ENTROPY BASED REDUNDANCY MEASURES IN WATER DISTRIBUTION NETWORK DESIGN

By

KOFI AWUMAH

A Thesis
Submitted to the Faculty of Graduate Studies
in Partial Fulfillment of the Requirements
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DOCTOR OF PHILOSOPHY

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Abstract

Reliability and flexibility in the face of failure conditions are implied aspects of redundancy in water distribution networks. Explicit and comprehensive measures which are computationally feasible have not yet been developed for either reliability or redundancy of water distribution networks. This lack of an acceptable measure or approach leads to the continued increase in the development of methodologies and surrogates to replace measures for system reliability and redundancy. In this thesis a measure of the redundancy inherent in the layout (geometric configuration) and component sizes of water distribution networks is developed using an approach based upon the desired properties of such a measure. Both local redundancy at a node and the redundancy measure for the whole network are developed. The measure is examined by an application to candidate layouts obtained from the solutions of a distribution layout design model. A comparison between redundancy measures and network performance, as indicated by percentage of flow supplied under a range of link failures, and network probabilistic reliability, as indicated by nodal pair reliability, demonstrates that an increase in the value of redundancy, as measured by the entropic parameter, increases the ability of the water distribution network to respond to failure problems in the network. The measure therefore holds value as a statement of redundancy. The value of the measure in design of water distribution networks is demonstrated by the use of the models in the least cost design and multi-objective analysis of example networks.

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Chapter 1

INTRODUCTION

The subject matter of this thesis is the design of water distribution networks with special emphasis on the robustness, resilience and reliability of the final design, from the standpoint of both the layout and the components that compose the network. A water distribution network is essentially a network of pipes of specific sizes carrying water to demand centers. Other components such as pumps, valves, and storage tanks are also part of the network but this work will focus principally on pipes and pumps since these elements account for a large portion of the network cost and the network reliability is highly dependent on their state or condition.

In order to handle the design process mathematically, water distribution networks are modelled as nodes connected by links. Consumer units of water are grouped together as nodes and assigned to the end of the nearest link, the links being pipe mains. Only the main pipes are considered in the design process, the smaller service pipes connecting the households to the main pipes being ignored. If the rate of water use by the demand nodes are known, the traditional design procedure becomes one of choosing the diameters of the main pipes to satisfy the nodal demands and specified minimum nodal pressure head standards.

The design of water distribution networks entails several problems. The hydraulic aspect of the design problem may be considered to have been adequately addressed since existing networks designed on the basis of the accepted developed formulae involving the hydraulic parameters (the relationship among the parameters of flow, diameter, and the headloss within a pipe) have been successful. The aspect of water distribution network design which has not been fully addressed, but is now receiving much attention, is reliability. This reliability is being assessed in terms of the failure rate of the components and their effect on the network and the failure of the distribution network to provide the required level of service during critical conditions. Factors contributing to the need for reliability consideration include uncertainties in demand by consumers, fire flow requirements and their locations, pipe failures and their locations, insufficient storage, and pumping failures. Consideration of reliability in water distribution networks is now receiving greater attention because of the fact that many portions of the existing water distribution systems are old resulting in increased levels of component failure and reduced capacities within the components themselves.

Due to the large costs involved in building and maintaining this essential utility, the aspect of the design that received the most attention in recent years was that which provided the desired level of performance at minimum cost. However, it was concluded that least cost solutions of networks can only be achieved at the expense of reliability. Looped layouts were therefore proposed to make municipal water distribution networks more redundant and therefore more reliable. Least cost optimization however drives looped layouts into branched networks. To prevent the networks from being driven into a branched condition, it was proposed that a constraint of minimum pipe sizes be included to ensure that loops remain in the network (Alperovits et al., 1977 and Quindry et al., 1981). The resulting solution is usually an implicit branched network with low capacity or weak connections consisting of very small diameter pipes between the major branches and is not really a looped network. It can be concluded that the joint consideration of network cost and network reliability is a multiobjective issue. This multiobjective nature of water distribution networks design has not been adequately addressed, however. The main problem of the multi-objective approach is the complexity of computing the probability of network failure given the failure rate of the components and variability of demands on the network. Another problem with the reliability issue is the fact that there are no comprehensive and generally acceptable measures available for network, as opposed to the complete (source-treatment-distribution) system reliability. A comprehensive reliability measure is one that includes all relevant characteristics of the network that contribute to its reliability. To be acceptable, the measure should be computationally feasible enough to be applicable to large networks.

Since there are no easy ways of incorporating all the uncertainties inherent in a water distribution network in the 'classical' design models (optimization design models that are non-iterative), a plausible approach might be the use of simulation models, such as that by Morgan and Goulter (1985). Simulation models are time consuming, however, and are therefore usually applied to a few final alternative layouts and demand patterns in the final design stage.

To this time, network reliability research has focussed on the hydraulic performance of the network under a range of assumed mechanical failures and demand conditions. These approaches address the reliability problem from a hydraulic perspective without recourse to general graph (network) theory which might help define the underlying robustness of the network. However, it has been argued that the shape or layout of a network determines how much reliability can be imposed on a network (Goulter, 1988). A measure which gives an indication of the underlying reliability of a network would be useful for defining inherently good designs. This layout issue of water distribution network design has however not been well addressed by researchers in this field.

Redundancy is an attribute of the network geometry that is closely related to its reliability, and may be considered as another form of a reliability measure reflecting resilience or flexibility of the network to imposed external conditions. A truly redundant network is inherently reliable or resilient; a truly redundant network ensures that if a single component fails, there is sufficient residual capacity in the network to provide all flow requirements. Furthermore, redundancy becomes more important as the network enters the 'old age' stage of its life span, when maintainance as well as reconstruction works become predominant. In this stage, therefore, the redundancy built into the system at the planning stage becomes very useful, and may be the only means by which consumers will receive uninterrupted service while repair or reconstruction works proceed.

In spite of this relevance to the reliabilty issue and the growing emphasis on reliability, very few measures have been developed to ensure adequate redundancy in water distribution network layouts. The measures that do exist incorporate, to varying degrees, the factors that contribute to reliability (e.g., Rowell and Barnes, 1982, Wagner et al., 1988b). Even though the measures are therefore incomplete, they still provide guidance as to the condition of the network; higher values of the reliability measure mean better reliability. Redundancy is a more difficult network characteristic to define completely and explicitly. A good measure for redundancy will be as useful as the existing reliability measures are to the network reliability problem and may even help overcome some of the problems associated with network reliability measure by looking at the geometric configuration of the network.

The objective of this thesis is therefore to focus on the reliability issue of the design of water distribution networks via its redundancy. A suitable measure will be developed for redundancy inherent in water distribution networks to help in the selection of a reliable network. This measure should be quantitative and be characterised by the following features;

1. The higher the value of redundancy measure, the more redundant and reliable is the network.

2. Changes to a network to improve redundancy will be reflected in the value of the redundancy measure

3. The measure will be able to distinguish between networks which contain subtle differences which cause them to be quite different in terms of redundancy

4. The measure would be capable of being incorporated into optimization design models for the purpose of either imposing a required level of redundancy on the network or allocating redundancy within the network.

Chapter 2

LITERATURE REVIEW

2.1 Introduction

Work on water distribution network design methods can be classified into two major categories, the first being development of classical pipe network design methods and the second being the development of pipe network optimization techniques. Perhaps the first of the classical methods was that of Cross (1936). This procedure is known as the Hardy-Cross method and is able to be employed using manual calculation. In this method, for a given pipe layout and demand pattern, pipe sizes are first assumed, flows balanced to satisfy continuity at the nodes, and pressure heads at the nodes determined. A check is then made to identify where the pressure heads are below minimum or above maximum desired values. The pipe sizes are then changed (diameters increased or decreased if pressures are below or above desired level respectively) and the flow balancing repeated until satisfactory results are obtained.

With the advent of the digital computers in the 1960's the analysis and design of water distribution networks underwent a major adjustment, most of which was directed at exploiting the newly available computing power. More specifically two new types of pipe network solution methods evolved out of the Hardy-Cross method.

These are the Newton-Raphson methods (Lui, 1969, Epp and Fowler, 1970, and Donachie, 1973) and Linearization methods (Wood and Charles, 1972, Fietz, 1973, and Collins and Johnson, 1975). The methods are essentially techniques for solving a set of non-linear equations, are iterative in nature and involve the use of an initial trial solution, followed by solution of a new problem which becomes an initial solution for the next iteration. The process is continued until there is no significant difference between two successive iterations. There can be convergence problems with these techniques, however. The approach is therefore essentially to use computer models that will do in a shorter time what was previously being done manually. However, the models are also able to handle considerably more complex systems than that normally able to be handled with the Hardy-Cross method.

The design approach taken in these models is to initially construct and calibrate a mathematical model, such as the model called KYPIPE (Wood, 1980). A simulation of emergency situations is then done using the model to derive alternative solutions using different pipe sizes, pumps, tanks and valves. The costs of the different alternatives may then be calculated and compared to arrive at a recommended solution.

The need or perhaps more specifically, the desirability for a modification to this design process arose because a network designed by the approaches

> "... usually consists of specifying a highly redundant layout, designing the individual arcs of the network very conservatively so that they are more than capable of meeting a single 'worst load case', and then simulating some actual extreme load cases on the network to ensure that the network is sufficiently flexible to meet them. When total network cost is not of prime importance, this process works satisfactorily. The ability to meet demand patterns other than the design demand is a result of the

choice of a highly redundant layout which gives the possibility of multiple flow paths and the oversizing of the arcs which gives these multiple paths spare capacity to carry extra flows. When total network cost is of prime importance, this design process is less satisfactory, and when cost optimization methods are used, the defects of the design process become very apparent." [1]

Prior to development of optimization approaches, the overall network design process had essentially become that of a step by step extension of existing water supply facilities in cities necessitated by the increasing populations in these areas. However, with the rapid urban growth in developing countries, the need for modernization of the facilities in developed countries due to their age, and high operational costs due to energy problems all put pressure on engineers to apply optimization principles to the design methodology. The application of these optimization principles/approaches did not occur without difficulty, however. Overviews of early water distribution network design using computer optimization models were provided by de Neufville et al. (1971) and Walski (1985). Both papers give a good account of the then current optimization models and problems associated with their application to real water distribution network design.

There are many ways of classifying the optimization models of water distribution networks. In this review, they are classified under two headings.

1. Models that are based on the minimum cost design of water distribution networks.
2. Models that consider both the cost and the reliability of water distribution networks.

[1] Templeman, A.B., "Discussion of Looped water distribution systems", *Journal of the Enviromental Engineering Division, ASCE*, pg 599, June 1982

The above two categories also reflect the changing emphasis in design philosophy of water distribution networks. Initially, cost minimization was the sole objective of optimization models, but recently, reliability of the network is being given equal consideration.

2.2 Models For Minimum Cost Design of Pipe Networks

These are models that are based on the relatively simple objective function of cost minimization subject to hydraulic constraints. The constraints do not usually include any measure of overall network performance or reliability. They ensure only the usual hydraulic considerations of flow continuity at nodes (the satisfaction of demand flows) and minimum pressure standards at selected nodes. Some complications that these models encounter include the occurence of loops in the networks, the very large number of variables arising from the natural complexity of the networks, the discrete nature of pipe sizes commercially available, approximation of discrete cost functions, stochastic nature of water demand (residential, commercial and fire fighting), the need for storage within the network, pump selection and operation, and topographical problems that may affect pressure profiles and which may cause a need for pressure reducing valves.

One of the first models developed that fall in this category is that of Karmeli et al. (1968). Their model, which is a linear programming model, is applicable to branched networks only, but it can also be adapted to handle multiple loadings. The decision variables are the lengths of discrete candidate pipe sizes which are determined so that the headlosses in paths from the source to each node are such that the minimum pressure requirements at the nodes are not violated. The cost function is easily formulated since for a given pipe size, the cost is a linear function of its length. It was shown later by Bhave (1979) and Fujiwara and Dey (1987) that only two adjacent pipe sizes in the candidates for each link will be chosen by the model, hence the dimensionality problem of having to specify a very large number of candidate pipe diameters to ensure feasibility and optimality was solved.

Jacoby (1968) was one of the first to propose a minimum cost optimization model for looped networks. His method involved the use of non-linear continous cost functions and included pumping costs. The solution strategy involved a merit function to move the solution towards a local optimum through the use of penalty costs for constraint violations.

Alperovits and Shamir (1977) also developed an approach to looped networks by extending the work of Karmeli et al. (1968) to looped networks and also addressed the following complications;

1. multiple sources
2. inclusion of pumping cost in the objective function
3. inclusion of pressure reducing valves
4. extension of an existing system
5. operation of an existing system

Their method is iterative, however, and involves the use of the dual variable from the constraint sets of the linear program (LP) to develop gradient functions that will indicate how to change the flows in the links in order to reduce the cost of the next LP run. Their gradient functions were later corrected by Quindry et al. (1979).

A graph theoretic matrix formulation of the Alperovits and Shamir (1977) model was later presented by Kessler and Shamir (1989) and the original steepest descent search procedure for improving the objective function value from one iteration to another was modified using a projected gradient method. In another extension to the work of Alperovits and Shamir (1977), Fujiwara and Khang (1990) presented a two-phase approach for the minimum cost design of looped water distribution networks. In phase 1, the gradient technique of Alperovits and Shamir (1977) is implemented, with flows and pumping head as the decision variables, giving a local optimum solution. In phase 2, the link headlosses from the phase 1 solution are fixed and the program solved again for the flows in the links and the pumping head. Iterating between phases 1 and 2 results in a minimum cost solution that converges to a local optimum solution.

Schaake and Lai (1969) developed a linear programming model in which the nodal pressure heads, rather than link flows and pipe diameters, are initially assumed. Their model is applicable to looped networks and uses continuous pipe diameters and cost functions. The objective function is nonlinear but could be piecewise linearized. The decision variables are the pipe diameters. Their model is also capable of handling multiple loads. Additional constraints of minimum pipe sizes are required for looped networks without which least cost designed networks will degenerate into implicit tree networks.

Quindry et al. (1981) extended the work of Schaake and Lai (1969) by using an iterative approach to obtain better optimal solutions. The iterative approach was similar to the general approach of Alperovits and Shamir (1977) and used dual variables of the demand constraints in a gradient expression to develop pressure distribution patterns which, when used as input to linear program, resulted in cheaper solutions. The major difference between their approach and that of Alperovits and Shamir (1977) is that they iterate by changing the heads at the nodes rather than the flows in the links.

The main problem with these gradient techniques is the search direction and step size to use from iteration to iteration. This problem is particularly serious with the Alperovits and Shamir model in which significant cost reduction was not obtained in the examples they provided. A recent paper by Fujiwara et al. (1987) looked at this problem and proposed a quasi-Newton search direction as opposed to the steepest descent method of Alperovits and Shamir. A backtracking line search method for the step size was also proposed instead of the fixed step size in the original paper. Their method resulted in cost reductions which are far greater than those obtained by Alperovits and Shamir.

Other models within this grouping include those of Schaake and Lai (1969), which included non-linear and dynamic programming models with linear models discussed earlier, Kally (1972), a linear programming model similar to that of Karmeli et al. (1968), Deb (1974), a linear programming for branched networks, and Deb (1976), [an extention of the Deb (1974)] model to looped networks by specifying minimum pipe sizes for all links. Shamir (1974) also presented a linear programming model, while Watanatada (1973) and Lansey and Mays (1987) proposed non-linear programming approaches for looped networks.

Bhave (1978) approached the minimum cost looped network system by first finding a minimum spanning tree of the network and then closing it into loops with minimum pipe sizes. Gessler (1982) on the other hand proposed a complete enumeration method to the cost minimization of such looped systems. Martin (1980) proposed a dynamic programming model to the minimization of network cost but the model was only applicable to serial (branched) networks.

Some researchers employed heuristics in conjunction with the classical network analysis. Cenedese and Mele (1978) used an iterative procedure in which a tree network is initially assumed and the network is designed to satisfy flow demand and minimum pressures. This tree network is assumed to be the most economical design. Loop forming links to produce the redundancy necessary for most urban networks are then introduced systematically. The addition of links which results in the network cost closest to the initial solution is taken as the optimal solution. Featherstone and El-Jumaily (1983) proposed another type of iterative method whereby an initial solution (set of pipe diameters) is assumed in the solutions. The Hardy-Cross method is then used at each step to determine the nodal pressure heads arising from that solution and those below the minimum required pressures are set to the minimum. The new pressure pattern is then used to re-estimate the pipe diameters. The Hardy-Cross method is then used again to re-calculate the nodal pressure heads and the process repeated until no further changes in the solution (of pipe sizes) are registered.

The major feature of all these cost optimization models for water distribution network design is that they tend to reduce the network cost at the expense of its reliability or redundancy. This situation is best stated by Templeman (1982);

> "...optimization tends to remove redundancy, and any spare capacity which is not immediately required by the design demand pattern is optimized out. Thus all flexibility is removed." [2]

In urban water distribution systems this removal of flexibility as it represents reliability or redundancy of the network is generally unacceptable. This is because the ability of the network to respond well (by satisfying water demands) to failure conditions is expected of the system by the public (e.g. Morgan and Goulter, 1985).

[2] Templeman, A.B., "Discussion of Looped water distribution systems", *Journal of the Enviromental Engineering Division, ASCE*, pg 599, June 1982

2.3 Models that Consider Both the Cost and the Reliability of the Network

This class of water distribution network design models represents the attempt to address the reliability issue through direct measures or by indirect approaches such as the inclusion of non-quantifiable 'redundancy'. Some models consider the overall water supply system reliability (source-treatment-distribution) and usually propose indices as a measure of the reliability. The first two units (source and treatment) are usually relatively easy to analyse. The most recent work in this area is by Hobbs and Biem (1985), (1988) and Biem and Hobbs (1988) who used a range of analytical and simulation techniques to determine the reliability of the supply system. However, the last unit, the distribution network, is very difficult to handle.

One of the early landmark works on reliability in water supply systems was that of de Neufville et al. (1971) in which they provided a performance index as a measure for network reliability. The overall performance of the network was taken as an average of the pressure above the minimum required at key points within the network, weighted by a factor, defined as the ratio of flow demand at the point (node) to the total flow demand in the network. This approach ignores the probability of failure of the system but focusses on what they termed "the quality of the failure mode".

Shamir and Howard (1981) also presented reliability indices for the whole water supply system in terms of the relative magnitude of the shortfall during a failure or the frequency of the occurrence of shortfalls. The failure could be that of the supply source, pumps, treatment plant or components within the distribution network. This work was followed by another in 1985 (Shamir and Howard, 1985), an extension of their 1981 model. More recently, Mays and Cullinane (1986) presented a review of the concepts of reliability that are applicable to water supply systems. They then proposed a method based on the use of time to failure and repair time data of the components of the distribution system to define its reliability. Cullinane (1986) (1987) proposed the concepts of hydraulic and mechanical availabilities as measures of reliability. In this work, Cullinane (1986) defined hydraulic availability as the percentage of time that the demand can be supplied at or above the minimum residual pressure. This approach can be applied to specified nodes to indicate nodal reliability and the average of these nodal reliabilities taken as a measure of whole water distribution networks hydraulic reliability. Mechanical availability was defined using the mean values of the time between failure and repair duration of the components of the water distribution network. In order to obtain the hydraulic availabilities, it is necessary to do extended period simulation on the network to obtain statistical values of the failure frequencies. It is interesting to note that in their work on the supply aspect of water supply systems, Hobbs and Biem (1986), (1988) also focussed on the unavailability, its frequency of occurrence and the expected volumes.

All the above approaches attempt to find a single measure for the whole water supply system and to use simple parameters such as the mean and standard deviation of the failure rates of the components. A single measure for the distribution network as a whole is not as easy to obtain due to the complexity of the analysis, a result of the complex interaction among the large number of links and nodes and the role of pumps and storage facilities within the network. Further, due to the spatial nature of the network layout and associated mechanical reliability, a single mean value for the reliability of the distribution network may be a gross misrepresentation of the actual situation. Rather there is a need for point reliabilities over the entire network from which reliability contour map can be plotted (Quimpo and Shamsi, 1988) and hopefully a generalized overall network reliability can be determined.

As shown in the material discussed above much of the research work on the reliability consideration of the network sections of water supply systems started in the begining of the 1980's, partly due to the age of such facilities in some major cities in North America. At this time the modelling philosophy of water distribution network design shifted from a strong emphasis on cost minimization without explicit regard for other factors to reliability maximization under cost constraint, or conversely, cost minimization under reliability constraint. The major problem encountered in this process is the definition of the measure of reliability. This issue has also been the problem in other network fields, such as electrical engineering, where research on network reliability has been underway on for decades.

In water distribution network reliability research, two types of reliabilities have been identified;

1. Mechanical reliability and

2. Hydraulic reliability.

Mechanical reliability is related to the failure rate of the network components such as pipes and pumps. This type of reliability depends on the structural strength and age of the components as well as the external environment in which they are located. Hydraulic reliability on the other hand refers to the ability of the network to satisfy the demands within the system. Hydraulic failure could be due to the inability of the network to deliver either the required quantity of water or to do so at the desired residual pressure level. Hydraulic failure can be caused by mechanical failure (failure of pipes, pumps or storage reservoirs) or by the actual demand exceeding that for which the system was actually designed. Branched layout networks are more susceptible to hydraulic failure than looped networks because the failure of a link in a branched network isolates all users downstream of the link. For this reason urban networks are almost invariably looped. Although many of the optimization design models that address the reliability issue consider only one type of reliability some do in fact consider both forms of it. In developing those optimization models which consider both types of reliability, the reliability aspect is either directly (explicitly) included, or an indirect or implicit approach is adopted.

Rowell and Barnes (1982) presented one of the first attempts to address the reliability issue. Their approach considered reliability indirectly by including redundancy in the layout of the network, in line with the recommendation of Templeman (1982). In their model to define the layout, a minimum spanning tree giving the primary branches is first designed. The tree is then closed to give a looped network using pipes sized such that some specified percentage of the demand at the node on which they are incident can be satisfied if the pipes in the primary branch supplying that node fails. However, their method was shown to have neglected hydraulic consistency required for looped water distribution networks (Goulter and Morgan, 1984).

Goulter and Morgan (1985) subsequently reported on a model in which an integrated approach to the layout and component designs was adopted. A degree of looping within the network was ensured through the use of the constraint that each node must be connected to at least two links. They noted, however, that this type of looping requirement may not neccessarily guarantee true redundancy and there is, therefore, the need to do a visual inspection of the layout and do alterations if required. Their model is however useful because the optimization technique used is computationally efficient Linear Programming, compared to the computationally intensive Integer and Non-linear Programming procedures used by Rowell and Barnes (1982).

Other models directed at reliability improvement through the use of redundancy include those of Awumah et al. (1989) in which a zero-one programming approach was adopted and that of Jacobs and Goulter (1989) where graph theory was used in conjunction with an integer programming formulation to maximize the redundancy in the network. The Awumah et al. (1989) work was a variation of the approach of Goulter and Morgan (1985) in which the emphasis was on the layout configuration of the network. The branches of the network in that study are the decision variables of the model formulation and a simple constraint ensures that the layout solution does not degenerate into a tree layout or a layout with weak redundancy at some sections. The Jacobs and Goulter (1989) work used fundamental graph theory definitions for reliability to define and control reliability aspects of network layout. Ormsbee and Kessler (1990a, 1990b) developed an approach that provides 'level one redundant' water distribution networks. This involves designing a network layout that is made up of two overlapping tree layouts and then providing a specified level hydraulic capacity within the trees using a linear programming formulation.

Another group of design models that considered mechanical reliability was initiated by the work of Kettler and Goulter (1983). In this approach statistical analysis of failure rates is used to obtain probability distributions of failures of components, in this case, of pipe failures. These distributions could then be used in formulating surrogate reliability constraints in design optimization models. The underlying principle in these approaches was that, if a strong correlation could be found between the pipe diameter and pipe failure rate, then the reliability constraint could be written in terms of pipe diameter, which happens to be the variable of the objective function in a number of the cost optimization models.

This overall approach may involve expressions for the probability that there is a continuous path between the source and the demand nodes, also known as Nodal Pair Reliability (Quimpo and Shamsi, 1988). For general networks, the calculation of such parameters this has been shown to be classified as non-polynomial hard (NP-Hard). In other words, the computational time required for these expressions is an exponential function of the network size [Ball (1980), Provan and Ball (1983), Jacobs and Goulter (1988)].

Many of the models that were developed to include mechanical reliability were, however, based on some form of surrogate of the exact reliability measure, or on heuristics. The need for heuristics or surrogates for reliability arises from the fact pointed out by Walters (1980) and Goulter (1987) that it is not yet possible to define a practical comprehensive measure of true network reliability. Goulter and Coals (1986), for example, considered the probability of failure of only the links directly connected to the demand node in place of the actual paths between the nodes and the source. Wagner et al. (1988a), (1988b) presented some methods to help reduce large networks into equivalent (from reliability point of view) smaller networks to help give simpler reliability expressions. Their approach appears to be of very limited practical use, however, because real networks are very well connected such that the series or parallel reductions proposed will not generally be applicable [Goulter and Jacobs (1989)]. Quimpo and Shamsi (1987), (1988) also proposed some analytical methods based on a minimum cut set algorithm as an approximation to the exact reliability measure. Mays et al., (1986), Su et al. (1987) and Shamsi (1990) also formulated models that incorporate mechanical reliability based on the minimum cut-set theory. A cut-set of a network is the set of links whose failure causes the network to fail. For source node to be connected to the demand nodes, there is a cut-set for each demand node paired with a source node. A minimum cut-set is the minimum number of links in the cut-set whose simultaneous failure results in the failure of the network. The methods based on minimum cut-set theory also suffer from impractically high levels of computational effort, however. Therefore, in spite of their further simplification through the adoption of a single-link-failure approach, the cut-set methods are still inapplicable to large networks (e.g., the solution of a simple network of 4 loops and 17 links by Su et al., 1987, based on minimum cut-sets required 200.5 minutes on CDC Cyber mainframe).

Other researchers considered mechanical reliability based upon issues other than those involving pipe failure probabilities for networks. Duan and Mays (1990) presented a reliability analysis of the pumping station by considering both mechanical and hydraulic reliabilities and modelling the availability of the pumps using a Markov process. Their concept was later implemented in an optimization framework by Duan et al. (1990) and included mechanical failure of storage tanks and hydraulic failure within the network.

The other type of reliability analysis that has received attention is hydraulic reliability. Morgan and Goulter (1985) proposed a model based on the use of multiple loads in conjunction with pipe failure to obtain a robust and reliable layout and pipe design. Tung (1986), Tung et al. (1987) and Lansey et al. (1989) developed chance constrained models to account for hydraulic reliability. In these three works, the stochastic nature of flow demands and pressure heads was explicitly recognised within the supply network. Wagner et al. (1988b) proposed a model that uses a capacitated network algorithm and simulated multiple link failures for improving the hydraulic reliability.

Both mechanical and hydraulic reliabilities were simultaneously considered by Goulter and Bouchart (1990) in the same model. The probabilities of pipe failure and 'demand exceedance' (demand exceedance is defined as actual demand exceeding design demand) are combined into a single reliability measure called "probability of no-node-failure". The probability of pipe failure is used to compute the probability of node isolation by multiplying the failure probabilities of links directly connected to this node, in the same manner as used by Goulter and Coals (1986). The demand exceedance aspect was considered by examining the magnitude of the design demand and estimating the probability that the actual demand would exceed the design value.

Due to the nature of the failure rates of pipes (smaller pipes fail more frequently [Kettler and Goulter, (1985)]), improvement in system capacity due to larger pipe sizes caused by larger design demands also resulted in lower pipe breakage. Hence using higher design demands which give larger pipe sizes, also improves the mechanical reliability and probability of node failure in the network.

Other researchers indicated that the network form (geometric configuration) plays a relatively important role in the amount of reliability or redundancy that can be imposed on the network. In this regard, Elms (1983) suggested a heuristic method for networks in general based on clustering procedures. The degree to which a network is connected is measured so that components of the network can be grouped into weakly connected and tightly connected sub-networks. Goulter (1988) pointed out that Elms' (1983) method has the potential use in water distribution networks for identifying weakly connected areas and then extended this to define measures of redundancy for water distribution networks.

2.4 Reliability in other related fields

In this section, reliability consideration in other fields which are similar to water distribution networks will be highlighted.

The first area of consideration is in the field of structural engineering. Templeman and Yates (1984) showed that there are mathematical similarities in these two engineering fields, in particular, between structural trusses and pipe networks. They both belong to the class of non-linear potentiated networks. The important feature of this observation is that both can be represented as networks which are sufficiently large that their reliability calculations will be difficult.

One of the most popular methods of assessing the reliability of large structural systems is the method by which the system is organised into a group of collapse modes (similar to reliability blocks). These collapse modes are then analysed using different types of simplifying assumptions. A package called PNET (Probabilistic Network Evaluation Technique) was developed by Ang et al. (1976) for use in structural system reliability analysis. In the method, the collapse modes that are highly correlated are assumed to be perfectly correlated and those that are weakly correlated are taken as statistically independent. Collapse modes are grouped and the failure probability of each group is taken as that of the collapse mode that has the largest collapse probability (weakest link assumption). The overall collapse probability is simplified as the sum of the collapse mode probabilities of the groups. Several researchers including Ishikawa and Ilzuka (1987) used this package to develop models for structural system reliability.

Other researchers have used some of the concepts from the collapse mode structure. The weakest link assumption has been used by Freudenthal (1956), Freudanthal et al. (1966), Ang and Amin (1968), and Ang and Cornell (1974). Basu and Templeman (1985) used maximum entropy to estimate the probability of failure of structural components. The method was justified on the assumption that the strength of the structural members and the imposed loads on them are random and the entropy approach permits these probabilities to be estimated without any prior analytical distribution assumptions. The overall system reliability was then computed using the weakest link concept.

Another approach to the reliability problem of structural systems is the use of approximations to the exact reliability measure. Cornell (1967) proposed a first-order bounds and Ditlevsen (1979) developed reliability bounds for use in structural systems. Frangopol (1985) proposed a reliability based optimum design of reinforced concrete structures in which he modelled the reliability as a system of individual collapse modes in series while the collapse modes themselves are modelled as a parallel system of plastic hinges. The complete system is therefore a series-parallel reliability system. He then used Cornell's reliability bounds for the analysis. The lower bound represents the probability of occurence of the most critical node and is obtained by assuming that the collapse mode failure events are perfectly dependent. The upper bound is obtained by assuming that these events are independent . Moses (1977) proposed finding the path of failure of a structural system, implying a series system regardless of the geometric configuration. A number of such paths can be identified and jointly considered as a parallel system.

The application of the above structural reliability metohds are possible with the same limitations explained in previous sections, such as computational time feasibility. Although the development and application of analytical approximations to reliability in water distribution networks may be worth pursuing, it must be kept in mind that there is more to reliability of water distribution networks than mechanical failure of components.

Another field that is related to water distribution networks is electrical networks. Electrical networks can also be considered as nonlinear potentiated networks and are also very large, and therefore their reliability analysis are of the same order of complexity as water distribution networks.

One of the numerous methods proposed for the reliability problem is the decomposition of the electrical network into sub-networks as a means of simplifying the computations. Rushdi (1984) developed an algorithm for the nodal pair reliability evaluation of complex systems. It involves the decomposition of the network into two or more sub-networks, after applying series-parallel reduction to the network, via a minimum cut-set. The reliability of these smaller networks can then be evaluated and that for the whole system derived using disjoint techniques. The method is based on the assumption that the components are a 2-state independent (good or failed only) and are not repairable. Several others developed algorithms for reliability assessment via decomposition and include Bodin (1970), Nakazawa (1976), de Mercado et al. (1976), Aggarwal et al. (1982) and Rushdi (1983).

Another popular approach was the use of algorithms for determining the bounds (lower and upper) for the exact reliability rather than estimating the exact reliability. Messenger and Shooman (1967), Jenson and Bellmore (1969), Zemel (1982), Ball and Provan (1983) and Provan (1986) are some of the researchers that proposed algorithms for the bounds on reliability of complex networks.

Fault tree analysis is another method found in the literature. Haasl (1965), Fussell et al. (1974), Bennettes (1975), Locks (1981) and Bojadjiev (1984) all developed methods using this approach. Fault tree analysis is a very exhaustive technique and is therefore suitable only to systems where failures have catastrophic consequences, such as in aircrafts and in nuclear systems.

All the methods for the reliability analysis in electrical networks do not appear to be suitable for use in water distribution networks. The decomposition methods, for example, are based on the assumption of non-repairable components which is not the case for pipe networks. Fault tree analysis methods are also too exhaustive to be applicable to large networks.

2.5 Summary and Conclusion

In this chapter, a survey and an overview of the design of water distribution networks is reported. The survey examined the traditional methods of the design, the advent of the use of digital computer and the initial emphasis on the development of models based on least cost design, and the current trend of reliability based designs. The review process then focussed on the methods proposed for the reliabilty analysis of water distribution networks and their limitations with regard to practical application to such complex networks. Finally, a short overview of the reliability assessment methods in other related large networks was presented and the potential applications to water distribution networks discussed.

The traditional computer method of designing water distribution networks can considered to be inadequate because they do not directly incorporate cost considerations, an issue which cannot be ignored entirely in the period of tight budgets and energy conservation. The attractiveness of these methods is that they provide the basis for efficient calculation of results which could only have been obtained formerly with painstaking manual calculation.

The least cost design models can be considered to have achieved some success from the point of view of cost alone since a great number of them are available to efficiently solve this problem, especially the Linear Programming models such as those of Alperovits and Shamir (1977) and Quindry et al. (1981). However, no models are yet available to give the exact global optimum solution and all models in the literature can give only locally optimal solutions, although it might be argued that seeking the global optimum design may be unneccesary and may in fact be undesirable for practical problems as long as the optimization models are able to provide, in reasonable time, solutions that are as effective but cost less than those obtained by traditional design methods.

When reliability becomes an issue in the design process, there appears to be general agreement that relatively little success has been achieved. There is the problem of defining what actually constitutes reliability in water supply systems and what level of reliability would be adequate. This is due in part to the complexity of the system, a multi-facet problem in which it is either impossible or extremely difficult to know which part is more important with respect to reliability. In addition, the complexity of the system also leads to the problem of dimensionality generally, to such an extent that methods such as network decomposition do not become helpful.

It is proposed that, due to the complexity of the process of reliability assessment in water distribution networks, other methods considering issues besides numerical probability of failure calculations will be the best approach to adopt. The approaches will have to focus on the effects of failure of components on the system and develop surrogate measures that will represent these effects. These measures should also be capable of being incorporated into design optimization models because the issue of cost will also have to be addressed and this cost minimization can best be done by means of "operation research techniques". In the words of Templeman (1982), research should be aimed at the "... development of 'quick but dirty' heuristic methods" and not rigorous analytical methods to locate the approximate solution. This work takes the approach recommended by Templeman and follows the works of Morgan and Goulter (1985), Mays et al., (1986) and other researchers, by using surrogates to drive the network design process to yield networks that would perform reliably.

Chapter 3

DEVELOPMENT OF ENTROPY BASED REDUNDANCY MEASURES

3.1 The Concept of Entropy

3.1.1 Introduction

The concept adopted as a basis for the development of redundancy measures for water distribution networks in this research is entropy. The idea of using entropy, which was first developed in classical thermodynamics (in the second law of thermodynamics), arose because entropy is concept that has found a wide application in many fields. In considering the use of entropy in water distribution reliability, it is important to recognise that entropy can be considered as "a measure of disorder, randomness or lack of information about the microscopic configuration of particles of which the system is comprised" (Sonntag and Van Wylen, 1966). Close comparison between entropy and redundancy in water distribution networks is being considered in this work because the concept of entropy is related to the 'configuration of a system'.

