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EFFECTS OF SOIL TYPE AND RISK AVERSION ON CROP PLANNING AND USE OF FERTILIZER

by

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ABSTRACT

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Farm income variability due to unstable and uncertain production and marketing has historically created anxiety among primary producers of agricultural products. The present study was undertaken to develop a method by which net farm income variation as associated with yield uncertainty only, was identified and included in the planning stage of future production plans.

It was felt that farmers react in different ways toward uncertainty. Different producers would in fact discount target yields more or less depending on their values in life and risk consciousness. In any event, no producer would discount planning values of yield (and therefore net income), beyond a point defined as a farm survival level of income. These basic ideas were encompassed into hypotheses and tested. The primary objectives included determination of: discounted average yields for wheat, oats, barley and flax at different levels of risk and fertilizer, optional crop plans and fertilizer use for alternative risk levels and, the

the effects of soil type, summerfallow, capital, labor, quota restrictions and risk aversion on crop planning.

The model is static but the planning period is five years corresponding to the five year date base from which yield production functions and variances of predicted yields were processed. The hypotheses were tested through a two part model. The first part maximized net income for alternative plans based on varying levels of exogenous variables and risk associated C_j values; the other part calculated the probability of any plan, previously determined, exceeding a specified value of income.

The technological matrix was based primarily on a case farm. Yields were not. As the procedure for discounting yields (by different values corresponding to alternative risk levels) required normal distributions, a source of many yield-nitrogen observations were required. This was fulfilled to a satisfactory degree by Manitoba Crop Insurance data. These yields were classified, weighted, and then summed to attain the required observations necessary to facilitate meaningul regression analysis and develop yield and variance of yield prediction equations.

An analysis of the primal model results indicated that as risk aversion increased, the amount of N used decreased. Also, no definite conclusions could be arrived at as to decreasing fertilization accompanying increasing levels of summerfallow. Capital restrictions had major effects upon net income, whereas quota restrictions had only minor effects. It was shown indirectly that the difference between actual N used and target N recommendation increased as certainty increased. The effect of risk aversion on longrun land resource decisions were also analyzed. Summerfallow proved not to be an economic means of supplying plant nutrients under the assumption of the model.

The secondary part of the model calculated the probability of plans exceeding a specified level of income. Fewer plans corresponding to increased levels of certainty surpassed the survival level of income than plans based on lower certainty values. Similarly plans based on increasing summerfallow levels and decreasing capital levels had lower probabilities of surpassing the specified level of income. The major finding indicated that there were many instances whereby future production plans that exhibited large expected net income and large variation of income could be subjected to discounting procedures with the result that expected net income decreased marginally whereas variation of income (risk) decreased significantly.

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Chapter 1

INTRODUCTION

This study determines feasible optimal crop plans and fertilizer use under conditions of subjective risk aversion for a selected farm business in the Carman District Farm Business Association. The effects of soil types, amount of summerfallow, available capital, available labor, market quota restrictions, and risk aversion on crop planning and fertilizer use are identified and evaluated through the development and operation of a linear programming model which maximizes net farm income. Soil type, summerfallow, and fertilizer use are translated into a set of constrained (endogenous) variables which produce wheat, barley, oats and flax. Available capital, available labor, quota restrictions and amounts of summerfallow are handled in the system as exogenous variables. Risk aversion is taken into account by determining the standard deviations of crop yields (independent and normally distributed random variables) and by discounting the corresponding mean yields by a given percentage of these standard deviations. In effect, the net prices in the objective function are discounted. Farm plans are obtained for alternative combinations of the exogenous variables as well as for alternative levels of risk acceptance, beginning at the 50 per cent level and declining to the 20 per cent level. For each of these farm plans, the probability of net farm income lying below a specified level is calculated. This specified level is the sum of annual fixed debt obligation plus personal withdrawals of income from the business.

NATURE AND SCOPE OF THE PROBLEM

Farm income variability due to unstable production, prices and marketings has historically created frustration, fear and anger among primary producers of agricultural products. Government price supports, assistance payments and marketing regulations all have been formulated to assist producers of a similar commodity at times of reduced farm income. Programs of crop insurance, hail insurance and research directed toward variety or breed improvement are maintained to ensure against unsatisfactory levels of production. These farm policies and programs are created with a view toward preventing farm income for a commodity or for the whole primary agricultural economy from falling below a certain level.

What does the individual farm manager do in the face of this variability of farm income? How does he plan his strategy to maximize profit and simultaneously plan against the ill effects of income instability which may often be a threat to farm survival? Regardless of the method used to determine farm plans under conditions of uncertainty, the farmers' first end is to survive on the farm.[1] The second end is to maximize profit.[2]

Farmers react in different ways toward uncertainty. If it were possible to measure accurately an individual farmer's attitude toward an uncertainty such as yield variation or quota limitation, then it would be feasible to build a farm planning model for any farm manager that would embody his attitudes toward risk.[3] Given similar conditions of technology, levels of working and capital inputs, and future expectations of prices, yields and marketings, farmers will react or plan in different ways. They will be guided by their own personal attitudes toward risk given their own conditions of debt, or equity and of satisfaction for money gains or prestige.

This study will endeavor to concern itself with net farm income variation as associated with grain production only. In this case income variability can arise from a variation in yield, in prices, and in marketings. Production inputs such as capital, labor and land can affect farm income and can add to income instability if

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[1] Farm survival is defined as a farmer's actions of minimizing the probability of an uncertain loss rendering him insolvent. The level of survival must cover the annual farm costs and the level of household consumption expenditures.

[2] Schickele, R. "Farmer's Adaptations to Income Uncertainty", Journal of Farm Economics, Vol. 32 (August, 1950) p. 362.

[3] There is a basic fundamental difference between risk and uncertainty. Any event is considered an uncertainty until it is given a probability of its happening. Once an event is given 'odds' of occuring, it is considered a risk. To incorporate any uncertainty into a planning model, some estimation of its chances of occuring must be determined; therefore, making it a risk.

their levels are uncertain or vary from one production period to the next.

Yield and market uncertainties create the strongest feelings of doubt among grain producers. For all grains controlled by the Canadian Wheat Board a prospect of expected prices and marketings is available to all farmers prior to the next production period. Price and quota uncertainties are reduced for the individual farmer and the farms economy as a whole.[4] However, crop yields for the individual are highly uncertain. Observation suggests that unpredictable weather conditions are the major cause of yield variability, besides their combined effects with soil types, soil nutrients, and management practises. Because of the very nature of weather, predictability of yield is next to impossible at this time.[5] Whether one considers the problem of optimal fertilization of crops or not as well as the associated costs of fertilizing, income variation is likely to be even larger than yield variation. Many costs are in effect fixed once the crop is seeded and therefore any variation in yield is nearly totally reflected in the same absolute amount of change in net return.

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[4] Farmers are aware of the initial price they will receive for Wheat Board controlled grains; however, there is still uncertainty attached to the final payment and therefore the overall net price. Similarly producers can not be certain if they may be allowed to deliver more grain than the guaranteed minimum.

[5] Yeh, M.H. and L.D. Black, Weather Cycles and Crop Productions. (The University of Manitoba, Faculty of Agriculture and Home Economics, Tech. Bulletin No. 8, November 1964) p. 8.

Efficient fertilizer use can yield great profits.[6] If incorrect use of fertilizer is made, profits may be considerably reduced, or net losses may result. Hence, precise information is required as to what quantities of applied nutrients are most profitable under various environmental and economic conditions. Due to conditions of price and yield uncertainty, this information is particularly imperative in decisions on whether or not to use fertilizer, and on the amounts and optimum nutrient combinations.

Interaction of the resource mix can also have a bearing on income variation through increased or reduced production. A limitation on capital can reduce fertilizer inputs, labor inputs, custom work, etc. which can reduce yield in one way or another. A fixed capital constraint confronted with increased costs of major inputs can cause the same result. An unstable labor situation can reduce production through fewer acres farmed, fewer acres cropped and lower amounts of fertilizer used.

Faced with income uncertainty, a farm manager must weigh all the uncertainties related to yield. On the basis of his own circumstances, his own attitudes toward the uncertainties spelled out, he will strive to make plans that satisfy his income objectives while minimizing the chances of not achieving these objectives. Because farmers realize that the chances of achieving some target net income

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[6] Veneyian, E. Economic Optima For Fertilizer Production Functions in Relation to Weather, Crop, Soil and Location Variables. Unplublished Ph.D. thesis, Iowa State, Ames Iowa, 1962.

goal are not certain, they wish to evaluate the trade-offs between expected income and risk acceptance of alternative plans before selecting a plan of operation.

This study considers aversion and indifference to risk only as it is illogical to suspect that any large number of farmers would gamble for a high income on the basis of high yields occuring at small odds. The farmer who is indifferent to risk would accept some optimal production point from the relevant production function in his planning system, while the farmer who is a risk averter would discount this point.[7] If it is this new discounted yield figure which is incorporated into the planning system then relative net returns per unit of activity change and, in effect, the corresponding farm plan increases the chances of actual net income exceeding some specified level, thus reducing income uncertainty. This in many cases may increase the chance of farm survival as the consequences of low yields and high costs of inputs on farm income have been planned against.

A risk averter will not discount his net income beyond a point defined earlier as the 'farm survival' level. In the discussion to follow in later sections, this level will be known as specified income. It will be specified in the sense that it is necessary to achieve this figure to insure farm survival by meeting all production costs and

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[7] The yield corresponding to the selected production point may very well be a target yield recommended through a soil test.

personal withdrawals over the long run. This suggests that for a risk averter there will be a trade-off point somewhere between the specified income and the net income generated by a farm plan corresponding to that farm manager's indifference toward risk. Similarly it would be possible for a risk averter to determine the probability of net farm income lying below specified income for any farm plan generated.

OBJECTIVES OF THE STUDY

The central theme is to identify and evaluate the effects of soil type, summerfallow, labor, capital, quota restrictions and risk averstion on crop planning and fertilizer use. The specific objectives which facilitate achievement of this central theme are:

1. To determine discounted average yields for wheat, oats, barley, and flax at different fertilizer rates on different soil types in accordance with selected levels of risk aversion.

2. To determine optimal crop plans and fertilizer use for alternative levels of the exogenous variables. ie. capital, quota restrictions and summerfallow.

3. To evaluate the effects of soil type, summerfallow, capital, labor, quota restrictions and risk aversion on crop planning, fertilizer recommendations and the probability of net farm income lying below a specified level.

HYPOTHESES

This study is guided by two hypotheses:

1. It is hypothesized that as risk aversion increases, the amount of fertilizer used decreases, and the differences between the target fertilizer recommendations and actual amounts used increases. It follows that these differences will be larger for crops grown under high variability and, correspondingly, that land associated with high yield variability might receive zero amounts of fertilizer. Further, under high risk aversion, land associated with high yield variability may not be competitive and, correspondingly, may not enter optimal solutions. Further, it is anticipated that summerfallow replaces fertilizer as a means of supplying plant nutrients when (a) the difference between the target fertilizer recommendation and actual amount used increases, (b) the level of capital declines, and (c) the more severely crop production is limited by quotas.

2. It is hypothesized that fixed debt obligations and personal withdrawals of income limit the degree to which risk acceptance can be reduced. It is implied that some risk must be accepted if net farm income is to be large enough to cover these obligations and personal withdrawals of income. Accordingly, this places bounds on the number of feasible cropping plans available to a risk averter.

PROCEDURES FOR ACHIEVING THE OBJECTIVES THROUGH TESTING HYPOTHESES

The hypotheses are tested through the operation of a linear programming model which maximizes net income. The effects of soil type, amount of summerfallow, available capital, market quota regulations, and risk aversion on crop planning and fertilizer use are evaluated through the operation of this model. Soil type, summerfallow and fertilizer use make up the basis of the constrained (endogenous) variables which produce wheat, oats, barley, and flax.

The model is static but the planning period is five years. It is assumed that all covariances between grains over the five year period are equal to zero. This corresponds to the five year data base (1964-68) from which production functions and variance of predicted yield functions were processed.

Farm plans are obtained for alternative combinations of the exogenous variables as well as three levels of risk aversion. Three relatively different soil types form the land base of the programming model in order to test the hypothesis that yield variation and associated effects are greater on one soil type than another. Four different summerfallow ratios are tested for each level of risk aversion in order to determine fertilizer use and risk aversion interaction at times of limited quota and/or limited capital. Similarly, five levels of capital and two levels of market quota restrictions are interacted with risk aversion to determine effects on crop plans and net income. Farm plans will be based solely on grain crop production where wheat on fallow, wheat on stubble, oats on stubble, barley on stubble, flax on stubble and flax on fallow for all three soil types are activities within the model.

Risk aversion enters into the model by way of yield values incorporated into the net prices of the productive activities available for solution. Risk aversion is taken into account by determining the standard deviations of crop yields at five points on the relevant production functions and by discounting the corresponding predicted mean yields by a given percentage of these standard deviations. The other components of net prices as determined in the model, mainly prices per bushel sold and all costs per acre of production, are considered as fixed in the short run and are held constant. In effect, the net prices in the objective function are discounted by a risk factor associated with yield variation.

Fertilizer use enters into the model by means of the activities listed above appearing at different levels of actual nitrogen. All crops grown on all three soil types are associated with five levels of actual N added to the soil with the exception of flax on fallow where no nitrogen is added and flax on stubble where three levels of actual N are considered. Yields and variances of yields for all crops grown at a specific added nitrogen level are determined through production function analysis.

For each of the farm plans obtained for alternative combinations of exogenous variables as well as for alternative levels of risk acceptance, the probability of net farm income for a particular plan lying below a specified income level is calculated. The probability of net farm income lying below this specified level is calculated under the statistical theorem that linear combinations of independent random variables which are normally distributed, are also normally distributed. The resulting distribution can be standardized and, correspondingly, probability calculated.

ORGANIZATION OF THE STUDY

Chapter 2 develops the framework for analysis based on theory of the firm under conditions of imperfect knowledge, and analyzes the effects of subjective risk on the planning models. Risk models used in other studies and the subjective risk model to be used in this study are presented at the end of the chapter.

The next chapter develops the model through (1) specifying the method of solution and (2) specifying the technological matrix, activities, and constraints to be used in the model.

Chapter 4 describes the empirical procedures used to render the model functional. These include obtaining, organizing, and projecting data as required by the model.

The fifth section reports on the tests of hypotheses. It examines the effect of risk aversion on cropping plans and fertilizer use as well as the trade-off between risk aversion and necessary income generated from a particular farm plan.

Chapter 6 summarizes the study and provides inference for further study.

Chapter 2

ANALYTICAL FRAMEWORK

CLASSICAL THEORY OF THE FIRM

Economic production is motivated by income incentives. Under perfect competition profit motivation provides the incentive for all firms based upon the role of a decision maker who maximizes net income when given a set of prices and a set of technical production functions. Perfect competition assumes: a perfectly elastic supply of inputs, a perfectly elastic demand for products, homogenity of product and perfect knowledge of market conditions and technical input-output relationships. A farm faced with these conditions would be able to attain optimal formation and use of resources for farm production.

Pure competition differs from perfect competition in that the assumption of perfect knowledge is not fulfilled. This situation is typical of general grain production. Other forms of industrial make-up including monopolistic competition, oligopolies and monopolies all disregard in varying degrees some or all of the remaining assumptions necessary for a perfect competitive industry.

DEVELOPMENT OF RISK ALLOWANCE

Uncertain income expectation resulting from imperfect knowledge was introduced into the theory of the firm by J.R. Hicks.[1] Hicks spoke of expected future prices as a subset of requirements for planning. He felt that plans are dependent not only on current prices but also the planner's expectations of future prices. Hicks says,

> Even if the most probable price expected to rule at some future date remains unchanged, a person's readiness to adopt a plan which involves buying or selling at that date may be affected, if he becomes less certain about the probability of that price, if the dispersion of possible prices is increased.2

Hicks is referring at this point to the degree of price variation--the increase in range of possible prices. He goes on to say that an increased dispersion of prices will have the same effect on planning as a reduction in the expected price.

> If we are to allow for uncertainty of expectations, in these problems of the determination of plans, we must not take the most probable price as the representative expected price, but the most probably price ± an allowance for the uncertainty of the expectation, that is to say, an allowance for risk.3

The allowance for risk is not determined solely by the

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[1] Hicks, J.R., Value and Capital; 2nd edition. London, England, Oxford; University Press 1946.

[2] Ibid, p. 125.

[3] Ibid, p. 126.

opinion of the planner about the degree of uncertainty but also by the willingness of the planner to accept risks which depends upon his tastes and preferences. Hicks concludes by suggesting that we be prepared to interpret these certain expectations as being those figures which best represent the uncertain expectations of reality.

Technological knowledge in terms of yields or productivity are introduced as uncertain in this study. Costs and physical input-output relationships are assumed to be known. The Hicksion approach of discounting general prices is applied to net prices.[4] By the discounting of yields and the holding constant of costs, in effect net prices are discounted.

The representative or discounted yield is the most probable or average yield minus an allowance for risk as attached to uncertain yield expectations. Given any grain yield which is normally distributed over time with mean (μ) and variance (σ^2) as parameters, it is possible to incorporate risk into an objective criterion capable of solution. This discounting procedure can be shown graphically as in Figure 2.1. It can be seen that the new discounted net price $\bar{p}_1$ is a function of the average net price (μ) minus the standard deviation (σ) of observed net prices times the absolute value of a subjective risk aversion factor (z). It

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[4] Net prices in this case are equal to 'Yield x Price - Costs (fixed and variable)'.

is important to note that in this study prices of grains are held constant, therefore variation of net prices of activities results from variation in yields.

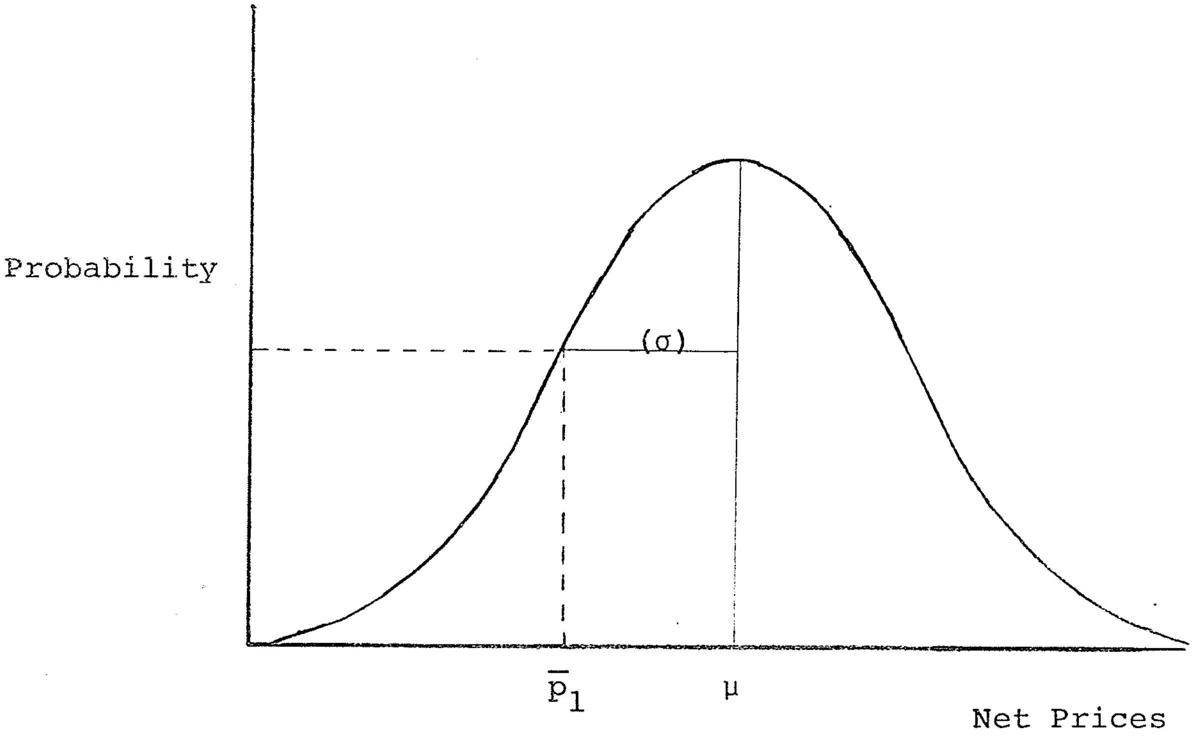

FIGURE 2.1

CALCULATION OF DISCOUNTED YIELD GIVEN A NORMAL DISTRIBUTION OF NET PRICES FOR A PARTICULAR GRAIN

THEORY ASSOCIATED WITH UNCERTAINTY

Decision making under conditions of uncertainty is a part of life. There is no way of building uncertainty into a planning model; the very existence of uncertainty guarantees that any decision in the present is not necessarily the correct one. It is a fact that if uncertainty did not exist, the decision making chore would be changed completely to mechanical means without the slightest regard for human managerial ability. As long as uncertainty exists man will make the final decisions regarding any economic or social plans; he will continue to do so using modern approaches to decision making or the longest used approach of appeal to the inner subjective feeling.

Planning models cover a wide spectrum. Decisions can be made on the basis of no information (just a subjective feeling) or on the basis of a systematic approach encompassing many sources of information. Availability of reliable data, soundness of decision tools and models, and the degree of belief in planning models are all factors which determine the degree of sophistication that a decision maker wishes to employ.

Pure Risk

The theory of risk is based upon the concepts of probability and its relationship to utility. An event that has occured a large enough number of times in the past lends to itself a probability that it will occur again in the future. As soon as some certain probability can be attached to an event that up to that time has been uncertain (no probability associated with it), then the uncertainty becomes a risk--something which can be insured against. A simple example of this being fire insurance on the home whereby insurance companies knowing the probability (based on past observation) of any house burning down, can set premiums accordingly and ensure that all individuals whose homes burn down can be compensated at no net loss of revenue to the insurance company. This is a case of pure risk. Crop insurance works on the same principle. Once an individual decides upon his course of action or cropping strategy, he then can guarantee himself a certain value of yield by insuring his various grains. It is important to note here, however, that the uncertainty associated with yield is not taken into account at the time of formulating cropping plans. In the examples cited the probability of any home burning or of a crop yielding below a particular value is known with certainty and therefore can be insured against.

Subjective Risk

Subjective risk differs from pure risk in that a probability of an event occuring is dependent not only on a distribution but on an attitude which is subjective. This attitude is based upon individual's opinions, their outlooks, their tastes and preferences. The components of subjective risk have been combined with income to explain the utility theory of production. Utility is the level of satisfaction received from an object or an event. A utility function can be described as some unmeasurable function which is a mathematical formulation of any person's feelings toward all qualities in life, tangible or intangible. This function is based upon an individual's tastes and preferences, his levels of satisfaction, his desires in life, and his differences or indifferences to the unknown--that is, his subjective probability or risk attitude. It is understandable why subjective risk and utility are in part one and the same.

Subjective Risk in the Realm of General Economic Theory

The most obvious of the extensions of orthodox economic theory which embraces the concept of uncertainty is also the most simple. It consists simply of revising the usual certainty model so that expected return or expected profit becomes the object to be maximized when uncertainty enters the picture. It is, of course, a perfectly symmetical expansion of its classical predecessor embracing the former as the special case of zero risk, or of expectations which are held with certainty. This concept is generally written as:

$$U = f\ (\mu,\sigma^2)$$

where U equals utility, μ equals expected return and σ^2 equals variance of expected return or risk. Should risk disappear then:

$$U = f\ (\mu,o)=\mu$$

Certain broad areas of agreement as to the nature of expected return-variance relationships exist. Most persons agree, for example, that increases in expected return, μ, tend to heighten its desirability. Similarly, increases in the dispersion of possible outcomes, σ^2, tend to soften a future plan's desirability. Also, movements along a single, risk-return indifference curve require additional units of risk to be compensated for by additions to expected return. These conditions may be expressed symbolically as:

$$\frac{\partial U}{\partial \mu} > 0; \quad \frac{\partial U}{\partial \sigma^2} < 0; \quad \text{and} \quad \frac{d\sigma^2}{d\mu} = \frac{\frac{\partial U}{\partial \mu}}{\frac{\partial U}{\partial \sigma^2}} > 0.$$

The last term, the marginal rate of substitution between the expected return and risk, follows directly from the first two.

The function:

$$U_o = f\ (\mu, \sigma^2)$$

then, can be likened to an indifference curve defined by the locus of μ, σ^2 points whose values are deemed to be equal to the certain receipt of U_o utility (dollars). Similarly, for each alternative plan described by points (μ_j, σ^2_j), a unique utility exists,

$$U_j = f\ (\mu_j, \sigma^2_j),$$

by which its desirability can be compared to other plans. Because utility functions are non-cardinal in nature, economists depend on ordinal numbering of related indifferences curves when considering utility. Considering expected income and income variance (risk acceptance) as variables, one would expect a farmer to be more satisfied or have a higher utility the greater his average (or <u>expected</u>) income and the lower his income variability. As

Markowitz[5] demonstrates, using an E-V indifference system,[6] satisfaction increases from southeast to northwest, while the indifference curves run from southwest to northeast[7] (Figure 2.2).

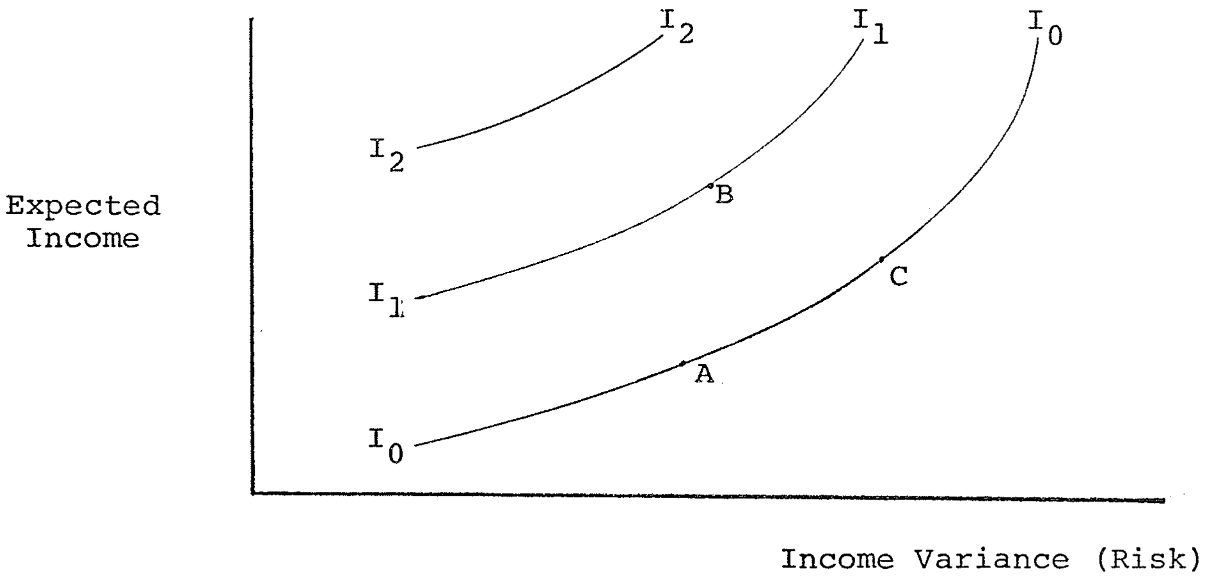

FIGURE 2.2

AN E-V INDIFFERENCE SYSTEM

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[5] Markowitz, Harry M.; Portfolio Selection: Efficient Diversification of Investments. Cowles Foundation Monograph 16, New York. John Wiley & Sons, 1959.

