AN APPARATUS FOR THE MEASUREMENT OF SMALL CHANGES OF MAGNETIZATION OF STRONGLY MAGNETIC MATERIALS

A thesis

submitted in partial fulfilment of the requirements

for the degree of

Master of Science

at the

University of Manitoba

by

Stamatis Patapis

June, 1970

THE UNIVERSITY OF MANITOBA
ELIZABETH DAFOE LIBRARY

C O N T E N T S

ACKNOWLEDGEMENTS

The help of my supervisor Dr. W. N. R. Stevens for the building of the apparatus and generally for the preparation of the thesis is acknowledged with thanks.

I would also like to thank Miss Ruth Hoehn and Mr. J. Kanellopoulos respectively for their help in proof reading and with one of the drawings, and Miss Sharon-Ann Ewens for the careful typing.

Finally, the financial assistance from the National Research Council of Canada is gratefully acknowledged.

A B S T R A C T

An apparatus, designed on the principle of the pyromagnetic technique, is described for measuring small magnetization changes by direct integration.

The magnetizing field of the specimen is provided by a superconducting solenoid. A test of error from nonlinear distortion of the specimen's field by interaction with the superconducting solenoid (image effect) is made by replacing the specimen by a small coil.

I N T R O D U C T I O N

Two types of techniques can be distinguished for magnetization measurements at different temperatures.

Either we measure the magnetization or we measure the change of the magnetization of the specimen (pyromagnetic technique), at different temperatures.

The basic principle of the pyromagnetic technique is to detect the change in flux enclosed by a pick-up coil resulting from a temperature change of a magnetized sample. The same principle has been used by Chynoweth[1] to measure Curie temperature of ferrites but in a different approach than that described below.

The method for making low temperature measurements of magnetization changes with temperature by using the pyromagnetic effect was first introduced by Pugh and Argyle[2] in 1961 and established in 1962 and 1963 with their measurements on Ni and Fe[3,4].

According to this method, the sample, spherical in shape, is located at the center of a pick-up coil. This sample is magnetized by a fixed field but by changing its temperature, through a heater, we change its magnetization. The latter results in change of the magnetic flux enclosed by the pick-up coil. This change of flux produces an emf at the terminals of the coil, which is fed to a ballistic galvanometer or an integrating circuit consisting of an electronic integrator. The deflection of the galvanometer or the output voltage of the integrator is proportional to the change of the magnetic flux penetrating the pick-up coil and consequently proportional to the change of the magnetization of the specimen.

The method has a sensitivity of one or two orders of magnitude better than previous methods in which the total magnetization is measured at each temperature.

We mention two examples of the method of direct measurements of magnetization.

Foner and Thompson[5] used a vibrating sample magnetometer in which the moment of the sample is compared with that of a similarly vibrating permanent magnet. The accuracy of the method was 1 or 2 parts in $2X10^4$.

Another technique is that according to which a specimen is moved between the centres of two coils connected in opposite sense in series with a ballistic galvanometer (e.g. Cooke et al[6]; Finnemore et al[7]). If the movement is performed in a short time compared with the galvanometer period, the deflection is proportional to the magnetization of the specimen.

An adaptation to the above method by Stevens[8] changes it to a method of measuring small changes of magnetization. According to that, two specimens are moved simultaneously between oppositely connected pairs of coils and the impulses are thus cancelled out. The obtained deflection depends on the difference of the magnetization of the specimens. So, thermally induced change of the magnetization of one of the two specimens results in proportional change of the galvanometer deflection.

The motivation of designing more accurate methods for magnetization measurements is the exact and detailed investigation of the relation between the intrinsic magnetization M of a specimen and its temperature T.

The Stoner collective electron model predicts a relation[9]

$$\left(\frac{M}{M_o}\right)^2 = 1 - \left(\frac{T}{T_c}\right)^2$$

where T_c is the Curie temperature and M_o is the magnetization at absolute zero.

On the contrary the simple spin wave theory predicts [10]

$$M = M_o\left(1 - CT^{3/2}\right)$$

where C is a constant factor.

More complete theoretical representation of the temperature dependence of the magnetization for low temperatures is [11]

$$\frac{M - M_o}{M_o} = -CT^{3/2} - DT^{5/2} - ET^{7/2} - \ldots\ldots$$

where C, D and E are constant factors and $CT^{3/2}$ is the dominant term.

Therefore, to the first approximation, the decrease in spontaneous magnetization with increasing temperature is predicted to follow a T^2 law by the collective electron model and a $T^{3/2}$ law by spin wave theory.

Experimentally it has been difficult to make an unambiguous selection between the T^2 and $T^{3/2}$ terms or to find higher or lower order terms. The problem has been further complicated by the fact that spin wave theory is not expected to be valid except at low temperatures where the sensitivity required for meaningful measurements is increased.

However, the experiments of Pugh and Argyle gave a decisive answer. Their results were well fitted to the spin wave model.

DESCRIPTION OF THE APPARATUS

In the design of our apparatus for measurements of small changes of magnetization, we follow the features described in the introduction.

The magnetizing field of the specimén is provided by a superconducting solenoid in the center of which the pick-up coil with the specimen are located.

Since we are interested in measurements in low temperatures where the spin wave terms are important and low temperatures are needed for the superconducting solenoid, the importance of the low temperature maintenance system is apparent.

As it is described in the next pages, the cryostat and the solenoid mounted on it are immersed in a bath of liquid helium contained by one Dewar while this Dewar is in a bath of liquid nitrogen contained by a second, larger Dewar.

The vacuum system, described later, helps in the thermal insulation of the specimen from the outside low temperatures by evacuating the space inside the cryostat where the specimen is located.

Finally the solenoid and its current supply system are described in detail.

F R A M E W O R K

A box-like framework BCFE of aluminum, approximately 3 feet in height, was built on a table as it is shown in figure 1.

The trap T, the diffusion pump D, and all the piping system of the vacuum system are suspended from the main body of the framework.

Two parallel arms (only one AB is shown in the figure) at the top of the framework, approximately 5 feet 7 inches from the floor, support one plate from which the "Helium Dewar" D_2 is suspended.

The "Nitrogen Dewar" D_1 is suspended from the same plate by six rods (not shown in this figure).

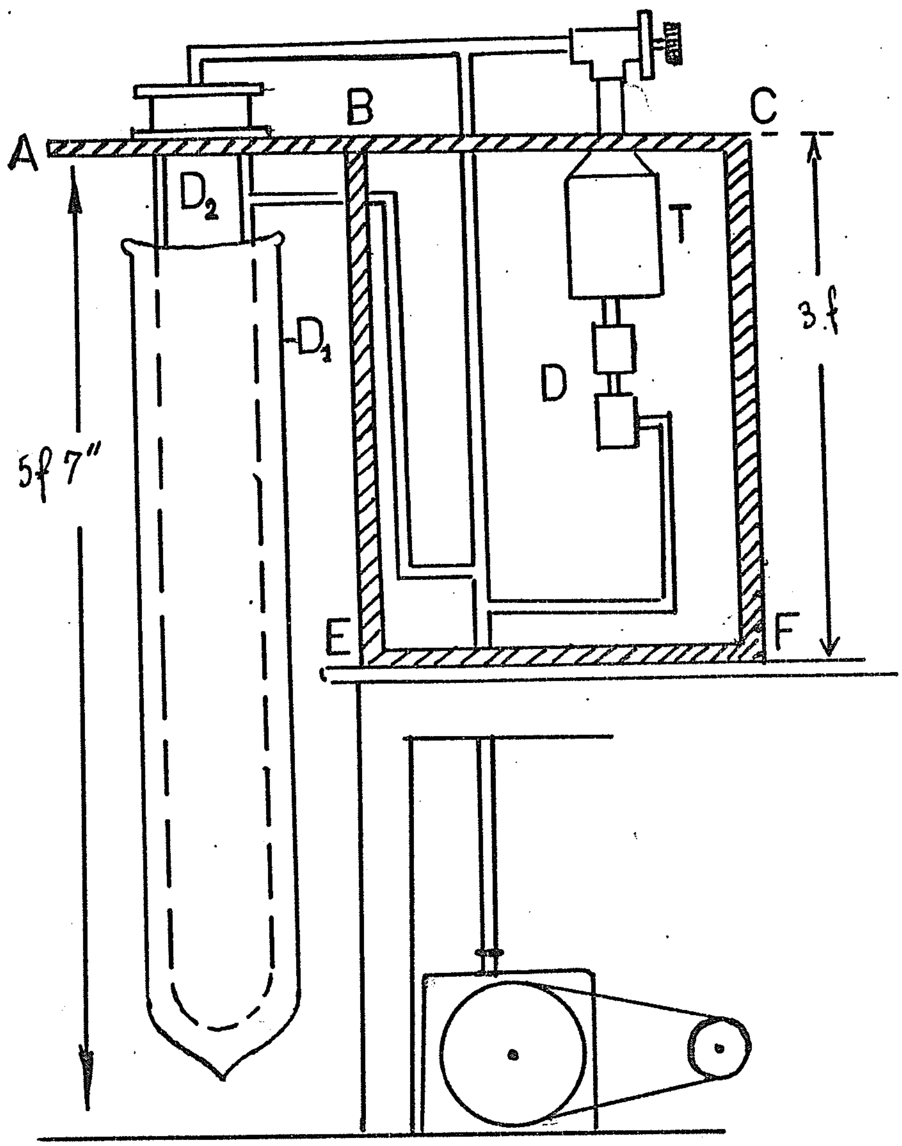


FIG. 1

D E W A R   S Y S T E M

The system for the maintenance of low temperatures consists basically of two Dewars. The outside Nitrogen Dewar D_1 and the inside Helium Dewar D_2. (Fig. 2)

The Nitrogen Dewar

The Dewar D_1 is suspended from the top metal plate AB by six equally spaced metal rods L and it is supported, through a "swelling" E, by the wooden ring W.

When this Dewar is filled with liquid Nitrogen, we have a temperature of about 77^o K inside it. The helium Dewar is immersed in this bath of liquid nitrogen.

The Helium Dewar

The helium Dewar D_2 is also held rigidly from the metal plate AB by a metal ring M. For thermal insulation, the space between the walls of this Dewar can be evacuated through the connection K to the vacuum system.

