ESSAYS ON ASSET PRICING WITH INCOMPLETE OR NOISY INFORMATION

BY

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ABSTRACT

This dissertation consists of two essays, in which I examine the effects of incomplete or noisy information on expected risk premium in equity markets. In the first essay I provide empirical evidence demonstrating that an information-quality (IQ) factor, built on accrual-based information precision measure, is priced. This result still stands after controlling for factors, such as size, Book-to-Market (B/M) ratio, and liquidity. To explain this empirical observation, I derive a continuous-time model in the spirit of Merton's (1973) Intertemporal Capital Asset Pricing Model (ICAPM) to examine how systematic IQ risk affects security returns. Unique to my model, imprecise information influences the pricing of an asset through its covariance with: (i) stock return; (ii) market return; and (iii) market-wide IQ. In equilibrium, the aggregate effect of these covariance terms (proportional to IQ-related betas) represents the systematic component of IQ risk and therefore requires a risk premium to compensate for it. My empirical test confirms that the aggregate effect of systematic IQ risk is significant and robust to the inclusion of other risk sources, such as liquidity risk.

In the second essay I extend a recent complete information stock valuation model with incomplete information environment. In practice, mean earnings-per-share growth rate (MEGR) is random and unobservable. Therefore, asset prices should reflect how investors learn about the unobserved state variable. In my model investors learn about MEGR in continuous time. Firm characteristics, such as stronger mean reversion and lower volatility of MEGR, make learning faster and easier. As a result, the magnitude of risk premium due to uncertainty about MEGR declines over learning horizon and

converges to a long-term steady level. Due to the stochastic nature of the unobserved state variable, complete learning is impossible (except for cases with perfect correlation between earnings and MEGR). As a result, the risk premium is non-zero at all times reflecting a persistent uncertainty that investors hold in an incomplete information environment.

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DEDICATION

To my Father and Mother, Xiandong Wang and Shuying Yin: thank you for the constant encouragement and support.

CHAPTER 1

Introduction

Traditional asset-pricing models such as Merton's (1973) Intertemporal Capital Asset Pricing model (ICAPM) are based on the assumption that the financial market is informationally efficient and that individuals are well informed. However, there is substantial evidence indicating that information releases are noisy and unreliable (see for example, Shapiro and Wicox, 1996; and Faust, Rogers, and Wright, 2000). Facing an imperfect information set, investors face information-quality risk. This is a deviation from the assumptions made in traditional asset-pricing models. It suggests that market prices may deviate from fundamental asset values, and result in the failure of standard asset-pricing models.

There is substantial empirical evidence in the extant literature showing that returns are related to the firm's information structure. [1] However, there is very little work laying the theoretical foundation for this empirical observation. The aim of the current dissertation is to fill this gap and to examine the pricing of information-quality (IQ) risk.

This dissertation consists of two essays. In the first essay, which is presented in Chapter 2, I examine the pricing of IQ risk by addressing the following two questions: (i) Is a market-wide IQ factor priced even after controlling for other known significant

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[1] See for example, Schipper (2002), Botosan, Plumlee, and Xie (2004), and Easley, Hvidkjaer, and O'Hara (2002).

market factors (such as size, value, and liquidity factors)? (ii) If so, does IQ risk affect security prices also through its factor sensitivity? To address these questions I construct two alternative measures to proxy for IQ-related cash flow noise based on the work of Barth et al. (2001). Using an ad-hoc IQ-adjusted Fama-French factor model, I find significant evidence supporting the notion that IQ is a priced market factor.

Motivated by this empirical finding, I derive a theoretical Information Quality Capital Asset Pricing Model (IQCAMP) based on Merton’s (1973) to further examine the association between systematic IQ risk and stock return. The precise analytical form of this risk allows me to perform a formal econometric study of the different components of systematic IQ and test whether they are priced. Unique to the current model, noisy information influences the pricing of assets through its impact not only on the factor loadings, but also on factor sensitivities. I show that IQ risk has systematic and idiosyncratic components, and that only the former is priced.

I further demonstrate that systematic IQ risk is priced through the extra betas of the asset. With imprecise information set, there are three additional systematic risk effects, related to information quality, that require a risk premium due to the additional risk investors will face. These effects stand for systematic information-quality risk.

The first source of systematic information-quality risk is measured by the covariance of the security’s noisy-return component with that of the overall market. Investors demand a return premium due to this commonality in IQ risk exposure. The second source of IQ risk is given by the covariance between the asset return and the market noisy-return component. This effect implies investors’ preference for securities with a higher return to offset a negative noise effect in the overall market. The last source of systematic IQ risk is measured by the covariation of the market return and the asset's noisy-return component. Investors prefer a security with negative covariance so that a positive noisy-return component for the asset will hedge against the risk of a bear market.

Based on these extra covariance terms (or betas), I derive a static unconditional version of the IQCAPM to explicitly demonstrate the link between the security return and its IQ-risk exposure. I then empirically test this version of the IQCAPM and find evidence supporting the pricing of systematic IQ risk at the cross section. Analytically, this model is similar to the liquidity-adjusted CAPM of Acharya and Pedersen (2005). Given the probable relation between security IQ and its market liquidity, it is possible that my IQ betas capture the effect of Acharya and Pedersen's (2005) illiquidity betas. A Pearson correlation test shows low and insignificant correlation between my IQ betas and illiquidity betas, implying that my IQ beta represents a source of systematic risk distinct of liquidity risk. Cross-sectional regression tests show that the IQ betas are significantly priced even after controlling for Acharya and Pedersen's (2005) liquidity betas.

In the second essay, which is presented in Chapter 3, I extend Bakshi and Chen (2005)'s earning-based stock valuation model (BC model) by allowing agents to learn. Although the true mean-reverting process of mean earnings growth rate (MEGR) in my model is unobservable, the continuous learning process allows agents to estimate it, and update estimates, based on available earnings information. I examine the posterior variance of learning-based estimates to track the dynamics of pricing errors and risk premiums over time. An extra risk premium on MEGR is demanded due to uncertainty about incomplete information, which can be reduced to a minimum level through learning. A closed-form solution to the equilibrium stock price is also provided for the incomplete-information environment.

One important finding in the second essay is that the faster the expected earnings growth rate reverts to its long-term mean, the smaller the required risk premium due to information incompleteness. This is because higher speed of mean reversion implies easier learning. Another finding in the second essay is that, ceteris paribus, higher uncertainty related to the unobservable (latent) mean of earnings growth rate, results in a larger required risk premium. This result is more pronounced for younger firms with shorter learning horizons for which, naturally, there is a short history of data available for learning. This finding is consistent with Pastor and Veronesi (2003), who predict that M/B declines over a typical firm’s lifetime, and younger firms should have higher M/B ratios than otherwise identical older firms since uncertainty about younger firms’ average profitability is greater.

In a perfect learning environment (e.g., the correlation between unobservable MEGR and earnings is one), the extra risk premium on MEGR declines and converges to zero in the long run. At the same time, the variance of the estimate of MEGR decreases over learning horizon and converges to zero. Perfect correlation implies that investors eventually have complete knowledge of the true process of the mean growth rate. However, in non-perfect learning environment, the extra risk premium on MEGR never vanishes regardless of the learning horizon. This long run risk premium reflects a persistent uncertainty that investors hold in an incomplete information environment.

For comparison, I also investigate the effects of firm characteristics (such as mean-reversion speed and volatility of MEGR) on price differential between my incomplete-information model and the BC complete-information model as well. I find that the price differential between my model and that of BC, defined as pricing error, can persist for years even under perfect learning conditions. The more volatile MEGR is, the longer the persistence. For an extreme incomplete-information environment, such as one with zero correlation between earnings and MEGR, investors basically learn nothing about state variable MEGR from earnings. In this case, pricing errors are largest on average. Finally, I show that pricing errors still exist after long learning horizon (e.g., eight years) with precisely estimated MEGR as long as the information environment is incomplete. The non-vanishing pricing errors reflect residual risk premium (not present in the complete information model) due to investors' imperfect forecasts of the underlying state variable.

CHAPTER 2

The IQCAPM: Asset Pricing with Information-Quality Risk

Traditional asset-pricing models are based on the assumption that the financial market is informationally efficient and that individuals are well informed (see for example, Sharpe, 1964; Lintner 1965; Mossin, 1966; and Merton, 1973). However, there is substantial evidence indicating that information releases are noisy (see for example, Faust, Rogers, and Wright, 2000; Shapiro and Wicox, 1999; and Wang 1993). With an imperfect information set, investors may face information-quality (IQ) risk. Ignoring IQ risk may lead to asset mispricing in traditional asset-pricing models. In this chapter I examine the pricing of IQ risk by addressing the following two questions: (i) Is a market-wide IQ factor priced even after controlling for other known significant market factors (such as size, value, and liquidity factors)? (ii) If so, does IQ risk affect security prices also through its factor sensitivity? To address these questions I construct two alternative measures to proxy for IQ-related cash flow noise based on the work of Barth et al. (2001).

2.1. Related Literature

Francis et al. (2005) define information risk as the likelihood of firm-specific information pertinent to pricing decision to be of poor quality. Recent studies show that information risk is a nondiversifiable risk factor. For instance, Francis et al. (2004, 2005) find a positive link between poorer accrual quality and larger cost of equity and debt, and they claim that accrual quality is a systematic priced risk factor. Easley and O'hara (2004) develop a rational-expectations asset-pricing model to examine the association between information structure (public versus private information) and cost of equity capital. They argue that private information increases the risk faced by uninformed investors since informed investors can adjust their portfolio weights upon the arrival of new information. Therefore they propose that the firm's cost of capital can be affected through the precision of information it provides to investors.[2]

There is substantial empirical evidence in the extant literature showing that returns are related to the firm's information structure.[3] However, there is very little theoretical work to explain this empirical observation. Huang and Liu (2003) assume an imperfect information structure and examine the optimal portfolio selection problem in the presence of periodically observable state variables (such as dividend yields, GDP growth rates, and inflation rates). The focus of their work is on the optimal investment strategy under imprecisely and periodically released information. They do not examine the impact of imperfect information on asset pricing. Related to my work, Hughes et al. (2007) examine the impact of both symmetric and asymmetric information on asset pricing within the Arbitrage Pricing Theory (APT) framework. They conclude that, at the limit, information impacts factor risk premiums, not factor sensitivities. This is consistent with Wang (1993) who presents a dynamic asset-pricing model under asymmetric information. He finds that information asymmetry among investors can increase price volatility, negative autocorrelation in returns, and the risk premium.

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[2] For more on the systematic nature of IQ risk see O'hara (2003) and Leuz and Verrecchia (2004).

[3] See for example, Wang (1993), Schipper (2002), Botosan, Plumlee, and Xie (2004), Easley, Hvidkjaer, O'Hara (2002), and Francis et al. (2004) and (2005).

The focus of the current chapter is on the effect of IQ on asset pricing rather than the effect of asymmetric information. I consider the case in which all (equally-informed) investors face an imprecise (noisy) information set. In most models for return predictability state variables are assumed to be perfectly and accurately observable at any point in time.[4] In practice, however, investors have to estimate state variables from available public information. I assume that cash flow is the fundamental element for pricing, and therefore poor accrual quality weakens pricing and increases information risk. In a related literature, Francis et al. (2005) construct accrual-based measures to proxy for information risk. They argue that the systematic effect of accrual quality on the cost of capital can be explained by a rational asset-pricing framework in which accruals quality captures information risk that cannot be diversified away. In support of this prediction of Francis et al. (2005), I show that an ad hoc factor model that considers an IQ factor significantly explains the cross-sectional variations of stock returns.

Market liquidity is affected by information asymmetry (see for example, Glosten and Milgrom, 1985; Easley and O'Hara, 1987; and Easley and O'Hara, 1992). One may expect that information quality problem is reflected in market liquidity as well. Pastor and Stambaugh (2003) show that market-wide liquidity is a priced factor for stock returns. I test whether the pricing of the IQ factor is subsumed by the Pastor and Stambaugh (2003) market-illiquidity factor. My ad hoc test shows that, even after controlling their liquidity measure and the Fama-French three factors, my IQ factor is still significantly priced. This result is robust for tests based on portfolios sorted on all other market factors.

[4] I see this assumption in discrete-time models, which assume that state variables are precisely observable at the beginning of every discrete-time interval (see for example, Kandel and Stambaugh, 1996; and Barberis, 2000). Similarly, in continuous-time models all state variables are assumed to be continuously and precisely observable (see for example, Samuelson, 1969; Merton, 1971 and 1973; and Breedon, 1979).

Note that the evidence I present for IQ being a priced market factor is based on an ad hoc empirical model. To conduct a formal econometric study, I need a theoretical description of IQ-adjusted risk premium. This calls for a theoretical asset-pricing model under imprecise information. To this end, I derive an IQ-adjusted intertemporal asset-pricing model to theoretically examine the pricing of IQ risk through its factor sensitivities. I model IQ impact on asset returns with an Ornstein-Uhlenbeck mean-reverting process under which the information-related return error fluctuates around its long-term mean. In my IQ-adjusted Capital Asset Pricing Model (IQCAPM) IQ risk has systematic and idiosyncratic components, and only the former is priced.

Our model demonstrates that systematic IQ risk is priced through extra asset betas as well as through the market risk premium. With an imprecise information set, there are three additional systematic risk effects measured by covariance (beta) terms between firm-specific information noise, asset’s fundamental return, and market-wide information noise. In equilibrium, these covariance terms determine the components of the systematic IQ risk and therefore require a risk premium to compensate for it.

The first component of IQ risk is the covariance of the security’s IQ-related return noise and market-wide IQ-related return noise, which I call *commonality in IQ*. Investors require higher expected returns for securities with positive covariance between the asset noise and the market noise. Securities with the negative covariance provide a hedge against the risk of a negative IQ return on the market portfolio, and therefore have lower expected returns. The second component is the covariance of the market IQ return with a security’s fundamental return. At times when overall market IQ return is negative, investors prefer to hold securities that pay a higher fundamental return. Thus, investors demand a premium for this covariance. The third component is the covariance between the market fundamental return and the security IQ return. Investors prefer a negative covariance as it corresponds to higher security IQ return in a declining market.

I empirically test a static version of the IQCAPM and finds evidence supporting the pricing of systematic IQ risk at the cross section. Analytically, my static version of IQCAPM is similar to the liquidity-adjusted CAPM of Acharya and Pedersen (2005). To check whether the IQ risk represented by my IQ betas is distinct of the liquidity risk measured by Acharya and Pedersen’s (2005), I estimate their liquidity beta and incorporate it into cross-sectional test of IQ betas as a control variable. My results of robustness check lend strong support to the significance of IQ betas even in the presence of liquidity berta of Acharya and Pedersen’s (2005).

Given the probable relation between security IQ and its market liquidity, it is possible that my IQ betas capture the effect of Acharya and Pedersen’s (2005) illiquidity betas. A Pearson correlation test shows low and insignificant correlation between my IQ betas and illiquidity betas implying that my IQ beta represents a source of systematic risk distinct of liquidity risk. My cross-sectional regression tests show that the IQ betas are significantly priced even after controlling for Acharya and Pedersen’s (2005) liquidity betas.

The remainder of this chapter is organized as follows: in Section 2.2, I conduct an ad hoc preliminary test to examine whether the IQ factor is a priced state variable, even after controlling for several widely used factors. Given the supportive evidence in Section 2.2, I derive an IQ-adjusted intertemporal asset-pricing model in Section 2.3 to obtain analytical form of asset prices in the presence of IQ factor. This model facilitates a formal empirical test of IQ pricing. Section 2.4 discusses the theoretical implication of noisy information for asset pricing, while distinguishing between the systematic and idiosyncratic components of IQ risk. Section 2.5 provides empirical evidence in support of my model. Robustness checks are provided in Section 2.6.

2.2. Is an IQ Factor a Priced State Variable? A Preliminary Test

Prior studies document that firms with relatively low (high) magnitudes of signed abnormal accruals, offer positive (negative) risk-adjusted returns (see for example, Sloan, 1996; Xie, 2001; and Chan et al., 2001). Francis et al. (2005) argue that the systematic effect of accrual quality on the cost of capital can be explained by a rational asset-pricing framework in which accruals quality captures an undiversifiable information risk. Motivated by the above literature, in this section, I perform an ad hoc empirical test to examine whether an IQ factor is a priced state variable after controlling for several widely used factors, such as market return, size, value, and liquidity.

2.2.a. An IQ-adjusted Factor Model

The basic intuition underlying standard asset-pricing theory is that expected stock returns are related to return’s sensitivity to state variables with market-wide effects on consumption and investment opportunities. Extra return is required to compensate investors for holding securities sensitive to these state variables. Fama and French (1992, 1993, and 1996) find evidence that expected excess market return, along with two other variables; size (market value equity) and B/M (book-to-market ratio); explain much of the variation in average stock returns.[5] In addition to the Fama-French three factors, liquidity is also important. There are two reasons to control for a liquidity factor when testing other factors. First, a large number of studies demonstrate that investors demand compensation for holding illiquid stocks and that market liquidity represents a priced state variable.[6] Second, illiquidity may be induced by asymmetric information.[7] To disentangle the liquidity and the IQ effects, I introduce a liquidity factor in my ad hoc model and estimate the following IQ-adjusted Fama-French factor model:

$$r_{it} - r_{ft} = \alpha_i + b_i MKT_t + s_i SMB_t + h_i HML_t + q_i IQF_t + l_i LIQ_t + \varepsilon_{it},$$

where r_{it} is the return of portfolio i in quarter t, r_{ft} is three-month T-bill rate for quarter t, MKT_t is the excess return on a broad market index, SMB_t is the return on a portfolio of small stocks minus the return on a portfolio of large stocks, HML_t is the return on a portfolio of stocks with high B/M ratio minus the return on a portfolio of stocks with low B/M ratio, IQF_t is the mimicking IQ factor constructed to capture IQ risk, and LIQ_t is the mimicking liquidity factor constructed based on Pastor and Stambaugh (2003) liquidity

[5] They argue that size and book-to-market represent risk factors missed by the capital asset pricing model of Sharp (1964) and Linter (1965). They propose a three-factor asset pricing model consisting of a market factor and risk factors related to size and B/M. In support of the model, Laknoishok, Shleifer and Vishny (1994) show that stocks with high book-to-market ratios (B/M) provide higher average returns than stocks with low ratios of B/M for U.S. stocks. They further show that the difference in returns on value stocks and growth stocks is the value premium associated with relative financial distress.

[6] See for example, Amihud and Mendlson (1987), Constantinides (1987), Heaton and Lucas (1996), Vayanos (1998), Chalmers and Kadlec (1998), Huang (2001), Lo, Mamaysky, and Wang (2001), Campbell, Grossman, and Wang (1993), Pastor and Stambaugh (2003), Acharya and Pedersen (2005), and Liu (2006).

[7] For example, Easley and O'hara (2004) find that asymmetric information increases uninformed investor's risk, because informed investors are able to adjust their portfolio proportions better and take advantage of their private information. Uninformed investors will require higher returns as compensation for the nondiversifiable information risk they face. Liu (2006) argues that when uninformed investors are aware of asymmetric information in the market they will choose not to trade, which will hurt market liquidity.

measure as a control variable. The factor loading q_i captures portfolio return's comovement to IQF_t that is distinct from its comovement with other factors: MKT_t, SMB_t, HML_t, and LIQ_t.

2.2.b. Data and Variable Construction

In this subsection I follow the model of Barth et al. (2001) in constructing an IQ measures to proxy for firm-specific IQ risk. The pricing process applied by capital-market participants uses aggregate earnings and accrual components of current earnings to predict the firm's future cash flow. I follow recent studies in which cash flow is the primitive element for pricing, and therefore poor accrual quality weakens pricing and increases information risk (see for example, Barth et al., 2001 and Francis et al., 2005).

The model developed by Barth et al. (2001) shows that aggregate cash flow and accrual components of current earnings have substantial predictive ability for future cash flows.[8] A larger deviation between accruals and cash flows represents a lower quality of accounting information and a lower IQ.[9] Based on the estimated residuals obtained from the model of Barth et al.'s (2001), I construct two alternative IQ measures for empirical test.[10] The forecast regression of future operating cash flows (OCF) uses previous period earnings components and is given below:

[8] Accrual components of current earnings include change in accounts receivable, change in accounts payable, change in inventory, depreciation, amortization, and other accruals.
[9] Similar to Barth et al. (2001), several theoretical and empirical studies treat the precision level of disclosed public information as a measure of IQ (see for example, Baiman and Verrecchia, 1996; Admati and Pleiderer, 2000; Easley and O'Hara, 2004; Gomes, Gorton and Madureira, 2004; and Lambert, Leuz , Verrecchia, 2006). Barth et al. (2001) suggest residuals obtained from their model as the basis for the construction of measures of information precision.
[10] For an additional accrual-based measure see Dechow and Dichev (2002), who use the standard deviation of residuals from regressions relating current accruals to cash flows. Other studies, such as Admati and Plaiderer (2000), Baiman and Verrecchia (1996), and Easley and O'hara (2004) use publicly disclosed information precision to measure information quality. Information precision is measured by the

$$CFO_{i,t+1} = \alpha_0 + \alpha_1 CFO_{i,t} + \alpha_2 \Delta AR_{i,t} + \alpha_3 \Delta INV_{i,t} + \alpha_4 \Delta AP_{i,t} + \alpha_5 DEPR_{i,t} + \alpha_6 OTHER_{i,t} + e_{i,t+1},$$

where $CFO_{i,t}$ represents cash flow from operations of firm *i* in quarter *t* (Compustat quarterly data item Q108), adjusted for the accrual portion of extraordinary items and discontinued operations (Compustat quarterly data item Q78); $\Delta AR_{i,t}$ represents the change in accounts receivable (Compustat Q103); $\Delta INV_{i,t}$ is the change in the inventory account (Compustat Q104); $\Delta AP_{i,t}$ is the change in accounts payable and the accrued liabilities account (Compustat Q105); $DEPR_{i,t}$ represents Depreciation and Amortization Expense (Compustat Q5); $OTHER_{i,t}$ is the net of all other accruals, calculated as income before extraordinary items and discontinued operations (Compustat Q8) minus ($CFO+\Delta AR+\Delta INV-\Delta AP-DEPR$). All variables are deflated by the average of total assets (Compustat Q44).

To empirically estimate the IQ proxies I use quarterly data from the COMPUSTAT data for quarter 1 of 1987 through quarter 4 of 2007. [11] Following Barth et al. (2001), I exclude firms in SIC codes 6000-6999 (financial institution, insurance, and real estate companies) because the empirical cash flow predictability model does not reflect their activities. I further exclude outliers, defined as observations within the upper and lower percentiles of my sample. After excluding firms with missing data for the variables used in the above regression model, I am left with 118,177 firm-quarter

predictability of expected future cash flows under the flexibility and discretion under Generally Accepted Accounting Principles (GAAP).

[11] Barth et al (2001) use Compustat data beginning in 1987 because cash flow from operations (Compustat annual data item #308) calculated from the statement of cash flows is only available since 1987, following the Statement of Financial Accounting Standard No. 95 (SFAS No. 95). Recent studies (such as Collins and Hribar, 2002, and Dechow, Cotharin, and Watts, 1998) find that using CFO derived from balance sheet accounts is likely to yield biased and noisy estimates. To minimize the impact of measurement error I use CFO reported in the statement of cash flows subsequent to the Statement of Financial Accounting Standard No. 95 (SFAS No. 95). Care should be taken when working with data from the quarterly statement of cash flow as it is reported on a cumulative basis (i.e., 2nd quarter statement of cash flows reports the cumulative amounts for quarter 1 and 2, not just the amount for quarter 2).

observations, representing 2,685 firms. The quarterly values of the three Fama-French factors: market factor (MKT), size factor (SMB), and value factor (HML), are constructed based on daily data downloaded from Kenneth French's website.[12] Below I detail the methodology for constructing the IQ factor (IQF_t) and the liquidity factor (LIQ_t).

The raw cash-flow regression residual, $e_{i,t}$, representing standardized future operating cash flow unexplained by disaggregated earnings components, is white noise and therefore could be negative or positive. A firm with a zero mean of $e_{i,t}$ may be interpreted as a high IQ firm, but in reality it is not necessarily the case. If variance of $e_{i,t}$ is large (even though the mean is zero), the firm's IQ is poor. Therefore, this raw estimate may not adequately represent IQ. Instead, I use two other dimensions of the estimated cash flow residual: the absolute magnitude of the residual and the residual variability, which better reflect the level of information precision.

The first IQ measure, $IQ1 = | e_{i,t} |$, is the absolute value of the estimated residual, which measures the magnitude of errors in predicting future operating cash flows. A higher $IQ1$ reflects a lower forecast quality of reporting earnings. The second IQ measure, $IQ2 = \sigma(e_i)_t$, is the standard deviation of estimated residuals over time *t*-4 through *t* which reflects the stability dimension of information quality. A lower $IQ2$ represents a higher predictability of earnings and therefore a higher quality of financial reporting.

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[12] See http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/. I thank Kenneth French for making these data available.

I use firm-specific IQ measures, to construct a market-wide IQ factor. The construction of the mimicking IQ factor is similar to the construction of *SMB* and *HML* in Fama and French (1993). The mimicking IQ factor, IQF_t, is the spread between the return on a portfolio of stocks with high IQ measures and the return on a portfolio of stocks with low IQ measures. Since each of the two IQ measures ($|e_{i,t}|$, and $\sigma(e_i)_t$) captures a different dimension of information quality, I build two alternative mimicking IQ factors based on each of measure, respectively.[13]

Next, I construct a market-wide liquidity factor as a control variable. Liquidity is an unobservable quantity. A variety of empirical-liquidity measures are proposed to capture different dimensions of liquidity: trading quantity, trading speed, trading cost, and price impact of trading.[14] To test my IQ factor, I adopt the liquidity measure of Pastor and Stambaugh (2003) as a control variable to disentangle the effects of liquidity and IQ. The liquidity measure of Pastor and Stambaugh (2003) captures the price impact of trading dimension of liquidity, which makes it ideal to compare the price impact of IQ.

Following Pastor and Stambaugh (2003), I use daily trading volume, daily return, the number of shares outstanding, and market value (MV) from CRSP for 1987-2007. Annual accounting data for calculating the book-to-market (B/M) ratio is obtained from COMPUSTAT. To be consistent, I exclude NASDAQ from my sample in constructing

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[13] Mimicking factors are widely used when examining economic factors. For example, Breedon (1979) shows that mimicking factors can substitute the state variable in Merton's (1973) intertemporal capital asset pricing model. Chen et al. (1987) construct mimicking portfolios to investigate several macroeconomic factors, and Breeden et al. (1989) use them in the context of aggregate-consumption growth. Fama and French (1996) construct SMB and HML as mimicking-size factor and mimicking-value factor to help explain variations of returns and to capture distress risk. Liu (2006) constructs a mimicking-liquidity factor to hedge the state risk of market-wide liquidity.

[14] For example, the bid-ask spread measure used in Amihud and Mendlson (1986) and Jones (2002), the turnover measure of Datar et al. (1998) and Lo and Wang (2000), the return-to-volume measure of Campbell, Grossman, and Wang (1993), Amihud (2002), and Pastor and Stambaugh (2005).

the liquidity measure because reported volumes on NASDAQ include inter-dealer trades, unlike the volumes reported on the NYSE and the AMEX. A stock's liquidity estimate is excluded if there are less than 15 consecutive observations in a given quarter; stocks with prices less than $5 and greater than $1000 are also excluded.

Following Pastor and Stambaugh (2003), the firm-specific liquidity measure is estimated by an ordinary least squares (OLS) coefficient on signed trading volume, denoted as $Illiquidity_t^i$ reflecting the liquidity level of firm *i*. [15] Note that, Pastor and Stambaugh (2003) estimate their liquidity measure using daily observation within each month. In this case, since the IQ measures are constructed on quarterly basis, my liquidity measure is estimated based on daily observations over a one-quarter interval.

The construction of the liquidity factor LIQ_t is similar to the construction of IQF_t. I sort stocks into two portfolios based on their liquidity level. The spread between the returns on a portfolio with high $Illiquidity_t^i$ and a portfolio with low $Illiquidity_t^i$, is the mimicking liquidity factor LIQ_t. I construct a mimicking LIQ factor instead of using the liquidity measure for market portfolio because of the concern raised in Liu (2006).[16]

2.2.c. Forming Portfolios

I use quarterly returns and market capitalization data for all common shares listed on NYSE and AMEX from CRSP for the period between Q1 1987 and Q4 2007. Book-

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[15] The details for estimating the Pastor and Stambaugh (2003) model are provided in Appendix B1.

[16] In Pastor and Stambaugh (2003), a market-wide liquidity measure is constructed, and then the innovation in market liquidity is used as the liquidity factor. Liu (2006) argues that the aggregate liquidity measure constructed in this way is problematic because it fails to distinguish illiquid stocks whose daily trading volumes are all equal to zero in the prior month. See Liu (2006) for a detailed discussion.

to-market value is calculated based on book values from COMPUSTAT.[17] I exclude records with missing IQ measure.

Next I double-sort stocks based on both IQ and Illiquidity so that my portfolios are adjusted for both effects. I first sort stocks into three IQ categories and then into three Illiquidity groups within each IQ category, and obtain 3 IQ × 3 Illiquidity portfolios. Note that with both IQ1 and IQ2, the higher the proxy the lesser the information quality. Thus, low values of IQ1 and IQ2 represent high information quality and as one moves from the low IQ1 or IQ2 portfolio to the high IQ1 or IQ2 portfolio, one actually moves from a high information-quality portfolio to a low information-quality portfolio.

I denote the three IQ portfolios with IQH, IQM, and IQL, where: IQH is the *high information-quality portfolio* (with low levels of IQ1 or IQ2); IQM is the *medium information-quality portfolio* (with medium levels of IQ1 or IQ2); and IQL is the *low information-quality portfolio* (with high levels of IQ1 or IQ2). When portfolios are formed, portfolio return, portfolio IQ measure, and portfolio illiquidity measure are computed as follows for time *t*:

$$r_t^p = \sum w_i \, r_{it} \ ,$$

$$IQ_t^p = \sum w_i \, IQ_{it} ,$$

$$Illiquidity_t^p = \sum w_i \, Illiquidity_t^i ,$$

where $p=1, 2 \ldots 9$ portfolios, $i=1, 2, \ldots x$ stocks within each portfolio, w_i is a value weight based on market capitalization for stock *i*, and *IQ* is *IQ1* or *IQ2*. The notation r_t^p is the

[17] Following Fama and French (1993), book value is defined to be the value of common stockholders' equity, plus deferred taxes and investment-tax credit, minus the book value of preferred stock. Book value is divided by market value on the day of firm's fiscal year-end.

portfolio return, IQ_t^p is portfolio IQ, and $Illiquidity_t^p$ is portfolio liquidity, all value weighted.[18]

Table 1 reports the descriptive statistics of nine value-weighted portfolios double sorted based on both IQ measures and Pastor and Stambaugh's (2003) liquidity measure. Panel A reports the statistics for 3 IQ1 $\times$ 3 Illiquidity portfolios. In this sample, portfolios range from the most informationally precise (IQH: lowest IQ1 portfolio) to least informationally precise (IQL: highest IQ1 portfolio). The average IQ1 measure is as low as 0.84% within the IQH category and as high as 13.68% within the IQL category.

In Panel B of Table 1, portfolios range from the most informationally stable (IQH: lowest IQ2 portfolio) to the least informationally stable (IQL: highest IQ2 portfolio), with the average IQ2 measure being as low as 1.29% within the IQH category and as high as 14.08% within the IQL category. In general, both IQ measures become more volatile as the portfolio becomes less informationally precise. Both panels of Table 1 show that, regardless of the IQ measure used to proxy for IQ, there is no clear relation between IQ and illiquidity, size, or B/M. Thus, it implies that informationally imprecise stocks do not possess a clear tendency to be less liquid, or have characteristics such as small size, and large B/M ratio.

Note that in both panels of Table 1, within each IQ category, the average portfolio Illiquidity ranges from a large negative value through a value close-to-zero, and to a large positive value. These negative illiquidity values are consistent with Pastor and Stambaugh (2003), who state that their firm-specific estimate of illiquidity is expected to

[18] Since I use the Fama-French value-weighted three factors provided on Kenneth French's website, to be consistent, I construct portfolio illiquidity measure and IQ measure using value weights rather than equal weight.

Table 1: Descriptive Statistics for Nine (3 IQ by 3 Liquidity) Portfolios

This table reports the descriptive statistics of nine portfolios double sorted based on the IQ measures and the liquidity measure following Pastor and Stambaugh (2003). At the beginning of each quarter from 1987 to 2007, eligible NYSE/AMEX stocks are first into sorted three groups according to estimated IQ measure and then sorted into three liquidity categories within each IQ group based on the Pastor and Stambaugh liquidity measure. Panel A reports the results for the sample where IQ1 ($|e_{i,t}|$, the absolute value of residual estimated using the Barth et al., 2001 model) is employed as the first ranking criteria. Panel B reports the results where IQ2 ($\sigma(e_i)_t$, the standard deviation of residuals estimated over time *t*-4 through *t* as in Barth et al., 2001) is employed as the first ranking criteria. Each panel documents the average and standard-deviation of quarterly IQ measures, quarterly portfolio returns, and quarterly liquidity measures for each portfolio. The portfolio Size (market capitalization) and B/M (Book-to-market ratio) are documented as well. I denote the three IQ portfolios with IQH, IQM, and IQL, where: IQH is the high information-quality portfolio (with low levels of IQ1 or IQ2); IQM is the medium information-quality portfolio (with medium levels of IQ1 or IQ2); and IQL is the low information-quality portfolio (with high levels of IQ1 or IQ2).

Panel A: IQ1 measure $= |e_{i,t}|$

IQ Portfolio	Illiquidity value	r^p Mean (%)	r^p Std.dev (%)	$IQ1^p$ Mean (%)	$IQ1^p$ Std.dev (%)	$Illiquidity^p$ Mean (%)	$Illiquidity^p$ Std.dev (%)	Size (bl$)	B/M
IQH	Low	6.23	8.72	0.84	0.13	-0.47	0.34	2.03	0.48
	Medium	4.91	6.78	0.85	0.12	-0.01	0.01	57.62	0.33
	High	5.41	8.22	0.85	0.16	0.36	0.33	4.11	0.44
IQM	Low	6.22	9.67	3.14	0.65	-0.50	0.35	2.09	0.47
	Medium	4.85	6.50	3.15	0.60	-0.001	0.002	54.68	0.32
	High	4.69	8.16	3.14	0.59	0.34	0.28	4.98	0.46
IQL	Low	6.25	9.55	13.18	4.84	-0.46	0.33	2.39	0.49
	Medium	4.91	6.95	13.68	5.50	-0.001	0.003	53.46	0.36
	High	5.31	8.54	13.65	4.33	0.36	0.30	3.69	0.46

Panel B: IQ2 measure $= \sigma(e_i)_t$

IQ Portfolio	Illiquidity value	r^p Mean (%)	r^p Std.dev (%)	$IQ2^p$ Mean (%)	$IQ2^p$ Std.dev (%)	$Illiquidity^p$ Mean (%)	$Illiquidity^p$ Std.dev (%)	Size (bl$)	B/M
IQH	Low	5.16	8.16	1.32	0.18	-0.33	0.21	3.02	0.50
	Medium	4.55	6.02	1.29	0.27	-0.001	0.002	50.90	0.36
	High	4.71	7.36	1.32	0.20	0.23	0.20	5.47	0.47
IQM	Low	6.14	9.09	3.18	0.58	-0.48	0.32	2.13	0.49
	Medium	5.11	6.92	3.14	0.55	-0.001	0.003	43.86	0.31
	High	5.54	8.81	3.16	0.58	0.39	0.33	3.84	0.44
IQL	Low	7.17	10.64	12.56	6.95	-0.74	0.61	1.44	0.47
	Medium	5.56	8.29	14.06	10.34	-0.003	0.007	72.93	0.36
	High	5.53	9.42	11.58	5.95	0.56	0.49	3.15	0.44

be larger in absolute value when liquidity is lower. Therefore, the medium Illiquidity measure group within each IQ category in Table 1 includes the most liquid stocks as evidenced by a close-to-zero value of $E(Illiquidity^p)$. In contrast, the low Illiquidity measure group within a each given IQ category consists of the least liquid stocks as evidenced by their large absolute values of $E(Illiquidity^{p})$. The highest Illiquidity measure group includes stocks with medium liquidity as evidenced by absolute values of $E(Illiquidity^{p})$, which are smaller relative to the absolute magnitude of estimated for low the Illiquidity group.

