

THE EFFECT OF TOPICS IN MATHEMATICS 305
ON FIRST-YEAR UNIVERSITY MATHEMATICS
ACHIEVEMENT

A Thesis
Submitted to the Faculty of Graduate Studies
at the University of Manitoba

In Partial Fulfillment
of the Requirements for the Degree
Master of Education

by
Wayne Arthur Johnson
July, 1978



THE EFFECT OF TOPICS IN MATHEMATICS 305
ON FIRST-YEAR UNIVERSITY MATHEMATICS
ACHIEVEMENT

BY

WAYNE ARTHUR JOHNSON

A dissertation submitted to the Faculty of Graduate Studies of
the University of Manitoba in partial fulfillment of the requirements
of the degree of

MASTER OF EDUCATION

© 1978

Permission has been granted to the LIBRARY OF THE UNIVER-
SITY OF MANITOBA to lend or sell copies of this dissertation, to
the NATIONAL LIBRARY OF CANADA to microfilm this
dissertation and to lend or sell copies of the film, and UNIVERSITY
MICROFILMS to publish an abstract of this dissertation.

The author reserves other publication rights, and neither the
dissertation nor extensive extracts from it may be printed or other-
wise reproduced without the author's written permission.

Acknowledgements

Appreciation is expressed for the advice and guidance given by Dr. Lars Jansson and Dr. Murray McPherson, both of whom acted as the chairman of my committee for a period of time, and to the committee members Professor Roy Dowling and Sister O'Neill.

The writer is grateful to Mr. Mark Bishop for his assistance with the statistical procedures and to Mr. Irvin Labovich for reading and for editing the original draft of the thesis.

A special thanks is extended to the members of the Department of Mathematics and Astronomy and the Department of Applied Mathematics at the University of Manitoba whose cooperation made it possible to obtain the information that was necessary for this study.

Abstract

The purpose of this study was to determine what effect the successful completion of Topics 305, in addition to Mathematics 300 in Grade 12, would have upon the marks obtained in first-year mathematics courses at university. A comparison of marks achieved at university was made between two groups. The first group had studied and completed both Mathematics 300 and Topics 305 in Grade 12. The second group completed Mathematics 300 in Grade 12 but not Topics 305.

A questionnaire was completed by those students who wrote the diagnostic tests administered by the Faculty of Science and the Faculty of Engineering in September of 1976. The questionnaire was used to acquire necessary high school background information in order to determine the group to which a student belonged. The Mathematics 300 mark was also obtained from the questionnaire. The university marks used in the comparisons were obtained directly from the records of the University of Manitoba.

Analysis of covariance was employed, using the Mathematics 300 mark as the covariate to determine if there were a statistical difference between the mean mark obtained by each of the two samples.

The results showed that the four comparisons which favoured the students who had taken both Mathematics 300 and Topics 305 in Grade 12 involved marks obtained neither on the

diagnostic tests at the beginning of the university year or on the calculus tests during the first term. There was not a significant difference in the results obtained on the remaining five comparisons of the study. These last comparisons involved either the first and second term marks obtained in the course consisting of algebra and geometry or the marks which were obtained in the calculus courses during the second term.

The evidence gathered in this study indicated that students who plan to study calculus at university should consider enrolling in Topics 305 in high school because it does give them an initial advantage in the period of transition from high school to university.

Table of Contents

	<u>Page</u>
List of Tables	vii
List of Appendices	ix
Chapter	
I STATEMENT OF THE PROBLEM	1
Purpose of the Study	1
Definitions	5
The Study	7
Hypotheses Tested	9
Assumptions and Limitations	11
Summary	12
II LITERATURE REVIEW	13
Predictors of Success in University Mathematics	13
Experimental Evaluation of Mathematics Programs	20
What Should be Taught in High School Mathematics	26
Summary	28
III EXPERIMENTAL DESIGN	30
Instrumentation and Data Collection	30
Sample	32
The Comparisons	37

Chapter		Page
IV	RESULTS OF THE EXPERIMENT	40
	Inferential Analysis of Data	40
	Summary	50
V	SUMMARY AND CONCLUSIONS	52
	The Problem Restated	52
	Summary of Procedure	54
	Discussion and Conclusions	58
	Recommendations	60
	References	62
	Appendices	68

List of Tables

Table		Page
1	Letter Grade Versus Percentage Mark Substituted	32
2	Number of Students Who Responded to the Questionnaire in Each Group and Faculty ...	33
3	Number of Students Eligible for Selection to Sample in Each Group and Faculty	34
4	Number of Students in Each Group and Faculty Eliminated Because of Unsigned Permission Form	34
5	Sample Size in Each Group and Faculty	35
6	Hypotheses Versus Marks Compared (Faculty of Science)	37
7	Hypotheses Versus Marks Compared (Faculty of Engineering)	38
8	Sample Size, Unadjusted Means and Adjusted Means for Diagnostic Test M	41
9	Analysis of Covariance for Achievement Differences on Diagnostic Test M	41
10	Sample Size, Unadjusted Mean and Adjusted Mean for Final Marks in Calculus 13.139	42
11	Analysis of Covariance for Achievement Differences on Final Marks in Calculus 13.139	42
12	Sample Size, Unadjusted and Adjusted Means for Final Marks in Calculus 13.140	43
13	Analysis of Covariance for Achievement Differences on Final Marks in Calculus 13.140	43
14	Sample Size, Unadjusted and Adjusted Means for Final Marks in Calculus 13.141	44
15	Analysis of Covariance for Achievement Differences on Final Marks in Calculus 13.141	44

Table		Page
16	Sample Size, Unadjusted and Adjusted Means for Diagnostic Test AM	45
17	Analysis of Covariance for Achievement Differences on Diagnostic Test AM	45
18	Sample Size, Unadjusted and Adjusted Means for Mid-Term Marks in Calculus 6.122	46
19	Analysis of Covariance for Achievement Differences on Mid-Term Marks in Calculus 6.122	46
20	Sample Size, Unadjusted and Adjusted Means for Final Marks in Calculus 6.122 ..	47
21	Analysis of Covariance for Achievement Differences on Final Marks in Calculus 6.122	47
22	Sample Size, Unadjusted and Adjusted Means for Mid-Term Marks in Algebra and Geometry 6.123	48
23	Analysis of Covariance for Achievement Differences on Mid-Term Marks in Algebra and Geometry 6.123	48
24	Sample Size, Unadjusted and Adjusted Means for Final Marks in Algebra and Geometry 6.123	49
25	Analysis of Covariance for Achievement Differences on Final Marks in Algebra and Geometry 6.123	49
26	Summary of Hypotheses and Results (Faculty of Science)	56
27	Summary of Hypotheses and Results (Faculty of Engineering)	57

List of Appendices

Appendix		Page
A	Topics in Mathematics 305 Course Outline	68
B	Course Descriptions From 1976-77 University of Manitoba Calendar For:	
	(a) Calculus 13.139	79
	(b) Calculus 13.140	79
	(c) Calculus 13.141	79
	(d) Calculus 6.122	79
	(e) Algebra and Geometry 6.123	79
C	Questionnaire	81
D	Letters	83

CHAPTER I

STATEMENT OF THE PROBLEM

The mathematics program in Manitoba has undergone changes in the past few years. Existing course have been altered and new courses have been introduced. However, little has been done to determine the effectiveness of the changes that have been made.

This study evaluated a new mathematics course that was introduced in Manitoba schools by the Manitoba Department of Education. Specifically, the study evaluated the high school mathematics courses Topics in Mathematics 305 (Topics 305) in terms of its value in preparing students for university mathematics.

Purpose of the Study

Mathematics, and indeed education in general, has undergone significant changes in the past. Shibli (1932) made the following observation which still has relevance today:

"The new philosophy of education has emphasized the changing character of our civilization and has demanded an education that is not static but dynamic. These developments have a marked influence upon the curriculum of the secondary school. The various courses of study have been altered or modified to meet the new demand. Teachers of mathematics have been reorganizing the courses of study and changing the methods of instruction in harmony with the spirit of the age" (p. 33).

The mathematics program in Manitoba has been modified in recent years. Curriculum planners need to evaluate present programs so that appropriate additions and modifications can be made. Ideally a change in curriculum should occur only if a positive effect is expected.

A review of the research entitled "Student Achievement and Attitude in a Modern and a Traditional Grade Ten Geometry Program" by Cross (1968) indicates that not all curriculum changes do produce this expected positive effect. "The results of this study show no consistent pattern of superiority for either the pilot or the traditional geometry programs in terms of student achievement and attitude" (Cross, 1968, p. Vii). Attitude and achievement were measured by two pre-tests consisting of the Mathematics Attitude Scale and the Sequential Test of Educational Progress - Form 2A and four post-tests consisting of the Cooperative Geometry, the Geometrical Achievement Measure Experiment, the Mathematics Attitude Scale, and the Geometrical Attitude Scale.

There is some doubt concerning whether the change in the curriculum, referred to in the Cross (1968) study, was beneficial to the students in terms of improved attitude and improved achievement in geometry.

The evaluation of an educational program is not a simple matter. Alkin (1972) states that "evaluation is a complex activity that involves the identification of many factors that contribute to educational outputs" (p. 110). Cronbach (1963) cites the following definition of evaluation.

He says that evaluation, in the broad sense, may be defined "... as the collection and use of information to make decisions about an educational program" (p. 672).

The reading of existing literature leaves little doubt about the need for sound evaluation of educational programs. Ahmann (1967, p. 84) claims that "... far too little attention is paid to curriculum evaluation..." and that much of the present evaluation "... can be regarded as less than top quality" (Ahmann, 1967, p. 84). Begle and Wilson (1970, p. 367), in referring to mathematics programs in particular, suggest evaluation is important because of the vast curriculum changes at all grade levels. Coleman (1969) agrees there is need for evaluation, but for a different reason. He says that "... because the effects of a program are not immediately and directly evident, that formal evaluation is necessary" (p. 6).

Clearly, there is need for research to determine if recent curriculum changes have been beneficial to the student. Johnson (1966) suggests a type of research that is needed in program evaluations. He states that "the type of research accepted by many researchers as having the greatest promise for finding definitive answers is scientific experimentation" (p. 420).

A legitimate objective of a mathematics program concerns the application of mathematics to other fields (Weaver, 1970, (p. 341). Bruner (1966) agrees but is more emphatic. He says, "The first object of any art of learning, over and

beyond the pleasure it may give, is that it should serve us in the future" (p. 17). A quotation from the 1965 publication of Dessart & Burns show general agreement, but refer specifically to mathematics. They ask, "... are the new programs effective in preparing students for the first college year of mathematics" (p. 141)?

The 1975 course outline for Topics 305, as printed by the Manitoba Department of Education and reproduced in Appendix A, states that "the topics in this course have been selected on the basis of their usefulness as background material for university studies in chemistry, physics, engineering, statistics, computer science, mathematics and commerce" (Topics 305 Course Outline, p. 2). In addition this outline states that "for students considering attending universities outside Manitoba, they will find that, in many cases, the background material provided in this course is necessary in the above fields" (Topics 305 Course Outline (p. 2).

