THE CONSTRUCTION AND EVALUATION OF A PROGRAMMED UNIT OF PLANE GEOMETRY

A Thesis

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by

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THESIS ABSTRACT

The Problem. The purpose of this study was to compare the effectiveness of a short programmed unit in plane geometry with conventional teaching methods. Specifically, the study was conducted to determine whether there were any significant differences in (1) mean achievement and (2) mean time between two groups of students; one group instructed by programmed materials and the other, for control purposes, by the writer. Student attitudes to programmed instruction were also investigated.

The study took place at St. Paul's High School in the fall of 1963 and lasted for a period of three weeks. The subjects, twenty-five pairs of grade eleven students distributed at random to two classrooms, were initially matched for mental ability and grade ten achievement, subject sequence, and sex.

Both groups studied the same subject matter, "The Areas of Polygons," as found in the prescribed grade eleven, university entrance course text book, A First Course in Plane Geometry. The writer wrote the program and it was revised and developed to its present form through intensive testing with three students.

The Method. The experimental group used only the programmed materials, although teacher help was available

to those students asking for it. Specific homework assignments were not given to the students of the experimental group; the students were instructed, rather, to answer approximately 140 frames per week in order to complete the program in three weeks. Three weeks is the normal time that was usually taken to teach this unit of geometry by conventional means.

Achievement was measured by a criterion test, constructed by the writer and validated by a competent authority.

Each student recorded, on prepared forms, the amount of time he devoted to geometry exercises during the experiment.

Student reaction to programmed instruction was measured by an attitudinal questionnaire.

Findings. No significant difference was found in mean achievement between the two groups. However, the programmed instruction group took significantly less time than the control group to complete the unit of work--approximately one and one-half hours per student.

Approximately two-thirds of the students preferred programmed to traditional instruction; compared to traditional instruction, programmed instruction made learning less difficult, and less homework was required.

Conclusions. The fact that the programmed instruction students did as well as their counterparts instructed conventionally by the writer, appears to indicate that short programmed units can be used in place of regular classroom instruction, and without changing the normal classroom routines. Although statistically significant, it is doubtful whether the difference in mean time, one-half hour per week per student, is educationally significant.

The favourable student reaction to programmed materials is encouraging for further experimentation. One drawback to such experimentation appears to be the lack of programs in which the subject matter closely corresponds with that of the prescribed courses.

ACKNOWLEDGMENTS

The writer is indebted to his advisor, Professor R. Hedley, who acted as a source of inspiration and provided valuable assistance in the compilation of this thesis.

Special thanks are due also to Rev. St. Claire Monaghan, S.J., who encouraged the writer to investigate the field of programmed learning.

Apologies are extended to those writers who spell programmed with one "m." For the sake of uniformity, in this thesis it has been spelt with two "m"s.

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CHAPTER I

THE INTRODUCTION

This study is concerned with the feasibility of employing a short programmed sequence in place of regular classroom instruction, without disrupting the normal "lockstep" classroom procedures. Two groups of students were taught the same unit of geometry by two different methods. One group was instructed by programmed instruction and the other group, for control purposes, by traditional methods.

I. THE PROBLEM

Statement of the problem. This study attempted to determine whether, in the handling of the same unit of geometry by the two groups, there were any significant differences between them in (1) mean achievement and (2) mean time.

Importance of the study. When programmed materials have been used for regular classroom instruction, the results obtained and reported were both positive and negative. A recent survey by the Canadian Teacher's Federation[1]

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[1] *A Survey of the Use of Programmed Instruction in Canadian Schools, 1962-3.* Research Memo No. 12. (Ottawa: Research Division Canadian Teachers' Federation, 1963), p. 7.

implies, however, that these results contradict each other because of the different ways in which the programmed materials were used. Thus, the materials may have been used exclusively or they may have been supplemented by the teacher; the students may have been allowed to take the programmed materials home or they may have worked at them in school only; the students may have had to complete the program by a predetermined date or they may have been permitted to work entirely at their own rate. In short, these and other conditions may not have been uniform; the evidence about them is incomplete.

A review of the literature indicated a dearth of detailed reports by teachers who employed programmed materials in the classroom. This study, therefore, attempted, by carefully delineating the conditions under which the program was employed, to supplement the existing knowledge on the use of programmed materials in the classroom.

II. DEFINITION OF TERMS

Achievement score refers to that score made by the student on a criterion test and reflects his mastery of the subject matter.

Mean achievement is the average of the set of achievement scores. In this study two such means were calculated: one for the programmed instruction group and the other for the traditional instruction (control) group.

Criterion test was the examination administered to the students after instruction in order to measure their knowledge of the subject matter presented by the two methods.

Subject matter was the content presented to both the control and experimental groups. In particular it was a unit of geometry dealing with the areas of plane rectilinear figures as it is found in Chapter II of the text, A First Course in Plane Geometry.[2]

Time or time spent is the total number of minutes during which the student was formally engaged in attempting to master the subject matter. For the programmed instruction group it meant the time which a student took to work through the program plus any other time which he spent studying the subject matter. The sum of these two mutually exclusive measurements constituted a time or time spent observation for each student of the programmed instruction group.

For each student of the control group, time or time spent, was also the sum of two mutually exclusive time measurements. One addend was the total time that each

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[2]W. J. Oliver, P. F. Winters, and F. A. Hodgkinson, A First Course in Plane Geometry, (Regina: School Aids and Text Book Publishing Co. 1954), pp. 189-202.

student spent in geometry classes during the experiment. The other addend was the time that each student spent studying or doing geometry assignments outside of geometry classes.

Mean time is the average of a set of time observations.

Programmed Instruction is a teaching method which uses systematically arranged materials in place of live instruction by a tutor. According to some authors, its distinguishing characteristics are the self-determination of pace, the presentation of small bits of information, active response by the learner, an immediate feedback for each response, and a low rate of error.

Program or programmed material is the subject matter arranged into a series of sequential steps.

Programming is the process of composing a program.

A frame is a single step in a program; it presents a small amount of information. In addition, the frame contains a statement which the student must complete or a question which he must answer. Both the statement and the question are related to the information already supplied by the frame. Thus the frame partially resembles a test item in its form but its purpose is different. It does not seek to discriminate between students (as does a test item) but rather to eliminate error in the student's progress through the whole program.

The stimulus is the technical name given to the information presented in the frame.

The response is the student's answer to the question posed in the frame. When he supplies the answer to the question or completes the statement, he is said to be responding to the stimulus.

Feedback informs the student about the correctness of his response and occurs immediately after he has responded. Feedback, or reinforcement, as it is called, increases the probability that the student will make the correct response to the same stimulus in the future. The cycle of presentation-answer-feedback, or technically, stimulus-response-reinforcement, repeats itself throughout the program.

Self-pacing is a characteristic of programmed instruction which allows each student to proceed through a sequence of frames at his own rate. The rapid learner is not held back and the slow learner is not left behind.

External-pacing refers to the outside regulation of the rate at which the student proceeds through a programmed sequence. This is in contradistinction to self-pacing where the student sets his own pace.

Traditional Instruction or conventional instruction implies no specific method of teaching. Where this study is concerned, traditional instruction included the use of lecture demonstration and the question-answer type methods, as well as private tutorial assistance to those students requesting it.

The University Entrance Course is a programme of studies set up by the Department of Education, and, when successfully completed by the student gives him a standing which is a prerequisite for entrance to the University of Manitoba. The geometry which was taught in order to form a basis for this study is a part of the Department's University Entrance course.

Ordinary or regular classroom practices were taken to mean those which are usually found in an average classroom. They are as follows: (1) All students proceed through the subject matter in a lockstep fashion, completing a unit of work at the same time in order that they may be examined on it at the same time; (2) the rate at which the subject matter is presented is determined by the teacher and is usually geared to the average student; (3) students who fail to master the work may receive remedial assistance.

Significant Differences. Differences between the means of the control and experimental groups were considered significant if they fell into that range of differences which by chance could occur less than five times out of a hundred as determined by a *t*-test. (See Chapter IV).

III. PLAN OF STUDY

The remainder of the chapter is devoted to a brief exposition of the procedure followed in the solution of the problem, and notes some limitations which arose as a result of following this procedure.

Subjects. The students of the experimental and control groups were selected from a total of 108 grade eleven students enrolled at St. Paul's High School, September 1963. From this total twenty-five pairs were matched for subject sequence, scholastic ability, and sex. Each of these matched students was then assigned at random to one or other of two classrooms; one class was designated as the control group, and the other as the experimental. The control group was taught by the writer and the experimental group by programmed instruction. A complete description of the matching procedure is found in Chapter IV.

The program. The program employed in this study was constructed by the writer, partly because he wanted to experiment with programming, and partly because it was more economical than buying a commercial program. Other advantages came as a result of this procedure. First, it helped to equate the material presented to both groups. As the writer was both the programmer and the teacher, he was able to structure his classroom presentation to resemble the program presentation. This was relatively easy, since the program evolved from his experience in teaching the subject matter. Secondly, the teacher variable was eliminated, because the same person was responsible for the instruction of both groups. Thirdly, the writer, by choosing to construct his own program was able to exercise greater freedom in the selection of the subject matter. Lastly, by describing the construction of the program, a feature which so many commercial programs lack, the probability of this study's making a contribution to the science of programming was increased. The construction of the program is described in Chapter III.

The procedure. Before instruction was begun, a standardized geometry test was administered to establish the previous knowledge of the subjects. Section I, Form A, of the "Cooperative Geometry Test" was employed. (See Appendix).

When instruction was begun, the subjects of the experimental group were permitted to work at programs at home, as well as during regular geometry periods. They were informed that approximately 140 frames should be done each week. At this rate the program instruction group would complete the program in three weeks--the time it would take the control group to cover the same unit of work.

Ordinary classroom practices were followed in instructing the control group.

After the completion of instruction, a criterion test, which measured achievement, was administered simultaneously to both the control and experimental groups. It was constructed by the writer and validated by a member of the Department of Education High School Examination Board.

During the day following the administration of the criterion test, an attitudinal questionnaire was administered in order to assess the student attitudes to programmed instruction. This questionnaire was administered to the students of the experimental group only.

Prior to writing the criterion test, the students submitted records of the time they had spent working on the programmed materials, or, in the case of the control group, the time they had spent doing out-of-class work on the subject matter.

<u>Hypotheses</u>. Using as a basis a short unit in geometry, this study attempted to determine whether, at the end of the experiment there were any significant differences between the control and experimental groups in (1) mean time and (2) mean achievement. Specifically the following null hypotheses were tested.

(1) The means of the criterion test scores for the control and experimental groups are not significantly different at the five per cent level of significance as measured by a *t*-test.

(2) The difference in mean time between the control and experimental groups is not significant at the five per cent level as determined by a *t*-test.

IV. ASSUMPTIONS AND LIMITATIONS

In following the procedure outlined above, several assumptions were made. It was assumed that:

1. each student kept a reliable record of his time;

2. in answering the attitudinal questionnaire, each student of the experimental group stated what he believed rather than what he thought he should believe;

3. the program taught those objectives that the criterion achievement test attempted to measure;

4. the students working on the program would do so in accordance with the instructions provided at the beginning of the program;

5. each student in grade eleven possessed the background knowledge of geometry demanded by the program;

6. the method used to construct the program would result in a program capable of satisfactorily teaching the intended population.

An apparent weakness in the last two assumptions suggests a possible limitation. All the students used to validate the program had a passing grade in grade ten geometry, but all the students using the program may not have had this qualification. A passing grade in mathematics is based on a combined geometry and algebra score; a student consequently could enter grade eleven having failed geometry in grade ten.

A second limitation concerns the applicability of this study. Although it may be pointed out that St. Paul's follows the same academic curriculum, writes the same external examinations, and is inspected by the same set of inspectors as the public schools, St. Pauls' is, nevertheless, a private boy's school, and thus the results of this study are applicable to it only.

A third limitation is the size of the sample used in the experiment; statistically speaking, it is small.

In summary of the chapter it may be said that the problem has been identified, the terms unique to the study defined, the hypotheses proposed and some limitations noted. The following chapter will review the programmed instruction literature pertinent to this study.

CHAPTER II

REVIEW OF THE LITERATURE

Programmed instruction is an ancient art, dating back to the time of Socrates, whose dialogues with his students took this form.[1] Pressy, in the 1930s, used a form of instruction which could be classified as programmed.[2] Most literature on the subject is, however, of recent origin. After the publication of Skinner's article, "The Science of Teaching and the Art of Learning," in the Harvard Educational Review (Spring 1954), there has been a small flood of articles and texts on the subject of programmed instruction. In fact, 1961 saw the publication of a journal devoted solely to that subject; "Journal of Programmed Instruction." Several source books containing research and other writings have been published. The most comprehensive of these is "Teaching Machines and Programmed Learning--a Source Book," edited by Lumsdaine and Glaser. Most of the articles in both the journal and the source books have been written by psychologists for psychologists, giving the impression that programmed

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[1] Jerome P. Lysaught and Clarence M. Williams, A Guide to Programmed Instruction, (New York: John Wiley and Sons, 1963), p. 3.

[2] A. A. Lumsdaine and Robert Glaser (eds.), A Guide to Programmed Instruction, (Washington: National Education Association of the United States, 1960), p. 47.

instruction is the exclusive domain of the psychologist; few if any of the articles deal with classroom applications of this teaching technique. On the other side of the ledger, Quakenbush[3] has made a small survey of the classroom applications of programmed learning; so has the Canadian Teachers' Federation in Canada, and the Center for Programmed Instruction in the United States. Several writers such as Lysaught and Williams, have published books aimed at instructing teachers in programming.

While the latter type of literature was of greater interest to this study, all the available literature was examined in an attempt to find answers to the following questions.

1. Does the method of programming the subject matter affect the ability of the program to teach?

2. Does the device employed to present the programmed materials affect the quality of instruction?

3. Does the ease with which the materials can be programmed vary with the nature of the subject matter?

4. Does supplementing programmed instruction by conventional instruction help or hinder the student?

5. Does external pacing adversely affect student achievement?

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[3] Jack Quakenbush, "How Effective are the New Auto-Instructional Materials and Devices?" IRE Transactions on Education, No. 4, December 1961, pp. 145 - 151.

I. METHODS OF PROGRAMMING

No one method of programming appeared to be superior. To the best of the writer's knowledge, no study exists which attempted to compare different methods of programming. Opinions, however, do exist.

For the purpose of the ensuing discussion, programming methods were taken to include the programming model and the rules or principles of programming.

The Programming Model. The programming model refers to the skeleton or framework of the completed program. It is independent of the subject content, or the order in which the content is presented. It refers to the way in which the frames are joined to each other (frame sequence); it also refers to the way the responses are made (mode of responding).

Student responses to the information presented in the frames can be either written-in (this is called the constructed response) or selected (this is called the multiple choice response).

The sequence of frames can be either linear or branching. In a linear program the sequence of frames is fixed and all the students work through the same sequence. In a branching program all the students do not follow the same sequence of frames; instead, each student follows a sequence determined by his responses.

Branching programs utilize the multiple-choice mode of responding, while linear programs are usually associated with the constructed response.

Figure 1 shows a schematic model of a linear program. The circles represent frames and the arrows indicate that there is only one path for the student to follow regardless of his response.

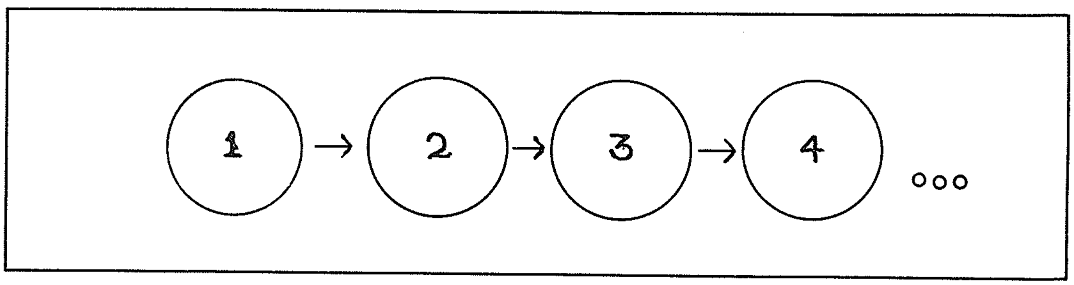

FIGURE 1

A LINEAR MODEL

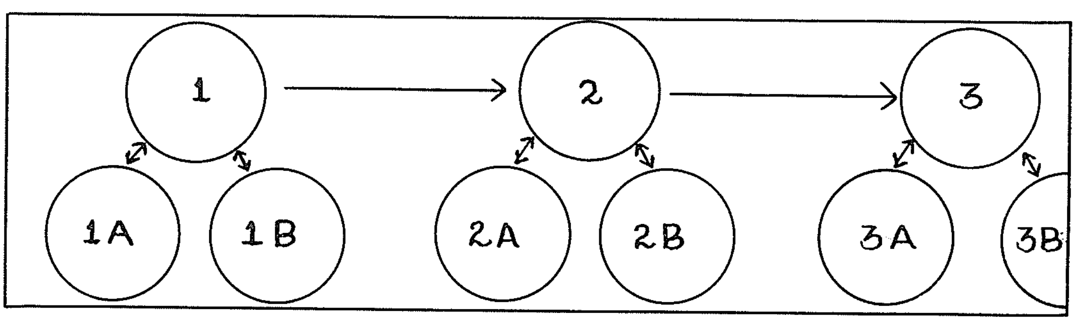

FIGURE 2

A BRANCHING MODEL

Figure 2 illustrates a model of a typical branching program. The student after reading the information in frame 1 tests his understanding of the information by selecting one of several answers (three in this case). A selection of the correct answer directs him to frame 2; an incorrect response directs him to either of the remedial frames, 1A of 1B. These are designed to help him understand the reason for his error. Suppose his initial response directed him to frame 1A. After reading the remedial information presented in frame 1A he would be redirected to frame 1 to make another selection. If he again made an incorrect response, he would be directed to frame 1B, and then back to frame 1. This cycle could repeat itself throughout the program.

Which model is superior? An examination of the literature revealed that opinions varied only slightly. The following statement made by Lysaught and Williams[4] is typical.

> ....most programmers would agree that more experimentation will be necessary before anyone can speak with true authority on the merits of various paradigms |programming models|. It is sufficient to point out that each of these paradigms has been used effectively in preparing programmed units for use in particular situations. For the teacher it becomes a matter of matching selection, assumptions and objectives to a desirable model.

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[4] Jerome P. Lysaught and Clarence R. Williams, A Guide to Programmed Instruction, (New York: John Wiley and Sons, 1963.) p. 89.

Although Hughes[5] feels it is unwise to become committed to one model to the exclusion of the other, he offers a concrete suggestion to programmers seeking a model.

> The kind of program you write should be geared to the principal type of behaviour you are trying to teach--recall or recognition. If the student must later recall the material verbatim without any prompting, the Skinner approach is generally more appropriate. |The design using a linear model and constructed response is credited to Skinner.|

The non-committal attitude of the programming experts to a particular model is indeed remarkable when one considers that in 1962, seventy-nine percent of the 122 commercial programs available for school use employed no branching.[6] The obvious reason for the preponderance of linear programs is that they are easier to write. Carr[7] is of the opinion that the construction of a branching program places a special burden on the author of the program, and on the teaching device used. He states, "Unfortunately no experiment has been done which compares the two methods

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[5] J. L. Hughes, Programmed Instruction for School and Industry, (Chicago: Science Research Associates, 1962) p. 15.

[6] The Center for Programmed Instruction, A Guide to Programmed Instructional Materials, (Washington: Government Printing Office, 1962) p. xii.

[7] Wendell I. Smith and J. William Moore, (eds) Programmed Learning, (Toronto: D. Van Nostrand Co., 1962) p. 68.

of programming directly." Faced with a lack of experimental evidence to the contrary, it is possible that the programmer will choose a linear model because it makes construction easier.

In a branching program, as in a linear one, each frame must contain a small amount of information which will cause the student to respond correctly. For the linear program a frame is considered adequate if it elicits the correct response most of the time. If the ratio of incorrect to total responses exceeds a certain limit, say ten percent, the frame is revised until this criterion is met. The branching program, on the other hand, assumes that errors are part of the normal learning process. Consequently, in constructing a certain frame, the programmer must envisage the most likely erroneous responses. He must then proceed to write remedial frames which explain why these responses are erroneous, and thus set the student back on the right track.

Moreover, the linear program requires a simpler device to present it than does the branching program. In a linear program all students proceed through the same sequence of steps. Therefore a device similar to the ordinary spools of a camera, where the film represents the paper on which the frames are written, is sufficient to present a linear program. It should be clear that a branching program, with its variety of paths which a student may take, requires a somewhat more complex mechanical contrivance to present it.

In spite of the fact that it appears easier to construct programs using a linear rather than a branching model, many branching programs have been constructed. This leads to the hypothesis that the selection of a particular programming model is a function of the programmer's individual preferences.

Two other factors which may lead to the selection of one programming model in preference to the other are: (1) the type of terminal behaviour desired (recognition or recall), and (2) the type of device available to present the program.

Rules or problems of programming. Since the programming literature contained no specific directions for selecting a programming model, it is not very surprising to find that no rules or principles for the construction of a _good_ program exist. This is not to say that no rules exist. They do, but, they do not guarantee their user a successful program. Carr has reviewed the rules of several authors and combined them into a single set of principles. He notes, however, that "these principles of programming... simply constitute problems which the programmer faces when he attempts to compose a program."[8]

The first step in writing a program is to specify precisely the terminal objectives of the program. The problems presented in the program should represent the kind of problem that the learner is expected to solve and the response in the program should approximate those that will be ultimately required of the student.

Secondly, the programmer must specify or assume the initial repertoire of the learner upon which he is to build the program. Good teaching begins with what the student knows.

Thirdly, he must decide the best order in which the individual frames are to be presented so that the student attains the desired terminal behaviours.

Fourthly, the programmer must now specify the size of step. The size of step can be defined operationally in two ways--as the number of steps in a program which takes the learner from the initial to the terminal behaviours, or, by the percentage of incorrect responses. The latter definition assumes that if the size of step is small the learner makes few errors. Evans, Glaser, and Hommes[9] have verified the fact, that, within limits, decreasing

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[8] A. A. Lumsdaine and Robert Glaser, (eds.), Teaching Machines and Programmed Learning--A Source Book, (Washington: National Education Association of the United States), p. 556.

[9] Ibid. pp. 447-451.

the size of step by increasing the number of frames has resulted in a decrease of errors in the program itself; the students also scored better on a post criterion test.

Fifthly, the programmer must decide on the amount of repetition necessary to guarantee adequate learning and maintenance of the behaviours already learned.[10]

It should be clear that the above steps serve only to indicate the problems that a programmer faces; they offer few or no solutions. Hence Green[11] comments, "In all honesty no one can prescribe a set of rules for successful writing of a program in a specific area." And Gilbert[12], speaking of the authors of programming principles, adds, "They may be mostly wrong. Use their principles only as a starting place.", and Mager[13] elucidates, "...a good program is one that works, rather than one that conforms to some particular writing style or strategy." Even Skinner,[14] the author of a set of principles admits, "...that a considerable measure

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[10] Ibid. pp. 447-451.

[11] Edward J. Green, The Learning process and Programmed Instruction, (New York: Holt, Rinehart and Winston, 1962.) p. 139.

[12] Lumsdaine and Glaser (eds) op. cit. p. 479.

[13] Robert F. Mager, Programming Methods, IRE Transactions on Education, December 1961, p. 151.

[14] Lumsdaine and Glaser (eds), op. cit. p. 151.

of art is needed in composing a program."

In constructing a program, it is essential that it be tested on a student. If he attains the desired terminal objectives then the program is on its way to becoming a good one. If the student does not attain the desired objectives, the programmer has failed and must start again. Once a program adequate for one student has been developed, it is tested on a second student, and then revised, and then tested on a third student and then revised, and so on, until the program is judged adequate for the student population which it is intended to teach. Gilbert[15] estimates that after fewer than ten trials and subsequent revisions a program will be produced that will teach ninety-eight percent of the students.

II. TEACHING MACHINES AND PROGRAMMED TEXTS

A teaching machine is a device employed to present the program. A device, it should be noted, is simply a means of communicating the programmed materials to the student; it should not be confused with the technique of programmed instruction, as defined in Chapter I.

Generally speaking, there are two ways by which the program may be presented to the student. They are by teaching machine or programmed text. In this study a

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[15] *Ibid*. p. 480.

teaching machine is understood to be a mechanical or electrical device that controls the presentation of the frames, keeps a record of the students' answers, and provides an immediate feedback by displaying a correct answer.

Teaching machines vary greatly in complexity. They range from the mechanical devices which present the program on sheets of paper, cards or disks, to the electrically operated machines which present the program on films or tapes or a combination of both.

Programmed texts on the other hand are relatively simple. On the surface they resemble an ordinary text or workbook. The frames are arranged in ordinal number sequence down a page. To the information presented in each frame, the student responds in a blank just below the frame or on a separate sheet of paper. The correct answer to each frame is provided, and it is placed just before the succeeding frame, or to the left or right of the succeeding frame. The student uses a shield to cover the answers. As he works each frame he moves the shield down to uncover the answer to that frame.

Variations are possible. For example, a horizontal, as opposed to the vertical arrangement just described, would have frame 1 on page one, frame 2 together with the answer to frame 1 on page three, frame 3 together with the answer to frame 2 on page five, and so on. If the book has a total of ten pages, frame 5 will fall on page nine, and frame 6 on page one just below frame 1. When the odd numbered pages are filled, the even numbered pages could be used in the same manner.

A comparison of teaching machines and programmed texts reveals that former have two basic disadvantages. The first is their high initial cost. Machines can cost from ten to 20,000 dollars (excluding the programmed materials which they present), whereas, the initial cost of the text book involves just the paper upon which the programmed materials are presented. Secondly, machines are prone to breakdowns. Breakdowns mean repairs, which could be costly. But even more serious is the fact that breakdowns mean an interruption of student training.

On the other hand, machine manufacturers claim that machines are cheat proof--a feature not possessed by the programmed text books. The program is locked inside the machine and mechanical controls ensure that only one frame is presented at a time. When the student responds to any one frame the machine proceeds to expose the next. In moving from one frame to the next, the machine covers, with a transparent plastic shield, the student's last answer. At the same time the correct answer is also exposed. The shield prevents the student from changing his answer and thus a reliable record of the student's performance is kept.

In a programmed text, however, the student is free to look at the answers before responding to the question; he can therefore copy the answers. He can erase answers and omit questions at will. The programmed text has no control over these actions. Nevertheless, according to Homme and Glaser[16], cheating occurs infrequently, and hence has little importance.

Proponents of machines have also stated that the machines, unlike programmed texts, maintain student interest because of their mechanical aspects. "The toylike quality of machines may have an enhancing effect upon motivation and thus contribute to an improvement in teaching," says Stolurow.[17] Several studies have compared machine teaching and programmed texts.

Studies conducted at Deer Park and South Huntington School Districts, Long Island,[18] the Hanover Junior High

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[16] Eugene Gallanter, (ed.), Automatic Teaching: The State of the Art, (New York: John Wiley and Sons, 1959), p.105.

[17] Lawrence M. Stolurow, Teaching by Machine, (Washington: U.S. Government Printing Office, 1961), p. 56.

[18] Lassar G. Gotkin and Leo S. Goldstein, Programmed Instruction for the Young Learner: A Comparison of Two Presentation Modes in Two Environments, (New York: The Center for Programmed Instruction, 1962), pp. 1-8.

School, New Hampshire,[19] the Collegiate School, New York,[20] and New York University,[21] involving students from the elementary to college levels, all found that the student mastery of subject matter is independent of mode of presentation. Equally effective learning resulted regardless of whether the student was taught by machine or text.

It appears that neither the motivational nor the cheat proof aspects of the machine is significant when related to student achievement.

Moreover, it is interesting to note that students using machines complained of breakdowns, and that those using programmed texts with a horizontal format complained of "too much turning of pages," while those students using texts with a vertical format made no complaints at all.[22]

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[19]Lewis D. Eigen, Robert T. Filep, Leo. S. Goldstein, and Bruce W. Angalet, A Comparison of Three Modes of Presenting a Programmed Instruction Sequence, (New York: Center for Programmed Instruction, 1962), pp. 1-40.

[20]Lewis D. Eigen and P.K. Komoski, Research Summary Number 1, (New York: The Center for Programmed Instruction, 1960), pp. 1-11.

[21]Millicent Alter and Robert E. Silverman, Response Mode, Pacing and Motivational Effects in Teaching Machines, (New York, Dept. of Psych., N. Y. U., 1961 cited by J. L. Hughes, <u>Programmed Instruction for Schools and Industry</u>), Chicago: Science Research Associates, 1962), p. 41.

[22]Eigen and others, <u>op</u>. <u>cit</u>. p. 8.

III. SUBJECT MATTER

Are some subjects easier to program than others? Apparently so, if the actual number of programs in a subject area can serve as an indicator. A survey conducted by the Center for Programmed Instruction revealed that sixty-two per cent of the programs available to educators were in mathematics or science, compared to fourteen per cent in the social sciences and language arts. No programs were available in the liberal arts area.[23]

Lysaught[24] agrees that the number of programs in a given subject area is an index of the ease of programming that subject area. He states, "Generally, the more logical the subject matter, the more easily it can be programmed. That is why there are several programs in algebra and so few in the social sciences."

Geometry, then, because it possesses an internal logic, should be easily programmed; the logic of the subject matter facilitates the ordering of the items. Nevertheless, only three per cent of the mathematics programs

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[23]Center for Programmed Instruction, (comp.) Programs '62 A Guide to Programmed Instructional Materials for Educators, (Washington: Government Printing Office, 1962), p.xxiii.

[24]Stuart Margulies and Lewis D. Eigen, (eds.) Applied Programmed Instruction, (New York: John Wiley and Sons, 1962), p. 39.

were in geometry as compared to twenty-two per cent in algebra.[25] The reason for this surprising wide difference seems to be nothing more profound than the personal choices made by the programmers. Many more of them were interested in algebra rather than in geometry.

Of course it is possible, or even probable, that the individual tastes of the programmers are a reflection of their conception of geometry as a difficult subject to program. Lewis, for example, imagines that it may be impossible to construct a good linear program in geometry, particularly one which teaches a student to solve geometry problems, for there are many kinds of procedures which will lead to a solution of a problem. The student must develop an ability to recognize those cues embedded in the problem which will enable him to select the kinds of procedures most likely to be successful.[26] The fact that a variety of different and potentially good procedures exist suggests that a branching model is better suited to teaching geometry.

On the other hand, to the best of this writer's knowledge, no geometry program utilizing the branching model has been published.

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[25]Center for Programmed Instruction, Op. cit., approximate calculations performed by the writer.

[26]Maurice Goldsmith (ed.) Mechanization in the Classroom, (Toronto: Ryerson Press: 1963), p. 91.

IV. ROLE OF THE TEACHER

When programmed materials are employed in place of regular classroom instruction, what is the role of the teacher? Various writers have commented on this question. While it appears that the teacher will enjoy more free time, his services are not dispensable.

Fusco[27] states,

> ...the classroom teacher may be freed from the burdensome and time consuming tasks of presenting materials and taking precious class time to repeat material for students who didn't get it the first time.

Instead, his free time will be utilized in giving personalized tutorial assistance to each pupil. Lysaught and Williams state, "...the teacher can devote a much larger part of his time to counselling, guiding, assisting and stimulating the individual learners."[28]

With the increased use of programmed materials, Ramo foresees the evolution of a new brand of teacher--a "teaching engineer." Highly specialized in both his subject field and programmed instruction, the teacher will make it his task to

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[27]Wendell I. Smith and J. William Moore, (eds.) Programmed Learning, (Toronto: D. Van Nostrand Company, 1962), p. 227.

[28]Jerome P. Lysaught and Clarence M. Williams, A Guide to Programmed Instruction, (New York: John Wiley and Sons, 1963), p. 154.

supplement, modify and revise teaching devices and programs.[29]

While it is obvious that the teacher will have more free time in which to render private assistance to individual students, is it possible that the rendering of such assistance will not result in increased learning? Is it possible that the students would learn equally well from programmed instruction alone? Several studies has investigated the effects of varying the degree of teacher interaction with programmed materials. One such study was the Roanoke experiment.

Using mathematics programs, it involved approximately 900 students. Three separate treatments were imposed. One group received conventional teaching; a second group used programmed materials with no help from the teacher; the third group worked on the programmed courses with help from the teacher being available at all times. Both the "help" and "no help" groups had a lower failure rate than the "traditional" group, but no consistent differences were found between them. Although teachers reported more opportunity to work with students of the "help" group, the availability of teacher assistance appears to have had no

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[29]A. A. Lumsdaine and Robert Glaser, (eds.) Teaching Machines and Programmed Learning--A Source Book, (Washington: National Education Association of the United States), p. 379.

effect on student achievement.[30] Availability does not imply utilization.

Klaus found that using a high school physics program to supplement regular classroom instruction resulted in higher achievement. He states, "...auto-instructional methods can produce increments in achievement even when substantial efforts have been made to maximize learning."[31] He further reports no significant difference in achievement between groups taught by program only, and those taught in the traditional manner. On the basis of this study it appears that maximum student achievement is had when both programmed and traditional instruction are employed conjointly.

The Canadian Teachers' Federation surveyed sixteen studies which compared programmed with regular instruction. Of these sixteen studies, eight reported that programmed instruction was superior to traditional instruction, and seven reported that traditional instruction was superior to programmed instruction. The survey notes, however, that all eight studies reporting on the inferiority of programmed

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[30] Jack Quackenbush, How Effective Are the New Auto-Instructional Machines and Devices? _IRE Transactions on Education_, No. 4, December 1961, p. 145.

[31] _loc. cit._

methods took place under conditions of minimal teacher assistance; and in four of the seven studies which indicated that programmed methods were superior to traditional instruction, programmed sessions were interspersed with conventional instruction.[32]

Assuming that the above studies are well designed, the following conclusion is suggested by the evidence which they submit. Help, in the form of normal instruction, may result in improved student achievement. In cases of minimal teacher assistance, there is a conflict in the results observed between the Roanoke experiment (superior to traditional) and the Canadian Teachers' Federation survey (inferior to traditional).

V. SELF-PACING

According to some writers, a chief feature of programmed instruction is that each student is allowed to "absorb" the subject content at his own pace. The rapid learner is not held back and the slow learner is not left behind. This feature is called self-pacing. It implies that a student may take as much time as he needs to read, assimilate and answer each frame of the program. Theoretically, there is no limit to the time he may take to pro-

[32]Research Division, A Survey of the Use of Programmed Instruction in Canadian Schools, 1962-3, (Ottawa: Canadian Teachers' Federation, 1963), p. 24.

ceed through a sequence of frames or answer an individual frame.

But this interpretation is hardly practical in a classroom situation, where courses must be completed within a certain time period. Nor would it be a satisfactory view if speed of performance were an element in the skills taught.[33] It appears reasonable to exert some sort of control over the self pacing aspect of programmed learning.

There are two ways in which a time limit could be imposed. If the speed of performance were a factor, a limit could be set on the length of time that each frame would be exposed. During a certain time period, two minutes for example, the student would be expected to read, assimilate and respond to the material presented in a particular frame. At the end of two minutes the frame would be replaced by its successor and the cycle repeated. Or, if gross rather than atomic control were desired, a time limit, such as ten class periods, could be set for the completion of the whole program or a unit of the whole program.

Control over the rate at which the subject matter is presented is called external pacing. Unlike self pacing, external pacing sets a limit on the length of time that each

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[33]Eugene Gallanter, (ed.) Automatic Teaching, The State of the Art, (New York: John Wiley and Sons, 1959), p. 9.

frame or a sequence of frames is exposed to the student.

Alter and Silverman[34] used an eighty-three frame program on basic electricity to compare the effects of self and external pacing. In the self paced group each student worked through the program at his own rate; in the externally paced group each frame was exposed to the students for a similar period of time. The length of time that each frame was exposed was geared to allow all students, including the slow workers, ample time in which to respond. A criterion test was administered to the students after they had completed the program. No significant differences in achievement were found although the students who worked at their own pace were generally finished sooner than those who were externally paced.

Using the same electricity program, Alter and Silverman compared self pacing with a rate of external pacing which allowed students an excessive amount of time on each frame. There were no significant differences in achievement.[35]

By an efficiency measure based on test score, test time, and training time, Follettie found self pacing to be superior to external pacing. The external pacing was based

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[34]Millicent Alter and Robert Silverman, "The Response in Programmed Instruction," The Journal of Programmed Instruction, Vol. I, pp. 55-78.

[35]Ibid. p. 73.

on the average reading rate of the group and hence was somewhat fast for the slow readers.[36]

On the basis of the above studies it was tenuously concluded that external pacing does not adversely affect student achievement unless the rate of presentation is too rapid. It should be noted that in the experiments described, control was exerted over single frames rather than groups of frames. The writer was not aware of any attempt to evaluate achievement under the later condition.

VI. SUMMARY

It should now be possible to answer the questions raised at the beginning of this chapter regarding the successful use of programmed materials in the classroom. The questions were: (1) Does the method of programming the subject matter affect the ability of the program to teach? (2) Does the device employed to present the programmed materials affect the quality of instruction? (3) Does the ease with which the materials can be programmed vary with the nature of the subject matter? (4) Does supplementing programmed instruction by conventional instruction help or hinder the student? (5) Does external pacing adversely affect student achievement?

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[36] J.F. Follettie, Effects of training response mode, test form and measure on acquisition of semi-ordered factual materials. Research Memorandum 24, Fort Benning, Ga., April 1961, Cited by Millicent Alter and Robert Silverman, Ibid,,p.74.

Based on the evidence reviewed the following conclusions are drawn.

1. A present no functional relationship exists between programming methods and the quality of a program. The choice of a particular programming strategy is a function of the programmer's own preferences. If it produces a successful program for him, it may or may not produce a successful program for someone else. The key to writing a good program is the effective utilization of student feedback. A program is tested on a student, then revised on the basis of his responses, and then tested again. This cycle is repeated until the program is judged adequate.

2. The quality of the instruction received, as measured by achievement tests, is independent of the device used to present the programmed materials. It is the materials themselves that determine the quality of instruction.

3. Certain subjects may be easier to program than others. Geometry is probably one of the easier subjects to program because its internal logic predetermines the order in which the frames are presented.

4. The conjoint use of programmed materials and conventional teaching techniques may result in increments in learning which would not occur if either method were used exclusively. No study has yet determined what combination of teacher and programmed instruction yields the most effective results.

5. When ample time is allowed for a student to respond to a frame, external pacing results in learning equivalent to that observed under conditions of self pacing.

This chapter has reviewed some aspects, pertinent to this study, of programmed instruction as found in the literature. The following chapter will deal with the construction of the program employed in this study. In it the writer will attempt to describe how he "followed the rules" or "solved the problems" which every programmer faces in constructing a program.

CHAPTER III

CONSTRUCTION OF THE PROGRAM

The construction of the program is described under the following headings: the selection of the subject matter, the delineation of objectives, and the selection of the programming model.

These headings are in fact the major problems faced in the construction of a program. The manner in which they are solved constitutes a particular programming strategy or method. What follows is then a description of the writer's strategy.

I. SUBJECT MATTER

A primary consideration in selecting the subject matter was the needs of the students. The possibility existed that the students would fail to learn from the programmed materials--as compared to the students receiving conventional instruction. If such a situation developed, the program instruction students would have to be re-taught by conventional means. Re-teaching too long a unit or one that was not self contained would place too great a burden on both the teacher and student. Consequently, it was desirable that the material to be programmed be reasonably short and independent of the remainder of the course.

It was particularly important that the unit chosen for programming be independent of the remainder of the course. Then re-teaching, in the case of failure, could take place at a more leisurely pace, and the students could proceed to the next unit of work unhampered by a lack of subject matter background.

It may appear from the foregoing that, in the interests of the students, the programmed unit be as short as possible. Nevertheless, it was desirable that the programmed materials receive a fair trial; Klaus[1] for example, suspects the validity of results based on programs of fewer than 100 frames. Tentatively, it was decided that a unit which took ten to twelve classes to teach traditionally was of sufficient length.

The unit selected was of this length. The Areas of Polygons[2] is a unit of grade eleven geometry and forms part of the university entrance program in Manitoba. Among the courses taught by the writer, it was the only unit which was of sufficient length and yet relatively independent of the course.