[6] An E-V indifference system is a set of mathematical curves each of which is formed by different combination of expected income (E) and the variance of risk surrounding the level of expected income (V) that yield the same utility or satisfaction.

[7] In Figure 2.2, the decision maker or farm planner is indifferent between producing at A or B. Point B corresponds to a production resulting in an increased expected income, but also an increased income variance. Point A corresponds to a production resulting in a reduced expected income and variance. The farm planner who can achieve a production on the I_0 level curve is indifferent between points A and B. However, point C which is on the next indifference curve if obtainable can yield more utility as expected income is increased with a small increase in income variance.

Subjective Risk in Production Economics

Heady and Candler[8] introduce this analysis into a production situation. They suggest that there is a minimum income variance that can be obtained for any given income, and alternatively for any given income variance there is a maximum attainable expected income. Thus in Figure 2.3, the area under OAB represents a feasible income level -- income variance combination.

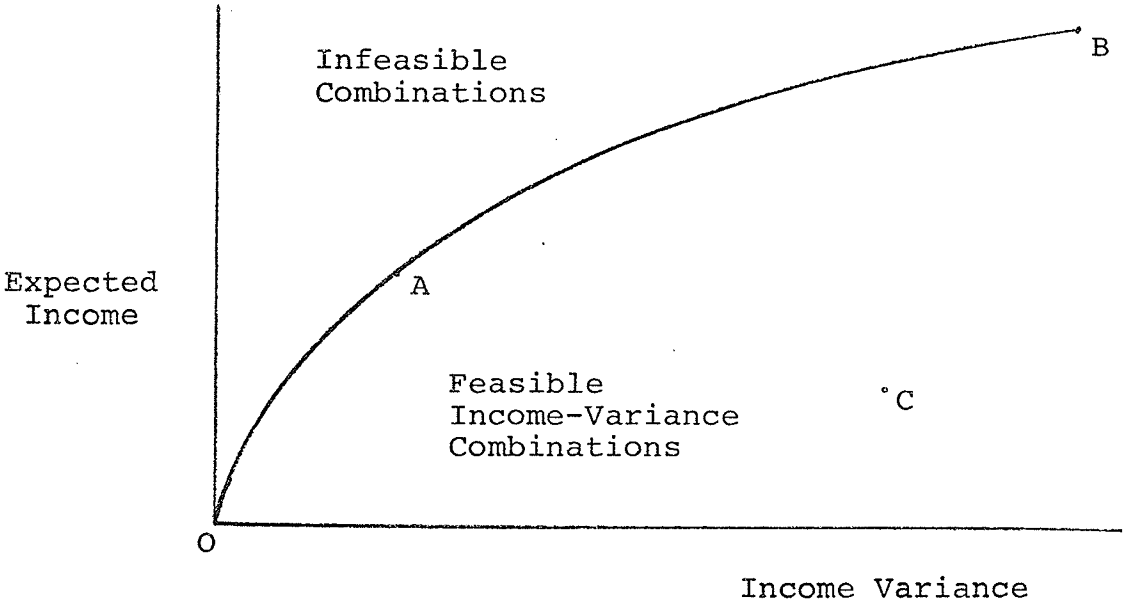

FIGURE 2.3

MINIMUM INCOME VARIANCE AS A FUNCTION OF EXPECTED INCOME

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[8] Heady, Earl O. and Candler, Wilfred. Linear Programming Methods. Ames, Iowa State University 1958 p. 558.

The problem becomes one of finding production plans that correspond to the upper boundary OAB--all feasible plans. Any plan on OAB is preferred to a plan under OAB (such as C) because it has greater income or smaller variance (risk) than any other feasible plan with the same variance or income.

Combining the E-V indifference system with the minimum income variance results in an illustration showing the decision maker's best choice of efficient alternative expected net incomes given his own indifference map[9] (Figure 2.4).

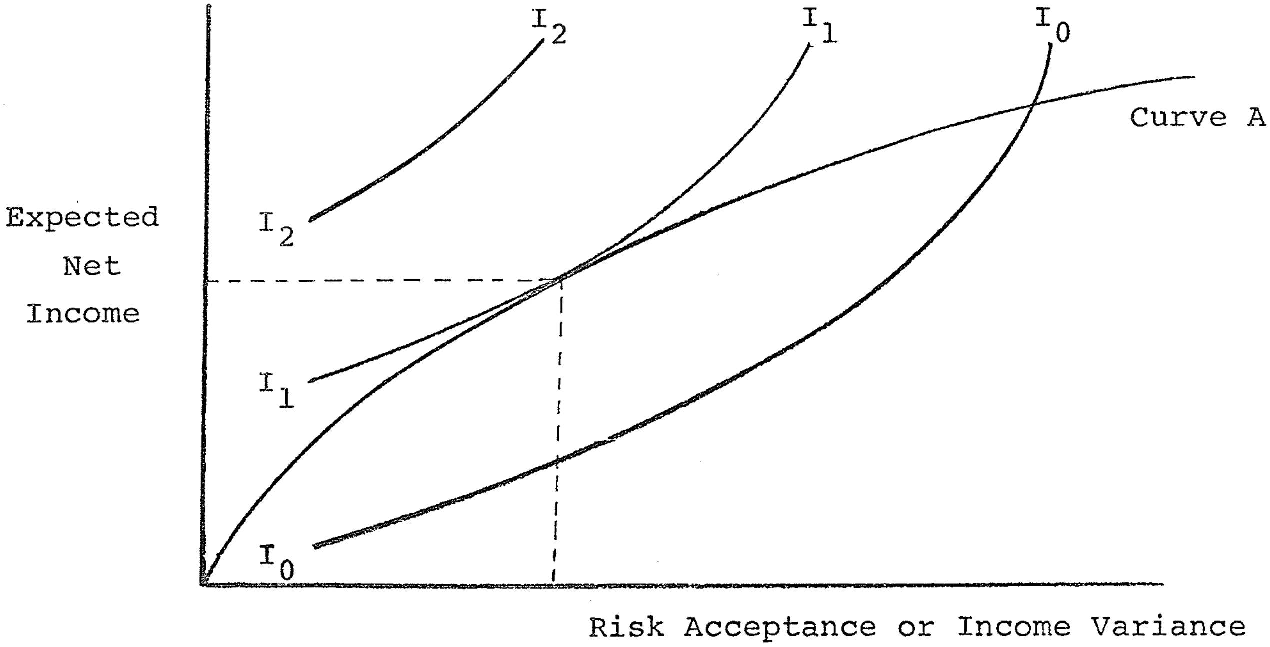

FIGURE 2.4

OPTIMAL INCOME-VARIANCE POINT GIVEN AN E-V INDIFFERENCE SYSTEM

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[9] Driver, H.C.; Tenure Forms and Instruments Impeding or Facilitating Farm Entry and Optimal Resource Efficiency. Unpublished Ph. D. thesis, Iowa State University, Agriculture Economics, 1969, p. 22.

The point at which a farm manager's highest indifference curve is tangent to the production surface of efficient combinations of expected income and minimum risk results in the optimal net income-risk situation for that individual. It is noteworthy that the more vertical the indifference curves the less willing the manager is willing to accept risk as the tangent point will be further to the west; the opposite is true regarding a more horizontal set of indifference curves. It is obvious that the slope of the indifference curves determine the degree of risk acceptance that a farm manager is willing to accept; and the slope of the indifference curves depend on that individual's tastes and preferences and attitudes toward income uncertainty.

Halter and Dean, [10] modern day authors in the area of risk and uncertainty, demonstrate a technique that makes it possible to determine the shape of any individual's utility curve. This bears mentioning at this time because the shape of a utility curve is directly related to an individual's attitude toward uncertainty. Halter and Dean illustrate three possible types of utility functions (Figure 2.5).

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[10] Halter, A.N. and G.W. Dean, Decisions Under Uncertainty With Research Application. Unpublished Manuscript, University of California, Davis, Agriculture Economics, 1970, p. 42.

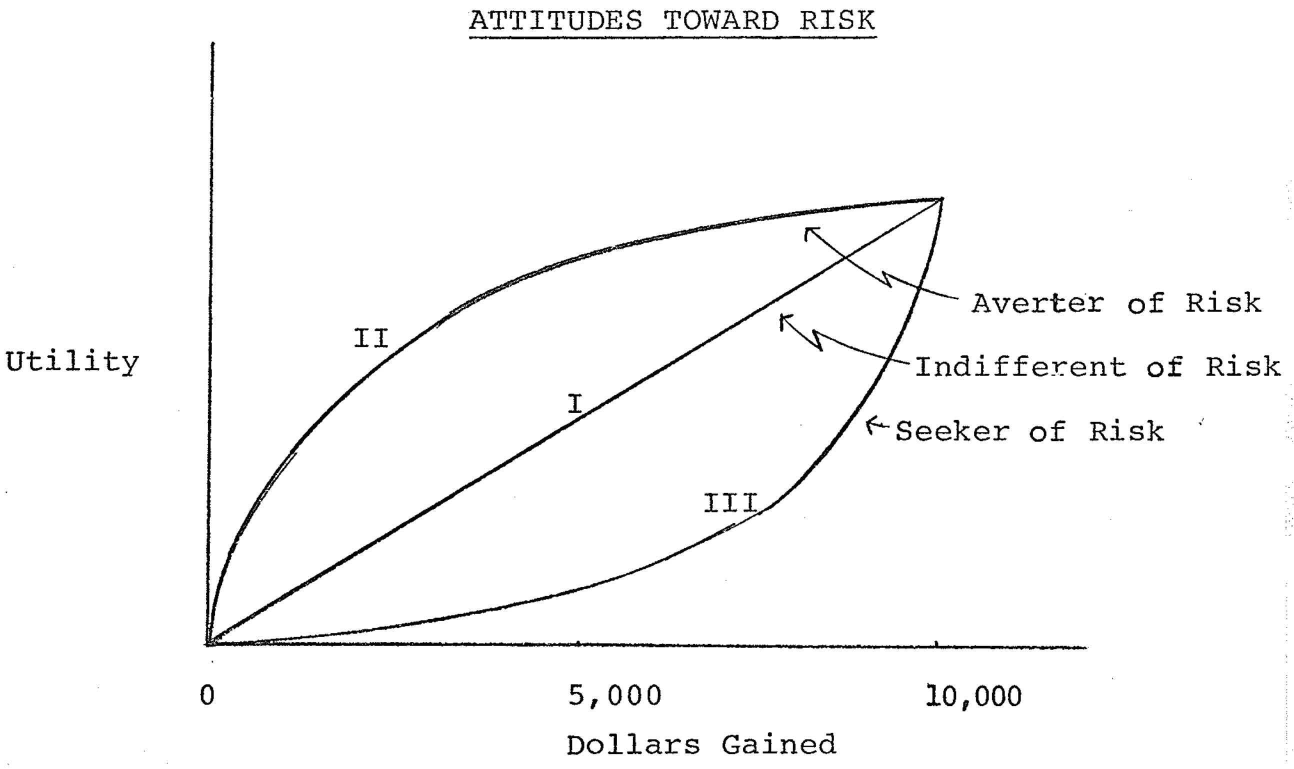

FIGURE 2.5

UTILITY FUNCTIONS

All three functions increase monotonically throughout, showing that all three individuals prefer more money to less. However, the marginal utility of an additional dollar of gain varies among the three cases. Individual I has a constant marginal utility of money, indicating he values the first or 5,000th dollar gain as highly as the 10,000th. Individual II has a decreasing marginal utility of money indicating that as dollar gains increase, they become subjectively less valuable; there is less glamour associated with taking a chance at higher levels of wealth.

The converse is true of Individual III. He has an increasing marginal utility of money, indicating that he does not mind accepting more risk for a chance of more money. This study assumes farmers would fall in the risk aversion category. Realizing that the farm can only survive if expected income is at least as large as all costs and living expenses in the long run, then it would only seem normal that the first dollars covering the necessary costs would be more highly valued than those dollars beyond the survival point.

In summary, the theory associated with uncertainty falls predominantly in the area of subjective risk. Pure risk is based on known probability distributions. Subjective risk is based on both probability distributions and attitudes which include personal tastes and preference. Given a situation where a manager can produce at a number of efficient production points which differ in regard to expected net income and associated risk, he will produce at the point that corresponds to his attitude toward risk. Theory suggests that this occurs at the point where the individual's highest attainable indifference curve in the E-V set is tangent to an efficient production point, in which case the shape of the indifference curve embodies the manager's attitudes toward risk.

This concept is consistent with Hick's allowance for uncertainty. Although Hicks felt that a manager would plan against uncertainty by discounting some average price or income by some subjective amount, he still arrives at the same result as the discussion on utility theory has just presented. The utility theory concept is somewhat more refined in that an individual's subjective attitudes toward risk enter into the analysis by means of indifference curves.

SUBJECTIVE RISK MODELS

Many attempts have been made to measure uncertainty, changing it to a risk concept, and then building that concept into an analytical model. Presented in the next few pages will be analytical models formulated exclusively for the purpose of incorporating subjective risk into the decision process.

Farrar's Model

Farrar[11] attempted to formulate a model for making the best investment portfolio decision under uncertainty. His model is based on the extension of orthodox economic theory embracing uncertainty. His revision of the certainty model requires maximization of expected returns or profit from a number of investment opportunities. Farrar feels that a rational investor can be induced to increase the risk which he will bear only if it is accompanied by a

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[11] Farrar, Donald Eugene. The Investment Decision Under Uncertainty. Englewood Cliff, N.J. Prentice-Hall Inc. 1962.

compensating rise in expected return. In a formal sense, therefore, an investment's expected value is discounted by some measure of its risk. Based on two assumptions:

1. An investor's utility of money function is positively sloped and concave downward - a risk averter, and
2. His investment strategy is the maximization of expected utility,

Farrar formulates his model in terms of Hicksian's allowance for risk. The model appears as:

$$E\ (U(x)) = \mu - A\sigma_x^2$$

where E (U(x)) represents the expected utility of some investment portfolio; μ equals the expected value; σ_x^2 equals the variance of expected value for the x^{th} investment, and A is some subjective risk factor. Farrar translates this into a programming model incorporating many investments which appears as:

$$E\ (U(P)) = \Sigma\gamma_i\mu_i - A\Sigma\Sigma\gamma_i\sigma_{ij}\gamma_j$$

where P, the portfolio, is a weighted sum $\Sigma\gamma_i X_i$ of individual securities whose indices, X_i, i=1, 2,---,n are treated as stochastic variates having unit value at the beginning of the present time period. Furthermore,

$$\gamma_i \geq 0$$

is the proportion of the portfolio which is invested in the i^{th} security ($\Sigma\gamma_i = 1$). The equation,

$$\mu_i = E(X_i)$$

stands for the expected value of security i. ($\Sigma\gamma_i\mu_i$ therefore represents the portfolio's expected value.) The identity,

$$\sigma_{ij} = E\ (X_i - \mu_i)\ (X_j - \mu_j)$$

is the covariance of securities i and j, and $\Sigma\Sigma\gamma_i\sigma_{ij}\gamma_j$ therefore, is the portfolio's variance when the vector $P = \Sigma\gamma_i X_i$ is substituted for the single variante, X, as the opportunity under consideration. Given a value of A (some risk aversion factor), the model is determinate by quadratic programming techniques.

Freund's Model

Freund[12] first solved a conventional linear program with various cropping activities and found that high risk crops would be grown on large acreages. The reasoning was that high risk crops are quite profitable in the long run, but most farmers are neither willing nor able to endure the extreme fluctuations of the net revenue of such crops. It was for this reason that Freund thought that the development of a risk programming procedure was important.

He begins by assuming that the net revenue of each process (activity) is normally distributed, therefore, that the net revenue for the whole model is normally distributed. Similarly, Freund assumes prices and quantities of all inputs are fixed, and that the formula for the entrepreneur's utility of money function is:

$$y(r) = 1 - e^{-ar} ,$$

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[12] Freund, R.J. "The Introduction of Risk into a Programming Model" <u>Econometrica</u> Vol. 24 P. 253-263 1956.

where y is the utility and r is the net revenue. This function is convex everywhere and represents the decision process of a conservative entrepreneur. The constant 'a' indicates the entrepreneur's aversion to risk. Maximization of expected utility can be accomplished by maximizing the function:

$$E\ (u^*) = \mu - a/2\ \sigma^2,$$

remembering the form of the parameters of the distribution of the net revenue in a quadratic programming model. Freund maximized expected utility by maximizing

$$E\ (u^*) = S'X - a/2X'\ \Sigma X$$

subject to

$$TX \leq V,$$

and

$$X \geq 0$$

where

S is a vector of net revenues of unit levels of a set of productive processes available to a firm,

X is a vector of the number of unit levels of each process in a productive program,

T is a matrix of the amount of certain scarce resources needed by the unit levels of the productive process, and

V is a vector of available amounts of the scarce resources.

Freund found in his solution that fewer high risk crops came into the solution than previously when risk was not built into the model. Net revenue had decreased but so had the standard deviation of net revenue.

Van Moeseke's Model

Van Moeseke[13] formulated a risk programming model in which technical conditions are know with perfect certainty, but net product prices are normally distributed with parameters μ_i and σ_i^2, and that net product prices are not independent so covariances σ_{ij} exist. He then defined his model as:

$$f(x) = \bar{C}'X - m(X'\ V\ X)^{\frac{1}{2}},$$

where $\bar{C}$ is an n-dimensional vector of net product prices, X is an n-dimensional vector of unit levels or processes or activities available for solution, V is the variance-covariance matrix of net product prices and $(X'\ V\ X)^{\frac{1}{2}}$ is the standard deviation of net income. Driver[14] developed the model to solve an empirical problem and maximized this objective function subject to the usual linear constraints and resource availabilities as specified by,

$$AX \leq b,$$

$$X \geq 0.$$

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[13] Van Moeseke, P. Stochastic Linear Programming. Yale Economic Essays. New Haven, Connecticut, Yale University Press. 1965.

[14] Driver, H.C. Tenure Farms and Instruments Impeding or Facilitating Farm Entry & Optimal Resources Efficiency. Unpublished PhD. thesis, Iowa State University, Agriculture Economics, 1969.

The objective function is non linear and convex in shape, but is homogenous of degree one. The risk aversion factor, m, is the amount of confidence or degree of probability by which most probably net income is likely to exceed representative net income or some risk accepted level of income. Thus, m, is the trade off between expected net income and risk acceptance. Its value (when known) is directly related to the cummulative probability of the normal variante, Z. Driver's revised model was solved by:

$$\min.\ (X'\ V\ X)$$

$$AX \leq b$$

$$\bar{C}'X = {}^{i}b$$

$$X \geq 0$$

where ${}^{i}b$ denotes alternative levels of expected net income.

As a result, a quadratic algorithm was used in solving the system for alternative levels of income. The original non linear objective function was then evaluated for each solution until a maximum on it was found.

The approach was costly as many runs were required to trace out the value of the non linear objective function and scan it for a maximum point.

SPECIFICATION OF RISK MODEL USED IN THIS STUDY

The risk programming model of Van Moeseke has major advantages over the models of Freund and Farrar. They include the following.

1. The Van Moeseke model fits more closely to the E-V indifference analysis. Whereas the other two models fall into the realm of Hicksian risk allowance, the former model specifies actual minimum variances for each optimum value of net expected income. By tracing out a function of income-risk values, it becomes possible to derive a trade off between risk and income on the basis of many solutions-- not just one as is given by the models of Freund and Farrar.
2. The whole process for the Van Moeseke model is reversible. Given a value for risk, it is possible to determine maximum net income, and vice-versa. Similarly given a value that the firm must achieve to survive, the risk surrounding it can be determined. This is not the case in the other two models.

The model to be used in this study provides the mechanism for making trade offs between expected income and risk as does the Van Moeseke model. It is derived into two parts. It is based on discounting of net prices by a subjective risk factor. The model is programmed linearly. It is assumed that there is no covariance between activities or processes, that net product prices are normally distributed and that costs of all inputs per unit are known with perfect certainty. Given a single productive activity, the objective is one of maximizing expected income, t(X). This appears as:

$$\text{Max. } t(X) = \bar{C}_1 X_1 - Z_j \sigma$$

Under a situation of many activities, the objective is similar, however, the function becomes

$$\text{Max. } t_j(X_j) = (\bar{C}_j - Z_j\sigma)'X_j \qquad 2.1$$

$$A_{ij}X_j \leq B_i \qquad 2.2$$

$$\text{All } X_j \geq 0. \qquad 2.3$$

In this model

X_j represents a vector of values for the unknown variables (cropping activities) of the j^{th} farm plan,

$(\bar{C} - Z_j\sigma)$ represents a vector of discounted net prices for the j^{th} plan,[15]

A_{ij} is the matrix of technical coefficients, associated with the cropping activities, and

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[15] Some concern may arise as to whether variance is in dollars or in physical terms (bushels) in this vector form. Variance is in dollars per unit level of activity. However, the following description shows that for equation 2.1, one needs to find only the variance in physical terms. The variance of the random variable C for 2.1 can be described as follows:

Let Y be yield per acre

P_y be price per unit

V be variance of the random variable C

$$C = P_y Y - \text{Costs}$$

$$V(C) = V(P_y Y) - V\ (\text{Costs})$$

$$V(C) = P_y 2\ V(Y) - 0 = P_y 2\ \sigma^2{}_y$$

$$\sqrt{V(C)} = P_y \cdot \sigma_y$$

$$C - Z\sqrt{V(C)} = P_y Y - ZP_y \cdot \sigma_y = P_y(Y - Z\sigma_y).$$

B_i is a vector of resource levels available for use in the j^{th} plan.

The second part of the model is specified as,

$$y_j = \frac{t_j - E(t_j)}{(X_j' \, V \, X_j)^{\frac{1}{2}}} \qquad 2.4$$

By this formula it is possible for any farm manager to determine the chances of expected income lying above the income generated by future production plans which include risk, or the chance of expected income exceeding a specified level of income. In this model

$E(t_j)$ is the expected income of pre-determined production plans not considering risk. In this sense, then

$E(t_j) = \bar{C}_j X_j$, and

t_j is the expected income from future production[16] plans which consider risk, where

$t_j = (\bar{C} - Z_j \sigma)' \, X_j$.[17]

Furthermore,

$(X_j' \, V \, X_j)^{\frac{1}{2}}$ is the income variance associated with the unit levels of acitivies within the plan. It is equivalent to:

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[16] t_j becomes $\hat{t}_s$, when a survival level of income or specified level is used, rather than some pre-determined income level developed from model 2.1.

[17] This explains how 2.1 and 2.4 are interrelated and form the over all complete model.

$$\sigma^2_{(t_j)} = \sum_{j=1}^{k} X_j^2 \, \sigma_j^2 \, p_j^2$$

where k is the number of activities and j is the j^{th} plan. y_j is the probability of income from a farm plan exceeding either the income generated by future production plans incorporating risk, or a survival level of income. This value must be negative if the corresponding plans are to be feasible.[18]

This model provides the mechanism for making trade offs between expected income and risk by selecting production points that correspond to different levels of risk aversion. Given these expected income solutions, knowing the variance of the cropping solution, and some survival level of income

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[18] Variance in this case is in terms of dollars. The random variable is t (i.e. a linear combination of the C_j's.)

$$t = c_1x_1 + c_2x_2 \quad \text{(objective function in terms of net income)}$$

$$t = (P_{Y_1} \; Y_1 - \text{Costs}) \; X_1 + (P_{Y_2} \; Y_2 - \text{Costs}) \; X_2$$

$$V(t) = (P^2_{Y_1} \sigma^2_{Y_1}) \; X_1^2 + (P^2_{Y_2} \sigma^2_{Y_2}) \; X_2^2$$

$$\sqrt{V(t)} = \{(P^2_{Y_1} \sigma^2_{Y_1}) \; X_1^2 + (P^2_{Y_2} \sigma^2_{Y_2}) \; X_2^2\}^{\frac{1}{2}}$$

$$= \{(\sigma^2_{c_1} \; X_1^2) + (\sigma^2_{c_2} \; X_2^2)\}^{\frac{1}{2}}$$

$$= (X'VX)^{\frac{1}{2}} \qquad \text{where}$$

$$V = \begin{bmatrix} \sigma^2_{c_1} & 0 \\ 0 & \sigma^2_{c_2} \end{bmatrix}$$

In equation 2.4 variance of income must be in dollars. Therefore, the yield variances associated with the various activities are multiplied by the appropriate unit product price squared.

of the cropping solution, and some survival level of income that must be met, it is possible for a farm manager to select a production point that is higher than the survival point. This ensures that an individual will not discount or avert risk to the point where he will not be able to meet his production costs and his personal withdrawals. This is shown in Figure 2.6.

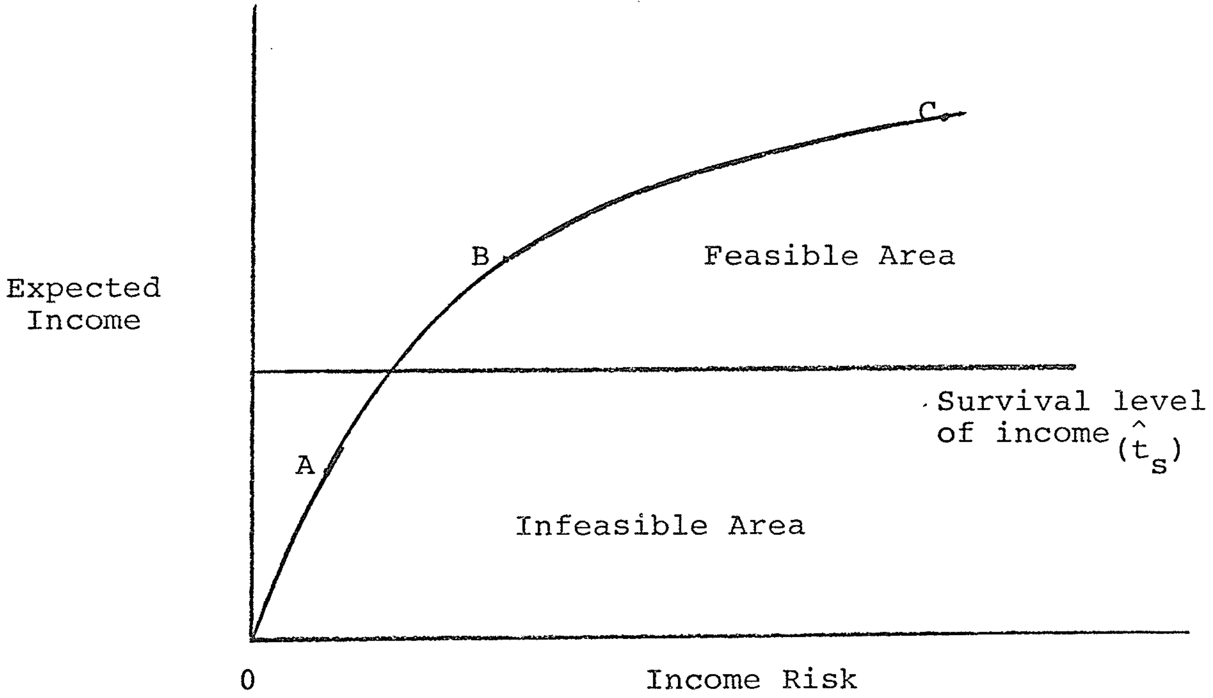

FIGURE 2.6

HYPOTHETICAL ILLUSTRATION OF DECISION MAKER'S TRADE-OFFS BETWEEN EXPECTED INCOME AND INCOME RISK (E-V FRONTIER).