> "... we conclude that the entropy should be directly related to the total number of states available to the system. ... in this sense, entropy can be considered as a measure of disorder, randomness, or lack of information about the microscopic configuration of the particles of which the system is comprised. A perfectly ordered system, with total number of quantum states equal to unity, corresponds to zero entropy and implies a complete knowledge of the microscopic state of the system." [3]

The application of entropy exploiting this characteristic has enjoyed a great deal of success in many fields where it has been used to measure many attributes of systems, particularly attributes giving a measure of system diversity (Kapur, 1983), as a measure of system complexity (Ferdinand, 1974), and as a measure of flexibility within manufacturing systems (Kumar, 1987).

3.1.2 Statistical Thermodynamics Entropy

In statistical thermodynamics, the concept of probability can be phrased in terms such as the 'mixed-upness' or the 'disorder' of the system. The greater the disorder of the system, the greater the thermodynamic entropy.

In terms of the particles of a gas, the greatest degree of order of the particles (i.e., minimum disorder) occurs when these particles are in a very small volume in ordinary space and are all travelling with the same velocity. The thermodynamic entropy of such a systems is zero. "... the more the particles spread out in ordinary space and the more their velocities spread out in velocity space, the greater the disorder and the greater the entropy... " (Lee et al., 1963).

[3] Sonntag and Van Wylen, "Fundamentals of Statistical Thermodynamics" *Series in Thermal and Transport Sciences*, John Wiley and Sons, Inc., New York, p. 90, 1966

The thermodynamic entropy of a system can be represented by:

$$S = -kN \sum_i \frac{N_i}{N} \ln \frac{N_i}{N} \tag{3.1}$$

Or,

$$S = -k' \sum_i p_i \ln p_i \tag{3.2}$$

where S = entropy of the system

p_i = fraction of particles in energy state i

N = total number of particles in system

N_i = number of particles in energy state i

k' = Boltzmann constant.

The above statements are based on the condition that the p_i's are distributed according to the most probable distribution for the given number of particles and energy of the system.

3.1.3 Communication Theory Entropy

The mathematical statement of thermodynamic entropy expressed in Equation 3.2 also represents the mathematical function of communication theory presented by Shannon (1948). The average information conveyed per symbol j when the probability of the occurence of symbol j in a message is P_j is given as

$$\frac{I}{N} = K \sum_j P_j \ln P_j \tag{3.3}$$

Or,

$$I = K' \sum_j P_j \ln P_j \tag{3.4}$$

where N = total number of symbols in the message

K'= a constant.

The value I can also be interpreted as the average uncertainty per symbol about the message before its reception (this is the amount of information unknown or missing which will be known as a result of the message being received). If this function is maximized, it is seen that the maximum average information per symbol results when the given symbols appear with equal frequencies. Entropy is, therefore, a convenient measure of the uncertainty or unpredictability of a system which involves some element of probability. Besides the similarity that occurs in the mathematical statements of entropy in thermodynamics and in communication theory, both are similar in concept because in communication theory, entropy is a measure of the uncertainty about the message before it is received, and in thermodynamics it is the measure of microscopic disorder, or the uncertainty about the microscopic state of the thermodynamic system.

3.1.4 Some Mathematical Expressions For Entropy

Different researchers developed different mathematical expressions for entropy besides those given for thermodynamic and communication entropies in Equations 3.2 and 3.4. This section highlights some of these expressions and the mathematical properties they exhibit. [For the sake of comparison, the entropy equation (Equation 3.4) due to Shannon (1948) is simplified as follows].

a) Type 1.

$$S_i = -\sum_{i=1}^{n} X_i \ln X_i \tag{3.5}$$

This expression is that due to Shannon (1948) and is also the thermodynamic entropy. It has mathematical property of being concave and S_i increases monotonically with the parameter n. The variable X_i is actually the probability of occurrence of symbol i in the message (in communication theory) or the fraction of particles in state i (in thermodynamics). The constant K' given in Equation 3.4 is taken as unity in these cases.

b) Type 2.

$$S_i = \frac{1}{(1-\alpha)} \left[\ln \sum_{i=1}^{n} (X_i^{\alpha}) \right] / \sum_{i=1}^{n} X_i \qquad (3.6)$$

$$\alpha \neq 1$$

This is the function due to Renyi (1961). It satisfies all the necessary and desirable axioms. The concavity and monotonicity properties of this expression were proven by Bessat and Raviv (1978) and Kapur (1986). This function has two advantages over Shannon's function. Firstly, it has the parameter α which permits the function to account for some additional factors in the use of entropy. Secondly, in Shannon's entropy function, the sum of the X_i's must equal unity. This condition is not necessary for the use of Renyi's entropy.

c) Type 3.

$$S_i = \frac{1}{(\beta-\alpha)} \cdot \ln \left[\sum_{i=1}^{n} X_i^{\alpha+\beta-1} \right] / \sum_{i=1}^{n} X_i^{\beta} \qquad (3.7)$$

$$\alpha \neq \beta$$

This entropy function is due to Kapur (1986) and has the two parameters, α and β. It has the same properties as Renyi's entropy, is more flexible to use due to the prescence of two parameters, but has problems with concavity and monotonicity.

d) Type 4.

$$S_i = -\sum_{i=1}^{n} u_i X_i \ln X_i \tag{3.8}$$

This entropy function was proposed by Belis and Guiasu (1968) and is called useful entropy. The parameter 'u_i' is a weight reflecting the 'usefulness' or 'effectiveness' of the received information. The function is therefore a modification of Shannon's entropy. It has both concavity and monotonicity properties. Furthermore, the sum of the parameters, u_i's, need not equal unity.

3.1.5 Application of the Concept of Entropy to Redundancy

In terms of uncertainty, entropy has characteristics which enable it to be used as a general and basic concept in science. Its evaluation and subsequent utility depends, however, on the constraints placed on the system in the particular field of investigation. The entropy concept was succesfully applied to many situations in fields such as statistics (Kullback and Leibler, 1951), transportation (Wilson, 1970), pattern recognition (Kapur et al., 1983), finance (Cozzoline and Zahner, 1973), operational research (Guiasu, 1977), and biological sciences (Tiwari and Hobbie, 1976).

The entropy concept is being applied as a measure of redundancy in this thesis because the properties postulated for the measure of information by Shannon (1948) are similar to some of those to be proposed in this chapter for a redundancy measure. This assertion implies that Shannon's entropy function can be adopted as a mathematical statement of the redundancy measure if it can satisfy certain specified properties. The variables in this redundancy function may be probabilities (P_j's in Equation 3.4) or any other variable relevant to the physical situation of water distribution networks.

Consider a water distribution network. Redundancy at a particular node of a water distribution network can be considered as a measure of the 'disorder' or the 'diversity' of how the required flow to the node is distributed in the incident links. This diversity or disorder is related to the number of incident links (or more exactly, the number of alternate flow paths) through which the water from the source reaches the node. As is the case with thermodynamic entropy, redundancy is zero for a perfectly ordered system, which is the system where there is exactly one flow path between the source and the node. This case implies a single link incident on the demand node. This can be considered as a perfectly ordered system because there is no diversity in this type of geometric configuration (a branched network or a network without loops). For any given demand pattern or design demands, flow rates in all pipes can simply be found working backwards from a demand node and accumulating demand flows as flows in each pipe, up to a source node. This implies a complete knowledge about the flow distribution of the system, hence it has zero entropy. As the system disorder increases (i.e., with the possibility of variation in flow rates in the links due to the presence of loops or an increase in the number of flow paths from a source to the nodes, and therefore lack of complete knowledge about the flow distribution in the system) the entropy increases and so does the redundancy measure at the node. Hence if it is necessay to maximize redundancy at a node then the disorder at the node is also to be maximised. Thus entropy maximisation is equivalent to redundancy maximisation.

3.2 The Concept of Redundancy

3.2.1 Definition of Redundancy

Redundancy is a general term used to describe a situation whereby a system, which is composed of components, remains useful even when one or more of its constituent components fail. Such usefulness remains due to the fact that the service normally provided by the components currently in the state of failure are taken over by other components. In other words, there are extra component units provided for this contingency situation of one or more components failing. The need for redundant components arises because no component can be made to be hundred percent reliable. Redundancy is therefore used to increase the reliability of a system to a desirable level by providing a 'back-up' capability. The capacity or efficiency of the 'replacement' component may be less than that of the regular component so that a reduced level of output of the system may result when the regular component fails. In most cases a range of back-up components are provided which together provide the total required level of capacity or efficiency should one component fail.

A multicomponent multistage system consists of a series of stages, each stage being made up of components in parallel. In general, redundancy is added to this type of system by connecting, in parallel, in a particular stage or stages additional components, which together provide in that stage (or stages), capacity in excess of that required. A series as opposed to parallel connection of such components in a stage does not result in redundancy as the failure of any one component cuts off that stage of the system and other stages 'downstream' resulting in the failure of the whole operation of the system 'downstream' of the failure. The number of a particular component to be connected in parallel for the purpose of providing redundancy depends on how vulnerable the system is to failure of that component and the reliability of the component itself.

The effect of adding redundancy to the system can be assessed by any measure of the system's reliability, the reliability improving with the addition of each redundant unit or capacity. Systems whose failure will be catastrophic will therefore need to have very high redundacy built into the stages considered critical in order to give a very high system reliability.

It is noteworthy that redundancy may be applied to a multicomponent multistage system at the component level or the stage level or at a subsystem level or even the system level. In the first case, redundant components are added in parallel to some components in a particular stage. In the second case, some stages are completely duplicated using parallel connections. In the third case, a group of stages forming a subsystem is duplicated while in the last case, the whole system is duplicated. No matter where the redundancy is applied, the objective is to improve the system reliability. It may, however, be necessary to determine where to apply the redundancy to give maximum improvement in system reliability for a fixed cost or 'level of effort'.

3.2.2 Types of Redundancies

Two types of redundancies can be distingished and are described below.

Passive Redundancy

This type of redundancy involves arranging the redundant components in parallel to the regular components such that they will be held in 'reserve'. The redundant components will not be used as long as the regular ones are not out of service due to failure. They are called into service either by automatic switching, manual switching, or even by the physical replacement of the defective component after a brief interruption in service. Examples of this type of redundancy include the spare tire of an automobile carried in the trunk, and the standby electric generating equipment in a hospital for emergency situation of failure of the city's electricity.

Active Redundancy

As opposed to passive redundancy, the redundant components in active redundancy are not held in 'reserve'. Rather, all components of the system function permanently even when some of them are not strictly necessary or are in a state of under-utilization, in the non-failed state of the components. Therefore all components mutually share the burden of keeping the system functioning. In the event of any one component failing, the others keep the system functioning, either at its normal level of output, or at a reduced level, until the defective component can be repaired or replaced. The important factor that makes this a redundancy situation is that the system does not cease to function completely as a result of the failure of any of the components. These components are therefore mutually redundant. An example of active redundancy can be found in an aircraft with four engines, all of which function normally when the aircraft is in flight. Although the aircraft was not provided with four engines simply for redundancy it, does not fall (or crash) because a single engine fails. The failure of any one of the engines does not result in a crash of the aircraft because the other three engines can keep it airborne.

3.2.3 Units of Measuring Redundancy

The reliability of a component or a system may be measured using a scale ranging from zero to one (unity). A very unreliable system will have a reliability measure close to zero while a very reliable one will have a reliability measure close to one. The scale of measurement of reliability is usually based on a time frame, and is a frequency measure, the measure being the fraction or the percentage of a given time span the component can be deemed not to have failed. The time span can be the useful life or the design life span of the system or component, or any particular desired interval in time.

Redundancy on the other hand cannot be measured in such relative terms. It is simply the number of alternative units of a component that can be placed in service in a given contingency situation. There is, however, flexibility in how a redundancy measure can be developed. The measure will, however, depend on the factors which must be considered to give an adequate statement of system performance.

Factors that might be considered in a redundancy measure are the reliability of the redundant component as compared to the regular component, the efficiency or the output of the system under the two conditions (of regular operating conditions and when the redundant component alone is in service), or the type of redundancy that is in place. Given a regular component which is in parallel to a redundant one, both of equal capacity (i.e., where the capacity of a single component is equal to that required by the system for full operational state), the system can be said to have redundancy of one unit with respect to this component, or as it is usually termed 'level one redundancy'. This measure can be modified if the fact that redundant component can be very unreliable compared to the regular component, or that the redundant component may be half as effective as the regular component. In the latter case, it might be possible to rate the redundancy as half (0.5) instead of unity. If the system can only deliver a fraction of its normal output under the emergency condition, then the redundancy measure can be taken as this fraction. All these factors may be ignored in the simplest case, however, and the redundancy condition developed simply by the presence of the two units may be taken as unity.

Redundancy may also be taken as the number of times (or the proportion of time) the system performance is considered satisfactory when each of the components fail in isolation , the failures occuring one at a time. The evaluation of redundancy under this definition can be done by removing a component from the system and assessing performance of the system. This process is repeated for every component. The system can be considered to have redundancy with respect to a particular component or stage when its failure does not degrade the system performance below the desired level.

In the specific case of passive redundancy, the measure of redundancy can simply be taken as the number of extra units of the components provided in reserve (or in parallel with the regular component). Usually, the redundant component is of equal rating as the regular one so that the system functions satisfactorily in the emergency mode, although this may not always be the case.

The situation is not that easy to evaluate if the redundancy is the active type because all components are in service simultaneously so that neither of them is totally redundant nor totally regular. It is therefore up to the designer to choose some properties of the system to arrive at a suitable measure of redundancy. This can be illustrated using the example of an aircraft with four engines. Assume that the loss of one engine results in the reduction in total engine power which is such that the craft can be kept airborne. If the aircraft can cruise at the normal speed under this condition, it is possible to say that a redundancy of one unit with respect to failure of an engine is provided. The other three engines will mutually bear the load of the failed engine, which is equivalent to saying that reserve power was available in the rest of the engines. If the aircraft cannot cruise at the normal speed but at a reduced level, then the redundancy built into the engines is not a full unit. The measure of redundancy in this case may be taken as the power delivered to the system when an engine fails divided by that normally provided when all the engines are operating. If two engines can fail simultaneously without the aircraft crashing then two units of redundancy or some percentage of two units of redundancy may be available. It is also possible to have three units of redundancy if the aircraft can survive on only a single engine, each of the single engines individually having sufficient reserve power built in to be able to cruise the plane at the normal speed.

Since any of the components can fail at any time (a reliability measure does not necessarily indicate when failure will occur), the measure of one full unit can only be realised, for example in the first situation descibed for the aircraft, if all the four engines have the same power rating. If the engines are not rated equally, then the measure will decrease below one unit since a situation may occur in which the largest engine fails and the three smaller engines will not be able to cope with the load (in other words, the aircraft will be very vulnerable to failure of the largest engine).

From the above discussion it can be concluded that there is no universal definition for redundancy nor are there any universal units for measuring it. A measure of redundancy for a system has to developed by considering the approapriate factors that contribute to its redundancy.

3.3 The Nature of Redundancy in Water Distribution Networks

3.3.1 Introduction

In water distribution network design, the design must include some amount of redundancy to ensure that the network would be reliable. Traditionally this redundancy is assumed to be added by providing looped rather than branched networks, or in other words by providing two independent paths from the source to each demand node (Rowell and Barnes, 1982), or by ensuring that each node be connected to the rest of the network by at least two links (Goulter and Morgan, 1985). Both the above cited approaches have been shown to have shortcomings in redundancy. The first on the basis of hydraulic exactitude (Goulter and Morgan, 1984) and the second on the basis of not having true alternative paths from source to demand point (Goulter, 1987).

In fact, provision of closed loops adds redundancy to the network. There is one redundant link for every closed loop in the network. However, to measure redundancy based solely on the number of loops is not adequate because when given a number of alternate layouts, it should be possible to differentiate between two layouts that have the same number of loops.

Thus, true redundancy in networks still remains essentially unquantified the literature. If an appropriate measure for redundancy can be developed, it can be used for the following purposes;

1. To compare different network layouts for the purpose of selecting the most redundant layout.

2. To provide a basis for selecting an appropriate principle for redundancy allocation within water distribution networks.

3. To be used in a multiobjective decision framework to identify the cost-redundancy frontier, which may be similar to the cost- reliability frontier.

4. Since a useful and explicit measure of reliability for large networks has not yet been developed (Goulter, 1987; Lansey et al., 1989), to be used in place of the use of exact system reliability measure to identify the most reliable networks.

3.3.2 Redundancy In Water Distribution Networks

A water supply network is a complex system and may include water reservoirs, treatment plants, pumps, pipe networks, valves, and elevated and underground storage tanks. An urban water supply system is typically composed of the following subsystems; water source, bulk transmission and treatment, finished water storage, and the water distribution system. Each of these subsytems can be taken as a system and studied separately. The present work will focus only on redundancy found in the the geometric configuration of the distribution network.

If the configuration of the network is not strictly that of a tree, a set of series-parallel paths from the source to each node can be traced. Any node connected to the source by a parallel set of links can be considered to have some amount of redundancy. This redundancy arises because the failure of one link or branch will not result in water being cut off to the node completely. Water will still reach the node, although possibly at a reduced pressure and flowrate. The system will thus continue to be useful.

Unless redundancy and reliability are an issue, it is not strictly necessary to provide parallel branches to the nodes as the network can be designed as a minimum spanning tree system. This will result in a series connection between the source and every node, and will be the most economical design (capital costs). The major reason for including parallel connections, usually through the use of loops, is to provide redundancy in the network. The question then is which type of redundancy exists in pipe networks.

The following discussion of the layout model by Rowell and Barnes (1982) is used to illustrate the explanation. In their model, Rowell and Barnes suggested the selection of an optimal minimum spanning tree, subject to all hydraulic constraints, as a first step. The second step involves adding 'redundant' links to the tree system based on demand constraints only, such that the redundant links should be able to supply some minimum required flow to the affected nodes, should a particular link fail but neglecting the other hydraulic constraints. This approach implies that an assumption that the redundant links will be held in 'reserve' until the main branch fails, whereupon they will be put into service. Although it is possible to design such a system using valves to control when a particular pipe is used, economic considerations alone will prevent this type of redundant system from being adopted for real water supply networks. Instead, all links will be in service permanently resulting in the reduction in the pipe sizes of the main branches and therefore in economic savings. (Note that for health reasons also, flow is normally maintained in all pipes in a distribution network). This type of redundacy is the active type and not the passive type implied by Rowell and Barnes (1982). The active redundancy type of design requires that the selection of both redundant and regular links be done together, subject to the hydraulic constraints of the network.

Another major component of water distribution system whose redundancy merits discussion is that of the pumps. Both forms of redundancy can be provided in the pump arrangement. For very large systems, active redundancy will be most feasible because to duplicate the large capacity pumps to provide passive redundancy will be too costly. Instead multiple relatively smaller, capacity pumps can be provided so that when one is out of service the others can keep the system running at an adequate level. It is noteworthy that pumps are usually selected based on peak flows. Therefore, they generally have very large redundancy with respect to average flow conditions. On the other hand, passive redundancy can be provided in the pump arrangements for smaller systems since this will be relatively inexpensive.

Storage tanks (both elevated and underground) are usually compartmentalized tanks. This adds redundancy to the system in the active form. The compartments provide redundancy because for cleaning or repair purposes, the unit is not completely taken out of service, but only on a compartment basis so that at least some portion of the storage continues to be useful.

The above discussions explain why large municipal water distribution systems do not fail completely due to the breakage of a pipe or a pump or other components. Their performance in the face of a failed component is due to the large amount of redundancy built into these systems. However, this redundancy is not quantified, but is added through the use of rules of thumb, visual inspection, intuition and personal judgement. The development of good or at least adequate measures of redundancy is therefore necessary if the problem of redundancy addition and assessment is to be approached in a scientific and rational manner. This situation may suggest the use of expert systems technology to the design problem. However, there is some need for more explicit statement, in this case on redundancy, for a rule base to be established.

3.3.3 Mechanical and Hydraulic Types of Redundancies In Water Distribution Networks

In this work, two types of redundancy in water distribution networks are identified. The first type will be termed 'mechanical redundancy' and the second type 'hydraulic redundancy'. Mechanical redundancy is a measure of the ability of the network to satisfy demand flows when component failure occurs and is a property of the layout (shape and size of components). Hydraulic redundancy, on the other hand, is a measure of how much degradation of network performance, in terms of the percentage of the demand flow that can be supplied at some minimum pressure heads, occurs when there is failure. Example of this type of redundancy is termed 'topological redundancy' by Ormsbee and Kessler (1990b). Hydraulic redundancy depends on factors other than the layout structure, such as the pumping head available, the availability of elevated storage tanks and their elevations, the time of occurrence of failure and the ability of the network to reverse flow directions in some links. Hydraulic redundancy can also be considered as a contributor to a measure of network reliability, the percentage of degradation of network performance being considered as the level of failure of the network.

Hydraulic redundancy can best be estimated by simulating pipe failures and estimating its effect on the network. This is a laborious exercise and several simplifying assumptions which may not be strictly valid have to be made. A network with increased inherent 'mechanical redundancy' will also exhibit improved hydraulic redundancy. Therefore, the fundamental objective of this work is to develop a good measure for the mechanical redundancy in water distribution networks. The use of the word 'redundancy' will therefore refer to the type 'mechanical redundancy', unless qualified with the word 'hydraulic'. This measure will, however, be evaluated using hydraulic redundancy measures derived from simulation.

3.3.4 The Difference Between Reliability and Redundancy

Redundancy of a system is related to its reliability in that redundancy is directed at ensuring that when there are failures of any of the components that make up the system, the system can still continue to perform the function for which it is designed. Reliability, on the other hand explicitly recognises risk and as such is a measure of the frequency of such failures.

Reliability incorporates risk but addresses in some way the failure of the system, the percentage of time that the system can be deemed not to have failed. Reliability is therefore directly related to probability while redundancy is related only to the ability of the system to perform in the face of failure conditions. Redundancy is not a measure of the frequency of occurrence of these failures. When added to a system, redundancy can improve the system reliability by reducing the frequency of failure of the system (which is different from the frequency of failure of the individual components) by ensuring that failure of components do not affect the system performance adversely. In summary, it should be recognised that reliability is positively correlated with redundancy but is not the same as reliability.

In water distribution networks, since water is carried by means of paths provided by a network of pipes, redundancy of the network would be a property of the network geometric configuration. This geometric configuration would determine whether in the event of pipe failures, other supply paths can be found to supply the demand points. Besides depending to some extent on configuration, a more comprehensive statement of redundancy also addresses the capacity of the supply paths. Reliability of the network on the other hand would also be a property of the pipe material, their strength, age, the soil environment, distribution of demand and other factors.

3.4 Development of the Measure for Mechanical Redundancy

3.4.1 Introduction

As defined earlier, mechanical redundancy of a water distribution network is a measure of the ability of the network to satisfy the hydraulic demands within the network when component failure, specifically pipe or pump failure, occurs. Hydraulic demand is the the amount of water to be delivered to a node at a specified minimum pressure head level. A functional form for a redundancy measure should therefore recognise both the layout structure of the network and hydraulic parameters such as the flow within the layout.

3.4.2 Model Representation of Water Distribution Networks for Redundancy Assessment

Water distribution networks are made up of pipe networks and other components such as pumps, storage tanks and valves. A water distribution network is conceptually presented as nodes connected by links, the nodes being demand centres which are fed with water through links made of pipes and valves. The source(s) of water are also denoted as node(s).

Storage tanks within the network can be considered as demand nodes because water flows into them (they are being filled during low demand periods in which case they have water demand) and water flows out of them (during high demand periods). They therefore have links carrying water from the source into them just as real demand points. Both pipes and pumps are considered as links, each pipe having a diameter and the pump having pumping capacity in terms of flow rate and static head. Their number and arrangement make up the configuration of the network and contribute to redundancy. Valves are located on the links and their function is to control flow magnitude and direction in the links, hence their presence is indicated by arrow directions on the links. Junctions of links where there are no demands are not considered as demand nodes (i.e., no redundancy measure would be assessed for these types of junctions).

Therefore all the elements that make up the water distribution network would be considered as links (arcs) or nodes (sinks) for the purpose of determining the mechanical redundancy. In the development of the redundancy measure, the hydraulic characteristic of the links to be considered would be pipes initially. The other elements which are modelled as links (pumps and valves) have similar hydraulic characteristics as pipes, and their inclusion in the redundancy measure will be addressed at a later stage.

3.4.3 Node-Link Configurations That Imply Redundancy

In its most fundamental sense, redundancy in water distribution network design implies that the demand points have alternate paths for water in the contingency situation of other links being out of service. The situations that give rise to a redundancy issue will be illustrated through specific examples rather than through abstract definitions. Consider the water distribution network itself as a directed network where the flow directions to the nodes are specified. As a simplifying step in the effort to develop a measure for redundancy, only those arcs directly incident on the node ,i.e., only those branches supplying water directly to the node, will be considered. Subsequently, the complexity that arises because of the interaction among the nodes of the network will be incorporated.

Consider the demand node, j, with the first two cases of arc-node configuration depicted in Figure 3.1. One or more arcs are required to deliver the desired level of flow to node j. In this case, the desired level of flow is set to 150 m^3/hr. Clearly, Case 2 has a higher redundancy than Case 1. In fact, Case 1 has zero redundancy as there are no alternative paths to serve that node should the supply link fail. However, the measure of redundancy does not depend only on the number of incident links or the degree of the node (in this case the number of links incident on a node is equal to the number of alternate paths to the node).

Consider the three other configurations in Figure 3.1. Case 3 has more than one incident link, and therefore has some measure of redundancy. However, in the event that link 1 is out of service, only $5m^3/hr$, or 3% of the required flow, can reach the demand node. Thus the node is very vulnerable to failure of link 1 (vulnerability being the magnitude of shortfall that will result when the link has failed) as the system will virtually be out of use if link 1 fails. Case 3 is therefore close to Case 1 and its redundancy measure differs from that of Case 2.

Hence a measure of redundancy should also be based, at least to some extent, on the ratio of the flow capacities of the links. Case 4 in Figure 3.1 has three incident links and therefore has greater redundancy than Cases 1, 2 and 3. Case 5 with its three incident arcs, each carrying equal amounts of flow, will be even more desirable since in the 'worst' situation of a single arc failure, it can supply $100m^3/hr$. Case 4 may only be able to supply $50m^3/hr$ under the failure of one of its arcs. Therefore Case 5 should have a higher measure of redundancy.

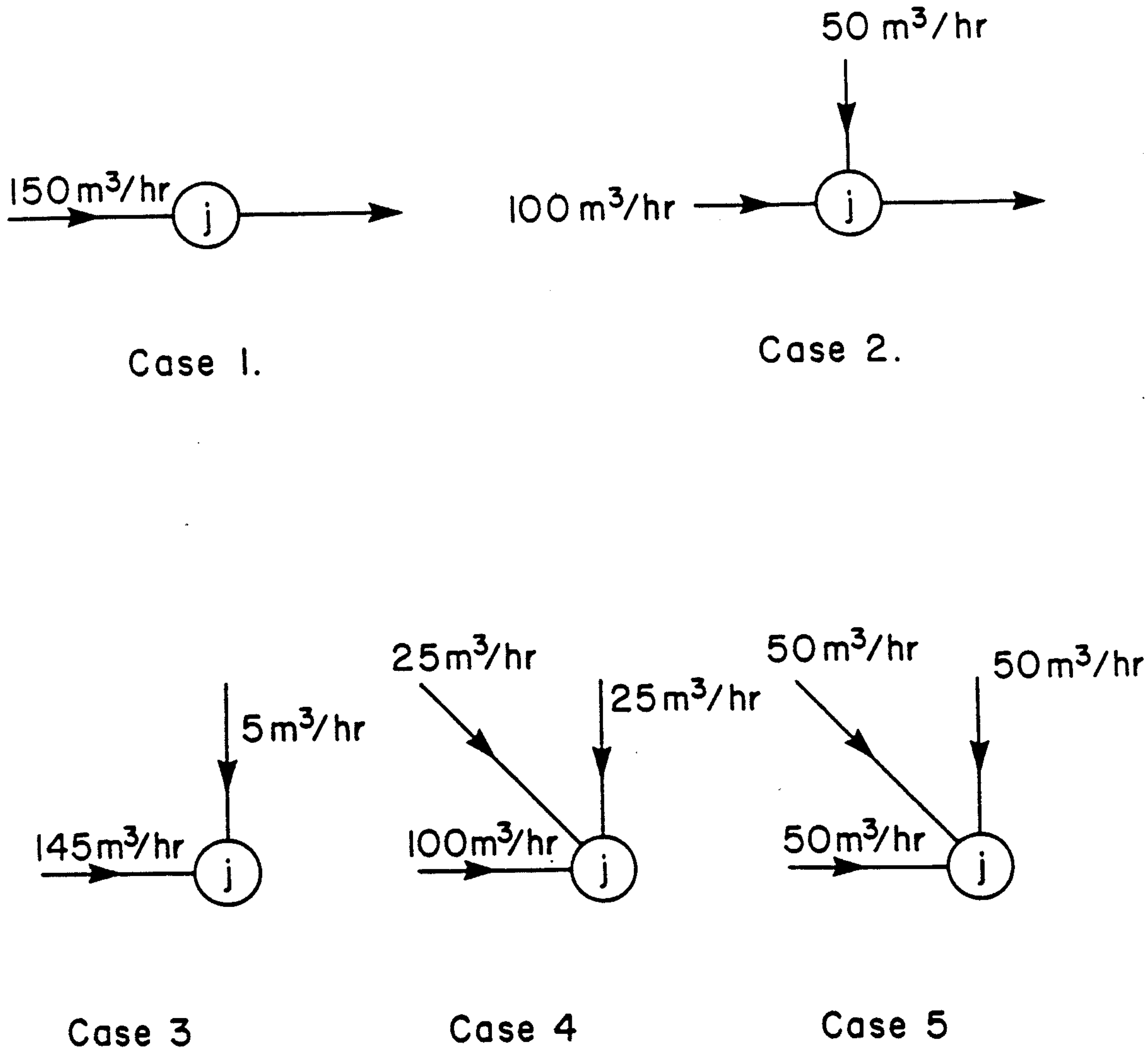


Figure 3.1: **Cases of Node-Link Configurations that Give Redundancy**

3.4.4 Theoretical Background

In order to develop the measure for redundancy, the following approaches are proposed.

a) The desired properties of the measure are postulated based on some intuitive considerations such as the physical motivation for the measure or its usefulness. The derivation of the functional form of the measure using strict mathematical techniques can then be performed.

b) A known fuctional form of the measure can be assumed and then its usefulness justified.

In this work, both approaches will be blended . The desired properties of the measure will first be postulated and then a suitable functional form adopted.

The principles for a redundancy measure for water distribution networks which can describe the inherent reliability of the network are most easily discussed in relation to an axiomatic approach to the formulation of the functions. The critical practical and theoretical requirement in design of redundant water distribution networks is that instead of carrying the necessary flow to a demand node by only one link, the network should utilize multiple links connected directly to that node. Such links may carry equal or unequal proportions of the flow. It is more advantageous from reliability/redundancy point of view for the links to carry equal proportions (Goulter and Coals, 1986; Walters, 1987). With unequal distribution of flow, failure of a link carrying the major portion of the flow to a node will have a major impact on the supply to that node.

The hydraulic principles behind the desirability of having equal flow capacity in the links incident on each node can be described as follows. The discharge q_{ij} in a pipe connecting nodes i and j can be expressed using the Hazen-Williams equation by:

$$q_{ij} = k \cdot C_{ij} \frac{h_{ij}^{0.54}}{L_{ij}^{0.54}} \cdot D_{ij}^{2.63} \tag{3.9}$$

where C_{ij} = Hazen-Williams roughness coefficient

L_{ij} = length of the pipe

h_{ij} = headloss through the pipe

D_{ij} = diameter of the pipe and

k = conversion factor for units.

For a given pipe network, the Hazen-Williams coefficients, the pipe diameters and the pipe lengths are all fixed. Hence only the discharge and headloss through the pipes are variable.

Consider an existing pipe network. For the fixed pipe sizes, the discharge q_{ij} in a pipe is proportional to the headloss, h_{ij}, encountered in that pipe, i.e., $q_{ij} \propto h_{ij}^{0.54}$. In other words, $h_{ij} \sim q_{ij}^2$. Hence for two or more links incident on a node (as is always the case in a redundant configuration), if one pipe has a small capacity, and if it is necessary to increase flow in it due to the failure of the other, larger pipe, the headloss in the smaller pipe will become prohibitively high thereby significantly decreasing the hydraulic performance of the network. It is the relative increase in q_{ij}, and associated increase in q_{ij}^2, that causes the rapid increase in headloss. Doubling a small q_{ij} quadruples the headloss in that pipe. Increasing the discharge in a larger capacity pipe by the same absolute (but smaller relative to the total flow into the node) amount does not cause the same increase in headloss. In order to reduce the impact of having to increase the flow in any link, it is desirable to have incident pipes with the same capacity, i.e., to have equal capacity links described previously.

Recognising the fundamental requirements of redundancy and the desirability of equal flow distribution, the properties the redundancy measure should exhibit are given below. These properties derived from the above interpretations and upon which the redundancy is based, can be divided into two groups. The first group embraces the qualitative observations made in the previous sections and is categorised as 'Necessary Properties'. The second group, the 'Desirable Properties', is based solely on mathemathical issues.

Necessary Properties

Consider a directed network with N nodes and let the redundancy measure at node j be S_j. Then

1. S_j at node j should be a function of $X_{1j}, X_{2j}, \ldots, X_{n(j)j}$ such that

$$\sum_{i=1}^{n(j)} X_{ij} = 1 \tag{3.10}$$

From the discussions in Section 3.4.1 based on Figure 3.1, the variable X_{ij} is given by:

$$X_{ij} = \frac{q_{ij}}{Q_j} \quad \text{where} \quad Q_j = \sum_{i=1}^{n(j)} q_{ij} \tag{3.11}$$

where X_{ij} = the variable of the redundancy measure

q_{ij} = flow in link ij incident on node j

Q_j = total flow into node j

$n(j)$= total number of links carring flow into node j.

This property arises out of the earlier discussion that the measure of redundancy should depend on the relative proportion of some physical property of the components of the system, such as the ratio of the power rating of the four engines of the aircraft in Section 3.2.2, or the ratio of the flow capacities of the incident links at the nodes of the networks in Section 3.4.3. In other words the measure of redundancy should be a property of the geometric configuration. Hence if there are two identical networks such that one is replica of the other but only different in 'size' (this can be taken as a case of a model and prototype), both networks should have the same measure of redundancy (given that both have the same flow ratios in their corresponding links.

2. S_j should be zero if n(j), the number of alternate paths from the source to node j, is exactly equal to one. At this stage of the work, it will be assumed that the number of links incident on node j represents the number of paths. This restriction is imposed because the node which has only one path from the source has no redundancy. In later developments this restriction will be lifted.

3. For a system with two or more incident links at the node, the contribution of redundancy by the link with the larger value of X_i should be less than the contribution of that with a smaller X_i, (i.e., if $X_i \geq X_j$ then $R_j \geq R_i$ where R_i and R_j are the relative contributions to redundancy by links i and j respectively). This requirement is due to the observation that the vulnerability of the system to failure of the larger capacity link is greater and since redundancy is the measure of how useful the system remains when a link fails, the system will be less useful when the the larger link fails than when the smaller link fails.

4. For a given number of incident links on node j, the measure of redundancy S_j should have its maximum value when all the X_{ij}'s are equal. This property was illustrated in Section 3.2.2 where it was explained that the closer the components are in all their physical properties, the better or the higher the redundancy.