Although this unit was relatively independent of the

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[1]Wendell I. Smith and J. William Moore, (eds.) Programmed Learning, (Toronto: D. Van Nostrand Co., 1962), p. 94.

[2]Oliver, W. J. (et al), A First Course in Plane Geometry, Regina: School Aids Publishing Co., 1954), pp. 189-202.

remainder of the grade eleven geometry course, it did require a prior knowledge of the subject. This prior knowledge was presumably part of the student's repertoire, and was acquired by him during the preceding year while he was in grade ten.

Since the program had to have a starting point, it was necessary to assume the specific concepts and facts that the student knew by virtue of successfully completing the grade ten course. In particular, it was felt that the student attempting the program knew the following:

1. an operational definition of "proof"--that he could, given certain facts, arrive at a required conclusion, using a chain of sound reasoning;

2. that the general enunciation of a theorem consists of two parts, the "if" and "then" clauses, which correspond respectively to the "given" and the "required to prove" of the particular enunciation;

3. the meaning of the word congruent, and could prove triangles congruent by "a.a.s.";

4. the properties of parallel lines and could identify corresponding and interior opposite angles;

5. the definitions of a rectangle and a parallelogram and their properties;

6. the compass and straight edge constructions for bisecting an angle, drawing an angle equal to a given angle,

drawing a line through a given point parallel to a given line, dropping a perpendicular to a given line from a given external or internal point, and drawing an angle of sixty degrees.

For the purpose of developing the program it was assumed that any person satisfactorily completing grade ten mathematics had a knowledge of the above mentioned concepts, definitions and skills. If it became evident (during the preliminary trials of the program) that students did not possess certain concepts, then these concepts would be worked in as part of the normal development of the program.

II. OBJECTIVES

The next step in constructing the program consisted in delineating the objectives of instruction. In order that the objectives be operational and reflect the emphasis suggested by the Department of Education, a survey was made of former final examinations prepared by the High School Examination Board.

This survey revealed that the student was expected to recite theorems and to perform and describe constructions. He was also expected to use these theorems and their corollaries as authorities in solving numerical and theoretical deductions. The emphasis was greatest on the students' ability to solve numerical and theoretical deductions.

On the final June examination in geometry, approximately 65 per cent of the marks were alloted to questions dealing with numerical or theoretical deductions. Twenty five per cent of the marks were alloted to questions which tested the students' ability to recite theorems, and approximately 10 per cent of the marks were assigned to questions which asked the student to solve construction problems.

If the program was to prepare the students for the final exam, it became apparent that the program must attempt to teach the required theorems and their application to numerical, theoretical and construction problems. In addition, emphasis was to be placed on the solution of problems rather than the recitation of theorems.

The next step was to list the specific objectives of the program. In effect these objectives enumerate the behaviour that a student upon the completion of the program should possess. This list would serve to guide the programmer in writing and ordering the individual frames of the program.

The specific objectives of this program are listed below.

The learner upon completion of the program should:

1. have the concept that area is the amount of surface bounded or enclosed by the sides of a polygon;

2. know, by giving examples, that a unit of area can

be arbitrarily determined;

3. know that the sizes of two polygons can be compared by counting the number of times each contains a unit or area;

4. know that two polygons equal in area need not be congruent;

5. realize, upon assuming that the area of a rectangle is equal to the product of its base and altitude, that the area of a parallelogram, as well as the area of a triangle can be calculated from the following theorems:

a) If a parallelogram and a rectangle stand on the same base and between the same parallels, (hence of equal altitude), then they are equal in area (Theorem I);
b) If a triangle and a rectangle stand on the same base and between the same parallels, then the area of the triangle equals one-half the area of the rectangle (Theorem III);

6. be able to recite the above two theorems;

7. in addition to the above two theorems, be able to recite and understand the proofs of the following theorems:

a) If two parallelograms stand on the same base and between the same parallels, then they are equal in area (Theorem II);
b) If two triangles stand on the same base and between the same parallels, then they are equal in area (Theorem IV);

8. be able to apply these theorems in the solution of theoretical, numerical or construction problems;

9. be able to apply the following corollaries of the

above theorems to the solution of theoretical problems;

a) Parallelograms on equal bases in the same straight line and between the same parallels are equal in area;
b) If two parallelograms stand on the same or equal bases and have equal altitudes, then they are equal in area;
c) If two parallelograms of equal area stand on the same base and on the same side of it, then they have equal altitudes and hence lie between the same parallel lines;
d) A triangle is equal to one-half a parallelogram on the same base and between the same parallels;
e) Triangles on equal bases in the same straight line and between the same parallels are equal in area;
f) The median of a triangle divides it into two triangles equal in area;
g) If triangles stand on the same base and on the same side of it, then they have equal altitudes and hence lie between the same parallel lines;

III. THE PROGRAMMING MODEL

Once the specific objectives were delineated, and before the construction of the frames could begin, it was necessary to choose a programming model. As described earlier, a model is the skeleton into which the individual frames are placed. Essentially two different models exist, the linear and the branching forms.

A linear model utilizing the constructed response was chosen in preference to a branching model for the following reasons:

1. The linear model was judged the easier to construct. It would not be necessary, as in a branching model, to estimate the student's response, for the linear, unlike the branching model, need not employ the selection mode of responding.

2. Although several linear programs exist, no branching program in geometry, to the writer's knowledge, has been constructed. Because he was a novice programmer, the writer felt that it would be more prudent to use the linear model.

3. The ultimate objective of the program seemed to favour the constructed mode of responding. The student upon completion of the program was expected to write out complete proofs. He would need practice at this skill. The constructed rather than the selected response would provide this

practice.

Writing and ordering of the frames. It was decided to let the subject matter guide the order in which the frames were written. This is a system particularly well suited to a subject matter such as geometry, whose internal logic predicts to a great extent the order in which its content may be presented. For example, once the theorem, "rectangle and a parallelogram standing on the same base and between the same parallels are equal in area" is proven, it may be used to prove the theorem, "two parallelograms standing on the same base and between the same parallels are equal in area."

While it is true that the order in which theorems are presented is often flexible, and in fact does vary somewhat from text to text, it is equally true that, once their order has been ascertained, the general order in which the frames are to be written has also been determined. The theorems in this program were presented in the same order in which they are found in the text book[3] prescribed for the course.

The following procedure was used in writing the sequences of frames dealing with the theorems. First, the key terms and concepts contained in the theorem were explained. Secondly, the entire theorem was reviewed; the

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[3] loc. cit.

student read it and then answered questions about it. Thirdly, the important steps in the theorem were summarized. Fourthly, the student was asked to write out the theorem on his own. Fifthly, the student was given practice in applying the theorem to the solution of other problems. Last, the corollaries were explained and their application to problems was illustrated.

Now that the general order of the frames was decided upon, the next step consisted in writing the items. To do this the list of specific objectives was consulted. Each objective was considered in turn and expanded if necessary. For example, the first objective states that the student will gain a concept of area as the amount of surface bounded by the sides of a polygon. The attainment of this concept depends upon the student; understanding what is meant by the terms "polygon," "bounded" and "surface." Ultimately, fourteen frames were written to develop this concept.

Another example of how the objectives were implemented into the actual writing of the frames is provided by the theorem, "If a rectangle and a parallelogram stand on the same base and lie between the same parallel lines, then they are equal in area." The three underlined phrases contain three concepts which were developed before the theorem was formally introduced.

Because these three concepts and others like them were required throughout the program, they were developed in the first part of the program, frames 1 - 137.

Initially, the frames were written on index cards, five by seven inches. The stimulus part of the frame was written on one side of the card and the response on the other. After 100 such items had been constructed, a very good geometry student was asked to work through them. The student looked at the item, wrote his response on a separate sheet of paper, and then flipped the card over and compared his response with the one on the back of the card. He was encouraged to think aloud, edit the items, and ask questions whenever he was puzzled.

The writer sat beside the student and observed his progress. Whenever the student made an error, the writer checked the item for faulty mechanics, sentence structure or grammar. The appropriate correction was then made and the student continued. If the error were due to the information presented in the frame, the writer immediately reconstructed the frame, or constructed additional frames which he felt might clear up the student's misunderstanding.

It should be noted that in attempting to clear up the student's difficulty, the writer was careful not to communicate orally with the student. It was desirable that the final version of the program be as independent of a human teacher as possible. Oral communication, while effective with the trial student, might lessen the effectiveness of the program to teach when a teacher was not present. Occasions arose, nevertheless, when the student's difficulty was not cleared up by the above method. It was then necessary for the writer to question the student. In such cases it was generally foundthat the sequence of frames leading up to the poor item had to be revised. Errors of this type occured infrequently.

In this manner the writer, with the assistance of the student, constructed what could be called the gross anatomy of the program. Now all that remained was to refine it by testing it with other students. This process continued until the program was judged adequate for a field trial with the student population for which it was intended.

The second student on which the program was tested was also a very good student generally, but did not have as good a grasp of geometry as the first student. He completed the program; no major revisions were made.

It should be noted that both these students were approximately half way through the grade ten geometry course when they tried the program. As a result they were handicapped by an incomplete background in geometry as defined earlier in this chapter. They were used however, because all the students who had a complete background were at that time in grade eleven and had already taken the unit "The Areas of

Polygons."

It was then decided to employ a student weak in geometry as a criterion for determining the adequacy of the program. He, unlike the others, worked through the program after he had completed the grade ten geometry course. When he completed the program, and then answered correctly three criterion questions from a prior grade eleven June examination, it was felt that the program was ready for a field trial.

The students used in the individual trials were selected on the basis of the writer's observations of their day to day work. It was interesting to note, therefore, that their I.Q.'s were respectively 128, 131 and 125. Their year end averages were respectively 86, 79 and 45 per cent and their final geometry marks were respectively 96, 84 and 59 per cent.

In preparation for the field trial, the items were recopied from the index cards onto standard size spirit duplicating masters. These were "run off" and the resulting materials collated and bound. The program was now in text book form. All the diagrams were reproduced in a separate book. The program and the book of diagrams can be found in Appendix.

Table I which follows was prepared for the purpose of comparing the objectives as listed earlier, with the items

TABLE I

AN INDEX TO THE DEVELOPMENT OF THE OBJECTIVES

FRAMES	OBJECTIVES
1 - 14	Area concept--surface bounded by polygon.
15 - 23	In order to compare the areas of two polygons we count the number of times each contains an arbitrary unit of area.
24 - 29	Instead of counting the number of times a rectangle contains a unit of area, we develop, but note that we do not prove, the formula for the area of a rectangle.
30 - 30	A review frame eliciting the definition of area.
31 - 56	The altitude of a triangle, including the case of the obtuse angled triangle. Noting that the base could be any side of the triangle.
57 - 83	Development of the concept of an altitude of a rectangle and a parallelogram, leading up to the....
84 - 87	...concept that "lying between the same parallel lines" means the same as having the same altitude.
88 - 90	Comparing the sides of rectangles and parallelograms with their altitudes. The altitude of a rectangle is equal to one of its sides; of a parallelogram it is shorter than one of its sides.
91 - 96	Introduction of the concept: standing on the same base.
96 - 119	A review of the concepts established.
120 - 130	Introduction of difference between equality and congruency symbols. Congruency implies equality in area, but the converse does not hold.

TABLE I (continued)

FRAMES	OBJECTIVES
131 - 136	An example of subtracting or taking away areas.
137 - 158	Theorem I and corollary.
159 - 170	Exercises utilizing theorem I as an authority.
171 - 189	A review.
190 - 203	Theorem II.
204 - 206	Corollary of theorem II.
207 - 210	Corollaries 2 and 3 of theorem II.
211 - 237	Exercises based on theorem II and corollaries.
238 - 245	To construct a parallelogram equal in area to a given parallelogram.
246 - 256	A second construction exercise, with reduced cues.
260 - 265	Preliminaries to theorem III. A diagonal bisects a parallelogram.
266 - 283	Theorem III and corollary.
284 - 295	Theorem IV and corollaries.
346 - 351	Questions on overview--a short enrichment passage on the postulational approach in geometry.
352 - 376	Exercises on numerical deductions involving areas of triangles, rectangles and parallelograms.
377 - 388	Application of theorems to novel problems.

of the final version of the program. It shows that the first 137 frames developed the terminology and concepts required for the introduction of the theorems. Frames 138 to 302 developed the first four theorems and their corollaries; examples of how the theorems are applied to problems were also provided. The remainder of the frames were composed of problems in which the number of cues or hints were gradually decreased.

SUMMARY

In summary, the caracteristics of this program are:

1. It was written for grade eleven students enrolled in the University Entrance Course and deals with the areas of polygons.

2. It has a linear paradigm.

3. The responses are constructed and every response is confirmed.

4. The ordering of the items was determined by the logic of the subject matter. Review frames were inserted where the trial students indicated a need for them.

5. Three preliminary trials were run with students of differing geometrical ability. The responses of these students were used as a basis for revising the program.

6. In the field trial the program was presented in text book form, using a vertical format. Diagrams were

reproduced under separate cover.

With reference to point 5 above, the writer wonders if the program would have been substantially different had he selected these students on the basis of their I.Q. rather than their class performance.

Some writers have stated that the programmer will learn a lot about the difficulties that students encounter in his subject. Consequently, programming a unit should shed some light also on how to teach it traditionally. This writer, however, was not aware of any such transfer from programmed to traditional instruction. The reason for this may be that the program as it was originally written, presented the subject matter in such a manner that the trial students were able to absorb it with little difficulty. The fact that these students made only a minimal number of errors due to a lack of understanding seems to substantiate this reasoning. However, this observation is based on the writer's experience with the trial students, and had the trial students been chosen on the basis of some criterion other than class performance, it is possible that the program would have differed.

In the following chapter the design of the experiment is described. The experiment was set up to assess the effects of employing the program in a normal classroom situation.

CHAPTER IV

THE EXPERIMENT

This study attempted to examine the feasibility of employing programmed instruction in place of regular classroom instruction. To determine this end two null hypotheses were proposed.

1. The means of the criterion test scores for the control and experimental groups are not significantly different.

2. The mean time for the control group does not differ significantly from the mean time of the experimental group.

Differences were considered significant at the five per cent level as measured by a *t*-test.

The procedure used in testing these hypotheses--the experiment--is presented under the following headings: obtaining the sample, treatments, measuring instruments, and tests of significance.

I. OBTAINING THE SAMPLE

The population. The statement of the hypotheses implies the identification of the population at which they are aimed. Once the population has been identified, the next step is to select a representative sample.

The population used in this study consisted of all the grade eleven students attending St. Paul's High School in September, 1963. Hence the results of the study will be generalized to these students; however, action based on these results can be taken with only future students. Therefore, assuming no extraordinary change in the type of student enrolling, the population for this study was identified as the present and future grade eleven students at St. Paul's.

The sample was chosen from this population. Circumstances, however, did not permit a purely random selection. Consequently, the generalization of results to the population may be limited. It could be pointed out--not in defence of the practice, but as an example of the over-riding influence of circumstances--that most educational research reported in journals employ samples which are not chosen by purely random selection.

The sample. In September 1963, a total of 108 grade eleven students were enrolled at St. Paul's. From this population twenty-five pairs of students were matched on the basis of their composite scores, and distributed at random to one of two classrooms. One class was designated as control and received traditional instruction; the other class was designated as experimental and received programmed instruction. The manner in which the sample was selected is described below.

First, eighteen students were automatically eliminated because, either they were repeating grade eleven, or, no mental ability test scores (Department of Education, Grade IX) were available for them. As a result the available population was reduced to ninety.

It was also noted that thirty-five students (including some of the eighteen mentioned above) were enrolled in an eight subject sequence compared to seven for the remainder. The extra subject was Latin. Administratively it was desirable to place these students in one classroom--Classroom B. Classroom B did not participate in the experiment.

The available population was now reduced to fifty-five students. It was from these fifty-five students that the twenty-five matched pairs were finally chosen.

Obtaining control. The experiment in this study was a comparative one. Two groups of students, "equal in all respects," were exposed to two different treatments (programmed versus traditional instruction) for the purpose of evaluating the relative effectiveness of each treatment. While it is impossible to have two identical groups, it is important that they be as similar as possible. Specifically, it was desirable that control be obtained over those background variables related to the learning that was to take place during the experiment.

These variables were identified as: I.Q., achievement (grade X average), prior knowledge of geometry, verbal ability, age, and sex.

There are several means of gaining control or equating these background variables. If the sample were large, control could be obtained simply by assigning the students at random to either the experimental or control groups. On the other hand, if the sample is small, random assignment need not generally produce "equated groups." To ensure that the mean of a specific variable is approximately the same for both groups, students are matched by pairs with respect to this variable.

Since the sample used in this study was small, the students used in the experiment were matched on the basis of a composite score reflecting both I.Q. and achievement. They were also matched for sex and subject sequence.

Matching for sex was unavoidable, since St. Paul's is a boy's school. Matching for subject sequence came as a result of an administrative decision to place all the Latin students in one classroom. However matching the other variables did present a problem.

A quick inspection revealed that matching on the basis of I.Q. as well as on the basis of achievement, while maintaining an adequate sample size, was impossible because of the variation between scores. Students who had the same I.Q. often had grade X averages which differed by as much as ten points.

Nevertheless, it was still deemed desirable to include achievement in matching, mainly because of its potential relationship to study habits. This suspected relationship assumed considerable importance, whether it was warranted or not, in the light of the hypothesis (stated earlier) concerning mean time.

Therefore, it was decided to construct a composite score consisting of the student's I.Q. and grade X average. The ninety available I.Q.s (excluding those of the repeaters) were converted to Z-scores; the grade X averages of these ninety students were also transformed to Z-scores. Converting to Z-scores has the same effect as changing a series of measurements with unlike units, to a series with a common unit in order that they may be summed. For example, it is incorrect to say that 3 yd. + 24 in. = 27, but it is correct to say that 9 ft. + 2 ft. = 11 ft.

The following formula was used to calculate the Z-scores. $Z = \frac{x - \bar{x}}{s} \times 10 + 50$, where x is an observation, $\bar{x}$ is the mean of that set of observations of which x is an element, and s is the standard deviation of that same set. A Z score is a convenient way of expressing the number of standard deviations a given score is from the mean. As such it becomes independent of the scale used in arriving at the initial scores.

The composite score for each student was calculated by summing his two Z scores. The writer could conceive no reason why the scores should not be given equal weight. The danger, rather, lies in the summing of the two scores. Summing may tend to obscure differences which would otherwise be present.

Now that each student had a composite score, the fifty-five students eligible to participate in the experiment were matched. First, those with equal scores were matched (thirteen pairs), then those whose scores differed by one (nine pairs), and lastly, those whose scores differed by two (four pairs). No match could be found for the other three students.

The matched students were then assigned at random to two classrooms, A and C. This was done by taking the first member of each pair and flipping a coin. If heads turned up he was placed in classroom A, and the second member of the pair in classroom C; if tails turned up, the first member was placed in C and the second in A.

After assignment, it was discovered that one of the students already matched wanted to take Latin. The pair to which he belonged was dropped from the experiment. The experimental sample now stood at twenty-five pairs: twelve whose scores were identical, nine whose scores differed by one, and four whose scores differed by two.

The remaining students (twenty-two) were assigned to classrooms A and C. These students were treated as though they were part of the experiment, but their scores were not used in the statistical analysis.

Checking and extending control. The purpose of matching followed by randomization was to obtain two equivalent groups of students. With what degree of certainty has this been accomplished? Certainly the use of the composite variable may be questioned on the grounds that it tended to obscure any difference which may have existed in its component variables. Furthermore, it was the only quantifiable variable controlled by matching. In view of the potential danger of non-equation when randomization is used exclusively, should not more variables have been matched? It was stated that verbal ability, prior knowledge of geometry, and age, as well as I.Q. and achievement were important variables related to learning.

The problem encountered in matching more than one variable has been mentioned; however, in order to settle the issue of equation, as well as to determine the type of statistical technique that would be necessary to complete the study, the means and variances of the variables were calculated, *ex post facto*. Table III, page 62, indicates that both groups are equivalent with respect to the background variables.

In every experiment, however, there are some variables which can not be controlled by matching or randomization. Control over these must be obtained by some other means.

Physical. The two classrooms A and C, used in this study were mirror images--the mirror running in a north south direction. Hence, the windows of the experimental group faced west and those of the control group towards the east. Instruction was given to both groups on Mondays, Wednesdays and Fridays--to the experimental group from 9:00 A.M., and to the control group from 9:38 to 10:15 A.M. Possibly because it was brighter, the writer thought he favoured the room with the eastern exposure. While this could constitute an advantage to the control group(the writer enjoyed teaching in this room), it should not constitute a disadvantage to the experimental group as they were taught by program (the writer enjoyed equally as much, not having to teach in this room).

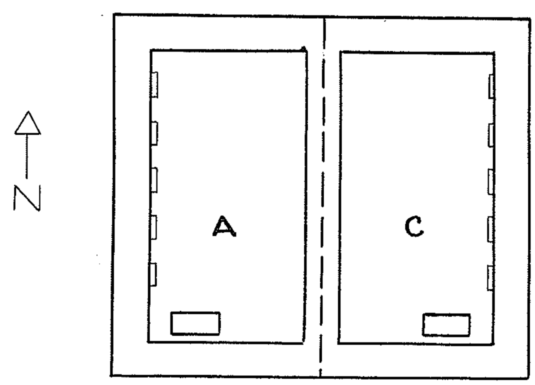

FIGURE 3

CLASSROOMS A AND C USED RESPECTIVELY BY THE EXPERIMENTAL AND CONTROL GROUPS

Psychological. The possibility existed that the experimental group, by virtue of being the experimental group, would over achieve. To equate this factor, an attempt was made to make the control group feel that it too was playing an important part in the experiment. Thus, when the time came to designate the two classes as either control or experimental, a representative from each class was present to "call" the coin that was flipped. Moreover, each student of the control group was periodically reminded of his participation in the experiment since he kept a record of the time that he spent doing geometry homework.

Teacher and subject matter. A single teacher was employed in the experiment. Rigid control, therefore, was maintained over the teacher variable in so far as the same teacher appeared in both classes. However, the danger inherent in a single teacher design is that the teacher may favour one technique over the other. The writer, for example, may have favoured programmed instruction and subconsciously let up on his teaching of the control group. Awareness of this danger is probably the best defence against it; although making certain that the same material is presented to both groups also reduces the potency of the danger.

Care was taken to ensure that the equation of the subject matter variable by giving the same topics, the same examples, and the same questions to both groups. The writer, because he was both the programmer and the teacher, was in an ideal position to effect this equation.

In so far as experience is related to the calibre of instruction, it may be felt, because the writer was an experienced teacher but a novice programmer, that the instruction received by the control group was "superior" to that received by the programmed instruction group. However, experience in one area is obviously related to performance in the other. The knowledge gained from teaching helps in the construction of the program, and conversely, the knowledge gained from writing the program may manifest itself in conventional classroom presentation. Experience in both areas thus becomes a vitiating variable and as such defies equation or inequation.

II. TREATMENTS

Control Group. The writer instructed the control group from 9:38 to 10:15 A.M. on Mondays, Wednesdays and Fridays. The method he employed was a combination of lecture, demonstration, and question-answer. The only departure from standard procedure was having the students keep a record of the time they spent doing geometry homework.

The students used the authorized text, A First Course in Plane Geometry; exercises from this text were supplemented by exercises from the program in order that subject matter be kept the same.

Based on his experience the writer knew that it would take him approximately nine class periods to complete the unit of work. Below is a Table of Specifications which indicates the approximate length of time that was allotted to each section of work. A comparison between Table I, which

TABLE II

TABLE OF SPECIFICATIONS

Topics	Number of Class Periods
Introduction	1/2
Theorem 18*	1/2
Exercises on theorem 18	1
Theorem 19	1
Exercises on theorem 19	1
Theorem 20	1/2
Theorem 21	1/2
Exercises on all theorems	4

*Theorems are numbered as they are found in the text. Theorems in the program are numbered 1,2,3,4, corresponding to 18,19,20, and 21. The general enunciations of these theorems may be found on page 43; students were not taught to label theorems with numerals; numerals are used in the table for the sake of neatness.

indicates the order in which the subject matter was developed in the program, and Table II, shows that the development of the subject matter follows the same pattern in both.

Experimental Group. During the first class with the experimental group the writer handed out the programmed materials and explained their use. The set of instructions with which every student was supplied may be found at the beginning of the program, appendix A. Most of these rules are standard; some however, are not, and deserve special comment.

Rule 10, "If you are desperately stuck seek the assistance of your teacher.", was included in an attempt to forestall a barrage of trivial questions. Adherence to this rule by the students would leave the teacher free to deal with more serious misunderstandings. It was felt that the inclusion of the word "desperately" would serve to accomplish this purpose.

This rule however, was not operative during the experiment. Using a concealed stop watch, the writer found that he spent less than seven minutes rendering individual assistance to the students of the experimental group. Moreover, this time was spent answering trivial questions, such as, "Is this where you want us to draw the circles?", or, "Am I doing this right?", et cetera. No questions indicating a lack of understanding of the subject matter were asked. In effect then this study served to measure the merits of programmed instruction exclusively.

Rule 6 states: "If your answer was wrong, circle the frame number in the answer column.". The purpose of this rule was to facilitate the proposed item analysis of the program.

Rule 9 was included for the purpose of helping the writer revise the program. It asked students to make comments about the program. It too may have been inoperative; less than one-tenth of one per cent of the frames had comments beside them. Of course, if it were operative, the lack of comments indicates that the program was satisfactory for most students.

There may be somewhat of a contradiction between Rule 7 which instructs students to work at their own speed and the verbal instructions given to the students regarding their rate of progress. Rule 7 suggests self pacing, whereas the instructions given suggest a form of external pacing.

After considering the length of time that it took to work through the program during preliminary testing, and at the same time noting that the control group would cover the subject matter in nine class periods spread over three weeks, it was decided that it would not be unreasonable to expect the experimental group to complete the program at the same time (and in the same amount of time) as the control group.

The reasons for this decision are obvious. First, in using a short programmed unit to replace regular instruction, it is important that regular instruction can be resumed without disruption or confusion; otherwise using the programmed sequence would hardly be worth the effort. Secondly, if students are allowed to work at their own rate entirely, a testing problem arises. Ideally the amount of time that elapses between the time that a student completes the program and writes a criterion test, should be the same for all students. If each of the students works at a different rate, twenty-five parallel forms of the same text may be required for this experiment if contamination of results is to be guarded against. Control over both the forgetting and contamination factors can be obtained by having the students complete the program at the same time, and then administering the test to the whole group. This implies, of course, a form of external pacing.

The external pacing imposed on the experimental group maintained control over groups of frames rather than individual frames. Since the program was almost 400 frames long, it was estimated that an initial rate of approximately 140 frames per week would result in the completion of the program in three weeks. This approximate rate of completion was suggested to the students. As the total available class time for geometry was limited to 114 minutes per week, it was assumed that students in order to maintain this rate would have to work on the programs at home.

To make sure that students were maintaining this rate, surveys were made of the students' progress at the end of the first, and again at the end of the second week. Slow students were warned that they were behind and told to catch up by working longer periods at home.

These "warnings" were not issued as threats, nor were there any sanctions attached to non-compliance with the rate of progress rule. A certain level of performance was set and the students were expected to adhere to it. The demands were made in a warm friendly way, and the students worked with no fear of reprisal. It may appear that making demands in a relaxed atmosphere represents two antithetical conditions, and as a result some question may be raised as to the motivation of the students. However, Reed,[1] using as a criterion pupils' science interest, has concluded as a result of a recent study, that the two variables of warmth and demand are not contradictory, and may coexist in multifarious combinations. While the reason for the use of external pacing has been noted earlier, it may be appropriate

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[1]Reed, Horace B., "Implications for Science Education of a Teacher Competence Research," Science Education, Vol. 46, No. 5, December 1962, p. 481.

to point out that the manner in which the students' rate-of-progress was controlled is partially compatible with self pacing as it was defined in Chapter I: "Self pacing permits a student to absorb the material at his own rate. The rapid learner is not held back, and the slow learner is not left behind." Consequently asking a student to catch up by putting in extra time is not a violation of the first part of the definition. The reason is clear; for a given rate of absorption, the amount of material absorbed is governed by the amount of time that the material is exposed. A student with a low absorption rate can cover, by spending more time, the same material a student with a high absorption rate can cover in less time.

On the other hand, the fact that a student is asked to catch up implies that he is being left behind--hence a violation of self pacing. The question arises: Will "forcing" a student to keep up by putting in additional time affect the amount that he will learn?, that is, will it affect his capacity to learn?

Undoubtedly asking a student to catch up places him under pressure. But this pressure is no greater than that to which he is exposed during regular instruction; in fact, it appears that the pressure is not as great. For in the regular class the rate of presentation is often too rapid for his rate of absorption, while programmed instruction rate of presentation is geared to his rate of absorption. He may have to spend long hours, but he does so confident that he will ultimately master the material.

However, as all students do not react in the same manner to external pressure, it was decided to quiz the students on this aspect of the experiment.

The programs were collected on the day of the criterion test. Everyone had completed his program. It may be noted that "completing" the program is not a difficult task if the student peeks ahead at the answers and then simply records them. Whether this sort of cheating occured to any great extent is quite impossible to determine with any degree of certainty. However, it was felt that the item analysis of the program might provide an indication. Cheating due to mounting pressure towards the end of the experiment, may be indicated by an error rate consistently above average during the first part of the program but below the average error rate for the later part of the program.

The time record cards were collected; and the criterion test administered to both groups at the same time. This guarded against contamination of results within and between groups. The scores of this test were called achievement scores.

III. MEASURING INSTRUMENTS

The instruments employed in this study can be separated into three classes according to the function that they served: first, to measure the criteria stated in the null hypotheses, time and achievement; secondly, to measure and hence determine whether certain background variables were equated, for their non-equation would have some bearing in the analysis of the first set of measurements; and thirdly, to measure the students' reactions to programmed instruction.

The particular instruments used are described below.

Achievement. Since the subject matter presented to the experimental and control groups formed part of the University Entrance Course prescribed by the Department of Education, it was felt that the criterion test should conform with the standards and format of the Department's annual geometry examination. This would tend to ensure that the instructional objectives being measured would tend to be the same as those set out by the High School Examination Board.

The examination questions were selected from prior departmental examinations. They were chosen on the basis of the following criteria: (1) The questions should be at varying levels of difficulty; (2) They should include a cross section of numerical, theoretical, and construction problems;

(3) The authorities (reasons) used to justify the steps in a solution of a problem should be different from problem to problem.

A time limit of one hour was placed on the criterion test. Since the departmental examination lasts three hours and has a total value of 100 marks, placing a one hour time limit on the criterion test implied that its total value should be approximately 33 marks--using the values that the department alloted to each question.

On the basis of the above criteria the criterion test was drafted. It was then sent to a member of the High School Examination Board for validation. (The High School Examination Board is responsible for setting departmental examinations). Her comments and suggestions were incorporated into the final version of the test. (See Appendix B for a sample of the test).

The final test contained seven questions with a total value of thirty-five marks. Eight marks were given to numerical deductions, sixteen marks were given to theoretical deductions, six marks to the recitation of a theorem, and five marks for a construction problem.

A scoring key was prepared and test was scored by the writer.

<u>Time</u>. Each student who participated in the experiment kept a record of his time on sheets specially prepared for

this purpose. For a student of the control group it involved keeping a record of only the time that he spent doing geometry homework outside of the regular geometry periods. For a student of the experimental group it meant keeping a record of the time he spent working at the program. Special "time record sheets" were prepared to help the students in this task.

Figure 4 shows a sample of part of a time record sheet used by the programmed instruction group. Each time

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Date:________

Starting frame:________	Starting time:________
Ending frame:________	Ending time:________

FIGURE 4

SAMPLE OF A TIME RECORD SHEET

that a student sat down to work at the program, he was to record the starting time and starting frame; after he had completed the work he set out to do, he recorded the ending frame and time. If he put in any additional time reviewing or the like, he recorded this time as such. The sum of all these sessions was a time observation for a particular student.

The students of the control group were given similar time record sheets except theirs made no mention of frames.

Obviously the control obtained over the time observations was not rigid. However it was an intention of this study to conduct the experiment under regular classroom conditions--this implies homework. It was hoped that supplying the time record sheets would encourage the students to keep a record of their time. In a further attempt to ensure that these records would be kept, students were reminded occasionally of their task. They were also informed that their records would not be held against them. The writer believes that the students did keep reasonably accurate records.

Background variables. Two variables were meaaured: prior knowledge of geometry, and verbal ability. To measure a student's prior knowledge of geometry a test from the Cooperative Mathematics Test series (Geometry, Form A) was administered prior to the commencement of the experiment.

The test is divided into two parts, each consisting of forty items and each with a time limit of forty minutes. Since Part I examined on the content given to Manitoba students in grade X, it alone was used. This part was normalized over 2143 students from various areas in the United States. The mean was 26.5, standard deviation was 5.36 and the reliability coefficient was 0.80. A sample of this test can be found in Appendix D.

As part of the schools' regular grade eleven fall testing program consisted of administrating the Grade XI

SCAT Tests, and since a students verbal ability may affect his ability to learn through programmed materials, the verbal ability scores as given by this test were recorded.

Student attitudes. An attitudinal questionnaire[2] was prepared by the writer for the purpose of measuring the students' general reaction to programmed instruction. The questionnaire asked students to compare programmed with traditional instruction in terms of effectiveness, appeal, boredom and pressure. The questionnaire was administered to the students of the experimental group after they had completed the program and had written the criterion achievement test. To encourage unbiased opinions, students were asked not to sign their names to the questionnaires. It should be noted that all the students of Classroom A, including those not in the experimental group, answered the questionnaire.

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[2]See Table XI, p. 224.

IV. STATISTICAL TECHNIQUES

Matching with randomization was the design employed in this experiment to obtain two "identical" samples. Each sample was then exposed to a different treatment (programmed versus traditional instruction). After the treatments, the same examination was administered to both groups. If both groups were equal to begin with, then any observed differences in the criterion scores could be attributed to the method of teaching.

However, due to sampling variation inherent in randomization (that is, due to chance), the background variables may not have been equated. In that case the difference in the mean criterion scores could be attributed to unequaled background variables as well as teaching. Consequently the means of the criterion scores would have to be corrected to account for these differences. In this study, the means of several background variables were calculated, and were found to be "equal"; hence, there was no need to employ more complicated statistical techniques, which would not have been the case had the means differed substantially.

A variation of the _t_ test was used to determine whether the mean criterion scores differed significantly.

The statistic _t_ can be used to determine how often, or with what probability, two samples with the observed mean criterion scores could be drawn at random from a common population of criterion scores. What are the chances of drawing from a common population, two samples whose criterion means differ by a certain amount? If chances are rare, say one in twenty or one in a hundred, then there is reason to doubt--since the treatments were different--that these samples were drawn from the same population of criterion scores. In this event it would be concluded that the differences in the means are due to the differences in the treatments--the two teaching methods employed.

While the statistic $\underline{t}$ indicates the probability with which the difference between the mean criterion scores of the two groups will occur, it does not indicate whether this probability is significant; this is the responsibility of the experimenter.

Two levels of significance frequently chosen by experimenters are the five and one per cent levels. If the difference between the mean criterion scores could occur by chance five or fewer times in a hundred, then it is said that the difference is statistically significant at the .05 level, or statistically significant at the .01 level if the difference could occur one or fewer times in a hundred. The latter is generally called highly significant. The statistic $\underline{t}$ was used to calculate the significance of the difference that occured. The formula used in the calculation was:

$$t = \frac{\bar{y}}{\sqrt{s^2/n}}$$, with n-1 degrees of freedom, and where $\bar{y}$ = the mean of the differences between the paired observations, s^2 = the variance of these differences, and n = the number of paired observations.[1]

The *t* test was used to test the following null hypotheses.

1. The achievement means of the experimental and control groups do not differ significantly at the .05 level.

2. The mean time scores of the experimental and control groups do not differ significantly at the .05 level.

Another statistical test employed in this study was the Chi-Square Test for Goodness of Fit. At the beginning of this Chapter, it was postulated that the population at which the study was aimed, was the future grade eleven students at St. Paul's High School--assuming, of course, no major changes in the type of students enrolling at St. Paul's. It was later noted that approximately one-third of the students did not participate in the experiment because they were enrolled in an eight as opposed to seven subject sequence.

Assuming that the population is normally distributed, and noting that a normally distributed sample provides an

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[1]Jerome C. R. Li, Introduction to Statistical Inference, (Ann Arbor: Edwards Brothers, 1957), p. 97.

inductive basis for making generalizations about a normally distributed population, it was decided to test the normalcy of the sample used in this experiment.

The formula employed was:

$$X^2 = \Sigma \frac{(f-h)^2}{h},$$

where f = actual frequency, and h = hypothetical frequency.[2] With one degree of freedom a value of 0.174 was calculated for X^2, which is not significant $p>.50$. That is, the sample was normally distributed.

In summary, it can be stated that: the population has been identified; a representative sample chosen; and hence the results of the study may be generalized to the population.

Control over the background variables has been established, and the treatments have been delineated.

Instruments have been designed for the measurement of the criterion variables; a possible limitation concerning the measurement of time was noted.

Finally the statistical techniques used in the analysis of the data were presented. The following chapter details the data pertinent to this study, indicates the results of the tests of significance, and then attempts through a further analysis of the data to seek explanations for the observed results.

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[2] *Ibid.* p. 432.

CHAPTER V

PRESENTATION AND ANALYSIS OF DATA

To assess the feasibility of using a short programmed unit in place of regular classroom instruction, two criteria were stated as null hypotheses and an experiment was designed to test them. The results of the experiment are presented in this chapter. Presented also, are analyses of data not directly related to the null hypotheses, but important nonetheless because of their connection with the general purpose of this study. Specifically these later analyses deal with the attitudinal questionnaire, the criterion test, and the program.

The data, for each student, relating to pre and post treatment variables is displayed in tabular form by Table IX.

Background variables. The difference between the means of the control and experimental groups, as shown by Table III, for each of the six background variables is not significant. This indicates that the procedure used to equate the background variables, matching with randomization did "work." No statistical "long shot" occured, and matching on the basis of the composite scores did not cancel out differences in I.Q. and grade ten averages to favour one group.

TABLE III

MEANS AND STANDARD DEVIATIONS OF THE BACKGROUND VARIABLES

Group	N	Variable	Mean	Std. Dev.
Experimental	25	Composite	94.52	13.99
Control	25	scores	94.64	14.13
Experimental	25	I.Q.	118.16	11.03
Control	25		119.04	11.02
Experimental	25	Grade X	59.64	8.27
Control	25	average.	59.72	7.80
Experimental	25	Previous	23.96	3.94
Control	25	knowledge.	24.16	4.17
Experimental	25	Verbal	294.49	7.62
Control	25	ability.	293.12	10.60*
Experimental	25	Age.	195.76	6.07
Control	25		197.42	6.42

*$F = 1.932$, $p > .05$[1]

[1]Sometimes it is possible to find that means, which are nearly equal, actually differ significantly because of a large difference between the standard deviations. The F-test was employed to determine whether the difference between the standard deviations of the verbal scores was significant. Since it was found to be non-significant, it could be safely concluded that the observed difference between the verbal means was also not significant.

The difference between the composite means of the two groups is due to the randomization process also. Some matched pairs were composed of members whose composite scores were not numerically equal, and random assignment weighted the control group.

<u>Criterion variables</u>. Table IV shows that the mean difference in achievement between the control and experimental groups was 0.72 in favour of the control group. Since the difference is not significant, ($p>.70$) the null hypothesis of equal mean achievement was accepted.

The mean difference in time of 94.45 minutes in favour of the control group was significant, ($p<.01$). Hence the null hypothesis of equal mean time was rejected.

The interpretation of the statistic is that the control group worked longer than the programmed instruction group to cover the same material, that is, the "average" student of the control group spent approximately 95 minutes more than his counterpart learning geometry. Since both groups spent equal time in class, he spent this time working at home over a period of three weeks. This amounts to one-half hour per week--certainly not a very large figure in terms of homework!

