A COMPARATIVE INVESTIGATION OF MATHEMATICS ACHIEVEMENT OF GRADE SEVEN STUDENTS UNDER AN INDEPENDENT STUDY SYSTEM AND A TRADITIONAL GROUP INSTRUCTION SYSTEM IN THE SAME JUNIOR HIGH SCHOOL

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by

ROBERT HENRY BURNELL

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ABSTRACT

This study was undertaken as a result of the writer's experience with a program of independent study and because of the expressed concern of educators for meeting individual needs. The study was set in one junior high school and was implemented by the grade 7 mathematics teacher in one of his classes, using one other class as a control group. Both of the classes were grouped heterogeneously. The purpose of the study was to determine if grade 7 students working on an independent study program would achieve as well in mathematics as those who were taught with the traditional group instruction method. A two-tailed t test was used with the .05 level of significance.

Independent study materials were produced by the writer in the form of self-instruction workbooks. The treatment group used these booklets from October to the end of the school year. The teacher's function was to act as a resource person, dealing with individuals, small groups, or the whole class as the situation arose. The control group was taught by means of the traditional group instruction method.

Pre-test information from I.Q. scores, mathematics ability scores (concepts and problem solving), and reading scores indicated the samples contained no real difference in ability. Post-test data was generated by the Canadian Test of Basic Skills mathematical concepts and problem solving, a teacher-prepared test, and the Canadian Test of Basic Skills reading ability test. At the conclusion of the experiment, treatment students were also polled with respect to their opinions about the experimental method.

Analysis of the data resulted in the null-hypothesis being accepted. The treatment group achieved as well as the control group. Further investigation was begun by looking at the extremes of both groups. The bright students in the treatment group seemed to do better and the weak students seemed to do worse than their counterparts in the control group. No formal analysis was attempted because such a course of action was thought to be beyond the scope of this study. The opinion poll indicated that most of the students in the treatment group found some merit in that method. Some students, however, did not like the experimental method.

The implications of the study are far-reaching. Independent study programs can be conducted in a traditional building, with traditional organization for study. The suggestion is that implementation of an independent study program will soon lead to a truly individualized program of diagnosis and prescription.

CHAPTER I

INTRODUCTION

Background

This study was motivated by a situation in which the writer was involved some five years ago and that lasted for a period of two years. This situation involved the teaching of a pair of grade 7 mathematics classes in a junior high school in suburban Winnipeg. These classes were especially arranged so that each contained two groups of students. One group in each class was made up of about twenty students whose past records demonstrated a superior ability in mathematics, while the other group in each class was made up of about ten students who had records of poor achievement in mathematics. The high achievers were placed on an independent study program; the smaller group of poor achievers were handled with a highly structured group instruction method. These two specially designed classes were made the responsibility of the writer, while the rest of the grade 7 classes were taught mathematics by two other teachers using the group instruction method. By the end of one full year on the independent study program, the group of high achievers had achieved as well on several term tests and on both the Christmas and Easter examinations as would have been expected otherwise.

Encouraged by the ability of the independent study group to attack a course of study on their own, the writer decided to try a similar approach the following year. This second attempt, however, was on a much larger scale. All the students in the school were involved as well as all the mathematics teachers. Only those students who were poor achievers in mathematics were taught in a group instruction situation. The rest of the student body was placed on an independent study program. Once again, the results were encouraging, even more than during the previous year, since many more students had experienced the independent study program. Unfortunately, these encouraging results did not constitute a basis for valid comment since no control group existed and no statistical analysis was attempted; hence, the effort to undertake the present study.

Purpose of the Study

The purpose of the study was to determine if there was a significant difference in achievement in mathematics between grade 7 students who were provided with an independent study program and grade 7 students whose mathematics program was presented by group instruction.

In order to ascertain whether or not this difference existed, this purpose was expressed as the following null-hypothesis:

> At the conclusion of the experimental teaching period, there is no significant difference between the achievement as expressed by post-test mean scores of the experimental group (independent study) and those of the control group (group instruction).

The alternative hypothesis was that there is a significant difference between the mean scores achieved on mathematics tests of the experimental group and the corresponding mean achievement of the control group.

A secondary purpose of the study was to prepare materials in workbook form which would provide a viable basis for an independent study program in mathematics at the seventh grade level.

As a sub-problem related to the major purpose, students were polled informally to determine their reaction to working in an independent study program. Students were asked to respond anonymously and were given little direction other than to state their feelings about the independent study method. The reason for anonymity and lack of direction was to elicit as honest an opinion as possible.

Need for the Study

The review of the literature reveals that many articles have been written and much research has been conducted on the topic of individualizing instruction. The main thrust of these articles and research studies has been directed at exposing certain weaknesses of the traditional practices and policies of the educational system. This critical analysis of the system suggests the need for an alternative approach. Programs bearing the titles "continuous progress" and "individualized instruction" frequently appear as possible alternatives. An overview of this critical analysis of the traditional system will be discussed in some detail in the review of the literature (Chapter II). The indication seems to be that the time has come to begin investigating suitable alternatives - alternatives that face squarely the problem of individual differences in the classroom. This is truly a central problem in education and one whose solution is sought in the name of improving the quality of instruction in our schools.

The literature indicates than an alternative to the traditional form of teaching may be individualized instruction. In light of the fact that many Manitoba schools have already implemented continuous progress, which, in a refined form has as its goal individualized instruction, it seems timely that some concern be shown for the vehicle through which this can be achieved. Methods for dealing with individual differences involve some form of independent study materials. These materials range from textbook assignments to elaborate workshop approaches.

In particular, this paper will be concerned with independent study achieved through the use of teacher-prepared units. It is believed that, since such units can be produced at little cost and are limited in form and content only by the energy and imagination of the teacher, they will become a popular vehicle for use in Manitoba schools.

Definitions

Continuous progress. Continuous progress refers to an educational system that allows students to follow courses of study that are tailored to their abilities at a rate that is compatible with their own capabilities. In John Goodlad's words:

> ...the sequence of content is determined by the inherent difficulty of the subject matter and the children's demonstrated ability to cope with it; materials are selected to match the spread of individual differences existing within the instructional group; and the children move upward according to their readiness to proceed. Promotion or nonpromotion does not exist as such.[1]

Individualized instruction. Individualized instruction refers to a learning situation flexible enough that instruction may be varied in form, content, and goals from individual to individual.

Traditional system. Traditional system refers to a graded school system characterized by group instruction, where course content is presented by the instructor in the form of a lecture.

Independent study. Independent study refers to a teaching method that utilizes some form of self-instruction. An example of self-instruction might simply be where a student is asked to learn a unit of work in a textbook by reading the discussions and explanations presented, studying the examples given and completing the assignments involved. Another example might be a set of activities that are presented in the manner of a workshop where a student is expected to follow a set of instructions (usually written) that cause him to manipulate materials, look for patterns and draw conclusions. The workbooks designed for this study also provide for self-instruction to take place. The student is asked to read through the material, answer questions, look for patterns, and draw conclusions. Any method utilizing self-instruction will involve the classroom teacher as a resource person to help students who encounter difficulties.

Vehicle. Vehicle refers to the materials and procedures that allow a particular method or approach to be achieved. In other words, the vehicle is a program of study.

Delimitations

The experiment was designed within the bounds of the following delimitations:

1. The only aspect of the classroom that was analysed formally in this study was mean achievement.

2. The study involved a single teacher in one junior high school.
3. The results can only be generalized to the grade 7 population in that one school.

Limitations

This study is subject to the following limitations:

1. Some of the post-test data were collected from grade 6 records which originated in different elementary schools. Therefore, the standardized tests that were used do not have exactly the same basis for comparison.
2. The materials used by the treatment group are restricted in scope by the fact that they were produced by a single person, namely the writer.
3. The sampling techniques may contain inadequacies.
4. The idiosyncracies of the participating teacher's personality in the classroom and the general atmosphere of the school must be considered.
5. Preparation of materials was limited by time restrictions.
6. The post-test measurement instruments may contain inherent inadequacies.

Summary

This study grew from a series of events experienced by the writer while teaching mathematics in a junior high school in suburban Winnipeg. These events centred on procedures developed by the staff of the school to meet the needs of individual pupils in learning mathematics. The purpose of the study was to determine if there is a significant difference in the mathematical achievement of students working on independent study programs from those whose mathematics is provided by the traditional group method.

The literature and research studies suggest that methods aimed at meeting individual differences in the classroom might involve a program of independent study in which major responsibility for progress is placed on the individual student. Methods which use independent study vary from simply giving students textbooks accompanied by written instructions to the use of elaborate workshop approaches. Because of this wide range in instructional techniques as well as the amply documented variations in student needs and abilities, these vehicles must be given serious attention. This paper deals with an independent study method that uses teacher-prepared self-instruction workbooks as a vehicle.

Within Chapter I the background and need for the study are discussed, the purpose stated, and the definitions, delimitations and limitations listed.

Chapter II deals with a review of the literature, the purpose of which is to underline the need for this study by describing some of the discussion and research on the topic that has developed over the years. Chapters III and IV describe the procedures and present the results respectively, while Chapter V provides a discussion of the implications of this study.

Footnotes

[1] John J. Goodlad, School, Curriculum, and the Individual (Waltham, Massachusetts: Blaisdell Publishing Company, 1966), p. 24.

CHAPTER II

REVIEW OF THE LITERATURE

Initially, the review of the literature will concern itself with readings related to a critical investigation of the traditional system. Following this, there will be an exposition of some of the research that deals with independent study. The writer believes that the scenario presented in this review will serve to underscore the need for studies such as this and to enlighten the reader with respect to the start that has already been made.

In the early half of the nineteenth century, the American educational system began to feel the pressure of rapid population growth due to the influx of immigrants. The need was felt for some unifying social agency, some form of educational system that was more efficient than the tutorial system then in use. Initiated by Horace Mann and Henry Barnard, an eight-year elementary school was introduced, patterned after the German Volkschule, and by 1870 nearly all the elementary schools in the United States were of this type. At this point the groundwork was laid for an educational system, the goal of which was to develop a homogeneous society in the United States. All children would be educated equally. This surely was a noble goal and well within keeping of America's founding documents - the Declaration of Independence, the Gettysberg Address, the writings of all the really great American thinkers.

> The eventual acceptance of the obligation to educate all the children of all the people was a direct outcome of initial egalitarian impulses.[1]

If the initial impulse was a noble one, it quickly became distorted, for a system of education evolved where the word "equal" became synonymous with the word "same", and the word "grade" served to ensure that all children in school would have to live up to a set of expectations that was derived in the uncertain world of the normal population. This system has grown from its modest beginnings in the late nineteenth century into an enormous machine capable of efficiently processing vast numbers of students. Recently, however, the machine has been very seriously examined to find out, firstly, if it is indeed doing the job it was designed to do, and, secondly, if the manner in which the fundamental philosophy has been interpreted is really valid.

The readings to be dealt with in the following discussion refer to some of the research that has been attempted to determine the degree to which the system has achieved its ends. As will be seen, the research has far-reaching implications that go beyond this first concern and lead into the critical and very difficult question of the validity of fundamental philosophy.

As was suggested earlier, the word "grade" implies a homogeneity. Textbooks, outlines of courses, and lesson plans are devised to this end. In fact, a whole set of expectations which includes attitudes and values, age level, and other aspects is laid out either explicitly or implicitly. It might be expected, then, that students in a particular grade would rate fairly closely if measured by a test that deals with achievement, intelligence, interest, etc. The literature, however, does not bear this out. In the <u>Sixty-First Yearbook of the National Society for the Study of</u>

Education, Fred T. Tyler provides a summary of research in this regard:

> The I.Q.'s for the seventh-graders in one junior high school in 1960 varied from the 60's to the 160's. And their grade equivalents on achievement tests in different school subjects ranged from the first through the eleventh grade. From numerous sources we know that scores at any grade level are remarkably variable for any test, whether it purports to measure intelligence, achievement, interest, values, or attitudes.[2]

The research is persuasive and the fact that not all students in the graded system meet the criterion for that grade is recognized by the system itself. Some students do not succeed and this is evidenced by the fact that machinery exists to remediate. One of the most obvious tools for this remediation is the practice of promotion or non-promotion. If a student, by the end of a specified time, is not living up to the expectations of the grade, he may be required to repeat in the hope that his being out of place in the previous year will be remedied. Here again the literature belies the attempts:

> Occasionally the retention of low-achieving students is recommended as a means of improving the emotional adjustment of pupils. Such reasoning implies that, after being retained, a student will achieve near the middle of the grade; and he will, therefore, "find himself" personally and socially because of the recognition he receives and the confidence he feels by being able to readily handle the classwork...Students who are retained do not find themselves in the middle of the achievement distribution. They are still in the very lowest part of the class.[3]

and again:

> ...the evidence is clear that the range of ability with which a teacher must deal is not materially influenced by the promotional policies.[4]

The results of this research provide the basis for arguments put forth by some of the well-known educationalists who have taken a critical stand. One such critic whose writings have gained a great deal of credence is John J. Goodlad:

> ...the notion of moving students upward through a graded system when the achievement distribution of each "graded" class so little resembles anything that might reasonably be defined as a grade is sheer nonsense. Grade, to have meaning, denotes graded subject matter, graded textbooks, graded teachers, graded students, and graded expectations. But students in today's graded schools are not graded, as the data...clearly reveal! If the students still aren't graded after more than one hundred years of perfecting a system of graded subject matter, graded textbooks, graded expectations, and non-promotion designed to thoroughly grade them, then perhaps it's about time to quit trying!"
>
> At this point one hears that quiet voice of protest, 'Good teachers don't try to fit children to the grade. They ignore grade level expectations in seeking to deal with individuals.' What better argument against the graded system than that good teachers seek to (in fact must) ignore it?"[5]

So goes the argument. A system, the structure of which encourages homogeneity, has not achieved its goal. The research points out that in spite of the influence of a graded system, children within the system display little if any common ability. In fact, the opposite is quite in evidence. Educators, then, will have to turn their energies towards dealing with individual differences. Token efforts (such as promotional and grading policies) to deal with differences have been, in fact, an attempt to eradicate these differences and have categorically failed.

That the graded system discussed here is indicative of the Canadian scene is attested to by the Hall-Dennis Report on education in the province of Ontario, entitled Living and Learning. The report states that the type of education offered in the first part of the nineteenth century, where a poor man's child received distinctly different treatment than did the child of a gentleman, was replaced with a lock-step graded system based on egalitarian principles:

> But in the United States and most of Canada the distinction was wiped out by a single-track system, through public schools for all and superimposed high schools for more and more and eventually all.[6]

The influence of American thought on things Canadian is a special concern for the writers of Living and Learning as well as for Canadians in general.

> One matter about which they are disturbed is the economic and cultural dependence on foreign countries, particularly the United States, that present Canadian circumstances reflect.[7]

In spite of the fact that there are obvious distinctions between Canadian citizens and American citizens, many of our institutions, including the educational system, have at worst been designed upon American models and have at best been permeated by American ideas. It is not the purpose of this paper to discuss the Americanization of Canada, but it is the purpose of this brief discussion to link the designs of the two countries' systems of education and to underline the undeniable influence of American thought on Canadian schools.

The literature reveals another aspect of the question. The grading practices of the past have implications for a vitally important area in the education of children - personal development or self-concept development:

> By way of a formal definition, self-concept is the person's total appraisal of his appearance, background and origins, abilities and resources, attitudes and feelings which culminate as a directing force in behaviour. We here hold that a person's conscious awareness, what he thinks and feels, is that which primarily guides, controls, and regulates his performance and action.[8]

A link between school environment and self-concept is recognized by one of the recommendations of the Hall-Dennis Report:

> The lock-step structure of past times must give way to a system in which the child will progress from year to year throughout the school system without the hazards and frustrations of failure. His natural curiosity and initiative must be recognized and developed. New methods of assessment and promotion must be devised. Counselling by competent persons should be an integral part of the educational process. The atmosphere within the classroom must be positive and encouraging. The fixed positions of pupil and teacher, the insistence on silence, the punitive approach must give way to a more relaxed pupil-teacher relationship which will encourage discussion, inquiry, and experimentation, and enhance the dignity of the individual.[9]

As the discussion has already emphasized, grouping, grading, and promotional practices establish a set of expectations. Students placed at a particular grade level are asked to meet that set of expectations and do so in a given period of time. The literature indicates, in fact, that very few students in a classroom will match that set of expectations. According to Labenne and Greene, the effect on self-concept is twofold. The student who is at the low end of the scale is being told that he does not measure up, leaving the distinct possibility that he will think himself incapable, i.e., not able to achieve the respect of his peers, parents, teachers, the system. This negative view, then, renders him incapable of measuring up. This twofold process is then enhanced, and the research substantiates it, by the fact that the teacher will behave towards the student in a manner which will support the student's own view of himself. The implication is that a negative view of self inhibits the possibility of personal growth. If there is some potential there, the door is effectively closed; if there is no greater potential there, the individual is made to feel that somehow or other this is not a good thing and he must go through life being dissatisfied with what he is.

> Finally, Gill found patterns of achievement significantly related to the perceived self in public-school students. He concluded his paper by stating: 'The results of this study support the conclusion with such convincing uniformity that the importance of the self-concept in the educational process seems to need more emphasis than is presently given to it.'[10]

This brings the discussion to the second question. Perhaps the initial egalitarian-motivated philosophy of an equal education for all can itself be questioned. Instead of saying that people have a right to the same education, perhaps we should be saying that they have a right to different educations. Research has shown that children in school have remained different in spite of a system designed to make them the same. Perhaps the nature of things is such that people must be different. Perhaps the educational system should use this as the basic premise and take action to exploit and enhance these individual differences.

Researchers and theorists are explicit. There must be a shift in emphasis in the system to a larger and more realistic concern for the individual and his uniqueness. The implication is that such a shift will allow the tapping of potential such as has not occurred in the past.