The fact that the successful completion of Topics 305 has a positive influence on achievement in the first year of university mathematics would be valuable information for high school students at the time of selection of their high school program. On the other hand, if the successful completion of Topics 305 is not a factor in higher achievement in the first year of university mathematics, then Manitoba educators will have to either reconsider the

rationale for Topics 305 or cease to offer it in the high school.

The need for research to determine if specific programs have met their objectives has been established. A legitimate objective of high school mathematics programs and the stated objective of Topics 305 is to prepare students for their first year of university mathematics. The aim of this study was to determine if Grade 12 students who had successfully completed Topics 305, in addition to Mathematics 300, were better prepared for university mathematics than Grade 12 students who took only Mathematics 300.

Definitions

Topics 305 - the high school half-course as described in the Manitoba Department of Education course outline for 1975-76 and designated as Topics in Mathematics 305. This course is normally studied in Grade 12.

Mathematics 300 - the high school mathematics course as described in the Manitoba Department of Education course outline for 1975-76. This course is normally studied in Grade 12.

First-year mathematics - the first-year mathematics courses as described in the 1976-77 University of Manitoba calendar and designated Calculus 13.139, Calculus 13.140, Calculus 13.141, Calculus 6.122, and Algebra and Geometry 6.123.

Diagnostic Test M (DTM) - the diagnostic test administered by the University of Manitoba Mathematics Department during the first class of Calculus 13.139 to determine if students have achieved a satisfactory standard of efficiency in high school mathematics.

Calculus 13.139* - the first-year, three-hour credit, first term calculus course as described in the 1976-77 University of Manitoba calendar and offered by the Department of Mathematics.

Calculus 13.140* - the first-year, three-hour credit, second term calculus course as described in the 1976-77 University of Manitoba calendar and offered by the Department of Mathematics.

Calculus 13.141* - the first-year, three-hour credit, second term calculus course as described in the 1976-77 University of Manitoba calendar and offered by the Department of Mathematics.

Diagnostic Test AM (DTAM)- the diagnostic test administered by the University of Manitoba, Department of Applied Mathematics, to all Engineering students during the first class of calculus 6.122 to determine if students have achieved a satisfactory standard of efficiency in high school mathematics.

* The course description, as given in the 1976-77 University of Manitoba calendar, appears in Appendix B.

Calculus 6.122* - the first-year, six-hour credit, calculus course for Engineering students, as described in the 1976-77 University of Manitoba calendar and offered by the Department of Applied Mathematics.

Algebra and Geometry 6.123* - the first-year, six-hour credit, algebra and geometry course for Engineering students, as described in the University of Manitoba 1976-77 calendar and offered by the Department of Applied Mathematics.

The Study

This study constituted an evaluation of Topics 305. Specifically, the study was designed to determine if the Grade 12 students who had successfully completed Topics 305 as well as Mathematics 300 (Group A) were better prepared for their first year of university mathematics than the Grade 12 students who completed only Mathematics 300 (Group B). The degree of preparation of these high school students for university mathematics was measured in terms of the marks achieved over the following year in the first-year mathematics courses at the University of Manitoba.

The comparison of marks achieved by the two groups was made between marks achieved by students in the first-year mathematics courses and on the diagnostic tests in both

* The course description, as given in the 1976-77 University of Manitoba calendar, appears in Appendix B.

the Faculty of Science and the Faculty of Engineering¹ at the University of Manitoba.

Because random assignment of students to the two groups in this study was impossible due to the elective nature of Topics 305, ANCOVA was used to determine if a significant difference existed between the marks achieved by the two groups. The Mathematics 300 mark was used as the covariate in the analysis. The effect of ANCOVA was to adjust the achievement means compared in order to compensate for initial differences in mathematical background.

Background information about students used to determine the two groups and the covariate scores were obtained by the questionnaire reprinted in Appendix C. This questionnaire was completed in September of 1976 by all the students in the Faculty of Science and the Faculty of Engineering who wrote the diagnostic tests administered by the Department of Mathematics and by the Department of Applied Mathematics respectively.

The marks achieved by each of the students in the courses that were pertinent to this study were obtained directly from the records of the Department of Mathematics and from the Department of Applied Mathematics during June of 1977.

¹ The first-year mathematics courses in the Faculty of Engineering which pertain to this study, are offered by the Department of Applied Mathematics.

Permission was granted by the Department of Mathematics and by the Department of Applied Mathematics to conduct the survey questionnaire to obtain the required student background information. Permission was also granted by the Committee on Research Involving Human Subjects to obtain the marks achieved by the students for the purposes of this study. The letters requesting and granting permission to carry out the procedures necessary to complete this study are reproduced in Appendix D.

Hypotheses Tested

A series of first-year university mathematics marks achieved by two groups of students at the University of Manitoba were compared in this study. For the purposes of statistical analysis, the hypotheses are stated in null form.

- 1) There will be no significant differences in marks obtained on Diagnostic Test M between students who have successfully completed Topics 305 in addition to Mathematics 300 and students who have not taken Topics 305 in Grade 12.
- 2) There will be no significant difference in the final marks obtained in Calculus 13.139 between students who have successfully completed Topics 305 in addition to Mathematics 300, and students who have not taken Topics 305 in Grade 12.

- 3) There will be no significant difference in the final marks obtained in Calculus 13.140 between students who have successfully completed Topics 305 in addition to Mathematics 300, and students who have not taken Topics 305 in Grade 12.
- 4) There will be no significant difference in the final marks obtained in Calculus 13.141 between students who have successfully completed Topics 305 in addition to Mathematics 300, and students who have not taken Topics 305 in Grade 12.
- 5) There will be no significant difference in marks obtained on Diagnostic Test AM between students who have successfully completed Topics 305 in addition to Mathematics 300, and students who have not taken Topics 305 in Grade 12.
- 6) There will be no significant difference in the mid-term marks obtained in Calculus 6.122 between students who have successfully completed Topics 305 in addition to Mathematics 300, and students who have not taken Topics 305 in Grade 12.
- 7) There will be no significant difference in the final marks obtained in Calculus 6.122 between students who have successfully completed Topics 305 in addition to Mathematics 300, and students who have not taken Topics 305 in Grade 12.

- 8) There will be no significant difference in the mid-term marks obtained in Algebra and Geometry 6.123 between students who have successfully completed Topics 305 in addition to Mathematics 300, and students who have not taken Topics 305 in Grade 12.
- 9) There will be no significant difference in the final marks obtained in Algebra and Geometry 6.123 between students who have successfully completed Topics 305 in addition to Mathematics 300, and students who have not taken Topics 305 in Grade 12.

Assumptions and Limitations

Marks obtained by students from various high schools, and consequently various high school mathematics teachers, were used in this study. For the purposes of this study, the assumption was made that marks obtained in Mathematics 300 by students in different high schools in Manitoba represent equivalent achievement in the prescribed course for Mathematics 300.

Similarly, equivalent marks obtained in the two Diagnostic Tests, Calculus 13.139, Calculus 13.140, Calculus 13.141, Calculus 6.122 and in Algebra and Geometry 6.123 were assumed represent equivalent achievement in those courses.

A limitation of the study was that random assignment of students to the two groups was impossible. This was because Topics 305 was an optional high school course and one does not have control over student selection of optional courses at the high school level.

Summary

Change in curriculum is on-going. There is a need to determine if new programs meet its stated objectives. This study compared the achievement in university mathematics to determine if students who successfully completed Topics 305 in Grade 12 in addition to Mathematics 300 were better prepared for mathematics in their first year at university than students who took only Mathematics 300 in Grade 12.

CHAPTER II

LITERATURE REVIEW

The review of the related literature is presented in three sections: the first section deals with factors that have been used to predict success in the first year of university mathematics; the second section reviews the experimental evaluation of mathematics programs as related to subsequent success in university mathematics; the final section deals with research on what should be taught in the high school mathematics programs in order to improve the preparation of students for mathematics at university.

Predictors of Success in University Mathematics

As students advance from one educational institution to another, or even from one grade level to another, the instructors often attempt to predict success or failure based on student background. In order to make accurate predictions of success or of failure the most important predictor variables must be known. Some research has been done to determine the most reliable and accurate predictors of success.

An article by Ivanoff and Evermade (1965) reports on some research that was carried out to determine which of the students who have completed Grade 8 mathematics should select Grade 9 algebra, and which of the students should select general mathematics in Grade 9. Two main conclusions

were reported. First, "the best two predictors of success in Grade 9 algebra are the mathematics mark obtained in Grade 8 mathematics and the arithmetic scores achieved on the High School Placement Test respectively" (p. 412). Second, they found that little practical value was gained by using a combination of more than two predictors (p. 416).

Another study by Lovet (1969) concluded that significant factors related to success in the first year of algebra were arithmetic level achieved, general school achievement, and initial algebra achievement.

Four other studies came to a similar conclusion. Comley (1959) concluded that the higher the mark achieved in high school mathematics, the higher the mark achieved in university mathematics. Hickman (1969) stated that calculus achievement is closely related to marks earned in algebra and trigonometry. Tuscher (1972) summarized that the single most important variable for predicting success in a first course in calculus is mathematical ability as measured by the Comparative Guidance and Placement Program and the Survey of Study Habits and Attitudes. Finally, Sommers (1973) concluded that the best predictor of success in calculus is the average mark achieved in high school mathematics.

Three additional studies dealing with prediction of success in university mathematics were written in somewhat more detail. In the first of these, Brasch (1972) reported that of the several variables tested, two of them

accounted for 87% of the weight of the prediction. These variables were the SAT-M (mathematics aptitude) score and high school class rank in quintiles. From this information a prediction equation was formulated and the critical value established.

Pugh (1969) attempted to determine the effectiveness of the College Entrance Exam Board (CEEB) Mathematics Achievement Test as an instrument for placing students in beginning college mathematics courses. The CEEB scores were compared with other variables as predictors of success. Other variables were the number of units of mathematics taken in high school, the high school class rank in mathematics and verbal mathematics (SAT-M). Based on several multiple regression analyses, it was concluded that the combination of SAT-M and the high school class rank in high school was the most efficacious predictor of success.

Vandruff (1973) selected factors from personal records to develop a regression equation to predict probable success or failure in calculus. Vandruff produced two regression equations. The first contained four variables. These were the high school grade point average (GPA) in mathematics, the high school class rank, the GPA in college before attempting calculus, and the spelling score on the Washington Precollege Test. The second equation contained only two variables. These variables were the high school GPA in mathematics and the mathematics score on the Washington Precollege Test. The most important conclusion reached was

that the best single prediction of success in calculus is the GPA in high school mathematics.

The final four studies also deal with predictors of success in university mathematics.

Akeju and Michael (1970) formulated a study to determine the degree of validity of selection devices in predictions of success for a sample of 109 college students at the Federal School of Science in Lagos, Nigeria. The subjects in which success was to be predicted were physics, chemistry, zoology, botany, pure mathematics and applied mathematics. The two predictor variables were the scores of the achievement tests of the West African School Certification Examinations (WASCE) in each of the subjects and the Internationally Developed Test (IDT) for African culture. The latter is a standardized test. The criterion of success was the student's obtaining a passing mark.