The cryostat and the superconducting solenoid R which is mounted on it are located inside the helium Dewar. The whole system is suspended vertically from the top flange F, by the stainless steel tube T. This cryostat-solenoid system can be removed from the helium Dewar and suspended through the flange F (soldered on the tube) from a nearby box-like framework similar to the one described before. In this way, we can easily work with the cryostat, e.g. change the specimen, etc.

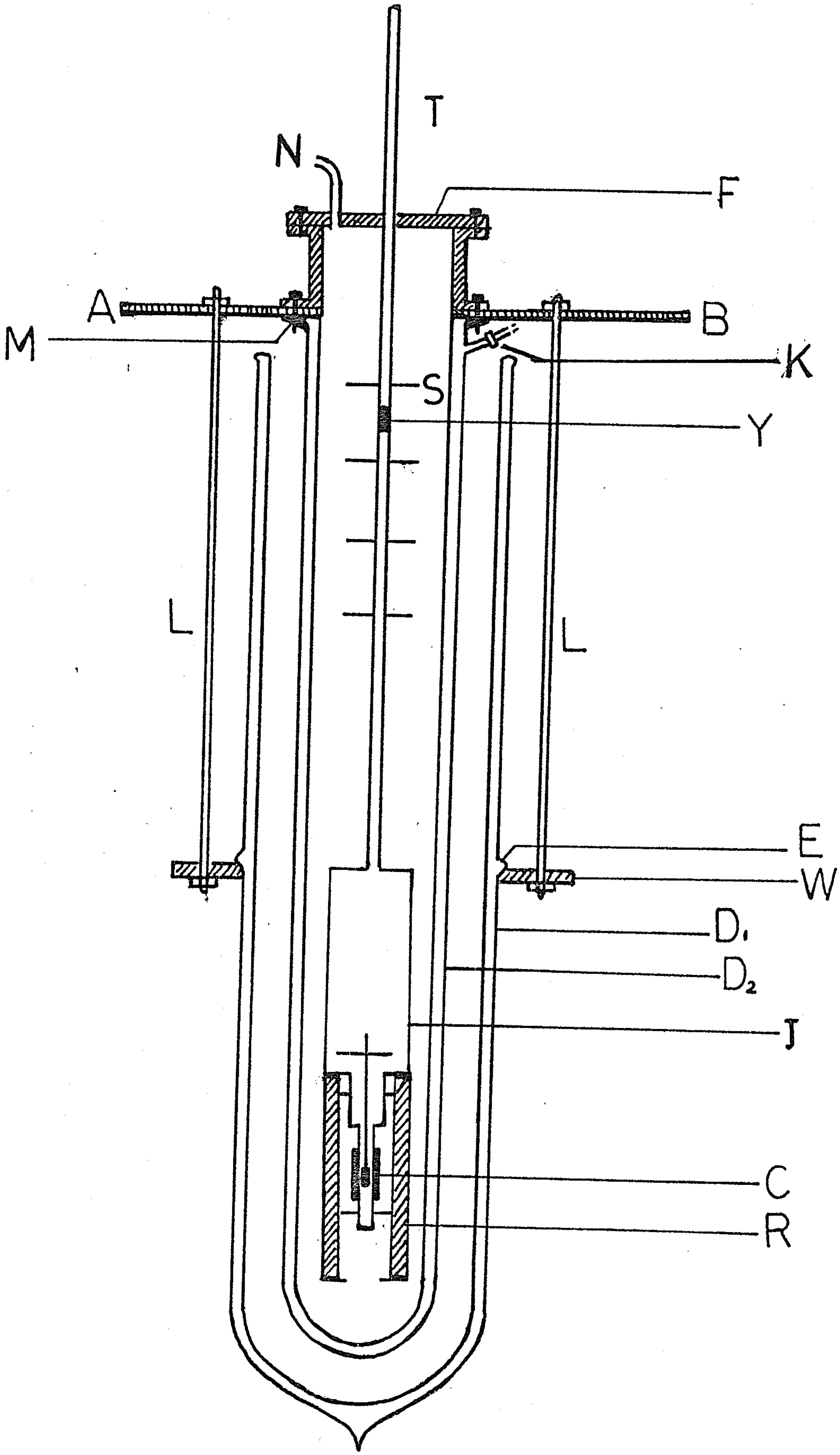


FIG. 2

Mounted on the tube T are four round copper shields S (radiation shields) which prevent the transfer of heat. The same role is played by the small cylindrical shield Y inside the tube. It consists of a cylinder, the two end surfaces of which are half covered in opposite ways. In this way it does not prevent the evacuation but prevents radiation from passing directly through.

The four shields S have small holes with rubber gromets through which pass electrical leads of find copper wire (0.005" in diameter).

The leads from the heater pass into a sealed tube, soldered on the flange F. The pick-up coil leads pass out of the vacuum space through a copper shielded rubber tube sealed with a clamp. These leads reach a box mounted on the framework inside of which, through a specially designed connection, they join the leads of a cable leading to the measuring system.

The connection between these two leads is shown in Fig. 3. The leads end with small copper plates P soldered on them with low thermal solder. These plates and the copper rings B around the screw S are pushed together by the spring R for better contact. Grease between their surfaces provides a better contact also. The cable of the measuring system passes through the hollow ceiling to assure constant temperature during an experimental run.

The vacuum chamber with the pick-up coil C (Fig. 2) at its lower end and the solenoid R are continuously in a liquid helium bath.

To introduce liquid helium into the helium Dewar we use a transfer tube which passes through a top opening and the radiation

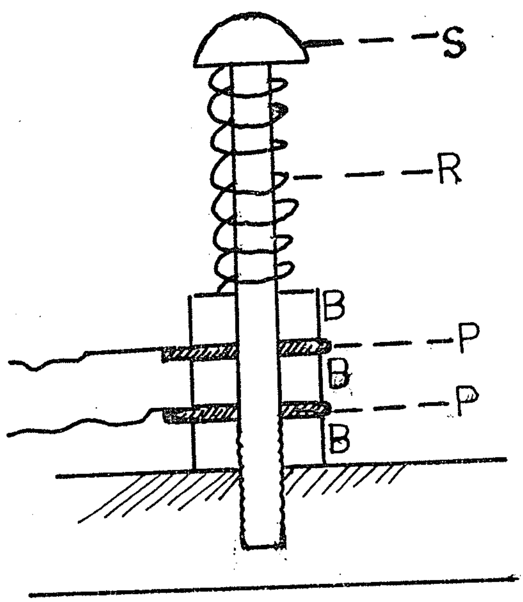


FIG. 3

shields to reach one end of a rubber tube (not shown). The other end of the rubber tube reaches the bottom of the helium Dewar at the center. Hence, the liquid helium is transfered into the Dewar at a point below the solenoid itself and the escaping vapour passes through it. Finally the vapour leaves the Dewar from the top tube N.

V A C U U M   S Y S T E M

The vacuum system can reach as low a pressure as 10^{-8} mm Hg. The basic elements of the system consist of a mechanical pump, a diffusion pump and a vapour trap, as it is shown in figure 4.

Mechanical pump: The mechanical pump M is a WELCH DUO SEAL 1405 H with a vented exhaust.

Diffusion pump: The diffusion pump D is an EDWARDS water cooled oil diffusion pump with a maximum backing pressure of 0.5 mm Hg.

Vapour trap: The vapour trap T is also an EDWARDS one and it was used with liquid nitrogen.

The system as it is shown in figure 4 consists of two parts; part A and part B.

Through the main part A, we evacuate the chamber in the cryostat. Hence the specimen is in a space of small pressure (10^{-8} mm Hg) and in this way it can be thermally insulated from the different temperature outside the jacket.

To facilitate the cool down of the specimen to helium temperature we introduce some helium gas from the filled bladder L by opening the valve V_L.

The second part B is a branch of the main system connected to the helium Dewar through the valve V. This part evacuates the space between the walls of the Dewar.

The different stages of pressure at different parts can be

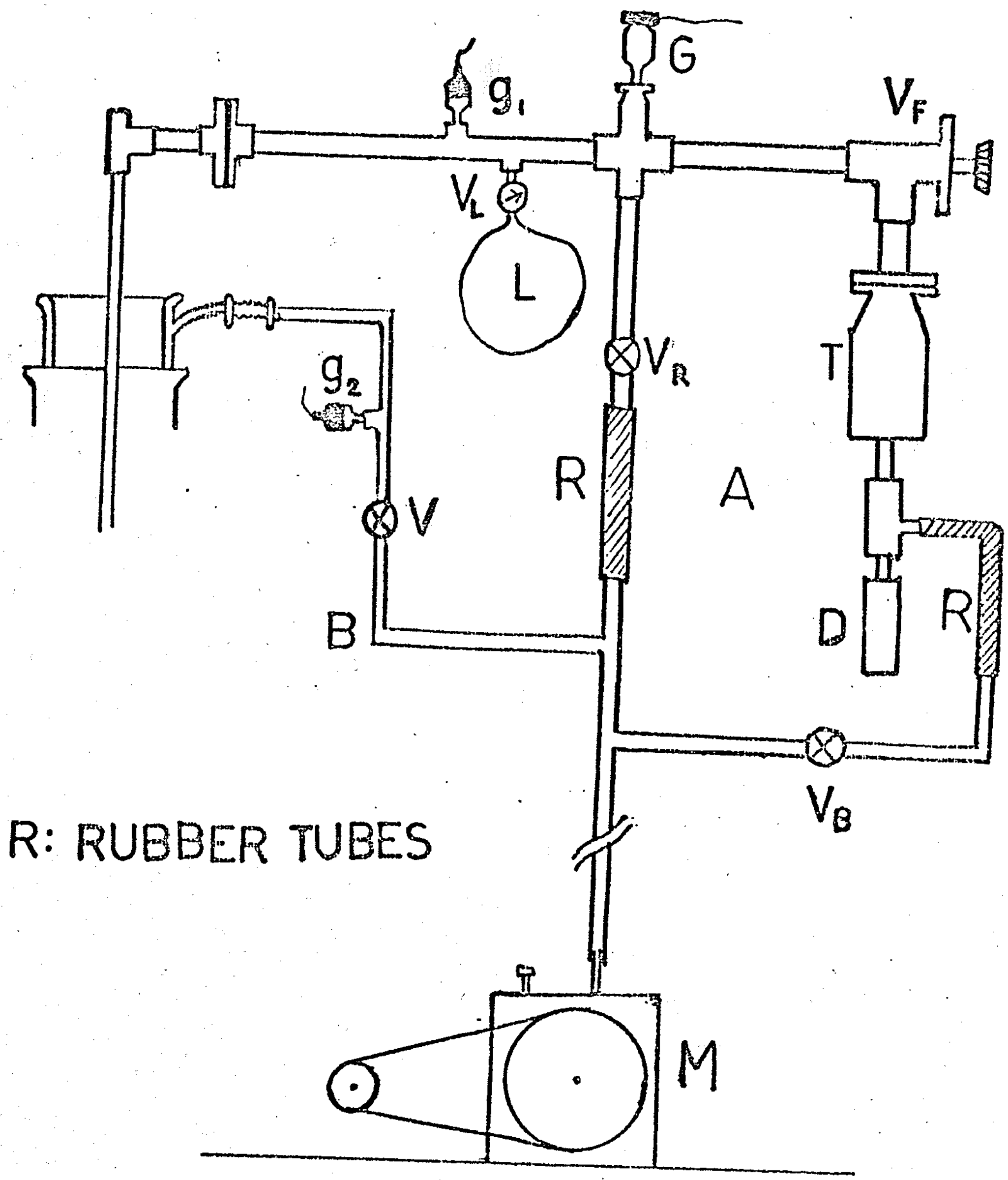


FIG. 4

checked through the thermocouple gauges g_1 and g_2 and the gauge G.

