

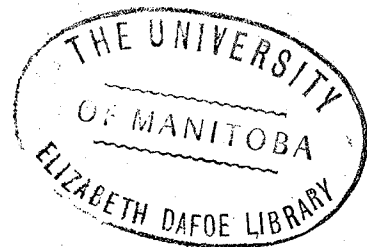
A STUDY  
OF THE BULK MODULUS AND SHEAR MODULUS  
OF A LAKE AGASSIZ CLAY

A THESIS  
Presented to  
The Departement of Civil  
Engineering  
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Master of Science

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## LIST OF SYMBOLS USED

Symbol	Definition
$\sigma_m$	Mean Normal Stress
$\sigma_1, \sigma_2, \sigma_3$	Principal Stress Components
$s_1, s_2, s_3$	Deviator Stress Components
$s_d$	Resultant Deviatoric Stress
$\epsilon$	Volumetric Strain
$\mathcal{E}$	% Volumetric Strain
$\epsilon_1, \epsilon_2, \epsilon_3$	Principal Strain Components
$\epsilon_m$	$(\epsilon_1 + \epsilon_2 + \epsilon_3)/3$
$\mathcal{E}_d$	Resultant Deviatoric Strain
$K$	Bulk Modulus
$G$	Shear Modulus
$e$	Void Ratio
$e_i$	Initial Void Ratio
$e_c$	Void Ratio at the Pre-consolidation Pressure
$V$	Sample Volume
$V_o$	Initial Sample Volume
$h$	Sample Height
$h_o$	Initial Sample Height
$A$	Sample Area
$A_o$	Initial Sample Area
$p_c$	Preconsolidation Pressure

## SUMMARY

The bulk modulus and shear modulus of the Lake Agassiz Clay were studied in this thesis. The approach used in the study was to separate the stress system into volumetric and deviatoric components of stress. The bulk modulus is related to the volumetric behavior of the soil. The shear modulus is related to the deviatoric stress and to the level of mean normal stress. Thus the bulk modulus and shear modulus can be related to separate physical stress components. These relationships vary with the soil investigated.

The bulk modulus of the Agassiz Clay was determined from isotropic compression tests. Sixteen isotropic compression tests were run from which mean normal stress versus volumetric strain curves were drawn. Curves were developed relating mean normal stress and bulk modulus for the range of soils investigated

The shear modulus of the Agassiz Clay was determined from drained triaxial tests which were run at a constant mean normal stress. A total of fifteen drained triaxial tests were run at various levels of mean normal stress. The slope of the deviatoric stress strain curve defines the shear modulus  $G$ . Expressions were developed for the stress dependent shear modulus  $G$ . The parameters for these expressions were determined for the material studied.

Curves of mean normal stress versus deviatoric stress at failure were plotted for samples from the same soil. These

curves constitute a unique failure theory . Expressions relating the mean normal stress and the deviatoric stress at failure were developed from this unique straight line relationship. It is concluded the the separation of volumetric and deviatoric components is a useful way of describing the stress system. The bulk modulus and shear modulus of a given soil can be satisfactorily determined by the use of the volumetric and deviatoric components.

## CHAPTER I

### INTRODUCTION

The deformation behavior of clay is not well understood. The lack of understanding arises from the complexity of the physical makeup of clays.

Biot<sup>1</sup> (1941) studied the elastic deformation of soils and obtained a rigorous mathematical solution for three dimensional consolidation of soils. DeWet<sup>2</sup> (1962) studied the effects of time and pore pressure and used Biot's solution for the application of three dimensional consolidation under footings. Saada<sup>3</sup> (1962) studied shear and consolidation by means of rheologic models. Kondner<sup>4</sup> (1963) studied the behaviour of cohesive soils under shearing stresses and found that the stress-strain relationship followed the general shape of a rectangular hyperbola.

The general approach by the aforementioned investigators was to analyze the experimental behaviour of soils in terms of theoretical models. This is particularly useful if the solution is in a closed form. An example of a solution in a closed form is the solution for the displacement of a continuum based on the theory of elasticity. The solutions however, are often too complicated for practical applications.

The theory of elasticity can be useful in soil mechanics problems if representative values of the elastic parameters

can be determined. The value of Young's Modulus  $E$  and Poisson's ratio  $\mu$  are not constant for a given soil. Both  $E$  and  $\mu$  can vary with the material and the stress level. In many cases the use of a non-constant  $E$  and  $\mu$  would be too complex to use in an analysis.

Young's Modulus  $E$  and Poisson's ratio  $\mu$  can be replaced by the bulk modulus  $K$  and the shear modulus  $G$ . The bulk modulus and shear modulus are related to separate components of stress. The shear and bulk moduli for a given soil may vary with the stress level, the stiffness of the material and physical properties of the material. However, since the bulk and shear moduli are associated with separate physical components of behavior they can be investigated separately and independent solutions may be obtained for each.

The purpose of this thesis is to investigate the bulk modulus and shear modulus of the Lake Agassiz clays. The need for determining elastic parameters has been demonstrated by the fact that present mathematical means of evaluating such analysis as settlement require an estimate of the elastic parameters.

## CHAPTER II

## THEORETICAL CONSIDERATIONS

Volumetric and Deviatoric Components of Stress

It has been shown by Domaschuk<sup>5</sup> (1968) that it is useful to separate the stress system into volumetric and deviatoric components of stress as illustrated diagrammatically in Figure 1.

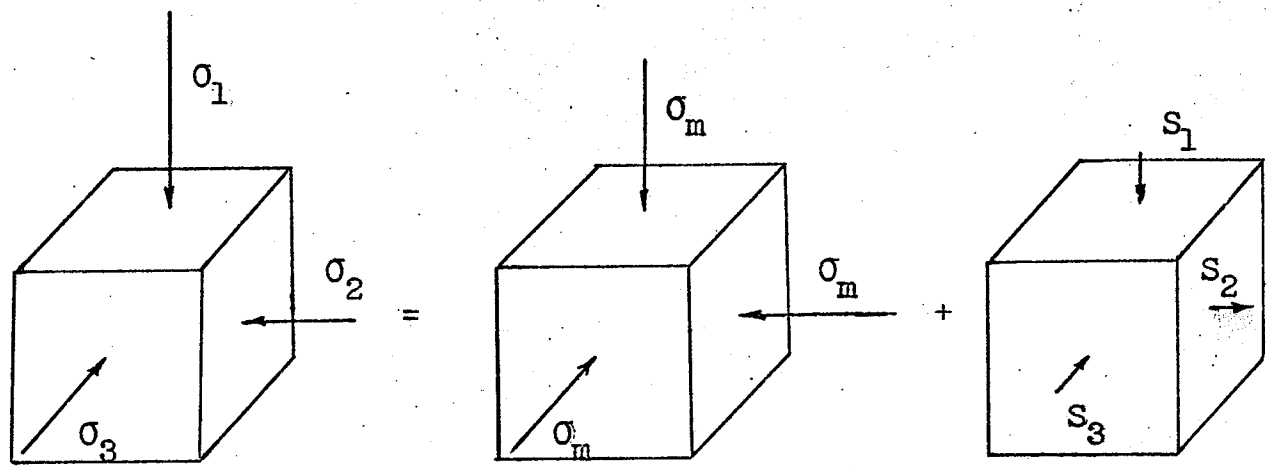
Figure 1(a) illustrates the general state of stress at a point in terms of the principal stresses,  $\sigma_1$ ,  $\sigma_2$  and  $\sigma_3$ . The principal stresses can be separated into two sets of components. The first set of components is equal to the mean normal stress  $\sigma_m$ , which is defined by the equation:

$$\sigma_m = (\sigma_1 + \sigma_2 + \sigma_3) / 3 \quad \dots (1)$$

The second set of components is defined by the deviatoric stresses  $S_1$ ,  $S_2$ , and  $S_3$  as shown in Figure 1(b) and 1(c). These components are given by:

$$\left. \begin{aligned} S_1 &= \sigma_1 - \sigma_m \\ S_2 &= \sigma_2 - \sigma_m \\ S_3 &= \sigma_3 - \sigma_m \end{aligned} \right\} \dots (2)$$

The system can be further simplified by representing the three mean normal stress components by a single volumetric component equal in magnitude to the mean normal stress. The deviatoric stresses can be represented by a single deviatoric



$$\sigma_m = 1/3(\sigma_1 + \sigma_2 + \sigma_3)$$

$$s = \sigma - \sigma_m$$

(a) General State of Stress.

(b) Mean Normal Stress.

(c) Deviatoric Stress Components.

Figure 1. Designation of the Stress Components at a Point.

component. This component, the resultant deviatoric stress represented by  $S_d$  is given by the following equation:

$$S_d = \sqrt{S_1^2 + S_2^2 + S_3^2} \quad \dots\dots (3)$$

For an isotropic elastic solid, the volumetric stress  $\sigma_m$  would bring about the same volume change as an equivalent hydrostatic stress. The volumetric stress component is associated with linear strains only, while the resultant deviatoric component is associated only with shear strain and dilatancy. Each component of stress is associated with different components of behaviour.

#### Volumetric and Deviatoric Components of Strain

The general state of strain corresponding to the stress system can be described by the principal strains  $\epsilon_1$ ,  $\epsilon_2$  and  $\epsilon_3$  and their direction cosines. This system can be separated into the strains caused by the volumetric component of stress and those strains resulting from shear or deviatoric stresses. These deviator strains will be the difference between the individual principal strains and the mean normal strain.

The mean normal strain  $\epsilon_m$ , can be represented by the expression:

$$\epsilon_m = 1/3 (\epsilon_1 + \epsilon_2 + \epsilon_3) \quad \dots\dots (4)$$

The resultant deviatoric strain can be represented by the following equation:



$$1/2 \mathcal{E}_d = \sqrt{(\varepsilon_1 - \varepsilon_m)^2 + (\varepsilon_2 - \varepsilon_m)^2 + (\varepsilon_3 - \varepsilon_m)^2} \quad \dots (5)$$

Strain component relationships are the same as stress component relationships, provided that one uses in place of normal stress, the linear strain, and in place of shear stress, one half the shearing strain.

### Stress-Strain Relationships

To simplify the relationship between stress and strain, the axis of stress and the axis of strain are assumed to coincide. Thus the material is assumed to be isotropic.

Volumetric strain can be related to mean normal stress by use of the bulk modulus  $K$  which is defined as:

$$K = \frac{\text{Limit}}{\varepsilon \rightarrow 0} \frac{\Delta \bar{\sigma}_m}{\Delta \varepsilon} \quad \dots (6)$$

in which  $\varepsilon$  is equal to the volumetric strain.

The resultant deviatoric component of stress and the resultant deviatoric strain can be related by the shear modulus  $G$ , which is defined as:

$$G = \frac{\text{Limit}}{\mathcal{E}_d \rightarrow 0} \frac{\Delta S_d}{\Delta \mathcal{E}_d} \quad \dots (7)$$

## CHAPTER III

## EXPERIMENTAL INVESTIGATION OF THE BULK MODULUS

In order to obtain solutions of the bulk modulus  $K$  defined by equation (6) as :

$$K = \frac{\text{Limit}}{\varepsilon \rightarrow 0} \frac{\Delta \sigma_m}{\Delta \varepsilon}$$

the value of  $\varepsilon$ , the volumetric strain, was measured at corresponding values of the volumetric stress  $\sigma_m$ . The incremental values of  $\Delta \varepsilon$  and  $\Delta \sigma_m$  were determined for each stress level and the value of  $K$  was determined corresponding to a level of the mean normal stress. The intention was to determine a solution of the bulk modulus  $K$  in terms of the mean normal stress for a given soil.

Soil Investigated

The study was carried out on undisturbed samples of Lake Agassiz clay. The Lake Agassiz clay is an extensive lacustrine deposit of glacial origin. The clay deposit usually consists of an upper level of varved chocolate brown clay and a lower deposit of massive darker clay underlain by a layer of till. The physical properties of Lake Agassiz clay have been described by Baracos <sup>6</sup> (1961).

Tests were carried out on four separate blocks of soil. The blocks designated as A, B and C were taken from a depth of 30 ft. in the North East area of Metropolitan Winnipeg. This material is a highly plastic grey silty clay of medium

stiffness. The clay contained numerous small limestone pebbles.

The block sample designated as Block D was taken from a depth of 16 ft. in the southern area of Metropolitan Winnipeg. This material is a chocolate brown clay of medium stiffness with medium plasticity.

The physical properties of the clays are shown in Table I. Figures 2 and 3 are photographs of samples taken from Block B and C, respectively, illustrating the physical make up of the soil.

TABLE I

	Block A		
	No. Tests	Range	Average
% Sand	1		9
% Silt			54
% Clay			37
Liquid Limit	2	59.3-64.4	61.3
Plastic Limit			32.0
Dry Density (pcf)	3	64.7-65.5	65.1
Void Ratio	3	1.59-1.62	1.60
Moisture Content	3	58.5-61.3	59.9
Saturation %	3	97.7-100	99.4
Specific Gravity	1		2.71
Compression Index	2	0.67-0.70	0.68
Preconsolidation Pressure (tsf)	2	2.08-2.12	2.10

TABLE I (continued)

Block B			
	No. Tests	Range	Average
% Sand	1		9.0
% Silt	1		54.0
% Clay	1		37.0
Liquid Limit	2	44.0-85.0	64.5
Plastic Limit	2	19.7-29.6	24.7
Dry Density (pcf)	7	63.2-80.5	68.0
Void Ratio	7	1.10-1.78	1.51
Moisture Content %	7	41.9-65.0	56.2
Saturation %	7	92.5-100	99.0
Specific Gravity	2	2.68-2.73	2.71
Compression Index	2	0.62-0.70	0.66
Preconsolidation Pressure (tsf)	2	2.10-2.18	2.14

Block C			
	No. Tests	Range	Average
% Sand	1		10
% Silt	1		34
% Clay	1		66
Liquid Limit	2	78.1-81.0	79.9
Plastic Limit	2	29.1-30.1	29.9
Dry Density	4	63.5-70.2	67.4
Void Ratio	4	1.47-1.51	1.51
Moisture Content %	4	51.5-63.7	57.3
Saturation %	4	98.9-100	100

TABLE I (continued)

Block C			
	No. Tests	Range	Average
Specific Gravity	1		2.71
Compression Index	2	0.60-0.70	0.65
Preconsolidation Pressure (tsf)	2	2.10-2.18	2.14
Block D			
	No. Tests	Range	Average
Liquid Limit	36	37-117	89*
Plastic Limit	36	14-40	30*
Dry Density	73	64-99	77*
Void Ratio	3	1.55-1.59	1.59
Moisture Content %	76	27-57	48*
Saturation %	73	86-100	97*
Specific Gravity	1		2.77
Preconsolidation Pressure (tsf)	2		2.52

\* Results taken from Baracos<sup>6</sup>

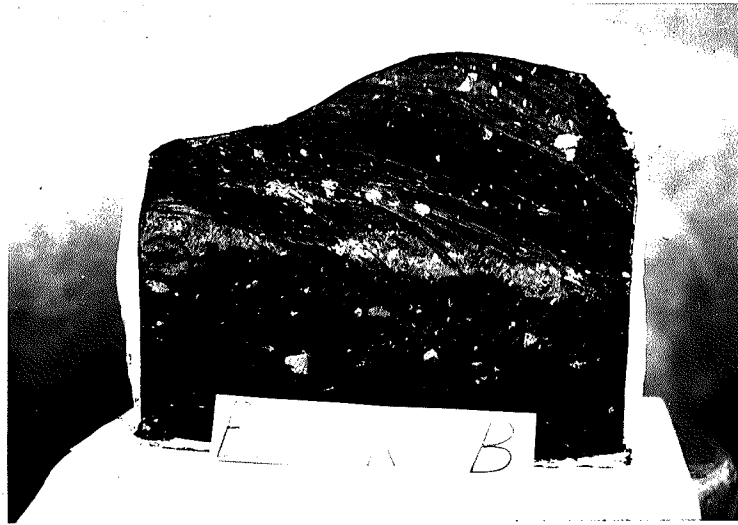


Figure 2. Clay Sample From Block B.

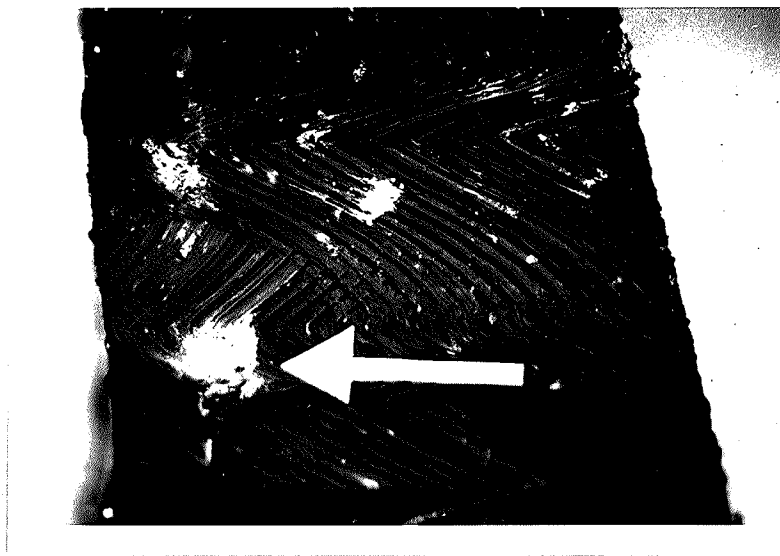


Figure 3. Clay Sample From Block C.

### Test Apparatus

The isotropic compression tests were run in a standard tri-axial cell. The samples were 1.5 in. in diameter and 3.0 in. in length. Each sample was set up with top and bottom drainage and with side drains of filter paper strips to reduce the time for consolidation. A perspex loading cap was used and the sample was encased in a triaxial membrane .005 inches in thickness. This rubber membrane was sealed against the loading cap and the bottom pedestal by rubber o-rings. Silicone grease was used on the cap and pedestal to ensure a complete seal between them and the membrane.

The water pressure in the cell was regulated by means of compressed air.

Back pressure ensured saturation of the sample. This necessitated the use of a volume change indicator working under back pressure. With the air driven into solution-accurate readings of volume change were obtained by measuring the water dispelled from a sample. One type of volume change device used consisted of a calibrated U-tube partly filled with mercury. This gauge is described by Bishop<sup>7</sup> (1962), and shown in Figure 16, page 35.

A second type of gauge was constructed which consisted of a burette enclosed inside of a plastic tube. The tube is filled almost to the top of the burette with water. About 3 to 4 ml. of water is placed in the burette. The remainder of the burette and plastic tube is filled with coloured kerosene.

As water is displaced from the sample, the water displaces kerosene up out of the burette. Thus the volume change can be measured. This device has been discussed by Bishop and Hankel<sup>7</sup> (1962) and is shown in the Figure 15, Page 34.

### Testing Procedure

Specimens used in the tests were obtained by pressing cutting tubes into the undisturbed block of clay. These samples were set up in the manner previously described.

The initial stage of the test was to completely saturate the sample. In order to do this, both the cell pressure and back pressure were increased simultaneously in 5 psi increments to a back pressure and cell pressure of 15 psi. At each increment, the pore pressure was allowed to reach equilibrium. Since the samples initially were very near 100% saturation only about 3 ml. of water was required to saturate a sample.

Domaschuk<sup>4</sup> (1968) has shown that at pore pressures of about 10 psi, the volumetric stress versus pore pressure curve becomes linear. This indicates that at this pore pressure the air in the sample has been driven into solution. For this reason back pressures in excess of 10 psi were used.

When the samples had been saturated, the cell pressure was increased in increments of 5 psi. After each increment the sample was allowed to consolidate. The volume change corresponding to each increment in cell pressure was recorded.

Five tests were run up to 30 psi, one test to 42 psi, seven tests to 60 psi and two tests to 90 psi. In one test



(No. 17), the sample was consolidated to 30 psi, allowed to rebound to zero stress and then reconsolidated to 60 psi.

In another test (No. 18), the sample was initially consolidated to 50 psi, allowed to rebound to zero stress and reconsolidated to 60 psi. This was done to ascertain the effects of stress cycling on the relationship between the bulk modulus and the mean normal stress.

### Test Results

A summary of the test results are given in Table II, Appendix II. The mean normal stress versus volumetric strain curves for the sixteen tests conducted are shown in Figures 4 through 10. The initial void ratio,  $e_i$ , and the void ratio at the preconsolidation pressure,  $e_c$ , are shown on the figures. The initial void ratio is the void ratio of the sample at the start of the laboratory test. The void ratio at the preconsolidation pressure is the void ratio of the sample after it had been reconsolidated isotropically to the preconsolidation pressure as determined by oedometer tests. The void ratio at the preconsolidation pressure  $e_c$ , is probably the better representation of the insitu void ratio. Due to stress relief and expansion upon sampling the void ratio of a soil sample at the start of a test was probably greater than it was under insitu conditions. The value of  $e_c$  may not be exactly equal to the insitu void ratio since isotropic reconsolidation in the laboratory may have induced some additional volumetric

strain.

The mean normal stress versus volumetric strain curves for Blocks A, B, C and D, are shown in Figures 4 to 7. The curves for Blocks A, B and C, show a steep, concave upward shape up to approximately 30 psi. There is a break in the curve at 30 psi, beyond which the curve changes to a concave upward shape again.

The curves for Block D do not show a distinct break but, there is a section of more or less constant slope in the area of 35 psi.

The preconsolidation pressure for Blocks A, B, and C ranges from 29 psi to 30 psi, as determined by one dimensional consolidation. The preconsolidation pressure as determined by triaxial compression was also equal to approximately 29 psi. One dimensional consolidation tests for Block D gave a preconsolidation pressure of 35 psi while triaxial consolidation gave a preconsolidation pressure of 8 psi. It is possible that this varved clay shows greater preconsolidation in the vertical direction than in the horizontal. This difference in stress history has the effect of obscuring the break in the curve at the preconsolidation pressure. Figure 8 shows the best fit curves for Blocks A, B, C and D. For Block A, B, and C it is evident that there is two distinct sections to the stress-strain relationship. The portion of the curve below the preconsolidation pressure depicts the behavior of the soil in an overconsolidated state. The portion of the curve above the preconsolidation pressure depicts the soil behavior in a

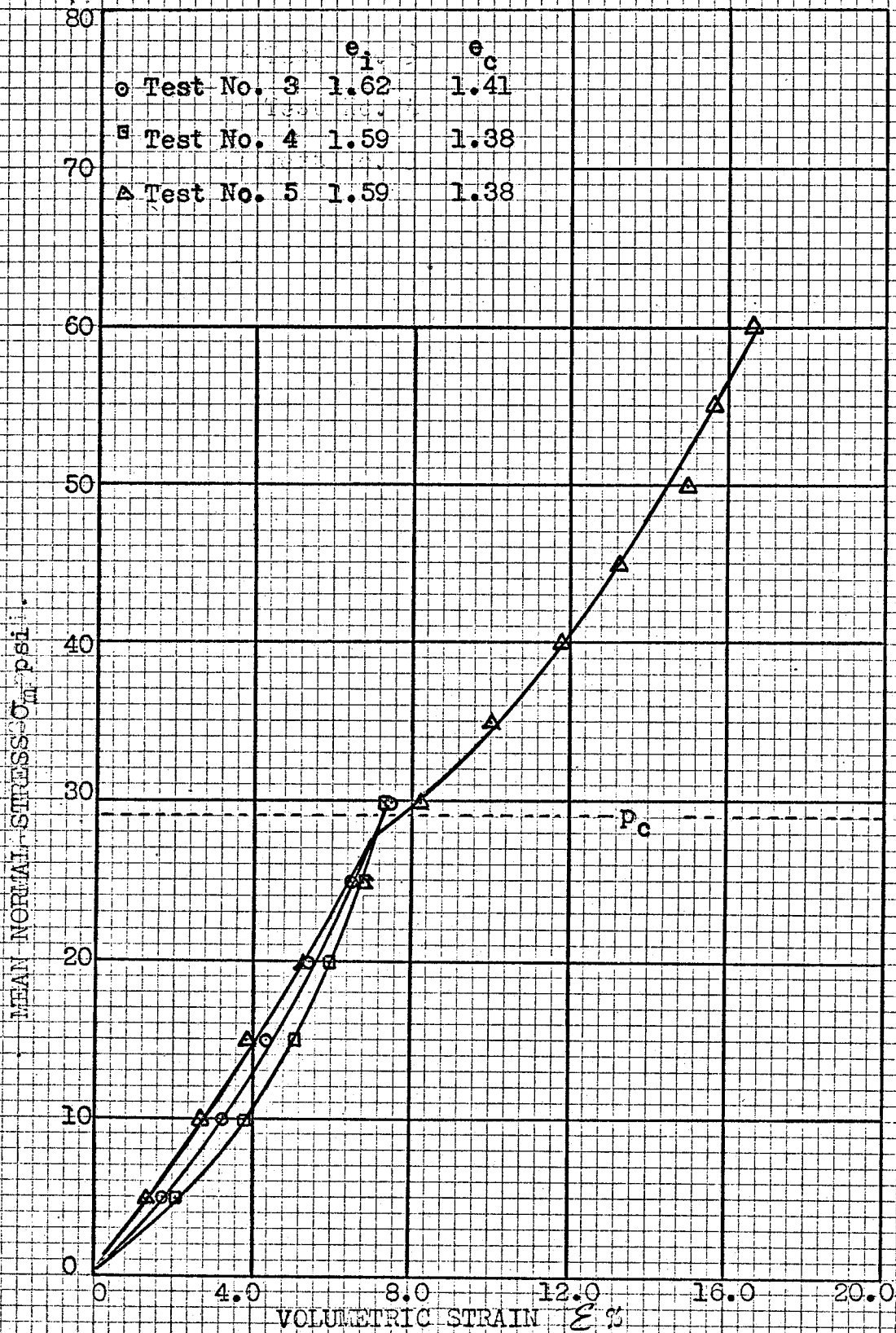


Figure 4. Volumetric Stress-Strain Relationship for Block A.

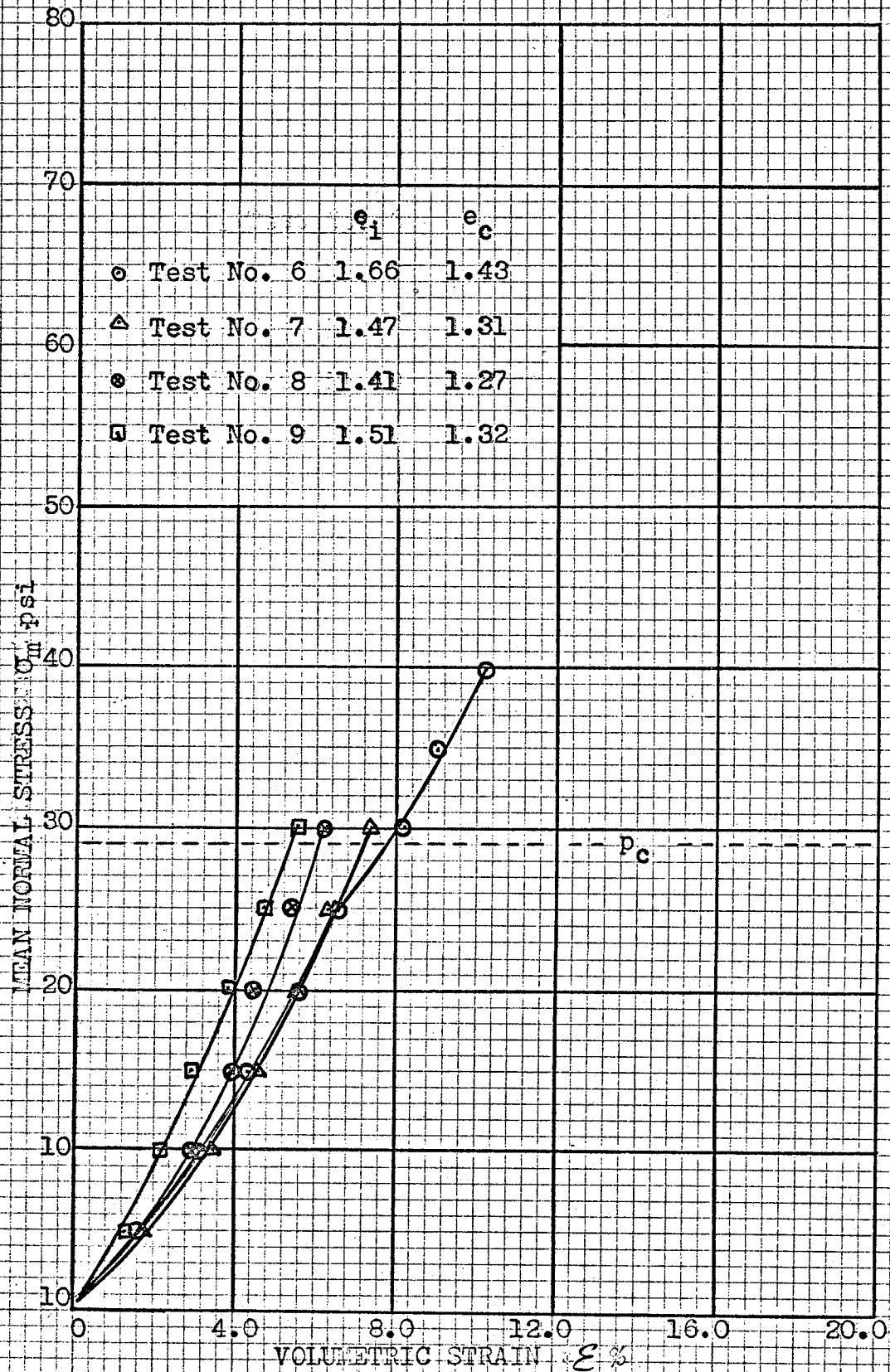


Figure 5. Volumetric Stress-Strain Relationship for Block C.

