ANALYSIS OF RIGID FRAMES
BY THE METHOD OF APPROXIMATION

A Dissertation

Presented to

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Master of Science

by

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CONTENTS

INTRODUCTION

A progressively increasing demand for multi-storey and multi-bay buildings makes it imperative that a simple as well as a reliable method is devised for the analysis and design of these complex structures. To cope with such an unprecedented demand the method must have, as its outstanding feature, the quality of being the least time-consuming.

Many factors contribute to this acute need for tall buildings, among which may be listed: population explosion, overcrowding in big cities, industrial expansion, a very rapid pace of development in underdeveloped countries etc. Skyscrapers seem to be the only solution at the moment to cope with an increasing demand for floor area, where the land area must remain the same. Also a certain school of city and town planners favour the construction of tall buildings with open spaces in between as a basis of the concept of the Garden Cities of the future. Thus it is obvious that any solution which would alleviate the above situation in the minimum possible time would be very welcome at this juncture.

The publication of the Hardy Cross method in 1932 enabled the structural engineers to tackle the complicated and till then, relatively forbidding problem of analysing complex multi-storey and multi-bay rigid frames on a rational and convenient basis. [illegible]

[illegible]

[illegible]

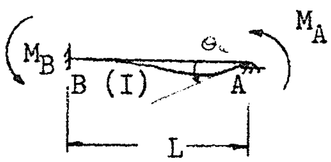
rational and convenient basis. Till then the analysis was confined to empirical formulas based on experience or experimental models which proved successful. Since then, these various other methods and modifications have been introduced, all designed to make the analysis and design of these structures relatively simple and as accurate as desired or designated. This has been instrumental in enabling architects to design bold and challenging structures unhampered by structural limitations thus making possible the modern day wonders in building construction too numerous and well known to be listed here.

This thesis deals with the same problem of analysis in yet another way, incorporating all the desirable features mentioned before. Broadly speaking, it is a combination of the Cantilever Moment Distribution Method and Morriss Method, and as the title suggests is essentially an approximation method of analysis.

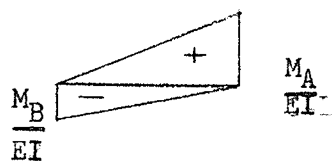
The second topic dealt with in this thesis is the analysis of the Vierendeel truss. Vierendell trusses are very useful for long spans and wherever greater openings for utilities are required. They find increasing use in garages and supermarkets and also in bridge construction to reduce the effect of lateral wind pressures.

GENERAL DESCRIPTIONS OF THE MOMENT DISTRIBUTION METHOD

Rotational stiffness = A counter-clockwise moment is applied at the simple support member of a straight member of constant cross section fixed at one end and simply supported at the other end.



In order to find the angle of rotation θ the conjugate-beam method can be applied.



$$\Delta_a = 0 = \frac{1}{EI}\left(\frac{M_A L}{2}\cdot\frac{L}{3} - \frac{M_B L}{2}\cdot\frac{2L}{3}\right)$$

$$= \frac{L^2}{3EI}\left(\frac{M_A}{2} - M_B\right)$$

$$M_A = 2M_B$$

$$\theta_a = \frac{1}{EI}\left(\frac{M_A L}{2}\cdot\frac{2}{3} - \frac{M_B L}{2}\cdot\frac{1}{3}\right)$$

$$= \frac{L}{3EI}\left(M_A - \frac{M_B}{2}\right)$$

$$= \frac{L}{3EI}\left(M_A - \frac{M_A}{4}\right) = \frac{M_A L}{4EI}$$

$$M_A = \frac{4EI}{L}\theta_a$$

when $\theta_a = 1$

$\frac{4EI}{L}$ is called stiffness factor

$$M_B = CM_A$$

$$C = \frac{M_B}{M_A} = \frac{1}{2}$$

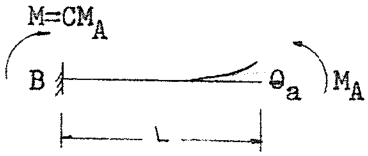
The factor "C" is called carry over factor

where I = moment of inertia of the uniform cross section of the beam

E = modulus of elasticity of the beam

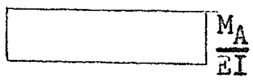
$\triangle_a$ = deflection at point A

θ_a = angle of rotation at point A



A counterclockwise moment is applied at the free end of a cantilever member of a straight member of constant cross section.

In order to find the angle of rotation the area-moment method can be applied.



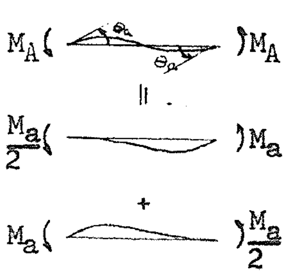
$\theta_a = \frac{M_A L}{EI}$ Therefore $\frac{EI\theta_a}{L} = M_A$ where $\frac{EI}{L}$ is the stiffness factor for the cantilever beam.

$M_B = -CM_A$

$C = -\frac{M_B}{M_A} = -1.$

Carry-over factor for cantilever beam

<u>Rotation stiffness for antisymmetry loading</u>.



$M_A = \frac{4EI}{L}\theta_a$ See page 3

$$M_{AB} = M_a + \frac{M_a}{2} = \frac{3M_a}{2}$$

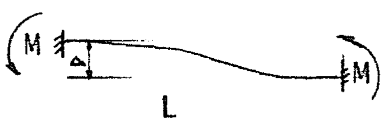
$$= \frac{4EI}{L}\cdot\frac{3}{2}\theta_a$$

$$M_{AB} = 6\frac{EL}{I}\theta_a$$

$6\frac{EL}{L}$ = rotational stiffness

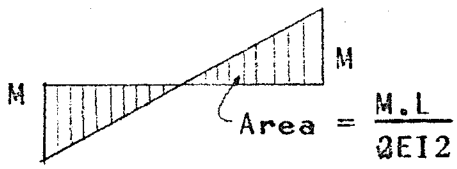
Lateral stiffness

$\triangle$ can be found by conjugate-beam method.



$$\triangle = \frac{ML}{4EI} \quad \frac{L}{6} - \left(\frac{L}{2} + \frac{2}{3}\,\frac{L}{2}\right)$$

$$= \frac{ML^2}{4EI}\left(\frac{1}{6} - \frac{5}{6}\right) = -\frac{ML^2}{6EI}$$



$$M = \frac{6EI}{L^2}\,\triangle$$

when $\triangle = 1$

$$\frac{6EI}{L^2} = \text{lateral stiffness}$$

Approximate analysis of wind stresses

Assumptions:

(1) Any one mult-story and multi-bay rigid frame can be modified to a symmetrical, single-bay, multi-story frame such as shown in Fig. 1 and Fig. 2.

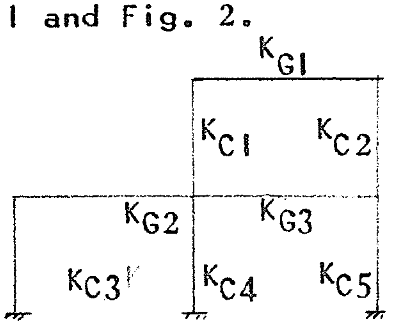


Fig. 1
Original frame

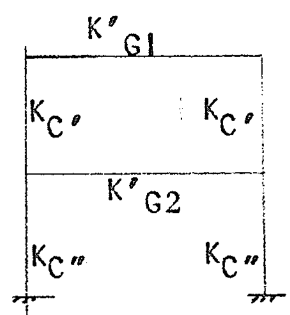


Fig. 2
Modified frame

(2) Any one K-value of any column of any story of the modified frame Fig. 2 is equal to the half of the sum of the K-values of the corresponding story in the original frame.

Namely $K'_C = \dfrac{K_{CI} + K_{C2}}{2}$

$$K''_C = \frac{K_{C3} + K_{C4} + K_{C5}}{2}$$

(3) Any one K-value of the girder of the modified frame is equal to the sum of the K-values of the girder of the corresponding floor.

Namely $K'_{G1} = K_{G1}$

$$K'_{G2} = K_{G2} + K_{G3}$$

(4) The sum of any two upper (or lower) columns' end moments of any story in the modified frame is equal to the sum of the upper (or lower) columns' end moments of the corresponding story in the original frame.

(5) The end moment of the left (or right) hand side of the girder of any floor of the modified frame is equal to the sum of the end moments of the left (or right) hand side of the girders in the corresponding floor of the original frame.

Cantilever moment distribution method

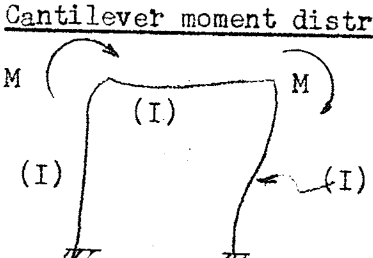


Fig. 1

Fig. 1 shows the distortion of a symmetrical bent that moves freely under applied corner moments Σ M. Each column has a moment diagram which corresponds to a free cantilever beam.

Since identical columns have the same lateral deflections under equally applied moments.

For such a cantilever the moment carry-over factor (C) to the fixed end is -1.

Fig. 2. The slope θ caused by a moment M, is $\frac{ML}{EI}$ as contrasted to $\frac{ML}{4EI}$ in Fig. 3.

L is the length of column

I is the moment of inertia of the uniform cross section of the column, and

E is the modulus of elasticity of the column.

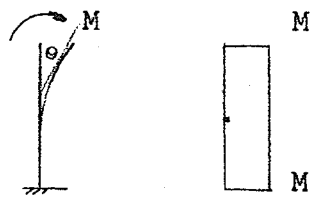


Fig. 2' Cantilever moment

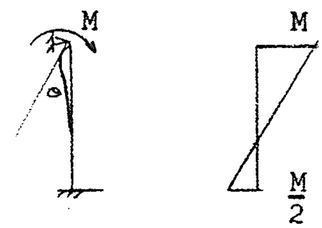


Fig. 3' Rotation without translation

The top member for anti-symmetrical moments caused by side sway has a stiffness, a resistance to equal end rotation, that is 1.5 times greater than its resistance to rotation at one end when the other end is fixed.

Morris method of moment distribution

After writing down the fixed end moment in the columns of each story, the steps, to be repeated through two or more cycles, are as follows:

(1) Distribute the moments at the joints.

(2) Carry-over the distributed moments at the ends of each member using a carry-over factor (c) of $+\frac{1}{2}$.

(3) Balance the columns moments in each story by making their sum equal the shear in the story times the story height. This assumes the joints to be fixed against rotation, but free to deflect laterally.

Steps 1, 2 and 3 complete a cycle, which may be repeated as many times as the desired degree of accuracy may require.

<u>Procedures</u>:

(1) Making any one irregular rectangular frame as a symmetrical regular rectangular frame.

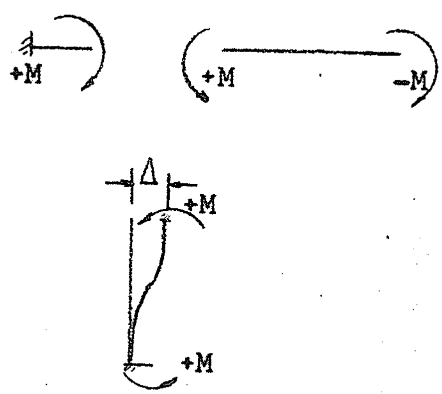
(2) Write down the fixed end moments by the columns using Cantilever moment distribution method to analyse the modified symmetrical frame.

(3) Using the final end moments from step (2), redistribute the end moments to the original frame in accordance with their relative stiffness which becomes the fixed end moments in the original frame.

(4) Applying Morris method to analyse the original frame. In this case only one cycle is needed.

The details of computation are shown in the following examples.

<u>Sign convention</u>: Clockwise moment acting on the joint being considered as positive.



Example 1. One-story frame as shown is fixed at the bottom and K-values are shown in brackets opposite the members. The frame is symmetrical about a vertical center line. The wind force acts toward the right as shown.

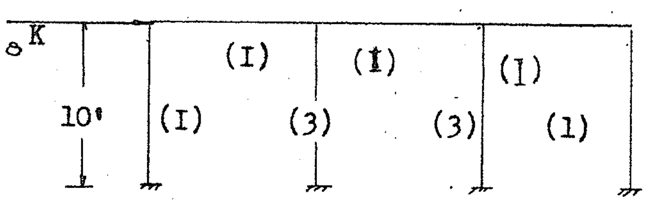


Fig. 1-A

Solve by Cross Method

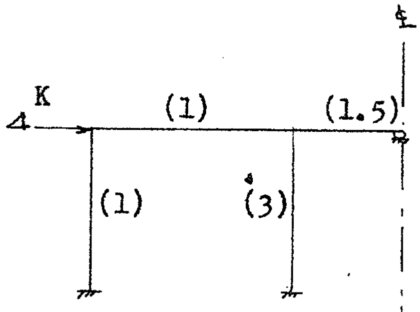


Fig. 1-B

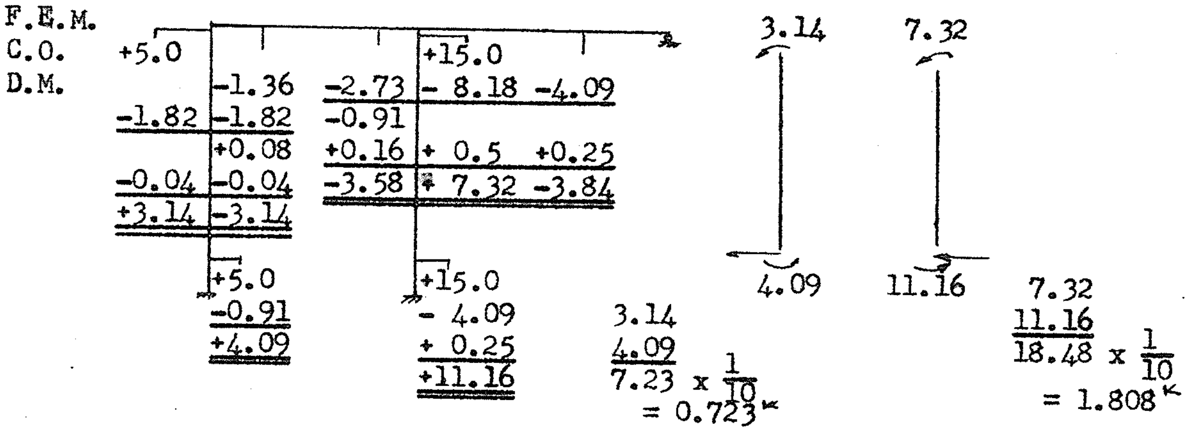


Fig. 1-C

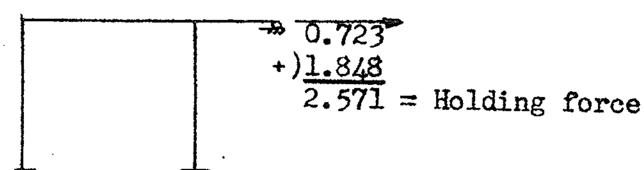


Fig. 1-D

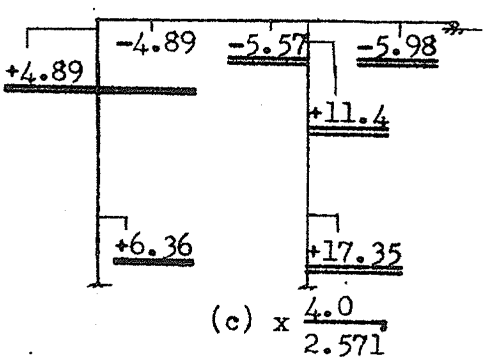


Fig. 1-E. Final End Moments

End moments are found by means of the approximate method of moment distribution.

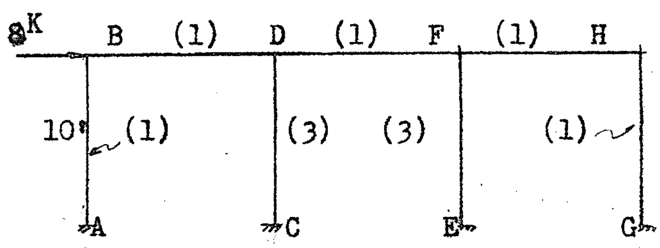


Fig. 1-A

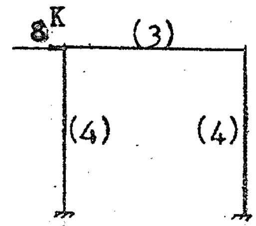


Fig. 1-F

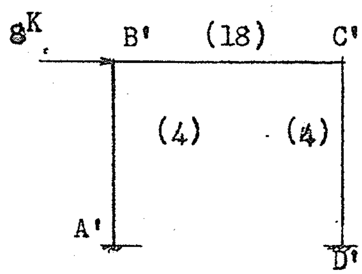


Fig. 1-G

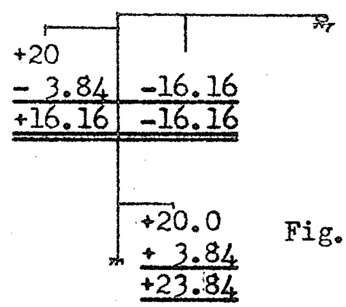


Fig. 1-H

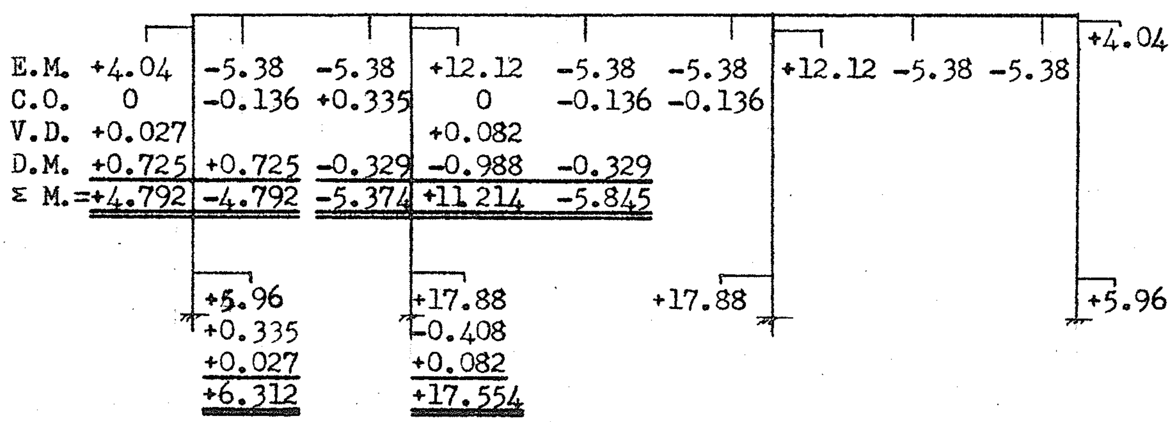


Fig. 1-I Shear and Moment distribution

(The moments resulting from one cycle moment distribution by "Morris Method")

End Mom. corresp. to K-values $\left(\begin{array}{l} +4.04 = 16.16 \times 2 \times \frac{1}{1+3+3+1} \\ \\ -5.38 = -16.16 \times \frac{1}{1+1+1} \end{array}\right.$

Carry-over $-0.136 = -\left(-5.38 + 12.12 - 5.38\right) \times \frac{1}{1+3+1} \times \frac{1}{2}$

Shear dist. $+0.027 = -\left(0+0.335+0-0.408\right) \times 2 \times 3 \times \frac{1}{1+3+3+1} \times \frac{1}{2}$

Details of Calculation

The actual numerical calculations are indicated in detail in Fig. 1-H and Fig. 1-I. The procedure is as follows:

(1) Modifying frame Fig. 1-A to Fig. 1-F which is a symmetrical frame. The girder is considered hinged at mid-span. Then one-half the structure can be considered.

(2) Recognizing the fact that the rotation at top of left column must be equal to that at top of right column in magnitude and direction, the K-value for girder is modified by multiplying by 6. The K-values are as shown in Fig. 1-G. On account of symmetry, the shear is equally distributed between the two columns, the end moments at the top (or bottom) of any column is determined by multiplying the shear in the column by one half the story height.

$$M = 4 \times 5 = 20^{K\text{-}ft.}$$

(3) Distribute the unbalanced moment at joint B' $-\frac{4}{4+18} \times 20 = -3.84$ will act on B'A', $-\frac{18}{4+18} \times 20 = -16.16$ will act on B'C'. When B'A' received a moment of -3.84, all of opposite sign or +3.84 is carried over to the fixed end column. There is nothing carried over along the girder from joint B' to C' (or C to B), on the right half of the frame, since on account of symmetry the rotation at the right end remains equal to the left.