5. For a given node the maximum value of $S_j = S(X_{1j}, X_{2j}, \ldots, X_{(nj)j})$ should monotonically increase with the number of incident links, n(j). This property arises out of the observation that the measure of redundancy should increase

anytime an 'extra' component is added in parallel to the system. Addition of an 'extra' link increases the number of supply paths available for the node, therefore its redundancy. However, if this 'extra' link is of a very small capacity relative to the other links, this increase in the redundancy will be very small and it is up to the designer or the design model to determine if it is worth adding this additional link.

6. S_j at node j should be a symmetrical function of $X_{1j}, X_{2j}, \ldots, X_{n(j)j}$ for symmetrical link configurations. This property arises out of the observation that if the values of the X variables are interchanged, the measure should remain unchanged. For example in Figure 3.1 case 2, if the link with 50 m^3/hr is made to have a flow of 100 m^3/hr and that of 100 m^3/hr is changed to 50 m^3/hr the redundancy measure should have the same value for both cases. This property is proposed on the assumption that the configuration of the links is symmetrical at the node. If the configuration of the links at a node is not symmetrical, then interchanging the variables, X, will result in a different redundancy measure which implies that the links will be weighted differently. This issue is discussed further in Desirable Properties, item 6 where the use of other parameters is introduced.

Desirable Properties

Since it is desirable to use the redundancy measure in a design optimization framework, the following mathematical properties are desirable.

1. S_j should be a continuous function of the $X'_{ij}s$. This property is desired because when there is a slight change in the flow distribution at the node the redundancy measure should respond accordingly in a continuous rather than a discrete fashion.

2. $S_j = S(X_{1j}, X_{2j}, \ldots, X_{n(j)})$ should be a differentiable function of $X_{1j}, X_{2j}, \ldots, X_{n(j)j}$. This property will be useful if the function is to be maximized.

3. $S_j = S(X_{1j}, X_{2j}, \ldots, X_{n(j)})$ should be a concave function of $X_{1j}, X_{2j}, \ldots, X_{n(j)j}$. This property will be of use if the function is to be maximized subject to linear constraints. In such a situation the local maximum of S_j will also be the global maximum.

4. The measure of redundancy should be mathematically tractable.

5. Let the redundancy of the overall network of N nodes be S_N. Since the network is a composite of many nodes, the measure of redundancy for the network, S_N, should be able to be decomposed to some extent into the measures of redundancies of the individual nodes or groups of nodes. Therefore, the overall measure of redundancy for the network should be some weighted function of the redundancies at the nodes.

6. The measure of redundancy may involve parameters other than the variable X_{ij} to account for some factors that may not otherwise be addressed.

3.4.5 Mathematical Statement of Mechanical Redundancy

There are several mathematical functions, including some of those presented in Section 3.1.4, that may satisfy some or all of the above necessary and desirable properties. The general expression that is adopted for this work is given by that developed by Shannon (1948) as follows:

$$S_j = -\sum_{i=1}^{M} X_i \ln X_i \tag{3.12}$$

where X_i = any variable of the system

M = number of sub-systems

S_j = entropic measure of the system

This particular entropy function is the basis of all other entropy functions. It was therefore selected so that modifications necessary to match it to the water distribution network problem (as described later in the chapter) could be performed on the fundamental equation rather than on expressions already modified for other purposes. Furthermore, Equation 3.12 satisfies all the properties desired in a redundancy measure. The first step in developing a specific redundancy measure for water distribution networks is to define the parameter X_i in Equation 3.12 such that the important physical conditions inherent in water distribution networks are included. Consider a network with N nodes where the nodes constitute the sub-systems. For a particular flow pattern under consideration let the i^{th} arc of the n(j) arcs incident on node j carry a flow of q_{ij}. The variable X_i was defined by Equation 3.11 in Section 3.4.4 and is repeated here as:

$$X_{ij} = \frac{q_{ij}}{Q_j} \quad \text{where} \quad Q_j = \sum_{i=1}^{n(j)} q_{ij}$$

where X_{ij} = the variable of the redundancy measure

q_{ij} = flow in link ij incident on node j

Q_j = total flow into node j

$n(j)$= total number of links carring flow into node j.

The variable X_{ij} now represents the contribution of the total flow to node j provided by the link between nodes i and j and provides the basis for incorporating relative flow capacity issues into the redundancy function. Thus X_{ij} is a measure of the relative capacities of links incident on node j and is therefore an indicator of the potential contribution of the link to the required demand to that node should another incident link fail. Equation 3.12 can now be restated as follows to give an entropic measure of local redundancy at node j:

$$S_j = -\sum_{i=1}^{n(j)} \left[\frac{q_{ij}}{Q_j}\right] \ln \left[\frac{q_{ij}}{Q_j}\right] \tag{3.13}$$

Maximizing S_j will maximize the redundancy of the node, where redundancy is represented by the extent to which the node receives water when a link incident on it fails. It is equivalent to maximizing entropy at the node. The maximum value of S_j for a given node j occurs when all (q_{ij}/Q_j) terms are equal. This condition occurs when the (q_{ij})'s are all equal, i.e., when each link incident on the node is carrying the same flow, which is consistent with the earlier discussion on the desirability of equal flow capacities.

Flows (q_{ij}) were chosen for the entropy term X_i in Equation 3.12 as it is the ability of the network to supply flow that is the important feature of redundancy. Although other hydraulic factors, such as pressure at a node or pressure drop along a link, are relevant they are not as important as the issue of flow itself.

Overall Network Redundancy

Redundancy for the network as a whole is a function of redundancies, (S_j)'s, of the individual nodes in the network. To obtain the overall network redundancy it is tempting to sum the redundancies at the individual nodes within the network. This approach, however, does not recognise that network redundancy is a measure of how well the *network* performs in terms of *total* flow in the network when a link fails. Therefore, it is the importance of a link relative to the total flow in all the links, not the importance of a link relative to the local flow, that is the important parameter in assessing <u>overall</u> network performance.

The following approach is taken to incorporate the individual redundancies into network wide redundancy measures. Let Q_o be the sum of flows in all links of the network, i.e., $Q_o = \sum_{j=1}^{N} Q_j$ where N is the number of nodes in the network. Note that Q_o is equal to the sum of flows in all the links rather than the total demand in the nework. As such it is greater than the total demand. The requirement of recognising Q_o suggests that (q_{ij}/Q_j) in Equation 3.13 be replaced by (q_{ij}/Q_o) This replacement gives rise to the following equation.

$$\widehat{S} = -\sum_{j=1}^{N}\left[\sum_{i=1}^{n(j)} \frac{q_{ij}}{Q_o} \ln \frac{q_{ij}}{Q_o}\right] \tag{3.14}$$

where $\widehat{S}$ is the network redundancy. In Equation 3.14 the summation is over all nodes in the network. However, it is the summation of the relative importance of links incident upon a node as opposed to the simple summation of the individual redundancies in the network. It should be noted that the maximum value of $\widehat{S}$ still occurs when the q_{ij} values are equal for each node j.

$\overline{S}_j$, the individual contribution to network redundancy from node j, is the term in square parentheses in Equation 3.14 ($\bar{S}_j$ is different from S_j because the former is the redundancy at a node considering the flow distribution at this node relative to the total network flow while the latter is redundancy at a node considering the flow distribution at that node only) and is written as:

$$\bar{S}_j = -\sum_{i=1}^{n(j)} \frac{q_{ij}}{Q_o} \ln \frac{q_{ij}}{Q_o} \tag{3.15}$$

Equation 3.15 can be decomposed as follows:

$$\bar{S}_j = -\sum_{i=1}^{n(j)} \frac{q_{ij}}{Q_j}\frac{Q_j}{Q_o} \ln \frac{q_{ij}}{Q_j}\frac{Q_j}{Q_o} \tag{3.16}$$

$$= -\frac{Q_j}{Q_o}\left[\sum_{i=1}^{n(j)} \frac{q_{ij}}{Q_j} \ln \frac{q_{ij}}{Q_j} + \sum_{i=1}^{n(j)} \frac{q_{ij}}{Q_j} \ln \frac{Q_j}{Q_o}\right] \tag{3.17}$$

$$= \frac{Q_j}{Q_o} S_j - \frac{Q_j}{Q_o} \ln \frac{Q_j}{Q_o} \tag{3.18}$$

Summing Equation 3.18 over the N nodes gives the network redundancy as:

$$\widehat{S} = \sum_{j=1}^{N} \left[\frac{Q_j}{Q_o} S_j \right] - \sum_{j=1}^{N} \left[\frac{Q_j}{Q_o} \right] \ln \left[\frac{Q_j}{Q_o} \right] \tag{3.19}$$

This overall network entropic measure of redundancy is now given in terms of weighted measures at the different nodes, $\sum_{j=1}^{N} \frac{Q_j}{Q_o} S_j$, plus another term. The weight $\frac{Q_j}{Q_o}$ on the (S_j)'s in the first term is the ratio of flow passing through the node j to the total flow in the network. Therefore, the first term represents the 'raw' nodal redundancy weighted by the relative importance of the node. More specifically it recognises the possible differences between two nodes with the same S_j. The form of this weight arises from the observation that redundancy at a node through which a very large proportion of flow passes should be valued, and therefore scaled, higher than one with a very low flow. It can be shown mathematically and through simulation that it is more difficult to re-allocate flow and maintain service at a node when the total flow into a node is quite small. A small total flow into a node generally indicates small capacity links incident on the node. Hence, nodes with smaller incident flows (link capacities) are more vulnerable to failure and therefore have lower contributions to redundancy (ensured by the smaller value of the weight $\frac{Q_j}{Q_o}$ which will be applied to this node's redundancy, S_j). Note that this lower redundancy associated with smaller incident links of a node is different from that discussed in Necessary Property 3 because the above statement refers to the overall network while Property 3 case refers only to the redundancy at an individual node. Thus, the first term of Equation 3.15 is associated with the redundancy of the nodes.

The second term can be considered as redundancy <u>among</u> the N nodes. More specifically the second term, $-\sum_{j=1}^{N} \frac{Q_j}{Q_o} \ln \frac{Q_j}{Q_o}$, is a measure of the distribution of flow to the nodes in the network and adds to the redundancy measure on the following basis. It has the same form as the general entropy expression. Improvement of redundancy at individual nodes occurs by equalizing the flow in each of the incident links on the node. Improvement of the network redundancy defined by the same type of expression implies the same general requirements, in this case, equality of demand distribution among the nodes. A network with a better flow distribution, namely one with the (Q_j/Q_o) for each node being closer in value to each other, will have a better inter-nodal measure of redundancy since these nodes will be less vulnerable to the impact of component (pipe) failures, i.e., none of the nodes will be connected very weakly due to the parity in pipe sizes at all nodes.

Note that, due to the values of the variables in Equation 3.19, all values of the second term as a whole are positive $[0 \leq Q_j/Q_o \leq 1$ and $-\ln(Q_j/Q_o) \geq 0]$ and hence the second negative term actually adds to the redundancy measure.

3.4.6 Some Properties of the Basic Redundancy Function

1. The basic function given by Equation 3.13 is strictly concave so that local maximum of the measure will necessarily be the global maximum if subject to no constraints, or constraint sets that are convex.

2. The function is symmetrical about the argument P as desired.

3. The units of the measure depend on the base of the logarithm chosen. The base can be chosen so as to give a maximum measure of one unit for any dimensional case. For a binary system, i.e., a node with exactly two alternate flow paths or incident links, the measure of redundancy when based on the logarithm to base 2 will have its maximum value of one unit (called bits).

This is given by;

$$S_j = -(1/2)\log_2(1/2) - (1/2)\log_2(1/2) = 1 \quad bit \tag{3.20a}$$

The units can be made universal for any number of dimensions by using the natural logarithm in which case we have

$$S_j = -k \sum p_i \ln p_i \qquad (3.20b)$$

where k is a transformation constant. In this case, the units of the measure are called 'nats'.

3.5 Extensions to the Basic Entropic Redundancy Function

3.5.1 Inclusion of the Alternate Number of Paths Between the Source and the Nodes

In Section 3.4, the redundancy at a node of a water distribution network was developed by considering the flow ratios in the incident links and the number of incident links at the node. The redundancy at a node is not completely represented by Equation 3.13, however. The ability of a network to respond to the failure of one of its links does not depend only on redundancy conditions in the immediate vicinity of the failure, i.e., at the nodes at either end of the failed link. Alternate paths for supplying nodes if a particular link fails may originate some distance from the nodes in the immediate vicinity of the link failure. The number of alternate paths contributes greatly to network redundancy and reliability and therefore need to be included in the redundancy function. It is the issue of what constitutes an alternate path that leads to this further refinement of Equation 3.13.

The entropy functions described in the previous sections have an implicit assumption that the number of alternate paths from a source to a demand point is equal to the number of links incident on the demand node. Such an assumption is unrealistic. Consider the case where one demand node has four incident links but flow to all four links from a source pass through the same two links some distance upstream and there are no other alternatives for the flow to pass through other than these two links. In this case there are obviously not four independent paths from source node to the demand node.

The contribution to the redundancy at a node by one of its incident links should therefore be a function not only of the percentage of flow that it brings to the node, but also of the true number of paths between the supply source and the node via that incident link. The ability of this incident link to continue an uninterrupted service to the node will be related to the number of the paths. The incident link with exactly one path from the source will not function if any of the links that form the path fails while a link with several paths to it may not cease to function. This situation arises because, for the latter type of incident link, if any of the paths is cut off by a link failure other than the incident link itself, other paths will be 'available' and keep this incident link in service. The two types of incident links are therefore not equal with regards to redundancy, the 'multiple path' link being more 'available' than the single path link. This concept is termed the 'availability' of the incident link and it is taken as being proportional to the number of paths between the source and the demand node in question through this incident link. The two types of incident links are illustrated in Figure 3.2.

Consider the redundancy of node 8 in the two layouts in Figure 3.2. The incident link 7-8 on node 8 in layout (a) is not equal qualitatively to link 7-8 in layout (b) because the former has exactly one path from the source to node 8 (1-4-6-7-8) while the latter has two paths (1-4-6-7-8) and (1-4-7-8). Hence when the two types of incident links are carring equal flow ratios to node 8, the one in layout (b) should contribute more redundancy to node 8 than the one in layout (a), and also the redundancy measure for node 8 in (b) should quantitatively be greater than that in (a). There is

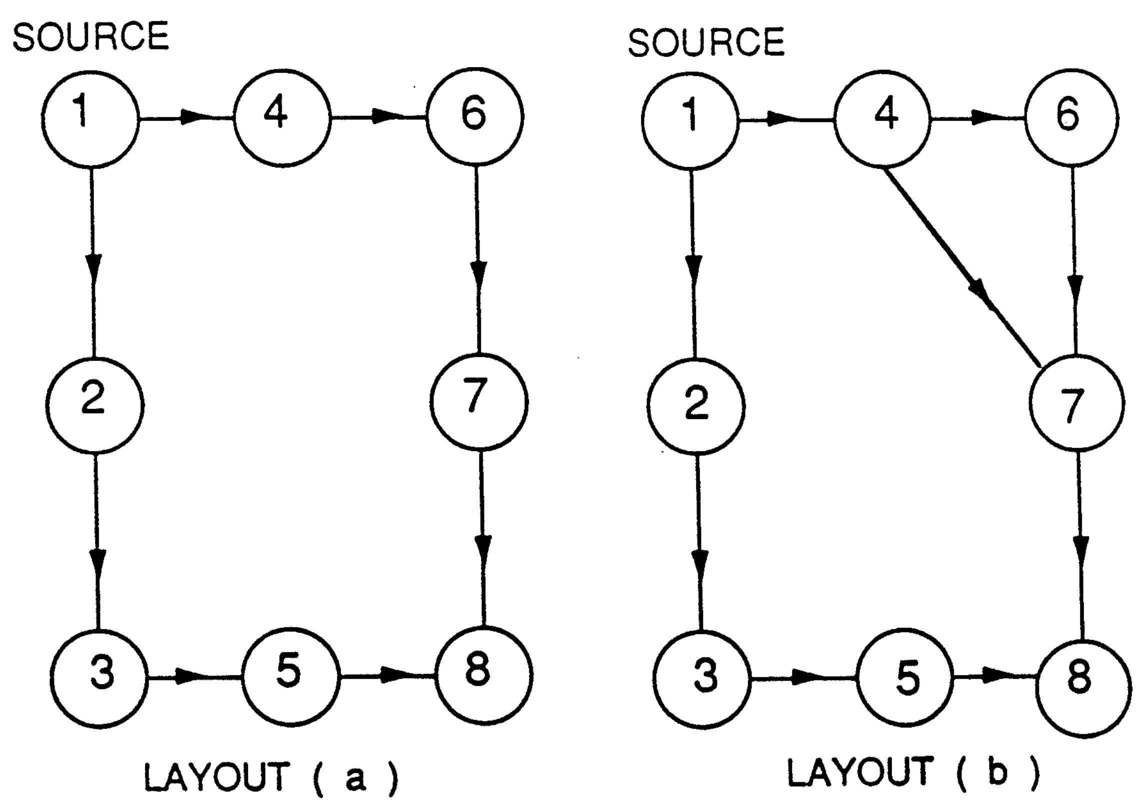


Figure 3.2: **Illustration of 'Availability' of the Incident Links**

therefore the need to modify the entropic measure of redundancy given by Equations 3.13 and 3.19 to account for the above observations.

This problem can be addressed by including a parameter which reflects the issues inherent in the desirable property number 6. Call this parameter the *path parameter.* Let the path parameter for node j be a_j. There are two important factors which must be considered in defining a_j. a_j must be quantitatively equal to the number of alternate <u>independent</u> paths between the source and the node in question. These paths are a function of the number of paths involved and the degree to which these paths interact with each other (or overlap). When assessing the number of independent paths it is necessary to determine whether there are any common links on all paths, i.e., are they dependent, and if common links exist, the extent to which the paths overlap, e.g., how many links on a particular path are used by other paths.

The number of paths in the network from the source to any demand node, j, can be determined using path enumeration algorithms, e.g., Misra (1970) and Aggarwal et al.(1973). These procedures do not identify dependence between paths however. The number of equivalent independent paths may be less than the value determined by the path enumerator algorithm and must reflect the possible dependency among the total number of paths. The number of equivalent independent paths is therefore less than the total number of paths if dependency exists or is equal to the total number of paths if there is no dependency. The development of the path parameter expression is elaborate and is described fully in the section below. An algorithm for its calculation is also presented in Appendix A.

Development of the Path Parameter

1. Definition of A True Alternate Path

If there are multiple number of paths between the source and the node in a distribution network, then two types of path systems can be identified. In one type of these paths, two paths which do not overlap can be traced between the source and the given node. In the second type, the two paths overlap at some point, which means that the two paths have 'common links' or branches. Although both provide alternate paths from the source to the demand nodes, the former type of paths are 'independent paths' while while the latter are 'dependent paths'.

The need to distinguish between the two types of paths arises because of the different effects they have on the redundancy at a node. When a node has two alternate and independent paths from the source and there is a link failure, there will be one path remaining to service the node. In the case of two alternate but dependent paths, the failure of some 'key' (common) links will result in the node being completely severed from the source. Hence the 'effective' redundancy at the node of the second case will be reduced. The parameter a_j is to be quantitatively equal to the number of alternate independent paths between the source and given node. Hence the first type of path system raises no problem. To obtain the a_j for the second type of path system, there is the need to develop an 'effective alternate independent' number of paths from the given number of dependent paths.

2. Quantifying The Effect of Common Links To The Number of Independent Paths

The difference between the two paths systems is that one has 'common links' that gives the dependency. Figure 3.3 illustrates how the addition of common links to a network affects its number of independent paths.

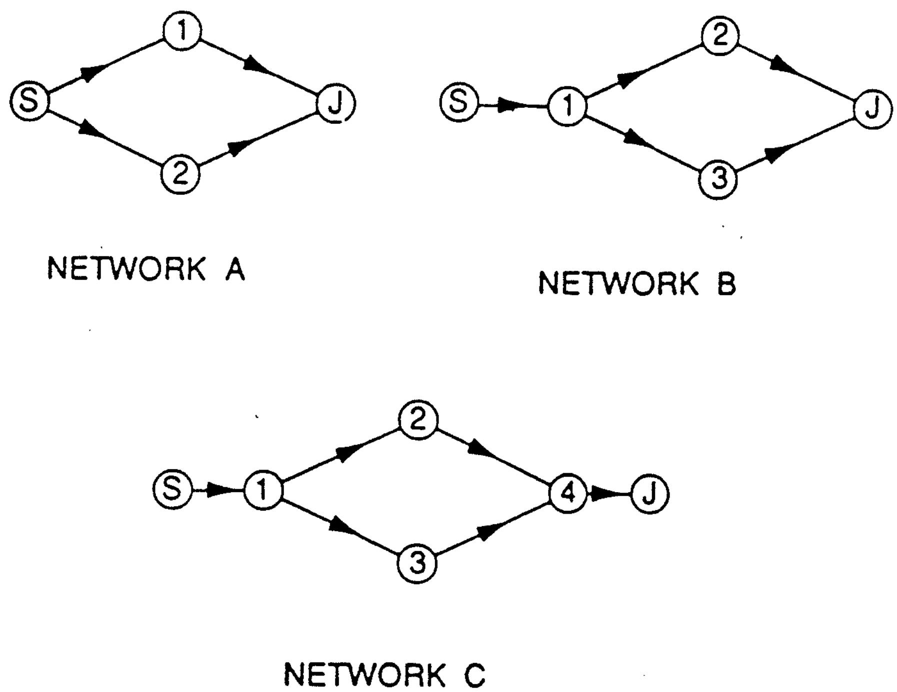


Figure 3.3: **Path Reduction By The Addition of Common Links**

Node j in Network A has 2 alternate independent paths from the source S. The addition of link (S-1), which is common to the original two paths in Network A, results in two alternate but dependent paths in Network B. The path system from source to node j in Network B is now 'reduced' compared to the path system in Network A. This is because in Network A each link belongs to exactly one path, while in Network B, link (S-1) belongs to the two paths.

Let the number of paths to which a link belongs be termed the DEGREE of that link. For any two paths between a given node and the source to be completely independent, every link in the two paths should have *ONE DEGREE.* If a link belongs to two paths, then it has a *DEGREE OF DEPENDENCY* of one unit. If the link belongs to three paths, then it has a degree of dependency of two units. Similarly four paths imply a degree of dependency of three units, and so on. Let the degree of link l be denoted by d_l. Then its degree of dependency is given by:

$$D_l = d_l - 1 \tag{3.21}$$

If there are nd number of alternate dependent paths from the source to the given node, the 'effective number of independent paths' can be obtained by removing the dependencies from the links. This adjusted number of independent paths is the required path parameter and is given for node j as:

$$a_j = nd \left[\frac{\sum_{l=1}^{M} d_l - \sum_{l=1}^{M} D_l}{\sum_{l=1}^{M} d_l} \right] \tag{3.22}$$

where M = the number of links in the nd number of paths.

Equation 3.22 can be factorized to:

$$a_j = nd \left[1 - \frac{\sum_{l=1}^{M} D_l}{\sum_{l=1}^{M} d_l} \right] \tag{3.23}$$

The term in the inner bracket is the factor that adjusts the number of dependent paths to equivalent independent paths. When there are no dependencies, this factor vanishes as the term $(\sum_{i=1}^{M} D_l)$ becomes equal to zero, and the path parameter a_j is exactly equal to nd. As the number of common links increases for a fixed number of dependent paths, the effective number of independent paths decreases. Thus the value of a_j for the Network B will be higher than that for Network C in Figure 3.3.

Since the entropy measure of redundancy is associated with a node, in order to include path parameter a_j in the entropy measure of redundancy, since the a_j is the total number of paths to node j and not the number of paths to a particular link, a new parameter specifying the number of effective independent paths from the source that go through the particular incident link to node j has to be determined.

Define this parameter as:

$$a_{ij} = nd_{ij}\left[1 - \frac{\sum_{l=1}^{M_{ij}} D_l}{\sum_{l=1}^{M_{ij}} d_l}\right] \tag{3.24}$$

where

a_{ij} = effective number of independent paths from the source through link ij from node i incident on node j

nd_{ij} =number of dependent paths from the source through link ij from node i incident on node j

M_{ij} =number of links in the nd_{ij} number of paths

The total number of effective independent paths for node j is therefore the sum of paths through the $n(j)$ incident links:

$$a_j = \sum_{i=1}^{n(j)} a_{ij} \tag{3.25}$$

An important feature to note in the use of Equation 3.25 to determine the number equivalent paths is that a_j can take on non-integer values. This property is intuitively attractive as common links between paths reduce the number of independent paths below the total number of paths. However, because two paths have some common links it does not necessarily mean that the number of equivalent paths is reduced to unity. In developing a means to handle the equivalent paths question it is important to recognise that the lower bound of a_j in all cases is unity representing a single (branch) path from the source to the demand point.

The Modified Redundancy Measure With Path Parameter

The basic entropy function of Equation 3.13 is now written to include the path parameter as

$$S_j = -\sum_{i=1}^{n(j)} \left[\frac{q_{ij}}{Q_j} \ln \left(\frac{q_{ij}}{a_{ij} Q_j} \right) \right] \tag{3.26}$$

The first term $\frac{q_{ij}}{Q_j}$ in the above expression does not include the parameter a_{ij} because the objective of the path parameter is to increase the basic redundancy measure if the number of independent paths between the source and the node is greater than one (unity). This is achieved by the division of the terms within the logarithm by a_{ij} $[\frac{q_{ij}}{Q_j} < \frac{q_{ij}}{a_{ij} Q_o} \leq 1$, therefore, $\ln \frac{q_{ij}}{a_{ij} Q_o} > \ln \frac{q_{ij}}{Q_j}]$. This increase will, however, be attenuated considerably if the term outside the logarithm is also divided by the parameter a_{ij} (which will be greater than unity for the node with more than one independent paths though incident link ij). Since the measure is qualitative at this time, attenuation of the function in this manner decreases its sensitivity. a_{ij} is therefore omitted from the terms outside the logarithm.

The entropy function of Equation 3.26 can be factorised to give

$$S_j = -\sum_{i=1}^{n(j)} \left[\frac{q_{ij}}{Q_j} \right] \ln \left[\frac{q_{ij}}{Q_j} \right] + \sum_{i=1}^{n(j)} \left[\frac{q_{ij}}{Q_j} \right] \ln a_{ij} \tag{3.27}$$

The first term in Equation 3.27 is the redundancy measure for the node using the assumption that each incident link constitutes exactly one path from the source to this node as defined previously in Equation 3.13. The second term is a function of the true number of alternate paths and represents a correction factor to reduce the number of alternate paths if some of the paths are dependent. The special cases of layout configurations for which application of the function is appropriate can be summarized as follows:

a) Nodes with one incident link but having several paths through the network upstream of the single incident link: If the equivalent number of paths is greater than unity, and thus $a_{ij} > 1.0$, the second term will contribute in a non-zero fashion to the measure of redundancy for this node. This is not possible under the use of the original function of Equation 3.13.

b) Nodes with two or more incident links where each incident link is exactly equal to one path from the source to this node: The second term will vanish since $a_{ij} = 1$ and the logarithm of one is zero. Hence the function will reduce into the original function given by Equation 3.13.

c) Nodes with several incident links such that the equivalent paths through some of these links are less than one: This may happen if some of the paths overlap, in which case they may have several common links. The a_{ij} will now be less than one and the second term of Equation 3.27 will become negative. This process will therefore reduce the measure below that given by the original function for those particular links considering them to be completely independent paths. It is important to note that the path parameter a_j will still never be less than one as it measures total equivalent paths rather than the value for a particular link. The network wide measure equivalent to Equation 3.19 for describing redundancy between nodes with this path parameter is the same as Equation

3.19 except that Equation 3.27 rather rather Equation 3.13 is used to give the values of S_j.

3.5.2 Inclusion of Age Factor of Pipes in the Measure

The Hazen-Williams formula for flow through pipes given by Equation 3.6, includes the friction coefficient, C_{ij} which is dependent on the material of the pipe as well as the age of the pipe. In general, as the pipe ages, it losses some of its carrying capacity. It can therefore be argued that the hydraulic redundancy inherent in the network decreases with time. This age factor can be introduced in the mechanical redundancy measure so that it can correlate more with hydraulic redundancy as the system deteriorates with age. Therefore the fundamental redundancy function can be modified include the age of the pipes (links). It should be noted that the inclusion of the age factor parameter under this circumstances is not a statement of the dynamics of the system but rather a statement of conditions at specified time intervals.

The concept of 'useful entropy' function given by Equation 3.8 in Section 3.1.4 is proposed to handle this problem. This function satisfies all the axioms and is essentially a modification of Shannon's entropy. The parameter 'u_i' in Equation 3.8 has to be modelled to account for the aging of the pipes, however.

Modelling of the Age Factor Parameter

Let u_i in Equation 3.11 be represented by u_{ij} for link ij and this be the age factor parameter for the pipe material in this link. The use of the age factor parameter, u_{ij}, is not to represent the age of the pipe in the network. It is to reflect the degree of deterioration of the pipe with age, or in other words the reduction in its carrying capacity and hence its contribution to redundancy. As C_{ij} is the only parameter (of the pipe parameters of diameter, length and smoothness) in the Hazen-William empirical equation that changes with the age of the pipe capacity, it is therefore proposed that the age factor parameter be derived from it.

The Hazen-Williams friction factor, C_{ij}, is a dimensionless parameter determined experimentally from laboratory studies which, for the pipes of the same structural material, will reflect their age and degree of deterioration. Note that since the Hazen-Williams friction coefficients also depend on the material of the pipe, use of the C_{ij} factor also permits a differentiation between the carrying capacity of two pipes of the same diameter, length and age but made from different materials. The rate of deterioration in carrying capacity of pipes also depends on the material from which they are made. Changes in the Hazen-Williams friction coefficient are able to reflect this change very well. Furthermore, data for pipe age and their corresponding friction coefficients can be obtained from the literature.

In general, values for the Hazen-William friction coefficient of new pipes vary from 100 to 150, e.g., values for steel and plastic pipes range between 140 and 150 while brick pipes have values around 100. In cast iron pipes, C values can deteriorate from about 130 to 75 over a period of 50 years. A plot of the age of pipe versus logarithm of the friction coefficient C_{ij}, for data obtained from Hwang (1981) is shown in Figure 3.4. A linear relationship was found to fit this data particularly well with regression coefficient (R^2) value of 0.935. Therefore the logarithm of the friction coefficient will be taken as being directly proportional to the age of the pipe in the links.

In this study a value of 150 of Hazen-William coefficient is taken as the upper reference point for the age factor parameter. All values are scaled down from this value. The reference point value for the age factor parameter is $\ln(150) = 5.0$. Dividing the parameter by 5.0 so that the age factor parameter for pipes with Hazen-Williams friction coefficient, $C_{ij} = 150$, is equal to unity, implies the use of the *scale factor* 0.2 to the general function. Hence the age factor parameter, which is time dependent, is

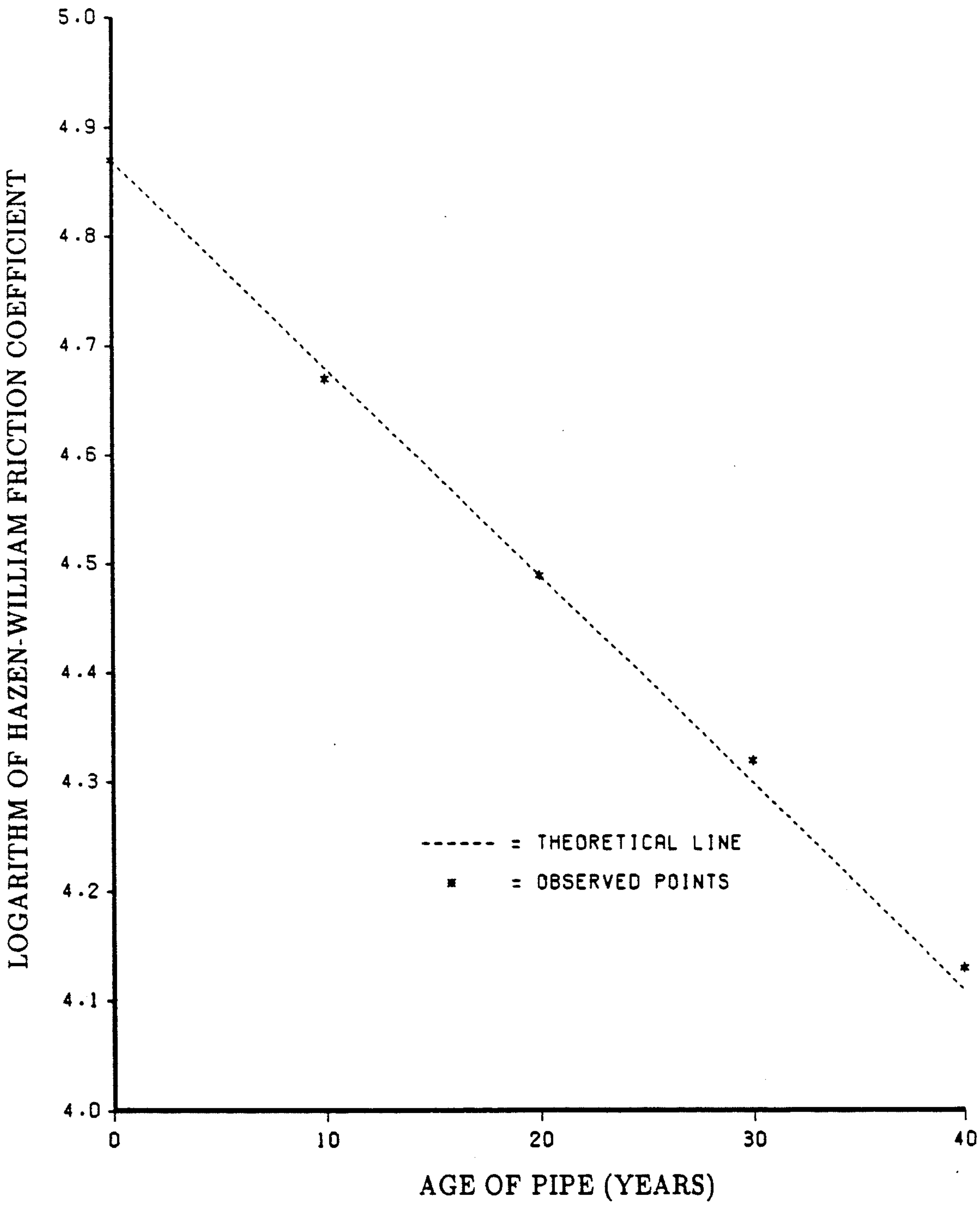


Figure 3.4: **Relationship Between Friction Coefficient and Age of Pipe**

given as;

$$u_{ij}(t) = 0.2 \ln C_{ij}(t) \tag{3.28}$$

where

$C_{ij}(t)$ = Hazen-Williams friction factor for pipe between node i and j after time t

$u_{ij}(t)$ = age factor parameter after time t

t = time after installation of new pipe in years.

The Modified Redundancy Measure With Parameters

Using the 'useful entropy' concept, the age factor parameter u_{ij} can be incorporated into the basic entropy function of Equation 3.13 to give;

$$S_j = -\sum_{i=1}^{n(j)} u_{ij} \left[\frac{q_{ij}}{Q_j}\right] \ln \left[\frac{q_{ij}}{Q_j}\right] \tag{3.29}$$

where u_{ij} = the age factor for link ij incident on node j.

The entropy function including both path parameter and age factor is now obtained by modifying Equation 3.26 as follows

$$S_j = -\sum_{i=1}^{n(j)} u_{ij} \left[\frac{q_{ij}}{Q_j} \ln \left(\frac{q_{ij}}{a_{ij} Q_j}\right)\right] \tag{3.30}$$

Equation 3.30 can be factorized to give

$$S_j = -\sum_{i=1}^{n(j)} u_{ij} \left[\frac{q_{ij}}{Q_j}\right] \ln \left[\frac{q_{ij}}{Q_j}\right] + \sum_{i=1}^{n(j)} u_{ij} \left[\frac{q_{ij}}{Q_j}\right] \ln a_{ij} \tag{3.31}$$

All variable definitions are as previously defined.

