Response of Cereals to Anhydrous Ammonia as Correlated with Tests for Available Soil Nitrogen

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HAYDEN E. TOLTON

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ABSTRACT

Anhydrous ammonia as a source of nitrogen alone and in combination with ammonium phosphate was applied to oats and barley seeded on fourteen stubble fields situated on seven genetic soil types. There was a basic phosphate requirement on all soils. Nitrogen applications were necessary to attain high yields. The soils with calcium carbonate near the surface gave increasing response to phosphate, the remaining farms showed diminishing total yield after the fifteen or twenty pound per acre application of phosphate. Eleven farms showed increases to nitrogen application up to 80 pounds per acre. The remaining three farms exhibited diminishing total yields at the high nitrogen applications. Phosphate and nitrogen response were closely associated with genetic soil types. Nitrogen response was also dependent on soil management practices.

The available soil nitrogen determined by the alkaline permanganate and incubation method did not correlate satisfactorily with nitrogen yield response. The correlation was erratic and little confidence could be placed in yield predictions from the methods at the present time. Further work will be needed in an attempt to adapt one or both of the methods to Manitoba soil conditions.

Quadratic and Cobb-Douglas functions were fitted to the yield data. It was proven that the Quadratic function response curves fitted the actual yield data better than the Cobb-Douglas function and greater confidence could be placed in the yield predictions made from the Quadratic function.

For methodological purposes Farm 8 was chosen to adapt a method to determine the most economic rates and combinations of phosphate and nitrogen fertilizers. The method showed that the optimum yield varied with various nutrient and crop price situations. This work on methodology can be applied to Manitoba fertilizers yield data and to future fertilizer investigation work.

Anhydrous ammonia was found to be a satisfactory source of nitrogen where available nitrogen is low such as in stubble fields.

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INTRODUCTION

During the past decade in Manitoba there has been a noticeable deficiency of nitrogen in cereal crops seeded on fields cropped the previous year. This problem necessitated the use of higher rates of nitrogen fertilizer. In order to apply these higher rates of nitrogen it was found necessary to use higher analysis nitrogen fertilizers.

Foreseeing this trend in fertilizer applications this investigation was under taken in the spring of 1955 with two main objectives. The first objective, to evaluate anhydrous ammonia as a nitrogen fertilizer alone, and in combination with phosphate fertilizer with special emphasis on yield increases with the different rates of fertilizer on oats and barley on several Manitoba soils. Included in this objective was an attempt to outline a method whereby the most economical rates or combinations of nitrogen and phosphate fertilizers could be easily and accurately determined. The second objective, to investigate methods of determining the quantity of nitrifiable nitrogen as outlined by other investigators and determine the correlation between nitrifiable nitrogen and the yield response of oats and barley to nitrogen fertilizer application.

In consideration of the complexity of the problem and the need for the greatest amount of information in the shortest possible time, the investigation was carried out on fourteen farms on seven genetic soil types during a two year period. More specific information could have been gained if the investigation had been undertaken on one and at the most two soils, but it was decided to obtain results that would have a wider application to Manitoba conditions. The investigation was carried out on a field trial basis which did not lend itself to randomization of fertilizer rates but the design did lend itself to statistical investigation such as analysis of variance, t-test and correlation and regression analysis.

REVIEW OF LITERATURE

Anhydrous Ammonia As A Source Of Nitrogen

The use of anhydrous ammonia as a nitrogen fertilizer has only recently become important and therefore, there is a scarcity of literature regarding its use and adaptability as a commercial source of nitrogen. Anhydrous ammonia is the cheapest source of nitrogen to manufacture because it is the primary product of the direct synthetic ammonia process. In early years it was thought that this ammonia must be converted to some solid product before it could be used as a fertilizer. Collings (5) stated that early experiments with easily hydrolyzed ammonium compounds appeared to show that a high percentage of free ammonia in the soils could be injurious to germinating seeds and plant roots. Recent research has shown that anhydrous ammonia can be satisfactorily used as a source of nitrogen for crops if proper safeguards are taken at the time of application.

Andrews et al (1) have probably carried out the most extensive work with anhydrous ammonia. These investigators explained that when anhydrous ammonia is applied to the soil, the ammonia goes into solution in the soil water and is almost immediately absorbed by the clay and organic matter forming ammonium clay and ammonium organic matter. In this form it does not leach out of the soil.

The form in which nitrogen can be absorbed by the plants has been a matter of consideration and some investigators have found that the ammonium nitrogen is absorbed as readily as nitrate nitrogen by some plants. Other plants appear to absorb ammonium nitrogen only in the seedling stage. Andrews et al (1) stated that young corn and cotton plants prefer ammonium nitrogen to nitrate nitrogen as evidenced by the fact that they grow more rapidly when supplied with ammonium nitrogen, although they did note that in general older plants prefer nitrate nitrogen to the ammonium form. While Collings (5) did not list any plants that utilize ammonium nitrogen, he did say that the ammonia in the ammonium clay and ammonium organic matter is available to those plants that utilize ammonia. This is also confirmed by Andrews et al (1). Meyers et al (12) reported that many species of plants when grown in sand or solution cultures develop as well or better when supplied with ammonium salts than when supplied with nitrate. They felt that this was not surprising since the nitrogen in ammonium compounds is in a highly reduced form. The statements of these investigators substantiate the fact that ammonia is a satisfactory source of nitrogen even before it is nitrified to the nitrate nitrogen form.

Andrews et al (1) stated that no sandy soil has been encountered with too little clay to absorb the ammonia applied at the rate of 32 pounds per acre at a depth of 4 inches but to apply very heavy rates of nitrogen as ammonia to these sandy soils it may be necessary to increase the depth of application to eight or ten inches or decrease the spacing of the ammonia applicators. The work reported by Jackson and Chang (11) showed that soils of neutral pH, medium texture and moisture content may absorb 60 lbs. of nitrogen per acre when released at a depth of 1 to 2 inches. They also found that soils containing six percent clay provided adequate absorption capacity for ammonia, and soils with a high pH value may retain as high as 600 lbs. of nitrogen released 2 to 4 inches below the surface.

In order to replace other nitrogen fertilizers, anhydrous ammonia must excel the other nitrogen fertilizers in crop response, ease of application and ability to reduce the nitrogen deficiency caused by the presence of undecomposed crop residue. Andrews et al (1) reported that fall applied anhydrous ammonia from the yield standpoint was twice as effective as fall applied ammonium nitrate. This difference in yield was attributed to less leaching of nitrogen where the anhydrous ammonia was applied. These same workers also reported that anhydrous ammonia was found to have crop producing value equal to or superior than ammonium nitrate for row crops.

The advantage of speed of application of anhydrous ammonia as a nitrogen source is revealed by the work of Andrews et al (1) in which they state that farmers usually apply anhydrous ammonia to about twice as many acres per day as with the solid sources of nitrogen. Harmsen and Van Schreven (9) concluded that ammonium fertilizers must be considered superior to nitrates for rapid decomposition of straw.

While anhydrous ammonia exhibits many advantages, some of the disadvantages are reported by Collings (5). He stated that the germinating seeds are usually killed by contact with the solution; tillage for a few days following the application will result in a loss of some of the ammonia; and heavy equipment is required for transportation, storage and application.

Application Of Statistical Equations To Yield Data

A great deal of investigation has been carried out on the problem of developing equations that will make it possible to predict the yield increase resulting from a given fertilizer application. Bear (2) mentioned that Liebig pioneered the work in this field when he developed his "law of the minimum" but Bear states that inter-relationships existing among various fertilizer compounds in their effects on plant growth are too complex to be explained by such a simple law. Bray (3) as quoted by Bear suggests that nitrogen does tend to follow Liebig's law of the minimum while phosphorus and potassium follow Baule's percentage yield concept. Tisdale and Nelson (16) stated that Mitscherlich and Spillman independently developed equations that take into account that when plants are supplied with adequate amounts of all nutrients save one, their growth is proportional to the amount of this limiting element which is supplied to the soil. Willcox (18) verified the percentage-yield concept developed by Mitscherlich and Baule, and in his work showed that there can be partial substitution of one element for another element.

This line of investigation has been greatly intensified during the last decade with the increase in the use of fertilizer, especially nitrogen fertilizer. Since fertilizer has become a major investment on many farms, Heady et al (10) have carried out some of the most intensive work in the field of applied statistics. They criticized the "law of the minimum" advanced by Liebig which supposed that fertility elements must be combined in fixed proportions; one element does not substitute for the other and a given crop yield cannot be maintained as a shift is made to more of one and less of another nutrient. These investigators used fertilizer yield data to show that the same yield can be maintained by replacing some of one nutrient in a fertilizer with more of another but less and less of the first nutrient will be replaced by each successive per pound increase in the second. They also showed that nutrient interaction was very important in fertilizer response as the yield resulting from a given combination of fertilizer nutrients was greater than the total yield of the two nutrients applied separately.

Heady et al (10) devoted considerable attention to deriving algebraic equations best suited to estimating the fertilizer yield response. These workers found that the Cobb-Douglas or logarithmic function could not be applied to diminishing total yield and was not satisfactory for experiments with high fertilizer applications. They found that the yield isoquants of this function have a constant slope along a fixed nutrient line in the nutrient plane, and therefore, the same nutrient combination should be used for all yield levels. It does not allow the range of substitution to narrow as higher yields are attained. Heady et al (10) selected the quadratic function because it allowed the yield isoquants to change in slope along a fixed nutrient line and they exhibited a higher coefficient of determination and could be considered as being the most efficient for predicting the response curves to fertilizer applications. It also allowed for diminishing total yield. These workers concluded that several different functions must be checked for best fit by examining the statistics for each function and by comparing the response curve and yield isoquants predicted from the two variable functions with scatter diagrams of the observations before a function can be selected for prediction for each crop.

Techniques For Determining Available Nitrogen

There is very little published information about the application of the incubation and chemical method for determining available nitrogen. Harmsen and Van Schreven (9) reported that as early as 1916 studies had been made to determine the percentage of nitrogen mineralized during incubation. This work has continued through the years and at the present time a great deal of work is being carried out in an effort to correlate the amount of nitrifiable nitrogen as determined in the laboratory with the yield response of crops to nitrogen fertilizer. A number of procedures have been used and conflicting results reported.

According to Fitts et al (6) the use of commercial nitrogen fertilizer is a relatively new practise and until 1943 there had been little research carried out with nitrogen fertilizers. They stated that accompanying this trend was a need for a method evaluating the soil nitrogen and predicting the magnitude of crop response from nitrogen applications. These workers also stated that the relationship between past cropping sequence and the degree of response of corn to nitrogen fertilization had been investigated but the use of past management data alone is unsatisfactory as a guide to the need for nitrogen fertilizer.

Prichett et al (14) began the present trend to correlate the mineralizable nitrogen with the response of crops to nitrogen fertilizers. These investigators determined the increase in ammonia, nitrite and nitrate nitrogen resulting from incubation.

Fitts et al (7) stated that nitrate production during incubation should give the most reliable results because of the similarity between the incubation and soil processes. It was also mentioned that nitrate production takes place in normal soils in the field under conditions which permit aerobic microbiological activity. They explain further that duplication of field environment in the laboratory has generally not been attempted and from the standpoint of soil testing, it is not likely desirable to do so. By air drying the sample and then imposing optimum conditions of incubation in the laboratory, it is possible to produce as much nitrate in two or three weeks as might be produced in one or two months in the field. The writers concluded that soils which produce only small amounts of nitrate nitrogen under optimum conditions in the laboratory are not likely to produce much under field conditions. These workers attempted to simplify the laboratory procedure so that it could be used on a mass production basis and at the same time maintain accurate results. They reduced the laboratory work by measuring only the nitrate nitrogen produced during the incubation period.

Conflicting opinions are reported concerning the suitability of the incubation method as a means of determining the available nitrogen so that predictions could be made on the crop response to nitrogen fertilizer. Prichett et al (14) concluded that a regression equation relating response of oats to nitrogen fertilization with mineralizable soil nitrogen can be used as a means of prediction provided the regression equation represents the average of several years data. They also stated that the mineralizable nitrogen content of the soil served as a better index of the probable response of oats to nitrogen fertilizers in Iowa than information on past management alone. Hanway and Dumenil (8) reported that regardless of the crop, a single set of samples from a field may provide a reliable indication of the potential nitrogen supplying power of the soil which will hold for a period of years. However, they qualified their statement saying that the interpretation of the incubation test as a basis for making fertilizer recommendations must differ, depending on the previous crop and the crop to be grown. The relationship developed in their study is limited to the nitrogen needs of corn which does not follow a leguminous meadow. Munson and Stanford (13) concluded that the nitrate determined by the incubation method most readily reflected the nitrate availability. They also found that nitrate nitrogen released by incubation was a more accurate measure of available nitrogen than was total nitrogen. These two workers compared the two week incubation method with the alkaline permanganate method recently introduced by Truog et al (17). Munson and Stanford (13) found that when the available nitrogen determined by the alkaline permanganate method was related to the nitrogen response of crops, only low correlation was obtained. This was in contrast to the high correlation obtained with the incubation method. They also stated that there was a low correlation between the available nitrogen determined by the alkaline permanganate method and the available nitrogen determined by the incubation method. Harmsen and Van Schreven (9) stated that reliable available nitrogen results sufficiently correlated with the nitrogen requirment of field crops can be expected only when the incubation technique is restricted to one soil type, one climatic zone, one farming system and when all samples are collected within one season, preferably during the early spring. They stated further that the results and their interpretation will vary from one year to another due to uncontrollable and unpredictable variations in the weather conditions.

FIELD INVESTIGATIONS

The field investigation was initiated to determine the yield response of oats and barley to anhydrous ammonia (82-0-0) alone and in combination with ammonium phosphate (11-48-0). The anhydrous ammonia was applied at rates to supply 0, 20, 40, 60 and 80 pounds of nitrogen per acre. The ammonium phosphate (11-48-0) was applied at rates of 0, 40 and 60 pounds per acre, supplying 0, 19.2 and 28.8 pounds of phosphate per acre. The ammonium phosphate (11-48-0) also supplied 4.4 and 6.6 pounds of nitrogen with the 40 and 60 pound applications, respectively. Ammonium phosphate sulphate (16-20-0) and ammonium nitrate phosphate (27-14-0) fertilizers were applied beside the ammonium phosphate (11-48-0) applications at rates shown in tables 4 and 6.

The anhydrous ammonia for the trials in 1955 was applied in the spring of 1955 while the anhydrous ammonia for the trials in 1956 was applied in the fall of 1955 with the exception of Farm 14 on which this fertilizer was applied in the spring of 1956. The machine used to apply the anhydrous ammonia fertilizer is shown in Figure 1. The eight applicators were spaced at one foot intervals. The anhydrous ammonia was applied at depths varying from four to six inches. The ammonium phosphate (11-48-0) was applied with the seed using the fertilizer attachment on the farmer co-operator's seed drill.

The field trials were placed on fourteen farms listed in Table 1. This table gives the name and location of the farmer co-operators on whose farms the trials were situated. The genetic soil type and crop grown is listed as well as the year in which the experiment was carried out. The experiments included seven oat and seven barley fields with 9 trials in 1955 and 5 trials in 1956. The experimental fields were located on different genetic soil types. The seven genetic soil types were included instead of only one or two genetic types in order that a great deal more information of practical use could be gained regarding anhydrous ammonia fertilizer.

The basis for choosing the individual fields were as follows:-

They were at least one-quarter mile in length

They were stubble fields on which the crop residue from the previous crop had not been burned or removed.

They were in a grain-fallow system, i.e. legumes had not been grown on the field for at least ten years.

The fields which were selected were representative of the dominant genetic soil type that occurred in the area.

Table 1: List Of Farms On Which The Experiments Were Conducted

Farm No.	Co-Operator	Location	Crop and Year	Genetic Type Texture and Association
1	Anderson, F.W.	NW-20-11-18W	Oats 1955	Solonetzic Black Clay Harding
2	Berquist, C.H.	SW-13-12-10W	Oats 1955	Black-Meadow Loamy fine sand Almassippi
3	Mitchell, E.T.	SW-35-11-17W	Oats 1955	Calcareous Black Clay loam Newdale
4	Sims, W.F.	NE-23-11-11W	Oats 1955	Black-Meadow Fine sandy loam Almassippi
5	Earl, H.	SW-25-5-1 E	Oats 1956	Solonetzic Black-Meadow Clay Red River
6	McFadden, D.J.	SE-6-12-20W	Oats 1956	Calcareous Black-Meadow Clay loam Carroll
7	Berquist, C.H.	SW-13-12-10W	Oats 1956	Black-Meadow Loamy fine sand Almassippi
8	Longstaffe, W.L.	SW-25-14-21W	Barley 1955	Black-Meadow Clay loam Newdale
9	Purdy, R.W.	SW-11-16-23W	Barley 1955	Calcareous Black-Meadow Clay loam Newdale
10	Remple, C.E.	NW-23-4-2 W	Barley 1955	Meadow Clay Osborne
11	Ruckle, A.W.	SE-21-11-14W	Barley 1955	Black Very fine sandy clay loam Wellwood
12	Russell, R.F.	NE-13-5-1 W	Barley 1955	Solonetzic Black-Meadow Clay Red River
13	Calvert, H.	SW-32-11-14W	Barley 1956	Black Very fine sandy clay loam Wellwood
14	Wilkie, A.	NW-29-6-4 W	Barley 1956	Black-Meadow Very fine sandy loam Almassippi

Figure 1: The machine used to apply the anhydrous ammonia fertilizer showing the applicators, metering pump and high pressure tank to carry the ammonia

Figure 2: PLOT DESIGN SHOWING ONE OF TEN REPLICATES IN THE EXPERIMENTAL FIELD

	Ammonium Phosphate 11–48–0 40 lbs./acre (← 7' or 10' →)	Check	Ammonium Phosphate 11–48–0 60 lbs./acre
Anhydrous Ammonia 25 lbs./acre (← 8' or 16' →)	N.= 24.4 lbs. P_2O_5=19.2 lbs.	N.= 20 lbs P_2O_5= 0 lbs	N = 26.6 lbs. P_2O_5 = 28.8 lbs.
Check	N.= 4.4 lbs. P_2O_5=19.2 lbs.	N.= 0 lbs. P_2O_5=0 lbs.	N = 4.6 lbs. P_2O_5=28.8 lbs.
Anhydrous Ammonia 50 lbs./acre	N=44.4 lbs. P_2O_5=19.2 lbs.	N=40.0 lbs. P_2O_5 = 0 lbs.	N=46.6 lbs P_2O_5=28.8 lbs.
Anhydrous Ammonia 75 lbs./acre	N.= 64.4 lbs. P_2O_5=19.2 lbs.	N.= 60 lbs. P_2O_5= 0 lbs.	N.= 66.6 lbs. P_2O_5=28.8 lbs.
Check	N = 4.4 lbs. P_2O_5=19.2 lbs.	N = 0 lbs. P_2O_5 = 0 lbs.	N = 6.6 lbs. P_2O_5=28.8 lbs.
Anhydrous Ammonia 100 lbs./acre	N.= 84.4 lbs. P_2O_5=19.2 lbs.	N.= 80 lbs. P_2O_5 = 0 lbs.	N.= 86.6 lbs. P_2O_5 =28.8 lbs.