By opening the baffle valve V_F, the roughing valve V_R and the backing valve V_B and after the backing pressure is 0.5 mm Hg, we actuate the system of the diffusion pump and vapour trap.

C R Y O S T A T

A schematic diagram of the cryostat is shown in figure 5.

The cryostat is suspended vertically by the stainless steel tube T through which it can be evacuated.

The vacuum chamber is enclosed by the top brass plate P, the base B and the brass jacket J. The two bases of the chamber are held rigidly by three brass rods E, so the jacket J can be removed from the whole system.

The solenoid R is suspended tightly against B by three brass supports from the top of the vacuum jacket (not shown) which end with a brass disk where the solenoid is held rigidly by three screws (only their washers W are shown in the figure). Two spacing disks Q help to maintain the lower part of the cryostat with the pick-up coil in the centre of the solenoid.

This lower part of the cryostat extends downwards into the interior of the solenoid from the base B. It consists of a cylinder F and one thin walled German silver tube U (8 mm in outside diameter) extending downwards from the cylinder base G so that it also forms part of the vacuum jacket. The tube is sealed at its lower end by a cap Y soldered on it. The pick-up coil C is directly wound on this tube.

In the interior of the cryostat, the three brass rods E hold one radiation shield X which is soldered on them. Under this shield, there is a suspension plate D. From this plate three thin wall (0.006") stainless steel tubes L are suspended to support a copper platform A. Through this copper plate a copper rod I, $\frac{1}{4}$ of an inch in diameter and about 14 inches in length is suspended vertically at the lower end of which the specimen S is mounted.

This isothermal copper rod slides inside the tube U but is prevented from touching the latter by two thin Mylar spacers. The lower end of this copper rod has a concave hemispherical surface machined to fit the specimen. A copper cap V fits from the outside to hold the specimen.

The specimen, spherical in shape, has a diameter of 0.22 of an inch almost equal to the inner diameter of the tube U in order to give maximum voltage impulse while leaving a sufficient gap between specimen and coil for thermal insulation of the former. A trace of grease is used to improve thermal contact between the specimen and its holder, the isothermal rod. This rod has at its upper end a cylindrical cavity Z where a semiconducting resistance thermometer is located to detect the temperature of the specimen at the other end.

Some semiconductors make excellent resistance thermometers in low temperatures where metallic elements and alloys are not sufficiently sensitive. Generally dopped Germanium and silicon are the base of these thermometers.

We use a Germanium thermometer (Crycal Inc. 4° K, resistance 1000 Ohm). Its position, at the upper end of the isothermal rod, is at a distance of 9.2 inches from the nearest end of the solenoid to avoid the influence of the magnetic field on its sensitivity.

The isothermal copper rod can be heated at a certain temperature by a heater located approximately at the centre of its length. This heater is mounted on a small copper rod (not shown) soldered on the plate A and has a mechanical heat switch actuated from outside by a mechanism mounted on the head of the cryostat. The heat through

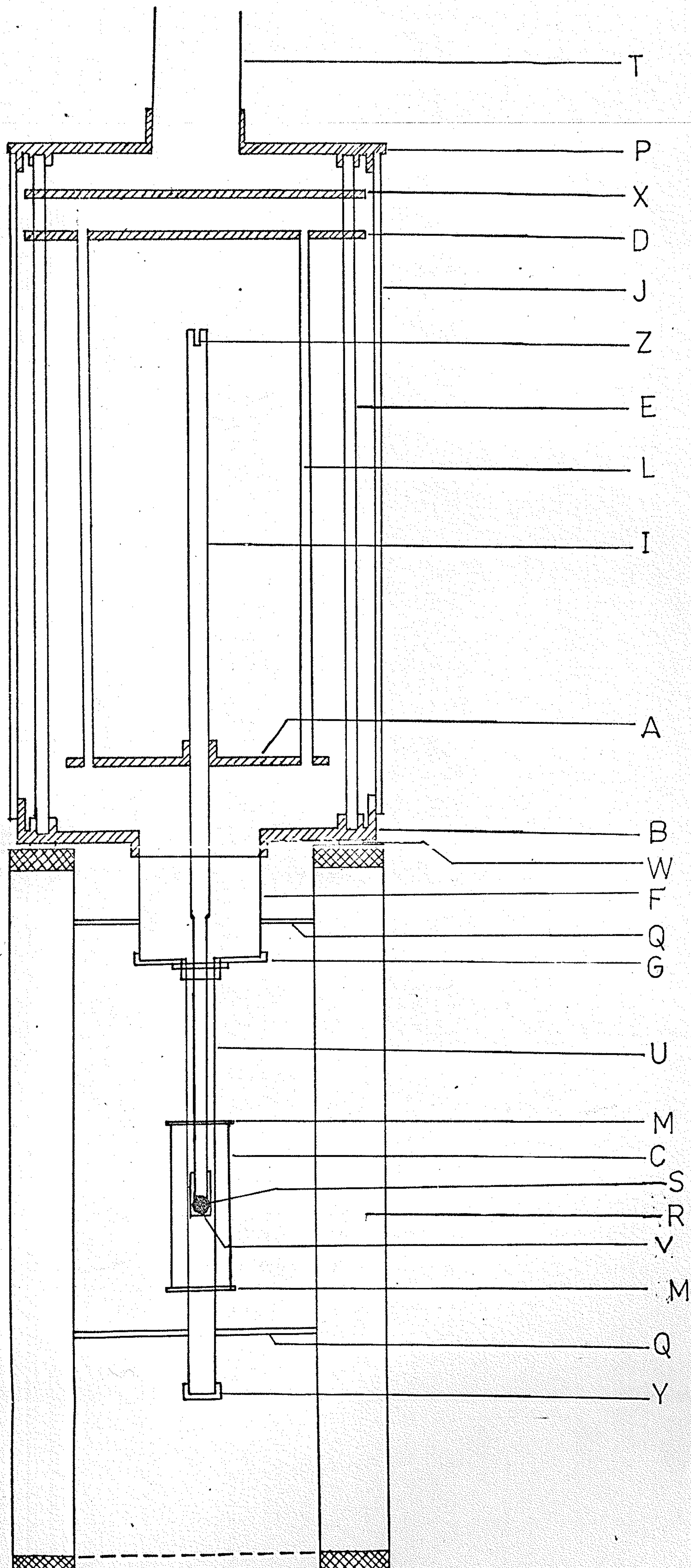


FIG. 5

the copper plate A passes to the isothermal rod and through it is transfered to its two extreme ends where the specimen and the thermometer are located.

The pick-up coil C of 10,000 turns of 0.002 inch in diameter copper wire is wound on the German silver tube. Firstly, small Tufnol rings M had been epoxied on to the tube to hold the coil in place. Next the coil was wound on the tube and then it was soldered into the base G. This coil has an exterior diameter of II mm and its length is 30 mm.

Its leads are carefully handled to assure maximum cancellation of the emf's along their length as they pass through the thermal gradient from the liquid helium to room temperature.

The coil is in a liquid helium bath to avoid coil thermal expansion resulting in a pick up voltage from the applied field.

Specimen Mounting

To mount the specimen, the cryostat, removed out of the helium Dewar, is suspended from a framework without the solenoid R, the jacket J and the small cap Y at the end of the tube U.

We slide the rod I with the attached specimen into the tube U and we find the exact position of the specimen, in the centre of the pick-up coil, by measuring its distance from the lower edge of the tube U. We solder the copper rod on the copper platform A and then we solder the cap Y and the jacket J in Wood's metal.

Finally we mount the solenoid and we put the whole system into the helium Dewar.

SOLENOID AND ITS CURRENT SUPPLY SYSTEM

The superconducting solenoid, of 2 inches bore, is made of insulated stabilized copper-clad miobium-titanium composite, a type II superconducting material. According to its specifications is has a critical current of 74 A, a d.c. inductance of 2 H and with a current of 66 A it produces a field of 40 KG.

As we mentioned before, the solenoid is mounted by three screws on a brass disk which is held tightly against the base of the vacuum jacket by three brass supports from the top of the jacket.

The screws have three washers W (fig. 5); the function of which is to leave a gap between the base B and the top surface of the solenoid so that the helium vapours escape through it out of the solenoid. In this way the vapours passing upwards through the center of the solenoid refrigerate it more quickly. The same role is played by the holes around on the spacing disks Q.

The current supply circuit of the solenoid is shown in fig. 6.

To introduce current into the solenoid we use a d.c. current supply, a battery B, and direct leads L running down the temperature gradient to the solenoid. But in this way we introduce heat into the cryostat.

As it can be shown (McFee[12], Mercuroff[13]), there is an irreducible minimum quantity of heat Q_{min} that must be introduced by a thermally insulated lead carrying a current i between temperatures T_1 and T_2 given by

$$Q_{min} = i\sqrt{L\ (T_1^2 - T_2^2)} \qquad (1)$$

where $L = 2.45\times10^{-8}\ W\Omega \deg K^{-2}$.