> In individualizing teaching, the emphasis is on the pupil as a person, the teacher as a person, and the interaction that takes place between them. In such an interpersonal relationship, the pupil can face the world and accept himself in a way which facilitates release of potential.[11]

It is that promise that leads to the topic of this study. If the promise is to be fulfilled, then concrete action must be taken intelligently, sincerely, and immediately. It is the writer's feeling that a very positive step will be taken if attention is paid to a vehicle which will allow for individualization of instruction. Such a vehicle, it is felt, will involve some form of independent study, and thus research in this area is vital. That independent study is of such importance is further indicated by a research conducted by Janet Smith entitled An Enquiry into Independent Study. In her study, which deals with a senior high school in Seattle, Washington, she states that the concern for independent study "re-affirms the student's responsibility in the learning process. To learn the student must resume an active rather than a passive role."[12] Such concern is very much in keeping with sound pedagogical thinking:

> Bruner's comment about reward and punishment is relevant at this point:'One of the great problems in teaching, which usually starts with the teacher being very supportive, is to give the rewarding function back to the learner and the task.'[13]

The soundness of independent study as a means for solving the problem as stated by Bruner is supported by the findings of William M. Alexander and Vynce A. Hines in a study published in 1966, of which the purpose was to systematize information about independent study. One of the results was the formulation of the following definition:

> Independent study is considered by us to be learning activity largely motivated by the learner's own aims to learn and largely rewarded in terms of its intrinsic values. Such activity as carried on under the auspices of secondary schools is somewhat independent of the class or other group organizations dominant in past and present secondary school instructional practices, and it utilizes the services of teachers and other professional personnel as resources for the learning.[14]

To suggest that independent study as a topic of concern for educators is strictly recent (within the last decade) would be incorrect, to say the least. It must be noted that although the decade of the sixties, and especially the later sixties, has witnessed a growing interest in this area and more and more research related to the topic has been carried out, the beginnings of the movement can be traced back over many decades prior to the 60's. In the early decades of the twentieth century, one of education's most influential philosophers was making the case for the individual:

> ...the important point is the fact that Dewey looks upon all social arrangements and conditions as means and agencies for creating individuals.[15]

In a 1926 publication entitled The Rise and Progress of the Dalton Plan, the author, A.J. Lynch, describes the purpose of the Dalton Plan:

> ...the Plan claims to provide a greater amount of freedom for each individual child than the old class method provides, and is thus a reaction against the rigidity and passivity of the ordinary classroom.[16]

In a 1914 publication entitled Individual Work in Primary Schools, author C.M. Fleming describes the Winnetka Technique:

> The pupils are allowed to work at the optimal rate at each portion of the subject. In order, therefore, to economize time and allow for variations in progress, self-instruction textbooks are prepared. These books supply the explanation and instructions, and indicate the amount of practice proved necessary to assist the progress of normal, diligent children. To increase independence, access to answers is allowed for all routine-work.[17]

Both the Dalton and Winnetka plans were attempts to account for individual differences and date back to the second decade of this century. The Winnetka Technique stems from the work of Frederic Burke and Mary Ward, which began in 1912. It was they who founded the movement towards individualized instruction. Their methods were eventually undertaken by the public school system in Winnetka, Illinois. In an article in the Twenty-Fourth Yearbook of the National Society for the Study of Education, Carleton W. Washburne reports on "Burke's individual system as developed at Winnetka":

> Public schools have recently been experimenting actively along this same line in Bronxville and Dunkirk, New York; in Miami, Florida; in Peru, Indiana; in Racine, Wisconsin; and to a greater or less degree in many other places.[18]

The Dalton Plan for individualizing instruction had its beginnings about the same time as the Winnetka Technique. It was inaugurated by Helen Parkhurst in the high school of Dalton, Massachusetts. This concern for individual differences generated enough interest in these early decades to motivate the National Society for the Study of Education to devote a major portion of its 1925 yearbook to the topic - Part II of the Twenty-Fourth Yearbook is entitled Adapting the School to Individual Differences. In fact, the Dalton Plan was implemented in the St. John's High School in Winnipeg during the period 1935-1950.

These early concerns did not lead to the abandoning of the graded school system, but the idea has persisted.

The concern has been renewed with a vigour in the decades of the sixties and early seventies. The literature abounds with contemporary articles arguing the case for individualizing instruction, and great energies have been turned towards research in this area. Studies have been made on individualizing instruction in many subject areas and at all grade levels. Vehicles ranging from programmed instruction to workshop approaches have been studied. One area that deserves much more attention is the vehicle that consists of teacher-prepared material. The literature indicates that some work has been done in this regard. Joseph T. Sutton[19] conducted a study for the two-year period 1961-1963, involving a traditional grade seven mathematics curriculum and the grade seven mathematics teachers of the Valusia County, Florida schools. The materials for the independent study groups were teacher-prepared. The results were inconclusive with some control groups achieving significantly higher results, possible reasons centering around the inadequacy of the materials and the heavier workload for the teacher in independent study classes. In a report by Joseph J. Lipson and others[20] an entire program is described. This program is designated Individually Prescribed Instruction (I.P.I.) and comprises a thorough set of specifically stated behaviourial objectives for each sequential unit of a subject area curriculum such as mathematics, along with a set of diagnostic tests based on these objectives. Teachers diagnose, then prescribe materials to be studied on an individual basis. The program was implemented for the mathematics curriculum, grades 1 - 6 in one elementary school in the Horseheads Central School District in New York, during the year of September 1968 to June 1969. William F. Mead and Lawrence M. Griffin[21] concluded from results that no significant differences occurred between the control and experimental groups with respect to achievement, but there were significant gains in attitudes. A comparable study, that is one based on the concept of individual prescription of the curriculum, was conducted by Thomas Alonzo Sinks[22]. This study dealt with four subject areas of the junior high curriculum. Students working independently were compared to students working in a traditional classroom. The students, all of the seventh grade, were given the Sequential Test of Educational Progress (S.T.E.P.). A combination of surveys, interviews, questionnaires, observations, and critiques were used to determine differences in behaviour, attitude, work habits and related factors. The results suggest independent study is significantly better for achievement and for desirable changes in behaviour, attitudes, and work habits.

Although the literature offers much in the way of writings based on secondary schools and colleges, there seems to be less material available on elementary education and very little on the junior high level. This comment is supported by a study conducted by the Vancouver Board of School Trustees, which investigated the individual study program in Vancouver secondary schools:

> There are relatively few independent study programs below grade 11 and almost none below grade 9.[23]

In an article in The Mathematics Teacher, January, 1962, M.J. Brannon suggests:

> The optimum age at which to begin such a study seems to be about 15; students are rarely mature enough to benefit from the adult treatment implied by this program until the middle or end of ninth grade, while those who enter the program as seniors sometimes experience considerable difficulty in overcoming the habit of being taught![24]

The wisdom of this remark is challenged by the writer on the basis of the following comments:

> It is a fact of great importance to educators that the psychobiological changes associated with puberty take place in the later elementary, junior and early high school years. For most boys and girls, the physical and psychological changes which cluster around puberty occur while they are attending junior high school.[25]

and, from an article on Winnetka's Skokie Junior High School by Sidney P. Marland:

> We believe that self-instruction, self-motivation, and indepen-

> dent enquiry are characteristics of education to be strongly supported and enhanced by teachers and the school organization. We believe that, typically, the preadolescent or early adolescent child is struggling to separate from adult authority, to rise above conventions and to flex his individuality and autonomy. The child is experiencing a period of healthy and normal rebellion. Traditionally, junior high schools have failed to exploit this burgeonning power in the child by containing him within classrooms, periods and prescribed curricula; and by surrounding him with adults.[26]

The scenario is completed. It is felt that the arguments and discussion derived from the review of the literature present a forceful case for the undertaking of a study of this nature. It fills an obvious gap since it deals with the junior high level, it deals with the Canadian situation (and specifically the Manitoba situation), and perhaps, most important, it deals with a traditional classroom in a traditional school organization.

Footnotes

[1] Robert W. Crary, Humanizing the School (New York: Alfred A. Knopf, Inc., 1969), p. 83.

[2] Fred T. Tyler, "Intra-Individual Variability", Individualizing Instruction; the Sixty-First Yearbook of the National Society for the Study of Education, Part I (Chicago, Illinois: National Society for the Study of Education, 1962), p. 164.

[3] Walter W. Cook and Theodor Clymer, "Acceleration and Retardation", Individualizing Instruction; the Sixty-First Yearbook of the National Society for the Study of Education, Part I (Chicago, Illinois: National Society for the Study of Education, 1962), p. 204.

[4] Ibid., p. 200.

[5] John J. Goodlad, "Individual Differences and the Vertical Organization of the School", Individualizing Instruction; the Sixty-First Yearbook of the National Society for the Study of Education, Part I (Chicago, Illinois: National Society for the Study of Education, 1962), p. 220.

[6] Mr. Justice E.M. Hall and others, Living and Learning; the Report of the Provincial Committee on Aims and Objectives of Education in the Schools of Ontario (Toronto, Ontario: Ontario Department of Education, 1968), p. 67.

[7] Ibid., p. 23.

[8] Wallace D. Labenne and Bert I. Greene, Educational Implications of Self Concept Theory (Pacific Pallisades, California: Goodyear Publishing Company, 1969), p. 10.

[9] Op. cit., p. 14.

[10] William Watson Purkey, Self Concept and School Achievement (New Jersey: Prentice-Hall, 1970), p. 18.

[11] Robert T. Dehaan and Ronald C. Doll, "Individualization and Human Potential", Individualizing Instruction; the 1964 Yearbook of the Association for Supervision and Curriculum Development (Washington, D.C.: National Education Association, 1964), p. 18.

[12] ERIC Document 001203
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[13] James B. Macdonald, "An Image of Man: the Learner Himself", Individualizing Instruction; the 1964 Yearbook of the Association for Supervision and Curriculum Development (Washington, D.C.: National Education Association, 1964), p. 41.

[14] ERIC Document 010043
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[15] Robert J. Roth, John Dewey and Self-Realization (New Jersey: Prentice-Hall, Inc., 1912), p. 11.

[16] A.J. Lynch, The Rise and Progress of the Dalton Plan (London: George Philip & Sons, 1926), p. 2.

[17] C.M. Fleming, Individual Work in Primary Schools (London: George C. Harrap & Company, 1934), p. 25.

[18] Carleton W. Washburne, "Burke's Individual System as Developed at Winnetka", Adapting the School to Individual Differences, the Twenty-Fourth Yearbook of the National Society for the Study of Education, Part II (Bloomington, Illinois: Public School Publishing Company, 1925), p. 78.

[19] ERIC Document 016609
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[20] Joseph J. Lipson and others, The Development of an Elementary School Mathematics Curriculum for Individualized Instruction (Pennsylvania: Pittsburgh University).

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[26] Sidney P. Marland, "Winnetka's Learning Laboratory", Educational Leadership (April, 1963), p. 459.

CHAPTER III

PROCEDURE

Setting

The study was conducted in a junior high school in a suburb of Winnipeg, Manitoba. The school is located in a middle-class community, with some affluent neighbourhoods in which many families have high incomes. The population of the school was approximately 600, with the grade 7 enrollment being about 180 students. The organization for instruction utilized by the school was of the standard graded form. Classes were grouped heterogeneously with classroom size averaging about 30 students. The school day was divided into eight periods of about 38 minutes for instructional purposes.

In the mathematics department, there were three teachers, each one taking responsibility for a complete grade level. This study involved the grade 7 mathematics teacher, who had been teaching for about ten years in this school and who had taught the new mathematics course at the grade 7 level for the six years since its inception. He was, therefore, an experienced teacher and was most eager to experiment with an alternative method. It might be added that this teacher was a competent mathematics teacher by any standard and remarkably adaptable as was to prove out in the course of events.

At the time the study was conducted, the principal had been in the school for one year and was beginning his second. The atmosphere created by the school's administrative personnel reflected a concern for orderly and businesslike conduct in the school.

Sample

The population from which the sample was selected was the grade 7 enrollment in the school. Two classrooms were chosen by the participating teacher, one designated the control group and one designated the treatment group. Since the grade 7 classes had already been set up, it was not possible to use a random process in conducting the sample. However, in this school the students were randomly assigned to classes in an attempt to devise a heterogeneous group. The students in each classroom ranged from low to high in terms of achievement. Although the control and treatment groups originally contained 31 and 32 students respectively, a lack of background information, transfers during the school year, and, in the treatment group, the failure of two students to take the post-test reduced the sample sizes to 23 for the control group and 24 for the treatment group.

In this study, it was necessary to collect data in the form of pre-test and background information. This was so, since it had to be established if there were any significant differences between the mean academic ability in mathematics, reading, and intelligence of the two groups. For the purpose of this comparison, the two mathematics scores (mathematical concepts and problem solving) from the Canadian Test of Basic Skills were used. The test had been administered to these students in their grade 6 year. In addition, the I.Q.'s, as measured at the beginning of grade 7 by the Otis Quick Scoring Mental Ability Test, Beta form, were recorded as were the reading scores from the language arts section of the Canadian Test of Basic Skills that was also administered at the beginning of the grade 7 year.

The Student's t distribution was used as a criterion in determining whether the following null-hypothesis would be accepted or rejected:

> At the beginning of the experiment there is no significant difference between the: -
>
> (i) intelligence, as expressed by pre-test mean I.Q. scores, of the treatment group (independent study) and the control group (group instruction);
>
> (ii) achievement, as expressed by pre-test mean C.T.B.S. mathematics scores, of the treatment group and the control group;
>
> (iii) reading ability, as expressed by pre-test mean C.T.B.S. reading scores, of the treatment group and the control group.

The alternative hypothesis was that there was a significant difference. For the purpose of testing this null-hypothesis, it was assumed that the samples were drawn from two different populations. If there were any differences in the two samples, it would then mean that the samples themselves contained the differences, since the fact that a single population was sampled means that there were no population differences. The statistic t was determined by the following formula. The .05 level of significance was used with a two-tailed test.

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{(n_1 - 1) + (n_2 - 1)} \cdot \frac{n_1 + n_2}{n_1 n_2}}}$$

1

A complete list of the formulae used in this study, the pre-test data, and the pre-test statistical computations are presented in Appendix A. The statistical information on the pre-test data is given in summary in the following table:

TABLE I

SUMMARY OF PRE-TEST DATA

Measure	Statistic	n = 23 Control		n = 26 Treatment
I.Q.	$\bar{x}$	110.4		110.8
	s	6.9		6.9
	t		.2031*	
Mathematical Concepts	$\bar{x}$	6.68		6.70
	s	1.4		1.5
	t		.0475*	
Problem Solving	$\bar{x}$	6.7		6.8
	s	1.1		1.4
	t		.2857*	
Reading	$\bar{x}$	7.2		7.1
	s	1.2		1.1
	t		.2940*	

t .05 with 47 degrees of freedom = 2.014
* not significant at the .05 level

As a result of statistical analysis, the null-hypothesis was not rejected. It was concluded that the two samples were not significantly different with respect to intelligence, mathematical ability, and reading skills. The reading score was considered here as a matter of interest, because the treatment group was required to do a great deal of reading in their mathematics class during the course of the year. In fact, the means and standard deviations are so close that the two distributions must be considered to provide a good basis for post-test comparison.

Vehicle

The grade 7 mathematics course was broken into the following topics or areas of interest: (1) fractions, (2) factoring whole numbers, (3) number sentences, (4) properties of whole numbers, (5) ratio and per cent, (6) geometric description, (7) geometric measurement, and (8) geometric construction.

Self-study units were then devised for each topic. These units were presented in the form of workbooks. Instruction within the workbook comes about partly through example plus explanation, partly through developmental questioning. A set of workbooks on a given topic is accompanied by six answer booklets. It would be very difficult to describe the workbooks with respect to how structured or unstructured they are. Appendix B contains a sample workbook on the topic of properties of whole numbers. This certainly is not purely representative of all the booklets, but it contains all of the elements of the various booklets. This particular workbook covers content that ranges from quite simple material with concrete examples to a rather complicated section entitled "Abstract Mathematical Systems".

The basic approach in writing the workbooks was simply to produce a very elaborate set of lesson plans which were worded such that they could be used by the students to teach themselves. The writer's experience in teaching the grade 7 course for four years allowed for anticipation of some problem areas; that is, areas that required a more detailed line of questioning, a good diagram, or some direct example or explanation. Of course, each part of a unit was produced with a specific purpose in mind, but no set of detailed behaviourally-stated objectives was given to the students. Instead, a list of questions accompanied each workbook. The student could determine if he or she had, in fact, learned what was expected to be learned in the workbook by completing this self-test.

Treatment

The treatment group was put on an independent study program. For these pupils, the initial exposure to the material was through self-instruction using the workbooks. The teacher acted as a resource person for individuals who came across problems. As time passed, he could deal with small groups who had similar problems or, if necessary, call for the attention of the entire class. Students were permitted to work through the workbooks at their own rate. A minimum amount of work was set to be covered in each workbook, usually the first half. This minimum amounted to the basic skills and concepts related to the topic covered in that workbook. The workbooks, however, contained enrichment for those who moved ahead; and, from time to time, a complete booklet of enrichment was prepared for those students who forged ahead. Answer booklets were provided and students were expected to check their own work, do the self-test, and be prepared for a test that would be administered when the last of the group had completed a topic. It was found that a very large gap soon opened up in terms of time required to cover the material, so that students were allowed to proceed to the next topic before the slowest students had completed the previous one. An anecdotal report is given by the participating teacher in Appendix C.

Method

The treatment was introduced to the experimental group at the end of October. The first two months were spent in finding a school where the study could be made, briefing the participating teacher, and preparing a backlog of materials. The participating teacher also expressed a desire to postpone the introduction of the treatment until he had an opportunity to develop a routine with the class. During this time he covered the topic of decimals. For the balance of the year, the treatment was in effect. Both control and treatment groups were exposed to the basic grade 7 mathematics content.

During the last week of classes in June, the treatment and control groups were post-tested with the following instruments: (1) Canadian Test of Basic Skills mathematical concepts, (2) Canadian Test of Basic Skills problem solving, (3) Canadian Test of Basic Skills reading, and (4) a teacher-prepared "new math" supplemental test for areas not covered well by the Canadian Test of Basic Skills (geometry, number sentences, properties of whole numbers, ratios). This test can be found in Appendix D. The two groups were compared with respect to these four measures, using the Student's t distribution as a criterion, the statistic t being computed with the formula below. Again the .05 level of significance was used with a two-tailed test to determine whether the null-hypothesis $H_0 : \mu_1 = \mu_2$ would be accepted or rejected in favour of the alternative hypothesis $H_1 : \mu_1 \neq \mu_2$. The use of the statistic t is further supported here in view of the fact that both groups achieved normal progress over the year as measured by the mathematical scores of the Canadian Test of Basic Skills. In effect, enough progress was made to allow for the possibility of a significant difference developing between the control and treatment groups on post-test measurements.

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{(n_1 - 1) + (n_2 - 1)} \cdot \frac{n_1 + n_2}{n_1 n_2}}}$$

2

A complete list of the formulae used in this study is presented in Appendix A. The post-test data and the post-test statistical computations are shown in Appendix E.

As a matter of interest, the students in the treatment group were asked to state their feelings about the method of studying mathematics they had experienced that year. This was accomplished by means of a simple verbal request from the participating teacher asking the students to simply state their feelings on paper. Anonymity was guaranteed. The poll was conducted in this manner because anything more formal was thought by the writer to be beyond the scope of this study. The poll was thought worth conducting, however, as a matter of interest.

