Erroneous Reasoning on Probability: Are Humans Poor Judges of the Probability of Dice Outcomes?

by

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A thesis submitted to the Faculty of Graduate Studies

in partial fulfillment of the requirements for the degree of Master of Arts

Department of Psychology

University of Manitoba

Winnipeg, Manitoba

2003

THE UNIVERSITY OF MANITOBA

FACULTY OF GRADUATE STUDIES
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ERRONEOUS REASONING ON PROBABILITY: ARE HUMANS POOR JUDGES OF THE PROBABILITY OF DICE OUTCOMES?

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Abstract

Research suggests that humans are poor statistical reasoners. When judging the probability of dice outcomes, people often ignore the distinguishability of the dice, and consider only combinations of dice, not permutations. One hundred and fourteen university students completed a Dice Outcome Questionnaire presenting the same two-dice problems to be solved (a) through intuition, (b) then through calculation prior to a demonstration, and (c) finally through calculation following the demonstration. Later, participants attempted to solve an additional problem to test generalization from two- to three-dice conditions and then completed a Demographics Questionnaire, which included a question about brain area involvement in solving probability problems. Frequency of correct answers was analyzed by a repeated measures three level ANOVA. Solution-orientation had a significant overall effect, and pairwise comparisons revealed no significant difference in the number of correct answers between intuition and pre-demonstration, but post-demonstration answers were more often correct than answers given in either of the other two orientations. These results suggest that students benefit from a demonstration of probability problems solutions. In addition, results of chi square analyses suggest that participants who get the highest number of correct answers to post-demonstration two-dice problems get the correct answer to a single post-demonstration three-dice problem but fail to correctly attribute problem solving activity to the left frontal brain area. These results suggest that the most capable introductory psychology students generalize from two- to three-dice problems, but lack the necessary physiological information to correctly attribute probability solving to the left frontal lobe. Implications of these findings were discussed.

Acknowledgements

The author would like to acknowledge with gratitude those who contributed to the completion of the thesis. First, I thank my advisor, Dr. Jim Nickels, for his advice, instruction, and valued comments and direction in the development of this thesis. Next, I would like to acknowledge my committee members. I thank Dr. Tammy Ivanco, whose suggestions contributed to the final version of the questionnaires developed for this project. Also, I thank Dr. Tim Schroeder for his valuable criticisms and suggestions.

The author would also like to thank two fellow graduate students in the Department of Psychology for their help. First, I want to thank Matthew J. Derksen for the time he gave to help in the collection of data for this project. Second, I thank Terri Lynn McKay for her suggestions on statistical analysis.

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Introduction

Many psychologists use probability theory in most of their research. Rarely, however, do they review historical incidents involving probability theory. The present study is an exception. Examining the history of probability theory, one will often come across the name of a seventeenth century French literary writer named Antoine Gombaud (or Gombault) Chevalier de Mere, sieur de Baussay (1607 - 1684). Often, he is mentioned because he had proposed dice problems to the noted mathematician Blaise Pascal. These dice problems may have arisen through de Mere's experiences at the gambling table. Many descriptions of de Mere state that he was a gambler - an accusation he would most likely have denied (Ore, 1960). De Mere was a French philosopher and he wrote literature. He considered himself an example of courtly behavior and sociability was his ideal; he believed that the key to good conversation was that it should be pleasant. He was charming, had good taste, and his conversation skills made him a popular guest among the elite of Paris. De Mere had received a classical education, and served briefly in the army. He spent half of his time at his estate in Poitou and the rest at the court in Paris. De Mere was an arbiter of conflicts and an advisor to King Louis XIV (Ore, 1960). In his writings, he explained the ethics of a noble life and emphasized pleasant consideration of others (de Mere, 1687, as cited in "Livre Rare Book" (2002), www.livre-rare-book.com/Matieres/dd/3940.html; Boudhors,1930).

De Mere's social gambling led him to several paradoxes probably well known even before his time. He presented these problems to the noted mathematician Blaise Pascal (1623 - 1662), who is one of the founders of probability theory. Pascal as a child would attend meetings of the Academie Libres with his father. When Pascal was sixteen, he published a treatise on conic sections. Two years later, he developed a calculating machine. Pascal was a scientist of math and physics, and he wrote about such topics in Lettres Provinciales and the Pensees. In 1651 or 1652, Pascal accompanied the Duke of Roannez on a trip to Poitou. De Mere was present on this trip, and this is how Pascal and de Mere first met (Ore, 1960). The two became acquainted through writing and discussing issues and they may have even gambled together. Pascal wrote about gamblers in the Pensees (Levi, 1995). During their exchanges, de Mere proposed probability problems to Pascal.

Erroneous Reasoning on Probability: de Mere

The first problem of de Mere to be discussed was uncovered in games of chance using dice. The paradox is outlined as follows: Two players use one die with one player betting that a 6 will appear within four throws of the die and the other player betting against this (Von Mises, 1939; Gani, 1982; Dale, 1998). De Mere noticed that there was a greater chance of the positive result, (that one 6 will likely appear within four throws of the die). A variation of this game uses two dice. Here, the pair of dice is thrown, and the bet is that at least one double 6 will occur within 24 throws of the dice. This time de Mere noticed that the win went to the player betting against a double six within 24 throws. These results confused de Mere and led him to believe that the arithmetic was wrong (Ore, 1960). De Mere's reasoning was that if the throw of one die can have six different results, then the throwing of 2 dice can have thirty-six different results, six times as many as with one die. Rolling a 6 is one of the six different possibilities in games with one die; rolling a 6-6 is one of the thirty-six probabilities with two dice. De Mere believed that the chance of getting one six when throwing one die four times is the same chance as getting 6-6 when throwing two dice 24 times. Through observation of actual dice-throwing, de Mere found he was wrong, because 25 throws were needed. He sought out Pascal's council. Pascal solved it for him and corresponded with Fermat, another noted scientist of that era, who solved the problem in a manner similar to that of Pascal (Szekely, 1986). The paradox was solved through a series of logical analyses and mathematical calculations, two topics that go beyond the scope of the present research.

The second problem that de Mere posed to Pascal involves a particular game in which two players each need a given number of points in order to win. If the game is, however, interrupted or ended before the required number of points is achieved, he wondered how the prize can be divided up according to the current score situation (Von Mises, 1964). Pascal solved this problem through the use of binomial coefficients, as illustrated by his Arithmetical Triangle (Ore, 1960). Pascal gave this problem to Fermat, who solved it the same way as Pascal had (Todhunter, 1949).

The third problem of de Mere (Kocherlabota, 1989), and the one that will be the focus of this research, is as follows: "In the long run, which number is more likely, the sum of 11 or the sum of 12, when throwing three dice?" There appear to be six ways a sum of 11 can occur: 6-4-1, 6-3-2, 5-5-1, 5-4-2, 5-3-3, and 4-4-3; and six ways a sum of 12 can occur: 6-5-1, 6-4-2, 6-3-3, 5-5-2, 5-4-3, and 4-4-4. Therefore, one might assume that sums of 11 and 12 have an equal chance of occurring, according to probability theory. de Mere noticed, however, that when he observed the actual throwing of three dice, the results did not turn out as he expected because 11 occurred more often. Later, de Mere presented this problem to Pascal.