Statistically, in the Akeju and Michael (1970) study, product-moment intercorrelations among prediction and criterion variables were determined. A multiple regression equation involving the prediction of each criterion measure from a weighted composite of predictor variables was then established. The predictions were compared with the actual outcomes.

The conclusion reached by Akeju and Michael (1970) was that the score on the achievement examination based on high school subjects (WASCE) given prior to admission to the

Federal School of Science was the most valid predictor of success in the college courses. In other words, prior achievement proved to be a better prediction of success than the standardized tests.

Morgan (1970) attempted to determine a discriminant equation for the prediction of success in an introductory mathematics course at the University of Toledo. Discriminant function analysis was performed using the score on the Cooperative Mathematics Test (CMT), the number of years of academic high school mathematics, the mean grade point in high school mathematics and the student's age in months beyond the seventeenth birthday. The critical score for the discriminant equation which resulted was considered to be the score midway between the mean score for a group of successful students and a group of unsuccessful students.

Success or failure predictions were made using the discriminant equation for each of the 50 students involved in the study and these were compared to their actual successes or failures. The equation was found to be 90% accurate. Morgan (1970) also determined that contributions to effective forecasting were as follows:

- | | |
|---|------|
| a) Cooperative Mathematics Test | 34%; |
| b) Mean grade point in high school | 27%; |
| c) Number of years of high school mathematics | 25%; |
| d) Age in months beyond seventeen | 14%. |

The study by Wilson & Gelso (1967) was an attempt to determine if marks in Mathematics 101 (a college algebra course at the University of California) could be predicted by two measures of student's high school performance in algebra. The first measure was the average mark obtained in the first high school algebra course (Algebra I) and the second high school algebra course (Algebra II). The second measure was the mark obtained in Algebra II.

The Pearson product-moment correlations were calculated between marks obtained in Mathematics 101 and the average marks in Algebra I and Algebra II. The same product-moment correlation was calculated between the Mathematics 101 mark and the mark in Algebra II alone. Although there was a substantial correlation in each case, the relationship was slightly stronger in the latter.

An investigation by Wick (1965) was designed to help answer the following two specific questions:

1. "What are the factors drawn from information normally available to college mathematics departments, which are most significantly associated with success in different types of first-year college mathematics courses" (p. 642)?
2. "How effectively can success in first-year college mathematics be predicted with the information normally available" (p. 643)?

A total of 1692 students in six Minnesota and Wisconsin colleges and universities were involved. Background information on students was obtained from the university and high school records as well as from a questionnaire. The criterion of success was the mark obtained in the first year of university mathematics.

The order in which success factors ranked with correlation to final exam results on Calculus and Analytical Geometry are as follows:

- | | |
|--|------|
| (a) mathematics average in Grade 12 | .46; |
| (b) mathematics aptitude | .43; |
| (c) mathematics average for Grade 11 and
Grade 12 | .40; |
| (d) mathematics average for Grade 10, Grade
11 and Grade 12 | .39; |
| (e) ratio of A's to number of courses in
Grade 10, Grade 11, and Grade 12 | .39. |

"An examination of the five best multiple regression equations for prediction in using predictor variables one, two and up to all five at once revealed that not all predictor variables were making a significant contribution to the predictors" (Wick, 1965, p. 646).

Two important conclusions were reached by Wick (1965). First, the high school mathematics record was consistently the source of the best predictor of success in first year university mathematics. The second conclusion was that a multiple regression prediction of success was an improvement

over single predictor variables (p. 647).

Experimental Evaluation of Mathematics Programs

Over the years many new mathematics programs have been introduced. These new programs have been evaluated in terms of how successful they were in terms of improving student knowledge in mathematics as measured by achievement on standardized tests or in terms of how well the programs prepared students for the study of further mathematics as measured by subsequent achievement in university mathematics courses.

In some cases an entirely new mathematics program with a new approach was introduced. In other cases only one or two new topics were altered in existing programs.

Grigsby (1973) compared achievements in the first course in university calculus between students with different calculus backgrounds in high school. The conclusion reached in this study was that there is not a strong relationship between entry knowledge of any particular topic in college calculus and the final achievement level reached. There was, however, an initial difference.

Austin (1975) came to the opposite conclusion. He found in his study that there was a significant difference between the mean achievement levels between students who had studied calculus in high school and students who had not studied any calculus in high school. The difference favored the students who had studied calculus in high school.

The study by Wick (1965) had a purpose in addition to the two stated in the previous section of this chapter. This additional question was, "Are the new experimental programs in secondary mathematics effective in preparing students for first-year college mathematics" (p. 642)?

After examination of the data the conclusion reached was that there was no significant difference in quality of preparation for first-year college mathematics as between students in the experimental School Mathematics Study Group (SMSG) and students in the traditional mathematics program in high school in terms of marks earned in college. (Wick, 1965, p. 647)

The purpose of the study by Shimizu (1969) was to determine a correlation of high school advanced mathematics courses with marks in first-year college mathematics at the University of Hawaii. Success of the advanced students was compared to success of a random sample of an equal number of students who did not take the advanced mathematics courses in high school. The findings were that there was no significant difference between marks obtained in the first year of college mathematics by the two groups.

Another study finding no significant difference in achievement in calculus between students who did take calculus in high school and students who did not take calculus in high school was conducted by McKillip (1965). In this study, regression equations were formulated using

high school mathematics marks, high school class ranks, CEEB results, SAT - mathematical and SAT - verbal scores. The equation was based on the university mathematics marks obtained by 753 students who had no calculus in high school. The marks of students with a calculus background were calculated with the equation to determine expected marks in university mathematics if the students would not have studied calculus in high school. The predicted marks were subtracted from the actual marks and the resulting signed differences were tested statistically.

Richert (1972) reported a study which compared the effects of four different types of high school backgrounds on achievement in a first course in calculus at the University of Kansas. The four groups, which were identified from information obtained via questionnaire, completed by the students at the university were:

- a) students who had studied trigonometry and analytic geometry in high school;
- b) students who had studied elementary functions as well as probability and statistics in high school;
- c) students who had studied elementary functions, analytic geometry and calculus in high school;
- d) students who had studied elementary functions in high school.

Because random assignment of students to the two groups was impossible, ANCOVA was employed in order to control scholastic aptitude and mathematical ability. The covariate representing scholastic aptitude was the composite score on the American College Testing Program. The covariate representing mathematical ability was the average mathematics mark obtained by the student during the previous three years of high school.

The results of this research showed that there was no significant difference at the .05 level between the four groups.

Tillotson (1962) compared marks achieved in a first-year calculus course by students who had studied calculus briefly in high school with students who had studied no calculus in high school. The two groups were identified from information obtained on a questionnaire which was completed by the students. Using random numbers Tillotson selected 48 cases from each group.

In order to adjust the achievement measures for the factor of scholastic ability and general mathematical background, Tillotson (1962) employed two variables. The first was the normalized rank in high school and the second was the score obtained on the mathematics placement test.

After the adjustments for scholastic ability and general mathematical background were made, there was no significant difference in achievement between the two groups.

Paul (1970) reported on a study that compared the achievement in calculus at a university between groups with different mathematical backgrounds. The groups were subdivided according to achievement expectancy as determined by regression analysis. The results of students with similar expectancy were compared. The information required to place the students in groups were obtained by questionnaire which was completed by students on the first day of classes at university. Additional information was obtained from a questionnaire mailed to the mathematics department chairmen of the high school from which the students graduated. The results of this study by Paul (1970) indicate that students, who had studied some calculus in high school, initially obtained results in university calculus which were significantly higher than the corresponding results of students who did not study calculus in high school. The amount of difference in achievement, however, decreased with time.

Another study attempted to determine if SMSG mathematics was a factor influencing success in freshman calculus at Oklahoma State University (Coon, 1963). Analysis of covariance was used to evaluate the calculus grade differences due to differences in high school preparation. It was found that SMSG students achieved significantly higher marks in calculus than students who studied the traditional mathematics program in high school.

The final study of this section is commonly known as The Eight-Year Study. This was an intensive study of "2108 graduates from the thirty schools which, under an agreement with more than 300 colleges, had been freed from the necessity of meeting the usual unit or examination requirements for college admission" (Chamberlin, Chamberlin, Drought & Scott, 1942, p. 207). Of these students 1475 were matched with graduates of conventional schools in terms of scholastic aptitude, interests and socio-economic background. The students involved in The Eight-Year Study were students who entered college in the years 1936, 1937, 1938 and 1939.

The definition of success in this study involved several areas and was in general representative of "success as judged by college standards, by the students' contemporaries, and by the students themselves" (Chamberlin, Chamberlin, Drought & Scott, 1942, p. 207). Data to determine if a student were successful was obtained from interviews, questionnaires, records of reading and activities, reports from instructors and others who had contact with the students.

Some of the findings are listed below. These findings showed that the graduates from the Thirty Experimental Schools:

- a) earned a slightly higher total grade average;
- b) specialized in the same academic fields as did the comparison students;

- c) received slightly more academic honors in each year;
- d) earned in each college year a higher percentage of non-academic honors.

(Chamberlin, Chamberlin, Drought & Scott, 1942, p. 208)

A basic conclusion reached in this study was that the "more experimental the school, the greater degree of success in college" (Chamberlin, Chamberlin, Drought & Scott, 1942, p. 209).

What Should Be Taught In High School Mathematics

A few articles and studies are presented here which provide some guidelines for the curriculum developer when proposing new programs.

Kline (1966) has some definite ideas on the general approach to curriculum development and the development of mathematics programs in particular. He states that "axioms that the curriculum should satisfy should be determined and then the curriculum produced" (p. 323). He goes on to say that "the main problem of teaching mathematics is motivation" (p. 329) and that educators "must present the value of mathematics" (p. 330).

A study by Comley (1958) was, in part, designed to determine attitudes of university students towards mathematics. A suggestion from the students regarding the high school mathematics curriculum which emerged from the

study was that calculus should be taught in the high school program (p. 12).

The main study related to this section of the literature review was conducted by Gilbert (1971). He compared how effectively high school mathematics programs prepared students for university calculus as viewed by three groups of students from six colleges. The three groups were:

- a) students with some calculus in high school;
- b) private school students with no calculus in high school;
- c) public school students with no calculus in high school.

The findings indicate no significant difference in the views of the three groups about calculus in first-year university.

The important aspect of the study by Gilbert (1971), however was the recommendations made by the students concerning objectives, course content, and pedagogical practices for a high school mathematics program. The five objectives recommended by the students were:

- a) to master algebraic manipulations and computations;
- b) to investigate thoroughly the concept of functions;
- c) to study, in depth, elementary functions and their graphs;

- d) to master manipulations and computations with inequalities;
- e) to introduce limits and continuity.