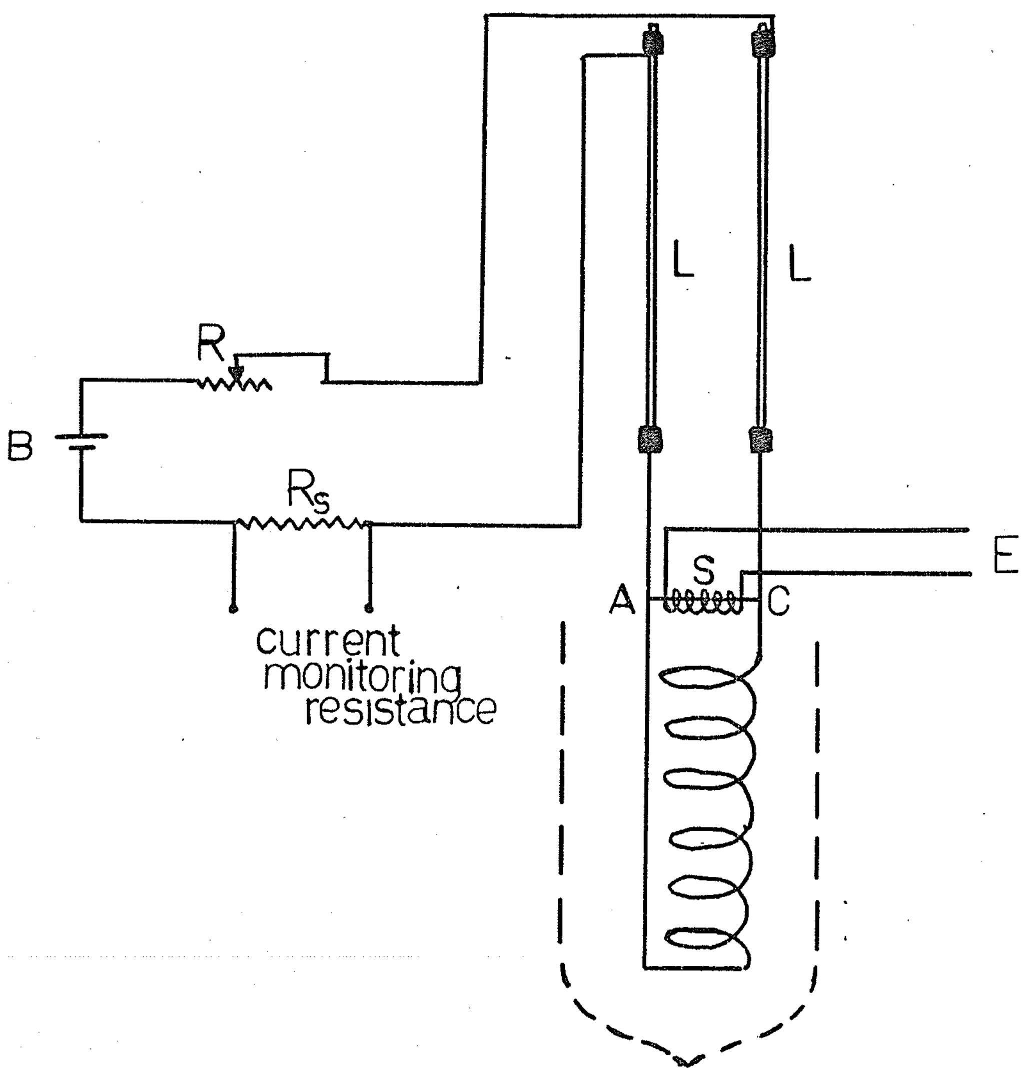


FIG. 6

Therefore for a lead between room temperature and 4.2° K and for a current of 25 A the heat input would be about 1.1 W which corresponds to an evaporation rate of 1100 cc of liquid helium per hour. Hence it is more convenient to refrigerate the leads with the returning helium vapour boiled off from the helium bath.

While the current flows, the Joule heating of the leads gives rise to a heat input of $\frac{dQ}{dt} = i^2R$ and for a certain current we can decrease the resistance R of the leads (increase electric conductivity). However, electrical conductivity is closely related to thermal conductivity. So the opposing demands of low thermal conduction along electrical leads and insignificant heat dissipation in these same leads present a problem. It is shown (Lock[14]) that the choice of the conducting metal is not as important as the optimization of the geometrical parameters of the lead in order to minimize the rate of evaporation from the liquid helium bath.

In our case we use two leads; the design of one is shown in fig. 7.

Each lead consists mainly of two parts, N and M which is a copper rod and the copper wires K.

The part of the lead under the Teflon bush T passes through a hole on the flange F which seals the helium Dewar and is suspended vertically along the tube T (fig. 2). The rod N with the brass sleeve Y, the stainless steel tube S (with the copper wires inside) and the lower part M can rotate by the top disk D in the Teflon bush T. Hence the lower part of the lead can be screwed into the stainless steel block B_2. Two pairs of blocks B_1 and B_2 are fixed on

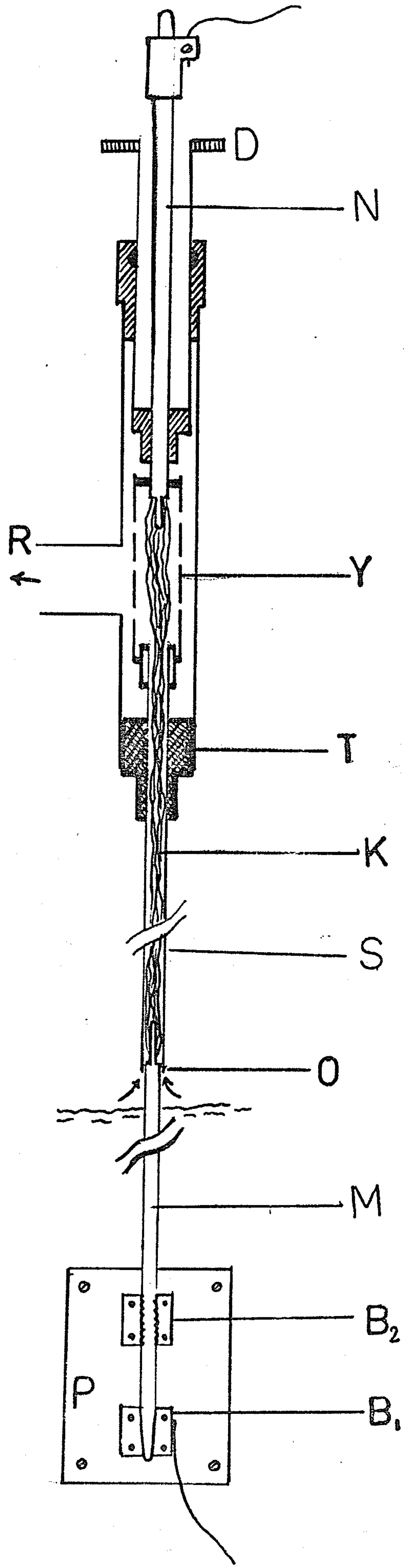


FIG.7

a plate of insulator P (only one pair is shown in the figure), which is mounted on the tube T and immersed in liquid helium. The upper block B_2, with the threads and a larger opening of its hole at the top, helps to lead the rod into the hole of the lower block B_1 where one lead of the solenoid is soldered on. This block has a conic hole which fits to the end of the rod which is conic also. The rod is in good contact with the block through an indium-tin plating.

The central part of the lead, of many thin copper wires, helps the heat transfer to the helium vapour which enters the openings O, between the stainless steel tube and the copper rod. The vapour passing upwards cools down the wires and leaves from R.

For better optimum conditions the cross sectional area A of all the wires was chosen from the plot of the optimized parameter il/A against the resistivity of the copper (Lock[14]). Their cross sectional area A is equal to 0.007 cm which corresponds to 36 wires of 0.0063 of an inch in diameter.

In a superconducting solenoid, once the desired current has been reached, it is possible to complete a superconducting short circuit between the terminals of the coil which traps the current in the coil and makes it possible to disconnect the source of the current.

The switch of the solenoid is one of the above type, a heater type persistent switch based on the above principle.

Thus to energize the solenoid we open the switch by switching on its heater S (fig. 6). It heats a section of the superconducting wire shunt AC, connected across the terminals, to keep its temperature above the critical one. A small power supply E of 150 mA is enough to hold the switch resistive.

The current supplied to the solenoid from the storage battery B through the leads L can be increased slowly to the desired value by the carbon-pile rheostat R while through the terminals of the shunt R_s we monitor its value. As soon as we reach the desired value, we switch off the heater. The shunt cools down below its critical temperature and so the switch is closed. A persistent current now flows in the closed superconducting circuit and the external supply may be disconnected.

MEASURING SYSTEM

The measuring system consists mainly of a potentiometer, a circuit with a battery and a standard resistance of high accuracy, through which we measure the exact resistance of the coil, and the integrating circuit. The latter consists of an electronic integrator, a voltmeter and an automatic recorder. The system with its circuits is illustrated in figure 8.

The pick-up coil's circuit consists of the pick-up coil C and one high accuracy standard resistance R_s of 1486 Ohm. The change of the magnetic flux Φ enclosed in this coil at a time t resulting from the change of the magnetic moment of the specimen induces a voltage V_C across the terminals of the coil C according to the relation[15]

$$V_C = - \frac{d\Phi}{dt} \qquad (2)$$

From this we have

$$\Delta\Phi = -\int_t V_C dt \qquad (3)$$

Hence we can detect the change of the magnetic flux if we know the integral

$$\int_t V_C \cdot dt \qquad (4)$$

The voltage V_C is fed through the switch I, in the switch box W, to the integrator In. As it will be shown, the voltage output V_o of the integrator is proportional to the integral (4) according to the relation

$$Vo = -\frac{1}{RC} \int_t V_C \, dt \quad (5) \quad \text{and so } \Delta\Phi = RCV_o \quad (6)$$

where R is resistance and C capacitance.