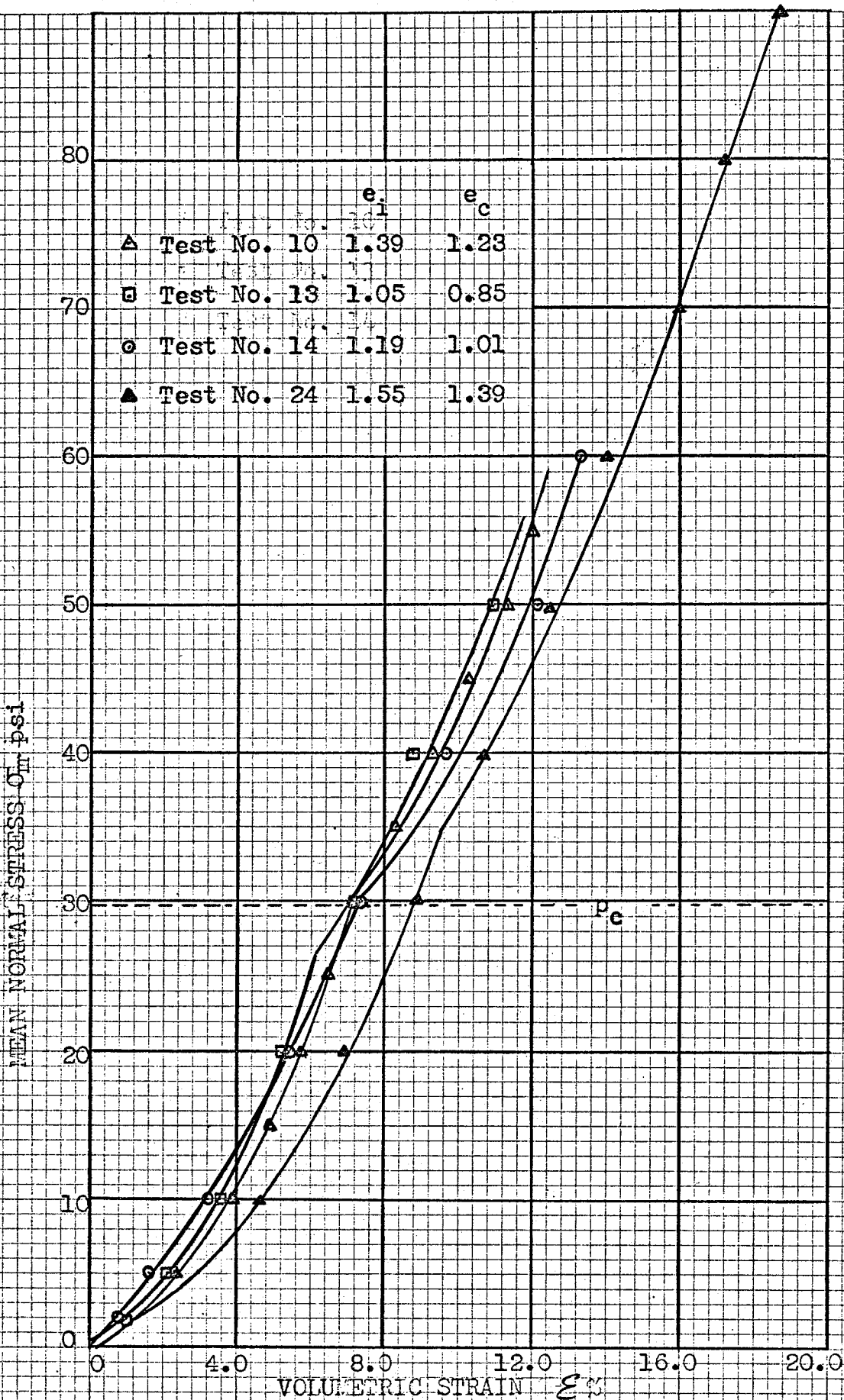


Figure 6. Volumetric Stress-Strain Relationship for Block B.

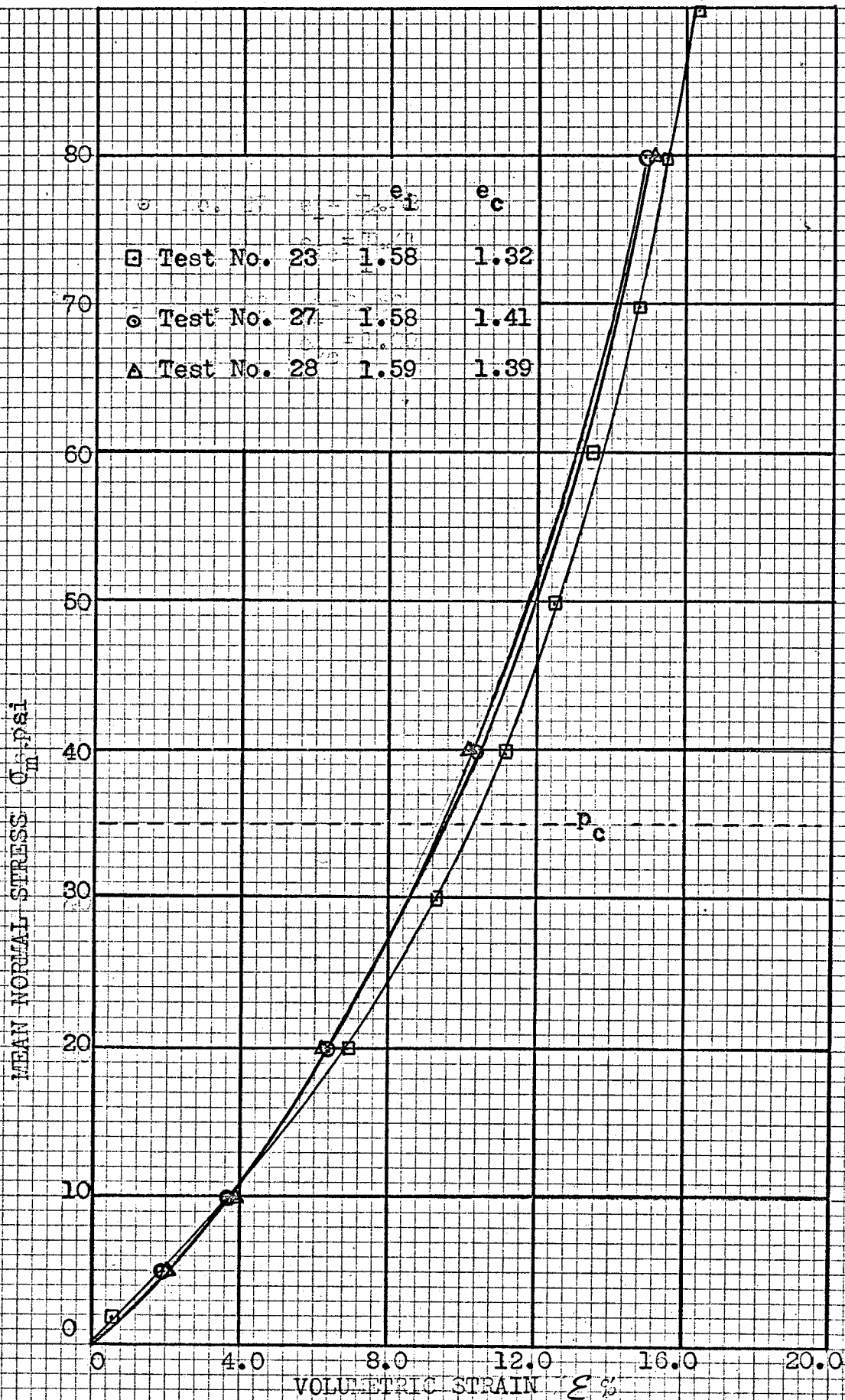


Figure 7. Volumetric Stress-Strain Relationship for Block D.



Figure 8. Comparison of Volumetric Stress-Strain Relationships for Blocks A, B, C and D.

normally consolidated state.

Further investigation into the effect of the pre-consolidation pressure on the stress-strain relationship was carried out in test numbers 17 and 18. In Test number 17, (Figure 9), the sample was isotropically consolidated to 30 psi and allowed to rebound to zero mean normal stress. Upon reconsolidation to 60 psi, a break in the curve at 30 psi exists, but it is not distinct. In test number 18, (Figure 10) the sample was consolidated to 50 psi, allowed to rebound to zero mean normal stress and reconsolidated to 60 psi. Reconsolidation showed the volumetric stress strain curve to be smooth and concave upward to 50 psi, at which point a slight change in the curves occurs and the slope decreases somewhat. The effect of the level of previous consolidation is not as apparent as in Figures 4, 5, and 6.

In these tests cyclic loading has apparently lessened the break in the curve. The material behaves essentially elastically since the reconsolidation branch is the same shape as the curves shown in Figures 4 to 6.

Tests 17 and 18 indicate that a permanent volumetric strain exists in the material after initial consolidation and subsequent stress release.

The bulk modulus was computed for small increments of mean normal stress and corresponding increments of volumetric strain for each test. The results were plotted in the form of bulk modulus versus mean normal stress curves in Figures 11 to 14. Since the bulk modulus - volumetric stress



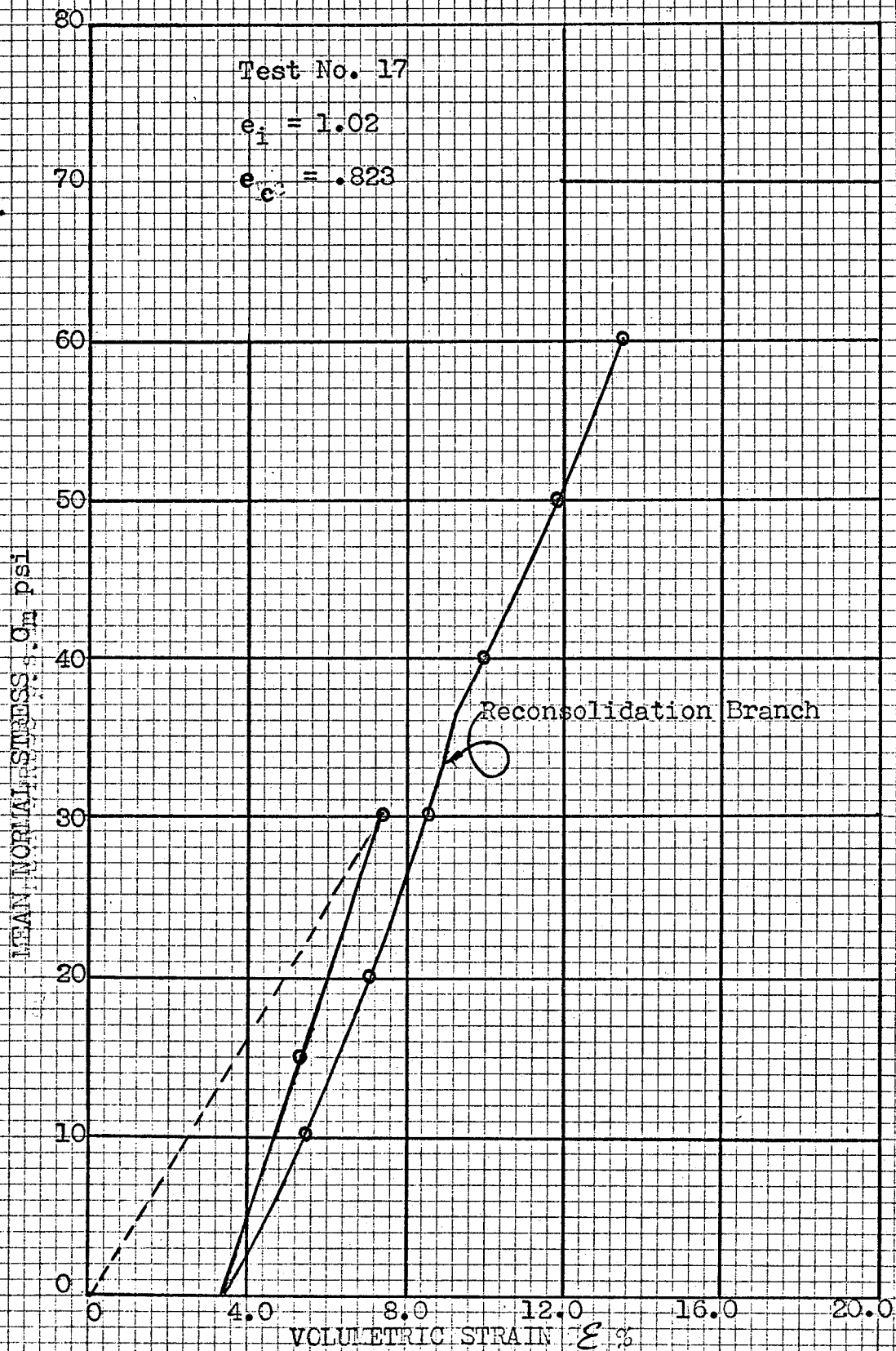


Figure 9. Reconsolidation of Sample 17, Block B.

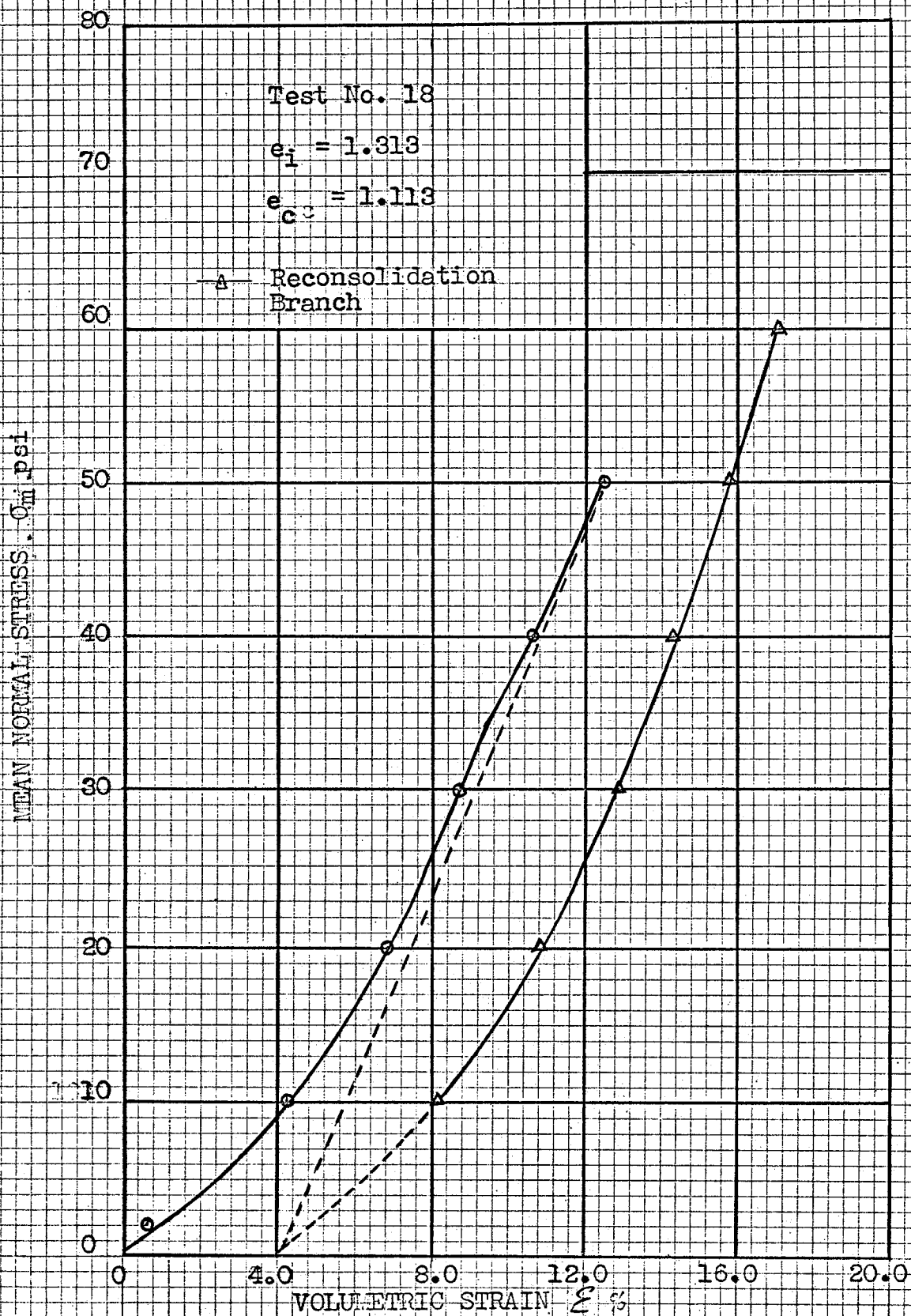


Figure 10. Reconsolidation of Sample 18, Block B.

relationship for individual blocks was very nearly the same the average curves for K from tests on Block A, B, C and D are shown in Figures 11 to 14. These figures exhibit the variation in bulk modulus with the variation in mean normal stress and soil properties.

Due to the affect of stress history and the variation with stress level there does not appear to be any simple mathematical relationship by which the bulk modulus can be defined. There does appear to be a consistency in the general shape of the bulk modulus versus mean normal stress curves.

#### Bulk Modulus Analysis

The solution for bulk modulus is given by:

$$K = \text{Limit}_{\epsilon \rightarrow 0} \frac{\Delta \sigma_m}{\Delta \epsilon}$$

and may be computed from the mean normal stress versus volumetric strain data. In the tests performed, the volume change was measured at each increment of mean normal stress.

In the isotropic compression tests the mean normal stress is equal to the cell pressure. The volumetric strain was computed from:

$$\epsilon = \Delta V / V_0 \quad \dots\dots(8)$$

in which  $\Delta V$  is the volume change and  $V_0$  is the initial volume of the sample.

Figures 11 to 14 show the bulk modulus for each block of soil studied. The bulk modulus for Block A is approximately 250 psi at low mean normal stress and rises to

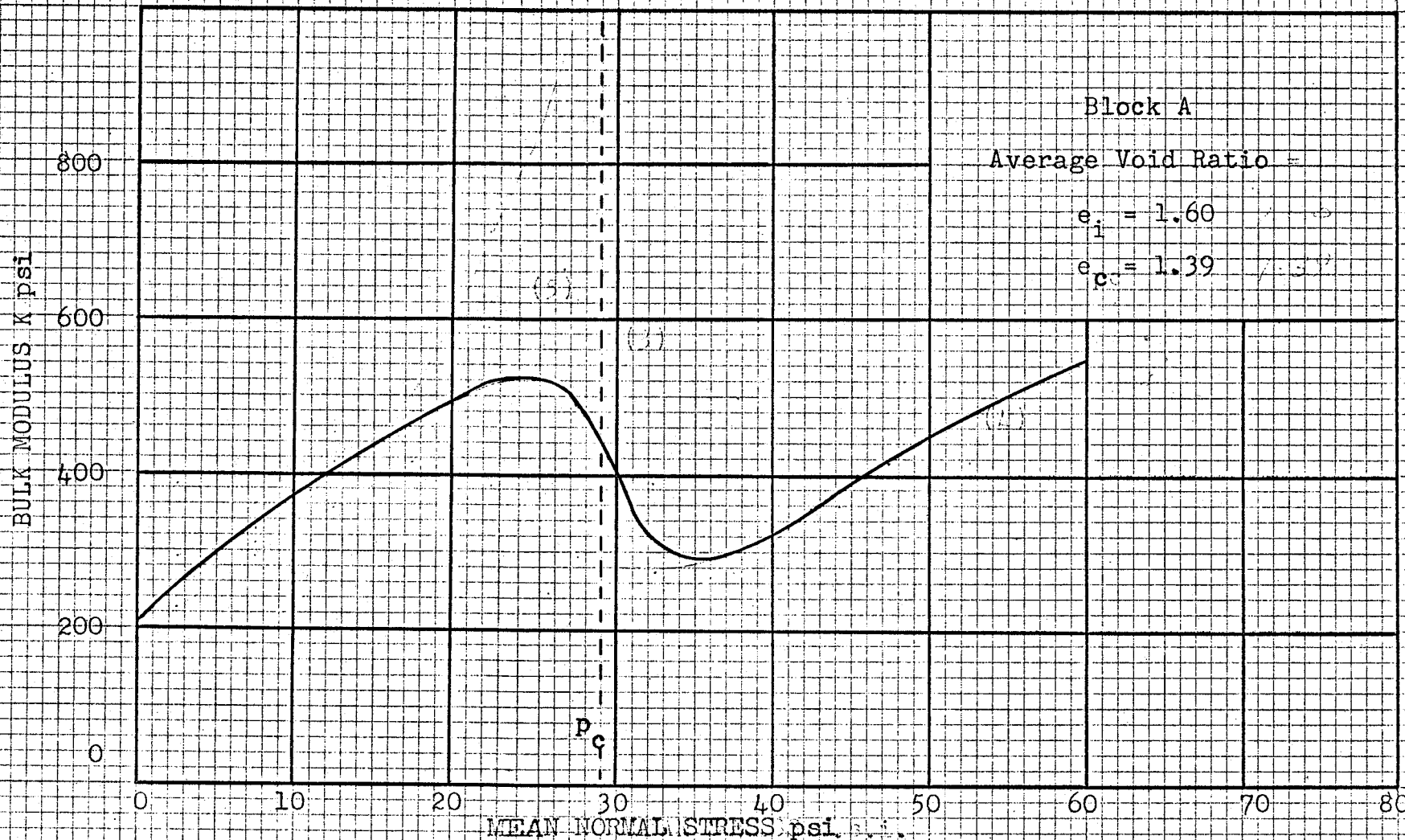


Figure 11. Bulk Modulus - Mean Normal Stress Relationship for Block A.

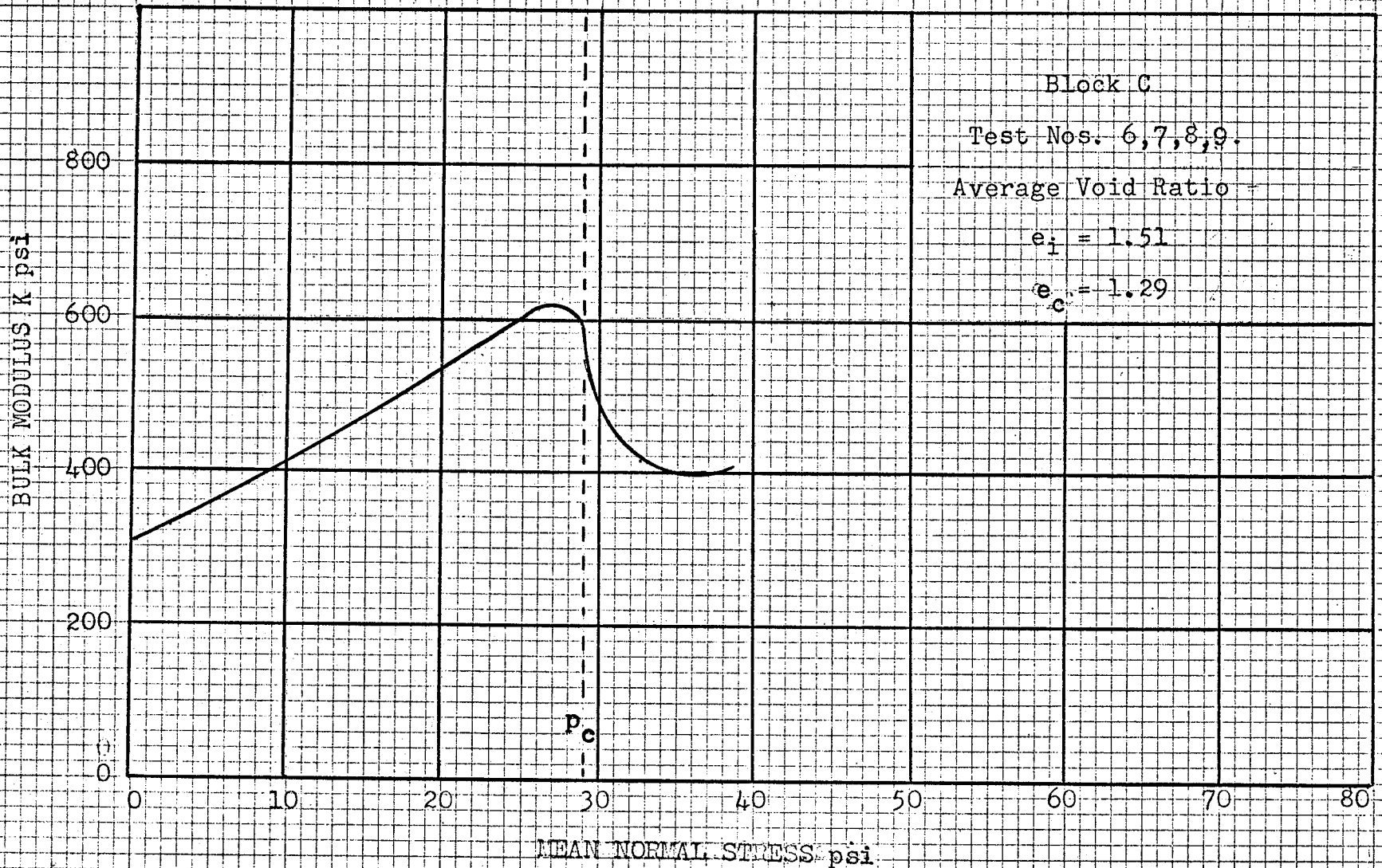


Figure 12. Bulk Modulus - Mean Normal Stress Relationship for Block C.

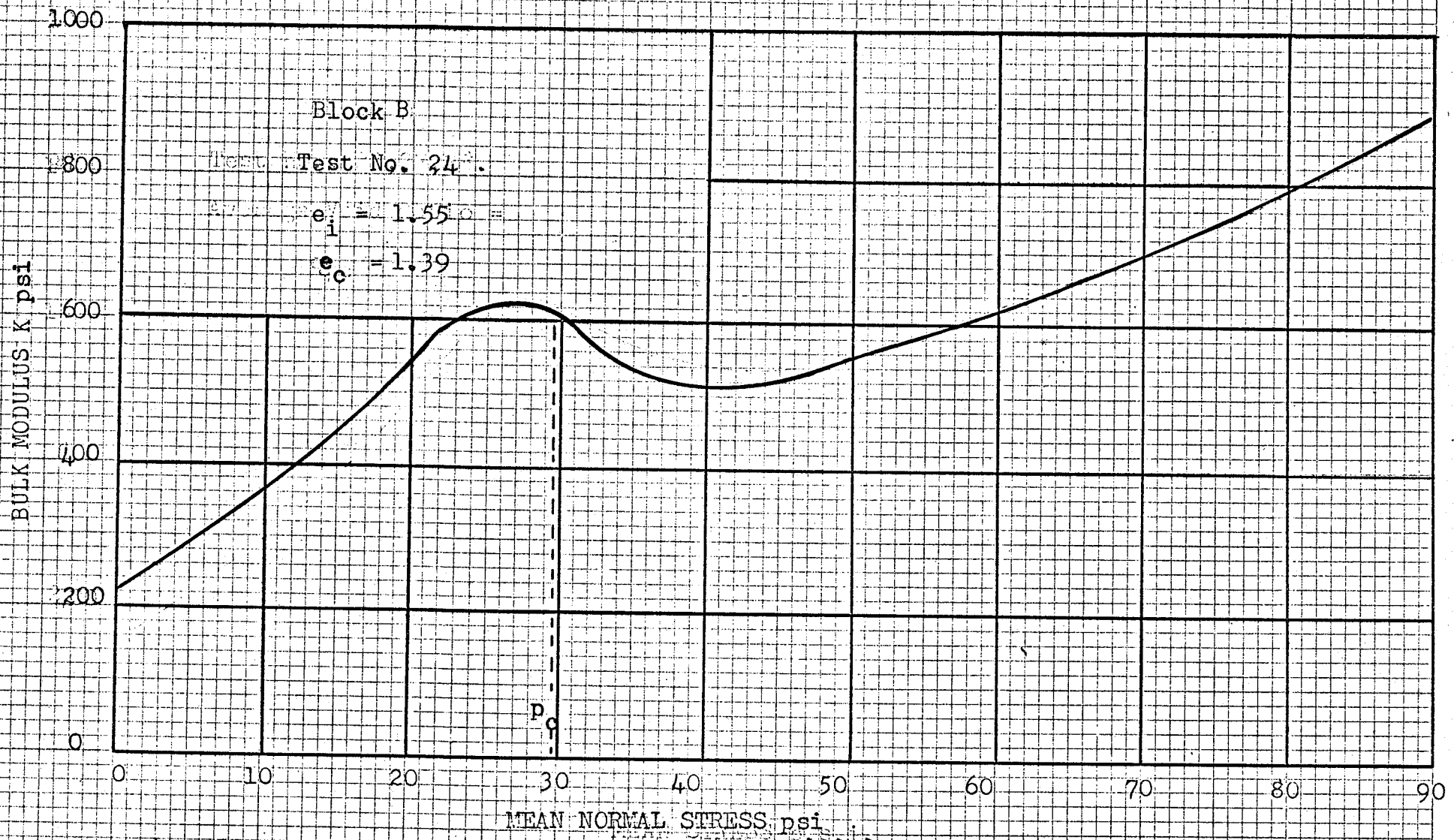


Figure 13. Bulk Modulus - Mean Normal Stress Relationship, for Block B

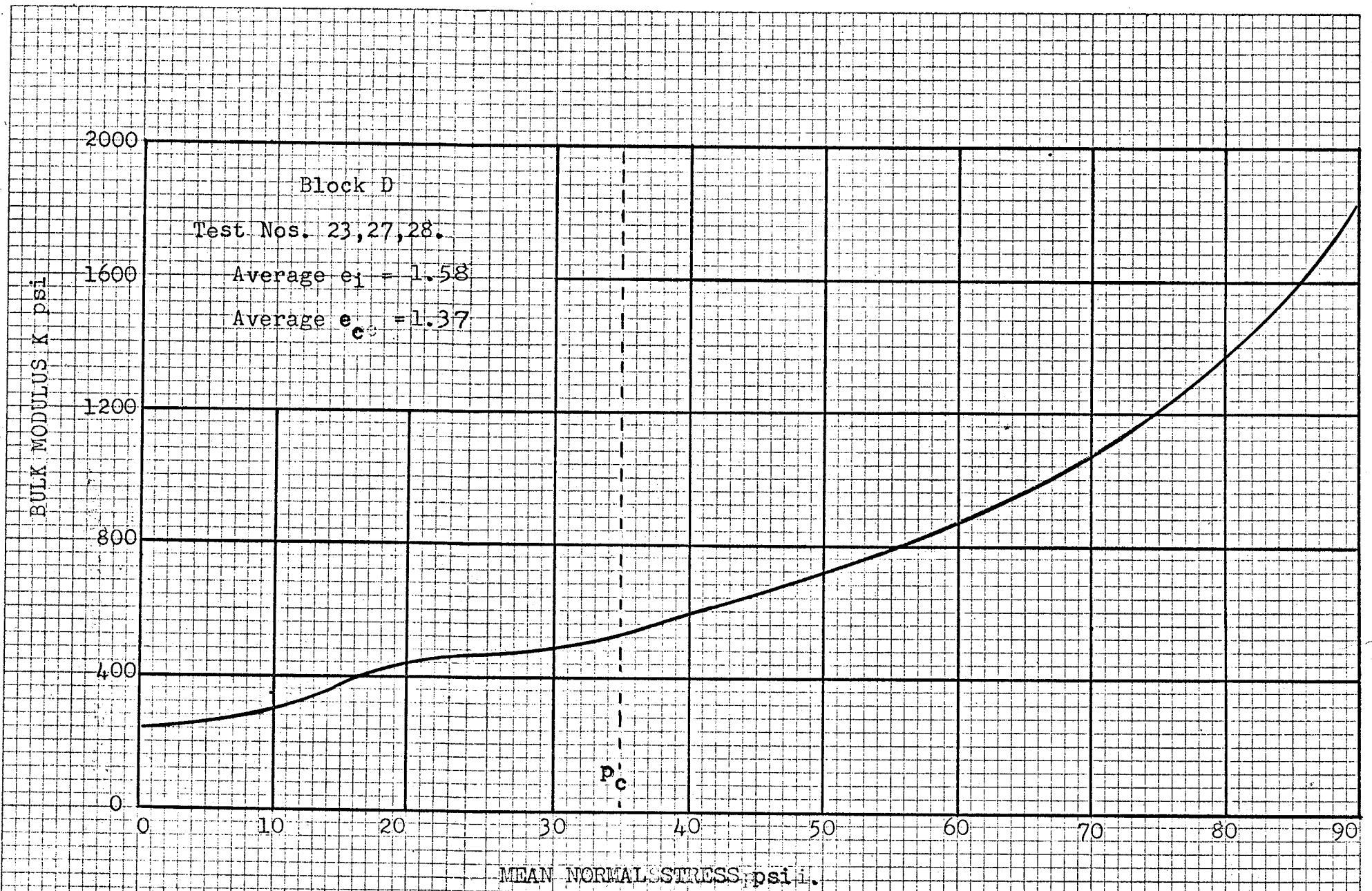


Figure 14. Bulk Modulus - Mean Normal Relationship for Block D.