These qualifications were adhered to very closely with the exception that farms 1, 3, 10 and 12 had been broken from forage crops within the last 10 years.

The design of the trials is shown in Figure 2. The ammonium phosphate (11-48-0) was applied the length of the field during the seeding operation. A check strip was left between the 40 and 60 pounds of ammonium phosphate (11-48-0). The strips varied in width from seven to ten feet depending on the size of the seed drill. The anhydrous ammonia was applied across the area on which the ammonium phosphate strips were to be placed. The ammonia was applied at 20, 40 60 and 80 pounds of nitrogen per acre with a check strip between the 20 and 40 pound rates of nitrogen and the 60 and 80 pound rates of nitrogen. This design resulted in a check strip adjacent to each rate and combination of rates of fertilizer. These combinations of anhydrous ammonia cross strips were replicated ten times on the ammonium phosphate strips at regular intervals down the length of the field. The applications of anhydrous ammonia were 8 feet wide in 1955 and 16 feet wide in 1956.

These field scale trials had a fixed design that did not allow randomization of the treatments but the fields chosen were quite uniform and any noticeable variations in soils or management such as dead furrows, drift soil ridges, old building sites etc. were avoided. Thus, any variation in the soils was random and effected each treatment equally.

At harvest time, one square yard of the test crop was cut from each replicate treatment resulting in ten square yards being harvested from each treatment. The samples were threshed, weighed and the yields were calculated in bushels per acre. The yields on the fourteen farms are shown in Tables 2 and 3.

Field Experimental Data And Results

Field Observations

During the growing season and at harvest time observations were made on the crop growth, color and straw strength. Figure 3 shows the difference between the check and 60 pounds per acre of nitrogen. The oats were much taller and the leaves were broader where the fertilizer had been applied. Figure 4 shows the check strip and 80 pounds per acre of nitrogen. The height and thickness of the oat crop is indicated by noting the stakes and signs. The comparison of 80 pounds per acre of nitrogen in combination with 40 pounds per acre of ammonium phosphate (11-48-0) and 40 pounds of ammonium phosphate (11-48-0) alone is shown in Figure 5. The dark green color resulting from the high nitrogen application is very noticeable as well as increased leaf width and crop height. The three figures all show that the nitrogen application increased tillering as was noted by the very thick stands on the nitrogen fertilized strips. While some difference in length of straw and leaf color was noted where 20 pounds of nitrogen had been applied this was not general on all farms. On all farms there was a very noticeable increase in length of straw, tillering and leaf color where 40, 60 and 80 pounds of nitrogen were applied. The leaves of the test crop were noticeably wider as the rates of nitrogen were increased. All anhydrous ammonia applications exhibited a darker green color than the adjacent rates of ammonium phosphate sulphate (16-20-0) and ammonium nitrate

Figure 3: Showing the check on the left and 60 pounds per acre of nitrogen on the right on Farm 4

Figure 4: Showing the check on the right and 80 pounds per acre of nitrogen on the left on Farm 1

Figure 5: Showing 80 pounds per acre of nitrogen in combination with 40 pounds per acre of ammonium phosphate (11-48-0) on the left and 40 pounds per acre of ammonium phosphate (11-48-0) on the right on Farm 13

phosphate (27-14-0).

The high rates of nitrogen application did not cause lodging or delayed maturity when phosphate was applied. The 80 pound application of nitrogen alone did cause lodging on some farms especially on farms 3, 9 and 10, but these farms had been broken from forage or native sod within the last seven years. Lodging was very serious on all rates of anhydrous ammonia on Farm 4 but this was also noticed with the 96 and 144 pound rates of ammonium phosphate sulphate (16-20-0). This field had a great deal of sweet clover plowed in as green manure in past fifteen years. The over abundance of nitrogen from the two sources caused the lodging observed on this field. These observations seem to show that lodging is not a problem if there is a balance between nitrogen and phosphate applied to the crop.

The Effect of Anhydrous Ammonia Fertilizer and Ammonium Phosphate 11-48-0 Fertilizer on Oat Yields

The oat yield data are shown in Table 2. The yield data shown in Table 2 has been processed from the original data in order that the yields from each fertilizer treatment can be compared to the mean check yield at the top of each farm column. The calculation used to obtain the yield figures in Table 2 is as follows:-

$$\frac{\text{actual treatment yield}}{\text{adjacent check yield}} \text{ X average check yield}$$

This allows one to compare any treatment yield to another treatment yield due to their common mean check yield. The mean yields for the seven farms for each treatment are shown in the last column in Table 2.

Table 2: Effect Of Anhydrous Ammonia And Ammonium Phosphate On Oat Yields On Stubble In 1955 And 1956

	P_2O_5 lbs/acre	Nitrogen lbs/acre	Yield Bushels Per Acre Farm 1	Farm 2	Farm 3	Farm 4	Farm 5	Farm 6	Farm 7	Average Yield
	0	0	32.3	22.6	42.5	49.4	36.5	59.1	46.2	41.22
	0	20	33.4	37.3*	49.5	54.2	47.1**	62.2	48.9	46.08
	0	40	36.4	30.2*	47.2	54.7	65.7**	75.9*	59.9*	52.85
	0	60	37.7	36.2**	42.8	67.0**	80.7**	81.4**	66.0**	58.82
	0	80	37.7	31.6**	44.8	57.0	91.0**	87.2**	63.5**	58.97
	19.2	4.4	33.3	26.0	44.1	57.5*	41.0	69.4*	46.3	45.37
	19.2	24.4	37.4	31.5*	53.6	62.4*	52.1**	79.5*	58.5*	53.57
	19.2	44.4	39.8*	34.6**	54.0*	58.8	70.7**	90.3**	68.5**	59.52
	19.2	64.4	46.8**	40.3**	48.1	62.9*	76.1**	96.4**	62.5*	61.87
	19.2	84.4	45.3*	42.7**	52.6*	66.8*	77.6**	89.2**	73.5**	63.95
	28.8	6.6	40.8**	31.5*	40.0	57.6	41.9**	63.5	50.5	46.54
	28.8	26.6	53.8**	35.9*	55.1	46.0	50.1**	71.1	57.2*	52.74
	28.8	46.6	48.1**	45.5**	46.3	60.8	67.1**	83.9**	77.1**	61.25
	28.8	66.6	48.9**	54.2**	49.1	65.5*	83.0**	79.8*	72.4**	64.70
	28.8	86.6	52.8**	49.0**	43.0	73.4**	88.8**	93.4**	81.6**	68.85
Granular fertilizer only Check			28.8	21.6	41.2	53.2	38.3	53.6	44.7	40.2
48# 16-20-0	9.6	7.7	29.3	30.0*	46.8	51.4	48.5**	72.3**	50.8	47.0
96# 16-20-0	19.2	15.4	37.7*	39.1**	53.1*	68.6**	53.1**	78.6**	57.6**	55.4
144# 16-20-0	28.8	23.0	39.9*	30.9**	48.9	63.5*				
57# 27-14-0	8.0	15.4					61.1**	71.0**	59.4**	
114# 27-14-0	16.0	30.8						71.3**	72.7**	

** Indicates that the yield of this treatment is significantly different from its adjacent check at the 1% level.

* Indicates that the yield of this treatment is significantly different from its adjacent check at the 5% level.

Before the yield data were processed as shown in Table 2, the yield data were analysed statistically by means of the t-test to determine whether the fertilizer treatments were significantly different from the adjacent check yield. The 1 percent and 5 percent levels of significance are indicated in Table 2.

The mean yield of each treatment showed that the yield increased up to and including the 80 pound application of nitrogen but at a decreasing rate with each added increment of fertilizer after the 40 pound application of nitrogen.

The mean yield indicated that the 28.8 pounds of phosphate produced only slightly higher yields than 19.2 pounds of phosphate when low rates of nitrogen were applied. As the nitrogen application was increased to the 60 and 80 pound rates, the yields were noticeably higher with the higher phosphate applications showing slight interaction between nitrogen and phosphate.

Farms 1 and 3 did not respond significantly to the nitrogen applications. On Farm 4, significant response to nitrogen occurred only at the 60 pound application. This lack of nitrogen response on these three farms is explained by the fact that farms 1 and 3 had been broken from alfalfa and Farm 4 had had sweet clover plowed in as green manure. Farms 3 and 4 exhibited very erratic yield increases even when phosphate was applied with the nitrogen.

Farms 4 and 6 were the only farms showing significant yield increases with the 40 pound application of ammonium phosphate (11-48-0) and these farms showed significant increases only at the 5 percent level.

Farms 1 and 5 showed significant difference in yield at the 1 percent level from the application of 60 pounds of ammonium phosphate (11-48-0) and Farm 2 showed a significant yield increase at only the 5 percent level with the same treatment. These soils apparently required a higher application of phosphate because the 40 pound application of ammonium phosphate (11-48-0) did not give significant yield increases. It would appear that on these three farms, yield increases could still be expected with higher phosphate rates.

The results of ammonium phosphate sulphate (16-20-0) and ammonium nitrate phosphate (27-14-0) fertilizers are included at the bottom of Table 2 for the purpose of comparing the results with the anhydrous ammonia and ammonium phosphate (11-48-0). These fertilizers gave the greatest yield increases on the farms where there was a good response to anhydrous ammonia fertilizer. There was a greater increase per pound of applied nutrient with these granular fertilizers than with the anhydrous ammonia. This suggests that fertilizer placement is important and more efficient use was being made of the fertilizer placed with the seed than with the nitrogen in the anhydrous ammonia fertilizer that was placed away from the seed.

The analysis of variance of the effect of nitrogen and phosphate on the oat yields on the seven farms in Table 3 showed that there was a highly significant difference among farms on different soils, with regard to their fertilizer response. The response to the different rates of phosphate and nitrogen on the seven farms was significantly different. The interaction between the nitrogen and phosphate was not significant.

Table 3: Analysis Of Variance Of The Effect Of Nitrogen And Phosphate On Oat Yields

Source of Sum of Squares	Sum of Squares	Degrees of Freedom	Variance	F
Farm	19,899.91	6	3,316.65	157.94**
P_2O_5	977.44	2	488.72	23.27**
N	5,497.59	4	1,374.40	65.45**
Farm X P_2O_5	762.67	12	63.56	3.03**
Farm X N	3,136.43	24	130.68	6.22**
P_2O_5 X N	105.70	8	13.21	<1
ERROR	1,007.80	48	21.00	
TOTAL	31,176.14	104		

** significant at the 1% level.

This lack of significant interaction was also shown in the regression analysis Table 8, although Farm 2 did exhibit significant interaction at the 5 percent level.

The Effect of Anhydrous Ammonia Fertilizer and Ammonium Phosphate (11-48-0) on Barley Yields

The barley yield data are shown in Table 4. The data has been processed similar to the oat yield data. The 1 percent and 5 percent levels of significance refer to the yield difference of each treatment from its adjacent check plot. The mean yields for each treatment shown in Table 4 gives some indication of the yield response to the different rates of fertilizer.

The mean yield column showed that the yields increased up to the 80 pound application of nitrogen. The largest yield increase due to nitrogen occurred between the 40 and 60 pounds per acre application except when 28.8 pounds of phosphate was applied, then the greatest increase due to nitrogen occurred between the 26.6 and 46.6 pound per acre application of nitrogen.

The 80 pound application of nitrogen increased the yield only one or two bushels over the 60 pound application.

There was a very definite response to phosphate shown in the mean yield column and the 40 pounds per acre of ammonium phosphate (11-48-0) gave higher yields than the 60 pounds per acre of ammonium phosphate (11-48-0). This indicates that diminishing total yields were showing up at the 60 pound rate of ammonium phosphate (11-48-0).

Table 4: Effect Of Anhydrous Ammonia And Ammonium Phosphate On Barley Yields On Subble In 1955 and 1956

	P_2O lbs/acre	Nitrogen lbs/acre	Yield Bushels Per Acre Farm 8	Farm 9	Farm 10	Farm 11	Farm 12	Farm 13	Farm 14	Average Yield
	0	0	17.5	29.3	40.6	10.1	28.9	14.9	15.3	22.37
	0	20	19.7	26.2	43.2	18.6**	27.8	16.9*	21.3**	24.81
	0	40	22.0	26.5	41.9	12.7*	31.9	24.0**	23.2**	26.02
	0	60	20.0	34.9	43.7	18.5**	37.5	30.3**	27.5**	30.34
	0	80	21.2	37.4*	40.5	21.7**	43.0*	29.1**	25.2**	31.15
	19.2	4.4	25.4**	33.6**	40.1	17.2**	39.9*	23.2**	23.8**	29.02
	19.2	24.4	31.4**	30.8	42.1	26.4**	37.6*	24.5**	31.9**	32.10
	19.2	44.4	35.3**	28.6	36.9	21.5**	34.8*	29.7**	32.0**	31.25
	19.2	64.4	32.0**	42.4**	41.9	27.8**	44.2**	37.4**	35.0**	37.24
	19.2	84.4	32.8**	44.0**	44.1	34.7**	47.9**	42.5**	30.0**	39.42
	28.8	6.6	25.2**	26.7	40.5	16.5**	44.5**	17.9**	21.7*	27.57
	28.8	26.6	26.9**	29.3	40.0	21.9**	40.6*	21.6*	26.5**	29.54
	28.8	46.6	37.5**	32.9	44.7	26.9**	42.3**	25.3**	29.8**	34.20
	28.8	66.6	34.2**	40.7**	42.3	31.4**	50.8**	37.6**	28.5**	37.92
	28.8	86.6	33.5**	42.6**	47.1*	35.4**	50.1*	36.3**	26.0**	38.71
Granular fertilizer only										
Check			22.3	30.3	36.9	7.9	40.2	15.6	19.3	24.6
48# 16-20-0	9.6	7.7	31.9**	32.1	45.8*	20.3**	43.8	17.1*	23.9*	30.7
96# 16-20-0	19.2	15.4	34.6**	33.7	44.7*	19.0**	43.8	19.6**	27.4**	31.8
144# 16-20-0	28.8	23.0	37.0**				47.6			
57# 27-14-0	8.0	15.4						24.9**	22.4	
114# 27-14-0	16.0	30.8						26.6**		

** Indicates that the yield of this treatment is significantly different from its adjacent check at the 1% level.

* Indicates that the yield of this treatment is significantly different from its adjacent check at the 5% level.

Farms 8, 9, 10 and 12 did not show significant responses to nitrogen applied alone except that farms 10 and 12 exhibited a significant response to 80 pounds of nitrogen. Farms 9, 10 and 12 had been broken from legume meadow or wasteland within the last seven years explaining the lack of nitrogen response on these farms. Farm 8 required phosphate in combination with the nitrogen to produce significant yield response as noted in Table 4.

Farms 11, 13 and 14 responded significantly to nitrogen applications.

All farms except Farm 10 showed significant response to the 40 pound per acre application of ammonium phosphate (11-48-0). The 60 pound per acre application of ammonium phosphate (11-48-0) did not produce a significant response on farms 9 and 10, while the response on Farm 14 was only significant at the 5 percent level. Five of the seven farms showed a lower response to 60 pounds per acre of ammonium phosphate (11-48-0) than to the 40 pounds per acre of ammonium phosphate (11-48-0). This showed that diminishing total yield had occurred with 60 pound per acre application of ammonium phosphate (11-48-0).

The results of the ammonium phosphate sulphate (16-20-0) and ammonium nitrate phosphate (27-14-0) fertilizers are included at the bottom of Table 4 for the purpose of comparing the results with the anhydrous ammonia and ammonium phosphate (11-48-0). The ammonium phosphate sulphate (16-20-0) gave significant response on all farms except farms 9 and 12. Farm 9 had been broken from waste land within the last seven years explaining the general lack of response but on Farm 12 the

Table 5: Analysis Of Variance Of The Effect Of Nitrogen And Phosphate On Barley Yields

Source of Sum of Squares	Sum of Squares	Degrees of Freedom	Variance	F
Farm	4,828.59	6	804.77	192.53**
P_2O_5	1,066.67	2	533.33	127.59**
N	1,528.67	4	382.17	91.43**
Farm x P_2O_5	444.72	12	37.06	8.87**
Farm X N	752.37	24	31.35	7.50**
P_2O_5 X N	76.41	8	9.55	2.28*
ERROR	200.51	48	4.18	
TOTAL	8,897.94	104		

** significant at the 1% level.

* significant at the 5% level.

check yield for these granular fertilizers was unaccountably high and this reflected on the significance of the fertilizer response. The optimum response to ammonium phosphate sulphate (16-20-0) lies between the 48 and 96 pound per acre application. The two farms, 13 and 14, seeded in 1956 were fertilized with ammonium nitrate phosphate (27-14-0). On Farm 14 the ammonium phosphate sulphate (16-20-0) appeared the most satisfactory while on Farm 13, the ammonium nitrate phosphate (27-14-0) was most satisfactory. This is understandable because the nitrogen in the form of anhydrous ammonia gave a greater response on Farm 13. There was a greater increase per pound of applied nutrient with these fertilizers than with the anhydrous ammonia - ammonium phosphate combination. This agrees with the results from these same fertilizers on the oats in Table 2. Farms 11, 13 and 14 showed significant response to all fertilizer treatments.