Summary

This study was conducted in a junior high school in a suburb of Winnipeg. This school was organized in the traditional group instruction manner. The experiment population was the grade 7 enrollment of about 180 students from which two classes of about 30 students each were chosen. One was used as a control group, the other underwent the experimental treatment. Each of these two heterogeneous classrooms was found to be statistically comparable through analysis of pre-test data which included: (1) the mathematical concepts and problem solving scores from the Canadian Test of Basic Skills retrieved from grade 6 records; (2) the I.Q. scores from the Otis Quick Scoring Mental Ability Test, Beta form, administered at the beginning of grade 7; and (3) the reading scores from the language arts portion of the Canadian Test of Basic Skills, administered at the beginning of grade 7.

The control group was taught from October through June with the traditional group instruction method of teaching, while the treatment group was set on an independent study program using the self-instruction booklets prepared by the writer. The students in the treatment group worked through each booklet completing at least the minimum amount of work that was specified by the teacher. This minimum was clearly indicated by the teacher. Each student was expected to mark his own work as he completed each assignment in the self-instruction booklet. A test was then administered to the group when the slowest of the group had completed that booklet. The teacher spent his time helping individual students or small groups with areas of difficulty that arose in each booklet. The course content for the two groups was identical, comprising the standard grade 7 topics prescribed by the Department of Education in this province. The same grade 7 teacher was in charge of both groups for the duration.

The length of time that the experiment ran was thought to provide for normal progress to be made. Post-test data was acquired through the use of the following instruments: (1) the mathematical concepts and problem solving portions of the Canadian Test of Basic Skills, (2) the reading portion of the Canadian Test of Basic Skills, and (3) a teacher-prepared "new math" test for areas not well covered by the Canadian Test of Basic Skills. The two groups were compared on the basis of these four measures using the Student's t distribution. A two-tailed t test was used with a .05 level of significance. As well, an informal poll was taken of the opinions of the students in the treatment group.

Chapter III has described the procedures used in this study. Chapter IV will deal with a description and explanation of the results. Implications of these results will then be discussed in the final chapter.

Footnotes

[1] Robert G. Steel and James H. Torrie, *Principles and Procedures of Statistics* (New York: McGraw-Hill, 1960), p. 73.

[2] *Ibid*., p. 73.

CHAPTER IV

RESULTS

The observations that were made in the experiment can be classified into two categories: formal and informal. The formal observation refers to the post-test data that pertains to the testing of the null-hypothesis:

> At the end of the experimental period there will be no significant difference, as expressed by post-test mean scores, between the control group and the treatment group.

This data includes the mean scores achieved by each group on each of the four measuring instruments, namely (1) the Canadian Test of Basic Skills mathematical concepts test, (2) the Canadian Test of Basic Skills problem solving test, (3) the Canadian Test of Basic Skills reading test, and (4) the teacher-prepared "new math" supplementary test. The informal observations refer to two items. One is the information gathered when the treatment group was polled for opinions on the experimental method; the other is a comparison that was made between the most able students of both groups and the least able students of both groups. The students with the top seven I.Q. scores from the treatment group were compared to their counterparts in the control group on the basis of the four post-test measurements that were taken. As well, the students with the lowest five I.Q. scores from the treatment group were compared to their counterparts in the control group on the basis of the four post-test measurements.

This second item in the informal observations brings to light interesting information about the relative post-test strength of the two extreme ability ranges.

Formal Observation: Data

The post-test data is presented in full in Appendix E. A summary of results is given in the table below:

TABLE II

SUMMARY OF POST-TEST DATA

Measure	Statistic	n = 23 Control	n = 24 Treatment
Teacher-prepared Supplemental Test	$\bar{x}$	32.2	33.0
	s	9.8	12.2
	t	.2581*	
Math Concepts	$\bar{x}$	8.02	7.99
	s	1.4	1.5
	t	.0698*	
Mathematical Problem Solving	$\bar{x}$	7.9	8.0
	s	1.3	1.4
	t	.2500*	
Reading	$\bar{x}$	7.84	7.75
	s	1.27	1.29
	t	.2368*	

t .05 with 45 degrees of freedom = 2.016
* not significant at the .05 level

Formal Observation: Conclusion

Values of t were not extreme; therefore, the null-hypothesis $H_0 : \mu_1 = \mu_2$ was accepted. In other words, at the end of the experiment the two samples contained no differences due to treatment. It can be concluded that the students in this study faired as well on independent study as in the traditional group-instructed classroom.

Informal Observations: Data

Two informal observations were made. Firstly, the extremes of the two groups were investigated with respect to the post-test measures, and, secondly, the treatment group was polled with regard to their opinion about the treatment they underwent.

A. The data from the investigation of the top seven I.Q.'s and the bottom five I.Q.'s of the control and treatment groups is presented in Appendix F. A summary is given in the following table:

TABLE III

SUMMARY OF POST-TEST DATA FOR TOP SEVEN STUDENTS

		Post-test	
		Control	Treatment
Math Concepts	$\bar{x}$	9.2	9.7
Problem Solving	$\bar{x}$	8.7	9.4
Reading	$\bar{x}$	9.1	8.9
Supplementary Test	$\bar{x}$	37.4	48.1

TABLE IV

SUMMARY OF POST-TEST DATA FOR BOTTOM FIVE STUDENTS

		Post-test	
		Control	Treatment
Math Concepts	$\bar{x}$	6.6	6.2
Problem Solving	$\bar{x}$	7.0	7.2
Reading	$\bar{x}$	6.3	6.0
Supplementary Test	$\bar{x}$	24	18

It was noticed that the top I.Q. group seemed to do better in the treatment than in the control group in three out of four tests. The mean mathematical concept scores was .5 grade level higher for the treatment group. The problem solving mean scores differed by .7 grade level in favour of treatment. Most interesting of all, the treatment group achieved 10.7 responses better on the average than did their counterparts in the control group on the teacher-prepared supplementary test.

Equally interesting was the pattern that seemed to emerge between the two groups on the lower extreme. The treatment group did not achieve as well as the control group on three out of four of the tests. The treatment group was .4 grade level behind the control group on mathematical concepts, .3 behind on the reading scores and 6 responses behind on the teacher-prepared supplement.

B. The students in the treatment group were asked to express an opinion about that method for studying mathematics. No exact direction was given and anonymity was guaranteed in the hopes of getting an original response. The responses were solicited during the post-test period. Responses were judged to be favourable or unfavourable by the investigator. The responses are presented in Appendix G. A summary of the responses is provided in the table that follows:

TABLE V

SUMMARY OF STUDENTS' OPINIONS

	Number of Responses
Favourable	15
Unfavourable	6
Mixed	3
Total	24

<u>Informal</u> <u>Observations</u>: <u>Conclusions</u>

A. No definite statement can be made with regards to conclusions since no statistical analysis was attempted because of the small number of students involved. The patterns that have emerged with the two extreme groups, however, do deserve close attention. The superior achievement of the top students in the treatment group may have been a matter of chance. If not, then this is a very important outcome. Of equal importance is the poor showing of the low students in the treatment group. Of special interest to the writer were the results of the teacher-prepared supplementary test. This is so since, by and large, such instruments form the basis for evaluation in the mathematics classroom. Such tests reflect very closely the content of the course as taught by a particular teacher. It was here that a real superiority seemed to show up in favour of the top seven students in the treatment group - in fact, 10.7 more responses out of a total possible of 61.

B. The treatment group for the most part seemed to find some merit in the experimental method. The favourable responses which include phrases such as "learned a lot", "interesting", "wasn't too boring", "if you did bad it's all your own fault", "work at your own speed", verify that even a limited attempt at individualizing such as this presses home quite forcefully the arguments in the review of the literature (Chapter II). The unfavourable responses must also be heeded. Some students obviously do not feel comfortable in a situation such as the one described here.

Summary

The results of this study were divided into two classifications: formal observations and informal observations. The formal observations dealt with that portion of the data that pertained to accepting or rejecting the null-hypothesis of the experiment, namely the mean achievement scores of the two groups on the four measuring devices. The informal observations dealt with the results of an opinion poll taken among the treatment group and a comparison between the top seven students and the bottom five students (as measured by I.Q. scores) of the control and treatment groups with regards to their achievement on the four post-tests that were administered.

The main purpose of this study was to determine whether there was a significant difference between the control and treatment groups as expressed by post-test mean scores. The data presented in the formal observation indicates that at the end of the experimental period no significant difference existed between the two groups in any of the four measurements. The null-hypothesis was accepted, leading to the conclusion that the treatment group faired as well as the control group over the duration of the experiment.

Two informal observations were made. One was an opinion poll taken among the students of the treatment group to try to determine their likes or dislikes about the program. The results indicated that most had something positive to say for the independent study method, but there were some who did not like it. The second informal observation dealt with a comparison between the two groups using just the seven most able students and the five least able students from each group. Using the results from the four post-tests that were administered as a comparison, it was seen that the top students of the treatment group seemed to achieve higher than their counterparts in the control group on three out of the four measurements, while the lower students in the treatment group did not achieve as well as did their counterparts in the control group on three out of the four tests. Although this outcome was not included in the formal analysis of the data, it is thought to be worth noting.

CHAPTER V

DISCUSSION

This study was motivated by a situation experienced by the writer in trying to deal with individual differences in the classroom. The apparent ability of some students at the junior high level to deal with a mathematics curriculum using an independent study program aroused the writer's interest in preparing this formal study for statistical analysis. The purpose of this study was expressed in the form of the following null-hypothesis:

> At the conclusion of the experimental teaching period, there is no significant differences between the achievement as expressed by post-test mean scores of the experimental group (independent study) and those of the control group (group instruction).

A secondary purpose of the study was to produce a set of materials in the form of workbooks that could be used as a vehicle for implementing an independent study program in the classroom. As a sub-problem the treatment group was polled informally to determine student opinion about this method.

The need for such a study was underscored in the review of the literature. As early as the turn of the century, educators were addressing themselves to the problem of individual differences in the classroom. The Dalton and Winnetka plans were early attempts to find a solution. The efforts of such attempts find their basis in the research that has indicated that the practice of using grade level demarkations simply has not been successful in its aims. The early research has been corroborated by the research that has occurred in recent decades as the question of individual differences has been attacked with renewed vigour. Although a great deal of attention has been paid to this problem, especially in the United States, there is a need for the present study. Firstly, it deals with the junior high level, an area that has not received the attention that has been shown to the elementary and senior high levels. Secondly, it deals with teacher-prepared materials rather than commercially-prepared materials; and finally, it deals with the Manitoba scene.

The study was conducted in one junior high school that was located in a basically middle-class community in suburban Winnipeg. This school could best be described as traditional in both physical structure and organization for instruction. The grade 7 mathematics teacher chose one of his heterogeneous classes as a control and another as a treatment group. Analysis of the pre-test data led to the conclusion that the two groups were, in fact, statistically comparable. The independent study workbooks were prepared by the writer and administered by the participating teacher to the treatment group. Meanwhile, the participating teacher used the traditional group instruction method with the control group. The experiment ran through the school year, and at the end of June post-test data were collected. The post-test measures were obtained with the following instruments: the Canadian Test of Basic Skills mathematical concepts test, problem solving test and the reading test from the language arts section, and a teacher-prepared "new math" supplementary test.

The post-test data were analysed using the Student's t distribution. No values of t were found to be significant at the .05 level for any of the four measures. This led to the accepting of the null-hypothesis and the conclusion that the treatment group faired as well as the control group. As a matter of interest a comparison was made between the most able students in each group in the first case and between the least able students in each group in the second case. The results of this comparison seemed to indicate that the top seven students in the treatment group achieved higher scores in three out of the four measures than their counterparts in the control group, while the lowest five students in the treatment group did not achieve as well as their counterparts in the control group on three out of the four measures. No formal analysis was made on the grounds of the very small sample these groups comprised. The opinion poll that was taken among the treatment group showed that, for the most part, the students involved found some merit in the independent study method, but there were some that felt it was a negative experience.

These results have important implications for teachers in the classroom as well as for any further research that might be carried on. The balance of this chapter will be devoted to a discussion of these implications and to a presentation of recommendations based on the outcomes of this study.

Implications

The major purpose of this study was to determine if students working on an independent study program would achieve as well as students who were taught in the traditional group-instruction, lecture-oriented method. Analysis of the post-test data has shown that the mean achievement of the control and treatment groups was, in fact, comparable for the four measures that were taken. Since both groups in the study comprised students ranging from low ability to high ability in mathematics, and since both groups faired equally well at the end of the experimental period, some important implications can be drawn. These implications are underscored by the fact that the study was conducted in a very traditional school building, where the organization for instruction was committed to the "graded" system.

I. The first implication relates to the fact that students can learn equally well in a situation where they are not totally and directly dependent upon the teacher for instruction. The implication is that some drastic alterations can occur in classroom organization (even within a standard mathematics curriculum and in a traditional school structure). Not all students in the class have to pay attention to the same lesson at the same time. This statement is of extreme importance since this is the very root and core of the problem of dealing with individual differences in the classroom.

The most immediate effect of the breaking of the lock-step nature of the lecture-oriented method of instruction is that capable students are free to move along in the work while weaker students can command the attention of the instructor for help in specific problem areas and can work at a pace that is compatible with their own abilities. When this happens the role of the teacher in the classroom changes. He becomes a resource person. As the students work through the materials, the teacher addresses himself to individuals with problems or groups of students with a similar problem. Thus, the normal situation is reversed. The students contact the work first, the teacher second.

Many other factors remain the same. The teacher still teaches (only within an altered organizational framework). The teacher still measures and evaluates progress. The teacher still is the responsible adult in the classroom; it is still his job to maintain control of the group. He must still keep records, make reports and meet with parents.

The alteration of instructional organization has obvious implications from the students' point of view. The students' role has changed from being by and large a passive one, to one that demands his or her active participation. The student must shoulder a more immediate part of the burden of responsibility for any learning that is to take place. He or she must be able to get started at a task and follow it through to completion. When difficulties arise, the student must aggressively seek the aid of the teacher. This suggests a certain set of values that the student must develop in order to achieve successfully. He or she will very quickly find that real learning must occur since the test results for that unit will expose laxity or copying. Since he or she is actively involved, the immediate blame for lack of progress lies with the student, even though the teacher is ultimately responsible.

The alteration of classroom organization has impact on the relationship between the teacher and student. The teacher must remain in close contact with all the various things that his students are doing. The most immediate pressure on him is one of keeping comprehensive, accurate, up-to-date records on each individual. There must be an understanding on the student's part that the teacher knows him, knows exactly what he is working on, and cares very much about how he is making out in his work.

II. The second implication pertains to the informal observation that was made about the achievement of the two extreme ability groups. The main thrust of this study dealt with the mean achievement of a heterogeneous group and certain conclusions were drawn. However, when the most able students in either group were compared and when the least able studends were compared, some interesting trends seemed to develop. The brighter students seemed to achieve higher results in the independent study group than in the control group, with the opposite occurring for the weaker students.

The implication here is that some students can do better when working on an independent study program, while others may, in fact, do worse. It seems to be correlated to relative intelligence; however, this study can not and does not draw any conclusions in this regard. If it is true, however, it seems that the teacher's reaction is obvious. Find those students in the group who will not succeed and provide an alternative approach. Perhaps much more structure and direction is needed in this group as well as a very much simplified program. For those students who do extremely well on independent study, the implication is that the teacher is faced with the responsibility of extending the program into an enrichment phase.

At any rate the situation becomes somewhat more complicated since the teacher is faced with the reality of the varied needs in his classroom. To have a successful program he must respond. To respond he needs to develop two skills that formerly he probably did not call upon - diagnosis and prescription. The teacher must develop the tools and skills of diagnosing weaknesses and strengths and must create the vehicles through which he can prescribe remediation and extension.

III. The third implication arises from the opinion poll that was taken among the treatment group. There seems to be a similar pattern here as was exhibited by the achievement of the two extremes of ability range in the treatment group. Some students liked the independent study method while some did not. There is no attempt on the part of this study to link these opinions to ability or achievement, but the implication is once again that there are some students better suited to this approach than others. In fact, there may be some students who definitely are not suited to this approach. The teacher must be cogniscent of such likes and dislikes amongst his students and must be prepared to take this into consideration when planning a program or set of programs for his students.

IV. The fourth implication relates to the materials that were prepared for the use of the treatment group. Teacher-prepared materials can provide a viable basis for an independent study program. The dollar cost of such materials is minimal, which should attract the attention of school administrations. The real cost, however, is found in the labour that is required of the teacher in creating these materials and putting them into a usable form. They are, on the other hand, limited in scope only by the initiative and imagination of the person creating them. Since such materials are inexpensive, and it has been shown that the set developed for this study is usable, it seems as if a teacher willing to put forth the effort could, over a period of time, develop a very comprehensive program which would include remedial as well as enrichment materials.

In summary, the outcome of this study implies that since students can follow a program in mathematics successfully on independent study, there can be some serious alterations made in classroom organization. There seems to be an indication that some students can do very well, while others are less successful than they might have been in a traditional classroom. Also, some students find the independent study method enjoyable, while others do not. The suggestion is that these discrepancies should be considered by any teacher implementing such a program. Although the set of materials developed for this study proved to be usable in the classroom, the implication is that the program could be extended to include remedial as well as enrichment materials.

Recommendations

The outcomes of this study led to the following recommendations. These recommendations are listed in two parts - those that pertain to using an independent study approach in the classroom and those that pertain to future research projects.

Classroom. One of the first things that was learned during the course of the experimental period was that a great deal of time and effort is involved in preparing the independent study workbooks. It was difficult for the writer to make sure that work was made available for those students who worked very hard and finished the topics ahead of their classmates. Therefore, it would be wise to have all of the materials to be used in the program prepared in advance, well in advance. It also became obvious very quickly that work presented at the "average" level was not sufficient for the better students. The remedy for this is, of course, to have ample material prepared for use as enrichment.

By the time that the experimental period drew to a close, another factor surfaced. The group really was heterogeneous and there were some students in the class who were not up to dealing with the content of a regular grade 7 program. It is recommended as a result of the real difficulty these students faced that steps be taken to include in such a program provision for those students with weak mathematics backgrounds. It is necessary to have materials prepared that are remedial in nature, and to use these materials in a different approach than the very independent one that was used in this study. Such students as these seem to need much more structure in their mathematics period and very much of the teacher's time and attention. One suggestion is to group the weaker students and devise appropriate activities for them as a group with the teacher taking a very prominant part in presenting the materials to them, in checking to see that work is completed in a given time period, in checking the accuracy and correctness of assignments, and in closely supervising tests.

Another recommendation pertaining to the materials themselves is to make sure that each workbook is accompanied by ample review materials for the pre-test period and that remedial materials be made available for that topic should the student not succeed on the test. It is also felt that since the students are primarily responsible for completing and marking their own work, that the teacher should be responsible for marking the pre-test review work that the students do. This will keep him in contact with how much is actually being learned by the students, give him an opportunity to talk with them, and will provide a good chance for the teacher to clear up difficulties with a short lesson and/or to assign further review work.

