AN ABSOLUTE MEASUREMENT OF THE PAIR PRODUCTION CROSS SECTION OF LEAD AT 2.76 MEV.

A Thesis
submitted in partial fulfilment of
the requirements for the degree of
Master of Science
at the
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by
R. D. Moore
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PREFACE

The work which comprises the subject matter of this thesis was carried out at the University of Manitoba between May 1955 and May 1956.

The author wishes to express his sincere appreciation of the efforts of the following persons:

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ABSTRACT

A value has been obtained for the absolute pair production cross section of lead for the 2.76 Mev. gamma rays of Na^{24}. A collimated beam of these gamma rays was made to fall on a specially constructed target. The positrons produced in the target were detected by counting 2 quanta annihilation events by means of two scintillation spectrometers in coincidence. The strength of the Na^{24} source was measured by a coincidence counting technique and the counter detection efficiency for annihilation radiation was measured with a calibrated Na^{22} source. The effect of absorption of the 0.511 Mev. annihilation radiation in the target was measured in a separate experiment. The value obtained for the cross section was 2.38 ± 0.62 barns.

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INTRODUCTION

The existence of a particle of charge +e, where -e is the charge on the electron, and mass equal to that of the electron was first observed experimentally by Anderson in 1932[1]. Dirac, in his relativistic quantum-mechanical theory of the electron (1928), showed that the possible energy states of a free electron consist of a set of closely spaced positive levels along with a complimentary set of states of negative total energy which are mirror images of the positive ones[2]. As the possibility of an electron existing in one of these negative energy states can not be excluded on quantum mechanical grounds, there was for a time some doubt as to their correct physical interpretation. With Anderson's discovery it was possible to associate the negative energy states with the positron. This was done in the well known "Dirac hole theory".[3]

According to this theory the "electron vacuum" is represented by the case in which all the positive states are empty and all the negative states are filled (by the Pauli exclusion principle a state containing one electron is "filled"). A filled positive state is observed as a negatron and an empty negative state as a positron. Although an electron in a negative energy state is not experimentally observable, it is possible for an external electromagnetic field to cause

1. C. D. Anderson, Phys. Rev. <u>44</u>, 406 (1933)
2. P.A.M. Dirac, Proc. Roy. Soc. A, <u>117</u>, 610, (1928)
3. P.A.M. Dirac, "The Principles of Quantum Mechanics" 3rd Ed. Chap. XI.

an electron in a negative energy state to make a transition to an unoccupied positive energy state. The result of this process, which is called "pair production", is the appearance of a pair of electrons, one positive and one negative. In order for momentum AND ENERGY to be conserved the presence of a third particle (e.g. a nucleus) is required.

Although pair production can take place in a variety of ways, the process of interest for the work of this thesis is the creation of pairs in the coulomb fields of atomic nuclei by photons. In order that it be able to produce a pair, the energy of the photon must be at least enough to account for the rest energy of the two electrons (1.022 Mev.). Any surplus appears as kinetic energy of the electrons. This energy (1.022 Mev.) is generally called the "threshold energy" for pair production.

The process of pair production by photons in nuclear fields is closely allied to the Bremsstrahlung process. In the case of the former a photon interacts with an electron in a negative energy state causing a transition to an unoccupied positive state. In the latter case an electron in a positive state undergoes a transition to a second positive state of lower energy with the emission of a photon. Because the transitions involved in the two processes are either reciprocals of one another (i.e. absorption or emission of a photon), or differ only in initial and final states, having worked out the various transition probabilities involved in one case, one can apply them in a straightforward fashion to the other. For this reason in theoretical papers the two are often treated together.

The first relativistic quantum mechanical treatment of the Bremsstrahlung - pair production problem was given by Bethe and Heitler in 1934.[4] This theory was based on the "Born approximation", i.e. plane wave functions were used for the electrons. This amounts to the condition that

$$\frac{Ze^2}{\hbar v} << 1 \quad \text{i.e.} \quad \frac{Z}{137} \cdot \frac{c}{v} << 1$$

where Z is the charge number of the nucleus, e is the electronic charge, $\hbar$ is Planck's constant divided by 2π, c is the velocity of light, $\frac{e^2}{\hbar c} = \frac{1}{137}$ is the fine structure constant and v is the electron velocity. (The inequality must be satisfied for both the positron and negatron). Since this will not be satisfied for elements of high Z (e.g. for Pb, Z = 82, Z/137 = 0.6), this theory therefore would be expected to break down in the region of high Z and low v, i.e. low incident photon energy. For low energies they considered only the simple coulomb field of the nucleus and for high energies took account of the effect of the orbital electrons by means of a screening correction. The expression obtained for the differential cross-section as well as a graph showing the values of the total cross-section, σpair, for various materials over a wide range of energies are given in Heitler's book.[5] They obtain the result that the cross-section for fixed photon energy is proportional to Z^2 for the case in which screening is neglected.

4. H. Bethe & W. Heitler, Proc. Roy. Soc. A, 146, 83, (1934)
5. W. Heitler, "The Quantum Theory of Radiation" 3rd Edition, Oxford 1954, p. 258, p 262.

For low photon energy and high Z the Born approximation, as pointed out above, is no longer valid. Jaeger and Hulme have developed a theory of pair production making use of exact (coulomb) wave functions and have made numerical computations of σpair for various high Z materials at low photon energies.[6] Values of σpair for Z = 82(Pb) for the two theories are given in the following table;

Photon Energy Mev	σpair in barns (10^{-24} cms^2) J.H.	B.H.
1.53	o.67	o.34
2.66	3.1	2.6

According to the Jaeger and Hulme theory the dependence of the cross-sections on Z is of the form: $aZ^2 + bZ^4$ where a and b are constants.

The process in which an electron pair is formed is called pair production: the process in which a negatron and positron interact and disappear is called pair annihilation. In terms of the Dirac hole theory the annihilation process involves the transition of an electron in a positive energy state to a negative energy state, the energy lost by the electron appearing as one or more photons.

In the most probable annihilation process the positron is reduced to thermal velocity by interaction with the atomic particles of the material through which it passes. It then interacts with a free electron, producing two photons. This process isreferred to as two quanta annihilation. Since the energy due to the electrons' motion is very small compared to their rest energy, the

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6. H. Hulme and J. Jaeger, Proc. Roy. Soc. 153, 443 (1936)

energy carried by the two photons must be very nearly 1.022 Mev. To satisfy conservation of momentum the energy must be shared equally between them. Thus the product of this mode of annihilation is two oppositely directed 511 Kev photons. These are usually referred to as the "annihilation radiation." Precision experiments have verified that this process does in fact occur.[8]

If the annihilation interaction occurs while the positron is moving with an energy which is not negligible compared to its rest energy, then the above statements are still true in the Lorentz system in which the centre of mass of the two electrons is at rest. However, when transformed to the system in which the negatron is at rest, resulting pairs of photons need no longer be oppositely directed or even of the same energy. The theory for this process has been given by Bethe and predicts that approximately 4% of a beam of 1 Mev positrons passing through lead will annihilate in flight.[9]

If the negatron is bound to a nucleus it is possible for a positron to annihilate with the emission of a single photon, the nucleus taking up the extra momentum. The theory for this process is given by Heitler who obtains a value for the cross-section proportional to Z^5.[10] Hence it is most important in heavy elements in which it may amount to as much as 20% of the two quanta annihilations.

There is one more significant process; that in which the annihilation produces three photons. This can

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8. D. E. Muller, H. C. Hoyt, D. J. Klein and J.W. DuMond, Phys. Rev. 88 (1952) 775
9. H. Bethe, Proc. Roy. Soc. A, 150 (1935) 129, 3rd Ed.

occur for positrons either in flight or at rest. From the point of view of perturbation theory this is a higher order process than the two quanta case, and hence much less likely to occur. The theoretical ratio of two to three quanta annihilations is given by Heitler as 1/370.[11] That the ratio in favour of the two quanta process is at least this great has been verified by De Benedetti.[12]

From the above it may be seen that it is feasible in an experiment to determine the rate at which positrons annihilate by determining the rate at which they annihilate at rest producing two quanta. The total rate of annihilation may then be obtained by accounting for the other processes through small corrections.

[10] W. Heitler, Quantum Theory of Radiation (3rd Ed. Oxford 1954) PPS (272-4)

[11] Ibid Page 278

[12] S. De Benedetti and R. Siegel, Phys. Rev. 94 (1954) 955.

THE EXPERIMENT

Although several experimental measurements of relative pair production cross sections have been made at low energy in order to determine its Z dependence, at the time this work was begun no absolute measurements had been made. The results of the relative measurements of Hahn et al[1] at 2.62 Mev. and Dayton[2] at 1.33 and 2.76 Mev. both agree with the Z dependence predicted by Jaeger and Hulme rather than that of Bethe and Heitler. As a further check on the theory an absolute measurement in this energy region was desirable.

The experiment which is the subject of this thesis is perfectly straightforward in that a target of the material being investigated is bombarded with an essentially monochromatic beam of gamma rays, for which the photon flux (no. of photons per square centimeter per second perpendicular to the beam) is known at the position of the target. The rate at which positrons annihilate in the target with the production of two oppositely directed 0.511 Mev. photons is measured by a coincidence counting technique using a pair of scintillation counters. From these data, along with certain subsidiary data discussed below, it is possible to calculate σ_{pair}.