(4) All final end moments will be added algebraically as shown in Fig. 1-H.

(5) Redistributing those end moments from Fig. 1-H to the original frame according to their relative K-values.

For columns:

$$\left.\begin{aligned} +4.04 &= +16.16 \times 2 \times \frac{1}{1+3+3+1} \\ +12.12 &= +16.16 \times 2 \times \frac{3}{1+3+3+1} \end{aligned}\right) \quad \text{For the top of columns}$$

$$\left.\begin{aligned} +5.96 &= +23.84 \times 2 \times \frac{1}{1+3+3+1} \\ +17.88 &= +23.84 \times 2 \times \frac{3}{1+3+3+1} \end{aligned}\right) \quad \text{For the bottom of columns}$$

For girders:

$$-5.38 = -16.16 \times \frac{1}{1+1+1} \qquad \text{At left end of girder}$$

$$-5.38 = -16.16 \times \frac{1}{1+1+1} \qquad \text{At right end of girder}$$

(6) After writing down the end moments in the girders and columns, distribute the unbalanced moments with the opposite sign at each joint, but for convenience write only the portion carried over to the other end of the rotating member. The amount carried over is one-half of the distributed moment. Such as $-0.136 = -(-5.38 + 12.12 - 5.38)\frac{1}{1+3+1} \times \frac{1}{2}$. Since the frame is fixed at the base, the rotation there is zero, hence there is no moment carried over to the top of the columns.

(7) The moments lost in the columns by distribution which should be added to adjust for shears, are the distributed moments plus the moments carried over, or three times the sum of the moments carried over, and distributing this sum, with sign reversed, in proportion to the $^{I}/L^{2}$ values half at

each end of a column. Such as $+0.027 = -3\left(0 + 0.335 + 0 - 0.408\right) \times 2 \times \frac{1}{1+3+3+1} \times \frac{1}{2}$.

At the end of the procedure we may find the totals of the original end moments and the carried over moments, and the moments of correction, and distribute the unbalanced total at each joint. All final end moments will be added algebraically as shown on Fig. 1-I.

Shear check

The sum of the final end moments in all columns of a story must equal the sum of the original fixed end moments in this story.

$$\begin{array}{l} +\ 4.792 \\ +\ 6.312 \\ +11.214 \\ \underline{+17.554} \\ +39.872 \times 2 \fallingdotseq -80^{K.FT.} = 8 \times 10 \end{array}$$

O.K.

The moments in Fig. 1-E compared with in Fig. 1-I within 4%.

The procedure in the following examples is the same.

Example 2

Two-story frame with lateral loads as shown is fixed at the bottom and the K-values are shown in brackets opposite the members. The frame is not symmetrical.[1]

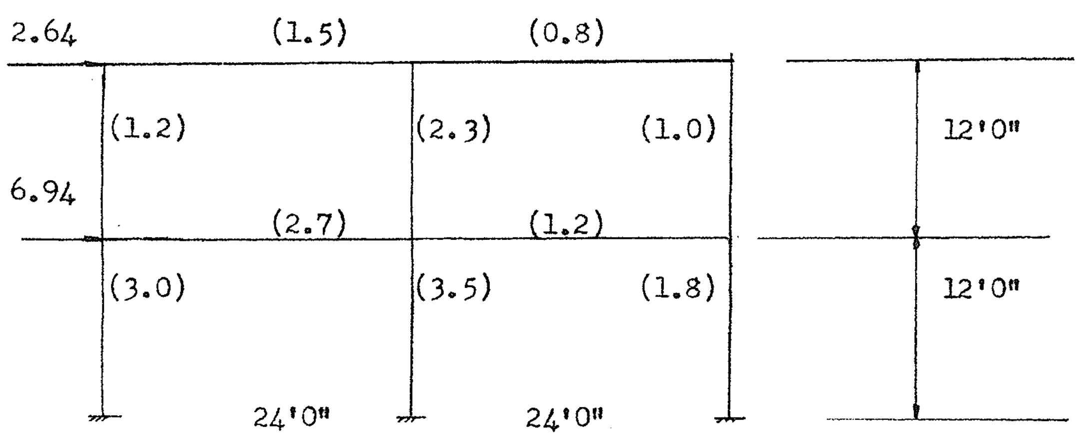
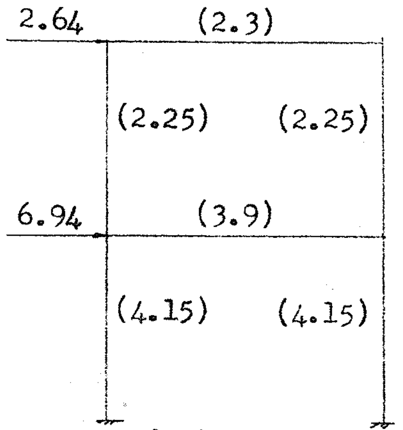


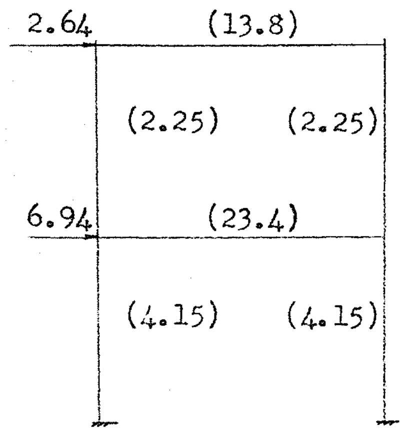
Fig. 2-A

(1) See Reference (11) "Analysis of statically indeterminate structures" By Parcel and Moorman, 1955, p.383-388.



Modified structure

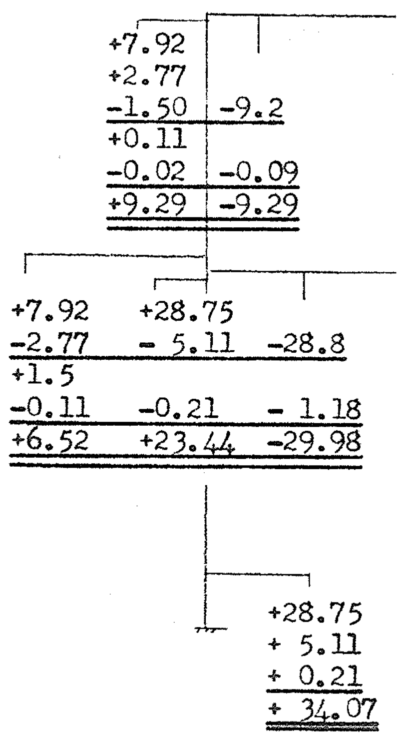
Fig.2-B



Modified K-values

Fig.2-C

Fig.2-D gives the moments resulting from two cycles of moment distribution. Using the Cantilever Moment distribution method.



$$\begin{array}{r} +9.29 \\ +6.52 \\ \hline +15.81 \end{array} = 2.64 \times \frac{12}{2} = 15.84$$

O.K.

$$\begin{array}{r} +23.44 \\ +34.07 \\ \hline +57.51 \end{array} = \frac{9.58}{2} \times 12 = 57.5$$

O.K.

Fig. 2-D

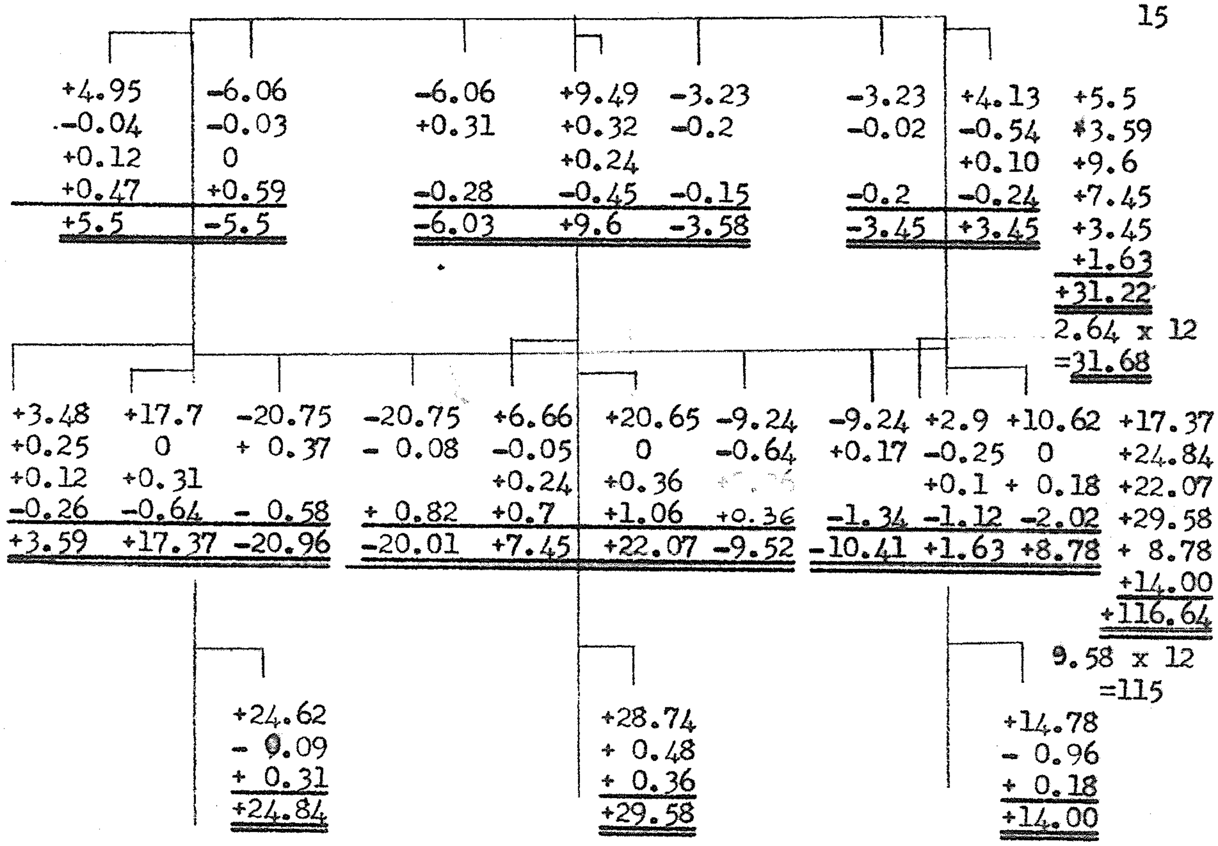
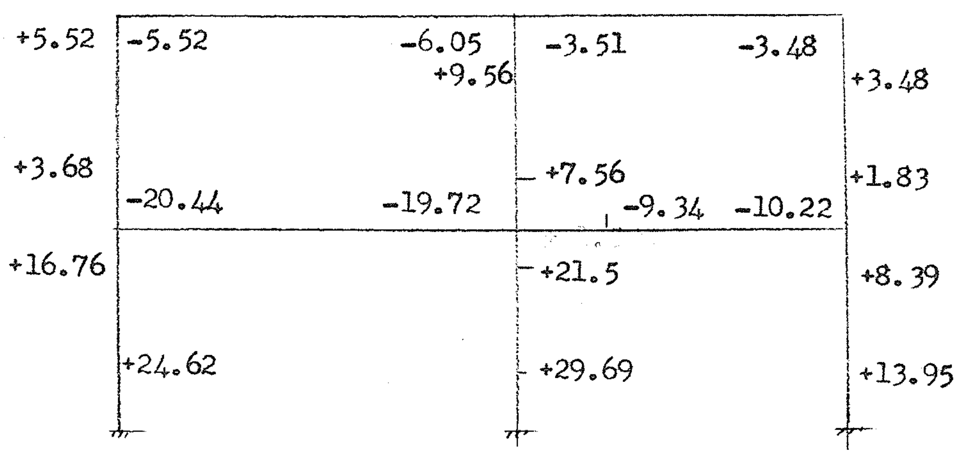


Fig. 2-E

Shear and Moment distribution

The resulting total moments in each story is obtained by adding moments at top and bottom of all columns in the story are shown alongside and equal to the shear multiplying by the story height.



Final Moments by equations

Fig. 2-F

All the end moments in Fig. 2-E compared with Fig. 2-F are within 4% of accuracy. Except one which is underlined is out of 12.57%. Actually the moment is of 0.23 K^{ft} difference with the exact result, which is acceptable.

Example 3

Three-story frame with lateral loads acting at the joints as shown. The frame is fixed at the bottom and is symmetrical.[2]

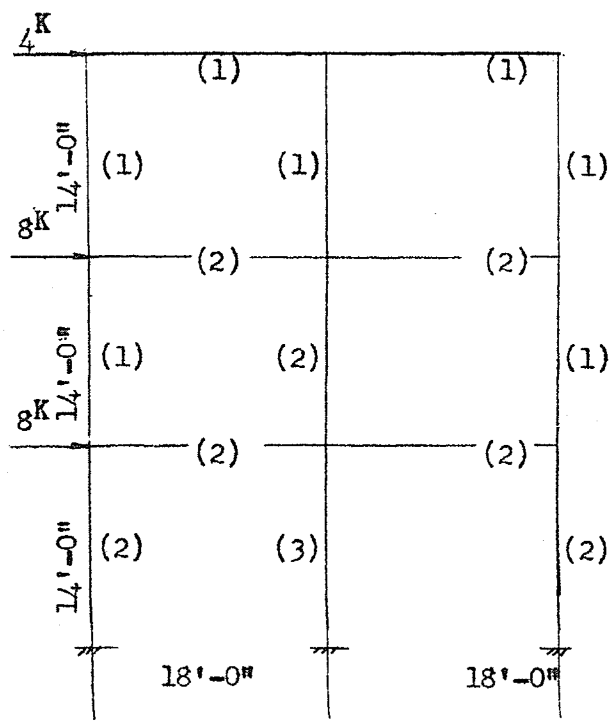
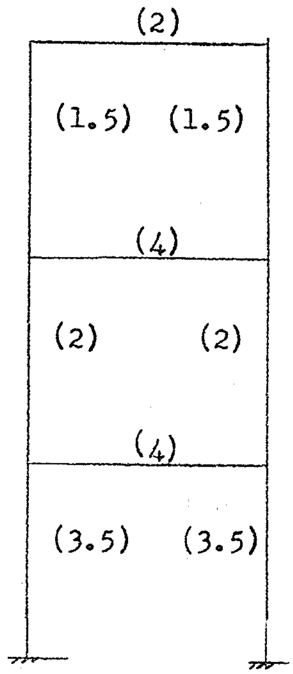


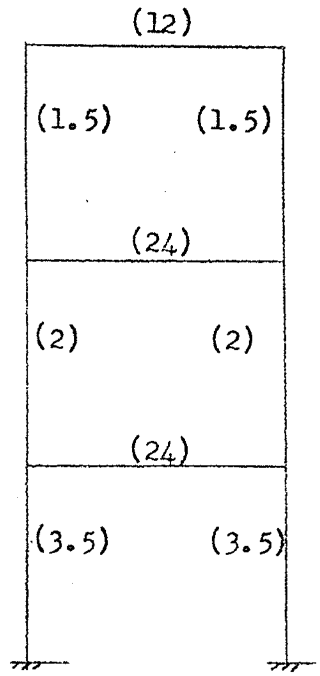
Fig. 3-A

(2) See reference (3) p.359.



Modified structure

Fig. 3-B



Modified K-values

Fig. 3-C

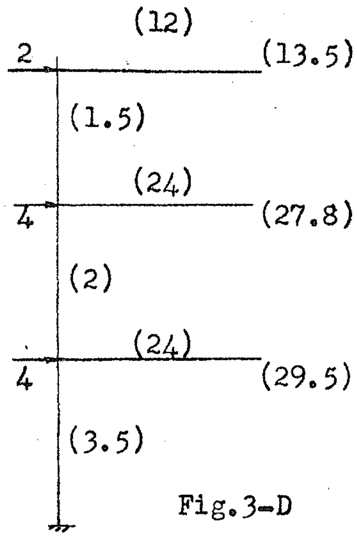


Fig.3-D

Relative K-values

Solve for moments

+70.0	+70.0	+42.0	+42.0	+14.0	+14.0
+13.3	-13.3	- 7.6	+ 7.6	+ 1.56	- 1.56
		+ 4.74	- 4.74	- 3.56	+ 3.56
+ 0.56	- 0.56	- 0.32	0.32	+ 0.39	- 0.39
+83.86	+56.14	+38.82	- 0.05	- 0.04	+15.61
			45.13	+12.45	

83.86
56.14
140.00
= 14 x 10

O.K.

38.82
45.13
83.95
6 x 14 = 84

O.K.

12.45
15.61
28.06
2 x 14 = 28

O.K.

Fig. 3-E

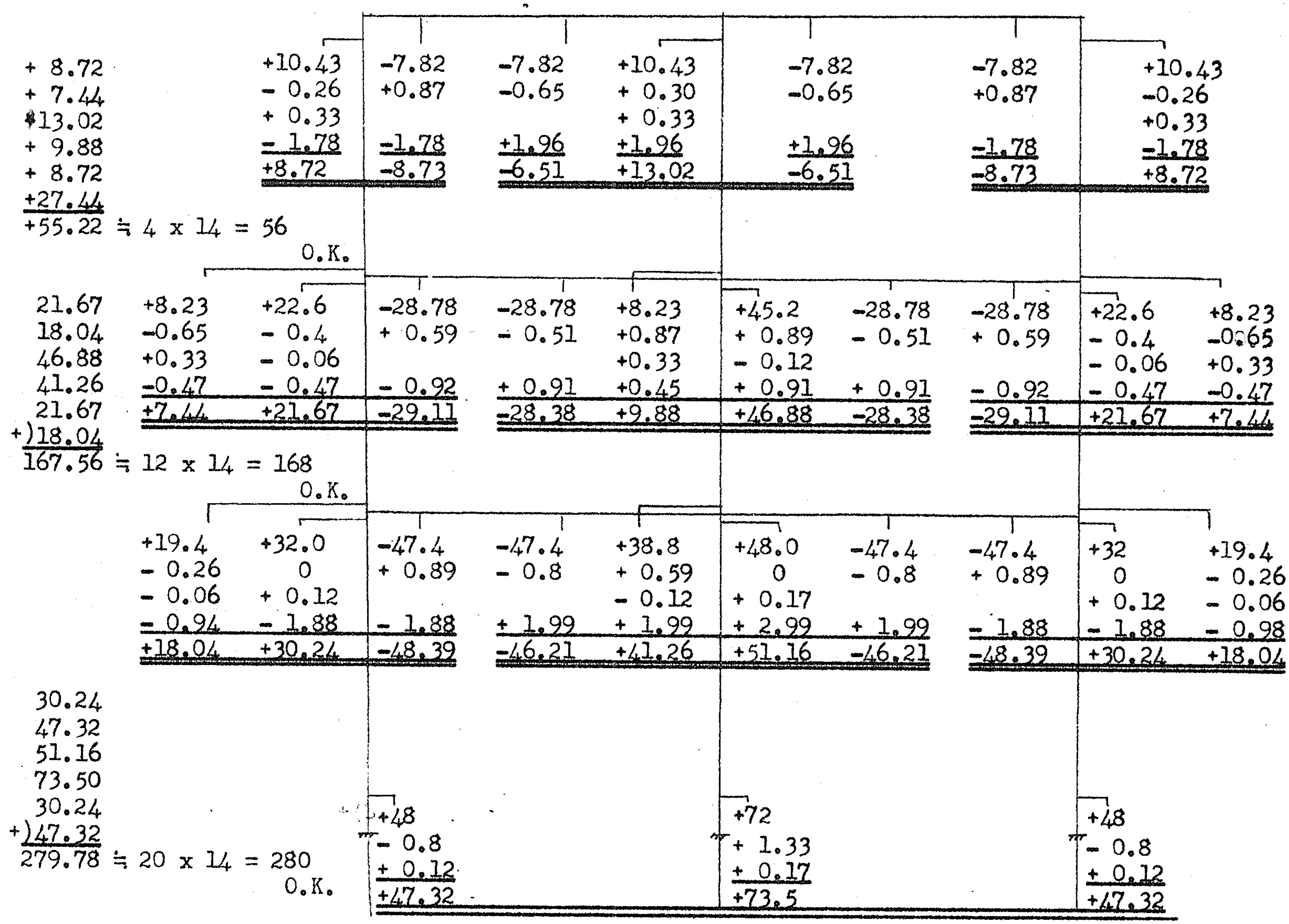
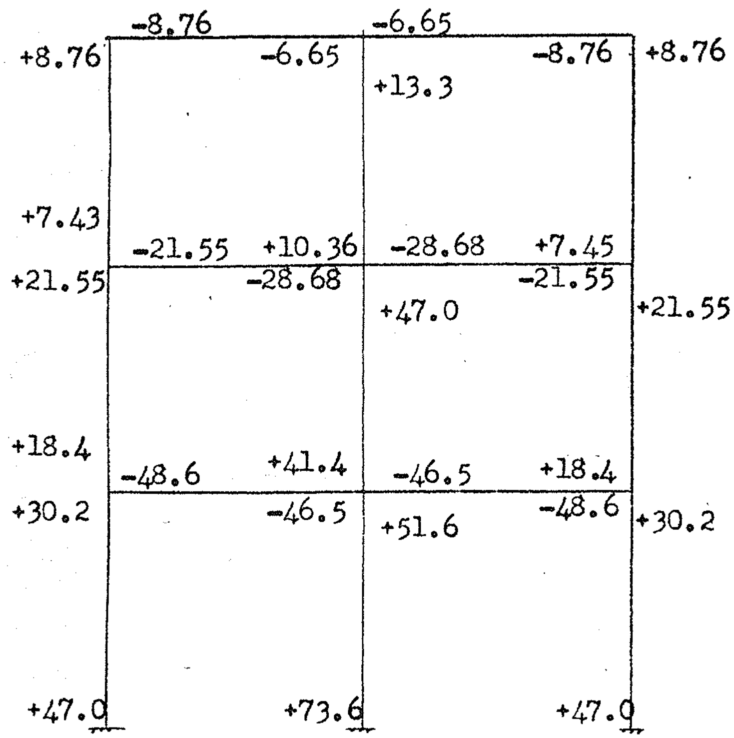
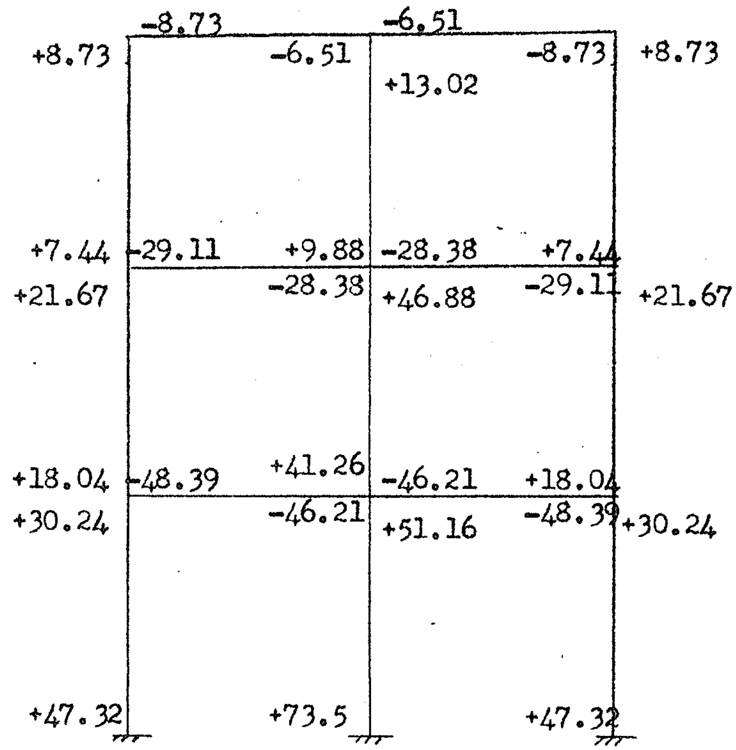


