THE UNIVERSITY OF MANITOBA

AN INVESTIGATION TO FIND THE UNDERSTANDING OF THE CONCEPT OF FUNCTIONALITY IN VERBAL STATEMENTS AND IN FORMULAS AMONG STUDENTS AT THE JUNIOR HIGH SCHOOL LEVEL

BEING A THESIS SUBMITTED TO THE COMMITTEE ON POSTGRADUATE STUDIES IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF EDUCATION

BY

LESLIE GEORGE MACKAY ROBINSON

WINNIPEG, MANITOBA

APRIL, 1936

ACKNOWLEDGMENTS

The writer wishes to express his heartfelt appreciation to Dr. D. S. Woods for his kind co-operation and assistance in the investigation and for his encouraging influence during the three summer sessions taken on the course, to Professor H. R. Low for his personal guidance during the investigation, to Mr. C. Cornish for his help and for the free use of his school during the preliminary work, and to all the principals and teachers who gave assistance in the administration of the series of tests used in the investigation.

Special acknowledgment is made to my wife without whose co-operation and encouragement during the past three years this work could not have been done.

TABLE OF CONTENTS

Page

LIST OF TABLES iv

LIST OF CHARTS xiii

Chapter

I. INTRODUCTION 1

Purposes of the Investigation
The Nature of the Investigation
Justification of the Study
The Importance of the Function Concept in Mathematics
The Poor Progress That is Being Made in Building Textbooks with the Function Concept as the Unifying Force
The Importance of the Formula
The Lack of Extensive Investigation with the Function Concept
The Abilities That Can be Developed by the Use of the Formula
Assumptions of the Investigation
Definitions of Certain Terms Used in the Investigation
Sources of Material for the Investigation

II. THE CONSTRUCTION, THE ADMINISTRATION, AND THE SCORING OF THE TESTS 23

The Construction of the Tests
The Administration of the Tests
The Marking of the Tests

III. THE EXTENT TO WHICH THE PUPILS ARE ABLE TO PUT TO USE THE ENTIRE GROUP OF FUNCTION CONCEPTS THAT ARE INCLUDED IN THE SET OF TESTS 32

IV. THE EXTENT TO WHICH THE PUPILS ARE ABLE TO PUT TO USE THE DIFFERENT FUNCTION CONCEPTS THAT ARE CONTAINED IN THE INDIVIDUAL TESTS 47

V. A DETAILED STUDY OF THE ERRORS MADE IN THE WORKING OF EACH QUESTION BY THE PUPILS OF THE JUNIOR HIGH SCHOOL GRADES 90

Chapter

Page

VI. SUMMARY AND CONCLUSIONS 157

A Summary of the Findings of the Investigation
Suggested Conclusions That Might be Drawn From the Findings of the Investigation
Lines Along Which Other Investigations Might Be Made

BIBLIOGRAPHY 166

Appendix

I. THE DIRECTIONS GIVEN TO THE TEACHERS AND TO THE PUPILS 169

II. INSTRUCTIONS FOR MARKING TESTS 171

III. THE SIX TESTS GIVEN IN THE INVESTIGATION 173

IV. THE STATISTICAL FORMULAS USED IN THE INVESTIGATION 180

LIST OF TABLES

Table		Page
1.	Showing the Schools, the Number of Groups from Each School, and the Number of Pupils, Girls and Boys, in Each Group	26
1.a	Showing the Number of Pupils in Each of the Junior High School Grades and How Many of These were Girls and How Many were Boys	27
2.	Showing the Medians for Boys and Girls and for Both for Each of the Junior High School Grades	33
3.	Showing the Means for Boys and Girls and for Both for Each of Junior High School Grades	33
4.	Comparative Tables of Gains Computed for the Medians and from the Means of the Distribution	36
5.	Summary of the Data Relative to the Distribution of Seventh Grade Scores on the Complete Set of Tests	39
6.	Summary of the Data Relative to the Distribution of Eighth Grade Scores on the Complete Set of Tests	39
7.	Summary of the Data Relative to the Distribution of Ninth Grade Scores of the Complete Set of Tests	40
8.	Summary of the Data Relative to the Distribution of the Scores of Grades Seven, Eight and Nine for the Complete Set of Tests	40
9.	Summary of the Data Relative to the Distribution of Tenth Grade Scores on the Complete Set of Tests	41
10.	Distribution of the Scores for the Seventh, Eighth, Ninth, and Tenth Grades and for the Combined Junior High School Grades	43
11.	The Range of Scores for Each Grade, the Standard Deviation and the Range Between the Central Tendencies of Successive Grades	46

Table		Page
12.	Grade Mastery Coefficients, Grade-to-Grade Gains and Ranks for the Five Tests	49
13.	The Grade Medians for Each of the Five Tests ...	50
14.	The Distribution of the Scores for Test I	53
15.	The Mastery Coefficients for the Questions of Test I for Each Grade and for the Junior High School Group Together with the Ranking of These Mastery Coefficients	55
16.	The Distribution of Scores for Test II	59
17.	The Mastery Coefficients for the Questions of Test II for Each Grade and for the Junior High School Group Together with the Ranking of These Mastery Coefficients	61
18.	The Distribution of the Scores for Test III	65
19.	The Mastery Coefficients for the Questions of Test III for the Grade Groups and for the Junior High School Group Together with the Ranking of These Mastery Coefficients for These Groups	67
20.	The Distribution of Scores for Test IV	72
21.	The Mastery Coefficients for the Questions of Test IV for Grade Groups and for the Junior High School Group Together with Ranking of These Mastery Coefficients for These Groups	73
22.	The Distribution of Scores for Test V	78
23.	The Mastery Coefficients for the Questions of Test V for the Grade Groups and for the Junior High School Group Together with the Ranking of These Mastery Coefficients for These Groups	81
24.	The Grade-Groups to Which both Test V and Test VI were Given with the Number of Pupils in Each Group	84
25.	The Median Score for Tests V and VI for the Groups That Took Both Tests	85

Table		Page
26.	The Distribution of Scores for the Special Groups for Test V and Test VI	85
27.	Showing a Comparison of the Mastery Coefficients for Tests V and VI for the Different Grade Groups Taking Both Tests	86
28.	Showing the Mastery Coefficients and the Error Coefficients for I(a) of Test I	94
28a.	Showing the Total Number of Errors for Each Grade and the Per Cent of This Number Due to Each Type of Error, Test I,(a)	94
29.	Showing the Mastery Coefficients and the Error Coefficients for 1 (b) of Test I	95
29a.	Showing the Total Number of Errors for Each Grade and the Per Cent of This Number Due to Each Type of Error, Test I, 1(b)	95
30.	Showing What Per Cent of the Total Questions for Test I (1) Had Both Parts Correct, (a) Part Alone Correct, (b) Part Alone Correct, and Neither Part Correct	96
31.	Showing the Mastery Coefficients and the Error Coefficients for Test I, 2 (a)	97
31a.	Showing the Total Number of Errors for Each Grade and the Per Cent of This Number Due to Each Type of Error, Test I, 2 (a)	98
32.	Showing the Mastery Coefficients and the Error Coefficients for Test I, 2 (b)	98
32a.	Showing the Total Number of Errors for Each Grade and the Per Cent of This Number Due to Each Type of Error, Test I 2 (b)	99
33.	Showing What Per Cent of the Total Questions for Test I (2) Had Both Parts Correct, (a) Part Alone Correct, (b) Part Alone Correct, and Neither Part Correct	99

Table		Page
34.	Showing the Mastery Coefficients and the Error Coefficients for Test I, 3 (a)	101
34a.	Showing the Total Number of Errors for Each Grade and the Per Cent of This Number Due to Each Type of Error, Test I, 3 (a)	101
35.	Showing the Mastery Coefficients and the Error Coefficients for Test I, 3 (b)	102
35a.	Showing the Total Number of Errors for Each Grade and the Per Cent of This Number Due to Each Type of Error, Test I, 3 (b)	102
36.	Showing the Per Cent of the Total Questions for Test I (3) Had Both Parts Correct, (a) Part Alone Correct, (b) Part Alone Correct, and Neither Part Correct	103
37.	Showing the Mastery Coefficients and the Error Coefficients for Test I, 4 (a)	104
37a.	Showing the Total Number of Errors for Each Grade and the Per Cent of This Number Due to Each Type of Error, Test I, 4 (a)	104
38.	Showing the Mastery Coefficients and the Error Coefficients for Test I, 4 (b)	105
38a.	Showing the Total Number of Errors for Each Grade and the Per Cent of This Number Due to Each Type of Error, Test I, 4 (b)	105
39.	Showing What Per Cent of the Total Questions for Test I, (4) Had Both Parts Correct, (a) Part Alone Correct, (b) Part Alone Correct, and Neither Part Correct	106
40.	Showing the Mastery Coefficients and the Error Coefficients for Test I, 5 (a)	108
40a.	Showing the Total Number of Errors for Each Grade and the Per Cent of This Number Due to Each Type of Error, Test I, 5 (a)	108
41.	Showing the Mastery Coefficients and the Error Coefficients for Test I, 5 (b).....	109

Table		Page
41a.	Showing the Total Number of Errors for Each Grade and the Per Cent of This Number Due to Each Type of Error, Test I, 5 (b)	109
42.	Showing What Per Cent of the total Questions for Test I, 5, Had Both Parts Correct (a) Part Alone Correct, (b) Part Alone Correct, and Neither Part Correct	110
43.	Showing the Combinations of Time, Rate, and Distance Found in Test I, 5 in the 907 Papers Examined	110
44.	Showing the Mastery Coefficients and Error Coefficients for Test I, 6 - Both Parts Combined	113
44a.	Showing the Total Number of Errors for the Two Parts of Question 6 Test I for Each Grade and the Per Cent of This Due to Each Type of Error.	114
45.	Showing the Per Cent of Answers of Each Combination That Appeared in Test I, 6	114
46.	Showing What Per Cent of the Total Number of Questions for Test I, 6 Had Two Parts Correct, What Per Cent Had Only One Part Correct, and What Per Cent Had Neither Part Correct	115
47.	Showing the Mastery Coefficients and the Error Coefficients for Test II, 1	118
47a.	Showing the Total Number of Errors for Each Grade and the Per Cent of This Number Due to Each Type of Error, Test II, 1.....	118
48.	Showing the Mastery Coefficients and the Error Coefficients for Test II, 2	119
48a.	Showing the Total Number of Errors for Each Grade and the Per Cent of This Number Due to Each Type of Error, Test II, 2	119
49.	Showing the Mastery Coefficients and the Error Coefficients for Test II, 3	120

Table		Page
49a.	Showing the Total Number of Errors for Each Grade and the Per Cent of This Number Due to Each Type of Error, Test II, 3.	121
50.	Showing the Mastery Coefficients and the Error Coefficients for Test II, 4	121
50a.	Showing the Total Number of Errors for Each Grade and the Per Cent of This Number Due to Each Type of Error, Test II, 4	122
51.	Showing the Mastery Coefficients and the Error Coefficients for Test II, 5	123
51a.	Showing the Total Number of Errors for Each Grade and the Per Cent of This Number Due to Each Type of Error, Test II, 5	123
52.	Showing the Mastery Coefficients and the Error Coefficients for Test II, 6	125
52a.	Showing the Total Number of Errors for Each Grade and the Per Cent of This Number Due to Each Type of Error, Test II, 6	125
53.	Showing the Mastery Coefficients and the Error Coefficients for Test III, 1	127
53a.	Showing the Total Number of Errors for Each Grade and the Per Cent of This Number Due to Each Type of Error, Test III, 1	128
54.	Showing the Mastery Coefficients and the Error Coefficients for Test III, 2	129
54a.	Showing the Total Number of Errors for Each Grade and the Per Cent of This Number Due to Each Type of Error, Test III, 2	129
55.	Showing the Mastery Coefficients and the Error Coefficients for Test III, 3	130
55a.	Showing the Total Number of Errors for Each Grade and the Per Cent of This Number Due to Each Type of Error, Test III, 3.	130

Table		Page
56.	Showing the Mastery Coefficients and the Error Coefficients for Test III, 4.	131
56a.	Showing the Total Number of Errors for Each Grade and the Per Cent of This Number Due to Each Type of Error, Test III, 4.	132
57.	Showing the Mastery Coefficients and the Error Coefficients for Test III, 5.	133
57a.	Showing the Total Number of Errors for Each Grade and the Per Cent of This Number Due to Each Type of Error, Test III, 5.	133
58.	Showing the Mastery Coefficients and the Error Coefficients for Test III, 6.	134
58a.	Showing the Total Number of Errors for Each Grade and the Per Cent of This Number Due to Each Type of Error, Test III, 6.	135
59.	Showing the Mastery Coefficients and the Error Coefficients for Test IV, 1.	136
59a.	Showing the Total Number of Errors for Each Grade and the Per Cent of This Number Due to Each Type of Error, Test IV, 1.	136
60.	Showing the Mastery Coefficients and the Error Coefficients for Test IV, 2.	137
60a.	Showing the Total Number of Errors for Each Grade and the Per Cent of This Number Due to Each Type of Error, Test IV, 2.	138
61.	Showing the Mastery Coefficients and the Error Coefficients for Test IV, 3.	139
61a.	Showing the Total Number of Errors for Each Grade and the Per Cent of This Number Due to Each Type of Error, Test IV, 3.	139
62.	Showing the Mastery Coefficients and the Error Coefficients for Test IV, 4.	140

Table		Page
62a.	Showing the Total Number of Errors for Each Grade and the Per Cent of This Number Due to Each Type of Error, Test IV, 4.	141
63.	Showing the Mastery Coefficients and the Error Coefficients for Test IV, 5.	142
63a.	Showing the Total Number of Errors for Each Grade and the Per Cent of This Number Due to Each Type of Error, Test IV, 5.	142
64.	Showing the Mastery Coefficients and the Error Coefficients for Test IV, 6.	143
64a.	Showing the Total Number of Errors for Each Grade and the Per Cent of This Number Due to Each Type of Error, Test IV, 6.	144
65.	Showing the Mastery Coefficients and the Error Coefficients for Test V, 1.	146
65a.	Showing the Total Number of Errors for Each Grade and the Per Cent of This Number Due to Each Type of Error, Test V, 1.	146
66.	Showing the Mastery Coefficients and the Error Coefficients for Test V, 2.	147
66a.	Showing the Total Number of Errors for Each Grade and the Per Cent of This Number Due to Each Type of Error, Test V, 2.	148
67.	Showing the Mastery Coefficients and the Error Coefficients for Test V, 3.	149
67a.	Showing the Total Number of Errors for Each Grade and the Per Cent of This Number Due to Each Type of Error, Test V, 3.	149
68.	Showing the Mastery Coefficient and the Error Coefficients for Test V, 4.	150
68a.	Showing the Total Number of Errors for Each Grade and the Per Cent of This Number Due to Each Type of Error, Test V, 4.	150

Table		Page
69.	Showing the Mastery Coefficients and the Error Coefficients for Test V, 5.	151
69a.	Showing the Total Number of Errors for Each Grade and the Per Cent of This Number Due to Each Type of Error, Test V, 5.	151
70.	Showing the Mastery Coefficients and the Error Coefficients for Test V, 6.	152
70a.	Showing the Total Number of Errors for Each Grade and the Per Cent of This Number Due to Each Type of Error, Test V, 6.	153
71.	Showing the Mastery Coefficients and the Error Coefficients for Test V, 7.	154
71a.	Showing the Total Number of Errors for Each Grade and the Per Cent of This Number Due to Each Type of Error, Test V, 7.	154
72.	Showing the Mastery Coefficients and the Error Coefficients for Test V, 8.	155
72a.	Showing the Total Number of Errors for Each Grade and the Per Cent of This Number Due to Each Type of Error, Test V, 8.	155

LIST OF CHARTS

Chart		Page
I.	To Show the Grade to Grade Advancement by Use of the Median Scores	38
II.	To Show the Percentage Distribution of Scores for Each Grade and for the Junior High School Group	44

CHAPTER I

INTRODUCTION

Purposes of the Investigation

The investigation which will be reported in the following pages was undertaken mainly for the purpose of finding to what extent the function concepts of mathematics could be put to use by representative groups of pupils at the beginning of the seventh, eighth and ninth grades. In the investigation the attempt has been made to adduce evidence bearing upon three main questions:

1. To what extent are the pupils commencing the seventh, eighth and ninth grades able to put to use the entire group of function concepts that are included in the set of tests to be given?

2. To what extent are the pupils commencing the seventh, eighth and ninth grades able to make use of the individual concepts within the separate tests?

3. What is the nature and extent of the errors made by the pupils in dealing with each question of the different tests?

The Nature of the Investigation

The investigation is an attempt to make a study of the conditions as they exist in the city and suburban

schools of Manitoba today. In no instance has there been any attempt to set up controlled groups. The groups tested were carefully selected in order to insure a representative cross-section of the junior high school grades in the city and suburban schools. Specific directions were given to the teacher who gave the tests to the different groups. In no case was the teacher to make any comment on any part of the tests nor was she to give any instructions to the pupils other than those that she was asked to read to the class before the writing of the tests. In this way the conditions under which the tests were written approximated standard conditions. The first marking of the tests was done by the students of the Faculty of Education under the close supervision of Dr. D. S. Woods and by three mathematics teachers of a city school. The second and third marking was done by the present writer and an efficient assistant with considerable experience in the teaching of mathematics. The writer did the final marking of Test I himself for all the papers.

Justification of the Study

There are four reasons why an investigation of this nature would be of considerable value. These are:

1. Because of the important place that the function concept is given in the teaching of mathematics by

authorities.

2. Because of the poor progress that is being made in the building of textbooks in mathematics with the function concept as the unifying force of the book.

3. Because of the very important place that the formula has been given in the teaching of mathematics today.

4. Because of the lack of extensive investigation in the field of functionality.

These will be discussed in the order named. Extensive use will be made of direct quotations taken from the available source material.

The Importance of the Functional Concept in Mathematics

This concept has within recent years been stressed as one of the most important aspects of mathematics in school. Klein of Germany should be and is given much of the credit for fathering the movement which has emphasised the importance of functionality in mathematics. To quote from Breslich:[1]

> In 1893 Professor Felix Klein of Germany in an address before the International Congress of Mathematicians at its meeting in Chicago directed force-

---

[1] E. H. Breslich, Measuring the Development of Functional Thinking in Algebra. (The Seventh Year Book of the National Council of Teachers of Mathematics), pp, 93-118. New York: Bureau of Publications Teachers' College, Columbia University, 525 West 120th St., 1932.

> fully the attention of the teachers of secondary school mathematics to the possibility and need of developing functional thinking in all their courses.

At the turn of the present century the recognition of the function concept showed itself in the leading countries of western Europe and in America. There followed a period in which articles were published, addresses were made, and committee reports were submitted on and about functionality in mathematics.[2] Today, as a result, it is the opinion of leading authorities in mathematics teaching that courses in mathematics should be built with functional thinking as the chief end in view.

Breslich in dealing with the mathematics of secondary schools writes:[3]

> It is readily conceded that the ability to think functionally is a major objective of mathematics.... So important is the function concept that mathematics has sometimes been defined as 'the science of serial, spacial, quantitative and magnitudinal relations.'

On page 6 of the Report of the Mathematical Association Committee 1910 may be seen the following statement:[4]

---

[2]H. R. Hamley, Relational and Functional Thinking, (The Ninth Year Book of the National Council of the Teaching of Mathematics), pp. 48-84. New York: Bureau of Publications, Teachers' College Columbia University, 1934.

[3]Breslich, The Administration of Mathematics in Secondary Schools, pp. 160-161. Chicago: The University of Chicago Press, 1933.

[4]Report of the Mathematical Association Committee 1910, The Teaching of Algebra and Trigonometry, p. 6. London: G. Bell and Sons Ltd., Portugal Street, Kingsway and Bombay, 1910.

> The idea of functionality is so important, socially as well as scientifically, that it should be fundamental even in an elementary course of mathematics.

The quotation given is from a British report. To show that the consensus of opinion was similar in America the following is taken from an American report submitted in 1923:[5]

> The one great idea which is best adapted to unify the course is that of functional relation. The concept of variable and of the dependence of one variable upon another is of fundamental importance to everyone.....
> .... The primary and underlying principle of the course should be the idea of relationship between variables, including the method of determining and expressing such relationships. The teacher should have this idea constantly in mind, and the pupil's advancement should be consciously directed along the lines which will present first one and then another of the ideas upon which finally the formation of the general concept of functionality depends.

So great was the demand for material on functionality that the Ninth Year Book of the National Council of Mathematics was devoted entirely to Dr. Hamley's article "Relational and Functional Thinking". This is undoubtedly one of the most extensive contributions on the subject that has been issued. He speaks of the importance of functional thinking as follows:[6]

---

[5] The Reorganization of Mathematics in Secondary Education, A report by the National Committee on Mathematical Requirements Under the Auspices of the Mathematical Association of America, Inc., pp. 15-16. Cambridge: Houghton Mifflin Company, The Riverside Press, 1923.

[6] H. R. Hamley, Op.cit., pp. 6-8.

> The world is becoming, as Klein expressed it, 'functionally minded.' Newspapers and magazines are using, to an increasing extent, not only functional language, but also the functional tools of the mathematician.....
> .... That being the case, it is our duty to take cognizance of the facts and to reorganize our mathematical teaching so that our youth may receive the knowledge and discipline that it needs to meet the changing time.
>
> Thoughtful teachers are now awakening to the fact that, if school mathematics is ever to meet the demands of modern life, or even to win the respect of the average man of affairs, it must be made more dynamic and functional.
> .....the purpose of this study is to examine the function concept in all its bearings and to justify the claim that has just been made by showing that the conception of function may be regarded as the natural co-ordinating principle of all school mathematics.

Further quotations[7] would but confirm what has been

---

[7]Percy T. Nunn, The Teaching of Algebra (Including Trigonometry), p. 51. London E.C.4,: Longman's Modern Mathematical Series, Longman's, Green and Co., 39 Paternoster Row, 1924.

[a]N. J. Lennes, The Function Concept in Elementary Algebra, (The Seventh Year Book of the National Council of Teachers of Mathematics), p. 55. New York: Bureau of Publications, Teachers' College, Columbia University, 1932.

[b]J. J. Jablonower, Recent and Present Tendencies in the Teaching of Algebra in the High Schools, (The Seventh Year Book of the National Council of Teachers of Mathematics), pp. 13-14. New York: Bureau of Publications, Teachers' College, Columbia University, 1932.

[c]Harry C. Barber, Teaching Junior High School Mathematics, p. 102. Cambridge: Houghton Mifflin Company, The Riverside Press, 1924.

[d]B. R. Buckingham, The Social Value of Arithmetic, (The Twenty-ninth Year Book.. Report of the Society's Committee on Arithmetic...National Society for the Study of Education), p. 54. Bloomington, Ill.: Public School Publishing Company, 1930.

said showing the great importance of this concept. Those given are sufficient to show that there is no field in the teaching of mathematics in junior and senior high schools that gives more justification for an investigation at this time than does that of the function concept.

The Poor Progress That is Being Made in Building Textbooks in Mathematics with the Function Concept as the Unifying Force

In 1928 Breslich[8] selected four typical textbooks, two geometries and two algebras, and made a very careful analysis of their content. He claims that the books chosen gave typical presentations of algebra and geometry and therefore the findings would indicate what was general practice at that time. He found that the one book of algebra associated the functional concept only with the graph while the other associated it with the graph and the formula. He writes:[9]

> Thus, even when authors express the intention of stressing functional thinking in algebra, they fail to make use of at least one out of nine opportunities offered.

The geometries failed more in this respect than did the

---

[8]E. H. Breslich, Developing Functional Thinking in Secondary School Mathematics, (The Third Year Book of The National Council of Teachers of Mathematics), pp. 42-43. New York: Bureau of Publications, Teachers' College, Columbia University, 1928.

[9]Ibid., p. 43.

algebras. Of them Breslich writes as follows:[10]

> In geometry the situation is even less satisfactory than in algebra. Although by far the greatest part of plane geometry deals with relationships, one of the two textbooks examined disregards the function concept entirely while in the other it is associated with the changes taking place in the formula corresponding to changes in geometric figures.

Lennes selected eight elementary algebras that had been published during the period 1928-1932. His purpose was to find to what extent textbooks dealt with functionality. He says of the books chosen:[11]

> Some of these are what may be called conservative books and include revisions of very widely used texts of long standing. Others are 'radical' or 'progressive' (according to the bias of the writer). They include all the newer books that are receiving serious attention.

He sums up his findings as follows:[12]

> In the majority of classes in elementary algebra, possibly sixty-five per cent or more, the ideas of function and of variation are introduced in a perfunctory way which cannot leave any lasting impression on the minds of the students. .... In no case can functionality or variation be said to be more than one of a large number of topics studied in the course. In the majority of cases these topics have no organic relation with the rest of the work and appear to be brought in for extraneous reasons..... ....Certain it is that in the vast majority of our classes in algebra, not only have we failed to make the function concept an all-pervading or unifying principle, but the consideration of it is so perfunctory that little would be lost if it were left out entirely.

---

[10] Ibid., p. 43.

[11] Lennes, Op.cit., p. 58.

[12] Ibid., p. 59.

The investigation made by Hamley[13] was much more extensive than those of Breslich and Lennes. He examined a large number of the leading textbooks of America, England, Germany, Austria, France and Hungary to find to what extent the functional concept was being made use of in these textbooks. He includes the reviews of more than forty algebras in the publication of his findings. He found that in each of these countries there was a marked attempt to give a prominent place to the functional concept in the textbooks the lead being taken by Germany and America. Further consideration of his findings will be confined to the books of America.

The books examined were published since 1923 and Hamley says that nearly all show the influence of the report of the National Committee on Mathematical Requirements. Of the books Hamley wrote:[14]

> In some cases the influence of the report has been a superficial one; chapters on graphical representation, variables and functions have been added in response to a demand, but their inclusion has not changed, in any essential feature, the formal character of the whole. In other cases, the influence has been more marked, for lately there have appeared books of entirely new form and embodying a new spirit. This is particularly true of books published since 1928, which, from the point of view of functional thinking, are a distinct advance on those of

---

[13] Hamley, Op.cit., pp. 85-110.

[14] Ibid., p. 103.

> the older type. The impression created by a close study of American textbooks is that in many cases the authors have tried to satisfy too many demands.