The experiment is divided into a series of subexperiments, each of which provides part of the data required. The first of these is an experiment in which the "strength" (no. of disintegrations per second) of the source is

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1. Hahn, Baldinger and Huber, Helv. Phys. Acta. 25, 505 (1952)
2. Irving E. Dayton, Phys. Rev. 89, 544, (1953)

measured. Once this is known it is possible to calculate the photon flux at the target from the experimental geometry. Next, the "coincidence rate" i.e. the number of two quanta annihilations detected per second must be measured for each target used. It is then necessary to determine the "annihilation rate" i.e. the number of two quanta annihilations per second occuring in the target. This is necessary since not all such annihilation events are detected by the counter. The ratio of the number of events detected to the number of events occuring is called the efficiency of the counter. Finally it was necessary to perform an experiment in order to determine a correction to be applied to the annihilation rate as abtained by the above procedure because of absorption of the 0.511 Mev photons in the target itself. Each of these sub-experiments will be described separately in detail in the order given above, following a brief description of the counting equipment.

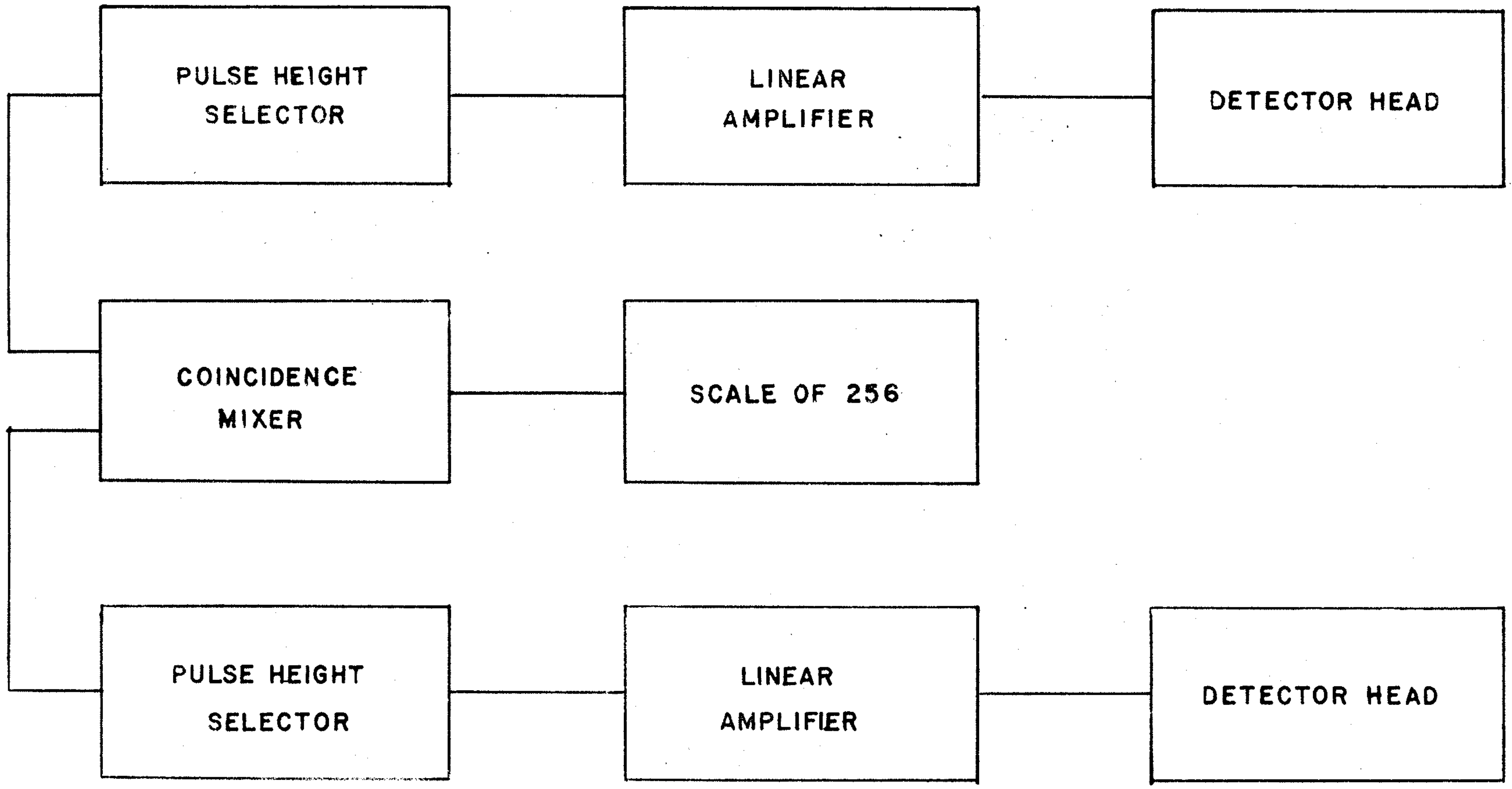
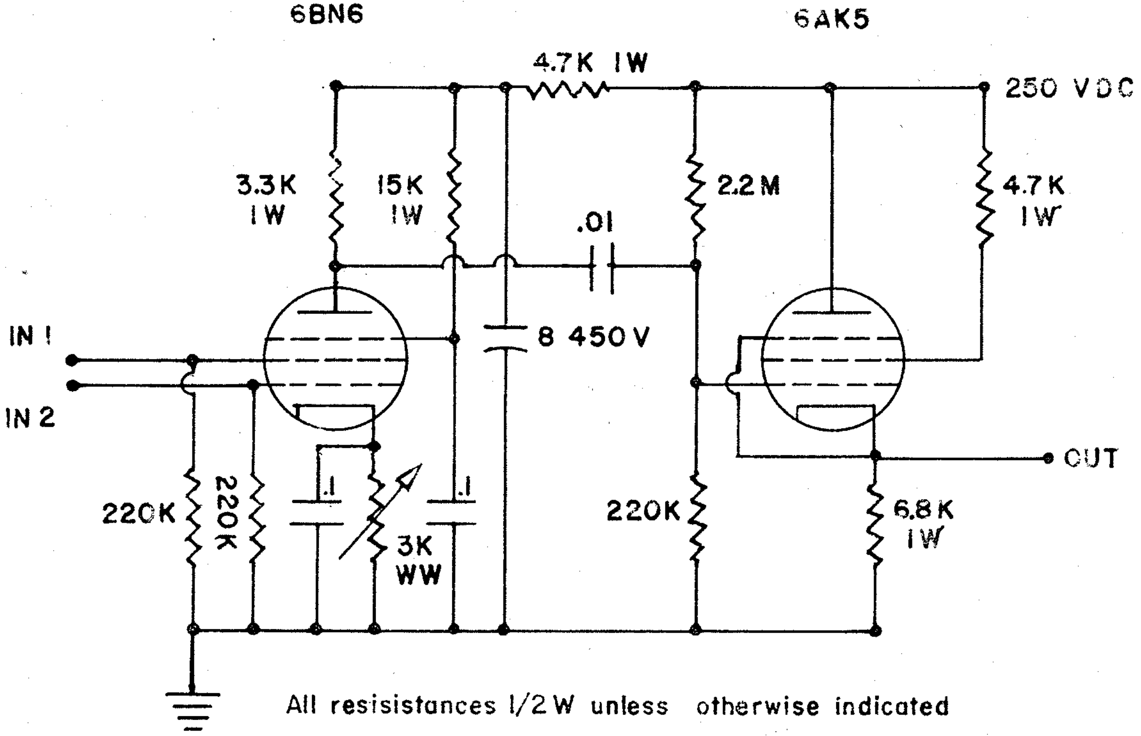


