# COMPARATIVE STUDY OF STRENGTH DESIGN METHODS FOR RECTANGULAR REINFORCED CONCRETE AND COMPOSITE STEEL-CONCRETE COLUMNS

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# COMPARATIVE STUDY OF STRENGTH DESIGN METHODS FOR RECTANGULAR REINFORCED CONCRETE AND COMPOSITE STEEL-CONCRETE COLUMNS

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## ABSTRACT

A comparative study of available design methods in predicting the ultimate strength determined from physical tests of rectangular normal-density concrete columns was undertaken. The physical tests included in the study involve reinforced concrete and encased composite (steel-concrete) columns. The design methods compared include ACI 318-95 (Building 1995) which is very similar to CSA A23.3 (Design 1984), the AISC-LRFD Specifications (1994), Eurocode 2 (Design 1992), and Eurocode 4 (Design 1994). The results of a finite element modelling (FEM) procedure were also compared by using a commercially available nonlinear FEM software (ABAQUS 1994a, 1994b).

The columns used for comparison in this study were braced and pin-ended and were constructed using normal strength concrete with a specified compressive strength between 2500 and 8250 psi. The columns were subjected to short-term loads producing pure axial force, combined axial force and single or double curvature bending, or pure bending. Major variables included the concrete strength, the end eccentricity ratio, the slenderness ratio, the reinforcing steel index, the structural steel index and the tie/hoop volumetric ratio. A total of 398 reinforced concrete and 221 composite steel-concrete columns were taken from the literature that formed the basis for a

comparative study of different design methods. This comparative study provided an insight for the variability and related statistics of the design methods examined. No further tests were conducted for this study.

Most of the design methods were affected to some degree by some or all of the major variables studied. Recommendations for improving the ACI 318-95 and the AISC-LRFD procedures are presented.

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# 1 - INTRODUCTION

A comparative study of available design methods in predicting the ultimate strength determined from physical tests of rectangular reinforced concrete columns and composite steel-concrete columns in which steel sections are encased in concrete was undertaken. Physical tests included in this study involve normal-density concrete. The design methods compared include ACI 318-95 (Building 1995) which is very similar to CSA A23.3 (Design 1984), the AISC-LRFD Specifications (1994), Eurocode 2 (Design 1992), and Eurocode 4 (Design 1994).

The ACI and CSA design methods are strongly influenced by the effective flexural rigidity (*EI*) of the column which varies due to cracking, creep, and the nonlinearity of the concrete stress-strain curve. In an attempt to account for these variables, Mirza (1990) and Tikka and Mirza (1992) proposed refined equations for calculating the flexural rigidity for use in ACI and CSA design procedures for reinforced concrete and composite steel-concrete columns, respectively. In addition, Tikka and Mirza (1992) reported that, in some cases, AISC-LRFD Specifications produced unconservative designs for composite steel-concrete columns subjected to minor axis bending. This is due to the fact that the AISC-LRFD Specifications permit a higher value of the radius of gyration of a composite cross-section subjected to minor axis bending than that justified by calculations. In this study, a new equation is proposed for the radius of gyration for use in the AISC-LRFD design procedure for composite columns. This equation plus those suggested by Mirza (1990) and Tikka and Mirza (1992) were also included in the comparative study reported here.

During the past 10 to 15 years, commercial FEM software has become more readily available and its use by design engineers has been steadily increasing. Presently, there are several FEM programs that are able to model the concrete column strength at ultimate limit state. In an attempt to examine the applicability of FEM in predicting the ultimate strength of reinforced concrete and composite steel-concrete columns, the results of a commercially available nonlinear FEM software (ABAQUS 1994a, 1994b) were also compared with the physical tests.

To determine the influence of a full range of variables on the design methods examined in this study, 384 reinforced concrete columns without moment gradient, 14 reinforced concrete columns with moment gradient, 75 composite steel-concrete columns subjected to major axis bending without moment gradient, 3 composite steel-concrete columns subjected to major axis bending with moment gradient, and 143 composite steel-concrete columns subjected to minor axis bending without moment gradient were taken from the literature. Due to practical implications, it was decided to exclude all columns with a specified concrete strength of less than 2500 psi. The remaining 521 columns were used for a comparative study of different design methods examined. No new tests were conducted for this study.

Major variables investigated in this study include the concrete strength, the end eccentricity ratio, the slenderness ratio, the reinforcing steel index, the structural steel index and the tie/hoop volumetric ratio. Based on the statistical analysis of the major variables that affect column design strength, an evaluation and comparison of each design method was conducted. Most of the design methods were affected to some degree by the variables studied. These evaluations and comparisons provided an insight for the variability and related statistics of different design methods examined, including FEM. These are discussed and presented in this report.

The columns investigated in this study were braced and pin-ended and were constructed using normal strength concrete with a specified compressive strength between 2500 and 8250 psi. The columns were subjected to short-term loads producing pure axial force, combined axial force and single or double curvature bending, or pure bending. The columns used in this investigation are graphically represented in Figure 1.1. Columns subjected to equal and opposite end eccentricities producing symmetric single curvature bending are depicted in Figure 1.1(a) while columns subjected to equal end eccentricities producing double curvature bending are depicted in Figure 1.1(e).

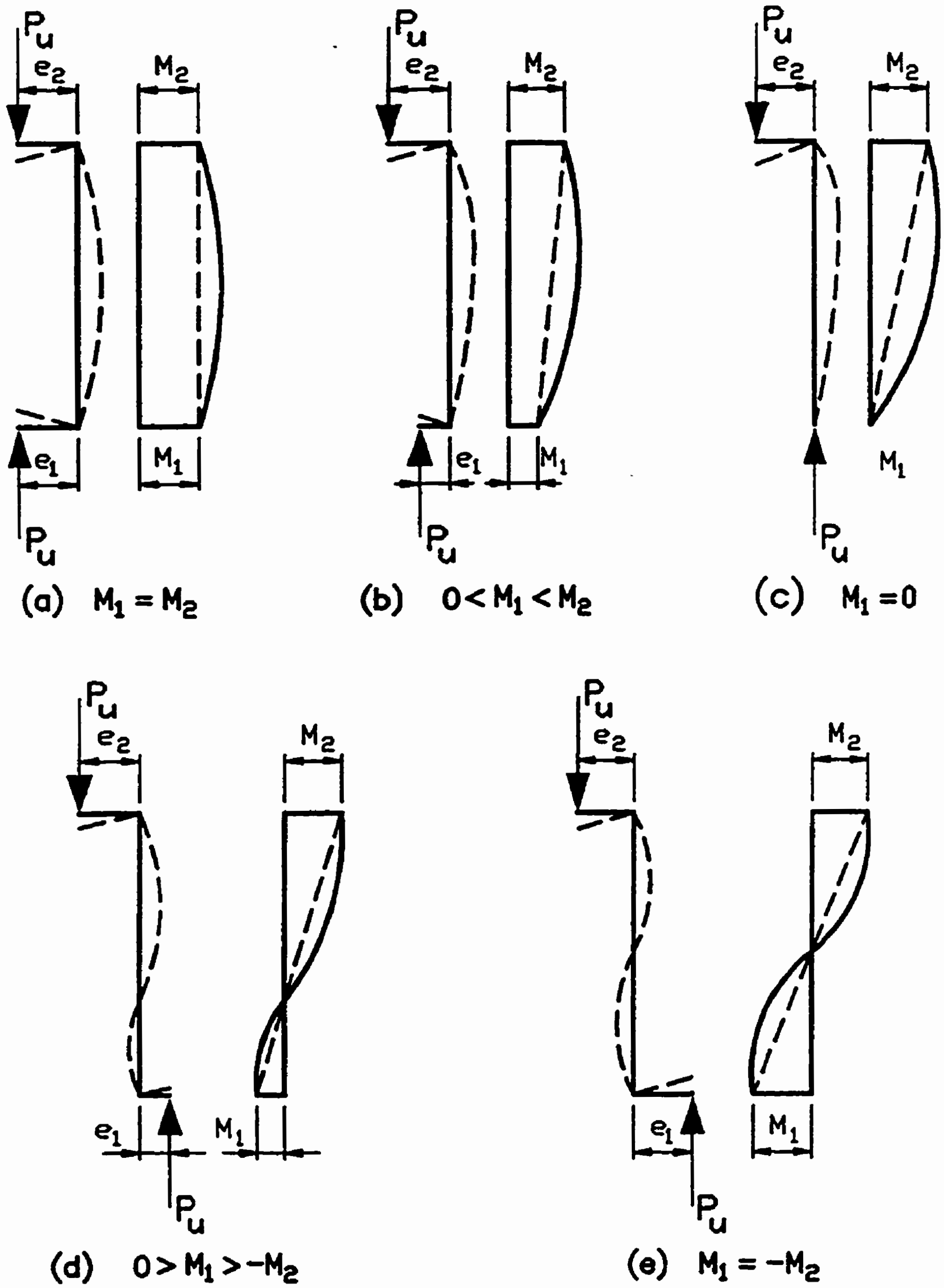



Figure 1.1 - Range of column load eccentricities and resulting second-order bending moment diagrams.

The columns in Figures 1.1(b), (c), and (d) represent the other load eccentricities examined in this study.

Due to the limited test data available, not all load eccentricities were available for each of the individual column types examined in this study. The range of load eccentricities for reinforced concrete columns without moment gradient include pure axial loading and the loading depicted in Figure 1.1(a) while reinforced concrete columns with moment gradient are represented by Figures 1.1(b), (c), (d) and (e). Composite steel-concrete columns subjected to major axis bending without moment gradient include cases of pure axial load, pure bending and those represented in Figure 1.1(a) while such columns with moment gradient are represented by Figures 1.1(c) and (d). Composite steel-concrete columns subjected to minor axis bending include pure axial loading, pure bending and the loading depicted in Figure 1.1(a).

# 2 - FINITE ELEMENT MODELLING OF REINFORCED CONCRETE AND COMPOSITE STEEL-CONCRETE COLUMNS

## 2.1 OVERVIEW OF THE FINITE ELEMENT MODELLING SOFTWARE USED

The finite element modelling (FEM) of reinforced concrete and composite steel-concrete columns was carried out by using commercially available nonlinear FEM software (ABAQUS 1994a, 1994b). The objective was to model the ultimate strength and load-deflection response of physical test specimens available in the literature. The FEM software was capable of static second-order nonlinear stress analysis which could include both material and geometric nonlinearity. This FEM software was chosen over others primarily for its ability to model the nonlinear stress-strain behavior of concrete under monotonic loading for low stress applications. Examples of low stress applications include structural components such as typical reinforced concrete beams, slabs, columns and shear walls.

This chapter summarizes the procedures and assumptions used in the modelling of reinforced concrete and composite steel-concrete cross-sections and columns using the FEM software. An overview of the concrete and steel stress-strain relations and the input procedures required by the FEM software are also presented.

## 2.2 CROSS-SECTION DISCRETIZATION AND MODELLING

The modelling of reinforced concrete and composite steel-concrete cross-sections was accomplished by using rebar elements and "beam" sections. The FEM software included an extensive library of pre-defined beam sections which were used to define the properties of the three-dimensional beam elements. Three separate sections were used to model the different materials that compose the cross-section. These materials include the unconfined concrete outside of the transverse tie reinforcement, the partially confined concrete within the transverse tie reinforcement, and the structural steel section. Rebar elements were used to model the longitudinal reinforcing steel.

The use of pre-defined beam sections greatly reduced the amount of data input required to model the cross-section. Only basic information on the section geometry was required; the FEM software automatically calculated the resulting section properties for use in the analysis. The pre-defined beam sections also had a default number of integration points used to discretize the section. The FEM software numerically integrates the cross-section to obtain the generalized force-moment/strain-curvature relations. Therefore, integration points define the mesh used in the numerical integration. A dense mesh will increase the accuracy of the solution at a cost of increased computation time. For this study, the number of integration points was increased from the default condition in an attempt to improve the accuracy of the FEM procedure used, as explained in the following section.

### 2.2.1 Overview of Beam Sections Used in Modelling of Cross-Sections

A pre-defined "box" beam section was used to model the unconfined concrete outside of the transverse tie reinforcement. Figure 2.1(a) illustrates this section as well as the location of the integration points. The default number of integration points was increased from 5 to 15 in each wall. The FEM software considers the box section to be thin walled and for this reason the integration points can only lie on the centerline of the section walls.

The pre-defined "rectangular" beam section was used to model the partially confined concrete within the transverse tie reinforcement. Figure 2.1(b) illustrates this section as well as the location of the integration points. The default number of integration points was increased from 5 to 15 in each direction.

There is no allowance made by the FEM software to account for the displaced concrete due to the structural steel section when modelling composite steel-concrete cross-sections. The FEM solution will, therefore, tend to predict slightly higher ultimate strengths for composite steel-concrete columns as compared to the physical tests. The effect will be most pronounced for columns tested under pure axial load and columns with large structural steel ratios ($\rho_{SS}$). However, practical limits on the maximum

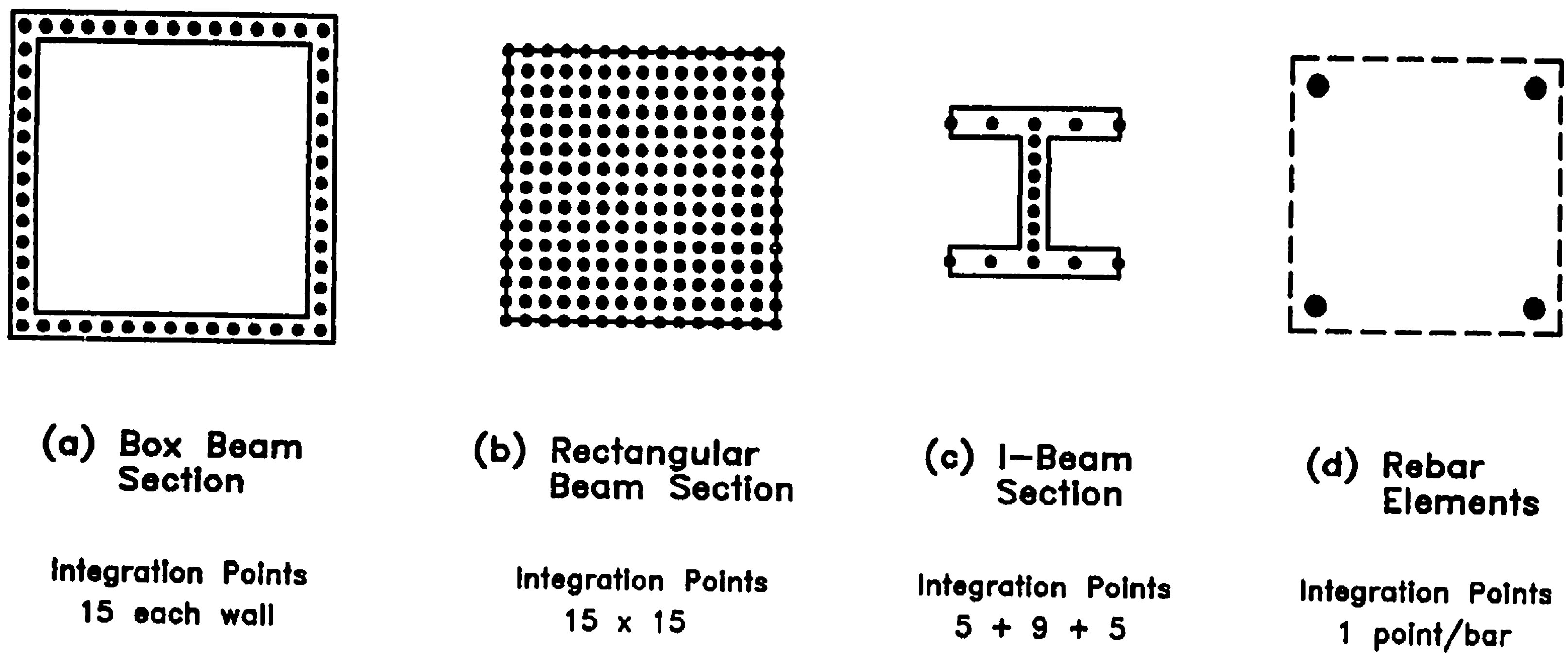



Figure 2.1 - Location of integration points for beam sections used for FEM.

structural steel ratios are between 10 and 15 percent. A slight inaccuracy of the FEM solution for predicting the ultimate strength of composite steel-concrete columns will result.

The pre-defined "I" beam section was used to model the structural steel section for composite columns. Figure 2.1(c) illustrates this section as well as the location of the integration points. The number of integration points was increased from 5 to 9 in the web while the default number of 5 points in the flanges was not changed. The FEM software considers the I-section to be thin walled and for this reason the integration points can only lie on the centerline of the section walls.

Rebar elements were used to model the longitudinal reinforcing steel. Figure 2.1(d) illustrates these elements within a typical cross-section. Each rebar element has one integration point. Since rebar elements can not exist as separate elements, they must be defined as being within other beam elements. In this study, the rebar elements were superimposed and imbedded into the partially confined concrete element mesh. The only input required was the location of the rebar element with respect to the local beam section axis and the name of the beam element to map it into. The FEM software automatically maps the rebar element into the beam element mesh and accounts for the displaced area of concrete.

### 2.2.2 Modelling of Reinforced Concrete Cross-Sections

For reinforced concrete cross-sections, three different materials have to be modeled: the unconfined concrete outside of the transverse ties, the partially confined concrete inside of the transverse ties, and the longitudinal reinforcing steel bars. The modelling of the cross-section was accomplished by superimposing two beam sections and rebar elements at common node points as is illustrated in Figure 2.2(d). The unconfined concrete was modeled by using a box section (Figure 2.2(a)). The inner wall of the box section coincides with the centerline of the transverse tie reinforcement. The partially confined concrete was modeled by using a rectangular section (Figure 2.2(b)). The outer edge of the rectangular section coincides with the centerline of the transverse tie reinforcement and the inner edge of the box section. The longitudinal reinforcing steel was modeled by superimposing rebar elements within the rectangular section mesh (Figure 2.2(c)).

### 2.2.3 Modelling of Composite Steel-Concrete Cross-Sections

For composite steel-concrete cross-sections, four different materials have to be modeled: the unconfined concrete outside of the transverse ties, the partially confined concrete inside of the transverse ties, the structural steel shape, and the longitudinal reinforcing steel bars. The modelling of the cross-section was

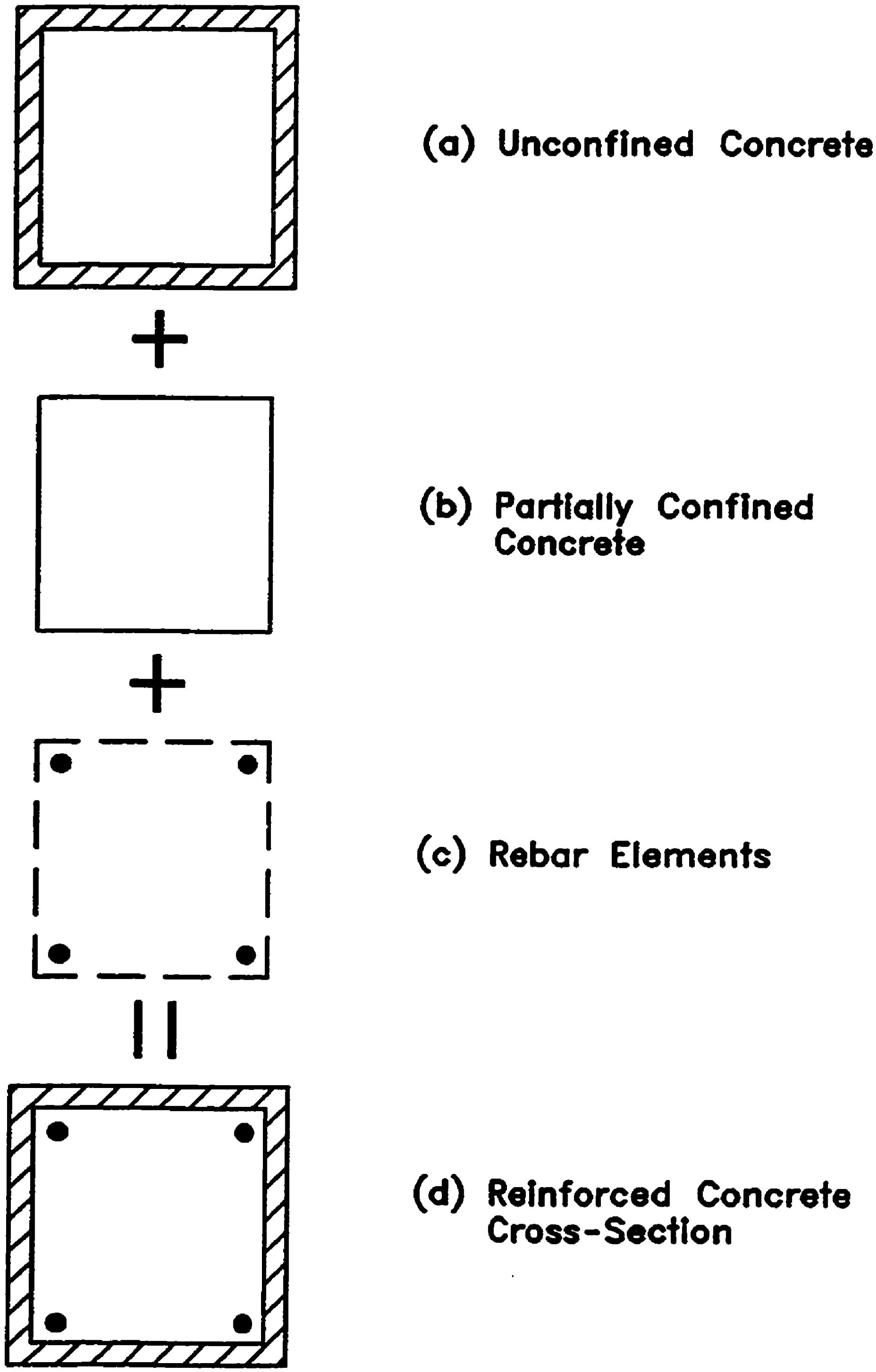



Figure 2.2 - FEM of reinforced concrete cross-section.

accomplished by superimposing three beam sections and rebar elements at common node points and is illustrated in Figures 2.3 and 2.4 for columns subjected to major axis and minor axis bending, respectively. The unconfined concrete was modeled by using a box section (Figures 2.3(a) and 2.4(a)). The inner wall of the box section coincides with the centerline of the transverse tie reinforcement. The partially confined concrete was modeled by using a rectangular section (Figures 2.3(b) and 2.4(b)). The outer edge of the rectangular section coincides with the centerline of the transverse tie reinforcement and the inner edge of the box section. The longitudinal reinforcing steel was modeled by superimposing rebar elements within the rectangular section mesh (Figures 2.3(c) and 2.4(c)). The structural steel shape was modeled by using an I-beam section (Figures 2.3(d) and 2.4(d)). Two different orientations of the I-beam section were used in order to model major and minor axis bending problems.

## 2.3 COLUMN DISCRETIZATION AND MODELLING

The modelling of reinforced concrete and composite steel-concrete columns was accomplished by using 3-node space beam elements. The length of the column was divided into a number of 3-node segments, each representing a beam element. Each beam element was connected to the adjacent elements at the outer two "common node" points. The

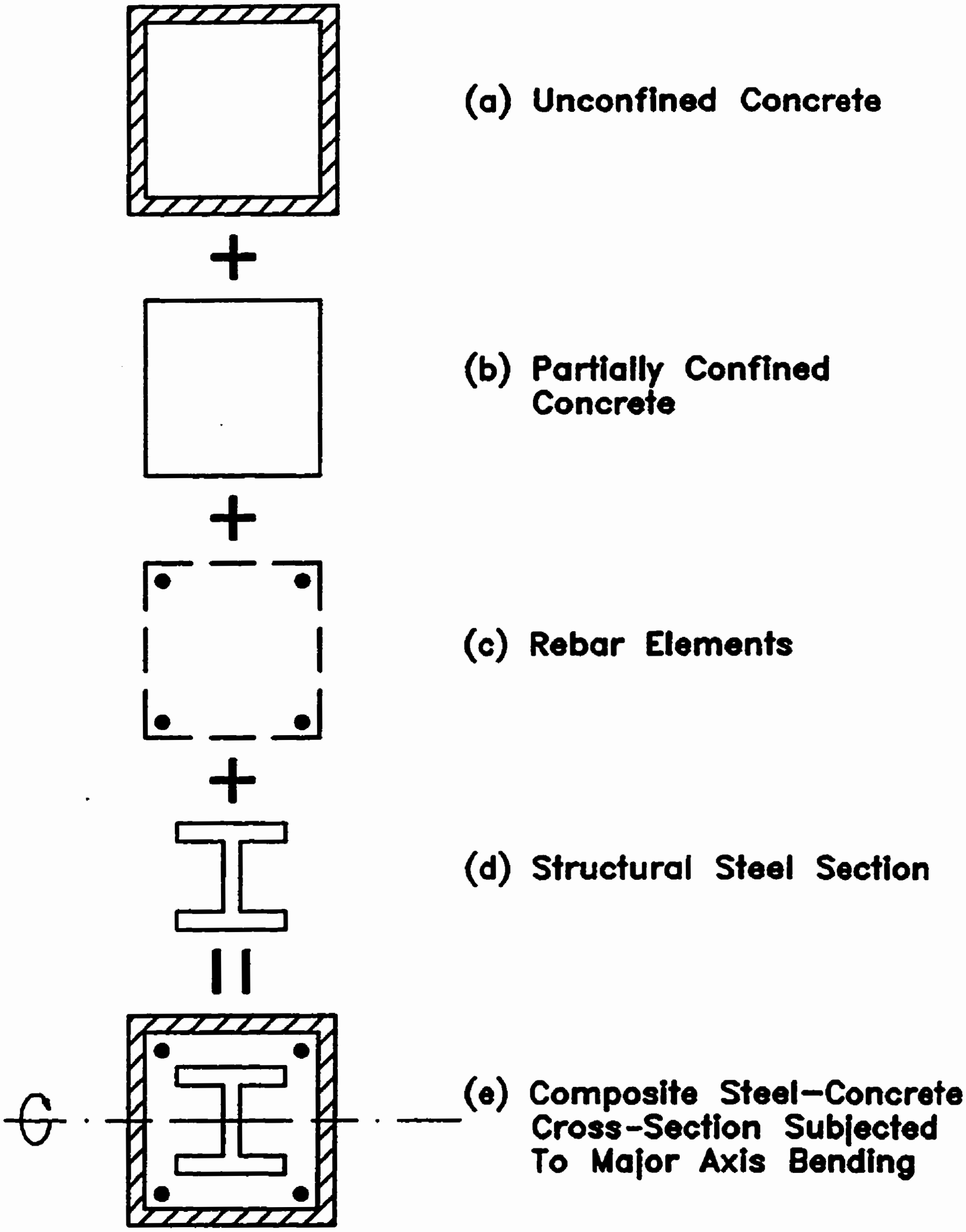



Figure 2.3 - FEM of composite steel-concrete cross-sections subjected to major axis bending.

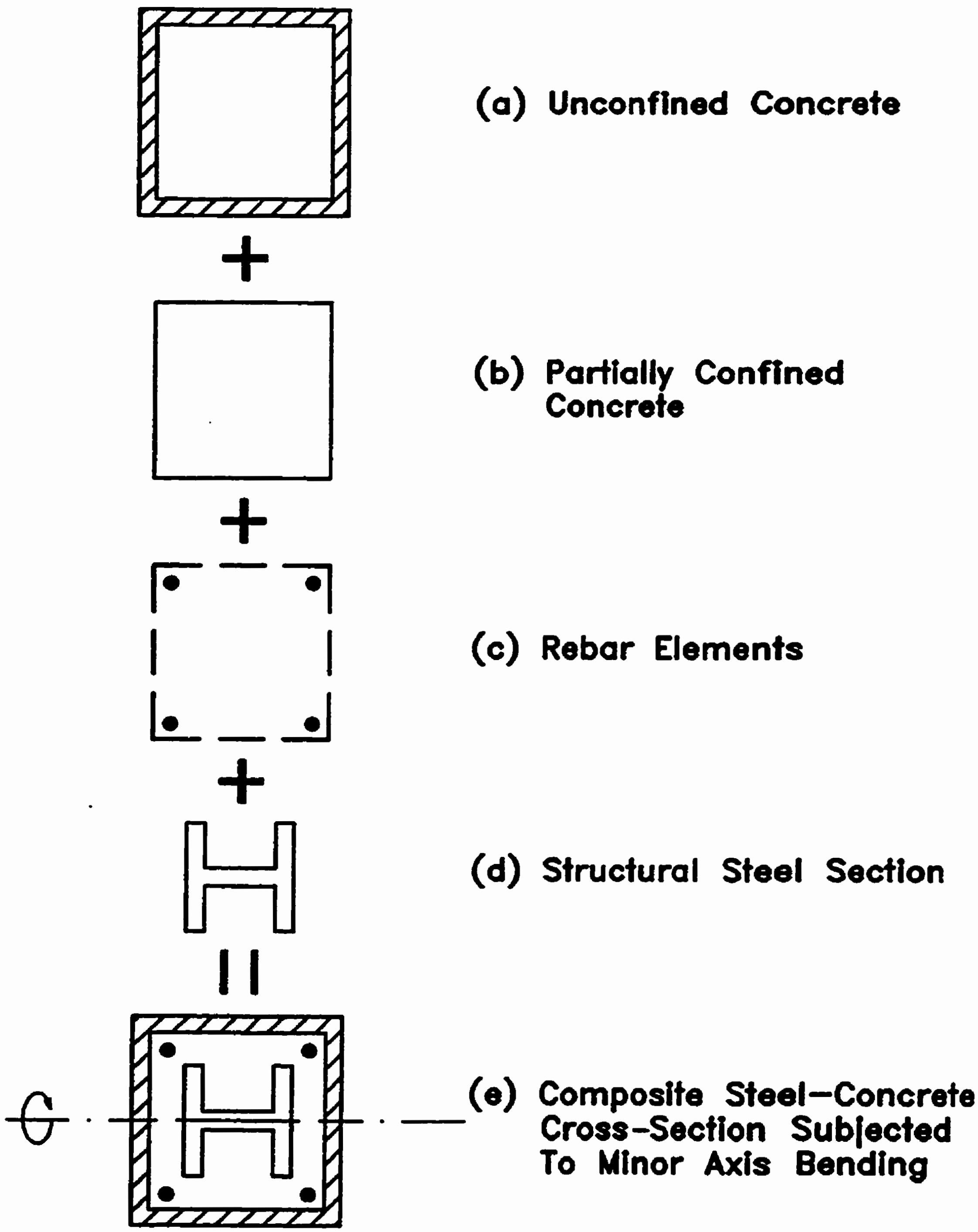


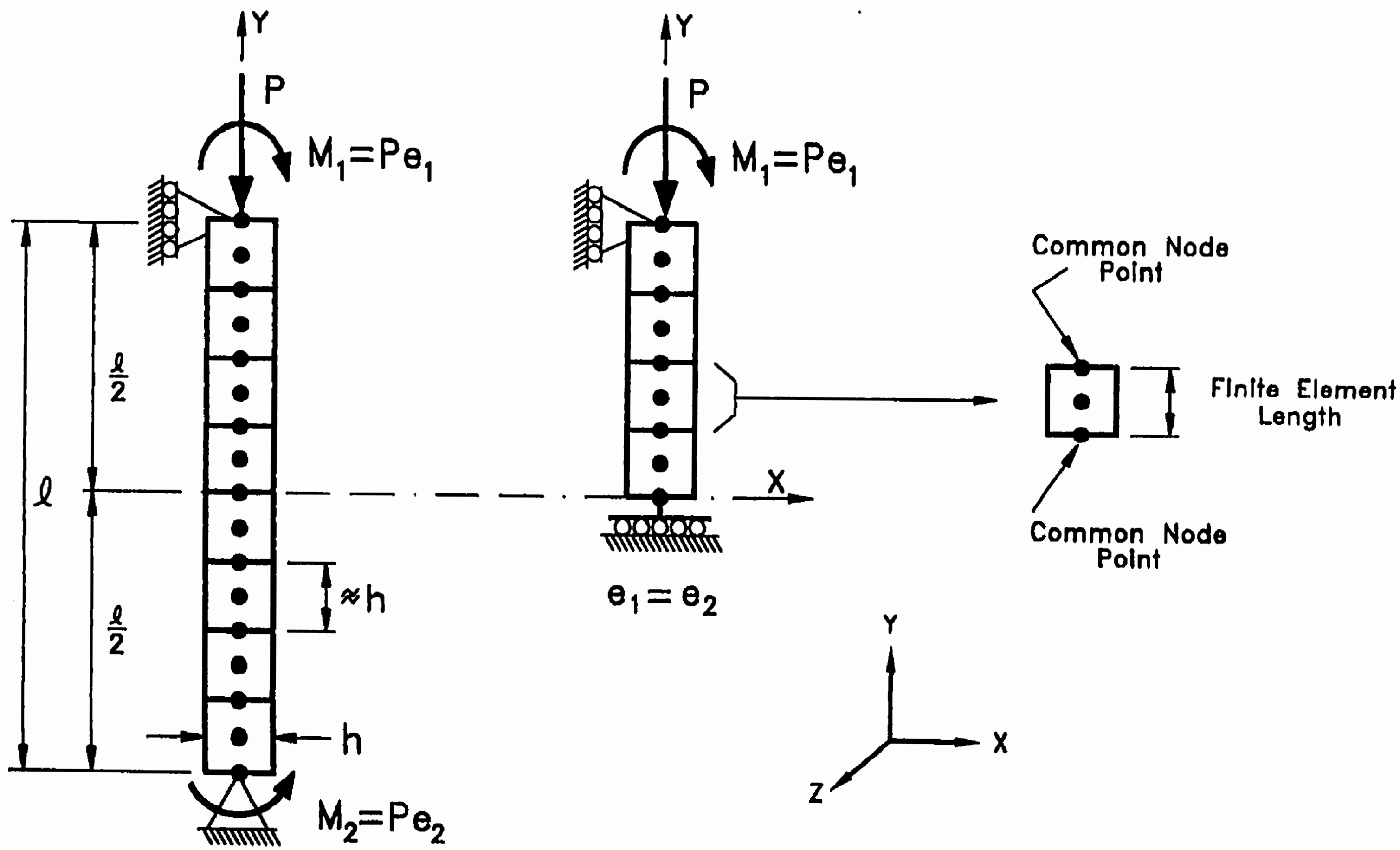

Figure 2.4 - FEM of composite steel-concrete cross-sections subjected to minor axis bending.

central point is used by the FEM software for integration purposes. A typical discretized column with unequal applied end moments is illustrated in Figure 2.5(a).

When using beam elements in an FEM analysis, the results will be sensitive to the chosen finite element length. The length of the three-node finite element segment is the distance between the outer two common node points (Figure 2.5(c)). Choosing a finite element length that is too small will result in the localization of the beam element curvature into a segment of small length. When the curvature is localized into a segment of small length, the element cannot be properly modeled using bending theory since the cross-section can no longer be assumed as being plane. To prevent the localization of beam element curvature, a finite element length equal to or slightly greater than the depth of the cross-section in the plane of bending was used in this study.

### 2.3.1 Boundary Conditions

In the FEM analysis, the column must be restrained in space by the use of boundary conditions. For the column shown in Figure 2.5(a) with bending about the z-axis, the top node is restrained from movement along the x- and z-axis and is restrained from rotation about the y-axis. The bottom node is restrained from movement along the x-, y- and z-axis and is restrained from rotation about the y-axis. These restraints modeled the end conditions used for




(a) FEM of Column when $e_1 \neq e_2$

(b) FEM of Column when $e_1 = e_2$

(c) Finite Element Segment

Figure 2.5 - Modeling and discretization of column for FEM.

loading the physical column specimens found in the literature.

### 2.3.2 Modelling Using Symmetry

For columns with equal and opposite applied end moments, symmetry can be used to reduce the number of elements required in the analysis. An equivalent cantilever column which is one-half the length of the original column can be modeled and is illustrated in Figure 2.5(b). The boundary conditions at the column mid-height restrain movement along the y- and z-axis and restrain rotation about the y- and z-axis. The boundary conditions at the top node of the column are the same as those used for columns with unequal applied end moments.

### 2.3.3 Proportional Column Loading

The column is loaded by introducing an applied joint load and moment at the top node of the column and an applied end moment at the bottom node of the column. The relative magnitude and sign of the applied end moments must reflect the specified end eccentricities used in the column test being simulated. Since the objective was to determine the FEM failure load of the column, the joint loads and moments had to be increased incrementally using a second-order analysis procedure, until failure occurred. Therefore, to begin the analysis, a small proportional loading had to be used. For this study, the initial loading was set at 10 percent of the failure load determined from the physical test. This load could then be increased incrementally until the column failed. To increase the efficiency of the solution, a higher initial loading, closer to the reported failure load, could have been used. This approach would not have had any affect on the final solution (failure load) since after each load increment, the FEM uses an iterative procedure to obtain the deflected shape of the column. However, it was decided that the response of the column (i.e. load-deflection curve) as the column was loaded from a small load to the failure load may be of interest.

### 2.3.4 Special Loading for Columns Under Pure Axial Load

For the analysis of columns under pure axial load, an imperfection is added to the initially ideally straight element model. This imperfection takes into account the possibility of loss of stability under the deflected condition of the column. The imperfection ensures a smooth transition from column stability to column instability. This is due to the fact that a perfectly straight column will remain straight until the critical load is reached and will then buckle suddenly. The large deflections associated with this sudden buckling can not be properly captured using the FEM software. For this study, the initial imperfection was approximated by applying a small uniformly distributed lateral load to the column. The magnitude of the uniformly distributed lateral load was equal to one percent of the self weight of the column and was applied over the entire column length.

## 2.4 STRESS-STRAIN RELATIONSHIP FOR CONCRETE

Two different concrete regions must be defined in the cross-section of both reinforced concrete and composite steel-concrete columns: the unconfined concrete outside of the transverse tie reinforcement and the partially confined concrete inside of the transverse tie reinforcement. Their distinction recognizes the differences in the stress-strain relationships due to the confining action of the rectangular transverse ties. Concrete confinement increases both the compressive strength and ductility of the concrete. Park et al. (1982), Sheikh and Uzemeri (1982), and Sheikh and Yeh (1986) developed methods to determine the effects of increased compressive strength and ductility due to lateral ties for reinforced concrete columns. No methods for determining the effects of confinement on the tensile stress-strain relationship for concrete are available. For this reason, the use of the same tensile stress-strain relation for both unconfined and partially confined concrete was assumed. The stress-strain relationships presented in this section are for columns subjected to monotonic loading.

Based on the recommendations of Skrabek and Mirza (1990), a modified version of the Kent and Park (1971)

Curve for unconfined concrete (Figure 2.6) was used to describe the stress-strain relation for concrete outside of the transverse ties. Equation 2.1 represents the curve between the origin and the peak stress, and Equation 2.2 represents the descending branch of the curve between the peak stress and the stress corresponding to the ultimate strain.

$$f_c = f'_c\left[\frac{2\varepsilon_c}{\varepsilon_o} - \left(\frac{\varepsilon_c}{\varepsilon_o}\right)^2\right] \tag{2.1}$$

$$f_c = f'_c\left[1 - Z(\varepsilon_c - \varepsilon_o)\right] \geq 0.2 f'_c \tag{2.2}$$

where $$Z = \frac{0.5}{\varepsilon_{50u} - \varepsilon_o} \tag{2.3}$$

and $$\varepsilon_{50u} = \frac{3 + \varepsilon_o f'_c}{f'_c - 1000} \tag{2.4}$$

where $f_c$ is the stress of concrete that corresponds to a given value of strain, $\epsilon_c$ (with $\epsilon_c \leq 0.004$); $f'_c$ is the peak compressive strength of concrete; $\epsilon_o$ is the strain of concrete corresponding to the peak stress. For SI conversion replace 3 by 0.0207 MPa and 1000 by 6.895 MPa in Equation 2.4. The strain at the peak stress ($\epsilon_o$) was allowed to vary as a function of the concrete strength (Equation 2.5) rather than using a constant value of 0.002 suggested by Kent and Park (1971):

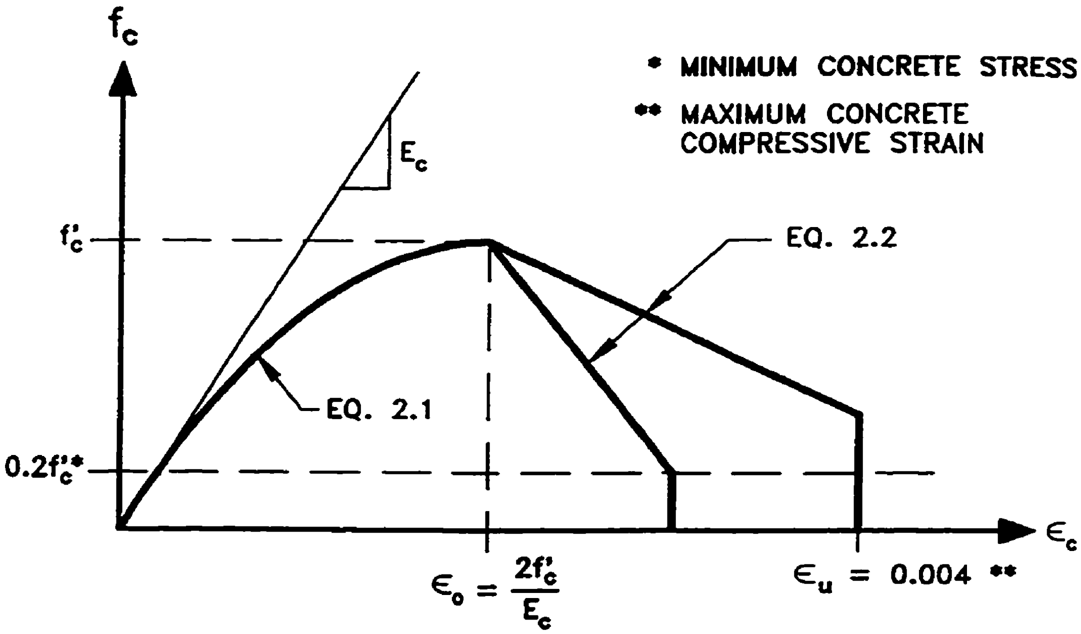



Figure 2.6 - Unconfined concrete compressive stress-strain relationship.

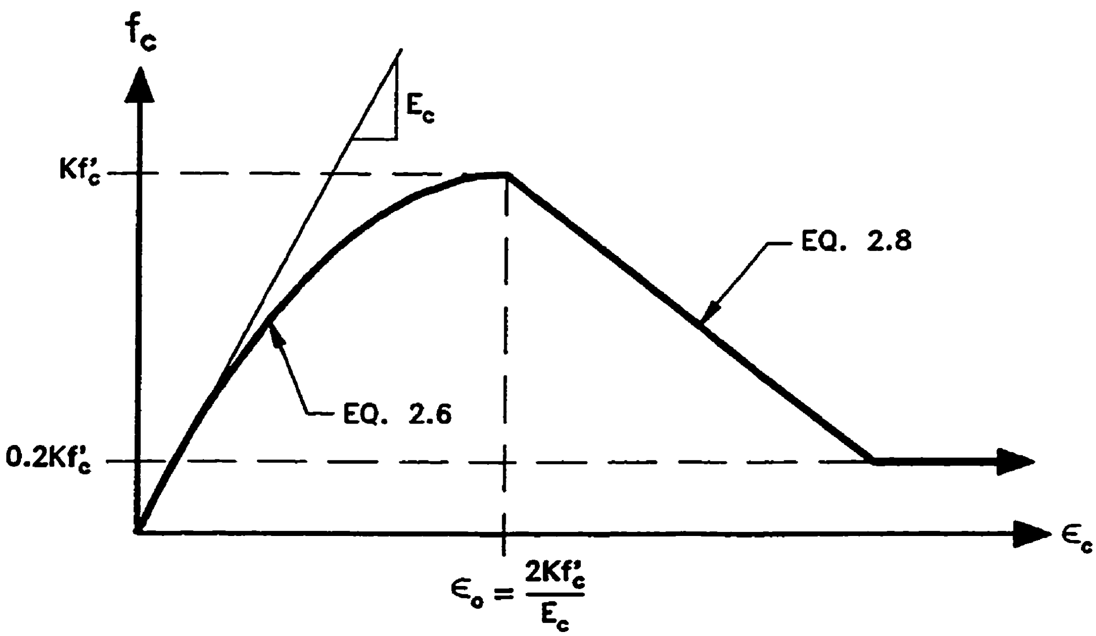



Figure 2.7 - Partially confined concrete compressive stress-strain relationship.

$$\epsilon_o = \frac{2f'_c}{E_c} \tag{2.5}$$

where $E_c$ is the modulus of elasticity of concrete in compression. Two descending branches for Equation 2.2 are shown in Figure 2.6 to illustrate the conditions where the concrete stress becomes equal to zero. The upper curve illustrates the upper limit on concrete strain of 0.004. The lower curve illustrates the lower limit on concrete stress as per Equation 2.2.

The Modified Kent and Park Curve (Park, Priestly and Gill 1982) was used in this study to describe the stress-strain relation for concrete inside of the transverse ties. The Modified Kent and Park Curve (Figure 2.7) was also used by Skrabek and Mirza (1990), and Tikka and Mirza (1992) for modelling partially confined concrete. This curve assumes that the concrete confinement is a function of the concrete cylinder strength $f'_c$, the vertical spacing of the ties $s_h$, the tie/hoop volumetric ratio which is the ratio of volume of transverse ties to the volume of concrete core $\rho''$, and the yield strength of the transverse ties $f_{yh}$. Equation 2.6 is used to describe the curve from the origin to the peak stress ($Kf'_c$), and Equation 2.8 is used to describe the descending branch of the curve.

$$f_c = Kf'_c\left[\frac{2\varepsilon_c}{K\varepsilon_o} - \left(\frac{\varepsilon_c}{K\varepsilon_o}\right)^2\right] \tag{2.6}$$

where
$$K = 1 + \frac{\rho'' f_{yh}}{f'_c} \tag{2.7}$$

$$f_c = Kf'_c\left[1 - Z\left(\varepsilon_c - K\varepsilon_o\right)\right] \geq 0.2Kf'_c \tag{2.8}$$

where
$$Z = \frac{0.5}{\varepsilon_{50u} + \varepsilon_{50h} - K\varepsilon_o} \tag{2.9}$$

and
$$\varepsilon_{50u} = \frac{3 + K\varepsilon_o f'_c}{f'_c - 1000} \tag{2.10}$$

and
$$\varepsilon_{50h} = \frac{3}{4}\rho''\sqrt{\frac{h''}{s_h}} \tag{2.11}$$

where $h''$ is the out-to-out width of the lateral ties. For SI conversion replace 3 by 0.0207 MPa and 1000 by 6.895 MPa in Equation 2.10.

The tensile stress-strain curve used in this study is shown in Figure 2.8. The relationship is assumed to be linear from the origin up to the modulus of rupture, $f_r$, with the elastic modulus for tension assumed to be equal to the initial tangent modulus of elasticity of concrete in compression. The work of Skrabek and Mirza (1990) shows that this simple model, as suggested by Park and Pauley (1975) and Mirza and MacGregor (1989), was sufficient. For the descending branch, the tangent strain softening modulus

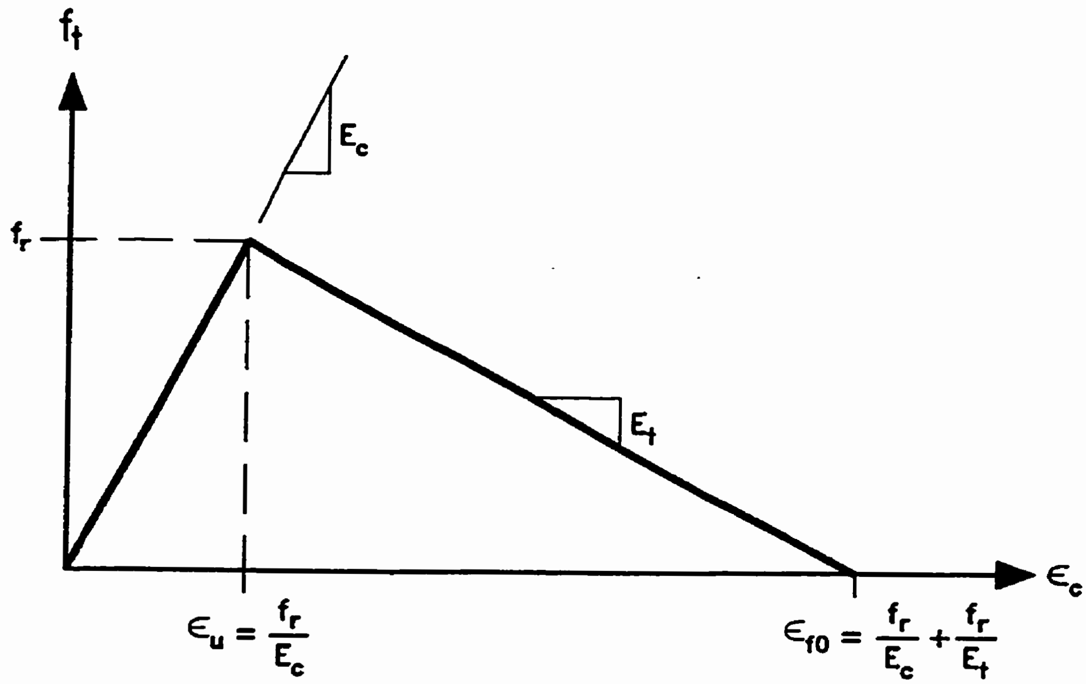



Figure 2.8 - Reinforced concrete tensile stress-strain relationship.

(Equation 2.12) as suggested by Bazant and Oh (1982) was used:

$$E_t = \frac{-70E_c}{57 + f_r} \tag{2.12}$$

where all units are in psi. For SI conversion replace -70 by -0.48 MPa and 57 by 0.39 MPa in Equation 2.12. The descending branch models "tension stiffening" resulting from the interaction of the concrete and longitudinal reinforcing steel after the concrete cracks. The FEM requires the descending branch of the curve to fully define the stress-strain curve for reinforced concrete as will be discussed later.

### 2.4.1 Modification to Physical Properties of Concrete for In-Situ Conditions and Rate of Loading Effect

In an attempt to simulate the actual behavior of the physical test columns available in the literature, the physical properties of concrete were modified to reflect in-situ conditions and account for rate of loading effects. Due to the variability of the concrete strength and stiffness, the material properties can not be accurately determined directly from standard concrete cylinder tests. Mirza, Hatzinikolas and MacGregor (1979) undertook an extensive investigation to statistically describe the strength of concrete. Their recommendations were used in this study. It was suggested that the in-situ strength of concrete could be approximated from the standard concrete cylinder strength by using Equation 2.13:

$$f_{ci} = 0.968 f'_c \tag{2.13}$$

where $f_{ci}$ is the in-situ compressive strength of concrete; and $f'_c$ is the concrete strength from standard cylinder tests. To account for the rate of loading effect, the concrete strength should be modified by using Equation 2.14:

$$f_{cr} = 0.89 f_{ci}\left(1 + 0.08\log\frac{f_{ci}}{t}\right) \tag{2.14}$$

where $f_{cr}$ is the concrete strength including the rate of loading effect; and $t$ is the testing time in seconds. For SI conversion, multiply $t$ by 0.0069 in Equation 2.14.

Equation 2.15 was suggested (Mirza, Hatzinikolas and MacGregor 1979) to account for the rate of loading effect on the modulus of rupture:

$$f_{rr} = 8.3\sqrt{f_{ci}}\left[0.96\left(1 + 0.11\log\frac{f_{ci}}{t}\right)\right] \tag{2.15}$$

where $f_{rr}$ is the modulus of rupture including the rate of loading effect. For SI conversion replace 8.3 by 0.69 and multiply $t$ by 0.0069 in Equation 2.15. Similarly, Equation

2.16 was suggested (Mirza, Hatzinikolas and MacGregor 1979) to account for the rate of loading effect on the modulus of elasticity of concrete:

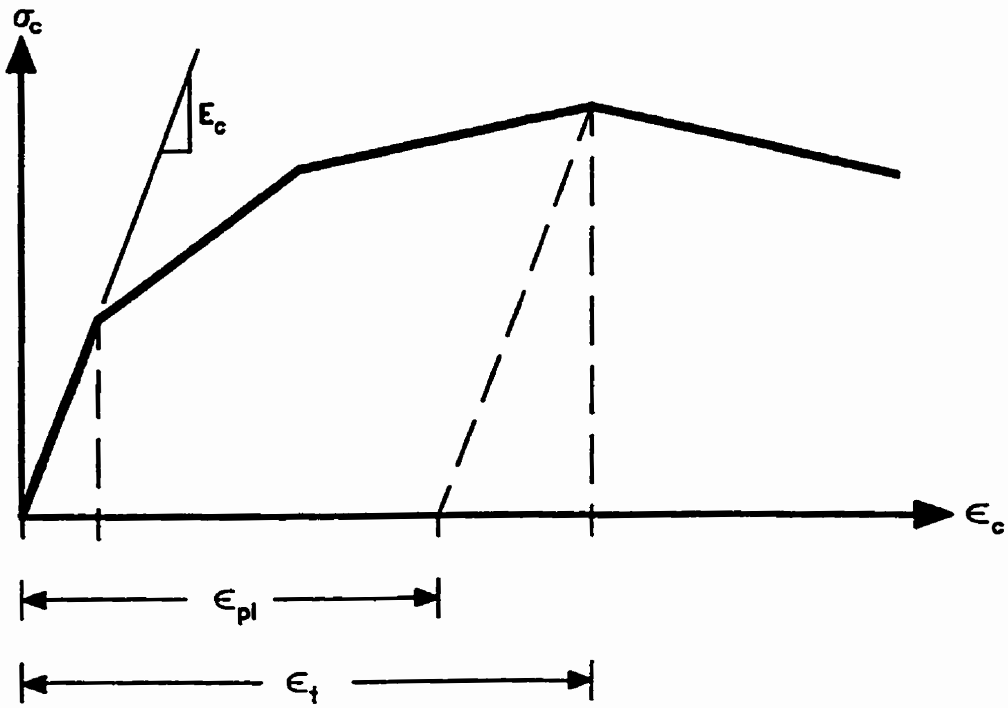
$$E_{cr} = 60400\sqrt{f_{cr}}\left[1.16 - 0.08\log(t)\right] \qquad (2.16)$$

where $E_{cr}$ is the modulus of elasticity of concrete including the rate of loading effect. For SI conversion replace 60400 by 5015 MPa in Equation 2.16.

### 2.4.2 Stress-Strain Curve of Concrete Used for FEM

The stress-strain curve is idealized by the FEM software using several finite segments. For this study, the entire stress-strain curve was divided into 17 segments: 8 equally spaced segments from the origin to the peak stress, and 9 equally spaced segments from the peak stress to the stress corresponding to the ultimate strain. Before the points on the curve could be input, the strain values had to be modified in terms of plastic strain values. Plastic strain values, not total strain values, are used in defining the softening behavior of concrete by the FEM software. The plastic strain is illustrated in Figure 2.9 and is defined by Equation 2.17:

$$\varepsilon_{pl} = \varepsilon_t - \frac{\sigma_c}{E_c} \qquad (2.17)$$




NOTE: Plastic strain values, not total strain values, are used in defining the stress-strain curve for FEM.

$$\epsilon_{pl} = \epsilon_t - \frac{\sigma_c}{E_c}$$

Figure 2.9 - Comparison of total strain and plastic strain values as used by FEM.

where $\epsilon_{pl}$ is the plastic strain; $\epsilon_t$ and $\sigma_c$ are respectively the strain and stress of the point under consideration; and $E_c$ is the elastic modulus of concrete.

In modelling the concrete behavior using the FEM software, special attention must be given to the concrete cracking behavior. The analysis of both reinforced concrete and composite steel-concrete columns using the FEM software requires the modelling of the concrete and longitudinal reinforcing steel (rebar) as well as their interaction. This modelling is accomplished by combining plain concrete elements with longitudinal reinforcing steel "rebar" elements. Rebar elements are superimposed or imbedded into the concrete element mesh. With this simplification, the behavior of the concrete can be considered to be independent of the longitudinal reinforcing steel. The effects associated with the concrete-rebar interface, such as bond slip and dowel action, are approximately modeled by introducing "tension stiffening" into the concrete model. In this way, the load transfer across cracks by the rebar can be simulated.

Instead of tracking each individual micro crack, a smeared crack model is used. In this way the constitutive calculations can be performed independently at each integration point of the finite element model. The presence of cracks enters into these calculations by the way in which the crack affects the stress and material stiffness associated with the integration point. That is, the model assumes that cracking causes damage in the sense that open cracks can be represented by a loss of elastic stiffness. Since the effects of the cracks are only considered at each integration point, the solution is mesh-sensitive for unreinforced concrete. For reinforced concrete, the interaction between the concrete and rebar significantly reduces this mesh sensitivity provided that a reasonable amount of "tension stiffening" is introduced in the concrete model to simulate these effects. For the FEM software, some degree of tension stiffening must be specified to prevent numerical instabilities.

Tension stiffening in the concrete model is dependent on the amount of reinforcement, the quality of the bond between the rebar and concrete, and the relative orientation of the crack with respect to the reinforcement. Tension stiffening in the FEM model accounts for the fact that, after reaching the rupture modulus of concrete, the tensile strength of reinforced concrete does not suddenly drop to zero but declines gradually as the strains increase. The tangent strain softening modulus, as defined in Equation 2.12, was used to model this behavior. The only input required by the FEM software is the strain at which the tensile strength of concrete becomes zero. It was calculated from Equation 2.18:

$$\varepsilon_{f0} = \frac{f_r}{E_c} + \frac{f_r}{E_t} \tag{2.18}$$

where $\epsilon_{t0}$ is the strain at which the tensile stress of concrete becomes zero. The value calculated from Equation 2.18 was assumed to provide a satisfactory approximation of the post-cracking behavior of the physical test columns.

The modelling of the interaction between the structural steel section and the surrounding concrete for composite steel-concrete columns does not use the same approach as for the longitudinal reinforcing steel bars. Although the steel section is imbedded into the surrounding concrete, the FEM does not have the capability of modelling this behavior as was done using rebar elements. However, as was noted earlier, the concrete and structural steel elements are connected at common node points. Since the common node points are located at a distance approximately equal to depth of the cross-section in the plane of bending, any "slip" between the concrete and structural steel section between the common node points will, therefore, be minimal and will not have any measurable effect on the FEM results.

## 2.5 STRESS-STRAIN RELATIONSHIP FOR STEEL

An elastic-perfectly plastic stress-strain curve was used to describe the behavior of both the longitudinal reinforcing steel and the structural steel section. Strain hardening and residual stresses were not included in this study. For composite steel-concrete columns, Skrabek and Mirza (1990) found that strain hardening had no effect on the strength ratios for slender columns ($\ell/h$>6.6) with end eccentricity ratios ($e/h$) between 0.05 and 4.0. However, strain hardening had a significant effect on the strength ratios for slender columns ($\ell/h$>6.6) subjected to pure bending. For composite steel-concrete cross-sections, strain hardening had no effect on the strength ratios for $e/h \leq 0.6$, some effect for $0.6 < e/h \leq 1.5$ and a significant effect for $e/h > 1.5$. The stress-strain curve for compression was assumed to be the same as used for tension and is illustrated in Figure 2.10.

### 2.5.1 Stress-Strain Curve of Steel Used for FEM

The measured yield strengths of the longitudinal reinforcing and structural steels were used for the FEM analysis. The FEM software is capable of modelling an elastic-plastic stress-strain curve. Two input variables are required to model this curve: the modulus of elasticity of the steel, $E_s$, and the yield stress, $f_y$.

## 2.6 SECOND-ORDER NONLINEAR ANALYSIS FOR FEM

Due to the nonlinear behavior of the material stress-strain curves and the nonlinear load-deflection response of reinforced concrete and composite steel-concrete columns, a

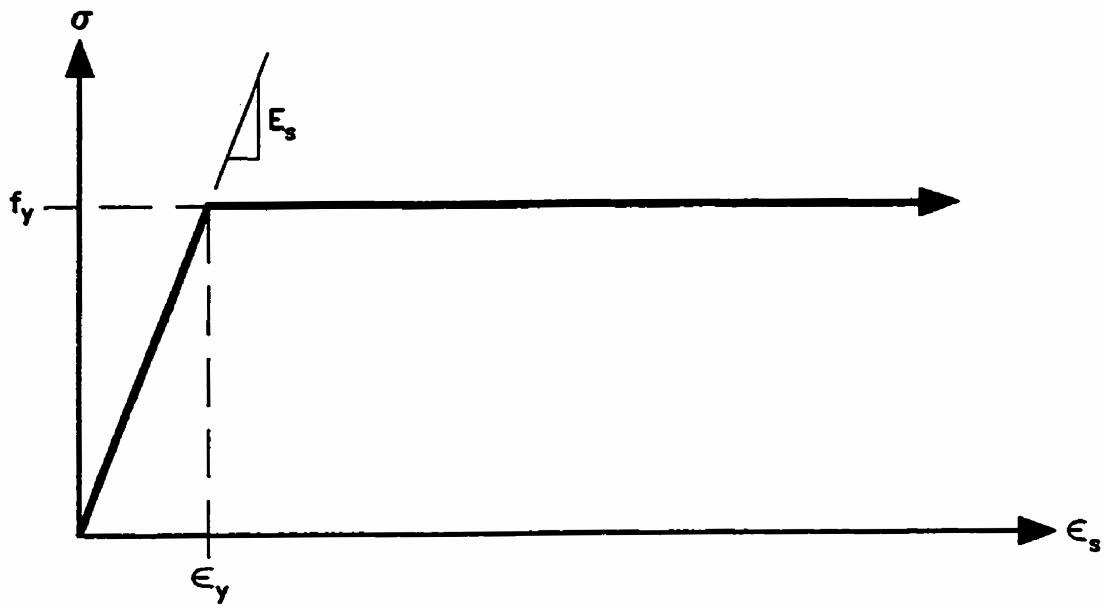



Figure 2.10 - Structural steel and reinforcing steel stress-strain relationships.

second-order nonlinear method was required. In a second-order analysis, the deflected shape of the member is constantly changing, thus requiring repeated updating of the stiffness matrix. For this reason an iterative process is required to determine a solution. The loads must be applied in a series of load increments. After a load increment is applied to the column, a linear analysis is performed using an updated stiffness matrix from the previous iteration. Therefore, the equilibrium state of the column at the end of one load increment is used to formulate the stiffness matrix that is used for the following load increment. A tolerance must be specified to indicate that satisfactory convergence has been reached.

The magnitude of the load increment used for the analysis has a significant effect on the solution time, accuracy and the convergence of the solution. Using a small load increment will increase the required computational time significantly. Using a large load increment may not properly capture the load-deflection response of the column and can also increase the computational time significantly if several analysis iterations are required to converge to a solution. The FEM software has two methods for solving the nonlinear equilibrium equations: the Newton-Raphson Load Control Method and the Modified Riks Method.

The Newton-Raphson Load Control method applies increasing load increments to a member and then iterates to an equilibrium condition. If the incremented load is higher than the maximum load, convergence is not ensured. The main disadvantage of the Newton-Raphson Load Control Method, therefore, is that it can break down completely when the maximum load is reached. The Modified Riks method, however, traces the load-deflection response up to and beyond the maximum load. The basis of this method is to use the load magnitude as an additional unknown and thus control the increments taken along the load-displacement response curve. Therefore, the FEM software automatically modifies the load increments in each step in an attempt to move equal arc lengths on the load-deflection response curve for the column. This method provides a solution regardless of whether the response is stable or unstable. Also, the behavior of the column up to and beyond the maximum load point on the load-deflection response curve can be obtained. For this study, the more robust Modified Riks Method was used.

## 3 - COMPARISON OF FEM METHOD WITH EXPERIMENTAL RESULTS

In this chapter, the ultimate strengths computed from FEM are compared to the ultimate strengths of physical tests obtained from the literature. The load cases studied for reinforced concrete columns, composite steel-concrete columns with major axis bending and composite steel-concrete columns with minor axis bending are discussed individually in Sections 3.1 to 3.3, respectively. No new physical tests were conducted for this study. Tests gathered from the literature included concentric loading, eccentric loading causing bending about one axis, and pure bending about an axis for columns with slenderness ratios $\ell/h$ (length to overall depth of the concrete cross-section) ranging from 2.0 to 40.0.

Problems were encountered during the process of interpreting the experimental results available from the literature. These problems are summarized below:

1) The specified length of some columns was not clearly defined. This occurred when columns contained haunches at the ends or when special testing apparatus were used. In the latter case, the actual column lengths were given but the location of the pin support or knife-edges, where the columns are allowed to rotate, were not clearly dimensioned or defined. This

pertains to tests conducted by Chang and Ferguson (1963), and Stevens (1965).

2) The location, quantity and yield strength of the longitudinal reinforcement were in some cases unclear or not provided. This pertains to tests conducted by Bunni (1975), Bondale (1966), Johnson and May (1978), and Stevens (1965).

3) The method of determining the concrete strength from cubes was unclear (cubes tested parallel or perpendicular to the direction of casting). This pertains to tests conducted by Gaede (1958), Ramu et al. (1969), Mehmel et al. (1969), Bunni (1975), Bondale (1966), Procter (1967), Johnson and May (1978), Roik and Mangerig (1987), Roik and Schwalbenhofer (1988), Stevens (1965), and Anslijn and Janss (1974).

4) In some cases, the test specimens were very small. This pertains to tests conducted by Kim and Yang (1995) and Stevens (1965).

5) The yield strength of the transverse tie reinforcement was in many cases not given. This pertains to the majority of tests examined in this study.

For some of the physical tests, 4-inch, 6-inch and 8-inch cubes were tested instead of the standard 6-inch diameter by 12-inch high cylinders to establish the concrete strength. In these cases the strength reported had to be converted to an equivalent standard cylinder strength.

Many different factors for obtaining an equivalent cylinder strength have been proposed by various authors. Roderick and Rogers (1969) and Roderick and Loke (1974) used Equation 3.1 as recommended by Evans (1943).

$$f'_c = 1.035u - 700 \tag{3.1}$$

in which the cube strength ($u$), and the equivalent cylinder strength ($f'_c$), are in pounds per square inch. Virdi and Dowling (1973) used a factor of 0.64 for converting the strength of a 6-inch cube to an equivalent cylinder. Furlong (1976) used a factor of approximately 0.8 to convert the strength of a 4-inch cube to obtain an equivalent 6-inch cylinder strength. Johnson and May (1978) used a factor of 0.76 for obtaining an equivalent cylinder strength from a 6-inch cube. Roik and Bergmann (1989) used a factor of 0.83 to convert the 4-inch cube strength and 0.85 to convert the 8-inch cube strength to an equivalent 6-inch cylinder strength.

For this study, two separate equations were used to convert cube strengths to an equivalent 6-inch cylinder strength. Equation 3.2 is based on the statistical theory of brittle fracture of solids (Bolotin 1969), as reproduced by Mirza, Hatzinikolas and MacGregor (1979):

$$f = f_o\left(0.58 + 0.42\left(\frac{v_o}{v}\right)^{\frac{1}{3}}\right) \tag{3.2}$$

in which $f_o$ and $v_o$ are the concrete strength and volume of a 4-inch cube, and $f$ and $v$ represent the concrete strength and volume of the given or desired size cube (6-inch cube strength is required).

Equation 3.2 accounts for the differences in strength due to volume differences of a cube with respect to a 4-inch cube. This equation was used to first convert the strength of a cube of a given size to the strength of a 4-inch cube, and then to convert the strength of the 4-inch cube to the strength of a 6-inch cube. Once an equivalent 6-inch cube strength is obtained, L'Hermite's (1955) equation (Equation 3.3) was used to convert the strength of the 6-inch cube to the strength of an equivalent 6-inch diameter by 12-inch high cylinder:

$$f'_c = \left(0.76 + 0.2\log\left(\frac{f_{cu}}{2840}\right)\right)f_{cu} \tag{3.3}$$

in which $f_{cu}$ is the 6-inch cube strength and $f'_c$ is the 6-inch by 12 inch cylinder strength in pounds per square inch. For SI units replace 2840 psi with 19.6 MPa.

In most cases steel coupons and bar samples were tested to determine the yield strength of the structural steel sections and longitudinal reinforcing bars. There were instances where only the nominal strengths were specified. As stated previously, the transverse tie reinforcement yield strengths were generally not given. In these instances, the yield strength of the transverse ties was assumed to be equal to the longitudinal reinforcing bar yield strength.

## 3.1 COMPARISON OF FEM METHOD WITH EXPERIMENTAL RESULTS FOR REINFORCED CONCRETE COLUMNS

The analysis of reinforced concrete columns was divided into two separate groups: columns with equal and opposite applied end moments (without moment gradient) and columns with unequal applied end moments (with moment gradient).

### 3.1.1 Reinforced Concrete Columns Without Moment Gradient

The ultimate strengths predicted by FEM were compared with the ultimate strengths of 384 physical tests taken from Hognestad (1951), Ernst et al. (1953), Viest et al. (1956), Gaede (1958), Bresler (1960), Bresler and Gilbert (1961), Chang and Ferguson (1963), Pfister (1964), Roy and Sozen (1964), Todeschini et al. (1964), Hudson (1965), Martin and Olivieri (1965), Mehmel et al. (1969), Ramu et al. (1969), Drysdale and Huggins (1971), Goyal and Jackson (1971), Bunni (1975), Green and Hellesland (1975), Heimdahl and Bianchini (1975), Sheikh and Uzumeri (1980), Scott et al. (1982), Razvi and Saatcioglu (1989), Cusson and Paultre (1992), Fang et al. (1994), and Kim and Yang (1995). Thirty of the physical tests were eventually removed from the comparison for reasons that will be discussed later in this section.

A description of these 384 physical tests used for the comparison of tested to FEM strength for reinforced concrete columns is given in Table 3.1.1. The table includes information on the geometric and material properties of test columns. Included in the table is the ratio of tested to FEM ultimate strength (strength ratio) for each of the 384 column specimens. The strength ratio was taken as the ratio of axial load capacities of a column.

The plot of the tested strength against the FEM strength (Figure 3.1.1(a)) shows a relatively narrow band of strength ratios. This indicates that the FEM model was able to predict the tested strength of the columns quite accurately with no apparent or significant outliers. Also, as the strengths of the columns increase, there is a proportional increase in the magnitude of error. This is expected since the percentage of error remains relatively constant. A histogram plotting the frequency, in percent, against the strength ratio (Figure 3.1.1(b)) shows a symmetric distribution of values about the mean. The mean strength ratio of all 384 test columns was 0.981 with a coefficient of variation of 12.6 percent (Figure 3.1.1(b)).

Table 3.1.1 - Description of Reinforced Concrete Columns without Moment Gradient Used for Comparison with FEM Ultimate Strength

| Author | Col. Desig. | h (in.) | b (in.) | $f'_c$ (psi) | $\rho_{rs}$ | $\frac{\rho_{rs} f_{yr}}{f'_c}$ | Tie/Hoop Vol. Ratio $\rho''$ | $\ell/h$ | e/h | TEST VALUES Applied Axial Load (kips) | TEST VALUES Applied BM at Ends (kip-in) | Strength Ratio | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Hognestad (1951) | B-6a | 10.0 | 10.0 | 4080 | 0.0248 | 0.2668 | 0.0043 | 7.5 | 0.000 | 456.0 | 0.0 | 0.9612 | |
| | B-6b | 10.0 | 10.0 | 4040 | 0.0248 | 0.2695 | 0.0043 | 7.5 | 0.000 | 420.0 | 0.0 | 0.8854 | |
| | C-6a | 10.0 | 10.0 | 2020 | 0.0248 | 0.5390 | 0.0043 | 7.5 | 0.000 | 225.0 | 0.0 | 0.7587 | ** |
| | C-6b | 10.0 | 10.0 | 1520 | 0.0248 | 0.7163 | 0.0043 | 7.5 | 0.000 | 202.0 | 0.0 | 0.7956 | ** |
| | A-7a | 10.0 | 10.0 | 5240 | 0.0248 | 0.2078 | 0.0043 | 7.5 | 0.325 | 274.0 | 890.5 | 1.1357 | |
| | A-7b | 10.0 | 10.0 | 5810 | 0.0248 | 0.1874 | 0.0043 | 7.5 | 0.250 | 284.0 | 710.0 | 0.9216 | |
| | B-7a | 10.0 | 10.0 | 4080 | 0.0248 | 0.2668 | 0.0043 | 7.5 | 0.250 | 256.0 | 640.0 | 1.0600 | |
| | B-7b | 10.0 | 10.0 | 4040 | 0.0248 | 0.2695 | 0.0043 | 7.5 | 0.250 | 248.0 | 620.0 | 1.0341 | |
| | C-7a | 10.0 | 10.0 | 1970 | 0.0248 | 0.5526 | 0.0043 | 7.5 | 0.250 | 141.0 | 352.5 | 0.8856 | ** |
| | C-7b | 10.0 | 10.0 | 1520 | 0.0248 | 0.7163 | 0.0043 | 7.5 | 0.250 | 126.8 | 317.0 | 0.8985 | ** |
| | A-8a | 10.0 | 10.0 | 5520 | 0.0248 | 0.1972 | 0.0043 | 7.5 | 0.500 | 162.0 | 810.0 | 0.9126 | |
| | A-8b | 10.0 | 10.0 | 5810 | 0.0248 | 0.1874 | 0.0043 | 7.5 | 0.500 | 152.0 | 760.0 | 0.8345 | |
| | B-8a | 10.0 | 10.0 | 4700 | 0.0248 | 0.2316 | 0.0043 | 7.5 | 0.500 | 156.0 | 780.0 | 0.9555 | |
| | B-8b | 10.0 | 10.0 | 4260 | 0.0248 | 0.2556 | 0.0043 | 7.5 | 0.500 | 146.0 | 730.0 | 0.9414 | |
| | C-8a | 10.0 | 10.0 | 1820 | 0.0248 | 0.5982 | 0.0043 | 7.5 | 0.500 | 99.0 | 495.0 | 0.9638 | ** |
| | C-8b | 10.0 | 10.0 | 1820 | 0.0248 | 0.5982 | 0.0043 | 7.5 | 0.500 | 99.0 | 495.0 | 0.9638 | ** |
| | A-9a | 10.0 | 10.0 | 5100 | 0.0248 | 0.2135 | 0.0043 | 7.5 | 0.750 | 89.0 | 667.5 | 0.9073 | |
| | A-9b | 10.0 | 10.0 | 5170 | 0.0248 | 0.2106 | 0.0043 | 7.5 | 0.750 | 91.2 | 684.0 | 0.9228 | |
| | B-9a | 10.0 | 10.0 | 4700 | 0.0248 | 0.2316 | 0.0043 | 7.5 | 0.750 | 94.0 | 705.0 | 0.9666 | |
| | B-9b | 10.0 | 10.0 | 4370 | 0.0248 | 0.2491 | 0.0043 | 7.5 | 0.750 | 89.5 | 671.3 | 0.9329 | |
| | C-9a | 10.0 | 10.0 | 1880 | 0.0248 | 0.5791 | 0.0043 | 7.5 | 0.750 | 73.0 | 547.5 | 0.9475 | ** |
| | C-9b | 10.0 | 10.0 | 1730 | 0.0248 | 0.6293 | 0.0043 | 7.5 | 0.750 | 65.5 | 491.3 | 0.8722 | ** |
| | A-10a | 10.0 | 10.0 | 5100 | 0.0248 | 0.2135 | 0.0043 | 7.5 | 1.250 | 46.1 | 576.3 | 0.9663 | |
| | A-10b | 10.0 | 10.0 | 5170 | 0.0248 | 0.2106 | 0.0043 | 7.5 | 1.250 | 44.0 | 550.0 | 0.9174 | |
| | B-10a | 10.0 | 10.0 | 4260 | 0.0248 | 0.2556 | 0.0043 | 7.5 | 1.250 | 43.5 | 543.8 | 0.9316 | |
| | B-10b | 10.0 | 10.0 | 4370 | 0.0248 | 0.2491 | 0.0043 | 7.5 | 1.250 | 44.0 | 550.0 | 0.9393 | |
| | C-10a | 10.0 | 10.0 | 2300 | 0.0248 | 0.4734 | 0.0043 | 7.5 | 1.250 | 44.5 | 556.3 | 1.0176 | ** |
| | C-10b | 10.0 | 10.0 | 1770 | 0.0248 | 0.6151 | 0.0043 | 7.5 | 1.250 | 45.0 | 562.5 | 1.0460 | ** |
| | B-11a | 10.0 | 10.0 | 3870 | 0.0480 | 0.5433 | 0.0043 | 7.5 | 0.000 | 500.0 | 0.0 | 0.8930 | |
| | B-11b | 10.0 | 10.0 | 4010 | 0.0480 | 0.5243 | 0.0043 | 7.5 | 0.000 | 485.0 | 0.0 | 0.8479 | |
| | C-11b | 10.0 | 10.0 | 2070 | 0.0480 | 1.0157 | 0.0043 | 7.5 | 0.000 | 353.0 | 0.0 | 0.8812 | ** |
| | A-12a | 10.0 | 10.0 | 4150 | 0.0480 | 0.5066 | 0.0043 | 7.5 | 0.250 | 315.0 | 787.5 | 1.0400 | |
| | A-12b | 10.0 | 10.0 | 5050 | 0.0480 | 0.4163 | 0.0043 | 7.5 | 0.250 | 325.0 | 812.5 | 0.9613 | |
| | B-12a | 10.0 | 10.0 | 4300 | 0.0480 | 0.4889 | 0.0043 | 7.5 | 0.250 | 303.0 | 757.5 | 0.9813 | |
| | B-12b | 10.0 | 10.0 | 4010 | 0.0480 | 0.5243 | 0.0043 | 7.5 | 0.250 | 284.0 | 710.0 | 0.9556 | |
| | C-12a | 10.0 | 10.0 | 2300 | 0.0480 | 0.9141 | 0.0043 | 7.5 | 0.250 | 252.0 | 630.0 | 1.0986 | ** |
| | C-12b | 10.0 | 10.0 | 2200 | 0.0480 | 0.9556 | 0.0043 | 7.5 | 0.250 | 230.0 | 575.0 | 1.0207 | ** |
| | A-13a | 10.0 | 10.0 | 5350 | 0.0480 | 0.3930 | 0.0043 | 7.5 | 0.500 | 220.0 | 1100.0 | 0.9737 | |
| | A-13b | 10.0 | 10.0 | 4850 | 0.0480 | 0.4335 | 0.0043 | 7.5 | 0.500 | 210.0 | 1050.0 | 0.9796 | |
| | B-13a | 10.0 | 10.0 | 3580 | 0.0480 | 0.5873 | 0.0043 | 7.5 | 0.500 | 180.0 | 900.0 | 0.9725 | |
| | B-13b | 10.0 | 10.0 | 4290 | 0.0480 | 0.4901 | 0.0043 | 7.5 | 0.500 | 206.0 | 1030.0 | 0.9129 | |
| | C-13a | 10.0 | 10.0 | 2300 | 0.0480 | 0.9141 | 0.0043 | 7.5 | 0.500 | 151.0 | 755.0 | 0.9753 | ** |
| | C-13b | 10.0 | 10.0 | 2070 | 0.0480 | 1.0157 | 0.0043 | 7.5 | 0.500 | 137.0 | 685.0 | 0.9181 | ** |
| | A-14a | 10.0 | 10.0 | 5350 | 0.0480 | 0.3930 | 0.0043 | 7.5 | 0.750 | 142.0 | 1065.0 | 0.8889 | |
| | A-14b | 10.0 | 10.0 | 5100 | 0.0480 | 0.4122 | 0.0043 | 7.5 | 0.750 | 153.0 | 1147.5 | 0.9701 | |
| | B-14a | 10.0 | 10.0 | 3580 | 0.0480 | 0.5873 | 0.0043 | 7.5 | 0.750 | 138.8 | 1041.0 | 0.9978 | |
| | C-14a | 10.0 | 10.0 | 1950 | 0.0480 | 1.0782 | 0.0043 | 7.5 | 0.750 | 115.5 | 866.3 | 1.0341 | ** |
| | C-14b | 10.0 | 10.0 | 2070 | 0.0480 | 1.0157 | 0.0043 | 7.5 | 0.750 | 104.0 | 780.0 | 0.9140 | ** |

Table 3.1.1 - Description of Reinforced Concrete Columns without Moment Gradient Used for Comparison with FEM Ultimate Strength

| Author | Col. Desig. | h (in.) | b (in.) | $f'_c$ (psi) | $\rho_{rs}$ | $\frac{\rho_{rs} f_{yr}}{f'_c}$ | Tie/Hoop Vol. Ratio $\rho''$ | $\ell/h$ | e/h | TEST VALUES Applied Axial Load (kips) | TEST VALUES Applied BM at Ends (kip-in) | Strength Ratio | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Hognestad (1951) | A-15a | 10.0 | 10.0 | 5100 | 0.0480 | 0.4122 | 0.0043 | 7.5 | 1.250 | 88.0 | 1100.0 | 1.0720 | |
| | A-15b | 10.0 | 10.0 | 4850 | 0.0480 | 0.4335 | 0.0043 | 7.5 | 1.250 | 79.0 | 987.5 | 0.9632 | |
| | B-15a | 10.0 | 10.0 | 3800 | 0.0480 | 0.5533 | 0.0043 | 7.5 | 1.250 | 74.0 | 925.0 | 0.9213 | |
| | B-15b | 10.0 | 10.0 | 4630 | 0.0480 | 0.4541 | 0.0043 | 7.5 | 1.250 | 84.5 | 1056.3 | 1.0386 | |
| | C-15a | 10.0 | 10.0 | 1950 | 0.0480 | 1.0782 | 0.0043 | 7.5 | 1.250 | 72.5 | 906.3 | 0.9836 | ** |
| | C-15b | 10.0 | 10.0 | 2070 | 0.0480 | 1.0157 | 0.0043 | 7.5 | 1.250 | 74.5 | 931.3 | 0.9997 | ** |
| | | | | | | | | | | | Mean | 0.9532 | (0.9570)*** |
| | | | | | | | | | | | Coeff. of Variation | 0.0745 | (0.0654)*** |
| Ernst, Hromadik & Riveland (1953) | 1 | 6.0 | 6.0 | 2920 | 0.0122 | 0.2166 | 0.0039 | 2.0 | 0.000 | 113.0 | 0.0 | 1.0004 | |
| | 2 | 6.0 | 6.0 | 2920 | 0.0122 | 0.2166 | 0.0039 | 5.0 | 0.000 | 97.0 | 0.0 | 0.8660 | |
| | 3 | 6.0 | 6.0 | 2920 | 0.0122 | 0.2166 | 0.0039 | 15.0 | 0.000 | 110.0 | 0.0 | 0.9463 | |
| | 4 | 6.0 | 6.0 | 2920 | 0.0122 | 0.2166 | 0.0039 | 25.0 | 0.000 | 101.0 | 0.0 | 0.8703 | |
| | 5 | 6.0 | 6.0 | 2920 | 0.0122 | 0.2166 | 0.0039 | 2.0 | 0.125 | 95.0 | 71.3 | 1.1706 | |
| | 6 | 6.0 | 6.0 | 2920 | 0.0122 | 0.2166 | 0.0039 | 5.0 | 0.125 | 92.0 | 69.0 | 1.1504 | |
| | 7 | 6.0 | 6.0 | 2920 | 0.0122 | 0.2166 | 0.0039 | 15.0 | 0.125 | 80.0 | 60.0 | 1.1449 | |
| | 8 | 6.0 | 6.0 | 2920 | 0.0122 | 0.2166 | 0.0039 | 25.0 | 0.125 | 65.0 | 48.8 | 1.2896 | |
| | 11 | 6.0 | 6.0 | 2920 | 0.0122 | 0.2166 | 0.0039 | 15.0 | 0.250 | 58.2 | 87.3 | 1.2127 | |
| | 12 | 6.0 | 6.0 | 2920 | 0.0122 | 0.2166 | 0.0039 | 25.0 | 0.250 | 38.7 | 58.1 | 1.1973 | |
| | 16 | 6.0 | 6.0 | 2920 | 0.0122 | 0.2166 | 0.0039 | 25.0 | 0.375 | 24.8 | 55.8 | 1.0742 | |
| | | | | | | | | | | | Mean | 1.0839 | |
| | | | | | | | | | | | Coeff. of Variation | 0.1326 | |
| Viest, Elstner & Hognestad (1956) | 20B1a | 5.0 | 5.0 | 2290 | 0.0320 | 0.6051 | 0.0063 | 8.0 | 0.726 | 27.0 | 98.0 | 1.1403 | ** |
| | 20B2a | 5.0 | 5.0 | 2550 | 0.0320 | 0.5434 | 0.0063 | 8.0 | 0.726 | 25.5 | 92.6 | 1.0908 | |
| | 20B1b | 5.0 | 5.0 | 2120 | 0.0320 | 0.6536 | 0.0063 | 8.0 | 0.762 | 22.1 | 84.2 | 0.9974 | ** |
| | 20B3b | 5.0 | 5.0 | 2300 | 0.0320 | 0.6024 | 0.0063 | 8.0 | 0.762 | 22.0 | 83.8 | 0.9681 | ** |
| | 35B1a | 5.0 | 5.0 | 4240 | 0.0320 | 0.3268 | 0.0063 | 8.0 | 0.500 | 49.3 | 123.3 | 1.1906 | |
| | 35B2a | 5.0 | 5.0 | 4410 | 0.0320 | 0.3142 | 0.0063 | 8.0 | 0.500 | 44.5 | 111.3 | 1.1317 | |
| | 35B1b | 5.0 | 5.0 | 4760 | 0.0320 | 0.2911 | 0.0063 | 8.0 | 0.450 | 48.0 | 108.0 | 1.0080 | |
| | 35B2b | 5.0 | 5.0 | 4770 | 0.0320 | 0.2905 | 0.0063 | 8.0 | 0.450 | 45.6 | 102.6 | 1.0490 | |
| | 35B3b | 5.0 | 5.0 | 4710 | 0.0320 | 0.2942 | 0.0063 | 8.0 | 0.450 | 43.1 | 97.0 | 0.9563 | |
| | 50B1a | 5.0 | 5.0 | 5370 | 0.0320 | 0.2580 | 0.0063 | 8.0 | 0.450 | 59.0 | 132.8 | 1.1544 | |
| | 50B1b | 5.0 | 5.0 | 4360 | 0.0320 | 0.3178 | 0.0063 | 8.0 | 0.500 | 50.0 | 125.0 | 1.1895 | |
| | 50B2b | 5.0 | 5.0 | 4540 | 0.0320 | 0.3052 | 0.0063 | 8.0 | 0.500 | 41.2 | 103.0 | 1.0041 | |
| | 20C1a | 5.0 | 5.0 | 2980 | 0.0320 | 0.4650 | 0.0063 | 8.0 | 0.250 | 57.6 | 72.0 | 1.0789 | |
| | 20C2a | 5.0 | 5.0 | 3130 | 0.0320 | 0.4427 | 0.0063 | 8.0 | 0.250 | 53.0 | 66.3 | 1.0665 | |
| | 20C1b | 5.0 | 5.0 | 1560 | 0.0320 | 0.8882 | 0.0063 | 8.0 | 0.350 | 34.0 | 59.5 | 1.0309 | ** |
| | 20C2b | 5.0 | 5.0 | 1810 | 0.0320 | 0.7655 | 0.0063 | 8.0 | 0.350 | 31.5 | 55.1 | 0.9706 | ** |
| | 20C3b | 5.0 | 5.0 | 1820 | 0.0320 | 0.7613 | 0.0063 | 8.0 | 0.350 | 32.4 | 56.7 | 0.9963 | ** |
| | 20C1c | 5.0 | 5.0 | 2440 | 0.0320 | 0.5679 | 0.0063 | 8.0 | 0.250 | 50.0 | 62.5 | 1.0384 | ** |
| | 35C1a | 5.0 | 5.0 | 3940 | 0.0320 | 0.3517 | 0.0063 | 8.0 | 0.250 | 70.0 | 87.5 | 1.1190 | |

Table 3.1.1 - Description of Reinforced Concrete Columns without Moment Gradient Used for Comparison with FEM Ultimate Strength

| Author | Col. Desig. | h (in.) | b (in.) | $f'_c$ (psi) | $\rho_{rs}$ | $\frac{\rho_{rs} f_{yr}}{f'_c}$ | Tie/Hoop Vol. Ratio $\rho''$ | $\ell/h$ | e/h | TEST VALUES Applied Axial Load (kips) | TEST VALUES Applied BM at Ends (kip-in) | Strength Ratio | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Viest, Elstner & Hognestad (1956) | 35C4a | 5.0 | 5.0 | 4230 | 0.0320 | 0.3276 | 0.0063 | 8.0 | 0.350 | 50.9 | 89.1 | 0.9611 | |
| | 35C1b | 5.0 | 5.0 | 4280 | 0.0320 | 0.3237 | 0.0063 | 8.0 | 0.350 | 54.0 | 94.5 | 1.0125 | |
| | 35C2b | 5.0 | 5.0 | 4670 | 0.0320 | 0.2967 | 0.0063 | 8.0 | 0.350 | 49.4 | 86.5 | 0.9788 | |
| | | | | | | | | | | | Mean | 1.0515 | (1.0661)*** |
| | | | | | | | | | | | Coeff. of Variation | 0.0715 | (0.0742)*** |
| Gaede (1958) | I / 1 | 3.9 | 6.1 | 3060 | 0.0100 | 0.1587 | 0.0012 | 29.4 | 0.200 | 17.0 | 13.4 | 0.7610 | |
| | I / 5 | 3.9 | 6.1 | 4085 | 0.0100 | 0.1024 | 0.0012 | 29.4 | 0.200 | 21.8 | 17.2 | 0.8633 | |
| | II / 4 | 3.9 | 6.1 | 3809 | 0.0100 | 0.1041 | 0.0012 | 29.4 | 0.500 | 8.1 | 16.0 | 0.7648 | |
| | II / 5 | 3.9 | 6.1 | 4016 | 0.0100 | 0.0983 | 0.0012 | 29.4 | 0.500 | 8.5 | 16.7 | 0.7808 | |
| | III / 1 | 3.9 | 6.1 | 4196 | 0.0100 | 0.1130 | 0.0012 | 35.4 | 0.500 | 7.5 | 14.8 | 0.7287 | |
| | III / 2 | 3.9 | 6.1 | 3603 | 0.0100 | 0.1310 | 0.0012 | 35.4 | 0.500 | 7.5 | 14.8 | 0.8486 | |
| | III / 3 | 3.9 | 6.1 | 3576 | 0.0100 | 0.1323 | 0.0012 | 35.4 | 0.500 | 7.5 | 14.8 | 0.8571 | |
| | III / 4 | 3.9 | 6.1 | 5048 | 0.0100 | 0.0902 | 0.0012 | 35.4 | 0.500 | 8.4 | 16.5 | 0.7821 | |
| | | | | | | | | | | | Mean | 0.7983 | |
| | | | | | | | | | | | Coeff. of Variation | 0.0638 | |
| Bresler (1960) | B-1 | 6.0 | 8.0 | 3700 | 0.0258 | 0.3735 | 0.0111 | 8.0 | 1.000 | 24.0 | 144.0 | 1.0068 | |
| | B-2 | 6.0 | 8.0 | 3900 | 0.0258 | 0.3544 | 0.0111 | 8.0 | 0.500 | 60.0 | 180.0 | 1.1563 | |
| | B-3 | 8.0 | 6.0 | 3700 | 0.0258 | 0.3735 | 0.0111 | 6.0 | 0.500 | 70.0 | 280.0 | 1.0729 | |
| | B-4 | 8.0 | 6.0 | 4600 | 0.0258 | 0.3005 | 0.0111 | 6.0 | 1.000 | 32.0 | 256.0 | 1.1000 | |
| | | | | | | | | | | | Mean | 1.0840 | |
| | | | | | | | | | | | Coeff. of Variation | 0.0573 | |
| Bresler & Gilbert (1961) | A-6 | 8.0 | 8.0 | 6000 | 0.0291 | 0.2155 | 0.0061 | 7.5 | 0.000 | 442.0 | 0.0 | 1.0815 | |
| | S-6 | 8.0 | 8.0 | 6000 | 0.0291 | 0.2155 | 0.0046 | 7.5 | 0.000 | 422.0 | 0.0 | 1.0145 | |
| | A-8 | 8.0 | 8.0 | 4000 | 0.0388 | 0.4311 | 0.0070 | 7.5 | 0.000 | 356.0 | 0.0 | 1.0677 | |
| | S-8 | 8.0 | 8.0 | 4000 | 0.0388 | 0.4311 | 0.0055 | 7.5 | 0.000 | 352.0 | 0.0 | 1.0640 | |
| | | | | | | | | | | | Mean | 1.0569 | |
| | | | | | | | | | | | Coeff. of Variation | 0.0277 | |
| Chang & Ferguson (1963) | 1 | 4.1 | 6.1 | 3385 | 0.0177 | 0.2534 | 0.0021 | 31.0 | 0.073 | 37.8 | 11.2 | 0.9640 | |
| | 2 | 4.1 | 6.1 | 5070 | 0.0177 | 0.1692 | 0.0021 | 31.0 | 0.389 | 15.5 | 24.5 | 0.8460 | |
| | 3 | 4.1 | 6.1 | 4190 | 0.0177 | 0.2047 | 0.0021 | 31.0 | 0.061 | 42.6 | 10.5 | 0.9200 | |
| | 4 | 4.1 | 6.1 | 4360 | 0.0177 | 0.1967 | 0.0021 | 31.0 | 0.382 | 16.3 | 25.3 | 0.9290 | |
| | 5 | 4.1 | 6.1 | 4750 | 0.0177 | 0.1806 | 0.0021 | 31.0 | 0.208 | 27.6 | 23.3 | 1.0070 | |
| | 6 | 4.1 | 6.1 | 4870 | 0.0177 | 0.2079 | 0.0021 | 31.0 | 0.064 | 44.4 | 11.6 | 0.8990 | |
| | | | | | | | | | | | Mean | 0.9275 | |
| | | | | | | | | | | | Coeff. of Variation | 0.0594 | |