It is necessary to keep in mind that the books selected were those that claimed to give a place of importance to the functional concept. In spite of this, a reading of the first sixteen reviews of American books shows that the author failed in every case to make functional thinking the main object of his work. In most cases the textbooks show that the idea of functionality has been missed entirely. In introducing the reviews of the books that succeeded best in their use of the functional concept, Hamley writes:[15]

> As we have already indicated, American mathematical textbooks published since 1928 show a striking contrast to those published before that date. In the most recent works the function concept which had previously been largely incidental, began to show signs of becoming constitutional. It may be claimed by the writer of the modern textbook, with greater justice than formerly, that the function concept is the unifying principle of the whole course.

Of the four books deserving special mention in this period Hamley says:[16]

> In each of these a praiseworthy attempt had been made to develop the function concept as the central theme of the book.

The findings of Hamley might seem to be contrary to

---

[15] Ibid., pp. 107-108.

[16] Ibid., p. 108.

those of Breslich and Lennes but such is not the case. They are in line with the others. It is well to keep in mind that of the twenty-one books of which reviews were given only four had succeeded in making the functional concept the central theme of the book. Secondly, these books were all selected because they claimed to be giving a definite place to the function concept. Thirdly, these books were only a small part of the total number of American textbooks examined for Hamley says:[17]

> Of the large number of books examined, comparatively few can be reviewed here.

It is therefore safe to conclude from the investigations of these three men that the function concept is only very slowly finding its way into the textbooks today.

The Importance of the Formula

There is no dearth of material to prove that the formula is very important in the algebra of the junior and senior high schools today. Smith and Reeve write in "The Teaching of Junior High School Mathematics":[18]

> It should be reiterated that the great objective in elementary algebra is the ability to use formulas.

Breslich places the formula in a place of high

---

[17] Ibid., p. 103.

[18] D. E. Smith and D. B. Reeve, The Teaching of Junior High School Mathematics. p. 67. Boston, New York, Chicago: Ginn and Company, 1927.

importance in the teaching of algebra for he writes:[19]

> The teaching of the formula calls for the united effort of junior high school teachers. The formula deserves a prominent place in all mathematical courses. The instruction in it should start early and never cease.

In the footnote to page eight of "The Reorganization of Mathematics in Secondary Education one may read:[20]

> The importance of the formula has received recognition by explicit inclusion in the C.E.E.B. requirements in Elementary Algebra.

To quote Godfrey and Siddons:[21]

> We have already said that one of our chief aims in algebra is the idea of generalization, and the construction and use of formulae.

Stella Stephens of Peabody College made a diagnostic study of the difficulties in formulas in first year algebra. She concludes the introductory chapter, in which she has been attempting to show the importance of the formula, by the use of the following statement:[22]

---

[19]Breslich, The Administration of Mathematics in Secondary Schools, p. 374.

[20]The Reorganization of Mathematics in Secondary Education. Op.cit., p. 8.

[21]Charles Godfrey and A. W. Siddons, The Teaching of Elementary Mathematics, p. 196. Cambridge: At the University Press, 1931.

[22]Stella Stephens, Diagnosis of Student Difficulties in Formulas in First Year Algebra. (An unpublished Masters Thesis) p. 9. Nashville, Tenn.: George Peabody College for Teachers, 1930.

> The formula, therefore, as judged from the standpoint of (1) college entrance requirements, (2) the views of the authors of modern textbooks, and (3) the opinions of prominent teachers of mathematics, occupies a very important and secure place in the field of secondary mathematics. An adequate teaching program in first year algebra should have as one of its objectives, the mastery of the unit of formulas.

One need but read articles of Nunn,[23] Durell,[24] Barber,[25] and Hamley[26] to remove any doubts that still remain concerning the importance of the formula.

It is possible though, as for the function concept, that the formula be given a high rank of importance by authorities and by the writers of textbooks in prefaces to the books and still not be given a place of prominence within the textbook itself. But such is not the case. The findings of Hamley on this matter are summed up as follows:[27]

> No discussion of the function concept in mathematics would be complete without some reference to the formula. We do not propose to deal at length on this aspect of elementary mathematics, however, because its importance seems to be generally recognized. Most modern textbooks follow the lead first given by Nunn and place the formula in the forefront of their teaching.

---

[23]Nunn, Op.cit., p. 63.

[24]Clement V. Durell, The Teaching of Elementary Algebra. London: G. Bell and Sons Ltd., 1931.

[25]Barber, Op.cit., p. 88.

[26]Hamley, Op.cit., pp. 119-120.

[27]Hamley, Ibid., p. 119.

A careful reading of the reviews given by Hamley[28] of more than forty algebras shows that the formula plays a very important place in many of these books. It is significant that the books that make the best use of the function concept have also made extensive use of the formula. It would seem that it would be difficult to teach the function concept in algebra with much success without using the formula.

The formula has still another claim to importance. It has not only been given a place of prominence in the textbooks but it has been one of the means used to change the introduction of the child to algebra from the old academic method, consisting of exercises with the fundamental operations, to the present system which makes use of the formula or the equation. Whether the formula or the equation should come first in algebra is still a disputed question but the majority of authorities favor the use of the formula. In the following quotation Durell makes reference to both the old and the new methods. He also states his preference for the use of the formula for the first steps in algebra. He writes:[29]

---

[28] Ibid., pp. 85-110.

[29] Durell, Op.cit., pp. 25-26.

> The abstract academic method of teaching algebra is dead, or nearly so; the subject is now developed either from the equation or the formula..... .....Algebra should be developed from the formula.

To quote from Smith and Reeve:[30]

> The most hopeful way to begin the study of algebra is with the formula; for not only is this the most important part of algebra and the most purposeful approach to the subject, ... It is for this reason that we have come to make this feature (the formula), instead of the equation, the essential part of algebra, -- the heart and brain of the subject.

Barber also gives his support to the formula for he writes:[31]

> The first steps in the new algebras are these: to understand simple formulas which state familiar rules, and to be able to translate from words to formulas and back again, and then to understand enough of substitution to make use of the formulas. The next steps should lead to an understanding of what equations are for and how they work in very simple cases.

Nunn takes a similar stand for he says:[32]

> In other words, the earliest lessons in the subject (algebra) should teach the use of the formula..... .....The gradual elaboration of the formulae as an instrument of description and investigation is, then, the first business of the course in algebra.

On the other hand, Godfrey and Siddons[33] are of the opinion that the equation and problem should be used to introduce the early algebra. They take a strong stand for

---

[30]Smith and Reeve, Op.cit., p. 174.

[31]Barber, Op.cit., p. 97.

[32]Nunn, Op.cit., pp. 17-18.

[33]Godfrey and Siddons, Op.cit., p. 167.

this and give definite reasons for their choice. Still, they would not allow either the formula or the equation to exclude the other. In a later part of their book they show how much they believe an understanding of the equation would be aided by work with the formula, for they write:[34]

> I am convinced that the solution of literal equations has more meaning for the boy after he has done some transformation of the formulae.

The Lack of Extensive Investigation With the Function Concept

Shortly before contributing his article on "Measuring the Development of Functional Thinking in Algebra,"[35] to the Seventh Year Book, Breslich constructed a series of tests with which he planned to measure the growth of functional thinking in grades nine to twelve. He had these tests given to 1,802 pupils at the various high school levels. From this first use of the tests he made a comparison between the results of the different grades. But he intended to carry his investigation further by using the pupils of the laboratory school of the University of Chicago and co-operating schools by giving the same pupils the same tests for two or three successive years to measure the growth of functional thinking within individ-

---

[34] Ibid., p. 212.

[35] Breslich, Measuring the Development of Functional Thinking, pp. 93-118.

uals over a period of time. He had not commenced this part of his investigation in 1932.

When Breslich decided to make his investigation, he found that there was a lack of material for this type of testing. In commenting on this in his article he says:[36]

> No published test was available for the purpose but some material was collected from mathematics textbooks. It was necessary to create additional suitable text material.

The source books used by the present writer make no mention of any investigation on the function concept other than that of Breslich.

It is therefore safe to say that the amount of investigation in this field is far from being extensive.

The Abilities That Can be Developed By the Use of the Formula

To show the wide use to which the formula lends itself as a function concept a short discussion will be devoted to the abilities that can be developed in the child by the use of the formula. A number of lists of abilities will be shown as submitted by selected authorities while other lists will be mentioned.

The list of abilities given by Smith and Reeve are:[37]

---

[36] Ibid., p. 102.

[37] Smith and Reeve, Op.cit., p. 67.

1. The ability to discover rules and to translate these into formulas.

2. The ability to translate formulas into rules.

3. The ability to evaluate formulas.

4. The ability to derive one or more formulas from a given formula.

5. The ability to understand the dependence of one quantity upon another.

6. The ability to work with ordinary simple formulas.

In the seventh chapter of his work on "Functional Thinking" Hamley sets up a course in mathematics in which the function concept is the central theme. In his treatment of the formula he would attempt to engender the following abilities:[38]

1. The ability to understand what a formula is.

2. The ability to see the functional relation expressed by a formula.

3. The ability to understand the dependence of the difference in terms of the formula upon each other.

4. The ability to understand the meaning of 'the subject of a formula'.

5. The ability to change the subject of the formula.

6. The ability to build formulas.

---

[38] Hamley, Op.cit., pp. 138-141.

The lists of Thorndike,[39] Stephens[40] and Breslich,[41] as well as that given in "The Reorganization of Mathematics in Secondary Education"[42] show slight differences but in each case the main concepts have been kept in mind.

Considerable use was made of these lists together with the tests of Breslich[43] and those of Smith and Reeve[44] in the construction of the series of tests used in the present investigation.

Assumptions of the Investigation

The fundamental assumptions that have been made for this investigation are:

(a) That the pupils who wrote the tests represent a typical cross-section of the seventh, eighth and ninth grade suburban and city school population.

(b) That the items used in the investigation are

---

[39]Edward L. Thorndike, The Psychology of Algebra, p. 309. The MacMillan Company, 1923.

[40]Stephens, Op.cit., pp. 14-18.

[41]Breslich, The Administration of Mathematics in Secondary Schools, pp. 270-273.

[42]The Reorganization of Mathematics in Secondary Education. Op.cit., p. 32.

[43]Breslich, Measuring the Development of Functional Thinking, pp. 93-117.

[44]Smith and Reeve, Op.cit., pp. 350-355.

suitable to test the understanding of the concepts for which they have been selected.

(c) That the child has an understanding of the concept if he is able to secure fifty per cent of the possible score for that test.

(d) That a correct answer to any given item will indicate an understanding of the concept in that special form to the extent of the pupil being able to put the concept to use.

(e) That an incorrect answer for any given item will indicate a lack of understanding of the concept in that special form to the extent of being able to put the concept to use.

(f) That the conditions under which the tests were given were uniform in all cases for specific directions as to the administration of the tests were given.

Definitions of Certain Terms Used in the Investigation

In the subsequent pages of the investigation the following terms will be used as defined below:

The Score Made by a Pupil: The total number of items for which the pupil gave the correct answer. This will be used in dealing with both the entire series of tests and the individual tests.

Mastery Coefficient: The per cent of the given group that has secured the correct solution to the item or to any given series of items.

Error Coefficient: The per cent of the given group that has failed to secure the correct solution to the item or to any given series of items.

Grade Group: The pupils of one grade. The grade seven pupils form one grade group, the grade eight pupils form another grade group, while the grade nine pupils form another grade group.

Junior High School Group: All the pupils of the junior high school grades (seven, eight, and nine) writing the particular test under consideration.

Sources of Material For the Investigation

The work of the investigation proper was original and was done by the present writer with the co-operation of the Faculty of Education of the University of Manitoba and of the principals and teachers of the schools in which the tests were written. To provide the proper ground work for the investigation use was made of the following:

1. Books and articles[45] on mathematical topics

---

[45]These are all included in the bibliography.

but more especially those dealing with the function concept and the formula.

2. Certain investigations[45] made by others in neighboring fields.

3. The available textbooks[45] on junior and senior high school mathematics.

4. All the available mathematical examinations that had been given to the Junior high school grades in the Winnipeg Public Schools from the School Board Offices during the period 1926-1936.

5. The results of the preliminary tests that were given to certain grades before the final series of tests was set up.

CHAPTER II

THE CONSTRUCTION, THE ADMINISTRATION, AND THE SCORING OF THE TESTS

It is the purpose of this chapter to show the methods used in the construction, administration, and scoring of the tests used in the investigation. A separate section of the chapter will be devoted to each of the given topics.

The Construction of the Tests.- With the co-operation of the principal of the city school used in the investigation the investigator constructed and gave a number of preliminary tests to a group of one hundred and twenty-one pupils. This group consisted of one grade seven class, one grade eight class, and one grade nine class.

A series of five tests was constructed and given to the one hundred and twenty-one pupils. The tests attempted to find to what extent the pupils could put to use five important function concepts included in the verbal statement and in the formula. The five concepts were:

1. Recognising Relationships
2. Writing Formulas
3. Substitution in Formulas

4. Deriving New Formulas or Changing the Subject of a Formula

5. Functional Relations or Variations in Formulas

The first tests given proved to be too difficult for any but the grade nine pupils. A second series was carefully constructed in which the individual items making up the tests were quite simple but still showed whether or not the child understood the concept. Each of the five tests was given separately with brief directions similar to those used in the investigation.[1] Each pupil handed in his test when he had completed it. The time required for each test was marked on his test paper. After the marking, separate score sheets were made for each grade for each test. These sheets showed an entry for each question for each pupil. Each therefore showed the score made by each pupil on the sheet, the number of pupils getting each question correct, and the time taken by each pupil for that test. By the use of all the score sheets it was possible to find the general results for the series of tests, the results for the separate tests, the results for the separate questions of the different tests, and the average time and range of time for each test. A careful study was made of the results shown for each item on the

---

[1] See the explanations preceding the tests in the Appendix.

tests. A suitable number of the best items was selected for each of the final tests.

The concepts tested in this investigation are concepts that had not been taught to the pupils of the junior high school grades early in the year. It was necessary to precede each test by a short explanation of the concept being tested and to accompany the explanation with illustrations. Except for Test V, each was headed by a sample question. It was thought best to give no sample for Test V.

The results secured from Test V led to the construction of Test VI which had for its purpose the testing of the understanding of similar item-concepts except that numbers and words were used in place of formulas and elements of formulas as had been the case in Test V. Test VI was given to a relatively small selection of pupils all of whom were in School I.

The Administration of the Tests.- Four of the five schools used in the investigation were suburban. Within the schools chosen use was made of seven grade sevens, eight grade eights, and eight grade nines. Every care was taken to make the selected groups as representative as possible of a cross section of the junior high school population of the city and suburban schools. Dr. D. S. Woods carefully supervised the selection of the schools

and the classes in the suburban area. There was only one city school used but about one-third of the pupils tested were from that school. It was thought better in this case

TABLE I

SHOWING THE SCHOOLS, THE NUMBER OF GROUPS FROM EACH SCHOOL, AND THE NUMBER OF PUPILS, GIRLS AND BOYS, IN EACH GROUP

School	Group		Number of Girls	Number of Boys	Total Number
I	A	VII	24	15	39
I	B	VII	19	23	42
II	C	VII	21	28	49
III	D	VII	25	13	38
IV	E	VII	16	15	31
V	F	VII	22	19	41
III	G	VII	19	21	40
I	H	VIII	20	23	43
I	I	VIII	24	15	39
I	J	VIII	19	22	41
IV	K	VIII	24	19	43
III	L	VIII	16	25	41
II	M	VIII	12	15	27
V	N	VIII	21	20	41
II	O	VIII	21	12	33
I	P	IX	12	28	40
I	Q	IX	16	15	31
I	R	IX	21	22	43
IV	S	IX	19	24	43
V	T	IX	20	7	27
III	U	IX	24	24	48
II	V	IX	19	28	47
II	W	IX	14	26	40
			448	459	907

The table reads: In School I is Group A a grade 7, consisting of 24 girls and 15 boys or a total of 39 pupils, etc.

to take a wide sampling of grades within one school than a smaller sampling in a number of schools.

Table I shows the schools used, the groups from each school, the number in each group and of that number, the number of boys and the number of girls. It also shows that 907 pupils in the junior high school were given the test in this investigation. Of this number 448 were girls and 459 were boys.

Table I (a) shows the number of pupils who wrote the tests in each of the junior high school grades and of these how many were girls and how many were boys.

TABLE I (a)

SHOWING THE NUMBER OF PUPILS IN EACH OF THE JUNIOR HIGH SCHOOL GRADES AND HOW MANY OF THESE WERE GIRLS AND HOW MANY WERE BOYS

Grade	Girls	Boys	Total
VII	146	134	280
VIII	157	151	308
IX	145	174	319
Total	448	459	907

The table reads: In grade 7 there were 146 girls and 134 boys, making a total of 280 pupils; in grade 8 there ... etc.

The investigation was chiefly concerned with the pupils and grades shown in Tables I and I(a) but use was

also made of four grade ten classes consisting of 28 girls and 76 boys. These pupils represented a sample of the finished product of the city junior high school. They were all taken from the same city high school and represented poor, average, and good classes.

The tests were given to the pupils of the suburban schools by fifteen carefully selected students of the Faculty of Education of the University of Manitoba under the supervision of Dr. D. S. Woods and Professor H. R. Low. The tests were given to the pupils of the one city school by the mathematics teachers of the classes used under the direction of the present writer. Specific instructions[2] were given to those giving the tests and every possible effort was made to make uniform the conditions under which the different classes wrote the tests. In no case was any person giving the tests to make any comments on them, nor was he to answer any questions of the pupils concerning the questions. The time given for each test, including the time required for reading the short preliminary explanation, was ten minutes. In the preliminary testing this had proved ample time to allow nearly every pupil to finish the test. The pupils were asked not to proceed with the next test until they had received instructions

---

[2]See the Appendix for the complete instructions.

to do so. The reason for this was that there was the danger of some rushing through the work in the test without having carefully read the instructions. They were allowed to turn back to check the work on those tests already written if they had any extra time at their disposal. At the end of ten minutes all the pupils were asked to turn to the next test. The teacher then read according to directions after which the pupils proceeded with the work of that test. This continued until five tests were completed. No extra time was to be given in order to check the work of all the tests.

The Marking of the Tests.- The first marking of the tests written by the pupils of the suburban schools was done by the students of the Faculty of Education under the personal supervision of Dr. D. S. Woods, while that for the tests written by the pupils of the city school used was done by the mathematics teachers of the classes writing the tests. Brief instructions and a key[3] were provided for the marking. The second and third markings were done by the present writer and an efficient assistant with considerable experience in the teaching of mathematics. The present writer did both the second and third markings of Test I himself. In the second marking the

---

[3]See the Appendix for complete instructions and the key.

accuracy of the first one was checked, while the third marking had for its purpose an analysis of the errors made. Each type of error that appeared quite frequently was given a designating letter. This letter was placed on the tests opposite the error. Errors appearing only a few times were classified as miscellaneous.

When the marking had been completed a pupil-class score sheet was prepared for each class that had written the test. This resulted in twenty-three such score sheets for the junior high school grades and four for grade ten. The girls and boys of each class were placed on the same score sheet but they were separated. Opposite to each pupil were squares to be filled with information concerning the results secured for different questions by that pupil as well as the results for each of the tests and for the entire series of tests. A blank square indicated that the question represented by that square was correct. The squares for the other questions had designating letters in them to show the nature of the error made. The vertical columns were then used to find the number of pupils who had each question correct in each class and the number who had each question wrong. The number of "wrongs" for each question was then subdivided into the number for each type of error that had been made. The class-pupil score sheets

formed the basis of the work in the three subsequent chapters of this investigation.

CHAPTER III

THE EXTENT TO WHICH THE PUPILS ARE ABLE TO PUT TO USE THE ENTIRE GROUP OF FUNCTION CONCEPTS THAT ARE INCLUDED IN THE SET OF TESTS

It is the purpose of this chapter to show the findings of the investigation in the entire field being tested. To make this possible it was necessary to make tables showing the distribution of the scores for girls alone, for boys alone, and for both girls and boys for each grade. Various combinations of these were used to make grade-sex distributions and general grade distributions for the entire group of junior high school pupils that took part in the investigation.

A Study of the Means and Medians

It will be desirable to examine the medians of the various grades to see if there is a grade to grade progress in achievement and if so, the extent of this progress. This will be done with the pupils as a complete group and also with the boys and girls separately. Table II shows a summary of the grade-medians and of the grade-sex medians.

TABLE II

SHOWING THE MEDIANS FOR BOYS AND GIRLS AND FOR BOTH FOR EACH OF JUNIOR HIGH SCHOOL GRADES

	Grade 7	Grade 8	Grade 9
Girls	19.27 ± .46	24.56 ± .36	28.09 ± .39
Boys	17.79 ± .48	24.85 ± .41	29.68 ± .33
Both	18.74 ± .31	24.72 ± .29	28.88 ± .25

The table reads: The median for girls is 19.27 in grade 7, 24.56 in grade 8, ... etc.

For the purpose of comparison a similar table is shown for the means of the same group. Table III is the table of means.

TABLE III

SHOWING THE MEANS FOR BOYS AND GIRLS AND FOR BOTH FOR EACH OF JUNIOR HIGH SCHOOL GRADES

	Grade 7	Grade 8	Grade 9
Girls	19.67 ± .37	24.85 ± .29	27.77 ± .31
Boys	18.62 ± .40	24.77 ± .33	29.44 ± .26
Both	19.16 ± .25	24.81 ± .23	28.67 ± .20

The table reads: The mean for the girls in grade 7 is 19.67, in grade 8 it is 24.85, ... etc.

Tables II and III show that there is a consistent grade-to-grade improvement in the understanding of the function concepts. They also show that there is a consider-

able difference between the median and mean scores of the girls and the boys in grades seven and nine. In grade seven the girls seem to be superior, in grade eight there is but little difference, while in grade nine the boys seem to be superior. It will be necessary to find whether or not the differences are statistically significant. Henry E. Garrett[1] states that if the ratio $D/P.E._{diff.}$ is greater than 4, then the difference is statistically significant.

Tables II and III show that the girls' median for grade seven is 1.48 score-points higher than that of the boys while their mean is 1.05 score-points higher than that of the boys. The probable error of the difference of the medians is found to be .665 which gives a value of 2.22 to the ratio $D/P.E._{diff.}$ This ratio indicates that there are ninety chances out of one hundred that the true difference is greater than zero. The results show that the differences are not statistically significant since the ratio in each case is less than 4. They also

---

[1]Henry E. Garrett, Statistics in Psychology and Education, pp. 128-137. New York: London: Toronto: Longmans, Green and Company, 1926.

[2]The probable error of the difference between the means (or medians) is found by the use of the formula

$$P.E._{diff.}^2 = P.E._{m.of\ boys}^2 - P.E._{m.of\ girls}^2$$

The D given in the ratio represents the difference between the measures being considered.

show that there are ninety or more chances out of one hundred that the true difference is greater than zero.

Tables II and III show also that there is little difference between the means and medians of the boys and girls in grade eight but that the boys did better than the girls in grade nine. The median of the boys is 1.59 score-points higher than that of the girls while their mean is 1.67 score-points higher than that of the girls in grade nine. The probable error of the difference of their medians is .51 while the ratio $D/P.E._{diff.} = 3.12$. This ratio indicates that there are ninety eight chances out of one hundred that the true difference is greater than zero. The probable error of the difference of the means is .404 while the ratio $D/P.E._{diff.} = 4.13$ for the means. This ratio indicates that it is a statistical certainty that a true difference exists between the mean score of the boys and the mean score of the girls in grade nine.

It seems to be quite significant to note that the girls had the higher mean scores in grade seven, that there was little difference in grade eight, and that the boys had the higher mean scores in grade nine. It is true that the difference between the mean scores in grade seven was not quite high enough to make the results statistically significant but it is also true that they

were high enough to show that there were at least ninety chances out of a hundred that the difference was greater than zero. The difference in the mean scores in grade nine is significant and shows that the boys of grade nine have a better understanding of the function concept than have the girls. The girls may have done better in grade seven because of earlier maturity. This advantage they lost in grade nine.

Table IV shows the gains made by girls, by boys, and by both together in the grade-to-grade intervals. Both the medians and the means have been used to secure the data in the table. In no case does a median differ-

TABLE IV

COMPARATIVE TABLES OF GAINS COMPUTED FOR THE MEDIANS AND FROM THE MEANS OF THE DISTRIBUTION

Average Gains in Score-points	From Medians			From Means		
	Girls	Boys	Both	Girls	Boys	Both
Gain of grade 8 over grade 7	5.29	7.06	5.98	5.18	6.15	5.65
Gain of grade 9 over grade 8	3.53	4.83	4.16	2.92	4.67	3.86

The table reads: The gain of grade 8 over grade 7 is 5.29 score-points for girls, 7.06 score-points for boys and 5.98 score-points for both if their medians are considered, while if the means are considered it is 5.18 for the girls, etc.

ence differ from a mean difference by as much as one score-point.

The table shows that the boys consistently master the material of the tests better than do the girls. The difference between the gains of the boys and the girls is quite marked when compared with the total grade-interval gain. The differences in the grade interval for both boys and girls are statistically significant as in each case the ratio $D/P.E._{diff.}$ is greater than four.

Chart I is an attempt to illustrate graphically the grade-to-grade improvement as shown by a comparison of the grade medians for the girls, the boys, and both girls and boys. It bears out the conclusions deduced from Tables II, III and IV.

A More Detailed Study of the Scores

Group medians, being merely central tendencies, fail to give the entire picture of what has resulted from the tests. They will be supplemented by tables which show the dispersion of scores in connection with the central tendencies. This data will be found in Tables V, VI and VII below.