3.5.3 Overall Network Redundancy After Function Extension To Include Dependent Paths and Aging Issue

With the inclusion of the new parameters a_{ij} and u_{ij}, there is the need to modify the overall network-wide redundancy function given by Equation 3.19. The modified expression is similar to Equation 3.14 with the parameters u_{ij} and a_{ij} included, as shown below as:

$$\widehat{S} = -\sum_{j=1}^{N}\left[\sum_{i=1}^{n(j)} \frac{u_{ij}q_{ij}}{Q_o} \ln \frac{q_{ij}}{a_{ij}Q_o}\right] \tag{3.32}$$

The individual contribution to network redundancy from node j, is the term in the square parenthesis, $\bar{S}_j$, as;

$$\bar{S}_j = -\sum_{i=1}^{n(j)} \frac{u_{ij}q_{ij}}{Q_o} \ln \frac{q_{ij}}{a_{ij}Q_o} \tag{3.33}$$

This contribution from a node can be decomposed as follows:

$$\bar{S}_j = -\sum_{i=1}^{n(j)} \frac{u_{ij}q_{ij}}{Q_j}\frac{Q_j}{Q_o} \ln \frac{q_{ij}}{a_{ij}Q_j}\frac{Q_j}{Q_o} \tag{3.34}$$

$$= -\frac{Q_j}{Q_o}\left[\sum_{i=1}^{n(j)} \frac{u_{ij}q_{ij}}{Q_j} \ln \frac{q_{ij}}{a_{ij}Q_j} + \sum_{i=1}^{n(j)} \frac{u_{ij}q_{ij}}{Q_j} \ln \frac{Q_j}{Q_o}\right] \tag{3.35}$$

Letting

$$\sum_{i=1}^{n(j)} u_{ij} = U_j \tag{3.36}$$

$$\bar{S}_j = \frac{Q_j}{Q_o}S_j - U_j\frac{Q_j}{Q_o} \ln \frac{Q_j}{Q_o} \tag{3.37}$$

Summing Equation 3.37 over the N nodes gives the overall network redundancy as:

$$\widehat{S} = \sum_{j=1}^{N}\left[\frac{Q_j}{Q_o}S_j\right] - \sum_{j=1}^{N}\left[U_j\frac{Q_j}{Q_o}\right] \ln \left[\frac{Q_j}{Q_o}\right] \tag{3.38}$$

Equation 3.38 is that of the basic function in Equation 3.19 but with U_j, which is the sum of age factor parameters of the links incident on node j, included in the second term. This second term is exactly the same as the type of entropy termed 'useful entropy' given in Equation 3.11. The network-wide redundancy can therefore be described as 'a sum of weighted nodal useful entropies plus the useful entropy among the nodes'.

3.5.4 Mathematical Properties of the Modified Functions

The modified entropy redundancy including all the parameters is given by Equation 3.31, which is the measure at individual nodes, and Equation 3.38, which is the overall network redundancy measure. The first term in Equation 3.31 is 'useful entropy' which satisfies all the desired mathematical properties. The second term is a product of a constant $(u_{ij} \cdot \ln a_{ij})$ and the variable $\frac{q_{ij}}{Q_j}$. The variable $\frac{q_{ij}}{Q_j}$ is a continuous variable $(0.0 \leq \frac{q_{ij}}{Q_j} \leq 1.0)$ and is linear and hence concave. The product of a constant $(u_{ij} \cdot \ln a_{ij})$ term and the variable $\frac{q_{ij}}{Q_j}$ is therefore concave. The sum of two concave functions is concave. Thus Equation 3.31 is a concave function.

The first term of Equation 3.38 is 'the sum of weighted S_j's'. Since each S_j is concave, the first term of Equation 3.38 is also concave. The second term is 'useful entropy', and as discussed before is also concave. Therefore the network redundancy function given by Equation 3.38 also satisfies all the necessary mathematical requirements. However, both these measures do not satisfy the symmetry property because of the presence of the parameters, u_{ij}, a_{ij}, and U_j. They will become symmetrical only if these parameters have the value of unity. This does not mean, however, that they are not adequate to give the desired result. The need for the parameters in the functional form can only be satisfied at the expense of the 'symmetrical' requirement which is not a rigorous requirement.

3.5.5 Further Complications Due to the Nature of Water Distribution Network Operation

Dual Flow Directions In Some Pipes

There is still , however, an issue not completely answered by the measures, Equation 3.30 and 3.38. In the development of the entropy based redundancy measures, links considered to contribute to reliability/redundancy at a node were those in which flow was towards the node in question. No consideration was given to how those links taking flow away from the node might contribute to redundancy. In the event of a link failure, *outflow* links from some nodes can become *inflow* links to the same nodes. This situation occurs if loops exist in the network, a normal requirement in urban distribution systems. If a link before (upstream of) node 'a' fails then flow can be provided to that same node by diverting it around the other portion of the loop. The outflow link from node 'a' in that loop could then become the inflow link to that node. This flow reversal is a critical aspect of permitting the system to adjust in an attempt to supply as much of the demanded flows as possible.

These outflow links therefore provide, at least implicitly, additional flow paths to a node and can contribute to the reliability of supply to the node. The entropy function of Equation 3.30 can then be modified to include all incident links rather than simply those which supply flow to the node under *normal* working condition. The modified expression proposed is

$$S'_j = -\sum_{i\in\overline{U}_j} u_{ij}\left[\frac{q_{ij}}{Q'_j}\ln\frac{q_{ij}}{a_{ij}Q'_j}\right] - \sum_{k\in\overline{L}_j} u_{jk}\left[\frac{q_{jk}}{Q'_j}\ln\frac{q_{jk}}{a_{jk}Q'_j}\right] \tag{3.39}$$

where Q'_j is the total of all flow leaving and entering node j by links contained in $\overline{U}_j$ and $\overline{L}_j$ and thus expressed as follows:

$$Q'_j = \sum_{i\in\overline{U}_j} q_{ij} + \sum_{k\in\overline{L}_j} q_{jk} \tag{3.40}$$

$\overline{L}_j$ = set of outflow links under normal flow conditions connected to node j in which the link (j-k) belongs to a loop containing node j.

$\overline{U}_j$ = set of nodes on the upstream ends under normal flow conditions of links incident on node j.

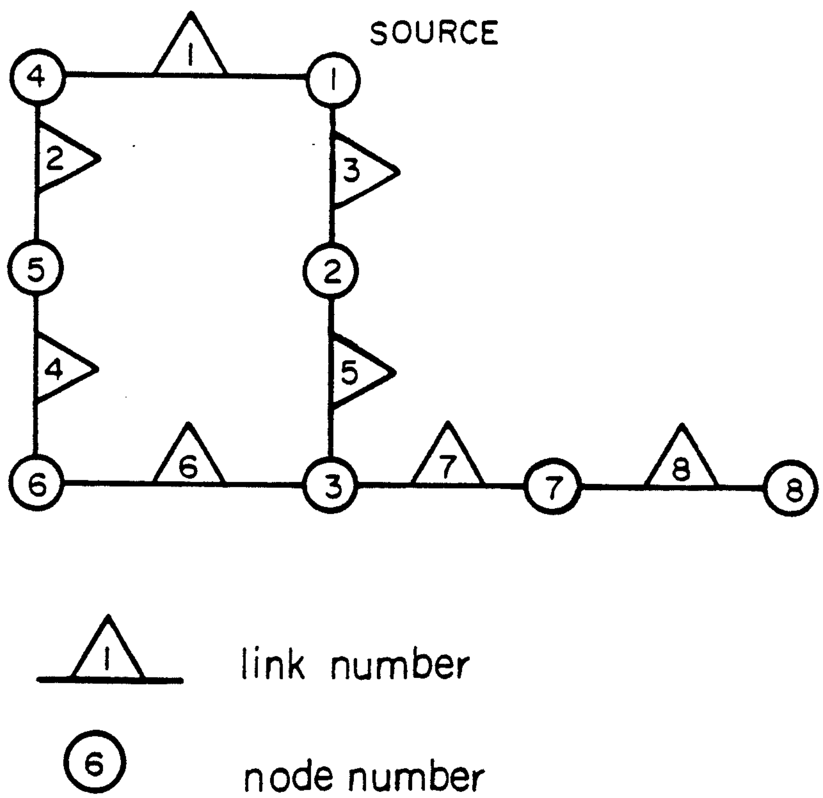


Figure 3.5: **Network Showing Links In Loop and Links Outside Loop**

Restrictions on outflow links to include only those which are part of a loop containing node j prevent the counting those outflow links from node j which are part of *pure* branches. As shown in Figure 3.5, pure branches cannot assist with supply to a node should one of the existing supply links fail. In Figure 3.5 it is clear that link 7, which is part of a pure branch, cannot contribute to supply of node 3 if a link on either arm of the main loop should fail. $\widehat{S}$, the network entropy, for this case is the same as $\widehat{S}$ in Equation 3.38 except S_j in Equation 3.30 is replaced by S_j' from Equation 3.39 to give:

$$\widehat{S} = \sum_{j=1}^{N} \left[\frac{Q_j}{Q_o} S_j'\right] - \sum_{j=1}^{N} \left[U_j \frac{Q_j}{Q_o}\right] \ln \left[\frac{Q_j}{Q_o}\right] \tag{3.41}$$

Multiple Source Networks

Some water distribution networks may have several source nodes to serve the demand nodes. This condition does not have any effect on the calculation procedures presented for the redundancy assessment. The simplicity of including this arises from the fact that, since all the nodes are interconnected, each node upstream of another node is a 'source node' to the node downstream to it. The incident links are those links connected to the upstream nodes which are the 'source nodes' to the downstream node whose redundancy is being measured. If there are multiple sources directly connected, i.e., connected by a single link, to a demand node, then this node may have a multiple incident links so that it will have a non-zero measure of redundancy. Therefore the measure will still be given by Equation 3.30 or 3.38 for both single source and multiple source networks.

Multiple Load Pattern Networks

In real water distribution networks, the flows in the links are not fixed at a single value but change with time due to changing demand at the nodes. Therefore one would question that since the redundancy measure is flow based (as given by Equation 3.30), which flow pattern gives the redundancy measure?

A single value of the flow magnitude such as the average value over a period of time may be used. If the network component sizing is done using peak demand pattern, then the corresponding peak flows in the links should be used to compute the redundancy measure. The question becomes one of defining the flow pattern by which the redundancy measure would be defined. Since the entropic redundancy measure is a function of the manner of supply for a particular demand pattern, the concept does not change simply because the demand pattern changes. Rather, the redundancy measure is defined in terms of the flow pattern for which the system is designed. Once the critical design conditions are defined, the entropy measure can be determined.

However, as is shown in a later section, the entropy function can be used to design networks for a range of demand conditions and flow patterns without having to explicitly evaluate all possible load patterns. This feature represents the most valuable aspect of the entropy functions.

Inclusion of Redundancy Contribution by Pumps in the Measure

As explained in Section 3.4.2, pumps are considered as links in the network. The arguments used to illustrate redundancy by virtue of the number of pipes and their relative carrying capacities apply to pumps. The hydraulic characteristic of pumps is given as:

$$h_{ij} = Aq_{ij}^2 + Bq_{ij} \tag{3.42}$$

where h_{ij} = pressure head of pump between nodes i and j

q_{ij} = pumping rate of pump between nodes i and j

A = constant

B = constant

In considering pumps as links, the pumping rate is therefore related to the square of the pumping head or $q_{ij} \propto h_{ij}^{0.5}$. Therefore if two pumps are provided in parallel, and one has a very small pumping capacity relative to the other and the larger pump fails, increasing the pumping rate (q_{ij}) of the smaller pump would result in substantial decrease in the pumping head that can be achieved, hence the system would not perform well. Therefore the closer the two pumps are in pumping capacity, the better the redundancy provided.

Therefore the redundancy provided by the use of multiple pumps connected in parallel in the network can be included in the entropy measure by considering them as links and their relative pumping rate capacities used in the place of relative flow rates of pipes in Equation 3.13.

Inclusion of Valves in the Measure

As explained in Section 3.4.2, valves are located on links (pipes) in the network. Their presence is therefore the same as the presence of pipes. Their hydraulic characteristic is the same as that of pipes, given by Hazen-Williams equation (Equation 3.9) but with the friction coefficient of the pipe on which it is located modified by that of the valve. This is given by:

$$q_{ij} = k(C_{ij} - CV_{ij})\frac{h_{ij}^{0.54}}{L_{ij}^{0.54}} \cdot D_{ij}^{2.63} \tag{3.43}$$

where CV_{ij} = Hazen-Williams roughness coefficient of valve

C_{ij} = Hazen-Williams roughness coefficient of the pipe on which the valve is located

L_{ij} = length of the pipe on which the valve is located

h_{ij} = headloss through the pipe on which valve is located

D_{ij} = diameter of the pipe on which valve is located

k = conversion factor for units.

Therefore, their presence in the network and their contribution to redundancy of the network is accounted for by the pipes on which they are located.

Inclusion of Redundancy Contribution by Tanks in the Measure

Storage tanks within the network are modelled as demand nodes within the network with the amount of water consumed at these nodes being equal to the volume of water stored at these nodes (difference in the inflow to the tanks and outflow from the tanks multiplied by the time of storage). Hence the redundancy of these nodes are obtainable in the same manner as the redundancy measure for other demand nodes. Their presence in the network therefore contributes to the redundancy measure for the whole network.

Chapter 4

EVALUATION OF THE ENTROPY BASED REDUNDANCY MEASURES

4.1 Introduction

In this chapter, the entropy based redundancy measures will be evaluated on the basis of their performance relative to other traditional methods of reliability assessment. As developed in the previous chapter, the entropy based redundancy measures are quantitative measures of network reliability . They are, however, relative quantitative measures. An obvious question to be answered in using the measure is what is an acceptable value of the entropic measure? Even more fundamental perhaps is what is the relationship between reliability (as assessed by some procedure) and the entropy based measures?

The answers to the two questions are related to similar questions for reliability itself. For example, what is an acceptable level of reliability? Since redundancy of a network is an essential contributor to reliability for that network, it may also be asked what is an acceptable level of redundancy? This last question is very difficult to answer as there are no procedures available for quantitative assessment of network redundancy. Since the entropic measure is an indicator of the contribution of redundancy to network reliability, the fact that numerical values have been determined for redundancy provides a quantitative basis for comparing networks.

The entropic measure must, therefore, be able to distinguish between subtle differences in network design, arising from variations both in layout and in component sizing. Inherently better network designs by any specified criteria must be able to be distinguished from less desirable designs on the basis of the same criteria. This thesis proposes a criteria for this evaluation. In order to determine whether the proposed entropic measures fulfill this requirement, they will be computed for a series of network layouts and designs. The water distribution networks used in this analysis should be alternative designs for the same network (demand pattern) that have inherently different levels of redundancy and reliability. A design model will be used to generate such alternative networks for the evaluation process. The measures will then be compared to a Nodal Pair Reliability (NPR) parameter and a Percentage of Demand Supplied at adequate Pressure (PSPF) parameter.

The NPR parameter has been used by Quimpo and Shamsi (1988) and Wagner et al.(1988a) in water distribution networks analysis. NPR measures the probability that a pair of nodes, in this case the source node and each of all other demand nodes, are successfully connected. The analysis requires some assessment of the probability of the links (pipes) failing. The algorithm used to determine the values of NPR is that developed by Kim et al.(1972). It is interesting to note that the entropic redundancy function contains the variable q_{ij}. The larger the value of q_{ij} generally the larger the pipe carrying the flow. (This assertion requires that the hydraulic gradient in each pipe be approximately equal. Such an assumption is not unreasonable. Rowell and Barnes (1982) assumed such a condition in their design model for looped networks). Examination of Table 4.1 shows that the larger diameter pipes have smaller failure probabilities. Hence the NPR parameter which directly considers probability of link failure will also have an indirect consideration of the q_{ij} terms in the entropic expression.

Table 4.1. Failure Rates of Pipes for the Determination of NPR

Diameter (m)	Average Rate of Failure ($Breaks/Km/Year$)†
0.102	0.316
0.152	0.191
0.203	0.137
0.254	0.109
0.305	0.091
0.381	0.075
0.508	0.059
0.762	0.045

† Data From Su et al.(1987)

The PSPF parameter on the other hand indicates the hydraulic redundancy of a water distribution network, and provides another means of assessing the flexibility or resilience of water distribution networks. It was developed to overcome a major shortcoming of the NPR parameter. The NPR parameter assumes that adequate supply can be maintained to a node as long as there is at least one connection or path between a source and that node. No consideration is given to whether there is sufficient capacity in the remaining path(s) to provide the necessary flow at adequate pressure. The PSPF parameter assesses the performance of the network given failure (removal) of a link in the network . Determination of the PSPF requires a hydraulic simulation of the network over a range of link removal situations. A different link is removed in each simulation and the proportion of demand that is supplied at adequate pressure is noted. The hydraulic simulation is required in each case of link removal in order to redistribute the flow through the remaining links in the network. The PSPF therefore assesses the hydraulic flexibility of the system through flow redistribution while the NPR recognises the probabilistic implications of pipe failure. As such, used jointly, the two parameters give a good basis upon which to assess the entropic function.

The actual evaluation of the entropy measure was as follows. A range of network solutions (layout and component size solutions) for a given design problem (source and demand situation) generated from a network design model of Awumah et al. (1989) are used in the comparison of the entropy measure and the traditional reliability measures. The model of Awumah et al. (1989) is able to generate both alternate layouts and optimal component sizing. The use of their approach therefore provides the opportunity to examine a range of alternative solutions with different costs and different levels of reliability and to observe or evaluate how the entropy measure performs for each candidate solution.

The design problem solved by the model of Awumah et al. (1989) is described in Figure 4.1, which shows all candidate links in the network, Table 4.2 which provides the demands and minimum pressure at each node and Table 4.3 which gives the relevant cost information. The eight candidate solutions for the model of Awumah et al. (1989) for this example network are shown in Figure 4.3.

Table 4.2. Demand at Nodes and Minimum Pressures for the Network

Node	Demand (m^3/h)	Minimum Pressure (m)
1	-1600	100
2	100	30
3	150	30
4	150	30
5	150	30
6	100	30
7	200	30
8	200	30
9	200	30
10	100	30
11	150	30
12	100	30

‡ Data From Awumah et al.(1989)

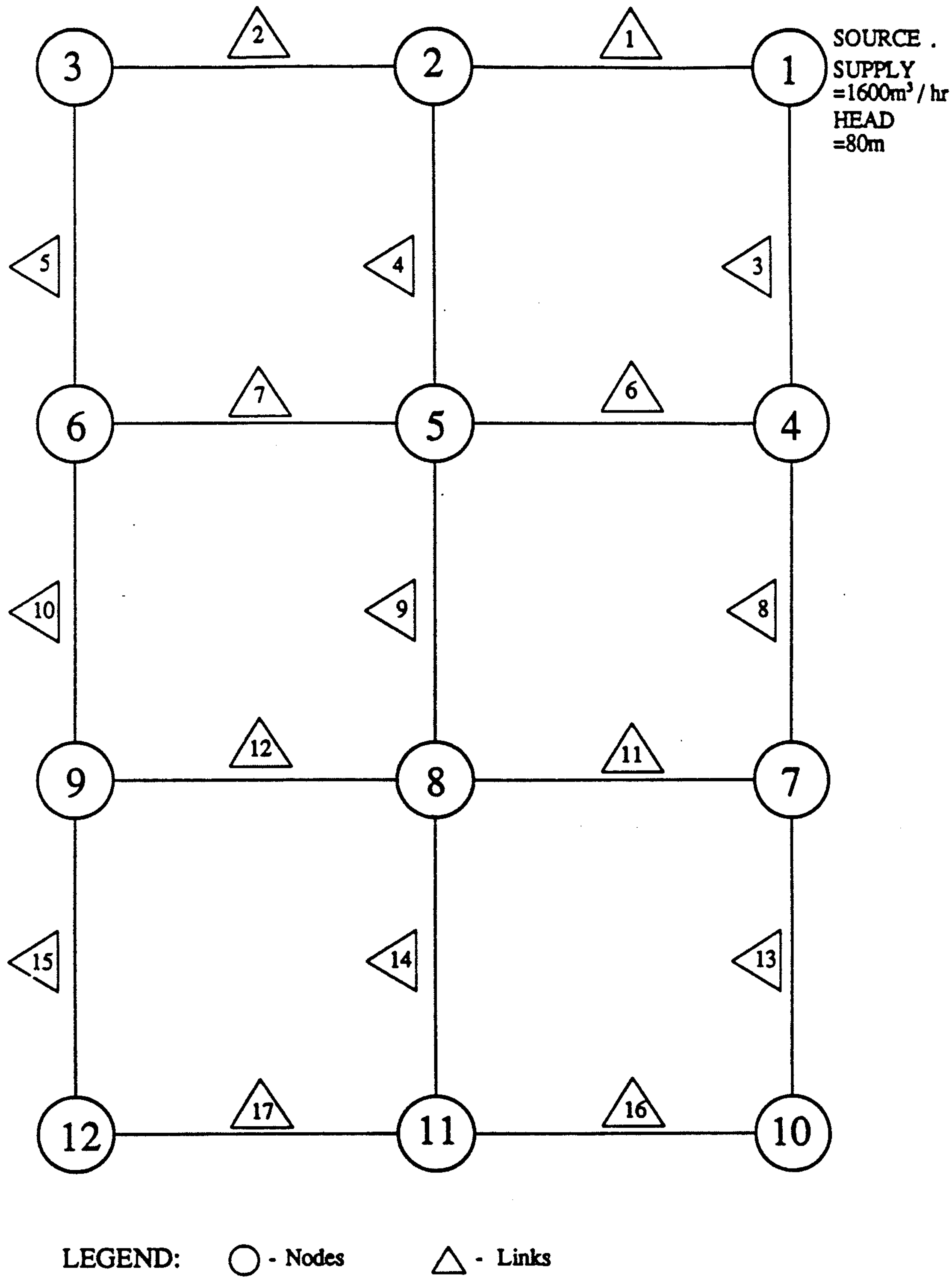


Figure 4.1: **Initial Network Layout Showing Link and Node Characteristics**

Table 4.3. Cost Data for Pipes

Pipe Diameter (m)	Cost Per Meter $(\$/m)$	Pipe Diameter (m)	Cost Per Meter $(\$/m)$
0.025	2	0.46	130
0.050	5	0.51	170
0.080	8	0.56	300
0.100	11	0.61	550
0.150	16	0.66	750
0.200	23	0.69	1050
0.250	32	0.71	1200
0.300	50	0.76	1500
0.360	60	0.81	1800
0.410	90	0.86	2200

§ Data From Awumah et al.(1989)

4.2 Entropy Based Redundancy Measures for the Candidate Networks

4.2.1 Introduction

The entropy based redundancy measures developed in Chapter 3 are used to compare the *mechanical redundancy* inherent in the networks given in Figure 4.2. In the first instance the basic entropic redundancy measure at a node in which no parameters were included (as given by Equation 3.13) was used to obtain redundancy at the nodes of the layouts. The overall network redundancy measure based on this S_j and given by Equation 3.19 was also calculated for the networks. The average values of the

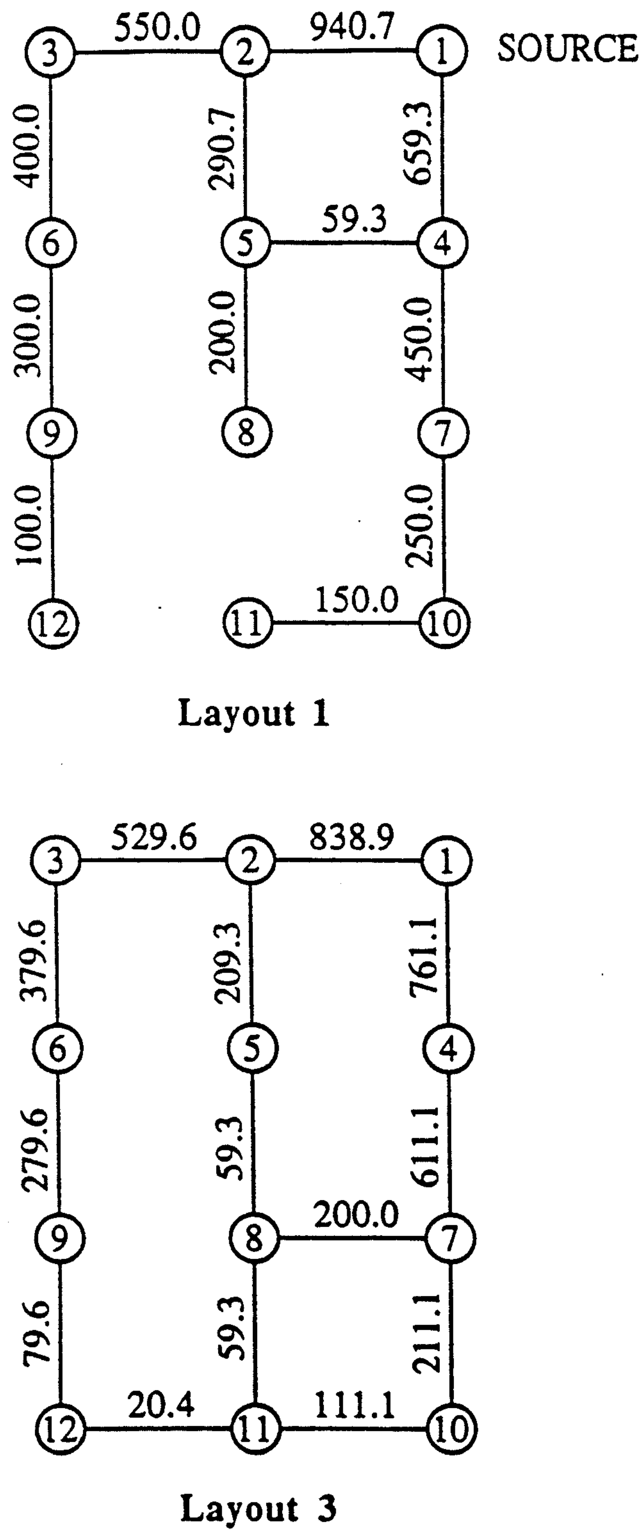


Figure 4.2: **Solution Networks for Redundancy Evaluation**

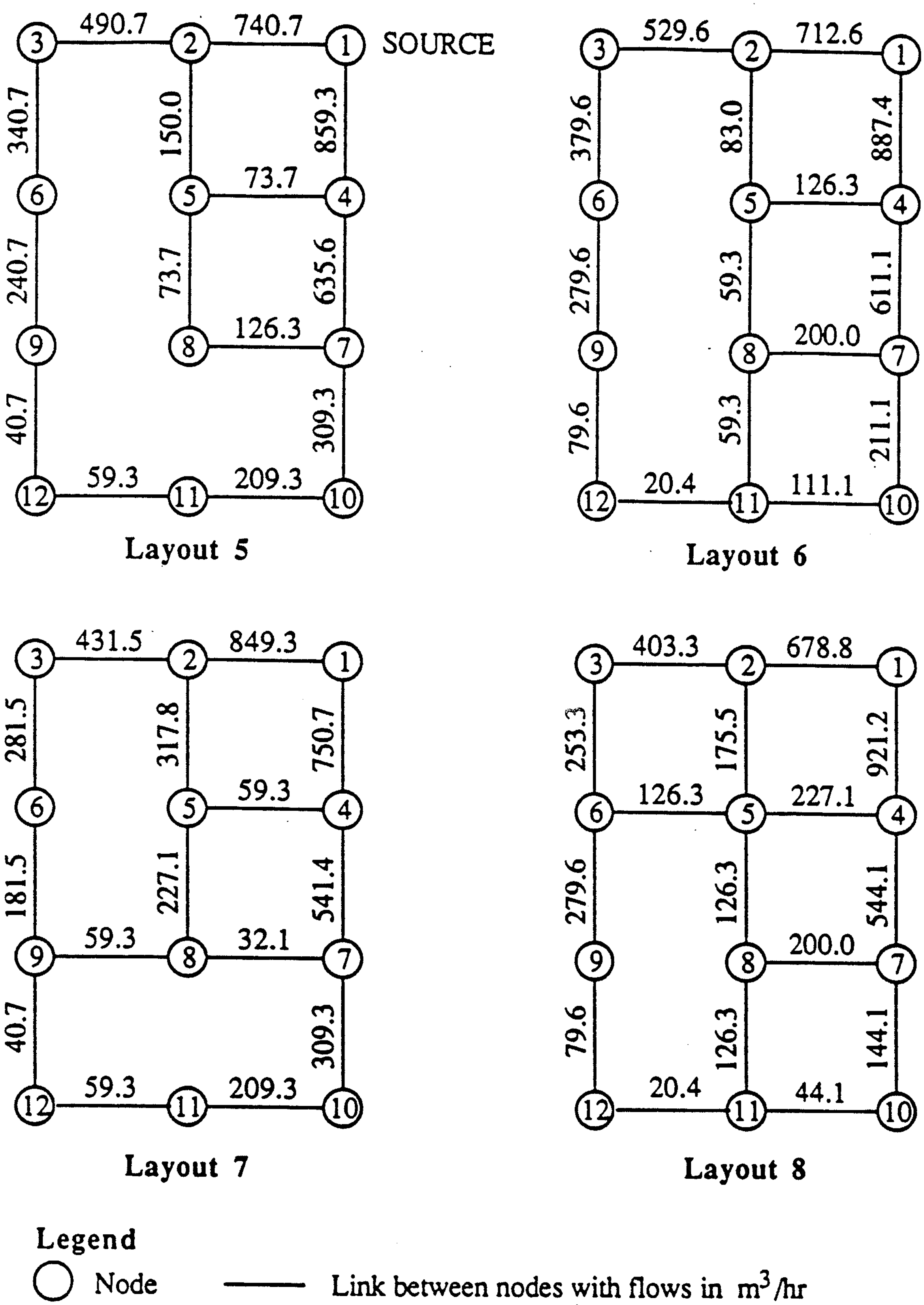


Figure 4.2: **Solution Networks for Redundancy Evaluation (Contd)**

nodal entropy measures and the maximum and minimum values for each candidate layout were also evaluated. These values are shown in Table 4.4.

The second type of redundancy evaluation for the network performed was based on an entropic measure that include the path parameter and age factor parameter. This nodal entropy measure is given by Equation 3.30 and the overall network redundancy measure is that given by Equation 3.38. The results of these measures, which are given in the same form as the first type above, are shown in Table 4.5.

The values of a third type of measure, based on the entropic redundancy measure that included the path parameter and age factor parameter and also considered the case where flow directions were allowed to change (i.e., links are bi-directional) are also provided. The nodal entropy measure in this third case is that given by Equation 3.39. The corresponding overall network redundancy measure is given by Equation 3.41. The values of these measures are summarized in Table 4.6. Note that in the last two types of entropy measures the age parameter was assumed the same for each pipe so it does not have any impact on the relative values of the measures for different nodes.

4.2.2 Discussion of the Results

The primary difference between the layouts in Figure 4.2 is the number and extent of loops contained in them, each loop representing at least one redundant link for a demand point. If a node has one or more redundant links, it has more than one path to the source. Network layouts with the same number of loops will therefore be expected to have redundancy measures that are close in value. The number of loops contained in the layouts are therefore included in the tables giving the entropic redundancy measures.

Table 4.4 Redundancy Measures Based on Basic Entropic Functions without any Parameters

Layout Number	No. of Loops	Nodal Entropy, $S_j^{\dagger}$			Overall Network Measure, $\widehat{S}^{\ddagger}$
		Average	Maximum	Minimum	
1	1	0.041	0.455	0.000	2.303
2	1	0.059	0.654	0.000	2.335
3	3	0.154	0.646	0.000	2.361
4	3	0.154	0.646	0.000	2.353
5	3	0.179	0.676	0.000	2.378
6	4	0.215	0.678	0.000	2.424
7	4	0.240	0.676	0.000	2.465
8	5	0.279	0.685	0.000	2.632

† Based on Equation 3.13

‡ Based on Equation 3.19

Table 4.4 shows the entropy measures derived from the flow patterns in Figure 4.2. The maximum and minimum values in this table represent the maximum and minimum values for the nodes in each of the layouts. Minimum value of zero imply that there are nodes with zero redundancy values. The average values are the average of the redundancy measures for all the nodes in the network.

Examination of Table 4.4 shows that the values of all entropic measures of redundancy increase with the number of loops in the network, a result which is consistent with the physical interpretation of redundancy upon which the various measures are based. In particular, layouts 1 and 2, which are poorly looped layouts, have the smallest measures of redundancy. As the number of redundant links in the layout increases, all the measures of redundancy increase despite the differences in the distribution of flow ratios at the nodes. The results also show that different redundancy measures are obtained for layouts that have the same number of loops. Layout 5 has a higher measure of redundancy than Layout 4 although both have three loops or redundant links. This difference can arise for two reasons. Firstly, a node having 'redundant' links may have flow ratios in its incident links which are closer in value to each other. One of the axioms upon which these redundancy measures are based is that the redundancy measure at a node increases as the flow capacities in the incident links become closer in value. This is the case for layout 5. A second explanation is that one layout may have a 'concentration' of redundancy occurring where it carries more weight, i.e., at a location with higher (Q_j/Q_o)'s. It was previously claimed that network redundancy measures derived from a weighted combination of nodal redundancies are more appropriate than those derived from unweighted combinations and should therefore include these factors. Intuitively, and on the basis of these weighted measures, it is therefore more advantageous to locate additional redundancy in the 'upstream' part of the network where its benefit will be passed on to more nodes downstream, than to locate it in the end zone (locations furthermost from the source node) of the network. Once again this is the case for layout 5 where redundant links occur at nodes 5 and 8, relative to layout 4 where redundant links occur at nodes 8 and 11. Hence, layout 5 has higher average nodal redundancy and also higher overall network redundancy $(\widehat{S})$ than layout 4.

The table also indicates that the change in the redundancy measures from one layout to another is not very pronounced. Increasing the loops in the layouts from one to eight resulted in an increase in overall network measure of only 0.329.

Table 4.5 Redundancy Measures Based on Entropic Functions that Include Parameters

Layout Number	No. of Loops	Nodal Entropy, $S_j^{\dagger}$ Average	Maximum	Minimum	Overall Network Measure, $\widehat{S}^{\ddagger}$
1	1	0.041	0.455	0.000	2.505
2	1	0.059	0.654	0.000	2.620
3	3	0.284	1.150	0.000	3.147
4	3	0.246	1.070	0.000	3.120
5	3	0.276	1.100	0.000	3.150
6	4	0.430	1.310	0.000	3.340
7	4	0.436	1.390	0.000	3.530
8	5	0.612	1.420	0.000	3.780

† Based on Equation 3.30

‡ Based on Equation 3.38

Table 4.5 shows the results for entropy redundancy calculations for the networks based on the functions that include path parameters (a_{ij}). These parameters are derived for each node within the network based on formulae presented in Chapter 3 (Equations 3.21 to 3.25). Age factor parameters (u_{ij}) are not used (they are set to unity) because the networks are considered as newly designed. The results shown in Table 4.5 indicate that the conclusions drawn for Table 4.4 also apply. However, the differences in the measure for the different layouts are now more pronounced. There are also notable increases in the values of the average nodal redundancies, the maximum nodal redundancies as well as the overall network redundancies redundancies. This increase in the value of the redundancy is due to the fact that the actual number of paths between the source and the nodes are included in the measure whilst in Table 4.4 the measures are based on a simplification of the actual situation or more specifically the number of links incident on a node. Consider node 8 and 12 of layout 1 in Figure 4.2. Node 12 has only one path from the source (1-2-3-6-9) while node 8 has two paths (1-4-5-8) and (1-2-5-8) indicating that node 8 has more flexibilty than node 12. Node 12 obviously has zero redundancy, and node 8 should have some amount of redundancy. Therefore, since node 8 has two paths (albeit partially overlapping), part of the redundancy at node 5 with the two independent paths (1-2-5) and (1-4-5) should be carried over to node 8. Values of redundancies in Table 4.5 based on path parameters (a_{ij}) are therefore greater than those in Table 4.4.

