THE EFFECTS OF SUMMER VACATION ON BASIC MATHEMATICAL FACTS

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ABSTRACT

The purpose of this study was to investigate the effects of summer vacation on the retention of learned basic facts in mathematics. Tests in basic addition, subtraction, multiplication and division facts were administered to students in grades three, four and five in June. Following the summer vacation the same tests were readministered to the same students.

A sample of two hundred seventy-four students from Wayoata School in the Transcona-Springfield School Division #12 was involved in the study.

The paired t-test was employed to determine the significance of differences in achievement between the June test scores and the September test scores. The analysis of covariance design and the Scheffé test were applied to determine significance of differences in retention between grade levels. Four correlation matrices involving eight variables were formed - one for the total sample and one for each grade level.

Although the findings were found to be statistically significant, the results were assessed in terms of their educational relevance. Significant differences did not necessarily imply that a student's loss of the basic facts would meaningfully affect his performance when

applying the facts to computational questions. If a difference was statistically significant but the actual difference was too small in the opinion of this investigator to have educational implications, then it was not considered meaningfully significant.

In this study, the results showed that differences in achievement between the June scores and the September scores were meaningfully significant in addition and in subtraction facts for students in grade three, but were not meaningfully significant for students in grade four or in grade five. For multiplication and division facts, differences between the June scores and September scores were meaningfully significant for both the grade three and the grade four students. This was not true for students in grade five.

Moreover, the results showed that at each grade level the retention of the basic facts increased with the grade level. However, when comparing differences between grade levels, it was shown that retention of addition facts was meaningfully significant between grades three and five, but not between grades three and four, or between grades four and five. For subtraction facts, there were no retention differences between any pair of grade levels. Differences were meaningfully significant in

retention of multiplication facts between grades three and five, and between grades three and four. This was not the case between grades four and five. In division facts, significant differences were found between each pair of grade levels.

The study concludes that statistically summer vacation has a detrimental affect on the retention of the basic mathematical facts, but that the results are not always educationally relevant.

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CHAPTER 1

INTRODUCTION

Recognizing that there is likely to be a decrease in the mastery of mathematical skills after the summer vacation, some teachers have modified the curriculum so that important areas which have been learned are reviewed to overcome this problem. This concern reveals itself in the numerous investigations beginning in the 1920's on retention of mathematical skills after the summer vacation. Several of these studies will be discussed specifically in the following chapter.

In general, past research has shown that losses in mathematical skills occur during the summer vacation.[1] The results of the retention studies reviewed are based on the analyses of standardized and non-standardized

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[1]Some studies which have shown that losses in mathematical skills have occurred during the summer vacation are:

M.A. Garfinkle, "The Effects of Summer Vacation on Ability in the Fundamentals of Arithmetic," Journal of Educational Psychology, X (January, 1919), 44-48.

Elizabeth Bruene, "Effects of the Summer Vacation on the Achievement of Pupils in the Fourth, Fifth, and Sixth Grades," Journal of Educational Research, XVIII (November, 1928), 309-314.

Sister Josephina, "Differences in Arithmetic Performances," The Arithmetic Teacher, VI (April, 1959), 152-153.

tests used to measure mathematical skills. Generally, these tests measure computational skills in addition, in subtraction, in multiplication and in division of whole numbers. Because the generality of these tests covered a variety of concepts and processes, the effect on a single concept or process was not determined.

Mathematics, Bidwell[2] claims, has a clear structure of concepts which is sequential and interdependent. Thus, if a student is to develop a knowledge of a mathematical topic, the concepts must be properly sequenced with each new concept properly related to what he already knows. The more meaningful mathematics is, the longer it is retained. However, if one or more dependent concepts is weak, the retention of mathematical skills will be affected. The development of mathematical competency is dependent on the conceptual understanding of a skill at each level of its structure.