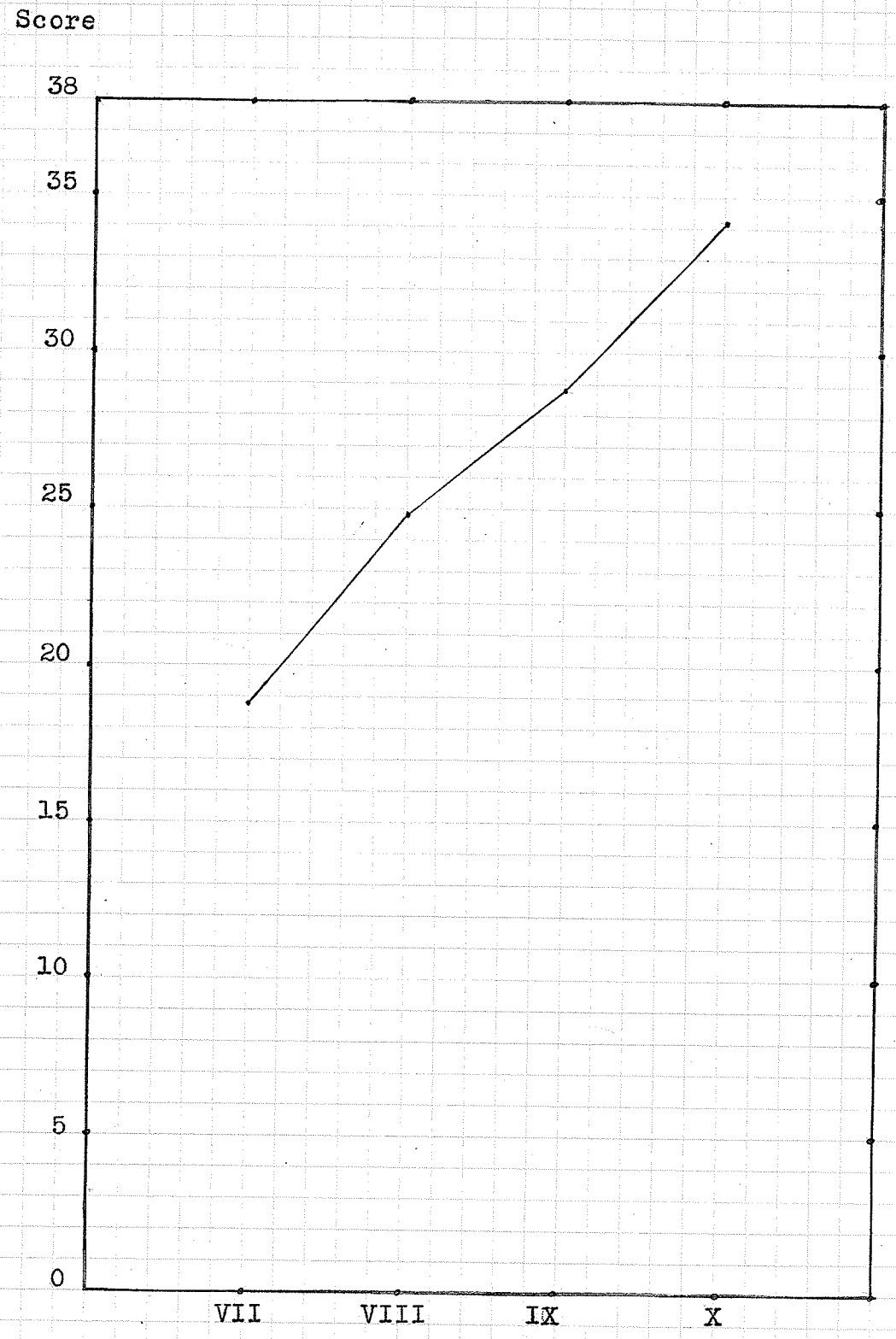

Chart I. - Showing the Grade-to-Grade Advancement by Use of the Median Scores.

TABLE V[1]

SUMMARY OF DATA RELATIVE TO THE DISTRIBUTION OF SEVENTH GRADE SCORES ON THE COMPLETE SET OF TESTS

	Girls	Boys	Both
Range of Scores	5 - 36	3 - 34	3 - 36
(Q_1 - Q_3) Range	14.69 - 26.20	13.88 - 23.82	14.40 - 23.93
Number of Cases	145	134	280
Mean	19.67 $\pm$ .37	18.62 $\pm$ .40	19.16 $\pm$ .25
Median	19.27 $\pm$ .46	17.79 $\pm$ .48	18.74 $\pm$ .31
$S.D._{dis.}$	6.63 $\pm$ .16	6.80 $\pm$ .28	6.72 $\pm$ .18
Skewness	- .257	- .426	- .178

The table reads: The range of scores is 5-36 for girls, 3-34 for boys, and 3-36 for both.

TABLE VI

SUMMARY OF DATA RELATIVE TO THE DISTRIBUTION OF EIGHTH GRADE SCORES ON THE COMPLETE SET OF TESTS

	Girls	Boys	Both
Range of Scores	11 - 36	11 - 36	11 - 36
(Q_1 - Q_3) Range	20.96 - 28.29	20.45 - 29.07	20.30 - 29.07
Number of Cases	157	151	308
Mean	24.85 $\pm$ .29	24.77 $\pm$ .33	24.81 $\pm$ .23
Median	24.56 $\pm$ .36	24.85 $\pm$ .41	24.72 $\pm$ .29
$S.D._{dis.}$	5.40 $\pm$ .20	6.10 $\pm$ .21	5.86 $\pm$ .16
Skewness	- .035	- .13	- .016

The table reads: The range of scores is 11-36 for girls, 11-36 for boys, and 11-36 for both.

[1] The statistical constants used have their usual meaning. The appendix contains the formulas used in this investigation to find any statistical constants used. The 'Range of Scores' means the range from the lowest to the highest score made by the pupils of the given group. The '(Q1-Q3) Range' means the interquartile range. It includes half the distribution of scores for the given group. $S.D._{dis.}$ means the standard deviation for the distribution.

TABLE VII

SUMMARY OF DATA RELATIVE TO THE DISTRIBUTION OF THE NINTH GRADE SCORES OF THE COMPLETE SET OF TESTS

	Girls	Boys	Both
Range of Scores	15 - 38	14 - 38	14 - 38
(Q_1 - Q_3) Range	23.23 - 31.97	26.25 - 33.12	25.13 - 32.62
Number of Cases	145	174	319
Mean	27.77 ± .31	29.44 ± .26	28.67 ± .20
Median	28.09 ± .39	29.68 ± .33	28.88 ± .25
$S.D._{dis.}$	5.50 ± .21	5.02 ± .18	5.37 ± .14
Skewness	-.224	- .0003	-.011

The table reads: The range of score is 15-38 for girls, 14-38 for boys, and 14-38 for both.

TABLE VIII

SUMMARY OF DATA RELATIVE TO THE DISTRIBUTION OF THE SCORES OF GRADES SEVEN, EIGHT, AND NINE FOR THE COMPLETE SET OF TESTS

	Girls	Boys	Both
Range of Scores	5 - 38	3 - 38	3 - 38
(Q_1 - Q_3) Range	19.42 - 28.91	19.23 - 30.73	19.34 - 29.84
Number of Cases	448	459	907
Mean	24.10 ± .14	24.74 ± .23	24.43 ± .16
Median	24.12 ± .17	25.69 ± .29	24.86 ± .20
$S.D._{dis.}$	6.79 ± .10	7.38 ± .16	7.07 ± .11
Skewness	-.019	-.210	-.016

The table reads: The range of score is 5-38 for girls, 3-38 for boys and 3-38 for both.

For comparative purposes data were secured from a limited number of cases of grade ten pupils. The results secured are arranged in Table IX.

TABLE IX

SUMMARY OF DATA RELATIVE TO THE DISTRIBUTIONS OF TENTH GRADE SCORES ON THE COMPLETE SET OF TESTS

	Girls	Boys	Both
Range of Scores	24 - 38	18 - 38	18 - 38
(Q_1 - Q_3) Range	29.00 - 36.50	29.50 - 36.50	29.17 - 36.50
Number of Cases	28	76	104
Mean	32.50 $\pm$ .53	32.84 $\pm$ .36	32.75 $\pm$ .30
Median	33.50 $\pm$ .63	34.50 $\pm$ .45	34.36 $\pm$ .38
$S.D._{dis.}$	4.15 $\pm$ .37	4.70 $\pm$ .25	4.58 $\pm$ .21
Skewness	-.400	-.860	-.829

The table reads: The range of score is 24-38 for girls, 18-38 for boys, and 18-38 for both.

An examination of the tables shows that nearly all the upper range of possible scores has been attained by some of the pupils in each grade. The lowest score made by a grade seven pupil was 3, by a grade eight pupil it was 11, and by a grade nine pupil, 14. The interquartile ranges vary from a low of 6.88 score-points to a high of 10.51 score-points. The range is greater as a rule for the lower grades. The $S.D._{dis.}$ varies from 6.72 to 5.37

within the totals for the junior high school grades while the S.D.dis for the three grades combined is 7.07. The amount of skewness is quite slight and may be either positive or negative.

Tables III - VI show that there is a large amount of over-lapping in the distributions of the scores for the three junior high school grades. To show this more clearly it is necessary to make use of the data contained in Table X and in Chart II. Table VIII shows the distribution of the scores for each grade and for the junior high school grades taken as a single group. Chart II was made from the data contained in Table X, taking intervals of three score-points and expressing the distribution of scores in those intervals as a percentage of the total number of pupils in the group for which the curve was made.

Chart II gives the percentage distributions of pupils in the named score intervals. One might conclude from the chart that an exceptionally large number of pupils had made the maximum score of 38 as the curves are not complete on the right but such is not the case. The lines are drawn to the line representing the interval 36-38. The percentage readings of the end points of the lines represent the percentage of pupils in each grade that made a score of 36 or 37 or 38 and not solely those that made a score of 38. It was thought that the chart would serve its

TABLE X

DISTRIBUTION OF THE SCORES FOR THE SEVENTH, EIGHTH, NINTH, AND TENTH GRADES AND FOR THE COMBINED JUNIOR HIGH SCHOOL GRADES

Score	Grade 7	Grade 8	Grade 9	J.H.S.Grades	Grade 10
38	0	0	8	8	11
37	0	0	15	15	15
36	1	7	11	19	11
35	2	7	14	23	14
34	1	11	16	28	7
33	2	7	19	28	7
32	7	16	27	50	4
31	5	8	15	28	5
30	2	18	22	42	3
29	4	7	20	31	3
28	9	10	23	42	6
27	10	29	23	62	4
26	13	20	21	54	8
25	5	18	14	37	2
24	16	26	13	55	1
23	12	22	12	46	1
22	10	13	15	38	0
21	12	15	6	33	0
20	13	15	7	35	0
19	21	13	6	40	1
18	13	13	5	31	1
17	24	10	4	38	0
16	15	7	1	23	
15	12	4	1	17	
14	10	2	1	13	
13	13	6		19	
12	14	0		14	
11	9	4		13	
10	6			6	
9	6			6	
8	4			4	
7	0			0	
6	4			4	
5	2			2	
4	2			2	
3	1			1	
2	0			0	
1	0			0	
0	0			0	
	280	308	319	907	104

The Table reads: Of pupils who received a score of 38 on the tests none were in grade 7, none in grade 8, 8 were grade 9, 8 were in junior high, and 11 were in grade 10.

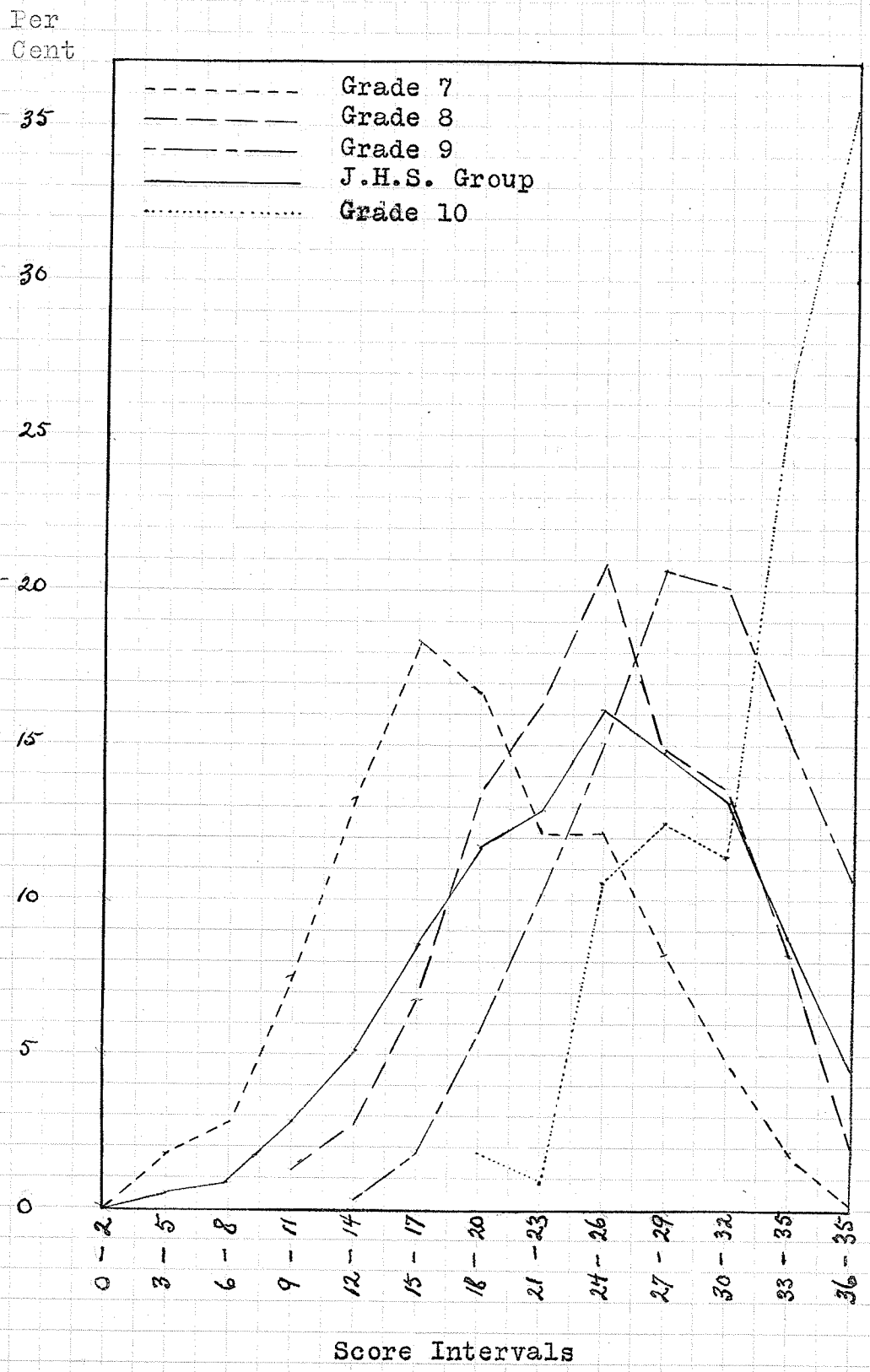

Chart II. - Showing the Per Cent of Each Group in Each Interval.

purpose better if at least a three score-point interval were used in order to make the curve more regular. An examination of the data of Table X will convince the reader of the wisdom of this choice. Table X will also show that, had an interval of one score-point been chosen, the curves for grades seven and eight would have been completed on zero per cent, that for grade nine on 2.51 per cent, that for the junior high school grades on .88 per cent and that for grade ten on 10.88 per cent. This would make all the curves nearly complete except that of grade ten. The latter is not of significance as this study is primarily concerned with the junior high school grades.

A more careful study of Table X in conjunction with earlier tables also reveals the following: There are 21 per cent of the grade seven pupils with scores above the grade eight median and 7 per cent with scores above the grade nine median. In grade eight 15 per cent have scores below the grade seven median while 24 per cent have scores above the grade nine median. The scores of grade nine show that 23 per cent of them are below the grade eight median and 4 per cent below the grade seven median. A comparison of the interquartile ranges shows that those of grades seven and nine do not overlap but are separated by 1.20 score-points. Those of grades seven and eight overlap 3.63 score-points while those of grades eight and nine

overlap 3.94 score-points.

Table XI shows the relation between the range of scores for each grade, the standard deviation for that grade, and the range between the central tendencies of of the grades.

TABLE XI

THE RANGE OF SCORES FOR EACH GRADE, THE STANDARD DEVIATION AND THE RANGE BETWEEN THE CENTRAL TENDENCIES OF SUCCESSIVE GRADES

Grade	Range of Scores	Standard Deviation	Range Between Medians	
VII	34	6.72	VII-VIII	5.98
VIII	26	5.86	VIII-IX	4.16
IX	25	5.37	IX-X	5.48

The table reveals that the range of scores is more than four times as great as the standard deviation in every case while the standard deviation is greater than the range between the central tendencies in every case except one, and in that case the difference is very small.

CHAPTER IV

THE EXTENT TO WHICH THE PUPILS ARE ABLE TO PUT TO USE THE DIFFERENT FUNCTION CONCEPTS THAT ARE CONTAINED IN THE INDIVIDUAL TESTS

The previous chapter dealt with the tests as a whole in order to find how well the entire group of questions was mastered by the pupils of the various grade levels and how the results of the different grades compared. This chapter will attempt to show how the work compared with respect to the different tests each of which is an attempt to test the ability of the pupil to make use of a different concept. The abilities that it was desired to test in the five tests were as follows:

1. The ability to understand the dependence that exists between quantities, i.e., upon what two quantities a given quantity depends.

2. The ability to write a formula from a general statement.

3. The ability to substitute numerical values for the members of the formula except the subject; and in this way find a numerical value for the subject for these specific values of the other members in the formula.

4. The ability to derive a new formula in which a value is given for an indicated member of the formula in terms of the other member or members, or the ability to

change the subject of the formula.

5. The ability to understand what effect a change in one or more members of a formula would have on the value of a given member of that formula.

Two methods were used to aid in making comparisons between the grades and between the tests in terms of the accomplishment of the pupils. The first was to find the single mastery coefficient for each grade for each test. This was done by finding the per cent that the total number of correct answers for the pupils of each grade was of the possible number. The results of this are shown in Table XII together with the grade-to-grade gains for each test and the ranking of the tests in the order of the extent to which they had been mastered by the combined groups of the junior high school grades. It will be noticed that the order of ranking does not change in a single instance when the ranks of the tests within each grade are compared with the junior high school group ranking.

It is very apparent that the tests were not mastered equally well for there is a wide range between the mastery coefficients within each grade. There is also a wide range between the grades of the mastery coefficients for any single test, this being much greater in the 7-8 interval except in the case of Test V in which case the gain in the

TABLE XII

GRADE MASTERY COEFFICIENTS, GRADE-TO-GRADE GAINS AND RANKS FOR THE FIVE TESTS

Test	Mastery Coefficients				Grade-to-Grade Gains		Ranks
	Grade 7	Grade 8	Grade 9	J.H.S. Grades	7-8	8-9	
I	60.00	72.02	78.92	70.74	12.02	6.90	2
II	52.94	67.32	77.75	66.54	14.38	10.43	3
III	62.50	83.87	93.26	80.58	21.37	9.39	1
IV	46.85	65.64	76.44	63.63	18.79	10.80	4
V	29.02	39.57	54.50	41.57	10.55	14.93	5

The table reads: The Mastery Coefficients for Test were 60.00 for grade 7, 72.02 for grade 8, 78.92 for grade 9, and 70.74 for junior high school grades. The grade-to-grade gain for Test I was 12.02 in the 7-8 intervals and 6.90 in the 8-9 interval. Test I ranked second easiest of the tests, etc.

8-9 interval is considerably greater. It is interesting and perhaps significant to note that Test V shows a low mastery coefficient for all the grades and ranks as the most difficult of the tests. This may be the reason that the grade-to-grade gain is greater in 8-9 interval than in 7-8 interval. The greatest difference in the grade-to-grade gains is seen in Test III which deals with the substitution of numbers to find a specific value for the remaining element of the formula. This is the only concept that grade 8 has been taught at this stage while grade 7 has not been taught any of the concepts. This would account for part of the

extreme spread in this test. Table XIII contains the medians for the different grades for each of the tests. Those of grade 10 are included for comparative purposes.

TABLE XIII

THE GRADE MEDIANS FOR EACH OF THE FIVE TESTS

Test	Possible Score	Grade 7	Grade 8	Grade 9	J.H.S.Grades	Grade 10
I	12	7.14	8.88	9.87	8.74	10.74
II	6	3.12	4.05	4.75	4.17	5.40
III	6	3.68	5.42	5.79	5.31	5.84
IV	6	2.74	3.99	5.16	3.99	5.89
V	8	2.27	3.30	4.49	3.39	6.82

The table reads: The possible score for Test I was 12, the median score for grade 7 was 7.14, the median score for grade 8 was 8.88, the median score for grade 9 was 9.87, for junior high school grades 8.74, for grade 10 10.74.

This table of central tendencies bears out what has been said about the data of Table XII.

Test I - Recognizing Relationships

The purpose of this test was to find to what extent the pupils of the junior high school grades had a mastery of the concept of understanding upon what quantities certain given quantity depends. A reading of the test given will show that the concept was briefly explained and

then demonstrated by the use of two illustrations. The pupils were asked to read the given sample carefully before commencing the test to fix clearly in mind how the work was to be done. They were then to proceed with the questions by giving two quantities upon which each of the given quantities depended. To do this properly it was necessary that the child should not only understand dependence but that he could distinguish between quality and quantity. The difficulties shown to exist in the individual questions will be considered later in the analysis of the errors. At this point the test as a whole will be explained first to find the distribution of the scores and secondly, to find the relative difficulties between the parts of the test. The first is shown in Table XIV and the second in Table XV. It is necessary to keep in mind that in scoring this test two score-points were given for each correct solution- i.e. for the two parts of the question. There were three possible solutions for question six, any two of which were accepted. In marking the question, if there was only one part correct, credit was given to part (a) while if there were two parts correct, credit was given to both parts. A separate table is made later to show how often the various possible combinations occurred in question six. The mastery coefficient of 6 (b) is relatively low for it represents the mastery coefficient for a knowledge of

TEST I RECOGNIZING RELATIONSHIPS

One quantity often depends on one or more other quantities for its value. Thus, the price paid for 3 baseballs depends on one other quantity, namely, the price per baseball. The price paid for a number of pencils depends on two other quantities, namely, the number of pencils, and the price for one pencil. Sometimes one quantity depends on three others.

To show how well you recognize relationships, fill in the blanks after each question giving two quantities on which each depends. Read the sample over carefully before you begin the questions to be certain that you understand what is wanted.

THE GIVEN QUANTITIES	THE OTHER QUANTITIES DEPENDED ON
Sample. The area of a rectangle depends on	1. The length of the rectangle 2. The width of the rectangle
1. The cost of a roast of meat depends on	1. ..... 2. .....
2. The time required to do a certain number of arithmetic questions depends on	1. ..... 2. .....
3. The area of a triangle depends on	1. ..... 2. .....
4. The amount of pay a man receives depends on ..	1. ..... 2. .....
5. If a train travels at the same rate all the time, the distance it travels depends on	1. ..... 2. .....
6. The simple interest on a loan depends on	1. ..... 2. .....

To facilitate reading of discussion a copy of each test is inserted where it will best serve these ends.

TABLE XIV

THE DISTRIBUTION OF THE SCORES FOR TEST I

Score	Grade 7	Grade 8	Grade 9	J.H.S.Grades	Grade 10
12	15	33	62	110	39
11	18	55	66	139	17
10	26	38	50	114	22
9	32	45	41	118	9
8	35	42	41	118	7
7	39	34	21	94	6
6	43	26	19	88	3
5	29	16	15	60	0
4	22	9	2	33	0
3	10	10	1	21	1
2	6	0	1	7	
1	4			4	
0	1			1	
Totals	280	308	319	907	104

The table reads: A score of 12 was obtained by 15 pupils in grade 7, 33 pupils in grade 8, 62 pupils in grade 9...etc.

both parts of the question combined.

Table XIV shows that the concept of one quantity depending upon another is well understood by the pupils of the junior high school grades. It also shows that there are many pupils in grade 7 that can do better than the median for grade 8 while over twenty per cent of the grade seven pupils have scores above the median score for grade 9. This would seem to indicate that a mastery of this concept does not depend as much upon age as it does upon general intelligence. There are many pupils in grade nine who have some understanding of the concept but their under-

standing is far from complete. The same is true for each of the other grades. Less than 14 per cent of the total junior high school pupils writing the test failed to get 50 per cent of the possible score, while over 12 per cent of the junior high school pupils showed a complete mastery of the questions on the test. The distribution of scores is quite well spread over the range interval for the grade but the grade nine distribution is slightly heavy for the higher scores. A reference to Table XIII reveals that the medians are 7.14, 8.88, and 9.87 for grades 7, 8 and 9 respectively. The median for the junior high school is 8.74. Table XII shows that the coefficients of mastery for the complete test are 60.00, 72.02 and 78.92 for these grades, and 70.74 for the combined junior high school group. Table XV shows the mastery coefficients for the different parts of Test I for each grade and for the junior high school as a whole. It also shows the ranking of the coefficients for the same groups.

The data of Table XV show that there is a wide range in the mastery coefficients for the same questions for the different grades and for the different questions within the same grade. The two questions having the highest mastery coefficients are I (a) and 6 (a). The first states that the cost of a roast of meat depends on ... Ans. The number of pounds of meat, while 6 (a) is answered by giving one

TABLE XV

THE MASTERY COEFFICIENTS FOR THE QUESTIONS OF TEST I FOR EACH GRADE AND FOR THE JUNIOR HIGH SCHOOL GROUP TOGETHER WITH THE RANKING OF THESE MASTERY COEFFICIENTS

Question	Mastery Coefficients				Ranking			
	Grade 7	Grade8	Grade9	J.H.S.Grades	VII	VIII	IX	J.H.S.
1.(a)	87.14	93.83	96.86	92.83	1	2	2	1
(b)	65.71	73.38	87.46	75.96	6	5	4	5
2.(a)	66.08	84.09	83.70	78.39	5	4	5	4
(b)	38.60	44.16	56.11	46.64	12	12	11	12
3.(a)	66.43	68.18	76.18	70.45	4	8	7	6
(b)	62.86	62.66	66.77	64.17	7	9	10	9
4.(a)	66.78	92.53	96.86	86.11	3	3	3	3
(b)	42.14	56.17	74.92	58.43	10	10	9	10
5.(a)	41.43	47.73	52.98	47.63	11	11	12	11
(b)	56.07	70.45	81.81	70.01	8	7	6	7
6.(a)	81.43	98.05	97.49	92.72	2	1	1	2
(b)	46.07	73.05	76.18	65.82	9	6	7	8

The table reads: The mastery coefficient for question 1 (a) for grade 7 is 87.14, for grade 8 is 93.83, for grade 9, 96.86, for junior high school grades it is 92.83. The ranking for 1(a) is 1 in Grade 7, 2 in grade 8, etc.

of rate, principal, or time as an answer to upon what the simple interest on a loan depends. Ranking third in every case is 4(a) which states that the amount of pay a man receives depends on .... the time he works. The three most

difficult questions if judged by their mastery coefficients are 2(b), 5(a) and 4(b) named in the order of their difficulty. Question 2(b) is answered by stating that the time required to do a certain number of arithmetic questions depends on .... the time required for one question. Question 5(a) should be, if a train travels at the same rate all the time, the distance it travels depends on .... the length of time it travels. Question 4(b) states, the amount of money a man receives depends on .... the amount he receives per unit of time (i.e. per hour, per day, ...etc.) The grade-to-grade gains are very varied. They are very slight for 1(a), 3(a) and 3(b). This is likely due to the fact that the situation comes close to the experience of all the pupils of the three grades. The greatest gain is seen in both parts of question 4. The ranking is very consistent and in no case is there a rank displacement of more than two from the rankings of the junior high school group as shown in the last column. In only one instance is there a rank displacement as great as four between any of the columns or groups. There are two rank displacements of three while the remainder range from 0 to 2. Question 4(a) is the only instance in which all the rankings are the same.