The testing procedure presents a problem that reflects the overall manner in which the program is organized. It is recommended as a result of the experience of this study that individuals be allowed to write a test when they are ready and have shown themselves to be so to the teacher's satisfaction. This allows for a very flexible approach since no one will have to wait for his classmates to catch up before writing a test. However, it then becomes necessary to have several different forms of the same test for each topic to avoid familiarity setting in. If this recommendation is followed, the program should flow smoothly along for each individual, assuming the materials have been prepared in advance and assuming that they are appropriate for the abilities of that student.

Another problem that arises has to do with the record keeping that is necessary. It is a recommendation of this study that if a teacher implements a similar approach to the one used in this study, then he should devise some detailed method of keeping records on each student's progress. What work has been completed with what kind of results, and, most important, in what period of time? The question of time presents or rather reflects a major problem. It is necessary that all students working independently be clearly aware of the work that has to be accomplished over the year, and that they be given some indication of a reasonable schedule for completing each topic in order to accomplish this.

The mathematics program, like any program of studies, can only be as good as its design. It is recommended that the materials for an independent study program be developed as a complete entity with end goals in mind. Perhaps it is redundent or even unnecessary to be commenting on this factor, but a great deal depends upon the nature of the materials if the students are going to make the kinds of connections from topic to topic that are necessary to produce a sound knowledge of mathematics. It is recommended that the topics for study be carefully linked or interwoven in a spiral effect leading to some ultimate end goal such as understanding a skill or concept. Such a discussion is more properly the domain of a course in curriculum construction, but the idea is so important to the basic foundation of an independent study program - namely the materials it uses - that it is worth mentioning here.

Further research. This study singled out one aspect of the classroom to investigate - mean achievement. During the course of the investigation, several interesting questions came to light, questions that deserve the attention of further research.

The recommendation is that a similar study be conducted at the junior high level, but that the base of the study be extended to include a larger sampling population. Several different schools around the city should be involved and one or more teachers in each school should handle a control and treatment group. In this manner, the results can be generalized to a larger population allowing for a more forceful statement of conclusion. As well, the analysis should be handled in such a manner that multiple variables can be compared both between the control and treatment groups, and within the treatment and control groups. Such a possibility exists with the statistical procedure of analysis of co-variance. Some attention should be paid to the measuring instruments, mainly the "new math" test. A well-validated and notedly reliable test should be secured for this part of the measuring process. As part of this study, several variables other than achievement could be considered. Of great interest would be information of the effect of reading skills, student attitude, student personality traits, teacher attitude, teacher personality, and physical structure of the school; and, of course, the independent study materials could be of a form other than the workbooks used in this study.

In summary, it is recommended that the set of materials be extended into remedial and enrichment phases and that ample review materials be provided for each unit. Students should work through the program individually, writing tests on each topic as they prove themselves to be ready. The teacher is charged with the task of keeping detailed records on student progress and with being in touch with his pupils to the degree that he can by marking the review materials and tests himself.

As a result of the outcomes and experience gained from this study, it is felt that a further study should be conducted. This study should be designed to include a much larger population and should also include several variables that could be compared both between and within the control and treatment groups.

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APPENDIX

APPENDIX A: List of Formulae, Pre-est Data, and Statistical Computations

List of Formulae

The following formulae are taken from Steel and Torrie: Principles and Procedures of Statistics:

n = sample size

$$\bar{x} = \text{mean} = \frac{\Sigma X}{n}$$

$$s = \text{standard deviation} = \sqrt{\frac{\Sigma X^2 - \frac{(\Sigma X)^2}{n}}{n - 1}}$$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s_{\bar{d}}}$$

$$s_{\bar{d}} = \sqrt{s^2 \ x \ \frac{n_1 + n_2}{n_1 n_2}}$$

$$s^2 = \text{pooled variance} = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{(n_1 - 1) + (n_2 - 1)}$$

Control Group

SUBJECTS	I.Q.	MATH CONCEPT	MATH PROBLEM SOLVING	READING
1	105	6.2	5.6	6.1
2	116	7.0	6.8	8.0
3	110	7.6	6.5	7.8
4	120	8.3	6.5	7.1
5	104	6.1	7.3	6.2
6	122	8.3	8.4	7.8
7	120	6.7	7.3	8.0
8	109	7.8	7.7	7.0
9	109	6.4	6.1	8.5
10	96	5.7	5.5	5.3
11	109	5.4	4.1	5.5
12	105	3.6	4.9	6.3
13	119	9.2	7.5	9.0
14	109	7.5	6.9	8.3
15	93	4.6	6.3	5.0
16	122	7.6	8.4	9.9
17	106	5.2	6.5	7.1
18	112	8.3	7.5	7.6
19	116	6.9	6.5	7.0
20	124	7.9	7.3	8.6
21	95	4.2	5.6	6.5
22	100	6.6	6.6	6.3
23	124	6.9	7.5	7.1
total	2539	154.0	153.3	166.0
$\bar{x}$	110.4	6.68	6.7	7.2
s	6.9	1.4	1.1	1.2

<u>Treatment Group</u>

SUBJECTS	I.Q.	MATH CONCEPT	MATH PROBLEM SOLVING	READING
1	104	5.6	5.8	7.2
2	108	6.9	6.5	6.9
3	119	7.9	7.5	8.1
4	107	6.7	5.6	6.7
5	114	6.6	4.9	7.4
6	110	7.3	6.1	7.4
7	115	7.8	6.1	7.7
8	95	6.9	6.1	5.0
9	103	7.3	6.8	7.5
10	122	8.9	9.2	8.8
11	110	4.6	5.8	6.9
12	108	6.9	7.1	7.5
13	90	3.2	5.3	5.0
14	102	4.2	4.1	6.5
15	123	6.7	7.8	9.4
16	112	7.6	6.8	7.4
17	128	8.2	9.4	9.4
18	124	8.9	9.4	8.3
19	103	6.6	6.8	7.2
20	95	5.4	5.8	7.1
21	116	6.2	7.1	8.2
22	142	8.5	9.2	9.2
23	101	4.4	5.8	5.7
24	108	7.3	6.8	7.9
25	107	7.1	7.1	7.4
26	114	6.7	7.1	8.1
total	2880	174.4	176.0	183.9
$\overline{x}$	110.8	6.70	6.8	7.1
s	6.9	1.5	1.4	1.1

I.Q.

Calculation of sample mean and standard deviation:

Control: - ($n_1 = 23$)

$$\bar{x}_1 = \frac{2539}{23} = 110.39 \approx 110.4$$

$$s_1 = \sqrt{\frac{281317 - \frac{(2539)^2}{23}}{22}}$$

$$= \sqrt{46.98} = 6.85 \approx 6.9$$

Treatment: - ($n_2 = 26$)

$$\bar{x}_2 = \frac{2880}{26} = 110.76 \approx 110.8$$

$$s_2 = \sqrt{\frac{320194 - \frac{(2880)^2}{26}}{25}}$$

$$= \sqrt{47.14} = 6.86 \approx 6.9$$

Calculation of t value for I.Q. scores:

$$(n_1 - 1)s_1^2 = \Sigma X^2 - \frac{(\Sigma X)^2}{n_1} = 281317 - 280283.5 = 1033.5$$

$$(n_2 - 1)s_2^2 = \Sigma X^2 - \frac{(\Sigma X)^2}{n_2} = 390194 - 319015.4 = 1178.6$$

$$t = \frac{110.8 - 110.4}{\sqrt{\frac{1178.6 + 1033.5}{25 + 22} \cdot \frac{26 + 23}{26 \text{ x } 23}}}$$

$$= \frac{110.8 - 110.4}{\sqrt{\frac{2212.1}{47} \cdot \frac{49}{598}}}$$

$$= \frac{.4}{\sqrt{47.1 \text{ x } \frac{49}{598}}}$$

$$= \frac{.4}{\sqrt{3.86}}$$

$$= \frac{.4}{1.97}$$

$$= .2031$$

Mathematical Concepts

Calculation of sample mean and standard deviation:

Control: - ($n_1 = 23$)

$$\bar{x}_1 = \frac{154.0}{23} = 6.68$$

$$s_1 = \sqrt{\frac{1075.96 - \frac{(154.0)^2}{23}}{22}}$$

$$= \sqrt{2.04} = 1.43 \approx 1.4$$

Treatment: - ($n_2 = 26$)

$$\bar{x}_2 = \frac{174.4}{26} = 6.70$$

$$s_2 = \sqrt{\frac{1221.18 - \frac{(1744)^2}{26}}{25}}$$

$$= \sqrt{2.21} = 1.48 \approx 1.5$$

Calculation of t value for mathematical concepts scores:

$$(n_1 - 1)s_1^{\,2} = \Sigma X^2 - \frac{(\Sigma X)^2}{n_1} = 1075.96 - 1031.13 = 44.83$$

$$(n_2 - 1)s_2^{\,2} = \Sigma X^2 - \frac{(\Sigma X)^2}{n_2} = 1221.18 - 1165.97 = 55.21$$

$$t = \frac{6.70 - 6.68}{\sqrt{\dfrac{44.83 + 55.21}{25 + 22} \cdot \dfrac{26 + 23}{26 \times 23}}}$$

$$= \frac{6.70 - 6.68}{\sqrt{\dfrac{110.04}{47} \times \dfrac{49}{598}}}$$

$$= \frac{.02}{\sqrt{2.13 \times \dfrac{49}{598}}}$$

$$= \frac{.02}{\sqrt{.17}}$$

$$= \frac{.02}{.42}$$

$$= .0475$$

Problem Solving

Calculation of sample mean and standard deviation:

Control: - ($n_1 = 23$)

$$\bar{x}_1 = \frac{153.3}{23} = 6.66 \approx 6.7$$

$$s_1 = \sqrt{\frac{1046.13 - \frac{(153.3)^2}{23}}{22}}$$

$$= \sqrt{1.11} = 1.05 \approx 1.1$$

Treatment: - ($n_2 = 26$)

$$\bar{x}_2 = \frac{176.0}{26} = 6.76 \approx 6.8$$

$$s_2 = \sqrt{\frac{1238.37 - \frac{(176.0)^2}{26}}{25}}$$

$$= \sqrt{1.87} = 1.35 \approx 1.4$$

Calculation of t value for problem solving scores:

$$(n_1 - 1)s_1^2 = \Sigma X^2 - \frac{(\Sigma X)^2}{n_1} = 1046.13 - 1021.78 = 24.35$$

$$(n_2 - 1)s_2^2 = \Sigma X^2 - \frac{(\Sigma X)^2}{n_2} = 1238.37 - 1191.38 = 46.99$$

$$t = \frac{6.8 - 6.7}{\sqrt{\frac{24.35 + 46.99}{22 + 25} : \frac{26 + 23}{26 \times 23}}}$$

$$= \frac{6.8 - 6.7}{\sqrt{\frac{71.34}{47} \times \frac{49}{598}}}$$

$$= \frac{.1}{\sqrt{1.52 \times \frac{49}{598}}}$$

$$= \frac{.1}{\sqrt{.12}}$$

$$= \frac{.1}{.35}$$

$$= .2857$$

Reading

Calculation of sample mean and standard deviation:

Control: - (n_1 = 23)

$$\bar{x}_1 = \frac{166.0}{23} = 7.21 \approx 7.2$$

$$s_1 = \sqrt{\frac{1231.4 - \frac{(166.0)^2}{23}}{22}}$$

$$= \sqrt{1.51} = 1.24 \approx 1.2$$

Treatment: - (n_2 = 26)

$$\bar{x}_2 = \frac{193.9}{26} = 7.07 \approx 7.1$$

$$s_2 = \sqrt{\frac{1477.23 - \frac{(193.9)^2}{26}}{25}}$$

$$= \sqrt{1.21} = 1.1$$

Calculation of t value for reading scores:

$$(n_1 - 1)s_1^2 = \Sigma X^2 - \frac{(\Sigma X)^2}{n_1} = 1231.30 - 1198.09 = 33.31$$

$$(n_2 - 1)s_2^2 = \Sigma X^2 - \frac{(\Sigma X)^2}{n_2} = 1477.23 - 1446.05 = 31.18$$

$$t = \frac{7.2 - 7.1}{\sqrt{\frac{33.31 + 31.18}{25 + 22} \cdot \frac{23 + 26}{23 \times 26}}}$$

$$= \frac{7.2 - 7.1}{\sqrt{\frac{64.49}{47} \cdot \frac{49}{598}}}$$

$$= \frac{.1}{\sqrt{1.37 \times \frac{49}{598}}}$$

$$= \frac{.1}{\sqrt{.11}}$$

$$= \frac{.1}{.34}$$

$$= .2940$$

APPENDIX B: Sample Workbook

SECTION I

PROPERTIES OF WHOLE NUMBERS

In this section we will consider some important features of our number system. We will be dealing only with the set of whole numbers $\{0, 1, 2, 3, 4, \ldots\}$. Often in the science lab you will investigate substances and try to find out what special properties these substances have. Some properties might be that a substance burns, that it is very light in weight, or that it is of a particular colour. All of these characteristics, then, tell us about that certain substance. In mathematics we can take a similar look at our number system to see if there are any special characteristics worth noting.

A. THE COMMUTATIVE PROPERTY:

Consider the following examples: -

$3 + 4 = 4 + 3$

$7 + 9 = 9 + 7$

$17 + 12 = 12 + 17$

$2 \times 6 = 6 \times 2$

$8 \times 5 = 5 \times 8$

$9 \times 11 = 11 \times 9$

All of the number sentences listed above are true. From the examples in the left-hand column it appears that the order in which two whole numbers are added does not affect the sum. 3 + 4 and 4 + 3 both result in the sum of 7. From the examples in the right-hand column it appears that the same is true for multiplication. 2 x 6 and 6 x 2 both result in the product 12.

This fact, that the order in which whole numbers are added or multiplied does not affect the outcome, is a special property of our number system. The word "commute" means to move about and so we call this property the commutative property. Since this property holds true for both addition and multiplication, we can speak of the COMMUTATIVE PROPERTY OF ADDITION (C.P.A.) and of the COMMUTATIVE PROPERTY OF MULTIPLICATION (C.P.M.). Here are some further examples: -

- Commutative Property of Addition (C.P.A.) -

$9 + 4 = 4 + 9$

$124 + 17 = 17 + 124$

$63 + 91 = 91 + 63$

- in general we can state the Commutative Property of Addition (C.P.A.) by saying $a + b = b + a$ where "a" and "b" can be replaced with any whole number.

<u>NOTE</u>:
- normally when asked to state the Commutative Property of Addition (C.P.A.), you use the form shown above, i.e. $a + b = b + a$.
- if, on the other hand, you were asked to give an example that shows the property, you would have to supply one using actual numbers, such as $4 + 1 = 1 + 4$ or $13 + 2 = 2 + 13$.

- Commutative Property of Multiplication (C.P.M.) -

7 x 9 = 9 x 7

14 x 23 = 23 x 14

172 x 19 = 19 x 172

- in general we can state the Commutative Property of Multiplication (C.P.M.) by saying ab = ba where "a" and "b" can be replaced with any whole number.

<u>NOTE</u>:
- normally when asked to state the Commutative Property of Multiplication (C.P.M.), you can use the form shown above, i.e. ab = ba.
- if, on the other hand, you were asked to give an example that shows this property, you would have to supply one using actual numbers, such as 3 x 9 = 9 x 3, 17 x 4 = 4 x 17, etc.

* Recall that ab means a x b.

<u>Summary</u>:
- Commutative Property of Addition (abbreviated C.P.A.) can be stated as a + b = b + a where "a" and "b" are any whole number.
- Commutative Property of Multiplication (abbreviated C.P.M.) can be stated as ab = ba where "a" and "b" are any whole number.

Assignment.

1. State the Commutative Property of Multiplication.

2. State the Commutative Property of Addition.

3. Make up two different examples that illustrate the C.P.A.

4. Make up two different examples that illustrate the C.P.M.

5. What is the answer to 5 - 2?

6. What is the answer to 2 - 5? (Be careful here! Is it possible to subtract five from two?)

7. Does the commutative property apply to the operation of subtraction? Explain.

8. Considering the examples 12 ÷ 4 and 4 ÷ 12, would you say that the commutative property applies to division? Explain.

* * * * * * * * * * * * *

B. <u>THE</u> <u>ASSOCIATIVE</u> <u>PROPERTY</u>:

Suppose we wish to find the sum of three whole numbers: 6, 4, and 8. You probably would add them as you looked at them. Try it. Did you add all three at the same time or did you combine two of them and then add the third? If you think you did it the first way, you would be mistaken. In fact, any time numbers are added the addition is done on pairs of numbers at a time. If a whole series is to be added, you combine the first two, then add that result to the third. Then the result is added to the next number and so the process continues.

In the addition you did above, you may have grouped the numbers in pairs as follows: -

(6 + 4) + 8		6 + (4 + 8)
10 + 8	or like this	6 + 12
18		18

Here the brackets show that we may associate the 4 with the 6 and then add 8, or we may associate the 4 with the 8 and then add 6. In either case the answer is 18. This demonstrates the ASSOCIATIVE PROPERTY OF ADDITION (A.P.A.).

<u>NOTE</u>: - If we grouped like this $(6 + 8) + 4$, that would also give us the same answer, but what have we done to the order of the numerals here? Mathematicians have agreed, when talking about the associative property, NOT TO CHANGE THE ORDER, so we will abide by this agreement.

Consider the following two lists of examples: -

$(7 + 8) + 3 = 7 + (8 + 3)$

$9 + (1 + 8) = (9 + 1) + 8$

$(3 + 4) + 6 = 3 + (4 + 6)$

$(6 \times 2) \times 3 = 6 \times (2 \times 3)$

$2 \times (9 \times 3) = (2 \times 9) \times 3$

$(8 \times 3) \times 4 = 8 \times (3 \times 4)$

All of the number sentences listed above are true. It can be seen that the associative property applies to both addition and multiplication. Now we may speak of another property for the operations of addition and multiplication; that is, the Associative Property of Addition (A.P.A.) and the Associative Property of Multiplication (A.P.M.). Here are some further examples: -

- Associative Property of Addition (A.P.A.) -

$(1 + 7) + 5 = 1 + (7 + 5)$

$(13 + 10) + 4 = 13 + (10 + 4)$

6 + (2 + 71) = (6 + 2) + 71

- in general we can state the Associative Property of Addition (A.P.A.) by saying (a + b) + c = a + (b + c) where "a", "b", and "c" can be any whole number.

NOTE:
- normally when asked to state the Associative Property of Addition (A.P.A.), you can use the form shown above, i.e. (a + b) + c = a + (b + c)
- if, on the other hand, you were asked to give an example that illustrates this property, you would have to supply one using actual numbers, such as (8 + 2) + 4 = 8 + (2 + 4), etc.