The knowledge of the basic mathematical facts which provide a foundation for computational growth is vital in developing mathematical competency. Previous

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[2] James K. Bidwell, "Learning Structures for Arithmetic," The Arithmetic Teacher, XVI (April, 1969), 263-268.

studies by Brueckner[3] and Cook[4] have focused on the retention of the basic facts during the summer vacation, but their studies are limited mainly to addition facts at the grade two level. While basic addition and subtraction facts are introduced and taught in grades one or two, basic multiplication and division facts are usually not introduced until the grade three level. Because all these facts are vital to computational competency, this investigator wishes to determine how effectively facts are retained after the summer vacation at the grade three, grade four and grade five levels. Moreover, this investigator wishes to explore the relationship of the retention of the basic facts among grade levels.

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[3]L. J. Brueckner, "Certain Arithmetic Ability of Second Grade Pupils," Elementary School Journal, XXVII (February, 1927), 433-444.

[4]Ruth Cathlyn Cook, "Vacation Retention of Fundamentals by Primary Grade Pupils," Elementary School Journal, LXIII (December, 1942), 214-219.

CHAPTER 2

RELATED LITERATURE

The review of literature will focus on two areas: first, the study of retention and forgetting; second, the research studies on the retention of mathematical skills after the summer vacation.

After retention and forgetting are defined in the first section, a discussion on how different learning materials affect the retention curve follows. Furthermore, a description of the ways retention is measured, and a discussion of the factors which affect retention and forgetting are included.

The next section reviews the research studies on retention of mathematical skills after the summer vacation. A summary of findings based on these retention studies concludes this chapter.

RETENTION AND FORGETTING

Retention refers to the amount of previously learned material which persists or has been retained by the subject, whereas forgetting refers to the amount which has been lost or has not been retained.[1] Re-

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[1]John F. Hall, The Psychology of Learning (New York: J. B. Lippincott Company, 1966), pp. 547-548.

tention and forgetting are different aspects of the same process or different ways of viewing the same data. A retention of 100 per cent implies no forgetting whereas a retention of 0 per cent implies 100 per cent forgetting.[2]

Retention is the basis for measuring effective learning, which is defined as a change in behavior resulting from experience.[3] This implies an improvement in performance resulting from practice. Thus, the efficiency of a learning situation can be measured by the degree of retention or forgetting experienced by the learner.

Retention Curves

Retention curves vary with the kind of materials learned. These may be classified into meaningless and meaningful materials.

Nonsense syllables are meaningless material which a student learns by rote. One of the initial studies on retention based on meaningless material was conduct-

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[2]John A. McGeogh and Arthur L. Irion, *The Psychology of Human Learning* (New York: David McKay Co. Ltd., 1952), p. 355.

[3]James Deese and Stewart H. Hulse, *The Psychology of Learning* (3rd ed.; New York: McGraw-Hill Book Company, 1958), p. 380.

ed by H. Ebbinghaus.[4] He first invented nonsense syllables in order to have learning tasks free of previous experience, and then used himself as a subject in the experiment. He found that forgetting was extensive and rapid, especially immediately after learning. Since his experiment, many similar studies in retention and forgetting support his findings. After reviewing many studies McGeogh states:

> It may be concluded that, over a wide range of conditions, the course of retention of nonsense syllables may be represented by a curve which has its most rapid fall during the time immediately after the cessation of practice and which declines more and more slowly with increasing intervals.[5]

Although some of the studies have shown some variations from the results of Ebbinghaus, in general, the curve of retention still conforms to that of Ebbinghaus.[6] The generally accepted retention curve of relatively meaningless material such as nonsense syllables is shown in Figure 1.

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[4]Hermann Ebbinghaus, Memory, Translated by Henry A. Ruger and Clara E. Bussenius, (New York: Dover Publications Inc., 1964).

[5]McGeogh and Irion, op. cit., pp. 356-357.

[6]William A. Kelley, Educational Psychology, (Milwaukee: The Bruce Publishing Co., 1965), p. 231.