Table 4.6 Redundancy Measures Based on Entropic Functions that Consider Bi-directional Links

Layout Number	No. of Loops	Nodal Entropy, $S_j'^{\dagger}$			Overall Network Measure, $\widehat{S}^{\ddagger}$
		Average	Maximum	Minimum	
1	1	0.117	0.540	0.000	2.707
2	1	0.283	0.690	0.000	3.120
3	3	1.109	1.470	0.600	4.850
4	3	0.971	1.350	0.630	4.640
5	3	1.111	1.330	0.860	4.880
6	4	1.383	1.680	1.090	5.340
7	4	1.411	1.640	1.090	5.690
8	5	1.571	1.960	1.270	6.230

† Based on Equation 3.39

‡ Based on Equation 3.41

The same conclusions for Table 4.5 apply to the results in Table 4.6 but with further increase in the values of all the redundancy measures. This increase in redundancy measure value occurs because the potential reversal in flow direction in the links is now recognised. This recognition increases the number of feasible paths between the nodes and the sources, thereby increasing the redundancy of the networks. It can also be observed that apart from Layouts 1 and 2, all layouts have a minimum nodal redundancy greater than zero and at least 0.60 reflecting the availability of alternate paths from the source to each node.

Conclusion

Three cases of the entropy based redundancy measures developed in Chapter 3 were applied to the networks designed using the model presented in the previous section. All three cases gave consistently good results in that they all showed increasing redundancy with increasing number of loops in the networks. The first case is a less accurate measure and is an approximation of the more refined measure of the second case. The last case is an extension of the second case. Differences that were obtained for the layouts that have the same number of loops were partly due to the location of the loops within the layouts and partly due to the variation in flow magnitudes (or variations in pipe sizes in the network) as described in the discussion in this section. The results indicate that the entropy based redundancy measure is capable of identifying subtle differences that may exist between two layouts that are very close in configuration, a property that is desirable for a good measure for redundancy/reliability of water distribution networks.

4.3 Hydraulic Redundancy Simulation Model

4.3.1 Introduction

This section examines the entropy based redundancy measures presented in the previous chapter in terms of how well they reflect the network performance as indicated by the extent to which the network is able to supply flow under a range of single link failure conditions. The basis for evaluating network performance in this context is the percentage of the total demand supplied at adequate pressure when the link has failed, i.e., has been removed from the system. This parameter, henceforth referred to as $PSPF$, shows not so much the performance at a specific location in the network as the performance of the network as a whole. As such it refers to network-wide redundancy. A hydraulic simulation is used to develop the $PSPF$ values for each of the cases examined. The $PSPF$ values so determined are then compared with entropy based redundancy measures.

4.3.2 Rationale and Scope of Simulation

The objective of the simulation is to measure the reduction in service that occurs in the network when a single pipe fails (in other words to evaluate the ability of the network to respond to a single link failure). Level of service within the network will be measured by the total percentage of the total demand required at all nodes that can be supplied such that the residual nodal pressure heads are not less than some minimum requirement. Links (pipes) will be failed or removed from the network singularly, resulting in each case in a new pipe layout configuration, which is then assessed on the extent to which it is able to satisfy the required level of service. Singular link failures are assumed because the joint failure of two or more links can be considered to be very small since the probability of failure of pipes are generally very small and most reliability or flexibility evaluation of water distribution networks are in practice performed on the basis of a single link failure. Furthermore, water distribution networks are usually complex. Hence going through all possible combinations of multiple link failures is practically impossible. Single link failure redundancy assessment is therefore widely applied and accepted in the literature (e.g., Wagner et al., 1988b, Lansey et al., 1989).

The simulation is deterministic rather than stochastic because the probability of pipe failures are not incorporated in the model. The pipes are simply removed one at a time and the effect of this removal on the network measured. This is because it is designed only to check if the measure reflects the ability of the networks to respond to failure conditions. This ability is termed mechanical redundancy which is essentially deterministic because it is a property of the geometric configuration and carrying capacity of the elements of the network. These properties are predetermined and fixed at the design stage, hence the measure of mechanical redundancy is deterministic. Furthermore, this simulation is not designed to evaluate the performance of the networks under all possible conditions (of link failure or change in loading patterns) but rather to check if the redundancy measure developed can differentiate between the performance of two networks under the same failure and demand conditions.

The following assumptions or simplifications are used in the analysis:

1. The simulation is performed on a designed network. A designed network is one in which the pipe sizes have been optimized so that the pipes can supply all the demands at the nodes at the minimum pressure head and there will be no 'surplus' pressure head in the system. The pipe sizes are therefore known and fixed so that when there is a link failure, the reduced layout also has its pipe sizes known.

2. The demands at the nodes are known and constant. Although these demands are not constant in practice, their daily fluctuations are smoothed out and averaged for this purpose. This assumption is consistent with current research practice in water distribution network design.

3. The source pressure head is assumed constant. This assumption implies a gravity fed network, or equivalently, a fixed level of head of a pumped supply. It

also implies that the flexibility within the network is measured and evaluated with respect to a single fixed level of hydraulic head.

4. In the event of a link failure, flow directions in the links can be reversed so that all nodes would have at least one supply path to the source (Flow reversal is defined as flow in a direction opposite in direction to that in the original layout). This assumption is consistent with reality in which flow directions in pipes are permitted to change in order to provide flow paths to a node.

4.3.3 Description of the Simulation Approach

Step 1: Design of the Original Network

Design the initial network and note the pipe sizes in each link and the heads at each node. The design of this network can be done using numerous models found in literature. In this work, the model of Awumah et al. (1989) is used. The final heads at the nodes, which are greater than or equal to the minimum required heads, are obtained from the design model and designated as the service heads, Hs_j, for all nodes j in the network. The pipe sizes for each link are also obtained from this model.

Step 2: Analysis of the Reduced Network For Hydraulic Feasibility

A link is removed from the network and the new layout is balanced by reversing flow directions and changing flow magnitudes in the links such that all demands at the nodes can be satisfied. (Note that this process does not guarantee supply at acceptable pressures). The Hardy-Cross network solver is then used to analyse this new network. The heads at each node j obtained from the Hardy-Cross procedure are designated as H_j. With the removal of a link, the pressure heads at some nodes will be less than the minimum required because, with the removal of a link, some links will now have to carry flows larger than that in the original design in order that all nodes can be fully supplied. This adjustment in flows results in higher headlosses with an associated lowering of nodal heads.

Step 3: Evaluation of Impact of Link Failure

With the lowering of pressure heads at some nodes below the minimum required, there is a need to improve the pressure at these nodes to give the required quality of flow at the nodes. One method is to increase the diameter of some of the pipes or to increase the static head rating of the pumps. These alternatives cannot be considered, however, because components are considered designed for the particular network. The objective of the simulation is to check the degradation of the given network when there is link failure. The components of the network can therefore be considered as having been installed. The only approach available to increasing the nodal heads is to reduce the flow that can can be supplied to the nodes. Lower discharges at the nodes result in lower flow magnitudes through the links, lower headlosses in their links and therefore improvement in the nodal pressure heads.

However, in this evaluation another more realistic approach is used. Since demands are imposed upon the system, they are assumed to be fixed. The policy for evaluating the impact of the failure of the links is therefore to determine how much of that total network demand is met at minimum pressure or greater.

Step 4: Iterations to Obtain The *PSPF* Parameter

The percentage of the total required network demand that was supplied with the failure of the specified link in the network is then calculated by dividing the total flow demand supplied at adequate pressure by the total demand in the complete network (This process assumes, of course, that the complete network supplies all demands at adequate pressure).

Each link is removed successfully (with the previously failed link being brought back into the network) and the process repeated. After all links have been sequentially removed, the average values of the parameter $PSPF$ for that network can be evaluated together with its standard deviation, the minimum and the maximum values for use in the redundancy assessment. The schematic for the model is given in Figure 4.4.

4.3.4 Application of Simulation to the Example Networks

This simulation model was applied to the networks shown in Figure 4.2. The network-wide entropy based redundancy measure for these networks were also computed using Equation 3.41 with the nodal redundancy measure, S_j', given by Equation 3.39, replacing the S_j in Equation 3.38. The redundancy measures in these equations reflect the case where flow direction reversal is included in the redundancy measure and this will be more appropriate for comparison with the hydraulic redundancy measure since in the simulation process, flow direction reversals were allowed.

In Equation 3.38, all the age factor parameters were set equal to unity, as this case reflects a new design and all pipe components are assumed to be of the same material and new (i.e., $u_{ij} = 1$ for all pipes). A summary of values obtained for the $PSPF$ of all networks are shown in Table 4.7 [col. 3-6]. Examination of this table shows clearly that increase in the value of the entropic redundancy measure [col. 2] corresponds to improved overall network performance as indicated by larger mean $PSPF$ values. It should be recognised that those layouts with smaller standard deviation of $PSPF$ do not have extreme shortfalls relative to the mean performance. Thus networks with larger mean and low standard deviation $PSPF$ values have more

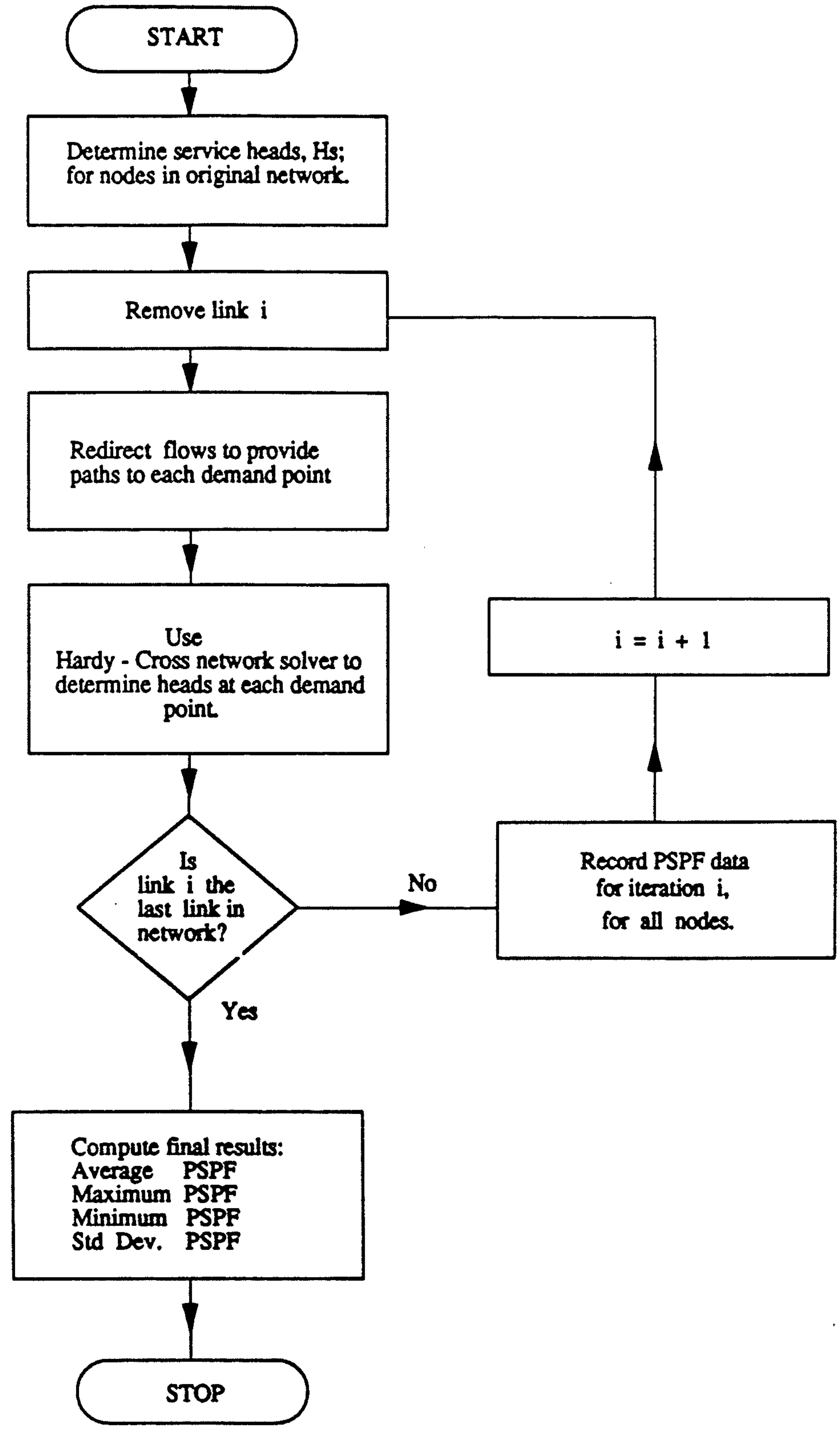


Figure 4.3: **Flow Chart for Determination of the PSPF Parameter**

flexibility in responding to link failures. This is clearly shown in layout 8 which has a high redundancy measure (a step larger than other layouts) corresponding to its high mean and low standard deviation $PSPF$ values.

It is also interesting to note that higher mean $PSPF$ values tend to be associated with lower standard deviations. This situation is due to some extent to the fact that as the mean increases the possible deviation above that mean decreases in range due to the upper limit of 100% on all values.

Similarly it is possible to differentiate between layouts which have similar mean $PSPF$ values on the basis of the $PSPF$ standard deviation values. The maximum and minimum $PSPF$ values for each layout also give an indication of how well the redundancy measure relates to network performance. Comparing layouts 3 and 4, it can be seen that layout 3 has a slightly larger redundancy measure reflecting the higher mean and smaller standard deviation in the $PSPF$ values of the layout. Layout 4 has a minimum $PSPF$ substantially lower than the minimum for layout 3. The minimum $PSPF$ value for layout 4 is, however, considerably lower than the next smallest $PSPF$ value for that layout (The next smallest value for layout 4 is 63.13%. This value is not shown in Table 4.7). Layout 3, on the other hand, has another $PSPF$ value quite close to its minimum value. (This other $PSPF$ value is 55.83 and is also not shown on the table). Layout 4 has, therefore, a single case of considerable weakness while layout 3 has two cases of relative weakness (relative to conditions in layout 4). Thus, although there are specific differences between the two networks, the performance on a network-wide basis is quite close and is reflected in the values of the redundancy measure.

Table 4.7. Entropy Measures of Network Wide Redundancy and PSPF Values for Layouts in Figure 4.2

Layout Number	Global Redundancy $\hat{S}^{\dagger} = \sum_{j=1}^{N} \frac{Q_j}{Q_o} S'_j - \sum_{j=1}^{N} \left[U_j \frac{Q_j}{Q_o}\right] \ln \left[\frac{Q_j}{Q_o}\right]$	PSPF Value (%) Mean	Maximum	Minimum	Standard Deviation
(1)	(2)	(3)	(4)	(5)	(6)
1	2.707	77.86	100.0	40.63	15.49
2	3.120	79.20	94.84	43.44	15.25
3	4.850	83.07	100.0	51.84	16.27
4	4.640	82.93	100.0	35.94	17.97
5	4.880	84.02	100.0	55.60	15.10
6	5.340	84.96	100.0	49.19	15.98
7	5.690	85.98	100.0	55.00	12.47
8	6.230	88.74	99.68	58.40	11.76

† Based on Redundancy Measure Including Flow Reversal, Equation 3.41

This similarity is useful because $PSPF$ measures redundancy indirectly and the ability of the network to respond to a single link failure (flexibility) with the possibility of flows to be redirected in the presence of such a failure.

4.4 Determination of Nodal Pair Reliability (NPR) Parameter

4.4.1 Introduction

The next stage of the evaluation process is to compare the entropy based redundancy measures to the probabilistic reliability index called Nodal Pair Reliability (NPR). Use of a probabilistic reliability measure is necessary because a fundamental objective of adding redundancy to a system is to improve its probabilistic reliability.

Several approaches for the computation of the reliability of stochastic networks exist and all vary in the degree of complexity. All suffer from the same problem of exponential order of compuational time, however. The approach used in this section to compute the NPR's for the networks is that of Kim et al. (1972) and is described below.

4.4.2 Method of Calculation of NPR

Phase I

Since water distribution networks are usually very complex, it will be appropriate to exploit any simplification techniques available which will not affect the final results of the calculations. Water distribution networks are composed of series-parallel configurations and non-series-parallel configurations. A series-parallel network is a network that can be reduced into a tree (network without loops) by performing series and parallel reductions (to be explained shortly). A non-series-parallel network is the type that cannot be reduced into a tree network.

A series reduction is performed by replacing two or more links, connected in series to any two nodes, by a single link. The reliability of this single link is given by Equation 4.2 below. The probability that these two nodes are connected is not affected by this replacement. In the same manner, a parallel reduction is performed by replacing two or more sets of links, each set consisting of links connected in series and all sets connected to the two nodes in parallel by a single link. The reliability of this new link is given by Equation 4.3 below. Once again this replacement does not affect the probability that the two nodes are connected. Both types of reductions are further illustrated in Figure 4.5.

The actual process of simplifying the network proceeds as follows. Perform series and parallel reduction of all components and replace each new set by a single component with the reliability of the set based on the following relationship:

a) The reliability of a series set, R_s of n serial members, each with reliability R_i in the set, is given by:

$$R_s = \prod_{i=1}^{n} R_i \tag{4.2}$$

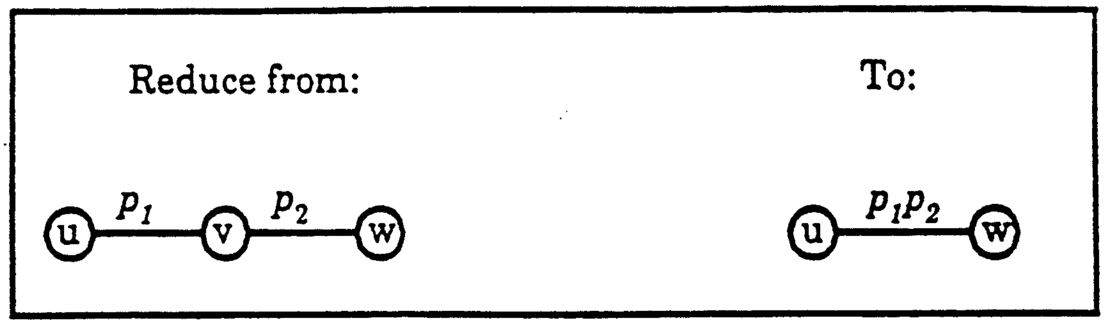
b) The reliability of a parallel set, R_p of n parallel members, each with relaibility R_i in the set, is given by:

$$R_p = 1 - \prod_{i=1}^{n} [1 - R_i] \tag{4.3}$$

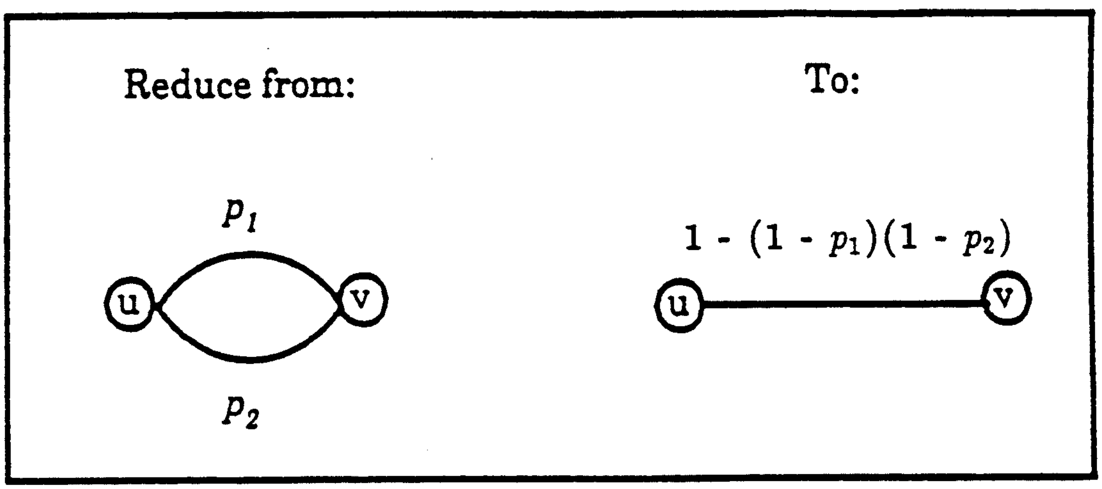
This process should be repeated for the network until a non-series-parallel system which is irreducible is obtained.

Phase II

For the reduced network, (the network developed after performing series and parallel reductions), enumerate all path sets from the source to the sinks (demand nodes) of the network. The path enumeration technique presented in Appendix A can be used for this enumeration.



Series Reduction



Parallel Reduction

Figure 4.4: **Illustration of Series and Parallel Reductions**

Phase III

Construct an equivalent reliability block diagram of all paths between the source and a sink. The block diagram is a parallel set of the enumerated paths. Each path has a set of components (series reduced). If a given component occurs more than once in the parallel set of paths, then an assumption of statistical independence between these paths cannot hold. The two parallel paths are dependent because with a given component occuring in both paths, the failure of the component results in the failure of the two paths.

The reliability block, which is a set of parallel paths, assumes that the failure of one path is independent of the failure of another, and this will no longer be a valid assumption. If every component appears in only one of the parallel paths, then the reliability function given by Eq. 4.3 will apply directly to these parallel sets of paths. If statistical dependence exists among the paths, the operation defined by Equations 4.4, 4.4a, and 4.4b is used to make the necessary correction.

$$\left[\prod_j p_j^{i_j}\right] \equiv \prod_j p_j \tag{4.4}$$

with the following properties holding:

$$\left[\sum_k \left(\prod_j p_j^{i_j}\right)\right]^* \equiv \sum_k \left[\prod_j p_j^{i_j}\right]^* \tag{4.4a}$$

$$\left[\prod_k \left(\prod_j p_j^{i_j}\right)\right]^* \not\equiv \prod_k \left[\prod_j p_j^{i_j}\right]^* \tag{4.4b}$$

where $i_1, i_2 \cdots, i_j$ are any non-negative integer powers of p_j and p_j is reliability of component j. The validity of and logic behind the development of Equation 4.4 is given in Appendix C.

The powers of p_j, i.e., $i_1, i_2 \cdots, i_j$, occur in an expression of reliability for a system because of the repetition of p_j in the parallel set of paths from source to the sink (e.g., Equation 4.3). For example, if the operator is applied to an expression $[p_1^3 p_2 p_3^4 p_4^2]$ the following holds:

$$[p_1^3 p_2 p_3^4 p_4^2]^{\times} = p_1 p_2 p_3 p_4$$

When this operator is applied to any set of terms, it results in a solution in which there will be no terms with powers greater than unity. The application of this operator to a system of paths with common components results in the correction of the probability of path failure to reflect statistical independence among the paths.

Consider the constructed reliability block from the source node s to a given demand node z. Let the number of parallel paths in the block be r. Define the components of the path in this set by $L_l(s,z)$. Let the components in a path be denoted by b_{ij}, each with reliability R_{ij} (defined as the probability that component b_{ij} has not failed). The reliability of this system, R_{SZ} (defined as the probability that the node z is connected to the source s) has been proven in Kim et al. (1972) to be equivalent to:

$$R_{SZ} = \left[1 - \prod_{l=1}^{r}\left(1 - \prod_{b_{ij}\in L_l(s,z)} R_{ij}\right)\right]^{\times} \tag{4.5}$$

where the operator $[\quad]^{\times}$ is as defined above.

The reliabilty given by Equation 4.5 for the pair of nodes (s,z) is the parameter NPR or nodal pair reliability.

4.4.3 Application of the Method to the Example Networks

The NPR was computed for all demand nodes in each of the networks shown in Figure 4.2. In order to perform the calculations, it is necessary to obtain the probability of failure and hence the reliability R_{ij}'s for the links of the network. The pipe failure rates given in Table 4.1 were used to obtain these probabilities. The breaks for pipe size were assumed to be distributed according to the *Poisson* distribution. The assumption of a *Poisson* distribution has been used previously by Goulter and Coals (1986), Lansey et al. (1987) and Quimpo and Shamsi (1988).

The *Poisson* parameter for the analysis is given by:

$$\lambda_j = \sum_{k=1}^{n(j)} r_{jk} L_{jk} \tag{4.6}$$

where λ_j = average number of failures per year for link j

r_{jk} = number of breaks per kilometer per year for pipe size k in link j

L_{jk} = length in kilometers of pipe size k in link j.

The probability of failure of a link is therefore given by:

$$p_j = \sum_{x=1}^{\infty} \left(\frac{e^{-\lambda_j} \lambda_j^x}{x!} \right) \tag{4.17}$$

where p_j = probability of one or more breaks in link j

x = the number of breaks.

In this work, a single failure condition was considered as the hydraulic redundancy measure given by $PSPF$ parameter evaluates the degradation of the network when one link fails. Hence $x = 1$ and Equation 4.17 reduces to:

$$p_j = \lambda_j e^{-\lambda_j} \tag{4.18}$$

The NPR values for the networks under these conditions are summarised in Table 4.8 below.

Table 4.8 NPR Parameters and Entropy Measures Obtained for the Layouts in Figure 4.2

Layout Number	Number of Loops	Nodal Pair Reliability (NPR)			Network Entropy
		Average	Maximum	Minimum	($\hat{S}^{\dagger}$)
1	1	0.8010	0.9529	0.6124	2.707
2	1	0.8081	0.9292	0.6110	3.120
3	3	0.8377	0.9240	0.6916	4.850
4	3	0.8338	0.9240	0.6840	4.640
5	3	0.8397	0.9442	0.6803	4.880
6	4	0.8741	0.9900	0.6870	5.340
7	4	0.8526	0.9561	0.6825	5.690
8	5	0.8851	0.9930	0.7092	6.230

† Based on Eq. 3.41

The variation in the value of the average NPR given in Table 4.8 indicates that significant increases in the average network reliability (as defined by the NPR) generally occur with increase in redundancy as specified by the entropic measures. Networks in the same class (same number of loops) have approximately the same average NPR, and there is a stepwise increase in this reliability measure as the number of loops in the network increases. Except for a few discrepancies, there is also a general increase in the maximum NPR values and minimum NPR values with increase in redundancy. The minimum NPR value in the network is also an indicator of how reliable is the network, particularly, in terms of the weakest portion or nodes in the system. Hence Layout 8 which has every NPR greater than or equal to 0.7092 can be taken as being more reliable than Layout 1 which has at least one NPR value as low as 0.6124. There also appears to be a strong relationship between the entropy based network redundancy measure, $\hat{S}$, and the average NPR measure. The only discrepancy that occurred was between Layout 6 and 7, where Layout 6 with a lower $\widehat{S}$ measure has the higher average NPR measure. However, Layout 6 also showed higher maximum and minimum NPR values than Layout 7. This discrepancy can be attributed to the fact that there is a basic difference between redundancy (which may be hydraulic or mechanical) and probabilistic reliability (or mechanical reliability). Comparing both layouts, it can be observed that layout 7 has the redundant link 8-9 connecting nodes 3, 6, 9 and 12 resulting in a more redundant layout while layout 6 has its redundant link 8-11 connecting only node 11, leaving nodes 3, 6, 9 and 12 isolated therefore weakly connected. The entropy based redundancy measures are able to identify such weaknesses while probabilistic reliability (given by average value of NPR) is not able to do so.

Chapter 5

ENTROPY BASED DESIGN OPTIMIZATION MODELS

5.1 Introduction

In this chapter the entropic redundancy measures which have the ability to assess the two aspects of reliability are formulated in such a manner that they can be included in mathematical optimization design approaches. The measures developed in Chapter 3 and evaluated in Chapter 4 are incorporated into water distribution network design models. In Chapter 4 it was asserted that the usefulness of the entropy based measures for the design of water distribution networks was related to their ability to consider simultaneously the redundancy aspects of both the layout of the network and the component sizes (pipe diameters). The general water distribution network optimization model incorporating mathematical statements of the entropic redundancy measure will be formulated. This model is then applied to network design problems.

5.2 General Entropy Based Models For the Design of Water Distribution Networks

5.2.1 Introduction

In developing an optimization model for the design of water distribution networks in which entropic redundancy measures can be recognised, two possible methods are available. One possible approach is an optimization formulation in which the objective is to minimize the cost of the network while imposing, in addition to the usual hydraulic constraints, a set of constraints of a minimum permissible level of entropy (redundancy) in the network. The alternate approach is to maximize the network redundancy subject to the necessary hydraulic constraints, and a constraint on the network cost (budget constraint). In this second approach, if the cost constraint is not included, then the solution obtained will be approximately equal to that which would be obtained if the network reliability was maximized, or more specifically, if the hydraulic redundancy of the network was maximized. Both approaches to the network design problem will be used for the design model.

In both approaches, it is assumed that the water demand at the nodes are known. Both the case of a single set of nodal demands and the case of multiple sets of nodal demands will also be included. The single set of demands case is henceforth referred to as the 'single demand pattern' while the multiple set of demands case as the 'multiple demand pattern'. The minimum permissible residual pressure head is assumed to have been specified and may be different for different nodes within the network. A variable permitting continuous pipe sizes is used in the model. This assumption is not realistic for practical purposes. However, a continuous pipe size solution can easily be converted into commercially discrete pipe size by converting the link into two lengths of pipe which provide the same hydraulic characteristics as the unavailable pipe size over the whole length of link. The diameters of the two 'replacement' pipes are the closest commercial sizes to the non-commercial pipe size in the link but with one being the next smallest and the other the next largest. The total length of these two diameters is equal to the total length of the link. This approach has been used by Quindry et al. (1981).

5.2.2 Model A: Cost Minimization Model

This model is a classical network optimization model with redundancy imposed as an additional constraint. Mathematically, the model is stated as follows:

a) Objective Function

The objective of this model is to minimize the total cost of the network. Two types of costs are identified, Capital Cost and Energy Cost (operational cost).

Capital Costs

The cost of installing pipes in the network is usually considered as the capital cost. This cost is a function of the pipe diameter and its length, and the following relationship is adopted (developed by U.S Army Corps, 1980).

$$CST_{ij} = 0.39 D_{ij}^{1.51} \tag{5.1}$$

where CST_{ij} = unit cost of installing pipe between nodes i and j, in \$ 10^6 per Km of pipe length.

D_{ij} = diameter of pipe in meters between nodes i and j.

The total capital cost for the network is therefore given by

$$C_p = \sum_{ij=1}^{NL} 0.39 L_{ij} D_{ij}^{1.51} \tag{5.2}$$

where L_{ij} = length of pipe between nodes i and j, in km.

NL= total number of links in the network

C_p = total capital cost for the network, in \$ 10^6.

Since the lengths of the links are fixed, the first two terms of Equation 5.2 are constants and can be replaced by a single constant term, α_{ij}, defined by;

$$\alpha_{ij} = 0.39 L_{ij} \tag{5.3}$$

Thus Equation 5.2 becomes;

$$C_p = \sum_{ij=1}^{NL} \alpha_{ij} D_{ij}^{1.51} \tag{5.4}$$

Energy Cost

The energy required to drive water through the network is a function of the flow rate of water and pressure heads in the network. This cost can be defined by;

$$C_e = \varepsilon \left[\sum_{ij=1}^{NL} q_{ij} h_{ij} + \sum_{j=1}^{N} \Delta_j H_j \right] \tag{5.5}$$

where C_e = energy cost for network

ε = price of unit quantity of energy

q_{ij} = flowrate of water in pipe between nodes i and j

h_{ij} = pressure headloss in pipe between nodes i and j

Δ_j = flow demand at node j

H_j = service pressure head at node j

N = total number of nodes in the network

NL= total number of links in the network

It is desirable to express the energy cost in terms of the pipe diameter, D_{ij}, and the headloss through the pipe, h_{ij} as it will reduce the number of variables in the objective function thereby reducing the complexity of the non-linear model. The following expression derived from the Hazen-William flow formula (Equation 3.6) can be used for this purpose;

$$q_{ij}h_{ij} = K_{ij}h_{ij}^{1.54}D_{ij}^{2.63} \tag{5.6}$$

where K_{ij} is a constant given by;

$$K_{ij} = \frac{kC_{ij}}{L_{ij}^{0.54}} \tag{5.7}$$

where C_{ij} = Hazen-William friction coefficient for link ij

k = conversion factor for units

Let the constant terms in the first term of Equation 5.5 be represented by β_{ij} where;

$$\beta_{ij} = \frac{\varepsilon kC_{ij}}{L_{ij}^{0.54}} \tag{5.8}$$

Δ_j in the second term of Equation 5.5 is the demand at the node which is a known constant. Therefore let γ_j represent the constant terms of this second term, where;

$$\gamma_j = \varepsilon\Delta_j \tag{5.9}$$

The service head, H_j, can also be expressed in terms of the headloss h_{ij} by the following expression;

$$H_j = H_s + (Z_s - Z_j) \sum_{\{ij\}\in P_{sj}} h_{ij} \tag{5.10}$$

where H_s = pressure head at the source node

Z_s = height of the source node above datum

Z_j = height of node j above datum

P_{sj} = set of links between the source node s and demand node j.

Z_s, Z_j, and H_s, are all constants. By setting

$$\lambda_j = H_s + Z_s - Z_j \tag{5.11}$$

Equation 5.5 becomes;

$$C_e = \sum_{ij=1}^{NL} \beta_{ij} h_{ij}^{1.54} D_{ij}^{2.63} + \sum_{j=1}^{N} \gamma_j \left(\lambda_j - \sum_{ij \in P_{sj}} h_{ij} \right) \tag{5.12}$$

where all variables and constants are as defined above.

The objective function is the minimization of the sum of all the costs, as defined by Equations 5.4 and 5.11 and is given by;

$$Minimize \quad C_t = \sum_{ij=1}^{NL} \left(\alpha_{ij} D_{ij}^{1.51} + \beta_{ij} h_{ij}^{1.54} D_{ij}^{2.63} \right) + \sum_{j=1}^{N} \gamma_j \left(\lambda_j - \sum_{ij \in P_{sj}} h_{ij} \right) \tag{5.13}$$

where C_t is the total network cost.

b) Constraints

1. Constraints to Define Flow in the Links

These constraints are required to ensure that flow in each link is correctly defined in terms of the headloss through that link.

Single Demand Pattern:

$$q_{ij} = K_{ij} h_{ij}^{0.54} D_{ij}^{2.63} \quad \forall \quad links \quad \{i,j\} \in NL \tag{5.14}$$

Multiple Demand Pattern:

$$q_{ijl} = K_{ij} h_{ijl}^{0.54} D_{ij}^{2.63} \quad \forall \ demands \quad l \in TD \qquad \forall \quad links \quad \{i,j\} \in NL \tag{5.15}$$

where TD = set of multiple demand patterns

q_{ijl} = flow in link from node i to node j for demand pattern l

h_{ijl} = pressure headloss in link from node i to node j for demand pattern l

and K_{ij} is as defined by Equation 5.7.

2. Flow Continuity Constraint

Continuity of flows must be observed at all nodes. Thus the difference between total inflow to and outflow from a node must equal the demand at that node.