The standard deviation for the control group was eighty minutes, compared to sixty-eight minutes for the experimental group. The lower figure for the experimental group suggests that programmed instruction, utilizing a linear program, caused a greater homogeneity in the time observation than traditional instruction. However the greater homogeneity may also have been due to the form of external pacing imposed on the experimental group. Lack of pertinent

TABLE IV

AN ANALYSIS OF THE CRITERION VARIABLES

Group	N	Variable	Mean	S. D.	F	t	Hypot.
Experimental	25	Achievement	20.48	9.17	--*	-.321, p>.70	acc.
Control	25	Achievement	21.20	8.06			
Experimental	24	Time	418.76	68.20	1.380 p>.05	-6.88. p<.01	rej.
Control	24	Time	513.21	80.24			

*Clearly there was no need to calculate the F-ratio, since it could not possibly exceed the F-ratio calculated for verbal ability, the number of degrees of freedom being the same.

data precluded further analyses along this line.

Near zero correlations between time and achievement and time and errors, 0.03 and -0.06 respectively, indicates that the rate at which a student worked through the program had no bearing on the number of errors that he made, or on how well he mastered the content. This appears to be part of a trend, for no significant correlations were found between time and each of the other variables.

A further analysis of achievement. An examination of the achievement scores disclosed that three scores of the experimental group were so low (0, 2, and 3) as to indicate practically no learning. An attempt was made to identify these under achievers by exploring their background variable scores.

Their scores, in terms of standard deviations from the mean, are recorded in Table V. For comparative purposes the scores of the three lowest achievers (scores of 7, 8, and 11) of the control group are included in the table.

TABLE V

IDENTIFYING THE UNDER-ACHIEVER

Group	N	Achv.	I.Q.	Gr. X	Comp.	Time	V.A.	P.K.	Age
Exper.	3	-2.0	-.01	-.75	-.4	.07	.3	-1.5	-.5
Control	3	-1.4	-.82	-1.0	-1.0	-.05	.7	-.25	.5

Grouped rather than individual scores are used in order to minimize the effects of individual differences. Taking a deviation in excess of one as significant, one fact stands out: the under achievers of the programmed instruction group, unlike those of the control group, all had a poor prior knowledge of geometry. This appears to indicate a weakness in the assumption stated in Chapter III, namely, that all students entering grade eleven had a certain prerequisite knowledge of geometry.

To explore this matter further, a survey was made of the grade ten final geometry examination marks obtained by the students of both groups. It was found that the above three students as well as one student of the control group had failing grades in this examination.[2] It is quite likely then that these students did not possess the prerequisite knowledge stated in Chapter III. Hence it is not very surprising that they failed to learn from the program, since the program was not written with them specifically in mind.

The reader may be interested to note that, excluding these four students along with their matched partners from the statistical analysis would cause the difference in mean

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[2] A passing grade in mathematics, and hence promotion to grade eleven mathematics, is determined by the basis of a combined geometry and algebra score. A student could fail one and pass the other and still receive a passing grade in mathematics.

achievement to favour the experimental group by 1.3 points. This difference is almost significant at the five per cent level, (t = 2.1).

The failure of the program to teach these students underlines the importance of accurately identifying, beforehand, the type of student that the program intends to teach. A low correlation between I.Q. and achievement (r = 0.26 for the experimental group, and r = 0.42 for the control group) precludes the use of I.Q. as a predictor of achievement. A correlation of 0.22 between verbal ability and achievement appears to indicate that those skills measured by the SCAT test of verbal ability are not a prerequisite to learning from this program. Better predictors were the composite scores, the grade ten average and the previous knowledge scores. They are summarized in Table VI below. Interestingly, the best predictor for the programmed instruction

TABLE VI

CORRELATIONS BETWEEN ACHIEVEMENT AND THE BACKGROUND VARIABLES FOR THE EXPERIMENTAL AND CONTROL GROUPS

Variable	Exp. Group	Control group
I.Q.	0.26*	0.42
Composite	0.49	0.72
Grade X Average	0.80	0.65
Previous knowledge	0.55	0.44

*r=.40 is significant at the 0.05 level, n = 25.

group are the grade X averages. This appears to suggest that general academic performance and performance on programmed materials are controlled by a common set of behaviours.

Attitudinal Questionnaire. Part of the attitudinal questionnaire was devoted to a comparison of programmed and traditional instruction. A five point rating scale was used to measure student opinion. Table VII summarizes the results; a complete analysis of the questionnaire can be found in Appendix B. The first four items reveal that a reasonable percentage of students were undecided. Unlike the first four items, item five implies action and very few students

TABLE VII

A SUMMARY OF STUDENT ATTITUDES TO PROGRAMMED VERSUS TRADITIONAL INSTRUCTION

Programmed Instruction:	Agree	Disagree	Undecided
1. is more effective	63 %	26 %	11 %
2. has more appeal	66	26	9
3. is less difficult	77	11	12
4. involves less home-study	49	17	34
5. for future mathematics course	60	37	3
6. is boring	20	69	11
7. exerts pressure	26	57	17

remained undecided; more than one-third of the students indicated that they would not prefer programmed instruction for future mathematics courses. Possible explanations for this attitude may be found in items six and seven: twenty per cent of the students found programmed instruction boring, while twenty-six per cent found that it exerted pressure. The pressure was probably due to the form of external pacing which was used in this study.

The external pressure, however, appeared to affect different students to different degrees. When the writer interviewed the three low scoring students mentioned earlier, he found that the pressure of having to complete the program by a certain time was compounded many times by their frustration at being unable to learn from programmed materials.

Why then did these students not ask for extra help from the teacher? Perhaps it was part of a pattern. In answer to the statement, "While working on the program, ___ occasions arose when I desired an additional explanation from the teacher," only twenty-three per cent of the students answered "no;" all the remaining students answered either "several" or "many." Yet, it has been noted that very few questions were asked, and those that were asked were of a trivial nature. On the surface the result appears contradictory. On the other hand, the student responses to the above item may simply be an indication of an emotional attachment to a habitual mode of instruction; the students did not really need explanations, they simply wanted to be reassured. Once the student becomes accustomed to programmed instruction, and the new role of the teacher under this form of instruction, it is possible that there will be greater consistency between his wants as expressed in the above item, and his actions as observed in the classroom.

<u>The achievement test and the program</u>. The criterion achievement test was analyzed in an attempt to determine the outcomes which the program failed to teach. Table VIII gives the results of this analysis. It shows that the program

TABLE VIII

AN ANALYSIS OF THE CRITERION ACHIEVEMENT TEST

Type	Question No.	Value	Sum of Values* Exper.	Sum of Values* Control	Difference by (a)ques.	Difference by (b)type
Numerical deduction	1	2	37	38	-1	+6
	2a	1	19	18	1	
	b	1	19	23	-4	
	c	2	39	40	-1	
	d	2	22	11	11	
Theorem	3	6	67	73	-6	-6
Theoretical deduction	4	4	80	79	1	+2
	5	6	74	94	-20	
	6	6	104	83	21	
Construction	7	5	51	71	-20	-20

*Out of a possible 25 times the value of the question.

instruction group did a little better than the control group in answering numerical and theoretical deductions, a little poorer in reciting the theorem, and much poorer in answering the construction question.

Consequently an examination was made of the subsequence of the program dealing with construction problems, frames 238 to 256. In this eighteen frame sequence, nineteen errors were committed. This gives an average error rate of approximately 5 per cent, which is below the average rate of 7.6 per cent for the whole program. This appears to support Jacob's contention that there is no empirical justification for considering low item difficulty per se as essential in a program.[1] A low error rate does imply mastery of the content.

The low error rate and the poor achievement portrayed by the above subsequence is not in agreement with the results obtained for the program as a whole. A negative correlation ($r = -0{,}64$), significant at the one per cent level, was obtained between errors and achievement. Generally, students with many errors tended to do poorly on the achievement test. A rank order correlation of 0.70, obtained between the number of errors at the end of thirty frames and the number of errors at the end of the program, suggests, moreover, that

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[1]Paul I. Jacobs, "Item Difficulty and Programmed Learning," The Journal of Programmed Instruction, Vol. II, No. 2, Summer 1963, p. 21.

an early identification of the poor achievers is possible.

Seven of the twenty-five program instruction students failed the criterion achievement test, that is, they scored less than fifty per cent. Of these seven, three had error rates in excess of 10.4 per cent, two had error rates of 9.6 per cent, and two had error rates of less than 6.5 per cent. That is, seventy-one per cent of the students with error rates in excess of 9 per cent failed the criterion test.

The item analysis of the program is summarized in Figures 5 and 6. Figure 5 express as proportions the number of frames on the vertical axis and the number of errors along the horizontal axis. Twenty-eight per cent of the frames had no errors, twenty-nine per cent had one error and fifteen per cent had two errors. The remaining twenty-eight per cent of the frames had three or more errors; since the total possible errors on any given frame was twenty-five, twenty-eight per cent of the frames had an error rate in excess of ten per cent. A closer investigation of these frames seemed in order.

Starting at frame 1, the program was subdivided into thirteen, 30 frame sub-sequences. Figure 6 shows that each sub-sequence except the first had an error rate below ten per cent. The curve itself is erratic, and suggests a crude program; the revision of a few frames should result in a generally smoother curve. Significant perhaps is the decreas-

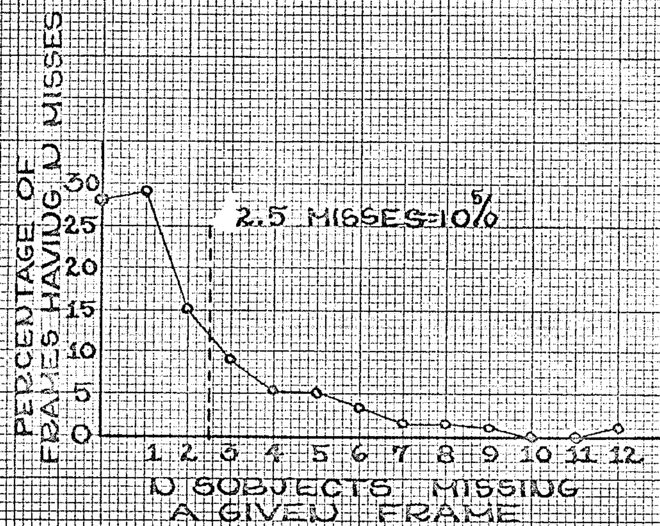

FIGURE 5

ERROR CONCENTRATION CURVE
The percentage of frames generating
a given number of mistakes is
plotted against that number

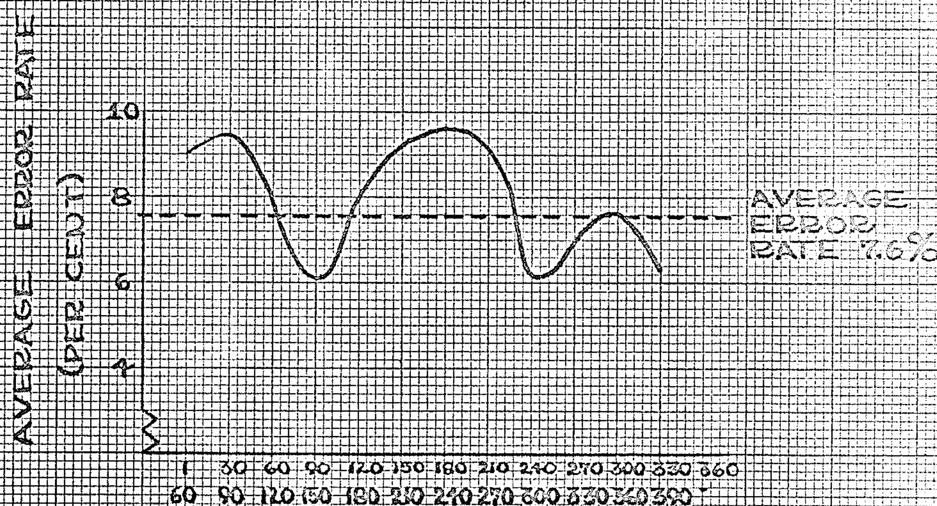

FIGURE 6

THE PER CENT ERROR RATE FOR
OVERLAPPING GROUPS
OF 30 FRAMES

ing error rate in the last sixty frames. The later part of the program was certainly the most difficult. For in this part of the program the student had to make use of all the concepts that he learned earlier; also, the responses that he was required to make were longer and more complex. The fact that fewer errors were committed in this section seems to indicate that students were achieving the standard of performance that was expected from them. However, this might also indicate an absence of "trick" questions.

Appearing throughout the program were instances of two or three consecutive frames having error rates in excess of ten per cent. In some cases the frames were faulty; in others however, the frames were properly constructed, but they may have required slightly more attention from the student, that is, they may be called "trick" questions. When two "trick" frames occured consecutively, students making an error on the first, generally did not make an error on the second. It appears that making an error on one frame raised the level of the students attention, causing him to make no errors on the frames immediately following. This speculation gives rise to other interesting speculations regarding the optimum error rate of a program, the relationship of errors to boredom, and the use of "trick" questions to build in an error rate.

CHAPTER VI

SUMMARY AND CONCLUSIONS

It is quite clear from available reports that students can learn by programmed methods. Most instances of negative results appear to come from classroom applications of these methods.

The purpose of this study was to determine the effects of employing a short programmed sequence in place of a unit of regular classroom instruction--without disrupting or changing the regular classroom routine. Three effects were considered: achievement, time spent and student reaction.

The unit of work employed was a section of the plane geometry course at the grade eleven level. Three weeks were spent teaching this unit. The study took place at Saint Paul's High School and involved fifty students--twenty-five matched pairs distributed randomly to two classrooms. One group received instruction by program, and the other, acting as a control group, received traditional instruction.

Two null hypotheses were tested. One stated that the mean achievement of each group was equal, the other, that the mean time for each group was equal. Further investigation consisted of assessing the student attitudes by means of a questionnaire, analyzing the program and the criterion test.

The results of the study showed that:

1. Student achievement was independent of instruction. The mean achievement of the students instructed by program was statistically equivalent to the mean achievement of those students instructed conventionally by the writer. Nevertheless, the data also revealed that for weak geometry students, that is, students with a poor prior background in geometry, programmed instruction was a less effective method of instruction. On the surface, this finding appears to support Lewis' contention (noted earlier), that a linear program in geometry may be able to teach most of the students and that a teacher be employed to tutor the stragglers.

2. There is evidence to suggest that external pacing was a factor in minimizing the amount learned by the programmed instruction group, and had the students been allowed to pace themselves, greater increments in learning would have resulted. Self pacing, however would necessitate a change in the "normal" classroom conditions; unless the increments in learning under self pacing are substantially greater than those obtained under conditions of external pacing, there appears to be no need to change existing classroom procedures. Further research in this area will be needed if programmed materials are to be used most efficiently in the classroom.

3. The amount of time saved by the programmed instruction group, while statistically significant, was certainly

not educationally significant--one-half hour per week per student. This is not to say that saving in time was not significant from the student's point of view. Most students of the experimental group noticed that they spent less time doing homework under programmed instruction than they did under conventional instruction.

4. The majority of students favoured programmed to traditional instruction. However the sharp cleavage between students favouring and disfavouring programmed instruction for further mathematics courses was an unexpected result--sixty per cent were and, thirty-seven per cent were not in favour, leaving only three per cent undecided. Other studies have reported a similar percentage of students in favour, and a much larger percentage of students were undecided. Whether the observed results were a function of the program itself, or a function of the conditions under which the program was employed, or a function of the student himself, is difficult to determine. As one of the students stated, "Learning from the program is all right, but I prefer being taught by you, sir." In general it appeared that the more mature student, capable of independent study, favoured programmed instruction. An interesting speculation may be: to what extent does programmed instruction promote the growth of independent study habits?

5. A significant negative correlation between errors

and achievement, appears to indicate that error rate is a good predictor of achievement. This suggests that a revision of the program to reduce the error rate may result in a corresponding increase in achievement. On the other hand an interesting result occured in this study which indicates the uncertain relationship between errors and achievement. A sub-sequence of the program dealing with construction problems had a below average error rate; yet an analysis of the criterion test revealed that the construction problem was answered least satisfactorily. This appears to support the contention that a low error rate is not essential in order that a student learn from a program. Since students indicated that their errors were usually due to carelessness or inattentiveness, rather than a lack of understanding, a problem of the programmer becomes: how to write a program that will maintain student interest.

6. Despite the fact that teacher assistance was available, very few students asked for help. A possible cause may have been that students were not encouraged to ask, but rather, were told to ask only when they were "desperately stuck;" or it is possible that the students' difficulties occured while he was working at the program at home where no teacher assistance was available. Neither of these explanations are adequate. In a subsequent trial of the same program with a different group of students few questions were asked although

the students were instructed to question the teacher at any time no matter how trivial the difficulty.

Apparently the onus of identifying the weak student who needs private tutorial assistance rests with the teacher, and the best criterion for such identification seems to be his initial error rate. For, by the end of the first thirty frames it was found that each student had established an error rate which in most cases continued throughout the program.

7. The form of external pacing employed in this study does not preclude the use of programmed materials in place of regular instruction. It appears that control over groups of frames does, however, place pressure on certain students; whether the achievement of these students would improve substantially under conditions of self pacing remains to be established empirically.

Conclusion. Despite the fact that students wanted additional assistance and did not receive it, and despite the fact that this program was not used under conditions ideal for maximum learning, namely self-pacing, the above evidence suggests that a short programmed unit can be used without disrupting the regular classroom routine and with minimal teacher assistance to achieve the same results as traditional instruction. This, coupled with the favourable student reaction is encouraging for further experimentation with programmed materials.

There seems to be little doubt that programmed materials can teach; this is a finding corroborated by many other studies. Perhaps surprisingly, this program has taught under conditions which may be termed unfavourable. An interesting study would be to compare achievement under these unfavourable conditions and those construed as more favourable to programmed instruction.

Of value too, would be a study which attempted to identify accurately, the type of student that succeeds with programmed materials. It has been suggested that intelligence and verbal ability are both poor predictors of achievement, while general scholastic performance, student maturity and study habits may be good indicators of the type of students that succeed with programmed materials.

An interesting experiment or series of experiments can be done in any study simply by making a complete analysis of the error rate. If an optimal error rate pattern can be described, it may be possible to revise a program so as to make it challenging for different levels of learners by introducing "trick" questions at different intervals, while still retaining the gross anatomy of the program.

Programmed learning is a fascinating medium of instruction. For the teacher interested in programming, it provides an opportunity to examine the atomic aspects of his presentation and the learning process; for the teacher interested in research, it contains many promising avenues of exploration; and, for the teacher concerned with stimulating his students by using a variety of instructional techniques, it provides a complete change for students accustomed to conventional presentation modes.

BIBLIOGRAPHY

SELECTED BIBLIOGRAPHY

A. BOOKS

Campbell, William Giles. Form and Style in Thesis Writing. Boston: Houghton Mifflin Co., 1954.

Coulson, John E. (ed.). Programmed Learning and Computer Based Instruction. New York: John Wiley and Sons, 1962.

Deterline, William A. An Introduction to Programmed Learning. Englewood Cliffs: Prentice Hall, 1962.

Galanter, Eugene (ed.). Automatic Teaching: The State of the Art. New York: John Wiley and Sons, 1959.

Goldsmith, Maurice (ed.). Mechanization in the Classroom. Toronto: Ryerson Press, 1963.

Good, Carter V., A. S. Barr, Douglas E. Scates. The Methodology of Educational Research. New York: Appleton-Century Crofts, 1941.

Green, Edward J. The Learning Process and Programmed Instruction. New York: Holt, Rinehart and Winston, 1962.

Hughes, J. L. Programmed Instruction for Schools and Industry. Chicago: Science Research Associates Inc., 1962.

Li, Jerome C. R. Introduction to Statistical Inference. Ann Arbor: Edwards Brothers, 1957.

Lumsdaine, A. A. and Robert Glaser (eds.). Teaching Machines and Programmed Learning--A Source Book. Washington: National Education Association of the United States, 1960.

Lysaught, Jerome P. and Clarence M. Williams. A Guide to Programmed Instruction. New York: John Wiley and Sons, 1963.

Margulies, Stuart and Lewis D. Eigen (eds.). Applied Programmed Instruction. New York: John Wiley and Sons, 1962.

Oliver, W. J. and others. A First Course in Plane Geometry. Regina: School Aids Publishing Co., 1954.

SELECTED BIBLIOGRAPHY (continued)

Rutherford, Gladys (ed.). Programmed Learning--and Its Future in Canada. Ottawa: Canadian Teachers' Federation, 1961.

Smith, Wendell I. and J. William Moore (eds.). Programmed Learning. Toronto: D. Van Nostrand Co., 1962.

Stolurow, Lawrence M. Teaching by Machine. Washington: United States Government Printing Office, 1961.

B. OTHERS

Programme of Studies for the Schools of Manitoba. Senior High Schools 1962-63. Department of Education: The Queens Printer for Manitoba, 1962.

Programs '62 A Guide to Programmed Instructional Materials Available to Educators by September 1962. The Center for Programmed Instruction, New York, in cooperation with the United States Department of Health Education and Welfare. Washington: United States Government Printing Office, 1962.

A Survey of the Use of Programmed Instruction in Canadian Schools, 1962-63. Research Memo No. 12. Ottawa: Research Division, Canadian Teachers' Federation, 1963.

Evaluating Programmed Instruction Materials Through Action Research. Ottawa: Research Division, Canadian Teachers' Federation, 1963.

Alter, Millicent and Robert Silverman. "The Response in Programmed Instruction." The Journal of Programmed Instruction, I, 1:55-78, 1962.

Eigen, Lewis D., and others. "A Comparison of Three Modes of Presenting a Programmed Instruction Sequence." New York: Center for Programmed Instruction, 1962.

Eigen, Lewis D. and P. K. Komoski. "Research Summary Number 1." New York: The Center for Programmed Instruction, 1960.

SELECTED BIBLIOGRAPHY (continued)

Gotkin, Lassar G. and Leo. S. Goldstein. "Programmed Instruction for the Young Learner: A Comparison of Two Presentation Modes in Two Environments." New York: The Center for Programmed Instruction, 1962.

Jacobs, Paul I. "Item Difficulty and Programmed Learning." The Journal of Programmed Instruction, 21-38. Vol. II, No. 2, Summer 1963.

Quakenbush, Jack. "How Effective Are the New Auto-Instructional Materials and Devices?" IRE Transcripts on Education, December 1961, 145-151.

Reed, Horace B. "Implications for Science Education of a Teacher Competence Research." Science Education, Vol. 46, No. 5, December 1962, 474-492.

APPENDIXES

APPENDIX A

THE PROGRAM AND AN ITEM ANALYSIS

OF THE PROGRAM

ITEM ANALYSIS OF THE PROGRAM

The errors which each student of the experimental group made in working the program are recorded on the following pages. The students were ranked according to their achievement scores, and are listed in descending order under the heading "S." The adjacent column lists the total number of errors that each student made; for example, student "14" ranked first in achievement and made eleven errors on the program. Errors occuring at a particular frame for a particular student are indicated by the symbol "o." The absence of this symbol indicates that the frame was answered correctly. Thus no students made an error on frame one, student "11" made an error on frame two, and three students "25," "20," and "12" each answered frame three incorrectly.

Altogether the students made 738 out of a possible "25 x 388" errors on the program; this results in an average error rate of 7.61 per cent.

S	T.E.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
14	11															o		o													
21	27				o																									o	
04	39				o																								o	o	o
24	13								o																						
25	24			o																											
23	33				o												o	o	o												
10	22				o																										
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22	9																														
16	18															o	o	o						o							o
07	29														o		o	o											o		
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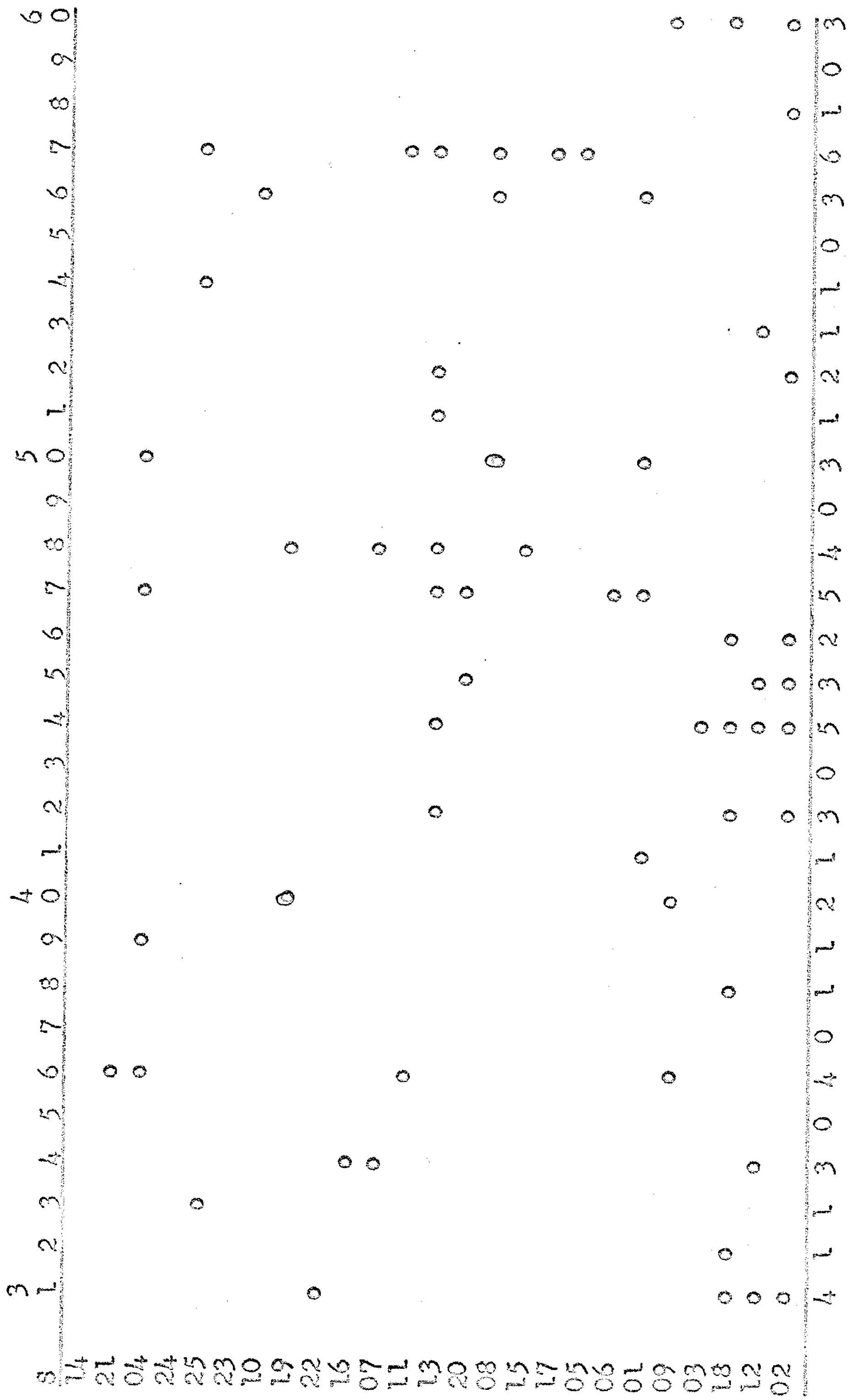
12	100			o	o	o				o	o	o	o	o	o			o		o							o		o		
02	63				o				o						o	o		o													o
	738	0	1	3	9	1	0	0	2	1	1	1	4	1	5	4	8	12	2	3	0	1	0	4	2	1	2	0	3	3	5



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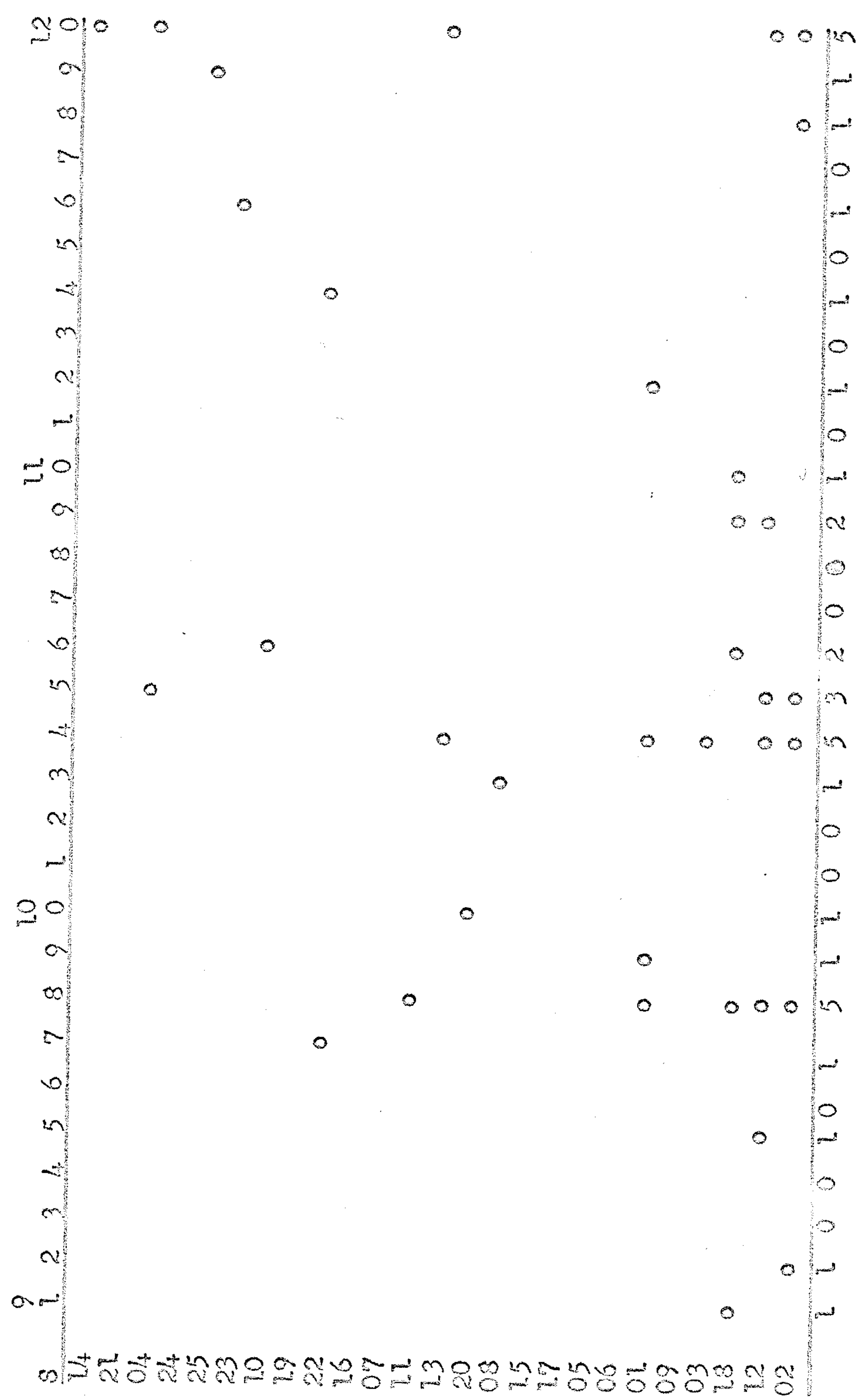

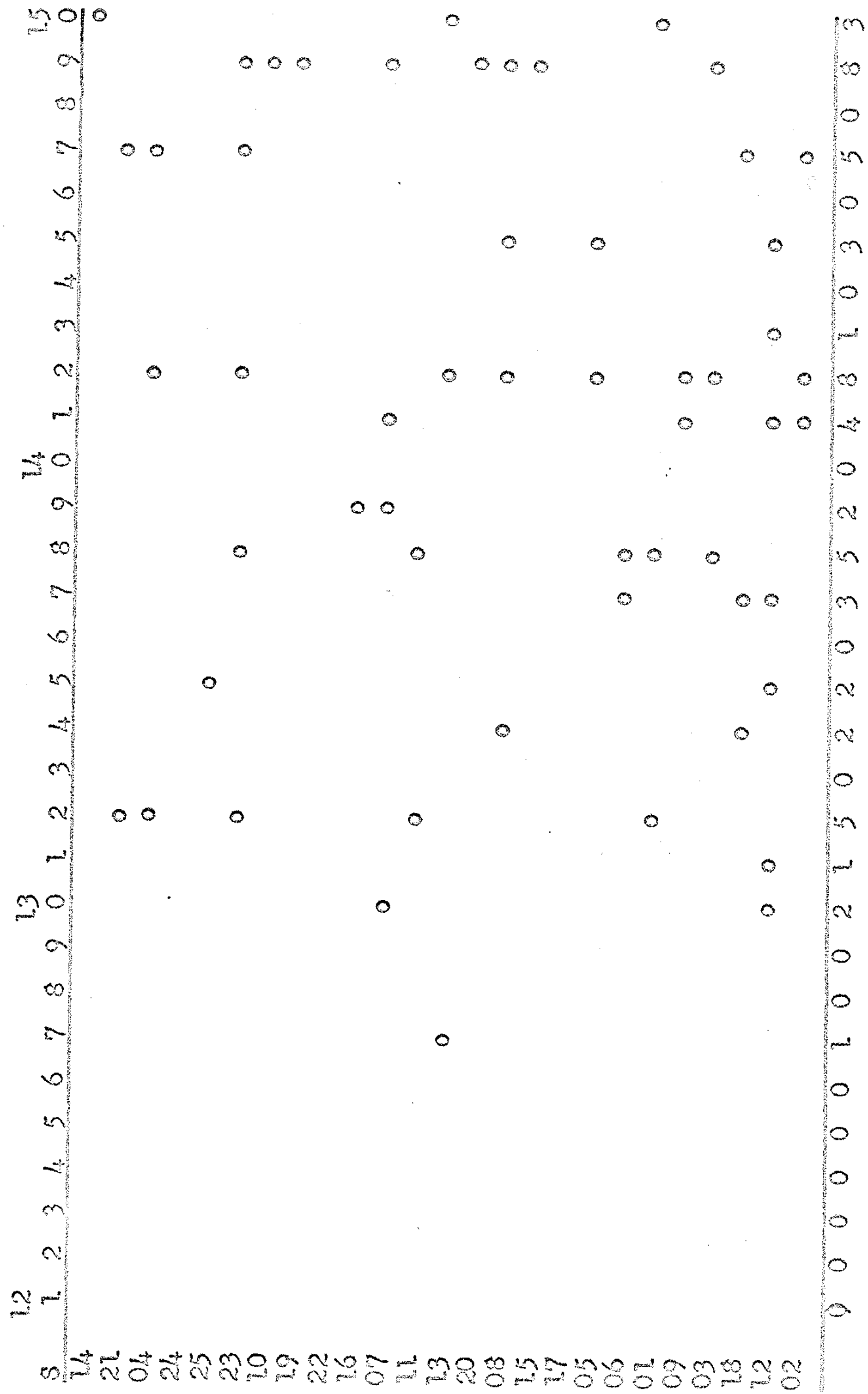

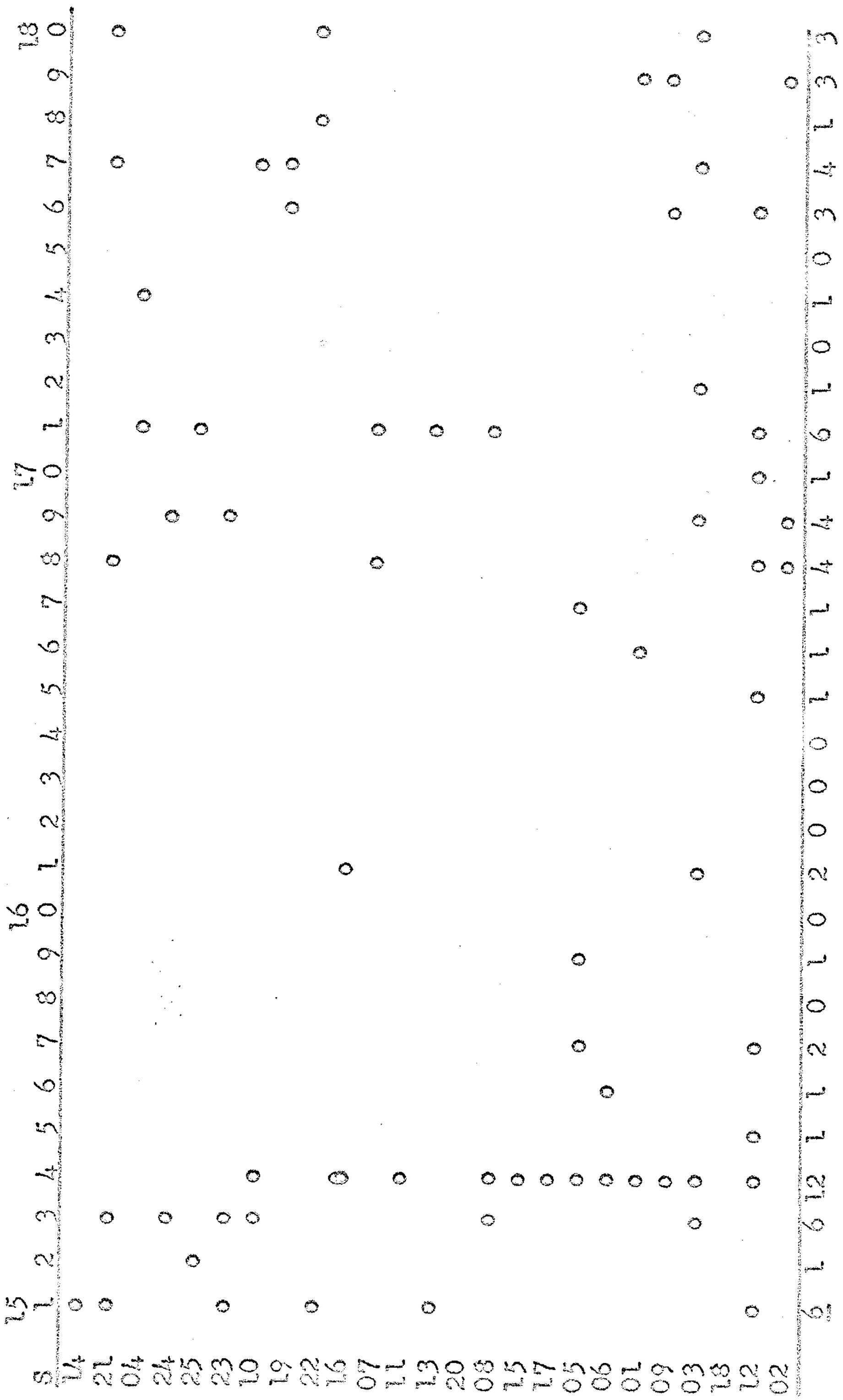

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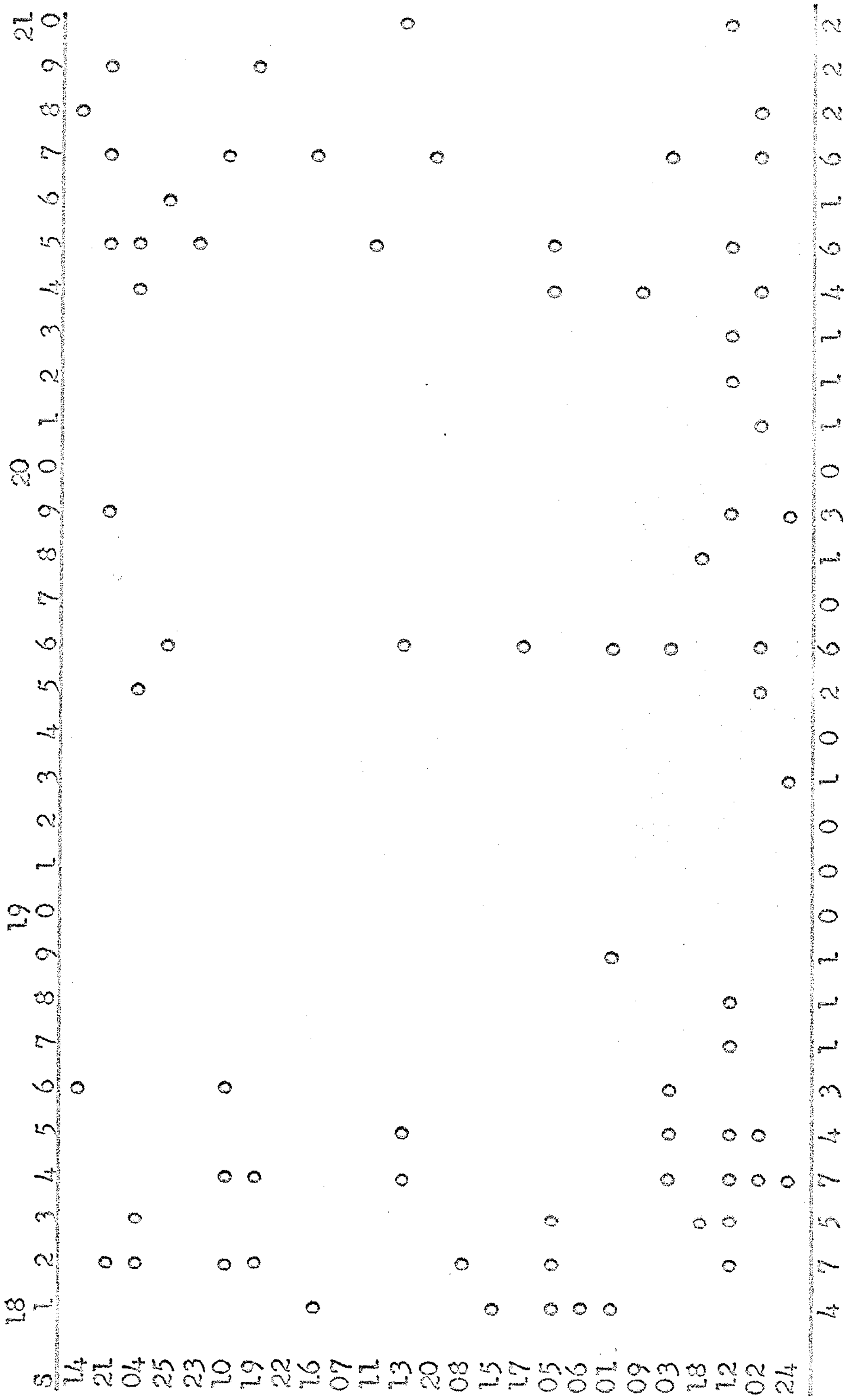