Fig. 3-F. Shear and Moment distribution



Final moments by Equations



Final moments by Approx. Method

The resulting end moments are well within 2.1% of accuracy

Example 4

The five-story frame with lateral loads is fixed at the bottom, and the K-values are shown in opposite the members. The upper three stories are set back so that there are only two columns, but the frame remains symmetrical about a vertical center line.(3)

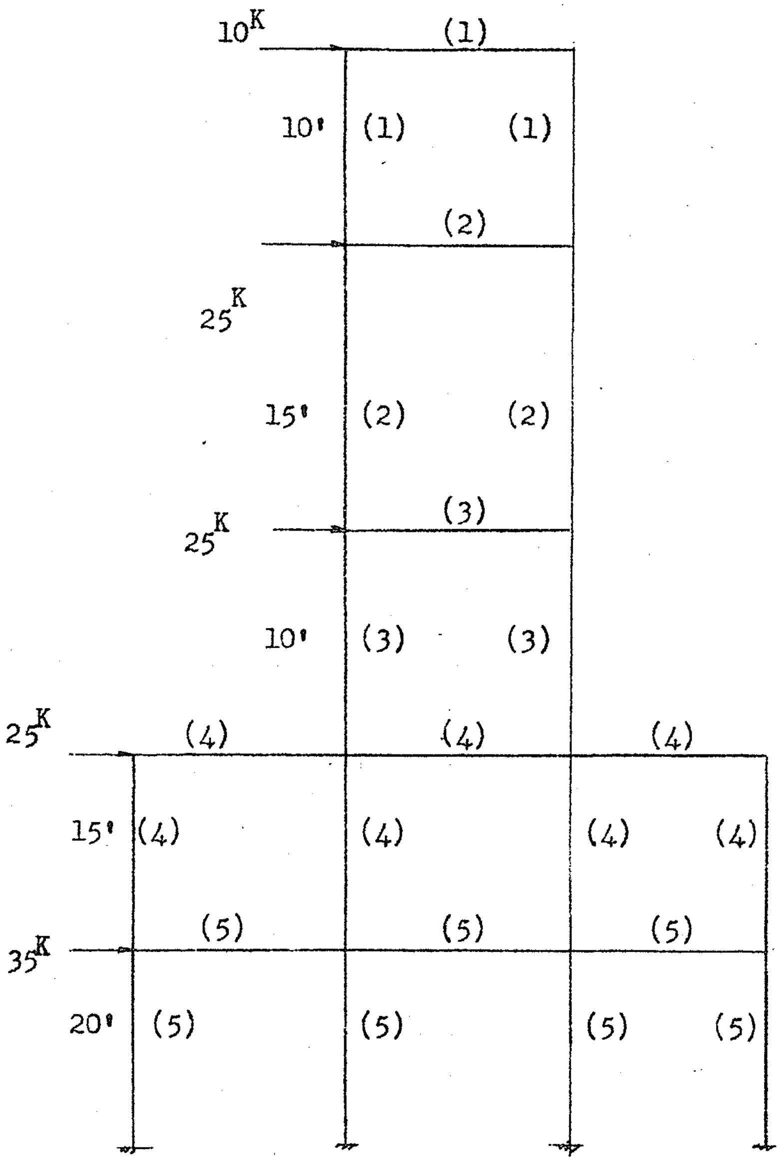
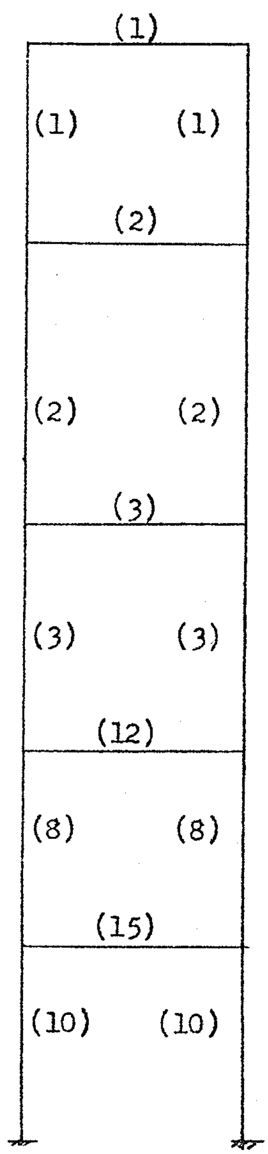


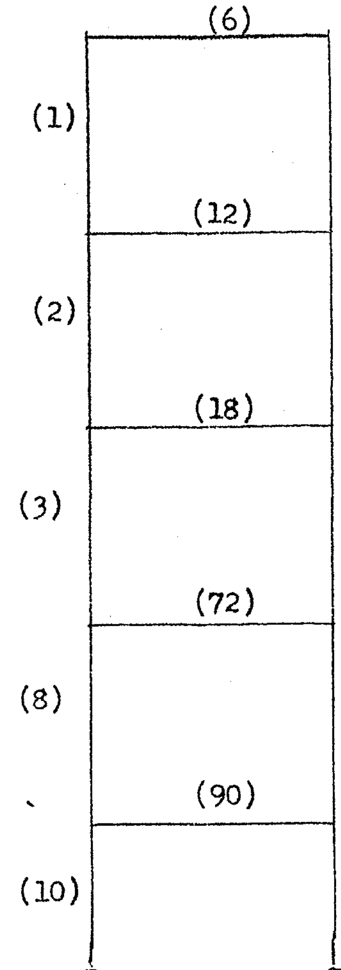
Fig. 4-A



Modified structure

Fig. 4-B

(3) See reference (2) p.45.



Modified K-values

Fig. 4- C

Solve for Moments using Cantilever moment distribution method

	$\frac{10}{108}$	$\frac{8}{108}$	$\frac{8}{83}$	$\frac{3}{83}$	$\frac{3}{23}$	$\frac{2}{23}$	$\frac{2}{15}$	$\frac{1}{15}$	$\frac{1}{7}$
+600	+600	+318.8	+318.8	+150	+150	+131.25	+131.25	+25	+25
+ 85	- 85	- 68	+ 68	+ 36.7	- 36.7	- 24.45	+ 24.45	+ 3.57	- 3.57
		+ 55.3	- 55.3	- 20.7	+ 20.7	+ 24.58	- 24.58	-12.3	+12.3
+ 5.12	- 5.12	- 4.1	- 4.1	+ 5.9	- 5.9	- 3.94	+ 3.94	+ 1.75	- 1.75
		0.96	- 0.96	- 0.36	+ 0.36	+ 0.76	- 0.76	- 0.38	0.38
	- 9.09	- 0.07			- 0.15	-0.10			- 0.05
+690.12	+509.79	+302.69	+334.64	+171.54	+128.31	+128.1	+134.3	+17.64	+32.36

690.12	302.69	171.54	128.1	32.86
509.79	334.64	128.31	134.3	17.64
1199.91	637.33	299.88	262.4	50.00
x 2 = 2399.82	x 2 = 1274.6	x 2 = 599.7	x 2 = 524.8	x 2 = 100.0
120 x 20	85 x 15	60 x 10	15 x 35	10 x 10
= 2400.0	= 1275.0	= 600.0	= 525.0	= 100.0
O.K.	O.K.	O.K.	O.K.	O.K.

Fig. 4-D

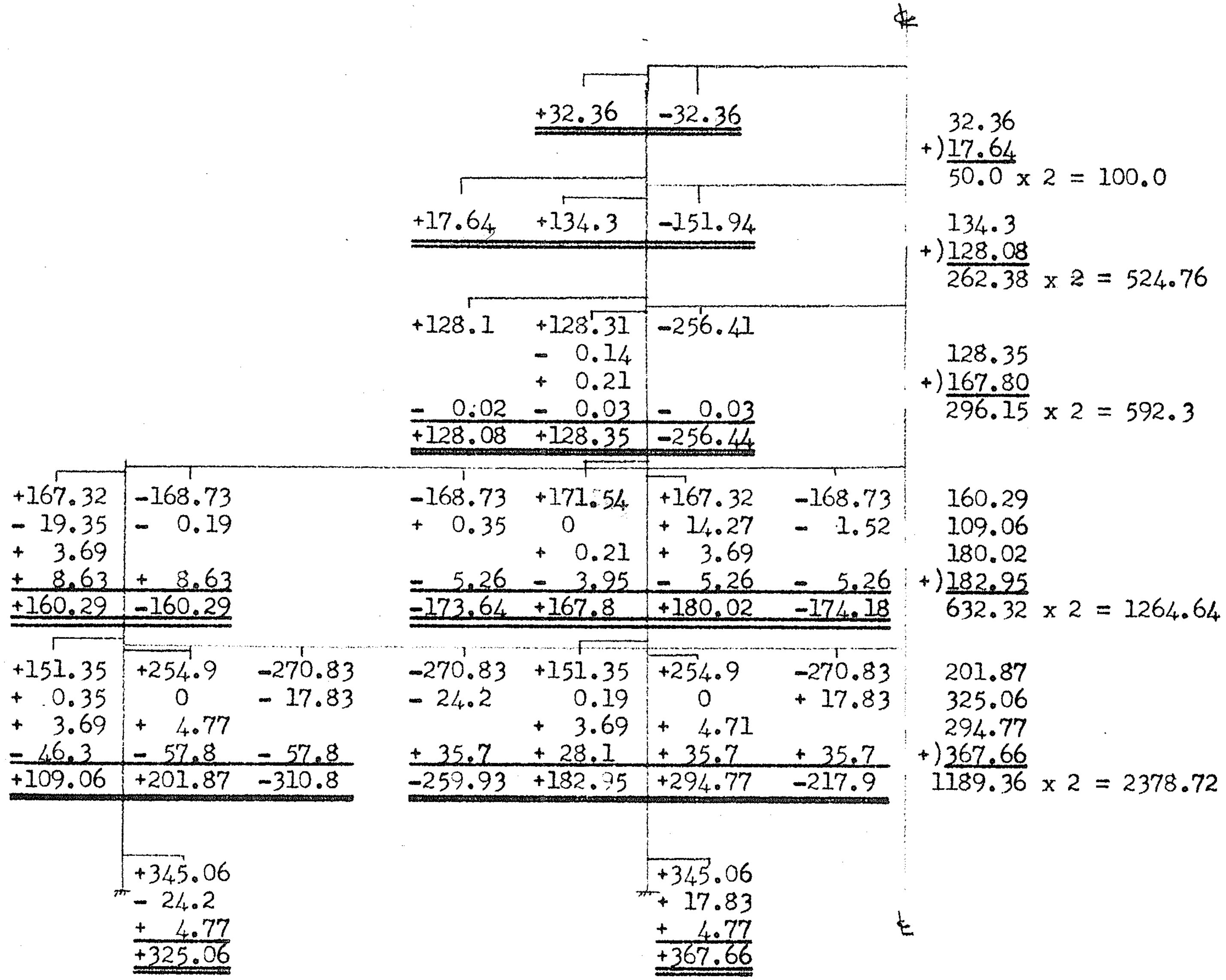
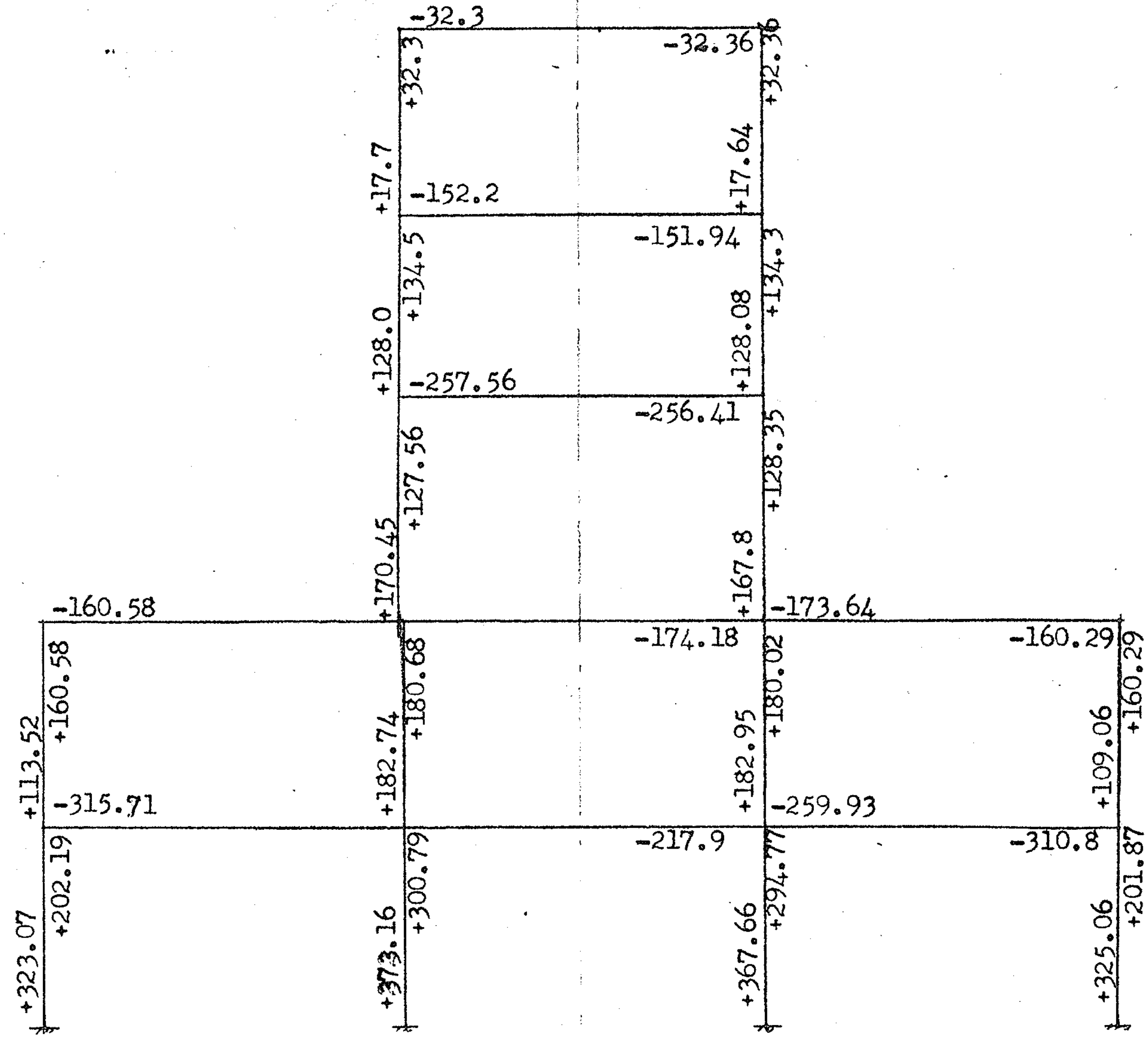


Fig. 4-E Shear and Moment distribution



Final moments by equations

℄

Final moments by approx. method

Fig. 4-F All end moments are within 4% of accuracy

Example 5

Six story frame with lateral loads.[4]

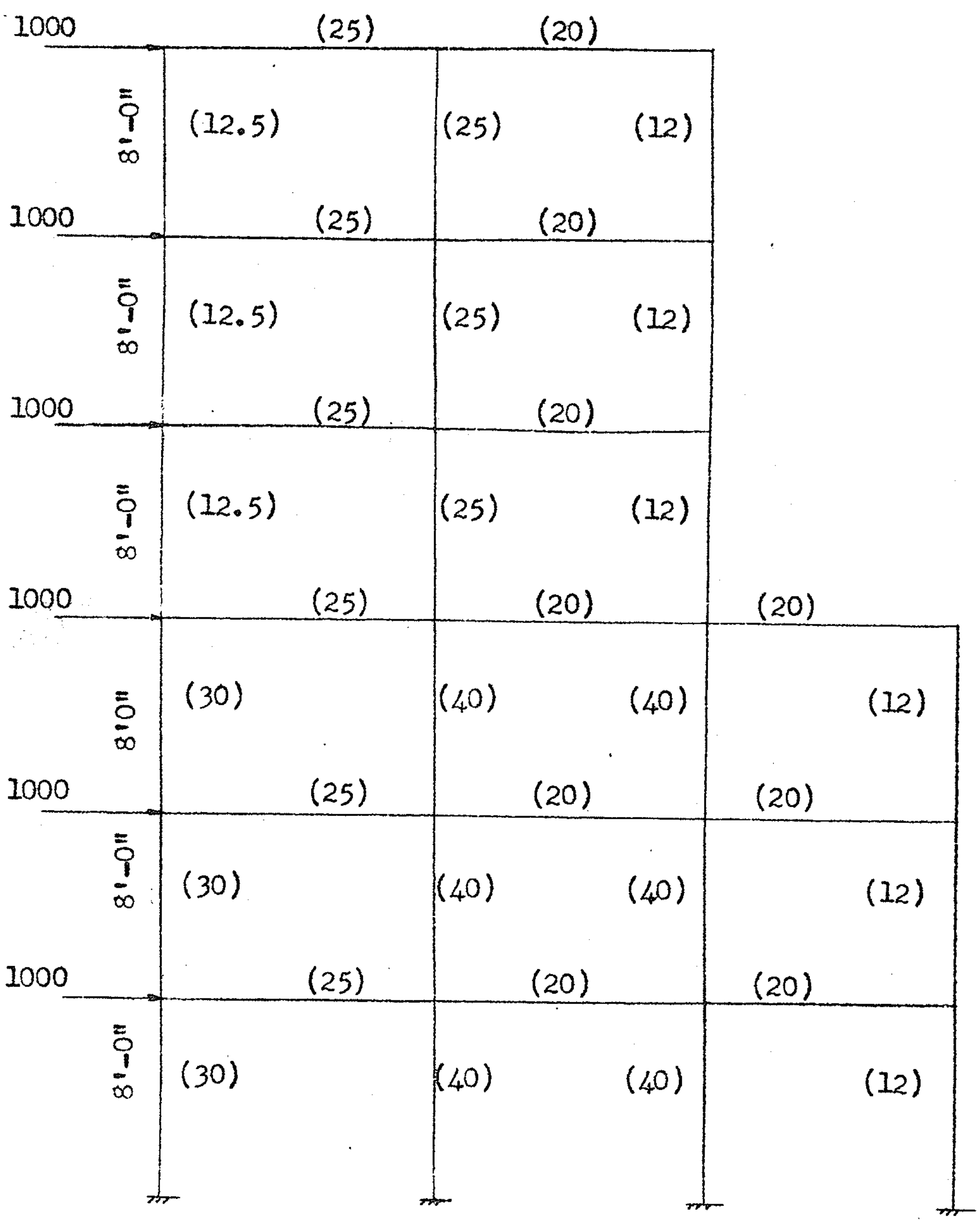


Fig. 5-A

(4) See reference (9)