The analysis of variance of the effect of nitrogen and phosphate on barley yields on the seven farms in Table 5 showed that there was a highly significant difference among farms on different soils with regard to their fertilizer response. The analysis of variance showed that there was a significant difference in response to the various rates of nitrogen and phosphate. The response to the different rates of phosphate on the seven farms was significantly different and response to the different rates of nitrogen on the seven farms, was also significantly different. There was significant interaction between the nitrogen and phosphate response at the 5 percent level of significance.

Methodological Approach to Yield Predictions and most Economical Fertilizer Nutrient Combinations

The main objectives of this section are as follows:-

(1) To show the physical relationships of nitrogen and phosphate and the corresponding yield.

(2) To determine the least cost combination of nutrients.

(3) To determine the most profitable level of nutrient applications.

In order to make predictions from the yield data, regression equations or production functions were derived. Production functions were derived separately for each farm due to the wide difference in fertilizer response amongst farms as shown by the analysis of variance tables.

Derivation of regression equations or yield functions

Preliminary inspection indicated that the Quadratic function would best fit the yield response data. In order to verify this it was decided to try to fit three different regression equations to the data. The correlation coefficients were calculated to derive the regression equations and are listed in Appendices (i), (ii) and (iii).

The regression equations fitted to the data were as follows:-

(1) Cobb-Douglas or Logarithmic function *

$$Y = aP^{b_{y1}}N^{b_{y2}}$$

Y = predicted yield

a = the ordinate of the point where the line crosses the Y axis or the check yield

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* The Cobb-Douglas equation is a linear function in terms of the logarithms of the original data. In a more general sense the Cobb-Douglas equation is referred to as a power function.

P = pounds phosphate applied per acre

N = pounds nitrogen applied per acre

by_1 = regression coefficient to estimate Y from P

by_2 = regression coefficient to estimate Y from N

(2) Quadratic function

$$Y = a + by_1P + by_2N + by_3P^2 + by_4N^2$$

(3) Quadratic function including an interaction term

$$Y = a + by_1P + by_2N + by_3P^2 + by_4N^2 + by_5NP$$

The symbols are the same for these latter two equations except the cross product term is added to the equation (3).

Y = predicted yield

a = the check yield or the ordinate of the point where the line crosses the Y axis

P = pounds phosphate applied per acre

N = pounds nitrogen applied per acre

by_1 = regression coefficient to estimate Y from P

by_2 = regression coefficient to estimate Y from N

by_3 = regression coefficient to estimate Y from P^2

by_4 = regression coefficient to estimate Y from N^2

by_5 = regression coefficient to estimate Y from NP

The regression coefficients were determined by the Fisher Modification of the Doolittle Method. An explanation of the regression coefficients is necessary in order that the practical application of the regression equations can be fully understood. The Quadratic function or equation (3) including interaction was chosen for discussion. The regression coefficient by_1 is a linear term and represents the slope of the yield response curve due to the applied phosphate. This regression coefficient by_1 permits positive fertilizer response. It is expected that with higher rates of fertilizer application the yield would increase. The regression coefficient by_2 represents the slope of the yield response curve due to the applied notrogen. This regression coefficient by_2 is also a linear term and permits positive fertilizer response for the same reason set forth for by_1. The regression coefficient by_3 was used to predict the yield from the squared term of phosphate application. This regression coefficient by_3 permits negative fertilizer response. Due to the nature of fertilizer response, it would be expected that as a nutrient is applied at higher and higher rates, its effectiveness is reduced or it may even cause an actual yield reduction. This term allows for a diminishing total product or yield. It is to be noted that with high rates of phosphate, this squared term can have a greater negative effect on predicted yield than the corresponding positive regression coefficient by_1 has on the predicted yield. The regression coefficient by_4 is used to estimate the yield from the squared term of applied nitrogen. This also permits negative fertilizer response. Diminishing total yield with high nitrogen applications can be explained by lodging, disease, susceptibility etc. due to the lush growth promoted by the application of this nutrient. The regression coefficients by_3 and by_4 permit a curvilinear response that will indicate diminishing or increasing yield. The regression coefficient by_5 represents the slope of the yield response curve due to the mutual effect of nitrogen and phosphate being applied simultaneously. It allows positive yield response because when nitrogen is increased there will be some increase in the phosphate requirement. If this requirement is met, a slight yield increase can be encountered.

The regression coefficients, coefficients of determination and regression equations are listed for the Cobb-Douglas function, Quadratic function and Quadratic function including the interaction term in Tables 6, 7 and 8 respectively. The individual regression coefficients were tested for significance with the t-test and the significance is indicated in each table. The coefficient of determination (R^2) shows the percentage of variance in yield explained by applications of the two nutrients. The significance of the coefficient of determination was determined and is indicated in each table. An example of the application of the coefficient of determination is shown in Table 8 with Farm 2, 94.37 percent of the variance in yield is explained by the application of nitrogen and phosphate.

Farms 3, 4 and 10 did not exhibit significant coefficients of determination with any of the three production functions, i.e. the regression equation was not considered satisfactory for predictions. This verifies Table 2 and 4 which show very erratic fertilizer responses on these three farms explained by the past management practises previously outlined.

The most important information revealed in Tables 6, 7 and 8 is that the coefficient of determination for each farm was highest with the

Table 6: The Coefficients Of Determination, Regression Coefficients, And Regression Equations For Nitrogen and P_2O_5 Applications

(Cobb-Douglas Function)

Farm	by_1	by_2	R^2	Regression Equation
1	.0661**	.0640*	.7016**	$Y = 28.89\ P^{.0661}\ N^{.0640}$
2	.0716**	.1307**	.7893**	$Y = 19.46\ P^{.0716}\ N^{.1307}$
3	.0143	.0257	.1855	$Y = 42.13\ P^{.0143}\ N^{.0257}$
4	.0163	.0525*	.3879	$Y = 48.03\ P^{.0163}\ N^{.0525}$
5	.0053	.1898**	.8028**	$Y = 32.85\ P^{.0053}\ N^{.1898}$
6	.0119	.0895	.6812**	$Y = 55.64\ P^{.0119}\ N^{.0895}$
7	.0343	.1154**	.7525**	$Y = 38.81\ P^{.0343}\ N^{.1154}$
8	.1279**	.0728**	.8874**	$Y = 16.12\ P^{.1279}\ N^{.0728}$
9	.0276	.0721	.3463	$Y = 24.64\ P^{.0276}\ N^{.0721}$
10	-.0016	.0174	.1456	$Y = 39.68\ P^{-.0016}\ N^{.0174}$
11	.1221**	.1828**	.8289**	$Y = 9.05\ P^{.1221}\ N^{.1828}$
12	.0533	.0636	.4568	$Y = 28.58\ P^{.0533}\ N^{.0636}$
13	.0484	.1886**	.7193**	$Y = 12.65\ P^{.0484}\ N^{.1886}$
14	.0573**	.1118**	.7631**	$Y = 14.80\ P^{.0539}\ N^{.1355}$

** significant at the 1% level.

* significant at the 5% level.

Table 7: The Coefficients of Determination, Regression Coefficients, And Regression Equations For Nitrogen And P_2O_5 Applications (Quadratic Function)

Farm	by1	by2	by3	by4	R^2	Regression Equation
1	-.1706	.2100*	.0211*	-.0011	.9074**	$Y = 29.7481 - .1706P + .2100N + .0211P^2 - .0011N^2$
2	-.1449	.3544	.0198	-.0017	.8987**	$Y = 19.4887 - .1449P + .3544N + .0198P^2 - .0017N^2$
3	.6966	.3010	-.0227	-.0032	.4725	$Y = 40.9709 + .6966P + .3010N - .0227P^2 - .0032N^2$
4	.4875	.0513	-.0131	.0012	.5791	$Y = 51.5341 + .4875P + .0513N - .0131P^2 - .0012N^2$
5	-.4149	.8635**	.0119	-.0028	.9576**	$Y = 36.4069 - .4149P + .8635N + .0119P^2 - .0028N^2$
6	1.4007**	.5849**	-.0450**	-.0028	.9084**	$Y = 56.4374 + 1.4007P + .5849N - .0450P^2 - .0028N^2$
7	-.0538	.5377**	.0124	-.0025	.8640**	$Y = 41.3831 - .0538P + .5377N + .0124P^2 - .0025N^2$
8	.9524**	.2632*	-.0199*	-.0021	.8800**	$Y = 14.5549 + .9524P + .2632N - .0199P^2 - .0021N^2$
9	.5012	-.0928	-.0143	.0029**	.8387**	$Y = 27.6027 + .5012P + .0928N - .0143P^2 + .0029N^2$
10	-.2222	-.0134	.0085	.0006	.3665	$Y = 41.0872 - .2222P - .0134N + .0085P^2 + .0006N^2$
11	.6953*	.1173	-.0133	.0006	.8668**	$Y = 10.1856 + .6953P + .1173N - .0133P^2 + .0006N^2$
12	.2541	.0913	.0043	.0026*	.8821**	$Y = 31.2670 + .2541P - .0913N + .0043P^2 + .0026N^2$
13	.9320**	.2177*	-.0286**	.0002	.9269**	$Y = 13.8699 + .9320P + .2177N - .0286P^2 + .0002N^2$
14	.9510**	.3943**	-.0289**	-.0035**	.9545**	$Y = 15.1165 + .9510P + .3943N - .0289P^2 - .0035N^2$

** significant at the 1% level.

* significant at the 5% level.

Table 8: The Coefficients Of Determination, Regression Coefficients, And Regression Equations For Nitrogen And P_2O_5 Applications

(Quadratic Function With Inter-Actions)

Farm	by_1	by_2	by_3	by_4	by_5	R^2	Regression Equation
1	-.2394	.1982	.0207	-.0013	.0019	.8755**	$Y = 30.6769 - .2394P + .1982N + .0207P^2 - .0013N^2 + .0019NP$
2	-.3465	.3198**	.0186*	-.0023*	.0055*	.9437**	$Y = 22.3098 - .3465P + .3198N + .0186P^2 - .0023N^2 + .0055NP$
3	.6079	.2857	-.0232	-.0035	.0024	.5032	$Y = 42.3166 + .6079P + .2857N - .0232P^2 - .0035N^2 + .0024NP$
4	.3907	.0346	-.0137	.0010	.0026	.5952	$Y = 52.6928 + .3907P + .0346N - .0137P^2 + .0010N^2 + .0026NP$
5	-.3084	.8817**	.0126	-.0026	-.0029	.9605**	$Y = 35.2072 - .3084P + .8817N + .0126P^2 - .0026N^2 - .0029NP$
6	1.4171**	.5877**	-.0450**	-.0028	-.0004	.9085**	$Y = 56.3454 + 1.4171P + .5877N - .0450P^2 - .0028N^2 - .0004NP$
7	-.2558	.5030*	.0112	-.0032	.0055	.8924**	$Y = 44.4864 - .2558P + .5030N + .0112P^2 - .0032N^2 + .0055NP$
8	.8114**	.2389*	-.0208*	-.0025*	.0039	.9201**	$Y = 16.4643 + .8114P + .2389N - .0208P^2 - .0025N^2 + .0039NP$
9	.4383	-.1038	-.0148	.0028*	.0017	.8475*	$Y = 28.3170 + .4383P - .1038N - .0148P^2 + .0028N^2 + .0017NP$
10	-.3171	-.0298	.0079	.0003	.0026	.5005	$Y = 42.4738 - .3171P - .0298N + .0079P^2 + .0003N^2 + .0026NP$
11	.5501	.0924	-.0143	.0002	.0040	.8989**	$Y = 12.1552 + .5501P + .0924N - .0143P^2 + .0002N^2 + .0040NP$
12	.4167	-.0635	.0054	.0031**	-.0044*	.9276**	$Y = 28.8736 + .4167P - .0635N + .0054P^2 + .0031N^2 - .0044NP$
13	.8607	.2053	-.0291	.0001	.0019	.9333**	$Y = 14.6334 + .8607P + .2053N - .0291P^2 + .0001N^2 + .0019NP$
14	.9926**	.4015**	-.0286**	-.0034**	-.0011	.9604**	$Y = 14.5499 + .9926P + .4015N - .0286P^2 - .0034N^2 - .0011NP$

** significant at the 1% level

* significant at the 5% level

Quadratic function including interaction and lowest with the Cobb-Douglas function with one exception. The Cobb-Douglas function will not permit diminishing total yield as shown by the work of Heady et al (10). While the regression coefficient for the interaction NP was significant only on Farms 2 and 12, there was sufficient interaction to show the highest coefficient of determination with the Quadratic function including interaction over the Quadratic function ignoring interaction. A comparison of the Quadratic function ignoring interaction. A comparison of the Quadratic function including interaction and the Cobb-Douglas function will be discussed in more detail with reference to Farm 8.

Discussion of the Quadratic function including interaction as applied to yield predictions

According to the nature of fertilizer response it is expected that by_1, by_2, and by_5 will be positive and by_3 and by_4 will be negative. On some of the farms studied however, variations occurred in the signs of the regression coefficients of the derivedequations because of differences in genetic soil type and soil management.

(a) Predicted oat yield response to applied nitrogen

The nitrogen response of oats was very significant at the 5% level on the majority of the farms. The regression coefficient by_2 was positive on all farms and the regression coefficient by_4 was negative in all cases except Farm 4. The predicted response curves in Figure 6 indicate the high nitrogen response on oat yields. These response curves show diminishing returns at the higher nitrogen applications except Farm 4 which shows increased response with the last nitrogen fertilizer increment. Farm 3 is the only farm showing diminishing total yield.

Table 8 and Figure 6 indicate that Farm 5 showed the greatest response to nitrogen applications. While the check yields varied on farms 2, 6 and 7, these farms exhibit similar nitrogen response curves (slopes). Farm 6 was on calcareous Black-Meadow soil and farms 2 and 7 were on Black-Meadow soils and the latter two farms were located on the same quarter section. All three farms also showed some degree of erosion. This shows that soils with similar genetic type and degree of erosion react similarly to nitrogen applications.

(b) Predicted barley yield response to applied nitrogen

The nitrogen response of barley was not as large as that of oats as shown by the regression coefficient by_2 was significant on only farms 8 and 14. All farms had a positive regression coefficient by_4 except farms 8 and 14. Therefore it would appear that the rates of nitrogen application were only carried far enough on farms 8 and 14 to give a maximum yield response. The regression equations in Table 8 and the nitrogen response curves in Figure 7 show that farms 11 and 13 which are situated in close proximity on Black soils have a very similar nitrogen response curve (slopes) and show a large nitrogen response. Farms 8 and 14 situated on Black-Meadow soils have a very similar nitrogen response curve (slopes). This is in keeping with the results obtained with oats on the same genetic soil type.

(c) Predicted oat and barley yield response to applied phosphate

The phosphate response varied considerably among farms as indicated by the regression coefficients by_1 and by_3 shown in Table 8. Oats and

Figure 6: PREDICTED OAT YIELD RESPONSE CURVES WITH VARYING RATES OF NITROGEN AND A CONSTANT RATE OF PHOSPHATE AT 20 POUNDS PER ACRE ON FARMS 1 TO 7