Single Demand Pattern:

$$\sum_{\{i,j\}\in[\overline{h}_i>\overline{h}_j]} q_{ij} - \sum_{\{j,k\}\in[\overline{h}_j>\overline{h}_k]} q_{jk} = \Delta_j \quad \forall \quad nodes \quad j \tag{5.16}$$

where $\overline{h}_i$ = head at node i

$[\overline{h}_i > \overline{h}_j]$ = set of links connected to demand node j and in which the head at node j is less than that at the node i at the other end of the link

Similarly,

$[\overline{h}_j > \overline{h}_k]$ = set of links connected to demand node j and in which the head at node j is greater than that at the node k at the other end of the link

Multiple Demand Pattern:

$$\sum_{\{i,j\}\in[\overline{h}_i>\overline{h}_j]} q_{ijl} - \sum_{\{j,k\}\in[\overline{h}_j>\overline{h}_k]} q_{jkl} = \Delta_{jl} \quad \forall \quad nodes \quad j \qquad \forall \quad demands \quad l \in TD \tag{5.17}$$

where all terms are the same as defined for the single demand pattern case but now qualified by the index l for all demand patterns in the set of multiple demand patterns.

3. Nodal Pressure Head Constraint

The pressure head at each node in the network must neither be below some minimum value nor above some maximum value specified.

Single Demand Pattern:

$$H_{jmax} \geq H_j \geq H_{jmin} \quad \forall \quad nodes \quad j \tag{5.18}$$

where H_{jmax} = maximum pressure head allowed at node j

H_{jmin} = minimum pressure head allowed at node j

H_j = service pressure head at node j as given by Equation 5.10

Multiple Demand Pattern:

$$H_{jmax,l} \geq H_j \geq H_{jmin,l} \quad \forall \quad nodes \quad j \qquad \forall \quad demands \quad l \in TD \tag{5.19}$$

where the index l refers to the particular demand pattern for the appropriate variable defined above.

4. Constraint to Ensure that Net Pressure Headloss is Zero

For hydraulic consistency, the net pressure headloss in the links of every loop must be equal to zero.

Single Demand Pattern:

$$\sum_{ij \in LP^+} h_{ij} - \sum_{jk \in LP^-} h_{jk} = 0 \quad \forall \quad LP \in LOOPS \tag{5.20}$$

where $LOOPS$ = total number of loops in network

LP^+ = set of links in loop LP in which the flow directions are positive (clockwise)

LP^- = set of links in loop LP in which the flow directions are negative (counterclockwise)

Multiple Demand Pattern:

$$\sum_{ij \in LP+} h_{ij,l} - \sum_{jk \in LP-} h_{jk,l} = 0 \quad \forall \quad LP \in LOOPS \qquad (5.21)$$
$$\forall \quad l \in TD$$

5. Nodal Entropic Redundancy Constraint

The entropy redundancy (as it represents a measure of reliability) at each demand node must be constrained to be above some minimum value. This restriction permits the model to act as both a layout and component design model. It also permits the model to eliminate links between nodes, i.e., select $D_{ij} = 0$, if its cheaper to do so while maintaining the desired level of redundancy.

Single Demand Pattern:

$$S_j = -\sum_{i=1}^{n(j)} u_{ij} \left[\frac{q_{ij}}{Q_j} \ln \left(\frac{q_{ij}}{a_{ij} Q_j} \right) \right] \geq S_{jmin} \quad \forall \quad nodes \quad j \qquad (5.22)$$

where S_{jmin} = minimum entropy allowed for node j (specified by the user)

S_j = entropy at node j, given by Equation 3.30

and all other variables as defined for Equation 3.30.

Multiple Demand Pattern:

$$S_j = -\sum_{i=1}^{n(j)} u_{ij} \left[\frac{q_{ijl}}{Q_{jl}} \ln \left(\frac{q_{ijl}}{a_{ij} Q_{jl}} \right) \right] \geq S_{jmin} \quad \forall \quad nodes \quad j \qquad (5.23)$$
$$\forall \quad demands \quad l \in TD$$

5.2.3 Model B: Entropy Maximization Model

a) Objective Function

The objective of this model is to maximize the overall network entropic redundancy. The entropic expression used for the objective given in Chapter 3 as $\widehat{S}$ in Equation 3.38. The objective function for this model is therefore:

$$Maximize \quad \widehat{S} = \sum_{j=1}^{N} \left[\frac{Q_j}{Q_o} S_j \right] - \sum_{j=1}^{N} \left[U_j \frac{Q_j}{Q_o} \right] \ln \left[\frac{Q_j}{Q_o} \right] \tag{5.24}$$

where $\widehat{S}$ = overall network entropic redundancy

S_j = entropic redundancy at node j given by Equation 3.30

U_j = age parameter for node j given by Equation 3.36

$Q_j = \sum_{i=1}^{n(j)} q_{ij}$

$Q_o = \sum_{j=1}^{N} Q_j$

N = total number of demand nodes in network

$n(j)$ = number of links incident on node j

The S_j values in Equation 5.24 are as defined in Equation 5.22.

b) Constraints

All constraint sets in Model A, given by Equations 5.13 up to 5.20, are valid for Model B, except constraint set number 5. This constraint set is replaced by the direct inclusion of the entropy measure into the objective function.

5. Budget Constraint

The following constraint is theoretically optional. It is, however, normally required in practical applications as budget is always an issue and it is not normally feasible to design a network for maximum redundancy/reliability. It is the network budget constraint and is necessary if it is desirable to obtain a network within budget limits. The total network cost is C_t given by Equation 5.11. Hence the budget constraint is:

$$\sum_{ij=1}^{NL} \left(\alpha_{ij} D_{ij}^{1.51} + \beta_{ij} h_{ij}^{1.54} D_{ij}^{2.63}\right) + \sum_{j=1}^{N} \gamma_j \left(\lambda_j - \sum_{ij \in P_{sj}} h_{ij}\right) \leq C_{tmax} \qquad (5.25)$$

where C_{tmax} = maximum total network cost allowed.

Either Model A or Model B can be applied to the design of network under redundancy restrictions. However, the choice of which model to use depends on what feature of the redundancy is of interest. In Model A it is possible to control the restriction at each node (Equation 5.22). It is also possible to place restrictions on the minimum allowable network-wide redundancy by adding the following constraint to the model:

$$\widehat{S} = \sum_{j=1}^{N} \left[\frac{Q_j}{Q_o} S_j\right] - \sum_{j=1}^{N} \left[U_j \frac{Q_j}{Q_o}\right] \ln \left[\frac{Q_j}{Q_o}\right] \geq \widehat{S}_m \qquad (5.26)$$

where $\widehat{S}_m$ is the minimum network-wide redundancy allowable.

Model B on the other hand is only able to optimize network-wide redundancy. However, the restrictions on the minimum allowable redundancy at any node can be imposed by including Equation 5.22 or 5.23 in the constraint set.

5.3 Application of the Models

In this section the application of the design models formulated is investigated. In one case, the design model is applied to examine the relationship, or more specifically the trade-off between system cost and system reliability and system redundancy. In the second case described in Section 5.4, the model is applied to the design of a network previously examined in the literature.

5.3.1 Application To a Network

As a first step, Model B without either a budget constraint or individual nodal redundancy constraint, was used to determine the maximum network cost for Layout 3 in Figure 4.2. The budget constraint was then included in Model B and decreased from the maximum determined above in step sizes which decreased as the 'distance' from the 'maximum network cost' increased (Note that the reverse procedure is possible, i.e., start with the minimum network cost and increase the right-hand side of the budget constraint up to the maximum network cost). The right-hand-side of the cost constraint was lowered until the network, which initially consisted of three loops, collapsed into a branched (tree) network without loops. This branched network represents the least expensive network layout and has the lowest reliability and redundancy values.

As expected at each lower budget limit, a solution with a lower overall network entropy redundancy resulted. The network reliability, as defined by the $\overline{NPR}$ value, was then computed for each network. The network hydraulic redundancy as denoted by the $PSPF$ parameter was also determined using the simulation process outline in Chapter 4 for the network solutions.

Table 5.1 below shows results obtained for the different model runs. Further details on the application of the model to this network, such as the cost and other coefficients used and the diameter obtained for the links, are given in Appendix D. Since the model is non-linear (both in objective function and some of the constraints), the formulation was solved using the non-linear optimization GRG2 package of Lasdon and Waren (1984).

Table 5.1 Results of the Cost Constrained Maximum Entropy Model Runs

Run Number (1)	Constrained Network Cost($ 10^6) (2)	Total Cost Savings ($ 10^6) (3)	Network Entropy ($\widehat{S}$) (4)	Average Network Reliability ($\overline{NPR}$) (5)	Average Network PSPF (%) (6)
1	0.682	0.000	2.079	0.843	83.02
2	0.675	0.007	2.077	0.842	82.50
3	0.670	0.012	2.052	0.841	82.20
4	0.665	0.017	1.967	0.838	81.70
5	0.660	0.022	1.867	0.833	79.00
6	0.655	0.027	1.683	0.825	75.10
7	0.650	0.032	1.465	0.817	71.80
8	0.645	0.037	1.300	0.809	67.90
9	0.640	0.042	1.117	0.804	63.80
10	0.630	0.052	0.844	0.795	57.30
11	0.620	0.062	0.567	0.788	54.50
12	0.604	0.078	0.000	0.776	50.50

The graphical presentation of these results are given in Figures 5.1, 5.2 and 5.3 below. Figure 5.4 shows the variation in network layout with the various levels of the network entropy.

Discussion of Results

It can be observed from Figures 5.1, 5.2 and 5.3 that the trade-off curve of the network cost savings versus network redundancy has the same general shape as the network cost savings versus its reliability as given by the average NPR parameter, and its hydraulic redundancy as given by the average $PSPF$ parameter. The 'kink' in all curves occur at the same cost level, the curves having a very steep slope cost savings between \$ 0.0 up to \$ 0.01 x 10^6. For cost savings above that level, the slopes are mild for all curves. However, in the case of the curve of network cost savings versus $PSPF$ parameter, another steep slope occurs for cost savings between \$0.05x10^6 and \$0.08x10^6. This steepness of slope is due to the fact that,in this region, the layout collapsed from the three looped network into branched network resulting in a sharp decrease in network cost but with a milder decrease in the hydraulic redundancy. The very small pipe diameters, in other words very small capacity links, occurring in the three loop network did not contribute significantly to redundancy. However, due to economies of scale their existence was quite expensive. The removal of these small diameter pipes in the subsequent steps allowed a relatively large decrease in cost with little effect on overall redundancy.

This steep slope is however not as steep as the first slope discussed previously. Since the best compromise solution is often located around the 'kink' of the trade-off curve, the solution set identified using the multi-objective entropic redundancy approach is likely be very similar to that obtained using the traditional multi-objective reliability approach. These results are consistent with the evaluation of the entropic

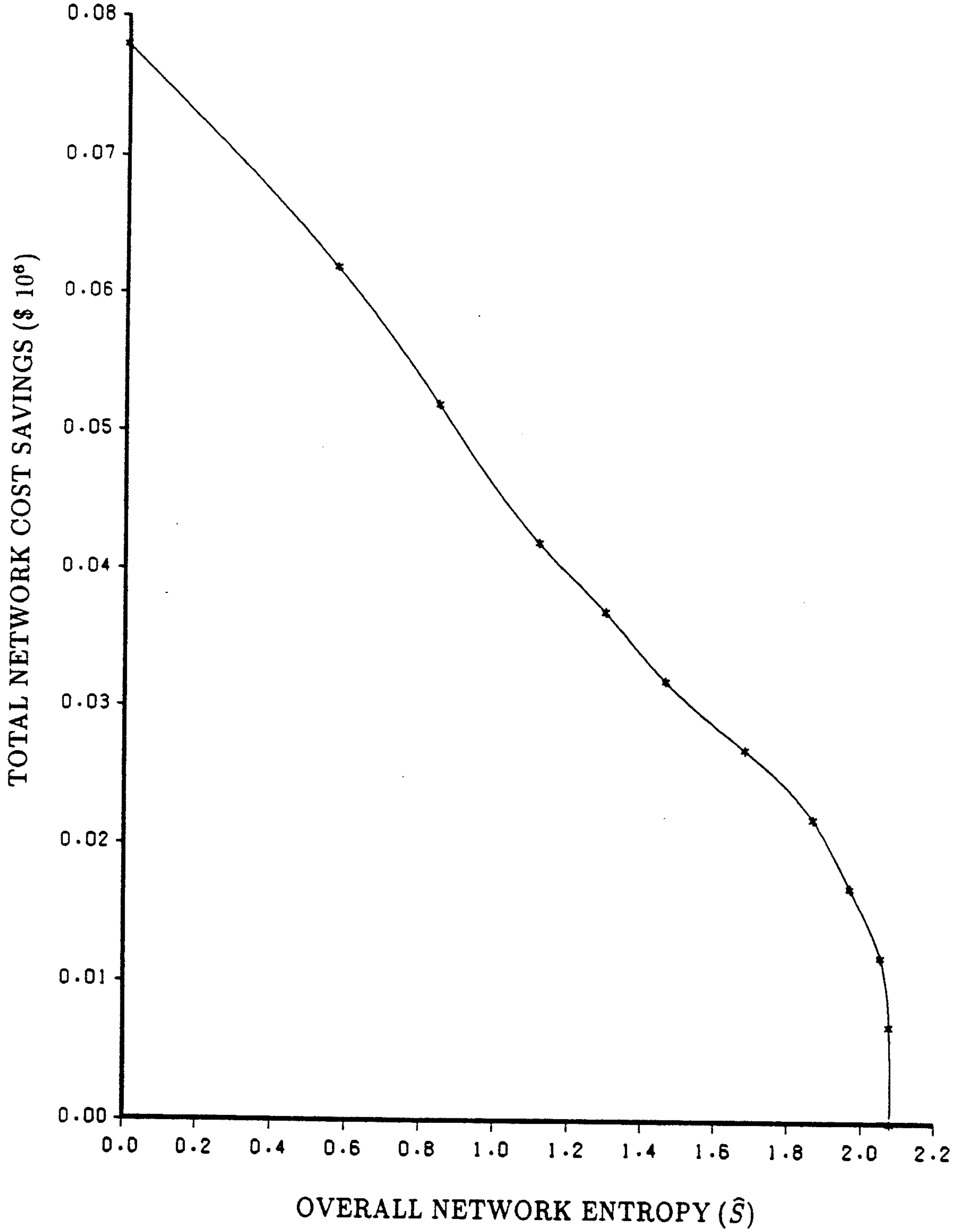


Figure 5.1: **Plot of Cost Savings vs. Network Entropy ($\hat{S}$)**

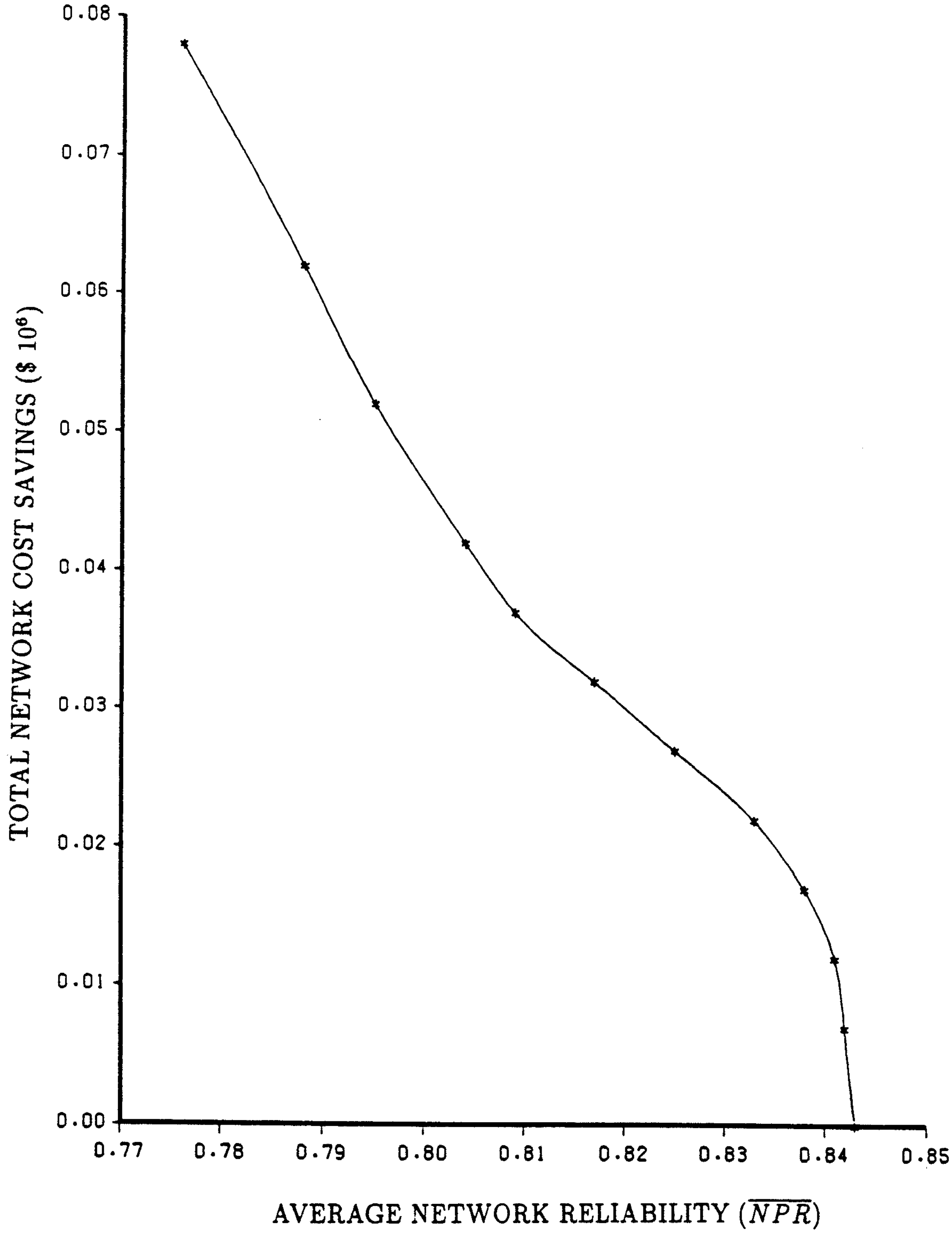


Figure 5.2: **Plot of Cost Savings vs. Network Reliability** ($\overline{NPR}$)

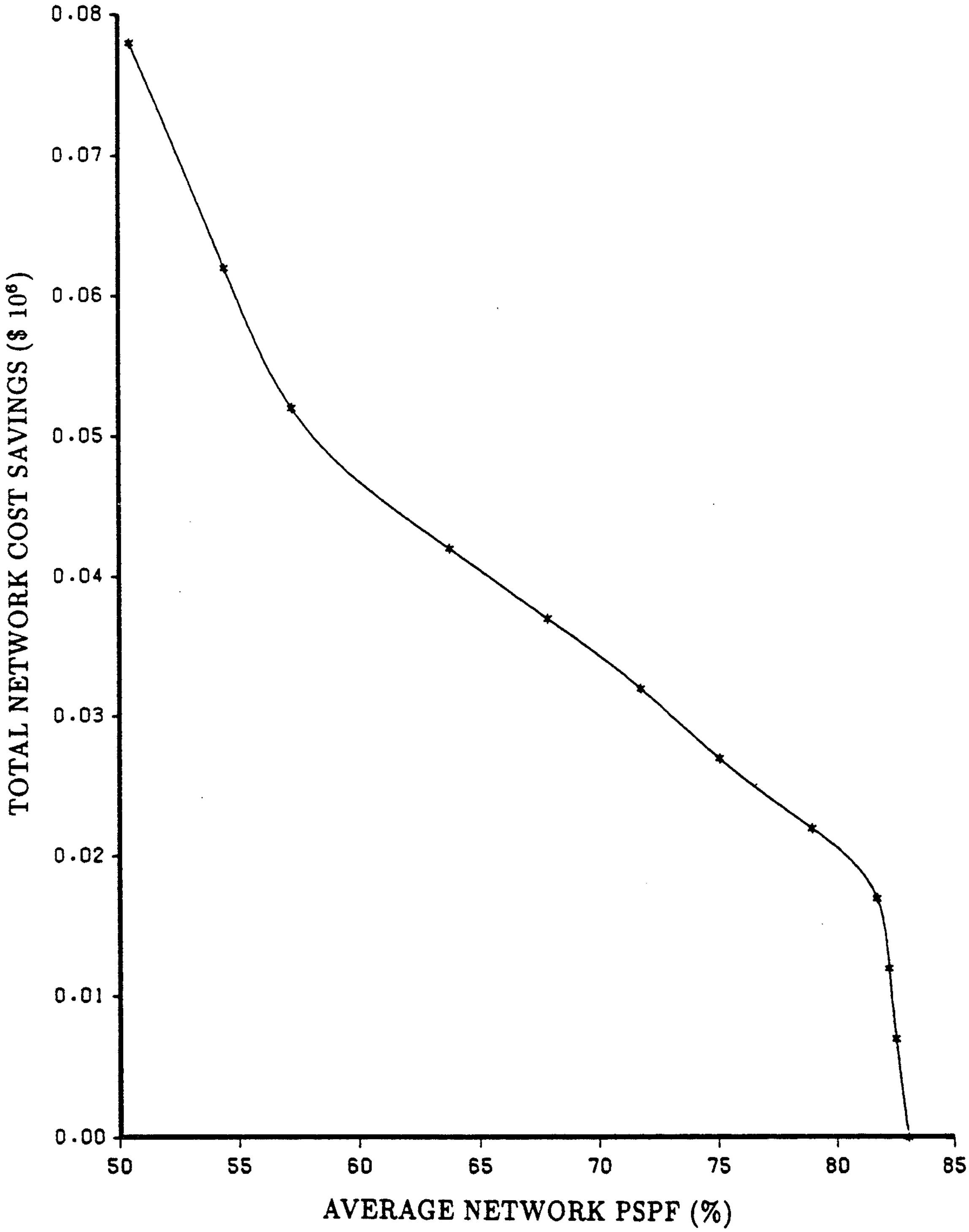


Figure 5.3: **Plot of Cost Savings vs. Network Hydraulic Redundancy** ($\overline{PSPF}$)

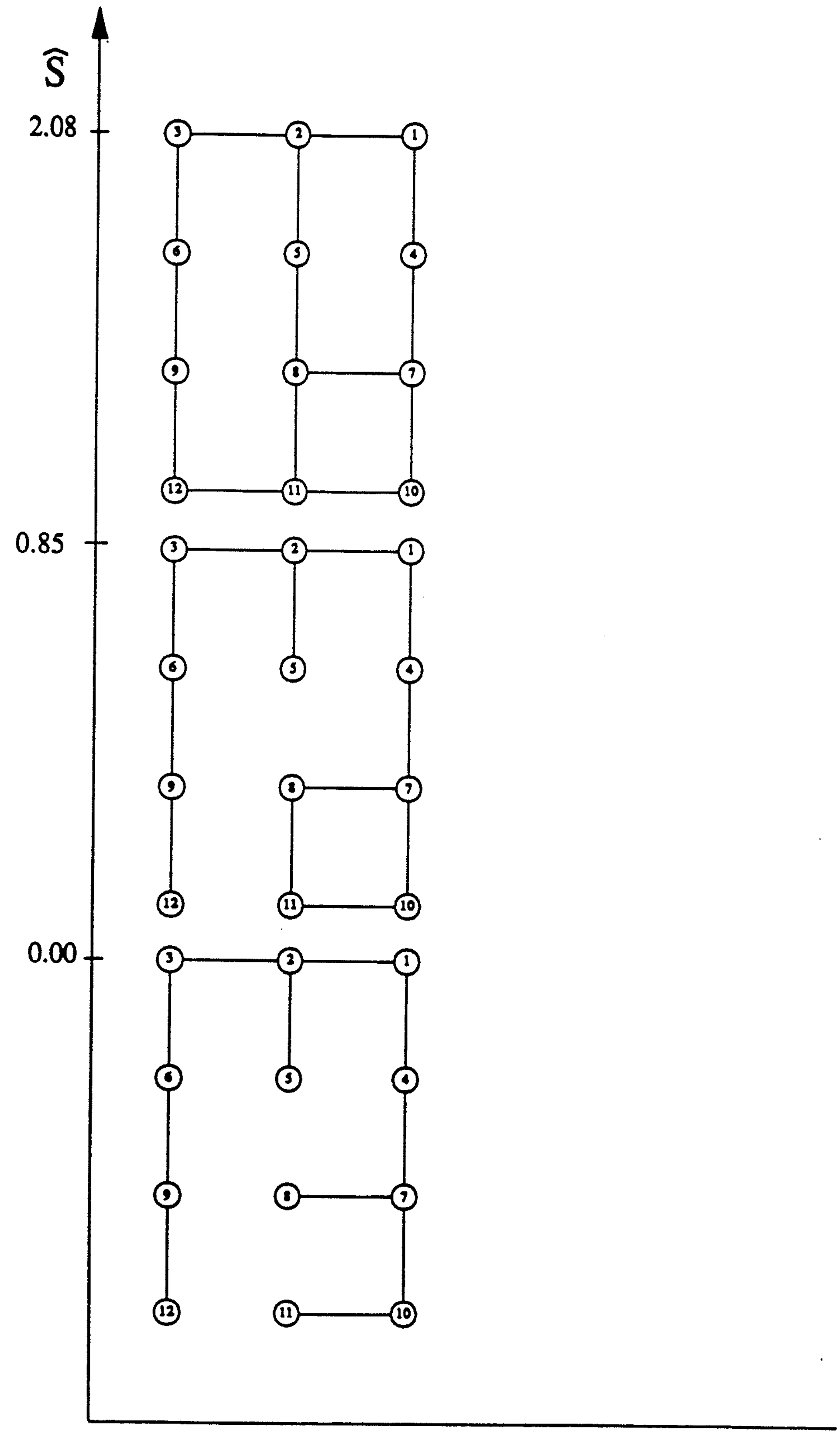


Figure 5.4: **Variation in Network Layout with Network Entropy ($\widehat{S}$)**

redundancy performed in the previous chapter and imply that, for large complex networks such as real water distribution systems where reliability calculations will be computationally impossible or hydraulic simulation of all solution networks on the trade-off curve will be too exhaustive to perform, a multi-objective analysis of network cost and reliability using the more computationally efficient entropy method (entropy method took 180 seconds on IBM PC 286 compactible compared to a similar network which used 200 minutes on a mainframe in Su et al., 1987) holds some promise. More work is needed , however, to ascertain whether the results are generally applicable to all networks.

The essence of the above work is that a formal statement of the redundancy (which is positively correlated to the hydraulic performance of the network under link failure conditions as well as to the network reliability) has been formulated in such a fashion that it can be incorported directly into classical optimization models.

5.4 Optimum Design of a Large Network Using the Entropy Based Models

5.4.1 Introduction

In this section, the entropy based optimization models developed at the begining of this chapter are used to optimally design a large network (a network larger than those normally used in the literature to evaluate reliability measures. A network of about 20 nodes, 30 links and 10 loops or greater can be considered as large). Either of the two models can be used to design a water distribution network. In this case Model A, in which the entropy is constrained at each node individually while minimizing network cost, is used. This option has the advantage that it prevents the network design for a particular cost having a good network-wide or mean redundancy measure at the expense of one or more relatively unreliable spots occurring in the network. This approach reflects classical system reliability methods in which it is generally desired to achieve good network reliability while maintaining some restriction on the reliability of the 'weakest' sector. In other words the system is 'only as good as its weakest link'. A similar approach was employed in the reliability analyses of Goulter and Coals (1986), Bouchart and Goulter (1990) and Goulter and Bouchart (1990).

Placing entropy restriction at the nodes while minimizing network cost still permits the model to eliminate links between nodes if it is cheaper to do so and still maintain the desired level of nodal redundancy. As such this option permits the model to act as both a layout and component design model.

5.4.2 Application To A Given Large Network Example

The model was applied to the network used by Morgan and Goulter (1985). This layout, showing all candidate links, is shown in Figure 5.5. The Morgan and Goulter network was selected as their model used to design the network considered a large number of demand cases and broken pipe combinations. The resulting network therefore has redundancy in terms of being able to handle a range of flows under different pipe failure conditions and provides a good basis by which the proposed entropy based redundancy measure can be evaluated.

This layout also has two sources. As noted previously multiple sources do not present any difficulties for the procedure as the entropy measure is based upon flow in the links incident on a node, rather than the original source of that flow. Furthermore the two sources imply added redundancy and reliability for the system relative to a single source system as a node has to be isolated from two sources before supply is completely cut off.

The demands at each node and the minimum pressure heads for the problem are

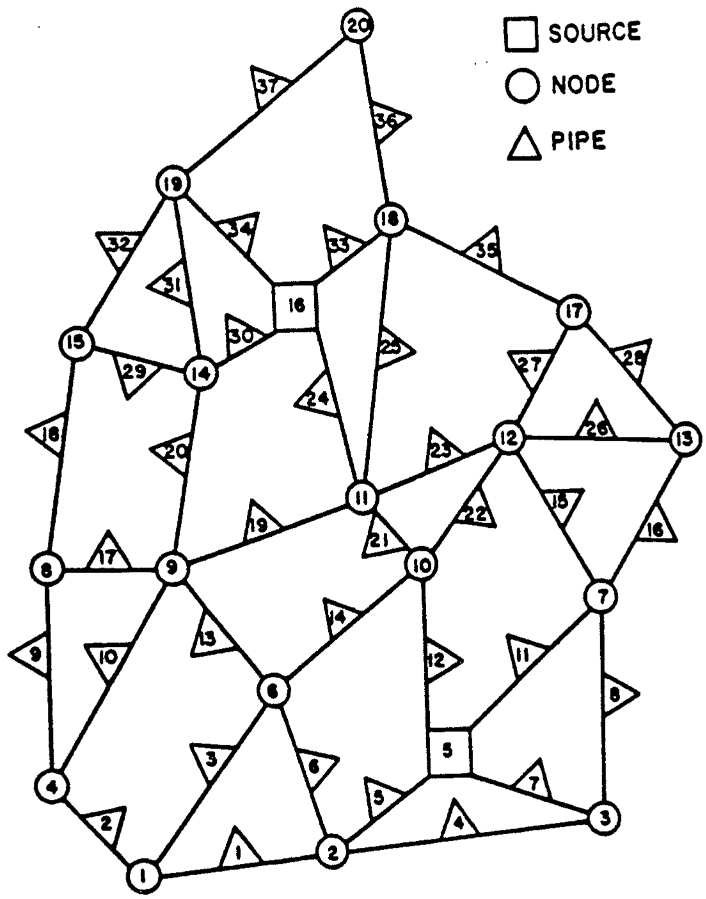


Figure 5.5: **Candidate Links for Network (Morgan and Goulter, 1985)**

the same as in the original problem of Morgan and Goulter (1985) and are shown in Table 5.2. While Morgan and Goulter (1985) had to use 37 loading cases to obtain their result, only 5 load cases were used here. Each of the 37 loading cases of Morgan and Goulter (1985) was an imposition of an emergency load at one node combined with the normal demand load at the remaining nodes. Five critical nodes representing the most vulnerable nodes to link failure under five loading pattern were identified. These critical nodes are 'terminal nodes' or nodes most downstream in network such that all nodes in the path connecting the source and the terminal node have higher nodal pressure than that at the terminal node. Imposing the emergency load at this terminal node as a design condition implies that emergency demand at any of the intermediate nodes can be satisfied given that emergency demands occur at one node at a time and the magnitude of this demand is less than or equal to that imposed at the terminal node. A different critical or terminal node was identified for each of the five loading cases in Table 5.2. This same concept of identifying critical nodes was employed in the model of Morgan and Goulter (1985). Five demand cases were used. The five demand cases were handled simultaneously by the model, i.e., the complete hydraulic constraints for all five load cases were incorporated in the constraint set.

The lengths of each link are as given in Morgan and Goulter (1985) and are shown in Table 5.3. Since the model is non-linear (in both objective function and some constraints), the formulation was again solved using the GRG2 non-linear package of Lasdon and Waren (1984). The solution of the formulation yielded continuous pipe sizes which were then converted into equivalent discrete commercial sizes of two lengths for each link.

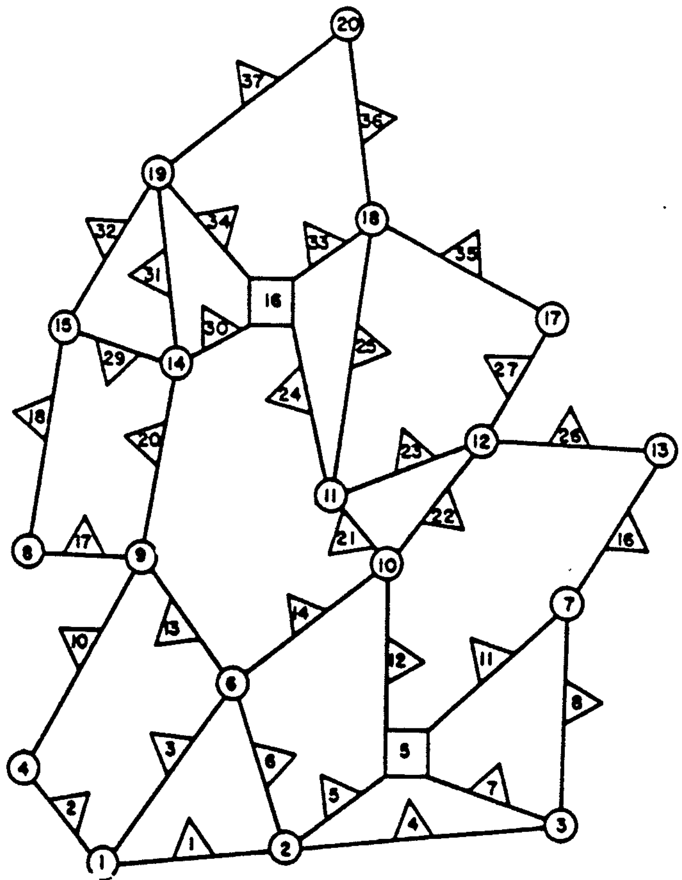
The design model was run for a series of restrictions on the minimum permissible value of the entropy based redundancy measure at each node. In the first instance, entropy values at each node were restricted to be greater than or equal to 0.70, this value being equivalent to the statement that each node should have at least two equal capacity incident links [mathematically, $-0.5 \ln 0.5 - 0.5 \ln 0.5 \approx 0.70$]. The entropy values were successively lowered, i.e., the redundancy (reliability) requirement became less restrictive, down to 0.50.

Application of the formulation produced the layout shown in Figure 5.6. Figure 5.7 gives the optimal layout determined by Morgan and Goulter (1985) under 37 load cases. A comparison of the pipe sizes obtained for the range of entropy constrained formulations and those obtained from the Morgan and Goulter procedure is shown in Table 5.3. The network wide global redundancy measure is given for each of the solutions delivered from the entropy approach. Due to the pipe failure basis of the Morgan and Goulter model, it is extremely difficult to generate this entropy measure for that network. For this reason, no network entropy value is given for the Morgan and Goulter solution.