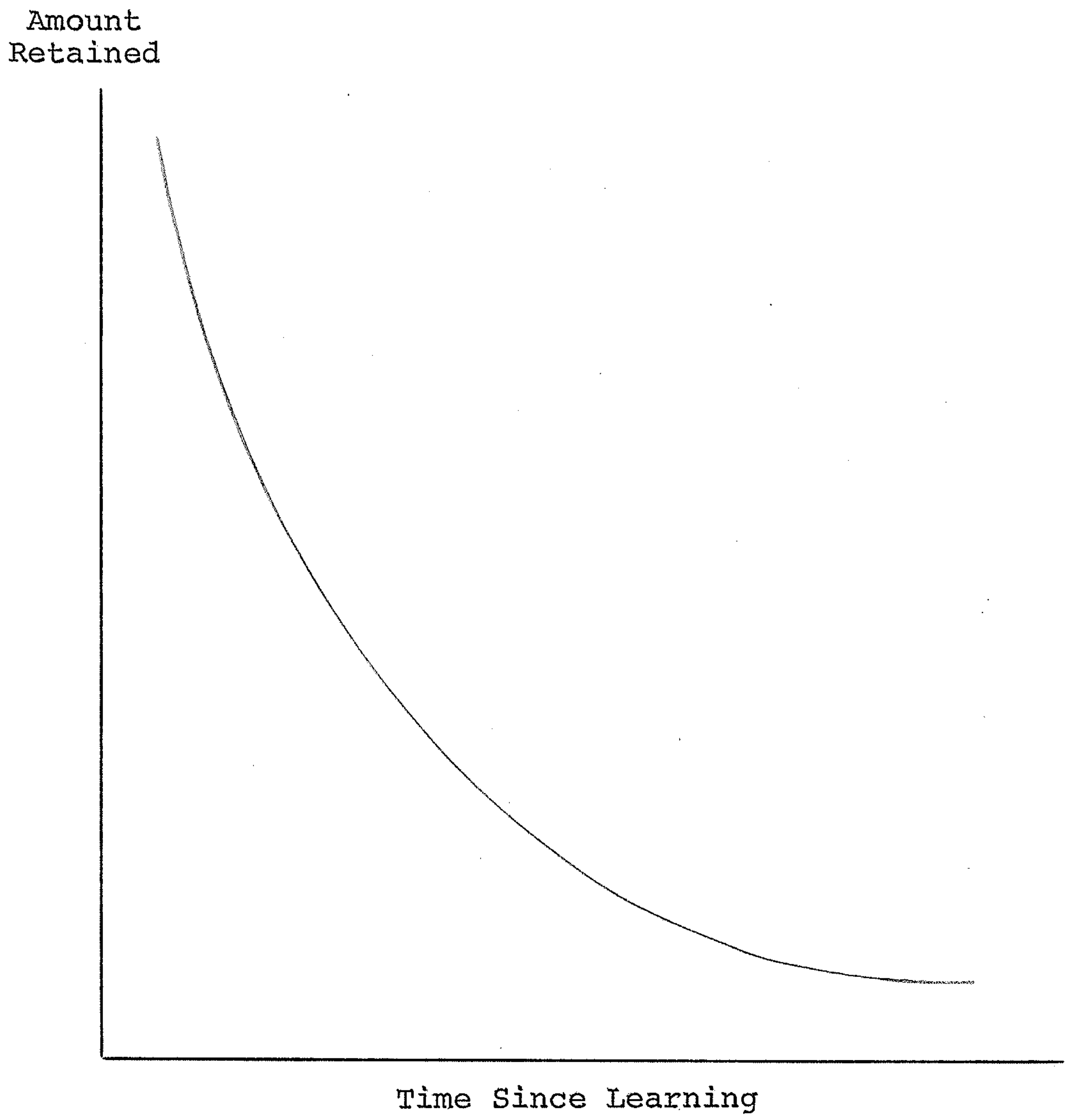

Figure 1 The Curve of Retention of Relatively Meaningless Material.

On the other hand, facts and ideas are examples of meaningful materials. Facts have meaning for the learner and are learned by rote, whereas ideas refer to a collection of related facts and contain highly associative values.

Briggs and Reed[7] conducted an experiment to determine the relationship of Ebbinghaus' curve of retention with meaningful materials. The results yielded a curve similar to the curve of retention, but the level of retention was higher than in studies published for nonsense materials. Figure 2 shows the relationship of the retention curve for material classified according to the degree of meaning.