FIG. I BLOCK DIAGRAM

FIG. 2

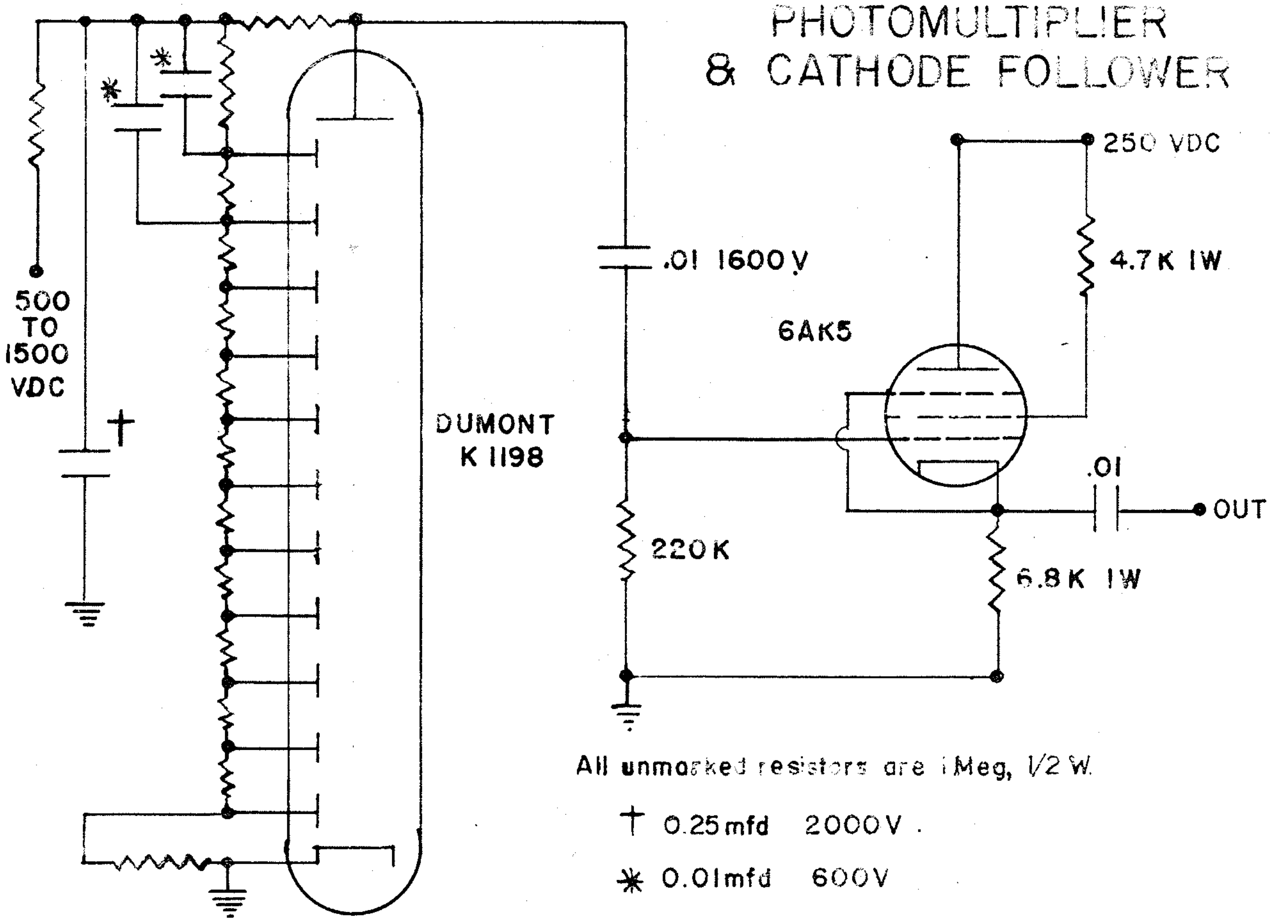
COINCIDENCE MIXER



All resisistances 1/2 W unless otherwise indicated

All capacities in mfd unless otherwise indicated

PHOTOMULTIPLIER & CATHODE FOLLOWER



All unmarked resistors are 1Meg, 1/2 W.

† 0.25mfd 2000V.

✳ 0.01mfd 600V

THE COUNTING EQUIPMENT

Figure 1 is a block diagram of the coincidence counter. Although this equipment is quite conventional a brief description of its operation at this point will serve to clarify later remarks.

Each detector head consisted of a five inch photomultiplier (Dumont 6364) on which was mounted a cylindrical NaI(Tl) crystal 9.6 centimeters in diameter and 6.3 centimeters long, along with a "base" consisting of the potential divider chain for the photomultiplier and a cathode follower. The circuit used in the bases is shown in Fig. 2.

The linear amplifiers were of a type in use at the University of Manitoba for several years[3] which had been modified for delay-line pulse shaping as shown in Fig. 3 below.

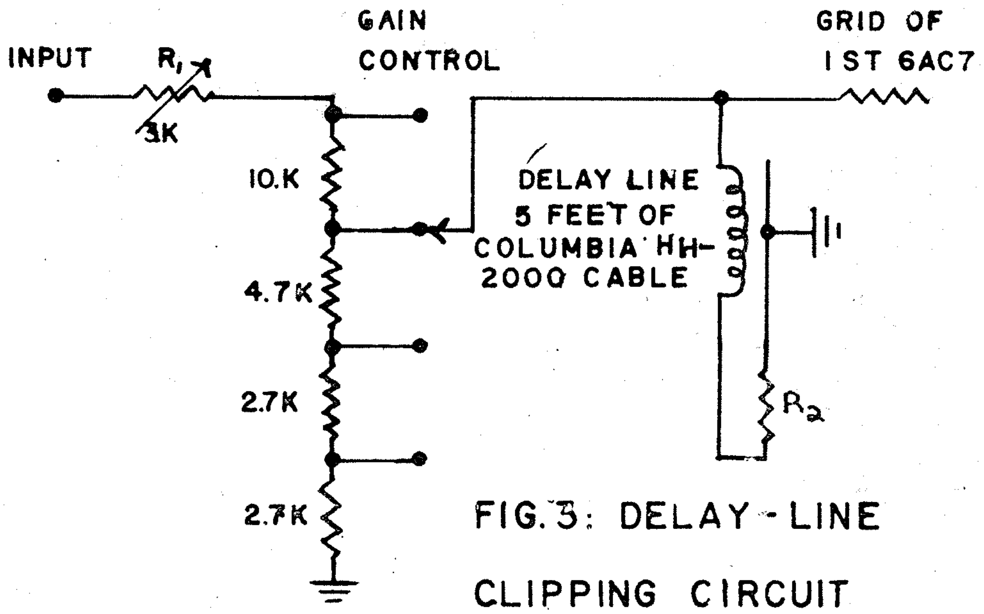


FIG. 3: DELAY-LINE CLIPPING CIRCUIT

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3. K.I. Roulston, PhD. Thesis, University of Man. March, 1952.

R_1 is used to provide correct termination of the delay line. R_2 is selected by trial and error to adjust the height of the reflected pulse to eliminate negative overshoot in the clipped pulses.

The single channel pulse height selectors were commercial units manufactured by the Dynatron Radio Company (model 101/A). These units have a resolving time of the order of 10^{-6} seconds and provide channel widths of 0, 0.5, 1, 2, 3, 5 and 7.5 volts over a bias range of 0 to 50 volts.

The coincidence mixer shown in Fig. 2 is a modification of a circuit designed by Fischer and Marshall[4]. The original circuit was designed to produce resolving times of the order of 10^{-10} seconds. To suit the purposes of this experiment the unit was re-designed to handle larger input pulses and to produce output pulses of the order of 10 volts. The characteristics of the 6BN6 tube and the theory of operation of the circuit are discussed in detail in the paper mentioned above. The unit proved extremely serviceable and provided resolving times of the order of 3.5×10^{-7} seconds.

The necessary D.C. voltages for the various units were obtained from regulated power supplies of standard design and the A.C. supply was provided by a Sorenson A.C. line regulator.

At this point a few remarks on the calibration of the counting equipment are in order. For energy calibration of the counters a Zn^{65} source was used. Since

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[4] Fischer and Marshall, Rev. Sci. Insts. 23, 417, (1952)

Zn^{65} is a positron emitter 0.511 Mev. annihilation radiation is present in its gamma ray spectrum and may easily be picked out by running a coincidence spectrum. The 0.511 Mev. photopeak so obtained was used to calibrate the counters through out the experiment. The only other calibration required was a periodic measurement of the resolving time τ of the coincidence mixer. If random pulses are fed into each of the two inputs of the mixer from two completely independent sources at rates N_1 and N_2 respectively, a chance coincidence rate given by:

$$N_c = 2\tau N_1 N_2$$

will be observed. In order to measure τ two separate gamma ray sources were used, one for each counter, with the counters isolated so that no genuine coincidences between the counts in the two channels could occur. τ could then be calculated from the result of a direct measurement of N_1, N_2, and N_c.

Hereafter, the assembly of equipment described above will be referred to simply as the coincidence counter.

MEASUREMENT OF SOURCE STRENGTH

The ideal source for this experiment would be one which emits a single gamma ray of energy greater than 1.022 Mev. It should also be a readily available isotope with a reasonably long half-life. After considerable investigation of available isotopes the one chosen was Na^{24}. As can be seen from the decay scheme shown below[5] its radiation consists almost entirely of two cascade gamma rays of energy 2.76 and 1.38 Mev respectively.

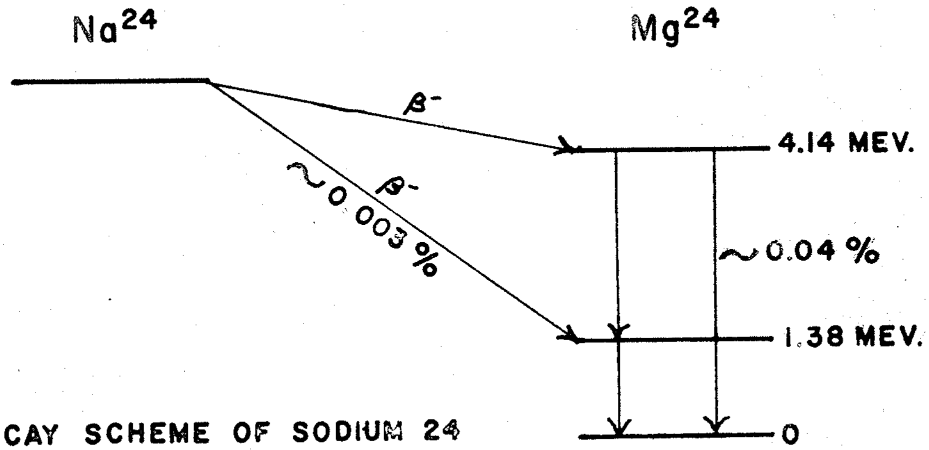

FIG. 4 DECAY SCHEME OF SODIUM 24

The small contribution to pair production in the target by the 1.38 Mev. gamma rays was corrected for by making use of the data of Griffiths and Warren[6] on the ratio of σpair at 2.76 Mev to σpair at 1.38 Mev. The half life of Na^{24} was found by Tobaillem[7] to be 14.90 ± 0.05 hours.

The source used in the experiment consisted of 0.5 grams of sodium carbonate irradiated to an initial activity of 450 millicuries in the Chalk River reactor. The source was sealed in a quartz phial, the active material being roughly in the form of a circular cylinder 1.5 centimeters long and o.5 centimeters in diameter.

