THE DISTRIBUTION OF STRESS

IN

RIVETED JOINTS

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THE DISTRIBUTION OF STRESS IN RIVETED JOINTS.

Definition of Scope of Title.

The title of the paper is somewhat general in that it covers a very large field of research and practical considerations.

Here, however, is meant specifically the manner in which stresses parallel to the direction of the application of load distribute themselves throughout the cover plates of double butt-plate riveted joints, as shown by strain measurements made between the rivets of the joints. A joint of this particular type is shown on plate. 1.

Bibliography.

Probably the most logical way to introduce such a paper as this is to make a rather brief summary of what has been done to date. It may be observed that the subject is hardly well known to professional engineers, as indicated by the fact that technical articles on it are not frequently met with. It goes without saying among investigators that before commencing upon a particular subject of inquiry, one should make himself thoroughly familiar with all available literature connected with what he proposes to do. The writer has gone through considerable records of those who have contributed to the general subject of stresses in riveted joints, and has given closer attention to those papers dealing with riveted butt plate joints, as will be seen later. But it would be out of place to reproduce these records here, and the purpose is equally served if mention is made of them, so that the reader may form his own opinions as to the extent of the work. It is also intended to indicate where more detailed information may be had along various paths of investigation.

Mention should be made at this point that several articles of European research have been translated and appended to this paper, for two reasons, first, they are of importance, second, the originals being in Swedish and German, few readers, whose native speech is English, could translate these articles to their own satisfaction.

Following, then, is a brief resume of some investigations which are more or less connected with this particular work.

One of the earliest reports of rivet stress analysis in plates is that of Professor R. Baumann, who investigated the injury of steel boiler plate at the Institute for Testing Materials in 1912, at Stuttgart, Germany.[1]

1. Zeitschrift des vereines deutscher ingenieure, Vol. 1912. p. 1890 ff.

The stresses in plates due to hot rivets expanding the metal around the holes were observed. The regions around the holes were compressed by pistons to approximate the driving stresses alone, and the latter measured by means of small holes drilled into the rivet holes. These holes were plugged with lead, and the change in size observed with suitable micrometers. It was found that steel rivets cooled rapidly on the edge of their heads as they were driven, and that an increasing amount of driving force was passed to the plate beneath the head as cooling and driving progressed, resulting in injury to the plates.

Shortly after, Professor E.G. Coker [2] investigated the stress distribution of a plate loaded by a rivet, his results coming out in 1913. Here the theory of interference of light was applied to specimens made of celluloid, wherein polarized light was turned on the specimen already subjected to load. The strain in the material caused varying intensities of this light to be allowed through it, and on to a photographic plate. The photograph thus indicated how stress was distributed throughout the specimen. As far as is known here, this was the first attempt to use light in reaching the solution of such a problem. Since, it has

2. Engineering, March 1913. "The Distribution of Stress in a Plate due to a Rivet".

been used to analyze the stresses in members of frames of dirigible air craft.

About the same time, Doctor D. Ruhl was preparing to test a steel plate loaded by two bolts at the ends, and to measure the strains resulting from this load with an instrument of his own invention, based on the principle of the Martens Extensometer, but twenty-five times more accurate. He divided his plate into a net of co-ordinates, and made his measurements both sidewise and lengthwise on the plate. The measurements were converted into stresses by means of suitable theory. Doctor Ruhl also made a test on a lead plate identical in dimensions with the steel plate, so as to learn in advance where the greatest strains in the latter would occur. At this point the work ceased (October 8, 1914) with the outbreak of war. They were resumed later by him, and made public in 1920. The article was translated from the German, and is appended to this paper.

In 1913 Professor Basquin[3] carried out some experiments on the increase in the length of structural members due to riveting. This, he showed caused initial

3. Journal of Western Society of Engineers, June 1913.

stresses of some consequence in the members under certain conditions. Professor Baumann, previously mentioned, also investigated in this direction.

Doctor Cyril Batho[4], McGill University, contributed to the general problem of riveted joints in 1916. He was interested in the amount of load taken by each rivet of an elementary strip of a continuous butt-plate joint, and the intensity of stress in the cover plates. He tested butt joints having a single line of rivets placed parallel with the direction of load, using Martens extensometers to measure the strain on the surface of the cover plates between the rivets. In his mathematical treatment of the problem, he considered the joints as statically indeterminate structures, and so designed his tests that they gave him certain numerical values which filled the openings in his theory, and allowed solution of a simple joint in this manner.

Parallel to the work just mentioned above is that of Professor A.E. Macdonald[5], who conducted work in 1922 similar to that of Doctor Batho. A butt plate joint with a staggered arrangement of rivets was used for testing. The cover plates were diamond shaped, and the number of rivets decreased as the distance from the

4. Journ. Franklin Inst. Nov.,1916."The Partition of Load in Riveted Joints".
5. Thesis, McGill Univ.,1922,"The Distribution of Stress in a Riveted Plate Joint of Variable Section".

edge of the main plates increased. It should be pointed out that this joint was not an element of a continuous joint, and so its solution could not be applied to the consideration of the joint of a tank, for instance.

Professor Maillard[6] also discussed a plate joint of five rivets in an article published in 1923, but the details of the article are not known to the writer. Then in the same year an enormous amount of work of an experimental nature was completed at Berlin by Doctor Theophyl Wyss[7] in which he completely investigated a truss structure with regard to the distribution of stress, both in the gusset plates and in the members themselves. He divided his gusset plates into a network of squares, and took strain measurements in two directions on each square, using extensometers for the purpose. The primary and secondary stresses of the members were measured and then checked against the theoretical values. Mention of this work is sufficient here, as a small summary of the paper presented by Doctor Wyss has been translated and appended to this paper.

6. Schweizerischen Bauzeitung,, July 28, 1923.
7. "Beitrag zur Spannungsuntersuchung au Knotenblechen eisern Fachwerke" - 283.

Following is a description in some detail of the experimental work conducted at the University of Manitoba.

The investigation of riveted joints was first commenced at the University of Manitoba in 1926 by senior undergraduate students working under faculty direction. Two groups of men were given the same object to work upon, namely, the characteristics of simple riveted butt-plate joints with regard to slip between the plates under load. Both groups made independent tests to determine slip on (1) a butt-plate joint having one three-quarter inch diameter rivet on each side of the splice, and (2) on a similar joint having two such rivets on one side of the splice, and three on the other side, all five rivets being in line. A Martens Extensometer was used on each edge of the joints to observe the relative motion between the main and cover plates of the joints. A gradually increasing load was applied, and readings were taken at regular intervals of loading. The results of tests of the two joints showed that the slip amounted to 0.003 inches average, commencing at a load of 8000 to 9000 pounds per rivet. The investigators felt that these rivets, while subject to allowable working stresses, carried their loads by friction alone.

Other tests by the same men were directed toward determining the amount of tension in the rivet shanks required for a given value of friction between the plates of the joints. Here it was found that a stress of about 20,000 pounds per square inch of the cross sectional area of the rivet must have existed to create sufficient friction to sustain loads of 8000 to 9000 pounds. This stress was said to be due to the driving and cooling of the rivet. The coefficient of friction of planed steel surfaces in contact was determined as 0.191. In order to find the tension in the rivets, bolts were substituted for them in the joint, which was then subjected to the same slip test as with the rivets in place. Several schemes were used to find the tension in the bolt corresponding to a known torque on the nut. It was assumed that a bolt and a rivet of the same diameters, placed and tested one after the other in the same joint would permit slip at the same loads when there was equal tension in their shanks.

Further tests were made by other students in 1927, two groups working along different lines. One group dealt with lap plate joints in some detail, as described herein. A lap joint having one three-quarter inch diameter rivet was fabricated and tested for slip characteristics as follows;

The joint was observed by means of two Martens extensometers (one on each edge of the plates) while an increasing load was applied to the joint. This test was made with two separate rivets in the same hole, one after the other. Then a bolt with a machined sleeve was made to replace the rivet. The tension in the bolt when turned up tight was found by measuring the compression in the sleeve with extensometers. The joint thus bolted was again tested for slip a number of times with varying bolt tensions. The results were; in the two tests on the one three-quarter inch rivet, slip commenced at about 3000 pounds load, and continued with increased loading. The tests stopped at about 14000 pounds load. The slip was recorded as 0.0066 and 0.0073 inches, with possibly further slip remaining in the joint. Then the bolt replaced the rivet, and a tension test of the joint was made with zero tension in the bolt. Next a test for slip was made with 20,000 pounds tension in the bolt. Slip started at approximately 5000 pounds load, continued at a uniform rate as the load increased to 8000 pounds, when rapid slip prevented further increase of load. A test was attempted with higher tension in the bolt, but the latter failed as it was being tightened.

The second group of tests of 1927 was made on butt-plate joints with one rivet only. Tests similar to those just described were made by loading the joints and observing slip with two Martens Extensometers. In the first series, one seven-eighth inch diameter rivet held the plates, and slip occurred at 17500, 16000, and 23000 pounds, with a value of 0.003 inches average. The test piece failed in tension in one test. Next, a joint having one one-inch diameter rivet was used, which showed slip starting at 11000 pounds, and which was not completed at 26500 pounds. As before, 0.003 inches was the amount of slip. An attempt was made to analyze the stress distributions on the face of the cover plates of the joints, but the results of the tests were not discussed by the authors.

The writer was a member of one of the groups of students who carried out some of the tests described above at the University of Manitoba in 1926. More extensive experimental work was undertaken in 1927 at the same institution, and on the same general problem of riveted joints. It was the intention to attempt to verify the conclusions reached by Doctor Batho at McGill University in 1916. What follows in this paper is the report of this later work.

Objects of the Work.

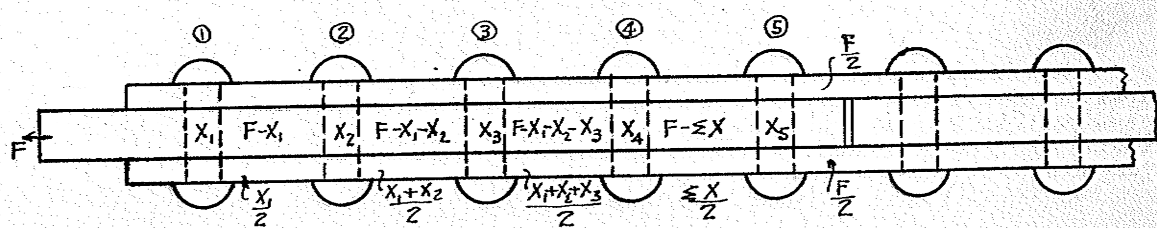
The object of this paper is two-fold: first, as mentioned above, to report the tests made by the writer, and second, to gather under one cover considerable information pertinent to the subject under consideration. In the first part, three riveted butt-strap joints were subjected to tests intended to reveal the distribution of stress in the cover plates of the joints, the proportion of the total load taking by each rivet for varying loads, and, in two instances, the magnitude of slip as well as the load on the joint causing same. "Slip" may be defined as the relative motion which takes place between the members of a structural joint when bolts or rivets which are smaller than their respective holes in the joint are forced by external load to bear on the hole walls.

The utility of work such as this could hardly be questioned, but its justification is not unreasonable at this point of writing. To many, a riveted joint is simply a structure that accomplishes the rigid connection of two pieces of material, the strength depending upon the number and size of rivets used, the physical properties of the material such as elasticity, etc., and the dimensions of the various members. As it is generally admitted that the critical parts of a structure, insofar as the strength is concerned, are the joints; and as these are bolted or riveted, usually riveted, then it is apparent that these joints are of no small importance, and worthy of close investigation.

THEORY:

As previously stated, Doctor Batho dealt with the problem from one viewpoint, in which he considered joints to be statically indeterminate structures which could be solved mathematically, if several assumptions were made, and certain values in the theory were determined from experiment. His solution stands without improvement, so far as the writer knows, and will be used in this paper.

Let the joint considered be as shown in the figure, the cover plates are each one-half of the thickness of the main plates, and of the same width. The rivets shall be placed in one line.



RIVETED BUTT JOINT

Let a_p represent the cross-sectional area of the main plate,

a_c represent the cross-sectional area of each cover plate,

L represent the pitch of the rivets,

n represent the number or rivets on each side of the junction of the main plates (in each structure).

F represent the load in tension carried by the joint,

X_1 represent the load carried by the first rivet, X_2 that carried by the second rivet, etc.,

Then between the first and second rivets the load carried by the main plate is $F - X_1$, and the load carried by each cover plate is $\frac{X_1}{2}$; between the second and third rivets the load carried by the main plate is $(F-X_1-X_2)$, and by each cover plate is $\frac{(X_1 - X_2)}{2}$, and between the $(n-1)^{th}$ and the n^{th} rivet, the load carried by the main plate is $(F - {}_1^{n-1}X)$, and by each cover plate is $({}_{\frac{1}{2}}^{n-1}X)$, where ${}_1^{n-1}X = X_1 + X_2 + \cdots X_{n-1}$.

The distribution of the load for five rivets is shown in the above figure, which represents one structure. If the load carried by any portion of the plate is P, the work stored in this portion is $\frac{P^2L}{2AE}$, where "L" and "A" are as above, and E is Young's Modulus for the material of the plate. It will be assumed that E is the same for both cover and main plates.

Then, if W represents the total work stored in the structure,

$$2EW = \frac{L}{ap}\left[(F-X_1)^2+(F-X_1-X_2)^2+(F-X_1-X_2-X_3)^2+ \text{----} +(F-\Sigma X)^2\right]$$

$$+\frac{2L}{ac}\left[\left\{\frac{X_1}{2}\right\}^2+\left\{\frac{X_1+X_2}{2}\right\}^2+\left\{\frac{X_1+X_2+X_3}{2}\right\}^2+ \text{----} \left\{\Sigma\frac{X}{2}\right\}^2\right]$$

$$+k\left[X_1^2+X_2^2+X_3^2+ \text{---} (F-\Sigma X)^2\right.$$

In agreement with the Theory of Least Work, the forces X_1, X_2, etc., take values to make W the smallest value possible.

Hence, $\frac{\partial W}{\partial X_1}=0,\ \frac{\partial W}{\partial X_2}=0,\ \frac{\partial W}{\partial X_3}=0,\ \text{-----}\ \frac{\partial W}{\partial X_{n-1}}=0.$

But, $$\frac{\partial W}{\partial X_1}=\frac{2L}{ap}\left[(F-X_1)+(F-X_1-X_2)+ \text{-----} +(F-\Sigma X)\right]$$

$$+\frac{L}{ac}\left[X_1+(X_1+X_2)+(X_1+X_2+X_3)+ \text{----} +\Sigma X\right]$$

$$+2k\left[X_1-(F-\Sigma X)\right]$$

Equate the above to zero, and multiply through by ac/L.

$$\left[(n-1)(1+2\frac{Ac}{ap})+4k\frac{ac}{L}\right]X_1+\left[(n-2)(1+2\frac{ac}{ap})+2k\frac{Ac}{L}\right]X_2+ \text{---}$$

$$+\left[(1+2\frac{Ac}{ap})+2k\frac{Ac}{L}\right]X_{n-1}=\left[(n-1)\ 2\frac{Ac}{ap}+\frac{2kac}{L}\right]F.$$

Let $C = 1+2\frac{Ac}{Ap}$., and $K = 2k\frac{Ac}{L}$, and let $F = 1$.

$$\frac{\partial W}{\partial X_1}=0,\quad [(n-1)C+2K]X_1+[(n-2)C+K]X_2+ \text{---} +(C+K)X_{n-1}=(n-1)(C-1+K).$$

$$\frac{\partial W}{\partial X_2}=0,\quad [(n-2)C+K]X_1+[(n-2)C+2K]X_2+[(n-3)C+K]X_3+ \ldots (C+K)X_{n-1}=(n-2)(C-1+K).$$

$$\frac{\partial W}{\partial X_3}=0,\quad [(n-3)C+K]X_1+[(n-3)C+K]X_2+[(n-3)C+2K]X_3+ \ldots (C+K)X_{n-1}=(n-3)(C-1+K).$$

$$\frac{\partial W}{\partial X_{n-1}}=0\quad [C+K]X_1+[C+K]X_2+[C+K]X_3+ \text{---} +[C+2K]X_{n-1}=C-1+K.$$

Thus we have a set of $(n-1)$ linear equations, whence we can find X_1, X_2, X_3 ---X_{n-1}; and $X_n = 1-\Sigma X$.

THE ABOVE EQUATIONS MAY BE MORE CLEARLY EXPRESSED -

X_1	X_2	X_3		X_n	
$(n-1)C+2K$	$(n-2)C+K$	$(n-3)C+K$	---	$C+K$	$= (n-1)(C-1)+K$
$(n-2)C+K$	$(n-2)C+2K$	$(n-3)C+K$	--	$C+K$	$= (n-2)(C-1)+K$
$(n-3)C+K$	$(n-3)C+K$	$(n-3)C+2K$	--	$C+K$	$= (n-3)(C-1)+K$
--	--	--	--	-	= --
$C+K$	$C+K$	$C+K$	--	$C+2K$	$= (C-1)+K$

TO SOLVE FOR FIVE RIVETS, PUT $n = 5$.

$4C+2K$	$3C+K$	$2C+K$	$C+K$	=	$4(C-1)+K$	(a)
$3C+K$	$3C+2K$	$2C+K$	$C+K$	=	$3(C-1)+K$	(b)
$2C+K$	$2C+K$	$2C+2K$	$C+K$	=	$2(C-1)+K$	(c)
$C+K$	$C+K$	$C+K$	$C+2K$	=	$(C-1)+K$	(d)

(a) - (b). $(C+K)X_1 - KX_2 = (C-1)$

or $KX_2 = (C+K)X_1 - (C-1)$ (1)

(b) - (c). $CX_1 + (C+K)X_2 - KX_3 = (C-1)$

or $KX_3 = CX_1 + (C+K)X_2 - (C-1)$ (2)

SUBSTITUTE IN (2) for X_2 FROM (1)

$$KX_3 = CX_1 + \frac{[(C+K)][(C+K)X_1 - (C-1)]}{K} - (C-1)$$

WHENCE $X_3 = \frac{CX_1}{K} + \frac{(C+K)[(C+K)X_1 - (C-1)]}{K^2} - \frac{C-1}{K}$ (3)

MULTIPLY (c) by $(C+2K)$

$$X_1(2C^2+5CK+2K^2) + X_2(2C^2+5CK+2K^2) + X_3(2C^2+6CK+4K^2)$$
$$+ X_4(C^2+3CK+2K^2) = 2C^2+5CK-2C+2K^2-4K \quad \text{..... (e)}$$

MULTIPLY (d) by $(C+K)$

$$X_1(C^2+2CK+K^2) + X_2(C^2+2CK+K^2) + X_3(C^2+2CK+K^2)$$
$$+ X_4(C^2+3CK+2K^2) = C^2+2CK+K^2-C-K \quad \text{..... (f)}$$

(e)-(f)

$$X_1(C^2+3CK+K^2)+X_2(C^2+3CK+K^2)+X_3(C^2+4CK+3K^2) = C^2+3CK+K^2-C-3K. \qquad (g)$$

IN (g), SUBSTITUTE FOR X_2 FROM (1)

AND FOR X_3 FROM (2)

WE GET:-

$$\overbrace{X_1(C^2+3CK+K^2)}^{(1)} + \overbrace{[(C+K)X_1-(C-1)][C^2+3CK+K^2]}^{(2)}$$
$$+\frac{1}{K^2}\Big[KCX_1+(C+K)\,[(C+K)X_1-(C-1)]-K(C-1)\Big]\,[(C^2+4CK+3K^2] \quad \cdots(3)$$
$$= C^2+3CK+K^2-C-3K \quad --(4)$$

TO SIMPLIFY AND SOLVE, MULTIPLY (1) BY K^2, (2) BY K, (4) BY K^2.

WHENCE

$$21C^2K^2X_1+20CK^3X_1+5K^4X_1+8C^3KX_1-7C^3K+7C^2K-14C^4K^2+14CK^2-7CK^3$$
$$+7K^3+C^4X_1-C^4+C^3 = C^2K^2+3CK^3+K^4-CK^2-3K^3.$$

$$\text{AND } X_1 = \frac{K^4+10K^3(C-1)+15K^2(C^2-C)+7K(C^3-C^2)+(C^4-C^3)}{5K^4+20CK^3+21C^2K^2+8C^3K+C^4}$$

FROM(1), $X_2 = \frac{1}{K}[(C+K)X_1-(C-1)]$

FROM(2), $X_3 = \frac{1}{K}[CX_1+(C+K)X_2-(C-1)]$

FROM(d), $X_4 = \dfrac{(C+K-1)-(C+K)(X_1+X_2+X_3)}{C+2K}$

AND $X_5 = 1-\Sigma X.$

C	X_1	X_2	X_3	X_4	X_5
1.5	0.235	0.086	0.068	0.151	0.460
2	0.368	0.105	0.054	0.105	0.368
3	0.528	0.112	0.034	0.061	0.265

TABLE - 1.   K = 1.

K	0	0.2	0.4	0.6	0.8	1.0	1.2	1.4
	0.5	0.46	0.427	0.405	0.383	0.368	0.358	0.345

TABLE 2 - RELATION OF X_1 to K FOR $C = 2$ FOR 5 RIVETS.

SOLUTION FOR FOUR RIVETS - SUBSTITUTE $n = 4$ IN THE GENERAL EQUATIONS.

	X_1	X_2	X_3			
	$3C+2K$	$2C+K$	$C+K$	$= 3(C-1)+K$	(1)	
	$2C+K$	$2C+2K$	$C+K$	$2(C-1)+K$	(2)	
	$C+K$	$C+K$	$C+2K$	$(C-1)+K$	(3)	
(1) -2	$C+K$	$-K$		$C-1$	(4)	WHENCE $X_2 = \dfrac{X_1(C+K)-C+1}{K}$
2 x (3) -(2)	K	$C+3K$		K	(5)	$X_3 = \dfrac{K(1-X_1)}{C+3K}$
(1) -(3)	$2C+K$	C	$-K$	$2(C-1)$	(6)	

SUBSTITUTE FOR X_2 FROM (4), AND X_3 FROM (5) in (6)

$$(2C+K)X_1 + \frac{CX_1(C+K) - C^2 + C}{K} - \frac{K^2(1-X_1)}{C+3K} = 2(C-1) \qquad (7)$$

CLEARING DENOMINATORS

$$K(C+3K)(2C+K)X_1 + CX_1(C+3K)(C+K) - (C^2-C)(C+3K) - K^3(1-X_1) = 2(C-1)(C+3K)K \qquad (8)$$

SIMPLIFY AND COLLECT LIKE TERMS

$$X_1(2KC^2 + 9K^2 + CK^2 + 4K^3 + C^2 + 4CK) = 2K(C-1)(C+3K) + K^3 + (C^2-C)(C+3K)$$

$$\text{WHENCE } X_1 = \frac{(2K+C)(C-1)(C+3K)+K^3}{(C+3K(K^2+3CK+C^2)+K^3}$$

C	X_1	X_2	X_3	X_4
1.5	0.247	0.1175	0.1672	0.468
2	0.375	0.125	0.125	0.375
3	0.530	0.120	0.0783	0.272

TABLE 3. K = 1.

K.	0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4
X_1	0.5	.458	.428	.406	.389	.375	.365	.355

TABLE 4. RELATION OF X_1 TO K FOR C = 2. FOUR RIVETS.

SOLUTION FOR THREE RIVETS.

SUBSTITUTE n = 3 IN THE GENERAL EQUATIONS.

$$(2C + 2K)X_1 + (C + K)X_2 = 2(C - 1) + K. \qquad (1)$$

$$(C + K)X_1 + (C + 2K)X_3 = (C - 1) + K \qquad (2)$$

2 x (2) - (1) $(C + 3K)X_2 = K$

OR $X_2 = \dfrac{K}{C + 3K},$

SUBSTITUTE FOR X_2 IN (1)

$$(2C + 2K)X_1 + \frac{(C + K)K}{C + 3K} = 2(C - 1) + K.$$

$$X_1(2C + 2K) = 2(C - 1) + K - \frac{(C + K)K}{C + 3K}.$$

$$X_1(2C + 2K)(C + 3K) = (2C + K - 2)(C + 3K) - K(C + K)$$

$$X_1 = \frac{C(C - 1) + K(3C + K - 3)}{(C + 3K)(C + K)}$$

AND $X_3 = 1 - X_1 - X_2.$

C	X_1	X_2	X_3
1.5	0.366	0.222	0.412
2	0.400	0.200	0.400
3	0.542	0.167	0.291

TABLE 5. K = 1.

K	0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4
X_1	0.5	.462	.437	.421	.409	.400	.393	.387

TABLE 6. RELATION OF X_1 TO K, C = 2.

THREE RIVETS

In the above analysis, the value of C depends on the relation of sectional area of cover plates to that of main plate, and in all three specimens tested, C = 2, i.e., the main plate was twice as large in section as each cover plate. If C is varied while K is held constant, at say = 1, it is seen that the load distribution is altered. Thus, when C = 1.5, or each cover has one quarter the sectional area of the main plate, X_1 carries 24.7% and X_4 carries 46.8% for four rivets, and X_1 carries 36.6%, X_3 - 41.2% for three rivets. In all cases, the first rivet (see Fig. 1.) carries less than the last rivet.

If $C = 2$, as in our case, the end rivets each carry equal parts of the load. If $C = 3$, in which case each cover plate has the same cross-sectional area as the main plate, the distribution is $X_1 = 52.8\%$, 53.0%, and 54.2%; $X_5 = 26.5\%$, $X_4 = 27.2\%$, $X_3 = 29.1\%$ for five, four and three rivets respectively. In a general way, this is the reverse of the case when $C = 1.5$.

Consider the formulae for the value of X_1 in all three cases solved. It is obvious that the rivets never receive the same load each, except if $K = \infty$, as in the case of perfectly elastic rivets. If the rivets were <u>rigid</u>, $(K = 0)$ the two end rivets of a joint would carry all the load, and those in between none at all. Neither extremes are ever true, and in the absence of definite information, K $(= 2kA_c/l)$ must be determined by experiment. Doctor Batho has investigated its values and variations rather completely. Sufficient to mention here that K depends on the way the work is stored, either in the rivets from shear action, or in the plate contacts by frictional resistance, or both. If shear is the case, K will vary, since the distribution of load on a rivet depends on the intensity of load. Whether K varies for each rivet, we do not attempt to conjecture. Our load distributions were irregular, and to fit them to the theory is unreasonable. The experiments of Prof. W. Weibull[2] on riveted joints with painted contact areas indicate that friction in them is negligible. On the other hand, with bare plates in contact, friction has a considerable value, but this would largely affect K only before the rivets were pulled into shear by the load on the joint.

The diagrams show theoretical division of load for five,, four and three rivets, and were plotted for the particular case of K = 1, C = 2. As it has been shown that K varies with the load, apparently in a linear relation, the diagrams would be true for <u>one load only</u>. and that unknown to us. The value of C is the correct one for the present tests. It is evident, then, that the diagrams serve only as a general indication of the load division to be expected at any stated load. The truth of this statement is more apparent after examining the actual load distribution curves.

2. See appendix.

THE APPARATUS:

The Joints.

Three joints (or more accurately, 2 joints and half of a third) were made to the shop details shown (See Plates 1, 2, 3). The principal factor controlling the shape of the joint was that of limitations imposed by the testing machine, to be discussed below:

The Dominion Bridge Company was asked to fabricate Specimen "A" and "B", while the Canadian Pacific Railway, Weston Shops, furnished Specimen "C".

Specimen "A" (see Plate 1) was composed of two main plates each $3\frac{1}{2}$" x $\frac{1}{2}$" at the test section, and 2 cover plates each $3\frac{1}{2}$" x $\frac{1}{4}$". The main plates were widened to $5\frac{1}{2}$" at the ends to allow a 1-13/16" diameter hole in each. Through these holes $1\frac{3}{4}$" diameter pins were slipped to hold the joint in the testing machine. All plates of the joint were planed all over, it being felt that if all joints tested had the plates planed where they made contact with another plate, there would be a better possibility of uniform friction between these plates, which would thus afford a better comparison of results of tests on different joints. Rivets used were $\frac{3}{4}$" diameter with button heads and were power driven. Three rivets were driven on one side of the splice, and four on the other, it being felt that the one joint could be used to obtain data on both three and four riveted structures. The width of the main plates at the ends, $5\frac{1}{2}$", was the greatest the testing machine would allow through the openings in its heads, while the $3\frac{1}{2}$" width at the test section was as large as could be turned around in the heads while mounting the joint for test. The $\frac{1}{2}$" thickness of the main plates was chosen arbitrarily.

Rivet holes were drilled 13/16" diameter. The tension value of the joint was calculated as $18000 \times (3\frac{1}{2} - \frac{13}{16}) \times \frac{1}{2} = 24000\#$.

The bearing value of 3 rivets = 23750#

The bearing value of 4 rivets = 45000#.