As meaning increases, therefore, the material is more readily retained and the rate of forgetting becomes slower. McGeogh points out that comparisons between different materials cannot be made legitimately because the conditions of original learning are not comparable.[8] However, he argues that there is good reason for concluding that meaningful materials are usually much better

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[7]Leslie J. Briggs and Homer S. Reed, "The Curve of Retention for Substance Material," Journal of Experimental Psychology, XXXII (June, 1943), 513-517.

[8]McGeogh and Irion, op. cit., p. 382.

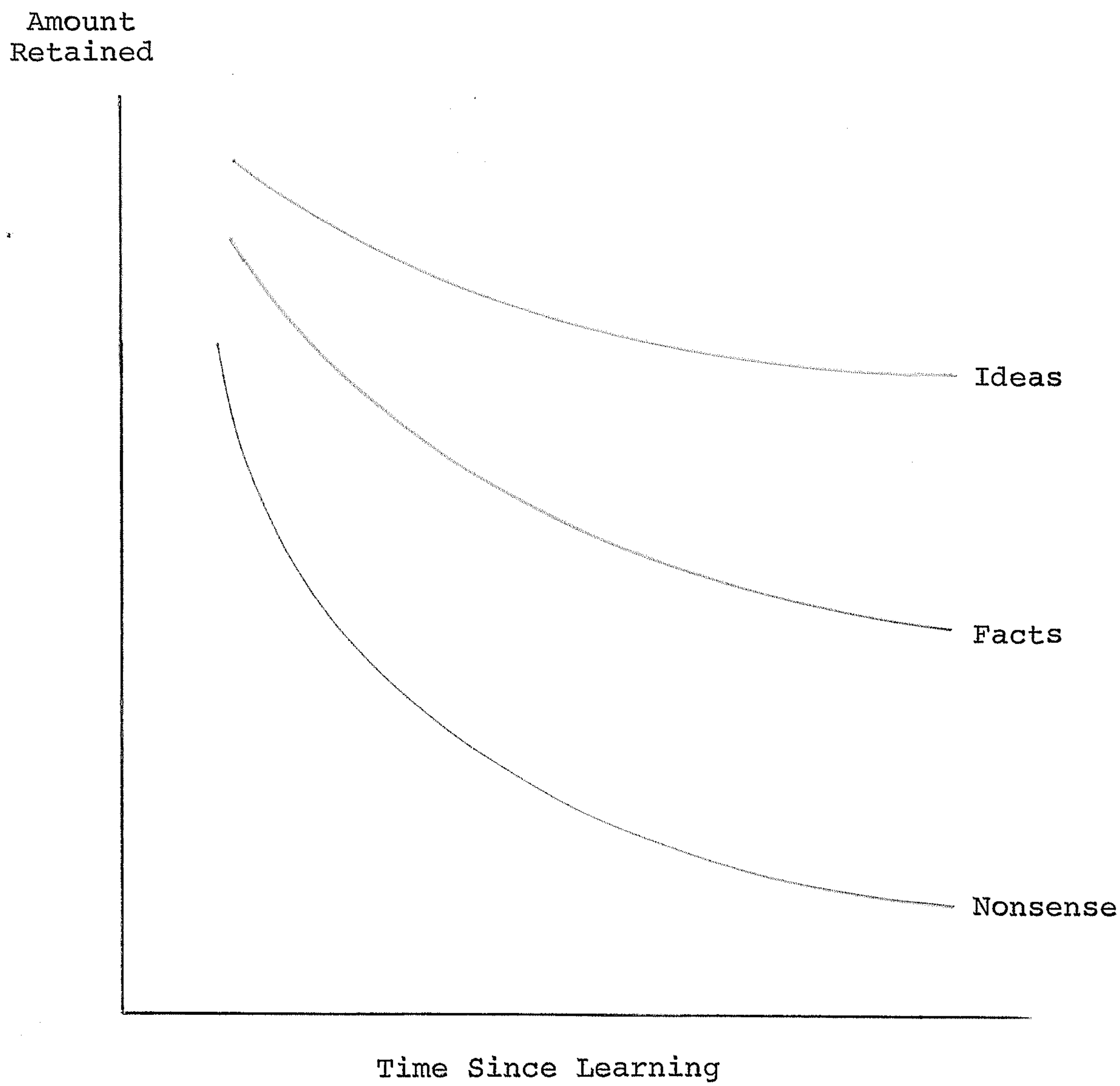

Figure 2 Traditional Hypothetical Retention Curves for Different Classes of Material

retained than nonsense materials.[9]

Ways of Measuring Retention

Another factor that must be considered is the manner in which retention is measured. The two ways of measuring retention are the direct method and the in-direct method.