5. Hollander, Perlman & Seaborg, Rev. Mod. Phys. 25, 469, (1953)
6. Griffiths and Warren, Proc. Phys. Soc. A, 55, 1050, (1952)
7. J. Tobaillem, J. Phys. et Radium , 16, 48, (1955).

Figure 5 shows the arrangement of the counters used in measuring the strength of the source.

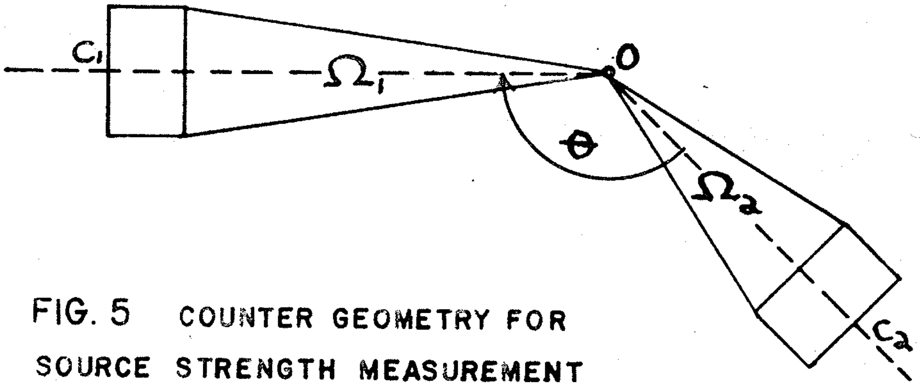

FIG. 5 COUNTER GEOMETRY FOR SOURCE STRENGTH MEASUREMENT

The source, its' axis of symmetry perpendicular to the paper, was located at O. C_1 and C_2 are the crystals of the counters in the coincidence counter. Ω_1 and Ω_2 are the solid angles subtended at the source by C_1 and C_2 respectively and θ is the angle between the axes of the crystals. After the counters had been calibrated the pulse height selectors in each channel were set with wide gates, channel 1 covering the 1.38 Mev photopeak and channel 2 that at 2.76 Mev. Let ϵ_2 be the probability that a 2.76 Mev. photon emitted into Ω_2 produces a pulse in the 2.76 Mev. photopeak, ϵ_1 be the probability that a 1.38 Mev. photon emitted into Ω_1 produces a pulse in the 1.38 Mev. Photopeak, and ϵ_3 be the probability that a 2.76 Mev. photon emitted into Ω_1 produces a pulse inside the gate of channel 1 pulse height selector by compton scattering in crystal 1. If N is the source strength then the counting rates in channels 1 and 2, N_1 and N_2 respectively, are given by:

$$N_1 = N(\epsilon_1\Omega_1 + \epsilon_3\Omega_1) \cdots\cdots \quad (1)$$

and $$N_2 = N\epsilon_2\Omega_2 \quad (2)$$

OR $$N_1 = N(\phi_1 + \phi_3) \quad (3)$$

and $$N_2 = N\phi_2 \quad (4)$$

where $\phi_1 = \epsilon_1\Omega_1$, $\phi_2 = \epsilon_2\Omega_2$ and $\phi_3 = \epsilon_3\Omega_1$

The chance coincidence rate Nc is given by:

$$N_c = 2\tau N_1 N_2 \quad (5)$$

Since each 2.76 Mev. photon is accompanied by a time-coincident 1.38 Mev. photon, there will be a real coincidence rate N_R given by:

$$N_R = N\phi_1\phi_2 \quad (6)$$

Let $$N_1' = N\phi_1 \quad (7)$$

THEN FROM (4), (6) & (7) WE GET:

$$N = \frac{N_1' N_2}{N_R} \quad (8)$$

The total coincidence rate η where

$$\eta = N_c + N_R \quad (9)$$

as well as N_1 and N_2 may be measured directly. From substitution for N_1 and N_2 in (5) we obtain N_c and hence N_r

from (9). N_1' must be obtained by a graphical method described below.

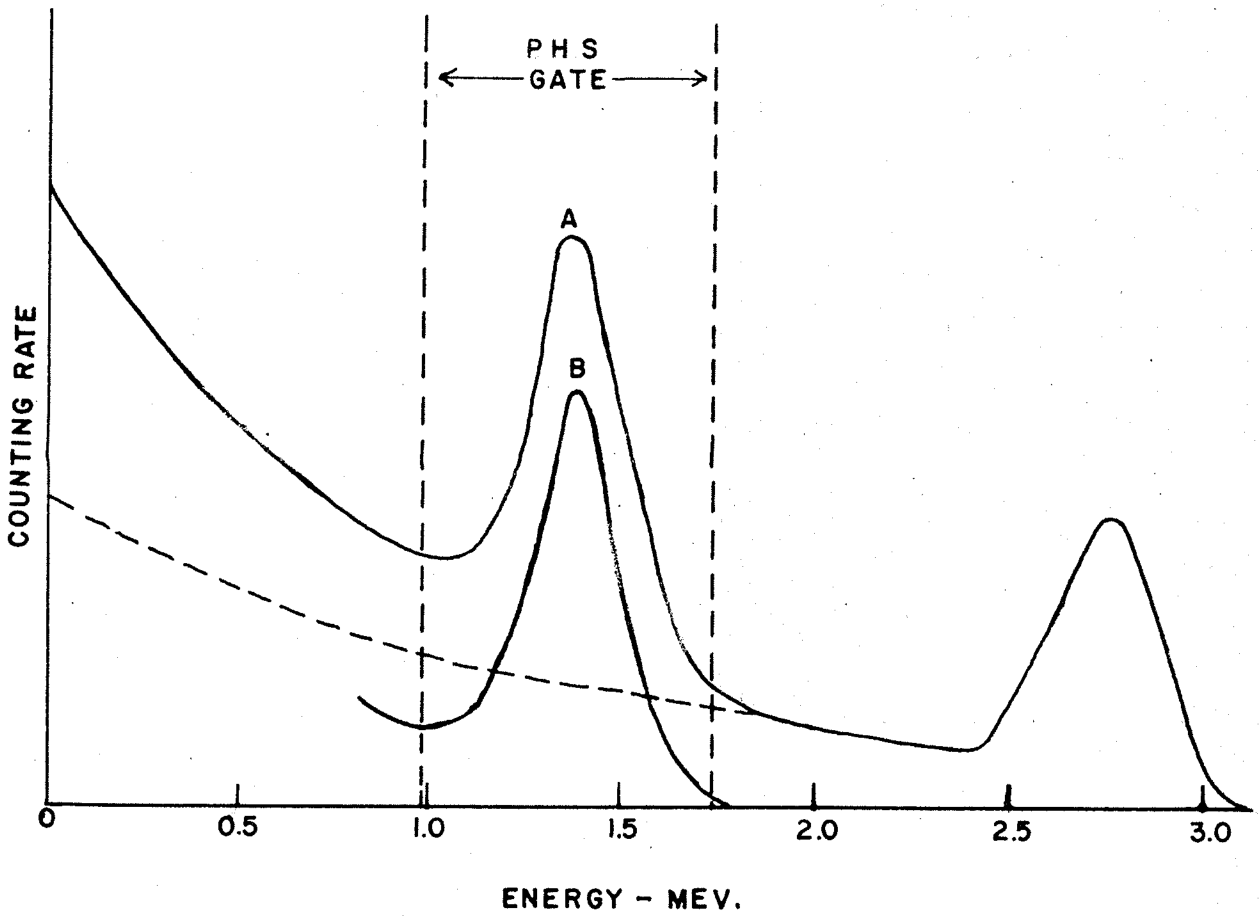

FIG. 6 GAMMA RAY SPECTRUM OF SODIUM 24

The solid line in Fig. 6 shows the appearance of a gamma ray spectrum of Na^{24} as obtained with a single channel spectrometer. The dashed line shows the contribution of the compton distribution due to 2.76 Mev. photons to the spectrum. Peak A is the 1.38 Mev. photopeak which is seen to be sitting on a background of compton from the 2.76 Mev.

photons. If the compton distribution is subtracted from curve A the result is curve B, which is the form the 1.38 Mev. photopeak would take in the absence of the compton background. The area under curve B which lies between the energy limits set by the pulse height selector gate is proportional to N_1'. To obtain N_1' it was necessary to run a single channel spectrum covering the 1.38 Mev. photopeak, thus obtaining curve A. The high energy tail of curve A was then extrapolated back, and this curve was subtracted from A. The exact shape of the extrapolated curve was adjusted until the subtraction yielded a curve B which had a peak to trough ratio and width at half maximum corresponding to the known performance of the counters at 1.38 Mev. The areas under each of curves A and B lying between the energy limits set by the pulse height selector gate were measured with a planimeter and N_1' was then calculated from

$$N_1' = \frac{\text{area under B} \times N_1}{\text{area under A}} \quad \ldots\ldots\ldots\ldots(10)$$

It was found that about 40% of the counts in channel 1 were due to this compton background, which means that the uncertainty (at least 10%) in the ratio obtained by the above procedure accounts for a large part of the error in the value finally obtained for the source strength.

Measurements of the source strength were made over a period of five days. Each day several runs were made in which the source to crystal distances and the angle θ were varied in order to detect any dependence of the results obtained on geometry. As no significant differences were found corresponding to these changes, the neglect of angular correlation in the derivation of equation (8) seems justified. The results obtained on each day were corrected to noon using the value of 14.90 hours for the half-life and these values averaged to obtain a mean value of N at noon of each of the five days.