For de Mere, each sum had six combinations, so he reasoned that each sum would have equal probabilities. To solve this paradox, however, the order of the cast must be taken into consideration. Otherwise, not all results will be equally probable (Szekely, 1986, p. 1-9). Taking the order of casting into consideration (permutations instead of combinations), 11 can be thrown 27 different ways with three dice; and 12 can be thrown 25 different ways with three dice (see Appendix A). Thus, the chance of 11 is greater than the chance of 12. Classical probability can be applied to this problem once the dice are considered distinguishable, that is, when the outcomes are viewed as ordered triplets. Only then can one talk about or appreciate the likelihood of occurrences.

In other words, a major flaw in thinking that creates this dice paradox is that one may consider the dice to be indistinguishable, that is, order information is ignored, and the dice are not considered different from each other (i.e., 6-4-1 = 6-1-4 = 4-6-1 = 4-1-6 = 1-6-4 = 1-4-6). Thus, the classical description of probability cannot be applied to it (Kocherlabota, 1989). The nature of the underlying problem is threefold: First, each die is distinguishable (a separate entity because order information is important). Second, often in nature, empirical results do not match theoretical expectations, so in choosing a probability model for describing natural phenomenon, one's everyday notions about the world are not sufficient (Kocherlabota, 1989). Third, one must consider the order of the cast (permutations) to ensure that all results are equally probable.

Erroneous Reasoning on Probability: Kahneman, Tversky, and Others

More recent research has demonstrated that humans are not particularly good at "probability calculations." Supposedly, errors in probability thinking are not limited to naive persons for "they are also found in the intuitive judgements of sophisticated psychologists." (Kahneman & Tversky, 1972, p. 433). Kahneman and Tversky (1972) published a paper examining how individuals evaluate the probabilities of uncertain events, especially in the context of probability learning and intuitive statistics. Kahneman and Tversky cite previous research that found that people do not follow principles of probability theory when judging the likelihood of uncertain events. These laws of probability and chance are not intuitive, nor are they always easy to apply. Many people use subjective probability, which is defined as the estimate of the probability of an event given by a subject or inferred from her or his behavior. Subjective probability judgements may lead to errors (e.g., heuristics) and are hard to eliminate (Kahneman & Tversky, 1972). Kahneman and Tversky (1972) investigated the use of heuristics instead of probability theory. Specifically, they examined the *representative* heuristic. Using this particular heuristic, probabilities of events are evaluated by the extent to which the events have similar properties to their parent population, and reflect the salient features of the process by which they are generated. The authors hypothesized that an event A is considered more probable than event B whenever A seems more representative than B. For example, the gender birth order of GBGBBG is judged as more probable than the birth order of BGBBBG, because the second event does not reflect the actual proportion of girls and boys in the population. Both examples can be correct, since each birth has a 50 % chance of resulting in either a boy or a girl and each birth sex is not influenced by the sex of the previous child. Ordering events by their subjective probabilities coincides with their ordering by representativeness. This hypothesis was tested in a questionnaire format. The results indicated that both high school and university students judged an event to be more likely than another, if its outcomes were more similar to the population it was drawn from. Students are not the only group of people to make intuitive mistakes. Experienced psychologists view statistical significance as a representation of scientific truth (Kahneman & Tversky, 1972). When one finds a significant result in a sample, it is expected to represent a real effect in the population, a belief which occurs with disregard for sample size. Analyses need an appropriate sample size to ensure that the study has sufficient power to reject the null hypothesis when it is false. Also, if an event appears more random, it will be judged as more likely. More representative events are given higher probabilities. Equally representative events are given equal probabilities.

When people are assessing the probability of uncertain events or quantities, they often make mistakes by using another heuristic called *availability*. If an event's likelihood is assessed by frequency or by the ease with which it is easily brought to mind or imagined, the individual is using the availability heuristic. For example, when assessing the likelihood of passing a math test, one may recall other easily retrieved instances of math tests. Representative and availability heuristics can be useful; however, they can also lead to systematic and severe errors. Using heuristics, people will ignore some factors that should be affecting probability judgements, such as the prior probability or base-rate frequency of outcomes. Another common mistake people make is the misconception of the meaning of chance (Tversky & Kahneman, 1974). The misconception of chance is that it is often viewed as a self-correcting process in which a deviation in one direction induces a deviation in the opposite direction to restore equilibrium. A popular example of this is the *gambler's fallacy*, in which the probability of a future event is mistakenly thought to be affected by past results. For example, a couple which had three girls in a row would expect the next pregnancy to be a boy. Often events have independent probabilities that are not affected by past results. This misconception of chance is found in naive subjects as well as in the statistical intuitions of experienced research psychologists (Tversky & Kahneman, 1971) which may lead to incorrect interpretations of data.

Erroneous Reasoning on Probability: Current Status

Psychologists have been arguing for some time about the definition of probability (Cosmides & Tooby, 1996). The Bayesian definition of probability states that probabilities refer to subjective degrees of confidence. The frequentist definition of probability is that probabilities refer to the frequencies of events in the world. Much of the literature in heuristics and biases concluded that humans do not use Bayesian probability when examining evidence (Kahneman & Tversky, 1972). Through a series of experiments examining use of Bayesian versus frequentist probability, however, frequentist problems can elicit Bayesian reasoning with only slight changes in the wording of the question (Cosmides & Tooby, 1996). This implies that humans may utilize a calculus of probability, not just rule of thumb heuristics. Often in the past, whenever a clash between intuition and probability theory occurred, the theory was considered wrong, not the intuition (Cosmides & Tooby, 1996). By the 1970s, however, whenever a clash between intuition and probability theory occurred, it was assumed that the intuition was wrong, not the probability theory. Cosmides and Tooby (1996) challenged the assumption that intuition is flawed; they present the possibility that intuition can have a sophisticated logic.

Much research suggests that statistically naive subjects draw incorrect probability inferences (Girotto & Gonzalez, 2001). Even experienced psychologists with training in statistics can draw incorrect probability inferences (Tversky & Kahneman, 1971). Originally these errors were assumed to be due to the use of heuristics instead of statistical probability reasoning. More recently, Cosmides and Tooby (1996) suggest that errors in probability reasoning may result because the probability information in the question is not presented in a manner similar to how information is acquired in natural settings. With this information Girotto and Gonzalez (2001) analyzed possible sources of error in probability reasoning problems. Next, Girotto and Gonzalez (2001) demonstrated that naive subjects solve problems when they can apply an informal principle but do not solve problems when they cannot use the principle. Finally, the implications of the results of the study allows for alternative views on the issue of human probabilistic reasoning.

Girotto and Gonzalez (2001) hypothesize that the form of the question and the structure of the problem information affect probability inferences. The usual problems examined require subjects to draw probability inferences on the basis of probability data. Newer versions of the problems require subjects to draw statistical inferences on the basis of statistical data. The two types of problems provide both similar data and similar questions; the difference lies in the presentation of the problem; probability (Bayesian) versus statistical (or frequentist) form. The results demonstrated that subjects performed better when presented with the frequency problem form than when presented with the probability problem form; 46% gave the correct answer compared to 16%, respectively (Girotto & Gonzalez, 2001).