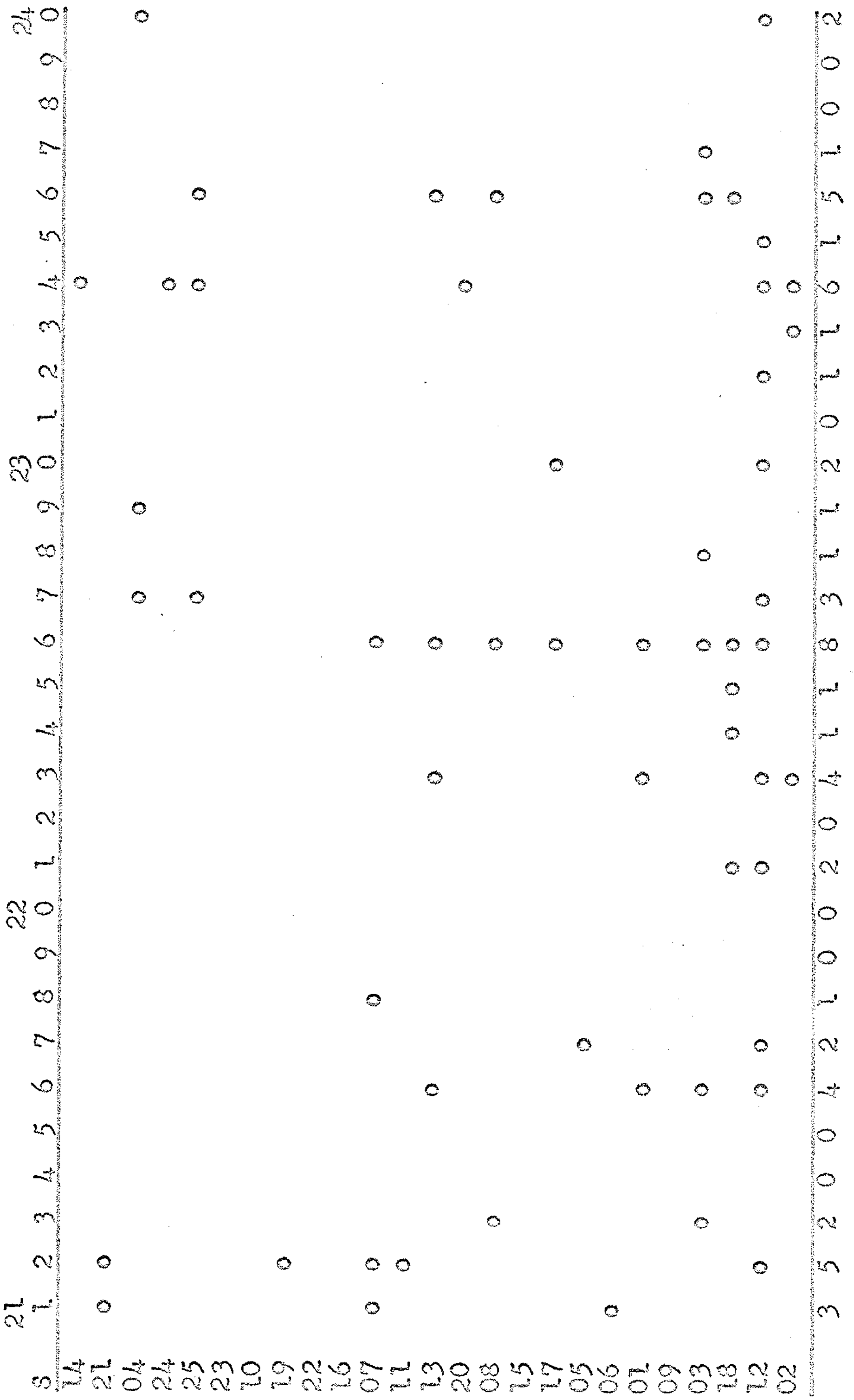

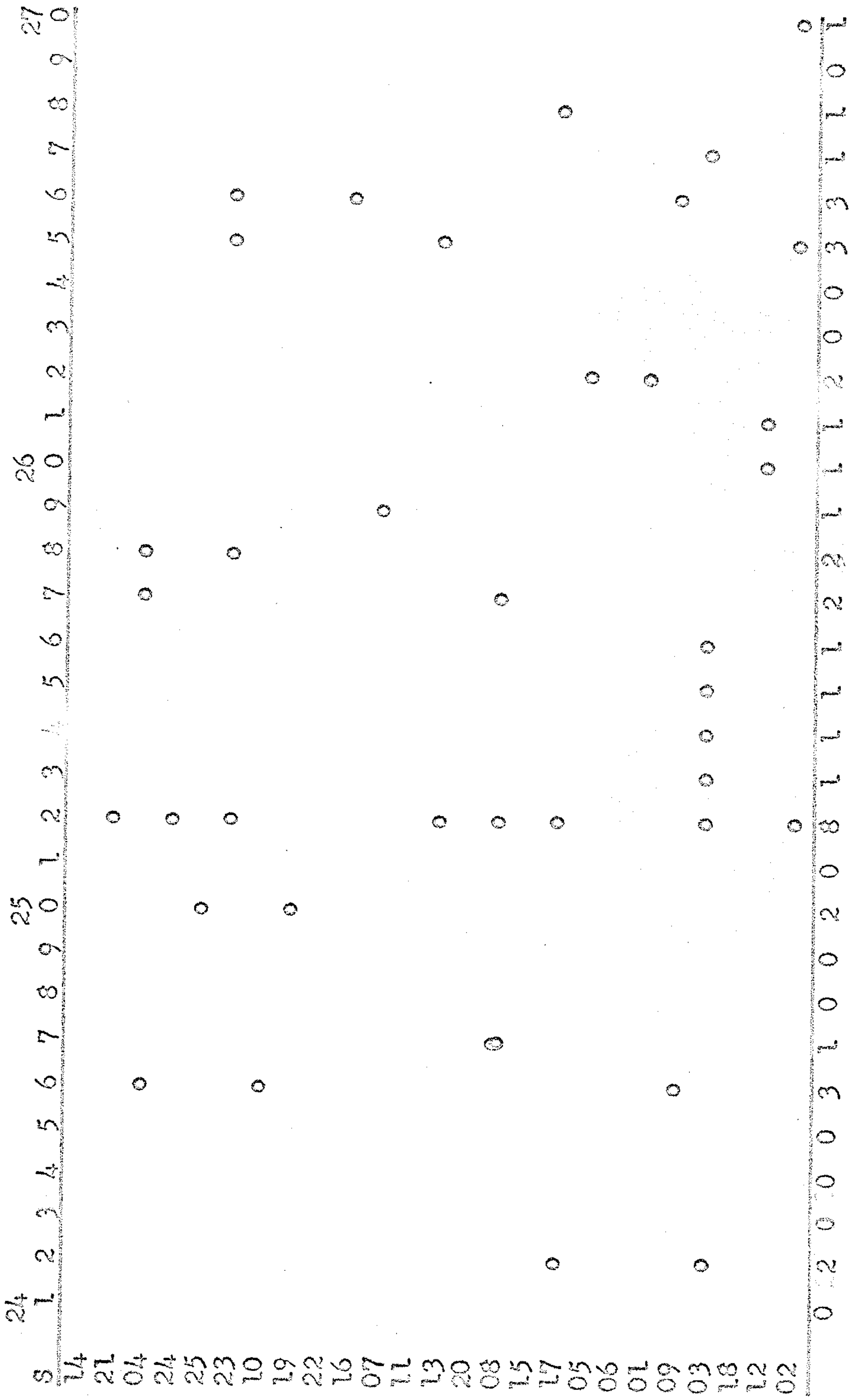

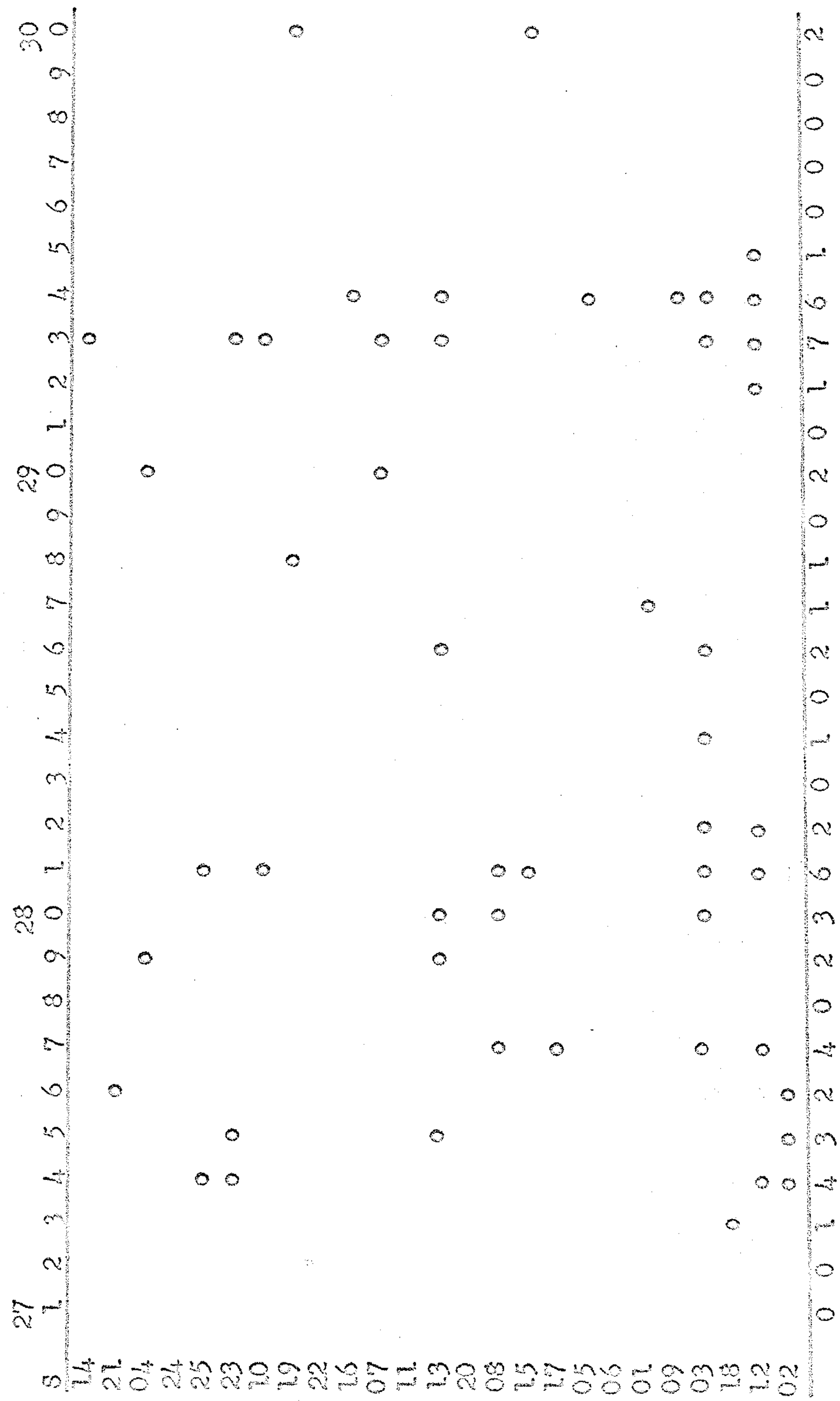
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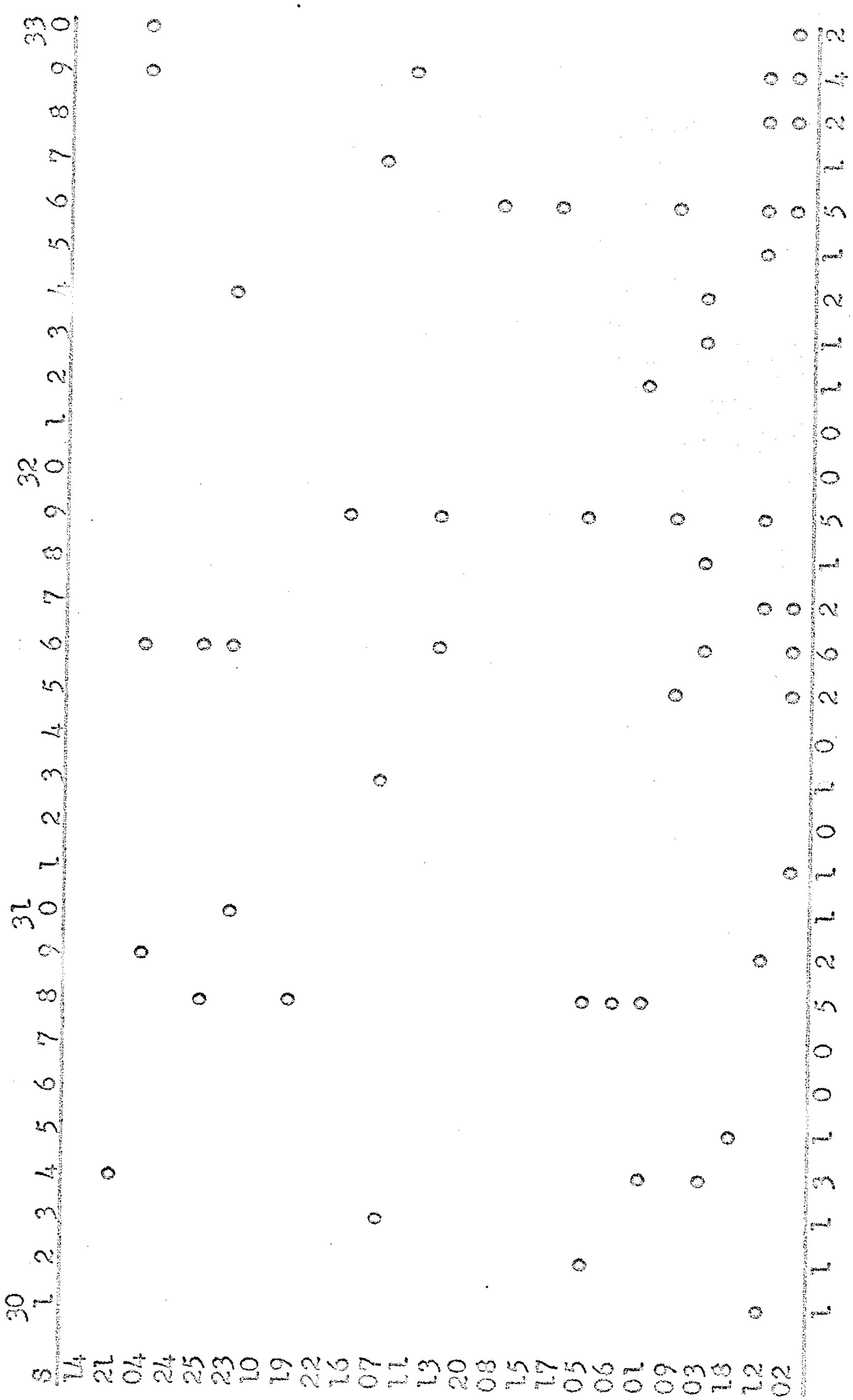

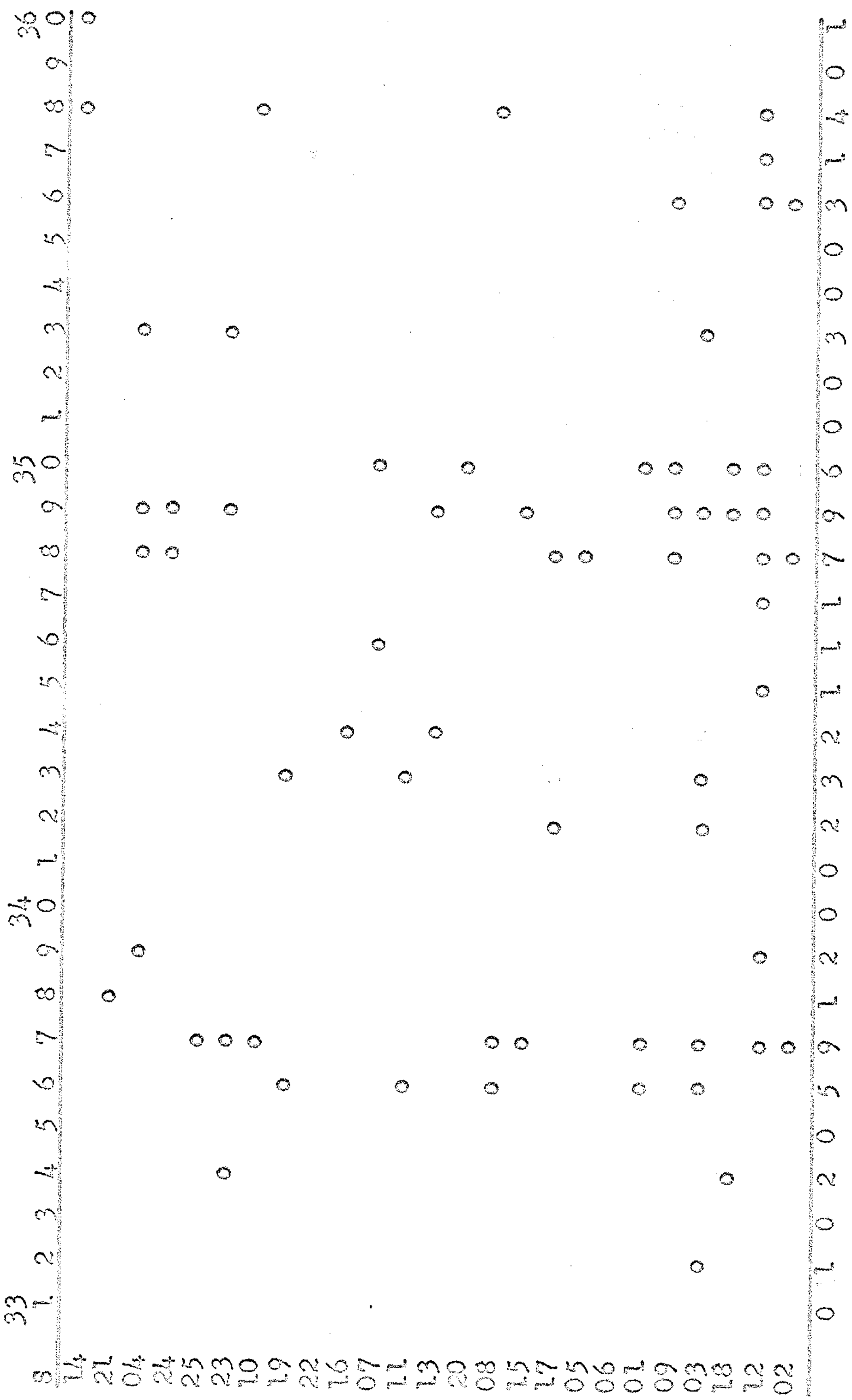
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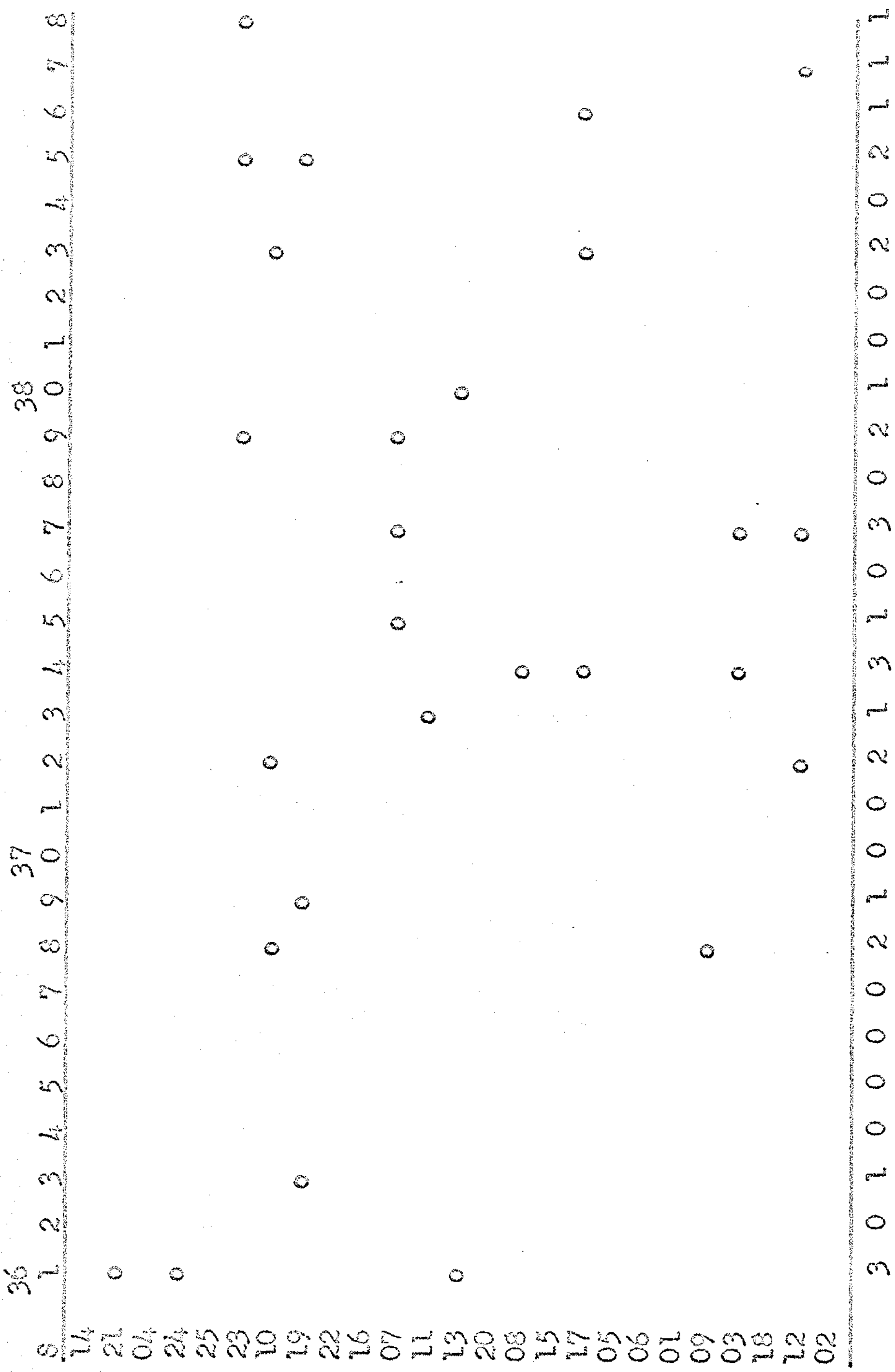
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GEOMETRY OF AREAS

A Programmed Text

by

Eugene Kristal

This booklet, THE GEOMETRY OF AREAS, looks quite different from your ordinary text books. This is because it is a programed text, which means that the topics are presented in small steps. Each numbered step is called a frame. Each frame requires that you either write or choose an answer. However this is not a test. To derive the greatest possible benifit from this program you should proceed strictly according to the following rules.

1. When you are ready to start studying place a slide -- a non-transparent sheet of paper will do -- over the right hand column which contains all the answers.

2. Read the frame and fill in the missing answer(s) in the blank(s) provided.

3. Then mov the slide down until you come to a heavy line like this " ________ ". You can now see the correct answer to the frame you have just done.

4. Sometimes the frame will make reference to a figure. You will find this figure in your supplementary booklet.

5. If your answer was correct, read the next frame, answer it, then move the slide down to uncover the answer.

6. If your answer was wrong, circle the frame number in the answer column. Then go back and re-read the necessary portions. Do not go ahead until you understand why you gave the incorrect answer.

7. Proceed from frame to frame at your own speed but do not waste any time. Whether or not you make errors will not affect your grade.

8. Each time you sit down to answer a few frames, record the frame number and the time in the appropriate form provided on the next page.

9. As participants in an experiment, your opinion is solicited. If you feel a frame is ambiguous, a dead give-away, or misleading, enter your criticism along side the frame. You may have other comments ... feel free to make them.

10. If you are desperately stuck, seek your teacher's assistance. After you are finished this program notify your teacher that you are ready to take a test.

11. This can be an interesting adventure in learning for you. Do not spoil it by peeking ahead at the answers. Good luck.

TIME RECORD NAME: ______________________

Date: ______________ 1963.

Starting frame: __________ Ending frame: __________

Starting time: __________ Ending time: __________

Date: ______________ 1963

Starting frame: __________ Ending frame: __________

Starting time: __________ Ending time: __________

Date: ______________ 1963

Staring frame: __________ Ending frame: __________

Starting time: __________ Ending time: __________

Date: ______________ 1963

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Date: ______________ 1963

Starting frame: __________ Ending frame: __________

Starting time: __________ Ending time: __________

Date: ______________ 1963

Starting frame: __________ Ending frame: __________

Starting time: __________ Ending time: __________

---

1. Figures 1, 2 and 3 are examples of closed figures because all their sides are joined to each other. Figure 2 is a four-sided closed figure. It is called a q________	
2. Tri" means 3. A three sided closed figure is called a ____________	1. quadrilateral
3. "poly" means many; "gon" means side. A polygon is a ____________ ____________ closed figure.	2. triangle
4. A four sided polygon is called a ____________.	3. many sided
5. The word "figure" is sometimes used instead of the word polygon. Thus we can say: triangle ABC, or polygon ABC, or ____________ ABC.	4. quadrilateral
6. The smallest number of sides that a polygon can have is three. Hence, when we speak of a polygon, we mean any figure which has ________ or more sides.	5. figure

7. A polygon may be defined as a closed figure with ________________ ________________ (sids)

6. three

8. The word ________________ is sometimes used instead of polygon.

7. three or more sides

9. Ref.to fig.1. Area is defined as t the amount of surface bounded by the sides of a polygon. Shade the area of polygon ABCDE.

8. figure

Refer to fig.2.
10. The ________________ is the amount of surface bounded by the sides of quadrilateral ABCD. Shade it.

9. 

11. From now on we will use the words figure and ________________ interchangeably.

10. area 

12. Refer to fig. 3. Shade the area of triangle ABC. This the amount of surface ________________

13. Area is defined as the ______ of surface ______ by the sides of a polygon.

14. Define area of a polygon: It is the amount of ______________________________

15. Inorder to give a numerical value to the area of a polygon, we find out how many times it contains a unit of *area*. If ▨ is a unit of *area*, then the area of the polygon in fig. 4 is ______ units of surface. [COUNT THEM]

16. If ▨ is a unit of AREA, then the area of the polygon in fig. 5 is ______ units of ______

17. Now that we know that it is possible to assign a numerical value to the area of a polygon, we can compare the size of two polygons

12. bounded

13. amount, bounded

14. surface bounded by the sides of the polygon.

15. 16

20. Refer to fig. 7. The area bounded by the sides of polygon abcdef is

_____ square units. If each sq. unit represents a square inch, then the area bounded by abcdef is

14 __________________ (wds).

19. square foot

---

21. If each unit of area in fig. 5 represents a sq. ft., then the area of the polygon in fig. 5 is 25_____

__________________ (wds)

20. 14, sq. inches

---

22. If each square unit represents a square mile, then the area of the polygon in fig. 4 is __________

__________________.

21. sq. feet

---

23. The area of the polygon in fig. 7

is _______ sq. units. We found it

by ______________ the number of square units it contained.

22. 16 square miles

---

24. There are two ways by which we may find the area of a rectangle. One

way be to c_______ the number of times it contained a unit of area. The other way is by using a formula we studied in grade school: Area = base X altitude.

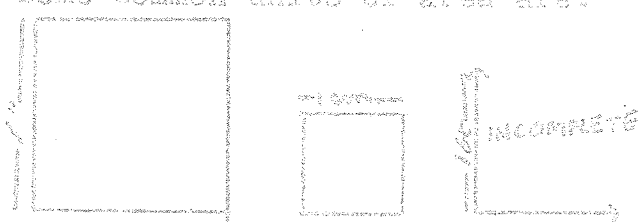
23. 14, counting

---

by comparing the number of times that they contain the same unit of area. For example, in Fig. 5, polygon ABCD contains the given unit of area 3 times, and polygon TUWXYZ contains the SAME unit of area 4 times. Hence we can SAY that figure ABCD is ____ area units ________ than fig. TUWXYZ.

16. 25, area

18. Some common units of area are:



1 sq. inch 1 sq. cm. 1 sq. foot

One square inch is the area bounded by a square whose sides each equal to one inch. A square centimeter is ________________________________

____

17. 1, smaller

19. The area bounded by a square one foot to a side is called a

____

18. Area bounded by a square centimeter to the side.

20.

25. Ref. fig. 8. Counting the square units, the area of rectangle ABCD is ________ (#) sq. units.

24. count

---

26. The altitude is ______ (#) units and the base is 6 units.

25. 18

---

27. Examing the figure we see that there are 6 columns, and each column contains 3 sq. units. Hence there are 6 columns X 3 sq. units per column or ______ (#) sq. uts.

26. 3

---

28. In effect then, to find the area of a rectangle we simply multiply the ____________ by the ____________

27. 18

---

29. State the formula for finding the area of a rectangle.

____

28. base altitude

---

30. Let us review briefly what we have done. Firstly we have defined the area of a polygon to be the ______

____________ bounded ____________

____________. Secondly, we have defined the area of a rectangle to be the product of its ________ and ________.

A = b a

29. area = base X altitude

31. Ref. to Fig. 9. ABC is a scalene triangle. AD and BE are altitudes. We define an altitude to be a line drawn from a vertex of a triangle p______________ to the opposite side.

32. Since a triangle may have three altitudes, the third altitude in trianlge ABC would be drawn from the vertex C, perpendicular to

the side ________ (lets).

33. Ref. fig. 9. Triangle ABC may have __________ (#) altitudes.

34. The altitudes are drawn from the vertices A, B and ______, perpendicular to the ____________ sides.

35. The side opposite A is _____ (lets)

Draw the altitude AX.

36. The altitude BY would be a line drawn from the vertx ____ (let),

______________________ to the

______________________ side AC.

37. Ref. gig. 10. Construct any two altitudes in triangle ABC.

amount of surf[illegible] by the sides [illegible]

base, altitude

30. BC.

31. perpendicular (⊥)

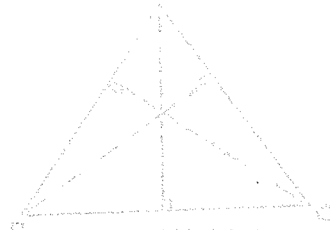
32. AB

33. three

34. C, opposite

35. BC

36, B, perpen[illegible]cular, [illegible] (⊥)



37. B any two

38. An altitude of a triangle is a line drawn from the ____________ of the triangle, perpendicualr to the opposite side or BASE.

38. vertex

39. In fig. 10, an altitude is drawn fr from the vertex ____ (let), to the ____________ ____________ or base AC

39. B, opposite side

40. Saying, "the base of a triangle" is another way of saying "the side ____________ THE vertex from which the altitude is drawn.

40. opposite

41. In fig. 10 the altitude associated with the base AB is drawn from the vertex ____ (let).

41. C

42. In our definiton of altitude we can substitute the words opposite side with one word, ________.

42. base

43. A triangle had 3 sides. How many bases can it have? how many altitudes?

____________ ____________

43. 3 3

44. Ref. fig. 11. Triangle ABC is obtuse. The altitude from B is drawn to the ba[illegible] ________ (lett.). The ____________ [illegible] is drawn [illegible] extended.

45. Ref. fig. 11. To draw the altitude from C, we have to extend the base ______ (lets).

44. AC, altitude

46. Ref. fig 12. Name the altitudes:

______, ______, ______

45. AB (not BA)

47. Refer to fig. 13. Underline the correct phrase in the following statements.
1. The altitude from B is drawn to a) base AC; b) base CA extended.

2. The altitude from A is drawn to a) base BC; b) base BC extended.

3. The altitude from C is drawn to a) base AB; b) base AB extended; c) base BA extended.

46. AY, BZ, CX.

48. We can now define the altitude of a triangle to be that line drawn from a vertex of the triangle, perpendicular to the ____ or the base ______

47. 1 is b; 2 is a; 3 is c

49. Ref. fig. 14. Select your answer, then turn to the frame indicated. In triangle ABC the altitude which is associated with the base AB is:

a) CR - turn to frame 50
b) [illegible] - turn to frame 51
c) [illegible] turn to frame 52.

48. BASE, EXTENDED

50. Your answer was C[illegible]. You are correct. With each base we associate only the [illegible] which is perpendicular to [illegible]

51. Your answer was AX. Remember that the altitude of a triangle was defined as the line drawn from the vertex of a triangle perpendicular to the base or base extended. The base is the side opposite the vertex from which the altitude is drawn. For example, the side opposite vertex A is BC. Thus with the base BC is associated the altitude ______.

52. Your answer was BS. Remember that the altitude of a triangle was defined as the line drawn from the vertex of a triangle perpendicular to the base or base extended. The base is the side opposite the vertex from which the altitude is drawn. For example, the side opposite the vertex B is CA extended. Thus with the base AC we associate the altitude ______.

53. Draw the three altitudes of triangle ABC, fig. 15.

54. Ref. fig. 13. In drawing the altitude to AC, the first step is to extend ______ A(let), and then drop the ⊥.

55. The base of a triangle is the side ______ the vertex from which the altitude is being drawn.

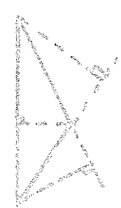
56. We define the altitude of a triangle to is ______

50. extended

51. AX, Now return to frame 49 and select the correct answer.

52. BS. Now return to frame 49 and select the correct answer.

53.



54.

C

55.

opposite

57. Another way of saying the same distance apart is equi________

56. the line drawn from the vertex of the triangle ... the base or base exten...

58. We define the distance between two parallel lines to be the perpendicular dis________ between them.

57. distant

59. Ref. fig. 16. XY//MN. The distance between these two lines is the length of the line segment;
a) AB; b) CD; C) neither of these.

58. tance

60. AB can also be called the altitude of MN//XY. Because a pair of parallel lines are everywhere equidistant, any other altitude drawn between MN and XY would equal ______(lets) in length.

59. a) AB

61. In fig. 17, OP is a line drawn from O ________________ to AB. Hence OP is an altitude.

60. AB

62. Ref. fig. 17. ∠OPA = 90°; therefore ∠POD equals ________ degrees.

Why? ____________________

61. perpendicualr

63. Since ∠POD = 90°, we can say that PO is an altitude drawn from P to *line segment* ______.(lets)

62. 90° - int. opposite ∠s are supplementary. (or other suitable reason)

64. Ref. fig. 18. ABCD is a quadrilateral with AB//CD. XY is a(n) ________

63. do

65. In a triangle an altitude is associated with a certain base. In fig. 18, the altitude XY is associated with two bases, because it is ⊥ to both. They are ________ and ______ (lets)

64. altitude

66. Ref. fig. 19. The two altitudes drawn to the base AD are _____ & _____ (lets

65. AB DC

67. "p" and "q" are altitudes for base ________ (lets) also.

66. p q

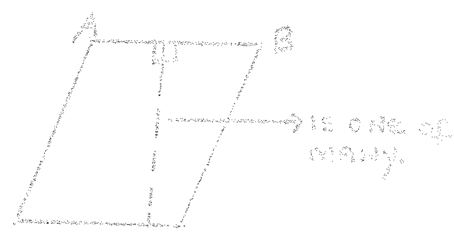
68. def. to fig. 19. Construct an altitude using BA as base. This is also an altitude for ______ DC, since it is perpendicular to DC as well as AB.

67. BC

69. Ref. fig. 18. Draw an altitude using BC extended as base.

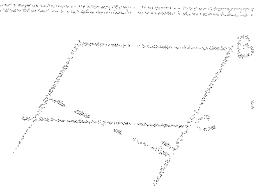
68.

base



70. In a //gm an altitude may be associated with ______ (#) bases.
↑ the ans. req'd is a number

69.



AN EXAMPLE.

71. Ref fig. 20. ABCD is a rectangle. Construct an altitude to base DC. Is it also an altitude for AB?

70. two

72. Ref. fig. 20. Draw an altitude using AD as base. Is this an altitude to base BC?

73. Ref. fig. 21. ABCD is a //gm. Draw two altitudes; one, to base DC; the other to base DC extended.

74. Ref. fig. 21. Construct an altitude using BC as base. Construct another using BC extnaded as base.

75. If we draw a(n) ______________ to one side of a //gm, then it is an ______________ to the opposite side as well.

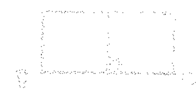
76. Construst an altitude using AD as base Fig. 22)

77. From the last few examples it is evident that when we speak of a base, whether it belong to a triangle, rectangle or parallelogram, we automatically associate an altitude with that base. For example, in triangle ABC (fig. 23) when we talk of the altitude associated with the base BA we mean the altitude ________(lets) only.

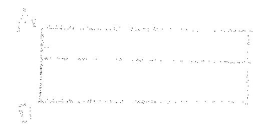
78. Ref. fig. 24. The altitude associated ~~ma~~ with the base BC is ______________

79. XY is an altitude to BC because it is

71.



72.



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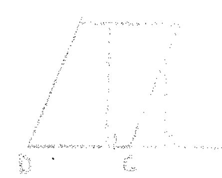
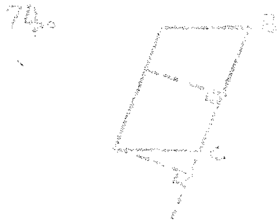
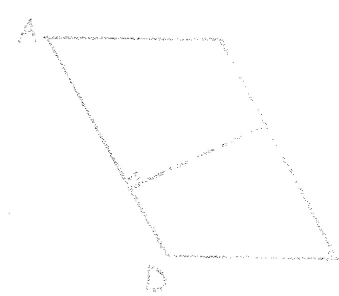


74.



75. Altitude, altitude

76.



77. CX

78. XY

QUIZ to be written after student has completed frame 81.

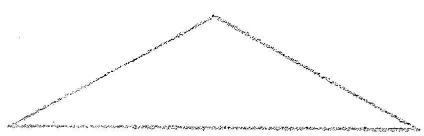
) Area is the ________________ ________________ bounded by the sides of a polygon.