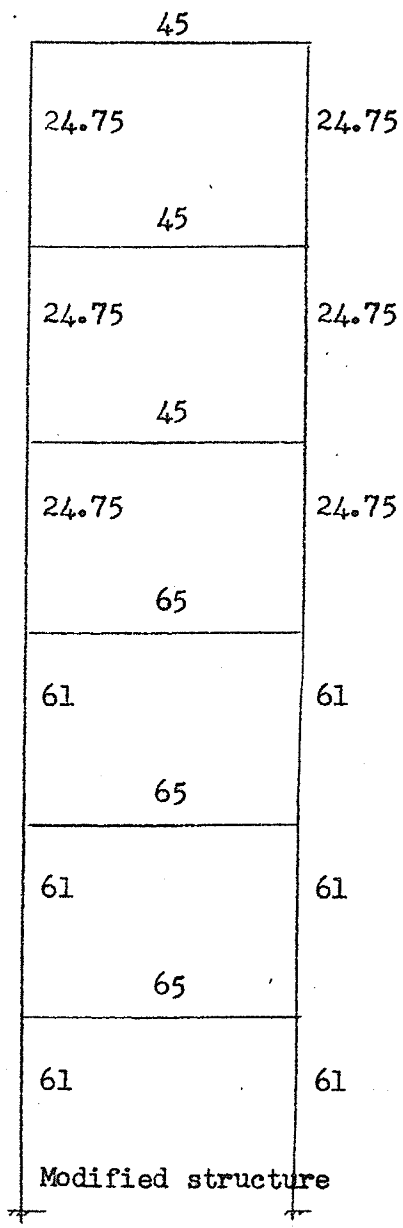

45
24.75
24.75
45
24.75
24.75
45
24.75
24.75
65
61
61
65
61
61
65
61
61
Modified structure

Fig. 5-B

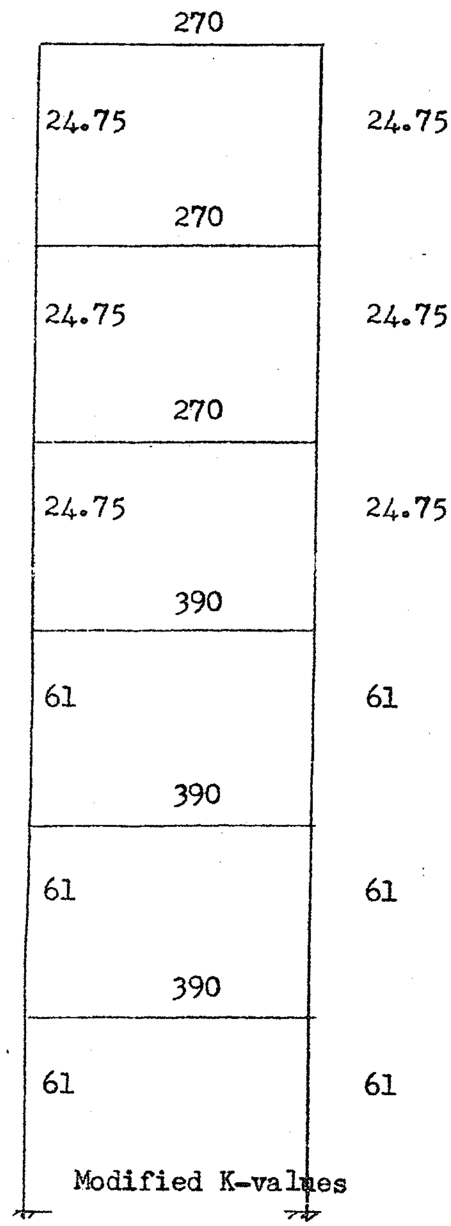

270
24.75
24.75
270
24.75
24.75
270
24.75
24.75
390
61
61
390
61
61
390
61
61
Modified K-values

Fig. 5-C

Solve for moments

	.119	.119	.119	.119	.126	.052	.0775	0.0775	0.0775	0.0775	.084	.916
12000	12000	10000	10000	8000	8000	6000	6000	4000	4000	2000	2000	
2620	-2620	-2620	2600	1764	-1764	-728	728	465	-465	-465	465	
		2660	-2660	-2660	2660	868	-868	-868	868	207	-207	
316	- 316	- 316	316	445	-445	-184	184	83.4	-83.4	-83.4	83.4	
		91	- 91	- 91	91	20.7	-20.7	-20.7	20.7	+7.0	-7.0	
10.8	-10.8	-10.8	10.8	14.1	-14.1	-5.8	5.8	2.1	-2.1	-2.1	2.1	
		3.0	- 3.0	- 3.0	3.0	+0.6	-0.6	-0.6	0.6			
0.35	-0.35	-0.35	0.35	0.5	- 0.5	-0.2	0.2					
+14947.15	+9052.85	+9806.85	+10193.15	+7469.6	8530.4	+5971.3	+6028.7	+3661.2	+4338.8	+1663.5	+2336.5	
		-18859.7		-17662.75		-14501.7		-9689.9		-6002.3	-2336.5	

14947.15	9806.85	7469.6	5971.3	3661.2	1663.5
9052.85	10193.15	8530.4	6028.7	4338.8	2336.5
+24000.00	+20000.00	+16000.0	12000.0	+8000.0	+4000.0

Fig. 5-D By cantilever moment distribution method

Fig. 5-E Shear and moment distribution

+1180	-1291	-1297	+2360	-1038	-1038	+1132
+37.5	-4.8	+39	-7.9	-29.4	-3.9	-32.3
+2.1	0	0	+4.2	0	0	+2.0
+27.4	+57.8	-11.0	-11.0	-8.8	-37.4	-22.4
+1247.0	-1247.0	-1269	+2345.3	-105.86	-1079.3	+1079.3

+84.0	+2190	-3330	-3330	+1680	+4380	-2670	-2670	+807	+2100
+19.5	+61.2	-7.9	+75	-4.8	-11.8	-53.8	-6.3	-17.6	-53.1
+2.1	2.4	0	0	+4.2	+4.9	0	0	+2.0	+2.3
+55.6	+55.6	+111.2	-19.4	-19.4	-19.4	-15.5	-73.3	-44.5	-44.5
+917.2	+2309.2	-3226.7	-3274.4	+1660	+4353.7	-2739.3	-2749.6	+745.9	+2004.7

+1850	+3040	-5380	-5380	+3700	+6080	-4310	-4310	+1780	+2920
+37.5	-151.5	-11.8	+122.5	-7.9	-181	-88.6	-9.5	-32.3	+27.8
+2.4	+113.0	0	0	+4.9	+226	0	0	+2.3	+108.3
+125	+125	+250	-43.6	-43.6	-4.36	-35	-221	-132.7	-132.7
+2014.9	+3126.5	-5641.8	-5301.1	+3653.4	+6081.4	-4433.6	-4540.5	+1617.3	+2923.4

+3015	+4200	-5580	-5580	+6030	+5600	-4460	-4460	+2894	+5600	-4460	-4460	+1680
+61.2	-334	-181	-303	-11.8	+86.5	+46.4	-14.5	-53.1	-153.2	+69.2	+46.4	+254
+1130	+69.0	0	0	+226	+92.0	0	0	+108.3	+92.0	0	0	+27.6
-253	-606	-505	-392	-392	-627	-314	-63.5	-38.1	-127.0	-63.5	+1532	+920
+2936.2	+3229	-6266	-6275	+5952.2	+5151.5	-4727.6	-4668.5	+2911.1	+5411.8	-3654.5	-2881.6	+2881.6

(Continued)

Fig. 5-E (Continued)

3670	+5010	-6790	-6790	+4900	+6680	-5330	-5330	+4900	+6680	-5330	-5330	+1470	+2000
-363	-359	+54.0	-278	-288	+107.2	-76.6	+43.2	+92.6	-130	+423	-76.6	+521	+285
+69.0	+86.0	0	0	+92.0	+114.7	0	0	+92.0	+114.7	0	0	+27.6	+34.3
-486	-486	-405	+172.5	+276	+276	+138	-259	-519	-519	-259	+485	+291	+291
+2890	+4251	-7141	-6895.5	+4980	+7177.9	-5262.6	-5545.8	+4565.6	+6145.7	-5166	-4921.6	+2309.6	+2610.3

+4825	+4460	-7250	-7250	+6440	+5940	-5800	-5800	+6440	+5940	-5800	-5800	+1930	+1782
-334	0	+67	-299	+86.5	0	-65	+53.6	-153.2	0	+475	-65	+254	0
+86.0	+36.7	0	0	+114.7	+47.6	0	0	+114.7	+47.6	0	0	+34.3	+14.3
-666	-666	-555	+157	+251	+251	+126	-386	-772	-772	-386	+831	+505	+505
+3911.0	+3830.7	-7738	-7392	+6892.2	+6238.8	-5739	-6132.4	+5629.5	+5215.6	-5711	-5034	+2723.3	+2301.3

5.65%

+7350	+9800	+9800	+2940
- 359	+ 107.2	- 130	+ 285
+ 36.7	+ 47.6	+ 47.6	+ 14.3
+7027.7	+9954.8	+9717.6	+3210.7

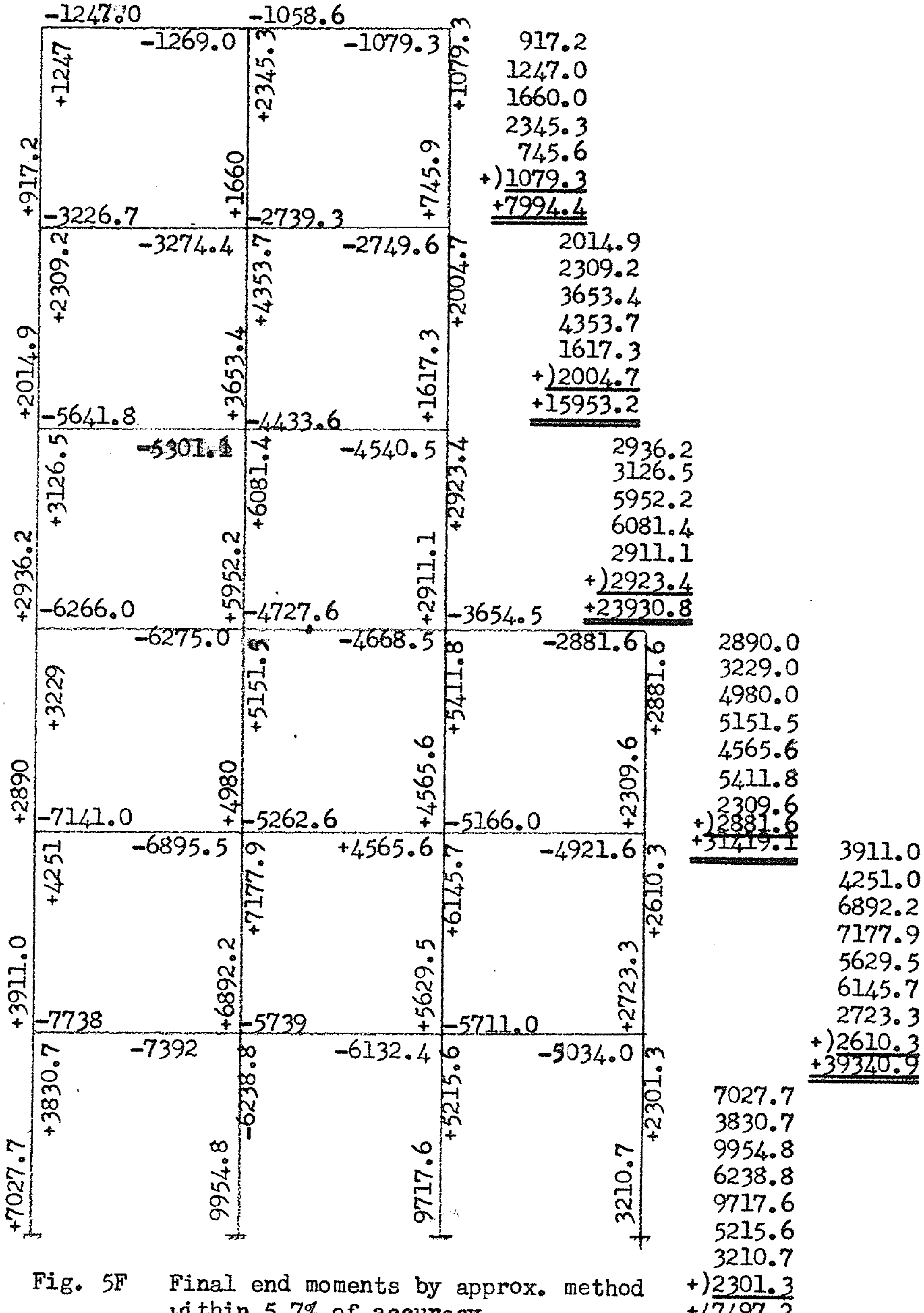


Fig. 5F Final end moments by approx. method within 5.7% of accuracy

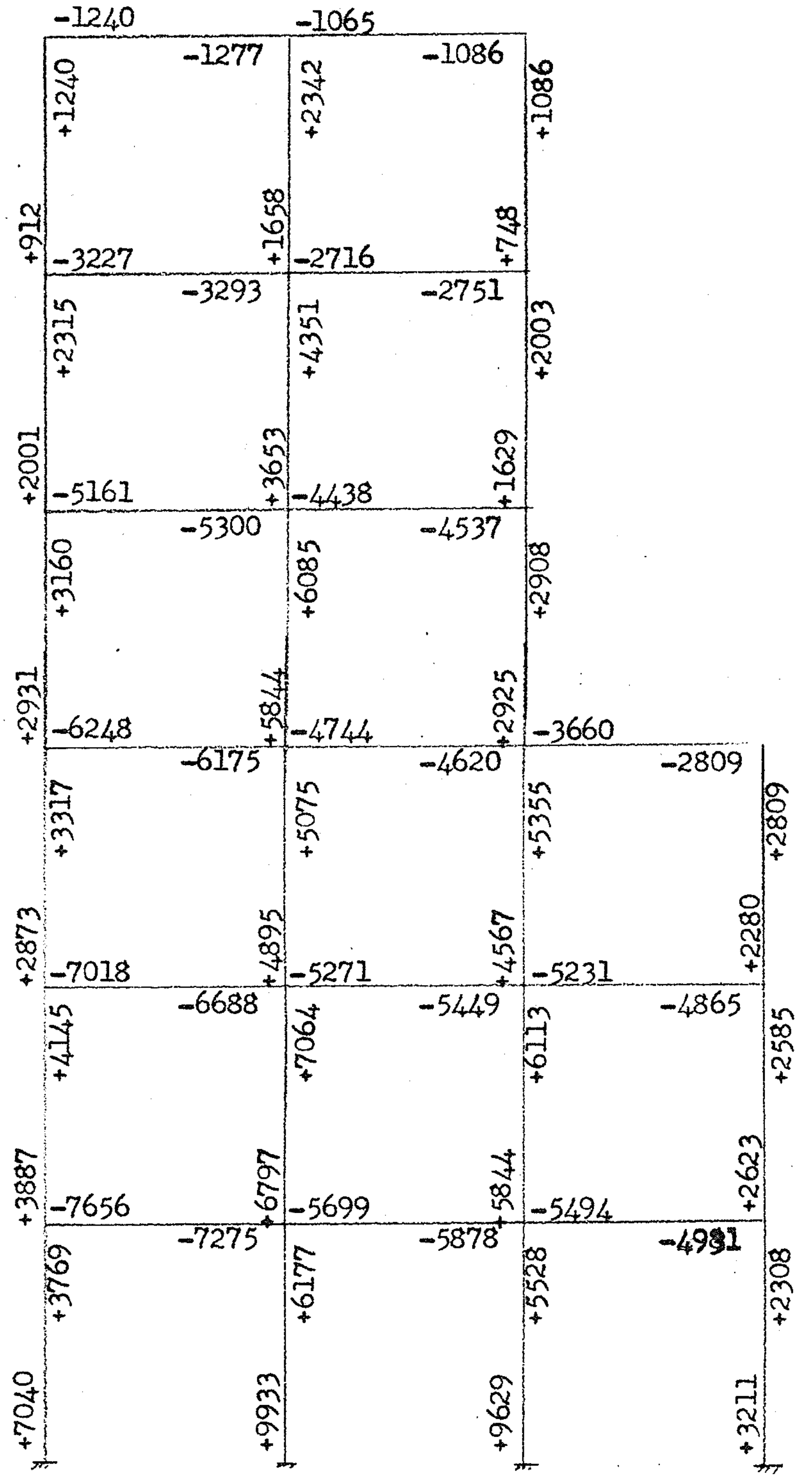
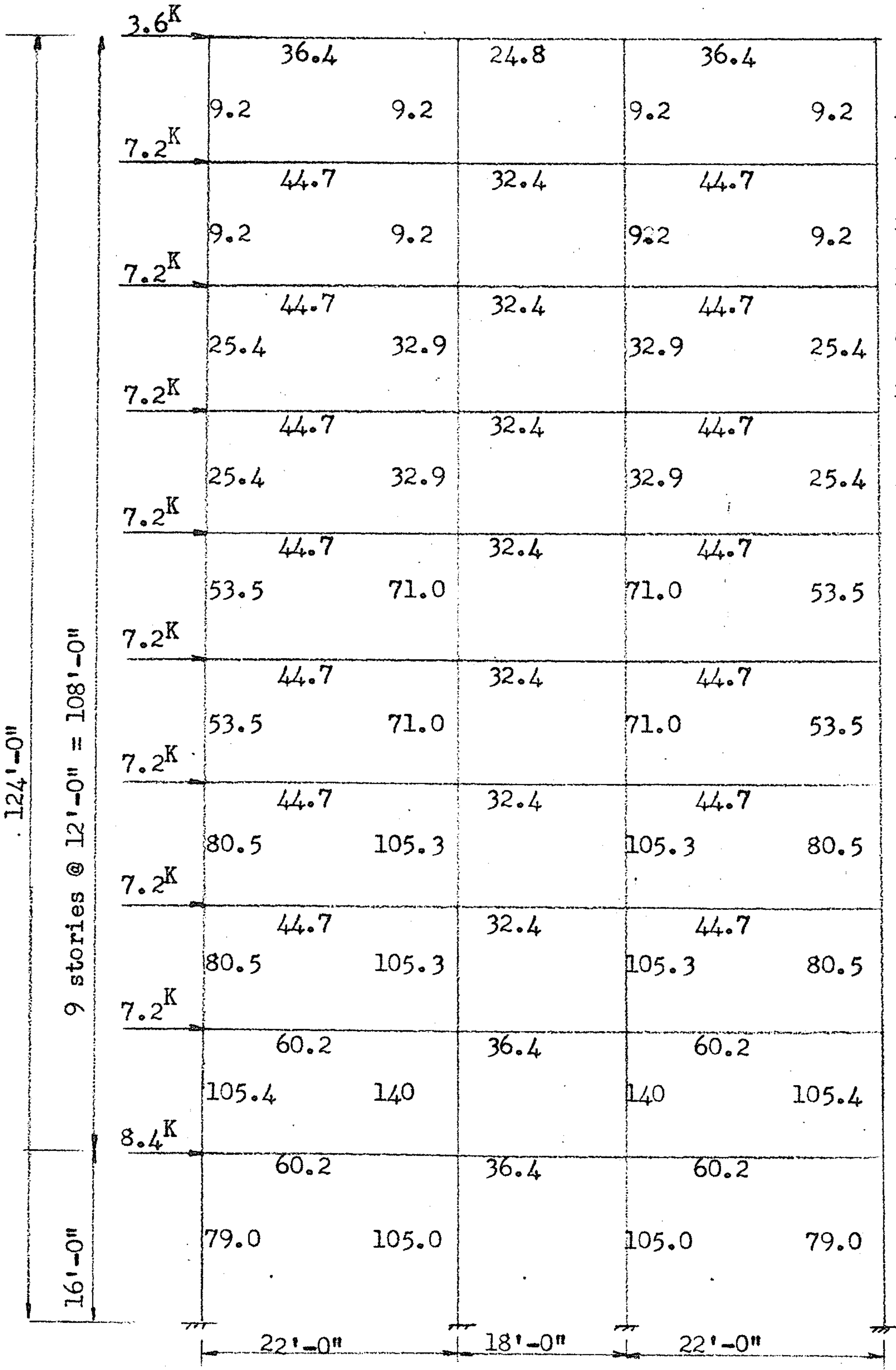