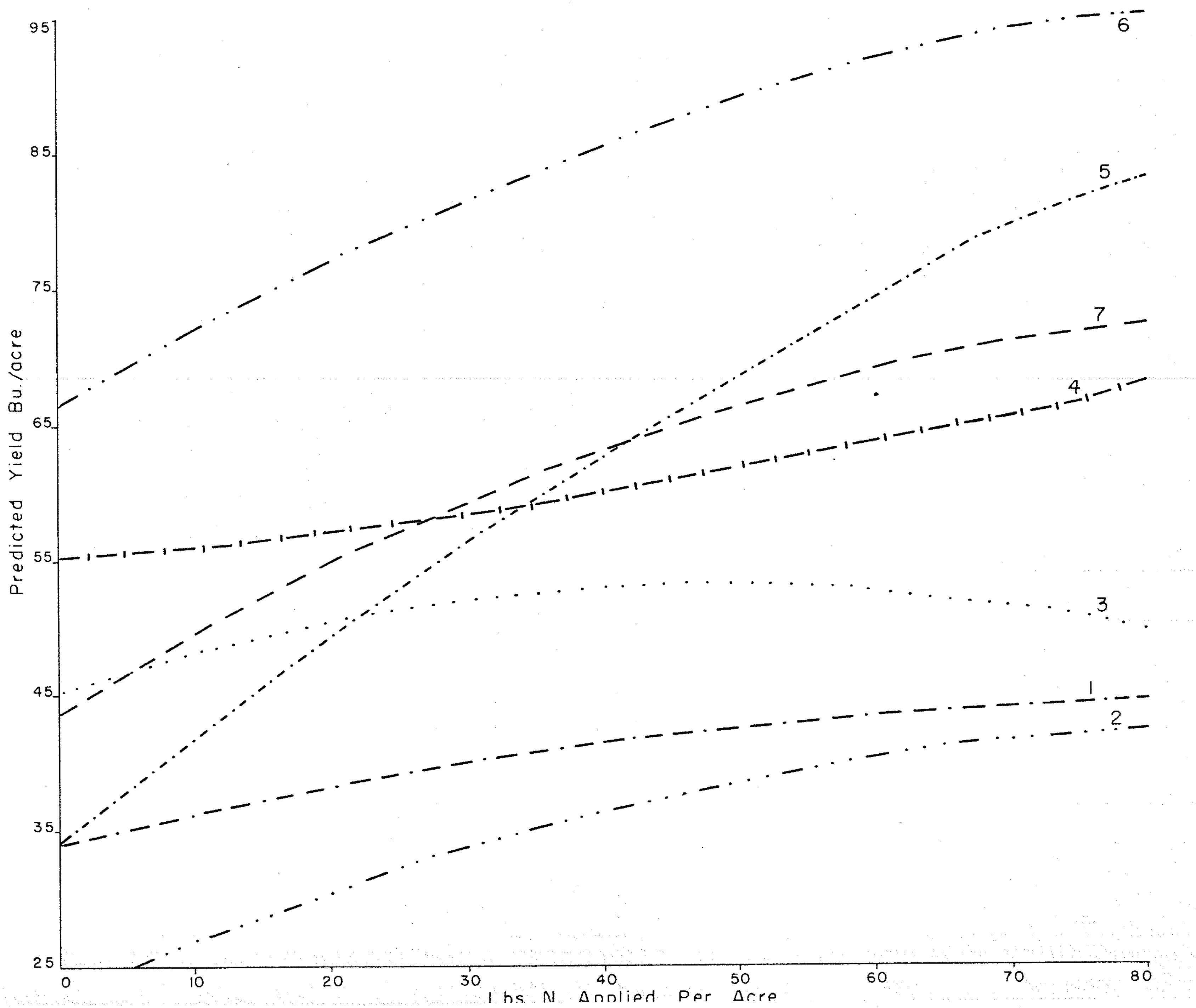
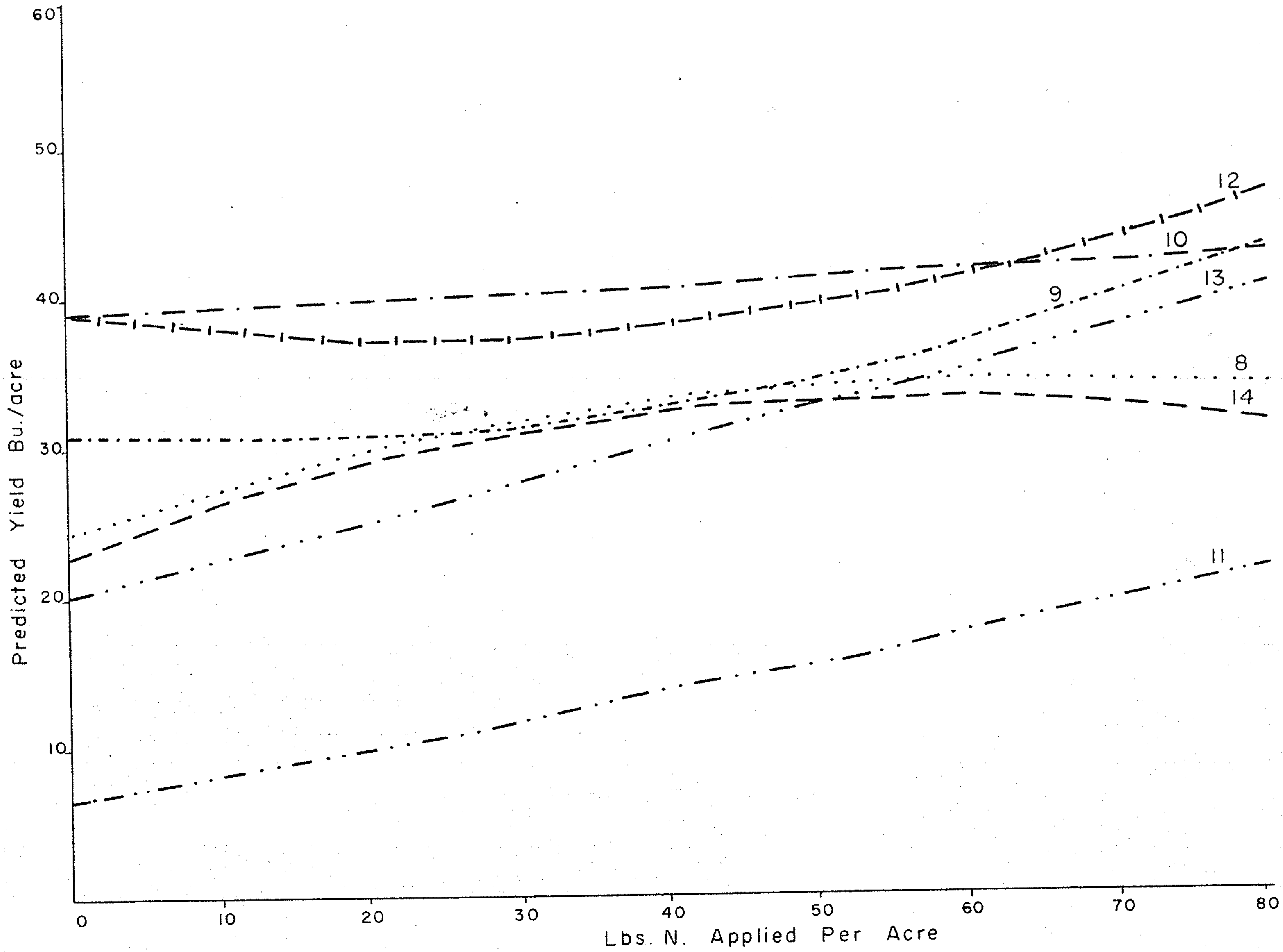


Figure 7: PREDICTED BARLEY YIELD RESPONSE CURVES WITH VARYING RATES OF NITROGEN AND A CONSTANT RATE OF 20 POUNDS PHOSPHATE PER ACRE ON FARMS 8 TO 14

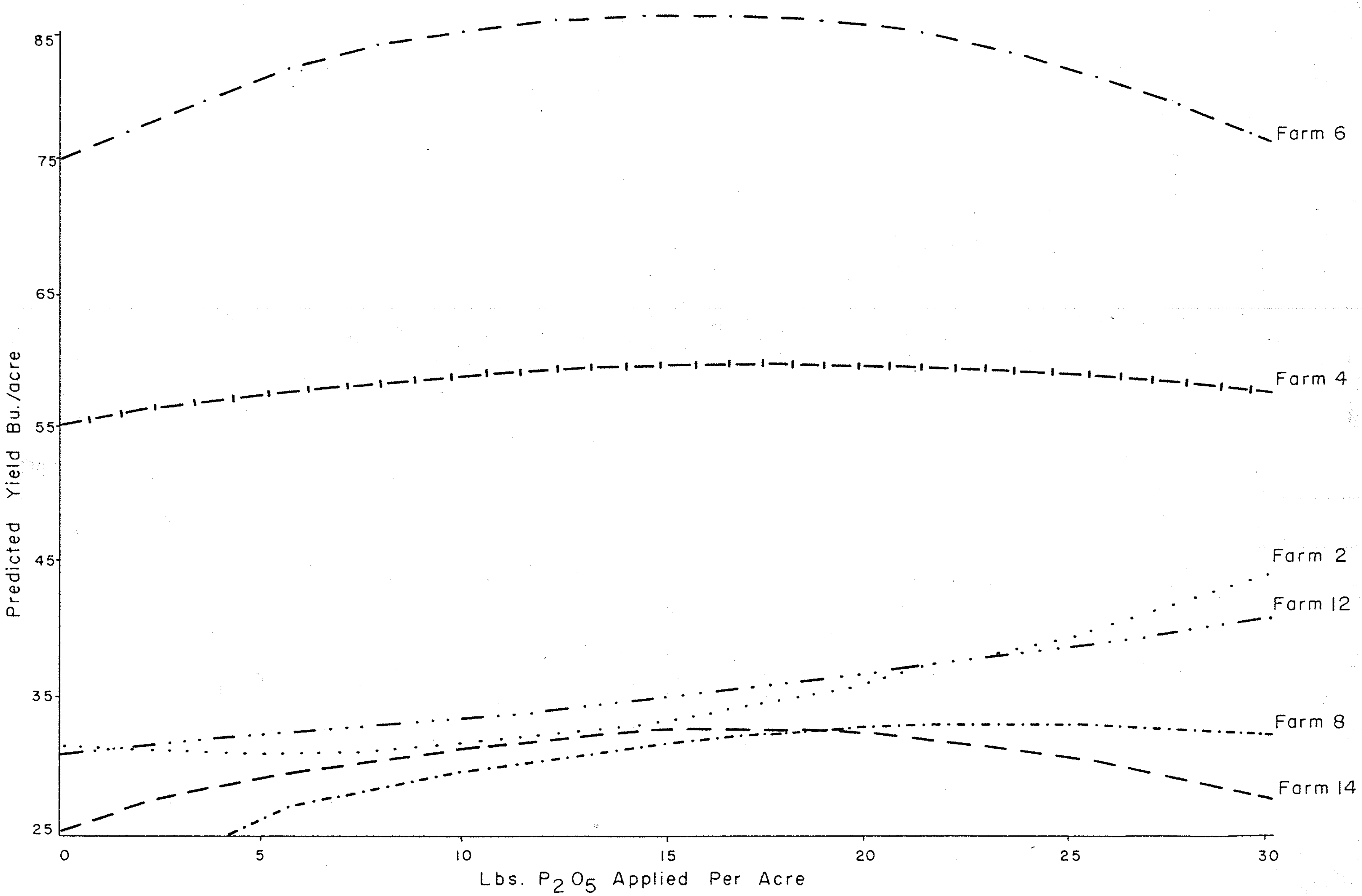


barley regression coefficients by_1 and by_3 will be discussed together due to the fact that there appeared to be little difference in the phosphate response pattern of the two species.

Only three farms exhibited significant positive regression coefficients by_1 and these three farms also had significant negative regression coefficients by_3 which show that total diminishing yields had occurred with the phosphate application. The only other farm that had a significant regression coefficient by_3 was Farm 2 and this coefficient was positive, due to the high phosphate requirement of this poorly drained clay soil.

The phosphate response was not graphed for all farms but it was found by studying Table 8 that the phosphate response curves could be divided into two groups, i.e. those on which diminishing total yield had occurred and those on which this was not the case. On farms 1, 2, 7, 5, 10 and 12, the maximum response had not been reached with the 30 pound application of phosphate and the response curves (slopes) were similar. Farm 12 did not show the initial yield reduction that was exhibited by Farm 2 in Figure 8. The other four farms show similar yield response curves (slopes) to Farm 2. Farms 2 and 7 were situated on Black-Meadow soils and the other four farms were situated on clay texture soils as indicated in Table 1. The former two farms had a high water table and the latter four were poorly drained therefore, it would be expected that calcium salts would be at or near the surface. These soils would be expected to show response with increasing phosphate application due to the tie-up of phosphate by the calcium.

Figure 8: PREDICTED YIELD RESPONSE CURVES WITH VARYING RATES OF PHOSPHATE AND CONSTANT RATE OF NITROGEN AT 40 POUNDS PER ACRE ON 6 FARMS



The remaining eight farms all showed diminishing total yield. Farms 3, 4, 9 and 11 showed very little response to phosphate while farms 13 and 14 exhibited intermediate response. Farms 6 and 8 showed the greatest response to phosphate fertilizer as shown by their response curves. Farms 6 and 8 were situated on calcareous Black-Meadow and Black-Meadow soils respectively which showed slight to moderate erosion. Soils that were moderate to well drained did not show high response to phosphate fertilizer. Therefore, the phosphate response was greatest where due to poor drainage, high water table or erosion, calcium carbonate and possibly other calcium salts were at or near the surface. The calculations for the farms shown in figures 6, 7 and 8 are shown in (iv), (v) and(vi).

Discussion of predicted yield response curves with the Cobb-Douglas function and Quadratic function including interaction

One farm was chosen to discuss the methods and principles involved in estimating economic fertilizer rates and nutrient combinations. The use of more than one farm in this section would complicate the discussion. Farm 8 was chosen because it was representative of Black-Meadow genetic soil type and four other farms of this same genetic soil type were included in the project. This farm also exhibited average response to the two fertilizer nutrients tested.

(a) Predicted yield response curves with the Cobb-Douglas function

Figure 9 shows the yield response curves predicted by the Cobb-Douglas function on Farm 8 with varying rates of nitrogen while phosphate was held constant and Figure 10 shows the yield response curves predicted by the same function with varying rates of phosphate while nitrogen was held constant. The tables showing the calculated values for these figures are included in Appendices (vii) and (viii).

Figures 9 and 10 show a definite yield pattern. While no specific interaction regression coefficient was used in the Cobb-Douglas function, interaction was evidenced by the increasing divergence between the curves as nitrogen was increased from 1 pound to 80 pounds. In Figure 9, an example of this interaction was noted by the yield difference between the yield curves where phosphate was held constant at 5 and 10 pounds per acre. The yield difference at 1 pound of nitrogen per acre was 1.83 bushels per acre while at 80 pounds of nitrogen per acre the yield difference was 2.52 bushels per acre. Therefore, the interaction accounted for approximately 0.7 bushels per acre.

In both figures 9 and 10, diminishing returns were taking place as the rates of application were increased as shown by the narrowing of the spread between the constant rate curves. In Figure 9, at the 40 pounds of nitrogen per acre the spread A to B between the curves shown by the constant rate of 5 and 10 pounds of phosphate per acre was 2.40 bushels per acre while the spread C to D between the curves shown by the constant rates of 25 and 30 pounds of phosphate per acre was 0.75 bushels per acre.

The Cobb-Douglas function does not permit diminishing total yield. It would appear from this function that if fertilizer rates were continually increased, the yields would continue to increase at a diminishing

Figure 9: PREDICTED YIELD RESPONSE CURVES WITH VARYING RATES OF NITROGEN AND CONSTANT RATES OF PHOSPHATE ON FARM 8 (Cobb—Douglas function)

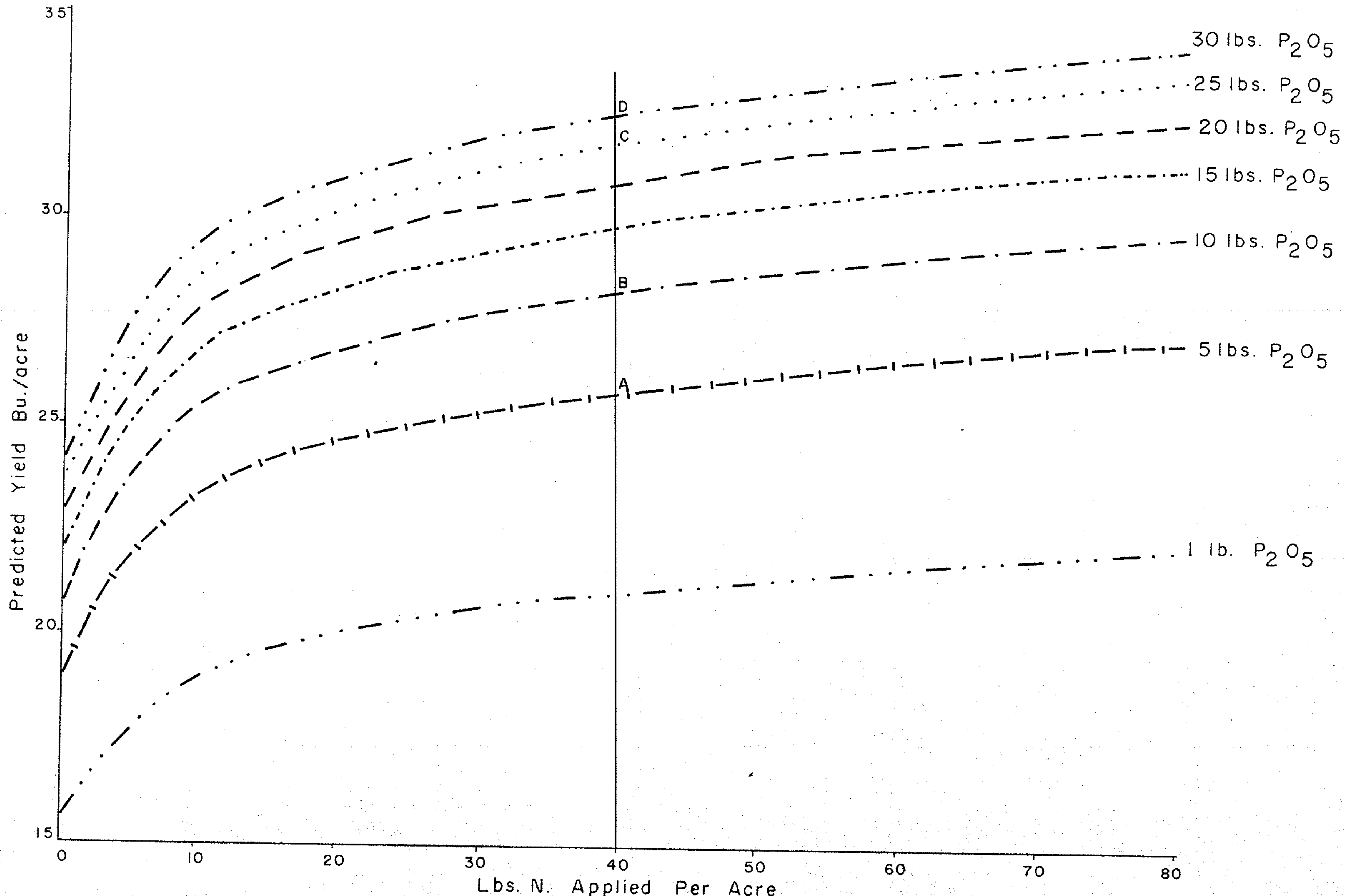
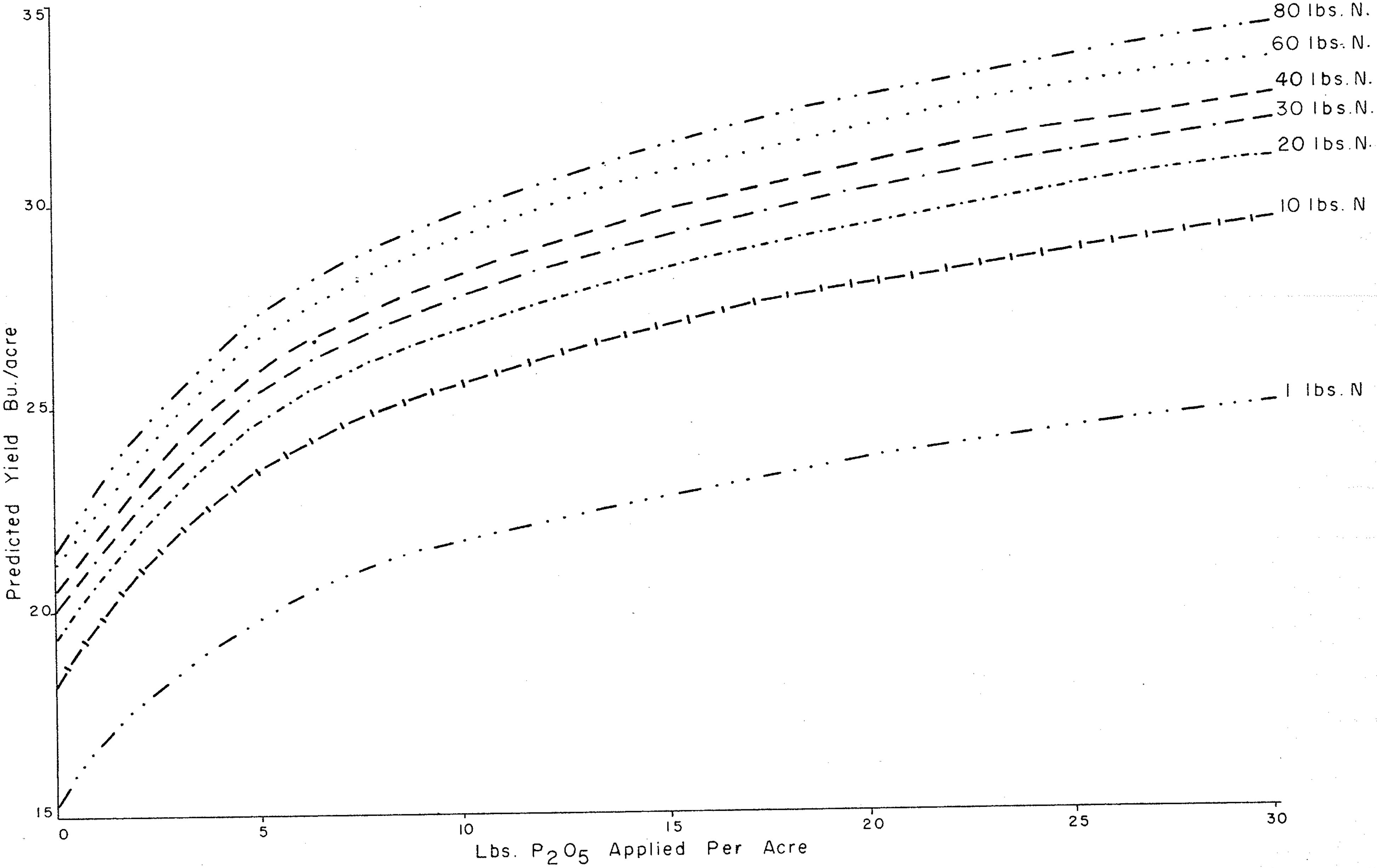


Figure 10: PREDICTED YIELD RESPONSE CURVES WITH VARYING RATES OF PHOSPHATE AND CONSTANT RATES OF NITROGEN ON FARM 8 (Cobb—Douglas function)



rate. The actual yield data showed that this was not the case.