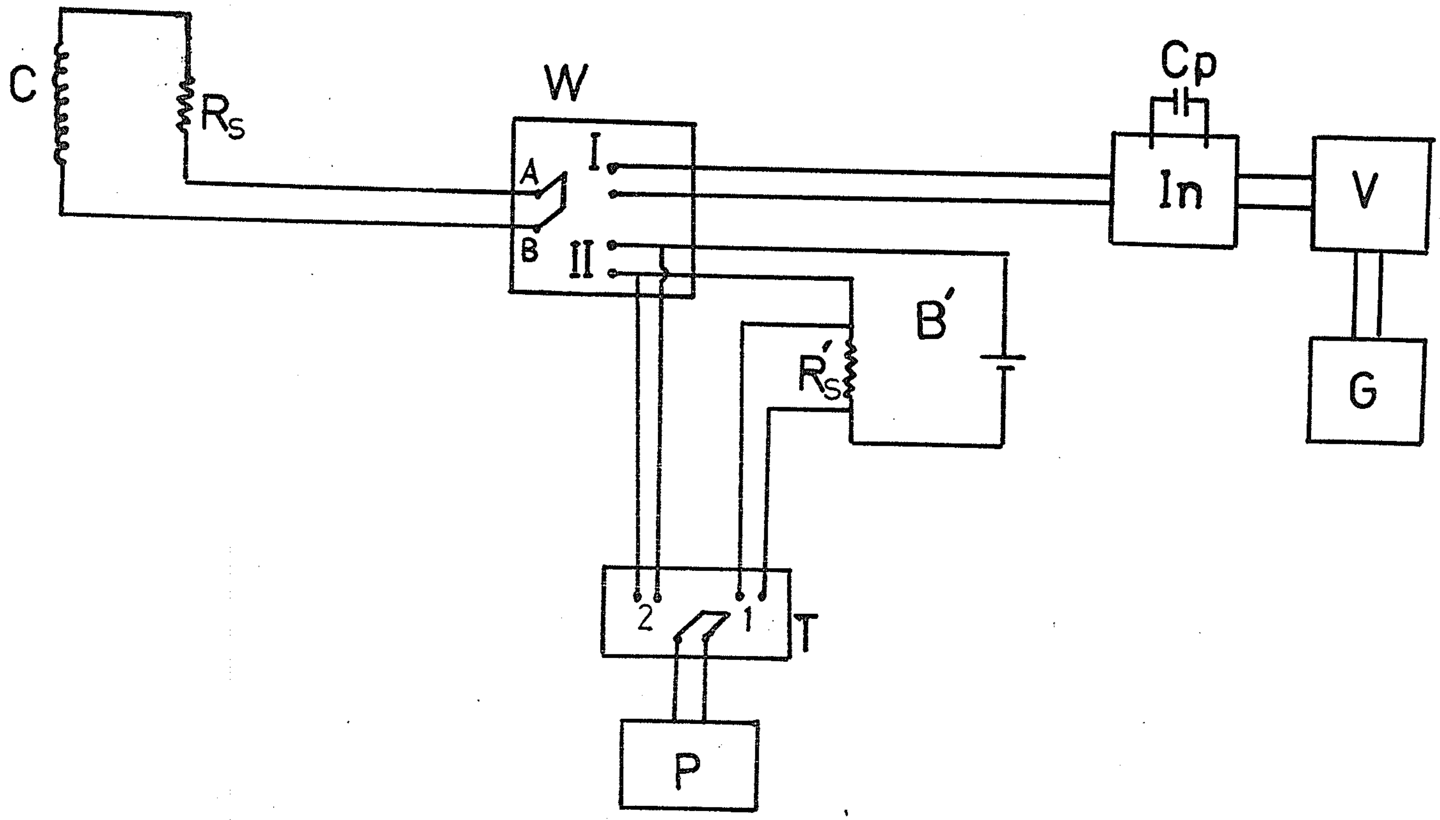


FIG.8

On the other hand the pick-up coil can be connected to a circuit B′ which consists of one of the same kind standard resistance R_s' and a battery. With the potentiometer P and through the selector switch T we can measure the voltage V_{AB} (across the points A and B) and the voltage across the R_s'.

To measure the resistance of the coil, we measure the voltage V_{AB} (position II of the switch in the switch box and position 2 of the selector) and the voltage across the resistance R_s' (position II and 1 respectively). So we calculate the current passing through the R_s' and consequently through the coil. From this current and the V_{AB} we find the resistance of the coil.

Integrator

An electronic integrator consists basically of an amplifier which through its special circuit performs the mathematical operation of integration.

The actual circuit and its equivalent one are shown in fig. 9 and 10. The amplifier A with input voltage V_i and output V_o has an external resistance R and a capacitor C.

The doubleheaded arrow (fig. 10) represents a short circuit (or ground) and implies that the voltage between the two points is zero and no current flows through this short circuit.

For the right branch of the equivalent circuit we take from Ohm's law

$$- V_o - \frac{q}{C} = 0 \qquad (7)$$

where C capacitance and q the electric charge of the capacitor.

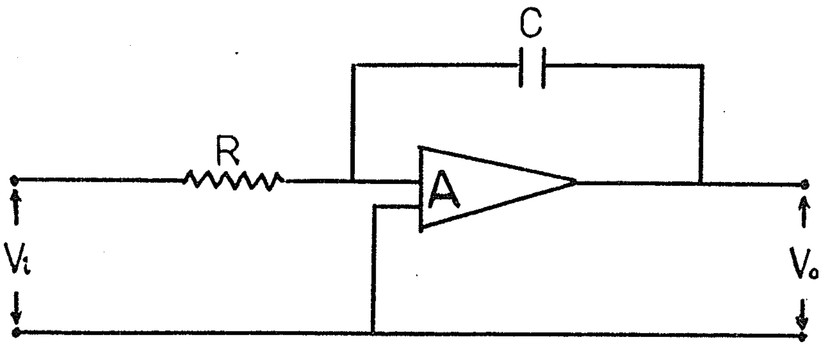


FIG. 9

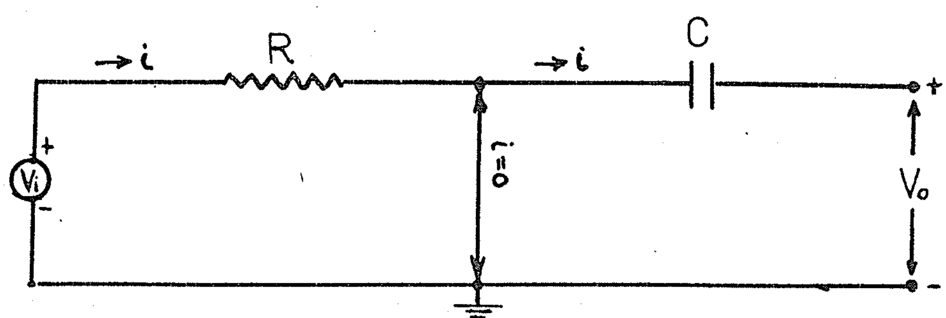


FIG. 10

From the (7) we have

$$q = - C V_o \qquad \text{and} \qquad dq = - C dV_o$$

and since $i = \frac{dq}{dt}$ we have

$$i = - C \frac{dV_o}{dt}$$

and

$$V_o = - \frac{1}{C} \int_t i \, dt \qquad (8)$$

But from the left branch of the equivalent circuit we have

$$i = \frac{V_i}{R}$$

and so

$$V_o = - \frac{1}{CR} \int_t V_i \, dt \qquad (9)$$

The amplifier therefore provides an output voltage proportional to the integral of the input voltage and consequently proportional to the change of the magnetic flux.

With our measuring system (fig. 8) the role of the resistance R of the above relation (9) is played by the resistance R_s and the resistance of the pick-up coil. A capacitor Cp of a capacitance $C = 0.5\ \mu F$ is used. It is obvious that the smaller the capacitance the greater is the value of the output and in this way it can be easily detected. The input voltage V_i is the voltage V_c across the pick-up coil. So the relation (9) is equivalent to the (5).

As it is shown in fig. 8 the integrator is connected to a voltmeter V (where we can read the values of the output voltages) and the latter to an automatic recorder R (where these values are recorded).

TEST FOR ERRORS FROM IMAGE EFFECT

The solenoid which produces the magnetizing field of the specimen is a type II superconductor.

As soon as the solenoid becomes superconducting and its field is smaller than H_{c1}, all the magnetic lines are expelled (Meissner effect) except in a very small depth in the surface (penetration depth).

But the superconducting solenoid works between H_{c1} and H_{c2} and at this state the magnetic flux penetrates it in a greater depth than before. This results in a change of the magnetic flux enclosed by the pick-up coil; the extent of this change and the error it may induce is tested by replacing the specimen by a coil of which the magnetic moment is well known.

SUPERCONDUCTORS AND SUPERCONDUCTING SOLENOIDS

Some Properties and Characteristics

The temperature of the transition from the natural state to the superconducting state is called c r i t i c a l   t e m p - e r a t u r e T_c. Even under this temperature the superconducting behaviour of a superconductor can be changed to the normal conductivity by the application of a value of an external magnetic field. This field is called c r i t i c a l   m a g n e t i c   f i e l d H_c and it varies with the temperature. This field may also be the result of the electric current which flows in the conductor. Actually, the c r i t i c a l   c u r r e n t i_c rather than the H_c is now considered to be fundamental quantity that induces the transition.

For a pure superconductor, the field distribution always corresponds to zero field inside its body independently of the initial conditions. This phenomenon is called M e i s s n e r   e f f e c t. Independently whether we cool down T_c a specimen or decrease the magnetic field below H_c, to reach superconductivity, all the flux penetrating the specimen is suddently expelled.

The magnetic field applied to a superconductor can only penetrate a fine distance into the surface, provided that the surface field nowhere exceeds H_c, according to the relation[16]

$$H_i = H_e . e^{-x/\lambda} \qquad (10)$$

where H_i is the field in the interior of the superconductor, H_e the external field parallel to the surface of the superconductor, x the axis taken in right angles to the surface and λ is known as the penetration length given by the expression

$$\lambda^2 = \frac{m\ c^2}{4\pi n e^2} \qquad (11)$$

where m is electron mass, c velocity of the light, n the density of the superconducting electrons and e the electron charge.

With the Meissner effect we can classify the superconductors into two groups:

i) Type I (or soft) superconductors, most of the pure superconducting elements and many alloys

ii) Type II (or hard) superconductors, usually alloys and intermetalic compounds.

With increasing applied magnetic field, in the Type I superconductors flux penetration takes place only when H_c is reached while in the Type II flux penetration starts at a value H_{c1} but full normal resistance is not restored until a very higher field H_{c2} (IO - IOO times H_{c1}) is reached. Between H_{c1} and H_{c2} the superconductor is in the mixed state where it breaks into a finite number of filaments of alternate superconducting and normal states parallel to the applied field.