Test II Writing Formulas

The purpose of this test was to find to what extent the pupils of the junior high school would show an understanding of how to write a formula in place of the word expression of a general relationship. A reading of the questions that were used in this test will show that in each question the child was given a general situation and was asked to express the general relation that exists in the form of a formula. All the letters or symbols that would be needed in the required formulas were also given. In question five there is no mention made of the number 7 but the child was allowed to draw on his own knowledge or experience for that information. The explanation preceding the questions explains that a formula is a short way of writing down a general rule for doing certain questions in mathematics. The explanation is illustrated by the use of the formula A = LW. The child was asked to read each question carefully and then write the desired formula. His attention was directed to the sample question in order that there would be no doubt as to how the questions were to be done. It is well to keep in mind that the pupils of the junior high school except in grade 9 have had little or no training in the writing of a formula to fit a general statement or a situation which can be expressed by a

TEST II

WRITING FORMULAS

A formula is just a short way of writing down a general rule for doing certain questions in mathematics. For example, we say that to find the area of a rectangle we multiply its length by its width. This is stated in a formula by writing A = L X W (In this formula A stands for the area of the rectangle, L for its length, and W for its width.

In each of the following questions you are asked to write a formula for some quantity. Read each question carefully and then write the formula. Read the sample first to see just what is wanted.

QUESTION OR STATEMENT	FORMULA
Sample: If (t) men take (g) days to do a piece of work, write a formula to show how many days it will take one man to do the same amount of work.	n = t x g or n = t g
1. A car travels (v) miles an hour for (t) hours. Write a formula for the distance (d) in miles covered by the car	d = ...
2. Write a formula for the amount remaining (R) of (C) dollars after spending (S) dollars	R = ...
3. Write the formula for the total number of cents (C) that (N) apples will cost at (L) cents each	C = ...
4. Jack is 16 years old. Henry is (x) years older than Jack. Write a formula to show Henry's age (n)	n = ...
5. Write a formula to show the number of days (n) in (m) weeks	n = ...
6. John has (y) marbles and this is 5 times as many as Tom has. Write a formula for the number of marbles Tom has	n = ...

formula. They may have done a little formula work in finding the area of rectangles, triangles, etc., but this would not have much effect in developing their ability to do the questions of this test. The pupils of grade 9 may have taken a small amount of this work in the early part of the year. The situations chosen are varied in character and in the nature of the formulas required for their solutions.

The method of studying the results of the work done on this test will be similar to that used with Test I, namely, making a study of the score distribution and of the mastery coefficients for all the questions included in the test while the analysis of the errors will be left for a later chapter. Table XVI shows the distribution of scores for the test and Table XVII the mastery coefficients for the questions that make up the test.

TABLE XVI

THE DISTRIBUTION OF SCORES FOR TEST II

Score	Grade 7	Grade 8	Grade 9	J.H.S.Grades	Grade 10
6	23	52	88	163	49
5	46	79	95	220	30
4	49	73	87	209	16
3	58	55	39	152	6
2	62	33	10	105	2
1	25	14	0	39	1
0	17	2	0	19	0
Total	280	308	319	907	104

The table reads: A score of 6 was made by 23 pupils in grade 7, 52 pupils in grade 8, 88 pupils in grade 9, 163 pupils in junior high school grades, and 49 in grade 10.

It is reasonable to assume that if the pupil obtains a score equal to or greater than fifty per cent of the possible score, that he has an understanding of the concept and that he is in a position to make use of this concept in mathematics. On this basis it is possible to conclude from the data of Table XVI that at least 63 per cent of the grade 7 pupils, 85 per cent of the grade 8 pupils and 97 per cent of the grade 9 pupils have an understanding of this concept. The table also shows that there is quite an even distribution of the grade 7 scores over the entire range of scores. The grade 8 scores also cover the entire range with the heaviest distribution at the 4 and 5 score levels. The scores of grade 9 range from 2 to 6 with the concentration at scores of 4, 5 and 6. Table XII shows that the mastery coefficients for the entire test are: Grade 7, 52.94; grade 8, 67.32; and grade 9, 77.75 while that for the junior high school group is 66.54. The median scores as shown by Table XIII are 3.12, 4.05, 4.75, and 4.17 for the same groups. This data shows that there is a wide range in the ability of the different grades to handle this concept. But Table XVI shows that 20 per cent of the pupils of grade 7 obtained a score above the median score of grade 9 while 33 per cent of the grade 7 pupils score higher than the grade 8 median. It might rightly be said that the grade 7 pupils had not received

any instructions in the handling of the concept tested by Test II and yet 8 per cent of the grade 7 pupils obtained a perfect score. On the other hand over 15 per cent of the grade 9 pupils failed to get a score greater than 3.

Table XVII shows the mastery coefficients for the different items in Test II for each grade and for the entire group of junior high school pupils. It also shows the ranking of the coefficients for each of the grades and for the junior high school group.

TABLE XVII

THE MASTERY COEFFICIENTS FOR THE QUESTIONS OF TEST II FOR EACH GRADE AND FOR THE JUNIOR HIGH SCHOOL GROUP TOGETHER WITH THE RANKING OF THESE MASTERY COEFFICIENTS

Question	Mastery Coefficients				Ranking			
	Grade7	Grade8	Grade9	J.H.S.Grades	VII	VIII	IX	J.H.S.
1	81.07	91.23	95.93	89.75	1	1	2	1
2	51.79	75.32	94.04	74.64	3	3	3	3
3	76.07	85.06	97.49	86.66	2	2	1	2
4	48.93	62.66	83.71	65.82	4	4	4	4
5	25.71	36.69	41.69	35.06	6	6	6	6
6	33.57	53.25	52.98	47.08	5	5	5	5

The table reads: The mastery coefficient for question 1 for grade 7 is 81.07, for grade 8 is 91.23, ...etc.... the ranking of question 1 for grade 7 is 1, for grade 8 is 2, etc.

The data contained in this table shows that the questions covered considerable range in difficulty and that, with one exception, the ranking of all the questions is exactly the same in all the grades. Questions 1 and 3 show the highest mastery coefficients which may be partially accounted for by the fact that the final formula obtained in these questions is similar to the sample formula. Question 3 ranks third in all cases. Question 5 ranks as the most difficult. It was the one question in which part of the data was not given, namely the number of days in a week. The pupils tested in the entire junior high school have a mastery coefficient of 47.08 for question 6. The concept of expressing n as a fraction of y seems to be very difficult compared with all the other questions except question 5. In every question except the 8-9 interval of 6 there is a grade-to-grade gain in the mastery coefficients this being more pronounced in some questions than in others. The range is greatest in questions 2 and 4.

Test III Substitution in Formulas

The purpose of this test was to find the concept that the junior high school pupils could put to use the substitution of numerical values for all members of the formula except the subject and then proceed to find a

TEST III SUBSTITUTION IN FORMULAS

The letters in all formulas stand for numbers or quantities. If you are given values for all but one of those letters, you can put those numbers in the place of the letters for which they stand and then work the question out to find the value of the other letter or symbol.

In the questions below you are given a number of formulas in the first column. In the second column is a list of values for certain letters in these formulas. In the third column you will show your work when you substitute (or put) these values for the letters in the formulas to find the answer. The answer must be written by itself in the last column. Look at the sample given to see that you understand. Be careful with your figuring.

Note (ck means c x k; $4r$ means $4 \times r$; $\frac{t}{e}$ means $t \div e$)

FORMULAS	GIVEN VALUES	WORK HERE	ANSWERS
Sample d = rt	r = 8 t = 6	d = 8 X 6 = 48	48
1. A = LW	L = 30 W = 9	A = ...	.....
2. n = 5g	g = 3.7	n = ...	.....
3. $h = \frac{t}{y}$	t = 54 y = 9	h = ...	.....
4. V = LWH	L = 8 W = 6 H = 4	V = ...	.....
5. r = 2s - 6	s = 26	r = ...	.....
6. p = 2L + 2W	L = 18 W = 11	p = ...	.....

numerical value for the subject of the formula by completing the computation. The pupils were told in a brief paragraph preceding the test that the letters in all formulas stand for numbers or quantities and that, if they were given values for all except one of these letters or symbols, it would be possible to proceed to find a numerical value for the remaining letter or symbol. They were also told that ck meant c x k, that 4r meant 4 x r and that t/c meant t ÷ c. They were asked to show their work in the proper column and to place the resulting answer apart from this work in the place provided at the extreme right. A reference to the test will show that the substitution required was as follows: substituting for two symbols which formed a product; substituting a decimal number (value) for a symbol which formed a product with the number 5; substituting for two symbols which formed a whole number quotient; substituting for three symbols which together formed a product; substituting for a symbol which formed a product with 2 which in turn was diminished by 6; and substituting for two symbols each of which formed a product with 2, after which it was necessary to find the sum of the two resulting quantities. It is safe to assume that the items are varied enough and representative enough of types of work in substitution to test whether the pupil has the ability to make use of the concept.

Table XII shows that Test III has the highest mastery coefficient of all the tests and that this is true not only for the combined junior high school group but also for each individual grade. It also shows that in the grade-to-grade gain it ranks with Test IV to show the greatest grade-to-grade gain of all the tests. The relatively high medians for the different grades are shown for Test III to be 3.68 for grade 7, 5.42 for grade 8 and 5.79 for grade 9. The median score for the junior high school group is 5.31.

The distribution of the scores for the different grades and for the junior high school group are shown in Table XVIII.

TABLE XVIII

THE DISTRIBUTION OF THE SCORES FOR TEST III

Score	Grade 7	Grade 8	Grade 9	J.H.S.Grades	Grade 10
6	44	148	224	416	76
5	49	75	70	194	22
4	57	46	18	121	5
3	73	27	3	103	1
2	43	10	4	57	
1	8	2	0	10	
0	6	0	0	6	
Totals	280	308	319	907	104

The table reads: A score of 6 was made by 44 pupils in grade 7, 148 pupils in grade 8, 224 pupils in grade 9, 416 pupils in junior high school grades, and 76 in grade 10.

The table shows that 16 per cent of the grade 7 pupils, 48 per cent of the grade 8 pupils and 70 per cent of the grade 9 pupils obtained a perfect score on this test. The remainder of the grade 7 distribution is quite evenly distributed over the entire range of scores. The grade 8 scores also have a wide range but the big concentration is at the high scores. The grade 9 pupils are largely concentrated at the 5 and 6 score levels. This test does not test the relative ability of the different grade groups as well as do the other tests as there is no doubt that the grade 9 group could do questions in substitution considerably harder than the ones contained in the test. Many of the pupils of grade 8 could do so also. It is not likely that over 8 or 10 per cent of the grade 7 pupils would advance much further if the questions were made much more complex at this stage. These assumptions would lead one to believe that if the test were doubled in length by the addition of graded questions of increasing difficulty the grade-to-grade gain would be made significantly greater in each case.

Table XIX shows the mastery coefficients for the different questions of Test III. These mastery coefficients are made for the grade groups and for the junior high school group. The ranking of the items is also given for each group.

TABLE XIX

THE MASTERY COEFFICIENTS FOR THE QUESTIONS OF TEST III FOR THE GRADE GROUPS AND FOR THE JUNIOR HIGH SCHOOL GROUP TOGETHER WITH THE RANKING OF THESE MASTERY COEFFICIENTS FOR THESE GROUPS

Question	Mastery Coefficients				Ranking			
	Grade7	Grade8	Grade9	J.H.S.Grades	VII	VIII	IX	J.H.S.
1	92.14	95.13	95.92	94.49	1	1	1	1
2	65.71	85.71	92.48	81.92	3	4	4	4
3	64.64	90.91	95.61	84.46	4	2	2	3
4	84.64	90.90	94.98	90.41	2	3	3	2
5	33.57	68.51	88.71	64.83	6	6	6	6
6	34.29	72.08	91.54	67.36	5	5	5	5

The table reads: The mastery coefficient for question 1 for grade 7 is 92.14, for grade 8 is 95.13, for grade 9 95.92 .... etc.

The data of the table show that the grade 9 group had an almost complete mastery of all the questions. The range in mastery coefficients is 88.71-95.92. The range for the grade 8 group in mastery coefficients for the first four questions is 85.71-95.13 while the entire range for the test in this group is 68.51-95.13. Questions 5 and 6 are found to be most difficult for the grade 8 group. The ranking of the mastery coefficients for grades 8 and 9 is identical. They are also identical with the ranking of the mastery coefficients for the junior high school group

with one exception. The grade 7 group found questions 1 and 4 to be the easiest, judging from their mastery coefficients. This may have been due partly to their similarity to the sample given which was also a product of two symbols or elements. This does not explain it all though, as question 4 represents a product of three symbols while the sample has but two. Grades 8 and 9 also found this the easiest and both grades had had experience in substitution involving the use of four fundamental operations. They both found question 3 slightly easier than question 2. Grade 7 found questions 2 and 3 considerably more difficult than questions 1 and 4 while questions 5 and 6 proved to be much more difficult than questions 2 and 3. The difficulty in question 2 may be in understanding the concept $\underline{5g}$. Precautions were taken against this in building the test by telling the pupils in the explanations that $\underline{4r}$ meant 4 x r. Another difficulty might have been in the fact that the number substituted contained a decimal. Question 3 contained very simple numbers to substitute and the child is given the meaning of the concept t/g in the explanation. Still the pupils of grade 7 succeeded in making less than two thirds of the possible score. This is true also of question 2. In questions 5 and 6 they were unable to get more than one third of the possible score. In each of these questions the concept contained in question 2 has to be used. In question 5

this quantity has to be diminished by 6 while in question 6 it is necessary to add the two resulting quantities. The two questions are of about equal difficulty.

Test IV - Changing Formulas to Obtain Other Formulas

This test might have been named "Changing the Subject of Formulas" or "Deriving New Formulas" but the meaning would not have been as clear to the child who has had little or no experience with this concept of the formula. The purpose of the test is to find to what extent the pupils of the junior high school are able to make use of this concept early in the year. At that time the pupils of grades 7 and 8 have had no teaching of this concept of the formula and it is not likely that many of the grade 9 pupils have had any teaching of it. In the explanation to the test the child is shown why it might be necessary or how it might be useful to get a new formula from one already known. The illustration used is that of finding a formula for W from the formula $A = LW$ in order that it might be possible to proceed directly to find the width of a rectangle if the area and the length are known. The child is not asked to show any work. He is reminded that the letters or symbols in formulas stand for quantities or numbers and can be added, subtracted, multiplied, and divided. The formulas express-

ing the correct answers to the chosen questions are quite varied including as they do a symbol diminished by a number, a quotient of two symbols, a symbol diminished by another symbol, a symbol increased by another symbol, a quotient resulting from dividing a symbol by a number, and a product of two symbols.

The mastery coefficients for this test as shown by Table X are 46.85 for grade 7, 65.64 for grade 8, 76.44 for grade 9 and 63.63 for the junior high school group. The grade-to-grade gain is 18.79 in the 7-8 interval and 10.80 in the 8-9 interval. It ranks fourth in every group having the lowest mastery coefficient of all the tests except Test V. In grade 9 its mastery coefficient does not differ much from those of Tests I and II while in grade 8 it is of about equal difficulty with Test II. In grade 7 it is shown to be considerably easier than Test V and noticeably harder than Test II. A comparison of the median scores for Test IV with those of the other tests as shown in Table XIII also shows these general relations. The medians are 2.74 for grade 7, 3.99 for grade 8, 5.16 for grade 9 and 3.99 for the junior high school group.

A study will now be made of the score distribution of this test as shown in Table XX and then of the mastery coefficients for the different questions for each of the different grade groups and for the junior high school

TEST IV CHANGING FORMULAS TO GET OTHER FORMULAS

Sometimes we want to change a formula from one form to another. If we want to find the area of a rectangle we use the formula A = LW but if we want to get the width when we know the area and the length we divide the area by the length. The new formula is $W = \frac{A}{L}$ which is a formula for W and not for A. Any formula can be changed to be a formula by which we can find another of the letters in that formula.

Look carefully at the sample given below and then do the questions that follow. You do not need to show any work. Just put the answer in the column under "New Formulas". Remember that the letters of symbols in the formulas stand for quantities or numbers and that you can add, subtract, multiply, and divide with them as I have shown in the formula above A = LW and as you see in the sample below.

GIVEN FORMULAS	TO FIND	NEW FORMULAS
Sample:		
$g = t \neq k$	Find a formula for t.	$t = g - k$
1. $k = n \neq 6$	Find a formula for n.	n = ...
2. $d = rt$	Find a formula for r.	r = ...
3. $S = c \neq G$	Find a formula for G	G = ...
4. $j = k - s$	Find a formula for k	k = ...
5. $n = 4s$	Find a formula for s	s = ...
6. $g = \frac{t}{m}$	Find a formula for t	t = ...

groups. The mastery coefficients will be shown in Table XXI.

TABLE XX

THE DISTRIBUTION OF SCORES FOR TEST IV

Score	Grade 7	Grade 8	Grade 9	J.H.S.Grades	Grade 10
6	35	77	139	251	85
5	23	51	60	134	11
4	43	51	41	135	3
3	51	59	33	143	1
2	48	41	26	115	4
1	41	24	14	79	0
0	39	5	6	50	0
Totals	280	308	319	907	104

The table reads: A score of 6 was made by 35 pupils in grade 7, 77 pupils in grade 8, 139 pupils in grade 9 ...etc.

The table shows that 12 per cent of grade 7, 25 per cent of grade 8, 43 per cent of grade 9, and 27 per cent of the junior high school group made a perfect score on this test. It shows also that in each grade of the junior high school the distribution includes the entire range of scores. It also shows that 54 per cent of the grade 7 group, 77 per cent of the grade 8 group, 85 per cent of the grade 9 group and 71 per cent of the junior high school group secured at least fifty per cent of the possible score. This would

indicate that the pupils of grades 8 and 9 and half of grade 7 have a good understanding of how to put this concept to use. In this it is assumed that the pupil who gets 3 of the 6 questions understands the concept. Table XX shows also that there is a very wide range in the abilities of the different pupils making up the individual groups. To compare the results of the individual items making up the test it is necessary to refer to Table XXI.

TABLE XXI

THE MASTERY COEFFICIENTS FOR THE QUESTIONS OF TEST IV FOR GRADE GROUPS AND FOR THE JUNIOR HIGH SCHOOL GROUP TOGETHER WITH RANKING OF THESE MASTERY COEFFICIENTS FOR THESE GROUPS

	Mastery Coefficients				Ranking			
Question	Grade7	Grade8	Grade9	J.H.S.Grades	VII	VIII	IX	J.H.S.
1	78.26	84.74	89.03	82.58	1	1	1	1
2	32.14	58.44	68.02	53.69	5	5	5	5
3	56.07	72.08	77.74	69.13	2	2	3	2
4	55.00	65.26	82.76	68.25	3	3	2	3
5	29.29	53.90	64.26	49.94	6	6	6	6
6	35.71	59.41	77.12	58.32	4	4	4	4

The table reads: The mastery coefficient for question 1 for grade 7 is 78.26, for grade 8 is 84.74, for grade 9 89.03, for junior high school grades is 82.58.

The contents of Table XXI show that there is considerable range in the difficulties presented by the different items making up Test IV. With one exception the ranks are the same in all groups. The difficulties are such that they are found to be common to all the grades. Questions 1 and 3 require the right hand member of the formula to be a difference for example (k - 6) and (S - c). At first glance question 1 might be considered to be the easier of the two as it involves but one symbol and a number while question 3 involves the manipulation of two symbols. Furthermore, the sample given at the beginning of the test is a type similar to question 3. The results as shown in Table XIX show that the number of pupils who failed to get question 1 was less than three-fifths of the number that failed to get question 3. The poorer results with question 3 may be due to the fact that the pupils are asked to find a formula for G which is the second element of the sum making up the right hand member of the original formula while in question 1 they are asked to find a formula for n which is the first element in the right hand member of the original formula. Question four involves expressing a value in the form of a sum and the results show that it is about equal in difficulty to question 3. Question 6, which involves a product, was mastered by a larger number than were questions 2 and 5 both of which involved the forming

of a quotient. The answer to question 2 is d/t while that to question 5 is n/4. More pupils answered question 2 correctly than answered question 5 correctly. This might be surprising after one has noticed the results of questions 1 and 3. The difficulty seems to be in understanding the concept 4s. The same trouble was seen in question 2, Test III.

Test V - Functional Relations or Variations

The purpose of Test V was to find to what extent the pupils of the junior high school grades understood the possible effect on any given member of a formula of making certain changes in one or more other members. The questions were confined to the two formulas A = LW and t = d/r. The meaning of the formulas was explained before any questions were asked. The test was divided into two parts: the first part consisting of four questions on the formula A = LW and the second part consisting of four questions based on the formula t = d/r. The questions in the first part involved changes in L or W or both L and W. The pupil was asked to state what change would take place in A in each case. It would not be sufficient for the pupil to say that A increased or that A got smaller but it would be necessary to say how many times larger or how many times smaller A became in each case. The questions

TEST V FUNCTIONAL RELATIONS OR VARIATIONS

The area of a rectangle is found by multiplying its length by its width. This can be expressed in the formula A = LW (A is the area, L the length, and W the width of the rectangle.)

In the questions below you are to tell just what changes take place in (A) if certain given changes take place in (L) or in (W) or in both (L) and (W). If it is larger, don't just say that it is larger but say that it is 4 times as large or whatever the number of times it is larger. If it is smaller say that it is 1/2 as large or 1/5 as large or whatever the fraction is. If there is no change, say 'A is not changed'.

Formula .....A = LW

CHANGES IN (L) OR (W) OR IN BOTH	CHANGES IN A
1. (L) is doubled and (W) is unchanged	A is .....
2. (W) is divided by 2 and (L) is unchanged	A is .....
3. Both (L) and (W) are doubled	A is .....
4. (L) is divided by 4 and (W) is multiplied by 2.	A is .....

The time (t) in hours, that it will take to go (d) miles at (r) miles an hour is found by dividing the distance by the rate. This is expressed by the formula $t = \frac{d}{r}$

In the questions below certain changes are made in (d) or in (r) or in both (d) and (r). In each case you are to look at the formula and tell what changes will be made in (t). (That is you are to tell how many times larger it will be, or how many times smaller it will be, or that it does not change .....just as in the questions with A = LW, above).

Formula ..... $t = \frac{d}{r}$

CHANGES IN (d) OR IN (r) OR IN BOTH	CHANGES IN (t)
5. (d) is doubled and (r) does not change	(t) is .....
6. (r) is multiplied by 3 and (d) does not change	(t) is .....
7. Both (d) and (r) are doubled	(t) is .....
8. (r) is divided by 3 and (d) is unchanged	(t) is .....

in the second part of the test involved changes in d or r. The pupil was asked to state what change, if any, took place in t in each case. Three of these questions involved changes in r. In order to answer the questions correctly the child must have some understanding of what is involved in inverse variation whether he has ever heard of the term or not.

It is quite safe to assume that few, if any, of the children had received any teaching of this concept. The results should indicate whether the child understands the concept or under what conditions he understands it. They may show that he understands direct variation if it is simple but in no other case. From the results it may not be easy to conclude just where the trouble lies but the broad general difficulties may be quite apparent. The results of the test will be studied by reference to the tables of test medians and test mastery coefficients as well as to tables showing the distribution of scores and the mastery coefficients of the different questions.

Table XII shows that the mastery coefficient of Test V is very low, ranking far below that of any other test. The mastery coefficient for the junior high school group is 41.57 which is 22.06 below the next highest coefficient. The mastery coefficient for grade

7 is 29.02, for grade 8 39.57 and for grade 9 54.50. If the results of the entire test are to be accepted as an indication of the understanding of the concept, it is quite clear that the understanding is not very great. The poor returns for this test are also shown by a study of the grade medians for Test V as shown in Table XIII. The median score for grade 7 is 2.27, for grade 8 3.30, for grade 9 is 4.49 and for the junior high school group 3.39. The possible score is 8 for Test V. Table XII also shows that the greatest grade-to-grade gain came in the 8-9 interval. This is the only test of the entire set in which such is the case.

Table XXII shows the distribution of scores for Test V for the different groups.

TABLE XXII

THE DISTRIBUTION OF SCORES FOR TEST V

Score	Grade 7	Grade 8	Grade 9	J.H.S.Grades	Grade 10
8	2	9	38	49	37
7	1	10	39	50	22
6	7	15	28	50	10
5	34	63	54	151	12
4	43	50	48	141	7
3	46	35	30	111	6
2	31	44	36	111	1
1	43	35	22	100	6
0	73	47	24	144	3
	280	308	319	907	104

The table reads: A score of 8 was made by 2 pupils in grade 7, 9 pupils in grade 8, 38 in grade 9, 49 in the junior high school grades and 37 in grade 10.