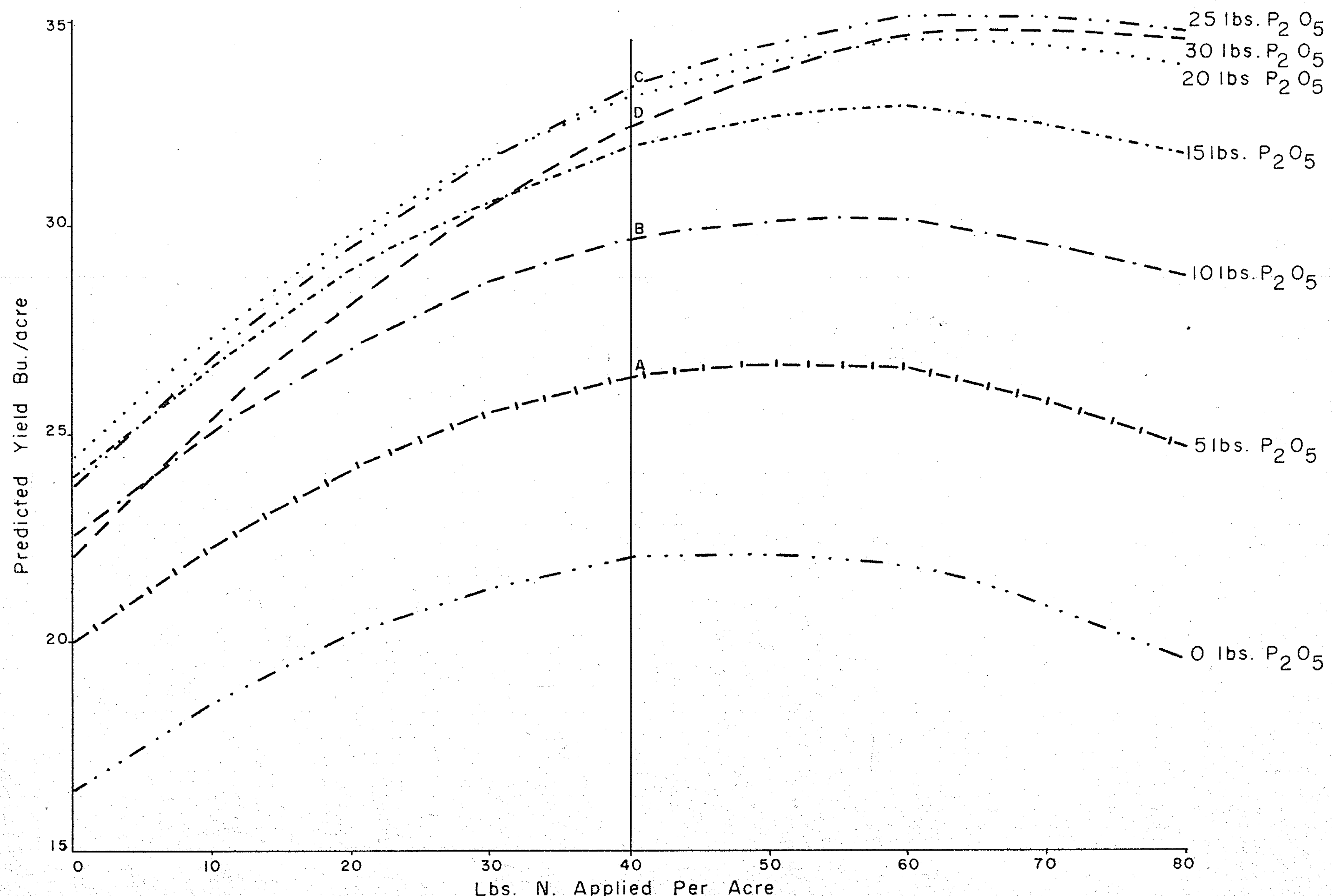
(b) Predicted yield response curves with the Quadratic function including interaction

Figure 11 shows the yield response curves predicted by the Quadratic function including interaction on Farm 8 with varying rates of nitrogen while phosphate was held constant and Figure 12 shows the yield response curves predicted by the same function with varying rates of phosphate while nitrogen was held constant. The tables showing the calculated values for these figures are included in Appendices (ix) and (x).

The predicted yield data pattern shown in figures 11 and 12 was considerably different from that in figures 9 and 10. The interaction of nutrients was very evident especially at the higher rates of fertilizer. In Figure 11 the interaction was shown by the greater divergence between the yield curves as the nitrogen was increased from 0 to 80 pounds per acre. The same example will be chosen as was used with the Cobb-Douglas function, i.e. yield difference between the yield response curves where phosphate was held constant at 5 and 10 pounds per acre. The yield difference at 0 pounds of nitrogen per acre was 2.49 bushels per acre, while at 80 pounds of nitrogen per acre the yield difference was 4.05 bushels per acre. Therefore, the interaction accounted for approximately 1.5 bushels per acre.

The Quadratic function including interaction shows that one element alone or the two elements in improper balance reduce the yield. As the two elements combine in more balanced ratio the yield increased due to the interaction of the elements. This was shown by the convergence

Figure 11: PREDICTED YIELD RESPONSE CURVES WITH VARYING RATES OF NITROGEN AND CONSTANT RATES OF PHOSPHATE ON FARM 8 (Quadratic function)



of the curves exhibited by the 25 and 30 pounds of phosphate per acre in Figure 11 and the 60 and 80 pounds of nitrogen per acre in Figure 12. Diminishing returns were taking place as the rates of application were increased as shown by the narrowing of the spread between the constant rate curves. In Figure 11 at the 40 pounds of nitrogen per acre, the spread A to B between the curves shown by the constant rates of 5 and 10 pounds of phosphate per acre was 3.27 bushels per acre while the spread D to C between the curves shown by the constant rates of 25 and 30 pounds of phosphate per acre was 0.88 bushels per acre but at this point the 25 pounds of phosphate per acre produced the higher yield.

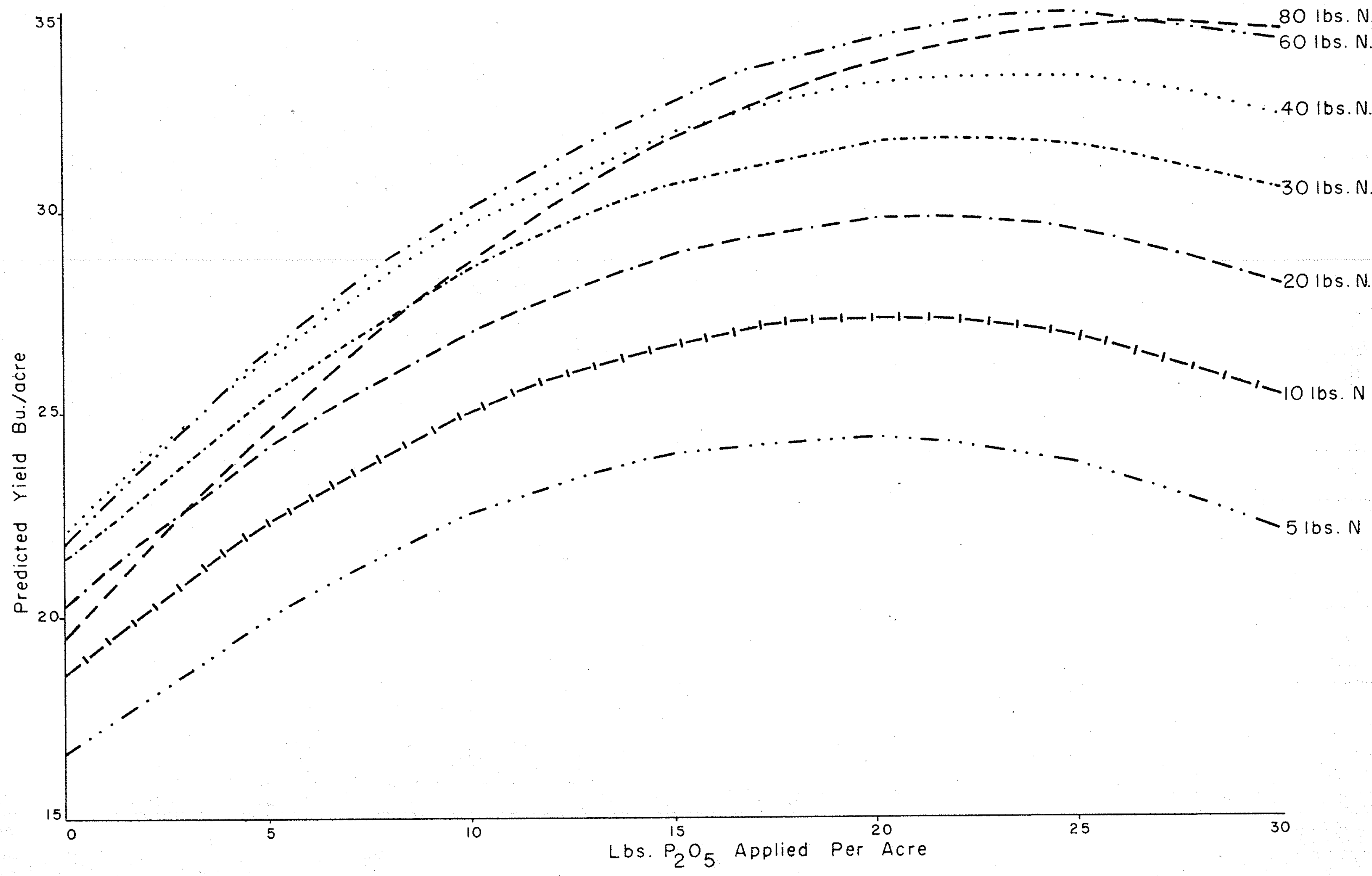
The Quadratic function indicated diminishing total yield as shown by all the curves in figures 11 and 12.

Therefore, the Quadratic function fits the actual data better than the Cobb-Douglas function. The coefficient of determination for the Quadratic function with interaction was 0.9201 while that for the Cobb-Douglas function was 0.8874. Therefore, greater confidence could be placed on yield predictions made from the Quadratic equation including interaction.

After comparing the response curves and the coefficients of determination of the two functions, the Quadratic function including interaction was chosen to determine the economic optima of fertilizer applications.

In order to prove that the Quadratic function or equation did fit the data, Figure 13 was made in which the mean yields of the actual square

Figure 12: PREDICTED YIELD RESPONSE CURVES WITH VARYING RATES OF PHOPHATE AND CONSTANT RATES OF NITROGEN ON FARM 8 (Quadratic function)



sample were compared with the predicted yield curve with the same quantity of fertilizer nutrients. The graph showed that the predicted yield curve fits very closely to the actual mean yields of the harvested replicates.

Yield isoquants and their significance

The method used in analysing the yield data was patterned after the work outlined by Heady et al (10). A yield isoquant is a contour line which indicates all of the possible combinations of the two fertility elements which will produce a given yield. In Figure 14, six yield isoquants are shown and the numbers on each isoquant represents the yield level. The yield isoquant equation was derived from the following regression equation:

$$(1) \quad Y = a + by_1P + by_2N + by_3P^2 + by_4N^2 + by_5NP$$

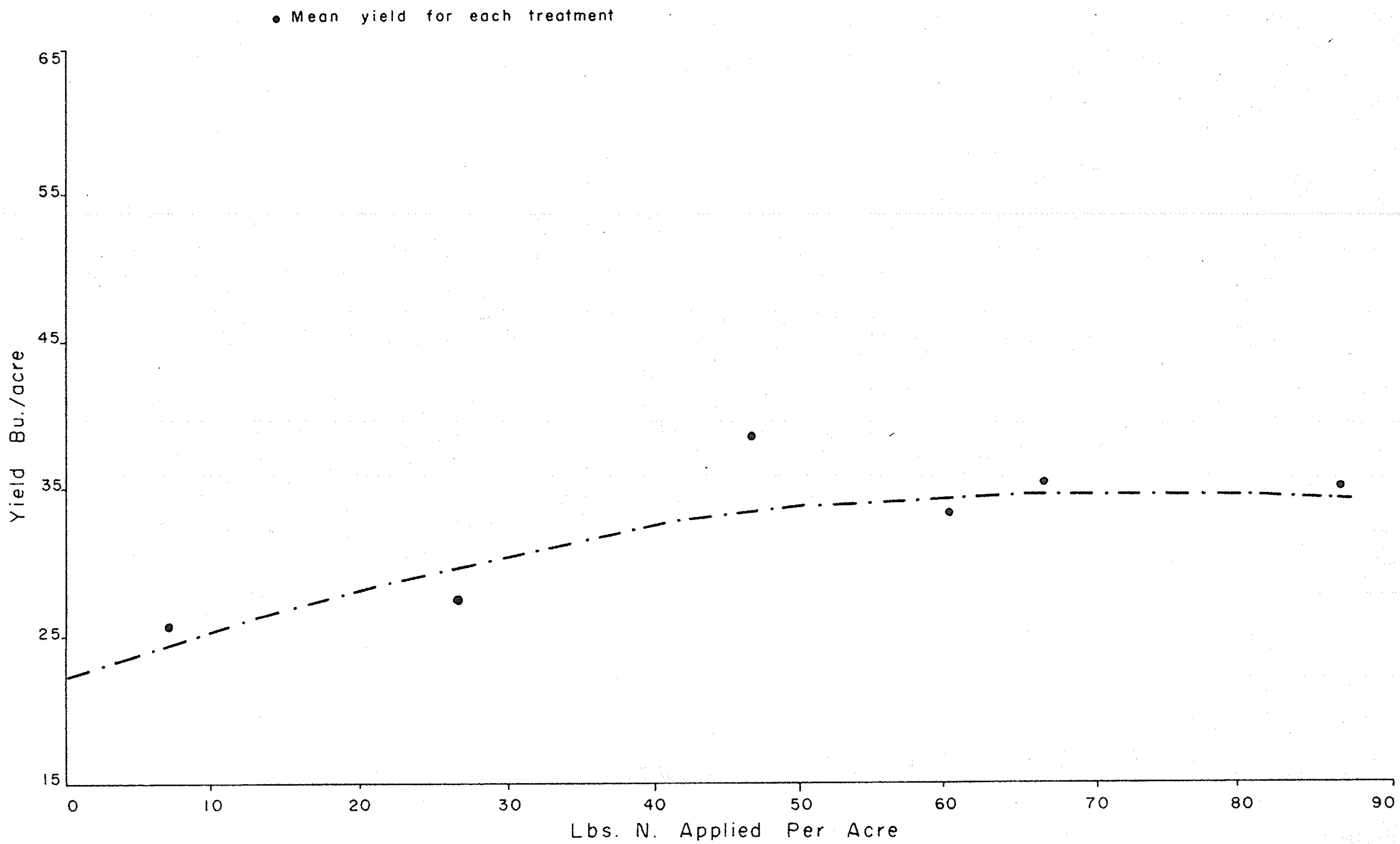
The isoquant equation was derived so that the amount of nitrogen required to combine with a given amount of phosphate to produce a specified yield could be calculated.

The following is the isoquant equation:

$$(2) \quad N = \frac{-(by_2 + by_5P) \pm \sqrt{(by_2^2 + 2by_2by_5P + by_5^2P^2) - 4by_4(a + by_1P + by_3P^2 - Y)}}{2\ by_4}$$

The regression coefficient (by) values for Farm 8 given in Table 8 along with the various yield isoquant values (Y) and varying rates of phosphate (P) were inserted in equation (2) to predict the corresponding rates of nitrogen required. The rate of application of nitrogen required

Figure 13: ACTUAL YIELD DATA FROM VARYING RATES OF NITROGEN AND A CONSTANT RATE OF PHOSPHATE AT 28.8 POUNDS PER ACRE COMPARED WITH THE PREDICTED YIELD CURVE HAVING THE SAME QUANTITY OF FERTILIZER NUTRIENTS



with a given application of phosphate to produce a given yield is shown in Table 9. The negative sign in front of the square root was disregarded because it did not apply in this case. Table 9 shows that a 25 bushel per acre yield could be obtained by the following combination of nitrogen and phosphate: 364 pounds of nitrogen per acre and 18 pounds of phosphate per acre or 36.84 pounds of nitrogen per acre and 3.5 pounds of phosphate per acre.