500 psi below the preconsolidated pressure. Above the preconsolidation pressure the value of  $K$  drops to about 300 psi. and then rises constantly with  $K$  equal to 550 psi at a mean normal stress equal to 60 psi. The bulk modulus for Blocks B and C is approximately 50 to 100 psi higher than Block A. Block D has a bulk modulus of about 350 psi at low mean normal stress and rises to a value of 1800 psi at  $\sigma_m$  equal to 90 psi.

The effect of the mean normal stress level is apparent in all tests. As the stress level in an overconsolidated sample increases the bulk modulus increases. There is an apparent reduction in the magnitude of the bulk modulus at a point just above the preconsolidation pressure. In a normally consolidated state the bulk modulus increases with an increase in stress level.

The soil structure plays an important part in the stress-strain behavior of the soil. The type of structure has a pronounced effect on the soils resistance to deformation and hence on the valence change which accompanies a change in the isotropic stress level.

Due to differences in soil structure the bulk modulus for the brown clay (Block D) is different from that determined for the grey clay (Blocks A, B, and C).

Stress history plays the most important role in the behavior of the cohesive Lake Agassiz Clay. The overconsolidation of the material by desiccation may cause a particle re-orientation. Stress relief permits rebound of



the soil so that initially, under reconsolidation the bulk modulus is low. The bulk modulus increases sharply, however, as the material returns to its preconsolidated state. Beyond the preconsolidation pressure a normally consolidated state is reached and with increased stress the material structure may be destroyed. The magnitude of the bulk modulus decreases initially due to the breakdown of the structure but continues to increase as the state of densification is increased.

The physical properties of the soil also affect the bulk modulus. As noted previously initial low void ratios result in a higher bulk modulus. This is shown in the Figures 11, 12 and 13 by comparing the bulk modulus of Block A with that determined for Blocks B and C at the same mean normal stress. Block A, having higher void ratios, exhibits lower initial values of bulk modulus. The void ratio appears to affect the bulk modulus more at lower values of mean normal stress.

### Conclusions

The bulk modulus study shows that the bulk modulus of the Agassiz Clay is dependent on the mean normal stress, the stress history, the soil structure and the void ratio.

The bulk modulus generally increases with an increase in mean normal stress. This behavior is only disrupted by the effect of the preconsolidation pressure which causes a slight decrease in bulk modulus at a mean normal stress just above the preconsolidation pressure.

For a given material, the variation in void ratio affects the bulk modulus at low values of mean normal stress. At higher stresses the effect of the void ratio is not very significant.

The soil structure is important in its effect on the bulk modulus. The chocolate brown varved clay (Block D) has a significantly higher bulk modulus than the massive grey clay (Blocks A, B and C) in the normally consolidated state. This indicates that the variations in clay type should be represented by a full set of curves. The total family of curves would represent the bulk modulus for the Agassiz clays.

## CHAPTER IV

## SHEAR MODULUS STUDY

The purpose of the shear modulus study was to determine the shear modulus of Lake Agassiz Clay and to determine the factors on which the shear modulus depends.

The shear modulus which is defined by equation(7):

$$G = \lim_{\epsilon_d \rightarrow 0} \frac{\Delta S_d}{\Delta \epsilon_d}$$

represents the slope of the deviatoric stress strain curve.

In order to obtain a deviatoric stress strain relationship for the Agassiz Clay, drained triaxial shear tests were run at constant values of the mean normal stress.

#### Soil Used In The Investigation

The soil used in the investigation was the gray massive clay which was used in the Bulk Modulus study and has been described fully in Chapter III. The tests were carried out on undisturbed samples taken from three block samples, Blocks A, B and C. The properties of these Blocks are shown in Table I, page 22.

#### Test Apparatus

The drained triaxial tests were run using standard triaxial cells. The samples used were 3 inches in length and 1.5 inches in diameter.

Water was used as the cell fluid and pressure was supplied

by a regulated air supply.

Volume changes in the samples were measured by a mercury U-tube device as described by Bishop and Henkel<sup>7</sup> (1962). The closed burette type of volume change device described by Bishop and Henkel<sup>7</sup> (1962) was also used.

Air cylinders were used to increase the vertical stresses on the samples during the shearing portion of the test. One air cylinder had a piston diameter of 1 inch, another had a piston diameter of  $2\frac{1}{4}$  inches. The air cylinders were calibrated in both a stationary and moving situation and were checked to ensure that the pressure load characteristics had not changed. The calibration curves for the cylinders are shown in Figures 43 and 44 in Appendix III.

Pressure was supplied to the air cylinder through a pressure regulator and was read from a bourdon gauge to the nearest 0.1 psi. The associated accuracy in vertical stress on the sample was  $\pm 0.08$  psi using the 1 inch diameter air cylinder and  $\pm 0.4$  psi using the  $2\frac{1}{4}$  inch diameter cylinder.

A schematic diagram of the testing apparatus for the shear modulus tests is shown in Figure 15 while Figure 16 shows the actual test setup.

#### Testing Procedure

Soil testing membranes were used with O-rings providing the seal at the ends of the sample. Before installing the membrane, silicone grease was applied to the base pedestal and

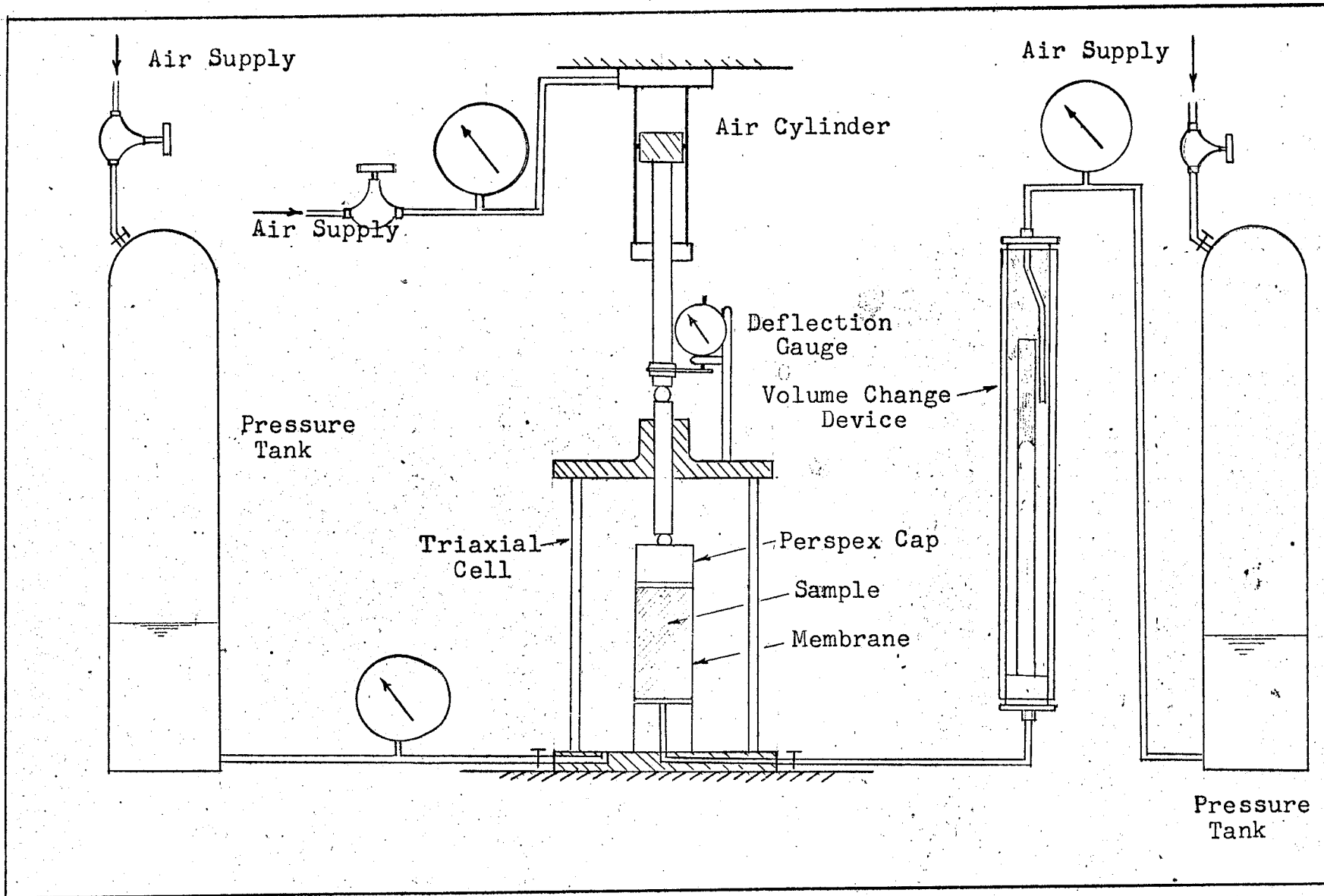


Figure 15. Test Apparatus

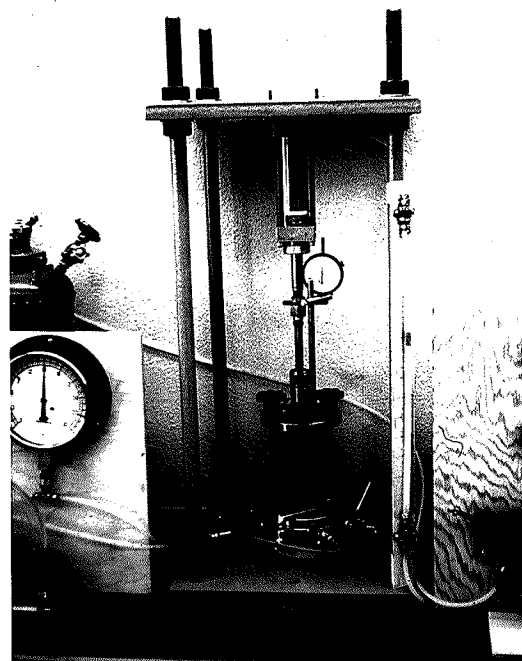


Figure 16. Test Apparatus.

the loading cap to help prevent leakage. A perspex loading cap was used and a porous stone was placed on either end of the sample. Bottom drainage was provided with filter paper strips placed vertically on the sides of the sample. Care was taken to ensure that the porous stones and filter strips were saturated and that no air was trapped between the membrane and the sample.

Initially an equal back pressure and cell pressure was applied to the sample to ensure saturation. The back pressure was provided through the volume change device. The back pressure and the cell pressure were raised in equal increments to the desired maximum level of back pressure, usually 15 psi. The test remained at this stage until the sample became completely saturated. The readings on the volume change device were plotted against time on a log scale to determine when the sample was saturated. The time period required to saturate a sample of Lake Agassiz Clay was generally 24 hours. Approximately two to three ml. of water were required to saturate each sample.

After saturation was complete the cell pressure was increased in increments of 5 psi until the mean normal stress acting on the sample was equal to the level of the preconsolidation pressure. At each increment of pressure, the sample was allowed to drain completely and thus consolidate fully at that stress. The volume change corresponding to each stress level was recorded. Having reached the preconsolidation pressure the cell pressure was increased or reduced to the desired level for the test.

In two tests the stress level was increased in increments of 5 psi to a mean normal stress of 42 psi and 60 psi. The shear test was run at these levels of constant mean normal stress. Four tests were run at a constant mean normal stress of 30 psi which is approximately equal to the preconsolidation pressure. In ten tests the mean normal stress was lowered, before starting the shear tests, placing the sample in an over-consolidated state. In each of these ten tests the sample was allowed to reach equilibrium before starting the shear test. The new volumes and the volume increases were recorded. The vertical displacement of each sample was measured and recorded to be used in computing the sample area.

In the shear test the vertical loads were applied in increments of 1 psi and the confining pressure was reduced by one half the amount of the vertical increment. The net effect was to keep the mean normal stress constant. The sample was allowed to drain and consolidate after each load increment. The volume change and vertical displacement of the sample was recorded after the sample had consolidated under each increment. This procedure was carried out until the sample failed.

### Test Results

Data from one complete shear test included the initial volume of the sample; the initial sample dimensions; the level of the mean normal stress; the net volume change caused in consolidating to the mean normal stress, the vertical stress



increments, cell pressure decrements and the resulting volume changes and vertical displacements. From this data the initial and final degree of saturation, initial and final void ratios, and the void ratio at the preconsolidation pressure were determined. This data is given in Table III, Appendix II. The deviatoric stresses were computed from equations (1) to (3). The deviator strains and the resultant deviatoric strains were computed using equations (5).

In the computations of stresses and strains the following sign convention was adopted. Compressive stresses and compressive strains are considered to be positive. An increase in volume is also considered to be positive with the implication that increases in dimensions are positive which is inconsistent with the assertion that compressive strains are positive. To achieve consistency, a negative sign was used in equations involving volume change or dimensional changes.

In a triaxial test the strain component in the direction of the major principle stress will be:

$$\epsilon_1 = - \frac{\Delta h}{h_0} \quad \dots\dots(9)$$

in which  $\Delta h$  is the change in height of the sample and  $h_0$  is the initial height of the sample.

The minor and intermediate principal strains are assumed to be equal. Hence  $\epsilon_2 = \epsilon_3$ .

The components  $\epsilon_2$  and  $\epsilon_3$  can be computed from the equation by Ladanyi<sup>8</sup> (1960)

$$\Delta d/d_0 = \sqrt{1 - h/h_0 (\Delta V/V_0 - \Delta h/h_0)} - 1 \quad \dots\dots(10)$$

in which  $d$  is the change in diameter of the sample and  $d_0$  is the initial diameter of the sample. This equation is derived in Appendix I. It also has been shown by Ladanyi (1960)<sup>8</sup> that for small strains the equation can be represented by:

$$\frac{\Delta d}{d_0} = \frac{1}{2} \frac{h}{h_0} \left( \frac{\Delta V}{V_0} - \frac{\Delta h}{h_0} \right)$$

The volumetric strain can be represented by:

$$\varepsilon = - \frac{\Delta V}{V_0} \quad \dots(11)$$

The following relationship is also true for volume change under imposed stresses

$$\frac{\Delta V}{V_0} = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \varepsilon_1 \varepsilon_2 + \varepsilon_1 \varepsilon_3 + \varepsilon_2 \varepsilon_3 + \varepsilon_1 \varepsilon_2 \varepsilon_3 \dots(12)$$

The higher orders are usually neglected and the equation is approximated by:

$$\frac{\Delta V}{V_0} = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 \quad \dots(13)$$

The derivation for this equation is shown in Appendix I.

The corrected area and volume for each test was computed before starting the test. Net volume change and vertical displacement were used in determination of the corrected area. In a saturated sample the new area is determined from the following equation:

$$A_c = A_0 \left( \frac{1 - \Delta V/V_0}{1 - \Delta h/h_0} \right) \quad \dots(14)$$

This corrected area was checked by computing the new area from the change in diameter as given by equation (10).

In order to determine the axial load that was to be applied to increase the resultant deviatoric stress while

simultaneously maintaining a constant mean normal stress, the area of the specimen under the newly applied load had to be predicted. The prediction was based on previously obtained This method gave satisfactory agreement with the areas computed after the application of each increment of loading.

All computations for the shear tests were done using a computer program designed for use on the IBM 360/65. The data is given in Table III, Appendix III.

The test data showing deviatoric shear stress versus deviatoric strain for all tests are shown in Figures 17 through 31. The volume change versus deviatoric strain is also shown on the figure for each test. The solid line represents the mathematical stress-strain relationship which will be discussed later.

Figures 32, 33 and 34 show a summary of the tests for Blocks A, B and C respectively. The results of tests conducted on soft samples are shown in Figure 35 and the results of a test conducted on a stiff sandy clay is shown in Figure 36.

The volume changes were plotted against the deviatoric strain for each test. The upward sloping curve denotes an increase in sample size or positive dilatancy. Examination of these curves gives some idea of the behavior under varying degrees of overconsolidation and the behavior of a normally consolidated clay. The normally consolidated samples show

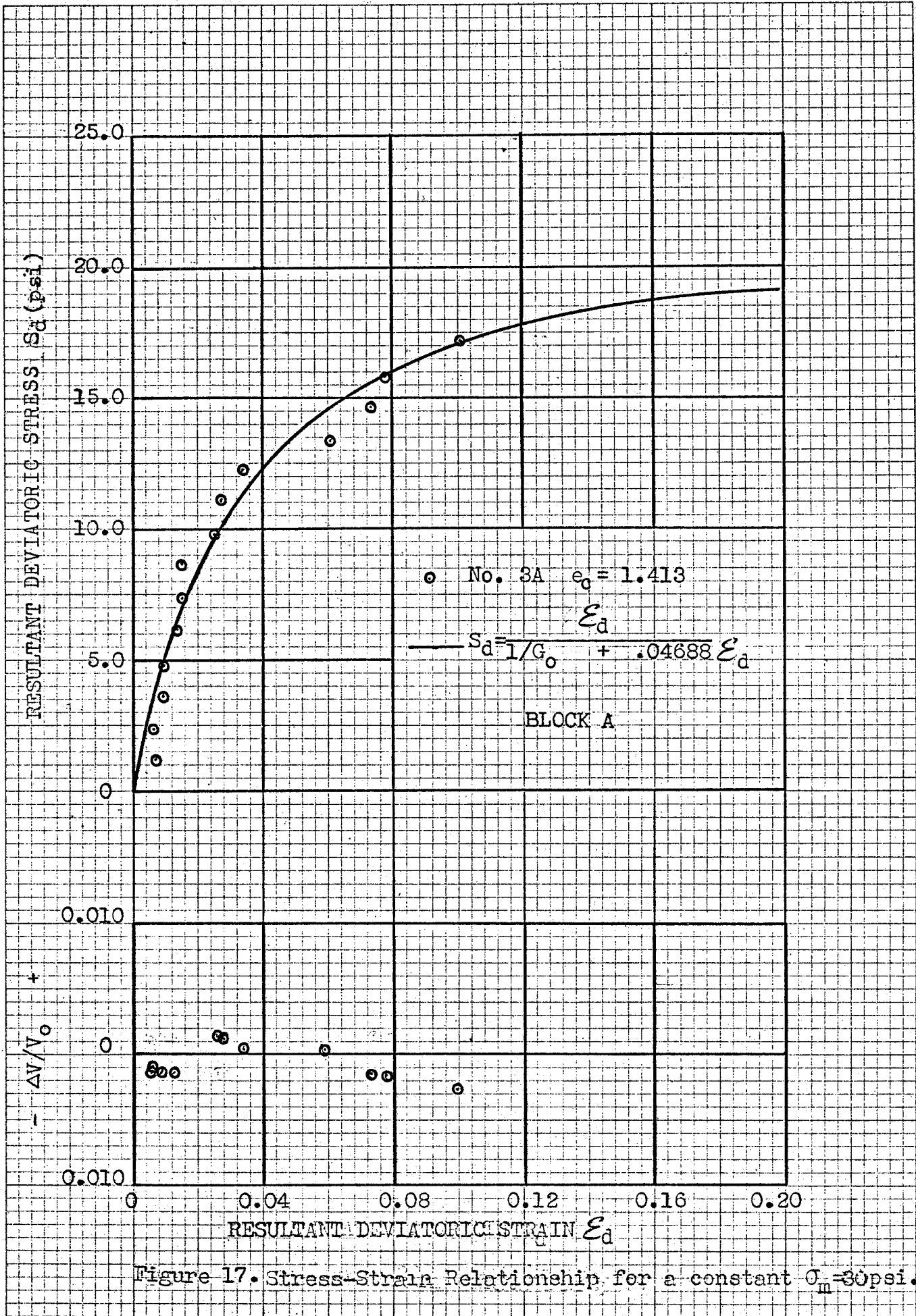


Figure 17. Stress-Strain Relationship for a constant  $\sigma_m = 30$ psi.

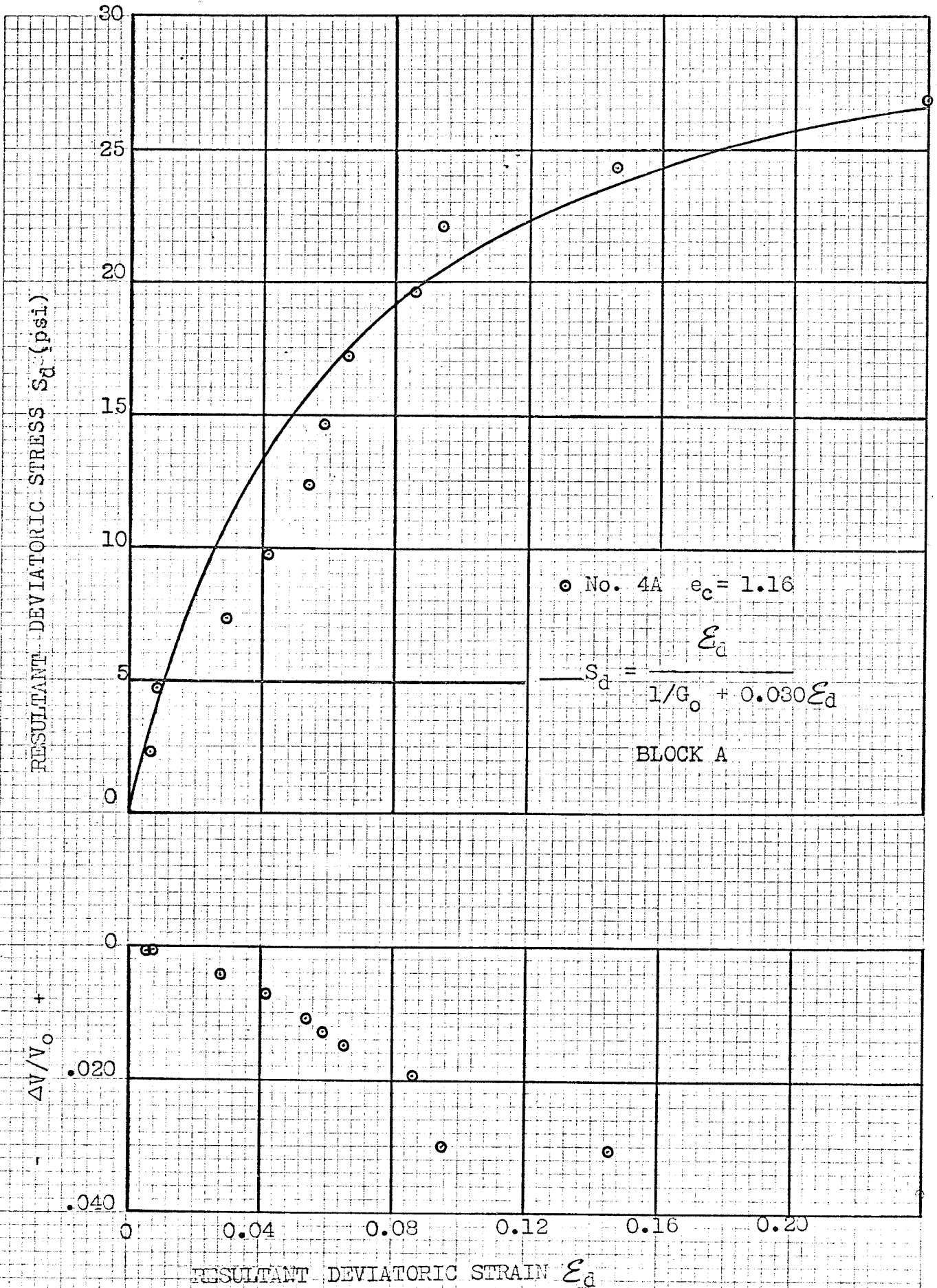


Figure 18. Stress-Strain Relationship for a Constant  $\bar{\sigma}_m = 30$  psi.

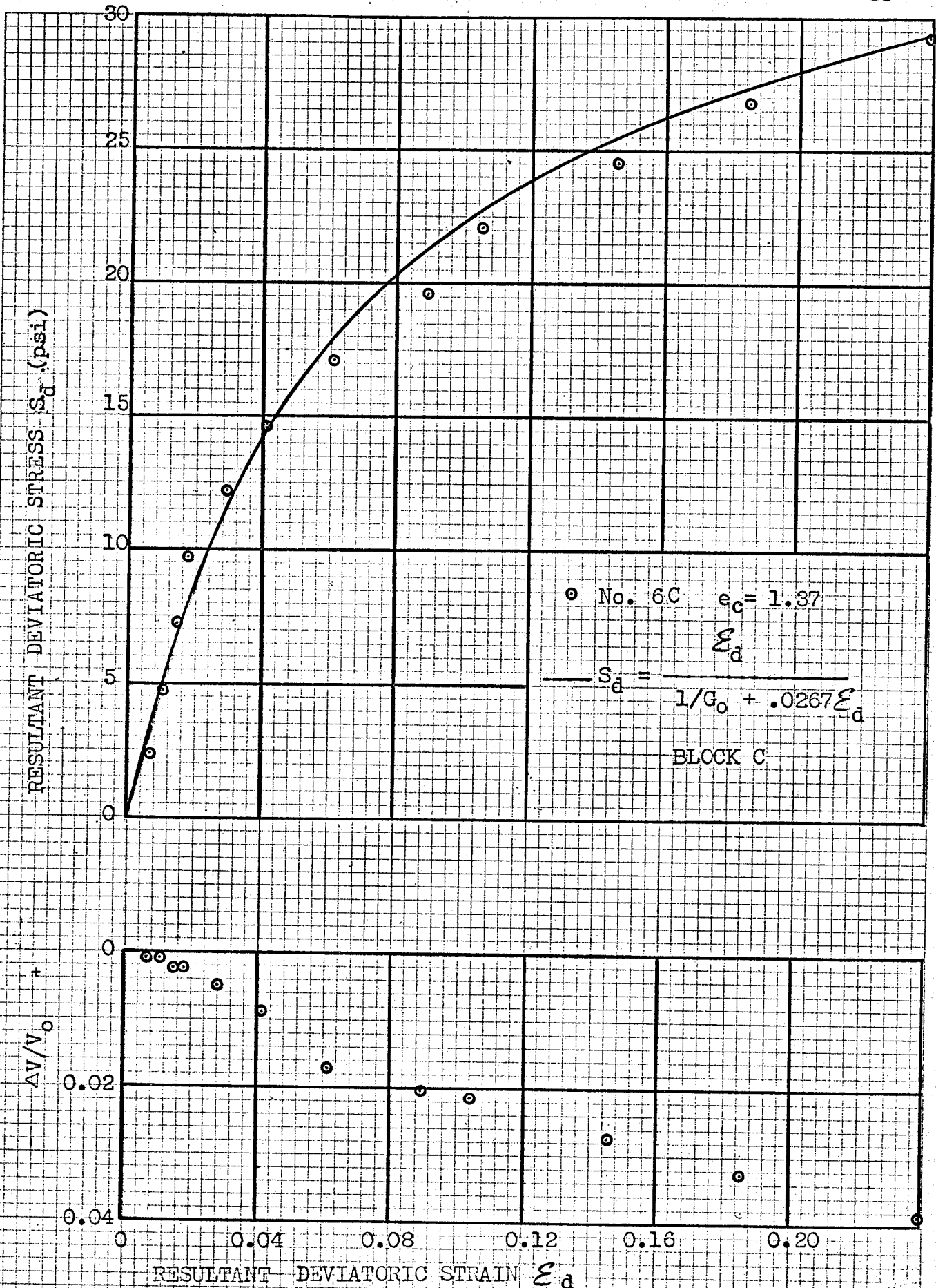


Figure 19. Stress- Strain Relationship for a Constant  $\sigma_m=42$  psi.