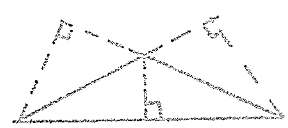
amount of surface

We can compare the areas of two polygons by finding out the number of times that each polygon contains a ______ of ______.

unit of area

Construct the three altitudes of the triangle below.

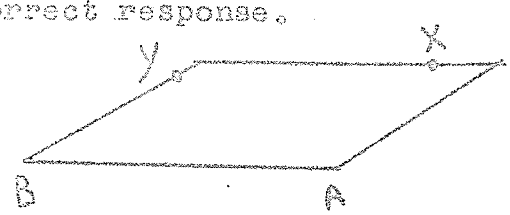




) An ________________ of a triangle is a line drawn from a vertex perpendicular to the base or the base ________________.

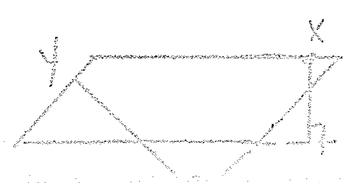
altitude, extended.

In the ||gm below it is obvious that an altitude drawn from X will intersect the base (BA) or (BA extended). Underline the correct response.



BA extended

) Construct an altitude from Y in the above diagram.



) Once you have completed this quiz tear out the page and hand it in to your teacher.

80. Every altitude drawn in a p________ ________ (one word) is an altitude to both bases because it is ⊥ to both

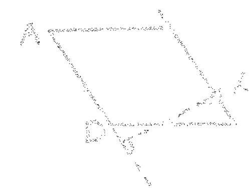
79. perpendicular (⊥)

81. Ref. fig. 25. Draw an altitude to AD from the point X. It is also an altitude to ________ (lets)

80. parallelogram

82. Ref. fig. 26. ABCD and EFGD are //gms which stand on the same or common base ________ (lets) and lie between the same parallel lines AF and DC.

81.



BC

83. These ~~//~~ parallelograms have (equal, unequal) altitudes, using CD as base.

82. DC

84. Ref. fig. 27. ABCD and EFGH are //gms which lie between the same parallel lines ________ and ________ (lets) but have different bases, DC and HG.

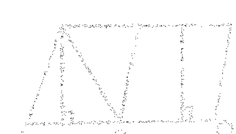
83. equal

85. Ref. fig. 28. Triangle ABC and //gm CDEF lie between the same parallel lines AE and BD. Using BC and CD as bases construct their altitudes. Are these altitudes equal? ________

84. AF DC

86. When two figures lie between the same ________ lines we say that their altitudes ________ are equal.

85.



yes

87. Since two parallel lines are everywhere equidistant, then any two figures which between the same parallel lines will have equal ______.

86. parallel

(is a rectangle) using
88. Ref. fig. 20. The altitude AD as base is longer, shorter, the same length as CD.

87. altitudes

89. WE conclude that the length of an altitude of a rectangle is the same as that of one of its(rect) ______.

88. the same

(a ||gm)
90. Ref. fig. 18. The altitude XY is longer than, shorter than, the same length as AD. [measure if necessary]

89. sides

91. Ref. fig. 29. Triangles ABC and DBC have a side in common; i.e., the side ______ (lts) is common to both.

90. shorter than

92. Using BC as base the altitude of triangle ABC is a line from the vertex ____ (let), perpendicular to the side ______ (lets).

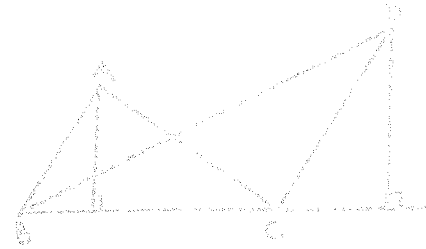
91. BC

93. Draw in the altitudes of triangles ABC and DBC using BC as base.

92. A BC

94. It is worthwhile to remember that whenever the word base is used an altitude is automatically associated with it. Thus, in the previous frame, with the base BC we immediately associated the TWO altitudes of Triangles ABC and BCD. Are these altitudes equal? ________

93.



95. Ref. fig. 30. Do these two //gms have the same base? ________

94. no

96. Name that base. ________ (lets)

95. yes

97. The altitudes associated with the common base DC of //gms ABCD and EFGC are equal, unequal. ________

96. DC

97. unequal

99. Ref. fig. 24. We have said that with a certain altitude you associate a certain base. In either a //gm or a rectangle one altitude can be associated with either of two bases. In fig. 24 the altitude XY is associated with the base ________ or the base ________

98.

100. And the altitude RS can be associated with ________ (#) of bases.

99. AD DC

[illegible] [illegible] are ______ of ______ (lets

100. [illegible]

[illegible] fig. 31. ABCD is a //gm. XY is an al[illegible] associated with base ______ or base ______. (lets)

101. AB, DC

---

103. RS is an altitude of //gm ______ (lets associated with base ______ or ______ (")

102. AD, BC

---

104. [illegible] lets) is an altitude of //gm ABCD associated with base DC (lets) or base AB extended.

103. EFCD, ED, FC

---

105. ML is also an altitude of //gm ______ (lets) associated with base ______ (lets) or base ______ (lets) ______ (word)

104. ML, DC

---

106. //gms ABCD and EFCD both have the same altitude ______ (lets)

105. EFCD, DC, FC extended

---

107. The above //gms also have the same base ______ (lets)

106. ML

[illegible] //gms ABCD and EFCD stand on the ______ base DC and lie between the same parallel lines AF and ______ (lets and hence have equal or the same altitudes.

107. DC

109. Ref. Fig. 32. ABCD is a ||gm, and ECDF is a rectangle. They have different bases BC and FD, but lie between the same parallel lines ______ (2 lts) and ______ (2 lts), and hence they have equal ______

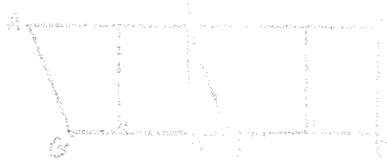
108. EH, same, D.

110. Draw in the equal altitudes.

109. AD? BC, [illegible]

111. If a parallelogram and a rectangle lie between the same parallel lines their altitudes are ______.

110.



112. Thus whenever we use the phrse "lie between the same parallel lines" we mean havin the same or equal ______.

111. equal

113. Ref. Fig 33. Treingles ABC and DBC have the same ______ BC.

112. altitudes

114. The above triangles lie between the same ______ lines, AD & BC.

113. base

115. Since they lie between the same parallel lines they must have ______ altitudes.

114. parallel

116. Lying between the same parallels means the same as having ______ (2 words)

115. equal

117. Ref. Fig. 34. Triangles ABC and DEF do, do not, have equal altitudes, hence they do, do not, lie between the same parallel lines.

116. equal altitudes

118. If two figures have equal altitudes then they must lie ______ the same ______

117. do not, do not

119. An altitude of a triangle is a line drawn form a ______ of the triangle perpendicular to the ______ ______, which we commonly call the b______.

118. between, parallel lines

120. Congruent means equal in all respects. When two figures are congruent, then the angles, sides and areas of one are equal to those of the other. We use the symbol " ≅ " to stand for congruency. The three lines may represent angles, ______ & ______

119. vertex, opposite side, base.

121. ______ is a symbol which means figures are equal in all respects.

120. sides, areas

122. When two figures are congruent, they are equal in three respects. Their corresponding ______ are equal; their corresponding ______ are equal, and their ______ are equal.

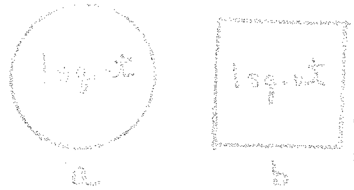
121. ≅

123. Write the underlined part in symbols. △[illegible] is congruent to △DEF

122. angles, sides, areas

124. To indicate that two figures are equal in area, but are not congruent, we use the " = " symbol.

123. $\cong$

125. 

Circle "a" is equal in area to square "b"
OR
Circle "a" ____ (symbol) square "B"

126. Rewrite that statement substituting the appropriate symbol for the underlined words.
$\triangle$ABC is equal in area to $\triangle$DEF

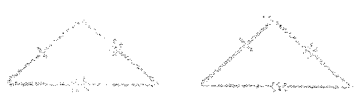
____

125. =

127. An equal sign means that two figures are equal in a__________, but are not __________.

125. =

128. These two triangles are c__________



Are they equal in area? __________

127. area, congruent

129. When two figures are congruent, we use the symbol ______. When two figures are equal only in area, we use the symbol ______.

128. congruent (s.s.s.)

yes

130. Area is the ______ of ______ bounded by the sides of a //gm.

131. Read fig. 35. ABCD is a rectangle; EFCD is a //gm. They stand on the same ______(word) ______(lets).

132. They lie between the same parallel ______(word) ______ and ______(lets)

133. Shade the area of triangle AED(fig. 35)

134. If we subtract the area of $\triangle$AED from figure AFCD, then we are left with the area of ______(unshaded part)

135. If we subtract the area of $\triangle$BFC from figure AFCD we are left with ______ ______.

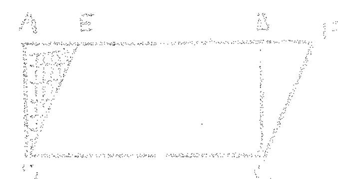
136. Complete the following proof, given that $\triangle$AED = $\triangle$BFC.
Proof:
fig. AFCD = fig. AFCD (qty. = itself)
$\triangle$ AED = $\triangle$ BFC (given)
//gm ______ = rect. ______ (ax. of = ...)

131. $\equiv$: =

132. amount, surface

131. base DC

132. lines AF and DC

133. 

134. figure or quad EDCF or //gm

135. fig. ABCD or rect.

137. Each of these propositions consists of a [illegible] consists of the given and what has to be proved (TBP). The "if" clause of the general enunciation tells us what is ______ and the "then" clause tells us ______

136. Given, [illegible]

138. Ref. fig. 36. Read the general enunciation and underline the "then" clause.

137. Given, [illegible] proved

139. The "if" clause tells us that we are given a ______ and a ______ standing on the same base and lying between the same parallel lines, hence of ______.

138. The [illegible]

140. Name the rectangle ______; the parallelogram ______; the same base ______

139. rectangle, parallelogram, equal altitude

141. GIVEN: //gm EFCD and rect. ABCD, ______ on the same base DC and ______ between the same // lines AF and DC.

140. ABCD, EFCD, DC.

142. The ____ caluse of the general enunciation corresponds to the given of the ______ enunciation.

141. standing, lying

143. The "then" clause of the general enunciation corresponds to the ______ [illegible]

142. If, [illegible]

143. We are supposed to prove that ______________________	143a. RTP
144. Read the proof carefully(fig. 36) and then answer the following questions. Refer back to the proof when ypu are in douot.	143b. rect ABCD = ||gm EFCD
145. The first major step in the proof was to prove ______________, and hence equal in area.	
146. The next step was to state that fig. AFCD = fig AFCD. The reason for this statement was: a __________ equals __________.	145. △ AED ≅ △ BFC
147. The last two lines of the proof indicated that we ______________ △ AED from fig. AFCD, and △ BFC from fig. AFCD.	146. quantity, itself
148. When we subtract △ AED from fig. AFCD we get ______________; and when we subtract △ BFC from fig. AFCD we get ______________.	147. subtracted
149. The reason why //gm EFCD and rect. ABCD were equal in area is the axiom of ___	148. ||gm EFCD, rect. ABCD
150. The axiom of subtraction states that "when equal are subbracted from ______ the results are __________.	149. subtraction
151. AFCD = fig. AFCD because __________	150. equals equal
152. △ AED = △ BFC because __________	151. a quantity equals itself.

[illegible]. △AED ≅ △BFC — reason ______

They are congruent

154. When equals (triangles AED and BFC) are subtracted from equals (______ and ______), the results (/7gm EFCD and rect. ABCD) are equal.

152. a a s

155. If you were to write this proof out, the first step would be to prove that ______ ≅ ______. Then you would subtract each of these triangles from figure ______.

154. fig AFCD [illegible] ABCE

156. Ref fig. 37. Write out the complete proof.

155. △ AED ≅ △ BFC, ABCD

157. We have prooved that a parallelogram and a rectangle ______ on the ______ base and lying between the same ______ ______ (hence of equal altitude) are equal in area.

158. The area of a rectangle is given by the formula: Area = Base X Altitude. Because of the theorem we have just proved, we can now state that the area of a /7gm = ______ X ______

157. standing, base
parallel lines

159. Ref. fig. 38. //gm ABCD and Rect. CDEF are equal in area because they fulfill the conditions required by Theorem 1, i.e., they ________

________ (the) base ____ (same)

and lie between the same parallels, AF//CD.

---

160. State the formula and calculate the area of the //gm in fig. 39.

---

161. The area of a //gm is 48 sq. inches Find the length of the base if its altitude is 5 inches.

---

162. Rewrite the underlined part using symbols. Rect. X is equal in area to //gm Y.

---

163. Ref. fig. 40. PQRS is a rectangle, and PTUS is a parallelogram. Do they stand on the same base?

If your answer is YES go to frame 165
" " " " NO " " " 164
If you DON'T KNOW " " " 164

---

164. Your answer was no. Do you remember that any one of the sides of a polygon can be considered as a base. Another way of asking whether they have the same base would be to ask: Do these

---

158. Base, Altitude

or [illegible]

---

159. stand on the same

CD

---

160. A = b X a

= 8" X 4"

= 32 sq. in.

---

161. A = b X a

48 = b X 5

48/5" = b

---

162. =

---

163. Go to appropriate

... to 163 and select the right answer.

165. ... answer was yes. Youare correct. ... that base. ________

166. Ref. fig. 40. Do //gm PTUS and rect. PQRS lie between the same parallels. Yes or no. ________

165. PS

167. Name the parallel lines between which the rectangle and the //gm. lie.

________ & ________ (lets)

166. yes

168. //gm PTUS (is equal to, not equal to) rect. PQRS in area. ________

Why? ________

167. PS and QU

169. Ref. fig. 41.
Given: //AFEB, and rect. ABCD

R.T.P.: //gm AFEB = rect. ABCD

Proof consists of one statement and one reason.

____

____

168. is equal to, because they stand on the same base PS and lie between the same parallels, PS//QU.

170. In Fig 62, Prove that //gm CDEF is equal in area to rect. ABCD.

169. //gm ABEF = [illegible]

Reason: They stand on the same base AB and lie between the same parallels, AB & DE

171. Define the area of a polygon

170. //gm CDEF = rect. ABCD because they stand on the same base CD and lie between the same parallels, AF & CD.

172. Congruent means:

171. is the amount of surface bounded by the sides of the polygon

173. The symbol ______ means equal in all respects.

172. equal in all respects.

174. Name the three respects in which congruent figures are the same.

173. ≡

175. "≡" does not mean the same thing as the symbol "=". When we say that two figures are "=" we mean only one thing, that they

174. sides, angles, areas.

176. We have proved one theorem to date Its general enunciation is:

175. they are equal in AREA

177. The formula for the area of a rectangle is : ______________

____

176. If a //gm and a rect. stand on the same base and lie between the same parallel lines, (hence of equal altitude), then they are equal in area.

178. The formula for the area of a //gm is ______________________________

177. A = ba

179. Ref. fig. 43. Rect. ABEF = //gm ABCD because (select the correct one)

a) they stand on the same base DE and lie between the same //s, AB & DE

b) they stand on the same base BE and lie between the same //s, AB & DE

c) they stand on the same baseAB and lie between the same //s, AD & BC

d) they stand on the same base AB and lie between the same //s, AB & DE

e) none of these are correct, and the correct reason is ______________

____

178. A = ba

180. Figures which lie between the same parallel lines must have the same or ______ altitudes. (5 letter word)

179. The correct answer is d.

181. So then we are faced with the problem of deciding whether two figures lie between the same parallels, we are faced with deciding whether these two figures have the ________ or ________

______________________? (2 wds.)

180. equal

182. Ref. fig. 44. Which of a, b, or c, are examples of polygons that lie between the same parallel lines? ____________

181. same or equal altitudes.

183. In fig. 44, which of a, b, or c, ~~xx~~ are examples of polygons having the same base? ____________

182. b only. Triangle ACD and //gm ABCD lie between AD & BC

184. Ref. fig. 45. Which of a, b, c, are examples of polygons lying between the same parallels? ____________

183. a, DC; b, AD; c, DC

185. Ref. fig. 46. How many altitudes ~~xx~~ which differ in length can you draw in this //gm ? ________. So do.

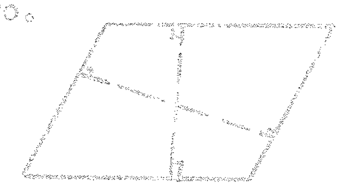
184. all three.

186. Ref. figs. 44 & 45. In which of the diagrams do the polygons not only stand on the same base, but also lie between the same parallel lines?

In fig. 44. ____________

In fig. 45. ____________

185. Two.



187. Ref. fig. 47. Given: //gm ABCD and rect. ABDX. Make a statement about their areas and give your reason.

____

____

186. In fig. 44, b
In fig. 45, all three.

188. Ref. fig. 48. Is //gm ABCF = rect. BCDE? ________. Why? ____________

They are equal in area because they stand on the same base AB and lie between the same ....

189. Ref. Fig. 49. This is your second theorem. Reading the general enunciation, we see that it states that under certain conditons two ______ ______ are equal in ______

188. yes. Because they stand on the same base BC and lie between the same parallels AD & BC.

---

190. Answer the following questions as you read through the theorem. We are given two parallelograms ______(lets) and ______(lets) which stand on the same base ______ (lets) and which lie between the same parallels ______ and ______(les)

189. parallelogram, area

---

191. We are asked to prove that: ______ ______

190. AFCD and RDCS
DC
DC & AS

---

192. In theorem 1, we showed that a rect. is equal in area to a ______ if they stand on the same base and lie between the same ______

191. //gm AFCD = //gm RDCS

---

193. Inorder to prove theorem 2, we must make use of theorem 1; This is why we construct ______ DCBE.

192. parallelogram, parallels

---

194. A rectangle by definition is a //gm with one ______ angle.

193. rectangle

---

195. Inorder to make a rectangle, we let draw from D a line perpendicualr to ______ (lets) and meeting ______ (same lets) at E.

194. right OR 90°

195. AS ? AS

196. First, we draw CB ______________ to EF. Hence DCBE is a //gm, because its opposite sides are ______________

196. parallel, parallel

197. DCBE is not only a //gm; it is also a rectangle, because it is a //gm with ______ ______ ______ (3 words)

197. with one right angle

198. //gm ADCF and Rect. DCBE stand on the ______ base ______ (lets), and lie between the same parallel lines, ______ & ______ (lets), Hence they are equal in ______________

198. same, DC
AS DC
area

199. For the same reason, //gm ______ (lets) and rect. ______ (lets) are equal in area, also.

199. BDCS DCBE

200. Since both //gms are equal in area to the same rectangle, then they must be equal to each other by the axiom of ______________

200. equality

201. In order to prove that the two //gms are equal in area we had to construct a ______________.

201. rectangle

202. We constructed this rectangle by first drawing DE ______________ to AS, and then drawing ______ (lets), parallel to ______ (lets).

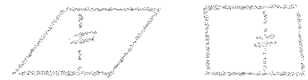
202. perpendicular
CB
DE

203. [illegible] Fig. 50. The proof of the theorem [illegible]

204. Theorem 2 has three corollaries. See fig. 49. The first one states that //gms in ________ bases in the [illegible] ________ ________ and between the same parallels are equal in area.	203. Check your proof with the original in fig. 49.
205. In fig. 51, //gms ABCD and EFGH stand on equal bases, ______(lets) = ______(lets), in the same straight line, and between the same parallel lines, DG & AF. Hence, they are equal in ________.	204. equal straight line
206. For two parallelograms to be equal in area it is necessary that they stand on the same base or equal bases in the ________ straight line, and lie between the same parallel lines.	205. AB EF area
207. However two parallelograms can be equal in area if they stand on the same base or on equal bases NOT in the same straight line only if they have equal ________.	206. same
208. Ref. fig. 52. //gms ABCD and ABEF are equal in area because they stand on the same base and have the ________ ________, "h"	207. altitudes
209. Draw two //gms with equal bases NOT in the same straight line, equal in area. Inorder to so do we must make certain that their ________ are equal.	208. same altitude. You may have used equal altitudes, but same altitutude is a better answer because the altitude (h) is spe[illegible]

210. If two //gms stand on the same side of the same base, and are equal in area, then they have ________________, and hence lie between the same parallel lines.

209. altitudes



211. To say that two figures lie between the same parallel lines or parallels is equivalent to saying that they have ________________ ________________.

210. equal altitudes

212. Ref. fig. 53. Given: AB//DF. DC = CF

PROVE that //gm ABCD = //gm EBCF.

The proof consists of One statemnt and one reason.

____

____

211. equal altitudes

213. Suppose we now wish to prove that fig. 1 (quad. AECD) = fig. 2. (△BCF)

The proof is as follows. Fill in the missing parts.

Statements	Reasons
//gm ABCD = ________________	on equal bases, DC & CF and between the same parallels, AB & DF.
________ = ________________	a qty. equals itself.
fig. 1 = ________________	axiom of subtraction?

212. //gm ABCD = //gm EBCF --- stand on equal bases in the same straight line, and lie between the same //s.

213b. Check your proof to the above by referring to the answer printed alongside this frame. Sorry its upside down!

213. Ans. //gm EBCF

△EBC = △EBC

fig. 2

214. [illegible] fig. [illegible] We are given [illegible] ABCD and ______	
215. The two //gms stand on the ______ base DC, and on the same side of it.	214. EFCD
216. Make a statemnt, giving your reason about //gms ABCD and EFCD. ______ ______	215. same
217. //gm ABCD and //gm EFCD <u>overlap</u>, i. e., they have some surface in common. The surface enclosed by ______ (lets) is common to both parallelograms.	216. they are equal in area, because they stand on the same base DC and lie between the same //s, AF & DC.
218. If we subtract (take away) the surface enclosed by $\triangle$ODC from //gm ABCD the remainder, or remaining surface, will be that enclosed by quadrilateral ______ (lets) Or simply: //gm ABCD − $\triangle$ODC = quad. ______.	217. triangle ODC
219. If we subtract $\triangle$ODC from fig. ______ (lets), the remainder is fig. EOCF.	218. ABOD, ABOD.
220. The axiom of subtraction states: if ______ are subtracted from equals the ______ are ______.	219. EFCD

220. equals, remainders, equal

221. Write out the formal proof for this question by completing the balnks.

_______________ = _______________ on the same base DC and between the same parallels AF & DC.

_______________ = _______________ a qty. equals itself.

_______________ = _______________ axiom of _____________.

221. //gm ABCD = //gm EFCD

△ODC = △ODC

fig. ABOD = fig. EOCF

subtraction

222. Parallelograms are equal in area if they stand on the same or _______________ (2 words) in the same straight line, and lie between the same parallel lines.

222. equal bases

223. Ref. fig. 55. Do parallelograms 1234 and 5678 have a common base (yes/no). Do they lie between the same parallel lines? (yes-nO).

223. NO
YES

224. Hence to prove that theses two //gms are equal in area we must prove ~~th~~ that they stand on _______________, and lie between the same parallel lines.

224. equal bases

226. A base of //gm 1234 is 4___. (let)
A base of //gm 5678 is ___8. (")

225. 3 7

226. Prove that 43 = 78 by using the axiom of addition. (continued ...)

cont.

1. 48 = 37 --- given

____ = ____ --- a qty. equals ..

____ = ____ --- ax. of addition

226. 83 = 83 itself
43 = 78

The proof of the problem is now left to the student. (see fig. 55).

227. Frames 224 and 226 are crucial to the solution.

Ref. fig. 54. Inorder to prove that fig. ABCD = fig. ECGF, we had to subtract a c____________ figure from both parallelograms.

228. common

Ref. fig. 55. The reason why //gm 1234 = //gm 5678 was: ____________

229. stand on equal bases in the same str. line and lie between the same //s. 16 &47.

Before we could prove that //gm 1234 = //gm 5678 we had to prove that: ____________

230. their bases were equal (or words to that effect)

We proved that base 43 was equal to base 87 by using the axiom of ____________.

232. Another way of proving that base AB equals 87 is to use the axiom of subtraction. The proof can be set up in 3 steps as follows:

47 = 47 -- ____________________

____ = ____ -- given

____ = ____ -- ax. of subt.

231. addition [illegible]

232.

qty. equals itself
48 37
43 87

---

233. Ref. fig. 56. Two //gms are given:

233. ____________(lets)&____________(lets)

233.

ABCD EFGH

---

234. Do they have a common base? ________

---

234. *NO*

235. Prove that they stand on equal bases in the same straight line. That is prove that DC = HG.

____

____

____

---

235. DH = CG -- given
HC = HC -- qty. equals [illegible]
DC = HG -- ax. of add.
OR ax. of = [illegible] could b[illegible]

---

236. Inorder to prove that fig.1 = fig.2 we would have to ____________ the common area of ____________ from the equal areas of parallelograms ABCD and EFGH.

236. subtract △ HKO

---

237. The proof is now left to the student (see fig. 56.)

238. Ref. fig. 57. Read but do not attempt the question tangibly.

239. To know that two //gms are equal in area if they stand on the same base and lie between the same parallels. The base we will use for the second //gm is ________(lets).

240. To make the second //gm lie between the same parallels as the first, it is necessary to extend the line segment ________(lets).

239. DC

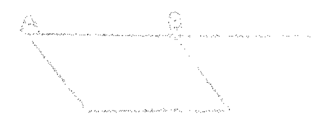
241. Although it is true that we could extend either AB or BA, in the diagram (fig. 57) it is more convenient to extend AB. Thus the first step in your construction is to "________ AB".

Do this and write it in the space provided in fig. 57.

240. AB or BA

242. Step 2 consists of drawing any line from D to meet AB or ________(lets) extended, at the point X.
Perform the const. and fill in 2.

241. extend

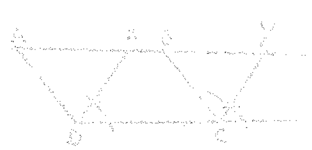


243. Our 3rd step is to draw from C a line parallel to ______(lets) and meeting AB at Y. We do this by making the corresponding angles equal.

242. AB

244. DXYC is a //gm because we have so constructed it, i.e., we made the opposite sides ________.

243. DX



245. There is considerable leeway in the terms used to describe a construction problem. The answer which follows serves only as an example.
1. Extend AB
2. Draw DX meeting AB at X.
3. Draw CY parallel to DX and meeting AB extended at Y.

244. parallel

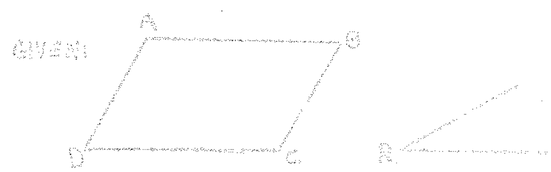
245. NOTE: Although it is sufficient to give a general description, it is necessary to showw all construction lines. (as in 243, ans)

246. Ref. fig. 58. Read the question, and now note the format that your answer should take.
We are given two things a __________ and an __________________.

246. parallelogram
angle

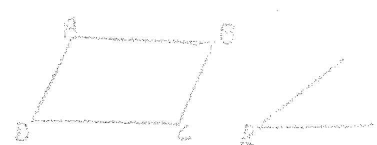
247. Opposite the "given" draw a //gm and an angle. Label the //gm ABCD and label the angle R.

247. This is a sample.



248. Now you may write beside your diagrams "//gm ABCD and $\angle$ R."

248.
GIVEN: //gm ABCD and $\angle$ R



249. In the "REQ'D" You may write: To construct a //gm equal in area to //gm __________ (lets), and with one angle equal to $\angle$ R.

249. ABCD.

250. In the space by "DIAGRAM" you will perform the construction, and in the space by "CONSTRUCTION" you will describe it. The 1st step is: Draw any line and on it mark off DX = DC.

251. Step 2. At W construct an angle equal to $\angle R$.

The 2 //gms have equal bases (step 1).

252. Inorder that the two //gms equal each other in area, they must have equal ______________.

253. Step3. On WX construct an altitude equal to the corresponding altitude of //gm ABCD. (Label the end remote from WX, Q.)

254. Step 4. At Q construct a line $\perp$ to the altitude and hence parallel to WX. Let that line intersect WZ at Z.

255. Step 5. At X construct XY // WZ, intersecting ZY at Y.

256. The //gm is now completed.

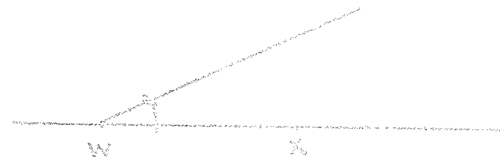
257. The diagonal of a //gm divides it into two __________ triangles.

258. If two triangles are congruent, they are equal in _ _ _ _ .

259. Draw a line form C parallel to BA and intersecting CR at X. [fig.59]

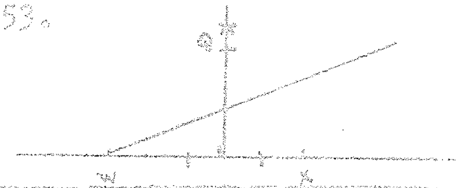
250. Perform and describe the construction

251.

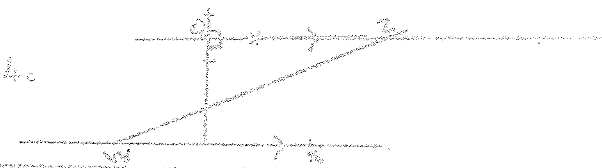


252. altitudes

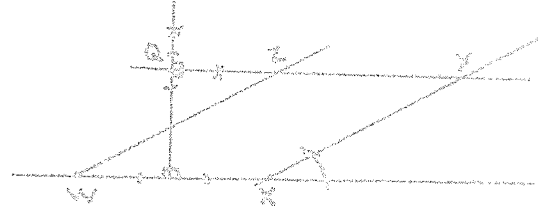
253.



254.



255.



257. congruent

258. area

260. Since AB //XC and AR // BC, what kind of a polygon is ABCX?

____

Why? __________________________

261. Ref. fig. 59. A side of $\triangle$ABC is now a diagonal of //gm ABCX. It is __________ (lets)

262. Ref. fig. 60. Diagonal BD divides //gm ABCD into two ______________ triangles, and hence they are equal in __________.

263. Since $\triangle$ABD = $\triangle$BDC, then $\triangle$ABD = __________ (fraction) of //gm ABCD.

264. Ref. fig. 61. Because a diagonal BISECTS a //gm, either $\triangle$ __________ (lets) or $\triangle$ __________ (lets) is equal to $\frac{1}{2}$ //gm ABCD.

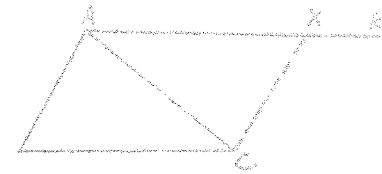
265. A diagonal _ _ _ _ _ _ _ a //gm.

266. Ref. fig. 62. Inorder to write the given we refer to the __________ clause of the general enunciation.

267. We are asked to prove that: __________

____

259.



260. //gm

def'n--opposite sides //.

261. AC

262. congruent
area

263. $\frac{1}{2}$

264. ABC; ADC

265. bisects

266. if

268. Inorder to prove this theorem we utilize the following facts: a) The area of a __________ equals the area of a rectangle if they stand on the same base, and lie between the same parallels b) a __________ bisects a //gm	267. $\triangle$ AXB = ½ rect. ABCD
269. Since we we were not given a //gm we have to __________ one.	268. //gm diagonal
270. We constructed the //gm by drawing ______ parallel to ______ (lets)	269. construc t
271. AYXB is a //gm because its __________	270. AY BX
272. The diagonal of //gm ABXY is ______	271. opposite sides are //.
273. Thus $\triangle$ AXB = ______ //gm ABXY. Why? __________	272. AX
274. A recapitulation of the steps involved in the proof: The 1st step was to construct a //gm ABXY by drawing __________ __________.	273. ½ -- a diagonal bisects a //gm.

275. The next step was to show that $\triangle ABX$ is equal to ________________

274. AY // BX, meeting DX at Y.

276. The 3rd step was to show that the ____________ and the ____________ are equal in area.

275. $\frac{1}{2}$ parallelogram ABXY

277. The last step employs the axiom of ____________ or ____________

276. //gm and the rectangle

278. The proof is now left to the student. See fig. 63.

277. equality ; substitution.

279. If the area of a rectangle is $32 in^2$ what is the area of a triangle having the same base and of equal altitude? ____________.

280. The wase of a triangle is 10 cm., & its altitude is 5 cm. What is its area. (Show your wokk.)

279. $16 in^2$

281. The area of a triangle is 42 sq. ft. Its base is 7 feet. What is its altitude. Show work.

280. $A = \frac{1}{2}ba$
$= \frac{1}{2} . 10 . 5$
$= 25$ sq. cm.

288. A rectangle by definition is a //gm with one right angle. BCYX is a rectangle because: a) it is a //gm. Why? ______ b) it has one right ∠. Why? ______	287. AD at f.
289. Once we know that BCXY is a ______ we can proceed with the remainder of the proof.	288. opposite sides are // ∠ BXY = 90° .. BX ⊥ AD
290. The two middle steps of the proof require the use of ______ 3. The last step the ax. of ______	289. rectangle
291. Ref. fig. 66. Proof is left for the student. Then read corollaries.	290. theorem equality
292. A corollary which follows from theorem four states: Triangles on ______ ______ in the same straight line, and between the same parallels are ______ (3 wds)	
293. Triangles of equal area standing on a common base must have *equal* ______ (1. wd.)	292. equal bases equal in area.
294. If these triangles not only stand on a common base, but also on the same side of it, then they must ______	293. equal altitudes

282. On the basis of Th.3, cor.,2, make a statement giving your reason about the diagram in fig. 64.

____

____

281. $A = \frac{1}{2}ba$
$42 = \frac{1}{2}\cdot 7\cdot a$
$\frac{42\cdot 2}{7} = a$
12 ft. = a

283. Ref. fig. 65. $\triangle$ABC and $\triangle$DBC stand on the same ______ (wd) ______ (lts)

282. $\angle$ EDC = $\frac{1}{2}$ //gm ABCD, on same base DC and between the same parallels, BE & DC.

284. We have to pove that: _______________

____

283. base BC

285. To prove that $\triangle$ABC = $\triangle$DBC we will use a theorem which states that a triangle is equal to one-half the area of a rectangle standing on the same base and lying between the same parallels. Looking at the diagram we see that $\triangle$ABC = $\frac{1}{2}$ rect. ______ (lets), and that $\triangle$DBC equals $\frac{1}{2}$ rect ______ (same lets).

284. $\triangle$ABC = $\triangle$DBC

286. However, we were not given rect. XYCB. Therefore we must construct it. The first step in constructing this rect. was to draw _______________

_______________ and meeting AD at X.

285. BXYC

287. The 2nd step was to draw CY // BX and meetimg _______________

286. BX $\perp$ AD

295. Ref. fig. 67. Is $\triangle ABC = \triangle BDE$?
____. Why? ____________________

294. lie between the same parallels lines
[turn back to review preceding frame 137 - & read over]

300. Ref. fig. 68. BX is a(n)________

295. Yes, because they stand on = bases etc.

301. An altitude of a triangle is the line drawn from a ________ of that triangle ________ to the ________ side.

300. altitude

302. We sometimes call the opposite side a BASE. In fig. 69, altitude AX is drawn to ________ BC?

301. vertex
perpendicualr
opposite

303. The altitude of a triangle is the line drawn________________
(do not use the words "opposite side!"________

302. base

304. So that our definition of an altitude will hold for an obtuse triangle, we say that it is a line drawn from the vertex of a triangle $\perp$ to the base or base ________

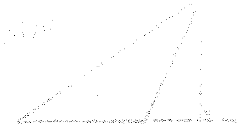
303. from a vertex of that $\triangle$ $\perp$ to the base.

305. Ref. fig. 70. Draw an altitude from the vertex A.

304. extended or produced.

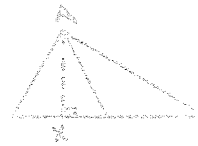
306.

307. Ref. fig. 71. Draw an altitude from the vertex A. Name it AX. Is this an altitude of triangle ABC? ______ If "yes" then name its base. ______



308. Ref. fig. 71. Is AX an altitude of triangle ACD? ______. If "yes", then what is the base? ______ (lets) ______ (word)

307.



yes
BC

309. Cor.1, Th.4, states: triangles on ______ ______ in the same straight line and between the same ______ are equal in area.

308. Yes.
DC extended

310. A median is a line drawn from the vertex of a triangle to the ______ ______ of the oppoiste side.

309. equal bases
parallels

311. Ref. fig. 72. There are two reasons why △ABC = △ACD.

1. They stand on ______ ______ and have the same ______
2. Cor.3, th.4, which states that a median of a triangle divides it into two triangles equal in area

310. mid-point

312. In fig. 72, the median AC divides △ ______ (lets) into two △s equal in area.

311. equal bases
altitude

313. Of the two reasons stated in 311, we will generally use the second. Thus the proof of that question would be written as follows:
Statement: △ABC = △ACD
Reason: median ______ a △ into ______ △s equal in area

312. ABD

314. A ________ divides a $\triangle$ into two triangles ________ in ________	313. divides two
315. Ref. fig. 73. AD si a median of triangle ABC. Therefore $\triangle$ ABD = $\triangle$ ADC. Why? ________	314 median equal area
316. Define: median. A line ________	315. a median divides a $\triangle$ into two triangles equal in area.
317. Ref. fig. 71. Given: BC = CD R.T.P.: $\triangle$ABC = $\triangle$ACD. Proof: AC is a median -- by def'n ________ -- ________	316. drawn from the vertex of a triangle to the mid-point of the opposite side.
318. Ref. fig. 74. 1 $\triangle$432 = $\triangle$ ________. Why? ________	317. $\triangle$ABC = $\triangle$ACD -- a median divides a triangle into two tirangles equal in area.
319. Instead of sayigg: "a diagonal divides a //gm into two congruent triangles" we prefer to say" a diagonal - - - - - - - a //gm."	318. 142 -- a median divides a $\triangle$ into 2 $\triangle$s = in area.

320. Similary, instead of saying: "a median divides a triangle into two △s equal in area", we prefer to say: The median ________ a △.

319. bisects

321. Thus in fig. 74, because a median bisects a triangle we can say that △432 = △412, and ALSO that

△412 = ______ (fraction) △ 132.

320. bisects

322. Since a median bisects a triangle, it follows that either smaller △ is ______ of the larger.

321. $\frac{1}{2}$

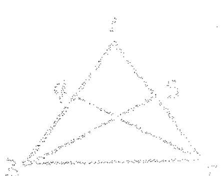
323. Ref. fig 74. Construct median 35. Is △352 = $\frac{1}{2}$ △132? ________.

Why? ________

322. $\frac{1}{2}$

324. Is △142 = $\frac{1}{2}$ △132? ________

Why? ________



323. Yes
median bisects triangle

325. Is △352 = △142 ? ________.

Why? Halves ________

324. Yes.
median bisects triangle

326. Ref. fig. 74. Given: 24 & 35 are medians.
R.T.P.: △342 = △352 [re-read 323-25 first]

Proof:

325. Yes.
of equals are equal

327. Triangles (fig. 75) ADC and BDC stand on the same base _______ (lets) and lie between the same parallels _______ (lets) & _______ (lets).

326. $\triangle 342 = \frac{1}{2}\triangle 132$ -- med. bis. $\triangle 352 = \frac{1}{2}\triangle 132$ -- cts $\triangle$. $\triangle 342 = \triangle 352$ -- ½s of =s.

328. Hence $\triangle$ ADC is(equal, not equal) to $\triangle$ DBC in area. Why? _______________ (fill in step 1)

327. DC; AB & DC

329. Traingles ADC and BDC overlap, that is, they have some surface in common. Name that surface _______ (lets) (fill in step 2)

328. $\triangle$ ADC = $\triangle$ BDC -- stand on same base DC and lie between the same //s, AB & DC

330. When $\triangle$ ODC is subtracted respectively from triangles ADC and BDC the remainders are rspectively $\triangle$ _______ and $\triangle$ _______. (fill in step 3)

329. $\triangle$ DOC = $\triangle$ DOC -- qty. equals itself

330. $\triangle 1 = \triangle 2$ -- ss. of =s.