The direct method consists of an end-test and a re-test. The end-test is administered after the cessation of a learning situation, and a re-test after an interval of time. One example of a direct method of measuring retention is recall, which requires the learner to reproduce as much of the material learned as possible. This method is commonly employed in retention studies.

Recognition is another direct method of measuring retention. This method involved the ability of the learner to differentiate among items presented to him. Multiple choice tests and true-false tests are ways of utilizing recognition to select the appropriate answer. For example, if a learner were presented with a list of words for recall, then during recall he would select the

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[9]McGeogh and Irion, op. cit., p. 383.

correct words from a set of all possible English words; whereas, in a recognition test, the learner may be asked to identify the correct words from a set of limited alternatives. Retention is more readily evidenced through recognition than through recall, "because in a recognition test people make their selection from a smaller set of alternatives than they do in recall".[10]

An indirect method of measuring retention is through a procedure called savings or relearning. After a period of time in which no review has taken place, the learner is asked to relearn the original material under the same conditions and the same specified criterion. The savings score consists of the difference between the amount of time required, or the number of trials attempted in the original learning and that needed for relearning. This score is converted to a percentage score by using the following formula:[11]

$$\text{Percent Savings Score} = \frac{\text{Original learning Score} - \text{Relearning Score}}{\text{Original Learning Score}} \times 100$$

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[10] Deese and Hulse, op. cit., p. 380.

[11] Ibid.

Once the savings score has been converted into a percentage, the retention score may be derived in the following manner:

Retention Score = 100 - Percent Savings Score

Thus, if a material were learned in ten trials and relearned in seven trials, a savings score of 30 per cent is obtained, or a retention score of 70 per cent.

Usually the indirect method is used in an experimental or laboratory setting. The comparison of the different measures of retention is shown in Figure 3. The difference between the methods is not the amount retained but the effectiveness of the tests used.[12]

Factors Affecting Retention

No one retains all the material which he learns. Over a period of time an individual forgets, often regardless of the conditions under which that learning took place. Kelley states:

> Forgetting which is a normal, everyday event denotes the gradual or rapid loss of material. . . . In early discussions of forgetting the most common explanation offered to account for it was decrease in recall due to the lack of use of material and to the passage of time. . . . However,

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[12]May V. Seagoe, The Learning Process and School Practice (Scranton: Chandler Publishing Co., 1970), p.159.