Table 5.2. Demand at Nodes and Minimum Pressures for Layout in Figure 5.6

Node	Min. Head (m)	Demand Patterns (flow in m^3/h)				
		(1)	(2)	(3)	(4)	(5)
1	75	165	165	165	165	165
2	75	220	220	220	220	220
3	73	145	145	145	145	145
4	72	165	165	165	165	165
5*	96	-	-	-	-	-
6	73	140	140	140	140	140
7	67	175	175	175	175	175
8	72	300	180	180	180	180
9	70	140	140	140	140	140
10	69	160	160	160	160	160
11	71	170	170	170	170	170
12	70	160	250	160	160	160
13	64	190	190	190	190	190
14	73	200	200	200	200	200
15	73	150	150	240	150	150
16*	96	-	-	-	-	-
17	67	165	165	165	285	165
18	70	140	140	140	140	140
19	70	185	185	185	185	185
20	67	165	165	165	165	285

* Source Nodes



Entropy Model Layout (For Entropy Levels $\geq$ 0.50-0.70)

Figure 5.6: **Entropy Based Model Layout Solution**

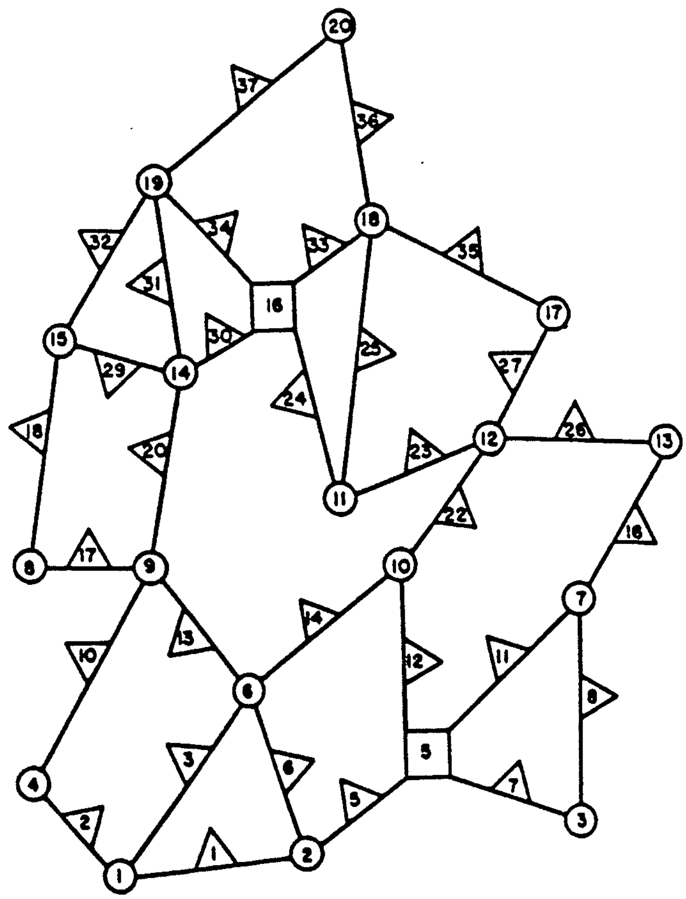


Figure 5.7: Morgan and Goulter (1985) Model Layout Solution

Table 5.3. Comparison of Solutions from Entropy Constrained Model and Morgan and Goulter (1985) Model

	Pipe Diameters (meters) and Lengths (meters)									
	Morgan and Goulter Model		Entropy Constrained Model (Nodal Entropy Levels)							
			0.50		0.55		0.60		0.70	
Link	Dia.	Lgth	Dia.	Lgth	Dia.	Lgth	Dia.	Lgth	Dia.	Lgth
1	0.25	760	0.20	577	0.20	280	0.20	739	0.20	760
			0.25	183	0.25	480	0.25	21		
2	0.15	113	0.20	520	0.20	520	0.20	520	0.20	520
	0.20	407								
3	0.25	797	0.15	309	0.20	25	0.15	300	0.25	890
	0.30	93	0.20	581	0.25	865	0.20	590		
4	-	-	0.25	1120	0.20	207	0.25	203	0.30	1120
					0.25	913	0.30	917		
5	0.30	371	0.30	610	0.30	610	0.30	610	0.20	46
	0.35	239							0.25	564
6	0.20	680	0.20	680	0.20	680	0.20	680	0.20	356
									0.25	324
7	0.20	474	0.30	236	0.30	680	0.35	680	0.35	472
	0.25	206	0.35	444					0.40	208
8	0.20	315	0.13	479	0.13	858	0.13	269	0.15	260
	0.25	555	0.15	391	0.15	12	0.15	601	0.20	610
9	-	-	-	-	-	-	-	-	-	-
10	0.20	520	0.13	980	0.13	980	0.13	635	0.15	127
	0.25	460					0.15	345	0.20	853

Table 5.3. Continued

	Pipe Diameters (meters) and Lengths (meters)									
	Morgan and Goulter Model		Entropy Constrained Model (Nodal Entropy Levels)							
			0.50		0.55		0.60		0.70	
Link	Dia.	Lgth	Dia.	Lgth	Dia.	Lgth	Dia.	Lgth	Dia.	Lgth
11	0.25	890	0.20	472	0.20	316	0.20	890	0.20	890
			0.25	418	0.25	574				
12	0.40	750	0.25	727	0.25	557	0.25	750	0.25	620
			0.30	23	0.30	193			0.30	130
13	0.25	620	0.15	620	0.15	620	0.15	620	0.15	620
14	0.35	541	0.15	109	0.15	165	0.15	42	0.20	355
	0.40	259	0.20	690	0.20	635	0.20	758	0.25	445
15	-	-	-	-	-	-	-	-	-	-
16	0.15	98	0.20	303	0.25	680	0.20	408	0.15	107
	0.20	582	0.25	377			0.25	272	0.20	573
17	0.15	42	0.15	154	0.15	47	0.15	232	0.15	285
	0.20	438	0.20	326	0.20	433	0.20	248	0.20	195
18	0.20	173	0.15	860	0.15	860	0.15	860	0.20	813
	0.25	687							0.25	47
19	-	-	-	-	-	-	-	-	-	-
20	0.20	770	0.25	37	0.30	770	0.25	287	0.20	770
			0.30	733			0.30	483		

Table 5.3. Continued-II

	Pipe Diameters (meters) and Lengths (meters)									
	Morgan and Goulter Model		Entropy Constrained Model (Nodal Entropy Levels)							
			0.50		0.55		0.60		0.70	
Link	Dia.	Lgth	Dia.	Lgth	Dia.	Lgth	Dia.	Lgth	Dia.	Lgth
21	-	-	0.25	350	0.25	350	0.25	350	0.25	350
22	0.20	36	0.30	620	0.25	620	0.25	543	0.30	620
	0.25	584					0.30	77		
23	0.15	345	0.13	22	0.13	311	0.15	615	0.20	226
	0.20	325	0.15	648	0.15	359	0.20	55	0.25	444
24	0.20	337	0.30	246	0.30	463	0.35	418	0.30	58
	0.25	453	0.35	544	0.35	327	0.40	372	0.35	732
25	0.20	1150	0.20	1150	0.15	196	0.20	1150	0.15	23
					0.20	954			0.20	1127
26	0.20	750	0.13	108	0.15	750	0.15	623	0.15	199
			0.15	642			0.20	127	0.20	551
27	0.15	99	0.20	533	0.20	164	0.15	17	0.15	71
	0.20	451	0.25	17	0.25	386	0.20	533	0.20	479
28	-	-	-	-	-	-	-	-	-	-
29	0.20	500	0.15	118	0.15	63	0.15	159	0.15	33
			0.20	382	0.20	437	0.20	341	0.20	467
30	0.25	6	0.35	240	0.35	124	0.40	450	0.30	283
	0.30	444	0.40	210	0.40	326			0.35	167

Table 5.3. Continued-III

	Pipe Diameters (meters) and Lengths (meters)									
	Morgan and Goulter Model		Entropy Constrained Model (Nodal Entropy Levels)							
			0.50		0.55		0.60		0.70	
Link	Dia.	Lgth	Dia.	Lgth	Dia.	Lgth	Dia.	Lgth	Dia.	Lgth
31	0.15	82	0.20	750	0.15	49	0.20	750	0.25	750
	0.20	668			0.20	701				
32	0.20	714	0.13	355	0.13	538	0.13	24	0.25	720
	0.25	6	0.15	365	0.15	182	0.15	696		
33	0.25	540	0.20	540	0.20	540	0.20	540	0.15	540
34	0.30	700	0.25	614	0.25	363	0.25	700	0.20	700
			0.30	86	0.30	337				
35	0.15	39	0.15	850	0.15	850	0.13	850	0.15	850
	0.20	810								
36	0.20	538	0.15	750	0.15	750	0.15	750	0.15	750
	0.25	212								
37	0.20	625	0.25	970	0.25	970	0.20	350	0.15	777
	0.25	345					0.25	620	0.20	193
COST	\$1,950,698		\$1,942,077		\$1,952,191		\$1,961,083		\$2,007,012	
$\hat{S}$	-		2.5212		2.5361		2.5585		2.5744	

5.4.3 Discussion of the Results

Both the entropy constrained and Morgan and Goulter approaches eliminated some of the candidate links. The Morgan and Goulter solution eliminated a total of 6 links (links 4, 9, 15, 19, 21 and 28). All solutions from the entropy models eliminated 4 links (links 9, 15, 19 and 28) all of which were also eliminated by the Morgan and Goulter approach. The two networks associated with nodal entropy ≥ 0.50 and nodal entropy ≥ 0.55 are closest in total cost, \$1,942,077 and \$1,952,191 respectively, to that of the solution of Morgan and Goulter whose cost was \$1,950,698. Direct comparison of the the approaches will therefore be performed using these three networks.

There is also a remarkable closeness in the pipe sizes for the remaining common links selected by the two models with nodal entropy ≥ 0.50 and ≥ 0.55 and the Morgan and Goulter model. This closeness in the selected pipe sizes together with the similarity in which links were eliminated shows that the entropy constrained approach is performing in a remarkably similar fashion to an accepted and more complex procedure for designing reliable networks. However, the entropy model required only 5 loading conditions and one solution from the optimization model compared to the 37 load patterns and the multiple iterations between the network solver and optimization formulation required for the Morgan and Goulter's approach. Morgan and Goulter's technique also required six iterations and 365 seconds on the AMDAHL 5850 mainframe compared to the 240 seconds used by the entropy method on an IBM PC 386 compactible. The ease with which these results were obtained from the entropy model indicates that the entropy constrained option is an efficient means of obtaining solutions comparable in cost and level of redundancy/reliability to the solution obtained by larger and more computationally intensive approaches.

The $\widehat{S}$ (overall network entropy) values for all solutions shown in Table 5.3 and Figure 5.6 suggest that the network entropy value is very insentitive to network design (both layout and component sizing) and may be a trivial indicator of network performance at this level of network complexity. It should be noted, however, that the reduction in $\widehat{S}$ value from the network design associated with nodal entropy ≥ 0.70 to that associated with nodal entropy ≥ 0.50 is 2%. The reduction in cost over the same two networks is just over 3%. Hence the change in redundancy is of the same order as the change in cost, which in this case is $65,000.

It should be noted that although the pipe sizes varied from one constrained entropy level to the other, the layout produced by the model does not. The sensitivity of the results lies mainly in the pipe sizes selected to meet the nodal redundancy requirements. The consistency in network layout was due in large part to the fact that the right hand side of the entropy constraints were not made sufficiently small. As the minimum acceptable nodal entropy redundancy measure approaches zero value, more links could be deleted and the network would degenerate into a branched network, which is not desirable for urban distribution systems. The fact that the layout does not change within the range of nodal entropy levels investigated suggests that the formulation is capable of identifying fundamentally reliable/redundant networks, at least in comparison with results from other well accepted models, without having to be too concerned with the level of required nodal redundancy specified at each node.

The question of exactly what nodal entropy value should be used in constraining the design can only be answered through a more complete understanding of what a particular level of redunadncy actually means. This issue is partially addressed in the example application, however. The value of 0.70 used to constrain the nodal redundancy in the first step requires that at least two links (and associated paths if the path parameter is included) of equal capacity be incident upon each node. Each decrease in required nodal redundancy below this value represents some further reduction in the ability of the network to respond to contingency conditions. If S_j values of 0.70 are met and they represent two equal capacity links (or paths) on a node there is a reasonable assumption of sufficient redundancy in the face of single link failure. Since reliability is based upon the ability of the network to perform adequately in the face of single component failure is the basis of current design practice, it appears unnecessary to require nodal redundancy levels higher than 0.70. More work is required, however, to obtain a better understanding of what specific values of entropic redundancy mean in terms of reliability, for specific layouts.

The overall performance of the entropy constraint approach in this example does suggest that the approach has some merit in designing reliable/redundant networks using simplified one-step optimization procedures.

Chapter 6

SUMMARY AND CONCLUSION

The definition of redundancy /flexibility of water distribution networks is a very difficult problem. It is difficult not only to formulate measures for these parameters but also to define what constitutes acceptable levels of these parameters for distribution networks. A modified procedure and parameter for quantitatively assessing and monitoring the reliability/redundancy/flexibilty of water distribution networks is proposed. The parameter is a relative measure and as such can be used at this stage only to compare redundancy among networks rather than to assess the absolute values of the reliability . It is the ability of the entropy parameter to recognise the intrinsic redundancy of network layouts caused by alternate paths and flow reversal possibilities and how such intrinsic redundancy contributes to system reliability that represents its contribution to the field of reliability analysis. The proposed measure does not overcome all the difficulties associated with stating and evaluating reliability in water distribution networks. However, it is shown to provide a reasonable statement of redundancy and therefore a surrogate for network reliability.

Evaluation of the parameter by comparison with an accepted and new measures of reliability , Nodal Pair Reliability and Percentage of Demand Supplied at adequate Pressure respectively, for a range of network layouts indicate that the procedure can identify important differences, in terms of reliability , among networks. However, the most promising feature of the parameter is perhaps the ease with which it can be incorporated into optimization design models for water distribution networks design. As yet there are no reliability or redundancy measures which satisfy this condition.

Use of the measure in design of redundant or reliable networks for an example problem demonstrates that it is capable of developing reliable layout and component designs without having to use the large numbers of load patterns or intensive iterative approaches normally required. Formulations embodying the measure also appear to have the capabilities of identifying reliable/redundant layouts which are quite insensitive to the entropy level constrained at individual nodes. As such the measure represents a first step in the development of computationally efficient formulations for incorporating reliability directly into the design of water distribution networks. Maximising the measure in a network has an effect equivalent to maximising the level of uniformity in capacities of the links incident upon the demand nodes. The use or target of equal capacities to achieve improved redundancy/reliabilty is consistent with recent developments in the design of reliable networks.

In a multi-objective framework between network cost and the computationally intensive network reliability or hydraulic redundancy, the more efficient entropy based redundancy measure can possibly be used for the generation of cost-reliability/redundancy trade-off curves.

Further work needs to be performed in determining what particular level of entropic redundancy actually means in terms of network performance. This work will also provide a basis for selecting the level of redundancy to be used for a particular situation. Since the provision of at least two paths to a node is a pre-requisite for basic reliability, a nodal entropy equal to 0.70, which corresponds to two links of equal capacity incident on the node, appears to be a basic starting point.

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Appendix A

An Algorithm For Determining The Path Parameter-a_j

A.1 Path Enumeration Step

The number of paths between the source and a node is determined by what is known as 'tie-set' method. A tie-set is a set of a system of components connected in series. A tie-set fails if any of the components fails. The number of tie-sets in the system (source to node) is given by the parallel number of tie-sets. The system fails if all the tie-sets in parallel fail. The number of paths to be determined in this section is therefore the determination of the number of parallel tie-sets, termed 'system tie-sets', all which must fail before the node in question to be cut off from the supply source.

In order to determine the number of paths from the source to the nodes in the network, it is unavoidable to do some form of path enumeration. Path enumeration is the first step in the determination of the path parameter a_j for node j. In this step, it is necessary to determine all the paths, dependent or independent, between the source to the node in question, and also all the links in each of these paths. After this is done, a matrix formulation is developed to obtain the equivalent number of independent paths and thus the path parameter a_j.

One method of doing the path enumeration is that due to Misra (1970). His method involves taking the various powers of the connection matrix of the network to obtain the paths of different step sizes. A solution of the square of the connection matrix will therefore give the two link paths between each node, and so on. Another method of path enumeration was proposed by Aggarwal et al. (1973) and modified by Billington and Alan (1983). The method presented by Billington and Alan will be discussed here and applied to this work. This is because their method is applicable to both unidirectional links (flow permitted in one direction only) and bidirectional links (flow permitted in either direction). The method can also be computerized.

The method involved the transformation of the network into a connection matrix which defines the transmission of flow between the source and the demand node of interest. In the connection matrix, zero is entered as an element if there is no connection between the two nodes, unity is entered where the node is connected to itself (the elements on the principal diagonal) and the label of link is entered if the two nodes are connected. The connection matrix of the network is built after doing series reduction of all connections between the links in the paths of the node and the source. This series reduction is done by replacing a number of links in series between any two nodes by a single link. The last row and the columns of this connection matrix are organised such that the source node corresponds to the first row and the demand node corresponds to the second row. The remaining nodes constitute the rest of the rows, with their arrangement arbitrary. The last row and the last column are then deleted after modifying the remaining entries of the matrix as follows:

$$C_{ij}(new) = C_{ij}(old) + C_{in} \cdot C_{nj} \quad \forall \quad i = 1, 2, \ldots (n-1) \qquad (A.1)$$

where the n^{th} row and column are the last ones in the matrix (to be deleted after this modification). This results in a new (n-1) by (n-1) matrix. The last row and column of the new matrix are then deleted [the $(n-1)^{th}$ row and $(n-1)^{th}$ column] to give a new $(n-2)$ by $(n-2)$ matrix. The process is continued until a 2x2 matrix of the source node and the designated node results. This matrix will contain all the paths between them. The process requires only $(n-2)$ steps for a network with n number of nodes, and in each successive step, the matrix size reduces rapidly. The order of the computation for the path enumeration for all nodes of a network of n nodes is $(n-1)\text{x}(n-1)$ or n^2. Hence this is a polynomial time.

In its application to water distribution networks, it is necessary to consider if any of the links is undirected. An undirected link can be replaced by two oppositely directed links, each given the same label in the enumeration process. The matrix iteration process is then done exactly as described above to give the solution.

A.2 A Matrix Method For Determining The Path Parameter a_{ij}

Step 1

First, do a series reduction of the paths from the source to the node for which redundancy is to be measured (series reduction is the conversion of a path that is made up of several links in series into one link path). This process is applicable because whether the path consists of only one link or several links in series does not change the fact that it is still one path. Therefore, with regards to the path parameter, this series reduction is required.

After the series reduction, use the enumeration method described above to do path enumeration of all paths for the node to obtain the 'system tie-sets'. The various paths obtained using the method above will consist of both independent and dependent paths. It is now necessary to determine which of them are dependent. Therefore the formula given in Eq. A.4 is used to convert the complete set of paths into equivalent independent paths.

Step 2

Develop a PATH MATRIX for the network out of the resulting paths in the path enumeration step. This process is done as follows:

- Let the links of the network be labelled as $j = 1, 2, \ldots L$ for L number of links.
- Let the paths from the source to the node be labelled $i = 1, 2, \ldots LP$ for LP number of paths.

The elements of the path matrix are therefore

$P_{ij} = 1$ if link j belongs to path i.
$P_{ij} = 0$ otherwise.

Step 3

Sum up all the entries in each row and each column. Let

$$SC_j = \sum_{i=1}^{LP} P_{ij} \quad \forall \quad links \quad j \tag{A.2}$$

and

$$SR_i = \sum_{j=1}^{L} P_{ij} \quad \forall \quad paths \quad i \tag{A.3}$$

SC_j is then the sum of entries in column j and this repesents the number of paths to which link j belongs and is the same as the degree, d_j, of link j. SR_i is the sum of entries in row i representing the number of links that make up path i.

Step 4

Determine if each path is independent. Then remove the dependencies and compute the path parameter for the path system. To do this, compute the following number for every path:

$$NP_i = \frac{1}{SR_i}\left[\sum_{j=1}^{L} P_{ij} \cdot SC_j\right] \quad \forall \quad paths \quad i \tag{A.4}$$

If $NP_i = 1$ for any path i, then this path is independent of other paths.

Consider an incident link on node j, labelled jx. Let the set of paths to which incident link jx belongs be IX. This is the set such that $P_{i,jx} = 1$ for all links in the paths.

There will be three different cases of path systems, depending on the values of SC_{ij} and NP_{IX}.

a) $SC_{jx} > 1$ and $NP_{IX} > 1$.

This means there is more than one path through incident link jx to node j. These paths must be dependent because they have at least one common link, link jx. If there are NX such paths in the set IX, define

$$SS_{j,jx} = \sum_{i \in IX} P_{ij} \quad \forall \quad j = 1, 2, \dots L \tag{A.5}$$

where $SS_{j,jx}$ represents the sum of degrees of all links in the set of paths $(i \in IX)$ to which the incident link jx belong. Let cx be the set of all links in the set of paths IX $(j \in cx)$ such that

$$S_{j,jx} > 0 \quad \forall \quad paths \in IX$$

Let NC represent the total number of such links in the set cx. The sum of the degree of dependencies in these particular links becomes:

$$DSC_{jx} = \sum_{j \in cx} [SS_{j,jx} - 1] \tag{A.6}$$

The total number of degrees of the links in the paths belonging to the set IX is given by:

$$TSR_{jx} = \sum_{i \in IX} SR_i \tag{A.7}$$

The equivalent terms for n_{ij}, $\sum_{l=1}^{M_{ij}} D_l$ and $\sum_{l=1}^{M_{ij}} d_l$ in Eq. A.4 are NX, DSC_{jx} and TSR_{jx} respectively. Therefore the path parameter for this case of path system is

$$a_{jx} = NX \left[1 - \frac{DSC_{jx}}{TSR_{jx}} \right] \tag{A.8}$$

This value must therefore be computed for all incident links for node j. The total number of effective independent paths at node j is therefore:

$$a_j = \sum_{jx \in L_j} a_{jx} \tag{A.9}$$

Where L_j is the set of incident links at node j.

b) $SC_{jx} = 1$ and $NP_{IX} = 1$.

In this case there is exactly one path from the source to the node through incident link jx and this path is independent of any other path in the network. Hence:

$$a_{jx} = 1 \tag{A.10}$$

c) $SC_{jx} = 1$ but $NP_{IX} > 1$.

In this case there is exactly one path from the source to the node through link jx, but this path is dependent on other paths in the path matrix. The path parameter will no longer be unity, but a fraction of this. There will therefore be the need to remove the dependency and obtain the effective number of independent paths.

In addition to the IX defined earlier, define another term IY as the set of paths which have links common to this path (for incident link jx). In order to identify this set of paths, let:

$$R_{ij} = P_{ij} \cdot P_{IX,j} \quad \forall i, j \tag{A.11}$$

For any path i, if R_{ij}=1, then $i \in IY$. On the other hand if $R_{ij} = 0$, then $i \notin IY$. Then:

$$SS_{j,jx} = \sum_{i \in IY} P_{ij} \quad \forall j = 1, 2, \ldots L_j \tag{A.12}$$

For every link j, if $SS_{j,jx} > 0$, then link j is a common link to the paths. Let CY be the set of such links. The sum of degree of dependencies is given as

$$DSC_{jx} = \sum_{j \in CY} [SS_{j,jx} - 1] \cdot P_{IX,j} \tag{A.13}$$

This term is the same as that for case (a) but is multiplied by the term $P_{IX,j}$. This is because while $SS_{j,jx}$ is to be the sum of the degrees of link j in paths of set IY, this sum may be for all paths link j belongs to other than the set IY. A multiplication of $(SS_{j,jx} - 1)$ by $P_{IX,j}$ will therefore ensure that only links belonging to path IX will be considered. The sum of degrees of the links for the set of paths IY is given as:

$$TSR_{jx} = \sum_{i \in IY} SR_i \tag{A.14}$$

Therefore, for the incident link jx with one path which is dependent on other paths, the equivalent number of independent paths is now:

$$a_{jx} = NX \left[1 - \frac{DSC_{jx}}{TSR_{jx}}\right] \tag{A.15}$$

with the nd term dropped since it is equal to unity.

A.3 Application To An Example

The matrix method described above is demonstrated by application to node 12 of layout number 4 in Figure 4.2. The matrix method given in this section illustrated through its application to one node of layout number 4. The node to which this is applied is node 12.

a) Step 1

The given Layout 4 is first reduced using series reduction and re-labelled as shown in Figure A.1.

A connection matrix is generated with node 1 in row 1 and node 12 in row 2 and the other nodes arranged arbitrarily as shown below;

	1	12	4	7	8	11
1	1	A	B	0	0	0
12	0	1	0	0	0	0
4	0	0	1	D	C	0
7	0	0	0	1	E	F
8	0	0	0	0	1	G
11	0	H	0	0	0	1

Delete row 11 and column 11 after the following operation; New entries are $C_{ij}(new) = C_{ij}(old) + C_{i,11} \cdot C_{11,j}$ giving the following reduced matrix;

	1	12	4	7	8
1	1	A	B	0	0
12	0	1	0	0	0
4	0	0	1	D	C
7	0	FH	0	1	E
8	0	GH	0	0	1

Delete node 8 and perform element modification step to obtain a new matrix;

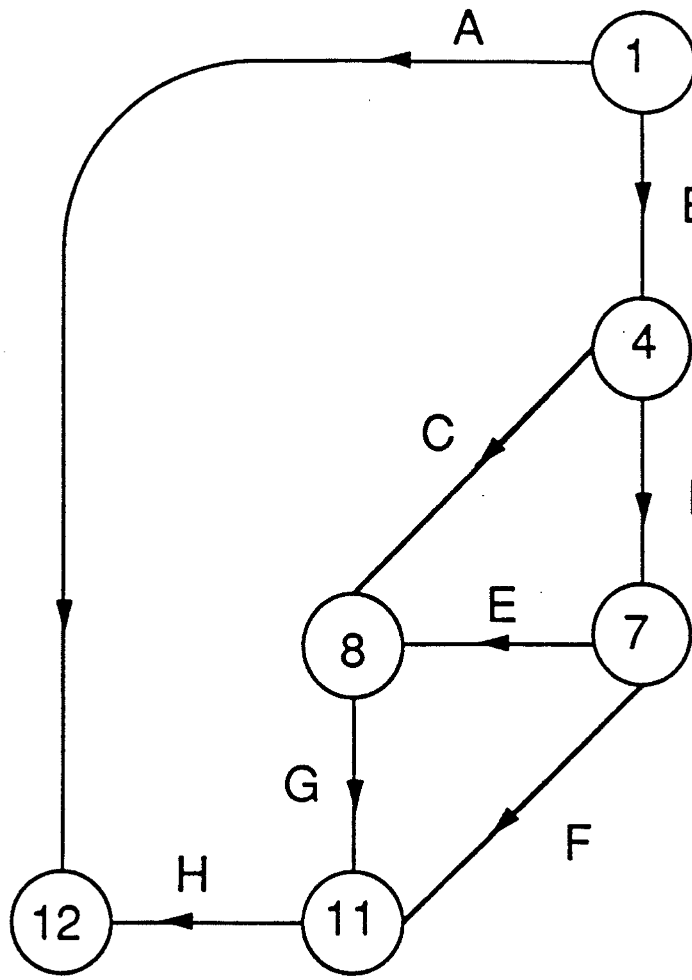


Figure A.1: **Series Reduction of Layout 4 for Path Enumeration**

	1	12	4	7
1	1	A	B	0
12	0	1	0	0
4	0	CGH	1	D
7	0	EGH+FH	0	1

Delete node 7 and perform the element modification step to obtain the new matrix below;

	1	12	4
1	1	A	B
12	0	1	0
4	0	CGH+DEGH+DFH	1

Finally, delete node 4 and perform the element modification step to obtain the solution matrix below;

	1	12
1	1	A + BDEGH + BDFH + BCGH
12	0	1

b) Steps 2 and 3

The last stage of the previous step shows there are four pathsets between node 1 and node 12, as summarised below (note that labels refer to the reduced layout);

Pathsets for Node 12 Layout 4

Path	Nodes In Path	Links In Path
1 (A)	1, 12	1
2 (BCGH)	1, 4, 8, 11, 12	2, 3, 7, 8
3 (BDEGH)	1, 4, 7, 8, 11, 12	2, 4, 5, 7, 8
4 (BDFH)	1, 4, 7, 11, 12	2, 4, 6, 8

The next step is to construct a path matrix out of the enumerated paths. For this example the path matrix is as follows;

Path Matrix For Node 12

	Link Number, j								
Path, i	1	2	3	4	5	6	7	8	SR_i
1	1	0	0	0	0	0	0	0	1
2	0	1	1	0	0	0	1	1	4
3	0	1	0	1	1	0	1	1	5
4	0	1	0	1	0	1	0	1	4
SC_j	1	3	1	2	1	1	2	3	-

c) Step 4

The incident links for node 12 are links 1 and 8. Check if the paths through these links to the source are dependent.

1. For incident link 1 ($jx = 1$) $SC_1 = 1$ and $NP_1 = 1$. This is path system (b), hence there is only one path to node 12 through link 1 and this path is independent of other paths. Hence the path parameter

$$a_{1,12} = 1$$

2. For incident link 8 ($jx = 8$) $SC_8 = 3$ and $NP_{IX} > 1$. This is path system (a) and there are three dependent paths through node 8.

i) The set of the dependent paths, belonging to the set IX are: $IX \in i = 2, 3, 4$ (from the column under link 8). Hence $NX = 3$.

ii) Compute $SS_{j,8}$ for all links j.

$$SS_{j,8} = \sum_{i \in IX} P_{ij} \quad \forall j$$

Link j	1	2	3	4	5	6	7	8
$SS_{j,8}$	0	3	1	2	1	1	2	3

iii) The set of links, cx, for which $SS_{j,8}$ >0 is is given by $cx \in j = 2, 3, 4, 5, 6, 7, 8$. Hence $NC = 7$.

iv) The sum of degree of dependencies is given by

$$DSC_8 = \sum_{j \in cx} (SS_{j,8} - 1)$$

This sums up to $DSC_8 = 6$.

v) The sum of the degrees of the links in path set IX is given by;

$$TSR_8 = \sum_{i \in IX} SR_i = 13$$

vi) Therefore, the path parameter is given by;

$$a_{8,12} = NX \left[1 - \frac{DSC_8}{TSR_8}\right]$$

$$a_{8,12} = 3 \left[1 - \frac{6}{13}\right] = 1.615$$

d) Step 5

The effective number of paths from the source to node 12 is therefore given as the sum of paths through links 1 and 8;

$$a_{12} = 1 + 1.615 = 2.615$$

Hence the 4 dependent paths to node 12 reduces to 2.615 independent paths.

Appendix B

Computer Program To Compute Entropy Based Redundancy Measures For any Network

Below is a listing of a computer program that will calculate the path parameters for the incident links of every node in any given network, use them in conjunction with flows in the links and their age factor parameters to calculate the redundancy at each node and the overall network redundancy given in Chapter 3. The computer language used to compile this program is 'C'. C is used because the algorithm used to evaluate the path parameter involves the algebraic manipulation of character strings (alphabets as labels for links which are multiplied and added in the path enumeration stage of the algorithm) and this cannot be done easily using the Fortran Computer Language. The program consists of six sub-programs which perform the following functions.

Sub-program Nodef

In this program, the dimensions of the problem are defined. This is done in terms of the maximum number of nodes in the network, the maximum number of links in the network, and the maximum number of paths in the network. The maximum number of paths actually refer to the possible maximum number of paths between the source and a node and not all the paths from all nodes to the source. Exact numbers are not required, only dimensions higher than those in the network to be considered need be stated.

Sub-program Enter

In this program, the network characteristics are input as data. The source node(s) are labelled and the links between each pair of nodes are defined and labelled. The flow direction is specified by entering the 'head node' for the link first and the 'tail node' last. If the link is bi-directional (flow is permitted in both directions), the link is defined twice, with the end nodes definition reversed in the second data input. The flow magnitude in each link as well as the age factor parameter of the link are entered as data. A name for this data is assigned by the user and the end of the data is indicated by entering the number '0' for a link (after defining the last link).

Sub-program Nodem

This is the main calling program for all sub-programs. The path parameters obtained from other sub-programs are used together with the flow data and age parameters to calculate the entropy based redundancy measures. All tables and solutions generated are also printed in this main program.

Sub-program Noder

In this program, the node removal algebra (given as Equation A.6) that leads to the solution of paths between a pair of nodes is executed.

Sub-program Counts

This sub-program constructs the 'Path Matrix' from the solution of paths between the source and the nodes. The sum of columns SC_j, the sum of rows, SR_i, of the path matrix are also calculated. The incident links for the node are indentified, the number of paths through the incident link, NX, is identified and the index NP for the path system is computed. All these are printed out as an output.

Sub-program Pathp

This sub-program checks for the three cases of dependency among the paths and calculates the appropriate path parameter for every incident link in the network. The path parameters acn be printed out or written into a data file for use in computing the entropy based redundancy measures.