Table 1 shows that within each IQ category highly liquid stocks with close-to-zero Illiquidity measure tend to have large size and low B/M ratio; while least liquid stocks tend to have small size and high B/M ratio.

2.2.d. Evidence of IQ Risk Premium

In this subsection I examine the correlation between the main variables used in the preliminary test and test for the significance of the IQ factor in explaining the variation in returns. At the beginning of each quarter from 1987 to 2007, stocks in my NYSE/AMEX sample are sorted first into three groups according to the estimated IQ measure and then sorted into three liquidity categories within each IQ group based on the illiquidity measure of Pastor and Stambaugh (2003).

Table 2 reports the Pearson correlations for the main variables used in the preliminary test. Panel A reports the results for IQ1-based mimicking IQ factor. This panel shows that the IQF factor is weakly correlated with the LIQ factor at 0.036 (with a *p*-value of 66.7%). This finding confirms that the LIQ factor is essentially orthogonal to IQ1-based market factor. Panel A further shows that the IQ1-based IQF factor is positively correlated with the market factor (0.067, *p*-value = 42.4%) and size factor

(0.063, *p*-value = 45.8%) both at insignificant levels. At the same time, there is a weak and negative correlation between IQF and HML factors (-0.140, *p*-value = 9.7%).

Table 2: Correlation Test

This table reports the Pearson correlations for the main variables used in the preliminary test. Data ranges from quarter 1 of 1987 to quarter 4 of 2007. Fama-French three factors (MKT, SMB, HML) are constructed based on the daily data downloaded from Kenneth French's website: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/. The mimicking liquidity factor (LIQ) is constructed based on liquidity measure of Pastor and Stambaugh (2003). The two panels refer to results based on the two alternative measures for the mimicking IQ factor. The *P*-value is reported in parentheses. ***, **, * denotes significance level at 1%, 5% and 10% , respectively.

Panel A: When the mimicking IQ factor is measured with IQ1 ($|e_{i,t}|$)

	Market (MKT)	Size (SMB)	Value (HML)	IQ Factor (IQF)	Liquidity (LIQ)
Market (MKT)	1				
Size (SMB)	0.403***	1			
	(<.0001)				
Value (HML)	-0.529***	-0.214**	1		
	(<.0001)	(0.011)			
IQ Factor (IQF)	0.067	0.063	-0.140*	1	
	(0.424)	(0.458)	(0.097)		
Liquidity (LIQ)	0.108	0.760***	0.322***	0.036	1
	(0.202)	(<.0001)	(0.0002)	(0.667)	

Panel B: When the mimicking IQ factor is measured with IQ2 ($\sigma(e_i)_t$)

	Market (MKT)	Size (SMB)	Value (HML)	IQ Factor (IQF)	Liquidity (LIQ)
Market (MKT)	1				
Size (SMB)	0.403***	1			
	(<.0001)				
Value (HML)	-0.529***	-0.214**	1		
	(<.0001)	(0.011)			
IQ Factor (IQF)	0.591***	0.271***	-0.605***	1	
	(<.0001)	(0.001)	(<.0001)		
Liquidity (LIQ)	0.072	0.728***	0.354***	-0.147*	1
	(0.391)	(<.0001)	(<.0001)	(0.080)	

Panel B of Table 2 reports the results for the IQ2-based mimicking IQ factor. In this panel, the mimicking IQ factor is negatively and weakly correlated with the mimicking liquidity factor (-0.147, *p*-value = 8%). The low correlation suggests that, in general, an informationally unstable market is not necessarily illiquidity and the IQ2-based factor captures information beyond the market liquidity factor. The mimicking IQ2-based factor is also weakly correlated with size factor (0.271, *p*-value = 0.10%), but it is highly correlated with value factor (-0.605, *p*-value < 0.01%). Panel B further shows that the mimicking IQ factor is highly correlated with market factor (0.591, *p*-value < 0.01%). This positive and high correlation implies that the market risk premium is higher during periods when the market suffers from a poor information quality.

Both panels of Table 2 show that the market liquidity factor is significantly correlated with size factor (for example, 0.760, *p*-value < 0.01% in Panel A) and value factor (for example, 0.322, *p*-value = 0.02% in Panel A). This result is consistent with the extant liquidity literature (see for example, Pastor and Stambaugh, 2003; and Liu, 2006). This confirms that small firms are less liquid and size can be a reasonable proxy for liquidity.

Next, I use GMM to estimate the IQ-adjusted Fama-French factor model with market liquidity as a control variable (see Section 2.1). I estimate the IQ-factor loading, q_i, for each of portfolio of those formed by double sorting based on IQ and liquidity for the 1987-2007 period. Table 3 reports estimated coefficients q_i (and the coefficients of the other market factors), together with their *t*-statistics.

Table 3: IQ by Liquidity Portfolios

This table reports the coefficient estimates of the IQ-adjusted Fama-French factor model with market liquidity as a control variable. At the beginning of each quarter from 1987 to 2007, eligible stocks are sorted first into 3 groups according to their estimated IQ measure and then sorted into 3 liquidity categories within each IQ group based on Pastor and Stambaugh's (2003) liquidity measure. GMM is used to estimate the coefficients for the following adjusted-factor model:

$$r_{it} - r_{ft} = \alpha_i + b_i MKT_t + s_i SMB_t + h_i HML_t + q_i IQF_t + l_i LIQ_t + \varepsilon_{it},$$

where r_{it} is the return of portfolio *i* in quarter *t*, r_{ft} is three-month T-bill rate for quarter *t*, the quarterly values of MKT_t , SMB_t, and HML_t are constructed based on daily values downloaded from Kenneth French's website, LIQ_t is the mimicking liquidity factor constructed based on Pastor and Stambaugh (2003) liquidity measure, and IQF_t is the mimicking IQ factor constructed based on each of the two IQ measures. Panel A reports the results for the sample where IQ1 ($|e_{i,t}|$, the absolute value of residual estimated using the Barth et al., 2001, model) is employed to construct the mimicking IQ factor. Panel B reports the results on the sample where IQ2 ($\sigma(e_i)_t$, the standard deviation of residuals estimated over time *t*-4 through *t* as in the Barth et al., 2001, model) is employed to construct the mimicking IQ factor. I denote the three IQ portfolios with IQH, IQM, and IQL, where: IQH is the high information-quality portfolio (with low levels of IQ1 or IQ2); IQM is the medium information-quality portfolio (with medium levels of IQ1 or IQ2); and IQL is the low information-quality portfolio (with high levels of IQ1 or IQ2).The *t*-statistic is documented in the parentheses. R^2 and adjusted-R^2 (in parentheses) are presented as well. ***, **, * denotes significance level at 1%, 5% and 10% respectively.

Panel A: IQ1 by Liquidity Portfolios (IQ1 = $|e_{i,t}|$)

IQ Portfolio	Liquidity value	α_i	b_i (*MKT*)	s_i (*SMB*)	h_i (*HML*)	q_i (*IQF*)	l_i (*LIQ*)	R^2
	Low	0.050**	0.903***	-0.206**	0.065	-0.468***	0.024	0.894
		(2.93)	(29.12)	(-2.49)	(1.24)	(-3.85)	(0.30)	(0.890)
IQH	Medium	0.045	0.993***	-0.118	0.221	-0.196	0.682***	0.896
		(0.46)	(25.65)	(-1.15)	(3.39)	(-1.30)	(6.76)	(0.893)
	High	0.052	0.834***	-0.063	0.137**	-0.280*	1.036***	0.897
		(1.50)	(20.36)	(-0.59)	(1.98)	(-1.75)	(9.69)	(0.894)
	Low	0.049	0.871***	-0.058	0.174***	-0.016	-0.101	0.864
		(1.97)	(25.82)	(-0.65)	(3.05)	(-0.12)	(-1.15)	(0.858)
IQM	Medium	0.035	0.937***	0.075	0.195**	0.275	0.646***	0.860
		(0.52)	(19.78)	(0.60)	(2.44)	(1.48)	(5.23)	(0.855)
	High	0.015	0.868***	0.360**	0.215**	0.213	0.684***	0.850
		(1.44)	(16.09)	(2.51)	(2.36)	(1.48)	(4.86)	(0.844)
	Low	0.037	0.885***	-0.101	0.084	0.491***	-0.011	0.851
		(0.37)	(23.34)	(-1.01)	(1.31)	(3.30)	(-0.11)	(0.845)
IQL	Medium	0.059	0.880***	-0.143	0.102	0.383**	0.913***	0.893
		(1.50)	(22.01)	(-1.35)	(1.51)	(2.44)	(8.75)	(0.889)
	High	0.060	0.863***	0.046	0.301***	0.299*	0.856***	0.901
		(2.12)	(21.88)	(0.44)	(4.51)	(1.94)	(8.32)	(0.898)

Panel B: IQ2 by Liquidity Portfolios (IQ2 = $\sigma(e_i)_t$)

IQ Portfolio	Liquidity value	α_i	b_i *(MKT)*	s_i *(SMB)*	h_i *(HML)*	q_i *(IQF)*	l_i *(LIQ)*	R^2
IQH	Low	0.052	0.907***	-0.013	0.143***	-0.646***	-0.083	0.887
		(1.21)	(29.89)	(-0.17)	(2.94)	(-8.48)	(-1.14)	(0.883)
	Medium	0.048	0.964***	0.056	0.266***	-0.598***	0.525***	0.892
		(0.73)	(24.83)	(0.61)	(4.29)	(-6.14)	(5.65)	(0.888)
	High	0.053	0.881***	0.029	0.052	-0.847***	0.848***	0.905
		(1.63)	(23.25)	(0.32)	(0.86)	(-8.90)	(8.86)	(0.902)
IQM	Low	0.053	0.946***	-0.022	0.138*	-0.312***	-0.039***	0.822
		(1.19)	(21.25)	(-0.21)	(1.95)	(-2.79)	(-0.37)	(0.815)
	Medium	0.063**	0.964***	0.097	0.096	-0.125	0.681***	0.882
		(2.79)	(20.33)	(0.86)	(1.27)	(-1.05)	(5.99)	(0.877)
	High	0.007	0.952***	0.110	0.254***	-0.051	0.962***	0.886
		(0.78)	(18.59)	(0.90)	(3.11)	(-0.4)	(7.84)	(0.882)
IQL	Low	0.054**	0.902***	-0.133	-0.132**	0.419***	-0.069	0.911
		(2.88)	(23.62)	(-1.46)	(-2.17)	(4.37)	(-0.75)	(0.908)
	Medium	0.043	1.004***	-0.013	0.101	0.277**	0.790***	0.867
		(1.00)	(18.31)	(-0.10)	(1.15)	(2.01)	(6.01)	(0.863)
	High	0.066**	0.774***	0.407***	0.157	0.537***	0.748***	0.849
		(3.20)	(12.66)	(2.78)	(1.61)	(3.50)	(5.10)	(0.844)

Panel A of Table 3 reports that five out of the nine factor-loading estimates for the IQ1-based factor are statistically significant. Among estimates of q_i for the nine portfolios, two are significant at the 1% significant level, one is significant at the 5% significance level, and two are significant at the 10% significance level. Panel B reports that seven out of the nine factor-loading estimates for the IQ2-based factor are statistically significant: six estimates are significant at the 1% significant level, and one estimate is significant at the 5% significance level.

The results in Table 3 clearly show that the IQ-adjusted Fama-French factor model (with market liquidity as a control variable) fits the data well. R^2 (adjusted-R^2) ranges from 0.850 (0.844) to 0.901 (0.998) for the IQ1-based model, and from 0.822 (0.815) to 0.911 (0.908) for IQ2-based model.

It is interesting to note that IQF gains higher significance when IQ2 is used as a proxy. Basically, IQ1 measure and IQ2 measure are complementary to each other. A larger IQ1 implies a lower precision of information quality in terms of the magnitude of forecast error, while a larger IQ2 corresponds to a greater uncertainty of forecast error reflecting a poorer level of reporting quality. The improved significance of IQ2 pricing effect on equity returns implies that unexpected uncertainty related to forecast error is more concerned by investors. Overall IQF provides incremental information about expected returns especially for informationally-imprecise (IQL) stocks and informationally-precise (IQH) stocks. Furthermore, the sign of the estimated IQF coefficient changes from negative to positive when moving from the IQH category to the IQL category (under both IQ measures).

This suggests that for IQH stocks, with more precise information, investors are willing to settle for a lower return when the overall quality of information in the market deteriorates (i.e., IQF increases). In other words, when the market as a whole suffers from poor IQ, investors are willing to pay more for stocks with more precise information set (such as stocks in the IQH portfolio). On the other hand, the positive and significant IQF coefficients estimated for the IQL portfolio indicate that, at times when the overall quality of information in the market deteriorates, investors demand a return premium (pay a lower price) for holding stocks with an imprecise information set (like stocks in the IQL portfolio).

2.2.e. Robustness Tests

The above results for the IQ-factor coefficient estimates may be sensitive to the portfolio sorting criteria. To test for the robustness of these results I re-estimate my model with portfolios sorted based on other firm characteristics, such as size, B/M, and Illiquidity, all by IQ, respectively.

First, to check whether my results are robust to the ranking sequence, in contrast to the 3 IQ $\times$ 3 Illiquidity portfolios used in Section 2.3, I reverse the ranking criteria and sort portfolios first into three Illiquidity categories and then into three IQ groups within the Illiquidity groups. Appendix A1 shows the model's fit of the reverse-sorting portfolios, in which the R^2 is still very high (close to 0.9) in both panels. Similar to the results in Section 2.3, IQF tends to get more significant when the IQ2 proxy is used. Panel A of the table in Appendix A1 reports that six out of the nine factor-loading estimates for the IQ1-based factor are statistically significant. Among estimates of q_i for the nine portfolios, two are significant at the 1% significant level, three are significant at the 5% significance level, and one is significant at the 10% significance level. Panel B reports that six out of the nine factor-loading estimates for the IQ2-based factor are statistically significant: All six estimates are significant at the 1% significant level. The table in Appendix A1 shows that IQF provides incremental information about expected returns, especially for IQH stocks, regardless of their illiquidity level.

Next, I check whether the significance of the IQ factor survives in B/M-sorted portfolios. I form nine B/M portfolios for each quarter over the period 1987 to 2007 by sorting stocks based on their previous-year book-to-market ratio. The table in Appendix A2 shows the model's fit of the B/M portfolios. Panel A of the table shows that for the IQ1-based regression, the R^2 for the nine B/M portfolios ranges between 0.48 and 0.81. based on the IQ1 measure. Similarly, Panel B shows that for the IQ2-based model, the R^2 for the nine B/M portfolios ranges between 0.46 and 0.81.

Panel A reports that of the nine factor-loading estimates for the IQ1-based factor, five are statistically significant. Among estimates of IQF coefficient for the nine portfolios, three are significant at the 1% significant level, one is significant at the 5% significance level, and one is significant at the 10% significance level. Panel B reports that five out of the factor loadings estimated for the nine B/M portfolios (based on IQ2) are statistically significant: one is significant at the 1% significant level, one is significant at the 5% significance level, and three are significant at the 10% significance level. These results suggest that IQF remains an important factor even after controlling for size, B/M, and liquidity factors for B/M-based portfolios.

Finally, I form nine size portfolios for each quarter during the sample period by ranking stocks based on their market capitalization at the beginning of the year and report the model fit in Appendix A3. The R^2 for the nine size portfolios ranges remains very high for both IQ measures. Similar to the results reported in Table 3, IQF tends to be more significant when IQ2 is used as a proxy. In Panel B, four IQ2-based estimates are significant at 1% significance level, and one is significant at 5% level. Panel A reports that of the nine factor-loading estimates for the IQ1-based factor, only three are statistically significant, all at the 5% significance level. Panel B reports that five out of the factor loadings estimated for the nine size portfolios (based on IQ2) are statistically significant: four are significant at the 1% significant level, and one is significant at the 5% significance level.

Note that the liquidity factor (LIQ) is always significant for all of nine size portfolios regardless of the IQ proxy used. The superior explanatory power of liquidity factor may be due to its strong correlation with size as reported in Table 2. Overall, the additional tests performed in this section show that that IQF is a robust and significant factor in explaining variation in returns.

2.3. An Intertemporal Asset-Pricing Model with Imprecise Information

In the previous section, I provide evidence supporting the notion that IQ is a priced market factor. To this end I used an ad-hoc IQ-adjusted Fama-French factor model with market liquidity as a control variable. In the current section I formulate a theoretical asset pricing model with IQ risk. The precise analytical form of this risk allows us to perform a formal econometric study of the different components systematic IQ and test whether they are priced.

Merton (1973) derives an intertemporal capital asset pricing model where investors maximize their expected utility of lifetime consumption. In his model trading in assets is assumed to take place continuously in time. In this section, I revisit this intertemporal model incorporating a noisy information structure. I model various channels through which information risk may affect security returns. Most of the standard assumptions of the intertemporal model still stand, with the extra assumption that investors face an imprecise information set.

I maintain Merton's (1973) assumptions of continuous trading, and that the returns and the changes in the opportunity set (the transition probabilities for returns on each asset over the next trading interval) are well explained by continuous-time stochastic processes. The vector set of stochastic processes describing the investment opportunity set and its changes is a time-homogeneous Markov process.[19] Below, I make four additional assumptions modifying Merton's framework to allow for imperfect IQ.

[19] Merton assumes that all assets have limited liability, and there are no transactions costs, taxes, or problems with indivisibilities of assets. There are a sufficient number of investors with comparable wealth levels so that each investor can buy and sell unlimited amounts at the market prices, and there exists an exchange market for borrowing and lending at the same rate of interest. Investors have homogeneous expectations with respect to asset returns. Short-sales of all assets, with full use of the proceeds are allowed. Finally, it is assumed that trading in assets takes place continually in time. For specific details see Merton (1973).

Assumption 1: There is an information-imprecision variable, ψ_i, which represents the information-related error in firm *i*'s instantaneous fundamental return.[20] This variable is given by the spread of the observed return and the true fundamental return on security *i*. I assume that ψ_i follows an Ornstein-Uhlenbeck process as follows: $d\psi_i = k(\mu_{\psi_i} - \psi_i)dt + \sigma_{\psi_i} dz_i$, for every asset *i* (*i* = 1, 2,…*n*), where k, μ_{ψ_i}, σ_{ψ_i} are constants, z_i is standard Wiener process, and $E[dz_i dz_j] = \rho_{\psi_i,\psi_j} dt$.

The drift term $k(\mu_{\psi_i} - \psi_i)$ gives the expected growth rate of information-related return error. With a positive speed of mean-reversion ($k > 0$), the level of information-related return error, ψ_i, fluctuates around a long-term steady-state mean μ_{ψ_i}, which is constant for security *i*. Parameter σ_{ψ_i} measures the magnitude of the innovation in ψ_i.

Assumption 2: Market participants observe the stochastic noisy instantaneous return $\tilde{r}_{it} \equiv r_{it} + \psi_{it}$, where r_{it} is the fundamental (precise) return on asset *i*. This fundamental return follows a Gaussian process: $dr_i = \mu_i dt + \sigma_i \, d\omega_i$, for every *i*, *i* = 1, 2,…*n*, and $E[d\omega_i d\omega_j] = \rho_{ij} dt$.

Applying Itô's lemma I write the mean of the instantaneous noisy as $\alpha_i = \mu_i + \mu_{\psi_i}$, which is the sum of the mean instantaneous fundamental return and mean noise. The instantaneous noisy return variance is given by: $\sigma^{*2}_i \equiv \sigma_i^2 + \sigma^2_{\psi_i} + 2\sigma_{i,\psi_i}$, where σ_{i,ψ_i} is the instantaneous covariance between the fundamental return on asset *i* and the information noise related to asset *i*: $\sigma_{i,\psi_i} = \rho_{i,\psi_i} \sigma_i \sigma_{\psi_i}$, where ρ_{i,ψ_i} denotes the

[20] For example, ψ_i could be related to imprecise information on a firm's cash flow.

instantaneous correlation between the fundamental return on asset *i* and the information noise for asset *i*.

I further denote the instantaneous correlation coefficient between $d\omega_i$ and $d\omega_j$ with ρ_{ij}, and the instantaneous correlation coefficient between $d\omega_i$ and dz_j with ρ_{i,ψ_j}. That is, $E(d\omega_i d\omega_j) = \rho_{ij} dt$, $E(d\omega_i dz_j) = \rho_{i,\psi_j} dt$. Note that a positive (negative) sign of ρ_{i,ψ_j} results in a positive (negative) intertemporal correlation in security *i*'s fundamental return if there is a positive shock in terms of IQ for security *j*. The instantaneous covariance between the noisy returns on any two assets *i* and *j* is given by: $\sigma^*_{ij} = Cov(r_i + \psi_i, r_j + \psi_j) = \sigma_{ij} + \sigma_{i,\psi_j} + \sigma_{j,\psi_i} + \sigma_{\psi_i,\psi_j}$.

The above assumptions imply the following Itô processes for the instantaneous noisy return on the asset *i* ($\tilde{r}_i$):

$$d\tilde{r}_i = (\mu_i + k(\mu_{\psi_i} - \psi_i))dt + \sqrt{\sigma_i^2 + \sigma^2_{\psi_i} + 2\sigma_{i,\psi_i}}\, d\varpi_i, \tag{1}$$

where ϖ_i is a standard Brownian Motion.[21] Equation (1) implies that $\{(\mu_i + k(\mu_{\psi_i} - \psi_i)), \sqrt{\sigma_i^2 + \sigma^2_{\psi_i} + 2\sigma_{i,\psi_i}}, \rho_{ij}\}$ is a sufficient set of statistics for the opportunity set at any given point in time.

Assumption 3: Following Merton (1971) and Merton (1973), I assume that there are *K* investors who maximize their expected lifetime utility of wealth:

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[21] Application of Itô's Lemma implies that: $d\varpi_i = \dfrac{\sigma_i\, d\omega_i + \sigma_{\psi_i} dz_i}{\sqrt{\sigma_i^2 + \sigma^2_{\psi_i} + 2\sigma_{i,\psi_i}}}$, and $E(d\varpi_i dz_i) = \dfrac{\sigma_i\, \rho_{i,\psi_i} + \sigma_{\psi_i}}{\sqrt{\sigma_i^2 + \sigma^2_{\psi_i} + 2\sigma_{i,\psi_i}}} dt$.

$$J[W(t),\psi(t),t] = \max E_0\left[\int_0^{T^k} U^k[c^k(s),s]ds + B^k[W^k(T^k),\psi(T^k),T^k]\right], \quad k = 1,2,...K, \quad (2)$$

where "E_0" is the expectation operator, conditional on the current value of the k^{th} investor's wealth. U^k is a von Neumann-Morgenstern utility function for consumption which is strictly concave. The initial value of an investor's wealth is given by $W^k(0) = W^k$. T^k is the k^{th} investor's horizon. Finally $c^k(t)$ is the instantaneous consumption flow at time *t*, and B^k denotes a strictly concave utility function of terminal wealth. The terminal value of lifetime utility in equation (2) is: $J[W(T),\psi(T),T] = B(W(T),\psi(T),T)$.

Assumption 4: There are *n* risky assets and one instantaneously riskless asset. With noisy information incorporated in (1), the wealth accumulation equation for the k^{th} investor is given by:

$$dW = \sum_{i=1}^{n+1} q_i W d\tilde{r}_i - cdt, \quad (3)$$

where q_i is the proportion of the investor's wealth invested in the i^{th} asset.

Theorem 1: Following Assumption 1 to Assumption 4, the wealth accumulation process is given by (see Appendix C for derivation),

$$dW = \left[\sum_{i=1}^{n} q_i(\mu_i + k(\mu_{\psi_i} - \psi_i) - r_f) + r_f\right]Wdt + \sum_{i=1}^{n} q_i W\sqrt{\sigma_i^2 + \sigma_{\psi_i}^2 + 2\sigma_{i,\psi_i}}\, d\varpi_i - cdt,$$

where r_f is an exogenous interest rate on a risk-free bond, and $\sum_1^{n+1} q_i = 1$. Using the above assumptions and theorem, I solve for an investor's consumption-investment optimal choice which results the following Hamilton-Jacobi-Bellman (HJB) equation:

$$\begin{aligned} 0 = \max[&U(c,t) + J_t + J_W[(\sum_1^n q_i(\mu_i + \mu_{\psi_i} - r_f) + r_f)W] \\ &+ \frac{1}{2} J_{WW} \sum_1^n \sum_1^n q_i q_j (\sigma_{ij} + \sigma_{\psi_i\psi_j} + \sigma_{i\psi_j} + \sigma_{j\psi_i}) W^2 \\ &+ \frac{1}{2} \sum_1^n \sum_1^n J_{\psi_i\psi_j} \sigma_{\psi_i} \sigma_{\psi_j} \rho_{\psi_i\psi_j} \\ &+ \sum_1^n \sum_1^n J_{W\psi_j} q_i W (\rho_{i,\psi_j} \sigma_i + \sigma_{\psi_j}) \end{aligned} \tag{4}$$

The n+1 first-order conditions for each investor derived from (4) are given by:

$$0 = U_c(c,t) - J_W(W,t,\psi),$$

$$\begin{aligned} 0 &= J_W(\mu_i + \mu_{\psi_i} - r_f)W + J_{WW} \sum_{j=1}^n q_j W^2 (\sigma_{ij} + \sigma_{\psi_i\psi_j} + \sigma_{i\psi_j} + \sigma_{j\psi_i}) \\ &+ \sum_{j=1}^n J_{W\psi_j} W (\rho_{i,\psi_j} \sigma_i + \sigma_{\psi_j}), \end{aligned}$$

$\forall\ i = 1,2,...n,$ where $c^* = c(W,t,\psi),$ $q_i^* = q_i(W,t,\psi)$ are optimal weights for consumption and assets in portfolio.

The assumption of constant risk-free rate in my model allows us to simplify my analysis and focus on the stock market. Using matrix notation, I rewrite (4) for the *n* risky assets:

$$0 = J_W(\mu + \mu_\psi - r_f I) + J_{WW} W \Sigma q + J_{W\psi} \vec{\sigma}, \tag{5}$$

where μ is the vector of mean of fundamental return of *n* securities, μ_ψ is the vector of long-term spread between fundamental return and observed return for each security, ψ is the vector of firm-specific information-related return error, Σ is the variance-covariance matrix of observed return with elements $\sigma_{ij}^* = \sigma_{ij} + \sigma_{\psi_i\psi_j} + \sigma_{i,\psi_j} + \sigma_{j,\psi_i}$, and $\vec{\sigma}$ is the vector $(\sigma_{1\psi}, \sigma_{2\psi}, ..., \sigma_{n\psi})$ of covariance terms between all noisy information variables and the observed return on asset *i*, $(i = 1,2,...n)$, with components given by $\sigma_{i\psi} = \sum_{j=1}^{n} (\rho_{i,\psi_j} \sigma_i + \sigma_{\psi_j}) \ \forall \ i = 1,2,...n.$. From (5), I obtain the vector of optimal portfolio weights,

$$q^* = -\frac{J_W}{WJ_{WW}} \Sigma^{-1} (\mu + \mu_\psi - r_f I) - \frac{J_{W\psi}}{WJ_{WW}} \Sigma^{-1} \bar{\sigma}, \tag{5.1}$$

For each asset *i*, I rewrite (5.1) as follows:

$$q_i^* = -\frac{J_W}{WJ_{WW}} \sum_{j=1}^{n} v_{ij} (\mu_i + \mu_{\psi_i} - r_f) - \frac{J_{W\psi}}{WJ_{WW}} \sum_{j=1}^{n} v_{ij} \sigma_{j\psi}, \tag{5.2}$$

$\forall \ i = 1,2,...n,$ and $j = 1,2,...n,$ where v_{ij} denotes the element in the inverse of the variance covariance matrix $\Sigma^{-1} = [v_{ij}]$.

Equations (5.1) and (5.2) give the optimal weights (demand) for assets *i* in the presence of noisy information. Portfolio weight q^* in (5.1) is the combination of the tangency (market) portfolio with a hedge portfolio, denoted by *h*. This last portfolio hedges against IQ risk, which causes the unfavourable changes in the fundamental return of assets in the investment opportunity set.

Theorem 2: The equilibrium security return is explained by security return's sensitivity to market return and its sensitivity to hedge portfolio return, as given below (proof is provided in Appendix C),

$$\mu_i + \mu_{\psi_i} - r_f = \beta_i^m (\mu_m + \mu_{\psi_m} - r_f) + \beta_i^h (\mu_h + \mu_{\psi_h} - r_f), \tag{6}$$

where
$$\beta_i^m = \frac{(\sigma_{im} + \sigma_{\psi_i\psi_m} + \sigma_{i\psi_m} + \sigma_{m\psi_i})\sigma_{h\psi} - \sigma_{i\psi}\sigma_{mh}^*}{(\sigma_m^2 + \sigma_{\psi_m}^2 + 2\sigma_{m\psi_m})\sigma_{h\psi} - \sigma_{mh}^*\sigma_{m\psi}}$$
$$= \frac{(\sigma_{im} + \sigma_{\psi_i\psi_m} + \sigma_{i\psi_m} + \sigma_{m\psi_i})\sigma_{h\psi} - \sigma_{i\psi}\sigma_{mh}^*}{\sum_{i=1}^n x_i[(\sigma_{im} + \sigma_{\psi_i\psi_m} + \sigma_{i\psi_m} + \sigma_{m\psi_i})\sigma_{h\psi} - \sigma_{i\psi}\sigma_{mh}^*]},$$

$$\beta_i^h = \frac{\sigma_{i\psi}(\sigma_m^2 + \sigma_{\psi_m}^2 + 2\sigma_{m\psi_m}) - (\sigma_{im} + \sigma_{\psi_i\psi_m} + \sigma_{i\psi_m} + \sigma_{m\psi_i})\sigma_{m\psi}}{\sigma_{h\psi}(\sigma_m^2 + \sigma_{\psi_m}^2 + 2\sigma_{m\psi_m}) - (\sigma_{mh} + \sigma_{\psi_h\psi_m} + \sigma_{h\psi_m} + \sigma_{m\psi_h})\sigma_{m\psi}}$$
$$= \frac{\sigma_{i\psi}(\sigma_m^2 + \sigma_{\psi_m}^2 + 2\sigma_{m\psi_m}) - (\sigma_{im} + \sigma_{\psi_i\psi_m} + \sigma_{i\psi_m} + \sigma_{m\psi_i})\sigma_{m\psi}}{\sum_{i=1}^n h_i[\sigma_{i\psi}(\sigma_m^2 + \sigma_{\psi_m}^2 + 2\sigma_{m\psi_m}) - (\sigma_{im} + \sigma_{\psi_i\psi_m} + \sigma_{i\psi_m} + \sigma_{m\psi_i})\sigma_{m\psi}]},$$

where $\sigma_{im}^* = \sigma_{im} + \sigma_{\psi_i\psi_m} + \sigma_{i\psi_m} + \sigma_{m\psi_i}$. The term $\mu_{\psi_m} = \sum_{i=1}^n x_i\mu_{\psi_i}$ represents the weighted return for market portfolio, and the term $\psi_m = \sum_{i=1}^n x_i\psi_i$ represents the total IQ noise inherent in the market portfolio. Similarly, the term $\mu_{\psi_h} = \sum_{i=1}^n h_i\mu_{\psi_i}$ represents the weighted return for hedge portfolio, and the term $\psi_h = \sum_{i=1}^n h_i\psi_i$ represents the total IQ noise inherent in hedge portfolio.

Equation (6) is the equilibrium IQ intertemporal capital asset pricing equation. It describes the equilibrium relation between the asset risk premium and two types of risk: market (systematic) risk and the risk of unfavorable shifts in the stochastic investment opportunity set. In the presence of imprecise information, the IQ risk has an impact on both the market beta and the hedge portfolio beta. Mean noise, μ_{ψ_i} , adds to the average risk premium. The variance of market portfolio σ_m^{*2} is boosted by $\sigma_{\psi_m}^2$, which accounts for the systematic IQ risk. Similarly, covariance between the hedge portfolio and the market portfolio σ_{mh}^* is affected as well by $\sigma_{\psi_h\psi_m}$, indicating the co-movement of IQ noise between the hedge portfolio and the market portfolio. Therefore, asset return's sensitivities to the market portfolio return and the hedge portfolio return, β_i^m and β_i^h, are adjusted to account for market-wide IQ risk, which are different from Merton's results under the IQ noise-free environment.

The adjustment in β_i^m and β_i^h shows the major difference from that of Hughes et al.'s (2007) APT model with imperfect information structure. In their model, risk related to imperfect information only affects factor risk premiums, not factor sensitivities. Unique to my model, IQ risk is shows up in factor sensitivities of the modified ICAPM, which implies that it has a systematic component. I further explore this point in the following section.

Next, I show that the pricing relation implied by Merton's (1973) three-fund separation theorem still holds with respect to the noisy opportunity set. I assume that there exists a (hedge) portfolio, *h*, whose return is perfectly negatively correlated with changes in the vector of state variables ψ.

Proposition 1: When the information structure is imperfect (noisy), the following pricing relation that is implied by Merton's (1973) Three-fund Separation Theorem holds. The hedge portfolio in my model hedges against the stochastic noisy IQ risk instead of instantaneous interest rate risk. However, both the market risk premium and the hedge portfolio risk premium are boosted by μ_{ψ_m} and μ_{ψ_h} to compensate for the information risk. The equilibrium model can be expressed to be,

$$\mu_i + \mu_{\psi_i} - r_f = \beta_i^m(\mu_m + \mu_{\psi_m} - r_f) + \beta_i^h(\mu_h + \mu_{\psi_h} - r_f), \tag{7}$$

where $\beta_i^m = \dfrac{\beta_{im}^* - \beta_{ih}^*\beta_{mh}^*}{1-\rho_{mh}{}^2}$, and $\beta_i^h = \dfrac{\beta_{ih}^* - \beta_{im}^*\beta_{mh}^*}{1-\rho_{mh}{}^2}$.

Proof: The proof of Proposition 1 is provided in the appendix C.

Equation (7) implies that for investors facing an investment opportunity set based on imprecise information, investing in any asset *i* is equivalent to investing in three mutual funds. Two funds (similar to the standard CAPM) allow the investor to match a risk-return profile comparable to asset *i* on an instantaneously efficient frontier. The third fund hedges against unfavourable intertemporal shifts in this imprecise frontier. In the presence of imprecise information structure, the proportion invested in each fund (which is a function of the betas) adjusts to reflect the additional IQ risk. In the following section I examine this risk more closely.

2.4. Implication of Imprecise Information for Asset Pricing

Similar to analysis in the standard framework, I can decompose the total risk of investing in any risky asset *i* into a systematic and an idiosyncratic component. To see this, I write the ex-ante version of equation (7):

$$\tilde{r}_i = r_f + \beta_i^m(\tilde{r}_m - r_f) + \beta_i^h(\tilde{r}_h - r_f) + \nu_i, \tag{8}$$

where $\tilde{r}_i$ is the observed return for security i, $\tilde{r}_m = \sum_{i=1}^{n} x_i\tilde{r}_i$, $\tilde{r}_h = \sum_{i=1}^{n} h_i\tilde{r}_i$, ν_i is the white noise, and x_i and h_i represent security *i's* weights in the market portfolio and the hedge portfolio, respectively. With imprecise information, the variance of the rate of return of security i is given by:

$$Var(\tilde{r}_i) = \beta_i^{m^2}[\sigma_m^2 + \sigma_{\psi_m}^2 + 2\sigma_{m\psi_m}] + \beta_i^{h^2}[\sigma_h^2 + \sigma_{\psi_h}^2 + 2\sigma_{h\psi_h}] + \sigma_{\nu_i}^2, \tag{9}$$

where $\sigma_{\psi_m}^2$ measures IQ risk for the market portfolio, $\sigma_{\psi_h}^2$ represents the same risk for hedge portfolio, and $\sigma_{\nu_i}^2$ is security *i*'s idiosyncratic risk. While idiosyncratic risk can be eliminated through diversification, the first two terms on the right-hand side of equation (9) are nondiversifiable. These terms imply that in the presence of imprecise information set, investors face an additional element of systematic risk - systematic IQ risk.