If solutions A, B, and C are found on the basis of different risk factors Z_j, production plans corresponding to points B and C are eligible for consideration by the farm planner to which they pertain. This example presumes one fixed level of exogenous variables for three different values of risk. Should a manager wish to change input-output coefficients or resource levels, a new E-V curve OABC would result. Regardless of the E-V curve finally selected, a manager will select a plan on it that exceeds the survival level and one that approximates most closely to his own subjective risk attitude.

This approach is closely related to subjective discounting procedures followed by farmers. A farmer who is given a set of solutions or plans determined through linear budgeting techniques applied to his farm based on different values of exogenous risk, will select the plan which meets the income objectives of the family while minimizing the chances of not achieving them. He will in effect choose a cropping plan that reflects his tastes and preferences, his attitude toward risk and money gains given his own debt-equity position.

Chapter 3

DEVELOPMENT OF THE MODEL

The model developed and applied to alternative levels of risk acceptance given varying levels of exogenous variables is based on the previous discussion given in the analytical framework.

METHODS OF SOLUTION

Given the model as described by:

$$\text{Max.} \quad t_j(X_j) = (\bar{C}_j - Z_j\sigma)' X_j \qquad 2.1$$

$$A_{ij}X_j \leq B_i \qquad 2.2$$

$$\text{All } X_j \geq 0 \qquad 2.3$$

and

$$y_j = \frac{t_j - E(t_j)}{(X_j' V X_j)^{\frac{1}{2}}} \qquad 2.4$$

the procedure involves the maximizing of 2.1. The technology matrix A is described and specified later in Table 3.1 as are the B_i levels.

The risk aversion factor (Z_j) is treated exogenously and given three alternative values for the purpose of determining changes in crop plans and net income t_j (X_j). All net prices of productive activities are discounted by a similar value of Z_j, however, the standard deviations of yield (σ) pertaining to the various activities are not the same.

Variance of yields (σ^2) are determined by regression techniques utilizing normal distributions of yields at various levels of nitrogen application. The resultant production functions are tested statistically and used to predict yield variances for all nitrogen levels. Associated mean yields for all distributions are predicted on the basis of the same production functions determined by regression.

Product prices are predicted averages over the five year range of the model.[1] These prices are assumed to have zero variance. As these prices are constant, net prices per activity are directly related to yield.

The second part of the model described in 2.4 determines the chances of expected income derived from any set of future plans exceeding a specified level of income. Using 2.4, plans derived from the solution of 2.1 for all levels of exogenous variables given the three different values of risk acceptance Z_j are tested to determine their chances of exceeding a specified level of income. As the technology matrix to be presented later is based on a case farm so will the specified level of income $\hat{t}_s$ used in 2.4. Expected income $E(t_j)$ is determined by multiplying expected net prices by the unit acre levels of activities contained

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[1] As the yield figures to be used are derived from five year's data, it is necessary that the corresponding results be interpreted as farm plans for the next five production periods.

in the solution vector (X_j), which are brought forward from the j^{th} plan generated by maximizing 2.1. The variance of net income associated with the plan to be tested is determined by multiplying the associated yield variance of a solution activity by the weight (acreage in the plan) of that activity squared, and the product prices squared, that is:

$$\sigma^2_{(t_j)} = \sum_{j=1}^{k} X_j^2 \, \sigma_j^2 \, P_j^2$$

The variances used are taken from the same source as described before. The solution to 2.4 is an area (probability) under the standard normal distribution which must not be less than 50 per cent of the area if the plans are to be judged as feasible.

THE MODEL IN DETAIL

Activities, resource and product constraints as well as corresponding input-output relationships are identified and illustrated in Table 3.1. This table specifies equation 2.2 and includes:

1. activities X_j listed in columns where $j=1, 2,---54$;
2. constraints b_i listed in rows where $i=1, 2,---18$, and
3. input-output relationships a_{ij} belonging to A.

Activities

A cash grain type farm presented later in Chapter 4 forms the basis of information and data used in the proposed model. Wheat seeded on fallow and on stubble, oats seeded on stubble, barley seeded on stubble, flax grown on stubble and fallow and present year's summerfallow are considered as endogenous variables or activities. Other possible grain activities such as rape and rye on fallow or stubble, and special crops including corn, sunflowers, sugar beets, etc. are omitted from this analysis as corresponding yield data are not available in sufficient number.

The wheat activities are broken down into quota and non-quota groups. Wheat sold on quota is activated in a different manner than that not sold on quota. As quota sales have historically resulted in higher net prices than non-quota sales, the wheat quota activities in this model will return more to net income than the wheat non-quota activities.

The case farm relies heavily on continuous cropping, high values of inputs, and low inventory of grain from one production period to the next. All grain produced is disposed of before the next harvest. Grain grown on a continuous crop base that can not be sold through Canadian Wheat Board channels, in the past has been sold to non-quota agencies e.g. local feed mills.

Dividing the grain activities by means of fertilizer levels creates some difficulty. It is the intention of the author to associate five levels of nitrogen application with all cropping activities, except flax. Wheat, oats, and barley are to have levels of nitrogen from zero to 80 pounds actual nitrogen. It is felt that this range of application covers the relevant area of fertilizer usage at this time. Flax grown on fallow is assumed to not require fertilizer as yield data in sufficient quantity for flax fertilized on fallow is not available. For flax grown on stubble, three levels of nitrogen are used in the analysis.

One of the major objectives of this study is to study the effects of different soil types on yield variation. The case farm is made up of three different types of soil. To bring the yield variations accruing to these soils into a position where their differences and the consequences of their differences may be examined, the crops discussed above are available to be grown on all three types of soil. In other words, differences in similar activities are due to landbase only.

Given the above discussion there are 105 activities (35 x 3 soil types).[2] However, due to limitations to be presented later, this number is reduced to 54. This number of activities forms the endogenous variable base of the planning model.

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[2] The 35 is made up of: (i) 6 groups of 5 activities each; quota wheat on fallow, wheat on fallow, quota wheat on stubble, wheat on stubble, oats on stubble, barley on stubble. (ii) 2 groups of flax; 3 activities of flax on stubble, 1 activity of flax on fallow. (iii) 1 activity for summerfallow.

Constraints

There are eighteen constraints in the planning model. They consist of nine land, three labor, one operating capital, one quota, and four acreage constraints.

Three different levels of each of three soil types are considered for the land base. It is assumed that all land included in the model is in fact owned (not rented). To determine the effect of summerfallow on net farm income and cropping plans, summerfallow will be forced into the solutions in a systematic way.[3] For a given percentage of summerfallow in each soil type, there is a corresponding constraint. It allows seeding of summerfallow at the same level. Similarly, there is a constraint limiting seeding on stubble land to the differences between the total acreage of the available soil type and the sum of the seeded summerfallow and the land left idle. Therefore, given three soil types and three classifications for each, there are nine land constraints which are treated as exogenous variables. Four different levels of summerfallow specification create a test for finding differences in crop planning. These levels cover a range from a requirement of no summerfallow to a certain given percentage for each soil. The specifications of summerfallow correspond to known percentages used in the area.

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[3] The reason for this will be presented in Chapter 5.

The levels of the three labor constraints pertaining to the case farm are employed. As this farm does not have livestock, only the high labor requirement periods[4] of seeding and harvest are considered. The months of May, June and July are treated as the period required to perform seeding, spraying and summerfallow cultivation functions. The months of August, September and October are specified as including the remainder of the summerfallow cultivation, harvest and fall work. The total labor resource includes all management and hired labor available to the case farm. The levels of these three restraints remain constant through all planning trials.

Short term credit available forms the operating capital requirement. The manager of the case farms feels he has a maximum line of short term credit established in his local community. This value is used as the initial constraint for the model. As the value is exceedingly high and is expected to have no restriction on farm plans, operating capital is given four other values which are lower than the one used initially. It is believed that a restriction of operating capital will have major effects on cropping plans, fertilizer use, and risk acceptance.

Another constraint is that of wheat quota. As discussed previously, wheat will be grown for either quota or for non-quota sale. Each quota bushel produced will

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[4] As related to a grain farm.

decrease available quota delivery. Determining in advance to a production period what the wheat quota will be in total bushels is impossible as this depends on acres seeded. A rough estimation of the total wheat quota allocated to the case farm will be presented later as will the technique used to determine that estimate. Two different totals are used as the effects of increasing or decreasing the quota are examined.

The other constraints are used as safety stops to prevent the most profitable activities from using all the available land restraints. They include acreage maximums for seeded wheat, oats, barley, and flax.

Matrix Coefficients

The coefficients pertaining to the nine land constraints produce no difficulty as use of one acre in any activity simply reduces the level of the corresponding restraint by one acre.

Labor hour coefficients require a more detailed analysis. Given such factors as tillage practises, size of machinery, grain hauling distances, soil texture, typical field size, and degree of stoniness, it is possible to calculate the number of hours of labor required to perform one acre's production of any activity. This total labor value is in turn broken down into spring and fall requirements.

It was decided that all variable or direct costs and all fixed costs would be allocated on a per acre basis to determine how much capital would be required in total to have an acre of any activity come into solution.[5] Variable costs include seed, seed preparation, soil tests, weed chemicals, fertilizer, crop insurance, machinery and equipment expenses, fuel, and custom work. Fixed costs are made up of depreciation, taxes and other overhead expenses.

The coefficients pertaining to the wheat quota constraint are based on discounted yields. When the planning model corresponds to a specific level of risk aversion, all quota wheat activities have yields which are discounted accordingly. As one acre of a quota activity enters into solution the available wheat quota is reduced by an amount equivalent to the yield ascertained by the discounting process.

This planning model is made complete with the determination of net prices. This is performed by a simple multiplication of product price and discounted yield followed by a substraction from that total of all costs. Product prices are average expectations of the case farm manager for the next five years. Yields are determined by a procedure discussed later.

This model is solved parometrically changing the levels of risk aversion, capital values and land constraints to determine the effects of these systematic changes on crop plans and net farm income. The model is outlined in Table 3.1.

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[5] Fixed costs could alternatively be placed on a total farm basis.

TABLE 3.1[1/]

IDENTIFICATION OF ACTIVITIES, CONSTRAINTS, AND CORRESPONDING MATRIX COEFFICIENTS

	$C_j \rightarrow$	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}	C_{12}	C_{13}	C_{14}	C_{15}	C_{16}
Resource (b_j) Used Or Activity Produced (X_j)	Constraint and Supply Level	$QWFB_0$	$QWFB_4$	$QWSB_0$	$QWSB_4$	WFB_0	WFB_4	WSB_0	WSB_4	OSB_0	OSB_4	TSB_0	TSB_4	XFB_0	XSB_0	XSB_2	B-Smf
Stubble B	$< b_1$			$A_{1,3}$	$A_{1,4}$			$A_{1,7}$	$A_{1,8}$	$A_{1,9}$	$A_{1,10}$	$A_{1,11}$	$A_{1,12}$		$A_{1,14}$	$A_{1,15}$	
Fallow B	$< b_2$	$A_{2,1}$	$A_{2,2}$			$A_{2,5}$	$A_{2,6}$							$A_{2,13}$			
Smfallow B	$= b_3$																$A_{3,16}$
Stubble C	$< b_4$																
Fallow C	$< b_5$																
Smfallow C	$= b_6$																
Stubble E	$< b_7$																
Fallow E	$< b_8$																
Smfallow E	$= b_9$																
Total Labor	$< b_{10}$	$A_{10,1}$	$A_{10,2}$	$A_{10,3}$	$A_{10,4}$	$A_{10,5}$	$A_{10,6}$	$A_{10,7}$	$A_{10,8}$	$A_{10,9}$	$A_{10,10}$	$A_{10,11}$	$A_{10,12}$	$A_{10,13}$	$A_{10,14}$	$A_{10,15}$	$A_{10,16}$
Spring Labor	$< b_{11}$	$A_{11,1}$	$A_{11,2}$	$A_{11,3}$	$A_{11,4}$	$A_{11,5}$	$A_{11,6}$	$A_{11,7}$	$A_{11,8}$	$A_{11,9}$	$A_{11,10}$	$A_{11,11}$	$A_{11,12}$	$A_{11,13}$	$A_{11,14}$	$A_{11,15}$	$A_{11,16}$
Fall Labor	$< b_{12}$	$A_{12,1}$	$A_{12,2}$	$A_{12,3}$	$A_{12,4}$	$A_{12,5}$	$A_{12,6}$	$A_{12,7}$	$A_{12,8}$	$A_{12,9}$	$A_{12,10}$	$A_{12,11}$	$A_{12,12}$	$A_{12,13}$	$A_{12,14}$	$A_{12,15}$	
Operating Capital	$< b_{13}$	$A_{13,1}$	$A_{13,2}$	$A_{13,3}$	$A_{13,4}$	$A_{13,5}$	$A_{13,6}$	$A_{13,7}$	$A_{13,8}$	$A_{13,9}$	$A_{13,10}$	$A_{13,11}$	$A_{13,12}$	$A_{13,13}$	$A_{13,14}$	$A_{13,15}$	$A_{13,16}$
Wheat Quota	$< b_{14}$	$A_{14,1}$	$A_{14,2}$	$A_{14,3}$	$A_{14,4}$												
Wheat Acreage	$< b_{15}$	$A_{15,1}$	$A_{15,2}$	$A_{15,3}$	$A_{15,4}$	$A_{15,5}$	$A_{15,6}$	$A_{15,7}$	$A_{15,8}$								
Barley Acreage	$< b_{16}$											$A_{16,11}$	$A_{16,12}$				
Oats Acreage	$< b_{17}$									$A_{17,9}$	$A_{17,10}$						
Flax Acreage	$< b_{18}$													$A_{18,13}$	$A_{18,14}$	$A_{18,15}$	

[1/] The abbreviation explanations are as follow:

Abbreviation	Explanation	Abbreviation	Explanation
B	Soil type B	T	Barley
C	Soil type C	X	Flax
E	Soil type E	Smf.	Summerfallow
Q	Quota	S	Stubble
W	Wheat	F	Fallow
O	Oats		

Numbers "0, 1, 2, 3 & 4" following the activities X_j indicate the zero, first, second, third & fourth levels of nitrogen application

TABLE 3.1 (continued)

$C_j \rightarrow$	C_{17}	C_{18}	C_{19}	C_{20}	C_{21}	C_{22}	C_{23}	C_{24}	C_{25}	C_{26}	C_{27}	C_{28}	C_{29}	C_{30}	C_{31}	C_{32}	C_{33}	C_{34}	C_{35}	C_{36}	C_{37}	C_{38}
Constraint and Supply Level	$QWFC_0$	$QWFC_1$	$QWFC_2$	$QWFC_3$	$QWFC_4$	$QWSC_0$	$QWSC_4$	WFC_0	WFC_1	WFC_2	WFC_3	WFC_4	WSC_0	WSC_4	OSC_0	OSC_4	TSC_0	TSC_4	XFC_0	XSC_0	XSC_2	C-Smf
Row 1																						
Row 2																						
Row 3																						
Row 4						$A_{4,22}$	$A_{4,23}$						$A_{4,29}$	$A_{4,30}$	$A_{4,31}$	$A_{4,32}$	$A_{4,33}$	$A_{4,34}$		$A_{4,36}$	$A_{4,37}$	
Row 5	$A_{5,17}$	$A_{5,18}$	$A_{5,19}$	$A_{5,20}$	$A_{5,21}$			$A_{5,24}$	$A_{5,25}$	$A_{5,26}$	$A_{5,27}$	$A_{5,28}$							$A_{5,35}$			
Row 6																						$A_{6,38}$
Row 7																						
Row 8																						
Row 9																						
Row 10	$A_{10,17}$	$A_{10,18}$	$A_{10,19}$	$A_{10,20}$	$A_{10,21}$	$A_{10,22}$	$A_{10,23}$	$A_{10,24}$	$A_{10,25}$	$A_{10,26}$	$A_{10,27}$	$A_{10,28}$	$A_{10,29}$	$A_{10,30}$	$A_{10,31}$	$A_{10,32}$	$A_{10,33}$	$A_{10,34}$	$A_{10,35}$	$A_{10,36}$	$A_{10,37}$	$A_{10,38}$
Row 11	$A_{11,17}$	$A_{11,18}$	$A_{11,19}$	$A_{11,20}$	$A_{11,21}$	$A_{11,22}$	$A_{11,23}$	$A_{11,24}$	$A_{11,25}$	$A_{11,26}$	$A_{11,27}$	$A_{11,28}$	$A_{11,29}$	$A_{11,30}$	$A_{11,31}$	$A_{11,32}$	$A_{11,33}$	$A_{11,34}$	$A_{11,35}$	$A_{11,36}$	$A_{11,37}$	$A_{11,38}$
Row 12	$A_{12,17}$	$A_{12,18}$	$A_{12,19}$	$A_{12,20}$	$A_{12,21}$	$A_{12,22}$	$A_{12,23}$	$A_{12,24}$	$A_{12,25}$	$A_{12,26}$	$A_{12,27}$	$A_{12,28}$	$A_{12,29}$	$A_{12,30}$	$A_{12,31}$	$A_{12,32}$	$A_{12,33}$	$A_{12,34}$	$A_{12,35}$	$A_{12,36}$	$A_{12,37}$	
Row 13	$A_{13,17}$	$A_{13,18}$	$A_{13,19}$	$A_{13,20}$	$A_{13,21}$	$A_{13,22}$	$A_{13,23}$	$A_{13,24}$	$A_{13,25}$	$A_{13,26}$	$A_{13,27}$	$A_{13,28}$	$A_{13,29}$	$A_{13,30}$	$A_{13,31}$	$A_{13,32}$	$A_{13,33}$	$A_{13,34}$	$A_{13,35}$	$A_{13,36}$	$A_{13,37}$	$A_{13,38}$
Row 14	$A_{14,17}$	$A_{14,18}$	$A_{14,19}$	$A_{14,20}$	$A_{14,21}$	$A_{14,22}$	$A_{14,23}$															
Row 15	$A_{15,17}$	$A_{15,18}$	$A_{15,19}$	$A_{15,20}$	$A_{15,21}$	$A_{15,22}$	$A_{15,23}$	$A_{15,24}$	$A_{15,25}$	$A_{15,26}$	$A_{15,27}$	$A_{15,28}$	$A_{15,29}$	$A_{15,30}$								
Row 16																	$A_{16,33}$	$A_{16,34}$				
Row 17															$A_{17,31}$	$A_{17,32}$						
Row 18																			$A_{18,35}$	$A_{18,36}$	$A_{18,37}$	

TABLE 3.1 (concluded)

$C_j \rightarrow$	C_{39}	C_{40}	C_{41}	C_{42}	C_{43}	C_{44}	C_{45}	C_{46}	C_{47}	C_{48}	C_{49}	C_{50}	C_{51}	C_{52}	C_{53}	C_{54}
Constraint and Supply Level	$QWFE_0$	$QWFE_4$	$QWSE_0$	$AWSE_4$	WFE_0	WFE_4	WSE_0	WSE_4	OSE_0	OSE_4	TSE_0	TSE_4	XFE_0	XSE_0	XSE_2	E-Smf
Row 1																
Row 2																
Row 3																
Row 4																
Row 5																
Row 6																
Row 7			$A_{7,41}$	$A_{7,42}$			$A_{7,45}$	$A_{7,46}$	$A_{7,47}$	$A_{7,48}$	$A_{7,49}$	$A_{7,50}$		$A_{7,52}$	$A_{7,53}$	
Row 8	$A_{8,39}$	$A_{8,40}$			$A_{8,43}$	$A_{8,44}$							$A_{8,51}$			
Row 9																$A_{9,54}$
Row 10	$A_{10,39}$	$A_{10,40}$	$A_{10,41}$	$A_{10,42}$	$A_{10,43}$	$A_{10,44}$	$A_{10,45}$	$A_{10,46}$	$A_{10,47}$	$A_{10,48}$	$A_{10,49}$	$A_{10,50}$	$A_{10,51}$	$A_{10,52}$	$A_{10,53}$	$A_{10,54}$
Row 11	$A_{11,39}$	$A_{11,40}$	$A_{11,41}$	$A_{11,42}$	$A_{11,43}$	$A_{11,44}$	$A_{11,45}$	$A_{11,46}$	$A_{11,47}$	$A_{11,48}$	$A_{11,49}$	$A_{11,50}$	$A_{11,51}$	$A_{11,52}$	$A_{11,53}$	$A_{11,54}$
Row 12	$A_{12,39}$	$A_{12,40}$	$A_{12,41}$	$A_{12,42}$	$A_{12,43}$	$A_{12,44}$	$A_{12,45}$	$A_{12,46}$	$A_{12,47}$	$A_{12,48}$	$A_{12,49}$	$A_{12,50}$	$A_{12,51}$	$A_{12,52}$	$A_{12,53}$	
Row 13	$A_{13,39}$	$A_{13,40}$	$A_{13,41}$	$A_{13,42}$	$A_{13,43}$	$A_{13,44}$	$A_{13,45}$	$A_{13,46}$	$A_{13,47}$	$A_{13,48}$	$A_{13,49}$	$A_{13,50}$	$A_{13,51}$	$A_{13,52}$	$A_{13,53}$	$A_{13,54}$
Row 14	$A_{14,39}$	$A_{14,40}$	$A_{14,41}$	$A_{14,42}$												
Row 15	$A_{15,39}$	$A_{15,40}$	$A_{15,41}$	$A_{15,42}$	$A_{15,43}$	$A_{15,44}$	$A_{15,45}$	$A_{15,46}$								
Row 16											$A_{16,49}$	$A_{16,50}$				
Row 17									$A_{17,47}$	$A_{17,48}$						
Row 18													$A_{18,51}$	$A_{18,52}$	$A_{18,53}$	

Chapter 4

DEVELOPMENT OF EMPIRICAL PROCEDURES FOR PROCESSING DATA INPUT

The model specified in equations 2.1, 2.2, 2.3 and 2.4 was employed in obtaining empirical results for the selected case farm. This chapter develops the empirical procedures used in obtaining, organizing, synthesizing and projecting data input required by the model. The assumptions underlying all procedures are identified.

SELECTION OF CASE FARM AND DATA SOURCES

The representative farm selected for use in this study is managed by a member of the Carman District Farm Business Association. The main reason this farm was selected is that it consists of three different but adjacently located soil types. On the basis of Manitoba Crop Insurance classifications, this farm consists of soil types:

1. B_{12} - known within the risk district in which the case farm resides as Sperling loam,
2. C_{32} - Red River clay, and

3. E_{32} - Osborne clay.[1]

The case farm was strictly a cash grain farm. It was assumed that all crops harvested were disposed of before the next harvest. The manager's records of all resources, inputs, credit arrangements, yields and costs (both fixed and variable), pertaining to his farm were employed where-ever possible.

In this model only nitrogen (within the fertilizer analysis) and its effects on crop yields on all three soil types was considered. As the case farm is located in a region where potassium in the form of potash is not required to be supplemented on any of the three soil types, its effects were not considered. Phosphorus was treated as being in ample supply for all crops on all soils. In discussion with a soil scientist at the University of Manitoba, it was felt that for the sake of reducing the analysis regarding fertilizer, it would be adequate to assume that phosphorus was always plentiful (non restricting), regardless of whether or not it was supplementarily added or the soil was inherently satiated.

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[1] Associated with these letters of classification is the concept that a higher productivity rating is given to beginning letters in the alphabet. Therefore, both B_{12} and C_{32} are rated higher than E_{32}; B_{12} is rated higher than C_{32}. The subscript '12' refers to a well internally drained soil in risk district twelve; '32' refers to a poorly internally drained soil in risk district twelve.

Most data input within the analytical model including expected prices, labor and capital restrictions, technical coefficients, fixed and variable costs were derived from records of the case farm. Tenure arrangements, crop rotation practises and yields were not. Farm management guides as well as other pertinent sources of data were used for specific areas where the model was formulated to use practises not followed by the representative farm, such as summerfallowing. Yield figures were determined solely from Manitoba Crop Insurance data.

The Manitoba Crop Insurance data employed consisted of crop yields over a five year range from 1964-1968[2] for all insurable soil types (risk areas) in Manitoba. Corresponding to each yield was a fertilization rate in actual pounds of nitrogen, phosphorus, and potash and, whether the particular crop was grown on fallow or stubble. On examination of the yield data it was apparent that to satisfy conditions requiring normal yield distributions (as related to the discounting process) it was necessary to increase the area from which the data were chosen. It was decided that the Rural Municipalities of MacDonald, Morris, Roland, Dufferin and Grey being located in risk district #12 would serve as the data source area. Within

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[2] Manitoba Crop Insurance began recording the yields of major crops for each policy held by them in 1964. By 1967 all insurable crops were recorded. The data at hand for major crops represent a five year range, 1964-68.

this risk district all of the Sperling loam and most of the Red River clay and Osborne clay exist. The shaded area indicates the data base, as shown in Figure 4.1.

Owing to lack of yield data many crops were still eliminated from the analysis even after the data area had been increased. For instance, cash crops including rape on stubble, peas, corn, and sugarbeets were not included. Another criterion required that every crop used in the analysis had to be grown on all soil types.[3] This condition eliminated growing oats, barley, and rye on summerfallow. This left the crops discussed in Chapter 3 for analysis which included wheat on fallow and stubble, oats, barley on stubble, and flax on fallow and stubble.

EMPIRICAL PROCEDURES USED IN PROCESSING DATA

Weighting of Yield Data

The raw data for each crop was broken down into arbitrary fertilizer groupings. These groupings were determined on the basis of a possible equal allocation to each group which had a consistent yield range. Using this criterion wheat grown on fallow was divided into three

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[3] A large enough number of rape on fallow observations for soil types B_{12} and C_{32} existed, but as there were too few for E_{32}, rape on fallow was not considered for the major analysis. It would be impossible to test for differences in yield variation as between different soils if each soil type was not represented.