Hansen and Helgeson (1996) examined the effects of statistical training on choices under uncertainty. The authors cite reports that show statistical training influences the way people reason about uncertain events in everyday life. The authors examined differences between statistically trained versus naive participants in strategies and preferences used to solve probability problems. With less statistical training, one would have a less involved method of decision making than statistically experienced people. When information on probabilities is lacking, decisions would be simplified by focusing on a single outcome. Hansen and Helgeson (1996) examined four hypotheses. The first was that statistically naive decision makers will prefer answers with minimal loss more so than the statistically experienced. Second, statistically naive decision makers (a) process decision information in a less compensatory manner, (b) spend more time on loss-related information, (c) access less information, and (d) spend less time arriving at a decision, compared to statistically experienced decision makers. The third hypothesis was that after statistically naive participants receive outcome distribution information, they prefer more risky alternatives as do the statistically experienced. The last hypothesis examined was that statistically naive participants receiving outcome distribution information (a) process information in the same compensatory manner, (b) spend similar amounts of time on gain and loss information, (c) access the same amount of information, and (d) spend the same amount of time making a decision as do statistically experienced participants.

The results supported the first hypothesis and the first three parts (a-c) of the second hypothesis (Hansen & Helgeson, 1996). Both statistically experienced and naive participants took about the same amount of time to solve the problems. The third hypothesis was not supported. Once statistically naive participants received outcome distribution information, they did not prefer more risky answers as do statistically experienced participants. There were no significant simple effects for the fourth hypothesis. There were, however, significant interaction effects between experience and distributional cue. Introducing the distributional cue made the statistically naive individuals approach more distributional, preferring risky alternatives more often compared to before they received the cue. Also, they used a more compensatory strategy similar to the statistically experienced participants. The implications of this research is that given appropriate cues, statistically naive decision makers perform similarly to statistically experienced decision makers. There are differences between these two types of people, with statistical training providing some advantage in solving these probability problems.

Brase, Cosmides, and Tooby (1998) state that "human thought processes are rational to the extent that they produce answers that conform to the strictures of normative theories drawn from mathematics, probability theory, or logic" (Brase et al, 1998, p. 3). If so, then the rational mind should be equipped with computational mechanisms that include normative principles drawn from probability theory and apply this to problems requiring statistical inference. Most research to date finds that human reasoning abilities are full of errors - heuristics, biases, and fallacious principles that violate the rules of mathematics or probability theory. These ideas led researchers to believe that the mind's reasoning faculties are full of errors, that cognitive structures have "mental limitations" (Brase et al., 1998). These limitations prevent people from using rational strategies, so heuristics are used because they are easier. Brase et al. (1998) cite many studies, however, demonstrating that nonhuman animals such as bumblebees and birds, who have small nervous systems, can make judgements under uncertainty during foraging that manifests as a well-calibrated statistical induction that human brains were considered too limited to do (Brase et al., 1998). The authors suggest this is so because the nonhuman animals were tested in an ecologically valid context, whereas humans were not tested in an ecologically valid context. Human reasoners' performance on probability problems are sensitive to the format that information is presented in and the kind of answers asked for. Presenting information in frequencies rather than in proportions or probabilities of single events improves human decision makers' performance. This information suggests that humans and other animals have inductive reasoning mechanisms that hold certain principles, however, their design requires a certain context (e.g., frequencies rather than probabilities) to perform properly. The authors stress the importance of context, that the human mind evolved to survive in a particular environment that presented information in a particular way, and that is why humans have trouble with problems requiring out of context information.

Shiloh, Salton, and Sharabi (2002) examined how individual differences in thinking styles influence the use of heuristics for judgements under uncertainty. Their results demonstrated that people with a high need for cognition - time to think and enjoyment of thinking - and have a low faith in intuition are more likely to make normative-statistical judgements, and avoid biases such as the gambler's fallacy. People who have great faith in intuition and a low need for cognition tend to make more heuristic judgements than did the high need for cognition group. In addition, Brugger, Regard, and Landis (1990) reported that belief in the paranormal is associated with errors in probability judgement. Undergraduates had to choose one of three possible answers regarding a game of pure chance using dice. Students had to choose which of two events was more probable, or if both events had an equally likelihood of occurring. Students who believed in extrasensory perception (ESP) attributed more personal involvement over randomly determined processes and made more errors in probability judgements than did students who did not believe in ESP. These results support the notion that there are individual differences in judgements under uncertainty.

Neural Substrates of Logical Reasoning

Not much is understood about the neural substrates of logical thinking. Having access to technology such as positron emission tomography (PET) and functional magnetic resonance imaging (fMRI) allows researchers to view the living brain as it is engaged in various activities, including cognitive tasks. PET is the use of a device that reveals the localization of the radioactive tracer in the living brain. PET measures metabolic activity of the brain, which is indicative of current brain activity (Carlson, 2001). An MRI images the brain by the interaction between radiowaves and strong magnetic fields. The MRI passes a strong magnetic field through a person's head. With this magnetic field, atoms in molecules in the head spin with a particular orientation. A radio frequency wave that is passed through the body at this time causes these spinning atoms to emit their own radio waves. Different molecules emit different energy at different frequencies. MRI's are tuned to detect radiation from hydrogen molecules, which exist in different concentrations in different brain regions. This radio wave information is then used to image the brain. An fMRI is a modified MRI procedure that permits the measurement of regional metabolism in the brain (Carlson, 2001). FMRI's detect oxygen levels in the brain's blood vessels before, during, and after working on a particular task.

Houde, Zago, Mellet, Moutier, Pineau, Mazoyer, and Tzourio-Mazoyer (2000) did PET scans of the brains of participants performing a logic task. First, participants performed the task as they would without instruction. Much of this performance was based on perceptual biases and other errors of reasoning mentioned above. Next, participants were trained to respond logically and then perform the same logic task. PET scans revealed that as participants shifted from a perceptual bias to a logical response, there was a shift in brain activity from the posterior brain (occipital and parietal lobes, which mediate sensory processing and responding) to the left prefrontal network, which includes the dentate gyrus, hippocampus, Broca's area, the anterior insula and the pre-sensory-motor association cortex (pre-SMA). The frontal cortex is associated with higher cognitive functions. The dentate gyrus is the site of activation during object and spatial memory, and cognitive inhibition of perceptual responses; Broca's area and the anterior insula are structures involved in so-called *inner speech* (Houde et al., 2000). The pre-SMA activation suggested to the authors a state of readiness to apply a motor response (Houde et al., 2000). Another study using fMRI to image the brain as participants answered mathematical reasoning problems found increased activity in bilateral frontal areas, similar regions to that mentioned above, and are involved in cognitive processing (Prabhakaran, Rypma, & Gabriell, 2001).

Parsons and Osherson (2001) used PET to view the neuroanatomy activated as participants solved deductive and probabilistic reasoning problems. Differences in instructions to the participants elicited either deductive reasoning or probabilistic reasoning. Deductive reasoning activates right brain areas, whereas probabilistic reasoning activates left brain areas including the inferior frontal, posterior cingulate, parahippocampal, medial temporal, and superior and medial prefrontal cortices. These areas mediate cognitive tasks and some memory tasks (Parsons and Osherson, 2001). The above data suggest that the neural substrates of logical reasoning reside in the prefrontal cortex and surrounding areas of the human brain. Knowledge and understanding of the neural substrates of logical reasoning is not common, however (Houde et al., 2000), so people may underestimate the importance of the left side of the frontal lobe for solving probability problems involving dice.