Several facts are noted by studying Figure 14. Diminishing returns were noted by the fact that isoquant lines representing equal increments of yield (20, 22.5, 25 etc.) move farther apart along any straight line through the origin e.g. line OX showing that increasingly larger quantities of a fixed fertilizer mixture are necessary to attain equal increments in crop yield. Line OX has a fixed nutrient N/P_2O_5 ratio of 2.5. The isoquants also show that as higher yields are attained, the marginal rates of substitution between phosphate and nitrogen changed along the fixed nutrient ratio line, e.g. line OX. In other words, the slopes of successively higher isoquants are different at the points where they are intersected by a straight line through the origin. This change in the slopes of the yield isoquants indicates that the combinations of nutrients which gave the lowest cost for one yield level are not the same as for a higher yield level. This will be discussed in the next section. The least cost combination would not be the same for yields of 25 and 32.5 bushels per acre. This is shown in Table 10.

The graph shows that the 20 bushel yield could be attained by the addition of only one nutrient either nitrogen or phosphate while the

Table 9 : Isoquant Combinations Of Nutrients For Producing Specified Yields And Corresponding Marginal Rates Of Substitution On Farm 8

Nitrogen lbs/acre	P_2O_5 lbs/acre	Marginal rate of substitution showing lbs. of nitrogen replaced by 1 lb. P_2O_5	Nitrogen lbs/acre	P_2O_5 lbs/acre	Marginal rate of substitution showing lbs. of nitrogen replaced by 1 lb. P_2O_5
	20 - bushel yield			27.5 - bushel yield	
0.06	5	-2.34	12.86	20	-0.12
2.52	4	-2.71	13.00	18	-0.46
5.44	3	-3.17	14.72	15	-1.09
8.90	2	-3.17	18.72	12	-2.00
18.32	0	-5.99	23.34	10	-3.02
			30.78	8	-5.15
			37.00	7	-8.18
	22.5 - bushel yield			30 - bushel yield	
0.40	10	-1.44	23.98	20	-0.37
3.62	8	-1.95	24.62	18	-0.85
10.88	5	-3.16	27.80	15	-1.87
24.12	2	-6.52	35.32	12	-4.12
32.44	1	-11.12	48.94	10	-17.66
	25 - bushel yield			32.5-bushel yield	
3.64	18	-0.26	39.32	22	-0.39
4.68	15	-0.75	39.88	20	-1.15
7.38	12	-1.37	42.22	18	-2.32
10.42	10	-1.92	56.66	15	-28.96
14.78	8	-2.73			
25.92	5	-5.47			
36.84	3.5	-11.79			

Figure 14: YIELD ISOQUANTS SHOWING ALL POSSIBLE NUTRIENT COMBINATIONS IN PRODUCING SPECIFIED YIELDS AND YIELD ISOCLINES FOR VARIOUS FERTILIZER NUTRIENT PRICE RELATIONSHIPS ON FARM 8

Pp = Price per lb. of P_2O_5 in cents
Np = Price per lb. of N. in cents

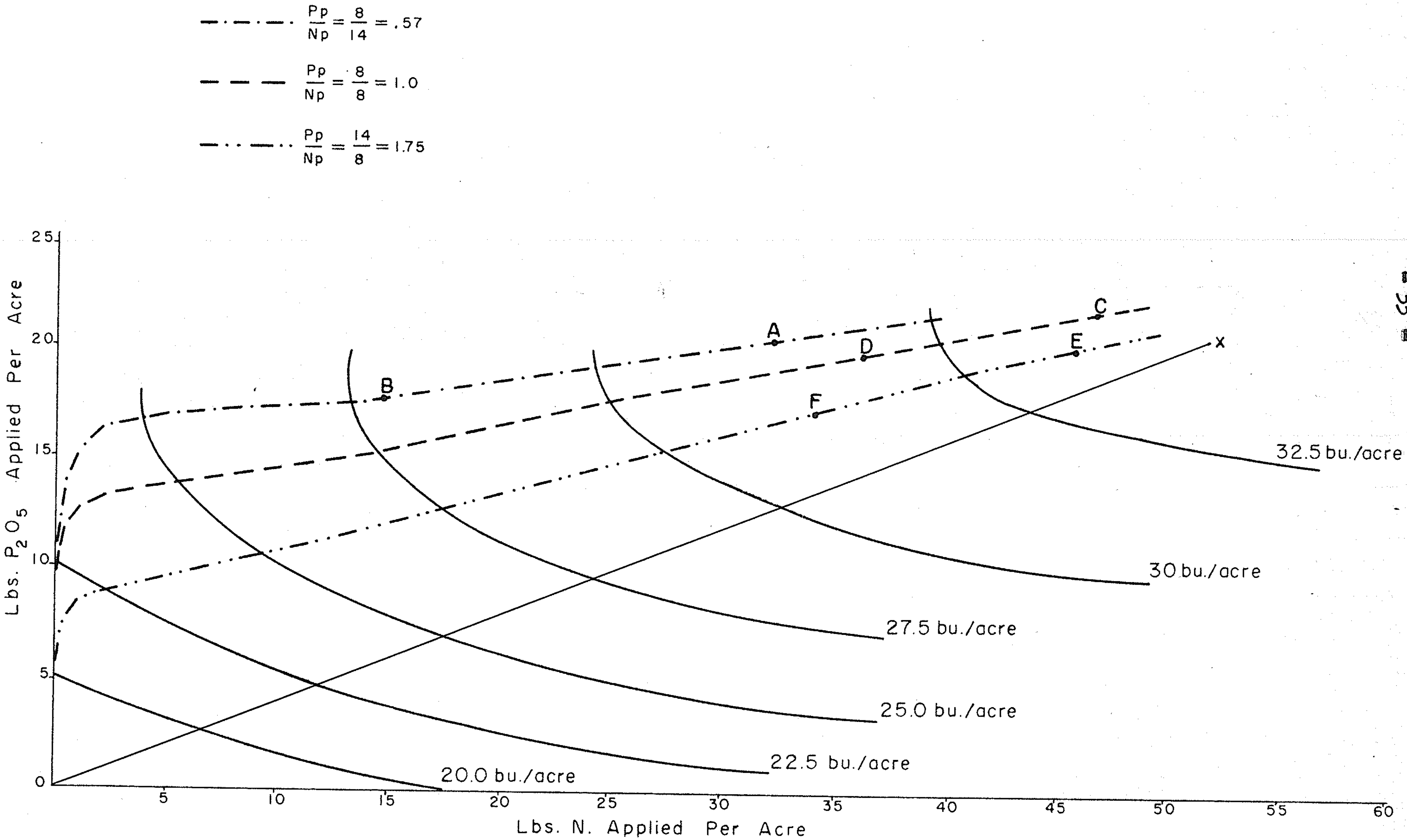


Table 10: Combinations Of Nitrogen And P_2O_5 To Minimize Fertilizer Costs Per Specified Yield Level For Different Price Ratios On Farm 8

Yield level	Total lbs/acre	Optimum lbs. nitrogen/acre	Optimum lbs. P_2O_5/acre	N/P_2O_5 ratio
		Price of 0.14 per lb. for nitrogen and 0.08 per lb. for P_2O_5 ($\frac{Pp}{Np}$ = ratio of .57)		
25.0 bus.	20.62	3.88	16.74	.23
27.5 bus.	30.63	13.14	17.49	.75
30.0 bus.	43.30	24.14	19.16	1.26
32.5 bus.	60.86	39.34	21.52	1.83
		Price of 0.08 per lb. for nitrogen and 0.08 per lb. for P_2O_5 ($\frac{Pp}{Np}$ = ratio of 1.0)		
25.0 bus.	19.32	5.52	13.80	.40
27.5 bus.	29.74	14.32	15.42	.93
30.0 bus.	42.45	24.90	17.55	1.42
32.5 bus.	60.02	39.62	20.40	1.93
		Price of 0.08 per lb. for nitrogen and 0.14 per lb. for P_2O_5 ($\frac{Pp}{Np}$ = ratio of 1.75)		
22.5 bus.	11.02	2.24	8.78	.25
25.0 bus.	19.94	9.32	10.62	.88
27.5 bus.	30.14	17.30	12.84	1.35
30.0 bus.	42.64	27.34	15.30	1.79
32.5 bus.	59.74	40.76	18.98	2.14

22.5 bushel yield can be reached by using phosphate only but no amount of nitrogen could produce this yield without the addition of at least 1 pound of phosphate. The other four yields in the figure can be obtained only with some minimum quantity of either nutrient. The ends of the yield isoquants give the limits in nutrient substitution. The graph shows that the yield isoquants become shorter with the higher yields indicating the range of nutrient substitution is much reduced. According to Heady et al (10) the maximum yield will only be a point and there can be no substitution of nitrogen and phosphate because only one combination of nutrients could produce this yield.

Derivation and nature of isoclines for Farm 8

In order that the economic optima of fertilizer application could be calculated, it was first necessary to calculate the marginal rate of substitution which shows the number of pounds of nitrogen replaced by one pound of phosphate. The marginal rates of substitution are shown in Table 9. The equation used to calculate the marginal substitution rates was the derivative of the regression equation.

The derivative equation is as follows:

$$\frac{dN}{dP} = - \frac{by_1 + 2\ by_3\ P + by_5\ N}{by_2 + 2\ by_4\ N + by_5\ P}$$

The phosphate and nitrogen application rates in Table 9 were inserted in the equation along with the regression coefficient values. The resulting negative figures are the substitution or replacement rates for these nutrient combinations. The term $\frac{dN}{dP}$ refers to the slope of the isoquant curve at each specific nutrient combination. To explain the derivative, an example will be used. Referring to Table 9, with 12 pounds of phosphate and 18.72 pounds of nitrogen, a yield of 27.5 bushels per acre could be produced. The derivative at this exact point is -2.00 or the slope of the isoquant curve is -2.00. This means that at this point one pound of phosphate can replace or substitute for two pounds of nitrogen in producing a yield of 27.5 bushels per acre.

Figure 14 shows three isoclines for barley. An isocline is a line indicating points of equal slope on successive yield isoquants. In other words, it really denotes points on all yield isoquants where one pound of phosphate will substitute for a constant quantity of nitrogen.

Economic optima for fertilizer application

This section is divided in two sections: First, the least cost combination of nutrients takes into consideration only the cost of fertilizer nutrient combinations; Second, the most profitable application of nutrients takes into account both the cost of fertilizer nutrient combinations and the price of the crop produced.

(a) Least cost combinations of nutrients

Table 10 shows the figures which make up the isoclines and represent the combinations of fertilizer nutrients that minimize the fertilizer costs per specified yield level for different price ratios. The most economic combination or least cost combination of nutrients occurred at the point on the isoquant where the marginal rate of substitution is equal to the inverse price ratio of nutrients. If phosphate is half the price of nitrogen, it would be logical to expect that the point to choose for the most economic combination of nutrients will be where two pounds of phosphate replace one pound of nitrogen. The isocline $\frac{Pp}{Np} = \frac{.08}{.14} = .57$ shows all points in the nutrient plane where 1 pound of nitrogen will replace .57 pounds of phosphate. Pp is the price per pound of phosphate and Np is the price per pound of nitrogen. The above is the actual present price situation of the two nutrients. If the price changed as in the situation $\frac{Pp}{Np} = \frac{.08}{.08} = 1.0$, the line so marked on Figure 14 will indicate all points where one pound of nitrogen will replace one pound of phosphate. The 1.75 isocline indicates all points where one pound of nitrogen will replace 1.75 pounds of phosphate. Table 13 verifies that the least cost combination of nutrients for a specific nutrient price ratio occurs at the point where the isocline intersects a particular yield isoquant. An example is shown in Table 13 when phosphate is $0.08 per pound and nitrogen is $0.14 per pound, the least cost application was $3.24. This was a combination of 13.14 pounds of nitrogen and 17.49 pounds of phosphate and it is noted in Figure 14 that this is the exact point where the 0.57 isocline intersects the 27.5 bushel yield isoquant. The other example in Table 13 also verifies the same point.

The practical implications of this section are very important. For profitable returns from fertilizer, the fertilizer mixture ratios must be altered with changes in soil conditions, changes in the cost of the individual nutrients as well as changes in the yield for which the farmer is aiming. The fertilizer mixture ratio refers to the ratio of nitrogen to phosphate in the fertilizer. For example, ammonium phosphate (11-48-0) has a nitrogen/phosphate mixture ratio of 0.23 and ammonium phosphate sulphate (16-20-0) has a nitrogen/phosphate mixture ratio of 0.80. Phosphate gives the greatest response at the lower yield levels but heavy applications of nitrogen are required to reach the high yield levels. This is very noticeable in Table 10 where the nitrogen/phosphate mixture ratio changes from 0.23 at the 25.0 bushel yield level to 1.83 at the 32.5 bushel yield when the nitrogen to phosphate inverse price ratio is 0.57. This same trend is noticeable with the other two price ratio situations. Another significant point in Table 10 is that the nitrogen/phosphate mixture ratio increased as the price of nitrogen is decreased in relation to the price of phosphate. The nitrogen/phosphate mixture ratio for 25 bushel per acre yield changes from 0.23 to 0.88 when the inverse price ratio of nitrogen to phosphate changes from 0.57 to 1.75 respectively. Table 10 also showed that approximately three times as much fertilizer is required to produce a 32.5 bushel per acre yield as a 25.0 bushel yield. The total pounds of fertilizer required to produce the same specified yield with the different nutrient price situations is nearly equal, eg. the total fertilizer to produce 27.5 bushels per acre in the 0.57, 1.0 and 1.75 inverse price ratio group are 30.63, 29.74 and 30.14 pounds per acre respectively.

(b) Most profitable application of nutrients

The quantities of fertilizer derived in Table 10 provide the basis for specifying the optimum nutrient combination for any yield level and the optimum rate of fertilizer application. This section specifies the quantities under various price ratios for a farmer who has unlimited capital. This section also outlines a method to simultaneously determine the optimum combination of nutrients and the optimum level of application. The exact fertilizer combination can be solved by setting the partial derivatives for both nutrients equal to their respective nutrient barley price ratios and solving simultaneously for the quantity of the nutrients to apply for maximum profits. The partial derivatives from the regression equation are as follows for nitrogen and phosphate respectively:

$$\frac{\delta Y}{\delta N} = by_2 + 2\ by_4\ N + by_5\ P$$

$$\frac{\delta Y}{\delta P} = by_1 + 2\ by_3\ P + by_5\ N$$

The partial derivative considering nitrogen holds the phosphate application constant and the partial derivative considering phosphate holds nitrogen constant. In this way when the two equations are solved simultaneously the optimum quantity of both nutrients in combination are determined.

The actual equations used in the simultaneous solution of the equations are as follows:

$$\frac{\delta Y}{\delta N} = 0.2389 + 2(-0.0025)N + 0.0039\ P = \frac{Np}{Bp}$$

$$\frac{\delta Y}{\delta P} = 0.8114 + 2(-0.0208)P + 0.0039\ N = \frac{Pp}{Bp}$$

Np = price nitrogen per pound

Pp = price phosphate per pound

Bp = price barley per bushel

The simultaneous solutions of these equations are given in Table 11 for six different price situations listed as situations A to F inclusive. The optimum yields for price situations A and B are found on the 0.57 isocline while the optimum yields for price situations C and D are found on the 1.0 isocline and that of E and F are located on the 1.75 isocline. The optimum rates of application of nitrogen and phosphate determined by the simultaneous equations were inserted in the regression equation to determine the optimum yields reported in Table 11.

With price situation A, the simultaneous solution of these equations shows that 53.02 pounds of fertilizer should be used including 32.60 pounds of nitrogen and 20.42 pounds of phosphate. The changes in fertilizer applications with the various price situations are shown in Table 11. When the price of barley decreased from 90 cents per bushel to 60 cents per bushel, the optimum yield moved from 32.08 bushels to 28.35 along the 0.57 isocline, in other words with the 33 percent drop in barley prices the optimum yield to aim for was reduced. By studying price situations A and B when the barley price declined 33 percent, the total usuage of fertilizer should decline by 38 percent. The input of nitrogen should decrease by 54 percent and the input of phosphate should decline by only 13 percent. A basic amount of phosphate was required and, being the cheaper element, phosphate would not be reduced as much as nitrogen. The decline of 33 percent in barley price from situation C to D shows that there should only be an 18 percent reduction in the total fertilizer usage. The input of nitrogen should be reduced by 22

Table 11: Optimum Quantities Of Fertilizer And Optimum Combinations Of Nutrients For Specific Price Relationships On Farm 8

Price situation	Optimum yield (bus.)	Optimum Fertilizer Use Per Acre			
		Total lbs.	lbs/N	lbs/P_2O_5	N/P_2O_5 ratio
A barley @ $.90 N @ .14 P_2O_5 @ .08	32.08	53.02	32.60	20.42	1.60
B barley @ $.60 N @ .14 P_2O_5 @ .08	28.35	32.62	14.92	17.70	.84
C barley @ $.90 N @ .08 P_2O_5 @ .08	34.01	68.75	46.98	21.77	2.16
D barley @ $.60 N @ .08 P_2O_5 @ .08	32.57	56.22	36.50	19.72	1.85
E barley @ $.90 N @ .08 P_2O_5 @ .14	33.63	65.68	45.64	20.04	2.28
F barley @ $.60 N @ .08 P_2O_5 @ .14	31.83	51.60	34.48	17.12	2.01

percent and the input of phosphate should decline 9 percent. With the reduction in the cost of nitrogen and thus total cost of fertilizer, the optimum yield did not need to be reduced as much between situation C and D as it needed to be between situations A and B. The fertilizer input also did not need to be reduced as much in the case of total pounds or in the case of each individual element. The ratio of nitrogen/phosphate decreased with the decrease in the price of barley but only at price situation B was the ratio less than 1.0 or, in other words, was the phosphate application greater than the nitrogen application. The nitrogen/phosphate ratio increased as the nitrogen became cheaper and phosphate more expensive as shown by the change in the ratio from 1.60 in price situation A to 2.28 in price situation E. The nitrogen application varied a great deal more than the phosphate application because the crop appeared to have a basic phosphate requirement and nitrogen application appeared to be the nutrient required in large amounts to induce the high yields. The price situation A represents the present price situation of phosphate, nitrogen and barley while price situation B represents the present price situation of phosphate and nitrogen but the price of barley was reduced by 33 percent to 60 cents per bushel.