- Associative Property of Multiplication (A.P.M.) -

(7 x 4) x 1 = 7 x (4 x 1)

18 x (3 x 7) = (18 x 3) x 7

(4 x 5) x 6 = 4 x (5 x 6)

- in general we can state the Associative Property of Multiplication (A.P.M.) by saying (ab)c = a(bc) where "a", "b", and "c" can be replaced with any whole number.

NOTE:
- normally when asked to state the Associative Property of Multiplication (A.P.M.), you use the form shown above, i.e. (ab)c = a(bc.
- if, on the other hand, you were asked to give an example

that illustrates this property, you would have to supply one using actual numbers, such as (3 x 8) x 9 = 3 x (8 x 9), etc.

* Recall that abc means a x b x c.

<u>Summary</u>:

- Associative Property of Addition (A.P.A.) can be stated as $(a + b) + c = a + (b + c)$ where "a", "b", and "c" are any whole numbers.
- Associative Property of Multiplication (A.P.M.) can be stated as $(ab)c = a(bc)$ where "a", "b", and "c" are any whole numbers.

<u>Assignment</u>.

1. State the Associative Property of Addition.

2. State the Associative Property of Multiplication.

3. Make up two different examples that illustrate the A.P.A.

4. Make up two different examples that illustrate the A.P.M.

5. What is the answer to (9 - 4) - 2?

6. What is the answer to 9 - (4 - 2)?

7. Does the associative property apply to the operation of subtraction? Explain.

8. What is the answer to (12 ÷ 6) ÷ 2?

9. What is the answer to 12 ÷ (6 ÷ 2)?

10. Does the associative property apply to the operation of division? Explain.

11. In the space to the right of each example, state which of the four properties it illustrates. Use the abbreviations. Here is a list of the properties and their abbreviations for you to refer to.

Commutative Property of Addition	C.P.A.
Commutative Property of Multiplication	C.P.M.
Associative Property of Addition	A.P.A.
Associative Property of Multiplication	A.P.M.

a) $3 \times 7 = 7 \times 3$ ____________________

b) $6 + (1 + 9) = (6 + 1) + 9$ ____________________

c) $19 + 47 = 47 + 19$ ____________________

d) $(3 \times 4) \times 5 = 3 \times (4 \times 5)$ ____________________

e) $6 + 1 = 1 + 6$ ____________________

f) $3 \times 15 = 15 \times 3$ ____________________

g) $(ab)c = a(bc)$ ____________________

h) $a + b = b + a$ ____________________

i) $(r + p) + q = r + (p + q)$ ____________________

j) $4 + (7 + 2) = (4 + 7) + 2$ ____________________

C. THE DISTRIBUTIVE PROPERTY:

This property will deal with two operations at once, namely multiplication and addition. Consider this example that contains a multiplication and an addition: -

$$3 \times (4 + 2)$$

$$3 \times \quad 6$$

$$18$$

one way to evaluate this phrase is to add the numbers in brackets, then multiply this result by three. Of course, this is the way you would normally expect to evaluate this phrase.

$$3 \times 4 \quad + \quad 3 \times 2$$

$$12 \quad + \quad 6$$

$$18$$

here, the multiplication is distributed to each number inside the bracket; then these two results are added. You would not normally evaluate this phrase in this manner, but you can see that in this instance you get the same answer.

Therefore, we can say $3 \times (4 + 2) = 3 \times 4 + 3 \times 2$. Consider these further examples: -

$$2(3 + 7) = 2 \times 3 + 2 \times 7$$

$$2 \times 10 = 6 + 14$$

$$20 = 20$$

$$4(5 + 2 + 6) = 4 \times 5 + 4 \times 2 + 4 \times 6$$

$$4(\ 7 \quad + 6) = 20 + 8 + 24$$

$$4 \times 13 = 52$$

$$52 = 52$$

$$(6 + 4)5 = 6 \times 5 + 4 \times 5$$

$$10 \times 5 = 30 + 20$$

$$50 = 50$$

Assignment.

1. Evaluate each side of the following equations to see if, in fact, both sides are equal.

   a) $4(7 + 8) = 4 \times 7 + 4 \times 8$

   b) $(2 + 10)7 = 2 \times 7 + 10 \times 7$

   c) $2(3 + 4 + 5) = 2 \times 3 + 2 \times 4 + 2 \times 5$

* * * * * * * * * * * * *

The above examples, then, illustrate another property of our number system. It is called the DISTRIBUTIVE PROPERTY OF MULTIPLICATION OVER ADDITION (D.P.M.A.). According to the property, multiplication can be distributed over addition, adding the resulting products. Further examples are: -

$$3(2 + 5) = 3 \times 2 + 3 \times 5$$

$$6(8 + 9) = 6 \times 8 + 6 \times 9$$

- in general we can state the Distributive Property of Multiplication over Addition (D.P.M.A.) by saying $a(b + c) = ab + ac$ where "a", "b", and "c" can be replaced with any whole numbers.

NOTE: - normally when asked to state the Distributive Property of Multiplication over Addition (D.P.M.A.), you can use the form shown above, i.e. $a(b + c) = ab + ac$.

- if, on the other hand, you were asked to give an example that illustrates this property, you would have to supply one using actual numbers, such as $4(3 + 2) = 4 \times 3 + 4 \times 2$, $8(10 + 19) = 8 \times 10 + 8 \times 19$, etc.

Assignment.

1. State the Distributive Property of Multiplication over Addition (D.P.M.A.).

2. Make up two different examples that illustrate the D.P.M.A.

3. One of the following is an <u>incorrect</u> illustration of the D.P.M.A. Underline the one that is incorrect.

   a) $6(2 + 4) = 6 \times 2 + 6 \times 4$

   b) $2 \times 3 + 2 \times 5 = 2(3 + 5)$

   c) $2(7 + 8) = 2 \times 7 \times 8$

   d) $(4 + 9)16 = 4 \times 16 + 9 \times 16$

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<u>Summary</u>: - The Distributive Property of Multiplication over Addition can be stated as $a(b + c) = ab + ac$ where "a", "b", and "c" can be replaced by any whole numbers.

D. <u>SOME PROPERTIES OF ZERO AND ONE</u>:

(i) Consider the following sets of patterns and see if you can provide the answer to the last two questions in each set.

I	II	III
$2 + 0 = 2$	$0 + 2 = 2$	$4 - 0 = 4$
$5 + 0 = 5$	$0 + 5 = 5$	$9 - 0 = 9$
$17 + 0 = 17$	$0 + 17 = 17$	$14 - 0 = 14$
$27 + 0 = 27$	$0 + 27 = 27$	$92 - 0 = 92$
$9 + 0 =$	$0 + 9 =$	$8 - 0 =$
$n + 0 =$	$0 + n =$	$n - 0 =$

IV	V	VI
$9 - 9 = 0$	$3 \times 0 = 0$	$0 \times 3 = 0$
$4 - 4 = 0$	$15 \times 0 = 0$	$0 \times 15 = 0$
$17 - 17 = 0$	$7 \times 0 = 0$	$0 \times 7 = 0$
$6 - 6 = 0$	$91 \times 0 = 0$	$0 \times 91 = 0$
$8 - 8 =$	$6 \times 0 =$	$0 \times 6 =$
$n - n =$	$n \times 0 =$	$0 \times n =$

The above are illustrations of the special properties of zero. You can see that adding zero to a given number does not change the number (patterns I and II), or, in other words, $a + 0 = a$ or $0 + a = a$. Similarly, subtracting zero from a given number does not change the number (pattern III), or, in other words, $a - 0 = a$. You can also see that subtracting a number from itself leaves zero (pattern IV), or, in other words, $a - a = 0$. And finally, multiplication of a given number by zero (patterns V and VI) produces a product of zero, or, in other words, $a \times 0 = 0$ or $0 \times a = 0$.

(ii) Now consider these further examples. As before, you will provide the answers for the last two questions in each set.

I

$4 \times 1 = 4$

$3 \times 1 = 3$

$17 \times 1 = 17$

$8 \times 1 =$

$n \times 1 =$

II

$1 \times 4 = 4$

$1 \times 3 = 3$

$1 \times 17 = 17$

$1 \times 8 =$

$1 \times n =$

III

$5 \div 1 = 5$

$17 \div 1 = 17$

$6 \div 1 = 6$

$8 \div 1 =$

$n \div 1 =$

IV

$8 \div 8 = 1$

$4 \div 4 = 1$

$7 \div 7 = 1$

$11 \div 11 =$

$n \div n =$

<u>NOTE</u>: - here $n \neq 0$ since from your work on fractions you will recall that 0 is not allowed as a denominator or divisor.

The above are illustrations of the special properties of one. You can see that multiplying a given number by 1 does not change the number (patterns I and II), or, in other words, $a \times 1 = a$ or $1 \times a = a$. Similarly, division by 1 does not change a number (pattern III), or, in other words, $a \div 1 = a$. And finally, a number divided by itself gives a quotient (answer) of 1 (pattern IV), or, in other words, $a \div a = 1$. Here we are careful to note that we cannot use $a = 0$, since zero as a divisor is not allowed.

Summary:

- $a + 0 = a$ and $0 + a = a$
  $a - 0 = a$ and $a - a = 0$ properties of zero
  $a \times 0 = 0$ and $0 \times a = 0$

- $a \times 1 = a$ and $1 \times a = a$
  $a \div 1 = a$ and $a \div a = 1$ properties of one (where $a \neq 0$)

Assignment.

1. State whether each of the following is true or false.

a) $5 - 5 = 0$
b) $16 \times 1 = 16 - 16$
c) $4 \times 3 \times 0 = 12$
d) $3 \times 1 = 3$
e) $0 \times 4 = 4$
f) $2 \times 1 = 6 - 3$
g) $0 \times 278 = 0$
h) $15 + 0 = 10 \times 15$
i) $48 + 0 = 49$
j) $5 + 2 + 0 = 7$

2. Evaluate each of the following phrases.

a) (2 + 7) - (8 - 0)

b) (7 + 8) x 0 + 9 - 9

c) (16 - 4) + (13 - 13)

d) (2 + 11) x (18 - 17)

e) 16 + 0 - 15 - 5

f) (1 + 0) x 18

g) 3 x 9 + 3 x 2 - 33

h) 2 x (5 - 5) x 3 + 9

i) 4 x (7 - 0) + 6

j) 3 x 9 x 8 x 0 x 4

k) [(6 + 2) x 0 + 6 - 6] x 0

l) 2(15 + 0) - 10 - 2

m) 0(6 + 3) + 6(4 + 0)

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E. USES FOR PROPERTIES OF WHOLE NUMBERS:

(i) Commutative and associative properties -

These properties can be used to make some calculations simpler. Add the following figures on a piece of scrap paper: -

23 + 56 + 77

Normally you would write the three numbers down in a vertical column and add one row at a time like this: -

$$\begin{array}{r} 23 \\ 56 \\ 77 \\ \hline 156 \end{array}$$

However, if we use the commutative and associative properties of addition to re-arrange the order and the manner in which the numbers are grouped, the task becomes simple enough to do mentally.

$23 + 77 + 56 \longleftarrow$ here the 56 and 77 have changed places (C.P.A.)

$(23 + 77) + 56 \longleftarrow$ here we are grouping the 23 and 77 rather than the 23 and 56 as in the first case (A.P.A.)

$100 + 56$

156

This procedure does not work for every question, but when it is possible calculation can be greatly simplified.

Here are some examples where the same simplifying technique of using the commutative and associative properties are applied to multiplication.

a) $4 \times 17 \times 25$

Can be rewritten as

$4 \times 25 \times 17 \longleftarrow$ here the 17 and 25 have changed their order (C.P.M.)

(4 x 25) x 17 ⟵ here the 4 and 25 are grouped instead of the 4 and 17 as in the original question (A.P.M.)

100 x 17

1700

c) 138 x 72 x 0 x 15

Can be rewritten as

0 x 138 x 72 x 15 ⟵ here the 0 has been moved to the front (C.P.M.)

0 x (138 x 72 x 15) ⟵ no matter what the product in the bracket is, it will be multiplied by 0; thus the final answer is 0.

0

Assignment.

All of the following can be done in a simpler manner. Make use of the commutative and associative properties to simplify the work that must be done. Show the changes you would make as well as giving the answer.

a) 16 + 42 + 84

b) 2 x 37 x 50

c) (19 x 72 x 0 x 4) + 16

d) 125 + 257 + 75

e) 25 x 52 x 4

f) 250 + 398 + 50

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(ii) The distributive property -

This property can be used to make some calculations simpler. Mentally multiply 3 x 29. Now look at the following: -

- 29 can be rewritten as 20 + 9
- 3 x 29 can be rewritten as 3 x (20 + 9)
- now, using the D.P.M.A. -

$$\begin{aligned} 3 \times (20 + 9) &= 3 \times 20 + 3 \times 9 \\ &= 60 + 27 \\ &= 87 \end{aligned}$$

In this manner some multiplications can be done very simply, simply enough in fact to be done mentally.

Further examples: -

a) $$\begin{aligned} 7 \times 108 &= 7 \times (100 + 8) \\ &= 7 \times 100 + 7 \times 8 \\ &= 700 + 56 \\ &= 756 \end{aligned}$$

b) $$\begin{aligned} 72 \times 9 &= (70 + 2) \times 9 \\ &= 70 \times 9 + 2 \times 9 \\ &= 630 + 18 \\ &= 648 \end{aligned}$$

Assignment.

Do the following multiplications mentally.

1. 6 x 13 =

2. 5 x 17 =

3. 2 x 39 =

4. 14 x 7 =

5. 6 x 27 =

6. 8 x 12 =

7. 47 x 3 =

8. 52 x 2 =

9. 4 x 109 =

10. 1007 x 7 =

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SELF-TEST

1. State each of the following properties: -

   a) Commutative Property of Addition (C.P.A.)

   b) Commutative Property of Multiplication (C.P.M.)

   c) Associative Property of Addition (A.P.A.)

   d) Associative Property of Multiplication (A.P.M.)

   e) Distributive Property of Multiplication over Addition (D.P.M.A.)

2. Give one example that illustrates each of the following: -

   a) C.P.A.

   b) C.P.M.

   c) A.P.A.

   d) A.P.M.

   e) D.P.M.A.

3. Show how you would use the commutative property to simplify the following calculations. In each case rewrite the phrase making the appropriate changes in order, then write the answer in the space at the right.

   a) 25 + 93 + 75 ____________________

   b) 4 + 317 + 96 ____________________

   c) 25 x 13 x 4 ____________________

4. Show how you would use the distributive property to simplify the following multiplication: -

3×35

5. State what property is illustrated by each of the following. Use the abbreviations given in the first question.

a) $7 + 4 = 4 + 7$

b) $xy = yx$

c) $3(2 + 5) = 3 \times 2 + 3 \times 5$

d) $a + b = b + a$

e) $(3 + 7) + 15 = 3 + (7 + 15)$

f) $78(a + b) = 78a + 78b$

6. Perform the following calculations: -

a) $17 \times 0 =$ b) $87 \div 1 =$

c) $32 \times 9 \times 0 =$ d) $4 + 0 + 8 =$

e) $0(85 + 39) =$ f) $4 \times 9 \times 1 =$

g) $0a =$ h) $1n =$

i) $x \div 1 =$

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SECTION II

F. INVERSE OPERATIONS:

We can think of the word "inverse" as meaning "undoing". Here are some examples of some actions and their inverses. You fill in the missing information.

ACTION	INVERSE ACTION
a) opening the door	closing the door
b) leaving the room	coming back into the room
c) building a building	tearing the building down
d) airplane taking off	
e) submarine diving	
f) going to the store	
g) adding money to your savings	
h) operation of addition of whole numbers	
i) operation of multiplication of whole numbers	

It is very convenient to say that we can "undo" something because we are able to "do" it in the first place. For example: -

It is possible to close the door because It is possible to open it.

NOTE: - think this through very carefully. If it were not possible for you to open the door, it would not be possible for you to close it. Now you might say, "But it is closed!" True, but you can not go through the action of closing it unless you first open it.

You can come back into the room	because	You could leave the room.

NOTE: - if you could not leave the room, it would not be possible to come back into the room. Admittedly you would be in the room, but that was because you were there to start with, not because you left and then came back. You can only come back in if first you are able to go out!

So you can now see that -

A building can be torn down	because	the building could be built.
An airplane can land	because	the airplane could take off.
A submarine can come to the surface	because	the submarine could dive below the surface.

In mathematics we think of subtraction as being the inverse of addition. So we can say: -

$8 - 3 = 5$	because	$5 + 3 = 8$
$19 - 7 = 12$	because	$12 + 7 = 19$
$9 - 2 = 7$	because	$7 + 2 = 9$
$a - b = c$	because	$c + b = a$

You could think of addition as the building up process; then subtraction becomes the tearing down process. You can tear down only that which could be built up in the first place.

In a similar fashion, we think of division as being the inverse of multiplication. So we can say -

$12 \div 3 = 4$ because $4 \times 3 = 12$

$18 \div 2 = 9$ because $9 \times 2 = 18$

$72 \div 24 = 3$ because $3 \times 24 = 72$

$a \div b = c$ because $cb = a$

Here you could think of multiplication as building up and of division as tearing down. Again you can tear down only that which could be built up in the first place.

<u>Assignment</u>.

1. To see if you truly understand the argument presented in the previous section called NOTE about inverses, try explaining it to some adult at your home.

2. Show why the following subtractions are true. The first one is done for you as an example.

   a) $7 - 5 = 2$ because $2 + 5 = 7$

   b) $13 - 9 = 4$

c) 4 - 0 = 4

d) 8 - 3 = 5

e) 27 - 9 = 18

3. Show why the following divisions are true. The first one is done for you as an example.

a) 14 ÷ 2 = 7 because 7 x 2 = 14

b) 12 ÷ 6 = 2

c) 15 ÷ 5 = 3

d) 16 ÷ 4 = 4

e) 42 ÷ 6 = 7

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G. ABOUT DIVISION WITH ZERO:

Consider the following statements: -

8 ÷ 2 = 4 because 4 x 2 = 8

0 ÷ 3 = 0 because 0 x 3 = 0

0 ÷ 7 = 0 because 0 x 7 = 0

Now look at the following which is a division by zero: -

$4 \div 0 = \underline{\qquad\quad}$ because $\underline{\qquad\quad} \times 0 = 4$

What number can be used to fill in the blanks? You might think 0 would work, but then you get: -

$4 \div 0 = \underline{\quad 0 \quad}$ because $\underline{\quad 0 \quad} \times 0 = 4$

But $0 \times 0 \neq 4$; thus, we can not say that $4 \div 0 = 0$. Do you think some other number, say 4, would work?

$4 \div 0 = \underline{\quad 4 \quad}$ because $\underline{\quad 4 \quad} \times 0 = 4$

But $4 \times 0 \neq 4$; thus we cannot say that $4 \div 0 = 4$. In fact, there is $\underline{\text{no number}}$ that will give you an answer to $4 \div 0$.