Specimen "B" (see Plate 2) was similar in many respects to Specimen "A". Both main and cover plates were the same width and thickness, both at the test section, and at the ends. Specimen "B", however, had four - $\frac{3}{4}$" diameter button head rivets on one side of the splice, and 5 similar rivets on the other, again with the object of making the joint useful for 2 different sets of tests. Plates were planed all over, as before. The strength of the joint was given as:-

Allowable Tension	-	24100#
Bearing 5 rivets	=	56250#
" 4 "	=	45000#

Specimen "C" (plate 3) owed its shape entirely to testing machine limitations. It was desired to make a structure that would show to some extent the variation in stress in the cover plates of a continuous joint, as in a tank plate splice. For this at least 2 lines of rivets parallel with the direction of load was required, so as to give some indication of the intensity of stress in the plates <u>between</u> rivet lines. The thickness of plates was made the same as in specimens "A" and "B", for purposes of comparison of results. On account of a definite maximum clearance between the moving and fixed heads of a testing machine, the length of the specimen was fixed, since the plates were too wide to fit the regular openings in the heads. In order to allow the stress to to distribute itself from $1\frac{3}{4}$" diameter pin holes, a certain length of plate was desirable, and after this was deducted from the maximum clearance in the machine, it became apparent that only half of such a joint could be accomodated in the machine. Hence the half-joint designated Specimen "C". The main plate was made 8"x $\frac{1}{2}$" at the test section, and both cover plates 8"x $\frac{1}{4}$". Contact surfaces were planed, and the rest of the surfaces left rough. The bevelled ends of the plates were burned to shape. Rivets were $\frac{3}{4}$" diameter, one side button head, the other side flattened to 3/8" high. This oddity was due to a shop error.

THE TESTING MACHINE.

The tests were carried out in a Riehlé Universal Testing Machine for tension and compression work (See plate 4). It is of 200,000 pounds capacity, and load is applied through a head moved by 2 square-threaded screws driven through a variable train of gears by an electric motor. The applied load is measured on a lever arm by a movable jockey weight balancing the load through a system of levers. The lever fulcrums are knife edges bearing on hardened smooth surfaces. There is a tendency for these knife edges to dull from shocks sustained when test pieces break in the machine, and thus the accuracy of measurement of load is impaired to some slight extent. A more serious defect from the point of view of these tests was that the centres of the bronze bushings in the movable head had been disturbed and were further apart than the proper distance for the 2 driving screws of the machine. In consequence, when the moving head was driven downward, there was a tendency to bow the screws, resulting in binding at the bushings. It was for this reason that the joints were made a definite maximum length so as not to injure the machine while testing them. Another defect (for extensometer work only) was that when the direction of the moving head was reversed, so as to decrease or increase load, the slack in the threads of the bushings gave rise to a distinct "jump" in the position of the test specimen, as observed by the extensometers.

The motor driving the testing machine was fitted with a resistance box to vary its speed, and by this means it was easily possible to reduce the rate of application of load to a point where it could be carefully watched by extensometers on the joint. At this low speed vibration was practically absent.

THE EXTENSOMETERS:

As it was intended to measure strain on the cover plates of the joints, extensometers were chosen for this purpose. The principle of the form of extensometer devised by Prof. Martens in 1884 was adopted, wherein the deflection of a material due to strain is so translated into rotation that a mirror reflecting a scale into a telescope is turned through a small angle which varies directly with the strain. The whole apparatus and its operation is described as follows: (See plate 6).

The surface on which strain is to be observed is scratched by 2 parallel lines, a, b, their distance apart being the distance in which it is desired to measure strain, and called the "gauge distance". Into one of these scratches the "distance piece" knife edge is fitted. The distance piece (c) is a small flat bar 1/16" x ¼" with one end turned at right angles and sharpened to a knife edge. On the flat of the bar, on the same side as this knife edge, a scratch is made parallel to the edge, and the same distance from it as the scratches on the test surface are apart. Then a bar of diamond-shaped cross section (d) with the edges at the acute angles ground sharp is fitted between the test surface, and the distance piece into the scratch in each. The long axis of the diamond-shaped bar is theoretically greater than the distance of the under surface of the distance piece to the test surface by the depth of the 2 scratches.

With this arrangement, when the test surface is strained the distance between the two parallel lines on it changes. But at the same time, the corresponding length on the distance piece remains the same in length, so that the two scratches occupied by the diamond piece move relative to each other. This gives rise to rotation of the diamond bar, to which is attached a small mirror (e) about ½" square. A small inverting telescope (f) is then trained on the mirror which is made to reflect the image of a graduated scale (g) usually mounted just beside the objective glass (h) of the telescope. This scale is placed a fixed distance (k) away from the mirror, known as the "calibrated distance". Thus when rotation of the mirror occurs, an observer sees the scale move over a certain number of its divisions. It may be shown mathematically, that for very small angles of rotation of the mirror, the intercept on the scale as observed in the telescope varies nearly directly with the strain on the test surface.

THE CALIBRATING MACHINE:

For each complete extensometer set of instruments there is a certain definite "calibrated distance" or distance from the mirror to the face of the scale. It may be seen that the further the scale is from the mirror, the larger the scale intercept for a constant angle of rotation, consequently, the finer, or more accurate the scale reading, subject to the power of the telescope.

It has been usual practice in the laboratory where this work was done to determine the calibrated distance so that with 20 divisions to the inch on the scale, 1 division passed over by the telescope cross hairs indicated 0.0001 inch strain. It should be remarked, that as the scale is flat, the reading should be at a place on it as close to directly opposite the centre of the objective glass of the telescope as possible, and that calibration should be carried out in this position.

The instrument used in the laboratory for determining the calibrating distance was modelled after the Whitworth Calibrating Instrument. (See plate 5).

It consists of a heavy steel base (a) made of plate $1\frac{1}{2}''$ thick, with a wooden upright standard (b) securely bolted to it. A small steel block (c) adjustable for height is clamped to a rod (d) fastened to the standard, and another larger block (e) above is moved vertically by means of a threaded rod (f) passing through a brass sleeve (g) near the top of the rod. Fastened to the top of the rod there is a circular plate (h) dial graduated to 250 equal divisions, with a small handle (k) for turning. A small upright bar (l) from the standard serves as a vernier. The threaded rod has 40 threads to the inch, and from this, if we turn the plate through one small division, or $\frac{1}{250}$ of one revolution, the threaded rod moves vertically $\frac{1}{40}$ of $\frac{1}{250}$ or $\frac{1}{10,000}$ of an inch. Now there is a horizontal scratch in both the upper and lower steel blocks, and when calibrating an extensometer the gauge piece (o) is placed so that its knife edge occupies one of these scratches, and the diamond bar (m) holding the mirror (p) occupies the scratch on the gauge piece and the scratch on the other steel block of the instrument. The gauge piece and mirror are secured in this position by passing a rubber band (t) around the standard and the gauge piece. The diamond bar is thus clamped in place. The mirror is now turned till it is in a vertical plane, and in the same plane as the axis of rotation of the diamond bar. The telescope is now placed approximately the correct distance from the mirror, and directed so that the mirror appears in the centre of the field of view. The telescope is then focused for <u>twice</u> the distance from the objective glass to the mirror. A little adjusting of the mirror will bring the scale into view in the telescope, and the cross hairs should read the location of the scale near or opposite the centre of the objective glass. Everything is now ready for calibration.

The dial of the calibrating machine should be turned back 20 or 30 divisions, then forward say 10 divisions, and a reading should be made, both on the dial and in the telescope. The dial should then be turned (in the same direction as it was turning before the stop was made for reading) for about 30 or 40 divisions when both dial and scale should be read again. If both intercept values coincide, the distance from the mirror to the scale is the calibrating distance, and should be recorded. If the scale intercept is the smaller, the distance from mirror to scale should be increased accordingly; if the dial intercept is the smaller, this distance should be decreased.

After several trials, the calibrated distance was found and checked several times. For both sets of instruments it was approximately 60 inches. The extensometer gauge pieces and mirrors were then ready for mounting on the test specimens.

THE METHOD OF TESTS:

There were two kinds of tests carried out in this work - (1) to determine the point of slip and also its magnitude, and (2) to determine the deformation of the cover plates of the joints under various loads. The method followed in testing was similar in both types of test.

The joint to be tested (A or B) was lowered through the stationary head of the testing machine until the lower end of the joint protruded past the moving head, and the plates or their edges, which ever was to be observed, faced in such a position as to be accessible for observation. $1\frac{3}{4}$" diameter pins, about 10 inches long were then slipped in the holes of each end of the joint and plate shims of hard steel $\frac{1}{2}$" and 5/8" thick were placed on each side of the joint, between the pins and the two heads of the machine, so that the tendency to bend the pins under load was partly avoided. Next, the moving head was brought downwards till it was bearing on the pin and shims, the joint of course being centred in the two heads.

In the slip tests, the load was then increased till the beam of the testing machine balanced at 1000 lbs. load. Two special gauge pieces were then mounted, one on each edge of the joint, directly opposite each other, and centred on the rivet farthest from the splice. It was thought that observations in this position would give the commencement of slip at the lowest load. The knife edges of the gauge pieces rested in fine scratches on the cover plates of the joint, while the mirror diamond rested in similar scratches on the main plate. Thus the extensometer showed relative motion between the two scratches 1" apart, this motion being composed of part slip and part elastic deformation. Since the latter is directly proportional to the load, and the former a variable, a curve of load displacement would reveal when slip occurred.

Next, the telescopes were set up on heavy wooden tables (to minimize vibration) at their calibrated distances, and the scales placed to show some convenient reading referred to as the "zero of the test".

In the test on 3 rivets for slip, the load was gradually applied by an operator of the testing machine, while an observor watched each telescope. Readings were taken simultaneously at 1000# intervals from 1000# to 40,000#, and back to 1000#. The test was not repeated to find if all slip was taken from the joint.

In the test on 5 rivets, readings were taken as above, 1000# being the initial, then at 5, 10, 15, 20,000#, then 3000# intervals to 53,000#. This test was not repeated; nor were readings taken as the load was released, on account of the mirrors of the extensometers being displaced.

Following these two tests the investigation into deformation of the plates was started. Dr. Batho, at McGill University, showed that readings of strain taken on plate surfaces were accurate indicators of strains within the metal, hence it was decided to observe the strain in the cover plates of the joints between the rivets, and locations for the gauge pieces of the extensometers were then plotted on the plates as shown on plates (1), (2), (3).

The two lines between each rivet, perpendicular to the rivet centre line, were placed 1" apart, and a fine scratch made along each line with a scriber. This enabled the gauge pieces to be placed at any location across the faces of the plates. The lines marked A, B, C, --

to G, represent the successive positions of the gauge piece during the readings. These positions were one half inch apart on specimens A and B, and as shown in plate 3 on specimen C.

In all these tests, the end of the joint under test was uppermost in the testing machine. Observations started with the section farthest from the splice, at position "A" on the one side, and "G" on the other, so that the gauge pieces were exactly opposite, this being for convenience in carrying out the tests.

The method of mounting the gauge pieces on the plates is illustrated in Plate 7, showing specimen "C" under test. Two 3/16" rods were placed, one of each side of the joint, and their ends were held together by elastic bands. The gauge pieces were slipped under the rods, the mirror diamond adjusted in place, and the rod held both in position. A small piece of bar under the other end of the rod let the latter bear evenly on the gauge piece. This arrangement permitted easy shifting of the gauge piece from one position to another across the same section.

In all tests except that made on 3 rivets, the mirror was placed at the upper end of the gauge piece, and only the readings in this position were taken. In the test made on 3 rivets, readings were taken with the mirror at the top, then at the bottom of the gauge piece.

In making a test, after everything was set up and adjusted, the procedure for each extensometer position was as follows: The load was run up to 1000#, and held there while the two extensometers were adjusted to give a "zero" reading. (It should be mentioned that three men were present at each test - one at each extensometer, and one to operate the testing machine). The load was then slowly and steadily applied at about 7000# per minute until a maximum previously agreed on was reached, extensometer readings being taken at intervals of several thousand pounds load. The machine was then reversed to decrease the load to less than at the initial reading, or to about 500#, then again reversed, and the load increased to 1000#. At this point the extensometers were required to read within 0.00002 of their original zero, or the set of readings was rejected, and re-taken, but this was very seldom necessary. The play in the bushings of the moving head caused the latter to throw over slightly when its direction was reversed, and this was one reason for taking the final check reading when the head was moving in the same direction as when the original reading was made. If this precaution was not observed, a check was not possible.

DISCUSSION OF EXPERIMENTAL RESULTS.

As previously stated, the first tests were performed with a view to determining the extent of the slip, as it is called, in two joints. The tests consisted of investigating the 3 rivet portion of specimen A, and 5 rivet portion of Specimen C. It was thought that after these two specimens had been subjected to the loads of tests mentioned, the remaining portions of each (both having 4 rivets) would probably have some of the slip removed, spoiling it for the test.

Curves were prepared from readings taken, in which relative movement between the plates (including some actual elongation of the metal) was plotted as abscissae against load as ordinates. These curves show what happened. In the three-rivet section, the total resistance to slip in the joint was greatest at small loads, decreasing more or less uniformly with the increase of load. Commencing about 13000# load, this resistance remained constant, or nearly so, up to about 35000# load, when it changed in magnitude to somewhat less, becoming constant once more up to the maximum load of the test or 40,000#. A second loading of the joint showed that the rivets were then in bearing, as the extensometers indicated no appreciable amount of displacement of the plates on comparing initial and final readings. There are no records of this test, since no readings were taken. The actual slip was 0.0047 inches.

Comparing the slope of the curve for increasing load with that for decreasing load the former is much less; also, parts of the first curve are nearly straight, while the second or release part appears to form a continuous and regular curve. An explanation is suggested. During that part of the curve showing increasing load with a large slope (tangent) the three plates and the rivets tend to act as one piece, but in diminishing degree till friction between the plates themselves is sufficiently overcome to permit relative motion, as shown by the sudden change of slope in the curve.

It is shown in Physics that the coefficient of friction decreases greatly between two surfaces when sliding is established between them, which is the case at this point. Hence the total resistance of the joint to motion is now lowered, and the curve indicates that slip is then continuous, bringing successive rivets into motion till all rivets are carrying load by bearing, shear and friction. Now when the load is diminished, friction again acts as a drag on the relative motion of the plates, in the opposite direction, and so the joint again tends to act as a single piece of metal at the start of decreasing load resulting in a high slope curve, but as the rivets start to free themselves of bearing and shear stress the resistance to relative motion of plates is once more over come, and motion occurs till there is zero load on the joint. On this assumption there would still be bearing and shear in the rivets held there by friction when the external load became zero.

The slip test on Specimen B on the five rivet section was in general similar in results to those just described. The detail is somewhat different. The initial reading was at 1000 pounds, and the load deflection curve traces the action following. There seemed to be a uniform rate of deflection with respect to load up to 35000 or 36000 pounds. The actual path of the curve is inclined to be irregular in places within the range just mentioned, and the theory is that in this range, motion or slip, is being forced to each rivet, starting at the two end rivets, then the two next the end rivets, and finally the centre rivet. When it is remembered that the plates are elastic members, the reason for this should be clear. At approximately 37000 lb. load it appeared that there was slip everywhere in the joint, and with the resultant lowered frictional coefficient, resistance to motion was at once lowered. In this test, the resistance became increasingly lower as the loading progressed. The test was discontinued at 53000 lbs. out of consideration for the other section of the specimen, which had but 4 rivets. This section might have been overstrained had much higher loads been imposed.

The following tests were concerned with the distribution of stress in the cover plates between the rivets.

Specimen "A" was first investigated, on the section of three rivets. A complete series of readings for both cover plates were taken, first with the extensometer mirrors at the top of the gauge pieces (Test "A") and then with mirrors reversed, or at the bottom (Test "B"). These readings are given in tables, and curves were prepared from them. In both tests, a series of readings across the main plate was included to allow comparison of the moduli "E" for centre and cover plates, and the readings plotted as stress-strain curves (Sheet 13). A glance at this sheet will show that the assumption of both moduli being the same in the theoretical treatment was not objectionable.

Sheet 15 shows the load distribution between the three rivets for the range of the tests. As a matter of interest, the results of Tests "A" and "B" were plotted separately, and also their average was plotted. From these diagrams, it would appear that an average of two tests was to be desired, though it should be remembered that each point plotted on this sheet was the average of 14 individual readings taken across the 2 cover plates at the specified load. Even this curve of averages of tests "A" and "B" appears unreliable at low loads, as it shows that the centre rivet of the group carries more load than the one nearest the splice. Theoretically, if the rivets were driven tight, as they seemed to be, this condition should not be the case; and the centre rivet should carry less load than either of the end rivets. The apparent unreliability is due to the very small strain values obtained. From sheet 66, summary of readings in Tests "A" and "B", it may be shown that a constant error of 0.00001" in section 2 at 3000 lbs. would result in about 7% difference in distribution, and 2% at 5000 lbs. This illustration is sufficient to show the possibilities of errors creeping in the results.

As the total load becomes larger, the proportion of load carried by each rivet tends toward equal amounts for all, but it is characteristic of this specimen that rivet No.1 bears much the greater part of the load, even

50% at 4000 lbs, if the curves reveal the true stress condition. It is suggested that mechanical inequalities in the rivets, and possibly in the plates, are responsible for this peculiarity. It is possible and likely that No. 1 rivet was driven tighter than the other two, giving it the relatively greater resistance to load that it shows.

The lower diagram on sheet 15 is intended to show in another manner the range in the proportions of load carried by each rivet in these two tests. Sheet 18 is a diagram showing the distribution of the strains over the faces of the two cover plates for a load of 25,000 lbs. The curves are built up from the average of Tests "A" and "B". From this sheet it becomes apparent that the two cover plates may differ considerably in the amount and distribution of the load carried by each. The compression strain on face No.2 is assumed to be caused by previous bending of the whole joint so that face No. 2 is convex, and face No. 1 concave. Applying external load in this case would cause compression shown as the plates straightened. It has been mentioned by Dr. Batho in his paper that the strains were consistently less directly between the rivets than at the edge of the cover plates. There seems to be confirmation of this shown on the strain distribution diagram, but not in every case. In contrast the two curves over the splice show relatively greater strain near the rivet line than further away.

The next tests were performed on Specimen "A" and "B", both on the sections having 4 rivets. The method of test was similar with that for three rivets, except that readings were taken only once at each position on the cover plates, with the mirror at the top of the gauge piece. That the average values of "direct" and "reversed" mirror tests were considerably different from either single test was not realized till the experimental work had been completed some time, and the results being worked into form for analysis. However, these results will be treated with the caution they merit, and no positive deductions should be based on them.

Sheet 14 is a comparison of the moduli of the inner and cover plates of Specimen "B", showing "E" for the main plates as 29.9 x 10^{6}#/□", and for the covers 28.7 x 10^{6}#/□", a difference of 4+ %.

Sheet 16 presents the distribution of load between the four rivets of each joint for the range of the tests. Whether the curves are true or not, on account of the one reading at each extensometer position, the proportion of load on each rivet seems to hold fairly constant through the tests. The theoretical distribution of 37.5% of total load for each of the end rivets of a straight line group of 4, arranged parallel with the direction of load, and 12.5% for each of the inner two rivets, is closely approached between 8000 and 12000 lb. in Test "A", specimen "B". Test "C" on specimen "A" shows one outer rivet taking as much as half the total load up to 12000 lb. load, decreasing to 40% at

32,000 lb. load. The other outer rivet holds consistently at 25% to 30% of load. As was mentioned, these results probably would be different if two tests were taken, as in the case of 3 rivets.

Sheets 19 and 20 show diagrams of the recorded strain on the cover plates. In both sheets it appears that there is less strain at the centre of the cover plate between the rivets than towards the edges. The section over the splice is the exception. A brief analysis of the action of rivets on the plates when under stress seems to agree in most places with the diagrams. When the joint is stressed in tension, that side of the rivet farthest from the splice line is putting compression on the cover plates, by bearing, shear and friction. This compression induced in the plate counteracts part of the tension already in the plate, so that the resulting strain in the plates is smaller between the rivets. Now directly over the splice, the two nearest rivets tend to pull away from the hole walls in the cover plates next to the splice, which would lead us to expect less strain between these 2 rivets and higher strain between the hole and the edge of the plate. Face No. 1, Specimen "A", and Face No. 2, Specimen "B", show this, but if bending is present in any degree, the sections may be different than shown.

Sheet 16 shows the variation in proportion of total load carried by each rivet in Test "A", Specimen "B". Note that the average value of each of the end rivets approaches the theoretical 37.5%, actually 37.4% and 36%. The inner rivets at 11.5% and 14% average are not far from perfect values. In short, considering the test as being incomplete, the agreement of theoretical and actual values is very satisfactory.

It is of interest to note the difference between the actual loads and the loads allowed by A.I.S.C. Specification, on the 4 rivet combination.

The allowable total load on the joint is 24,100#(tension at 18,000#.sq.in.). Divided between four rivets, this is 6025# each, on the usual assumptions. Actually, at this load the outer rivets of the group in Specimen "A" carry 10420# and 7030#, and the inner rivets 3140#

and 3510#. In Specimen "B", the outer two carry 3740# and 7800#, and the inner 2 carry 4120# and 3520#.

At the working load of the rivets, or 4 x 11250 = 45000#, the direction of the load curves inddcate that there would probably be a more equal division of the load between the rivets. It has been observed that as the total load is increased the division of load among the rivets tends to become equalized between them.

Following the investigation of four-riveted structures, it was decided to examine the five-riveted end of Specimen "B"(Test "B"). Again, readings with the two extensometers were made only in the "direct" position, or with the mirror topmost. As was done previously, curves were plotted for purposes of analysis and discussion.

Considering the distribution of load among the rivets,(sheet 17) the test showed better results than previous tests. Except for the "break" at 10,000#, each rivet holds within 12% to its share of the load at the start of the test. This "break" in continuity of the two curves is suspected of resulting from erroneous readings of strain. As before, with increasing load, all 4 rivets display the tendency to decrease their share of load if carrying more than 20% of the total load, and increase it if below that amount. One feature to be noted is the disproportionate share of load carried by one end rivet, roughly 47% to 40% of the total load, and also that the two end rivets carry from about 37% at 5000# to 70% at 45000#, leaving the inner three rivets bearing little load. It is suggested that this arises from driving conditions of the various rivets. For example, a tightly-driven rivet will have considerably more value in friction than one loosely driven. From this it follows that the tight rivet, permitting less slip than the loose rivet for the same load, has a greater proportionate value of resistance in any group of rivets. Its actual value will vary with its position relative to other rivets in the group.

To illustrate the distribution of strain in the cover plates, sheet 21 was prepared, showing the strain as read at each extensometer position. The readings on Face No. 1 being larger than those on Face No. 2 indicates bending of the plates, as explained before in a previous test.

Compression exists on Face No. 2 between the two rivets farthest from the splice. It should be noted that the characteristic decrease of strain directly between rivets is present, as well as increased strain between the two rivets adjacent to the splice. It is to be observed that in the latter place, the strain is comparatively uniform across the plate, so that when the plates are calculated to carry a certain limit stress, there is no considerable variation from the calculated stress.

To return to load distribution, an interesting condition is seen. At the maximum load of 46,000# one rivet bears approximately 40% or 18,400#. This means 17,750#/□". shear, and 45,300#/□" bearing. These values are high;

but not beyond the elastic limit. But, previous to this test, the joint withstood 53,000# in a slip test, in which case, either the percentage of load on this rivet must have dropped below 40% say to 35%, or else the rivet carried 21200#, equivalent to 24400#/□" shear, and 52,200#/□" bearing, the latter being beyond the elastic limit.

Since the joint showed no sign of failure, it is assumed that either the value of the rivet in friction added to its bearing, and shear value gave strength enough to carry the load without exceeding the elastic limit, or, the rivet yielded sufficiently to pass a part of the load to other rivets. Which condition existed is difficult to say, and harder to prove. In this connection, it has been shown by other investigators that $\frac{3}{4}$"ϕ power-driven rivets have a value of about 8000# in friction, and this may increase under favorable driving conditions.

The final test of the series was attempted on Specimen "C", along the same general lines as those tests performed on Specimens "A" and "B". It was seen after the test that readings on one plate showed compression which increased as the applied tension load increased. This suggested that the method of mounting the joint in the testing machine was at fault, i.e. that the straps were forcing bending into the joint. In other words, the test was of no use in the present connection. The readings have been given, to show the fault, but nothing further done. With this type of mounting in the testing machine proven useless, there was no other convenient way of dealing with the specimen.

SUMMARY AND CONCLUSIONS.

Following is a summary of the principal contents of this paper;

(1) A general outline of some of the developments in the investigation of riveted structures has been given in brief form.

(2) A riveted plate joint has been considered as a statically indeterminate structure, and a series of equations have been developed for one type of joint by means of the Method of Least Work, giving the loads carried by each rivet in the joint in terms of a quantity "K" which depends on the manner in which work is stored in, or by the action of the rivets.

(3) A series of tests have been made on butt plate joints to investigate "slip", and to determine the stress distribution in their cover plates by means of readings taken on the surface of the latter. It was shown that two extensometer readings averaged gave a better result than one single reading at any location, on account of initial bending in the plates and other possible factors.

(4) The tests bore out the theory in that the end rivets of all groups of rivets invariably carried larger parts of the total load than the inner rivets. While the distribution of load was not uniform, nor even symmetrical in a group, it was indicated, but not proven that the driving of the rivets would be a great factor in determining the proportion of load that each would bear, since their value in friction depended directly on this.

(5) Within the limits of the observations herein reported, it would appear that the stress in the cover plates of butt-plate joints is somewhat decreased in the region lying between rivets in the same line[1], and on the same side of the splice, as compared to the stress at other points in the same cross-section of the plates. That this is to be expected has been pointed out elsewhere, and has been borne out by Dr. Cyril Batho.

1. In this discussion it will be assumed that a "line" of rivets is parallel to the direction of load, and a "row" of rivets is parallel to the splice line.

Several tests showed an increase of stress on the rivet line between the two rivets adjacent to the splice, but theoretical justification on this point is rather more difficult. If both rivets carry their load by friction, which is possible, then the rivet heads, by their clamping action will exert tension in the region in question, so that the stress will be greater there than on either side. This theory would support the stresses as observed. But if we assume that bearing and shear in the rivet shanks is to bear most of the load, then the stress will be higher on each side of the rivets, with a consequent decrease of stress between rivets on the rivet line. This is contrary to what was observed in the present tests, but appears to be entirely reasonable, especially after the joints have been subjected to heavy loads which would tend to force the rivets into contact with the hole walls.

(6) One illustration served to suggest that rivets when loaded to their elastic limit as shown by calculated values have two possible actions, first, resisting the load with the help of friction on the surfaces of the members held, and second, yielding to the load until other rivets in the group bear enough load to relieve the over-stressed rivet.

C. W. Carr
April/16/29.

Test made to determine slip on 3 rivets.

Load increasing - read down.				Load decreasing - read up.			
Load	Reading on Edge "A".	Reading on Edge "B".	Average.	Load	Reading on Edge "A".	Reading on Edge "B".	Average.
1	0	0	0	1	500	438	469
2	10	1	5	2	510	451	480
3	16	3	9	3	525	464	494
4	20	8	14	4	540	478	509
5	25	14	19	5	552	489	520
6	30	21	25	6	563	500	531
7	35	29	32	7	579	511	540
8	41	38	39	8	590	522	556
9	50	47	48	9	600	532	576
10	58	57	57	10	610	542	576
11	66	68	67	11	621	551	586
12	78	78	78	12	631	562	596
13	100	91	95	13	642	570	606
14	122	110	116	14	649	580	614
15	142	130	136	15	654	590	622
16	172	158	165	16	670	597	633
17	194	178	186	17	679	602	640
18	219	201	210	18	688	613	650
19	242	227	234	19	694	620	657
20	269	250	259	20	700	628	664
21	291	272	281	21	707	634	670
22	313	296	304	22	712	642	677
23	338	318	328	23	719	649	384
24	360	341	350	24	725	658	691
25	382	368	375	25	730	663	696
26	404	388	396	26	734	670	702
27	429	412	420	27	738	675	706
28	450	433	441	28	747	682	714
29	477	458	467	29	747	688	717
30	499	482	490	30	759	692	721
31	523	502	512	31	754	697	725
32	549	530	539	32	757	700	728
33	572	550	561	33	757	705	732
34	597	575	586	34	764	710	737
35	621	588	604	35	770	717	743
36	649	622	635	36	772	720	746
37	675	648	661	37	775	724	749
38	700	680	690	38	769	740	754
39	735	705	720	39	769	735	752
40	769	735	752	40	769	735	752

The load given is the actual total load on the joint in thousands of pounds, and the readings are measured deflections from a starting point at one thousand pounds load, shown in one-hundred-thousandths of an inch. There are no reverse readings, since a joint slips but once.

Test made to determine slip on 5 rivets.

Load.	Reading on Edge A.	Reading on Edge B.	Average of Readings.
1	0	0	0
5	9	15	12
10	16	27	21
15	28	38	33
20	40	51	46
23	50	60	55
25	52	64	58
27	59	68	64
29	63	73	68
31	69	78	74
33	74	84	79
35	80	90	85
37	86	98	92
39	101	117	109
41	127	141	134
43	153	171	162
45	191	210	201
47	229	259	245
49	251	318	284
51	305	361	333
53	373	413	393

The load is given in thousands of pounds, and the readings are in one-hundred-thousandths of an inch.

Readings Recorded in Test "A", Specimen "A", Face No. 1.

Applied load in thousands of pounds.

This section taken to determine the modulus of the centre plate.

osition of nstr't.	1	3	5	7	9	11	13	15	17	19	21	23	25
A.	100	90	82	79	77	75	74	70	69	66	63	60	58
B.	100	78	71	68	64	60	59	57	55	54	52	51	50
C.	100	85	80	77	72	70	68	65	63	62	61	60	59
D.	100	82	78	74	72	69	68	65	64	63	62	61	60
E.	100	82	77	72	69	67	64	62	61	60	59	59	57
F.	100	78	72	69	66	62	60	59	59	58	57	56	54
G.	100	85	80	78	75	72	70	70	69	68	67	67	63

Section 2. Taken between the two rivets farthest from the splice.