Table 12 on economic production levels was included to show that the information included in Table 11 could be worked out by finding the point where marginal cost equals marginal revenue. In Table 12 the nitrogen applications were varied while phosphate applications were held constant. The most economic production is where the value of the production of the last increment of fertilizer is equal to the cost of the last increment of fertilizer. By studying this economic production table, the most economic production levels occur with each constant rate of phosphate application or in other words there are seven combinations of fertilizer that can be considered most economic. When barley is worth 90 cents per bushel an economic yield is between 31.63 and 33.05 bushels per acre and the nitrogen application lies between 30 and 40 pounds per acre in combination with 20 pounds of phosphate. This corresponds very closely to the fertilizer combination and yield in price situation A in Table 11. When barley is worth 60 cents per bushel an economic yield is between the 26.67 and 28.90 bushel per acre and the nitrogen application lies between 10 and 20 pounds per acre in combination with 15 pounds of phosphate. This corresponds very closely to the fertilizer combination and yield in price situation B in Table 11. The method using the partial derivatives in simultaneous equations is a more direct method and lends itself to price changes of both nutrients and crops without calculating tables such as the table of Economic Production levels.

Table 13 shows that the least cost combinations of fertilizer nutrients for a specified yield verify the results in Table 10. When phosphate and nitrogen are 8 and 14 cents per pound respectively, a combination of 13.14 and 17.49 pounds per acre of nitrogen and phosphate respectively is the least costly. When phosphate and nitrogen are 14 and 8 cents per pound respectively, a combination of 17.30 and 12.84 pounds per acre of nitrogen and phosphate respectively is the least cost combination.

Table 12: Economic Production Levels On Farm 8
(Cost Of Nitrogen = $0.14 per lb.)

Nitrogen lbs/acre	Additional Nitrogen lbs/acre	P_2O_5 lbs/acre	Total Yield bus/acre	Additional Yield bus/acre	Marginal Cost	Marginal Revenue @ .90/bus	@ .60/bus
0		0	16.46				
10	10		18.60	2.14	$1.40	$1.93	$1.28
20	10		20.24	1.64	1.40	1.47	.98
30	10		21.38	1.14	1.40	1.03	.68
40	10		22.02	0.64	1.40	.58	.38
60	20		21.79		2.80		
80	20		19.57		2.80		
0		5	21.00				
10	10		23.33	2.33	1.40	2.10	1.40
20	10		25.17	1.84	1.40	1.66	1.10
30	10		26.50	1.33	1.40	1.20	.80
40	10		27.34	0.84	1.40	.76	.50
60	20		27.50	0.16	2.80	.14	.10
80	20		25.67		2.80		
0		10	22.49				
10	10		25.02	2.53	1.40	2.28	1.52
20	10		27.05	2.03	1.40	1.83	1.22
30	10		28.58	1.53	1.40	1.38	.92
40	10		29.61	1.03	1.40	.93	.62
60	20		30.16	0.55	2.80	.50	.33
80	20		28.72		2.80		
0		15	23.95				
10	10		26.67	2.72	1.40	2.45	1.63
20	10		28.90	2.23	1.40	2.01	1.34
30	10		30.62	1.72	1.40	1.54	1.03
40	10		31.85	1.23	1.40	1.11	.74
60	20		32.79	0.94	2.80	.85	.56
80	20		31.74		2.80		
0		20	24.37				
10	10		27.29	2.92	1.40	2.63	1.75
20	10		29.71	2.42	1.40	2.18	1.45
30	10		31.63	1.92	1.40	1.73	1.15
40	10		33.05	1.42	1.40	1.28	.85
60	20		34.38	1.33	2.80	1.20	.80
80	20		33.72		2.80		
0		25	23.74				
10	10		26.85	3.11	1.40	2.80	1.87
20	10		29.47	2.62	1.40	2.36	1.57
30	10		31.58	2.11	1.40	1.90	1.27
40	10		33.20	1.62	1.40	1.46	.97
60	20		34.92	1.72	2.80	1.55	1.03
80	20		34.65		2.80		
0		30	22.08				
10	10		25.39	3.31	1.40	2.98	1.98
20	10		28.20	2.91	1.40	2.62	1.74
30	10		30.51	2.31	1.40	2.08	1.39
40	10		32.32	1.81	1.40	1.63	1.09
60	20		34.43	2.11	2.80	1.90	1.27
80	20		34.55	.12	2.80	.11	.07

Table 13: Least Cost Combination Of Fertilizer Nutrients Required For A Specified Yield Determined By The Use Of Mathematics

27.5 Bushels Yield			
Nutrient Combinations		Cost of Fertilizer	
Nitrogen lbs/acre	P_2O_5 lbs/acre	P_2O_5 = \$0.08/lb. N = \$0.14/lb.	P_2O_5 = \$0.14/lb. N = \$0.08/lb.
12.86	20.00	\$3.40	\$3.83
13.00	18.00	3.26	3.56
13.14	17.49	<u>\$3.24</u>	3.50
14.72	15.00	3.26	3.28
17.30	12.84	3.45	<u>\$3.17</u>
18.72	12.00	3.58	3.18
23.34	10.00	4.07	3.27
30.78	8.00	4.95	3.58
37.00	7.00	5.74	3.94

Fertilizer rates cannot be recommended from the analysis carried out on Farm 8 but the main purpose of this analysis was to show that this method will work with Manitoba conditions and crops. Heady et al (10) stated that the practical application of this type of analysis is important due to the fact that the fertilizer recommendations need to vary with the capital level of the farmer as well as the soil. While the economic optima is for conditions of unlimited capital, the data derived is of the kind that is useful to farmers regardless of their capital position. The data are more useful to farmers with limited capital than for those with unlimited capital. The general recommendations can be more satisfactorily used for farmers aiming for high yields because the isoclines tend to converge at the high yields and the specifications of numerous possible nutrient combinations for yield of 32.5 bushels of barley gave somewhat similar costs because in Table 9 there is only a range of 16 pounds of nitrogen per acre and 6 pounds of phosphate per acre to produce this yield. However, for medium yields, the isoclines are farther apart and the isoquants have greater curvature. In Table 9, for a yield of 25 bushels per acre there is a range of 33 pounds of nitrogen per acre and 15 pounds of phosphate per acre. Therefore, choosing different combinations of nutrients to produce a medium specified yield could make a very great difference in the cost of fertilizer. Therefore, exact principles outlined in this method can give considerable gain over general recommendations which might lead to nutrient combinations near the end of the isoquants. Therefore, the optimum quantities of fertilizer and optimum combinations of nutrients as shown in Table 11 can be of greatest value to farmers with limited capital.

This section points out that economic fertilizer recommendations must be based on the soil conditions, price of nutrients and price of crops. Economic nutrient combinations and rates of application will change with changes in soil conditions and price fluctuations of both nutrients and crops.

LABORATORY INVESTIGATIONS

Surface soil samples were taken from the experimental fields during the summer for the laboratory investigations. The soil samples were taken to a depth of six inches at ten locations on each side of the fertilized strips. Two composite samples were made representing each side of the fertilized strips. These composite soil samples were air dried in the laboratory. The samples were then crushed to pass through a 20 mesh sieve and placed in glass jars. The available nitrogen was determined by two methods which are described below.

Incubation Method

The available nitrogen was determined by the incubation method outlined by Stanford and Hanway (15) with minor modifications. The method is described in detail. Plastic vials, 24 mm. in diameter and 95 mm. in length were used for incubation of the samples. A hole, 1/16 inch in diameter was drilled in the bottom of each vial, and a hole 1/8 inch in diameter in each metal cap to permit aeration of the soil during incubation. Pyrex wool pads approximately 5 mm. in thickness were placed in the bottom of the vial to cover the hole. About 1/2 inch of plaster grade vermiculite was placed on top of the pyrex wool and tapped down gently. Ten grams of air dry soil, mixed with an approximately equal volume of vermiculite was placed in the vial and tapped gently. The sample was then leached with 20 ml. of a Krilium 6 solution and allowed to stand for at least 15 minutes before applying suction to complete this leaching as outlined by Munson and Stanford (13). Leaching was then continued with two 20 ml. portions of distilled water allowing each portion to leach through before adding the next portion. Suction was again applied to remove the excess water prior to incubation. The vials were incubated in a humid incubator controlled at 35° C. for 14 days and then oven dried at 70° C. for 24 hours. The soil and vermiculite mixture was shaken out of the vial and suspended by vigorous stirring in 100 ml. of 0.1 percent $Ca(OH)_2$ solution. The suspension was filtered and the nitrate content was determined colorimetrically using phenoldisulphonic acid. The results were expressed as P.P.M. nitrogen as nitrates and also pounds of nitrogen per acre.

The leaching procedure was very slow with all soils, especially the clay soils and therefore, suction was applied to leach the samples. The addition of the Krilium 6 resulted in a clear non-turbid leachate. Suspending the soil-vermiculite mixture in distilled water instead of leaching, was more rapid unless specialized equipment was adapted as outlined by Stanford and Hanway (15). In order to be able to shake the soil-vermiculite mixture from the vials after incubation, the vials and soil had to be dried in the oven at 70° C. Besides enabling the dried soil-vermiculite mixture to be removed from the vial, it permitted storage of the samples without further nitrification taking place until they could be analysed.

Alkaline Permanganate Method For Determining Available Nitrogen

The available nitrogen was determined by the alkaline permanganate method outlined by Truog (17) with minor modifications. This method is described in detail. One-half gram of the 20 mesh soil was placed in an 800 c.c. Kjeldahl distilling flask. One-quarter of a teaspoon of a mixture consisting of 20 parts of potassium permanganate and 80 parts of anhydrous sodium carbonate was added. The method was also carried out with one-half teaspoon of the mixture and the results with both quantities of the reagent mixture are shown in Table 14. Then 150 ml. of ammonia free water were added washing down any soil or reagent mixture sticking in the neck of the flask. A small piece of wax (pea size) was added to prevent foaming. The flask was then placed on the Kjeldahl distillation rack. The electric elements had been previously heated to enable the contents of the flask to come to a boil in exactly five minutes. The boiling was continued for exactly five minutes. The distillate, containing the liberated ammonia, was caught in 20 ml. of distilled water. The distillate was diluted to approximately 50 ml. with distilled water. Two ml. of Nessler's solution was added and the solution was made up to 100 ml. The P.P.M. of available nitrogen in the form of ammonia was determined colorimetrically. The available nitrogen was also converted to pounds per acre.

The soil sample size was reduced from one gram as outlined by Truog (17) to one-half gram. This was necessary due to the high nitrogen content of Manitoba soils as compared to Wisconsin soils where the test was developed. The color developed by the Nessler's reagent was too dark with the larger sample size. For the same reason it was necessary to dilute the distillate to 100 ml. It was found necessary to maintain the same quantity of oxidizing agent with the smaller sample size because the quantity of oxidizing agent outlined by Truog (17) was reduced before the distillation was completed. This resulted in similar quantities of available nitrogen for all soils. In order to ensure that there was sufficient oxidizing agent, the double quantity was used for comparison.

It was found necessary to have all equipment free from ammonia because a very small amount of contamination caused erratic results. The method could only be carried out in the laboratory when ammonia compounds were not being used.

Correlation Of Yield Response And Available Nitrogen Determined In The Laboratory

The results of the laboratory methods are shown in Table 14. The available nitrogen was reported in terms of pounds of available nitrogen per acre and this was calculated on the basis of 2,000,000 pounds of soil per acre to the depth of six inches. The results shown for the alkaline permanganate method 1 and 2 refer to the quantity of available nitrogen released by one-quarter and one-half teaspoons of oxidizing agent respectively. Farms one to seven on which oats were grown were

Table 14: Relationship Between Check Yield, Yield Ratio And Available Nitrogen

Farm No.	Check Yield bus/acre	Yield Ratio	Pounds Of Available Nitrogen Per Acre: Incubation Method	Alkaline Permanganate Method 1 *	Alkaline Permanganate Method 2 **
2	22.6	69.7	58	141	155
1	32.3	78.7	128	294	277
5	36.5	59.3	169	286	362
3	42.5	84.6	181	228	304
7	46.2	70.5	130	186	374
4	49.4	91.7	105	297	242
6	59.1	78.1	179	245	418
11	10.1	62.3	83	222	309
13	14.9	69.2	239	307	448
14	15.3	73.9	173	262	402
8	17.1	77.2	145	248	288
12	28.9	97.1	165	294	330
9	29.3	92.3	194	321	360
10	40.6	97.3	58	254	256

* 1 teaspoon of anhydrous Na_2CO_3 and $KMnO_4$

** 2 teaspoons of anhydrous Na_2CO_3 and $KMnO_4$

dealt with separately from farms eight to fourteen inclusive on which barley was grown. This was done because the growing conditions favored the oat crop and a common basis of comparison was difficult to establish. The available nitrogen as determined by the laboratory method was correlated with the check yields. The check yield was chosen because it was felt that this should be a measure of the nitrogen available to the crop. The response to nitrogen on these farms was determined by the yield ratio suggested by Bray (3). These yield ratios are shown in the third column of Table 14 and were included to confirm the fact that the check yield gave a satisfactory indication of the available nitrogen in the fields. The yield ratio was calculated as follows:-

$$\text{Yield Ratio} = \frac{\text{Yield in bus/acre in 19.2 lbs. } P_2O_5 \text{ treatment}}{\text{Mean Yield in bus/acre of 19.2 lbs. } P_2O_5 + 20, 40, 60 \text{ and } 80 \text{ lb. N treatment}} \times 100$$

The incubation method showed that there was a wide variation in the amount of available nitrogen in the soils chosen. The results range from a low value of 58 pounds to a high value of 239 pounds of available nitrogen per acre.

The laboratory results revealed that the two tests showed different amounts of available nitrogen. The incubation test showed lower values for each farm than the alkaline permanganate method. The alkaline permanganate method1showed smaller quantities of available nitrogen than the alkaline permanganate method 2 except in the case of farms 1 and 4. This was probably because the greater concentration of oxidizing with the method 2 would release more nitrogen. There was a very wide variation

Table 15: Correlation OF Available Nitrogen As Determined By Two Methods With Each Other And Yield Response

Factor to Correlate	Yield Ratio	Incubation Method	Alkaline Permanganate Method 1	Alkaline Permanganate Method 2
Oat Fields				
Check Yield	.4199	.5921	.3326	.8731**
Yield Ratio		-.0190	.2894	-.2136
Incubation Method			.4392	.8352**
Alkaline Permanganate Method 1				.2920
Barley Fields				
Check Yield	.9079**	-.3054	.2999	-.4779
Incubation Method			.7862	.8681**
Alkaline Permanganate Method 1				.5772

** significant at the 1% level

Alkaline Permanganate Method 1 - 1 teaspoon of Anhydrous Na_2CO_3 + $K MnO_4$

Alkaline Permanganate Method 2 - 2 teaspoons of Anhydrous Na_2CO_3 + $K MnO_4$

in the amount of nitrogen released by the double quantity of oxidizing agent which was shown by largest increase in Farm 7 from 186 to 374 pounds of available nitrogen per acre, while in Farm 10 the change was only from 254 to 256 pounds of available nitrogen per acre. This was possibly explained by the fact that in some soils the one-quarter teaspoon of oxidizing agent released nearly all the available nitrogen while in some other soils there was sufficient nitrogen that the increased concentration of oxidizing agent could release more nitrogen. It must be admitted that this explanation is not verified in all cases because Farm 2, which was situated on the same quarter section as Farm 7 did not show a great increase of available nitrogen with the additional oxidizing agent.

The tests for available nitrogen should correlate with the check yield before any degree of confidence could be placed in them. The correlation results are shown in Table 15. The correlation in Table 15 indicates that the check yield was related to the amount of available nitrogen. In the barley fields, the correlation of the yield ratio and check yield was 0.9079. In the oat fields the correlation of yield ratio and check yield was only 0.4199 but the check yield exhibited higher correlation values with the laboratory methods than the yield ratio exhibited. The correlation of oat check yields and the incubation method results was not significant as was the case with the correlation of the oat check yields and alkaline permanganate method 1 results. The check yield and available nitrogen released by alkaline permanganate method 2 were correlated at the one percent level of significance. The results of the methods did not correlate significantly except that the incubation method and alkaline permangante method 2 had a significant correlation. The correlation of the barley check yield and the results of incubation method as well as the results of the alkaline permanganate method 2 gave a negative coefficient while the correlation coefficient for barley check yield and alkaline permanganate method 1 was positive but not significant.

The two alkaline permanganate methods did not correlate significantly. The negative correlation for two methods and low correlation for the alkaline permanganate method 1 in the barley fields are due to the results from farms 10 and 13. Farm 10 had the highest check yield and lowest response to fertilizer but also had the least available nitrogen as shown by all methods. This could possibly be explained by the fact that this field had been broken out of pasture only five years prior to the experiment. The fertility of the soil was high as far as yield was concerned but the laboratory methods did not reflect this fertility. Farm 13 showed the highest available nitrogen with two methods and second highest with the other but its check yield was second lowest and its response to nitrogen was second highest. No explanation can be found for this situation. The field had been broken out of crested wheat grass sod ten years previously but this grass would not have added any nitrogen to the soil. Farm 11 on the same genetic soil type and situated across the highway from Farm 13 exhibited low availability of nitrogen in all methods but reacted similarly to Farm 13 with respect to fertilizer response and check yield. The other barley fields fell into a satisfactory sequence that could show high correlation coefficients. However, these were not calculated because it was felt that due to the unexplained conditions in Farms 10 and 13, coefficients omitting them would be biased.