Table 3.1.1 - Description of Reinforced Concrete Columns without Moment Gradient Used for Comparison with FEM Ultimate Strength

| Author | Col. Desig. | h (in.) | b (in.) | $f'_c$ (psi) | $\rho_{rs}$ | $\frac{\rho_{rs} f_{yr}}{f'_c}$ | Tie/Hoop Vol. Ratio $\rho''$ | $\ell/h$ | e/h | TEST VALUES Applied Axial Load (kips) | TEST VALUES Applied BM at Ends (kip-in) | Strength Ratio |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Pfister (1964) | 1A | 12.0 | 12.0 | 3790 | 0.0367 | 0.4644 | 0.0039 | 6.0 | 0.000 | 684.0 | 0.0 | 0.9025 |
| | 2A | 12.0 | 12.0 | 3820 | 0.0367 | 0.4607 | 0.0019 | 6.0 | 0.000 | 695.0 | 0.0 | 0.9174 |
| | 3A | 12.0 | 12.0 | 3820 | 0.0367 | 0.4607 | 0.0007 | 6.0 | 0.000 | 700.0 | 0.0 | 0.9320 |
| | 4A | 12.0 | 12.0 | 3840 | 0.0367 | 0.4583 | 0.0004 | 6.0 | 0.000 | 650.0 | 0.0 | 0.8608 |
| | 1B | 8.0 | 18.0 | 4310 | 0.0367 | 0.4084 | 0.0053 | 9.0 | 0.000 | 762.0 | 0.0 | 0.9178 |
| | 2B | 8.0 | 18.0 | 4350 | 0.0367 | 0.4046 | 0.0036 | 9.0 | 0.000 | 774.0 | 0.0 | 0.9398 |
| | 3B | 8.0 | 18.0 | 4350 | 0.0367 | 0.4046 | 0.0004 | 9.0 | 0.000 | 751.0 | 0.0 | 0.9439 |
| | 1C | 10.0 | 12.0 | 3600 | 0.0395 | 1.0094 | 0.0032 | 7.2 | 0.000 | 654.0 | 0.0 | 0.9219 |
| | 2C | 10.0 | 12.0 | 3680 | 0.0395 | 0.9875 | 0.0027 | 7.2 | 0.000 | 646.0 | 0.0 | 0.9072 |
| | 3C | 10.0 | 12.0 | 3780 | 0.0395 | 0.9614 | 0.0008 | 7.2 | 0.000 | 624.0 | 0.0 | 0.8834 |
| | 4D | 10.0 | 12.0 | 3690 | 0.0395 | 0.9848 | 0.0004 | 7.2 | 0.000 | 622.0 | 0.0 | 0.8613 |
| | | | | | | | | | | | Mean | 0.9080 |
| | | | | | | | | | | | Coeff. of Variation | 0.0317 |
| Roy & Sozen (1964) | A1.222 | 5.0 | 5.0 | 3080 | 0.0080 | 0.1299 | 0.0199 | 5.0 | 0.000 | 95.0 | 0.0 | 0.9581 |
| | A1.223 | 5.0 | 5.0 | 3080 | 0.0176 | 0.2857 | 0.0199 | 5.0 | 0.000 | 109.0 | 0.0 | 0.9780 |
| | A2.222 | 5.0 | 5.0 | 2980 | 0.0080 | 0.1342 | 0.0199 | 5.0 | 0.000 | 94.5 | 0.0 | 0.9811 |
| | A3.222 | 5.0 | 5.0 | 3700 | 0.0080 | 0.1081 | 0.0199 | 5.0 | 0.000 | 108.6 | 0.0 | 0.9625 |
| | A3.223 | 5.0 | 5.0 | 3700 | 0.0176 | 0.2378 | 0.0199 | 5.0 | 0.000 | 118.7 | 0.0 | 0.9513 |
| | B1.242 | 5.0 | 5.0 | 3480 | 0.0080 | 0.1149 | 0.0199 | 5.0 | 0.000 | 93.3 | 0.0 | 0.8619 |
| | B1.243 | 5.0 | 5.0 | 3480 | 0.0176 | 0.2529 | 0.0199 | 5.0 | 0.000 | 102.9 | 0.0 | 0.8561 |
| | B2.242 | 5.0 | 5.0 | 3480 | 0.0080 | 0.1149 | 0.0199 | 5.0 | 0.000 | 98.8 | 0.0 | 0.9342 |
| | B2.243 | 5.0 | 5.0 | 3480 | 0.0176 | 0.2529 | 0.0199 | 5.0 | 0.000 | 110.0 | 0.0 | 0.9147 |
| | B3.242 | 5.0 | 5.0 | 3370 | 0.0080 | 0.1187 | 0.0199 | 5.0 | 0.000 | 99.4 | 0.0 | 0.9395 |
| | B3.243 | 5.0 | 5.0 | 3370 | 0.0176 | 0.2611 | 0.0199 | 5.0 | 0.000 | 110.0 | 0.0 | 0.9357 |
| | C1.342 | 5.0 | 5.0 | 3320 | 0.0080 | 0.1205 | 0.0224 | 5.0 | 0.000 | 95.0 | 0.0 | 0.8983 |
| | C1.343 | 5.0 | 5.0 | 3320 | 0.0176 | 0.2651 | 0.0224 | 5.0 | 0.000 | 112.1 | 0.0 | 0.9508 |
| | C2.342 | 5.0 | 5.0 | 3440 | 0.0080 | 0.1163 | 0.0224 | 5.0 | 0.000 | 94.5 | 0.0 | 0.8722 |
| | C2.343 | 5.0 | 5.0 | 3440 | 0.0176 | 0.2558 | 0.0224 | 5.0 | 0.000 | 108.0 | 0.0 | 0.8966 |
| | C3.342 | 5.0 | 5.0 | 3390 | 0.0080 | 0.1180 | 0.0224 | 5.0 | 0.000 | 96.7 | 0.0 | 0.9012 |
| | C3.343 | 5.0 | 5.0 | 3390 | 0.0176 | 0.2596 | 0.0224 | 5.0 | 0.000 | 114.0 | 0.0 | 0.9544 |
| | D1.262 | 5.0 | 5.0 | 3150 | 0.0080 | 0.1270 | 0.0199 | 5.0 | 0.000 | 90.0 | 0.0 | 0.8944 |
| | D1.263 | 5.0 | 5.0 | 3150 | 0.0176 | 0.2794 | 0.0199 | 5.0 | 0.000 | 105.0 | 0.0 | 0.9339 |
| | D2.262 | 5.0 | 5.0 | 3200 | 0.0080 | 0.1250 | 0.0199 | 5.0 | 0.000 | 90.0 | 0.0 | 0.8837 |
| | D2.263 | 5.0 | 5.0 | 3200 | 0.0176 | 0.2750 | 0.0199 | 5.0 | 0.000 | 107.8 | 0.0 | 0.9458 |
| | D3.262 | 5.0 | 5.0 | 3380 | 0.0080 | 0.1183 | 0.0199 | 5.0 | 0.000 | 90.0 | 0.0 | 0.8508 |
| | D3.263 | 5.0 | 5.0 | 3380 | 0.0176 | 0.2604 | 0.0199 | 5.0 | 0.000 | 106.9 | 0.0 | 0.9073 |
| | E1.282 | 5.0 | 5.0 | 3330 | 0.0080 | 0.1201 | 0.0199 | 5.0 | 0.000 | 90.0 | 0.0 | 0.8579 |
| | E1.283 | 5.0 | 5.0 | 3330 | 0.0176 | 0.2643 | 0.0199 | 5.0 | 0.000 | 101.7 | 0.0 | 0.8719 |
| | E2.282 | 5.0 | 5.0 | 3410 | 0.0080 | 0.1173 | 0.0199 | 5.0 | 0.000 | 90.0 | 0.0 | 0.8446 |
| | E2.283 | 5.0 | 5.0 | 3410 | 0.0176 | 0.2581 | 0.0199 | 5.0 | 0.000 | 111.3 | 0.0 | 0.9401 |
| | E3.282 | 5.0 | 5.0 | 3460 | 0.0080 | 0.1156 | 0.0199 | 5.0 | 0.000 | 89.0 | 0.0 | 0.8280 |
| | E3.283 | 5.0 | 5.0 | 3460 | 0.0176 | 0.2543 | 0.0199 | 5.0 | 0.000 | 106.9 | 0.0 | 0.8935 |
| | | | | | | | | | | | Mean | 0.9103 |
| | | | | | | | | | | | Coeff. of Variation | 0.0473 |

Table 3.1.1 - Description of Reinforced Concrete Columns without Moment Gradient Used for Comparison with FEM Ultimate Strength

| Author | Col. Desig. | h (in.) | b (in.) | $f'_c$ (psi) | $\rho_{rs}$ | $\frac{\rho_{rs} f_{yr}}{f'_c}$ | Tie/Hoop Vol. Ratio $\rho''$ | $\ell/h$ | e/h | TEST VALUES Applied Axial Load (kips) | TEST VALUES Applied BM at Ends (kip-in) | Strength Ratio |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Todeschini, Bianchini & Kesler (1964) | 5B0 | 11.0 | 11.0 | 4654 | 0.0102 | 0.2279 | 0.0025 | 8.1 | 0.000 | 550.0 | 0.0 | 1.0073 |
| | 9B0F | 11.0 | 11.0 | 3767 | 0.0331 | 0.7766 | 0.0025 | 8.1 | 0.000 | 625.0 | 0.0 | 0.9480 |
| | 9A0 | 11.0 | 11.0 | 3951 | 0.0331 | 0.7196 | 0.0025 | 8.1 | 0.000 | 632.0 | 0.0 | 0.9321 |
| | 9B0 | 11.0 | 11.0 | 4254 | 0.0331 | 0.6776 | 0.0025 | 8.1 | 0.000 | 575.0 | 0.0 | 0.7999 |
| | 9C0 | 11.0 | 11.0 | 6321 | 0.0331 | 0.4519 | 0.0025 | 8.1 | 0.000 | 900.0 | 0.0 | 0.9338 |
| | 9C0R | 11.0 | 11.0 | 7549 | 0.0331 | 0.3749 | 0.0025 | 8.1 | 0.000 | 1050.0 | 0.0 | 0.9416 |
| | 11B0 | 11.0 | 11.0 | 4662 | 0.0516 | 0.9458 | 0.0025 | 8.1 | 0.000 | 890.0 | 0.0 | 0.9648 |
| | 5B1 | 11.0 | 11.0 | 5008 | 0.0102 | 0.2118 | 0.0025 | 8.1 | 0.136 | 425.0 | 637.5 | 1.1156 |
| | 9B1 | 11.0 | 11.0 | 5151 | 0.0331 | 0.5609 | 0.0025 | 8.1 | 0.136 | 534.0 | 801.0 | 1.0378 |
| | 5A3 | 11.0 | 11.0 | 3716 | 0.0102 | 0.2857 | 0.0025 | 8.1 | 0.318 | 200.0 | 700.0 | 1.1409 |
| | 5B3 | 11.0 | 11.0 | 5242 | 0.0102 | 0.1935 | 0.0025 | 8.1 | 0.318 | 275.0 | 962.5 | 1.2205 |
| | 5C3 | 11.0 | 11.0 | 6816 | 0.0102 | 0.1571 | 0.0025 | 8.1 | 0.318 | 310.0 | 1085.0 | 1.1127 |
| | 9A3 | 11.0 | 11.0 | 3962 | 0.0331 | 0.7242 | 0.0025 | 8.1 | 0.318 | 300.0 | 1050.0 | 1.0502 |
| | 9B3 | 11.0 | 11.0 | 4382 | 0.0331 | 0.6578 | 0.0025 | 8.1 | 0.318 | 312.0 | 1092.0 | 1.0494 |
| | 9C3 | 11.0 | 11.0 | 6130 | 0.0331 | 0.4703 | 0.0025 | 8.1 | 0.318 | 375.0 | 1312.5 | 1.0694 |
| | 9C3R | 11.0 | 11.0 | 7708 | 0.0331 | 0.3684 | 0.0025 | 8.1 | 0.318 | 450.0 | 1575.0 | 1.0970 |
| | 5B5 | 11.0 | 11.0 | 4729 | 0.0102 | 0.2189 | 0.0025 | 8.1 | 0.500 | 160.0 | 880.0 | 1.1594 |
| | 9B5 | 11.0 | 11.0 | 4243 | 0.0331 | 0.6864 | 0.0025 | 8.1 | 0.500 | 220.0 | 1210.0 | 1.0295 |
| | 11B5 | 11.0 | 11.0 | 5578 | 0.0516 | 0.8598 | 0.0025 | 8.1 | 0.500 | 250.0 | 1375.0 | 0.8525 |
| | | | | | | | | | | | Mean | 1.0243 |
| | | | | | | | | | | | Coeff. of Variation | 0.1050 |
| Hudson (1965) | 11 | 4.0 | 4.0 | 3600 | 0.0250 | 0.2778 | 0.0040 | 8.0 | 0.000 | 60.0 | 0.0 | 0.9158 |
| | 12 | 4.0 | 4.0 | 3600 | 0.0250 | 0.2778 | 0.0027 | 8.0 | 0.000 | 60.0 | 0.0 | 0.9193 |
| | 13 | 4.0 | 4.0 | 3600 | 0.0250 | 0.2778 | 0.0020 | 8.0 | 0.000 | 67.3 | 0.0 | 1.0368 |
| | 14 | 4.0 | 4.0 | 3600 | 0.0250 | 0.2778 | 0.0005 | 8.0 | 0.000 | 59.3 | 0.0 | 0.9170 |
| | 21 | 4.0 | 4.0 | 3900 | 0.0250 | 0.2564 | 0.0040 | 8.0 | 0.000 | 70.0 | 0.0 | 0.9842 |
| | 22 | 4.0 | 4.0 | 3900 | 0.0250 | 0.2564 | 0.0027 | 8.0 | 0.000 | 65.0 | 0.0 | 0.9173 |
| | 23 | 4.0 | 4.0 | 3900 | 0.0250 | 0.2564 | 0.0020 | 8.0 | 0.000 | 69.5 | 0.0 | 0.9954 |
| | 24 | 4.0 | 4.0 | 3900 | 0.0250 | 0.2564 | 0.0005 | 8.0 | 0.000 | 70.0 | 0.0 | 0.9949 |
| | 31 | 4.0 | 4.0 | 4100 | 0.0250 | 0.2439 | 0.0040 | 8.0 | 0.000 | 69.0 | 0.0 | 0.9315 |
| | 32 | 4.0 | 4.0 | 4100 | 0.0250 | 0.2439 | 0.0027 | 8.0 | 0.000 | 70.0 | 0.0 | 0.9525 |
| | 33 | 4.0 | 4.0 | 4100 | 0.0250 | 0.2439 | 0.0020 | 8.0 | 0.000 | 65.0 | 0.0 | 0.8857 |
| | 34 | 4.0 | 4.0 | 4100 | 0.0250 | 0.2439 | 0.0005 | 8.0 | 0.000 | 64.7 | 0.0 | 0.8866 |
| | 41 | 4.0 | 4.0 | 3700 | 0.0250 | 0.2703 | 0.0040 | 8.0 | 0.000 | 65.0 | 0.0 | 0.9497 |
| | 42 | 4.0 | 4.0 | 3700 | 0.0250 | 0.2703 | 0.0027 | 8.0 | 0.000 | 68.9 | 0.0 | 1.0172 |
| | 43 | 4.0 | 4.0 | 3700 | 0.0250 | 0.2703 | 0.0020 | 8.0 | 0.000 | 68.9 | 0.0 | 1.0408 |
| | 44 | 4.0 | 4.0 | 3700 | 0.0250 | 0.2703 | 0.0005 | 8.0 | 0.000 | 69.1 | 0.0 | 1.0579 |
| | 11 | 4.0 | 4.0 | 4700 | 0.0250 | 0.2128 | 0.0053 | 8.0 | 0.298 | 35.0 | 41.7 | 1.0696 |
| | 12 | 4.0 | 4.0 | 4700 | 0.0250 | 0.2128 | 0.0035 | 8.0 | 0.298 | 44.0 | 52.4 | 1.3558 |
| | 13 | 4.0 | 4.0 | 4700 | 0.0250 | 0.2128 | 0.0026 | 8.0 | 0.298 | 37.0 | 44.0 | 1.1362 |
| | 14 | 4.0 | 4.0 | 4700 | 0.0250 | 0.2128 | 0.0007 | 8.0 | 0.298 | 40.0 | 47.6 | 1.2339 |
| | 21 | 4.0 | 4.0 | 5200 | 0.0250 | 0.1923 | 0.0053 | 8.0 | 0.298 | 37.0 | 44.0 | 1.0483 |
| | 22 | 4.0 | 4.0 | 5200 | 0.0250 | 0.1923 | 0.0035 | 8.0 | 0.298 | 44.0 | 52.4 | 1.2574 |
| | 23 | 4.0 | 4.0 | 5200 | 0.0250 | 0.1923 | 0.0026 | 8.0 | 0.298 | 39.0 | 46.4 | 1.1105 |
| | 24 | 4.0 | 4.0 | 5200 | 0.0250 | 0.1923 | 0.0007 | 8.0 | 0.298 | 39.0 | 46.4 | 1.1123 |

Table 3.1.1 - Description of Reinforced Concrete Columns without Moment Gradient Used for Comparison with FEM Ultimate Strength

| Author | Col. Desig. | h (in.) | b (in.) | $f'_c$ (psi) | $\rho_{rs}$ | $\frac{\rho_{rs} f_{yr}}{f'_c}$ | Tie/Hoop Vol. Ratio $\rho''$ | $\ell/h$ | e/h | TEST VALUES Applied Axial Load (kips) | TEST VALUES Applied BM at Ends (kip-in) | Strength Ratio |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Hudson (1965) | 31 | 4.0 | 4.0 | 5600 | 0.0250 | 0.1786 | 0.0053 | 8.0 | 0.298 | 45.0 | 53.6 | 1.2106 |
| | 32 | 4.0 | 4.0 | 5600 | 0.0250 | 0.1786 | 0.0035 | 8.0 | 0.298 | 45.0 | 53.6 | 1.2206 |
| | 33 | 4.0 | 4.0 | 5600 | 0.0250 | 0.1786 | 0.0026 | 8.0 | 0.298 | 40.0 | 47.6 | 1.0756 |
| | 34 | 4.0 | 4.0 | 5600 | 0.0250 | 0.1786 | 0.0007 | 8.0 | 0.298 | 38.0 | 45.2 | 1.0247 |
| | 41 | 4.0 | 4.0 | 6300 | 0.0250 | 0.1587 | 0.0053 | 8.0 | 0.298 | 45.0 | 53.6 | 1.1039 |
| | 42 | 4.0 | 4.0 | 6300 | 0.0250 | 0.1587 | 0.0035 | 8.0 | 0.298 | 45.0 | 53.6 | 1.1130 |
| | 43 | 4.0 | 4.0 | 6300 | 0.0250 | 0.1587 | 0.0026 | 8.0 | 0.298 | 45.0 | 53.6 | 1.1062 |
| | 44 | 4.0 | 4.0 | 6300 | 0.0250 | 0.1587 | 0.0007 | 8.0 | 0.298 | 42.0 | 50.0 | 1.0399 |
| | 11 | 6.0 | 6.0 | 2100 | 0.0444 | 0.8466 | 0.0112 | 8.0 | 0.000 | 99.0 | 0.0 | 0.7135 ** |
| | 12 | 6.0 | 6.0 | 2100 | 0.0444 | 0.8466 | 0.0112 | 8.0 | 0.000 | 100.0 | 0.0 | 0.7207 ** |
| | 13 | 6.0 | 6.0 | 2100 | 0.0444 | 0.8466 | 0.0015 | 8.0 | 0.000 | 111.0 | 0.0 | 0.8569 ** |
| | 14 | 6.0 | 6.0 | 2100 | 0.0444 | 0.8466 | 0.0015 | 8.0 | 0.000 | 118.5 | 0.0 | 0.9207 ** |
| | 21 | 6.0 | 6.0 | 2600 | 0.0689 | 1.0598 | 0.0109 | 8.0 | 0.000 | 147.0 | 0.0 | 0.7829 |
| | 22 | 6.0 | 6.0 | 2600 | 0.0689 | 1.0598 | 0.0109 | 8.0 | 0.000 | 137.0 | 0.0 | 0.7244 |
| | 23 | 6.0 | 6.0 | 2600 | 0.0689 | 1.0598 | 0.0014 | 8.0 | 0.000 | 150.0 | 0.0 | 0.8313 |
| | 24 | 6.0 | 6.0 | 2600 | 0.0689 | 1.0598 | 0.0014 | 8.0 | 0.000 | 170.0 | 0.0 | 0.9597 |
| | 31 | 6.0 | 6.0 | 3100 | 0.0444 | 0.5735 | 0.0112 | 8.0 | 0.000 | 155.0 | 0.0 | 0.9178 |
| | 32 | 6.0 | 6.0 | 3300 | 0.0444 | 0.5387 | 0.0112 | 8.0 | 0.000 | 120.0 | 0.0 | 0.6862 |
| | 33 | 6.0 | 6.0 | 3100 | 0.0444 | 0.5735 | 0.0015 | 8.0 | 0.000 | 165.5 | 0.0 | 1.0528 |
| | 34 | 6.0 | 6.0 | 3300 | 0.0444 | 0.5387 | 0.0015 | 8.0 | 0.000 | 133.0 | 0.0 | 0.7921 |
| | 41 | 6.0 | 6.0 | 4200 | 0.0689 | 0.6561 | 0.0109 | 8.0 | 0.000 | 195.0 | 0.0 | 0.8170 |
| | 42 | 6.0 | 6.0 | 3800 | 0.0689 | 0.7251 | 0.0109 | 8.0 | 0.000 | 167.0 | 0.0 | 0.7392 |
| | 43 | 6.0 | 6.0 | 4200 | 0.0689 | 0.6561 | 0.0014 | 8.0 | 0.000 | 150.0 | 0.0 | 0.6587 |
| | 44 | 6.0 | 6.0 | 3800 | 0.0689 | 0.7251 | 0.0014 | 8.0 | 0.000 | 190.0 | 0.0 | 0.8739 |
| | 11 | 6.0 | 6.0 | 3000 | 0.0444 | 0.5926 | 0.0112 | 8.0 | 0.333 | 72.0 | 144.0 | 1.0633 |
| | 12 | 6.0 | 6.0 | 3400 | 0.0444 | 0.5229 | 0.0112 | 8.0 | 0.333 | 80.0 | 160.0 | 1.1156 |
| | 13 | 6.0 | 6.0 | 3000 | 0.0444 | 0.5926 | 0.0015 | 8.0 | 0.333 | 70.5 | 141.0 | 1.0888 |
| | 14 | 6.0 | 6.0 | 3400 | 0.0444 | 0.5229 | 0.0015 | 8.0 | 0.333 | 78.0 | 156.0 | 1.1284 |
| | 21 | 6.0 | 6.0 | 2700 | 0.0689 | 1.0206 | 0.0109 | 8.0 | 0.333 | 78.0 | 156.0 | 0.9548 |
| | 22 | 6.0 | 6.0 | 3100 | 0.0689 | 0.8889 | 0.0109 | 8.0 | 0.333 | 80.0 | 160.0 | 0.9329 |
| | 23 | 6.0 | 6.0 | 2700 | 0.0689 | 1.0206 | 0.0014 | 8.0 | 0.333 | 80.0 | 160.0 | 1.0326 |
| | 24 | 6.0 | 6.0 | 3100 | 0.0689 | 0.8889 | 0.0014 | 8.0 | 0.333 | 87.0 | 174.0 | 1.0637 |
| | 31 | 6.0 | 6.0 | 4000 | 0.0444 | 0.4444 | 0.0112 | 8.0 | 0.333 | 100.0 | 200.0 | 1.2872 |
| | 32 | 6.0 | 6.0 | 4200 | 0.0444 | 0.4233 | 0.0112 | 8.0 | 0.333 | 100.0 | 200.0 | 1.2541 |
| | 33 | 6.0 | 6.0 | 4000 | 0.0444 | 0.4444 | 0.0015 | 8.0 | 0.333 | 100.8 | 201.6 | 1.3362 |
| | 34 | 6.0 | 6.0 | 4200 | 0.0444 | 0.4233 | 0.0015 | 8.0 | 0.333 | 99.5 | 199.0 | 1.2800 |
| | 41 | 6.0 | 6.0 | 4200 | 0.0689 | 0.6561 | 0.0109 | 8.0 | 0.333 | 100.0 | 200.0 | 1.0313 |
| | 42 | 6.0 | 6.0 | 4300 | 0.0689 | 0.6408 | 0.0109 | 8.0 | 0.333 | 106.0 | 212.0 | 1.0825 |
| | 43 | 6.0 | 6.0 | 4200 | 0.0689 | 0.6561 | 0.0014 | 8.0 | 0.333 | 107.5 | 215.0 | 1.1437 |
| | 44 | 6.0 | 6.0 | 4300 | 0.0689 | 0.6408 | 0.0014 | 8.0 | 0.333 | 117.5 | 235.0 | 1.2368 |

Mean 1.0109 (1.0248)***

Coeff. of Variation 0.1624 (0.1546)***

Table 3.1.1 - Description of Reinforced Concrete Columns without Moment Gradient Used for Comparison with FEM Ultimate Strength

| Author | Col. Desig. | h (in.) | b (in.) | $f'_c$ (psi) | $\rho_{rs}$ | $\frac{\rho_{rs} f_{yr}}{f'_c}$ | Tie/Hoop Vol. Ratio $\rho''$ | $\ell/h$ | e/h | TEST VALUES Applied Axial Load (kips) | TEST VALUES Applied BM at Ends (kip-in) | Strength Ratio |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Martin & Olivieri | 402-1 | 3.5 | 5.0 | 4350 | 0.0248 | 0.2284 | 0.0036 | 40.0 | 0.000 | 33.0 | 0.0 | 0.9993 |
| (1965) | 402-2 | 3.5 | 5.0 | 3530 | 0.0248 | 0.2814 | 0.0036 | 40.0 | 0.000 | 28.0 | 0.0 | 0.9393 |
| | | | | | | | | | | | Mean | 0.9693 |
| | | | | | | | | | | | Coeff. of Variation | 0.0438 |
| Mehmel, | 0.1 | 6.3 | 10.0 | 4837 | 0.0111 | 0.1515 | 0.0054 | 8.8 | 0.082 | 211.9 | 108.5 | 0.8914 |
| Schwarz, | 0.2 | 6.1 | 10.0 | 5288 | 0.0112 | 0.1401 | 0.0055 | 9.0 | 1.000 | 30.9 | 189.6 | 1.1505 |
| Kasparek | 1.2 | 8.0 | 10.0 | 4894 | 0.0122 | 0.1544 | 0.0038 | 16.8 | 0.475 | 71.9 | 271.7 | 0.9378 |
| & Makovi | 2.1 | 8.0 | 9.9 | 4823 | 0.0123 | 0.1578 | 0.0037 | 22.3 | 0.178 | 132.3 | 187.6 | 0.9101 |
| (1969) | 2.2 | 8.0 | 9.9 | 5317 | 0.0122 | 0.1424 | 0.0038 | 22.2 | 0.478 | 58.2 | 222.4 | 0.9387 |
| | 3.1 | 6.0 | 9.9 | 4964 | 0.0123 | 0.1641 | 0.0051 | 22.4 | 0.164 | 105.9 | 104.2 | 0.9234 |
| | 3.3 | 6.3 | 10.0 | 4543 | 0.0110 | 0.1687 | 0.0056 | 21.4 | 0.082 | 176.0 | 90.1 | 1.0771 |
| | 3.4 | 6.2 | 10.0 | 5614 | 0.0112 | 0.1314 | 0.0056 | 21.5 | 1.000 | 22.9 | 142.7 | 1.0640 |
| | 4.1 | 5.9 | 10.0 | 5288 | 0.0125 | 0.1568 | 0.0050 | 30.0 | 0.163 | 82.7 | 79.8 | 0.9900 |
| | 4.2 | 5.8 | 10.0 | 5430 | 0.0127 | 0.1548 | 0.0051 | 30.4 | 0.493 | 32.6 | 93.8 | 1.0422 |
| | 5.1 | 6.2 | 10.0 | 5302 | 0.0319 | 0.3448 | 0.0043 | 21.5 | 0.165 | 165.4 | 169.3 | 1.0174 |
| | 5.2 | 6.3 | 9.9 | 4781 | 0.0318 | 0.3815 | 0.0040 | 21.4 | 0.503 | 83.1 | 261.9 | 0.9739 |
| | | | | | | | | | | | Mean | 0.9930 |
| | | | | | | | | | | | Coeff. of Variation | 0.0794 |
| Ramu, | 41 | 5.9 | 9.8 | 3809 | 0.0166 | 0.2894 | 0.0047 | 28.9 | 0.033 | 115.8 | 22.8 | 0.9665 |
| Grenacher, | 14 | 5.9 | 9.8 | 4085 | 0.0166 | 0.2699 | 0.0047 | 28.9 | 0.100 | 85.8 | 50.7 | 0.9138 |
| Baumann | 53 | 5.9 | 9.8 | 6157 | 0.0166 | 0.1791 | 0.0047 | 28.9 | 0.100 | 105.4 | 62.3 | 0.8985 |
| & Thurlimann | 24 | 5.9 | 9.8 | 4002 | 0.0166 | 0.2755 | 0.0047 | 28.9 | 0.250 | 53.2 | 78.5 | 0.9655 |
| (1969) | 31 | 5.9 | 9.8 | 3344 | 0.0166 | 0.3296 | 0.0047 | 28.9 | 1.000 | 17.6 | 104.2 | 0.8949 |
| | 73 | 5.9 | 9.8 | 4502 | 0.0166 | 0.2449 | 0.0047 | 14.4 | 0.033 | 198.5 | 39.1 | 0.8761 |
| | | | | | | | | | | | Mean | 0.9192 |
| | | | | | | | | | | | Coeff. of Variation | 0.0415 |
| Drysdale | D-1-A | 5.0 | 5.0 | 4400 | 0.0320 | 0.4079 | 0.0041 | 31.2 | 0.200 | 38.9 | 38.9 | 1.0310 |
| & Huggins | D-1-B | 5.0 | 5.0 | 4400 | 0.0320 | 0.4079 | 0.0041 | 31.2 | 0.200 | 38.6 | 38.6 | 1.0231 |
| (1971) | D-2-C | 5.0 | 5.0 | 4230 | 0.0320 | 0.4242 | 0.0041 | 31.2 | 0.200 | 39.7 | 39.7 | 1.0692 |
| | D-2-D | 5.0 | 5.0 | 4230 | 0.0320 | 0.4242 | 0.0041 | 31.2 | 0.200 | 40.5 | 40.5 | 1.0901 |
| | | | | | | | | | | | Mean | 1.0533 |
| | | | | | | | | | | | Coeff. of Variation | 0.0301 |

Table 3.1.1 - Description of Reinforced Concrete Columns without Moment Gradient Used for Comparison with FEM Ultimate Strength

| Author | Col. Desig. | h (in.) | b (in.) | $f'_c$ (psi) | $\rho_{rs}$ | $\frac{\rho_{rs} f_{yr}}{f'_c}$ | Tie/Hoop Vol. Ratio $\rho''$ | $\ell$/h | e/h | TEST VALUES Applied Axial Load (kips) | TEST VALUES Applied BM at Ends (kip-in) | Strength Ratio |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Goyal & Jackson (1971) | A1 | 3.0 | 3.0 | 3900 | 0.0244 | 0.3197 | 0.0290 | 24.0 | 0.500 | 7.5 | 11.2 | 0.9019 |
| | A2 | 3.0 | 3.0 | 3900 | 0.0244 | 0.3197 | 0.0290 | 24.0 | 0.500 | 7.5 | 11.3 | 0.9080 |
| | C1 | 3.0 | 3.0 | 4250 | 0.0244 | 0.2933 | 0.0290 | 24.0 | 0.333 | 10.0 | 10.0 | 0.8662 |
| | C2 | 3.0 | 3.0 | 4250 | 0.0244 | 0.2933 | 0.0290 | 24.0 | 0.333 | 10.5 | 10.5 | 0.9112 |
| | E1 | 3.0 | 3.0 | 4450 | 0.0244 | 0.2801 | 0.0290 | 24.0 | 0.167 | 15.0 | 7.5 | 0.8179 |
| | E2 | 3.0 | 3.0 | 4450 | 0.0244 | 0.2801 | 0.0290 | 24.0 | 0.167 | 14.7 | 7.4 | 0.8015 |
| | G1 | 3.0 | 3.0 | 4540 | 0.0244 | 0.2746 | 0.0290 | 24.0 | 0.250 | 12.5 | 9.3 | 0.8674 |
| | G2 | 3.0 | 3.0 | 4540 | 0.0244 | 0.2746 | 0.0290 | 24.0 | 0.250 | 11.9 | 8.9 | 0.8305 |
| | I1 | 3.0 | 3.0 | 4600 | 0.0171 | 0.1674 | 0.0210 | 24.0 | 0.167 | 13.5 | 6.8 | 0.8221 |
| | I2 | 3.0 | 3.0 | 4600 | 0.0171 | 0.1674 | 0.0210 | 24.0 | 0.167 | 12.9 | 6.5 | 0.7856 |
| | K1 | 3.0 | 3.0 | 4700 | 0.0171 | 0.1638 | 0.0210 | 24.0 | 0.250 | 10.5 | 7.9 | 0.8501 |
| | K2 | 3.0 | 3.0 | 4700 | 0.0171 | 0.1638 | 0.0210 | 24.0 | 0.250 | 10.3 | 7.7 | 0.8323 |
| | M1 | 3.0 | 3.0 | 4650 | 0.0171 | 0.1656 | 0.0210 | 24.0 | 0.333 | 8.4 | 8.4 | 0.8735 |
| | M2 | 3.0 | 3.0 | 4650 | 0.0171 | 0.1656 | 0.0210 | 24.0 | 0.333 | 8.3 | 8.3 | 0.8704 |
| | O1 | 3.0 | 3.0 | 4790 | 0.0171 | 0.1608 | 0.0210 | 16.0 | 0.167 | 18.5 | 9.3 | 0.7973 |
| | O2 | 3.0 | 3.0 | 4790 | 0.0171 | 0.1608 | 0.0210 | 16.0 | 0.167 | 20.8 | 10.4 | 0.8951 |
| | P1 | 3.0 | 3.0 | 4790 | 0.0171 | 0.1608 | 0.0210 | 16.0 | 0.250 | 14.5 | 10.9 | 0.8124 |
| | P2 | 3.0 | 3.0 | 4790 | 0.0171 | 0.1608 | 0.0210 | 16.0 | 0.250 | 16.4 | 12.3 | 0.9160 |
| | Q1 | 3.0 | 3.0 | 3900 | 0.0171 | 0.1974 | 0.0210 | 16.0 | 0.333 | 11.6 | 11.6 | 0.9144 |
| | Q2 | 3.0 | 3.0 | 3900 | 0.0171 | 0.1974 | 0.0210 | 16.0 | 0.333 | 11.0 | 11.0 | 0.8708 |
| | R1 | 3.0 | 3.0 | 4350 | 0.0171 | 0.1770 | 0.0210 | 36.0 | 0.167 | 7.5 | 3.8 | 0.7896 |
| | R2 | 3.0 | 3.0 | 4350 | 0.0171 | 0.1770 | 0.0210 | 36.0 | 0.167 | 7.0 | 3.5 | 0.7350 |
| | S1 | 3.0 | 3.0 | 4240 | 0.0171 | 0.1816 | 0.0210 | 36.0 | 0.250 | 5.2 | 3.9 | 0.7076 |
| | S2 | 3.0 | 3.0 | 4240 | 0.0171 | 0.1816 | 0.0210 | 36.0 | 0.250 | 5.5 | 4.1 | 0.7487 |
| | T1 | 3.0 | 3.0 | 4200 | 0.0171 | 0.1833 | 0.0210 | 36.0 | 0.333 | 4.4 | 4.4 | 0.7467 |
| | T2 | 3.0 | 3.0 | 4200 | 0.0171 | 0.1833 | 0.0210 | 36.0 | 0.333 | 4.6 | 4.6 | 0.7894 |
| | | | | | | | | | | | Mean | 0.8331 |
| | | | | | | | | | | | Coeff. of Variation | 0.0720 |
| Bunni (1975) | A-1 | 5.0 | 5.0 | 3606 | 0.0314 | 0.4093 | 0.0103 | 4.0 | 0.000 | 130.0 | 0.0 | 1.0575 |
| | A-2 | 5.0 | 5.0 | 3484 | 0.0314 | 0.4236 | 0.0103 | 8.0 | 0.000 | 130.0 | 0.0 | 1.0771 |
| | A-3 | 5.0 | 5.0 | 3484 | 0.0314 | 0.4243 | 0.0103 | 12.0 | 0.000 | 132.5 | 0.0 | 1.0974 |
| | A-4 | 5.0 | 5.0 | 3606 | 0.0314 | 0.4093 | 0.0103 | 16.0 | 0.000 | 130.0 | 0.0 | 1.0464 |
| | A-5 | 5.0 | 5.0 | 3606 | 0.0314 | 0.4093 | 0.0103 | 20.0 | 0.000 | 129.5 | 0.0 | 1.0407 |
| | B-6 | 5.0 | 5.0 | 3476 | 0.0490 | 0.6056 | 0.0103 | 4.0 | 0.000 | 135.0 | 0.0 | 0.9881 |
| | B-7 | 5.0 | 5.0 | 3476 | 0.0490 | 0.6056 | 0.0103 | 16.0 | 0.000 | 132.5 | 0.0 | 0.9692 |
| | B-8 | 5.0 | 5.0 | 3423 | 0.0490 | 0.6208 | 0.0103 | 20.0 | 0.000 | 135.0 | 0.0 | 0.9917 |
| | C-9 | 5.0 | 5.0 | 3537 | 0.0706 | 0.8148 | 0.0103 | 4.0 | 0.000 | 165.0 | 0.0 | 1.0677 |
| | C-10 | 5.0 | 5.0 | 3537 | 0.0706 | 0.8148 | 0.0103 | 16.0 | 0.000 | 165.0 | 0.0 | 1.0592 |
| | C-11 | 5.0 | 5.0 | 3438 | 0.0706 | 0.8383 | 0.0103 | 20.0 | 0.000 | 165.0 | 0.0 | 1.0749 |
| | D-12 | 5.0 | 5.0 | 3408 | 0.0314 | 0.4423 | 0.0158 | 4.0 | 0.000 | 150.0 | 0.0 | 1.1873 |
| | D-13 | 5.0 | 5.0 | 3499 | 0.0314 | 0.4128 | 0.0158 | 8.0 | 0.000 | 150.0 | 0.0 | 1.1918 |
| | D-14 | 5.0 | 5.0 | 3499 | 0.0314 | 0.4128 | 0.0158 | 12.0 | 0.000 | 150.0 | 0.0 | 1.1890 |
| | D-15 | 5.0 | 5.0 | 3408 | 0.0314 | 0.4423 | 0.0158 | 16.0 | 0.000 | 150.0 | 0.0 | 1.1843 |
| | D-16 | 5.0 | 5.0 | 3606 | 0.0314 | 0.4154 | 0.0158 | 20.0 | 0.000 | 150.0 | 0.0 | 1.1497 |
| | E-17 | 5.0 | 5.0 | 3423 | 0.0490 | 0.6265 | 0.0158 | 4.0 | 0.000 | 180.0 | 0.0 | 1.2774 |

Table 3.1.1 - Description of Reinforced Concrete Columns without Moment Gradient Used for Comparison with FEM Ultimate Strength

| Author | Col. Desig. | h (in.) | b (in.) | $f'_c$ (psi) | $\rho_{rs}$ | $\frac{\rho_{rs} f_{yr}}{f'_c}$ | Tie/Hoop Vol. Ratio $\rho''$ | $\ell/h$ | e/h | TEST VALUES Applied Axial Load (kips) | Applied BM at Ends (kip-in) | Strength Ratio |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Bunni (1975) | E-18 | 5.0 | 5.0 | 3423 | 0.0490 | 0.6179 | 0.0158 | 8.0 | 0.000 | 165.0 | 0.0 | 1.1796 |
| | E-19 | 5.0 | 5.0 | 3423 | 0.0490 | 0.6208 | 0.0158 | 12.0 | 0.000 | 165.0 | 0.0 | 1.1747 |
| | E-20 | 5.0 | 5.0 | 3423 | 0.0490 | 0.6265 | 0.0158 | 16.0 | 0.000 | 165.0 | 0.0 | 1.1681 |
| | E-21 | 5.0 | 5.0 | 3287 | 0.0490 | 0.6495 | 0.0158 | 20.0 | 0.000 | 150.0 | 0.0 | 1.0878 |
| | F-22 | 5.0 | 5.0 | 3491 | 0.0706 | 0.8397 | 0.0158 | 4.0 | 0.000 | 205.5 | 0.0 | 1.2754 |
| | F-23 | 5.0 | 5.0 | 3613 | 0.0706 | 0.8280 | 0.0158 | 8.0 | 0.000 | 187.0 | 0.0 | 1.1279 |
| | F-24 | 5.0 | 5.0 | 3613 | 0.0706 | 0.8280 | 0.0158 | 12.0 | 0.000 | 180.0 | 0.0 | 1.0867 |
| | F-25 | 5.0 | 5.0 | 3491 | 0.0706 | 0.8397 | 0.0158 | 16.0 | 0.000 | 180.5 | 0.0 | 1.1194 |
| | F-26 | 5.0 | 5.0 | 3613 | 0.0706 | 0.8113 | 0.0158 | 20.0 | 0.000 | 180.0 | 0.0 | 1.1014 |
| | | | | | | | | | | | Mean | 1.1142 |
| | | | | | | | | | | | Coeff. of Variation | 0.0722 |
| Green & Hellesland (1975) | S1 | 5.0 | 7.1 | 4990 | 0.0124 | 0.1389 | 0.0025 | 15.0 | 0.101 | 112.8 | 57.1 | 0.9867 |
| | S5 | 4.9 | 7.0 | 4870 | 0.0358 | 0.4706 | 0.0025 | 15.2 | 0.091 | 139.6 | 62.8 | 0.9720 |
| | | | | | | | | | | | Mean | 0.9793 |
| | | | | | | | | | | | Coeff. of Variation | 0.0106 |
| Heimdahl & Bianchini (1975) | AR1 | 5.0 | 5.0 | 4633 | 0.0320 | 0.4945 | 0.0098 | 3.2 | 0.206 | 78.0 | 80.4 | 0.8587 |
| | AR2 | 5.0 | 5.0 | 4633 | 0.0320 | 0.4945 | 0.0098 | 3.2 | 0.212 | 81.5 | 86.4 | 0.9056 |
| | AR3(4) | 5.0 | 5.0 | 5376 | 0.0320 | 0.4262 | 0.0098 | 6.0 | 0.560 | 42.3 | 118.4 | 0.9167 |
| | AR4(4) | 5.0 | 5.0 | 5376 | 0.0320 | 0.4262 | 0.0098 | 6.0 | 0.545 | 46.0 | 125.4 | 0.9529 |
| | AR5(4) | 5.0 | 5.0 | 5376 | 0.0320 | 0.4262 | 0.0098 | 6.0 | 1.050 | 23.6 | 123.9 | 0.9803 |
| | AR6(4) | 5.0 | 5.0 | 5376 | 0.0320 | 0.4262 | 0.0098 | 6.0 | 1.048 | 23.7 | 124.2 | 0.9823 |
| | AW1 | 5.0 | 5.0 | 4433 | 0.0320 | 0.5335 | 0.0098 | 3.2 | 0.208 | 70.0 | 72.9 | 0.7904 |
| | AW2 | 5.0 | 5.0 | 4433 | 0.0320 | 0.5335 | 0.0098 | 3.2 | 0.213 | 79.0 | 84.0 | 0.9013 |
| | AW3(4) | 5.0 | 5.0 | 4934 | 0.0320 | 0.4793 | 0.0098 | 6.0 | 0.556 | 39.0 | 108.5 | 0.8633 |
| | AW4(4) | 5.0 | 5.0 | 4934 | 0.0320 | 0.4793 | 0.0098 | 6.0 | 0.573 | 40.3 | 115.5 | 0.9014 |
| | AW5(4) | 5.0 | 5.0 | 4934 | 0.0320 | 0.4793 | 0.0098 | 6.0 | 1.056 | 14.8 | 78.2 | 0.6316 |
| | AW6(4) | 5.0 | 5.0 | 4934 | 0.0320 | 0.4793 | 0.0098 | 6.0 | 1.064 | 15.8 | 84.1 | 0.6751 |
| | DR1 | 5.0 | 5.0 | 3666 | 0.0320 | 0.6250 | 0.0098 | 6.0 | 0.546 | 42.6 | 116.3 | 1.0321 |
| | DR2 | 5.0 | 5.0 | 3666 | 0.0320 | 0.6250 | 0.0098 | 6.0 | 1.055 | 19.7 | 103.9 | 0.8944 |
| | DW1 | 5.0 | 5.0 | 3666 | 0.0320 | 0.6451 | 0.0098 | 6.0 | 0.542 | 39.4 | 106.8 | 0.9430 |
| | DW2 | 5.0 | 5.0 | 3666 | 0.0320 | 0.6451 | 0.0098 | 6.0 | 1.060 | 18.3 | 97.0 | 0.8212 |
| | | | | | | | | | | | Mean | 0.8782 |
| | | | | | | | | | | | Coeff. of Variation | 0.1219 |
| Sheikh & Uzumeri (1980) | 2A1-1 | 12.0 | 12.0 | 5440 | 0.0172 | 0.1706 | 0.0078 | 6.4 | 0.000 | 768.0 | 0.0 | 0.9973 |
| | 2A1H-2 | 12.0 | 12.0 | 5370 | 0.0172 | 0.1729 | 0.0078 | 6.4 | 0.000 | 755.0 | 0.0 | 0.9899 |
| | 4C1-3 | 12.0 | 12.0 | 5280 | 0.0344 | 0.3516 | 0.0063 | 6.4 | 0.000 | 850.0 | 0.0 | 0.9554 |
| | 4C1H-4 | 12.0 | 12.0 | 5320 | 0.0344 | 0.3490 | 0.0063 | 6.4 | 0.000 | 860.0 | 0.0 | 0.9609 |
| | 4C6-5 | 12.0 | 12.0 | 5070 | 0.0344 | 0.3662 | 0.0187 | 6.4 | 0.000 | 1058.0 | 0.0 | 1.2250 |
| | 4C6H-6 | 12.0 | 12.0 | 4980 | 0.0344 | 0.3728 | 0.0187 | 6.4 | 0.000 | 950.0 | 0.0 | 1.1223 |

Table 3.1.1 - Description of Reinforced Concrete Columns without Moment Gradient Used for Comparison with FEM Ultimate Strength

| Author | Col. Desig. | h (in.) | b (in.) | $f'_c$ (psi) | $\rho_{rs}$ | $\frac{\rho_{rs} f_{yr}}{f'_c}$ | Tie/Hoop Vol. Ratio $\rho''$ | $\ell/h$ | e/h | TEST VALUES Applied Axial Load (kips) | Applied BM at Ends (kip-in) | Strength Ratio |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Sheikh & Uzumeri (1980) | 4A3-7 | 12.0 | 12.0 | 5930 | 0.0333 | 0.3137 | 0.0161 | 6.4 | 0.000 | 960.0 | 0.0 | 0.9940 |
| | 4A4-8 | 12.0 | 12.0 | 5920 | 0.0333 | 0.3142 | 0.0156 | 6.4 | 0.000 | 992.0 | 0.0 | 1.0405 |
| | 4A5-9 | 12.0 | 12.0 | 5880 | 0.0333 | 0.3163 | 0.0231 | 6.4 | 0.000 | 921.0 | 0.0 | 0.9570 |
| | 4A6-10 | 12.0 | 12.0 | 5900 | 0.0333 | 0.3153 | 0.0226 | 6.4 | 0.000 | 975.0 | 0.0 | 1.0212 |
| | 4C3-11 | 12.0 | 12.0 | 5900 | 0.0344 | 0.3444 | 0.0132 | 6.4 | 0.000 | 958.0 | 0.0 | 0.9605 |
| | 4C4-12 | 12.0 | 12.0 | 5920 | 0.0344 | 0.3433 | 0.0125 | 6.4 | 0.000 | 1105.0 | 0.0 | 1.1105 |
| | 4A1-13 | 12.0 | 12.0 | 4540 | 0.0333 | 0.4670 | 0.0078 | 6.4 | 0.000 | 852.0 | 0.0 | 1.0208 |
| | 2A5-14 | 12.0 | 12.0 | 4570 | 0.0172 | 0.2205 | 0.0231 | 6.4 | 0.000 | 725.0 | 0.0 | 1.0847 |
| | 2A6-15 | 12.0 | 12.0 | 4600 | 0.0172 | 0.2190 | 0.0226 | 6.4 | 0.000 | 780.0 | 0.0 | 1.1475 |
| | 2C1-16 | 12.0 | 12.0 | 4720 | 0.0222 | 0.2825 | 0.0063 | 6.4 | 0.000 | 779.0 | 0.0 | 1.0726 |
| | 2C5-17 | 12.0 | 12.0 | 4770 | 0.0222 | 0.2795 | 0.0192 | 6.4 | 0.000 | 792.0 | 0.0 | 1.0526 |
| | 2C6-18 | 12.0 | 12.0 | 4800 | 0.0222 | 0.2778 | 0.0187 | 6.4 | 0.000 | 1004.0 | 0.0 | 1.3234 |
| | 4B3-19 | 12.0 | 12.0 | 4850 | 0.0367 | 0.4294 | 0.0142 | 6.4 | 0.000 | 920.0 | 0.0 | 1.0632 |
| | 4B4-20 | 12.0 | 12.0 | 5030 | 0.0367 | 0.4140 | 0.0138 | 6.4 | 0.000 | 982.0 | 0.0 | 1.1084 |
| | 4B6-21 | 12.0 | 12.0 | 5150 | 0.0367 | 0.4044 | 0.0194 | 6.4 | 0.000 | 1038.0 | 0.0 | 1.1415 |
| | 4D3-22 | 12.0 | 12.0 | 5150 | 0.0367 | 0.4044 | 0.0158 | 6.4 | 0.000 | 967.0 | 0.0 | 1.0724 |
| | 4D4-23 | 12.0 | 12.0 | 5200 | 0.0367 | 0.4005 | 0.0165 | 6.4 | 0.000 | 1015.0 | 0.0 | 1.1218 |
| | 4D6-24 | 12.0 | 12.0 | 5200 | 0.0367 | 0.4005 | 0.0220 | 6.4 | 0.000 | 1062.0 | 0.0 | 1.1585 |
| | | | | | | | | | | | Mean | 1.0709 |
| | | | | | | | | | | | Coeff. of Variation | 0.0847 |
| Scott, Park & Priestly (1982) | 2 | 17.7 | 17.7 | 3670 | 0.0186 | 0.3194 | 0.0193 | 2.7 | 0.000 | 1589.5 | 0.0 | 1.0258 |
| | 4 | 17.7 | 17.7 | 3670 | 0.0186 | 0.3194 | 0.0193 | 2.7 | 0.109 | 1234.3 | 2381.1 | 1.0878 |
| | 6 | 17.7 | 17.7 | 3670 | 0.0179 | 0.2783 | 0.0182 | 2.7 | 0.000 | 1510.8 | 0.0 | 1.0104 |
| | 8 | 17.7 | 17.7 | 3670 | 0.0179 | 0.2783 | 0.0182 | 2.7 | 0.073 | 1245.5 | 1618.2 | 1.0400 |
| | | | | | | | | | | | Mean | 1.0410 |
| | | | | | | | | | | | Coeff. of Variation | 0.0321 |
| Razvi & Saatcioglu (1989) | 3 | 6.3 | 6.3 | 4641 | 0.0312 | 0.4590 | 0.0243 | 2.9 | 0.000 | 256.5 | 0.0 | 0.9130 |
| | 4 | 6.3 | 6.3 | 4641 | 0.0312 | 0.4590 | 0.0121 | 2.9 | 0.000 | 230.0 | 0.0 | 0.8864 |
| | 5 | 6.3 | 6.3 | 4641 | 0.0312 | 0.4590 | 0.0121 | 2.9 | 0.000 | 217.6 | 0.0 | 0.8300 |
| | 6 | 6.3 | 6.3 | 5657 | 0.0156 | 0.1923 | 0.0251 | 2.9 | 0.000 | 258.1 | 0.0 | 0.9485 |
| | 7 | 6.3 | 6.3 | 5657 | 0.0156 | 0.1923 | 0.0125 | 2.9 | 0.000 | 234.3 | 0.0 | 0.9106 |
| | 15 | 6.3 | 6.3 | 4206 | 0.0312 | 0.5065 | 0.0121 | 2.9 | 0.000 | 231.1 | 0.0 | 0.9335 |
| | 16 | 6.3 | 6.3 | 4206 | 0.0312 | 0.5065 | 0.0243 | 2.9 | 0.000 | 251.1 | 0.0 | 0.9431 |
| | | | | | | | | | | | Mean | 0.9093 |
| | | | | | | | | | | | Coeff. of Variation | 0.0451 |

Table 3.1.1 - Description of Reinforced Concrete Columns without Moment Gradient Used for Comparison with FEM Ultimate Strength

| Author | Col. Desig. | h (in.) | b (in.) | $f'_c$ (psi) | $\rho_{rs}$ | $\rho_{rs} f_{yr} / f'_c$ | Tie/Hoop Vol. Ratio $\rho''$ | $\ell$/h | e/h | TEST VALUES Applied Axial Load (kips) | TEST VALUES Applied BM at Ends (kip-in) | Strength Ratio |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Cusson & Paultre (1992) | 8 B | 9.3 | 9.3 | 7629 | 0.0362 | 0.3317 | 0.0473 | 6.0 | 0.000 | 1018.4 | 0.0 | 1.0719 |
| | 8 D | 9.3 | 9.3 | 8065 | 0.0362 | 0.3135 | 0.0387 | 6.0 | 0.000 | 1018.9 | 0.0 | 1.0644 |
| | | | | | | | | | | | Mean | 1.0682 |
| | | | | | | | | | | | Coeff. of Variation | 0.0050 |
| Fang, Hong & Wu (1994) | MA1-1 | 9.9 | 9.9 | 6356 | 0.0180 | 0.2013 | 0.0229 | 4.0 | 0.000 | 801.9 | 0.0 | 1.0416 |
| | MA1-2 | 9.9 | 9.9 | 8246 | 0.0180 | 0.1552 | 0.0229 | 4.0 | 0.000 | 862.4 | 0.0 | 0.9179 |
| | MA2-1 | 9.9 | 9.9 | 7088 | 0.0180 | 0.1805 | 0.0183 | 4.0 | 0.000 | 895.5 | 0.0 | 1.0936 |
| | MA2-2 | 9.9 | 9.9 | 7088 | 0.0180 | 0.1805 | 0.0183 | 4.0 | 0.000 | 884.4 | 0.0 | 1.0798 |
| | MA3-1 | 9.9 | 9.9 | 6356 | 0.0180 | 0.2013 | 0.0092 | 4.0 | 0.000 | 784.2 | 0.0 | 1.0928 |
| | MA3-2 | 9.9 | 9.9 | 8246 | 0.0180 | 0.1552 | 0.0092 | 4.0 | 0.000 | 850.3 | 0.0 | 0.9541 |
| | MB1-3 | 9.9 | 9.9 | 7658 | 0.0254 | 0.2288 | 0.0275 | 4.0 | 0.000 | 963.8 | 0.0 | 1.0137 |
| | MB1-4 | 9.9 | 9.9 | 7658 | 0.0254 | 0.2288 | 0.0275 | 4.0 | 0.000 | 911.0 | 0.0 | 0.9583 |
| | MB2-3 | 9.9 | 9.9 | 7658 | 0.0254 | 0.2288 | 0.0191 | 4.0 | 0.000 | 886.7 | 0.0 | 0.9634 |
| | MB2-4 | 9.9 | 9.9 | 7658 | 0.0254 | 0.2288 | 0.0191 | 4.0 | 0.000 | 884.4 | 0.0 | 0.9609 |
| | NB1 | 9.9 | 9.9 | 4785 | 0.0254 | 0.3662 | 0.0275 | 4.0 | 0.000 | 659.4 | 0.0 | 0.9485 |
| | NB2 | 9.9 | 9.9 | 4785 | 0.0254 | 0.3662 | 0.0191 | 4.0 | 0.000 | 594.4 | 0.0 | 0.8995 |
| | NB3 | 9.9 | 9.9 | 4785 | 0.0254 | 0.3662 | 0.0137 | 4.0 | 0.000 | 545.9 | 0.0 | 0.8498 |
| | | | | | | | | | | | Mean | 0.9826 |
| | | | | | | | | | | | Coeff. of Variation | 0.0780 |
| Kim & Yang (1995) | 10L4-1 | 3.1 | 3.1 | 3699 | 0.0403 | 0.6120 | 0.0076 | 3.0 | 0.300 | 24.6 | 23.3 | 1.0115 |
| | 10L4-2 | 3.1 | 3.1 | 3699 | 0.0403 | 0.6120 | 0.0076 | 3.0 | 0.300 | 24.6 | 23.2 | 1.0098 |
| | 60L2-1 | 3.1 | 3.1 | 3699 | 0.0202 | 0.3060 | 0.0076 | 18.0 | 0.300 | 14.3 | 13.5 | 1.0087 |
| | 60-L2-2 | 3.1 | 3.1 | 3699 | 0.0202 | 0.3060 | 0.0076 | 18.0 | 0.300 | 14.8 | 14.0 | 1.0412 |
| | 100L2-1 | 3.1 | 3.1 | 3699 | 0.0202 | 0.3060 | 0.0076 | 30.0 | 0.300 | 8.6 | 8.1 | 1.0342 |
| | 100L2-2 | 3.1 | 3.1 | 3699 | 0.0202 | 0.3060 | 0.0076 | 30.0 | 0.300 | 7.9 | 7.4 | 0.9484 |
| | 100L4-1 | 3.1 | 3.1 | 3699 | 0.0403 | 0.6120 | 0.0076 | 30.0 | 0.300 | 11.0 | 10.4 | 0.8819 |
| | 100L4-2 | 3.1 | 3.1 | 3699 | 0.0403 | 0.6120 | 0.0076 | 30.0 | 0.300 | 10.6 | 10.0 | 0.8459 |
| | | | | | | | | | | | Mean | 0.9727 |
| | | | | | | | | | | | Coeff. of Variation | 0.0753 |

Note: The strength ratio is defined as the tested strength divided by the FEM strength.

h = depth of concrete cross-section perpendicular to the axis of bending.
b = width of the concrete cross-section parallel to the axis of bending.

Table 3.1.1 - Description of Reinforced Concrete Columns without Moment Gradient Used for Comparison with FEM Ultimate Strength

$\rho'' = 2(b''+d'')A_t/b''d''s$
$b''$ = outside width of ties/hoops.
$d''$ = outside depth of ties/hoops.
$A_t$ = area of cross-section of a tie/hoop bar.
$s$ = spacing of ties/hoops.

** Excluded from the final analysis on the basis of concrete strength ($f'_c$) being lower than the practical value of 2500 psi, as explained in the text.

*** Revised statistics after the removal of tests identified with a double asterisk (**).

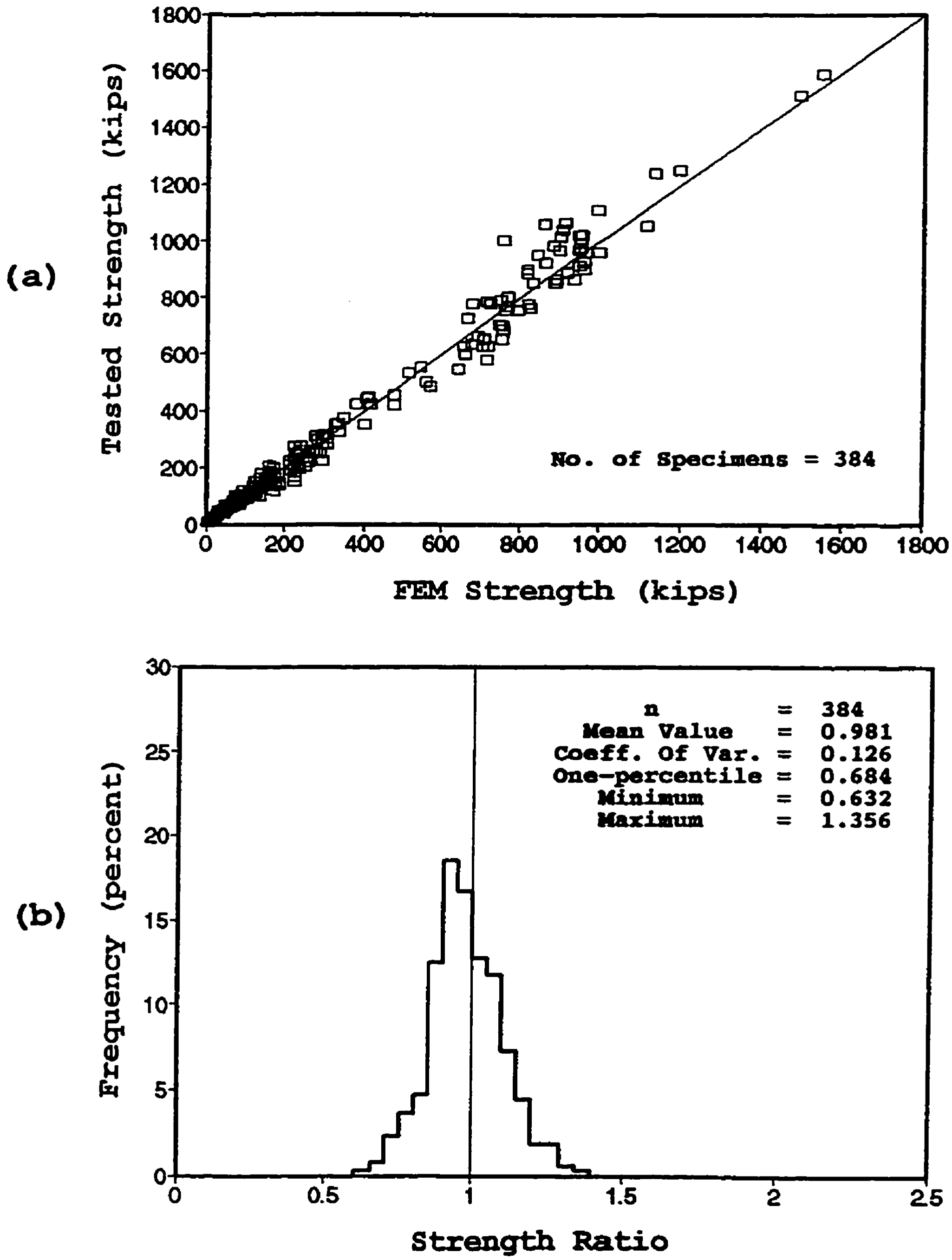



Figure 3.1.1 - Comparison of tested strength to FEM strength for reinforced concrete columns without moment gradient (all $f'_c$).

The calculated mean, coefficient of variation, minimum and maximum values of strength ratios for all test columns listed in Table 3.1.1 are shown in Table 3.1.2. The strength ratio statistics shown in Table 3.1.2 were divided into five categories, based on the slenderness ratio ($\ell/h$). Columns with $\ell/h$ less than or equal to 3 are assumed to be pedestals, short columns are assumed to have $\ell/h$ greater than 3 but less than 6.6, slender columns are assumed to have $\ell/h$ greater than or equal to 6.6 but less than or equal to 30, super-slender columns are assumed to have $\ell/h$ greater than 30, and ACI-permitted columns are assumed to have $\ell/h$ greater than 3 but less than or equal to 30. The data were further categorized into five ranges of end eccentricity ratio ($e/h$) as shown in Table 3.1.2.

Differences in the statistics for four different ranges of end eccentricity ratios were observed (Table 3.1.2 Columns 3,4,5 and 6). This was particularly evident for super-slender columns. Super-slender columns with a low end eccentricity ratio have a low coefficient of variation (4.4 percent for $e/h$=0 and 3.6 percent for 0<$e/h$<0.1) while the same columns with higher end eccentricity ratios have a relatively high coefficient of variation (15.3 percent for 0.1≤$e/h$≤0.7). The overall minimum strength ratio (0.632) was found to occur in a

Table 3.1.2 - Strength Ratio Statistics of Reinforced Concrete Columns without Moment Gradient for Different Ranges of e/h and $\ell/h$, using FEM (all $f'_c$)

| Column Type (1) | (2) | $e/h = 0$ (3) | $0 < e/h < 0.1$ (4) | $0.1 \le e/h \le 0.7$ (5) | $0.1 \le e/h \le 1.5$ (6) | $e/h = \infty$ (7) |
|---|---|---|---|---|---|---|
| Pedestal $\ell/h \le 3$ | No. | 16 | 1 | 4 | 4 | * |
| | Mean | 0.973 | 1.040 | 1.070 | 1.070 | * |
| | CV | 0.125 | | 0.071 | 0.071 | * |
| | Min | 0.830 | 1.040 | 1.010 | 1.010 | * |
| | Max | 1.290 | 1.040 | 1.171 | 1.171 | * |
| Short $3 < \ell/h < 6.6$ | No. | 79 | * | 12 | 19 | * |
| | Mean | 0.992 | * | 0.941 | 0.914 | * |
| | CV | 0.110 | * | 0.106 | 0.143 | * |
| | Min | 0.828 | * | 0.790 | 0.632 | * |
| | Max | 1.323 | * | 1.150 | 1.150 | * |
| Slender $6.6 \le \ell/h \le 30$ | No. | 79 | 9 | 133 | 164 | * |
| | Mean | 0.956 | 1.083 | 1.015 | 1.009 | * |
| | CV | 0.133 | 0.150 | 0.128 | 0.121 | * |
| | Min | 0.659 | 0.876 | 0.761 | 0.761 | * |
| | Max | 1.192 | 1.277 | 1.356 | 1.356 | * |
| Super Slender $\ell/h > 30$ | No. | 2 | 3 | 18 | 18 | * |
| | Mean | 0.969 | 0.928 | 0.876 | 0.876 | * |
| | CV | 0.044 | 0.036 | 0.153 | 0.153 | * |
| | Min | 0.939 | 0.899 | 0.708 | 0.708 | * |
| | Max | 0.999 | 0.964 | 1.090 | 1.090 | * |
| ACI Permitted $3 < \ell/h \le 30$ | No. | 158 | 9 | 145 | 183 | * |
| | Mean | 0.974 | 1.083 | 1.009 | 0.999 | * |
| | CV | 0.123 | 0.150 | 0.128 | 0.126 | * |
| | Min | 0.659 | 0.876 | 0.761 | 0.632 | * |
| | Max | 1.323 | 1.277 | 1.356 | 1.356 | * |

* No data available

Note: CV stands for the coefficient of variation.

column with $\ell/h$=6.0 and $e/h$=1.056, while the overall maximum strength ratio (1.356) was found to occur in a column with $\ell/h$=8.0 and $e/h$=0.298, as indicated by Table 3.1.1. The probability distribution of the strength ratios computed for the 384 test columns is plotted on a normal probability scale in Figure 3.1.2 and is compared to a normal probability distribution with a mean value of 0.98 and a coefficient of variation of 12.5 percent. The data closely follow the normal curve and can be assumed to be normally distributed.

Considering current construction practice, it was decided to exclude all columns with a specified concrete cylinder strength less than 2500 psi. For concrete strengths reported by cube strengths, the equivalent standard cylinder (6 inch diameter by 12 inch high) strength was computed, and this value was used as the basis for excluding the column from this study. Using this criterion resulted in the removal of thirty columns from the data base. These columns are identified by a double asterisk (**) in Table 3.1.1. The removal of the thirty columns affected the overall statistics by slightly increasing both the mean and coefficient of variation from 0.981 and 12.6 percent to 0.984 and 12.7 percent, respectively. The overall minimum and maximum strength ratios did not change. Revised statistics are also included in Table 3.1.1 for authors whose columns were

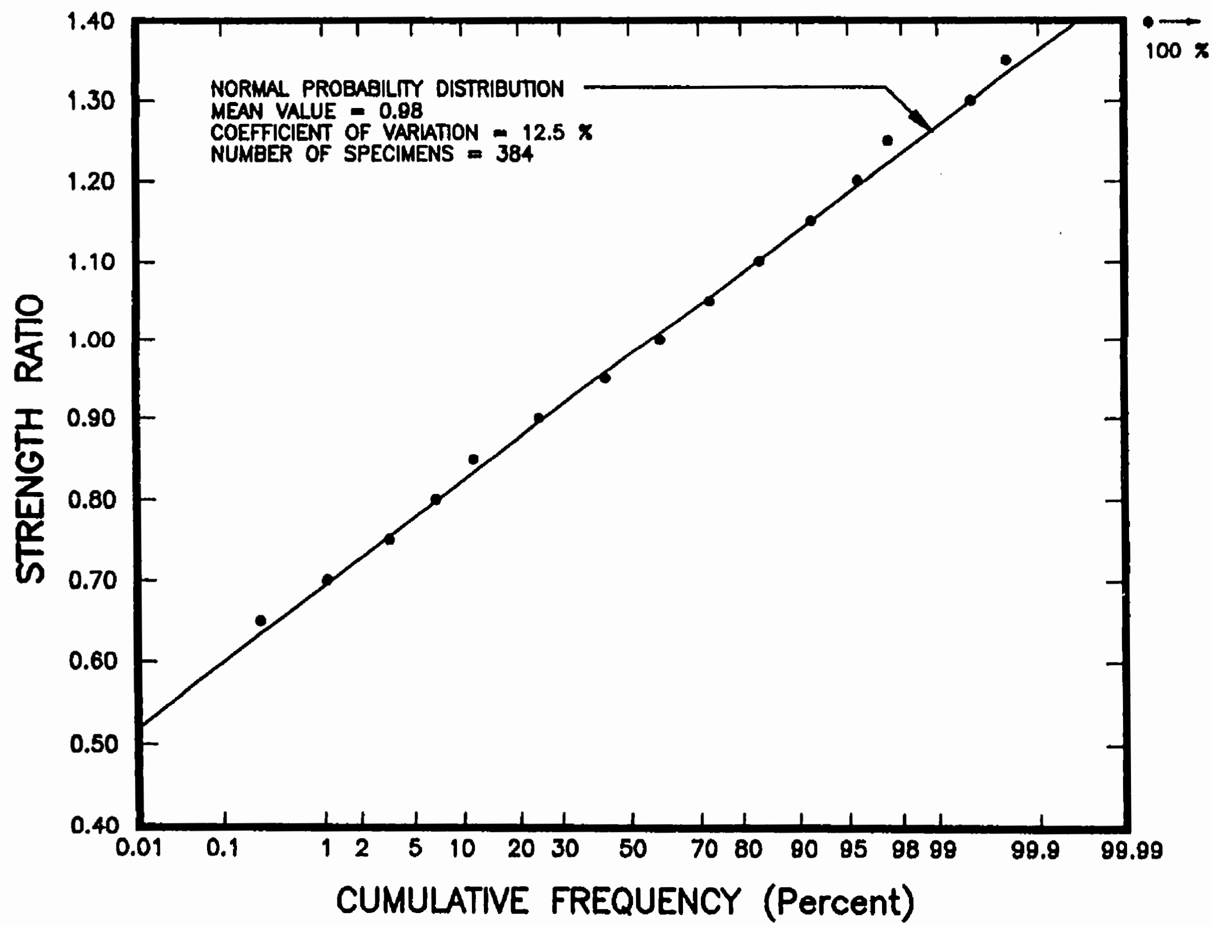



Figure 3.1.2 - Probability distribution of strength ratios using FEM of reinforced concrete columns without moment gradient (all $f'_c$).

removed from the data base. Columns in Table 3.1.1 that are not identified by a double asterisk represent the 354 reinforced concrete test columns that were used in the comparative study.

A plot of tested strength against the FEM strength (Figure 3.1.3(a)) for the 354 test columns shows a relatively narrow band of strength ratios. A histogram plotting the frequency, in percent, against the strength ratios (Figure 3.1.3(b)) shows a symmetric distribution of values about the mean. The strength ratio statistics for the 354 test columns in Table 3.1.3 do not show any significant differences over those obtained for the data given in Table 3.1.2 which included all 384 test columns.

The probability distribution of the strength ratios computed for the 354 test columns is plotted on a normal probability scale in Figure 3.1.4 and is compared to a normal probability distribution with a mean value of 0.98 and a coefficient of variation of 12.5 percent. The data closely follow the normal curve and can be assumed to be normally distributed.

### 3.1.2 Reinforced Concrete Columns With Moment Gradient

The ultimate strengths computed by FEM were compared with the ultimate strengths of 14 physical tests taken from MacGregor and Barter (1965), Martin and Olivieri (1965), and Mehmel et al. (1969).

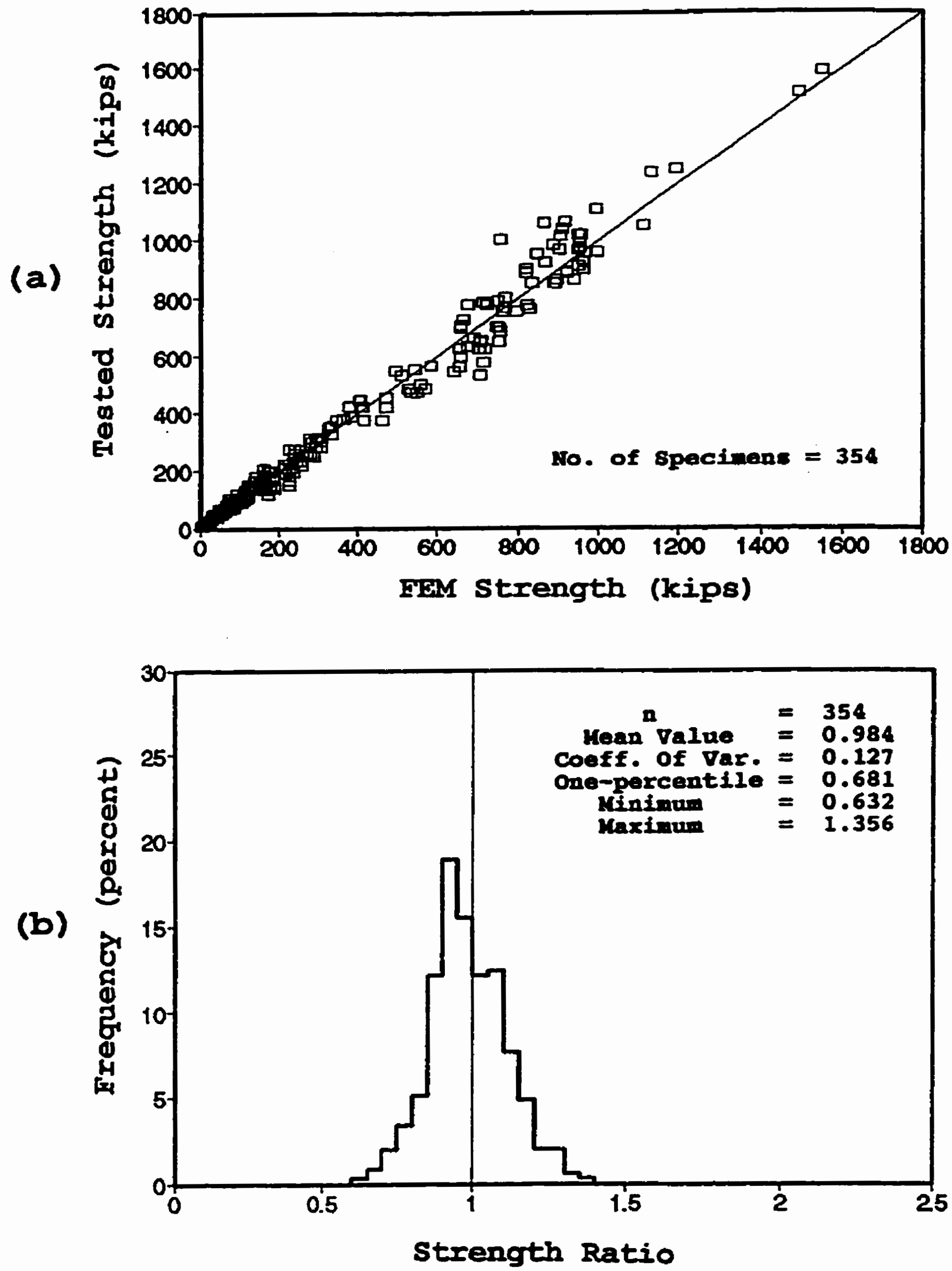



Figure 3.1.3 - Comparison of tested strength to FEM strength for reinforced concrete columns without moment gradient ($f'_c \geq 2500$ psi).

Table 3.1.3 - Strength Ratio Statistics of Reinforced Concrete Columns without Moment Gradient for Different Ranges of e/h and $\ell/h$, using FEM ($f'_c \geq 2500$ psi).

| Column Type (1) | (2) | $e/h = 0$ (3) | $0 < e/h < 0.1$ (4) | $0.1 \leq e/h \leq 0.7$ (5) | $0.1 \leq e/h \leq 1.5$ (6) | $e/h = \infty$ (7) |
|---|---|---|---|---|---|---|
| Pedestal $\ell/h \leq 3$ | No. | 10 | 1 | 4 | 4 | * |
| | Mean | 0.940 | 1.040 | 1.070 | 1.070 | * |
| | CV | 0.064 | | 0.071 | 0.071 | * |
| | Min | 0.830 | 1.040 | 1.010 | 1.010 | * |
| | Max | 1.026 | 1.040 | 1.171 | 1.171 | * |
| Short $3 < \ell/h < 6.6$ | No. | 79 | * | 12 | 19 | * |
| | Mean | 0.992 | * | 0.941 | 0.914 | * |
| | CV | 0.110 | * | 0.106 | 0.143 | * |
| | Min | 0.828 | * | 0.790 | 0.632 | * |
| | Max | 1.323 | * | 1.150 | 1.150 | * |
| Slender $6.6 \leq \ell/h \leq 30$ | No. | 72 | 5 | 121 | 141 | * |
| | Mean | 0.971 | 0.957 | 1.019 | 1.013 | * |
| | CV | 0.125 | 0.084 | 0.132 | 0.127 | * |
| | Min | 0.659 | 0.876 | 0.761 | 0.761 | * |
| | Max | 1.192 | 1.077 | 1.356 | 1.356 | * |
| Super Slender $\ell/h > 30$ | No. | 2 | 3 | 18 | 18 | * |
| | Mean | 0.969 | 0.928 | 0.876 | 0.876 | * |
| | CV | 0.044 | 0.036 | 0.153 | 0.153 | * |
| | Min | 0.939 | 0.899 | 0.708 | 0.708 | * |
| | Max | 0.999 | 0.964 | 1.090 | 1.090 | * |
| ACI Permitted $3 < \ell/h \leq 30$ | No. | 151 | 5 | 133 | 160 | * |
| | Mean | 0.982 | 0.957 | 1.012 | 1.001 | * |
| | CV | 0.118 | 0.084 | 0.132 | 0.132 | * |
| | Min | 0.659 | 0.876 | 0.761 | 0.632 | * |
| | Max | 1.323 | 1.077 | 1.356 | 1.356 | * |

* No data available

Note: CV stands for the coefficient of variation.

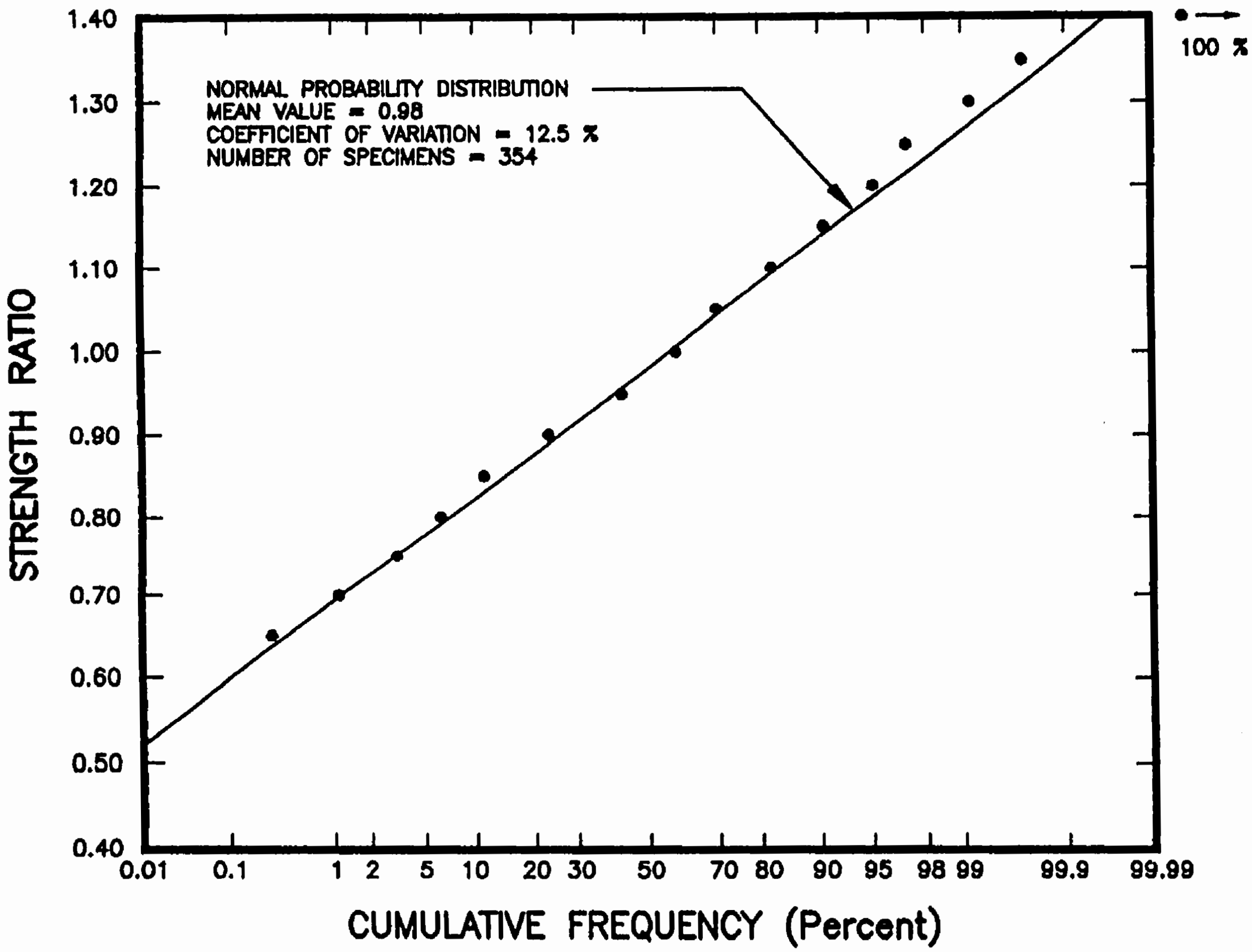



Figure 3.1.4 - Probability distribution of strength ratios using FEM of reinforced concrete columns without moment gradient ($f'_c \geq 2500$ psi).

A description of these 14 physical tests used for the comparison of tested to FEM strength of reinforced concrete columns with moment gradients is given in Table 3.1.4. The table includes information on the geometric and material properties of test columns. Included in the table is the ratio of tested to FEM ultimate strength (strength ratio) for each of the 14 column specimens. The strength ratio was taken as the ratio of axial load capacities of a column.

The plot of tested strength against the FEM strength (Figure 3.1.5(a)) shows a relatively narrow band of strength ratios. This indicates that the FEM model was able to predict the tested strength of the columns quite accurately with no apparent or significant outliers. Also, as the strengths of the columns increase, there is a proportional increase in the magnitude of error. This is expected since the percentage of error remains relatively constant. A histogram plotting the frequency, in percent, against the strength ratio (Figure 3.1.5(b)) shows a slightly non-symmetric distribution of values about the mean.

The mean strength ratio of all 14 test columns was 1.003 with a coefficient of variation of 9.8 percent (Figure 3.1.5(b)). This is comparable to a mean value of 0.984 and a coefficient of variation of 12.7 percent obtained for 354 reinforced concrete columns without moment gradient (Figure 3.1.3(b)).

Table 3.1.4 - Description of Reinforced Concrete Columns with Moment Gradient Used for Comparison with FEM Ultimate Strength

| Author | Col. Desig. | h (in.) | b (in.) | $f'_c$ (psi) | $\rho_{rs}$ | $\frac{\rho_{rs} f_{yr}}{f'_c}$ | Tie/Hoop Vol. Ratio $\rho''$ | $\ell/h$ | e/h | TEST VALUES Applied Axial Load (kips) | Applied BM at Ends (kip-in) $M_1$ | $M_2$ | Strength Ratio |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| MacGregor | A-1 | 2.5 | 4.4 | 4880 | 0.0400 | 0.3664 | 0.0078 | 27.3 | 0.200 | 38.0 | -19.0 | 19.0 | 0.9471 |
| & Barter | A-2 | 2.5 | 4.4 | 4740 | 0.0400 | 0.3772 | 0.0078 | 27.3 | 0.200 | 38.0 | -19.0 | 19.0 | 0.9658 |
| (1965) | B-1 | 2.5 | 4.4 | 4210 | 0.0400 | 0.4247 | 0.0078 | 27.3 | 1.500 | 7.5 | -27.9 | 27.9 | 1.2494 |
| | B-2 | 2.5 | 4.4 | 4730 | 0.0400 | 0.3780 | 0.0078 | 27.3 | 1.500 | 7.1 | -26.5 | 26.5 | 1.1585 |
| | | | | | | | | | | | | Mean | 1.0802 |
| | | | | | | | | | | | | Coeff. of Variation | 0.1368 |
| Martin | 412-1 | 3.5 | 5.0 | 4880 | 0.0248 | 0.2036 | 0.0036 | 40.0 | 0.211 | 26.5 | -9.9 | 19.8 | 1.0351 |
| & Olivieri | 412-2 | 3.5 | 5.0 | 3630 | 0.0248 | 0.2737 | 0.0036 | 40.0 | 0.211 | 20.0 | -7.5 | 15.0 | 0.8993 |
| (1965) | 422-1 | 3.5 | 5.0 | 5060 | 0.0248 | 0.1963 | 0.0036 | 40.0 | 0.388 | 21.0 | -14.4 | 28.9 | 1.0596 |
| | 422-2 | 3.5 | 5.0 | 3730 | 0.0248 | 0.2664 | 0.0036 | 40.0 | 0.388 | 17.0 | -11.7 | 23.4 | 0.9854 |
| | 432-1 | 3.5 | 5.0 | 5410 | 0.0248 | 0.1836 | 0.0036 | 40.0 | 0.282 | 21.5 | -10.7 | 21.5 | 0.8895 |
| | 432-2 | 3.5 | 5.0 | 3830 | 0.0248 | 0.2594 | 0.0036 | 40.0 | 0.282 | 21.0 | -10.5 | 21.0 | 1.0243 |
| | | | | | | | | | | | | Mean | 0.9822 |
| | | | | | | | | | | | | Coeff. of Variation | 0.0735 |
| Mehmel, | 1.1 | 8.0 | 10.0 | 5105 | 0.0122 | 0.1487 | 0.0038 | 16.7 | 0.177 | 192.8 | 261.3 | 273.2 | 0.9665 |
| Schwarz, | 3.2 | 5.9 | 9.9 | 5373 | 0.0125 | 0.1539 | 0.0051 | 22.5 | 0.503 | 39.7 | 117.2 | 118.8 | 0.9445 |
| Kasparek | 6.1 | 6.3 | 10.0 | 5572 | 0.0111 | 0.1316 | 0.0056 | 14.5 | 0.170 | 211.3 | 0.0 | 224.6 | 0.9532 |
| & Makovi | 6.2 | 6.2 | 10.0 | 5814 | 0.0112 | 0.1271 | 0.0057 | 21.7 | 0.503 | 77.2 | 0.0 | 240.1 | 0.9679 |
| (1969) | | | | | | | | | | | | | |
| | | | | | | | | | | | | Mean | 0.9581 |
| | | | | | | | | | | | | Coeff. of Variation | 0.0117 |

Note: The strength ratio is defined as the tested strength divided by the FEM strength.

h = depth of concrete cross-section perpendicular to the axis of bending.

b = width of the concrete cross-section parallel to the axis of bending.

$\rho'' = 2(b''+d'')A_t/b''d''s$

b″ = outside width of ties/hoops.

d″ = outside depth of ties/hoops.

Table 3.1.4 - Description of Reinforced Concrete Columns with Moment Gradient Used for Comparison with FEM Ultimate Strength

$A_t$ = area of cross-section of a tie/hoop bar.
s = spacing of ties/hoops.
$M_1$ = smaller end moment, positive if member is bent in single curvature, negative if bent in double curvature.
$M_2$ = larger end moment, always positive.

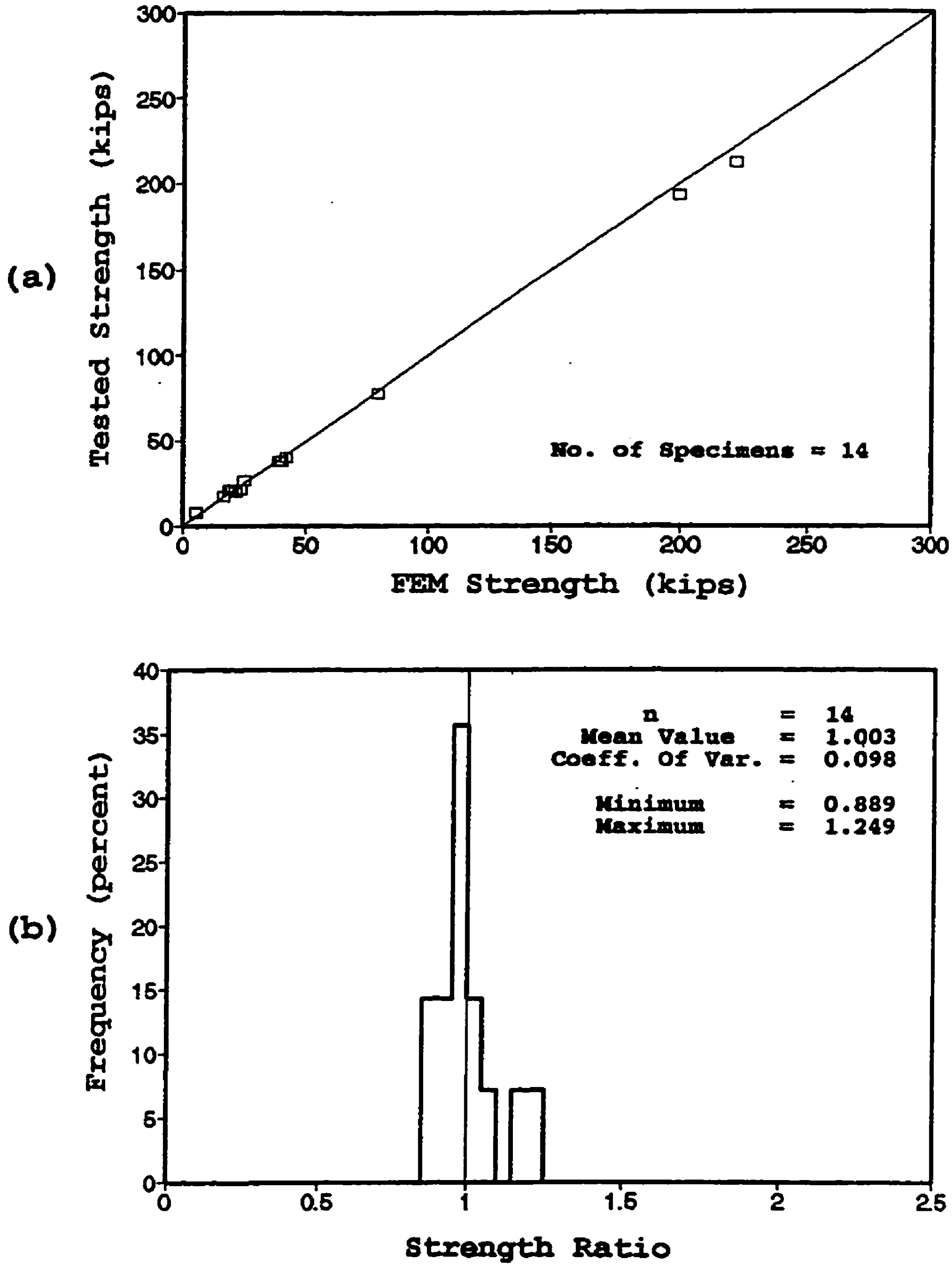



Figure 3.1.5 - Comparison of tested strength to FEM strength for reinforced concrete columns with moment gradient.

The calculated mean, coefficient of variation, minimum and maximum values of strength ratios for all test columns listed in Table 3.1.4 are given in Table 3.1.5. The strength ratio statistics shown in Table 3.1.5 were divided into five categories, based on the slenderness ratio ($\ell/h$). The data were further categorized into five ranges of end eccentricity ratio ($e/h$) as shown in Table 3.1.5.

Differences in statistics of two different ranges of end eccentricity ratios (Table 3.1.5 Columns 5 and 6) were observed. Slender columns with an end eccentricity ratio less than or equal to 0.7 have a low coefficient of variation (1.1 percent) while slender columns with end eccentricity ratios greater than 0.7 have a relatively high coefficient of variation (11.5 percent). The overall minimum strength ratio (0.889) was found to occur in a column with $\ell/h$=40 and $e/h$=0.282, while the overall maximum strength ratio (1.249) was found to occur in a column with $\ell/h$=27.3 and $e/h$=1.5, as indicated in Table 3.1.4.

The probability distribution of the strength ratios computed for the 14 test columns is plotted on a normal probability scale in Figure 3.1.6 and is compared to a normal probability distribution with a mean of 1.0 and a coefficient of variation of 10 percent. The data follow the normal curve with a slight scatter, however, it can be assumed to be normally distributed.

Table 3.1.5 - Strength Ratio Statistics of Reinforced Concrete Columns with Moment Gradient for Different Ranges of e/h and ℓ/h using FEM

| Column Type (1) | (2) | e/h = 0 (3) | 0 < e/h < 0.1 (4) | 0.1 ≤ e/h ≤ 0.7 (5) | 0.1 ≤ e/h ≤ 1.5 (6) | e/h = ∞ (7) |
|---|---|---|---|---|---|---|
| Pedestal ℓ/h ≤ 3 | No. | * | * | * | * | * |
| | Mean | * | * | * | * | * |
| | CV | * | * | * | * | * |
| | Min | * | * | * | * | * |
| | Max | * | * | * | * | * |
| Short 3 < ℓ/h < 6.6 | No. | * | * | * | * | * |
| | Mean | * | * | * | * | * |
| | CV | * | * | * | * | * |
| | Min | * | * | * | * | * |
| | Max | * | * | * | * | * |
| Slender 6.6 ≤ ℓ/h ≤ 30 | No. | * | * | 6 | 8 | * |
| | Mean | * | * | 0.958 | 1.019 | * |
| | CV | * | * | 0.011 | 0.115 | * |
| | Min | * | * | 0.945 | 0.945 | * |
| | Max | * | * | 0.968 | 1.249 | * |
| Super Slender ℓ/h > 30 | No. | * | * | 6 | 6 | * |
| | Mean | * | * | 0.982 | 0.982 | * |
| | CV | * | * | 0.073 | 0.073 | * |
| | Min | * | * | 0.889 | 0.889 | * |
| | Max | * | * | 1.060 | 1.060 | * |
| ACI Permitted 3 < ℓ/h ≤ 30 | No. | * | * | 6 | 8 | * |
| | Mean | * | * | 0.958 | 1.019 | * |
| | CV | * | * | 0.011 | 0.115 | * |
| | Min | * | * | 0.945 | 0.945 | * |
| | Max | * | * | 0.968 | 1.249 | * |

* No data available

Note: CV stands for the coefficient of variation.