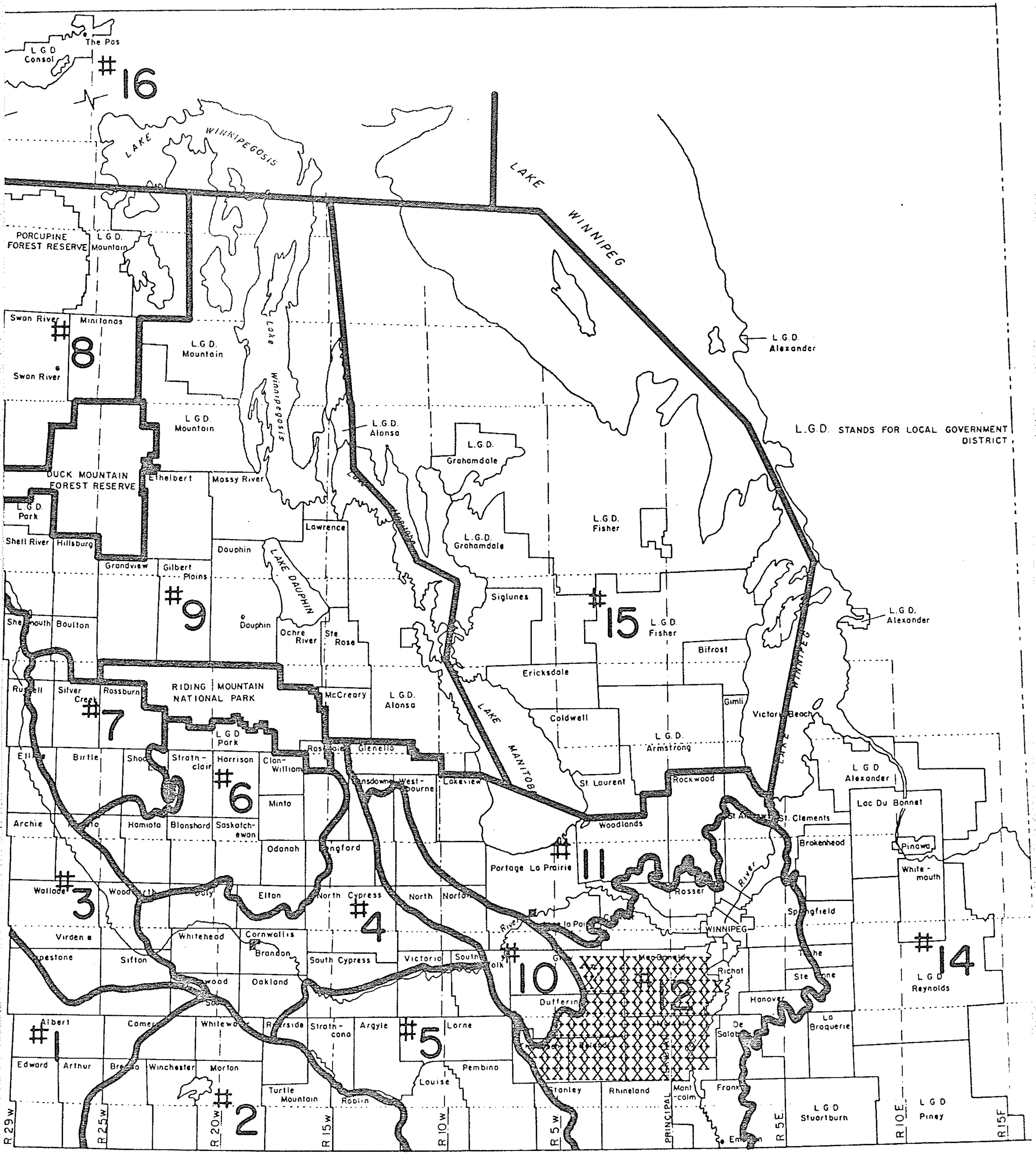

FIGURE 4.1

PROVINCE OF MANITOBA CROP INSURANCE RISK AREAS nitrogen groups--no nitrogen added, 1-8 pounds actual N, and 9 pounds and over actual N. The raw data pertaining to wheat, oats, and barley all grown on stubble was divided into five actual N levels--no nitrogen added, 1-10 pounds, 11-20 pounds, 21-30 pounds and 31 pounds and over. Flax grown on stubble was divided into two groups--no nitrogen and all rates nitrogen application. As discussed previously flax on fallow was analyzed as being non fertilized only, therefore, only the raw data exhibiting that quality was collected.

Considering the six crops, the nitrogen divisions corresponding to each, and the fact that all these groupings occur on all three soil types, there were 63 classifications that required yield data from the source defined earlier.

As previously discussed, the model makes use of normally distributed net revenues for all productive activities. As the product prices were assumed to be constant, then it was necessary that the yield observations be normally distributed. The raw data or actual observations in the grain-fertilizer classifications did not fit a normal distribution.[4] It was decided that introduction of the Central Limit Theorem would ensure normality of mean yields;

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[4] In eight out of 63 classifications tested, six were skewed to the right, and two skewed to the left. It was concluded that normality existed in very few instances.

and that thirty[5] samples of five observations within each grain-fertilizer classification would guarantee normality.[6] Each of the five observations within one sample was selected from one year of the raw data by means of a random digit table.

On further examination of the data, it was apparent that no uniform distribution of yields over the five year range existed. For example, the classification of stubble wheat grown on soil type E[7] fertilized at a rate of 11-20 pounds of actual N had 715 observations for the years 1964-1968 for the chosen data base area. However, each year does not have one-fifth of that total. The year 1964 is recorded as having 52 observations while the years 1965, 1966, 1967 and 1968 show 86, 182, 207 and 188 observations respectively.[8] Although more year's data, if available, might average out the effects of the trends of observations,

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[5] Snedecor, George, W. and Cochran, W.G.; Statistical Methods; Iowa State Press, Ames, Iowa. 6th edition, p. 51.

[6] In five randomly selected classifications tested after sampling, all were normal.

[7] Hence E_{32} will be known as E; B_{12} as B; and C_{12} as C.

[8] This is the case for all classifications. Those classes based on a higher rate of nitrogen application show a tendency of more observations toward the latter years. The opposite being true in classifications based on lesser or zero rates of application.

one is faced with the possibility of incorporating large technological changes into the data. If fewer years data was used the model would become based on the yield results of one of two years, a highly unrealistic situation. For these reasons the five year data range was decided upon. Technology is assumed to remain unchanged during this time.[9]

To determine a representative distribution of sample yields for each grain-fertilizer grouping based on the five year range decided upon, it was necessary not to bias any particular year. If average sample yields were determined from the total number of observations within any classification, this would occur. To make meaningful comparisons between classifications it was necessary to weight all observations within each sample of five by a value equivalent to the percentage of the total observations that occur in the year from which any observation was drawn. Using the example previously cited, the value of any observation drawn in 1966 was weighted by 182/715 or .25455. All classifications have thirty samples. Each observation within every sample was weighted by a factor corresponding to the makeup of the raw data. All corresponding nitrogen

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[9] The number of farms using fertilizer increased during 1964-68. However, fertilizer rates increased enormously. Increased fertilizer usage due to adoption of fertilizer practises by farms not previously using them is a change in technology; increased rates on farms already using fertilizer is not.

rates were similarly weighted.

For all randomly selected samples in all 63 grain-fertilizer classifications, each observation of yield and corresponding nitrogen level was weighted. Each newly weighted yield sample and corresponding nitrogen level was then added resulting in a weighted mean yield and a weighted nitrogen mean level. At the same time the yield variance for each sample was calculated. Within each classification, therefore, there were now 30 weighted mean yields, 30 weighted nitrogen mean levels, and 30 variance of weighted[10] yields.

Regression of Mean Yields and Variances

In order to include activities based on high rates of nitrogen application in the programming matrix, it was necessary to predict yields at high rates of actual N. Very few instances of high nitrogen application (40 to 80 pounds actual N) existed in the raw data. Mean levels were regressed on nitrogen levels for the various crops which were grown on three different types of soil.

In the case of wheat grown on fallow of all soil types, the sample mean yields and fertilizer means from the three fertilizer classifications of zero N, 1-8 pounds N and 9 pounds and over N were all lumped together. A regression based on 90 observations of mean yields versus

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[10] From now on the word "weighted" will be understood to precede the means of both yield and nitrogen.

corresponding fertilizer mean levels provided a prediction equation for all levels of nitrogen to be used in the model. This occurred for all three soil types.

Regression results for wheat, oats and barley all grown on stubble were all based on 150 yield-fertilizer mean levels. The 30 samples from each of the five fertilizer classifications (zero N, 1-10 pounds N, 11-20 pounds N, 21-30 pounds N and 31 pounds and over N) pertaining to each grain were all used in the regression analysis equation which was used to predict yields for all N rates of that crop. This was done for all three soil types.

Regression results for flax grown on stubble were based on the 60 yield-fertilizer mean levels determined from the zero nitrogen and added nitrogen classifications.[11]

Prediction equations for yield variances versus nitrogen levels for all crops were similarly determined by regression analysis.

It was decided that the data would be tested for linear and quadratic relationships. Criteria formulated before the actual regression results were determined acted as a yardstick for indicating which equation to use for various cropping classifications. The criteria were:

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[11] Flax grown on fallow (non-fertilized) could not be subjected to regression analysis. Yield and variance were based on the raw data.

1. Within each cropping classification that there be consistency in selection of a prediction equation for yield and for variance. If the linear equation was to be used in predicting yields at all rates of nitrogen, then the linear equation also be used to predict variance at all rates of nitrogen.

2. Within each cropping classification that the equation which exhibited regression (b_i) values significant at the 5% level in both yield versus nitrogen and variance versus nitrogen regressions be chosen.

3. Within each cropping classification that the highest multiple R^2 equation (yield versus nitrogen) be chosen, after the first two criteria had been fulfilled.

The regression results for both the linear and quadratic equations are shown in Appendix A. The equations actually used for each cropping activity appear in Table 4.1.

Using the pre-determined criteria, a linear prediction equation was fitted to fourteen out of the fifteen crops. Only wheat grown on fallow on soil type C (WFC) required a quadratic equation for best fit.

Based on the prediction equations of yield and variance, five levels of nitrogen corresponding to 0, 20, 40, 60 and 80 pounds N were used to determine yield and variance for all crops except flax on stubble. Variance predictions were made in the form $(\text{Variance}/n)^{\frac{1}{2}}$[12] to facilitate the discounting process. The prediction

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[12] $(\text{Variance}/n)^{\frac{1}{2}} = \frac{\sigma}{\sqrt{n}}$

TABLE 4.1

PREDICTION EQUATIONS AND CORRESPONDING YIELDS
YIELD = f(NITROGEN)

Grain	Soil Type	Equation	Form	Yield at N=80 (For Flax N=40)
Fallow wht.	B	$35.15+.183\overline{N}$	linear	49.77
Stubble wht.	B	$27.85+.103\overline{N}$	linear	36.08
Stubble oats	B	$55.71+.435\overline{N}$	linear	90.49
Stubble bly.	B	$36.42+.202\overline{N}$	linear	52.57
Stubble flax	B	$11.94+.111\overline{N}$	linear	16.38
Fallow wht.	C	$26.23+.161\overline{N}+.00018\overline{N}^2$	quadratic[a/]	37.93
Stubble wht.	C	$21.34+.071\overline{N}$	linear	27.04
Stubble oats	C	$43.19+.316\overline{N}$	linear	68.51
Stubble bly.	C	$28.63+.109\overline{N}$	linear	37.34
Stubble flax	C	$9.79+.057\overline{N}$	linear	12.06
Fallow wht.	E	$24.33+.229\overline{N}$	linear	42.65
Stubble wht.	E	$18.47+.074\overline{N}$	linear	24.36
Stubble oats	E	$40.52+.383\overline{N}$	linear	71.16
Stubble bly.	E	$21.77+.185\overline{N}$	linear	36.59
Stubble flax	E	$7.66+.099\overline{N}$	linear	11.63

[a/] This function is increasing at an increasing rate. However, in tracing the function out, the values appear to be almost perfectly linear. This result would not be normally expected although some soil scientists believe that farmers using high levels of N have better management and therefore, attain beyond proportionally better yields at those higher levels.

equations selected and the corresponding yields and variance for the crops subjected to regression analysis are presented in detail in Appendix A. It should be noted that although five levels of nitrogen were used for prediction, in all cases except wheat grown on fallow on soil type C, which requires a quadratic prediction equation, only the 0 and the 80 pounds levels of nitrogen were tied to the various cropping activities in the model. Realistically a farmer who planned on the basis of a linear production function would only consider two possible points on that function--the lowest or the highest attainable. If it were profitable to fertilize, the decision would be made to fertilize at a maximum, restraints permitting. In the case of the quadratic production function, all five levels of N were used as activities. The yield values used at the 80 pounds $\bar{N}$ rate are shown in Table 4.1.

Discounting Yields

Previously the raw data were sampled in a manner consistent with the Central Limit Theorem which resulted in normal distributions of mean yields for the various crop-fertilizer classifications. As stated earlier, normality of yields is required for discounting procedures. The Central Limit Theorem was of fundamental importance in this study as its use guaranteed normality.

The required normal distributions, characterized by means (μ) and variances (σ^2), were standardized to μ=o and σ=1 for purposes of finding area under the normal curve between any two values of yield. The area under the normal distribution can be obtained by converting the units of measurement into standard units using the formula,

$$Z = \frac{x - \mu}{\sigma}$$

If the original population is not normal, however, the sampling distribution of the mean can be approximated closely with a normal distribution, if n is large. In other words if the number of observations per sample is large, the sampling distribution of the statistic,

$$Z = \frac{\bar{X} - U}{\sigma_{\bar{X}}}, \text{ where } \bar{X} = \text{average of a sample, } U = \bar{\bar{X}} \text{ and } \sigma_{\bar{X}} = \sigma/\sqrt{n} = (\sigma^2 / n)^{\frac{1}{2}},$$

can be approximated closely with the standard normal distribution. It is difficult to state precisely how large n must be. Unless the distribution of the original population has a very unusual shape the approximation will be good even if n is relatively small-certainly if n is 30 and quite possibly if n is as small as five.[13]

On the basis of the sets of normal distributions, for all grain-fertilizer classifications, prediction equations were formulated by means of regression techniques. Predicted yields and variances were consequently determined at five levels of nitrogen application. A necessary under-

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[13] Freund, John E.; Modern Elementary Statistics Prentice-Hall, Inc., Englewood Cliffs, New Jersey. 3rd edition.

standing at this point, however, is that the predicted yield be one and the same as the average yield of all normal yield distribution. Theoretically whether the actual production function is linear, quadratic or any other form, it should pass through a distribution of yields at the average or mean value of the normal distribution corresponding to all levels of nitrogen.

Figure 4.2 shows a three dimensional graph which exhibits this understanding.

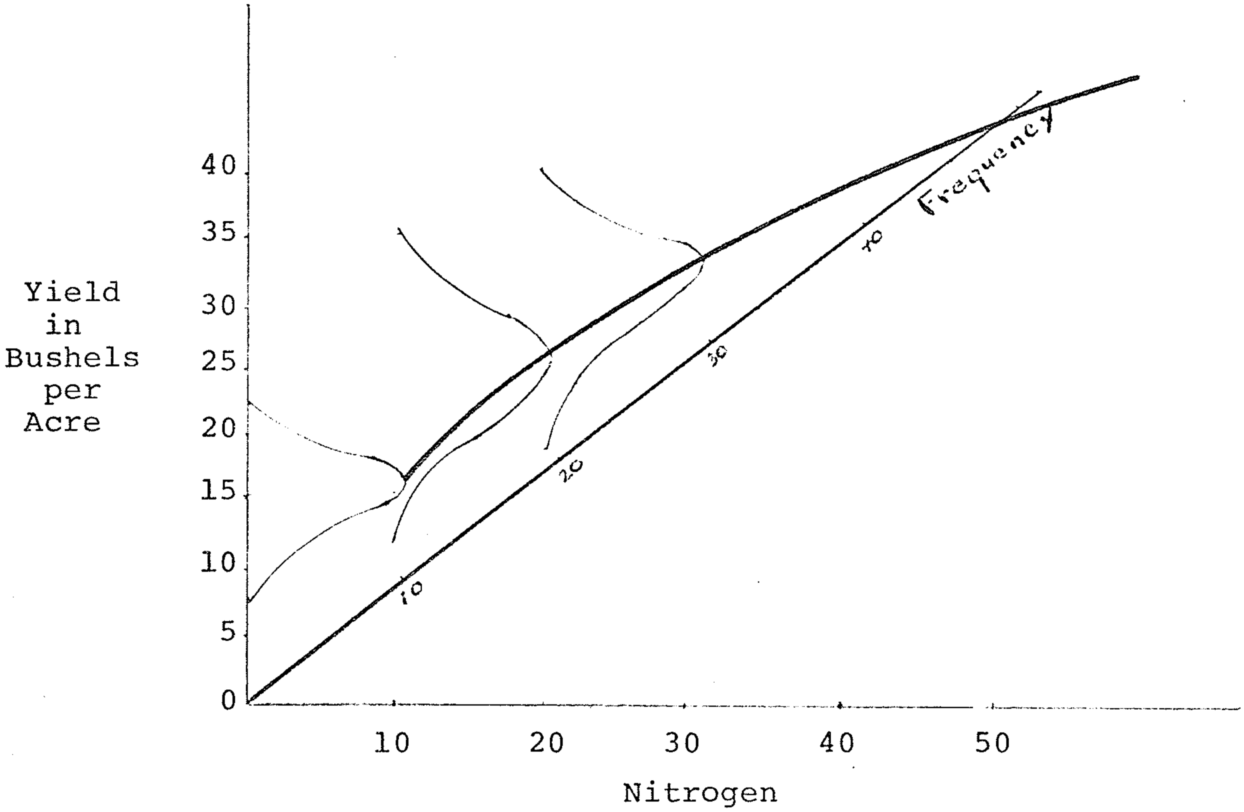

FIGURE 4.2

A QUADRATIC PRODUCTION FUNCTION
PASSING THROUGH THE MEANS OF THE NORMAL DISTRIBUTIONS

The discounting process was described in detail in Chapter 2. Given a normal distribution of grain yields with average yield ($\bar{\bar{Y}}$) and a standard deviation ($\sigma_{\bar{y}}$), a new discounted yield ($\bar{Y}$) becomes a function of the average yield ($\bar{\bar{Y}}$) minus the standard deviation ($\sigma_{\bar{y}}$) times the value of a subjective risk aversion factor (Z). Given, for example, that at a nitrogen level of 10 pounds actual N the average yield of a particular crop is 20 bushels per acre and knowing that $\sigma_{\bar{y}} = 5$ it is possible to discount from 20 by any risk factor. As all observations of yield ($\bar{Y}$) lie somewhere under the normal curve, a farmer who is faced with this production situation at the planning stages of his next production period has a 50 per cent chance of being above or below the average yield of 20. If the farmer is indifferent to risk, he will enter the value of 20 into any planning or forecasting model and budget; if he is a risk averter he will discount from the value 20 by an amount that corresponds to a subjective risk factor. If the producer wants to be 80 per cent sure of achieving some yield value or greater, he will discount to a yield corresponding to 80 per cent of the area of the normal distribution of yields being to the right. It is possible by means of a standard normal distribution table to find a value of -Z which corresponds to 80 per cent of the distribution falling to the right.[14] From that table[15] -Z=.84. Entering the values of $\bar{\bar{Y}} = 20$; $\sigma_{\bar{y}} = 5$; and $-Z = .84$ into

$$-Z = \frac{\bar{Y} - \bar{\bar{Y}}}{\sigma_{\bar{y}}} \qquad 4.1$$

a value of $\bar{Y} = 15.8$ results. The farmer using this new discounted yield in his planning model is 80 per cent sure that at least 15.8 bushels per acre will result.

Three levels of yields corresponding to three levels of subjective risk were determined for each grain-fertilizer category. All yields listed in Table 4.1 were altered by a risk factor. The three levels of risk used are shown in Table 4.2.

TABLE 4.2

RISK-CERTAINTY LEVELS

Risk Levels	Degree of Certainty	-Z Value
1	50%	0
2	65%	.39
3	80%	.84

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[14] It was stated in the beginning chapters that only indifference and aversion of risk would be considered in this model; hence, the use of the -Z variate which pertains to the left half of the normal curve.

[15] See Appendix E4.

The model was tested with the three different levels of yield for all productive activities. The differences in yield between the three levels were incorporated into the C_{ij} values and formed the basis of any change in net prices.

SPECIFICATION OF MATRIX COEFFICIENTS

Activities

Chapter 3 developed the productive activities for use in the model. Wheat on fallow and stubble, oats on stubble, barley on stubble, and flax on fallow and stubble acted as endogenous variables available for solution. The wheat activities were further broken down into quota and non-quota restrictions. The activities were then made available on three different soil types. Fertilizer levels broke the activities into greater numbers. It was first anticipated that five levels of nitrogen application consisting of 0, 20, 40, 60 and 80 pounds actual N per acre would be used. However, due to linear production functions, in most cases the values of 0 and 80 only were considered.

Constraints

The land base on the case farm summed to 2110 acres. The farm made up of the three soil types discussed earlier consists of approximately:

1. 460 acres of B
2. 1500 acres of C
3. 150 acres of E

Four different sets of ratios of summerfallow for each soil type were considered. They were:

1. Continuous cropping (0 per cent summerfallow)
   - 460 acres of B, 1500 acres of C, and 150 acres of E were available for crop.
2. 20 per cent summerfallow
   - 368 acres of stubble B; 46 acres of fallow B, 1200 acres of stubble C; 150 acres of fallow C, 120 acres of stubble E; 15 acres of fallow E were available for crop.
3. 40 per cent summerfallow
   - 92 acres of stubble B; 184 acres of fallow B, 300 acres of stubble C; 600 acres of fallow C, 30 acres of stubble C; 60 acres of fallow E were available for crop.
4. Different ratios fixed in relation to soil type
   - 460 acres of stubble B
     900 acres of stubble C; 300 acres of fallow C
     50 acres of stubble E; 50 acres of fallow E
     were available for crop.

The periods of seeding and harvest were considered as the high labor requirement times. Total labor available during these periods was 4853 hours--the sum of all manager and hired labor. This figure was broken into a spring total labor value and a fall total labor value of 2751 and 2102 hours, respectively. The levels of these three constraints remained constant through all parametric changes in the model.

The manager of the case farm felt that he had a short term line of credit of $60,000 at his local bank and another $9,000 with agri-business firms with which he does business. The value $69,000 then became the model's initial operating capital maximum level. As this level appeared exceedingly high, four lower levels of available operating capital were also employed in the model to determine the effects of a capital restriction on net profit and cropping plans. The other levels included capital values of:

1. \$35,000
2. \$30,000
3. \$25,000
4. \$20,000

The value of the wheat quota constraint available to this farm was determined by the following rough estimation technique. At the time this model was formulated a figure of 500 million bushels of wheat could be realistically expected to be used domestically and for export;[16] also, there are approximately 55 million acres of land available for cropping in Western Canada.[17] Given these two facts, the wheat quota that could pertain to the case farm's 2110 acres is found by solving:

$$\frac{2{,}110 \text{ case farm acres}}{55{,}000{,}000 \text{ total acres}} = \frac{X \text{ case farm bushels}}{600{,}000{,}000 \text{ total bushels}}$$

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[16] According to the Report of the Federal Task Force on Agriculture, "Canadian Agriculture in the Seventies" Dec. 1969, pp. 86-95, by year 1980, 20 million acres of wheat averaging between 25-32 bushels per acre will be required of Canadian wheat growers. In 1967-68 demand amounted to a total of 518 million bus. Weighing these considerations and the uncertainty of purchases by the Soviet Union and China in 1969-70, the author proposes to use 600 million bushels as a base for the model.

[17] pp. 110 Ibid. There are 85 million acres of improved land in Western Canada. 11 million acres of this number are forage. An average 26 per cent of the remaining 74 million acres has been historically summerfallowed. This leaves approximately 55 million acres for crop production.

where X = 23,018 bushels. Based on this estimate two values of wheat quota corresponding to 25,000 and 20,000 bushels were used throughout the model trials.

The final constraints, those of the safety stops, were arbitrarily set at values of 1200 total acres for both wheat and barley, and 600 total acres for both oats and flax. The case farm manager reported that he would consider seeding these crops past these levels as a matter of poor rotation. These stops prevent solutions as regards acreages from surpassing the limits set.

Matrix Coefficients

Labor hour coefficients required a detailed analysis. Knowing case farm factors such as tillage practises, size of machinery, hauling distances, degree of clay or sand in soil, typical field size, and degree of stoniness, it was possible by means of a mathematical program[18] to calculate the number of hours required to perform one acres work of any activity. For all cropping activities it was found that a labor requirement of approximately 1.0 hours per acre was needed to complete all the operations necessary to seed, spray, harvest, and prepare the same acre for next years crop regardless of type of grain grown. This total was divided

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[18] Model developed and used by W.J. Craddock in "Interregional Competition in Canadian Cereal Production" prepared for Economics Council of Canada, Special Study No. 12. This part of that model was used to determine labor and operating costs coefficients.

into values of approximately 0.3 and 0.7 for spring and fall respectively. For the cultivation practises necessary to summerfallow properly, approximately 0.6 hours of labor were required per acre. This was allocated to spring labor.

Determining operating capital coefficients proved to be rigorous. It was decided that all variable or direct costs and all fixed costs would be allocated on a per acre basis. This would help determine how much capital would be required in total to have an acre of any activity come into solution. The calculation of the operating capital coefficients for wheat fertilized at 80 pounds N grown on fallow on soil type C will be used as an example. First, however, the main presentation of the costs that make up the capital requirement coefficients are presented.

Variable Costs. The case farm does not, as yet, soil test to determine the level of nutrients in the soil. Seed costs were determined in the following manner. Using:

1. a seeding rate of:
   wheat - 1¼ bushels per acre,
   oats - 2 bushels per acre,
   barley - 1½ bushels per acre, and
   flax - 40 pounds per acre,
2. certified seed one out of three years at prices of:
   wheat - $1.60 per bushel,
   oats - $1.00 per bushel,
   barley - $1.30 per bushel, and
   flax - $4.00 per bushel,
3. his own seed the other two out of three years at a cost of:
   wheat - $1.40 per bushel,
   oats - $ .70 per bushel,
   barley - $ .90 per bushel, and
   flax - $3.00 per bushel,

the seed replacement cost per year pertaining to this farm would be:

for wheat - $1.83 per acre,
oats - $1.60 per acre,
barley - $1.55 per acre, and
flax - $2.36 per acre.

Seed cleaning at a custom cleaning plant in the local area costs:

wheat - 1¼ bushels at 6¢ = $ .08 per acre,

oats - 2 bushels at 5¢ = $ .10 per acre,

barley - 1½ bushels at 6¢ = $ .09 per acre, and

flax - 40/56 bushels at 25¢ = $ .18 per acre.

Seed treatment costs were computed at a value of 5¢ per acre for wheat, oats, and barley basis Panogen and 3¢ per acre for flax basis Panogen at the recommended rate.