Now look at the following in which zero is divided by zero: -

$0 \div 0 = \underline{\qquad\quad}$ because $\underline{\qquad\quad} \times 0 = 0$

What number can be used to fill in the blank? You might say that 0 would work, so: -

$0 \div 0 = \underline{\quad 0 \quad}$ because $\underline{\quad 0 \quad} \times 0 = 0$

And you see that it does work.

But consider that a number divided by itself should equal one.

$0 \div 0 = \underline{\quad 1 \quad}$ because $\underline{\quad 1 \quad} \times 0 = 0$

And you can see that the number 1 is also an answer.

In fact, other numbers will also give an answer that works: -

$0 \div 0 = 2$	because	$2 \times 0 = 0$
$0 \div 0 = 7$	because	$7 \times 0 = 0$

etc.

Here we cannot give an answer to $0 \div 0$, since in fact <u>all</u> numbers will work.

Since division by 0 gives no exact answer, we say that it is <u>undefined</u> (doesn't work). Thus, you may not use 0 as a divisor.

H. THE DISTRIBUTIVE PROPERTY AND PAIRS OF OPERATIONS OTHER THAN ADDITION AND MULTIPLICATION:

Previously we considered the D.P.M.A. where multiplication could be distributed over a following addition.

i.e. $3 \times (2 + 4) = 3 \times 2 + 3 \times 4$

Here the multiplication by 3 is distributed to both parts of the addition 2 + 4.

Now let's consider other pairs of operations such as multiplication and subtraction. Keep the order in which these operations are mentioned in mind since this indicates which operation is to be distributed over the other. Here multiplication will be distributed over subtraction. Will this work? Look at the following and try to answer the questions mentally as you go along.

- what is the answer to 3(8 - 2)?
- what is the answer to 3 x 8 - 3 x 2?
- can we say, then 3(8 - 2) = 3 x 8 - 3 x 2?

Assignment.

Rewrite the following applying the Distributive Property of Multiplication over Subtraction. Then evaluate the phrases on each side of the (=) sign to see if, in fact, you get the same answer both ways. The first one is done for you as an example.

1. 2(7 - 5) = 2 x 7 - 2 x 5

   2 x 2 = 14 - 10

   4 = 4

2. 5(9 - 5) =

3. 7(6 - 4) =

4. 8(11 - 7) =

5. State the Distributive Property of Multiplication over Subtraction (that is, using placeholders like "a", "b", and "c").

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Now let's try some other possible pairs. Answer these questions as you proceed, making sure to check with the answer booklet once you reach the end of each part.

<u>Division</u> <u>over</u> <u>Addition</u>

- what is the answer to 12 ÷ (2 + 4)?
- what is the answer to 12 ÷ 2 + 12 ÷ 4?
- can we say, then 12 ÷ (2 + 4) = 12 ÷ 2 + 12 ÷ 4?

<u>Division</u> <u>over</u> <u>Subtraction</u>

- what is the answer to 12 ÷ (4 - 2)?
- what is the answer to 12 ÷ 4 - 12 ÷ 2?
- can we say, then 12 ÷ (4 - 2) = 12 ÷ 4 - 12 ÷ 2?

Assignment.

Can we say that:

1. Multiplication is distributive over multiplication? Show work necessary to get your answer.

2. Addition is distributive over subtraction? Show work necessary to get your answer.

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I. COMMUTATIVE AND ASSOCIATIVE PROPERTIES REVIEWED:

In the following example, consider the bracket to contain a certain number.

$$(\quad) + 9 = 9 + (\quad)$$

This, then, is an example that illustrates the Commutative Property of Addition. Now, if we place some number in the bracket, it should not change the fact that this is an example of the C.P.A. So placing 3 x 4 in the bracket: -

$$(3 \times 4) + 9 = 9 + (3 \times 4)$$

Now consider the example if it is written so that the order of the *addition* is *not* altered. That is, it is not an example of the C.P.A.

$$(3 \times 4) + 9 = (3 \times 4) + 9$$

What if, in this case, some alteration is done to the expression within the bracket? For example: -

$$(3 \times 4) + 9 = (4 \times 3) + 9$$

Is this an example of the C.P.A.? No, since the order of addition has not changed. What property is illustrated here? Well, since there was a change in the order for the multiplication part of the larger expression, we could say that this is an example of the Commutative Property of Multiplication (C.P.M.).

It is possible to conceive of this example illustrating both the C.P.A. and the C.P.M.

$$(3 \times 4) + 9 = 9 + (4 \times 3)$$

Here both the order of the addition and the order of the multiplication have changed. This, then, is an example of both the C.P.A. and C.P.M.

Here is another example where two properties might be illustrated in a single situation: -

$$(3 + 4) + 5 = 4 + 3 + 5 \qquad \text{order changed}$$

$$= 4 + (3 + 5) \qquad \text{grouping changed}$$

You can see that the above illustrated the C.P.A. since the order of the addition changes. However, since the grouping also changed as indicated by the bracket enclosing a different pair of numbers on each side of the (=) sign, it also illustrates the Associative Property of Addition (A.P.A.).

On the left-hand side below is a list of examples. Some illustrate one property, others more than one. To the right of each is stated what property or properties are being illustrated. Study each example carefully until you are certain you understand why each illustrates the indicated properties.

a)	$3 + 4 = 4 + 3$	C.P.A
b)	$3 \times 7 = 7 \times 3$	C.P.M.
c)	$a + (b + c) = (a + b) + c$	A.P.A.
d)	$(3 \times 2) \times 9 = 3 \times (2 \times 9)$	A.P.M.
e)	$(4 + 1) + 6 = 1 + (4 + 6)$	C.P.A. and A.P.A.
f)	$(9 \times 7) + a = a + (9 \times 7)$	C.P.A.
g)	$(7 \times 9) + a = (9 \times 7) + a$	C.P.M.
h)	$4 + (3 \times 2) = (2 \times 3) + 4$	C.P.A. and C.P.M.

<u>Assignment</u>.

In a manner similar to the above examples, write which property or properties are illustrated by each of the following: -

a) $ab = ba$

b) $n + m = m + n$

c) $2 + (7 + 9) = (7 + 2) + 9$

d) $(ab)c = (ba)c$

e) $(4 + 7) + 3 = (7 + 4) + 3$

f) $8 \times (2 + 9) = (2 + 9) \times 8$

g) $14 + (7 \times 3) = (3 \times 7) + 14$

h) $abcd = dabc$

i) $a(x + y) = (x + y)a$

j) $a(x + y) = (y + x)a$

k) $(3 + 2) + 4 = (2 + 3) + 4$

l) $3(a + b) = 3a + 3b$

* * * * * * * * * * * * *

J. <u>THE PROPERTY OF CLOSURE</u>:

Consider the set of whole numbers, that is, the set $\{0, 1, 2, \ldots\}$ Recall that the three dots mean "and so on". In other words, this set also contains the rest of the whole numbers, i.e. 5, 6, 7, etc. Now mentally add 4 and 2. Is the sum in the above set? Add 9 and 3. Is the sum in the above set? Add 0 and 56. Is the sum in the above set? Try to find two whole numbers, that is, two numbers from the above set, whose sum is not in the set. Would we ever be able to find two numbers from this set such that their sum is not in this set? The answer is no.

Now consider the following set: $A = \left\{0, 1, 2, 3, 4, 5, 6\right\}$. Add 2 and 3. Is the sum in this set? Add 5 and 1. Is the sum in this set? Add 6 and 3. Is the sum in this set? Can you find other pairs of numbers from <u>this</u> <u>set</u> that give a sum not in this set? Of course you can. Some pairs would be 3 + 5, 2 + 6, 4 + 3, etc. All these produce sums which do not appear in the given set.

We now have the following definitions: -

<u>CLOSED</u> - we say that a given set is closed for an operation like addition, multiplication, etc., if when performing that operation on pairs of numbers from the given set, the answer is always a number of the set.

<u>NOT</u> <u>CLOSED</u> - we say that a given set is not closed for an operation like addition, multiplication, etc., if when performing that operation on pairs of numbers from the given set, the answer is not in the given set for <u>at</u> <u>least</u> <u>one</u> such pair. <u>NOTE</u>: - here the answer may be in the set for some pairs from the set, but if <u>one</u> or more pairs can be found that give an answer not in the set, the set is not closed.

This is referred to as the CLOSURE PROPERTY. Considering the very first example in this section, we can say that for the set of whole numbers the set is closed for the operation of addition.

Further examples: -

$$A = \{1, 3, 5, 7, 9, \ldots\} \quad \text{i.e., all odd whole numbers}$$

- This set is closed for multiplication since for all pairs, the product is also in the set (that is, the product is also an odd number). We see that 3 x 5 = 15 and 15 is an odd number; 7 x 11 = 77 and 77 is an odd number. Try a few more examples yourself. The product of any two numbers will be odd, so this set is closed for multiplication.

- This set is not closed for addition since for at least one pair the sum is not in the set (that is, the sum is not an odd number). Here 5 and 3 are a pair of numbers from the set, but 5 + 3 = 8 and 8 is not a number from that set. There are other examples of pairs from this set that give sums not in the set, but all that is necessary is to find <u>one</u> pair. Thus, we can say that this set is <u>not</u> closed for addition.

<u>Assignment</u>.

In each case state whether the set given is closed or not closed for the operations indicated. If a set is not closed, write down an example that shows it is not closed.

1. $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

   addition

   multiplication

2. $\{2, 4, 6, 8, \ldots\}$

   addition

   multiplication

3. $\{0, 1, 2, 3, \ldots\}$

   multiplication

   division

4. $\{1, 2, 3, 4, \ldots, 100\}$

   addition

   multiplication

5. $\{0, 1\}$

   multiplication

6. $\{0, 1, 2, 3, \ldots\}$

   subtraction

   addition

7. Check back over questions 3 and 6. Here the given set is the set of

whole numbers. For what operations is the set of whole numbers closed?

8. If a set goes on forever (infinite set), can we automatically say it is closed for all operations? (See question 6.)

9. If a set contains a definite number of members, can we automatically say it is not closed for all operations? (See question 5.)

* * * * * * * * * * * * *

SELF-TEST

1. Complete the following: -

a) $9 - 4 = 5$ because $5 + 4 = 9$

b) $18 - 7 = 11$ because

c) $5 - 3 = 2$ because

d) $25 - 8 =$ because

e) $13 - 5 =$ because

f) $12 \div 4 = 3$ because

g) $21 \div 3 =$ because

h) $a \div b = c$ because

2. Use an example of division by 0 to show why we cannot use 0 as a divisor.

3. Work out an example to show that multiplication is distributive over subtraction.

4. State which properties or property is illustrated by each of the following: -

   a) $(2 \times 5) + 8 = 8 + (2 \times 5)$

   b) $19 + (6 \times 3) = 19 + (3 \times 6)$

   c) $a(b + c) = (c + b)a$

   d) $4 \times (2 \times 7) = (4 \times 2) \times 7$

   e) $5(x + y) = 5x + 5y$

   f) $7(8 + 4) = (4 + 8)7$

   g) $4 + 5 = 5 + 4$

   h) $5 + (6 + 7) = (6 + 5) + 7$

5. State whether the following sets are closed or not closed for the operation(s) indicated. If not closed, give a counter example.

a)	$\{1, 3, 5, 7, \ldots\}$	addition
b)	$\{0, 1\}$	addition multiplication
c)	$\{2, 4, 6, \ldots\}$	addition

SECTION III

ABSTRACT MATHEMATICAL SYSTEMS

A. Definition: - an abstract mathematical system is a set of elements together with some operation defined for these elements.

As an example of an abstract system, we can use the modular arithmetic you studied earlier. For the sake of this discussion, we will consider modulo 3. Thus, the set of elements for this system is the set of bar numbers $\left\{\bar{0},\ \bar{1},\ \bar{2}\right\}$. For an operation, we can consider the addition of bar numbers. (Recall that this meant something different than addition in the ordinary sense.) Addition was defined as taking a pair of numbers from any of the bar numbers, adding that pair in the normal way, but using as the answer the set that this sum appears in.

Example: $\bar{1} + \bar{2} = \bar{0}$

$$\text{a number from } \bar{1} \longrightarrow 4 + \underset{\substack{\uparrow \\ \text{a number from } \bar{2}}}{5} = 9 \longleftarrow \text{a number from } \bar{0}$$

From this point on, we will use the following as a symbol for modular addition: $\oplus$. Here we are purposely making the symbol different than that used for ordinary addition, since modular addition is not exactly the same as ordinary addition.

Now consider the following table of values arrived at by performing the operation of modular addition on the various pairs of elements of this abstract system: -

⊕	$\bar{0}$	$\bar{1}$	$\bar{2}$
$\bar{0}$	$\bar{0}$	$\bar{1}$	$\bar{2}$
$\bar{1}$	$\bar{1}$	$\bar{2}$	$\bar{0}$
$\bar{2}$	$\bar{2}$	$\bar{0}$	$\bar{1}$

Having completed the above table, we now have a fairly good picture of this particular abstract mathematical system: -

- the set of elements is $\left\{\bar{0}, \bar{1}, \bar{2}\right\}$.
- the operation is modular addition which means you add pairs of numbers from the bar numbers, using the set in which this sum appears as the answer.
- the symbol for the operation is ⊕.
- a table for the values obtained using this operation in this set of elements is shown above.

Now we can ask a number of questions about this abstract mathematical system. They are the same questions we asked about our own familiar system of whole numbers. Is this system commutative for the operation ⊕? Is the system associative for the operation ⊕? Is there a distributive property? Is there a property similar to the property of zero for the operation ⊕? Is the system closed for the operation ⊕? Let's take these questions one at a time.

Is the system commutative for the operation ⊕?

How can this question be answered? Look at the table of values given. What is the answer to $\bar{1} \oplus \bar{2}$? What is the answer to $\bar{2} \oplus \bar{1}$? Is the answer the same in both cases? What about $\bar{0} \oplus \bar{1}$ and $\bar{1} \oplus \bar{0}$? Will this happen for all possible pairs of bar numbers? If it is always true, then we can say that in this particular abstract mathematical system the operation of $\oplus$ is commutative. Is this system commutative? Write "yes" or "no" in the blank, then check the answer booklet.

____

Is this system associative for the operation $\oplus$?

- write the answer for $(\bar{1} \oplus \bar{2}) \oplus \bar{1}$ ________________________
- write the answer for $\bar{1} \oplus (\bar{2} \oplus \bar{1})$ ________________________
- try a few other examples of your own making in this space: -

Is this system associative for the operation $\oplus$? Write "yes" or "no" in the blank, then check the answer booklet. ____________________

Is there a distributive property?

Recall that for the distributive property we spoke of one operation being distributed over the second operation. Here we only have the one operation (modular addition), not two different ones. However, we can still ask whether this operation ⊕ is distributive over itself. Follow the steps outlined to see if there is a distributive property for this system.

$$\bar{1} \oplus (\bar{0} \oplus \bar{2}) \overset{?}{=} \bar{1} \oplus \bar{0} \quad \oplus \quad \bar{1} \oplus \bar{2}$$

$$\bar{1} \oplus \bar{2} \overset{?}{=} \bar{1} \oplus \bar{0}$$

$$\bar{0} \neq \bar{1}$$

Does the distributive property work here? Answer "yes" or "no" in the blank, then check the answer booklet. ______________________________

Is there a property like the property of zero?

NOTE: - recall that the property of zero for addition stated that n + 0 = n. In other words, adding 0 did not cause the other number to change. Because 0 causes no change in addition questions, 0 is often called the IDENTITY ELEMENT of addition in our number system. The word "identity" here refers to the fact that things remain identically as they were even after the 0 is added. Thus, the above question could be restated as: - Is there an identity element for the operation ⊕ in this abstract mathematical system?

Check back in the table of values for this system to see if there is any one element that produces no change when paired with some other element of the system using the operation ⊕. What is the identity element for the operation ⊕ in this particular abstract system? __________

Is this system closed for the operation ⊕?

Considering the set of elements for this system $\{\bar{0}, \bar{1}, \bar{2}\}$ and all of the possible pairs using the operation ⊕ as shown in the table of values, do you ever get an answer that is not one of the elements of the set? Is this set closed for the operation ⊕? Answer "yes" or "no", then check the answer booklet. ______________________________

B. A NEW CHARACTERISTIC - INVERSE ELEMENTS:

In a previous unit we talked about inverse operations. Now we will use a similar idea to talk about inverse elements.

A. Definition: - the inverse elements in a given mathematical system are those elements that produce, when paired with any other particular element, the identity element. In this system, as an example, the inverse element for $\bar{1}$ is $\bar{2}$ since $\bar{1} \oplus \bar{2} = \bar{0}$.

$\bar{1}$ element	⊕	$\bar{2}$ inverse element	=	$\bar{0}$ identity element

Similarly, the inverse element for $\bar{2}$ is $\bar{1}$ since -

$$\underset{\text{element}}{\bar{2}} \quad \oplus \quad \underset{\substack{\text{inverse}\\\text{element}}}{\bar{1}} \quad = \quad \underset{\substack{\text{identity}\\\text{element}}}{\bar{0}}$$

Now for the purpose of discussing these inverse elements further, let's take a different abstract mathematical system. Here we will use modulo 5. It will be very similar to the previous example, except that the set of elements will be larger.

- set of elements $\left\{\bar{0}, \bar{1}, \bar{2}, \bar{3}, \bar{4}\right\}$.
- operation is modular addition (same as examined in the previous example).
- symbol for the operation is ⊕.
- table of value to be filled in by you right now: -

⊕	$\bar{0}$	$\bar{1}$	$\bar{2}$	$\bar{3}$	$\bar{4}$
$\bar{0}$					
$\bar{1}$					
$\bar{2}$					
$\bar{3}$					
$\bar{4}$					

<u>NOTE</u>: - check the answer booklet right now to see if you have filled in the table <u>correctly</u>.

Of course, all of the properties hold true in modulo 5 that were true in modulo 3. The identity element for the operation ⊕ in this system is also $\bar{0}$. Look at the following and answer the questions as you proceed.

$\bar{1} \oplus \bar{4} = \bar{0}$ what is the inverse element of $\bar{1}$? Here you see that $\bar{4}$ fits the definition since under the operation $\oplus$ it produces the identity element $\bar{0}$.

$\bar{4} \oplus \bar{1} = \bar{0}$ what is the inverse element of $\bar{4}$? Explain why.

$\bar{1} \oplus \bar{4} = \bar{0}$ inverse element of $\bar{1}$ is $\bar{4}$.
$\bar{2} \oplus$ ___ $= \bar{0}$ inverse element of $\bar{2}$ is ___.
$\bar{3} \oplus$ ___ $= \bar{0}$ inverse element of $\bar{3}$ is ___.
$\bar{4} \oplus$ ___ $= \bar{0}$ inverse element of $\bar{4}$ is ___.

<u>Assignment</u>.

For the purpose of this assignment we will use the abstract system described below.