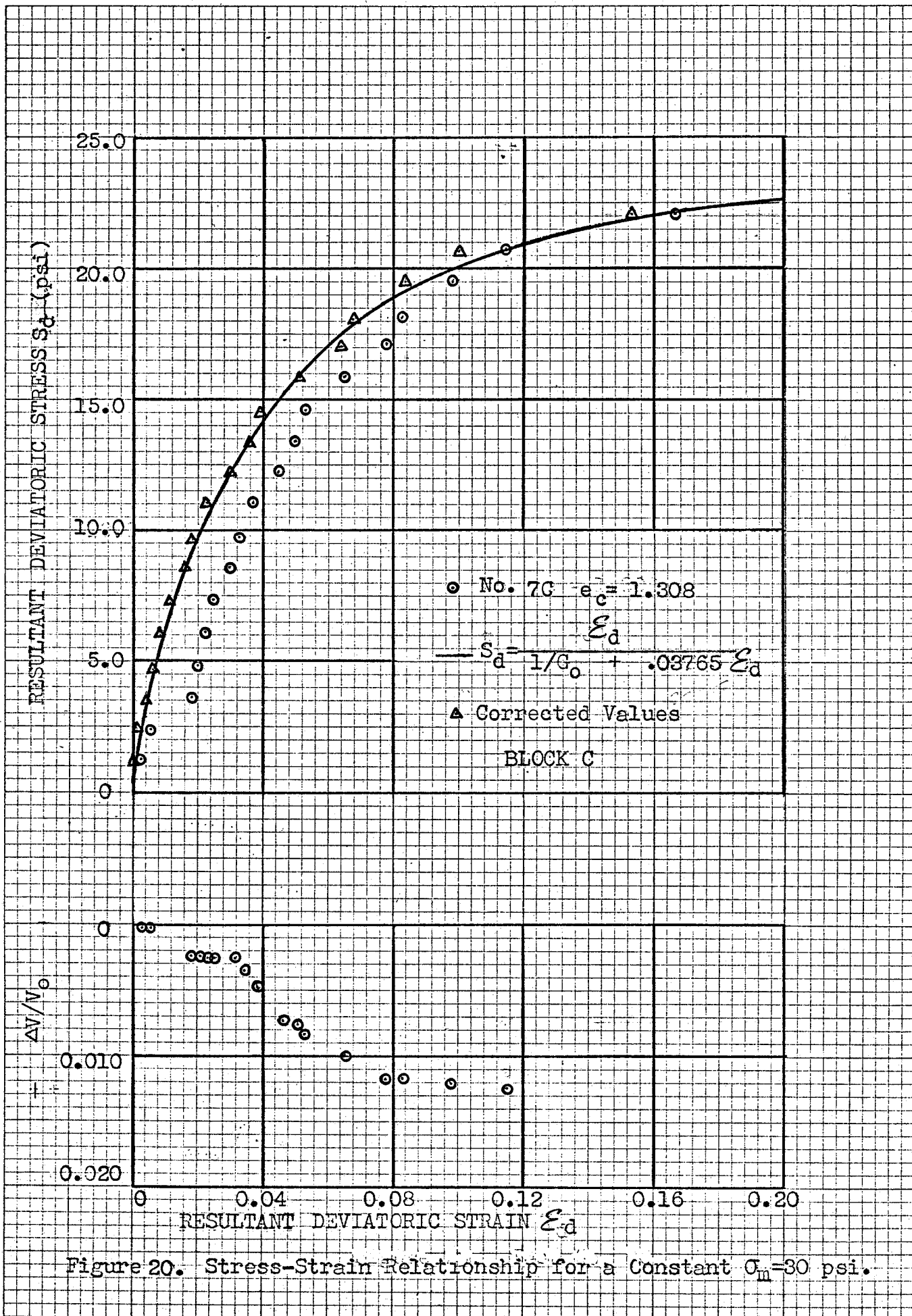


Figure 20. Stress-Strain Relationship for a Constant  $\sigma_m = 30$  psi.

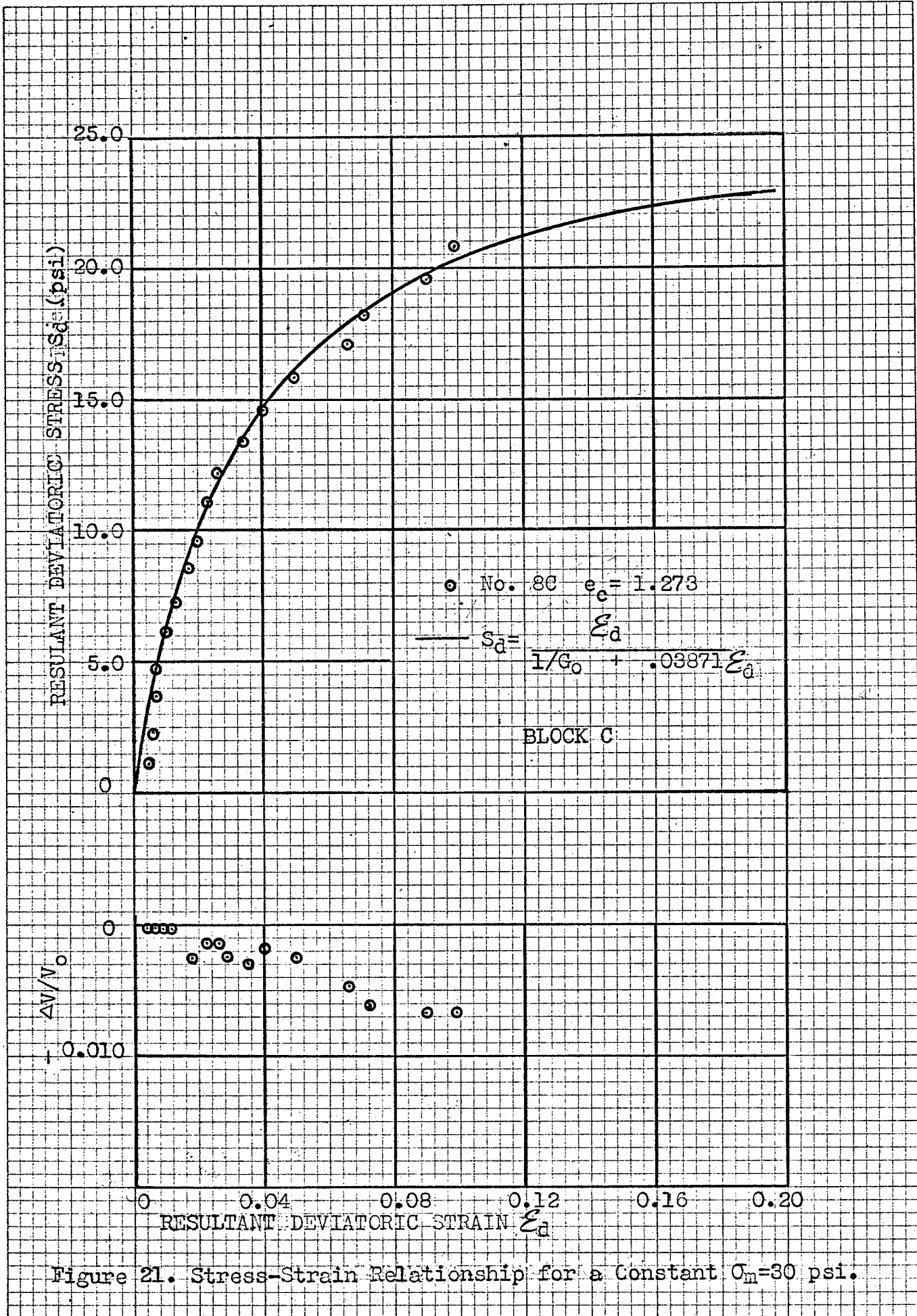


Figure 21. Stress-Strain Relationship for a Constant  $\sigma_m = 30$  psi.



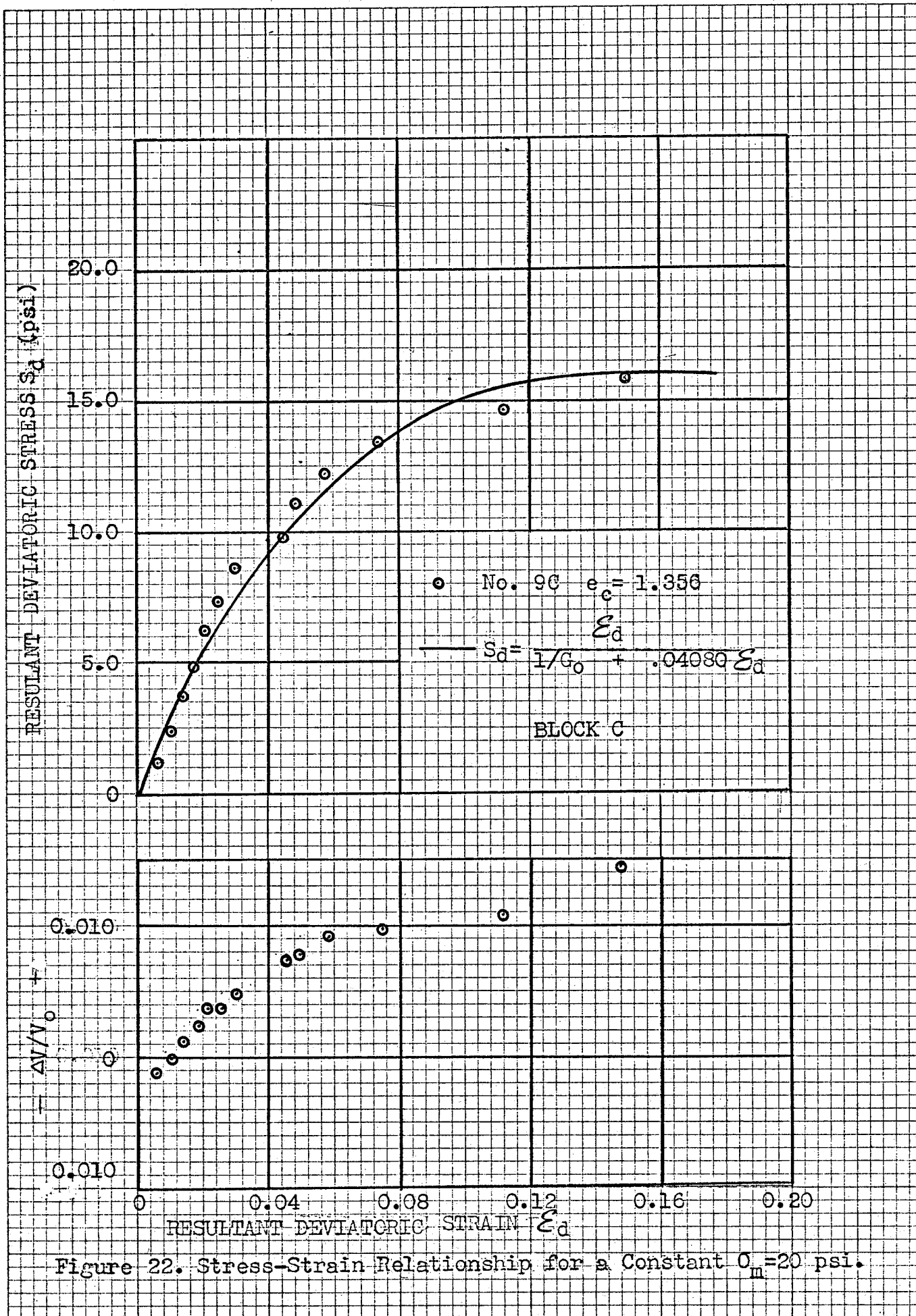


Figure 22. Stress-Strain Relationship for a Constant  $\sigma_m = 20$  psi.

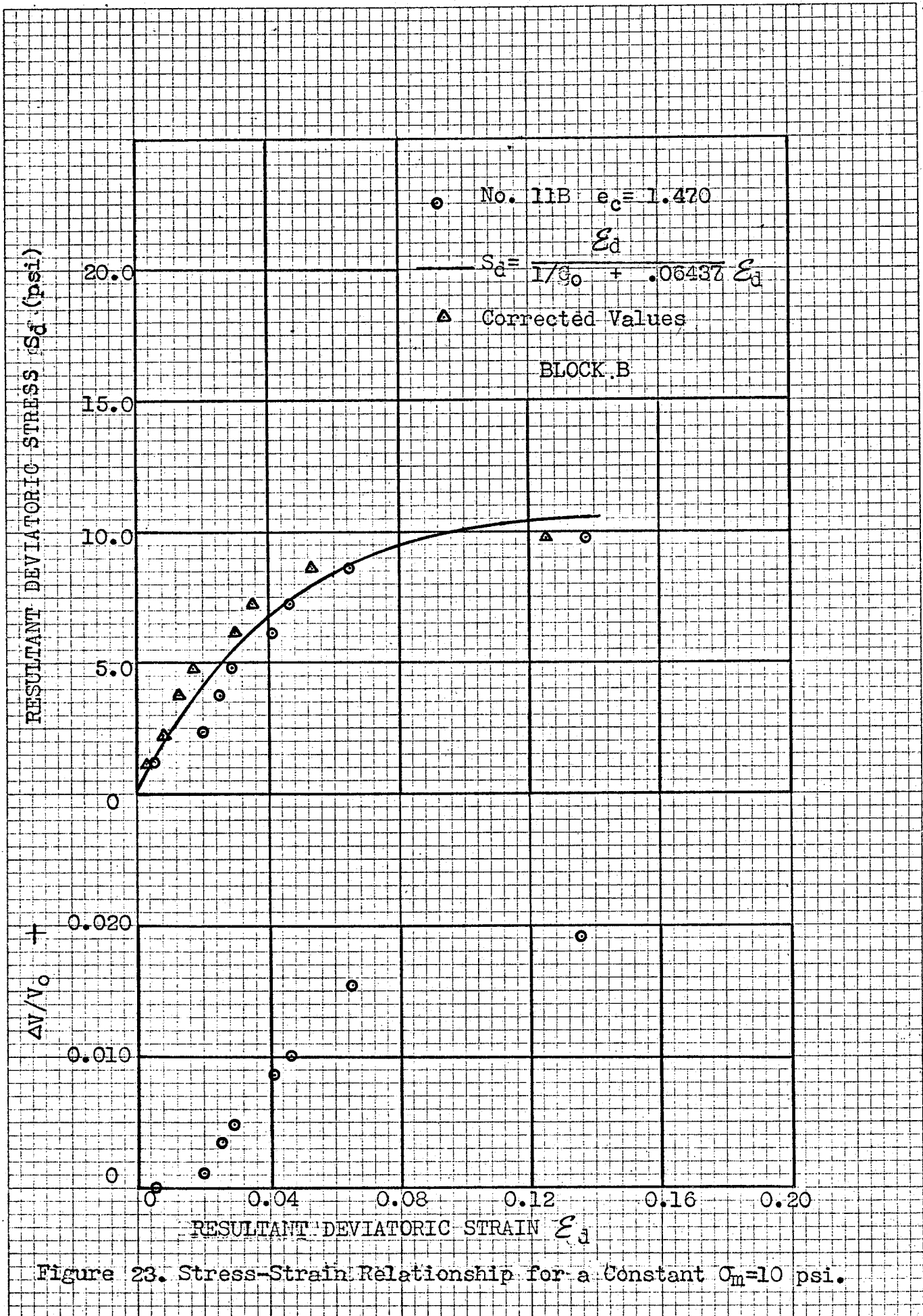


Figure 23. Stress-Strain Relationship for a Constant  $C_m = 10$  psi.

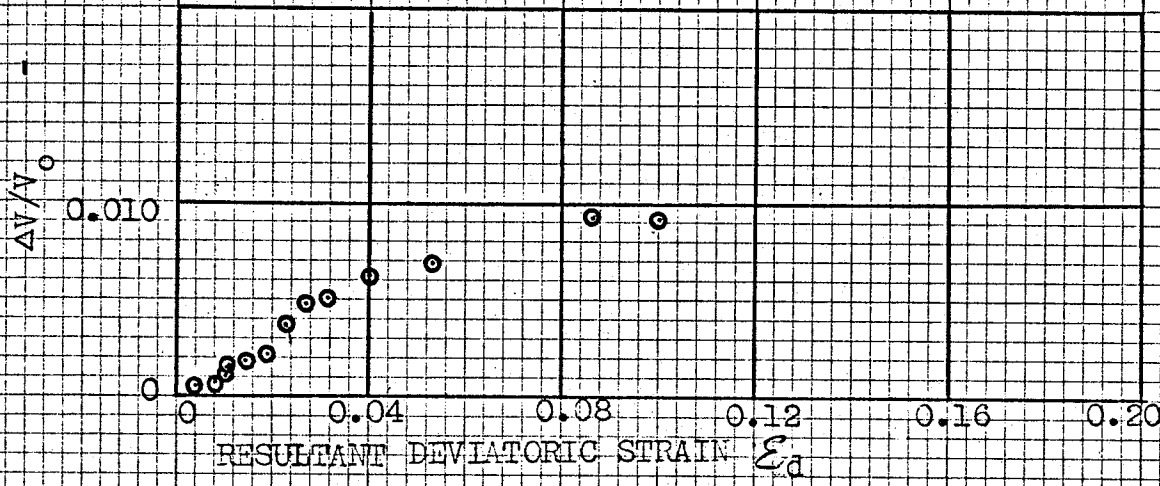
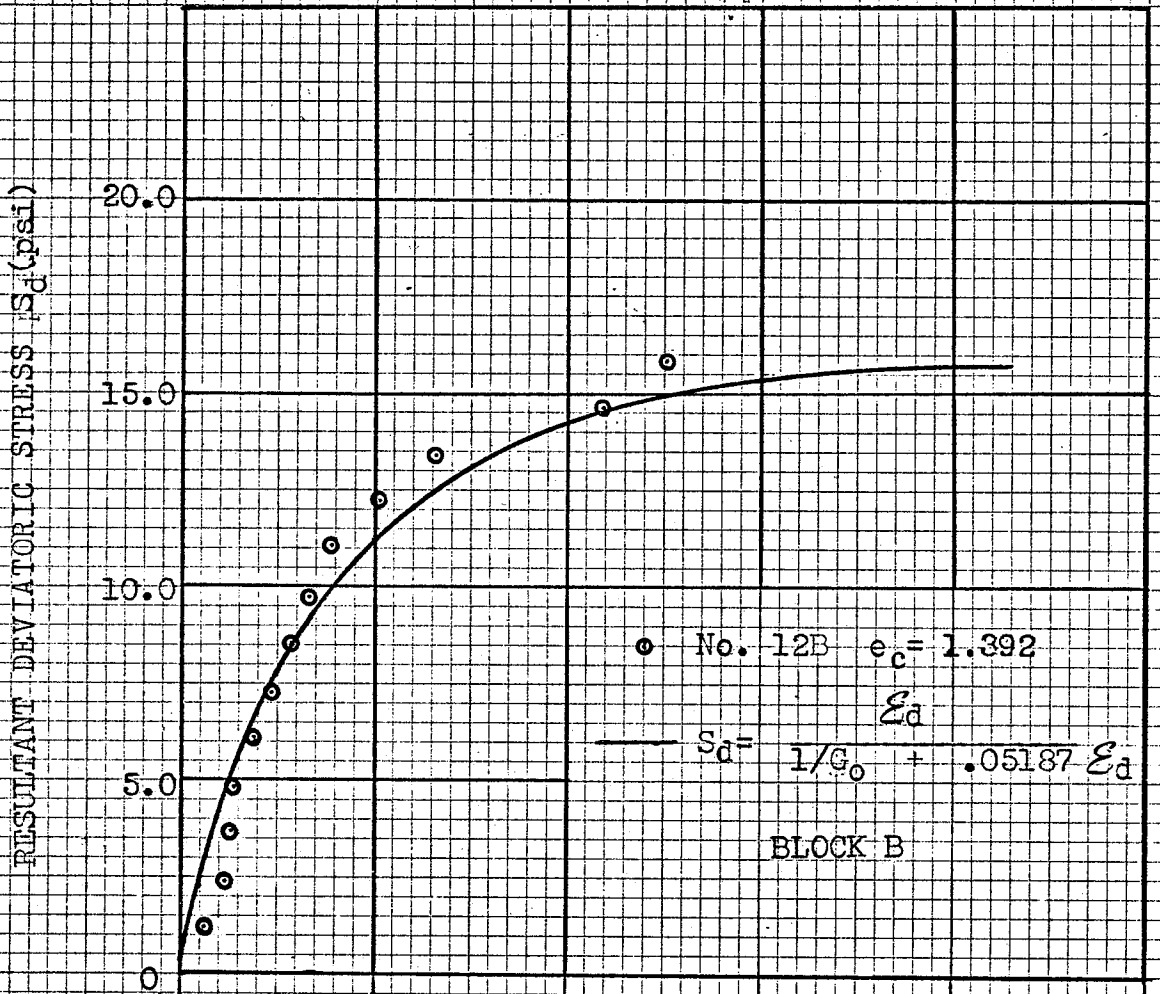
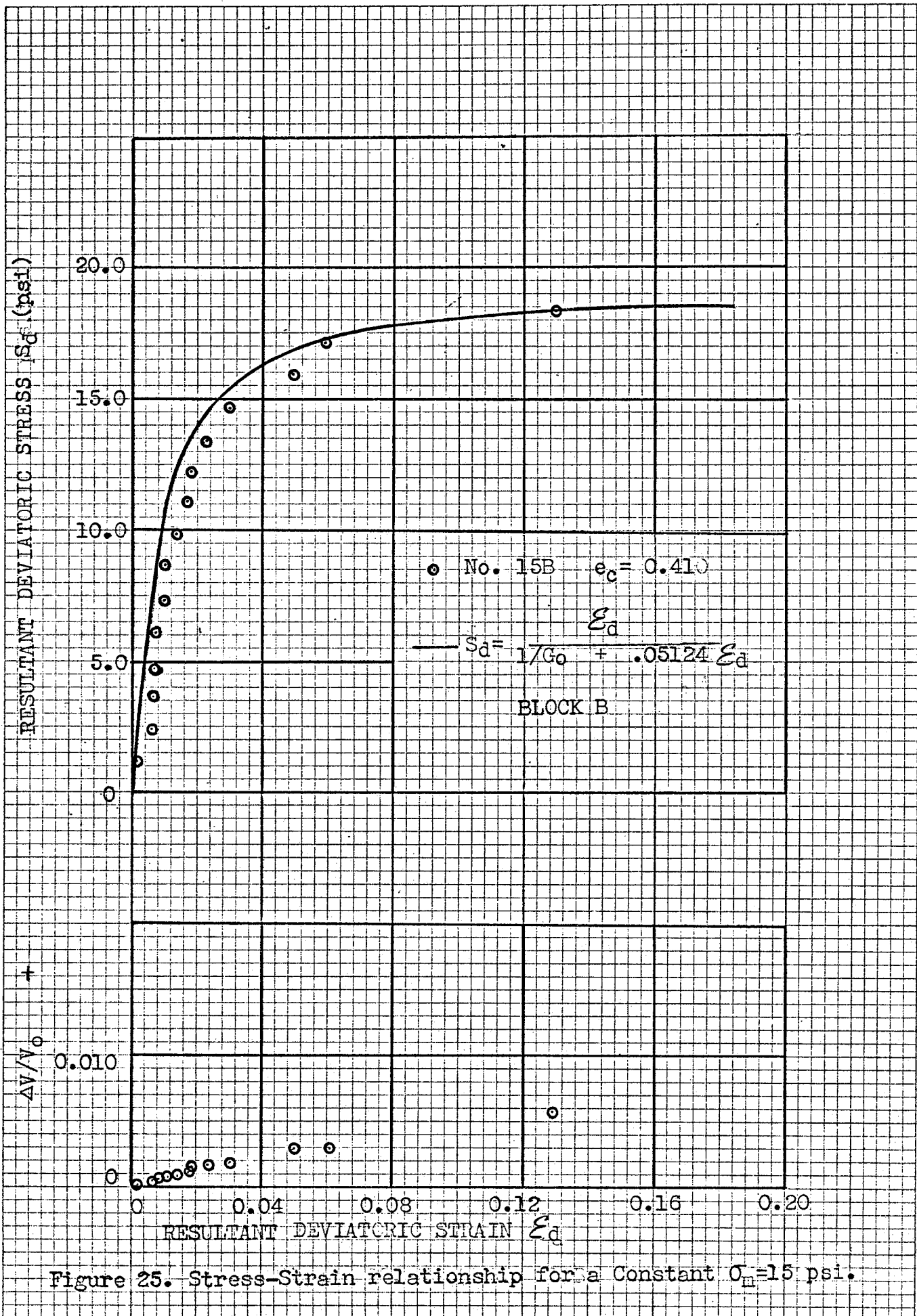


Figure 24. Stress-Strain Relationship for a Constant  $\sigma_m = 15$  psi.



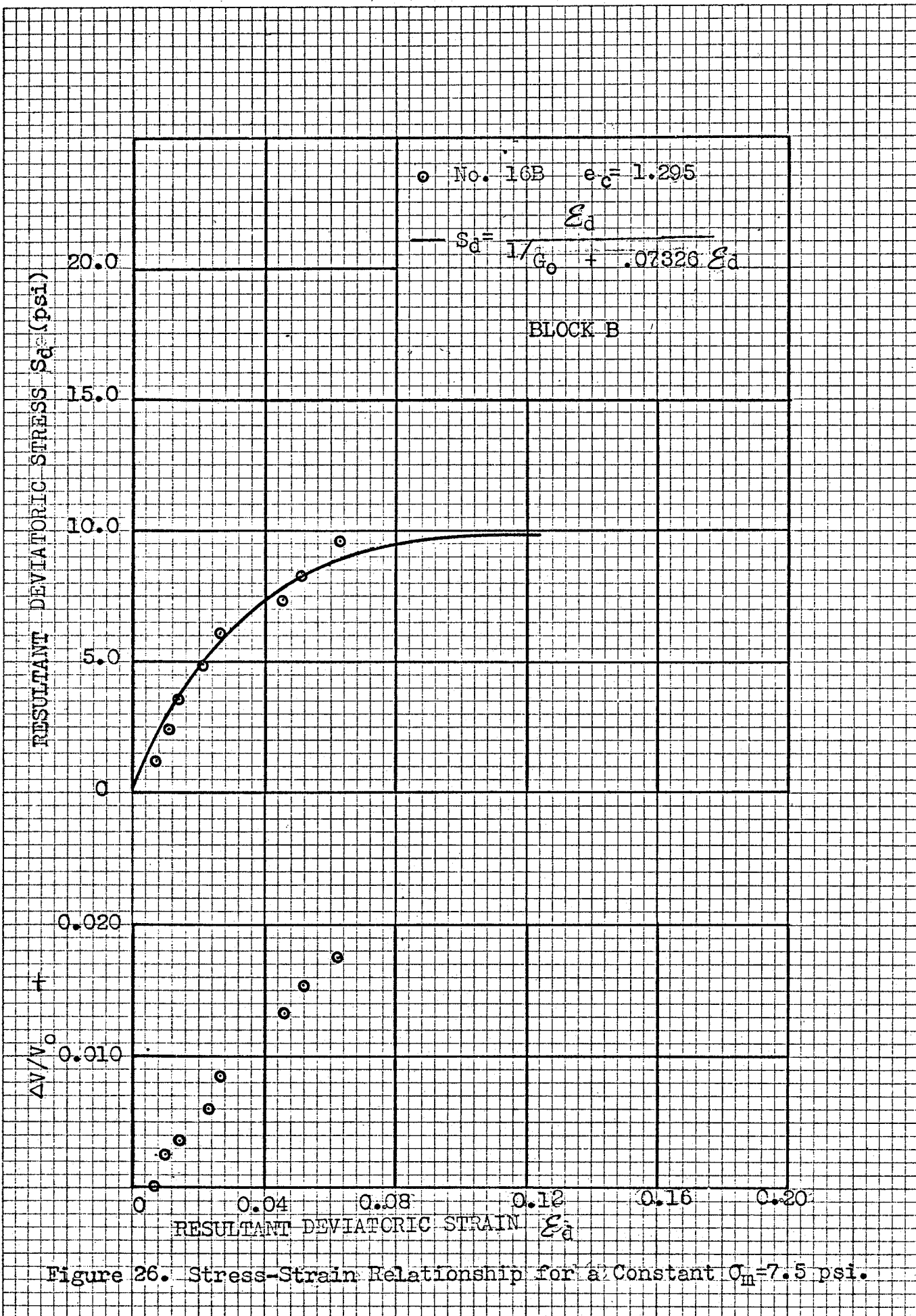


Figure 26. Stress-Strain Relationship for a Constant  $C_m = 7.5$  psi.