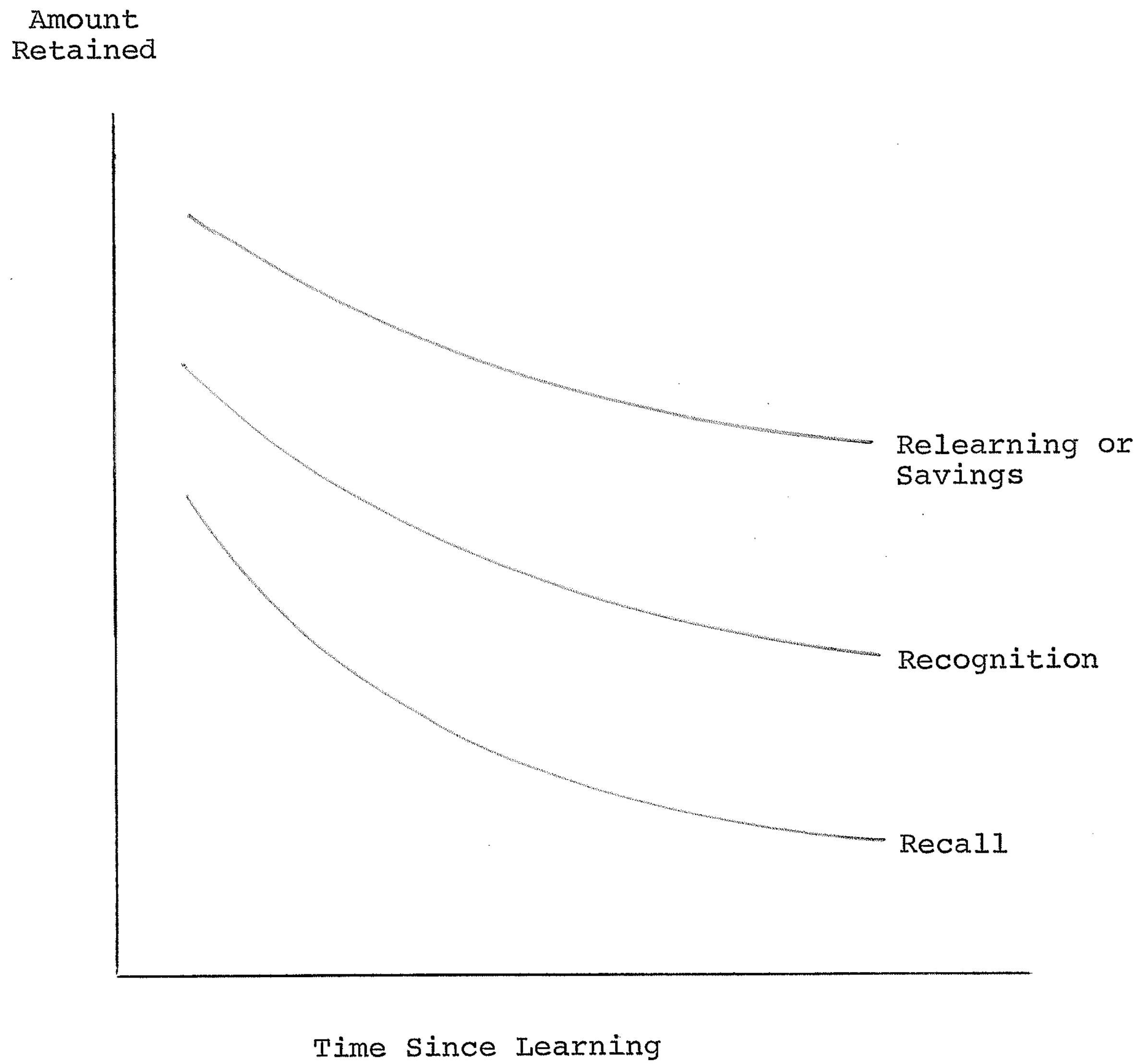

Figure 3 Measures of Retention

> disuse and the passage of time are not sufficient . . . to explain forgetting. . . . There is a tendency to forget that which does not seem important or essential; that to which no personal interest is attached and also that which is unpleasant. Forgetting may also be a means of solving some external conflicts and likewise of satisfying important emotional needs.[13]

McGeogh[14], Hankins[15], Deese and Hulse[16] and others agree that the major cause of forgetting is interference from other learnings. When something being learned is affected by something already learned, the effect is called proactive; when something already learned is affected by something learned subsequently, the effect is called retroactive.

An experimental design for measuring proactive effects is as follows:

Proaction

Experimental Group	Learn Task B	Learn Task A	Recall Task A
Control Group	Rest	Learn Task A	Recall Task A

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[13]Kelley, op. cit., p. 228.

[14]McGeogh, op. cit., pp. 395-406.

[15]Norman E. Hankins, Psychology for Contemporary Education, (Columbus: Charles E. Merrill Publishing Co.), pp. 196-198.

[16]Deese and Hulse, op. cit., pp. 398-414.

In this design the experimental group learns Task B while the control group rests. Then, both groups learn Task A followed by a test of retention for Task A. Frequently when new materials or skills are learned, they compete with earlier learnings with the result that the new learning is disturbed.[17] This type of interference which prior learning exercises upon later learning is called proactive inhibition.