The data of this table stand in contrast to that of corresponding tables for the other tests in a number of ways. In the first place only a very small percentage of the pupils in any group obtained a perfect score: less than one per cent in grade 7, 3 per cent in grade 8, 12 per cent in grade 9 and 5 per cent of the junior high school group. Secondly, a large number of pupils obtained a zero score: 26 per cent of grade 7, 15 per cent of grade 8, 7 per cent of grade 9 and 16 per cent of the junior high school group. In the third place, the scores are quite evenly distributed over the entire range of scores in all the grades with the heaviest concentration at the lower score levels. The proportion receiving at least 50 per cent of the possible score was 31 per cent for grade 7, 47 per cent for grade 8, 63 per cent for grade 9 and 48 per cent for the junior high school group. This, together with the other facts mentioned concerning the data of Table XX, is sufficient to show that the junior high school pupils have a poor understanding of the concept to be tested. In order to see whether the difficulties are common to both indirect and direct variation or whether the results show that the pupil has an understanding of the one but not the other, it will be necessary to study the data of Table XXIII. This

table includes the mastery coefficients for the items of Test V together with the ranking of each item for each group considered.

The data of the table show that there are two distinct divisions in the test when judged by the mastery coefficients of the separate items of the test. The first includes questions 1 to 5. These questions possess the common characteristic of direct variation. The second division includes the other three questions all of which deal with indirect variation. The range of the coefficients for the first division for the junior high school group is 42.56-65.27. This does not indicate a high degree of mastery but is much better than the results secured for the other three questions. The range of coefficients for questions 6, 7, and 8 for the junior high school group is 18.30-27.56. These results show that the concept is much more fully understood by the junior high school pupil if it is embodied in direct rather than in indirect variation. The data of Table XXIII also shows that there is considerable grade-to-grade gain in each item but that this gain is much more marked in some cases than in others. It shows also that there is a slight variation in the ranking of the items for 2nd, 3rd, and 4th positions but the other rankings are consistent. Questions 6 and 8 are of about equal difficulty and are decidedly the hardest questions in the

test. Question 2 is much the easiest.

TABLE XXIII

THE MASTERY COEFFICIENTS FOR THE QUESTIONS OF TEST V FOR THE GRADE GROUPS AND FOR THE JUNIOR HIGH SCHOOL GROUP TOGETHER WITH THE RANKING OF THESE MASTERY COEFFICIENTS FOR THESE GROUPS

	Mastery Coefficients				Ranking			
Question	Grade7	Grade8	Grade9	J.H.S.Group	VII	VIII	IX	J.H.S.
1	48.57	54.87	66.46	57.00	2	2	3	2
2	52.14	67.21	74.92	65.27	1	1	1	1
3	38.57	49.03	54.86	47.85	3	4	4	4
4	28.93	47.08	53.29	42.56	5	5	5	5
5	37.50	52.27	67.71	53.14	4	3	2	3
6	6.43	12.99	36.99	19.40	7	7	7	7
7	13.57	20.78	46.39	27.56	6	6	6	6
8	6.07	12.01	35.11	18.30	8	8	8	8

The table reads: The mastery coefficient for question 1 for grade 7 is 48.57, for grade 8 is 54.87, for grade 9 is 66.46, and the junior high school group 57.00.

A survey of the data given up to this point on Test V raises a number of problems that need investigation. Do the poor scores made in the first four questions indicate that (1) the child does not understand this concept of the formula or (2) do they indicate that the child does not understand the relationship existing between the dimensions of a rectangle and its area or (3) do they indicate he does

not understand either? Are the poor results of the last four questions in the test due to a lack of understanding the rate-time-distance concept or to the lack of understanding of the formula concept? To what extent would the difficulties due to indirect variation be removed if the situation were presented in a concrete form instead of in the formula? These and other questions suggested the preparation of another test, which will henceforth be referred to as Test VI, in which each question paralleled the corresponding question of Test V except that numbers were used in the place of formulas. At the beginning of each section the formula was given as it had been given in Test V in order that there should be no need to guess the relationships existing in the area of a rectangle and in the rate-time-distance situation. The child was not asked to make any use of the formulas. A reading of the questions in the test shows that in the first four questions the quantity given was always the area and in each case the child was to write down the new area that would result from certain stated changes in the length or the width of the rectangle or in both the length and width. In the last four questions the quantity given in each case was the time required for a trip in which either the rate or the distance or both the rate and the distance have been changed. The test was then given to 264 pupils from the seven grade-

TEST VI

INSTRUCTIONS

Read each question carefully. Do any figuring that you need to do in the space under work. Write nothing but the answer to the questions under Answers.

QUESTIONS	WORK	ANSWERS
The area of a rectangle is found by multiplying its length by its width. This can be expressed in the formula A = LW (A is the area, L the length, and W the width of the rectangle.		
1. The area of a rectangle is 400 square feet. What would be the area of another rectangle if it is twice as long and the same width as this one?		.....
2. The area of a rectangle is 300 square feet. What would be the area of another rectangle if it is the same length but one fifth as wide as this one?		.....
3. The area of a rectangle is 800 square feet. What would be the area of another rectangle that was twice as long and twice as wide as this one?		.....
4. The area of a rectangle is 1200 square feet. What would be the area of another rectangle that was one-fourth as long and twice as wide as this one?		.....
The time (t) in hours, that it will take to go (d) miles at (r) miles an hour is found by dividing the distance by the rate. This is expressed by the formula $t = \frac{d}{r}$		
5. Tom takes 12 hours to make a certain trip in his car. How long would he take to make a trip 4 times as far at the same rate of speed?		.....
6. John takes 14 hours to make a certain trip in his car. How long would he take to make the same trip if he went twice as fast?		.....
7. Jack takes 20 hours to make a certain trip in his car. How long would he take to make the trip 3 times as far if he travelled 3 times as fast as in his first trip?		.....
8. Jim takes 30 hours to make a certain trip in his car. How long would he take to make the same trip if he went one third as fast as in his first trip?		.....

groups that had been used in School I for the other tests. All the pupils who had written the former test were not present for Test VI while there were some pupils who wrote Test VI who had not written Test V. It was necessary to include in the comparison only those pupils who had written both tests. The data for Test V was obtained from the grade-pupil work-sheets for those concerned. New score-sheets were prepared for these pupils to show the results for each question for each pupil writing the test. Table XXIV shows the groups to which both Test V and Test VI were given with the number of pupils in each group taking both tests.

TABLE XXIV

THE GRADE-GROUPS TO WHICH BOTH TEST V AND TEST VI WERE GIVEN WITH THE NUMBER OF PUPILS IN EACH GROUP

School	Grade 7	Grade 8	Grade 9	Totals
I	31-A	39-H	28-P	
	31-B	35-I	27-Q	
		38-J	35-R	
	62	112	90	264

The table reads: In grade 7 there were 31 pupils from group A, and 31 pupils from group B making a total of 62 pupils from grade 7; in grade 8 there were 39 pupils from group H, ... etc.

The median scores for each grade are given in Table XXV. They show that in the case of every grade the results were considerably better for Test VI than for Test V when judged by the median scores.

TABLE XXV

THE MEDIAN SCORE FOR TESTS V AND VI
FOR THE GROUPS THAT TOOK BOTH TESTS

Test	Grade 7	Grade 8	Grade 9	All the Grades
V	3.50	3.74	4.81	3.78
VI	4.50	4.58	6.29	5.49

The table reads: The median score made in Test V for grade 7 is 3.50, for grade 8 is 3.74, for grade 9 is 4.81 and for all the grades is 3.78.

Table XXVI shows the distribution of scores for both tests.

TABLE XXVI

THE DISTRIBUTION OF SCORES FOR THE
SPECIAL GROUPS FOR TEST V AND TEST VI

Score	Grade 7		Grade 8		Grade 9		All the Grades	
	Test V	Test VI	Test V	Test VI	Test V	Test VI	Test V	Test VI
8	1	9	3	16	13	27	17	52
7	0	3	3	17	14	15	17	35
6	3	7	7	11	7	14	17	32
5	7	10	30	13	16	9	53	32
4	10	7	17	21	12	10	39	38
3	10	8	8	22	8	6	26	36
2	5	10	16	6	6	5	27	21
1	10	6	13	2	6	0	29	8
0	16	2	15	4	8	4	39	10
Totals	62	62	112	112	90	90	264	264

The data of this table bears out that of Table XXV in showing that the general scores were higher in Test VI than in Test V. It shows also many more of the pupils obtained scores in the upper score levels in Test VI than in Test V. If the comparison were to stop here the conclusions formed would likely be erroneous. To make a true comparison of the results it is necessary to turn to the data of Table XXVII which shows the mastery coefficients for each item in each test side by side for the same groups as shown in Table XXVI.

TABLE XXVII

SHOWING A COMPARISON OF THE MASTERY COEFFICIENTS FOR TESTS V AND VI FOR THE DIFFERENT GRADE GROUPS TAKING BOTH TESTS

Question	Grade 7		Grade 8		Grade 9		All the Grades	
	Test V	Test VI	Test V	Test VI	Test V	Test VI	Test V	Test VI
1	46.77	54.84	63.39	66.96	65.56	76.67	60.23	67.42
2	53.23	40.32	66.96	41.07	77.78	66.67	67.42	49.62
3	40.32	41.94	53.57	41.96	61.11	63.33	53.03	49.24
4	25.81	30.64	47.32	41.07	62.22	63.33	47.35	46.21
5	43.55	75.81	54.46	82.14	67.78	86.67	56.44	82.20
6	8.06	83.87	16.96	91.07	40.00	90.00	22.73	89.02
7	17.74	61.29	22.32	81.25	45.56	82.22	29.17	76.89
8	8.06	35.48	14.29	38.39	40.00	46.67	21.59	40.53

The table reads: The mastery coefficient for question 1 for grade 7 was 46.77 for Test V and 54.84 for Test VI, for grade 8 was 63.39 for Test V and 66.96 for Test VI, for grade 9 was ...etc.

The data shown in Table XXVII reveal that in the first four questions one test seems to have given as much difficulty as another. In question 1 every group did better on Test VI than on Test V but the difference is not great. In question 2 they all did better on Test V than on Test VI and the difference is quite marked. In question 3 grades 8 and 9 did better on Test V than on Test VI but the difference in the results is not as great as in question 2. In question 4 grade 7 did better on Test VI, grade 8 on Test V and grade 9 slightly better on Test VI. It seems quite safe to conclude that the difficulty in these questions is not due to the formula so much as it is to the lack of an understanding of the relationship between the area of a rectangle and its dimensions. It is not likely that many of these pupils would be unable to find the area of a rectangle if they were given the dimensions but they fail to see how a definite change in one dimension or in both would tend to affect the area.

The results for the second part of Test VI are seen to be much better than those for the second part of Test V. There is almost no comparison in the results. It can be seen from the table that for grade 7, over one and a half times as many pupils succeeded in Test VI than in Test V when dealing with question 5. In the other questions the differences are still more marked. They did ten times

as well in question 6, three times as well in question 7, and four times as well in question 8 in Test VI as they did in Test V. The chief difficulty in doing questions 5 to 8 in Test V is therefore due to the lack of ability to use that formula in the way required to get the solutions to the questions. This difficulty is more marked in dealing with indirect variation than in dealing with direct variation. This can be shown by comparing the mastery coefficients of questions 5, 6 and 7 in Test V and then doing the same in Test VI. The mastery coefficient of question 8 Test VI is still surprisingly low. This may be due to the fraction concept. Questions 2 and 4 of Test VI also give results that are relatively lower than might be expected. Part of this may be due to the fraction concept which must be used in both questions. In Test V the fraction expression is not used but rather the wording is, W is divided by 2, L is divided by 4, and r is divided by 3. In Test VI the wording is one-fifth as wide, one-fourth as long, and one-third as fast, for the corresponding questions. The wording in Test V may be suggestive of the operation that should be performed. The reader may suggest that in Test VI there would be room for considerable loss due to errors in calculation. If this is so it was due to mistakes in mental calculation for there were only four mistakes in written calculation in the 2112 questions that

were to be done. Of the remaining 887 mistakes made, 110 were due to the use of the wrong variation (i.e. multiplying in place of dividing or vice versa) and 777 were miscellaneous errors.

This concludes the study of the individual tests. The following chapter will be devoted to an analysis of the types of errors made in the individual questions of the tests. The results may help to form conclusions concerning some of the findings already discussed.

CHAPTER V

A DETAILED STUDY OF THE ERRORS MADE IN THE WORKING OF EACH QUESTION BY THE PUPILS OF THE JUNIOR HIGH SCHOOL GRADES

It is the purpose of this chapter to make a detailed examination of the questions that the pupils failed to get correct and to find the number and the nature of the errors. In each grade-group score-sheet the vertical columns were added to find the number of correct solutions that had been given by the pupils of that group for each question. Then the total number of errors was counted for each question. When the error had been entered on the work sheet the type of error was indicated by a designating letter. When the total of the errors had been found the different designating letters were counted and the number of each type of error entered in the proper place. The results were transferred from the individual grade-group score-sheets to a special error sheet for each question. On this sheet the number of each type of error for each question was computed for each grade and for the junior high school group. From the results the mastery coefficient and the error coefficient for each question were computed. This was done for each grade. The next step was to find what per cent of the total errors in each case was of each type recorded. This

chapter will be devoted to the study of the findings of this portion of the investigation. For each question there will be made a small table containing the mastery coefficients and the error coefficients. The number of pupils in each group will be included at the head of each table for convenience in interpreting the table. A sub-table will be made to follow each question table for the purpose of showing the per cent of the total number of errors that were of each type. It is thought to be quite essential to include in the sub-table the number of errors made by each group on that question. In the tables the type of error will be indicated by the designating letter. This will be the case for Tests I, III and V. The nature of the other tests gives rise to quite distinct errors in each question. Because of this the form actually taken by the error will be included in the table.

In order to provide a reference list that will be convenient for following the discussion of the types of errors a list of designating letters will be given at this point with the meaning applied to each. They are as follows:

R Meaning a correct solution of the question

X Meaning an incorrect answer

O Meaning an omission of the question

m Meaning a miscellaneous answer. As a rule it

seems to have no connection with the question or with a possible correct answer.

q Meaning that a quality is given that is pertinent to the question. A quantity is required though, not a quality.

r Meaning some quality that is not pertinent to the solution of the question.

s Meaning that a former answer has been repeated in other words. Credit has been given for the first answer and therefore cannot be given again.

t Meaning that the pupil has said that the area of a triangle depends on the sides of the triangle. Of course this is true but the child does not know why. No credit is given for it as an answer to the question.

h Stating that the area of a triangle depends on the length and width of the triangle. A carry over from the sample.

d Stating that the distance a train goes depends on how far it is going or upon the distance.

i Stating that the amount of interest received on a loan depends upon the amount of interest or simply upon the interest.

f Meaning an error in calculation or figuring.

w Used only for Test V and meaning that the wrong

variation has been taken .... direct in place of indirect or vice versa.

g Used only in Test V and meaning that the pupil has written down again simply the changes that were said to take place in L or W or r or d. This seems to be due to a misunderstanding of what is wanted in the question.

Test I. Writing Formulas

In dealing with the questions of Test I, each question will be considered in its two parts separately. When the two parts have been treated a table will be given to show the per cent of the total number of times that both answers were given, that part (a) alone was correct, that part (b) alone was correct, and that neither was correct. This data will be given for the three different grades and also for the junior high school group. The questions will now be dealt with in the order of their appearance in the tests.

Test I. Question 1 The cost of a roast of meat depends on .....

(a) the number of pounds of meat.

(b) the price per pound.

TABLE XXVIII

SHOWING THE MASTERY COEFFICIENTS AND THE ERROR COEFFICIENTS FOR 1 (a) OF TEST I

	Grade 7	Grade 8	Grade 9	J.H.S. Group
Pupils	280	308	319	907
R	87.14	93.83	96.86	92.83
X	12.86	6.17	3.14	7.17

The table reads: The mastery coefficient for grade 7 is 87.14 for grade 8 ..... The error coefficient for grade 7 is 12.86, for grade 8 is 6.17, for ..... etc.

TABLE XXVIII (a)

SHOWING THE TOTAL NUMBER OF ERRORS FOR EACH GRADE AND THE PER CENT OF THIS NUMBER DUE TO EACH TYPE OF ERROR, TEST I,(a)

Type	Grade 7	Grade 8	Grade 9	J.H.S. Group
O	2.78	0	0	1.54
m	50.00	31.58	30.00	41.54
q	27.28	42.10	40.00	33.85
r	11.11	26.32	30.00	18.46
s	8.33	0.00	0.00	4.61
No. of Errors	36	19	10	65

The table reads: The per cent of the total number of errors for this question due to omissions was 2.78 for grade 7, 0.00 for grade 8, 0.00 for grade 9 and 1.54 for the J.H.S. group ... etc.

TABLE XXIX

SHOWING THE MASTERY COEFFICIENTS AND THE ERROR COEFFICIENTS FOR 1 (b) OF TEST I

	Grade 7	Grade 8	Grade 9	J.H.S. Group
No. of Pupils	280	308	319	907
R	65.71	73.38	87.46	75.96
X	34.29	26.62	12.54	24.04

The table reads: Like Table XXVIII.

TABLE XXIX (a)

SHOWING THE TOTAL NUMBER OF ERRORS FOR EACH GRADE AND THE PER CENT OF THIS NUMBER DUE TO EACH TYPE OF ERROR, TEST I, 1(b)

Type	Grade 7	Grade 8	Grade 9	J.H.S. Group
O	6.25	0.00	0.00	0.00
m	31.25	20.73	20.00	25.23
q	50.00	68.29	60.00	58.72
r	2.08	0.00	2.50	1.38
s	10.42	10.98	17.50	11.92
No. of Errors	96	82	40	218

The table reads: The same as Table XXVIII (a)

TABLE XXX

SHOWING WHAT PER CENT OF THE TOTAL QUESTIONS FOR TEST I (1) HAD BOTH PARTS CORRECT, (a) PART ALONE CORRECT, (b) PART ALONE CORRECT, AND NEITHER PART CORRECT

Results	Grade 7	Grade 8	Grade 9	J.H.S. Group
Both Parts	57.86	70.13	85.90	71.88
(a) only	29.28	23.70	10.97	20.95
(b) only	7.86	3.25	1.57	4.08
Neither Part	5.00	2.92	1.56	3.09

The table reads: The per cent of the questions having both parts correct in grade 7 is 57.86, in grade 8 is 70.13, in ..... etc.

A study of the data contained in the five tables dealing with question 1 in Test I shows that the question was quite well mastered by all the grades. Part (a) was done better than part (b). This was partly due to the use of a quality in place of a quantity in expressing a dependence. Many pupils said that the price depended on the kind of meat or on the time of the year or gave other answers which were quite pertinent to the question but which were not quantities. Table XXIX (a) shows that only 33.85 per cent of the errors in part (a) were due to giving a quality in place of a quantity. Miscellaneous errors and unrelated qualities (r) rank high among the errors for part (a). Table XXX shows that in only 3 per

cent of the total number of questions neither part of the question was correct, while in 72 per cent of the questions both parts were correct. It also shows that the grade-to-grade gain for having the two parts is quite pronounced.

Test I. Question 2. The time required to do a certain number of arithmetic questions depends on:

(a) The number of questions

(b) The time required for one question

TABLE XXXI

SHOWING THE MASTERY COEFFICIENTS AND THE ERROR COEFFICIENTS FOR TEST I, 2 (a)

	Grade 7	Grade 8	Grade 9	J.H.S.Group
No. of Pupils	280	308	319	907
R	66.08	84.09	83.70	78.39
X	33.92	15.91	16.30	21.61

Table reads: Like Table XXVIII.

TABLE XXXI (a)

SHOWING THE TOTAL NUMBER OF ERRORS FOR EACH GRADE AND THE PER CENT OF THIS NUMBER DUE TO EACH TYPE OF ERROR, TEST I, 2 (a)

Type	Grade 7	Grade 8	Grade 9	J.H.S. Group
o	0.00	0.00	3.85	1.03
m	35.79	26.53	25.00	30.61
q	56.85	63.27	67.30	61.22
r	3.15	10.20	3.85	5.10
s	4.21	0.00	0.00	2.04
No. of Errors	95	49	52	196

The table reads: Like XXVIII (a)

TABLE XXXII

SHOWING THE MASTERY COEFFICIENTS AND THE ERROR COEFFICIENTS FOR TEST I, 2 (b)

	Grade 7	Grade 8	Grade 9	J.H.S. Group
No. of Pupils	280	308	319	907
R	38.60	44.16	56.11	46.64
X	61.40	55.84	43.89	53.36

The table reads: Like Table XXVIII.

TABLE XXXII (a)

SHOWING THE TOTAL NUMBER OF ERRORS FOR EACH GRADE AND THE PER CENT OF THIS NUMBER DUE TO EACH TYPE OF ERROR, TEST I, 2 (b)

Type	Grade 7	Grade 8	Grade 9	J.H.S. Group
0	4.65	2.90	2.86	3.51
m	41.86	30.82	41.42	37.82
q	43.60	60.47	50.00	51.44
r	.58	2.33	2.86	1.86
s	9.31	3.48	2.86	5.37
No. of Errors	172	172	140	484

The table reads: Like Table XXVIII (a)

TABLE XXXIII

SHOWING WHAT PER CENT OF THE TOTAL QUESTIONS FOR TEST I (2) HAD BOTH PARTS CORRECT, (a) PART ALONE CORRECT, (b) PART ALONE CORRECT, AND NEITHER PART CORRECT

Results	Grade 7	Grade 8	Grade 9	J.H.S.Group
Both Parts	30.72	40.59	53.92	42.23
(a) only	35.35	43.51	29.78	36.16
(b) only	7.86	3.57	2.19	4.41
Neither Part	26.07	12.33	14.11	17.20

The table reads: Like Table XXX.

The tables show that the mastery of Part (a) is much better than that of Part (b), that quality answers play a big part in the errors of both parts, that about one-third of the errors were miscellaneous, that the omissions were not numerous and that there were a relatively large number of (s) errors in Part (b). They also show that less than 50 per cent of the pupils obtained the correct answer to Part (b). The type of quality answers that were frequently made were .... the kind of questions, how long they were, how quickly you could do arithmetic, how accurate you were, etc. Table XXXIII shows that only 42 per cent of the pupils obtained both parts of the question correct. It also shows that there is a spread of 10.00 in percentage measure between the successive grades. There were eight times as many cases in which Part (a) was given alone than in which Part (b) was given alone. In nearly one-fifth of all the cases neither part of the question was correct.

Test I - Question 3. The area of a triangle depends on ...

(a) The base of the triangle

(b) The altitude (or height) of the triangle.

TABLE XXXIV

SHOWING THE MASTERY COEFFICIENTS AND THE ERROR COEFFICIENTS FOR TEST I, 3 (a)

	Grade 7	Grade 8	Grade 9	J.H.S.Group
No. of Pupils	280	308	319	907
R	66.43	68.18	76.18	70.45
X	33.57	31.82	23.82	29.55

The table reads: Like Table XXVIII.

TABLE XXXIV (a)

SHOWING THE TOTAL NUMBER OF ERRORS FOR EACH GRADE AND THE PER CENT OF THIS NUMBER DUE TO EACH TYPE OF ERROR, TEST I, 3 (a)

Type	Grade 7	Grade 8	Grade 9	J.H.S. Group
o	9.57	3.06	6.58	6.34
m	10.64	4.08	13.16	8.96
s	7.45	0.00	0.00	1.50
t	4.26	12.25	14.47	11.19
h	68.08	80.61	65.79	72.01
No. of Errors	94	98	76	268

TABLE XXXV

SHOWING THE MASTERY COEFFICIENTS AND THE ERROR COEFFICIENTS FOR TEST I, 3 (b)

	Grade 7	Grade 8	Grade 9	J.H.S. Group
No. of Pupils	280	308	319	907
R	62.86	62.66	66.77	64.17
X	37.14	37.34	33.23	35.83

The table reads: Like Table XXVIII.

TABLE XXXV (a)

SHOWING THE TOTAL NUMBER OF ERRORS FOR EACH GRADE AND THE PER CENT OF THIS NUMBER DUE TO EACH TYPE OF ERROR, TEST I, 3 (b)

Type	Grade 7	Grade 8	Grade 9	J.H.S. Group
0	10.57	6.09	9.44	8.62
m	10.57	3.48	22.64	12.00
s	5.77	2.60	.94	3.08
t	9.62	20.00	19.81	16.61
h	63.47	67.83	47.17	59.69
No. of Errors	104	115	106	325

The table reads: Like Table XXVIII (a)

TABLE XXXVI

SHOWING WHAT PER CENT OF THE TOTAL QUESTIONS FOR TEST I (3), HAD BOTH PARTS CORRECT, (a) PART ALONE CORRECT, (b) PART ALONE CORRECT, AND NEITHER PART CORRECT

Results	Grade 7	Grade 8	Grade 9	J.H.S. Group
Both Parts	60.72	61.36	65.83	62.73
(a) only	5.72	6.82	10.35	7.72
(b) only	2.14	1.30	.94	1.43
Neither Part	31.42	30.52	22.88	28.12

The table reads: Like Table XXXX.

A survey of the tables shows that about two-thirds of the pupils tested understand upon what the area of a triangle depends. There is very little difference between the grades. The two parts are known about equally well with a slight lead given to Part (a). The outstanding error (h) is stating that the area of a triangle depends on its length and its width. The second major error is that of saying that the area of the triangle depends on the sides. There are also a considerable number of omissions and miscellaneous errors. Table XXXVI shows that over 25 per cent of the questions did not have either part correct. There are more cases in which Part (a) was answered alone than there are in which Part (b) was answered alone.

Test I. - Question 4. The amount of pay a man receives depends on .....

(a) The length of time he works

(b) The amount he receives per unit of time

TABLE XXXVII

SHOWING THE MASTERY COEFFICIENTS AND THE ERROR COEFFICIENTS FOR TEST I, 4 (a)

	Grade 7	Grade 8	Grade 9	J.H.S. Group
No. of Pupils	280	308	319	907
R	66.78	92.53	96.86	86.11
X	33.22	7.47	3.14	13.89

The table reads: Like Table XXVIII.

TABLE XXXVII (a)

SHOWING THE TOTAL NUMBER OF ERRORS FOR EACH GRADE AND THE PER CENT OF THIS NUMBER DUE TO EACH TYPE OF ERROR, TEST I, 4 (a)

Type	Grade 7	Grade 8	Grade 9	J.H.S. Group
0	4.30	4.35	0.00	3.97
m	22.58	26.08	0.00	21.43
q	63.44	60.87	80.00	64.28
r	3.23	4.35	20.00	4.76
s	6.45	4.35	0.00	5.56
No. of Errors	93	23	10	126

This table reads: Like Table XXVIII (a)

TABLE XXXVIII

SHOWING THE MASTERY COEFFICIENTS AND THE ERROR COEFFICIENTS FOR TEST I, 4 (b)

	Grade 7	Grade 8	Grade 9	J.H.S.Group
No. of Pupils	280	308	319	907
R	42.14	56.17	74.92	58.43
X	57.86	43.83	25.08	41.57

The table reads: Like Table XXVIII.