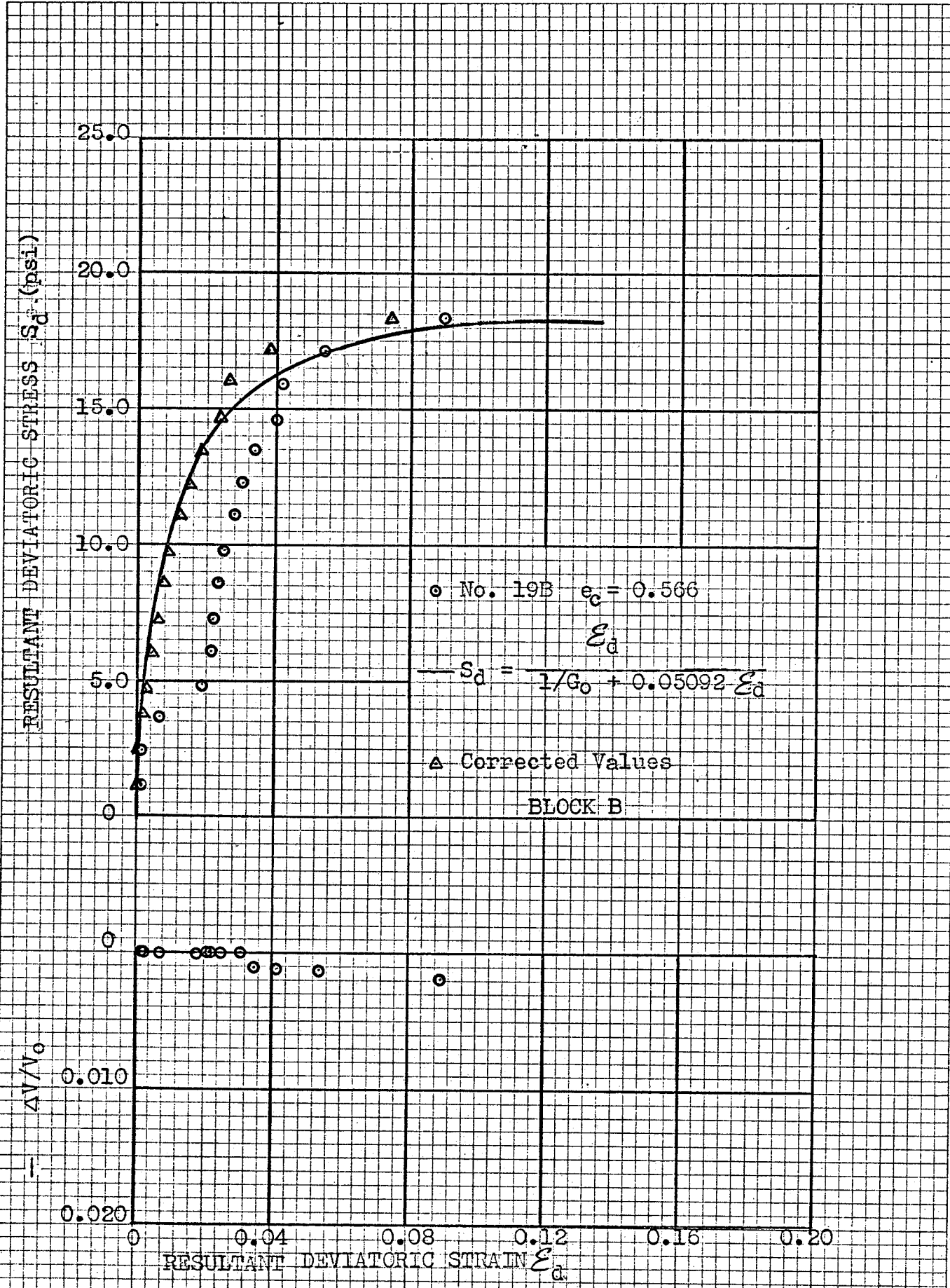


Figure 27. Stress-Strain Relationship for a Constant  $\sigma_m = 22.5$  psi.

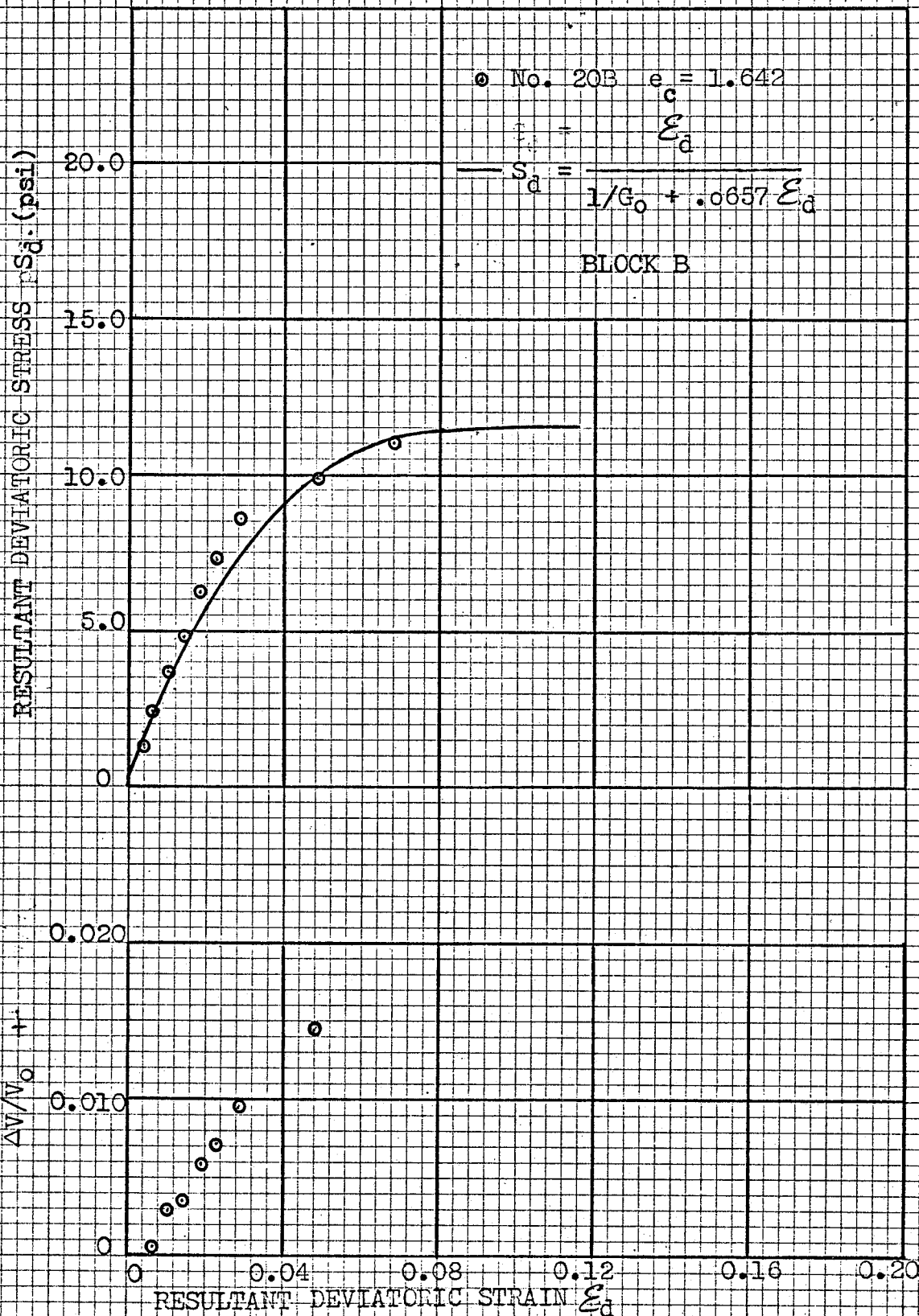


Figure 28. Stress-Strain Relationship for a Constant  $\sigma_m = 7.5$  psi.

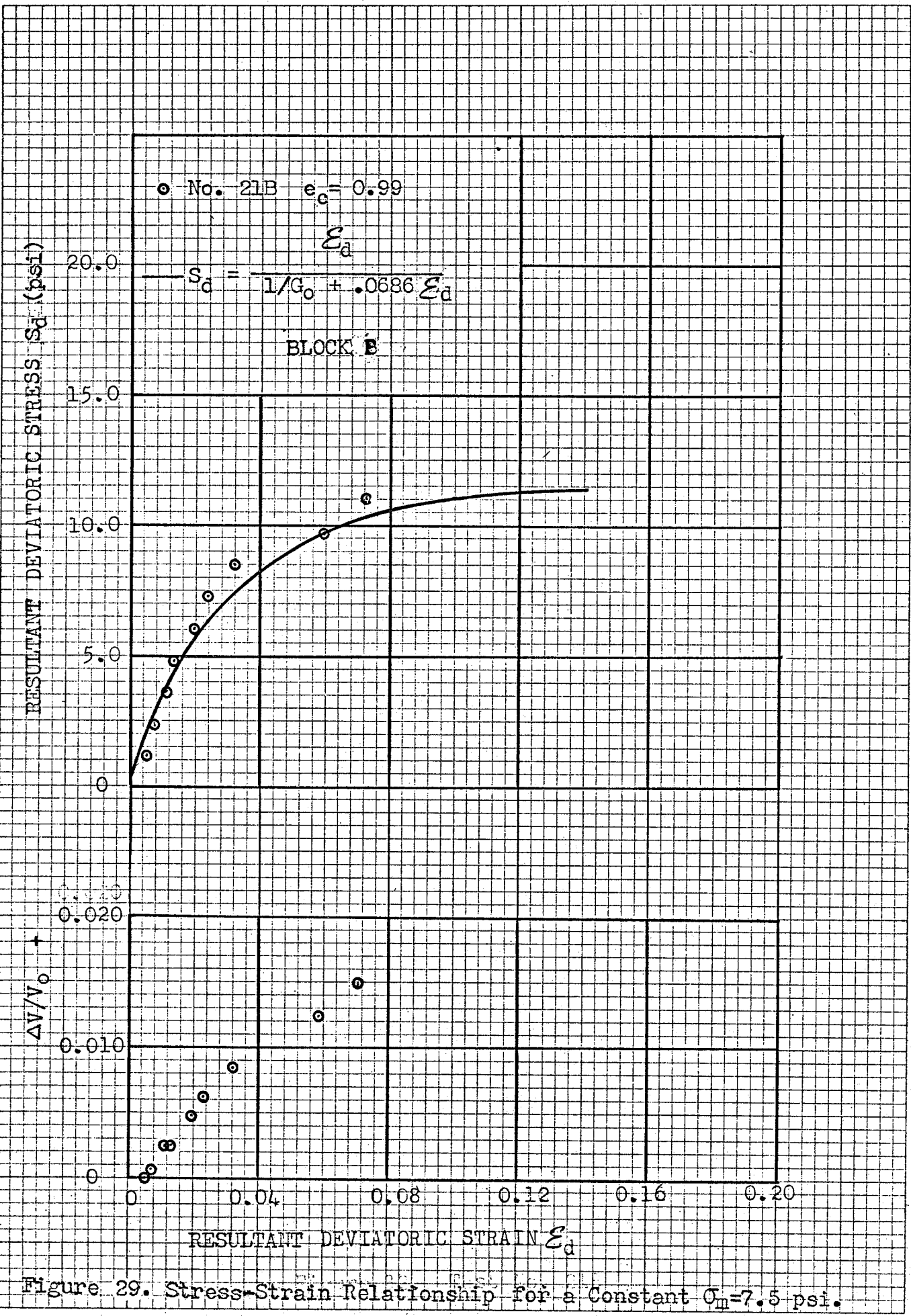


Figure 29. Stress-Strain Relationship for a Constant  $G_m = 7.5$  psi.



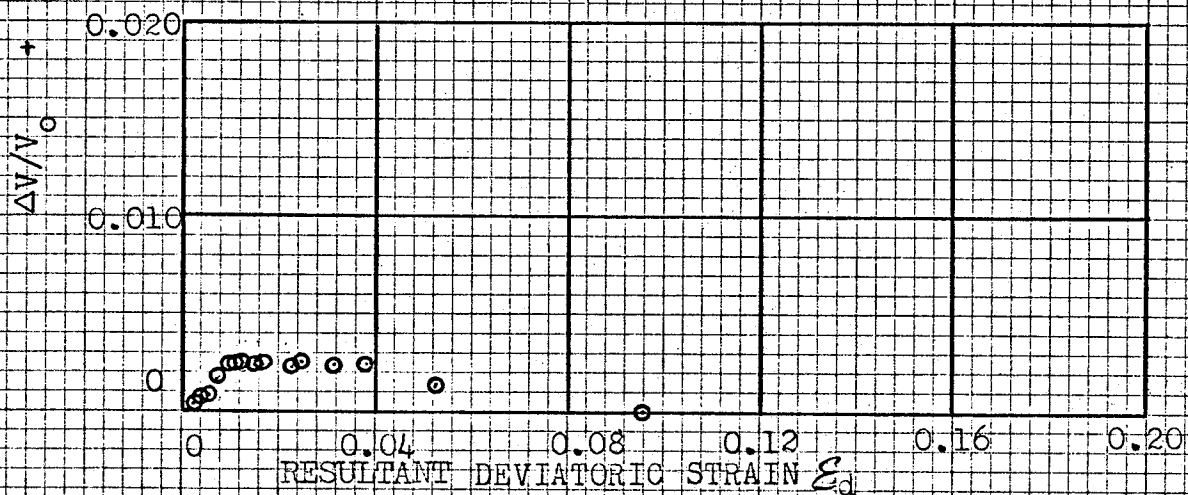
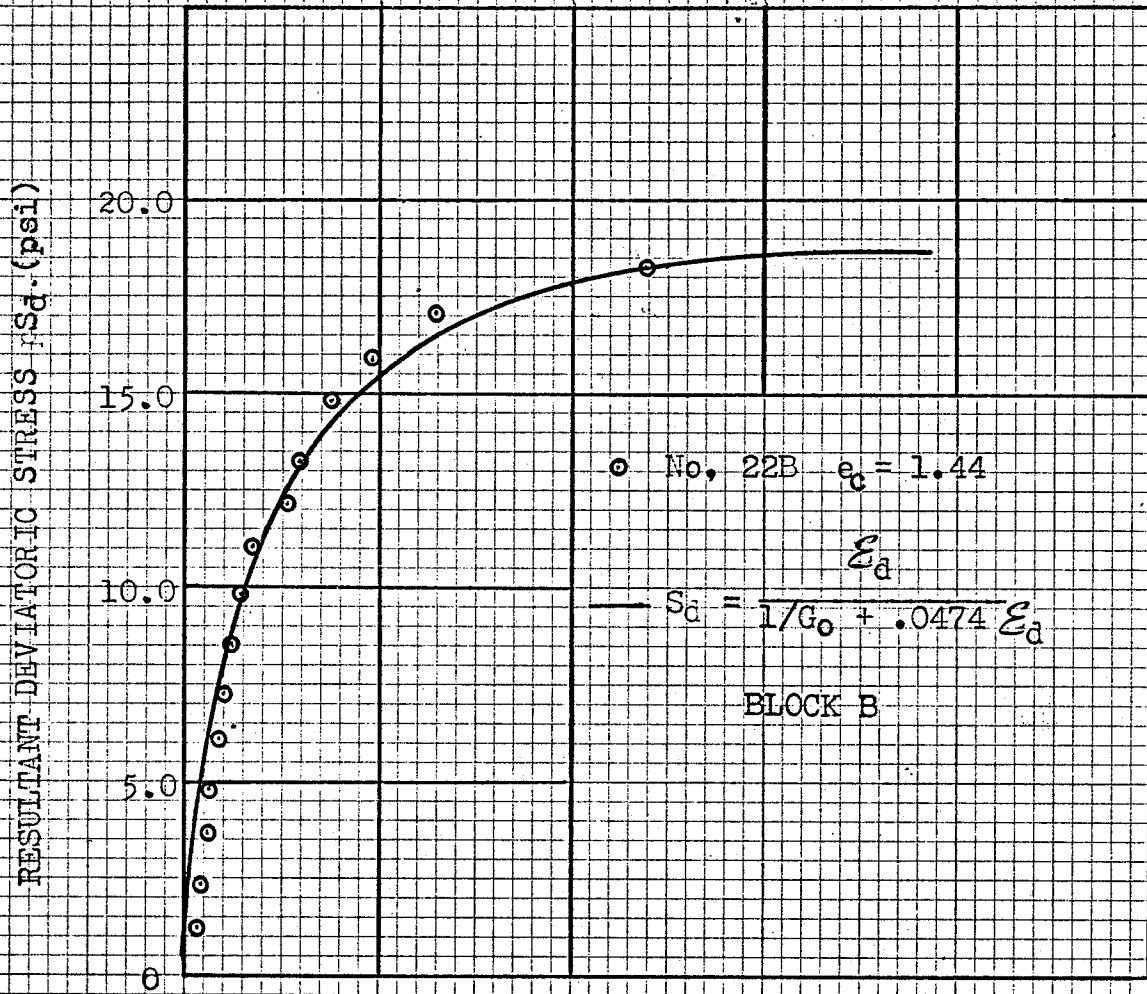


Figure 30. Stress-Strain Relationship for a Constant  $\sigma_m = 22.5$  psi.

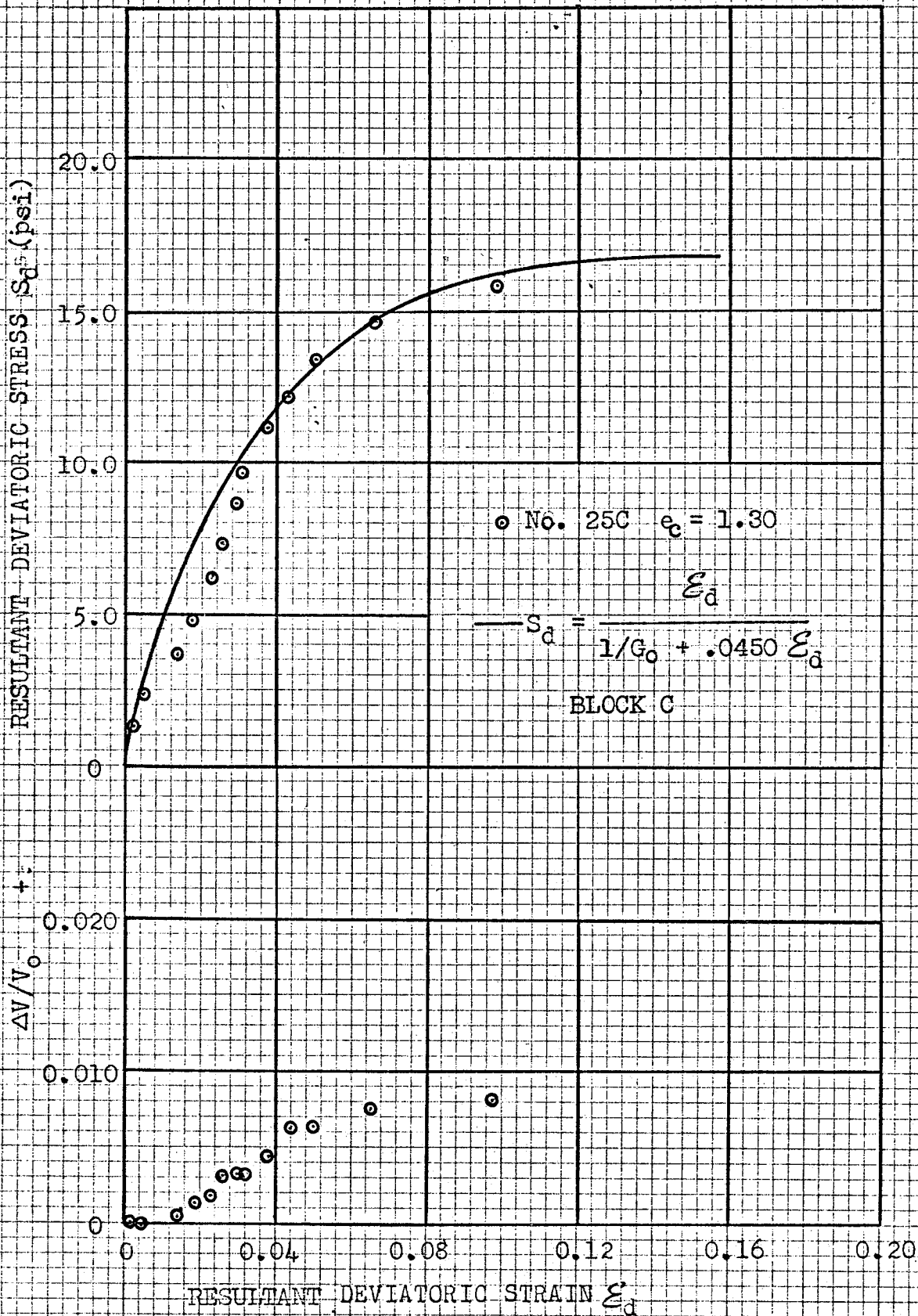


Figure 31. Stress-Strain Relationship for a Constant  $C_m = 15$  psi.

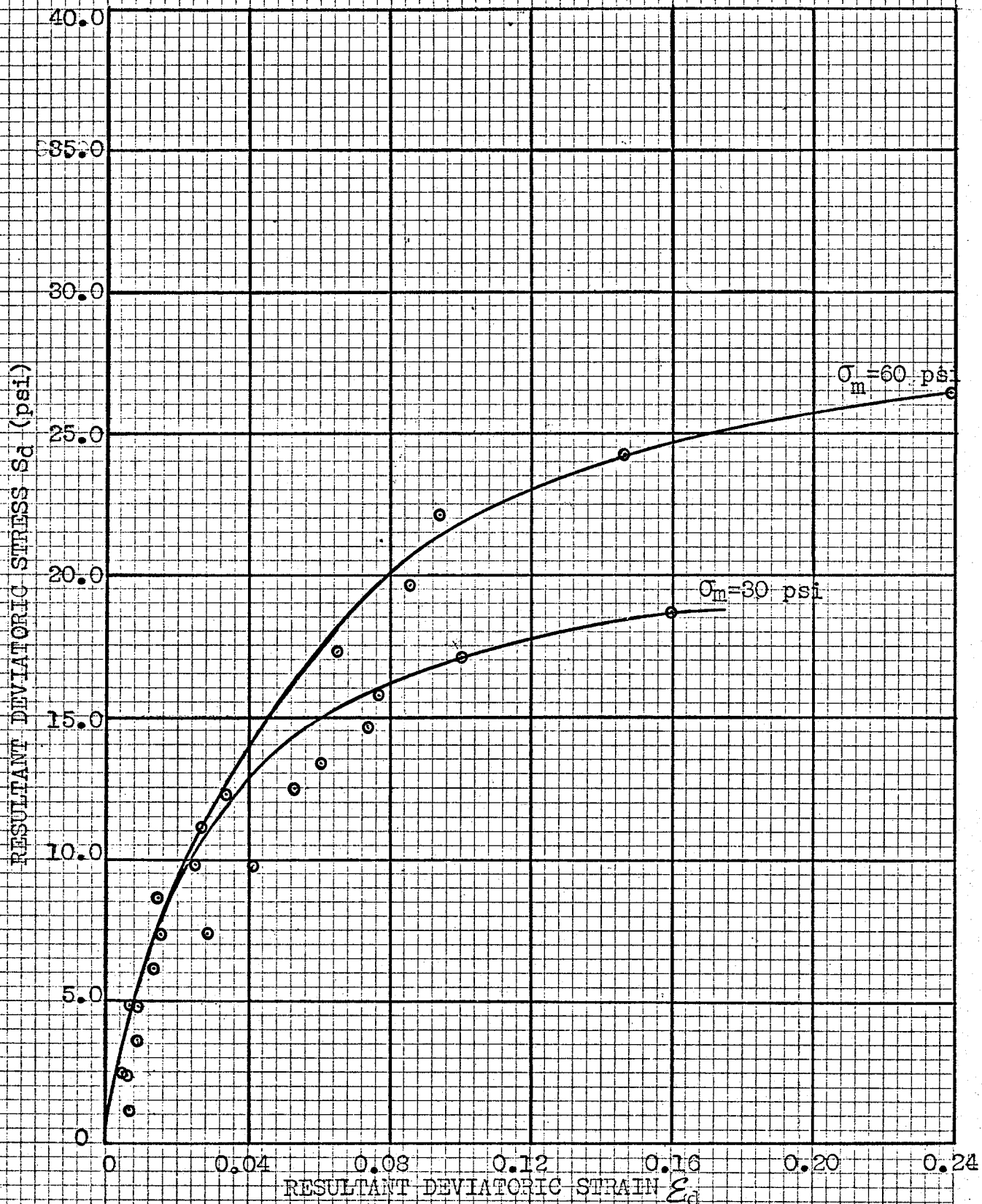


Figure 32. Stress-Strain Relationship for Block A.

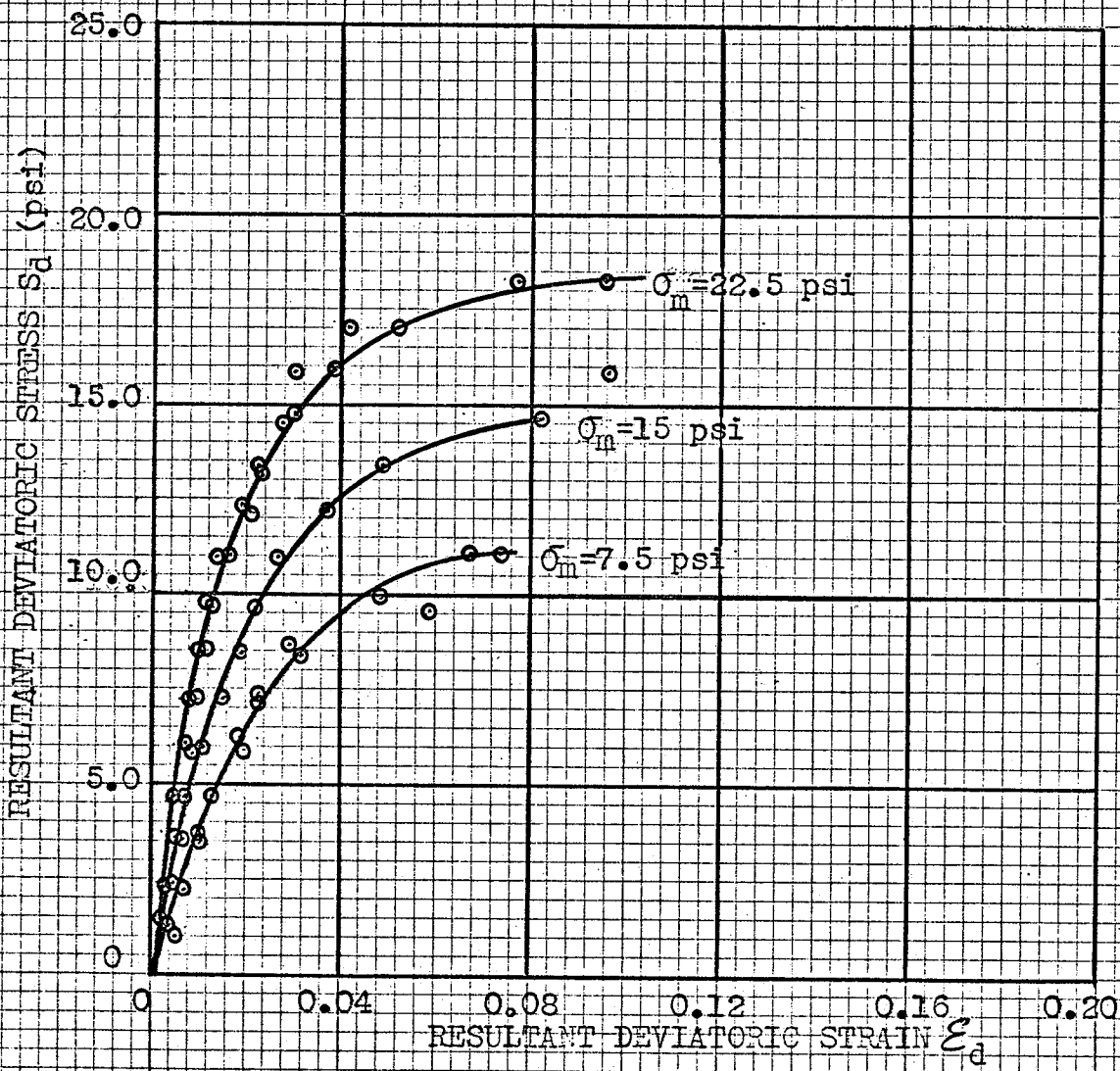


Figure 33. Stress-Strain Relationship for Block B.