The strength N of a source at time τ whose strength at $\tau = 0$ was N_0 is given by:

$$N = N_0 \exp\left[-\frac{\log_e 2}{T}\tau\right] \quad \ldots\ldots(11)$$

where T is the half-life. This may be put in the form:

$$\log_e N = -\frac{\log_e 2}{T}\tau + \log_e N_0 \quad \cdot\cdot(12)$$

from which we see that a plot of $\log_e N$ against τ should be a straight line of slope $-\log_e 2/T$.

Such a plot is shown in Fig. 7 for the data obtained in this experiment. The solid line is a least squares fit to the points. From the slope of this line we obtain

$$T = 15.97 \pm 1.6 \text{ hours}$$

the error being estimated graphically. This is in agreement with the value reported by Tobaillem. Since the value 14.90 is considerably more precise than the one obtained in this experiment, it is preferable to use it in calculating N at various times from these data. Making a least squares fit of a straight line of slope corresponding to $T = 14.90$

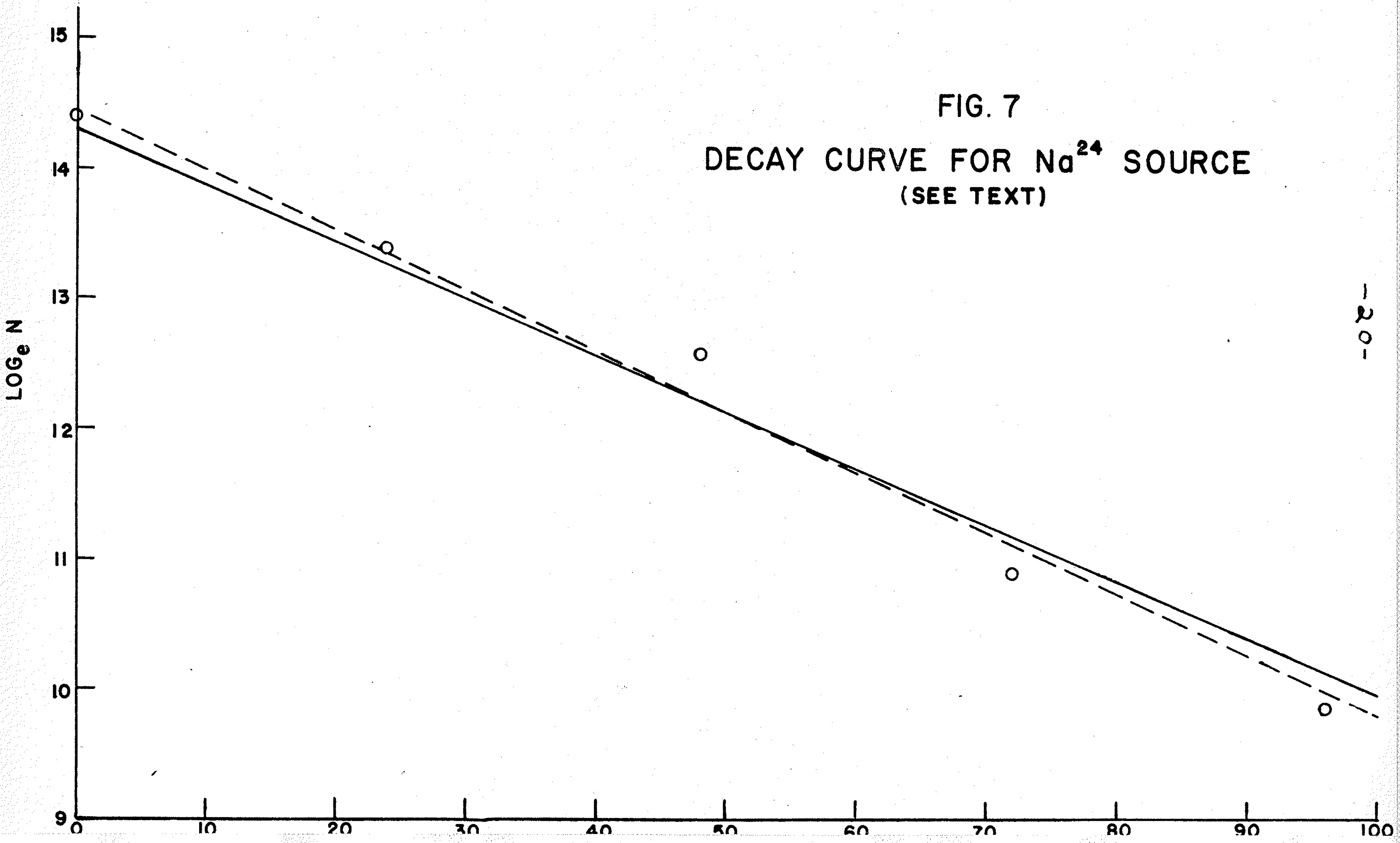

FIG. 7

DECAY CURVE FOR Na^{24} SOURCE
(SEE TEXT)

hours, we obtain

$$\log_e N = -0.04652\tau + 14.45 \ldots\ldots\ldots\ldots (13)$$

which is the dashed line in Fig. 7. This equation was used to obtain N at various times in later calculations. From a consideration of the scattering of the points on Fig. 7 an error of 10% has been attached to values of N calculated from equation (13).

MEASUREMENT OF PAIR PRODUCTION IN THE TARGET

In this part of the experiment a specially designed target was irradiated with a collimated beam of gamma rays from the Na^{24} source. The annihilation of positrons created in the target was detected by means of the coincidence counter.

The target consisted of a lead disc 11/16 inches in diameter and 1 millimeter thick cemented to a graphite disc of the same diameter and 4 millimeters thick. If the lead disc had been used alone a high percentage of the positrons produced therein would leave it with enough kinetic energy to move out of the sensitive region of the coincidence counter before annihilating. Rather than accept this loss and attempt to correct for it, the graphite disc, thick enough to stop the most energetic positrons leaving the lead disc, was added. In this way most of the annihilation events were confined to a small volume entirely within the sensitive region of the counter. In this arrangement pairs were also produced in the graphite target. Since σpair is known to vary at least as Z^2 and since the mass of carbon is small compared to that of the lead, the contribution of the graphite is slight compared to that of the lead. The effect of the graphite was accounted for by making use of a graphite disc of the same dimensions as the one incorporated in the target. (Hereafter the lead and graphite combination will be referred to as the "target", the graphite disc as the "blank"). The coincidence counting rate was determined for the blank, and subtracted from that obtained with the target.

Since most of the positrons move in the forward direction as defined by the incident photon direction, the loss due to positrons coming out the front and around the edges of the lead disc could be accounted for by a small correction. In order to obtain this correction a second target was used in which the lead disc was completely surrounded by graphite of sufficient thickness to stop any positrons leaving the lead. In this case, as above, the effect of the graphite was accounted for by making use of a graphite blank. A comparison of the counting rates obtained with this target and blank gives the correction to be applied to the counting rate obtained with the original target. Here the amount of graphite is enough that its contribution is comparable to that of the lead. This means the difference obtained is fairly inaccurate unless the two counting rates are known with high precision. However, since the correction was found to be only 8% a fairly high error can be tolerated in its value. In the case of the target first described the effect of the lead is much greater than that of the carbon, and it is easier to obtain a reliable difference, so it was considered preferable to use this type of target in the experiment.

The experimental arrangement used is shown schematically in Fig. 8.

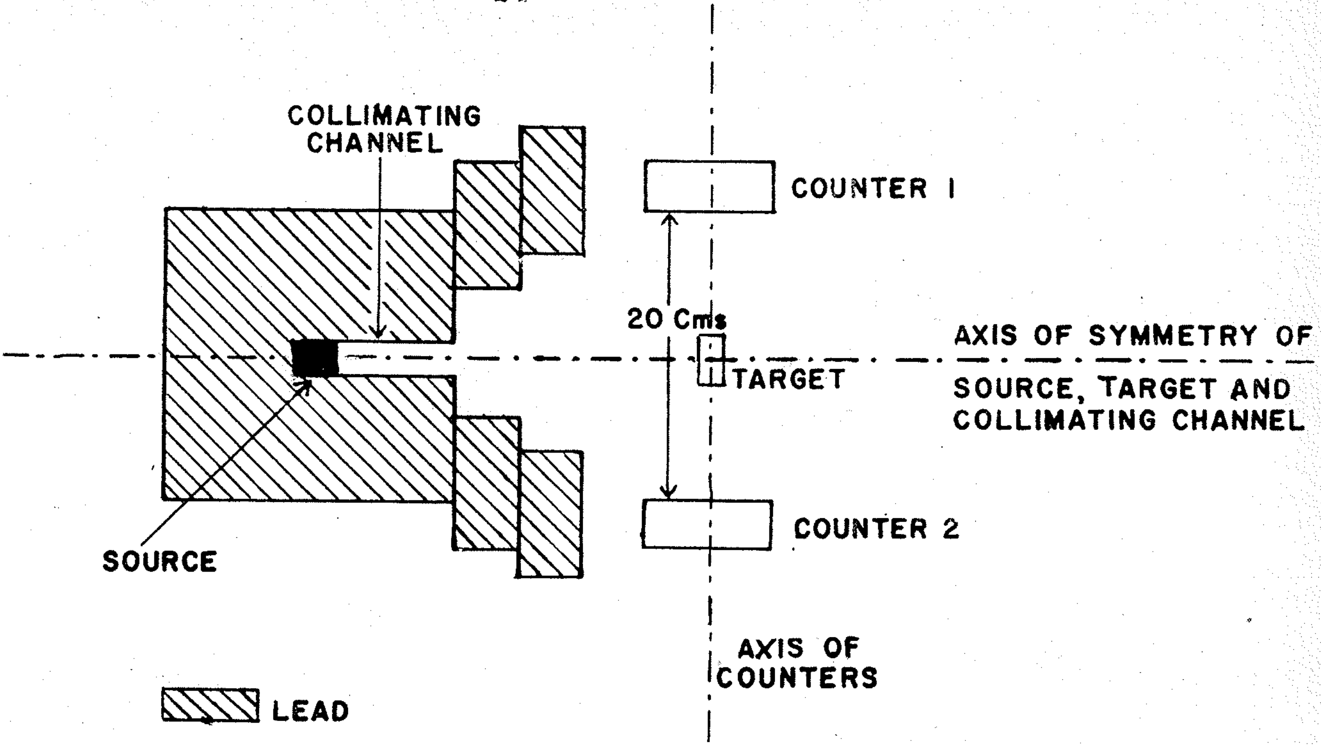

FIG. 8 PLAN VIEW OF APPARATUS (NOT TO SCALE)