331. Fill in the three steps for the question in fig. 75.

332. Ref. fig. 76. A trapezium is a four sided figure with one pair of sides _______.

331. As above

323

(AOD is A1, ΔBOC is A2)

333. Ref. fig. 76. ΔABD and Δ ______ stand on the same base AB and lie between the same parallels, AB & DC. (fill in step 1) hence they are ______

332. parallel

334. Δs ABD and ABC have a surface in common, namely, Δ ____________.
(fill in step 2)

333. ABC, equal in area

335. To get Δ1 = Δ2 we must subtract the common triangle AOB from Δs
____________ & ____________ respectively
(fill in step 3)

334. AOB

336. Ref. fig. 76. the completed proof is below:

ΔABD = ΔABC – on same base & between same ‖s

Δ AOB = ΔAOB -- qty. equals itself

Δ1 = Δ2   ax. of – n.

335. ABD; ABC.

337. We have just proven Δ1 = Δ2, in two different ways. In the first (fig 75) we subtracted Δ AOB from the equal Δs ADB and ABC; in the (fig 76) second we subtracted the common Δ ________ from the equal Δs ____ and ________.

336. ✓

338. Ref. fig. 77. Since the last reason states the ax of addition, this implies that we must add ________ to ____________ to get equals.
Can you complete the proof?

337. DOC   ADC
BDC

329. HINT: First statement is: ΔADE = ΔBDE

338. equals; equals

340. To prove that a line is a median we must show that it fulfills the conditions given by the ______________ of a median, namely: that it is a line drawn from ______________________________

____

339. [illegible]

341. Ref. fig. 78. We are given that AD is a median for △ ________.

340. definition; the vertex of a △ to the mid-point of opp. side.

342. Is PD a median for △PDC? ____________

Why? ______________________________

____

341. ABC

343. Make a statement about about △ABD and △ADC. Give your reason.

____

____

342. Yes. D is mid-point of opp. side BC. (by def'n)

344. Make a statement about triangles PBD and PDC. Give your reason.

____

____

343. △s ABD and ADC are equal because a med. bisects a △

345. In order to get △1 = △2, we must subtract △s PBD and PDC from:

a) △s ABD and ADC respectively

b) △s ADC and ABD respectively

344. They are equal, because med. bisects △ PBC.

346. [illegible] the exercise page 22-[illegible], [illegible] answering these questions.

We [illegible] measure [illegible] of any [illegible] by ______. The number [illegible] it contains a unit of ______

______?

345. a. △ABD = △ADC --
△PBD = △PDC --
△1 = △2 --

347. We can measure the area of a rect. in two ways:

______

______

346. counting
area

348. [illegible]

347. counting the no. of units
using the formula bh

349. The formulas for finding the area of a rectangle and //gm are true only if ______

______

348. assumption

350. All mathematics is based on

______

349. the formula for finding the area of rect. is true

351. Do you think that " A of rect. = bh" is an obvious assumption. ______

350. assumptions - axioms [illegible]

352. The formula for finding the area of a rect. is ______

a //gm is ______

a △ is ______

351. You are entitled to your opinion

353. Substituting in the correct formula find the area of $\triangle$ ABC fig. 79.	352. b·a b·a $\frac{1}{2}$·b·a
354. Ref. fig. 80. The base is ______ in. The alt. is ______ in. The area is ______ ______ ______	353. A = $\frac{1}{2}$ X 6 X 4 = 12 sq. " Note: not 4 X 6
355. Ref. fig. 81. Area of ABCD is equal to ______ X ______ = ______	354. 5" 2" 10 sq. in.
356. Ref. 81. ABCD is a rectangle. Is AD an altitude? ______. Why? ______	355. 6 X 3 = 18 sq. in.
357. Ref. fig. 82. ABCD is a //gm. Is AD an altitude? ______. Why? ______	356. Yes. It is perpendicular.
358. Hence, can you find the area of //gm ABCD using only the information that is given. ______ Why? ______	357. No. It is not perpendicular
359. Ref. fig. 83. Find the area of //gm AACD. ______ X ______ = ______	358. No, no alti tude is given.

360. Ref. fig. 84. The base that we would use in finding the area of $\triangle$ABC given that CX is an altitude is ______ (lets).

359. 3 X 4 = 12 sq. in.

361. Ref. fig. 85. The base associated with altitude AX is ______

360. AB

362. Ref. fig. 86. We know that the area of a triangle is equal to $\frac{1}{2}$ba.

A of $\triangle$ ABC = $\frac{1}{2}$ X $\underset{\text{(base)}}{\underline{\text{AB}}}$ X $\underset{\text{(alt)}}{\underline{\qquad}}$ (lets)

361. CB extended

363. Ref. fig. 86. The area of triangle ABC can be found by using a different base and altitude.

A of $\triangle$ABC = $\frac{1}{2}$ X $\underset{\text{(base)}}{\underline{\qquad}}$ X $\underset{\text{(alt)}}{\underline{\qquad}}$

362. CR

364. In any triangle we can have ______ bases and an equal number of alts.

363. BC AM

365. Ref. fig. 87. Draw in the 3 altitudes

364. three

366. The area of $\triangle$ABC (fig. 88) can be ~~fm~~ found in three different ways, depending on which base we use. Using base BC we have:
$\triangle$ ABC = $\frac{1}{2}$ X BC X AQ.

Using base AB we have:
$\triangle$ ABC = $\frac{1}{2}$ X ______ X ______

365. See fig. 88 for example

367. Using AC as base we have:

△ABC = ½ X ________ X ________

368. Ref. fig. 89. To find the area of △ABC with the information given we would use:
a) △ABC = @½ X AB X CX -- turn to 369

b) " = ½ X BC X AR -- " " 370

c) " = ½ X BC X CX -- " " 371

369. You chose a and your choice is correct as we are given the values of both AB and CX. Now you may proceed to calculate the area.

370. You chose b. Note that the problem asks you to find the area. In that case you must chose a base and its altitude for which the values are given. BC has a value, but does AR? Return to 368 and chose right ans.

371. You chose c. To every base their is a particular altitude. The altitude that corresponds with base BC is not CX, it is AR. Now go bask to 368, and try again.

372. Ref. fig. 86. Suppose the area of △ABC is 24 sq. in. and AM is 4in. Find the length of BC.

____

366. AB CM

367. AC BP

XXX

369. A = ½ X 6 X [illegible] = [illegible] sq in.

373. Ref. Fig. 90. Find the area of $\triangle$ ABC

= ½ X ________ X ________
(base) (alt)

= ________

372. A = ½ · b · a
24 = ½ · b · 4
12in. = b

374. Now that we know the area of $\triangle$ABC we can find the length of AX.

Do so. ________

373. A = AB X CR X ½
= 8 X 3 X ½
= 12 sq. uts.

375. Given that the area of a //gm is 48 sq. uts. and its base 9 uts. Find its altitude. ________

374. 12 = ½ X 6 X AX
4 uts. = AX

376. Ref. Fig. 91. Find XY.

375. Altitude is 48/9 uts.

377. Ref. Fig. 92. ABCD is a //gm.

$\triangle$DEC = ________ //gm ABCD

Why? ________

376. A = DC x EF = 8x5 = 40 ...
40 = BC x XY
= 6 x XY
40/6 uts = XY

378. Since $\triangle$DEC = ½ //gm ABCD, we can make the following statement:

$\triangle$... + $\triangle$... = ________ //gm ABCD, because a whole is equal to the sum ...

377. ½. stand on same base ... and lie between same //s AB ...

379. Ref. fig. 92. Prove that $\triangle$EDC = $\triangle$AED + $\triangle$EBC.
Proof:

378. $\frac{1}{2}$

379. $\triangle$EDC = $\frac{1}{2}$ //gm ABCD
$\triangle$AED + $\triangle$EBC = $\frac{1}{2}$ //gm ABCD
$\triangle$EDC = $\triangle$AED + $\triangle$EBC -- ax. of =

See frames 377 & 378 for other reasons.

380. Ref. fig. 93. Complete the proof. Having done this proceed to solve problems 94 and 95. The answers to them are given below.

381. Your answer for 93.

$\triangle$EDC = $\triangle$ADC -- on the same base DC and between the same //s AB & DC

or in 3 steps:

$\triangle$EDC = $\frac{1}{2}$ //gm ABCD //--- you should know the reason.
$\triangle$ADC = $\frac{1}{2}$ //gm ABCD --- diagonal bisects //gm or same as above.
___ ax. of equality.

382. Your answer for 94, could look like this
$\triangle$EBC = $\frac{1}{2}$ --gm ABCD --- on same base and betweensame //s.
$\triangle$AED $\triangle$EBC = $\frac{1}{2}$ //gm -- sum of parts equals whole

$\triangle$BDC = $\frac{1}{2}$ //gm ABCD --- diagonal bisects //gm

___ ax. of eq.

383. Your answer to 95.

$\triangle$ 3 = $\triangle$2 -- median bisects triangle

$\triangle$ADC = $\triangle$DBC --

$\triangle$ODC = $\triangle$ODC --

$\triangle$ 1 = $\triangle$ 2 --

___ -- ax. of eq.

384. Ref. fig. 97. $\triangle$ARD = ______ //gm ABCD
Why? ______

______

385. $\triangle$1 + $\triangle$ 2 = ______ //gm ABCD.
Why? ______

386. The conclusion you may draw from the statements in 384 and 385 is:

______

Why? ______

387. Ref. fig. 98. Make a statement ~~xxx~~ about $\triangle$ EDC and //gm ABCD. Give a reason.

______

______

388. Now attempt problem fig. 99.

389. You have now completed the formal program. In view of the fact that you have done very few problems, an additional sheet has been provided.

384. ½ -- on same base AD and between same //s, AD & BC.

385. ½ -- a whole is equal to the sum of its parts

386. $\triangle$ARD = $\triangle$ 1 + $\triangle$ 2

ax. Of. eq.

387. $\triangle$EDC = ½ //gm ABCD -- on same base and between same //s. BE and DC.

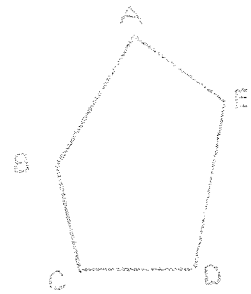


Fig 1

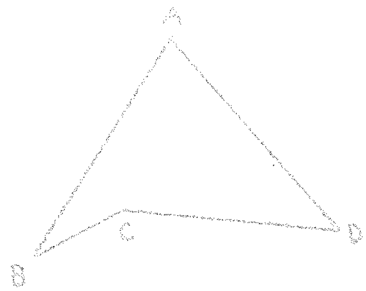


Fig. 2

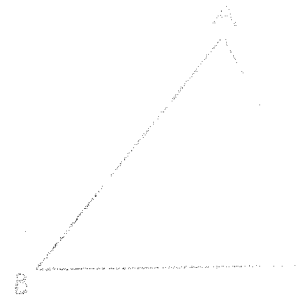


Fig. 3

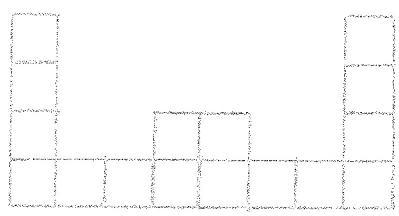


Fig. 4

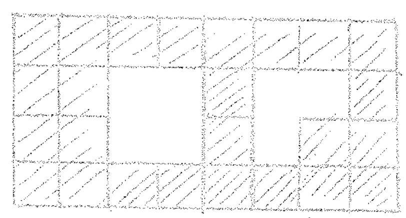


Fig. 5

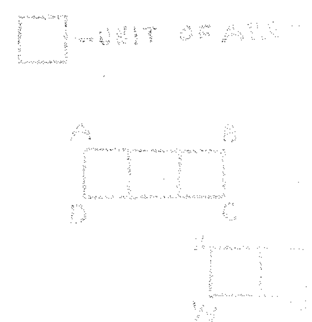


Fig. 6

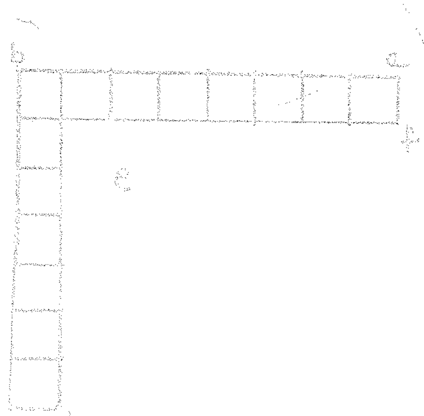


Fig. 7

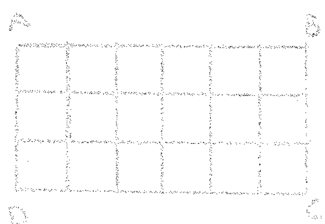


Fig. 8

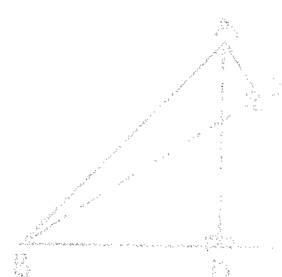


Fig. 9

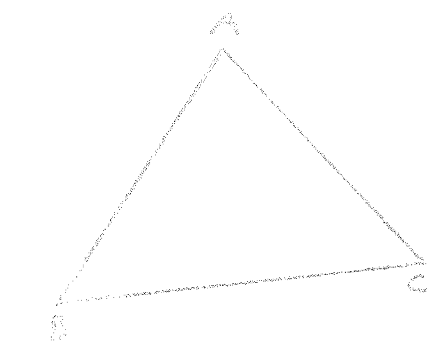


fig. 10

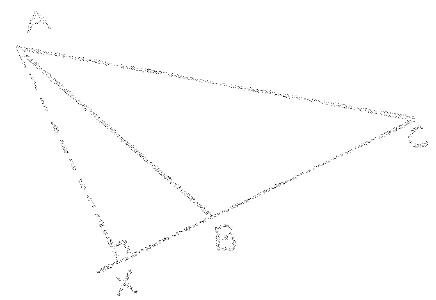


fig. 11

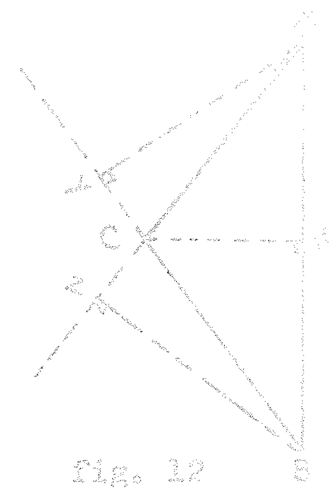


fig. 12

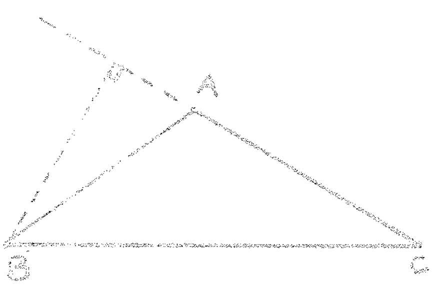


fig. 13

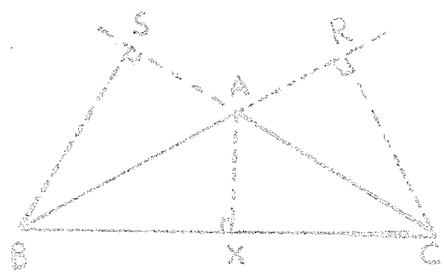


fig. 14

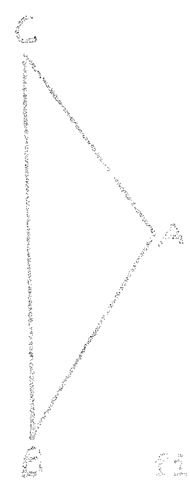


fig. 15

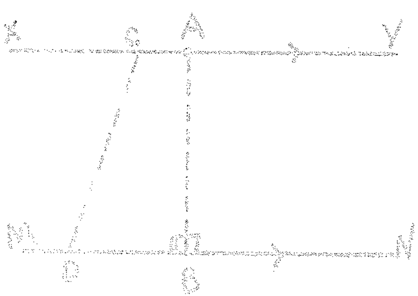


fig. 16

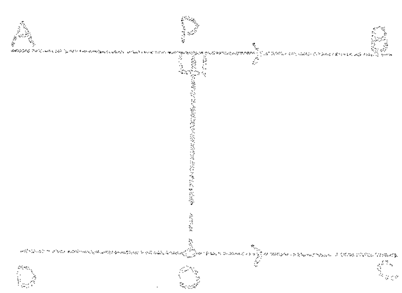


fig. 17

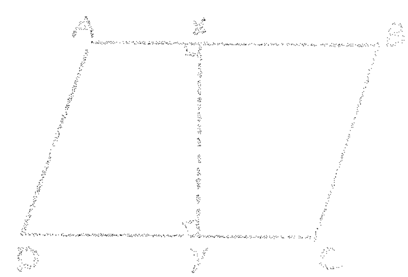


fig. 18

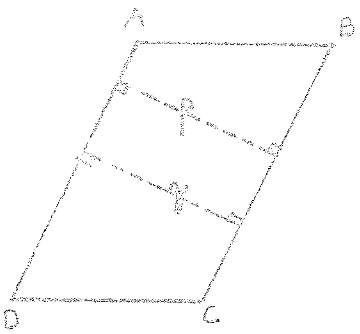


fig. 19

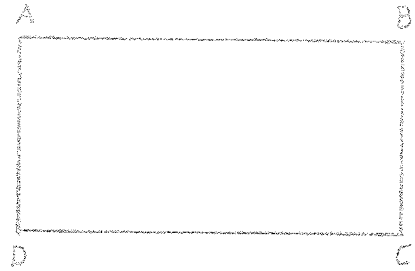


fig. 20

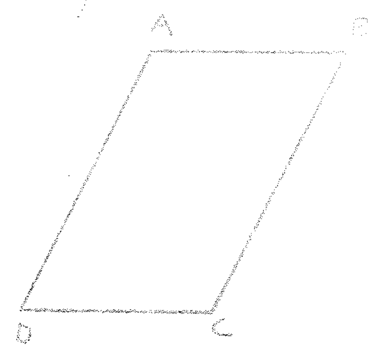


fig. 21

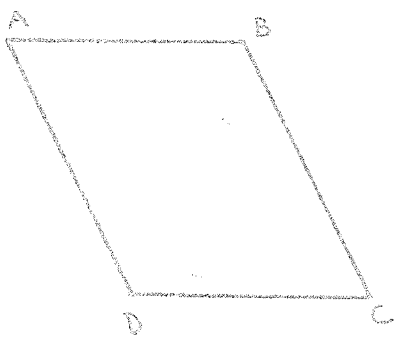


fig. 22

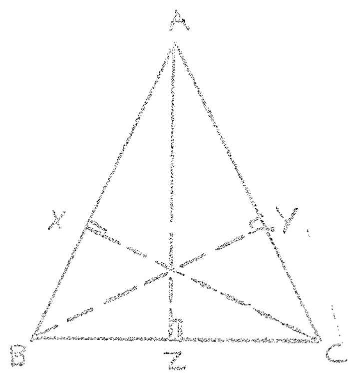


fig. 23

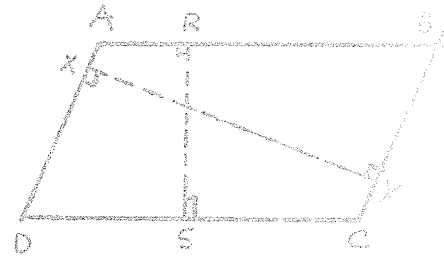


fig. 24

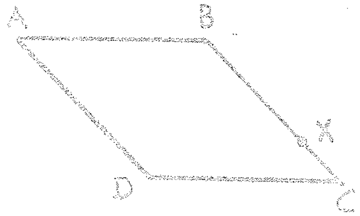


fig. 25

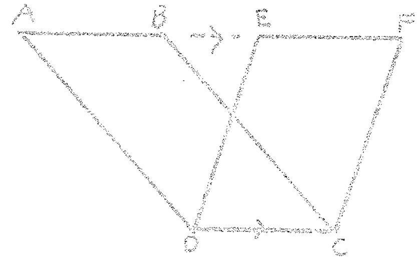


fig. 26

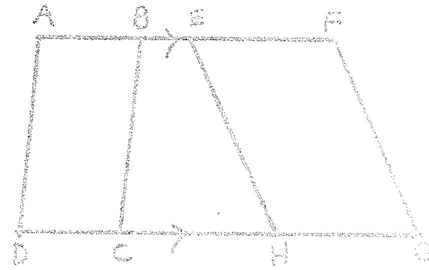


fig. 27

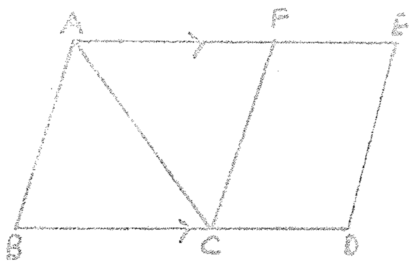


fig. 28

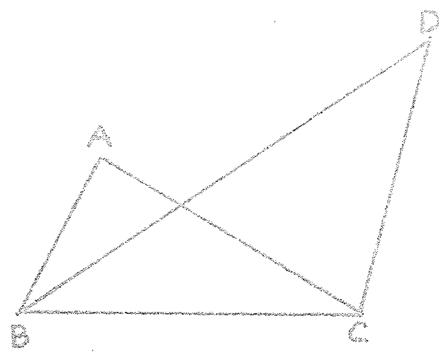


fig. 29

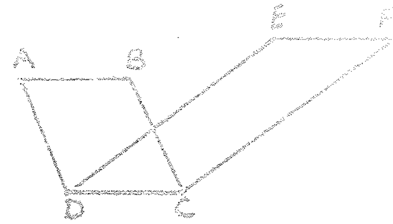


fig. 30

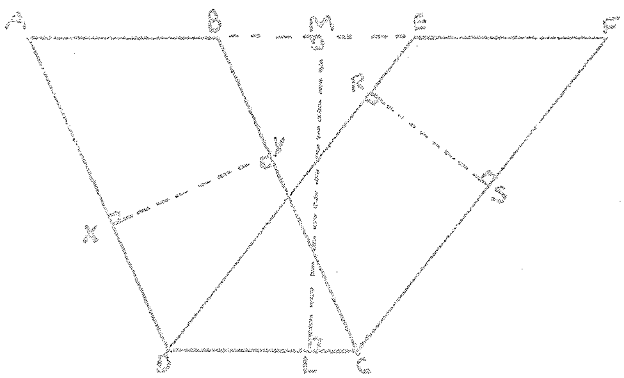


fig. 31

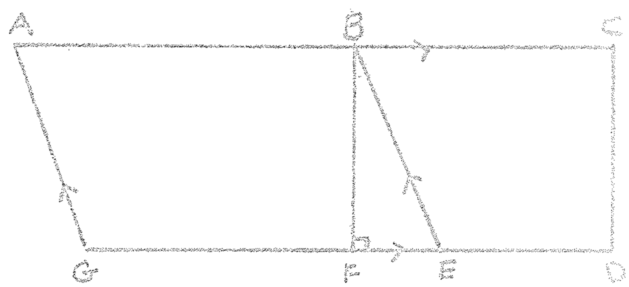


fig. 32

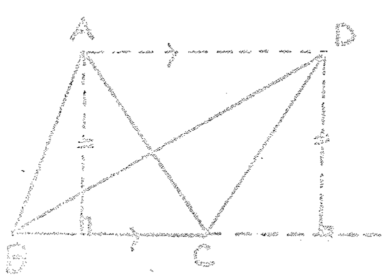


fig. 33

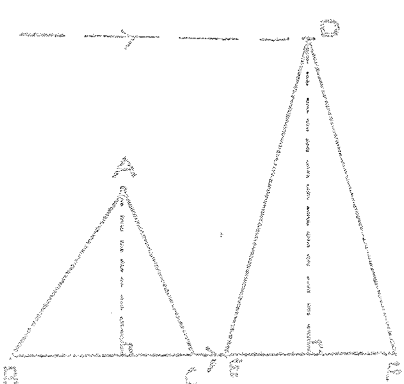


fig. 34

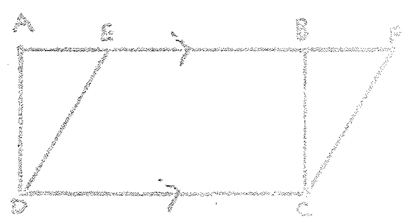
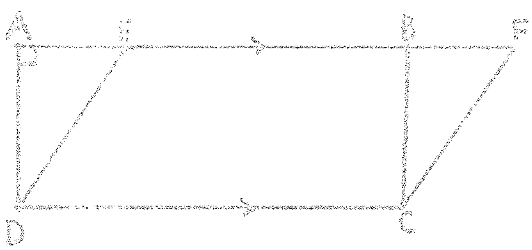


fig. 35

- 5 - Fig. 36.

If a parallelogram and a rectangle stand on the same base and lie between the same parallel lines, (hence of equal altitude), then they are equal in area.

DIAGRAM



particular enunciation GIVEN: //gm EFCD and rect. ABCD on the same base DC and between the same parallels AF and CD.

R. T. P. //gm EFCD = rect. ABCD.

PROOF:

Statements	Reasons
In $\triangle$s AED, BFC	
ED = FC	opposite sides of //gm.
$\angle$ A = $\angle$ CBF	corr. $\angle$s for AD//BC
$\angle$ AED = $\angle$ BFC	corr. $\angle$s for ED//FC
$\triangle$AED $\cong$ $\triangle$BFC	a.a.s.
fig. AFCD = fig. AFCD	quantity equals itself
$\triangle$AED = $\triangle$BFC	proven
//gm EFCD = rect. ABCD	axiom of -n.

COROLLARY: The area of a parallelogram is equal to the product of its base and height.

- 6 - Fig. 37.

If a parallelogram and a rectangle stand on the same base and lie between the same parallel lines then they are equal in area.

DIAGRAM:

GIVEN:

R.T.P.:

PROOF:

Statements	Reasons

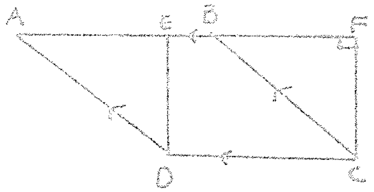


fig. 38.

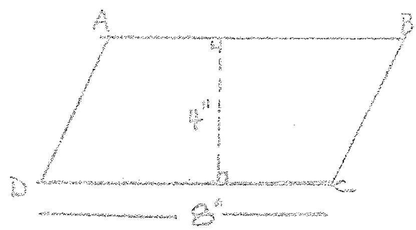


fig. 39

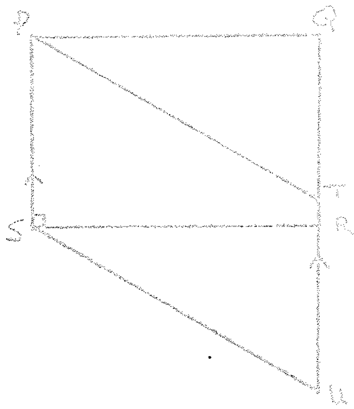


fig. 40

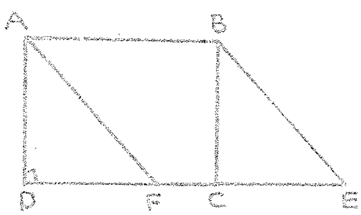


fig. 41

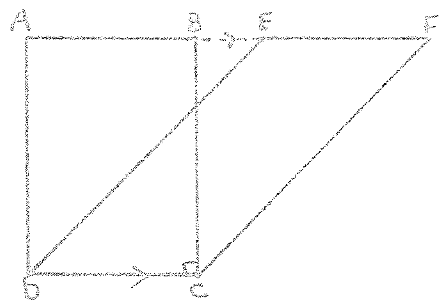


fig. 42

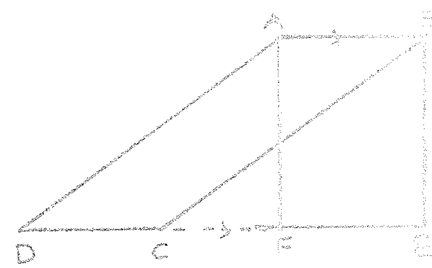
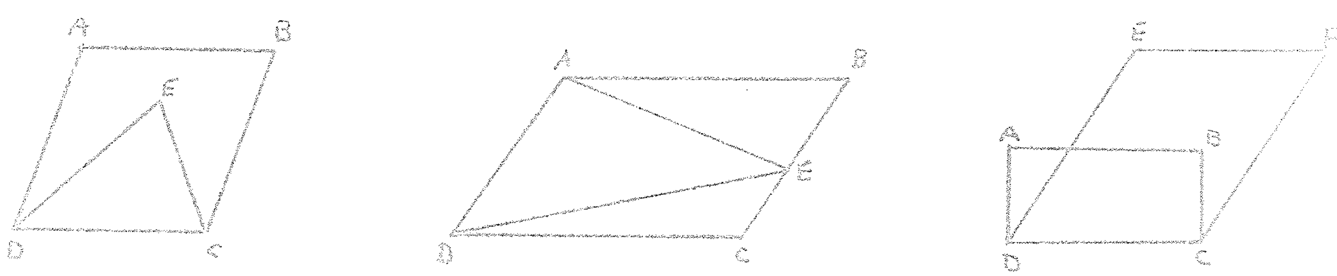


fig. 43



(a) (B) (c)

fig. 44

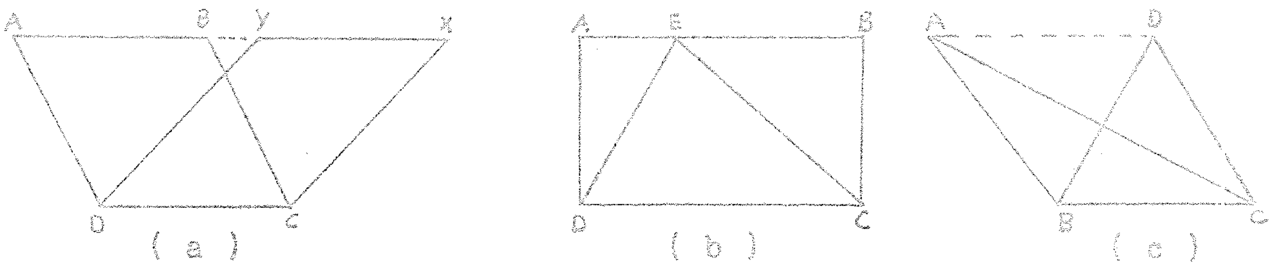


fig. 45

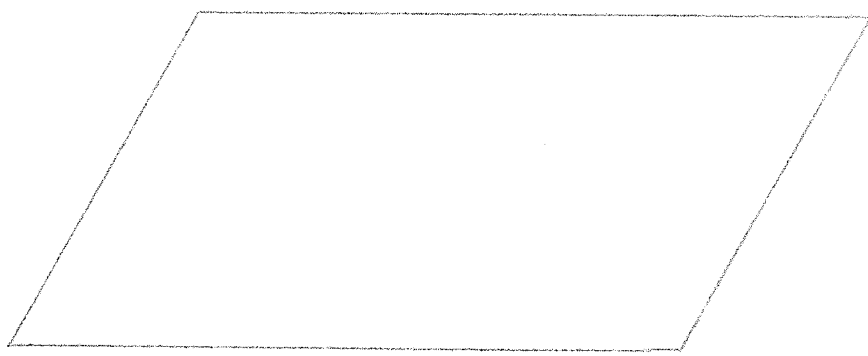


fig. 46

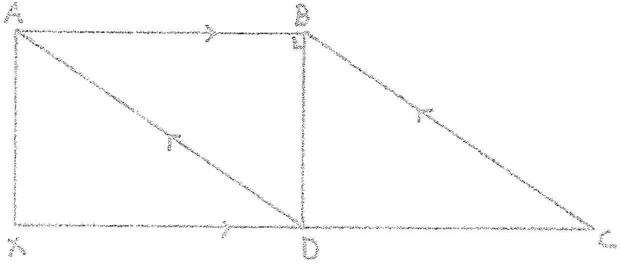


fig. 47

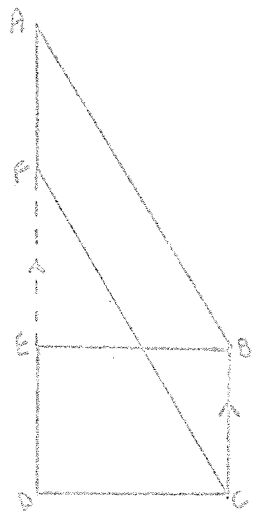
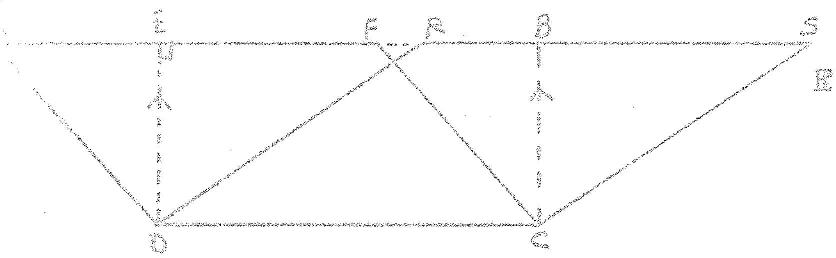


fig. 48

THEOREM - 9 - fig. 49

If two parallelograms stand on the same base and lie between the same parallel lines then they are equal in area.

DIAGRAM:



GIVEN: //gm AFCD and //gm RDCS on the same base DC, and between the same parallels AS and DC.

R.T.P.: //gm AFCD = //gm RDCS

CONSTRUCTION: Construct DE⊥AS and CB parallel to DE.

PROOF:

Statements	Reasons
1. EDCB is a rectangle	const.
2. //gm AFCD = rect. DCBE //gm RSCD = rect. DCBE	stand on the same base DC and lie between the same parallels AS & DC.
3. //gm AFCD = //gm RSCD	ax. of equality.

COROLLARY 1. //gms on eqql bases in the same straight line and between the same parallels are equal in area.

COROLLARY 2. If two parallelograms stand on the same or equal bases and have the same altitude, then they are equal in area

COROLLARY #3. IF two parallelograms of equal area stand on the same base and on the same side of it, then they have equal altitudes, and hence lie between the same parallel lines.

- 10 - fig. 50.

If two parallelograms stand on the same base and lie between the same parallels, then they are equal in area.

DIAGRAM:

GIVEN:

RTP:

CONSTRUCTION:

PROOF:

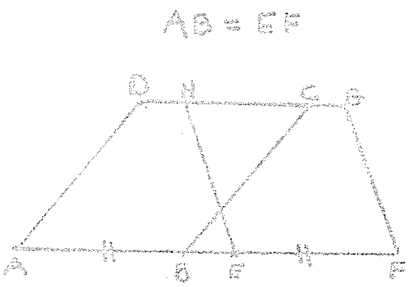


fig. 51

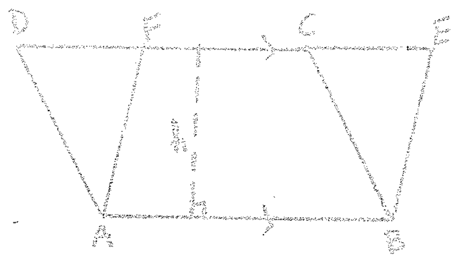


fig. 52

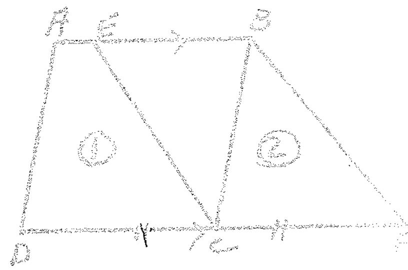
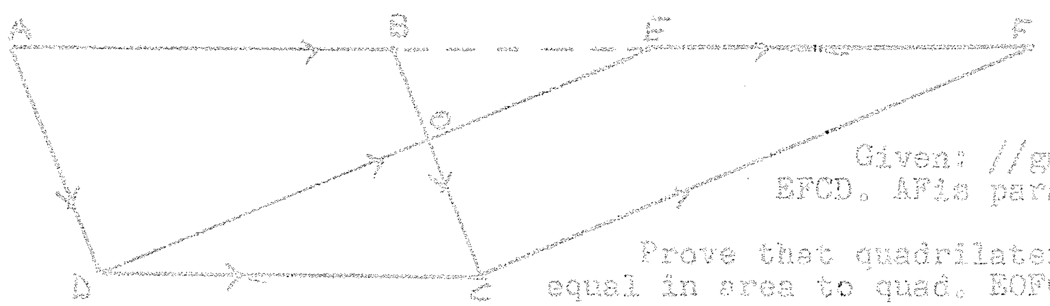


fig. 53



Given: //gms ABCD and EFCD. AF is parallel to DC.

Prove that quadrilateral ABOD is equal in area to quad. EOFC.

fig. 54

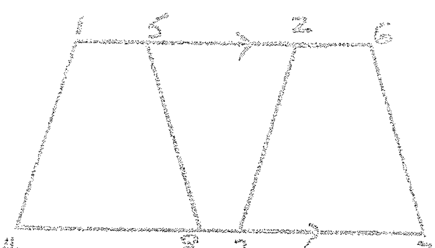


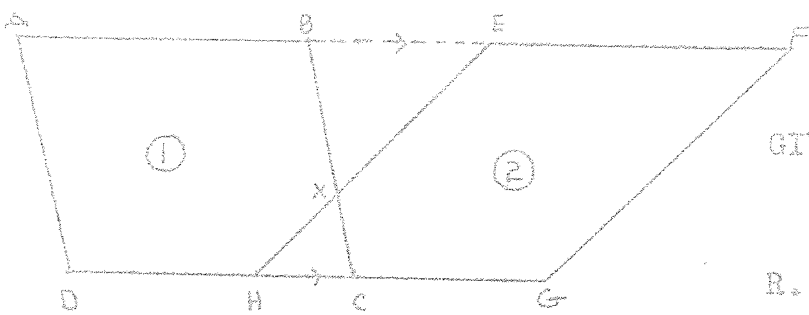
fig. 55

Given: 1234 is a //gm. 5678 is a //gm
48 = 37

R.T.P: //gm 1234 = //gm 5678.

Proof: Statements	Reasons

Fig. 56.



GIVEN: //gms ABCD & EFGH
DH = CG; AF//DG.

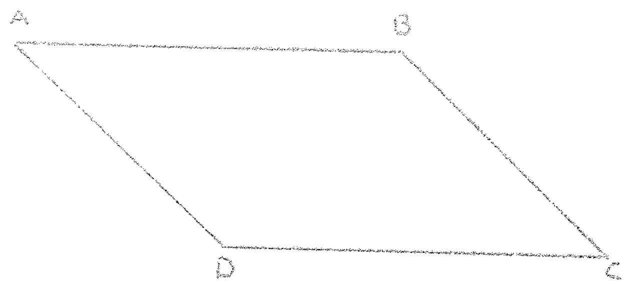
R. T. P. :

fig.1 (ABXHD) = fig.2 (EXCGF)

PROOF:

1. DG = DG -- a qty. equals itself
2. DH = GC -- given
3. DC = HG -- ax. of -n
4. //gm ABCD = //gm EFGH -- on equal bases, DC & HG, and lie the same //s, AF & DG.
6. $\triangle$HXC = $\triangle$HXC -- a qty. equals itself
6. fig.1 = fig. 2 -- ax. of -n

Fig. 57.



GIVEN: //gm ABCD

Req'd: To constructa second parallelogram equal in area to //gm ABCD and standing on the same base DC. (Use compass and str. edge Only)

Construction:

1.

2.

3.

Proof; DXYC is a //gm because ______

//gm DXYC = //gm ABCD because ______

______

Fig. 58.

QUES: Construct a parallelogram equal in area to a given parallelogram and with one angle equal to a given angle.

GIVEN:

REQ'D:

DIAGRAM:

CONSTRUCTION:

step 1.

step 2.

step 3.

step 4.

step 5.

step 6. WXYZ is the required //gm. It is equal in area to //gm ABCD and $\angle W$ equals $\angle R$.