TABLE XXXVIII (a)

SHOWING THE TOTAL NUMBER OF ERRORS FOR EACH GRADE AND THE PER CENT OF THIS NUMBER DUE TO EACH TYPE OF ERROR, TEST I, 4 (b)

Type	Grade 7	Grade 8	Grade 9	J.H.S. Group
0	10.49	1.48	1.25	5.30
m	17.90	14.08	11.25	15.12
q	53.71	59.25	57.50	56.50
r	0.00	.74	2.50	.80
s	17.90	24.45	27.50	22.28
No. of Errors	162	135	80	377

This table reads: Like Table XXVIII (a)

TABLE XXXIX

SHOWING WHAT PER CENT OF THE TOTAL QUESTIONS FOR TEST I, (4) HAD BOTH PARTS CORRECT, (a) PART ALONE CORRECT, (b) PART ALONE CORRECT, AND NEITHER PART CORRECT

Results	Grade 7	Grade 8	Grade 9	J.H.S. Group
Both Parts	32.50	54.22	74.29	54.58
(a) only	34.29	38.31	22.57	31.53
(b) only	9.64	1.95	.62	3.86
Neither Part	23.57	5.52	2.52	10.03

This table reads: Like Table XXX.

A study of the tables that present the data for this question shows that there were about one and a half times as many correct answers to the first part of this question as there were to the second part. Nearly two-thirds of the errors on the first part of the question were due to the use of qualities in place of quantities while over half of the errors in the second part were due to the use of qualities. Only 13.89 per cent of the questions for Part (a) were wrong while 41.57 per cent of the questions for Part (b) were wrong. About one-sixth of the errors were miscellaneous. Over one-fifth of the errors to Part (b) were due to a restatement in different words of the answer given in Part (a). There is a large grade-to-grade gain not only when those who obtained both parts correct are considered but

also when either part of the question is considered. About one-third of the grade 7 group, one half of the grade 8 group, and three-quarters of the grade 9 group obtained both answers to question 4. More than one-fifth of the grade 7 group failed to get either answer. Eight times as many got Part (a) alone than got Part (b) alone. Most of the answers that expressed a dependence on a quality in place of on a quantity gave qualities that were very pertinent to the question. The answers of this kind that predominated were those stating that the amount of pay a man receives depends on: the nature of the work, the speed of the worker, the ability of the worker, ... etc. There was little or no mention made of the firm for which the man was working nor of the generosity of the employer. A study of the answers that were given in terms of qualities would lead one to conclude that the child has a good understanding of the meaning of dependence in the situation presented but he has failed to follow directions. This failure may have been due to poor reading habits in attacking a question or to the crowding out of what he has read by his experiences in connection with the situation. In his own experiences the amount of pay a man receives for his work may have become strongly associated with those qualities mentioned.

Test I. - Question 5. If a train travels at the same rate all the time, the distance it travels depends on:

(a) The length of time it travels

(b) The rate per unit of time

TABLE XL

SHOWING THE MASTERY COEFFICIENTS AND THE ERROR COEFFICIENTS FOR TEST I, 5 (a)

	Grade 7	Grade 8	Grade 9	J.H.S. Group
No. of Pupils	280	308	319	907
R	41.43	47.73	52.98	47.63
X	58.57	52.27	47.02	52.37

This table reads: Like Table XXVIII.

TABLE XL (a)

SHOWING THE TOTAL NUMBER OF ERRORS FOR EACH GRADE AND THE PER CENT OF THIS NUMBER DUE TO EACH TYPE OF ERROR, TEST I, 5 (a)

Type	Grade 7	Grade 8	Grade 9	J.H.S. Group
O	7.32	6.83	6.67	6.94
m	20.73	14.29	14.67	16.65
q	7.92	8.69	1.33	6.10
r	3.05	4.35	.67	2.74
s	5.48	4.97	8.00	6.10
d	55.50	60.87	68.66	61.47
No. of Errors	164	161	150	475

The table reads: Like Table XXVII (a)

TABLE XLI

SHOWING THE MASTERY COEFFICIENTS AND THE ERROR COEFFICIENTS FOR TEST I, 5 (b)

	Grade 7	Grade 8	Grade 9	J.H.S. Group
No. of Pupils	280	308	319	907
R	56.07	70.45	81.81	70.01
X	43.93	29.55	18.19	29.99

The table reads: Like Table XXVIII.

TABLE XLI (a)

SHOWING THE TOTAL NUMBER OF ERRORS FOR EACH GRADE AND THE PER CENT OF THIS NUMBER DUE TO EACH TYPE OF ERROR, TEST I, 5 (b)

Type	Grade 7	Grade 8	Grade 9	J.H.S.Group
o	19.51	14.29	24.14	14.75
m	26.02	41.76	29.31	31.99
q	11.38	7.69	6.90	9.19
r	5.69	3.30	0.00	3.67
s	6.50	3.30	3.45	4.78
d	30.90	29.66	36.20	31.62
No. of Errors	123	91	58	272

TABLE XLII

SHOWING WHAT PER CENT OF THE TOTAL QUESTIONS FOR TEST I, 5, HAD BOTH PARTS CORRECT, (a) PART ALONE CORRECT, (b) PART ALONE CORRECT, AND NEITHER PART CORRECT

Results	Grade 7	Grade 8	Grade 9	J.H.S. Group
Both Parts	18.57	33.12	43.89	32.42
(a) only	22.86	14.61	9.09	15.21
(b) only	37.50	37.34	37.93	37.60
Neither Part	21.07	14.93	9.09	14.77

The table reads: Like Table XXX.

TABLE XLIII

SHOWING THE COMBINATIONS OF TIME, RATE, AND DISTANCE FOUND IN TEST I, 5 IN THE 907 PAPERS EXAMINED

	Used With				
Answer	Time	Rate	Distance	Otherwise	Total Times Used
Time	X	294	73	65	432
Rate	294	X	248	93	635
Distance	73	248	X	57	378

The table reads: Time was used 294 times with rate, 73 times with distance and 73 times in some other way than those mentioned, making a total of 432 times in which it was used. Rate was used 294 times with time, 248 times ..... etc.

The tables on Test I, 5 show that the child sees the dependence of "distance travelled" on rate more frequently

than he does on time. The distance-time concept seems to be quite weak as the mastery coefficient for it is 47.63 while that for the distance-rate concept is 70.01. The number of omissions is large in both parts of the question but is twice as great for Part (b) as it is for Part (a). The miscellaneous errors form 16.65 per cent of Part (a) and 31.99 per cent of those of Part (b). The answers given under this classification were varied in nature and showed little or no understanding of what was involved in the question. The (q) errors are well illustrated by the following answers: the nature of the railway, the nature of the country ... level or hilly, the kind of engine, the class of train. All such answers show that the child understands the dependence relation up to a certain point. The (r) errors included such answers as: the engineer, whether you were in a hurry, the kind of coal, etc. The outstanding error in this question was "that the distance a train goes depends on such things as the following: the length of the journey, the distance, how far it is going, etc." These are all classed as (d) errors. It would seem that the child had failed to read the question properly. On the other hand in the big majority of cases when the (d) error was made, it was the second answer given having been preceded by either "rate" or "time" for the first answer. It is difficult to tell what part of

these errors was due to a poor reading of the question and what part to the laws of association. The child writes the word "time" while he is thinking of the rate-time-distance concept as a situation. Association will likely call to his mind either "rate" or "distance" or both "rate and distance". It is possible that if it is "distance" that comes to his mind, he will neglect to reread the question and simply write the word "distance" or an expression having the same implication. Further study of the different combinations of "time", "rate" and "distance" was needed. To get the data concerning "distance" combinations the pupil-grade work sheets were used. A distribution sheet was made for the different combinations of which "distance" formed a part. These data were then used in conjunction with data already available on the use of "time" and "rate" to make Table XLIII, which shows the number of times the different combinations of the three answers appeared in the 907 papers examined. It shows that "time" was used 432 times, "rate" 635 times, and "distance" 378 times. Of these there were 248 "rate-distance" combinations, 73 "time-distance" combinations and 294 "time-rate" combinations. One might conclude that the "rate-distance" combination would come to the child's mind more than three times as often as would the "time-distance" combination. A study of Table XLIII will show that this

ratio is possibly too high. In the 432 times that "time" appeared, 294 times it appeared with "rate". Of the remaining 138 times it appeared 73 times with "distance". On the other hand, in the 635 times that "rate" appeared, 294 times it appeared with "time". Of the remaining 341 times it appeared 248 times with "distance". The first ratio is 73 out of 138 or .5 while the second is 248 out of 341 or .7. This still shows that the "rate-distance" combination is more likely to take place than is the "time-distance" combination.

Test I - Question 6. The simple interest on a loan depends on .....

To give two of: The rate of interest
The time for which it is loaned
The amount of the loan

TABLE XLIV

SHOWING THE MASTERY COEFFICIENTS AND ERROR COEFFICIENTS FOR TEST I, 6 - BOTH PARTS COMBINED

	Grade 7	Grade 8	Grade 9	J.H.S.Group
No. of Pupils	280 X 2	308 X 2	319 X 2	907 X 2
R	63.75	85.55	86.83	79.27
X	36.25	14.45	13.17	20.73

The table reads: Like Table XXVIII.

TABLE XLIV (a)

SHOWING THE TOTAL NUMBER OF ERRORS FOR THE TWO PARTS OF QUESTION 6 TEST I FOR EACH GRADE AND THE PER CENT OF THIS DUE TO EACH TYPE OF ERROR

Type	Grade 7	Grade 8	Grade 9	J.H.S. Group
0	33.00	13.48	20.24	25.53
m	46.31	10.11	11.90	30.05
s	1.97	5.62	1.19	2.66
i	18.72	70.79	66.67	41.76
No. of Errors	203	89	84	376
No. of Pupils	280 X 2	308 X 2	319 X 2	907 X 2

The table reads: Like Table XXVIII (a).

TABLE XLV

SHOWING THE PER CENT OF ANSWERS OF EACH COMBINATION THAT APPEARED IN TEST I, 6

Combination	Grade 7	Grade 8	Grade 9	J.H.S. Group
Principal-Rate	26.43	42.21	37.62	35.72
Rate - Time	2.50	10.06	10.66	7.94
Principal-Time	24.64	19.81	22.26	22.16
Principal	25.71	19.81	16.93	20.62
Rate	3.23	1.95	1.25	2.09
Time	1.78	3.24	3.76	2.98
Principal-Rate-Time	0.00	1.30	5.01	2.21
Neither	15.71	1.62	2.51	6.28

The table reads: The per cent of the total questions in which the principal-rate combinations occurred was: .....

TABLE XLVI

SHOWING WHAT PER CENT OF THE TOTAL QUESTIONS FOR TEST I, 6 HAD TWO PARTS CORRECT, WHAT PER CENT HAD ONLY ONE PART CORRECT, AND WHAT PER CENT HAD NEITHER PART CORRECT

Results	Grade 7	Grade 8	Grade 9	J.H.S. Group
Both Parts	53.57	73.38	75.55	68.03
One Part	30.72	25.00	21.94	25.69
Neither Part	15.71	1.62	2.51	6.28

The table reads: The per cent of the pupils that had both parts correct is 53.57 for grade 7, 73.38 for grade 8, 75.55 for grade 9, and 68.03 for the junior high school group.

Table XLIV shows that the pupils of grades eight and nine have a good understanding of this question. About two-thirds of the grade seven pupils also got both parts of the question correct. Table XLIV (a) shows that one-third of the 203 errors made in grade seven were due to omissions, almost one-half to miscellaneous answers and nearly all the rest to the "i" error. Under the "i" error were included those answers that stated that the amount of interest depended upon "the interest", "the amount of interest", "how much the interest was", etc. This is likely due to a carelessness in expression on the part of the pupil more than to a lack of understanding of the fact that the amount of interest depends on the rate of interest. There may be some, though, who fail to see "the amount of interest" and

"the rate of interest" as two distinct concepts. There were more errors of this type made in grades eight and nine than in grade seven. Over two-thirds of the errors made in these two grades were the "i" error.

Table XLV shows the per cent of the total answers for each grade made up of each of the possible combinations of "principal", "rate", and "time" of these taken singly. It shows that over one-third of the answers consisted of the "principal-rate" combination and over one-fifth of the "principal-time" combination, while less than one-twelfth of the answers consisted of the "rate-time" combination. This would indicate that it is much easier for the child to think of the "principal-rate" combination. This is more marked in grades eight and nine. In grade seven the "principal-rate" and the "principal-time" combinations were used with nearly equal frequencies. In all cases the "rate-time" combination was seldom used. Its absence is most marked in grade seven. In those cases in which only one of the three concepts was given the use of the "principal" concept outnumbered by far that of the other concepts. It would seem that the junior high school pupils are much more familiar with this concept than with the other. It may be that it is a much easier concept for a child of that age to understand. The table also shows that only a very small part of the pupils gave the three concepts as an answer.

None of these was in grade seven.

Table XLVI shows that about two-thirds of the junior high school pupils had at least two of the concepts given in their answers. There was only about one-half of the grade seven pupils that got two parts. Only a very small part of the grade eight and grade nine pupils failed to get at least one of the concepts.

Test II. Writing Formulas

The purpose of this test was to find the ability of pupils of the junior high school level to write general relationships that are expressed in words in the form of formulas. The results obtained for each question will be analyzed to find the nature of the errors. In the tables showing the errors it is necessary to show the form that the error took in those cases where certain definite errors occurred quite frequently. If the error did not occur very often, it was classed as miscellaneous. Omissions, mistakes, and corrects are to be designated by the use of (O), (X) and (R) respectively as in Test I.

Test II. Question I. A car travels (v) miles an hour for (t) hours. Write the formula for the distance (d) in miles covered by the car.

Answer: .... d = vt

TABLE XLVII

SHOWING THE MASTERY COEFFICIENTS AND THE ERROR COEFFICIENTS FOR TEST II, 1

	Grade 7	Grade 8	Grade 9	J.H.S.Group
No. of Pupils	280	308	319	907
R	81.07	91.23	95.93	89.75
X	18.93	8.77	4.07	10.25

The table reads: The Per cent of the pupils that obtained the correct solution for question 1 Test II was for grade 7, 81.07, for grade 8, 91.23, for .... etc.

TABLE XLVII (a)

SHOWING THE TOTAL NUMBER OF ERRORS FOR EACH GRADE AND THE PER CENT OF THIS NUMBER DUE TO EACH TYPE OF ERROR, TEST II, 1.

Type	Grade 7	Grade 8	Grade 9	J.H.S. Group
0	16.98	11.11	0.00	12.90
m	83.02	88.89	100.00	87.10
No. of Errors	53	27	13	93

The table reads: The per cent of the total number of errors due to omissions was, for grade 7, 16.98, for grade 8, 11.11; for grade 9, ...etc.

The tables show that in all groups the mastery coefficients are high for this question. The errors are few and miscellaneous in character. The total number of errors is 93 in which there are only 12 omissions.

Test II. Question 2. Write a formula for the amount remaining (R) of (C) dollars after spending (S) dollars.

Answer....R = C - S

TABLE XLVIII

SHOWING THE MASTERY COEFFICIENTS AND THE ERROR COEFFICIENTS FOR TEST II, 2

	Grade 7	Grade 8	Grade 9	J.H.S.Group
No. of Pupils	280	308	319	907
R	51.79	75.32	94.04	74.64
X	48.21	24.68	5.96	25.36

The table reads: Like Table XLVII.

TABLE XLVIII (a)

SHOWING THE TOTAL NUMBER OF ERRORS FOR EACH GRADE AND THE PER CENT OF THIS NUMBER DUE TO EACH TYPE OF ERROR, TEST II, 2

Type	Grade 7	Grade 8	Grade 9	J.H.S. Group
0	11.11	9.21	5.26	10.00
m	45.18	44.74	63.16	46.52
cs	37.78	35.53	31.58	36.52
s-c	5.93	10.52	0.00	6.96
No. of Errors	135	76	19	230

The table reads: Like Table XLVII (a).

Table XLVIII shows that there is a large grade-to-grade gain in this question. The grade seven results are weak while the grade nine results are very strong. The big portion of the errors are miscellaneous but over one-third are due to writing R = CS. In sixteen cases the answer given was R = S - C.

Test II. Question 3. Write the formula for the total number of cents (C) that (N) apples will cost at (L) cents each.
Answer .... C = NL.

TABLE XLIX

SHOWING THE MASTERY COEFFICIENTS AND THE ERROR COEFFICIENTS FOR TEST II, 3.

	Grade 7	Grade 8	Grade 9	J.H.S. Group
No. of Pupils	280	308	319	907
R	76.07	85.06	97.49	86.66
X	23.93	14.94	2.51	13.34

The table reads: Like Table XLVII.

TABLE XLIX (a)

SHOWING THE TOTAL NUMBER OF ERRORS FOR EACH GRADE AND THE PER CENT OF THIS NUMBER DUE TO EACH TYPE OF ERROR, TEST II, 3.

Type	Grade 7	Grade 8	Grade 9	J.H.S. Group
O	31.34	13.04	0.00	22.31
M	68.66	86.96	100.00	77.69
No. of Errors	67	46	8	121

The table reads: Like table XLVII (a)

The data of the tables show that all the grades had a high coefficient of mastery for this question. All the errors were due to omissions or were miscellaneous in character. The total number of errors made was 121.

Test II. Question 4. Jack is 16 years old. Henry is (x) years older than Jack. Write a formula to show Henry's age (n).

Answer: .... n = x - 16

TABLE L

SHOWING THE MASTERY COEFFICIENTS AND THE ERROR COEFFICIENTS FOR TEST II, 4.

	Grade 7	Grade 8	Grade 9	J.H.S. Group
No. of Pupils	280	308	319	907
R	48.93	62.66	83.71	65.82
X	51.07	37.34	16.29	34.18

The table reads: Like Table XLVII.

TABLE L (a)

SHOWING THE TOTAL NUMBER OF ERRORS FOR EACH GRADE AND THE PER CENT OF THIS NUMBER DUE TO EACH TYPE OF ERROR, TEST II, 4

Type	Grade 7	Grade 8	Grade 9	J.H.S. Group
0	13.28	3.48	1.92	7.74
m	34.97	35.66	26.92	33.87
16x	25.87	26.95	32.70	27.42
nx	7.70	1.74	0.00	4.19
16-x	8.39	10.43	11.54	9.69
x-16	9.79	21.74	26.92	17.09
No. of Errors	143	115	52	310

The table reads: Like Table XLVII (a).

The two tables show that the results were not so good for question 4 as they were for the first three questions of this test. The mastery coefficient for the junior high school group is 65.82 which is quite low. Less than one-half of the grade seven pupils found the correct solution. There were 310 errors made of which 85 were in the form n = 16x, 30 in the form n = 16 - x and 53 in the form n = x - 16.

Test II. Question 5. Write a formula to show the number of days (n) in (m) weeks.

Answer: .... n = 7m.

TABLE LI

SHOWING THE MASTERY COEFFICIENTS AND THE ERROR COEFFICIENTS FOR TEST II, 5.

	Grade 7	Grade 8	Grade 9	J.H.S.Group
No. of Pupils	280	308	319	907
R	25.71	36.69	41.69	35.06
X	74.29	63.31	58.31	64.94

The table reads: Like Table XLVII.

TABLE LI (a)

SHOWING THE TOTAL NUMBER OF ERRORS FOR EACH GRADE AND THE PER CENT OF THIS NUMBER DUE TO EACH TYPE OF ERROR, TEST II, 5.

Type	Grade 7	Grade 8	Grade 9	J.H.S. Group
0	15.87	8.20	2.69	9.17
m	25.48	25.13	10.21	20.54
nm	33.66	34.36	41.40	36.34
m÷n	21.63	25.64	36.56	27.67
m÷7	3.36	6.67	9.14	6.28
No. of Errors	208	195	186	589

The table reads: Like Table XLVII (a).

A study of Table LI shows that the concept contained in this question is understood very poorly since the mastery coefficient for the junior high school group is 35.06 meaning that only about one-third of the pupils understood it. Only about two-fifths of the grade nine pupils got the correct answer. An examination of the types of errors made shows over three-fifths were due to the answers (nm) or (m ÷ n) in which there is no attempt to use the seven. In the other questions of Test II the pupils were given the two parts for the right hand member of their formula while in question 5 they have only one part. The majority of them are lost and do not know what to do. It seems natural for them to try some combination of (n) and (m). About four hundred pupils made one of the two errors mentioned. Fewer than forty of the 589 making the error made an attempt to use the number 7 in any way. The number of omissions was not relatively high.

Test II. Question 6. John has (y) marbles and this is 5 times as many as Tom has. Write a formula for the number of marbles Tom has.

Answer: .... n = y/5

TABLE LII

SHOWING THE MASTERY COEFFICIENTS AND THE ERROR COEFFICIENTS FOR TEST II, 6.

	Grade 7	Grade 8	Grade 9	J.H.S.Group
No. of Pupils	280	308	319	907
R	33.57	53.25	52.98	47.08
X	66.43	46.75	47.02	52.92

The table reads: Like Table XLVII.

TABLE LII (a)

SHOWING THE TOTAL NUMBER OF ERRORS FOR EACH GRADE AND THE PER CENT OF THIS NUMBER DUE TO EACH TYPE OF ERROR, TEST II, 6.

Type	Grade 7	Grade 8	Grade 9	J.H.S.Group
0	14.52	5.56	0.00	7.29
m	22.58	25.00	26.00	24.38
5y	62.90	69.44	74.00	68.33
No. of Errors	186	144	150	480

The table reads: Like Table XLVII (a).

Table LII shows that number 6 is another question that the junior high school pupils do not understand very well as less than fifty per cent of the total number attempting the question were able to get the correct answer for it. It is surprising to see that a slightly larger per

cent of the grade eight pupils got the correct answer than there were of the grade nine pupils. There was only one outstanding error made in the question which was to write $n = 5y$ in place of $n = y/5$. About 350 of the 480 errors made were of this type. It would be difficult to state definitely the reasons for this error, but it would seem that a large number must have been due to a poor reading of the question. If it were due largely to the difficulty inherent in the concept $y/5$, there should have been a large number of the errors in the form of $(y + 5)$ or $(y-5)$, etc., in an attempt to give the correct answer. This was not the case.

Test III.

The errors made by the pupils in the questions in Test III will be classified under three headings, namely omissions (O), miscellaneous (m), and mistakes in calculation (f). The miscellaneous errors will be those in which the pupil has attempted to make some substitution of the given values but has not understood the operations that should be performed in order to get the correct answer. This lack of understanding is generally due to a misunderstanding of the concept in which they are making the substitution. For example, if they are to substitute a value 3.7 for g in the expression 5g, and then proceed

to get the correct value for the quantity 5g, they must understand that 5g means 5 times g. Mistakes in calculation will be those in which the pupil has substituted properly, has put in the proper signs to show the operations to be performed, and has then made some mistake in his figures. The number of errors in Test III is not relatively high.

Test III. Question I.

$A = LW$ Given $L = 30$, $W = 9$ To find a value for A

TABLE LIII

SHOWING THE MASTERY COEFFICIENTS AND THE ERROR COEFFICIENTS FOR TEST III, 1.

	Grade 7	Grade 8	Grade 9	J.H.S. Group
No. of Pupils	280	308	319	907
R	92.14	95.13	95.92	94.49
X	7.86	4.87	4.08	5.51

The table reads: The mastery coefficients are 92.14 for grade 7, 95.13 for grade 8, 95.92 for grade 9, ... etc.

TABLE LIII (a)

SHOWING THE TOTAL NUMBER OF ERRORS FOR EACH GRADE AND THE PER CENT OF THIS NUMBER DUE TO EACH TYPE OF ERROR, TEST III, 1.

Type	Grade 7	Grade 8	Grade 9	J.H.S. Group
0	31.82	0.00	0.00	14.00
m	27.27	53.33	0.00	28.00
f	40.91	46.67	100.00	58.00
No. of Errors	22	15	13	50

The table reads: The per cent of the errors due to omissions in grade 7 was 31.82, in grade 8 was 0.00 and grade 9 was 0.00 .... etc.

The two tables show that this question was well understood, there being only 50 errors. In grade seven the errors were quite evenly divided among omissions, miscellaneous errors, and errors in calculation. There were no omissions in grades eight and nine. All the errors of grade nine were in calculation.

Test III Question 2.

$n = 5g$ Given $g = 3.7$ To find a value for n.

Answer: $n = 18.5$

TABLE LIV

SHOWING THE MASTERY COEFFICIENTS AND THE ERROR COEFFICIENTS FOR TEST III, 2.

	Grade 7	Grade 8	Grade 9	J.H.S. Group
No. of Pupils	280	308	319	907
R	65.71	85.71	92.48	81.92
X	34.29	14.29	7.52	18.08

The table reads: Like Table LIII.

TABLE LIV (a)

SHOWING THE TOTAL NUMBER OF ERRORS FOR EACH GRADE AND THE PER CENT OF THIS NUMBER DUE TO EACH TYPE OF ERROR, TEST III, 2.

Type	Grade 7	Grade 8	Grade 9	J.H.S. Group
0	26.04	0.00	12.50	17.07
m	51.04	45.45	25.00	45.73
f	22.92	54.55	62.50	37.20
No. of Errors	96	44	24	164

The table reads: Like Table LIII(a).

The data show that the understanding of this question is good, for over four-fifths of the pupils got the correct answer. There is a much greater difference between grade seven and grade eight than between grade eight and grade nine. This characteristic is quite marked for all the

questions of Test III except question 1. There were 164 errors, almost half of which were miscellaneous while over one-third were due to calculation.

Test III Question 3.

$h = t/y$ Given $t = 54$, $y = 9$ To find a value for h.

Answer: $h = 6$

TABLE LV

SHOWING THE MASTERY COEFFICIENTS AND THE ERROR COEFFICIENTS FOR TEST III, 3

	Grade 7	Grade 8	Grade 9	J.H.S.Group
No. of Pupils	280	308	319	907
R	64.64	90.91	95.61	84.46
X	35.36	9.09	4.39	15.54

The table reads: Like Table LIII.