The magnetic flux penetrates the superconductor in filaments parallel to the external field which are arranged in a triangular pattern corresponding to minimum free energy. The depth penetration L of the magnetic field H is given by the relation[17]

$$0.4\pi J_c\ L = H \qquad (12)$$

where J_c is the critical current density (A/cm^2). This is in contrast with the field independent penetration depth λ of the relation (10) for the Type I superconductor (or Type II superconductor under the field H_{c_1}). For a critical current density of 10^4 A/cm^2 the above equation gives[18] a penetration depth of approximately 1 mm per KG while the penetration depth from the relation (13) is 10^{-5} to 10^{-4} cm.

The importance of this formation, between H_{cI} and H_{c2}, is that the critical field of these filaments is much greater than that of the bulk superconductor. But the flux penetration is allowed at fields far below the critical field H_{c2}. The properties of the Type II superconductors found application in the superconducting solenoids.

The first wire for superconducting magnet made of Nb_3Sn was developed by Kunzler and his colleagues at the Bell Telephone Laboratories in 1961. Some characteristic effects which arose during the development of the superconducting solenoids are the following.

As the current is increased for the first time the coil will "quench" i.g. return to the normal state at a value of current far below the value expected from measurements on short samples. Secondly the current does not reach its expected critical current and this phenomenon is called "degratiation". Both of the above effects are cured by copper plating and a combination of polyester and epoxy insulation of the wire.

Another pecularity is the phenomenon that the field produced rises in a series of small abrupt steps which are known as "flux jumps". Also in some superconducting coils the current required to "quench" the coil can be made to increase by repeatedly cycling the current from zero to the quenching value. This behaviour is known as "training".

Generally there are different approaches to the above problems which help to solve them.

THE EXPERIMENT

In the experiment, in order to test the error induced by the superconducting solenoid (image effect) we replace the specimen with a small coil (simulator coil) of which we know the exact magnetic moment from the current which passes through it.

The replacement does not change many things in the cryostat, as it is shown in figure 11. The copper rod I is replaced by another shorter in length because we do not use a thermometer. The rod ends with two disk-shaped "swallows" Z in the middle space of which its diameter is 3/22 of an inch. These "swallows" hold the simulator coil S which is wound on the rod directly.

This coil consists of 2,500 turns of 0.002 of an inch diameter copper wire and its length is $\frac{1}{4}$ of an inch. Its exterior diameter of 7.11 mm is almost equal to the inside diameter of the tube U (without touching it) for better inductive coupling. The coil is aligned so that its axis coincides with the common axis of the pick-up coil and the solenoid, and its center is the common center of the latter two.

The circuit of this coil and its connection to the measuring system is shown in fig. 12. Its circuit is connected to the selector switch T so that through the potentiometer we can find the exact current i_s which passes through it. It consists of a battery, a variable resistance R and two standard resistances R_1 and R_2 of 1 ohm and 10 ohm respectively (the fact that we use two depends on the ability of the potentiometer to read voltages up to 100 mV).

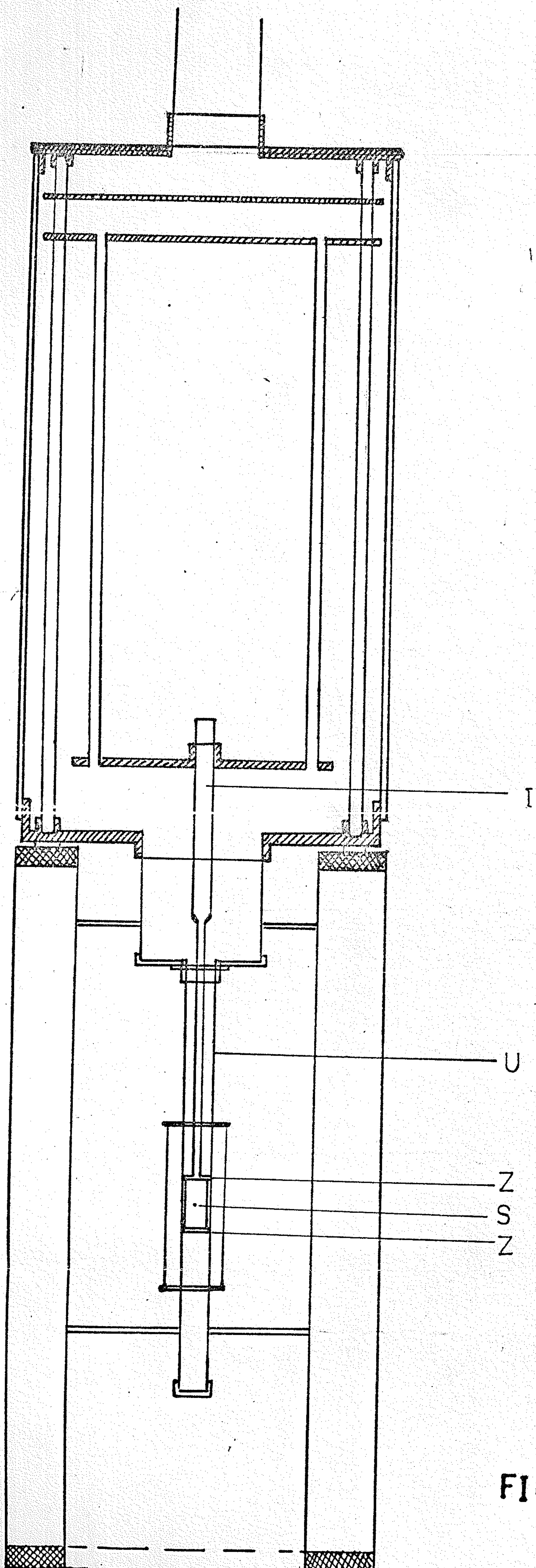


FIG. 11

Using the variable resistance R we can change the current i_s passing through the coil and consequently the magnetic moment of it. Therefore, for different values of current i_s we take different values of voltage V_{AB} (across the terminals A and B) and consequently different values of integrator voltage output V_o.

We measure the current i_s by measuring the voltage across the standard resistance R_1 or R_2 with the potentiometer P (position 3 or 4 of the selector switch T).

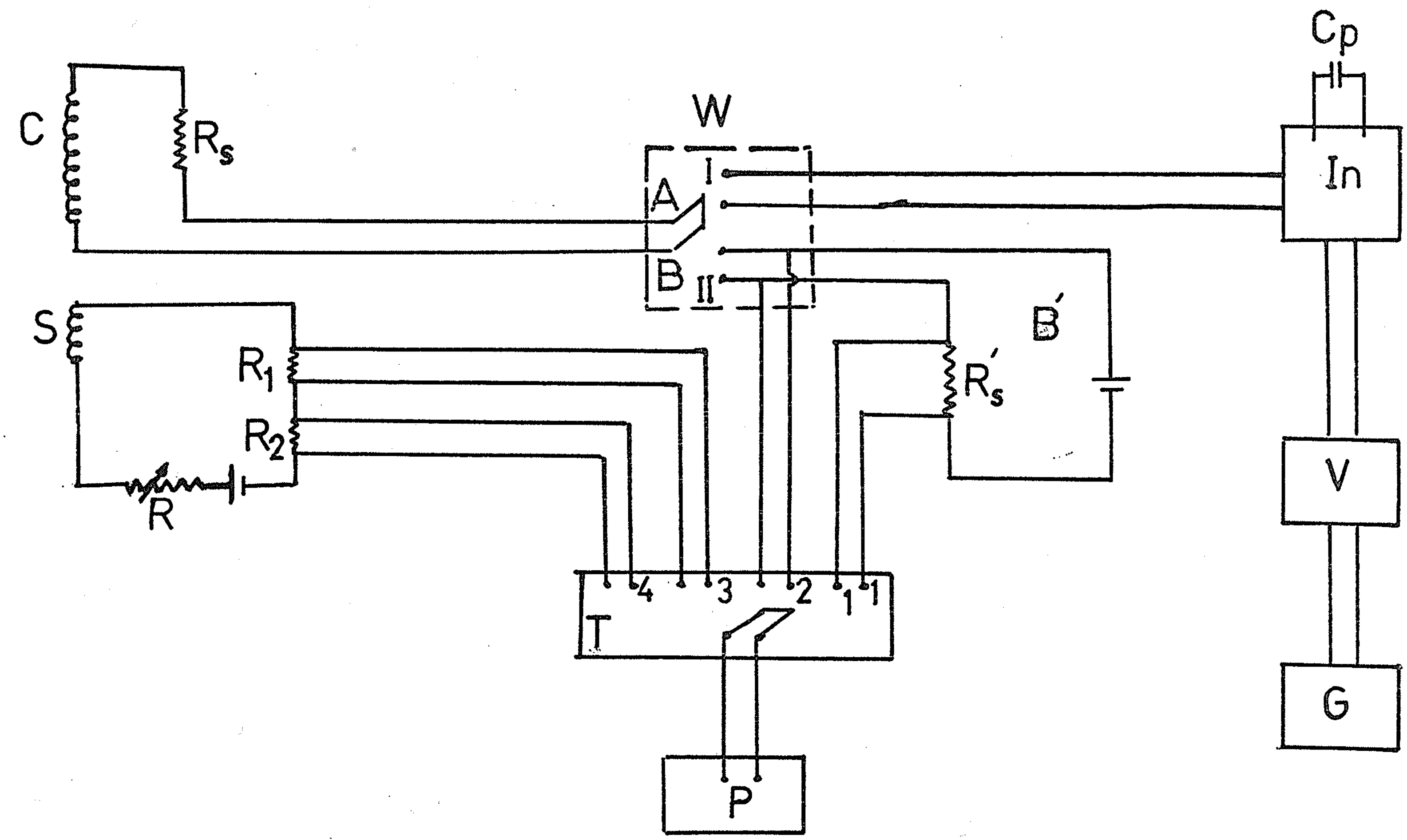


FIG. 12

MEASUREMENTS AND RESULTS

For a certain value of the current i_s passing through the simulator coil we have a magnetic moment m given by the relation

$$m = \frac{i_s \times n \times \pi}{12} \times (d_1^2 + d_1 d_2 + d_2^2) \qquad (13)$$

where d_1 and d_2 the inside and outside diameter of the coil, n the number of turns.

Every change of the current i_s results in a change of the magnetic flux penetrating the pick-up coil and consequently induces a voltage impulse V_o detected by the integrator.

The experimental values of i_s and the voltage output V_o of the integrator are tabulated in table I. The relation $V_o' = f(i_s)$ is illustrated in fig. 13, for room temperature, and in fig. 14 for helium temperature, where $V_o' = \Sigma V_o$.