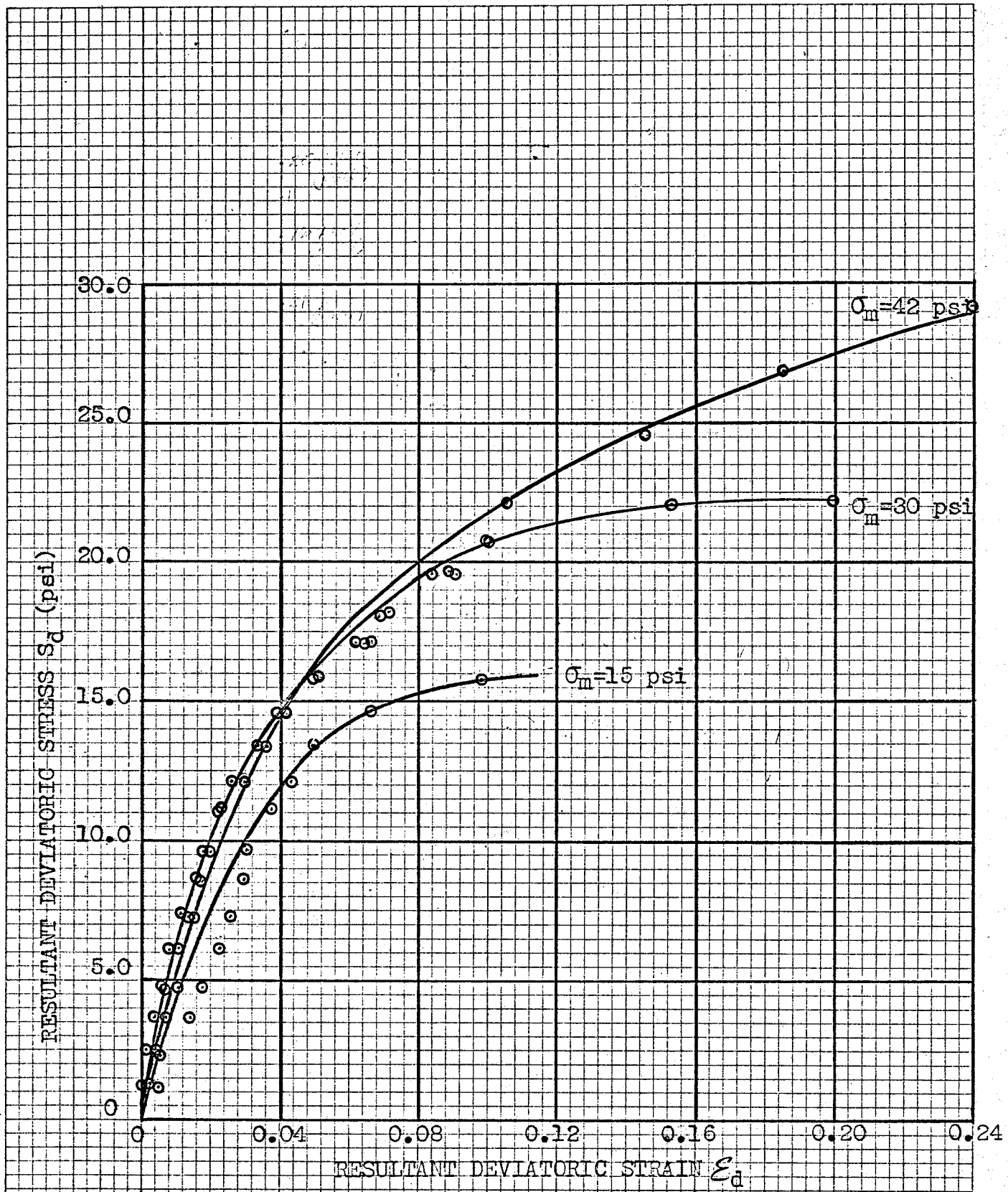


Figure 34. Stress-Strain Relationship for Block C.

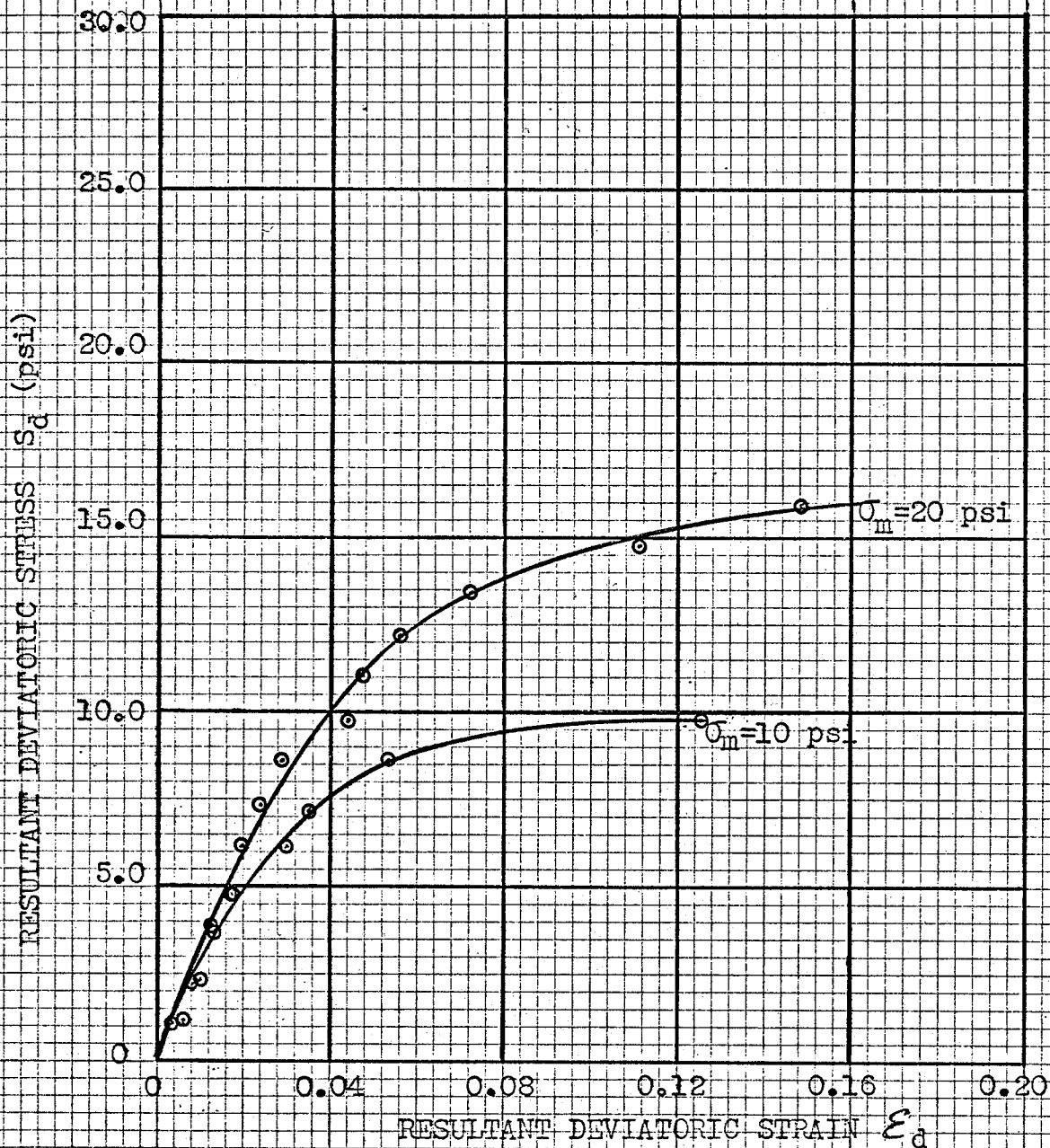


Figure 35. Stress-Strain Relationship for Softer Samples, Block B.

From Block B and Block C.

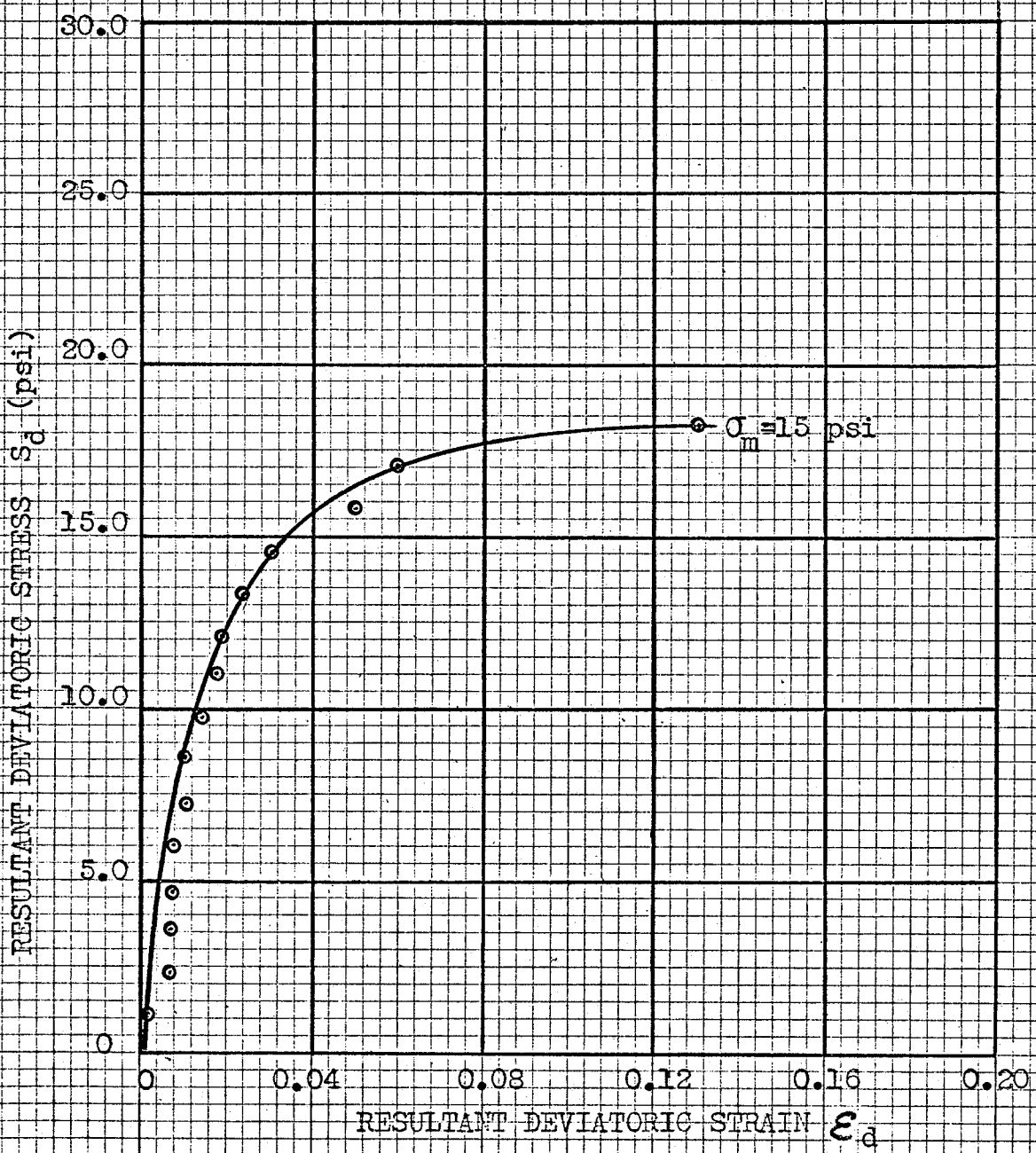


Figure 36. Stress-Strain Relationship for Stiff, Sandy Sample  
Taken From Block B.

decrease in volume or negative dilatancy while slightly overconsolidated samples, with an overconsolidation ratio of 1.33 or greater, show very small increase in volume or positive dilatancy.

Two samples tested from Block B exhibited different behavior than the other samples taken from the same block. These samples ; numbers 15B and 19B, contained mainly silty, sandy material with numerous small stones and the void ratios were much lower. Both tests show a much steeper stress-strain relationship than the other samples.

Initial conditions were difficult to define in some of the deviatoric stress-strain curves. It can be seen that on some of the curves a concave upwards shape exists initially. Since it is a smooth curve it would not appear to be an irregular seating error. When seating error was noted during a test the curve was generally similar to that depicted by test No. 12B, Figure 24. These curves have been corrected as shown. Initial effects such as this may not be significant in the overall picture. The behavior could be inherent in the laboratory triaxial test due to some softening because of the use of back pressure. Kondner<sup>4</sup> (1963) noted this effect and suggested that it is not necessarily due to a seating error.

#### Shear Modulus Analysis

It has been suggested by Kondner<sup>4</sup> (1963) that the stress-strain curve for a cohesive soil follows a rectangular hyperbola curve



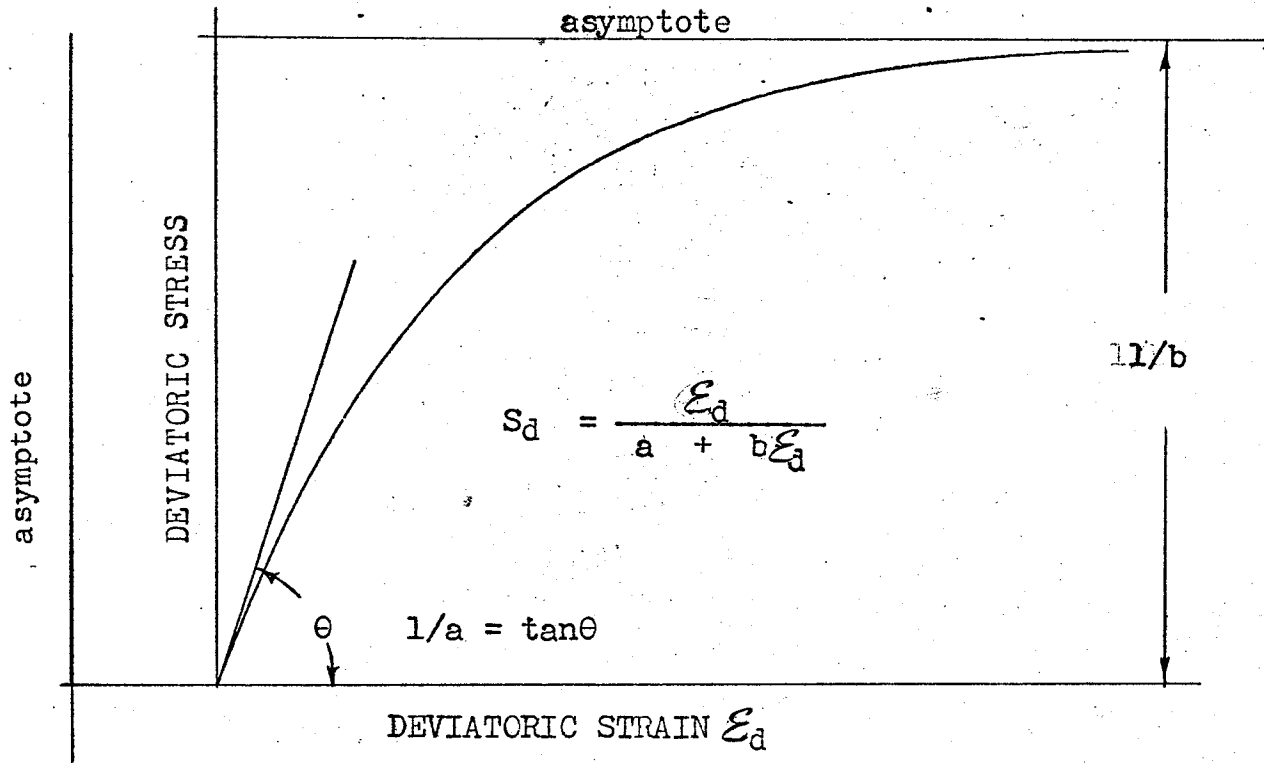


Figure 37. Rectangular Hyperbolic Representation as Applied to Deviatoric Stress-Strain Relationship.

A description of this shape of curve, modified for deviatoric stress and strain is shown in Figure (37)

The initial slope of this curve is equal to  $1/a$  which equals  $\tan \theta$ . The value of  $1/a$  is equal to the initial tangent shear modulus  $G_0$ .  $G_0$  is defined as the shear modulus when the deviator stress components are all equal to zero.

The upper limit of stress reached by the curve is defined by the parameter  $1/b$ . The equation of the curve then becomes:

$$S_d = \frac{\epsilon_d}{a + b\epsilon_d} \quad \dots (15)$$

The initial shear modulus can be determined from the stress strain curves. In order to define the parameter "b" the deviatoric stress and strain at failure along with the parameter "a" can be used in equation (15) to compute "b".

The last strain increment is sometimes difficult to define. In order to get a better value to use in the computation of "b" the points on the stress strain curve can be extended to verify that the last reading taken during failure is part of a smooth curve. This final strain can be used in the computation of "b" and the theoretical curve can be determined from equation (15). Several computations of each curve had to be made because the initial tangent is also difficult to determine thus several attempts will give the best fit of the theoretical curve to the actual data. The computations were done using a computer program so that a number of trials could easily be run. The curves determined in this manner are shown plotted as solid lines in Figures 18 through 32. The equation of each curve is given

on the figure.

The values of 'a' and 'b' were plotted against the mean normal stress for the tests and are shown in Figures 38 and 39. It is apparent that these parameters vary according to the mean normal stress. These parameters also appear to be dependent on the physical characteristics of the soil. The values obtained appear to agree over a range of void ratios for each individual block of material. There is a difference in the parameters obtained for the different blocks of material. This is probably due to variation in the stiffness and void ratios of the different blocks.

The slope of the computed curves is the first derivative of equation (15) and represents the shear modulus. The shear modulus  $G$ , is given by the following equation:

$$G = G_0(1 - b s_d)^2 \quad \dots(16)$$

### Conclusions

The data from the shear modulus study is sufficient to make some definite conclusions about the shearing behavior of the Lake Agassiz Clay. The deviatoric stress strain behavior can be adequately described by use of the following rectangular hyperbola curve.

$$s_d = \frac{\mathcal{E}_d}{a + b\mathcal{E}_d}$$

This equation is a slight modification of the one proposed by Kondner. The advantage of this form of representing the deviatoric stress-strain behavior is that the first derivative of the equation represents the slope of the curve which defines the

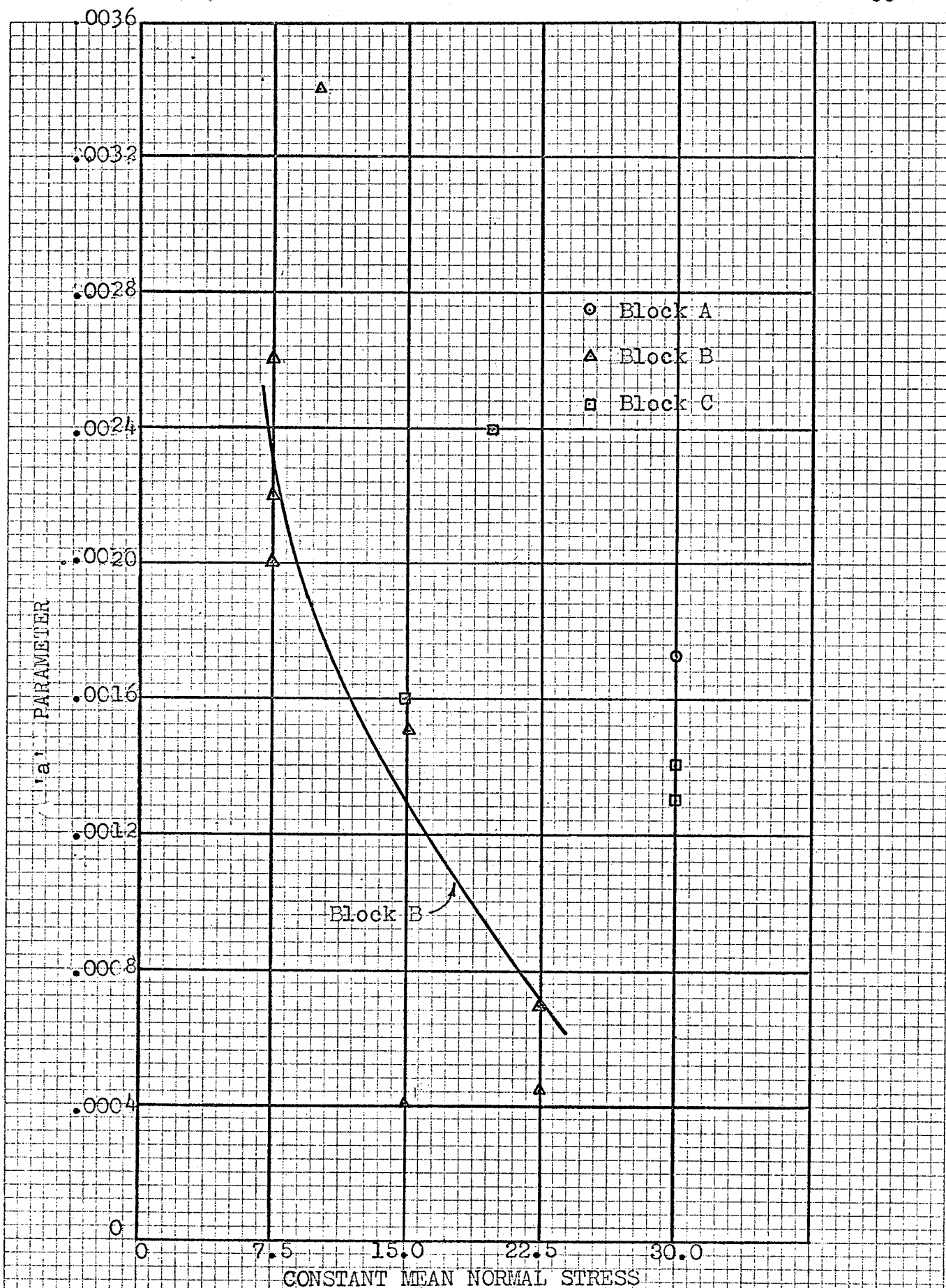


Figure 38. Relationship Between 'a' and  $\sigma_m$

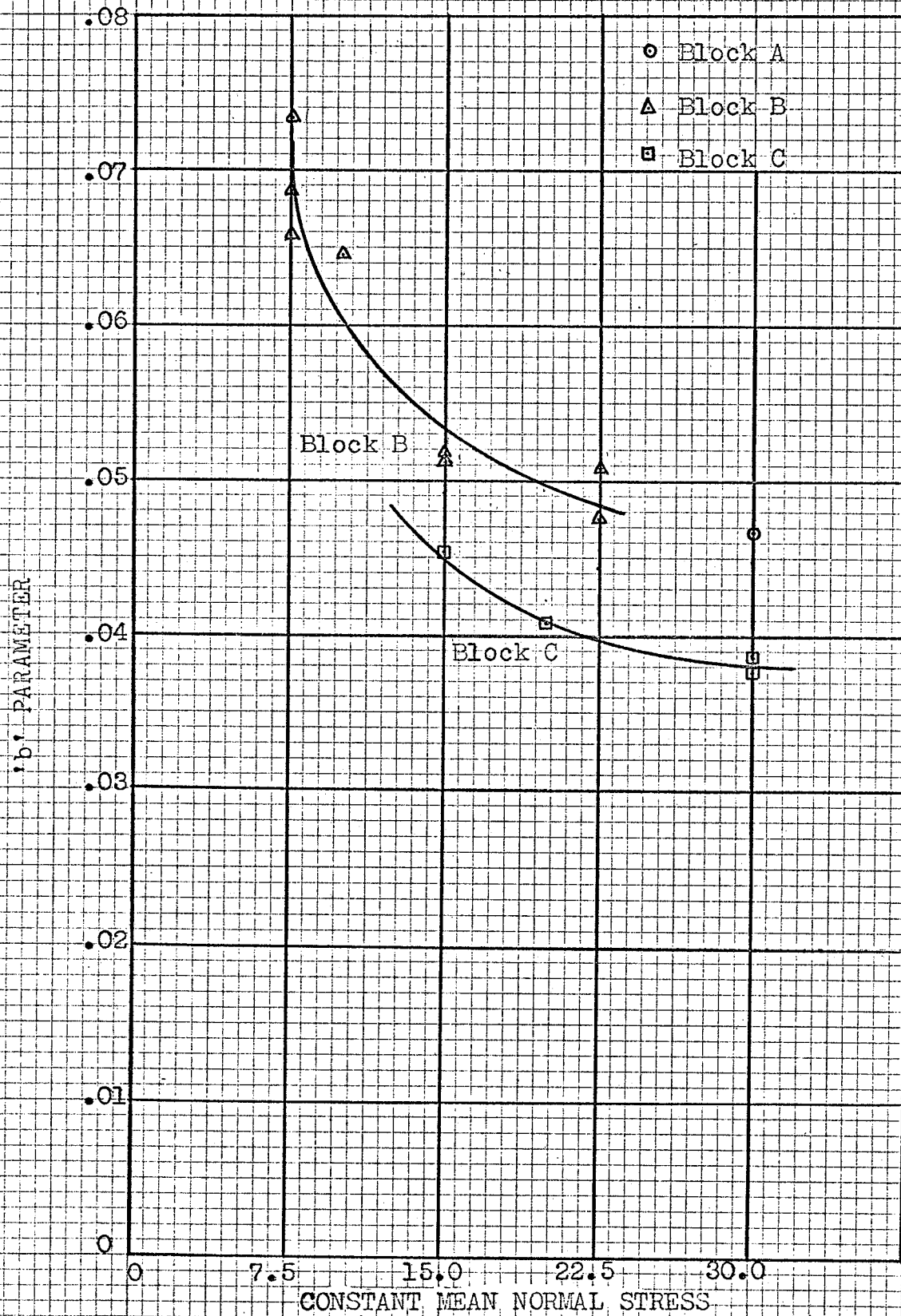


Figure 39. Relationship Between 'b' and  $\sigma_m$

shear modulus  $G$ . The equation is:

$$G = G_0(1 - b s_d)^2$$

The parameters "a" and "b" can be described as functions of the mean normal stress, the soil type, the void ratio, the stress history and the soil fabric. Thus using the soil properties and knowing the stresses applied, the value of the shear modulus can be determined.

CHAPTER V  
FAILURE CRITERIA

The drained shear strength of clay is usually described in terms of  $c'$ , the cohesion and  $\phi'$ , the angle of shearing resistance of the soil. In a triaxial test run at constant  $\sigma_3$  the shear stress can be defined as  $\frac{1}{2}(\sigma_1 - \sigma_3)$  in which  $\sigma_1$  and  $\sigma_3$  are the major and minor principal stresses respectively. The failure envelope is described by  $c'$  and  $\phi'$ .

In a triaxial test run at constant mean normal stress the shear stress at failure can be described by the resultant deviatoric stress  $S_d$ . Domaschuk (1968)<sup>5</sup> investigated the relationship between the mean normal stress and the resultant deviatoric stress at failure. His investigation was based on the results of consolidated undrained triaxial tests. The relationship between the mean normal stress and resultant deviatoric stress at failure was found to be linear.

The resultant deviatoric stress at failure was plotted against the mean normal stress for each drained triaxial test performed in the shear modulus study. The relationship is shown Figure 40 for Block A, Figure 41 for Block B and Figure 42 for Block C. These figures exhibit the apparent straight line relationship. The value of resultant deviatoric stress at a mean normal stress equal to zero was computed by trial and error for each set of data.