A.	0	7	10	11	14	18	19	20	21	22	24	27	28
B.	0	2	5	8	10	12	15	18	20	21	22	25	28
C.	0	5	8	10	12	15	17	19	20	22	23	25	27
D.	0	3	6	8	9	11	12	15	17	19	20	22	23
E.	0	6	8	11	13	17	19	21	23	25	28	29	30
F.	0	7	8	11	12	15	18	19	20	22	24	27	29
G.	0	6	7	9	11	12	15	18	19	20	21	24	25

Section 3. Taken between the two rivets nearest to the splice.

A.	0	8	10	15	19	21	24	28	31	32	34	36	39
B.	0	5	9	11	15	19	20	23	27	29	31	34	38
C.	0	6	9	11	13	17	20	21	24	27	30	32	37
D.	0	6	9	10	12	13	15	18	19	20	20	21	24
E.	0	4	7	10	11	13	17	19	20	20	22	24	28
F.	0	5	9	12	14	18	20	21	23	25	28	30	32
G.	0	4	7	10	12	15	18	20	22	23	27	29	31

Section 4. Taken directly over the splice.

A.	0	2	6	9	12	15	19	21	24	28	31	37	41
B.	0	2	7	10	13	19	22	27	29	32	39	42	50
C.	0	2	7	10	12	17	20	22	26	30	34	39	43
D.	0	2	7	10	12	18	20	23	27	30	33	38	42
E.	0	5	9	11	15	19	22	26	29	31	36	40	45
F.	0	6	9	11	17	19	22	26	28	31	34	39	42
G.	0	3	8	10	13	17	19	21	23	26	29	31	37

The instrument positions A, B, C, -- etc., are shown on Plates 1, 2 and 3.

Readings Recorded in Test "A", Specimen "A". Face No. 2.

Position of Instr't.	1	3	5	7	9	11	13	15	17	19	21	23	25
	Applied Load in thousands of pounds.												
A.	100	102	101	97	90	83	77	71	67	61	57	51	48
B.	100	110	108	104	99	94	89	81	75	67	61	55	48
C.	100	104	102	99	94	89	82	77	71	64	59	51	48
D.	100	110	106	102	98	93	88	81	74	68	62	57	53
E.	100	108	104	100	96	91	85	79	73	67	60	53	49
F.	100	111	109	104	101	95	88	83	77	71	64	59	53
G.	100	104	102	99	93	88	81	75	69	62	64	49	43

Section 2. Taken between the two rivets farthest from the splice.

A.	0	-1	0	0	1	2	3	4	6	7	9	10	11
B.	0	-4	-3	-3	0	0	0	2	3	3	4	6	7
C.	0	-3	-2	-1	0	0	1	2	3	4	4	4	6
D.	0	0	1	1	2	3	3	3	4	4	5	6	6
E.	0	-2	0	1	1	2	3	4	6	7	8	9	10
F.	0	-2	0	1	1	2	3	4	5	6	8	9	10
G.	0	-1	1	1	1	2	3	4	5	6	7	9	10

Section 3. Taken between the two rivets nearest to the splice.

A.	0	-2	-1	0	2	5	8	11	15	20	21	25	27
B.	0	0	1	4	6	9	11	15	18	20	24	27	29
C.	0	0	1	2	4	7	9	12	15	18	21	23	25
D.	0	-2	-1	0	1	2	4	6	9	11	13	16	17
E.	0	-1	1	1	3	5	9	10	12	16	19	20	21
F.	0	0	1	4	6	10	13	16	20	23	26	30	31
G.	0	2	4	9	11	14	17	21	25	29	31	35	37

Section 4. Taken directly over the splice.

A.	0	2	4	8	11	16	19	24	28	32	37	42	46
B.	0	4	8	12	16	21	25	29	34	38	43	46	49
C.	0	3	8	12	16	21	25	31	35	41	45	47	51
D.	0	2	4	7	11	15	20	26	31	36	40	46	51
E.	0	3	7	10	13	18	24	30	33	40	45	49	51
F.	0	2	5	8	11	15	20	24	30	33	39	42	47
G.	0	3	5	9	11	15	20	24	29	32	36	41	45

Readings recorded in Test "B", Specimen "A". Face No. 1.
Gauge piece reversed from position used in Test "A".

Applied load in thousands of pounds.

This section taken to determine the Modulus of the centre plate.

Position of Instr't.	1	3	5	7	9	11	13	15	17	19	21	23	25
A.	0	10	16	18	20	22	25	26	27	29	30	31	32
B.	0	18	22	24	27	30	31	33	35	36	38	39	41
C.	0	14	17	20	21	23	25	27	28	29	30	30	31
D.	0	13	16	18	20	21	22	24	25	26	27	27	29
E.	0	16	20	21	23	26	27	29	30	30	31	32	34
F.	0	15	19	21	23	25	27	28	30	30	31	32	34
G.	0	11	14	16	19	20	21	23	23	24	26	27	30

Section 2. Taken between the two rivets farthest from the splice.

Position of Instr't.	1	3	5	7	9	11	13	15	17	19	21	23	25
A.	100	80	72	69	64	61	59	56	54	52	50	49	49
B.	100	78	71	68	64	61	60	57	56	53	51	50	49
C.	100	78	70	68	64	61	60	60	60	59	55	54	53
D.	100	79	74	71	68	65	62	62	61	60	60	59	57
E.	100	80	73	70	67	63	61	60	60	59	57	56	55
F.	100	79	72	69	65	63	60	60	59	57	55	54	52
G.	100	80	74	71	69	65	64	63	61	60	59	57	55

Section 3. Taken between the two rivets nearest to the splice.

Position of Instr't.	1	3	5	7	9	11	13	15	17	19	21	23	25
A.	100	79	72	67	63	60	56	51	49	43	37	34	31
B.	100	75	70	66	63	59	55	50	49	43	40	35	32
C.	100	78	73	70	68	65	61	60	58	53	50	47	46
D.	100	78	71	70	68	66	66	64	63	60	56	54	53
E.	100	79	74	70	67	64	60	58	53	50	45	41	39
F.	100	74	70	65	62	57	52	49	45	40	38	34	32
G.	100	75	69	64	61	59	54	51	48	44	40	36	34

Section 4. Taken directly over the splice.

Position of Instr't.	1	3	5	7	9	11	13	15	17	19	21	23	25
A.	100	72	63	58	52	47	39	34	30	21	14	10	6
B.	100	73	65	60	53	49	42	36	30	27	22	19	15
C.	100	70	60	54	49	41	36	30	24	20	13	7	1
D.	100	75	67	60	52	45	40	35	30	22	18	13	10
E.	100	74	66	60	54	49	41	39	34	30	25	21	19
F.	100	80	70	62	57	53	49	44	39	35	30	27	23
G.	100	80	70	65	60	55	50	46	41	39	33	30	26

Readings Recorded in Test "B", Specimen "A". Face No. 2.
Gauge Piece reversed from position used in Test "A".

Position of Instr't.	Applied load in thousands of pounds.											
	1	3	5	7	9	11	13	15	17	19	21	23	25
This section taken to determine the Modulus of the centre plate.													
A.	0	0	3	9	12	18	24	30	37	45	51	58	62
B.	0	-8	-2	1	7	10	17	22	29	36	42	49	53
C.	0	-8	-2	2	6	11	13	19	23	29	34	39	46
D.	0	-7	-1	3	9	13	20	23	30	37	42	48	52
E.	0	-10	-7	0	3	9	13	18	23	30	37	42	49
F.	0	-10	-7	0	3	10	17	21	28	33	40	47	51
G.	0	0	2	9	12	20	25	31	39	43	51	58	62
Section 2. Taken between the two rivets farthest from the splice.													
A.	0	-20	-22	-25	-27	-28	-28	-28	-28	-28	-26	-24	-21
B.	0	-20	-22	-23	-24	-24	-23	-23	-22	-21	-21	-19	-18
C.	0	-20	-21	-22	-22	-22	-22	-22	-21	-20	-19	-18	-16
D.	0	-19	-19	-19	-20	-20	-21	-20	-20	-19	-19	-18	-17
E.	0	-16	-18	-19	-19	-18	-18	-17	-15	-14	-12	-11	-10
F.	0	-20	-21	-21	-21	-20	-19	-18	-17	-15	-13	-11	-10
G.	0	-18	-18	-18	-19	-18	-18	-16	-15	-13	-11	-19	-9
Section 3. Taken between the two rivets nearest to the splice.													
A.	0	-15	-18	-18	-18	-18	-17	-16	-13	-13	-12	-10	-9
B.	0	-19	-20	-19	-19	-18	-17	-15	-14	-12	-11	-10	-7
C.	0	-19	-19	-19	-18	-17	-16	-15	-14	-12	-11	-10	-9
D.	0	-18	-18	-18	-18	-17	-16	-15	-14	-12	-12	-11	-10
E.	0	-17	-17	-17	-15	-13	-12	-11	-10	-9	-8	-7	-4
F.	0	-19	-18	-17	-15	-13	-11	-10	-9	-7	-5	-2	1
G.	0	-19	-19	-18	-16	-13	-11	-10	-9	-7	-3	-1	2
Section 4. Taken directly over the splice.													
A.	0	-20	-21	-21	-20	-20	-18	-17	-14	-12	-10	-8	-6
B.	0	-19	-20	-19	-17	-15	-12	-10	-9	-7	-3	0	5
C.	0	-16	-15	-13	-10	-9	-8	-6	-2	0	3	8	10
D.	0	-15	-14	-11	-9	-8	-4	-1	2	8	11	18	20
E.	0	-12	-12	-11	-9	- 7	-4	-2	1	3	8	11	16
F.	0	-11	-11	-10	-9	- 7	-4	-1	0	3	8	11	16
G.	0	-10	-10	-9	-8	- 5	-2	0	1	8	11	15	20

Readings Recorded in Test "C", Specimen "A". Face No. 1.

Position of Instr't.	Applied load in thousands of pounds.									
	1	5	9	13	17	21	25	29	33	27
Section 2. Taken between the two rivets farthest from the splice.										
A.	0	9	12	13	13	12	12	13	15	14
B.	0	9	14	14	13	13	14	14	15	17
C.	0	6	10	13	13	10	10	12	15	16
D.	0	6	10	12	11	11	12	13	13	14
E.	0	5	10	11	11	10	10	11	12	13
F.	0	6	9	11	11	11	12	14	15	16
G.	0	6	8	7	7	7	8	8	8	9
Section 3. Taken between the two centre rivets.										
A.	0	8	10	13	16	19	21	26	30	33
B.	0	7	9	11	14	17	20	22	26	30
C.	0	8	10	12	14	16	19	21	24	28
D.	0	7	9	10	11	12	15	18	20	23
E.	0	7	10	11	13	15	17	20	23	26
F.	0	8	10	12	14	17	20	22	26	29
G.	0	8	10	10	11	14	16	19	21	24
Section 4. Taken between the two rivets nearest the splice.										
A.	0	10	14	17	20	24	27	31	38	43
B.	0	10	12	15	19	22	28	33	39	44
C.	0	7	10	10	13	17	20	23	28	32
D.	0	7	10	10	10	12	15	18	20	23
E.	0	7	10	11	12	15	18	21	27	30
F.	0	9	10	13	15	18	20	23	29	34
G.	0	8	12	15	19	20	25	30	33	37

Note: There was no Section 5 taken over the splice, since the Section 4 of Test "A" for the same specimen was taken there. The values obtained in the test mentioned will be used in conjunction with the above values to give the distribution of load over the various sections.

Readings recorded in Test "C", Specimen "A". Face No. 2.

Position of Instr't.	1	5	9	13	17	21	25	29	33	27
Section 2. Taken between the two rivets farthest from the splice.										
A.	0	-2	0	7	12	19	23	28	32	35
B.	0	0	1	9	16	21	28	31	34	38
C.	0	3	5	11	20	25	30	34	39	41
D.	0	0	2	9	15	21	26	30	32	37
E.	0	1	5	10	18	23	29	32	36	40
F.	0	4	8	13	19	23	29	32	37	40
G.	0	0	4	10	16	20	24	29	32	37
Section 3. Taken between the two centre rivets.										
A.	0	1	7	14	21	28	32	39	43	49
B.	0	1	8	15	21	29	34	40	46	50
C.	0	0	4	11	19	23	30	34	40	44
D.	0	2	7	13	18	27	31	36	41	45
E.	0	2	7	13	20	27	32	38	42	48
F.	0	3	9	16	22	30	36	41	47	51
G.	0	3	9	17	21	30	34	40	45	50
Section 4. Taken between the two rivets nearest the splice										
A.	0	4	11	20	30	40	49	57	65	71
B.	0	2	8	17	25	33	42	50	56	63
C.	0	2	8	17	24	32	40	47	55	61
D.	0	1	6	13	20	26	31	36	42	48
E.	0	3	10	18	26	33	41	50	58	63
F.	0	4	11	20	28	37	44	52	61	70
G.	0	2	10	18	27	35	42	50	58	63

See note on Set No. 1 regarding Section 5.

Summary of readings taken on three rivets. Specimen "A".

Test "A"

Net Load.	Section 2	3	4	M
0	0	0	0	0
2	23	35	41	71
4	49	65	94	128
6	68	99	137	178
8	87	129	183	234
10	110	168	245	292
12	130	205	298	347
14	153	241	354	405
16	172	280	406	509
18	188	293	460	509
20	207	347	521	562
22	237	382	579	611
24	247	416	640	657

Test "B"

0	0	0	0	0
2	13	36	54	73
4	53	72	110	135
6	67	102	162	187
8	87	139	205	241
10	111	161	258	290
12	125	196	307	351
14	138	225	354	399
16	157	252	407	451
18	170	295	457	509
20	192	332	510	573
22	210	368	559	628
24	229	394	606	681

Average results of Tests "A" and "B".

0	0	0	0	0
2	18	35	47	72
4	51	68	102	132
6	67	100	149	183
8	87	134	194	237
10	110	165	251	291
12	127	200	302	349
14	145	233	354	402
16	164	266	406	452
18	179	294	459	509
20	199	339	515	566
22	223	375	569	620
24	238	405	623	669

The net load is given in thousands of pounds, and the readings given are the average of 14 readings taken at the load and section indicated. Thus the average of Tests "A" and "B" is actually the average of 28 readings for the same section at the load shown.

Readings Recorded in Test "A", Specimen "B", Face No. 1.

Position of Instr't.	Applied load in thousands of pounds. 1	5	9	13	17	21	25	29	33
	This section taken to determine the Modulus of the centre plate.								
A.	100	94	90	87	82	79	77	71	67
B.	100	94	90	87	82	80	78	73	70
C.	100	93	87	84	82	79	78	74	69
D.	100	96	92	88	84	81	79	75	71
E.	100	92	89	86	82	80	77	73	69
F.	100	97	92	90	85	81	79	77	71
G.	100	97	92	89	87	81	80	74	71
	Section 2. Taken between the two rivets farthest from the splice.								
A.	100	96	92	91	89	89	89	89	90
B.	100	97	92	90	89	88	87	87	88
C.	100	97	92	90	89	88	88	88	88
D.	100	97	92	90	89	88	88	88	88
E.	100	96	90	89	88	87	84	84	84
F.	100	92	89	87	84	82	82	81	81
G.	100	97	94	92	91	90	89	88	88
	Section 3. Taken between the two centre rivets.								
A.	100	95	90	85	81	78	73	70	68
B.	100	94	89	84	81	78	74	71	69
C.	100	96	90	87	82	79	77	73	71
D.	100	97	91	89	86	82	80	78	76
E.	100	97	92	90	87	82	80	78	77
F.	100	95	90	87	82	79	75	71	69
G.	100	98	92	89	83	80	77	72	70
	Section 4. Taken between the two rivets nearest the splice.								
A.	100	97	90	84	79	72	69	60	67
B.	100	98	91	87	80	75	69	61	58
C.	100	98	92	89	83	79	72	67	61
D.	100	97	92	89	85	79	75	71	68
E.	100	97	92	89	85	80	73	69	62
F.	100	95	90	85	79	74	68	60	56
G.	100	96	90	85	80	74	68	61	58
	Section 5. Taken directly over the splice.								
A.	100	91	83	79	72	66	59	52	47
B.	100	91	85	79	70	64	58	52	47
C.	100	91	84	78	70	62	55	49	40
D.	100	90	82	74	68	60	52	47	40
E.	100	92	85	78	70	62	56	50	40
F.	100	91	84	79	70	64	57	49	41
G.	100	92	88	82	77	70	62	57	50

Readings Recorded in Test "A", Specimen "B". Face No. 2.

Position of Instr't.	Applied load in thousands of pounds.								
	1	5	9	14	17	21	25	29	33
This section taken to determine the Modulus of the centre plate.									
A.	0	10	21	33	47	60	72	86	97
B.	0	10	21	33	46	59	72	83	95
C.	0	7	20	31	44	57	70	81	92
D.	0	12	23	33	45	59	70	81	92
E.	0	7	17	27	39	50	64	74	85
F.	0	10	20	32	44	57	70	83	93
G.	0	10	21	33	45	56	70	83	93
Section 2. Taken between the two rivets farthest from the splice.									
A.	0	4	7	10	13	18	21	26	31
B.	0	5	8	11	15	19	23	28	31
C.	0	3	6	10	15	18	22	26	30
D.	0	4	6	10	13	18	21	25	29
E.	0	2	4	9	13	18	21	25	30
F.	0	-1	0	4	9	12	17	20	24
G.	0	2	6	9	12	17	20	23	27
Section 3. Taken between the two centre rivets.									
A.	0	4	8	12	18	23	29	35	40
B.	0	3	8	12	19	25	30	38	41
C.	0	5	9	12	17	21	26	31	36
D.	0	5	7	10	15	18	22	27	30
E.	0	4	9	12	19	21	27	33	37
F.	0	4	9	14	20	25	30	35	41
G.	0	5	9	14	18	22	29	33	39
Section 4. Taken between the two rivets nearest to the splice.									
A.	0	8	14	20	28	35	44	51	60
B.	0	9	14	20	27	34	42	51	60
C.	0	5	9	13	20	26	33	40	48
D.	0	5	8	10	16	20	26	33	40
E.	0	4	10	14	20	26	35	43	50
F.	0	4	10	16	22	30	37	45	53
G.	0	7	10	17	22	30	36	43	50
Section 5. Taken directly over the splice.									
A.	0	9	18	26	34	43	52	63	74
B.	0	7	15	23	32	41	50	63	73
C.	0	9	18	27	37	46	57	66	78
D.	0	9	17	27	35	45	55	65	74
E.	0	10	19	26	36	45	54	65	75
F.	0	8	16	25	35	43	52	64	74
G.	0	9	16	23	32	41	51	60	70

Readings recorded in Test "B", Specimen "B". Face No. 1.

Applied load in thousands of pounds.

Section 2. Taken between the two rivets farthest from the splice.

Position of Instr't.	1	6	11	16	21	26	31	36	41	46
A.	0	9	16	23	30	35	41	48	52	59
B.	0	9	14	22	30	35	41	47	51	57
C.	0	11	17	24	31	37	43	49	52	59
D.	0	9	15	21	29	34	42	49	52	58
E.	0	9	16	23	30	39	44	51	58	62
F.	0	10	16	23	30	37	43	49	54	59
G.	0	9	15	20	27	33	39	44	49	54

Section 3. Taken between the two rivets second farthest from the splice.

Position of Instr't.	1	6	11	16	21	26	31	36	41	46
A.	0	11	20	28	36	45	54	61	70	76
B.	0	11	21	28	37	44	54	62	68	77
C.	0	10	18	27	35	42	49	57	65	70
D.	0	8	13	20	26	32	39	45	51	56
E.	0	11	17	25	31	40	46	53	60	66
F.	0	10	18	25	32	40	47	53	61	68
G.	0	9	16	24	30	38	45	52	58	65

Section 4. Taken between the two rivets second nearest to the splice.

Position of Instr't.	1	6	11	16	21	26	31	36	41	46
A.	0	11	20	29	38	46	56	65	75	83
B.	0	10	20	27	36	44	53	62	71	79
C.	0	11	18	25	32	41	50	59	65	72
D.	0	10	15	21	28	35	42	50	57	64
E.	0	10	16	23	30	38	47	54	62	70
F.	0	10	16	23	30	37	46	53	61	70
G.	0	10	17	24	31	40	50	57	64	71

Section 5. Taken between the two rivets nearest to the splice.

Position of Instr't.	1	6	11	16	21	26	31	36	41	46
A.	0	10	20	30	39	50	60	70	81	93
B.	0	10	20	29	36	46	56	66	77	87
C.	0	10	16	24	31	40	50	59	69	76
D.	0	9	14	22	30	38	45	53	62	70
E.	0	9	15	21	30	38	48	56	64	73
F.	0	10	18	25	34	43	53	63	73	81
G.	0	9	13	22	30	40	50	60	70	80

No section was taken directly over the splice in this test, since readings were already taken at that section in Test "A". The latter readings are combined with those given above in order to produce the necessary data.

Readings Recorded in Test "B", Specimen "B". Face No. 2.

Position of Instr't.	Applied load in thousands of pounds.									
	1	6	11	16	21	26	31	36	41	46
Section 2. Taken between the two rivets farthest from the splice. All the readings in this section are negative, indicating compression.										
A.	0	2	1	3	4	4	7	8	8	8
B.	0	1	0	2	2	3	5	6	7	7
C.	0	2	1	2	2	3	5	7	7	7
D.	0	1	0	1	0	2	3	4	5	5
E.	0	1	0	0	0	2	2	5	6	6
F.	0	1	0	0	0	1	2	2	2	2
G.	0	1	1	1	1	2	2	3	3	2
Section 3. Taken between the two rivets second farthest from the splice. These readings are positive, except where shown.										
A.	100	-1	99	98	97	94	92	91	89	87
B.	100	100	97	93	91	89	87	85	82	79
C.	100	100	98	97	95	93	92	90	89	86
D.	100	100	98	97	95	92	92	91	90	89
E.	100	100	99	96	93	90	89	88	86	83
F.	100	100	97	94	91	90	88	87	84	81
G.	100	100	98	95	92	90	90	89	86	84
Section 4. Taken between the two rivets second nearest to the splice.										
A.	100	99	95	92	90	87	84	80	77	72
B.	100	100	96	93	90	88	86	82	79	75
C.	100	100	97	94	91	90	88	85	82	79
D.	100	100	99	97	95	93	92	90	89	87
E.	100	99	96	92	90	88	85	81	79	77
F.	100	99	94	91	88	85	81	79	75	74
G.	100	99	95	91	88	85	81	79	75	71
Section 5. Taken between the two rivets nearest to the splice.										
A.	100	97	91	87	81	78	73	70	66	61
B.	100	99	93	90	86	82	79	75	71	68
C.	100	100	97	93	90	89	88	85	82	81
D.	100	99	97	94	91	90	90	89	87	85
E.	100	99	96	92	91	89	88	84	82	79
F.	100	99	94	90	88	84	81	78	72	69
G.	100	98	91	88	82	79	75	70	66	60

See footnote on set No. 1 of this test.

Summary of distribution of load between rivets for Specimen "A", 3 rivets.

Net Load	Test "A" 1	2	3	Test "B" 1	2	3	Average 1	2	3	Net Load
2	56.1	29.3	14.6	24.1	42.5	33.4	38.3	36.1	25.6	2
4	52.1	17.0	30.9	48.1	17.3	34.6	50.0	16.7	33.3	4
6	49.6	22.6	27.8	41.4	21.6	37.0	45.0	22.2	32.8	6
8	47.5	23.0	29.5	42.4	25.4	37.2	44.8	24.2	30.9	8
10	44.8	23.7	35.5	43.0	19.4	37.6	43.8	21.9	34.3	10
12	43.6	25.1	31.2	40.7	23.1	36.2	42.0	24.2	33.8	12
14	43.2	24.9	31.9	39.0	24.6	36.4	40.9	24.8	34.2	14
16	42.4	26.6	31.0	38.6	23.3	38.1	40.3	25.1	34.6	16
18	40.8	22.9	36.3	37.2	27.3	35.4	39.1	25.1	33.8	18
20	39.7	26.9	33.4	37.7	27.5	34.8	38.6	27.2	34.2	20
22	40.8	25.0	34.2	37.5	28.2	34.3	39.2	26.7	34.1	22
24	38.6	26.4	35.0	37.8	27.2	35.0	38.2	26.8	35.0	24

(Percentage of total load on rivet number:)

Summary of distribution of load for four rivets.

Net Load.	Test "C", Specimen "A". 1	2	3	4	Test "A", Specimen "B". 1	2	3	4	Net Loa
4	52.0	11.8	10.8	25.4	38.2	8.9	4.9	48.0	4
8	50.6	10.8	11.8	26.8	42.1	12.7	5.7	39.5	8
12	49.7	9.3	11.9	29.1	40.8	12.5	7.3	39.5	12
16	48.0	9.9	13.1	29.0	38.5	16.4	9.0	36.1	16
20	43.8	17.1	11.6	29.5	37.4	16.0	12.7	33.9	20
24	42.8	14.4	13.6	29.2	35.4	17.7	15.2	31.7	24
28	41.6	15.9	14.7	27.8	33.9	19.1	17.5	29.5	28
32	39.3	17.5	16.3	26.9	32.3	18.5	19.4	29.8	32

(Percentage of total load on rivet number)

Summary of distribution of load for five rivets.
Test "B", Specimen "B".

Net Load.	Percentage of total load on rivet number. 1	2	3	4	5
5	40.4	8.5	5.0	0	46.1
10	22.4	26.7	4.3	2.5	44.1
15	34.8	14.2	3.5	4.0	43.5
20	35.2	13.3	3.5	5.0	43.0
25	33.1	15.7	3.1	5.5	42.6
30	31.7	16.3	5.1	4.8	42.1
35	30.7	16.3	6.3	5.3	41.4
40	29.4	18.1	5.9	6.3	40.3
45	29.3	18.0	5.9	6.6	40.2

In all cases the net load is given in thousands of pounds.

Summary of readings taken on 4 and 5 Rivets.

4 Rivets. Test "A", Specimen "B".

Net Load.	Section. 2	3	4	5	M
0	0	0	0	0	0
4	47	58	64	123	103
8	96	125	138	228	211
12	134	175	199	328	311
16	171	244	284	444	426
20	208	297	368	556	537
24	238	357	459	672	640
28	268	419	557	790	754
32	295	464	641	913	859

4 Rivets. Test "C", Specimen "A".

4	53	65	76	102
8	98	119	142	194
12	150	178	214	302
16	195	235	288	406
20	226	304	364	515
24	267	357	442	623
28	301	416	521	723
32	335	474	609	828
36	367	530	682	933

5 Rivets. Test "B", Specimen "B".

Net Load.	2	3	4	5	6
5	57	69	76	76	141
10	63	138	150	157	281
15	147	207	222	239	422
20	198	273	293	321	563
25	233	343	365	404	703
30	267	404	447	488	843
35	302	462	524	576	984
40	330	533	599	670	1124
45	371	599	674	757	1265

Col. 6 is taken by interpolation from sheet M 45.
Col. 5 Test "C", Specimen "A", is taken by interpolation from values given for that section in the tests for 3 rivets.
The net loads given are in thousands of pounds.
The values are the sum of 14 readings for the load and section indicated.

Readings Recorded in Test "A", Specimen "C". Face No. 1.

Position of Instr't.	6	11	16	21	26	31	36	41	46
	Applied load in thousands of pounds.								
Section 1. For location see drawing of Specimen "C"									
A.	0	9	14	17	13	10	7	2	-2
B.	0	9	13	13	10	6	0	-2	-5
C.	0	5	8	7	3	0	-3	-9	-11
D.	0	8	10	9	5	0	-5	-10	-12
E.	0	8	10	8	4	0	-5	-11	-14
F.	0	8	10	8	2	-1	-8	-12	-16
G.	0	8	10	8	2	-1	-8	-12	-17
H.	0	8	11	9	2	-2	-10	-13	-18
K.	0	5	8	6	1	-2	-9	-13	-18
Section 2.									
A.	0	8	10	10	8	4	0	-5	-8
B.	0	9	10	10	8	3	-1	-5	-9
C.	0	8	10	10	8	3	-1	-7	-10
D.	0	10	13	13	11	9	5	0	-2
E.	0	8	10	10	9	5	1	-4	-9
F.	0	9	12	12	10	8	2	-2	-4
G.	0	8	10	10	7	2	-2	-7	-10
H.	0	8	11	10	9	5	0	-2	-6
K.	0	8	11	10	8	3	0	-6	-10
Section 3.									
A.	0	6	8	3	0	-4	-10	-14	-19
B.	0	7	8	6	0	-2	-8	-12	-17
C.	0	7	8	5	1	-2	-8	-13	-17
D.	0	6	8	6	2	-1	-7	-11	-15
E.	0	5	7	5	2	-2	-8	-11	-16
F.	0	7	9	8	2	-2	-8	-13	-18
G.	0	3	3	0	-4	-10	-16	-21	-27
H.	0	3	2	-1	-7	-10	-18	-24	-30
K.	0	3	2	-1	-7	-10	-18	-24	-30

The above readings are given in one-hundred-thousandths of an inch. The minus sign denotes compression at the location shown.

....

Summary of Readings, Test "A", Specimen "C".

Section.	6	11	16	21	26	31	36	41	46
	Applied Load in thousands of pounds.								
1	0	154	298	419	531	627	720	818	909
2	0	57	93	135	190	239	296	347	394
3	0	39	57	73	98	133	157	187	211

The figures given represent the sum of nine readings taken at the one section on the two outer plates, and are thus approximately eighteen times the actual average deflection at that section.