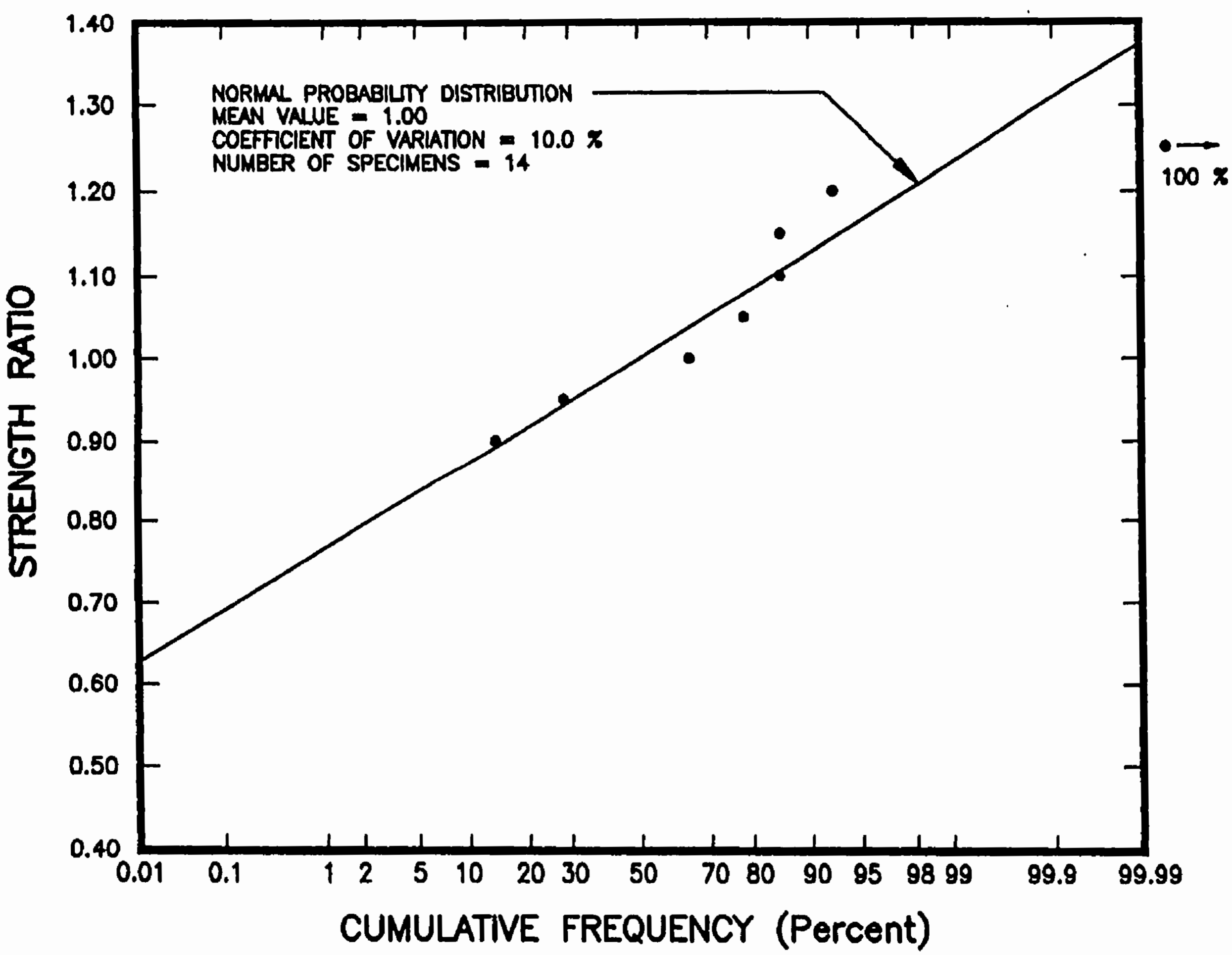



Figure 3.1.6 - Probability distribution of strength ratios using FEM of reinforced concrete columns with moment gradient.

## 3.2 COMPARISON OF FEM METHOD WITH EXPERIMENTAL RESULTS FOR COMPOSITE STEEL-CONCRETE COLUMNS SUBJECTED TO MAJOR AXIS BENDING

The analysis of composite steel-concrete columns subjected to major axis bending was divided into two separate groups: columns with equal and opposite applied end moments (without moment gradient) and columns with unequal applied end moments (with moment gradient).

### 3.2.1 Composite Steel-Concrete Columns Subjected to Major Axis Bending Without Moment Gradient

The ultimate strengths computed using FEM were compared with the ultimate strengths of 75 physical tests taken from Bondale (1966), Procter (1967), Johnson and May (1978), Morino et al. (1984), Suzuki et al. (1984), Roik and Mangerig (1987), and Roik and Schwalbenhofer (1988). Six of the physical tests were eventually removed from the comparison for reasons that will be discussed later in this section.

A description of these 75 physical tests used for the comparison of tested to FEM strength for composite steel-concrete columns subjected to major axis bending without moment gradient is given in Table 3.2.1. The table includes information on the geometric and material properties of test columns. Included in the table is the ratio of tested to FEM ultimate strength (strength ratio) for each of the 75 column specimens. The strength ratio

Table 3.2.1 - Description of Composite Steel-Concrete Columns Subjected to Major Axis Bending without Moment Gradient Used for Comparison with FEM Ultimate Strength

| Author | Col. Desig. | h (in.) | b (in.) | $f'_c$ (psi) | $\rho_{ss}$ | $\rho_{rs}$ | $\frac{\rho_{ss} f_{yss}}{f'_c}$ | Tie/Hoop Vol. Ratio $\rho''$ | $\ell/h$ | e/h | TEST VALUES Applied Axial Load (kips) | TEST VALUES Applied BM at Ends (kip-in) | Strength Ratio |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Bondale (1966) | R.S.100.1 | 6.0 | 3.8 | 4192 | 0.0653 | 0.0062 | 0.698 | 0.0064 | 17.6 | 0.167 | 92.3 | 92.3 | 1.2324 * |
| | R.S.80.2 | 6.0 | 3.8 | 4313 | 0.0653 | 0.0062 | 0.679 | 0.0064 | 14.3 | 0.333 | 70.1 | 140.2 | 1.2379 * |
| | R.S.60.3 | 6.0 | 3.8 | 4435 | 0.0653 | 0.0062 | 0.660 | 0.0064 | 11.0 | 0.500 | 55.8 | 167.3 | 1.1778 * |
| | | | | | | | | | | | | Mean | 1.2160 |
| | | | | | | | | | | | | Coeff. of Variation | 0.0273 |
| Procter (1967) | 1 | 11.3 | 8.0 | 4722 | 0.0473 | - | 0.422 | - | 11.7 | 0.533 | 132.2 | 793.0 | 0.9676 |
| | 2 | 11.3 | 8.0 | 4722 | 0.0473 | - | 0.422 | - | 11.7 | 0.800 | 87.4 | 786.2 | 0.9899 |
| | 3 | 11.3 | 8.0 | 4722 | 0.0473 | - | 0.422 | - | 11.7 | 0.000 | 470.4 | 0.0 | 0.8534 |
| | 4 | 11.3 | 8.0 | 4722 | 0.0473 | - | 0.422 | - | 11.7 | 0.533 | 143.4 | 860.2 | 1.0510 |
| | 5 | 11.3 | 8.0 | 5407 | 0.0473 | - | 0.369 | - | 11.7 | 0.800 | 91.8 | 826.6 | 1.0088 |
| | 6 | 12.0 | 8.0 | 5407 | 0.0521 | - | 0.410 | - | 11.0 | 0.750 | 129.9 | 1169.3 | 1.1099 |
| | 7 | 12.0 | 8.0 | 5407 | 0.0521 | - | 0.410 | - | 11.0 | 0.500 | 199.4 | 1196.2 | 1.1154 |
| | 8 | 12.0 | 8.0 | 5407 | 0.0521 | - | 0.410 | - | 11.0 | 0.000 | 560.0 | 0.0 | 0.8503 |
| | 9 | 11.0 | 8.0 | 6007 | 0.0484 | - | 0.339 | - | 12.0 | 0.273 | 268.8 | 806.4 | 1.0522 |
| | 10 | 11.0 | 8.0 | 6007 | 0.0484 | - | 0.339 | - | 12.0 | 0.273 | 250.9 | 752.6 | 0.9823 |
| | 11 | 12.0 | 8.0 | 6007 | 0.0521 | - | 0.369 | - | 11.0 | 0.000 | 533.1 | 0.0 | 0.7504 |
| | 12 | 12.0 | 8.0 | 6007 | 0.0521 | - | 0.369 | - | 11.0 | 0.250 | 315.8 | 947.5 | 1.0133 |
| | S1 | 11.0 | 8.0 | 4722 | 0.0484 | - | 0.432 | - | 2.2 | 0.000 | 470.4 | 0.0 | 0.8765 |
| | S2 | 11.0 | 8.0 | 4722 | 0.0484 | - | 0.432 | - | 2.2 | 0.000 | 481.6 | 0.0 | 0.9101 |
| | S3 | 12.0 | 8.0 | 5407 | 0.0521 | - | 0.410 | - | 2.0 | 0.000 | 698.9 | 0.0 | 1.0537 |
| | S4 | 12.0 | 8.0 | 5407 | 0.0521 | - | 0.410 | - | 2.0 | 0.000 | 703.4 | 0.0 | 1.0598 |
| | | | | | | | | | | | | Mean | 0.9778 |
| | | | | | | | | | | | | Coeff. of Variation | 0.1058 |
| Johnson & May (1978) | RC1 | 7.9 | 7.9 | 3620 | 0.0745 | 0.0028 | 0.866 | 0.0019 | 8.1 | 0.112 | 301.3 | 265.5 | 1.0912 * |
| | RC3 | 7.9 | 7.9 | 2847 | 0.0745 | 0.0028 | 1.101 | 0.0019 | 8.1 | 0.136 | 305.8 | 327.5 | 1.2957 * |
| | RC4 | 7.9 | 7.9 | 4363 | 0.0745 | 0.0028 | 0.718 | 0.0019 | 14.8 | 0.197 | 191.1 | 296.5 | 0.8692 * |
| | | | | | | | | | | | | Mean | 1.0853 |
| | | | | | | | | | | | | Coeff. of Variation | 0.1965 |
| Morino, Matsui & Watanabe (1984) | A4-90 | 6.3 | 6.3 | 3060 | 0.0870 | 0.0044 | 1.481 | 0.0025 | 5.8 | 0.250 | 166.4 | 262.0 | 1.1486 |
| | B4-90 | 6.3 | 6.3 | 3394 | 0.0870 | 0.0044 | 1.302 | 0.0025 | 14.4 | 0.250 | 114.5 | 180.3 | 0.9145 |
| | C4-90 | 6.3 | 6.3 | 3380 | 0.0870 | 0.0044 | 1.176 | 0.0025 | 21.7 | 0.250 | 93.9 | 147.8 | 0.9759 |
| | D4-90 | 6.3 | 6.3 | 3075 | 0.0870 | 0.0044 | 1.474 | 0.0025 | 28.9 | 0.250 | 64.7 | 101.8 | 0.8309 |
| | A8-90 | 6.3 | 6.3 | 4874 | 0.0870 | 0.0044 | 0.953 | 0.0025 | 5.8 | 0.469 | 118.0 | 348.3 | 1.0166 |
| | B8-90 | 6.3 | 6.3 | 4830 | 0.0870 | 0.0044 | 0.956 | 0.0025 | 14.4 | 0.469 | 93.9 | 277.3 | 0.9541 |
| | C8-90 | 6.3 | 6.3 | 3568 | 0.0870 | 0.0044 | 1.305 | 0.0025 | 21.7 | 0.469 | 68.0 | 200.7 | 0.9302 |
| | D8-90 | 6.3 | 6.3 | 3322 | 0.0870 | 0.0044 | 1.398 | 0.0025 | 28.9 | 0.469 | 50.1 | 147.9 | 0.8823 |
| | | | | | | | | | | | | Mean | 0.9566 |
| | | | | | | | | | | | | Coeff. of Variation | 0.1005 |

Table 3.2.1 - Description of Composite Steel-Concrete Columns Subjected to Major Axis Bending without Moment Gradient Used for Comparison with FEM Ultimate Strength

| Author | Col. Desig. | h (in.) | b (in.) | $f'_c$ (psi) | $\rho_{ss}$ | $\rho_{rs}$ | $\frac{\rho_{ss} f_{yss}}{f'_c}$ | Tie/Hoop Vol. Ratio $\rho''$ | $\ell$/h | e/h | TEST VALUES Applied Axial Load (kips) | TEST VALUES Applied BM at Ends (kip-in) | Strength Ratio |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Suzuki, | LH-000-C | 8.3 | 8.3 | 4781 | 0.0290 | 0.0021 | 0.275 | 0.0010 | 2.9 | 0.000 | 380.0 | 0.0 | 1.0339 |
| Takiguchi, | LH-020-C | 8.3 | 8.3 | 4781 | 0.0290 | 0.0021 | 0.275 | 0.0291 | 2.9 | 0.000 | 374.3 | 0.0 | 0.8056 |
| Ichinose | LH-040-C | 8.3 | 8.3 | 4781 | 0.0290 | 0.0021 | 0.275 | 0.0146 | 2.9 | 0.000 | 374.3 | 0.0 | 0.8949 |
| & Okamoto | LH-100-C | 8.3 | 8.3 | 4781 | 0.0290 | 0.0021 | 0.275 | 0.0058 | 2.9 | 0.000 | 385.8 | 0.0 | 0.9922 |
| (1984) | RH-000-C | 8.3 | 8.3 | 4852 | 0.0546 | 0.0021 | 0.625 | 0.0010 | 2.9 | 0.000 | 547.0 | 0.0 | 1.0952 |
| | RH-020-C | 8.3 | 8.3 | 4852 | 0.0546 | 0.0021 | 0.625 | 0.0291 | 2.9 | 0.000 | 561.4 | 0.0 | 0.9573 |
| | RH-040-C | 8.3 | 8.3 | 4852 | 0.0546 | 0.0021 | 0.625 | 0.0146 | 2.9 | 0.000 | 521.1 | 0.0 | 0.9575 |
| | RH-100-C | 8.3 | 8.3 | 4852 | 0.0546 | 0.0021 | 0.625 | 0.0058 | 2.9 | 0.000 | 521.1 | 0.0 | 1.0030 |
| | HT60-000-C | 8.3 | 8.3 | 4852 | 0.0600 | 0.0021 | 1.037 | 0.0010 | 2.9 | 0.000 | 598.8 | 0.0 | 1.0091 |
| | HT60-020-C | 8.3 | 8.3 | 4852 | 0.0600 | 0.0021 | 1.037 | 0.0291 | 2.9 | 0.000 | 656.4 | 0.0 | 0.9092 |
| | HT60-040-C | 8.3 | 8.3 | 4852 | 0.0600 | 0.0021 | 1.037 | 0.0146 | 2.9 | 0.000 | 662.2 | 0.0 | 0.9820 |
| | HT60-100-C | 8.3 | 8.3 | 4852 | 0.0600 | 0.0021 | 1.037 | 0.0058 | 2.9 | 0.000 | 627.6 | 0.0 | 0.9806 |
| | HT80-000-C | 8.3 | 8.3 | 4852 | 0.0633 | 0.0021 | 1.482 | 0.0010 | 2.9 | 0.000 | 716.9 | 0.0 | 1.1836 |
| | HT80-020-C | 8.3 | 8.3 | 4852 | 0.0633 | 0.0021 | 1.482 | 0.0291 | 2.9 | 0.000 | 734.2 | 0.0 | 0.8597 |
| | HT80-040-C | 8.3 | 8.3 | 4852 | 0.0633 | 0.0021 | 1.482 | 0.0146 | 2.9 | 0.000 | 728.4 | 0.0 | 0.9139 |
| | HT80-100-C | 8.3 | 8.3 | 4852 | 0.0633 | 0.0021 | 1.482 | 0.0058 | 2.9 | 0.000 | 711.1 | 0.0 | 0.9405 |
| | HT-80-000-CB | 8.3 | 8.3 | 4425 | 0.0423 | 0.0021 | 1.059 | 0.0007 | 3.8 | 0.873 | 110.4 | 797.1 | 1.0812 |
| | HT-80-020-CB | 8.3 | 8.3 | 4425 | 0.0423 | 0.0021 | 1.059 | 0.0291 | 3.8 | 1.062 | 110.4 | 969.3 | 0.8584 |
| | LH-000-B | 8.3 | 8.3 | 4297 | 0.0290 | 0.0021 | 0.306 | 0.0010 | 2.9 | inf. | 0.0 | 328.8 | 1.0857 |
| | LH-020-B | 8.3 | 8.3 | 4596 | 0.0290 | 0.0021 | 0.286 | 0.0291 | 2.9 | inf. | 0.0 | 352.8 | 1.1459 |
| | LH-040-B | 8.3 | 8.3 | 4525 | 0.0290 | 0.0021 | 0.290 | 0.0146 | 2.9 | inf. | 0.0 | 338.4 | 1.1049 |
| | LH-100-B | 8.3 | 8.3 | 4368 | 0.0290 | 0.0021 | 0.301 | 0.0058 | 2.9 | inf. | 0.0 | 338.4 | 1.1125 |
| | RH-000-B | 8.3 | 8.3 | 4852 | 0.0546 | 0.0021 | 0.625 | 0.0010 | 2.9 | inf. | 0.0 | 586.8 | 0.9663 |
| | RH-020-B | 8.3 | 8.3 | 4852 | 0.0546 | 0.0021 | 0.625 | 0.0291 | 2.9 | inf. | 0.0 | 654.0 | 1.0571 |
| | RH-040-B | 8.3 | 8.3 | 4852 | 0.0546 | 0.0021 | 0.625 | 0.0146 | 2.9 | inf. | 0.0 | 639.6 | 1.0410 |
| | RH-100-B | 8.3 | 8.3 | 4852 | 0.0546 | 0.0021 | 0.625 | 0.0058 | 2.9 | inf. | 0.0 | 610.8 | 0.9999 |
| | HT60-000-B | 8.3 | 8.3 | 4809 | 0.0600 | 0.0021 | 1.046 | 0.0010 | 2.9 | inf. | 0.0 | 825.6 | 0.9141 |
| | HT60-020-B | 8.3 | 8.3 | 4809 | 0.0600 | 0.0021 | 1.046 | 0.0291 | 2.9 | inf. | 0.0 | 950.4 | 1.0194 |
| | HT60-040-B | 8.3 | 8.3 | 4809 | 0.0600 | 0.0021 | 1.046 | 0.0146 | 2.9 | inf. | 0.0 | 926.4 | 1.0372 |
| | HT60-100-B | 8.3 | 8.3 | 4809 | 0.0600 | 0.0021 | 1.046 | 0.0058 | 2.9 | inf. | 0.0 | 864.0 | 0.9355 |
| | HT80-000-B | 8.3 | 8.3 | 4767 | 0.0633 | 0.0021 | 1.509 | 0.0010 | 2.9 | inf. | 0.0 | 1122.0 | 1.0681 |
| | HT80-020-B | 8.3 | 8.3 | 4767 | 0.0633 | 0.0021 | 1.509 | 0.0291 | 2.9 | inf. | 0.0 | 1250.4 | 1.0467 |
| | HT80-040-B | 8.3 | 8.3 | 4767 | 0.0633 | 0.0021 | 1.509 | 0.0146 | 2.9 | inf. | 0.0 | 1212.0 | 1.1160 |
| | HT80-100-B | 8.3 | 8.3 | 4767 | 0.0633 | 0.0021 | 1.509 | 0.0058 | 2.9 | inf. | 0.0 | 1174.8 | 1.0937 |
| | | | | | | | | | | | | Mean | 1.0059 |
| | | | | | | | | | | | | Coeff. of Variation | 0.0886 |
| Roik | 23 | 11.8 | 11.8 | 6596 | 0.0868 | 0.0050 | 0.515 | 0.0029 | 16.7 | 0.300 | 525.9 | 1863.3 | 1.1046 |
| & Mangerig | 24 | 11.8 | 11.8 | 6596 | 0.0868 | 0.0050 | 0.515 | 0.0029 | 16.7 | 0.500 | 368.0 | 2173.4 | 1.0548 |
| (1987) | 25 | 11.8 | 11.8 | 6596 | 0.0868 | 0.0050 | 0.515 | 0.0029 | 26.7 | 0.300 | 377.5 | 1337.5 | 1.1069 |
| | | | | | | | | | | | | Mean | 1.0888 |
| | | | | | | | | | | | | Coeff. of Variation | 0.0270 |

Table 3.2.1 - Description of Composite Steel-Concrete Columns Subjected to Major Axis Bending without Moment Gradient Used for Comparison with FEM Ultimate Strength

| Author | Col. Desig. | h (in.) | b (in.) | $f'_c$ (psi) | $\rho_{ss}$ | $\rho_{rs}$ | $\frac{\rho_{ss} f_{yss}}{f'_c}$ | Tie/Hoop Vol. Ratio $\rho''$ | $\ell/h$ | e/h | TEST VALUES Applied Axial Load (kips) | TEST VALUES Applied BM at Ends (kip-in) | Strength Ratio |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Roik | V22 | 11.0 | 11.0 | 6994 | 0.0495 | 0.0079 | 0.350 | 0.0028 | 12.4 | 0.571 | 213.6 | 1345.4 | 0.8813 |
| & Schwarr | V23 | 11.0 | 11.0 | 6994 | 0.0495 | 0.0079 | 0.350 | 0.0028 | 12.4 | 0.214 | 436.8 | 1031.9 | 0.8804 |
| (1988) | V31 | 11.0 | 11.0 | 7780 | 0.0996 | 0.0079 | 0.484 | 0.0028 | 12.4 | 0.357 | 383.8 | 1510.9 | 0.7654 |
| | V32 | 11.0 | 11.0 | 7780 | 0.0996 | 0.0079 | 0.484 | 0.0028 | 12.4 | 0.214 | 506.5 | 1196.5 | 0.7652 |
| | V33 | 11.0 | 11.0 | 7483 | 0.0996 | 0.0079 | 0.503 | 0.0028 | 12.4 | 0.571 | 294.1 | 1852.4 | 0.8107 |
| | V41 | 11.0 | 11.0 | 7483 | 0.1445 | 0.0079 | 0.952 | 0.0028 | 12.4 | 0.357 | 477.3 | 1879.1 | 0.7495 |
| | V42 | 11.0 | 11.0 | 8093 | 0.1445 | 0.0079 | 0.881 | 0.0028 | 12.4 | 0.571 | 344.6 | 2171.0 | 0.7103 |
| | V43 | 11.0 | 11.0 | 8093 | 0.1445 | 0.0079 | 0.812 | 0.0028 | 12.4 | 0.214 | 614.4 | 1451.4 | 0.7349 |
| | | | | | | | | | | | | Mean | 0.7872 |
| | | | | | | | | | | | | Coeff. of Variation | 0.0819 |

Note: The strength ratio is defined as the tested strength divided by the FEM strength.

h = depth of concrete cross-section perpendicular to the axis of bending.

b = width of the concrete cross-section parallel to the axis of bending.

$\rho'' = 2(b''+d'')A_t/b''d''s$

$b''$ = outside width of ties/hoops.

$d''$ = outside depth of ties/hoops.

$A_t$ = area of cross-section of a tie/hoop bar.

s = spacing of ties/hoops.

The term $f_{yss}$ was taken as the web yield strength for computing the $\rho_{ss} f_{yss}/f'_c$ ratio.

* Excluded from the final analysis on the basis of incomplete or insufficient information, as explained in the text.

was taken as the ratio of bending moment strengths for columns with $e/h=\infty$ and the ratio of axial load capacities for columns with all other $e/h$ values.

A plot of tested strength against the FEM strength (Figure 3.2.1(a)) shows a relatively narrow band of strength ratios. This indicates that the FEM method was able to predict the tested strength of the columns quite accurately with no apparent or significant outliers. Also, as the strengths of the columns increase, there is a proportional increase in the magnitude of error. This is expected since the percentage of error remains relatively constant. A histogram giving the frequency, in percent, against the strength ratio (Figure 3.2.1(b)) shows a relatively symmetric distribution of values about the mean. The mean strength ratio of all 75 test columns was 0.986 with a coefficient of variation of 12.7 percent (Figure 3.2.1(b)).

The calculated mean, coefficient of variation, minimum and maximum values of strength ratios for all test columns listed in Table 3.2.1 are shown in Table 3.2.2. The strength ratio statistics shown in Table 3.2.2 were divided into five categories, based on the slenderness ratio ($\ell/h$). Columns with $\ell/h$ less than or equal to 3 are assumed to be pedestals, short columns are assumed to have $\ell/h$ greater than 3 but less than 6.6, slender columns are assumed to

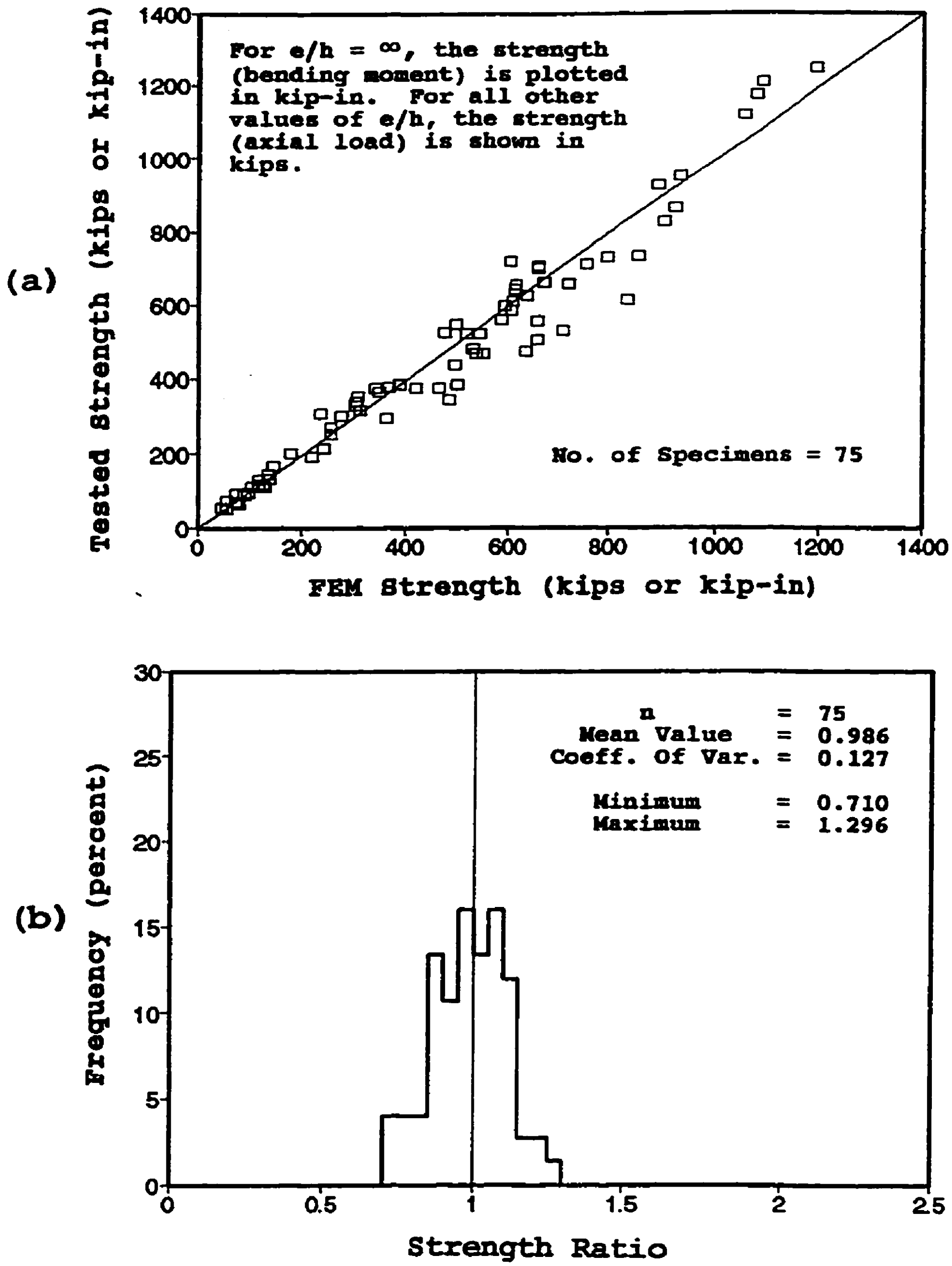



Figure 3.2.1 - Comparison of tested strength to FEM strength for composite steel-concrete columns subjected to major axis bending without moment gradient (all columns).

Table 3.2.2 - Strength Ratio Statistics of Composite Steel-Concrete Columns Subjected to Major Axis Bending without Moment Gradient for Different Ranges of e/h and ℓ/h, using FEM (all columns)

| Column Type (1) | (2) | e/h = 0 (3) | 0 < e/h < 0.1 (4) | 0.1 ≤ e/h ≤ 0.7 (5) | 0.1 ≤ e/h ≤ 1.5 (6) | e/h = ∞ (7) |
|---|---|---|---|---|---|---|
| Pedestal ℓ/h ≤ 3 | No. | 20 | * | * | * | 16 |
| | Mean | 0.971 | * | * | * | 1.047 |
| | CV | 0.091 | * | * | * | 0.063 |
| | Min | 0.806 | * | * | * | 0.914 |
| | Max | 1.184 | * | * | * | 1.146 |
| Short 3 < ℓ/h < 6.6 | No. | * | * | 2 | 4 | * |
| | Mean | * | * | 1.083 | 1.026 | * |
| | CV | * | * | 0.086 | 0.121 | * |
| | Min | * | * | 1.017 | 0.858 | * |
| | Max | * | * | 1.149 | 1.149 | * |
| Slender 6.6 ≤ ℓ/h ≤ 30 | No. | 3 | * | 29 | 32 | * |
| | Mean | 0.818 | * | 0.970 | 0.976 | * |
| | CV | 0.072 | * | 0.166 | 0.159 | * |
| | Min | 0.750 | * | 0.710 | 0.710 | * |
| | Max | 0.853 | * | 1.296 | 1.296 | * |
| Super Slender ℓ/h > 30 | No. | * | * | * | * | * |
| | Mean | * | * | * | * | * |
| | CV | * | * | * | * | * |
| | Min | * | * | * | * | * |
| | Max | * | * | * | * | * |
| ACI Permitted 3 < ℓ/h ≤ 30 | No. | 3 | * | 31 | 36 | * |
| | Mean | 0.818 | * | 0.978 | 0.982 | * |
| | CV | 0.072 | * | 0.162 | 0.154 | * |
| | Min | 0.750 | * | 0.710 | 0.710 | * |
| | Max | 0.853 | * | 1.296 | 1.296 | * |

* No data available

Note: CV stands for the coefficient of variation.

have $\ell/h$ greater than or equal to 6.6 but less than or equal to 30, super-slender columns are assumed to have $\ell/h$ greater than 30, and ACI-permitted columns are assumed to have $\ell/h$ greater than 3 but less than or equal to 30. The data were further categorized into five ranges of end eccentricity ratio ($e/h$) as shown in Table 3.2.2.

Differences in the statistics for four different ranges of end eccentricity ratios were observed (Table 3.2.2 Columns 3,5,6 and 7). Slender columns under pure axial load ($e/h$=0) have a low mean of 0.818 as compared to the overall mean of 0.986 while slender columns with $e/h>0$ have a relatively high coefficient of variation (16.6 and 15.9 percent) as compared to the overall value (12.7 percent). Columns tested under pure bending ($e/h=\infty$) have the lowest coefficient of variation. The overall minimum strength ratio (0.710) and maximum strength ratio (1.296) were found to occur for the column with $\ell/h$=12.4 and $e/h$=0.571 and for the column with $\ell/h$=8.1 and $e/h$=0.136, respectively (Table 3.2.1). The probability distribution of the strength ratios computed for the 75 test columns is plotted on a normal probability scale in Figure 3.2.2 and is compared to a normal probability distribution with a mean value of 0.99 and a coefficient of variation of 12.5 percent. The data closely follow the normal curve and can be assumed to be normally distributed.

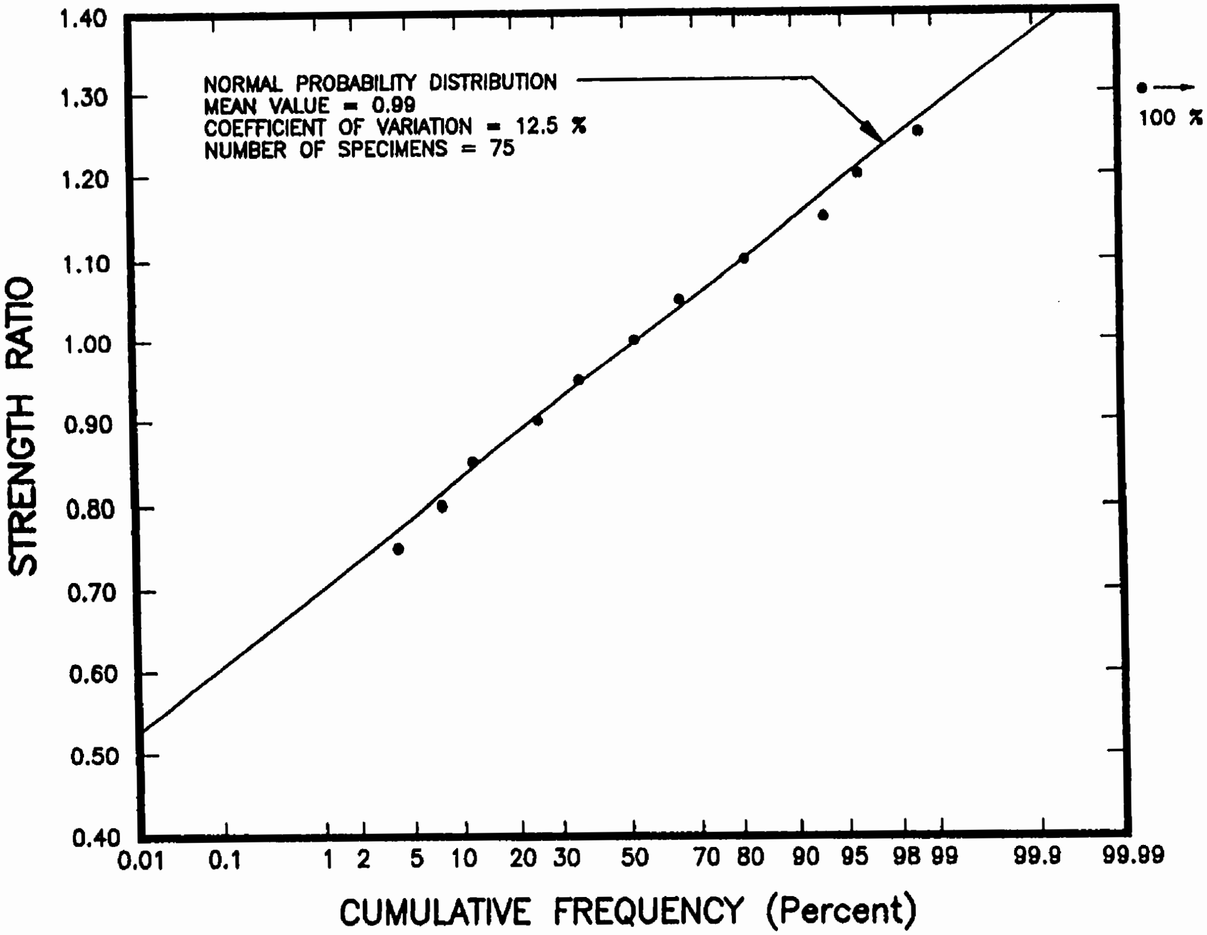


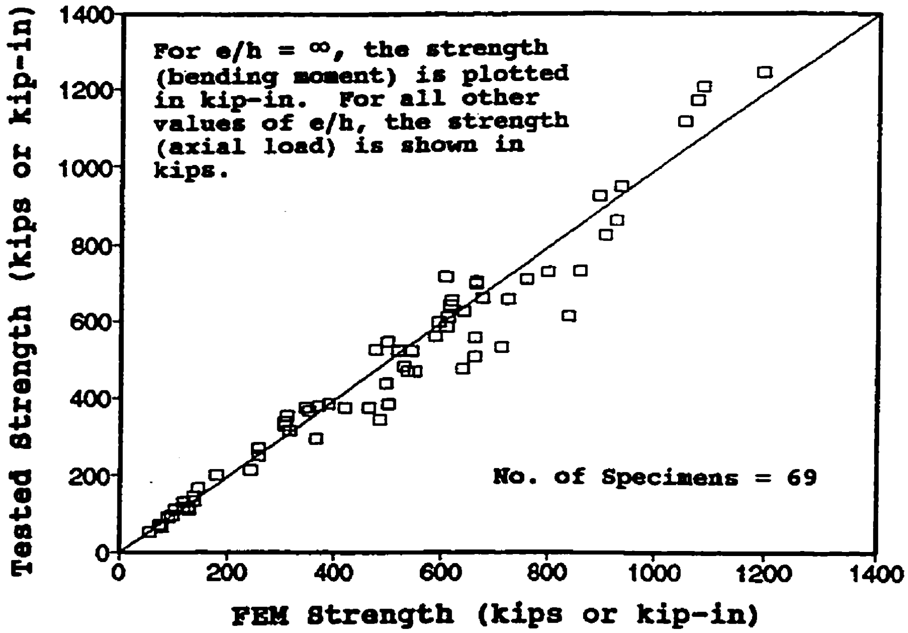

Figure 3.2.2 - Probability distribution of strength ratios using FEM of composite steel-concrete columns subjected to major axis bending without moment gradient (all columns).

After a reexamination of the three tests by Bondale (1966), and the three tests by Johnson and May (1978), it was decided to drop these tests from the comparative study. These six columns are identified by an asterisk (*) in Table 3.2.1. In the analysis of the columns of these two studies, several assumptions were made because of the lack of sufficient information on geometric or material properties. Bondale (1966) did not give the yield strength of the longitudinal reinforcing steel and there were conflicting concrete strengths reported by Bondale (1966) and Basu (1967) for the same tests. For the columns tested by Johnson and May (1978), the location and yield strength of the longitudinal reinforcing steel was not given. Also, the columns were reported as part of a test frame. The equivalent effective lengths of the columns were given, however, there was no indication on how these values were obtained. The assumptions made for the data of these two studies could have affected the computed ultimate strengths.

The removal of the six columns affected the overall statistics by slightly decreasing both the mean and coefficient of variation from 0.986 and 12.7 percent to 0.972 and 11.7 percent, respectively. The overall minimum strength ratio did not change, however, the maximum strength ratio was reduced from 1.296 to 1.184.

The plot of tested strength against the FEM strength (Figure 3.2.3(a)) for the 69 test columns shows a

(a)




(b)

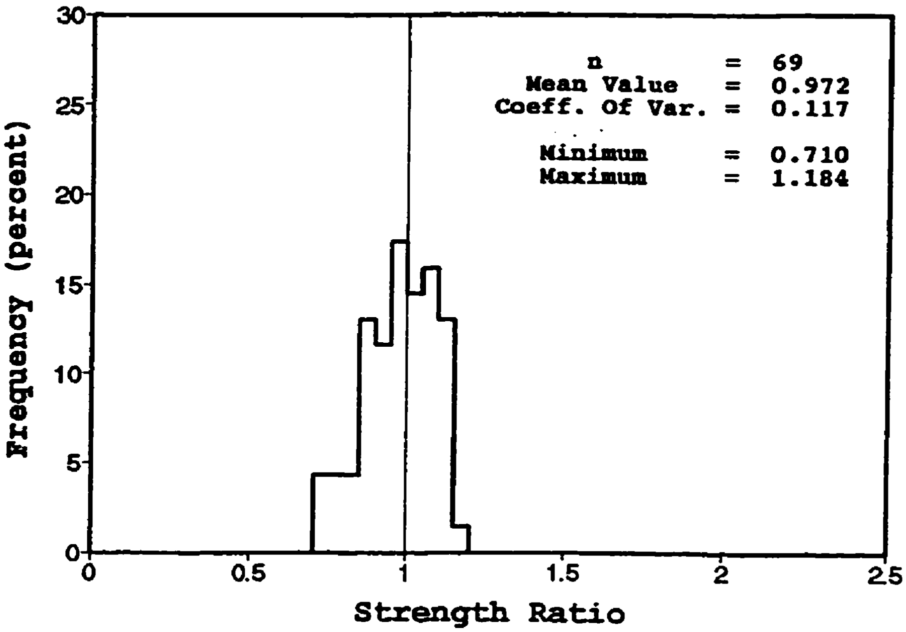



Figure 3.2.3 - Comparison of tested strength to FEM strength for composite steel-concrete columns subjected to major axis bending without moment gradient (some data removed due to incomplete or insufficient information).

relatively narrow band of strength ratios. A histogram showing the frequency, in percent, against the strength ratios (Figure 3.2.3(b)) demonstrates a relatively symmetric distribution of values about the mean. The strength ratio statistics for the 69 test columns in Table 3.2.3 do not show any significant differences over those in Table 3.2.2 which included all 75 test columns.

The probability distribution of the strength ratios computed for the 69 test columns is plotted on a normal probability scale in Figure 3.2.4 and is compared to a normal probability distribution with a mean value of 0.97 and a coefficient of variation of 11.5 percent. The data closely follow the normal curve and can be assumed to be normally distributed.

### 3.2.2 Composite Steel-Concrete Columns Subjected to Major Axis Bending With Moment Gradient

The ultimate strengths computed using FEM were compared with the ultimate strengths of 3 physical tests taken from Roik and Schwalbenhofer (1988).

A description of these 3 physical tests used for the comparison of tested to FEM strength for composite steel-concrete columns subjected to major axis bending with moment gradient is given in Table 3.2.4. The table includes information on the geometric and material properties of test columns. Included in the table is the ratio of tested to FEM ultimate strength (strength ratio)

Table 3.2.3 - Strength Ratio Statistics of Composite Steel-Concrete Columns Subjected to Major Axis Bending without Moment Gradient for Different Ranges of e/h and ℓ/h, using FEM (some data removed due to incomplete or insufficient information)

| Column Type (1) | (2) | e/h = 0 (3) | 0 < e/h < 0.1 (4) | 0.1 ≤ e/h ≤ 0.7 (5) | 0.1 ≤ e/h ≤ 1.5 (6) | e/h = ∞ (7) |
|---|---|---|---|---|---|---|
| Pedestal ℓ/h ≤ 3 | No. | 20 | * | * | * | 16 |
| | Mean | 0.971 | * | * | * | 1.047 |
| | CV | 0.091 | * | * | * | 0.063 |
| | Min | 0.806 | * | * | * | 0.914 |
| | Max | 1.184 | * | * | * | 1.146 |
| Short 3 < ℓ/h < 6.6 | No. | * | * | 2 | 4 | * |
| | Mean | * | * | 1.083 | 1.026 | * |
| | CV | * | * | 0.086 | 0.121 | * |
| | Min | * | * | 1.017 | 0.858 | * |
| | Max | * | * | 1.149 | 1.149 | * |
| Slender 6.6 ≤ ℓ/h ≤ 30 | No. | 3 | * | 23 | 26 | * |
| | Mean | 0.818 | * | 0.923 | 0.936 | * |
| | CV | 0.072 | * | 0.138 | 0.135 | * |
| | Min | 0.750 | * | 0.710 | 0.710 | * |
| | Max | 0.853 | * | 1.116 | 1.116 | * |
| Super Slender ℓ/h > 30 | No. | * | * | * | * | * |
| | Mean | * | * | * | * | * |
| | CV | * | * | * | * | * |
| | Min | * | * | * | * | * |
| | Max | * | * | * | * | * |
| ACI Permitted 3 < ℓ/h ≤ 30 | No. | 3 | * | 25 | 30 | * |
| | Mean | 0.818 | * | 0.936 | 0.948 | * |
| | CV | 0.072 | * | 0.140 | 0.135 | * |
| | Min | 0.750 | * | 0.710 | 0.710 | * |
| | Max | 0.853 | * | 1.149 | 1.149 | * |

* No data available

Note: CV stands for the coefficient of variation.

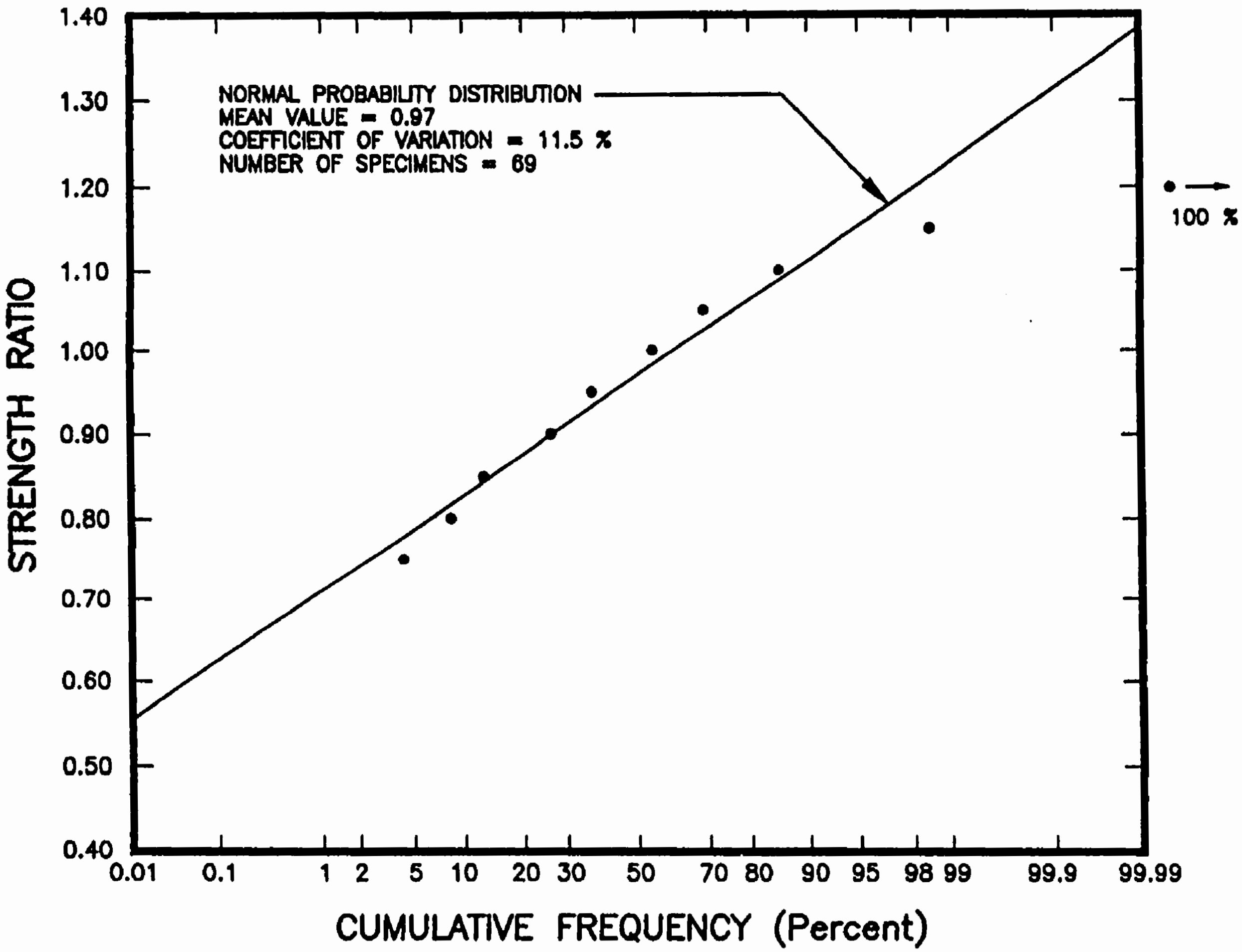



Figure 3.2.4 - Probability distribution of strength ratios using FEM of composite steel-concrete columns subjected to major axis bending without moment gradient (some data removed due to incomplete or insufficient information)

Table 3.2.4 - Description of Composite Steel-Concrete Columns Subjected to Major Axis Bending with Moment Gradient Used for Comparison with FEM Ultimate Strength

| Author | Col. Desig. | h (in.) | b (in.) | $f'_c$ (psi) | $\rho_{ss}$ | $\rho_{rs}$ | $\frac{\rho_{ss} f_{yss}}{f'_c}$ | Tie/Hoop Vol. Ratio $\rho''$ | $\ell/h$ | e/h | TEST VALUES Applied Axial Load (kips) | Applied BM at Ends (kip-in) $M_1$ | $M_2$ | Strength Ratio |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Roik | V71 | 11.0 | 11.0 | 7423 | 0.1445 | 0.0079 | 0.885 | 0.0028 | 10.7 | 0.714 | 360.4 | -2269.3 | 2837.7 | 0.8084 |
| & Schwa'r | V72 | 11.0 | 11.0 | 7989 | 0.1445 | 0.0079 | 0.822 | 0.0028 | 10.7 | 0.571 | 441.3 | -2430.7 | 2780.0 | 0.8117 |
| (1988) | V73 | 11.0 | 11.0 | 7989 | 0.1445 | 0.0079 | 0.822 | 0.0028 | 10.7 | 0.357 | 607.5 | 0.0 | 2391.6 | 0.8264 |
| | | | | | | | | | | | | | Mean | 0.8155 |
| | | | | | | | | | | | | | Coeff. of Variation | 0.0118 |

Note: The strength ratio is defined as the tested strength divided by the FEM strength.

h = depth of concrete cross-section perpendicular to the axis of bending.
b = width of the concrete cross-section parallel to the axis of bending.
$\rho'' = 2(b''+d'')A_t/b''d''s$
$b''$ = outside width of ties/hoops.
$d''$ = outside depth of ties/hoops.
$A_t$ = area of cross-section of a tie/hoop bar.
s = spacing of ties/hoops.
$M_1$ = smaller end moment, positive if member is bent in single curvature, negative if bent in double curvature.
$M_2$ = larger end moment, always positive.

The term $f_{yss}$ was taken as the web yield strength for computing the $\rho_{ss} f_{yss}/f'_c$ ratio.

for each of the 3 column specimens. The strength ratio was taken as the ratio of axial load capacities for the column.

The mean strength ratio of all three test columns was 0.816 with a coefficient of variation of 1.2 percent. These are significantly lower than the mean value of 0.972 and the coefficient of variation of 11.7 percent obtained for composite steel-concrete columns subjected to major axis bending without moment gradient (Figure 3.2.3(b)).

## 3.3 COMPARISON OF FEM METHOD WITH EXPERIMENTAL RESULTS FOR COMPOSITE STEEL-CONCRETE COLUMNS SUBJECTED TO MINOR AXIS BENDING

The ultimate strengths computed using FEM were compared with the ultimate strengths of 143 physical tests taken from Stevens (1965), Bondale (1966), Anslijn and Janss (1974), Roderick and Loke (1974), Johnson and May (1978), Morino et al. (1984), Roik and Mangerig (1987), and Roik and Schwalbenhofer (1988). The columns were bent in symmetric single curvature (i.e. without moment gradient) when subjected to bending moments. Sixty-two of the physical tests were eventually removed from the comparison for reasons that will be discussed later in this section.

A description of these 143 physical tests used for the comparison of tested to FEM strength for composite steel-concrete columns subjected to minor axis bending is given in Table 3.3.1. The table includes information on the geometric and material properties of test columns.

Table 3.3.1 - Description of Composite Steel-Concrete Columns Subjected to Minor Axis Bending Used for Comparison with FEM Ultimate Strength

| Author | Col. Desig. | h (in.) | b (in.) | $f'_c$ (psi) | $\rho_{ss}$ | $\rho_{rs}$ | $\frac{\rho_{ss} f_{yss}}{f'_c}$ | Tie/Hoop Vol. Ratio $\rho''$ | $\ell/h$ | e/h | TEST VALUES Applied Axial Load (kips) | TEST VALUES Applied BM at Ends (kip-in) | Strength Ratio |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Stevens (1965) | A1 | 6.5 | 7.0 | 1867 | 0.1292 | - | 2.914 | - | 2.0 | 0.000 | 358.4 | 0.0 | 1.2149 ** |
| | A2 | 6.5 | 7.0 | 1653 | 0.1292 | - | 3.292 | - | 7.1 | 0.000 | 313.6 | 0.0 | 1.0424 ** |
| | A3 | 6.5 | 7.0 | 1867 | 0.1292 | - | 2.914 | - | 12.6 | 0.000 | 322.6 | 0.0 | 1.0379 ** |
| | A4 | 6.5 | 7.0 | 2012 | 0.1292 | - | 2.705 | - | 12.6 | 0.000 | 302.4 | 0.0 | 0.9600 ** |
| | A5 | 6.5 | 7.0 | 1831 | 0.1292 | - | 2.972 | - | 18.2 | 0.000 | 293.4 | 0.0 | 1.1543 ** |
| | A6 | 6.5 | 7.0 | 2179 | 0.1292 | - | 2.498 | - | 23.7 | 0.000 | 235.2 | 0.0 | 1.2272 ** |
| | B1 | 3.5 | 5.0 | 2084 | 0.0674 | - | 1.334 | - | 13.1 | 0.000 | 82.9 | 0.0 | 1.2235 * |
| | B2 | 3.5 | 5.0 | 1441 | 0.0674 | - | 1.928 | - | 18.3 | 0.000 | 61.2 | 0.0 | 1.3664 * |
| | B3 | 3.5 | 5.0 | 1796 | 0.0674 | - | 1.548 | - | 23.4 | 0.000 | 64.1 | 0.0 | 1.8868 * |
| | B4 | 3.5 | 5.0 | 1582 | 0.0674 | - | 1.757 | - | 28.6 | 0.000 | 44.4 | 0.0 | 1.6386 * |
| | B5 | 3.5 | 5.0 | 2048 | 0.0674 | - | 1.357 | - | 33.7 | 0.000 | 51.5 | 0.0 | 2.1611 * |
| | B6 | 3.5 | 5.0 | 1760 | 0.0674 | - | 1.579 | - | 38.9 | 0.000 | 36.7 | 0.0 | 2.1328 * |
| | B7 | 3.5 | 5.0 | 2266 | 0.0674 | - | 1.226 | - | 44.0 | 0.000 | 34.5 | 0.0 | 2.2306 * |
| | FA1 | 12.0 | 16.0 | 1831 | 0.0996 | - | 1.790 | - | 3.0 | 0.000 | 1070.7 | 0.0 | 1.1537 ** |
| | FA2 | 12.0 | 16.0 | 1975 | 0.0996 | - | 1.660 | - | 6.0 | 0.000 | 1008.0 | 0.0 | 1.0626 ** |
| | FA3 | 12.0 | 16.0 | 1724 | 0.0996 | - | 1.902 | - | 9.0 | 0.000 | 943.0 | 0.0 | 1.0335 ** |
| | FA4 | 12.0 | 16.0 | 1939 | 0.0996 | - | 1.691 | - | 12.0 | 0.000 | 954.2 | 0.0 | 1.0086 ** |
| | FA5 | 12.0 | 16.0 | 1939 | 0.0996 | - | 1.691 | - | 15.0 | 0.000 | 949.8 | 0.0 | 1.0262 ** |
| | RE1a | 6.5 | 7.0 | 1975 | 0.1292 | - | 2.814 | - | 18.2 | 0.000 | 300.2 | 0.0 | 1.1493 ** |
| | RE1b | 6.5 | 7.0 | 1760 | 0.1292 | - | 3.158 | - | 18.2 | 0.000 | 280.0 | 0.0 | 1.1145 ** |
| | RE2a | 6.5 | 7.0 | 1867 | 0.1292 | - | 2.976 | - | 18.2 | 0.000 | 275.5 | 0.0 | 1.0375 ** |
| | RE2b | 6.5 | 7.0 | 2266 | 0.1292 | - | 2.453 | - | 18.2 | 0.000 | 268.8 | 0.0 | 0.9537 ** |
| | RE3a | 6.5 | 7.0 | 2193 | 0.1292 | 0.0044 | 2.534 | 0.0055 | 18.2 | 0.000 | 313.6 | 0.0 | 1.0385 ** |
| | RE3b | 6.5 | 7.0 | 1867 | 0.1292 | 0.0044 | 2.976 | 0.0055 | 18.2 | 0.000 | 277.8 | 0.0 | 0.8039 ** |
| | RE4a | 6.5 | 7.0 | 1939 | 0.1292 | - | 2.866 | - | 18.2 | 0.000 | 271.0 | 0.0 | 1.0330 ** |
| | RE4b | 6.5 | 7.0 | 1796 | 0.1292 | - | 3.095 | - | 18.2 | 0.000 | 284.5 | 0.0 | 1.1056 ** |
| | S1G | 8.0 | 10.0 | 1724 | 0.1288 | - | 2.526 | - | 10.5 | 0.000 | 537.6 | 0.0 | 1.1665 * |
| | S2G | 10.0 | 12.0 | 1975 | 0.0858 | - | 1.470 | - | 8.4 | 0.000 | 649.6 | 0.0 | 1.2060 * |
| | S3G | 12.0 | 14.0 | 2230 | 0.0613 | - | 0.930 | - | 7.0 | 0.000 | 815.4 | 0.0 | 1.2525 * |
| | S1E | 8.0 | 10.0 | 2895 | 0.1288 | - | 1.504 | - | 10.5 | 0.000 | 629.4 | 0.0 | 1.2118 * |
| | S2E | 10.0 | 12.0 | 3120 | 0.0858 | - | 0.930 | - | 8.4 | 0.000 | 851.2 | 0.0 | 1.3288 * |
| | S3E | 12.0 | 14.0 | 2933 | 0.0613 | - | 0.707 | - | 7.0 | 0.000 | 851.2 | 0.0 | 1.1410 * |
| | S1S | 8.0 | 10.0 | 2193 | 0.1288 | - | 1.986 | - | 10.5 | 0.000 | 577.9 | 0.0 | 1.1740 * |
| | S2S | 10.0 | 12.0 | 2524 | 0.0858 | - | 1.150 | - | 8.4 | 0.000 | 716.8 | 0.0 | 1.2177 * |
| | S3S | 12.0 | 14.0 | 2858 | 0.0613 | - | 0.726 | - | 7.0 | 0.000 | 947.5 | 0.0 | 1.3041 * |
| | CV2 | 6.5 | 7.0 | 1095 | 0.1292 | - | 4.257 | - | 12.6 | 0.115 | 134.4 | 100.8 | 1.0815 ** |
| | CV3 | 6.5 | 7.0 | 1867 | 0.1292 | - | 2.496 | - | 12.6 | 0.115 | 161.3 | 121.0 | 1.1356 ** |
| | CV4 | 6.5 | 7.0 | 2450 | 0.1292 | - | 1.902 | - | 12.6 | 0.115 | 179.2 | 134.4 | 1.1462 ** |
| | CV5 | 6.5 | 7.0 | 3008 | 0.1292 | - | 1.550 | - | 12.6 | 0.115 | 201.6 | 151.2 | 1.1863 |
| | CV6 | 6.5 | 7.0 | 3613 | 0.1292 | - | 1.290 | - | 12.6 | 0.123 | 228.5 | 182.8 | 1.2804 |
| | AE1 | 6.5 | 7.0 | 2012 | 0.1292 | - | 2.317 | - | 4.4 | 0.154 | 165.8 | 165.8 | 1.0209 ** |
| | AE2 | 6.5 | 7.0 | 2635 | 0.1292 | - | 1.769 | - | 7.0 | 0.154 | 163.5 | 163.5 | 0.8956 |
| | AE3 | 6.5 | 7.0 | 2524 | 0.1292 | - | 1.847 | - | 12.6 | 0.154 | 141.1 | 141.1 | 1.0411 |
| | AE4 | 6.5 | 7.0 | 2858 | 0.1292 | - | 1.631 | - | 18.2 | 0.154 | 118.7 | 118.7 | 1.0535 |
| | AE5 | 6.5 | 7.0 | 2266 | 0.1292 | - | 2.057 | - | 23.6 | 0.154 | 98.6 | 98.6 | 1.2246 ** |
| | AE6 | 6.5 | 7.0 | 1975 | 0.1292 | - | 2.359 | - | 7.0 | 0.000 | 291.2 | 0.0 | 1.0232 ** |
| | AE7 | 6.5 | 7.0 | 2048 | 0.1292 | - | 2.276 | - | 7.0 | 0.077 | 224.0 | 112.0 | 1.0973 ** |
| | AE8 | 6.5 | 7.0 | 2120 | 0.1292 | - | 2.198 | - | 18.2 | 0.077 | 161.3 | 80.6 | 1.1581 ** |

Table 3.3.1 - Description of Composite Steel-Concrete Columns Subjected to Minor Axis Bending Used for Comparison with FEM Ultimate Strength

| Author | Col. Desig. | h (in.) | b (in.) | $f'_c$ (psi) | $\rho_{ss}$ | $\rho_{rs}$ | $\frac{\rho_{ss} f_{yss}}{f'_c}$ | Tie/Hoop Vol. Ratio $\rho''$ | $\ell/h$ | e/h | TEST VALUES Applied Axial Load (kips) | TEST VALUES Applied BM at Ends (kip-in) | Strength Ratio | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Stevens (1965) | AE9 | 6.5 | 7.0 | 1441 | 0.1292 | - | 3.234 | - | 23.6 | 0.231 | 78.4 | 117.6 | 1.3356 | ** |
| | AE10 | 6.5 | 7.0 | 1867 | 0.1292 | - | 2.496 | - | 23.6 | 0.308 | 72.8 | 145.6 | 1.3555 | ** |
| | AE11 | 6.5 | 7.0 | 2268 | 0.1292 | - | 2.057 | - | 16.6 | inf. | 0.0 | 250.9 | 1.0101 | ** |
| | FE1 | 12.0 | 16.0 | 2048 | 0.0996 | 0.0042 | 1.601 | 0.0028 | 15.0 | 0.000 | 985.6 | 0.0 | 0.9656 | ** |
| | FE2 | 12.0 | 16.0 | 2230 | 0.0996 | 0.0042 | 1.471 | 0.0028 | 15.0 | 0.000 | 1055.0 | 0.0 | 1.0525 | ** |
| | FE3 | 12.0 | 16.0 | 2048 | 0.0996 | 0.0042 | 1.601 | 0.0028 | 15.0 | 0.083 | 672.0 | 672.0 | 1.1412 | ** |
| | FE4 | 12.0 | 16.0 | 1903 | 0.0996 | 0.0042 | 1.723 | 0.0028 | 15.0 | 0.167 | 486.1 | 972.2 | 1.1891 | ** |
| | FE5 | 12.0 | 16.0 | 2413 | 0.0996 | 0.0042 | 1.359 | 0.0028 | 15.0 | 0.167 | 515.2 | 1030.4 | 1.1573 | ** |
| | FE6 | 12.0 | 16.0 | 2193 | 0.0996 | 0.0042 | 1.495 | 0.0028 | 15.0 | 0.250 | 360.6 | 1081.9 | 1.0638 | ** |
| | FE7 | 12.0 | 16.0 | 2193 | 0.0996 | 0.0042 | 1.495 | 0.0028 | 15.0 | 0.333 | 295.7 | 1182.7 | 1.0575 | ** |
| | FE8 | 12.0 | 16.0 | 2303 | 0.0996 | 0.0042 | 1.424 | 0.0028 | 15.0 | 0.417 | 262.1 | 1310.4 | 1.0793 | ** |
| | FE9 | 12.0 | 16.0 | 2230 | 0.0996 | 0.0042 | 1.471 | 0.0028 | 15.0 | 0.500 | 230.7 | 1384.3 | 1.0983 | ** |
| | FE10 | 12.0 | 16.0 | 2560 | 0.0996 | 0.0042 | 1.281 | 0.0028 | 15.0 | 0.583 | 199.4 | 1395.5 | 1.0232 | |
| | FE11 | 12.0 | 16.0 | 2487 | 0.0996 | 0.0042 | 1.319 | 0.0028 | 15.0 | 0.667 | 168.0 | 1344.0 | 0.9648 | ** |
| | FE12 | 12.0 | 16.0 | 2487 | 0.0996 | 0.0042 | 1.319 | 0.0028 | 10.0 | inf. | 0.0 | 1577.0 | 0.9417 | ** |
| | | | | | | | | | | | | Mean | 1.1838 | (1.0800)*** |
| | | | | | | | | | | | | Coeff. of Variation | 0.2334 | (0.1249)*** |
| Bondale (1966) | R.W.120.0 | 3.8 | 6.0 | 3864 | 0.0653 | 0.0062 | 0.758 | 0.0064 | 33.5 | 0.000 | 52.9 | 0.0 | 1.0275 | * |
| | R.W.100.1 | 3.8 | 6.0 | 4417 | 0.0653 | 0.0062 | 0.663 | 0.0064 | 28.2 | 0.267 | 20.8 | 20.8 | 1.0146 | * |
| | R.W.80.2 | 3.8 | 6.0 | 5471 | 0.0653 | 0.0062 | 0.535 | 0.0064 | 22.9 | 0.533 | 21.7 | 43.5 | 1.1524 | * |
| | R.W.60.3 | 3.8 | 6.0 | 4592 | 0.0653 | 0.0062 | 0.637 | 0.0064 | 17.5 | 0.800 | 17.9 | 53.8 | 1.2142 | * |
| | | | | | | | | | | | | Mean | 1.1022 | |
| | | | | | | | | | | | | Coeff. of Variation | 0.0881 | |
| Anslijn & Janss (1974) | 1.1 | 9.4 | 9.4 | 6042 | 0.0747 | 0.0079 | 0.512 | 0.0021 | 17.5 | 0.000 | 483.0 | 0.0 | 0.6224 | |
| | 1.2 | 9.4 | 9.4 | 5543 | 0.0747 | 0.0079 | 0.558 | 0.0021 | 17.5 | 0.000 | 489.6 | 0.0 | 0.7837 | |
| | 1.3 | 9.4 | 9.4 | 5288 | 0.0747 | 0.0079 | 0.560 | 0.0021 | 17.5 | 0.000 | 469.8 | 0.0 | 0.7811 | |
| | 2.1 | 9.4 | 9.4 | 5288 | 0.0747 | 0.0079 | 0.601 | 0.0021 | 14.2 | 0.000 | 527.1 | 0.0 | 0.7274 | |
| | 2.2 | 9.4 | 9.4 | 4529 | 0.0747 | 0.0079 | 0.701 | 0.0021 | 14.2 | 0.000 | 489.6 | 0.0 | 0.7550 | |
| | 2.3 | 9.4 | 9.4 | 5543 | 0.0747 | 0.0079 | 0.573 | 0.0021 | 14.2 | 0.000 | 580.0 | 0.0 | 0.7963 | |
| | 3.1 | 9.4 | 9.4 | 5985 | 0.0747 | 0.0079 | 0.500 | 0.0021 | 10.0 | 0.000 | 591.1 | 0.0 | 0.7769 | |
| | 3.2 | 9.4 | 9.4 | 6042 | 0.0747 | 0.0079 | 0.495 | 0.0021 | 10.0 | 0.000 | 502.9 | 0.0 | 0.6774 | |
| | 3.3 | 9.4 | 9.4 | 5288 | 0.0747 | 0.0079 | 0.565 | 0.0021 | 10.0 | 0.000 | 527.1 | 0.0 | 0.7445 | |
| | 4.1 | 9.4 | 9.4 | 5288 | 0.0747 | 0.0079 | 0.565 | 0.0021 | 5.0 | 0.000 | 573.4 | 0.0 | 0.8116 | |
| | 4.2 | 9.4 | 9.4 | 4529 | 0.0747 | 0.0079 | 0.660 | 0.0021 | 5.0 | 0.000 | 555.8 | 0.0 | 0.8632 | |
| | 4.3 | 9.4 | 9.4 | 5600 | 0.0747 | 0.0079 | 0.534 | 0.0021 | 5.0 | 0.000 | 617.5 | 0.0 | 0.8439 | |
| | 5.1 | 9.4 | 9.4 | 4894 | 0.0747 | 0.0079 | 0.840 | 0.0021 | 14.2 | 0.000 | 529.3 | 0.0 | 0.6945 | |
| | 5.2 | 9.4 | 9.4 | 5302 | 0.0747 | 0.0079 | 0.775 | 0.0021 | 14.2 | 0.000 | 591.1 | 0.0 | 0.7394 | |
| | 5.3 | 9.4 | 9.4 | 5006 | 0.0747 | 0.0079 | 0.821 | 0.0021 | 14.2 | 0.000 | 555.8 | 0.0 | 0.7183 | |
| | 6.1 | 9.4 | 9.4 | 4894 | 0.0747 | 0.0079 | 1.111 | 0.0021 | 17.5 | 0.000 | 529.3 | 0.0 | 0.6118 | |
| | 6.2 | 9.4 | 9.4 | 5302 | 0.0747 | 0.0079 | 1.026 | 0.0021 | 17.5 | 0.000 | 485.2 | 0.0 | 0.5296 | |
| | 6.3 | 9.4 | 9.4 | 5020 | 0.0747 | 0.0079 | 1.083 | 0.0021 | 17.5 | 0.000 | 558.0 | 0.0 | 0.6438 | |
| | 7.1 | 9.4 | 9.4 | 4992 | 0.0747 | 0.0079 | 1.060 | 0.0021 | 14.2 | 0.000 | 555.8 | 0.0 | 0.6537 | |

Table 3.3.1 - Description of Composite Steel-Concrete Columns Subjected to Minor Axis Bending Used for Comparison with FEM Ultimate Strength

| Author | Col. Desig. | h (in.) | b (in.) | $f'_c$ (psi) | $\rho_{ss}$ | $\rho_{rs}$ | $\frac{\rho_{ss} f_{yss}}{f'_c}$ | Tie/Hoop Vol. Ratio $\rho''$ | $\ell/h$ | e/h | TEST VALUES Applied Axial Load (kips) | TEST VALUES Applied BM at Ends (kip-in) | Strength Ratio |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Anslijn | 7.2 | 9.4 | 9.4 | 5317 | 0.0747 | 0.0079 | 0.995 | 0.0021 | 14.2 | 0.000 | 588.9 | 0.0 | 0.6782 |
| & Janss | 7.3 | 9.4 | 9.4 | 5020 | 0.0747 | 0.0079 | 1.054 | 0.0021 | 14.2 | 0.000 | 577.8 | 0.0 | 0.6850 |
| (1974) | 8.1 | 9.4 | 9.4 | 5288 | 0.0747 | 0.0079 | 1.024 | 0.0021 | 10.0 | 0.000 | 547.0 | 0.0 | 0.5963 |
| | 8.2 | 9.4 | 9.4 | 6042 | 0.0747 | 0.0079 | 0.897 | 0.0021 | 10.0 | 0.000 | 531.5 | 0.0 | 0.5425 |
| | 8.3 | 9.4 | 9.4 | 5985 | 0.0747 | 0.0079 | 0.905 | 0.0021 | 10.0 | 0.000 | 573.4 | 0.0 | 0.5881 |
| | 9.1 | 8.3 | 12.6 | 4529 | 0.0497 | 0.0067 | 0.434 | 0.0019 | 16.2 | 0.000 | 513.9 | 0.0 | 0.8883 |
| | 9.2 | 8.3 | 12.6 | 5985 | 0.0497 | 0.0067 | 0.325 | 0.0019 | 16.2 | 0.000 | 569.0 | 0.0 | 0.7341 |
| | 9.3 | 8.3 | 12.6 | 5317 | 0.0497 | 0.0067 | 0.364 | 0.0019 | 16.2 | 0.000 | 463.2 | 0.0 | 0.7750 |
| | 10.1 | 8.3 | 12.6 | 5288 | 0.0497 | 0.0067 | 0.666 | 0.0019 | 16.2 | 0.000 | 518.3 | 0.0 | 0.7813 |
| | 10.2 | 8.3 | 12.6 | 4851 | 0.0497 | 0.0067 | 0.726 | 0.0019 | 16.2 | 0.000 | 608.7 | 0.0 | 0.7414 |
| | 10.3 | 8.3 | 12.6 | 5006 | 0.0497 | 0.0067 | 0.709 | 0.0019 | 16.2 | 0.000 | 531.5 | 0.0 | 0.8279 |
| | 11.1 | 9.4 | 9.4 | 5416 | 0.0747 | 0.0079 | 0.573 | 0.0021 | 14.2 | 0.167 | 251.4 | 395.9 | 0.8453 |
| | 11.2 | 9.4 | 9.4 | 5600 | 0.0747 | 0.0079 | 0.554 | 0.0021 | 14.2 | 0.167 | 264.7 | 416.8 | 0.8732 |
| | 11.3 | 9.4 | 9.4 | 4795 | 0.0747 | 0.0079 | 0.647 | 0.0021 | 14.2 | 0.167 | 240.4 | 378.6 | 0.8704 |
| | 12.1 | 9.4 | 9.4 | 5416 | 0.0747 | 0.0079 | 0.975 | 0.0021 | 14.2 | 0.167 | 264.7 | 416.8 | 0.8767 |
| | 12.2 | 9.4 | 9.4 | 5232 | 0.0747 | 0.0079 | 1.009 | 0.0021 | 14.2 | 0.167 | 251.4 | 395.9 | 0.8507 |
| | 12.3 | 9.4 | 9.4 | 4795 | 0.0747 | 0.0079 | 1.101 | 0.0021 | 14.2 | 0.167 | 222.8 | 350.8 | 0.7952 |
| | 13.1 | 8.3 | 12.6 | 5600 | 0.0497 | 0.0067 | 0.351 | 0.0019 | 11.4 | 0.190 | 269.1 | 423.7 | 0.8701 |
| | 13.2 | 8.3 | 12.6 | 5232 | 0.0497 | 0.0067 | 0.370 | 0.0019 | 11.4 | 0.190 | 233.8 | 368.2 | 0.7951 |
| | 13.3 | 8.3 | 12.6 | 5119 | 0.0497 | 0.0067 | 0.379 | 0.0019 | 11.4 | 0.190 | 229.4 | 361.2 | 0.7926 |
| | | | | | | | | | | | | Mean | 0.7483 |
| | | | | | | | | | | | | Coeff. of Variation | 0.1300 |
| Roderick | SE 1 | 7.0 | 8.0 | 3690 | 0.0505 | - | 0.581 | - | 12.0 | 0.000 | 273.0 | 0.0 | 0.9410 |
| & Loke | SE 2 | 7.0 | 8.0 | 4280 | 0.0505 | - | 0.501 | - | 12.0 | 0.057 | 211.0 | 84.4 | 0.9748 |
| (1974) | SE 3 | 7.0 | 8.0 | 3910 | 0.0505 | - | 0.548 | - | 12.0 | 0.114 | 129.0 | 103.2 | 0.8439 |
| | SE 4 | 7.0 | 8.0 | 3880 | 0.0516 | - | 0.541 | - | 12.0 | 0.000 | 264.0 | 0.0 | 0.8880 |
| | SE 5 | 7.0 | 8.0 | 3710 | 0.0516 | - | 0.566 | - | 12.0 | 0.057 | 195.0 | 78.0 | 0.9916 |
| | SE 6 | 7.0 | 8.0 | 3280 | 0.0509 | - | 0.708 | - | 12.0 | 0.114 | 108.0 | 86.4 | 0.8111 |
| | SE 7 | 7.0 | 8.0 | 4200 | 0.0516 | - | 0.483 | - | 12.0 | 0.214 | 88.0 | 132.0 | 0.9353 |
| | SE 8 | 7.0 | 8.0 | 4140 | 0.0516 | - | 0.491 | - | 17.1 | 0.000 | 290.0 | 0.0 | 0.9262 |
| | SE 9 | 7.0 | 8.0 | 4580 | 0.0520 | - | 0.448 | - | 17.1 | 0.029 | 201.0 | 40.2 | 0.9346 |
| | SE 10 | 7.0 | 8.0 | 4310 | 0.0516 | - | 0.472 | - | 17.1 | 0.057 | 135.0 | 54.0 | 0.7589 |
| | SE 11 | 7.0 | 8.0 | 3250 | 0.0502 | - | 0.659 | - | 17.1 | 0.114 | 88.0 | 70.4 | 0.8668 |
| | SE 12 | 7.0 | 8.0 | 4280 | 0.0520 | - | 0.480 | - | 17.1 | 0.214 | 67.0 | 100.5 | 0.9195 |
| | SE 13 | 7.0 | 8.0 | 3070 | 0.0270 | - | 0.378 | - | 12.0 | 0.000 | 180.0 | 0.0 | 0.8675 |
| | SE 14 | 7.0 | 8.0 | 2890 | 0.0270 | - | 0.401 | - | 12.0 | 0.057 | 116.0 | 46.4 | 0.8884 |
| | SE 15 | 7.0 | 8.0 | 3810 | 0.0270 | - | 0.304 | - | 12.0 | 0.114 | 108.0 | 86.4 | 0.8366 |
| | | | | | | | | | | | | Mean | 0.8923 |
| | | | | | | | | | | | | Coeff. of Variation | 0.0707 |
| Johnson | RCS | 7.9 | 7.9 | 3609 | 0.0745 | 0.0028 | 0.868 | 0.0019 | 14.3 | 0.100 | 185.5 | 146.0 | 0.8510 * |
| & May | | | | | | | | | | | | | |
| (1978) | | | | | | | | | | | | Mean | 0.8510 |
| | | | | | | | | | | | | Coeff. of Variation | N/A |

Table 3.3.1 - Description of Composite Steel-Concrete Columns Subjected to Minor Axis Bending Used for Comparison with FEM Ultimate Strength

| Author | Col. Desig. | h (in.) | b (in.) | $f'_c$ (psi) | $\rho_{ss}$ | $\rho_{rs}$ | $\frac{\rho_{ss} f_{yss}}{f'_c}$ | Tie/Hoop Vol. Ratio $\rho''$ | $\ell/h$ | e/h | TEST VALUES Applied Axial Load (kips) | TEST VALUES Applied BM at Ends (kip-in) | Strength Ratio |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Morino, Matsui & Watanabe (1984) | A4-00 | 6.3 | 6.3 | 3060 | 0.0870 | 0.0044 | 1.481 | 0.0025 | 5.8 | 0.250 | 112.9 | 177.8 | 1.0324 |
| | B4-00 | 6.3 | 6.3 | 3394 | 0.0870 | 0.0044 | 1.302 | 0.0025 | 14.4 | 0.250 | 83.5 | 131.5 | 1.0264 |
| | C4-00 | 6.3 | 6.3 | 3380 | 0.0870 | 0.0044 | 1.176 | 0.0025 | 21.7 | 0.250 | 61.7 | 97.2 | 1.0435 |
| | D4-00 | 6.3 | 6.3 | 3075 | 0.0870 | 0.0044 | 1.474 | 0.0025 | 28.9 | 0.250 | 46.4 | 73.0 | 1.1108 |
| | A8-00 | 6.3 | 6.3 | 4874 | 0.0870 | 0.0044 | 0.953 | 0.0025 | 5.8 | 0.469 | 77.4 | 228.4 | 1.0120 |
| | B8-00 | 6.3 | 6.3 | 4830 | 0.0870 | 0.0044 | 0.956 | 0.0025 | 14.4 | 0.469 | 59.4 | 175.5 | 0.9086 |
| | C8-00 | 6.3 | 6.3 | 3568 | 0.0870 | 0.0044 | 1.305 | 0.0025 | 21.7 | 0.469 | 39.6 | 117.0 | 0.9461 |
| | D8-00 | 6.3 | 6.3 | 3322 | 0.0870 | 0.0044 | 1.398 | 0.0025 | 28.9 | 0.469 | 30.3 | 89.5 | 0.9554 |
| | | | | | | | | | | | | Mean | 1.0044 |
| | | | | | | | | | | | | Coeff. of Variation | 0.0643 |
| Roik & Mangerig (1987) | 7 | 11.8 | 11.8 | 6596 | 0.0868 | 0.0050 | 0.515 | 0.0029 | 10.0 | 0.100 | 1022.3 | 1207.4 | 1.1947 |
| | 8 | 11.8 | 11.8 | 6596 | 0.0868 | 0.0050 | 0.515 | 0.0029 | 10.0 | 0.300 | 501.6 | 1777.2 | 1.1371 |
| | 9 | 11.8 | 11.8 | 6596 | 0.0868 | 0.0050 | 0.515 | 0.0029 | 16.7 | 0.100 | 824.0 | 973.2 | 1.2357 |
| | 10 | 11.8 | 11.8 | 6596 | 0.0868 | 0.0050 | 0.515 | 0.0029 | 16.7 | 0.300 | 410.5 | 1454.6 | 1.2103 |
| | 11 | 11.8 | 11.8 | 6596 | 0.0868 | 0.0050 | 0.515 | 0.0029 | 26.7 | 0.100 | 454.6 | 536.9 | 1.1780 |
| | 12 | 11.8 | 11.8 | 6596 | 0.0868 | 0.0050 | 0.515 | 0.0029 | 26.7 | 0.300 | 223.7 | 792.6 | 1.0283 |
| | | | | | | | | | | | | Mean | 1.1640 |
| | | | | | | | | | | | | Coeff. of Variation | 0.0638 |
| Roik & Schwal'r (1988) | V102 | 11.0 | 11.0 | 7572 | 0.0495 | 0.0079 | 0.302 | 0.0028 | 12.6 | 0.357 | 252.0 | 992.2 | 0.8758 |
| | V111 | 11.0 | 11.0 | 7646 | 0.0495 | 0.0314 | 0.293 | 0.0028 | 12.6 | 0.357 | 394.6 | 1553.4 | 0.9408 |
| | V112 | 11.0 | 11.0 | 7646 | 0.0495 | 0.0314 | 0.293 | 0.0028 | 12.6 | 0.214 | 565.4 | 1335.6 | 0.9604 |
| | V113 | 11.0 | 11.0 | 7646 | 0.0495 | 0.0314 | 0.293 | 0.0028 | 12.6 | 0.000 | 1031.9 | 0.0 | 0.7727 |
| | V121 | 11.0 | 11.0 | 7646 | 0.0434 | 0.0314 | 0.199 | 0.0028 | 12.6 | 0.571 | 255.8 | 1611.6 | 0.9372 |
| | V122 | 11.0 | 11.0 | 7646 | 0.0434 | 0.0314 | 0.199 | 0.0028 | 12.6 | 0.714 | 182.8 | 1439.2 | 0.8268 |
| | V123 | 11.0 | 11.0 | 7646 | 0.0434 | 0.0314 | 0.199 | 0.0028 | 12.6 | 0.357 | 345.1 | 1358.7 | 0.8657 |
| | | | | | | | | | | | | Mean | 0.8828 |
| | | | | | | | | | | | | Coeff. of Variation | 0.0773 |

Note: The strength ratio is defined as the tested strength divided by the FEM strength.

h = depth of concrete cross-section perpendicular to the axis of bending.

b = width of the concrete cross-section parallel to the axis of bending.

$\rho'' = 2(b''+d'')A_t/b''d''s$

b" = outside width of ties/hoops.

d" = outside depth of ties/hoops.

Table 3.3.1 - Description of Composite Steel-Concrete Columns Subjected to Minor Axis Bending Used for Comparison with FEM Ultimate Strength

$A_t$ = area of cross-section of a tie/hoop bar.
s = spacing of ties/hoops.

The term $f_{yss}$ was taken as the web yield strength for computing the $\rho_{ss} f_{yss}/f'_c$ ratio.

* Excluded from the final analysis on the basis of incomplete or insufficient information, as explained in the text.

** Excluded from the final analysis on the basis of concrete strength ($f'_c$) being lower than the practical value of 2500 psi, as explained in the text.

*** Revised statistics after the removal of tests identified with an asterisk (*) as well as those identified with a double asterisk (**).

Included in the table is the ratio of tested to FEM ultimate strength (strength ratio) for each of the 143 column specimens. The strength ratio was taken as the ratio of bending moment strengths for columns with $e/h=\infty$ and as the ratio of axial load capacities for columns with all other $e/h$ values.

A plot of tested strength against the FEM strength (Figure 3.3.1(a)) shows a relatively wide band of strength ratios and a number of possible outliers. These outliers are in the non-conservative region indicating an overestimation of the column strength by the FEM method. Also, as the strengths of the columns increase, there is a proportional increase in the magnitude of error. However this should be expected if the percentage of error remains relatively constant. A histogram giving the frequency, in percent, against the strength ratio (Figure 3.3.1(b)) shows a relatively wide and scattered distribution of values about the mean. The mean strength ratio of all 143 test columns was 1.004 with a coefficient of variation of 26.9 percent (Figure 3.3.1(b)).

The calculated mean, coefficient of variation, minimum and maximum values of strength ratios for all test columns listed in Table 3.3.1 are shown in Table 3.3.2. The strength ratio statistics shown in Table 3.3.2 were divided into five categories, based on the slenderness ratio ($\ell/h$).

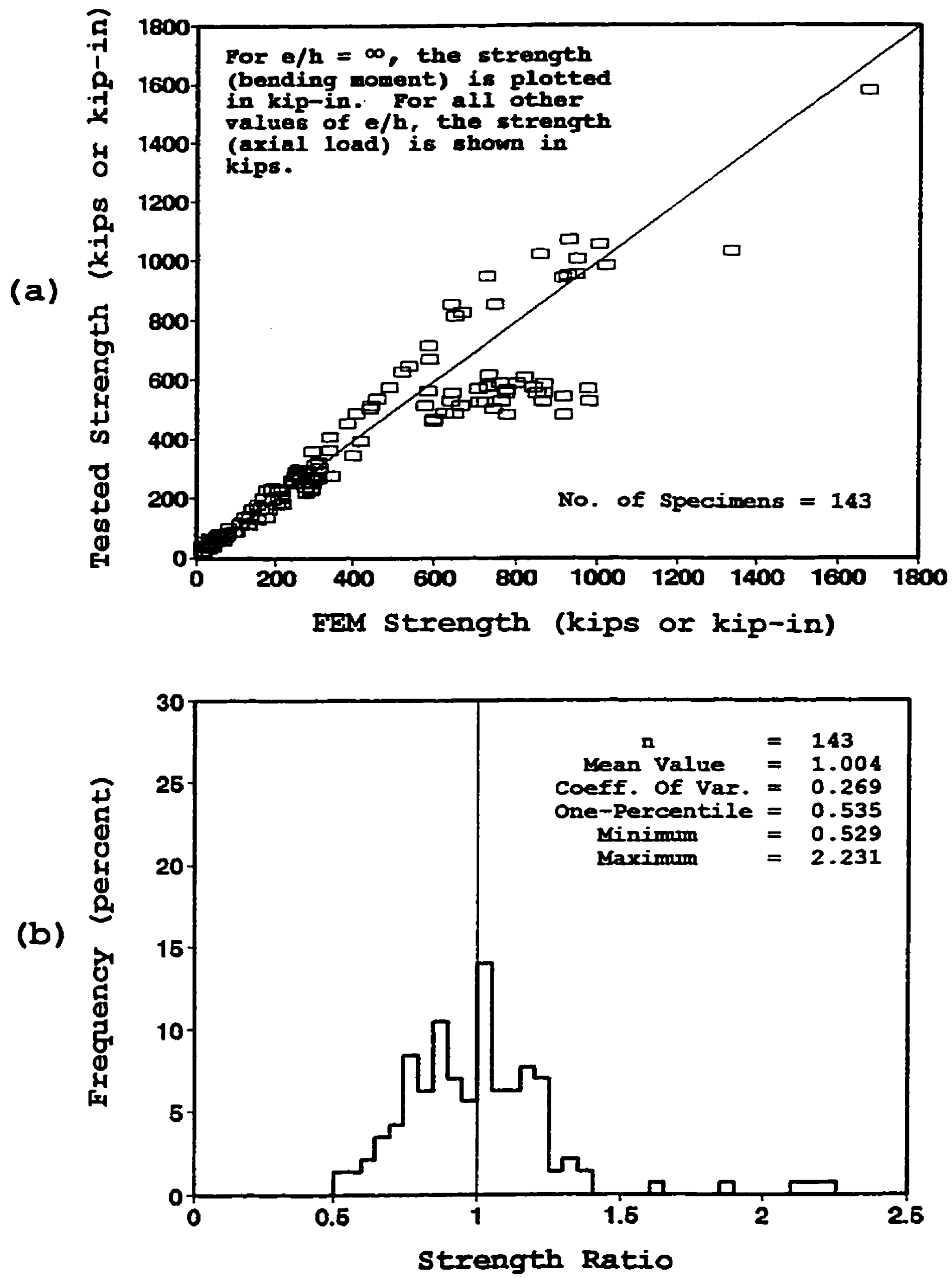



Figure 3.3.1 - Comparison of tested strength to FEM strength for composite steel-concrete columns subjected to minor axis bending (all columns).

Table 3.3.2 - Strength Ratio Statistics of Composite Steel-Concrete Columns Subjected to Minor Axis Bending for Different Ranges of e/h and $\ell/h$, using FEM (all columns)

| Column Type (1) | (2) | $e/h = 0$ (3) | $0 < e/h < 0.1$ (4) | $0.1 \leq e/h \leq 0.7$ (5) | $0.1 \leq e/h \leq 1.5$ (6) | $e/h = \infty$ (7) |
|---|---|---|---|---|---|---|
| Pedestal $\ell/h \leq 3$ | No. | 2 | * | * | * | * |
| | Mean | 1.184 | * | * | * | * |
| | CV | 0.037 | * | * | * | * |
| | Min | 1.154 | * | * | * | * |
| | Max | 1.215 | * | * | * | * |
| Short $3 < \ell/h < 6.6$ | No. | 4 | * | 3 | 3 | * |
| | Mean | 0.895 | * | 1.022 | 1.022 | * |
| | CV | 0.127 | * | 0.010 | 0.010 | * |
| | Min | 0.812 | * | 1.012 | 1.012 | * |
| | Max | 1.063 | * | 1.032 | 1.032 | * |
| Slender $6.6 \leq \ell/h \leq 30$ | No. | 64 | 8 | 54 | 56 | 2 |
| | Mean | 0.943 | 0.993 | 1.013 | 1.014 | 0.976 |
| | CV | 0.281 | 0.137 | 0.148 | 0.150 | 0.050 |
| | Min | 0.530 | 0.759 | 0.793 | 0.793 | 0.942 |
| | Max | 1.887 | 1.158 | 1.355 | 1.355 | 1.010 |
| Super Slender $\ell/h > 30$ | No. | 4 | * | * | * | * |
| | Mean | 1.888 | * | * | * | * |
| | CV | 0.305 | * | * | * | * |
| | Min | 1.028 | * | * | * | * |
| | Max | 2.231 | * | * | * | * |
| ACI Permitted $3 < \ell/h \leq 30$ | No. | 68 | 8 | 57 | 59 | 2 |
| | Mean | 0.941 | 0.993 | 1.014 | 1.014 | 0.976 |
| | CV | 0.275 | 0.137 | 0.144 | 0.146 | 0.050 |
| | Min | 0.530 | 0.759 | 0.793 | 0.793 | 0.942 |
| | Max | 1.887 | 1.158 | 1.355 | 1.355 | 1.010 |

* No data available

Note: CV stands for the coefficient of variation.

The data were further categorized into five ranges of end eccentricity ratio ($e/h$) as shown in Table 3.3.2.

Differences in the statistics for the different ranges of end eccentricity ratios (Table 3.3.2 Columns 3,4,5,6 and 7) were observed. Slender and super-slender columns with $e/h=0$ have a very high coefficient of variation (28.1 and 30.5 percent). The coefficient of variation of slender columns decreases significantly with $e/h>0$ but remains relatively high at approximately 15 percent. The mean value of strength ratios for super-slender columns (1.888) is extremely high compared to the overall mean value (1.004). The mean value of strength ratios for short columns is low (0.895) compared to the overall mean value (1.004). The overall minimum strength ratio (0.529) was found to occur for a column with $\ell/h=17.5$ and $e/h=0$, while the overall maximum strength ratio (2.231) was found to occur in a column with $\ell/h=44$ and e/h=0, as indicated in Table 3.3.1. The probability distribution of the strength ratios computed for the 143 test columns is plotted on a normal probability scale in Figure 3.3.2 and is compared to a normal probability distribution with a mean value of 1.0 and a coefficient of variation of 27.0 percent. The data do not follow the normal probability distribution and skew significantly as shown in Figure 3.3.2.