To simplify analysis, I assume that the return on all assets is uncorrelated with the return on the hedge portfolio ($\sigma_{ih}^* = 0, \forall i$). This condition implies that $\beta_{ih}^* = \beta_{mh}^* = 0$, and equation (7) becomes a static version of the IQ-adjusted Capital Asset Pricing Model (IQCAPM):

$$\mu_i + \mu_{\psi_i} - r_f = \beta_i^m (\mu_m + \mu_{\psi_m} - r_f), \tag{10}$$

where $\beta_i^m = \dfrac{\sigma_{im} + \sigma_{\psi_i \psi_m} + \sigma_{i \psi_m} + \sigma_{m \psi_i}}{\sigma_m^2 + \sigma_{\psi_m}^2 + 2\sigma_{m \psi_m}}$.

Under the above assumption, the three-fund separation relation collapses to the standard equation that reflects a two-fund separation, adjusted for noisy information. The two mutual funds (the riskless asset and the market portfolio) allow the investor to match a risk-return profile comparable to asset *i* on an instantaneously efficient frontier. Thus, the beta in equation (10) measures the risk contribution of asset *i* (σ_{im}^*) to the total risk of holding the market portfolio (σ_m^{*2}), which consists of systematic component of IQ risk.

After expanding the covariance term in (10) I can write the asset pricing equation as follows:

$$\mu_i + \mu_{\psi_i} = r_f + \lambda \frac{\sigma_{im}}{\sigma_m^{*2}} + \lambda \frac{\sigma_{\psi_i \psi_m}}{\sigma_m^{*2}} + \lambda \frac{\sigma_{i \psi_m}}{\sigma_m^{*2}} + \lambda \frac{\sigma_{m \psi_i}}{\sigma_m^{*2}}, \tag{11}$$

where $\lambda = \mu_m + \mu_{\psi_m} - r_f$, which is the IQ-adjusted risk premium on the market. Equation (11) is the long-term IQCAPM relation. The IQCAPM provides the framework for understanding the various channels through which IQ risk may affect asset returns.

The systematic risk consists of four components: $\sigma_{im}, \sigma_{\psi_i\psi_m}, \sigma_{i\psi_m}$ and $\sigma_{m\psi_i}$. The first component, σ_{im}, is the covariance of precise returns like in the standard CAPM. With an imprecise information set there are three additional systematic risk effects, $\sigma_{\psi_i\psi_m}$, $\sigma_{i\psi_m}$ and $\sigma_{m\psi_i}$. Therefore investors demand a higher risk premium due to the additional risk they face. I call these effects systematic IQ risk.[22]

Component $\sigma_{\psi_i\psi_m}$ is the covariance between the security's information-related noise and the overall market information-related noise. Investors demand a return compensation for a security whose information noise positively co-varies with the market noise. To hedge against the market IQ risk, investors prefer to hold a security with negative $\sigma_{\psi_i\psi_m}$ in their portfolios.

The third component, $\sigma_{i\psi_m}$, is the covariance between a security's return and the overall market return due to information imprecision. Investors prefer to hold securities whose returns are negatively correlated with the overall market portfolio noise. Therefore, investors demand a risk premium for positive $\sigma_{i\psi_m}$.

The last effect comes from $\sigma_{m\psi_i}$, the covariance between the market fundamental return and security's information-related noise. Investors prefer a security with negative

[22] Note that, analytically, my model is similar to that of Acharya and Pedersen (2005) which focuses on liquidity costs rather than imprecise information.

$\sigma_{m\psi_i}$ to hedge against a down market, and require compensation for positive levels of $\sigma_{m\psi_i}$.

Note that, based on the static CAPM adjusted to an imprecise information set, ceteris paribus, the excess return of security i increases as its systematic IQ risk ($\sigma_{\psi_i\psi_m}$) increases. This result has important implications for asset pricing with imprecise information. It suggests that investors demand a premium for systematic IQ risk that cannot be diversified away. This risk is priced because, even when one holds security i within the market portfolio, one still faces the systematic IQ risk that security i contributes to the market portfolio. This result is unique to my model, and demonstrates that in my model systematic IQ risk is priced through the beta of the asset as well as through the market risk premium.

Two-fund separation implies that investors hold the market portfolio. The risk involved in holding the market when information is imprecise is given by: $\sigma_m^{*2} = \sigma_m^2 + \sigma_{\psi_m}^2 + 2\sigma_{m\psi_m}$. This means that the total systematic risk consists of three components: that of standard CAPM beta (σ_m^2), the systematic component of IQ risk ($\sigma_{\psi_m}^2$), and the comovement of market return and market IQ risk ($\sigma_{m\psi_m}$). The presence of $\sigma_{\psi_m}^2$ and $\sigma_{m\psi_m}$ explicitly show that IQ risk impacts the pricing of securities by altering their betas (sensitivity) with respect to the IQ-adjusted market portfolio.

2.5. An Empirical Test of the Static IQCAPM

In this section, I follow three steps to test the empirical fit of the static version of my IQCAPM: (i) I form a market portfolio and two sets of 25 IQ portfolios based on the two IQ proxies I use; (ii) I estimate the four betas of the static IQCAPM for each of the 25 IQ portfolios; and (iii) finally, I run cross-sectional regressions for average return across the 25 portfolios to test the empirical fit of the static IQCAPM. Robustness tests are provided based on several alternative specifications of the regression model.

2.5.a. Forming Portfolios

To reduce noise related to the IQ proxy estimation for individual stocks, I form a market portfolio and two alternative sets of 25 IQ portfolios based on the two different IQ measures I estimate.

When portfolios are formed, portfolio return, portfolio IQ measure, market return, and market IQ measure are computed as follows for time *t*:

$$r_t^p = \sum w_i \, r_{it} \ ,$$

$$IQ_t^p = \sum w_i \, IQ_{it} \, ,$$

$$r_t^m = \sum w_j r_{jt} \, , \text{ and } IQ_t^m = \sum w_j IQ_{jt} \ ,$$

where *p=1, 2…25 portfolio,* and *i =1, 2,...x* stocks within IQ portfolio, *j = 1, 2...y* stocks in the market portfolio, w_i is either an equal-weight or a value-weight based on market capitalization of stock *i*, and *IQ* is *IQ1* or *IQ2*.. The notation r_t^p represents the weighted average return for each IQ portfolio, and r_t^m is the weighted average return for market portfolio at time *t*. IQ_t^p and IQ_t^m represent weighted average IQ measures for portfolio and for market portfolio, respectively.

2.5.b. Estimating IQ Betas for Portfolios

For the empirical test, in the spirit of Acharya and Pedersen (2005), the unconditional version of my static IQCAPM is as follows,

$$E(\tilde{r}_t^{\,p} - r_{ft}) = \lambda\beta^{Market} + \lambda\beta_1^{IQ} + \lambda\beta_2^{IQ} + \lambda\beta_3^{IQ}, \tag{12}$$

where

$$\beta^{Market} = \frac{\sigma_{pm}}{\sigma_m^{*2}} = \frac{Cov(r_t^p, r_t^m - E_{t-1}(r_t^m))}{Var(r_t^m - E_{t-1}(r_t^m) + (IQ_t^m - E_{t-1}(IQ_t^m)))},$$

$$\beta_1^{IQ} = \frac{\sigma_{p\psi_m}}{\sigma_m^{*2}} = \frac{Cov(r_t^p, IQ_t^m - E_{t-1}(IQ_t^m))}{Var(r_t^m - E_{t-1}(r_t^m) + (IQ_t^m - E_{t-1}(IQ_t^m)))},$$

$$\beta_2^{IQ} = \frac{\sigma_{\psi_p m}}{\sigma_m^{*2}} = \frac{Cov(IQ_t^p - E_{t-1}(IQ_t^p), r_t^m - E_{t-1}(r_t^m))}{Var(r_t^m - E_{t-1}(r_t^m) + (IQ_t^m - E_{t-1}(IQ_t^m)))},$$

$$\beta_3^{IQ} = \frac{\sigma_{\psi_p\psi_m}}{\sigma_m^{*2}} = \frac{Cov(IQ_t^p - E_{t-1}(IQ_t^p), IQ_t^m - E_{t-1}(IQ_t^m))}{Var(r_t^m - E_{t-1}(r_t^m) + (IQ_t^m - E_{t-1}(IQ_t^m)))},$$

where $\lambda = E(r_t^m + IQ_t^m - r_{ft})$, represents the average IQ-adjusted market risk premium. The unconditional static IQCAPM expressed in equation (12) provides the framework for understanding the various channels through which IQ risk may affect asset returns. Given the above four expressions for betas in the unconditional IQCAPM, three sets of innovations are required to obtain IQ betas: (i) $IQ_t^m - E_{t-1}(IQ_t^m)$, the market portfolio IQ innovations; (ii) $IQ_t^p - E_{t-1}(IQ_t^p)$, the IQ innovations for portfolio *p*, where *p = 1, 2... 25*; and (iii) $r_t^m - E_{t-1}(r_t^m)$, the market portfolio return innovations.

Since seasonality is pervasive in quarterly data, the AR(4) regression is ideal for estimating market portfolio IQ innovations:[23]

$$IQ_t^m = a + a_1 IQ_{t-1}^m + a_2 IQ_{t-2}^m + a_3 IQ_{t-3}^m + a_4 IQ_{t-4}^m + u_t^m,$$

where $u_t^m = IQ_t^m - E_{t-1}(IQ_t^m)$ is the innovation in the market IQ measure. Similarly, the innovation in the portfolio IQ measure is estimated with the following AR(4) process:

$$IQ_t^p = a + a_1 IQ_{t-1}^p + a_2 IQ_{t-2}^p + a_3 IQ_{t-3}^p + a_4 IQ_{t-4}^p + u_t^p,$$

where p = 1, 2... 25, and $u_t^p = IQ_t^p - E_{t-1}(IQ_t^p)$ is the portfolio IQ innovation estimated at time t for each of the 25 portfolios.

Table 4 reports the empirical fit of the AR(4) specification for estimating innovations in the market IQ measure. The table shows that there is a strong seasonal pattern in the $IQ1_t^m$ measure and in the $IQ2_t^m$ measure. In Panel A of Table 4 I see that the equally-weighted market IQ measure, $IQ1_t^m$, is highly correlated with its fourth lag measure ($IQ1_{t-4}^m$) at the 1-percent significance level (0.582, *t*-statistic = 3.14). The IQ-stability measure, $IQ2_t^m$, is negatively correlated with its first lag measure ($IQ2_{t-1}^m$) at the 5-percent significance level (-0.305, *t*-statistic = -2.51), negatively correlated with its second lag measure ($IQ2_{t-2}^m$) at the 10-percent significance level (-0.205, *t*-statistic = -

[23] Acharya and Pedersen (2005) use the AR(2) specification to extract innovations in their liquidity measure and market returns, which is suitable for their monthly data.

1.67), and negatively correlated with its third lag measure ($IQ2^m_{t-3}$) at the 5-percent significance level (-0.296, *t*-statistic = -2.42). Overall, the R^2 of the AR(4) specification for $IQ1^m_t$ and $IQ2^m_t$ is 0.568 and 0.778, respectively. The resulting innovations in the market IQ measures appear stationary. Similarly, Panel B shows that for value-weighted $IQ1^m_t$ and $IQ2^m_t$ the R^2 for the AR(4) specification is 0.605 and 788, respectively.

Table 4: Seasonality Test of IQ Measures in Market Portfolio

In this table, I show the empirical fit of an *AR(4)* specification for three alternative market IQ measures during time period from quarter 1 of 1987 to quarter 4 of 2007. The AR(4) specification is given as follows:

$$IQ^m_t = a + a_1 IQ^m_{t-1} + a_2 IQ^m_{t-2} + a_3 IQ^m_{t-3} + a_4 IQ^m_{t-4} + u^m_t ,$$

where $u^m_t = IQ^m_t - E_{t-1}(IQ^m_t)$, which is the innovation in market IQ measures. The R^2 is obtained for each single regression, and the adjusted R^2 is reported in the parentheses. The *t*-statistic is reported in the parentheses as well. Panel A focuses on equally-weighted market IQ measures and Panel B focuses on value-weighted market IQ measures.

Panel A: Equally-weighted Market IQ Measures

Market IQ measures	Alpha	IQ^m_{t-1}	IQ^m_{t-2}	IQ^m_{t-3}	IQ^m_{t-4}	R^2 for AR(4)
$IQ1^m_t = \sum_i w_i \mid e_{i,t} \mid$	0.015 (1.17)	0.204 (1.19)	-0.207 (-1.24)	0.015 (0.09)	0.582*** (3.49)	0.568 (0.542)
$IQ2^m_t = \sum_i w_i \sigma(e_i)_t$	0.003 (1.35)	-0.305** (-2.51)	-0.204* (-1.67)	-0.296** (-2.42)	-0.151 (-1.24)	0.778 (0.764)

Panel B: Value-weighted Market IQ Measures

Market IQ measures	Alpha	IQ^m_{t-1}	IQ^m_{t-2}	IQ^m_{t-3}	IQ^m_{t-4}	R^2 for AR(4)
$IQ1^m_t = \sum_i w_i \mid e_{i,t} \mid$	0.011 (1.64)	0.521*** (4.27)	-0.014 (-0.10)	0.219 (1.62)	0.133 (1.10)	0.605 (0.581)
$IQ2^m_t = \sum_i w_i \sigma(e_i)_t$	0.009** (2.31)	1.209*** (9.81)	-0.538*** (-2.67)	0.240 (1.16)	-0.071 (-0.49)	0.788 (0.775)

Because of the seasonality effect in the quarterly data, employing a higher order of the autoregressive specification, such as an AR(5), yields little improvement in terms of explanatory power. As a result, an AR(4) specification is most suitable to estimate innovations in market portfolio IQ measures. For similar reason, the innovations in the market portfolio return are also computed using an AR(4) specification, adjusted for market characteristics at the beginning of each quarter (market volatility, log of one-quarter lagged market capitalization and lagged book-to-market ratio). The resulting innovations in market portfolio return appear stationary as well.

Following equation (12), I obtain four betas: $\beta^{Market}, \beta_1^{IQ}, \beta_2^{IQ},$ and β_3^{IQ}, based on innovations in the IQ measures and return innovations for the market portfolio and 25 IQ portfolios. Table 5 documents the descriptive statistics of equally-weighted portfolios ranked based on the two IQ measures (each presented in a separate panel) for the quarterly sample from 1987 to 2007. The estimates of four IQCAPM betas: standard CAPM beta (β^{Market}) and three IQ-related betas ($\beta_1^{IQ}, \beta_2^{IQ}$ and β_3^{IQ}) are presented. The average and standard-deviation of the two alternative IQ measures and portfolio returns, the average of market capitalization of portfolios (Size, in billion dollars), and Book-to-market ratio (B/M) for each portfolio are documented as well.[24]

Table 5 shows that for both IQ measures, as expected, there is a tendency for high-IQ (informationally-imprecise) stocks to have higher return volatility. This pattern suggests that informationally-imprecise stocks tend to be more volatile. For example the 25th $IQ1$ portfolio (the portfolio with the highest $IQ1$ value) has the highest standard-deviation of portfolio return ($\sigma(r^p) = 14.38\%$). The monotonic relation between the IQ level and portfolio size is also observed in Table 5. Both Panels show that portfolio size generally declines as the portfolio $IQ1$ value (or $IQ2$ value) increases implying that firms with informationally-imprecise stocks tend to be small. There is no clear association between B/M and IQ level in both Panels of Table 5.

[24] I reproduce the same statistics for value-weighted portfolios and the results are similar.

Table 5: Descriptive Statistics for Information-Quality Sorted Portfolios

This table consists of two panels, in which I report the descriptive statistics of the odd-numbered equally-weighted portfolios ranked on the IQ measures for the quarterly sample from 1987 to 2007. In Panel A, 25 IQ portfolios are formed based on $IQ1 = |\,e_{i,t}\,|$, which is the absolute value of residual estimated using the Barth et al. (2001) model. For Panel B, IQ portfolios are ranked based on $IQ2 = \sigma(e_i)_t$, the standard deviation of residuals over time *t*-4 through *t* as used in Barth et al. (2001) model. In each panel, the estimates of the four IQCAPM betas: standard market beta (β^{Market}) and three IQ betas ($\beta_1^{IQ}, \beta_2^{IQ}$ and β_3^{IQ}) are presented. The average and standard-deviation of the two alternative IQ measures and portfolio returns, the average of market capitalization of the portfolios (Size), and Book-to-market ratio for each portfolio are documented as well.

Panel A: 25 IQ1 $(|\,e_{i,t}\,|)$ Portfolios

Rank p	β^{Market} (%)	β_1^{IQ} (%)	β_2^{IQ} (%)	β_3^{IQ} (%)	$IQ1_t^p$ Mean (%)	$IQ1_t^p$ Std.dev (%)	r_t^p Mean (%)	r_t^p Std.dev (%)	Size (bl$)	B/M ratio
1	75.80	-1.59	-1.39	3.90	2.78	2.76	5.89	10.11	3.38	0.55
3	66.48	-1.14	-2.17	2.54	2.99	1.90	4.36	9.38	2.73	0.61
5	74.22	-1.35	-2.15	3.90	3.76	3.13	4.54	9.78	3.11	0.58
7	76.68	-1.20	-2.50	4.61	3.88	3.07	5.22	10.43	2.94	0.49
9	76.91	-1.38	-2.20	3.68	4.08	2.52	5.20	10.27	2.58	0.53
11	85.22	-1.35	-2.86	4.14	4.35	2.69	5.17	11.24	2.22	0.51
13	90.09	-1.68	-1.83	4.38	4.70	2.76	5.08	11.97	2.69	0.52
15	86.27	-2.61	-3.57	7.92	5.44	4.90	5.47	11.39	2.47	0.52
17	89.41	-1.84	-3.68	6.55	5.85	4.09	5.80	11.61	2.61	0.51
19	93.85	-1.68	-2.98	3.28	6.08	2.63	5.91	12.28	2.83	1.36
21	95.74	-3.49	-1.38	3.49	6.67	3.05	5.92	12.85	2.71	0.47
23	95.06	-3.76	-2.46	3.08	8.13	7.09	6.24	12.70	2.71	0.43
25	107.98	-4.01	-2.25	3.59	9.86	4.57	7.59	14.38	2.61	0.44

Panel B: 25 IQ2 $(\sigma(e_i)_t)$ Portfolios

Rank p	β^{Market} (%)	β_1^{IQ} (%)	β_2^{IQ} (%)	β_3^{IQ} (%)	$IQ2_t^p$ Mean (%)	$IQ2_t^p$ Std.dev (%)	r_t^p Mean (%)	r_t^p Ste.dev (%)	Size (bl$)	B/M
1	68.18	-0.33	-0.04	0.002	0.54	0.08	5.29	8.94	4.03	0.55
3	72.73	-0.50	-0.02	0.001	1.23	0.16	5.07	9.07	3.56	0.82
5	77.65	-0.77	-0.01	0.001	1.71	0.22	5.17	9.53	2.97	0.53
7	81.93	-1.05	0.03	0.001	2.17	0.31	5.37	10.04	2.62	0.51
9	91.82	-0.18	0.04	0.006	2.64	0.41	5.91	11.23	2.28	0.53
11	91.47	-1.24	0.07	0.006	3.15	0.53	6.53	11.37	2.42	0.53
13	95.40	-1.19	0.08	0.009	3.74	0.68	6.36	11.58	2.07	0.68
15	105.71	-1.45	0.16	0.017	4.46	0.91	7.67	13.02	1.98	0.50
17	106.63	-1.44	0.27	0.027	5.38	1.23	6.73	12.73	1.74	0.47
19	107.51	-1.68	0.42	0.049	6.67	1.70	7.54	13.20	1.83	0.50
21	111.76	-1.56	0.48	0.059	8.60	2.33	8.82	13.92	2.45	0.47
23	116.43	-1.07	1.12	-0.016	12.38	3.53	8.85	14.70	1.58	0.45
25	134.79	-1.48	16.28	3.558	50.32	43.28	10.80	17.80	2.29	0.43

2.5.c. Empirical Fit of the Static IQCAPM

In this subsection, I test the empirical fit of the static version of IQCAPM (Equation 12), and examine how IQ risk affects return through the IQ betas: $\beta_1^{IQ}, \beta_2^{IQ}, and\ \beta_3^{IQ}$. Alternative specifications of the model are designed to identify the potential effect of IQ betas in total and each IQ beta effect separately.

The first specification constraints the beta risk premium to be identical for all four IQCAPM betas:

$$E(\tilde{r}_t^{\,p} - r_{ft}) = \alpha + \lambda \beta^{all}, \tag{12.1}$$

where $\tilde{r}_t^{\,p} = r_t^p + IQ_t^p$ representing the observed portfolio return, and $\beta^{all} = \beta^{Market} + \beta_1^{IQ} + \beta_2^{IQ} + \beta_3^{IQ}$ representing the overall magnitude of systematic risk for each portfolio. The parameter λ stands for the average market risk premium, which includes the market-wide IQ related component of risk premium.

To isolate IQ risk from the standard market risk, and to examine the aggregate effect of systematic IQ risk, I test the second specification as follows:

$$E(\tilde{r}_t^{\,p} - r_{ft}) = \alpha + \lambda_1 \beta^{Market} + \lambda_2 \beta_{net}^{IQ}. \tag{12.2}$$

The first beta factor, β^{Market}, in specification (12.2) is the standard CAPM beta reflecting the portfolio return sensitivity relative to market return. The second beta, $\beta_{net}^{IQ} = \beta_1^{IQ} + \beta_2^{IQ} + \beta_3^{IQ}$, represents an aggregate magnitude of systematic portfolio IQ risk.

To compare the effects of different dimensions of systematic IQ risk on return, I decompose the aggregate IQ beta into three betas $\beta_1^{IQ}, \beta_2^{IQ} and\ \beta_3^{IQ}$, as derived in equation (12) of Section 5.3. Thus, I allow for a unique risk premium for each beta in the following equation:

$$E(\tilde{r}_t^{\,p} - r_{ft}) = \alpha + \lambda_1 \beta^{Market} + \lambda_2 \beta_1^{IQ} + \lambda_3 \beta_2^{IQ} + \lambda_4 \beta_3^{IQ}. \qquad (12.3)$$

where $\beta^{all} = \beta^{Market} + \beta_1^{IQ} + \beta_2^{IQ} + \beta_3^{IQ}$, $\beta_{net}^{IQ} = \beta_1^{IQ} + \beta_2^{IQ} + \beta_3^{IQ}$, for $p=1,2,\ldots 25$ portfolio.

Regression model (12.3) represents my static IQCAPM with four the four betas spelled out, with or without the average IQ level, $E(IQ_t^p)$. The total systematic IQ risk affects asset returns through three channels. The first channel is expressed by β_1^{IQ}, reflecting the sensitivity of portfolio return relative to market-wide IQ. The second channel, β_2^{IQ}, is the result of association between portfolio IQ noise and the market return. The last one, β_3^{IQ}, is the IQ commonality beta reflecting the co-movement between individual portfolio IQ noise and market-wide IQ noise.

Next I am interested in the total and relative significance of the IQ risk effect on returns with the above specifications. I use the 25 equally-weighted portfolios (ranked on both IQ measures over the 1987-2007 period) to estimate the regression models (12.1) – (12.3) cross sectionally over the 25 portfolios. Table 6 documents the coefficient estimates from GMM estimation.

Panel A of Table 6 shows that for the 25 *IQ*1-sorted portfolios the R^2 for specification (12.2) is 0.761, and the coefficient estimate for the aggregate IQ beta, β_{net}^{IQ}, is significant at 5 percent level as (-0.097, t = -2.20). This finding suggests that systematic IQ risk is priced. For specification (12.3) the R^2 is 0.768 with β^{Market} significant at 1 percent level, and two of the three IQ betas, β_1^{IQ} and β_3^{IQ}, being significant at 10 percent level. This result confirms that, alongside the standard CAPM beta, the first IQ beta (the sensitivity of portfolio return relative to market IQ) and the second IQ beta (commonality in IQ beta) are priced as well.

Table 6: Asset Pricing Tests of IQ risk (Equally-weighted Portfolios)

This table reports the coefficient estimates from cross-sectional regressions of the static IQCAPM for 25 equally-weighted portfolios using quarterly data for the 1987-2007 period, with an equally-Weighted market portfolio. I use GMM to obtain the coefficient estimates based on the following models,

CAPM $E(\tilde{r}_t^{\,p} - r_{ft}) = \alpha + \lambda\beta^{Market}$,

(12.1) $E(\tilde{r}_t^{\,p} - r_{ft}) = \alpha + \lambda\beta^{all}$,

(12.2) $E(\tilde{r}_t^{\,p} - r_{ft}) = \alpha + \lambda_1\beta^{Market} + \lambda_2\beta_{net}^{IQ}$,

(12.3) $E(\tilde{r}_t^{\,p} - r_{ft}) = \alpha + \lambda_1\beta^{Market} + \lambda_2\beta_1^{IQ} + \lambda_3\beta_2^{IQ} + \lambda_4\beta_3^{IQ}$,

where $\beta^{all} = \beta^{Market} + \beta_1^{IQ} + \beta_2^{IQ} + \beta_3^{IQ}$, and $\beta_{net}^{IQ} = \beta_1^{IQ} + \beta_2^{IQ} + \beta_3^{IQ}$. Panel A reports the results for 25 portfolios sorted on $IQ1$. Panel B reports the results on 25 IQ2-sorted portfolios. The R^2 and the adjusted-R^2 (in parentheses) are reported. The *t*-statistic is reported (in parentheses) as well.

Panel A: 25 Equally Weighted IQ1 ($|e_{i,t}|$) Portfolios

Model	alpha	β^{Market}	β_1^{IQ}	β_2^{IQ}	β_3^{IQ}	β^{all}	β_{net}^{IQ}	R^2
CAPM	-0.007	0.060***						0.732
	(-0.89)	(6.41)						(0.720)
	0.031		-0.633***					0.658
	(14.54)		(-5.49)					(0.643)
	0.043			-0.048				0.004
	(12.59)			(-0.34)				(-0.039)
	0.046				-0.071			0.013
	(8.23)				(0.528)			(-0.030)
12.1	0.008					0.062***		0.679
	(-0.98)					(5.77)		(0.665)
12.2	-0.001	0.052***					-0.097**	0.761
	(-0.10)	(6.19)					(-2.20)	(0.740)
12.3	0.003	0.043***	-0.212*	-0.077	-0.071*			0.768
	(0.37)	(3.31)	(-1.73)	(-0.76)	(-1.74)			(0.722)

Panel B: 25 Equally Weighted IQ2 ($\sigma(e_i)_t$) Portfolios

Model	alpha	β^{Market}	β_1^{IQ}	β_2^{IQ}	β_3^{IQ}	β^{all}	β_{net}^{IQ}	R^2
CAPM	-0.023	0.083***						0.865
	(-2.78)	(9.50)						(0.859)
	0.038		-1.956					0.439
	(9.65)		(-4.54)					(0.414)
	0.054			0.300***				0.408
	(19.23)			(6.51)				(0.383)
	0.056				1.135***			0.280
	(7.43)				(9.90)			(0.249)
12.1	-0.015					0.075***		0.885
	(-3.12)					(14.20)		(0.881)
12.2	-0.016	0.076***					0.065***	0.886
	(-2.12)	(9.03)					(3.06)	(0.876)
12.3	-0.007	0.065***	-0.032	0.512***	-1.867***			0.915
	(-0.85)	(5.53)	(-0.10)	(3.98)	(-3.81)			(0.898)

Panel B of Table 6 reports the empirical fit of the static IQCAPM using *IQ*2. As expected, the model fits the data better when *IQ*2 is used as a proxy for IQ. The R^2 for specification (12.2) is now higher (0.886) with both the market beta, β^{Market}, (0.076, *t*=9.03) and the aggregate IQ beta, β^{IQ}_{net}, (0.065, *t*=3.06) being significant at 1 percent level. This result lends strong support for the pricing of systematic IQ risk in returns. Similarly, if I decompose the aggregate IQ beta into three IQ-related betas, as shown in Panel B for specification (12.3), β^{IQ}_{2} (0.512, t=3.98) and β^{IQ}_{3} (-1.866, t=-3.81) are both significant at 1 percent level (alongside the market beta), while β^{IQ}_{1} is insignificant. These results lend further evidence for the pricing of systematic IQ risk in favour of the hypothesis that market-wide IQ represents a priced source of risk.

Note the relatively high regression R^2 in regression model (12.3). This is could be due to multicollinearity between the four betas as demonstrated. I test for multicollinearity in Table 7 that shows that some of the betas are collinear to a certain degree.[25] The existence of collinearity is unavoidable due to the correlation among the market-IQ innovations, portfolio-IQ innovations, and market-return innovations that I estimate to compute the four betas. In general, for regression models (12.1) and (12.2), β^{IQ}_{net} and β^{all} are significant, with high regression R^2's indicating that my model fits the data well. In these regression models there is no issue with multicollinearity. Overall, Table 6 lends strong support for my static IQCAPM.

[25] Acharya and Pederson (2005) face the same problem with their liquidity betas. See their discussion about the multicollinearity problem.

Table 7: Correlation Coefficients between the Four IQCAPM Betas

This table reports the Pearson correlations between the market beta and IQ betas for the 25 equally-weighted IQ portfolios formed each quarter. At the beginning of each quarter from 1987 to 2007, eligible NYSE/AMEX stocks are sorted into 25 portfolios according to firm-specific IQ measures. Panel A reports the results for 25 *IQ*1-sorted portfolios. Panel B reports the results for 25 portfolios sorted on *IQ*2. The P-value is documented in parentheses.

Panel A: 25 IQ1 ($|e_{i,t}|$) Portfolios

	β^{Market}	β_1^{IQ}	β_2^{IQ}	β_3^{IQ}
β^{Market}	1	-0.856*** (<.0001)	-0.031 (0.881)	-0.009 (0.962)
β_1^{IQ}		1	-0.045 (0.83)	0.125 (0.552)
β_2^{IQ}			1	-0.396** (0.049)
β_3^{IQ}				1

Panel B: 25 IQ2 ($\sigma(e_i)_t$) Portfolios

	β^{Market}	β_1^{IQ}	β_2^{IQ}	β_3^{IQ}
β^{Market}	1	-0.753*** (<.0001)	0.550*** (0.004)	0.460** (0.021)
β_1^{IQ}		1	-0.254 (0.219)	-0.195 (0.348)
β_2^{IQ}			1	0.977*** (<.0001)
β_3^{IQ}				1

To test robustness of the results presented in support of the static IQCAPM in Table 6, I estimate my model with alternative specifications for value-weighted portfolios based on the two IQ measures. The table in Appendix A4 shows that the results based on value-weighted portfolios are qualitatively similar to the results obtained for the equally-weighted portfolios. The results confirm that IQ systematic risk is still significantly priced, whether I use an equally-weighted or a value-weighted portfolio.

2.6. Robustness Tests - IQ Risk vs. Illiquidity Risk

As discussed in Section 2.5, I find that portfolio size generally declines as portfolio IQ proxy increases, implying that firms with informationally-imprecise stocks tend to be small (see Table 5). This observation generally implies a monotonic association between portfolio IQ and portfolio size. Note that size and illiquidity are highly correlated (as shown in Table 2). Given the strong support provide by Acharya and Pedersen (2005) for their liquidity-adjusted CAPM, I tests whether the support my tests lend to the IQCAPM are robust in the presence of Acharya and Pedersen's (2005) liquidity betas. This will allow us to test whether: (i) systematic IQ risk is significantly correlated with systematic illiquidity risk and other firm characteristics, such as size and B/M; and (ii) the explanatory power of IQ beta is captured by illiquidity beta.

Below I follow three procedures in this section: (i) I estimate liquidity betas; (ii) I form portfolios stratified by IQ level, liquidity level, size, and B/M to control for firm characteristics; and (iii) I examine the performance of my empirical asset-pricing model with market beta, IQ beta, and liquidity beta.

2.6.a. The IQCAPM with Systematic Illiquidity Risk

Based on the normalized illiquidity measure of Amihud (2002), Acharya and Pedersen (2005) provide a unified theoretical model to explain how asset returns are affected by liquidity risk and commonality in liquidity.[26] To test whether the explanatory power of my IQ beta is robust to inclusion of liquidity risk, I run a three-beta model as follows,

$$E(r_t^p - r_t^f) = \alpha_0 + \alpha_1 \beta^{Market} + \alpha_2 \beta_{net}^{IQ} + \alpha_3 \beta_{net}^{ILLIQ},$$

[26] Prior studies examine the systematic nature of liquidity. Chordia, Subrahmanyam, and Anshuman (2000) show that stocks returns are cross sectionally related to the variability in liquidity, where liquidity is proxied by measures such as trading volume and turnover. Chordia, Roll, and Subrahmanyam (2000), Huberman and Halka (1999), and Hasbrouck and Seppi (2000) find that individual stock liquidity co-moves with the market-wide liquidity, which is known as "commonality in liquidity".

where $r_t^p - r_t^f$ is excess return on portfolio *p*, β^{Market} is market beta, β_{net}^{IQ} is the net IQ beta discussed in Section 5, and β_{net}^{ILLIQ} is the net illiquidity beta of Acharya and Pedersen (2005).[27]

The net illiquidity beta, β_{net}^{ILLIQ}, is the sum of Acharya and Pedersen's (2005) three liquidity betas. The illiquidity cost used in Acharya and Pedersen (2005) is constructed based on the absolute return-to-volume measure of Amihud (2002), which captures the price-impact dimension of liquidity. Following Amihud (2002), a stock's illiquidity level at time *t* is defined as

$$ILLIQ_t^i = \frac{1}{Days_t^i} \sum_{d=1}^{Days_t^i} \frac{\left|R_{td}^i\right|}{V_{td}^i}, \tag{13}$$

where R_{td}^i and V_{td}^i are the return and dollar volume (in millions) on day *d* in quarter *t*, respectively, and $Days_t^i$ is the number of observation days in quarter *t* for stock *i*.

Note that in Amihud (2002), $ILLIQ_t^i$ is monthly average of daily data return-to-volume measure. For comparison to the quarterly estimates of IQ measures in my paper, I compute $ILLIQ_t^i$ as quarterly average of absolute return-to-volume using daily observations within quarter *t* for stock *i*. The more illiquid a stock is, the greater a price movement corresponding to little volume, which suggests a higher value of $ILLIQ_t^i$. Amihud's (2002) illiquidity measure has often been used in the empirical microstructure literature.[28]

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[27] See Appendix B2 for details about the construction of Acharya and Pedersen's (2005) illiquidity beta.
[28] Amihud (2000) and Jones (2002) find that expected market returns are significantly related to time-series measures of market liquidity. Based on microstructure data for NYSE, AMEX, and NASDAQ stocks, Hasbrouck (2002) computes a measure of Kyle's lambda and finds that its Spearman (Pearson) correlation

2.6.b. Data and Portfolios

To estimate $ILLIQ_t^i$ and β_{net}^{ILLIQ}, I sample all eligible stocks over the period 1987 to 2007 corresponding to the sample period used with the IQ measures. Daily return and volume data are obtained from CRSP. To be consistent with Acharya and Pedersen (2005), I exclude NASDAQ since the volume data includes interdealer trades; and I exclude stocks with less than 15 quarterly observations. Following equation (13), I estimate $ILLIQ_t^i$ in quarter *t* for every stock *i*, and match it with both IQ measures (IQ1 $= | e_{i,t} |$, and IQ2 $= \sigma(e_i)_t$) for each stock. Stocks with missing $ILLIQ_t^i$ or missing IQ measures are excluded from my sample.