Readings Recorded in Test "A", Specimen "C". Face No. 2.

Position of Instr't.	6	11	16	21	26	31	36	41	46
	Applied Load in thousands of pounds.								
Section 1. For location see drawing of Specimen "C".									
A.	0	10	20	36	50	65	83	98	115
B.	0	10	23	40	59	76	90	108	121
C.	0	10	28	42	62	77	92	110	125
D.	0	10	24	40	55	73	90	102	117
E.	0	10	20	37	56	66	80	96	110
F.	0	9	20	35	50	66	81	96	110
G.	0	10	23	37	52	68	86	100	112
H.	0	10	20	37	51	67	84	98	111
K.	0	7	16	30	44	59	75	90	101
Section 2.									
A.	0	-1	0	6	14	25	36	47	56
B.	0	-2	0	7	15	24	34	45	54
C.	0	-2	0	3	10	20	31	41	50
D.	0	-2	0	6	14	23	34	43	51
E.	0	-4	-3	1	10	20	29	39	46
F.	0	-2	0	7	14	23	34	46	55
G.	0	-2	-1	3	11	20	31	40	50
H.	0	-2	0	4	12	21	33	43	51
K.	0	-2	0	3	12	21	30	41	49
Section 3.									
A.	0	-3	-3	0	9	16	24	33	40
B.	0	-1	1	8	14	23	31	40	48
C.	0	-1	0	4	11	20	30	37	45
D.	0	-1	0	4	14	20	30	38	46
E.	0	-1	0	5	11	21	30	36	47
F.	0	0	1	7	14	20	30	39	46
G.	0	0	1	6	12	20	29	36	43
H.	0	-2	-1	2	10	17	24	32	40
K.	0	-1	0	4	11	19	28	35	42

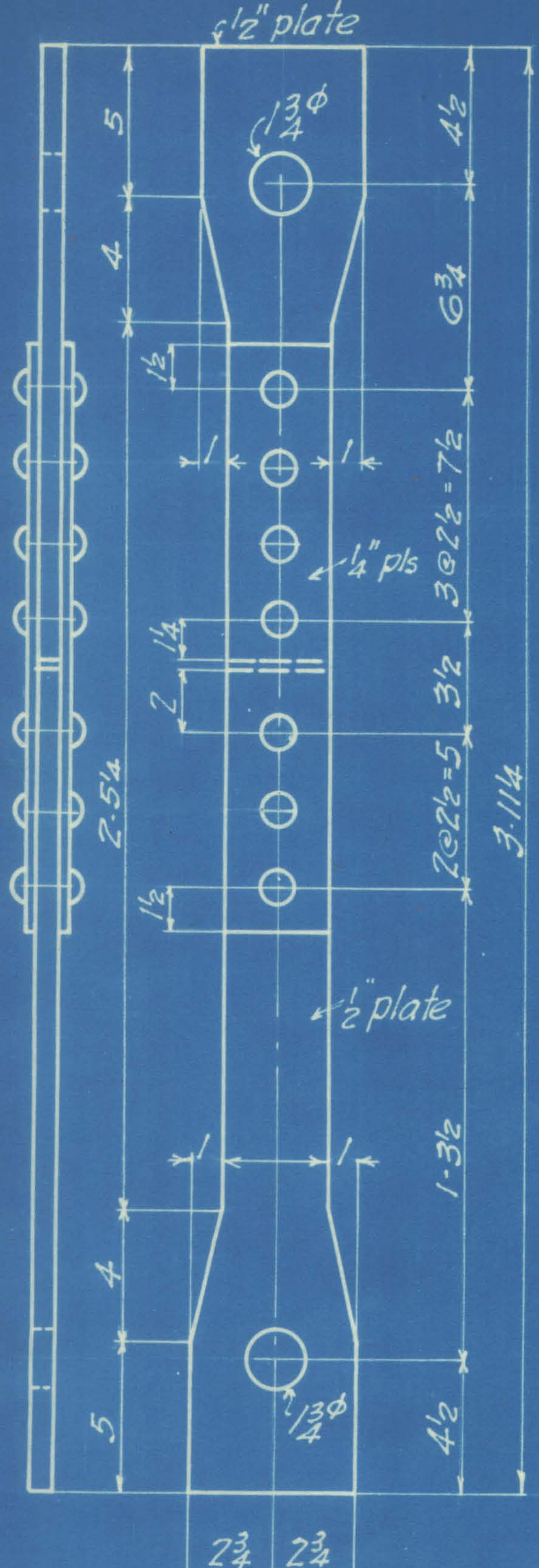
The above readings are given in one-hundred-thousandths of an inch. The minus sign denotes compression at the location shown.

....

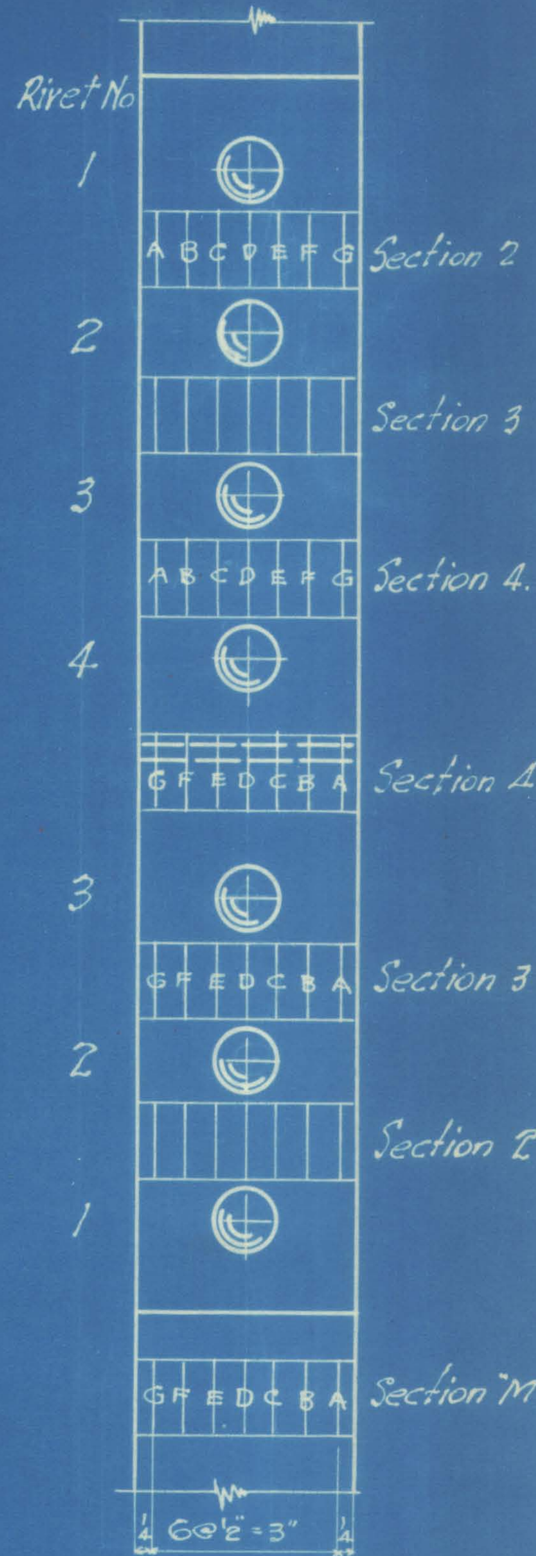
Summary of Distribution of Load between rivets for Specimen "C".

Row Number.	11	16	21	26	31	36	41	46
	Actual Load on joint - thousands of pounds.							
1	63.0	68.7	67.7	65.3	61.8	58.9	57.6	56.7
2	11.7	12.1	14.8	17.3	16.9	19.3	19.5	20.1
3	25.3	19.2	17.4	18.4	21.3	21.8	22.9	23.2

The figures given are percentages of the total actual load indicated.

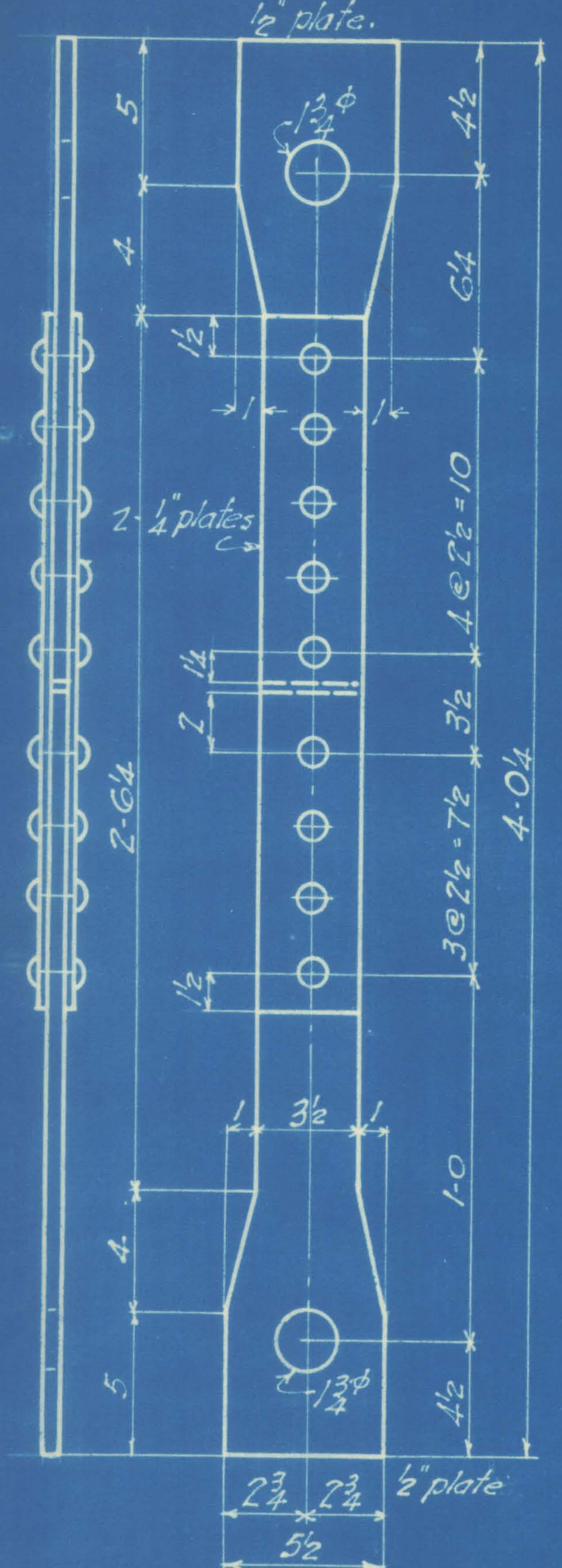


DETAIL OF SPECIMEN "A"



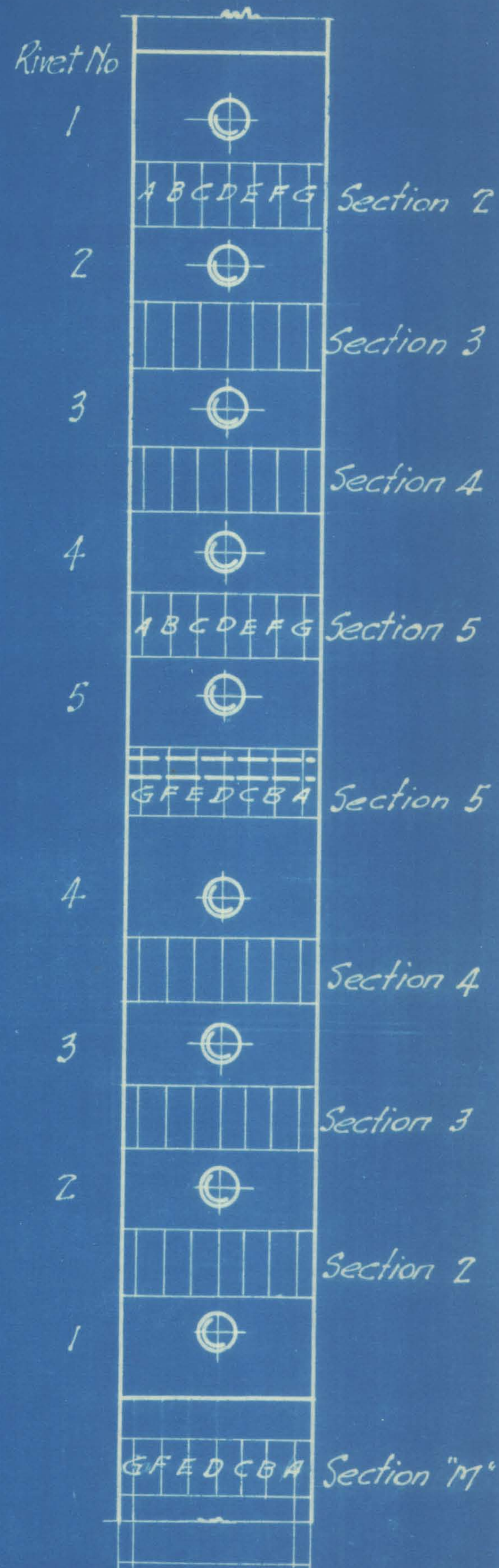
Extensometer Positions.

Rivets : 3/4 φ
Holes : drill 13/16 φ except shown.

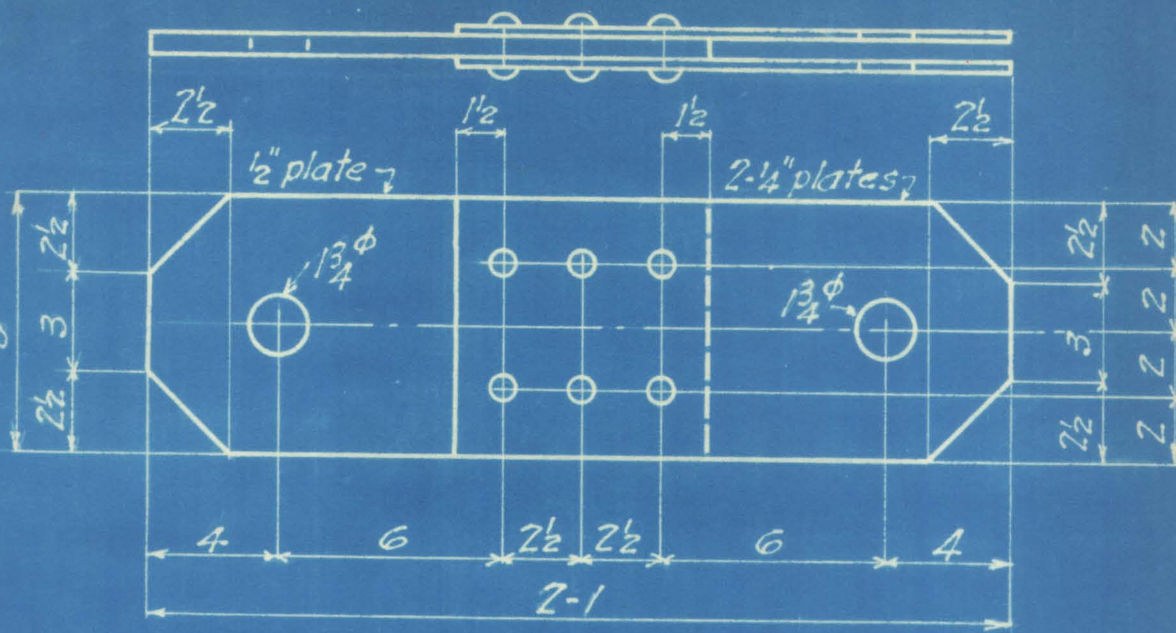


DETAILS OF SPECIMEN "B"

Rivets ¾ φ. Holes - drill 13/16 φ except noted.

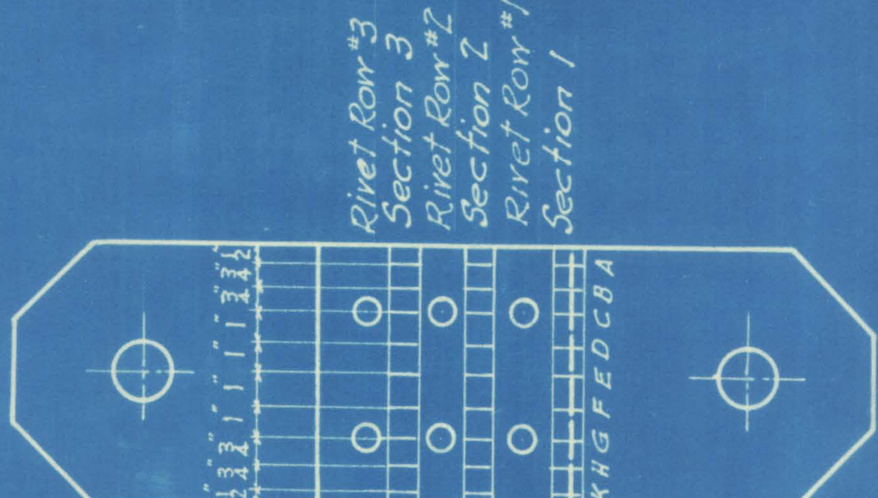


Extensometer Positions

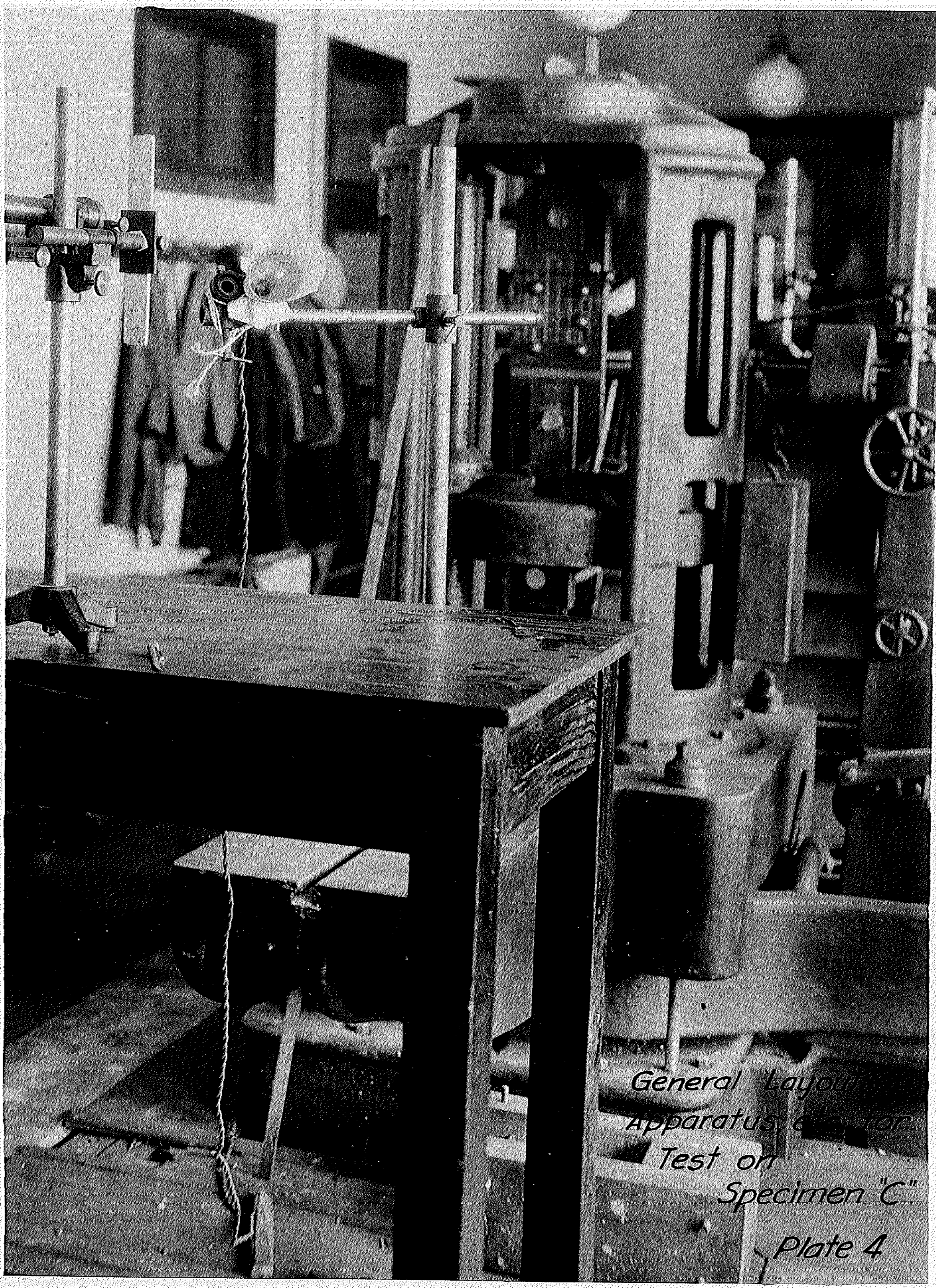


DETAILS FOR SPECIMEN "C" - 6 RIVETS.

Rivets - $\frac{3}{4}\phi$. Holes: drill $\frac{13}{16}\phi$ except as shown.
Surfaces in contact in joint to be planed.

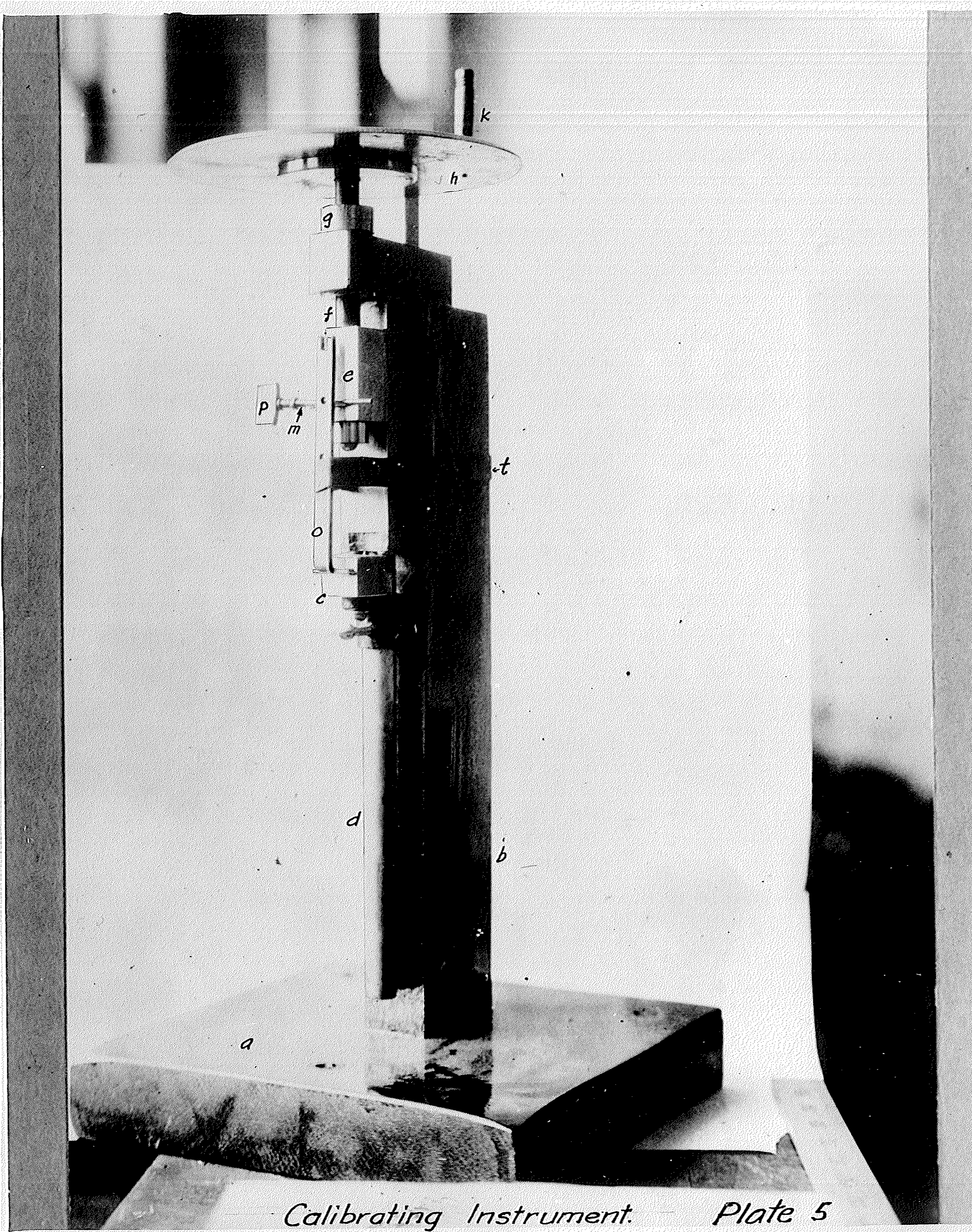


Drawing of Specimen "C" to show positions of extensometer for readings taken.



General Layout of Apparatus, etc. for Test on Specimen "C".

Plate 4



Calibrating Instrument. Plate 5

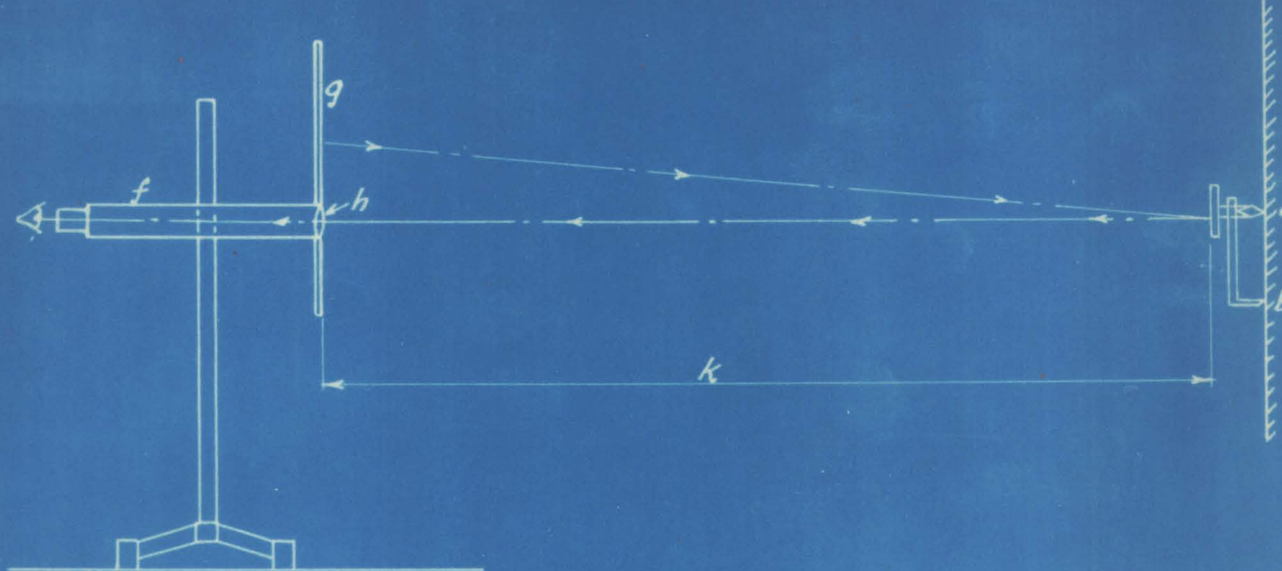
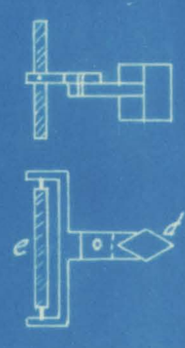


DIAGRAM SHOWING ARRANGEMENT FOR OBSERVATIONS WITH EXTENSOMETER.