After a reexamination of the data from Bondale (1966) and Johnson and May (1978), it was decided to drop these

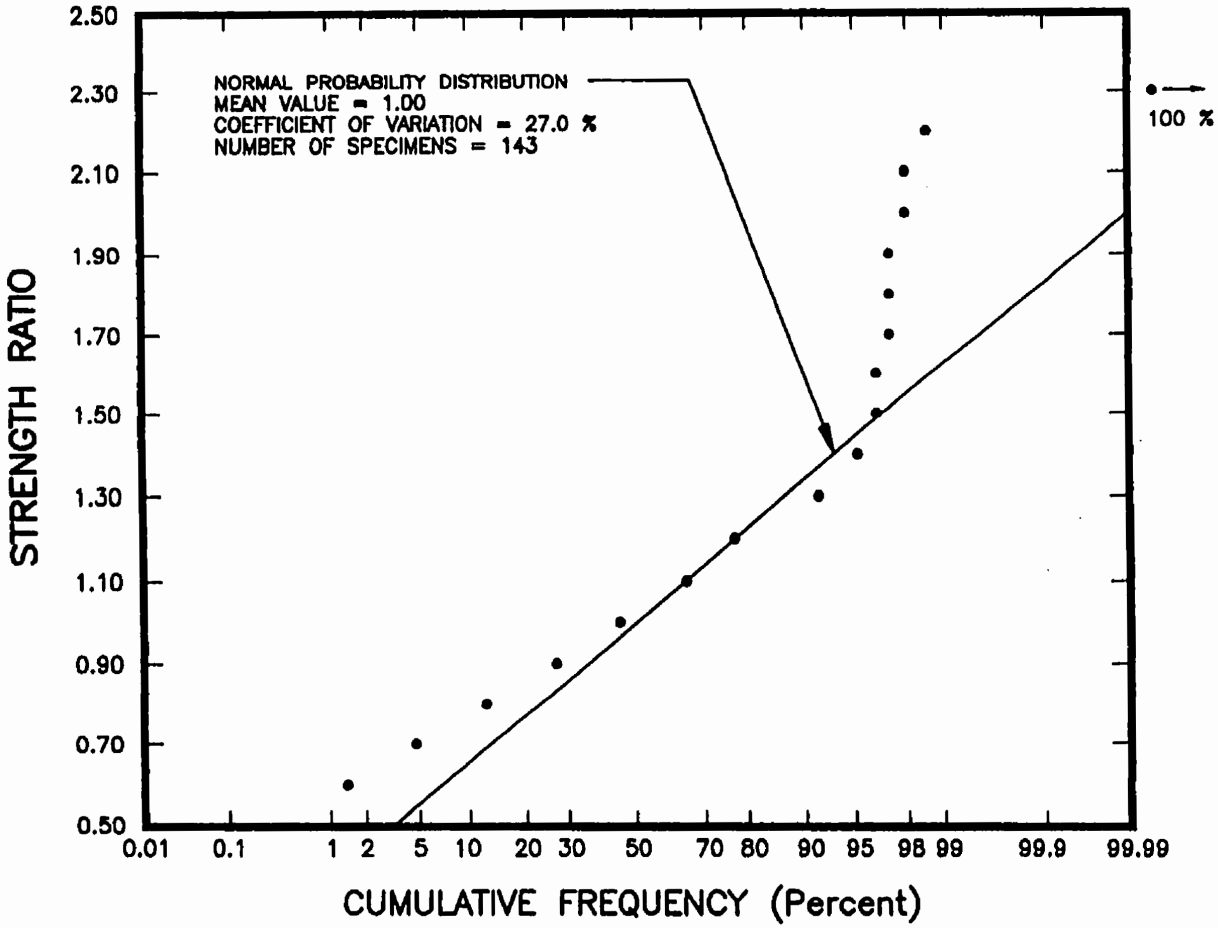



Figure 3.3.2 - Probability distribution of strength ratios using FEM of composite steel-concrete columns subjected to minor axis bending (all columns).

test series altogether from the comparative study. Similarly, after a reexamination of the data from Stevens (1965), it was decided to remove a selected number of test columns of this test series from the comparative study. All of the above-noted columns are identified by an asterisk (*) in Table 3.3.1.

In the analysis of the columns from Bondale (1966) and Johnson and May (1978), several assumptions were made because of the lack of sufficient information on geometric and/or material properties. Bondale (1966) did not give the yield strength of the longitudinal reinforcing steel and there were conflicting concrete strengths reported by Bondale (1966) and Basu (1967) for the same tests. For the column tested by Johnson and May (1978), the location and yield strength of the longitudinal reinforcing steel were not given. Also, the column was reported as part of a test frame. The equivalent effective length of the column was given, however, there was no indication on how this value was obtained. The assumptions that were made for these two studies could have affected the computed ultimate capacities.

For the tests by Stevens (1965), columns in Series S examined the effect of using different types of aggregates in the concrete mixture on the overall strength of the column. The types of aggregates examined include river gravel and sand, expanded clay, and foamed slag and river sand with extremely high water/cement ratios of 0.77, 0.85, and 0.7 by weight, respectively. The stress-strain curve for concrete used for the FEM method is not able to properly account for these types of concrete (Mirza, Hatzinikolas and MacGregor 1979). Columns in Series B by Stevens (1965) used an extremely small width of cross-section (3.5 inches) with high slenderness ratios ($\ell/h$ from 13.1 to 44) tested under pure axial load. Such columns are likely to be highly sensitive to slight imperfections in fabrication or to a misalignment of the column within the testing apparatus and, hence, were excluded from the comparative study.

The removal of the 21 columns affected the overall statistics by decreasing both the mean and coefficient of variation to 0.94 and 18.9 percent, respectively. The overall minimum strength ratio did not change, however, the maximum strength ratio was reduced from 2.231 to 1.355.

The plot of tested strength against the FEM strength (Figure 3.3.3(a)) for the 122 test columns still shows a relatively wide band of strength ratios and a number of possible outliers. A histogram giving the frequency, in percent, against the strength ratio (Figure 3.3.3(b)) shows a wide but relatively symmetric distribution of values about the mean.

The strength ratio statistics for the 122 test columns in Table 3.3.3 show a significant improvement over those given in Table 3.3.2 which included all 143 test columns.

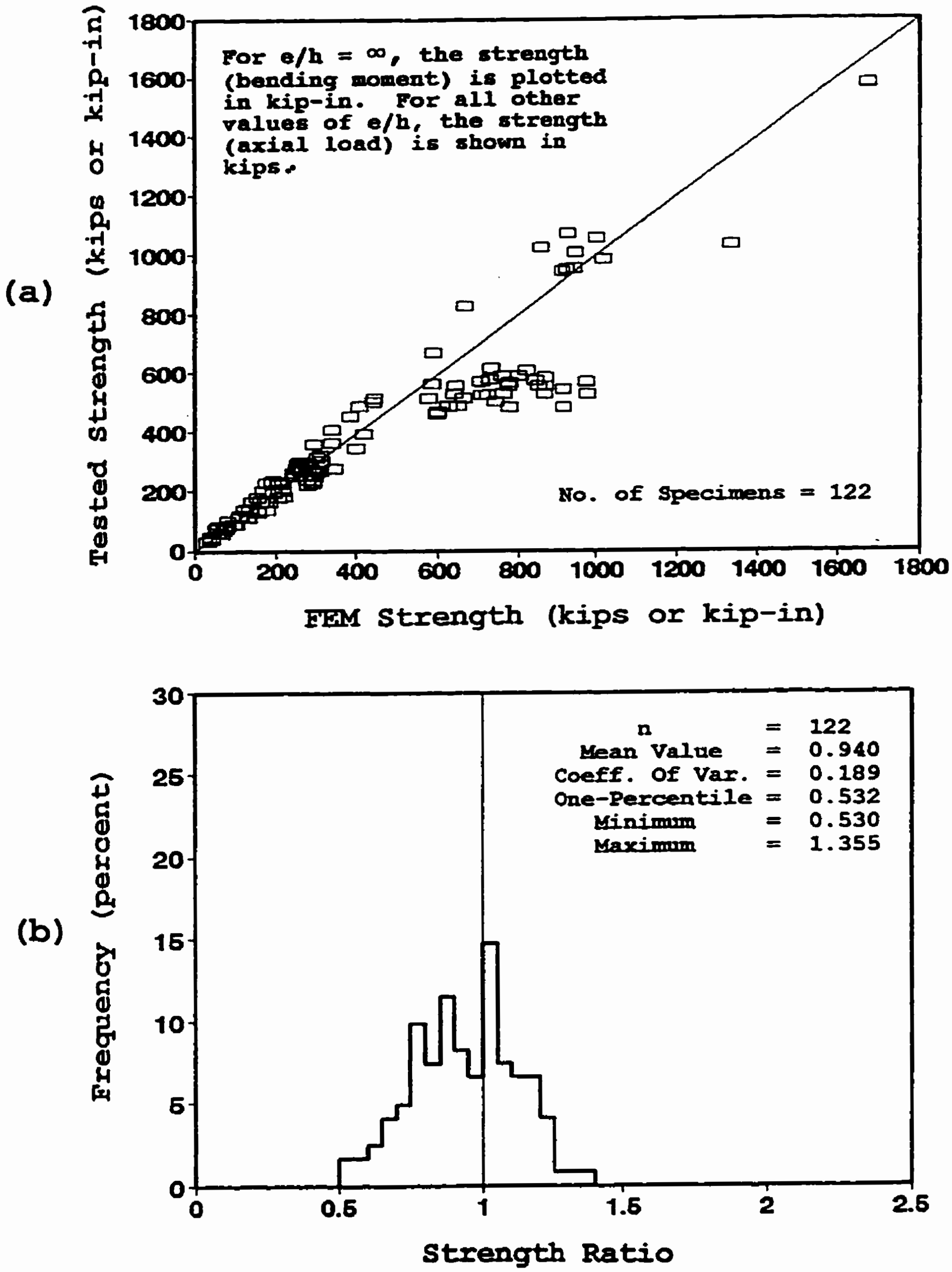



Figure 3.3.3 - Comparison of tested strength to FEM strength for composite steel-concrete columns subjected to minor axis bending (some data removed due to incomplete or insufficient information).

Table 3.3.3 - Strength Ratio Statistics of Composite Steel-Concrete Columns Subjected to Minor Axis Bending for Different Ranges of e/h and ℓ/h, using FEM (some data removed due to incomplete or insufficient information)

| Column Type (1) | (2) | e/h = 0 (3) | 0 < e/h < 0.1 (4) | 0.1 ≤ e/h ≤ 0.7 (5) | 0.1 ≤ e/h ≤ 1.5 (6) | e/h = ∞ (7) |
|---|---|---|---|---|---|---|
| Pedestal ℓ/h ≤ 3 | No. | 2 | * | * | * | * |
| | Mean | 1.184 | * | * | * | * |
| | CV | 0.037 | * | * | * | * |
| | Min | 1.154 | * | * | * | * |
| | Max | 1.215 | * | * | * | * |
| Short 3 < ℓ/h < 6.6 | No. | 4 | * | 3 | 3 | * |
| | Mean | 0.895 | * | 1.022 | 1.022 | * |
| | CV | 0.127 | * | 0.010 | 0.010 | * |
| | Min | 0.812 | * | 1.012 | 1.012 | * |
| | Max | 1.063 | * | 1.032 | 1.032 | * |
| Slender 6.6 ≤ ℓ/h ≤ 30 | No. | 51 | 8 | 51 | 52 | 2 |
| | Mean | 0.848 | 0.993 | 1.014 | 1.010 | 0.976 |
| | CV | 0.211 | 0.137 | 0.150 | 0.151 | 0.050 |
| | Min | 0.530 | 0.759 | 0.793 | 0.793 | 0.942 |
| | Max | 1.227 | 1.158 | 1.355 | 1.355 | 1.010 |
| Super Slender ℓ/h > 30 | No. | * | * | * | * | * |
| | Mean | * | * | * | * | * |
| | CV | * | * | * | * | * |
| | Min | * | * | * | * | * |
| | Max | * | * | * | * | * |
| ACI Permitted 3 < ℓ/h ≤ 30 | No. | 55 | 8 | 54 | 55 | 2 |
| | Mean | 0.852 | 0.993 | 1.014 | 1.011 | 0.976 |
| | CV | 0.206 | 0.137 | 0.145 | 0.147 | 0.050 |
| | Min | 0.530 | 0.759 | 0.793 | 0.793 | 0.942 |
| | Max | 1.227 | 1.158 | 1.355 | 1.355 | 1.010 |

* No data available

Note: CV stands for the coefficient of variation.

The probability distribution of the strength ratios computed for the 122 test columns is plotted on a normal probability scale in Figure 3.3.4 and is compared to a normal probability distribution with a mean value of 0.94 and a coefficient of variation of 19.0 percent. The data closely follow the normal curve and can be assumed to be normally distributed.

Considering current construction practice, it was decided to further exclude all columns with a specified concrete cylinder strength less than 2500 psi. For concrete strengths reported by cube strengths, the equivalent standard cylinder (6 inch diameter by 12 inch high) strength was computed, and this value was used as the basis for excluding the column specimens from this study. Using this criterion resulted in the removal of a further 41 columns from the data base. These columns are identified by a double asterisk (**) in Table 3.3.1. The removal of these 41 columns affected the overall statistics further by decreasing both the mean and coefficient of variation to 0.867 and 18.7 percent, respectively. The overall minimum strength ratio did not change. However, the maximum strength ratio was reduced from 1.355 to 1.280. After the removal of the 41 columns, statistics were recalculated and are shown in Table 3.3.1 for each of the studies from which data were removed. Columns in Table 3.3.1 that are not identified by a single or double asterisk represent the 81 composite steel-concrete test

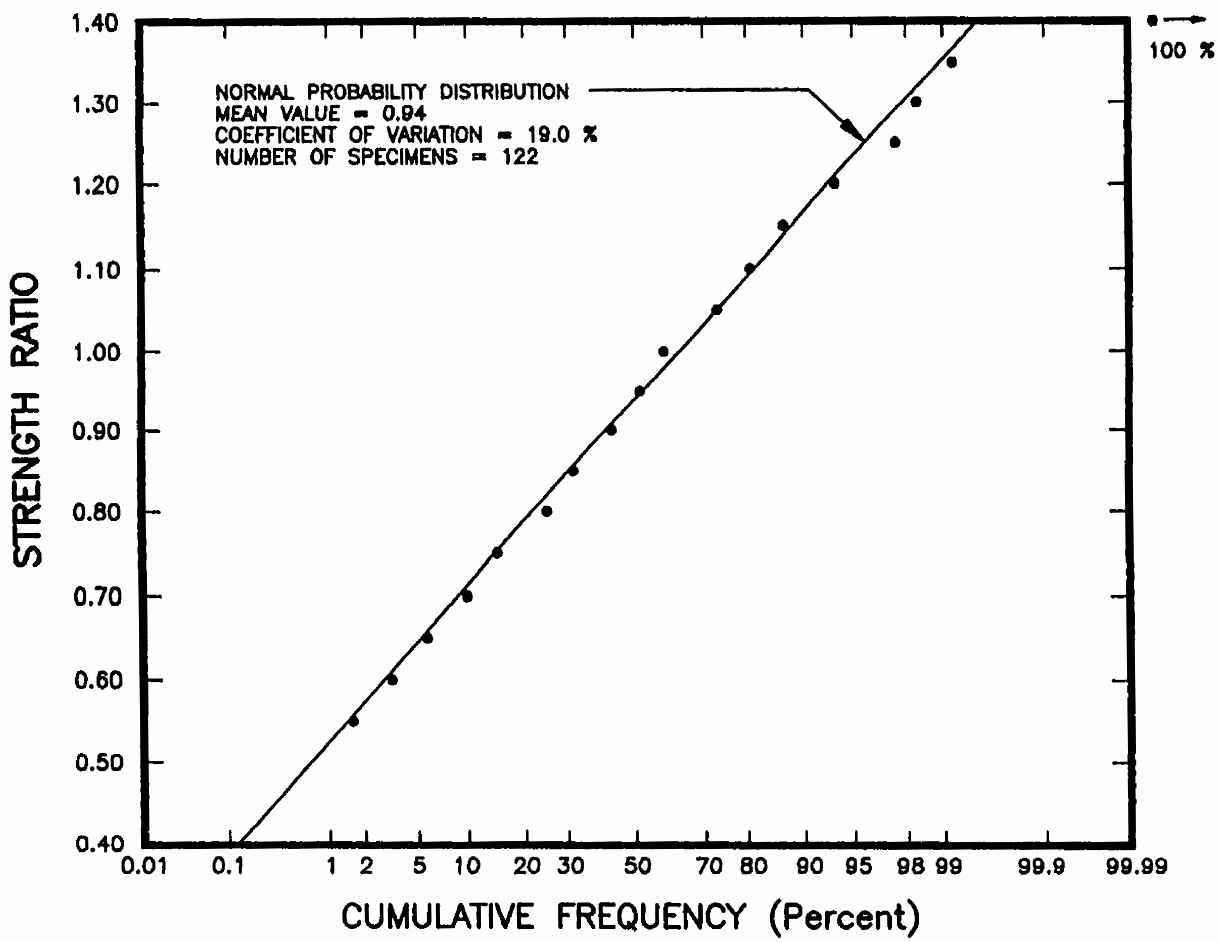



Figure 3.3.4 - Probability distribution of strength ratios using FEM of composite steel-concrete columns subjected to minor axis bending (some data removed due to incomplete or insufficient information).

columns subjected to minor axis bending that were finally used for the comparative study.

The plot of tested strength against the FEM strength (Figure 3.3.5(a)) for the 81 test columns still shows a relatively wide band of strength ratios with a number of points showing non-conservative values. A histogram giving the frequency, in percent, against the strength ratio (Figure 3.3.5(b)) shows a slightly non-symmetric distribution of values about the mean.

The strength ratio statistics for the 81 test columns in Table 3.3.4 show significant improvements over those given in Table 3.3.3 which included 122 test columns and Table 3.3.2 which included 143 test columns. However, the mean value for all columns reduced to 0.867, as indicated by Figure 3.3.5(b). Note the overall mean value for columns subjected to combined axial load and bending moment (Table 3.3.4 Column 4 plus Column 6) is 0.962.

The probability distribution of the strength ratios computed for the 81 test columns is plotted on a normal probability scale in Figure 3.3.6 and is compared to a normal probability distribution with a mean value of 0.87 and a coefficient of variation of 18.5 percent. The data closely follow the normal curve and can be assumed to be normally distributed.

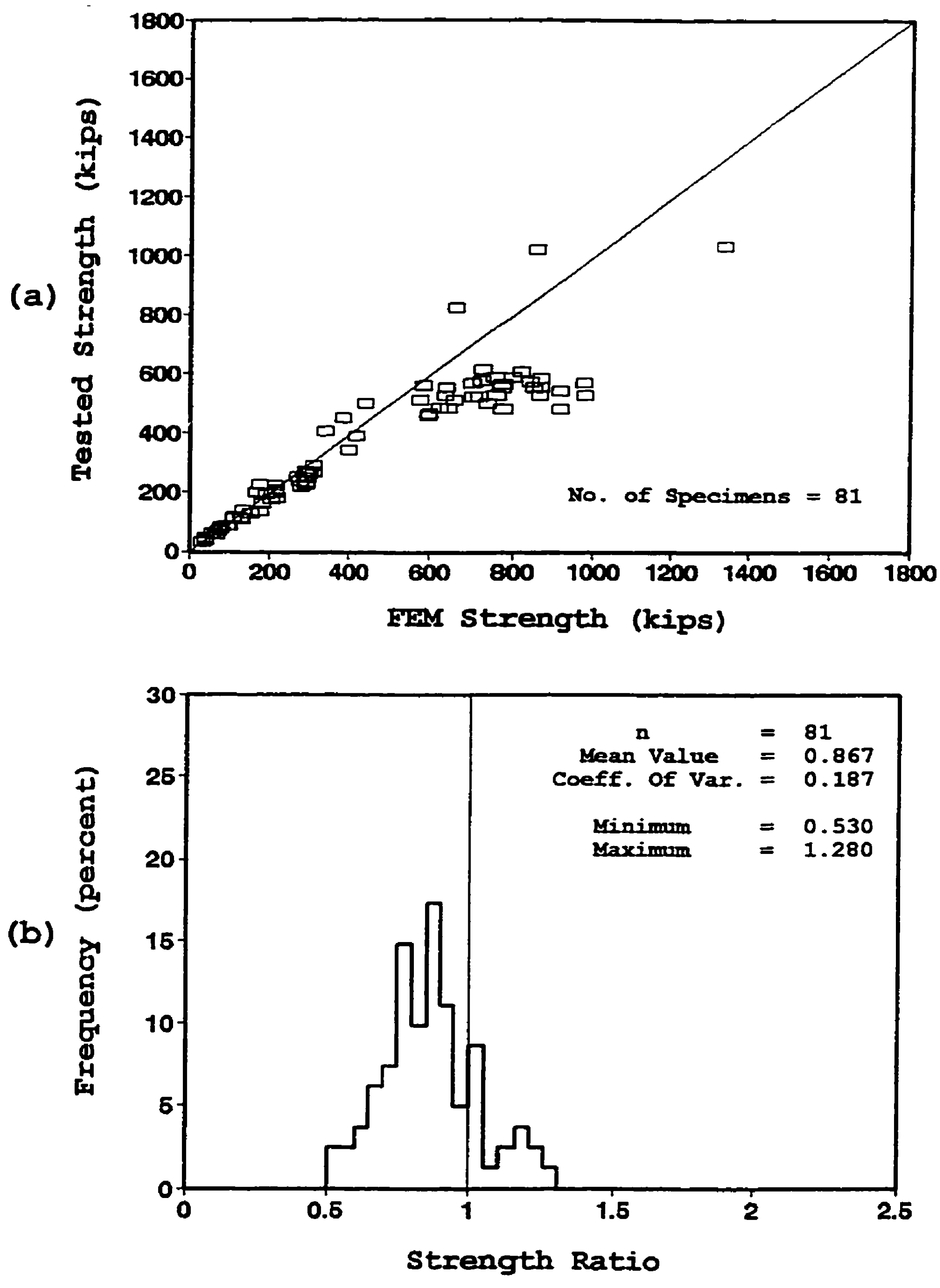



Figure 3.3.5 - Comparison of tested strength to FEM strength for composite steel-concrete columns subjected to minor axis bending (some data removed due to incomplete or insufficient information and due to $f'_c$<2500 psi).

Table 3.3.4 - Strength Ratio Statistics of Composite Steel-Concrete Columns Subjected to Minor Axis Bending for Different Ranges of e/h and $\ell/h$, using FEM (some data removed due to incomplete or insufficient information and due to $f'_c$<2500 psi).

| Column Type (1) | (2) | $e/h = 0$ (3) | $0 < e/h < 0.1$ (4) | $0.1 \le e/h \le 0.7$ (5) | $0.1 \le e/h \le 1.5$ (6) | $e/h = \infty$ (7) |
|---|---|---|---|---|---|---|
| Pedestal $\ell/h \le 3$ | No. | * | * | * | * | * |
| | Mean | * | * | * | * | * |
| | CV | * | * | * | * | * |
| | Min | * | * | * | * | * |
| | Max | * | * | * | * | * |
| Short $3 < \ell/h < 6.6$ | No. | 3 | * | 2 | 2 | * |
| | Mean | 0.840 | * | 1.022 | 1.022 | * |
| | CV | 0.031 | * | 0.014 | 0.014 | * |
| | Min | 0.812 | * | 1.012 | 1.012 | * |
| | Max | 0.863 | * | 1.032 | 1.032 | * |
| Slender $6.6 \le \ell/h \le 30$ | No. | 32 | 5 | 38 | 39 | * |
| | Mean | 0.734 | 0.910 | 0.969 | 0.965 | * |
| | CV | 0.143 | 0.102 | 0.143 | 0.143 | * |
| | Min | 0.530 | 0.759 | 0.793 | 0.793 | * |
| | Max | 0.941 | 0.992 | 1.280 | 1.280 | * |
| Super Slender $\ell/h > 30$ | No. | * | * | * | * | * |
| | Mean | * | * | * | * | * |
| | CV | * | * | * | * | * |
| | Min | * | * | * | * | * |
| | Max | * | * | * | * | * |
| ACI Permitted $3 < \ell/h \le 30$ | No. | 35 | 5 | 40 | 41 | * |
| | Mean | 0.743 | 0.910 | 0.972 | 0.968 | * |
| | CV | 0.141 | 0.102 | 0.139 | 0.140 | * |
| | Min | 0.530 | 0.759 | 0.793 | 0.793 | * |
| | Max | 0.941 | 0.992 | 1.280 | 1.280 | * |

*** No data available**

Note: CV stands for the coefficient of variation.

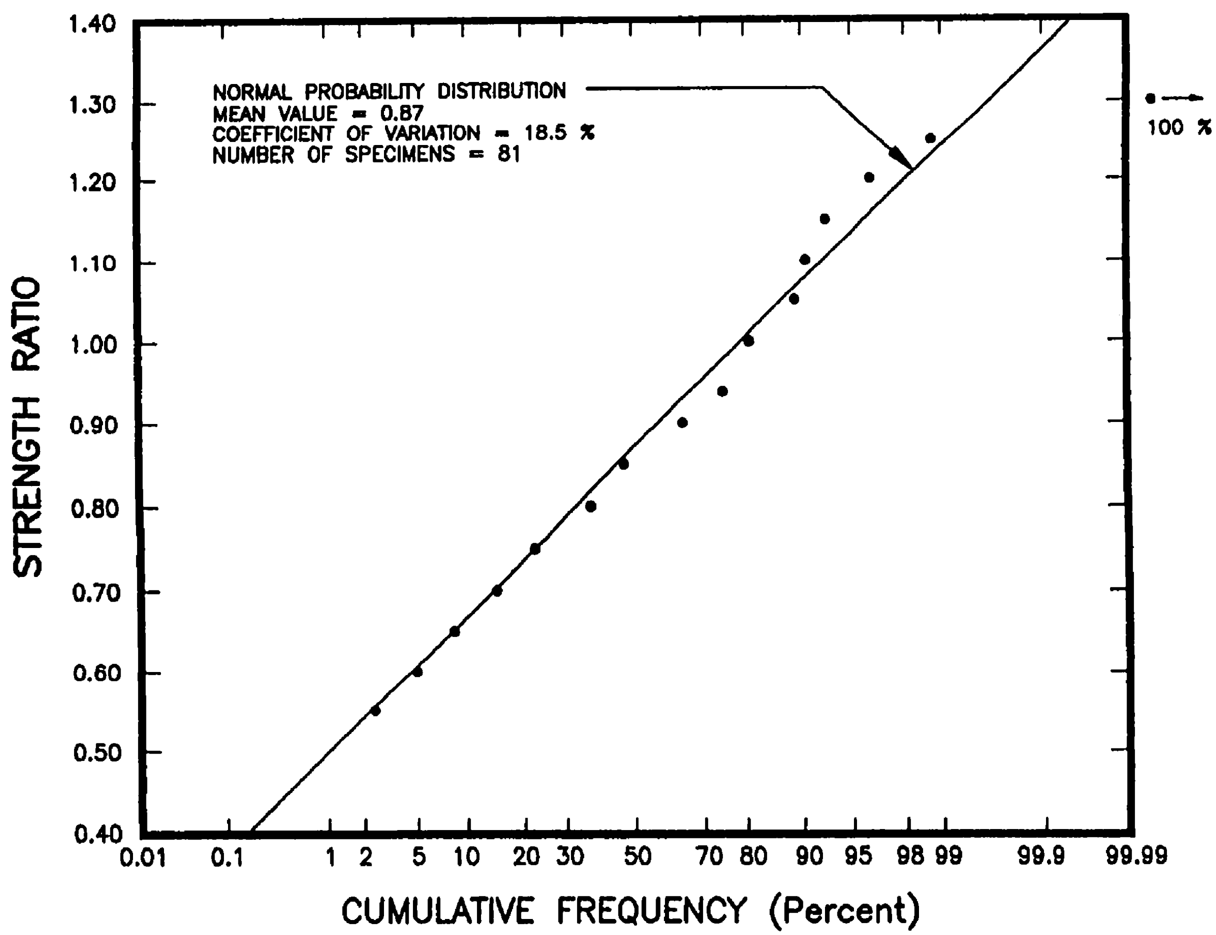



Figure 3.3.6 - Probability distribution of strength ratios using FEM of composite steel-concrete columns subjected to minor axis bending (some data removed due to incomplete or insufficient information and due to $f'_c$<2500 psi).

# 4 - OVERVIEW OF DESIGN METHODS COMPARED IN THIS STUDY

An overview of the design procedures given in ACI 318-95 (Building 1995) which is very similar to CSA A23.3-M84 (Design 1984), the AISC-LRFD Specifications (1994), Eurocode 2 (Design 1992), and Eurocode 4 (Design 1994) is presented in this chapter. Mirza (1990) and Tikka and Mirza (1992) proposed refined equations for calculating the flexural rigidity for use in ACI and CSA design procedures of reinforced and composite steel-concrete columns, respectively. A refined equation for calculating the radius of gyration of composite steel-concrete columns is proposed in this study for use in the AISC-LRFD design procedure. This equation plus those suggested by Mirza (1990) and Tikka and Mirza (1992) were also included in the comparative study and are discussed in this chapter.

A computer program was developed to compute the nominal axial load resistance ($P_{des}$) and/or bending moment resistance ($M_{des}$) of each test column using different procedures. Figure 4.1 lists the calculation procedures used for strength analysis of reinforced concrete and composite steel-concrete columns. In an attempt to compare the nominal column strengths determined from each of the design methods, all material resistance, performance, and safety factors were set equal to 1.0. The columns used in this study were pin-ended in braced, non-sway frames.

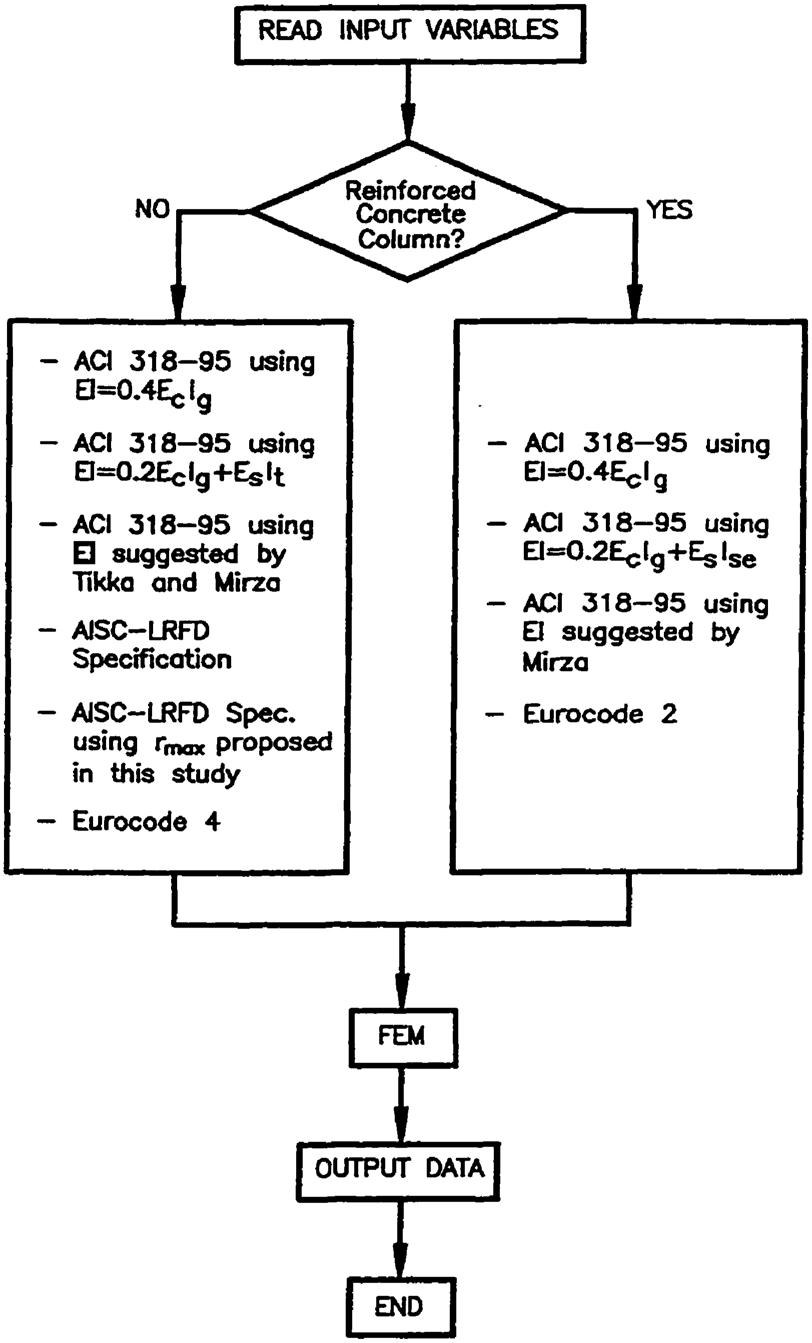



Figure 4.1 - Calculation procedures used for strength analysis of test columns.

## 4.1 ACI 318-95 PROVISIONS

The design equations given in ACI 318-95 (Building 1995) and CSA A23.3-M84 (Design 1984) for the design of reinforced concrete and composite steel-concrete columns are discussed in this section. All of the design equations apply equally to both reinforced concrete and composite steel-concrete columns unless stated otherwise. As the refined *EI* equations by Mirza (1990) and Tikka and Mirza (1992) were developed for ACI 318-95 and CSA A23.3-M84, these equations are also discussed here. Since CSA A23.3-M84 is very similar to ACI 318-95, the discussions are provided only for ACI 318-95 to prevent repetition, but are applicable to both codes.

### 4.1.1 Limitations of ACI 318-95 Provisions

For the design of both reinforced concrete and composite steel-concrete columns, ACI 318-95 has established several limitations. For reinforced concrete columns, these limitations include:

- The applied axial load acting on tied columns is limited to 80 percent of the pure axial load capacity.
- The area of longitudinal reinforcing bars shall not be less than 0.01 nor more than 0.08 times the gross area of the cross-section.
- For rectangular sections, the minimum number of longitudinal bars is four.

- Lateral ties shall be a minimum size of No. 3 for longitudinal bars No. 10 or smaller, and at least No. 4 in size for No. 11, No. 14 and No. 18 longitudinal bars.
- Lateral ties shall have a vertical spacing not exceeding 16 longitudinal bar diameters, 48 tie bar diameters, or the least dimension of the column.
- Ties shall be arranged so that every corner and alternate longitudinal bar shall have lateral support provided by the corner of a tie with an included angle of not more than 135 degrees and no bar shall be farther than 6 inches clear on each side along the tie from a laterally supported bar.

For composite steel-concrete columns, the limitations include:

- The applied axial load acting on tied columns is limited to 85 percent of the pure axial load capacity.
- The specified concrete strength, $f'_c$, shall not be less than 2500 psi.
- The design yield strength of the structural steel core shall be the specified minimum yield strength for the grade of structural steel but is not to exceed 50,000 psi.
- The area of longitudinal reinforcement shall not be less than 0.01 nor more than 0.08 times the net area of the concrete section.

- A longitudinal bar shall be located at every corner of a rectangular cross-section with other bars not spaced farther than one-half the least side dimension of the composite member.
- Lateral ties must extend completely around the structural steel core.
- Lateral ties must have a minimum diameter of not less than 1/50 times the greatest side dimension of the composite member, but not less than No. 3 and not greater than No. 5 bar.
- Lateral ties shall have a vertical spacing not exceeding 16 longitudinal bar diameters, 48 tie bar diameters, or one-half times the least dimension of the column.

For both reinforced concrete and composite steel-concrete columns, the limitations include:

- The upper limit on the slenderness ratio ($\ell/h$) is 30.

Many column test specimens did not meet some of the limits noted above. In such cases, however, these limits were ignored for computing the ACI (and CSA) strengths.

### 4.1.2 Calculation of Cross-Section Capacity

The determination of the cross-section capacity for both reinforced concrete and composite steel-concrete columns is based on the following assumptions:

(a) concrete and steel strains are compatible and no slip occurs;

(b) strain is linearly proportional to the distance from the neutral axis;

(c) residual stresses in the rolled steel section (for composite columns) are neglected.

The maximum useable strain at the extreme concrete compression fiber is equal to 0.003. An equivalent rectangular stress block with a stress ordinate of $0.85f'_c$ is used for calculation. The equivalent stress block is uniformly distributed over a zone bounded by the edges of the cross-section and a straight line located parallel to the neutral axis at a calculated depth from the extreme compression fiber. Equation 4.1 is used to determine the depth of the equivalent concrete stress block:

$$a = \beta_1 c \tag{4.1}$$

where $a$ is the depth of the equivalent rectangular stress block; $c$ is the distance from the extreme compression fiber to the neutral axis; and $\beta_1$ is a numerical coefficient. The coefficient $\beta_1$ is dependent on the concrete strength and is computed using Equation 4.2 or 4.3:

For $f'_c \leq 4000$ psi $\qquad \beta_1 = 0.85 \qquad$ (4.2)

For $f'_c > 4000$ psi $\qquad \beta_1 = 0.85 - 0.05\left(\frac{f'_c - 4000}{1000}\right) \geq 0.65 \qquad$ (4.3)

For SI conversion replace 4000 by 30 MPa, 1000 by 10 MPa and 0.05 by 0.08 in Equation 4.3. The equivalent stress block is illustrated in Figure 4.2. Note that the tensile strength of concrete is ignored.

The reinforcing and structural steels are assumed to be elastic-perfectly plastic. Therefore, at strains less than the yield strain, the stress in the steel can be computed as the modulus of elasticity of steel, $E_s$, times the strain. For strains greater than the yield strain, the stress in the steel is the specified yield stress, $f_y$, of the steel. The strain hardening is neglected. Note that the displaced area of concrete in the compression zone is considered.

The strength of the cross-section can be represented by an axial load-bending moment interaction curve similar to the one shown in Figure 4.3. Due to the large number of calculations required in developing the axial load-bending moment interaction curve, a computer program was written. Details of the computer program analysis procedure will be presented in the following section.

ACI 318-95 imposes an upper limit on the maximum design axial load, $P_{n(max)}$, permitted for rectangular reinforced concrete cross-sections:

$$P_{n(\max)} = 0.8\left(0.85f'_c\left(A_g - A_{st}\right) + f_y A_{st}\right) \tag{4.4}$$

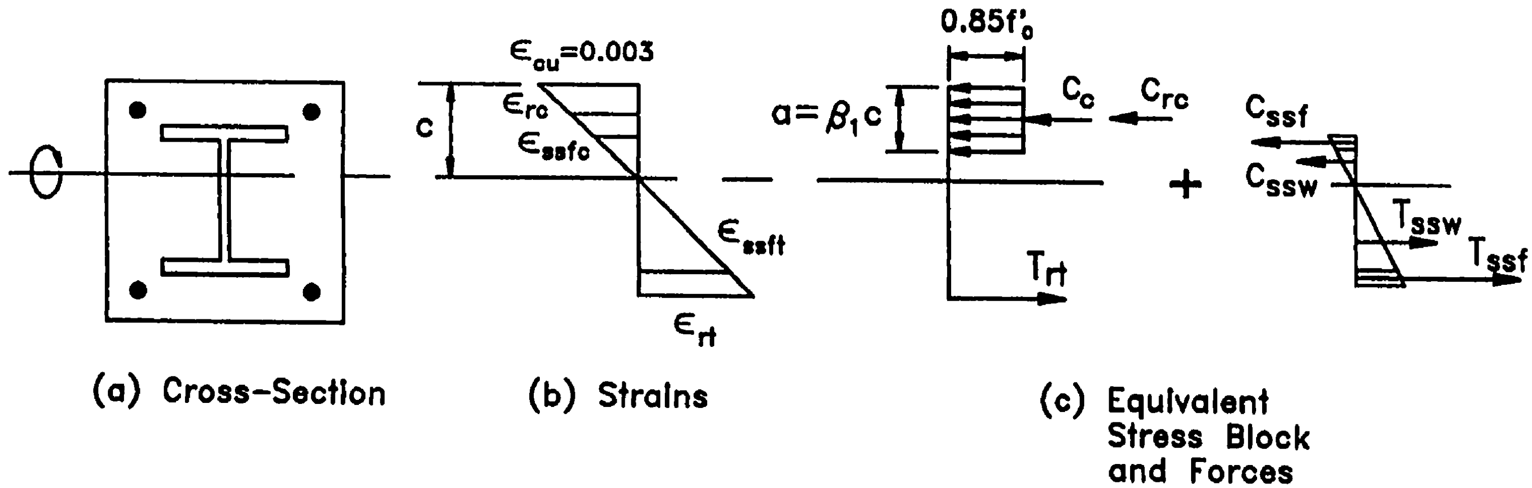



Figure 4.2 - Equivalent stress block specified by ACI.

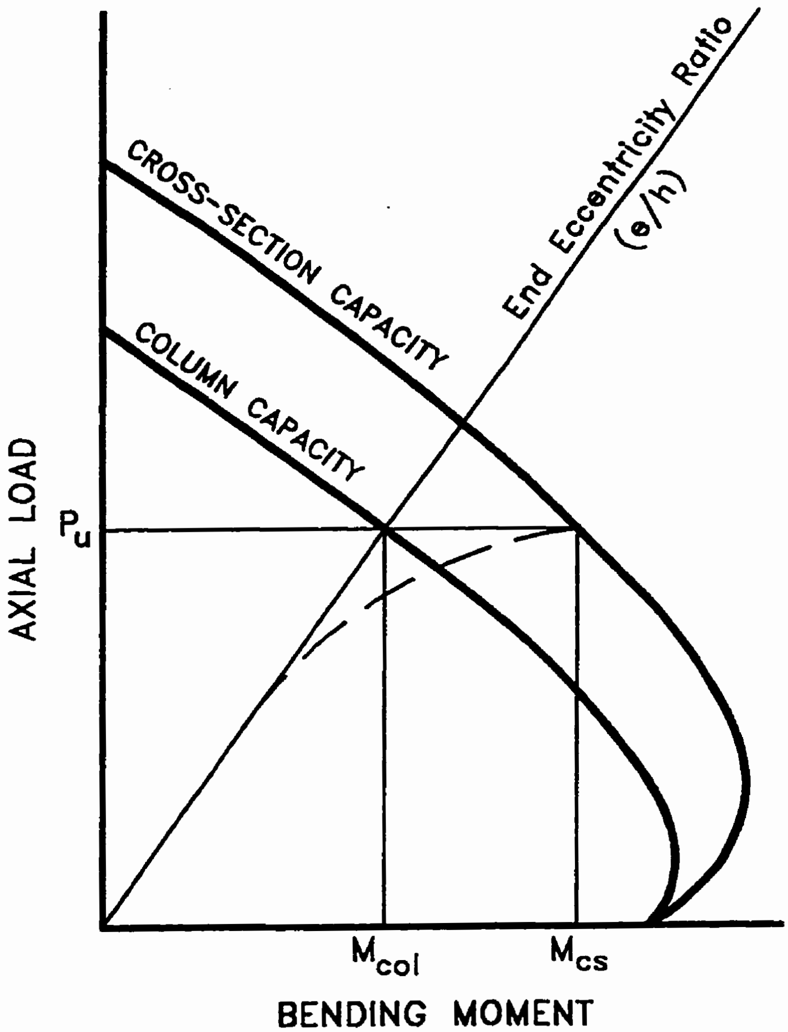



Figure 4.3 - Schematic cross-section and column axial load-bending moment interaction diagrams.

where $A_g$ is the gross area of the cross-section; and $A_{st}$ is the total area of longitudinal reinforcement. For composite steel-concrete cross-sections, the upper limit is increased from 80 percent to 85 percent of the nominal cross-section capacity and $A_{st}$ is the total area of longitudinal reinforcement plus the area of the structural steel core.

The upper limit is provided to account for accidental eccentricities not considered in the analysis. The value of 80 or 85 percent of the nominal strength is meant to approximate an axial load strength at an $e/h$ ratio of approximately 0.10. Since the columns in this study have been prepared and tested in a controlled laboratory environment under short time loading, this upper limit was not used for the comparison.

### 4.1.3 Computer Analysis of Cross-Section Capacity

The computer program developed for this study uses the equations and assumptions as outlined in the previous section to compute the cross-section axial load-bending moment interaction curve. The analysis procedure used and presented in this section is summarized in the flow chart in Figure 4.4. For determining the cross-section strength, ACI 318-95 uses the same analysis approach for both reinforced concrete and composite steel-concrete columns.

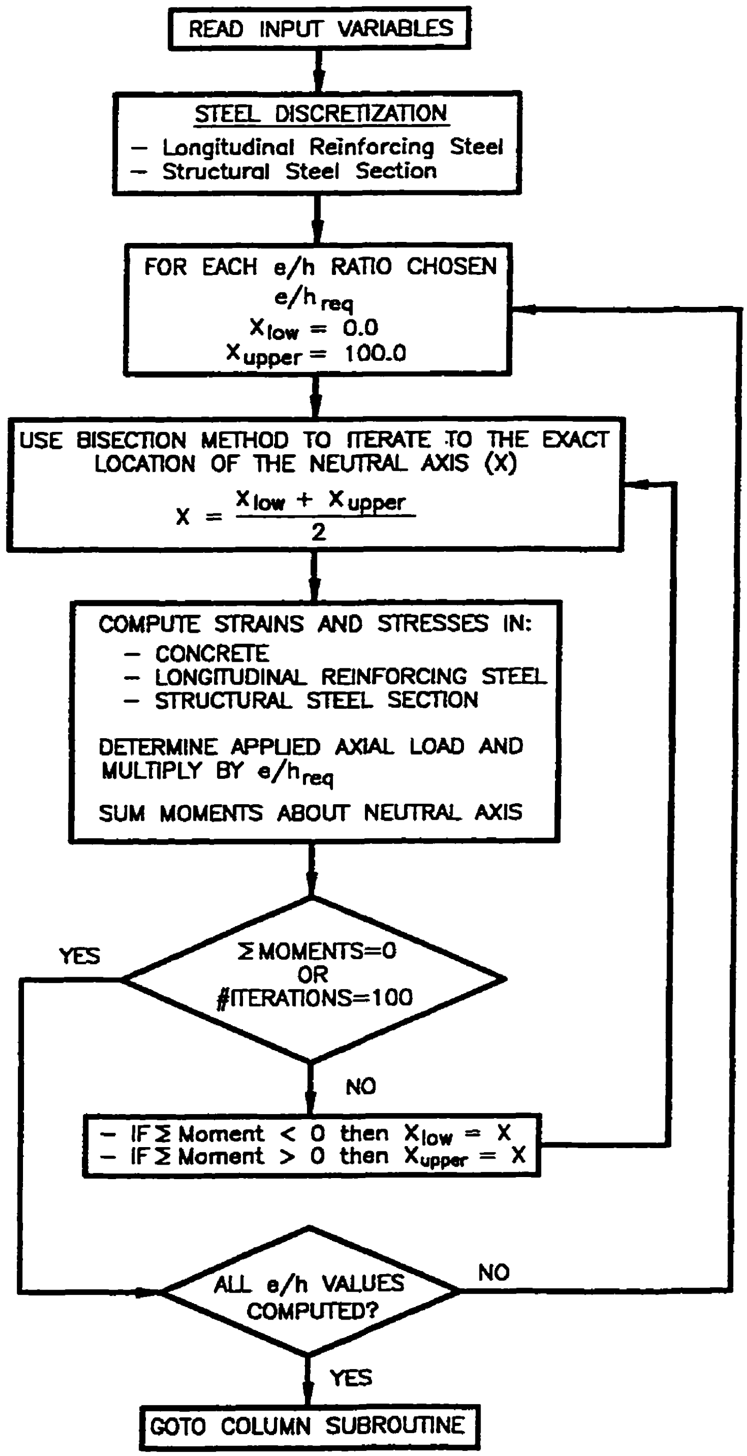
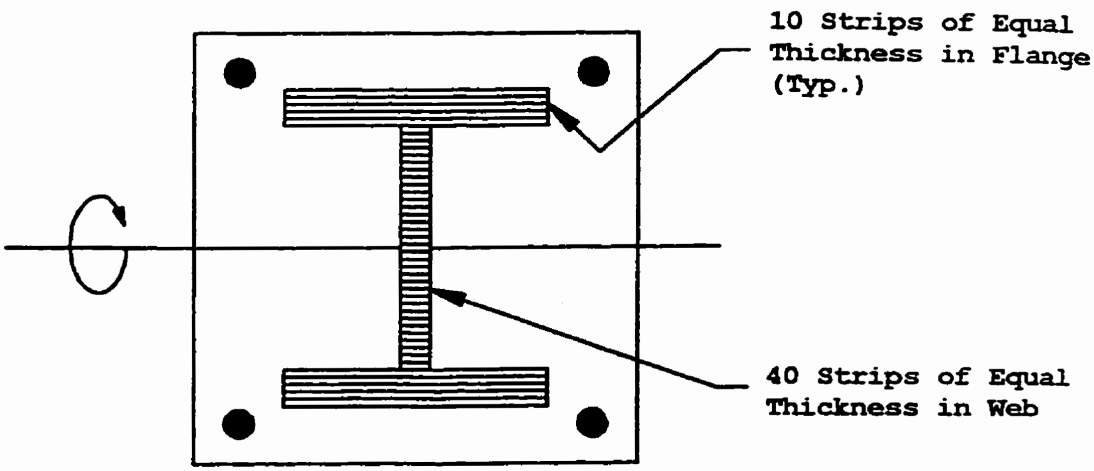


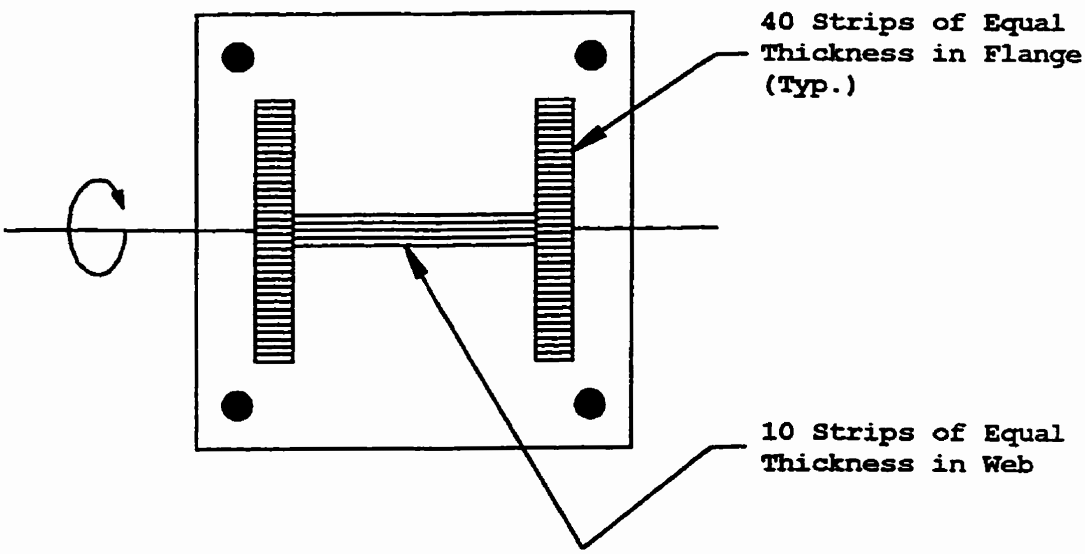

Figure 4.4 - ACI cross-section interaction curve flowchart.

Therefore, to avoid repetition, both column types will be discussed together.

The cross-section can consist of up to three materials (concrete, longitudinal reinforcing steel and structural steel), each possessing a unique stress-strain relationship. In order to distinguish among these three materials in the analysis, the cross-section was first discretized. Since ACI 318-95 uses an equivalent rectangular stress block, the concrete was not discretized. No distinction is made between the concrete outside the transverse reinforcement (unconfined concrete) and the concrete inside the transverse reinforcement (confined concrete). Each longitudinal reinforcing bar was represented by one element with a specified area and distance from the plastic neutral axis (measured perpendicular to the axis of bending). The structural steel section required the discretization of both the flanges and web (Figure 4.5). For major axis bending, the flanges were discretized into 10 strips with the element width being equal to the flange width. The web was divided into 40 strips with the element width being equal to the web thickness. For minor axis bending, the flanges were discretized into 40 strips with the element width being equal to the flange thickness. The web was divided into 10 strips with the element width being equal to the web depth. The area of each structural steel element and the distance from the plastic neutral axis to the centroid of the




(a) Structural steel section for composite columns subjected to major axis bending.




(b) Structural steel section for composite columns subjected to minor axis bending.

Figure 4.5 - Discretization of structural steel section for composite columns. (The number of strips was doubled for computing the Eurocode 4 strength)

element (measured perpendicular to the axis of bending) were computed.

The cross-section axial load-bending moment interaction curve represents points of axial load and corresponding bending moment that ensure strain compatibility and also satisfy conditions of equilibrium. For this study, the cross-section interaction curve was represented by 102 points. To ensure the points on the interaction curve were equally distributed along its length, the criterion used in determining the points was based on the end eccentricity ratio ($e/h$). The first point was determined for $e/h$=0 (pure axial load) and the last point was determined for $e/h=\infty$ (pure bending). The remaining 100 points were distributed along the cross-section interaction curve.

For the analysis procedure, a lower limit ($X_{low}$) and upper limit ($X_{upper}$) on the distance of the neutral axis from the plastic centroid ($X$) was established (Figure 4.6). Using a maximum concrete strain of 0.003 and a given location of the neutral axis, the strains and stresses in the concrete, longitudinal reinforcing steel and structural steel can be computed (Figure 4.2). The location of the neutral axis was varied and the bisection method was used to converge to a solution (for $X$) for each of the required end eccentricity ratios ($e/h_{req}$). The convergence

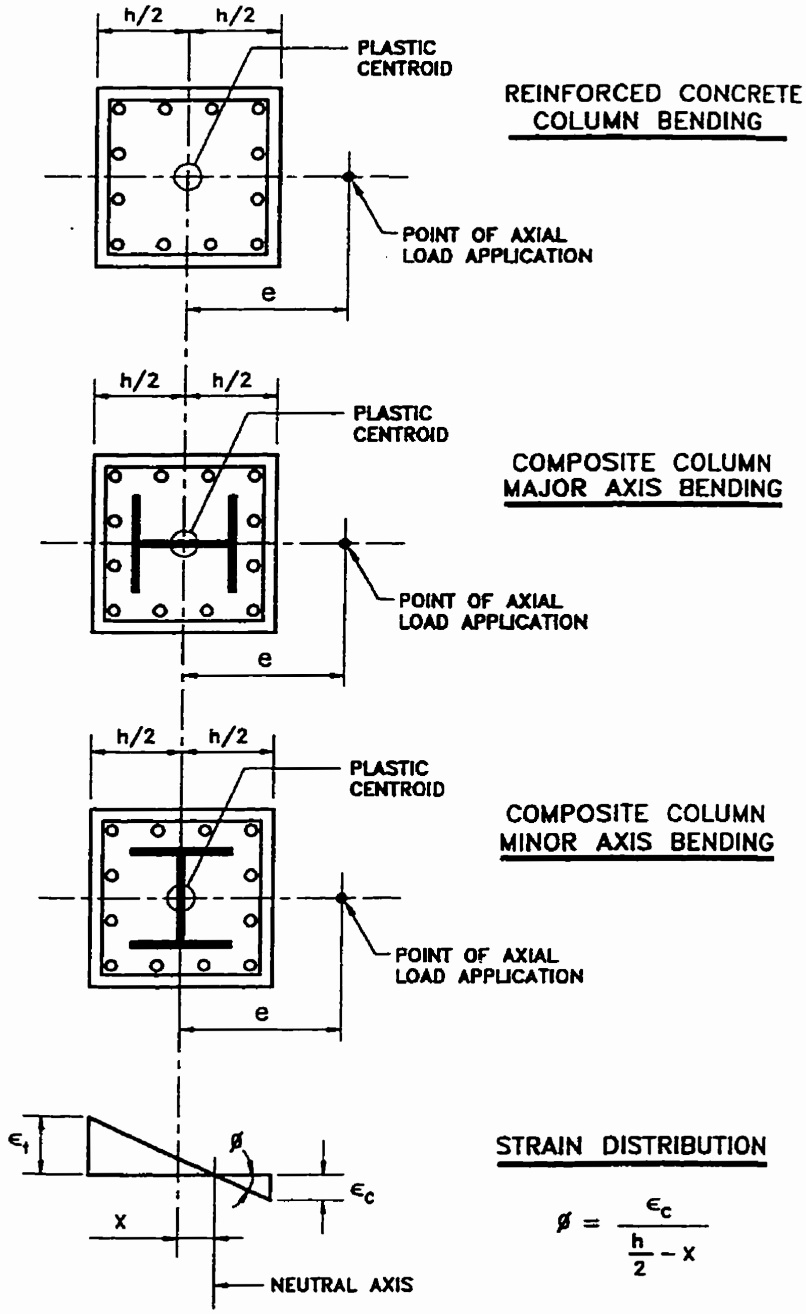



Figure 4.6 - Strain distribution for columns studied.

tolerance was limited to 100 iterations for numerical stability.

For a given location of the neutral axis, the depth of the equivalent rectangular stress block was computed using Equations 4.1 to 4.3. The resulting concrete compressive force was then established. Using the assumption that the strain is linearly proportional to the distance from the neutral axis, the strain in each of the longitudinal reinforcing steel bars and structural steel elements was computed. The steel was assumed to be elastic-perfectly plastic, as discussed in the previous section, and residual stresses in the structural steel section were ignored. Figure 4.2 shows a linear-elastic stress distribution for the structural steel section, however, for different positions of the neutral axis, the stress distribution may be elastic-plastic. For steel elements within the equivalent rectangular stress block, the displaced area of concrete was considered. The resulting force in each of the steel elements was then determined.

To determine the applied axial load for the given location of the neutral axis, the concrete compressive force was added to the summation of all steel element forces. The net value is, therefore, equal to the applied axial load on the section. Finally, a sum of moments of forces about the neutral axis was performed. The distance from the applied axial load to the plastic centroid was assumed to be equal to the required end eccentricity. If the actual end eccentricity ratio ($e/h_{actual}$) is equal to the required end eccentricity ratio ($e/h_{req}$), the sum of moments about the neutral axis will equal zero. If the summation does not equal zero, the location of the neutral axis is not correct and another iteration is required. To obtain a solution using the bisection method the following conditions apply (Figure 4.4): (a) if the summation is less then zero, $X_{low}=X$, and (b) if the summation is greater than zero, $X_{upper}=X$.

Once the 102 points on the cross-section axial load-bending moment interaction curve were computed, the analysis of the column capacity was performed.

### 4.1.4 Calculation of Column Capacity

For short columns, the column capacity is equal to the cross-section capacity. For slender columns, ACI 318-95 permits the use of a moment magnifier approach to determine the column capacity. This approach uses the axial load obtained from a first-order elastic analysis and a magnified moment that includes the second-order effects caused by the lateral displacement of the column.

For columns braced against sway, ACI 318-95 defines a limit between short and slender columns. Slenderness effects can be neglected (i.e. the column is a short column) if Equation 4.5 is satisfied:

$$\frac{k\ell_u}{r} \leq 34 - 12\frac{M_1}{M_2} \tag{4.5}$$

where $k$ is the effective length factor; $\ell_u$ is the unsupported length of the column; $r$ is the radius of gyration; $M_1$ is the smaller end moment and is positive if the column is bent in single curvature and negative if bent in double curvature; and $M_2$ is the larger end moment, always positive. For pin-ended columns, $k$ is equal to 1.0. ACI 318-95 defines the radius of gyration, $r$, for rectangular columns as equal to 0.3 times the overall dimension in the direction stability is being considered. For composite steel-concrete columns, there is a further limitation that the radius of gyration shall not be greater than the value computed by Equation 4.6:

$$r_{max} = \sqrt{\frac{(0.2E_cI_g) + E_sI_t}{(0.2E_cA_g) + E_sA_t}} \tag{4.6}$$

where $E_c$ is the modulus of elasticity of concrete; $I_g$ is the moment of inertia of the gross concrete cross-section about the centroidal axis; $I_t$ is the moment of inertia of the structural steel shape about the centroidal axis of the composite member; and $A_t$ is the area of the structural steel shape. ACI 318-95 provides Equation 4.7 to calculate the modulus of elasticity of concrete:

$$E_c = 57000\sqrt{f'_c} \tag{4.7}$$

For SI conversion replace 57000 by 4733 in Equation 4.7. For values of $k\ell_u/r$ greater than 100, the moment magnifier approach is not permitted by ACI 318-95. However, this upper limit on $k\ell_u/r$ was not used for the comparative study presented in the later part of this report. Equation 4.8 is used by ACI 318-95 to calculate the magnified moment:

$$M_c = \delta_{ns} M_2 \tag{4.8}$$

where $M_c$ is the moment to be used for design of the compression member; and $\delta_{ns}$ is the moment magnification factor for columns in frames braced against sway and is computed using Equation 4.9:

$$\delta_{ns} = \frac{C_m}{1 - \dfrac{P_u}{0.75 P_c}} \geq 1.0 \tag{4.9}$$

where $C_m$ is an equivalent moment diagram factor; $P_u$ is the design axial load for the given eccentricity; and $P_c$ is the critical column load. The coefficient of 0.75 in Equation 4.9 represents a stiffness reduction factor which was set equal to unity for this study. Hence, Equation 4.9 is modified to:

$$\delta_{ns} = \frac{C_m}{1 - \dfrac{P_u}{P_c}} \geq 1.0 \tag{4.10}$$

The equivalent moment diagram factor is used to account for moment gradients in the column and is calculated using Equation 4.11:

$$C_m = 0.6 + 0.4\frac{M_1}{M_2} \geq 0.4 \tag{4.11}$$

The critical column load used in Equations 4.9 and 4.10 is computed using Equation 4.12.

$$P_c = \frac{\pi^2 EI}{(k\ell_u)^2} \tag{4.12}$$

where *EI* is the flexural rigidity of the compression member. The ACI moment magnifier approach is strongly influenced by the flexural rigidity, *EI*, of the column which varies due to cracking, creep, and nonlinearity of the concrete stress-strain curve, among other factors. ACI 318-95 provides Equations 4.13 and 4.14 for calculating the flexural rigidity of a reinforced concrete column:

$$EI = \frac{0.4E_cI_g}{1+\beta_d} \tag{4.13}$$

$$EI = \frac{(0.2E_cI_g + E_sI_{se})}{1+\beta_d} \tag{4.14}$$

where $I_{se}$ is the moment of inertia of the reinforcing steel about the centroidal axis; and $\beta_d$ is the ratio of maximum axial dead load to total axial load. For composite steel-concrete columns, *EI* can be taken as either the value obtained from Equation 4.13 or the value computed by Equation 4.15:

$$EI = \frac{(0.2E_cI_g)}{1+\beta_d} + E_sI_t \tag{4.15}$$

In this study, short-term loads are used. Hence, the ratio of maximum axial dead load to total axial load, $\beta_d$, is equal to zero. This results in the following simplified equations for the calculation of flexural rigidity according to ACI 318-95:

(i) for both reinforced concrete and composite steel-concrete columns:

$$EI = 0.4E_cI_g \tag{4.16}$$

(ii) for reinforced concrete columns:

$$EI = 0.2E_cI_g + E_sI_{se} \tag{4.17}$$

(iii) for composite steel-concrete columns:

$$EI = 0.2E_cI_g + E_sI_t \tag{4.18}$$

In an attempt to take into account the cracking and nonlinearity of the concrete stress-strain curve in determining *EI*, Mirza (1990) proposed the following design equation for calculating the flexural rigidity for reinforced concrete columns:

$$EI = (0.3 - 0.3e/h)E_c I_g + E_s I_{se} \geq E_s I_{se} \quad (4.19)$$

The use of Equation 4.19 is subject to the following limitations:

$$f'_c \leq 6000 psi$$
$$\rho_{rs} \geq 1\%$$
$$\ell / h \leq 30$$
$$e / h \geq 0.1$$

Similarly, Tikka and Mirza (1992) proposed the following design equation for calculating the flexural rigidity of composite steel-concrete columns:

$$EI = (0.3 - 0.2e/h)E_c\left(I_g - I_t\right) + 0.8E_s\left(I_t + I_{se}\right) \geq E_s I_t \quad (4.20)$$

The use of Equation 4.20 is subject to the following limitations:

$$f'_c \leq 8000 psi$$
$$\rho_{rs} \geq 1\%$$
$$4\% \leq \rho_{ss} \leq 10\%$$
$$\ell / h \leq 30$$
$$e / h \geq 0.1$$

Both Equations 4.19 and 4.20 were statistically developed from a theoretical computer analysis of approximately 9500 and 12000 columns, respectively. The influence of a full range of variables on the flexural rigidity of slender tied columns was undertaken in developing these equations.

For the comparative study, the ACI slender column strengths were computed in three different ways:

- using Equations 4.16, 4.17, and 4.19 for reinforced concrete columns; and
- using Equations 4.16, 4.18, and 4.20 for composite columns.

Many column test specimens did not meet some of the limits related to Equation 4.19 and 4.20 noted in the previous paragraph. In such cases, however, these limits were ignored when computing the ACI (and CSA) strengths using Equation 4.19 or 4.20.

### 4.1.5 Computer Analysis of Column Capacity

If a column is defined as being short, that is if Equation 4.5 is satisfied, the cross-section and column axial load-bending moment interaction curves are equal. When Equation 4.5 is not satisfied, slenderness effects are considered using the moment magnifier approach. The computer program developed for this study uses the equations given in the previous section to compute the ACI 318-95 column axial load-bending moment interaction curve. The analysis procedure used and presented in this section is summarized in the flow chart in Figure 4.7.

The column axial load-bending moment interaction curve is developed from the cross-section axial load-bending moment interaction curve. The computer program stores the

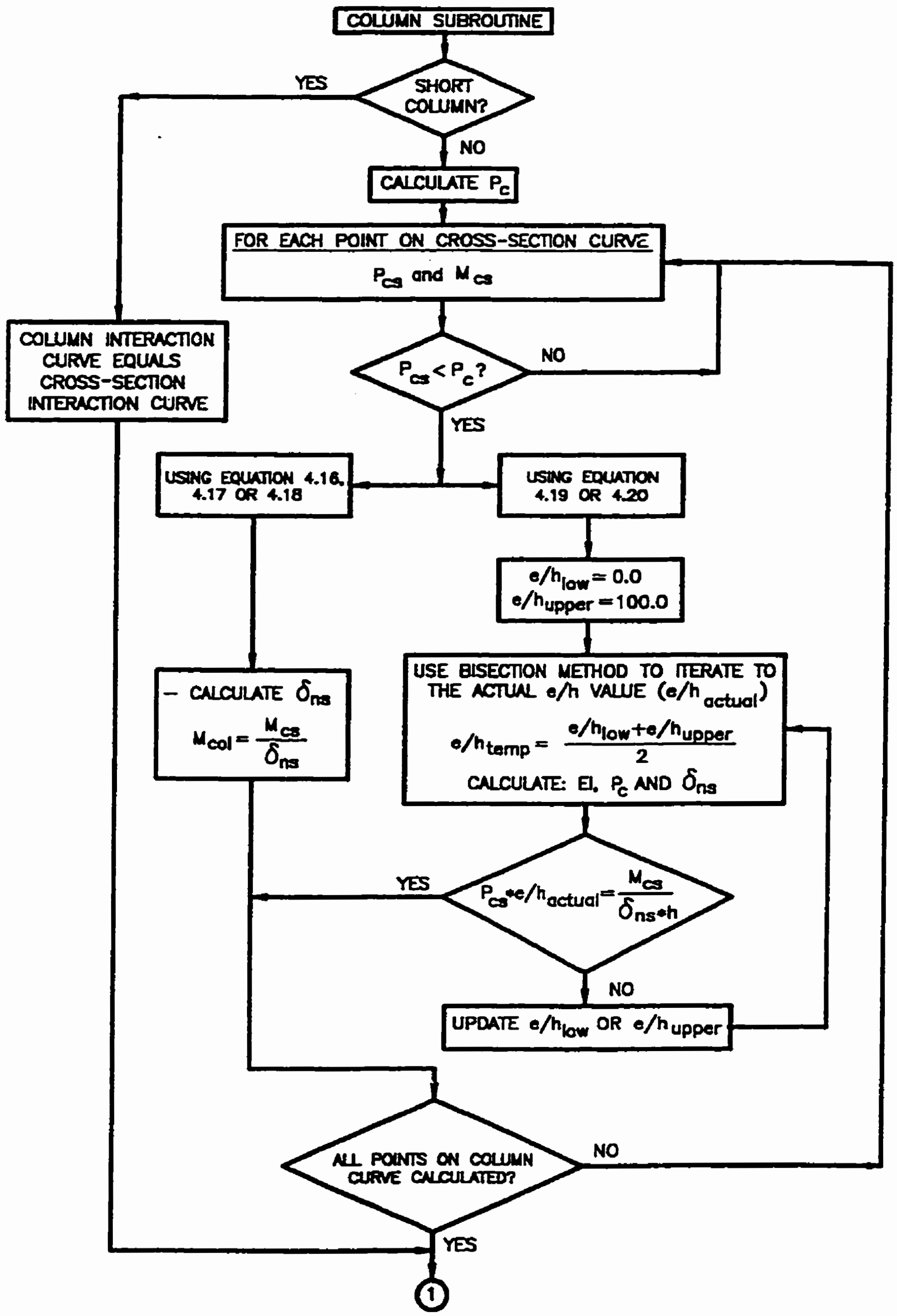



Figure 4.7 - ACI column interaction curve flowchart.

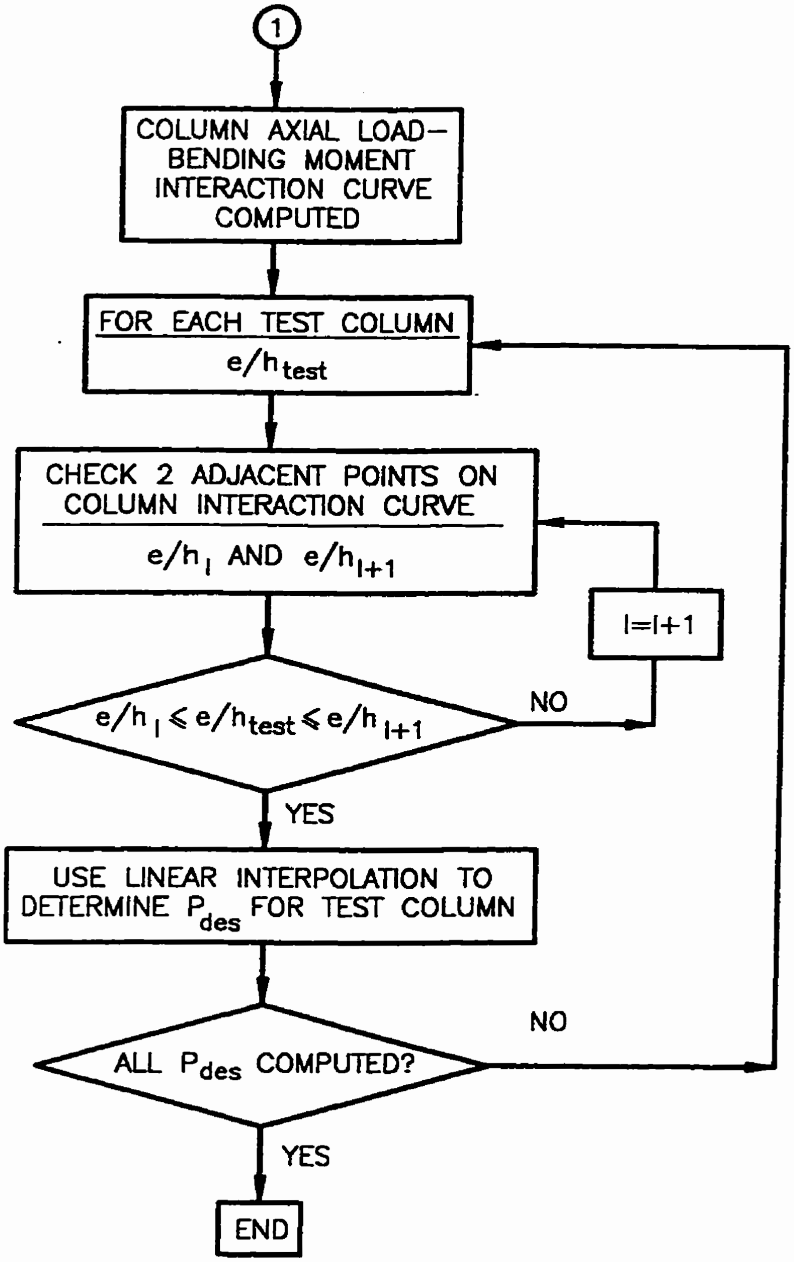



Figure 4.7 (continued) - ACI column interaction curve flowchart.

values of the axial load and corresponding bending moment for each of the 102 points used to define the cross-section interaction curve. The only difference between the cross-section and column interaction curve for a given axial load level is the moment capacity (Figure 4.3). The relationship between the moment capacity of the cross-section and column is represented by Equation 4.8 where $M_C$ is the cross-section moment capacity and $M_2$ is the column moment capacity. Since the cross-section moment capacity, $M_C$, has been previously calculated and stored, the column moment capacity can be obtained by simply dividing $M_C$ by the moment magnification factor, $\delta_{ns}$. The corresponding axial load and column moment capacity represent one point on the column interaction curve.

The critical column load, $P_C$, is first calculated using Equation 4.12 (for $e/h$=0) and compared to the pure axial load capacity of the column cross-section. The pure axial load capacity of the column cross-section is calculated using $P_{n(max)}/0.8$ for reinforced concrete columns and $P_{n(max)}/0.85$ for composite steel-concrete columns, where $P_{n(max)}$ is taken from Equation 4.4 and the related description. The lower of the two values (critical column load and pure axial load capacity of the column cross-section) was used to establish the column pure axial load capacity. Any points on the cross-section interaction curve that are greater than the column pure axial load capacity are not considered in developing the column interaction curve. For points with axial loads less than the column pure axial load capacity, each cross-section moment capacity ($M_C$) is divided by the moment magnification factor ($\delta_{ns}$) to obtain the column moment capacity.

The procedure as outlined in the previous paragraph is applicable to ACI 318-95 using Equations 4.16, 4.17 and 4.18. For ACI 318-95 using Equations 4.19 and 4.20, a more complex analysis approach is required, as indicated in Figure 4.7. The reason for this is that these flexural rigidity ($EI$) equations are dependent on the end eccentricity ratio ($e/h$) of the column. The variable flexural rigidity affects the column critical load ($P_C$) which in turn affects the moment magnification factor ($\delta_{ns}$). Therefore, an iterative approach similar to that used for developing the cross-section interaction curve was used. The bisection method was used to iterate to a temporary end eccentricity ratio ($e/h_{temp}$) of the column such that Equation 4.8 is satisfied. By establishing a temporary end eccentricity ratio ($e/h_{temp}$), the flexural rigidity could be calculated and the resulting critical column load and moment magnification factor can be determined. The column moment capacity is calculated by dividing the cross-section moment by the moment magnification factor. The actual end eccentricity ratio

($e/h_{actual}$) is computed by dividing the column moment capacity by the axial load capacity and the depth of the column ($h$). If the temporary end eccentricity ratio ($e/h_{temp}$) is equal to the actual end eccentricity ratio ($e/h_{actual}$), the correct solution has been obtained, otherwise, further iterations are required. The convergence tolerance was limited to 25 iterations for numerical stability.

Once the column axial load-bending moment interaction curve was computed, the design axial load ($P_{des}$) of the column could be determined. The end eccentricity ratio of the test column ($e/h_{test}$) was successively compared to the end eccentricity ratio of two adjacent points on the column interaction curve (Figure 4.8). Once the test column end eccentricity ratio ($e/h_{test}$) fell between two adjacent points, linear interpolation was used to calculate the design axial load ($P_{des}$) of the test column.

## 4.2 AISC-LRFD PROVISIONS

The design equations given in the AISC-LRFD Specifications (1994) for the design of composite steel-concrete columns are discussed in this section. As the refined equation for the radius of gyration of composite steel-concrete columns was developed for AISC-LRFD Specifications, this equation is also discussed here. The AISC-LRFD Specifications have no provisions for the design

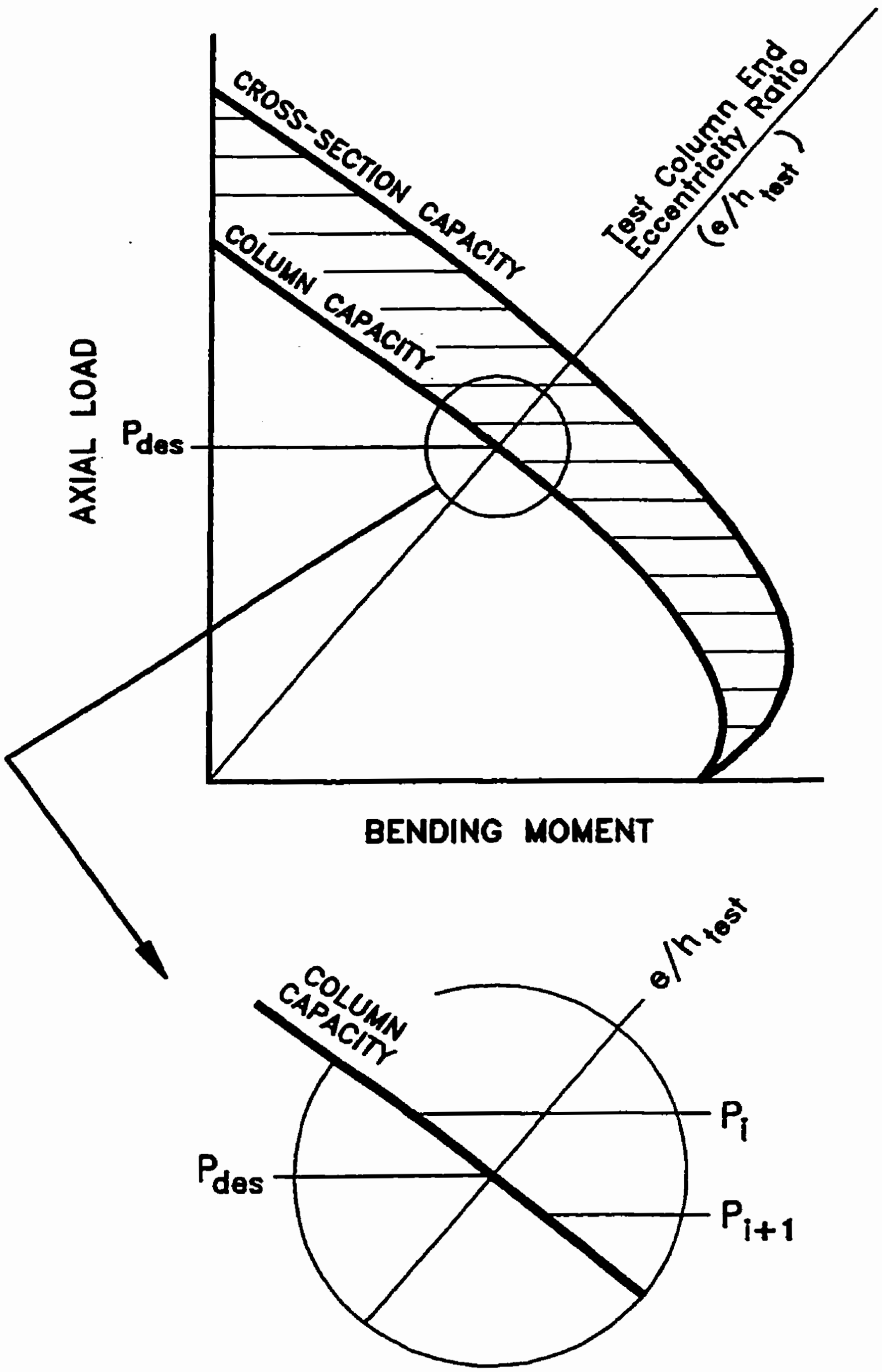



Figure 4.8 - Linear interpolation technique for determining $P_{des}$.

of reinforced concrete columns. The AISC-LRFD approach involves converting the composite cross-section into an equivalent steel cross-section. Once converted, the column is designed using the AISC-LRFD design equations for steel columns.

### 4.2.1 Limitations of the AISC-LRFD Provisions

For the design of composite steel-concrete columns, the AISC-LRFD Specifications (1994) have several limitations that must be checked. To qualify as a composite column, the following limitations must be satisfied:

- The cross-sectional area of the structural steel shape must be at least four percent of the total composite cross-section.
- The spacing of ties must not exceed two-thirds the least dimension of the composite cross-section.
- The cross-sectional area of the tie reinforcement shall be at least 0.007 square inch per inch of tie spacing.
- The encasement must provide at least 1.5 inches of clear cover outside of both the tie and longitudinal reinforcement.
- Concrete must have a specified compressive strength, $f'_c$, of not less than 3000 psi nor more than 8000 psi for normal weight concrete.

- The specified minimum yield stress of structural steel and reinforcing bars used in calculating the strength of the composite column shall not exceed 55,000 psi.

For many column test specimens used for the comparative study, the limits noted above were not satisfied. For such cases, however, these limits were ignored when computing the AISC-LRFD strengths.

### 4.2.2 Calculation of the Cross-Section Capacity

The AISC-LRFD Specifications limit the strength interaction of structural steel sections subjected to axial load and bending moment according to Equations 4.21 and 4.22:

For $\frac{P_u}{P_n} \geq 0.2$

$$\frac{P_u}{P_n} + \frac{8}{9}\left(\frac{M_{ux}}{M_{nx}} + \frac{M_{uy}}{M_{ny}}\right) \leq 1.0 \qquad (4.21)$$

For $\frac{P_u}{P_n} < 0.2$

$$\frac{P_u}{2P_n} + \left(\frac{M_{ux}}{M_{nx}} + \frac{M_{uy}}{M_{ny}}\right) \leq 1.0 \qquad (4.22)$$

where $P_u$ is the required compressive strength; $P_n$ is the nominal compressive strength without bending moment; $M_u$ is the required flexural strength; $M_n$ is the nominal flexural strength without axial load; and the subscripts $x$ and $y$ refer to strong and weak axis bending, respectively. In this study, bending moments about the strong and weak axes are considered separately which leads to the following simplified equations:

For $\frac{P_u}{P_n} \geq 0.2$

$$\frac{P_u}{P_n} + \frac{8}{9}\left(\frac{M_u}{M_n}\right) \leq 1.0 \tag{4.23}$$

For $\frac{P_u}{P_n} < 0.2$

$$\frac{P_u}{2P_n} + \left(\frac{M_u}{M_n}\right) \leq 1.0 \tag{4.24}$$

Equations 4.23 and 4.24 apply to steel sections and modified composite steel-concrete cross-sections. The modifications for composite steel-concrete cross-sections will be discussed later. Essentially, Equation 4.23 and 4.24 can be used to describe the axial load-bending moment interaction curve for a column of any length. Equation 4.25 is used to determine the nominal compressive strength, $P_n$, of the column of any length, including the cross-section:

$$P_n = A_g F_{cr} \tag{4.25}$$

where $A_g$ is the gross area of the steel shape; and $F_{cr}$ is the critical buckling stress which is determined by using Equations 4.26 or 4.27:

For $\lambda_c \leq 1.5$

$$F_{cr} = \left(0.658^{\lambda_c^2}\right) F_y \tag{4.26}$$

For $\lambda_c > 1.5$

$$F_{cr} = \left(\frac{0.877}{\lambda_c^2}\right) F_y \tag{4.27}$$

where

$$\lambda_c = \frac{K\ell}{r\pi}\sqrt{\frac{F_y}{E}} \tag{4.28}$$

where $F_y$ is the specified yield stress of the steel section; $\lambda_c$ is the column slenderness parameter; $E$ is the modulus of elasticity of the steel section; $K$ is the effective length factor; $\ell$ is the laterally unbraced length of the column; and $r$ is the governing radius of gyration about the axis of buckling. In this study the effective length factor, $K$, is equal to 1.0.

The above equations apply to steel columns and must be modified to incorporate the design of composite steel-concrete columns. The first modification involves determining a modified radius of gyration, $r_m$, that replaces $r$ in Equation 4.28. The modified radius of gyration is equal to the radius of gyration of the steel shape except that it shall not be less than 0.3 times the overall depth of the composite cross-section in the plane of buckling.

The second modification involves determining a modified yield stress, $F_{my}$, that replaces the yield stress, $F_y$, in Equations 4.26, 4.27 and 4.28. The modified yield stress is computed using Equation 4.29:

$$F_{my} = F_y + c_1 F_{yr}\left(A_r / A_s\right) + c_2 f'_c\left(A_c / A_s\right) \tag{4.29}$$

where $F_{yr}$ is the specified minimum yield stress of the longitudinal reinforcing bars; $A_r$ is the area of the longitudinal reinforcing bars; $A_s$ is the area of the steel section; $A_c$ is the area of concrete; and $c_1$ and $c_2$ are numerical coefficients equal to 0.7 and 0.6, respectively.

The third modification involves determining a modified modulus of elasticity, $E_m$, that replaces the modulus of elasticity, $E$, in Equation 4.28. The modified modulus of elasticity is computed using Equation 4.30:

$$E_m = E + c_3 E_c\left(A_c / A_s\right) \tag{4.30}$$

where $E_c$ is the modulus of elasticity of concrete; and $c_3$ is a numerical coefficient equal to 0.2. The modulus of elasticity of concrete is calculated using Equation 4.31:

$$E_c = w^{1.5}\sqrt{f'_c} \tag{4.31}$$

where $E_c$ and $f'_c$ are in ksi; $w$ is the unit weight of concrete and for this study was assumed to be 145 lbs/ft$^3$. For SI conversion use $w$ equal to 2350 kg/m$^3$ and divide Equation 4.31 by 25.

The nominal flexural strength of the column is determined from a plastic stress distribution on the composite cross-section. The AISC-LRFD Specifications provide Equation 4.32 as an approximate simplified method for determining the nominal flexural strength:

$$M_n = ZF_y + \frac{1}{3}\left(h_2 - 2c_r\right)A_rF_{yr} + \left(\frac{h_2}{2} - \frac{A_wF_y}{1.7f'_c h_1}\right)A_wF_y \qquad (4.32)$$

where $Z$ is the plastic section modulus of the steel section; $h_1$ is the depth of the composite cross-section perpendicular to the plane of bending; $h_2$ is the width of the composite cross-section parallel to the plane of bending; $c_r$ is the average distance from the compression face to longitudinal reinforcement near that face and the distance from the tension face to the longitudinal reinforcement near that face; and $A_w$ is the web area of the encased steel shape.

When determining the cross-section capacity, the column length, $\ell$, is set equal to zero in Equation 4.28. Equations 4.23 and 4.24 can then be used to determine the axial load-bending moment interaction curve for the composite cross-section.

### 4.2.3 Calculation of Column Capacity

As previously stated, Equations 4.23 and 4.24 are used for computing both the cross-section and column axial load-bending moment interaction curves. The nominal compressive strength, $P_n$, and the required flexural strength, $M_u$, must be modified to account for length effects.

In determining the nominal compressive strength of the column for the case of pure axial load, the actual column length is substituted into Equation 4.28. In determining the required flexural strength, the moments must be modified to account for second-order effects using a moment magnifier approach. The AISC-LRFD Specifications provide Equation 4.33 for computing the magnified moments:

$$M_u = B_1 M_{nt} + B_2 M_{lt} \tag{4.33}$$

where $B_1$ is a moment magnification factor for non-sway moments; $M_{nt}$ is the required flexural strength in the member assuming there is no lateral translation of the frame; $B_2$ is a moment magnification factor for sway moments only; and $M_{lt}$ is the required flexural strength in the member as a result of lateral translation of the frame only. In this study, the columns are considered to be braced against sway. Therefore, Equation 4.33 can be simplified to:

$$M_u = B_1 M_{nt} \tag{4.34}$$

The moment magnification factor, $B_1$, is computed using Equation 4.35:

$$B_1 = \frac{C_m}{1 - \frac{P_u}{P_{e1}}} \geq 1.0 \tag{4.35}$$

where $C_m$ is a factor that accounts for moment gradients in the column and is calculated using Equation 4.36:

$$C_m = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) \tag{4.36}$$

where $M_1/M_2$ is the ratio of smaller to larger end moments and is positive when the column is bent in reverse curvature and negative when bent in single curvature. The elastic buckling load, $P_{e1}$, is determined from Equation 4.37:

$$P_{e1} = \frac{A_s F_{my}}{\lambda_c^2} \tag{4.37}$$

The column slenderness parameter, $\lambda_c$, in Equation 4.37 is calculated from Equation 4.28 where the effective length factor, $K$, and the radius of gyration, $r_m$, are to be taken in the plane of bending being considered.

Once the nominal compressive strength and the required flexural strength have been modified for length effects and second-order effects, respectively, Equations 4.23 and 4.24 are used to determine whether the composite column is able to resist the applied loading.

**4.2.4 Proposed Modification to AISC-LRFD Provisions**

Tikka and Mirza (1992) examined the AISC-LRFD Specifications using a full range of variables that affect composite steel-concrete column strength. They concluded that the AISC-LRFD method produces a safe design for columns subjected to major axis bending but is, in some cases, unconservative when designing columns subjected to minor axis bending. The unconservative design was found to occur in columns with low reinforcing steel ratios, $\rho_{rs}$.

In this study, the AISC-LRFD Specifications were again examined in an attempt to determine the cause of the unconservative design for composite columns subjected to minor axis bending. After a more comprehensive parametric study using the same variables as those used by Tikka and Mirza (1992), it was further found that the AISC-LRFD Specifications tend to produce unconservative designs for composite steel-concrete columns subjected to minor axis bending when the following variables increase:

- *structural steel index,* $\rho_{ss}$

- *end eccentricity ratio, e/h*

- *slenderness ratio,* $\ell/h$

- *the yield stress of the steel section,* $F_y$

and when the following variables decrease:

- *reinforcing steel index,* $\rho_{rs}$

- *concrete strength,* $f'_c$

Through an extensive evaluation of the AISC-LRFD Specifications, it was found that the computed value of the radius of gyration significantly affected the design strength of a composite column. It was also interesting to note that the AISC-LRFD method has no upper limit on the usable radius of gyration as does ACI 318-95 (Equation 4.6). Several equations for the radius of gyration of composite columns were examined and the following equation is proposed:

$$r_{max} = \sqrt{\frac{0.3E_c I_g + 0.8E_s\left(I_{ss} + I_{rs}\right)}{0.3E_c A_g + 0.8E_s\left(A_s + A_r\right)}} \qquad (4.38)$$

where $I_g$ is the moment of inertia of the gross composite cross-section; $I_{ss}$ is the moment of inertia of the steel section; $I_{rs}$ is the moment of inertia of the longitudinal reinforcing bars; and $A_g$ is the gross area of the composite section. The use of Equation 4.38 is subject to the following limitations:

$$f'_c \leq 8000 psi$$
$$\rho_{rs} \geq 1\%$$
$$4\% \leq \rho_{ss} \leq 10\%$$
$$\ell / h \leq 30$$
$$e / h \geq 0.1$$

Many column test specimens did not meet some of these limits related to Equation 4.38. However, these limits were ignored when computing the AISC-LRFD strengths using Equation 4.38.

Equation 4.38 was developed based on a theoretical study of 11880 columns bending about the major axis and 11880 columns bending about the minor axis. These are the same columns studied by Tikka and Mirza (1992). Equation 4.38 was chosen because it is similar to Equation 4.6 used by ACI 318-95 (1995). Equation 4.38 has little effect on the prediction of column strength when the column is subject to bending about the major axis of the steel section but improves the prediction of column strength when the bending is applied about the minor axis of the steel section. This is consistent with the conclusions of Tikka and Mirza (1992), where the AISC-LRFD Specifications were found to be unconservative for minor axis bending and conservative for major axis bending. The use of Equation 4.38 does not require any changes to the design approach of AISC-LRFD Specifications, except that the existing definition of the radius of gyration is proposed to be modified to include an upper limit to be used for composite steel-concrete column design.

Equation 4.38 allows the inclusion of the longitudinal reinforcing steel bars in determining the moment of inertia of the steel in the composite cross-section as opposed to ACI 318-95 which does not permit this. The 0.8 coefficient related to the steel contribution indicates "softening" of reinforcing and structural steel and is the result of the elastic-plastic nature of the stresses developed in the reinforcing and structural steel at ultimate load. This softening effect was also observed by Tikka and Mirza (1992).

For the comparative study, the AISC-LRFD column strengths were computed in two different ways:

- using the radius of gyration as specified in the AISC-LRFD Specifications; and
- using the upper limit of Equation 4.38 on the radius of gyration.

### 4.2.5 Computer Analysis of Column Capacity

The computer program developed for this study uses the equations and assumptions given in the previous sections to compute the design axial load ($P_{des}$) of each composite test column. The analysis procedure used is presented in this section.

Instead of generating the cross-section and column axial load-bending moment interaction curves and interpolating for the given test column end eccentricity ratio ($e/h_{test}$), as was done for ACI 318-95, a closed form solution was used. Substituting Equations 4.34 and 4.35 into Equations 4.23 and 4.24 yields:

For $\frac{P_u}{P_n} \geq 0.2$

$$\frac{P_u}{P_n} + \frac{8}{9}\left( \frac{M_{nt}C_m}{M_n\left(1 - \frac{P_u}{P_{e1}}\right)} \right) = 1.0 \tag{4.39}$$

For $\frac{P_u}{P_n} < 0.2$

$$\frac{P_u}{2P_n} + \left( \frac{M_{nt}C_m}{M_n\left(1 - \frac{P_u}{P_{e1}}\right)} \right) = 1.0 \tag{4.40}$$

In the present form, Equations 4.39 and 4.40 can not be solved directly since each equation has two unknowns, $M_{nt}$ and $P_u$ (equal to $P_{des}$). However, the test column end eccentricity ratio ($e/h_{test}$) is known and the value of $M_{nt}$ is equal to $P_u$ times $e_{test}$ which leaves only one unknown variable ($P_u$) in Equations 4.41 and 4.42:

For $\frac{P_u}{P_n} \geq 0.2$

$$\frac{P_u}{P_n} + \frac{8}{9}\left( \frac{P_u e_{test} C_m}{M_n\left(1 - \frac{P_u}{P_{e1}}\right)} \right) = 1.0 \tag{4.41}$$

For $\frac{P_u}{P_n} < 0.2$

$$\frac{P_u}{2P_n} + \left( \frac{P_u e_{test} C_m}{M_n\left(1 - \frac{P_u}{P_{e1}}\right)} \right) = 1.0 \tag{4.42}$$

Both sides of Equations 4.41 and 4.42 were then multiplied by $(1-P_u/P_{e1})$ to give:

For $\frac{P_u}{P_n} \geq 0.2$

$$\frac{P_u\left(1 - \frac{P_u}{P_{e1}}\right)}{P_n} + \frac{8 P_u e_{test} C_m}{9 M_n} = \left(1 - \frac{P_u}{P_{e1}}\right) \tag{4.43}$$

For $\frac{P_u}{P_n} < 0.2$

$$\frac{P_u\left(1 - \frac{P_u}{P_{e1}}\right)}{2P_n} + \frac{P_u e_{test} C_m}{M_n} = \left(1 - \frac{P_u}{P_{e1}}\right) \tag{4.44}$$

Multiplying through Equation 4.43 by $-(P_n P_{e1})$ and Equation 4.44 by $-(2P_n P_{e1})$, rearranging and gathering terms of $P_u$ results in the following expressions:

For $\frac{P_u}{P_n} \geq 0.2$

$$P_u^2 - \left(P_{e1} + \frac{8P_n P_{e1} e_{test} C_m}{9M_n} + P_n\right)P_u + P_n P_{e1} = 0 \tag{4.45}$$

For $\frac{P_u}{P_n} < 0.2$

$$P_u^2 - \left(P_{e1} + \frac{2P_n P_{e1} e_{test} C_m}{M_n} + 2P_n\right)P_u + 2P_n P_{e1} = 0 \tag{4.46}$$

in which $e_{test}$ is calculated from the test column end eccentricity ratio ($e/h_{test}$) and $P_n$, $P_{e1}$, $C_m$ and $M_n$ are values that can be readily calculated using the equations presented earlier. Equations 4.45 and 4.46 are in the form of a general quadratic equation: $ax^2 + bx + c = 0$, where $x = P_u$ and $a$, $b$ and $c$ are the constants indicated in Equations 4.45 and 4.46. The solution for a general quadratic equation was used to determined $P_u$:

$$P_u = x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{4.47}$$

Equation 4.47 gives two solutions due to the plus and minus signs. It was determined that the minus sign gives the correct solution since the positive solution for $P_u$ is greater than the pure axial load capacity of the cross-section. Note that $P_u$ is equal to the design axial load ($P_{des}$).