The alkaline permanganate method 2 appeared satisfactory with the oat yield data but it was not satisfactory with the barley yield data. The other two methods did not correlate significantly with either oat or barley yield data. The alkaline permanganate method is very rapid and would enable a great many samples to be analysed in a laboratory but it was noted from the results that the two methods with varying amounts of oxidizing agent did not correlate.

Hanway and Dumenil (8) found that past soil management and crop rotations could not be overlooked in making nitrogen fertilizer recommendations. Farms 3, 4 and 10 showed low availability of nitrogen but their check yields were high. These farms had legume forage crops in their rotation which showed in the yield but was not reflected in the available nitrogen determined by the laboratory methods.

While certain trends are shown in the yield response to nitrogen and available nitrogen in Table 15, the erratic results indicated by the correlation coefficients suggest that further field and laboratory work must be carried out before these tests can assume any practical importance under Manitoba soil and climatic conditions.

GENERAL DISCUSSION OF FIELD AND LABORATORY INVESTIGATIONS

The yield response data obtained from the field investigation showed that anhydrous ammonia fertilizer is a satisfactory source of nitrogen for stubble crops. The oat fields exhibited a large response to nitrogen. Only one oat field showed diminishing total yield with nitrogen application but diminishing yields occurred on all farms with only one exception. The barley fields exhibited a moderate response to applied nitrogen. Only two barley fields exhibited diminishing total yield.

Farms that had recently been broken from leguminous forage did not exhibit significant yield increase to applied nitrogen as in the case of farms 3, 4 and 9.

The phosphate response was not as great as the nitrogen response but a definite phosphate requirement was noted on all farms. Phosphate application was necessary to obtain the maximum nitrogen response. On the basis of phosphate response the farms were divided into two major groups. In the first group, comprising six farms, maximum yield response had not been reached with the thirty pound application of phosphate. On these farms, due to the high water table, imperfect drainage or soil erosion, calcium carbonate or other calcium salts were at or near the surface. On the remaining eight farms, which were situated on quite well drained medium to light textured soils, diminishing total yields occurred.

The t-test showed that the various treatments gave significant response when compared to their adjacent check but the regression equations or production functions reflected the fertilizer response in a more satisfactory manner. The use of these equations facilitate making yield predictions because they smooth out the minor variations encountered in the original yield response pattern. When regression equations have been calculated, the coefficients of determination can be used to determine the percentage of the yield variation that is due to the applied fertilizer.

The regression equation was used to calculate the marginal rate of substitution of nitrogen for phosphate. There was a definite substitution of one element for another, particularly at lower yields i.e. lowere rates of fertilizer application. Less substitution took place at higher yields(higher rates of fertilizer application) indicating that each element was required for a specific function. This also suggests that maximum yield could be obtained by only one combination of nutrients.

The optimum rates and combinations of nitrogen and phosphate were determined for the various nutrient and barley price situations. Fertilizer recommendations based on the method outlined herein would be more useful to farmers with limited capital who are aiming at medium yields because there would be a wide variation in nutrient combinations to produce this yield. These wide variations could make a great difference in the cost of fertilizer.

The laboratory investigation showed that there was significant correlation between the yield response to applied nitrogen on the oat fields and the available nitrogen determined by the alkaline permanganate method 2 but the available nitrogen determined by this method did not correlate with the yield response to applied nitrogen on the barley fields. The available nitrogen determined by the incubation method and alkaline permanganate method 1 did not correlate significantly with the yield response to applied nitrogen on the oat or barley fields. The correlation between the methods to determine available nitrogen was inconsistent. Further field and laboratory work will be necessary to adapt the methods for practical use in determining available nitrogen. Further investigation should be confined to one soil type over a period of several years.

This would remove many of the variables from the investigation and allow the investigators to concentrate on specific problems in the method such as, time of taking the soil samples, size of soil samples and length of incubation period. The yield response data would be more reliable when the yields were determined for several years on the same soil.

The investigational work on optimum rates and combinations of fertilizer could be continued on the same plots used for the available nitrogen study. The regression equations developed to determine the physical relationships between varying combinations of applied nutrients and yield could very well include the available nitrogen determined in the laboratory. Such an investigation would therefore bring together the laboratory, field and economic investigations.

This investigation would add considerable accuracy to the yield predictions on any particular farm or soil.

SUMMARY AND CONCLUSIONS

The studies undertaken in this investigation may be grouped under three major headings:-

(1) The evaluation of anhydrous ammonia as a nitrogen fertilizer alone and in combination with phosphate fertilizer.

(2) The application of a statistical method whereby the most economical rates and combinations of nitrogen and phosphate fertilizers can be easily and accurately determined.

(3) The investigation of methods for determining the quantity of available nitrogen in the soil.

The field investigation was undertaken on seven oat fields and seven barley fields. Seven genetic soil types were represented on the fourteen fields. Anhydrous ammonia was applied at 0, 20, 40, 60 and 80 pounds per acre of nitrogen alone and in combination with 0, 40 and 60 pounds per acre of ammonium phosphate (11-48-0). Yield data was obtained from the fertilizer treatments as a measure of the fertilizer response on the different genetic soil types. The yield data was analysed statistically to determine the significance of the fertilizer response and to determine the most economical rates and combinations of fertilizer application. Available nitrogen was determined in the laboratory investigation by the alkaline permanganate method and the incubation method. The crop response to nitrogen fertilizer was correlated with the available soil nitrogen determined in the laboratory.

Conclusions were as follows:-

1. The mean oat and barley yields increased up to the 80 pound application of nitrogen but at a diminishing rate after the 40 pound application on oats and after the 60 pound application on barley.
2. The mean oat yields increased up to the 28.8 pound application of phosphate but the mean yields of barley showed the greatest increase at the 19.2 pound application of phosphate and showed some reduction in mean yield with 28.8 pound application of phosphate.
3. The high rates of nitrogen application did not cause lodging or delayed maturity when phosphate was applied to give a proper nitrogen-phosphate balance.
4. Nitrogen response was found to be closely associated with genetic soil types.
5. Soil management and past cropping history, such as the growing of forage crops and soil erosion, altered the response to applied nitrogen.

6. The regression equations or production functions were very useful in indicating the fertilizer response pattern on the various soils.

7. The Quadratic function including interaction fitted the yield data better than the Cobb-Douglas function or the Quadratic function ignoring interaction.

8. Diminishing total yield was permitted with the Quadratic function but was not permitted with the Cobb-Douglas function.

9. The Quadratic function showed that all of the oat fields exhibited a large response to nitrogen. Only one oat field showed diminishing total yield with nitrogen application but diminishing yields occurred on all farms with only one exception.

10. The barley fields exhibited a moderate response to applied nitrogen as shown by the Quadratic function. Only two farms showed diminishing total yield.

11. The Quadratic function showed that the phosphate response was not as great as the nitrogen response but a definite phosphate requirement was noted on all farms.

12. The yield response curves showed that genetic soil types are more important in making fertilizer recommendations than the location of soil types with regard to climatic regions.

13. The yield data on Farm 8 showed that there was a definite substitution of one element for another and that the yield could be maintained by replacing some nitrogen with more phosphate. Less and less of the nitrogen would be replaced by each successive one pound

increase in the phosphate at higher and higher levels of phosphate application, i.e. high yield levels.

14. The investigation points out that fertilizer recommendations must be based on the soil conditions, price of fertilizer nutrients and price of crops. Nutrient combinations and rates of application will change with various soil conditions and price fluctuations of both nutrients and crops.

15. There was significant correlation between the yield response on the oat fields and the available nitrogen determined by alkaline permanganate method 2.

16. Before the laboratory methods can assume any practical importance for making fertilizer recommendations, further field and laboratory investigations will be necessary.

17. The accuracy of further investigation in this type of project could be increased by using rod row rather than field strip experiments. The randomized rod rows minimize the soil variations within the test as well as soil management factors that are encountered in the field strips. It would be much easier to obtain a satisfactory rod row plot than a complete field for fertilizer strips.

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Appendix (i)

The Correlation Coefficients For The Cobb-Douglas Function

Farm	ry_1	ry_2
1	.6883**	.5884*
2	.5743*	.7664**
3	.2802	.3703
4	.3030	.5881*
5	.1831	.8955**
6	.2636	.8158**
7	.3478	.8426**
8	.8658**	.5143
9	.3191	.5419*
10	.0206	.3790
11	.6420*	.7464**
12	.5201	.5145
13	.3727	.8145**
14	.5482*	.7662**

** significant at the 1% level

* significant at the 5% level

Appendix (ii)

The Correlation Coefficients For The Quadratic Function

Farm	Crop	ry1	ry2	ry3	ry4
1	Oats	.7568*	.5334	.7983*	.4841
2	"	.6232	.7298*	.6527	.6663
3	"	.2040	.1161	.0900	-.0194
4	"	.2856	.6986	.2354	.7071
5	"	.0361	.9689**	.0492	.9044**
6	"	.2559	.8553**	.1618	.7773*
7	"	.3989	.8524**	.4106	.7778*
8	Barley	.7960*	.4237	.7024	.3462
9	"	.2830	.8020*	.2235	.8620**
10	"	.1065	.4832	.1819	.5134
11	"	.6039	.7433*	.5452	.7329*
12	"	.7046	.6266	.6960	.6726
13	"	.3031	.8861**	.2164	.8601**
14	"	.4400	.5449	.2974	.3918

** significant at the 5% level

* significant at the 1% level

Appendix (iii)

The Correlation Coefficients For The Quadratic Function Including Interactions

Farm	ry1	ry2	ry3	ry4	ry5
1	.7568	.5334	.7983	.4841	.8272
2	.6232	.7298	.6527	.6663	.9006**
3	.2040	.1161	.0900	-.0194	.2134
4	.2856	.6986	.2354	.7071	.6466
5	.0361	.9689**	.0492	.9044**	.5478
6	.2559	.8553*	.1618	.7773	.6254
7	.3989	.8524*	.4106	.7778	.8034*
8	.7960*	.4237	.7024	.3462	.8110*
9	.2830	.8020*	.8020*	.2235	.8620**
10	.1065	.4832	.1819	.5134	.5234
11	.6039	.7433	.5452	.7329	.8931**
12	.7046	.6266	.6960	.6726	.7515
13	.3031	.8861**	.2164	.8601*	.7326
14	.4400	.5449	.2974	.3918	.5039

** significant at the 1% level

* significant at the 5% level

Appendix (iv)

Predicted Oat Yields With Varying Rates Of Nitrogen And Rate Of Phosphate Constant At 20 Pounds Per Acre

(Quadratic Function)

Farm No.	Nitrogen lbs/acre	Predicted yield bus/acre	Farm No.	Nitrogen lbs/acre	Predicted yield bus/acre
1	0	34.17	5	0	34.08
	10	36.40		10	42.06
	20	38.37		20	49.51
	30	40.09		30	56.45
	40	41.54		40	62.87
	60	43.66		60	74.14
	80	44.75		80	83.34
2	0	22.82	6	0	66.68
	10	26.89		10	72.20
	20	30.50		20	77.15
	30	33.64		30	81.55
	40	36.33		40	85.39
	60	40.33		60	91.87
	80	42.48		80	95.14
3	0	45.20	7	0	43.85
	10	48.19		10	49.66
	20	50.47		20	54.83
	30	52.06		30	59.36
	40	52.95		40	63.25
	60	52.62		60	69.11
	80	49.50		80	72.41
4	0	55.02			
	10	55.99			
	20	57.15			
	30	58.52			
	40	60.08			
	60	63.82			
	80	68.35			

Appendix (v)

Predicted Barley Yields With Varying Rates Of Nitrogen And Rate Of Phosphate Constant At 20 Pounds Per Acre

(Quadratic Function)

Farm No.	Nitrogen lbs/acre	Predicted yield bus./acre	Farm No.	Nitrogen lbs/acre	Predicted yield bus./acre
8	0	24.37	12	0	39.36
	10	27.29		10	38.16
	20	29.71		20	37.57
	30	31.63		30	37.61
	40	33.05		40	38.26
	60	34.38		60	41.43
	80	33.72		80	47.08
9	0	31.17	13	0	20.20
	10	30.75		10	22.64
	20	30.89		20	25.11
	30	31.60		30	27.59
	40	32.86		40	30.09
	60	37.06		60	35.16
	80	43.51		80	40.30
10	0	39.29	14	0	22.96
	10	39.54		10	26.41
	20	39.85		20	29.19
	30	40.23		30	31.28
	40	40.66		40	32.60
	60	41.70		60	33.49
	80	42.99		80	31.56
11	0	6.43			
	10	8.17			
	20	9.96			
	30	11.78			
	40	13.65			
	60	17.49			
	80	21.50			

Appendix (vi)

Predicted Yields On Six Farms With Varying Rates Of Phosphate And Rate Of Nitrogen Held Constant At 40 Pounds Per Acre

(Quadratic Function)

Farm No.	Phosphate lbs/acre	Predicted yield bus/acre	Farm No.	Phosphate lbs/acre	Predicted yield bus/acre
2	0	31.42	8	0	22.02
	5	31.25		5	26.36
	10	32.02		10	29.65
	15	33.70		15	31.91
	20	36.33		20	33.12
	25	39.88		25	33.30
	30	44.37		30	32.44
4	0	55.67	14	0	25.17
	5	57.80		5	29.20
	10	59.25		10	31.80
	15	60.01		15	32.97
	20	60.08		20	32.70
	25	59.48		25	31.01
	30	58.18		30	27.89
6	0	75.37	12	0	31.29
	5	81.25		5	32.57
	10	84.88		10	34.01
	15	86.26		15	35.60
	20	85.39		20	37.34
	25	82.28		25	39.25
	30	76.90		30	41.30

Appendix (vii)

Predicted Yields With Varying Rates Of Nitrogen And Constant Rates Of Phosphate On Farm 8

(Cobb-Douglas Function)

P_2O_5 lbs/acre	Nitrogen lbs/acre	Predicted Yield bus/acre	P_2O_5 lbs/acre	Nitrogen lbs/acre	Predicted Yield bus/acre
1	1	16.12	20	1	23.65
	10	19.06		10	27.96
	20	20.05		20	29.41
	30	20.65		30	30.29
	40	21.09		40	30.93
	60	21.72		60	31.86
	80	22.18		80	32.53
5	1	19.81	25	1	24.33
	10	23.42		10	28.77
	20	24.63		20	30.26
	30	25.37		30	31.17
	40	25.91		40	31.83
	60	26.68		60	32.78
	80	27.25		80	33.47
10	1	21.64	30	1	24.91
	10	25.59		10	29.45
	20	26.91		20	30.97
	30	27.72		30	31.90
	40	28.31		40	32.58
	60	29.15		60	33.55
	80	29.77		80	34.26
15	1	22.79			
	10	26.95			
	20	28.35			
	30	29.19			
	40	29.81			
	60	30.70			
	80	31.35			

Appendix (viii)

Predicted Yields With Varying Rates Of Phosphate And Constant Rates Of Nitrogen On Farm 8

(Cobb-Douglas Function)

Nitrogen lbs/acre	P_2O_5 lbs/acre	Predicted yield bus/acre	Nitrogen lbs/acre	P_2O_5 lbs/acre	Predicted yield bus/acre
1	1	16.12	40	1	21.09
	5	19.81		5	25.91
	10	21.64		10	28.31
	15	22.79		15	29.81
	20	23.65		20	30.93
	25	24.33		25	31.83
	30	24.91		30	32.58
10	1	19.06	60	1	21.72
	5	23.42		5	26.68
	10	25.59		10	29.15
	15	26.95		15	30.70
	20	27.96		20	31.86
	25	28.77		25	32.78
	30	29.45		30	33.55
20	1	20.05	80	1	22.18
	5	24.63		5	27.25
	10	26.91		10	29.77
	15	28.35		15	31.35
	20	29.41		20	32.53
	25	30.26		25	33.47
	30	30.97		30	34.26
30	1	20.65			
	5	25.37			
	10	27.72			
	15	29.19			
	20	30.29			
	25	31.17			
	30	31.90			

Appendix (ix)

Predicted Yields With Varying Rates Of Nitrogen And Constant Rates Of Phosphate on Farm 8

(Quadratic Function)

P_2O_5 lbs/acre	Nitrogen lbs/acre	Predicted yield bus/acre	P_2O_5 lbs/acre	Nitrogen lbs/acre	Predicted yield bus/acre
0	0	16.46	20	0	24.37
	10	18.60		10	27.29
	20	20.24		20	29.71
	30	21.38		30	31.63
	40	22.02		40	33.05
	60	21.79		60	34.38
	80	19.57		80	33.72
5	0	20.00	25	0	23.74
	10	22.33		10	26.85
	20	24.17		20	29.47
	30	25.50		30	31.58
	40	26.34		40	33.20
	60	26.50		60	34.92
	80	24.67		80	34.65
10	0	22.49	30	0	22.08
	10	25.02		10	25.39
	20	27.05		20	28.20
	30	28.58		30	30.51
	40	29.61		40	32.32
	60	30.16		60	34.43
	80	28.72		80	34.55
15	0	23.95			
	10	26.67			
	20	28.90			
	30	30.62			
	40	31.85			
	60	32.79			
	80	31.74			

Appendix (x)

Predicted Yields With Varying Rates Of Phosphate And Constant Rates of Nitrogen On Farm 8 (Quadratic Function)

Nitrogen lbs/acre	P_2O_5 lbs/acre	Predicted yield bus/acre	Nitrogen lbs/acre	P_2O_5 lbs/acre	Predicted yield bus/acre
0	0	16.46	40	0	22.02
	5	20.00		5	26.36
	10	22.49		10	29.65
	15	23.98		15	31.91
	20	24.36		20	33.12
	25	23.74		25	33.30
	30	22.08		30	32.44
10	0	18.60	60	0	21.76
	5	22.34		5	26.45
	10	25.03		10	30.09
	15	26.69		15	32.70
	20	27.30		20	34.26
	25	26.88		25	34.79
	30	25.42		30	34.28
20	0	20.25	80	0	19.57
	5	24.19		5	24.66
	10	27.08		10	28.70
	15	28.94		15	31.71
	20	29.75		20	33.67
	25	29.53		25	34.60
	30	28.27		30	34.49
30	0	21.38			
	5	25.52			
	10	28.61			
	15	30.67			
	20	31.68			
	25	31.66			
	30	30.60			