The six topics that the students recommended be a part of the high school mathematics program were:

- a) function;
- b) relations;
- c) series;
- d) sequences and limits;
- e) continuity;
- f) differentiation.

The three pedagogical practices recommended by the students were that teachers should:

- a) make students responsible for doing homework;
- b) encourage students to do independent study;
- c) encourage student self-enrichment.

Summary

The literature was definite about which factors were most reliable in predicting success in first-year university mathematics. The use of more than one variable in a regression equation provided the most reliable predictions. The best single predictor variable, was the mark achieved in the previous mathematics course.

The studies which experimentally evaluate mathematics programs were of two main types. The first was the type in

which students were selected in high school and followed through all or part of their university mathematics courses. Marks of the groups in question were compared. In the second type of study, students who attended university were placed in groups according to their high school background. The marks of the two groups were subsequently compared employing ANCOVA to adjust for uncontrolled variables.

The literature was not as clear in terms of the results of the evaluation of the mathematics programs. Some studies indicate that a particular mathematics program had a positive effect on achievement in university mathematics. Other studies may have indicated that this same mathematics program provides no significant effect on achievement in university mathematics.

Only one study was found which provided a detailed outline of what should be taught in high school mathematics programs. There was, however, general agreement by the calculus students at university. They felt that calculus should be introduced at the high school level.

CHAPTER III

EXPERIMENTAL DESIGN

Chapter 3 is divided into three sections. First, the instrument used in the study is described. Also explained in this section is the method of obtaining the required data. Second, a description of the sample is given. Third, the specific comparisons that were made are indicated and the statistical method used in making each comparison is stated.

Instrumentation and Data Collection

The data required for this study fell into three categories. The first category consisted of the high school background information which was required to determine the two groups of students that were compared in the study. The second category consisted of the marks obtained by the students on the university mathematics courses and diagnostic tests used for the actual comparisons. The final category consisted of the Mathematics 300 mark obtained in high school which was used as the covariate in the analysis of covariance procedure employed in this study.

The high school background information and the Mathematics 300 marks were obtained via the questionnaire which is reprinted in Appendix C and was completed by two groups of students at the University of Manitoba in September of 1976. The first group, consisting of those students

enrolled in the Faculty of Science, wrote the Diagnostic Test M which was administered by the Department of Mathematics; the second group consisting of those students enrolled in the Faculty of Engineering, wrote the Diagnostic Test AM which was administered by the Department of Applied Mathematics. The high school background information obtained, included the year of graduation, the student's age, the high school from which the student graduated, and whether or not the student completed Topics 305. In addition the questionnaire provided the Mathematics 300 mark. Permission to obtain and to use the student's university results for the study was requested from each student who completed the questionnaire.

The marks achieved by the students on the diagnostic tests and the university mathematics courses were obtained directly from the records of the Department of Mathematics and the Department of Applied Mathematics in June of 1977.

Percentage marks were used for each of the comparisons of the study and for the covariate. When the marks were reported as percentage marks, the actual percentage marks were used. If the marks were reported as letter grades, percentage marks were substituted in accordance with the scale in Table 1.

Table 1

Letter Grade Versus Percentage Mark Substituted

Letter	Percentage
A+	95
A	85
B+	77
B	72
C+	67
C	62
D	55
F	25

Any student in the Faculty of Engineering who did receive a mid-term mark but who did not receive a final mark in either Calculus 6.122 or Algebra and Geometry 6.123, was assigned the same final mark as was obtained at mid-term.

Sample

For the purposes of this study, the students who completed the questionnaire were divided into two groups according to a specific factor in their high school background. This factor which determined the group into which each student was placed was whether or not the student had successfully completed Topics 305 in high school. The group of students were:

- a) Group A - students who had successfully completed both Mathematics 300 and Topics 305 in Grade 12.
- b) Group B - students who had successfully completed Mathematics 300 but did not enroll in Topics 305 in Grade 12.

The number of students who responded to the questionnaire in each of the two groups and in each faculty of concern to the study is shown in Table 2 below.

Table 2

Number of Students
Who Responded to the Questionnaire
in Each Group and Faculty

	Science	Engineering
Group A	52	40
Group B	533	287
Total A & B	585	327

In order to be considered eligible for selection to the sample of either of the two groups, the students had to meet certain qualifications. Specifically, each student had

- a) to have signed the permission form on the questionnaire indicating his willingness to participate in the study;
- b) to have graduated from a Manitoba high school in June of 1976;

- c) to have been not more than nineteen years old
at the time that the questionnaire was completed.

Table 3 shows the number of students who met the
above conditions in order to be eligible for selection to
the sample from each group.

Table 3

Number of Students
Eligible for Selection to Sample
in Each Group and Faculty

	Science	Engineering
Group A	47	36
Group B	296	154
Total A & B	343	190

The number of students who were not eligible for
selection to the sample because they did not sign the
permission form on the questionnaire was small. The actual
numbers are shown in Table 4.

Table 4

Number of Students
in Each Group and Faculty Eliminated
Because of Unsigned Permission Form

	Science	Engineering
Group A	1	0
Group B	5	3

The remaining non-eligible students were not eligible for selection to the samples because they did not meet the last two eligibility criteria listed previously.

Because there were only 47 students in the Faculty of Science eligible to be a part of the sample of Group A students, all 47 were considered to be members of the sample of that group. Using random numbers, approximately an equal number of eligible Group B students in the Faculty of Science was selected to become the sample of Group B students in that faculty. Similarly, because there were only 36 students in the Faculty of Engineering eligible to be a part of the sample of Group A students, all 36 were considered to be members of the sample. Again, using random numbers, approximately an equal number of eligible Group B students in the Faculty of Engineering were selected to become the sample of Group B students in that Faculty.

The number of students in the sample of each group and faculty is stated in Table 5 below.

Table 5
Sample Size in Each Group and Faculty

	Science	Engineering
Sample A	47	36
Sample B	52	39
Total A & B	99	75

The marks achieved on the diagnostic tests and on the university mathematics courses of concern to this study were sought for the 99 students in the Faculty of Science and for the 75 students in the Faculty of Engineering in Table 5.

For the purpose of the analysis, students enrolled in the Faculty of Science who completed the diagnostic test but did not receive a mark in any of the other university mathematics courses pertaining to this study, were considered not to be a part of the sample. Also, students in the Faculty of Science who did not receive a mark in a particular university mathematics course of concern to this study were considered not to be a part of the sample for that particular comparison. The result was that the number of students in the final sample for each individual comparison could be less than the numbers indicated in Table 5 and could vary in each of the nine comparisons.

Students in the Faculty of Engineering who completed the diagnostic test but received no other marks on the courses of concern to this study were considered not to be a part of the samples of Engineering students. Thus, the number of students in the final sample of Engineering students was less than indicated in Table 5.

In the Faculty of Science all of the calculus students first wrote the Diagnostic Test M and then studied Calculus 13.139. After Calculus 13.139 was completed this group was separated with some of the students studying Calculus 13.140

and with others studying Calculus 13.141. In the Faculty of Engineering all students were required to write the Diagnostic Test AM. All students were also required to study both Calculus 6.122 and Algebra and Geometry 6.123 which were offered throughout the entire university year.

The Comparisons

Nine separate comparisons were made in this study between the major marks obtained by the students in the samples of Group A and Group B in mathematics courses over the first-year of university in the Faculty of Science and the Faculty of Engineering. Each of the nine comparisons corresponds to one of the hypotheses stated on pages 9, 10 and 11 in chapter 1.

Table 6 below illustrates which hypothesis corresponds to each course in the Faculty of Science on which marks obtained by students in Group A and in Group B were compared. Table 7 serves the same purpose for the Faculty of Engineering.

Table 6

Hypotheses Versus Marks Compared
(Faculty of Science)

Hypotheses	Grades Compared
H1	Diagnostic Test M
H2	Calculus 13.139 (final)
H3	Calculus 13.140 (final)
H4	Calculus 13.141 (final)

Table 7

Hypotheses Versus Marks Compared
(Faculty of Engineering)

Hypotheses	Grades Compared
H5	Diagnostic Test AM
H6	Calculus 6.122 (mid-term)
H7	Calculus 6.122 (final)
H8	Algebra & Geometry 6.123 (mid-term)
H9	Algebra & Geometry 6.123 (final)

Because Calculus 13.139, Calculus 13.140 and Calculus 13.141 were three-credit hour courses, no mid-term marks were reported to the students and thus only the final marks were compared in this study. Calculus 6.122 and Algebra & Geometry 6.123, however, were six-credit hour courses. Both mid-term and final marks were reported to the students. Hence both the mid-term and final marks were compared in this study.

In order to test each hypothesis, the earned marks corresponding to that hypothesis, as shown in Table 6 and Table 7, were compared between the students in Sample A and the students in Sample B. Analysis of covariance (ANCOVA) using the Mathematics 300 mark as the covariate was employed to determine if there were a significant difference between the marks earned by the students in Sample A and the students in Sample B. The Mathematics 300 mark achieved in

high school was chosen as the covariate because of the great importance the literature placed upon achievement in high school mathematics as a predictor of success in the first year of university mathematics.

CHAPTER IV

RESULTS OF THE EXPERIMENT

The intention of this study was to determine what effect the successful completion of Topics 305 would have upon the marks obtained by students in their first year of university mathematics at the University of Manitoba.

The procedure adopted for this chapter was to treat each hypothesis on a separate basis.

To test each hypothesis, the mean mark obtained by the students in Sample A and Sample B was determined. The ANCOVA was used to determine if the difference between the mean marks obtained by the two samples were significant. The Mathematics 300 mark was used as the covariate.

Two tables were used in the presentation of the results of the nine hypotheses of the study. The first table states the sample size, the values of the unadjusted mean, and adjusted mean for each group. The second table provides the ANCOVA results from which inferences were made.

Inferential Analysis of Data

H1 There will be no significant difference in the marks obtained on the Diagnostic Test M between students in Group A and students in Group B.

Table 8

Sample Size, Unadjusted Means and
Adjusted Means for Diagnostic Test M

	No. in Sample	Mean (Unadjusted)	Mean (Adjusted)
Group A	47	83.43	81.80
Group B	45	72.80	74.50

Table 9

Analysis of Covariance For Achievement
Differences on Diagnostic Test M

Source of Variation	df	SS	MS	F
Between Groups A and B	1	1159.840	1159.840	6.635*
Error within Groups A and B	89	15558.270	174.812	

* Significant at the .05 level.

The F-ratio, reported in Table 9, was significant at the .05 level and thus the null hypothesis was rejected. When this was considered together with the mean scores reported in Table 8, the conclusion reached was that students in the sample of Group A did significantly better on the Diagnostic Test M than students in the Group B sample.

H2 There will be no significant difference in the final marks obtained in Calculus 13.139 between students in Group A and students in Group B.