The failure criteria as described by Domaschuk (1968)<sup>5</sup> is:

$$S_{df} = k + m\sigma_m \quad \dots\dots\dots(17)$$

in which  $S_{df}$  is the resultant deviatoric stress at failure,  $k$

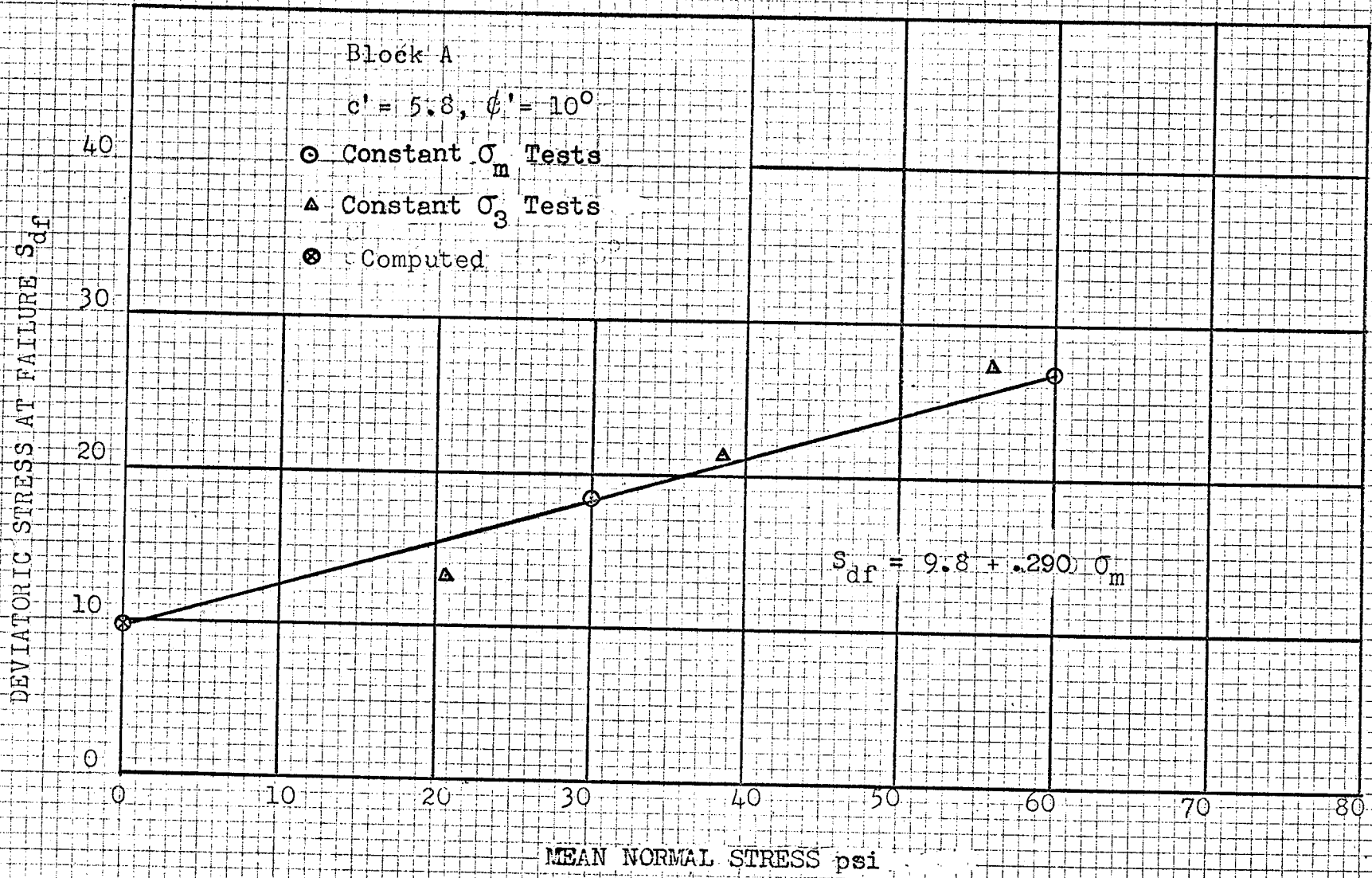


Figure 40. Relationship Between  $\sigma_m$  and  $S_{df}$



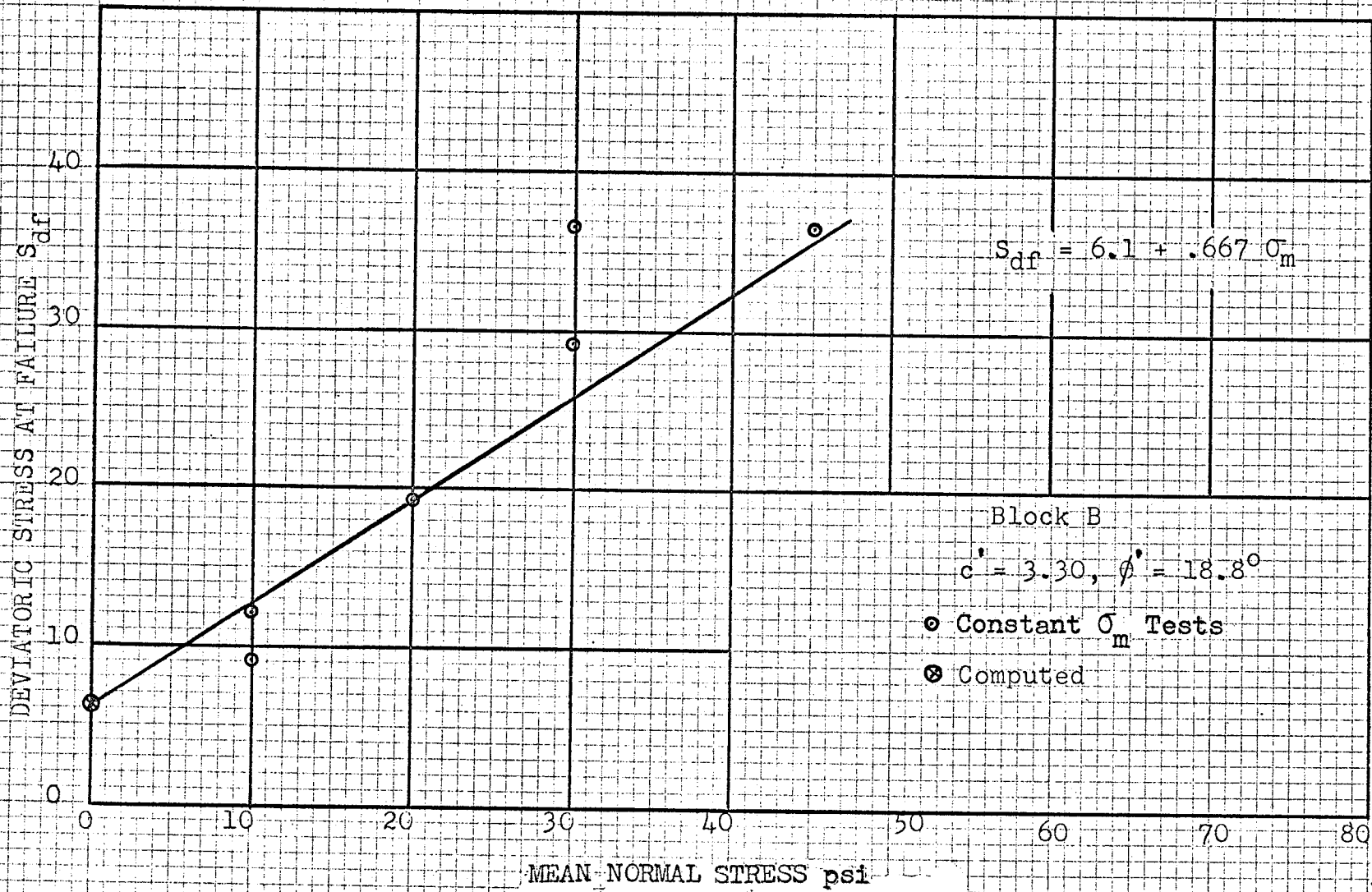


Figure 41. Relationship Between  $\sigma_m$  and  $S_{df}$

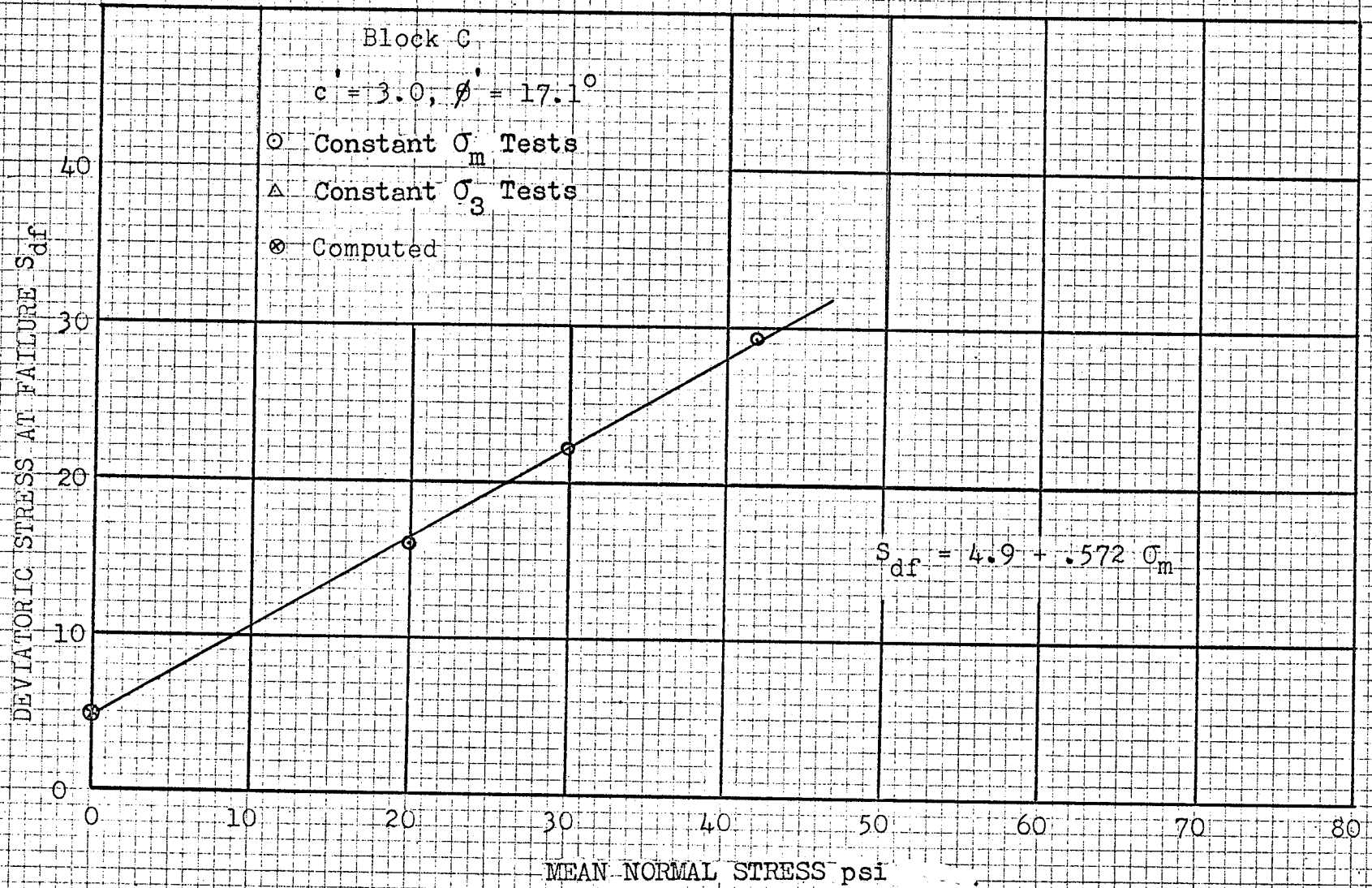


Figure 42. Relationship Between  $\sigma_m$  and  $S_{df}$

is the intercept at a mean normal stress equal to zero, and  $m$  is the slope of the line.

The failure criteria for each set of data is shown on the figures. The conventional drained shear strength parameters are also shown on each figure.

The data for Block A contains the results from four drained triaxial tests run in the conventional manner with  $\sigma_3$  constant. These points appear to fit the straight line relationship well.

### Conclusions

The failure criteria describes well the relationship between mean normal stress and resultant deviatoric stress at failure. The straight line failure criteria described by equation (17) can be useful in the solution of certain stability problems analyzed on the bases of mean normal stress and resultant deviatoric stress.

CHAPTER VI  
GENERAL CONCLUSIONS AND  
RECOMMENDATIONS FOR FURTHER STUDY

The elastic parameters, bulk modulus and shear modulus are dependent on the mean normal stress level for a given soil. These parameters can be determined by using the drained triaxial test run at constant mean normal stress.

Bulk modulus can be determined from the volumetric stress-strain relationship determined from an isotropic compression test. Mean normal stress versus bulk modulus can be developed for given soils.

The shear modulus can be determined from constant mean normal stress drained triaxial tests. The shear modulus may be described by equation (16) if parameters for an average curve equation describing the deviatoric stress-strain relationship are determined.

The relationship between the mean normal stress and resultant deviatoric shear stress at failure forms a unique failure theory.

Recommendations for Further Study

The study of bulk modulus and shear modulus should be extended through a variety of soil profiles which would then give complete coverage of the Lake Agassiz area. These studies should include soils at higher and lower preconsolidation pressure than the ones already studied.

The use of the bulk modulus and shear modulus should be extended to calculating settlements in clay similar to that

studied by Domaschuk<sup>13</sup> (1965) for sand.

Field observations should be made of settlements produced by a boundary load and these observations should be compared with an analytical solution based on the use of bulk modulus and shear modulus.

A computer program will have to be developed to calculate settlements at various points.

The above recommendations involve a great deal of work but would be very worthwhile. The study undertaken by the author forms only a small portion of this larger project.

APPENDIX I  
EQUATIONS

Derivation of the Equation for the Computation of  
Minor Principal Strain in a Triaxial Test

For a soil sample of initial volume  $V_0$ , and initial height  $h_0$ , the initial area is given by:

$$A_0 = \frac{V_0}{h_0}$$

In which  $A_0$  is equal to the initial area of the sample.

During the the test:

$$A = \frac{V_0 - \Delta V}{h_0 - \Delta h}$$

The difference between the area at any time during the the test and the initial area is:

$$A - A_0 = \frac{V_0 - \Delta V}{h_0 - \Delta h} - \frac{V_0}{h_0}$$

Or:

$$A = \frac{\pi}{4}(d^2 - d_0^2) = \frac{\pi}{4} \frac{d_0^2 h_0}{h_0} \left[ \frac{1 - \Delta V/V_0}{1 - \Delta h/h_0} - 1 \right]$$

In which  $d$  is the diameter of the sample at any time during the test and  $d_0$  is the initial diameter of the sample.

Rearrange this equation:

$$\frac{d^2 - d_0^2}{d_0^2} = \frac{1 - \Delta V/V_0}{1 - \Delta h/h_0} - 1$$

or:

$$\left[ \frac{d_0 + \Delta d}{d_0} \right]^2 = \frac{1 - \Delta V/V_0}{1 - \Delta h/h_0}$$

$$1 + \Delta d/d_0 = \sqrt{\frac{1 - \Delta V/V_0}{1 - \Delta h/h_0}}$$

$$\Delta d/d_0 = \sqrt{\frac{1 - \Delta V/V_0}{1 - \Delta h/h_0}} - 1$$

Which also equals:

$$\sqrt{1 - h/h_0 (\Delta V/V_0 - \Delta h/h_0)} - 1$$

Derivation of the Equation Relating Volumetric Strain and  
Principal Strain

The ratio of the change in volume  $\Delta V$  to the initial volume  $V_0$  is given by the following relationship.

$$\Delta V/V_0 = \frac{V_0 - V}{V_0}$$

In an element of material:

$$\varepsilon_1 = \frac{\Delta x}{dx} \quad \varepsilon_2 = \frac{\Delta y}{dy} \quad \varepsilon_3 = \frac{\Delta z}{dz}$$

Substituting:

$$\Delta V/V_0 = (1 + \varepsilon_1)(1 + \varepsilon_2)(1 + \varepsilon_3) - 1$$



Rearranging:

$$\Delta V/V_0 = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \varepsilon_1 \varepsilon_2 + \varepsilon_1 \varepsilon_3 + \varepsilon_2 \varepsilon_3 + \varepsilon_1 \varepsilon_2 \varepsilon_3$$

Neglecting the higher orders this equation can be approximated.

by:

$$\Delta V/V_0 = \varepsilon_1 + \varepsilon_2 + \varepsilon_3$$

APPENDIX II  
DATA SHEETS

TABLE II

## DATA SUMMARY OF ISOTROPIC COMPRESSION TESTS

Test No. 3A			Test No. 4A		
	$e_i = 1.62$			$e_i = 1.59$	
	$e_c = 1.41$			$e_c = 1.38$	
$\sigma_m$	$\Delta V$	$V/V_0$	$\sigma_m$	$\Delta V$	$V/V_0$
psi	ml.	%	psi	ml.	%
0	0	0	0	0	0
5	2.05	2.26	5	1.10	1.25
10	3.41	3.76	10	2.30	2.61
15	4.43	4.89	15	3.32	3.77
20	5.40	5.96	20	4.53	5.15
25	6.35	7.00	25	5.90	6.70
30	7.27	8.02	30	7.10	8.07
			35	8.70	9.88
			40	10.30	11.70
			45	11.60	13.18
			50	13.15	14.94
			55	13.80	15.68
			60	14.60	16.58

---

Test No. 5A			Test No. 6C		
	$e_i = 1.59$			$e_i = 1.66$	
	$e_c = 1.38$			$e_c = 1.43$	
$\sigma_m$	$\Delta V$	$V/V_0$	$\sigma_m$	$\Delta V$	$V/V_0$
psi	ml.	%	psi	ml.	%
0	0	0	0	0	0
5	2.70	3.08	5	2.18	2.46
10	4.29	4.89	10	3.63	4.10
15	5.35	6.10	15	4.78	5.39
20	6.10	6.95	20	6.07	6.85
25	6.80	7.75	25	7.10	8.01
30	7.29	8.31	30	7.72	8.71
			35	8.48	9.57
			40	9.56	10.79

TABLE II(continued)

Test No. 7C			Test No. 8C		
	$e_i=1.47$			$e_i=1.41$	
	$e_c=1.31$			$e_c=1.27$	
$\sigma_m$	$\Delta V$	$V/V_0$	$\sigma_m$	$\Delta V$	$V/V_0$
psi	ml.	%	psi	ml.	%
0	0	0	0	0	0
5	0.92	1.04	5	0.95	1.09
10	2.42	2.73	10	2.55	2.93
15	3.42	3.86	15	3.05	3.50
20	4.27	4.82	20	3.65	4.19
25	5.07	5.72	25	4.35	4.99
30	5.94	6.70	30,	4.95	5.68

Test No.9C			Test No. 10B		
	$e_i=1.51$			$e_i=1.39$	
	$e_c=1.32$			$e_c=1.23$	
$\sigma_m$	$\Delta V$	$V/V_0$	$\sigma_m$	$\Delta V$	$V/V_0$
psi	ml.	%	psi	ml.	%
0	0	0	0	0	0
5	2.70	3.07	5	1.20	1.38
10	3.50	3.98	10	2.70	3.10
15	4.20	4.77	15	3.40	3.91
20	5.00	5.68	20	4.20	4.83
25	5.70	6.47	25	4.90	5.64
30	6.50	7.38	30	5.60	6.45
			35	6.40	7.36
			40	7.40	8.51
			45	8.10	9.32
			50	9.05	10.40
			55	9.75	11.21
			60	11.05	12.70

TABLE II (continued)

Test No. 13B			Test No. 14B		
	$e_i=1.05$			$e_i=1.19$	
	$e_c=0.85$			$e_c=1.01$	
$\sigma_m$	$\Delta V$	$V/V_0$	$\sigma_m$	$\Delta V$	$V/V_0$
psi	ml.	%	psi	ml.	%
0	0	0	0	0	0
2	0.90	1.04	2	1.10	1.27
5	1.80	2.07	5	2.15	2.48
10	3.05	3.51	10	3.55	4.09
20	4.70	5.41	20	5.45	6.27
30	6.20	7.14	30	7.15	8.23
40	7.70	8.86	40	8.80	10.13
50	9.60	11.05	50	10.90	12.58
60	10.60	12.20	60	11.95	13.77

Test No. 17B			Test No. 18B		
	$e_i=1.02$			$e_i=1.31$	
	$e_c=0.82$			$e_c=1.02$	
$\sigma_m$	$\Delta V$	$V/V_0$	$\sigma_m$	$\Delta V$	$V/V_0$
psi	ml.	%	psi	ml.	%
0	0	0	0	0	0
30	6.40	7.37	2	0.60	0.69
15	5.30	6.11	10	3.70	4.27
30	-	-	20	5.90	6.80
10	4.70	5.41	30	7.50	8.65
20	6.15	7.10	40	9.20	10.60
30	7.48	8.62	50	10.80	12.45
40	8.65	9.97	0	-	-
50	10.20	11.75	10	7.05	8.13
60	11.65	13.41	20	9.40	10.81
			30	11.20	12.90
			40	12.40	14.28
			50	13.70	15.80
			60	14.90	17.15

TABLE II (continued)

Test No. 23D			Test No. 24B		
$e_i = 1.58$			$e_i = 1.55$		
$e_c = 1.32$			$e_c = 1.39$		
$\sigma_m$	$\Delta V$	$V/V_o$	$\sigma_m$	$\Delta V$	$V/V_o$
psi	ml.	%	psi	ml.	%
0	0	0	0	0	0
2	0.40	0.46	2	0.50	0.66
5	1.65	1.90	5	1.90	2.54
10	3.40	3.92	10	3.60	4.75
20	6.00	6.83	20	5.30	6.98
30	8.10	9.33	30	6.80	8.96
40	9.70	11.18	40	8.10	10.68
50	10.80	12.43	50	9.35	12.32
60	11.75	13.51	60	10.65	14.05
70	12.80	14.72	70	12.50	16.50
80	13.45	15.47	80	13.40	17.70
90	14.40	16.32	90	14.60	19.29
Test No. 27D			Test No. 28D		
$e_i = 1.58$			$e_i = 1.59$		
$e_c = 1.41$			$e_c = 1.39$		
$\sigma_m$	$\Delta V$	$V/V_o$	$\sigma_m$	$\Delta V$	$V/V_o$
psi	ml.	%	psi	ml.	%
0	0	0	0	0	0
5	0.71	0.91	5	0.50	0.65
10	2.11	2.72	10	1.91	2.47
20	4.30	5.55	20	3.75	4.85
40	7.30	9.42	40	6.75	8.77
80	10.80	13.92	80	10.70	13.80

TABLE III

Computer Print Out of Data From Constant Mean Normal Stress  
Shear Tests.

Explanation of Tables

Note: Constant Mean Effective Stress In Tables Refers To  
Constant Mean Normal Stress.

Initial and Final Dry Density = Rdg.(62.4)

Initial and Final % Saturation = Rdg.(100%)

Initial and Final Moisture Content = Rdg.(100%)

Sample Volume In  $\text{cm}^3$ .

Sample Area In sq. inches.

Symbols Used

S1 = Major Principal Stress

S2 = Minor Principal Stress

E1 = Strain In Direction of Major Principal Stress

E2 = Strain In Direction of Minor Principal Stress

E MEAN =  $1/3(E1 + 2E3)$

DEV STRESS = Resultant Deviatoric Stress

DEV STRAIN = Resultant Deviatoric Strain

TABLE III

## SUMMARY OF CONSTANT MEAN NORMAL STRESS SHEAR TESTS

TEST NUMBER 34 CONSTANT MEAN EFFECTIVE STRESS

INITIAL DRY DENSITY= 1.033

INITIAL VOID RATIO= 1.624

INITIAL SATURATION= 0.977

INITIAL VOLUME OF SAMPLE= 90.654

INITIAL MOISTURE CONTENT= 0.585

FINAL MOISTURE CONTENT= 0.500

VOLUME AFTER ISOTROPIC COMPRESSION= 83.384

AREA AFTER ISOTROPIC COMPRESSION= 1.800

J	S		E		SHEAR TEST MEAN STRESS CONSTANT		
	S1	S3	E1	E3	F	MEAN DEV STRESS	DEV STRAI
2	1.00	0.50	0.0033	-0.0012	0.0003	1.2247	0.0037
3	2.00	1.00	0.0028	-0.0008	0.0004	2.4495	0.0030
4	3.00	1.50	0.0038	-0.0013	0.0004	3.6742	0.0042
5	4.00	2.00	0.0041	-0.0014	0.0004	4.8990	0.0045
6	5.00	2.50	0.0059	-0.0024	0.0004	6.1237	0.0068
7	6.00	3.00	0.0062	-0.0031	-0.0000	7.3485	0.0076
8	7.00	3.50	0.0062	-0.0031	-0.0000	8.5732	0.0076
9	8.00	4.00	0.0096	-0.0056	-0.0005	9.7980	0.0124
10	9.00	4.50	0.0109	-0.0062	-0.0005	11.0227	0.0139
11	10.00	5.00	0.0134	-0.0071	-0.0002	12.2474	0.0168
12	11.00	5.50	0.0246	-0.0128	-0.0003	13.4722	0.0305
13	12.00	6.00	0.0299	-0.0146	0.0003	14.6969	0.0363
14	13.00	6.50	0.0325	-0.0159	0.0003	15.9217	0.0395
15	14.00	7.00	0.0416	-0.0202	0.0004	17.1464	0.0504
16	15.00	7.50	0.0835	-0.0419	-0.0001	18.3712	0.1024

FINAL DRY DENSITY= 1.129 FINAL SATURATION= 0.977

FINAL SATURATION COMPUTED FROM FINAL MOISTURE=



## TEST NUMBER 4A CONSTANT MEAN EFFECTIVE STRESS

INITIAL DRY DENSITY= 1.046

INITIAL VOID RATIO= 1.590

INITIAL SATURATION= 1.022

INITIAL VOLUME OF SAMPLE= 88.033

INITIAL MOISTURE CONTENT= 0.600

FINAL MOISTURE CONTENT= 0.517

VOLUME AFTER ISOTROPIC COMPRESSION= 73.433

AREA AFTER ISOTROPIC COMPRESSION= 1.579

## SHEAR TEST MEAN STRESS CONSTANT

J	S1	S3	E1	E3	E	MEAN DEV STRESS	DEV STRAIN
2	2.00	1.00	0.0031	-0.0012	0.0002	2.4495	0.0035*****
3	4.00	2.00	0.0035	-0.0014	0.0002	4.3990	0.0040
4	6.00	3.00	0.0142	-0.0051	0.0013	7.3435	0.0158
5	8.00	4.00	0.0190	-0.0062	0.0022	9.7930	0.0206
6	10.00	5.00	0.0254	-0.0074	0.0035	12.2474	0.0268
7	12.00	6.00	0.0280	-0.0077	0.0042	14.6969	0.0292
8	14.00	7.00	0.0315	-0.0085	0.0048	17.1464	0.0326
9	16.00	8.00	0.0414	-0.0112	0.0063	19.5959	0.0429
10	18.00	9.00	0.0481	-0.0095	0.0097	22.0454	0.0470
11	20.00	10.00	0.0696	-0.0207	0.0094	24.4949	0.0738
12	22.00	11.00	0.1091	-0.0393	0.0098	26.9444	0.1216

FINAL DRY DENSITY= 1.302 FINAL SATURATION= 1.032

FINAL SATURATION COMPUTED FROM FINAL MOISTURE= 1.295

## TEST NUMBER 6 CONSTANT MEAN EFFECTIVE STRESS

INITIAL DRY DENSITY= 1.017

INITIAL VOID RATIO= 1.664

INITIAL SATURATION= 1.037

INITIAL VOLUME OF SAMPLE= 88.612

INITIAL MOISTURE CONTENT= 0.637

FINAL MOISTURE CONTENT= 0.539

VOLUME AFTER ISOTROPIC COMPRESSION= 78.652

AREA AFTER ISOTROPIC COMPRESSION= 1.628

		SHEAR TEST MEAN STRESS CONSTANT								
J	S1	S3	E1	E3	E	MEAN	DEV	STRESS	DEV	STRAIN
2	2.00	1.00	0.0027	-0.0011	0.0002	2.4495	0.0031			
3	4.00	2.00	0.0045	-0.0019	0.0002	4.8990	0.0053			
4	6.00	3.00	0.0068	-0.0021	0.0008	7.3485	0.0072			
5	8.00	4.00	0.0079	-0.0027	0.0008	9.7980	0.0086			
6	10.00	5.00	0.0133	-0.0042	0.0017	12.2474	0.0143			
7	12.00	6.00	0.0193	-0.0053	0.0029	14.6969	0.0201			
8	14.00	7.00	0.0302	-0.0067	0.0056	17.1464	0.0302			
9	16.00	8.00	0.0428	-0.0116	0.0065	19.5959	0.0444			
10	18.00	9.00	0.0491	-0.0147	0.0066	22.0454	0.0521			
11	20.00	10.00	0.0678	-0.0215	0.0083	24.4949	0.0729			
12	22.00	11.00	0.0852	-0.0281	0.0097	26.9444	0.0925			
13	24.00	12.00	0.1214	-0.0463	0.0096	29.3939	0.1369			

FINAL DRY DENSITY= 1.191 FINAL SATURATION= 1.048

FINAL SATURATION COMPUTED FROM FINAL MOISTURE= 1.146

## TEST NUMBER 7C CONSTANT MEAN EFFECTIVE STRESS

INITIAL DRY DENSITY= 1.096

INITIAL VOID RATIO= 1.473

INITIAL SATURATION= 1.073

INITIAL VOLUME OF SAMPLE= 88.612

INITIAL MOISTURE CONTENT= 0.583

FINAL MOISTURE CONTENT= 0.521

VOLUME AFTER ISOTROPIC COMPRESSION= 82.672

AREA AFTER ISOTROPIC COMPRESSION= 1.679

J	S1	S3	E1	E3	E	MEAN DEV STRESS	CONSTANT	DEV STRAIN
2	1.00	0.50	0.0010	-0.0005	0.0000	1.2247	0.0012	
3	2.00	1.00	0.0020	-0.0010	-0.0000	2.4495	0.0024	
4	3.00	1.50	0.0081	-0.0029	0.0008	3.6742	0.0090	
5	4.00	2.00	0.0090	-0.0033	0.0008	4.8990	0.0100	
6	5.00	2.50	0.0100	-0.0038	0.0008	6.1237	0.0112	
7	6.00	3.00	0.0107	-0.0042	0.0008	7.3485	0.0121	
8	7.00	3.50	0.0133	-0.0054	0.0008	8.5732	0.0153	
9	8.00	4.00	0.0143	-0.0054	0.0012	9.7980	0.0161	
10	9.00	4.50	0.0163	-0.0058	0.0016	11.0227	0.0181	
11	10.00	5.00	0.0204	-0.0067	0.0023	12.2474	0.0221	
12	11.00	5.50	0.0229	-0.0077	0.0025	13.4722	0.0250	
13	12.00	6.00	0.0240	-0.0079	0.0027	14.6969	0.0261	
14	13.00	6.50	0.0295	-0.0101	0.0031	15.9217	0.0323	
15	14.00	7.00	0.0355	-0.0124	0.0036	17.1464	0.0390	
16	15.00	7.50	0.0373	-0.0133	0.0036	18.3712	0.0413	
17	16.00	8.00	0.0436	-0.0165	0.0036	19.5959	0.0491	
18	17.00	8.50	0.0508	-0.0201	0.0036	20.8206	0.0579	
19	18.00	9.00	0.0710	-0.0309	0.0031	22.0454	0.0832	

FINAL DRY DENSITY= 1.190 FINAL SATURATION= 1.084

FINAL SATURATION COMPUTED FROM FINAL MOISTURE= 1.105

## TEST NUMBER 8C CONSTANT MEAN EFFECTIVE STRESS

INITIAL DRY DENSITY= 1.124

INITIAL VOID RATIO= 1.410

INITIAL SATURATION= 0.989

INITIAL VOLUME OF SAMPLE= 87.149

INITIAL MOISTURE CONTENT= 0.515

FINAL MOISTURE CONTENT= 0.481

VOLUME AFTER ISOTROPIC COMPRESSION= 82.199

J	SHEAR TEST					MEAN STRESS CONSTANT	
	S1	S3	E1	E3	E	MEAN DEV STRESS	DEV STRAIN
2	1.00	0.50	0.0018	-0.0009	-0.0000	1.2247	0.0022
3	2.00	1.00	0.0025	-0.0012	-0.0000	2.4495	0.0031
4	3.00	1.50	0.0028	-0.0014	-0.0000	3.6742	0.0035
5	4.00	2.00	0.0029	-0.0014	-0.0000	4.8990	0.0035
6	5.00	2.50	0.0042	-0.0021	-0.0000	6.1237	0.0051
7	6.00	3.00	0.0053	-0.0027	-0.0000	7.3485	0.0065
8	7.00	3.50	0.0075	-0.0026	0.0008	8.5732	0.0082
9	8.00	4.00	0.0085	-0.0037	0.0004	9.7980	0.0099
10	9.00	4.50	0.0099	-0.0044	0.0004	11.0227	0.0117
11	10.00	5.00	0.0115	-0.0046	0.0008	12.2474	0.0131
12	11.00	5.50	0.0150	-0.0060	0.0010	13.4722	0.0171
13	12.00	6.00	0.0171	-0.0077	0.0005	14.6969	0.0203
14	13.00	6.50	0.0208	-0.0093	0.0007	15.9217	0.0246
15	14.00	7.00	0.0285	-0.0121	0.0014	17.1464	0.0331
16	15.00	7.50	0.0311	-0.0129	0.0018	18.3712	0.0360
17	16.00	8.00	0.0392	-0.0168	0.0019	19.5959	0.0457
18	17.00	8.50	0.0424	-0.0185	0.0018	20.8206	0.0497
19	18.00	9.00	0.0862	-0.0417	0.0010	22.0454	0.1044