The other form of interference is retroaction. A design for measuring retroactive effects is as follows:

Retroaction

Experimental Group	Learn Task A	Learn Task B	Recall Task A
Control Group	Learn Task A	Rest	Recall Task A

Both groups learn Task A to some criterion. While the experimental group is learning Task B, the control group rests. After, both groups are tested for the retention of Task A. In this design Task B is the interfering condition which inhibites the recall of Task A. This condition described by McGeogh as a "decrement in retention resulting from activity, usually a learning activity,

[17] Kelley, op. cit., p. 229.

interpolated between an original learning and a later measurement of retention",[18] is given the name retroactive inhibition. McGeogh considers the interference which later learning exerts upon the retention of prior learning as the basic factor contributing to forgetting.

Many other factors influence forgetting. Blair states some of these factors that determine the rate of forgetting.[19]

1. The kind of test of retention used makes a difference. A test of pure recall would show a more rapid drop than a test of recognition.

2. The kind of material which is learned affects the shape of the retention curve. The more meaningful the material, the less rapid the drop.

3. The thoroughness of the original learning is also a factor. Overlearning produces a retention curve of an entirely different shape, one which may remain at a high level for an indefinite period of time.

4. The kinds of activity which occurred after the original learning partially determine how rapidly forgetting occurs.

5. Active involvement of the learner in the learning situation also retards forgetting.

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[18] McGeogh and Irion, op. cit., p. 404.

[19] Glenn M. Blair, R. Stewart Jones, and Raymond H. Simpson, Educational Psychology (New York: The MacMillan Co., 1962). p. 271.

Summary

Retention refers to the extent to which material originally learned is still retained, and forgetting refers to the portion which is not retained.

Retention curves vary with the degree of meaning contained in what is being learned, but tend to resemble the curve of retention first established by Ebbinghaus. In general, the more meaningful the material, the better it is retained.

There are three methods of measuring retention: recall, recognition and relearning. Recall and recognition are the methods most commonly used in the classroom.

The major cause of forgetting is the interference between earlier and later learnings. In proactive inhibition, what has been learned interferes with what is being learned; whereas, in retroactive inhibition, what is being learned interferes with what has been learned.

Some factors which affect retention and forgetting that were discussed are the kind of test, the type of material learned and the thoroughness with which material is learned. Furthermore, the active involvement of the learner and the type of activity the learner engages in after the original learning also influence the degree of retention and forgetting.

The purpose of this section was to provide an overview of retention and forgetting. The following section will be devoted to the discussion of previous studies related to the retention of mathematical skills after the summer vacation.

STUDIES IN RETENTION OF MATHEMATICAL SKILLS AFTER THE SUMMER VACATION

Most retention studies in mathematics follow a simple pattern. Towards the end of the school year the students are tested, and following the summer vacation, the same students are given either the same test or a similar test. The first test establishes a measure of achievement at the end of the school year. When the score from the second test is compared to the first score, the amount of retention after the summer vacation can be established. Many researchers, interested in subject areas other than mathematics, use batteries of standardized tests as criterion measures. When this occurs, only the results of the mathematics tests will be considered.

One of these was conducted by Garfinkle[20] who

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[20] M. A. Garfinkle, "The Effects of Summer Vacation on Ability in the Fundamentals of Arithmetic," Journal of Educational Psychology, X (January, 1919), 44-48.

studied the loss of arithmetic ability during the summer vacation for the purpose of locating weaknesses that need to be relearned in September. Standardized tests stressing fundamentals in addition, in subtraction, in multiplication and in division were administered to 747 fifth-, sixth-, and seventh-grade pupils. Form 1 of the Courtis Test in Fundamentals, Series B was given in June, and Form 2 of the same test was given in September. Speed and accuracy were the two factors tested. Except for addition, there was a slight decrease in speed from June to September, but there was a decided decrease in accuracy for all groups. Moreover, the loss of accuracy in some of the skill areas for the seventh grade pupils amounted to more than the equivalent of two year's work. For example, June median for addition was 81 for the seventh grade pupils and 75 for the fifth grade pupils. However, the September median for the seventh grade pupils was 69, which is below the June median for fifth grade pupils. Similar results were found for multiplication and subtraction.