According to the relation (6) where $\Delta\phi = RCV_o$ and taking into account that we have $R = 3845.2\,\Omega$ in room temperature, and $R_o = 1522.0\,\Omega$ for helium temperature, and $C = 0.5\,\mu F$ we take

$RC = 3845.2 \times 0.5 \times 10^{-6} = 1923 \times 10^{-6}$ sec - room temperature

$R_oC = 1522.0 \times 0.5 \times 10^{-6} = 761 \times 10^{-6}$ sec - helium temperature

The slopes of the straight lines of the graphs (fig. 13, 14) were found, by the method of leasts squares, equal to

$$\left(\frac{dV_o'}{di_s}\right)_r = 10.01 \text{ mV/mA} \qquad (14) \text{ - room temperature}$$

$$\left(\frac{dV_o'}{di_s}\right)_o = 26.22 \text{ mV/ma} \qquad (15) \text{ - helium temperature}$$

and from (6) and (14), (15) we get

$$\left(\frac{d\phi}{di_s}\right)_r = RC \frac{dV_o}{di_s} = 1923 \times 10^{-6} \times 10.01 \text{ mV.sec/mA} = 19.24 \times 10^2 \text{emu-flux/mA} \quad (16)$$

and

$$\left(\frac{d\phi}{di_s}\right)_o = 19.95 \times 10^2 \text{ emu-flux/ma} \quad (17)$$

From (13) we have

$$\frac{dm}{di_s} = \frac{n \times \pi}{12} \times (d_1^2 + d_1 d_2 + d_2^2)$$

and for n = 2,500 turns and $d_1 = 0.238$ cm and $d_2 = 0.711$ cm, we get

$$\frac{dm}{di_s} = 0.048 \text{ emu-moment/mA} \quad (18)$$

and dividing (16) and (17) by (18) we find

$$\left(\frac{d\phi}{dm}\right)_v = 401.80 \times 10^2 \text{ emu-flux/emu-moment}$$

$$\left(\frac{d\phi}{dm}\right)_o = 417.01 \times 10^2 \text{ emu-flux/emu-moment}$$

and

$$\left(\frac{d\phi}{dm}\right)_o - \left(\frac{d\phi}{dm}\right)_r = 15.21 \times 10^2 \text{ emu-flux/emu-moment}$$

and

$$\frac{\left(\frac{d\phi}{dm}\right)_o - \left(\frac{d\phi}{dm}\right)_r}{\left(\frac{d\phi}{dm}\right)_r} = 0.04$$

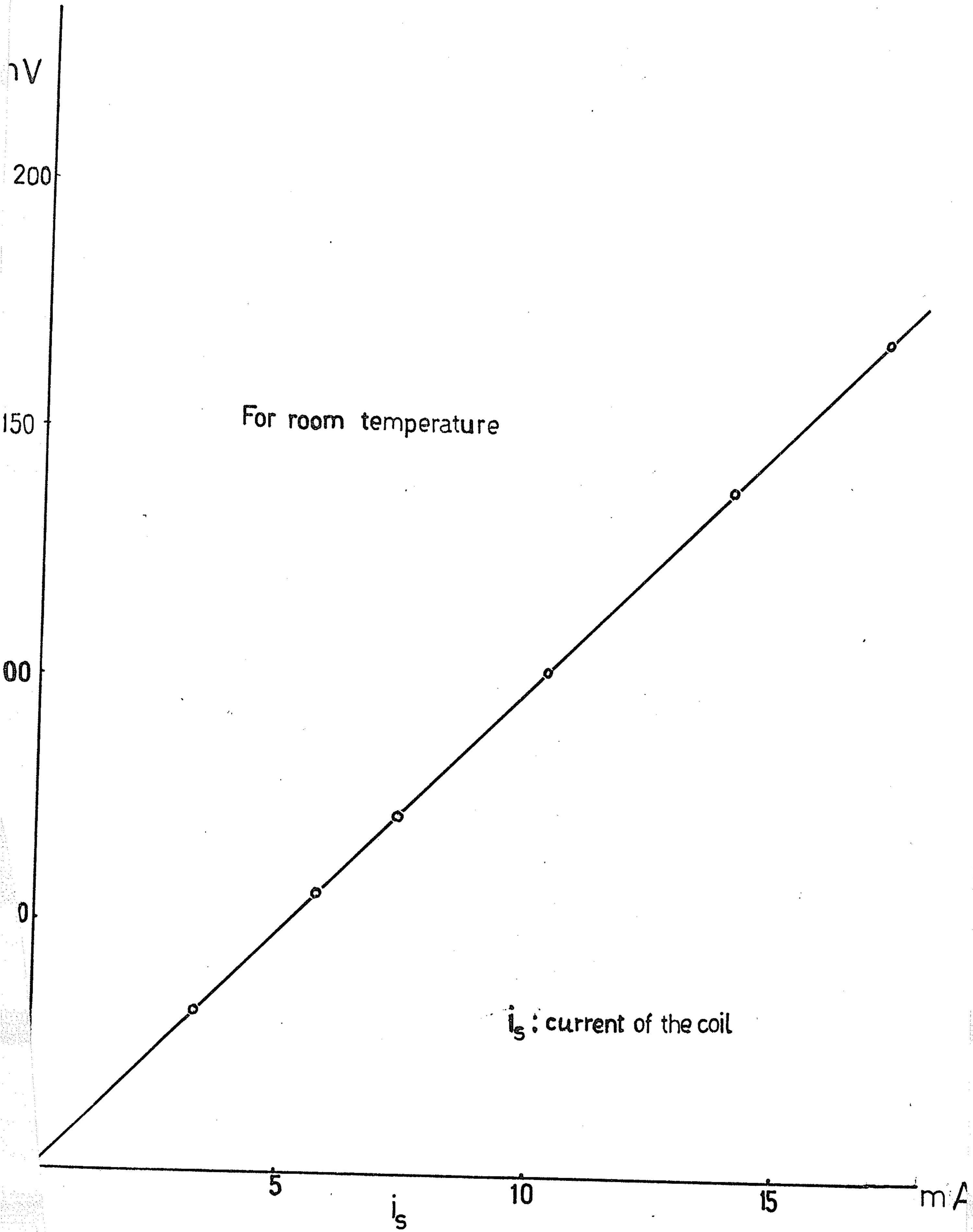
TABLE I

Room Temperature

i_s mA	V_o mV	V_o' mV
3.285	32.07	32.07
5.718	24.35	56.42
7.350	16.20	72.62
10.278	29.10	101.72
13.990	37.54	139.26
17.055	30.54	169.84

Helium Temperature

i_s mA	V_o mV	V_o' mV
3.825	100.70	100.70
7.603	95.60	196.30
10.803	84.00	280.30
15.006	110.25	390.55
19.580	120.10	510.65
24.580	131.80	642.45
29.245	122.80	765.25
33.000	98.70	863.95



nV
200
150
00
0
For room temperature
i_s : current of the coil
5
10
15
mA
i_s

FIG. 13

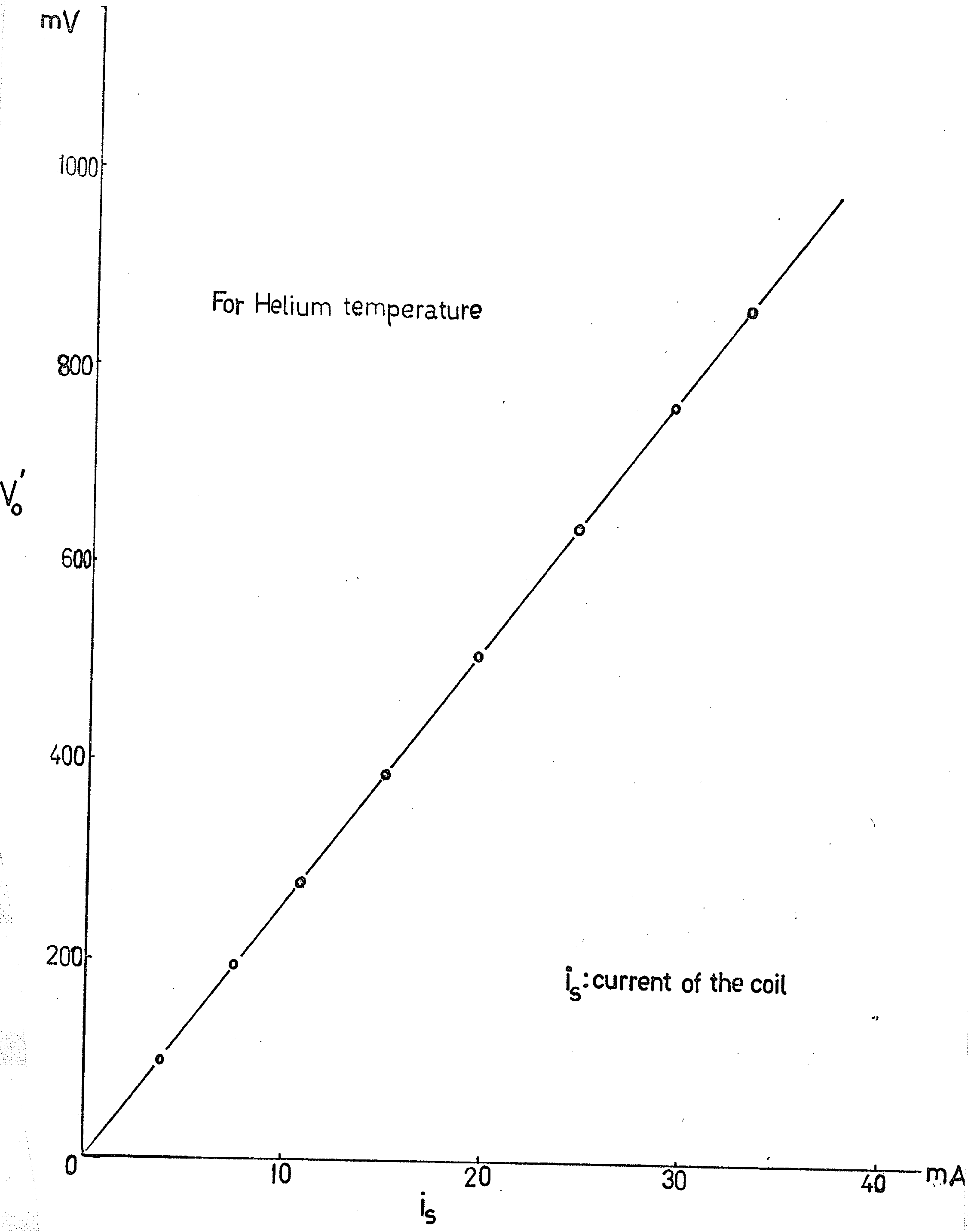

mV
1000
800
600
400
200
0
V_o'
For Helium temperature
i_s: current of the coil
10
20
30
40
mA
i_s

FIG. 14

Therefore a change 4% of the magnetic flux penetrating the pick-up coil per moment is the result of the expulsion of a part of the flux from the solenoid when it is superconducting.