Table 10

Sample Size, Unadjusted Mean and Adjusted Mean
for Final Grades in Calculus 13.139

	No. in Sample	Mean Unadjusted	Mean Adjusted
Group A	47	80.62	78.90
Group B	45	69.16	71.95

Table 11

Analysis of Covariance for Achievement Differences
on Final Marks in Calculus 13.139

Source of Variation	df	SS	MS	F
Between Groups A and B	1	1048.082	1048.082	6.845**
Error within Groups A and B	89	13626.656	153.108	

** Significant at the .01 level.

The F-ratio stated in Table 11 was significant at the .01 level and thus the null hypothesis was rejected. When this result was considered together with the mean scores reported in Table 10 the conclusion was that students in the Group A sample performed significantly better in



Calculus 13.139 than students in the sample of Group B students.

H3 There will be no significant difference in the final marks obtained in Calculus 13.140 between the students in Group A and the students in Group B.

Table 12

Sample Size, Unadjusted and Adjusted Means
for Final Marks in Calculus 13.140

	No. in Sample	Mean Unadjusted	Mean Adjusted
Group A	39	69.57	68.31
Group B	23	64.48	66.44

Table 13

Analysis of Covariance for Achievement Differences
on Final Marks in Calculus 13.140

Source of Variation	df	SS	MS	F
Between Groups A and B	1	55.141	55.141	.224
Error within Groups A and B	59	14541.770	246.471	

The F-ratio revealed in Table 13 was not significant at the .05 level and, in turn, the null hypothesis was not rejected. The interpretation of these results must be that, even though there was a difference in the final marks

achieved in Calculus 13.139 between the students in Group A and the students in Group B, the difference was not sufficiently large to be statistically significant.

H4 There will be no significant difference in the final marks obtained in Calculus 13.141 between students in Group A and students in Group B.

Table 14

Sample Size, Unadjusted and Adjusted Means
For Final Marks in Calculus 13.141

	No. in Sample	Mean Unadjusted	Mean Adjusted
Group A	3	79.66	82.75
Group B	10	72.40	71.57

Table 15

Analysis of Covariance for Achievement Differences
on Final Marks in Calculus 13.141

Source of Variation	df	SS	MS	F
Between Groups A and B	1	251.107	251.107	1.525
Error within Groups A and B	10	1646.408	164.641	

The F-ratio reported in Table 15 was not significant at the .05 level and thus the hypothesis was not rejected.

Again, the students in the Group A sample achieved a higher mean than the students in the Group B sample but the difference was not statistically significant.

H5 There will be no significant difference in the marks obtained on the Diagnostic Test AM between the students in Group A and the students in Group B.

Table 16

Sample Size, Unadjusted and Adjusted Means
for Diagnostic Test AM

	No. in Sample	Mean Unadjusted	Mean Adjusted
Group A	30	63.40	63.70
Group B	37	45.67	45.40

Table 17

Analysis of Covariance for Achievement Differences
on Diagnostic Test AM

Source of Variation	df	SS	MS	F
Between Groups A and B	1	5559.965	5559.965	12.070**
Error within Groups A and B	64	29481.809	460.053	

** Significant at .01 level

Because the F-ratio was significant at the .01 level, the null hypothesis was rejected. Table 16 reports that the

students in the Group A sample achieved the higher means. This information led to the conclusion that the students in Group A achieved significantly higher marks on the Diagnostic Test AM than the students in Group B.

H6 There will be no significant difference in the mid-term marks obtained in Calculus 6.122 between the students in Group A and the students in Group B.

Table 18

Sample Size, Unadjusted and Adjusted Means
for Mid-Term Marks in Calculus 6.122

	No. in Sample	Mean Unadjusted	Mean Adjusted
Group A	30	69.80	70.20
Group B	37	58.21	57.89

Table 19

Analysis of Covariance for Achievement Differences
on Mid-Term Marks in Calculus 6.122

Source of Variation	df	SS	MS	F
Between Groups A and B	1	2510.734	2510.734	7.967**
Error within Groups A and B	64	20169.602	315.150	

** Significant at the .01 level

The null hypothesis was rejected because the F-ratio was significant at the .01 level. Table 18 indicates that the students in the sample of Group A achieved a higher mean than the sample of Group B students. These facts led to the conclusion that the students in Group A perform significantly better at mid-term in Calculus 6.122 than students in Group B.

H7 There will be no significant difference in the final marks obtained in Calculus 6.122 between students in Group A and students in Group B.

Table 20

Sample Size, Unadjusted and Adjusted Means
for Final Marks in Calculus 6.122

	No. in Sample	Mean Unadjusted	Mean Adjusted
Group A	30	69.16	69.61
Group B	37	60.21	59.85

Table 21

Analysis of Covariance for Achievement Differences
on Final Marks in Calculus 6.122

Source of Variation	df	SS	MS	F
Between Groups A and B	1	1575.859	1575.859	3.809
Error with Groups A and B	64	26475.852	413.685	

The significance of the F-ratio in Table 21 was calculated to be .055. Because this value was greater than the accepted .05 level, the null hypothesis was not rejected. Therefore, the conclusion was that there was not a significant difference in achievement in the final marks in Calculus 6.122 between the two groups.

Table 22

Sample Size, Unadjusted and Adjusted Means for
Mid-Term Marks in Algebra and Geometry 6.123

	No. in Sample	Mean Unadjusted	Mean Adjusted
Group A	30	70.50	70.82
Group B	37	63.35	63.08

Table 23

Analysis of Covariance for Achievement Differences
on Mid-Term Marks in Algebra and Geometry 6.123

Source of Variation	df	SS	MS	F
Between Groups A and B	1	991.809	991.809	3.647
Error within Groups A and B	64	17404.133	271.939	

The null hypothesis was not rejected at the .05 level because the significance of the F-ratio, reported in Table 23 was calculated to be .061. The result was that this

study did not find a significant difference between the two groups in achievement on the mid-term mark in Algebra and Geometry 6.123.

H9 There will be no significant difference in the final marks obtained in Algebra and Geometry 6.123 between students in Group A and students in Group B.

Table 24

Sample Size, Unadjusted and Adjusted Means
for Final Marks in Algebra and Geometry 6.123

	No. in Sample	Mean Unadjusted	Mean Adjusted
Group A	30	67.83	68.21
Group B	37	62.81	62.50

Table 25

Analysis of Covariance for Achievement Differences
on Final Marks in Algebra and Geometry 6.123

Source of Variation	df	SS	MS	F
Between Groups A and B	1	539.355	539.355	1.587
Error within Groups A and B	64	21747.652	339.807	

Table 25 revealed an F-ratio which was calculated to have a significance of .212. This was far beyond the

acceptable .05 level and thus the null hypothesis was not rejected. This study, therefore, did not find a significant difference in achievement between Group A and Group B on the final mark in Algebra and Geometry 6.123.

Summary

Nine comparisons were made in this study between the marks achieved by first-year university mathematics students in Group A and in Group B. Each comparison related to a single hypothesis in this study. Of these nine comparisons, four were found to have significant results at the .05 level of confidence and hence the corresponding hypothesis were rejected. The remaining five comparisons did not have significant results at the .05 level of confidence and hence the corresponding hypotheses were not rejected.

Four of the comparisons of this study show that students who had taken both Mathematics 300 and Topics 305 in Grade 12 obtained significantly higher results than the students who had taken only Mathematics 300 in Grade 12. The significantly higher results occurred in Diagnostic Test M, the final marks in Calculus 13.139, the results in Diagnostic Test AM, and the mid-term marks in Calculus 6.122. A significant difference was not found in results obtained between the two groups on the final marks in Calculus 13.140, the final marks in Calculus 13.141, the final marks in Calculus 6.122, the mid-term marks in Algebra

and Geometry 6.123, and the final marks in Algebra and Geometry 6.123.

All four of the comparisons in which a significant difference was found favoured the students who had taken both Mathematics 300 and Topics 305 in high school (Group A). None of the comparisons favoured the students who had taken only Mathematics 300 in high school (Group B).

Each of the comparisons on which a significant difference was found involved marks that were reported either at the beginning of the university year or at mid-year. The significantly different results reported at the beginning of the university year were the marks obtained in the two diagnostic tests; the significantly different results reported at mid-year were the final marks in Calculus 13.139 and the mid-term marks in Calculus 6.122. A significant difference in results was not found for any of the marks which were reported at the end of the university year.

A significant difference in results between the two groups was not found in either the mid-year or final marks in Algebra and Geometry 6.123.

In the comparisons related to the first three hypotheses, there was not as great a difference between the adjusted means as there was between the unadjusted means for the two groups. In the comparisons related to the remaining six hypotheses, the opposite was found to be true. In these cases there was a greater difference between adjusted means than there was between the unadjusted means. Further reference is made to this in Chapter 5.

CHAPTER V

SUMMARY AND CONCLUSIONS

The Problem Restated

The purpose of this study was to ascertain whether students who successfully completed both Mathematics 300 and Topics 305 in Grade 12 (Group A) achieved higher marks in their first-year university mathematics courses than the students who completed only Mathematics 300 in Grade 12 (Group B).

Specifically, this study compared the marks achieved by two groups of students on nine sets of first-year university mathematics scores, two of which were diagnostic tests and seven of which were regular courses. For the purposes of statistical analysis each comparison was written in the form of a null hypothesis. These are listed below:

- 1) There will be no significant differences in marks obtained on Diagnostic Test M between students who have successfully completed Topics 305 in addition to Mathematics 300, and students who have not taken Topics 305 in Grade 12.
- 2) There will be no significant difference in the final marks obtained in Calculus 13.139 between students who have successfully completed Topics 305 in addition to Mathematics 300, and students who have not taken Topics 305 in Grade 12.

- 3) There will be no significant difference in the final marks obtained in Calculus 13.140 between students who have successfully completed Topics 305 in addition to Mathematics 300, and students who have not taken Topics 305 in Grade 12.
- 4) There will be no significant difference in the final marks obtained in Calculus 13.141 between students who have successfully completed Topics 305 in addition to Mathematics 300, and students who have not taken Topics 305 in Grade 12.
- 5) There will be no significant difference in marks obtained on Diagnostic Test AM between students who have successfully completed Topics 305 in addition to Mathematics 300, and students who have not taken Topics 305 in Grade 12.
- 6) There will be no significant difference in the mid-term marks obtained in Calculus 6.122 between students who have successfully completed Topics 305 in addition to Mathematics 300, and students who have not taken Topics 305 in Grade 12.
- 7) There will be no significant difference in the final marks obtained in Calculus 6.122 between students who have successfully completed Topics 305 in addition to Mathematics 300, and students who have not taken Topics 305 in Grade 12.

- 8) There will be no significant difference in the mid-term marks obtained in Algebra and Geometry 6.123 between students who have successfully completed Topics 305 in addition to Mathematics 300, and students who have not taken Topics 305 in Grade 12.
- 9) There will be no significant difference in the final marks obtained in Algebra and Geometry 6.123 between students who have successfully completed Topics 305 in addition to Mathematics 300, and students who have not taken Topics 305 in Grade 12.