Hypotheses

Given the above information, it was hypothesized that (a) participants who try to solve probability problems through intuition will give fewer correct answers than those who try to solve such problems through calculation, (b) participants who try to solve probability problems through calculation but before a demonstration of how to do the problems will give fewer correct answers than those who try to solve such problems through calculation but after the demonstration, (c) participants who give the highest number of correct answers on post-demonstration two-dice problems will be more likely to give a correct answer (i.e., to generalize) to a single three-dice problem, and (d) participants will mistakenly attribute responsibility for probability-solving activity to areas of the brain other than the left frontal lobe.

Method

Participants

Participants were drawn from the University of Manitoba's Introductory Psychology subject pool. One-hundred and fourteen students received course credit for their in-class participation during a 60 minute class period. They ranged in age from 18 to 52 years of age (M = 23.1). Sixty-three females and 49 males (plus two participants who did not indicate their gender) made up the sample. Based on data obtained from a demographics questionnaire (see *Materials*), participants reported moderate experience in statistics or math (M = 2.8 on a four point scale) and had little knowledge of statistics or math (M= 4.2 on a 7 point scale). Participants also had little experience in brain physiology (M = 2.1 on a four point scale) and little knowledge of brain physiology (M = 3.0 on a seven point scale). Participants indicated their proficiency in gambling and games of chance as being only moderate (M = 3.4 on a 7 point scale) but were slightly confident in their ability to solve probability problems similar to those with which they were being presented (M = 5.0 on a seven point scale). Participants judged both experience and knowledge of brain physiology as important for solving probability problems with dice (M = 4.4 on a seven point scale).

Materials

A Dice Outcome Questionnaire (DOQ) contained white, blue, yellow, and green pages (see Appendix B). The first three colored sections compared the likelihood of two specific dice-sides occurring in the long run when the number of possible sides varied from two through six. In all questions, the two sums differed by only one number (e.g., 11 and 12). In some problems, the higher sum was more likely to occur than the lower sum (e.g., 5 instead of 4), whereas in other problems the lower sum was more likely to occur (e.g., 9 instead of 10). In this questionnaire, the lower sum always appeared first. In each pairing, the choices were equally probable if the dice were considered indistinguishable, but not so if the dice were considered distinguishable. In this latter case, one sum was always more probable than the other. In five cases, the odd sum was the lower number and occurred first; in three cases, the even sum was the lower number and occurred first. For example, in one problem, a 7 was presented before an 8; in another problem, 4 was presented before a 5.

For each pairing, the odd sum had the higher probability of occurrences. To avoid students seeing this by running several calculations and catching on to a pattern, they were initially told that they were to answer the questions without doing any mental calculations and later that they were not to go back to check their previous answers until the end of the experiment. Given this set of instructions, the students should not have been able to detect the pattern that odd sums are always the correct answers.

The section containing the green page contained a single question which asked for an answer to a three- rather than a two-dice problem (see Appendix B). This question tested the capability of participants to generalize beyond two-dice problems.

A Demographics Questionnaire (see Appendix C) was administered after completion of the DOQ. It asked the participants to give their age and gender. The questionnaire also asked participants about their experience with the biology or physiology of the brain, about their training and proficiency in math or statistics, and asked them how confident they were in their abilities to answer the type of probability problems they just attempted to solve. Finally, this questionnaire asked the participants about the location of the brain area most involved in solving probability problems with dice, and how important is the brain compared to knowledge in solving these problems.

Procedure

Participants were told that this experiment was studying people's intuition regarding the likelihood that certain dice-sums will come up more often than other dice-sums. When participants were ready to start the experiment, they were asked to assume that they are playing a dice game against an opponent to win more money than their opponent (see Appendices B and C for the instructions given throughout the duration of the study). The game was explained to them, and they were asked the following question, "When two different sums are compared, which sum would you bet on as coming up more often in the long run?"

Participants first were asked to answer by intuition or "feel." They were told not to perform any mental or manual calculations to get the answer. Then they were asked to complete the white pages of the DOQ (see Appendix B). The answers to all questions are given in Appendix D, which was not available to the participants.

Once the white pages were completed, participants were asked to answer the same problems through manual calculations on the blue pages.

Participants were then given a demonstration of solving the dice problem using two 2-sided dice (see Appendix E). During the demonstration, the experimenter did not use the following words: "distinguishable," "undistinguishable," "order of casts," "permutations," or "combinations." Not saying these words was necessary so that the solution would not be given away directly.

All participants were then asked to calculate on the yellow pages the answers to the same problems.

Participants were then asked to answer a three-dice problem on a green page. If the participants gave a high number of correct answers to the green questions, they not only would have demonstrated that they understand the distinguishability concept, but also would have demonstrated generalization from a two- to a three-dice problem.

Participants were then given the Demographics Questionnaire (see Appendix C), which included a question about the location of the brain region most involved in solving probability problems with dice.

Finally, participants were debriefed. The debriefing informed the students about the purpose of the experiment.

Results

A repeated measures ANOVA was used to determine if there were any significant differences in number of correct answers between the 3 sections of the DOQ. These three sections were titled "the intuitive solution (Intuit)," "the pre-demonstration calculation solution (Pre-Demo)," and "the post-demonstration calculation solution (Post-Demo)." The means and standard deviations for each solution are: $M = 3.31$, $SD. = 2.32$; $M = 3.31$, $SD = 2.85$; and $M = 4.32$, $SD = 2.88$, respectively (see Figure 1 for a plot of the means and their standard errors). The effect of solution-orientation was evaluated by the Greenhouse-Geisser test, $F\,(1.8, 207.8) = 10.688$, $p < .001$, $\eta^2 = .086$ (see Table 1), corrected for a significant Mauchly's test of sphericity. Pairwise comparisons revealed no significant differences between Intuit and Pre-Demo; however, Post-Demo was significantly different from both Intuit ($p < .001$) and Pre-Demo ($p < .001$). Therefore, hypothesis (a) was not confirmed (the Pre-Demo section did not have higher scores than the Intuit section), but hypothesis (b) was confirmed (after viewing a demonstration, participants gave more correct answers).[1]

When Post-Demo scores were divided at the median, 56 participants had low scores (0-4), and 58 participants had high scores (5-8). Forty five participants correctly answered the three-dice generalization question (green page of the DOQ), and 69 participants answered incorrectly. The resulting 2 X 2 Chi-Square (see Table 1) yielded a significant association between Post-Demo correctness with two-dice and generalization correctness with three-dice. Therefore, hypothesis (c) was confirmed (those participants scoring high in the Post-Demo section correctly generalized the solution to a three-dice problem).