## 4.3 EUROCODE 2 PROVISIONS

The design equations given in Eurocode 2 (Design 1992) for the design of reinforced concrete columns are discussed in this section. Eurocode 2 has no provisions for the design of composite steel-concrete columns. Presently, there is a move to unify the design codes of most European countries to a unified standard. Eurocode 2 is a European Prestandard and was approved as a prospective standard for provisional application in 1991.

### 4.3.1 Limitations of Eurocode 2

For the design of reinforced concrete columns, Eurocode 2 has established several limitations. These are summarized below:

- The nominal concrete strength shall not be less than 1750 psi nor greater than 7300 psi unless its use is appropriately justified.
- The minimum amount of longitudinal reinforcement shall be 0.15 times the design axial load divided by the yield strength of the longitudinal bars ($0.15N_{sd}/f_{yd}$) but shall not be less than 0.003 times the gross area of concrete cross-section.
- The maximum amount of longitudinal reinforcement shall be limited to 0.08 times the gross area of the concrete cross-section.

- For short columns ($\ell/h<7.5$), the minimum design end eccentricity ratio ($e/h$) shall be 0.05.
- For columns subject to slenderness effects ($\ell/h>7.5$), the minimum design end eccentricity ratio ($e/h$) shall be 0.1.
- The maximum slenderness ratio ($\ell/h$) shall be 42.
- The larger dimension of the columns shall not exceed four times the smaller dimension.
- The minimum transverse dimension of a column cross-section is 8 inches (200 mm).
- The minimum longitudinal bar diameter is 1/2 inch (12 mm).
- A minimum of one longitudinal bar must be placed in each corner of a column having a polygonal cross-section.
- The minimum size of the ties shall be 1/4 inch (6 mm) but not less than one-quarter the diameter of the longitudinal bars. The maximum spacing of ties shall be the smallest of: (a) 12 times the diameter of the longitudinal bars, (b) the least dimension of the column, or (c) 12 in. (300mm).

Many column test specimens did not meet some of the limits noted above. In such cases, however, these limits were ignored for computing the Eurocode 2 strengths.

### 4.3.2 Calculation of the Cross-Section Capacity

The determination of the cross-section capacity is based on the following assumptions:

(a) strains between concrete and longitudinal reinforcing steel are compatible and no slip occurs;

(b) strain is linearly proportional to the distance from the neutral axis.

The maximum useable strain at the extreme concrete compression fiber is equal to 0.0035. An equivalent rectangular stress block with a stress ordinate of $0.85f_{cd}$ is used for calculation. The design concrete strength, $f_{cd}$, is equal to the specified nominal compressive strength (measured cylinder strength for this study) of concrete. The equivalent stress block is assumed to be uniformly distributed over a zone bounded by the edges of the cross-section and a straight line located parallel to the neutral axis at a depth of 0.8 times the distance from the extreme compression fiber to the neutral axis. The equivalent rectangular stress block is illustrated in Figure 4.9. Note that the tensile strength of concrete is ignored.

The longitudinal reinforcing steel is assumed to be elastic-perfectly plastic. Therefore, at strains less than the design yield strain, the stress in the steel can be computed as the modulus of elasticity of steel, $E_s$, times the strain. For strains greater than the design yield strain, the stress in the steel is the design yield stress,

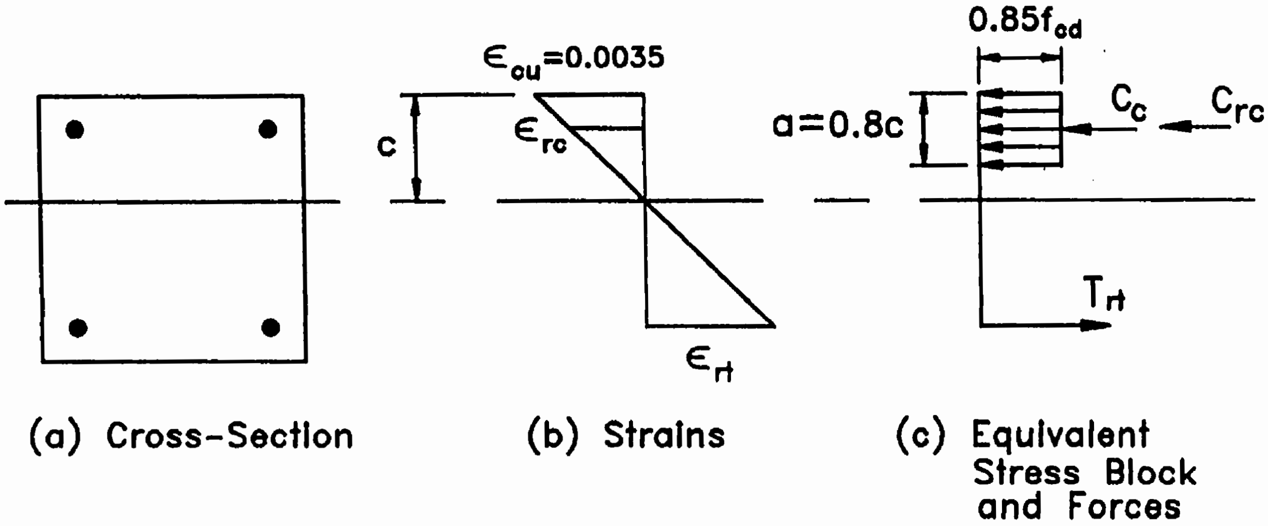



Figure 4.9 - Equivalent stress block specified by Eurocode 2.

$f_{yd}$, of the steel. In this study, the design yield stress of the reinforcing steel is equal to the measured yield stress. The strain hardening is neglected. Note that the displaced area of concrete in the compression zone is considered.

The strength of the cross-section can be represented by an axial load-bending moment interaction curve similar to the one shown in Figure 4.3. Due to the large number of calculations required in developing the axial load-bending moment interaction curve, a computer program was written. Details of the computer program analysis procedure will be presented in the following section.

There is no explicit upper limit on the maximum design axial load permitted on a cross-section compared to that provided by ACI 318-95 using Equation 4.4. However, there are provisions in Eurocode 2 that take this into consideration and will be discussed later.

### 4.3.3 Computer Analysis of Cross-Section Capacity

The computer program developed for this study uses the equations and assumptions outlined in the previous section to compute the cross-section axial load-bending moment interaction curve. The analysis procedure used and presented in this section is summarized in the flow chart in Figure 4.10.

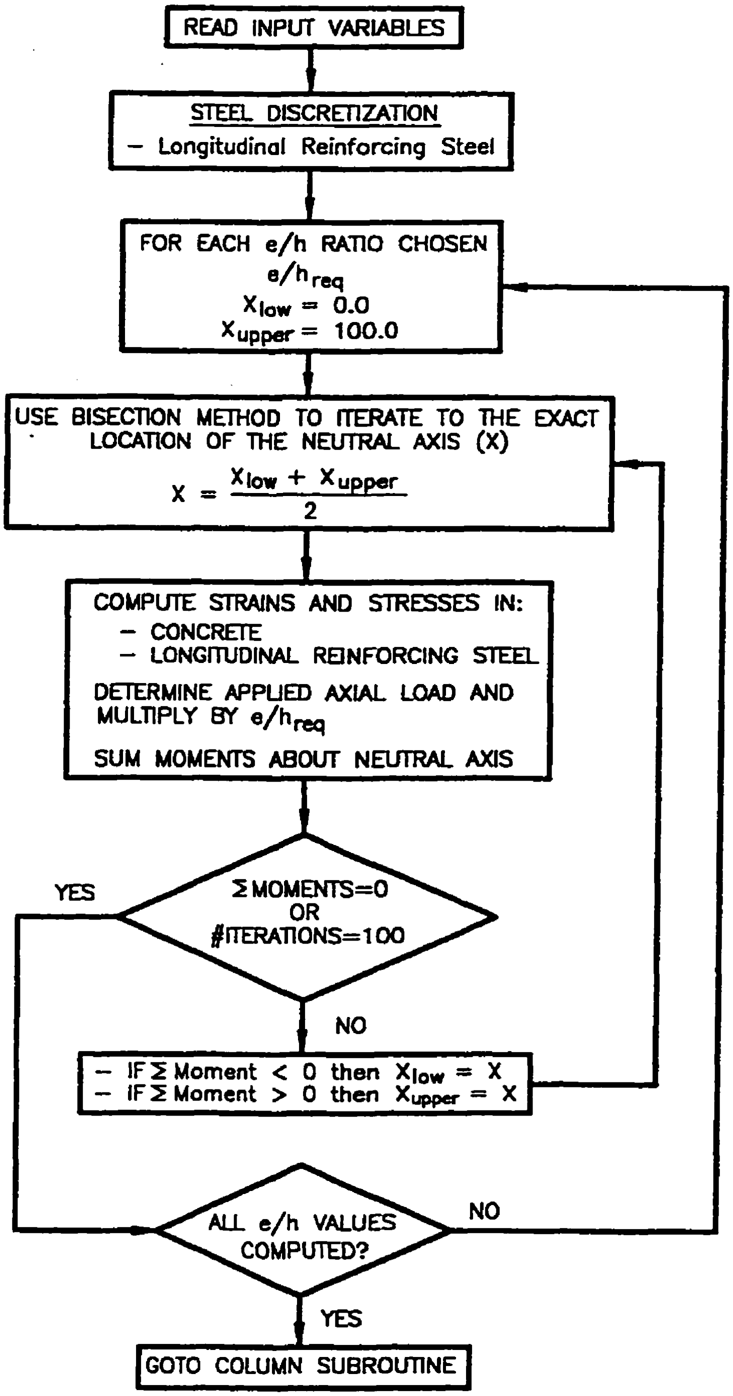



Figure 4.10 - Eurocode 2 cross-section interaction curve flowchart.

The cross-section consists of two materials (concrete and longitudinal reinforcing steel), each possessing a unique stress-strain relationship. Since Eurocode 2 uses an equivalent rectangular stress block, the concrete is not discretized. No distinction is made between the concrete outside the transverse reinforcement (unconfined concrete) and the concrete inside the transverse reinforcement (confined concrete). Each longitudinal reinforcing bar was represented by one element with a specified area and distance from the plastic neutral axis (measured perpendicular to the axis of bending).

The cross-section axial load-bending moment interaction curve represents points of axial load and corresponding bending moment that ensure strain compatibility and also satisfy conditions of equilibrium. For this study, the cross-section interaction curve was represented by 102 points. To ensure the points on the interaction curve were equally distributed along its length, the criterion used in determining the points was based on the end eccentricity ratio ($e/h$). The first point was determined for $e/h=0$ (pure axial load) and the last point was determined for $e/h=\infty$ (pure bending). The remaining 100 points were distributed along the cross-section interaction curve.

For the analysis procedure, a lower limit ($X_{low}$) and upper limit ($X_{upper}$) on the distance of the neutral axis from the plastic centroid ($X$) was established (Figure 4.6). Using a maximum concrete strain of 0.0035 and a given location of the neutral axis, the strains and stresses in the concrete and longitudinal reinforcing steel can be computed (Figure 4.9). The location of the neutral axis was varied and the bisection method was used to converge to a solution (for $X$) for each of the required end eccentricity ratios ($e/h_{req}$). The convergence tolerance was limited to 100 iterations for numerical stability.

For a given location of the neutral axis, the depth of the equivalent rectangular stress block was determined and the resulting concrete compressive force was then established. Using the assumption that the strain is linearly proportional to the distance from the neutral axis, the strain in each of the longitudinal reinforcing steel bars was computed. The steel was assumed to be elastic-perfectly plastic, as discussed in the previous section. For reinforcing bars within the equivalent rectangular stress block, the displaced area of concrete was considered. The resulting forces in each of the steel reinforcing bars was then determined.

To determine the applied axial load for the given location of the neutral axis, the concrete compressive force was added to the summation of forces on all reinforcing steel bars. The net value is, therefore, equal to the applied axial load on the section. Finally, a sum of moments of forces about the neutral axis was performed.

The distance from the applied axial load to the plastic centroid was assumed to be equal to the required end eccentricity. If the actual end eccentricity ratio ($e/h_{actual}$) is equal to the required end eccentricity ratio ($e/h_{req}$), the sum of moments about the neutral axis will equal zero. If the summation does not equal zero, the location of the neutral axis is not correct and another iteration is required. To obtain a solution using the bisection method the following conditions apply (Figure 4.10): (a) if the summation is less then zero, $X_{low}=X$, and (b) if the summation is greater than zero, $X_{upper}=X$.

Once the 102 points on the cross-section axial load-bending moment interaction curve were computed, the analysis of the column capacity was performed.

### 4.3.4 Calculation of the Column Capacity

In determining the design column capacity, Eurocode 2 modifies the first-order eccentricities to account for initial imperfections and second-order effects, if necessary.

Initial imperfections account for the dimensional inaccuracies and uncertainties in the position of the line of action of the axial loads. These effects are accounted for by increasing the first-order eccentricities by an additional eccentricity, $e_a$, acting in the most unfavorable direction. The additional eccentricity is computed using Equation 4.48:

$$e_a = \nu\left(\frac{\ell_o}{2}\right) \tag{4.48}$$

where $$\nu = \frac{1}{100\sqrt{0.0254\ell_o}} \geq \frac{1}{200} \tag{4.49}$$

where $\ell_o$ is the effective length of the column, in.; and $\nu$ is the assumed inclination of the column to account for initial imperfections. For SI conversion replace 0.0254 by 1.0 meter in Equation 4.49.

Slenderness effects must be considered if the slenderness ratio, $\lambda$, is greater than or equal to the critical slenderness ratio, $\lambda_{crit}$, of the column computed using Equations 4.50 and 4.51, respectively:

$$\lambda = \frac{\ell_o}{i} \tag{4.50}$$

$$\lambda_{crit} = 25\left(2 - \frac{e_{o1}}{e_{o2}}\right) \tag{4.51}$$

where $i$ is the radius of gyration of the uncracked column in the plane of bending; $e_{o1}$ is the smaller first-order end eccentricity, positive if the column is bent in single curvature, negative if the column is bent in double curvature; and $e_{o2}$ is the larger first-order end eccentricity, always positive.

If the slenderness ratio is less than the critical slenderness ratio, second-order effects can be neglected. However, the minimum design end eccentricity including imperfections is taken greater than or equal to *h/20* where *h* is the depth of the cross-section in the plane of bending. This limit corresponds to an end eccentricity ratio (*e/h*) of 0.05, which places an upper limit on the design axial load similar to the one provided by ACI 318-95 using Equation 4.4. Equation 4.4 was based on a minimum end eccentricity ratio of approximately 0.10 which is slightly more conservative than the limit used in Eurocode 2. This limit of minimum eccentricity was not used for computing the strength of columns for which $\lambda<\lambda_{crit}$ since the study dealt with the physical tests on columns.

Eurocode 2 provides a simplified design method for determining second-order effects in columns. A moment magnifier approach is not used in Eurocode 2. A column must be designed to account for the total eccentricity attributed to it which can be calculated using Equation 4.52:

$$e_{tot} = e_o + e_a + e_2 \qquad (4.52)$$

where $e_{tot}$ is the total design eccentricity; $e_o$ is the first-order eccentricity; $e_a$ is the additional eccentricity to account for imperfections; and $e_2$ is the second-order eccentricity.

Equation 4.52 is applicable to columns with equal and opposite first-order end eccentricities. To account for moment gradients in the column, $e_o$ in Equation 4.52 is replaced by an equivalent eccentricity, $e_e$, as calculated using Equation 4.53:

$$e_e = 0.6e_{o2} + 0.4e_{o1} \geq 0.4e_{o2} \tag{4.53}$$

where $e_{o1}$ is the smaller first-order end eccentricity, positive if the column is bent in single curvature, negative if the column is bent in double curvature; and $e_{o2}$ is the larger first-order end eccentricity, always positive.

For this study the second-order eccentricity, $e_2$, was calculated by using the "model column" approach. This approach is applicable to columns with a slenderness ratio, $\lambda$, less than 140, with rectangular or circular cross-sections, and with the minimum first-order end eccentricity greater than 0.1 times the depth of the cross-section in the plane of bending. This minimum eccentricity requirement was not used for the comparative study in order to study the applicability of Eurocode 2 for columns with smaller end eccentricity ratios ($e/h<0.1$).

The model column is an isolated cantilever column which is fixed at the base and free at the top and is illustrated in Figure 4.11. The model column is assumed to be bent in single curvature under loads and moments which give the maximum moment at the base. The maximum deflection, which equals the second-order eccentricity, $e_2$, of such a column is calculated using Equation 4.54:

$$e_2 = k_1 \frac{\ell_o^{\,2}}{10}\left(\frac{1}{r}\right) \tag{4.54}$$

where

for $15 \leq \lambda \leq 35$

$$k_1 = \frac{\lambda}{20} - 0.75 \tag{4.55}$$

for $\lambda > 35$

$$k_1 = 1.0 \tag{4.56}$$

and $1/r$ is the curvature of the critical section at the base. The curvature is derived from the equilibrium of internal and external forces. In cases where great accuracy is not required, the curvature in Equation 4.54 is computed using Equation 4.57:

$$\frac{1}{r} = 2k_2 \frac{\varepsilon_{yd}}{(0.9d)} \tag{4.57}$$

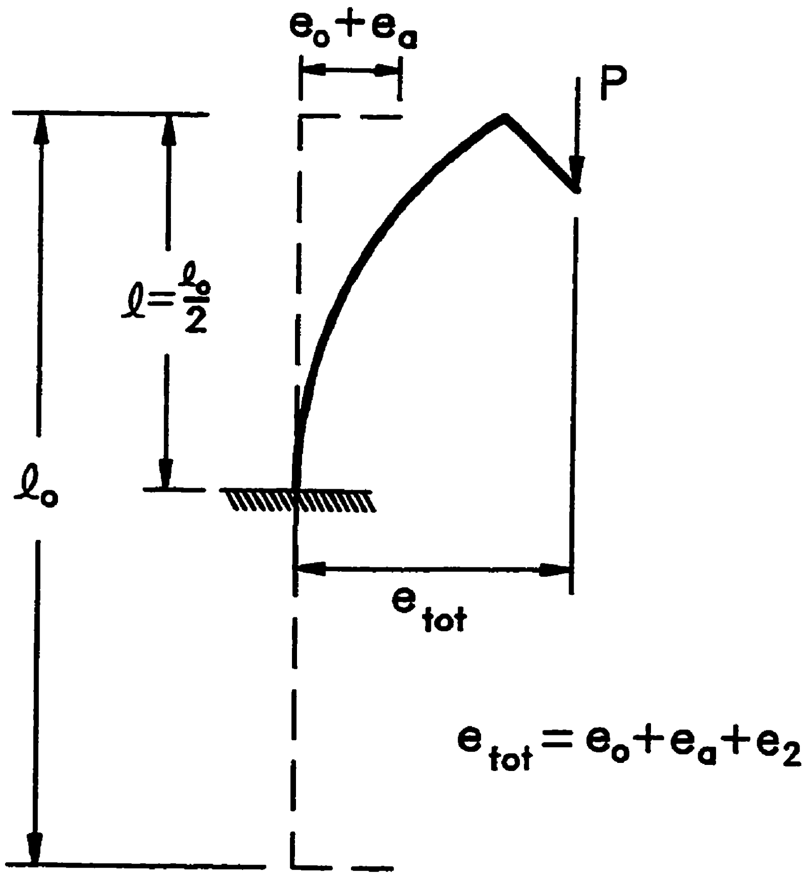



Figure 4.11 - Model column used by Eurocode 2 for determining second-order eccentricities.

where $k_2$ is a coefficient that takes into account the decrease in curvature with increasing axial force; $\epsilon_{yd}$ is the design yield strain of the steel reinforcement; and $d$ is the effective depth of the cross-section in the expected direction of stability failure. The coefficient $k_2$ can be calculated using Equation 4.58:

$$k_2 = \frac{N_{ud} - N_{sd}}{N_{ud} - N_{bal}} \leq 1.0 \tag{4.58}$$

where $N_{ud}$ is the ultimate capacity of the cross-section subjected to pure axial load only; $N_{sd}$ is the design axial load and $N_{bal}$ is the axial load which maximizes the ultimate moment capacity of the cross-section. It will always be conservative to assume a value of $k_2$ equal to 1.0. Equations 4.59 and 4.60 are provided to calculate $N_{ud}$ and $N_{bal}$, respectively:

$$N_{ud} = 0.85 f_{cd} A_c + f_{yd} A_s \tag{4.59}$$

$$N_{bal} = 0.4 f_{cd} A_c \tag{4.60}$$

where $A_c$ is the net area of the concrete cross-section; and $A_s$ is the area of the longitudinal reinforcing bars.

Once the total design eccentricity, $e_{tot}$, is found, it is multiplied by the design axial load to obtain the column design moment which takes into account initial imperfections, moment gradients and second-order effects. The column design moment is compared with the axial load-bending moment interaction curve at the corresponding design axial load level. If the column design moment lies within the cross-section axial load-bending moment interaction curve, the column is acceptable for the design loads.

### 4.3.5 Computer Analysis of Column Capacity

If a column is defined as being short, that is if $\lambda$ (Equation 4.50) is less than or equal to $\lambda_{crit}$ (Equation 4.51), the cross-section and column axial load-bending moment interaction curves are equal. Otherwise, slenderness effects are considered and the first-order end eccentricities are modified. The computer program developed for this study uses the equations given in the previous section to compute the Eurocode 2 column axial load-bending moment interaction curve. The analysis procedure used and presented in this section is summarized in the flow chart in Figure 4.12.

The column axial load-bending moment interaction curve is developed from the cross-section axial load-bending moment interaction curve. The computer program stores the

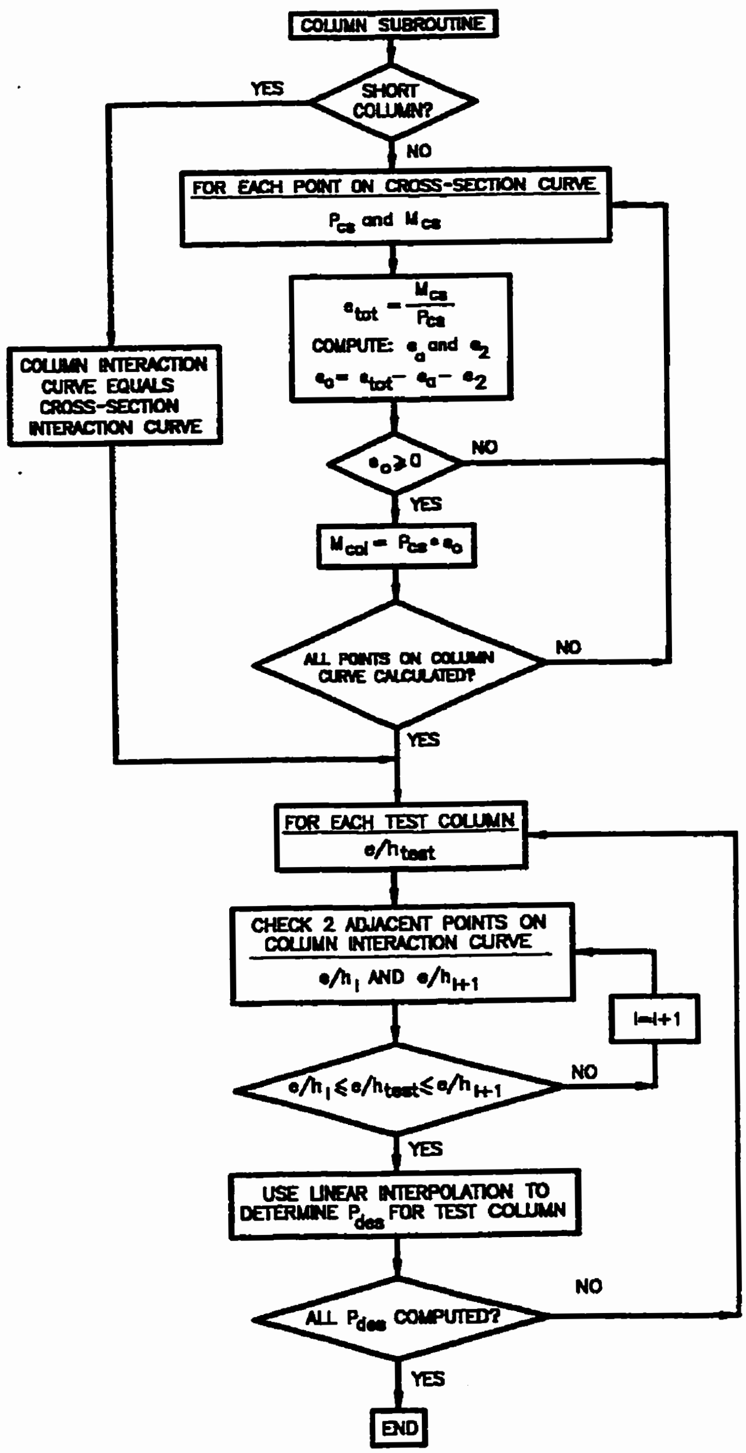



Figure 4.12 - Eurocode 2 column interaction curve flowchart.

value of the axial load and corresponding bending moment for each of the 102 points used to define the cross-section interaction curve. The only difference between the cross-section and column interaction curve for a given axial load level is the moment capacity (Figure 4.3). The relationship between the moment capacity of the cross-section and that of the column is represented by Equation 4.52 where $e_{tot}$ times the axial load is the cross-section moment capacity and $e_o$ times the axial load is the column moment capacity. Since the cross-section moment capacity has been previously calculated and stored, the column moment capacity can be obtained by simply rearranging Equation 4.52 to:

$$e_o = e_{tot} - e_a - e_2 \qquad (4.61)$$

For columns with moment gradients, the right hand side of Equation 4.61 is divided by the value obtained from Equation 4.62:

$$C_m = 0.6 + 0.4\frac{e_{o1}}{e_{o2}} \geq 0.4 \qquad (4.62)$$

The value of $e_a$ is calculated using Equation 4.48 and $e_2$ is calculated using Equation 4.54. For each point on the cross-section curve, the additional eccentricity, $e_a$, and second-order eccentricity, $e_2$, are subtracted from the cross-section eccentricity, $e_{tot}$, to obtain the column eccentricity. Any points on the cross-section curve that have a column eccentricity of less than zero are not considered in developing the column interaction curve. For points with column eccentricities greater than or equal to zero, the value is multiplied by the axial load to obtain the column moment capacity.

Once the column axial load-bending moment interaction curve was computed, the design axial load ($P_{des}$) of the column could be determined. The end eccentricity ratio of the test column ($e/h_{test}$) was successively compared to the end eccentricity ratio of two adjacent points on the column interaction curve (Figure 4.8). Once the test column end eccentricity ratio ($e/h_{test}$) fell between two adjacent points, linear interpolation was used to calculate the design axial load ($P_{des}$) of the test column.

No equation is provided in Eurocode 2 to determine the pure axial load capacity of a column. For Eurocode 2, the column pure axial load capacity in this study is defined as the condition where the column eccentricity, calculated using Equation 4.61, equals zero. To determine this point, the computer program stores the axial load and negative column eccentricity values until a positive column eccentricity is obtained. Linear interpolation is then used between these two points to determine the pure axial load capacity of a column.

## 4.4 EUROCODE 4 PROVISIONS

The design equations given in Eurocode 4 (Design 1994) for the design of composite steel-concrete columns are discussed in this section. Eurocode 4 has no provisions for the design of reinforced concrete columns. Presently, there is a move to unify the design codes of most European countries to a unified standard. Eurocode 4 was ratified as a European Prestandard in 1992.

### 4.4.1 Limitations of Eurocode 4

Eurocode 4 provides two methods for designing composite columns: a general method and a simplified method. The simplified method discussed in this section was used for the comparative study. The use of the simplified design method is applicable to columns that meet the following limitations:

- The column cross-section must have double symmetry and must be uniform over the column length.
- The nominal concrete strength shall not be less than 1750 psi nor greater than 7300 psi unless its use is appropriately justified.
- The structural steel contribution ratio, $\delta$ (defined later in Equation 4.64), should be between 0.2 and 0.9. The steel members may be rolled or welded.

- The cross-sectional area of longitudinal reinforcement shall not be less than 0.003 nor greater than 0.04 times the gross area of the concrete cross-section.
- The non-dimensional slenderness parameter, $\lambda$ (defined later in Equation 4.65), should not exceed 2.0.
- The concrete cover to a flange of a composite column should not be less than 1.5 inches (40 mm) or one-sixth the width of the flange.
- For fully-encased steel sections, limits to the thickness of concrete cover are:

  For the y-direction, 1.5 in.$\leq c_y \leq 0.4b$

  For the z-direction, 1.5 in.$\leq c_z \leq 0.3h$

  The terms of $c_y$, $c_z$, $b$ and $h$ are shown in Figure 4.13. Greater cover can be used but is considered to be ineffective when considering column cross-section strength and is ignored in calculations.
- Longitudinal shear resistance shall be provided by bond stresses and friction at the concrete and steel section interface or by mechanical shear connection, such that no significant slip occurs.
- The larger dimension of the columns shall not exceed four times the smaller dimension.
- The minimum transverse dimension of a column cross-section is 8 inches (200 mm).

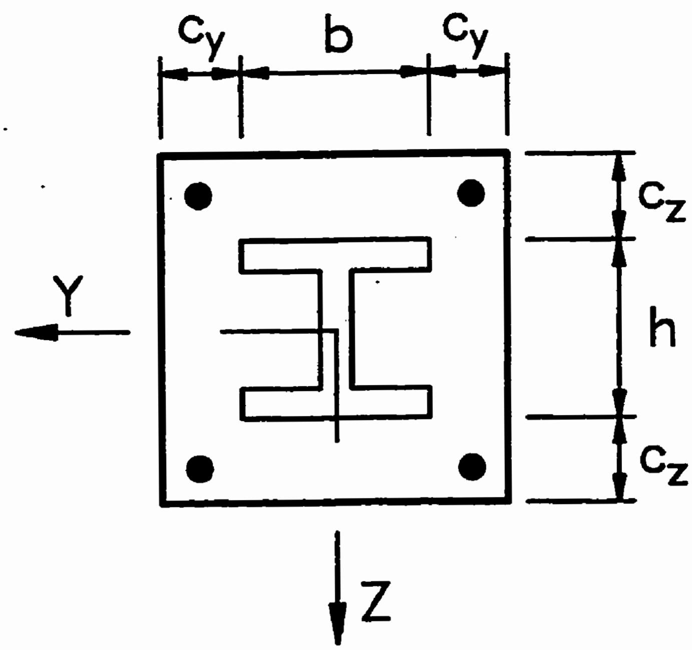



Figure 4.13 - Cross-section notation specified by Eurocode 4.

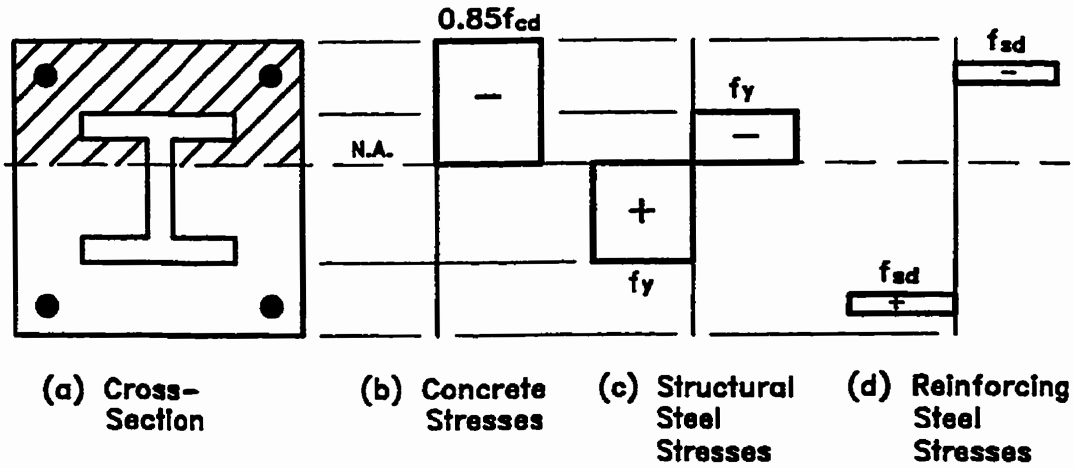



Figure 4.14 - Plastic stress blocks specified by Eurocode 4.

- The minimum longitudinal bar diameter is 1/2 inch (12 mm).
- A minimum of one longitudinal bar must be placed in each corner of a column having a polygonal cross-section.
- The minimum size of the ties shall be 1/4 inch (6 mm) but not less than one-quarter the diameter of the longitudinal bars. The maximum spacing of ties shall be the smallest of: (a) 12 times the diameter of the longitudinal bars, (b) the least dimension of the column, or (c) 12 in. (300mm).

Many column test specimens did not meet some of these limitations. For such test columns, however, these limitations were not taken into consideration for calculating the Eurocode 4 strengths.

### 4.4.2 Calculation of Cross-Section Capacity

The determination of the cross-section capacity is based on the assumption of using full plastic stress blocks for concrete, longitudinal reinforcing steel and structural steel. The strains in the concrete, reinforcing and structural steels are not necessarily directly proportional to the distance from the neutral axis. There is no limit on the maximum usable concrete strain since strain compatibility is not required. An equivalent concrete stress block with a stress ordinate of $0.85f_{cd}$ is used for the analysis. The design concrete strength, $f_{cd}$, is equal to the specified nominal compressive strength (measured cylinder strength for this study) of concrete. The concrete stress block is assumed to be uniformly distributed over a zone bounded by the edges of the cross-section and a straight line parallel to and located at the neutral axis. The reinforcing and structural steels are assumed to be yielded in compression or tension on adjacent sides of the neutral axis. The strain hardening of the structural steel section and reinforcing bars is neglected and the displaced area of concrete in the compression zone is considered. A typical plastic stress distribution is illustrated in Figure 4.14.

The strength of a cross-section is represented by an axial load-bending moment interaction curve similar to the one shown in Figure 4.3. The simplified method of Eurocode 4 allows the interaction curve to be approximated by a four point polygon for major axis bending and a five point polygon for minor axis bending (Figure 4.15). For major axis bending, points A, C, D and B shown in Figure 4.15 must be calculated. For minor axis bending, an additional point, E, must be determined. In this study the simplified polygon interaction curve was used since this method reflects the approach that would be used by design engineers.

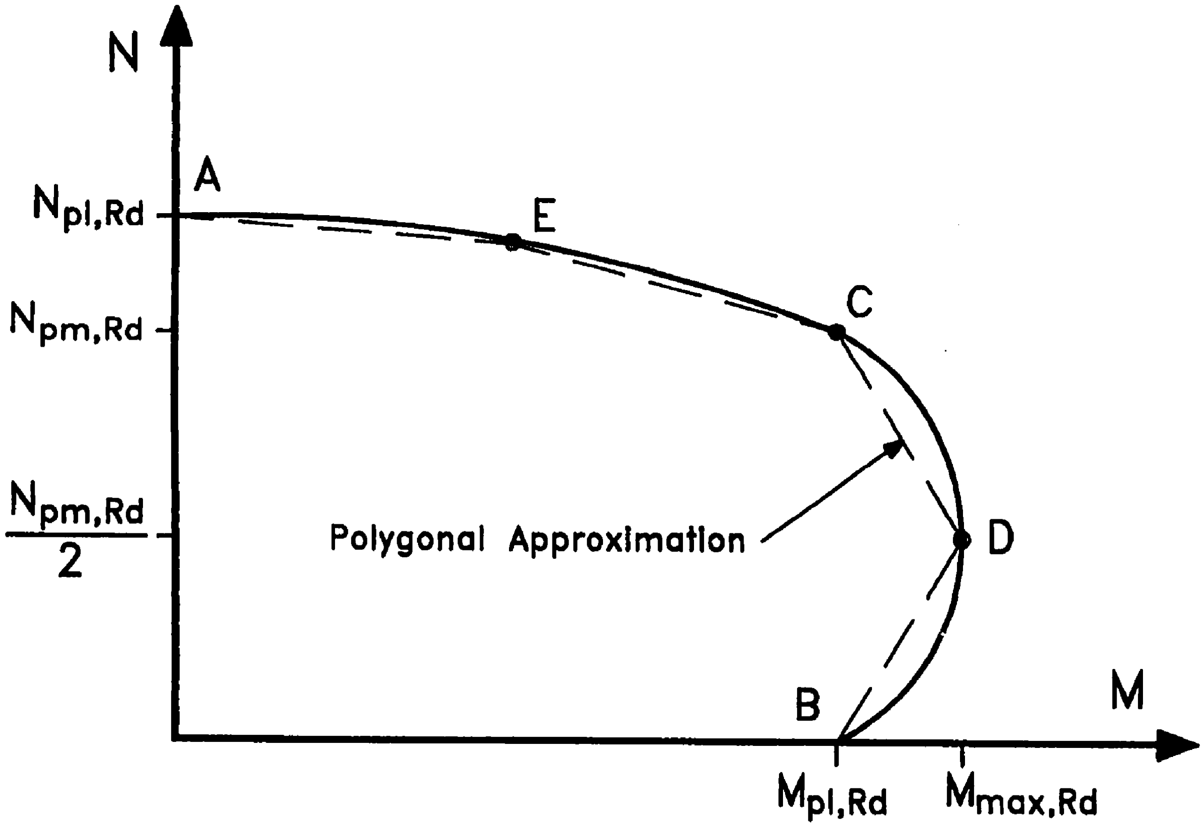



**Figure 4.15** - Axial load-bending moment interaction curve with polygonal approximation specified by Eurocode 4 for composite cross-section.

Point A corresponds to the plastic resistance of the cross-section in pure compression, $N_{pl,Rd}$, and is calculated using Equation 4.63:

$$N_{pl,Rd} = A_a f_y + A_c\left(0.85 f_{cd}\right) + A_s f_{sd} \tag{4.63}$$

where $A_a$ and $f_y$ are the area and design strength of the structural steel shape, respectively; $A_c$ and $f_{cd}$ are the area and design strength of concrete, respectively; and $A_s$ and $f_{sd}$ are the area and design strength of the reinforcing steel, respectively. In this study, the partial safety factors for material strengths are set to equal unity, therefore, the design strengths are equal to the nominal (measured) strengths.

Point B corresponds to the plastic resistance of the cross-section under pure bending, $M_{pl,Rd}$. Point C, $N_{pm,Rd}$, corresponds to the axial load capacity of the cross-section under a moment equal to the plastic resistance of the cross-section under pure bending. Point D corresponds to an axial load that is half of the axial load obtained for point C and resulting approximate maximum moment, $M_{max,Rd}$. Point E is an arbitrary point located between points A and C. For this study, Point E was calculated at half the end eccentricity ratio as that used for Point C. Eurocode 4 provides approximate formulas for calculating points A to

E, however, in this study a computer program was written to solve for these points. Details of the computer program analysis procedure are presented in the following section.

Eurocode 4 specifies that the structural steel contribution ratio, $\delta$ as computed from Equation 4.64, must lie between 0.2 and 0.9:

$$\delta = \frac{A_a f_y}{N_{pl,Rd}} \tag{4.64}$$

If the steel contribution ratio is less than 0.2, the column should be designed as a reinforced concrete column. If the $\delta$ ratio is greater than 0.9, the column should be designed as a steel column. In this study, three composite steel-concrete columns subjected to minor axis bending (Roik and Schwalbenhofer (1988) columns V121, V122 and V123) had a steel contribution ratio of 0.161 which is outside of these limits. However, these limits were not taken into consideration for calculating the Eurocode 4 strengths for these columns.

### 4.4.3 Computer Analysis of Cross-Section Capacity

The computer program developed for this study uses the equations and assumptions outlined in the previous section to compute the cross-section axial load-bending moment interaction curve. The simplified approach of Eurocode 4 requires the computation of four or five points on the cross-section axial load-bending moment interaction curve. The first point calculated was for the resistance of the cross-section to pure axial load as determined using Equation 4.63 (Point A in Figure 4.15). The analysis procedure used for the remaining points (Points B to E in Figure 4.15) and presented in this section is summarized in the flow chart in Figure 4.16.

The cross-section consists of three materials (concrete, longitudinal reinforcing steel and structural steel), each possessing a unique stress-strain relationship. In order to distinguish among these three materials in the analysis, the cross-section was first discretized. Since Eurocode 4 uses a rectangular plastic stress block, the concrete was not discretized. No distinction is made between the concrete outside the transverse reinforcement (unconfined concrete) and the concrete inside the transverse reinforcement (confined concrete). Each longitudinal reinforcing bar was represented by one element with a specified area and distance from the plastic neutral axis (measured perpendicular to the axis of bending). The structural steel section required the discretization of both the flanges and web (Figure 4.5). For major axis bending, the flanges were discretized into 20 strips with the element width being equal to the flange width. The web was divided into 80 strips with the element width being equal to the web thickness. For minor axis bending, the flanges were

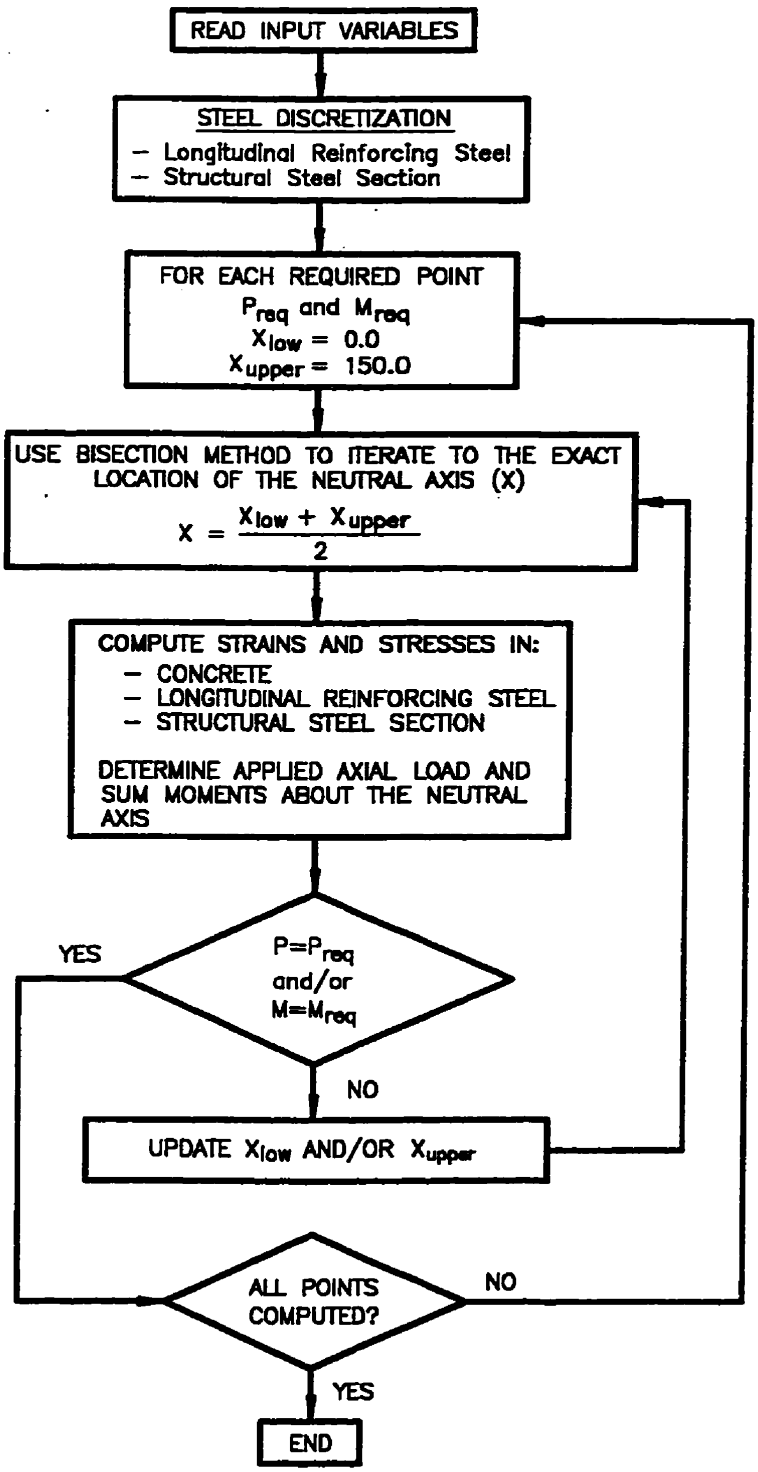



**Figure 4.16 - Eurocode 4 cross-section interaction curve flowchart.**

discretized into 80 strips with the element width being equal to the flange thickness. The web was divided into 20 strips with the element width being equal to the web depth. The area of each structural steel element and the distance from the plastic neutral axis to the centroid of the element (measured perpendicular to the axis of bending) were computed.

For the analysis procedure, a lower limit ($X_{low}$) and upper limit ($X_{upper}$) on the distance of the neutral axis from the plastic centroid ($X$) was established. For a given location of the neutral axis, the stresses in the concrete, longitudinal reinforcing steel and structural steel can be computed (Figure 4.14). The location of the neutral axis was varied and the bisection method was used to converge to a solution (for $X$) for each of the required points (Points B to E) on the simplified interaction curve. The convergence tolerance was limited to 100 iterations for numerical stability.

For a given location of the neutral axis, the depth of the rectangular concrete plastic stress block is known. The resulting concrete compressive force was then established. The stress in each of the longitudinal reinforcing steel bars and structural steel elements was computed. The steel was assumed to be fully yielded in compression or tension, as discussed in the previous section, and residual stresses were ignored. For steel elements within the rectangular concrete stress block, the displaced area of concrete was considered. The resulting force in each of the reinforcing bars and structural steel elements was then determined.

To determine the applied axial load for the given location of the neutral axis, the concrete compressive force was added to the summation of forces in all reinforcing bars and structural steel elements. The net value is, therefore, equal to the applied axial load on the cross-section. Finally, a sum of moments of forces about the neutral axis was performed to establish the applied moment on the section.

Once the 4 or 5 points on the cross-section axial load-bending moment interaction curve were computed, the analysis of the column capacity was performed.

#### 4.4.4 Calculation of Column Capacity

For computing the resistance of a column under pure axial compression, the non-dimensionalized slenderness parameter, $\lambda$, in the plane of bending, is first computed using Equation 4.65:

$$\lambda = \sqrt{\frac{N_{pl,Rd}}{N_{cr}}} \tag{4.65}$$

where $N_{pl,Rd}$ is the plastic resistance of the cross-section in pure compression and is computed from Equation 4.63 with the partial safety factors set equal to unity; and $N_{cr}$ is the elastic critical load for the column. The elastic critical load is calculated using Equation 4.66:

$$N_{cr} = \frac{\pi^2 (EI)_e}{\ell^2} \tag{4.66}$$

where $(EI)_e$ is the effective elastic flexural rigidity; and $\ell$ is the so-called buckling length of the column. In Eurocode 4, the buckling length, $\ell$, includes the effective length factor. For this study the buckling length is equal to the column length.

Eurocode 4 provides Equation 4.67 to compute the effective elastic flexural rigidity under short term loads:

$$(EI)_e = E_a I_a + 0.8 E_{cd} I_c + E_s I_s \tag{4.67}$$

where $E_a$ and $I_a$ are the modulus of elasticity and moment of inertia of the structural steel shape, respectively; $E_{cd}$ and $I_c$ are the modulus of elasticity and moment of inertia of the gross (uncracked) concrete section, respectively; and $E_s$ and $I_s$ are the modulus of elasticity and moment of inertia of the longitudinal reinforcing steel, respectively. The modulus of elasticity of concrete is calculated using Equation 4.68:

$$E_{cd} = 262{,}250 \left( f_{cd} + 1160 \right)^{\frac{1}{3}} \tag{4.68}$$

where $E_{cd}$ is the secant modulus of elasticity of concrete in psi. For SI conversion replace 262,250 psi by 9500 MPa and 1160 psi by 8 MPa.

Once the non-dimensionalized slenderness parameter, $\lambda$, is computed, the resistance of the column under pure axial load can be determined using Equation 4.69:

$$N_{\chi} = \chi\left(N_{pl.Rd}\right) \tag{4.69}$$

where $\chi$ is a reduction coefficient for the relevant buckling curve that accounts for any initial imperfections in the column. In determining the reduction coefficient, the European buckling curves for steel sections are used. Hence, the reduction coefficient can be described by Equation 4.70:

$$\chi = \frac{1}{\Phi + \sqrt{\Phi^2 - \lambda^2}} \leq 1.0 \tag{4.70}$$

with $$\Phi = 0.5\left[1 + \alpha(\lambda - 0.2) + \lambda^2\right] \tag{4.71}$$

where $\alpha$ is a factor of imperfection for the relevant European buckling curve. For composite columns bending about the major axis, $\alpha$ is taken equal to 0.34 and for composite columns bending about the minor axis, $\alpha$ is taken equal to 0.49.

For columns subjected to combined axial load and bending moments, the moments determined from a first-order elastic analysis must be modified to include second-order effects. However, a column need not be checked for second-order effects if one of the following equations is satisfied:

$$\frac{N_{sd}}{N_{cr}} \leq 0.1 \tag{4.72}$$

$$\lambda \leq \lambda_{crit} \quad \text{with } \lambda_{crit} = 0.2(2-r) \tag{4.73}$$

where $N_{sd}$ is the design axial load; and $r$ is the ratio of smaller to larger end moments in the column, positive if the member is bent in single curvature and negative if the member is bent in double curvature.

If neither Equation 4.72 nor Equation 4.73 is satisfied, Eurocode 4 provides a simplified moment magnifier approach to increase the larger first-order design bending moment, $M_{sd}$, by a correction factor, $k$:

$$kM_{sd} = \left( \frac{\beta}{1 - \dfrac{N_{sd}}{N_{cr}}} \right) M_{sd} \geq M_{sd} \tag{4.74}$$

where $\beta$ is an equivalent moment factor to account for moment gradients and can be calculated using Equation 4.75:

$$\beta = 0.66 + 0.44r \geq 0.44 \tag{4.75}$$

Once the magnified moment has been calculated, the resistance of the member in combined compression and bending must be checked using the following steps:

The first step involves non-dimensionalizing the cross-section axial load-bending moment interaction curve (Figure 4.17). The axial loads are divided by the plastic resistance of the cross-section under pure compression, $N_{pl,Rd}$. The bending moments are divided by the plastic resistance of the cross-section under pure bending, $M_{pl,Rd}$.

The design axial load ($N_{sd}$) divided by $N_{pl,Rd}$ is then plotted on the interaction curve and corresponds to point $\chi_d$ in Figure 4.17. The associated value for bending, $\mu_d$, on the cross-section curve is determined.

The resistance of the column under pure axial load ($N_X$) divided by $N_{pl,Rd}$ is then plotted on the interaction curve ($\chi$ in Figure 4.17) and the corresponding value for bending, $\mu_k$, is determined. The value, $\mu_k$, is known as the moment of imperfection of the column. The influence of this imperfection is assumed to decrease linearly to the value $\chi_n$. Where the variation in bending moment along the column length is approximately linear, the value of $\chi_n$ may be calculated using Equation 4.76:

$$\chi_n = \chi\left(\frac{(1-r)}{4}\right) \quad \text{but} \quad \chi_n \leq \chi_d \tag{4.76}$$

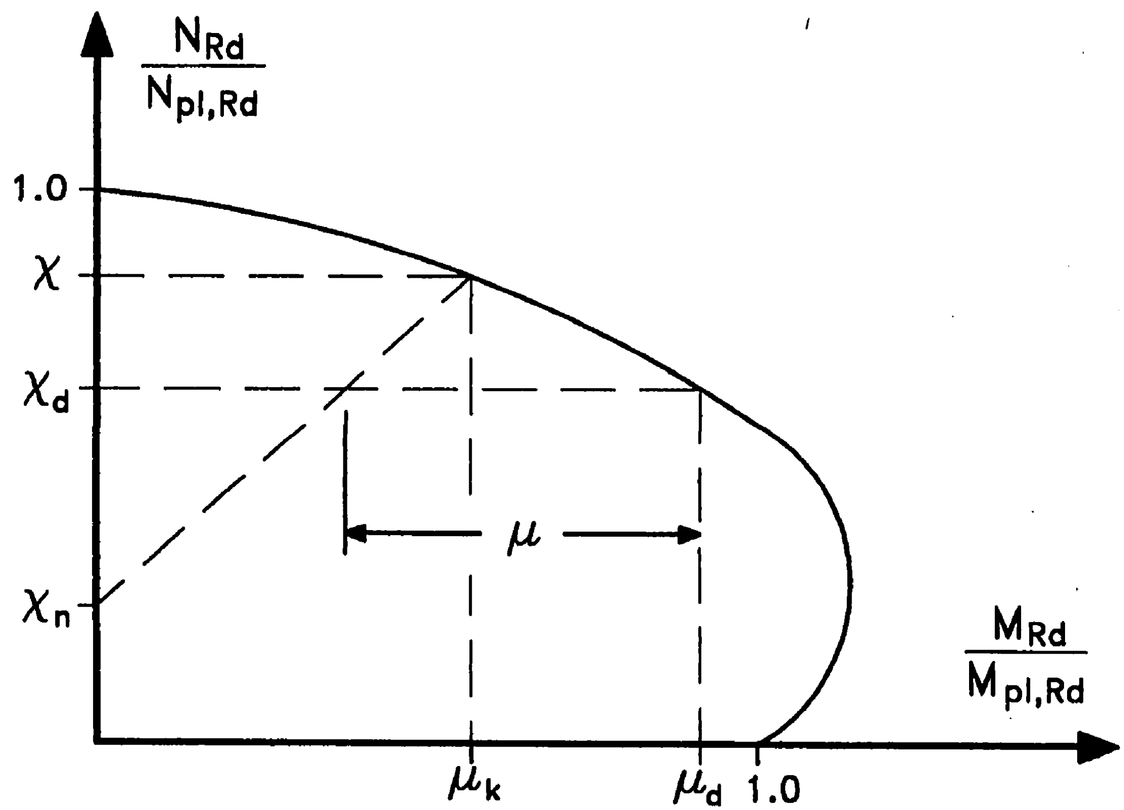



Figure 4.17 - Schematic design procedure used to check column capacity as specified by Eurocode 4.

The magnitude of $\mu$, which represents the remaining available moment resistance of the column, as shown in Figure 4.17, can be computed using Equation 4.77:

$$\mu = \mu_d - \mu_k \frac{(\chi_d - \chi_n)}{(\chi - \chi_n)} \tag{4.77}$$

The value of $\mu$ should not be taken greater than 1.0 unless the design bending moment, $M_{sd}$, is due solely to the action of the eccentricity of the force, $N_{sd}$ (i.e. in an isolated columns without transverse loads acting between the column ends). In this study, the above condition is satisfied, therefore, the value $\mu$ was allowed to be greater than 1.0.

Finally, the member has sufficient resistance if the design bending moment satisfies Equation 4.78:

$$kM_{sd} \leq 0.9\,\mu M_{pl,Rd} \tag{4.78}$$

where $kM_{sd}$ is the maximum design bending moment within the column length including second-order effects, if any. The 0.9 factor in Equation 4.78 accounts for the simplifications in computing the cross-section axial load-bending moment interaction curve. The curve was developed using a plastic stress distribution with no limitation on the concrete strain. Also, the moment according to second-order theory is determined with the effective flexural rigidity $(EI)_e$ using the gross (uncracked) area of the concrete section. The 0.9 coefficient is not considered to be a safety factor and was not set to unity in this study.

### 4.4.5 Computer Analysis of Column Capacity

If a column is defined as being short, that is if either Equation 4.72 or Equation 4.73 is satisfied, the cross-section and column axial load-bending moment interaction curves are equal. When neither Equation 4.72 nor Equation 4.73 is satisfied, slenderness effects are considered and the moment magnifier approach is used. The computer program developed for this study uses the equations and procedure given in the previous section to compute the Eurocode 4 column axial load-bending moment interaction curve. The analysis procedure used and presented in this section is summarized in the flow chart in Figure 4.18.

The column axial load-bending moment interaction curve is developed from the cross-section interaction curve. The resistance of the column under pure axial compression, $N_\chi$, is first calculated using Equation 4.69 (for $e/h$=0) to determine the upper limit on the column axial load capacity. Any points on the cross-section interaction curve that have a greater axial load capacity are not considered in developing the column interaction curve.

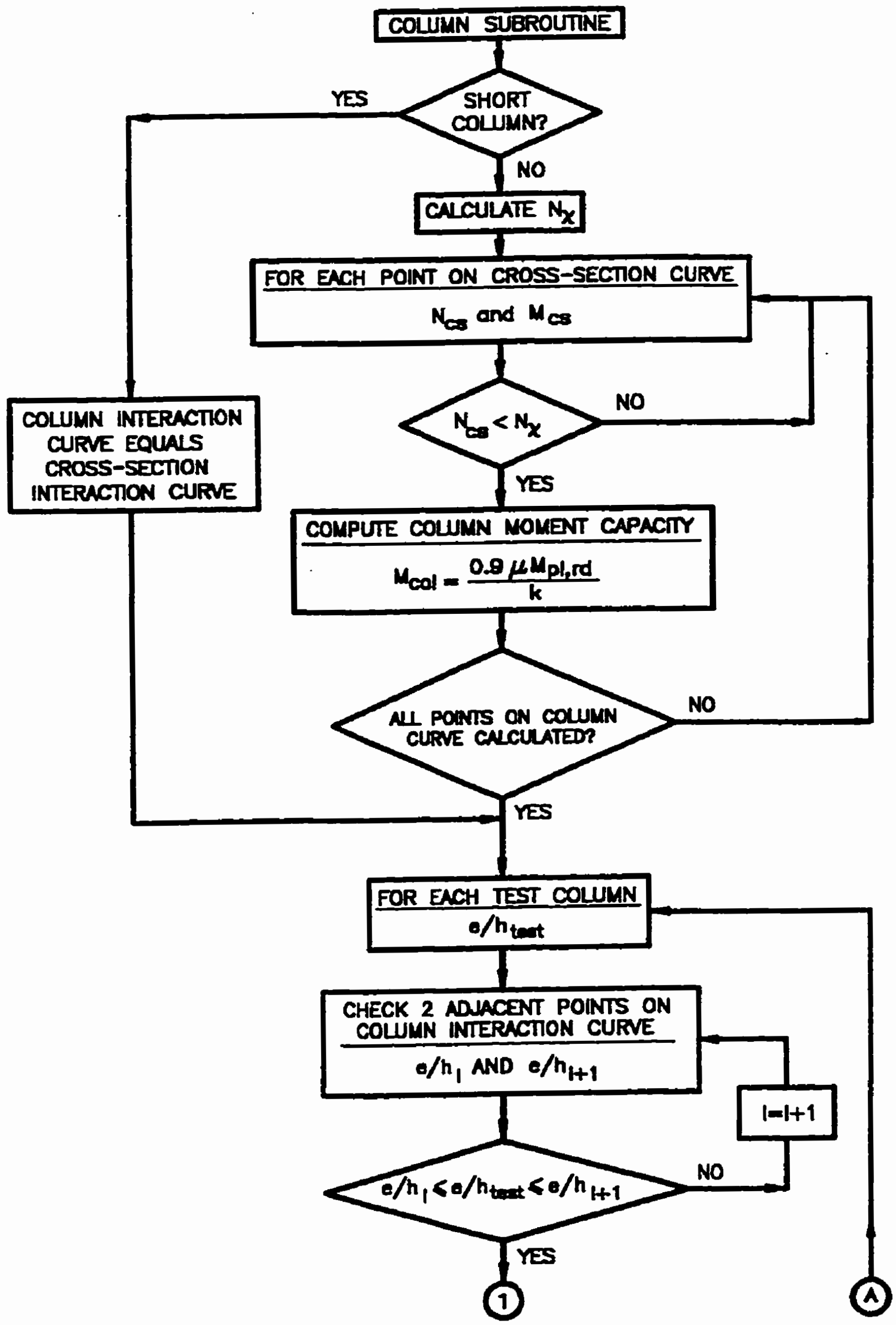



**Figure 4.18 - Eurocode 4 column interaction curve flowchart.**

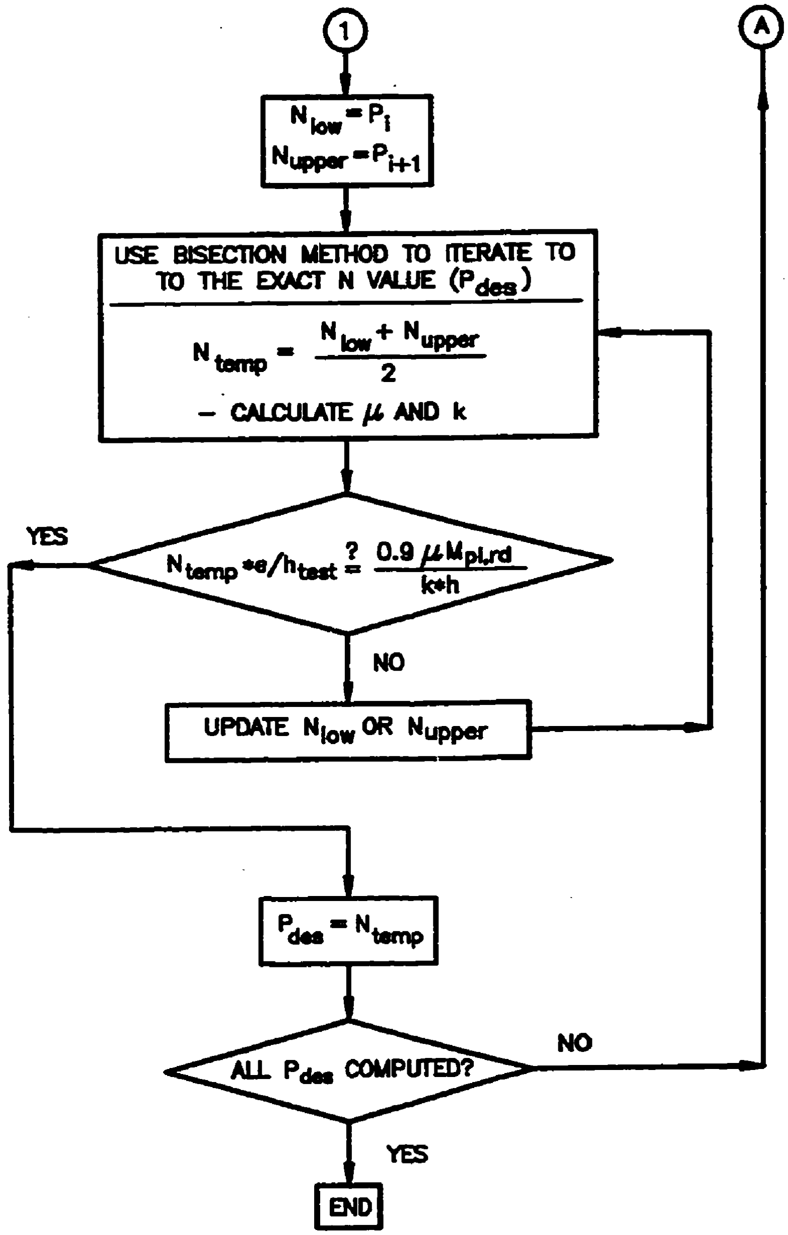



Figure 4.18 (continued) - Eurocode 4 column interaction curve flowchart.

The computer program stores the value of the axial load and corresponding bending moment for each of the four or five points used to define the cross-section interaction curve. The only difference between the cross-section and column interaction curve for a given axial load level is the moment capacity (Figure 4.3). The relationship between the moment capacity of the cross-section and that of the column can be obtained by re-arranging Equation 4.78 to Equation 4.79:

$$M_{sd} = \frac{0.9\,\mu M_{pl,Rd}}{k} \tag{4.79}$$

where $\mu$ relates the moment capacity for a given axial load level to the moment capacity of the column under pure bending ($M_{pl,Rd}$). The value of $\mu$ is calculated using Equation 4.77 where the relevant terms have been defined and are also illustrated in Figure 4.17. For each of the four or five points on the cross-section curve, Equation 4.79 is used to determine the column moment capacities. Each of the new points are stored and represent a polygonal approximation of the column interaction curve.

Once the polygonal approximation of the column axial load-bending moment interaction curve has been computed, the design axial load ($P_{des}$) of the column can be determined. To obtain more accurate results, the approximate column interaction curve was not used directly.

The end eccentricity ratio of the test column ($e/h_{test}$) was successively compared to the end eccentricity ratio of two adjacent points on the column interaction curve. Once the test column end eccentricity ratio ($e/h_{test}$) fell between two adjacent points, an iterative approach was used to determine the design axial load ($P_{des}$).

The two adjacent points on the column interaction curve were used to establish an upper and lower limit on the axial load which was used as the variable in the iterative approach. The bisection method was used to iterate to a required axial load such that Equation 4.79 is satisfied. For the given axial load level, linear interpolation of the cross-section interaction curve was used to establish the cross-section moment capacity. If Equation 4.79 is satisfied for the given axial load and end eccentricity ratio, the correct solution ($P_{des}$) has been obtained, otherwise, further iterations are required. The convergence tolerance was limited to 25 iterations for numerical stability.

# 5 - COMPARISON OF DESIGN METHODS WITH TEST RESULTS FOR REINFORCED CONCRETE COLUMNS

For the comparative study, reinforced concrete columns were divided into two separate groups: columns with equal and opposite applied end moments (i.e. symmetrical single curvature bending) and columns with unequal applied end moments (i.e. moment gradient).

## 5.1 REINFORCED CONCRETE COLUMNS WITHOUT MOMENT GRADIENT

In an attempt to study a full range of variables, 354 reinforced concrete test columns without moment gradient found in the literature were used to compare the test strengths with those obtained from ACI 318-95, Eurocode 2 and FEM methods. Note that ACI 318-95 strengths were computed in three different ways: (a) using Equation 4.16, (b) using equation 4.17, and (c) using Equation 4.19.

### 5.1.1 Description of Column Tests Available in the Literature

A complete description of 384 reinforced concrete test columns without moment gradient found in the literature is presented in Chapter 3. Thirty of these columns were not used for comparison because the specified nominal concrete strength for these columns was less than 2500 psi. These columns are identified with a double-asterisk in Table 3.1.1.

Each column used had a different combination of geometric and material properties. The maximum and minimum values of the overall cross-section dimensions ($bxh$), the nominal concrete strength ($f'_c$), the end eccentricity ratio ($e/h$), the slenderness ratio ($\ell/h$), the reinforcing steel yield stress ($f_{yr}$), the reinforcing steel ratio ($\rho_{rs}$), and the tie/hoop volumetric ratio ($\rho''$) are listed in Table 5.1.1. The values shown in the table represent the maximum and minimum values of the data; a detailed description of each column is given in Table 3.1.1.

In order to study the effects of variables over a broad range, several of the limitations specified by ACI 318-95 and Eurocode 2 design methods were not imposed in this study. ACI 318-95 imposes a lower limit on the reinforcing steel ratio of one percent. The minimum value in this study is 0.8 percent (Table 5.1.1) which is not significantly lower than the ACI 318-95 limit. ACI 318-95 also specifies an upper limit of 30 on $\ell/h$ ratio and 80% of the pure axial load capacity on applied axial loads acting on tied columns. The maximum value of $\ell/h$ used in this study is 40 (Table 5.1.1) which is significantly higher than 30. The use of Equation 4.19 for ACI 318-95 has two further limitations: (a) the concrete strength be less than or equal to 6000 psi, and (b) the end eccentricity ratio be greater than or equal to 0.1. Both of these limits were

Table 5.1.1 - Summary of Geometric and Material Properties of Reinforced Concrete Column Specimens without Moment Gradient*

| Properties | Minimum Values | Maximum Values |
|---|---|---|
| bxh (in. x in.) | 3.0 x 3.0 | 17.7 x 17.7 |
| $f'_c$ (psi) | 2550 | 8246 |
| e/h | 0.00 | 1.25 |
| $\ell/h$ | 2.0 | 40.0 |
| $f_{yr}$ (psi) | 39503 | 104500 |
| $\rho_{rs}$ (%) | 0.80 | 7.06 |
| $\rho''$ (%) | 0.035 | 4.734 |

* Number of Specimens = 354; h = depth of the concrete cross-section perpendicular to the axis of bending; b = width of the concrete cross-section parallel to the axis of bending.

Note: 1.0 in. = 25.4 mm; 1000 psi = 6.895 MPa

not applied in this study. Eurocode 2 specifies that the maximum nominal concrete strength shall not be greater than 7300 psi (50 MPa) unless its use is appropriately justified. The maximum $f'_c$ used in this study is 8246 psi (Table 5.1.1) which is not significantly higher than 7300 psi. For short and slender columns, Eurocode 2 specifies the minimum end eccentricity ratio ($e/h$) of 0.05 and 0.1, respectively. The minimum end eccentricity ratio of zero in this study does not meet the Eurocode 2 limit. Also, Eurocode 2 imposes a lower limit on the minimum transverse dimension of a column cross-section of 8 in. The minimum column size of 3.0 x 3.0 in. (Table 5.1.1) used in this study is significantly lower than the Eurocode 2 limit. The size and spacing of transverse ties and the cover limitations imposed by ACI 318-95 and Eurocode 2 were not satisfied for some of the test columns studied.

### 5.1.2 Comparison of Design Methods with Test Results

ACI 318-95, Eurocode 2 and FEM methods were compared to the results of 354 reinforced concrete test columns. The comparison was made using the strength ratio which was defined as the ratio of the tested ultimate axial load strength to the computed ultimate axial load strength ($P_{test}/P_{des}$). The computed strengths ($P_{des}$) were based on the resistance factors or partial safety factors taken equal to unity and the measured strengths of the concrete and reinforcing steel.

A summary of the strength ratio ($P_{test}/P_{des}$) statistics computed for three different design methods for columns with $\ell/h$=3-30 and $e/h$=0-1.5 is given in Table 5.1.2. The design methods compared in this table include ACI 318-95, Eurocode 2 and FEM. Three different equations for flexural rigidity, $EI$, were used for computing ACI 318-95 column strengths (Equations 4.16, 4.17 and 4.19).

Table 5.1.2 gives the coefficient of variation, mean, maximum and minimum values of strength ratios for each of the different design methods. For the statistical analysis, the columns studied were divided into two groups: Group 1 considered all columns with a slenderness ratio greater than 3 but less than or equal to 30 that are subjected to pure axial load (end eccentricity ratio of zero); and Group 2 included all columns with a slenderness ratio greater than 3 but less than or equal to 30 that have an end eccentricity ratio greater then or equal to 0.1 but less than or equal to 1.5. Table 5.1.2 includes the statistics for a total of 311 columns. Columns having a slenderness ratio less than 3 (pedestals) or greater than 30 (super slender columns) or having an end eccentricity ratio greater than zero but less than 0.1 are not included in this table.

Table 5.1.2 - Summary of Strength Ratio Statistics for Different Design Methods for Reinforced Concrete Columns without Moment Gradient.

| Group Number (1) | Slenderness Ratio $\ell/h$ (2) | End Eccentricity Ratio e/h (3) | Number of Specimens (4) | ACI using Eq. 4.16 (5) | ACI using Eq. 4.17 (6) | Eurocode 2 (7) | ACI using Eq. 4.19 (8) | FEM (9) |
|---|---|---|---|---|---|---|---|---|
| **(a) Coefficient of Variation** | | | | | | | | |
| 1 | 3 - 30 | 0 | 151 | 0.245 | 0.154 | 0.170 | 0.144 | 0.118 |
| 2 | 3 - 30 | 0.1 - 1.5 | 160 | 0.125 | 0.134 | 0.148 | 0.119 | 0.132 |
| **(b) Mean Strength Ratio** | | | | | | | | |
| 1 | 3 - 30 | 0 | 151 | 1.137 | 1.093 | 1.150 | 1.090 | 0.982 |
| 2 | 3 - 30 | 0.1 - 1.5 | 160 | 1.042 | 1.042 | 1.067 | 1.037 | 1.001 |
| **(c) Maximum Value of Strength Ratio** | | | | | | | | |
| 1 | 3 - 30 | 0 | 151 | 2.555 | 1.958 | 1.811 | 1.527 | 1.323 |
| 2 | 3 - 30 | 0.1 - 1.5 | 160 | 1.561 | 1.702 | 1.814 | 1.521 | 1.356 |
| **(d) Minimum Value of Strength Ratio** | | | | | | | | |
| 1 | 3 - 30 | 0 | 151 | 0.685 | 0.685 | 0.746 | 0.685 | 0.659 |
| 2 | 3 - 30 | 0.1 - 1.5 | 160 | 0.627 | 0.627 | 0.596 | 0.627 | 0.632 |

Tables A1.1 to A1.5 in Appendix A provide a more detailed analysis of the strength ratio statistics for each of the design methods. The tables examine 5 groups of slenderness ratios as well as 5 groups of end eccentricity ratios for a total of 25 separate combinations. The tables include the number of test specimens in the group and the mean, coefficient of variation, minimum and maximum values of the strength ratios. All 354 columns are included in the statistics for these tables.

A review of Table 5.1.2 and Tables A1.1 through A1.5 leads to the following observations:

(1) The columns tested under pure axial load tended to have a higher coefficient of variation than columns tested with eccentric loading (Table 5.1.2). The columns under pure axial load are very sensitive to slight imperfections in the column or misalignments of the testing apparatus. Any resulting eccentricities can greatly affect the resulting test ultimate strength. Columns that are tested under eccentric loads tend to be less sensitive to these slight imperfections or misalignments.

(2) The coefficient of variation of the strength ratio using ACI 318-95 tends to be affected by the flexural rigidity equation used. The more simplified equation (Equation 4.16) produced a coefficient of variation of 24.5 and 12.5 percent for columns of Group 1 ($e/h$=0) and Group 2 ($e/h$=0.1–1.5), respectively (Column 5 of

Table 5.1.2). The more complex equation (Equation 4.19), suggested by Mirza (1990), produced a coefficient of variation of 14.4 and 11.9 percent for the same column groups (Column 8 of Table 5.1.2).

(3) ACI 318-95 uses the flexural rigidity, *EI*, only for slender column design. Hence, for short columns, ACI 318-95 with all three EI equations will produce identical results. This implies that slender columns under pure axial load would have a significantly higher coefficient of variation than those given in Columns 5, 6, and 8 of Table 5.1.2. This can be observed from Tables A1.1, A1.2 and A1.4. When Equations 4.16, 4.17 and 4.19 are used, ACI 318-95 produces a coefficient of variation of strength ratios equal to 33.9, 19.2 and 17.2 percent, respectively, for slender columns under pure axial load ($6.6 \leq \ell/h \leq 30$ and $e/h=0$), and a constant coefficient of variation of 10.5 percent with all three equations for short columns under pure axial load ($3<\ell/h<6.6$ and $e/h=0$). This is indicated by column 3 of Tables A1.1, A1.2 and A1.4.

(4) Eurocode 2 produces coefficients of variation of strength ratios which are similar to those produced by ACI 318-95 using Equation 4.17. FEM has consistently low and the least variable coefficients of variation

of strength ratios. This can be observed from Table 5.1.2 and Tables A1.3 and A1.5.

(5) All design methods produced mean strength ratios closer to or higher than 1.0, although FEM produced the lowest mean strength ratios.

Figure 5.1.1 shows the cumulative frequency distribution of strength ratios ($P_{test}/P_{des}$) for the different design methods plotted on a normal probability scale. The curves in Figure 5.1.1 represent the data for all 354 test columns. The curves for ACI 318-95 using Equations 4.16, 4.17, and 4.19, and Eurocode 2 follow one another fairly closely from the one-percentile to the 90-percentile values of the strength ratios, but become progressively more conservative beyond the 90-percentile values. The FEM produces the least conservative results, followed by ACI 318-95 using Equation 4.19 for $EI$. This is expected since the FEM and Equation 4.19 take into account cracking of the concrete and the nonlinear behavior of the concrete and steel at ultimate strength.

The one-percentile value for all methods is approximately 0.70 while the five-percentile value is approximately 0.79 for FEM and 0.85 for all other methods. Note that for establishing safety in design equations, the five-percentile and one-percentile values are more important than the mean value (Mirza 1990).

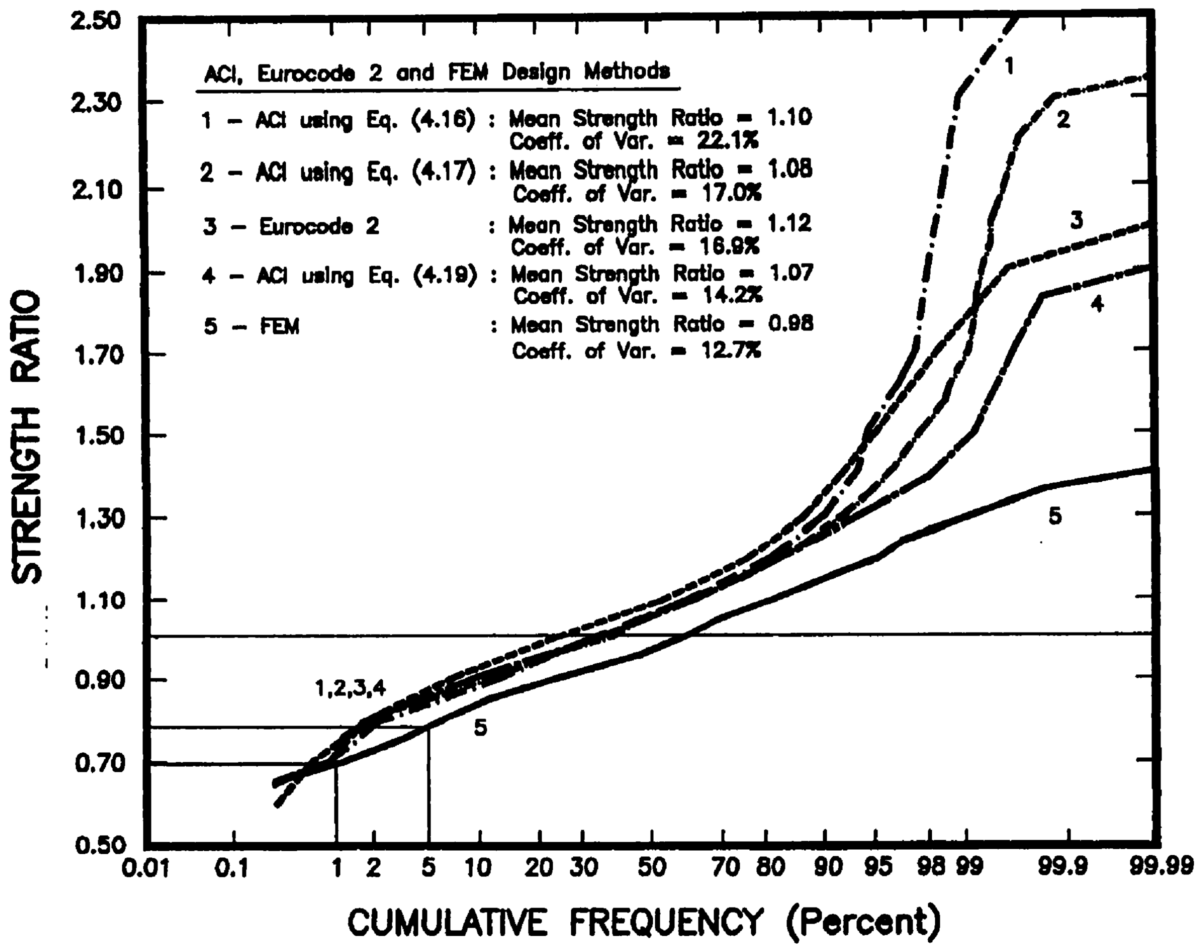



Figure 5.1.1 - Probability distribution of strength ratios of reinforced concrete columns without moment gradient for different design methods (n=354).

### 5.1.3 Effects of Major Variables on Strength Ratios

The effects of concrete strength ($f'_c$), end eccentricity ratio ($e/h$), slenderness ratio ($\ell/h$), reinforcing steel index ($\rho_{rs}f_{yr}/f'_c$), and tie/hoop volumetric ratio ($\rho''$) on the maximum, mean and minimum values of strength ratios ($P_{test}/P_{des}$) obtained using ACI 318-95, Eurocode 2 and FEM are examined in this section.

Since the values of concrete strength, end eccentricity ratio, slenderness ratio, reinforcing steel index and tie/hoop volumetric ratio used in this study were not controlled variables, strength ratios had to be grouped into ranges. Up to ten separate ranges were used in plotting each of the figures presented in this section. The maximum, mean and minimum strength ratios were determined for each of the ranges. Grouping the strength ratios resulted in having a significantly different number of columns in some of the ranges. This may explain the jaggedness of the lines associated with some of the figures presented in this section.

Figures 5.1.2.a and 5.1.2.b examine the effect of concrete strength ($f'_c$) on the maximum, mean, and minimum strength ratios. Because of the many different values of concrete strength used, ten ranges of concrete strength were set at 2500-2650, 2650-3300, 3300-3950, 3950-4600, 4600-5250, 5250-5900, 5900-6550, 6550-7200, 7200-7850, and

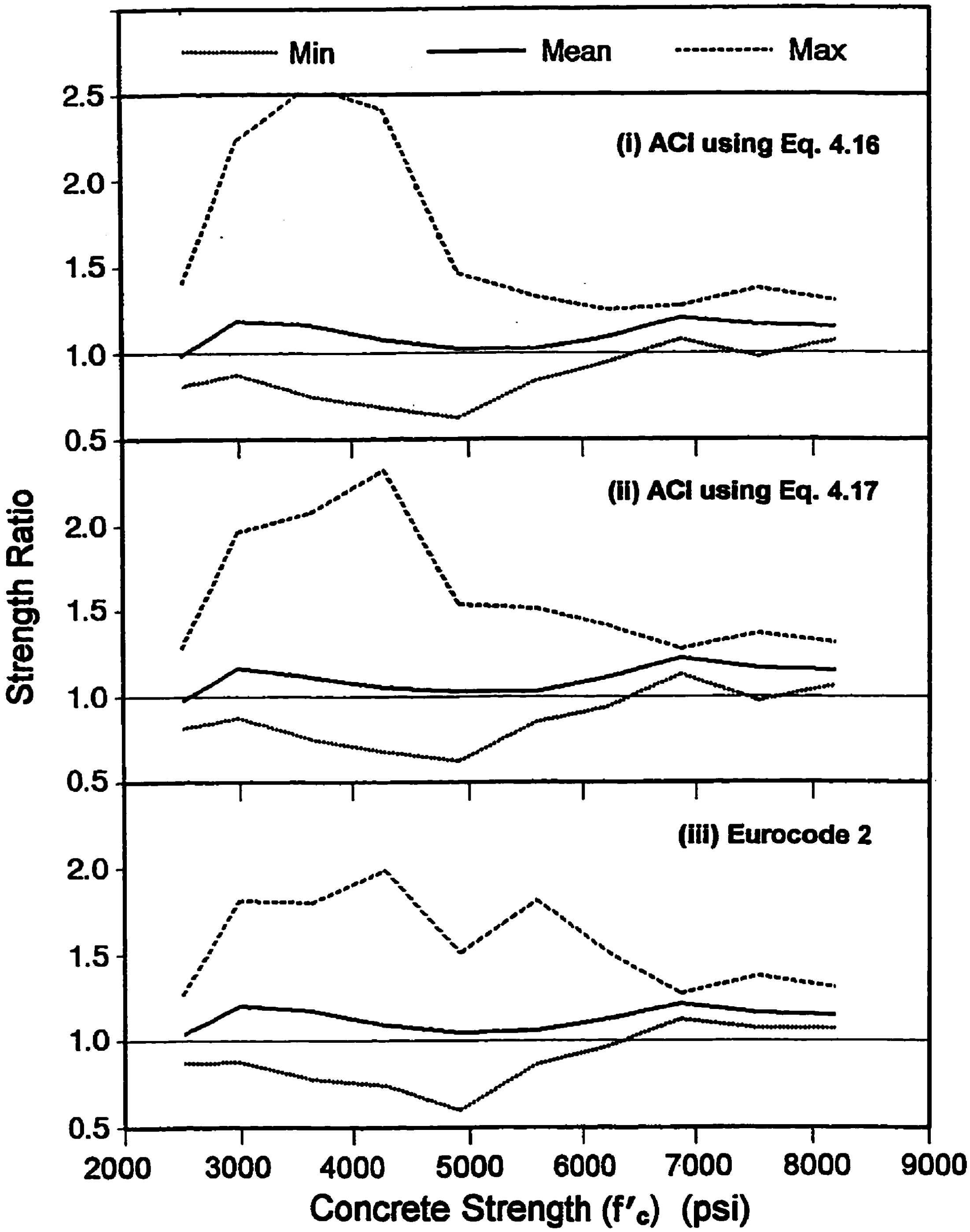



Figure 5.1.2.a - Effect of concrete strength on strength ratios obtained from different design methods for reinforced concrete columns without moment gradient (n varies for each $f'_c$; total number of specimens=354).

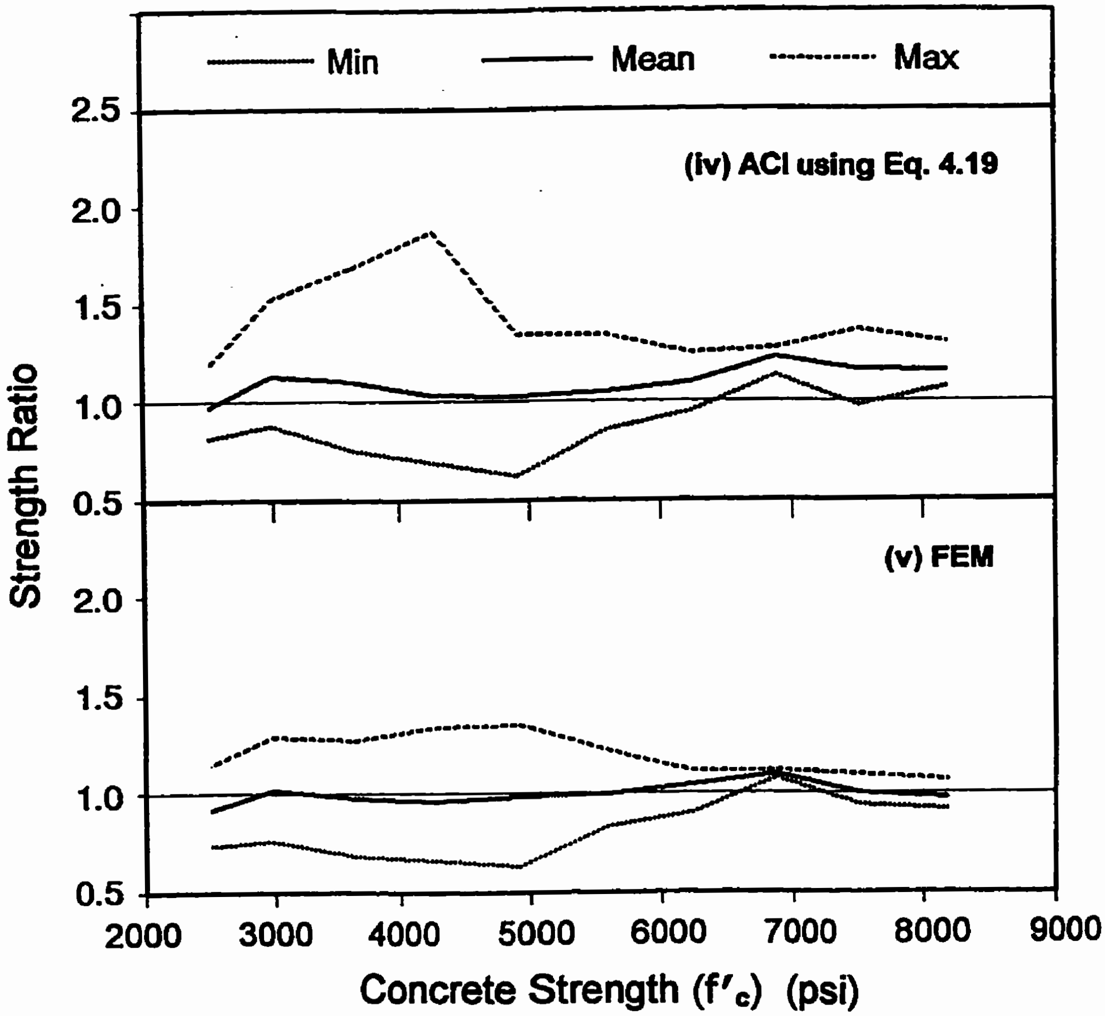



Figure 5.1.2.b - Effect of concrete strength on strength ratios obtained from different design methods for reinforced concrete columns without moment gradient (n varies for each $f'_c$; total number of specimens=354).

7850-8300 psi. It can be seen from these figures that, as the concrete strength increases, the differences between the maximum and minimum strength ratios decrease for all design methods. ACI 318-95 using both Equation 4.16 and 4.17 tends to produce the greatest differences between the maximum and minimum strength ratios for concrete strengths $f'_c$<5000 psi. The differences between the maximum and minimum values of strength ratios for Eurocode 2 and ACI 318-95 using Equation 4.19 were less significant. The smallest differences between the maximum and minimum strength ratios over the entire range of concrete strengths were obtained for FEM. Also, the mean strength ratio for FEM tended to follow closely a value of 1.0 over the entire range of $f'_c$ as opposed to the other methods that had mean strength ratios generally above 1.0. The minimum strength ratio curve was similar in shape and magnitude over the entire range of concrete strengths for all design methods.

Figures 5.1.3.a and 5.1.3.b examine the effect of the end eccentricity ratio $(e/h)$, on the maximum, mean, and minimum strength ratios. Because of the many different end eccentricity ratios used, eight ranges of end eccentricity ratio were set at 0, 0-0.2, 0.2-0.4, 0.4-0.6, 0.6-0.8, 0.8-1.0, 1.0-1.2 and 1.2-1.3. All methods, except FEM, produced large differences between maximum and minimum strength ratios at low end eccentricity ratios (Figures 5.1.3.a and 5.1.3.b). ACI 318-95 using Equation 4.19

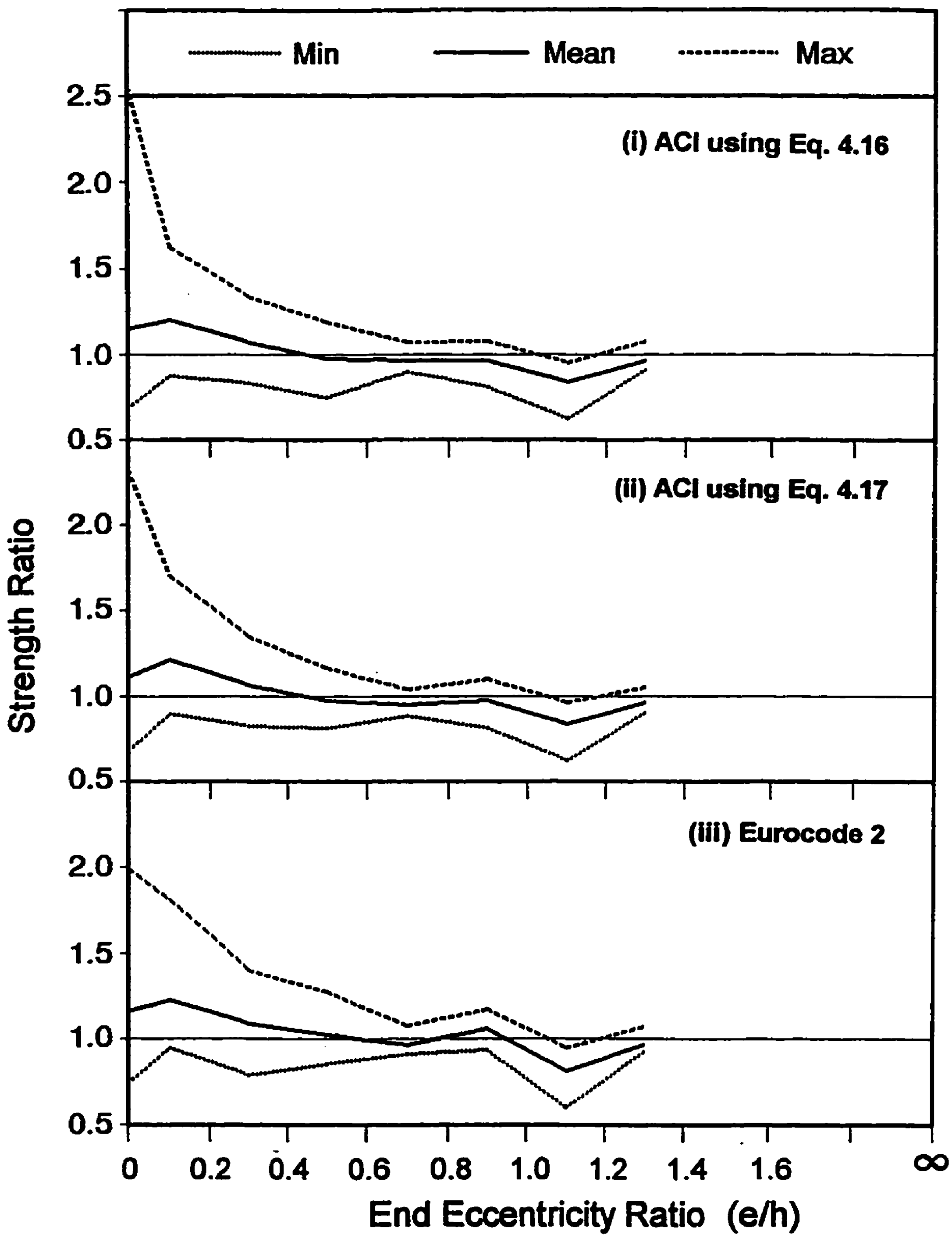



Figure 5.1.3.a - Effect of end eccentricity ratio on strength ratios obtained from different design methods for reinforced concrete columns without moment gradient (n varies for each e/h ratio; total number of specimens=354)

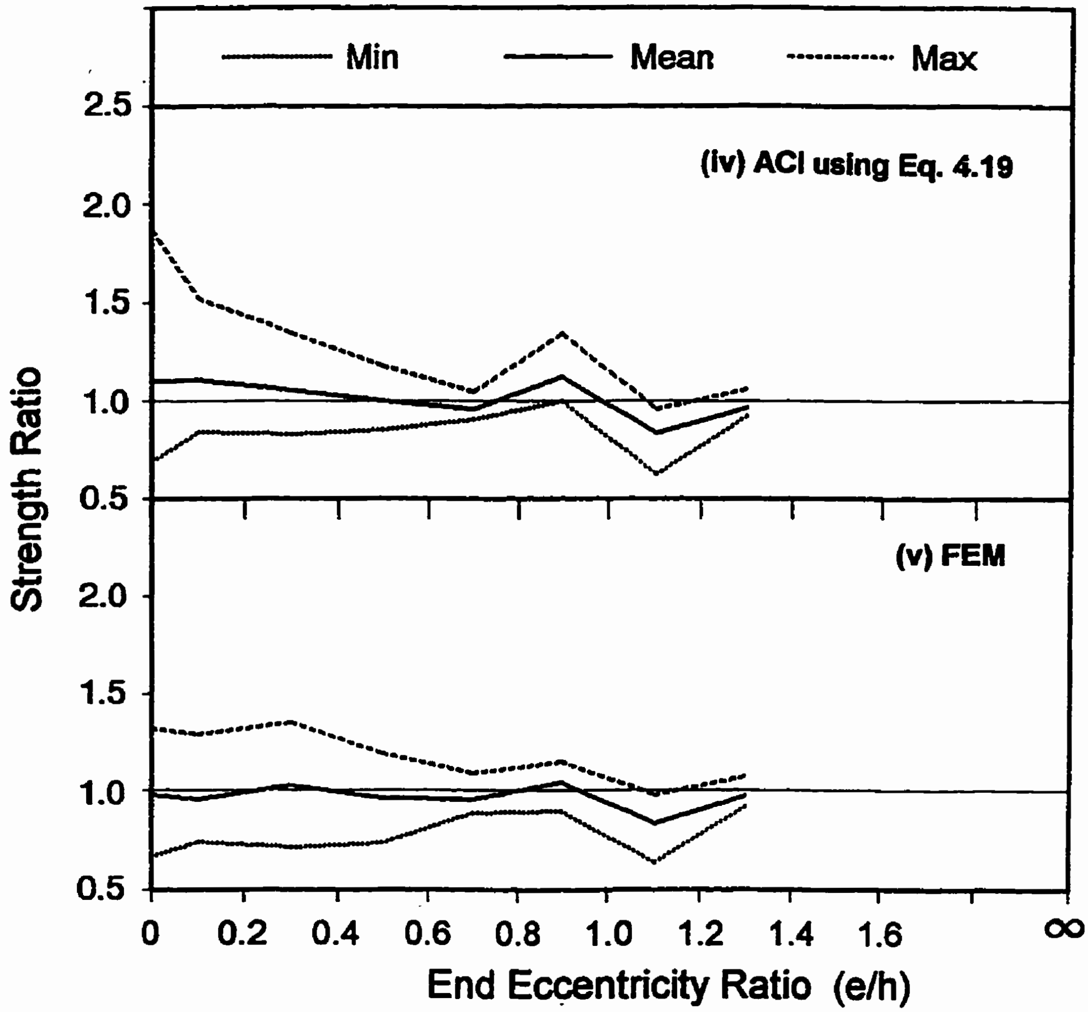



Figure 5.1.3.b - Effect of end eccentricity ratio on strength ratios obtained from different design methods for reinforced concrete columns without moment gradient (n varies for each e/h ratio; total number of specimens=354)

reduced the difference between the maximum and minimum strength ratios significantly in this region of *e/h* since Equation 4.19 includes the effect of *e/h* on *EI*. As the end eccentricity ratio increased, the difference between the maximum and minimum strength ratios tended to decrease for all methods. However, the mean strength ratios tended to become less conservative for increasing *e/h* values. Note that the FEM method produced the most consistent results over the entire *e/h* range as can be seen from Figures 5.1.3.a and 5.1.3.b. The minimum strength ratio curve was similar in shape and magnitude over the entire range of end eccentricity ratios for all design methods.