About 600 pounds of lead was used in collimating the gamma ray beam and shielding the counters. This amount was found necessary to prevent prohibitively high single channel counting rates. With this arrangement the chance coincidence rate, as calculated from the singles rates and the resolving time of the coincidence mixer, amounts to only a few percent of the genuine coincidence rate due to positron annihilations in the target. Fig. 9 shows the appearance of a single channel spectrum of the radiation from the target. The target was supported between the counters by a holder made of light cardboard. (The contribution to the total coincidence rate of the cardboard was found to be negligible compared to that of the target.) The source,

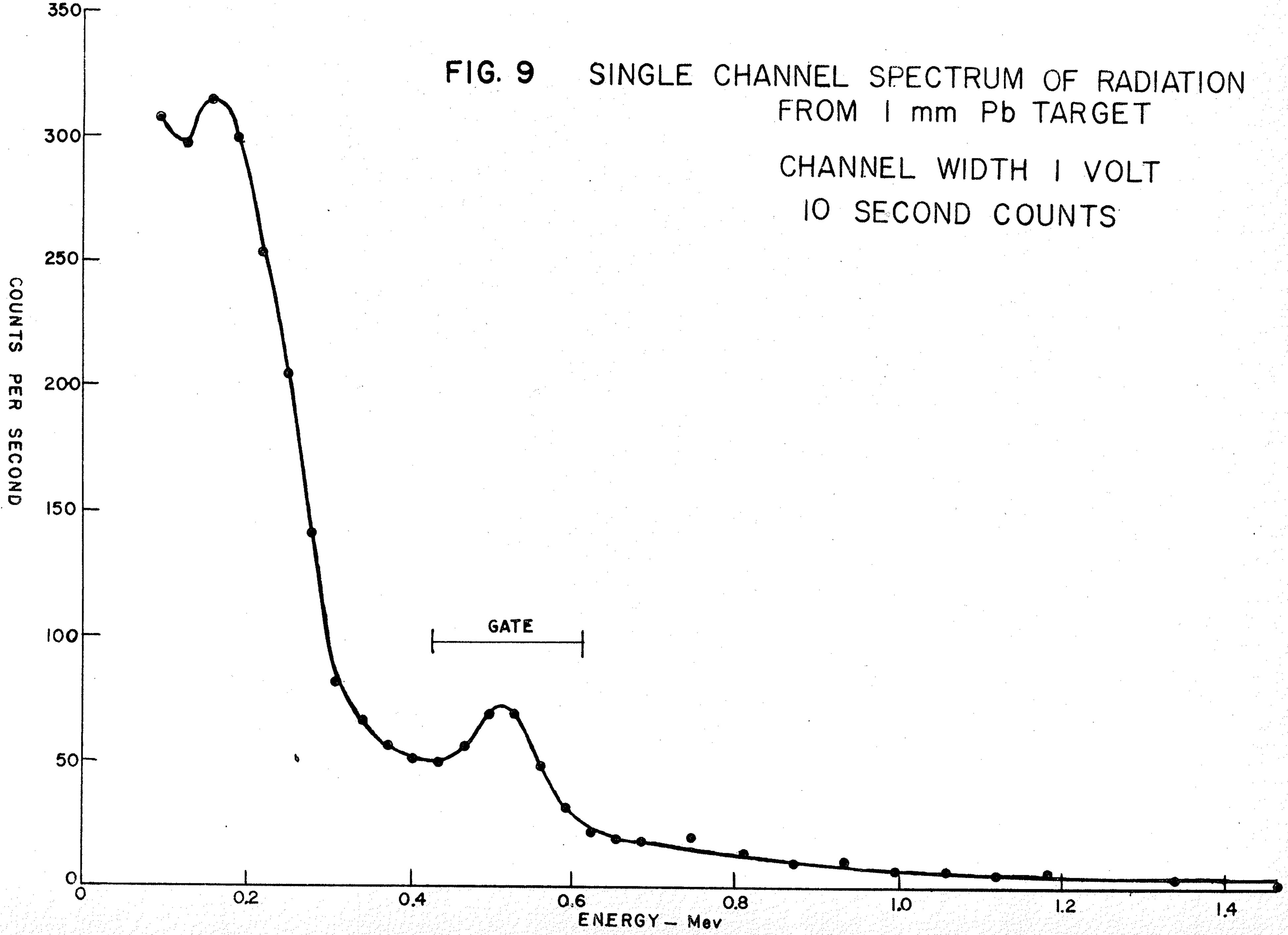

FIG. 9 SINGLE CHANNEL SPECTRUM OF RADIATION FROM 1 mm Pb TARGET

CHANNEL WIDTH 1 VOLT
10 SECOND COUNTS

collimating channel and target shared a common axis of symmetry as shown in Fig. 8. The target was placed symmetrically on the common axis of the two counters to insure that oppositely directed annihilation photons coming from the target were detected with maximum efficiency. The source to target distance had to be decreased during the experiment to compensate for decay of the source, hence no value is given for this distance in Fig. 8.

To measure the coincidence counting rate, the two channels of the counter were calibrated with annihilation radiation as described previously, and each one set with a wide gate covering the 0.511 Mev. photopeak as shown in Fig. 9. The coincidence rates were measured with the target and the blank alternately in position until over 10,000 counts had been obtained on the former and about 4,000 on the latter. This represents a statistical accuracy of the order of 1% for the target and 1.5% for the blank. Since the results obtained here require considerable correction before they can be applied to the calculation of σpair, they will not be given until after a description of those portions of the experiment which are the sources of these corrections.

MEASUREMENT OF DETECTION EFFICIENCY

As was mentioned previously, the coincidence counting rate measured by the above method does not give the rate at which two quanta annihilations take place in the target. To begin with, the solid angle subtended by the counter crystals at the target is obviously not 4π radians. This means that a substantial number of the annihilation photons never enter the crystals. Further, a large number of those that do enter either produce no pulses at all, or else produce pulses which do not fall within the gates of the pulse height analysers. Consequently, in order to obtain the annihilation rate defined previously the coincidence counting rate must be multiplied by an experimentally determined correction factor.

In determining the correction factor, a calibrated source of Na^{22} obtained from Atomic Energy of Canada, Limited, was used. The decay scheme of Na^{22} is shown in Fig. 10.[8]

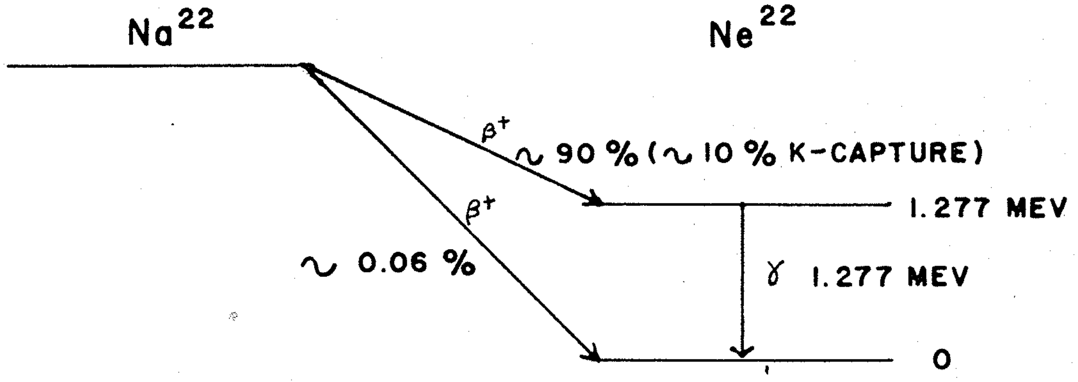

FIG. 10 DECAY SCHEME OF SODIUM 22

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[8] Hollander, Perlman and Seaborg, Rev. Mod Phys. 25, 469, (1953)

The energy of the positron feeding the 1.277 Mev. level in Ne^{22} is given by Macklin[9] as 0.542 Mev. Sherr[10] reports 9.9 ± 0.6% K-capture.

The source received from A.E.C.L. consisted of an aqueous solution of NaCl, the total activity being about five microcuries. For the measurement, two much weaker sources were prepared by sealing small amounts of the original solution in lucite capsules of the form shown in Fig. 11.