Herbicide spraying consisted of the use of the general broad leaf killing M.C.P.A. Sodium Salt. The case farm in normal conditions reported using levels of 10 ounces of this per acre for wheat, barley, and oats; and, 8 ounces per acre for flax. At a cost of 6¢ per ounce of acid this added a cost of:

wheat - 60¢ per acre,
oats - 60¢ per acre,
barley - 60¢ per acre, and
flax - 48¢ per acre to the total.

Fertilizer created the largest increase to cost for those activities using it. The case farm manager suggested that a fertilizer brand analysis of 11-55-0 was applied to a maximum of 60 pounds gross with the seed and then 28-0-0 was broadcasted over the land surface to bring the nitrogen level to the specified target. Given budgeting prices of

11-50-0 at $90 per ton and 28-0-0 at $42 per ton, bulk basis, the cost of fertilizer amounted to the totals shown in Table 4.3 for the respective nitrogen levels regardless of the grain type.

TABLE 4.3

FERTILIZER COSTS AT VARIOUS NITROGEN LEVELS

Nitrogen Level	Formulation	Pounds Req'd	Pounds of N Added	Cost Per Ton	Total Cost Per N Level
20 lbs.N	11-55-0	60	6.6	$90	$2.70
	28-0-0	49	13.4	42	1.03
			20.0		3.73
40 lbs.N	11-55-0	60	6.6	$90	$2.70
	28-0-0	120	33.4	42	2.52
			40.0		5.22
60 lbs.N	11-55-0	60	6.6	$90	$2.70
	28-0-0	191	53.4	42	4.01
			60.0		6.71
80 lbs.N	11-55-0	60	6.6	$90	$2.70
	28-0-0	262	73.4	42	5.50
			80.0		8.20

The farm purchased $139 worth of building repairs and supplies in the year before. Dividing this figure by 2110 acres results in a building improvement and repair charge of $0.67 per acre.

The machinery and equipment expense included all farm fuel, grease, repairs, and licenses. This value was determined by means of the mathematical program referred to in footnote 18 and appeared at the following levels for each activity regardless of level of fertilizer:

1. Wheat on fallow - $2.22 per acre,
2. Wheat on stubble - $2.19 per acre,
3. Oats on stubble - $2.26 per acre,
4. Barley on stubble- $2.21 per acre, and
5. Flax on stubble - $2.16 per acre.

The final variable cost arose from custom bulk fertilizer spreading of 28-0-0 at a cost of $.65 per acre.[19]

Fixed Costs. The other component of the operating capital coefficient was that of fixed costs. This included taxes, depreciation, and miscellaneous fixed costs. It did not include any opportunity costs of investment or labor.

Building and machinery depreciated a book value of $13,170 the year previous. This figure was not expected to change significantly for the next five years. A depreciation cost per acre, therefore, of $13,170/2110 acres or $6.25 resulted and was used for all activities.

The tax cost per acre varied depending on the soil type. As the assessments per soil type were different, tax levies varied as follows:

1. Soil B - $2.20 per acre,
2. Soil C - $2.00 per acre, and
3. Soil E - $1.65 per acre.

The miscellaneous fixed costs included hydro and telephone (business portion) of $726, miscellaneous overhead expense of $1152 and building and equipment insurance

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[19] As the case farm does not crop insure, no cost was allocated to its use.

of $358. The total miscellaneous cost of $2236 resulted in a per acre charge of $1.05.

Operating Capital Coefficient for Activity - Non Quota Wheat on Fallow, 80 pounds N (WFC_4). Adding all variable costs and fixed costs results in a value of $22.97 which became the operating capital coefficients for activity WFC_4. All costs are reviewed in Table 4.4.

TABLE 4.4

SUMMARY OF ALL COSTS ASSOCIATED WITH WFC_4

Variable Factors	Cost per Acre	
Soil Test	-	
Seed Replacement	$1.83	
Seed Cleaning	.08	
Seed Treatment	.05	
Chemicals	.60	
Fertilizer	8.20	
Crop Insurance	-	
Bldg. Improvement & Repair	.07	
Machinery & Equipment Expense	2.19	
Custom Work	.65	
TOTAL		$13.67
Fixed Factors	**Cost per Acre**	
Depreciation	$6.25	
Taxes	2.00	
Miscellaneous	1.05	
TOTAL		$ 9.30
	Capital Coefficient =	$22.97

Determination of Net Prices

The planning model was made complete with the determination of C_{ij} values. As discussed before, this was performed by a simple multiplication of predicted product price and discounted yield followed by a subtraction from that product of the operating capital value.

Product prices were subject to future expectations for the next five years by the manager of the case farm. He felt that prices in the future would level off at the values listed below:

1. Quota wheat - initial price plus adjustment for final payment would bring a net price of approximately $1.50 per bushel basis dry #3 C.W.R.S. Wheat.[20]
2. Non-quota wheat - could command a price of $1.25 per bushel basis dry #3 C.W.R.S. sold privately and in the spring season.
3. Non-quota oats - could sell for $.60 per bushel given that they were at least 40 pounds per bushel in weight, sold toward spring. This figure would compare quite favourably to a net payment per bushel through Wheat Board channels.
4. Non-quota barley - case farm can sell all barley to a private feedlot. This feedlot was willing to purchase barley at $.88 per bushel.
5. Flax - the 1969 average price for all Manitoba was $2.60 per bushel.[21] It was expected that this price would not hold. However, a value of $2.50 per bushel was used in the planning model.

This completed the preparation for the planning model. This was then presented to the computer for calculation

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[20] Canadian Western Red Spring.

[21] Manitoba Crop Production Yearbook, 1969.

and solution. Levels of the exogenous variables were varied systematically in conjunction with alternative levels of net prices corresponding to different values of risk to determine effects on net farm income and cropping plans.

Appendix B shows solutions of expected incomes $t(X_j)$ and corresponding unit acre levels (X_j). These solutions meet the specifications called for by equations 2.1, 2.2 and 2.3, and form the basis for the answers determined by equation 2.4 presented in the next section.

Table 4.5 illustrates a systems flow chart that was followed in analyzing the data yield input. It is broken into five main phases. The first phase begins with the raw data which is analyzed through four parts to a point where calculated weighted means for yield and for fertilizer use and calculated weighted variance of yield exist for all samples. Phase 2 follows these weighted numbers through regression techniques, choosing of the appropriate prediction equation, and predicting of yield and variance at different levels of nitrogen. Phase 3 discounts from the newly predicted yields by three different risk associated values. The fourth phase selects all those activities having been discounted by the same risk value and prepares them for presentation into the planning model. The final phase calls for accumulation of all data - yields that have been discounted as well as all resource and technical data.

TABLE 4.5
OPERATIONS FLOW CHART

Phase 1			
Part 1 -group raw yield data by year and nitrogen	Part 2 -draw 30 samples of 5 yields and nitrogen for each group from each of the 5 years.	Part 3 -take each sample S_i and multiply by weighted values W_j.	Part 4 -for each S_i find weighted mean value $\overline{WY_i}$ and weighted nitrogen mean $\overline{WN_i}$. Also for each S_i find variance of W_jY_{ij} using $\sigma^2_{W_jY_{ij}} = \frac{\sum(W_jY_{ij} - \overline{WY_i})^2}{n-1}$
a $WFB(C)(E)_0$ $WFB(C)(E)_{1-8}$ $WFB(C)(E)_{9+}$	$\underline{S_1}$ $\underline{S_2}$ ... $\underline{S_{30}}$ Y_{11} N_{11} Y_{21} N_{21}... $Y_{30.1}$ $N_{30.1}$ Y_{12} N_{12} Y_{22} N_{22}... $Y_{30.2}$ $N_{30.2}$ . . . . Y_{15} N_{15} Y_{25} N_{25}... $Y_{30.5}$ $N_{30.5}$ as above for $WFB(C)(E)_{1-8}$ as above for $WFB(C)(E)_{9+}$	a $\underline{S_i}$ ... $\underline{S_{30}}$ W_1Y_{11} W_1N_{11}... $W_1Y_{30.1}$ $W_1N_{30.1}$ W_2Y_{11} W_2N_{12}... $W_2Y_{30.2}$ $W_2N_{30.2}$ . . .. W_5Y_{15} W_5N_{15}... $W_5Y_{30.5}$ $W_5N_{30.5}$ as above for $WFB(C)(E)_{1-8}$ as above for $WFB(C)(E)_{9+}$	a 30 calculated $\overline{WY_i}$'s 30 calculated $\overline{WN_i}$'s 30 calculated $\sigma^2_{W_jY_{ij}}$'s as above for $WFB(C)(E)_{1-8}$ as above for $WFB(C)(E)_{9+}$
b $WSB(C)(E)_0$ $WSB(C)(E)_{1-10}$ $WSB(C)(E)_{11-20}$ $WSB(C)(E)_{21-30}$ $WSB(C)(E)_{31+}$	b similarly for the whole group of $WSB(C)(E)_i$	b "	b "
c $OSB(C)(E)_0$ $OSB(C)(E)_{1-10}$ $OSB(C)(E)_{11-20}$ $OSB(C)(E)_{21-30}$ $OSB(C)(E)_{31+}$	c "	c "	c "
d $TSB(C)(E)_0$ $TSB(C)(E)_{1-10}$ $TSB(C)(E)_{11-20}$ $TSB(C)(E)_{21-30}$ $TSB(C)(E)_{31+}$	d "	d "	d "
e $XSB(C)(E)_0$ $XSB(C)(E)_{1+}$	e "	e "	e "
f $XFB(C)(E)_0$	f "	f "	f find grand mean of sample means; also variance - carry to phase 3.

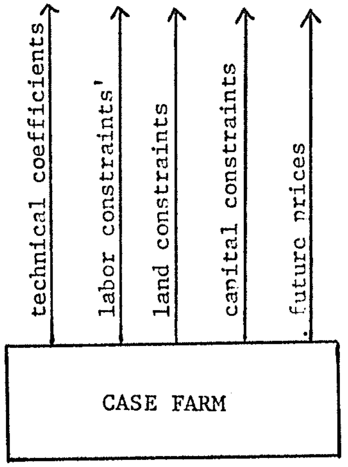
* Yield data input into planning model is prepared in Phases 1,2,&3.

TABLE 4.5 (continued)

Phase 2		
Part 1	Part 2	Part 3
-Run regressions using all the weighted values of WFB(C)(E)$_i$. First regress linearly and quadraticaly $\overline{WY}_i$ vs $\overline{WN}_i$; then similarly $\sigma^2_{W_jY_{ij}}$ vs $\overline{WN}_i$ and $\frac{\sigma_{W_jY_{ij}}}{\sqrt{n}}$ vs $\overline{WN}_i$	-On the basis of decision criteria discussed before choose applicable yield and variance equations.	-for chosen equations find levels of yield and variance corresponding to:
a enter 90 pairs of values into each regression test.	a linear for WFB linear for WFE quadratic for WFC	a 0 lbs. actual N. 20 lbs. actual N. 40 lbs. actual N. 60 lbs. actual N. 80 lbs. actual N.
b enter 150 pairs of values.	b linear for WSB linear for WSC linear for WSE	b "
c "	c linear for OSB linear for OSC linear for OSE	c "
d "	d linear for TSB linear for TSC linear for TSE	d "
e enter 60 pairs of values.	e linear for XSB linear for XSC linear for XSE	e 0 lbs. actual N. 20 lbs. actual N. 40 lbs. actual N.
f carry to Phase 3	f carry to Phase 3	f carry to Phase 3

TABLE 4.5 (concluded)

Phase 3	Phase 4	Phase 5
-for each corresponding predicted pair of yield ($\bar{\bar{Y}}$) and variance ($\sigma/\sqrt{n}$) enter into $-Z_1 = \frac{\bar{Y} - \bar{\bar{Y}}}{\sigma/\sqrt{n}}$ and solve for new discounted yield given three values of Z_1.	-Select all the new discounted yields ($\bar{Y}$'s) that correspond to the three levels of risk aversion. Therefore from a,b,c,d,e & f, select all the yields ($\bar{\bar{Y}}$) discounted by the same value of Z. The three sets of yield values are now ready to be entered into the planning model.	-accumulation of data input for planning model.
a $Z_1 = .00 \rightarrow 50\%$ $Z_2 \doteq .39 \rightarrow 65\%$ $Z_3 = .84 \rightarrow 80\%$	i -collect all the new discounted yields from a,b,c, d, e & f corresponding to Z_1.	Planning Model 1. Variation of exogenous variables for Z_1, discounted yields.
b "	ii -as above corresponding to Z_2.	2. as above for Z_2 discounted yields
c "	iii -as above corresponding to Z_3.	3. as above for Z_3 discounted yields.
d "		
e "		
f "		



PROCEDURES USED TO DETERMINE CHANCES OF EXPECTED INCOME EXCEEDING A SPECIFIED LEVEL OF INCOME

As previously described in detail in Chapter 3, the second part of the model specified as equation 2.4 determines the chances of expected income derived from any set of future plans (for which the variance is known) exceeding a specified level of income. Expected income $E(t_j)$ in the case was determined by multiplying expected net prices $\bar{C}_{ij}$ (used for the 50 per cent certainty level) by the unit acre levels of activities contained in the solution vector X_j (of all certainty levels), which was brought forward from the j^{th} plan generated by maximizing equation 2.1. Consequently the net incomes (t_j) associated with the 50 per cent certainty level were not modified before they were entered into equation 2.4; however, the net incomes associated with the 65 per cent and 80 per cent certainty levels were modified. The unit level solutions determined in the models corresponding to the latter certainty levels were kept as the risk solution component of equation 2.1, but were then multiplied by average net prices $(\bar{C}_{ij})$ instead of $(C - Z_j\sigma)$. This operation was performed following the reasoning that a farm manager who is a risk averter will determine his acreage of various crops to be grown using a discounting process, but once he has determined a set of plans corresponding to his own risk attitude, he will use average net prices for purposes of budgeting or for making trade-offs with a survival level of income. The new values of $E(t_j)$ for the 65 per cent and 80 per cent certainty levels are presented in Appendix C.

The specified income ($\hat{t}_s$) was the minimum yearly net revenue necessary to ensure survival of the case farm. This was determined by considering the factors indicated in Table 4.6.

TABLE 4.6

DETERMINATION OF SPECIFIED INCOME

Income and Cost Factors	Amount
Household and Personal Expenses	$12,000
Interest on Farm Debt	5,500
Principle Repayment (Land Debt)	9,250
TOTAL	26,750
Non-farm Labor Income	1,750
TOTAL	$25,000

A figure of $25,000 was used in the second part of model as specified in equation 2.4.

Corresponding to the solutions of equations 2.1, 2.2 and 2.3 is the set of income standard deviations for the j^{th} plan, shown in Appendix D. The variance of income associated with the plan to be tested was determined by multiplying the associated yield variances of a solution activity by the weights (acreages in the plan) of those activities squared, and by the the square of the product prices, that is:

$$\sigma^2_{(t_j)} = \Sigma X_j^2 \sigma_j^2 p_j^2 .$$

Taking the square root of this value gave the desired standard deviation.

For instance, $\sigma_{(t_j)}$ for the cropping plan derived using exogenous variables of $25,000 capital, 40 per cent summerfallow ratio, 20,000 bushels wheat quota at the 80 per cent certainty level was $10,279. This was attained by multiplying the variance of the individual crops (Appendix A, Table A2) both by the unit levels squared of the same crops solved for in equation 2.1 (given in Appendix B) and the product prices squared (page 77); and, then, taking the square root of the summed products. Therefore, for the plan indicated above, the variance equalled:

$$\begin{aligned}
&(\text{184 acres of } QWFB_0)^2 \cdot (\text{Variance of } WFB_0 \text{ or } 172) \cdot (\$1.50)^2 + \\
&(\text{92 acres of } QWSB_0)^2 \cdot (207) \cdot (\$1.50)^2 + \\
&(\text{508 acres of } QWFC_0)^2 \cdot (145) \cdot (\$1.50)^2 + \\
&(\text{92 acres of } WFC_0)^2 \cdot (145) \cdot (\$1.25)^2 + \\
&(\text{193 acres of } OSC_0)^2 \cdot (99) \cdot (\$.60)^2 + \\
&(\text{60 acres of } QWFE_0)^2 \cdot (145) \cdot (\$1.50)^2 = 105,654,371
\end{aligned}$$

while $\sigma_{(t_j)}$ equalled $(105,654,371)^{\frac{1}{2}} = \$10,279$.

Model 2.4 was then operational for making tradeoffs between expected income and risk by specifying probabilities of the generated income, from any set of plans, exceeding the specified level. Given expected income $E(t_j)$, knowing the standard deviation of the cropping solution $\sigma_{(t_j)}$ and a survival level of income $\hat{t}_s$, it was possible to determine the probability of income from a farm plan exceeding the survival level of income. This value had to be negative if the corresponding plan was to be feasible.

Appendix Table E denotes the probabilities (y_j) solved for by means of,

$$y_j = \frac{\hat{t}_s - E(t_j)}{(X_j' \, V \, X_j)^{\frac{1}{2}}} \quad . \qquad 2.4$$

In the case mentioned above of 80 per cent certainty $25,000 capital, 40 per cent summerfallow ratio, and 20,000 bushel quota, the value of y_j was found to be,

$$y_j = \frac{\$25,000 - \$18,753}{\$10,279} = +.6077.$$

This value corresponds to a probability of 27 per cent as determined from Appendix Table E4. This value indicates that the plan determined by discounting and then subjected to average net prices has at least a 27 per cent chance of surpassing the survival income of $25,000. All other probabilities as shown in Appendix Tables E1, E2 and E3 were similarly calculated.

Chapter 5

ANALYSIS OF OPTIMAL CROP PLANS AND USE OF FERTILIZER

The previous chapters have developed a theoretical model built to specifications presented by the analytical framework and made operational by the develpment of empirical procedures for processing data input. In this chapter, empirical results generated by similation of the model are presented as tests for the hypotheses stated in Chapter 1.

HYPOTHESIS NO. 1

This study was quided by two hypotheses. The first hypothesis read:

> It is hypothesized that as risk aversion increases, the amount of fertilizer used decreases, and the differences between the target fertilizer recommendations and actual amounts used increases. It follows that these differences will be larger for crops grown under high variability and, correspondingly, that land associated with high yield variability might receive zero amounts of fertilizer. Further, under high risk aversion, land associated with high yield variability may not be competitive and, correspondingly, may not enter optimal solutions. Further, it is anticipated that summerfallow replaces fertilizer as a means of supplying plant nutrients when (a) the difference between the target fertilizer recommendation and actual amount used increases, (b) the level of capital declines, and (c) the more severely crop production is limited by quotas.

Results of Model Simulation

Cropping Plans and Use of N. The first part of the hypothesis is accepted. The results generated indicated that as risk aversion increased, the amount of N used decreased. Tables 5.1a and 5.1b show that as risk aversion increased from 50 per cent to 65 per cent then 80 per cent certainty, total N for each cropping plan was less. At the 80 per cent certainty level, the cropping plans called for no grains to be grown using N.

TABLE 5.1a

TOTAL ADDED N FOR CROPPING PLANS CORRESPONSING TO ALTERNATIVE RISK - SUMMERFALLOW LEVELS[a/]

	Summerfallow Levels			
Certainty	0%	20%	40%	Fixed As To Soil Type[b/]
	(in actual pounds)			
50%	48,800	81,760	101,280	76,000
65%	48,000	55,680	52,560	68,800
80%	-	-	-	-

a/ For $69,000 available capital and 25,000 bushels available quota.

b/ As earlier presented, 0% of Soil B, 25% of Soil C and 33.3% of Soil E.

TABLE 5.1b

TOTAL ADDED N FOR CROPPING PLANS CORRESPONDING TO ALTERNATIVE RISK - SUMMERFALLOW LEVELS[a]

	Summerfallow Levels			
Certainty	0%	20%	40%	Fixed As To Soil Type[b]
	(in actual pounds)			
50%	48,000	39,040	29,280	44,800
65%	8,288	18,520	27,896	16,952
80%	-	-	-	-

a/ For $30,000 available capital and 20,000 bushels available quota.
b/ 0% for Soil B, 25% for Soil C and 33.3% for Soil E.

In reviewing the cropping plans presented in detail in the appendix, it was evident that under the assumptions of the model, wheat and oats were grown on the largest acreage. Regardless of degree of certainty, level of summerfallow, or capital restriction wheat was never grown on fewer than 730 acres and most often was grown within the 950-1150 acre range. Oats came into solution nearly every cropping plan and was grown most often in the 200-400 acre range.

In a few cropping plans barley came into the solution when the capital level was non restricting and continuous cropping was in force. Flax was grown in a few isolated instances when certainty was increased to 80 per cent and continuous cropping again was in force.

On further examination of the results, it was obvious that a crop was grown either at the highest level of nitrogen fertilization (80 pounds N) or at the zero level. As previously discussed in Chapter 4 the nature of the generated production functions were such that activities corresponding to levels of fertilization in between these values were not in use. The linear functions in part explain the decrease in fertilization from the 65 per cent to 80 per cent certainty levels as presented in Tables 5.1a and 5.1b.

The nonsimilar yield variations associated with the different grains explain for the most part the diversity of the cropping plans and fertilizer use. As is evident by the regression results for variance prediction[1], variation as related to yield changed drastically from grain to grain grown on the same and on different soil types.[2] Although crop variation was not taken into account at the 50 per cent certainty level (C_j values were not discounted), variation became a factor in the next two levels. Consequently wheat grown on fallow and on stubble, which was profitable to grow at the 50 per cent level of certainty, continued to be grown at the 65 and 80 per cent levels (C_j values were discounted by factors relating yield variance and risk). This trend continued as the net price of wheat was discounted less, relative to barley and flax.

Similarly as this explains why wheat and oats started out by being grown on most of the available acres and con-

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[1] Shown in Appendix Table A2.

[2] For example, WFB_0, WFB_4, OSB_4 and OSE_4 have variances per acre of 172, 2760, 7052 and 8692 bushels respectively.

tinued throughout the other certainty levels, the reason for the change in the levels of fertilization accompanying the cropping plans is closely related. At the first level the activities corresponding to the highest level of N yielded the highest net price and consequently formed the basis of solution except in those which were restricted by capital. However, due to the presence of linear production and variance functions the standard deviation associated with a higher net price resulting from increased fertilizer use was relatively higher than the standard deviation associated with a low net price (an activity not fertilized). Therefore, at the 80 per cent certainty level all activities based on a rate of 80 pounds N either had net prices lower than the same activity at zero pounds N, or had negative net prices in which case they were unavailable for solution. This explains why no fertilizer was used at all for any of the cropping plans at the 80 per cent certainty level.

Limitations imposed by the yield data source accounted for most of the problems involving nitrogen application. The actual data revealed few instances of high fertilizer usage.[3] Consequently, in many of the grains selected for use in the model, the prediction equation in use was greatly affected by yield figures corresponding to high rates of N application which would not be assumed to be in themselves

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[3] This might be explained by the fact that individuals who fertilize their crops at high rates of N may not crop insure because they feel that there is no chance of yield falling below the insurable level.

realistic. This had a tremendous effect on locating the parameters of the expected quadratic production function. The parameters of the quadratic equation could not be estimated without large errors (Appendix Table A1). This meant selecting linear functions, with their inherent limitations and lack of correspondence to reality, in accordance with criteria previously presented on page 61.

Undoubtedly in years to come, crop insurance yield figures as a source of data will become more extensive and will allow a more realistic analysis of yield responses to fertilizers on different soils. Response equations in the form of quadratic, or Cobb-Douglas, or any curve linear type over the relevant yield-nitrogen levels are essential for improving the analysis of the effects of risk aversion on fertilizer use. Unquestionably an interdisciplinary approach, in particular between soil scientists and agricultural economists, concerning accurate, realistic yield response functions would serve an analysis such as this with very meaningful results.

It is noticeable, referring back to Tables 5.1a and 5.1b that no consistent increase or decrease of nitrogen use occurs horizontally across the Tables. Reviewing in detail the physical cropping plans as presented in detail in Appendix B, it is apparent that theplans corresponding to Table 5.1a, where capital is non restricting, were affected by the wheat activities grown on soil type C. The net prices or revenue per acre of the various wheat quota activities on

Soil C are presented in Table 5.2.

TABLE 5.2

NET REVENUES PER ACRE FOR QUOTA WHEAT ON SOIL C, NON-RESTRICTING CAPITAL, 20% SUMMERFALLOW, 25,000 BUSHELS AVAILABLE QUOTA AND 65% CERTAINTY

Activity	Net Revenues in Dollars
$QWFC_0$	22.43
$QWFC_1$	21.01
$QWFC_2$	22.33
$QWFC_3$	23.52
$QWFC_4$	24.56
$QWSC_0$	15.64
$QWSC_4$	9.68

Graphing these figures as shown in Figure 5.1 indicates that net revenue associated with the summerfallow activity $QWFC_4$ (Point A) was considerably higher than that of the stubble activity $QWSC_0$ (Point B). However, by the values used in this plan, activity $QWSC_0$ in turn has a higher net revenue than the same activity fertilized, $QWSC_4$ (Point C). Consequently in comparing this plan to one in which there was 40 per cent summerfallow it becomes obvious that along with increased summerfallow follows increased fretilization. This occurance rejects a part of the first hypothesis. When summerfallow is reduced and stubble land increased, the stubble activity requiring no fertilizer produces a larger net revenue than the same activity fertilized.

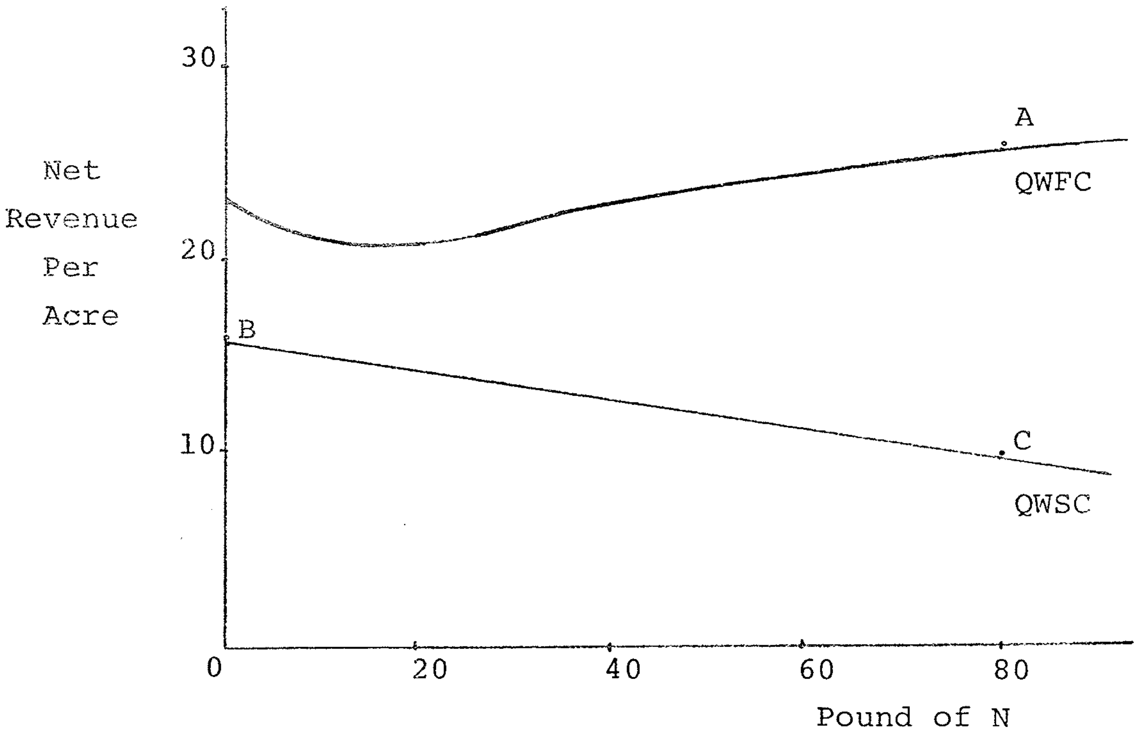

FIGURE 5.1

NET REVENUES PER ACRE FOR VARYING NITROGEN LEVELS AND ALTERNATIVE SUMMERFALLOW AND STUBBLE SITUATIONS[a]

[a] For $69,000 available capital, 25,000 bushels available quota and 65% Certainty.