Participants were again divided at the median into high and low Post-Demo scorers. On

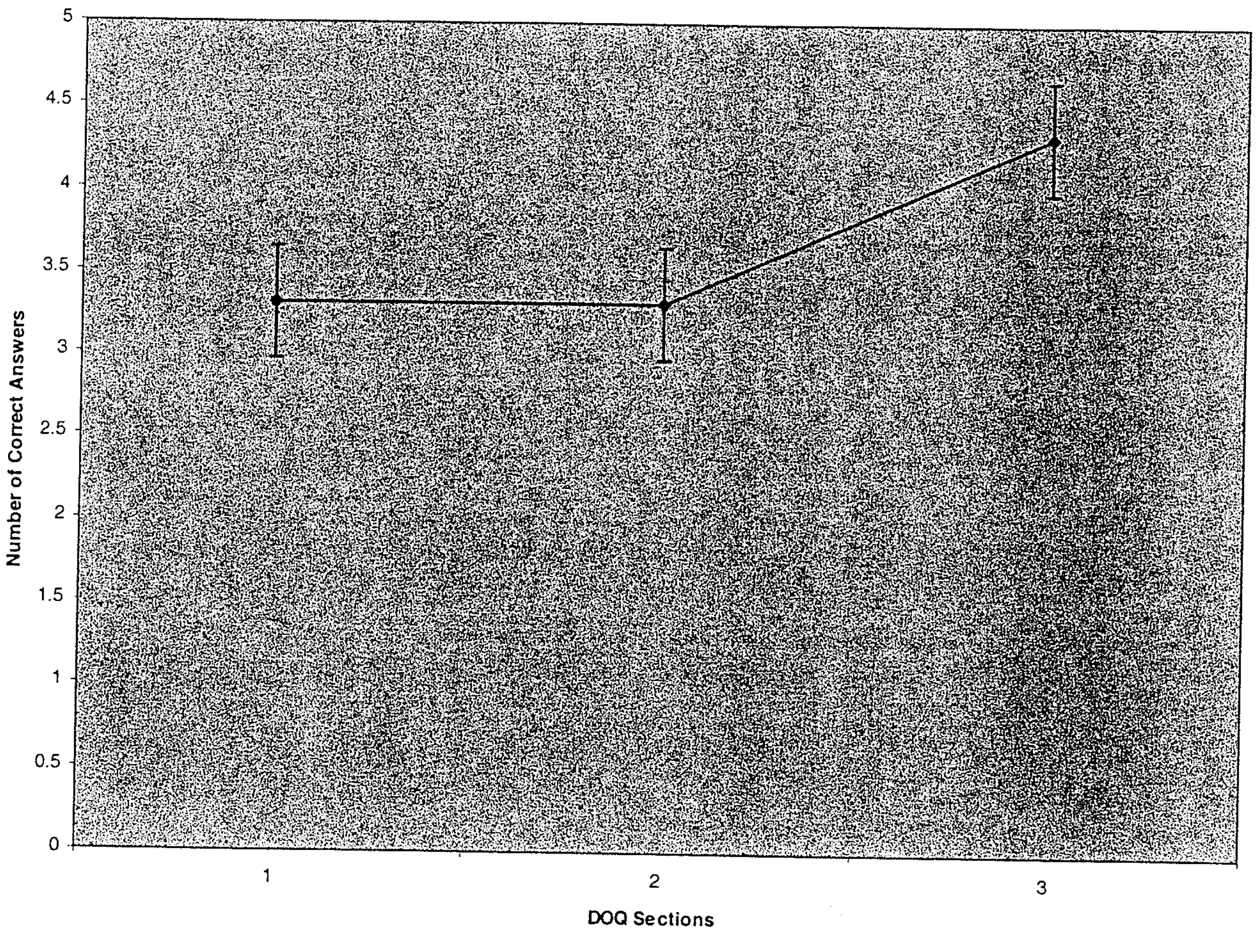

Figure1. Mean Scores for Each Section of the Dice Outcome Questionnaire DOQ). Codes for DOQ Sections are: 1 = Intuit, 2 = Pre-Demo, 3 = Post-Demo.

Table 1

Comparison of Post-Demonstration Correctness and Generalization Correctness

	Generalization Correctness		
Post-Demo Correctness	Correct	Incorrect	Total
Low scores	15	41	56
High scores	30	28	58
Total	45	69	114

Note. χ^2 (1, N = 114) = 7.42, p = .01.

the other hand, 25 participants answering Question H on the Demographics Questionnaire gave the correct attribution of brain activity involved while solving probability problems to the left frontal lobe, whereas 89 participants listed one of the other seven brain areas. To compare these two numbers directly would seriously inflate the number of incorrect responses, because there were seven opportunities to select an incorrect answer by chance and only one opportunity to select the correct answer by chance. Accordingly, the 25 correct answers were compared to the average incorrect answer by dividing the 89 misattributions by 7. The average number of misattributions was 89/7 = 12.7 = 13. The resulting 2 X 2 Chi-Square (see Table 2) yielded an insignificant association between Post-Demo correctness and Brain Area Involvement. Therefore, hypothesis (d) was not confirmed (participants did not attribute less responsibility for probability-solving activity to the left frontal lobe than to the average of the other areas of the brain).

Table 2

Comparison of Post-Demonstration Correctness and Brain Attribution Correctness

	Brain Attribution Correctness		
Post-Demo Correctness	Left Frontal Lobe	Other Brain Areas	Total
Low scores	12	5	17
High scores	13	8	21
Total	25	13	38

Note. χ^2 (1, N = 38) = 1.12, p = .29.

Discussion

Humans typically are poor statisticians, and they do not intuitively apply the laws of probability theory. What one naively intuits about the probability of events does not generally match the objective probability. The aim of this study was to examine the nature of erroneous reasoning about human probability and demonstrate that certain counterintuitive probability laws can be taught by demonstration and will generalize to more complex problems.

The results of the present study suggest that although manually calculated solutions to probability problems involving two dice fail to yield more correct answers than initial intuitive solutions, people given the opportunity to view a demonstration of a correct solution have improved chances of correctly solving such problems. Hypothesis (a) was thus not confirmed, but hypothesis (b) was confirmed. A speculation to explain this result is that viewing a demonstration will promote more probabilistic thinking than will written instructions. Having extra time to think and write out solutions is not enough to promote probabilistic thinking. This is in line with the results of research (Girotto & Gonzalez, 2001; Hansen and Helgeson, 1996) that suggests that only information presented in an appropriate manner will elicit rational thinking.

Results also showed that people getting a high number of probability problems correct after getting a demonstration tend to generalize from a two dice to a three dice problem. Apparently, the participants understand the distinguishability concept and so can generalize to more complex problems. This supports hypothesis (c) and the idea that humans may not be such poor intuitive statisticians as is commonly believed (Cosmides & Tooby, 1996).

Finally, results showed that participants failed to attribute responsibility for problem solving activity with dice to the left frontal lobe of the brain as compared to the average of all other seven brain areas. Such a finding does not support hypothesis (d) but rather indicates that people generally lack the physiological information required to solve this problem. It was hypothesized that participants would choose other areas over the left frontal lobe because knowledge and understanding of the neural substrates of logical reasoning is not well known (Houde et al., 2000). Participants in the present study apparently did not have the appropriate knowledge of brain physiology to correctly identify the left frontal lobe as being responsible for probability reasoning with dice.

All of these results taken together suggest that presenting probability information in the form of (a) demonstrations of dice outcomes and (b) learning the special functions of each brain area will promote more effective probability reasoning.