Similar to the procedure for forming portfolios in Section 2.5, I form an equally-weighted market portfolio and a value-weighted market portfolio by market capitalization for each quarter *t* during the sample period, in which market return, market IQ measures, and market ILLIQ are computed.

To avoid the potential problem that IQ-sorted portfolios might present a sample biased in favour of the IQ effect but against the Illiquidity effect, I double-sort stocks into IQ by ILLIQ portfolios. Specifically, I sort stocks into five IQ quintiles first and then into five ILLIQ quintiles for each IQ proxy. For comparison, I also reverse the ranking criteria and form portfolios sorted first into five ILLIQ quintiles and then into five IQ quintiles within every ILLIQ groups.

To control for firm characteristics, such as size, B/M ratio, and liquidity (ILLIQ), I form portfolios sorted on each of them. Specifically, I form 25 ILLIQ portfolios for

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with ILLIQ is high and equal to 0.737 (p-value=0.473). Hasbrouck (2002) states that "[a]mong the proxies considered here, the illiquidity measure [ILLIQ] appears to be the best."

each quarter during the 1987 to 2007 period by sorting stocks based on their previous-year ILLIQ levels. I form 25 size portfolios for each quarter during the sample period by sorting stocks based on their market capitalization at the beginning of the year. Lastly, I form 25 B/M (book-to-market ratio) portfolios for each quarter during the sample period by ranking stocks based on their previous-year book-to-market ratio. Finally, I form 25 IQ portfolios for each quarter by sorting stocks based on their previous-year IQ levels (based on both IQ proxies).

For each portfolio, I estimate market beta and net IQ beta following the procedures described in Section 2.5. In line with Acharya and Pedersen (2005), I estimate the net illiquidity beta as the sum of Acharya and Pedersen's (2005) three liquidity betas. Note that in Acharya and Pedersen (2005), an AR(2) is employed to estimate innovations in variables (portfolio illiquidity cost, market illiquidity cost, and market return). Since seasonality effect is pervasive in quarterly IQ measures (as documented in Section 2.5), I use an AR(4) specification to test for the autocorrelation pattern of market portfolio ILLIQ measure. As shown in Table 8, there is no seasonality effect in quarterly ILLIQ data for market portfolio, therefore I employ the AR(2) specification to compute market liquidity innovation. The resulting innovations in market portfolio ILLIQ appear stationary.

Table 8: Empirical Fit of an *AR*(4) Specification for the Market Portfolio ILLIQ measures

Following Acharya and Pederson (2005), I employ the illiquidity measure of Amihud (2002) to construct a net liquidity beta to be included in the IQCAPM test. In this table, I show the estimated coefficients based on *AR*(4) regression for the market ILLIQ measure for the period starting at quarter 1, 1987 and ending at quarter 4, 2007. The AR(4) specification is as follows:

$$ILLIQ_t^m = a + a_1 ILLIQ_{t-1}^m + a_2 ILLIQ_{t-2}^m + a_3 ILLIQ_{t-3}^m + a_4 ILLIQ_{t-4}^m + u_t^m,$$

where $u_t^m = ILLIQ_t^m - E_{t-1}(ILLIQ_t^m)$, which is the innovation in the market-wide ILLIQ measure following Amihud (2002). The R^2 is obtained for each regression for both equally-weighted and value-weighted market portfolios. The adjusted-R^2 is reported in parentheses. The *t*-statistic is also documented in the parentheses.

Market AP ILLIQ	Alpha	$ILLIQ_{t-1}^m$	$ILLIQ_{t-2}^m$	$ILLIQ_{t-3}^m$	$ILLIQ_{t-4}^m$	R^2 for *AR*(4)
ILLIQ	0.038	0.412***	0.213**	0.132	0.136	0.767
(*Equally-weighted*)	(1.31)	(3.61)	(1.75)	(1.12)	(1.24)	(0.753)
ILLIQ	0.025**	0.935***	-0.161	0.144	-0.017	0.835
(*Value-weighted*)	(2.89)	(11.20)	(-1.45)	(1.32)	(-0.22)	(0.830)

Table 9 reports the Pearson correlations coefficients between market beta, net IQ beta, and net illiquidity beta for 25 (5 IQ by 5 Liquidity) equally-weighted portfolios as well as correlation coefficients between each beta, size, and B/M. The two panels of the table report results based on the two alternative IQ measures. Table 9 shows that the correlation coefficients between β_{net}^{IQ} (estimated based on both IQ1 and IQ2), and β_{net}^{ILLIQ} are low and statistically insignificant. The consistency of low correlation implies that systematic IQ risk represents a risk source different from Acharya and Pedersen's (2005) systematic liquidity risk. In Panel B, β_{net}^{ILLIQ} is highly correlated with size of portfolio (0.727, $p < 0.01\%$) confirming that smaller firms tend to have illiquid stocks.[29]

[29] The table in Appendix A5 shows that the results based on value-weighted portfolios are qualitatively similar to the results obtained for the equally-weighted portfolios.

Table 9: **Correlation Coefficients between Market Beta, Net IQ Beta, and Illiquidity Beta (Equally-weighted Portfolios)**

This table reports the Pearson correlations between market beta, net IQ beta, and net liquidity beta for 25 (5 IQ by 5 Liquidity) equally-weighted portfolios. At the beginning of each quarter from 1987 to 2007, eligible NYSE/AMEX stocks are sorted first into 5 groups according to estimated IQ measure and then sorted again into 5 liquidity categories within each IQ group. The Pearson correlations between each of betas, portfolio size, and portfolio book-to-market ratio are documented for each panel. *p*-values are reported in parentheses.

Panel A: 25 Equally-weighted (5 IQ1 by 5 Liquidity) Portfolios

	β^{Market}	β_{net}^{IQ}	β_{net}^{ILLIQ}	Size	B/M
β^{Market}	1	0.109	-0.483**	0.015	0.234
		(0.602)	(0.014)	(0.945)	(0.258)
β_{net}^{IQ}		1	-0.053	0.275	-0.106
			(0.802)	(0.184)	(0.613)
β_{net}^{ILLIQ}			1	-0.185	-0.200
				(0.375)	(0.337)
Size				1	0.174
					(0.404)
B/M					1

Panel B: 25 Equally-weighted (5 IQ2 by 5 Liquidity) Portfolios

	β^{Market}	β_{net}^{IQ}	β_{net}^{ILLIQ}	Size	B/M
β^{Market}	1	0.506**	-0.625***	-0.847***	0.134
		(0.011)	(0.001)	(<.0001)	(0.524)
β_{net}^{IQ}		1	0.042	-0.164	0.127
			(0.842)	(0.434)	(0.544)
β_{net}^{ILLIQ}			1	0.727***	-0.095
				(<.0001)	(0.832)
Size				1	-0.045
					(0.832)
B/M					1

2.6.c. Testing the IQCAPM with Systematic Illiquidity Risk

In this subsection, I run the following regression to further examine whether systematic IQ risk is priced. To this end, I use the following model:

$$E(\tilde{r}_t^{\,p} - r_t^{\,f}) = \alpha_0 + \lambda_1 \beta^{Market} + \lambda_2 \beta_{net}^{IQ} + \lambda_3 \beta_{net}^{ILLIQ}. \tag{14}$$

I use net betas for both IQ and liquidity, to avoid problems of multicollinearity that can arise in a seven-beta model (one market beta, three IQ betas, and three liquidity betas).

Table 10 reports the coefficient estimates from a cross-sectional estimation of regression model (14) for 25 equally-weighted portfolios using quarterly data during the

1987-2007 period. I run regressions based on 5 IQ by 5 ILLIQ portfolios, 5 ILLIQ by 5 IQ portfolios, 25 IQ portfolios, 25 ILLIQ portfolios, 25 size portfolios and 25 B/M (book-to-market ratio) portfolios and document the results for each of the two estimated IQ measures in separate panels.

Panel A shows that when IQ1 ($|e_{i,t}|$) is employed to estimate net IQ beta, the regression coefficient of β_{net}^{IQ} is statistically significant for the 5 IQ1 by 5 ILLIQ portfolios, 5 ILLIQ by 5 IQ1 portfolios, 25 IQ1 Portfolios, and 25 B/M Portfolios, all at the 1 percent level of significance. At the same time, the regression coefficient for β_{net}^{ILLIQ} is statistically significant only for 25 Size Portfolios and 25 B/M Portfolios, both at the 1 percent level of significance.

Similarly, Panel B of Table 10 reports that for the net IQ beta estimated based on IQ2 ($\sigma(e_i)_t$), the regression coefficient is statistically significant and robust to the inclusion of the net liquidity beta. Panel B further shows that when IQ2 is used to estimate the net IQ beta the estimated coefficient of the net liquidity beta is statistically significant (at least at the 10 percent significance level) for four out of the six portfolio formation schemes. This implies that investors price both systematic IQ risk and systematic liquidity risk. Note that the net IQ beta is consistently insignificant for the ILLIQ and size portfolios. This finding can be explained by the results of correlation reported in Table 9. In Particular, Panel B of Table 9 shows that the ILLIQ beta is highly correlated with size factor at 1% significance level, while the correlation between IQ beta and Size factor is very weak. The significance of IQ beta might be subsumed by ILLIQ beta either in ILLIQ portfolio or size portfolio.

Table 10
Testing the Static IQCAPM with the Net Liquidity Beta
(Equally-weighted Portfolios)

This table reports the results for robustness tests based on 5 ILLIQ by 5 IQ, 25 IQ portfolios, 25 ILLIQ portfolios, 25 size portfolios and 25 B/M (book-to-market ratio) portfolios. The coefficient estimates from cross-sectional regressions of the IQCAPM are documented for equally-weighted portfolios using quarterly data during the 1987-2007 period, while controlling for Acharya and Pedersen's (2005) net liquidity beta. GMM is used to estimate the coefficients from the following model:

$$E(\tilde{r}_t^{\,p} - r_t^{\,f}) = \alpha_0 + \lambda_1 \beta^{Market} + \lambda_2 \beta_{net}^{IQ} + \lambda_3 \beta_{net}^{ILLIQ},$$

where β^{Market} is market beta, β_{net}^{IQ} denotes the net IQ beta, and β_{net}^{ILLIQ} is the net illiquidity beta estimated following Acharya and Pedersen (2005). Results based on each of the two alternative IQ measure are given in separate panel. The R^2 is reported for each cross-sectional regression for different portfolios, and the adjusted-R^2 and *t*-statistic are reported in parentheses.

Panel A: Equally-weighted Portfolios (IQ1 = $|e_{i,t}|$)

Portfolios	alpha	β^{Market}	β_{net}^{IQ}	β_{net}^{ILLIQ}	R^2
5 IQ1 by 5 ILLIQ	0.010***	0.023***	0.035***	-0.020	0.735
	(2.92)	(5.06)	(4.77)	(-0.92)	(0.697)
5 ILLIQ by 5 IQ1	0.017***	0.016***	0.037***	-0.042	0.676
	(2.73)	(2.18)	(4.47)	(-1.45)	(0.630)
25 IQ1 Portfolios	-0.023	0.057***	0.028***	-0.070	0.583
	(-1.65)	(3.49)	(6.07)	(-1.11)	(0.524)
25 ILLIQ Portfolios	0.022***	0.012	-0.005	-0.039*	0.599
	(4.20)	(1.82)	(-0.17)	(-1.96)	(0.542)
25 Size Portfolios	0.015	0.033**	-0.037	0.172***	0.470
	(1.27)	(2.10)	(-0.90)	(3.02)	(0.394)
25 B/M Portfolios	0.127***	-0.082*	-0.255***	0.447***	0.817
	(2.92)	(-1.79)	(-2.96)	(3.70)	(0.791)

Panel B: Equally-weighted Portfolios (IQ2 = $\sigma(e_i)_t$)

Sample	alpha	β^{Market}	β_{net}^{IQ}	β_{net}^{ILLIQ}	R^2
5 IQ2 by 5 ILLIQ	0.018*	0.021*	0.134**	-0.093**	0.622
	(1.83)	(1.95)	(2.67)	(-2.38)	(0.568)
5 ILLIQ by 5 IQ2	0.023***	0.013*	0.205***	-0.130***	0.732
	(3.84)	(2.05)	(4.44)	(-4.97)	(0.694)
25 IQ2 Portfolios	-0.012	0.052***	0.139***	0.014	0.722
	(-1.21)	(4.52)	(5.77)	(0.22)	(0.682)
25 ILLIQ Portfolios	0.039***	0.002	-0.055	-0.167***	0.535
	(3.85)	(0.20)	(-0.58)	(-3.34)	(0.469)
25 Size Portfolios	0.025***	0.020***	-0.028	0.065*	0.146
	(3.89)	(3.05)	(-0.89)	(2.01)	(0.024)
25 B/M Portfolios	0.047	0.041	-0.581**	0.322**	0.566
	(1.32)	(1.04)	(-2.28)	(2.55)	(0.504)

Focusing on the static version of IQCAPM, the results presented in Panel A and Panel B of Table 10 show that the model's net IQ beta estimated based on both IQ measures is a priced source of systematic risk, even after adjusting for liquidity risk. This lends strong support for the validity of the IQCAPM.[30]

[30] Table A6 in Appendix A shows that the results based on value-weighted portfolios are qualitatively similar to the results obtained for the equally-weighted portfolios.

CHAPTER 3

A Generalized Earning-Based Stock Valuation Model with Learning

3.1. Introduction

This chapter extends the earnings-based stock valuation model of Bakshi and Chen (2005) (BC hereafter) by relaxing the complete information assumption and allowing for a market with incomplete information. To this end, I assume as in the BC model that earnings growth is observed by investors. However, they do not observe the instantaneous mean of earnings growth rate (thereafter, MEGR). The MEGR is an additional state variable, and I model it as a mean-reverting process. My model allows for continuous learning about the unobserved state variable, and asset prices reflect this learning process. I investigate the effects of firm characteristics, such as mean-reversion speed and volatility of earnings growth, on differences in asset pricing between my incomplete-information and the BC complete-information models as well.

My results indicate that the faster the earnings-growth mean reverts to its long-term value, the smaller the mispricing attributed to information incompleteness. This effect results from the fact that the higher speed of reversion towards the constant long-term mean leads to a faster exponential decay of any initial deviation from this mean and, therefore, faster learning. Ceteris paribus, the higher volatility of the unobservable MEGR results in larger mispricing. This result is more pronounced for younger firms with shorter learning horizons for which, naturally, there is a short history of data available for learning. This finding is consistent with Pastor and Veronesi (2003), who predict that Market-to-Book ratio (M/B) declines over a typical firm's lifetime, and younger firms should have higher M/B ratios than otherwise identical older firms since uncertainty about younger firms' average profitability is greater.

In my model the mean squared error of MEGR estimate, a measure of the degree of learning, persists and remains especially large for short learning horizons. The persistent uncertainty of the MEGR estimate generates an extra risk premium beyond what is accounted for in the complete information model. Over time both the uncertainty about MEGR estimate and extra risk premium decline to equilibrium levels as more information becomes available. In a perfect learning environment (e.g., unobservable MEGR is perfectly correlated with earnings), the extra risk premium on MEGR declines and converges to zero in the long run. At the same time, the variance of the estimate of MEGR decreases over learning horizon and converges to zero.[1] Perfect correlation implies that investors eventually have complete knowledge of the true process of the mean growth rate.

However, in non-perfect learning environment, the extra risk premium on MEGR never vanishes regardless of learning horizon. This long run risk premium reflects a persistent uncertainty that investors hold in an incomplete information environment.

For comparison, I compute the risk premiums based on my incomplete-information model and the complete-information model of BC. First, MEGR risk premium in incomplete information case is always bigger than that under complete

[1] When the correlation between earnings and their latent MEGR is perfectly negative, this result holds as long as the speed of mean reversion is not too small relative to the volatility of MEGR. This condition is the consequence of measuring the long-term uncertainty of MEGR by the ratio of the earnings volatility to the speed of mean reversion. See Proposition 1 below.

information environment. They are the same only if the correlation between earnings and MEGR is perfect. Second, The difference in MEGR risk premiums declines with learning horizon faster for firms with larger correlation between earnings and underlying MEGR. Third, for 20 technology stocks used in BC, I find that the difference in risk premiums can be as high as 40%-50% for short learning horizons of several months. Given BC parameter values the difference declines to a steady state level after 6-11 months. Finally, the level of incomplete information premium can reach up to 7 percent for firms with short learning horizons and weaker mean reversion even if their earnings are perfectly correlated with MEGR.

The equilibrium stock prices computed based on my model have patterns similar to those of risk premiums. With perfect correlation between earnings growth and MEGR, investors perfectly learn about MEGR within ~ 11 months (based on 20 technology stock data of BC). By this time there is no longer any difference in prices between BC model and my model. Further, average price differential between my model and BC model ranges from 0 percent for perfect learning case (the correlation between earnings and MEGR is perfect) to -15.5% for zero-learning case (the correlation between earnings and MEGR is zero), with incomplete information price being lower on average. The lower stock price based on my incomplete-information model is corresponding to the extra risk premium on MEGR that investors demand implying that investors' uncertainty about MEGR should be compensated.

I find that the price differential between my model and that of BC, defined as pricing error, can persist for years even under perfect learning conditions. The more volatile MEGR is, the longer the persistence. I also show that fast mean-reversion speed of MEGR facilitates learning in that pricing errors are small in magnitude even after short learning process; while with low mean-reversion speed of MEGR, pricing errors are reduced substantially only after long learning process. Holding MEGR's volatility and mean-reversion speed constant, I find that there is a negative association between long-term pricing errors and degree of incompleteness of information environment as reflected by correlation between earnings and MEGR (in absolute value). For an extreme incomplete-information environment, such as one with zero correlation between earnings and MEGR, investors basically learn nothing about state variable MEGR from earnings. In this case, pricing errors are largest on average. Finally, I show that pricing errors still exist after long learning horizon (e.g., eight years) with precisely estimated MEGR as long as the information environment is incomplete. The non-vanishing pricing errors reflect residual risk premium (not present in the complete information model) due to investors' imperfect forecasts of the underlying state variable.

The remainder of the chapter is organized as follows. The next section discusses related literature. Section 3.3 extends the complete information stock valuation model by modeling investors' inference about an unobserved state variable. Section 3.4 compares risk premiums and prices in the incomplete and complete information models. Section 3.5 concludes the chapter.

3.2 Related Literature

Prior studies, such as Grossman and Shiller (1981), have found that the volatility of stock return is too high relative to the volatility of its underlying dividends and consumption.[2] The discrepancy between the high volatility of stock return and low volatility of dividends and consumption is viewed as the basic reason for the equity premium puzzle in recent work such as Campbell (1996) and Brennan and Xia (2001). To reconcile the discrepancy, learning about an unobservable state variable, such as the dividend growth rate, has been introduced to stock valuation (see, for example, Timmermann, 1993; Brennan, 1998; Brennan and Xia, 2001; Veronesi, 1999 and 2001, and Lewellen and Shanken, 2002).

Most of traditional stock valuation models neglect the learning process and implicitly assume that state variables for return predictability are known to investors (see, for example, Merton, 1971, and 1973; Samuelson, 1969, Breedon, 1979, and Bakshi and Chen, 2005). However there is substantial evidence indicating that market information is incomplete (see, for example, Faust, Rogers, and Wright, 2000; and Shapiro and Wicox, 1996). With an incomplete information set, investors may face an estimation risk because they are unable to observe many of state variables characterizing financial markets. This limitation is recognized by recent studies, (see, for example, Williams, 1977; Dothan and Feldman, 1986; Detemple, 1986; Gennotte, 1986; Timmerman, 1993; Brennan, 1997; and Feldman, 2007), which examine the role of learning with incomplete information in equilibrium.

For example, Timmermann (1993) provides a simple learning model, in which average dividend growth is unknown, to account for the fact that agents may not observe the true data-generating process for dividends. The model of Timmermann (1993) shows

[2] Among others, Brennan and Xia (2001) state that the standard deviation of real annual continuously compounded stock returns in the U.S. was 17.4 % from 1871 to 1996, while the standard deviation for dividend growth was only 12.9 %, and 3.44 % for consumption growth. Pastor and Veronesi (2009) document that the postwar volatility of market returns was 17% per year while volatility of dividend growth was 5%.

that dividend surprise affects stock price not only through current dividends but also through the effect on expected dividend growth rate, which also changes expected future dividends. The latter effect also explains why return volatility is much higher than that of dividend growth.

Instead of using price-to-dividend ratio (P/D), Pastor and Veronesi (2003) assume that M/B is the only observed state variable but its long term mean (a constant) is not. Their learning model predicts that the uncertainty of the estimate declines to zero hyperbolically. In the end, the case is identical to complete information. In a later study, Pastor and Veronesi (2006) calibrate their 2003 model to value stocks at the peak of the Nasdaq "bubble" in March 2000. They find a positive link between uncertainty about average dividend growth and the level and variance of stock prices. Pastor and Veronesi (2006) argue that the observed Nasdaq bubble is associated with the time-varying nature of uncertainty about technology firms' future productivity, and can be explained by learning model. Pastor and Veronesi (2009) extend Timmermann (1993) and show the positive association between the volatility of stock returns and its sensitivity to the uncertainty of average dividend growth.

The calibration of Pastor and Veronesi (2003) model to annual data from the CRSP/COMPUSTAT database shows that it takes about 10 years with learning to revert to complete information case under their parameter values. Further, once their model reverts back to complete information case, eventually there is no risk premium associated with uncertainty about latent state variable (mean of dividend growth rate). This result is the artifact of the long term mean being a constant (although unknown). In contrast, MEGR in my model is an additional state variable. Complete learning is impossible (except for perfect correlation cases) and therefore risk premium is non-zero at all times. The non-vanishing risk premium in my model reflects a persistent uncertainty that investors hold in an incomplete information environment. The greater risk premium on MEGR results in lower stock price as a compensation to investors for remaining uncertainty about the state variable.

In a more sophisticated framework, Brennan and Xia (2001) provide a dynamic equilibrium model of stock prices in which representative agents learn about time-varying mean of dividend growth rate. They claim that the non-observability of expected dividend growth demands a learning process which increases the volatility of stock prices. The calibration of their model matches the observed aggregate dividend and consumption data for the U.S. capital market. Unlike us, they assume a constant risk-less interest rate in their dynamic model. Similarly, Pastor and Veronesi (2003) do not model risk free rate as random. In contrast, my model incorporates a stochastic interest rate into a pricing-kernel process to discount future risky payoff. The dynamic interest rate is consistent with a single-factor Vasicek (1977) interest-rate process which makes the model arbitrage-free as in Harrison and Kreps (1979).

Bakshi and Chen (2005) derive an earnings-based stock valuation model which is directly related to my work. The model of Bakshi and Chen (2005) makes a more realistic assumption about the stochastic nature of risk-free interest rate. They adopt a stochastic pricing kernel process together with a mean-reverting process of earnings. Based on a sample of stocks and S&P 500 index, they show that the empirical performance of their model produces significantly lower pricing errors than existing models.[3]

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[3] However, the applicability of Bakshi and Chen (2005) model is limited to stocks with zero or negative earnings. To address this issue, Dong and Hirshleifer (2005) introduce an alternative earnings adjustment

In contrast to Bakshi and Chen (2005), in my model I recognize that the state variable, MEGR, is uncertain and subject to learning. In my model investors estimate MEGR based on earnings growth observations. My incomplete-information model shows that the uncertainty about MEGR declines exponentially over time. Complete information case of Bakshi and Chen (2005) is a special case of my model with perfect correlation between MEGR and earnings growth in the limit of very long learning horizons. In addition, in my model estimates of state variable are imprecise resulting in an incremental risk premium not present in complete information models.

3.3 A Generalized Earnings-Based Model with Incomplete-information

In this section, I introduce an incomplete-information stock valuation model, in which investors estimate the latent state variable, MEGR. I retain several desirable features in the BC model.

Assumption 1: *The basic building block for pricing is earnings rather than dividends.* $D(\tau)d\tau$ *is dividend-per-share paid out over a time period* $d\tau$*, and it is assumed to be equal, on average, to a fraction of the firm's earning-per-share (EPS), denoted by* $Y(\tau)$*, with white noise that is uncorrelated with the pricing kernel,*

$$D(t)dt = \delta Y(t)dt + dw_d(t), \tag{15}$$

parameter to the earnings process of BC model. The models of both Bakshi and Chen (2005) and Dong and Hirshleifer (2005) implicitly assume that information is complete about the mean of earnings growth rate. However, they do not recognize that the state variable, mean of earnings growth rate, is unobservable and has to be learned by observing realized earnings data.

where $0 \le \delta \le 1$, *which is a constant dividend-payout ratio, and* $dw_d(t)$ *is the increment to a standard Wiener process that is orthogonal to everything else.*[4]

The constant dividend-payout-ratio assumption is widely used in equity literature (eg. Lee et al. 1999; and Bakshi and Chen, 2005).[5] Consistent with Bakshi and Chen (2005), the inclusion of $dw_d(t)$ allows firm's paid dividend to randomly deviate from a fixed percentage of earnings. In practice, many firms do not pay cash dividends and therefore the implementation of dividend-based valuation model is limited (e.g., Gordon model and its variants).[6] To avoid this problem, the specification in equation (15) allows us to value stocks based on firm's earnings, instead of cash dividends directly.

Assumption 2: *Earnings growth is assumed to follow an arithmetic Brownian motion as follows:*

$$\frac{dY(t)}{Y(t)} = G(t)dt + \sigma_y dw_y(t). \tag{16}$$

MEGR, denoted by G(t), follows an Ornstein-Uhlenbeck mean-reverting process:

$$\begin{aligned} dG(t) &= k_g(\mu_g^0 - G(t))dt + \sigma_g d\omega_g(t) \\ &= k_g(\mu_g^0 - G(t))dt + \sigma_g(\rho_{gy} dw_y(t) + \sqrt{1-\rho_{gy}^2}\, dw_0(t)), \end{aligned} \tag{17}$$

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4 The white noise process of $dw_d(t)$ is uncorrelated with other variables, (eg., earnings growth, MEGR, risk-less interest rate, and pricing kernel), and therefore not a priced risk factor.

5 In practice, many aspects are exogenous (eg. firm's production plan, operating revenues and expenses, target dividend-payout-ratio) to net earnings process and any deviation from the fixed exogenous structure will affect the earnings process. To simplify the valuation of cash flow, Bakshi and Chen (2005) assume that the earnings process indirectly incorporates these aspects reflecting firm's investment policy and growth opportunities.

6 Fama and French (2001) find that, in recent years, many firms (especially technology firms) repurchase outstanding shares or reinvest in new projects with earnings, instead of paying cash dividends. As shown in the bottom panel of Figure 7 of their paper, the fraction of firms that pay no dividend rises from 27 percent in 1963 drastically to 68 percent in 2000. Similarly, while only 31 percent of firms neither pay dividends nor repurchase shares in 1971 (when repurchase data is available), the fraction grows to 52 in 2000.

where k_g, μ_g^0, σ_y, and σ_g *are constants, and* $dw_y(t)$ *and* $d\omega_g(t)$ *are increments to standard Wiener processes. Shocks to G(t), the MEGR, are correlated with shocks to EPS growth with an instantaneous correlation coefficient* ρ_{gy}. *The orthogonal part of* $d\omega_g(t)$ *is denoted by* $dw_0(t)$. *The long-term mean for* $G(t)$, *under the actual probability measure, is* μ_g^0, *and the speed at which* $G(t)$ *reverts to* μ_g^0 *is governed by* k_g.

The specification in equation (16) provides a link between actual EPS growth and expected EPS growth. Both EPS growth (actual and expected), as Bakshi and Chen (2005) analyze that, could be positive or negative reflecting firm's transition stages in its growth cycle. The mean-reverting process for expected EPS growth *G(t)* in equation (17) implies that any deviations of *G(t)* from its long-term mean μ_g^0 decline exponentially over time.

Assumption 3: *The pricing kernel follows a geometric Brownian motion, which makes the model arbitrage-free as in Harrison and Kreps (1979):*

$$\frac{dM(t)}{M(t)} = -R(t)dt - \sigma_m dw_m(t),$$

where σ_m *is a constant, and* $R(t)$ *is the instantaneous riskless interest rate.*

Assumption 4: *The instantaneous riskless interest rate,* $R(t)$, *follows an Ornstein-Uhlenbeck mean-reverting process:*

$$dR(t) = k_r(\mu_r^0 - R(t))dt + \sigma_r dw_r(t),$$

where k_r, μ_r^0 *and* σ_r *are constants. This process is consistent with a single-factor Vasicek (1977) interest-rate process.*

Shocks to earnings growth, denoted by $w_y(t)$ in equation (16), is correlated with systematic shocks $w_m(t)$ and interest rate shocks $w_r(t)$ with their respective correlation coefficients, denoted by ρ_{my} and ρ_{yr}. In addition, $w_g(t)$ is correlated with $w_m(t)$ and $w_r(t)$ with correlation coefficients ρ_{mg} and ρ_{gr}, respectively. Consistent with BC, both actual and expected EPS growth shocks are priced risk factors.

Following the BC model I consider a continuous-time, infinite-horizon economy with an exogenously specified pricing kernel, $M(t)$. For a firm in this economy, its shareholders receive infinite dividend stream $\{D(t): t \geq 0\}$ as specified in equation (15). The per-share price of firm's equity, P_t, for each time $t \geq 0$, is determined by the sum of expected present value of all future dividends, as given by

$$P_t = \int_t^{\infty} E_t[\frac{M(\tau)}{M(t)} D(\tau)] d\tau \,, \tag{18}$$

where $E_t(\cdot)$ is the time-*t* conditional expectation operator with respect to the objective probability measure.

Following assumptions 1 to 4, the equilibrium stock price at time *t* is determined by three state variables: *Y(t)*, *G(t)*, and *R(t)*. Note that, EPS and risk-less interest rate, *Y(t)* and *R(t)*, are observable at time *t*. However, the mean EPS growth, *G(t)*, is unobservable in any point of time in practice. Bakshi and Chen (2005) use analyst estimates as unobserved *G(t)* to implement their valuation formula, in which the uncertainty about estimates is neglected, and the associated risk premium is missing in asset prices. In contrast, I recognize the fact that investors cannot observe *G(t)* and have to learn it by observing available relevant information, such as earnings. The learning process in my model affects risk premium and equilibrium prices reflecting investors' uncertainty about estimates of *G(t)*. In the next subsection, I describe the dynamic learning process for the unobserved MEGR. The time-varying nature of uncertainty about estimates is explored as well.

3.3.a. Learning about unobserved MEGR

In practice analysts use past observations of EPS growth to build their forecasts of MEGR into the future. To be consistent with this observation I model the best (in the mean square sense) estimate of the unobserved MEGR as an expectation conditional on previous observations on earnings growth. Due to the Markovian nature of the model a representative agent takes as given the estimate of MEGR (Genotte, 1986; and Dothan and Feldman, 1986) when pricing assets.

Theorem 1: *Following standard results from one-dimensional linear filtering (see, for example, Liptser and Shiryaev, 1977 and 1978), the processes for* $Y(t)$ *and the MEGR estimate,* $\hat{G}(t)$*, based on the information set available to the agents, are given by*

$$\frac{dY(t)}{Y(t)} = \hat{G}(t)dt + \sigma_y dw_y^*,$$

$$d\hat{G}(t) = k_g(\mu_g^0 - \hat{G}(t))dt + \Sigma_t dw_y^*, \tag{19}$$

where $\Sigma_t = \dfrac{S(t)+\sigma_{gy}}{\sigma_y}$, $\sigma_{gy} = \rho_{gy}\sigma_y\sigma_g$, and $dw_y^* = \dfrac{1}{\sigma_y}\left(\dfrac{dY(t)}{Y(t)} - \hat{G}(t)dt\right)$. $S(t)$ *is the posterior variance of the agent's estimate of G(t) given earnings information accumulated until time t, which is defined as,* $S(t) \equiv E[(G(t)-\hat{G}(t))^2 \mid Y(t)]$. *If an initial forecast error variance is* $S(0)$*, S(t) is given by,*

$$S(t) = S_2 + \frac{S_1 - S_2}{1 - Ce^{\gamma(S_1 - S_2)t}}, \qquad \text{when } S(0) \in [S_1, \infty), \tag{20}$$

where $S_1 = \frac{-\eta}{2} + \sqrt{\frac{\eta^2}{4} - \alpha}$, $S_2 = \frac{-\eta}{2} - \sqrt{\frac{\eta^2}{4} - \alpha}$, $C = \frac{S(0) - S_1}{S(0) - S_2}$, $\alpha = -\sigma_g^2 \sigma_y^2 (1 - \rho_{gy}^2)$,

$\eta = 2\sigma_y^2 (\frac{\sigma_{gy}}{\sigma_y^2} + k_g)$, *and* $\gamma = -\frac{1}{\sigma_y^2}$.

Proof. See Appendix D.

The term dw_y^* represents an increment of the standard Wiener process given earnings information available to investors. σ_{gy} is an instantaneous covariance between the innovations in MEGR and earnings. *S(t)* quantifies the forecast error of $\hat{G}(t)$ reflecting the degree of information incompleteness. For example, *S(t)* of zero implies perfect knowledge of the underlying state variable.

Note that $\gamma < 0$ and $S_1 > S_2$. Hence, equation (20) implies that in the long run as more information becomes available, $S(t)$ declines and eventually converges to S_1, which is always nonnegative. In addition to S_1, another bound for $S(t)$ is denoted by S_2, which is always non-positive and lower than S_1. Therefore, S_2 is irrelevant to my analysis of the long-term value of $S(t)$. Nevertheless, S_2 is one of the parameters determining the speed of convergence of $S(t)$ to S_1.

Next, I change the parameters in SDE (19) to reflect the agent's information set:

$$d\hat{G}(t) = \left(k_g + \beta + \frac{S(t)}{\sigma_y^2} \right) \left[\hat{\mu}_g^0 - \hat{G}(t) \right] dt + \left(\frac{S(t)}{\sigma_y^2} + \beta \right) \frac{dY(t)}{Y(t)}, \tag{21}$$

where $\beta = \frac{\sigma_{gy}}{\sigma_y^2}$. Note that, under this representation of the process for the MEGR estimate, the speed of mean reversion is governed by $\left(k_g + \beta + \frac{S(t)}{\sigma_y^2} \right)$ and its long-term mean is given by $\hat{\mu}_g^0 = \dfrac{k_g}{k_g + \beta + \dfrac{S(t)}{\sigma_y^2}} \mu_g^0$. Since in the long run $S(t)$ converges to S_1, I define the long-run speed of mean reversion, k_g^*, as $k_g^* = \left(k_g + \beta + \dfrac{S_1}{\sigma_y^2} \right)$. Substituting for S_1 and rearranging the terms I get the following expression for the long-run speed of mean reversion: $k_g^* = \sqrt{(\beta + k_g)^2 + \dfrac{\sigma_g^2}{\sigma_y^2}(1 - \rho_{gy}^2)}$. The last expression for k_g^* is intuitive. In my model, investors learn about the true MEGR from historical changes in EPS. Specifically, investors update the latent mean growth rate based on an OLS-type relation between the "explanatory variable", $\frac{dY(t)}{Y(t)}$, and the "dependent variable", $d\hat{G}(t)$. This is very similar to the case of hedging a short position in an underlying asset with futures contracts. In both cases, the hedge ratio is the OLS slope coefficient, or β. In my model, β is the sensitivity of MEGR to the percentage change in EPS.