Fig. 5-G Final moments by equations

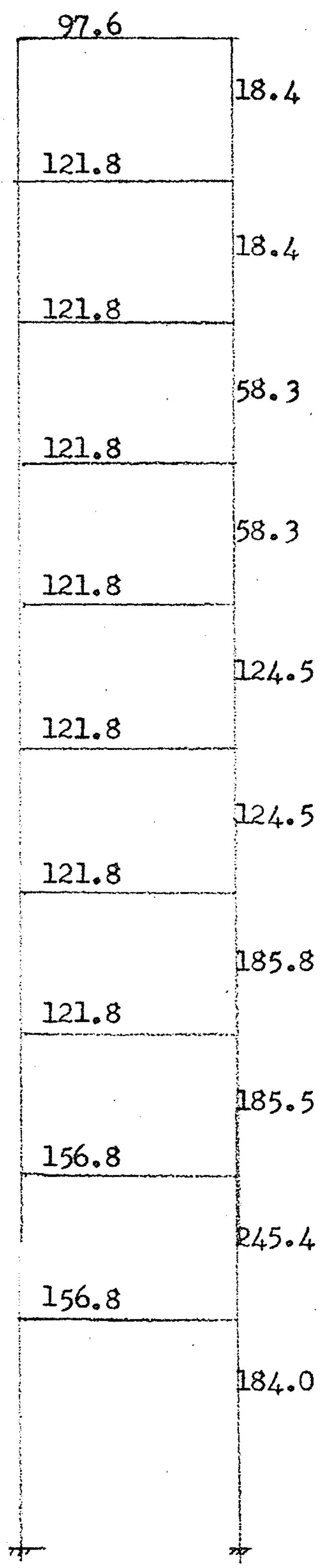
Example 6



A ten-story building is shown in Fig. 6-A. Dimensions, loads, and K-values are included.(5)

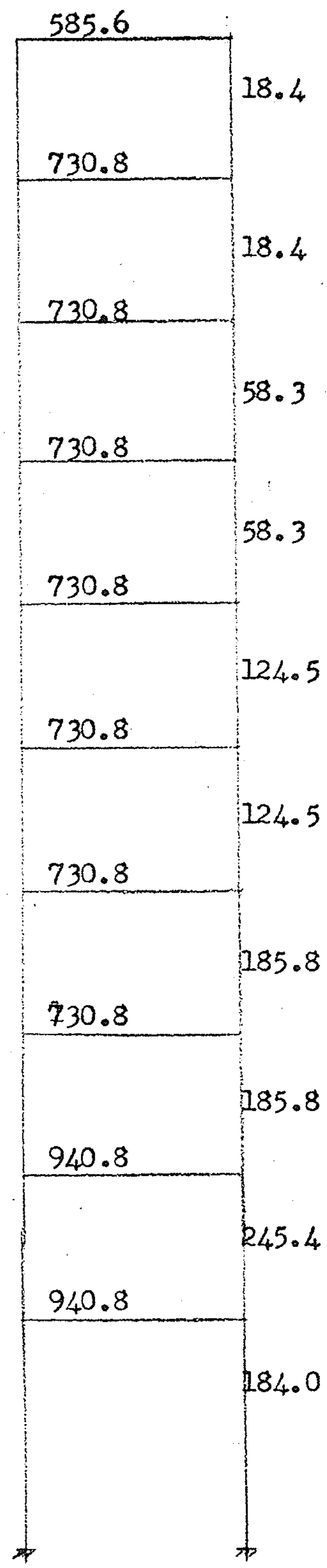
Fig. 6-A

(5) See Reference (3) p.364



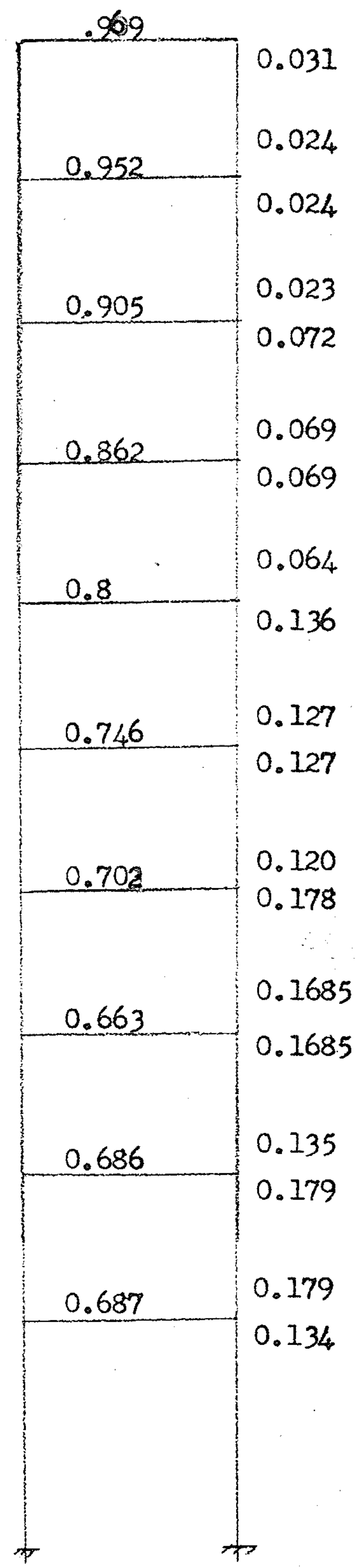
Modified structure
K-values

Fig.6-B



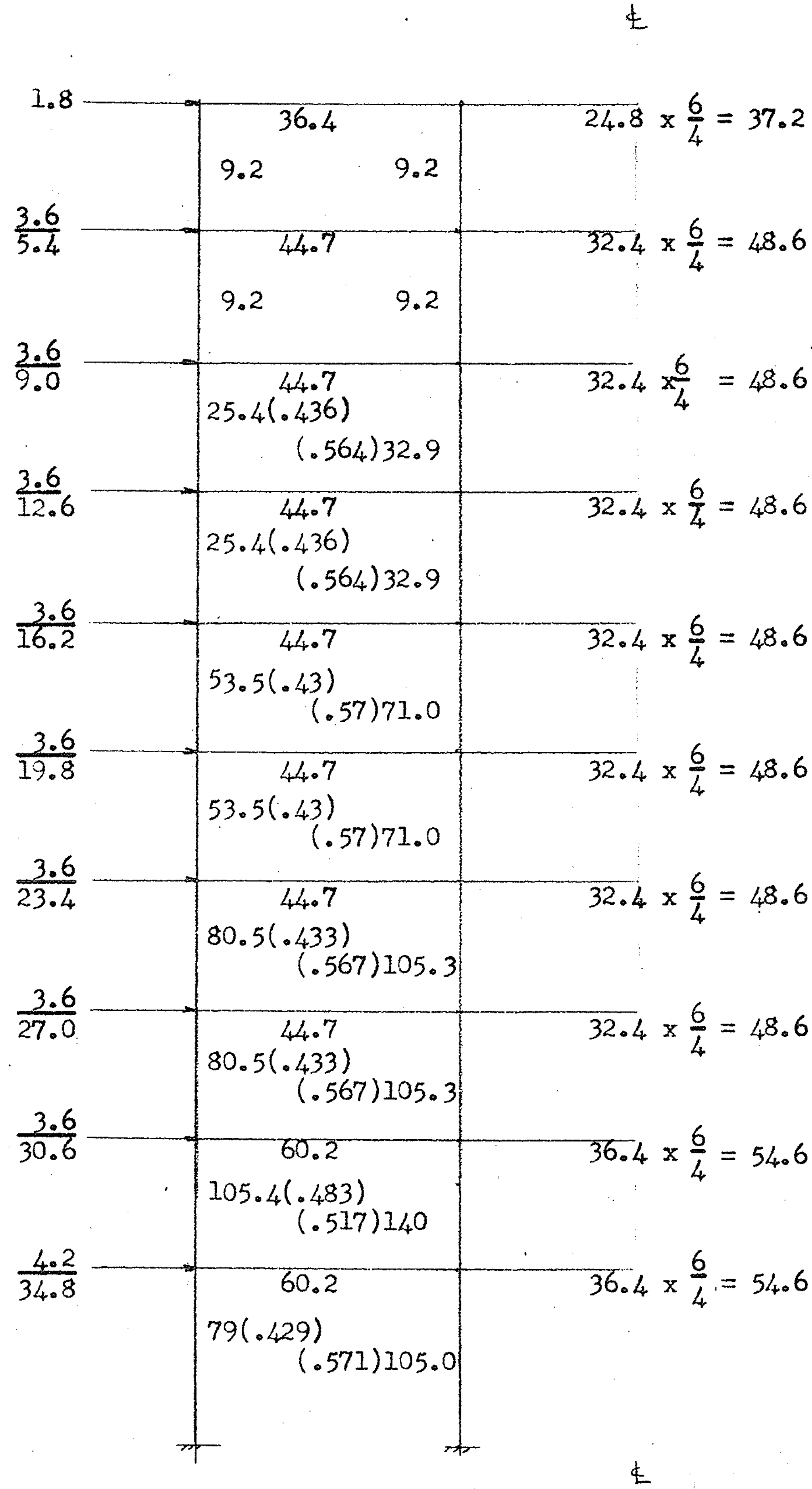
Modified
K-values

Fig. 6-C



Distribution
Factors

Fig. 6-D



Relative K-values

Fig. 6-E

			0.179	0.135			0.178	0.120			0.136	0.064			0.072	0.023			0.031
	0.134	0.179			0.1685	0.1685			0.127	0.127			0.069	0.069			0.024	0.024	
278.4	278.4	183.6	183.6	162	162	140.4	140.4	118.8	118.8	97.2	97.2	75.6	75.6	54	54	32.4	32.4	10.8	10.8
62.0	-62.0	-82.6	82.6	51	-51	-51	51	27.4	-27.4	-27.4	27.4	8.95	-8.95	-8.95	8.95	1.04	-1.04	-1.0	1.04
		85.8	-85.8	-64.6	64.6	60.1	-60.1	-40.5	+40.5	28.45	-28.45	-13.4	13.4	6.95	-6.95	-2.22	2.22	0.37	-0.37
11.4	-11.4	-15.36	15.36	21.0	-21.0	-21.0	21.0	8.76	-8.76	-8.76	8.76	1.4	-1.4	-1.4	1.4	0.06	-0.06	-0.06	0.06
		6.5	-6.5	-4.9	4.9	5.3	-5.3	-3.57	3.57	1.29	-1.29	-0.65	.65		-0.1	-0.03	0.03		
0.87	-.87	-1.17	1.17	1.72	-1.72	-1.72	1.72	0.62	-0.62	-0.62	0.62								
		0.52	-0.52	-0.39	0.39		-0.42	-0.28	0.28										
352.67		177.29			158.17	132.08			126.37	90.16			79.24	50.6			+33.55	10.07	
	204.13		189.91	165.83			147.9	111.23			104.2	71.86			57.4	31.25			+11.53
	-381.42		-355.74		-290.25		-259.13		-216.53		-176.06		-129.84		-88.65		-43.62		-11.53

352.67	177.29	165.83	132.08	111.23	90.16	71.86	50.6	31.25	10.07
204.13	189.91	158.17	147.9	126.37	104.2	79.24	57.4	33.55	11.53
556.80	367.2	324.0	279.98	237.6	194.36	151.10	108.0	64.80	21.6
34.8 x 16	30.6 x 12	27 x 12	23.4 x 12	19.8 x 12	16.2 x 12	12.6 x 12	9 x 12	5.4 x 12	1.8 x 12
	367.4	324	280.8	237.6	194.4	151.2	108.0	64.8	21.6

Fig. 6-G. Final end moments by Cantilever moment distribution method

Distribution Factors

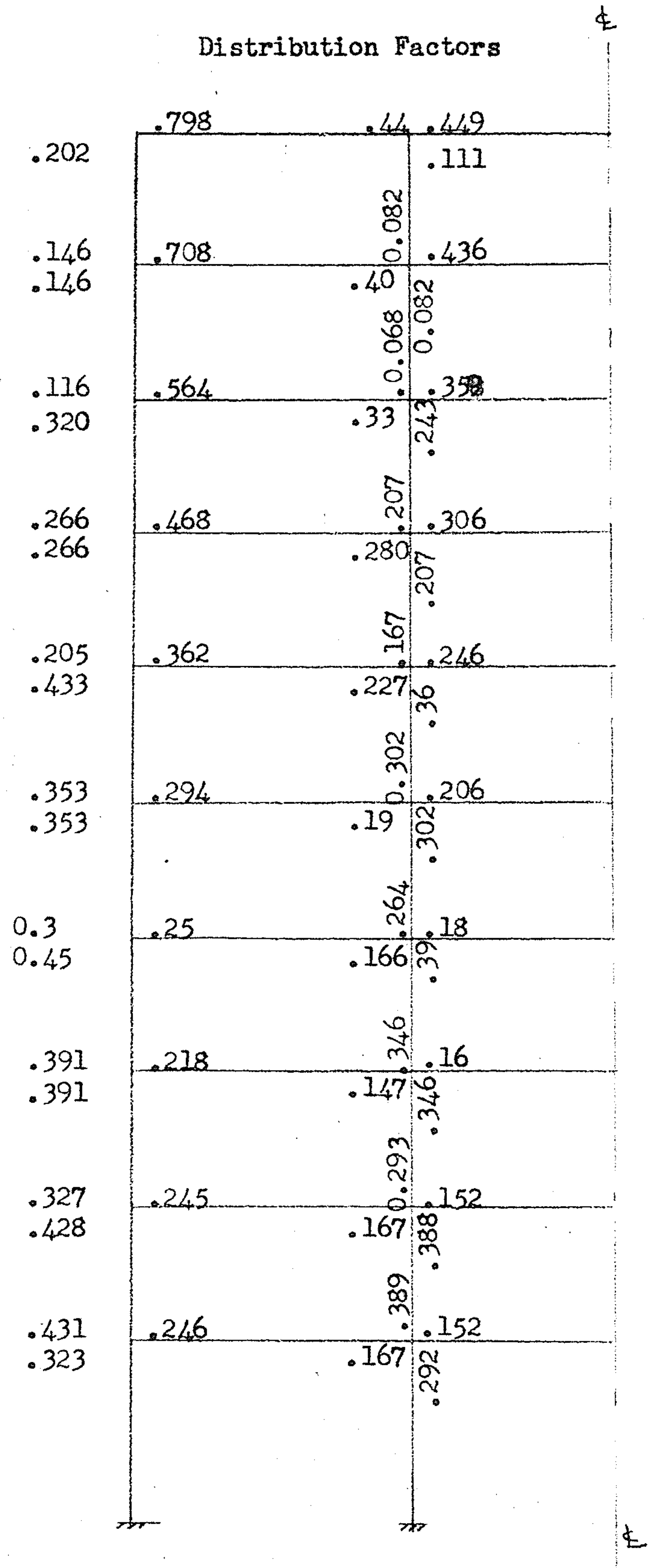


Fig. 6-H

Fig. 6-I. Shear and moment distribution

	+ 5.76	- 4.3		- 4.3	+ 5.76	- 2.93	+5.22	
	- 0.42	+ 0.32		- 0.58	+ 0.24		+4.08	
	+ 0.19				+ 0.19		+6.37	
	- 0.31	- 1.24		+ 0.71	+ 0.18	+ 0.73	+5.79	
	+ 5.22	- 5.22		- 4.17	+ 6.37	- 2.2	+21.46	1.8 x 12 = 21.6
+ 5.04	+ 16.76	- 16.0	- 16.0	+ 5.04	+ 16.76	- 11.6	+15.62	
- 0.15	- 0.49	+ 1.16	- 2.05	+ 0.08	+ 0.22		+14.64	
+ 0.19	+ 0.35			+ 0.19	+ 0.35		+17.91	
- 1.0	+ 1.0	+ 4.86	+ 2.85	+ 0.58	+ 0.58	+ 3.1	+16.53	
+ 4.08	+ 15.62	- 19.7	- 15.2	+ 5.79	+ 17.91	- 8.5	+64.70	5.4 x 12 = 64.8
+ 15.64	+ 25.0	- 32.2	- 32.2	+ 15.64	+ 32.4	- 22.3	+20.99	
- 0.42	- 3.87	+ 1.06	- 2.38	+ 0.24	+ 0.75		+19.92	
+ 0.35	+ 2.42			+ 0.35	+ 3.12		+37.34	
- 0.93	- 2.56	- 4.5	+ 1.45	+ 0.3	+ 1.07	+ 1.57	+31.76	
+ 14.64	+ 20.99	- 35.64	- 33.13	+ 16.53	+ 37.34	- 20.73	+110.01	9 x 12 = 108.0
+ 22.1	+ 34.6	- 47.6	- 47.6	+ 28.6	+ 44.7	- 3300	+32.37	
- 1.35	- 1.18	+ 1.02	- 6.81	+ 0.78	+ 0.94		+26.27	
+ 2.42	+ 2.2			+ 3.12	+ 2.84		+47.74	
- 3.25	- 3.25	- 5.72	- 1.0	- 0.74	- 0.74	- 1.09	+43.27	
+ 19.92	+ 32.37	- 52.3	- 45.41	+ 31.76	+ 47.74	- 34.09	+149.65	12.6 x 12 = 151.2

(Continued)

Fig. 6-I (Continued)

+ 31.30	+ 44.8	- 64.6	- 64.6	+ 40.5	+ 59.4	- 46.8	+39.72	
- 3.87	- 2.42	+ 1.28	- 2.09	+ 0.75	+ 2.07		+32.89	
+ 0.35	+ 0.53			+ 0.45	+ 0.7		+65.56	
- 1.51	- 3.19	- 2.67	+ 2.14	+ 1.57	+ 3.39	+ 2.32	+57.25	
+ 26.27	+ 39.72	- 65.99	- 64.35	+ 43.27	+ 65.56	- 44.48	+195.42	16.2 x 12 = 194.4
+ 38.8	+ 54.4	- 79.5	- 79.5	+ 51.4	+ 72.1	- 57.7	+48.58	
- 2.5	- 2.27	+ 1.3	- 2.02	+ 2.03	+ 2.02		+42.23	
+ 0.53	+ 0.39			+ 0.7	+ 0.64		+77.88	
- 3.94	- 3.94	- 3.28	+ 1.96	+ 3.12	+ 3.12	+ 2.54	+68.56	
+ 32.89	+ 48.58	- 81.48	- 79.56	+ 57.25	+ 77.88	- 55.16	+237.25	19.8 x 12 = 237.6
+ 47.6	+ 64.0	- 96.5	- 96.5	+ 63.1	+ 83.9	- 65.8	+55.79	
- 2.42	- 3.78	+ 1.27	- 1.89	+ 2.07	+ 3.29		+48.79	
+ 0.39	+ 0.58			+ 0.64	+ 0.77		+92.02	
- 3.34	- 5.01	- 2.79	+ 1.73	+ 2.75	+ 4.06	+ 1.88	+83.13	
+ 42.23	+ 55.79	- 98.02	- 96.66	+ 68.56	+ 92.02	- 63.92	+279.73	23.4 x 12 = 280.8
+ 57.2	+ 68.5	-106.4	-106.4	+ 75.0	+ 89.6	- 77.2	+59.32	
- 3.4	- 4.38	+ 1.4	- 2.1	+ 2.98	+ 3.65		+61.77	
+ 0.58	+ 0.79			+ 0.77	+ 1.04		+98.67	
- 5.59	- 5.59	- 3.12	+ 1.86	+ 4.38	+ 4.38	+ 2.02	+104.77	
+ 48.79	+ 59.32	-108.12	-106.64	+ 83.13	+ 98.67	- 75.18	+324.53	27 x 12 = 324.0