PROGRAM LISTING

```
/* ******** N O D E F *************** */
/* */
/* This file defines the sizes of data structures */
#define HEAP_ SIZE 16384
#define MAX_ NODES 40
#define MAX_ LINKS 50
#define MAX_ PATHS 200
/* ********* E N T E R *************** */
/* */
#include <stdio.h>
#include <math.h>
#include <string.h>
#define MAT_ SIZE 40
/* */
FILE *sysio;
char a[MAT_ SIZE][MAT_ SIZE];
double u[MAT_ SIZE][MAT_ SIZE];
double q[MAT_ SIZE][MAT_ SIZE];
char instring[20];
char file_ name[20];
/* */
void main(int, char*[2]);
/* */
void main
(
```

```
int argc,
char *argv[2] )
)
{
int num, i, j;
char link;
/* */
if (argc == 1) {
printf("Enter the name of the file you wish to create ⟶ " );
gets(file_name);
printf("\n\n");
} else
strcpy(file_name, argv[1]);
printf("Input the number of nodes ⟶ ");
gets(instring);
num = atoi(instring);
for (i = 0; i< num; ++i)
for (j = 0; j< num; ++j) {
if (i == j) a[i][j] = 49;
else a[i][j] = 48;
q[i][j] = 0.0
u[i][j] = 0.0
}
printf("Enter links (enter 0 to exit)\n\n");
for (;;) {
printf("Link name ⟶");
```

```
gets(instring);
link = instring[0];
if (link == 48) break;
printf("Source node —→");
gets(instring);
i = atoi(instring);
printf("Destination node —→");
gets(instring);
j = atoi(instring);
printf("Input the age parameter —→");
gets(instring);
u[i][j] = atof(instring);
printf("input the flow —→");
gets(instring);
q[i][j] = atof(instring);
printf("\n\n");
a[i][j] = link;
}
sysio = fopen(file_name, "w");
fprintf(sysio, "%d\n", num);
for (i = 0; i < num; ++i) for (j = 0; j < num; ++j) fprintf(sysio, "%c\n", a[i][j]);
for (i = 0; i < num; ++i) for (j = 0; j < num; ++j) fprintf(sysio, "%f\n", q[i][j]);
for(i = 0; i< num; ++i)
for(j = 0; j< num; ++j)
fprintf(sysio, "%f\n", q[i][j]);
fclose(sysio);
```

```
    return;
}
/ * * /
/* ********** N O D E M ************* */
/* */
#include <stdio.h>
#include <string.h>
#include <math.h>
#include "nodef.c"
/* */
char a[MAX_NODES][MAX_NODES][2];
char *b[MAX_NODES][MAX_NODES];
double uu[MAX_NODES][MAX_NODES];
double qq[MAX_NODES][MAX_NODES];
double Q0;
int nodes[MAX_NODES];
char links[MAX_LINKS];
int table[MAX_PATHS][MAX_LINKS];
int path;
double np[MAX_PATHS];
int num_inc;
int inc_link[MAX_PATHS];
double ajx[MAX_PATHS];
double Q[MAX_NODES];
double U[MAX_NODES];
double ss[MAX_NODES];
```

```
double s;
char charheap[HEAP_SIZE];
int heaptop;
char *new_string;
/* */
void main(int, char *[ ]);
void noder(char *, char *, char *);
void count_string(char *, char *,
int [MAX_PATHS][MAX_LINKS], int *, double[MAX_PATHS],
int*, int[MAX_PATHS]);
double pathp(int, int, int, int [MAX_PATHS][MAX_LINKS],
int, double);
/* */
void main
(
int argc,
char *argv[3]
)
{
int num_nodes, num, num_links, dest, remove_node, i, j, k;
double temp, t1, t2;
/* */
FILE *sysin;
FILE *sysout;
static char infile[15], outfile[15];
static char instring[20];
```

```
/* */
switch (argc) {
case 1 :
printf("Input the name of the input file ⟶ ");
gets(infile);
case 2 :
printf("Input the name of the output file ⟶ ");
gets(outfile);
break;
case 3 :
strcpy(infile, argv[1]);
strcpy(outfile, argv[2]);
}
/* */
/* get input data */
printf("\n\n");
sysin = fopen(infile, "r");        /* Open input file */
fgets(instring, 20, sysin); /* Read in the number of nodes */
num_nodes = atoi(instring);
printf("%d\n", num_nodes);
for(i = 0; i<num_nodes; ++i) /* Read in connection matrix */
for(j = 0; j<num_nodes; ++j) {
fgets(instring, 20, sysin);
a[i][j][0] = instring[0]; /* Only first character is */
a[i][j][1] = 0; /* needed. Second character is */
}          /* a delimiter. */
```

```
for(i = 0; i<num_nodes; ++i)        Read in age parameters
for(j = 0; j<num_nodes; ++j) {
fgets(instring, 20, sysin);
uu[i][j] = atof(instring);
}
for(i = 0; i<num_nodes; ++i)        Read in flows
for(j = 0; j<num_nodes; ++j) {
fgets(instring, 20, sysin);
qq[i][j] = atof(instring);
fclose(sysin);
/* */
/* echo input data */
sysout = fopen(outfile, "w");
for(i = 0; i<num_nodes; ++i)
for(j = 0; j<num_nodes; ++j)
printf("a(%2d,%2d)—→%c\n", i, j, a[i][j][0]);
printf("\n\n");
fprintf(sysout,"\n\n");
/* */
/* perform node removal */
for (dest = 1; dest<num_nodes; ++dest) {
/* */
/* initialize table (path matrix) */
for (i = 0; i<MAX_PATHS; ++i)
for (j = 0; j<MAX_LINKS; ++j)
table[i][j] = 0;
```

```
path = 0;
for (i = 0; i<MAX_LINKS; ++i) {
links[i] = 0;
np[i] = 0.0;
}
num_inc = 0;
/* */
printf("Source node —→0\n");
printf("Destination node —→%2d\n", dest);
/* */
/* copy a (connection matrix) to b */
num = num_nodes;
for (i = 0; i<num_nodes; ++i)
for (j = 0; j<num_nodes; ++j)
b[i][j] = a[i][j];
/* */
/* initialize heap */
heaptop = 0;
/* */
/* initialize pointers rows and columns in connection matrix */
for (i = 0; i<num; ++i) nodes[i] = i;
/* */
while (num>2) {
/* Remove row and column from connection matrix for a node */
i = 1;
if (nodes[i] == dest) ++i;
```

```
remove_node = nodes[i];
printf("\n\nRemoving node %2d\n", remove_node);
printf ("Nodes remaining:\n");
for (j = i; j<num - 1; ++j) {
nodes[j] = nodes[j + 1];
printf("%2d\n", nodes[j]);
}
-num;
/* Perform node removal on each element remaining in the */
/* connection matrix */
for (i = 0; i<num; ++i)
for (j = 0; j<num; ++j) {
new_string =& charheap[heaptop];
strcpy(new_string, b[nodes[i]][nodes[j]]);
noder(b[nodes[i]][remove_node],        /* B(i,k) * */
b[remove_node][nodes[j]],              /* B(k,j) + */
new_string);                   /* B(i,j) */
heaptop += strlen(new_string) + 1;
b[nodes[i]][nodes[j]] = new_string;
}
}
printf("\n\n");
for (i = 0; i<num; ++i)
for (j = 0; j<num; ++j) {
printf("b(%2d,%2d) —→%s\n",
nodes[i], nodes[j], b[nodes[i]][nodes[j]]);
```

```
}
/* */
fprintf(sysout, "0 to %2d ⟶ %s\n\n",
dest, b[0][dest]);
/* */
/* Create table (path matrix) */
count_string(b[0][dest], links, table, &path, np, &num_inc, inc_link);
/* */
/* Print out path matrix */
printf("\n\n\n—");
fprintf(sysout, "\n\n\n—");
for (num_links = 0; links[num_links] != 0; ++num_links) {
printf(" %c", links[num_links]);
fprintf(sysout, " %c", links[num_links]);
}
printf(" — SR        NP\n");
fprintf(sysout, " — SR        NP\n");
for (i = 0; i≤ num_links + 1; ++i) {
printf("—");
fprintf(sysout, "—");
}
printf("—\n");
fprintf(sysout, "—\n");
for (i = 0; i≤ path; ++i) {
printf("%3d —", i);
fprintf(sysout, "%3d —", i);
```

```
for (j = 0; j<num_links; ++j) {
printf("%3d", table[i][j]);
fprintf(sysout, "%3d", table[i][j]);
}
printf(" —%3d", table[i][j]);
fprintf(sysout, " —%3d", table[i][j]);
printf("        %8.4f\n", np[i]);
fprintf(sysout, "        %8.4f\n", np[i]);
}
for (i = 0; i≤ num_links + 1; ++i) {
printf("—");
fprintf(sysout, "—");
}
printf("—\n");
fprintf(sysout, "—\n");
printf("SC —");
fprintf(sysout, "SC —");
for (j = 0; j<num_links; ++j) {
printf("%3d", table[path + 1][j]);
fprintf(sysout, "%3d", table[path + 1][j]);
}
for (j = 0; j<num_inc; ++j) {
i = 0;
for (;;) {
if (table[i][inc_link[j]] == 1) break;
++i;
```

```
}
ajx[j] = pathp(inc_link[j],num_links, path, table,
table[path + 1][inc_link[j]], np[i]);
}
Q[dest] = 0.0;
U[dest] = 0.0;
printf(
" \n\nIncident links     No. of Paths     Path
param     Age param     Flow \n");
fprint(sysout
" \n\nIncident links     No. of Paths     Path
param     Age param     Flow \n");
for (i = 0; i < num_inc; ++i) {
j=0;
for(;;) {
if (links[inc_link[i]] == a[j][dest][0]) break;
++j;
}
Q[dest] += qq[j][dest];
U[dest] += uu[j][dest];
printf(
%c     %3d     %7.3f     %5.3f     %5.0f\n",
links[inc_link[i]], table[path + 1][inc_link[i]]'
ajx[i], uu[j][dest], qq[j][dest]);
fprintf(sysout
%c     %3d     %7.3f     %5.3f     %5.0f\n",
```

```
links[inc_link[i]], table[path + 1][inc_link[i]]'
ajx[i], uu[j][dest], qq[j][dest]);
}
ss[dest] = 0.0 for (i = 0; i < num_inc; ++i) {
j=0;
for(;;) {
if (links[inc_link[i]] == a[j][dest][0]) break;
++j;
}
temp = log(qq[j][dest] / (ajx[i] * Q[dest]));
ss[dest] -= (uu[j][dest] * qq[j][dest] * temp) / Q[dest];
}
printf(" \nSum of age parameters     %8.3f\n", U[dest]);
printf("Sum of flowss              %8.3f\n", Q[dest]);
printf(" Entropy of node           %8.3f\n", ss[dest]);
fprint(sysout,
" \nSum of age parameters     %8.3f\n", U[dest]);
fprint(sysout
"Sum of flowss              %8.3f\n", Q[dest]);
fprint(sysout
" Entropy of node           %8.3f\n", ss[dest]);
printf("\n\n\nHeap used %d bytes\n\n\n", heaptop);
fprintf(sysout, "\n\n\nHeap used %d bytes\n\n\n", heaptop);
}
Q0 = 0.0;
for (j = 0; j < num_nodes; ++j)
```

```
Q0 += qq[0][j];
printf("\n\nTotal flow of system    %8.2f\n", Q0);
fprintf(sysout, "\n\nTotal flow of system    %8.2f\n", Q0);
t1 = 0.0;
t2 = 0.0;
for (i = 0; i < num_nodes; ++i) {
t1 += (Q[i] * ss[i]) / Q0;
t2 += (U[i] * Q[i] * log(Q[i] / Q0 )) / Q0;
}
s = t1 - t2;
printf("Entropy of system    %8.5f\n", s);
fprintf(sysout, "Entropy of system    %8.5f\n", s);
fclose(sysout);
return;
}
/* */
/* *************** N O D E R **************** */
/* */
/* This function performs the node removal algebra */
/* */
#include "nodef.c"
/* */
void noder
(
char *a,
char *b,
```

```
char *c
)
{
int as, ae, bs, be, cs, cross, i, j;
char one;
/* */
/* Table of ASCII codes used */
/* 0 = End of string */
/* 43 = '+' */
/* 48 = '0' */
/* 49 = '1' */
/* */
as = 0;
bs = 0;
cs = 0;
while (c[cs] != 0) ++cs;
c[cs++] = 43;
one = 48;
if (c[0] == 48) cs = 0;
if (c[0] == 49) {
one = 49;
cs = 0;
}
if ((a[0] == 49) && (b[0] == 49)) {
if (cs == 0) {  c[0] = 49;
++cs;
```

```
}
} else {
if ((a[0] != 48) && (b[0] != 48)) {
for (;;) {
ae = as;
while ((a[ae] != 43) && (a[ae] != 0)) ++ae;
for (;;) {
be = bs;
while ((b[be] != 43) && (b[be] != 0)) ++be;
cross = 0;
for (i = as; i<ae; ++i)
for (j = bs; j<be; ++j)
if (a[i] == b[j]) cross = 1;
if (cross == 0) {
if (a[as] != 49)
for (i = as; i<ae ; ++i)
c[cs++] = a[i];
if (b[bs] != 49)
for (i = bs; i<be ; ++i)
c[cs++] = b[i];
}
if (b[be] == 43) {
bs = ++be;
if (cs != 0)
if (c[cs - 1] != 43) c[cs++] = 43;
} else break;
```

```
}
if (a[ae] == 43) {
bs = 0;
as = ++ae;
if (cs != 0)
if (c[cs - 1] != 43) c[cs++] = 43;
} else break;
}
}
}
if (cs == 0) {
c[cs++] = one;
} else {
if (c[cs - 1] == 43) --cs;
}
c[cs] = 0;
return;
}
/* */
/* ************ C O U N T S ****************** */
/* */
/* This function constructs the path matrix */
#include "nodef.c"
/* */
void count_string
(
```

```
char *path,
char links[MAX_LINKS],
int count[MAX_PATHS][MAX_LINKS],
int *p,
double np[MAX_PATHS],
int *num_inc,
int inc_link[MAX_PATHS]
)
{
int i, j, l, k, sum;
/* */
l = 0;
i = 0;
/* Do not perform if no paths exist in the connection matrix */
if ((path[0] != '0') —— (path[0] != '1'))
for (;;) {
if (path[i] == 0) { /* Quit at end of string */
k = 0;
for (;;) {
if (k == *num_inc) {
inc_link[k] = j;
++*num_inc;
break;
}
if (inc_link[k] == j) break;
++k;
```

```
}
break;
}
if (path[i] == '+') { /* Increase path count */
++*p; /* if plus sign is found */
k = 0;
for (;;) {
if (k == *num_inc) {
inc_link[k] = j;
++*num_inc;
break;
}
if (inc_link[k] == j) break;
++k;
}
} else {
j = 0;
for (;;) {
if (path[i] == links[j]) break; /* Is this a previous link? */
if (links[j] == 0) { /* If this is a new link */
links[j] = path[i]; /* make a new column */
break;
}
++j;
if (j>l ) l = j;
}
```

```
++count[*p][j]; /* Count the path */
}
++i; /* Move to next character */
} /* in the string */
/* Sum the rows of the path matrix */
for (i = 0; i ≤ *p; ++i) {
sum = 0;
for (j = 0; j ≤ l; ++j)
sum += count[i][j];
count[i][l + 1] = sum;
}
/* Sum the columns of the path matrix */
for (j = 0; j ≤ l; ++j) {
sum = 0;
for (i = 0; i ≤ *p; ++i)
sum += count[i][j];
count[*p + 1][j] = sum;
}
/* Calculate the NP values */
for (i = 0; i ≤ *p; ++i) {
sum = 0;
for (j = 0; j ≤ l; ++j)
if (count[i][j]) sum += count[*p + 1][j];
np[i] = ((double) sum) / ((double) count[i][l + 1]);
}
return;
```

```
}
/* */
/* ************** P A T H P ******************** */
/* */
#include <stdio.h>
#include "nodef.c"
/* */
double pathp
(
int inc_link,
int num_links,
int paths,
int table[MAX_PATHS][MAX_LINKS],
int sc,
double np
)
{
int num_rows;
static int *stable[MAX_PATHS];
int cond;
int sum1, sum2;
int tsr;
double ajx;
int i, j, k;
/* */
if ((sc>1) && (np>1)) cond = 1;
```

```
else if ((sc == 1) && (np == 1)) cond = 2;
else if ((sc == 1) && (np > 1)) cond = 3;
/* */
switch (cond) {
/* */
case 1 :
/* */
num_rows = 0;
for (i = 0; i ≤ paths; ++i)
if (table[i][inc_link] == 1)
stable[num_rows++] = table[i];
sum2 = 0;
for (j = 0; j<num_links; ++j) {
sum1 = 0;
for (i = 0; i<num_rows; ++i)
sum1 += stable[i][j];
if (sum1 ≥ 2) sum2 += -sum1;
}
tsr = 0;
for (i = 0; i<num_rows; ++i)
tsr += stable[i][num_links];
ajx = ((double) num_rows) * (1.0 - (((double)sum2) / ((double) tsr)));
break;
/* */
case 2 :
/* */
```

```
ajx = 1.0;
break;
/* */
case 3 :
i = 0;
for (;;) {
if (table[i][inc_link] == 1) break;
++i;
}
sum2 = 0;
for (j = 0; j<num_links; ++j)
sum2 += (table[paths + 1][j] - 1) * table[i][j];
tsr = 0;
for (j = 0; j ≤ paths; ++j) {
for (k = 0; k<num_links; ++k) {
if (table[j][k] * table[i][k] != 0) {
tsr += table[j][num_links];
break;
}
}
}
ajx = 1.0 - (((double) sum2) / ((double) tsr));
}
/* */
return(ajx);
}
```

Appendix C

Proof of Reliability Operator [ ]*

This appendix gives the proof for the operator [ ]* as used to compute the reliability (NPR) from the reliability block diagram. This can be found in the paper by Kim et al. (1972).

The method is best explained by its application to a simple network. Consider the network whose series-parallel system for node 1 paired to node 4 is given in Figure C.1 (top figure) the corresponding reliability block diagram is shown (bottom figure). The components in the paths are B_{12} and B_{24} in path a and B_{13} and B_{24} in path b. Component B_{24} is common to both paths hence the two paths are not independent. Let component B_{24} of path b be denoted by B'_{24}. Let E_{ij} and p_{ij} respectively denote the event of the successful operation of B_{ij} and the probability of E_{ij}. The the reliability of the system is:

$$P_s = Pr[(E_{12} \cap E_{24}) \cup (E_{13} \cap E'_{24})] \tag{C.1}$$

$$P_s = Pr[(E_{12} \cap E_{24})] + Pr[(E_{13} \cap E'_{24})] - Pr[E_{12} \cap E_{13} \cap E_{24} \cap E'_{24}] \tag{C.2}$$

If the components in the paths are assumed to fail independently, then Equation C.2 becomes:

$$P_s = p_{12}p_{24} + p_{13}p'_{24} - p_{12}p_{13}p_{24}p'_{24} \tag{C.3}$$

However, the failures of components B_{24} and B'_{24} are dependent because they are the same component. Hence the assumption of independence between the two paths is incorrect. To correct this, the term $p_{24}p'_{24}$ in Equation C.3 must be replaced by $Pr[E_{24} \cap E'_{24}]$. However, the following relationship holds;

$$Pr[E_{24} \cap E'_{24}] = p_{24} \neq p_{24}^2 \qquad (C.4)$$

This expression arises because E_{24} and E'_{24} are events denoting the successful operation of components B_{24}. Hence the reliability function given by Equation C.2 reduces to:

$$P_s = p_{12}p_{24} + p_{13}p_{24} - p_{12}p_{13}p_{24} \qquad (C.5)$$

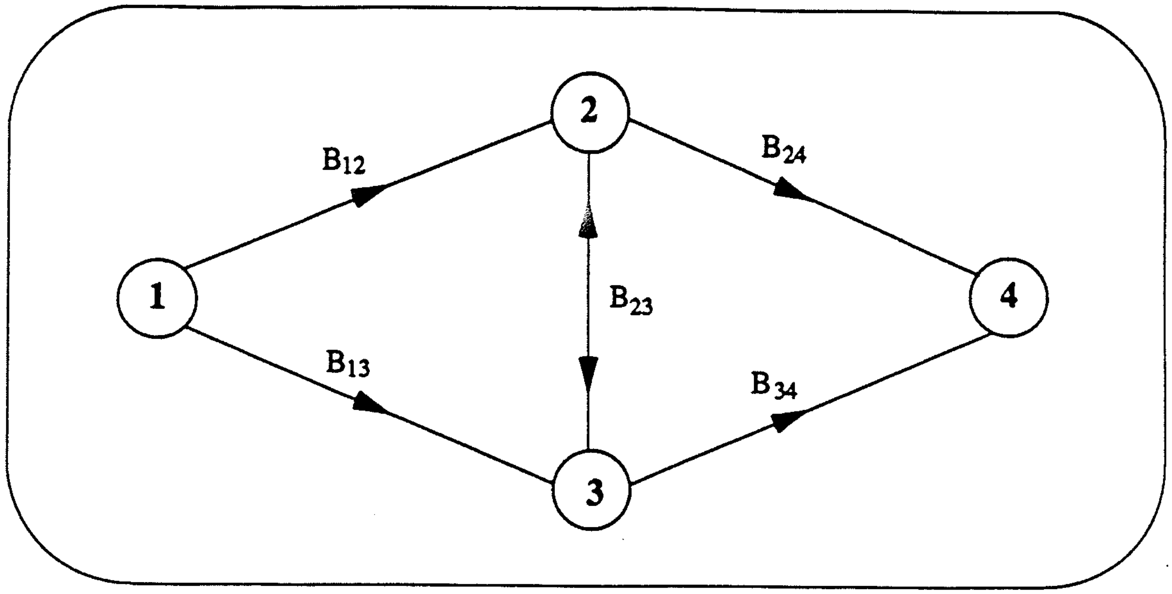
On the other hand, applying the series-parallel reduction formulae of Equations 4.2 and 4.3 to the reduced system in Figure C.2, the system reliability is obtained as:

$$P_s = p_{24}[1 - (1 - p_{12})(1 - p_{13})] \qquad (C.6)$$

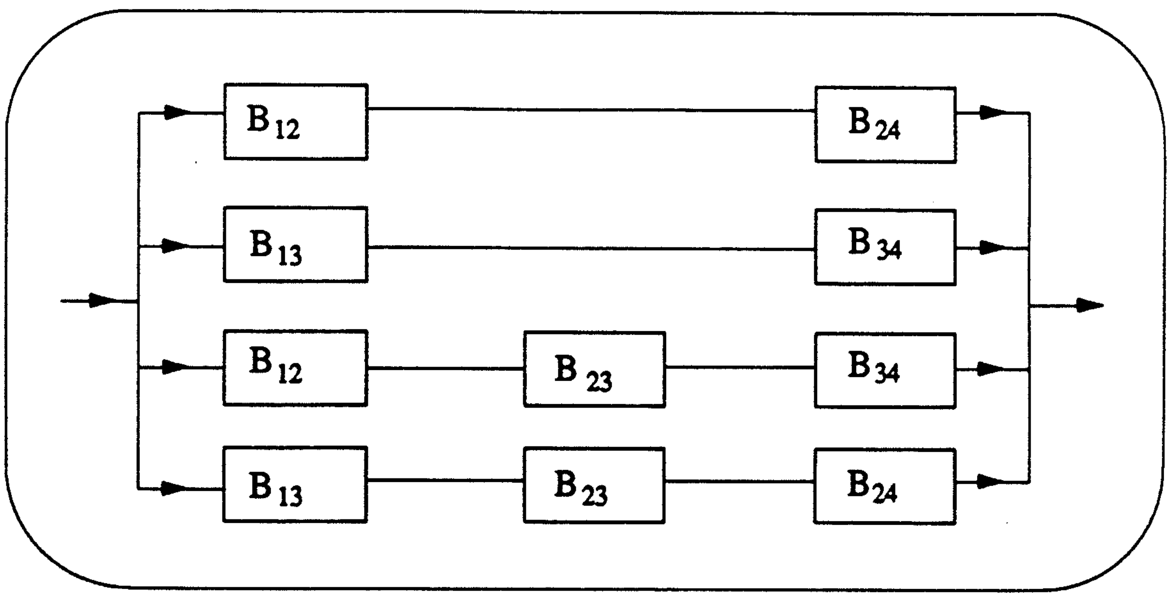
When expanded Equation C.6 becomes:

$$P_s = p_{12}p_{24} + p_{13}p_{24} - p_{12}p_{13}p_{24} \qquad (C.7)$$

giving the same result as that in Equation C.5 obtained using the operator.



Non series-parallel network



Equivalent reliability block diagram

Figure C.1: **A Network and its Equivalent Reliability Block**

Appendix D

Data for Model Application and the Results

D.1 Data for Layout 3 in Figure 4.2

The following are the values of the constants and coefficients used for the application of Model B in Chapter 5 on Layout 3 in Figure 4.2. The layout showing the link numbers are given in Figure D.1.

Capital cost coefficient, CST_{ij} = Given by Equation 5.1

Energy cost coefficient, ε =\$ 10^{-3} per m^3/h.

Hazen-William Coefficient, C_{ij} = 100 for all links

Height of point above datum, Z= 0.0 for all nodes.

Nodal demands = Given in Table 4.2

Minimum Pressure heads = Given in Table 4.2

Age factor parameter, u_{ij} = 1.0 for all links.

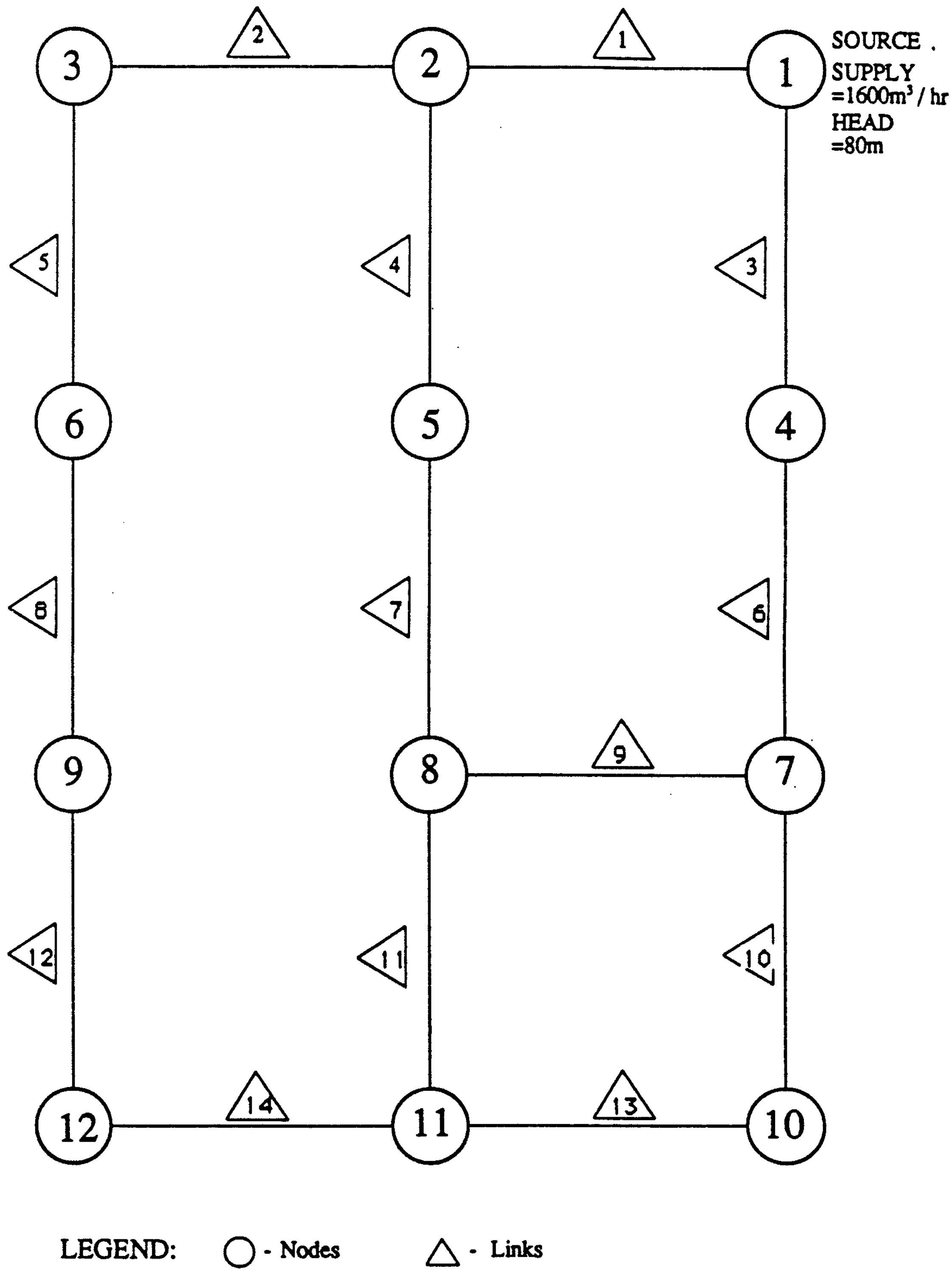


Figure D.1: **Layout 3 in Figure 4.2**

D.2 Results of Application of Model B to Layout

The following tables contain the result of the runs of Model B on Layout 3 of Figure 4.2. The continuous pipe diameter solution obtained directly from the model is then converted into two adjacent commercially available pipe sizes and their corresponding lengths.

Table D.1. Results of Run 1

	Pipe Diameters (m) and Lengths (m)				
	Model Solution	Equivalent Commercial Sizes			
Link	Diameter	Diameter	Length	Diameter	Length
1	0.373	0.36	625	0.41	375
2	0.316	0.30	782	0.36	218
3	0.338	0.30	532	0.36	468
4	0.260	0.25	835	0.30	165
5	0.276	0.25	586	0.30	414
6	0.327	0.30	567	0.36	433
7	0.200	0.20	1000		
8	0.264	0.25	816	0.30	184
9	0.200	0.20	1000		
10	0.223	0.20	517	0.25	483
11	0.186	0.15	436	0.20	564
12	0.143	0.10	122	0.15	878
13	0.186	0.15	361	0.20	639
14	0.143	0.10	128	0.15	872

Table D.2. Results of Run 2

Link	Pipe Diameters (m) and Lengths (m)				
	Model Solution	Equivalent Commercial Sizes			
	Diameter	Diameter	Length	Diameter	Length
1	0.379	0.36	608	0.41	392
2	0.313	0.30	795	0.36	205
3	0.339	0.30	529	0.36	471
4	0.251	0.25	917	0.30	83
5	0.277	0.25	573	0.30	427
6	0.320	0.30	615	0.36	385
7	0.195	0.15	83	0.20	917
8	0.261	0.25	864	0.30	136
9	0.195	0.15	81	0.20	919
10	0.221	0.20	501	0.25	499
11	0.183	0.15	463	0.20	537
12	0.154	0.15	918	0.20	82
13	0.183	0.15	388	0.20	612
14	0.152	0.15	955	0.20	45

Table D.3. Results of Run 3

Link	Pipe Diameters (m) and Lengths (m)				
	Model Solution	Equivalent Commercial Sizes			
	Diameter	Diameter	Length	Diameter	Length
1	0.386	0.36	585	0.41	415
2	0.308	0.30	899	0.36	101
3	0.340	0.30	481	0.36	519
4	0.234	0.20	430	0.25	570
5	0.278	0.25	515	0.30	485
6	0.317	0.30	705	0.36	295
7	0.190	0.15	133	0.20	867
8	0.253	0.25	912	0.30	88
9	0.186	0.15	184	0.20	816
10	0.220	0.20	575	0.25	425
11	0.181	0.15	513	0.20	487
12	0.166	0.15	805	0.20	195
13	0.181	0.15	415	0.20	585
14	0.157	0.15	910	0.20	90

Table D.4. Results of Run 4

Link	Pipe Diameters (m) and Lengths (m)				
	Model Solution	Equivalent Commercial Sizes			
	Diameter	Diameter	Length	Diameter	Length
1	0.387	0.36	543	0.41	457
2	0.305	0.30	915	0.36	85
3	0.345	0.30	422	0.36	578
4	0.231	0.20	482	0.25	518
5	0.275	0.25	563	0.30	437
6	0.315	0.30	731	0.36	269
7	0.188	0.15	175	0.20	825
8	0.250	0.25	1000		
9	0.186	0.15	214	0.20	786
10	0.218	0.20	618	0.25	382
11	0.179	0.15	571	0.20	429
12	0.169	0.15	783	0.20	217
13	0.177	0.15	485	0.20	515
14	0.140	0.10	925	0.15	75

Table D.5. Results of Run 5

Link	Pipe Diameters (m) and Lengths (m)				
	Model Solution	Equivalent Commercial Sizes			
	Diameter	Diameter	Length	Diameter	Length
1	0.388	0.36	512	0.41	488
2	0.311	0.30	893	0.36	107
3	0.341	0.30	442	0.36	558
4	0.230	0.20	501	0.25	499
5	0.279	0.25	511	0.30	489
6	0.312	0.30	752	0.36	248
7	0.186	0.15	228	0.20	772
8	0.259	0.25	915	0.30	85
9	0.186	0.15	235	0.20	765
10	0.211	0.20	637	0.25	363
11	0.175	0.15	603	0.20	397
12	0.181	0.15	728	0.20	272
13	0.171	0.15	498	0.20	502
14	0.139	0.10	908	0.15	92

Table D.6. Results of Run 6

Link	Pipe Diameters (m) and Lengths (m)				
	Model Solution	Equivalent Commercial Sizes			
	Diameter	Diameter	Length	Diameter	Length
1	0.389	0.36	498	0.41	502
2	0.315	0.30	871	0.36	129
3	0.335	0.30	464	0.36	536
4	0.228	0.25	522	0.30	478
5	0.285	0.25	493	0.30	507
6	0.107	0.30	769	0.36	231
7	0.182	0.15	243	0.20	757
8	0.262	0.25	889	0.30	111
9	0.185	0.15	250	0.20	750
10	0.210	0.20	652	0.25	348
11	0.171	0.15	623	0.20	377
12	0.187	0.15	702	0.20	298
13	0.169	0.15	512	0.20	488
14	0.135	0.10	917	0.15	83

Table D.7. Results of Run 7

Link	Pipe Diameters (m) and Lengths (m)				
	Model Solution	Equivalent Commercial Sizes			
	Diameter	Diameter	Length	Diameter	Length
1	0.391	0.36	473	0.41	527
2	0.319	0.30	805	0.36	195
3	0.331	0.30	478	0.36	522
4	0.226	0.20	542	0.25	458
5	0.290	0.25	468	0.30	532
6	0.308	0.30	778	0.36	222
7	0.180	0.15	255	0.20	745
8	0.267	0.25	867	0.30	133
9	0.184	0.15	265	0.20	735
10	0.208	0.20	668	0.25	332
11	0.168	0.15	639	0.20	361
12	0.193	0.15	122	0.20	878
13	0.165	0.15	526	0.20	474
14	0.000	-	-	-	-

Table D.8. Results of Run 8

Link	Pipe Diameters (m) and Lengths (m)				
	Model Solution	Equivalent Commercial Sizes			
	Diameter	Diameter	Length	Diameter	Length
1	0.383	0.36	491	0.41	509
2	0.319	0.30	805	0.36	195
3	0.338	0.30	458	0.36	542
4	0.201	0.20	560	0.25	440
5	0.291	0.25	451	0.30	549
6	0.319	0.30	763	0.36	237
7	0.175	0.15	508	0.20	492
8	0.266	0.25	876	0.30	124
9	0.198	0.15	251	0.20	749
10	0.207	0.20	817	0.25	183
11	0.168	0.15	639	0.20	361
12	0.193	0.15	878	0.20	122
13	0.163	0.15	505	0.20	495
14	0.000	-	-	-	-

Table D.9. Results of Run 9

Link	Pipe Diameters (m) and Lengths (m)				
	Model Solution	Equivalent Commercial Sizes			
	Diameter	Diameter	Length	Diameter	Length
1	0.375	0.36	512	0.41	488
2	0.320	0.30	789	0.36	211
3	0.345	0.30	433	0.36	567
4	0.192	0.15	165	0.20	835
5	0.291	0.25	251	0.30	749
6	0.326	0.30	748	0.36	252
7	0.121	0.10	378	0.15	622
8	0.267	0.25	858	0.30	142
9	0.215	0.20	662	0.25	338
10	0.206	0.20	891	0.25	109
11	0.168	0.15	639	0.20	361
12	0.194	0.15	122	0.20	878
13	0.161	0.15	639	0.20	361
14	0.000	-	-	-	-

Table D.10. Results of Run 10

Link	Pipe Diameters (m) and Lengths (m)				
	Model Solution	Equivalent Commercial Sizes			
	Diameter	Diameter	Length	Diameter	Length
1	0.368	0.36	825	0.41	175
2	0.320	0.30	789	0.36	211
3	0.353	0.30	420	0.36	580
4	0.173	0.15	589	0.20	411
5	0.292	0.25	186	0.30	814
6	0.332	0.30	567	0.36	433
7	0.000	-	-	-	-
8	0.268	0.25	835	0.30	165
9	0.231	0.20	329	0.25	671
10	0.205	0.20	817	0.25	183
11	0.167	0.15	653	0.20	347
12	0.195	0.15	122	0.20	878
13	0.161	0.15	361	0.20	639
14	0.000	-	-	-	-

Table D.11. Results of Run 11

Link	Pipe Diameters (m) and Lengths (m)				
	Model Solution	Equivalent Commercial Sizes			
	Diameter	Diameter	Length	Diameter	Length
1	0.365	0.36	912	0.41	82
2	0.320	0.30	789	0.36	211
3	0.353	0.30	420	0.36	580
4	0.165	0.15	600	0.20	400
5	0.292	0.25	151	0.30	849
6	0.332	0.30	548	0.36	452
7	0.000	-	-	-	-
8	0.269	0.25	758	0.30	242
9	0.215	0.20	645	0.25	355
10	0.228	0.20	593	0.25	407
11	0.000	-	-	-	-
12	0.195	0.15	182	0.20	818
13	0.185	0.15	361	0.20	639
14	0.000	-	-	-	-

Table D.12. Results of Run 12

Link	Pipe Diameters (m) and Lengths (m)				
	Model Solution	Equivalent Commercial Sizes			
	Diameter	Diameter	Length	Diameter	Length
1	0.364	0.36	820	0.41	180
2	0.321	0.30	670	0.36	330
3	0.353	0.30	123	0.36	877
4	0.158	0.15	889	0.20	111
5	0.292	0.25	176	0.30	824
6	0.333	0.30	433	0.36	567
7	0.000	-	-	-	-
8	0.269	0.25	605	0.30	395
9	0.196	0.15	101	0.20	899
10	0.239	0.20	322	0.25	678
11	0.000	-	-	-	-
12	0.195	0.15	120	0.20	880
13	0.206	0.20	918	0.20	82
14	0.000	-	-	-	-