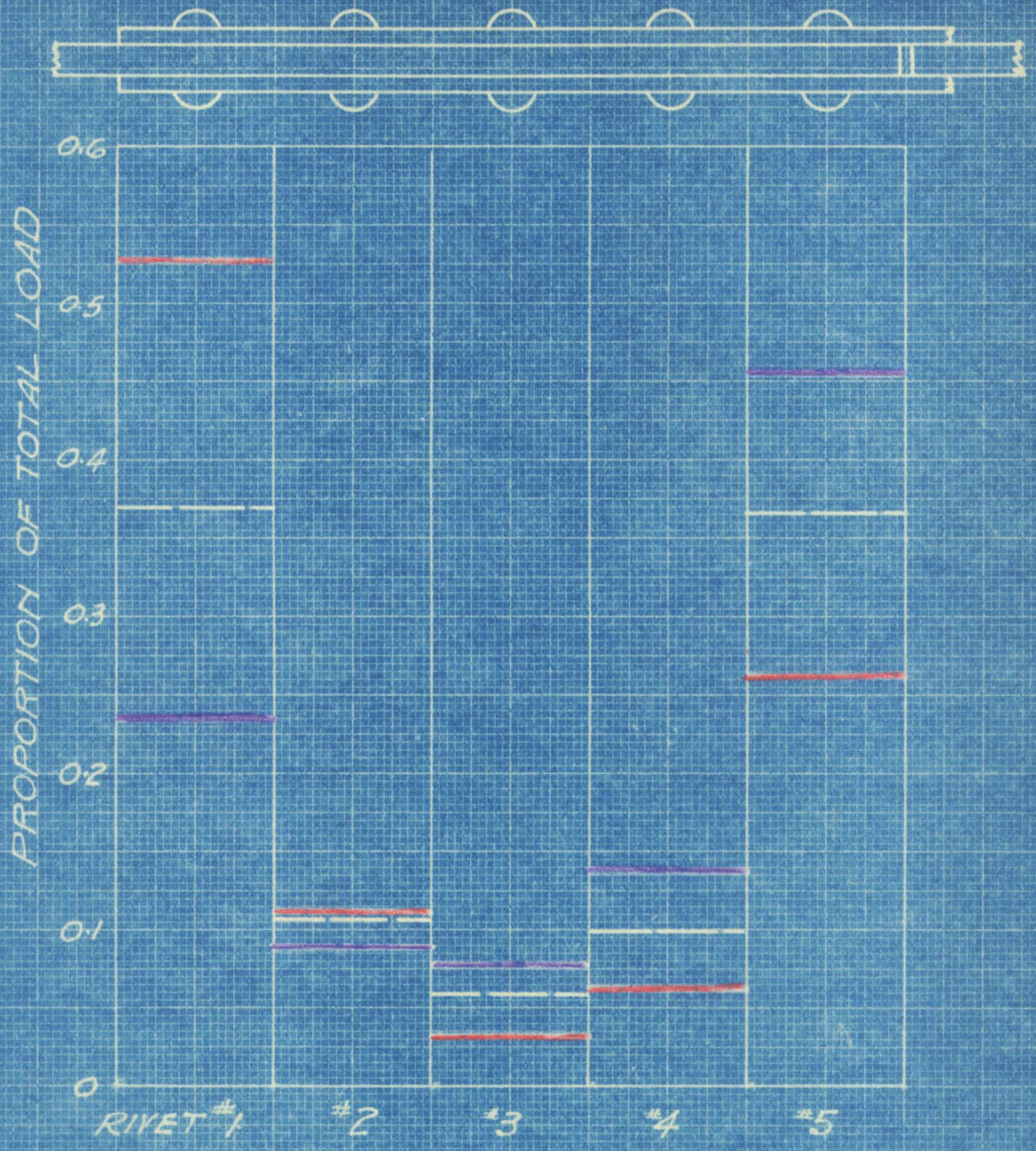
DISTANCE PIECE (c)



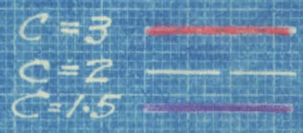
MIRROR

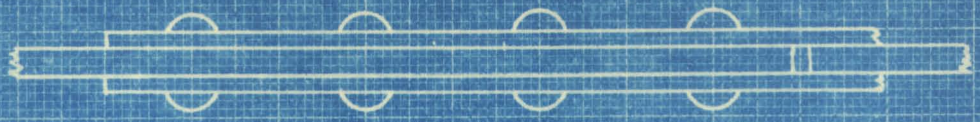
EXTENSOMETER DETAILS

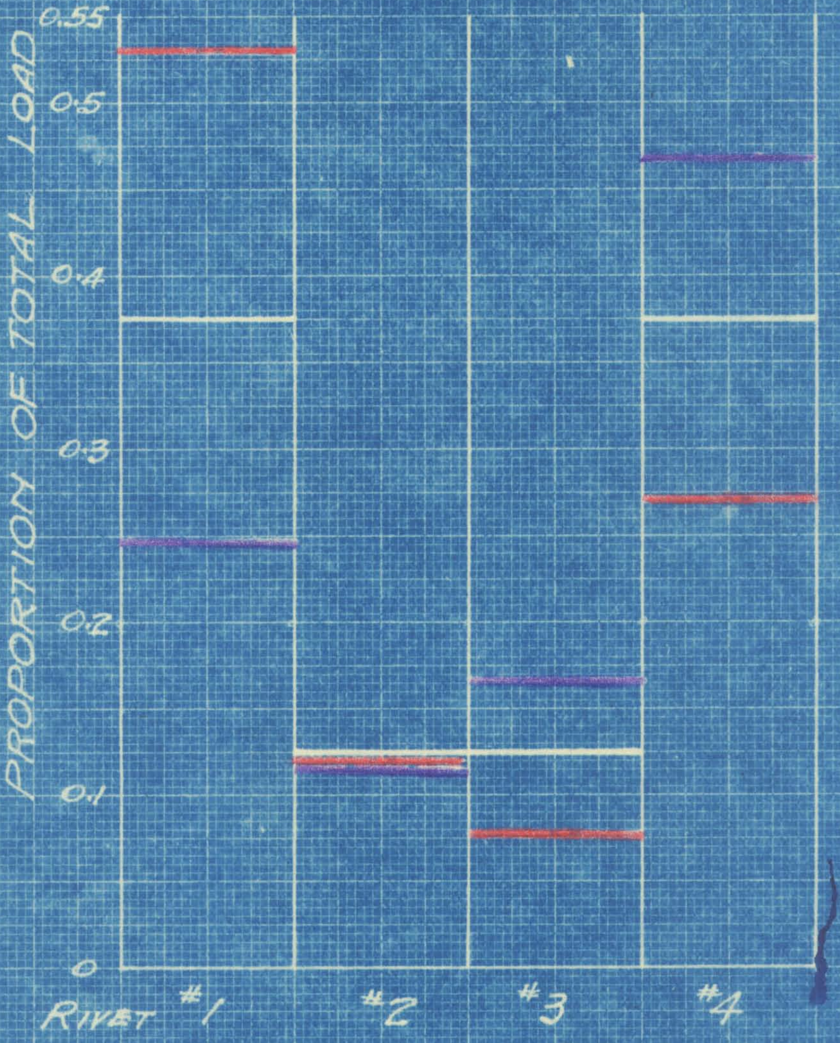
PLATE 6



THEORETICAL DIVISION OF LOAD
BETWEEN FIVE RIVETS
FOR "K" = 1.0

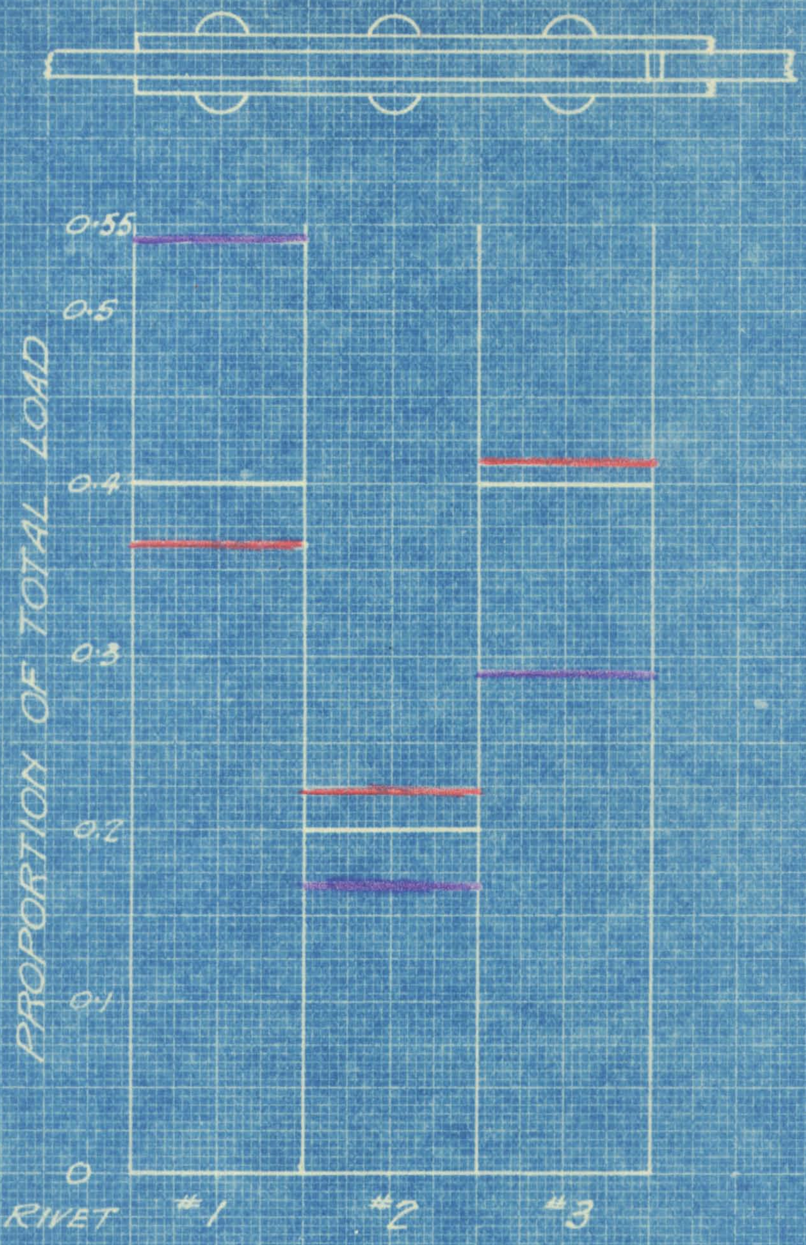






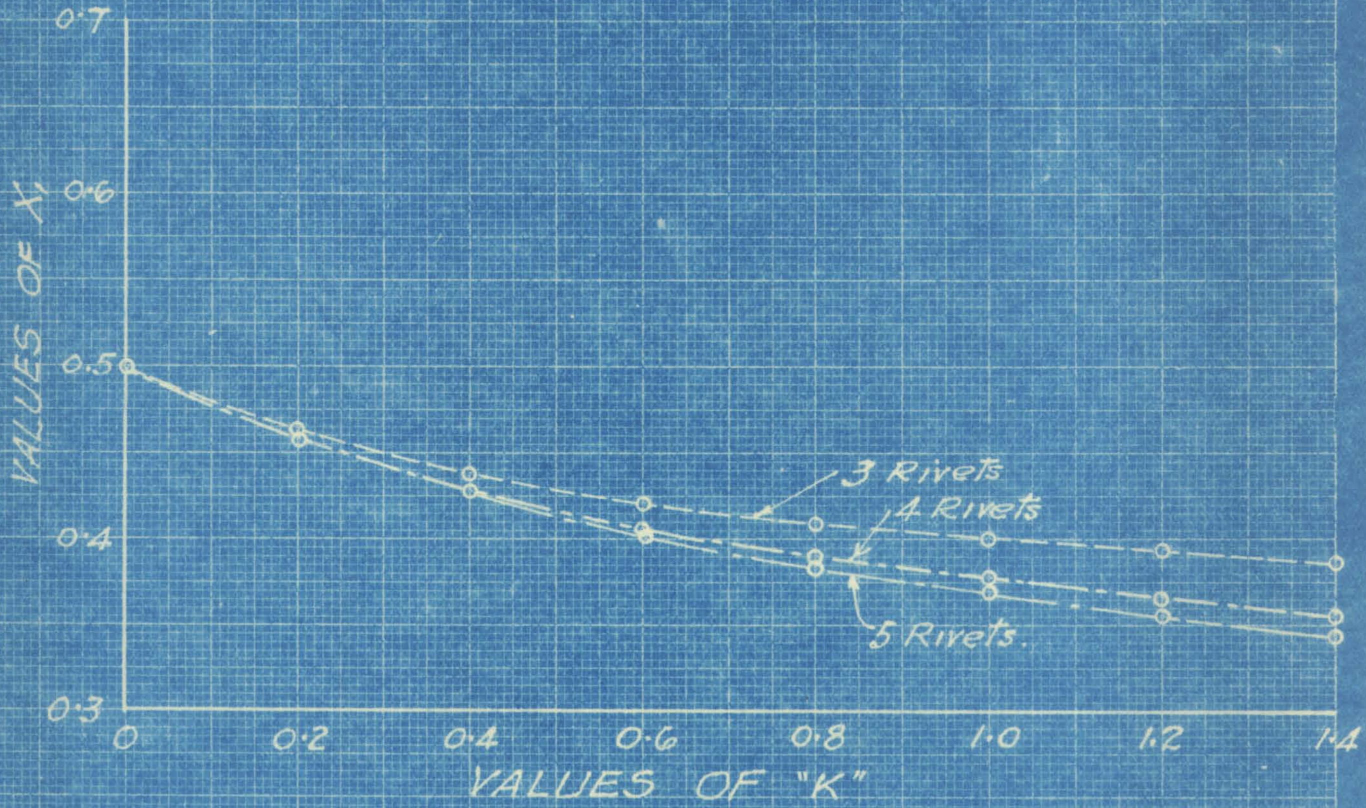
THEORETICAL DIVISION OF LOAD
BETWEEN FOUR RIVETS.
FOR "K"="1"

C = 1.5
C = 2.0
C = 3.0

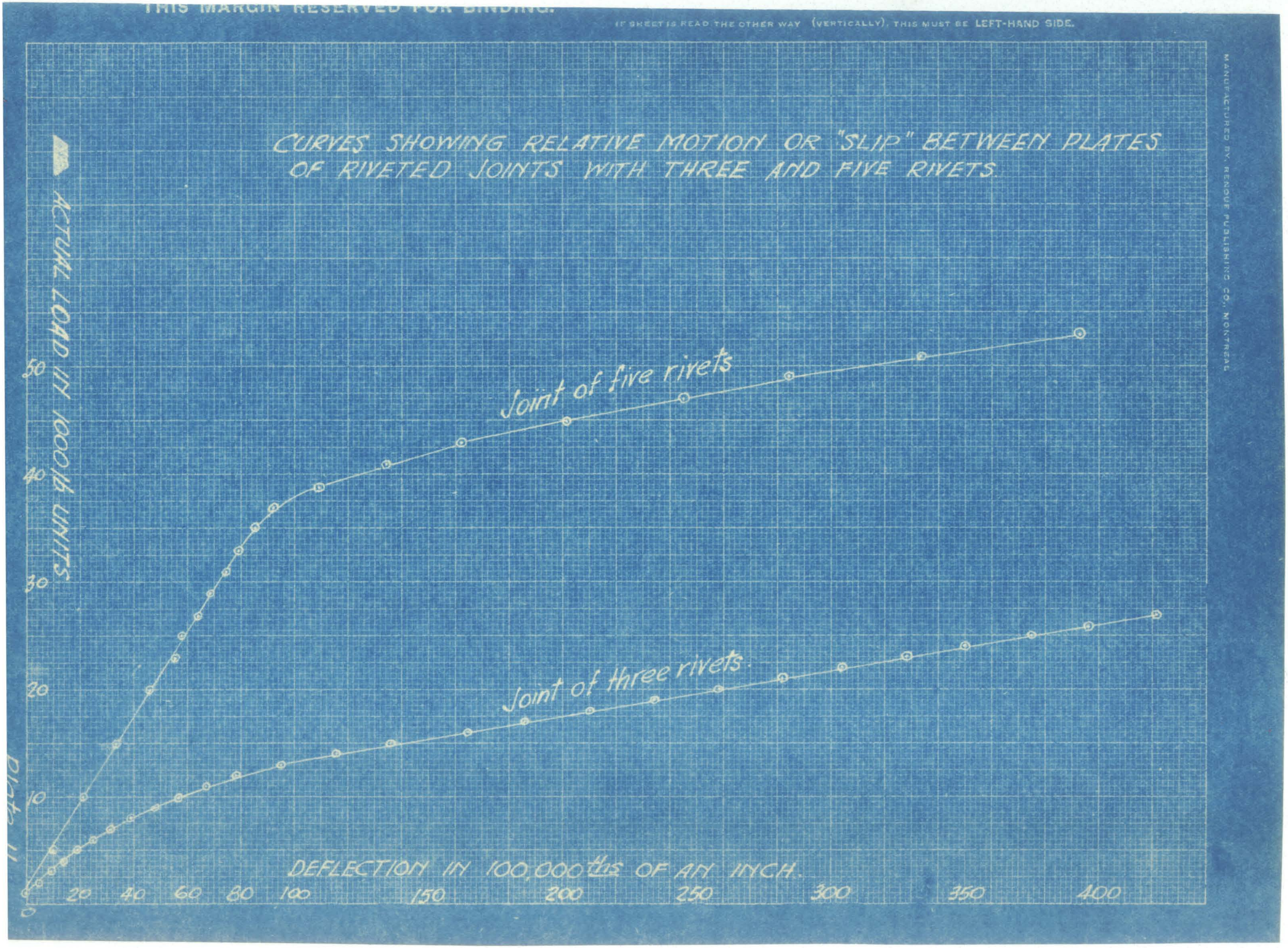


THEORETICAL DIVISION OF LOAD
BETWEEN THREE RIVETS.
FOR "K" = 1.

C = 3
C = 2
C = 1.5



CURVES SHOWING RELATION BETWEEN X, and K
FOR THREE, FOUR, and FIVE RIVETS.
WHERE C = 2.



THIS MARGIN RESERVED FOR BINDING.
IF SHEET IS READ THE OTHER WAY (VERTICALLY), THIS MUST BE LEFT-HAND SIDE.
MANUFACTURED BY, RENOUF PUBLISHING CO., MONTREAL
CURVES SHOWING RELATIVE MOTION OR "SLIP" BETWEEN PLATES
OF RIVETED JOINTS WITH THREE AND FIVE RIVETS.
ACTUAL LOAD IN 1000 lb UNITS
Joint of five rivets
Joint of three rivets.
DEFLECTION IN 100,000ths OF AN INCH.
0
10
20
30
40
50
20
40
60
80
100
150
200
250
300
350
400
Plate 11

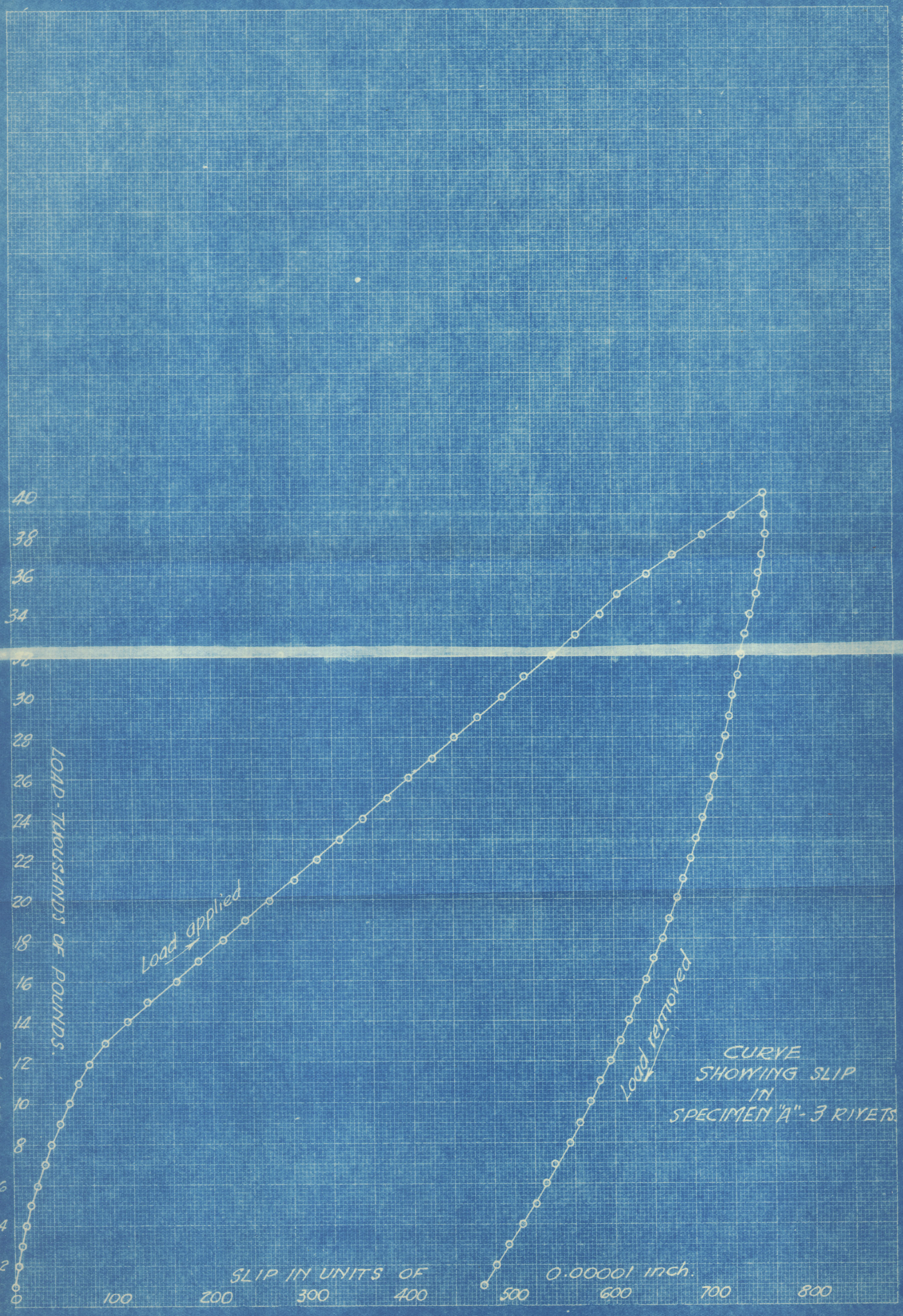
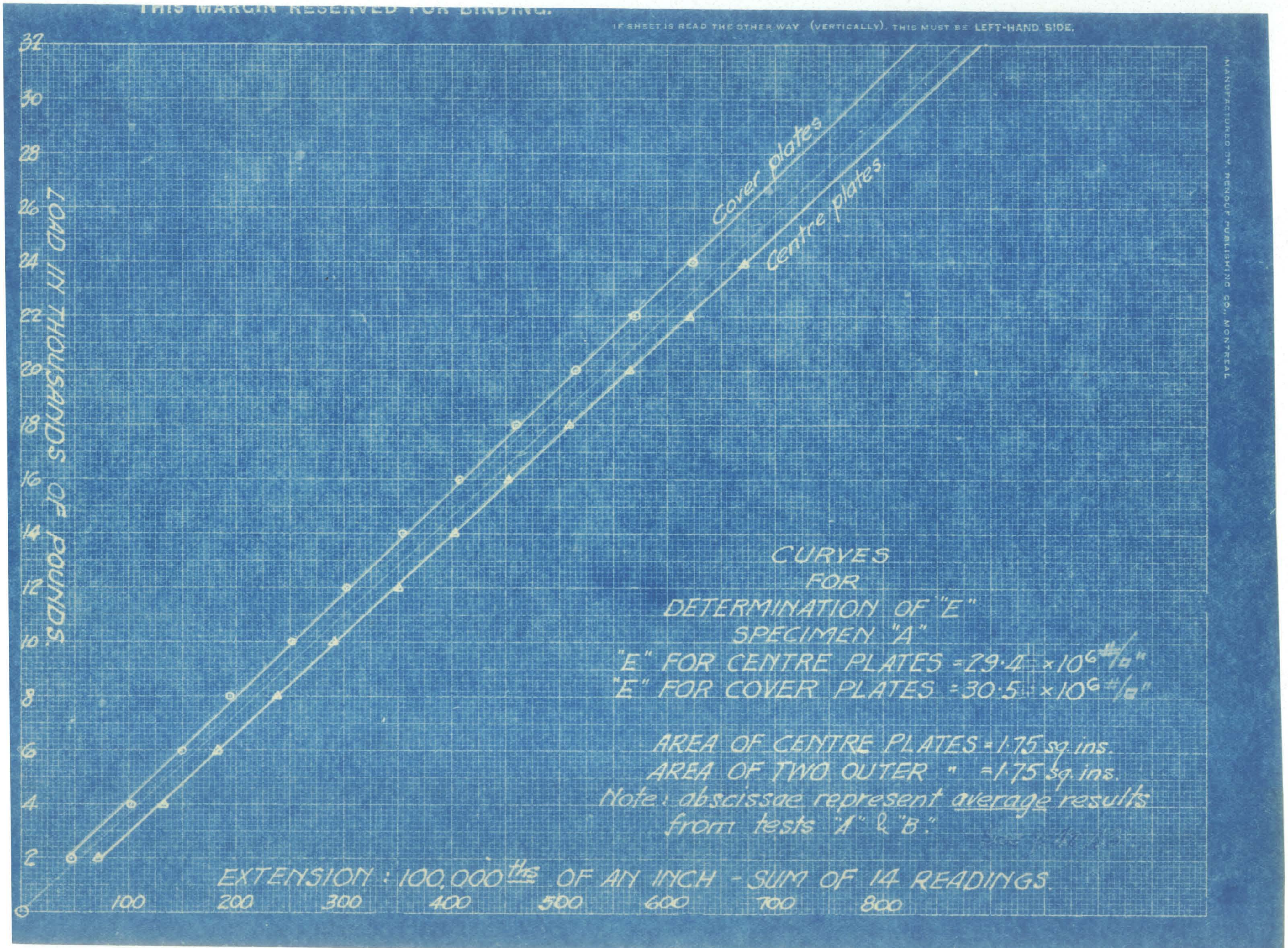
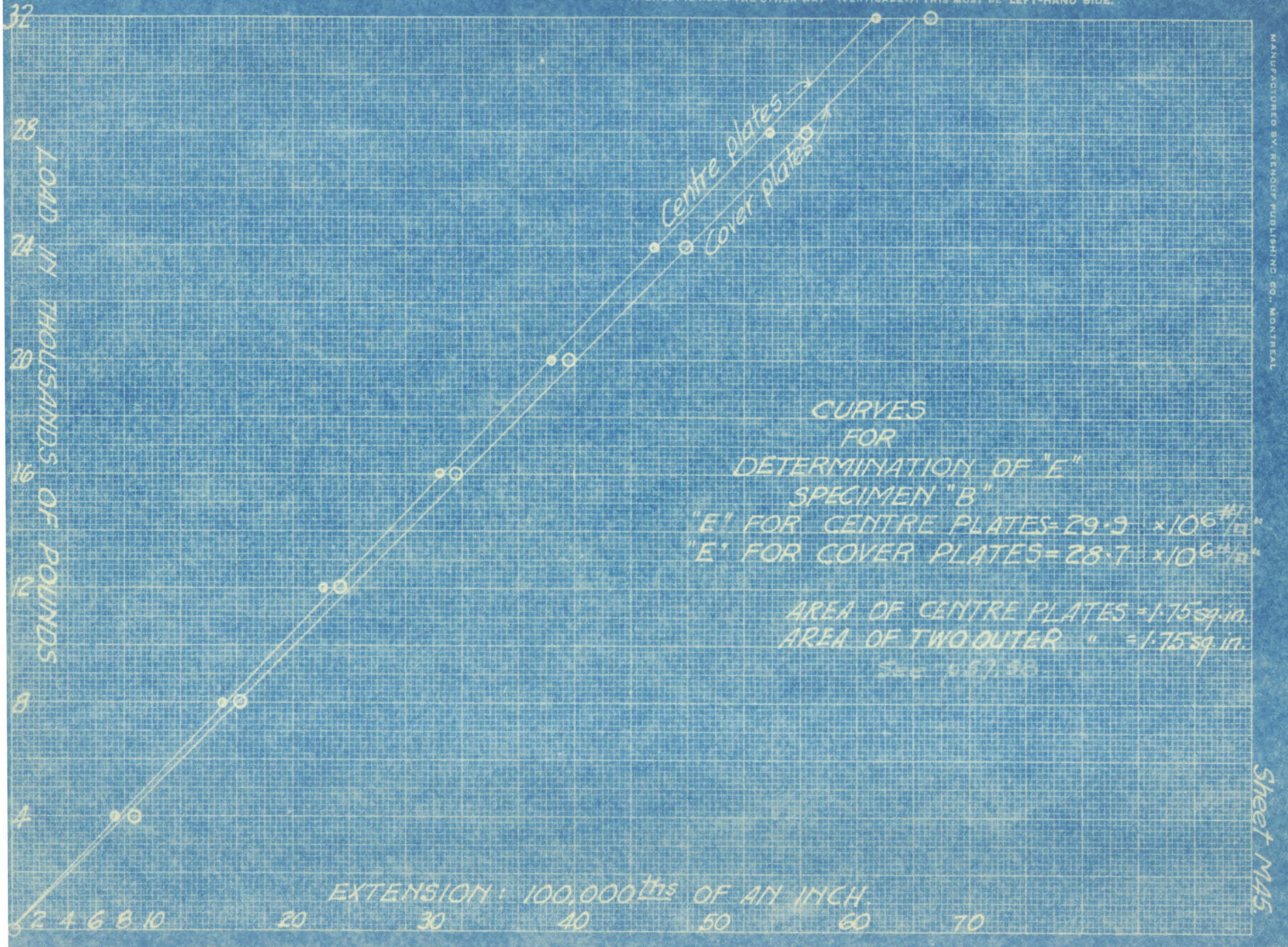


Plate 12

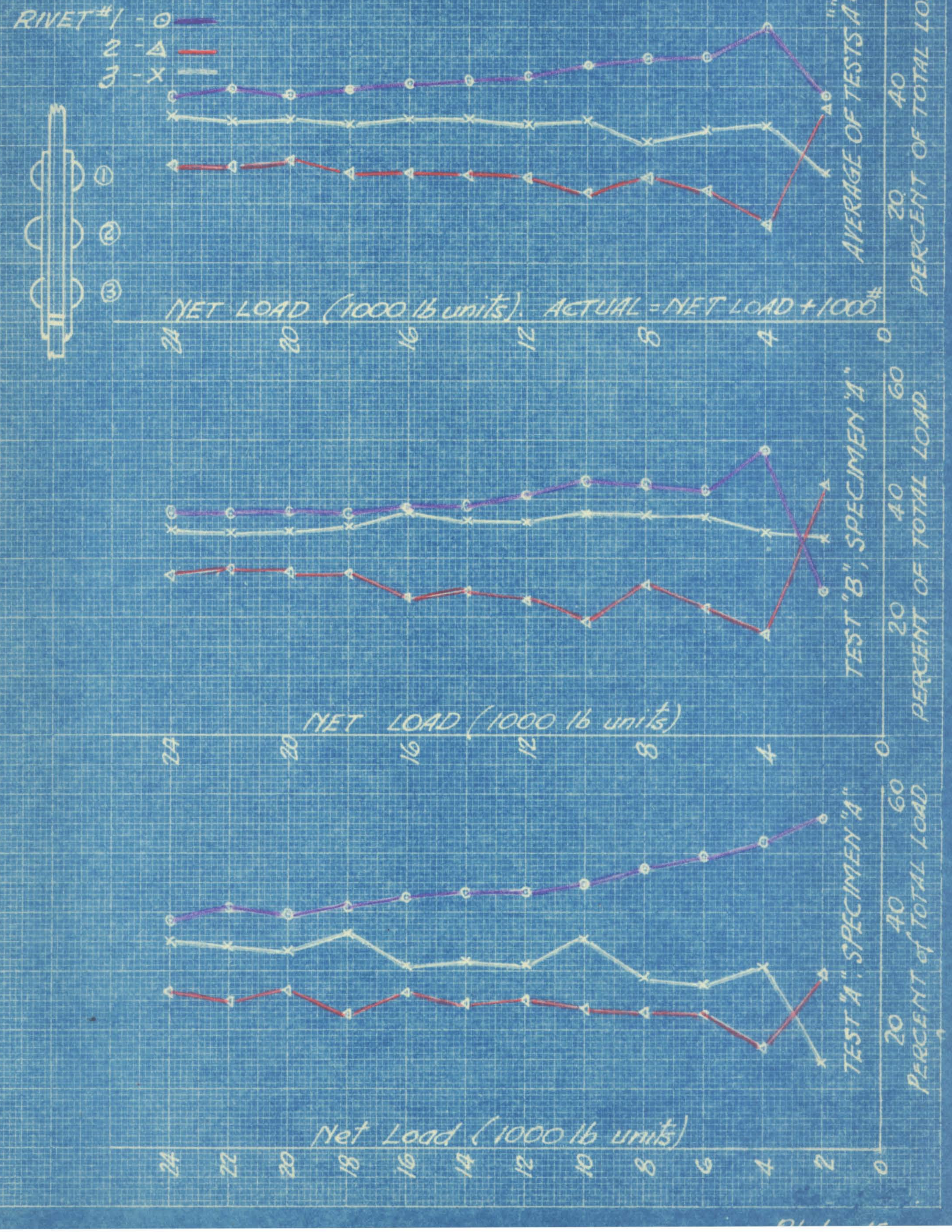
MANUFACTURED BY RENOUF PUBLISHING CO. MONTREAL


THIS MARGIN RESERVED FOR BINDING.
IF SHEET IS READ THE OTHER WAY (VERTICALLY), THIS MUST BE LEFT-HAND SIDE.
MANUFACTURED BY RENOUF PUBLISHING CO., MONTREAL
LOAD IN THOUSANDS OF POUNDS.
32
30
28
26
24
22
20
18
16
14
12
10
8
6
4
2
0
Cover plates
Centre plates
CURVES
FOR
DETERMINATION OF "E"
SPECIMEN "A"
"E" FOR CENTRE PLATES = 29·4 × 10⁶ #/□"
"E" FOR COVER PLATES = 30·5 × 10⁶ #/□"
AREA OF CENTRE PLATES = 1·75 sq. ins.
AREA OF TWO OUTER " = 1·75 sq. ins.
Note: abscissae represent average results from tests "A" & "B".
EXTENSION: 100,000ths OF AN INCH - SUM OF 14 READINGS
100
200
300
400
500
600
700
800


LOAD IN THOUSANDS OF POUNDS
4
8
12
16
20
24
28
32
EXTENSION: 100,000ths OF AN INCH.
2 4 6 8 10
20
30
40
50
60
70
Centre plates
Cover plates
CURVES
FOR
DETERMINATION OF "E"
SPECIMEN "B"
"E" FOR CENTRE PLATES = 29·9 $\times 10^6$ #/□"
"E" FOR COVER PLATES = 28·7 $\times 10^6$ #/□"
AREA OF CENTRE PLATES = 1·75 sq. in.
AREA OF TWO OUTER " = 1·75 sq. in.