Summary of Procedure

In September of 1976, a questionnaire was distributed to each of the students who had written the mathematics diagnostic tests administered by the Faculty of Science and by the Faculty of Engineering at the University of Manitoba. The purpose of the questionnaire was to determine which students enrolled in first-year mathematics courses at the university had completed Topics 305 in high school. This information was required for the placement of the students into either Group A or Group B. Also obtained from the questionnaire were additional high school background information and permission from the student to use his university marks for the study. This information was needed to determine which students qualified to participate in the

study. Finally, the Mathematics 300 mark, which was used as the covariate in the analysis, was obtained from the questionnaire.

Because the number of students who completed both Mathematics 300 and Topics 305 was relatively small, the sample of this group (Sample A) was the entire group. The number of students selected to be the sample of Group B (Sample B) was approximately the same as the number of students in Sample A. Sample B was selected at random from the Group B students who met the required qualifications.

The university marks, related to each of the nine hypotheses, were obtained directly from the records of the University of Manitoba. These marks were then analyzed, using analysis of covariance, to determine if there was a significant difference between the mean for each of the samples. In each analysis the Mathematics 300 mark was used as the covariate.

Summary of Results

The results of each of the comparisons was summarized in Table 26 and Table 27. These tables relate the hypothesis, the university mark which corresponded to that hypothesis, the time of reporting of the corresponding university mark, acceptance or rejection of the hypothesis, and a brief conclusion for each of the hypotheses. The Faculty of Science results are given in Table 26 and the Faculty of Engineering results are given in Table 27.

Table 26

Summary of Hypotheses and Results
(Faculty of Science)

Hypothesis	Corresponding Mark	Time of Reporting	Action	Conclusion
H1	Diagnostic Test M	Beginning of university year	rejected	Students with Topics 305 obtained higher marks in the Diagnostic Test M
H2	Calculus 13.139 (final)	middle of university year	rejected	Students with Topics 305 obtained higher marks in Calculus 13.139 (final)
H3	Calculus 13.140 (final)	end of university year	not rejected	There was no difference in final marks obtained in Calculus 13.140 between students with and without Topics 305
H4	Calculus 13.141 (final)	end of university year	not rejected	There was no difference in final marks obtained in Calculus 13.141 between students with and without Topics 305

Of interest is the fact that there was a very small number of students in the sample corresponding to the fourth hypothesis. As a result there may be some lack of confidence in the corresponding findings even though the statistical procedure took the size of the sample into account.

Table 27

Summary of Hypotheses and Results
(Faculty of Engineering)

Hypothesis	Corresponding Mark	Time of Reporting	Action	Conclusion
H5	Diagnostic Test AM	beginning of university year	rejected	Students with Topics 305 obtained higher marks in the Diagnostic Test AM
H6	Calculus 6.122 (mid-term)	middle of university year	rejected	Students with Topics 305 obtained higher marks in Calculus 6.122 at mid-term
H7	Calculus 6.122 (final)	end of university year	not rejected	There was no difference in the final marks obtained in Calculus 6.122 between students with and without Topics 305
H8	Algebra and Geometry 6.123 (mid-term)	middle of university year	not rejected	There was no difference in the mid-term marks obtained in Algebra and Geometry 6.122 between students with and without Topics 305
H9	Algebra and Geometry 6.123 (final)	end of university year	not rejected	There was no difference in the final marks obtained in Algebra and Geometry 6.123 between students with and without Topics 305

Although the Hypothesis 7 was formally rejected at the .05 level, the actual level of significance of the F-ratio was calculated to be .055. The margin of difference between the accepted level and the actual level was very small.

Discussion and Conclusions

Before the present study was undertaken, the author's belief was that the successful completion of Topics 305 in high school would have a positive effect upon the marks that students obtained in their first-year of university mathematics. The findings, however, revealed a significant difference in favour of the students who completed Topics 305 in high school in only four of the nine comparisons which make up the study. These somewhat indefinite results were consistent with the results of studies found in the review of the relevant literature.

Although the statistical analysis revealed a significant difference in the mean mark obtained by each of the two groups in four of the nine comparisons the actual mean was greater for the group with Topics 305 in every one of the comparisons. In their report on The Eight-Year Study Chamberlin, Chamberlin, Drought, and Scott (1942) imply that when one finds even small margins of difference for a number of groups consistently favours the same group, it is apparent that "the probability greatly increases that the difference cannot be due to chance alone" (p. 208).

Also important in the results of this study were the

values of the unadjusted and adjusted means in each of the comparisons. When these were considered a greater difference was found between unadjusted means in six of the comparisons and a greater difference between the adjusted means in only three of the comparisons. This occurred because in six of the comparisons the mean mark in Mathematics 300 was greater for the students who had studied Topics 305 in Grade 12, and in the other three comparisons the mean mark in Mathematics 300 was greater for the students who had studied Topics 305 in Grade 12. These results, combined with the fact that the Mathematics 300 mark was used as the covariate, tended to dispel the argument that the group of students who did complete Topics 305 in Grade 12 would have a greater interest and ability in mathematics than the students who did not complete Topics 305, and thus would be expected to obtain higher marks in university mathematics.

The information thus far in the discussion supports the author's original belief that the successful completion of Topics 305 in high school indeed does have a positive effect on the marks obtained in first-year university mathematics.

Before a final conclusion could be formulated, however, two important patterns in the results warranted consideration. First, the only comparisons which produced significantly different results were comparisons of the marks earned on the diagnostic tests or on calculus courses relevant to this study. None of the comparisons of the marks earned on the course Algebra and Geometry 6.122 showed a significant

difference between the two groups. Second, the only comparisons which produced significantly different results were comparisons of marks which were reported either at the beginning or at the middle of the university year. None of the marks reported at the end of the university year showed a significant difference between the two groups.

These last two patterns, unexpectedly suggested that the successful completion of Topics 305 in high school did not have a statistically significant positive effect on any of the marks obtained in the algebra and geometry course selected for this study. There was, however, a statistically significant positive effect on the initial marks obtained in calculus at university.

The overall conclusion of this study, based on the statistical evidence, was that students who have successfully completed Topics 305 in addition to Mathematics 300 in high school initially performed better in first-year calculus than students who took only Mathematics 300 in high school.

Recommendations

On the basis of the findings of this study, a number of recommendations seem defensible:

- 1) Students are recommended to enroll in Topics 305 in Grade 12 if they intend to study mathematics, in general, and calculus in particular, at university.

- 2) A study to verify the Mathematics 300 mark as a strong predictor of success in university mathematics is recommended.
- 3) The characteristics of students who select Topics 305 in high school are at present unknown. A study to determine differences in characteristics, including mathematical ability and attitude toward mathematics between students who do, and students who do not study Topics 305 in Grade 12 is recommended.
- 4) A study similar to the present study is recommended to determine if the same conclusions would be obtained.

References

- Ahamann, S., Aspects of Curriculum Evaluation: A Synopsis. Perspective of Curriculum Evaluation (AERA Monograph Series No. 1). Chicago: Rand McNally & Co., 1967, 39 - 83.
- Akeju, S., & Michael, W., Predicting success in the Federal School of Science, Lagos, Nigeria. Educational and Psychological Measurement, 1970, 30, 483 - 486.
- Alkin, M., Towards an Evaluation Model: A Systems Approach. In P. Taylor & D. Cowley (Eds.), Readings in Curriculum Evaluation, Dubuque Iowa: W.C. Brown Co., 1972.
- Austin, H., The effects of high school calculus on achievement during the first semester in college calculus for first-year students (Doctoral dissertation, University of Virginia, 1975). Dissertation Abstracts International, 1975, 36, 2728A.
- Begle, E., & Wilson, J., Evaluation of Mathematics Programs. Mathematics Education (The sixty-ninth Yearbook of the National Society of the Study of Education). Chicago: The University of Chicago Press, 1974, 367 - 404.
- Brasch, B., An evaluation of some predictors of student success in calculus (Doctoral dissertation, University of Denver, 1972). Dissertation Abstracts International, 1972, 33, 27A - 29A.

Bruner, J., The Process of Education, Cambridge, Harvard University Press, 1966.

Chamberlin, D., Chamberlin, E., Drought, N., & Scott, W.,
Did They Succeed in College. New York: Harper and Brothers, 1942.

Colman, J., Evaluating Educational Programs. The Urban Review, 1969, 3, 6 - 8.

Comley, R., A follow up of VICSM students who started Course I in 1958 and 1959. University of Illinois, 1965.
(Eric Document Reproduction Service No. ED 039 124).

Coon, L., SMSG mathematics as a factor influencing success in freshman calculus (Doctoral dissertation, Oklahoma State University, 1963). Dissertation Abstracts International, 1963, 25, 4475A.

Cronbach, L., Course Improvement Through Evaluation.
Teachers College Record, 1963, 64, 672 - 683.

Cross, R., Student achievement and attitude in a modern and a traditional grade ten geometry program.
Unpublished Master of Education thesis, University of Manitoba, 1968.

Dessart, D., & Burns, P., Summary of investigations relating to mathematics in secondary education: 1965. School Science and Mathematics, 1967, 67, 135 - 144.

Gilbert, R., A study in the transition from twelfth year mathematics to the calculus as viewed by students and their instructors (Doctoral dissertation, State University of New York at Buffalo, 1971). Dissertation Abstracts International, 1971, 32, 2504A.

Grigsby, C., An investigation into the nature of current calculus experiences in high school and the relationship between these experiences and final achievement level reached in the introductory calculus courses at the university (Doctoral dissertation, University of North Carolina, 1973). Dissertation Abstracts International, 1973, 35, 280A.

Hickman, J., A study of various factors related to success in first semester college calculus (Doctoral dissertation, University of Southern Mississippi, 1969). Dissertation Abstracts International, 1969, 30, 2252A.

Ivanoff, J., & Evermade, T. Use of discriminant analysis for selecting students for ninth-grade algebra or general mathematics. The Mathematics Teacher, 1965, 58, 412 - 416.

Johnson, D. A pattern for research in the mathematics classroom. The Mathematics Teacher, 1966, 59, 418 - 425.

- Kline, M. A proposal for the high school mathematics curriculum. The Mathematics Teacher, 1966, 59, 322 - 330.
- Lovet, C. An analysis of the relationship of several variables to achievement in first-year algebra. (Doctoral dissertation, University of Texas, 1969). Dissertation Abstracts International, 1969, 30, 1470A.
- McKillip, W. The effects of secondary school analytic geometry and calculus on students' first semester calculus grades at the University of Virginia (Doctoral dissertation, University of Virginia, 1965). Dissertation Abstracts International, 1965, 26, 5920A.
- Morgan, W. Prediction of success in junior college mathematics. The Mathematics Teacher, 1970, 63, 260 - 263.
- Paul, H. The relationship of various high school mathematics programs to achievement in the first course in college calculus (Doctoral dissertation, The Ohio State University, 1970). Dissertation Abstracts International, 1970, 31, 3396A - 3397A.
- Pugh, R. An analysis of achievement behaviour in selected mathematics courses. Bloomington, Indiana: Indiana University, 1969. (Eric Document Reproduction Service No. ED 013 371).