This ironical trend of increased fertilizer use associated with increased summerfallow became less frequent as capital became more restrictive. Referring back to Table 5.1b, it is noticeable that once capital is reduced to

$30,000 that at the 50 per cent certainty level increased summerfallow percentage results in reduced nitrogen addition. This is explained by a general reduction in cropping acres of soils C and E due to capital restrictions, and specifically fewer acres of soil type B available for crop because of increased summerfallow specifications. As capital is reduced even further the 65 and 80 per cent certainty levels begin to demonstrate reduced nitrogen addition as summerfallow is forced into the plan.

Effects of Capital Levels and Quotas on the Use of N and Cropping Plans

Capital. Consistent with Tables 5.3a and 5.3b was the trend of decreasing fertilizer use at lower levels of capital for the 50 per cent and 65 per cent certainty levels. Once capital became restricting any further decrease in its availability caused a significant reduction in use of N, as more activities were forced into solution that required no nitrogen. The effect of diminishing capital on cropping plans resulted in a larger percentage of the cropping area being planted to wheat. In cases where capital was severely limiting not all of soil type C available for cropping was used and in many cases no E soil was sown at all.

Two major conslusions result from the analysis of the effects of varying capital on nitrogen use. First, it appears that relatively speaking, wheat responds less to nitrogen than does barley and oats, since wheat acreage percentage wise decreased as capital and nitrogen increased.[4] Secondly, when capital becomes resticting not only does nitrogen use decrease, but the soil types that do not yield well relative to type B are the first to be uneconomical and thus would not be seeded if these planning results were actually used. This second conclusion validates part of the hypothesis.

Quotas. The level of wheat quota had some effects on fertilizer use as shown in Tables 5.4a and 5.4b. Quotas had no effect on the use of N until capital dropped to the restricting level of $25,000. At this point only the 50 per cent certainty level made use of fertilizer activities and exhibited increasing use of N for most decreases in quota at all ratios of summerfallow. As quota activities, which tended to be wheat non-fertilized at these restricted capital levels, were reduced by quota limitations, they were replaced by activities yielding the next highest value of net profit. In all cases quota restrictions reduced quota wheat acreages, forcing acreage into the next best alternative, in most instances oats, both fertilized and non-fertilized depending on certainty level.

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[4] Observation of the different generated production functions makes this obvious.

TABLE 5.3a

TOTAL ADDED N FOR CROPPING PLANS CORRESPONDING TO ALTERNATIVE RISK - CAPITAL LEVELS[a]

	Capital Levels				
Certainty	$69,000	$35,000	$30,000	$25,000	$20,000
	(in actual pounds)				
50%	48,800	48,800	48,000	31,920	16,864
65%	48,000	48,000	8,288	-	-
80%	-	-	-	-	-

a/ For quota level equal to 25,000 bushels and summer-fallow percentage equal to 0%.

TABLE 5.3b

TOTAL ADDED N FOR CROPPING PLANS CORRESPONDING TO ALTERNATIVE RISK - CAPITAL LEVELS[a]

	Capital Levels				
Certainty	$69,000	$35,000	$30,000	$25,000	$20,000
	(in actual pounds)				
50%	101,280	74,096	29,280	23,880	4,800
65%	52,560	52,560	27,896	-	-
80%	-	-	-	-	-

a/ For quota level equal to 25,000 bushels and summer-fallow percentage equal to 40%.

TABLE 5.4a

TOTAL ADDED N FOR CROPPING PLANS CORRESPONDING TO ALTERNATIVE SUMMERFALLOW, QUOTA AND RISK LEVELS[a]/

	Summerfallow Levels							
	0%		20%		40%		Fixed As To Soil Type[b]	
	Quota Levels (000's bushels)							
Certainty	25	20	25	20	25	20	25	20
	(in actual thousands of pounds)							
50%	31.7	42.7	31.3	39.0	23.9	23.9	28.0	37.7
65%	-	-	-	-	-	-	-	-
80%	-	-	-	-	-	-	-	-

a/ For $25,000 available capital.
b/ 0% of Soil B, 25% of Soil C and 33.3% of Soil E.

TABLE 5.4b

TOTAL ADDED N FOR CROPPING PLANS CORRESPONDING TO ALTERNATIVE SUMMERFALLOW, QUOTA AND RISK LEVELS[a]/

	Summerfallow Levels							
	0%		20%		40%		Fixed As To Soil Type[b]	
	Quota Levels (000's bushels)							
Certainty	25	20	25	20	25	20	25	20
	(in actual thousands of pounds)							
50%	16.9	26.6	16.2	26.0	4.8	4.8	12.9	22.7
65%	-	-	-	-	-	-	-	-
80%	-	-	-	-	-	-	-	-

a/ For $20,000 available capital.
b/ 0% of Soil B, 25% of Soil C and 33.3% of Soil E.

Comparison of Target N Recommendations and Actual N Used

Unfortunately, at the time this study was completed, there was not a target yield recommendation service for the individual soils used as a base for this project. As the case farm did not soil test, it did not have physical target recommendations specific to its own soils, that could be compared to the economic target recommendations which would be made following the analysis presented in this study. This prevented testing the hypothesis that the difference between target N recommendations and actual N used increased as risk aversions increased.

Under the present soil testing program, a set of target yields are specified for a particular field given the actual level of nutrients in the soil and suggested added levels required to bring about alternative target yields. This predicting of yields is determined from a curve linear yield response to fertilizer (both nitrogen and phosphorous) function specific to various general soil classifications (clays, loams, sandy loams, etc.). These functions have been developed by the Soil Testing Laboratory, Soil Science Department, University of Manitoba, from test plot data, over a number of years on these various soil classifications. These target yields and levels of suggested fertilizer addition would help form the basis of any planning program. Necessary conditions for acceptance of a target yield and associated fertilizer level are that the production function from which the target yield was derived pertains or is similar to the field or soil of the farm manager who is using it, and that the farm manager is indifferent to risk believing he has a 50-50 chance of achieving the target level. However, if he wants to be more certain of achieving a certain target yield, he will discount the target yield he ultimately decides upon. In any event as shown in an earlier chapter, when certainty was increased, yields used in the planning model were reduced as was the level of nitrogen application. Consequently, the author feels that the hypothesis under test can not be rejected as it has been indirectly shown that the difference between actual N used and target N recommendations increased as certainty increased.

These findings can be of significance to a farmer planning on the basis of soil testing results. As stated before, a planner indifferent to risk will accept and plan with soil test values. However, introducing factual yield variation into a farm planning program and a wish to reduce the effects of yield uncertainty will result in a target yield being discounted by some subjective value. This thesis has attempted to exhibit a procedure, being methodologically sound, that demonstrates what could happen under one type of modern planning system. This procedure is <u>not</u> necessary, however, for farm managers to discount any yield given to them from any source. Whether yield variation is considered in a formal sense or not farmers often discount yields which they feel have not realistically taken into account all the consequences of undesirable weather conditions. Evidence of this is the fact that very few farmers budget with target yields as indicated by a soil test. They invariably use a lower yield determined by some historical or subjective rule of thumb discounting procedure.[5] Consequently it is the opinion of this author that target recommendations although a most important facet of farm planning, in most cases need to be altered by some factor related to yield variation. This will provide a more realistic approach to a farm planner, both in economic terms and in response to his personal risk attitude.

Effect of Risk Aversion on Land Resource Decision

At the highest level of risk aversion all land received zero amounts of N, and some land was left idle. This implies that if risk aversion is an important decision variable, then the manager essentially discounts returns more on land with high yield variability. This has very meaningful consequences on long run decisions of buying or selling or renting land.

Over a period of unproductive years a farmer will dispose of uneconomical land given no contractual restrictions. Similarly during or prior to years of favourable markets and prices, a farm unit with access to available labor and capital resources may desire to farm more land. Decisions regarding buying or selling or renting of land are subject to risk and uncertainty just as short run decisions of what to plant and at what level to fertilize. Using a risk aversion analysis

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[5] For example discount all yields by 1/3 or 1/4, etc.

such as presented above in conjunction with shadow prices[6] (the amount one can afford to pay for one more unit of a scarce resource - paying less would imply additional profit) as generated by the MPS 360 program can result in a beneficial aid to the problem of risk aversion concerning long run land resource decisions. The shadow prices will indicate to the farm manager how much he can afford to pay for more units of different kinds of land inputs in terms of prices or rents offered. In addition the same analysis regarding increasing risk aversion can be applied to shadow prices. It was expected that as certainty increased, shadow prices would correspondingly decrease. Shadow prices taken from the program results bear this out as indicated in Table 5.5.[7]

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[6] Shadow prices represent the marginal value products of the corresponding resources. The marginal value products are of interest since they indicate possible gains in income through acquistion of scarce resources. For example given the set of constraints, activities, and matrix coefficients used to generate a set of planning results, a positive value of 50 for the shadow price of a restricting land restraint would indicate that a one acre increase in the land restraint would add an extra $50 (less the annual costs) to income.

[7] In this instance only one example will be presented; however, all sets of plans indicate the same findings.

TABLE 5.5

MARGINAL VALUE PRODUCTS FOR SCARCE RESOURCES AT VARIOUS LEVELS OF CERTAINTY[a]/

	Certainty Levels		
Restraints	50%	65%	80%
		(in dollars)	
Stubble B	31.79	21.96	14.72
Sumfllw. B	39.27	27.06	24.30
B Fallow	- 11.26[b]	- 11.26	- 11.26
Stubble C	18.76	10.72	8.59
Sumfllw. C	24.67	16.67	13.81
C Fallow	- 11.06	- 11.06	- 11.06
Stubble E	20.70	9.41	7.02
Sumfllw. E	30.92	16.10	11.52
E Fallow	- 10.71	- 10.71	- 10.71
Total Lab. SS.	-	-	-
Spring Lab. SS.	-	-	-
Fall Lab. SS.	-	-	-
Operating Cap.	-	-	-
Wheat Quota	.25	.25	.25
Wheat Acreage	-	-	-
Barley Acreage	-	-	-
Oat Acreage	-	-	-
Flax Acreage	-	-	-

a/ For a summerfallow ratio of 40%; $69,000 available capital; and, 25,000 bushels quota.

b/ Shadow prices can not be negative unless resources are forced into the program which occurred in these cases.

As is evident in Table 5.5, shadow prices decreased as certainty increased. For instance, the shadow prices of

Stubble C associated with 50 per cent, 65 per cent and 80 per cent certainty were \$18.76, \$10.72 and \$8.59, respectively. This indicated that for the production period for which the assumptions of the model were formulated, one extra acre of Stubble C could return an income of the values listed above. A farmer could afford to rent extra acres of Stubble C up to these values. Therefore, if the manager was one who planned on 65 per cent certainty, and was able to rent additional acres of Stubble C at \$8.00 per acre, he could budget for additional profit of \$10.72 - \$8.00 = \$2.72 per acre. He would not consider paying rent of more than \$10.72. If the shadow price for a certain soil was zero, the manager would either not produce on that soil, dispose of it, or accept more risk.

The whole analysis of determining whether to buy land over a period of years must include the time horizon over which the decision applies. This is facilitated by use of a capitalization equation,

$$P.V. = \sum_{t=1}^{n} \frac{R}{(1 + r)^t}$$

which takes into account the present worth of all future revenues from the acre of land under consideration. For this formula R represents the estimated annual income for that acre (in this case the shadow price), r is the interest rate the manager would have to pay if he mortgaged the purchase, and t represents the mortgage period. Using an interest rate of 6 per cent for a period of 25 years, the present values for the land shadow prices as shown in Table 5.5 are calculated and presented in Table 5.6.

TABLE 5.6

PRESENT VALUES[a] FOR SCARCE LAND RESOURCES AT VARIOUS LEVELS OF CERTAINTY[b]

	Certainty Levels		
Restraints	50%	65%	80%
		(in dollars per acre)	
Stubble B	406	281	188
Sumfllw. B	502	346	311
B Fallow	144	144	144
Stubble C	240	137	110
Sumfllw. C	315	213	177
C Fallow	141	141	141
Stubble E	265	120	90
Sumfllw. E	395	206	147
E Fallow	137	137	137

a/ Capitalized at 6 per cent over 25 years.
b/ For summerfallow ratio of 40 per cent; $69,000 available capital; and, 25,000 bushels quota.

Comparing the present values to the market price of land in this district, a farm manager can determine whether or not to buy land. If the present value as determined by the capitalization procedure is greater than the market price in the area he may well decide to purchase additional land.

His next step will be to decide which soil type to purchase. At the 50 per cent certainty level he would select the soil type exhibiting the largest net present value. If he wished to purchase on the basis of 65 per cent certainty of achieving some plan, he would select the land type having the largest net present value in this case.

The procedures for renting additional land or selling land would be based on the same reasoning.

Effect of Summerfallow on Profit and Cropping Plans

Summerfallow, in this analysis, proved not to be an economic means of supplying plant nutrients under the assumptions of the model. It was hypothesized that summerfallow would replace fertilizer as a means of supplying plant nutrients when (a) the difference between the target fertilizer recommendation and actual amount used increased; (b) the level of capital declined, and (c) the more severely crop production was limited by quotas.

The model assumed four different levels of the summerfallow variable. It was felt that if summerfallow was considered as an endogenous variable available for solution within the model it would be necessary to introduce alternative five year cropping rotations. This would be necessary to allocate the benefits of summerfallowing among the various years.[8] Although this procedure would result in a more

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[8] For example, 2/3 of the expected increased yield over stubble might accrue to the first year after summerfallow, 1/4 to the second year, and 1/12 to the third year.

realistic analysis and a more sufficient test of the minor hypothesis, time limitations and a lack of agreement as to the allocation of the benefits of summerfallowing accruing to following years' crops caused the author to abandon the idea of letting the model decide the optimal level of summerfallow. Instead, the variable was treated as exogenous and fixed at the levels indicated in Chapter 4. This assumption placed limitations on the results as it was impossible to draw conclusions regarding summerfallow as a competitive alternative to nitrogen.

Presented below in Tables 5.7a and 5.7b, however, are results which indicate the effect of summerfallow on net farm profit. They indicate that as more summerfallow was forced into solution, a decrease in total acres cropped was directly related to a decrease in net revenue from the farm plan as a whole when capital was an effective constraint. Although more revenue was derived from grain grown on summerfallow, the increase did not match the decrease in revenue from fewer acres being cropped as the penalty for forcing in summerfallow often amounted up to $8,000 and higher.

Cropping plans and corresponding net incomes are given in Appendix B. They point out the interrelationships of capital levels, quotas, and risk aversion on profit for given summerfallowing practises as discussed below.

<u>Capital Levels</u>. Capital had no effect on net farm income at the first level of $69,000. It became restricting at a capital level of $35,000 for the 50 per cent certainty

TABLE 5.7a

NET FARM INCOME FOR PLANS CORRESPONDING TO ALTERNATIVE RISK - SUMMERFALLOW LEVELS[a]

Certainty	Summerfallow Levels			
	0%	20%	40%	Fixed As To Soil Type[b]
	(in dollars)			
50%	39,048	31,808	24,564	33,925
65%	31,347	25,626	18,190	27,045
80%	24,954	20,198	13,949	21,412

a/ For $30,000 available capital and 25,000 bushels available quota.
b/ 0% for Soil B, 25% for Soil C and 33.3% for Soil E.

TABLE 5.7b

NET FARM INCOME FOR PLANS CORRESPONDING TO ALTERNATIVE RISK - SUMMERFALLOW LEVELS[a]

Certainty	Summerfallow Levels			
	0%	20%	40%	Fixed As To Soil Type[b]
	(in dollars)			
50%	28,104	21,057	11,017	22,703
65%	23,613	17,326	10,160	18,610
80%	19,913	14,215	7,314	15,245

a/ For $20,000 available capital and 20,000 bushels available quota.
b/ 0% for Soil B, 25% for Soil C and 33.3% for Soil E.

level; at the $30,000 capital figure for 65 per cent certainty; and, at $25,000 for the 80 per cent certainty level. From this observation, it appears that as certainty increases less capital is required to generate a corresponding net farm income value. This bears mentioning as it was previously stated that the level of equity, debt position, and availability of operating capital have a bearing on farm plans. Farmer attitude toward certainty is related to capital availability.

Quota Levels. Quota levels had only minor effects upon net income as the quota concept was developed in this model. At the 50 per cent certainty level decreasing quota from 25,000 to 20,000 bushels caused a reduction of net income in the range of $1150 - $1250 at all summerfallow ratios at the first four capital levels.[9]

As capital became more restricting ($20,000 level) a reduction in the quota restraint caused a decrease in net income of $800 - $900 in most cases. This reduction differed from the $1150 - $1250 result in the former case as the new capital restriction prevented some of the available acreage from being employed. This caused fewer acres to be sown to wheat, therefore, fewer bushels to be sold at a $.25 difference per bushel.

At the 65 per cent certainty level, decreasing quota from 25,000 to 20,000 bushels resulted in a reduction of net

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[9] This is explained by 5,000 bushels being sold off quota at $.25 per bushel less or $1,250

income in the range of \$1150 - \$1250 at all summerfallow ratios and at all capital levels.

At the 80 per cent certainty level decreasing quota caused net income to reduce between \$600 - \$1250. Restricting capital levels at the 0 per cent summerfallow stage placed the income difference in the lower end of the above range. This was due to only 60 per cent of the now limited acreage being sown to wheat--the remainder to oats. Increasing the summerfallow requirement allowed more acreage to be sown into wheat, and thus the difference approached the upper part of the range i.e. the \$1250 difference.

Risk Aversion. The level of risk aversion had major effects upon net income of plans generated at 50, 65 and 80 per cent certainty in this model. Regardless of capital level or quota limitation, as indicated in Tables 5.7a and 5.7b net income decreased as net prices $\bar{C}_j$ were subsequently discounted by $Z_j\sigma$. As the whole model was constructed on the concept that increased certainty necessarily resulted in decreased income, the result could not be different.

HYPOTHESIS NO. 2

The second hypothesis as developed read:

> It is hypothesized that fixed debt obligations and personal withdrawals of income limit the degree to which risk acceptance can be reduced. It is implied that some risk must be accepted if net farm income is to be large enough to cover these obligations and personal withdrawals of income. Accordingly, this places bounds on the number of feasible cropping plans available to a risk averter.

As described earlier, the solution vectors (activity acreages) for the different certainty levels formed the basis of testing the hypothesis that some risk must be accepted if net farm income is to be large enough to cover fixed debt obligations and personal withdrawals of income. Model 2.4 provided the mechanism for making trade-offs between expected income and risk, given some expected income solution, a specified or survival level of income, and the variance of the cropping solution. It was hypothesized that plans originally constructed on the basis of increased certainty would in fewer cases exceed the specified level of income. This is apparent as presented in the results of Model 2.4 found in Appendix E. Appendix Table E3 (based on 80 per cent certainty) exhibits fourteen plans each having a probability of less than 50 per cent of exceeding $25,000, whereas Table E2 (based on 65 per cent certainty) has only ten.

Three tables listed as 5.8a, 5.8b and 5.8c are presented below to show the effect of risk aversion, summerfallow ratio, and capital upon the probabilities as specified by Model 2.4. It is essential to note that only those plans having a probability of greater than or equal to 50 per cent would be considered as feasible in the terms of a rational farm planner as described in this study. Although it was found that some plans were infeasible at the 80 per cent certainty level, it occurred that the remaining feasible plans at this level had the highest probability of occurring. This resulted from a lower net income variance of the cropping plan as a whole compared to the net income variance of the cropping plans of the lower certainty levels.

TABLE 5.8a

PROBABILITY OF EXPECTED INCOME FROM CROPPING PLANS, ORIGINALLY FORMULATED AT 50% CERTAINTY, EXCEEDING $25,000[a]

	Summerfallow Levels			
Certainty	0%	20%	40%	Fixed As To Soil Type[b]
	(in percent)			
50%	73	68	56	68
65%	73	68	53	68
80%	77	68	40	70

a/ For $69,000 available capital and 25,000 bushels available quota.

b/ 0% for Soil B, 25% for Soil C and 33.3% for Soil E.

TABLE 5.8b

PROBABILITY OF EXPECTED INCOME FROM CROPPING PLANS, ORIGINALLY FORMULATED AT 65% CERTAINTY, EXCEEDING $25,000[a]

	Summerfallow Levels			
Certainty	0%	20%	40%	Fixed As To Soil Type[b]
	(in percent)			
50%	68	62	46	62
65%	79	64	45	68
80%	77	64	33	70

a/ For $30,000 available capital and 20,000 bushels available quota.

b/ 0% for Soil B, 25% for Soil C and 33.3% for Soil E.

TABLE 5.8c

PROBABILITY OF EXPECTED INCOME FROM CROPPING PLANS, ORIGINALLY FORMULATED AT 80% CERTAINTY, EXCEEDING $25,000[a]

	Summerfallow Levels			
Certainty	0%	20%	40%	Fixed As To Soil Type[b]
50%	60	39	16	45
65%	61	38	10	45
80%	61	38	11	45

a/ For $20,000 available capital and 25,000 bushels available quota.
b/ 0% for Soil B, 25% for Soil C and 33.3% for Soil E.

The standard deviations of net income for the various plans are lower for the plans based on 80 per cent certainty as indicated by Table 5.9. (All values shown in Appendix D). As these values were entered into the denominator of Model 2.4, it is understandable why probabilities of plans at the 80 per cent certainty level would in cases where the expected income just surpassed the specified income level show a very large probability of surpassing that specified level. Since the original plan had a very small variance of net income, chances of the expected income occurring were very high.

Summerfallow ratios had major affects upon the cropping plan probabilities. Tables 5.8a, 5.8b and 5.8c all indicate that increased summerfallow resulted in reduced probability. Although the variances of the plans decreased (in most cases) when summerfallow was decreased, the change

TABLE 5.9

STANDARD DEVIATION FOR CROPPING PLANS CORRESPONDING TO ALTERNATIVE RISK - SUMMERFALLOW LEVELS[a]

	Summerfallow Levels			
Certainty	0%	20%	40%	Fixed As To Soil Type[b]
	(in dollars)			
50%	28,687	21,902	24,220	26,657
65%	28,829	22,917	30,169	28,685
80%	14,452	10,546	11,858	12,688

a/ For $35,000 available capital and 25,000 bushels available quota.

b/ 0% for Soil B, 25% for Soil C and 33.3% for Soil E.

was proportionally less than the decrease in expected net income. Hence, probabilities also decreased. It was obvious that the lowest probabilities always occurred at the 40 per cent summerfallow ratio regardless of capital or certainty levels. These corresponding plans had a relatively large risk attached to them considering their generated income was too low. At this ratio too little land was cropped to enable income to exceed $25,000 when capital was restricting. Although summerfallow did not always replace fertilizer (Hypothesis No. 1), it is evident now that summerfallow is not consistent with risk reduction.

Capital, as seen by comparing Tables 5.8a, 5.8b and 5.8c with one another, had major effects upon the probabilities of the various plans. In almost all cases a further restriction in capital caused a further reduction in the probability of any plan exceeding $25,000. Again in this instance, the decrease in risk of the plan brought about by the decrease in total acres cropped was not compensated by a less relative decrease in expected income. Therefore the probabilities decreased as capital declined.

The E-V Frontier

The major finding of this study centers around the concept described earlier as the E-V frontier - the locus of expected income - variance combinations. The first part of this model, specified by equations 2.1, 2.2 and 2.3, generated plans X_j under alternative relative net prices for given levels of exogenous variables. The expected income of these plans were found and entered into Equation 2.4 to determine the probability of exceeding a specified level of income given the variation of the plan as a whole. Taking the net income variation of the cropping plans (in this case the standard deviation) and the corresponding expected incomes as generated for equation 2.4 and graphing them leads to a major conclusion and to a deeper understanding as to how the two parts of the model are related.

Figures 5.2a and 5.2b each exhibit three combinations of standard deviation (σ) and expected income $E(t_j)$ as determined for employment into 2.4. Each figure demonstrates an estimated $E \cdots (V)^{\frac{1}{2}}$ space with three locations, one corresponding to the three previously mentioned certainty levels.[10]

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[10] This provides only an estimate of the E-V frontier. To test for the true frontier, one would need to do quadratic programming as Driver has done (page 32).

Figure 5.2a demonstrates a case where capital is not limiting. It is significant by this example that plans resulting from discounted net prices have nearly the same $E(t_j)$ as plans from non-discounted net prices. The plan corresponding to 50 per cent certainty has $E(t_j)$ and σ of \$37,075 and \$26,038 respectively; for 65 per cent - \$35,592 and \$22,917; and, for 80 per cent - \$29,858 and \$10,546. This suggests that a farm planner selecting plans on the basis of these results could reduce his standard deviation (risk) \$3,121 by giving up only \$1,483 expected income if he considered the 65 per cent certainty level. This becomes even more explicit when it is realized that by using a plan formulated at 80 per cent certainty and, subjected to average prices, would result in a decrease in $E(t_j)$ of \$7,217, but a large decrease in σ of \$15,492 as compared to the 50 per cent plan. This suggests that a plan chosen strictly because it exhibits the highest $E(t_j)$ may in fact not be the "best" plan. Taking risk into account may result in a plan with a marginally smaller expected income but with a greatly reduced variation of occurance, i.e. risk attached to it.