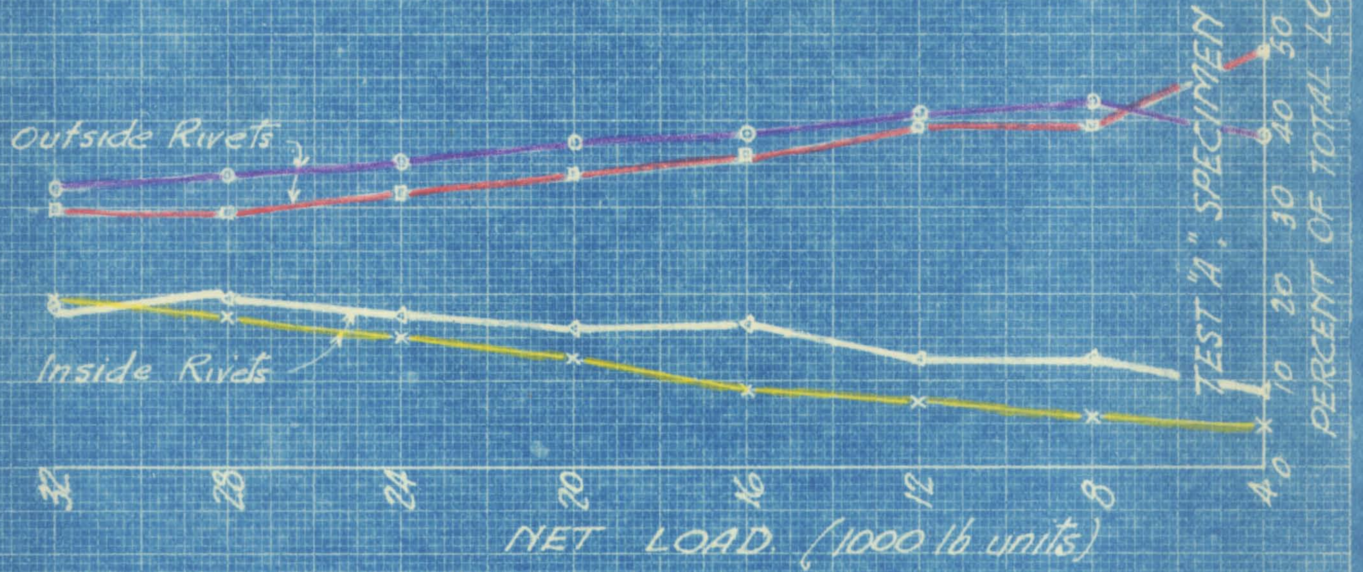
Sheet M45.

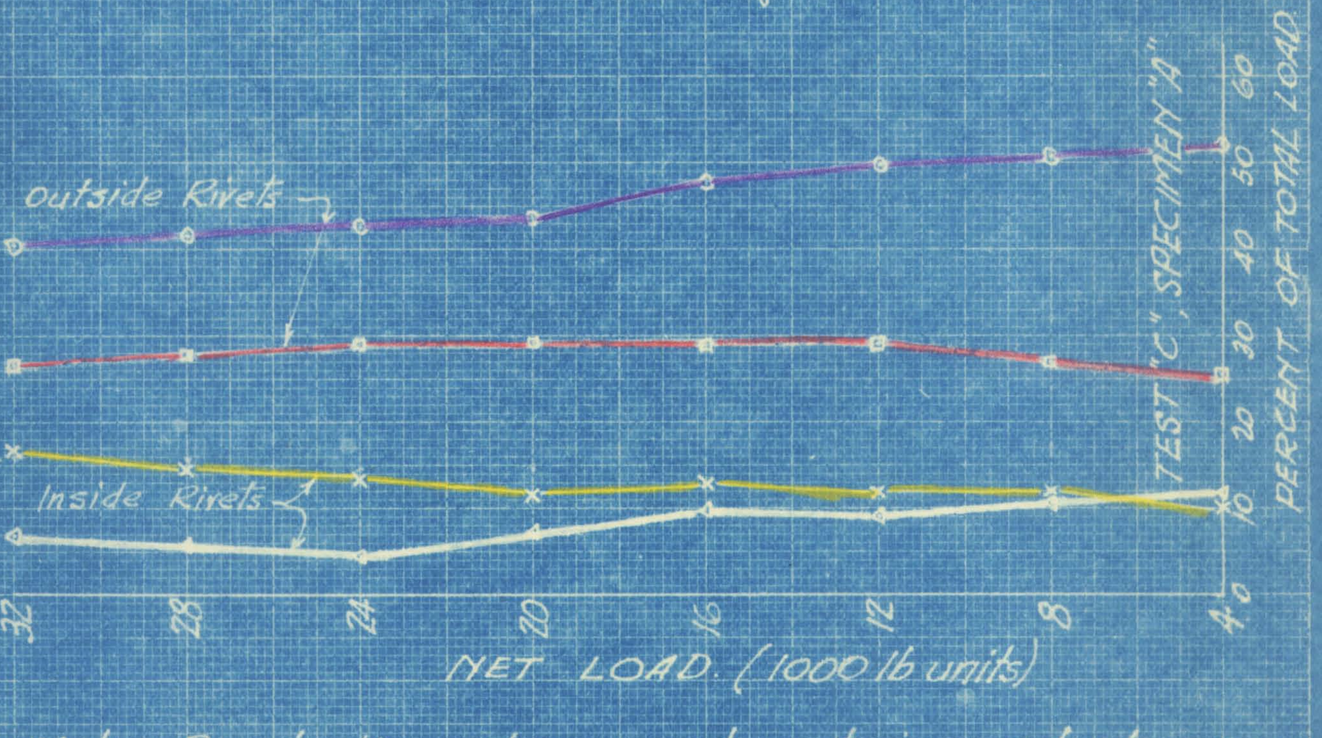
LOAD CURVES SHOWING DISTRIBUTION FOR VARIOUS LOADS
IN A RIVETED JOINT OF THREE RIVETS
TAKEN FROM TESTS AS NOTED.



LOAD CURVES SHOWING DISTRIBUTION FOR VARIOUS LOADS IN TWO RIVETED JOINTS OF FOUR RIVETS.
TAKEN FROM
TEST "C", SPECIMEN "A" and TEST "A", SPECIMEN "B"

Rivet #1 - ⊙ (purple)
2 - △ (white)
3 - X (yellow)
4 - ⊡ (red)





Note: Due to the extensometers being set at zero at an initial load of 1000 lb, the actual load is 1000# greater than the net load.

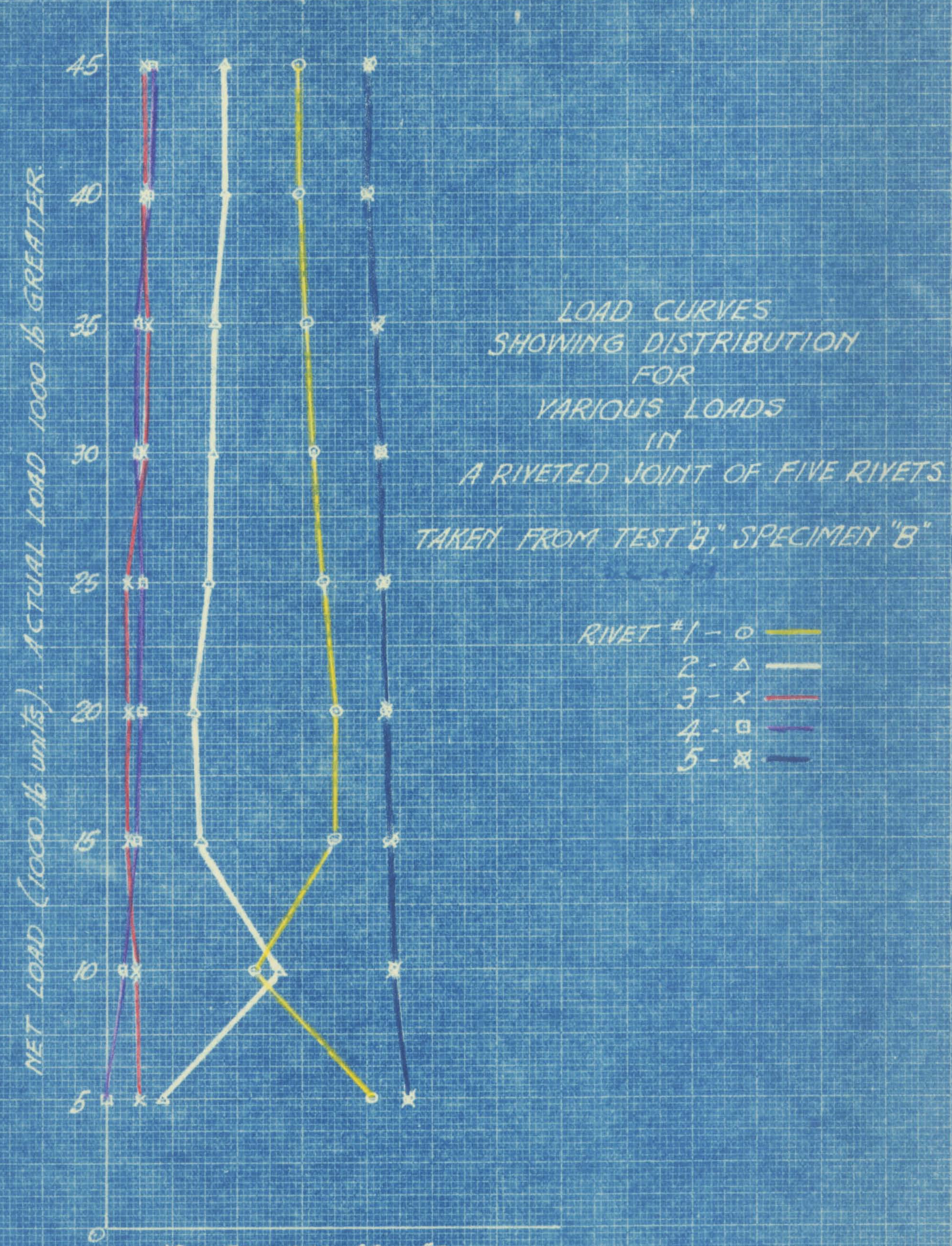
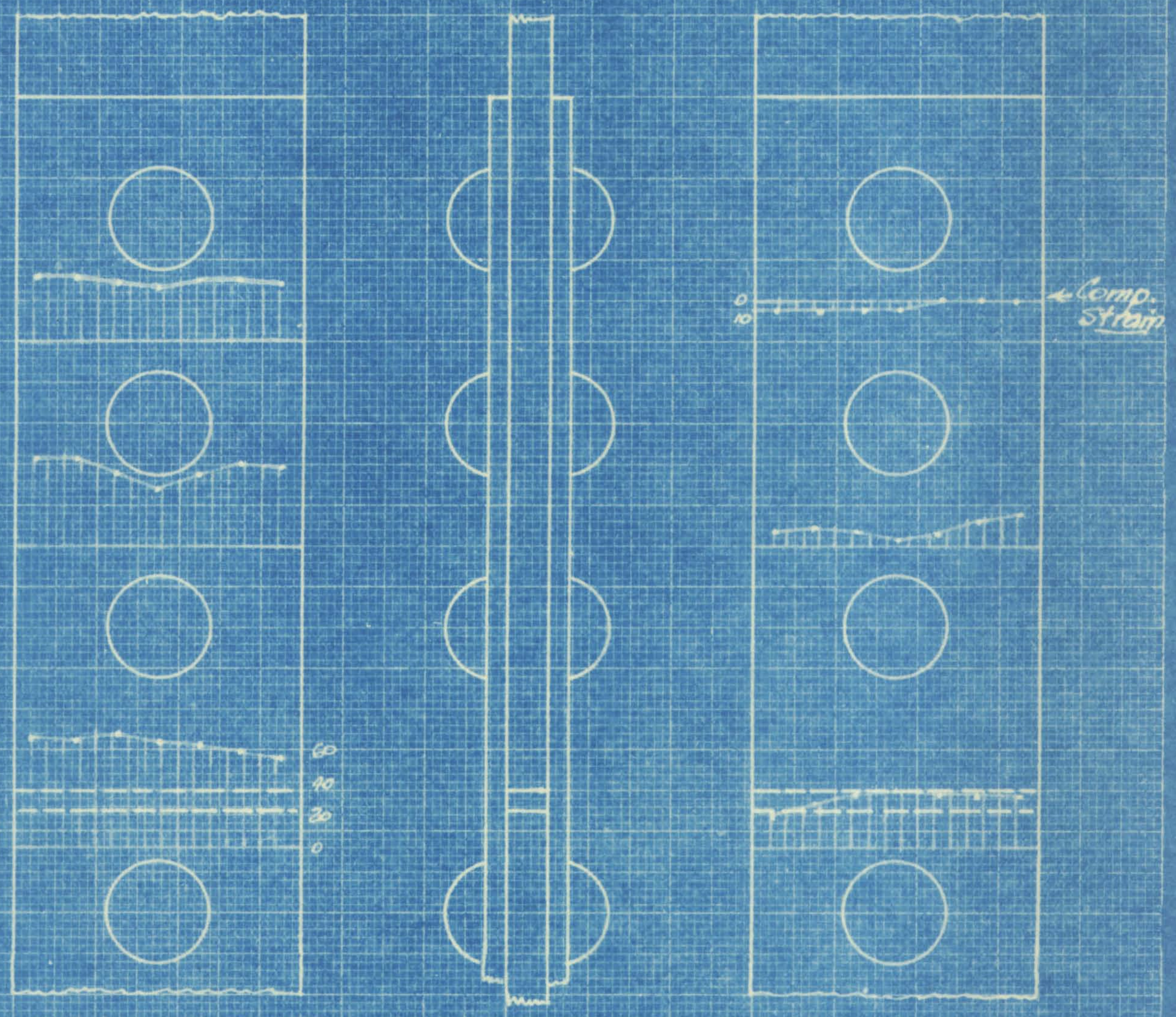

LOAD CURVES
SHOWING DISTRIBUTION
FOR
VARIOUS LOADS
IN
A RIVETED JOINT OF FIVE RIVETS
TAKEN FROM TEST "B," SPECIMEN "B"
RIVET #1 - ○
2 - △
3 - ×
4 - ▣
5 - ⊠
45
40
35
30
25
20
15
10
5
0
10 20 30 40 50
PERCENT OF TOTAL LOAD.
NET LOAD (1000 lb units). ACTUAL LOAD 1000 lb GREATER.

Plate 17



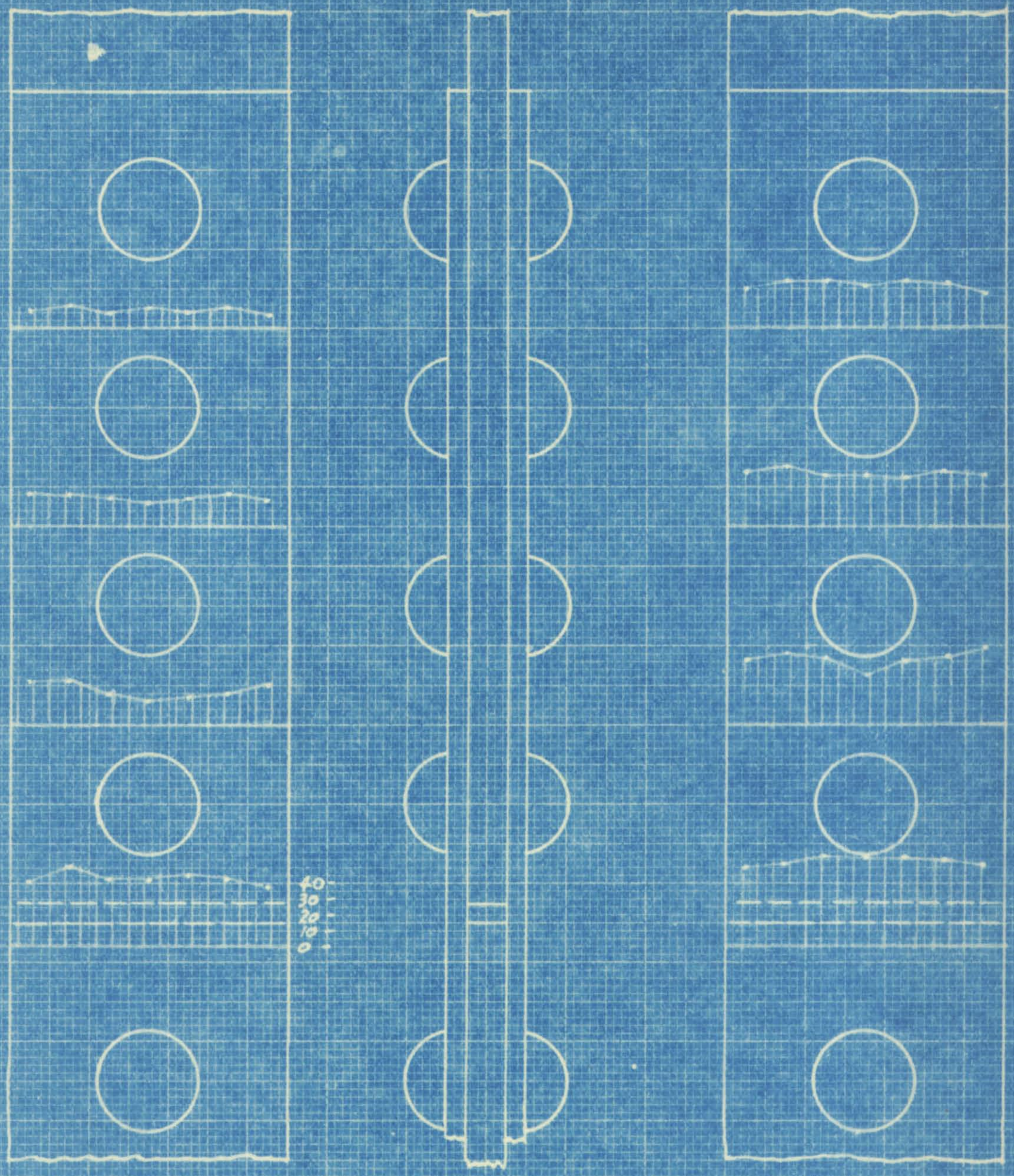
Face No. 1.
From Set No. 1.

Face No. 2.
From Set No. 2.

THE DISTRIBUTION OF STRAIN IN COVER PLATES.
SPECIMEN "A" — LOAD 25000#.
3-RIVET SECTION

Each point plotted represents the average of Tests A & B.

Scales: Outlines 0 1 2 3 inches
Strains: 30 – 20 – 10 – 0 – in units of 0.00001 inch



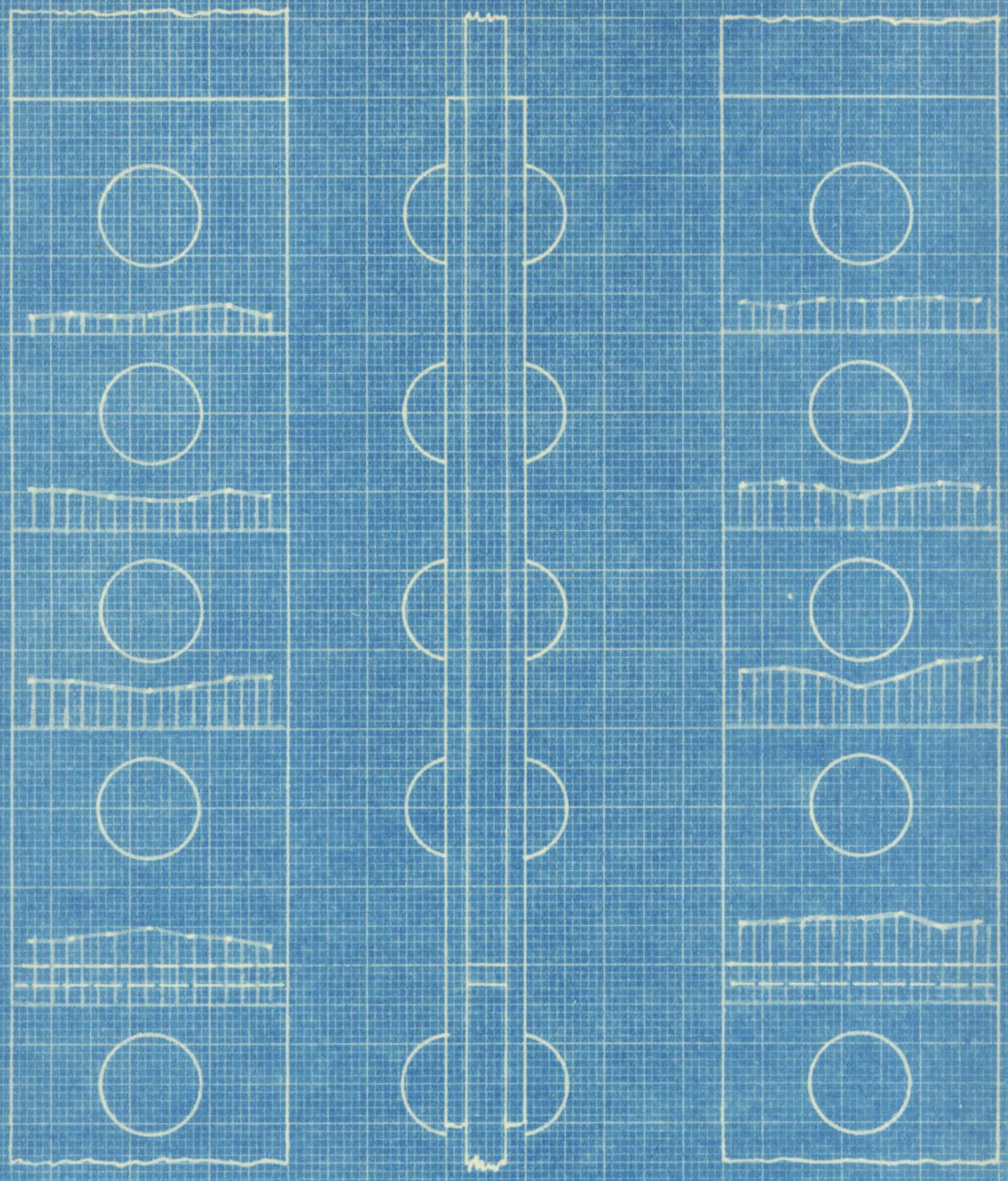
Face No 1
From Set No 1

Face No 2
From Set No 2

THE DISTRIBUTION OF STRAIN IN COVER PLATES.
SPECIMEN "A" - 4-RIVET SECTION
LOAD - 25000#

Scales: Outlines 0 1 2 3 inches
Strains 30 – 20 – 10 – 0 – in units of 0.00001 inch.

Taken From Test "C", Specimen "A"



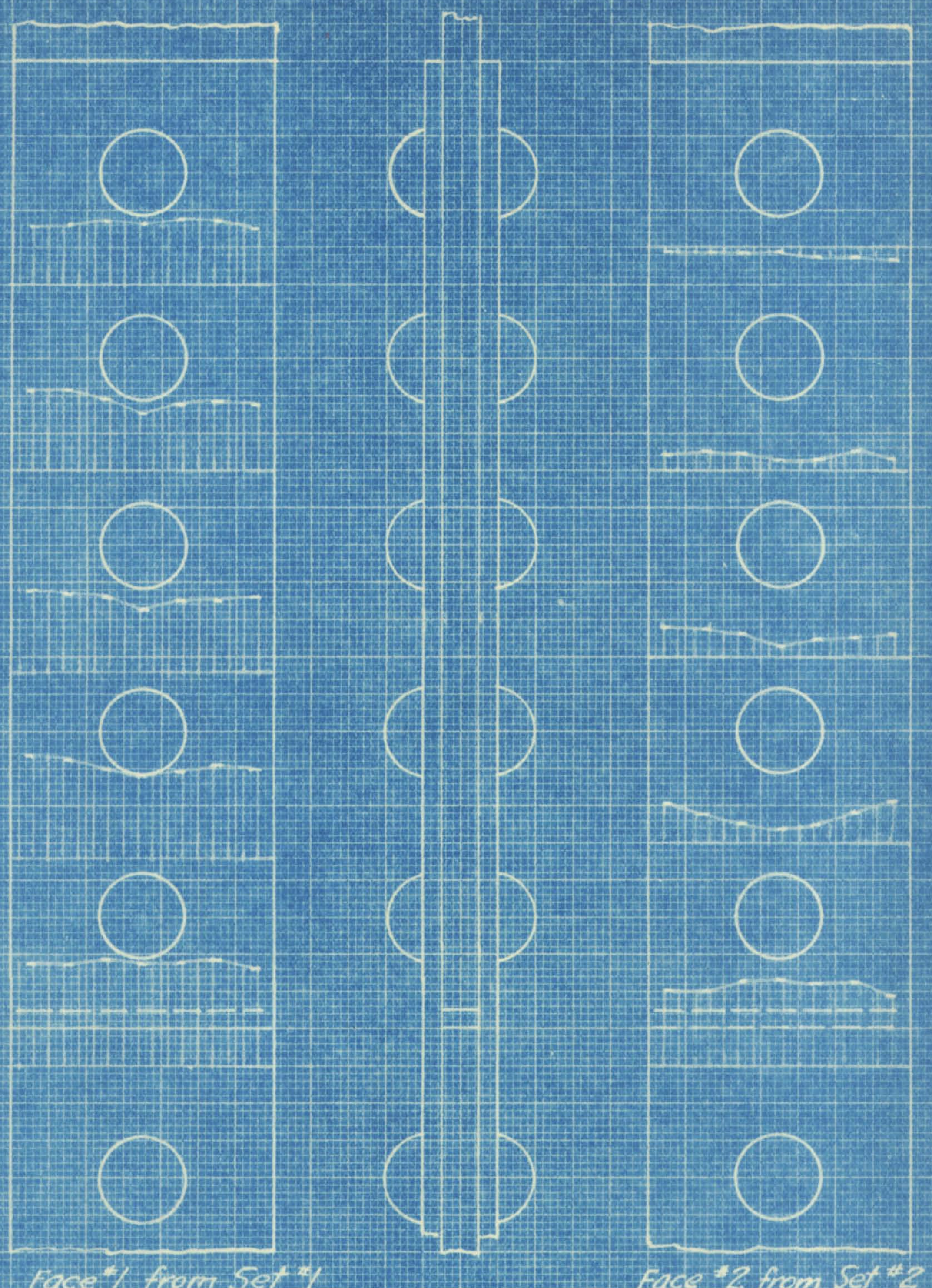
Face №1
From Set №1

Face №2
From Set №2

THE DISTRIBUTION OF STRAIN IN COVER PLATES
SPECIMEN "B" - 4-RIVET SECTION
LOAD - 25,000#

Scales: Outlines 0 1 2 3 inches
Strains 30- 20- 10- 0- in units of 0.00001 inch

Taken From Test "A" - Specimen "B"



Face #1 from Set #1 Face #2 from Set #2

THE DISTRIBUTION OF STRAIN IN COVER PLATES.
SPECIMEN "B" - 5-RIVET SECTION.
LOAD - 31000#

Scales: Outlines 0 1 2 3 inches.
Strains 30 20 10 0 in units of 0.00001 inch

Taken from Test "B"-Specimen "B"

-- A P P E N D I X --

DETERMINATION OF SLIP RESISTANCE IN RIVETED AND BOLTED JOINTS.