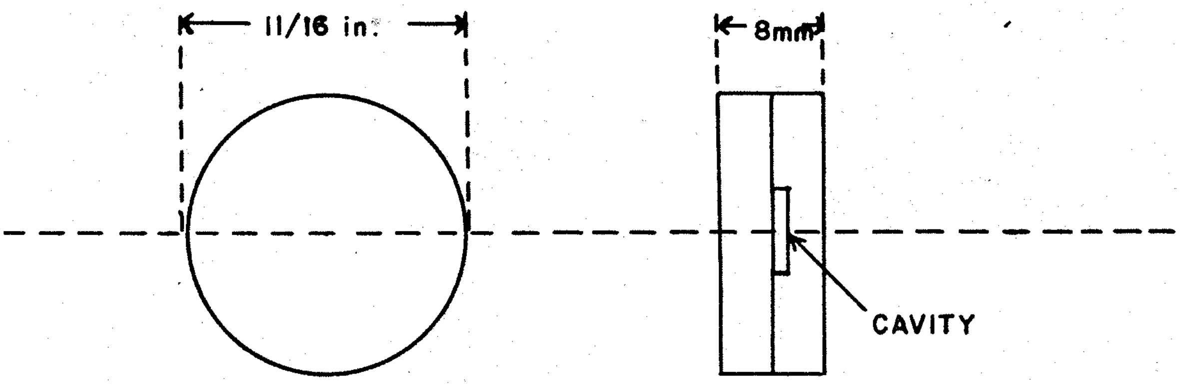

FIG. 11 LUCITE CAPSULE USED WITH Na^{22} SOURCE IN DETECTION EFFICIENCY MEASUREMENT

The capsules were made up of two lucite discs cemented together. A small cavity was turned in one of the discs and the weighed quantity of solution placed in it. After the water had been allowed to evaporate, the discs were cemented together. The thickness of lucite surrounding the source was sufficient to insure that all of the positrons emitted by the Na^{22} annihilated before escaping from the

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[9] Macklin, Ledofsky and Wu, Phys. Rev. 77, 137, (1950)
[10] Sherr and Miller, Phys. Rev. 93, 1076, (1954)

lucite. Thus these sources provided a source of annihilation radiation similar to that provided by the target in the foregoing experiment. To determine the detection efficiency one of these sources was placed in the position previously occupied by the target. The counters were calibrated and the gates positioned exactly as before and the coincidence counting rate determined. The number of positrons emitted per second was computed from the weight of the solution that had been placed in the lucite container and the known total activity of the original solution. From the results the ratio:

$$\frac{\text{Number of positrons emitted per second}}{\text{coincidence rate}}$$

was calculated. In calculating the strength of the two sources the K-capture was allowed for, using Sherr's figure. Runs were made with both the sources, and a mean value of $1.046 \pm 0.05 \times 10^2$ was obtained for the ratio.

Two possible effects of the 1.277 Mev. gamma ray which is emitted in time coincidence with the 0.542 Mev. positron on the above result were investigated. Since the gamma ray energy is more than 1.022 Mev. it could produce pairs in the lucite, the annihilation radiation of which would then contribute to the total coincidence rate. A rough theoretical estimate was made, which showed this effect to be negligible compared to the 5% error in the value of the source strength given by A.E.C.L. The second effect is a little more complicated. Since the 1.277 Mev. gamma rays are in time coincidence with the annihilation gamma rays, it is possible for an annihilation photon and a 1.277 Mev. photon to be detected in the same crystal at the same time.

If this happens, a pulse is produced whose height is the sum of the pulse heights produced by each photon alone. Two things may happen as a result. First, a 0.511 Mev. photon falling below the pulse height analyser gate could be "boosted in" by a pulse of the appropriate size produced by a 1.277 Mev. photon. Second, a 0.511 Mev. Photon landing within the gate could be "boosted out" in a similar way. The first of these effects tends to increase the coincidence rate, the second to decrease it. A careful experimental check on these effects showed they produced a negligible effect.

MEASUREMENT OF THE EFFECT OF ABSORPTION OF 0.511 MEV. PHOTONS IN THE TARGET.

In order for a two quanta annihilation event to be detected both 0.511 Mev. photons must enter the detecting crystals. Consequently the coincidence rate obtained with the target must be corrected to account for absorption of 0.511 Mev. photons within it. Although the absorption due to the graphite is negligibly small, that due to the lead is not.

Because of the target structure and the geometry of the counters, in over 50% of the annihilation events which occur in the target at least one of the photons has to pass through some lead in order to be counted. A theoretical estimate indicated that a correction amounting to more than 30% was involved. Although in principle it is possible to calculate a correction for this effect from theory, such a calculation would be extremely difficult. Fortunately, it is not difficult to obtain a value experimentally.

For this part of the experiment, the gamma ray source was 50 milligrams of radium borrowed from the Manitoba Cancer Relief Institute. The arrangement of the target, source, lead shielding and counters was as shown previously in Fig. 8. Ten different targets were used, each of which consisted of a lead disc 11/16 inches in diameter cemented to a graphite disc of the same diameter. The mass of lead in the targets covered a range from 0.1 to 3.3 grams. The coincidence rate for each target was measured as before, making use of a graphite blank.

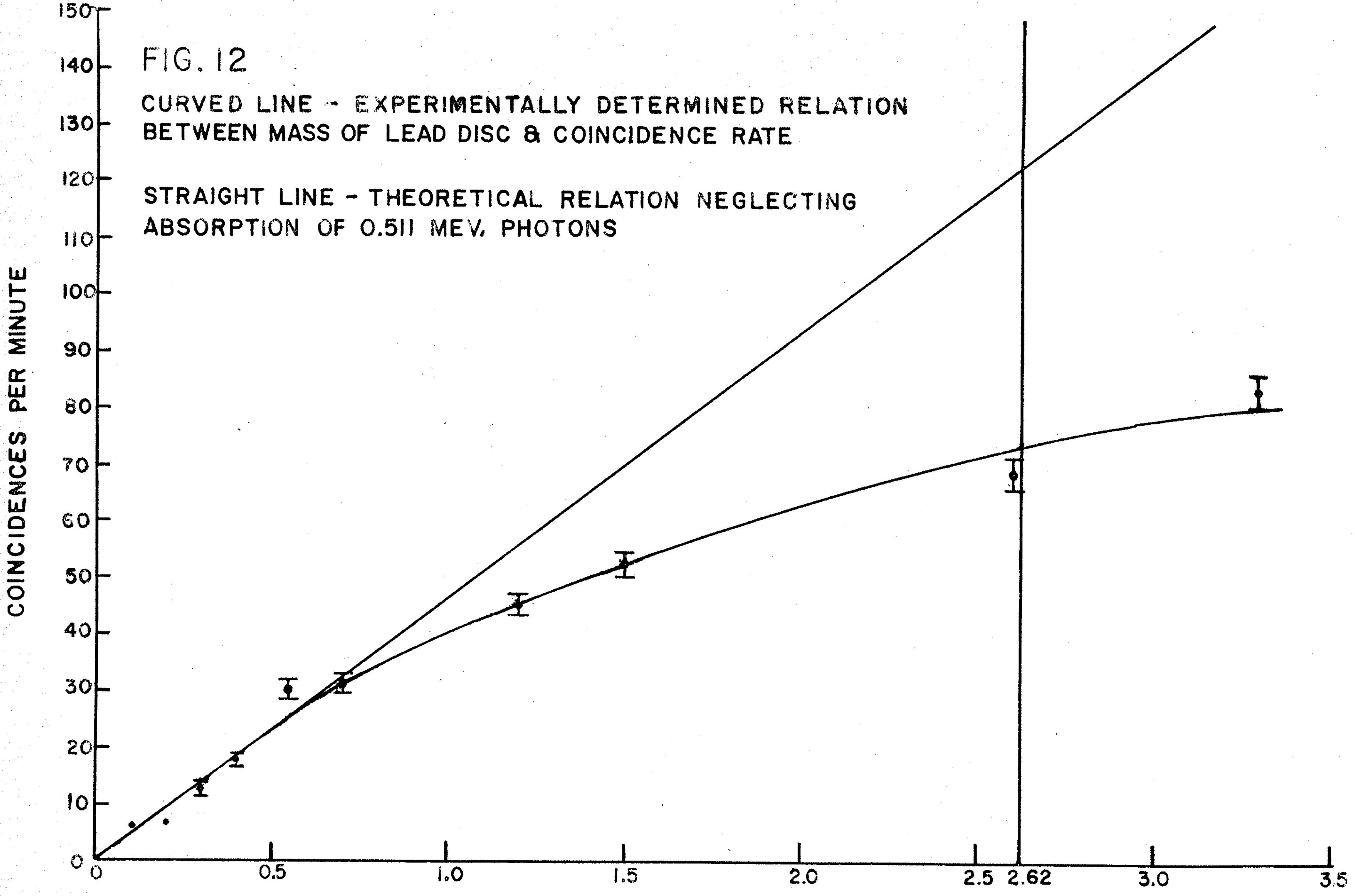

FIG. 12

CURVED LINE - EXPERIMENTALLY DETERMINED RELATION BETWEEN MASS OF LEAD DISC & COINCIDENCE RATE

STRAIGHT LINE - THEORETICAL RELATION NEGLECTING ABSORPTION OF 0.511 MEV. PHOTONS

Neglecting absorption, the coincidence rate for a given target is simply proportional to the mass of the lead therein, and a plot of coincidence rate against mass of lead for the 10 targets would be a straight line through the origin. The effect of absorption will be to reduce the counting rate in each case, the effect increasing as the mass of the lead increases. Thus one would expect the experimental points to lie on a curve which is asymptotic to the straight line at the origin and departs further from it as the mass of the lead is increased.