DISCUSSION

At room temperature the magnetic lines of the specimen (or the coil) penetrate the pick-up coil and the solenoid (not superconducting).

The relation between the magnetic flux Φ enclosed by the pick-up coil and the magnetic moment m of the specimen is given by the linear relation

$$\Phi = \frac{m}{2}\frac{n}{b-a}\log\left\{\frac{b+(b^2+l^2)^{1/2}}{a+(a^2+b^2)^{1/2}}\right\}$$

where n is the number of the turns , l the length of the pick-up coil and a and b the inside and outside diameter of it. But since Φ and m are related to the quantities V_0 and i_s through the linear relations (6) and (13) respectively, we expect the relation $V_0' = f(i_s)$ to be linear too.

We have applied the method of least squares in order to determine the deviations of the experimental points from the straight line and the results for room temperature are tabulated in table II. As we see the maximum deviation from the straight line is $\delta V_0' = -0.257$ mV which gives a maximum deviation of magnetic moment equal to $\delta m = 12.30 \times 10^{-4}$ emu of magnetic moment.

The main source of error during our measurements was this induced by the electronic noise which influenced the measured values of the output V_0 of the integrator. The error was estimated equal to 0.2 mV for room temperature and 0.7 mV for helium temperature. For both measurements this error is no greater than 1% of the measured value of impulse voltage.

Hence, for room temperature, the magnitude of the deviations of the experimental points (as they are shown in table II show that they can easily be justified as experimental errors.

For helium temperature, when the solenoid is superconducting, a part of the magnetic flux of the specimen (or the coil replacing it) is expelled from the solenoid and penetrates it only in a depth L the value of which depends on the value of the field and it is given by the relation (12). Since we are interested in differences of magnetic moment this does not influence the linearity of the relation $V_o' = f(i_s)$.

But the relation (12) is valid with the restriction that the magnetic lines outside of the superconductor are parallel to its surface, something which does not happen in our case. On the contrary the magnetic field penetrates the superconductor's surface in different angles.

The results from the application of the method of leasts squares to test the linearity of the relation $V_o' = f(i_s)$ for helium temperature are tabulated in table III and the plot of the deviations of the different points against the values of V_o' is shown in fig. 15.

We see that for the first value of current $i_s = 3.825$ mA we have a deviation $\delta V_o = 2.385$ mV, of larger magnitude than these which follow. We believe that this large deviation was caused by the experimental way in which this value was obtained. While the circuit was open ($i_s = 0$) we closed it in a manual way by connecting two terminals and the current $i_s = 3.825$ mV passed through. On the contrary, the other values of current were obtained by using a variable resistance.

The deviations of the other points follow a regular change as is

TABLE II

Room Temperature

i_s mA	m emu-moment	V'_o mV	$\delta V'_o$ mV	m emu-moment
3.285	0.157	32.07	0.078	3.7×10^{-4}
5.718	0.274	56.42	0.079	3.7×10^{-4}
7.350	0.352	72.62	-0.053	-2.5×10^{-4}
10.278	0.492	101.72	-0.257	-12.3×10^{-4}
13.990	0.669	139.26	0.133	-6.3×10^{-4}
17.055	0.816	169.84	0.038	1.8×10^{-4}

TABLE III

Helium Temperature

i_s mA	m emu-moment	V'_o mV	$\delta V'_o$ mV	m emu-moment
3.825	0.183	100.70	+2.385	4.3×10^{-3}
7.603	0.364	196.30	-1.066	-1.9×10^{-3}
10.803	0.517	280.30	-0.964	-1.8×10^{-3}
15.006	0.718	390.55	-0.908	-1.6×10^{-3}
19.580	0.937	510.65	-0.729	-1.3×10^{-3}
24.580	1.176	642.45	-0.019	0
29.245	1.399	765.25	+0.474	0.8×10^{-3}
33.000	1.579	863.95	+0.725	1.[illegible] $\times 10^{-3}$

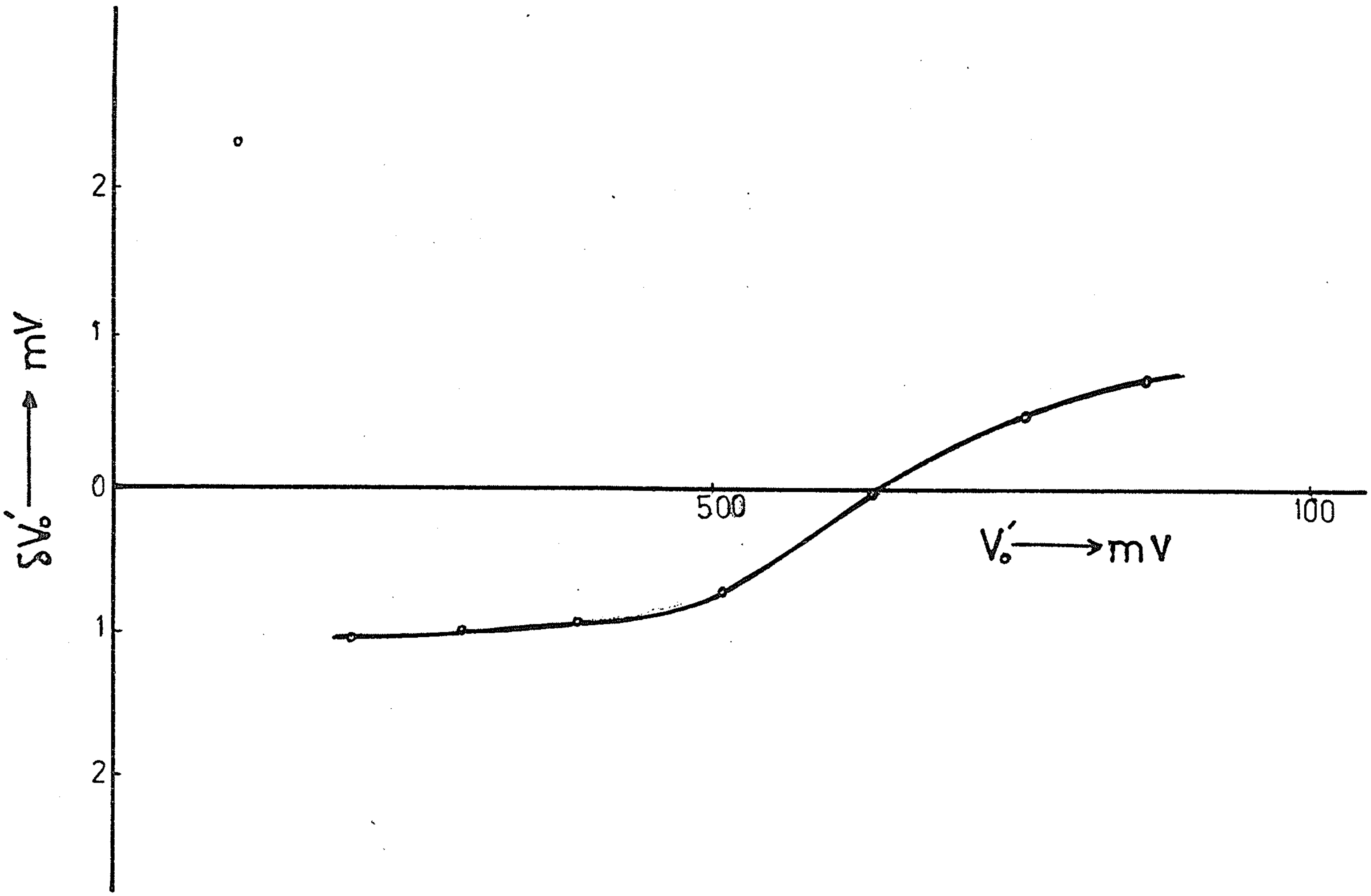


FIG. 15

shown in fig. 15. But in spite of this regularity no conclusions can be derived about the exact relation between V_o' and i_s and consequently between the magnetic flux enclosed by the pick-up coil and the magnetic moment of the small coil (or specimen) when the solenoid is superconducting because the values of the last four experimental points lie in the range of the experimental error while the deviations of the first three (except the first one) are very close to this range.

REFERENCES

1) A. G. Chynoweth, J. Appl. Physics, 29, 503 (1958)

2) E. W. Pugh and B. E. Argyle, Suppl. J. Appl. Phys., 32, 334 S (1961)

3) E. W. Pugh and B. E. Argyle, Suppl. J. Appl. Phys., 33, 1178 (1962)

4) B. E. Argyle, S. H. Charap and E. W. Pugh, Phys. Rev., 132, 2051 (1963)

5) S. Foner and E. D. Thomson, J. App. Phys., 30, 229 S (1959)

6) A. H. Cooke, K. A. Gehring and R. Lanzeby, Proc. Phys. Soc. 85, 967 (1965)

7) D. K. Finnemore, T. F. Stromberg and C. A. Swenson, Phys. Rev., 149, 231 (1966)

8) W. N. R. Stevens, J. of Sc. Instr. Vol I, Ser. 2, 1188 (1968)

9) F. Keffer, Encycl. of Phys., Vol XVIII/2, p. 9.

10) F. Keffer, Encycl. of Phys., Vol XVIII/2, p. 24.

11) F. Keffer, Encycl. of Phys., Vol XVIII/2, p. 24.

12) R. McFee, Rev. Sc. Instrum. 30, 98 (1959)

13) W. Mercourof, Cryogenics, 3, 171 (1963)

14) J. M. Lock, Cryogenics, 9, 48 (1969)

15) H. E. Duckworth, "Electricity and Magnetism" (1960), p. 232.

16) V. L. Newhouse, "Applied Superconductivity" (1964), p. 19.

17) V. L. Newhouse, "Applied Superconductivity" (1964), p. 213.

18) V. L.Newhouse, "Applied Superconductivity" (1964), p. 285.