Professor W. Weibull. (See "TEKNISK TIDSKRIFT", Jan. 12, 1924)

Translated for C. W. Carry, University of Manitoba, Dec. 1927.

Part 1.

In modern practice it is usually admitted that a stress transmitted from one plate to another by a rivet gives rise to shear stress in the rivet shank. In this case it must be assumed that the rivet shaft is in direct contact with the hole wall; but with hot riveting, this is not quite true. The general opinion is that the principal force keeping the plates in place is the friction between the plates due to the tension in the rivet shank, but in spite of this, design calculations are made by bearing and shear. The two points of view lead to the same conclusion, namely, that the total cross-sectional area of the rivets is an important factor in the strength of riveted joints. This conformity is quite superficial in nature, and the two points of view present different appreciations of the importance of the pressure on the hole wall, the strain in the weakened plate with one row of rivets, and the stresses in the rivet shank.

A different idea, which seems to be of more importance, uses the old formulae and disregards the force of friction in the contact area. It has been suggested by Prof. Forsell, (T.T.V.V. 1912, pamphlet 12) that on account of poor rivet driving the tension stresses in the rivet shank and consequently the resistance to slip may drop to a very small value, and ultimately we must depend on the shear stress in the rivets. At the same time it should be pointed out that from the point of rigidity it would be better if a joint could be made in such a way that no sliding could occur, and further, that it is now in order to try to form some idea of the extent of the slip which normally occurs, and also of the action due to the slip.

Investigation of this problem has been conducted by Prof. Magnell, (T.T.V.V. 1912, pamphlet 10) working on an ordinary riveted joint, but it was very difficult to determine whether and when the rivet shaft started to carry load. One method of avoiding influence of the rivet shaft was used by Fremont (Revue de Metalurgie 1910-1, p.135-154), and referred to in the work mentioned in connection with Prof. Forssell. This method was to turn a riveted plate joint around the rivet shaft, and thus determine the moment necessary to start sliding. To accurately determine the resistance to slip, Fremont has supposed a uniform pressure on the whole contact area between the plates. However, the actual pressure is at its greatest close to the rivet and becomes smaller as the distance out from the rivet increases. Thus it may be possible that the values obtained by Fremont are not quite true. Further, it is not certain that the coefficient of friction in twisting is equal to that of parallel sliding, since the different stress distribution in the plates gives rise to unequal cross-contraction within them. Fremont's method is certainly limited to experiment with only one rivet since it is not possible to turn the plates if more than one rivet is used. To determine the slip resistance in multiple row riveting is impossible with this method. Still another method and a practical one to avoid the influence of the rivet shaft seems to me to dispense with the rivet shaft and compensate the internal tension forces in rivets with external compression forces on the head of the rivets as shown in fig. 1.

From an experimental point of view, this method gives distinct advantages. The slip resistance in each contact area can be considered as the product of the tension stress P in the rivet shaft, and the coefficient of friction μ. μ for the same material can be determined from accessible data , and may be settled by the designer himself. On the other hand the force P depends in considerable degree on the riveting work, and very often will avoid the control of the designer.

In ordinary riveted joints, this made very large differences in the resistance to slip. (In Fremont's measurements the resistance varied between 0 to 1300 kg/cm^2 in exactly similar joints). Thus the slip is dependent on the differences in P to the greatest extent. By separating P and μ we have an opportunity to find irregularities in which conceal the large variation of P in a real riveted joint. Then we can compute the coefficient of friction for a certain case with sufficient accuracy, and by means of a static inquiry into normal joints we may finally make the comparisons of data necessary to fix the value of P. This proposed method made it possible to vary the characteristics desired on a plate and to inquire into them in a simple manner; e.g. the polishing of the contact area by repeated sliding, treatment of the contact areas, the age of the joint, etc.,

The course of the investigation started with determining the coefficient of friction μ

for unequal conditions. With knowledge of its value, the resistance to slip in a riveted joint can immediately be determined from the usual formulae. By means of installing bolts we can now tell more exactly than before when the bolt starts to move on account of the large bearing stresses in the hole wall.

To determine the slip resistance in a riveted joint, it is necessary to know (in addition to μ) about the tension stresses in the rivet shaft resulting from ordinary riveting, and the inquiry about this must be made on ordinary joints. This point is treated in the second part of this paper.

Determining the frictional coefficient.

Arrangement of the experiment. The arrangement is shown in Fig. 2.

The required pressure on the heads of the rivets is obtained with the arm of a lever. The force is produced with the screw (a) and is multiplied 10 times on the head of the rivets with the lever. The screw (b) serves to adjust the arms of the lever. The force (P) is measured with a manometer designed by Prof. Kreuger, who had the kindness to place one of them at my disposal. The manometer consists of a calibrated membrane where the deflection could be read, greatly magnified on a graduated glass tube which contained mercury. This arrangement was very convenient to handle. The plates were subjected to tension in an Alfa tension testing machine. The displacement of the plates relative to each other was measured with an instrument graduated to 0.01 mm. A displacement of about 0.003 mm. could easily be estimated. The instrument was connected to one plate with the movable pin centred on one of the double plates as seen in fig. 3. By means of a block the lever arms were kept at a fixed distance from the double plates. This arrangement is shown in fig. 3. and 4. The force W. to apply tension to the plates was obtained from an electric motor giving a motion between the blocks of the machine of 1 mm. per minute. With large loads this drive was used up to half of the slip resistance with a speed of about 2 mm. per minute and after that the slower speed was used, thus keeping the load in equilibrium with the resistance while the measurements were made.

How experiments were carried out:

It was very easy to determine when sliding started. Just before, an elastic displacement of about 0.01 to 0.02 mm. took place, and this disappeared by removal of the force. In some cases, and especially in 15 mm. plates permanent set took place before sliding occurred, possibly on account of straightening out small bends in the plates. This usually occurred with the first application of tension after placing a new specimen in the blocks. The point where sliding starts was marked in all cases, usually by a little knock. As soon as the sliding started, the motor drive was released. The amount of displacement was usually about 0.1 mm. The measurements were made in such a way than when

$d_1 = 0$, (i.e. the centre plate without rivet hole) the whole series of measurements for joints with the same dimensions were carried through with different rivet pressures without new installing in the blocks, and without adjusting the plates to the original starting point.

The total displacement in this manner amounted to 1 or 2 mm. for each group of measurements. In the series with $d_1 = 0$, adjustment of the plates to the starting conditions was made as soon as the total sliding was found to be equal to the distance between the rivet and the hole wall in an ordinary riveted joint. In hot riveting it is supposed that this distance is 1 to 2% of the diameter of the rivet, which in this case would be 0.1 to 0.2 mm. In each case the load was reloaded till sliding re-occurred.

Results of Measurements:

The results of measurements on resistance to slip are given in Tables 1-5. The dimensions and loads stated in Table 5 will be referred to again. The following marks are used.

P - The total force of friction = pressure on the head of the rivets in kg.
W - Total slip resistance in kg.

$f = \frac{\pi d^2}{4} =$ area of the rivet hole in mm^2.

$\delta_n = P/f =$ unit force of friction in kg/mm^2.

$\delta_f = W/2f =$ unit slip resistance, or shear (rivet area) in kg/mm^2.

$\delta =$ value of the slip in 10^{-2} mm.

$\mu = \frac{W}{P} =$ coefficient of friction.

Tables 1, 2, 3, 4, and 5 follow.

In the series 1 - 3, only the contact area between the plates was rough. In order to investigate the influence of the contact area, experiments were made, some with machine oil between the contact areas, and some with those areas painted and dried for about 14 days. These results are given in Table 4. All those measurements where force P was applied directly before the test are denoted in the tables with "age = 0". To inquire what influence the "age" had on friction, tension tests were made after the pressure P had been acting for some time. These results are stated in Table 5. Part of the results are shown graphically in fig. 5 and 6. The marks on the different curves is in reference to the dimensions of the riveted joints; so that the noted dimensions are in the order S x d x d_2. The marks 10 x 10 x 13 thus means S = 10, d = 0, d_2 = 13. The curves for 10 x 6 x 6 and 10 x 13 x 13 are computed after the method of least squares under assumption of linear relation between P and W.

Discussion of the results;

The measured values of the resistance to slip in some series are quite without any order. Possibly this variation depends to some extent not on the variation of the coefficient of friction, but on the difficulty of exactly measuring the force W, which in the testing machine used was obtained with a beam counter-balance, the latter requiring to be brought in equilibrium with W. Particularly with large forces was it hard to obtain exact balance at the instant of slip. The average results plainly show some connection, nevertheless. Table 6 shows adopted corrected values for the slip resistance taken from the curves. The curve for 10 x 10 x 13 is thus corrected for a constant error of plus 100 kg.

A comparison between the curves $d_1 = 0$ and $d_1 = d_2$. Shows that for small pressures the force of frdction and the frictional coefficient remain constant.

With higher pressures the coefficient increases in those joints which have no hole in the centre plate. It may be that the material in the centre plate is pressed into the holes in the exterior plates, and on this account makes the sliding more difficult. This phenomenom does not occur when the centre plate has a hole. On the contrary, the coefficient of friction shows itself to advantage, depending on the pressure. In an actual joint we can thus calculate with a constant coefficient of friction, independent of the tension stresses in the rivet shaft.

For the materials tested, the coefficient of friction for wrought iron against wrought iron is 0.35.

Influence of the thickness of the plates:

The actual distribution of the pressure in the contact area between the plates depends on the thickness of the plates. In very thin plates the pressure is concentrated upon a relatively small area around the rivet shaft. In thicker plates the pressure becomes more uniformly distributed over the whole contact area.

Now, if the frictional coefficient depended on the unit pressure, the plate thickness would influence the average value of the coefficient. The measurements show that the coefficient is constant for all pressures usually met with, and that the thickness of the plates has no influence on the calculation of the coefficient.

Influence of the diameter of the rivet:

The measurements show that within the limit of observation error, the slip resistance increases with the unit pressure calculated over the area of the rivet holes as the square of the diameter, or in other words that the slip resistance in an actual riveted joint may be considered as the product of the total stress in rivet shaft and a friction coefficient which is independent of the diameter of the rivet hole. In those joints which have no hole in the centre plate, (an arrangement which has more theoretical interest,) it appears to be more convenient to calculate with the unit pressure in the rivet hole as an independent variable, and not as in the preceding case with the total pressure, i.e. though the influence of the hole edge increases the coefficient only by an amount corresponding to the diameter of the hole, practically the same result is obtained if the unit pressure were taken instead.

The influence of treating the contact areas.

It may be seen in Table 3 that with placing machine oil between the contact areas there is a characteristic increase in the resistance the first time the joint is pulled apart. After this the coefficient seems to decrease. The difference between the two cases is very small, so that the coefficient can be regarded as unchanged in practical cases. Im any case we need have no fear that oil between the plates has any bad influence. The matter was quite changed when the contact areas were painted, which is commonly done to prevent rust. The paint used was certainly not the best, and on analysis consisted of a large amount of lead sulphate; but, however, was bought for the purpose, and might be equal to that used in most cases in practice. The painting was done so that quite a heavy coat was given, and this dried in about 14 days. The resistance to slip was so small that it could not be measured. With a slip rate of 1 mm/min. the resistance mounted from 30 to 40 kg. with a compression at the rivet heads of $W = 1694$ kg; or the coefficient was in this case less than 0.05. Should the same relation generally occur in painted areas, this treatment would mean an essential weakening of the joint, which may have further possibilities if the rivet shaft has an initial stress which lies close to the yield point of the material.

Influence of the age of the joint;

Tests were made to ascertain the change in slip resistance when the force on the rivet has acted for some time, and showed an increase in the slip resistance of 5% to 10% (Table 5). In any case there seems to be an increase in the friction coefficient. The values stated, which are based upon "age = 0" seems thus to keep one on the safe side of the computed values in an actual joint.

Influence of polishing the contact areas by sliding;

From Fremont's test already mentioned, it was shown that the resistance in the first rotation was remarkably larger than in the following rotations with the same joint.

This phenomenom was explained by considering that the contact areas became polished through sliding, and thus the coefficient decreased. In a check series on Fremont's results given below, I found this phenomenom clearly confirmed, but even experimentally it could be proven that polishing of the contact areas was not the cause for the occurrence. This will be explained below. In this case the same joint had been used many times without any systematic decrease in the coefficient being observed. In the joints 10 x 13 x 13 the coefficient showed itself a little higher than the average after 45 slidings. It is impossible to give any proof that the polishing of the contact areas had any infleunce on the coefficient of friction.

....

In the preceding part mentioned, the author conducted tests of riveted joints, some made by others and some by himself, concerning the friction coefficient in joints made by the "Tikniska Hogskola". In the following part of this paper the author discusses the stresses in the rivet shaft, and the magnitude of the slip resistance.

PART 11

(a) Rotation about the rivet shaft.

Values from Fremont are used in the first method, these being obtained by turning the plates around the rivet shaft. These are shown in Table 7, comepnsated with regard to the time of pressure of the rivet, and for the average aalculated values. The measured slip resistance is computed from Fremont on the assumption that the pressure caused by the rivets is uniform over the whole area of the plates in contact, but this is not true, while the unit pressure is higher around the rivet shaft. Without any knowledge of this distribution we can still determine the tension stress in the rivet shaft if we measure the turning moment necessary. With knowledge of the stress in the rivet shaft we can thus provide ourselves with a table which will show directly the relation between that stress and the turning moment. I have determined this empirical relation with a joint having the same rivet-head dimensions as those used by Fremont. The tension stress has here been substituted for a compression force at the head of the rivets as in the preceding inquiry, and the rivet shaft has been removed in part. The measured values are given in Table 8, and graphically in fig. 10. To obtain an entire equality with the values of Fremont the unit frictional resistance in proportion to the area of the rivet hole has been calculated with an assumed uniform distribution of compression over the whole contact area between the plates. The joint and its dimensions are given in fig. 9. All measurements were made with one and the same joint so that the influence of polishing of the contact area could be tested at the same time. The measurements were made in the order shown in the table. We may assume now that the coefficient is known, and equal to that in the preceding test, so that the actual pressure distribution mainly around the rivet can be calculated.

Discussion of the measured results:

The observation made by Fremont that the second and following measurements gave a smaller frictional resistance than the first one for each group was very plainly confirmed. The supposed reason for this is that the contact area has been polished, and on account of this the coefficient becomes smaller, but such is not the case, since the values for $P = 8000$ kg shown later, for after 23 slidings the force actually became somewhat higher. This peculiar condition might be explained in two different ways, after the most plausible explanation is impossible (that of polishing). It might be thought that it takes a certain time for the static friction to extend itself. Between the first test with a given value for P and the following group's last test there elapsed some minutes. The measurements within each group were made about 15 sec. apart. On the other hand it is even possible that after the first rotation, stresses within the joint may be altered on account of the friction forces formed, and thus the original pressure distribution in the contact area is changed. With a dependable pressure distribution, the friction coefficient varies with $\frac{W}{P}$.

It is worthy of note that the values from the first rotation gave a constant value for the coefficient of friction (within the limit of observation error $\frac{W}{P} = 1.21$ [illegible]) while on the other hand the other values gave a decreasing coefficient with increasing pressure (from 1.08 to 0.93).

Calculation of the stress in the rivet shaft;

In computing the stresses the first values of the group are more reliable than those following, because we are not sufficiently certain of what happens after the first rotation. If we now suppose some of the coefficients from the joints of Fremont to be reproduced by the present joint, we can find the average stress in the rivet shaft.

With 3 sec. compression time, $= 22.5$ kg/mm^2.
With 60sec. compression time, $= 27.5$ kg/mm^2.

The latter test is close to the yield point of the material used.

In both cases the rivet was driven at 950°C with 50 tons pressure.

Computing the Pressure Distribution between the Plates (contact area), I determined the coefficient for the material used (= 0.35). This value was calculated on Fremont's assumption of uniformly distributed compression, and gave me smaller values, which in this case is:

$$P = Po \times \pi (R^2 - r^2)$$

and $M = \frac{4 \pi \mu p_o}{3} (R^2 - r^2)$

$$= \frac{3(R^2 - r^2)}{4(R^3 - r^3)} \cdot \frac{M}{P}$$, and when $R = 5$ cm, $r = 1.3$ cm, $\frac{M}{P} = 1.21$.

$$\mu = 0.17$$

This is less than half the values measured before. Thus the compression cannot be uniformly distributed, but must be heavily concentrated around the rivet shaft. To get some idea about the real distribution we assumed that the unit pressure in the plates followed the law $p = p_0 r^a$, and from the measured values of M and P we calculated "a"

$$M = \frac{4 \pi p_o \mu (R^{a+3} - r,^{a+3})}{a + 3}, \quad P = \frac{2 \pi Po(R^{a+2} - r,^{a+2})}{a + 2}$$

Using the above values

$a = -5$, $P = Po^{r-5}$

which means an especially marked concentration of the unit pressure in the area directly around the rivet shaft.

In the preceding calculation of the stress in the rivet shaft the pressure distribution is supposed equal to that in Fremonts and determined here in the joints reproduced. In such thickness as we have here ($s = 20$ mm) the compression distribution is still highly dependent on the smoothness of the contact areas. The average friction moment can thus be assumed as some value between 5 and 1.3 cm. A certain insecurity lies in this method of reasoning, and on this account I checked over the results after a method stated below, and which in the main confirmed the values already calculated. The slip resistance determined from Fremont seemed to be too small, considerably smaller than those stated by Bach, a result easy to explain through distinguishing between the actual distribution and that stated by Fremont. Fremont's values of the slip resistance appeared in round numbers to be about half of that resistance arising from actual tension.

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Pulling straight against the rivet shaft:

As a check on the preceding values of tension strwsses in the rivet shaft, tests have been made with 10 riveted joints which were made and kindly placed at my disposal by the Atlas Diesel Company. The size agreed with that used before (see part I) and were 10 x 13 x 13. Using these joints, 5 tests were carried out with the contact areas treated with oil, and 5 with the contact areas painted according to the shop practice. The riveting was done in a compression riveting machine with a total compression of 15 tons, requiring 15 seconds.

The tests were made as in Part I with an Alpha tension testing machine whose lowest speed was 1 mm/min. The displacement was measured with a Zeiss Measuring instrument placed on the centre plate, and the needle placed on one of the outer plates.

Results of measurements:

The results are given in Table 9, and graphically in fig. 11, Joints 1 to 5 having untreated contact areas gave very similar results, which is an indication that the workmanship and materials must have been of the best. The joints with painted contact areas showed variations, departing from uniformity in change of the coefficient of friction. All ten rivet shafts showed a very clearly defined breach area.

Discussion of the results of measurements:

In the tests with the rivet shafts removed (part I) the moments at the instant of sliding could be determined with a very high degree of accuracy. In this test it was quite different. In all the joints the sliding was quite continuous, so it was impossible to tell when the force of friction was overcome. This, is the proof of the statement made by Prof. Forsell that it is difficult im an actual joint in tension to cut out the direct resistance of the rivet shaft, and thus determine in what proportion the external force divides itself into slip resistance and pressure on the hole wall. This condition arises from the fact that we do not know how the rivet shaft is placed in the hole from the start, and thus in certain cases a slight displacement appears. The preceding measurements have shown quite clearly that the frictional coefficient is lowered with contact areas painted compared with those not painted. This is plainly confirmed through the systematic difference in the curve for the tension stresses for both kinds of joints, but most when the relative displacement of the plates exceeded 0.05 mm. When the slip carried the load to the rivet shaft, the pressure on the hole wall, H, is equal to the tension force K, minus the resistance W, or $H = K - W$.

The force is mainly spent on bending the rivet shaft, even in the total displacement.

With two equal joints, but with different slip resistance, W_1 and W_2, we have $H = K_1 - W_1 = K_2 - W_2$ or $K_1 - K_2 = W_1 - W_2$. Assume that the slip resistance in both cases is constant, thus for each displacement, $K_1 = K_2 =$ constant. This is shown to be the case with desirable accuracy. The mean value of joints 1 - 5 and 6 - 10 shows nearly a practically constant difference of 1.05 tons up to the elastic limit. For one of the joints this difference went up to 2.5 tons (No. 6) and on the other hand, another one (No. 9) had a difference of practically zero. The results show quite a distinct decrease in strength with painted joints. The painting of contact areas is seen to be very bad from point of rigidity, so that if possible another method should be used to prevent rust. The preceding results seem to me to be capable of expression in the following forms. Untreated contact areas have a frictional coefficient of 0.35. Those parts of the contact areas which have a considerable coating of paint have a coefficient of zero. Usually the pressure of the rivet has caused metallic contact in some parts of the joint ($\mu = 0.35$) but in some parts the coat of paint remains between the surfaces ($\mu = 0$). A painted joint thus has an average coefficient lying between zero and 0.35.

Assume that in Joint No. 6 a good coat of paint was obtained and that $\mu = 0$, so that the total stress P in the rivet shaft is found from

$$P = \frac{2500}{2 \times 0.35} = 3500 \text{ Kg.}$$

Then $\delta = 26.5$ kg/mm^2.

which value agrees quite well with both measurements of Fremont, namely 22.5 and 27.5 kg/mm^2. These values agree even more closely with 3 and 60 seconds time of pressure; the last one 15 seconds. We take this value δ_η as good enough, and obtain an average friction coefficient for painted areas.

$$\mu = 0.35 - \frac{1050}{2 \times 3500} = 0.20$$

How long the low value will hold, allowing a longer time for the paint to dry, is an open question.

PART III - The value of the slip resistance, and its influence on the strength of the joints:

With a knowledge of the preceding measurements, we can now obtain some idea of the slip resistance and its influence on the strength of the joint. Let us assume an initial stress in the rivet of 20 to 25 kg/mm^2 in tension for good riveting, so with the untreated areas of contact the shear is 7.0 to 8.5 kg/mm^2, which is about equal, or rather higher than we are accustomed to use. With painted contact areas we cannot calculate more than 4 or 5 kg slip resistance per mm^2, and often much less, sometime even 0. So long as the external force does not exceed the slip resistance the rivet shaft is without load, which with forces changing from + to - is very desirable. This proves the existence of a large initial stress in the rivet shaft. On the other hand should the external force be so large that sliding occurs, the rivet shaft is without its initial stress in tension, and has shear stress. In the case of a combination of these two, the rivet may be unsafe if the tension stress is near the elastic limit. We arrive at the conclusion that with untreated contact areas the initial stress should be near the elastic limit, but with painted contact areas the stresses should not be higher than to keep the plates together. We can calculate this mathematically as follows;

δ_f is the initial stress of the rivet.

K is half the external force.

W (constant) is the slip resistance in shear.

f is the area of the rivet (shaft).

so that the shear stresses in the rivet shaft (τ) is assumed uniformly distributed over the whole area.

$$\tau = \frac{K - W}{f}, \quad \text{if } K > W$$

$$\tau = 0, \quad \text{if } K \leqq W$$

The two stresses δ_f and τ can be combined as two forces at right angles.

The tension $\delta = \frac{\delta_f}{2}\left(1 \pm \sqrt{1 + \frac{4\tau^2}{\delta_f^2}}\right)$

It is assumed that the largest of these (δ_{max}) determines the ultimate strength, so δ_{max} at this limit takes a certain value independent of δ_f and τ.

$$\delta_{max} = \frac{\delta_f}{2}\left[1 + \sqrt{1 + \frac{4(K-W)^2}{f^2 . \delta_f^2}}\right] \text{ or, if } \frac{K}{f} = k, \text{ and } \frac{W}{f} = W.$$

$$\delta_{max} = \frac{\delta_f}{2}\left[1 + \sqrt{1 + \frac{4(k-W)^2}{\delta_f^2}}\right] \text{and since } W = \mu . \delta_f$$

$$\frac{\delta_{max}}{\delta_f} = \frac{1}{2}\left[1 + \sqrt{1 + 4\left(\frac{k}{\delta_f} - \mu\right)^2}\right]$$

N.B. 1 Kg/mm^2 = 1380#/□"

Let us check these formulae with the measurements from Table 9.

Column 1 is the average value of joints 1 to 5
" 2 " " " " " " 6 to 10
" 3 is the value for joint 6.

	1	2	3
K	5500	4965	3875 Kg
k	41	37	29 Kg/mm^2
k/δ_f	1.55	1.40	1.10
μ	0.35	0.20	0
δ_{max}	47.5	47.5	45.5 Kg/mm^2

These values agree fairly well with the ultimate strength for this material. The last results are only approximate, and further inquiry remains to be done.

SUMMARY.

(1) In order to find the slip resistance in riveted and bolted joints, the frictional coefficient has to be determined. With these values the slip resistance for bolted joints can be calculated from ordinary formulae. To calculate the slip resistance in riveted joints the tension stress in the rivet shaft must be known. This stress is determined from two different methods, and values are obtained which agree.

(2) For riveted joints made in the ordinary way with untreated contact areas, the slip resistance can be calculated as 6 to 8 kg/mm^2 for shear computed over the whole area of the rivet hole. For painted contact areas we can usually compute with a slip resistance of 4 to 5 kg/mm^2 for shear, but because this resistance is often very small, we do not consider this resistance at all.

(3) Painting should never be used. Some other material to preserve steel against rust should be considered.

(4) In calculating the stress in the material of the rivet shaft a formula is given which will give quite fair results when checked.

STRESS IN RIVETED JOINTS IN STEEL CONSTRUCTION.

From "Zentralblatt der Bauverwaltung" No. 101, 102, pp607ff. December 1923. By: Mullenhoff, Sterkrade, Germany.

The statics of steel construction has not improved noticeably in the last 40 years. Although many fine pieces of work have been done, they have not greatly changed the fundamental work of Schwedler, Culman, Winkler, Muller-Breslau, Ritter, Engesser, and Mohr. Many engineers connected with this type of construction do not know how many approximations have been adopted, and how many shop influences have been cut away. Today, more than ever before, the necessity of economy forces us to enlarge our knowledge of the true stresses. Particularly is this true in riveted joints. The German Steel Association has, as the first one, started investigation into this problem, and solutions have been found in many investigations. I can recall only the works of Ruhl and Fimdesen[1]. Two more valuable contributions have just been published in Pamphlet 252 under the title "Experiments to determine the deflections in plates caused by rivets," by R. Baumann; and "The determination of stresses in plates

(1. Pamphlets 221,229. Research Works Association of German Engineers).

caused by rivets, "Pamphlet 262, by the same author. Another contribution to investigations into stress distribution in the gussets of a steel frame has been made by Dr. Theophil Wyss, whose investigations concerned the stresses in the members from external forces. But before we discuss these works, we should consult a small essay by Maillard in "Schweiserischen Bauzeitung," July 28, 1923, in which the author deals with a jointed steel plate with five rivets. Such a joint may not be solved by ordinary methods, where it is usually assumed that each of the rivets bears an equal part of the load. For the assumption mentioned to hold true, the sectional area of the main and cover plates must bear the same proportion to the load carried at any given section, and there must be an equal number of rivets on each side of the joint, for only so can the deformation of both plates be equal.