Figures 5.1.4.a and 5.1.4.b examine the effect of the slenderness ratio ($\ell/h$), on the maximum, mean and minimum strength ratios. Because of the many different slenderness ratios used, nine ranges of slenderness ratios were set at 2-4.5, 4.5-9, 9-13.5, 13.5-18, 18-22.5, 22.5-27, 27-31.5, 31.5-36 and 36-40. From the figures it can be seen that the slenderness ratio affects the strength ratios for all design methods. ACI 318-95 using Equation 4.16 and 4.17 tends to produce strength ratios that become more variable as the slenderness ratio increases, whereas ACI 318-95 using Equation 4.19 produces significantly better results. Eurocode 2 produced strength ratios that did not show any significant improvements over the strength ratios obtained from ACI 318-95. However, the strength ratios obtained

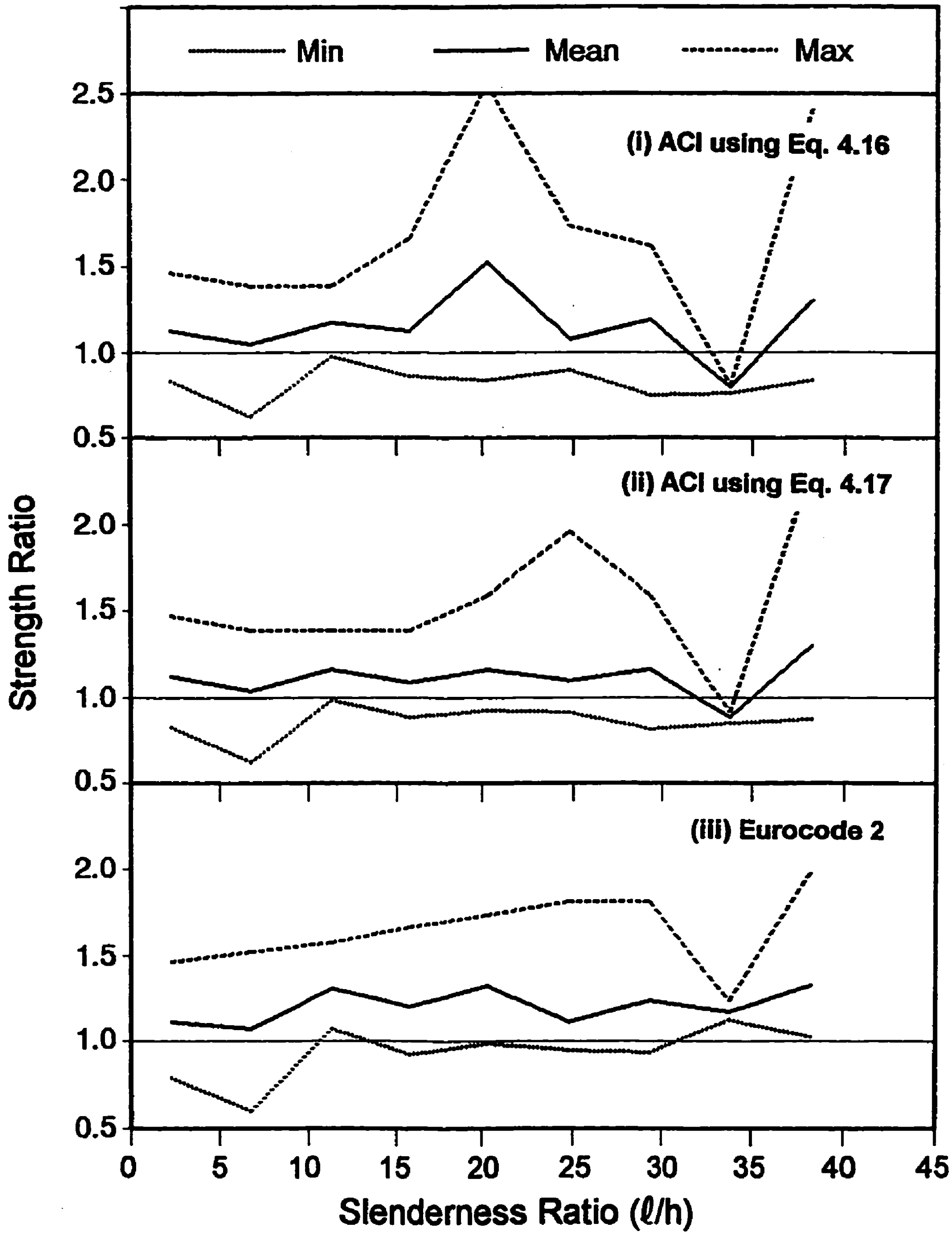



Figure 5.1.4.a - Effect of slenderness ratio on strength ratios obtained from different design methods for reinforced concrete columns without moment gradient (n varies for each ℓ/h ratio; total number of specimens=354).

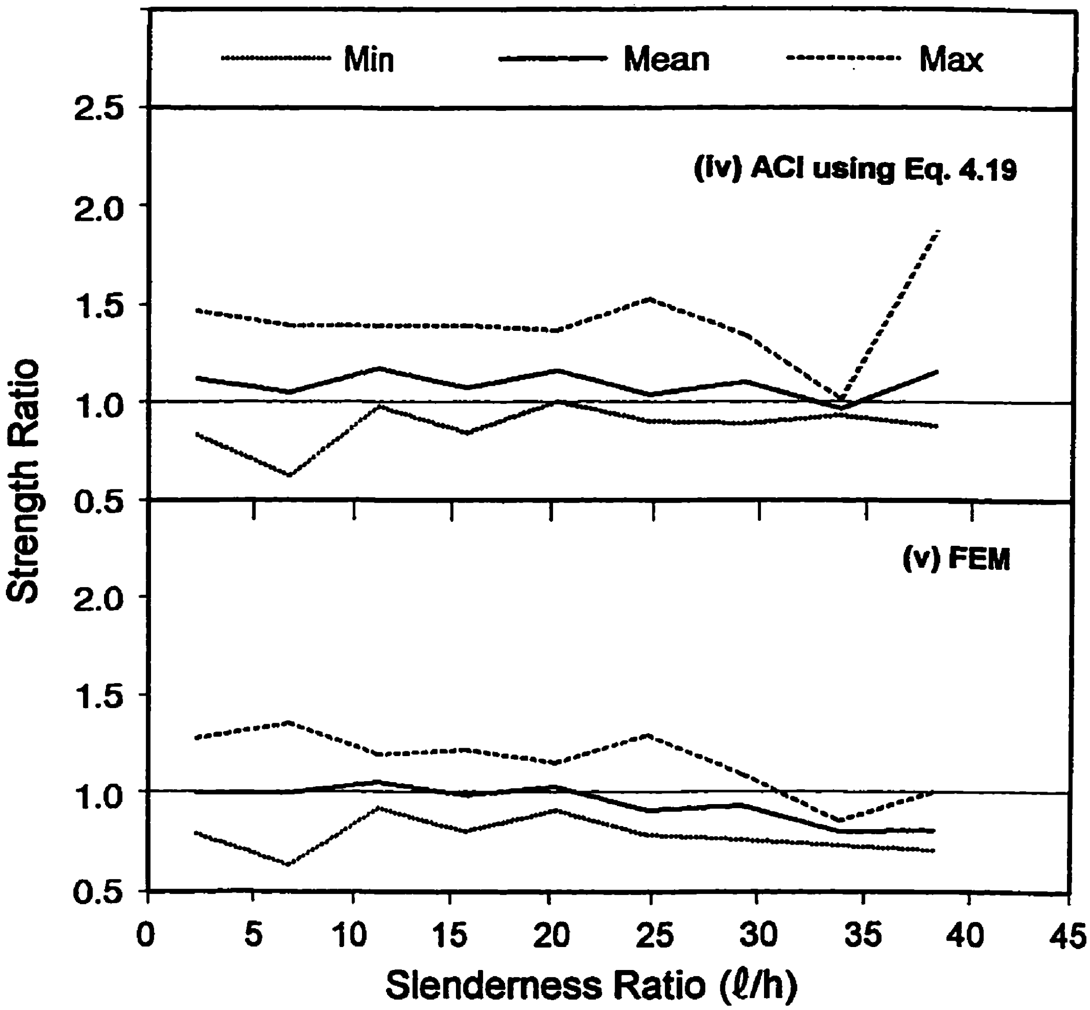



Figure 5.1.4.b - Effect of slenderness ratio on strength ratios obtained from different design methods for reinforced concrete columns without moment gradient (n varies for each ℓ/h ratio; total number of specimens=354).

from FEM demonstrated low variations over the entire range of $\ell/h$ values. For slenderness ratios less than 30, all design methods, except FEM, produce conservative values of mean strength ratios. The mean strength ratios determined from FEM tended to become less conservative as the slenderness ratio increased and were less than 1.0 at $\ell/h \geq 20$. The minimum strength ratio curve was similar in shape over the entire range of slenderness ratios for all design methods.

Figures 5.1.5.a and 5.1.5.b examine the effect of reinforcing steel index ($\rho_{rs} f_{yr}/f'_c$) on the maximum, mean, and minimum strength ratios. Because of the many different reinforcing steel index values used, ten ranges of reinforcing steel indices were set at 0-0.11, 0.11-0.22, 0.22-0.33, 0.33-0.44, 0.44-0.55, 0.55-0.66, 0.66-0.77, 0.77-0.88, 0.88-0.99, 0.99-1.1. The zigzag nature of the plots for strength ratios produced by ACI 318-95 using Equation 4.16 is, probably, caused by the grouping of the strength ratios and due to the fact that the contribution of the reinforcing steel is not included in Equation 4.16 used for calculating the flexural rigidity, *EI*. Equation 4.17 used in ACI 318-95 for computing *EI* includes the contribution of the reinforcing steel and can be seen to improve the results plotted in Figure 5.1.5.a. The results for Eurocode 2 and ACI 318-95 using Equation 4.19 improved as the reinforcing steel index increased. FEM had the most

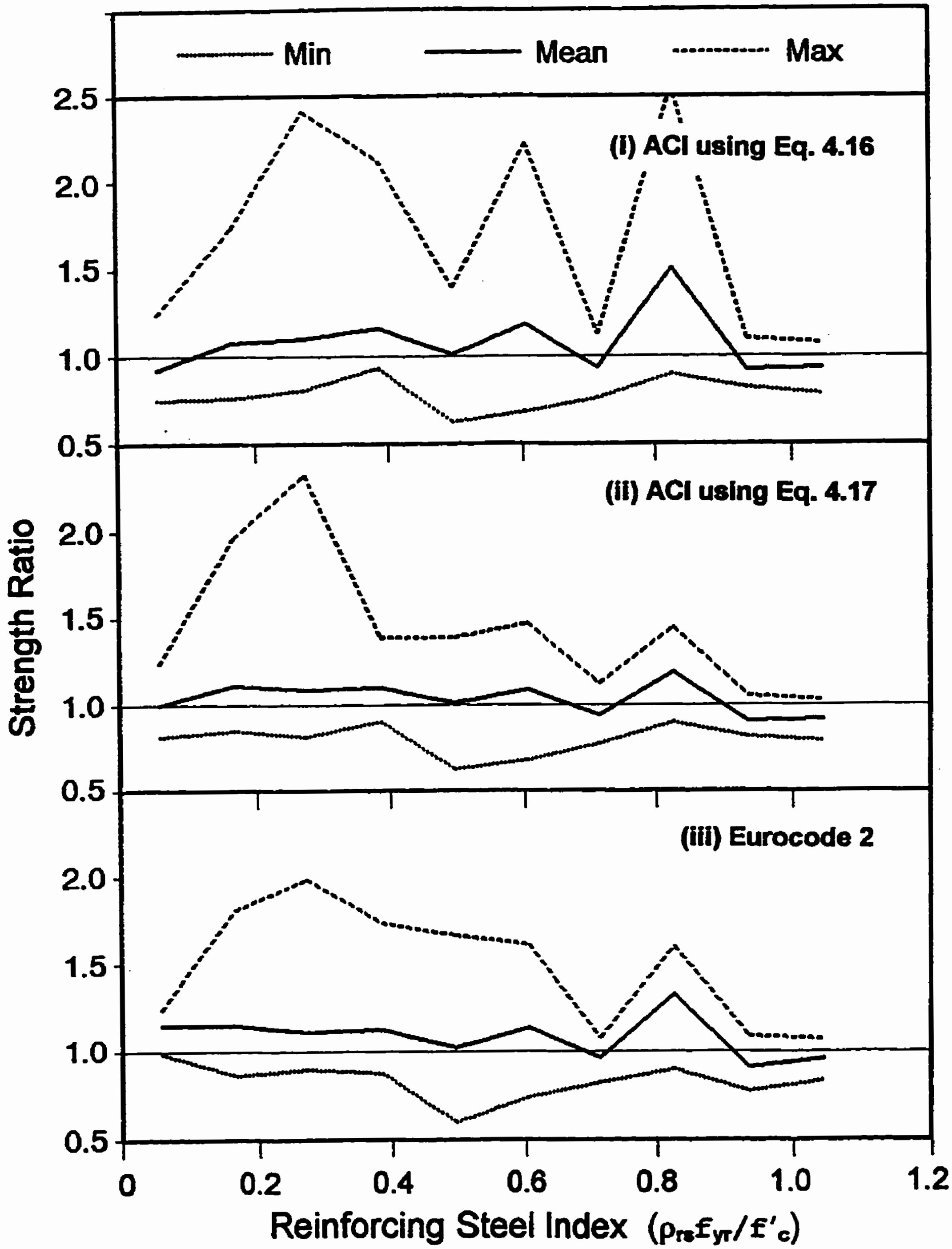



Figure 5.1.5.a - Effect of reinforcing steel index on strength ratios obtained from different design methods for reinforced concrete columns without moment gradient (n varies for each $\rho_{rs}f_{yr}/f'_c$ ratio; total number of specimens=354)

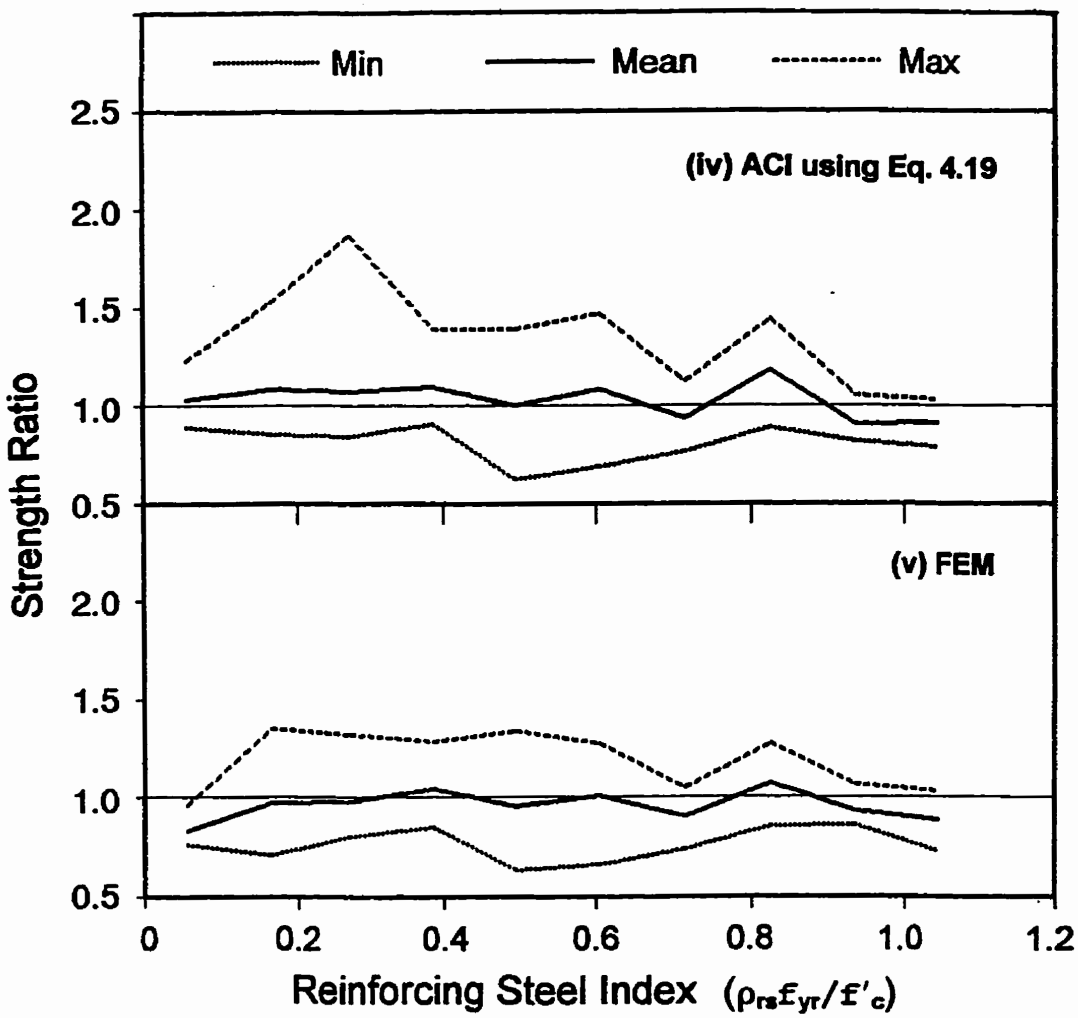



Figure 5.1.5.b - Effect of reinforcing steel index on strength ratios obtained from different design methods for reinforced concrete columns without moment gradient (n varies for each $\rho_{rs}f_{yr}/f'_c$ ratio; total number of specimens=354)

consistent and narrow range of strength ratios over the entire range of $\rho_{rs}f_{yr}/f'_c$ values. The minimum value curve was similar in shape and magnitude over the entire range of the reinforcing steel index for all design methods.

Figures 5.1.6.a and 5.1.6.b examine the effect of the tie/hoop volumetric ratio ($\rho''$) on the maximum, mean, and minimum strength ratios. This ratio is defined as the volume of transverse ties or hoops divided by the volume of concrete core (out-to-out of ties or hoops) over a unit length of a column. The equation used to compute the tie/hoop volumetric ratio is presented as a footnote in Table 3.1.1. A tie or hoop is the transverse reinforcement used to tie and restrain the longitudinal reinforcing steel and has a confining effect on the concrete core it surrounds. Ties and hoops are used interchangeably in this report in order to remain consistent with the literature. Because of the many different tie/hoop volumetric ratios used, eight ranges of tie/hoop volumetric ratios were set at 0-0.005, 0.005-0.01, 0.01-0.015, 0.015-0.02, 0.02-0.025, 0.025-0.03, 0.03-0.035, 0.035-0.04. It can be seen that as the tie/hoop volumetric ratio increases, the difference between the maximum and minimum strength ratios decreases for all design methods. This may be due, perhaps, to the limited number of test columns with high tie/hoop volumetric ratios. ACI 318-95 using Equation 4.16 and 4.17 tends to produce the greatest differences between the

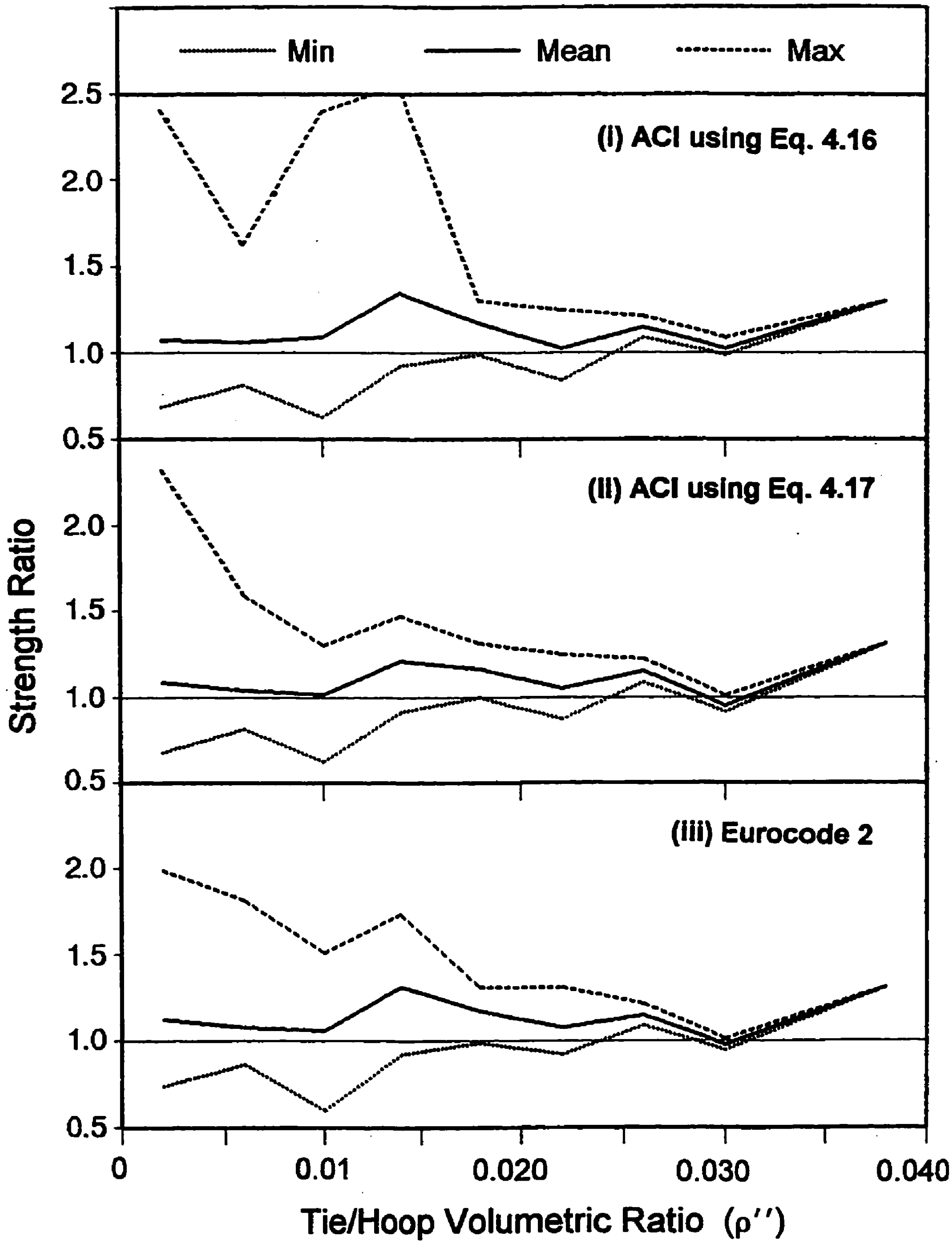



Figure 5.1.6.a - Effect of tie/hoop volumetric ratio on strength ratios obtained from different design methods for reinforced concrete columns without moment gradient (n varies for each ρ'' ratio; total number of specimens=354

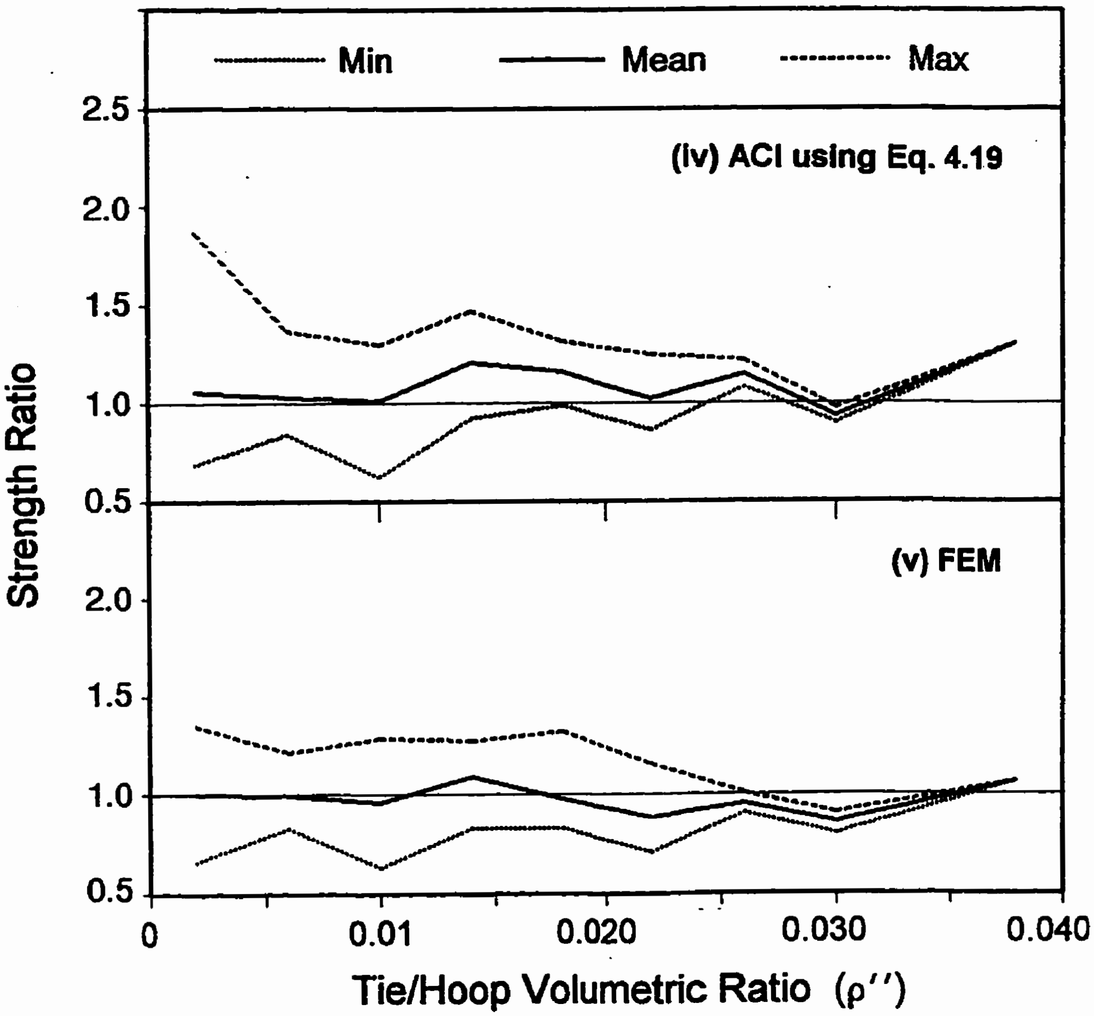



Figure 5.1.6.b - Effect of tie/hoop volumetric ratio on strength ratios obtained from different design methods for reinforced concrete columns without moment gradient (n varies for each ρ'' ratio; total number of specimens=354

maximum and minimum strength ratios for tie/hoop volumetric ratios $\rho'' \leq 0.02$. Less significant differences between the maximum and minimum strength ratios were observed for Eurocode 2 and for ACI 318-95 using Equation 4.19. The smallest differences between the maximum and minimum strength ratios over the entire range of tie/hoop volumetric ratios were observed for FEM. However, the mean strength ratio determined from FEM tended to be less than 1.0 over almost the entire range of $\rho''$ plotted. The minimum value curve was similar in shape and magnitude for all design methods.

The following conclusions can be summarized from the data plotted in Figures 5.1.2 to 5.1.6 and the related discussion:

(1) The strength ratios produced by ACI 318-95 using Equation 4.16 demonstrated a pronounced effect of most of the major variables investigated.

(2) ACI 318-95 using Equation 4.17 demonstrated an improved effect on the strength ratio variations due to all of the major variables examined when compared to the results obtained from ACI 318-95 using Equation 4.16.

(3) The strength ratios produced by ACI 318-95 using Equation 4.19 demonstrated significantly improved statistics as compared to those obtained using both Equations 4.16 and 4.17. This was expected since

Equation 4.19 for *EI* takes into account the effect of *e/h* ratio as a function of cracking in concrete caused by the presence of bending moment.

(4) The strength ratios obtained for Eurocode 2 were affected by most of the variables investigated. The results for Eurocode 2 tended to follow similar trends as those observed for ACI 318-95 using Equation 4.17.

(5) The FEM produced the most consistent results that tended to be least affected by the variables examined. However, at slenderness ratios $\ell/h>20$ the mean strength ratios decreased below 1.0.

(6) The minimum strength ratio curves for all methods tended to follow a similar trend for all variables examined in this study. No significant differences in this region were noticeable for any of the design methods.

## 5.2 REINFORCED CONCRETE COLUMNS WITH MOMENT GRADIENT

Fourteen reinforced concrete test columns with moment gradient found in the literature were used to compare the test strengths with those obtained from ACI 318-95, Eurocode 2 and FEM methods.

### 5.2.1 Description of Column Tests Available in the Literature

A complete description of the 14 reinforced concrete test columns with moment gradient found in the literature is presented in Chapter 3.

Each column used had a different combination of geometric and material properties. The maximum and minimum values of the overall cross-section dimensions (*bxh*), the nominal concrete strengths ($f'_c$), the end eccentricity ratio ($e/h$), the slenderness ratio ($\ell/h$), the reinforcing steel yield stress ($f_{yr}$), the reinforcing steel ratio ($\rho_{rs}$), the tie/hoop volumetric ratio ($\rho''$), and the ratio of smaller to larger end moment ($M_1/M_2$) are listed in Table 5.2.1. The values shown in the table represent the maximum and minimum values of the data; a detailed description of each column is given in Table 3.1.4.

Due to the limited number of test columns available in the literature, several of the limitations specified by ACI 318-95 and Eurocode 2 were not used in this study. ACI 318-95 specifies an upper limit of 30 on $\ell/h$ ratio which is significantly lower than the maximum value of 40 used in this study (Table 5.2.1). ACI 318-95 also imposes a limit of 80% of the pure axial load capacity on the applied axial loads acting on tied columns. Eurocode limits the minimum transverse dimension of a column to 8.0 in. which is

Table 5.2.1 - Summary of Geometric and Material Properties of Reinforced Concrete Column Specimens with Moment Gradient*

| Properties | Minimum Values | Maximum Values |
|---|---|---|
| bxh (in. x in.) | 4.4 x 2.5 | 10.0 x 8.0 |
| $f'_c$ (psi) | 3630 | 5814 |
| e/h | 0.17 | 1.50 |
| $\ell/h$ | 14.5 | 40.0 |
| $f_{yr}$ (psi) | 40000 | 66112 |
| $\rho_{rs}$ (%) | 1.11 | 4.00 |
| $\rho''$ (%) | 0.358 | 0.781 |
| $M_1 / M_2$ | -1.00 | 0.99 |

* Number of Specimens = 14; h = depth of the concrete cross-section perpendicular to the axis of bending; b = width of the concrete cross-section parallel to the axis of bending; $M_1$ = the smaller end moment, positive if member is bent in single curvature, negative if bent in double curvature; $M_2$ = the larger end moment, always positive.

Note: 1.0 in. = 25.4 mm; 1000 psi = 6.895 MPa

greater than the minimum column size of 4.4 x 2.5 in. used in this study (Table 5.2.1).

### 5.2.2 Comparison of Design Methods with Test Results

ACI 318-95, Eurocode 2 and FEM methods were compared to the results of 14 reinforced concrete test columns. The comparison was made based on the strength ratio which is defined as the ratio of the tested ultimate axial load strength to the computed ultimate axial load strength ($P_{test}/P_{des}$). The computed strengths ($P_{des}$) were based on the resistance factors or partial safety factors taken equal to unity and the measured strengths of the concrete and reinforcing steel. A summary of the strength ratio ($P_{test}/P_{des}$) statistics computed for the three different design methods is given in Table 5.2.2. Note that three different equations for flexural rigidity, *EI*, were used for computing ACI 318-95 strengths (Equations 4.16, 4.17 and 4.19).

The eight Group 3 columns had a slenderness ratio greater than 14 but less than 30 with an end eccentricity ratio greater than 0.1 but less than or equal to 1.5. An examination of statistics for these columns in Table 5.2.2 (Group 3) leads to the following observations:

(1) The coefficient of variation of the strength ratios for ACI 318-95 was significantly affected by the flexural rigidity equation used: 42.5 percent when

Table 5.2.2 - Summary of Strength Ratio Statistics for Different Design Methods for Reinforced Concrete Columns with Moment Gradient.

| Group Number (1) | Slenderness Ratio $\ell/h$ (2) | End Eccentricity Ratio e/h (3) | Number of Specimens (4) | ACI using Eq. 4.16 (5) | ACI using Eq. 4.17 (6) | Eurocode 2 (7) | ACI using Eq. 4.19 (8) | FEM (9) |
|---|---|---|---|---|---|---|---|---|
| | | | | (a) Coefficient of Variation | | | | |
| 3 | 14 - 30 | 0.1 - 1.5 | 8 | 0.425 | 0.217 | 0.171 | 0.181 | 0.115 |
| 4 | 40 | 0.2 - 0.4 | 6 | 0.090 | 0.100 | 0.093 | 0.085 | 0.073 |
| | | | | (b) Mean Strength Ratio | | | | |
| 3 | 14 - 30 | 0.1 - 1.5 | 8 | 1.346 | 1.259 | 1.271 | 1.247 | 1.019 |
| 4 | 40 | 0.2 - 0.4 | 6 | 1.776 | 1.751 | 1.711 | 1.716 | 0.982 |
| | | | | (c) Maximum Value of Strength Ratio | | | | |
| 3 | 14 - 30 | 0.1 - 1.5 | 8 | 2.260 | 1.673 | 1.616 | 1.587 | 1.249 |
| 4 | 40 | 0.2 - 0.4 | 6 | 2.037 | 2.059 | 1.980 | 1.908 | 1.060 |
| | | | | (d) Minimum Value of Strength Ratio | | | | |
| 3 | 14 - 30 | 0.1 - 1.5 | 8 | 0.804 | 0.893 | 1.057 | 0.972 | 0.945 |
| 4 | 40 | 0.2 - 0.4 | 6 | 1.611 | 1.535 | 1.525 | 1.556 | 0.889 |

Equation 4.16 was used to 21.7 percent when Equation 4.17 was employed for computing *EI*. Equation 4.19, proposed by Mirza (1990), showed an improvement over the other two equations with a coefficient of variation of strength ratios equal to 18.1 percent.

(2) The coefficient of variation of the strength ratios for Eurocode 2 (17.1%) was slightly lower than that for ACI 318-95 using Equation 4.19, while FEM produced the lowest coefficient of variation of the strength ratios (11.5%).

(3) The mean values of strength ratios for all design methods, except FEM, were relatively high. There was no significant difference in the mean values of the strength ratios for Eurocode 2 and ACI 318-95 using Equation 4.17 and 4.19.

For the six columns with $\ell/h$=40 and $e/h$=0.2-0.4 (Group 4), there appears to be no significant difference between strength statistics for columns obtained for different design methods, with the exception of FEM. The statistics obtained using FEM were similar for both Group 3 and Group 4 columns, as indicated in Table 5.2.2.

The observations noted in the foregoing paragraphs cannot be considered conclusive because of the limited number of tests (merely 14) available from the literature for reinforced concrete columns subjected to moment gradient.

# 6 - COMPARISON OF DESIGN METHODS WITH TEST RESULTS FOR COMPOSITE STEEL-CONCRETE COLUMNS SUBJECTED TO MAJOR AXIS BENDING

For the comparative study, composite steel-concrete columns subjected to major axis bending were divided into two separate groups: columns with equal and opposite applied end moments (i.e. symmetric single curvature bending) and columns with unequal applied end moments (i.e. moment gradient).

## 6.1 COMPOSITE STEEL-CONCRETE COLUMNS SUBJECTED TO MAJOR AXIS BENDING WITHOUT MOMENT GRADIENT

In an attempt to study a full range of variables, 69 composite steel-concrete test columns subjected to major axis bending without moment gradient and found in the literature were used to compare the test strengths with those obtained from ACI 318-95, AISC-LRFD Specifications, Eurocode 4 and FEM methods. Note that ACI 318-95 strengths were computed in three different ways: (a) using Equation 4.16, (b) using Equation 4.18, and (c) using Equation 4.20. Similarly, the AISC-LRFD strengths were computed in two different ways: (a) using the radius of gyration as specified in the AISC-LRFD Specifications, and (b) using the radius of gyration with the upper limit of Equation 4.38.

### 6.1.1 Description of Column Tests Available in the Literature

A complete description of 75 composite steel-concrete test columns subjected to major axis bending without moment gradient and found in the literature is presented in Chapter 3. Six of these columns were not used for comparison because of the incomplete or insufficient information available. These columns are identified with an asterisk in Table 3.2.1.

Each column used had a different combination of geometric and material properties. The maximum and minimum values of the overall cross-section dimensions ($bxh$), the nominal concrete strength ($f'_c$), the end eccentricity ratio ($e/h$), the slenderness ratio ($\ell/h$), the structural steel section yield stress ($f_{yss}$), the reinforcing steel yield stress ($f_{yr}$), the structural steel ratio ($\rho_{ss}$), the reinforcing steel ratio ($\rho_{rs}$), and the tie/hoop volumetric ratio ($\rho''$) are listed in Table 6.1.1. The values shown in the table represent the maximum and minimum values of the data; a detailed description of each column is given in Table 3.2.1.

In order to study the effects of variables over a broad range, several of the limitations specified by ACI 318-95, AISC-LRFD and Eurocode 4 design methods were not used in this study. ACI 318-95 has a limit of 85 percent

Table 6.1.1 - Summary of Geometric and Material Properties of Composite Steel-Concrete Column Specimens Subjected to Major Axis Bending without Moment Gradient *

| Properties | Minimum Values | Maximum Values |
|---|---|---|
| bxh (in. x in.) | 6.3 x 6.3 | 11.8 x 11.8 |
| $f'_c$ (psi) | 3060 | 8093 |
| e/h | 0.00 | 1.06 ** |
| $\ell/h$ | 2.0 | 28.9 |
| $f_{yss}$ (psi) | 37813 | 113686 |
| $f_{yr}$ (psi) | 48520 | 60919 |
| $\rho_{ss}$ (%) | 2.90 | 14.45 |
| $\rho_{rs}$ (%) | 0.00 | 0.79 |
| $\rho''$ (%) | 0.000 | 2.915 |

* Number of Specimens = 69; h = depth of the concrete cross-section perpendicular to the axis of bending; b = width of the concrete cross-section parallel to the axis of bending.

** 16 specimens were tested under pure bending ($e/h=\infty$)

Note: 1.0 in. = 25.4 mm; 1000 psi = 6.895 MPa

of the pure axial load capacity on the applied axial loads acting on composite columns. ACI 318-95 also imposes an upper limit on the structural steel section yield strength at 50,000 psi which is significantly lower than the maximum value of 113,686 psi used in this study. In addition, ACI 318-95 has a lower limit on the longitudinal reinforcing steel ratio of one percent which is above the minimum value of zero (Table 6.1.1). Similarly, AISC-LRFD Specifications have a lower limit on the structural steel ratio of 4 percent and an upper limit on the useable structural steel and longitudinal reinforcing steel yield strength of 55,000 psi. The use of Equation 4.20 for ACI 318-95 and Equation 4.38 for the AISC-LRFD Specifications have two further limitations which include: (a) the structural steel ratio must be between 4 and 10 percent, and (b) the end eccentricity ratio be greater than or equal to 0.1. Both of these limits were not applied in this study. Eurocode 4 imposes an upper limit on the concrete strength of 7300 psi which is not significantly different from the maximum value of 8093 psi used in this study. The minimum reinforcing steel ratio imposed by Eurocode 4 is 0.3 percent which is above the minimum value of zero (Table 6.1.1). Eurocode 4 limits the minimum transverse dimension of a column at 8 in. which is not significantly different from the minimum column size of 6.3 x 6.3 in. used in this study. The size and spacing of transverse ties for ACI 318-95, AISC-LRFD

Specifications and Eurocode 4 were not satisfied for some of the test columns used in this study.

### 6.1.2 Comparison of Design Methods with Test Results

ACI 318-95, AISC-LRFD, Eurocode 4 and FEM methods were compared to the results of 69 composite steel-concrete test columns subjected to major axis bending. The comparison was made using the strength ratio which was defined as the ratio of the tested ultimate strength to the computed ultimate strength. For columns with $e/h<\infty$, the ultimate strength was taken as the ultimate axial load strength, and for columns with $e/h=\infty$, the ultimate strength was taken as the ultimate bending moment strength. The computed strengths were based on the resistance factors or partial safety factors taken equal to unity and the measured strengths of the concrete, reinforcing steel, and structural steel.

A summary of the strength ratio statistics computed for four different design methods for columns with $\ell/h$=2.9-30 and $e/h$=0-$\infty$ is given in Table 6.1.2. The design methods compared in this table include ACI 318-95, AISC-LRFD Specifications, Eurocode 4 and FEM. Three different equations for flexural rigidity, *EI*, were used for computing ACI 318-95 column strengths (Equations 4.16, 4.18 and 4.20). Two different values for the radius of

Table 6.1.2 - Summary of Strength Ratio Statistics for Different Design Methods for Composite Steel-Concrete Columns Subjected to Major Axis Bending without Moment Gradient.

| Group Number (1) | Slenderness Ratio $\ell/h$ (2) | End Eccentricity Ratio e/h (3) | Number of Specimens (4) | ACI using Eq. 4.16 (5) | ACI using Eq. 4.18 (6) | Eurocode 4 (7) | AISC-LRFD (8) | AISC-LRFD using Eq. 4.38 (9) | ACI using Eq. 4.20 (10) | FEM (11) |
|---|---|---|---|---|---|---|---|---|---|---|
| **(a) Coefficient of Variation** | | | | | | | | | | |
| 1 | 11-12 | 0 | 3 | 0.071 | 0.071 | 0.071 | 0.056 | 0.056 | 0.071 | 0.072 |
| 2 | 3 - 30 | 0.1 - 1.5 | 30 | 0.193 | 0.153 | 0.171 | 0.226 | 0.218 | 0.150 | 0.135 |
| 3 | 2.9 | $\infty$ | 16 | 0.116 | 0.116 | 0.091 | 0.080 | 0.080 | 0.116 | 0.063 |
| **(b) Mean Strength Ratio** | | | | | | | | | | |
| 1 | 11-12 | 0 | 3 | 0.858 | 0.858 | 0.942 | 1.438 | 1.438 | 0.858 | 0.818 |
| 2 | 3 - 30 | 0.1 - 1.5 | 30 | 1.160 | 1.006 | 1.083 | 1.354 | 1.382 | 1.020 | 0.948 |
| 3 | 2.9 | $\infty$ | 16 | 1.147 | 1.147 | 1.106 | 0.904 | 0.904 | 1.147 | 1.047 |
| **(c) Maximum Value of Strength Ratio** | | | | | | | | | | |
| 1 | 11-12 | 0 | 3 | 0.898 | 0.898 | 0.990 | 1.485 | 1.485 | 0.898 | 0.853 |
| 2 | 3 - 30 | 0.1 - 1.5 | 30 | 1.781 | 1.503 | 1.441 | 1.951 | 1.972 | 1.503 | 1.149 |
| 3 | 2.9 | $\infty$ | 16 | 1.417 | 1.417 | 1.295 | 1.039 | 1.039 | 1.417 | 1.146 |
| **(d) Minimum Value of Strength Ratio** | | | | | | | | | | |
| 1 | 11-12 | 0 | 3 | 0.788 | 0.788 | 0.866 | 1.346 | 1.346 | 0.788 | 0.750 |
| 2 | 3 - 30 | 0.1 - 1.5 | 30 | 0.908 | 0.777 | 0.813 | 0.902 | 0.906 | 0.790 | 0.710 |
| 3 | 2.9 | $\infty$ | 16 | 0.973 | 0.973 | 0.966 | 0.777 | 0.777 | 0.973 | 0.914 |

gyration, $r$, were used for computing the AISC-LRFD column strengths.

Table 6.1.2 gives the coefficient of variation, mean, maximum and minimum values of the strength ratios. For the statistical analysis, the columns studied were divided into three groups: Group 1 considered all columns having an end eccentricity ratio of zero (pure axial load) and slenderness ratio of 11-12 (i.e. $3<\ell/h\leq30$); Group 2 included all columns with an end eccentricity ratio greater than or equal to 0.1 but less than or equal to 1.5 and with a slenderness ratio greater than 3 but less than or equal to 30; and Group 3 considered all columns having an end eccentricity ratio of infinity (pure bending) and a slenderness ratio of 2.9 (i.e. $\ell/h<3$). Table 6.1.2 includes the statistics for a total of 49 columns. The remaining 20 columns having an end eccentricity ratio of zero and a slenderness ratio less than 3 (pedestals) are not included in this table.

Tables B1.1 to B1.7 in Appendix B provide a somewhat more detailed analysis of the strength ratio statistics for each of the design methods. The tables examine 5 groups of slenderness ratios as well as 5 groups of end eccentricity ratios for a total of 25 separate combinations. The tables include the number of test specimens in the group and the mean, coefficient of variation, minimum and maximum values of the strength ratios. All 69 columns are included in the statistics for these tables.

A review of Table 6.1.2 leads to the following observations:

(1) The columns tested under pure axial load and pure bending (Groups 1 and 3) tended to have a lower coefficient of variation than columns tested with eccentric loading (Group 2 in Table 6.1.2). This is opposite to what was observed for reinforced concrete columns (Table 5.1.2).

(2) The coefficient of variation of the strength ratios using ACI 318-95 tends not to be significantly affected by the flexural rigidity equation used (Group 2 values in Columns 5, 6 and 10 of Table 6.1.2).

(3) The coefficient of variation of the strength ratios using the AISC-LRFD Specifications (Columns 8 and 9 of Table 6.1.2) was low for Group 1 and Group 3 columns ($e/h$=0 and $\infty$) but was significantly higher for Group 2 columns ($e/h$=0.1-1.5) as compared to all other methods.

(4) The coefficient of variation of the strength ratios for Eurocode 4 was similar to those produced by ACI 318-95 using all three $EI$ equations (Equation 4.16, 4.18 and 4.20). FEM has the lowest coefficient of variation of the strength ratios for Group 2 ($e/h$=0.1-1.5) columns.

(5) The mean value of the strength ratios for the AISC-LRFD Specifications (Column 8 and 9 of Table 6.1.2) was significantly higher for Group 1 and Group 2 columns than for all other methods. Also, the mean value of the strength ratios for the AISC-LRFD Specifications tends to decrease as the end eccentricity ratio increases. This is opposite to that observed for all other design methods. This indicates that the simplified equation for determining the nominal flexural strength (Equation 4.32) may be slightly unconservative.

(6) There were no significant differences in mean values of the strength ratios for Eurocode 4 and ACI 318-95 using all three EI equations (Equation 4.16, 4.18 and 4.20).

(7) The maximum and minimum values of the strength ratios for the AISC-LRFD Specifications were significantly higher for Group 1 columns and were significantly lower for Group 3 columns when compared to all other methods.

Figure 6.1.1.a shows the cumulative frequency distribution of strength ratios for four different design methods plotted on a normal probability scale. The four methods compared include ACI 318-95 using Equation 4.18, the AISC-LRFD Specifications, Eurocode 4 and FEM. The curves in the figure represent the data for all 69 test columns. The curves for ACI 318-95 using Equation 4.18 and

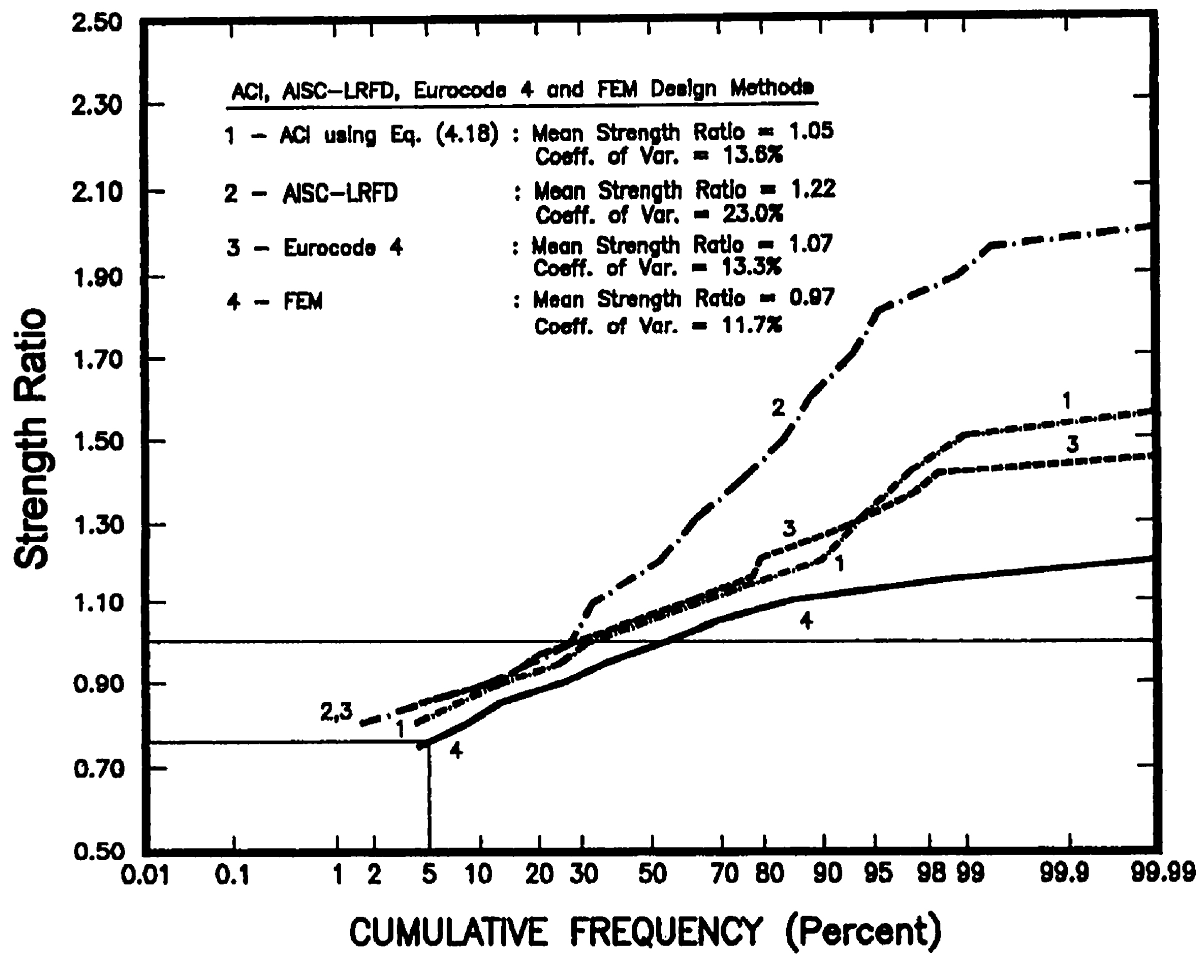



Figure 6.1.1.a - Probability distribution of strength ratios of composite steel-concrete columns subjected to major axis bending without moment gradient for different design methods (n=69).

Eurocode 4 follow one another closely. The FEM produces the least conservative results, while AISC-LRFD produces the most conservative results over the entire range of strength ratios. The five-percentile value is approximately 0.77 for FEM, 0.84 for ACI 318-95 using Equation 4.18 and 0.87 for both the AISC-LRFD Specifications and Eurocode 4.

Figure 6.1.1.b shows the cumulative frequency distribution of strength ratios plotted on a normal probability scale for ACI 318-95 using Equations 4.16, 4.18 and 4.20. The curve for ACI 318-95 using Equation 4.20 follows closely the curve for ACI 318-95 using Equation 4.18 over the entire range of strength ratios. However, the curve for ACI 318-95 using Equation 4.16 is more conservative than that for Equation 4.18, which was also the case for reinforced concrete columns. The five percentile values are 0.84 for ACI 318-95 using both Equations 4.18 and 4.20 and 0.94 for ACI 318-95 using Equation 4.16.

Figure 6.1.1.c shows the cumulative frequency distribution of strength ratios plotted on normal probability scale for the AISC-LRFD Specifications and the AISC-LRFD Specifications using Equation 4.38. From the figure it can be seen that there is very little difference between the two curves. This is expected since Equation 4.38 was chosen to improve the predicted strength of composite steel-concrete columns subjected to minor axis

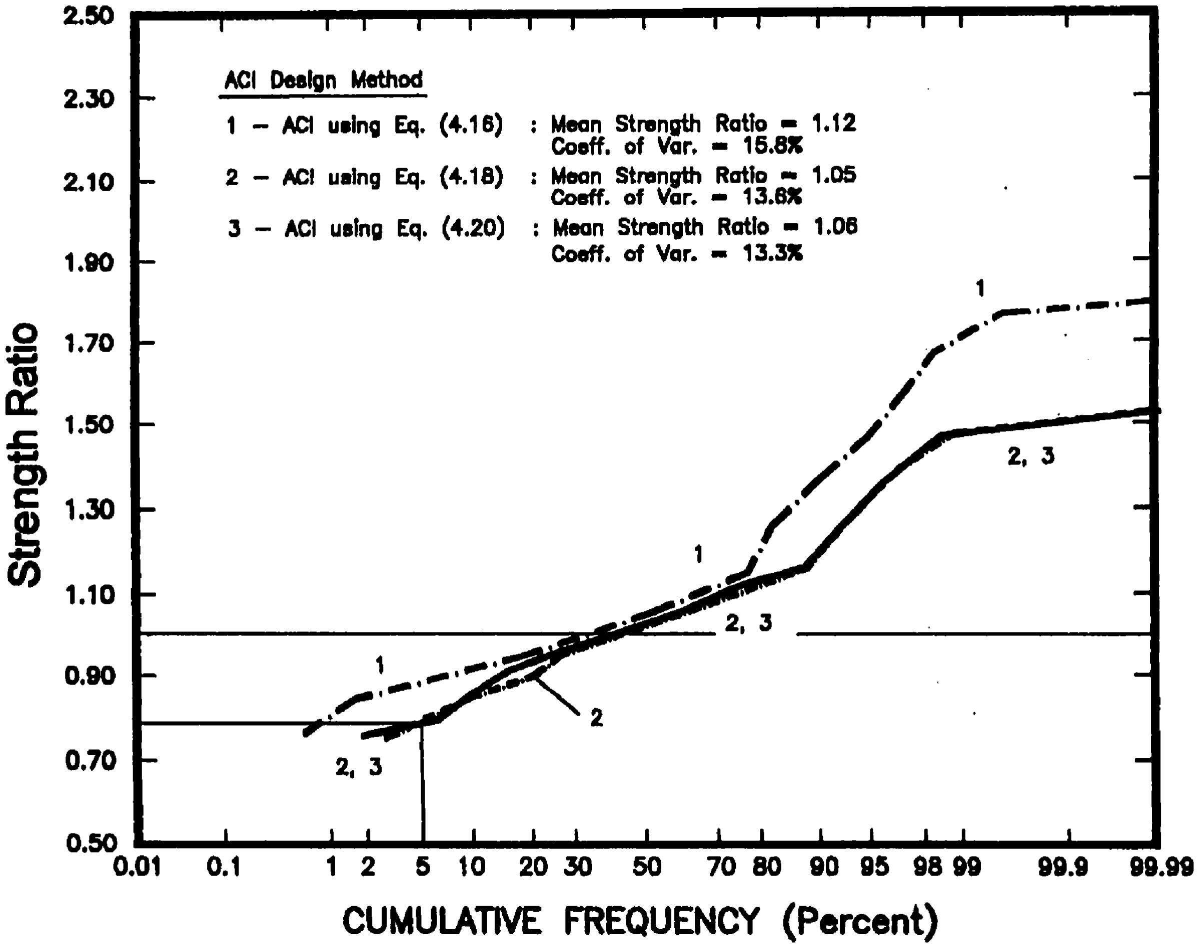



Figure 6.1.1.b - Probability distribution of strength ratios of composite steel-concrete columns subjected to major axis bending without moment gradient for different design methods (n=69).

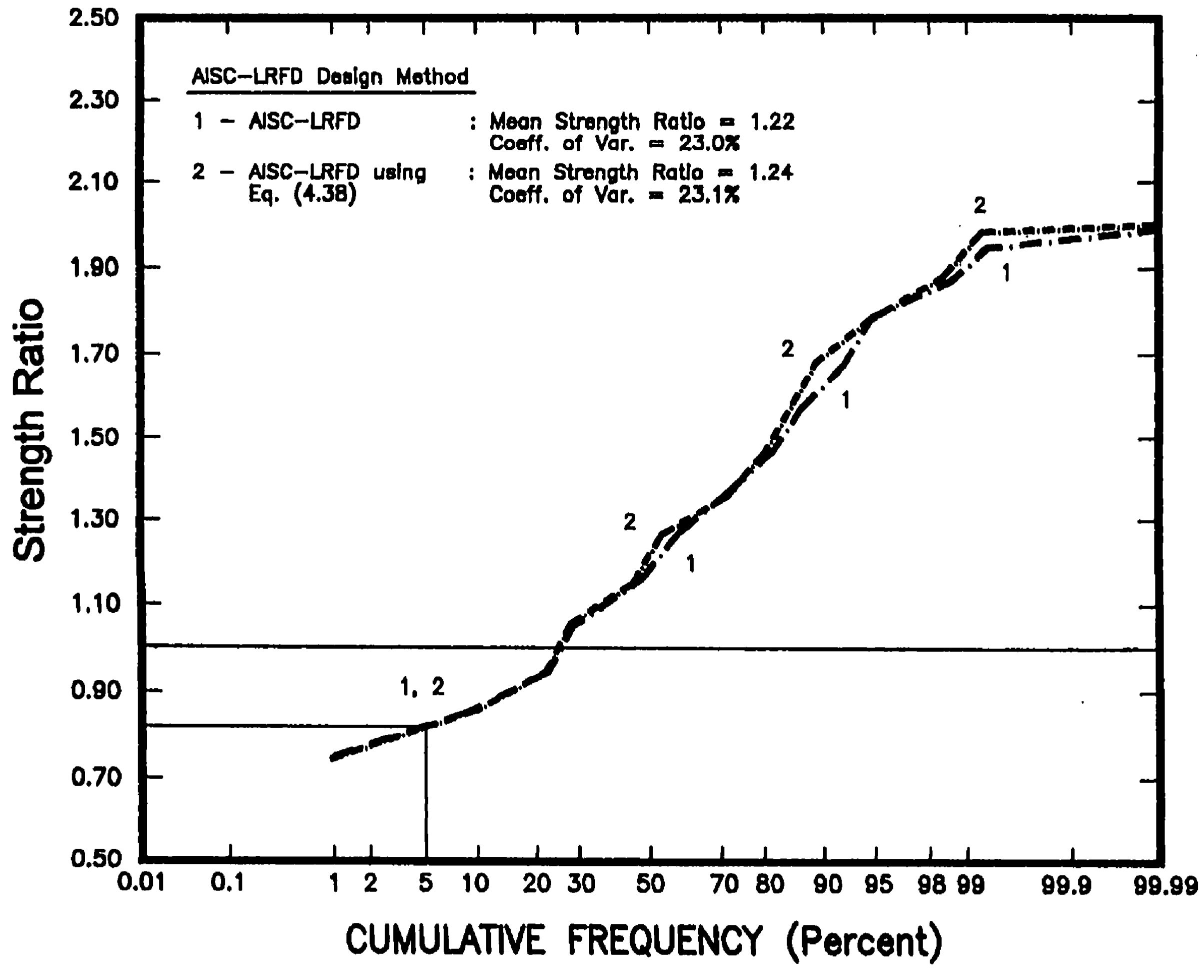



**Figure 6.1.1.c** - Probability distribution of strength ratios of composite steel-concrete columns subjected to major axis bending without moment gradient for different design methods (n=69).

bending and was not intended to improve the predicted strength for composite columns subjected to major axis bending. The five percentile values are 0.87 for AISC-LRFD and 0.86 for AISC-LRFD using Equation 4.38.

### 6.1.3 Effects of Major Variables on Strength Ratios

The effects of concrete strength ($f'_c$), end eccentricity ratio ($e/h$), slenderness ratio ($\ell/h$), structural steel index ($\rho_{ss}f_{yss}/f'_c$), and tie/hoop volumetric ratio ($\rho''$) on the maximum, mean and minimum values of strength ratios obtained using ACI 318-95, AISC-LRFD Specifications, Eurocode 4 and FEM are examined in this section.

Since the values of concrete strength, end eccentricity ratio, slenderness ratio, structural steel index and the tie/hoop volumetric ratio used in this study were not controlled variables, strength ratios had to be grouped into ranges. Up to ten separate ranges were used in plotting each of the figures presented in this section. The maximum, mean and minimum strength ratios were determined for each of the ranges. Grouping the strength ratios resulted in having a significantly different number of columns in some of the ranges. This may explain the jaggedness of the lines associated with some of the figures presented in this section.

Figures 6.1.2.a to 6.1.2.c examine the effect of concrete strength ($f'_c$) on the maximum, mean, and minimum strength ratios. Because of the many different values of concrete strength used, nine ranges of concrete strength were set at 3000-3300, 3300-3950, 3950-4600, 4600-5250, 5250-5900, 5900-6550, 6550-7200, 7200-7850, and 7850-8100 psi. It can be seen from these figures that the maximum, mean and minimum strength ratios remains relatively constant over the entire range of concrete strength for all design methods, except for the AISC-LRFD Specifications. However, the mean value of the strength ratio tends to become less conservative as the concrete strength increases. The AISC-LRFD Specifications and the AISC-LRFD Specifications using Equation 4.38 are significantly affected by the concrete strength. For concrete strengths between approximately 5500 and 7000 psi, both curves produce significantly high values for the maximum, mean and minimum strength ratios (Figure 6.1.2.b). FEM produced the most consistent results over the entire range of concrete strength as can be seen by Figure 6.1.2.c.

Figures 6.1.3.a to 6.1.3.c examine the effect of the end eccentricity ratio ($e/h$), on the maximum, mean, and minimum strength ratios. Because of the many different end eccentricity ratios used, eight ranges of end eccentricity ratio were set at 0, 0-0.2, 0.2-0.4, 0.4-0.6, 0.6-0.8, 0.8-1.0, 1.0-1.1 and $\infty$. Note that no test data were available

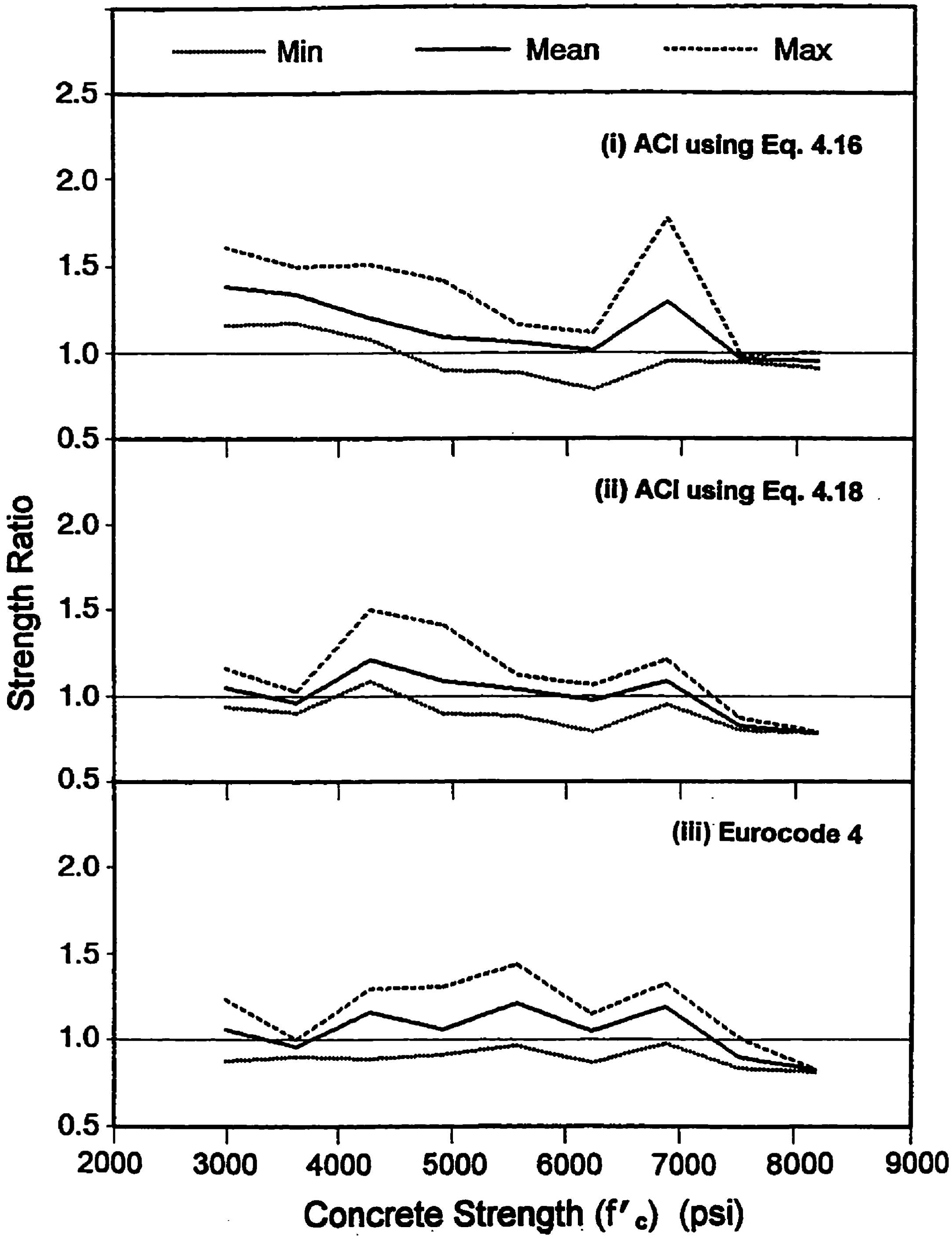



Figure 6.1.2.a - Effect of concrete strength on strength ratios obtained from different design methods for composite steel-concrete columns subjected to major axis bending without moment gradient (n varies for each $f'_c$; total number of specimens=69).

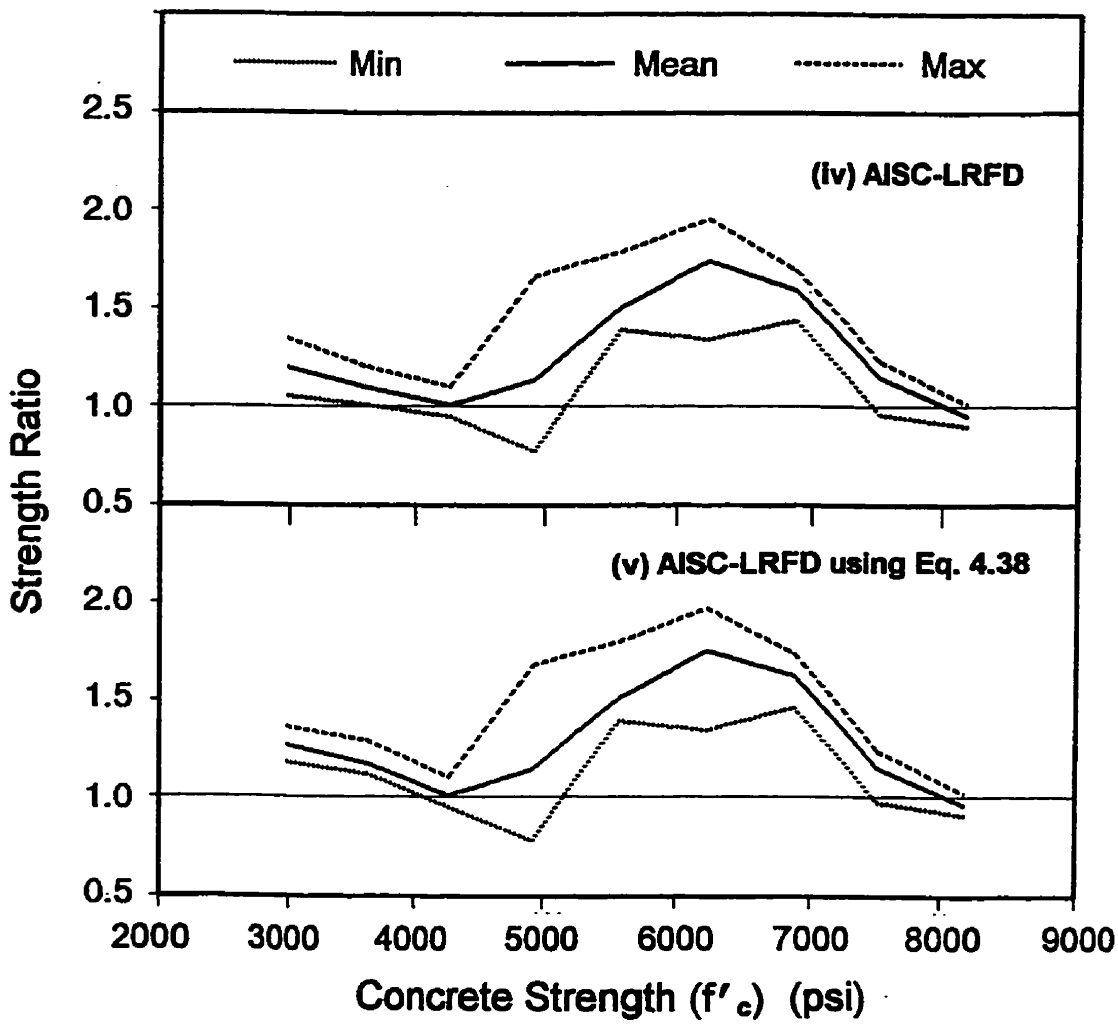



Figure 6.1.2.b - Effect of concrete strength on strength ratios obtained from different design methods for composite steel-concrete columns subjected to major axis bending without moment gradient (n varies for each $f'_c$; total number of specimens=69).

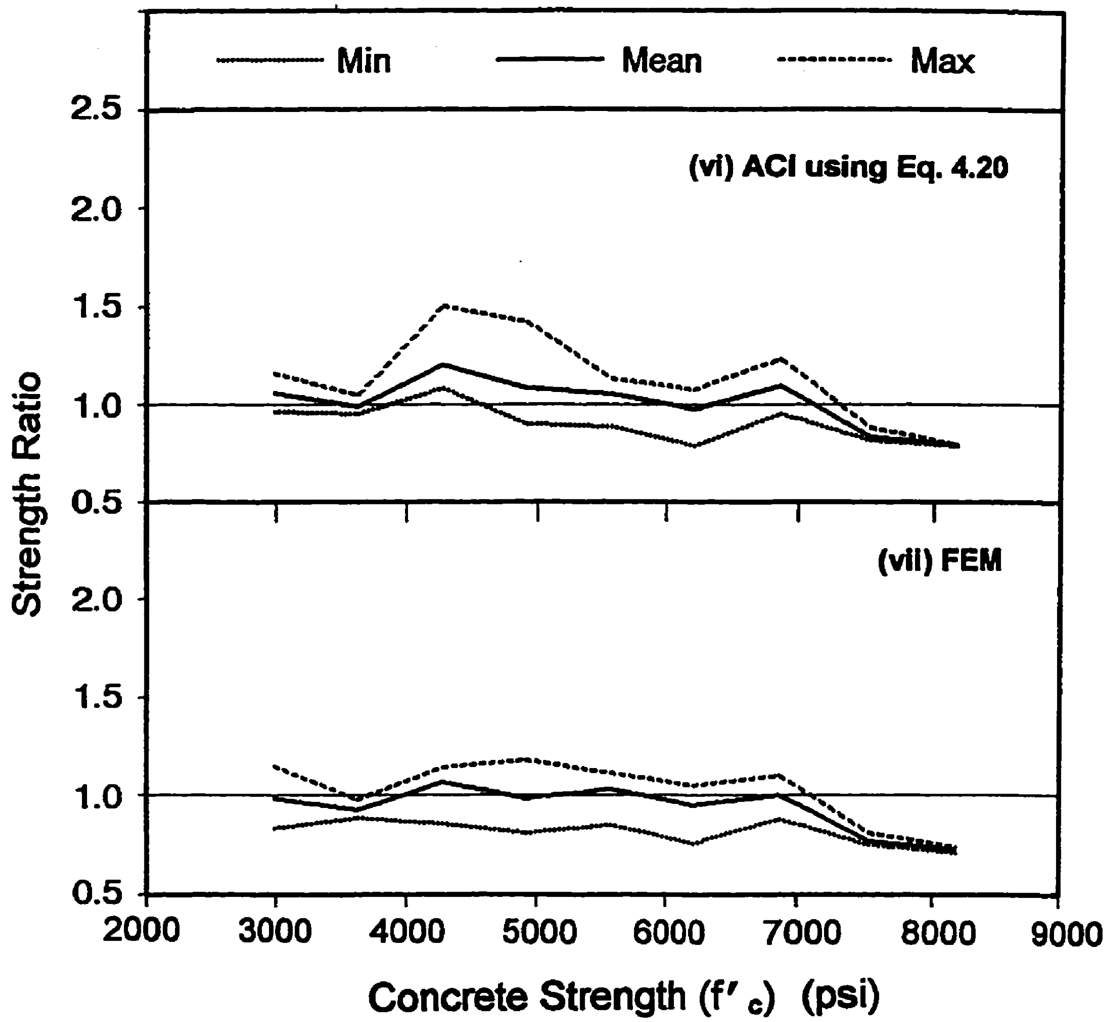



Figure 6.1.2.c - Effect of concrete strength on strength ratios obtained from different design methods for composite steel-concrete columns subjected to major axis bending without moment gradient (n varies for each $f'_c$; total number of specimens=69).

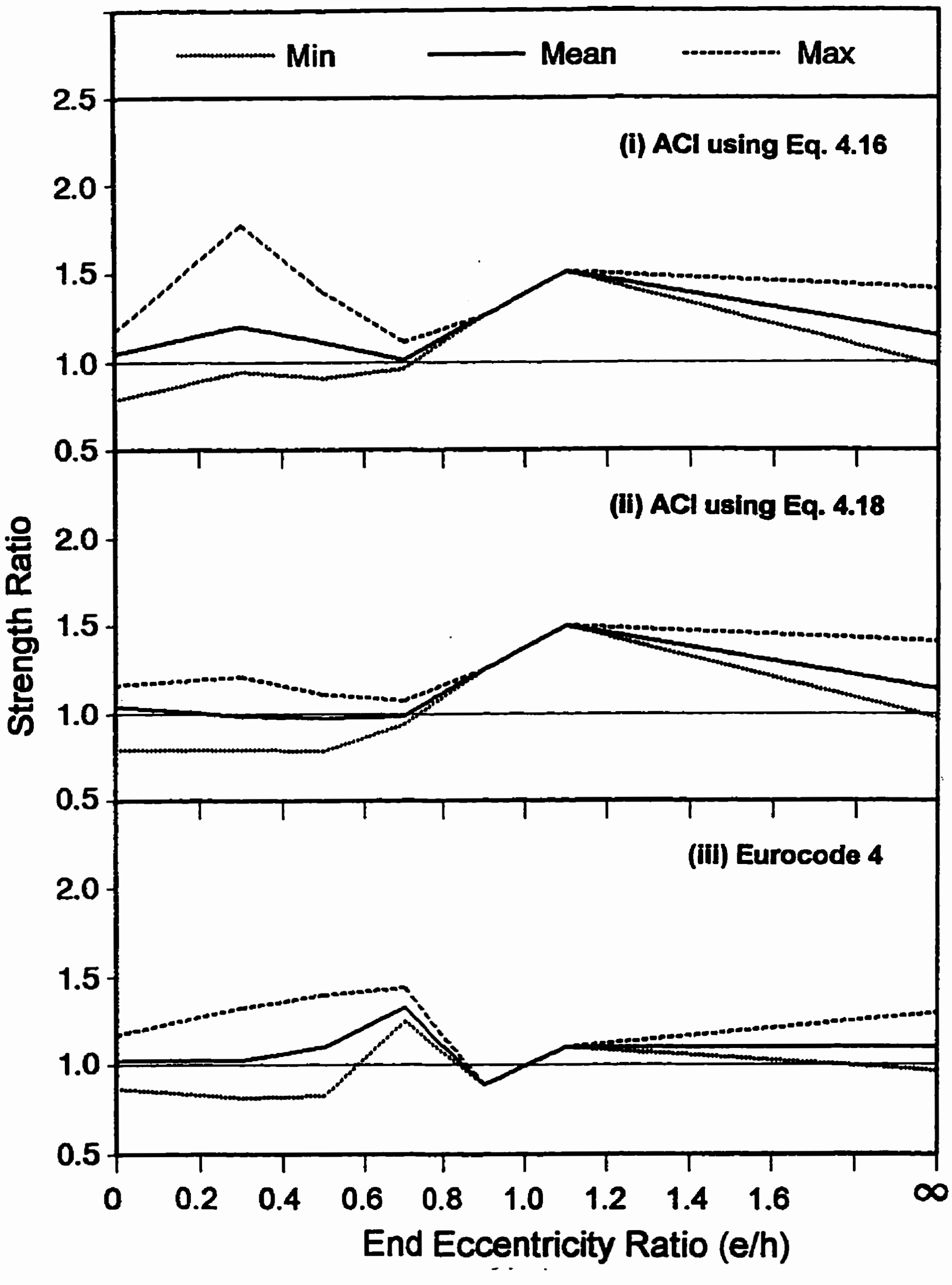



Figure 6.1.3.a - Effect of end eccentricity ratio on strength ratios obtained from different design methods for composite steel-concrete columns subjected to major axis bending without moment gradient (n varies for each e/h ratio; total number of specimens=69).

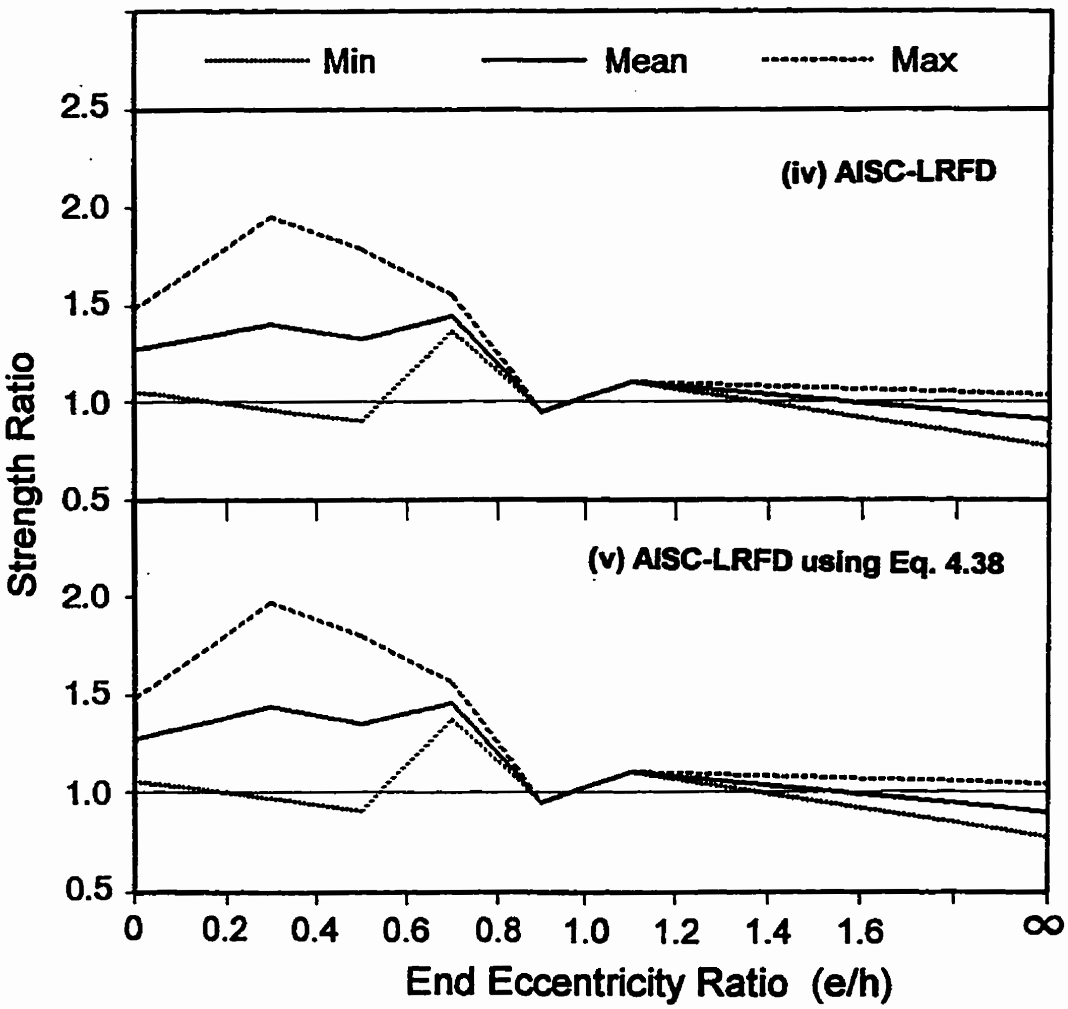



Figure 6.1.3.b - Effect of end eccentricity ratio on strength ratios obtained from different design methods for composite steel-concrete columns subjected to major axis bending without moment gradient (n varies for each e/h ratio; total number of specimens=69).

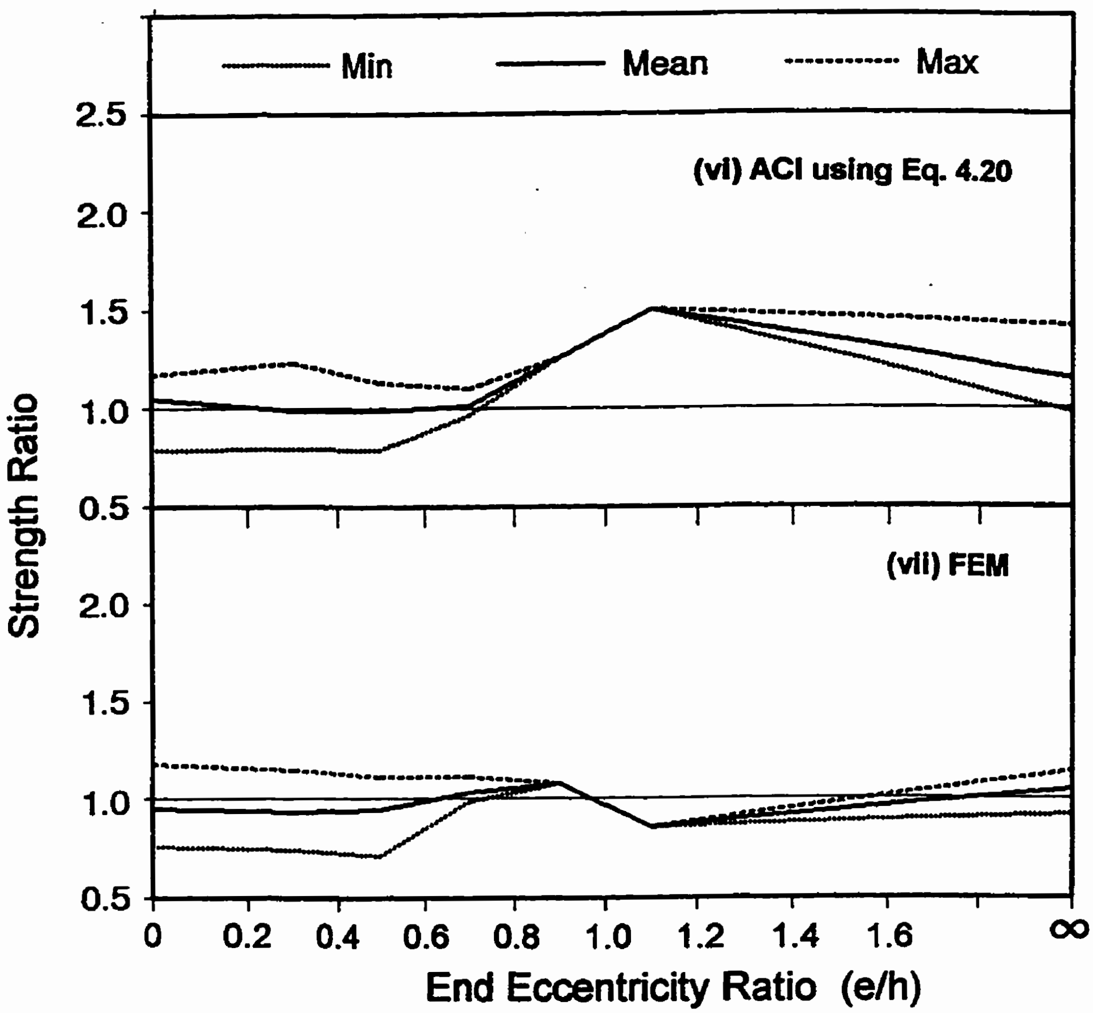



Figure 6.1.3.c - Effect of end eccentricity ratio on strength ratios obtained from different design methods for composite steel-concrete columns subjected to major axis bending without moment gradient (n varies for each e/h ratio; total number of specimens=69).

for end eccentricity ratios in the range greater than 1.06 and less than $\infty$. The curves in the figures, therefore, were extended from the $e/h$ value of 1.06 to the case of pure bending ($e/h=\infty$). The effect of the end eccentricity ratio can not be clearly established from Figures 6.1.3.a to 6.1.3.c due, perhaps, to the lower number of test data available for certain ranges of $e/h$ ratios plotted in these figures. However, the strength ratios for ACI 318-95 using Equation 4.16, AISC-LRFD, and AISC-LRFD using Equation 4.38 appear to be more affected by the end eccentricity ratio than those for ACI 318-95 using Equation 4.18, Eurocode 4, ACI 318-95 using Equation 4.20, and FEM. All methods, except FEM, have mean strength ratios greater than 1.0 for $e/h$ ratios over almost the entire range between 0 and 1.06. FEM had a mean strength ratio below 1.0 for most of the $e/h$ ratios examined.

Figures 6.1.4.a to 6.1.4.c examine the effect of the slenderness ratio ($\ell/h$) on the maximum, mean and minimum strength ratios. Because of the many different slenderness ratios used, seven ranges of slenderness ratios were set at 2-4.5, 4.5-9, 9-13.5, 13.5-18, 18-22.5, 22.5-27 and 27-29. The strength ratios obtained by ACI 318-95 using Equation 4.16 were significantly affected by high slenderness ratios ($\ell/h>15$). ACI 318-95 using Equations 4.18 and 4.20 and Eurocode 4 were not significantly affected by the

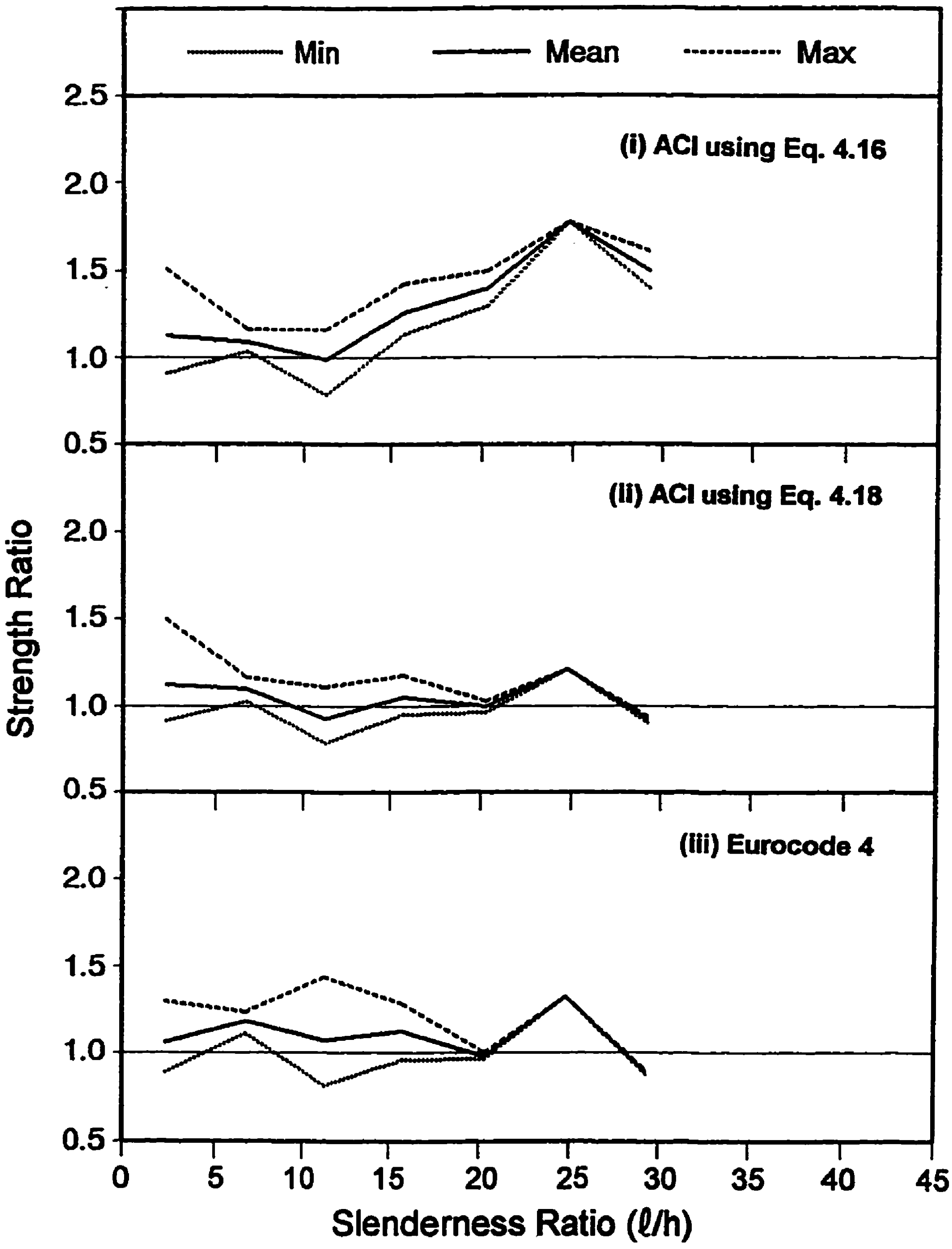



Figure 6.1.4.a - Effect of slenderness ratio on strength ratios obtained from different design methods for composite steel-concrete columns subjected to major axis bending without moment gradient (n varies for each ℓ/h ratio; total number of specimens=69).