TABLE LV (a)

SHOWING THE TOTAL NUMBER OF ERRORS FOR EACH GRADE AND THE PER CENT OF THIS NUMBER DUE TO EACH TYPE OF ERROR, TEST III, 3.

Type	Grade 7	Grade 8	Grade 9	J.H.S. Group
0	6.06	0.00	0.00	4.25
m	89.90	89.29	50.00	85.82
f	4.04	10.71	50.00	9.93
No. of Errors	99	28	14	141

The table reads: Like Table LIII (a)

The pupils have a good understanding of this question, more especially in grades eight and nine. Two thirds of the total errors made were by pupils in grade seven. There were very few omissions of this question. The chief errors were miscellaneous in character, showing that the concept was not understood by that small group of pupils.

Test III Question 4.

V = LWH Given L = 8 To find a value for V
W = 6
H = 4

Answer: V = 192

TABLE LVI

SHOWING THE MASTERY COEFFICIENTS AND THE ERROR COEFFICIENTS FOR TEST III, 4

	Grade 7	Grade 8	Grade 9	J.H.S.Group
No. of Pupils	280	308	319	907
R	84.64	90.90	94.98	90.41
X	15.36	9.10	5.02	9.59

The table reads: Like Table LIII.

TABLE LVI (a)

SHOWING THE TOTAL NUMBER OF ERRORS FOR EACH GRADE AND THE PER CENT OF THIS NUMBER DUE TO EACH TYPE OF ERROR, TEST III, 4

Type	Grade 7	Grade 8	Grade 9	J.H.S.Group
0	13.95	0.00	0.00	6.90
m	32.56	50.00	12.50	34.48
f	53.49	50.00	87.50	58.62
No. of Errors	43	28	16	87

The table reads: Like Table LIII (a).

The tables show that the pupils have a good understanding of this question. There is not a great deal of difference between the grades as shown by the results in this question. There were only 87 errors and almost three-fifths of these were due to mistakes in calculation. There were very few omissions.

Test III Question 5.

$r = 2s - 6$ Given $s = 26$ To find a value for r.

Answer: $r = 46$

TABLE LVII

SHOWING THE MASTERY COEFFICIENTS AND THE ERROR COEFFICIENTS FOR TEST III, 5

	Grade 7	Grade 8	Grade 9	J.H.S.Group
No. of Pupils	280	308	319	907
R	33.57	68.51	88.71	64.83
X	66.43	31.49	11.29	35.17

The table reads: Like Table LIII.

TABLE LVII (a)

SHOWING THE TOTAL NUMBER OF ERRORS FOR EACH GRADE AND THE PER CENT OF THIS NUMBER DUE TO EACH TYPE OF ERROR, TEST III, 5

Type	Grade 7	Grade 8	Grade 9	J.H.S.Group
0	15.06	11.34	5.56	12.85
m	81.18	68.04	52.78	73.99
f	3.76	20.62	41.66	13.16
No. of Errors	186	97	36	319

The table reads like Table LIII (a).

The understanding of this question was very poor in grade seven and only fair in grade eight. There were omissions in all the grades. A large part of the errors were due to a misunderstanding of the concept (2s - 6). There seemed to be two outstanding difficulties. The first

was to understand the meaning of (2s). This same difficulty showed itself in question 2 of Test III. The other difficulty was to know at what point the 6 was to be subtracted. This question proved to be the most difficult for each grade of all the questions in Test III.

Test III Question 6.

$p = 2L + 2W$ Given $L = 18$, $W = 11$ To find a value for p.

Answer: $p = 58$

TABLE LVIII

SHOWING THE MASTERY COEFFICIENTS AND THE ERROR COEFFICIENTS FOR TEST III, 6.

	Grade 7	Grade 8	Grade 9	J.H.S.Group
No. of Pupils	280	308	319	907
R	34.29	72.08	91.54	67.36
X	65.71	27.92	8.46	32.74

The table reads like Table LIII.

TABLE LVIII (a)

SHOWING THE TOTAL NUMBER OF ERRORS FOR EACH GRADE AND THE PER CENT OF THIS NUMBER DUE TO EACH TYPE OF ERROR, TEST III, 6

Type	Grade 7	Grade 8	Grade 9	J.H.S. Group
0	4.35	5.81	0.00	4.38
m	91.30	91.86	70.37	89.56
f	4.35	2.33	29.63	6.06
No. of Errors	184	86	27	297

The table reads like Table LIII (a).

This question ranked as the second most difficult of Test III. Grades eight and nine handled it fairly well but grade seven did poorly with it. Nearly all the mistakes were classed as miscellaneous as they were due to lack of an understanding of how to substitute in the expression (2L + 2W). The difficulties were found in understanding the concept (2L) and (2W) and then in knowing what to do with the results of substituting in each of these. The mistakes made were varied and covered a considerable range.

Test IV.

The purpose of this test is to find how well the junior high school pupils are able to get new formulas from given formulas. Each question will be dealt with to show the mastery coefficients and the error coefficients

for each grade. Each question will have its own particular type of errors. These will be shown in the second table for the question.

Test IV Question 1.

$k = n + 6$ To find a formula for n.

Answer: $n = k - 6$

TABLE LIX

SHOWING THE MASTERY COEFFICIENTS AND THE ERROR COEFFICIENTS FOR TEST IV, 1.

	Grade 7	Grade 8	Grade 9	J.H.S.Group
No. of Pupils	280	308	319	907
R	72.86	84.74	89.03	82.58
X	27.14	15.26	10.97	17.42

The table reads: The mastery coefficient for grade 7 is 72.86, for grade 8 is 84.74, for grade 9 is ....etc.

TABLE LIX (a)

SHOWING THE TOTAL NUMBER OF ERRORS FOR EACH GRADE AND THE PER CENT OF THIS NUMBER DUE TO EACH TYPE OF ERROR, TEST IV, 1.

Type	Grade 7	Grade 8	Grade 9	J.H.S. Group
0	7.90	0.00	0.00	3.80
m	65.79	72.34	68.57	68.35
k + 6	26.31	27.66	31.43	27.85
No. of Errors	76	47	35	158

The table reads: The per cent of errors due to omissions is 7.90 in grade 7, 0.00 in grade 8, ...etc.

The tables show that the question is quite well understood. There is only one outstanding type of error which is (k + 6). This error was made over forty times. Over two-thirds of the errors were miscellaneous but they did represent attempts by the pupils. There were very few omissions.

Test IV. Question 2.

d = rt To find a formula for r.

Answer: r = d/t

TABLE LX

SHOWING THE MASTERY COEFFICIENTS AND THE ERROR COEFFICIENTS FOR TEST IV, 2

	Grade 7	Grade 8	Grade 9	J.H.S.Group
No. of Pupils	280	308	319	907
R	32.14	58.44	68.02	53.69
X	67.86	41.56	31.98	46.31

The table reads like Table LIX.

TABLE LX (a)

SHOWING THE TOTAL NUMBER OF ERRORS FOR EACH GRADE AND THE PER CENT OF THIS NUMBER DUE TO EACH TYPE OF ERROR, TEST IV, 2

Type	Grade 7	Grade 8	Grade 9	J.H.S. Group
0	5.79	0.78	0.00	2.86
m	22.63	23.44	31.37	25.00
dt	21.58	25.78	33.34	25.71
d - t	42.63	38.28	25.49	37.14
d + t	7.37	11.72	9.80	9.29
No. of Errors	190	128	102	420

The table reads like Table LIX (a).

An examination of the data of the tables shows that the pupils have a poor understanding of this question. The question ranks as the second most difficult in Test IV. Only two-thirds of the grade nine pupils did it correctly, while less than one-third of the grade seven students got the correct solution. Of the three errors shown to have appeared most frequently the (d - t) and the (dt) errors occurred over 250 times. One quarter of the errors were miscellaneous.

Test IV. Question 3.

$S = c + G$ To find a formula for G.

Answer: $G = S = c$

TABLE LXI

SHOWING THE MASTERY COEFFICIENTS AND THE ERROR COEFFICIENTS FOR TEST IV, 3

	Grade 7	Grade 8	Grade 9	J.H.S.Group
No. of Pupils	280	308	319	907
R	56.07	72.08	77.74	69.13
X	43.92	27.92	22.26	30.87

The table reads like Table LIX.

TABLE LXI (a)

SHOWING THE TOTAL NUMBER OF ERRORS FOR EACH GRADE AND THE PER CENT OF THIS NUMBER DUE TO EACH TYPE OF ERROR, TEST IV, 3.

Type	Grade 7	Grade 8	Grade 9	J.H.S. Group
0	8.94	3.49	4.22	6.07
m	34.15	46.51	8.45	31.43
s + c	21.14	26.74	29.58	25.00
s - g	13.82	9.30	7.04	10.71
cs	13.82	6.98	2.82	8.93
c - s	8.13	6.98	47.89	17.86
No. of Errors	123	86	71	280

The table reads like Table LIX (a)

The tables on question 3 show that the pupils of grades eight and nine did almost equally well in handling this concept. Over half of the grade seven pupils got the correct answer. There were 280 errors made and almost two-thirds of these were one or other of the four outstanding forms listed in Table LX (a). It will be noticed that 3 of the 4 forms shown are combinations of s and c.

Test IV. Question 4.

$j = k - s$ To find a formula for k.

Answer: $k = j + s$

TABLE LXII

SHOWING THE MASTERY COEFFICIENTS AND THE ERROR COEFFICIENTS FOR TEST IV, 4.

	Grade 7	Grade 8	Grade 9	J.H.S. Group
No. of Pupils	280	308	319	907
R	55.00	65.26	82.76	68.25
X	45.00	34.74	17.24	31.75

The table reads like Table LIX.

TABLE LXII (a)

SHOWING THE TOTAL NUMBER OF ERRORS FOR EACH GRADE AND THE PER CENT OF THIS NUMBER DUE TO EACH TYPE OF ERROR, TEST IV, 4.

Type	Grade 7	Grade 8	Grade 9	J.H.S. Group
0	12.70	3.74	3.63	7.64
m	29.36	41.12	27.27	33.33
j - s	39.68	39.25	54.55	42.36
js	14.29	11.21	3.64	11.11
s - j	3.97	4.67	10.91	5.55
No. of Errors	126	107	55	288

The table reads like Table LIX (a).

Question 4 is well understood by most of grade nine and by two-thirds of grade eight. Over one-half of the grade seven pupils got the correct solution. The outstanding error was writing (j - s) in place of (j + s). This error occurred 122 times. One-third of the errors were miscellaneous.

Test IV. Question 5.

$n = 4s$ To find a formula for s.

Answer: $s = n/4$.

TABLE LXIII

SHOWING THE MASTERY COEFFICIENTS AND THE ERROR COEFFICIENTS FOR TEST IV, 5

	Grade 7	Grade 8	Grade 9	J.H.S.Group
No. of Pupils	280	308	319	907
R	29.29	53.90	64.26	49.94
X	70.71	46.10	35.74	50.06

The table reads like Table LIX.

TABLE LXIII (a)

SHOWING THE TOTAL NUMBER OF ERRORS FOR EACH GRADE AND THE PER CENT OF THIS NUMBER DUE TO EACH TYPE OF ERROR, TEST IV, 5.

Type	Grade 7	Grade 8	Grade 9	J.H.S. Group
0	9.59	2.82	5.26	6.39
m	26.26	35.92	31.58	30.62
n - 4	32.82	24.65	17.54	26.43
4n	21.21	25.34	26.32	23.79
n + 4	10.10	11.27	19.30	12.77
No. of Errors	198	142	114	454

The table reads like Table LIX (a).

Question 5 ranks as the most difficult in Test IV. There seem to be at least two reasons for this. The first

is the difficulty in understanding the concept (4s), while the second is in knowing that the answer should be in the form of a quotient. Perhaps the two cannot be separated in this manner. It is true that the first is a difficulty regardless of what is required in the question. If the concept (4s) were understood completely, this understanding might suggest the necessary method of procedure for the question. The form taken by the main errors made can be seen in Table LXIII (a).

Test IV Question 6.

$g = t/m$ To find a formula for t.

Answer: $t = gm$

TABLE LXIV

SHOWING THE MASTERY COEFFICIENTS AND THE ERROR COEFFICIENTS FOR TEST IV, 6

	Grade 7	Grade 8	Grade 9	J.H.S. Group
No. of Pupils	280	308	319	907
R	35.71	59.41	77.12	58.32
X	64.29	40.59	22.88	41.68

The table reads like Table LIX.

TABLE LXIV (a)

SHOWING THE TOTAL NUMBER OF ERRORS FOR EACH GRADE AND THE PER CENT OF THIS NUMBER DUE TO EACH TYPE OF ERROR, TEST IV, 6

Type	Grade 7	Grade 8	Grade 9	J.H.S. Group
0	11.11	7.20	8.22	9.26
m	22.78	12.80	9.59	16.93
g ÷ m	38.89	48.80	63.01	44.97
m ÷ g	6.11	5.60	8.22	6.35
g + m	6.77	12.00	6.85	8.44
g - m	14.44	13.60	4.11	12.17
No. of Errors	180	125	73	378

The table reads like Table LIX (a)

This is another question that is not well understood by the pupils writing the test. There is a large grade-to-grade gain shown in the results which is generally a characteristic of a difficult question. Nearly half of the errors were in the form g + m in place of gm. The other three common errors also showed attempts to express a value for t in terms of g and m. The concept t/m seems to be difficult for the child to understand.

Test V.

The treatment of this test will show that there were

a very large number of errors in all the questions but more especially in the last three. The errors are grouped under four classifications. The first classification is "omissions" (O) and refers to questions in which there is absolutely no work shown and no writing put down for the question. The second classification is "miscellaneous errors". These play a big part in Test V, for all questions which cannot be classified as one of the other three types of errors are put in the miscellaneous class. The third classification is "the wrong variation" (w) in which the pupil has multiplied in place of dividing or vice versa. An example of this would be found in dealing with question 1 and the formula A = LW where (L) is doubled and (W) is unchanged. The child making the (w) error would say that (A) is half as large instead of (A) is twice as large. If he had said that (A) is one-third as large, the error would have been classified as miscellaneous for the unit of change is not correct, making the answer wrong in two respects. The fourth and final classification of errors for Test V is the (g) classification. In such cases the child has simply recopied the "given part" of the question. For example, if the child were doing question 1, he would write his answer in the form "(L) is doubled and (W) is unchanged" and would give no information concerning changes in (A).

Test V. Question 1.

A = LW Given (L) is doubled and (W) is unchanged A is .....

Answer: A is twice as large.

TABLE LXV

SHOWING THE MASTERY COEFFICIENTS AND THE ERROR COEFFICIENTS FOR TEST V, 1

	Grade 7	Grade 8	Grade 9	J.H.S. Group
No. of Pupils	280	308	319	907
R	48.57	54.87	66.46	57.00
X	51.43	45.13	33.54	43.00

The table reads: The mastery coefficients for grade 7 is 48.57, for grade 8 is 54.87, for grade 9 .....etc.

TABLE LXV (a)

SHOWING THE TOTAL NUMBER OF ERRORS FOR EACH GRADE AND THE PER CENT OF THIS NUMBER DUE TO EACH TYPE OF ERROR, TEST V, 1.

Type	Grade 7	Grade 8	Grade 9	J.H.S. Group
O	20.14	10.79	4.67	12.57
m	54.86	42.45	64.49	53.07
w	14.58	32.37	20.56	22.57
g	10.42	14.39	10.28	11.79
No. of Errors	144	139	107	390

The table reads: The per cent of the errors due to omissions for grade 7 is 20.14, for grade 8 is 10.79, for ....etc.

This question ranks the second easiest in Test V but the pupils certainly do not understand the concept well. The miscellaneous type forms more than half of the errors while there are omissions in all grades. Over one-fifth of the pupils made a mistake by taking the wrong variation while over one-tenth of the pupils simply wrote "L is doubled and W is unchanged" which was the "given" part of the question. An added difficulty must have arisen in understanding the word "doubled". This is the only reason that suggests itself for the results of question 1 being poorer than those of question 2. One would have expected the results of question 1 to have been even better than those of question 2 as the concept "increasing in size" seems simpler than the concept "decreasing in size".

Test V. Question 2.

A = LW Given (W) is divided by 2 (L) in unchanged A is .....

Answer: A is $\frac{1}{2}$ as large.

TABLE LXVI

SHOWING THE MASTERY COEFFICIENTS AND THE ERROR COEFFICIENTS FOR TEST V, 2

	Grade 7	Grade 8	Grade 9	J.H.S.Group
No. of Pupils	280	308	319	907
R	52.14	67.21	74.92	65.27
X	47.86	32.79	25.08	34.73

The table reads like Table LXV.

TABLE LXVI (a)

SHOWING THE TOTAL NUMBER OF ERRORS FOR EACH GRADE AND THE PER CENT OF THIS NUMBER DUE TO EACH TYPE OF ERROR, TEST V, 2

Type	Grade 7	Grade 8	Grade 9	J.H.S. Group
0	23.13	9.90	12.50	16.19
m	58.96	59.40	57.50	58.73
w	7.46	10.90	13.75	10.16
g	10.45	19.80	16.25	14.92
No. of Errors	134	101	80	315

The table reads like Table LXV (a).

The data of the tables show that this question is better understood than is question 1 but even at that only about two-thirds of the pupils were able to get the question. However, this is much better than was done on any other question in Test V. Nearly three-fifths of the errors are miscellaneous. There were about 50 pupils who omitted the question and about the same number who simply wrote down the "given" portion of the question. More than 30 pupils took the wrong variation.

Test V. Question 3.

Both (L) and (W) are doubled. A is .....

Answer: A is 2 times as large.

TABLE LXVII

SHOWING THE MASTERY COEFFICIENTS AND THE ERROR COEFFICIENTS FOR TEST V, 3

	Grade 7	Grade 8	Grade 9	J.H.S. Group
No. of Pupils	280	308	319	907
R	38.57	49.03	54.86	47.85
X	61.43	50.97	45.14	52.15

The table reads like Table LXV.

TABLE LXVII (a)

SHOWING THE TOTAL NUMBER OF ERRORS FOR EACH GRADE AND THE PER CENT OF THIS NUMBER DUE TO EACH TYPE OF ERROR, TEST V, 3.

Type	Grade 7	Grade 8	Grade 9	J.H.S. Group
O	12.79	3.18	4.17	6.98
m	77.91	86.62	86.11	83.30
w	1.16	0.00	2.08	1.06
g	8.14	10.20	7.64	8.66
No. of Errors	172	157	144	473

The table reads like Table LXV (a)

Less than half the pupils got the correct answer for this question. Of the 473 errors 394 were miscellaneous. The results show a very poor understanding of the question.

Test V. Question 4.

A = LW (L) is divided by 4 and
(W) is multiplied by 2 A is .....

Answer: A is one-half as large.

TABLE LXVIII

SHOWING THE MASTERY COEFFICIENTS AND THE ERROR COEFFICIENTS FOR TEST V, 4.

	Grade 7	Grade 8	Grade 9	J.H.S. Group
No. of Pupils	280	308	319	907
R	28.93	47.08	53.29	42.56
X	71.07	52.92	46.71	56.34

The table reads like Table LXV.

TABLE LXVIII (a)

SHOWING THE TOTAL NUMBER OF ERRORS FOR EACH GRADE AND THE PER CENT OF THIS NUMBER DUE TO EACH TYPE OF ERROR, TEST V, 4.

Type	Grade 7	Grade 8	Grade 9	J.H.S. Group
O	21.10	7.36	10.07	13.50
m	62.31	74.85	68.46	68.10
w	8.55	7.36	13.42	9.60
g	8.04	10.43	8.05	8.80
No. of Errors	199	163	149	511

The table reads like Table LXV (a)

Barely two-fifths of the pupils got the correct answer for this question. There were 348 miscellaneous errors and 69 omissions. The question is very poorly understood even by grade nine pupils.

Test V. Question 5.

$t = d/r$ (d) is doubled and (r) does not change. (t) is .....

Answer: (t) is twice as large.

TABLE LXIX

SHOWING THE MASTERY COEFFICIENTS AND THE ERROR COEFFICIENTS FOR TEST V, 5.

	Grade 7	Grade 8	Grade 9	J.H.S. Group
No. of Pupils	280	308	319	907
R	37.50	52.27	67.71	53.14
X	62.50	47.73	32.29	46.86

The table reads like Table LXV.

TABLE LXIX (a)

SHOWING THE TOTAL NUMBER OF ERRORS FOR EACH GRADE AND THE PER CENT OF THIS NUMBER DUE TO EACH TYPE OF ERROR, TEST V, 5.

Type	Grade 7	Grade 8	Grade 9	J.H.S. Group
o	32.57	25.85	18.45	26.82
m	52.00	42.18	55.34	49.41
w	8.57	23.13	18.45	16.00
g	6.86	8.84	7.76	7.77
No. of Errors	175	147	103	425

This question is understood by slightly more than half of the pupils. There were 114 omissions and 210 miscellaneous errors. There were 68 pupils who took the wrong variation and 33 pupils who simply repeated the "given part". The results are poor for this question but they are far better than those for the other three questions on this formula.

Test V. Question 6.

$t = d/r$ (r) is multiplied by 3 and (d) does not change. (t) is .....

Answer: t is one-third as large

TABLE LXX

SHOWING THE MASTERY COEFFICIENT AND THE ERROR COEFFICIENTS FOR TEST V, 6.

	Grade 7	Grade 8	Grade 9	J.H.S. Group
No. of Pupils	280	308	319	907
R	6.43	12.99	36.99	19.40
X	93.57	87.01	63.01	80.60

The table reads like Table LXV.

TABLE LXX (a)

SHOWING THE TOTAL NUMBER OF ERRORS FOR EACH GRADE AND THE PER CENT OF THIS NUMBER DUE TO EACH TYPE OF ERROR, TEST V, 6.

Type	Grade 7	Grade 8	Grade 9	J.H.S. Group
0	22.90	16.05	15.42	18.33
m	41.22	42.54	39.30	41.18
w	31.68	36.56	41.30	36.11
g	4.20	4.85	3.98	4.38
No. of Errors	262	268	201	731

The table reads like Table LXV (a).

This question is very poorly understood as there are 731 errors of which 134 are omissions, 301 are miscellaneous, 264 are the wrong variation, and 32 are repeating the "given" part of the question. The pupils certainly do not understand the concept of Test V as it is found in the indirect variation of the formula t = d/r. This is shown just as strikingly in questions 7 and 8.

Test V. Question 7.

t = d/r Both (d) and (r) are doubled. (t) is ...

Answer: (t) is not changed.

TABLE LXXI

SHOWING THE MASTERY COEFFICIENTS AND THE ERROR COEFFICIENTS FOR TEST V, 7.

	Grade 7	Grade 8	Grade 9	J.H.S. Group
No. of Pupils	280	308	319	907
R	13.57	20.78	46.39	27.56
X	86.43	79.22	53.61	72.44

The table reads Like Table LXV.

TABLE LXXI (a)

SHOWING THE TOTAL NUMBER OF ERRORS FOR EACH GRADE AND THE PER CENT OF THIS NUMBER DUE TO EACH TYPE OF ERROR, TEST V, 7.

Type	Grade 7	Grade 8	Grade 9	J.H.S. Group
O	24.80	22.13	0.00	17.35
m	47.52	65.99	11.12	44.90
w	23.55	7.79	84.79	33.65
g	4.13	4.09	4.09	4.10
No. of Errors	242	244	177	657

The table reads like Table LXV (a).

The pupils have a very poor understanding of this question. There are 657 errors of which 114 are omissions, 295 miscellaneous, 221 the wrong variation, and 27 repeating "given" parts of the question. The grade nine pupils

have a much better understanding of the question than have the other grades.

Test V. Question 8.

$t = d/r$ (r) is divided by 3 and (d) is unchanged. (t) is .....

Answer: (t) is three times as large.

TABLE LXXII

SHOWING THE MASTERY COEFFICIENTS AND THE ERROR COEFFICIENTS FOR TEST V, 8

	Grade 7	Grade 8	Grade 9	J.H.S. Group
No. of Pupils	280	308	319	907
R	6.07	12.01	35.11	18.30
X	93.93	87.99	64.89	81.70

The table reads: Like Table LXV.

TABLE LXXII (a)

SHOWING THE TOTAL NUMBER OF ERRORS FOR EACH GRADE AND THE PER CENT OF THIS NUMBER DUE TO EACH TYPE OF ERROR, TEST V, 8.

Type	Grade 7	Grade 8	Grade 9	J.H.S. Group
O	28.90	26.18	17.39	24.67
m	47.91	40.98	36.24	42.11
w	20.15	28.41	42.51	29.42
g	3.04	4.43	3.86	3.80
No. of Errors	263	271	207	741

The table reads like Table LXV (a).

Question 8 ranks as the most difficult of all the questions in Test V. Less than one-fifth of the pupils got the correct answer for the question. Of the 741 errors, 183 were omissions, 312 miscellaneous, 218 the wrong variation, and 28 repeating of the "given" part of the question. The greater portion of the pupils have no understanding of the question.

CHAPTER VI

SUMMARY AND CONCLUSIONS

The final chapter of this investigation is to serve the following three purposes:

1. To make a concise summary of the most important findings of the investigation.

2. To suggest certain conclusions that might be drawn from these findings, and

3. To suggest lines along which other studies might be made in neighboring fields with the purpose of making more extensive findings than those of the present investigation.

These three topics will receive treatment in the order in which they have been given.

A Summary of the Findings of the Investigation

1. There is a consistent grade-to-grade gain across the junior high school level in the general knowledge of the concepts contained in the series of tests used in the investigation. This gain is quite uniform, being slightly greater in the seven-eight interval than in the eight-nine interval. Nor is this grade-to-grade gain confined to the results of the entire series but it is also true for each

of the five tests. There is only one exception, in the thirty-eight items used in the tests, in which there was not a quite uniform grade-to-grade gain as shown by the mastery coefficients for each question for the different grade groups.

2. The mean scores made by the different grades were as follows: 19.16 for grade seven, 24.81 for grade eight, and 28.67 for grade nine.

3. The grade-to-grade gain as shown by the results for the complete series of tests is considerably greater for the boys than it is for the girls.

4. The mean score of the girls in grade seven is higher than that of the boys. The difference is not great enough to make it a statistical certainty that a true difference does exist, but it is great enough that there are over ninety chances out of one hundred that the true difference is greater than zero.