Note that β is an imperfect "hedge ratio" due to the less than perfect correlation in general between EPS and latent MEGR. Analogous to the case of hedging with futures, in my model this imperfect correlation translates into "basis risk" measured as $\frac{\sigma_g^2}{\sigma_y^2}(1 - \rho_{gy}^2)$, and serves as an adjustment for an imperfect forecast $\hat{G}(t)$. Another adjustment for the latent MEGR comes from parameter k_g, the strength of latent mean growth rate reversion towards its long-term mean. In the following propositions I consider two special cases for the correlation, ρ_{gy}, between EPS and the mean of earnings growth rate, MEGR.

Proposition 1.a: *When the correlation,* ρ_{gy}*, between EPS growth and MEGR is perfectly positive, the posterior error variance of MEGR estimate, S(t), declines with time and converges to zero, which suggests that complete learning is obtained eventually in this case.*

Proof: see Appendix D.

Proposition 1.b: *When the correlation,* ρ_{gy}*, between EPS growth and MEGR is perfectly negative, the posterior variance of the MEGR estimate, S(t), converges to* S_1*.* S_1 *could be either positive or zero, depending on the sign of* $(k_g + \beta)$*, which is the long-run speed of mean reversion for the latent MEGR in this case.*

Proof: see Appendix D.

The intuition behind Proposition 1 is that a perfect and positive correlation between earnings and MEGR eventually allows investors to estimate the true mean growth rate with perfect accuracy, which implies perfect learning. When the correlation is perfect negative, the learning is perfect as long as the speed of mean reversion of the true process for the mean growth rate, k_g, is not too small relative to the absolute value of β, which measures the relative variability of MEGR and EPS growth.[7] In other words, learning is perfect in this perfect-negative-correlation case as long as the long-run speed of mean reversion for the process of MEGR, k_g^*, is positive. I can think of this situation as interplay of two effects. First, absent uncertainty, mean reversion represented by k_g,

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[7] In this case, $\beta = -\frac{\sigma_g}{\sigma_y}$ for $\rho_{gy} = -1$.

implies an exponential decay of any initial forecast error facilitating learning in this case. The second effect, representing the inverse of the signal-to-noise ratio, $\frac{\sigma_y}{\sigma_g}$, counteracts learning due to noise in the latent variable. The signal is the volatility of EPS growth, and the noise is the standard deviation of MEGR. In this case, the signal is too weak (β is large in absolute value), and complete learning is not possible in the long run despite the perfect negative correlation. The long-run result is determined by relative magnitudes of k_g and β.

To illustrate Proposition 1, I demonstrate the evolution of the learning process for MEGR estimate, $\hat{G}(t)$, in an incomplete-information environment. By using Euler approximation, I discretize the continuous processes for EPS growth rate, *Y*, its true mean, *G(t)*, and its mean estimate, $\hat{G}(t)$, which are given by:

$$
\begin{gathered}
Y(t) = Y(t-1)\left(1 + G(t-1)\Delta t + \sigma_y \sqrt{\Delta t}\varepsilon_y\right), \\
G(t) = G(t-1) + k_g(\mu_g^0 - G(t-1))\Delta t + \sigma_g\left(\rho_{gy}\varepsilon_y + \sqrt{1-\rho_{gy}^2}\varepsilon_0\right)\sqrt{\Delta t}, \\
\hat{G}(t) = \hat{G}(t-1) + k_g(\mu_g^0 - \hat{G}(t-1))\Delta t + \left(\frac{S(t)}{\sigma_y^2} + \beta\right)\left(\frac{Y(t) - Y(t-1)}{Y(t-1)} - \hat{G}(t-1)\Delta t\right),
\end{gathered}
$$

where Δt is discrete time interval, which is set to be 1/12 for monthly observations. Parameters ε_y and ε_0 are independent random variables following standard normal distribution.

The base case parameter values are chosen to closely match the corresponding values of 20 technology stocks analyzed in Bakshi and Chen (2005).[8] In particular, I

[8] The 20 technology stocks used in Bakshi and Chen (2005) includes firms under ticker ADBE, ALTR, AMAT, CMPQ, COMS, CSC, CSCO, DELL, INTC, KEAN, MOT, MSFT, NNCX, NT, ORCL, QNTM, STK, SUNW, TXN and WDC.

assume the following annualized initial values: $Y(0)=2$; $G(0)=0.5$;[9] $\hat{G}(0)=0.2$; and $S(0)=0.5$. Further, base case parameter values are: $\delta = 4\%$, $k_g = 3$; $\mu_g^0 = 0.3$; $\sigma_y = 0.5$; $\sigma_g = 0.5$.[10] To examine a perfect learning case, I assume that EPS and its unobservable MEGR are negatively but perfectly correlated, that is $\rho_{gy} = -1$. In this case, $\beta = \frac{\sigma_{gy}}{\sigma_y^2} = -1$ and $k_g^* = (k_g + \beta) = 2$, corresponding to the case of Proposition 1.b. Based on these values, the lower bound for $S(t)$ is $S_1=0$ suggesting perfect learning in the long run.

Based on the base parameter values, I plot three processes in Figure 1: the process for the true MEGR, $G(t)$, the process for the MEGR estimate, $\hat{G}(t)$, and the process for the posterior variance of the estimate, $S(t)$. As time progresses, the MEGR estimate, $\hat{G}(t)$, converges to the true MEGR, $G(t)$, as expected in the complete learning case. At the same time, the forecast error variance of the estimate, $S(t)$, converges to its lower bound of $S_1=0$. Thus, all uncertainty about the MEGR estimate is eventually eliminated by learning.

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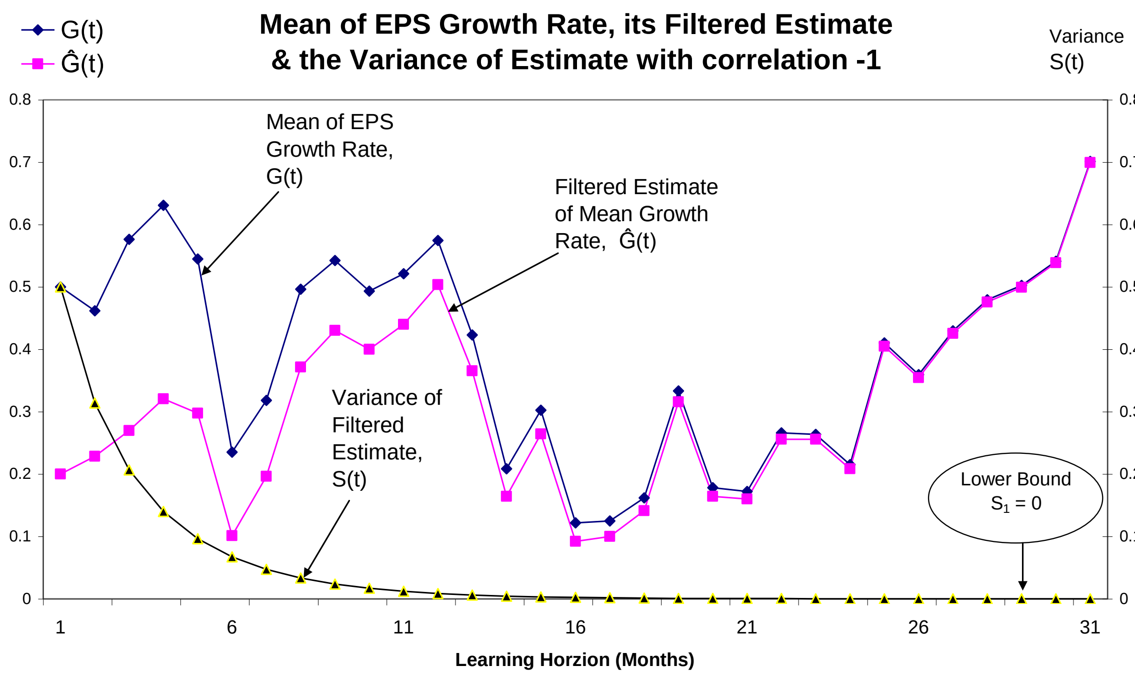
[9] Consistent with Table 1 of Bakshi and Chen (2005), in which the expected earnings growth (*G(t)*) is reported to be 0.4923 for 20 technology stocks.

[10] BC estimates the parameter values under the objective probability measure, which are given below for reference: $k_g = 2.688\,(0.485)$; $\mu_g^0 = 0.296\,(0.044)$; $\sigma_g = 0.425\,(0.083)$; $\rho_{yr} = -0.02\,(0.02)$; and $\delta = 4\%$. The market-implied estimate of σ_y is reported to be 0.345. The values in parentheses are cross-sectional standard errors. δ is obtained by regressing dividend yield on the earnings yield (without a constant). Average dividend divided by average net-earnings per share yields a similar δ. Note that throughout the empirical exercise, BC fixes two parameters to be that $\rho_{gy} = 1$, and $\rho_{gr} = \rho_{yr}$ to reduce estimation burden.

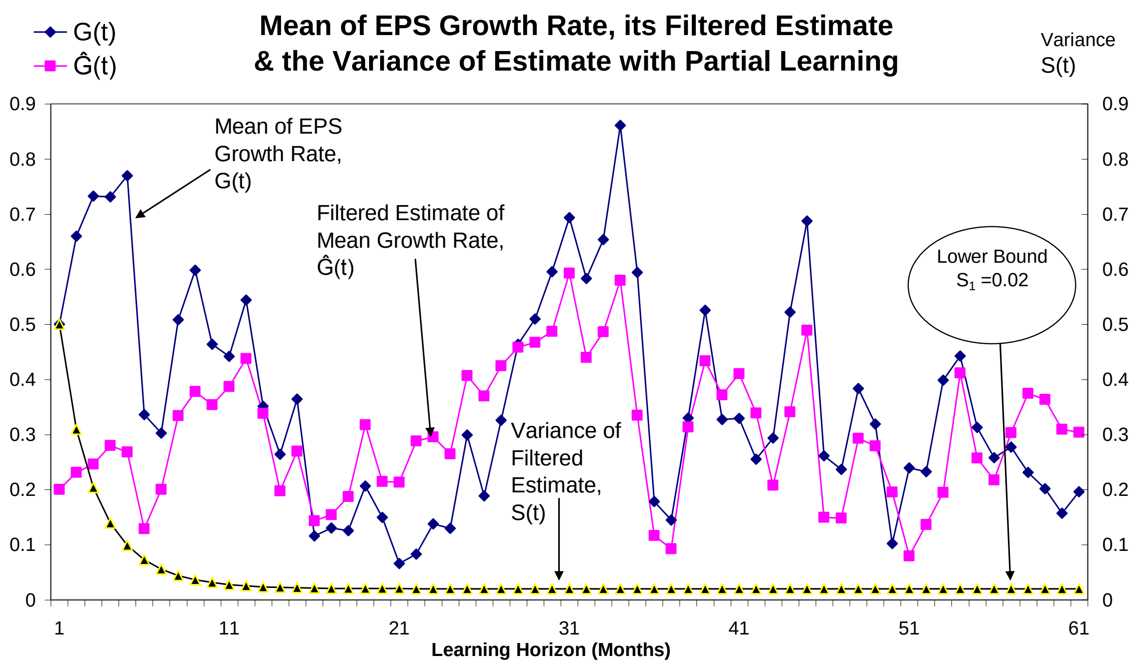
Figure 1: Case of Perfect Learning



In this figure I plot three processes: the process for the true MEGR, $G(t)$; the process for the MEGR estimate, $\hat{G}(t)$; and the process for the posterior variance of the estimate, $S(t)$. To generate the figure I assume the following initial values: $Y(0)=2$; $G(0)=0.5$; $\hat{G}(0)=0.2$; and $S(0)=0.5$. Parameters values for the assumed stochastic processes take the following values: $k_g = 3; \mu_g^0 = 0.3; \sigma_y = 0.5; \sigma_g = 0.5;$ and $\rho_{gy} = -1.$ Based on these values, the lower bound for $S(t)$ is $S_1=0$, which suggests that complete learning is obtained eventually.

Next, I consider the case of imperfect correlation. I assume that $\rho_{gy} = -0.8$, while maintaining all other parameters at the same base case level as used in Figure 1. Figure 2 shows that although the MEGR estimate, $\hat{G}(t)$, does not converge to the true mean growth rate, $G(t)$, the difference between the two decreases with time. At the same time, the forecast error variance of the estimate, $S(t)$, converges to its positive lower bound of $S_1 = 0.02008$.[11] Thus, investors can only partially learn about the true mean growth rate.

Figure 2: Case of Partial Learning



In this figure I plot three processes: the process for the true MEGR, $G(t)$; the process for the estimated MEGR, $\hat{G}(t)$; and the process for the posterior variance of the filtered estimate, $S(t)$. To generate the figure I assume the following initial values: $Y(0)=2$; $G(0)=0.5$; $\hat{G}(0)=0.2$; and $S(0)=0.5$. Parameters values for the assumed stochastic processes take the following values: $k_g = 3; \mu_g^0 = 0.3; \sigma_y = 0.5; \sigma_g = 0.5;$ and $\rho_{gy} = -0.8$. Based on these values, the lower bound for $S(t)$: $S_1 = 0.02008$.

The learning speed at which $S(t)$ converges to its long-run value S_1 is affected by the speed of mean reversion of MEGR, the volatilities of MEGR and EPS growth, and

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[11] Using $\rho_{gy} = -0.8$ along with the base parameter values in the formula $S_1 = \frac{-\eta}{2} + \sqrt{\frac{\eta^2}{4} - \alpha}$, where $\eta = 2\sigma_y^2(\frac{\sigma_{gy}}{\sigma_y^2} + k_g)$ and $\alpha = -\sigma_g^2\sigma_y^2(1-\rho_{gy}^2)$, I obtain that $S_1 = 0.02008$.

the correlation between them. From the solution for $S(t)$ in equation (20), the speed of its convergence, which I denote by K, is given by:

$$K = \left|\gamma(S_1 - S_2)\right| = 2k_g^*. \tag{22}$$

Recall that $k_g^* = \sqrt{(\beta + k_g)^2 + \frac{\sigma_g^2}{\sigma_y^2}(1-\rho_{gy}^2)}$. Note that β is a function of parameters σ_g, σ_y, and ρ_{gy}. In the following propositions, I examine the impact of these parameters on the speed of learning.

Proposition 2: *The learning speed at which the posterior forecast error variance $S(t)$ converges to its lower bound, S_1, increases in ρ_{gy}, the correlation between EPS growth and MEGR.*

Proof: see Appendix D.

The intuition behind Proposition 2 is that the information from EPS growth receives smaller weight if the correlation between EPS growth and its unobservable MEGR is smaller. In such case, learning the true MEGR from EPS data is slower.

Proposition 3: *The learning speed at which the posterior forecast error variance $S(t)$ converges to its lower bound, S_1, increases in k_g if $(k_g + \beta)$ is positive, where $\beta = \frac{\sigma_{gy}}{\sigma_y^2}$.*

Proof: see Appendix D.

Information about the true MEGR, $G(t)$, comes from two sources: (i) mean-reverting nature of the unobservable mean process; and (ii) continuous observations on change in EPS, $\frac{dY(t)}{Y(t)}$. Even in the absence of observations on earnings growth I know from equations (17) and (19) that regardless of the initial value of $\hat{G}(t=0)$, in the long term $\hat{G}(t)$ converges to the true MEGR, $G(t)$. The speed of this convergence is governed by k_g. A higher value of k_g means that $\hat{G}(t)$ will be close to its mean more often, making it easier to learn the value of the latter. However, investors' learning by observing actual EPS growth, $\frac{dY(t)}{Y(t)}$ can increase or decrease the speed of this convergence depending on the correlation between MEGR and earnings growth. If correlation between $\frac{dY(t)}{Y(t)}$ and $G(t)$ is negative and large enough in absolute value, learning may become slower simply because the updates of $\hat{G}(t)$ become less sensitive to new information, $\left(\frac{dY(t)}{Y(t)} - \hat{G}(t)dt\right)$.

3.3.b. The Valuation Equation

In this section I derive share price using standard SDE arguments based on stochastic discount factor (SDF) approach (see, e.g., Cochrane, 2005). The implicit assumption here is that any shock responsible for the difference between dividends $D(t)$ and $\delta Y(t)$ is not priced:

$$E_t^*\left[d\left(MP\right)\right] + M\delta Y dt = 0,$$

where operator E_t^* represents an expectation with respect to investors' information set.

Under standard assumptions (see Dothan and Feldman, 1986; Detemple, 1986; Gennotte, 1986; and Feldman, 2007), the equilibrium price at time t is given in the following form:

$$P(t, Y, \hat{G}(t), R) = \delta Y Z(t, \hat{G}(t), R), \quad \text{subject to } P(t) < \infty, \tag{23}$$

where δY represents dividends-per-share. The time-t price-dividend ratio, $Z(t, \hat{G}, R)$, is given below,

$$Z(t, s, \hat{G}, R) = \int_t^{\infty} \exp^{[\varphi(t,s)+\psi(t,s)\hat{G}(t)-\nu(t,s)R(t)]}\, ds, \tag{24}$$

which represents the expected present value of a continuous stream of future dividends arriving at a unit rate. The functions under the integral $Z(\hat{G}, R, t)$ have the following form (see Appendix E for details of derivation):

$$\begin{aligned} &\varphi(t,s) = \frac{1-e^{-k_g(s-t)}}{k_g}, \; \upsilon(t,s) = \frac{1-e^{-k_r(s-t)}}{k_r}, \text{ and} \\ &\psi(t,s) = -\lambda_y \tau + \int_t^s \left(\frac{\Sigma_t^2}{2}\varphi^2 + k_g \mu_g^* \varphi + \frac{\sigma_r^2}{2}\upsilon^2 - \rho_{gr}\sigma_r \Sigma_t \varphi \upsilon - k_r \mu_r^* \upsilon \right) du, \end{aligned} \tag{25}$$

where $\Sigma_t = \dfrac{S(t) + \sigma_{gy}}{\sigma_y}$, $\lambda_y \equiv \rho_{my}\sigma_m \sigma_y$ representing the risk premium for firm's earnings shocks, $\mu_r^* \equiv \mu_r^0 - \dfrac{\rho_{mr}\sigma_m \sigma_r - \rho_{yr}\sigma_y \sigma_r}{k_r}$ and $\mu_g^* \equiv \mu_g^0 - \dfrac{(\rho_{my}\sigma_m - \sigma_y)\Sigma_t}{k_g}$ are, respectively, the long-term means of $\hat{G}(t)$ and $R(t)$ under the risk-neutral probability measure defined by the pricing kernel $M(t)$. I denote $\lambda_g \equiv (\rho_{my}\sigma_m - \sigma_y)\Sigma_t$ as the risk premium for $\hat{G}(t)$ in my incomplete-information model.

For the integral in equation (24) to exist, the integrand should be declining with time *s* sufficiently fast. Since functions $\varphi(t,s)$ and $\upsilon(t,s)$ in equations (25) are bounded, this requirement implies that function $\psi(t,s)$ should be negative and unbounded at large time *s*. The latter restriction implies certain constraint on model parameters, called a transversality condition as given below (see Appendix E for proof):

$$-\lambda_y+\frac{\sigma_r^2}{2k_r^2}-\mu_r^*+\mu_g^0+\frac{(2\sigma_{gy}-\eta)S_1+\sigma_{gy}^2-\alpha}{2(k_g\sigma_y)^2}+\left((\sigma_y-\rho_{my}\sigma_m)k_r-\rho_{gr}\sigma_r\right)\frac{(S_1+\sigma_{gy})}{k_rk_g\sigma_y}<0.$$

(26)

In the following proposition I show that the risk premium on MEGR based on BC full-information model is only a special case of my model. Following BC, I define $\lambda_g^{BC}=\rho_{mg}\sigma_g\sigma_m-\sigma_{gy}$, as the risk premium on MEGR under BC complete-information model.

Proposition 4: *The magnitude of difference in risk premium on MEGR between my incomplete-information model and BC model is given by* $\Delta\lambda_g=\lambda_g-\lambda_g^{BC}=(\rho_{my}\sigma_m-\sigma_y)\dfrac{S(t)}{\sigma_y}+\sigma_g\sigma_m(\rho_{my}\rho_{gy}-\rho_{mg})$ *at time t. The difference in risk premiums declines with learning and converges to a long-run level equal to* $(\rho_{my}\sigma_m-\sigma_y)\dfrac{S_1}{\sigma_y}+\sigma_g\sigma_m(\rho_{my}\rho_{gy}-\rho_{mg})$. *When EPS growth and MEGR are perfectly correlated, the long-run difference in risk premium vanishes. Similarly, the risk-neutral long-term mean of MEGR, defined as* μ_g^* *in my model, converges to that of the complete-information (BC) model.*

Proof: see Appendix E.

A higher value of posterior variance $S(t)$ results in less precise pricing. As a result, stocks with higher $S(t)$ are considered relatively risky in the market. As $S(t)$ is reduced by learning, risk premium due to information incompleteness is reduced as well. The lower bound of posterior variance, S_1, determines the minimum level of information risk premium investors demand to compensate for the uncertainty in an incomplete-information environment.

In Figure 4, I demonstrate this result. I plot information-related risk premium on MEGR for firms with varying levels of correlation between EPS growth and MEGR: $\rho_{gy} = -1, \rho_{gy} = 0, \rho_{gy} = 0.5, \rho_{gy} = 1.$ Holding the other parameters constant, according to proposition 4, the only two special cases in which information-related risk premium on MEGR is zero in the long run are the cases of perfect correlation, $\rho_{gy} = 1$ and $\rho_{gy} = -1$. These are the instances in which complete learning is possible. The only difference between the two cases is that the curve of information-related risk premium for $\rho_{gy} = 1$ is much steeper than that for $\rho_{gy} = -1$ reflecting a quicker learning process. Note that the case of $\rho_{gy} = 0$ has the largest long-run risk premium. In fact, zero correlation implies that learning about MEGR is most difficult because the unobservable state variable is independent of available earnings observations. As a result, investors will demand the highest information-related risk premium on MEGR in the zero-correlation case among all cases with varying correlations.

3.4. Comparison of the Incomplete and Complete Information Models

In this section I examine the differences between my learning-based model and the complete-information (BC) model. The purpose of this section is to investigate the properties of my estimates of latent mean growth rate, examine how different firm characteristics affect the learning process, and compare the time series of price differentials in my incomplete-information model to those in complete-information model.

To simplify discussion, I assume deterministic risk-less interest rate, i.e., $\sigma_r = k_r = 0,\ \mu_r = r_f$ for both models. To understand the major differences between the two models, I focus on the difference in risk premium on MEGR and price difference in equilibrium which are functions of the parameter vector, $\Omega = \{k_g, \sigma_g, \rho_{gy}\}$ and learning horizons. The difference in risk premium is computed following Proposition 4. The per-share price in equilibrium with incomplete-information is computed following equations (23) to (25). The stock price with complete-information is computed based on the price formula in Bakshi and Chen (2005). Lastly, the pricing error in equilibrium between two models is defined as (Price with incomplete-information - BC price)/BC price, in percentage format.

Two issues are explored in this section. First, I examine the time series behaviour of risk premium difference based on varying parameter values. Next, I examine the dynamic change of percentage price errors observed at different learning horizons, such as short-term (4 months), intermediate-term (10 months), and long-term horizons (25 months), respectively, for varying parameter values.

In Figure 3, I plot three processes: the process for the risk premium based on true mean EPS growth rate, $G(t)$; the process for the risk premium based on the filtered mean growth rate, $\hat{G}(t)$; and the process for the variance of the filtered estimate, $S(t)$. To generate the figure I use similar base parameter values as used in Figure 1 and Figure 2 with minor adjustment, that is $\rho_{gy} = 1$. With perfect correlation between EPS and its MEGR, the lower bound for $S(t)$, given by S_1, is equal to zero. While complete information risk premium is flat at 4%, the risk premium based on MEGR estimate, $\hat{G}(t)$, is substantially higher than 4% during the initial period. As posterior variance of estimate $S(t)$ reaches its minimum (in this figure, the minimum bound S_1=0), the risk premium based on MEGR estimate, $\hat{G}(t)$, drops over time and reaches 4% in the long term limit. This figure suggests that the investors demand an extra risk premium to compensate their estimation risk due to incomplete-information. As learning progresses, the extra risk premium declines over time.

Figure 4 demonstrates the impact of change in the correlation between EPS and its MEGR on $\Delta\lambda_g$, the risk premium difference between my incomplete-information model and the complete-information model (BC). Holding other parameters constant, I change the correlation coefficient to be: $\rho_{gy} = -1$, $\rho_{gy} = 0$, $\rho_{gy} = 0.5$, and $\rho_{gy} = 1$, respectively. Based on these values, I compute the lower bound for $S(t)$ as given below: when $\rho_{gy} = -1$ or $\rho_{gy} = 1$, S_1= 0; when $\rho_{gy} = 0.5$, S_1=2.63%; and when $\rho_{gy} = 0$, S_1=4.05%. Following Proposition 4, I compute the difference of risk premium on MEGR based on my incomplete-information model and BC model. Figure 4 shows that when $\rho_{gy} = -1$ or $\rho_{gy} = 1$, both $\Delta\lambda_g$ decline and eventually converge to zero in agreement with propositions 1.a, 1.b, and 4. The minor difference between the two perfect learning cases $(\rho_{gy} = 1$ and $\rho_{gy} = -1)$ is in the speed at which $\Delta\lambda_g$ converges to zero. As demonstrated in Figure 4, for perfect positive correlation $(\rho_{gy} = 1)$, $\Delta\lambda_g$ declines much faster and converges to zero after nine months, while for $\rho_{gy} = -1$, it takes around sixteen months for $\Delta\lambda_g$ to converge to zero. This finding implies that with the same degree of learning (ρ_{gy} equals one in absolute value), extra risk premium for positive ρ_{gy} case diminishes much faster than that for negative ρ_{gy} case as corresponding posterior variance *S(t)* declines faster. Slower learning in the case of negative correlation reflects the conflict between the mean-reverting nature of the MEGR process and new information coming from earnings growth as described in Proposition 3. For partial learning case, I find that when $\rho_{gy} = 0.5$, risk premium difference $\Delta\lambda_g$ declines at a medium speed which is faster than that for $\rho_{gy} = -1$, but slower than that for $\rho_{gy} = 1$, in support of Proposition 2.

Note that in Figure 4, for $\rho_{gy} = 0.5$, $\Delta\lambda_g$ converges to 1.58%, which is not equal to zero any more, implying that partial learning process results in compensation for the fact that the posterior variance of estimate *S(t)* cannot be eliminated completely even for long-term learning horizons ($S_1 > 0$). For the case of $\rho_{gy} = 0$, $\Delta\lambda_g$ converges to 2.43%, which is the highest one among all of the risk premium differences in Figure 4. Note that the highest long-term $\Delta\lambda_g$ in this case is corresponding to its posterior variance of MEGR estimate equal to S_1=4.05% for $\rho_{gy} = 0$, which is largest among all of that in Figure 4 (S_1= 0 for both $\rho_{gy} = 1$ and -1; and S_1=2.63% for $\rho_{gy} = 0.5$). The presence of S_1

affects the risk-neutral drift of $\hat{G}(t)$ process and stock price in equilibrium reflecting the systematic nature of uncertainty about MEGR estimate.

Consistent with Proposition 4, the magnitude of S_1 positively affects the long-term magnitude of extra risk premium demanded by learning process. Recall that in Proposition 4, the long-term risk premium difference $\Delta\lambda_g$ is parameterized to be:

$\Delta\lambda_g = \lambda_g - \lambda_g^{BC} = (\rho_{my}\sigma_m - \sigma_y)\dfrac{S_1}{\sigma_y} + \sigma_g\sigma_m(\rho_{my}\rho_{gy} - \rho_{mg})$. I further find that the additional risk premium on MEGR, $\Delta\lambda_g$, declines faster with learning for firms with higher k_g, which governs mean-reversion speed. This result is demonstrated in Figure 5. Holding parameters at base case levels and $\rho_{gy} = 1$, I let the mean-reversion speed take three different values: $k_g = 2$, $k_g = 3$, and $k_g = 4$, respectively. For BC model, the risk premium on *G(t)* remains flat at 4% level regardless of mean-reversion speed. While for incomplete-information model, risk premium curves for each k_g start with different magnitude and declines at varying speed, but eventually converge to complete-information premium of 4% due to perfect learning. I see that before converging to its long run level, the risk premium on $\hat{G}(t)$ is highest for the case with the smallest speed of mean-reversion ($k_g = 2$), while lowest for the case with the largest speed of mean-reversion ($k_g = 4$). This phenomenon is in line with Proposition 3. In this case with $\rho_{gy} = 1$, learning speed, *K,* is positively correlated with k_g, implying that the uncertainty *S(t)* declines faster if MEGR reverts to its long-term mean at a larger speed. At the same time, the faster decline of *S(t)* is associated with a lower risk premium at the same point in time during learning process.

In addition to examining the impact of k_g on risk premium, I examine its impact on stock price in equilibrium as well. In Figure 6, I plot time series of pricing errors between my model and BC model in percentage terms with respect to, respectively, low speed, medium speed, and high speed of k_g. The mean-reversion speed of MEGR is assumed to be $k_g = 2$; $k_g = 3$; to $k_g = 4$, respectively, for each time series. I find that pricing errors are most volatile for low speed k_g, but small in magnitude and stable for high speed. This is consistent with my proposition 3, because the higher speed k_g implies larger learning speed *K*. For example, in Figure 6 when $k_g = 2$, the percentage pricing errors decline slowly until below 1% after 37 months of learning; when $k_g = 3$, the percentage pricing errors decline relatively fast until below 1% after 15 months of learning; while when $k_g = 4$, the percentage pricing errors decline faster to reach 1% only after 5 months of learning.

In Figure 7, I further examine whether the pricing errors decline faster with learning for firms with lower σ_g which implies a less noisy MEGR process. For comparison, I plot three time series of percentage pricing errors with respect to relatively low uncertainty $(\sigma_g = 0.5)$, medium uncertainty $(\sigma_g = 0.65)$, and high uncertainty $(\sigma_g = 0.8)$. I find that the magnitude of pricing errors is reduced more when MEGR is less volatile during the same learning horizon (e.g. 15 months). That is, the less uncertainty about MEGR, the smaller magnitude the percentage pricing error will decline to. This result follows from my proposition 3, in which I show that the learning speed K is inversely related to the level of σ_g. Intuitively, less noisy MEGR process makes learning easier and quicker to learn about it. Results in Figures 6 and 7 reveal that parameters σ_g and k_g have opposing effects on learning.

I also find that the effect of k_g on pricing errors is stronger for a young firm. Young firm is interpreted as a firm with short history of observations on earnings implying short learning horizon. Similarly, I find that prices are much less sensitive to learning horizon when k_g is large. These results are demonstrated in Figure 8 which presents the paths of pricing errors for k_g varying from a low level of 1.8 to a high level of 5.8, for short learning horizon (t=4 months), intermediate learning horizon (t=10 months), and long learning horizon (t=25 months), respectively. For relatively low k_g ranging from 1.8 to 3.0, pricing errors are most sensitive to learning horizon. For example, on average, pricing error for short learning horizon is around -8%, which is most volatile; pricing error for medium-learning-horizon is around -5%; and pricing error for long-learning-horizon is around -2%, which is lowest in absolute value but non-zero. For medium k_g ranging from 3.0 to 4.6, pricing errors for long learning horizon converge to zero, and pricing errors for the other two learning horizons are substantially lower than those with low k_g. For high k_g ranging from 4.6 to 5.8, pricing errors for both long and medium learning horizons are zero, on average, while producing pricing errors of -1% for short learning horizon. This phenomenon observed in Figure 8 reveals that large mean-reversion speed of MEGR facilitates learning in that pricing error is small in magnitude even after short learning process; while with low mean-reversion speed of MEGR, pricing errors are reduced substantially only after long learning process.

In Figure 9, I examine the impact of precision of MEGR $(1/\sigma_g)$ on pricing errors at different observation times. I make σ_g range from 0.80 to 0.48 in the direction of improving precision of MEGR process. Similar to Figure 8, I choose three observation times (learning horizons) for comparison, which are: t = 4 Months; t = 10 Months; and t = 25 Months. I find that for all three horizons the pricing errors decrease as σ_g declines in general. With a relatively low precision of MEGR (high σ_g ranging from 0.80 to 0.66), the pricing error for the long learning horizon varies around zero but does not vanish; the average pricing error for the medium learning horizon is -4%; and the pricing error for the short learning horizon varies widely and averages at -7%. In comparison, with a relatively high precision of MEGR (low σ_g ranging from 0.64 to 0.48), the pricing errors for the long learning horizon converge to zero, those for the medium learning horizon vary around zero, and decline substantially and approach zero for the short learning horizon. The pattern in Figure 9 suggests that high precision level of MEGR makes learning easier in that it facilitates in reducing pricing errors even in the short learning horizon case. Increasing precision of the MEGR process is equivalent to increasing its mean-reversion speed, k_g .

In Figure 10, I examine the impact of parameter ρ_{gy} on the long-term level of pricing errors with incomplete information. I assume that the estimated $\hat{G}(t)$ and the true $G(t)$ are the same to examine whether pricing error still exists in an incomplete information environment (e.g., $|\rho_{gy}| \neq 1$). Parameter ρ_{gy} determines how well investors can eventually learn about the state variable, MEGR. To see price variation as a function of learning environment I let the correlation take four different values: $\rho_{gy} = 0$; $\rho_{gy} = 0.5$; $\rho_{gy} = 0.9$; $\rho_{gy} = 1$. The sample period covers eight years (96 months). I find that for perfect correlation such as $\rho_{gy} = 1$, the pricing errors are largely around -10% at the beginning of learning horizon, but converge at zero over fourteen-month learning period. For non-perfect learning cases, the magnitude of long-term pricing errors for $\rho_{gy} = 0.9$ is 1.21%, increasing to 7.05% for $\rho_{gy} = 0.5$, and finally to 15.48% for $\rho_{gy} = 0$ (all numbers are in absolute value).

These findings in Figure 10 have two implications. First, there is a negative association between long-term pricing errors and degree of incompleteness of information environment as reflected by absolute value of ρ_{gy}. Intuitively, the magnitude of correlation between earnings and MEGR determines how well investors learn about MEGR and consequently how well they price as evidenced by the long-term level of pricing errors. Secondly, pricing errors still exist after long learning horizon (e.g., eight years) with precisely estimated $\hat{G}(t)$ as long as the information environment is incomplete.

Since long-term pricing errors never vanish in an imperfect learning environment, I examine whether faster learning affects the magnitude of long-term pricing errors. Following Proposition 3, faster learning can be achieved at higher mean-reversion speed, k_g. Figure 11 presents the relation between long-term pricing errors and mean-reversion speed k_g in an imperfect learning environment. To generate the figure, I assume that correlation $\rho_{gy} = 0.9$, and the mean-reversion speed k_g takes on the following values: $k_g = 2$, $k_g = 3$, and $k_g = 4$, respectively. The magnitude (absolute value) of long-term pricing error is 3.42% for $k_g = 2$, decreasing to 1.32% for $k_g = 3$, and again decreasing to 1.14% for $k_g = 4$. This result implies that larger speed of mean-reversion leads to a reduction in the magnitude of long-term pricing errors, holding the other parameters constant. As before, the long run pricing errors are not zero. Similar to the intuition imlied by Figure 10, the non-vanishing pricing errors reflect residual risk premium (not present in the complete information model) due to investors' imperfect forecasts of the underlying state variable.

Figure 3: Risk Premium on Estimate of MEGR in Perfect Learning Case

In this figure I plot three processes: the process for the risk premium based on true MEGR, $G(t)$; the process for the risk premium based on the estimated MEGR, $\hat{G}(t)$; and the process for the variance of the filtered estimate, $S(t)$. To generate the figure I assume the following initial values: $Y(0)=2$; $G(0)=0.5$; $\hat{G}(0)=0.2$; and $S(0)=0.5$. Parameters values for the assumed stochastic processes are given by: $k_g = 3$; $\mu_g^0 = 0.3$; $\sigma_y = 0.5$; $\sigma_g = 0.5$; $\sigma_m = 0.8$; $\rho_{gy} = 1$; $\rho_{my} = 0.1$; and $\rho_{mg} = 0.1$. Based on these values, I get the following lower bound for $S(t)$: $S_1= 0$.