- the set of elements is $\{\bar{0}, \bar{1}, \bar{2}, \bar{3}\}$, i.e., modulo 4.
- the operation is modular <u>multiplication</u>. That is, you take pairs of numbers from the bar numbers and the answer is the bar number to which their product belongs.

example: $\bar{2} \otimes \bar{3} = \bar{2}$ since

a number from $\bar{2}$ → $2 \times 7 = 18$ ← a number from $\bar{2}$
a number from $\bar{3}$

- the symbol for the operation is $\otimes$.
- the table of values to be filled in by you: -

<u>Remember</u>: here
you multiply.
Do not add!

$\otimes$	$\bar{0}$	$\bar{1}$	$\bar{2}$	$\bar{3}$
$\bar{0}$				
$\bar{1}$				
$\bar{2}$				
$\bar{3}$				

Check your answers in the answer booklet to make sure you have filled the above table in correctly.

1. Is this system commutative for the operation $\otimes$? Try several examples in the space below.

2. Is this system associative for the operation $\otimes$? Try several examples in the space below.

3. Is the operation $\otimes$ distributive over itself? Try at least one example.

4. Find the bar number that belongs in each of the blanks.

a) $\bar{0} \otimes \underline{\qquad} = \bar{0}$

b) $\bar{1} \otimes \underline{\qquad} = \bar{1}$

c) $\bar{2} \otimes \underline{\qquad} = \bar{2}$

d) $\bar{3} \otimes \underline{\qquad} = \bar{3}$

5. Considering the previous question, what is the identity element for the operation $\otimes$ in this system?

6. Can you find the inverse element for each of the following? Recall that the inverse element is the one that, when paired with a particular element, produces the identity element.

a) $\bar{0} \otimes \underline{\qquad} = \bar{1}$ inverse element of $\bar{0}$ is $\underline{\qquad}$.

b) $\bar{1} \otimes \underline{\qquad} = \bar{1}$ inverse element of $\bar{1}$ is $\underline{\qquad}$.

c) $\bar{2} \otimes \underline{\qquad} = \bar{1}$ inverse element of $\bar{2}$ is $\underline{\qquad}$.

d) $\bar{3} \otimes \underline{\qquad} = \bar{1}$ inverse element of $\bar{3}$ is $\underline{\qquad}$.

7. Is this system closed or not closed for the operation $\otimes$? Explain your answer.

C. OTHER ABSTRACT SYSTEMS:

Here is the description of an abstract mathematical system: -

- set of elements {○, □, △}
- operation is described by the table of values below. You may not understand how the results are arrived at, but this does not matter as long as you can see that any given pair does give the answer indicated in the table.
- symbol for the operation is *.
- table of values: -

*	○	□	△
○	○	□	△
□	□	☆	○
△	△	○	☆

thus, for example:

△ * □ = ○

Answer the following questions as you proceed. Check your answers in the answer booklet frequently to make sure you are on the right track.

- what is the answer to ○ * △ ? ______________________________

Check the appropriate row and column of the table of values to get the proper answer.

- what is the answer to △ * ○? ____________________

- are the two the same? ____________________

- will this be so for all possible pairs? ____________________

- can you say, then, that the operation * is commutative for this system? ____________________

- what is the answer to ○ * (□ * □)? ____________________

- what is the answer to (○ * □) * □? ____________________

- are the two the same? ____________________

- can you say, then, that the operation * is associative for this system? ____________________

- is the operation * distributive over itself? Work out the last step of this example to see if it is distributive: -

○ * (□ * △) ≟ ○ * □ * ○ * △

○ * ○ ≟ □ * △

≟

- now observe another example: -

○ * (□ * □) ≟ ○ * □ * ○ * □

○ * ☆ ≟ □ * □

NO ANSWER ≟ ☆

NOTE: - there is no answer to ○ * ☆ since the ☆ is in fact not even an element of the set for this system.

- can we say that the operation * is distributive over itself? ________________________

- what is the identity element for this system? ________________________

- write the inverse element for each of the members of the set of elements of this system: -

a) ○, inverse element is ________.

b) □, inverse element is ________.

c) △, inverse element is ________.

- is this system closed or not closed for the operation *? Explain.

Assignment.

Part I. The following abstract mathematical system will be referred to by the questions in this assignment.

- set of elements $\{ |, \ulcorner, \sqcap \}$
- operation is defined by the table of values below.
- symbol for the operation is ⊡

- table of values: -

⊡	│	┌	⊓
│	⊓	│	┌
┌	│	┌	⊓
⊓	┌	⊓	│

1. In each of the following questions support your answer with one or more examples as is necessary.

   a) Is the operation ⊡ commutative?

   b) Is the operation ⊡ associative?

   c) Is the operation ⊡ distributive over itself?

d) What is the identity element for the operation $\boxed{\bullet}$?

e) Test the three elements and name the inverse of each.

Part II. Here is a system that you can make up and investigate yourself. Consider the three primary colours - red, blue, and yellow - and think of the operation of mixing them two at a time in equal proportions. Also consider the colour "clear" as being included. Could you describe this as an abstract mathematical system? There are a few blank pages at the end of this booklet. Use them to do the following: -

- using small circles coloured with coloured pencil or felt pen, list the set of elements of the system.
- describe the operation of "mixing" in your own words, (i.e., here you are defining the operation of the system).

- make up a symbol for the operation.
- develop a table of values showing the results of "mixing" various pairs of primary colours. (Don't forget to include "clear" as a colour.)
- make a heading <u>commutative property</u> and show whether it holds true for this system using examples.
- make a heading <u>associative property</u> and do the same for it.
- identify the identity element using several examples to show that it is indeed the identity element.
- list the elements of the system and indicate what the inverse element is for each. If there is no inverse element for one in particular, give a reason.

* * * * * * * * * * * * *

D. INVERSE ELEMENTS IN THE WHOLE NUMBERS:

The set of whole numbers $\{0, 1, 2, 3, 4, \ldots\}$ also forms a system for which all of the previous properties hold. There is one exception. What is the identity element for the set of whole numbers under the operation of addition? Look at these examples to help you recall some facts about addition with 0: -

$$1 + 0 = 1$$

2 + 0 = 2

3 + 0 = 3

thus, the identity element for addition in this system is 0.

etc.

Now let's see if there is an inverse element for each of the whole numbers under addition. Can you find the elements of the set of whole numbers that belong in the spaces below?

1 + ___ = 0 inverse element of 1 is ___.

2 + ___ = 0 inverse element of 2 is ___.

3 + ___ = 0 inverse element of 3 is ___.

etc.

Of course, there is no inverse elements for the set of whole numbers. In that case let's expand the set of whole numbers to include inverse elements for each whole number. Consider this set: -

$$\{\ldots -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, \ldots\}$$

Notice the inclusion of elements with a minus sign attached. Call these the negative whole numbers. Then we can say: - (negative numbers are placed in brackets to avoid confusing the minus sign with the operation sign +) -

1 + (-1) = 0 inverse element of 1 is -1.

2 + (-2) = 0 inverse element of 2 is -2.

3 + (-3) = 0 inverse element of 3 is -3.

Fill in the blanks: -

4 + (-4) = 0 inverse element of 4 is ___.

5 + _____ = 0 inverse element of ___ is ___.

___ + (-8) = 0 inverse element of ___ is ___.

<u>Assignment</u>.

Consider the set of whole numbers, including the negative whole numbers $\{\ldots, -2, -1, 0, 1, 2, \ldots\}$. Write the inverse element of each of the following: -

1. 1, _____
2. 7, _____
3. 18, _____
4. -5, _____
5. -19, _____
6. 0, _____

* * * * * * * * * * * * *

If we include the negative whole numbers, then there are inverse elements for the operation of addition. What is the identity element for the operation of multiplication? Consider these examples: -

$$\left.\begin{array}{ccccc} 1 & \text{x} & 1 & = & 1 \\ 2 & \text{x} & 1 & = & 2 \\ 3 & \text{x} & 1 & = & 3 \\ 4 & \text{x} & 1 & = & 4 \end{array}\right\}$$

thus, the identity element for multiplication in this system is 1.

etc.

Now let's see if there are inverse elements for each of the whole numbers under multiplication. Can you find the elements of the set of whole numbers that belong in the spaces below?

2 x ___ = 1

3 x ___ = 1

4 x ___ = 1

etc.

Of course, there are no inverse elements for the set of whole numbers under multiplication. Let's expand the set of whole numbers so that it includes all fractions also: -

$$\{0, 1/4, 1/3, 1/2, 2/3, 5/6, 1, 3/2, 2, \ldots\}$$

Now look at these examples and recall your work with fractions.

2 x 1/2 = 1 inverse element of 2 is 1/2.

3 x 1/3 = 1 inverse element of 3 is 1/3.

4 x 1/4 = 1 inverse element of 4 is 1/4.

Fill in the blanks: -

5 x ___ = 1 inverse element of 5 is ___.

6 x ___ = 1 inverse element of 6 is ___.

17 x ___ = 1 inverse element of 17 is ___.

Assignment.

Consider the set of whole numbers, including all fractions. Write the inverse element of each of the following under multiplication.

1. 5, _____ 2. 9, _____ 3. 25, _____

4. 1/7, ___ 5. 1/11, ___ 6. 1/47, ____

* * * * * * * * * * * * *

So you can see that the set of whole numbers $\{0, 1, 2, 3, \ldots\}$ has no inverse elements for addition or multiplication. We can include such inverse elements, however, by EXPANDING the set of whole numbers to include all of the negative numbers and all of the fractions.

APPENDIX C: Teacher's Resume

TEACHER'S RESUME

From the beginning, the project as outlined by Bob Burnell seemed to have promise of interest and educational value. Beforehand, I had often thought in terms of contract-type work set up in units whereby students could work independently from any set rate; those normally established by the classroom teacher following the needs of the slowest students. In comparing two groups following two different methods of presentation, this project would give some idea of the advantages and disadvantages of this continuous contract-type method. Two classes which appeared to be fairly similar in overall class achievement were selected, one to be the experimental group - 7D, one to be the control group - 7E. Both groups contained strong and weak students; that is, going towards the two extremes of ability as well as what might be termed "average students." The phases of work the two classes went through included work on: -

1. Fractions
2. Number sentences and introduction to algebra
3. Properties of whole numbers
4. Primes, H.C.F. and L.C.M.
5. Introductory geometry
    - point, line, plane, types of angles, measuring, types of polygons
6. Geometric construction
    - use of ruler, protractor, and compass
    - construction of angles and triangles

7. Geometric measurement
    - linear
    - square
    - area of rectangles, parallelograms, and triangles

8. Ratio and per cent

The experimental group began the project very enthusiastically and went through the first set of work very quickly. This presented us with the first type of problem - making sure that new work or extra work or some type of project was ready and at hand for the students as they finished.

As the year progressed, it became quite noticeable that the weak students gradually slowed down and their enthusiasm for the new method weakened. By the end of the year it had become quite a task to keep them going, to have them do the minimum amount necessary. Also, I feel that a good deal of work was being copied, which, of course, really defeated the whole purpose of this method of presentation. The extremely capable students, however, were able to continue at a steady rate, in some cases going more rapidly than new work could be made available for them.

The method of presentation should enable the teacher to give much more work or a broader scope or greater complexity to more advanced students without affecting the work load or difficulty for the weaker students. Major problems appear to be: -

1. the great need for teacher time in preparation of materials;
2. the difficulty of wording to make the work comprehensible for the student. Since much reading is involved, it is necessary to keep explanations simple;

3. the number of work hours needed for the physical presentation on paper (i.e., typing);

4. the copious amounts of paper needed for each unit;

5. the amount of time needed for collating and stapling;

6. room needed for storing units of work -
    - ones to be completed
    - ones completed

7. preparation of sufficient numbers of answer booklets.

This was just for one class of approximately 27 students; one might well imagine the problems for six classes. Problems 1, 2, and 7 would not be affected, but 3, 4, 5, and 6 most certainly would be.

The results of tests given at the end of the year showed little difference between the two groups. Although it would appear that the year's work can be completed by the contract-type method in a much shorter time than by normal teacher presentation, one must always beware of the actual depth of learning taking place. Can we be sure that there is any retention of what supposedly has been learned or is there going to be an even greater loss over a period of two or three or four months than what might be found with the more conventional method?

It would appear that this method might be implemented for the top 20 - 25% of the students, while an increasingly larger amount of teacher time would be given over to the weaker students. There is also a special advantage in having this material on hand for students who enter the school at odd times during the school year, since it enables them to catch up on work missed much more easily in terms of class time for teacher and student.

A further recommendation would be the use of projects that would bring in the practical use of the skills and concepts learned in that particular unit or being combined with preceeding units.

Having gone through this experiment, I know that my own methods will be greatly influenced and my scope of presentation broadened by what has been experienced through this last year.

APPENDIX D: Teacher-prepared "New Math" Supplemental Test

MATHEMATICS TEST

NAME: TOTAL POSSIBLE RESPONSES = 61 ____.

ROOM NUMBER: ____________________.

1. List all of the replacements for "n" from the set of whole numbers $\{0, 1, 2, 3, 4, 5, \ldots\}$ that will make the following number sentences true.

a) $n < 3$ ____________________

b) $n + 3 = 12$ ____________________

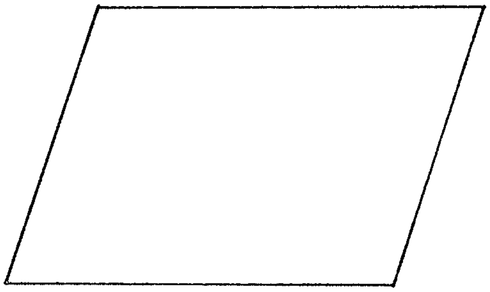
c) $25 = 4n + 5$ ____________________

d) $n > 17$ ____________________

e) $n \leqq 4$ ____________________

2. Each geometric figure below is accompanied by a list of names. Underline the name that <u>best</u> describes each figure.

a)



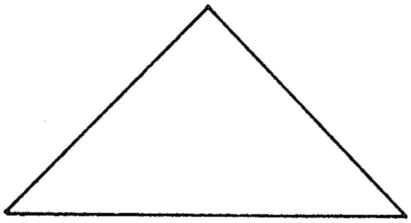
rectangle

trapezoid

square

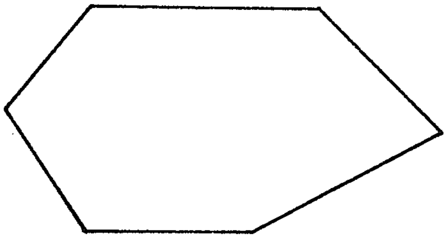
parallelogram

b)



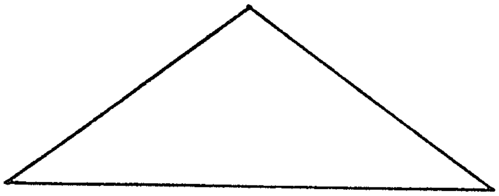
equilateral triangle
isosceles triangle
scalene triangle
obtuse triangle

c)



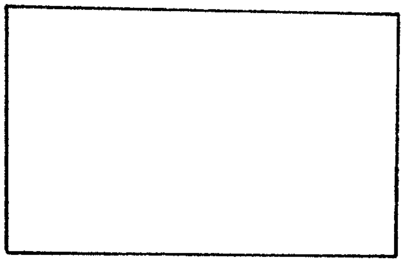
polygon
pentagon
septagon
hexagon

d)



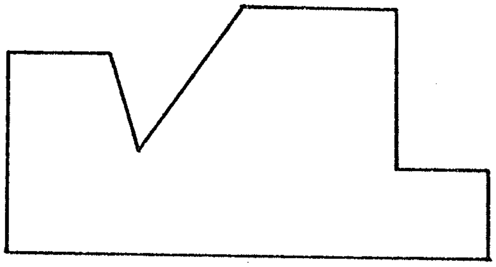
obtuse triangle
scalene triangle
polygon
equilateral triangle

e)



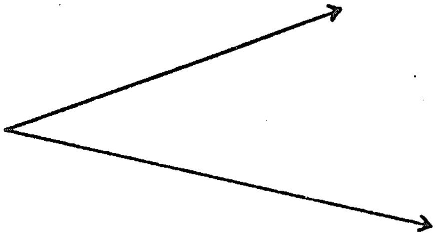
square
rhombus
rectangle
parallelogram

f)



simple closed curve
polygon
pentagon
hexagon

g)



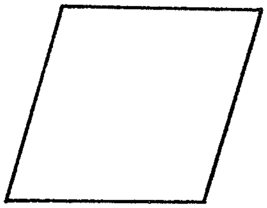
acute angle

obtuse angle

right angle

straight angle

h)



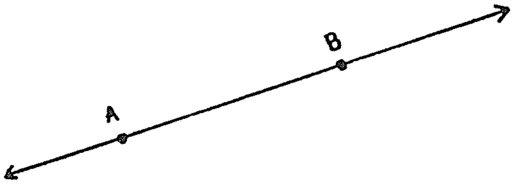
square

quadrilateral

rhombus

parallelogram

i)



$\overline{AB}$

$\overrightarrow{AB}$

$\overleftarrow{AB}$

$\overleftrightarrow{AB}$

j)

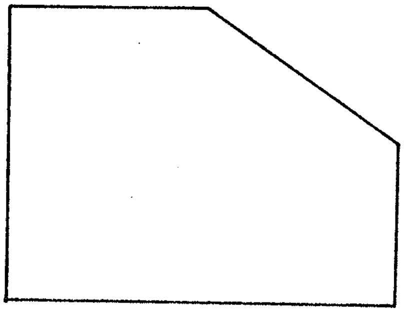
$\overline{PQ}$

$\overrightarrow{PQ}$

$\overline{RP}$

$\overleftrightarrow{PR}$

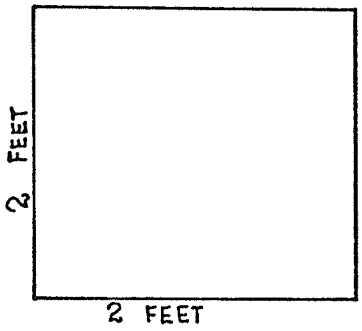
3. Use your ruler to find the perimeter of the following figure. All measurements are to be taken to the nearest 1/4 inch.



P = ____________________.