(Continued)

Fig. 6-I (Continued)

+ 71.8	+ 91.6	-136.6	-136.6	+ 94.0	+ 98.2	- 82.5	+ 77.85	
- 3.78	- 5.75	+ 2.08	- 3.28	+ 3.29	+ 5.15		+ 70.38	
+ 0.79	+ 1.09			+ 1.04	+ 1.16		+111.71	
- 6.94	- 9.09	- 5.2	+ 3.1	+ 5.44	+ 7.2	+ 2.82	+106.90	
+ 61.77	+ 77.85	-139.72	-136.78	+104.77	+111.71	- 79.68	+366.84	30.6 x 12 = 367.2
+ 85.6	+ 87.5	-146.4	-146.4	+ 91.7	+116.6	- 88.4	+ 79.85	
- 5.74		+ 2.21	- 3.15	+ 4.84			+149.17	
+ 1.09	+ 0.28			+ 1.16			+123.50	
- 10.57	- 7.93	- 6.04	+ 3.95	+ 9.2	+ 6.9	+ 3.59	+205.65	
+ 70.38	+ 79.85	-150.23	-145.6	+106.9	+123.5	- 84.81	+558.17	34.8 x 16 = 556.0

+151.2	+201.4
- 4.31	+ 3.87
0.28	+ 0.38
+149.17	+205.65

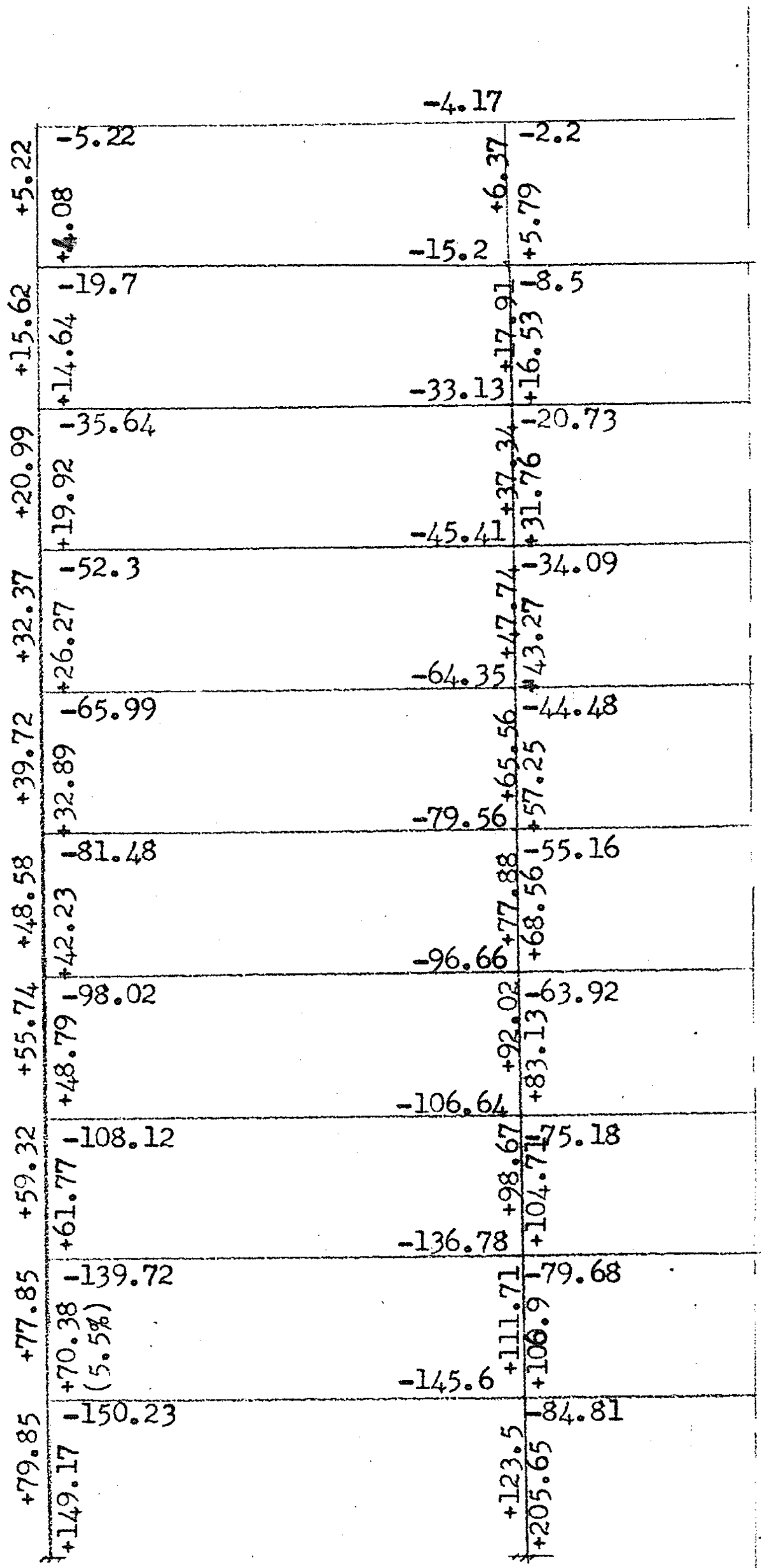


Fig. 6-J Final end moments by approx. method within 7.8% of accuracy. As indicated, of course, which is acceptable. $2.23^{K'}$ is not a great number.

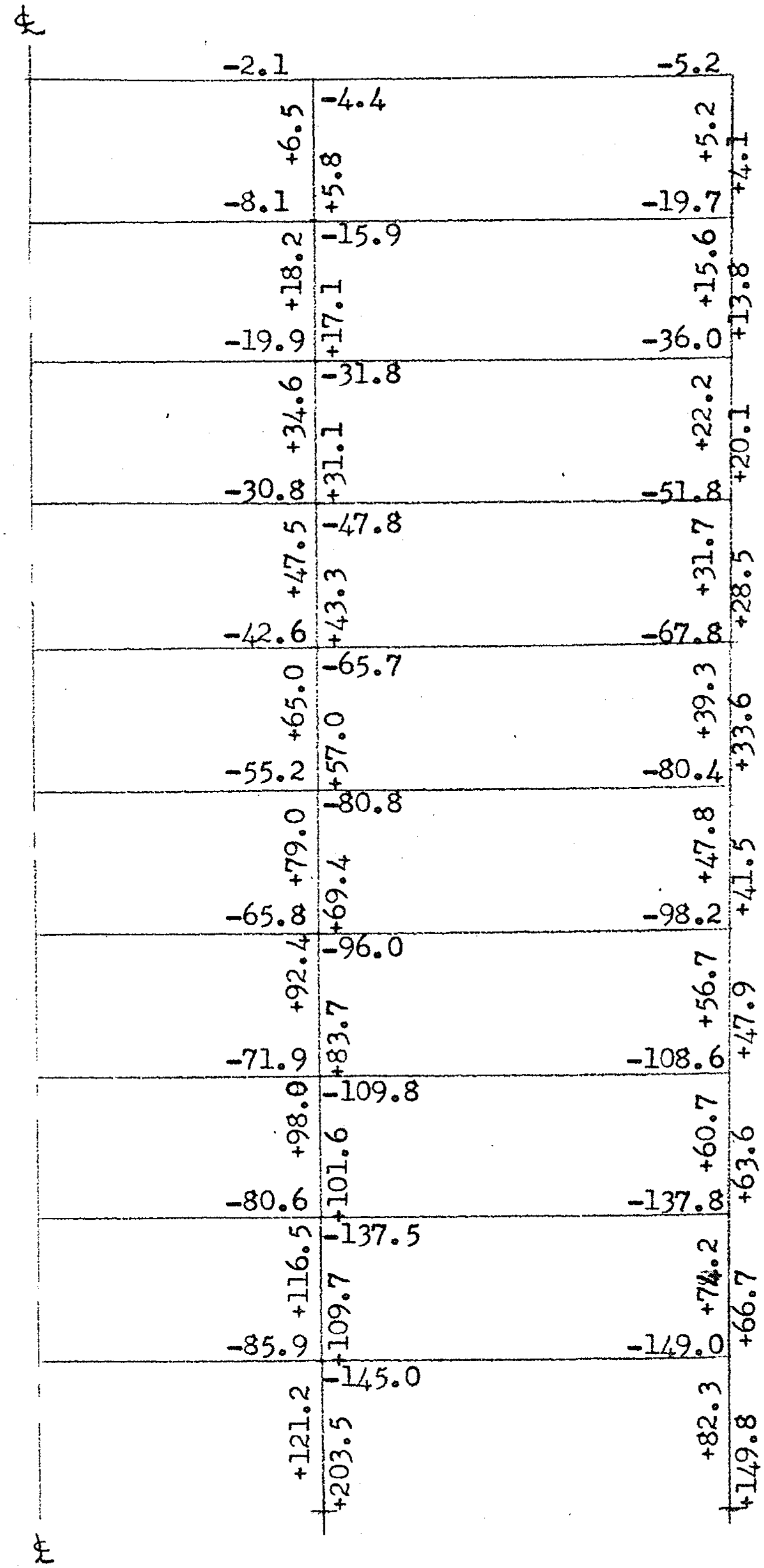
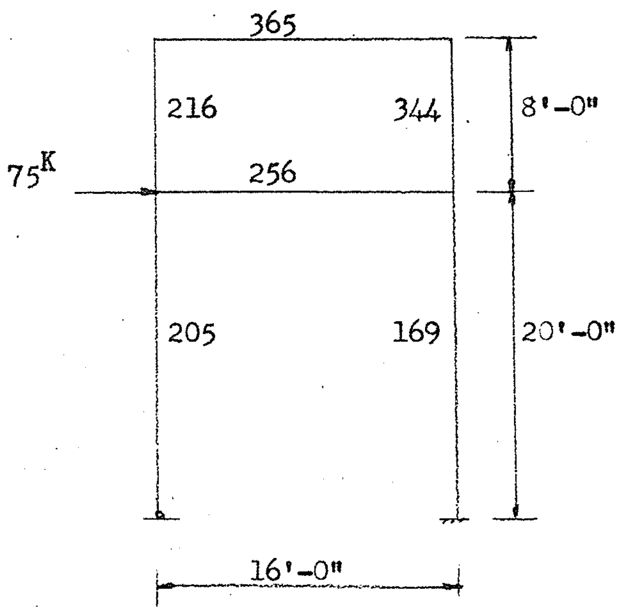
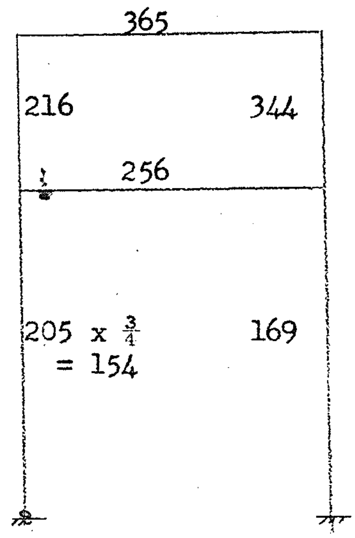


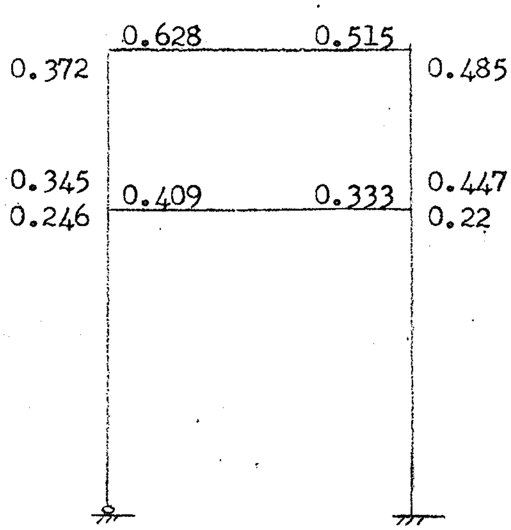
Fig. 6-K Final moments by solution of equations.

Rigid frame with different supporting conditions and the loading condition is shown in the figure[6]



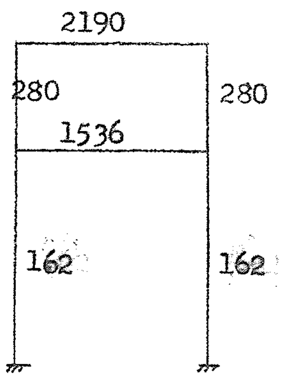


Modifying K-value of the column with pin support

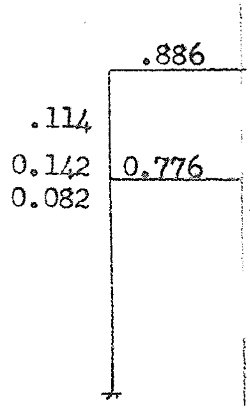


Distribution factor of the original structure

(6) See reference (4) p.149



Modified structure



Distribution Factors of modified structure

F.E.M. by Shear distribution

	0.082	0.142	0.114
+312	+438		
+ 36.0	− 36.0	−62.4	+62.4
		+ 7.1	− 7.1
+ 0.58	− 0.58	− 1.1	
+348.58	+401.42	−56.4	+55.3

− 345.02

+ 42.6	− 55.3	− 55.3	+ 67.9	+48.89
+ 1.12	− 3.25	+ 3.99	− 1.27	−39.74
− 0.55	0	0	− 0.875	+58.745
+ 5.72	+ 9.66	− 7.45	− 7.01	−74.61
+ 48.89	− 48.89	− 58.76	+ 58.745	−6.715 ≠ 0

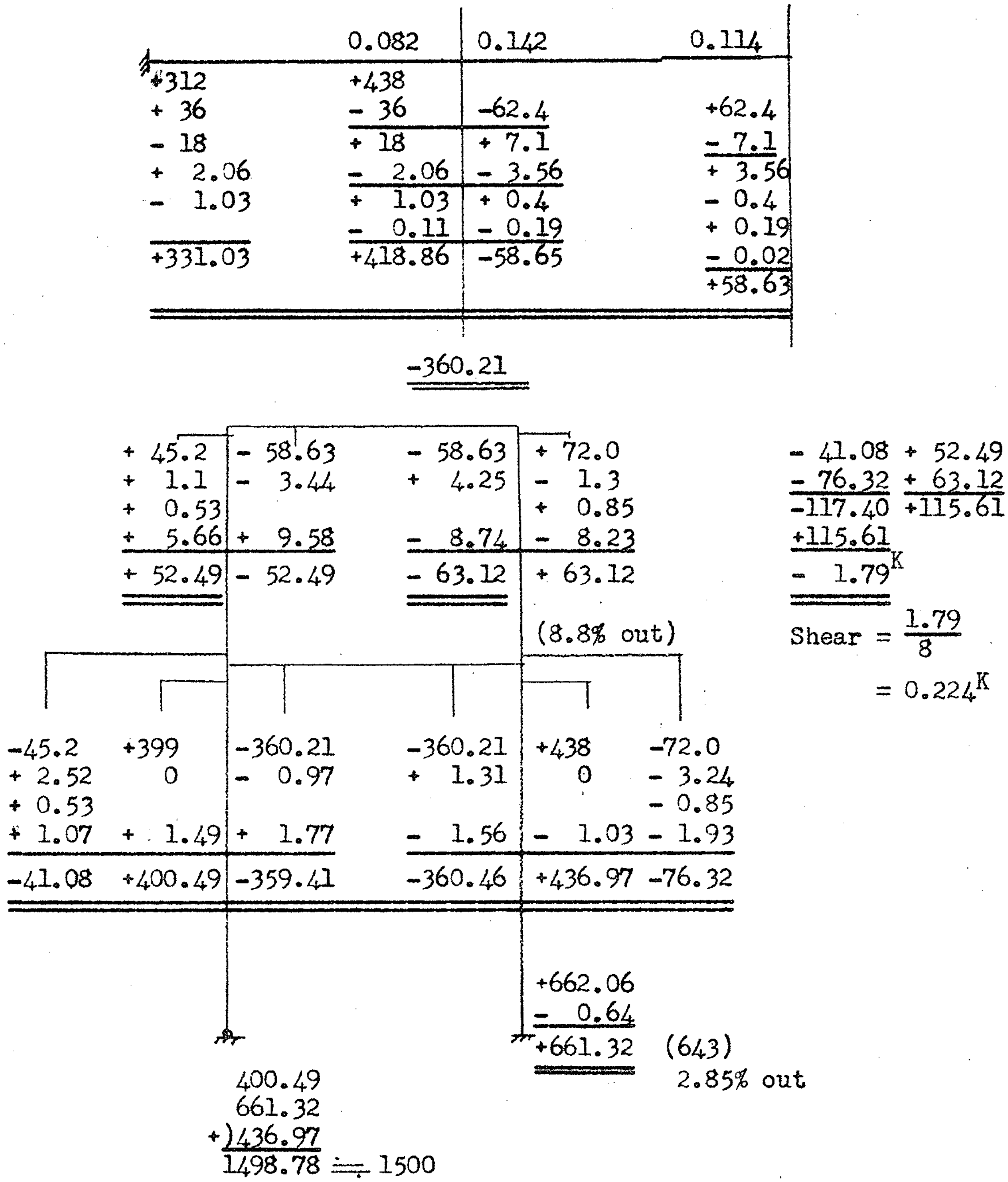
Shear = $\frac{6.715}{8}$

= $0.84^{K} \neq 0$

−43.5	+382	−345.02	−345.02	+420	−69.3	+383.39
+ 2.36	0	− 0.95	+ 1.33	0	− 3.06	+419.33
− 0.55	0	0	0	0	− 0.875	+696.54
+ 1.95	+ 1.39	+ 2.32	− 1.02	− 0.67	− 1.37	
−39.74	+383.39	−343.65	−344.71	+419.33	−74.61	+1499.26 ≒ 1500

+697.16
− 0.62
(+643) +696.54 (85% out)

The resulting end moments are within an error of 10%. But the shearing force is not equal to zero. It might be acceptable.



Actually, the carryover factor from the top column of the bottom story is not equal to -1, because it is no longer a cantilever beam. The fixity at the base is partially restrained. Finally, the resulting end moments is still within 10% of error and the static equilibrium condition is much better.

P. 46

Use the same method for solving Vierendeel Trusses.

A Vierendeel girder is similar to a truss in that it has top and bottom chords with vertical web members, but differs from a truss in that it has no diagonal web system.

Vierendeel girders are used as bridges or parts of a building.

The Vierendeel girder is very useful for building construction either as the roof system or as a long span girder requiring rectangular openings.

The parallel chord Vierendeel girder may be considered as a special closed box section of multiple span but with point reactions, or it may be considered as similar to a single span multiple story building frame loaded with horizontal loads.