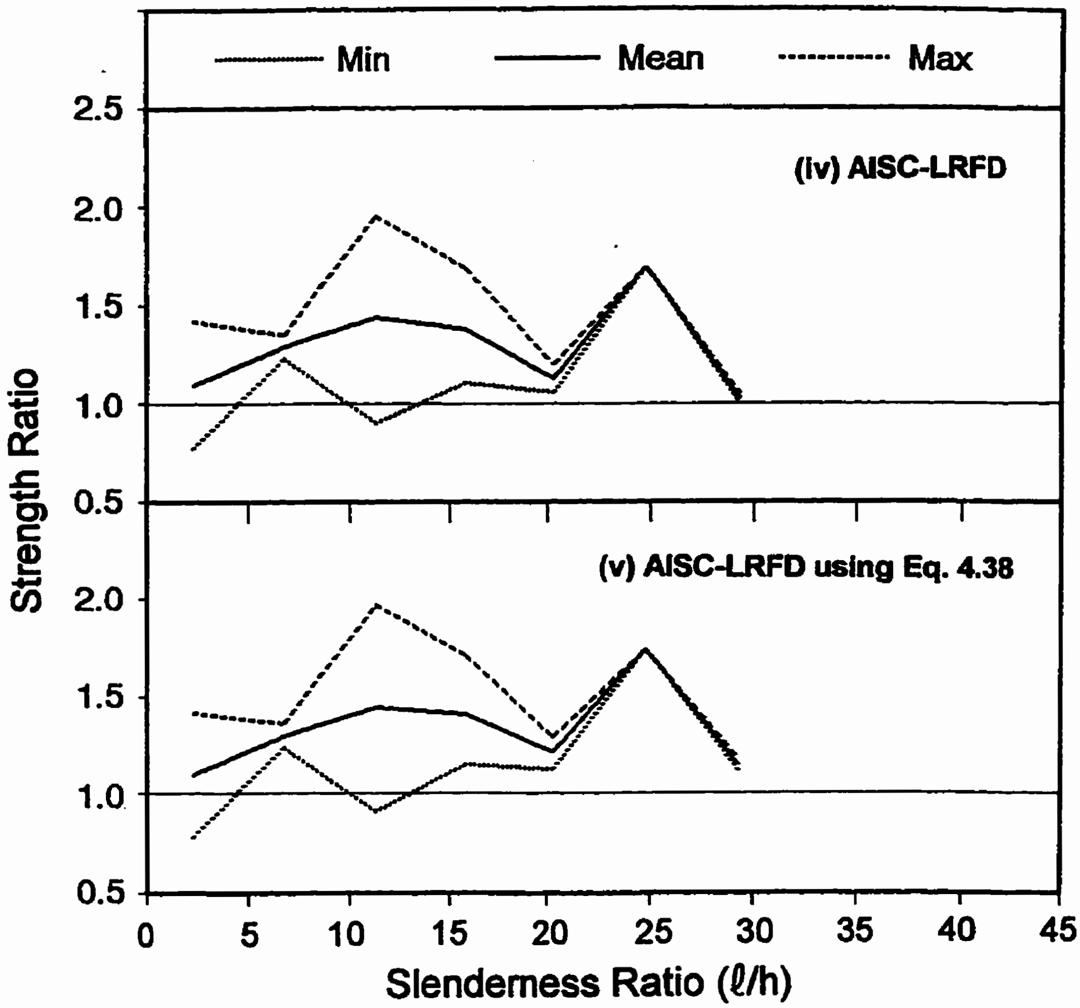



Figure 6.1.4.b - Effect of slenderness ratio on strength ratios obtained from different design methods for composite steel-concrete columns subjected to major axis bending without moment gradient (n varies for each ℓ/h ratio; total number of specimens=69).

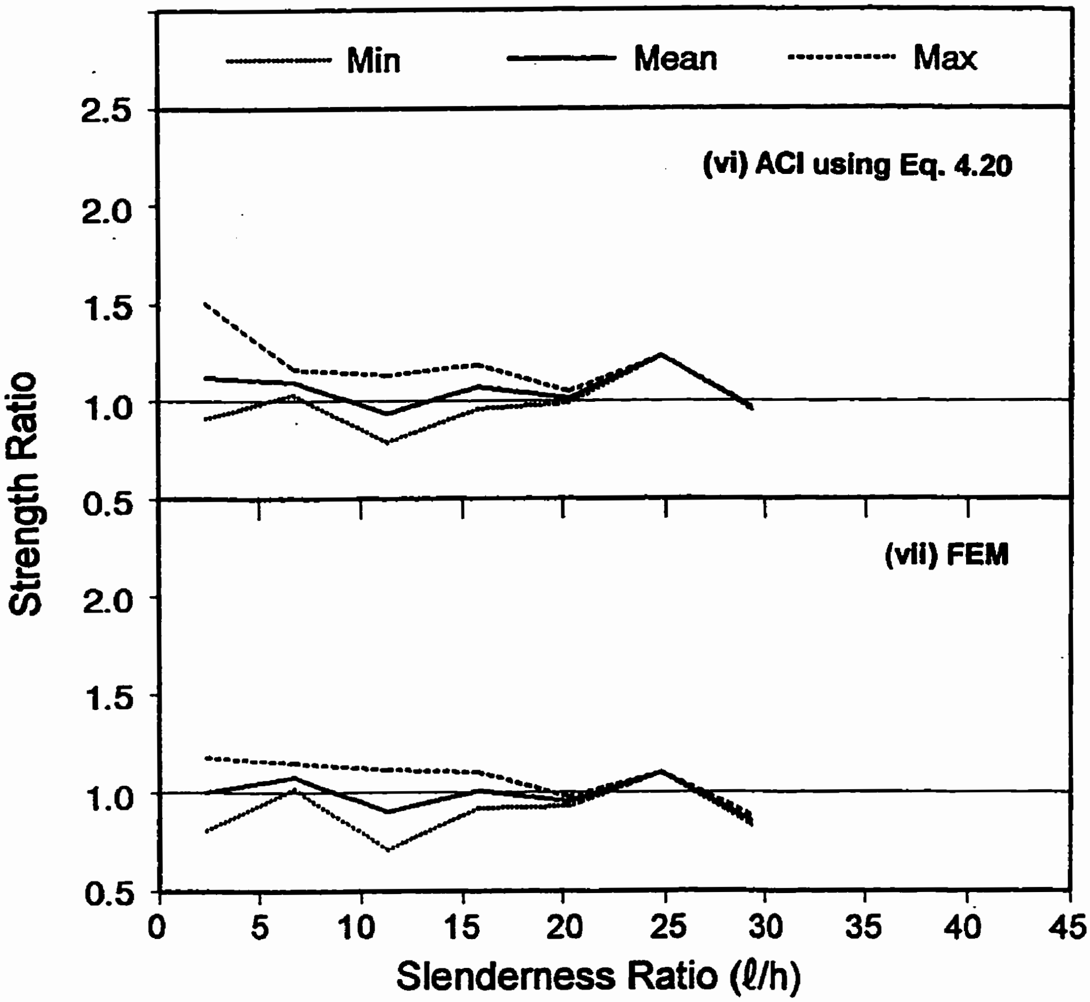



Figure 6.1.4.c - Effect of slenderness ratio on strength ratios obtained from different design methods for composite steel-concrete columns subjected to major axis bending without moment gradient (n varies for each ℓ/h ratio; total number of specimens=69).

slenderness ratio and showed a significant improvement over the results obtained from ACI 318-95 using Equation 4.16, although the difference between the maximum and minimum strength ratios for Eurocode 4 was somewhat high for slenderness ratios ranging from 8 to 15. The strength ratios for the AISC-LRFD Specifications and the AISC-LRFD Specifications using Equation 4.38 were significantly affected by the slenderness ratio, particularly for slenderness ratios ranging from 8 to 15. FEM had the most consistent and narrow range of strength ratios over the entire range of slenderness ratios.

Figures 6.1.5.a to 6.1.5.c examine the effect of the structural steel index ($\rho_{ss}f_{yss}/f'_c$), on the maximum, mean, and minimum strength ratios. Because of the many different structural steel index values used, seven ranges of structural steel index were set at 0.2-0.4, 0.4-0.6, 0.6-0.8, 0.8-1.0, 1.0-1.2, 1.2-1.4 and 1.4-1.6. For all methods, the difference between the maximum and minimum strength ratios reduced as the structural steel index increased. The zigzag nature of the plots for the strength ratios produced by ACI 318-95 using Equation 4.16 is, probably, caused by the grouping of the strength ratios and due to the fact that the contribution of the structural steel is not included in Equation 4.16 used for calculating the flexural rigidity, *EI*. Equations 4.18 and 4.20 used with ACI 318-95 for computing *EI* include the contribution

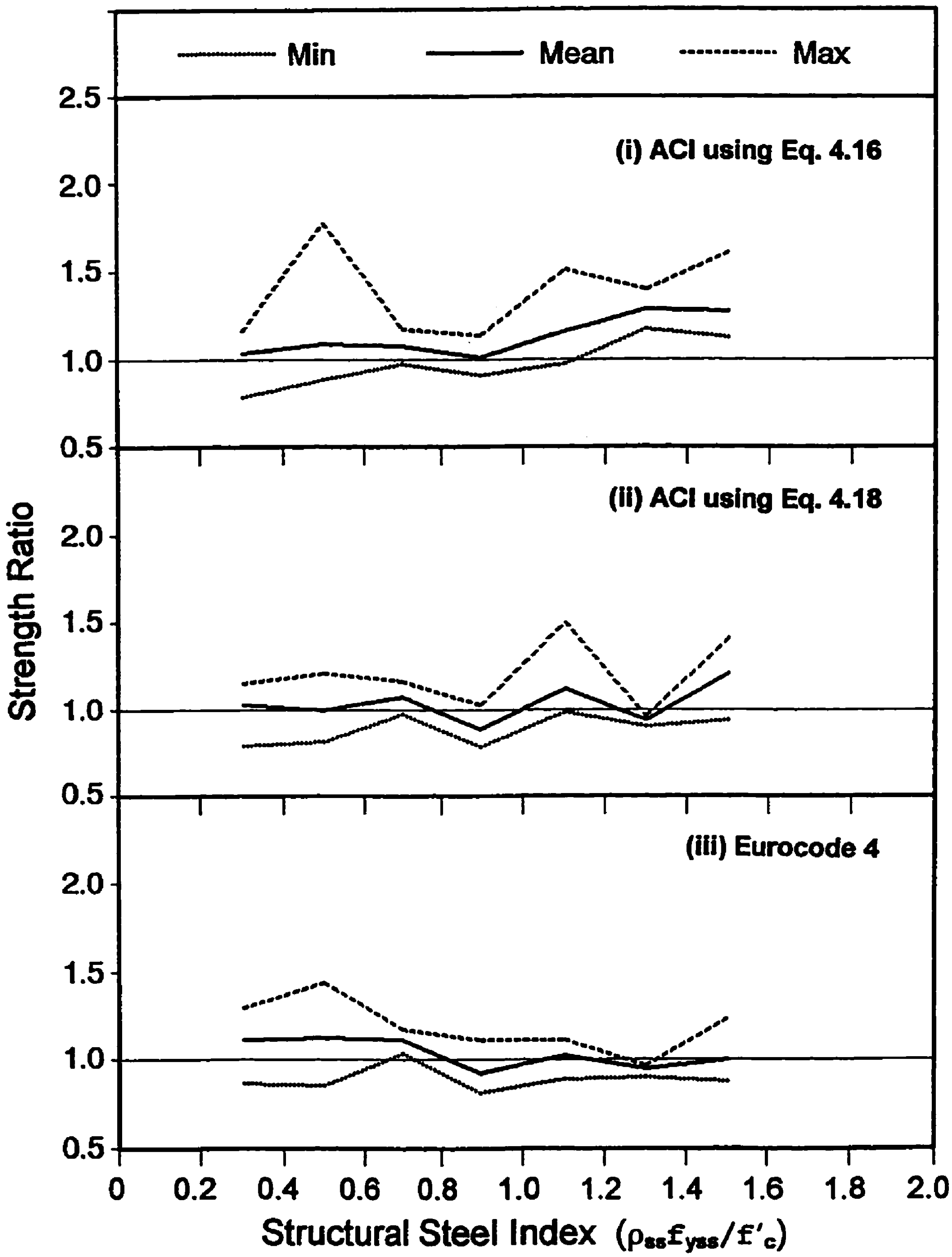



Figure 6.1.5.a - Effect of structural steel index on strength ratios obtained from different design methods for composite steel-concrete columns subjected to major axis bending without moment gradient (n varies for each $\rho_{ss}f_{yss}/f'_c$ ratio; total number of specimens=69).

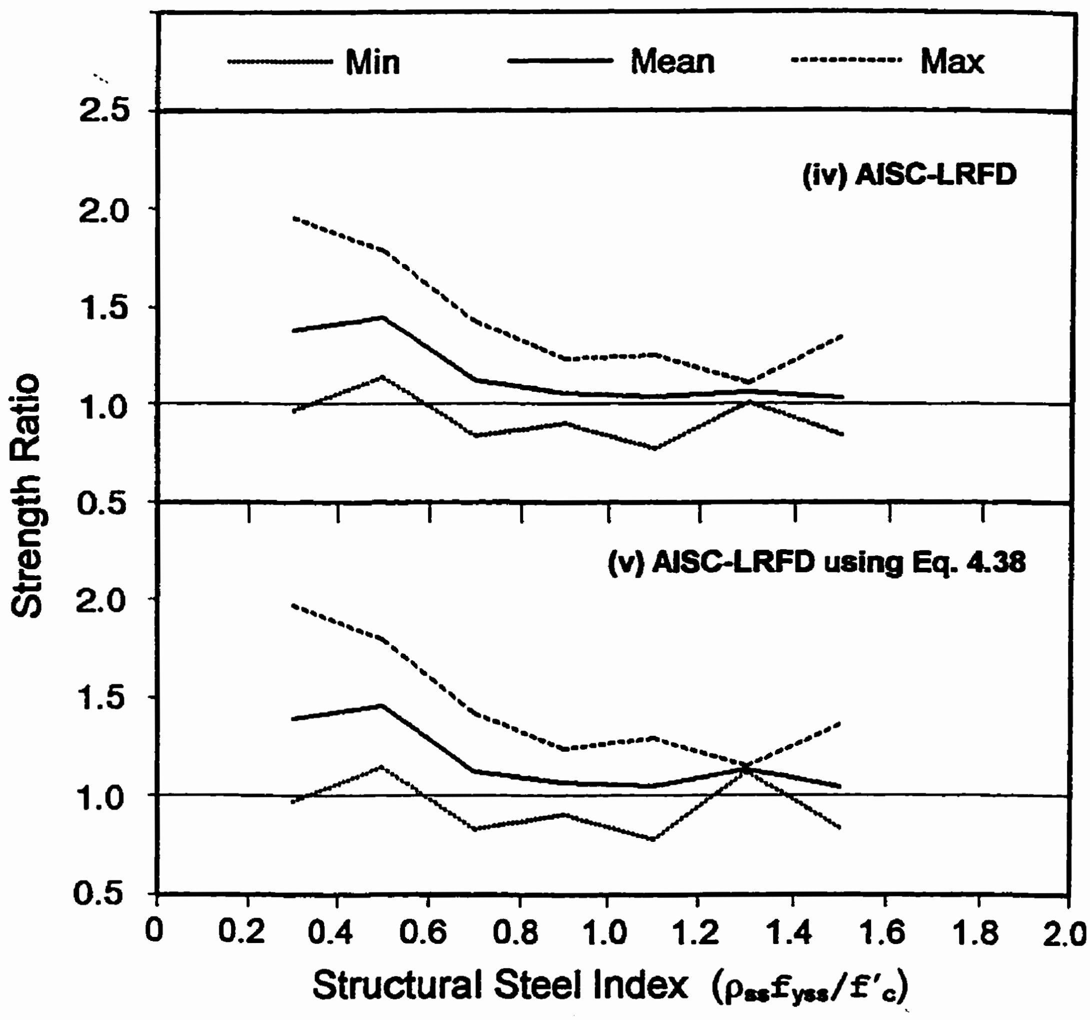



Figure 6.1.5.b - Effect of structural steel index on strength ratios obtained from different design methods for composite steel-concrete columns subjected to major axis bending without moment gradient (n varies for each $\rho_{ss}f_{yss}/f'_c$ ratio; total number of specimens=69).

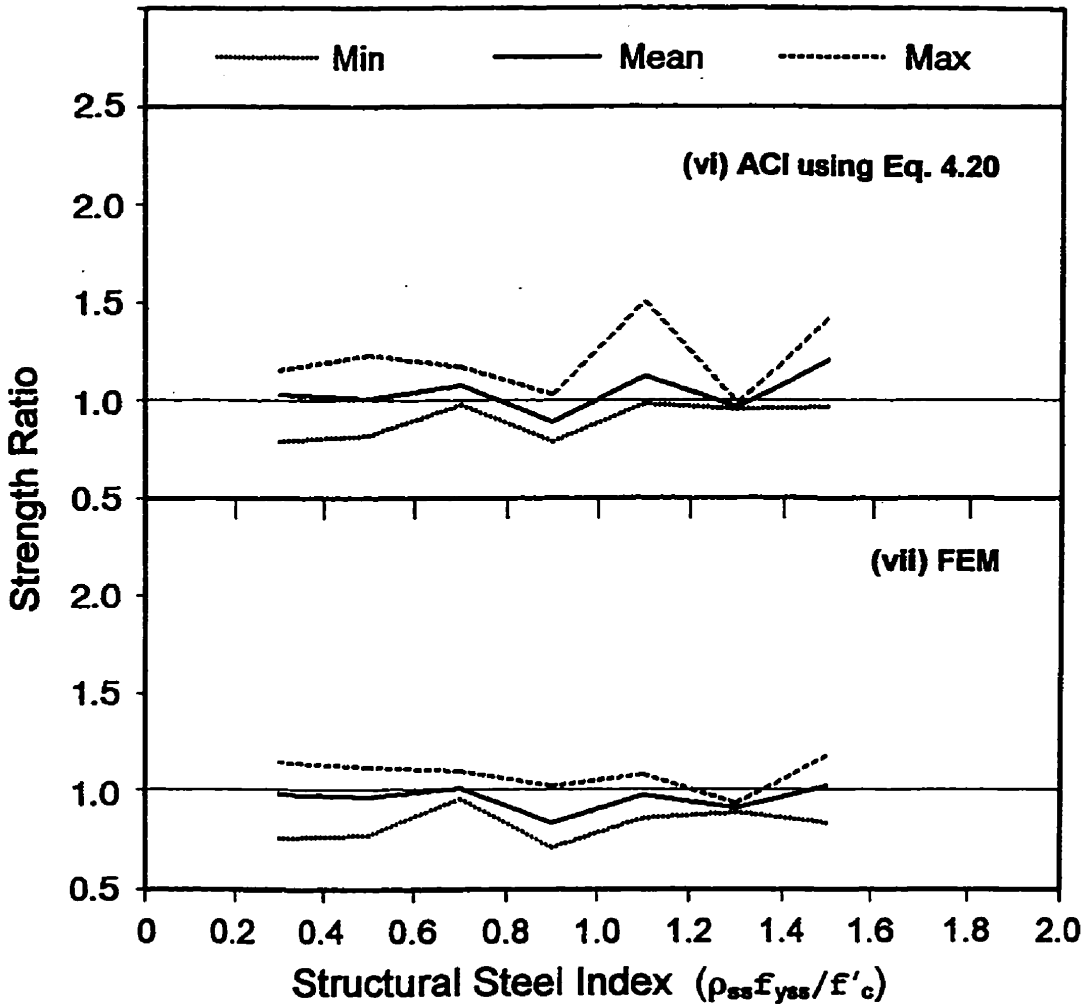



Figure 6.1.5.c - Effect of structural steel index on strength ratios obtained from different design methods for composite steel-concrete columns subjected to major axis bending without moment gradient (n varies for each $\rho_{ss}f_{yss}/f'_c$ ratio; total number of specimens=69).

of the structural steel and can be seen to improve the results plotted in Figures 6.1.5.a and 6.1.5.c. The strength ratios obtained for Eurocode 4 show a trend similar to the results obtained from ACI 318-95 using Equations 4.18 and 4.20. The maximum, mean and minimum values of the strength ratios for the AISC-LRFD and the AISC-LRFD using Equation 4.38 tend to decrease as the structural steel index increases. The strength ratios for FEM are not significantly affected by the structural steel index, however, the mean strength ratio is less than 1.0 for most of the $\rho_{ss}f_{yss}/f'_c$ values examined.

Figures 6.1.6.a to 6.1.6.c examine the effect of the tie/hoop volumetric ratio ($\rho''$) on the maximum, mean, and minimum strength ratios. Because of the many different values of the tie/hoop volumetric ratio used, ten ranges of tie/hoop volumetric ratio were set at 0, 0-0.0033, 0.0033-0.0066, 0.0066-0.0099, 0.0099-0.0132, 0.0132-0.0165, 0.0165-0.0198, 0.0198-0.0231, 0.0231-0.0264 and 0.0264-0.0297. The plots of the strength ratios for all methods, except for FEM, are similar over almost the entire range of $\rho''$ ratios examined. The difference between the maximum and minimum strength ratios for all design methods is large for $\rho''\leq 0.005$ but reduces and becomes consistent for $\rho''>0.005$. Note that the majority of test columns had tie/hoop volumetric ratios in the range of 0.0 to 0.005.

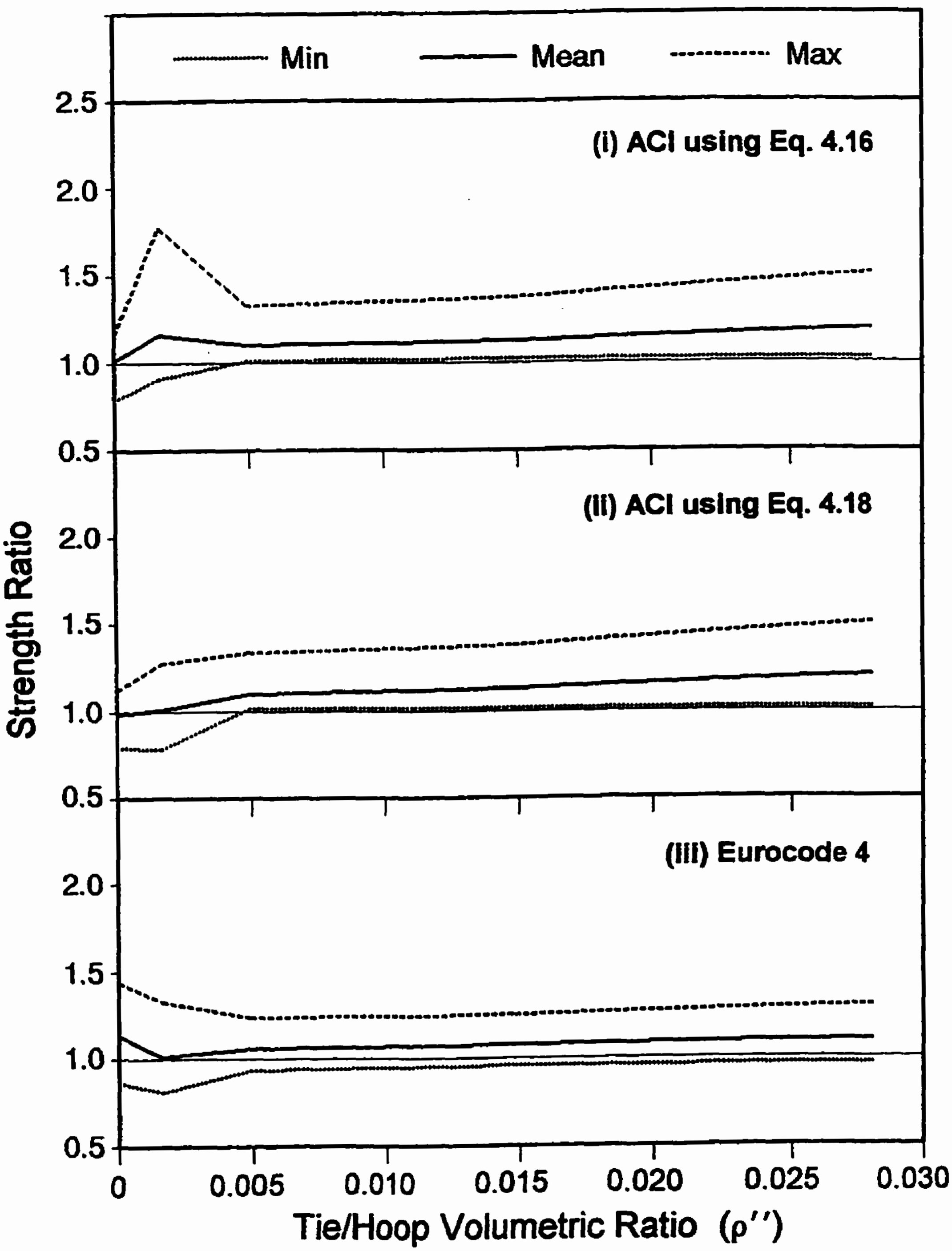



Figure 6.1.6.a - Effect of tie/hoop volumetric ratio on strength ratios obtained from different design methods for composite steel-concrete columns subjected to major axis bending without moment gradient (n varies for each ρ'' ratio; total number of specimens=69).

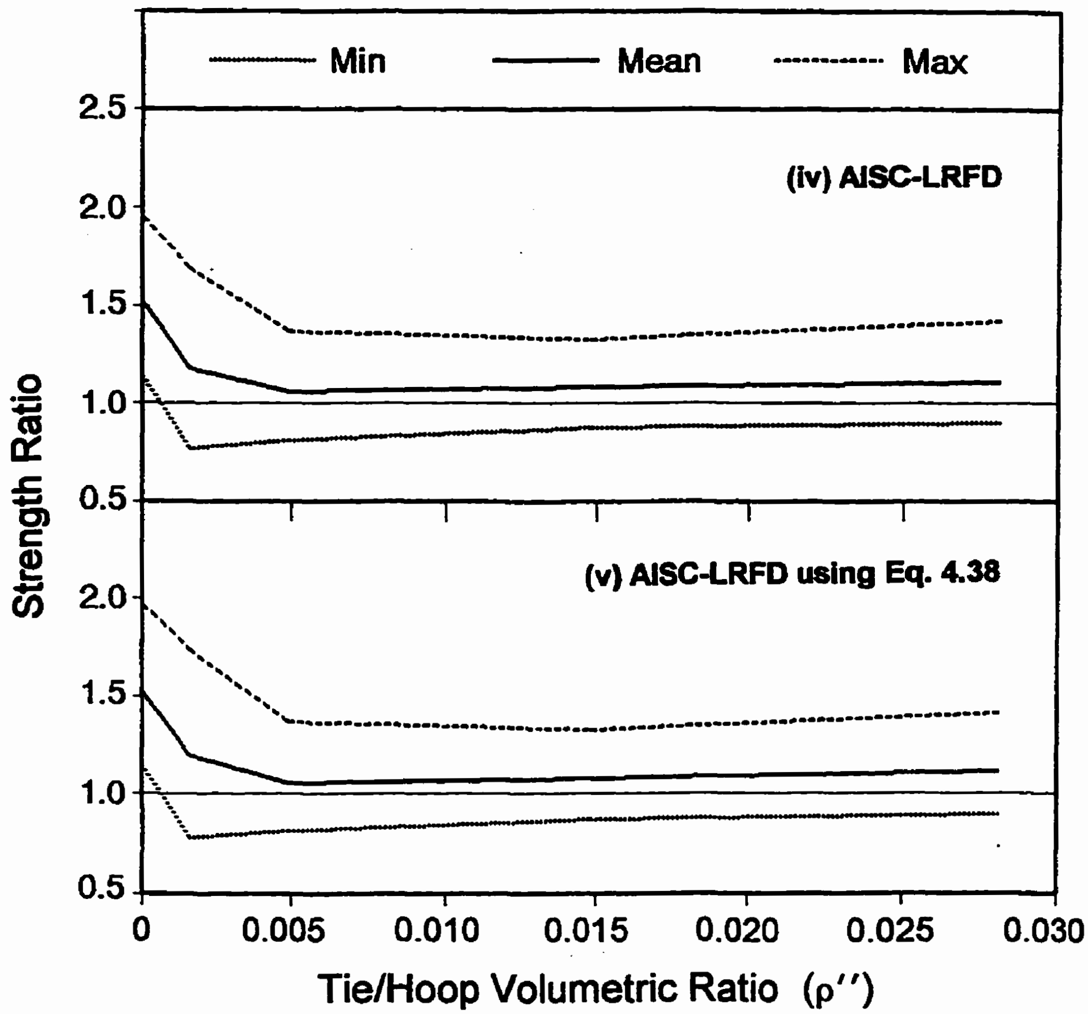



Figure 6.1.6.b - Effect of tie/hoop volumetric ratio on strength ratios obtained from different design methods for composite steel-concrete columns subjected to major axis bending without moment gradient (n varies for each ρ'' ratio; total number of specimens=69).

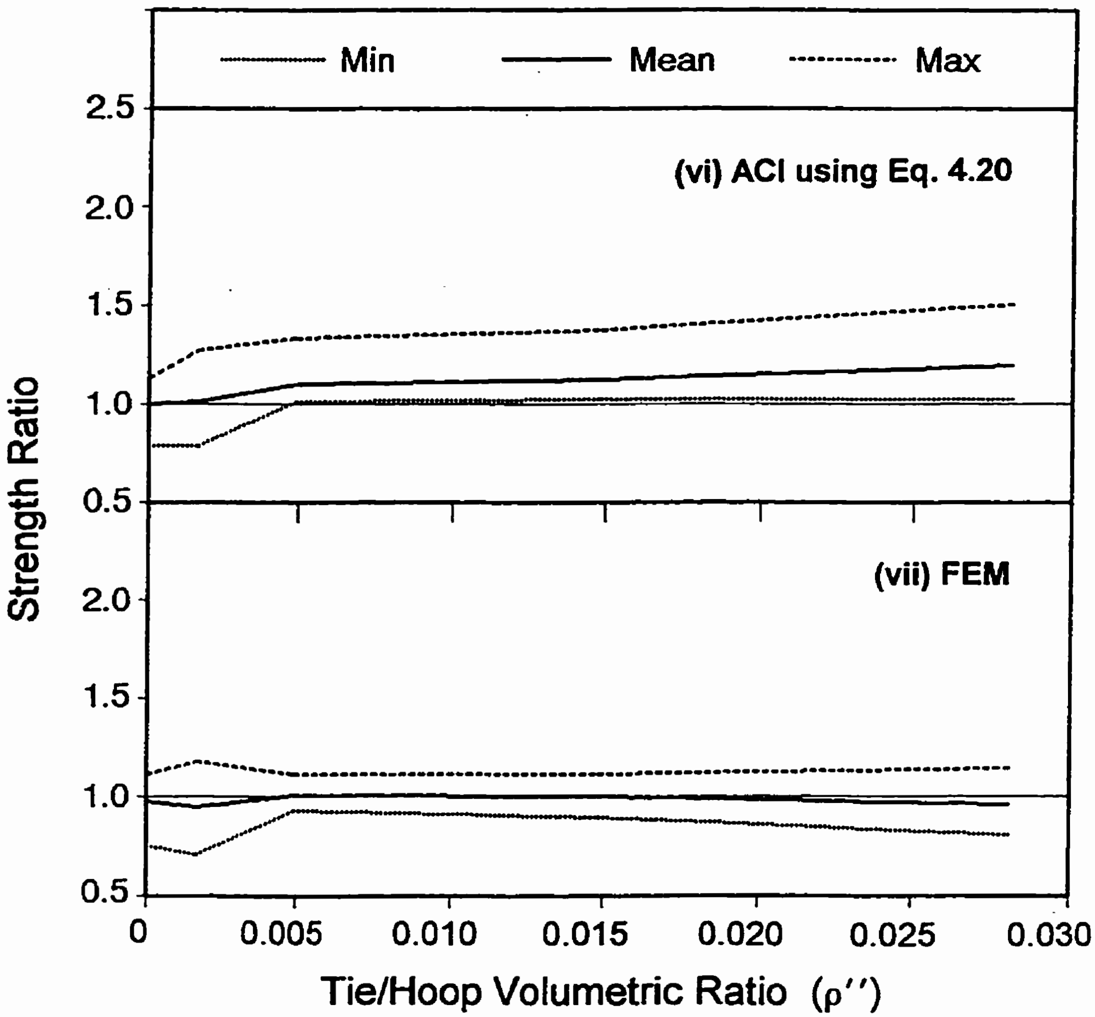



Figure 6.1.6.c - Effect of tie/hoop volumetric ratio on strength ratios obtained from different design methods for composite steel-concrete columns subjected to major axis bending without moment gradient (n varies for each ρ'' ratio; total number of specimens=69).

It is for this reason that the high variability of strength ratios is occurring over this small range of $\rho''$ values, and is not spreaded out uniformly as was the case for the strength ratio curves for other variables discussed previously. The effect of $\rho''$ on strength ratios obtained for FEM (Figure 6.1.6.c) is minimal because of the fact that the lateral confinement of concrete provided by ties/hoops was included in FEM analysis of strength.

The following conclusions can be summarized from the data plotted in Figures 6.1.2 to 6.1.6 and the related discussion:

(1) The strength ratios produced by ACI 318-95 using Equation 4.16 demonstrated a pronounced effect of most of the major variables investigated. The strength ratios become less conservative as the concrete strength increases while becoming more conservative as the slenderness ratio and structural steel index increase.

(2) ACI 318-95 using Equation 4.18 demonstrated an improved effect on the strength ratio variations due to all of the major variables examined when compared to the results obtained from ACI 318-95 using Equation 4.16.

(3) The strength ratios produced by ACI 318-95 using Equation 4.20 gave statistics similar to those obtained from ACI 318-95 using Equation 4.18. Note

that Equation 4.20 takes into account the effect on *EI* of *e/h* ratio as a function of cracking of concrete caused by the presence of bending moment.

(4) The strength ratios produced by the AISC-LRFD Specifications demonstrated a pronounced effect of all of the major variables investigated, particularly the concrete strength, the slenderness ratio, and the structural steel index.

(5) The strength ratios produced by the AISC-LRFD Specifications using Equation 4.38 produced similar results as the existing AISC-LRFD Specification. This was expected since Equation 4.38 was chosen to improve the predicted strengths of composite steel-concrete columns subjected to minor axis bending and was not intended to improve the predicted strength for composite columns subjected to major axis bending.

(6) The strength ratio statistics for Eurocode 4 were similar to those obtained from ACI 318-95 using Equations 4.18 and 4.20.

(7) The FEM produced the most consistent results that tended to be least affected by the variables examined. However, in most cases, the mean strength ratios were below 1.0 for the FEM procedure used.

## 6.2 COMPOSITE STEEL-CONCRETE COLUMNS SUBJECTED TO MAJOR AXIS BENDING WITH MOMENT GRADIENT

Three composite steel-concrete test columns subjected to major axis bending with moment gradient found in the literature were used to compare the test strengths with those obtained from ACI 318-95, AISC-LRFD Specifications, Eurocode 4 and FEM design methods.

### 6.2.1 Description of Column Tests Available in the Literature

A complete description of the 3 composite steel-concrete test columns subjected to major axis bending with moment gradient and found in the literature is presented in Chapter 3.

Each column used had a different combination of geometric and material properties. The maximum and minimum values of the overall cross-section dimensions ($bxh$), the nominal concrete strength ($f'_c$), the end eccentricity ratio ($e/h$), the slenderness ratio ($\ell/h$), the structural steel section yield stress ($f_{yss}$), the reinforcing steel yield stress ($f_{yr}$), the structural steel ratio ($\rho_{ss}$), the reinforcing steel ratio ($\rho_{rs}$), the tie/hoop volumetric ratio ($\rho''$), and the ratio of smaller to larger end moment ($M_1/M_2$) are listed in Table 6.2.1. A few more details are given in Table 3.2.4.

Table 6.2.1 - Summary of Geometric and Material Properties of Composite Steel-Concrete Column Specimens Subjected to Major Axis Bending with Moment Gradient*

| Properties | Minimum Values | Maximum Values |
|---|---|---|
| bxh (in. x in.) | 11.0 x 11.0 | 11.0 x 11.0 |
| $f'_c$ (psi) | 7423 | 7989 |
| e/h | 0.36 | 0.71 |
| $\ell$/h | 10.7 | 10.7 |
| $f_{yss}$ (psi) | 45457 | 45457 |
| $f_{yr}$ (psi) | 60919 | 60919 |
| $\rho_{ss}$ (%) | 14.45 | 14.45 |
| $\rho_{rs}$ (%) | 0.79 | 0.79 |
| $\rho''$ (%) | 0.283 | 0.283 |
| $M_1 / M_2$ | -0.88 | 0.00 |

* Number of Specimens = 3; h = depth of the concrete cross-section perpendicular to the axis of bending; b = width of the concrete cross-section parallel to the axis of bending; $M_1$ = the smaller end moment, positive if member is bent in single curvature, negative if bent in double curvature; $M_2$ = the larger end moment, always positive.

Note: 1.0 in. = 25.4 mm; 1000 psi = 6.895 MPa

Due to the limited number of available test columns in the literature, several of the limitations imposed by ACI 318-95, AISC-LRFD and Eurocode 4 design methods were not used in this study. ACI 318-95 imposes a lower limit on the longitudinal reinforcing steel ratio of one percent, which is not significantly different from the value of 0.79 percent used in this study. The use of Equation 4.20 in ACI 318-95 and Equation 4.38 in the AISC-LRFD Specifications requires that the structural steel ratio be limited to a maximum of 10 percent which is below the value of 14.45 percent used for these tests (Table 6.2.1). The AISC-LRFD Specifications also have an upper limit on the useable reinforcing steel yield strength of 55,000 psi, which is not significantly different from 60,919 psi used. Eurocode 4 imposes an upper limit on the concrete strength of 7300 psi which is not significantly different than 7423-7989 psi used for these tests.

### 6.2.2 Comparison of Design Methods with Test Results

ACI 318-95, AISC-LRFD, Eurocode 4 and FEM methods were compared to the results of 3 composite steel-concrete test columns subjected to major axis bending. The comparison was made based on the strength ratio which is defined as the ratio of the tested ultimate axial load strength to the computed ultimate axial load strength ($P_{test}/P_{des}$). The computed strengths ($P_{des}$) were based on the resistance factors or partial safety factors taken equal to unity and the measured strengths of the concrete, structural steel and reinforcing steels.

Table 6.2.2 gives the coefficient of variation, mean, maximum and minimum values of the strength ratios ($P_{test}/P_{des}$) for each of the design methods studied. An examination of Table 6.2.2 leads to the following observations:

(1) The coefficient of variation of the strength ratios for all design methods is low. This is probably due to the fact that all columns were taken from the same investigation.

(2) The strength ratio statistics computed for ACI 318-95 using all three EI equations (Equations 4.16, 4.18 and 4.20) were identical. This was expected since two of the three column specimens were below the limiting value and the third one was barely above the limiting value of $k\ell_u/r$ computed from Equation 4.5 for short columns. Hence, the effect of flexural rigidity, EI, was negligible.

(3) The mean, maximum and minimum strength ratios for AISC-LRFD and AISC-LRFD using Equation 4.38 tend to be more conservative than those for all other methods.

(4) The mean, maximum and minimum strength ratios for Eurocode 4 and FEM are similar to but somewhat lower than those obtained for ACI 318-95.

Table 6.2.2 - Summary of Strength Ratio Statistics for Different Design Methods for Composite Steel-Concrete Columns Subjected to Major Axis Bending with Moment Gradient.

| Group Number (1) | Slenderness Ratio $\ell/h$ (2) | End Eccentricity Ratio e/h (3) | Number of Specimens (4) | ACI using Eq. 4.16 (5) | ACI using Eq. 4.18 (6) | Eurocode 4 (7) | AISC-LRFD (8) | AISC-LRFD using Eq. 4.36 (9) | ACI using Eq. 4.20 (10) | FEM (11) |
|---|---|---|---|---|---|---|---|---|---|---|
| **(a) Coefficient of Variation** | | | | | | | | | | |
| 4 | 10.7 | 0.3 - 0.8 | 3 | 0.008 | 0.008 | 0.010 | 0.034 | 0.035 | 0.008 | 0.012 |
| **(b) Mean Strength Ratio** | | | | | | | | | | |
| 4 | 10.7 | 0.3 - 0.8 | 3 | 0.926 | 0.926 | 0.905 | 1.124 | 1.126 | 0.926 | 0.815 |
| **(c) Maximum Value of Strength Ratio** | | | | | | | | | | |
| 4 | 10.7 | 0.3 - 0.8 | 3 | 0.934 | 0.934 | 0.914 | 1.161 | 1.164 | 0.934 | 0.826 |
| **(d) Minimum Value of Strength Ratio** | | | | | | | | | | |
| 4 | 10.7 | 0.3 - 0.8 | 3 | 0.921 | 0.921 | 0.896 | 1.084 | 1.086 | 0.921 | 0.808 |

The observations noted in the foregoing paragraph cannot be considered conclusive because of the limited number of tests (merely 3) available from the literature for composite steel-concrete columns subjected to major axis bending with moment gradient.

# 7 - COMPARISON OF DESIGN METHODS WITH TEST RESULTS FOR COMPOSITE STEEL-CONCRETE COLUMNS SUBJECTED TO MINOR AXIS BENDING

For the comparative study, only composite steel-concrete columns subjected to minor axis bending without moment gradient were examined. This is because no test results are available in the literature searched for composite steel-concrete columns subjected to minor axis bending with moment gradient.

## 7.1 COMPOSITE STEEL-CONCRETE COLUMNS SUBJECTED TO MINOR AXIS BENDING WITHOUT MOMENT GRADIENT

In an attempt to study a full range of variables, 81 composite steel-concrete test columns subjected to minor axis bending without moment gradient and found in the literature were used to compare the test strengths with those obtained from ACI 318-95, AISC-LRFD Specifications, Eurocode 4 and FEM methods. Note that ACI 318-95 strengths were computed in three different ways: (a) using Equation 4.16, (b) using Equation 4.18, and (c) using Equation 4.20. Similarly, the AISC-LRFD strengths were computed in two different ways: (a) using the radius of gyration as specified in the AISC-LRFD Specifications, and (b) using the radius of gyration with the upper limit of Equation 4.38.

### 7.1.1 Description of Column Tests Available in the Literature

A complete description of 143 composite steel-concrete test columns subjected to minor axis bending without moment gradient and found in the literature is presented in Chapter 3. Twenty-one of these columns were not used for comparison because of the incomplete or insufficient information available. These columns are identified with an asterisk in Table 3.3.1. In addition, another set of 41 columns was not used for comparison because the specified nominal concrete strength was less than 2500 psi. These columns are identified with a double-asterisk in Table 3.3.1.

Each column used had a different combination of geometric and material properties. The maximum and minimum values of the overall cross-section dimensions ($bxh$), the nominal concrete strength ($f'_c$), the end eccentricity ratio ($e/h$), the slenderness ratio ($\ell/h$), the structural steel section yield stress ($f_{yss}$), the reinforcing steel yield stress ($f_{yr}$), the structural steel ratio ($\rho_{ss}$), the reinforcing steel ratio ($\rho_{rs}$), and the tie/hoop volumetric ratio ($\rho''$) are listed in Table 7.1.1. The values shown in the table represent the maximum and minimum values of the

Table 7.1.1 - Summary of Geometric and Material Properties of Composite Steel-Concrete Column Specimens Subjected to Minor Axis Bending*

| Properties | Minimum Values | Maximum Values |
|---|---|---|
| bxh (in. x in.) | 6.3 x 6.3 | 16.0 x 12.0 |
| $f'_c$ (psi) | 2524 | 7646 |
| e/h | 0.00 | 0.71 |
| $\ell/h$ | 5.0 | 28.9 |
| $f_{yss}$ (psi) | 32928 | 72852 |
| $f_{yr}$ (psi) | 31910 | 60919 |
| $\rho_{ss}$ (%) | 2.70 | 12.92 |
| $\rho_{rs}$ (%) | 0.00 | 3.14 |
| $\rho''$ (%) | 0.000 | 0.295 |

* Number of Specimens = 81; h = depth of the concrete cross-section perpendicular to the axis of bending; b = width of the concrete cross-section parallel to the axis of bending.

Note: 1.0 in. = 25.4 mm; 1000 psi = 6.895 MPa

data; a detailed description of each column is given in Table 3.3.1.

In order to study the effects of variables over a broad range, several of the limits imposed by ACI 318-95, AISC-LRFD and Eurocode 4 design methods were not used in this study. These limits are discussed in section 6.1.1 and will not be repeated here.

**7.1.2 Comparison of Design Methods with Test Results**

ACI 318-95, AISC-LRFD Specifications, Eurocode 4 and FEM methods were compared to the results of 81 composite steel-concrete test columns subjected to minor axis bending. The comparison was made using the strength ratio which was defined as the ratio of the tested ultimate axial load strength to the computed ultimate axial load strength ($P_{test}/P_{des}$). The computed strengths ($P_{des}$) were based on the resistance factors or partial safety factors taken equal to unity and the measured strengths of the concrete, reinforcing steel and structural steel.

A summary of the strength ratio ($P_{test}/P_{des}$) statistics computed for four different design methods for columns with $\ell/h$=5-30 and $e/h$=0.1-1.5 is given in Table 7.1.2. The design methods compared in this table include ACI 318-95, AISC-LRFD Specifications, Eurocode 4 and FEM. Three different equations for flexural rigidity, $EI$, were used for computing ACI 318-95 column strengths (Equations 4.16,

Table 7.1.2 - Summary of Strength Ratio Statistics for Different Design Methods for Composite Steel-Concrete Columns Subjected to Minor Axis Bending.

| Group Number (1) | Slenderness Ratio $\ell/h$ (2) | End Eccentricity Ratio e/h (3) | Number of Specimens (4) | ACI using Eq. 4.16 (5) | ACI using Eq. 4.18 (6) | Eurocode 4 (7) | AISC-LRFD (8) | AISC-LRFD using Eq. 4.38 (9) | ACI using Eq. 4.20 (10) | FEM (11) |
|---|---|---|---|---|---|---|---|---|---|---|
| **(a) Coefficient of Variation** | | | | | | | | | | |
| 1 | 5 - 18 | 0 | 35 | 0.201 | 0.301 | 0.127 | 0.144 | 0.179 | 0.193 | 0.141 |
| 2 | 5 - 30 | 0.1 - 1.5 | 41 | 0.222 | 0.174 | 0.174 | 0.283 | 0.248 | 0.169 | 0.140 |
| **(b) Mean Strength Ratio** | | | | | | | | | | |
| 1 | 5 - 18 | 0 | 35 | 0.964 | 1.173 | 0.977 | 1.137 | 1.335 | 0.890 | 0.743 |
| 2 | 5 - 30 | 0.1 - 1.5 | 41 | 1.075 | 1.122 | 1.049 | 1.299 | 1.483 | 1.035 | 0.968 |
| **(c) Maximum Value of Strength Ratio** | | | | | | | | | | |
| 1 | 5 - 18 | 0 | 35 | 1.441 | 2.051 | 1.342 | 1.561 | 2.086 | 1.506 | 0.941 |
| 2 | 5 - 30 | 0.1 - 1.5 | 41 | 1.815 | 1.871 | 1.566 | 2.032 | 2.355 | 1.579 | 1.280 |
| **(d) Minimum Value of Strength Ratio** | | | | | | | | | | |
| 1 | 5 - 18 | 0 | 35 | 0.582 | 0.582 | 0.683 | 0.770 | 0.845 | 0.582 | 0.530 |
| 2 | 5 - 30 | 0.1 - 1.5 | 41 | 0.765 | 0.925 | 0.746 | 0.761 | 1.009 | 0.827 | 0.793 |

4.18 and 4.20). Two different values for the radius of gyration, $r$, were used for computing the AISC-LRFD column strengths.

Table 7.1.2 gives the coefficient of variation, mean, maximum and minimum values of the strength ratios. For the statistical analysis, the columns studied were divided into two groups: Group 1 considered all columns having an end eccentricity ratio of zero (pure axial load) and slenderness ratio of 5-18 (i.e. $3<\ell/h\leq30$); and Group 2 included all columns with an end eccentricity ratio greater than or equal to 0.1 but less than or equal to 1.5 and with a slenderness ratio greater than 5 but less than or equal to 30. Table 7.1.2 includes the statistics for a total of 76 columns. The remaining 5 columns having an end eccentricity ratio greater than zero but less than 0.1 are not included in this table.

Tables C1.1 to C1.7 in Appendix C provide a somewhat more detailed analysis of the strength ratio statistics for each of the design methods. The tables examine 5 groups of slenderness ratios as well as 5 groups of end eccentricity ratios for a total of 25 separate combinations. The tables include the number of test specimens in the group and the mean, coefficient of variation, minimum and maximum values of the strength ratios. All 81 columns are included in the statistics for these tables.

A review of Table 7.1.2 leads to the following observations:

(1) The coefficient of variation of the strength ratios using ACI 318-95 tends to be significantly affected by the flexural rigidity equation used (Group 2 values in Columns 5, 6 and 10 in Table 7.1.2).

(2) The coefficient of variation of the strength ratios for the AISC-LRFD Specifications (Column 8 in Table 7.1.2) was low for Group 1 columns and high for Group 2 columns. The AISC-LRFD Specifications using Equation 4.38 (Column 9 in Table 7.1.2) demonstrated similar results but showed an improvement for Group 2 columns.

(3) The coefficient of variation of strength ratios for Eurocode 4 showed an improvement over ACI 318-95 using all three *EI* equations (Equations 4.16, 4.18 and 4.20). FEM had the lowest coefficient of variation of strength ratios for Group 2 columns.

(4) The mean values of the strength ratios for the AISC-LRFD Specifications and the AISC-LRFD Specifications using Equation 4.38 (Column 8 and 9 of Table 7.1.2) were significantly higher for Group 2 columns than those for all other methods.

Figure 7.1.1.a shows the cumulative frequency distribution of strength ratios ($P_{test}/P_{des}$) for four different design methods plotted on a normal probability

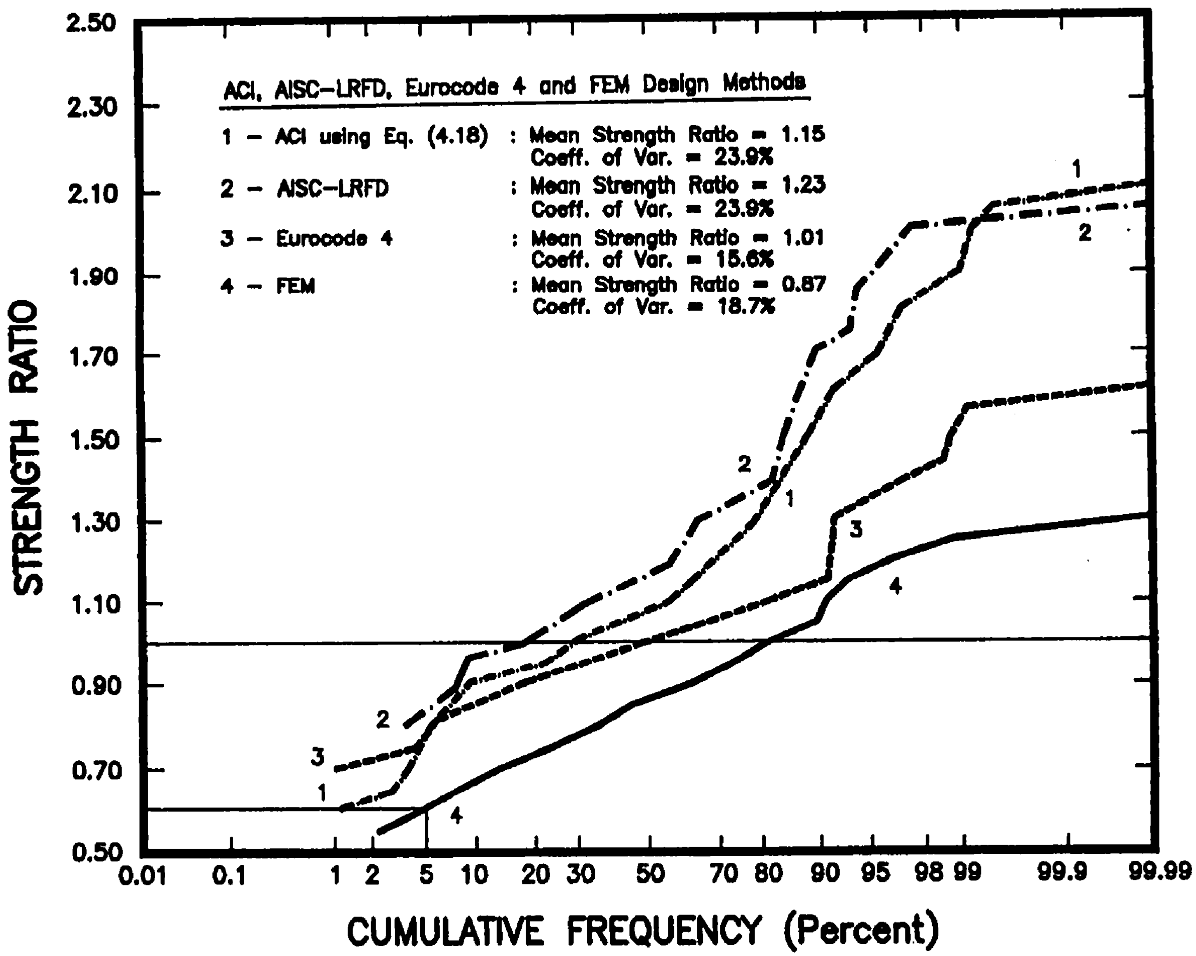



Figure 7.1.1.a - Probability distribution of strength ratios of composite steel-concrete columns subjected to minor axis bending for different design methods (n=81).

scale. The four design methods compared include ACI 318-95 using Equation 4.18, the AISC-LRFD Specifications, Eurocode 4 and FEM. The curves in the figure represent the data for all 81 test columns. The curves for ACI 318-95 using Equation 4.18 and the AISC-LRFD Specifications follow one another closely. The curve for Eurocode 4 is less conservative than the ACI 318-95 and AISC-LRFD methods with FEM producing the least conservative results over the entire range of strength ratios. The five-percentile value is approximately 0.60 for FEM, 0.80 for both ACI 318-95 using Equation 4.18 and Eurocode 4, and 0.85 for the AISC-LRFD.

Figure 7.1.1.b shows the cumulative frequency distribution of strength ratios ($P_{test}/P_{des}$) plotted on a normal probability scale for ACI 318-95 using Equations 4.16, 4.18 and 4.20. The curve for ACI 318-95 using Equation 4.20 follows the curve for ACI 318-95 using Equation 4.16 over almost the entire range of strength ratios examined. However, the curve for ACI 318-95 using Equation 4.18 is more conservative than that for Equation 4.16, which is opposite to what was observed for both reinforced concrete columns and composite steel-concrete columns bending about the major axis. This is also opposite to what was expected since Equation 4.18 includes the effect of the structural steel in calculating the flexural rigidity, *EI*. The five percentile values are 0.73

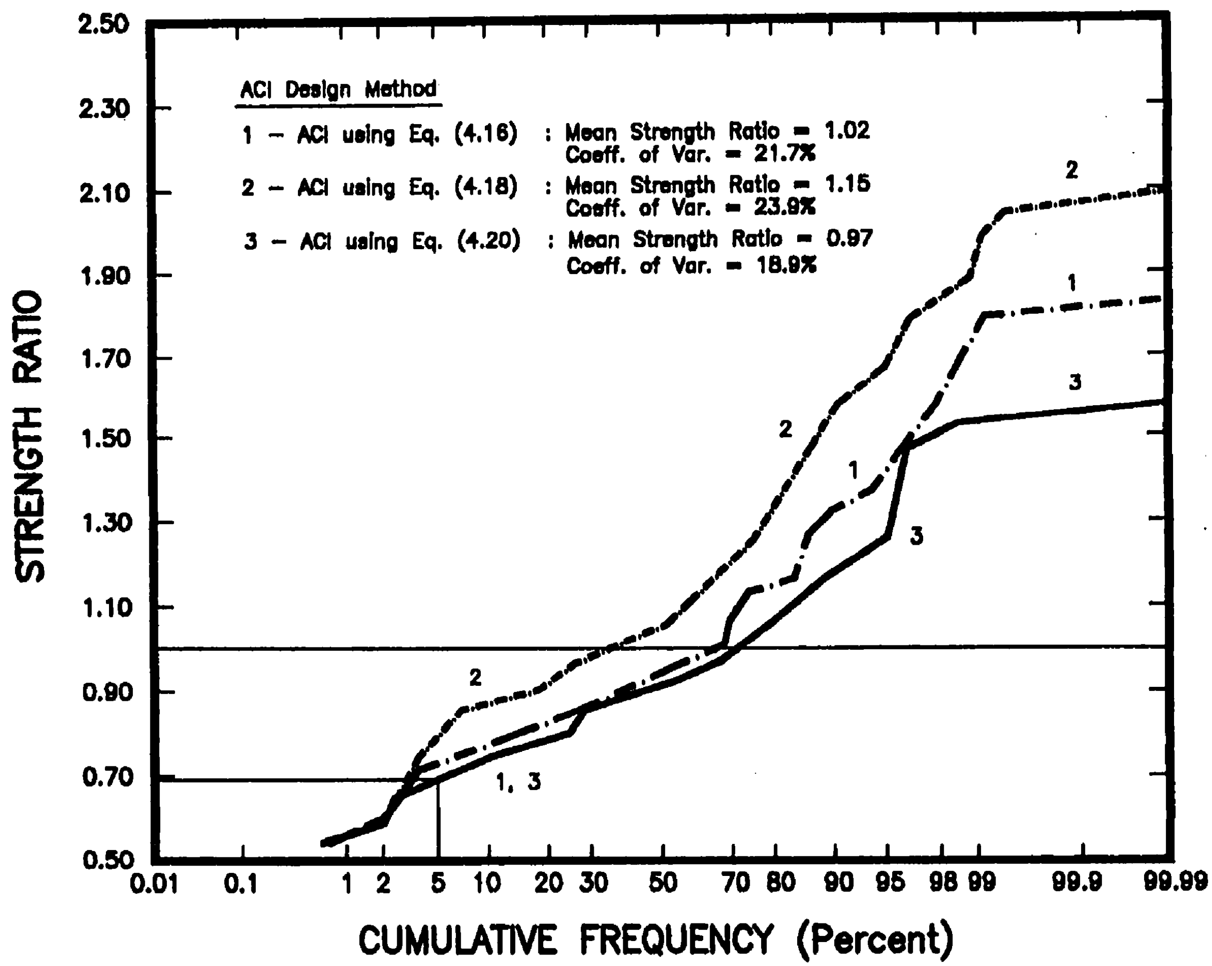



Figure 7.1.1.b - Probability distribution of strength ratios of composite steel-concrete columns subjected to minor axis bending for different design methods (n=81).

for ACI 318-95 using Equation 4.20, 0.77 for ACI 318-95 using Equation 4.16 and 0.80 for ACI 318-95 using Equation 4.18.

Figure 7.1.1.c shows the cumulative frequency distribution of strength ratios ($P_{test}/P_{des}$) plotted on a normal probability scale for the AISC-LRFD Specifications and the AISC-LRFD Specifications using Equation 4.38. From the figure it can be seen that there is a significant difference between the two curves. This is expected since Equation 4.38 was chosen to improve the predicted strength of composite steel-concrete columns subjected to minor axis bending. The use of using Equation 4.38 shifts the results of the AISC-LRFD Specifications upwards to a more conservative region but does not significantly reduce the variability of the strength ratios. The five percentile values are 0.85 for the AISC-LRFD and 1.04 for AISC-LRFD using Equation 4.38.

### 7.1.3 Effects of Major Variables on Strength Ratios

The effects of concrete strength ($f'_c$), end eccentricity ratio ($e/h$), slenderness ratio ($\ell/h$), structural steel index ($\rho_{ss}f_{yss}/f'_c$), and the tie/hoop volumetric ratio ($\rho''$) on the maximum, mean and minimum values of strength ratios ($P_{test}/P_{des}$) obtained using ACI

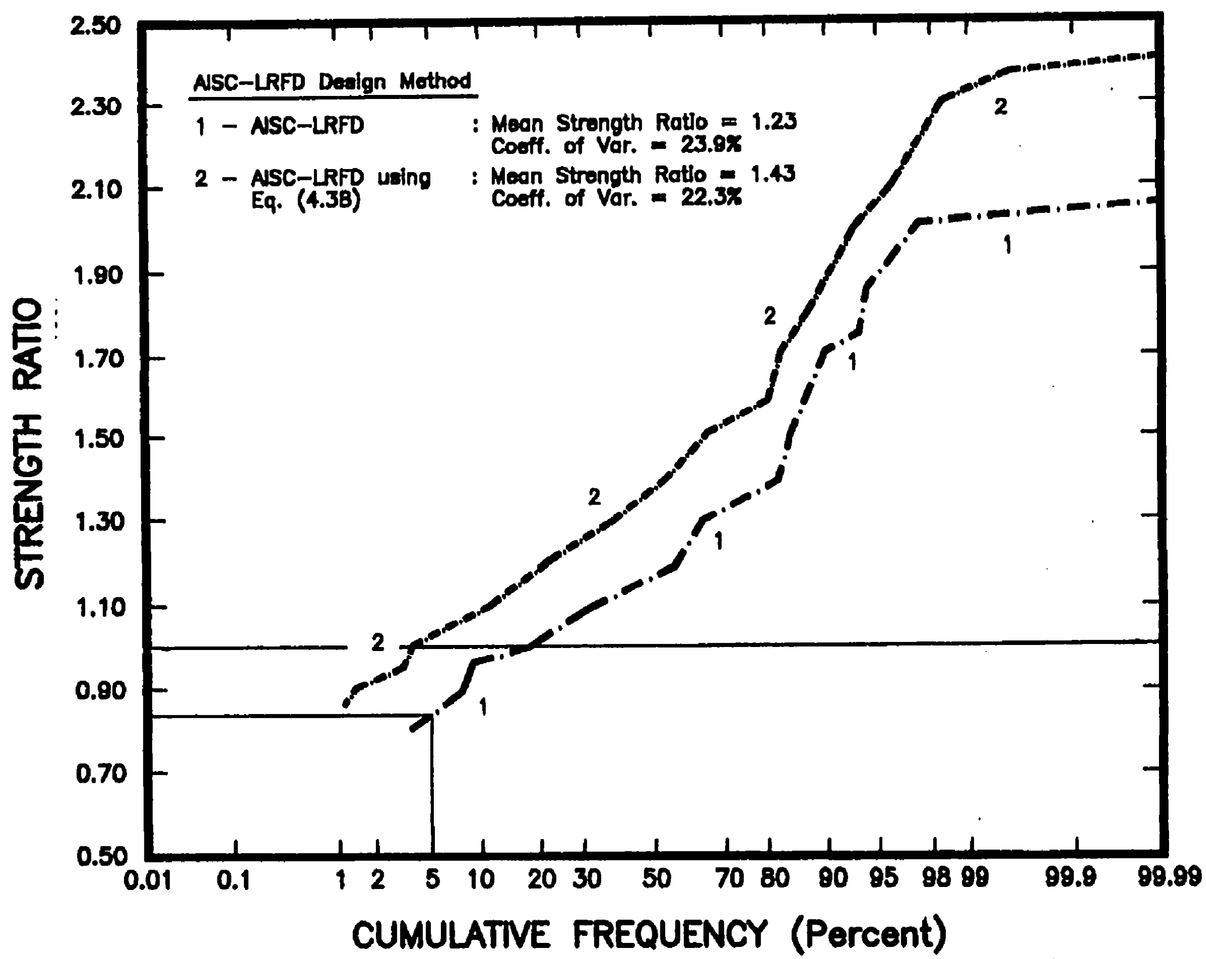



Figure 7.1.1.c - Probability distribution of strength ratios of composite steel-concrete columns subjected to minor axis bending for different design methods (n=81).

318-95, AISC-LRFD Specifications, Eurocode 4 and FEM are examined in this section.

Since the values of concrete strength, end eccentricity ratio, slenderness ratio, structural steel index and tie/hoop volumetric ratio used in this study were not controlled variables, strength ratios had to be grouped into ranges. Up to ten separate ranges were used in plotting each of the figures presented in this section. The maximum, mean and minimum value of strength ratios were determined for each of the ranges. Grouping the strength ratios resulted in having a significantly different number of columns in some of the ranges. This may explain the jaggedness of the lines associated with some of the figures presented in this section.

Figures 7.1.2.a to 7.1.2.c examine the effect of concrete strength ($f'_c$) on the maximum, mean, and minimum strength ratios. Because of the many different values of concrete strength used, nine ranges of concrete strength were set at 2500-2650, 2650-3300, 3300-3950, 3950-4600, 4600-5250, 5250-5900, 5900-6550, 6550-7200 and 7200-7700 psi. It can be seen from these figures that the maximum, mean and minimum strength ratios for all methods vary over the entire range of the concrete strength. For concrete strengths greater than 6500 psi, all curves tend to deviate upward towards the conservative region. This is due, probably, to the limited test data available for concrete

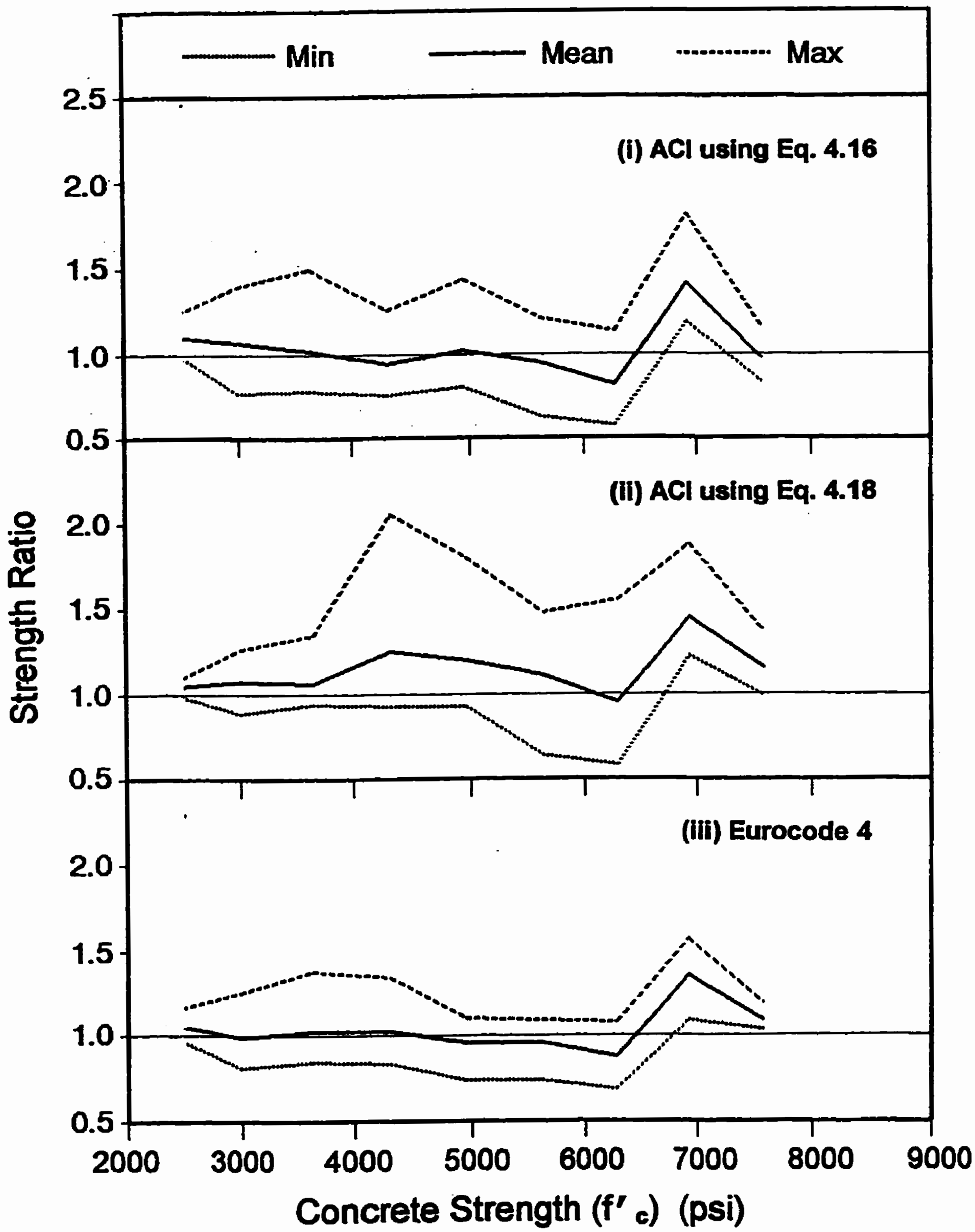



Figure 7.1.2.a - Effect of concrete strength on strength ratios obtained from different design methods for composite steel-concrete columns subjected to minor axis bending (n varies for each $f'_c$; total number of specimens=81).

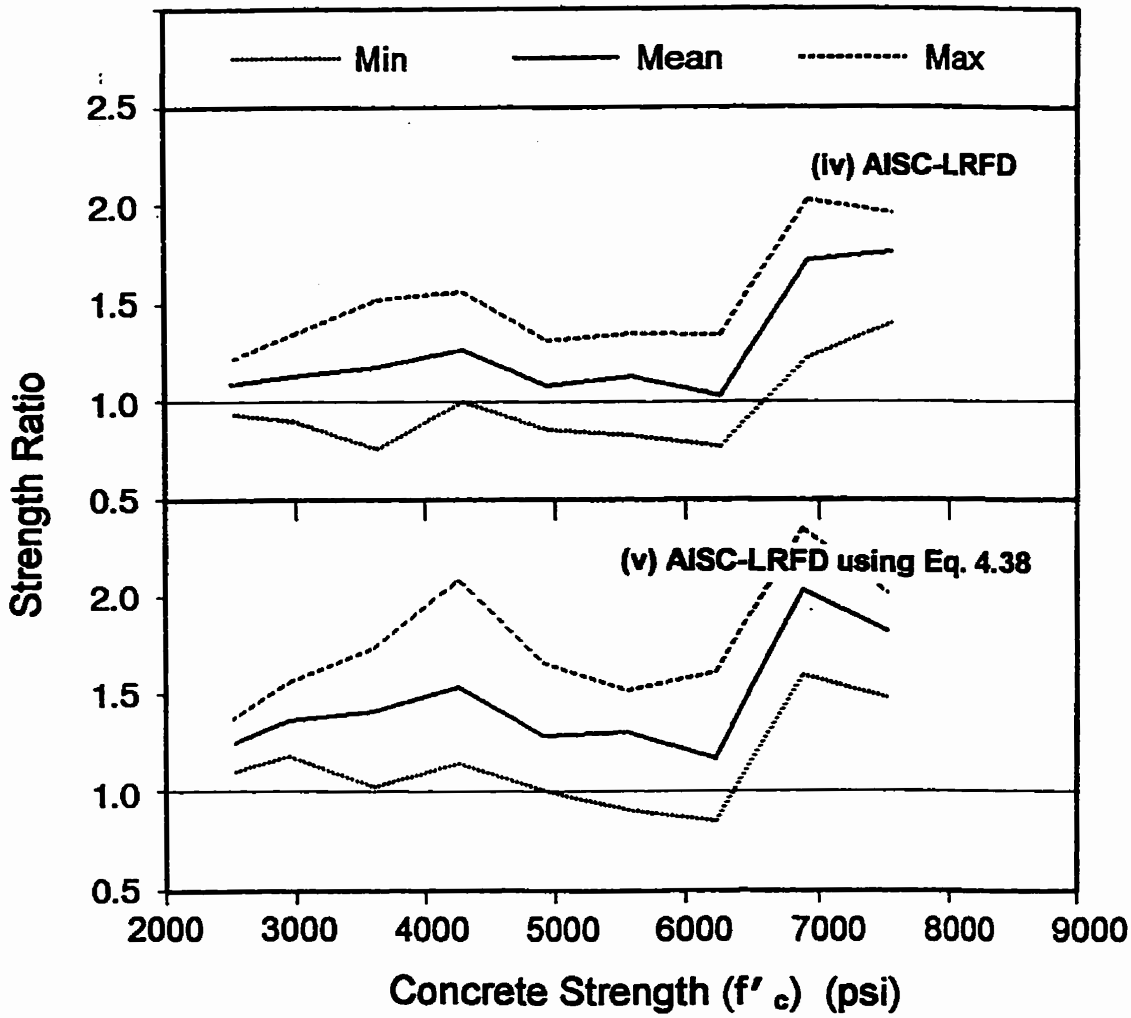



Figure 7.1.2.b - Effect of concrete strength on strength ratios obtained from different design methods for composite steel-concrete columns subjected to minor axis bending (n varies for each $f'_c$; total number of specimens=81).

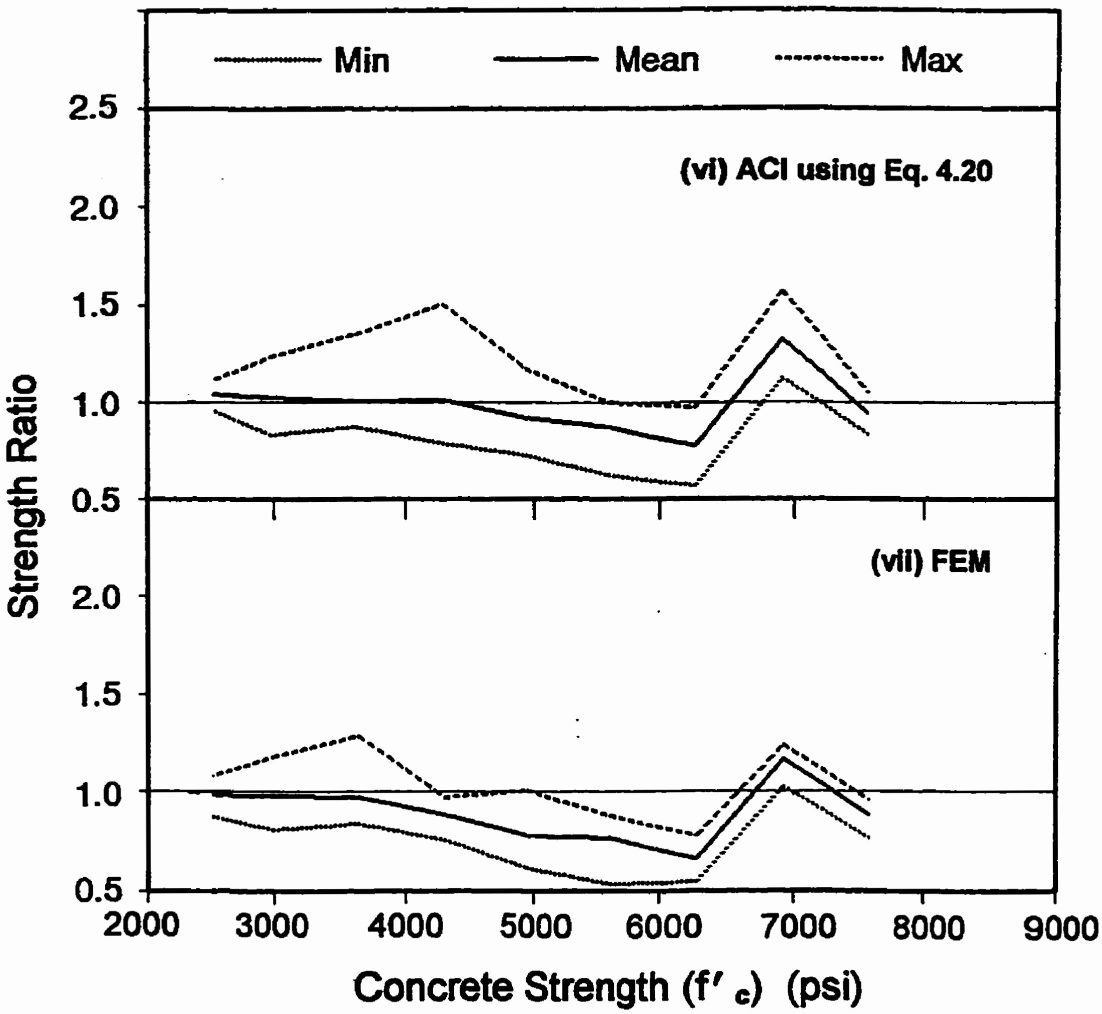



Figure 7.1.2.c - Effect of concrete strength on strength ratios obtained from different design methods for composite steel-concrete columns subjected to minor axis bending (n varies for each $f'_c$; total number of specimens=81).

strengths greater than 6500 psi. The strength ratios for ACI 318-95 using Equation 4.20 show an improvement over the strength ratios obtained for ACI 318-95 using Equations 4.16 and 4.18. The strength ratio curves for Eurocode 4 also show an improvement over the results obtained for ACI 318-95 using Equations 4.16 and 4.18. The strength ratios for the AISC-LRFD and the AISC-LRFD using Equation 4.38 tend to be more conservative than those obtained for all other design methods. FEM produced the most consistent results, however, the mean strength ratios were below 1.0 for almost all of the concrete strengths examined (Figure 7.1.2.c).

Figures 7.1.3.a to 7.1.3.c examine the effect of the end eccentricity ratio ($e/h$), on the maximum, mean, and minimum strength ratios. Because of the many different end eccentricity ratios used, five ranges of end eccentricity ratio were set at 0, 0-0.2, 0.2-0.4, 0.4-0.6 and 0.6-0.75. The effect of the end eccentricity ratio can not be clearly established from Figures 7.1.3.a to 7.1.3.c due, perhaps, to the lower number of test data available for certain ranges of $e/h$ ratios plotted in these figures. However, the strength ratios for ACI 318-95 using Equations 4.16 and 4.18, AISC-LRFD and AISC-LRFD using Equation 4.38 appear to be more affected by the end eccentricity ratio than those for ACI 318-95 using Equation 4.20, Eurocode 4 and FEM. All methods, except FEM, have mean strength ratios greater than 1.0 over almost the entire range of $e/h$ between 0 and

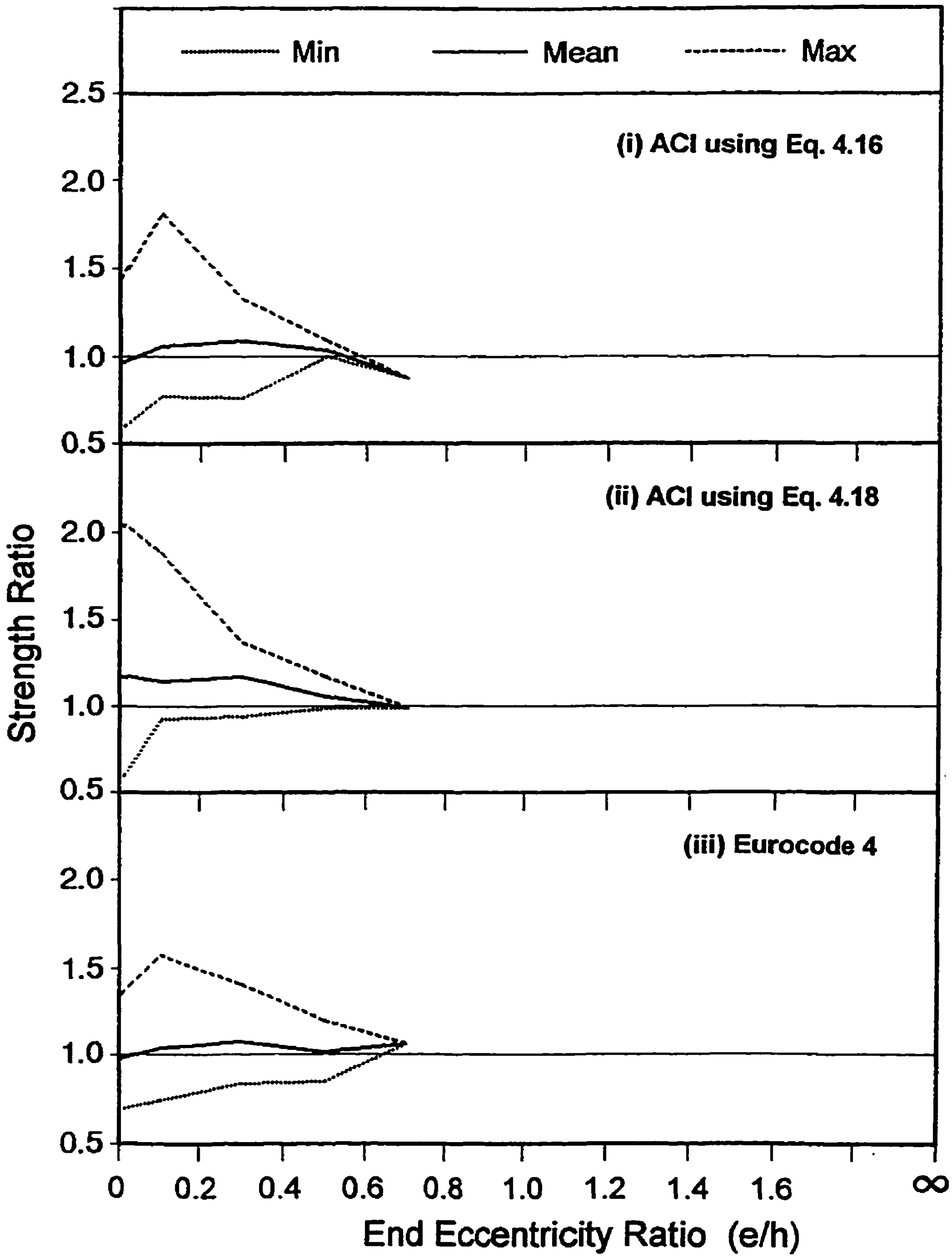



Figure 7.1.3.a - Effect of end eccentricity ratio on strength ratios obtained from different design methods for composite steel-concrete columns subjected to minor axis bending (n varies for each e/h ratio; total number of specimens=81).

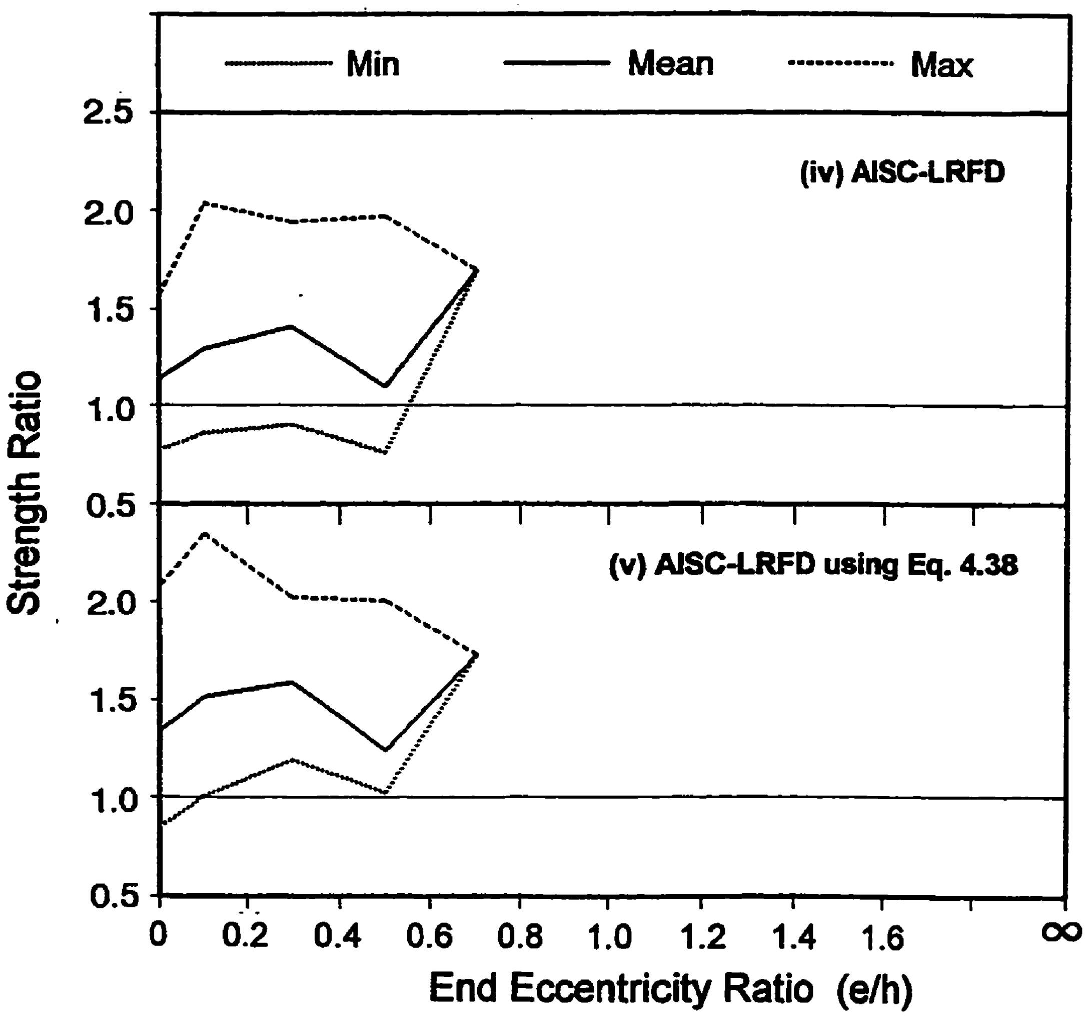



Figure 7.1.3.b - Effect of end eccentricity ratio on strength ratios obtained from different design methods for composite steel-concrete columns subjected to minor axis bending (n varies for each e/h ratio; total number of specimens=81).

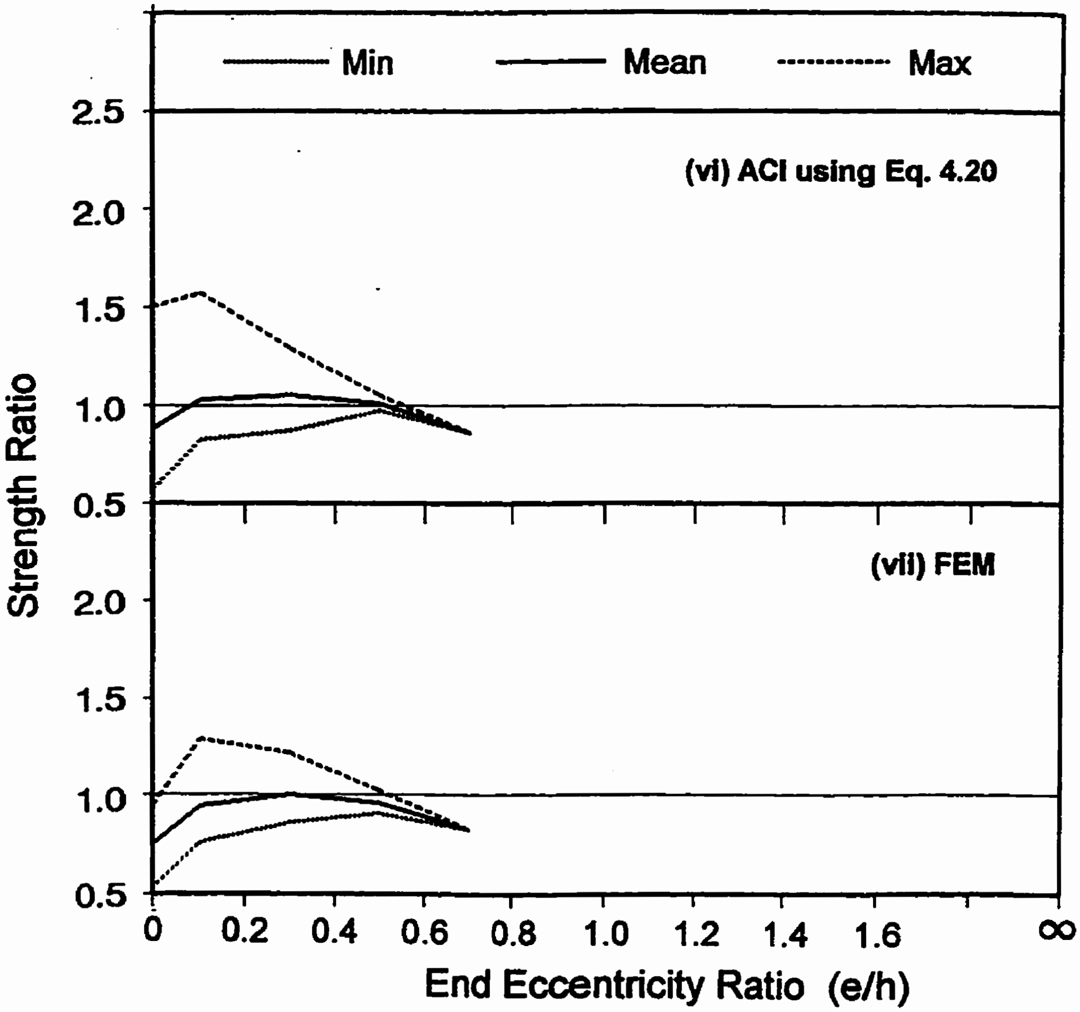



Figure 7.1.3.c - Effect of end eccentricity ratio on strength ratios obtained from different design methods for composite steel-concrete columns subjected to minor axis bending (n varies for each e/h ratio; total number of specimens=81).

0.71. FEM had a mean strength ratio below 1.0 for most of the $e/h$ ratios examined.

Figures 7.1.4.a to 7.1.4.c examine the effect of the slenderness ratio ($\ell/h$) on the maximum, mean and minimum strength ratios. Because of the many different slenderness ratios used, six ranges of slenderness ratios were set at 5-9, 9-13.5, 13.5-18, 18-22.5, 22.5-27 and 27-29. From the figures it can be seen that the slenderness ratio affects the strength ratios for all design methods. As the slenderness ratio increases, the mean strength ratio tends to increase for all design methods. The strength ratios obtained for ACI 318-95 using Equation 4.20 and Eurocode 4 showed an improvement over the strength ratios for ACI 318-95 using both Equations 4.16 and 4.18. The strength ratios for the AISC-LRFD and the AISC-LRFD using Equation 4.38 were most significantly affected by the slenderness ratio. FEM had the most consistent results over the entire range of slenderness ratios examined.

Figures 7.1.5.a to 7.1.5.c examine the effect of the structural steel index ($\rho_{ss}f_{yss}/f'_c$) on the maximum, mean, and minimum strength ratios. Because of the many different structural steel index values used, ten ranges of structural steel index were set at 0-0.2, 0.2-0.4, 0.4-0.6, 0.6-0.8, 0.8-1.0, 1.0-1.2, 1.2-1.4, 1.4-1.6, 1.6-1.8 and 1.8-2.0. For all methods, the difference between the maximum and minimum strength ratios tended to reduce as the

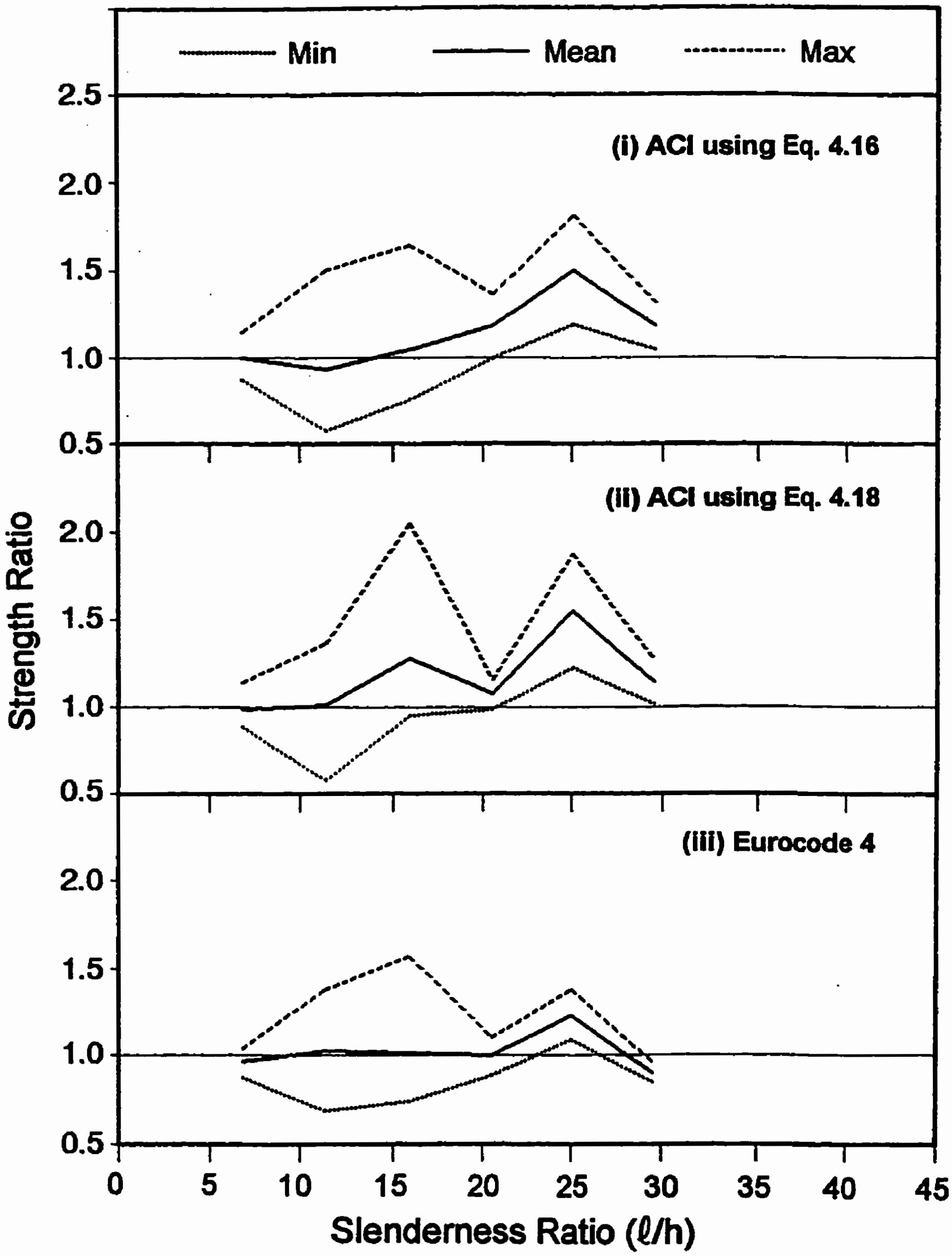



Figure 7.1.4.a - Effect of slenderness ratio on strength ratios obtained from different design methods for composite steel-concrete columns subjected to minor axis bending (n varies for each ℓ/h ratio; total number of specimens=81).