Richert, A. The effect of different types of twelfth-grade mathematics courses on achievement in a first course in university calculus (Doctoral dissertation, University of Kansas, 1972). Dissertation Abstracts International, 1972, 33, 6072A.

Shibli, J. Recent Developments in the Teaching of Geometry. Pennsylvania: J. Shibli Publishing, 1932.

Shimizu, M. Achievement in senior advanced mathematics and first-year college mathematics. The Mathematics Teacher, 1969, 57, 311 - 315.

Sommers, D. A study of selected factors predictive of success in calculus at Hope College (Doctoral dissertation, The Ohio State University, 1973). Dissertation Abstracts International, 1973, 34, 6885A.

Tillotson, D. The relationship of an introductory study of calculus in high school to achievement in a university calculus course (Doctoral dissertation, University of Kansas, 1962). Dissertation Abstracts International, 1962, 24, 577A - 576A.

Tuscher, M. A proposed model for predicting success in a first course of college calculus in a community junior college (Doctoral dissertation, University of Southern California, 1972). Dissertation Abstracts International, 1972, 33, 1468A - 1469A.

- Vandruff, J. Prediction of success of community college students in calculus in the state of Washington (Doctoral dissertation, Arizona State University, 1973). Dissertation Abstracts International, 1973, 34, 1055A.
- Weaver, J. Evaluation and the Classroom Teacher. Mathematics Education (The sixty-ninth Yearbook of the National Society for the Study of Education). Chicago: The University of Chicago Press, 1970, 335 - 366.
- Wick, M. A study of factors associated with success in first-year college mathematics. The Mathematics Teacher, 1965, 58, 642 - 648.
- Wilson, R., & Gelso, C. The prediction of grades in college algebra --- a continuation and extension. Los Angeles, California: University of California, 1967. (Eric Document Reproduction No. ED 011 778).

APPENDIX A

Course Outline for Topics in Mathematics 305

(Topics 305)

PROVINCE OF MANITOBA
DEPARTMENT OF EDUCATION

TOPICS IN MATHEMATICS 305

HALF CREDIT
PILOT COURSE

DRAFT - HALF COURSE

TOPICS IN MATHEMATICS 305

(Pilot Course)

NOTE: Mathematics 305 is not to be substituted for Mathematics 300. A student enrolled in Mathematics 305 should have completed or be currently enrolled in Mathematics 300.

Rationale

Most of the topics in this course have been selected on the basis of their usefulness in background material for university studies in chemistry, physics, engineering, statistics, computer science, mathematics and commerce.

For students considering attending universities outside Manitoba, they will find that, in many cases, the background material provided in this course is necessary in the above fields.

REFERENCES

Primary References

Since no single text covers the complete basic outline, it is recommended that each teacher have a copy of the six texts listed below. It is not suggested that class sets be purchased.

Del Grande et al. Calculus. Agincourt, Ontario: Gage Educational Publishers.

Edwards, R. Introduction to the Theory of Numbers. Boston, Mass.: Houghton Mifflin Publishers.

Elliott et al. Calculus, Complex Numbers and Polar Co-ordinates. Toronto, Ontario: Holt, Rinehart and Winston of Canada Ltd.

Fisher, R.C. Integrated Algebra and Trigonometry. 3rd ed. Scarborough, Ontario: Prentice-Hall of Canada, Ltd.

Hart, W.L. Algebra - Elementary Functions and Probability. Toronto, Ontario: D.C. Heath Publishers, 1965.

Shanks, M.E. Pre-Calculus Mathematics. 2nd ed. Don Mills, Ontario: Addison-Wesley Publishing Co., 1972.

Supplementary References:

- Ayres. First Year College Mathematics : Schaum's.
Scarborough, Ontario: McGraw-Hill Ryerson Ltd.
- Brink. (out of print). A First Year of College Mathematics.
- Friedberg, R. An Adventurer's Guide to Number Theory.
Toronto, Ontario. McGraw-Hill Ryerson Ltd.
- Glecksman, A.M. An Introduction to Linear Programming and
the Theory of Games. New York, N.Y.: John Wiley and
Sons Publishers, 1963.
- Nichols, D. Modern Intermediate Algebra. Revised edition.
Toronto, Ontario: Holt, Rinehart and Winston, 1970.
- Protter and Morrey. Calculus With Analytic Geometry; A
First Course. Addison-Wesley.
- Vance, E.P. An Introduction to Modern Mathematics. Don Mills,
Ontario: Addison-Wesley Publishing Co., 1968.

COURSE OUTLINE

Unit I Complex Numbers (Suggested Time: 4 hours)

1. Definition of $a + bi$
2. Absolute value
3. Graphical representation
4. Addition and subtraction of complex numbers
5. Multiplication of complex numbers
6. Conjugate complex numbers
7. Division of complex numbers

Unit II Theory of Equations (Suggested time: 15 hours)

1. Solving quadratic equations involving complex coefficients and roots
2. Synthetic division
3. Remainder and factor theorems
4. Rational root theorem
5. Limits to real roots
6. Fundamental theorem of algebra
7. Complex roots occur in conjugate pairs
8. Solving word problems including:
 - a) linear equations
 - b) quadratic equations
 - c) non-linear systems

Unit III Calculus of Polynomials (Suggested time: 14 hours)

1. Intuitive definition of a limit of a function at a point. (do not stress the epsilon - delta definition). Emphasize algebraic examples (e.g.: rational expressions, square root expressions).
2. Definition of a derivative at a point as the limit of the slope of the secant. i.e., $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$
3. A second interpretation of a derivative as a rate of change.
4. Differentiation techniques
 - a) power rule
 - b) Sum
 - c) Product
 - d) Quotient
 - e) Chain rule

5. Applications

- a) equations of tangent lines to a point on the curve
- b) velocity and acceleration
- c) maximum and minimum word problems

Unit IV Polar Co-ordinates (Suggested time: 6 hours)

- 1. Polar co-ordinate system
- 2. Polar graphs
- 3. Transformation: polar/cartesian
- 4. Complex numbers in polar form

Unit V Number Theory (Suggested time: 6 hours)

- 1. Principle of mathematical induction
- 2. Division algorithm
- 3. Divisibility properties of integers
 - a) greatest common divisor (Euclidean algorithm)
 - b) least common multiple
 - c) Fundamental Theorem of Arithmetic
- 4. Prime numbers
 - Sieve of Eratosthenes
 - infinitude of primes

5. Divisibility criteria

Unit VI Three-Dimensional Geometry (Suggested Time: 6 hours)

- 1. Plotting points
- 2. Distance formula
- 3. Direction cosines
- 4. Direction numbers
- 5. Parametric equations of lines
- 6. Standard equation of the plane
($Ax + By + Cz + D = 0$)
- 7. Distance from a point to a plane
- 8. Visualizing quadric surfaces

Unit VII Linear Programming (Suggested time: 4 hours)

- 1. Review of linear inequalities
- 2. Linear programming in two variables using analytic geometry.

A GUIDE TO TEACHERS

1. In each of the units the construction of mathematical models (word problems) should be emphasized, where possible.
2. A survey is included. It is to be completed as the course is being piloted. It will include comments on the course, amount of time spent on each topic, the problems that arise, the resources used, etc.
3. The number of teaching (including testing) hours is 55-60.
4. The following material should be incorporated into the course wherever possible:

Nature of Proof

- a) Counterexample: its purpose and use in proof
 - b) Qualifiers
 - universal qualifier "all"
 - existential qualifier "some"
 - negation of qualifiers
 - how qualifiers and counterexample are related
 - c) Definitions
 - built from undefined words
 - using undefined terms to form defined terms
 - what constitutes a good definition
 - d) Proof
 - deductive proof
 - inductive proof
 - direct proof
 - indirect proof
 - e) Necessary and sufficient conditions
5. Teachers are encouraged to show students the proofs of as many theorems as possible within the limit of the time allotted for each particular unit.

EVALUATION SHEET

305 Mathematics

NAME _____

SCHOOL _____

TOPICS

STATISTICS

	1	2	3	4	5	6	7		1	2	3	4	5	6
A. <u>Teacher reaction:</u>														
1. Should this unit be included?														
2. Were you interested in this unit?														
3. Class time spent on unit (# hours)?														
4. Test time (hours)?														
5. Time spent on teacher preparation														
6. Class time spent on teacher preparation														
B. <u>Student reaction:</u>														
1. Were the students interested in this unit?														
2. Did students learn the material?														
3. Level of difficulty of material?														
4. Out-of-class time spent by students?														

C. Please list and comment on main textbooks and references used.

- a. Additional references which you used but which are not on the reference list included with the outline.

- b. Which (if any) of the references in (a) would you recommend be added to the reference list?

D. Suggestions:

- a. Other subject matter which should be included:

- b. Subject matter which should be removed from course:

- c. Subject matter which should be given more emphasis:

- d. Subject matter which should be given less emphasis:

- E. Please comment and make suggestions for improvement re the over-all content and continuity of each half course with which you have been involved.

Topics In Math 305:

Statistics and Probability 305:

F. Initial Enrollment _____

Final Enrollment _____

Please return this form upon completion of the half-course(s) to:

Mathematics Consultant
Program Development Secretariat
404-1181 Portage Avenue
Winnipeg, Manitoba
R3G 0T3

APPENDIX B

Course Descriptions From the
University of Manitoba Calendar 1975-76
for:

- a) Calculus 13.139
- b) Calculus 13.140
- c) Calculus 13.141
- d) Calculus 6.122
- e) Algebra and Geometry 6.123

13.139 Introduction to Calculus (0-0:3-2)3 or (3-2:0-0)3

Differentiation and integration of elementary functions, with applications to maxima and minima, rates of change, area, and volume. Students may not hold credit for more than one of: 139, 6.122, or the former courses 120, 134, or 136.

Efficiency in high-school mathematics will be measured by a diagnostic test administered during the first class. Students failing to achieve a satisfactory standard will be expected to register concurrently in 90.

13.140 Calculus for the Physical and Mathematical Sciences (0-0:3-2)3 or (3-2:0-0)3.

Theory and techniques of integration, curve sketching, arc length, and surface area with applications to the mathematical and physical sciences. Students may not hold credit for more than one of: The former courses 120 or 134, 140, 141, or 6.122. Prerequisite: 139.

13.141 Calculus for the Biological and Social Sciences (0-0:3-2)3 or (3-2:0-0)3.

Techniques of integration, curve sketching, Taylor's formula, partial derivatives, with applications to the social and biological sciences. Students may not hold credit for more than one of: The former courses 120 or 134, 140, 141, or 6.122. Prerequisite: 139.

6.122 Calculus (3-L:3-L)6

Calculus of functions of one variable, treated strictly, but with stress on applications. For students in Engineering only. Students may not hold credit for 122 and any other first year calculus course. Corequisite: 123.

6.123 Algebra and Geometry (3-L:3-L)6

A basic course in real and complex algebra, matrices, two- and three-dimensional geometry, and vectors, with stress on techniques and applications. For students in Engineering only. Students may not hold credit for more than one of 123, 13.223, 13.235, or the former courses 13.135 or 13.244.

