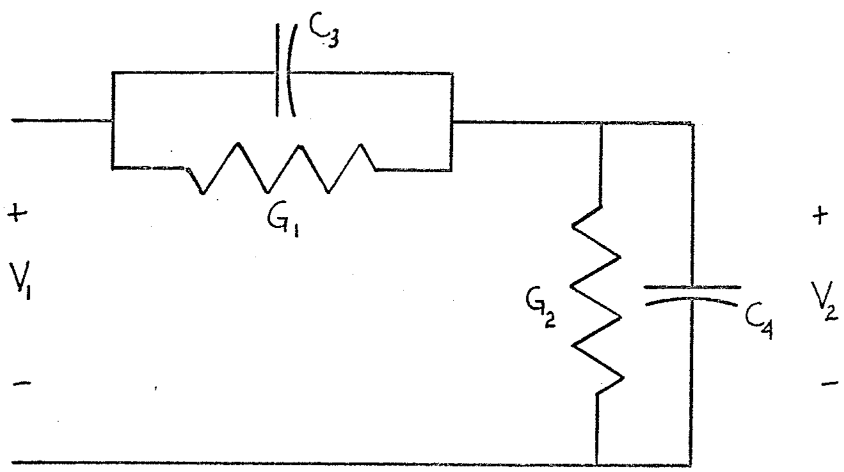
The identification procedure simply involves checking to determine which function, $F_k(x)$, that a given coefficient-signature vector, A, will satisfy, and the rules for checking are:

<u>Rule 1.</u> Eliminate element e_k unless each constant function entry in $F_k(x)$ is equal to the corresponding entry in A.

<u>Rule 2.</u> Eliminate element e_k unless each non-constant function entry in $F_k(x)$ has a solution for x which is non-negative. This follows from equation (4).

<u>Rule 3.</u> Eliminate element e_k unless every non-constant function entry in $F_k(x)$ has identical solutions for x. This follows from the assumption of a single fault.

At this point an example is given to clarify the procedure:



$G_1 = 1$ mho $G_2 = 10$ mhos $C_3 = 10$ f. $C_4 = 1$ f.

Figure 3. Example network

The nominal network in Figure 3 is given along with a coefficient-signature vector A:

$$A^T = \begin{bmatrix} 22/21 & 20/21 & 2/21 \end{bmatrix} \tag{10}$$

The problem is to determine which network element is faulty and what is the approximate value of that element.

The first step is to generate the coefficient functions whose origin is best understood by first deriving a symbolic transfer function:

$$\frac{V_2}{V_1} = \frac{a_2 s + a_3}{a_0 s + a_1} \tag{11}$$

$$= \frac{\dfrac{C_3}{C_3 + C_4}\, s + \dfrac{G_1}{C_3 + C_4}}{s + \dfrac{G_1 + G_2}{C_3 + C_4}}$$

The coefficient functions may now be written as:

$$F_1^T(x) = \left[\frac{G_1 x + G_2}{C_3 + C_4} \quad \frac{C_3}{C_3 + C_4} \quad \frac{G_1 x}{C_3 + C_4}\right] \qquad (12)$$

$$= \left[\frac{x + 10}{11} \quad \frac{10}{11} \quad \frac{x}{11}\right]$$

$$F_2^T(x) = \left[\frac{G_2 x + G_1}{C_3 + C_4} \quad \frac{C_3}{C_3 + C_4} \quad \frac{G_1}{C_3 \div C_4}\right] \qquad (13)$$

$$= \left[\frac{10x + 1}{11} \quad \frac{10}{11} \quad \frac{1}{11}\right]$$

$$F_3^T(x) = \left[\frac{G_1 + G_2}{C_3 x + C_4} \quad \frac{C_3 x}{C_3 x + C_4} \quad \frac{G_1}{C_3 x + C_4}\right] \qquad (14)$$

$$= \left[\frac{11}{10x + 1} \quad \frac{10x}{10x + 1} \quad \frac{1}{10x + 1}\right]$$

$$F_4^T(x) = \left[\frac{G_1 + G_2}{C_4 x + C_3} \quad \frac{C_3}{C_4 x + C_3} \quad \frac{G_1}{C_4 x + C_3}\right] \qquad (15)$$

$$= \left[\frac{11}{x + 10} \quad \frac{10}{x + 10} \quad \frac{1}{x + 10}\right]$$

Now the given coefficient-signature vector is compared with each coefficient function in turn and the previously described rules of elimination are applied.