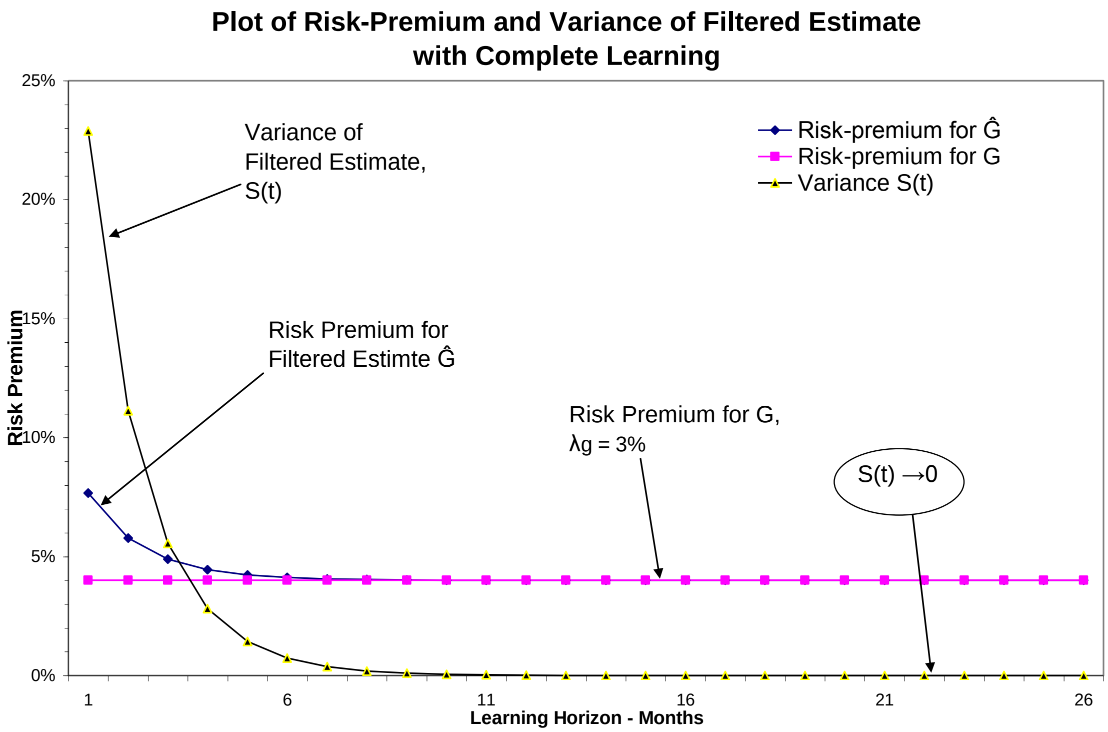

Figure 4: Risk Premium on Estimate of MEGR vs. Correlation

This figure demonstrates the curves of noise-related risk premium for four firms with different degree of correlation between EPS and its MEGR, holding the other parameters constant. The correlation is assumed to be: $\rho_{gy} = -1$, $\rho_{gy} = 0$, $\rho_{gy} = 0.5$, and $\rho_{gy} = 1$, respectively. To generate the figure I assume the following initial values for each firm: $Y(0)=2$; $G(0)=0.5$; $\hat{G}(0)=0.2$; and $S(0)=0.5$. Parameters values for the assumed stochastic processes take the following values: $k_g = 3$; $\mu_g^0 = 0.3$; $\sigma_y = 0.5$; $\sigma_g = 0.5$; $\sigma_m = 0.8$; $\rho_{my} = 1$; $\rho_{mg} = \rho_{my}\rho_{gy}$; $r = 3\%$; and $\delta = 4\%$. Based on these values, I obtain the following lower bounds for $S(t)$: when $\rho_{gy} = 1$ or -1, S_1= 0; when $\rho_{gy} = 0.5$, S_1=2.63%; and when $\rho_{gy} = 0$, S_1=4.05%. Let $\Delta\lambda_g$ denote the information-related risk premium on MEGR, I obtain the convergence level of noise-related risk premium for each firm: $\Delta\lambda_g(\rho_{gy} = -1 \text{ or } \rho_{gy} = 1) \to 0$; $\Delta\lambda_g(\rho_{gy} = 0.5) \to 1.58\%$; and $\Delta\lambda_g(\rho_{gy} = 0) \to 2.43\%$, respectively.

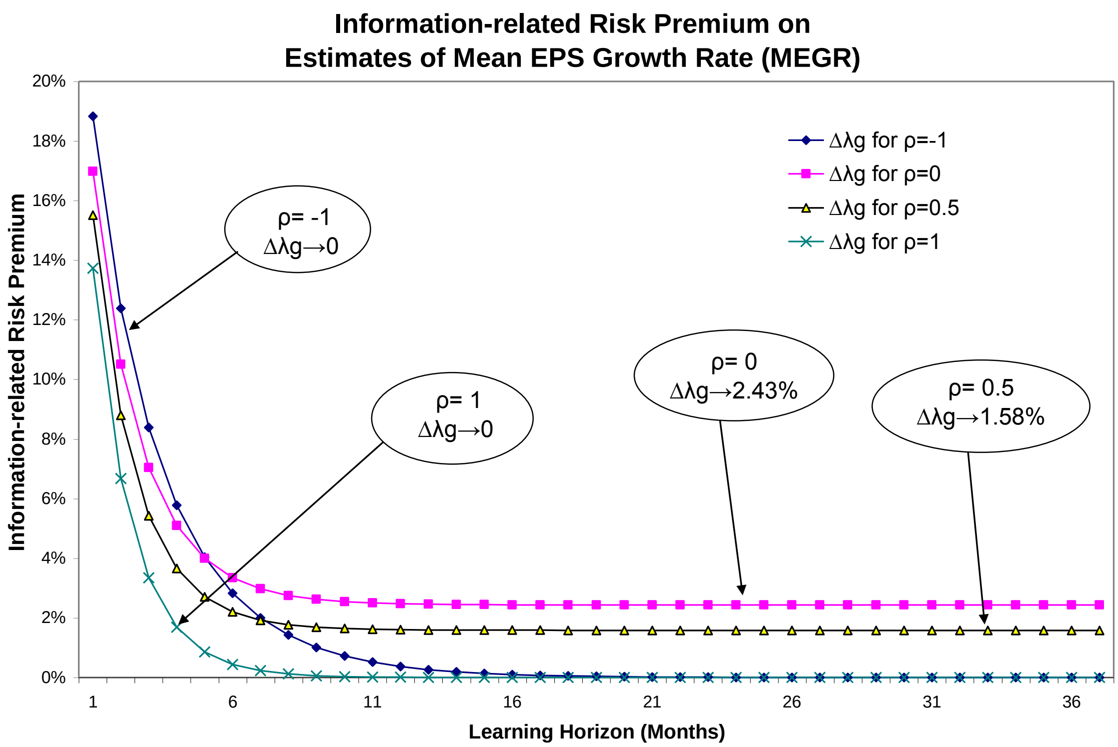

Figure 5: Risk Premium on Estimate of MEGR vs. Mean-reversion Speed of MEGR

In this figure, I examine the impact of change in mean-reversion speed (k_g) of MEGR on the risk premium under my incomplete-information model and the complete-information model. To generate the figure I assume the following initial values: $Y(0)=2$; $G(0)=0.5$; $\hat{G}(0)=0.2$; and $S(0)=0.5$. Parameters values for the assumed stochastic processes take the following values: $\mu_g^0 = 0.3$; $\sigma_y = 0.5$; $\sigma_g = 0.5$; $\sigma_m = 0.8$; $\rho_{gy} = 1$; $\rho_{my} = 0.1$; and $\rho_{my} = 0.1$. The speed of mean-reversion of MEGR is assumed to be, $k_g = 2$, $k_g = 3$, and $k_g = 4$, respectively. Based on these values, I obtain the following lower bound for $S(t)$: $S_1 = 0$. The constant risk premium on $G(t)$ under complete-information model is 4 per cent.

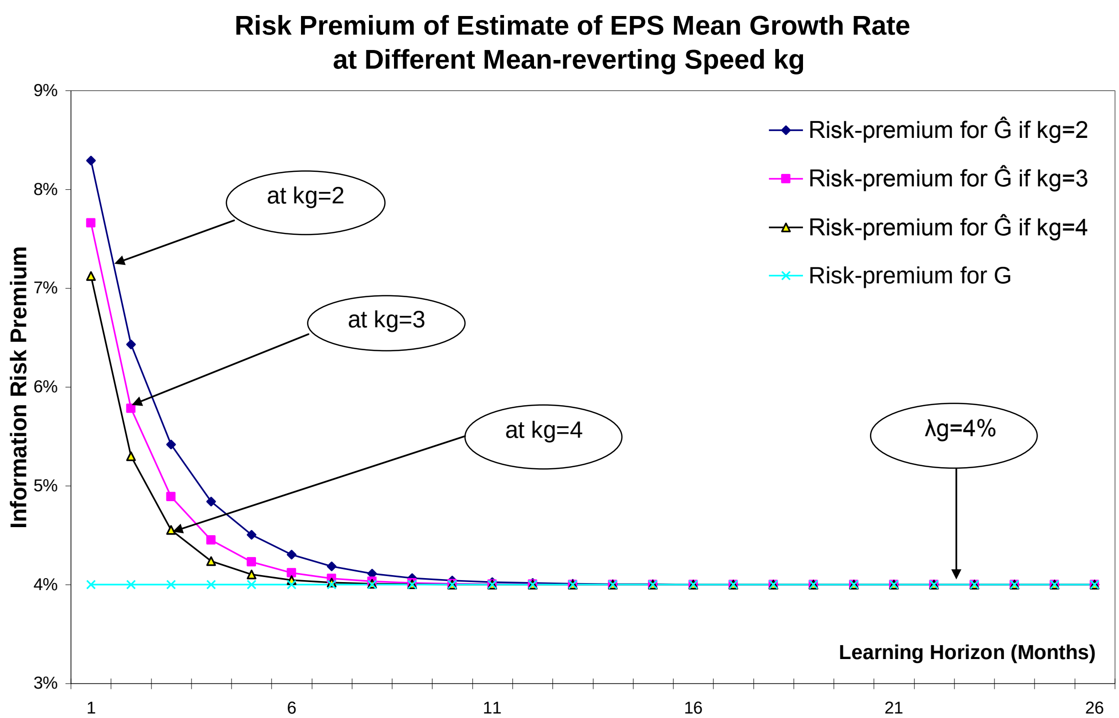

Figure 6: Percentage Pricing Errors vs. Mean-Reversion Speed of MEGR

In this figure, I examine the impact of change in mean-reversion speed (k_g) of MEGR on the pricing performance based on my incomplete-information model and the complete-information model. At each time during the sample period for each level of speed, k_g, prices are computed by the learning model respectively. Percentage pricing error is defined as the ratio of (Incomplete-Information model price – BC Complete-Information model price)/ BC Complete-Information model price**.** This chart show the time series of pricing errors for each level of speed, k_g. To generate the figure I assume the following initial values: Y(0)=2; G(0)=0.5; $\hat{G}$(0)=0.2; and S(0)=0.5. The mean-reversion speed of MEGR for each series is assumed to be, $k_g = 2$, $k_g = 3$, and $k_g = 4$, respectively. The other parameters for the assumed stochastic processes take the following values: $\mu_g^0 = 0.3$; $\sigma_y = 0.5$; $\sigma_g = 0.5$; $\sigma_m = 0.8$; $\rho_{gy} = -1$; $\rho_{my} = 1$; $\rho_{mg} = -1$; $r = 3\%$; and $\delta = 4\%$.

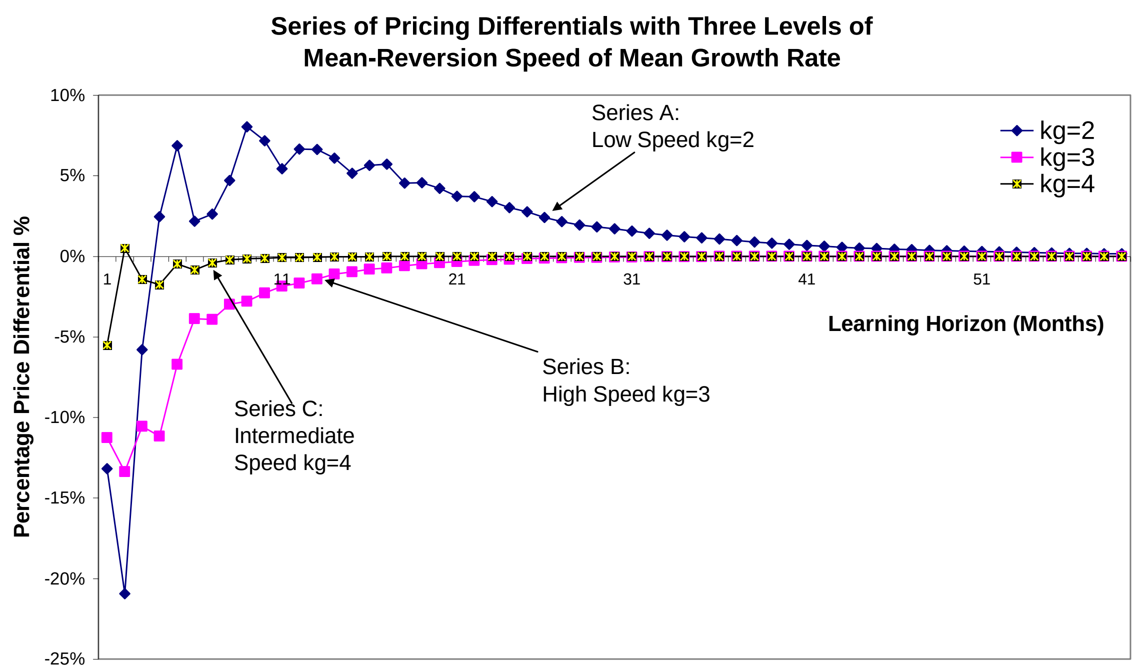

Figure 7: Percentage Pricing Errors vs. Volatility of MEGR

In this figure, I examine how the percentage pricing errors are influenced by the precision level of the MEGR. To generate the figure I assume the following initial values: $Y(0)$=2; $G(0)$=0.5; $\hat{G}(0)$=0.2; and $S(0)$=0.5. Parameters for the assumed stochastic processes take the following values: $k_g = 3;\ \mu_g^0 = 0.3;\ \sigma_y = 0.5;\ \sigma_g = 0.5;\ \sigma_m = 0.8;\ \rho_{gy} = -1;\ \rho_{my} = 1;\ \rho_{mg} = -1;$ r = 3%; and $\delta = 4\%$. I assume that the volatility of the mean EPS growth rate for each series are: $\sigma_y = 0.5, \sigma_y = 0.65,$ and $\sigma_y = 0.8,$ respectively. With each level of volatility, σ_y, percentage pricing errors are computed for each month during the sample period respectively. Percentage pricing error is defined as the ratio of (Incomplete-Information model price – BC Complete-Information model price)/ BC Complete-Information model price**.**

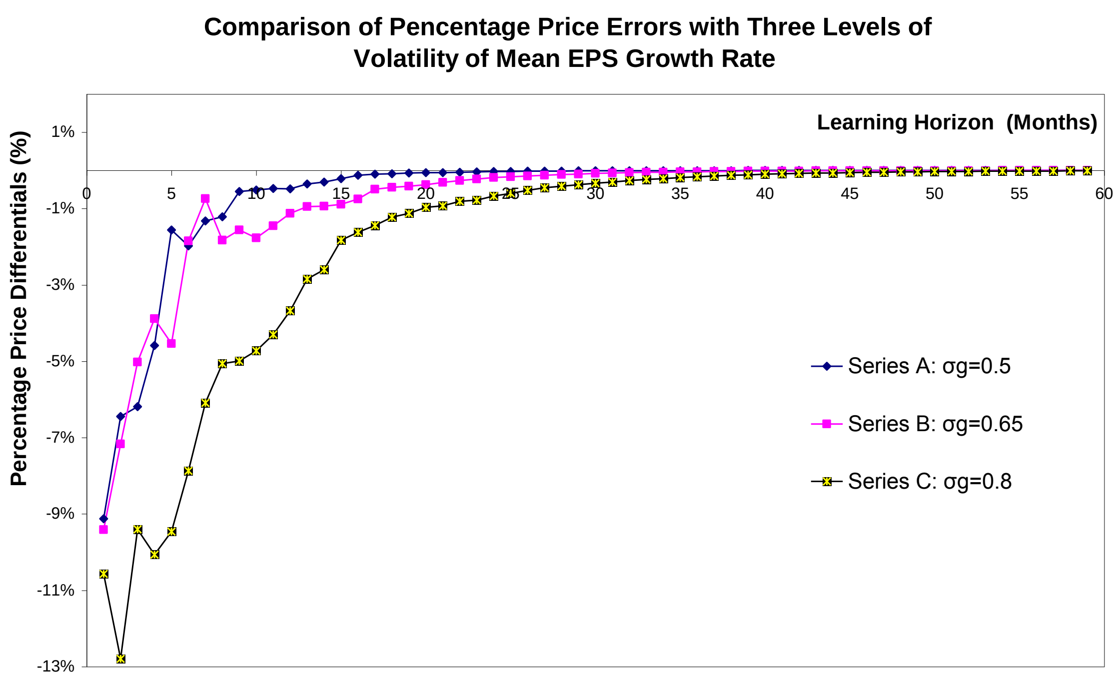

Figure 8: Learning Horizon vs. Mean-Reversion Speed of MEGR

In this figure, I observe the pricing errors (percentage) between my incomplete-information model and BC complete-information model by increasing the speed of MEGR. I focus on the pricing errors at three observation times (learning horizon), which are: t = 4 Months (short-learning horizon); t = 10 Months (intermediate-learning horizon); and t = 25 Months (long-learning horizon), respectively. Percentage pricing error is defined as the ratio of (Incomplete-Information model price – BC Complete-Information model price)/ BC Complete-Information model price**.** To generate the figure I assume the following initial values: $Y(0)$=2; $G(0)$=0.5; $\hat{G}(0)$=0.2; and $S(0)$=0.5. Parameters for the assumed stochastic processes take the following values: $\mu_g^0 = 0.3$; $\sigma_y = 0.5$; $\sigma_g = 0.5$; $\sigma_m = 0.8$; $\rho_{gy} = -1$; $\rho_{my} = 1$; $\rho_{mg} = -1$; $\mathrm{r} = 3\%$; and $\delta = 4\%$. The value of speed k_g increases from 1.8 to 5.8 gradually.

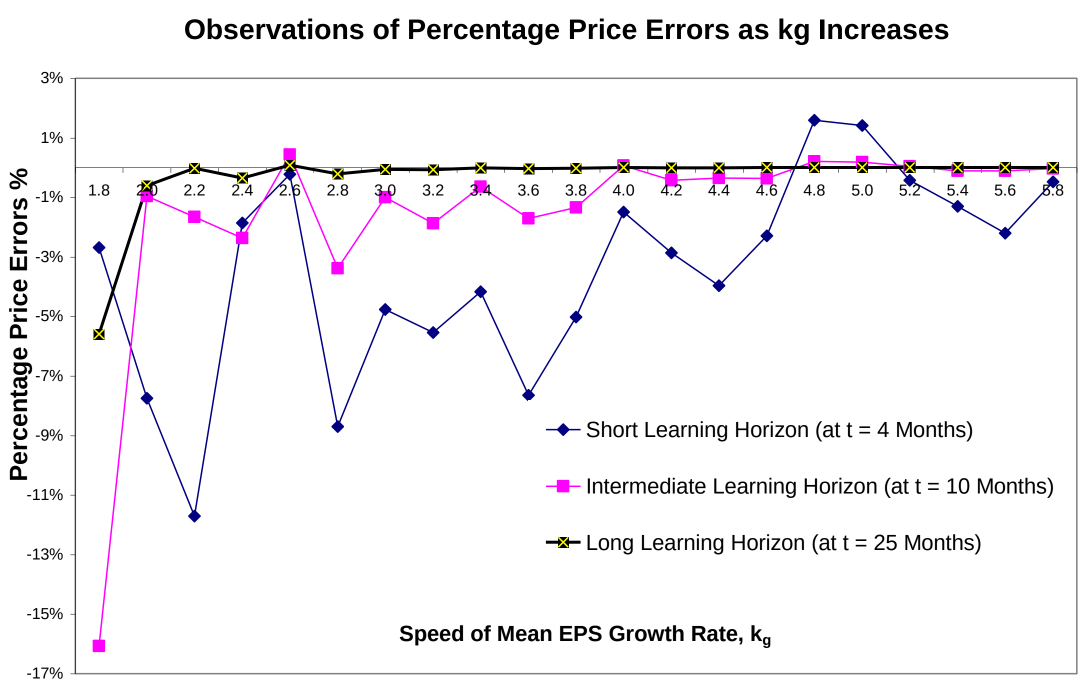

Figure 9: Learning Horizon vs. Volatility of MEGR

In this figure, I observe the percentage pricing errors by decreasing the volatility of MEGR. I focus on the pricing errors at three observation times (learning horizon), which are: t = 4 Months (short-learning horizon); t = 10 Months (intermediate-learning horizon); and t = 25 Months (long-learning horizon), respectively. Percentage pricing error is defined as the ratio of (Incomplete-Information model price – BC Complete-Information model price)/ BC Complete-Information model price**.** To generate the figure I assume the following initial values: $Y(0)=2$; $G(0)=0.5$; $\hat{G}(0)=0.2$; and $S(0)=0.5$. Parameters for the assumed stochastic processes take the following values: $k_g = 3$; $\mu_g^0 = 0.3$; $\sigma_y = 0.5$; $\sigma_m = 0.8$; $\rho_{gy} = -1$; $\rho_{my} = 1$; $\rho_{mg} = -1$; r = 3%; and $\delta = 4\%$. The volatility of MEGR decreases from 0.80 to 0.48 gradually.

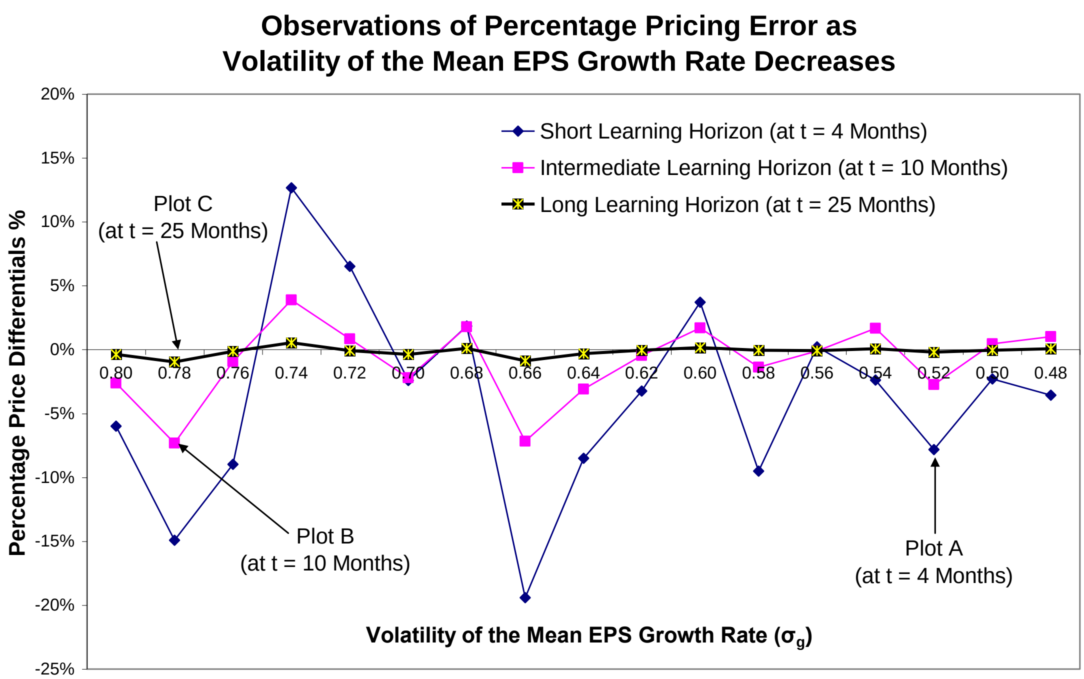

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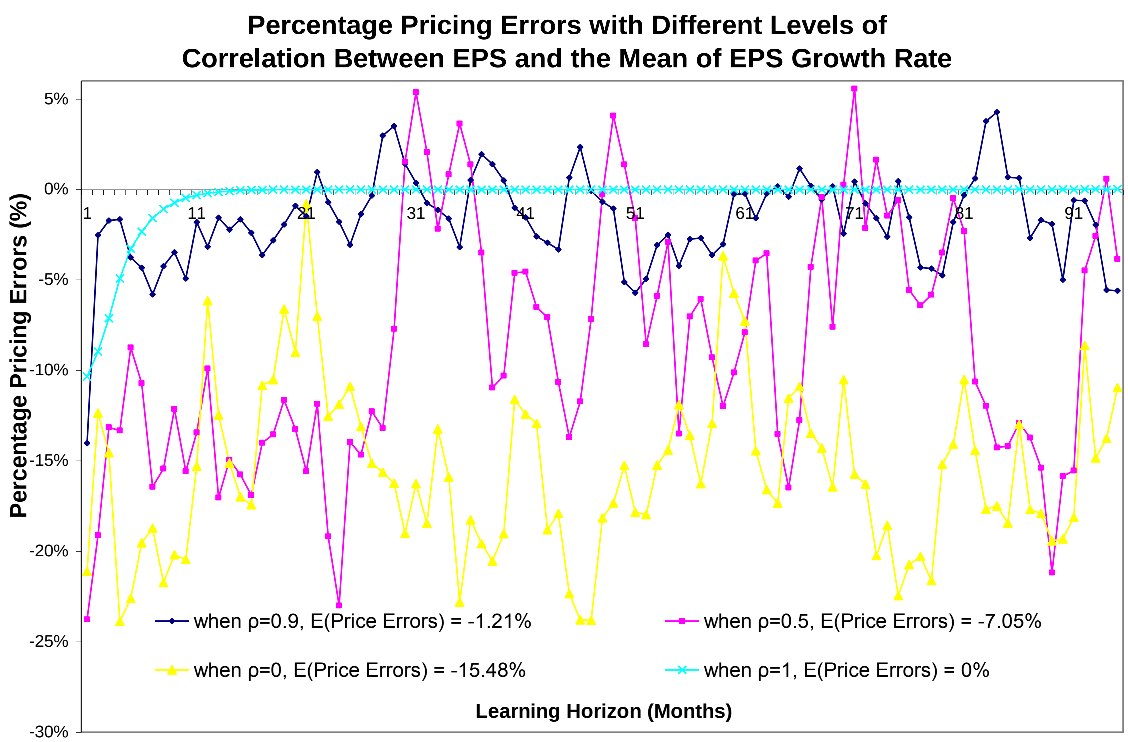
Figure 10: Long Term Mean of Percentage Pricing Errors vs. Correlation

In this figure, I examine that in an incomplete-information environment, how pricing errors are influenced by a parameter, ρ, the correlation between EPS and its MEGR. The value of parameter ρ determines the degree to which learning on the true MEGR can be achieved by using available data on EPS. I assume the correlation parameter ρ_{gy} to take the following four different levels: $\rho_{gy}=0$; $\rho_{gy}=0.5$; $\rho_{gy}=0.9$; and $\rho_{gy}=1$, respectively. The sample period covers eight years (96 months). Percentage pricing error is defined as the ratio of (Incomplete-Information model price – BC Complete-Information model price)/ BC Complete-Information model price**.** To generate the figure I assume the following initial values: $Y(0)=2$; $G(0)=0.5$; $\hat{G}(0)=0.2$; and $S(0)=0.5$. Parameters for the assumed stochastic processes take the following values: $k_g=3$; $\mu_g^0=0.3$; $\sigma_y=0.5$; $\sigma_g=0.5$; $\sigma_m=0.8$; $\rho_{my}=1$; $\rho_{mg}=\rho_{my}\rho_{gy}$; $r=3\%$; and $\delta=4\%$.



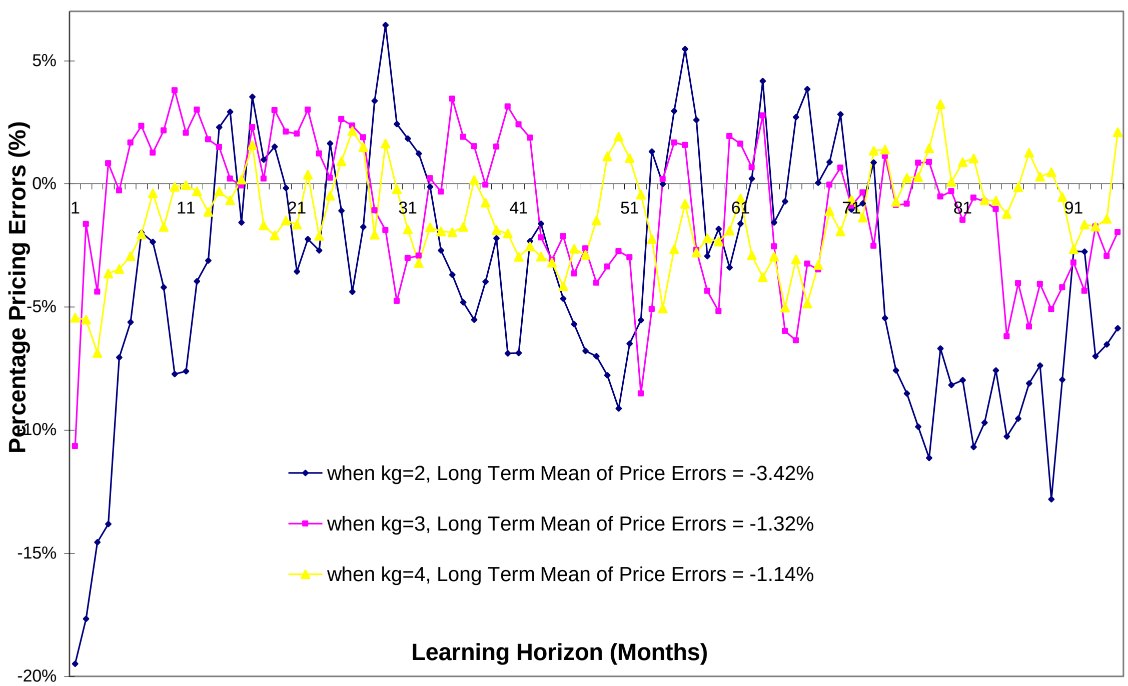
Sample of Data for Figure 10

Long Term Mean of Pricing Errors		Percentage Pricing Errors over Learning Horizon (Months) t=3	t=6	t=9	t=12	t=15	t=18	t=21	t=24	t=27	t=30
$\rho_{gy}=1$	**0%**	-0.071	-0.023	-0.007	-0.002	-0.001	0	0	0	0	0
$\rho_{gy}=0.9$	**-1.21%**	-0.017	-0.043	-0.035	-0.032	-0.017	-0.028	-0.015	-0.018	-0.003	0.014
$\rho_{gy}=0.5$	**-7.05%**	-0.131	-0.107	-0.121	-0.099	-0.158	-0.135	-0.156	-0.230	-0.123	0.015
$\rho_{gy}=0$	**-15.48%**	-0.145	-0.195	-0.202	-0.062	-0.170	-0.105	-0.008	-0.119	-0.151	-0.19

Figure 11: Long Term Mean of Percentage Pricing Errors vs. Mean-Reversion Speed of MEGR

In this figure, I examine that in an imperfect learning environment (eg., $\rho_{gy} = 0.9$), how long-term steady level of pricing errors are affected by boosting the speed of MEGR, k_g, to a higher level. The sample period covers eight years (96 months). Due to imperfect learning, pricing errors will decrease but never converge to zero regardless of the level of MEGR mean-reversion speed. But the long-term mean of pricing errors would sustain at a relatively lower level (at absolute value) with a higher speed k_g. To generate the figure I assume the following initial values: $Y(0)$=2; $G(0)$=0.5; $\hat{G}(0)$=0.2; and $S(0)$=0.5. The other parameters for the assumed stochastic processes take the following values: $\mu_g^0 = 0.3$; $\sigma_y = 0.5$; $\sigma_g = 0.5$; $\sigma_m = 0.8$; $\rho_{my} = 1$; $\rho_{mg} = 0.9$; r = 3%; and $\delta = 4\%$. The value of mean-reversion speed k_g is assumed to be: $k_g = 2$, $k_g = 3$, and $k_g = 4$, respectively.

Percentage Pricing Errors with Different Levels of Speed Kg



Sample of Data for Figure 11

Long Term Mean of Pricing Errors		Percentage Pricing Errors over Learning Horizon (Months) t=1	t=11	t=21	t=31	t=41	t=51	t=61	t=71	t=81	t=91
$k_g = 2$	-3.42%	-0.195	-0.076	-0.036	0.018	-0.069	-0.065	-0.016	-0.010	-0.080	-0.027
$k_g = 3$	-1.32%	-0.107	0.021	0.020	-0.030	0.024	-0.030	0.016	-0.009	-0.015	-0.032
$k_g = 4$	-1.14%	-0.054	-0.001	-0.017	-0.018	-0.030	0.010	-0.006	-0.007	0.009	-0.027

CHAPTER 4

Conclusions

Most models for return predictability assume that state variables are perfectly and precisely observable at any point in time with precise and complete information. In this dissertation I examine the effect of incomplete or noisy information on the risk premium and pricing performance based on traditional asset pricing models. In both essays, I show that an extra risk premium is demanded by investors due to the presence of imperfect information.

In Chapter 2, I examine the pricing of IQ risk. I present strong evidence showing that a mimicking IQ market factor is priced in a context of an IQ factor-adjusted Fama-French factor model with liquidity as a control variable. The evidence in favour of a market IQ factor is robust with respect to test portfolio formation and model specification.

Motivated by these results, I derive a continuous-time model, in the spirit of Merton's (1973) intertemporal model, in which information on state variables is continuously imprecise. This allows us to derive an analytical expression and examine the impact of systematic IQ risk on asset pricing.

With an imprecise information set, Merton's three-fund separation theorem still holds in my model except that the third portfolio is acting to hedge against exposure to IQ risk. In my IQCAPM, the market risk premium and hedge portfolio risk premium are boosted by μ_{ψ_m} and μ_{ψ_h}, respectively. These terms represent the expected IQ related return on these portfolios. Although the true return is obscured by the presence of imprecise information, an asset excess return is still linear in the excess returns on the market portfolio and that on a portfolio designed to hedge against unfavourable shifts in the stochastic investment opportunity set. I show that, unique to my IQ-adjusted model, imprecise information influences the pricing of an asset through its impact on its betas with respect to the two portfolios.

Based on my theoretical framework, I further show that IQ risk has systematic and idiosyncratic components and that only the former, which is nondiversifiable, is priced. This risk component of the IQ risk is priced because, even when an investor holds an individual security within the market portfolio, s/he still faces the systematic IQ risk that this security contributes to the IQ risk inherited in the market portfolio. Therefore, investors demand a higher premium for holding an asset with a higher systematic IQ risk.

I derive a static unconditional version of the IQCAPM to empirically test the validity of the model. The total systematic IQ risk affects asset returns through three IQ betas. The first IQ beta reflects the sensitivity of asset return relative to market-wide IQ. The second IQ beta manifests the relation between asset IQ noise and the market return. The last beta is a commonality IQ beta, reflecting the co-movement between individual portfolio IQ noise and market-wide IQ noise.