Figure 5.2b indicates that as capital becomes more restricting (compared to Figure 5.2a), the $E\cdots(V)^{\frac{1}{2}}$ combinations in relation to each other begin to become more horizontal. This suggests that limiting capital in the original development of the plans caused a plan regardless at what certainty level it had been formulated to seek those activities demonstrating the highest net return. The variations around the plans were not significantly different (particularly when cap-

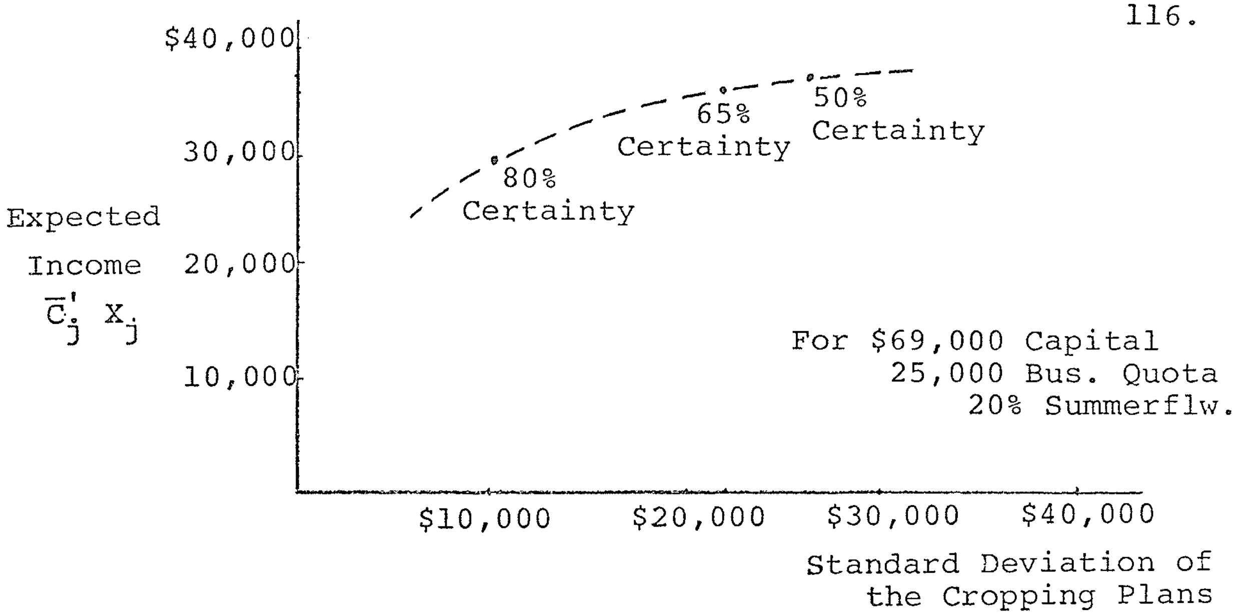

FIGURE 5.2a

AN $E \cdots (V)^{\frac{1}{2}}$ SET OF ESTIMATED POINT COMBINATIONS BASED ON PLANS DETERMINED FROM ALTERNATIVE RISK LEVELS

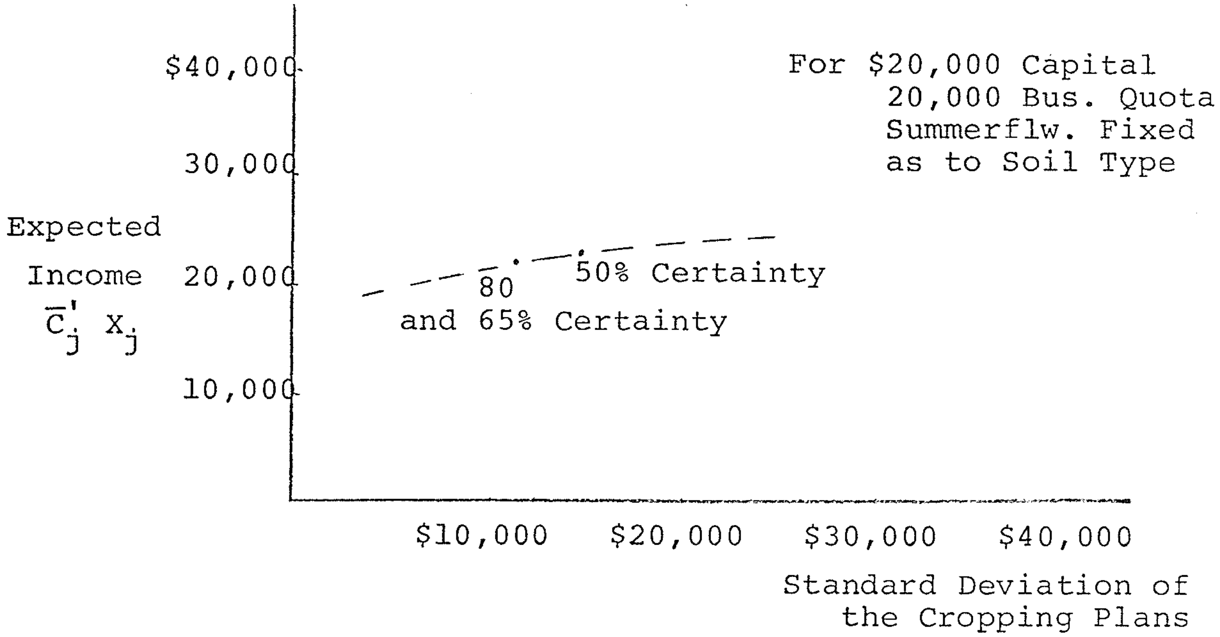

FIGURE 5.2b

AN $E \cdots (V)^{\frac{1}{2}}$ SET OF ESTIMATED POINT COMBINATIONS BASED ON PLANS DETERMINED FROM ALTERNATIVE RISK LEVELS ital was \$20,000) as there was simply not enough capital to bring those activities having large yield variations into solution.

The significance of the description above is twofold. Firstly, as related to this project, it provides the binding between equations 2.1, 2.2 and 2.3, and equation 2.4. It does this by showing the relationship between expected income and risk of plans originally formulated under conditions of discounted net prices as associated with yield variation. Secondly, as related to farm management decision tools in general, the $E \cdots (V)^{\frac{1}{2}}$ results as graphed in Figures 5.2a and 5.2b indicate that when risk is considered, the results eminating from a conventional linear programming model may not be the "best" solution. Although the plans may yield the highest expected income, this analysis has demonstrated that under certain circumstances a small sacrifice in expected income can be associated with a large reduction in variance (risk). This has very major consequences upon farm managers who want to and who must plan on the basis of plans developed with risk-income trade-offs in mind.

Chapter 6

SUMMARY AND INFERENCES FOR FURTHER STUDY

It was the objective of this study to identify and evaluate the effects of soil type, summerfallow, labour, capital and quota restrictions and risk aversion on crop planning and fertilizer use. This lead to specific objectives of determining discounted average yields for wheat, oats, barley and flax at different fertilizer rates on different soil types in accordance with selected levels of risk aversion. It also led to determining optimal crop plans and fertilizer use for alternative levels of the exogenous variables, and of evaluating the effects of soil type, summerfallow, capital, labour, quota restrictions and risk aversion on the probability of net farm income lying above a specified level.

A conceptual framework was developed for achieving these objectives. Hypotheses were formulated (page 8) and analytical as well as empirical procedures were developed for testing these hypotheses.

An underlying premise in the development of the conceptual framework was that farm managers be considered either as risk averters or those who were indifferent to risk since it was felt that very few managers would plan on the basis of excessively high yields at low odds. For a given case farm, a set of optional grain activities were specified and translated into a planning model made operational to cover a five year planning period.

For this given case farm, resources for producing products were translated into a set of constrained variables or activities (Table 3.1). The model was developed in two parts. The first part maximized net farm income subject to linear constraints given varying levels of net prices as determined by different levels of risk aversion as attached to uncertain yield values. Expected net income was expected to be inversely related to risk aversion. The degree of risk aversion was arbitrary selected at the levels described earlier (Table 4.2). The second part of the model determined the probability of net farm income for a particular plan exceeding some specified income level. This probability was calculated under the statistical theorem that linear combinations of independent random variables which are normally distributed, are also normally distributed. The resulting distribution was standardized and corresponding probabilities calculated.

It was assumed that alternative levels of exogenous variables, within the first part of the model, had no affect on the input-output relationship where specified over the duration of the model. The planning model did not handle payments to resource owners either outside or inside the case farm in terms of opportunity costs for land, capital or labour.

Data required by the model was obtained through:

(1) personal interview with the case farm manager,

(2) Manitoba Crop Insurance Corporation yield - fertilizer values, and

(3) farm management planning guides.

The premise underlying the processing of data inputs required by the model involved the development of production and variance functions consistent with normal yield distributions of all grains on all three soil types. This required regression analysis for yield prediction beyond the range of sufficient available yield data.

Input-output coefficients required by the model were determined from special studies, farm management guides and case farm procedures where applicable.

The significant findings of the study are summerized in terms of hypotheses formulated for achieving objectives.

It was concluded that as risk aversion increased the amount of N used decreased. It was evident that as net prices were discounted by successively larger risk values in conjunction with the yield variation of the corresponding grain activity, that fewer activities based on nitrogen came into solution. This is explained by a larger variance of yield being associated with a higher yield. Therefore, the new discounted net price of an activity at the 80 pounds N level was discounted at a relatively higher degree than the associated net price of an activity at 0 pounds N. Nitrogen levels between these two values were not presented for solution as linear production functions and their unreality ruled intermediate values out.

It was evident that as capital restrictions increased a larger percentage of the farm plans required no nitrogen and was sown to wheat. This suggested that relatively speaking, wheat responds less to nitrogen than do barley and oats. As hypothesized, high risk aversion in association with restricting capital resulted in land associated with high yield variability not being competitive, and therefore not entering optimal solutions.

The level of wheat quota had only minor effects upon fertilizer use when capital was severely limited. In all cases quota restrictions reduced quota wheat acreages, forcing acreages into the next best alternative, in most cases oats.

It was expected that the difference between target N recommendations and actual N used would increase as risk aversion increased. This could not be shown directly, but as was evident by the fact that when certainty increased, yields and nitrogen levels used in the planning model decreased, the author feels that indirectly the hypothesis has been verified.

Summerfallow in this analysis proved not to be an economic means of supplying plant nutrients under the assumption of the model. As indicated in the results, as more summerfallow was forced into solution, a decrease in total acres cropped was not compensated by an equivalent value increase in net income from wheat grown on summerfallow. Quota level changes had only minor effects upon net income.

The second major hypothesis implied that some risk must be accepted if net farm income is to be large enough to cover fixed debt obligations and personal withdrawals of income. Results verified this reasoning as it was evident that more plans formulated under 80 per cent certainty fell below a probability of 50 per cent of exceeding the survival of $25,000 than plans formulated under 50 per cent certainty. However, of special note, it was observed that when plans based on increased certainty had expected incomes larger than $25,000, the probability of achieving the expected income was often higher than the probability of expected income surpassing $25,000 when the corresponding plan was originally formulated at 50 per cent certainty. This was due to a lower variation of net income as concerning the plan as a whole. Capital restrictions resulted in decreased probabilities as did an increase in the specified ratio of summerfallow.

The most noteworthy finding of this study dealt with the E - V frontier. As demonstrated in Chapter 5, the plan exhibiting the highest expected net income need not necessarily be the best plan. Building yield variation into the certainty model and then comparing farm plans with respect to two criteria, expected income and risk, allowed a more meaningful analysis of future production plans. As shown, taking risk into account may result in a plan with a marginally smaller expected income but with a greatly reduced variation of occurrence.

There are numerous areas of this study where more refinement in the sense of data input would prove to be beneficial. The basic source of yield data proved to be reliable for low yields only. A source exhibiting numerous realistic high yields could result in better detailed results of the analysis. A better situation would be a population of yields specific to the farm planning unit.

A more detailed model considering concepts such as crop insurance, phosphorus fertilization, and in particular a wider array of available crops would be most useful.

In any event there are many areas to which the concept of building risk into a farm planning model may lead. There have been in the past a sufficient number of empirical studies to show that decision makers adopt different management practices because they have different attitudes toward risk, and that these attitudes can be measured and compared. This study has not attempted to measure an individual's attitude in utility terms, but has strived to demonstrate a technique of formulating alternative sets of plans based on different predetermined values of risk aversion. It would be expected that given sets of plans based on an individual's own productive situation, he would select the one that corresponds most closely to his attitudes toward risk. In this way the problem of defining the actual risk aversion value is circumvented although risk has still been taken into account.

In conclusion the author issues a final warning. If decision makers use the concept of planning formulated in this thesis, all of their decisions will be "good" in the sense of maximizing expected income. Unfortunately, uncertainty still exists, and a carefully reasoned farm manager might still have a bad outcome in any particular instance. This procedure does not guarantee good outcomes - hopefully, just good decisions!

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APPENDIX A

REGRESSION RESULTS FOR YIELD, VARIANCE AND VARIANCE/n PREDICTIONS

The linear and quadratic forms of the equation tested were respectively:

$Y = a + b_1x$, and

$Y = a + b_1x + b_2x$

where,

Y = mean yield prediction (Table A1), the mean variance prediction (Table A2), and the mean variance/n prediction (Table A3) based on all the sample yields and sample variances respectively,

a = a constant,

b_1= the first regression coefficient associated with nitrogen,

b_2= the second regression coefficient associated with nitrogen, and

x = the mean nitrogen levels of the samples used.

For Tables A1, A2 and A3 the T value(s) associated with the b_i values are:

\+ significant at 10 per cent

* significant at 5 per cent

TABLE A1

REGRESSION RESULTS FOR YIELD PREDICTION

Grain-Soil Class	Equation Tested	Constant	b Value(s)	T Value(s)	R^2 Value(s)
WFB	linear	35.15	.1828	3.313*	.1109
	quadratic	33.96	.7198	4.701*	.2332
			-.0246	3.724*	
WSB	linear	27.85	.1029	5.390*	.1641
	quadratic	28.50	.0011	.019	.1825
			.0021	1.822+	
OSB	linear	55.71	.4347	10.587*	.4309
	quadratic	56.65	.3057	2.506*	.4358
			.0024	1.124	
TBS	linear	36.42	.2019	5.212*	.1551
	quadratic	36.34	.2132	1.846+	.1551
			-.0002	.105	
XSB	linear	11.94	.1112	5.865*	.3723
	quadratic	12.02	.0662	1.199	.3805
			.0014	.869	
WFC	linear	27.48	-.0075	1.510	.0253
	quadratic	26.23	.1607	2.554	.0996
			-.0002	2.681	
WSC	linear	21.34	.0713	3.981*	.0967
	quadratic	21.01	.1227	2.131*	.1021
			-.0010	.939	
OSC	linear	43.19	.3164	6.535*	.2238
	quadratic	40.50	.7473	5.264*	.2750
			-.0093	3.216*	
TSC	linear	28.63	.1090	2.457*	.0392
	quadratic	27.06	.3476	2.423*	.0587
			-.0046	1.748+	
XSC	linear	9.79	.0568	2.433*	.0926
	quadratic	9.81	.0379	.402	.0933
			.0007	.207	
WFE	linear	24.33	.2290	4.118*	.1616
	quadratic	23.58	.6308	3.664*	.2160
			-.0217	2.458*	
WSE	linear	18.47	.0736	3.695*	.0845
	quadratic	18.44	.0789	1.254	.0845
			-.0001	.088	
OSE	linear	40.52	.3829	9.879*	.3974
	quadratic	41.25	.2562	2.130*	.4024
			.0029	1.113	
TSE	linear	21.77	.1852	5.466*	.1680
	quadratic	21.49	.2327	2.228*	.1693
			-.0010	.481	
XSE	linear	7.66	.0992	3.911*	.2087
	quadratic	8.06	-.1542	2.241*	.3754
			.0093	3.901*	

TABLE A2

REGRESSION RESULTS FOR VARIANCE PREDICTION

Grain-Soil Class	Equation Tested	Constant	b Value(s)	T Value(s)	R^2 Value(s)
WFB	linear	172.38	32.3416	6.481*	.3231
	quadratic	110.44	60.1904	4.131*	.3537
			-1.2766	2.030*	
WSB	linear	207.42	8.0938	6.065*	.1991
	quadratic	412.28	-24.1823	7.854*	.5632
			.6532	11.070*	
OSB	linear	1716.63	66.6914	5.914*	.1912
	quadratic	3090.37	-122.4580	4.183*	.3873
			3.4423	6.860*	
TSB	linear	912.91	31.2866	3.709*	.0850
	quadratic	1642.75	-74.9234	3.205*	.2101
			2.0333	4.835*	
XSB	linear	34.73	8.5324	8.464*	.5526
	quadratic	40.28	5.6988	1.947+	.5608
			.0852	1.031	
WFC	linear	234.71	.3260	1.429	.0227
	quadratic	144.72	12.4031	4.578*	.2053
			-.0132	4.471*	
WSC	linear	67.31	10.3867	10.893*	.4450
	quadratic	150.91	-2.4943	.871	.5184
			.2534	4.733*	
OSC	linear	-98.67	102.2424	9.994*	.4005
	quadratic	365.05	27.9641	.917	.4265
			1.5991	2.580*	
TSC	linear	446.98	23.5466	5.646*	.1772
	quadratic	554.53	7.2769	.537	.1860
			.3119	1.262	
XSC	linear	31.15	2.7161	3.385*	.1650
	quadratic	30.61	3.2140	.992	.1653
			-.0177	.159	
WFE	linear	145.10	28.1764	7.659*	.4000
	quadratic	149.18	26.0164	2.209*	.4002
			.1166	.193	
WSE	linear	140.17	7.1181	7.014*	.2495
	quadratic	140.60	7.0427	2.199*	.2495
			.0017	.025	
OSE	linear	-50.49	109.2803	16.089*	.6362
	quadratic	734.40	-28.1241	1.611	.7526
			3.1741	8.315*	
TSE	linear	300.31	9.8908	4.855*	.1374
	quadratic	349.56	1.6116	.258	.1488
			.1780	1.402	
XSE	linear	15.62	4.8893	7.048*	.4613
	quadratic	29.08	-3.6008	2.055*	.6316
			.3125	5.133*	

TABLE A3

REGRESSION RESULTS FOR VARIANCE/n PREDICTION

Grain-Soil Class	Equation Tested	Constant	b Value(s)	T Value(s)	R^2 Value(s)
WFB	linear	5.71	.3304	8.028*	.4228
	quadratic	5.28	.5245	4.336*	.4414
			-.0089	1.704+	
WSB	linear	6.06	.1073	6.489*	.2215
	quadratic	8.73	-.3136	8.608*	.6132
			.0085	12.202*	
OSB	linear	16.53	.3165	6.077*	.1997
	quadratic	22.77	-.5420	3.987*	.3872
			.0156	6.706*	
TSB	linear	10.79	.2216	4.647*	.1273
	quadratic	15.54	-.4691	3.645*	.2852
			-.0132	5.698*	
XSB	linear	2.69	.1511	10.182*	.6412
	quadratic	2.69	.1518	3.492*	.6412
			.0000	.018	
WFC	linear	6.27	.0045	1.529	.0259
	quadratic	5.15	.1536	4.327*	.1909
			-.0002	4.211*	
WSC	linear	4.27	.1213	11.008*	.4502
	quadratic	5.25	-.0294	.890	.5247
			.0030	4.801*	
OSC	linear	7.60	.4625	12.412*	.5100
	quadratic	9.25	.1981	1.791*	.5305
			.0060	2.532*	
TSC	linear	9.15	.1737	6.364*	.2149
	quadratic	9.64	.0994	1.118	.2190
			.0014	.878	
XSC	linear	2.28	.0654	4.095*	.2242
	quadratic	2.26	.0875	1.358	.2259
			.0008	.354	
WFE	linear	5.38	.3127	8.133*	.4291
	quadratic	5.50	.2470	2.010*	.4312
			.0035	.563	
WSE	linear	4.99	.1122	8.885*	.3478
	quadratic	5.04	.1037	2.604*	.3481
			.0002	.224	
OSE	linear	8.22	.5001	15.516*	.6193
	quadratic	11.42	-.0592	.674	.7089
			.0129	6.726*	
TSE	linear	7.13	.1310	6.661*	.2306
	quadratic	7.74	.0291	.484	.2471
			.0022	1.795+	
XSE	linear	1.93	.1127	7.508*	.4929
	quadratic	2.16	-.0307	.744	.5905
			.0053	3.686*	

APPENDIX B

CROPPING RESULTS

TABLE B1

CROPPING RESULTS FOR 50% CERTAINTY, $69,000 CAPITAL

	Summerfallow Percentages							
	0%		20%		40%		Mixed Percentages	
Quota levels '000 bus.→	20	25	20	25	20	25	20	25
t_j -qta.=20	$42,659		$37,075		$29,960		$39,066	
t_j -qta.=25		$41,411		$35,826		$28,709		$37,817
Activities				In Acres				
$QWFB_0$								
$QWFB_4$			92	92				
$QWSB_0$								
$QWSB_4$								
WFB_0								
WFB_4					184	184		
WSB_0								
WSB_4								
OSB_0								
OSB_4	450	450	276	276	92	92	460	460
TSB_0								
TSB_4	10	10						
XFB_0								
XSB_0								
XSB_2								
B-smf			92	92	184	184		
$QWFC_0$								
$QWFC_1$								
$QWFC_2$								
$QWFC_3$								
$QWFC_4$			300	300	600	527	300	300
$QWSC_0$	1172	937	364	129			538	304
$QWSC_4$								
WFC_0								
WFC_1								
WFC_2								
WFC_3								
WFC_4						73		
WSC_0	28	263	302	537			272	506
WSC_4								
OSC_0								
OSC_4			234	234	300	300	90	90
TSC_0	300	300						
TSC_4								
XFC_0								
XSC_0								
XSC_2								
C-smf			300	300	600	600	300	300
$QWFE_0$								
$QWFE_4$			30	30	53	60	50	50
$QWSE_0$								
$QWSE_4$								
WFE_0								
WFE_4					7			
WSE_0								
WSE_4								
OSE_0								
OSE_4	150	150	90	90	30	30	50	50
TSE_0								
TSE_4								
XFE_0								
XSE_0								
XSE_2								
E-smf			30	30	60	60	50	50

TABLE B2

CROPPING RESULTS FOR 50% CERTAINTY, $35,000 CAPITAL

	Summerfallow Percentages						Mixed Percentages	
	0%		20%		40%			
Quota levels '000 bus. →	20	25	20	25	20	25	20	25
$t_j(X_j)$-qta.=20	$42,659		$35,797		$27,995		$38,074	
$t_j(X_j)$-qta.=25		$41,411		$34,548		$26,745		$36,825
Activities				In Acres				
$QWFB_0$								
$QWFB_4$			92	92	74			
$QWSB_0$								
$QWSB_4$								
WFB_0								
WFB_4					110	184		
WSB_0								
WSB_4								
OSB_0								
OSB_4			276	276	92	92	460	460
TSB_0								
TSB_4								
XFB_0								
XSB_0	no change in either plan from $69,000 available capital at 50% certainty							
XSB_2								
B-smf			92	92	184	184		
$QWFC_0$			221	221	340	289	172	172
$QWFC_1$								
$QWFC_2$								
$QWFC_3$								
$QWFC_4$			79	79	260	260	128	128
$QWSC_0$			485	251			632	398
$QWSC_4$								
WFC_0						51		
WFC_1								
WFC_2								
WFC_3								
WFC_4								
WSC_0			181	415			178	412
WSC_4								
OSC_0								
OSC_4			234	234	300	300	90	90
TSC_0								
TSC_4								
XFC_0								
XSC_0								
XSC_2								
C-smf			300	300	600	600	300	300
$QWFE_0$								
$QWFE_4$			30	30	60	60	50	50
$QWSE_0$								
$QWSE_4$								
WFE_0								
WFE_4								
WSE_0								
WSE_4								
OSE_0								
OSE_4								
TSE_0								
TSE_4								
XFE_0								
XSE_0								
XSE_2								
E-smf			30	30	60	60	50	50

TABLE B3

CROPPING RESULTS FOR 50% CERTAINTY, $30,000 CAPITAL

	Summerfallow Percentages							
	0%		20%		40%		Mixed Percentages	
Quota levels '000 bus.→	20	25	20	25	20	25	20	25
$t_j(X_j)$-qta.=20	$39,048		$31,808		$24,564		$33,926	
$t_j(X_j)$-qta.=25		$37,799		$30,559		$23,314		$32,683
Activities				In Acres				
QWFB$_0$								
QWFB$_4$			92	92	135	34		
QWSB$_0$								
QWSB$_4$								
WFB$_0$								
WFB$_4$					49	150		
WSB$_0$								
WSB$_4$								
OSB$_0$								
OSB$_4$	460	460	276	276	92	92	460	460
TSB$_0$								
TSB$_4$								
XFB$_0$								
XSB$_0$								
XSB$_2$								
B-smf			92	92	184	184		
QWFC$_0$			300	300	600	600	300	300
QWFC$_1$								
QWFC$_2$								
QWFC$_3$								
QWFC$_4$								
QWSC$_0$	1172	937	528	478			703	469
QWSC$_4$								
WFC$_0$								
WFC$_1$								
WFC$_2$								
WFC$_3$								
WFC$_4$								
WSC$_0$	21	256	205	255	289	289		208
WSC$_4$								
OSC$_0$								
OSC$_4$								
TSC$_0$								
TSC$_4$								
XFC$_0$								
KSC$_0$								
XSC$_2$								
C-smf			300	300	600	600	300	300
QWFE$_0$								
QWFE$_4$			30	30	60	60	50	50
QWSE$_0$								
QWSE$_4$								
WFE$_0$								
WFE$_4$								
WSE$_0$								
WSE$_4$								
OSE$_0$								
OSE$_4$	140	140	90	90	30	30	50	50
TSE$_0$								
TSE$_4$								
XFE$_0$								
XSE$_0$								
XSE$_2$								
E-smf			30	30	60	60	50	50

TABLE B4

CROPPING RESULTS FOR 50% CERTAINTY, $25,000 CAPITAL

	Summerfallow Percentages							
	0%		20%		40%		Mixed Percentages	
Quota levels '000 bus.→	20	25	20	25	20	25	20	25
$t_j(X_j)$-qta.=20	$34,181		$27,134		$19,888		$28,781	
$t_j(X_j)$-qta.=25		$33,156		$25,941		$18,638		$27,924
Activities				In Acres				
$QWFB_0$					38	37		
$QWFB_4$			92	92	108	8		
$QWSB_0$	61		7				161	38
$QWSB_4$								
WFB_0								
WFB_4					38	139		
WSB_0								
WSB_4								
OSB_0								
OSB_4	399	460	269	276	92	92	299	422
TSB_0								
TSB_4								
XFB_0								
XSB_0								
XSB_2								
B-smf			92	92	184	184		
$QWFC_0$			300	300	600	600	300	300
$QWFC_1$								
$QWFC_2$								
$QWFC_3$								
$QWFC_4$								
$QWSC_0$	1092	937	520	294			494	419
$QWSC_4$								
WFC_0								
WFC_1								
WFC_2								
WFC_3								
WFC_4								
WSC_0				79				
WSC_4								
OSC_0								
OSC_4								
TSC_0								
TSC_4								
XFC_0								
XSC_0								
XSC_2								
C-smf			300	300	600	600	300	300
$QWFE_0$								
$QWFE_4$			30	30	60	60	50	50
$QWSE_0$								
$QWSE_4$								
WFE_0								
WFE_4								
WSE_0								
WSE_4								
OSE_0								
OSE_4		74		90				
TSE_0								
TSE_4								
XFE_0								
XSE_0								
XSE_2								
E-smf			30	30	60	60	50	50