This fundamental principle lends itself to connections such as shown in figs. 1 to 3. In fig. 1, the width is constant and the thickness is variable (the stress in the hole wall is not considered), while in fig. 2 and 3, there is constant thickness and variable width. These simple examples are sufficient to show the difficulty of the problem. It was this problem that Dr. Wyss tried to solve when he attempted to determine the stress distribution in various gussets in a steel frame. Very careful stress measurements in at least three directions were made in a large number of small squares 2 cm to each side marked on the gussets. A rather simple theoretical consideration, but an enormous piece of experiment and calculation had to be done to give us for the first time some idea of the internal stress conditions in gussets. Fig. 4. shows one of the gussets he used for the experiment, showing the lines of principal stress in tension and compression. The tension lines in the larger area of the gusset are catenaries which run diagonally from one rivet to another, but are bent downward by the compression lines radiating from the vertical post. The plates 170 X 8 and 70 X 8 mm. riveted to the bottom chord are subject not only to tension but also to large moments from secondary stresses. Because of this, the rivets in the top of the plates are compressed so that one part of the tension lines which come from the rivet farthest out on the diagonal members are turned down to enclose these rivets. Another group of lines of principal stress shows the flow from the tension forces in the centre plate of the bottom chord to the rivets, angles and cover plates. The intersection point E of the principal stress lines can easily be seen. Fig. 5 shows the measured stresses from direct forces and moments in the members riveted to the gussets. The gradual falling off of the diagonal force is well shown, and the distribution of forces over the rivets is worthy of note.

Fig. 6 is the same gusset, showing a comparison of the measured normal stress and shear at the same section with the calculated values for a corresponding frictionless framework (shown dotted), and the same frame with elastic gussets (shown in full lines). The figures reproduced give only a small part of the results of this fine piece of work. Every engineer working on structural steel should study this paper.

From quite a different point of view, but none the less worthy, is the work of Professor Baumann, who made tests for the Institute of Testing Materials at Stuttgart to determine the reasons behind the injury of boiler plates, particularly determining the stress in the plates due to driving the rivets. The work was first thought to be of use to boiler work only, but the results are of course valuable to structural steel problems. Much of this information was published in the Journal "Zeitschrift des Vereines deutscher Ingenieure", Vol. 1212, page 1890 ff. When driving a red-hot rivet, the rivet shank presses on the hole wall, and thus the plate about the hole becomes warm, and expands outward, which gives rise to new stresses in the plate farther out from the hole. The cup and header of the gun load the plate around the rivet with more or less load in a circular area. This causes the plates to arch, and the action is extended so far that in a boiler, where the rivet pitch is small, the steel around the rivet is subject to forces piling up, as it were, from the driving of the other rivets near it. To determine the deflection from the pressure in the hole wall, two lead plugs of suitable diameter were used, while the hole was compressed between two pistons. The plugs were observed with a special micrometer. The holes were placed normal and parallel to the rivet edge and the rivet, and between the open hole under compression and the first open hole.

The test gave small values on account of low compression in the first group of measurements.

The compression of the circular area occupied by the head of the rivet was performed with two pistons instead of the rivet, and the lowest load just fitted the plates together. With higher compression the plates started to arch or vault, and commenced to gap at some small distance from the hole. Cold lead rivets served to approximate the deflection from driving without the deflection supplied by the heat. Driving experiments with equal bodies of lead and steel gave more than twice as large driving stresses for the lead than for the steel. The hot steel rivets cool very rapidly on the edge of the head, and cause harder driving. On this account a larger proportion of the force from the riveting machine is passed over to the plates, and a smaller proportion to the rivet shank.

To study the working stresses, holes were punched in a plate 12.7 mm thick, the diameter of the punched hole being 20.6 mm, and reamed to 22.2 mm. Rivets 19.6 mm diameter were driven, and the plate deflections were measured. Table 1 gives the average values for rivets driven in the same direction, the front side being the side from which the rivets were driven and the plates punched.

Fig. 10 Gauge lines on the test plate.

Fig. 11 Gauge lines on the cover plates.

Plates bent slightly by punching were curved by riveting about 5 mm in a distance of 60 mm. The distance from the edge of the plate to the centre of the hole was 37 mm, but the rivet forced the edge so much that the plate gained about $1\frac{1}{4}$ mm in that distance. The deflections were quite large for part of the tests, possibly because only one plate permitted measurement on both sides, and the rivets were placed very close together. The deflection in i-k has not been measured. After this test the heads of the rivets were drilled away, and the shanks struck out, when it was seen that the rivet heads had pressed themselves about 0.4 mm into the plate. The holes were then drilled to 45 mm diameter ($1\frac{3}{4}$ inch). The table shows the change in length after each operation was completed.

TABLE 1.

After	Holes Front	Holes back	Holes average	Rivets Front	Rivets back	Rivets average	Rivets struck out average	Drilled to 45mm dia. average

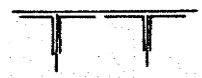
The last decimal place is not quite certain. The reason for the extension of the plate in striking out the rivet is not clear, and may be an error in measurement. Note the changes in the line e-i after drilling. Later two plates 12.7 mm thick were riveted together in the same manner, and here the measurements were quite easy. Before measuring the two plates, the gauge lines were checked, and the plates held together with bolts to be certain that no curvature from the plate would be included. The plates were punched, and the two faces from which the punching was done were bolted together, then riveted, the pressure being about 40 tons, and the resulting change in length as shown in Table 2. After riveting, the thickness of the plates was measured in a number of places, and the results are given in fig. 13, being the increase in thickness in thousandths of an inch.

If we assume that the curvature in a given distance is uniform, we can compute the distortion of the surface from the difference in thickness at the ends and the centre of the given distance. These values are given in col. 9 of Table 2. The extension in the centre of each plate is equal to the difference between the observed values (col.7) and the amount of curvature (col.10). The extensions are then drawn up, and in this manner we obtain the lengths m,n,o,p,q,r,s,t, in the plane of their base line, as shown in fig. 12. The dotted lines are the corresponding values after punching to a scale 10 times that of the full lines. In the centre of the plate "t", the apparent stress between the rivets is 650 kg/cm^2, while close to the rivet (r) a stress of 2600 kg/cm^2 exists, and this is greater than the elastic limit. It is probable that in these places there is compression, and in spite of this there exists considerable tension, the sum total being zero. For the same reason, compression cannot exist throughout the whole length of the line m-n-o-p-q. The elastic limit has been exceeded on this line, and the tension stresses which are naturally present have not been able to overcome the elastic deformation. The curious conduct of the lines e and i of the first test might also be explained thus.

TABLE 2.

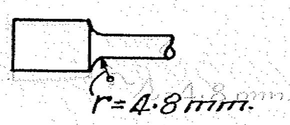
A bulge on the edge of the plates may be seen in front of the rivets, caused by low compression, mainly on the internal edge where the two plates touch, also from larger compression near the external edge of the rivet head. In order to measure the internal deflection in the contact area of the plates, four holes 4 mm diameter were drilled in the upper plate. Four wires were soldered to the lower plate, and passed freely through these holes. The movement of marks on these wires was measured, and hence the vertical motion and rotation of the bases of wires was recorded, as in fig. 8. In addition deflections were computed for a cylindrical hole of equal diameter, and wall thickness equal to the distance from the edge of the plates to the rivet hole. The permanent set in the plates was small when lead rivets were used with low compression (about 20,000 kg). But with higher compression (30,000 kg), plastic deformation lines were visible, and at 40,000 kg, the lines could be seen at the edge of the plate. The elastic limit was exceeded at this point. Because these lines are clearly visible when red-hot steel rivets are used, they may be the result of higher temperatures rather than higher compression.

The temperature was measured with pyrometer elements placed in holes drilled tangential to the edge of the hole from the edge of the plate, and the decrease of temperature was greatest under the header, being over 500° Celsius, or 932°Fahr. The riveting was done in a machine, and required 10 seconds to form the heads of the rivets, and it is possible that smaller changes of temperature would result from machines which would work faster. The change of temperature with respect to the time was determined very carefully from many tests and diagrams. Often only one minute after driving a rivet, there is a temperature of barely 300° cel. close to the header, and by fast work, a rivet can be placed in very much less time. These tests have shown that very large tension and compression stresses tend to occur in the same region. At such places the brittleness is very great, especially when the compression of the riveting is greater than necessary. With the stresses arising later from load, fracture may easily result, particularly when the hole wall was previously injured from punching and reaming. Attention should be paid to these features so that rivets are not made hotter than necessary for driving, so as to avoid excessive heat in the rivets and plates. The compression of the riveting machine should not be too large. Rivet rows of small pitch should not be continuous, but rather first three rivets, then two, then one, so that the yellow and blue temper colors on the steel plate may be avoided. If none of these colors are seen, it is not sufficient proof that the steel has not reached high temperatures, since the air has no admission to these places, and cannot form the thin skin of oxide to which these colors are due. Sometimes the plates are not clean enough for these colors to appear. The experiments of Professor Baumann went further than those of Professor Basquin of Northwestern University, Evanston, Illinois, U.S.A., published in the Journal of the Western Society of Engineers, June, 1913, parts of which are given here.

On account of riveting, structural pieces are longer than normal. Hence, the true length is made shorter by about 1/32 of one inch for every six feet of length (see Prof. Basquin). From this it is evident there are often very large initial stresses before working loads are applied. This is true in the case of such a plate as this , where tension occurs in the centre, and compression on both sides. These must be equal, and the elastic limit is soon reached if the former stresses are high. Experiments made by C.L. Strobel (Trans. Am. Soc. Civ. Eng. 1888, p.103.) showed that columns made from zee bars riveted only on the outstanding leg at the ends of the section were kinked under high compression.

We know that in a wide plate the stresses on the edge of a single hole will be two to three times that of the average stress across the section. Also it is true that in a smaller plate these stresses will exceed the elastic limit in a small area. Flusteel has a fatigue limit about 45% of the ultimate strength for continuous stress without reversal. Thus it seems important to inquire about the influence of such over-stresses in the region of the fatigue limit. The tests were made with a machine designed by Haigh for the purpose, and specimens of one-sixth actual working size were used. A test bar without holes ("A") was tested, having an ultimate strength of 34.7 kg/mm^2, and elastic limit of 20 kg/mm^2. It carried a base load of 10.25 kg/mm^2 with 2352×10^6 stress reversals of ± 9.45 kg/mm^2. The stresses changed also between +19.7 kg/mm^2 and +0.80 kg/mm^2. The base load was then made 11.8 kg/mm^2 and the maximum stress about 21.3 kg/mm^2. Under these conditions the bar slowly but constantly stretched.

A bar "C" of equal cross section, but with only 4.8 mm radius on the shoulder of the test section began to tear where the stresses had not equally distributed themselves, after 529×10^6 reversals. This was the result of local overstress. This was also shown by Professor Coker in an examination of celluloid plates under polarized light.



Another test bar "D", similar to the first one, but 19 mm wide, and having a hole 0.6 mm diameter in the centre was loaded in this manner.

A microphotographic examination gave the diameter to about 0.05 to 0.1 mm, this being much smaller than the hole. This experiment calls forth the important statement that the local change in stress has not such a bad influence as we might expect. Series of test bars were subjected to load, and the results are given in the table following:

Further, these experiments seem to point out that local overstress does not lower the ultimate strength for continuous stress as we might have expected. A definite judgment on this matter is possible from experiments made only on structural pieces of working size. It appears to me that even when all this information is pieces together, the inquiry into riveted joints is still a rich field for research. Our present knowledge does not allow us to effect much saving in material, but this may be effected by the use of higher allowable stresses or different methods of designing joints.

THE DETERMINATION OF STRESS AND STRAIN CONDITIONS IN ONE PLANE WITH REFERENCE TO A PLATE STRETCHED BY TWO RIVETS.

Doctor Dietrich Ruhl, Dortmund.

See "Zeitschrift des Vereines deutscher Ingenieure"
Vol. 64, No. 29. July 1920.

Translation by C.W. Carry, March 1928.

PREFACE: Apart from the general consideration of force distribution in riveted joints, there is one point which is particularly important in steel construction problems. This is the transmission of forces from a rivet to a plate, considered with respect to the stress and strain conditions therein.

For the solution of this problem it was necessary to build a new apparatus for strain measurement, of which the two main parts were the micrometer and the guage.

The extensometer has about 25 times the accuracy of measurement obtained in the apparatus of either Bauschinger or Martens. As far as the author knows, this is a new method of experimental inquiry into stresses, and the use of graphic methods on the measured strain is employed.

Because of the mathematical treatment, it is always possible to determine the stress condition in one plane with this method. These observations show that immediately in front of the rivet there are large stresses, which, combined with relatively small loads, will exceed the elastic limit. The local disturbances in stress conditions thus caused set up a special kind of force transmission from the rivet to the hole wall in a very small area, such that in a short distance from the hole the disturbances can hardly be detected. The increase of stress in a cross section through the centre of the rivets at the hole edge is 1.82 times the average stress found by assuming uniform distribution in the gross section. The experimental apparatus may be used for determining the stress conditions in a system with two axes. The determination of the amount of distortion normal to the principal force is possible in a very simple manner with this extensometer.

At the present time particular attention is being paid to forming butt joints and riveting in bridge building, and there is a very strong tendency to investigate this phase of construction on theoretical ground as well as experimental. Theoretically, it is possible to compute exactly the force distribution in a joint when we know the deflection in all its parts. But, considering only the first row of rivets in a joint, for example, to find the deflection across the whole plate, made up of that due to punching, and that due to each rivet, is not a simple matter. Further, we must clearly separate that part of the deflection in the rivets which is due to shear and bending stresses. A joint must be regarded here as a statically indeterminate system whose solution is possible. One part of the main task before us has been experimentally and theoretically solved, but one of the heaviest problems - the transmission of forces from the rivets into the plate, which is attempted here, has not previously been tried in Germany.

Late in March, 1913, there appeared an article in "Engineering" by Prof. E.G. Coker, entitled "The Distribution of Stress in a Plate Due to a Rivet." The work of Prof. Coker appeared in the Journal at a time when some part of the apparatus designed by the author was already built.

From this it may be seen that both works are quite independent of each other. The actual experiments are different, and particular attention must be called to the fact that Prof. Coker used the optical interference method of stress determination, in which the body under examination must be made of transparent material such as glass, Xylonite, or Nitrocellulose. At a later place the measurements obtained by Coker are shown in comparison with the present results.(See fig. 10). For utilization of the experimental results in computing the distribution of forces in a joint, the distortion of the rivets must be represented by means of mathematical formulae.

Specifically, the problem requiring solution is this:- The stress and strain conditions in a plate loaded by a rivet.

The force transmitted by the plate shall be a tension force, corresponding to the cases already treated, and the rivet shall be on the end of the plate. If the force was one of compression, the stress distribution would run in quite another manner, but the general course of the experiment would follow the same principle. The rivet is normal to the neutral axis of the plate. It may be taken for granted that the stresses which run out from the rivet distribute themselves uniformly throughout the thickness of the plate. The problem can be considered as in one plane, or of stress in two axes, while the distortion normal to the plane of the plate is neglected. This is a major problem which is not only of importance in riveting. There is a similar problem in the stress distribution in eye-bars, as used in old American bridges, also in the links of a chain belt drive. In practice, eyebars are used frequently in wire rope, which is anchored by means of steel bolts or clamps, as in airplane or dirigible rigging. The stress distribution is knownfor a flat plate from the Elastic Theory when the displacement ξ and η in the direction x and y are known for every point through two functions in x and y.

If we consider the displacements ξ and μ drawn on the coordinate planes x, y, we obtain two areas which we may call the displacement areas. (See fig. 1 and 2). The stresses σx and σy, and τ allow solution from the derivatives of these displacement functions.

Thus, $\xi = F(x, y)$ --- ①, and $\mu = F(x, y.)$ ---- ②

Then $$\sigma x = \frac{mE}{m^2-1}\left(m\frac{\partial\xi}{\partial x} + \frac{\partial\mu}{\partial y}\right) \quad \text{---- ③}$$

$$\sigma y = \frac{mE}{m^2-1}\left(\frac{\partial\xi}{\partial x} + m\frac{\partial\mu}{\partial y}\right) \quad \text{---- ④}$$

$$\tau = G\left(\frac{\partial\mu}{\partial x} + \frac{\partial\xi}{\partial y}\right) \quad \text{--- ⑤}$$

As it is better stated in the detailed paper, it is not possible to master this work from analysis of similar cases. Better prospects are offered in an approximate solution after the method of Ritzschen. (See Lorenz, 1913, page 543.)

The author tried to obtain a starting point from the method just mentioned, but such complicated expressions resulted that further work appeared useless. In the same manner a start by the trigonometric series of the method of Hager brought chaos. If we are to make a simple start at the solution, an experiment is necessary to show that the results of this approximation are close enough to the true results. Thus, it was absolutely necessary to learn from experiment the shape of the displacement area ξ and μ. When the curves ξ and μ is known for a general case, then it is possible to try the curves on the general conditions before us, and there is a far better possibility of arriving at a useful result. But in all cases the experiment gave an absolute solution for any given case free from objection. The publication of the results of these experiments and the hint of the importance of the problems gives rise to the hope that it may be possible to find a serviceable mathematical statement for them.

Fig.1. Displacement area ξ of the plate loaded by a rivet.

Fig.2. Displacement area μ of the plate loaded by a rivet.

In preparation for the experiment, the principal thought was to carry out the tests in such a manner as to give the results which would be necessary for a later mathematical treatment. Corresponding exactly to the mathematical method, the displacement areas ξ and μ were determined, and the tangent then drawn to the curves ξ and μ. From the tangent values obtained, the stresses $\sigma x, \sigma y, \tau$, (from Eq. 1 to 5) were found.

In order to justify the method chosen, the special circumstances and technical difficulty of the experiment must be pointed out, also the fact that the solution was not previously attempted in an exact manner. For symmetrical forces, we must have at least three plates. (See fig.3.)

Fig. 3. General view of the joint.

The symmetrically loaded plate is placed between the two outer covers, which are accessible on one side only. But to carry out the measurement of deflection, it was necessary to compensate the errors always present by measuring both sides at the same time. It was necessary to place the plates apart from one another, as shown in fig.4, so that the symmetrically loaded

Fig. 4.

centre plate would be accessible for observation. With regard to the bemding stresses in the rivet, the space "e" between the plates used for measurement must be made as small as possible. The tension grip is shown in figs. 5 and 6; one end holds the plate, and the other fits into the machine. The distance "e" is 16 mm. On account of the rigidity and the deflection of the rivet, the load did not exceed 3000 kg. The methods used up till now, and the apparatus for determining the stresses, namely those of Pruss, Rudeloff and Coker were of no account for this work, which is well explained in Research Book No. 221.

On account of the anticipated small deflections, it was necessary to build an extensometer of great accuracy and fine measurement. Translation of motion was necessary, and the mechanism had to be placed outside the forks on account of the small clearance width. The apparatus has two main parts, one of which was the gauge piece, of very small dimensions, which was placed between the plates and the forks, and which carried the motion of the points to the outside of the forks where magnifying apparatus was placed (see fig.5. and 6.)

Figs. 5 and 6.

As so much depends on the adjustment of such a mechanism, there is a fear that because of a fault in the transfer of motion, the desired accuracy may be lost, and particular attention is required to avoid this unfavorable condition. From a special experiment with this instrument, it was found that the accuracy compared to that of Martens apparatus was exactly as computed. Because of the intercept on the scale being either the n^{th} part of the load, or the n^{th} part of the measured distance, the accuracy of the instrument is n times as great as the simple apparatus of Martens. The magnifying power is 25 times that of Martens'. To utilize this accuracy fully, it is obviously necessary that the applied load be measured with equal accuracy. It may be pointed out that it is possible to determine the amount of lateral distortion with the instrument, hence the gauge piece was placed laterally and longitudinally at a given section.

HOW THE EXPERIMENT WAS CONDUCTED.

First, the properties of the material was determined from a similar test piece. Martens' apparatus was used to determine the elastic limit, the Modulus of Elasticity, and the yield point. The elastic limit and the yield point were found from the stress strain diagram, and gave:

Elastic Limit.	2190 kg/cm^2.
Yield Point.	2580 " "
Ultimate Strength.	3780 " "
Elastic Modulus.	2,050,000 " " .

Percent Elongation at Ultimate Strength 27.3

Area of Fracture Reduced to 72.5% of original area.

The bolt was made of the best Reibahlen steel of 7000-9000 kg/cm^2 ultimate strength, and other properties were not known for the bolt. The net of coordinates was drawn on the plate, and the intersection points marked with a centre punch. The system of points and the notation employed is shown in fig. 7.

Fig. 7. Experimental Plate, and General Notation.

The range of the observation was 2000 kg. For some series, the divisions were read at 500 kg intervals. The actual load varied from 600 to 2600 kg. Readings were commenced with the initial load, proceeded to the top of the range and back, but the top reading was not taken so as to eliminate the errors in the machine itself, the load indicators and the extensometer. The measurement of the plate started May 5th, 1914, and the initial point is marked "1" on fig.7. This point was in front of the rivet, and was the hardest to reach. The displacement in cross section was determined as far as possible with the extensometer in its simplest arrangement (see fig 5). The tower was placed on the lateral edge of the plate. In this way, all lateral deflections were referred to the edge of the plate, and in order to get the deflections referred to the X-axis, certain values had to be subtracted from those read, and the X-axis itself appeared to move to the edge of the plate on this account. Due to a proposal made by Dr. Lorenz, an experiment was made with a lead plate of the same dimensions as the steel plate, and an identical net of coordinates. The lead plate under load gave a distinct deflection in permanent set. The plate is shown after set in fig.8.

Fig. 8. Lead plate after loading.

It may be seen clearly how the lead is piled up in front of the ri vet, and how it is distorted in cross section through the centre of the rivet.

Fig. 9. Stresses σ_x in the length and cross section for a load of 2000 kg.

In a general manner the lateral and longitudinal lines agree with those of the steel plate, so that from this we can make a rough picture of the deflection of the steel plate. But it is hardly possible from these curves to conclude anything about the stresses in the steel plate, because the distortions follow a somewhat different law, as becomes evident through direct measurements on the lead plate. With these measurements, which were made on August 8th, 1914, the work ceased.

WORKING OUT THE EXPERIMENTAL RESULTS, AND DETERMINING THE STRESSES.

The displacements ξ and μ were drawn up for the lateral and longitudinal sections, and the tangents were drawn on the curves at each point required. In this way $\frac{\partial \xi}{\partial x}, \frac{\partial \xi}{\partial y}, \frac{\partial \mu}{\partial x}, \frac{\partial \mu}{\partial y}$ were derived. Figs. 1 and 2 gives a summary sketch concerning the displacement just in front of the rivet. Here, some difficulty was encountered in determining the stresses by direct measurement, for in front of the rivet they became so high that the elastic limit was passed and the stresses could not be computed from the Elastic Theory. The probable course of the displacement curves ξ and μ for an imaginary case of an elastic condition, supplemented by actual knowledge was established, and the stresses found from this method were used as trial values, and found to be accurate. The stresses $\sigma x, \sigma y$ and τ were determined from equations 3 to 5, aided by the displacement curves. Before the stresses can be determined, the above tangents must be found. The value of the Elastic Modulus is required at this point. The value of the Modulus "E" is found from the plate at the point where the stresses are uniformly distributed over the cross section, this location being not far from the Y-axis. "E" was found to be 2060000 kg/cm^2. In a similar manner the value of "m" with reference to the lateral extension was determined, and was 3.77. From "E" and "m" the value for the displacement follows:

$$G = \frac{m\ E}{2(m+1)} = \frac{3.77 \times 2060000}{2(3.77+1)} = 815000 kg/cm^2.$$

From the above relation the stresses in Tables 1 to 3 are found.

Table 1. Stresses σ_x in kg/cm^2 for P = 2000kg.

Table 2. Stresses σ_y in kg/cm^2 for p = 2000kg.

Stresses τ for P = 2000kg. -- Table 3.

The dimensions of the plate are given in fig.7, and the combined sections of the two outer plates were made equal to the section of the main plate, as in practice. The edge distance of the plate in the direction of the force and normal to the force was made 2 rivet diameters. The stress in the X-axis had three main components. This is illustrated in fig. 9. In the section through the Y-axis the stresses are uniformly distributed, and amounted to 249kg/cm^2. Closer to the hole a definite "piling up" of the stresses is evident, and this declines very rapidly toward the hole edge, until at section 13-144 the stress distribution may be called uniform. However, the influence of the hole was perceptible as far as section 176-17. The distance of the rivet influence is about three hole diameters, measured from the centre of the hole. For the present case, where there are two holes in the tension member, Rudeloff found that the distance of the influence of the hole was 5 hole diameters this being very large. That the present case showed a smaller distance is probably due to the fact that in consequence of the symmetrical distribution of the stresses in the cross sectional area through the Y-axis, the latter could not develop freely on account of the influence of the hole. The present plate was too short for such an investigation.

In section 104-8, $\sigma_x = 0$, $y = 0$, must be true, because at this point none of the force can be carried over to the wall of the hole. The three sections 96, 89, 92, show a gradual increase in the tension stresses on the edge of the hole. The greatest stresses at point 83 is 941 kg/cm^2, and decreases on the edge of the hole to only 116 kg/cm^2. Under the assumption of uniform distribution, the stress in the smallest section can be computed to be

$$\sigma_m = \frac{2000}{6} = 334 \text{ kg/cm}^2$$

Thus the increase of the tension stresses on the edge of the hole through the centre of the rivet is 941/334 = 2.82 times. The increase of tension stress in this case by transmitting load through a rivet is thus much worse than in the case of a member with only one hole. For the last case, the increase of stress as determined by Pruss was 2.1 to 2.3, being 2% to 9% too small, according to Leon and Zedlicky. In the present case of transmission of load through a bolt, the stress in the cross section was about 1.3% greater than in the case of a member with only one hole. Coker's values for this case of stress transmission is given in Fig. 10.

"Comparison of σ_x and σ_y for longitudinal and cross sections through the centre of rivets with values obtained from Coker (shown dotted)."p" is the stress uniformly distributed over the whole cross section".

We recognize that the stresses σ_x in the cross section through the rivet centre, and those in a longitudinal section are about equal. Note that the stress curve determined by Coker reached only to within 5 mm of the hole edge, while in this work the stresses are found right to the edge. A place of change in the character of the stress curve is seen between sections 82-76 and 27-1. Up to this point only tension stresses were apparent, but here tension and compression stresses appear together. Section 27-1 shows the most remarkable case on the plate. In this section the main part of the load leaving the rivet must be compression, and equal to the total tension stress for equilibrium. The tension area of 27-21 lies outside the location of the plastic deformation which occurs around the hole edge. Up to this point the stresses can be easily determined without difficulty. The area of the stress curve reaches approximately 2000 kg, hence it follows that from point 21 the curve must fall very quickly to zero and become negative. The negative area must reach the value of 2000 kg. Exception appears only insofar as the lateral walls of the hole must also take a part of the rivet force. An average of the plus and minus areas gave a force of about 1875kg.

The sections farther from the rivet towards the edge of the plate show a similar character of stress curve, but the values in the direction of the edge of the plate fall off rather rapidly, and on the edge of the plate they must disappear. Calculation gave the largest stresses on the edge of the plate as 23 kg/cm^2. We may assume that the stresses σ_x on the average suffer from errors of this size. The stresses σ_y in table 2 on the lateral edge are seen, and disappear on the Y-axis. The σ_x main stresses are uniformly distributed on the Y-axis. Along the lateral edge the stresses have only small values. The thrust stresses must be in all lateral areas of the plate, and in symmetrical lines. They disappear on the X- and Y-axis. Further, shear stresses must exist at point 83, and at the edge of the hole in the section through the rivet they must be zero. The computed values at the edge of the plate was 26 kg/cm^2. The error is about the same size as for σ_x and σ_y. To compare this with other cases, or to use it as a reference for other experiments is not possible, since shear stresses have never been previously determined. To complete the picture of the stresses, the known relation is followed out from the known stresses.

σ_x, σ_y, τ, are the major stresses computed for certain points of the plate, and are found from the direction of the major strains. After the computed stress lines are drawn, as in fig. 11, the major stress lines may be drawn in.

Regarding stresses $\sigma_{min.}$, it is to be noted that some of them in the series 68-193 and 69-194 show considerably larger stresses than σ_x in front of the rivet. $\sigma_{min.}$ lines radiate clearly from the rivet (see fig. 11). The remaining part of the plate up to the area directly behind the rivet hole has only relatively small compression forces. $\sigma_{max.}$ shows the path of the tension forces which carry the rivet load to the plate. The major stress lines corresponding to the highest stresses are at the lateral edge of the hole. The major shear stresses give considerably higher values than the values of τ in the direction of the coordinate axes; however, these values in the direction away from point 1 are hardly so large that they can be any more dangerous than σ_{max} and σ_{min}. It should be noted that the $\tau_{max.}$ lines disappear neither at the edge of the plate, nor in the symmetrical lines shown.

TO REPRESENT THE DISPLACEMENT OF THE RIVET BY AN EMPIRICAL EXPRESSION.

The transmission of the load for this case may be connected with the results of Rudeloff. The influence of rivet holes on distortions was determined from the method of Zimmerman, in which the rivet is substituted by a rectangle of height "d" and width "nd", as designated in fig.12.

Let

$\frac{l}{2}$ = distance from the Y-axis to the centre of the rivet.

b = width of the plate.

δ = thickness of the plate.

d = diameter of the hole.

$Z = \frac{P}{b\delta}$ = uniform stress in the whole cross section.

E = Elastic Modulus.

Then we obtain, shown in the detailed paper, the following relation, under the assumption that the rivet itself is rigid. This gives the rivet displacement with reference to the Y-axis in approximation only.

$$\xi r = \frac{Z}{2E}\left(l - d + \frac{bd}{b-nd}\right) + \frac{HP}{\pi d \delta K}$$

Constants $n = 2.72$, $K = 18 \times 10^5\ Kg/mm^2$

The relation is only approximate. For similar cases we obtain from the above a value close to that of the rivet displacement, which figure is more accurate if the case in question is nearly identical with the above.