Research on judgements under uncertainty and people's ability to accurately use rational probability rules (e.g., Kahneman & Tversky, 1982; Girotto & Gonzalez, 2001) has been conducted under a single premise. This premise is that human cognitive processes are rational to the extent that the rules humans use conform to the foundations of normative theories drawn from mathematics, probability theory, and logic (Brase et al., 1998). Proponents of this viewpoint assume that human cognitive structures should be able to correctly apply normative principles taken from probability theory to accurately calculate problems of statistical inference. The conclusions drawn from this kind of research suggests that humans are poor statistical or probabilistic reasoners (e.g., Kahneman & Tversky, 1972; Kahneman & Tversky, 1982).

More recent research found some puzzles in the premise. For example, evolutionary biologists studying animal behavior (specifically judgments under uncertainty to test various mathematical models taken from optimal foraging theory) find that even animals with very small nervous systems, such as the bumblebee or birds, can still accurately make judgments under uncertainty when they are foraging that the human's brain is considered "too limited" to calculate (Brase et al, 1998). Are bumblebees really more rational than humans? The reply is that the bumblebees were tested in ecologically valid conditions, whereas humans were not. Testing humans in an ecologically valid context demonstrates that they are as rational as bumblebees and birds. The research by Hansen and Helgeson (1996) and Girotto and Gonzalez (2001) described earlier supports this hypothesis in that presenting information to humans in an appropriate manner elicits correct probability judgements. Humans can rationally solve probability problems when information is presented in frequencies rather than in probabilities. Under a frequency format, people use base-rate information, producing answers that conform to the principles of Bayes' rules (Cosmides & Tooby, 1996). In this way, the conjunction fallacy and the overconfidence bias errors (present when information is presented in probabilities) are eliminated.

Thus, it seems as if humans, similar to other animals, have inductive reasoning mechanisms using rational principles; and these mechanisms require representations of frequencies to function properly. The human mind does embody a calculus of probability. The current research suggests that viewing a demonstration of the solution is very effective method of conveying the distinguishability concept, perhaps even superior to written instructions. Presenting information in an appropriate manner elicits rational thinking. Individuation is the process of viewing the world as made of discrete entities. Perhaps at first, the participants did not individuate the dice. However, they may have individuated the dice after seeing a demonstration of the solution, so each die was finally viewed as a discrete entity, thus demonstrating an understanding of the distinguishability concept.

A shortcoming of the present study is that the participants were never asked explicitly, at any time whether they could explain or verbalize the distinguishability concept or whether they intuitively understood the concept. To do so might have either biased their responses or provided inaccurate perceptions due to a "hindsight bias." Still, further research should examine this issue. Another shortcoming is the potential for practice effects. Each of the three sections of the DOQ used the same questions (problems). To minimize this confounding, the experimenter instructed participants not to return to previous pages and not to look ahead in the questionnaire. The data analyses revealed that there was no significant difference between Intuit and Pre-demo conditions, suggesting that practice effects probably played little role in the results.

Practical implications of this study suggests that certain counterintuitive probability laws can be taught by demonstration. In fact, these laws may be applied and generalized to a more complex problem. Replicating these results with different types of questions or problems and different methods of demonstration will further uncover the true nature of humans reasoning abilities. It is the researcher's contention that humans may not be as 'irrational' as once thought. Perhaps other types of probability solutions can be taught by offering simple, non-explicit demonstrations.

The major theoretical implication of the current study is that judgement under uncertainty can be examined from a different viewpoint using a different approach. The premise of this research may be changing back to the old view -- if there is a clash between intuition and theory, then maybe the theory should be reviewed for its validity. Although at first people are not good probability reasoners, presenting a demonstration or example can reduce these errors. Humans can quickly understand and apply counterintuitive probability laws, reducing erroneous reasoning on probability. They just need a small amount of help, structure, or priming.

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Appendix A

Combinations and Permutations of Three Dice

Combinations of Three Dice

Sum of 11

6-4-1
6-3-2
5-5-1
5-4-2
5-3-3
4-4-3

Sum of 12

6-5-1
6-4-2
6-3-3
5-5-2
5-4-3
4-4-4

Permutations of Three Dice

Sum of 11	6-4-1	6-1-4	4-6-1	4-1-6	1-6-4	1-4-6
	6-3-2	6-2-3	3-6-2	3-2-6	2-6-3	2-3-6
	5-5-1	5-2-4	4-5-2	5-1-5	2-5-4	1-5-5
	5-4-2		3-5-3	4-2-5		2-4-5
	5-3-3			4-3-4		3-3-5
	4-4-3					3-4-4
Sum of 12	6-5-1	6-1-5	5-6-1	5-1-6	1-6-5	1-5-6
	6-4-2	6-2-4	4-6-2	4-2-6	2-6-4	2-4-6
	6-3-3	5-3-4	3-6-3	5-2-5		3-3-6
	5-5-2		4-5-3	4-3-5		2-5-5
	5-4-3		3-5-4			3-4-5
	4-4-4					

Appendix B

DICE OUTCOME QUESTIONNAIRE

Assume you are playing a dice game against an opponent. The winner will receive a large amount of money. The game involves **two fair dice** that will be "tossed" over 1000 times by a random number generator. Your task is to select which of two sums of these two dice you will "bet on" in order to win the prize money. You are to choose one and only one sum as your bet on all trials. Fortunately, YOU get to choose first. This choice will be YOUR bet for all dice-tosses with that specific pair of dice. Your opponent will have to take the sum you did not choose. (Remember, neither of you can "change" sums once the initial selection is made.) Also, assume that you will be playing with different pairs of dice (for example, a pair may have six sides or five sides or four sides or three sides or two sides).

The question is: **When two different sums of dice are compared, which sum would you bet on as coming up more often in the long run?** Please indicate your selection in each comparison below by marking an "X" in the appropriate box. Please mark one and only one box in each comparison and do NOT perform any mental or manual calculations to get the answer, for we want your intuition or "feel" for the answer at this point. Later you will have an opportunity to check the accuracy of your initial selection.

TWO DICE, EACH WITH SIX SIDES NUMBERED FROM 1 TO 6

1. Which sum would you bet on expect to come up MORE OFTEN in the long run?

/__/ the sum of the two dice = 6

/__/ the sum of the two dice = 7

/__/ both sums are equally probable

2. Which sum would you bet on expect to come up MORE OFTEN in the long run?

/__/ the sum of the two dice = 9

/__/ the sum of the two dice = 10

/__/ both sums are equally probable

3. Which sum would you bet on expect to come up MORE OFTEN in the long run?

/__/ the sum of the two dice = 11

/__/ the sum of the two dice = 12

/__/ both sums are equally probable

Please turn to the next page and indicate your bets for a different pair of dice. Do NOT turn back to previous pages and bets.

TWO DICE, EACH WITH <u>FIVE</u> SIDES NUMBERED FROM 1 TO 5

4. <u>Which sum would you expect to come up MORE OFTEN in the long run?</u>

/__/ the sum of the two dice = 2

/__/ the sum of the two dice = 3

/__/ both sums are equally probable

5. <u>Which sum would you bet on expect to come up MORE OFTEN in the long run?</u>

/__/ the sum of the two dice = 7

/__/ the sum of the two dice = 8

/__/ both sums are equally probable

Please turn to the next page and indicate your bet for a different pair of dice. Do NOT turn back to previous pages and bets.