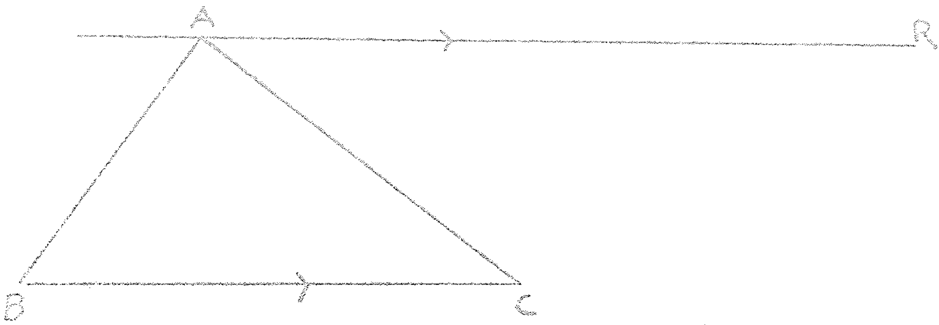
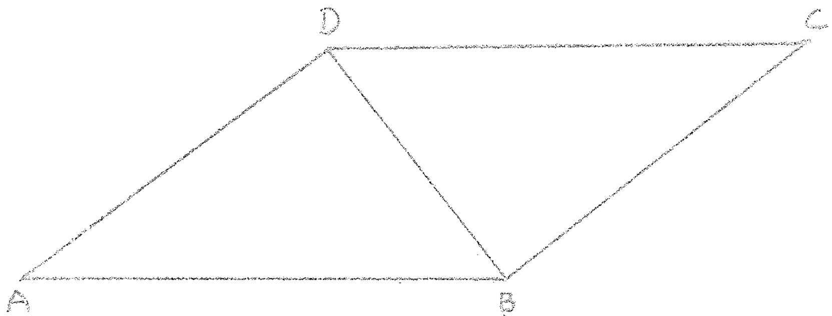


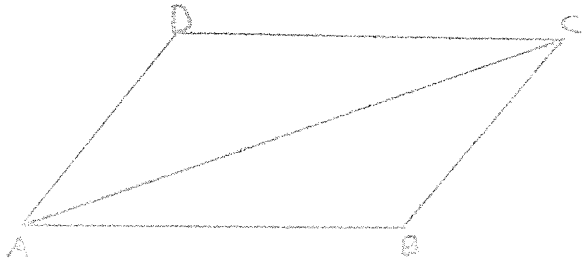
fig. 59



ABCD is a //gm

DB is a diagonal

fig. 60



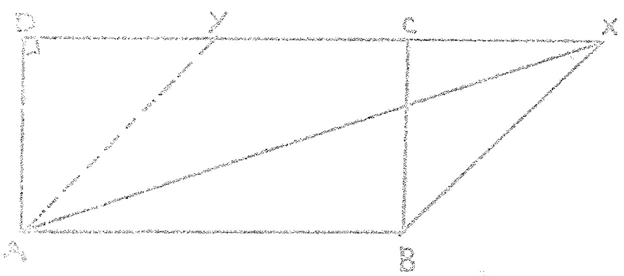
ABCD is a //gm.

fig. 61

THEOREM 3 -- Fig. 62.

If a triangle and a rectangle stand on the same base and lie between the same parallel lines, (hence of equal altitude), then the area of the triangle is equal to one-half the area of the rectangle.

DIAGRAM:



GIVEN: Rect. ABCD and $\triangle$AXB on the same base AB, and between the same parallels, DX & AB.

R. T. P. : $\triangle$AXB = $\frac{1}{2}$ rect. ABCD.

CONSTRUCTION: Draw AY // BX, meeting DX at Y.

Proof:

Statements	Reasons
ABXY is a //gm	by const.
$\triangle$XAB = $\frac{1}{2}$ //gm ABXY	diagonal bisects //gm.
//gm ABXY = rect. ABCD	on the same base AB, and between same //s, DX & AB.
$\triangle$XAB = $\frac{1}{2}$ rect ABCD	ax. of equality.(subst.)

Corollary 1. The area of a triangle is $\frac{1}{2}$ the base times the altitude.

$$A = \tfrac{1}{2}ba$$

Corollary 2. A triangle is equal to one-half of any parallelogram on the same base and between the same parallels.

Fig. 63.

If a triangle and a rectangle stand on the same base and lie between the same parallel lines, hence of equal altitude, then the area of the triangle is equal to one-half the area of the rectangle.

DIAGRAM: GIVEN:

R. T. D.:

CONSTRUCTION:

PROOF:

Statements	Reasons

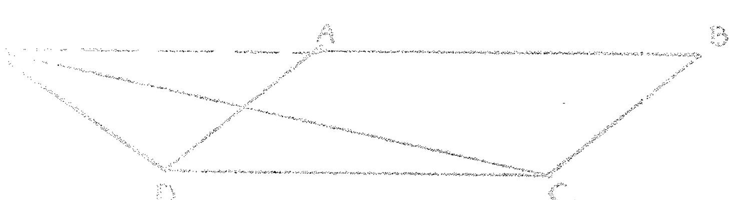


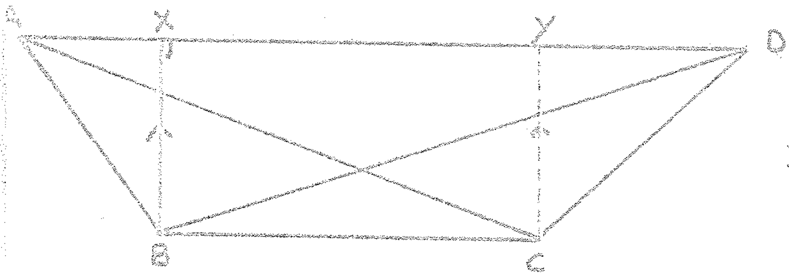
fig. 64

Given: //gm ABCD with BA extended to E. EC and ED are joined.

Theorem 4 -- Fig. 65.

Iftwo triangles stand on the same base and lie between the same parallel lines, then they are equal in area.

DIAGRAM:



GIVEN: $\triangle$s ABC & DBC standing on base BC, and lying between same parallels AD & BC.

R. T. P.: $\triangle$ABC = $\triangle$DBC.

Construction: Draw BX $\perp$ AD to meet AD at X.

Draw CY // BX to meet AD at Y.

PROOF:

Statements	Reasons
figure BCYX is a rect.	by const.
$\triangle$ABC = $\frac{1}{2}$ rect. BCYX	$\triangle$= $\frac{1}{2}$ rect. if they stand on the same base BC and lie between the same //s, AD,BC.
$\triangle$DBC = $\frac{1}{2}$ rect BCYX	
$\triangle$ABC = $\triangle$DBC	ax. of equality.

Corollary1. Triangles on equal bases in the same straight line and between the same parallels are equal in area.

Corollary 2. If triangles stand on the same base and on the same side of it, then they have equal altitudes and hence lie between the same parallel lines.

Corollary 3. The median of a triangle divides it into two trinales equal in area.

Fig. 66.

If two trinagles stand on the same base and lie between the same parallel lines, then thev are equal in area.

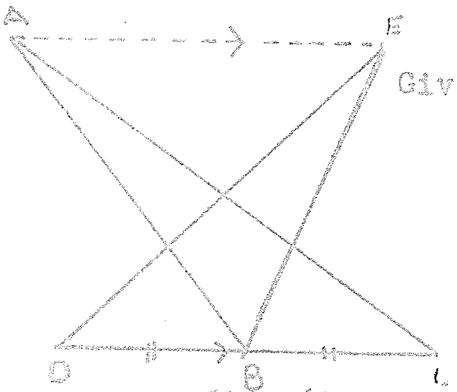
DIAGRAM: GIVEN:

R.T.P.:

CONSTRUCTION:

PROOF:

Statements	Reasons



Given: DB = BC
AE // DC

fig. 67

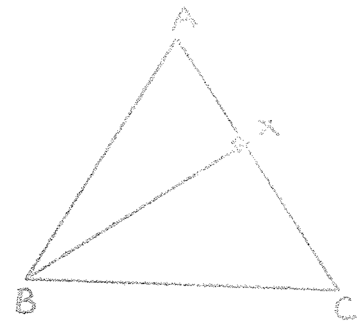


fig. 68

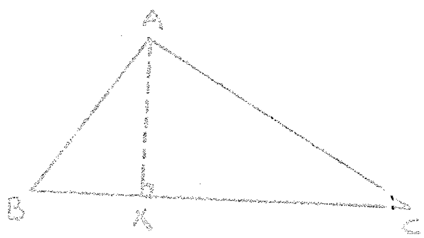


fig. 69

fig. 70

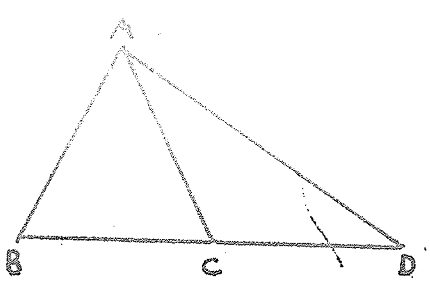


fig. 71

In triangle ABD, AC is median

Porve that $\Delta ABC = \Delta ACD$

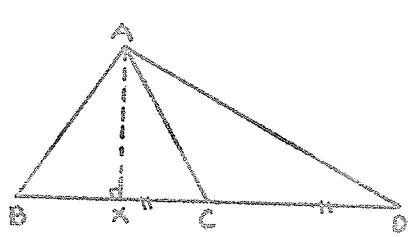
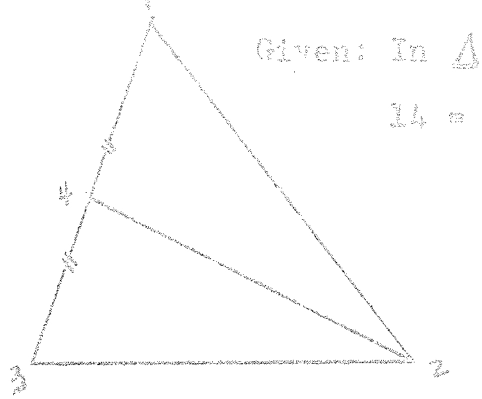


fig. 72

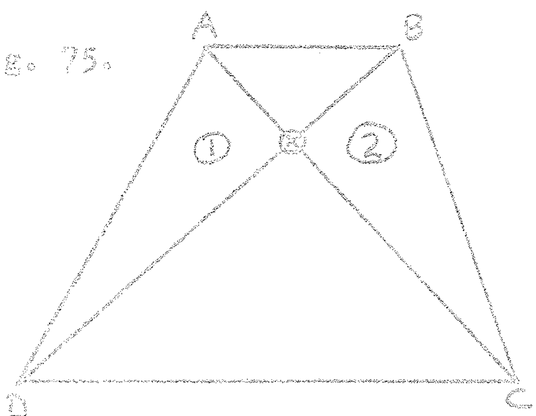
fig. 73



Given: In $\triangle 132$,
$14 = 43$.

fig. 74

Fig. 75.

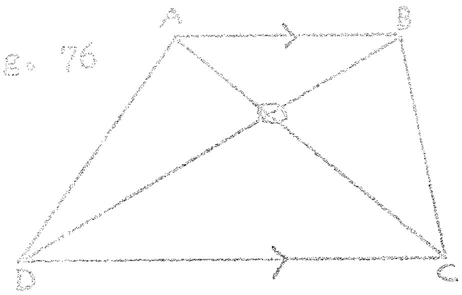


Given: Triangles ADC and BDC
AB // DC

R.T.P.: $\triangle 1 = \triangle 2$.

PROOF: Statements	reasons
Step 1.	
" 2.	
" 3.	

Fig. 76

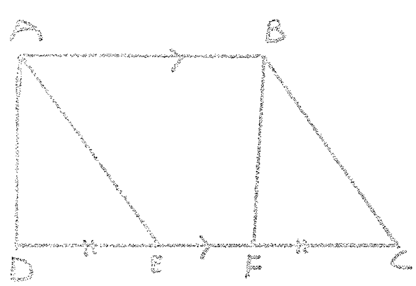


Given: Trapezium ABCD with diagonals AC and BD.

R.T.P.: $\triangle 1 = \triangle 2$

Proof: Statements	Reasons
Step 1.	
Step 2.	
Step 3.	

Fig. 77

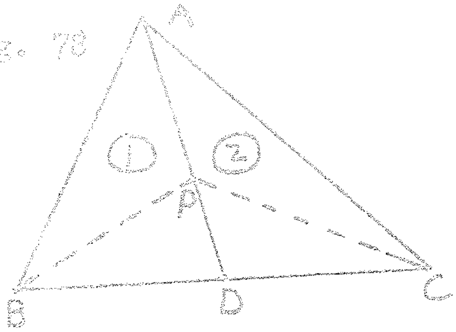


Given: Trapezoid ABCD with AB // DC and DE = FC

R.T.P.: Fig. ABFD = fig. ABCE

Proof: Statements	Reasons
Step 1.	
Step 2.	
Step 3	- - - - - - - ax. of addition.

Fig. 78



Given: ABC is a triangle with median AD

P is any point on AD.

R.T.P.: $\triangle 1 = \triangle 2$

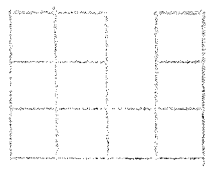
Proof:	Statements	Reasons
Step 1.		
Step 2.		
Step 3.		

AN OVERVIEW

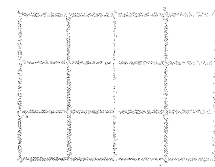
Up to this point we have proven four theorems and solved sveral problems utilizing these theorems or their corollaries. And, of course, it follows that the likelyhood of success in problem solving is greatly enhanced by a thorough understanding of these theorems and their corollaries. It would be expedient, then, to make a list of these theorems and corollaries to facilitate the solution of new area problems.

However, we have done something more than learn how to solve a new type of problem, that is, we have, <u>without the use of intuition</u> verified two formulas which we have been using for many years. The first is that the area of //gm equals $b \cdot a$, and the second that the area of triangle equals $\frac{1}{2} \cdot b \cdot a$. You will notice that we have omitted the formula for the area of a rectangle. Why? Lets statt at the begining.

We can measure the area of a polygon by counting the [illegible] of times that it contains a unit of area. Thus the area of F1 is 10 sq. units. We can find the area of a rectangle also by counting the number of times that it contains a unit of area. For example, the area of the rectangle F2 is 12 area units. We can imagine a person with no knowledge of formulae counting the area of many rectangles until, at some time or other, he realizes that each ~~mmx~~ column contains 3 area units and there are 4 columns, qnd hence the area of the rectangle is simply 3 X 4 or 12 area units. Since then millions of areas of rectangles have been calcualted by this method (b·a) and the results are the same as though each unit had been counted. This of course does not mean that some day someone will NOT come across a rectangle where the results of counting and using the formula so not correspond. Hence this is why we ASSUME that the area of a rectangle is the product of its base and altitude.



F1



F2

We have, on the other hand, PROVED that the area of //gm is equal to the area of a rectangle if they have equal bases and altitudes. Thus we can state with complete certainty that the area of a //gm = b·a, if it is granted that the area of a rectangle is b·a. Similary the formula for the area of a triangle is based on the 3rd theorem which proves that the area of $\triangle$ is ½ the area of a rectangle.

<u>A word on assumptions in mathematics.</u> All mathematics is based on assumptions, or as they are usually called, postulates or axioms. Some of these assumptions are obvious. For example, the area of a rectangle formula; or the fact that two points can be joined by one and only one straingt line. There are some assumptions that are not so obvious.

[illegible] ... two parallel lines are [illegible] ... two lines are parallel [illegible] ... Moreover, since the advent of Einstein's Theory of Relativity, it is now generally agreed that this geometry [illegible] of our world.

Perhaps a more familiar example will make this clearer. We have always assumed that "2 + 2 = 4" is a true statement [illegible] "2 + 2 = 11" is equally valid depending on the base in which we are working.

In summary: we have defined the area of a rectangle [illegible]. If it is true then the formulas for the areas of [illegible] are also true. If it is false then they too are false.

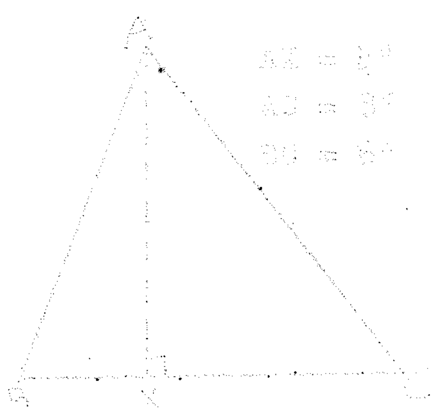


Fig. 70

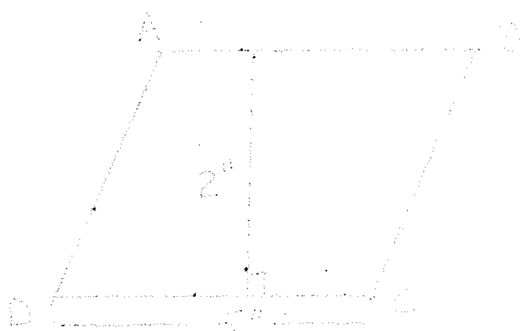


Fig. [illegible]

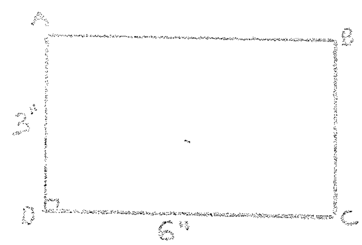


fig. 81

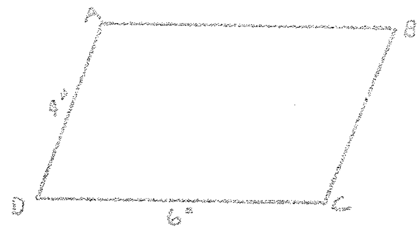


fig. 82

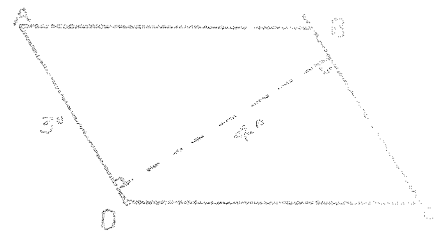


fig. 83

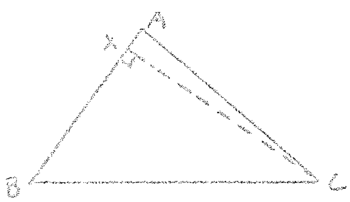


fig. 84

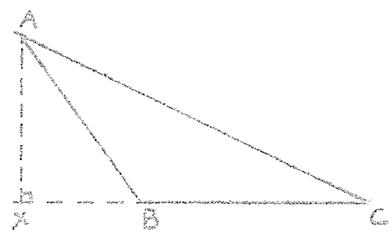


fig. 85

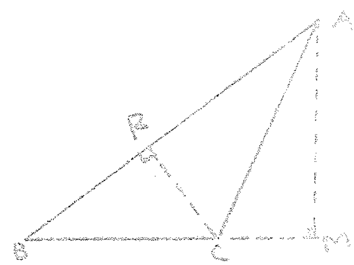


fig. 86

fig. 87

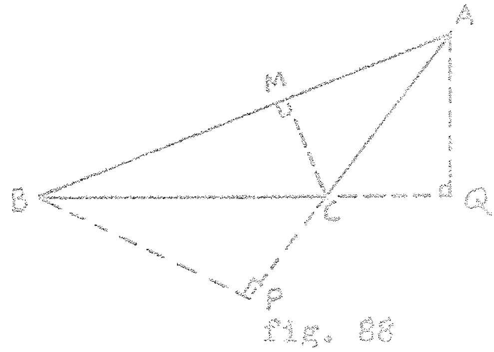


fig. 88

AB = 6 in.
BC = 8 in.
CX = 4 in.

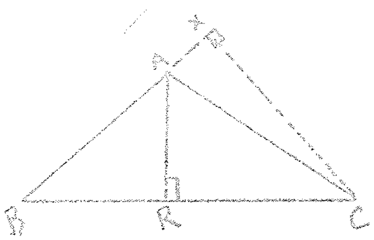


fig. 89

Given: AB = 8 uts.
CR = 3 uts.
BC = 6 uts.

Find: AX = ________

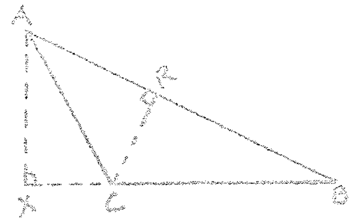


fig. 90

Given: //gm ABCD with
DC = 8 uts.
BC = 6 uts
EF = 5 uts
XY = ?

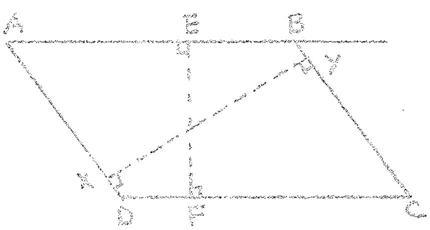


fig. 91

Given: ABCD is a //gm.

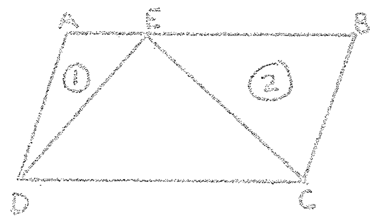
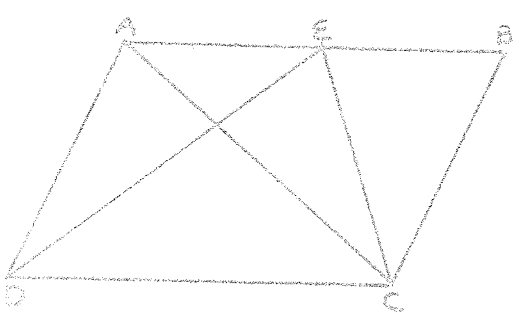


fig. 92



Given: ABCD is a //gm.

R.T.P.: $\triangle$ EDC = $\triangle$ ADC

Proof: ________

fig. 93

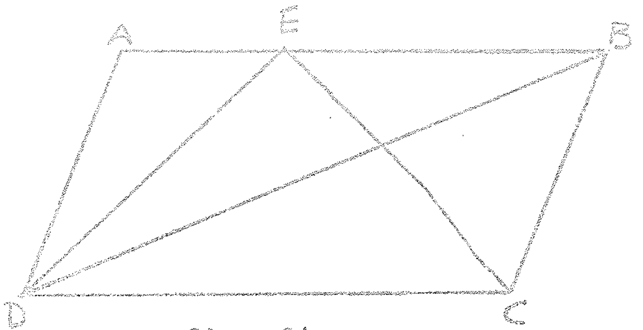


fig. 94

Given: ABCD is a //gm.

R.T.P.: a) △EDC = △BDC

b) △AED + △EBC = △BDC.

statements	reasons

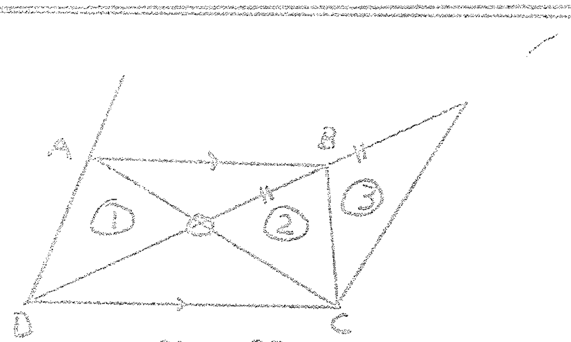


fig. 95

Given: Trapezoid ABCD with AB // CD
BC is a median of ∠OEC.

R.T.P.: △1 = △3

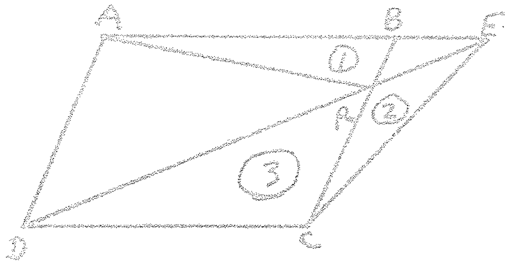


fig. 98

Given: ABCD sis a //gm with AB extended to E. DR is joined and extended to E. AR is is joined.

R.T.P.: $\triangle 1 = \triangle 2$

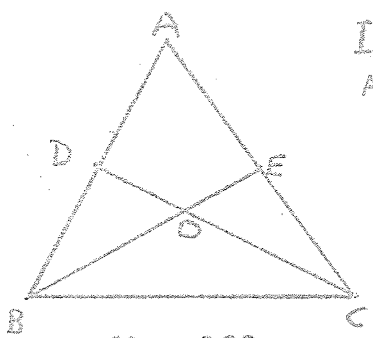


fig. 100

In $\triangle ABC$,
$AD = DB$; $AE = EC$

Prove: $\triangle BOC$ = quad. ADOE

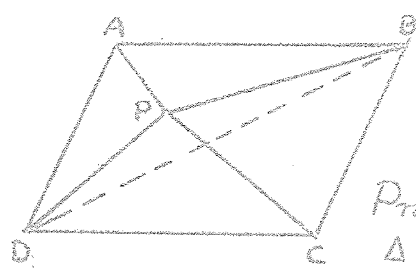


fig. 102

ABCD is a llgm.
P is any point in side it.

Prove: $\triangle APD + \triangle PBC = \triangle BDC$

CONST:

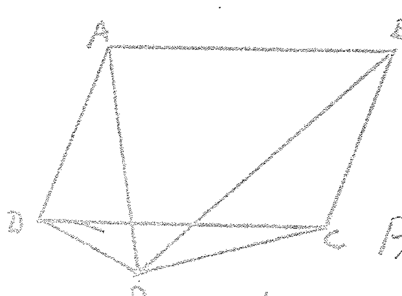


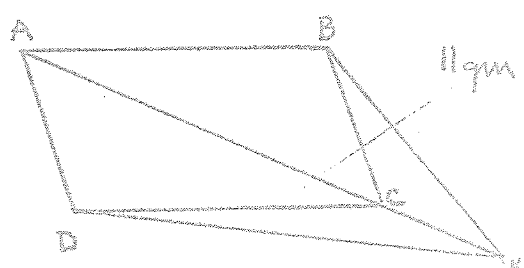
fig. 101

P is any point outside llgm ABCD.

Prove:

$\triangle APD + \triangle BPC = \frac{1}{2}$ llgm ABCD

Const: Through P draw a line ll to BC meeting



llgm ABCD.

fig. 103

Diagonal AC is produced to X. BX & DX are joined.

Prove: $\triangle BCX = \triangle DCX$.

APPENDIX B

CRITERION ACHIEVEMENT TEST AND ITS ITEM ANALYSIS

ITEM ANALYSIS OF CRITERION ACHIEVEMENT TEST FOR

EXPERIMENTAL AND CONTROL GROUPS

Q.*	Th.	P.	01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
1	37	.74	1	0	0	1	1	2	2	2	2	2	2	0	1	2	2	1	1	1	2	2	2	2	2	2	2
2a	19	.76	0	0	1	1	1	0	1	1	1	1	1	1	1	1	1	0	0	0	1	1	1	1	1	1	1
b	19	.76	1	0	1	0	0	1	1	1	1	0	1	1	1	1	1	1	1	1	0	1	0	1	1	1	1
c	39	.78	0	0	2	2	2	2	1	2	2	2	0	0	2	2	2	2	2	0	2	2	2	2	2	2	2
d	22	.44	0.	0	0	1	1	1	1	0	2	2	0	0	0	2	1	2	1	0	0	2	0	2	2	2	2
3	67	.45	0	0	2	6	1	2	1	0	0	6	0	0	4	6	1	2	5	0	6	2	5	6	5	2	5
4	80	.80	4	0	0	4	4	4	4	4	4	4	4	0	2	4	4	4	1	1	4	4	4	4	4	4	4
5	74	.50	4	0	1	6	3	1	2	4	1	6	5	0	6	6	0	2	5	0	0	2	6	0	2	6	6
6	104	.69	2	0	5	6	6	2	6	6	0	4	6	0	3	6	6	6	4	0	4	5	6	4	5	6	6
7	51	.41	2	0	0	3	0	0	3	1	1	0	3	0	2	5	3	3	0	0	5	3	5	4	4	4	0
	512	.59	14	0	12	30	19	15	22	21	14	27	22	2	22	35	21	23	20	3	24	22	33	24	28	30	29

1**	38	.76	0	0	2	2	2	2	1	2	0	1	0	2	2	2	1	2	2	2	2	2	1	2	2	2	2
2a	18	.72	0	0	1	1	1	1	0	1	1	0	0	1	1	1	1	1	1	0	1	1	0	1	1	1	1
b	23	.92	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1
c	40	.80	2	2	2	2	0	2	2	2	2	0	0	0	2	2	2	2	2	2	2	2	0	2	2	2	2
d	11	.22	0	0	0	0	0	0	0	2	0	0	0	0	0	0	1	1	2	0	2	0	0	0	1	2	0
3	73	.49	1	0	0	0	1	0	4	5	4	0	0	5	3	0	6	1	6	2	3	5	5	6	5	5	6
4	79	.79	0	2	4	0	4	4	3	4	0	1	4	4	3	4	4	4	4	4	4	4	4	4	4	2	4
5	94	.63	1	2	5	5	1	1	6	5	2	5	4	5	1	1	0	5	4	4	5	6	4	6	4	6	6
6	83	.55	1	3	5	1	0	6	5	6	0	0	0	6	6	1	0	6	6	1	4	6	0	5	3	6	6
7	71	.57	1	2	1	1	4	4	4	3	4	0	2	3	3	3	3	2	3	2	3	5	5	4	0	4	5
	530	.61	7	12	21	12	14	21	26	31	14	8	11	27	22	15	19	25	31	18	27	32	19	31	23	31	33

*The total number of points awarded for each item is expressed as a raw score as well as a proportion of the possible points for the experimental group.

**The same...except for the control group.

TEST NAME: ______________

CLASS: ______________

TIONS: Answer all questions on question paper. There is a time limit of ONE HOUR.

Answer as many questions as you can;

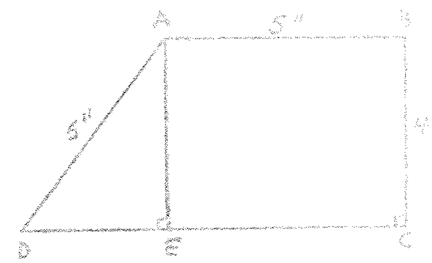
S

1. Given: Quadrialteral ABCD with

AB // CD, AE $\perp$ DC, BC $\perp$ DC

AB = 5 inches, AD = 5 in.,

BC = 4 in. , DC = 8 in.



Find: the area of ABCD. — where appropriate show your reasonigg

2. ABCD is a parallelogram with side BC = 8 uts.,

AC = 13 uts., BE = 4 uts., BE $\perp$ AC.

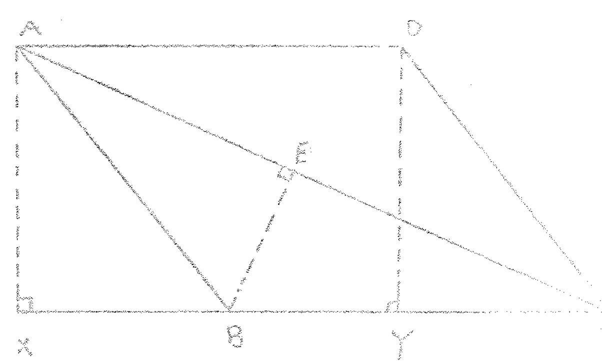
$\perp$ s to BC from A and D meet BC at X and Y.

Find: a) area of triangle ABC

b) area of //gm ABCD

c) area of rectangle AXYD

d) length of AX.



~~Show your reasoning~~

NAME: ______________________

CLASS: ______________________

3. The area of a triangle is equal to one-half the area of the rectangle on the same base and between the same parallels.

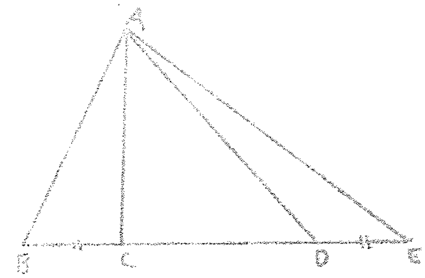
* #3 *

NAME: ______________________

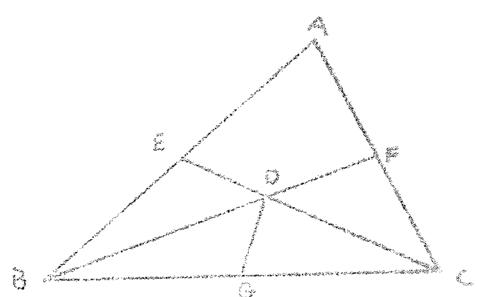
CLASS: ______________________

4. In triangle ABE, BC = DE . Prove that

$\triangle ABD = \triangle ACE$

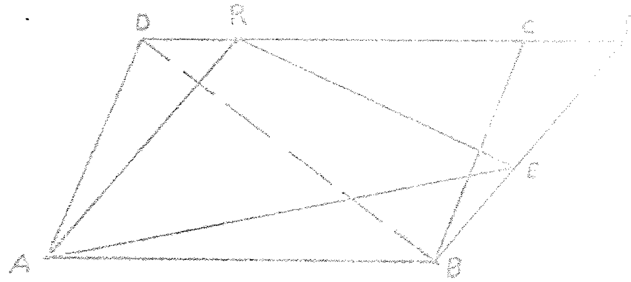


5. In triangle ABC, E, F, and G are mid-points of AB, AC, and BC respectively. EC and FB intersect at D. DG is joined. Prove: Quad. EDGB = FDGC in area.



6. Parallelogram ABCD and Parallelogram ABPR stand on the same base AB and lie between the same parallels AB and DP. E is any point on BP.

Prove: $\Delta ABD = \Delta ABE + \Delta EPR$



7. Construct a parallelogram equal in area to a given triangle. Describe the construction, but omit the proof. (Use compass and straight edge only.)

APPENDIX C

TABLES

TABLE IX

RAW DATA FOR EXPERIMENTAL AND CONTROL GROUPS

Pairs	Composite	I.Q.	Gr. X Av.	Pre. Know.	Ver. Ab.	Age	Achv.	Time Spent	Errors
1E	64	97	45	24	287	197	14	527	37
1C	63	91	50	14	294	190	7	575	
2E	73	105	46	19	298	205	00	379	63
2C	73	102	52	17	291	196	12	675	
3E	79	105	55	24	285	192	12	401	63
3C	78	108	52	24	274	200	21	505	
4E	81	105	56	26	294	192	30	369	39
4C	83	114	52	21	280	200	12	419	
5E	84	110	56	21	288	200	19	402	28
5C	84	107	60	22	283	210	14	610	
6E	84	110	56	28	291	197	15	358	12
6C	84	115	52	28	282	205	21	550	
7E	86	117	53	26	290	196	22	406	29
7C	86	112	57	28	292	190	26	515	
8E	86	112	57	20	289	196	21	524	32
8C	87	116	55	23	276	205	31	480	
9E	88	121	50	24	290	195	14	379	24
9C	88	111	60	21	280	197	14	419	
10E	89	114	58	26	295	210	27	514	22
10C	88	116	56	29	309	206	8	450	
11E	90	121	53	16	290	207	22	431	16
11C	90	123	52	27	309	206	11	527	
12E	91	114	59	20	291	194	2	344	100
12C	91	108	65	24	295	195	27	510	

TABLE IX (continued)

Pairs	Composite	I.Q.	Gr. X Av.	Prev.Know.	Ver. Ab.	Age	Achv.	Time Spent	Errors
13E	93	115	60	22	298	196	22	435	39
13C	93	123	55	18	285	195	22	410	
14E	95	105	71	25	300	196	35	415	11
14C	96	130	51	25	291	192	15	465	
15E	98	122	61	26	292	198	21	451	9
15C	98	124	59	26	292	193	19	465	
16E	98	127	57	22	295	195	23	403	18
16C	98	123	60	27	297	196	25	540	
17E	100	127	59	25	297	183	20	394	18
17C	101	112	72	23	288	190	31	615	
18E	102	135	54	15	301	181	3	543	37
18C	103	128	61	22	301	207	18	460	
19E	104	119	69	29	292	194	24	485	25
19C	104	129	61	30	314	196	27	---*	
20E	104	125	64	23	284	192	22	362	10
20C	106	136	57	28	298	194	32	575	
21E	107	122	70	25	300	194	33	448	27
21C	108	130	64	21	294	193	19	585	
22E	111	137	62	32	316	194	24	312	9
22C	111	128	69	29	293	192	31	490	
23E	113	125	72	27	311	195	28	446	33
23C	113	131	68	25	304	201	23	545	
24E	120	128	78	28	301	198	30	430	13
24C	118	128	76	30	302	200	31	427	
25E	121	136	72	26	287	197	29	311	24
25C	122	131	77	22	304	187	33	505	

*No record submitted

TABLE X

CORRELATION MATRIX OF PRE AND POST TREATMENT VARIABLES FOR THE EXPERIMENTAL GROUP

VARIABLE	COMPOSITE	ACHIEVEMENT	TIME	ERRORS	VERBAL ABILITY	PREV. KNOW.
Composite	xx	.49*	-.20	-.40*	.46*	.11
Achievement		xx	0.03	-.64**	.22	.55**
Time			xx	-.06	-.11	-.32
Errors				xx	-.16	-.40*
Verbal Ability					xx	.31
Prev. Knowledge						xx

*significant at .05 level

**significant at .01 level

TABLE XI

SUMMARY OF RESULTS OF ATTITUDINAL QUESTIONNAIRE ADMINISTERED TO THE EXPERIMENTAL GROUP

Compared to regular classroom methods:	
1. how well has PI taught you?	
a) much less effective	8.57 %
b) somewhat less effective	17.14
c) the same	11.43
d) somewhat more effective	51.42
e) much more effective	11.43
2. how do you like PI?	
a) much less	8.57
b) somewhat less	17.14
c) the same	8.57
d) somewhat more	28.57
e) much more	37.15
3. how difficult was learning under PI?	
a) much more difficult	2.86
b) somewhat more difficult	8.57
c) the same	11.43
d) somewhat less difficult	48.57
e) much less difficult	28.57
4. how much more home study under PI?	
a) much more	2.86
b) somewhat more	14.29
c) the same	34.28
d) somewhat less	25.71
e) much less	22.86
5. In future Mathematics courses I would prefer PI.	
a) strong objection	14.29
b) some objection	22.85
c) no preference	2.86
d) some preference	22.85
e) strong preference	37.15
6. PI is a boring method of learning.	
a) strongly agree	17.14
b) agree	2.86
c) uncertain	11.44
d) disagree	34.28
e) strongly disagree	34.28

TABLE XI (continued)

7. PI is a good method because there is no pressure on the student.

a) strongly agree	20.00
b) agree	37.14
c) uncertain	17.14
d) disagree	22.86
e) strongly disagree	2.86

8. While working on the program, _____ occasions arose when I desired an additional explanation from the teacher.

a) no	22.86
b) several	68.57
c) many	8.57

9. I _____ peeked ahead to see the answer, before writing my own response.

a) always	00.00
b) almost always	00.00
c) sometimes	14.29
d) rarely	62.85
e) never	22.86

APPENDIX D

THE COOPERATIVE GEOMETRY TEST

227 - 242

FORM A
GEOMETRY

CTD

GENERAL DIRECTIONS

This test has two 40-minute parts. Do not spend too much time on any one question. If a question seems to be too difficult, make the most careful guess you can, rather than waste time over it. Do not worry if you do not finish the test. Your score is the number of correct answers you mark.

Use scratch paper to work problems. Do not make any marks in your test booklet.

Mark all answers on the separate answer sheet. Make your answer marks heavy and black. Mark only one answer for each question. If you make a mistake or wish to change an answer, be sure to erase your first choice completely.

Note how the answer to the EXAMPLE below is marked on your answer sheet.

EXAMPLE

In $\triangle PQR$, if $P = 40°$ and $Q = 50°$, then $R = (?)$

A 40°
B 50°
C 90°
D 130°
E 140°

Do not turn this page until you are told to.