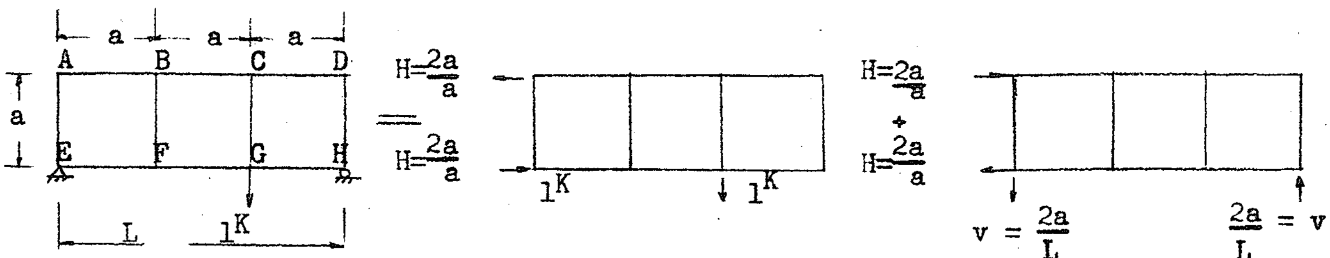


Fig. 1 Fig. 2 Fig. 3

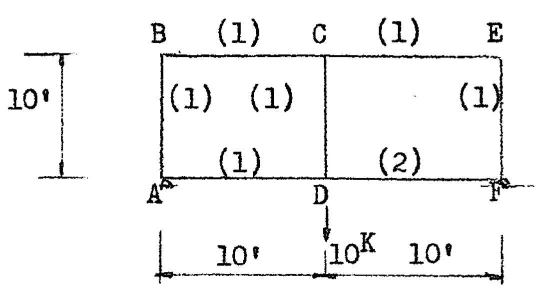
Fig. 1 illustrates how the parallel chords Vierendeel girder may be considered as a single span multiple story frame, subjected to a unit vertical force at one intermediate panel point and to the simple beam reaction at the end as shown. The effect of the two forces may be computed separately and then added as shown.

Note from Fig. 2 that the two "H" reactions are equal, but opposite in sign, due to two loading cases shown. Although the sum of these two "H" reactions will always be zero we must not make the mistake of assuming the members, AE, BF, etc. have no deflection. The similarity of the one span building frame and the Vierendeel girder fails in this one respect because in the building frame joints "A" and "E" have no linear deflection with respect to each other, while in the Vierendeel girder joint "A" does not remain vertically over joint "E" except for symmetrical girders with symmetrical loading.

Example. Two-bay unsymmetrical Vierendeel Trusses is simply supported at A and F. The K-values are shown in opposite the members. A 10^K vert. load is applying at joint D as shown in Fig. 1

$$M = 6\ E\ \frac{1}{L^2} = \frac{6K}{L}\ E$$

$$\left.\begin{array}{l} M_{BC} = M_{CB} = M_{AD} = M_{DA} = \frac{6}{10} \\ M_{DF} = M_{FD} = -6\ \frac{x2}{10} \\ M_{LE} = M_{E} = -6\ \frac{x1}{10} \end{array}\right\}\ \text{Fixed end moments for girders}$$



Solve by Cross Method

Fig. 1

<u>Holding Joint E</u> to prevent sidesway

0	+100	+100	0	-100	-100	0
-50	- 50	- 25	+ 6.5	+ 16.6		+33.4
-18.7	+ 0.3	+ 0.6	+ 0.6	+ 0.6	+ 33.3	+33.3
+ 9.2	+ 9.2	+ 4.6	+ 1.55	+ 1.17	+ 0.3	- 4.98
- 2.77	- 1.22	- 2.44	- 2.44	- 2.44	+ 2.34	- 2.34
+ 1.99	+ 1.99	+ 1.0	+ 0.41	+ 0.48	- 1.22	- 0.71
- 0.64	- 0.31	- 0.63	- 0.63	- 0.63	+ 0.96	+ 0.96
+ 0.47	+ 0.47	+ 0.24	+ 0.11	+ 0.13	- 0.31	- 0.22
-60.45	+ 60.45	- 0.16	- 0.16	- 0.16	+ 0.26	- 0.26
		+ 78.25	+ 5.94	- 84.25	- 64.36	+64.35

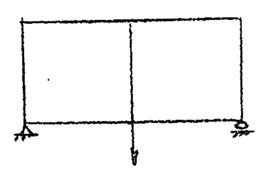
0	+100	+100	0	-200	-200	0
-25		- 18.7		+ 66.7	+133.3	+66.7
-37.5	- 37.5	+ 13.0	+13.0	+ 26.0	+ 13.0	+16.6
+ 4.6	+ 6.5	- 2.77	+ 0.3	- 9.86	- 19.74	- 9.86
- 5.55	- 5.55	+ 3.09	+ 3.09	+ 6.16	+ 3.08	+ 1.17
+ 1.0	+ 1.55	- 0.64	- 1.22	- 1.42	- 2.82	- 1.42
- 1.27	- 1.27	+ 0.82	+ 0.82	+ 1.64	+ 0.82	+ 0.48
+ 0.24	+ 0.41	- 0.16	- 0.31	- 0.43	- 0.87	- 0.43
- 0.32	- 0.32	+ 0.22	+ 0.22	+ 0.44	- 73.23	+73.24
-63.80	+ 63.82	+ 94.86	+15.9	-110.77		

Fig. 2

Solve for vertical holding force.

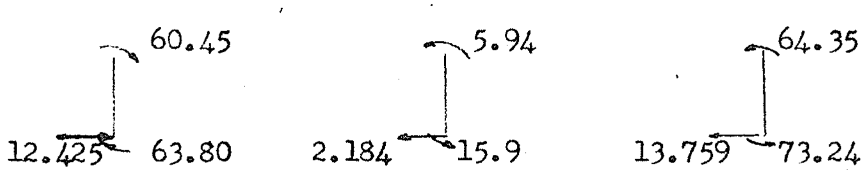
60.45	- 84.25
63.82	-110.77
78.25	- 64.36
+94.86	- 73.23
+ 297.38	-332.61

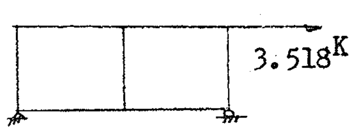
29.738K   33.281K

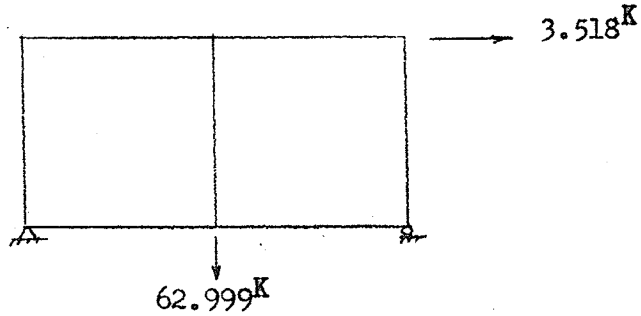


33.261
+29.738
+62.999K

Solve for horizontal holding force.







Holding forces diag.

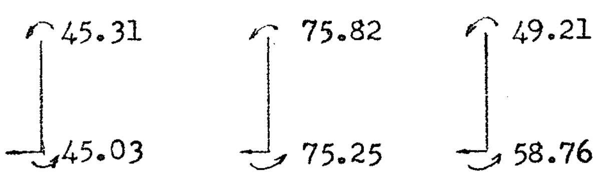
Holding Joint D to prevent vertical deflection

+100	0	0	+100	0	0	+100
- 50.0	-50.0	-25.0	- 8.6	-25.0	- 50.0	- 50.0
- 18.7	- 6.9	-13.8	- 13.8	-13.8	- 6.9	- 12.5
+ 12.8	+12.8	+ 6.4	+ 0.3	+ 4.8	+ 9.7	+ 9.7
+ 0.5	- 1.92	- 3.84	- 3.84	- 3.84	- 1.92	+ 2.1
+ 0.71	+ 0.71	+ 0.36	+ 2.8	- 0.04	- 0.09	- 0.09
+ 45.31	-45.31	- 1.04	- 1.04	- 1.04	- 49.31	+ 49.21
		-36.92	+ 75.82	-38.92		

+100	0	0	+100	0	0	+100
- 25.0						- 25.0
- 37.5	-37.5	-18.7		-12.5	- 50.0	- 25.0
+ 6.4	- 8.6	-17.2	- 17.2	-34.4	- 17.2	+ 4.8
+ 1.1	+ 1.1	+ 0.5	- 6.9	+ 4.1	+ 8.2	+ 4.2
+ 0.36	+ 0.3	+ 0.6	+ 0.6	+ 1.1	+ 0.55	- 0.04
- 0.33	- 0.33	- 0.16	- 1.92	- 0.20	- 0.40	- 0.20
+ 45.03	-45.03	+ 0.57	+ 0.57	+ 1.14	-58.75	+ 58.76
		-34.39	+ 75.25	-40.76		

Fig. 3

Solve for Horizontal Force.



45.31
45.03
75.82
75.25
49.21
58.76
349.38

$$349.38 \times \frac{1}{10} = 34.938^{K}$$

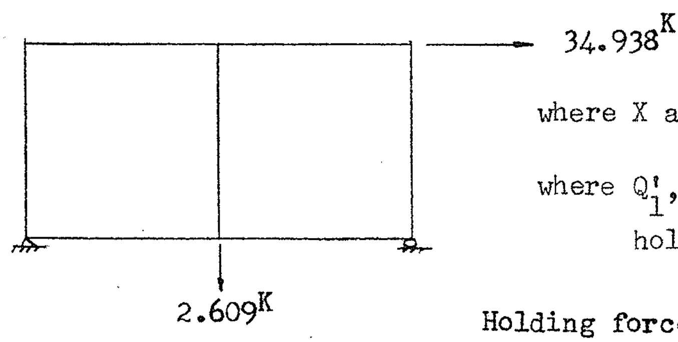
Solve for Vertical Holding Force.

45.31 (———) 36.92   38.92 (———) 49.31

45.03 (———) 34.39   40.76 (———) 58.75

90.34 (———) 71.31   79.68 (———) 108.06

16.165   18.774



where X and Y are correction factors

where Q_1', Q_1'', Q_2' and Q_2'' are holding forces

Holding forces diag.

$$YQ_1' + X\,Q_1'' - 10 = 0$$

$$YQ_2' + X\,Q_2'' + 0 = 0$$

$$62.999Y + 2.609X - 10 = 0 \text{ - - - - - - - - - - } (1)$$

$$+3.518Y + 34.928X + 0 = 0 \text{ - - - - - - - - - - } (2)$$

From (2)

$$Y = \frac{-34.938}{3.518}\,X \text{ - - - - - - - - - - - - - - - } (3)$$

Sub (3) in (1)

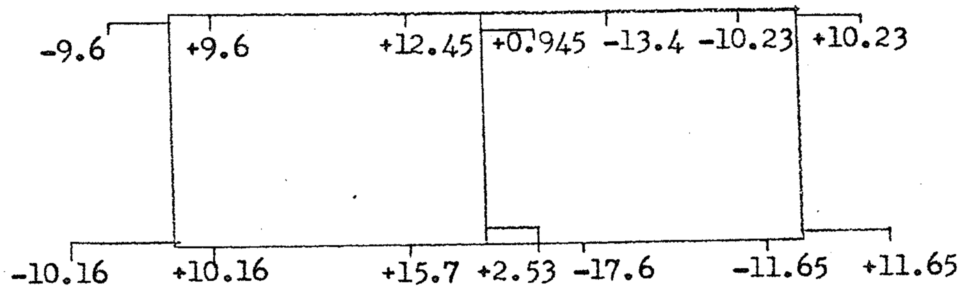
$$+63.0\left(-\frac{34.938}{3.518}\,X\right) + 2.609\,x - 10 = 0$$

$$-627\,x + 2.609\,X - 10 = 0$$

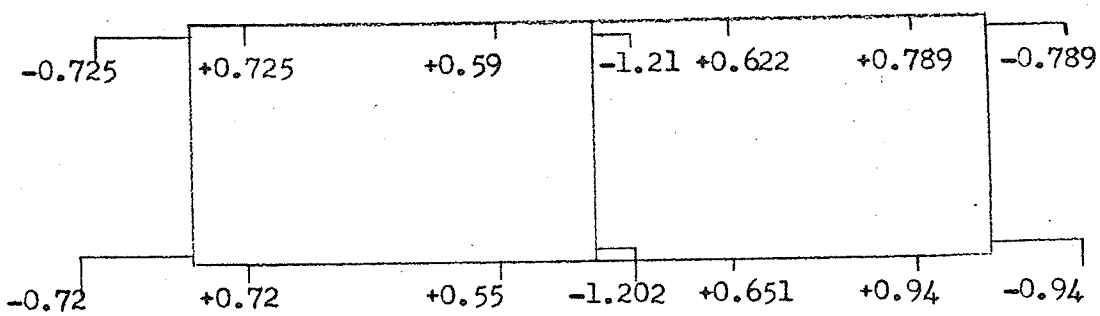
Solve for X and Y

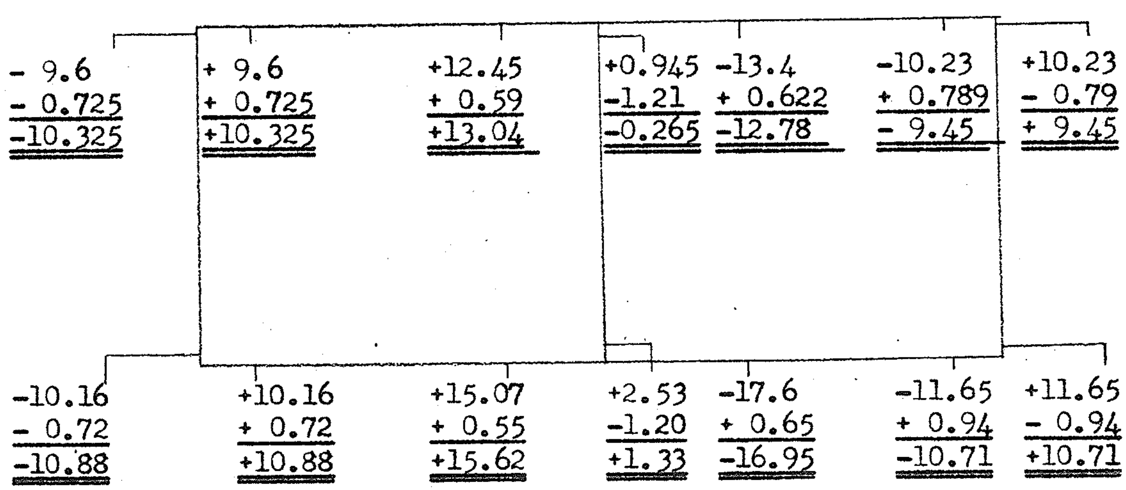
$$\therefore \begin{cases} X = -\dfrac{10}{624.39} = -0.016 \\ Y = 0.159 \end{cases}$$

Multiplying Fig. 2 by $Y = 0.159$



Multiplying Fig. 3 by $X = -0.016$





Final end moments by exact method

Solving the same Vierendeel Trusses by Approx. Method. Case 1.

This structure can be split into two structures as shown in the following figure. The results are then obtained by the use of the principle of superposition.

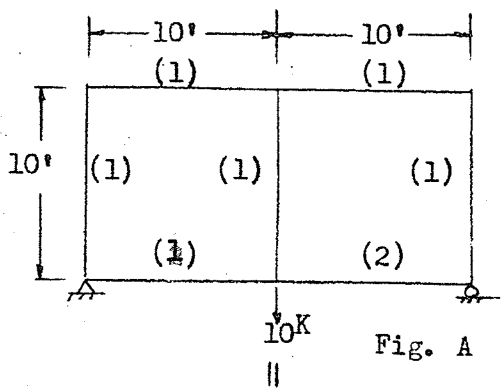


Fig. A

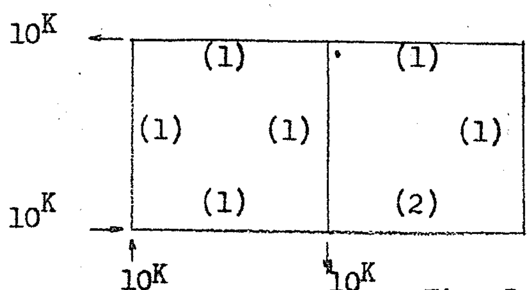


Fig. B

+

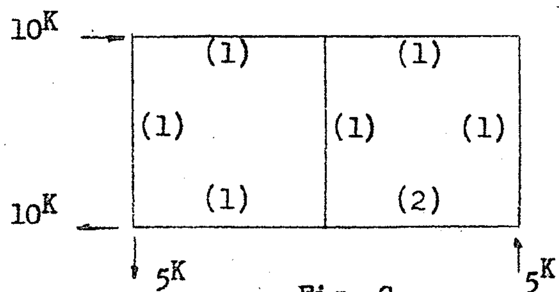


Fig. C

Analysing Fig. B and Fig. C (use the same procedures as the preceding examples).

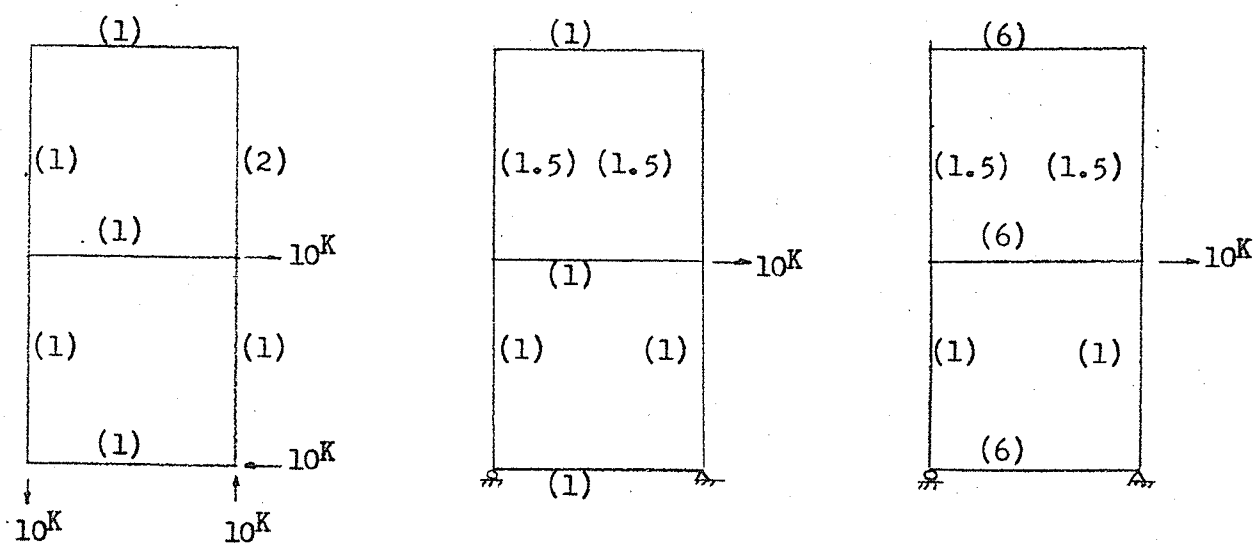
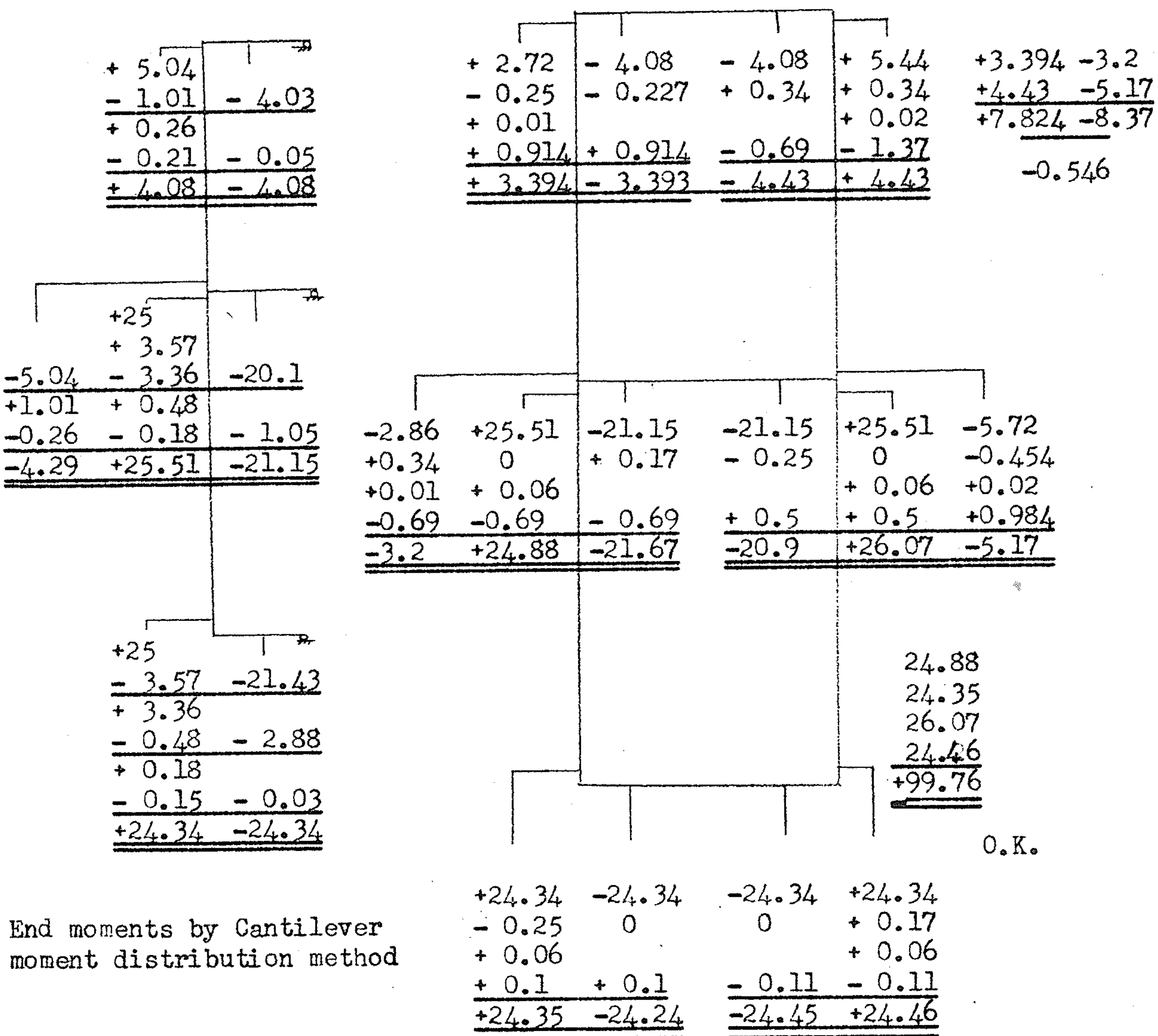