TWO DICE, EACH WITH FOUR SIDES NUMBERED FROM 1 TO 4

6. Which sum would you expect to come up MORE OFTEN in the long run?

/__/ the sum of the two dice = 4

/__/ the sum of the two dice = 5

/__/ both sums are equally probable

Please turn to the next page and indicate your bet for a different pair of dice. Do NOT turn back to previous pages and bets.

TWO DICE, EACH WITH THREE SIDES NUMBERED FROM 1 TO 3

7. Which sum would you expect to come up MORE OFTEN in the long run?

/__/ the sum of the two dice = 5

/__/ the sum of the two dice = 6

/__/ both sums are equally probable

Please turn to the next page and indicate your bet for a different pair of dice. Do NOT turn back to previous pages and bets.

TWO DICE, EACH WITH TWO SIDES NUMBERED FROM 1 TO 2

8.Which sum would you expect to come up MORE OFTEN in the long run?

/__/ the sum of the two dice = 3

/__/ the sum of the two dice = 4

/__/ both sums are equally probable

Please STOP and wait for further instructions. Do NOT turn back to previous pages and bets.

The previous two-dice problems are repeated below, and you have the same task as before. When two different sums of dice are compared, which sum would you bet on as coming up more often in the long run? Please indicate your selection in each comparison below by marking an "X" in the appropriate box. Please mark one and only one box in each comparison. However, **for these problems you are asked to write out EACH way you can obtain each sum.** (You can later check your calculated answers with the intuitive answers you gave before.) As you list each way to obtain a specific sum, be sure to identify the number each die-face shows, for example, **1 & 2.**

TWO DICE, EACH WITH SIX SIDES NUMBERED FROM 1 TO 6

9. Which sum would you bet on expect to come up MORE OFTEN in the long run?

/__/ the sum of the two dice = 6

/__/ the sum of the two dice = 7

/__/ both sums are equally probable

10. Which sum would you bet on expect to come up MORE OFTEN in the long run?

/__/ the sum of the two dice = 9

/__/ the sum of the two dice = 10

/__/ both sums are equally probable

11. Which sum would you bet on expect to come up MORE OFTEN in the long run?

/__/ the sum of the two dice = 11

/__/ the sum of the two dice = 12

/__/ both sums are equally probable

Please turn to the next page and indicate your bets for a different pair of dice. Do NOT turn back to previous pages and bets.

TWO DICE, EACH WITH FIVE SIDES NUMBERED FROM 1 TO 5

12. Which sum would you expect to come up MORE OFTEN in the long run?

/__/ the sum of the two dice = 2

/__/ the sum of the two dice = 3

/__/ both sums are equally probable

13. Which sum would you bet on expect to come up MORE OFTEN in the long run?

/__/ the sum of the two dice = 7

/__/ the sum of the two dice = 8

/__/ both sums are equally probable

Please turn to the next page and indicate your bet for a different pair of dice. Do NOT turn back to previous pages and bets.

TWO DICE, EACH WITH FOUR SIDES NUMBERED FROM 1 TO 4

14. Which sum would you expect to come up MORE OFTEN in the long run?

/__/ the sum of the two dice = 4

/__/ the sum of the two dice = 5

/__/ both sums are equally probable

Please turn to the next page and indicate your bet for a different pair of dice. Do NOT turn back to previous pages and bets.

TWO DICE, EACH WITH THREE SIDES NUMBERED FROM 1 TO 3

15. Which sum would you expect to come up MORE OFTEN in the long run?

/__/ the sum of the two dice = 5

/__/ the sum of the two dice = 6

/__/ both sums are equally probable

Please turn to the next page and indicate your bet for a different pair of dice. Do NOT turn back to previous pages and bets.

TWO DICE, EACH WITH <u>TWO</u> SIDES NUMBERED FROM 1 TO 2

<u>16. Which sum would you expect to come up MORE OFTEN in the long run?</u>

/__/ the sum of the two dice = 3

/__/ the sum of the two dice = 4

/__/ both sums are equally probable

Please STOP and wait for further instructions. Do NOT turn back to previous pages and bets.

The previous two-dice problems are repeated below, and you have the same task as before. When two different sums of dice are compared, which sum would you bet on as coming up more often in the long run? Please indicate your selection in each comparison below by marking an "X" in the appropriate box. Please mark one and only one box in each comparison. For these problems you are again asked to write out EACH way you can obtain each sum, However, **now you have the advantage of having seen a demonstration of how to ensure you get the correct answer to each problem**. As you list each way to obtain a specific sum, be sure to identify the number each die-face shows, for example, **1 & 2.**

TWO DICE, EACH WITH SIX SIDES NUMBERED FROM 1 TO 6

17. Which sum would you bet on expect to come up MORE OFTEN in the long run?

/__/ the sum of the two dice = 6

/__/ the sum of the two dice = 7

/__/ both sums are equally probable

18. Which sum would you bet on expect to come up MORE OFTEN in the long run?

/__/ the sum of the two dice = 9

/__/ the sum of the two dice = 10

/__/ both sums are equally probable

19. Which sum would you bet on expect to come up MORE OFTEN in the long run?

/__/ the sum of the two dice = 11

/__/ the sum of the two dice = 12

/__/ both sums are equally probable

Please turn to the next page and indicate your bets for a different pair of dice. Do NOT turn back to previous pages and bets.

TWO DICE, EACH WITH FIVE SIDES NUMBERED FROM 1 TO 5

20. Which sum would you expect to come up MORE OFTEN in the long run?

/__/ the sum of the two dice = 2

/__/ the sum of the two dice = 3

/__/ both sums are equally probable

21. Which sum would you bet on expect to come up MORE OFTEN in the long run?

/__/ the sum of the two dice = 7

/__/ the sum of the two dice = 8

/__/ both sums are equally probable

Please turn to the next page and indicate your bet for a different pair of dice. Do NOT turn back to previous pages and bets.

TWO DICE, EACH WITH FOUR SIDES NUMBERED FROM 1 TO 4

22. Which sum would you expect to come up MORE OFTEN in the long run?

/__/ the sum of the two dice = 4

/__/ the sum of the two dice = 5

/__/ both sums are equally probable

Please turn to the next page and indicate your bet for a different pair of dice. Do NOT turn back to previous pages and bets.

TWO DICE, EACH WITH <u>THREE</u> SIDES NUMBERED FROM 1 TO 3

<u>23. Which sum would you expect to come up MORE OFTEN in the long run?</u>

/__/ the sum of the two dice = 5

/__/ the sum of the two dice = 6

/__/ both sums are equally probable

Please turn to the next page and indicate your bet for a different pair of dice. Do NOT turn back to previous pages and bets.

TWO DICE, EACH WITH TWO SIDES NUMBERED FROM 1 TO 2

24. Which sum would you expect to come up MORE OFTEN in the long run?

/__/ the sum of the two dice = 3

/__/ the sum of the two dice = 4

/__/ both sums are equally probable

Please STOP and wait for further instructions. Do NOT turn back to previous pages and bets.