To test the validity of my static IQCAPM, I employ two alternative proxies for IQ based on firm-specific cash flow residual estimated using the model of Barth et al. (2001). The empirical result shows that the three IQ betas, particularly the last commonality in IQ beta, are significant in explaining variation in returns. This result is robust with respect to test-portfolio formation and remains significant after adjustment for Acharya and Pedersen's (2005) systematic liquidity risk.

In Chapter 3, I formulate a dynamic framework for valuing stocks which allows for learning about a stochastic but unobservable MEGR (mean of earnings growth rate) in an incomplete-information environment. The instantaneous MEGR is a state variable in my model, and investors can learn about it from continuously released earnings information.

I have shown in this chapter that the posterior variance of MEGR estimate generates extra risk premium on MEGR beyond what is accounted for in the complete information model. I further show that the time-varying nature of posterior variance of MEGR leads to a dynamic change in risk premium and more volatile stock prices. As learning reduces the posterior variance of estimate, extra risk premium declines to an equilibrium level over time. I parameterize the risk premium on MEGR and find that the time-varying magnitude of risk premium is not only affected by posterior error variance of estimate but also affected by firm characteristics, such as volatility of earnings, volatility of MEGR, mean-reversion speed of earnings, and correlation between earnings and the unobservable MEGR.

My results indicate that the faster the MEGR reverts to its long-term value, the smaller the magnitude of risk premium attributed to information incompleteness. This effect results from the fact that the higher speed of reversion towards the constant long-term mean leads to a faster exponential decay of any initial deviation from this mean and, therefore, faster learning. With a lower mean-reversion speed, risk premium on MEGR and posterior variance of MEGR estimate decline slowly but essentially constant over time if learning horizon is long enough. I also find that the effect of mean-reversion speed on pricing errors is stronger for a young firm with short history of information. By increasing the speed of mean-reversion, pricing errors due to information-incompleteness can be reduced substantially and quickly even learning horizon is short.

Lower volatility on MEGR has similar effect of higher effective speed of mean-reversion process of latent variable on learning. Both facilitate faster learning process about the true unobservable state variable, which is shown by the fast reduction in the posterior variance of MEGR estimate.

I have also shown that larger correlation (in absolute value) between earnings and latent MEGR leads to more complete learning about the true unobservable variable. With a perfect correlation (1 or -1), complete learning is achievable which leads to the same magnitude of risk premium and equilibrium prices in the long run as those in complete-information environment. In such case, the extra risk premium due to information-incompleteness vanishes eventually. In contrast, with an imperfect correlation (between -1 and 1), complete learning is impossible and therefore extra risk premium is non-zero at all times. The non-vanishing risk premium in my model reflects a persistent uncertainty that investors hold in an incomplete information environment. The additional long-term risk premium on MEGR results in lower equilibrium price as a compensation to investors for remaining uncertainty about the state variable.

My finding is consistent with that learning can generate higher equity premium when investors are ambiguity averse (e.g., Cagetti et al. 2002; Leippold et al. 2008; and Epstein and Schneider 2008). As Pastor and Veronesi (2009) predict that when investors are cautious of model misspecification in incomplete-information environment, model uncertainty is penalized and risk premium rises as compensation.

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Appendix

Appendix A – Results of Robustness Tests for Chapter 2

Table A1 - Robustness Test for the IQ Factor with Reverse Sorting

This table reports the coefficient estimates of the IQ factor-adjusted Fama-French factor model with liquidity as a control variable. At the beginning of each quarter from 1987 to 2007, eligible stocks are sorted first into 3 liquidity groups and then into 3 IQ category within the liquidity groups. GMM is used to estimate the coefficients for the following special relation:

$$r_{it} - r_{ft} = \alpha_i + b_i(r_{mt} - r_{ft}) + s_i SMB_t + h_i HML_t + q_i IQF_t + l_i LIQ_t + \varepsilon_{it},$$

where r_{it} is the return of portfolio *i* in quarter *t*, r_{ft} is three-month T-bill rate for quarter *t*, the values of $(r_{mt} - r_{ft})$, SMB_t, and HML_t are obtained from Kenneth French's website, LIQ_t is the market liquidity factor constructed following Pastor and Stambaugh (2003), and IQF_t is the mimicking-IQ-factor in quarter *t*. Panel A documents the results on 9 (3 Liquidity by 3 IQ) portfolios when IQ1 is used to proxy for IQ risk. Panel B documents the results on 9 (3 Liquidity by 3 IQ) portfolios when IQ2 is used to proxy for IQ risk. I denote the three IQ portfolios with IQH, IQM, and IQL, where: IQH is the high information-quality portfolio (with low levels of IQ1 or IQ2); IQM is the medium information-quality portfolio (with medium levels of IQ1 or IQ2); and IQL is the low information-quality portfolio (with high levels of IQ1 or IQ2). The *t*-statistic is reported in the parentheses. R^2 and adjusted-R^2 are documented as well.

Panel A: Nine Portfolios (IQ1 =| $e_{i,t}$ |)

Liquidity level	IQ level	α_i	b_i *(MKT)*	s_i *(SMB)*	h_i *(HML)*	q_i *(IQF)*	l_i *(LIQ)*	R^2
	IQH	0.010	0.904***	-0.174*	0.076	-0.435***	0.002	0.896
		(1.12)	(25.16)	(-1.89)	(0.95)	(-3.14)	(0.985)	(0.892)
Low	IQM	0.009	0.871***	-0.059	0.180	-0.073	-0.112	0.860
		(0.55)	(22.10)	(-0.45)	(1.42)	(-0.44)	(-0.87)	(0.855)
	IQL	0.009	0.877***	-0.104	0.071	0.507***	-0.008	0.845
		(0.65)	(16.18)	(-0.63)	(0.61)	(3.13)	(-0.05)	(0.840)
	IQH	0.007	1.017***	-0.117	0.264	-0.126	0.685***	0.889
		(0.61)	(21.69)	(-0.69)	(1.51)	(-0.80)	(4.21)	(0.884)
Medium	IQM	0.005	0.908***	-0.001	0.129	0.189	0.734***	0.863
		(0.73)	(21.67)	(-0.00)	(0.76)	(0.93)	(3.77)	(0.858)
	IQL	0.008	0.921***	-0.146	0.130	0.508**	0.916***	0.902
		(1.06)	(19.24)	(-1.15)	(1.11)	(2.59)	(7.70)	(0.898)
	IQH	0.013**	0.816***	-0.009	0.089	-0.460**	1.022***	0.897
		(2.24)	(15.26)	(-0.07)	(0.94)	(-2.26)	(8.03)	(0.893)
High	IQM	0.017	0.839***	0.425*	0.262**	0.509*	0.634**	0.838
		(0.59)	(18.51)	(1.71)	(2.30)	(1.72)	(2.36)	(0.832)
	IQL	0.010	0.855***	0.114	0.315***	0.212**	0.781*	0.894
		(0.71)	(16.57)	(0.59)	(3.30)	(2.30)	(1.65)	(0.889)

Panel B: Nine Portfolios (IQ2 = $\sigma(e_i)_t$)

Liquidity level	IQ level	α_i	b_i *(MKT)*	s_i *(SMB)*	h_i *(HML)*	q_i *(IQF)*	l_i *(LIQ)*	R^2
	IQH	0.010	0.905***	-0.045	0.122	-0.626***	-0.021	0.899
		(1.89)	(28.53)	(-0.49)	(1.29)	(-7.00)	(-0.23)	(0.895)
Low	IQM	0.015**	0.915***	0.025	0.132	-0.423***	-0.095	0.805
		(2.26)	(20.00)	(0.24)	(1.15)	(-2.83)	(-0.86)	(0.798)
	IQL	0.011	0.923***	-0.169*	-0.066	0.382***	-0.084	0.923
		(1.44)	(23.73)	(-1.77)	(-0.88)	(2.89)	(-0.89)	(0.920)
	IQH	0.008	0.979***	-0.049	0.212*	-0.656***	0.687***	0.905
		(0.86)	(22.67)	(-0.38)	(1.97)	(-5.33)	(6.30)	(0.902)
Medium	IQM	0.012	0.966***	0.075	0.050	-0.217	0.748***	0.883
		(0.58)	(15.50)	(0.60)	(0.37)	(-1.57)	(5.34)	(0.878)
	IQL	0.006	1.034***	0.005	0.101	0.171	0.678***	0.850
		(0.16)	(15.30)	(0.03)	(0.63)	(0.81)	(3.67)	(0.845)
	IQH	0.016**	0.846***	0.076	0.110	-0.649***	0.797***	0.899
		(2.25)	(18.19)	(0.76)	(1.60)	(-5.01)	(8.69)	(0.895)
High	IQM	0.009	0.856***	0.048	0.208**	0.145	1.088***	0.900
		(0.38)	(17.31)	(0.29)	(2.05)	(0.79)	(6.53)	(0.896)
	IQL	0.012*	0.845***	0.425**	0.202**	0.551***	0.668***	0.896
		(1.91)	(15.74)	(2.50)	(2.11)	(3.60)	(3.45)	(0.892)

Table A2 - Robustness Test for the IQ Factor for B/M portfolios

This table reports the coefficient estimates of the Information factor-adjusted Fama-French factor model with liquidity as a control variable. At the beginning of each quarter from 1987 to 2007, eligible stocks are sorted into 9 groups based on book-to-market value. GMM is used to estimate the coefficients for the following special relation:

$$r_{it} - r_{ft} = \alpha_i + b_i(r_{mt} - r_{ft}) + s_i SMB_t + h_i HML_t + q_i IQF_t + l_i LIQ_t + \varepsilon_{it},$$

where r_{it} is the return of portfolio *i* in quarter *t*, r_{ft} is three-month T-bill rate for quarter *t*, the values of $(r_{mt} - r_{ft})$, SMB_t, and HML_t are obtained from Kenneth French's website, LIQ_t is the mimicking liquidity factor constructed based on liquidity measure of Pastor and Stambaugh (2003), and IQF_t is the mimicking IQ factor in quarter *t*. The *t*-statistic is reported in the parentheses. R^2 and adjusted-R^2 are documented for each panel as well.

Panel A: Nine B/M Portfolios
(IQ1 = $|e_{i,t}|$)

B/M Portfolios	b_i *(MKT)*	s_i *(SMB)*	h_i *(HML)*	q_i *(IQF)*	l_i *(LIQ)*	R^2
1 (Lowest)	0.976***	-0.829***	-0.124	0.946***	0.332*	0.534
	(10.51)	(-4.27)	(-0.59)	(4.56)	(1.76)	(0.521)
2	0.945***	-0.318***	0.070	0.325**	0.272**	0.685
	(15.24)	(-2.74)	(0.48)	(2.28)	(2.18)	(0.676)
3	0.975***	-0.375**	0.138	0.007	0.438***	0.802
	(15.32)	(-2.29)	(1.40)	(0.04)	(2.81)	(0.796)
4	0.917***	-0.232	0.216	-0.201	0.349*	0.745
	(14.23)	(-1.31)	(1.21)	(-1.61)	(1.97)	(0.737)
5	0.897***	-0.238**	0.146	-0.012	0.441***	0.808

	(16.36)	(-2.05)	(1.49)	(-0.10)	(3.23)	(0.803)
6	0.984***	-0.065	0.244**	-0.415***	0.263*	0.809
	(18.54)	(-0.44)	(2.09)	(-2.80)	(1.69)	(0.803)
7	0.848***	0.443***	0.325***	-0.216	0.097	0.770
	(14.31)	(2.63)	(2.92)	(-1.25)	(0.51)	(0.763)
8	0.874***	0.332	0.371**	-0.609***	0.408*	0.706
	(9.22)	(1.43)	(2.22)	(-2.76)	(1.84)	(0.697)
9 (Highest)	0.936***	1.080***	0.093	-0.734*	0.041	0.484
	(5.73)	(3.31)	(0.40)	(-1.79)	(0.11)	(0.469)

Panel B: Nine B/M Portfolios
(IQ2 = $\sigma(e_i)_t$)

B/M Portfolios	b_i *(MKT)*	s_i *(SMB)*	h_i *(HML)*	q_i *(IQF)*	l_i *(LIQ)*	R^2
1 (Lowest)	1.007***	-0.833***	-0.099	0.359	0.919***	0.569
	(10.11)	(-3.46)	(-0.41)	(1.56)	(3.66)	(0.544)
2	0.927***	-0.388**	0.040	0.343**	0.370**	0.706
	(14.16)	(-2.60)	(0.23)	(2.26)	(2.38)	(0.688)
3	0.973***	-0.475**	0.063	0.544***	-0.049	0.812
	(13.77)	(-2.51)	(0.55)	(3.03)	(-0.27)	(0.800)
4	0.916***	-0.238	0.199	0.348*	-0.251*	0.748
	(12.41)	(-1.16)	(0.99)	(1.75)	(-1.75)	(0.732)
5	0.907***	-0.196	0.161	0.388*	-0.099	0.805
	(13.56)	(-1.31)	(1.22)	(2.30)	(-0.65)	(0.793)
6	0.963***	-0.132	0.165	0.330*	-0.487*	0.811
	(15.56)	(-0.75)	(1.16)	(1.81)	(-2.38)	(0.800)
7	0.830***	0.476**	0.326**	0.065	-0.184	0.768
	(12.45)	(2.33)	(2.27)	(0.29)	(-0.82)	(0.755)
8	0.851***	0.422	0.403**	0.296	-0.527*	0.707
	(7.48)	(1.62)	(2.04)	(1.25)	(-1.83)	(0.689)
9 (Highest)	0.880***	1.277	0.176	-0.184	-0.251	0.462
	(4.55)	(2.73)	(0.62)	(-0.35)	(-0.44)	(0.430)

Table A3 - Robustness Test for Testing of IQ Factor for Size Portfolios

This table reports the coefficient estimates of the IQ factor-adjusted Fama-French factor model with liquidity as a control variable. At the beginning of each quarter from 1987 to 2007, eligible stocks are sorted into 9 groups based on size (market capitalization). GMM is used to estimate the coefficients for the following special relation:

$$r_{it} - r_{ft} = \alpha_i + b_i(r_{mt} - r_{ft}) + s_i SMB_t + h_i HML_t + q_i IQF_t + l_i LIQ_t + \varepsilon_{it},$$

where r_{it} is the return of portfolio *i* in quarter *t*, r_{ft} is three-month T-bill rate for quarter *t*, the values of $(r_{mt} - r_{ft})$, SMB_t, and HML_t are obtained from Kenneth French's website, LIQ_t is the market liquidity factor constructed in Pastor and Stambaugh (2003), and IQF_t is mimicking-IQ-factor in quarter *t*. The *t*-statistic is reported in the parentheses. R^2 and adjusted-R^2 are documented for each panel as well.

Panel A: Nine Size Portfolios (IQ1 = $|e_{i,t}|$)

Size Portfolios	b_i *(MKT)*	s_i *(SMB)*	h_i *(HML)*	q_i *(IQF)*	l_i *(LIQ)*	R^2
1 (Smallest)	0.894***	0.600***	0.490***	0.602**	0.591***	0.868
	(17.73)	(4.61)	(5.60)	(2.32)	(4.34)	(0.864)
2	0.918***	0.363***	0.292***	0.263	1.014***	0.911
	(16.90)	(2.65)	(3.24)	(1.31)	(8.65)	(0.908)
3	0.965***	0.350**	0.333***	-0.204	0.786***	0.895
	(19.61)	(2.21)	(3.40)	(-0.85)	(5.25)	(0.892)
4	0.976***	-0.054	0.087	-0.158	1.093***	0.910
	(25.72)	(-0.43)	(0.80)	(-0.78)	(8.54)	(0.908)
5	0.956***	-0.101	0.051	0.168	1.054***	0.878
	(21.39)	(-0.84)	(0.44)	(0.73)	(7.77)	(0.874)
6	1.038***	-0.054	0.034	0.393**	0.777***	0.841
	(18.68)	(-0.37)	(0.17)	(2.02)	(4.96)	(0.836)
7	0.967***	-0.152	-0.092	0.376**	0.774***	0.848
	(25.43)	(-1.44)	(-0.55)	(2.43)	(6.93)	(0.844)
8	0.953***	-0.210	-0.033	0.209	0.615***	0.818
	(19.77)	(-1.44)	(-0.19)	(1.16)	(4.38)	(0.813)
9 (Largest)	0.963***	-0.666***	-0.039	0.155	0.390***	0.836
	(23.37)	(-5.22)	(-0.34)	(1.16)	(3.20)	(0.831)

Panel B: Nine Size Portfolios (IQ2 = $\sigma(e_i)_t$)

Size Portfolios	b_i *(MKT)*	s_i *(SMB)*	h_i *(HML)*	q_i *(IQF)*	l_i *(LIQ)*	R^2
1 (Smallest)	0.800***	0.671***	0.577***	0.677***	0.570***	0.845
	(14.91)	(5.50)	(8.03)	(4.95)	(4.32)	(0.871)
2	0.938***	0.387***	0.291***	0.018	1.017***	0.901
	(15.92)	(3.21)	(3.35)	(0.13)	(9.22)	(0.898)
3	0.971***	0.401***	0.373***	0.065	0.739***	0.889
	(19.14)	(2.62)	(3.75)	(0.35)	(4.79)	(0.886)
4	0.983***	0.042	0.139	0.063	1.008***	0.899
	(19.87)	(0.30)	(1.27)	(0.41)	(6.84)	(0.896)
5	0.950***	-0.040	0.090	0.144	1.011***	0.861
	(20.28)	(-0.32)	(0.82)	(0.90)	(6.71)	(0.857)
6	0.965***	-0.032	0.102	0.341***	0.782***	0.839
	(14.14)	(-0.25)	(0.59)	(2.87)	(5.45)	(0.834)
7	0.925***	-0.126	-0.109	0.233**	0.784***	0.843
	(18.36)	(-1.34)	(-0.71)	(1.98)	(7.87)	(0.839)
8	0.911***	-0.068	0.058	0.266***	0.488***	0.811
	(14.44)	(-0.56)	(0.37)	(2.65)	(4.02)	(0.806)
9 (Largest)	0.923***	-0.590***	0.021	0.304***	0.314***	0.825
	(18.71)	(-5.11)	(0.19)	(2.73)	(2.96)	(0.820)

Table A4 - Robustness Test for Asset Pricing Tests of the Static IQCAPM (Value-weighted Portfolios)

This table reports the coefficient estimates from cross-sectional regressions of the static IQCAPM for 25 value-weighted portfolios using quarterly data during 1987-2007 with a value-weighted market portfolio. I use GMM to obtain the coefficient estimates based on the following models,

CAPM $\quad E(\tilde{r}_t^{\,p} - r_{ft}) = \alpha + \lambda \beta^{Marketl}$,

(12.1) $\quad E(\tilde{r}_t^{\,p} - r_{ft}) = \alpha + \lambda \beta^{all}$,

(12.2) $\quad E(\tilde{r}_t^{\,p} - r_{ft}) = \alpha + \lambda_1 \beta^{Market} + \lambda_2 \beta_{net}^{IQ}$,

(12.3) $\quad E(\tilde{r}_t^{\,p} - r_{ft}) = \alpha + \lambda_1 \beta^{Market} + \lambda_2 \beta_1^{IQ} + \lambda_3 \beta_2^{IQ} + \lambda_4 \beta_3^{IQ}$,

where $\beta^{all} = \beta^{Market} + \beta_1^{IQ} + \beta_2^{IQ} + \beta_3^{IQ}$, and $\beta_{net}^{IQ} = \beta_1^{IQ} + \beta_2^{IQ} + \beta_3^{IQ}$. Panel A reports the results for 25 portfolios sorted on $IQ1$. Panel B reports the results on 25 IQ2-sorted portfolios. The R^2 and the adjusted-R^2 (in parentheses) are reported. The t-statistic is reported (in parentheses) as well.

Panel A: 25 Value-weighted IQ1 ($|e_{i,t}|$) Portfolios

	alpha	β^{Market}	β_1^{IQ}	β_2^{IQ}	β_3^{IQ}	β^{all}	β_{net}^{IQ}	R^2
CAPM	0.006	0.042***						0.610
	(1.05)	(6.90)						(0.593)
	0.046		-0.058					0.020
	(19.65)		(-0.82)					(-0.022)
	0.046			0.026				0.009
	(19.74)			(0.35)				(-0.034)
	0.041				0.066***			0.249
	(15.27)				(5.28)			(0.216)
12.1	0.005					0.038***		0.710
	(1.16)					(8.09)		(0.698)
12.2	0.006	0.038***					0.041***	0.711
	(1.06)	(6.17)					(4.10)	(0.685)
12.3	0.003	0.040***	0.086	0.019	0.044***			0.733
	(0.52)	(5.73)	(1.59)	(1.10)	(3.63)			(0.680)

Panel B: 25 Value-weighted IQ2 ($\sigma(e_i)_t$) Portfolios

	alpha	β^{Market}	β_1^{IQ}	β_2^{IQ}	β_3^{IQ}	β^{all}	β_{net}^{IQ}	R^2
CAPM	0.017***	0.033***						0.589
	(4.13)	(8.12)						(0.571)
	0.046		-0.059					0.020
	(19.65)		(-0.82)					(-0.022)
	0.046			0.025				0.008
	(19.74)			(0.35)				(-0.034)
	0.041				0.066***			0.249
	(15.27)				(5.28)			(0.216)
12.1	0.022***					0.027***		0.523
	(3.84)					(4.81)		(0.502)
12.2	0.015***	0.034***					-0.014*	0.597
	(3.44)	(7.42)					(-1.91)	(0.560)
12.3	0.016**	0.033***	-0.023	-0.084	0.218			0.598
	(2.73)	(4.78)	(-0.22)	(-0.46)	(0.35)			(0.518)

Table A5 - Robustness Tests for the Correlation Coefficients between Market Beta, Net IQ Beta, and Illiquidity Beta (Value-weighted Portfolios)

This table reports the Pearson correlations between market beta, net IQ beta, and net liquidity beta for 25 (5 IQ by 5 Liquidity) value-weighted portfolios. At the beginning of each quarter from 1987 to 2007, eligible NYSE/AMEX stocks are sorted first into 5 groups according to estimated IQ measure and then sorted again into 5 liquidity categories within each IQ group. The Pearson correlations between each of betas, portfolio size, and portfolio book-to-market ratio are documented for each panel. *p*-values are reported in parentheses.

Panel A: Value-weighted 25 (5 IQ1 by 5 Liquidity) Portfolios (IQ1 = $|e_{i,t}|$)

	β^{Market}	β_{net}^{IQ}	β_{net}^{ILLIQ}	Size	B/M
β^{Market}	1	0.199	-0.094	0.050	-0.418**
		(0.338)	(0.654)	(0.811)	(0.038)
β_{net}^{IQ}		1	0.087	0.355*	-0.054
			(0.676)	(0.081)	(0.797)
β_{net}^{ILLIQ}			1	0.565***	-0.405**
				(0.003)	(0.044)
Size				1	-0.679***
					(0.001)
B/M					1

Panel B: Value-weighted 25 (5 IQ2 by 5 Liquidity) Portfolios (IQ2 = $\sigma(e_i)_t$)

	β^{Market}	β_{net}^{IQ}	β_{net}^{ILLIQ}	Size	B/M
β^{Market}	1	0.232	-0.174	0.063	-0.327
		(0.265)	(0.404)	(0.762)	(0.110)
β_{net}^{IQ}		1	-0.265	-0.247	-0.006
			(0.201)	(0.234)	(0.976)
β_{net}^{ILLIQ}			1	0.415**	-0.243
				(0.039)	(0.242)
Size				1	-0.597***
					(0.002)
B/M					1

Table A6 - Robustness Test for Testing the Static IQCAPM with the Net Liquidity Beta (Value-weighted Portfolios)

This table reports the results for robustness tests based on 5 ILLIQ by 5 IQ, 25 IQ portfolios, 25 ILLIQ portfolios, 25 size portfolios and 25 B/M (book-to-market ratio) portfolios. The coefficient estimates from cross-sectional regressions of the IQCAPM are documented for value-weighted portfolios using quarterly data during the 1987-2007 period, while controlling for Acharya and Pedersen's (2005) net liquidity beta. GMM is used to estimate the coefficients from the following model:

$$E(\tilde{r}_t^{\,p} - r_t^{\,f}) = \alpha_0 + \lambda_1 \beta^{Market} + \lambda_2 \beta_{net}^{IQ} + \lambda_3 \beta_{net}^{ILLIQ},$$

where β^{Market} is market beta, β_{net}^{IQ} denotes the net IQ beta, and β_{net}^{ILLIQ} is the net illiquidity beta estimated following Acharya and Pedersen (2005). Results based on each of the two alternative IQ measure are given in separate panel. The R^2 is reported for each cross-sectional regression for different portfolios, and the adjusted-R^2 and *t*-statistic are reported in parentheses.

Panel A: Value-weighted Portfolios (IQ1 = $|e_{i,t}|$)

Portfolios	Alpha	β^{Market}	β_{net}^{IQ}	β_{net}^{ILLIQ}	R^2
5 IQ1 by 5 ILLIQ	0.003	0.031***	0.014	0.466***	0.582
	(0.43)	(3.91)	(0.82)	(-5.32)	(0.522)
5 ILLIQ by 5 IQ1	0.002	0.031***	0.016	-2.775***	0.595
	(0.28)	(5.40)	(1.12)	(-5.01)	(0.537)
25 IQ1 Portfolios	0.036	0.117	1.078***	30.045**	0.866
	(0.51)	(1.45)	(11.11)	(2.23)	(0.847)
25 ILLIQ Portfolios	0.034	0.007	-0.009	-1.051	0.210
	(2.51)	(0.56)	(-0.59)	(-1.29)	(0.097)
25 Size Portfolios	0.028	0.066	0.496**	16.611***	0.757
	(0.37)	(0.84)	(3.47)	(3.92)	(0.722)
25 B/M Portfolios	0.051	-0.008	-0.005	7.035***	0.516
	(1.34)	(-0.18)	(-0.72)	(4.51)	(0.447)

Panel B: Value-weighted Portfolios (IQ2 = $\sigma(e_i)_t$)

Portfolios	alpha	β^{Market}	β_{net}^{IQ}	β_{net}^{ILLIQ}	R^2
5 IQ2 by 5 ILLIQ	0.010	0.036**	0.006	-2.187	0.359
	(0.58)	(2.12)	(0.07)	(-1.67)	(0.267)
5 ILLIQ by 5 IQ2	0.025	0.016	0.175**	-2.992	0.555
	(1.70)	(1.39)	(2.11)	(-1.32)	(0.491)
25 IQ2 Portfolios	0.019**	0.023***	-0.053***	0.527	0.260
	(2.38)	(2.91)	(-2.84)	(0.20)	(0.154)
25 ILLIQ Portfolios	0.250**	-0.131	-0.275	20.622	0.246
	(2.43)	(-1.72)	(-1.12)	(1.05)	(0.138)
25 Size Portfolios	0.012	0.032**	-0.074*	0.716	0.264
	(0.89)	(2.16)	(-2.04)	(1.25)	(0.159)
25 B/M Portfolios	0.039	0.011	0.003	12.009***	0.557
	(1.02)	(0.26)	(0.10)	(4.36)	(0.493)

Appendix B – Construction of Liquidity Measures

Part B1 – The Liquidity Measure of Pastor and Stambaugh (2003)

This appendix provides the details for the construction of the Pastor and Stambaugh (2003) liquidity measure used in the preliminary test in Chapter 2. The aggregate liquidity measure is employed to describe overall market liquidity, which captures the price reaction to trading volume.

The liquidity measure in Pastor and Stambaugh (2003) for stock *i* in month *t* is the ordinary-least-square (OLS) estimate of $\gamma_{i,t}$ in the following regression model:

$$r^e_{i,d+1,t} = \theta_{i,t} + \phi_{i,t} r_{i,d,t} + \gamma_{i,t} \mathrm{sign}(r^e_{i,d,t}) \cdot v_{i,d,t} + \varepsilon_{i,d+1,t}, \quad d = 1, \ldots, D,$$

where:

$r_{i,d,t}$: the return on stock *i* on day *d* in month *t*,

$r^e_{i,d,t}$: $r_{i,d,t} - r_{m,d,t}$, where $r_{m,d,t}$ is the return on the CRSP value-weighted market return on day *d* in month *t*, and

$v_{i,d,t}$: the dollar volume for stock *i* on day *d* in month *t*.

Then the market-wide liquidity measure is constructed by $\hat{\gamma}_t = \frac{1}{N}\sum_{i=1}^{N} \hat{\gamma}_{i,t}$, where *N* ranges from 2,744 to 3,844 for each quarter from 1987 through 2007. The market-wide average liquidity is tested to be stationary.

The basic intuition behind the liquidity measure of Pastor and Stambaugh (2001) is that stocks tend to have larger $\gamma_{i,t}$ in absolute magnitude when liquidity is lower.

Part B2 – Liquidity Beta of Acharya and Pedersen (2005)

This appendix provides the details about construction of illiquidity betas used in Section 2.6 following the method of Acharya and Pedersen (2005). Similar to the aggregate IQ beta, the aggregate liquidity beta of Acharya and Pedersen (2005) consists of three specific liquidity betas as illustrated below,

$$\beta_{net}^{ILLIQ} = \beta_1^{ILLIQ} - \beta_2^{ILLIQ} - \beta_3^{ILLIQ},$$

where

$$\beta_1^{ILLIQ} = \frac{Cov(r_t^i, c_t^m - E_{t-1}(c_t^m))}{Var(r_t^m - E_{t-1}(r_t^m) - (c_t^i - E_{t-1}(c_t^i)))},$$

$$\beta_2^{ILLIQ} = \frac{Cov(c_t^i - E_{t-1}(c_t^i), r_t^m - E_{t-1}(r_t^m))}{Var(r_t^m - E_{t-1}(r_t^m) - (c_t^i - E_{t-1}(c_t^i)))},$$

$$\beta_3^{ILLIQ} = \frac{Cov(c_t^i - E_{t-1}(c_t^i), c_t^m - E_{t-1}(c_t^m))}{Var(r_t^m - E_{t-1}(r_t^m) - (c_t^i - E_{t-1}(c_t^i)))},$$

where c_t^i represents illiquidity cost for stock *i* at time *t*, c_t^m is the market illiquidity, and r_t^i and r_t^m stand for returns on stock *i* and on market portfolio, respectively.

To estimate the relative illiquidity cost for stocks, Acharya and Pedersen (2005) employ the return-to-volume measure of Amihud (2002), which capture the price-impact dimension of liquidity. The illiquidity of stock *i* in time *t* is defined as:

$$ILLIQ_t^i = \frac{1}{Days_t^i} \sum_{d=1}^{Days_t^i} \frac{\left|R_{td}^i\right|}{V_{td}^i},$$

where R_{td}^i and V_{td}^i are the return and dollar volume (in millions) on day *d* in quarter *t*, and $Days_t^i$ is the number of observation days in quarter *t* for stock *i*.

Appendix C - Proof of Theorem and Propositions of Chapter 2

Proof of Theorem 1: Substituting for $d\tilde{r}$ from equation (2) of Chapter 2 and using the budget constraint $\sum_{1}^{n+1} q_i = 1,$ I rewrite equation (3) as:

$$dW = \left[\sum_{i=1}^{n} q_i (\mu_i + k(\mu_{\psi_i} - \psi_i) - r_f) + r_f\right] Wdt + \sum_{i=1}^{n} q_i W \sqrt{\sigma_i^2 + \sigma_{\psi_i}^2 + 2\sigma_{i,\psi_i}}\, d\varpi_i - cdt. \qquad \text{(C1)}$$

where r_f is an exogenous interest rate on a risk-free bond. The assumption of constant risk-free rate in my model allows me to simplify my analysis and focus on the stock market.

The necessary instantaneous optimality condition for solving for an investor's consumption-investment optimal choice is as follows:

$$0 = \max_{c,q} [U(c,t)dt + J_t dt + J_W \mathrm{E}_t(dW) + \frac{1}{2} J_{WW} E(dW)^2 + \sum_{i=1}^{n} J_{\psi_i} \mathrm{E}_t(d(\psi_i)) \qquad \text{(C2)}$$

$$+ \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} J_{\psi_i \psi_j} E(d\psi_i d\psi_j) + \sum_{i=1}^{n} J_{W\psi_i} WE(dWd\psi_i) + o(dt)].$$

From equation (C2) I have the Gaussian process of wealth accumulation, thus the variance and covariance of the instantaneous change in wealth and the instantaneous change in the observable noisy return is given by:

$$E(dW) = \left[\sum_{1}^{n} q_i (\mu_i + \mu_{\psi_i} - r_f) + r_f - c\right] Wdt, \qquad \text{(C3)}$$

$$E(dW)^2 = \sum_{1}^{n} \sum_{1}^{n} q_i q_j W^2 (\sigma_{ij} + \sigma_{\psi_i \psi_j} + \sigma_{i\psi_j} + \sigma_{j\psi_i}) dt, \qquad \text{(C4)}$$

$$E(d\psi_i d\psi_j) = \sigma_{\psi_i} \sigma_{\psi_j} \rho_{\psi_i \psi_j} dt, \qquad \text{(C5)}$$

$$E(dWd\psi_j) = \sum_{i=1}^{n} q_i W (\rho_{i,\psi_j} \sigma_i + \sigma_{\psi_j}) dt. \qquad \text{(C6)}$$

Substitute (C3), (C4), (C5) and (C6) into (C2), I get the following equation,

$$
\begin{aligned}
0 = \max[&U(c,t) + J_t + J_W[(\sum_1^n q_i(\mu_i + \mu_{\psi_i} - r_f) + r_f)W] \\
&+ \frac{1}{2} J_{WW} \sum_1^n \sum_1^n q_i q_j (\sigma_{ij} + \sigma_{\psi_i \psi_j} + \sigma_{i\psi_j} + \sigma_{j\psi_i}) W^2 \\
&+ \frac{1}{2} \sum_1^n \sum_1^n J_{\psi_i \psi_j} \sigma_{\psi_i} \sigma_{\psi_j} \rho_{\psi_i \psi_j} \\
&+ \sum_1^n \sum_1^n J_{W\psi_j} q_i W(\rho_{i,\psi_j} \sigma_i + \sigma_{\psi_j})
\end{aligned}
\tag{C7}
$$

The n+1 first-order conditions for each investor derived from (C7) are given by:

$$
0 = U_c(c,t) - J_W(W,t,\psi), \tag{C8}
$$

$$
\begin{aligned}
0 = &J_W(\mu_i + \mu_{\psi_i} - r_f)W + J_{WW} \sum_{j=1}^n q_j W^2 (\sigma_{ij} + \sigma_{\psi_i \psi_j} + \sigma_{i\psi_j} + \sigma_{j\psi_i}) \\
&+ \sum_{j=1}^n J_{W\psi_j} W(\rho_{i,\psi_j} \sigma_i + \sigma_{\psi_j}),
\end{aligned}
\tag{C9}
$$

$\forall\ i = 1,2,...n,$ where $c^* = c(W,t,\psi),\ q_i^* = q_i(W,t,\psi)$ are optimal solutions for (C8) and (C9) as functions of the perceived state variables.