The results obtained are shown in Fig. 12. As can be seen, the first five points (mass of lead $\leq$ 0.55 grams) determine the asymptotic straight line unambiguously. It may also be seen that the following points deviate from this line by successively greater amounts. Two corrections were applied to these data because of the absorption of the primary photon beam in the lead discs. The first effect is that, because of the presence of the lead, the gamma ray flux seen by the graphite in one of the targets is smaller than that seen by the blank. Hence the counting rate to be subtracted as the contribution of the graphite to the total rate for a given target is slightly smaller than that obtained for the blank. The second effect is a result of the diminution in intensity of the primary photon beam as it passes through the lead disc. That is, if the disc is thought of as broken up into transverse layers, the photon flux at a given layer will depend on its' depth within the lead. This effect tends to lower the coincidence rate, and is greater for the heavier discs. Hence it would tend to distort the shape of the curve. Corrections were calculated making use of the values given by Davisson[11] for total gamma ray absorption coefficients in lead. They were found to amount to about 10% and 4% respectively for the heaviest target. The errors shown on the points in Fig. 12 are statistical counting errors. In the case of the first two points, the errors are of the order of the size of the dots.

The vertical line in Fig. 12 intersects the mass axis at the mass of the lead disc used in the measurements with Na^{24}. The ratio of the two coincidence rates (i.e. straight line to curve) at this mass value is 1.667 ± 0.08 from the graph. The error is also estimated from the graph. The net coincidence rate, due to the lead disc, obtained in the experiment with Na^{24} is therefore multiplied by this number to correct for the absorption of the 0.511 Mev. photons in the target.

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[11] Kai Siegbohn, "Beta and Gamma Ray Spectroscopy" (1st edition, Interscience Publishers Inc., 1955) P.869

FINAL RESULTS

Calculation of the Gamma Ray Flux

The coincidence rate due to the lead is calculated in the following section. The data used in the calculation were obtained from a number of separate runs made over a period of five days. The mean value given is that for noon, Feb. 20, 1956. It was obtained by extrapolating the separate runs to this time using the value 14.90 hours for the half-life of Na^{24}. Two different source to target distances, R_1 (54.8 and 34.1 centimeters) were used, the shorter distance being used toward the end of the experiment to compensate for the decay of the source. All the data taken at the shorter distance were converted to the larger one, assuming the coincidence rate to be proportional to R_1^{-2}. All the data so obtained were averaged to give the mean coincidence rate for this time and for the larger geometry.

The photon flux corresponding to this geometry was calculated assuming the source to be concentrated at a point 54.8 ± 0.5 cms. from the front face of the target. Since the dimensions of the source were small compared to the source to target distance, any error introduced by this simplification will be small. To less that 1% error, the solid angle subtended by the target disc at the source is given by:

$$\Omega = \frac{\pi r^2}{R_1^2}$$

where r is the radius of the disc. Hence if N photons per second are emitted by the source, the photon flux through the target disc is given by:

$$\phi \doteq \frac{N\Omega}{4\pi} = \frac{N}{4}\frac{r^2}{R_1^2}$$

Using the value for N obtainedpreviously, we get

$$\phi = (1.19 \pm 0.17) \times 10^2 /\text{sec.}$$

Effects due to the presence of the collimator have been omitted from the above calculation. First the aperture of the collimating channel was large enough so that all parts of the front face of the target were exposedto the primary photon beam. Hence the solid angle is, in fact, determined by the target diameter as assumed above. Second, it is possible that small angle compton scattering near the surface of the collimating channel could result in a large number of degraded photons which were still energetic enough to produce pairs, striking the target. In order to check this effect,the curves shown in Figure 13 were obtained. The detector used was a 1 X 1 inch cylindrical NaI(Tl) crystal mounted on a 2 inch Dumont photomultiplier (type 6292). The apparatus was set up with the lead shielding positioned as shown in Figure 8. The crystal was placed with its front face in the position previously occupied by the front face of the target. A single channel pulse height specrum yielded the solid curve in Figure 13. The lead was then removed and the spectrum repeated with the source in the same position relative to the crystal to obtain the dashed curve in Figure 13. The 1 inch crystal was used because, of those available, it was closest to the target diameter. It is only necessary to consider the portion of the curves lying above 1.022 Mev. The contribution of the scattered photons to the total pair production in the target is a function of the area between the curves. It may be seen from Fig. 13 that this area amounts to less than 10% of the area under the dashed curve which represents the primary photon flux. Furthermore, the biggest difference occurs for energies less than 1.4 Mev. Since the ratio of σpair at 2.76 Mev. to σpair at 1.38 Mev. is given by West[12] as 15.0 ±0.8, the ratio of σpair at 2.76 Mev. to σpair at energies less than 1.4 Mev. will be at least 15. Thus the contribution of this effect cannot be more than 1%.

Calculation of the rate of production of pairs in the target.

In the part of the experiment in which the target and blank were alternately irradiated with gamma rays from Na^{24}, 12,796 counts were obtained with the target, and 8,396 with the blank. These data lead to a value for the net coincidence rate due to the lead of

$$(5.54 \pm 0.08) \times 10^{-3} \text{ per second,}$$

The error given is the net statistical counting error. This value has been corrected for absorption of the 2.76 Mev. photons by the lead, and for loss of positrons around the edges and through the front face of the disc.

Applying the corrections given previously for

[12] H. J. West, Jr. Phys. Rev. 101, 915, (1956)

absorption of the 0.511 Mev. photons, and for detection efficiency to the above figure yields

0.966 ± 0.11 per second

for the 2 quanta annihilation rate.

Since the 1.38 Mev. gamma rays also produce pairs in the target, their effect must be accounted for. Using West's value for the ratio $\frac{\sigma\text{pair } 2.76 \text{ Mev.}}{\sigma\text{pair } 1.38 \text{ Mev.}}$ we obtain

0.906 ± 0.1 per second

for the 2 quanta annihilation rate of positrons produced by the 2.76 Mev. gamma ray.

At this point possible corrections because of 1 and 3 quanta annihilation processes which could not be detected by the coincidence counter must be considered. As was mentioned previously, the 3 quanta processes amount to no more than 1/370 of the 2 quanta processes and hence their effect is negligible compared to other errors in the experiment. The calculations of Jaeger and Hulme[13] give the ratio of 1 to 2 quanta annihilations as 0.01 in lead for positrons of the energy being dealt with here. The relative magnitude of the 1 quanta process is even smaller in carbon. Hence this effect may likewise be neglected. Actually these effects are to some extent accounted for automatically in the detection efficiency measurement. 1 and 2 quanta annihilations do occur for the positrons emitted by the Na^{22} source although to a somewhat different extent due to difference in positron energy. Thus in view of the smallness of the effect it is of little value to attempt further correction for it.

13. Jaeger and Hulme, Proc. Cambridge Phil. Soc. <u>32</u>, 158, (1936)

Calculation of σpair.

Let N be the rate at which pairs are produced in the lead.

σ be the total pair production cross section.

A be the chemical atomic weight of lead.

L be Avodadro's number.

a be the area of the lead disc.

m be the mass of the lead disc.

ϕ be the total photon flux through the disc.

$$\text{Then} \quad N = \frac{\phi m L \sigma}{aA}$$

$$\text{i.e.} \quad \sigma = \frac{aAN}{\phi m L}$$

Substituting in this equation we obtain:

σPair = 2.38 ± 0.62 barns.

Discussion of Results

The value obtained for σpair in this experiment favors the prediction of the Bethe-Heitler theory (2.85 barns) rather than that of Jaeger and Hulme (3.3 barns, obtained by extrapolating the value given previously for 2.66 Mev). This is at variance with the results of Hahn et al and Dayton, the result of whose Z dependence experiments favored the theory of Jaeger and Hulme.

During the course of this experiment, an experiment in which the same measurement was made was published by Schmid and Huber[14]. They give a value of σpair in lead of 3.12 ± 0.18 barns for 2.76 Mev. This represents an experimental error of about 6% as compared to some 25% in the present experiment. Their value is in agreement with that of Jaeger and Hulme.

Their experiment was similar to the present one

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[14] P. Schmid and P. Huber, Helv. Phys. Acta. 28, 369, (1955)

in that Na^{22} was used to determine the detection efficiency, and the source strength was measured by a coincidence technique (this technique was not described in the paper but was presumably similar to that used in the present experiment). The method used by Schmid and Huber to produce and measure the pairs was, however, quite different. They made use of a "sandwich" consisting of a thin disc shaped gamma ray source between two lead discs of the same diameter, the whole being housed in an aluminum "pill-box". This assembly wasplaced between a pair of scintillation counters in a coincidence circuit so that not only annihilation radiation but also primary photons from the source in large numbers entered the scintillators. Since liquid scintillators were employed the background radiation could not be separated from the annihilation radiation by differential pulse height discrimination as was done in the present experiment. The net result was that annihilation radiation constituted only about 25% of the total coincidence rate. In calculating σ_{pair} it was necessary to make large theoretical corrections. Since the error given is just the statistical counting error they have assumed that no error is implicit in these theoretical corrections.

It is the opinion of the author that an experiment which depends on large corrections calculated from one theory does not constitute a reliable check of another theory. Further, in view of personal experience, it is difficult to believe that the measurements involved in the experiment of Schmid and Huber could be made to the accuracy they claim, with their equipment.

In the present experiment all errors were estimated from a consideration of the worst possible case. For this reason, barring the existence of some instrumental error (a highly unlikely thing) it is felt that the value of σ_{pair} obtained is reliable within the stated limits of error.