APPENDIX C

Questionnaire

QUESTIONNAIRE

For All First Year University Mathematics Students

INTRODUCTION:

As a high school mathematics teacher I am concerned about the adequacy of the present high school mathematics courses in preparing students for university mathematics. Consequently, I am conducting a study to determine which high school mathematics course(s) best prepare students for mathematics at university.

Your assistance is requested by completing the following questionnaire, regarding your background in high school mathematics. Permission is also requested to obtain and use the university mathematics marks obtained by you during the 1976-77 academic year. Please indicate your willingness to allow your marks to be used for the study outlined by completing the questionnaire and signing your name in the appropriate space on the opposite side of this page.

- NOTE: 1) All information received will be kept in STRICT CONFIDENCE and will be used ONLY for the purpose of the study outlined above.
- 2) Information received will in no way effect the grades obtained in ANY of your present or future courses.
- 3) Names will NOT be published in reporting the results of this study.

Thank you for your cooperation and assistance.

W.A. Johnson

QUESTIONNAIRE:

NOTE: Topics 305 is a one-half credit, grade XII mathematics course, developed by the Manitoba Department of Education and offered, in addition to the regular Math 300, on an experimental basis, in a limited number of Manitoba high schools during the 1975-76 school year.

Please answer the questions as accurately as possible by either circling the correct response or clearly printing your response in the space provided.

NAME _____ Sex: M _ F (circle one)

University of Manitoba student number _____ Age _____

1. Faculty in which you are registered. a) Arts _ b) Engineering c) Science
d) Other (Specify) _____ (circle one)
(Engineering students ONLY should omit questions 2 and 3 and go directly to question 4.)
2. Intended major area of study, if known. a) Math b) Physics c) Chemistry
d) English e) History f) Biology g) Other (specify) _____
h) Undecided (circle one)
3. Mathematics course you intend to take in the second term of this year.
a) 13:140 b) 13:141 c) other (specify) _____ (circle one)
4. High School attended _____
Address of high school (city or town only) _____
5. Year of graduation from high school. _____

-2-

6. Did you take Topics 305 in grade XII? a) Yes b) No (circle one)
If "Yes" state final mark _____
7. Please indicate the regular mathematics course that you took in grade XII by stating the final mark obtained in the space provided.
a) Math 300 _____ b) Math 301 _____
c) Other (specify) _____
(course) (mark)
8. Did you write a final exam on the entire course in the regular mathematics program in grade XII? a) yes b) no (circle one)
If "Yes" state time length of the exam. _____
9. (To be completed only by students who did NOT take Topics 305)
Why did you not take Topics 305 in grade XII? (circle one or more)
a) not offered in my school
b) did not know about the course
c) prevented by timetable conflicts
d) was not interested
e) other (specify) _____

Permission:

Please indicate your willingness to allow your marks to be used for the purposes of the study outlined by signing your name in the space below.

Date _____

Signature _____

APPENDIX D

Letters

Wayne Arthur Johnson,
14 Brunlea Key,
Winnipeg, Manitoba,
R2G 2C6.
June 23, 1976.

Dr. L.C. Jansson,
Graduate Studies and Research Committee,
Chairman,
Faculty of Education,
University of Manitoba,
Winnipeg, Manitoba.

Dear Dr. Jansson:

As a mathematics teacher and a graduate student in the Faculty of Education working under the direction of Dr. A.M. McPherson, I am interested in the adequacy of preparation of high school students for university mathematics. For my Masters thesis I propose to relate achievement in first year Calculus to participation in an enrichment course in mathematics in grade XII (Topics 305).

Specifically, I wish to compare the marks obtained in first year Calculus by students in each of the following two groups:

- a) students who have taken Topics 305 in addition to the regular grade XII mathematics program.
- b) students who have taken only the regular grade XII mathematics program.

The purpose of this letter, to you as chairman of the committee acting for the committee on Research on Human Subjects, is to request permission to gain access to the marks obtained by the students in each of the above two groups on each of the following:

- a) diagnostic test (administered by Dept. of Math and Astronomy)
- b) Calculus 13:139
- c) Calculus 13:140
- d) Calculus 13:141
- e) diagnostic test (administered by Dept. of applied Math)
- f) Calculus 6:122 (mid term mark)
- g) Calculus 6:122 (final mark)

The information obtained would be kept in strict confidence and used only for the purposes of the study outline above. Note, also, student names would not be published in the thesis.

I would appreciate the requested marks for the 1976-77 academic year.

Yours truly,

W.A. Johnson.

Wayne Arthur Johnson,
14 Brunlea Key,
Winnipeg, Manitoba,
R2G 2C6,
July 6, 1976.

Dr. N. S. Mendelsohn,
Mathematics Department Head,
University of Manitoba,
Winnipeg, Manitoba,
R3T 2N2.

Dear Dr. Mendelsohn:

As a graduate student, working under the direction of Dr. A. M. McPherson, in the Faculty of Education, I am interested in the adequacy of preparation of high school students for first year university Calculus. (i.e. diagnostic test, 13:139, 13:140, 13:141) Specifically, I would like to determine if students, who have taken a particular enrichment course in mathematics are better prepared for first year calculus than the students who have taken only the regular mathematics program.

The purpose of this letter is to request permission to conduct a survey of first year calculus students, via questionnaire, to determine background information of the students for the study outlined above. I would like the questionnaire to be completed by the students as early as possible in the 1976-77 academic year. The time to complete the questionnaire would be about 5 minutes.

Enclosed is a draft of the questionnaire that I wish to have completed by the students.

Your cooperation would be very much appreciated.

Yours truly,

W.A. Johnson.

Wayne Arthur Johnson,
14 Brunlea Key,
Winnipeg, Manitoba,
R2G 2C6,
July 6, 1976.

Dr. F. M. Arscott,
Applied Mathematics Department Head,
University of Manitoba,
Winnipeg, Manitoba,
R3T 2N2.

Dear Dr. Arscott:

As a graduate student, working under the direction of Dr. A. M. McPherson, in the Faculty of Education, I am interested in the adequacy of preparation of high school students for first year university mathematics. (i.e. diagnostic test, 6:122, 6:123). Specifically, I would like to determine if students, who have taken a particular enrichment course in mathematics are better prepared for first year mathematics than the students who have taken only the regular mathematics program.

The purpose of this letter is to request permission to conduct a survey of first year mathematics students, via questionnaire, to determine background information of the students for the study outlined above. I would like the questionnaire to be completed by the students as early as possible in the 1976-77 academic year. The time to complete the questionnaire would be about 5 minutes.

Enclosed is a draft of the questionnaire that I wish to have completed by the students.

Your cooperation would be very much appreciated.

Yours truly,

W.A. Johnson.

Wayne Arthur Johnson,
14 Brunlea Key,
Winnipeg, Manitoba,
R2G 2C6,
July 15, 1976.

Dr. F. M. Arscott,
Applied Mathematics Department Head,
University of Manitoba,
Winnipeg, Manitoba,
R3T 2N2.

Dear Dr. Arscott:

Further to my letter of July 6, 1976 requesting permission to circulate a questionnaire to all 6:122 students, I am enclosing, for your inspection, a copy of the final version of the questionnaire.

The questionnaire has been approved by all three members of my thesis committee.

Again, thank-you for your cooperation.

Yours truly,

W.A. Johnson

Wayne Arthur Johnson,
14 Brunlea Key,
Winnipeg, Manitoba,
R2G 2C6,
July 15, 1976.

Dr. N. S. Mendelsohn,
Mathematics Department Head,
University of Manitoba,
Winnipeg, Manitoba,
R3T 2N2.

Dear Dr. Mendelsohn:

Further to my letter of July 6, 1976 requesting permission to circulate a questionnaire to all 13:139 students, I am enclosing, for your inspection, a copy of the final version of the questionnaire.

The questionnaire has been approved by all three members of my thesis committee.

Again, thank-you for your cooperation.

Yours truly,

W.A. Johnson



The University of Manitoba

-89-

Faculty of Education
Graduate Studies Office
Telephone: 474-9012

Winnipeg, Manitoba R3T 2N2

16 July 1976.

Mr. Wayne Johnson,
14 Brunlea Key,
Winnipeg, Manitoba.
R2G 2C6.

Dear Wayne,

In response to your letter of 23 June 1976, I am pleased to inform you that the Faculty of Education Ethics Review Subcommittee of the Committee on Research Involving Human Subjects (CRIHS) has approved your plans for research related to your Master's program.

It is my understanding that you will be working closely with the Departments of Mathematics and Applied Mathematics and I will forward copies of this approval to them. While not stated explicitly in your June 23rd letter, it has also been agreed that permission will be sought from students to use their course grades.

I wish you success in this project.

Sincerely,

Lars C. Jansson,
Chairman,
Graduate Studies and Research Committee,
Ethics Review Subcommittee (CRIHS).

LCJ/lp.

c.c. A. M. McPherson, Head, Curriculum: Mathematics and Natural Sciences.
N. S. Mendelsohn, Head, Mathematics and Astronomy.
F. M. Arscott, Head, Applied Mathematics.
R. Quackenbush, Assistant Head, Mathematics and Astronomy.



The University of Manitoba

Winnipeg, Manitoba
Canada R3T 2N2

Faculty of Science
Department of Applied Mathematics

-90-

Room 553 Machray Hall,
July 26, 1976.

Mr. Wayne Arthur Johnson,
14 Brunlea Key,
Winnipeg, Manitoba
R2G 2C6.

Dear Mr. Johnson:

In response to your letter of July 6th, I am happy to authorize the giving of your questionnaire to first year students of mathematics in the Faculty of Engineering. Approval has been given by the Associate Dean of Engineering.

We shall be glad to collaborate with you in this project and will be interested to learn the results of your investigation.

Yours sincerely,

F. M. Arscott,
Head of Department.

FMA/gh



The University of Manitoba

Department of Mathematics and Astronomy

-91-

Winnipeg, Canada R3T 2N2

August 6, 1976

Mr. W. A. Johnson
14 Brunlea Key
Winnipeg, Manitoba
R2G 2C6

Dear Mr. Johnson:

The Department of Mathematics is most pleased to be able to co-operate with you in your study of the mathematical preparation of first year calculus students.

Sincerely yours,

R. W. Quackenbush,
Assistant Head,
Department of Mathematics.

RWQ/lg



The University of Manitoba

Department of Mathematics and Astronomy

-92-

Winnipeg, Canada R3T 2N2

August 17, 1976

Mr. Wayne A. Johnson
14 Brunlea Key
Winnipeg, Manitoba
R2G 2C6

Dear Mr. Johnson:

You have the permission of the Department of Mathematics to administer to our First Year Calculus Classes the Questionnaire regarding preparation of high school students for university mathematics, as discussed with Prof. Quackenbush.

You will also have access to the marks obtained in the diagnostic tests and the first year calculus results.

Yours truly,

N. S. Mendelsohn, Head
Department of Mathematics

NSM/lg