Fig. B Modified K-values



End moments by Cantilever
moment distribution method

Fig. B-1

End Moments by Approx. Shear and Moment Method

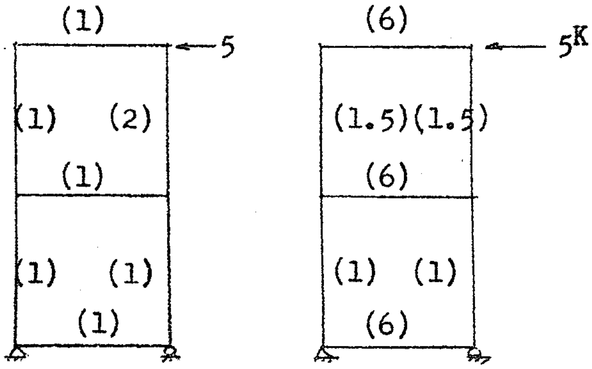


Fig. C Modified structure

-12.5	
- 4.41	
+ 3.38	+13.53
- 0.986	
+ 0.197	+ 0.788
-14.319	+14.318

-14.319
-10.484
-24.803

-12.5	-12.5	
+ 4.41	+ 2.94	+17.65
- 3.38	- 2.21	
+ 0.986	+ 0.658	+ 3.95
-10.484	-11.112	+21.60

-11.112
-13.794
-24.906

-12.5	
- 2.94	
+ 2.21	+13.23
- 0.658	
+ 0.094	+ 0.564
-13.794	+13.794

End Moments by Cantilever moment distribution method.

- 9.55	+14.32	+14.32	-19.1
- 0.58	+ 0.80	- 1.19	+ 0.88
- 0.35	0	0	- 0.70
- 2.32	- 2.32	+ 1.93	+ 3.86
-12.80	+12.80	+15.06	-15.06

-12.80
- 9.37
-15.06
-11.84
-48.07 ≒ 50

O.K.

- 7.0	-11.11	+21.6	+21.6	-11.11	-14.0
- 1.19	0	+ 0.44	- 0.58	0	+ 1.59
- 0.35	+ 0.11	0	0	+ 0.11	- 0.7
- 0.83	- 0.83	- 0.83	+ 0.64	+ 0.64	+ 1.26
- 9.37	-11.83	+21.21	+21.66	- 9.82	-11.84

11.83
14.03
9.82
13.53
-49.21 ≒ 50

-13.8	+13.8	+13.8	-13.8
- 0.58	0	0	+ 0.44
+ 0.11	0	0	+ 0.11
+ 0.24	+ 0.24	- 0.27	- 0.27
-14.03	+14.03	+13.53	-13.53

O.K.

Fig. C' End moments by Approx. Shear and Moment Distribution Method

Fig. D = Fig. B' + Fig. C'

+ 3.394	- 3.394	- 4.43	+ 4.43
-12.80	+12.8	+15.06	+15.06
- 9.406	+ 9.406	+10.63	-10.63

- 3.2	+24.88	-21.67	-20.9	+26.07	- 5.17
- 9.37	-11.83	+21.21	+21.66	- 9.82	-11.84
-12.57	+13.05	- 0.46	+ 0.76	+16.25	-17.01

+24.35	-24.24	-24.45	+24.46
-14.03	+14.03	+13.53	-13.53
+10.32	-10.32	-10.93	+10.93

Final End Moments

Check shear in horizontal direction

10.32 — 13.05 12.57 — 9.406

10.93 — 16.25 17.01 — 10.63

21.25 — 29.30 29.58 — 20.036

$$\begin{array}{r} 21.25 \\ \underline{29.30} \\ 50.55 \end{array} \times \frac{1}{10} = 5.055^K \qquad \begin{array}{r} 29.58 \\ \underline{20.036} \\ 49.616 \end{array} \times \frac{1}{10} = 4.9616^K$$

$$\text{Total vertical forces} = \begin{array}{r} 5.055 \\ +)\underline{4.916} \\ 9.971 \end{array} \doteqdot 10^K$$

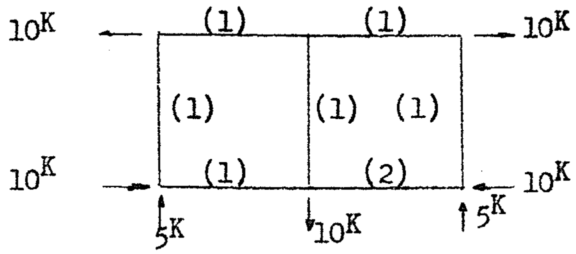
O.K.

Check shear in vertical direction

$$\begin{array}{rr} +\ 9.406 & -\ 0.46 \\ +10.630 & -10.32 \\ \underline{+\ 0.76} & \underline{-10.93} \\ +20.796 & -21.71 \end{array} = 0.914 \times \frac{1}{10} = 0.0914^K \doteqdot 0$$

Negligible

Fig.A can be solved as Fig. 2 by approx. moment distribution method



Case 2

Fig.A. The external loadings are combined (As in Fig. B and Fig. C in the preceding example)

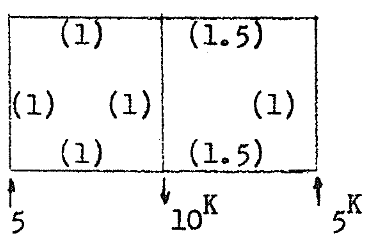


Fig. 1

Modified structure

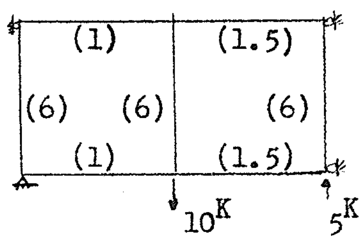


Fig. 2

Relative K-values

0	+12.5	+12.5	0	-12.5	-12.5	0
-10.7	- 1.78	+ 1.78	0	- 2.5	+ 2.5	+10
-10.7	+10.7	+ 0.08	+ 0.51	+ 0.13	-10.0	+10.0
		+14.36	+ 0.51	-14.87		

Fig. 3 Moment distribution by Cantilever M.D. Method

-10.7	+10.7	+14.36	+ 0.51	- 9.91	- 6.67	+10.0
0	- 0.83	0	+ 0.62	- 0.83	- 0.83	+ 0.55
	+ 0.16	+ 0.16		- 0.34	- 0.34	
+ 0.33	+ 0.33	- 1.52	- 1.52	- 1.52	- 1.35	- 1.35
-10.37	+10.37	+13.0	- 0.4	-12.6	- 9.19	+ 9.2

-10.7	+10.7	+14.36	+ 0.51	-19.83	-13.3	+10.0
0	+ 0.62	0	- 0.83	+ 1.1	+ 1.24	- 0.83
	+ 0.16	+ 0.16		- 0.68	- 0.68	
- 0.39	- 0.39	+ 1.28	+ 1.28	+ 2.55	+ 2.38	+ 1.19
-11.09	+11.09	+15.8	+ 1.06	-16.86	-10.36	+10.36

Fig. 4 Shear and Moment Distribution

Check vertical shear

$$
\begin{array}{r}
+10.37 \\
+11.09 \\
+13.00 \\
\underline{+15.80} \\
+50.26 \\
\times \frac{1}{10} = 5.026
\end{array}
\qquad\qquad
\begin{array}{r}
-12.60 \\
-16.86 \\
-\ 9.19 \\
\underline{-10.36} \\
-48.91 \\
\times \frac{1}{10} = 4.891
\end{array}
$$

Total Vertical Loads

$$
\begin{array}{l}
= 5.026 \\
\ \ \underline{4.891} \\
\ \ 9.917^{K} \doteq 10^{K}
\end{array}
$$

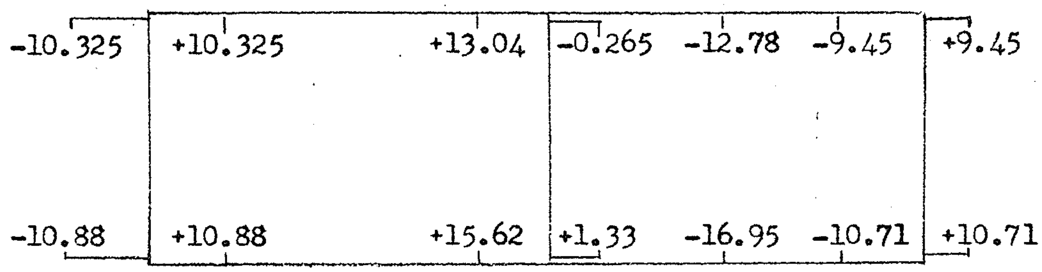
O.K.

Check horizontal shear

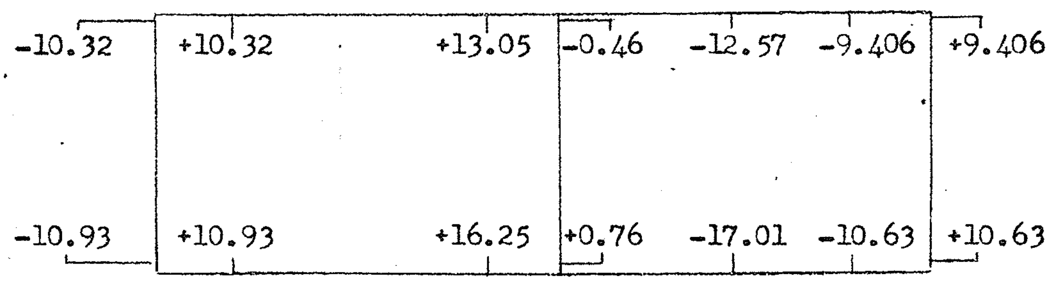
$$
\begin{array}{r}
-10.37 \\
-11.09 \\
\underline{-\ 0.40} \\
-21.86
\end{array}
\qquad\qquad
\begin{array}{r}
+\ 9.2 \\
+\ 1.06 \\
\underline{+10.36} \\
+20.62
\end{array}
= 1.24^{K'} \times \frac{1}{10} = 0.124^{K} \neq 0
$$

<u>Negligible</u>

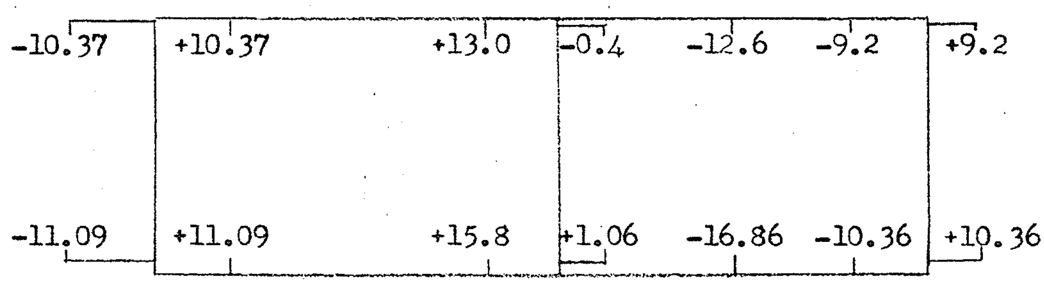
The following diagrams give the end moments resulting from different methods which can be compared with the exact method.



By Exact Method



Case 1. By approx. method using principle of superposition



Case 2. By approx. method but modifying supporting condition. The end moments in the middle web member is out of 51% $\left(\frac{0.4 - 0.265}{0.265} = \frac{0.135}{0.265} = 51\%\right)$. This amount of moment is negligible. But the other members moments are very satisfactory.

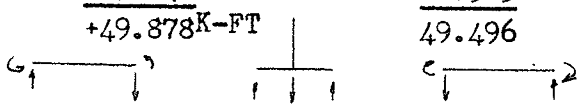
Re-calculate end moments in Fig. 4 of Case II.

-10.7	+10.7	+14.36	+ 0.51	- 9.91	- 6.67	+10.0
0	- 0.83	0	+ 0.62	- 0.83	- 0.83	+ 0.55
+ 0.245	+ 0.16	+ 0.16	+ 0.245	- 0.34	- 0.34	+ 0.245
+ 0.208	+ 0.208	- 1.602	- 1.602	- 1.602	- 1.472	- 1.472
-10.25	+10.25	+12.918	- 0.237	-12.68	- 9.312	+ 9.312

-10.7	+10.7	+14.36	+ 0.51	-19.83	-13.3	+10.0
0	+ 0.62	0	- 0.83	+ 1.1	+ 1.24	- 0.83
+ 0.245	+ 0.16	+ 0.16	+ 0.245	- 0.68	- 0.68	0.245
- 0.512	- 0.512	+ 1.22	+ 1.22	+ 2.429	+ 2.217	+ 1.108
-10.97	+10.97	+15.74	+ 1.245	-16.981	-10.523	+10.523

Shear for chord members

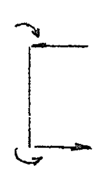
10.250	12.680
12.918	16.981
10.970	9.312
+15.740	10.523
+49.878 K-FT	49.496



4.9878
4.9496
$+9.9374^{K} \doteq 10^{K}$

Shear for web member

-10.250
-10.970
- 0.230
+ 9.312
+ 1.245
+10.523
- 0.077 K-FT



0.0077^{K}

$$\frac{0.265 - 0.237}{0.265} = 10.57\%$$

The above calculation procedures are the same as the preceding examples. Except one step which is the shear adjustment in correcting the vertical and horizontal directions. And the final end moments is reduced from 51% to 10.6% of inaccuracy.

DISCUSSION :

For very high buildings, in general, the large number of equations (one for each story) makes a direct solution rather cumbersome and time consuming.

As a result of the extreme complexity of the problem and the high degree of statical indeterminacy an exact analysis for wind loads on multistory rigid frame is generally impractical.

Now let us take a look at the Cantilever moment distribution method. It is a direct method of permitting joint rotation, and joint translation to take place as an integral part of the process of method distribution.

In many structures involving identical columns or chords (for Vierendeel trusses), complete freedom of sidelurch is permitted by Cantilever moment distribution method without any special sidesway correction. This is the unique characteristic of producing no change in shear during moment distribution. It is an excellent device for solving a symmetrical, single-bay, multistory frame with anti-symmetrical lateral loads which is giving final results to a high degree of accuracy. Continuing this proceedure would give results of converging towards any desired degree of accuracy. Cantilever distribution for a symmetrical, multi-story structure as shown in Fig. 1.

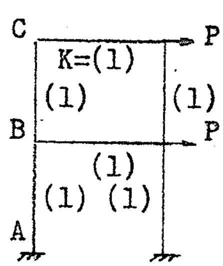


Fig. 1

Columns AB and BC meeting at joint B are considered to act as a cantilever which its free end joint B and its far ends, A and C, are considered fixed against rotation. To maintain rotational continuity at joint B, the total fixed-end moment is distributed in accordance with their relative stiffness (K-values), and to avoid a linear discontinuity at joint B, two of the three joints A, B and C must provide freedom for joint translation laterally.

When any joint in a symmetrical multistory frame is being balanced, the two columns meeting the joint are dealt with as if they were cantilevers with fixed far ends that require freedom of side-lurch at the joint being balanced and at the next joint above, to permit double cantilever deflection. All other joints beyond the next joint above the one being balanced translate the same amount as does the next joint above the one being balanced.

Note: Cantilever moment distribution method can not be applied to solve unsymmetrical structures. However, we have found it to be simple, convenient and sufficiently accurate in solving for a symmetrical rectangular rigid frame.

Morris method is simple to apply and it is satisfactory for regular frames, but it is slow in convergency when the stiffness of a column is large in comparison with that of the adjoining girder. The moments are not self-checking as in the case of rotation, neither is the sidesway directly determined. If we require a high degree of accuracy the method must be repeated for several cycles.

Using the approximate moment distribution method the desired end moments are obtained. A close check is obtained at each joint, and there is almost a perfect agreement with the exact solution, given in each example. The above method has been found to be convenient and sufficiently accurate in many cases to determine end moments in the rectangular rigid frame. It is especially useful in the analysis of a complicated unsymmetrical framework.

The disadvantage of this method is the columns' length must be equal and the supporting conditions must be the same.

This method also requires a certain amount of preliminary work, such as modifying the structure and using the Gantilever moment distribution method to get the first approximation of deformation. Then the Morriss method is applied.

In the preceding examples, some of the final end moments exceed 12% compared with the exact solution. Minor discrepancies are not serious and may be disregarded.

However, this method is a greatly simplified process compared to the analysis by elastic equations.

The above analysis, deformations due to axial forces are generally neglected.

In a tall building, the wind moments may be seriously affected by the changes of the column lengths due to their axial loads. Especially when the building is very high and has not many bays. It is usually necessary to take the secondary moments into account. The lengthening of the windward columns and the shortening of the leeward columns are computed, and the fixed end moments can be determined in each girder. Then distribute and balance, and finally add the resulting moments to the original end moments.

Rectangular Vierendeel trusses can be solved by the above method. The details of calculation are shown on the example. Vierendeel truss is usually in symmetry. In our example, we are considering an unsymmetrical Vierendeel truss with vertical load applying at the middle joint. The resulting end moments are very satisfactory. Unsymmetrical, multi-panel Vierendeel truss can be solved in the same way.

The Vierendeel truss allows the use of one or more stories to accommodate its depth over a large span without interfering with internal communication or external openings.

A Vierendeel truss in which the lower chord forms the ceiling of the ground story while the top chord forms the roof support of the second story is ideal in the construction of markets and garages which require a long clear span for the first floor and office space on the second floor.

Welding is especially useful for this construction, as large moments are concentrated at the joints of horizontal and vertical members.

CONCLUSION:

The Approximation Method cannot be applied to an irregular structure or a regular frame with irregular loading condition. It can only be applied to a rectangular framework with equal length of columns. In a one bay, multistory frame, the loading condition must be anti-symmetrical. In a multi-bay framework, the loading condition must be acting at the joints.

The carry over factor in the bottom story of frames with hinged and fixed ends is uncertain. In the example (p.43) a frame with two columns, we release 1/2 of the carry over moments from the top of the columns at the base because of the hinge. The final end moments are within 10% of inaccuracy. If a frame with three columns, one is pinned and the rest are fixed ends, the release end moment may be approximately equal to 1/3 of the carry over moments from the top of the columns to the base.

In (p.60) the Vierendeel truss, those end moments are shown very satisfactory with shear adjustment in two directions.

There are several excellent methods of wind stress analysis. They are: (a) Morris Method, (b) Method of Shear Adjustment or The Witmer Method of K-percentages etc. These approximation methods have a high percent of accuracy. The decision on which is the best and most convenient method depends on the individual.

From the examples which have been solved on Vierendeel Truss, it can be seen that we can apply the Morris Method or K-percentages Method for solving the same type of Vierendeel Trusses. Very good results would be obtained from these methods.

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