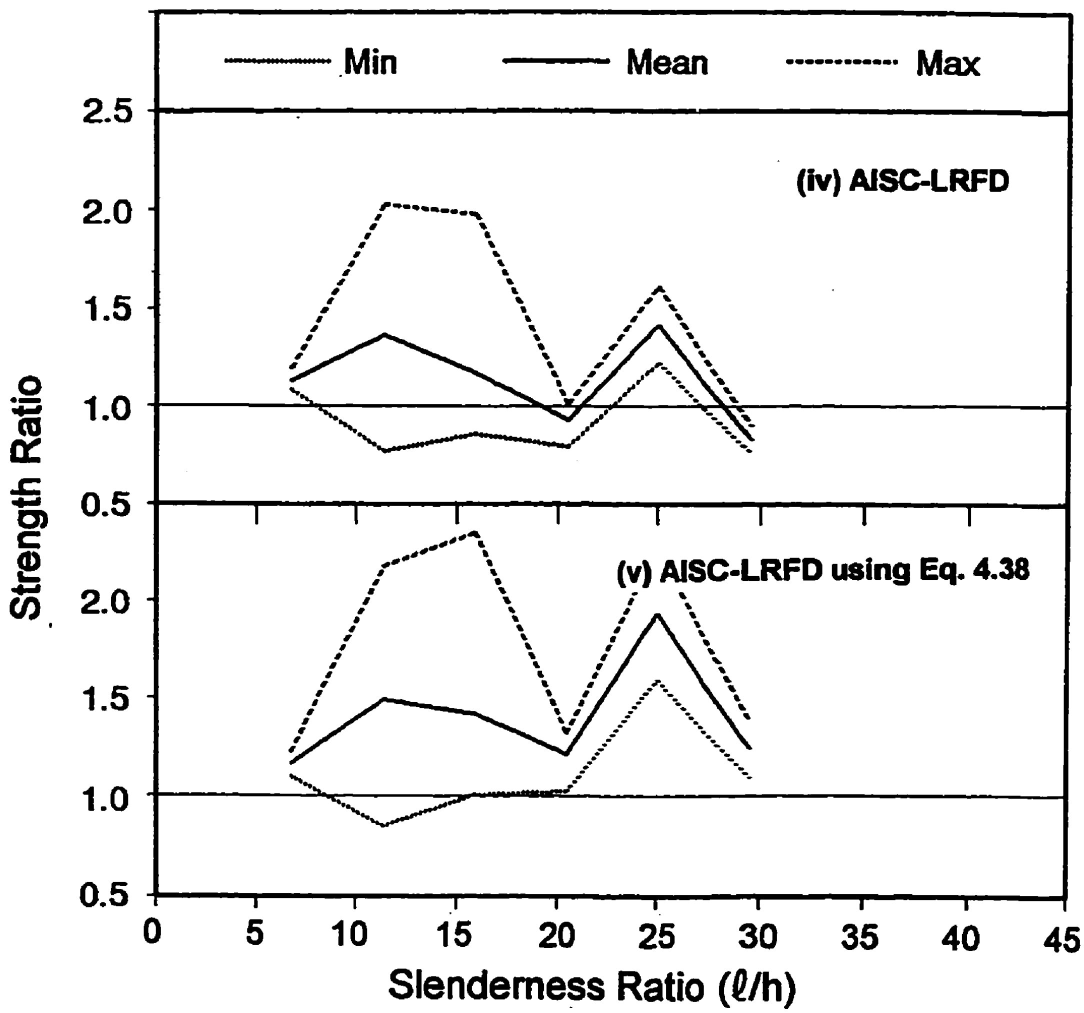



Figure 7.1.4.b - Effect of slenderness ratio on strength ratios obtained from different design methods for composite steel-concrete columns subjected to minor axis bending (n varies for each ℓ/h ratio; total number of specimens=81).

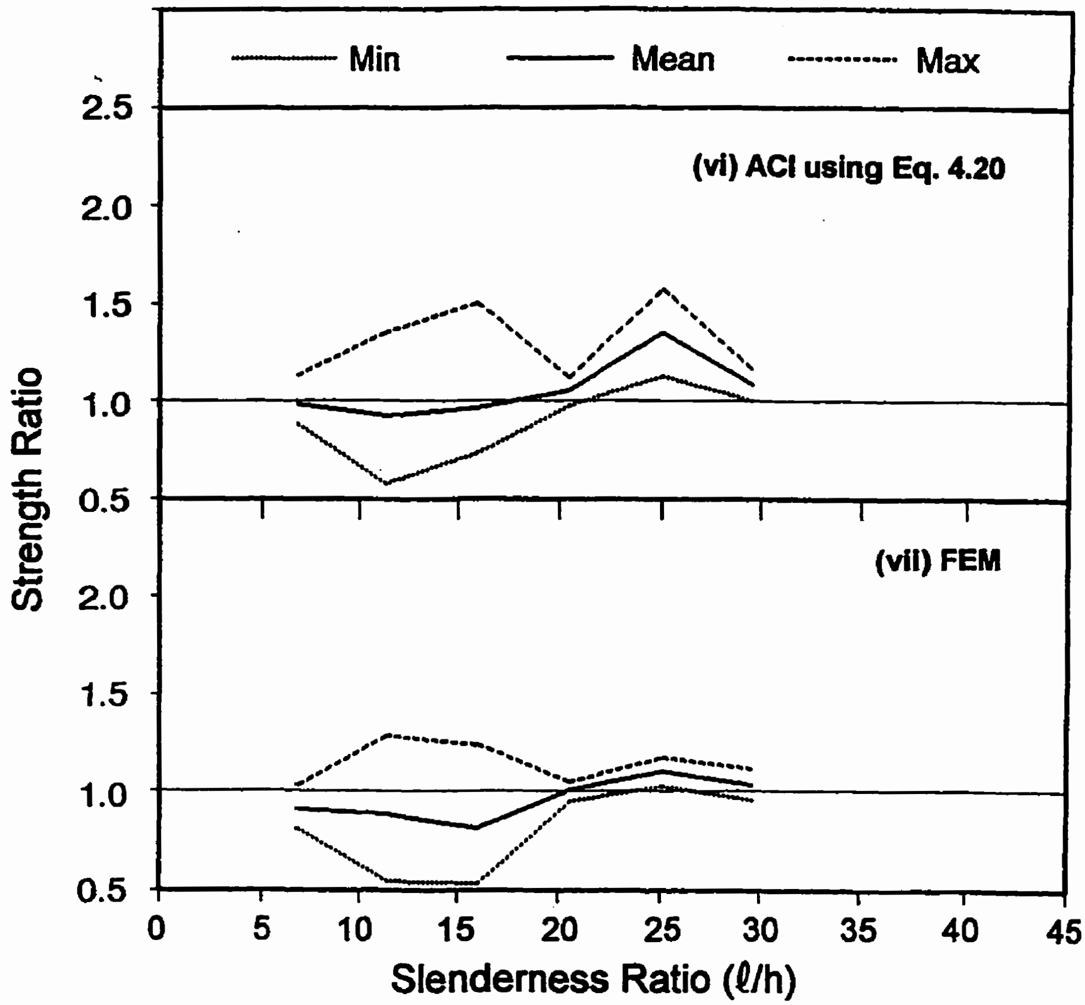



Figure 7.1.4.c - Effect of slenderness ratio on strength ratios obtained from different design methods for composite steel-concrete columns subjected to minor axis bending (n varies for each ℓ/h ratio; total number of specimens=81).

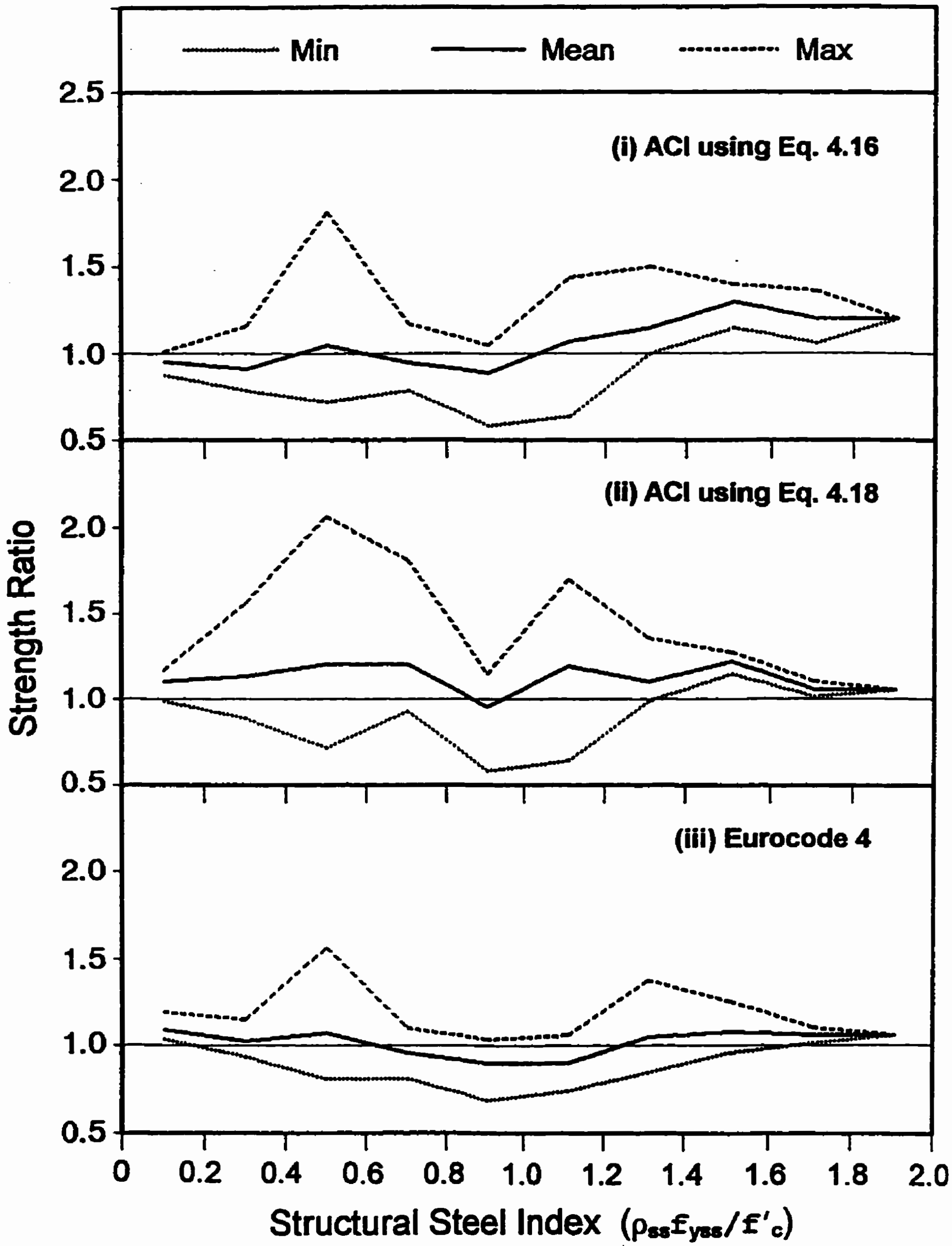



Figure 7.1.5.a - Effect of structural steel index on strength ratios obtained from different design methods for composite steel-concrete columns subjected to minor axis bending (n varies for each $\rho_{ss}f_{yss}/f'_c$ ratio; total number of specimens=81).

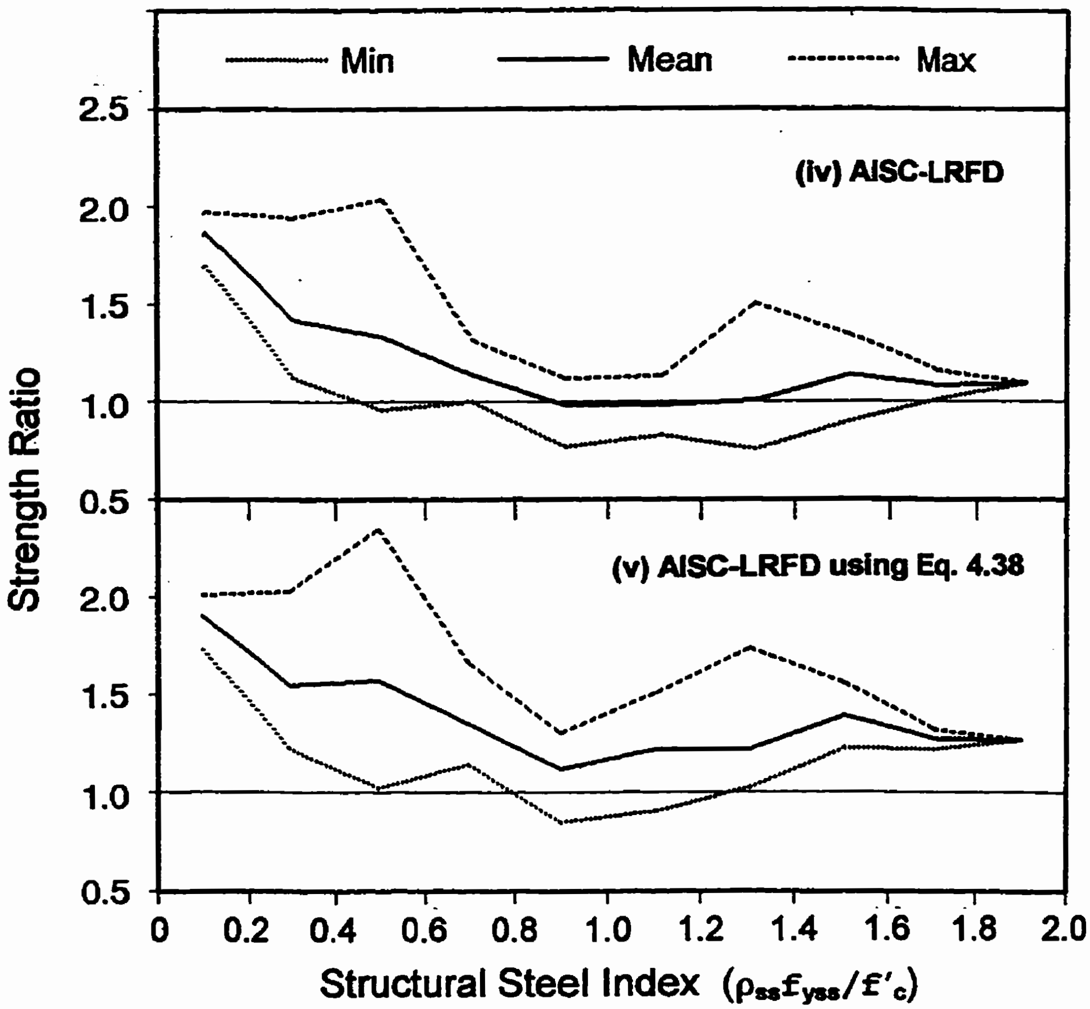



Figure 7.1.5.b - Effect of structural steel index on strength ratios obtained from different design methods for composite steel-concrete columns subjected to minor axis bending (n varies for each $\rho_{ss}f_{yss}/f'_c$ ratio; total number of specimens=81).

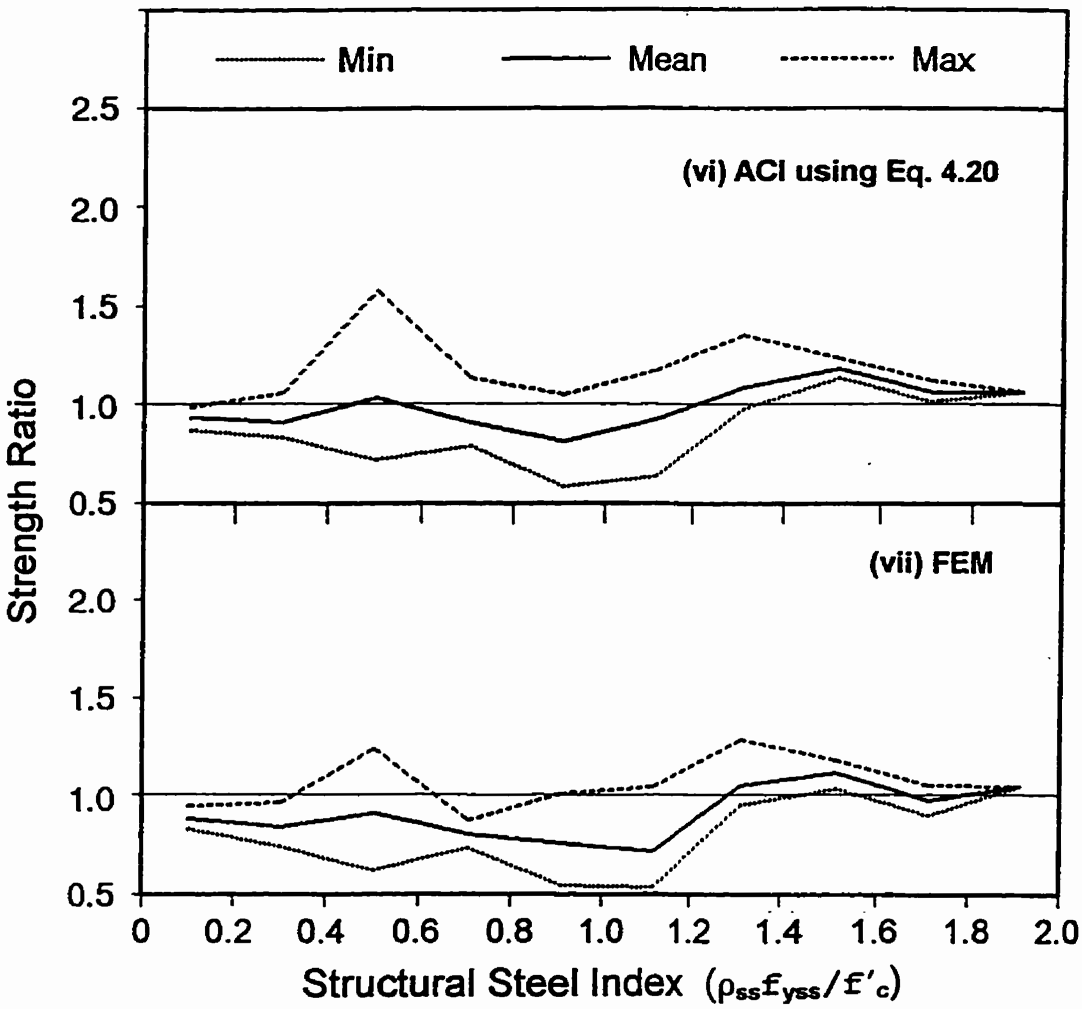



Figure 7.1.5.c - Effect of structural steel index on strength ratios obtained from different design methods for composite steel-concrete columns subjected to minor axis bending (n varies for each $\rho_{ss}f_{yss}/f'_c$ ratio; total number of specimens=81).

structural steel index increased. The strength ratios obtained from ACI 318-95 using Equation 4.20 showed an improvement over the strength ratios obtained from ACI 318-95 using both Equations 4.16 and 4.18. Note that Equation 4.16 does not include the effect of the structural steel in calculating the flexural rigidity, *EI*. The strength ratios obtained for Eurocode 4 were similar to those obtained from ACI 318-95 using Equation 4.20. The strength ratios for the AISC-LRFD and the AISC-LRFD using Equation 4.38 gradually become less conservative as the structural steel index increases. FEM produced the most consistent results, however, the mean strength ratio was less than 1.0 for most of the $\rho_{ss}f_{yss}/f'_c$ values examined.

Figures 7.1.6.a to 7.1.6.c examine the effect of the tie/hoop volumetric ratio ($\rho''$) on the maximum, mean, and minimum strength ratios. Because of the many different values of the tie/hoop volumetric ratio used, ten ranges of tie/hoop volumetric ratio were set at 0, 0-0.00033, 0.00033-0.00066, 0.00066-0.00099, 0.00099-0.00132, 0.00132-0.00165, 0.00165-0.00198, 0.00198-0.00231, 0.00231-0.00264 and 0.00264-0.00297. Note that the majority of test columns had tie/hoop volumetric ratios in the range of 0.0017 to 0.0030. It is for this reason that the high variability of strength ratios is occurring over this small range of $\rho''$ values, and is not spread out uniformly as was the case for the strength ratio curves for other variables

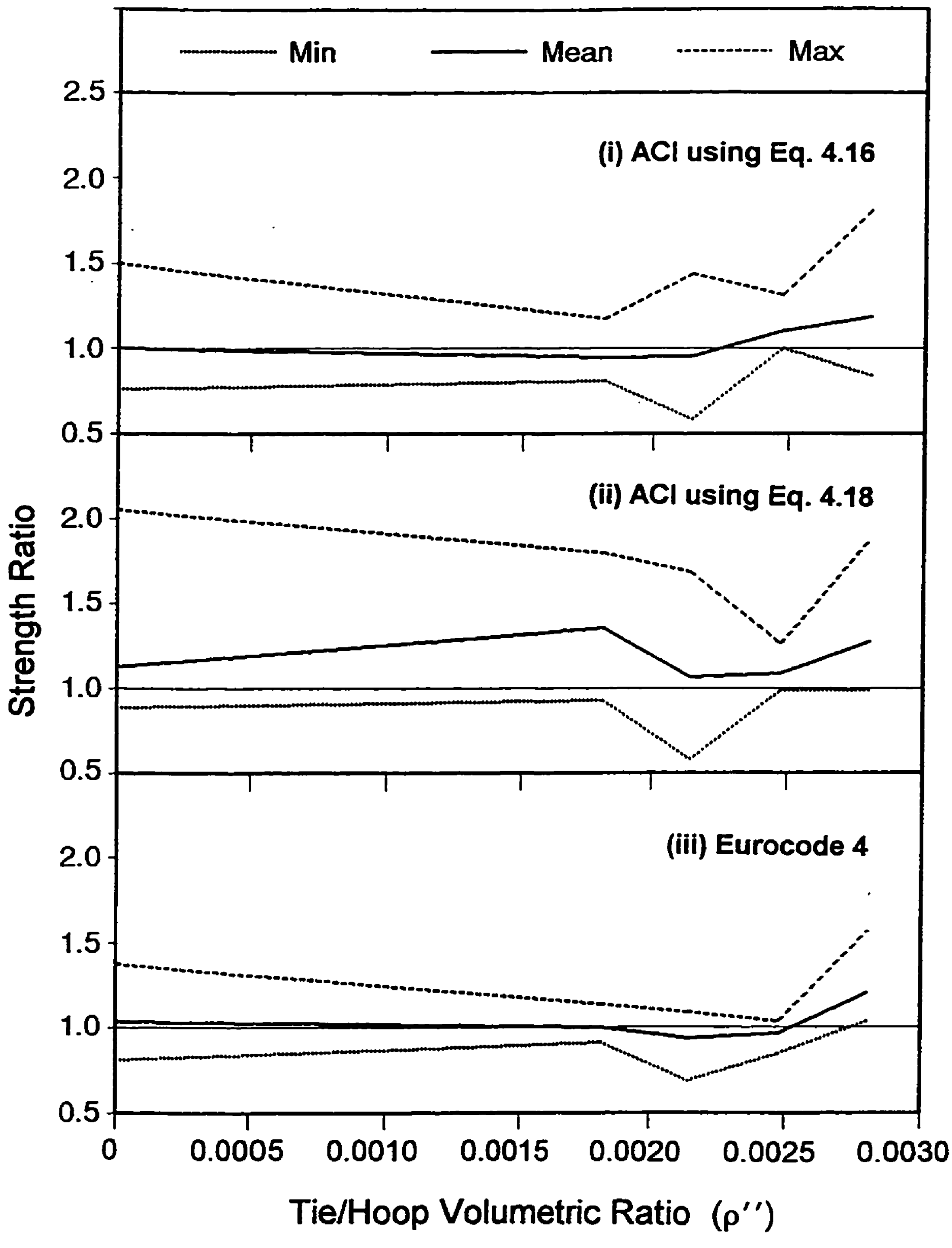



Figure 7.1.6.a - Effect of tie/hoop volumetric ratio on strength ratios obtained from different design methods for composite steel-concrete columns subjected to minor axis bending (n varies for each ρ'' ratio; total number of specimens=81).

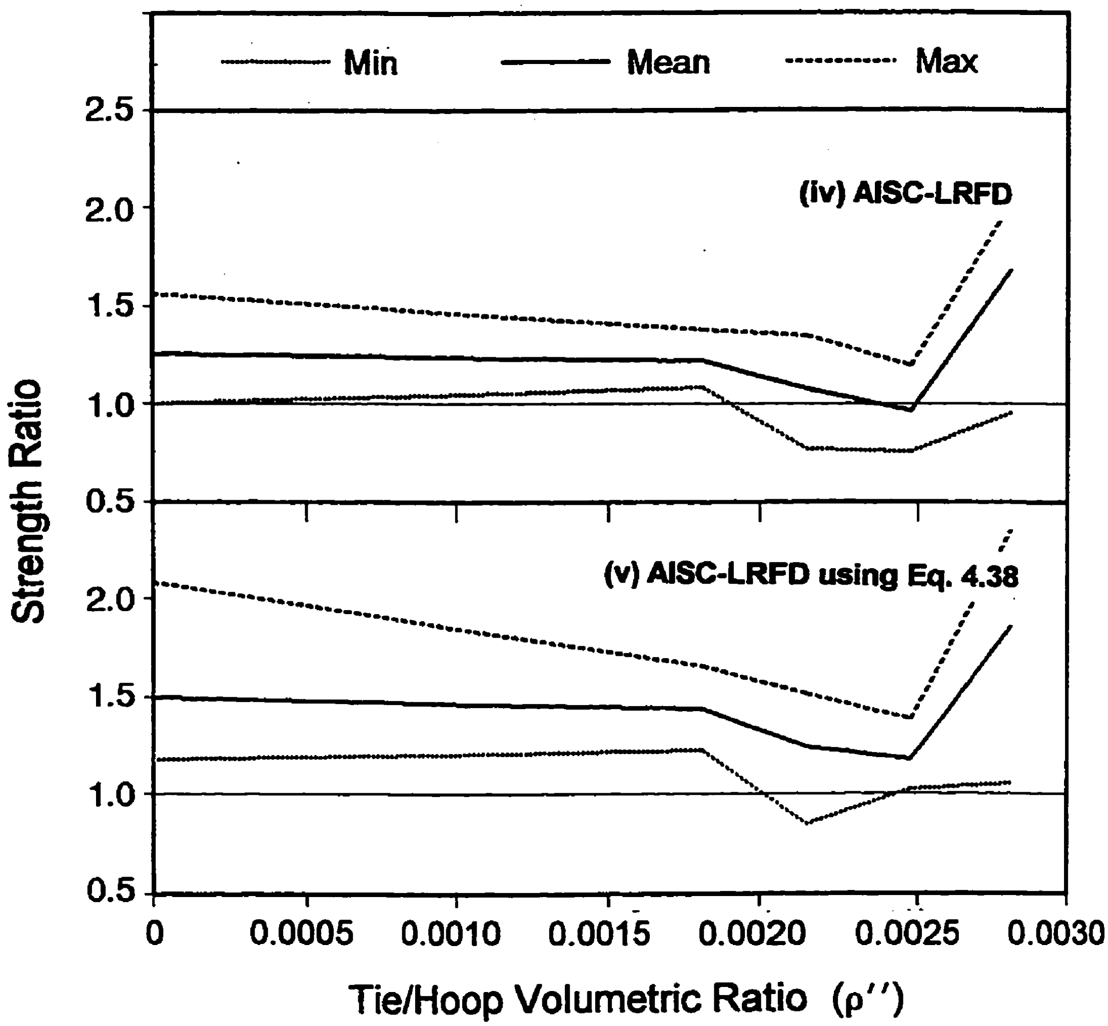



Figure 7.1.6.b - Effect of tie/hoop volumetric ratio on strength ratios obtained from different design methods for composite steel-concrete columns subjected to minor axis bending (n varies for each $\rho''$ ratio; total number of specimens=81).

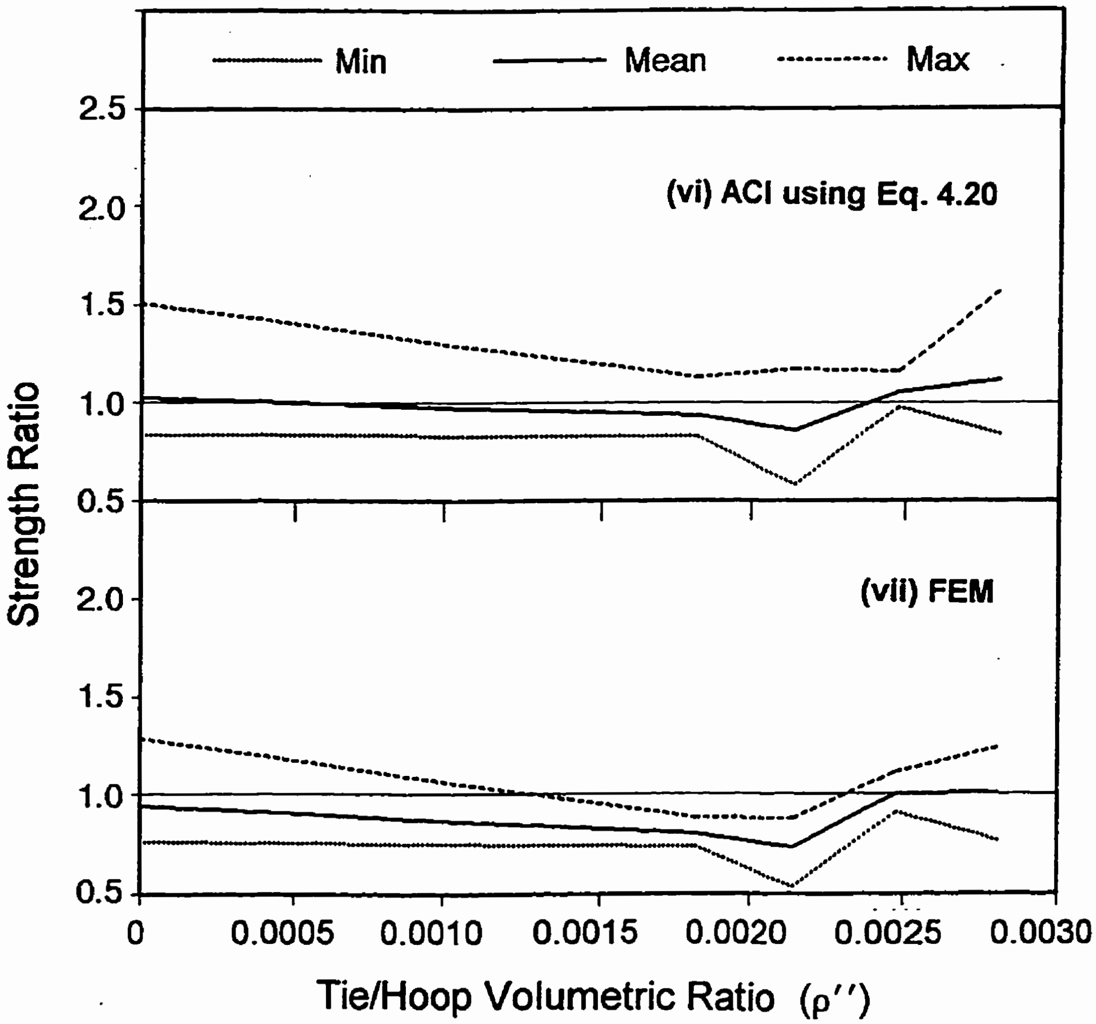



Figure 7.1.6.c - Effect of tie/hoop volumetric ratio on strength ratios obtained from different design methods for composite steel-concrete columns subjected to minor axis bending (n varies for each $\rho''$ ratio; total number of specimens=81).

discussed previously. For all methods, the difference between the maximum and minimum strength ratios tends to decrease as the tie/hoop volumetric ratio increases. The effect of $\rho''$ on strength ratios obtained for FEM (Figure 7.1.6.c) is minimal because of the fact that the lateral confinement of concrete provided by ties/hoops was included in FEM analysis of strength.

The following conclusions can be summarized from the data plotted in Figures 7.1.2 to 7.1.6 and the related discussion:

(1) The strength ratios produced by ACI 318-95 using Equation 4.18 and Equation 4.16 were affected by most of the major variables investigated. The strength ratios tended to become less conservative as the end eccentricity ratio increased and more conservative as the slenderness ratio increased.

(2) The strength ratios produced by ACI 318-95 using Equation 4.20 and Eurocode 4 gave statistics somewhat better than those obtained from ACI 318-95 using Equations 4.16 and 4.18. Note that Equation 4.20 takes into account the effect on *EI* of *e/h* ratio as a function of cracking of concrete caused by the presence of bending moment.

(3) The strength ratios produced by the AISC-LRFD Specifications demonstrated a pronounced effect of all

of the major variables investigated, particularly the end eccentricity ratio and structural steel index.

(4) The strength ratio statistics obtained for the AISC-LRFD Specifications using Equation 4.38 showed a similar trend as those computed for the AISC-LRFD Specifications. However, the strength ratio statistics for the AISC-LRFD Specifications using Equation 4.38 produced an overall shift in the results towards the conservative side. This was expected since Equation 4.38 was chosen to do just that for steel-concrete composite columns subjected to minor axis bending.

(5) The FEM produced the most consistent results that tended to be least affected by the variables examined. However, in most cases, the mean strength ratios were below 1.0 for the FEM procedure used.

# 8 - SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

## 8.1 SUMMARY

This study presents a statistical evaluation and comparison of the effects of different parameters on the ultimate strength of rectangular reinforced concrete columns and composite steel-concrete columns (structural steel shapes encased in concrete subjected to major and minor axis bending). The columns studied involved normal-density, normal-strength concrete, were pin-ended, and had both equal and unequal load eccentricities acting at the column ends. To study the full range of variables, 384 reinforced concrete columns without moment gradient, 14 reinforced concrete columns with moment gradient, 75 composite steel-concrete columns subjected to major axis bending without moment gradient, 3 composite steel-concrete columns subjected to major axis bending with moment gradient, and 143 composite steel-concrete columns subjected to minor axis bending without moment gradient were taken from the literature. No new tests were conducted for this study.

The results of the physical tests found in the literature were compared against the ultimate column strengths computed using ACI 318-95, the AISC-LRFD Specifications, Eurocode 2, Eurocode 4, and FEM. For ACI 318-95, proposed equations by Mirza (1990) and Tikka and Mirza (1992) for determining the flexural rigidity of reinforced concrete columns and composite steel-concrete columns, respectively, were also included in this study. A new equation for computing the radius of gyration of encased composite column cross-sections is proposed for use in the AISC-LRFD procedure and its evaluation was also included in the study.

Various combinations of the specified concrete strength, the end eccentricity ratio, the slenderness ratio, the reinforcing steel index, the structural steel index and the tie/hoop volumetric ratio were used to study the effects of these variables on the computed column strengths.

Based on the statistical analysis of the major variables that affect column strength, a comparison of different design methods is presented. Most of the design methods were affected to some degree by some or all of the variables studied. The variability of each of the design methods used for computing the ultimate strength of reinforced concrete and composite steel-concrete columns is documented.

## 8.2 CONCLUSIONS RELATED TO REINFORCED CONCRETE COLUMNS WITHOUT MOMENT GRADIENT

From the discussion, tables and plots given in Chapter 5 for reinforced concrete columns without moment gradient, the following conclusions seem to be valid:

(1) ACI 318-95 using Equation 4.16, in most cases, produces the most conservative results for the mean strength ratios but also has the highest variability when compared to ACI 318-95 using both Equations 4.17 and 4.19. Note that Equation 4.16 is the simplified flexural rigidity equation which only accounts for the concrete contribution.

(2) ACI 318-95 using Equation 4.17 demonstrated an improved effect on the strength ratio statistics due to all of the major variables examined when compared to the results obtained from ACI 318-95 using Equation 4.16. This is due, probably, to the fact that Equation 4.17 accounts for the contribution of both the concrete and longitudinal reinforcing steel.

(3) ACI 318-95 using Equation 4.19 was not significantly affected by most of the major variables examined. The strength ratio statistics showed an improvement over those obtained for ACI 318-95 using both Equations 4.16 and 4.17. This is due, probably, to the fact that Equation 4.19 accounts for the contributions of concrete, longitudinal reinforcing steel, and $e/h$ ratio. Note that the $e/h$ ratio is taken as a function of cracking of concrete caused by the presence of bending moment.

(4) The strength ratio statistics for Eurocode 2 were affected by most of the major variables examined and

tended to show a trend similar to that obtained from ACI 318-95 using Equation 4.17.

(5) FEM produced the most consistent results and was not significantly affected by the variables examined. This was demonstrated by the relatively small differences between the maximum and minimum strength ratios over the full range of variables studied. However, the mean strength ratios were below 1.0 in many cases.

## 8.3 CONCLUSIONS RELATED TO REINFORCED CONCRETE COLUMNS WITH MOMENT GRADIENT

From the discussion given in Chapter 5 for reinforced concrete columns with moment gradient, the following conclusions seem to be valid:

(1) The strength ratio statistics for ACI 318-95 were significantly affected by the flexural rigidity equation used: Equation 4.16 demonstrated the highest variability of strength ratios followed by ACI 318-95 using Equations 4.17 and 4.19.

(2) The strength ratio statistics for Eurocode 2 were similar to those obtained from ACI 318-95 using Equations 4.17 and 4.19.

(3) Again, the FEM produced the most consistent strength ratio statistics with a significant improvement over those obtained for all other methods.

## 8.4 CONCLUSIONS RELATED TO COMPOSITE STEEL-CONCRETE COLUMNS SUBJECTED TO MAJOR AXIS BENDING WITHOUT MOMENT GRADIENT

From the discussion, tables and plots given in Chapter 6 for composite steel-concrete columns subjected to major axis bending without moment gradient, the following conclusions seem to be valid:

(1) The strength ratios from ACI 318-95 using Equation 4.16 were affected by most of the variables examined.

(2) Improved strength ratio statistics were obtained for ACI 318-95 using Equations 4.18 and 4.20 and for Eurocode 4. The statistics were similar for the three procedures. Note that Equation 4.20 for *EI* takes into account the effect of $e/h$ ratio as a function of cracking of concrete caused by the presence of bending moment.

(3) The strength ratios produced by the AISC-LRFD Specifications demonstrated a pronounced effect of all of the major variables examined, particularly, the concrete strength. The AISC-LRFD Specifications using Equation 4.38 produced similar results. Note that Equation 4.38 was chosen to improve the predicted strength of composite steel-concrete columns subjected to minor axis bending and was not intended to improve the predicted strength for composite columns subjected to major axis bending.

(4) The strength ratio statistics for FEM tended to be least affected by the variables examined. The differences between the maximum and minimum strength ratios were relatively small as compared to all other methods. However, the mean strength ratios fell below 1.0 in most cases.

## 8.5 CONCLUSIONS RELATED TO COMPOSITE STEEL-CONCRETE COLUMNS SUBJECTED TO MINOR AXIS BENDING WITHOUT MOMENT GRADIENT

From the discussion, tables and plots given in Chapter 7 for composite steel-concrete columns subjected to minor axis bending without moment gradient, the following conclusions seem to be valid:

(1) The strength ratios for ACI 318-95 using Equation 4.16 and Equation 4.18 were affected by most of the variables examined.

(2) The strength ratio statistics for ACI 318-95 using Equation 4.20 and for Eurocode 4 showed an improvement over those obtained for ACI 318-95 using Equations 4.16 and 4.18. Note that Equation 4.20 takes into account the effect of *e/h* as a function of cracking of concrete caused by the presence of bending moment.

(3) The strength ratio statistics produced by AISC-LRFD demonstrated a pronounced effect of all of the major variables examined. The AISC-LRFD using Equation 4.38 produced similar but more conservative results. This was expected since Equation 4.38 was chosen to improve

the predicted strength of composite steel-concrete columns subjected to minor axis bending.

(4) The strength ratio statistics for FEM tended to be the least affected by the variables examined. However, the mean strength ratios were below 1.0 in most of the cases examined.

## 8.6 RECOMMENDATIONS

For final (more accurate) designs, Equation 4.19 proposed by Mirza (1990) and Equation 4.20 proposed by Tikka and Mirza (1992), are recommended for use in ACI 318-95 for determining the flexural rigidity of reinforced concrete and composite steel-concrete columns, respectively. The existing ACI 318-95 equations (Equations 4.16, 4.17 and 4.18) may be used as a substitute in most cases, particularly for initial sizing of members.

Equation 4.38 for determining the maximum useable radius of gyration for composite steel-concrete columns is recommended for use in the AISC-LRFD procedure. This equation tends to increase the minimum strength ratios for composite steel-concrete columns subjected to minor axis bending.

Due to the limited number of physical test columns subjected to moment gradient available in the literature, it is suggested that further experimental research be focused in this area.

The relatively low mean strength ratios found for FEM for composite steel-concrete columns subjected to minor axis bending raise some concerns. Further studies to determine the cause of this observation are recommended.

# LIST OF SYMBOLS

| | |
|---|---|
| $a$ | depth of the equivalent rectangular stress block. |
| $b$ | overall width of flexural rigidity taken parallel to the axis of bending. |
| $c$ | distance from the extreme compression fiber to the neutral axis. |
| $c_r$ | average distance from the compression face to the longitudinal reinforcement in that face and the distance from the tension face to the longitudinal reinforcement in that face (AISC). |
| $c_y$ | perpendicular distance from the outer concrete face to the tip of the structural steel flange in a direction parallel to the axis of bending (Eurocode 4). |
| $c_z$ | perpendicular distance from the outer concrete face to the tip of the structural steel flange in a direction perpendicular to the axis of bending (Eurocode 4). |
| $d$ | effective depth of the composite flexural rigidity in the expected direction of stability failure (Eurocode 2). |
| $e$ | end eccentricity of axial load at column ends. |
| $e_a$ | additional eccentricity to account for initial imperfections (Eurocode 2). |
| $e_e$ | equivalent eccentricity to account for moment gradients (Eurocode 2). |
| $e_o$ | initial first-order eccentricity (Eurocode 2). |
| $e_{o1}$ | the smaller applied end eccentricity, positive if member is bent in single curvature, negative if bent in double curvature (Eurocode 2). |
| $e_{o2}$ | the larger applied end eccentricity, always positive (Eurocode 2). |
| $e_{tot}$ | total design eccentricity (Eurocode 2). |
| $e_2$ | second-order eccentricity (Eurocode 2). |
| $e/h$ | end eccentricity ratio. |

| | |
|---|---|
| $f_c$ | stress of concrete that corresponds to a given value of strain. |
| $f_{cd}$ | design concrete strength (Eurocode 2 and Eurocode 4). |
| $f_{ci}$ | in-situ strength of concrete. |
| $f_{cr}$ | in-situ strength of concrete accounting for the rate of loading. |
| $f_{cu}$ | strength of a 6-inch concrete cube. |
| $f_o$ | strength of a 4-inch concrete cube. |
| $f_r$ | modulus of rupture of concrete. |
| $f_{rr}$ | modulus of rupture of concrete accounting for the rate of loading. |
| $f_{sd}$ | design strength of longitudinal reinforcing steel (Eurocode 4). |
| $f_y$ | specified yield strength of structural steel. |
| $f_{yd}$ | design yield strength of reinforcing steel (Eurocode 2). |
| $f_{yh}$ | specified yield strength of transverse ties. |
| $f_{yr}$ | specified yield strength of reinforcing steel. |
| $f_{yss}$ | specified yield strength of structural steel. |
| $f'_c$ | specified strength of concrete. |
| $h$ | overall depth of the flexural rigidity taken perpendicular to the axis of bending. |
| h″ | out to out width of the lateral ties. |
| $i$ | radius of gyration of the uncracked concrete flexural rigidity (Eurocode 2). |
| $k$ | effective column length factor (ACI); moment correction factor (Eurocode 4). |
| $\ell$ | column length. |
| $\ell_o$ | effective length of column (Eurocode 2). |
| $\ell_u$ | unsupported column length. |

| | |
|---|---|
| $\ell/h$ | slenderness ratio. |
| $n$ | number of test specimens. |
| $r$ | radius of gyration; ratio of smaller to larger end moments in the column (Eurocode 4). |
| $r_m$ | modified radius of gyration (AISC). |
| $r_{max}$ | maximum useable radius of gyration (ACI and AISC). |
| $s_h$ | vertical spacing of transverse ties. |
| $t$ | testing time in seconds. |
| $u$ | cube strength of concrete. |
| $v_o$ | volume of a 4-inch cube. |
| $A_a$ | area of structural steel shape (Eurocode 4). |
| $A_c$ | area of concrete (AISC); net area of concrete (Eurocode 2). |
| $A_g$ | gross area of flexural rigidity (ACI); gross area of steel section (AISC). |
| $A_r$ | area of longitudinal reinforcing steel (AISC). |
| $A_{rs}$ | area of longitudinal reinforcing steel. |
| $A_s$ | total area of the structural steel section (AISC); area of longitudinal reinforcement (Eurocode 2 and Eurocode 4). |
| $A_{ss}$ | area of structural steel section. |
| $A_{st}$ | total area of the longitudinal reinforcement. |
| $A_t$ | total area of the structural steel section in the composite flexural rigidity. |
| $A_w$ | web area of structural steel section (AISC). |
| $B_1$ | moment magnification factor for non-sway moments (AISC). |
| $B_2$ | moment magnification factor for sway moments only (AISC). |
| $C_c$ | compressive force developed in equivalent stress block. |

$C_m$ factor relating the actual bending moment diagram to an equivalent uniform bending moment diagram.

$C_{rc}$ compressive force developed in reinforcing steel.

$C_{ssf}$ resultant compressive force in the structural steel section flange.

$C_{ssw}$ resultant compressive force in the structural steel section web.

$E$ modulus of elasticity of structural steel (AISC).

$E_a$ modulus of elasticity of structural steel section (Eurocode 4).

$E_c$ initial tangent modulus of elasticity of concrete.

$E_{cd}$ design modulus of elasticity of concrete (Eurocode 4).

$E_{cr}$ initial tangent modulus of elasticity of concrete accounting for rate of loading.

$E_m$ modified modulus of elasticity of structural steel section (AISC).

$E_s$ modulus of elasticity of structural steel.

$E_t$ tangent strain softening modulus of concrete.

$EI$ effective flexural rigidity of reinforced concrete and composite steel-concrete column.

$F_{cr}$ critical buckling stress (AISC).

$F_{my}$ modified yield stress for structural steel section (AISC).

$F_y$ yield stress for structural steel section (AISC).

$F_{yr}$ yield stress for reinforcing steel (AISC).

$I_a$ moment of inertia of structural steel section (Eurocode 4).

$I_c$ gross moment of inertia of concrete flexural rigidity (Eurocode 4).

$I_g$ gross moment of inertia of concrete flexural rigidity.

$I_{rs}$ moment of inertia of longitudinal reinforcing steel.

$I_s$ moment of inertia of structural steel section (Eurocode 4).

$I_{se}$ moment of inertia of reinforcing steel taken about the centroidal axis of the flexural rigidity.

$I_{ss}$ moment of inertia of structural steel section.

$I_t$ moment of inertia of structural steel section taken about the centroidal axis of the composite flexural rigidity.

$K$ effective length factor (AISC).

$M_c$ magnified moment to be used for design of the compression member (ACI).

$M_{col}$ moment capacity of column at an axial load equal to $P_u$.

$M_{cs}$ moment capacity of flexural rigidity at an axial load equal to $P_u$.

$M_n$ nominal flexural strength of composite section (AISC).

$M_{lt}$ required flexural strength in the member as a result of lateral translation of the frame only (AISC).

$M_{nt}$ required flexural strength in the member assuming there is no lateral translation of the frame (AISC).

$M_{pl,Rd}$ plastic resistance of composite flexural rigidity under pure bending (Eurocode 4).

$M_{sd}$ design bending moment including second-order effects (Eurocode 4).

$M_u$ magnified moment for use in column design (AISC).

$M_1$ the smaller applied end moment.

$M_2$ the larger applied end moment.

$N_{bal}$ axial load which maximizes the ultimate moment capacity of the section.

| | |
|---|---|
| $N_{cr}$ | elastic critical composite column load (Eurocode 4). |
| $N_{pl,Rd}$ | plastic resistance of composite flexural rigidity in pure compression (Eurocode 4). |
| $N_{sd}$ | design axial load (Eurocode 2 and Eurocode 4). |
| $N_{ud}$ | design ultimate capacity of the flexural rigidity subjected to axial load only (Eurocode 2). |
| $N_{\chi}$ | resistance of column under pure axial load (Eurocode 4). |
| $P_c$ | critical column load (ACI). |
| $P_{el}$ | critical column load (AISC). |
| $P_n$ | nominal compressive strength. |
| $P_{n(max)}$ | maximum nominal flexural rigidity compressive strength (ACI). |
| $P_u$ | ultimate compressive strength; required compressive strength (AISC). |
| $T_{rt}$ | tensile force developed in reinforcing steel. |
| $T_{ssf}$ | resultant tensile force in the structural steel section flange. |
| $T_{ssw}$ | resultant tensile force in the structural steel section web. |
| $Z$ | plastic section modulus of structural steel section. |
| $\beta$ | factor relating the actual bending moment diagram to an equivalent uniform bending moment diagram (Eurocode 4). |
| $\beta_d$ | ratio of maximum axial dead load to total axial load (ACI). |
| $\nu$ | inclination of column due to initial imperfections (Eurocode 2). |
| $\delta$ | structural steel section contribution factor (Eurocode 4). |
| $\delta_{ns}$ | moment magnification factor (ACI). |
| $\epsilon_{cu}$ | strain in concrete. |

| | |
|---|---|
| $\epsilon_{f0}$ | strain where tensile strength of concrete equals zero. |
| $\epsilon_o$ | strain in unconfined concrete at peak compressive stress. |
| $\epsilon_{pl}$ | plastic strain of concrete. |
| $\epsilon_{rc}$ | strain of reinforcing steel in compression. |
| $\epsilon_{rt}$ | strain of reinforcing steel in tension. |
| $\epsilon_{ssfc}$ | compressive strain in the flange of the structural steel section. |
| $\epsilon_{ssft}$ | tensile strain in the flange of the structural steel section. |
| $\epsilon_t$ | total strain of concrete. |
| $\epsilon_u$ | ultimate strain of concrete. |
| $\epsilon_y$ | yield strain of steel. |
| $\epsilon_{yd}$ | design yield strain of the reinforcing steel. |
| $\lambda$ | column slenderness ratio (Eurocode 2); non-dimensional slenderness parameter (Eurocode 4). |
| $\lambda_{crit}$ | critical column slenderness ratio (Eurocode 2 and Eurocode 4). |
| $\rho_{rs}$ | ratio of area of longitudinal reinforcing bars to gross flexural rigidity area (reinforcing steel ratio). |
| $\frac{\rho_{rs} f_{yr}}{f'_c}$ | reinforcing steel index. |
| $\rho_{ss}$ | ratio of area of structural steel to gross flexural rigidity area (structural steel ratio). |
| $\frac{\rho_{ss} f_{yss}}{f'_c}$ | structural steel index. |
| $\rho''$ | tie/hoop volumetric ratio. |

| | |
|---|---|
| $\sigma_c$ | stress in concrete for a given value of total strain, $\epsilon_t$. |
| $\chi$ | reduction coefficient that accounts for initial imperfections (Eurocode 4). |

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# APPENDIX A

Table A1.1 - Strength Ratio Statistics for Reinforced Concrete Columns without Moment Gradient for Different Ranges of e/h and ℓ/h using ACI Code with Equation 4.16.

| Column Type (1) | (2) | e/h = 0 (3) | 0 < e/h < 0.1 (4) | 0.1 ≤ e/h ≤ 0.7 (5) | 0.1 ≤ e/h ≤ 1.5 (6) | e/h = ∞ (7) |
|---|---|---|---|---|---|---|
| Pedestal ℓ/h ≤ 3 | No. | 10 | 1 | 4 | 4 | * |
| | Mean | 1.068 | 1.151 | 1.093 | 1.093 | • |
| | CV | 0.084 | 0.000 | 0.085 | 0.085 | * |
| | Min | 0.921 | 1.151 | 1.014 | 1.014 | * |
| | Max | 1.196 | 1.151 | 1.201 | 1.201 | * |
| Short 3 < ℓ/h < 6.6 | No. | 79 | • | 12 | 19 | * |
| | Mean | 1.135 | • | 0.979 | 0.936 | * |
| | CV | 0.105 | * | 0.089 | 0.133 | * |
| | Min | 0.874 | * | 0.831 | 0.627 | * |
| | Max | 1.466 | * | 1.126 | 1.126 | * |
| Slender 6.6 ≤ ℓ/h ≤ 30 | No. | 72 | 5 | 121 | 141 | • |
| | Mean | 1.139 | 1.162 | 1.071 | 1.056 | • |
| | CV | 0.339 | 0.246 | 0.118 | 0.119 | • |
| | Min | 0.685 | 0.872 | 0.752 | 0.752 | * |
| | Max | 2.555 | 1.527 | 1.561 | 1.561 | * |
| Super Slender ℓ/h > 30 | No. | 2 | 3 | 18 | 18 | * |
| | Mean | 2.340 | 1.499 | 1.059 | 1.059 | * |
| | CV | 0.042 | 0.021 | 0.285 | 0.285 | • |
| | Min | 2.270 | 1.463 | 0.759 | 0.759 | * |
| | Max | 2.410 | 1.521 | 1.623 | 1.623 | * |
| ACI Permitted 3 < ℓ/h ≤ 30 | No. | 151 | 5 | 133 | 160 | * |
| | Mean | 1.137 | 1.162 | 1.063 | 1.042 | • |
| | CV | 0.245 | 0.246 | 0.119 | 0.125 | • |
| | Min | 0.685 | 0.872 | 0.752 | 0.627 | * |
| | Max | 2.555 | 1.527 | 1.561 | 1.561 | * |

• No data available

Note: CV stands for the coefficient of variation.

Table A1.2 - Strength Ratio Statistics for Reinforced Concrete Columns without Moment Gradient for Different Ranges of e/h and ℓ/h using ACI Code with Equation 4.17.

| Column Type (1) | (2) | e/h = 0 (3) | 0 < e/h < 0.1 (4) | 0.1 ≤ e/h ≤ 0.7 (5) | 0.1 ≤ e/h ≤ 1.5 (6) | e/h = ∞ (7) |
|---|---|---|---|---|---|---|
| Pedestal ℓ/h ≤ 3 | No. | 10 | 1 | 4 | 4 | * |
| | Mean | 1.068 | 1.151 | 1.093 | 1.093 | * |
| | CV | 0.084 | 0.000 | 0.085 | 0.085 | * |
| | Min | 0.921 | 1.151 | 1.014 | 1.014 | * |
| | Max | 1.196 | 1.151 | 1.201 | 1.201 | * |
| Short 3 < ℓ/h < 6.6 | No. | 79 | * | 12 | 19 | * |
| | Mean | 1.135 | * | 0.979 | 0.936 | * |
| | CV | 0.105 | * | 0.089 | 0.133 | * |
| | Min | 0.874 | * | 0.831 | 0.627 | * |
| | Max | 1.466 | * | 1.126 | 1.126 | * |
| Slender 6.6 ≤ ℓ/h ≤ 30 | No. | 72 | 5 | 121 | 141 | * |
| | Mean | 1.046 | 1.208 | 1.073 | 1.056 | * |
| | CV | 0.192 | 0.295 | 0.128 | 0.128 | * |
| | Min | 0.685 | 0.894 | 0.818 | 0.812 | * |
| | Max | 1.958 | 1.596 | 1.702 | 1.702 | * |
| Super Slender ℓ/h > 30 | No. | 2 | 3 | 18 | 18 | * |
| | Mean | 2.201 | 1.511 | 1.020 | 1.020 | * |
| | CV | 0.081 | 0.025 | 0.138 | 0.138 | * |
| | Min | 2.074 | 1.468 | 0.844 | 0.844 | * |
| | Max | 2.328 | 1.534 | 1.229 | 1.229 | * |
| ACI Permitted 3 < ℓ/h ≤ 30 | No. | 151 | 5 | 133 | 160 | * |
| | Mean | 1.093 | 1.208 | 1.064 | 1.042 | * |
| | CV | 0.154 | 0.295 | 0.128 | 0.134 | * |
| | Min | 0.685 | 0.894 | 0.818 | 0.627 | * |
| | Max | 1.958 | 1.596 | 1.702 | 1.702 | * |

* No data available

Note: CV stands for the coefficient of variation.

Table A1.3 - Strength Ratio Statistics for Reinforced Concrete Columns without Moment Gradient for Different Ranges of e/h and $\ell$/h using Eurocode 2.

| Column Type (1) | (2) | $e/h = 0$ (3) | $0 < e/h < 0.1$ (4) | $0.1 \le e/h \le 0.7$ (5) | $0.1 \le e/h \le 1.5$ (6) | $e/h = \infty$ (7) |
|---|---|---|---|---|---|---|
| Pedestal $\ell/h \le 3$ | No. | 10 | 1 | 4 | 4 | * |
| | Mean | 1.068 | 1.140 | 1.089 | 1.089 | * |
| | CV | 0.084 | 0.000 | 0.083 | 0.083 | * |
| | Min | 0.921 | 1.140 | 1.012 | 1.012 | * |
| | Max | 1.196 | 1.140 | 1.189 | 1.189 | * |
| Short $3 < \ell/h < 6.6$ | No. | 79 | * | 12 | 19 | * |
| | Mean | 1.135 | * | 0.930 | 0.894 | * |
| | CV | 0.105 | * | 0.099 | 0.140 | * |
| | Min | 0.874 | * | 0.785 | 0.596 | * |
| | Max | 1.466 | * | 1.107 | 1.107 | * |
| Slender $6.6 \le \ell/h \le 30$ | No. | 72 | 5 | 121 | 141 | * |
| | Mean | 1.165 | 1.240 | 1.107 | 1.090 | * |
| | CV | 0.218 | 0.270 | 0.133 | 0.133 | * |
| | Min | 0.746 | 0.949 | 0.861 | 0.861 | * |
| | Max | 1.811 | 1.644 | 1.814 | 1.814 | * |
| Super Slender $\ell/h > 30$ | No. | 2 | 3 | 18 | 18 | * |
| | Mean | 1.899 | 1.439 | 1.149 | 1.149 | * |
| | CV | 0.071 | 0.040 | 0.077 | 0.077 | * |
| | Min | 1.804 | 1.405 | 0.994 | 0.994 | * |
| | Max | 1.995 | 1.505 | 1.308 | 1.308 | * |
| ACI Permitted $3 < \ell/h \le 30$ | No. | 151 | 5 | 133 | 160 | * |
| | Mean | 1.150 | 1.240 | 1.091 | 1.067 | * |
| | CV | 0.170 | 0.270 | 0.139 | 0.146 | * |
| | Min | 0.746 | 0.949 | 0.785 | 0.596 | * |
| | Max | 1.811 | 1.644 | 1.814 | 1.814 | * |

* No data available

Note: CV stands for the coefficient of variation.

Table A1.4 - Strength Ratio Statistics for Reinforced Concrete Columns without Moment Gradient for Different Ranges of e/h and $\ell/h$ using ACI Code with Equation 4.19.

| Column Type (1) | (2) | $e/h = 0$ (3) | $0 < e/h < 0.1$ (4) | $0.1 \le e/h \le 0.7$ (5) | $0.1 \le e/h \le 1.5$ (6) | $e/h = \infty$ (7) |
|---|---|---|---|---|---|---|
| Pedestal $\ell/h \le 3$ | No. | 10 | 1 | 4 | 4 | * |
| | Mean | 1.068 | 1.151 | 1.093 | 1.093 | * |
| | CV | 0.084 | 0.000 | 0.085 | 0.085 | * |
| | Min | 0.921 | 1.151 | 1.014 | 1.014 | * |
| | Max | 1.196 | 1.151 | 1.201 | 1.201 | * |
| Short $3 < \ell/h < 6.6$ | No. | 79 | * | 12 | 19 | * |
| | Mean | 1.135 | * | 0.979 | 0.936 | * |
| | CV | 0.105 | * | 0.089 | 0.133 | * |
| | Min | 0.874 | * | 0.831 | 0.627 | * |
| | Max | 1.466 | * | 1.126 | 1.126 | * |
| Slender $6.6 \le \ell/h \le 30$ | No. | 72 | 5 | 121 | 141 | * |
| | Mean | 1.040 | 1.083 | 1.059 | 1.051 | * |
| | CV | 0.172 | 0.220 | 0.111 | 0.112 | * |
| | Min | 0.685 | 0.844 | 0.855 | 0.855 | * |
| | Max | 1.527 | 1.366 | 1.521 | 1.521 | * |
| Super Slender $\ell/h > 30$ | No. | 2 | 3 | 18 | 18 | * |
| | Mean | 1.782 | 1.285 | 1.012 | 1.012 | * |
| | CV | 0.074 | 0.014 | 0.093 | 0.093 | * |
| | Min | 1.689 | 1.263 | 0.875 | 0.875 | * |
| | Max | 1.875 | 1.297 | 1.162 | 1.162 | * |
| ACI Permitted $3 < \ell/h \le 30$ | No. | 151 | 5 | 133 | 160 | * |
| | Mean | 1.090 | 1.083 | 1.051 | 1.037 | * |
| | CV | 0.144 | 0.220 | 0.112 | 0.119 | * |
| | Min | 0.685 | 0.844 | 0.831 | 0.627 | * |
| | Max | 1.527 | 1.366 | 1.521 | 1.521 | * |

* No data available

Note: CV stands for the coefficient of variation.

Table A1.5 - Strength Ratio Statistics for Reinforced Concrete Columns without Moment Gradient for Different Ranges of e/h and $\ell$/h using FEM.

| Column Type (1) | (2) | $e/h = 0$ (3) | $0 < e/h < 0.1$ (4) | $0.1 \le e/h \le 0.7$ (5) | $0.1 \le e/h \le 1.5$ (6) | $e/h = \infty$ (7) |
|---|---|---|---|---|---|---|
| Pedestal $\ell/h \le 3$ | No. | 10 | 1 | 4 | 4 | * |
| | Mean | 0.940 | 1.040 | 1.070 | 1.070 | * |
| | CV | 0.064 | 0.000 | 0.071 | 0.071 | * |
| | Min | 0.830 | 1.040 | 1.010 | 1.010 | * |
| | Max | 1.026 | 1.040 | 1.171 | 1.171 | * |
| Short $3 < \ell/h < 6.6$ | No. | 79 | * | 12 | 19 | * |
| | Mean | 0.992 | * | 0.941 | 0.914 | * |
| | CV | 0.110 | * | 0.106 | 0.143 | * |
| | Min | 0.828 | * | 0.790 | 0.632 | * |
| | Max | 1.323 | * | 1.150 | 1.150 | * |
| Slender $6.6 \le \ell/h \le 30$ | No. | 72 | 5 | 121 | 141 | * |
| | Mean | 0.971 | 0.957 | 1.019 | 1.013 | * |
| | CV | 0.125 | 0.084 | 0.132 | 0.127 | * |
| | Min | 0.659 | 0.876 | 0.761 | 0.761 | * |
| | Max | 1.192 | 1.077 | 1.356 | 1.356 | * |
| Super Slender $\ell/h > 30$ | No. | 2 | 3 | 18 | 18 | * |
| | Mean | 0.969 | 0.928 | 0.876 | 0.876 | * |
| | CV | 0.044 | 0.036 | 0.153 | 0.153 | * |
| | Min | 0.939 | 0.899 | 0.708 | 0.708 | * |
| | Max | 0.999 | 0.964 | 1.090 | 1.090 | * |
| ACI Permitted $3 < \ell/h \le 30$ | No. | 151 | 5 | 133 | 160 | * |
| | Mean | 0.982 | 0.957 | 1.012 | 1.001 | * |
| | CV | 0.118 | 0.084 | 0.132 | 0.132 | * |
| | Min | 0.659 | 0.876 | 0.761 | 0.632 | * |
| | Max | 1.323 | 1.077 | 1.356 | 1.356 | * |

* No data available

Note: CV stands for the coefficient of variation.

# APPENDIX B

Table B1.1 - Strength Ratio Statistics for Composite Steel-Concrete Columns Subjected to Major Axis Bending without Moment Gradient for Different Ranges of e/h and ℓ/h using ACI Code with Equation 4.16.

| Column Type (1) | (2) | $e/h = 0$ (3) | $0 < e/h < 0.1$ (4) | $0.1 \le e/h \le 0.7$ (5) | $0.1 \le e/h \le 1.5$ (6) | $e/h = \infty$ (7) |
|---|---|---|---|---|---|---|
| Pedestal $\ell/h \le 3$ | No. | 20 | * | * | * | 16 |
| | Mean | 1.072 | * | * | * | 1.147 |
| | CV | 0.068 | * | * | * | 0.116 |
| | Min | 0.912 | * | * | * | 0.973 |
| | Max | 1.169 | * | * | * | 1.417 |
| Short $3 < \ell/h < 6.6$ | No. | * | * | 2 | 4 | * |
| | Mean | * | * | 1.094 | 1.237 | * |
| | CV | * | * | 0.065 | 0.162 | * |
| | Min | * | * | 1.029 | 1.029 | * |
| | Max | * | * | 1.160 | 1.503 | * |
| Slender $6.6 \le \ell/h \le 30$ | No. | 3 | * | 23 | 26 | * |
| | Mean | 0.858 | * | 1.165 | 1.148 | * |
| | CV | 0.071 | * | 0.203 | 0.199 | * |
| | Min | 0.788 | * | 0.908 | 0.908 | * |
| | Max | 0.898 | * | 1.781 | 1.781 | * |
| Super Slender $\ell/h > 30$ | No. | * | * | * | * | * |
| | Mean | * | * | * | * | * |
| | CV | * | * | * | * | * |
| | Min | * | * | * | * | * |
| | Max | * | * | * | * | * |
| ACI Permitted $3 < \ell/h \le 30$ | No. | 3 | * | 25 | 30 | * |
| | Mean | 0.858 | * | 1.160 | 1.160 | * |
| | CV | 0.071 | * | 0.196 | 0.193 | * |
| | Min | 0.788 | * | 0.908 | 0.908 | * |
| | Max | 0.898 | * | 1.781 | 1.781 | * |

* No data available

Note: CV stands for the coefficient of variation.

Table B1.2 - Strength Ratio Statistics for Composite Steel-Concrete Columns Subjected to Major Axis Bending without Moment Gradient for Different Ranges of e/h and $\ell/h$ using ACI Code with Equation 4.18.

| Column Type (1) | (2) | $e/h = 0$ (3) | $0 < e/h < 0.1$ (4) | $0.1 \leq e/h \leq 0.7$ (5) | $0.1 \leq e/h \leq 1.5$ (6) | $e/h = \infty$ (7) |
|---|---|---|---|---|---|---|
| Pedestal $\ell/h \leq 3$ | No. | 20 | * | * | * | 16 |
| | Mean | 1.072 | * | * | * | 1.147 |
| | CV | 0.068 | * | * | * | 0.116 |
| | Min | 0.912 | * | * | * | 0.973 |
| | Max | 1.169 | * | * | * | 1.417 |
| Short $3 < \ell/h < 6.6$ | No. | * | * | 2 | 4 | * |
| | Mean | * | * | 1.094 | 1.237 | * |
| | CV | * | * | 0.085 | 0.162 | * |
| | Min | * | * | 1.029 | 1.029 | * |
| | Max | * | * | 1.160 | 1.503 | * |
| Slender $6.6 \leq \ell/h \leq 30$ | No. | 3 | * | 23 | 26 | * |
| | Mean | 0.858 | * | 0.969 | 0.971 | * |
| | CV | 0.071 | * | 0.123 | 0.117 | * |
| | Min | 0.788 | * | 0.777 | 0.777 | * |
| | Max | 0.898 | * | 1.206 | 1.206 | * |
| Super Slender $\ell/h > 30$ | No. | * | * | * | * | * |
| | Mean | * | * | * | * | * |
| | CV | * | * | * | * | * |
| | Min | * | * | * | * | * |
| | Max | * | * | * | * | * |
| ACI Permitted $3 < \ell/h \leq 30$ | No. | 3 | * | 25 | 30 | * |
| | Mean | 0.858 | * | 0.979 | 1.006 | * |
| | CV | 0.071 | * | 0.123 | 0.153 | * |
| | Min | 0.788 | * | 0.777 | 0.777 | * |
| | Max | 0.898 | * | 1.206 | 1.503 | * |

* No data available

Note: CV stands for the coefficient of variation.

Table B1.3 - Strength Ratio Statistics for Composite Steel-Concrete Columns Subjected to Major Axis Bending without Moment Gradient for Different Ranges of e/h and ℓ/h using Eurocode 4.

| Column Type (1) | (2) | $e/h = 0$ (3) | $0 < e/h < 0.1$ (4) | $0.1 \leq e/h \leq 0.7$ (5) | $0.1 \leq e/h \leq 1.5$ (6) | $e/h = \infty$ (7) |
|---|---|---|---|---|---|---|
| Pedestal $\ell/h \leq 3$ | No. | 20 | * | * | * | 16 |
| | Mean | 1.031 | * | * | * | 1.106 |
| | CV | 0.073 | * | * | * | 0.091 |
| | Min | 0.912 | * | * | * | 0.966 |
| | Max | 1.169 | * | * | * | 1.295 |
| Short $3 < \ell/h < 6.6$ | No. | * | * | 2 | 4 | * |
| | Mean | * | * | 1.178 | 1.087 | * |
| | CV | * | * | 0.078 | 0.134 | * |
| | Min | * | * | 1.114 | 0.890 | * |
| | Max | * | * | 1.243 | 1.243 | * |
| Slender $6.6 \leq \ell/h \leq 30$ | No. | 3 | * | 23 | 26 | * |
| | Mean | 0.942 | * | 1.050 | 1.082 | * |
| | CV | 0.071 | * | 0.170 | 0.178 | * |
| | Min | 0.866 | * | 0.813 | 0.813 | * |
| | Max | 0.990 | * | 1.391 | 1.441 | * |
| Super Slender $\ell/h > 30$ | No. | * | * | * | * | * |
| | Mean | * | * | * | * | * |
| | CV | * | * | * | * | * |
| | Min | * | * | * | * | * |
| | Max | * | * | * | * | * |
| ACI Permitted $3 < \ell/h \leq 30$ | No. | 3 | * | 25 | 30 | * |
| | Mean | 0.942 | * | 1.060 | 1.083 | * |
| | CV | 0.071 | * | 0.165 | 0.171 | * |
| | Min | 0.866 | * | 0.813 | 0.813 | * |
| | Max | 0.990 | * | 1.391 | 1.441 | * |

* No data available

Note: CV stands for the coefficient of variation.

Table B1.4 - Strength Ratio Statistics for Composite Steel-Concrete Columns Subjected to Major Axis Bending without Moment Gradient for Different Ranges of e/h and ℓ/h using AISC-LRFD Specifications.

| Column Type (1) | (2) | e/h = 0 (3) | 0 < e/h < 0.1 (4) | 0.1 ≤ e/h ≤ 0.7 (5) | 0.1 ≤ e/h ≤ 1.5 (6) | e/h = ∞ (7) |
|---|---|---|---|---|---|---|
| Pedestal ℓ/h ≤ 3 | No. | 20 | * | * | * | 16 |
| | Mean | 1.250 | * | * | * | 0.904 |
| | CV | 0.099 | * | * | * | 0.080 |
| | Min | 1.058 | * | * | * | 0.777 |
| | Max | 1.417 | * | * | * | 1.039 |
| Short 3 < ℓ/h < 6.6 | No. | * | * | 2 | 4 | * |
| | Mean | * | * | 1.292 | 1.158 | * |
| | CV | * | * | 0.064 | 0.150 | * |
| | Min | * | * | 1.233 | 0.947 | * |
| | Max | * | * | 1.350 | 1.350 | * |
| Slender 6.6 ≤ ℓ/h ≤ 30 | No. | 3 | * | 23 | 26 | * |
| | Mean | 1.438 | * | 1.377 | 1.384 | * |
| | CV | 0.056 | * | 0.240 | 0.226 | * |
| | Min | 1.346 | * | 0.902 | 0.902 | * |
| | Max | 1.485 | * | 1.951 | 1.951 | * |
| Super Slender ℓ/h > 30 | No. | * | * | * | * | * |
| | Mean | * | * | * | * | * |
| | CV | * | * | * | * | * |
| | Min | * | * | * | * | * |
| | Max | * | * | * | * | * |
| ACI Permitted 3 < ℓ/h ≤ 30 | No. | 3 | * | 25 | 30 | * |
| | Mean | 1.438 | * | 1.370 | 1.354 | * |
| | CV | 0.056 | * | 0.232 | 0.226 | * |
| | Min | 1.346 | * | 0.902 | 0.902 | * |
| | Max | 1.485 | * | 1.951 | 1.951 | * |

* No data available
Note: CV stands for the coefficient of variation.

Table B1.5 - Strength Ratio Statistics for Composite Steel-Concrete Columns Subjected to Major Axis Bending without Moment Gradient for Different Ranges of $e/h$ and $\ell/h$ using AISC-LRFD Specifications with Equation 4.38.

| Column Type (1) | (2) | $e/h = 0$ (3) | $0 < e/h < 0.1$ (4) | $0.1 \le e/h \le 0.7$ (5) | $0.1 \le e/h \le 1.5$ (6) | $e/h = \infty$ (7) |
|---|---|---|---|---|---|---|
| Pedestal $\ell/h \le 3$ | No. | 20 | * | * | * | 16 |
| | Mean | 1.251 | * | * | * | 0.904 |
| | CV | 0.099 | * | * | * | 0.080 |
| | Min | 1.059 | * | * | * | 0.777 |
| | Max | 1.418 | * | * | * | 1.039 |
| Short $3 < \ell/h < 6.6$ | No. | * | * | 2 | 4 | * |
| | Mean | * | * | 1.300 | 1.162 | * |
| | CV | * | * | 0.065 | 0.153 | * |
| | Min | * | * | 1.241 | 0.948 | * |
| | Max | * | * | 1.360 | 1.360 | * |
| Slender $6.6 \le \ell/h \le 30$ | No. | 3 | * | 23 | 26 | * |
| | Mean | 1.438 | * | 1.410 | 1.415 | * |
| | CV | 0.056 | * | 0.229 | 0.215 | * |
| | Min | 1.346 | * | 0.906 | 0.906 | * |
| | Max | 1.485 | * | 1.972 | 1.972 | * |
| Super Slender $\ell/h > 30$ | No. | * | * | * | * | * |
| | Mean | * | * | * | * | * |
| | CV | * | * | * | * | * |
| | Min | * | * | * | * | * |
| | Max | * | * | * | * | * |
| ACI Permitted $3 < \ell/h \le 30$ | No. | 3 | * | 25 | 30 | * |
| | Mean | 1.438 | * | 1.401 | 1.382 | * |
| | CV | 0.056 | * | 0.222 | 0.218 | * |
| | Min | 1.346 | * | 0.906 | 0.906 | * |
| | Max | 1.485 | * | 1.972 | 1.972 | * |

* No data available

Note: CV stands for the coefficient of variation.

Table B1.6 - Strength Ratio Statistics for Composite Steel-Concrete Columns Subjected to Major Axis Bending without Moment Gradient for Different Ranges of e/h and ℓ/h using ACI Code with Equation 4.20.

| Column Type (1) | (2) | $e/h = 0$ (3) | $0 < e/h < 0.1$ (4) | $0.1 \le e/h \le 0.7$ (5) | $0.1 \le e/h \le 1.5$ (6) | $e/h = \infty$ (7) |
|---|---|---|---|---|---|---|
| Pedestal $\ell/h \le 3$ | No. | 20 | * | * | * | 16 |
| | Mean | 1.072 | * | * | * | 1.147 |
| | CV | 0.068 | * | * | * | 0.116 |
| | Min | 0.912 | * | * | * | 0.973 |
| | Max | 1.169 | * | * | * | 1.417 |
| Short $3 < \ell/h < 6.6$ | No. | * | * | 2 | 4 | * |
| | Mean | * | * | 1.094 | 1.237 | * |
| | CV | * | * | 0.085 | 0.162 | * |
| | Min | * | * | 1.029 | 1.029 | * |
| | Max | * | * | 1.160 | 1.503 | * |
| Slender $6.6 \le \ell/h \le 30$ | No. | 3 | * | 23 | 26 | * |
| | Mean | 0.858 | * | 0.983 | 0.987 | * |
| | CV | 0.071 | * | 0.125 | 0.119 | * |
| | Min | 0.788 | * | 0.790 | 0.790 | * |
| | Max | 0.898 | * | 1.237 | 1.237 | * |
| Super Slender $\ell/h > 30$ | No. | * | * | * | * | * |
| | Mean | * | * | * | * | * |
| | CV | * | * | * | * | * |
| | Min | * | * | * | * | * |
| | Max | * | * | * | * | * |
| ACI Permitted $3 < \ell/h \le 30$ | No. | 3 | * | 25 | 30 | * |
| | Mean | 0.858 | * | 0.992 | 1.020 | * |
| | CV | 0.071 | * | 0.124 | 0.150 | * |
| | Min | 0.788 | * | 0.790 | 0.790 | * |
| | Max | 0.898 | * | 1.237 | 1.503 | * |

* No data available

Note: CV stands for the coefficient of variation.

Table B1.7 - Strength Ratio Statistics for Composite Steel-Concrete Columns Subjected to Major Axis Bending without Moment Gradient for Different Ranges of e/h and ℓ/h using FEM.

| Column Type (1) | (2) | $e/h = 0$ (3) | $0 < e/h < 0.1$ (4) | $0.1 \leq e/h \leq 0.7$ (5) | $0.1 \leq e/h \leq 1.5$ (6) | $e/h = \infty$ (7) |
|---|---|---|---|---|---|---|
| Pedestal $\ell/h \leq 3$ | No. | 20 | * | * | * | 16 |
| | Mean | 0.971 | * | * | * | 1.047 |
| | CV | 0.091 | * | * | * | 0.063 |
| | Min | 0.806 | * | * | * | 0.914 |
| | Max | 1.184 | * | * | * | 1.146 |
| Short $3 < \ell/h < 6.6$ | No. | * | * | 2 | 4 | * |
| | Mean | * | * | 1.083 | 1.026 | * |
| | CV | * | * | 0.086 | 0.121 | * |
| | Min | * | * | 1.017 | 0.858 | * |
| | Max | * | * | 1.149 | 1.149 | * |
| Slender $6.6 \leq \ell/h \leq 30$ | No. | 3 | * | 23 | 26 | * |
| | Mean | 0.818 | * | 0.923 | 0.936 | * |
| | CV | 0.072 | * | 0.138 | 0.135 | * |
| | Min | 0.750 | * | 0.710 | 0.710 | * |
| | Max | 0.853 | * | 1.115 | 1.115 | * |
| Super Slender $\ell/h > 30$ | No. | * | * | * | * | * |
| | Mean | * | * | * | * | * |
| | CV | * | * | * | * | * |
| | Min | * | * | * | * | * |
| | Max | * | * | * | * | * |
| ACI Permitted $3 < \ell/h \leq 30$ | No. | 3 | * | 25 | 30 | * |
| | Mean | 0.818 | * | 0.936 | 0.948 | * |
| | CV | 0.072 | * | 0.140 | 0.135 | * |
| | Min | 0.750 | * | 0.710 | 0.710 | * |
| | Max | 0.853 | * | 1.149 | 1.149 | * |

* No data available

Note: CV stands for the coefficient of variation.

# APPENDIX C

Table C1.1 - Strength Ratio Statistics for Composite Steel-Concrete Columns Subjected to Minor Axis Bending for Different Ranges of e/h and $\ell$/h using ACI Code with Equation 4.16.

| Column Type (1) | (2) | e/h = 0 (3) | 0 < e/h < 0.1 (4) | 0.1 ≤ e/h ≤ 0.7 (5) | 0.1 ≤ e/h ≤ 1.5 (6) | e/h = ∞ (7) |
|---|---|---|---|---|---|---|
| Pedestal $\ell/h \le 3$ | No. | * | * | * | * | * |
| | Mean | * | * | * | * | * |
| | CV | * | * | * | * | * |
| | Min | * | * | * | * | * |
| | Max | * | * | * | * | * |
| Short $3 < \ell/h < 6.6$ | No. | 3 | * | 2 | 2 | * |
| | Mean | 0.910 | * | 1.095 | 1.095 | * |
| | CV | 0.028 | * | 0.064 | 0.064 | * |
| | Min | 0.882 | * | 1.045 | 1.045 | * |
| | Max | 0.930 | * | 1.145 | 1.145 | * |
| Slender $6.6 \le \ell/h \le 30$ | No. | 32 | 5 | 38 | 39 | * |
| | Mean | 0.969 | 0.909 | 1.079 | 1.074 | * |
| | CV | 0.208 | 0.110 | 0.228 | 0.228 | * |
| | Min | 0.582 | 0.776 | 0.765 | 0.765 | * |
| | Max | 1.441 | 1.028 | 1.815 | 1.815 | * |
| Super Slender $\ell/h > 30$ | No. | * | * | * | * | * |
| | Mean | * | * | * | * | * |
| | CV | * | * | * | * | * |
| | Min | * | * | * | * | * |
| | Max | * | * | * | * | * |
| ACI Permitted $3 < \ell/h \le 30$ | No. | 35 | 5 | 40 | 41 | * |
| | Mean | 0.964 | 0.909 | 1.080 | 1.075 | * |
| | CV | 0.201 | 0.110 | 0.222 | 0.222 | * |
| | Min | 0.582 | 0.776 | 0.765 | 0.765 | * |
| | Max | 1.441 | 1.028 | 1.815 | 1.815 | * |

* No data available

Note: CV stands for the coefficient of variation.

Table C1.2 - Strength Ratio Statistics for Composite Steel-Concrete Columns Subjected to Minor Axis Bending for Different Ranges of e/h and $\ell/h$ using ACI Code with Equation 4.18.

| Column Type (1) | (2) | $e/h = 0$ (3) | $0 < e/h < 0.1$ (4) | $0.1 \le e/h \le 0.7$ (5) | $0.1 \le e/h \le 1.5$ (6) | $e/h = \infty$ (7) |
|---|---|---|---|---|---|---|
| Pedestal $\ell/h \le 3$ | No. | * | * | * | * | * |
| | Mean | * | * | * | * | * |
| | CV | * | * | * | * | * |
| | Min | * | * | * | * | * |
| | Max | * | * | * | * | * |
| Short $3 < \ell/h < 6.6$ | No. | 3 | * | 2 | 2 | * |
| | Mean | 0.910 | * | 1.094 | 1.094 | * |
| | CV | 0.028 | * | 0.060 | 0.060 | * |
| | Min | 0.882 | * | 1.048 | 1.048 | * |
| | Max | 0.930 | * | 1.141 | 1.141 | * |
| Slender $6.6 \le \ell/h \le 30$ | No. | 32 | 5 | 38 | 39 | * |
| | Mean | 1.197 | 1.204 | 1.127 | 1.123 | * |
| | CV | 0.301 | 0.176 | 0.179 | 0.178 | * |
| | Min | 0.582 | 0.965 | 0.925 | 0.925 | * |
| | Max | 2.051 | 1.546 | 1.871 | 1.871 | * |
| Super Slender $\ell/h > 30$ | No. | * | * | * | * | * |
| | Mean | * | * | * | * | * |
| | CV | * | * | * | * | * |
| | Min | * | * | * | * | * |
| | Max | * | * | * | * | * |
| ACI Permitted $3 < \ell/h \le 30$ | No. | 35 | 5 | 40 | 41 | * |
| | Mean | 1.173 | 1.204 | 1.125 | 1.122 | * |
| | CV | 0.301 | 0.176 | 0.175 | 0.174 | * |
| | Min | 0.582 | 0.965 | 0.925 | 0.925 | * |
| | Max | 2.051 | 1.546 | 1.871 | 1.871 | * |

* No data available

Note: CV stands for the coefficient of variation.

Table C1.3 - Strength Ratio Statistics for Composite Steel-Concrete Columns Subjected to Minor Axis Bending for Different Ranges of $e/h$ and $\ell/h$ using Eurocode 4.

| Column Type (1) | (2) | $e/h = 0$ (3) | $0 < e/h < 0.1$ (4) | $0.1 \le e/h \le 0.7$ (5) | $0.1 \le e/h \le 1.5$ (6) | $e/h = \infty$ (7) |
|---|---|---|---|---|---|---|
| Pedestal $\ell/h \le 3$ | No. | * | * | * | * | * |
| | Mean | * | * | * | * | * |
| | CV | * | * | * | * | * |
| | Min | * | * | * | * | * |
| | Max | * | * | * | * | * |
| Short $3 < \ell/h < 6.6$ | No. | 3 | * | 2 | 2 | * |
| | Mean | 0.907 | * | 1.036 | 1.036 | * |
| | CV | 0.027 | * | 0.003 | 0.003 | * |
| | Min | 0.879 | * | 1.034 | 1.034 | * |
| | Max | 0.924 | * | 1.039 | 1.039 | * |
| Slender $6.6 \le \ell/h \le 30$ | No. | 32 | 5 | 38 | 39 | * |
| | Mean | 0.984 | 0.992 | 1.050 | 1.050 | * |
| | CV | 0.130 | 0.107 | 0.181 | 0.179 | * |
| | Min | 0.683 | 0.864 | 0.746 | 0.746 | * |
| | Max | 1.342 | 1.069 | 1.566 | 1.566 | * |
| Super Slender $\ell/h > 30$ | No. | * | * | * | * | * |
| | Mean | * | * | * | * | * |
| | CV | * | * | * | * | * |
| | Min | * | * | * | * | * |
| | Max | * | * | * | * | * |
| ACI Permitted $3 < \ell/h \le 30$ | No. | 35 | 5 | 40 | 41 | * |
| | Mean | 0.977 | 0.992 | 1.049 | 1.049 | * |
| | CV | 0.127 | 0.107 | 0.177 | 0.174 | * |
| | Min | 0.683 | 0.864 | 0.746 | 0.746 | * |
| | Max | 1.342 | 1.069 | 1.566 | 1.566 | * |

* No data available

Note: CV stands for the coefficient of variation.

Table C1.4 - Strength Ratio Statistics for Composite Steel-Concrete Columns Subjected to Minor Axis Bending for Different Ranges of e/h and $\ell/h$ using AISC-LRFD Specifications.

| Column Type (1) | (2) | $e/h = 0$ (3) | $0 < e/h < 0.1$ (4) | $0.1 \le e/h \le 0.7$ (5) | $0.1 \le e/h \le 1.5$ (6) | $e/h = \infty$ (7) |
|---|---|---|---|---|---|---|
| Pedestal $\ell/h \le 3$ | No. | * | * | * | * | * |
| | Mean | * | * | * | * | * |
| | CV | * | * | * | * | * |
| | Min | * | * | * | * | * |
| | Max | * | * | * | * | * |
| Short $3 < \ell/h < 6.6$ | No. | 3 | * | 2 | 2 | * |
| | Mean | 1.115 | * | 1.151 | 1.151 | * |
| | CV | 0.025 | * | 0.049 | 0.049 | * |
| | Min | 1.083 | * | 1.111 | 1.111 | * |
| | Max | 1.135 | * | 1.192 | 1.192 | * |
| Slender $6.6 \le \ell/h \le 30$ | No. | 32 | 5 | 38 | 39 | * |
| | Mean | 1.139 | 1.351 | 1.296 | 1.306 | * |
| | CV | 0.150 | 0.098 | 0.289 | 0.287 | * |
| | Min | 0.770 | 1.132 | 0.761 | 0.761 | * |
| | Max | 1.561 | 1.466 | 2.032 | 2.032 | * |
| Super Slender $\ell/h > 30$ | No. | * | * | * | * | * |
| | Mean | * | * | * | * | * |
| | CV | * | * | * | * | * |
| | Min | * | * | * | * | * |
| | Max | * | * | * | * | * |
| ACI Permitted $3 < \ell/h \le 30$ | No. | 35 | 5 | 40 | 41 | * |
| | Mean | 1.137 | 1.351 | 1.289 | 1.299 | * |
| | CV | 0.144 | 0.098 | 0.284 | 0.283 | * |
| | Min | 0.770 | 1.132 | 0.761 | 0.761 | * |
| | Max | 1.561 | 1.466 | 2.032 | 2.032 | * |

* No data available

Note: CV stands for the coefficient of variation.

Table C1.5 - Strength Ratio Statistics for Composite Steel-Concrete Columns Subjected to Minor Axis Bending for Different Ranges of e/h and $\ell$/h using AISC-LRFD Specifications with Equation 4.38.

| Column Type (1) | (2) | $e/h = 0$ (3) | $0 < e/h < 0.1$ (4) | $0.1 \leq e/h \leq 0.7$ (5) | $0.1 \leq e/h \leq 1.5$ (6) | $e/h = \infty$ (7) |
|---|---|---|---|---|---|---|
| Pedestal $\ell/h \leq 3$ | No. | * | * | * | * | * |
| | Mean | * | * | * | * | * |
| | CV | * | * | * | * | * |
| | Min | * | * | * | * | * |
| | Max | * | * | * | * | * |
| Short $3 < \ell/h < 6.6$ | No. | 3 | * | 2 | 2 | * |
| | Mean | 1.134 | * | 1.181 | 1.181 | * |
| | CV | 0.025 | * | 0.055 | 0.055 | * |
| | Min | 1.102 | * | 1.135 | 1.135 | * |
| | Max | 1.154 | * | 1.227 | 1.227 | * |
| Slender $6.6 \leq \ell/h \leq 30$ | No. | 32 | 5 | 38 | 39 | * |
| | Mean | 1.354 | 1.662 | 1.492 | 1.498 | * |
| | CV | 0.178 | 0.091 | 0.250 | 0.247 | * |
| | Min | 0.845 | 1.508 | 1.009 | 1.009 | * |
| | Max | 2.086 | 1.870 | 2.355 | 2.355 | * |
| Super Slender $\ell/h > 30$ | No. | * | * | * | * | * |
| | Mean | * | * | * | * | * |
| | CV | * | * | * | * | * |
| | Min | * | * | * | * | * |
| | Max | * | * | * | * | * |
| ACI Permitted $3 < \ell/h \leq 30$ | No. | 35 | 5 | 40 | 41 | * |
| | Mean | 1.335 | 1.662 | 1.477 | 1.483 | * |
| | CV | 0.179 | 0.091 | 0.251 | 0.248 | * |
| | Min | 0.845 | 1.508 | 1.009 | 1.009 | * |
| | Max | 2.086 | 1.870 | 2.355 | 2.355 | * |

* No data available

Note: CV stands for the coefficient of variation.

Table C1.6 - Strength Ratio Statistics for Composite Steel-Concrete Columns Subjected to Minor Axis Bending for Different Ranges of e/h and $\ell$/h using ACI Code with Equation 4.20.

| Column Type (1) | (2) | $e/h = 0$ (3) | $0 < e/h < 0.1$ (4) | $0.1 \leq e/h \leq 0.7$ (5) | $0.1 \leq e/h \leq 1.5$ (6) | $e/h = \infty$ (7) |
|---|---|---|---|---|---|---|
| Pedestal $\ell/h \leq 3$ | No. | * | * | * | * | * |
| | Mean | * | * | * | * | * |
| | CV | * | * | * | * | * |
| | Min | * | * | * | * | * |
| | Max | * | * | * | * | * |
| Short $3 < \ell/h < 6.6$ | No. | 3 | * | 2 | 2 | * |
| | Mean | 0.910 | * | 1.089 | 1.089 | * |
| | CV | 0.028 | * | 0.056 | 0.056 | * |
| | Min | 0.882 | * | 1.046 | 1.046 | * |
| | Max | 0.930 | * | 1.132 | 1.132 | * |
| Slender $6.6 \leq \ell/h \leq 30$ | No. | 32 | 5 | 38 | 39 | * |
| | Mean | 0.888 | 1.010 | 1.036 | 1.032 | * |
| | CV | 0.203 | 0.130 | 0.173 | 0.173 | * |
| | Min | 0.582 | 0.837 | 0.827 | 0.827 | * |
| | Max | 1.506 | 1.202 | 1.579 | 1.579 | * |
| Super Slender $\ell/h > 30$ | No. | * | * | * | * | * |
| | Mean | * | * | * | * | * |
| | CV | * | * | * | * | * |
| | Min | * | * | * | * | * |
| | Max | * | * | * | * | * |
| ACI Permitted $3 < \ell/h \leq 30$ | No. | 35 | 5 | 40 | 41 | * |
| | Mean | 0.890 | 1.010 | 1.039 | 1.035 | * |
| | CV | 0.193 | 0.130 | 0.169 | 0.169 | * |
| | Min | 0.582 | 0.837 | 0.827 | 0.827 | * |
| | Max | 1.506 | 1.202 | 1.579 | 1.579 | * |

* No data available

Note: CV stands for the coefficient of variation.

Table C1.7 - Strength Ratio Statistics for Composite Steel-Concrete Columns Subjected to Minor Axis Bending for Different Ranges of e/h and $\ell$/h using FEM.

| Column Type (1) | (2) | $e/h = 0$ (3) | $0 < e/h < 0.1$ (4) | $0.1 \le e/h \le 0.7$ (5) | $0.1 \le e/h \le 1.5$ (6) | $e/h = \infty$ (7) |
|---|---|---|---|---|---|---|
| Pedestal $\ell/h \le 3$ | No. | * | * | * | * | * |
| | Mean | * | * | * | * | * |
| | CV | * | * | * | * | * |
| | Min | * | * | * | * | * |
| | Max | * | * | * | * | * |
| Short $3 < \ell/h < 6.6$ | No. | 3 | * | 2 | 2 | * |
| | Mean | 0.840 | * | 1.022 | 1.022 | * |
| | CV | 0.031 | * | 0.014 | 0.014 | * |
| | Min | 0.812 | * | 1.012 | 1.012 | * |
| | Max | 0.863 | * | 1.032 | 1.032 | * |
| Slender $6.6 \le \ell/h \le 30$ | No. | 32 | 5 | 38 | 39 | * |
| | Mean | 0.734 | 0.910 | 0.969 | 0.965 | * |
| | CV | 0.143 | 0.102 | 0.143 | 0.143 | * |
| | Min | 0.530 | 0.759 | 0.793 | 0.793 | * |
| | Max | 0.941 | 0.992 | 1.280 | 1.280 | * |
| Super Slender $\ell/h > 30$ | No. | * | * | * | * | * |
| | Mean | * | * | * | * | * |
| | CV | * | * | * | * | * |
| | Min | * | * | * | * | * |
| | Max | * | * | * | * | * |
| ACI Permitted $3 < \ell/h \le 30$ | No. | 35 | 5 | 40 | 41 | * |
| | Mean | 0.743 | 0.910 | 0.972 | 0.968 | * |
| | CV | 0.141 | 0.102 | 0.139 | 0.140 | * |
| | Min | 0.530 | 0.759 | 0.793 | 0.793 | * |
| | Max | 0.941 | 0.992 | 1.280 | 1.280 | * |

* No data available

Note: CV stands for the coefficient of variation.

# IMAGE EVALUATION TEST TARGET (QA-3)

150mm

6"

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