Network elements G_1 and G_2 are both eliminated from consideration because the first rule is not satisfied; the constant entry in the coefficient function is not equal to the corresponding entry in the coefficient-signature vector. Network element C_3 is eliminated because the third rule is not satisfied; the solutions for x are not identical.

The remaining network element, C_4, satisfied all three rules and is clearly the fault. The solution of $x = 0.5$ means that the fault is 50% of nominal value.

The coefficient functions were generated directly from a symbolic transfer function for simplicity in this example, but in practice the symbolic transfer function is avoided. The parameters of each bilinear function may be obtained from a set of three equations which are formed by setting x equal to zero, one and infinity (i.e. the network element is open-circuited, set to nominal value and short-circuited). The method of generating the transfer function in order to obtain known values for the coefficients, is arbitrary; however, in this thesis the state equation approach is favored for reasons to be discovered in the third chapter.

II. PRAGMATIC SOLUTION

Modification of rules of elimination

It was previously assumed that the coefficient-signature vector was known exactly; however, in practice a degree of uncertainty is always involved in determining the coefficient-signature vector of an electrical network. As was stated in the previous chapter, it is assumed that signal levels will be of sufficient magnitude that noise will not cause any difficulty in measurement, but the coefficient-signature vector will still be corrupted by measurement error, and numerical error in computation.

In order to achieve a pragmatic solution, the emphasis must be shifted from determining the coefficient functions that perform an exact mapping to determining the coefficient function that comes closest to an exact mapping. Thus a modified set of rules for network element elimination are given:

<u>Rule 1</u>: Eliminate element e_k unless each constant function entry in $F_k(x)$ contains the corresponding entry in A within a prescribed tolerance. The tolerance is determined by the accuracy of measurements and the size of network being considered.

<u>Rule 2.</u> Eliminate element e_k unless each non-constant function entry in $F_k(x)$ has a solution for x which is non-negative. This follows from equation (4).

<u>Rule 3.</u> Consider the set of network elements whose coefficient functions contain two or more non-constant function entries. Eliminate all of

these network elements except the element e_k whose solutions for x contain the minimum deviation. In the limit, as the coefficient-signature vector becomes exact, the solutions for x would become identical.

Error reduction

The coefficient function $F_k(x)$ is in general an array of bilinear functions which tend towards a zero slope for large values of x. In the region of near zero slope a slight deviation in the measured value of a coefficient can cause gross deviations in the evaluations of x. Thus the third rule, which relies on selecting the coefficient function which exhibits a minimum deviation in the solutions of x, becomes useless.

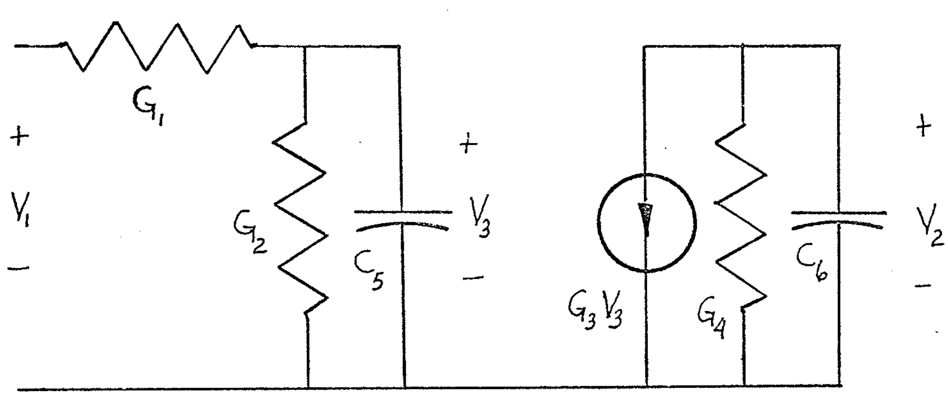
Since large values of x correspond to near zero slope, the difficulty can be alleviated by evaluating x for small values, and otherwise considering the element e_k to be short-circuited. This simply means that an element which is nearly short-circuited can be considered to have the same effect as being completely short-circuited.