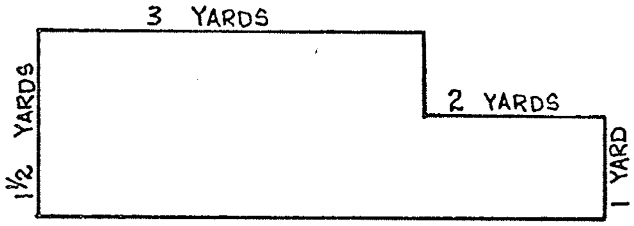
4. What is the area of the following figures?

a)



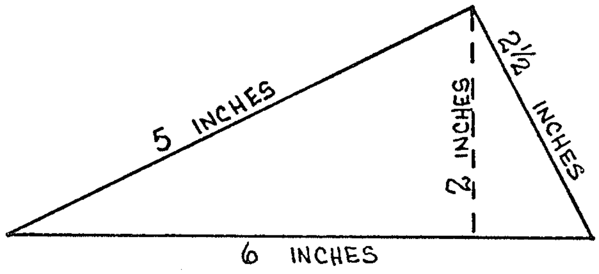
A = ____________________

b)



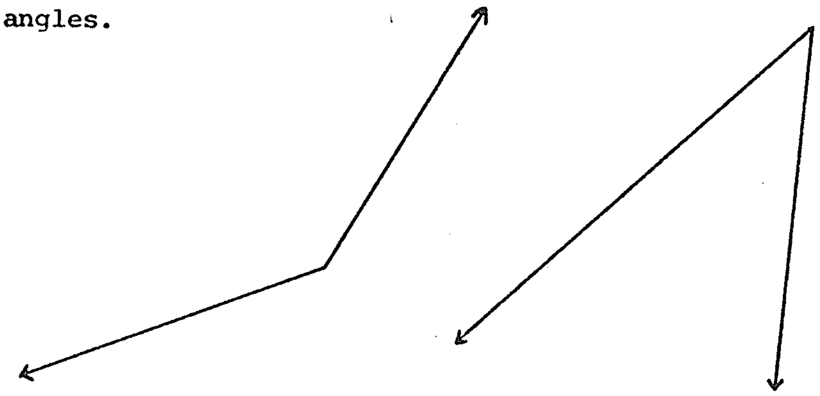
A = ____________________

c)

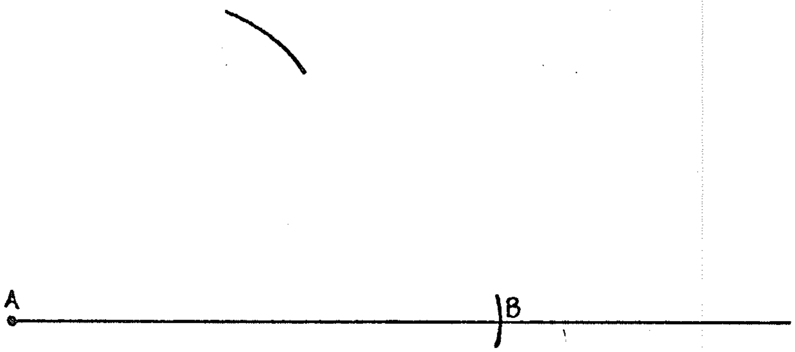


A = ____________________

5. Use your protractor to measure the size of each of the following angles.



6. Below is pictured the construction of a triangle with sides AB = 3 inches, AC = 2 inches, and BC 2 1/2 inches. The construction has been started. Complete it.



7. a) Rewrite the following ratio in three different ways: -

6 to 10 ________________, ________________, ________________

b) A classroom has 8 windows along one 24-foot wall.

i) Express as a ratio the number of windows to the length of the wall. ____

ii) How many windows are there for every 6 feet of wall space?

____

c) In a school of 600 students there are 30 teachers. Express in lowest terms the number of teachers to the number of students.

____

d) A picture frame is 4 feet, 8 inches long by 2 feet, 11 inches wide.

i) What is the ratio of length of this picture frame to its width?

____

ii) Express this ratio in its lowest terms.

____

e) Express two other equivalent ratios for each of the following: -

i) 5/2 ______________________________

ii) 21/35 ______________________________

f) How would the following ratios be expressed using only whole numbers for each of the terms of the ratio?

i) $\frac{3\ 1/2}{1}$

ii) $\frac{1\ 1/2}{2}$

8. Solve for the unknown in each of the following proportions: -

a) $\frac{5}{8} = \frac{x}{24}$ x = ________________________

b) $\frac{x}{3} = \frac{12}{9}$ x = ________________________

c) $\frac{6}{x} = \frac{12}{18}$ x = ____________________

d) $\frac{3}{5} = \frac{x}{11}$ x = ____________________

e) $\frac{7}{4} = \frac{9}{x}$ x = ____________________

f) $\frac{55}{x} = \frac{33}{10}$ x = ____________________

9. Solve the following problems by: -

i) forming a proportion containing an unknown;

ii) solving for the unknown (x = ?);

and iii) expressing the answer in the form of a statement.

a) A tree 30 feet high casts a shadow 40 feet long. What will be the height of a tree that casts a shadow 20 feet long?

PROPORTION: ____________________

STATEMENT: ____________________

b) In an election for school president, John received 3 votes for every 2 votes that Peter received. If John received 291 votes, how many votes did Peter get?

PROPORTION: ______________________________

STATEMENT: ______________________________

c) A map has a scale of 5 miles to the inch. What length on the map will represent a distance of 12 miles?

PROPORTION: ______________________________

STATEMENT: ______________________________

10. Using the list of properties on the following page, indicate which property is being illustrated by each of the following equations. Use the abbreviations provided.

a) $3 + (4 + 7) = (3 + 4) + 7$ ____________________

b) $3 \times 23 = 69$ ____________________

c) $4 + 7 = 7 + 4$ ____________________

d) $(3 \times 4) + (3 \times 8) = 3(4 + 8)$ ____________________

e) $14 + 8 + 0 = (14 + 8)$ ____________________

f) $14 \times 0 = 0$ ____________________

g) $4 \times 8 \times 1 = (4 \times 8)$ ____________________

h) $3 \times (4 + 2) = (4 + 2) \times 3$ ____________________

i) $5\ 1/2 \times 5 = (5 \times 5) + (1/2 \times 5)$ ____________________

j) $0 \div 14 = 0$ ____________________

CPA	Commutative
CPM	
APA	Associative
APM	
DPMA	Distributive
Cl. P.	Closure Property
P1M	Property of 1 in Multiplication (Property of Unity)
PZA	Property of Zero in Addition
PZM	Property of Zero in Multiplication
PZD	Property of Zero in Division

APPENDIX E: Post-test Data and Statistical Computations

Control Group

SUBJECT	MATH CONCEPT	MATH PROBLEM SOLVING	MATH TEST	READING
1	5.8	4.6	17	—
2	8.8	9.7	34	8.3
3	7.6	6.9	31	7.7
4	9.2	8.1	38	8.3
5	7.4	8.6	30	7.1
6	9.2	9.7	39	8.7
7	8.6	8.3	33	8.6
8	9.3	8.3	45	7.5
9	7.4	7.6	28	9.3
10	6.4	6.6	23	6.1
11	6.4	7.2	23	6.7
12	7.8	8.9	41	6.9
13	11.2	9.1	44	9.7
14	9.5	8.3	41	8.2
15	4.8	7.4	14	5.7
16	10.0	9.5	40	10.2
17	7.4	8.3	24	6.5
18	8.5	7.9	28	8.0
19	8.1	8.1	42	8.0
20	8.9	9.7	40	10.0
21	7.0	7.2	25	6.5
22	7.6	5.5	34	6.8
23	7.6	6.6	27	7.8
total	184.5	182.1	741	172.6
$\bar{x}$	8.02	7.9	32.2	7.84
s	1.4	1.3	9.8	1.3

Treatment Group

SUBJECT	MATH CONCEPT	MATH PROBLEM SOLVING	MATH TEST	READING
1	5.0	5.9	21	6.8
2	8.2	8.3	33	7.1
3	9.9	8.3	47	9.2
4	8.0	7.6	29	8.1
5	8.8	7.4	36	8.8
6	7.4	5.9	28	8.0
7	6.0	5.9	15	5.5
8	8.2	9.3	26	8.4
9	10.0	9.7	43	8.1
10	7.8	6.9	38	7.8
11	5.3	6.9	8	4.1
12	6.0	6.9	18	6.8
13	9.0	9.1	45	9.1
14	8.2	7.2	33	6.9
15	9.3	10.1	47	9.4
16	10.2	10.1	45	8.8
17	7.6	8.6	35	7.8
18	6.6	7.6	21	7.7
19	8.6	7.9	52	8.8
20	10.8	10.9	58	9.1
21	7.0	8.6	29	6.0
22	8.9	7.9	28	7.3
23	7.2	5.9	30	7.7
24	7.8	8.1	28	8.8
total	191.8	191.0	793	186.1
$\overline{x}$	7.99	8.0	33.0	7.75
s	1.5	1.4	12.2	1.3

Mathematical Concepts

Calculation of sample mean and standard deviation:

Control: - (n_1 = 23)

$$\bar{x}_1 = \frac{184.5}{23} = 8.02$$

$$s_1 = \sqrt{\frac{1525.93 - \frac{(184.5)^2}{23}}{22}}$$

$$= \sqrt{2.08} = 1.44 \approx 1.4$$

Treatment: - (n_2 = 24)

$$\bar{x}_2 = \frac{191.8}{24} = 7.99$$

$$s_2 = \sqrt{\frac{1584.20 - \frac{(191.8)^2}{24}}{23}}$$

$$= \sqrt{2.23} = 1.49 \approx 1.5$$

Calculation of t value for mathematical concepts scores:

$$(n_1 - 1)s_1^2 = \mathcal{E}X^2 - \frac{(\mathcal{E}X)^2}{n_1} = 1525.93 - 1480.01 = 45.92$$

$$(n_2 - 1)s_2^2 = \mathcal{E}X^2 - \frac{(\mathcal{E}X)^2}{n_2} = 1584.20 - 1532.80 = 51.4$$

$$t = \frac{8.02 - 7.99}{\sqrt{\frac{45.92 + 51.4}{23 + 22} \circ \frac{24 + 23}{24 \times 23}}}$$

$$= \frac{8.02 - 7.99}{\sqrt{\frac{97.32}{45} \circ \frac{47}{552}}}$$

$$= \frac{.03}{\sqrt{2.16 \quad x \quad \frac{47}{552}}}$$

$$= \frac{.03}{\sqrt{.18}}$$

$$= \frac{.03}{.43}$$

$$= .0698$$

Problem Solving

Calculation of sample mean and standard deviation:

Control: - (n_1 = 23)

$$\bar{x}_1 = \frac{182.1}{23} = 7.91 \approx 7.9$$

$$s_1 = \sqrt{\frac{1480.03 - \frac{(182.1)^2}{23}}{22}}$$

$$= \sqrt{1.74} = 1.34 \approx 1.3$$

Treatment: - (n_2 = 24)

$$\bar{x}_2 = \frac{191.0}{24} = 7.95 \approx 8.0$$

$$s_2 = \sqrt{\frac{1564.74 - \frac{(191.0)^2}{24}}{23}}$$

$$= \sqrt{2.10} = 1.44 \approx 1.4$$

Calculation of t value for problem solving scores:

$$(n_1 - 1)s_1^2 = \Sigma X^2 - \frac{(\Sigma X)^2}{n_1} = 1480.03 - 1441.76 = 38.27$$

$$(n_2 - 1)s_2^2 = \Sigma X^2 - \frac{(\Sigma X)^2}{n_2} = 1564.74 - 1544.49 = 44.74$$

$$t = \frac{8.0 - 7.9}{\sqrt{\frac{44.74 + 38.27}{23 + 22} \cdot \frac{24 + 23}{24 \times 23}}}$$

$$= \frac{8.0 - 7.9}{\sqrt{\frac{83.01}{45} \cdot \frac{47}{552}}}$$

$$= \frac{.1}{\sqrt{1.84 \times \frac{47}{552}}}$$

$$= \frac{.1}{\sqrt{.16}}$$

$$= \frac{.1}{.4}$$

$$= .2500$$

Supplemental "New Math" Test

Calculation of sample mean and standard deviation:

Control: - ($n_1 = 23$)

$$\bar{x}_1 = \frac{741}{23} = 32.2$$

$$s_1 = \sqrt{\frac{25555 - \frac{(741)^2}{23}}{22}}$$

$$= \sqrt{96} = 9.79 \approx 9.8$$

Treatment: - ($n_2 = 24$)

$$\bar{x}_2 = \frac{793}{24} = 33.0$$

$$s_2 = \sqrt{\frac{29633 - \frac{(793)^2}{24}}{23}}$$

$$= \sqrt{149.17} = 12.23 \approx 12.2$$

Calculation of t value for supplemental test scores:

$$(n_1 - 1)s_1^2 = \varepsilon X^2 - \frac{(\varepsilon X)^2}{n_1} = 25555 - 23873.1 = 1671.9$$

$$(n_2 - 1)s_2^2 = \varepsilon X^2 - \frac{(\varepsilon X)^2}{n_2} = 29633 - 26202 = 3431$$

$$t = \frac{33.0 - 32.2}{\sqrt{\frac{1671.9 + 3431}{23 + 22} \cdot \frac{24 + 23}{24 \times 23}}}$$

$$= \frac{33.0 - 32.2}{\sqrt{\frac{5102.9}{45} \cdot \frac{47}{552}}}$$

$$= \frac{.8}{\sqrt{9.65}}$$

$$= \frac{.8}{3.1}$$

$$= .2581$$

Reading

Calculation of sample mean and standard deviation:

Control: - (n_1 = 22)*

$$\bar{x}_1 = \frac{172.6}{22} = 7.84$$

$$s_1 = \sqrt{\frac{1387.57 - \frac{(172.6)^2}{22}}{21}}$$

$$= \sqrt{1.60} = 1.27 \approx 1.3$$

Treatment: - (n_2 = 24)

$$\bar{x}_2 = \frac{186.1}{24} = 7.75$$

$$s_2 = \sqrt{\frac{1481.27 - \frac{(186.1)^2}{24}}{23}}$$

$$= \sqrt{1.66} = 1.29 \approx 1.3$$

* data not available for one subject, reducing sample size to 22.

Calculation of t value for reading scores:

$$(n_1 - 1)s_1^2 = \Sigma X^2 - \frac{(\Sigma X)^2}{n_1} = 1387.57 - 1354.13 = 33.44$$

$$(n_2 - 1)s_2^2 = \Sigma X^2 - \frac{(\Sigma X)^2}{n_2} = 1481.27 - 1443.05 = 38.22$$

$$t = \frac{7.84 - 7.75}{\sqrt{\frac{38.22 + 33.44}{23 + 21} \circ \frac{24 + 22}{24 \times 22}}}$$

$$= \frac{7.84 - 7.75}{\sqrt{\frac{71.66}{44} \circ \frac{46}{528}}}$$

$$= \frac{.09}{\sqrt{3.26 \times \frac{23}{528}}}$$

$$= \frac{.09}{\sqrt{.14}}$$

$$= \frac{.09}{.38}$$

$$= .2368$$

APPENDIX F: Data for Upper and Lower Extremes of Control and Treatment Groups

Top Seven I.Q. Subjects - Post-test Results

CONTROL SUBJECT	MATH CONCEPT	MATH PROBLEM SOLVING	MATH TEST	READING
1	9.2	8.1	38	8.3
2	9.2	9.7	39	8.7
3	8.6	8.3	33	8.7
4	11.2	9.1	44	9.7
5	10.0	9.5	40	10.2
6	8.9	9.7	40	10.0
7	7.6	6.6	27	7.8
total	64.7	61.0	261	63.4
$\bar{x}$	9.2	8.7	37.43	9.1

TREATMENT SUBJECT	MATH CONCEPT	MATH PROBLEM SOLVING	MATH TEST	READING
1	9.9	8.3	47	9.2
2	10.0	9.7	43	8.1
3	9.0	9.1	45	9.1
4	9.3	10.1	47	9.4
5	10.2	10.1	45	8.8
6	8.6	7.9	52	8.8
7	10.8	10.9	58	9.1
total	67.8	66.1	337	62.5
$\bar{x}$	9.7	9.4	48.14	8.9

<u>Bottom Five I.Q. Subjects - Post-test Results</u>

CONTROL SUBJECTS	MATH CONCEPT	MATH PROBLEM SOLVING	MATH TEST	READING
1	6.4	6.6	23	6.1
2	4.8	7.4	14	5.7
3	7.4	8.3	24	6.5
4	7.0	7.2	25	6.5
5	7.6	5.5	34	6.8
total	33.2	35.0	120	31.6
$\bar{x}$	6.6	7.0	24	6.3

TREATMENT SUBJECT	MATH CONCEPT	MATH PROBLEM SOLVING	MATH TEST	READING
1	6.0	5.9	15	5.5
2	5.3	6.9	8	4.1
3	6.0	6.9	18	6.8
4	6.6	7.6	21	7.7
5	7.0	8.6	29	6.0
total	30.9	35.9	91	30.1
$\bar{x}$	6.2	7.2	18	6.0

APPENDIX G: Students' Opinions

STUDENTS' OPINIONS

Favourable

"I thought that the work in the booklet was interesting and I learned a lot."

"I think this method of teaching is satisfactory for the students who are generally good in math. I find this is better for me because we work at our own speed."

"I think the idea of these booklets was really a good idea. They helped me and describe many things better than the text. I think they should be continued."

"I thought the method of teaching was good because it gave people a chance to work at their own speed. I thought the explaining in writing in the book was better than just talking, because you could go back and look it up if you forgot something."

"I think this is a good way to teach it except I think you should have checked the books to see if they contained what was in the test."

"It was a pretty good method. I learned alot in it."

"This system this year was most enjoyable and improvement over the books. The booklets are not dragged out as the work in the books are."

"I thought that this years work wasn't bad even though some parts were harder than others but I enjoyed it any way. I learned new ways to divide too."

"I liked this way of doing it better this way because you can work at your own speed."

"I thought the method of math was quite good. It let you be more on your own. One of the good things is that if you do bad in a test it is all your own fault so you can't blame it on the teacher."

"I thought that it was interesting and I learned quite a lot."

"I think this method is better than taking it right from the text book and it explains better."

"I think this system was quite good. I think it was better than just papers or work from the book."

"I liked this years math class more than any others. I thought it wasn't too boring and the work in the booklets more than I would in the books."

"I thought it was a lot easier than"

Unfavourable

"I think when a kid can't do something the teacher (not saying any names) should try to be patient and help the kid out as best he can. like in grade 6 I couldn't do something the teacher helped me out he told me to work it out how I think it is to be done, then he would check it and tell me what I did wrong."

"I didn't like the booklet very much. Cause I liked it better when the teacher explained it on the board. I didn't understand half as good as last year."

"I don't like the method we had this year. I think it would have been better if the teacher had explained it. It's ok for the smart one but no for us slow ones."

"Some booklets were okay"

"To much homework"

"Math was O.K. But I would rather have had a teacher teach us instead of booklets."

Mixed

"I liked the booklets except that one or two sections could have been explained more clearly, ex. the latest book on percent. I think it was better when you"

"I think that the method was O.K. but some of the harder work should have gone over it together."

"It was good enough better than the text except that you can cheat too much if your no good at math and otherwise was fair."