FINAL DRY DENSITY= 1.202 FINAL SATURATION= 0.988

FINAL SATURATION COMPUTED FROM FINAL MOISTURE= 1.040

## TEST NUMBER 9C CONSTANT MEAN EFFECTIVE STRESS

INITIAL DRY DENSITY= 1.081

INITIAL VOID RATIO= 1.507

INITIAL SATURATION= 1.001

INITIAL VOLUME OF SAMPLE= 88.037

INITIAL MOISTURE CONTENT= 0.557

FINAL MOISTURE CONTENT= 0.534

VOLUME AFTER ISOTROPIC COMPRESSION= 82.737

AREA AFTER ISOTROPIC COMPRESSION= 1.707

SHEAR TEST MEAN STRESS CONSTANT							
J	S1	S3	E1	E3	E	MEAN DEV STRESS	DEV STRAIN
2	1.00	0.50	0.0036	-0.0012	0.0004	1.2247	0.0039
3	2.00	1.00	0.0048	-0.0024	-0.0000	2.4495	0.0059
4	3.00	1.50	0.0055	-0.0034	-0.0004	3.6742	0.0072
5	4.00	2.00	0.0064	-0.0044	-0.0008	4.8990	0.0088
6	5.00	2.50	0.0071	-0.0054	-0.0012	6.1237	0.0102
7	6.00	3.00	0.0090	-0.0063	-0.0012	7.3485	0.0125
8	7.00	3.50	0.0112	-0.0081	-0.0016	8.5732	0.0157
9	8.00	4.00	0.0159	-0.0117	-0.0025	9.7980	0.0225
10	9.00	4.50	0.0172	-0.0127	-0.0027	11.0227	0.0244
11	10.00	5.00	0.0203	-0.0149	-0.0032	12.2474	0.0287
12	11.00	5.50	0.0267	-0.0185	-0.0034	13.4722	0.0369
13	12.00	6.00	0.0418	-0.0271	-0.0041	14.6969	0.0562
14	13.00	6.50	0.0556	-0.0365	-0.0058	15.9217	0.0752

FINAL DRY DENSITY= 1.134 FINAL SATURATION= 1.001

FINAL SATURATION COMPUTED FROM FINAL MOISTURE= 1.040

## TEST NUMBER 11B CONSTANT MEAN EFFECTIVE STRESS

INITIAL DRY DENSITY= 1.058

INITIAL VOID RATIO= 1.561

INITIAL SATURATION= 1.040

INITIAL VOLUME OF SAMPLE= 86.875

INITIAL MOISTURE CONTENT= 0.599

FINAL MOISTURE CONTENT= 0.609

VOLUME AFTER ISOTROPIC COMPRESSION= 83.775

AREA AFTER ISOTROPIC COMPRESSION= 1.720

J	SHEAR TEST			MEAN STRESS CONSTANT			DEV STRAIN
	S1	S3	E1	E3	E	MEAN DEV STRESS	
2	1.00	0.50	0.0025	-0.0013	-0.0000	1.2247	0.0031*****
3	2.00	1.00	0.0079	-0.0046	-0.0004	2.4495	0.0102
4	3.00	1.50	0.0090	-0.0063	-0.0012	3.6742	0.0125
5	4.00	2.00	0.0102	-0.0076	-0.0016	4.8990	0.0145
6	5.00	2.50	0.0136	-0.0112	-0.0029	6.1237	0.0203
7	6.00	3.00	0.0157	-0.0130	-0.0035	7.3485	0.0234
8	7.00	3.50	0.0210	-0.0185	-0.0053	8.5732	0.0323
9	8.00	4.00	0.0490	-0.0352	-0.0071	9.7980	0.0687
10	9.00	4.50	-0.0099	0.0049	-0.0000	11.0227	0.0121

FINAL DRY DENSITY= 1.097 FINAL SATURATION= 1.043

FINAL SATURATION COMPUTED FROM FINAL MOISTURE= 1.123

TEST NUMBER 12B CONSTANT MEAN EFFECTIVE STRESS

INITIAL DRY DENSITY= 1.061

INITIAL VOID RATIO= 1.554

INITIAL SATURATION= 1.033

INITIAL VOLUME OF SAMPLE= 86.875

INITIAL MOISTURE CONTENT= 0.592

FINAL MOISTURE CONTENT= 0.582

VOLUME AFTER ISOTROPIC COMPRESSION= 81.375

AREA AFTER ISOTROPIC COMPRESSION= 1.682

J	S1	S3	SHEAR TEST					MEAN STRESS CONSTANT	
			E1	E3	E	MEAN	DEV	STRESS	DEV
2	1.00	0.50	0.0017	-0.0012	-0.0002	1.2247	0.0023		
3	2.00	1.00	0.0035	-0.0020	-0.0002	2.4495	0.0045		
4	3.00	1.50	0.0038	-0.0025	-0.0004	3.6742	0.0051		
5	4.00	2.00	0.0043	-0.0030	-0.0006	4.8990	0.0059		
6	5.00	2.50	0.0055	-0.0037	-0.0006	6.1237	0.0075		
7	6.00	3.00	0.0068	-0.0045	-0.0008	7.3485	0.0093		
8	7.00	3.50	0.0082	-0.0060	-0.0012	8.5732	0.0115		
9	8.00	4.00	0.0092	-0.0070	-0.0016	9.7980	0.0132		
10	9.00	4.50	0.0112	-0.0081	-0.0017	11.0227	0.0158		
11	10.00	5.00	0.0143	-0.0103	-0.0021	12.2474	0.0201		
12	11.00	5.50	0.0190	-0.0130	-0.0024	13.4722	0.0261		
13	12.00	6.00	0.0322	-0.0212	-0.0034	14.6969	0.0436		
14	13.00	6.50	0.0378	-0.0241	-0.0035	15.9217	0.0505		

FINAL DRY DENSITY= 1.123 FINAL SATURATION= 1.036

FINAL SATURATION COMPUTED FROM FINAL MOISTURE= 1.115

## TEST NUMBER 15B CONSTANT MEAN EFFECTIVE STRESS

INITIAL DRY DENSITY= 1.829

INITIAL VOID RATIO= 0.482

INITIAL SATURATION= 1.102

INITIAL VOLUME OF SAMPLE= 86.875

INITIAL MOISTURE CONTENT= 0.196

FINAL MOISTURE CONTENT= 0.189

VOLUME AFTER ISOTROPIC COMPRESSION= 83.075

AREA AFTER ISOTROPIC COMPRESSION= 1.701

J	S1	S3	SHEAR TEST MEAN STRESS CONSTANT					E3	E MEAN DEV STRESS	CONSTANT DEV STRAIN
			E1	E2	E3	E4	E5			
2	1.00	0.50	0.0007	-0.0004	0.0000	1.2247	0.0009			
3	2.00	1.00	0.0023	-0.0014	-0.0002	2.4495	0.0030			
4	3.00	1.50	0.0027	-0.0016	-0.0002	3.6742	0.0034			
5	4.00	2.00	0.0028	-0.0017	-0.0002	4.8990	0.0037			
6	5.00	2.50	0.0030	-0.0018	-0.0002	6.1237	0.0039			
7	6.00	3.00	0.0036	-0.0021	-0.0002	7.3485	0.0046			
8	7.00	3.50	0.0039	-0.0023	-0.0002	8.5732	0.0050			
9	8.00	4.00	0.0050	-0.0030	-0.0003	9.7980	0.0066			
10	9.00	4.50	0.0064	-0.0037	-0.0003	11.0227	0.0082			
11	10.00	5.00	0.0068	-0.0043	-0.0006	12.2474	0.0090			
12	11.00	5.50	0.0089	-0.0054	-0.0006	13.4722	0.0117			
13	12.00	6.00	0.0112	-0.0066	-0.0006	14.6969	0.0146			
14	13.00	6.50	0.0189	-0.0110	-0.0010	15.9217	0.0244			
15	14.00	7.00	0.0228	-0.0131	-0.0011	17.1464	0.0294			
16	15.00	7.50	0.0500	-0.0289	-0.0026	18.3712	0.0644			

FINAL DRY DENSITY= 1.902 FINAL SATURATION= 1.116

FINAL SATURATION COMPUTED FROM FINAL MOISTURE= 1.205



## TEST NUMBER 16B CONSTANT MEAN EFFECTIVE STRESS

INITIAL DRY DENSITY= 1.130

INITIAL VOID RATIO= 1.397

INITIAL SATURATION= 0.925

INITIAL VOLUME OF SAMPLE= 86.875

INITIAL MOISTURE CONTENT= 0.477

FINAL MOISTURE CONTENT= 0.493

VOLUME AFTER ISOTROPIC COMPRESSION= 83.175

AREA AFTER ISOTROPIC COMPRESSION= 1.712

J	SHEAR TEST						MEAN STRESS CONSTANT			
	S1	S3	E1	E3	E	MEAN	DEV	STRESS	DEV	STRAIN
2	1.00	0.50	0.0032	-0.0016	-0.0000	1.2247	0.0039			
3	2.00	1.00	0.0036	-0.0030	-0.0008	2.4495	0.0054			
4	3.00	1.50	0.0044	-0.0040	-0.0012	3.6742	0.0069			
5	4.00	2.00	0.0062	-0.0061	-0.0020	4.8990	0.0101			
6	5.00	2.50	0.0080	-0.0083	-0.0028	6.1237	0.0133			
7	6.00	3.00	0.0141	-0.0138	-0.0045	7.3485	0.0228			
8	7.00	3.50	0.0161	-0.0154	-0.0049	8.5732	0.0257			
9	8.00	4.00	0.0198	-0.0188	-0.0059	9.7980	0.0316			

FINAL DRY DENSITY= 1.161 FINAL SATURATION= 0.922

FINAL SATURATION COMPUTED FROM FINAL MOISTURE= 1.000

TEST NUMBER 198 CONSTANT MEAN EFFECTIVE STRESS

INITIAL DRY DENSITY= 1.611

INITIAL VOID RATIO= 0.682

INITIAL SATURATION= 1.047

INITIAL VOLUME OF SAMPLE= 82.821

INITIAL MOISTURE CONTENT= 0.263

FINAL MOISTURE CONTENT= 0.235

VOLUME AFTER ISOTROPIC COMPRESSION= 77.121

AREA AFTER ISOTROPIC COMPRESSION= 1.666

J	SHEAR TEST					MEAN STRESS CONSTANT		
	S1	S3	E1	E3	E	MEAN	DEV	STRESS DEV. STRAIN
2	1.00	0.50	0.0004	-0.0002	0.0000	1.2247	0.0004	*****
3	2.00	1.00	0.0007	-0.0004	0.0000	2.4495	0.0009	
4	3.00	1.50	0.0028	-0.0014	-0.0000	3.6742	0.0035	
5	4.00	2.00	0.0079	-0.0040	-0.0000	4.8990	0.0096	
6	5.00	2.50	0.0084	-0.0042	-0.0000	6.1237	0.0103	
7	6.00	3.00	0.0088	-0.0044	-0.0000	7.3435	0.0108	
8	7.00	3.50	0.0096	-0.0048	-0.0000	8.5732	0.0118	
9	8.00	4.00	0.0100	-0.0050	-0.0000	9.7980	0.0123	
10	9.00	4.50	0.0114	-0.0057	-0.0000	11.0227	0.0140	
11	10.00	5.00	0.0124	-0.0063	-0.0000	12.2474	0.0152	
12	11.00	5.50	0.0142	-0.0067	0.0003	13.4722	0.0170	
13	12.00	6.00	0.0166	-0.0079	0.0003	14.6969	0.0200	
14	13.00	6.50	0.0170	-0.0080	0.0003	15.9217	0.0204	
15	14.00	7.00	0.0223	-0.0108	0.0003	17.1464	0.0270	
16	15.00	7.50	0.0368	-0.0179	0.0003	18.3712	0.0447	

FINAL DRY DENSITY= 1.734 FINAL SATURATION= 1.057

FINAL SATURATION COMPUTED FROM FINAL MOISTURE= 1.132

## TEST NUMBER 20B CONSTANT MEAN EFFECTIVE STRESS

INITIAL DRY DENSITY= 0.975

INITIAL VOID RATIO= 1.780

INITIAL SATURATION= 0.989

INITIAL VOLUME OF SAMPLE= 88.621

INITIAL MOISTURE CONTENT= 0.650

FINAL MOISTURE CONTENT= 0.644

VOLUME AFTER ISOTROPIC COMPRESSION= 84.221

AREA AFTER ISOTROPIC COMPRESSION= 1.747

J	SHEAR TEST					MEAN STRESS CONSTANT		
	S1	S3	E1	E3	E	MEAN	DEV	STRESS DEV STRAIN
2	1.00	0.50	0.0017	-0.0009	-0.0000	1.2247	0.0021	*****
3	2.00	1.00	0.0024	-0.0015	-0.0002	2.4495	0.0032	
4	3.00	1.50	0.0034	-0.0032	-0.0010	3.6742	0.0054	
5	4.00	2.00	0.0046	-0.0041	-0.0012	4.8990	0.0071	
6	5.00	2.50	0.0055	-0.0057	-0.0020	6.1237	0.0092	
7	6.00	3.00	0.0068	-0.0070	-0.0024	7.3485	0.0113	
8	7.00	3.50	0.0112	-0.0104	-0.0032	8.5732	0.0176	
9	8.00	4.00	0.0150	-0.0148	-0.0048	9.7980	0.0243	
10	9.00	4.50	0.0170	-0.0247	-0.0108	11.0227	0.0340	

FINAL DRY DENSITY= 0.994 FINAL SATURATION= 0.989

FINAL SATURATION COMPUTED FROM FINAL MOISTURE= 1.010

## TEST NUMBER 218 CONSTANT MEAN EFFECTIVE STRESS

INITIAL DRY DENSITY= 1.289

INITIAL VOID RATIO= 1.102

INITIAL SATURATION= 1.030

INITIAL VOLUME OF SAMPLE= 86.875

INITIAL MOISTURE CONTENT= 0.419

FINAL MOISTURE CONTENT= 0.373

VOLUME AFTER ISOTROPIC COMPRESSION= 82.175

AREA AFTER ISOTROPIC COMPRESSION= 1.696

J	SHEAR TEST		MEAN STRESS		CONSTANT		
	S1	S3	E1	E3	E MEAN	DEV STRESS	DEV STRAIN
2	1.00	0.50	0.0023	-0.0012	-0.0000	1.2247	0.0029*****
3	2.00	1.00	0.0027	-0.0017	-0.0002	2.4495	0.0036
4	3.00	1.50	0.0038	-0.0031	-0.0008	3.6742	0.0056
5	4.00	2.00	0.0047	-0.0036	-0.0008	4.8990	0.0068
6	5.00	2.50	0.0062	-0.0055	-0.0016	6.1237	0.0096
7	6.00	3.00	0.0077	-0.0069	-0.0020	7.3435	0.0120
8	7.00	3.50	0.0100	-0.0093	-0.0029	8.5732	0.0158
9	8.00	4.00	0.0203	-0.0164	-0.0042	9.7980	0.0299
10	9.00	4.50	0.0250	-0.0198	-0.0049	11.0227	0.0366

FINAL DRY DENSITY= 1.344 FINAL SATURATION= 1.033

FINAL SATURATION COMPUTED FROM FINAL MOISTURE= 0.994

## TEST NUMBER 22B CONSTANT MEAN EFFECTIVE STRESS

INITIAL DRY DENSITY= 1.014

INITIAL VOID RATIO= 1.674

INITIAL SATURATION= 1.020

INITIAL VOLUME OF SAMPLE= 86.875

INITIAL MOISTURE CONTENT= 0.630

FINAL MOISTURE CONTENT= 0.571

VOLUME AFTER ISOTROPIC COMPRESSION= 79.275

AREA AFTER ISOTROPIC COMPRESSION= 1.653

J	S1	S3	SHEAR TEST		MEAN STRESS CONSTANT		E MEAN DEV STRESS	DEV STRAIN
			E1	E3	E			
2	1.00	0.50	0.0020	-0.0010	-0.0000	1.2247	0.0024*****	
3	2.00	1.00	0.0026	-0.0014	-0.0001	2.4495	0.0033	
4	3.00	1.50	0.0038	-0.0022	-0.0002	3.6742	0.0049	
5	4.00	2.00	0.0044	-0.0025	-0.0002	4.8990	0.0056	
6	5.00	2.50	0.0051	-0.0035	-0.0006	6.1237	0.0070	
7	6.00	3.00	0.0056	-0.0041	-0.0009	7.3485	0.0080	
8	7.00	3.50	0.0071	-0.0048	-0.0009	8.5732	0.0098	
9	8.00	4.00	0.0089	-0.0057	-0.0008	9.7980	0.0119	
10	9.00	4.50	0.0103	-0.0065	-0.0009	11.0227	0.0137	
11	10.00	5.00	0.0166	-0.0096	-0.0008	12.2474	0.0214	
12	11.00	5.50	0.0185	-0.0106	-0.0009	13.4722	0.0238	
13	12.00	6.00	0.0238	-0.0134	-0.0010	14.6969	0.0303	
14	13.00	6.50	0.0312	-0.0172	-0.0011	15.9217	0.0395	
15	14.00	7.00	0.0415	-0.0221	-0.0009	17.1464	0.0519	
16	15.00	7.50	0.0769	-0.0408	-0.0016	18.3712	0.0961	
17	16.00	8.00	-0.0256	0.0126	-0.0002	19.5959	0.0312	

FINAL DRY DENSITY= 1.111 FINAL SATURATION= 1.023

FINAL SATURATION COMPUTED FROM FINAL MOISTURE= 1.074

## TEST NUMBER 25C CONSTANT MEAN EFFECTIVE STRESS

INITIAL DRY DENSITY= 1.094  
 INITIAL VOID RATIO= 1.478  
 INITIAL SATURATION= 1.042  
 INITIAL VOLUME OF SAMPLE= 86.875  
 INITIAL MOISTURE CONTENT= 0.569  
 FINAL MOISTURE CONTENT= 0.537  
 VOLUME AFTER ISOTROPIC COMPRESSION= 80.625  
 AREA AFTER ISOTROPIC COMPRESSION= 1.669

J	SHEAR TEST MEAN STRESS CONSTANT						
	S1	S3	E1	E3	E	MEAN DEV STRESS	DEV STRAIN
2	1.00	0.50	0.0009	-0.0005	0.0000	1.2247	0.0011
3	2.00	1.00	0.0021	-0.0010	-0.0000	2.4495	0.0025
4	3.00	1.50	0.0054	-0.0030	-0.0002	3.6742	0.0069
5	4.00	2.00	0.0067	-0.0040	-0.0004	4.8990	0.0088
6	5.00	2.50	0.0088	-0.0052	-0.0006	6.1237	0.0115
7	6.00	3.00	0.0095	-0.0063	-0.0011	7.3485	0.0129
8	7.00	3.50	0.0105	-0.0068	-0.0011	8.5732	0.0142
9	8.00	4.00	0.0115	-0.0073	-0.0011	9.7980	0.0154
10	9.00	4.50	0.0140	-0.0092	-0.0015	11.0227	0.0190
11	10.00	5.00	0.0156	-0.0110	-0.0021	12.2474	0.0217
12	11.00	5.50	0.0187	-0.0126	-0.0022	13.4722	0.0255
13	12.00	6.00	0.0242	-0.0161	-0.0027	14.6969	0.0329
14	13.00	6.50	0.0370	-0.0231	-0.0031	15.9217	0.0491

FINAL DRY DENSITY= 1.169 FINAL SATURATION= 1.048

FINAL SATURATION COMPUTED FROM FINAL MOISTURE= 1.104

APPENDIX III  
CALIBRATION CURVES

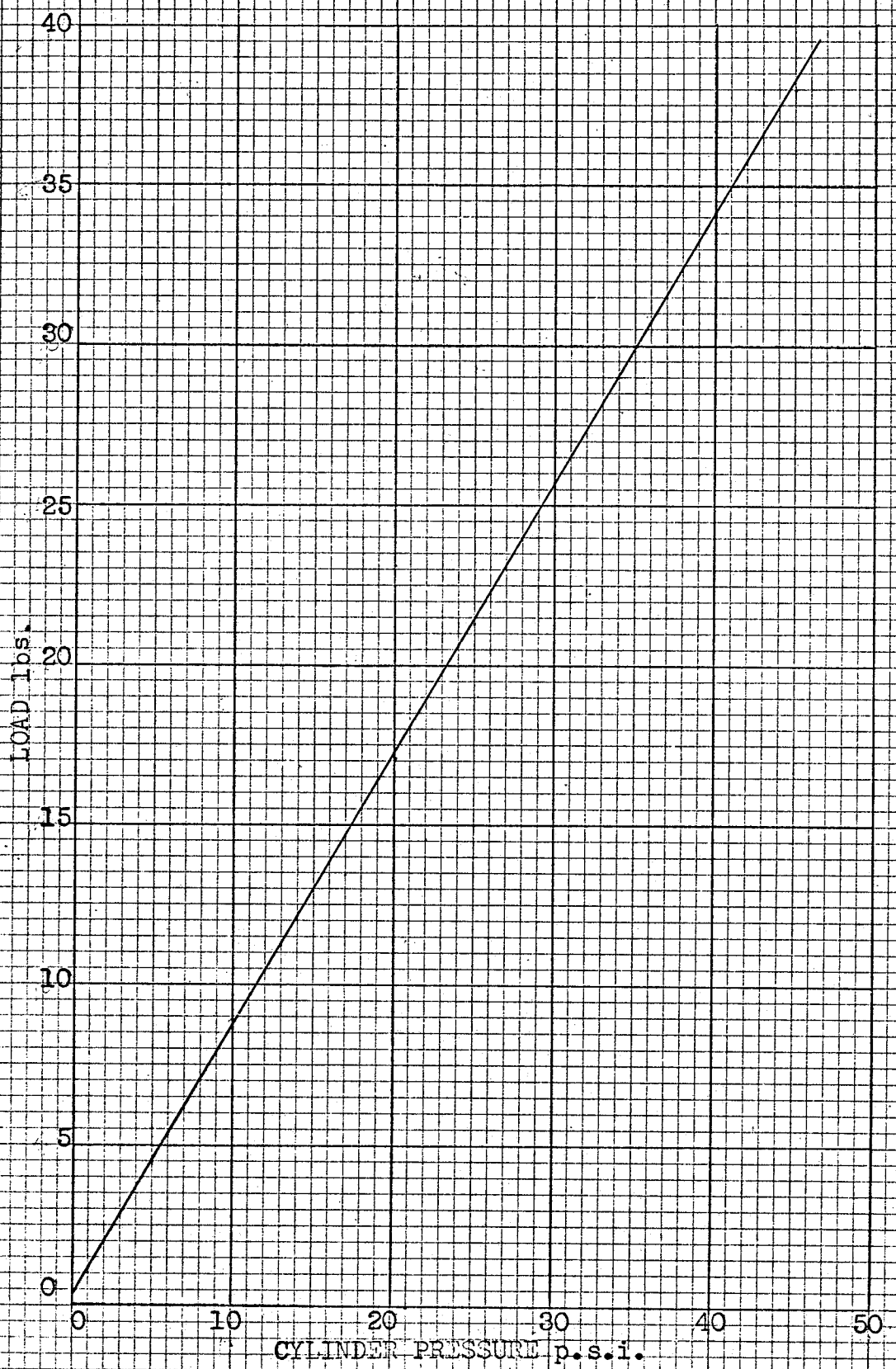


Figure 43. Calibration Curve for 1 in. Air Cylinder.



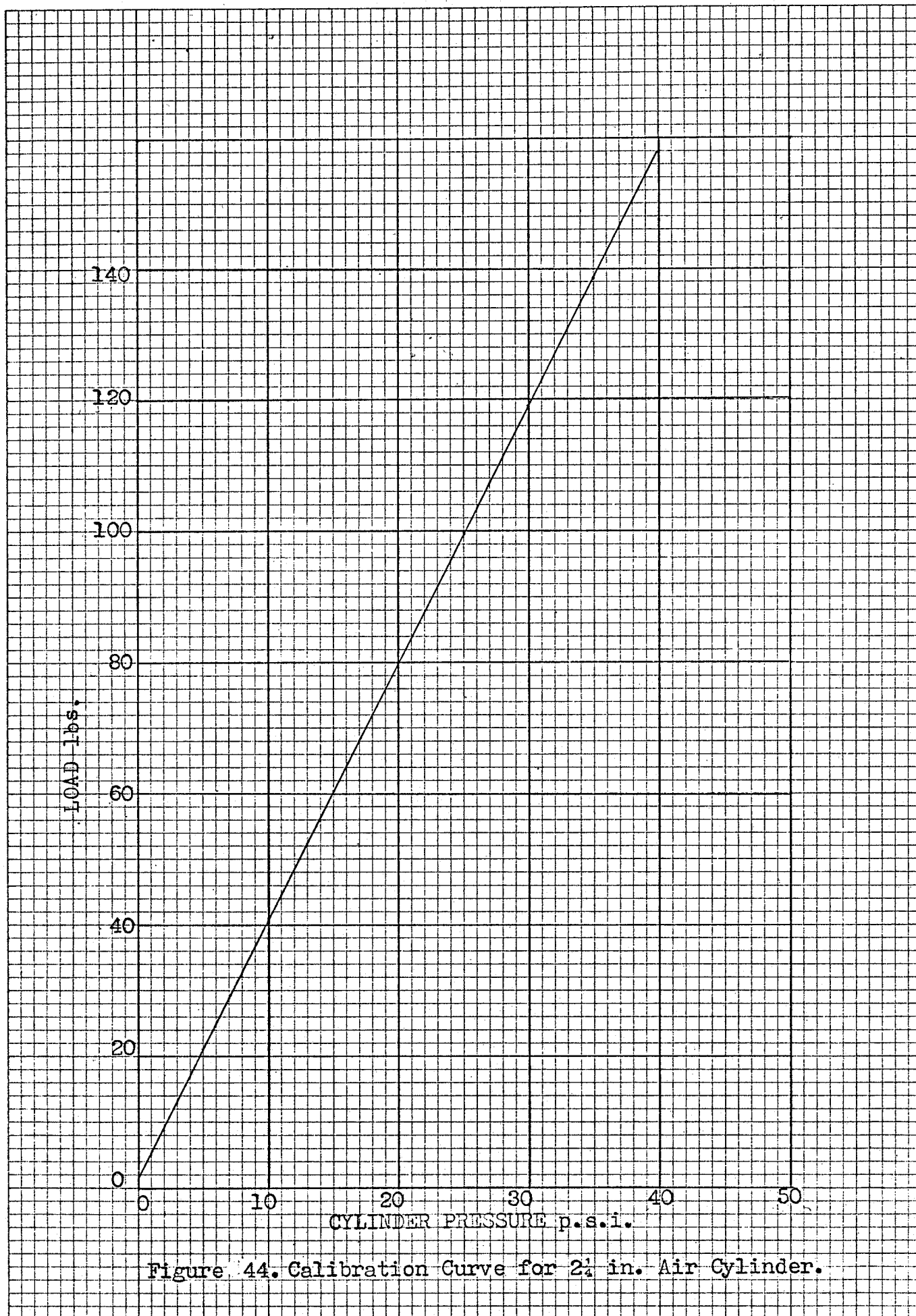


Figure 44. Calibration Curve for 2 $\frac{1}{4}$  in. Air Cylinder.

## BIBLIOGRAPHY

1. Boit L. A. "General Theory of Consolidation", Journal of Applied Physics 12, 155, 1941.
2. Dewet J. W. "Three Dimensional Consolidation" Highway Research Board Bulletin 342, 1962
3. Saada A. S. "A Rheological Analysis of Shear and Consolidation of Saturated Clays". Highway Research Board Bulletin 342, 1962.
4. Kondner R. L. "Hyperbolic Stress Strain Response in Cohesive Soils". Journal of the Soil Mechanics and Foundations Division. Proceeding A. S. C. E. vol. 89, No. SM1 Feb. 1963.
5. Domaschuk L. "An Analysis of Pore Water Stress Through a Separation of the Volumetric and Deviatoric Components of Stress". Presented to the 21st. Canadian Soil Mechanics Conference, 1968
6. Baracos A. "The Stability of River Banks In The Metropolitan Winnipeg Area". Proceedings Fourteenth Canadian Soil Mechanics Conference, National Research Council T.M. No. 69, 1961.
7. Bishop A.W. and Henkel D.J. "The Measurement of Soil Properties in the Triaxial Test". Edward Arnold and Co., London, 1962.
8. Ladanyi B. "Etude Des Relations Entre Les Contraintes et Les Deformations Lors Du Cisaillement Des Sols Pulveruleux." Annales Des Travaux Publics De Belgique, 1960.
9. Ahlvin R.G. and Ulery H.H. "Tabulated Values For Determining the Complete Pattern of Stresses, Strains and Deflections Beneath a Uniform Circular Half Space". Highway Research Board Bulletin 342, 1962.
10. Kondner R. L. and Horner J. M. "Triaxial Compression With Effective Octahedral Normal Stress Control." Canadian Geotechnical Journal Vol. II, No. 1, Feb. 1965.
11. Vesic A. S. "Behavior of Granular Materials Under High Stresses". Journal of the Soil Mechanics and Foundations Division Proceedings A. S. C. E. Vol. 94, May 1968.
12. Scott R. F. "Principles of Soil Mechanics". Addison Wesley, 1963.
13. Domaschuk L. "A Study Of The Static Stress-Deformation Characteristics of a Sand". Ph. D Thesis , Georgia Institute of Technology, December, 1965.