Seshu and Waxman [2] chose to investigate element variations of zero to three hundred percent of nominal value. Using this precedent, evaluations of x in the range of zero to three will be considered; otherwise, the element will be considered short-circuited.

Upon examination of equation (6) of the general coefficient function, it becomes apparent that the choice of element dimension (i.e. admittance or impedance) determines whether a function be hyperbolic or straight line. Since the straight line function with its constant slope is more advantageous, that dimension which maximizes the number of straight line functions should be chosen.

III. EXAMPLE

To demonstrate the feasibility of locating a single fault in an active network by means of the coefficient-signature method, consider the nominal network in Figure 4.



$G_1=G_2=G_3=G_4=10^{-3}$mho $\qquad C_5=C_6=1000$pf.

Figure 4. Example network

In order for the coefficient-signature method to be verified, assume that an arbitrary fault exists (e.g. element C_6 reduced to 50% of nominal value).

Using the technique proposed by J. E. Valstar [3], the coefficient-signature vector of the network with the faulty element C_6, is determined. In practice the entries of this vector are not exact but fall within a tolerance determined by the size of the network and the accuracy of the measurements. A reasonable tolerance to assign in this example would be $\pm$ 5%. Thus the coefficient-signature vector corresponding to the faulty element C_6, would be:

$$A^T = \left[4.0\text{x}10^6 \pm 5\% \quad 4.0\text{x}10^{12} \pm 5\% \quad -2.0\text{x}10^{12} \pm 5\% \right] \qquad (16)$$

A typical measurement result might be:

$$A^T = \left[4.20\text{x}10^6 \quad 3.80\text{x}10^{12} \quad -1.90\text{x}10^{12} \right] \qquad (17)$$

With the knowledge of the nominal network, the coefficient functions can be computed. Each coefficient function is an array of bilinear functions each of which requires three constants to describe it. As an example of how these constants are determined, consider the particular bilinear function $f_{1,1}(x)$ which is associated with coefficient a_1 and is a member of the array of the coefficient function $F_1(x)$:

$$f_{1,1}(x) = \frac{c_{1,1} \; x + 1}{c_{1,2} \; x + c_{1,3}} \qquad (18)$$

or $\qquad a_1(x) = f_{1,1}(x)$

A computer is used to generate the coefficients of the standard voltage transfer function for three different values of the element G_1 where all the other elements are held at their nominal value such that three equations result:

$$a_1(x=0) = \frac{1}{c_{1,3}} \quad (19)$$

$$a_1(x=1) = \frac{c_{1,1} + 1}{c_{1,2} + c_{1,3}} \quad (20)$$

$$a_1(x=2) = \frac{2c_{1,1} + 1}{2c_{1,2} + c_{1,3}} \quad (21)$$

Note that in equation (21), x is set equal to two whereas in previous parts of this thesis it was set equal to infinity. The reason for this change is that computers are limited to finite numbers.