COOPERATIVE MATHEMATICS TESTS

Geometry | 40 minutes

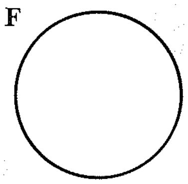
PART I

1 Which of the following are measures of a pair of supplementary angles?

A 180°, 180°
B 40°, 50°
C 60°, 60°
D 60°, 120°
E 90°, 270°

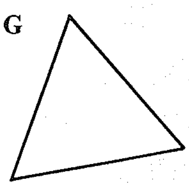
2 The area formula A = bh, where b is the base and h is the height, applies to which of the following figures?

F



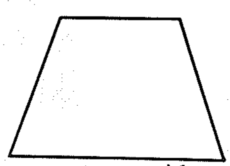
circle

G



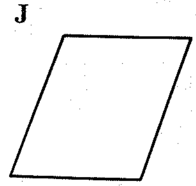
triangle

H



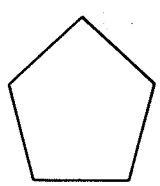
trapezoid

J



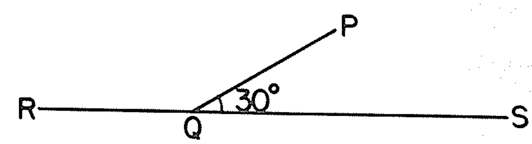
parallelogram

K



pentagon

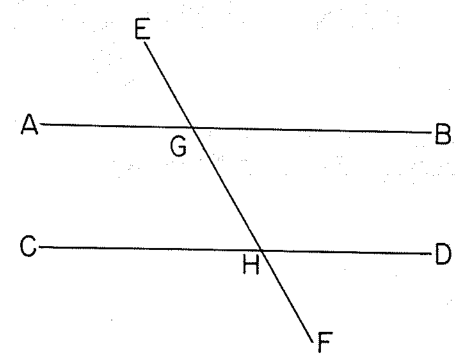
3



In the figure above, if RS is a straight line and ∠PQS = 30°, then ∠PQR = (?)

A 15°
B 30°
C 60°
D 90°
E 150°

4



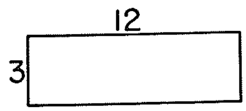
In the figure above, lines AB and CD are crossed by line EF. ∠EGB and ∠EHD are known as

F vertical angles
G complementary angles
H alternate interior angles
J corresponding angles
K exterior angles

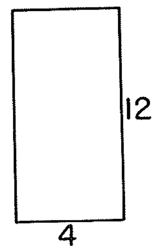
Go on to the next page.

All of the following rectangles have equal areas **except**

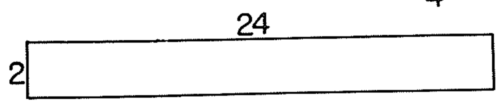
A



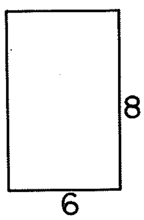
B



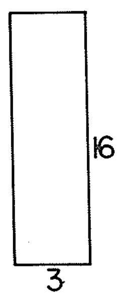
C

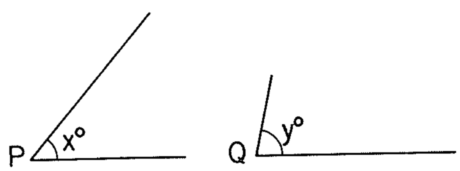


D



E

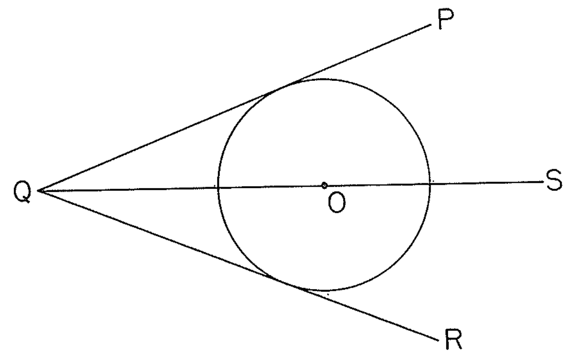




If, in the figure above, the measure of $\angle Q$ is 3 times the measure of $\angle P$ and if $x = 24$, then $y = (?)$

F 8
G 12
H 24
J 36
K 72

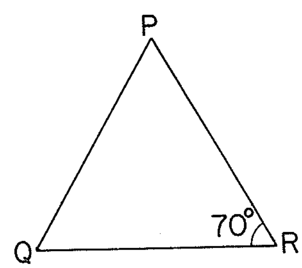
7



In the figure above, QP and QR are tangent to a circle with center at O. If $\angle PQR = 70°$, then $\angle SQR = (?)$

A 20°
B 30°
C 35°
D 45°
E 140°

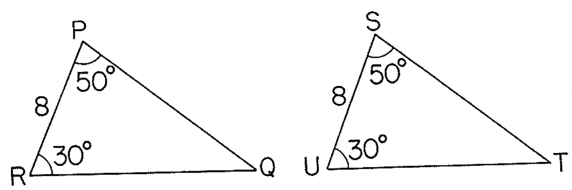
8



In $\triangle PQR$ above, $PQ = PR$, and $\angle R = 70°$. $\angle Q = (?)$

F 20°
G 35°
H 40°
J 70°
K 110°

9



If only the facts above are given, by what authority is $\triangle PQR$ congruent to $\triangle STU$?

A SAS
B ASA
C SSS
D AAA
E SSA

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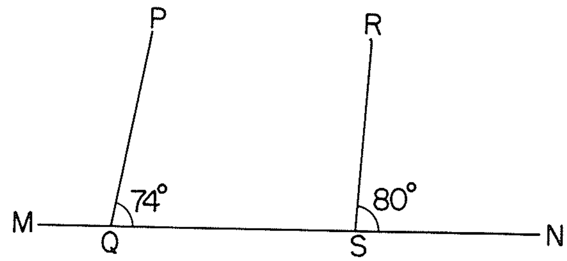
10



On straight line MN above, ∠RSN = 80° and ∠PQN = 74°. By which of the following amounts must angle PQN be increased in order that PQ will be parallel to RS?

F 6°
G 10°
H 16°
J 80°
K 106°

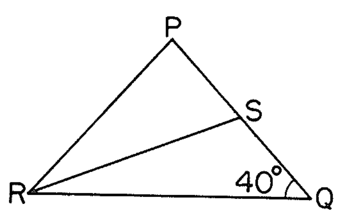
11 Following are the distances, in inches, of five points from the center of a circle:

Point A - 1.75
Point B - 2.01
Point C - 1.01
Point D - 2.00
Point E - 1.50

If the radius of the circle is 2 inches, which point lies outside the circle?

A Point A
B Point B
C Point C
D Point D
E Point E

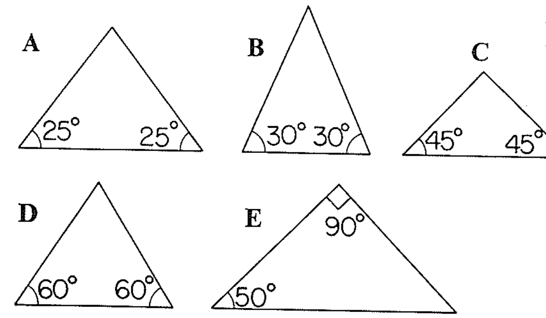
12



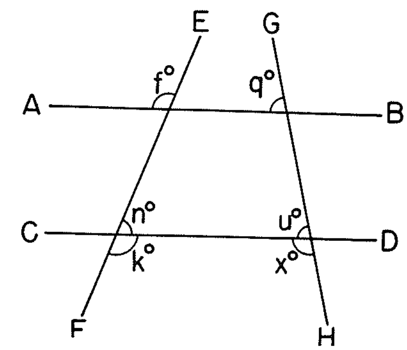
In △PQR above, PR = PQ, angle Q = 40°, and RS bisects angle PRQ. ∠SRQ = (?)

F 10°
G 20°
H 25°
J 40°
K 50°

13 Which of the following is an isosceles ri triangle?



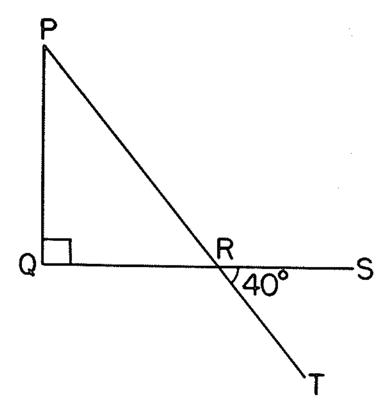
14



In the figure above, AB || CD, and EF and GH ar straight lines. Which of the following is true?

F f = q
G f = u
H f = n
J f = x
K f = k

15



In the figure above, ∠Q = 90°, QS and PT are straight lines, and ∠SRT = 40°. ∠P = (?)

A 40°
B 50°
C 80°
D 90°
E 140°

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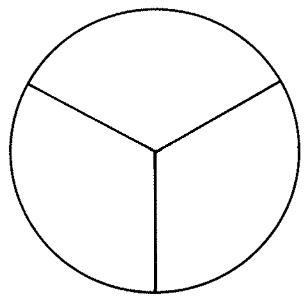
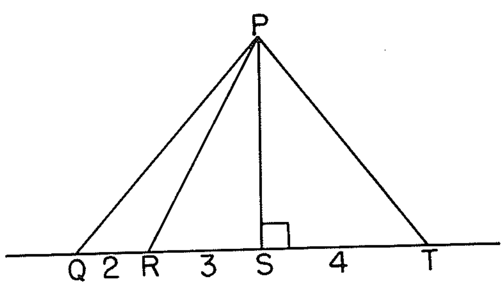


Shown above are three spokes from the center of a wheel. The sum of the lengths of these spokes is how many times the length of the diameter of the wheel?

F 1
G 1.5
H 2
J 2.5
K 3

7



In the figure above, PS ⊥ QT, QR = 2, RS = 3, and ST = 4. Arrange PQ, PR, and PT in order of size, beginning with the shortest.

A PQ, PT, PR
B PT, PQ, PR
C PR, PT, PQ
D PR, PQ, PT
E PT, PR, PQ

18 If two angles of a quadrilateral are supplementary, the other two angles are

F acute
G obtuse
H complementary
J supplementary
K equal and supplementary

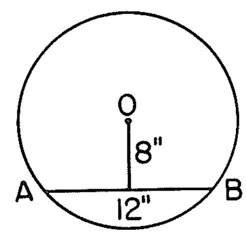
19 At 4 o'clock, the size of the angle formed by the minute hand and the hour hand of a clock is

A 30°
B 45°
C 60°
D 90°
E 120°

20 The statement, "A figure is a triangle if and only if it is a closed broken line figure having three sides," is

F a definition
G a theorem
H an axiom
J a conclusion
K a falsehood

21

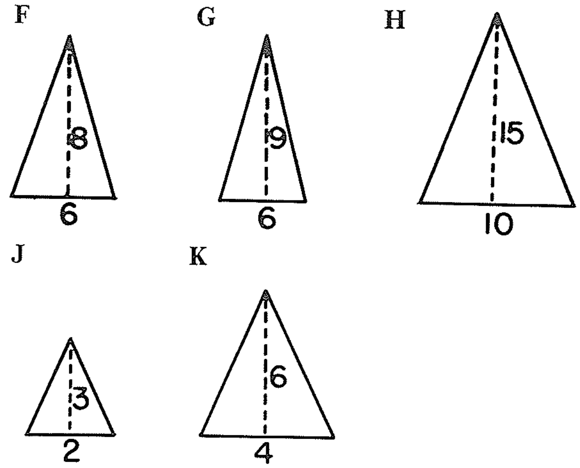


In the circle above, chord AB is 12 inches long and 8 inches from center O. What is the length, in inches, of the radius of the circle?

A $\sqrt{80}$
B 10
C $\sqrt{208}$
D 16
E 20

Go on to the next page.

22 The length of the base and of the altitude is given in each of the following isosceles triangles. The vertex angles in all the triangles are equal **except** in

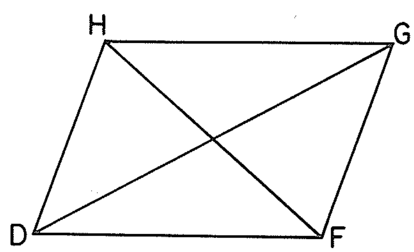


23 Which of the following statements concerning the diagonals of a square is (are) true?

I. The diagonals are equal.
II. The diagonals are perpendicular.
III. The diagonals bisect each other.

A II only
B I and II only
C I and III only
D II and III only
E I, II, and III

24



In the figure above, FGHD is a parallelogram. Which of the following statements is a condition which implies that FGHD is a rectangle?

F DF = GH
G ∠HDG = ∠DGF
H ∠HDF = ∠DHG
J ∠HDF and ∠DHG are supplementary.
K HF and DG are perpendicular bisectors of each other.

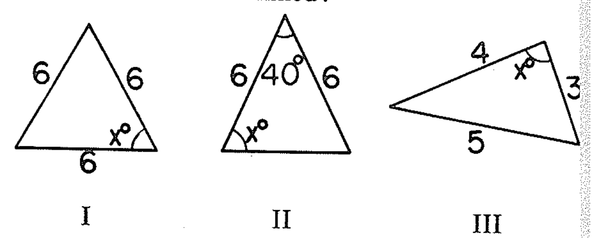
25 A line is drawn from the origin through each the following points. The steepest line go through which of these points?

A (2, 7)
B (4, 7)
C (3, 3)
D (6, 2)
E (10, 1)

26 What is the perimeter of a rectangle if the distanc around three of its sides is 8?

F 6
G 8
H 9
J 12
K It cannot be determined from the informatio given.

27 For which of the following triangles can th value of x be determined?



A I only
B II only
C III only
D I and II only
E I, II, and III

28 Major premise: Two lines in the same plane are parallel if and only if they have no point in common.

Minor premise: Line AB is parallel to line CD.

Conclusion: ?

F AB and CD have no point in common.
G AB and CD have only one point in common.
H AB and CD have two points in common.
J If another line RQ crosses AB, then RQ cannot be parallel to CD.
K There are many lines in space parallel to CD.

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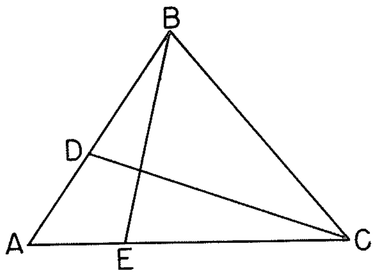
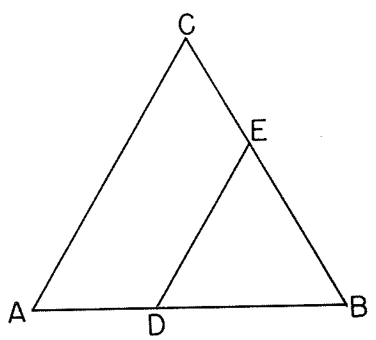


In the figure above, ABC is a triangle and BD = CE. Triangles BCD and CBE are

A congruent by SSS
B congruent by SAS
C congruent by ASA
D similar by SAS
E not necessarily congruent or similar

30



In the figure above, if CA = CB and ED = EB, then which of the following can be concluded?

F CA must be parallel to ED
G CA cannot be parallel to ED
H △ABC is equilateral
J △BDE is equilateral
K AD = CE

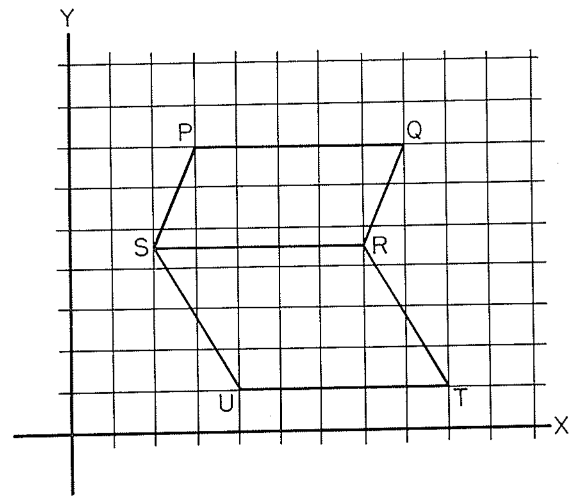
31 Which of the following should be proved equal in order to show that two parallelograms are congruent?

A One pair of corresponding angles
B One pair of corresponding sides
C Two pairs of adjacent sides and the included angles
D A pair of diagonals
E Two pairs of diagonals

32 Which of the following statements most directly supports the assertion, "The hypotenuse of a right triangle is longer than either leg"?

F Two distinct points determine one and only one straight line.
G The distance from a point to a line is the length of the perpendicular from the point to the line.
H The shortest line segment from a point to a line is the perpendicular from the point to the line.
J The shortest distance between two points is a straight line.
K There is one and only one perpendicular from a point to a line.

33



If each division of the grid in the figure above represents one foot and if SR is parallel to the X-axis, what is the area, in square feet, of PQRTUS?

A 15
B 30
C 36
D 42
E 60

34 Two regular polygons having the same number of sides have areas whose ratio is 9 to 4. What is the ratio of their perimeters?

F 3 to 2
G 9 to 4
H 27 to 12
J 81 to 16
K It cannot be determined from the information given.

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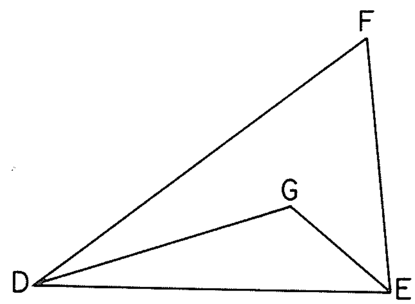
35



In the figure above, the bisectors of angles EDF and FED intersect at G. If the number of degrees in $\angle F$ is n, then the number of degrees in $\angle G$ is

A $\frac{n}{2}$

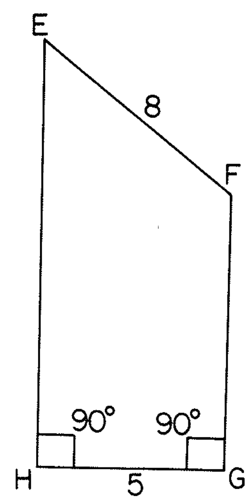
B $90 - \frac{n}{2}$

C $90 + n$

D $90 + \frac{n}{2}$

E $180 - \frac{n}{2}$

36



In the trapezoid above, the perimeter equals 37, EF = 8, and HG = 5. Find the area.

F $\frac{13}{2}$

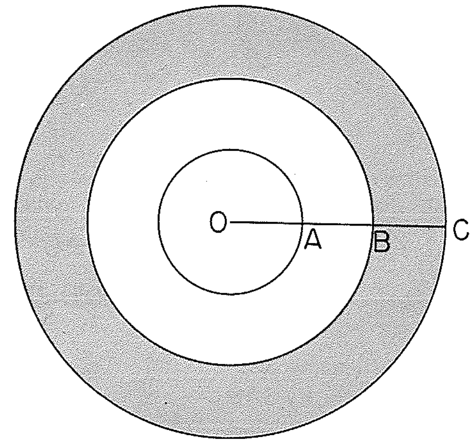
G 12

H 60

J 120

K 370

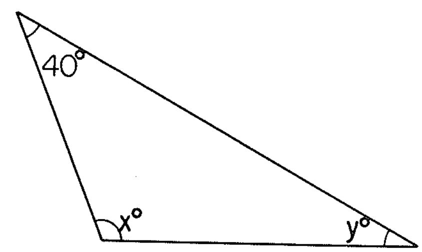
37



In the figure above, OA = AB = BC = 1. What is the area of the shaded ring?

A 9π
B 5π
C 4π
D 3π
E π

38



In the triangle above, if $60 \leqq y \leqq 100$, then

F $0 < x < 60$
G $40 \leqq x \leqq 80$
H $60 < x < 100$
J $60 \leqq x \leqq 100$
K $80 < x < 120$

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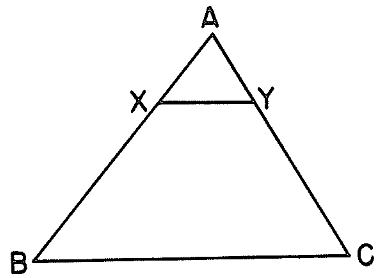


In the figure above, $AX = \frac{1}{3}AB$ and $AY = \frac{1}{3}AC$. Which of the following statements are true?

I. $XY = \frac{1}{3}BC$

II. XY is parallel to BC

III. Area $\triangle AXY = \frac{1}{3}$ area $\triangle ABC$

IV. Area $\triangle AXY = \frac{1}{9}$ area $\triangle ABC$

A I and II only
B II and III only
C I and III only
D I, II, and III only
E I, II, and IV only

40 How many sides has a regular polygon if each of its interior angles has a measure of 170°?

F 10
G 34
H 36
J 144
K 170

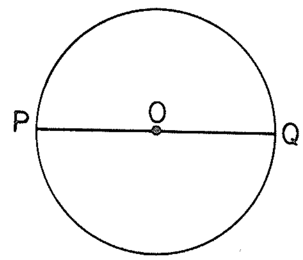
STOP!

If you finish before time is called, look over your work on this part. Do not go on to Part II until you are told to.

Geometry | 40 minutes

PART II

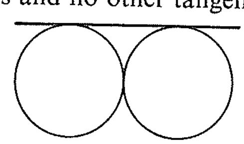
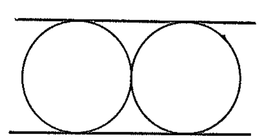
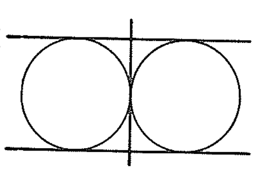
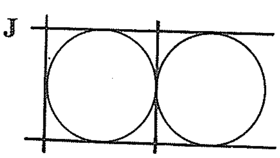
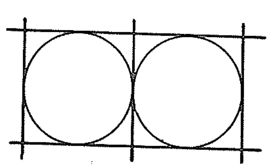
1



The figure above is a circle with center at O. The path once completely around the circle from P back to P is about how many times the path from P straight across through O to Q?

A 1
B $\frac{4}{3}$
C $\frac{3}{2}$
D $\frac{5}{2}$
E $\frac{22}{7}$

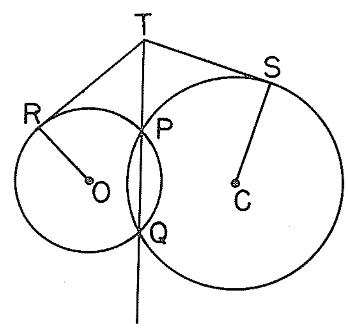
2 Which of the following figures shows all of the possible common tangents to two touching circles and no other tangents?

F

G

H

J

K


3 If a straight line is drawn from one vertex of a hexagon (six-sided polygon) to another vertex, which of the following pairs of polygons **could** be produced?

A Two quadrilaterals
B Two triangles
C Two pentagons
D A quadrilateral and a pentagon
E A triangle and a quadrilateral

4



In the figure above, circles O and C intersect at P and Q. If TR and TS are tangent to circles O and C respectively, which of the following line segments has the same length as TR?

F TS
G TP
H TQ
J SC
K RO

5 If the hypotenuse of a right triangle is 10 inches long and one acute angle measures 60°, then one leg must have a length, in inches, of

A $3\frac{1}{3}$
B 5
C $5\sqrt{2}$
D 6
E 9

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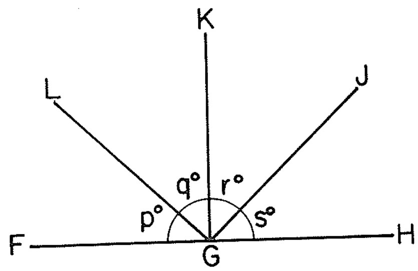
5



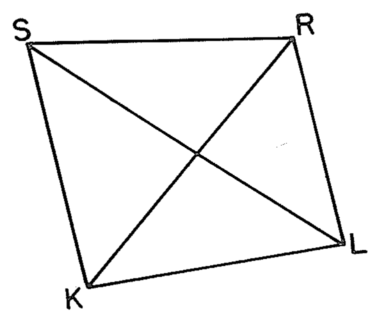
In the figure above, GJ and GL are bisectors of the supplementary adjacent angles FGK and KGH, respectively. GK is **not** perpendicular to FH. Which of the following statements is **false**?

F $r + q = 90$
G $p + r = q + s$
H $p + q = r + s$
J $r \neq q$
K $p + s = 90$

7 If one of the three sides of an isosceles right triangle equals the corresponding side of another isosceles right triangle, the two triangles

A are congruent
B may be congruent
C are not congruent
D are equal in area but not congruent
E are equal in area and may be congruent

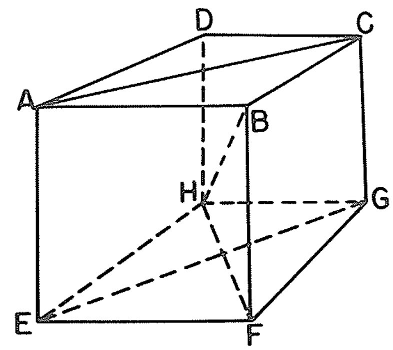
8



In the figure above, KR and LS are diagonals of quadrilateral KLRS. KLRS would be a square if

F KR = LS and KR ⊥ LS
G KR and LS bisect each other, KR = LS, and KR ⊥ LS
H KR and LS bisect each other
J KR and LS bisect each other and KR = LS
K KR and LS bisect each other and KR ⊥ LS

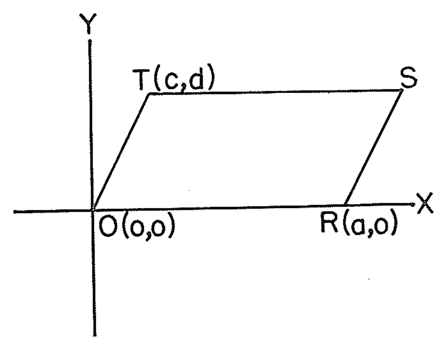
9



The figure above is a cube; AC, HF, and EG are diagonals of two of the faces, and HB is an internal diagonal of the cube. Which of the following is true?

A BH and CG are parallel lines.
B AE and BF are skew lines.
C △HFB is an isosceles triangle.
D △EFH is an equilateral triangle.
E None of these

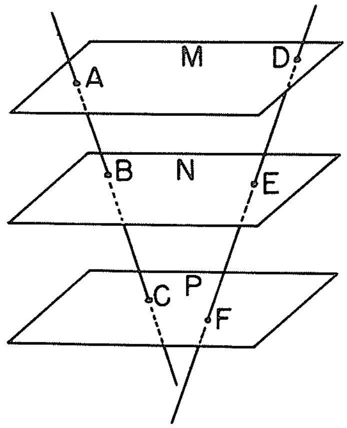
10



In the graph above, if quadrilateral ORST is a parallelogram, the coordinates of vertex S must be

F (c, d)
G (a − c, d)
H (a + c, d)
J (c − a, d)
K (d, a + c)

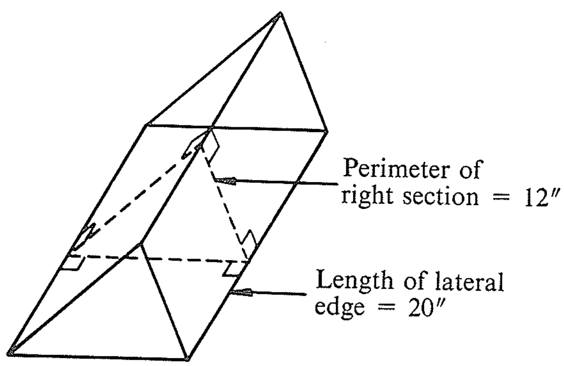
11



In the figure above, planes M, N, and P are parallel. ABC and DEF are straight lines. If AB = 8, BC = 6, and DF = 21, find DE.

A 8 **B** 10 **C** 11
D 12 **E** 28

12



The perimeter of a right section of a prism is 12 inches and the length of a lateral edge is 20 inches. The lateral area of the prism in square inches is

F 64 **G** 120 **H** 240
J 320 **K** 480

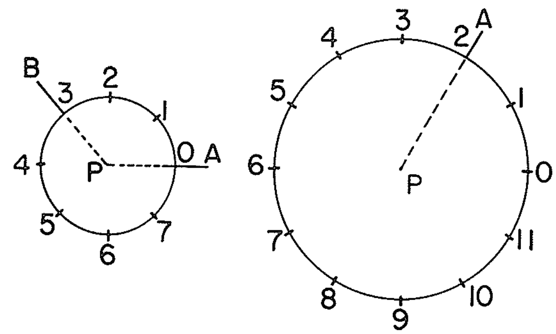
13 When the circumference of a circle is increased from 100π inches to 150π inches, by how many inches is the radius increased?

A 25 **B** 50 **C** 75
D 100 **E** 200

14 The statement "p implies q and q implies p" means exactly the same as all of the following **except**

F "if p then q and conversely"
G "p if and only if q"
H "p and q are equivalent"
J "p and q are unrelated"
K "p is necessary and sufficient for q"

15

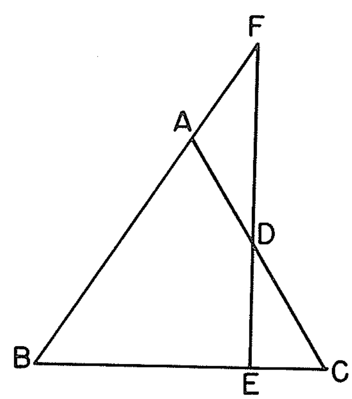


Circle I Circle II

In the figure above, $\angle APB$ has its vertex at the center of Circle I. If the same angle were similarly placed at the center of Circle II but with side PA crossing the 2-mark, what number corresponds to the point at which PB would cross Circle II?

A $4\frac{1}{2}$ **B** $4\frac{5}{7}$ **C** 5
D $6\frac{1}{2}$ **E** $6\frac{5}{7}$

16



In $\triangle ABC$ above, AB = AC, FE $\perp$ BC, and BF is a straight line. $\triangle DAF$ is isosceles because

F $\angle F = \angle ADF$, since both are complements of the equal angles B and C
G DA = FA, since both equal AC − DC
H its sides are parallel to the sides of $\triangle ABC$
J its sides are perpendicular to the sides of $\triangle ABC$
K $\angle F = \angle ADF$, since both equal one-half the supplement of angle C

Go on to the next page.

17 What is the converse of the statement, "If two angles are vertical, then they are equal"?

A If two angles are vertical, then they are not equal.
B If two angles are equal, then they are vertical.
C If $\angle x$ and $\angle y$ are vertical angles, then $\angle x = \angle y$.
D If two angles are not vertical, then they are not equal.
E If two angles are not equal, then they are not vertical.

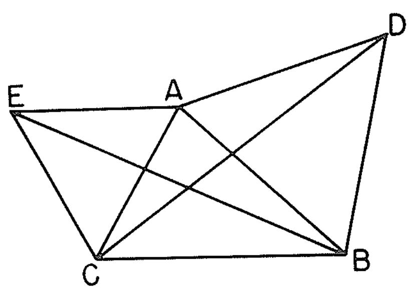
18 Which of the following is true for any parallelogram ABCD which has an acute angle at B and diagonals AC and BD?

F AB < BC
G AB = BC
H AB > BC
J AC < BD
K AC > BD

19 Chords of the same length are drawn in two circles of unequal radii. Which of the following is true?

A The chord in the larger circle could be equal to the radius of the smaller circle.
B The chord in the smaller circle could not be a diameter.
C The distance from the center to the chord is less in the larger circle.
D The minor arc intercepted on the larger circle is longer.
E The minor arc intercepted on the larger circle contains the greater number of degrees.

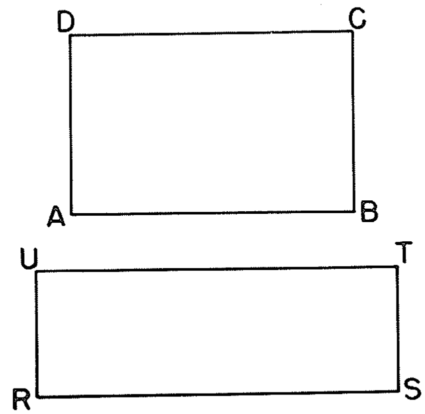
20



In the figure above, equilateral triangles AEC and ABD were drawn on AC and AB, as shown. We can prove triangle AEB congruent to triangle ACD by which of the following authorities?

F AAA
G SAS
H ASA
J SAA
K HL

21



In the figure above, ABCD and RSTU are rectangles. If the length of RS is $1\frac{1}{4}$ times that of AB and the length of RU is $\frac{4}{5}$ that of AD, how do the areas of the rectangles compare?

A Area ABCD = area RSTU
B Area ABCD = $\frac{4}{5}$ area RSTU
C Area ABCD = $\frac{5}{4}$ area RSTU
D Area ABCD = $\frac{16}{25}$ area RSTU
E Area ABCD = $\frac{25}{16}$ area RSTU

22 Two distinct planes x and y are each perpendicular to plane t. Which of the following statements is true?

F Plane x is perpendicular to plane y.
G The line of intersection of x and t is parallel to the line of intersection of y and t.
H The line of intersection of x and t is perpendicular to the line of intersection of y and t.
J If x and y intersect, their line of intersection is perpendicular to t.
K If x and y intersect, their line of intersection is parallel to t.

23 If $\triangle ABC$ is inscribed in a circle of diameter 10 and $\angle A$ is acute, then what can be concluded about the length of BC?

A BC < 5
B BC = 5
C BC < 10
D BC = 10
E BC > 10

Go on to the next page.

24 If an angle of a regular polygon is greater than 100°, then how many sides has the polygon?

F 4
G At most 4
H 5
J At most 5
K At least 5

25 Suppose that after measuring a dozen or more angles of various sizes that are inscribed in several different circles, as well as measuring the arcs which they intercept, you conclude that the degree measure of any angle inscribed in a circle is the same as one-half the degree measure of the intercepted arc. This is an example of

A indirect proof
B deductive proof
C inductive reasoning
D proof by exhaustion of all possible cases
E arriving at false conclusions

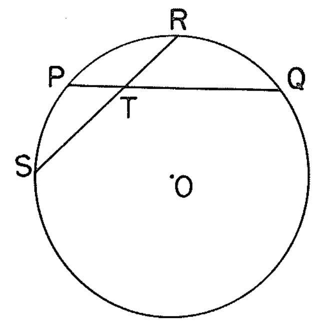
26 Which three of the points P(2, 4), Q(3, 6), R(4, 7), and S(5, 10) lie in a straight line?

F P, Q, and R
G P, Q, and S
H P, R, and S
J Q, R, and S
K No three of these points lie in a straight line.

27 Of the following, which must be shown equal in order to prove that two regular polygons with the same number of sides are congruent?

A Corresponding vertex angles
B Corresponding central angles
C The sums of their exterior angles
D The ratios of corresponding angles
E Their perimeters

28



In the figure above, O is the center of the circle and PQ and RS are chords which intersect at T. In order to know the length of TR, it is sufficient to know the lengths of

F PQ and RS
G PQ and ST
H PQ and radius of circle O
J PT, TQ, and radius of circle O
K PT, TQ, and ST

29 The radii of two concentric circles are 5 and 13 inches. The length of a chord of the larger circle which is tangent to the smaller circle is

A 8
B 9
C 13
D 24
E 65

30 The median drawn to the hypotenuse of a right triangle forms two triangles which are

F congruent
G scalene
H right
J equilateral
K isosceles

31 The distance between points A and B is 4 inches. Point P is 5 inches from A and 2 inches from B. The locus of P in space is

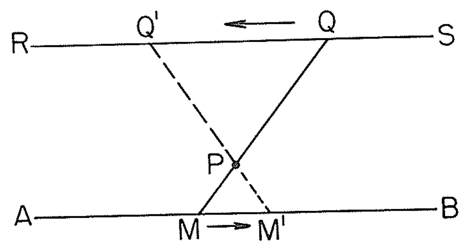
A a point
B a line
C a circle
D a plane
E two planes

Go on to the next page.

Two similar polygons have corresponding sides whose ratio is 1 to 2. If the area of the smaller polygon is 100 square inches, then the area in square inches of the larger one is

F 150
G 200
H 250
J 400
K 500

3



In the figure above, man M is walking toward B at a rate of 3 miles per hour. Train Q is traveling toward R at such a rate that man M, train Q, and pole P are always in a straight line. Q′ and M′ are the positions of the train and the man after 1 hour. If AB and RS are parallel and 160 feet apart, and if P is 10 feet from AB, what is the speed of the train, in miles per hour?

A 30

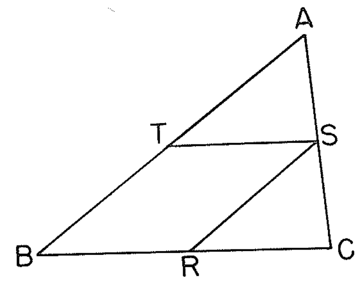
B 45

C 50

D $\frac{160}{3}$

E 160

34

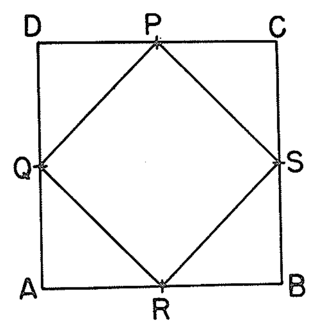


In the figure above, R, S, and T are midpoints of the sides of $\triangle ABC$. Which of the following statements is (are) true?

I. If $\triangle ABC$ is equilateral, then BRST is a rhombus.
II. If $AB = BC$, then BRST is a rhombus.
III. If BRST is a rhombus, then $\triangle ABC$ is isosceles.

F None
G II only
H III only
J I and II only
K I, II, and III

35



In the figure above, P, Q, R, and S are the midpoints of the sides of square ABCD. If a side of ABCD is 1, what is the perimeter of PQRS?

A $\frac{1}{4}$

B $\frac{1}{2}$

C $\frac{\sqrt{2}}{2}$

D $\sqrt{2}$

E $2\sqrt{2}$

Go on to the next page.

36 The height of a rectangle is 7 inches. The diagonal is 3 inches longer than the base. What is the length of the base in inches?

F 10

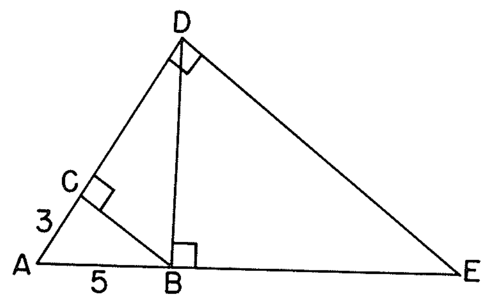
G 20

H 26

J $6\frac{2}{3}$

K $4\sqrt{13}$

37



If, in the figure above, $AD \perp DE$, $DB \perp AE$, $BC \perp AD$, $AC = 3$ and $AB = 5$, then $BE = (?)$

A $\frac{32}{9}$

B $\frac{4\sqrt{10}}{3}$

C $\frac{16}{3}$

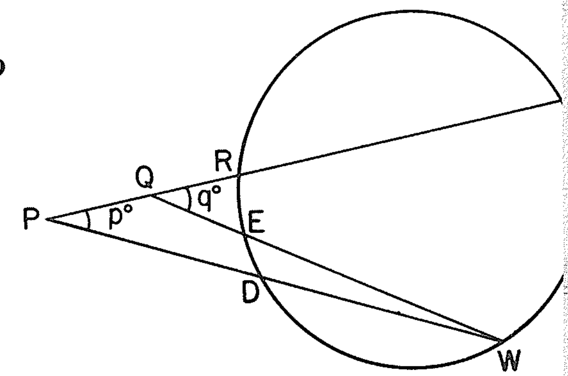
D $\frac{20}{3}$

E $\frac{80}{9}$

38 The ratio of the volumes of two similar cones is 8 to 27. The ratio of their total surface areas is

F 2 to 3
G 4 to 9
H 8 to 27
J $2\sqrt{2}$ to $3\sqrt{3}$
K 16 to 81

39



In the figure above, points R, M, W, D, and I are on the circle, P and Q are outside the circle and PM, PW, and QW are straight lines. I minor arc ED has n degrees, then what is p – (in terms of n?

A $-n$

B $-\frac{n}{2}$

C 0

D $\frac{n}{2}$

E n

40 Each **exterior** angle of a regular polygon can **not** be

F 120°
G 90°
H 80°
J 72°
K 60°

Look over your work on this part.
Do <u>not</u> go back to Part I.