5. There is very little difference in the mean scores of the boys and girls in grade eight.

6. The mean score of the boys in grade nine for the complete series of tests is enough greater than that of the girls to make the difference statistically significant.

7. There is a very wide overlapping of the score distribution for the different grades. In each grade

there is a wide range of scores for the complete series of tests and there are scores all through the upper range of possible scores. Many of the grade seven pupils have scores greater than the median score for grade nine pupils. There are also grade nine pupils having scores under the median score for the grade seven pupils. The overlapping is still more marked for adjacent grades, i.e. for grades seven and eight and for grades eight and nine. This is what one would expect. The interquartile ranges of grades seven and nine do not overlap but are separated by only 1.20 score-points. The interquartile range of grade eight overlaps almost entirely with those of the other two grades. The range of scores in each grade is more than four times as great as the standard deviation of the distribution for that grade. In every case, except one, the standard deviation of the distribution is greater than the range between the medians of adjacent grades.

8. The different tests ranked in the same order of difficulty for each grade without a single exception.

9. The different items within each test came very close to ranking in the same order of difficulty for each of the grades. Where there was a difference, the rank displacement was always very small.

10. Within each test there were pupils in each grade who made a perfect score. In some cases this number was

quite high. There were also pupils in each grade who made a very low score. In other words, within each test there were scores in every grade that were distributed over nearly the entire range of scores for that test.

11. In Tests I, II, III and IV there was only one instance in which a grade-group failed to have over more than sixty per cent of the possible score for that Test. The exception was in Test IV in which only fifty-four per cent of the grade seven pupils obtained at least fifty per cent of the possible score. In Test V, the proportion of the pupils securing at least fifty per cent of the possible score for the test was thirty-one per cent of the grade sevens, forty-seven per cent of the grade eights and sixty-three per cent of the grade nines.

12. There were higher scores made in all the grades in Test VI than in Test V. For three of the four questions dealing with the formula A = LW the results were slightly better in Test V than in Test VI. In the four questions dealing with the formula t = d/r, the results were very superior for Test VI.

The outstanding errors in the different questions were found to be common to all the grades. In other words, the different grades did not have errors peculiar to themselves but they had errors in common with the other grades. This was true with respect to all the tests. Not only were

the chief errors common across the grades but generally, if a certain error made up a high percentage of all the errors for one grade, it also made up a high percentage of all the errors for the other grades.

Suggested Conclusions That Might be Drawn From the Findings of the Investigation

The following conclusions are suggested as a result of the findings of the investigation:

1. That there is a uniform increase in the amount of understanding of the function concepts tested in this investigation across the junior high school grades. This increase in ability to make use of the given concepts is not confined to the results of the entire series of tests but it is shown in the results of each of the five tests and also in the results of the individual items or questions that make up the tests. It is safe to conclude, then, that in any wide sampling of a cross section of the pupils of the junior high school level, the amount of understanding of the concepts will increase as you move up the junior high school grades.

2. That the average pupils of the junior high school grades have an understanding of series of concepts as shown by the median scores of the grades. This understanding is slightly over fifty per cent for grade seven over sixty-five per cent for grade eight and over seventy-five per cent for

grade nine.

3. That the boys master the concepts more rapidly than do the girls. The difference in the grade-to-grade gains of the boys and the girls may not seem so great if simply compared with the mean or median scores of the different grades, but it seems quite large when compared with the grade-to-grade gains of the combined boys and girls group.

4. That there are more than ninety chances out of one hundred that the girls in grade seven would make a higher score than would the boys. Therefore, the chances are greater that the girls of grade seven have a better understanding of the function concepts used in this investigation than have the boys.

5. That the understanding of the function concepts is about the same for the boys and girls in grade eight.

6. That the boys in grade nine have a better understanding of the function concepts than have the girls. The difference between their mean scores is sufficiently great to make it a statistical certainty that a difference does exist.

7. That there is a very wide range in the ability to understand the function concepts among the pupils of each grade. This range covers almost the complete range of scores for grade seven and at least the upper two-

thirds of the range of scores for the other two grades. This state of affairs indicates that the range of abilities among the pupils of a single grade, if measured by the scores of the pupils, is more than four times as great as the difference between the median scores of adjacent grades. Considerable of this range of ability to understand the function concept must be due to a difference of intelligence. It would therefore be safe to conclude that an understanding of the function concept depends considerably on the intelligence of the child.

8. That there is a wide overlapping in the range of abilities for the different grades. This is shown by the fact that the range of scores for any single grade is more than four times as great as the range between the medians of adjacent grades. It must be concluded, then, that there are pupils in grade seven with a greater understanding of the function concept than have many pupils in grades eight and nine while there are pupils in grade eight who have a greater understanding of the function concepts than have many pupils in grade nine.

9. That the difficulties of the five concepts tested by the different tests ranked in the same order of magnitude for each of the grades. There must be common difficulties to all the grades in these concepts.

10. That there must be difficulties common to all

the grades in the different items or questions making up the tests, as the ranking of the different questions was either the same, or very close to the same, for each of the grades.

11. That there is a wide range of ability to understand the concepts within each grade within each test for in nearly every case the scores cover the entire range of scores for that test.

12. That over sixty per cent of the pupils of each grade have an understanding of the concepts of the first four tests except for grade seven in Test IV.

13. That the junior high school pupils have a very poor understanding of the concept of Test V. That this lack of understanding for the questions on the formula $A = LW$ is largely due to the lack of understanding of the relationship existing between the area of a rectangle and its dimensions. That the lack of understanding of the questions on the formula $t = d/r$ is largely due to the lack of ability to understand the dependence between the members of the formula. That the pupils understand direct variation in a formula much better than they do indirect variation.

14. That the errors made were similar in the different grades, both in nature and in relative frequency of occurrence. Therefore, the same concept will present

similar difficulties to the different grades.

Lines Along Which Other Investigations Might Be Made

There is room for much investigation in the field of the function concept, but only a few lines along which future investigations might be made will be suggested. They are as follows:

1. An investigation should be made in the elementary school grades dealing with as many phases of dependence as possible.

2. A study, similar to the present investigation, could be made to good advantage in the senior high school grades. The questions would necessarily be made more difficult and perhaps the tests should be longer. It would be well to include questions on graphs and tables for such a test.

3. In the senior high school grades it would be well to make a somewhat similar investigation in which the items chosen for the different tests were graded in difficulty ranging from quite easy to very difficult. Such a set of tests would make it possible to provide even the best pupils in each grade with items on each test difficult enough to demand their very best effort in order to secure the correct response.

BIBLIOGRAPHY

Books

Barber, Harry C. Teaching Junior High School Mathematics. Cambridge: Houghton Mifflin Company, The Riverside Press, 1924. Pp. 137.

Breslich, E. H. Developing Functional Thinking in Secondary School Mathematics. The Third Year Book of the National Council of Teachers of Mathematics. New York: Bureau of Publications, Teachers' College, Columbia University, 1928. Pp. 42.56.

Breslich, E. H. Measuring the Development of Functional Thinking in Algebra. The Seventh Year Book of the National Council of Teachers of Mathematics, New York: Bureau of Publications, Teachers' College, Columbia University, 1932. Pp. 93-118.

Breslich, E. H. The Administration of Mathematics in Secondary Schools. Chicago, Ill.: The University of Chicago Press, 1933. Pp. 407.

Durell, V. Clement. The Teaching of Elementary Algebra. London: G. Bell and Sons Ltd., 1931. Pp. 136.

Crocker, L. J. Changing Objectives in the Teaching of Higher Mathematics. An Unpublished Master's Thesis, University of Manitoba, 1931. Pp. 151.

Garret, Henry E. Statistics in Psychology and Education. New York-London-Toronto: Longmans, Green and Company, 1926. Pp. 128-137.

Hamley, H. R. Relational and Functional Thinking. The Ninth Year Book of the National Council of Teachers of Mathematics. New York: Bureau of Publications, Teachers' College Columbia University, 1934. Pp. 215.

Hazelwood, Lucye Linwood. A Study of Changes in Test-Books and Methods in Algebra Since 1800 with Indications of the Present Trend. An Unpublished Master's Thesis, Duke University, Virginia, 1932.

Holzinger, Karl J. Statistical Methods for Students in Education. Boston-New York-Chicago-London: Ginn and Company, 1928. Pp.

Jablowner, J. J. Recent and Present Tendencies in the Teaching of Algebra in the High Schools. The Seventh Year Book of the National Council of Teachers of Mathematics. New York: Bureau of Publications, Teachers' College, Columbia University 1932. Pp. 1-18.

Lennes, N. J. The Function Concept in Elementary Algebra. The Seventh Year Book of the National Council of Teachers of Mathematics. New York: Bureau of Publications, Teachers' College, Columbia University 1932. Pp. 52-73.

Nunn, T. Percy. The Teaching of Algebra (Including Trigonometry) Longmans Modern Mathematical Series, Longmans, Green and Co., 39 Paternoster Row, London, E.C. 4, 1924.

Report of the Mathematical Association Committee 1910. The Teaching of Algebra and Trigonometry. London: G. Bell and Sons, Ltd., Portugal Street, Kingsway and Bombay, 1910. Pp. 8.

Smith, D. E. and Reeve, D. B. The Teaching of Junior High School Mathematics. Boston-New York- Chicago: Ginn and Company, 1927. Pp. 411.

Stephens, Stella. Diagnosis of Student Difficulties in Formulas in First Year Algebra. An Unpublished Master's Thesis, George Peabody College for Teachers, Nashville, Tennessee, 1930.

The Reorganization of Mathematics in Secondary Education. A Report by the National Committee on Mathematical Requirements Under the Auspices of the Mathematical Association of America, Inc., Cambridge: Houghton Mifflin Company, The Riverside Press, 1923. Pp. 171.

Thorndike, Edward L. The Psychology of Algebra. Toronto: The MacMillan Company, 1923. Pp. 483.

A LIST OF THE MOST IMPORTANT OF THE ALGEBRA TEXTBOOKS USED IN THE INVESTIGATION

Barber, Harry C. Junior High School Mathematics Book I. Pp. 231. Book II, Pp. 267. Houghton Mifflin Company, Boston-New York-Chicago: The Riverside Press, Cambridge, 1927.

Betz, William. Junior Mathematics for Today. Book I, Pp. 406, Book II, Pp. 438, Book III, Pp. 562. Ginn and Company, Boston-New York-Chicago-London: 1933.

Breslich, E. R. Eighth Year Mathematics, Pp. 296. Ninth Year Mathematics, Pp. 319. New York: The MacMillan Company, 1932.

Brueckner, L. J., Anderson, C. J., Banting, G. O., Everyday Mathematics. Toronto: The John C. Winston Co. Ltd., 1935. Grade VII, Pp. 229, Grade VIII, Pp. 261.

Gugle, Marie. Modern Junior Mathematics, Book I, Pp. 307, Book II, Pp. 339, The Gregg Publishing Company, New York-Chicago-London; 1920.

Nunn, T. Percy. Exercises in Algebra, (Including Trigonometry), Part I, Pp. 421. New York-London-Toronto: Longmans Green and Company, 1924.

Rugg, Harold O., and Clark, John R. The Fundamentals of High School Mathematics. Yonkers-on-Hudson, New York, World Book Company, Chicago, 1921.

Schorling, R., and Clark, J. R. Modern Algebra, First Course Pp. 391, Second Course, Pp. 460, The World Book Company, Yonkers--on-Hudson, New York: 1929.

Stone, J. C., Mills, C. N., Mallory, V. S., Unit Mastery Mathematics, Books I, II, and III, Chicago-New York-Boston: Benj. H. Sanborn and Co., 1934.

Thorndike, E. L. Thorndike's Junior Mathematics, Book I, Pp. 196, Book II, Pp. 198, Book III, Pp. Toronto: W. J. Gage and Company Ltd., 1928.

Wells, S. B. and Hart, W. W. Modern Second Course in Algebra. (Revised). Toronto: The Copp. Clark Co., Ltd., 1931. Pp. 273.

APPENDIX I

INSTRUCTIONS FOR GIVING TESTS

1. Have a watch to time the tests. Give 10 minutes for each of the 5 tests. This includes the time it takes the teacher to read the instructions preceding each test and also the sample given for each test.

2. Do not make any comments on the paper and do not answer any questions raised by the pupil.

3. When the pupils are all ready (with good pencils), give each pupil a paper telling him to place it face downward on his desk and not to look at it.

4. READ THE FOLLOWING TO THE PUPILS (clearly and slowly, making no comments.)

" Boys and Girls, you have on your desks a set of 5 tests, each of which contains about 6 questions. These tests are being given to about 800 pupils in grades seven, eight and nine, to find how well they can do these questions. You have not been taught much of this work and perhaps not any of it but if you listen carefully while the instructions are being read before each test, you should be able to do most of the questions. The questions have been carefully selected from a larger number of questions which were tried out on over 100 pupils of grades seven, eight and nine. The average pupil in each grade did well on these questions. I want you to do your very best to make a good mark for yourself and to help your room make a good average on the tests. Remember that other pupils in your grade have done well on these questions so it is possible for you to do well also. I am going to read the rules of the tests to you now and I want you to listen carefully. Here they are:

a. Write your name, the name of your school, and your grade in the proper place on each page.
b. Use a pencil to do the tests. Do all your writing on the paper and put your work where you are told to put it in the instructions before each test. Write your best.
c. Do not ask any questions about the paper or about any of the questions. Do your best all by yourself. If you can not answer a certain question, leave it and go on to the next question.
d. There are 5 tests and you will be given 10 minutes for each test. This will give you plenty of time if you keep working.
e. When you finish a test check it over carefully. Do not turn the page over to see the next test until I tell you to do so. If you are finished before I tell you to turn

to the next test, you can check the test you have finished or go back to any other test that you have done before that one. When I say that the time is up for any test you must turn on to the next test even if you are not finished. You may get time later to go back to finish it.

f. When the time is up the monitors will collect the papers carefully.

Are you ready? ..... Then turn over your papers and follow carefully while I read the instructions for Test I. I am now keeping the time and we have only 10 minutes for this test" (this ends the part spoken to the pupils)

5. The teacher will then mark down the time...read the instructions for Test I., and then let the pupils do the 6 questions. At the end of 10 minutes, time is called and all the pupils must turn over the page to look at the next test and to follow the reading of the instructions down to the end of the sample. The teacher must make no comments and must not do any writing on the board.

6. The teacher will keep the papers from the different rooms in different parcels (i.e. keep the rooms separate.)

Note: The time given for the tests is sufficiently long. Most of the pupils will have time to spare. The teacher must not lose time in reading the instructions. There should be only one reading and this should be given slowly and distinctly.

REMEMBER: NO COMMENTS AND NO ANSWERING OF QUESTIONS RAISED BY THE PUPILS.

APPENDIX II

INSTRUCTIONS FOR MARKING TESTS

1. Put a mark only for those that are correct and put this mark at the extreme right of the paper. Make it quite prominent.

2. Count the number of corrects for each test and enter this in the proper place at the top of the page.

3. Transfer all these to the bottom of the first page to show the total score made in the 5 tests.

4. In Test I put a small sign on those questions in which there is one of the two answers correct but do not total this in the score. It will make the later analysis easier.

KEY FOR MARKING

TEST I

1. The number of pounds of meat.
   The price per pound.

2. The number of questions.
   The time required for one question.

3. The base of the triangle.
   The altitude (or height) of the triangle.

4. The length of time he works. (days, hours, months, etc.)
   The amount he receives per unit of time (same unit as above)

5. The length of time it travels (hours, minutes, etc.)
   The rate per unit of time (per hour, minute, etc.)

6. The rate of interest.
   The time for which it is loaned (years).

TEST II

1. $d = vt$ 2. $R = C - S$ 3. $C = NL$

4. $n = x + 16$ 5. $n = 7m$ 6. $n = \frac{y}{5}$

TEST III

1. A = 270	2. n = 18.5	3. h = 6
4. V = 192	5. 5 = 46	6. p = 58

TEST IV

1. n = k - 6	2. $r = \frac{d}{t}$	3. G = S - C
4. k = j - s	5. $3 = \frac{n}{4}$	6. t = gm

TEST V

1. a. A is twice as large
   b. A is one half as large
   c. A is four times as large
   d. A is one half as large

2. a. t is twice as large
   b. t is one third as large
   c. t is not changed
   d. t is three times as large.

APPENDIX III

THE SIX TESTS

NAME ..... SCHOOL .....GRADE.....

SCORE.....

TEST I RECOGNIZING RELATIONSHIPS

One quantity often depends on one or more other quantities for its value. Thus, the price paid for 3 baseballs depends on one other quantity, namely, the price per baseball. The price paid for a number of pencils depends on two other quantities, namely, the number of pencils, and the price for one pencil. Sometimes one quantity depends on three others.

To show how well you recognize relationships, fill in the blanks after each question giving two quantities on which each depends. Read the sample over carefully before you begin the questions to be certain that you understand what is wanted.

THE GIVEN QUANTITIES	THE OTHER QUANTITIES DEPENDED ON
Sample. The area of a rectangle depends on	1. The length of the rectangle 2. The width of the rectangle
1. The cost of a roast of meat depends on	1. ..... 2. .....
2. The time required to do a certain number of arithmetic questions depends on	1. ..... 2. .....
3. The area of a triangle depends on	1. ..... 2. .....
4. The amount of pay a man receives depends on..	1. ..... 2. .....

5.	If a train travels at the same rate all the time, the distance it travels depends on	1. ..... 2. .....
6.	The simple interest on a loan depends on	1. ..... 2. .....

NAME..... SCHOOL..... GRADE .....

SCORE .....

TEST II

WRITING FORMULAS

A formula is just a short way of writing down a general rule for doing certain questions in mathematics. For example, we say that to find the area of a rectangle we multiply its length by its width. This is stated in a formula by writing A = L X W (In this formula A stands for the area of the rectangle, L for its length, and W for its width.

In each of the following questions you are asked to write a formula for some quantity. Read each question carefully and then write the formula. Read the sample first to see just what is wanted.

QUESTION OR STATEMENT	FORMULA
Sample: If (t) men take (g) days to do a piece of work, write a formula to show how many days it will take one man to do the same amount of work.	n = t x g or n = t g
1. A car travels (v) miles an hour for (t) hours. Write a formula for the distance (d) in miles covered by the car	d = ...
2. Write a formula for the amount remaining (R) of (C) dollars after spending (S) dollars	R = ...
3. Write the formula for the total number of cents (C) that (N) apples will cost at (L) cents each	C = ...
4. Jack is 16 years old. Henry is (x) years older than Jack. Write a formula to show Henry's age (n)	n = ...
5. Write a formula to show the number of days (n) in (m) weeks	n = ...
6. John has (y) marbles and this is 5 times as many as Tom has. Write a formula for the number of marbles Tom has	n = ...

NAME ..... SCHOOL .....GRADE .....

SCORE .....

TEST III SUBSTITUTION IN FORMULAS

The letters in all formulas stand for numbers or quantities. If you are given values for all but one of these letters, you can put these numbers in the place of the letters for which they stand and then work the question out to find the value of the other letter or symbol.

In the questions below you are given a number of formulas in the first column. In the second column is a list of values for certain letters in these formulas. In the third column you will show your work when you substitute (or put) these values for the letters in the formulas to find the answer. The answer must be written by itself in the last column. Look at the sample given to see that you understand. Be careful with your figuring.

Note (ck means c x k; 4r means 4 x r; $\frac{t}{e}$ means t ÷ e)

FORMULAS	GIVEN VALUES	WORK HERE	ANSWERS
Sample d = rt	r = 8 t = 6	d = 8 x 6 = 48	48
1. A = LW	L = 30 W = 9	A = ...	.....
2. n = 5g	g = 3.7	n = ...	.....
3. $h = \frac{t}{y}$	t = 54 y = 9	h = ...	.....
4. V = LWH	L = 8 W = 6 H = 4	V = ...	.....
5. r = 2s - 6	s = 26	r = ...	.....
6. p = 2L + 2W	L = 18 W = 11	p = ...	.....

NAME ..... SCHOOL ..... GRADE .....

SCORE .....

TEST IV CHANGING FORMULAS TO GET OTHER FORMULAS

Sometimes we want to change a formula from one form to another. If we want to find the area of a rectangle we use the formula $A = LW$ but if we want to get the width when we know the area and the length we divide the area by the length. The new formula is $W = \frac{A}{L}$ which is a formula for W and not for A. Any formula can be changed to be a formula by which we can find another of the letters in that formula.

Look carefully at the sample given below and then do the questions that follow. You do not need to show any work. Just put the answer in the column under 'New Formulas'. Remember that the letters or symbols in the formulas stand for quantities or numbers and that you can add, subtract, multiply, and divide with them as I have shown in the fomula above $A = LW$ and as you see in the sample below.

GIVEN FORMULAS	TO FIND	NEW FORMULAS
Sample: $g = t + k$	Find a formula for t.	$t = g - k$
1. $k = n + 6$	Find a formula for n.	n = ...
2. $d = rt$	Find a formula for r.	r = ...
3. $S = c + g$	Find a formula for G	G = ...
4. $j = k - s$	Find a formula for k	k = ...
5. $n = 4s$	Find a formula for s	s = ...
6. $g = \frac{t}{m}$	Find a formula for t	t = ...

NAME ..... SCHOOL ..... GRADE.....
SCORE .....

TEST V FUNCTIONAL RELATIONS OR VARIATIONS

The area of a rectangle is found by multiplying its length by its width. This can be expressed in the formula A = LW (A is the area, L the length, and W the width of the rectangle.)

In the questions below you are to tell just what changes take place in (A) if certain given changes take place in (L) or in (W) or in both (L) and (W). If it is larger, don't just say that it is larger but say that it is 4 times as large or whatever the number of times it is larger. If it is smaller say that it is 1/2 as large or 1/5 as large or whatever the fraction is. If there is no change, say 'A is not changed'.

Formula ..... A = LW

CHANGES IN (L) OR (W) OR IN BOTH	CHANGES IN A
1. (L) is doubled and (W) is unchanged	A is .....
2. (W) is divided by 2 and (L) is unchanged	A is .....
3. Both (L) and (W) are doubled	A is .....
4. (L) is divided by 4 and (W) is multiplied by 2.	A is .....

The time (t) in hours, that it will take to go (d) miles at (r) miles an hour is found by dividing the distance by the rate. This is expressed by the formula $t = \frac{d}{r}$

In the questions below certain changes are made in (d) or in (r) or in both (d) and (r). In each case you are to look at the formula and tell what changes will be made in (t). (That is you are to tell how many times larger it will be, or how many times smaller it will be, or that it does not change ..... just as in the questions with A = LW, above.)

Formula ..... $t = \frac{d}{r}$

CHANGES IN (d) OR IN (r) OR IN BOTH	CHANGES IN (t)
5. (d) is doubled and (r) doesn't change	(t) is .....
6. (r) is multiplied by 3 and (d) doesn't change	(t) is .....
7. Both (d) and (r) are doubled	(t) is .....
8. (r) is divided by 3 and (d) is unchanged.	(t) is .....

NAME ..... GRADE.....
SCHOOL ..... AGE.....

TEST VI

INSTRUCTIONS

Instructions to the teachers: Read the instructions to the pupils and the two sections on the formulas. Make no comments and answer no questions.

Read each question carefully. Do any figuring that you need to do in the space under work. Write nothing but the answer to the question under Answers.

QUESTIONS	WORK	ANSWERS
The area of a rectangle is found by multiplying its length by its width. This can be expressed in the formula A = LW (A is the area, L the length, and W the width of the rectangle).		
1. The area of a rectangle is 400 square feet. What would be the area of another rectangle if it is twice as long and the same width as this one?		.....
2. The area of a rectangle is 300 square feet What would be the area of another rectangle if it is the same length but one-fifth as wide as this one?		.....
3. The area of a rectangle is 800 square feet What would be the area of another rectangle that was twice as long and twice as wide as this one?		.....
4. The area of a rectangle is 1200 square feet What would be the area of another rectangle that was one-fourth as long and twice as wide as this one?		.....
The time (t) in hours, that it will take to go (d) miles at (r) miles an hour is found by dividing the distance by the rate. This is expressed by the formula $t = \frac{d}{r}$		
5. Tom takes 12 hours to make a certain trip in his car. How long would he take to make a trip 4 times as far at the same rate of speed.		.....
6. John takes 14 hours to make a certain trip in his car. How long would he take to make the same trip if he went twice as fast?		.....
7. Jack takes 20 hours to make a certain trip in his car. How long would he take to make the trip 3 times as far if he travelled 3 times as fast as in his first trip?		.....
8. Jim takes 30 hours to make a certain trip in his car. How long would he take to make the same trip if he went one third as fast as in his first trip ?		.....

THE STATISTICAL FORMULAS USED IN THE INVESTIGATION

1. Median

$$Md = l.l. + \left(\frac{\frac{N}{2} - f_{up}}{f_{md}} \right) h$$

p. 87

$$Md = u.l. - \left(\frac{\frac{N}{2} - f_{do}}{f_{md.}} \right) h$$

2. S. D. $= \left(\sqrt{\frac{\Sigma f d^2}{N} - \left(\frac{\Sigma f d}{N} \right)^2} \right) h$ p. 109

3. $Q_3 = u.l. - \left(\frac{\frac{N}{4} - f_{do}}{f_3} \right) X \; h$

p. 112

4. $Q_1 = l.l. + \left(\frac{\frac{N}{4} - f_{up}}{f_1} \right) X \; h$

5. $S_k = \frac{Q_1 + Q_3 - 2 M_d}{Q}$ p. 122

6. $Q = \frac{Q_3 - Q_1}{2}$ p. 110

7. $P.E._m = .6745 \frac{S.D.}{\sqrt{N}}$ p. 233

8. $P.E._{Md.} = .84535 \frac{S.D.}{\sqrt{N}} = 1.2533 \; P.E._{m.}$ p. 238

9. $P.E._{s.d.} = .7071 \; P.E._{M.}$ p. 238

10. $P.E._{diff.}^2 = P.E._{m.of\ boys}^2 + P.E._{m.of\ girls}^2$ ~~238.~~ 128-137 *

Holzinger, Karl J. "Statistical Methods for Students in Education " Boston-New York-Chicago: Gunn & Co., 1928.

* Garrett, Henry E. Op.cit., pp. 128-137.