Equations (19), (20) and (21) are solved for the unknown constants and the bilinear function is now completely described as:

$$f_{1,1}(x) = (x+2)\ 10^6 \quad (22)$$

Proceeding in this manner, all the coefficient functions may be listed as:

$$F_1^T(x) = \left[\ (x + 2)10^6 \quad (x + 1)10^{12} \quad -(x)10^{12}\ \right] \quad (23)$$

$$F_2^T(x) = \left[\ (x + 2)10^6 \quad (x + 1)10^{12} \quad -10^{12 \pm 5\%}\ \right] \quad (24)$$

$$F_3^T(x) = \left[\ (3)10^{6 \pm 5\%} \quad (2)10^{12 \pm 5\%} \quad -(x)10^{12}\ \right] \quad (25)$$

$$F_4^T(x) = \left[\ (x + 2)10^6 \quad (2x)10^{12} \quad -10^{12 \pm 5\%}\ \right] \quad (26)$$

$$F_5^T(x) = \left[\frac{(x+2)10^6}{x} \quad \frac{(2)10^{12}}{x} \quad \frac{-10^{12}}{x} \right] \tag{27}$$

$$F_6^T(x) = \left[\frac{(x+2)10^6}{x} \quad \frac{(2)10^{12}}{x} \quad \frac{-10^{12}}{x} \right] \tag{28}$$

Notice that the coefficient functions are modified in keeping with the first rule of elimination such that each constant entry is associated with the prescribed tolerance of $\pm 5\%$.

The elimination procedure is initiated by comparing the measured coefficient-signature vector in equation (17) with each of the coefficient functions in equations (23) to (28). Elements G_2, G_3 and G_4 are eliminated because the first rule is not satisfied; the entries in the measured coefficient-signature vector corresponding to the constant entries in the coefficient function do not fall within the prescribed tolerance.

The second rule does not eliminate any elements; all the solutions for x are non-negative.

The difference between the maximum and the minimum solutions for x in $F_1(x)$, $F_5(x)$ and $F_6(x)$ are 0.9, 0.1 and 0.08, respectively. Thus rule three eliminates elements G_1 and C_5. The remaining element C_6 is the fault, viz. the fault that was assumed at the beginning of this example. The approximate value of the fault is taken as the mean value of the solutions for x in the coefficient function $F_6(x)$, and is not significantly different from the assumed value of $x = 0.5$.

CHAPTER III

DISCUSSION AND CONCLUSION

Having demonstrated the feasibility of the coefficient-signature method, a comparison should be drawn to determine what features recommend this particular method. The comparison is begun by first examining Figure 5 and Figure 6 which epitomize the computational steps for the frequency-signature method and the coefficient-signature method.

The proposed procedure avoids the difficulty of having to compute a symbolic transfer function which requires the listing of 2-trees. Even for a medium sized network (e.g. 20 edges and 7 nodes) the required list of 2-tress could heavily tax the memory of a large computer, and consequently a severe restriction is placed on the size of network that may be considered by the frequency-signature method.

The standard voltage transfer function in this thesis is derived from the state equations instead of topological formulas, but could have been derived from any of a number of types of network equations. The choice of the state equations was influenced by the ease with which they lend themselves to computer programming [5], and by their flexibility in being used for alternate forms of analysis (e.g. time response).

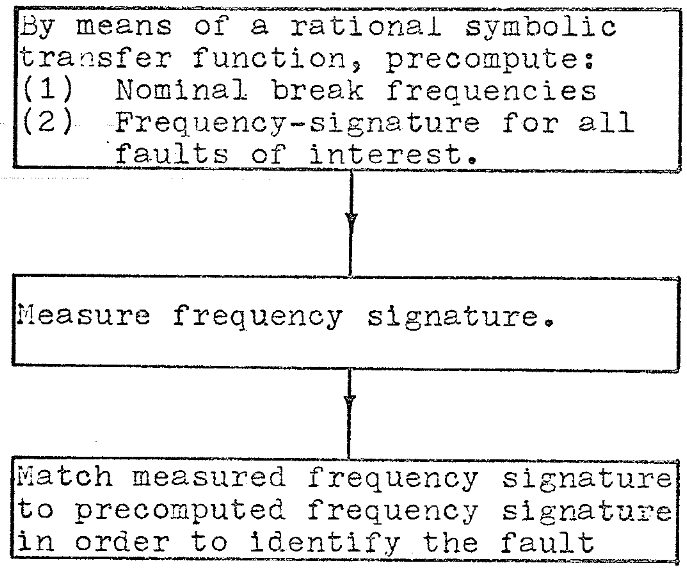


Figure 5. Flow diagram of the frequency-signature procedure.