We would now like you to do ONE more thing before you finish the study. We would like you to make one more comparison of sums EXCEPT THAT NOW THREE DICE WILL BE USED (rather than two). Again, **for these problems you are asked to <u>write out EACH way you can obtain each sum</u>**, As you list each way to obtain a specific sum, be sure to identify the number each die-face shows, for example, **1 & 2 & 3.**

THREE DICE, EACH WITH <u>SIX</u> SIDES NUMBERED FROM 1 TO 6

<u>25. Which sum would you expect to come up MORE OFTEN in the long run?</u>

/__/ the sum of the three dice = 11

/__/ the sum of the three dice = 12

/__/ all three sums are equally probable

Please STOP and wait for further instructions. Do NOT turn back to previous pages and bets.

Appendix C
Demographics Questionnaire

A. **Age**: __________ **Gender**: ___________

B. How **confident are you in your ability to solve other probability problems similar to those you just finished solving?**
(Circle the MOST appropriate number.)

Not at all confident 1 2 3 4 5 6 7 Very confident

C. **How much exposure to information on <u>statistics or math</u> have you had (through books, courses, etc. whether in high school, university, or elsewhere)?**
(Check the MOST appropriate space.)

___1. I have had no exposure to information on statistics or math
___2. I have had very little exposure to information on statistics or math (e.g., one book read or one course taken)
___3. I have had a fair amount of exposure to information on statistics or math (e.g., two books read or two courses taken)
___4. I have had quite a lot of exposure to information on statistics or math (e.g., more than two books read or two courses taken)

D. **How much knowledge do you feel you have in <u>statistics or math</u>?**
(Circle the MOST appropriate number.)

No knowledge 1 2 3 4 5 6 7 Considerable knowledge

E. **How much proficiency do you feel you have in <u>gambling and games of chance</u>?** (Circle the MOST appropriate number.)

Not at all proficient 1 2 3 4 5 6 7 Very proficient

Please continue on the reverse side of this page.

F. **How much exposure to information on brain physiology have you had (through books, courses, etc. whether in high school, university or elsewhere)?**
(Check the MOST appropriate space.)

___1. I have had no exposure to information on brain physiology
___2. I have had very little exposure to information on brain physiology (e.g., one book read or one course taken)
___3. I have had a fair amount of exposure to information on brain physiology (e.g., two books read or two courses taken)
___4. I have had quite a lot of exposure to information on brain physiology (e.g., more than two books read or two courses taken)

G. **How much knowledge do you feel you have in brain physiology?**
(Circle the MOST appropriate number.)

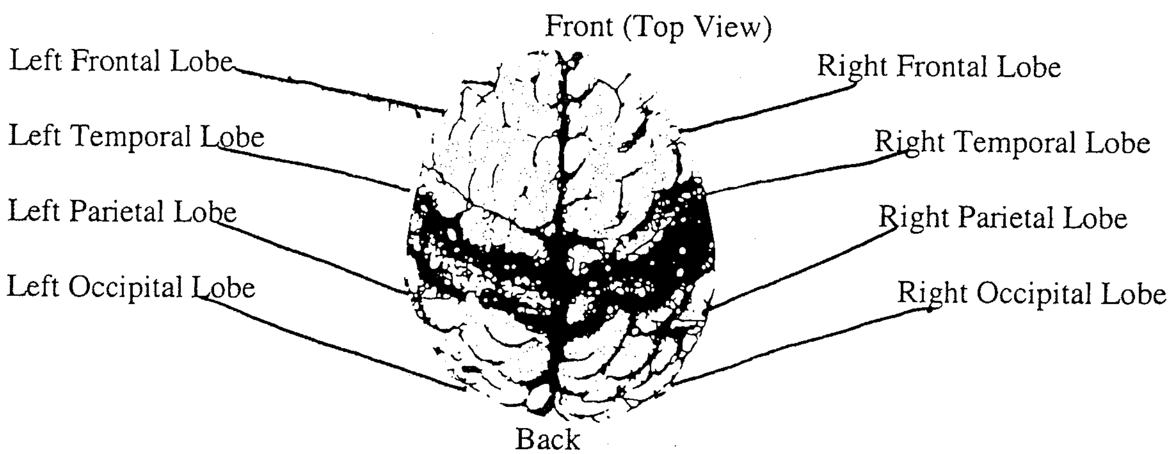
No knowledge 1 2 3 4 5 6 7 Considerable knowledge

H. **Which area of the brain do you think is MOST involved in solving probability problems with dice?**
The diagram of the brain at the bottom of the page may assist you in giving an answer. (Circle only ONE answer.)

1. Right frontal lobe
2. Left frontal lobe
3. Right temporal lobe
4. Left temporal lobe
5. Right parietal lobe
6. Left parietal lobe
7. Right occipital lobe
8. Left occipital lobe

I. **How important is the brain as compared to knowledge for solving probability problems with dice?** (Circle the MOST appropriate number.)

Brain most important 1 2 3 4 5 6 7 Experience most important



Appendix D

Correct Answers to the Dice Outcome Questionnaire

2 dice, 6 sides Page 1, 6, 11	2 dice, 5 sides Page 2, 7, 12	2 dice, 4 sides Page 3, 8, 13	2 dice, 3 sides Page 4, 9, 14	2 dice, 2 sides Page 5, 10, 15
7 = 7-1, 1-7, 4-3, 3-4, 5-2, 2-5 6 = 5-1, 1-5, 3-3, 2-4, 4-2 9 = 6-3, 3-6, 4-5, 5-4 10 = 6-4, 4-6, 5-5 11 = 5-6, 6-5 12 = 6-6	3 = 1-2, 2-1 6 = 1-1 7 = 5-2, 2-5, 4-3, 3-4 8 = 4-4, 5-3, 3-5	4 = 4-1, 1-4, 2-3, 3-2 5 = 3-1, 1-3, 2-2	5 = 3-2, 2-3 6 = 3-3	3 = 2-1, 1-2 4 = 2-2

Appendix E

Demonstration

How many ways can you get a sum of 3 using two 2-sided dice?

Possibility #1 2, 1 = 3

Possibility #2 1, 2 = 3

Therefore, with two 2-sided dice there are only two ways to get a sum of 3.

Author Note

This manuscript was submitted in partial fulfillment of the requirements for the degree of Master of Arts in Psychology under the supervision of Dr. J. B. Nickels.

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Footnotes

[1]The experimenter wanted to ensure that the demonstration was beneficial but at the same time did not improve post-demonstration scores merely because the question in the demonstration was identical to question eight in the post-demonstration. In other words, improvement in post-demonstration scores could be due to memory of what was just presented rather than to learned-distinguishability of how to solve dice problems. Therefore, a second ANOVA was run on number of correct answers for only the first seven questions of each of the three sections of the DOQ. The new intuitive, pre-demonstration, and post-demonstration means and standard deviations for each solution were: $M = 2.90$, $SD. = 2.04$; $M = 2.94$, $SD = 2.48$; and $M = 3.74$, $SD = 2.57$, respectively. The effect of solution-orientation was evaluated by the Greenhouse-Geisser test, $F\ (2,\ 112) = 6.587$, $p < .002$, $\eta^2 = .071$, corrected for a significant Mauchly's test of sphericity. Pairwise comparisons revealed no significant differences between Intuit and Pre-Demo; however, Post-Demo was significantly different from both Intuit ($p = .001$) and Pre-Demo ($p = .001$). These results based on seven questions are comparable to those based on all eight questions.