Proof of Theorem 2: Equations (C8) and (C9) give the optimal weights (demand) for assets i in the presence of noisy information. Portfolio weight q^* in (C9) is the combination of the tangency (market) portfolio with a hedge portfolio, denoted by h. This last portfolio hedges against IQ risk, which causes the unfavourable changes in the fundamental return of assets in the investment opportunity set. I solve for equilibrium market weights for risky assets from (C7) as follows:

$$
a^k(\mu + \mu_\psi - r_f I) = \Sigma q^k W^k - b^k \vec{\sigma}\ , \tag{C10}
$$

where $a^k = -\frac{J_W}{J_{WW}}$ and $b^k = -\frac{J_{W\psi}}{J_{WW}}$, for investor k, k = 1, 2,…, K. Summing across the K investors and dividing by $\sum_1^K a^k$, I obtain: $(\mu + \mu_\psi - r_f I) = A\Sigma x - B\vec{\sigma}$, and

$$\mu_i + \mu_{\psi_i} - r_f = A\sigma^*_{im} - B\sigma_{i\psi}, \tag{C11}$$

where $A = \frac{\sum_1^K W^k}{\sum_1^K a^k}$ and $B = \frac{\sum_1^K b^k}{\sum_1^K a^k}$, $x = \frac{\sum_1^K q^k W^k}{\sum_1^K W^k}$. x is the vector of equilibrium market weights for risky assets, and $\sigma^*_{im} = \sigma_{im} + \sigma_{\psi_i\psi_m} + \sigma_{i\psi_m} + \sigma_{m\psi_i}$. Multiplying both sides of equation (C11) by the transpose of x, I get: $x^{'}(\mu + \mu_\psi - r_f I)) = x^{'} A\Sigma x - x^{'} B\vec{\sigma}$. Given that x is the vector of market portfolio weights in equilibrium, $\mu_{\psi_m} = \sum_{i=1}^{n} x_i \mu_{\psi_i}$, I can write equation (C11) for the market portfolio:

$$\mu_m + \mu_{\psi_m} - r_f = A\sigma^{*2}_m - B\sigma_{m\psi}. \tag{C12}$$

Similar to the construction of equation (4.5) for the market portfolio, I first write: $h^{'}(\mu + \mu_\psi - r_f I) = h^{'} A\Sigma x - h^{'} B\vec{\sigma}$. Defining $\mu_{\psi_h} = \sum_{i=1}^{n} h_i \mu_{\psi_i}$ and $\psi_h = \sum_{i=1}^{n} h_i \psi_i$, I obtain:

$$\mu_h + \mu_{\psi_h} - r_f = A\sigma^*_{mh} - B\sigma_{h(\psi)}. \tag{C13}$$

Solving for A and B from (C12) and (C13), I have:

$$A = \frac{(\mu_m + \mu_{\psi_m} - r_f)\sigma_{h\psi} - (\mu_h + \mu_{\psi_h} - r_f)\sigma_{m\psi}}{\sigma^{*2}_m \sigma_{h\psi} - \sigma^*_{mh}\sigma_{m\psi}},$$

$$B = \frac{(\mu_m + \mu_{\psi_m} - r_f)\sigma^*_{mh} - (\mu_h + \mu_{\psi_h} - r_f)\sigma_{m\psi}}{\sigma^{*2}_m \sigma_{h\psi} - \sigma^*_{mh}\sigma_{h\psi}},$$

where $\sigma^{*2}_m = \sigma^2_m + \sigma^2_{\psi_m} + 2\sigma_{m,\psi_m}$ and $\sigma^*_{mh} = \sigma_{mh} + \sigma_{\psi_h\psi_m} + \sigma_{h\psi_m} + \sigma_{m\psi_h}$. Finally, I arrive at the expected return premium by substituting for A and B in (C13):

$$\mu_i + \mu_{\psi_i} - r_f = \frac{(\sigma_{im} + \sigma_{\psi_i\psi_m} + \sigma_{i\psi_m} + \sigma_{m\psi_i})\sigma_{h\psi} - \sigma_{i\psi}\sigma^*_{mh}}{(\sigma^2_m + \sigma^2_{\psi_m} + 2\sigma_{m\psi_m})\sigma_{h\psi} - \sigma^*_{mh}\sigma_{m\psi}}(\mu_m + \mu_{\psi_m} - r_f)$$

$$+\frac{\sigma_{i\psi}(\sigma_m^2+\sigma_{\psi_m}^2+2\sigma_{m\psi_m})-(\sigma_{im}+\sigma_{\psi_i\psi_m}+\sigma_{i\psi_m}+\sigma_{m\psi_i})\sigma_{m\psi}}{(\sigma_m^2+\sigma_{\psi_m}^2+2\sigma_{m\psi_m})\sigma_{h\psi}-\sigma_{mh}^*\sigma_{m\psi}}(\mu_h+\mu_{\psi_h}-r_f).$$

The above equation can be simplified further:

$$\mu_i+\mu_{\psi_i}-r_f=\beta_i^m(\mu_m+\mu_{\psi_m}-r_f)+\beta_i^h(\mu_h+\mu_{\psi_h}-r_f),$$

where

$$\beta_i^m=\frac{(\sigma_{im}+\sigma_{\psi_i\psi_m}+\sigma_{i\psi_m}+\sigma_{m\psi_i})\sigma_{h\psi}-\sigma_{i\psi}\sigma_{mh}^*}{(\sigma_m^2+\sigma_{\psi_m}^2+2\sigma_{m\psi_m})\sigma_{h\psi}-\sigma_{mh}^*\sigma_{m\psi}}$$
$$=\frac{(\sigma_{im}+\sigma_{\psi_i\psi_m}+\sigma_{i\psi_m}+\sigma_{m\psi_i})\sigma_{h\psi}-\sigma_{i\psi}\sigma_{mh}^*}{\sum_{i=1}^n x_i[(\sigma_{im}+\sigma_{\psi_i\psi_m}+\sigma_{i\psi_m}+\sigma_{m\psi_i})\sigma_{h\psi}-\sigma_{i\psi}\sigma_{mh}^*]},$$

$$\beta_i^h=\frac{\sigma_{i\psi}(\sigma_m^2+\sigma_{\psi_m}^2+2\sigma_{m\psi_m})-(\sigma_{im}+\sigma_{\psi_i\psi_m}+\sigma_{i\psi_m}+\sigma_{m\psi_i})\sigma_{m\psi}}{\sigma_{h\psi}(\sigma_m^2+\sigma_{\psi_m}^2+2\sigma_{m\psi_m})-(\sigma_{mh}+\sigma_{\psi_h\psi_m}+\sigma_{h\psi_m}+\sigma_{m\psi_h})\sigma_{m\psi}}$$
$$=\frac{\sigma_{i\psi}(\sigma_m^2+\sigma_{\psi_m}^2+2\sigma_{m\psi_m})-(\sigma_{im}+\sigma_{\psi_i\psi_m}+\sigma_{i\psi_m}+\sigma_{m\psi_i})\sigma_{m\psi}}{\sum_{i=1}^n h_i[\sigma_{i\psi}(\sigma_m^2+\sigma_{\psi_m}^2+2\sigma_{m\psi_m})-(\sigma_{im}+\sigma_{\psi_i\psi_m}+\sigma_{i\psi_m}+\sigma_{m\psi_i})\sigma_{m\psi}]}.$$

***Proof of Proposition 1**:* If I assume that the noisy information variable ψ is the single state variable, and that there exists an asset (hedge portfolio) whose return is perfectly negatively correlated with changes in ψ, then: $\rho_{h\psi}=-1$, $\rho_{m\psi}=-\rho_{mh}$, $\rho_{i\psi}=-\rho_{ih}$, and I can simplify β_i^m as follows:

$$\beta_i^m=\frac{(\sigma_{im}+\sigma_{\psi_i\psi_m}+\sigma_{i\psi_m}+\sigma_{m\psi_i})\sigma_{h\psi}-\sigma_{i\psi}\sigma_{mh}^*}{(\sigma_m^2+\sigma_{\psi_m}^2+2\sigma_{m\psi_m})\sigma_{h\psi}-\sigma_{mh}^*\sigma_{m\psi}}$$

$$=\frac{\sigma_{im}^*\sigma_{h\psi}-\sigma_{i\psi}\sigma_{mh}^*}{\sigma_m^{*2}\sigma_{h\psi}-\sigma_{mh}^*\sigma_{m\psi}}$$

$$=\frac{\beta_{im}^*-\sigma_{i\psi}\sigma_{mh}^*/\sigma_m^{*2}\sigma_{h\psi}}{1-\sigma_{mh}^*\sigma_{m\psi}/\sigma_m^{*2}\sigma_{h\psi}}$$

$$= \frac{\beta_{im}^* - (-\rho_{ih})\sigma_i^* \sigma_\psi \sigma_{mh}^* / \sigma_m^{*2} (-1)\sigma_h \sigma_\psi}{1 - \sigma_{mh}^* (-\rho_{mh})\sigma_m \sigma_\psi / \sigma_m^{*2} (-1)\sigma_h \sigma_\psi}$$

$$= \frac{\beta_{im}^* - \beta_{ih}^* \beta_{mh}^*}{1 - \rho_{mh}{}^2}.$$

Similarly, β_i^h can be simplified as follows:

$$\beta_i^h = \frac{\sigma_{i\psi}(\sigma_m^2 + \sigma_{\psi_m}^2 + 2\sigma_{m\psi_m}) - (\sigma_{im} + \sigma_{\psi_i \psi_m} + \sigma_{i\psi_m} + \sigma_{m\psi_i})\sigma_{m\psi}}{(\sigma_m^2 + \sigma_{\psi_m}^2 + 2\sigma_{m\psi_m})\sigma_{h\psi} - \sigma_{mh}^* \sigma_{m\psi}}$$

$$= \frac{\beta_{ih}^* - \beta_{im}^* \beta_{mh}^*}{1 - \rho_{mh}{}^2}.$$

Therefore, under the single state variable assumption, if there exists a hedge portfolio whose return is perfectly negatively correlated with changes in the single state variable, then my result in equation 7 of Chapter 2 can be simplified to be the following,

$$\alpha_i - \mu_{\psi_i} - r = \beta_i^m (\alpha_m - \mu_{\psi_m} - r) + \beta_i^h (\alpha_h - \mu_{\psi_h} - r),$$

where $\beta_i^m = \dfrac{\beta_{im}^* - \beta_{ih}^* \beta_{mh}^*}{1 - \rho_{mh}{}^2}$, *and* $\beta_i^h = \dfrac{\beta_{ih}^* - \beta_{im}^* \beta_{mh}^*}{1 - \rho_{mh}{}^2}$;

$$\beta_{im}^* = \frac{\sigma_{im} + \sigma_{\psi_i \psi_m} + \sigma_{i\psi_m} + \sigma_{m\psi_i}}{\sigma_m^2 + \sigma_{\psi_m}^2 + 2\sigma_{r_m, \psi_m}} = \frac{\sigma_{im}^*}{\sigma_m^{*2}},$$

$$\beta_{ih}^* = \frac{\sigma_{ih} + \sigma_{\psi_i \psi_h} + \sigma_{i\psi_h} + \sigma_{h\psi_i}}{\sigma_h^2 + \sigma_{\psi_h}^2 + 2\sigma_{r_h, \psi_h}} = \frac{\sigma_{ih}^*}{\sigma_h^{*2}},$$

$$\beta_{mh}^* = \frac{\sigma_{hm} + \sigma_{\psi_h \psi_m} + \sigma_{h\psi_m} + \sigma_{m\psi_h}}{\sigma_m^2 + \sigma_{\psi_m}^2 + 2\sigma_{r_m, \psi_m}} = \frac{\sigma_{hm}^*}{\sigma_m^{*2}}$$

$$\rho_{mh}{}^2 = \frac{(\sigma_{hm} + \sigma_{\psi_h \psi_m} + \sigma_{h\psi_m} + \sigma_{m\psi_h})^2}{(\sigma_m^2 + \sigma_{\psi_m}^2 + 2\sigma_{r_m, \psi_m})(\sigma_h^2 + \sigma_{\psi_h}^2 + 2\sigma_{r_h, \psi_h})} = \frac{\sigma_{hm}^{*2}}{\sigma_m^{*2} \sigma_h^{*2}}.$$

QED.

Appendix D – Proof of Theorem and Propositions of Chapter 3

Proof of Theorem 1:

EPS, denoted by *Y*, follows an Ito processes:

$$\frac{dY(t)}{Y(t)} = G(t)dt + \sigma_y dw_y \ . \tag{D1}$$

The mean of EPS growth rate follows an Ornstein-Uhlenbeck mean-reverting process:

$$dG(t) = k_g(\mu_g^0 - G(t))dt + \sigma_g d\omega_g . \tag{D2}$$

According to standard results from one-dimensional linear filtering (see, for example, Liptser and Shiryaev, 1977 and 1978), the solution for the filtered estimate of mean growth rate $\hat{G}(t)$, specialized in equations (D1) and (D2), is given by the following stochastic differential equation (SDE):

$$d\hat{G}(t) = k_g(\mu_g^0 - \hat{G}(t))dt + E_t[G(t)\frac{(G(t)-\hat{G}(t))Y(t)}{\sigma_y^2 Y(t)^2} + \rho_{gy}\frac{\sigma_g}{\sigma_y Y(t)}](dY(t) - \hat{G}(t)Y(t)dt),$$

that is,

$$d\hat{G}(t) = k_g(\mu_g^0 - \hat{G}(t))dt + \Sigma_t dw_y^*, \tag{D3}$$

where $\Sigma_t = \dfrac{S(t)+\sigma_{gy}}{\sigma_y}$, $\sigma_{gy} = \rho_{gy}\sigma_y\sigma_g$, and $dw_y^* = \dfrac{1}{\sigma_y}\left(\dfrac{dY(t)}{Y(t)} - \hat{G}(t)dt\right)$. The posterior variance of the agent's estimate of $G(t)$, $S(t) \equiv E[(G(t)-\hat{G}(t))^2 \mid Y(t)]$, satisfies the following Riccati ordinary differential equation (ODE):

$$\frac{dS(t)}{dt} = \gamma(S(t)^2 + \eta S(t) + \alpha), \tag{D4}$$

where $\alpha = -\sigma_g^2\sigma_y^2(1-\rho_{gy}^2)$, $\eta = 2\sigma_y^2(\dfrac{\rho_{gy}\sigma_g}{\sigma_y} + k_g)$, , and $\gamma = -\dfrac{1}{\sigma_y^2}$.

Equation (D4) is equivalent to the following,

$$\frac{dS(t)}{(S(t)-S_1)(S(t)-S_2)} = \gamma dt . \tag{D5}$$

Re-arranging equation (D5), I obtain the following ODE:

$$\frac{1}{(S_1 - S_2)}\left(\frac{dS(t)}{S(t)-S_1} - \frac{dS(t)}{S(t)-S_2}\right) = \gamma dt\,. \tag{D6}$$

Taking integral with respect to time t on both sides of equation (D6), I get,

$$\ln\left(\frac{S(t)-S_1}{S(t)-S_2}\right) = \gamma(S_1 - S_2)t + c\,, \tag{D7}$$

where c denotes a constant. I can think of *S(t)* as a variance of forecast error based on all relevant information up to time *t*. If an initial forecast error variance is $S(0)$, then solving equation (D7), I obtain:

$$S(t) = S_2 + \frac{S_1 - S_2}{1 - Ce^{\gamma(S_1 - S_2)t}}, \qquad \text{when } S(0) \in [S_1, \infty), \tag{D8}$$

where $S_1 = \frac{-\eta}{2} + \sqrt{\frac{\eta^2}{4} - \alpha}$, $S_2 = \frac{-\eta}{2} - \sqrt{\frac{\eta^2}{4} - \alpha}$, $C = \dfrac{S(0)-S_1}{S(0)-S_2}$.

Note that $\gamma < 0$ and $S_1 > S_2$. Hence, equation (A8) implies that in the long run as more information becomes available $S(t)$ converges to S_1, which is always nonnegative. Another bound for $S(t)$ is denoted by S_2, which is always non-positive and lower than S_1. Therefore, S_2 is not relevant to my analysis of the long-term value of $S(t)$. Nevertheless, S_2 is one of the parameters determining the speed of convergence of $S(t)$ to S_1.

Proof of Proposition 1.a:

When $\rho_{gy} = 1$ I have $\beta = \dfrac{\rho_{gy}\sigma_y\sigma_g}{\sigma_y^2} = \dfrac{\sigma_g}{\sigma_y} > 0$ and $\eta = 2\sigma_y^2(\beta + k_g) > 0$. Therefore, ODE (4) of Chapter 3 becomes

$$\frac{dS(t)}{S(t)\left[S(t)+\eta\right]} = \gamma dt\,,$$

Thus the two bounds for $S(t)$ are $S_1 = 0$ and $S_2 = -\eta < 0$. Further, the solution for $S(t)$ is given by $S(t) = \dfrac{\eta C e^{\gamma\eta t}}{1 - Ce^{\gamma\eta t}}$. In the limit, S(t) converges to $S_1 = 0$ as $t \to \infty$.

Q.E.D.

Proof of Proposition 1.b:

When $\rho_{gy} = -1$, then $\beta = \frac{\rho_{gy}\sigma_y\sigma_g}{\sigma_y^2} = -\frac{\sigma_g}{\sigma_y} < 0$, $\alpha = 0$, $k_g^* = (\beta + k_g)$, and $\eta = 2\sigma_y^2(\beta + k_g)$. I consider three cases:

(i) when $k_g > |\beta|$, the effective speed of mean reversion for the process of the latent mean growth rate, k_g^* is positive and I have: $\eta > 0$. Similar to Proposition 1.a, the two bounds for $S(t)$ become $S_1 = 0$ and $S_2 = -\eta < 0$. Therefore, the solution for $S(t)$ is $S(t) = \frac{\eta C e^{\gamma\eta t}}{1 - Ce^{\gamma\eta t}} \to 0$ as $t \to \infty$.

(ii) when $k_g < |\beta|$, then $\eta < 0$, $S_1 = -\eta > 0$, $S_2 = 0$, and $k_g^* < 0$. If the initial value of $S(t)$ satisfies $S(0) > S_1 = |\eta|$, then $S(t)$ is given by $S(t) = \frac{-\eta C e^{\gamma\eta t}}{1 - Ce^{\gamma\eta t}}$, where $C = \frac{S(0) - |\eta|}{S(0)}$. In this case, $S(t)$ will decrease over time and will converge to $|\eta|$.

If $S(0) < S_1 = |\eta|$, then $S(t) = \frac{-\eta}{1 + Ce^{\gamma|\eta|t}}$, where $C = \frac{|\eta|}{S(0)} - 1$. Therefore, $S(t)$ will increase over time and will converge to $|\eta|$ from below.

Finally, if $S(0) = |\eta|$, then $S(t)$ will remain at $|\eta|$ for every *t*.

(iii) when $k_g = |\beta|$, I have: $\eta = 0$, $S_1 = 0$, $S_2 = 0$, and $k_g^* = 0$. This means that ODE (4) becomes:

$$\frac{dS(t)}{S(t)^2} = \gamma dt,$$

and the solution to this ODE is:

$$S(t) = \frac{S(0)}{1 + S(0)|\gamma|t}.$$

Therefore, $S(t)$ will approach zero hyperbolically as $t \to \infty$, and thus slower than in case (i), in which learning is exponential.

Q.E.D.

Proof of Proposition 2:

We differentiate K with respect to ρ_{gy}. Note that k_g^* is positive, therefore:

$$\frac{\partial K}{\partial \rho_{gy}} = \frac{\partial |\gamma(S_1 - S_2)|}{\partial \rho_{gy}} = \frac{\partial 2k_g^*}{\partial \rho_{gy}} > 0 .$$

Specifically, since $\frac{\partial K}{\partial \rho_{gy}} = \frac{2k_g \sigma_g}{\sigma_y}[(\beta + k_g)^2 + \frac{\sigma_g^2}{\sigma_y^2}(1 - \rho_{gy}^2)]^{-1/2} = \frac{2k_g \sigma_g}{\sigma_y k_g^*}$ is positive, therefore the speed of convergence is positively related to ρ_{gy}.

Q.E.D.

Proof of Proposition 3:

We differentiate K with respect to k_g and get:

$$\frac{\partial K}{\partial k_g} = \frac{\partial (2k_g^*)}{\partial k_g} = \frac{2}{k_g^*}(k_g + \beta) .$$

Since k_g^* is positive I have: $\frac{\partial K}{\partial k_g} > 0$, if $(k_g + \beta) > 0$. Otherwise, if $(k_g + \beta) \le 0$, then $\frac{\partial K}{\partial k_g} \le 0$.

Q.E.D.

Appendix E - Derivation of the Asset Price and Proof of Proposition 4 of Chapter 3

Derivation of the Asset Price:

Our model of learning unobserved state variables is consistent with evidence that analysts use past observations of EPS growth to build their forecasts. Due to the Markovian nature of the model the valuation procedure by a representative agent takes as given the filtered estimate of the mean EPS growth (Genotte, Dothan and Feldman) when pricing assets.

Given the information set available to the agents, the processes for $Y(t)$ and the MEGR estimate are given in Theorem 1,

$$\frac{dY(t)}{Y(t)} = \hat{G}(t)dt + \sigma_y dw_y^*$$

$$d\hat{G}(t) = k_g(\mu_g^0 - \hat{G}(t))dt + \Sigma_t dw_y^* \quad \text{(E1)}$$

where $\Sigma_t = \dfrac{S(t) + \sigma_{gy}}{\sigma_y}$ and $\sigma_{gy} = \rho_{gy}\sigma_y\sigma_g$.

Now I derive share price using standard SDE arguments based on stochastic discount factor (SDF) approach (see, e.g., Cochrane, 2005). The implicit assumption here is that any shock responsible for the difference between D_t and δY is not priced:

$$E_t^*[d(MP)] + M\delta Ydt = 0 \quad \text{(E2)}$$

Evaluating the differential and dividing through by $M\delta Y$ I obtain:

$$E_t^*\left(\frac{dM}{M}\frac{P}{\delta Y} + \frac{dP}{\delta Y} + \frac{dM}{M}\frac{dP}{\delta Y}\right) + dt = 0 \quad \text{(E3)}$$

We now guess a solution for the price in the following form:

P(t, Y, $\hat{G}(t)$, R) = δYZ(t, $\hat{G}(t)$, R) (E4)

where δY represents dividends-per-share. Operator E_t^* represents an expectation with respect to dw_y^*, investors' information set. $Z(t, \hat{G}, R)$ is the time-*t* price-dividend ratio.

The second and the third terms under the expectation in equation (E3) follow from a simple application of the Itô rule:

$$\frac{dP}{\delta Y} = dZ + Z\frac{dY}{Y} + dZ\frac{dY}{Y} = Z_t dt + Z_{\hat{G}} d\hat{G} + Z_R dR + \frac{1}{2}\left(Z_{\hat{G}\hat{G}} d\hat{G}^2 + 2Z_{\hat{G}R} d\hat{G}dR + Z_{RR} dR^2\right) +$$
$$Z\left(\hat{G}dt + \sigma_y dw_y^*\right) + \left(\sigma_y Z_{\hat{G}} \Sigma_t + Z_R \sigma_{yr}\right)dt, \text{ where } \sigma_{yr} = \rho_{yr}\sigma_y\sigma_r,$$
$$\frac{dM}{M}\frac{dP}{\delta Y} = -\left(\rho_{my}\sigma_m Z_{\hat{G}} \Sigma_t + \lambda_y Z + \lambda_r Z_R\right)dt, \text{ where } \lambda_y = \rho_{my}\sigma_m\sigma_y \text{ and } \lambda_r = \rho_{mr}\sigma_m\sigma_r.$$

Collecting all the terms in (E3), taking the expectation, and dividing through by *dt*, I obtain the PDE for the share price:

$$Z_t + k_g(\mu_g^* - \hat{G}(t))Z_{\hat{G}} + k_r(\mu_r^* - R)Z_R + \frac{\Sigma_t^2}{2} Z_{\hat{G}\hat{G}} + \frac{\sigma_r^2}{2} Z_{RR} +$$
$$\rho_{gr}\sigma_r\Sigma_t Z_{\hat{G}R} - (R + \lambda_y - \hat{G})Z + 1 = 0, \qquad \text{(E5)}$$
$$\text{where } \mu_g^* = \mu_g^0 + \frac{(\sigma_y - \rho_{my}\sigma_m)\Sigma_t}{k_g}, \text{ and } \mu_r^* = \mu_r^0 + \frac{\sigma_{yr} - \lambda_r}{k_r}.$$

The above PDE satisfies Feynman-Kac conditions, and therefore allows us to write the solution which can be written as follows:

$$Z = \int_t^\infty E_t^* \exp\left(\int_t^s (\hat{G}(u) - R(u) - \lambda_y)du\right)ds.$$

The integrand solves the same equation as Z with the free term 1 deleted from the equation. I look for an integrand solution as

$$\exp(\varphi(t,s) + \psi(t,s)\hat{G}(t) - \upsilon(t,s)R(t)).$$

Equivalently, I are looking for price-dividend ratio in the form:

$$Z(t,s,\hat{G},R) = \int_t^\infty \exp^{[\varphi(t,s)+\psi(t,s)\hat{G}(t)-\nu(t,s)R(t)]} ds. \qquad \text{(E6)}$$

Inserting the proposed expression for the integrand into its PDE and recognizing that the resulting ordinary differential equation (ODE) must hold for arbitrary values of $\hat{G}(t)$ and *R(t)*, I arrive at the following ODEs for functions $\varphi(t,s)$, $\psi(t,s)$, and $\upsilon(t,s)$ (prime denotes $\partial/\partial t$ derivative):

$$\begin{aligned} &\psi' - k_g\psi + 1 = 0 \\ &\upsilon' - k_r\upsilon + 1 = 0 \qquad \text{(E7)} \\ &\varphi' + \frac{\Sigma_t^2}{2}\psi^2 + k_g\mu_g^*\psi + \frac{\sigma_r^2}{2}\upsilon^2 - \rho_{gr}\sigma_r\Sigma_t\psi\upsilon - k_r\mu_r^*\upsilon + \lambda_y = 0 \end{aligned}$$

When $s=t$ (or $\tau \equiv s\text{-}t = 0$), the integrand is equal to zero. Therefore, I have the following initial conditions for functions $\varphi(t,s)$, $\psi(t,s)$, and $\upsilon(t,s)$:

$$\varphi(s,s) = \psi(s,s) = \upsilon(s,s) = 0.$$

Subject to these initial conditions, the solution to the decoupled system (E7) is:

$$\psi(t,s) = \frac{1 - e^{-k_g(s-t)}}{k_g}, \quad \upsilon(t,s) = \frac{1 - e^{-k_r(s-t)}}{k_r}, \text{ and}$$

$$\varphi(t,s) = -\lambda_y \tau + \int_t^s \left(\frac{\Sigma_t^2}{2}\psi^2 + k_g \mu_g^* \psi + \frac{\sigma_r^2}{2}\upsilon^2 - \rho_{gr}\sigma_r \Sigma_t \psi\upsilon - k_r \mu_r^* \upsilon \right) du. \tag{E8}$$

The requirement that the integral (E6) exist places certain restrictions on function φ(t,s). For the integral in (E6) to exist, the integrand should be declining with *s* sufficiently fast. Since functions ψ and υ are bounded, this requirement implies that function φ should be negative and unbounded at large *s*. The latter restriction implies certain constraint on model parameters (a transversality condition), which I now derive. I need three auxiliary results to complete the derivation of the transversality condition:

a) For any bounded positive function *f(u)* and positive constant *k*:

$$\sup_{u\in[0,\tau]} f(u) = M < \infty, \int_0^\tau f(u) e^{-k(\tau-u)} du \le M \int_0^\tau e^{-k(\tau-u)} du = \frac{M}{k}\left(1 - e^{-k\tau}\right) \le \frac{M}{k} \tag{E9}$$

In what follows, I ignore non-growing integrals such as (E9) and keep only the leading terms that are unbounded in τ.

b) In equation (6) for the posterior variance of the MEGR estimate, *S(t)*, I assume that $S(0) > S_1$. This condition also implies that constant $C < 1$. Therefore, the solution for variance *S(t)* is

$$S(t) = S_2 + \frac{S_1 - S_2}{1 - Ce^{\gamma(S_1 - S_2)t}}$$

$$\int_t^s S_u du \overset{\tau=s-t}{=} \int_0^\tau \left(S_2 + \frac{S_1 - S_2}{1 - Ce^{\gamma(S_1 - S_2)u}} \right) du = S_1\tau - \frac{1}{\gamma} \ln \frac{1 - Ce^{\gamma(S_1 - S_2)\tau}}{1 - C} \xrightarrow{leadingterms} S_1\tau \tag{E10}$$

c) Using equation (6) for the posterior variance *S(t)* in Chapter 3, I have:

$$\int_t^s S_u^2 du = \int_t^s \left(\frac{1}{\gamma} S_u^{'} - \eta S_u - \alpha \right) du = \frac{1}{\gamma}\big(S(s) - S(t)\big) - \eta \int_t^s S_u du - \alpha(s-t)$$
$$\xrightarrow{leadingterms} -(\eta S_1 + \alpha)\tau \qquad \text{(E11)}$$

Using results (E10) and (E11) as well as (E8) to eliminate non-growing integrals I obtain the following leading terms in function φ:

$$\varphi(t,s) = -\lambda_y \tau + \int_t^s \left(\frac{\Sigma_t^2}{2}\psi^2 + k_g \mu_g^* \psi + \frac{\sigma_r^2}{2}\upsilon^2 - \rho_{gr}\sigma_r \Sigma_t \psi \upsilon - k_r \mu_r^* \upsilon \right) du \xrightarrow{leadingterms}$$
$$\xrightarrow{=} -\lambda_y \tau + \frac{1}{2(k_g \sigma_y)^2} \int_t^s \left(S_u^2 + 2\sigma_{gy} S_u + \sigma_{gy}^2 \right) du + \int_t^s \left(\mu_g^0 + \frac{\sigma_y - \rho_{my}\sigma_m}{k_g} \frac{S_u + \sigma_{gy}}{\sigma_y} \right) du +$$
$$+ \frac{\sigma_r^2}{2k_r^2}\tau - \frac{\rho_{gr}\sigma_r}{k_r k_g \sigma_y} \int_t^s \left(S_u + \sigma_{gy} \right) du - \mu_r^* \tau \xrightarrow{leadingterms}$$
$$\left(-\lambda_y + \frac{\sigma_r^2}{2k_r^2} - \mu_r^* \right)\tau + \frac{\tau}{2(k_g \sigma_y)^2}\left(-\eta S_1 - \alpha + 2\sigma_{gy} S_1 + \sigma_{gy}^2 \right) +$$
$$\left(\mu_g^0 + \frac{\sigma_y - \rho_{my}\sigma_m}{k_g \sigma_y}\left(S_1 + \sigma_{gy} \right) \right)\tau - \frac{\rho_{gr}\sigma_r}{k_r k_g \sigma_y}\left(S_1 + \sigma_{gy} \right)\tau$$
$$= \left[-\lambda_y + \frac{\sigma_r^2}{2k_r^2} - \mu_r^* + \mu_g^0 + \frac{-\eta S_1 - \alpha + 2\sigma_{gy} S_1 + \sigma_{gy}^2}{2(k_g \sigma_y)^2} + \left(\frac{\sigma_y - \rho_{my}\sigma_m}{k_g \sigma_y} - \frac{\rho_{gr}\sigma_r}{k_r k_g \sigma_y} \right)\left(S_1 + \sigma_{gy} \right) \right]\tau.$$

Finally, the transversality condition states that the leading terms must be negative for the price integral to exist:

$$-\lambda_y + \frac{\sigma_r^2}{2k_r^2} - \mu_r^* + \mu_g^0 + \frac{(2\sigma_{gy} - \eta)S_1 + \sigma_{gy}^2 - \alpha}{2(k_g \sigma_y)^2} + \left((\sigma_y - \rho_{my}\sigma_m)k_r - \rho_{gr}\sigma_r \right)\frac{(S_1 + \sigma_{gy})}{k_r k_g \sigma_y} < 0.$$

(E12)

Applying Itô's lemma to $P(t)$, I obtain a stochastic differential equation (SDE) for $P(t)$, subject to $P(t) < \infty$:

$$\frac{dP(t)}{P(t)} = [\frac{1}{Z} Z_t + \hat{G} + k_g(\mu_g^0 - \hat{G})\frac{1}{Z} Z_{\hat{G}} + \frac{\Sigma_t^2}{2Z}\frac{1}{Z} Z_{\hat{G}\hat{G}} + \Sigma_t \sigma_y \frac{1}{Z} Z_{\hat{G}} + \rho_{yr}\sigma_r \sigma_y \frac{1}{Z} Z_R +$$
$$\rho_{gr}\sigma_r \Sigma_t \sigma_y \frac{1}{Z} Z_{\hat{G}R} + k_r(\mu_r^0 - R)\frac{1}{Z} Z_R + \frac{\sigma_r^2}{2Z} Z_{RR}]dt + [\sigma_y + \Sigma_t \frac{1}{Z} Z_{\hat{G}}]dw_y^* + \sigma_r \frac{1}{Z} Z_R dw_r.$$

Plugging the SDE for $\frac{dP(t)}{P(t)}$ into equation (E3), I get the following risk-neutral drift of stock return in an incomplete-information environment,

$$E_t^*\left(\frac{dP}{P}\right)+\frac{dY}{Y}dt=Rdt+\sigma_m\left(\sigma_1\rho_{my}+\sigma_2\rho_{mr}\right)dt,$$

where $\sigma_1=\sigma_y+\Sigma_t\frac{1}{Z}Z_{\hat{G}}$, and $\sigma_2=\sigma_r\frac{1}{Z}Z_R$.

QED.

Proof of Proposition 4:

The risk premium for *G(t)* based on complete-information BC model is defined by λ_g^{BC}, given below,

$$\lambda_g^{BC}=\rho_{mg}\sigma_g\sigma_m-\sigma_{gy},$$

The risk premium for $\hat{G}(t)$ based on my incomplete-information model is defined by λ_g, given below,

$$\lambda_g=(\rho_{my}\sigma_m-\sigma_y)\Sigma_t=(\rho_{my}\sigma_m-\sigma_y)\left(\frac{S(t)+\sigma_{gy}}{\sigma_y}\right)$$
$$=(\rho_{my}\sigma_m-\sigma_y)\frac{S(t)}{\sigma_y}+(\rho_{my}\rho_{gy}\sigma_g\sigma_m-\sigma_{gy}),$$

Therefore, the difference in risk premiums between my incomplete-information model and complete-information model (BC) is given by,

$$\Delta\lambda_g=\lambda_g-\lambda_g^{BC}=(\rho_{my}\sigma_m-\sigma_y)\frac{S(t)}{\sigma_y}+\sigma_g\sigma_m(\rho_{my}\rho_{gy}-\rho_{mg}).$$

In the long-run limit as $S(t)\to S_1$=0, I obtain the following result:

$$\Delta\lambda_g\to\sigma_g\sigma_m(\rho_{my}\rho_{gy}-\rho_{mg})=\sigma_g\sigma_m\rho_{mg}\left(\frac{\rho_{my}\rho_{gy}}{\rho_{mg}}-1\right).$$

For special cases such as $\rho_{gy}=1$ or $\rho_{gy}=-1$, I get $\frac{\rho_{my}\rho_{gy}}{\rho_{mg}}=|\rho_{gy}|=1$. Therefore, the term in parentheses above disappears, which implies a zero difference in risk premiums. That is $\Delta\lambda_g\to 0$.

Let μ_g^{BC} denote the risk-neutral long-term mean of earnings growth under BC model. Following BC model, $\mu_g^{BC} = \mu_g^0 + \frac{\sigma_{gy} - \rho_{mg}\sigma_g\sigma_m}{k_g}$. The parameter μ_g^* denotes the risk-neutral long-term mean of EPS growth rate under my incomplete-information model, as given below:

$$\mu_g^* = \mu_g^0 - \frac{\lambda_g}{k_g} = \mu_g^0 + \frac{\sigma_y - \rho_{my}\sigma_m}{k_g}\left(\frac{S(t) + \sigma_{gy}}{\sigma_y}\right).$$

According to Proposition (1), if $\rho_{gy} = 1$ or $\rho_{gy} = -1$, $S(t)$ declines over time and converges to zero.[1] Therefore, as $S(t) \to 0$

$$\mu_g^* \equiv \mu_g^0 - \frac{\lambda_g}{k_g} \xrightarrow{S(t)\to 0} \mu_g^0 + \frac{\sigma_{gy} - \rho_{my}\rho_{gy}\sigma_g\sigma_m}{k_g} = \mu_g^0 + \frac{\sigma_{gy} - \rho_{mg}\sigma_g\sigma_m}{k_g} = \mu_g^{BC}.$$

QED.

[1] The only exception is the case of $k_g < |\beta|$.