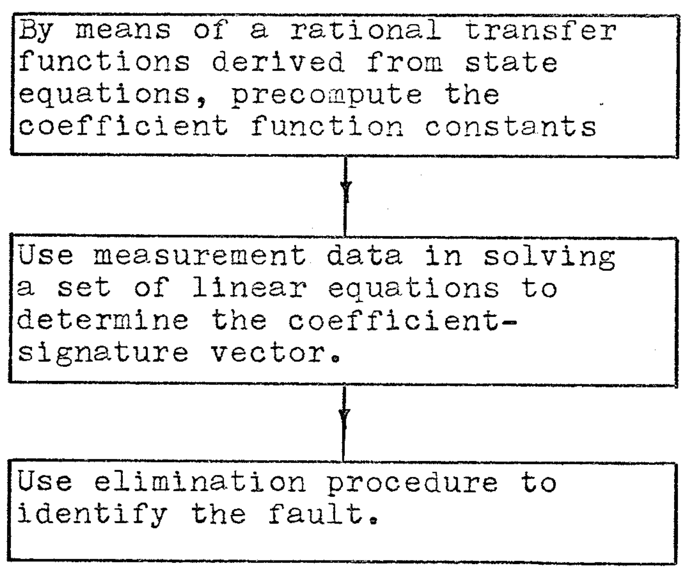


Figure 6. Flow diagram of the coefficient-signature procedure.

Another desirable feature of the coefficient-signature method, is that the computation of the nominal poles and zeros is avoided. In the frequency-signature method, the accurate search for a large number of poles and zeros (e.g. ten or more) requires a large investment in computation time and further prohibits the size of network that may be considered.

The mathematical model used by the coefficient-signature method requires less information to be precomputed and stored in memory in order to identify a fault.

In the coefficient-signature method there are p coefficient functions each of which contains an array of m+n+1 bilinear functions. Each bilinear function needs three constants to be described and therefore a total of 3(m+n+1)p pieces of information have to be precomputed and stored.

In the case of the frequency signature method, each frequency signature contains at least m+n+1 pieces of information where m+n+1 equals the number of poles plus zeros plus one, but in practice this figure should be doubled or even tripled in order to identify a fault with any accuracy. There are ph frequency signatures where ph equals the number of network elements and h equals the number of quantized step variations in a network element. Thus it can be seen that the frequency-signature method must precompute and store 3h(m+n+1)p pieces of information as compared to $3(m+n+1)p$ pieces of information in the case of the coefficient-signature method.

The critical factor is h and in the case of Seshu and Waxman [2] a figure of fifteen (i.e. $h = 300\% / 20\% = 15$) was chosen in demonstrating the frequency-signature method on a simple common-emitter transistor amplifier. If a fault is to be accurately identified in a large network, the size of the quantized variations in element value must be reduced to smaller variations which in turn increases the value of h. The greater the discrimination desired in identifying a fault with the frequency-signature method, the larger the value of h must become, whereas the coefficient-signature method suffers from no such handicap.

Thus in requiring less information to accurately identify a fault, the coefficient-signature method makes more efficient use of a computer's memory and this feature becomes more significant as the size of networks being considered beomes larger.

It must be conceded that the frequency-signature method is slightly faster in identifying a fault. Both methods utilize elimination procedures which are basically equivalent with respect to computation time. Whereas the frequency-signature method can use the measurement data directly in the elimination procedure, the coefficient-signature method must first determine the coeffieicnt-signature vector by solving a set of linear equations before beginning the elimination procedure.

It might be concluded that the frequency signature method would be preferable for on-line testing of a large number of small networks. The coefficient-signature method would be better suited for testing larger networks where the efficient use of a computer is more important than a fast test procedure.

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