TABLE B5

CROPPING RESULTS FOR 50% CERTAINTY, $20,000 CAPITAL

	Summerfallow Percentages							
	0%		20%		40%		Mixed Percentages	
Quota levels '000 bus. →	20	25	20	25	20	25	20	25
$t_j(X_j)$-qta.=20	$28,961		$21,914		$12,720		$23,561	
$t_j(X_j)$-qta.=25		$28,104		$21,057		$11,017		$22,703
Activities				In Acres				
$QWFB_0$					184	184		
$QWFB_4$			92	92				
$QWSB_0$	249	127	195	73	92		349	226
$QWSB_4$								
WFB_0						27		
WFB_4								
WSB_0						65		
WSB_4								
OSB_0								
OSB_4	211	333	81	203			111	234
TSB_0								
TSB_4								
XFB_0								
XSB_0								
XSB_2								
B-smf			92	92	184	184		
$QWFC_0$			300	300	390	390	300	300
$QWFC_1$								
$QWFC_2$								
$QWFC_3$								
$QWFC_4$								
$QWSC_0$	846	772	274	199			248	173
$QWSC_4$								
WFC_0								
WFC_1								
WFC_2								
WFC_3								
WFC_4								
WSC_0								
WSC_4								
OSC_0								
OSC_4								
TSC_0								
TSC_4								
XFC_0								
XSC_0								
XSC_2								
C-smf			300	300	600	600	300	300
$QWFE_0$								
$QWFE_4$			30	30	60	60	50	50
$QWSE_0$								
$QWSE_4$								
WFE_0								
WFE_4								
WSE_0								
WSE_4								
OSE_0								
OSE_4								
TSE_0								
TSE_4								
XFE_0								
XSE_0								
XSE_2								
E-smf			30	30	60	60	50	50

TABLE B6

CROPPING RESULTS FOR 65% CERTAINTY, $69,000 CAPITAL

	Summerfallow Percentages							
	0%		20%		40%		Mixed Percentages	
Quota levels '000 bus.→	20	25	20	25	20	25	20	25
$t_j(X_j)$ -qta.=20	$32,506		$25,925		$18,287		$28,300	
$t_j(X_j)$ -qta.=25		$31,549		$24,759		$17,040		$27,081
Activities				In Acres				
$QWFB_0$								
$QWFB_4$								
$QWSB_0$	10							
$QWSB_4$								
WFB_0			92	92	184	184		
WFB_4								
WSB_0								
WSB_4								
OSB_0								
OSB_4	450	460	276	276	92	92	460	460
TSB_0								
TSB_4								
XFB_0								
XSB_0								
XSB_2								
B-smf			92	92	184	184		
$QWFC_0$								
$QWFC_1$								
$QWFC_2$								
$QWFC_3$								
$QWFC_4$			300	300	600	600	300	300
$QWSC_0$	1190	1008	778	536	300	55	790	536
$QWSC_4$								
WFC_0								
WFC_1								
WFC_2								
WFC_3								
WFC_4								
WSC_0		182		242		245	60	314
WSC_4								
OSC_0			122	122			50	50
OSC_4								
TSC_0	310	310						
TSC_4								
XFC_0								
XSC_0								
XSC_2								
C-smf			300	300	600	600	300	300
$QWFE_0$								
$QWFE_4$			8		6			
$QWSE_0$		10						
$QWSE_4$								
WFE_0								
WFE_4			22	30	54	60	50	50
WSE_0								
WSE_4								
OSE_0								
OSE_4	150	140	90	90	30	30	50	50
TSE_0								
TSE_4								
XFE_0								
XSE_0								
XSE_2								
E-smf			30	30	60	60	50	50

TABLE B7

CROPPING RESULTS FOR 65% CERTAINTY, $35,000 CAPITAL

	Summerfallow Percentages							
	0%		20%		40%		Mixed Percentages	
Quota levels '000 bus. →	20	25	20	25	20	25	20	25
$t_j(X_j)$ -qta.=20							$28,278	
$t_j(X_j)$ -qta.=25								$27,060
Activities			In Acres					
$QWFB_0$	no change in either plan from $69,000 available capital at 65% certainty		no change in either plan from $69,000 available capital at 65% certainty		no change in either plan from $69,000 available capital at 65% certainty			
$QWFB_4$								
$QWSB_0$								
$QWSB_4$								
WFB_0								
WFB_4								
WSB_0								
WSB_4								
OSB_0								
OSB_4							460	460
TSB_0								
TSB_4								
XFB_0								
XSB_0								
XSB_2								
B-smf								
$QWFC_0$								
$QWFC_1$								
$QWFC_2$								
$QWFC_3$								
$QWFC_4$							267	267
$QWSC_0$							843	589
$QWSC_4$								
WFC_0							33	33
WFC_1								
WFC_2								
WFC_3								
WFC_4								
WSC_0							7	261
WSC_4								
OSC_0							50	50
OSC_4								
TSC_0								
TSC_4								
XFC_0								
XSC_0								
XSC_2								
C-smf							300	300
$QWFE_0$								
$QWFE_4$								
$QWSE_0$								
$QWSE_4$								
WFE_0								
WFE_4							50	50
WSE_0								
WSE_4								
OSE_0							50	50
OSE_4								
TSE_0								
TSE_4								
XFE_0								
XSE_0								
XSE_2								
E-smf							50	50

TABLE B8

CROPPING RESULTS FOR 65% CERTAINTY, $30,000 CAPITAL

	Summerfallow Percentages							
	0%		20%		40%		Mixed Percentages	
Quota levels '000 bus.→	20	25	20	25	20	25	20	25
$t_j(X_j)$ -qta.=20	$31,347		$25,626		$18,190		$27,045	
$t_j(X_j)$ -qta.=25		$30,010		$24,381		$16,957		$25,794
Activities				In Acres				
$QWFB_0$			46		39			
$QWFB_4$								
$QWSB_0$	330	134	44	44			248	248
$QWSB_4$								
WFB_0			46	92	145	184		
WFB_4								
WSB_0	27	223						
WSB_4								
OSB_0	104	104	232	232				
OSB_4					92	92	212	212
TSB_0								
TSB_4								
XFB_0								
XSB_0								
XSB_2								
B-smf			92	92	184	184		
$QWFC_0$			300	156	403	250	236	30
$QWFC_1$								
$QWFC_2$								
$QWFC_3$								
$QWFC_4$					197	197		
$QWSC_0$	844	844	734	734	300	300	602	602
$QWSC_4$								
WFC_0				144		153	64	270
WFC_1								
WFC_2								
WFC_3								
WFC_4								
WSC_0								
WSC_4								
OSC_0	346	346	167	167			298	298
OSC_4								
TSC_0	310	310						
TSC_4								
XFC_0								
XSC_0								
XSC_2								
C-smf			300	300	600	600	300	300
$QWFE_0$			30	30			50	50
$QWFE_4$					60	60		
$QWSE_0$								
$QWSE_4$								
WFE_0								
WFE_4								
WSE_0								
WSE_4								
OSE_0	150	150	90	90	30	30	50	50
OSE_4								
TSE_0								
TSE_4								
XFE_0								
XSE_0								
XSE_2								
E-smf			30	30	60	60	50	50

TABLE B9

CROPPING RESULTS FOR 65% CERTAINTY, $25,000 CAPITAL

	Summerfallow Percentages							
	0%		20%		40%		Mixed Percentages	
Quota levels '000 bus. →	20	25	20	25	20	25	20	25
$t_j(X_j)$ -qta.=20	$28,602		$22,415		$16,130		$23,736	
$t_j(X_j)$ -qta.=25		$27,362		$21,168		$14,882		$22,467
Activities				In Acres				
$QWFB_0$			5		94			
$QWFB_4$								
$QWSB_0$	410	214	276	276	92	92	460	460
$QWSB_4$								
WFB_0			87	92	90	184		
WFB_4								
WSB_0	50	246						
WSB_4								
OSB_0								
OSB_4								
TSB_0								
TSB_4								
XFB_0								
XSB_0								
XSB_2								
B-smf			92	92	184	184		
$QWFC_0$			300	300	600	600	300	195
$QWFC_1$								
$QWFC_2$								
$QWFC_3$								
$QWFC_4$								
$QWSC_0$	740	740	502	256	188	91	249	125
$QWSC_4$								
WFC_0								105
WFC_1								
WFC_2								
WFC_3								
WFC_4								
WSC_0								
WSC_4								
OSC_0	600	600	267	520		100	472	600
OSC_4								
TSC_0	12	12						
TSC_4								
XFC_0								
XSC_0								
XSC_2								
C-smf			300	300	600	600	300	300
$QWFE_0$			30	30	60	60	50	50
$QWFE_4$								
$QWSE_0$								
$QWSE_4$								
WFE_0								
WFE_4								
WSE_0								
WSE_4								
OSE_0								
OSE_4								
TSE_0								
TSE_4								
XFE_0								
XSE_0								
XSE_2								
E-smf			30	30	60	60	50	50

TABLE B10

CROPPING RESULTS FOR 65 % CERTAINTY, $20,000 CAPITAL

	Summerfallow Percentages							
	0%		20%		40%		Mixed Percentages	
Quota levels '000 bus.→	20	25	20	25	20	25	20	25
$t_j(X_j)$ -qta.=20	$24,820		$18,554		$10,193		$19,873	
$t_j(X_j)$ -qta.=25		$23,613		$17,326		$10,160		$18,610
Activities				In Acres				
QWFB$_0$			65		184	179		
QWFB$_4$								
QWSB$_0$	460	460	276	276	92	92	460	460
QWSB$_4$								
WFB$_0$			27	92		5		
WFB$_4$								
WSB$_0$								
WSB$_4$								
OSB$_0$								
OSB$_4$								
TSB$_0$								
TSB$_4$								
XFB$_0$								
XSB$_0$								
XSB$_2$								
B-smf			92	92	184	184		
QWFC$_0$			300	300	486	486	300	296
QWFC$_1$								
QWFC$_2$								
QWFC$_3$								
QWFC$_4$								
QWSC$_0$	675	421	401	256			249	
QWSC$_4$								
WFC$_0$								4
WFC$_1$								
WFC$_2$								
WFC$_3$								
WFC$_4$								
WSC$_0$								
WSC$_4$								
OSC$_0$	309	570		150			102	358
OSC$_4$								
TSC$_0$								
TSC$_4$								
XFC$_0$								
XSC$_0$								
XSC$_2$								
C-smf			300	300	600	600	300	300
QWFE$_0$			30	30			50	50
QWFE$_4$								
QWSE$_0$								
QWSE$_4$								
WFE$_0$								
WFE$_4$								
WSE$_0$								
WSE$_4$								
OSE$_0$								
OSE$_4$								
TSE$_0$								
TSE$_4$								
XFE$_0$								
XSE$_0$								
XSE$_2$								
E-smf			30	30	60	60	50	50

TABLE B11

CROPPING RESULTS FOR 80% CERTAINTY, $69,000 / $35,000 / $30,000 CAPITAL

	Summerfallow Percentages							
	0%		20%		40%		Mixed Percentages	
Quota levels '000 bus.→	20	25	20	25	20	25	20	25
$t_j(X_j)$ -qta.=20	$24,954		$20,198		$13,949		$21,412	
$t_j(X_j)$ -qta.=25		$24,051		$18,986		$12,763		$20,184
Activities				In Acres				
$QWFB_0$			92	92	184	184		
$QWFB_4$								
$QWSB_0$	460	302	276	276	92	92	460	460
$QWSB_4$								
WFB_0								
WFB_4								
WSB_0		158						
WSB_4								
OSB_0								
OSB_4								
TSB_0								
TSB_4								
XFB_0								
XSB_0								
XSB_2								
B-smf			92	92	184	184		
$QWFC_0$			300	156	600	508	300	106
$QWFC_1$								
$QWFC_2$								
$QWFC_3$								
$QWFC_4$								
$QWSC_0$	740	740	494	390	169		390	350
$QWSC_4$								
WFC_0				144		92		194
WFC_1								
WFC_2								
WFC_3								
WFC_4								
WSC_0								
WSC_4								
OSC_0	450	450	406	510	131	300	510	550
OSC_4								
TSC_0								
TSC_4								
XFC_0								
XSC_0	310	310						
XSC_2								
C-smf			300	300	600	600	300	300
$QWFE_0$			30	30	60	60	50	50
$QWFE_4$								
$QWSE_0$								
$QWSE_4$								
WFE_0								
WFE_4								
WSE_0								
WSE_4								
OSE_0	150	150	90	90	30	30	50	50
OSE_4								
TSE_0								
TSE_4								
XFE_0								
XSE_0								
XSE_2								
E-smf			30	30	60	60	50	50

TABLE B12

CROPPING RESULTS FOR 80% CERTAINTY, $25,000 CAPITAL

	Summerfallow Percentages							
	0%		20%		40%		Mixed Percentages	
Quota levels '000 bus.→	20	25	20	25	20	25	20	25
$t_j(X_j)$ -qta.=20	$23,619		$18,454		$12,765		$19,529	
$t_j(X_j)$ -qta.=25		$22,729		$17,327		$11,638		$18,357
Activities			In Acres					
$QWFB_0$			92	92	184	184		
$QWFB_4$								
$QWSB_0$	460	302	276	276	92	92	460	460
$QWSB_4$								
WFB_0								
WFB_4								
WSB_0		158						
WSB_4								
OSB_0								
OSB_4								
TSB_0								
TSB_4								
XFB_0								
XSB_0								
XSB_2								
B-smf			92	92	184	184		
$QWFC_0$			300	300	600	508	300	289
$QWFC_1$								
$QWFC_2$								
$QWFC_3$								
$QWFC_4$								
$QWSC_0$	740	740	494	212	169		390	125
$QWSC_4$								
WFC_0						92		11
WFC_1								
WFC_2								
WFC_3								
WFC_4								
WSC_0								
WSC_4								
OSC_0	600	600	276	565	20	193	327	600
OSC_4								
TSC_0								
TSC_4								
XFC_0								
XSC_0	11	11						
XSC_2								
C-smf			300	300	600	600	300	300
$QWFE_0$			30	30	60	60	50	50
$QWFE_4$								
$QWSE_0$								
$QWSE_4$								
WFE_0								
WFE_4								
WSE_0								
WSE_4								
OSE_0								
OSE_4								
TSE_0								
TSE_4								
XFE_0								
XSE_0								
XSE_2								
E-smf			30	30	60	60	50	50

TABLE B13

CROPPING RESULTS FOR 80% CERTAINTY, $20,000 CAPITAL

	Summerfallow Percentages							
	0%		20%		40%		Mixed Percentages	
Quota levels '000 bus. →	20	25	20	25	20	25	20	25
$t_j(X_j)$ -qta.=20	$20,542		$14,919		$17,313		$16,174	
$t_j(X_j)$ -qta.=25		$19,913		$14,215		$7,313		$15,243
Activities				In Acres				
$QWFB_0$			92	92	184	184		
$QWFB_4$								
$QWSB_0$	460	460	276	276	92	92	460	460
$QWSB_4$								
WFB_0								
WFB_4								
WSB_0								
WSB_4								
OSB_0								
OSB_4								
TSB_0								
TSB_4								
XFB_0								
XSB_0								
XSB_2								
B-smf			92	92	184	184		
$QWFC_0$			300	300	484	484	300	300
$QWFC_1$								
$QWFC_2$								
$QWFC_3$								
$QWFC_4$								
$QWSC_0$	740	537	401	212			358	111
$QWSC_4$								
WFC_0								
WFC_1								
WFC_2								
WFC_3								
WFC_4								
WSC_0								
WSC_4								
OSC_0	242	450		195			138	243
OSC_4								
TSC_0								
TSC_4								
XFC_0								
XSC_0								
XSC_2								
C-smf			300	300	600	600	300	300
$QWFE_0$			30	30			50	50
$QWFE_4$								
$QWSE_0$								
$QWSE_4$								
WFE_0								
WFE_4								
WSE_0								
WSE_4								
OSE_0								
OSE_4								
TSE_0								
TSE_4								
XFE_0								
XSE_0								
XSE_2								
E-smf			30	30	60	60	50	50

APPENDIX C

EXPECTED INCOME $E(t_j)$ FOR THREE LEVELS OF CERTAINTY

TABLE C1

EXPECTED INCOME $E(t_j)=\bar{C}_jX_j$ FOR 50 PER CENT CERTAINTY

		Summerfallow Percentages			
Capital Level	Quota Level (bushels)	0%	20%	40%	Fixed As To Soil Type
$69,000	25,000	$42,659	$37,075	$29,960	$39,066
	20,000	41,411	35,826	28,709	37,817
35,000	25,000	42,659	35,797	27,995	38,074
	20,000	41,411	34,548	26,745	36,825
30,000	25,000	39,048	31,808	24,564	33,926
	20,000	37,799	30,599	23,314	32,683
25,000	25,000	34,181	27,134	19,888	28,781
	20,000	33,156	25,941	18,638	27,924
20,000	25,000	28,961	21,914	12,720	23,561
	20,000	28,104	21,057	11,017	22,703

TABLE C2

EXPECTED INCOME $E(t_j)=\bar{C}_jX_j$ FOR 65 PER CENT CERTAINTY

		Summerfallow Percentages			
Capital Level	Quota Level (bushels)	0%	20%	40%	Fixed As To Soil Type
$69,000	25,000	$42,659	$35,592	$27,540	$39,066
	20,000	41,411	34,220	26,172	37,817
35,000	25,000	42,659	35,592	27,540	38,074
	20,000	41,411	34,225	26,172	36,825
30,000	25,000	37,777	31,767	24,663	33,302
	20,000	36,412	30,419	23,313	31,947
25,000	25,000	33,407	26,506	19,573	28,056
	20,000	32,041	25,141	18,226	26,695
20,000	25,000	28,822	21,894	12,686	23,456
	20,000	27,457	20,541	11,017	22,088

NOTE: $E(t_j) \neq (C-Z_j\sigma)'X_j$ in this case.

TABLE C3

EXPECTED INCOME $E(t_j)=\bar{C}_jX_j$ FOR 80 PER CENT CERTAINTY

		Summerfallow Percentages			
Capital Level	Quota Level (bushels)	0%	20%	40%	Fixed As To Soil Type
$69,000	25,000	$35,704	$29,858	$21,980	$31,642
	20,000	34,602	28,319	20,414	31,070
35,000	25,000	35,704	29,858	21,980	31,642
	20,000	34,602	28,319	20,414	31,070
30,000	25,000	35,704	29,858	21,980	31,642
	20,000	34,602	28,319	20,414	31,070
25,000	25,000	33,407	26,506	18,883	28,056
	20,000	32,041	25,141	18,753	26,695
20,000	25,000	28,822	22,132	12,635	23,456
	20,000	27,457	21,113	11,017	22,080

NOTE: $E(t_j) \neq (C-Z_j\sigma)'X_j$ in this case.

APPENDIX D

STANDARD DEVIATION OF CROPPING PLANS (σ_{t_j}) FOR THREE LEVELS OF CERTAINTY

Presented below are the standard deviations to be used in the equation,

$$y_j = \frac{\hat{t}_s - E(t_j)}{(X'VX)^{\frac{1}{2}}}$$

where,

$$\sigma_{t_j} = (X'VX)^{\frac{1}{2}} = \left(\sum_{j=1}^{k} X_j^2 \sigma_j^2 p_j^2\right)^{\frac{1}{2}}$$

TABLE D1

STANDARD DEVIATION OF CROPPING PLANS FOR 50 PER CENT CERTAINTY (in dollars)

Capital Level	Quota Level (bushels)	Summerfallow Percentages			
		0%	20%	40%	Fixed As To Soil Type
$69,000	25,000	$28,687	$26,038	$36,045	$29,463
	20,000	27,483	26,081	33,148	28,836
35,000	25,000	28,687	21,902	24,220	26,657
	20,000	27,483	21,647	25,226	25,537
30,000	25,000	28,384	18,760	17,059	25,428
	20,000	27,169	18,622	16,138	24,682
25,000	25,000	24,200	17,643	13,176	17,852
	20,000	26,205	17,864	12,986	22,850
20,000	25,000	15,805	11,509	12,200	11,838
	20,000	19,465	14,137	12,003	14,497

TABLE D2

STANDARD DEVIATION OF CROPPING PLANS FOR 65 PER CENT CERTAINTY (in dollars)

Capital Level	Quota Level (bushels)	Summerfallow Percentages			
		0%	20%	40%	Fixed As To Soil Type
$69,000	25,000	$28,829	$22,917	$30,169	$29,356
	20,000	28,078	22,052	30,095	28,652
35,000	25,000	28,829	22,917	30,169	28,685
	20,000	28,078	22,052	30,095	27,839
30,000	25,000	14,952	15,841	14,338	14,849
	20,000	14,036	15,333	13,607	14,771
25,000	25,000	13,211	10,378	11,554	12,084
	20,000	11,675	9,323	11,552	11,378
20,000	25,000	13,063	9,955	9,701	11,766
	20,000	11,698	8,837	9,665	11,512

TABLE D3

STANDARD DEVIATION OF CROPPING PLANS FOR 80 PER CENT CERTAINTY (in dollars)

Capital Level	Quota Level (bushels)	Summerfallow Percentages			
		0%	20%	40%	Fixed As To Soil Type
$69,000	25,000	$14,452	$10,546	$11,858	$12,688
	20,000	12,823	9,166	10,371	11,868
35,000	25,000	14,452	10,546	11,858	12,688
	20,000	12,823	9,166	10,371	11,868
30,000	25,000	14,452	10,546	11,858	12,688
	20,000	12,823	9,166	10,371	11,868
25,000	25,000	13,927	10,388	11,832	12,469
	20,000	12,081	9,304	10,279	11,909
20,000	25,000	13,535	9,626	9,668	12,196
	20,000	12,216	8,749	9,668	11,519

APPENDIX E

PROBABILITY OF CROPPING PLANS ORIGINALLY FORMULATED AT THREE LEVELS OF CERTAINTY, EXCEEDING $25,000

Probability of pre-determined cropping plans subjected to average net prices $\bar{C}_j$ exceeding a specified level of income, in this case $25,000, is determined by,

$$y_j = \frac{t_s - E(t_j)}{(X'VX)^{\frac{1}{2}}} .$$

TABLE E1

PROBABILITY OF CROPPING PLANS, ORIGINALLY FORMULATED AT 50 PER CENT CERTAINTY, EXCEEDING $25,000

		Summerfallow Percentages			
Capital Level	Quota Level (bushels)	0%	20%	40%	Fixed As To Soil Type
$69,000	25,000	-.6155(73)	-.4637(68)	-.1376(56)	-.4774(68)
	20,000	-.5971(73)	-.4150(66)	-.1118(54)	-.4444(67)
35,000	25,000	-.6155(73)	-.4929(69)	-.1237(55)	-.4904(69)
	20,000	-.5971(73)	-.4410(67)	-.0691(53)	-.4630(68)
30,000	25,000	-.4949(69)	-.3628(64)	+.0255(49)	-.3510(64)
	20,000	-.4710(68)	-.2985(62)	+.1044(46)	-.3112(62)
25,000	25,000	-.3793(65)	-.1209(55)	+.3879(35)	-.2117(58)
	20,000	-.3112(62)	-.0526(52)	+.4899(31)	-.1279(55)
20,000	25,000	-.2506(60)	+.2681(39)	+1.0065(16)	+.1215(45)
	20,000	-.1594(56)	+.2789(39)	+1.1562(12)	+.1584(44)

NOTE: Probability expressed as percentages in brackets. Determined by entering value of y_j in Table E4 and finding area under normal curve.

TABLE E2

PROBABILITY OF CROPPING PLANS, ORIGINALLY FORMULATED AT 65 PER CENT CERTAINTY, EXCEEDING $25,000

		Summerfallow Percentages			
Capital Level	Quota Level (bushels)	0%	20%	40%	Fixed As To Soil Type
$69,000	25,000	-.6125(73)	-.4621(68)	-.0841(53)	-.4791(68)
	20,000	-.5692(72)	-.4181(66)	-.0389(52)	-.4473(67)
35,000	25,000	-.6125(73)	-.4621(68)	-.0841(53)	-.4557(68)
	20,000	-.5692(72)	-.4181(66)	-.0389(52)	-.4247(66)
30,000	25,000	-.8545(80)	-.4271(67)	+.-235(49)	-.5590(71)
	20,000	-.8130(79)	-.3534(64)	+.1239(45)	-.4703(68)
25,000	25,000	-.6363(74)	-.1451(56)	+.4697(32)	-.2523(60)
	20,000	-.6030(73)	-.0151(51)	+.5862(28)	-.1489(56)
20,000	25,000	-.2925(61)	+.3120(38)	+1.2693(10)	+.1312(45)
	20,000	-.2100(58)	+.5045(31)	+1.4467(7)	+.2529(40)

TABLE E3

PROBABILITY OF CROPPING PLANS, ORIGINALLY FORMULATED AT 80 PER CENT CERTAINTY, EXCEEDING $25,000

		Summerfallow Percentages			
Capital Level	Quota Level (bushels)	0%	20%	40%	Fixed As To Soil Type
$69,000	25,000	-.7406(77)	-.4606(68)	+.2546(40)	-.5234(70)
	20,000	-.7488(77)	-.3620(64)	+.4421(33)	-.5114(70)
35,000	25,000	-.7406(77)	-.4606(68)	+.2546(40)	-.5234(70)
	20,000	-.7488(77)	-.3620(64)	+.4421(33)	-.5114(70)
30,000	25,000	-.7406(77)	-.4606(68)	+.2546(40)	-.5234(70)
	20,000	-.7488(77)	-.3620(64)	+.4421(33)	-.5114(70)
25,000	25,000	-.6036(73)	-.1449(56)	+.5169(30)	-.2450(60)
	20,000	-.5763(72)	-.0151(51)	+.6077(27)	-.1423(57)
20,000	25,000	-.2823(61)	+.2979(38)	+1.2789(11)	+.1265(45)
	20,000	-.2011(58)	+.4442(33)	+1.4463(7)	+.2534(40)

NOTE: Probability expressed as percentages in brackets. Determined by entering value of y_j in Table E4 and finding area under normal curve.

TABLE E4

Area under the Normal Curve, F(y)†

y	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9294	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Percentage Points of the Normal Distribution†

F(y)	.75	.90	.95	.975	.99	.995	.99	.995	.99995	.99999
α =2 1-F(y)	.50	.20	.10	.05	.02	.01	.002	.001	.0001	.00001
T_α	0.67	1.28	1.65	1.96	2.33	2.58	3.09	3.29	3.891	4.417

†F(y) is the area under the normal curve from $-\infty$ to y: is twice the area from y to ∞ (area from $-\infty$ to $-y$ plus the area from y to ∞).

Source: Anderson, R.L. and Bancroft, T.A.; Statistical Theory in Research.
New York, New York, McGraw-Hill Book Co., Inc. 1952 P. 382.