In another study Dix[21] investigated summer loss

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[21]William L. Dix, "An Investigation of Vacation Loss," Eighth Yearbook of the Department of Elementary School Principals, (Washington: Department of Elementary School Principals of the National Education Association, 1929), 245-249.

in arithmetic fundamentals with about 150 grade five students. They were given the Woody-McCall Mixed Fundamentals test in June before the summer recess, and again on the first day of school in September. When first tested, the students were only 0.9 points below the standard for the sixth grade or about two months below the norm. However, in September the same students were 2.0 points below the standard for sixth grade or equivalent to a half a year below the norm. The loss was equivalent to approximately four months in arithmetic skills. The same experiment was repeated the following year with almost identical results. The loss for that group was 2.3 points. Further data gave the breakdown of the percentage losses in the four basic operations: division 16 per cent; subtraction 15 per cent; multiplication 13 per cent, and; addition 7 per cent.

Likewise, Patterson[22] investigated the effects of summer vacation on the retention of arithmetic and reading skills, but unlike Garfinkle and Dix, she also took into account the mental ability of the children.

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[22]Mildred V. W. Patterson, "The Effects of Summer Vacation on Children's Mental Ability and on Their Retention of Arithmetic and Reading," Education, XLVI (December, 1925), 222-228.

For arithmetic skills, 147 students from grades four to eight were given a battery of standardized tests in the middle of June. The students of grades four to six were given the Woody-McCall Mixed Fundamentals Test, Form 1, and the students of grades seven and eight were given the Woody Multiplication Test. The same tests were given to the same students by the same person in mid-September. The Binet-Simon classification was used to group each grade into three intelligence groups: Supernormal, I.Q. 110 and upwards; Normal, I.Q. 90-109; Subnormal, I.Q. below 90. All grades from four to eight showed losses in arithmetic ability. In comparing the intelligence groups, all groups from four to six showed summer losses, but the supernormal group showed the greatest loss. For grades seven and eight, however, the normal and subnormal groups showed losses, whereas the supernormal group showed gains. Patterson concluded that summer vacation was detrimental to the student's arithmetic ability. However, assessment of the results is difficult because different tests were used with different groups, and losses were not determined statistically.

About the same time Kramer[23] investigated forget-

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[23] G.A. Kramer, "Do Children Forget During the Vacation?" Baltimore Bulletin of Education, VI (December, 1927), 56-60.

ting of arithmetic skills during the summer vacation with 150 grade five pupils. Based on the Illinois Examination, students were classified into three equal ability groups: Group X, I.Q. 110 and over; Group Y, I.Q. 88-109; Group Z, I.Q. less than 88. For the group of 150, Kramer obtained a median score of 39.7 in June and a median score of 37 in September resulting in a loss of 7 per cent. The percentage losses for the ability groups were: Group X, 5 per cent; Group Y, 4½ per cent; Group Z, no loss. Kramer stated that the coefficient of correlation of .79 between the June and September arithmetic scores showed a relatively high probability that a student would retain very nearly the score obtained in June. According to Kramer, the slight losses were insignificant, and they did not conform to the generally accepted theory that there were dramatic retention losses over the summer vacation. However, these students had been involved in an intensive remediation program in arithmetic prior to testing in June.

Sister Irmina Saelinger's retention study[24] show-

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[24]Sister Mary Irmina Saelinger, The Effects of Summer Vacation Upon the Retention of the Elementary School Subjects, Doctoral Dissertation, The Sisters of Saint Benedict, 1928.

ed a marked loss in rate and accuracy of computational skills in addition, in subtraction, in multiplication and in division during the summer vacation. Approximately 1000 students in grades two to seven were involved in her study. The Munroe Diagnostic Tests in Arithmetic, Diagnostic Computational Scale and the Stanford Achievement, Arithmetic Computation Test 4 were used at the end of June to test the ability of the students in arithmetic. The same students were retested in the middle of September and again at the beginning of October. The October tests were used to determine the amount of loss that would be regained in the two week period following the mid-September tests. Although all grades suffered losses, the loss was greater for students in grades two, three and four. However, students in the upper grade levels showed less power to regain the rate of work acquired in June before the vacation. Also, the amount of loss differed with the four fundamental operations. Multiplication suffered the greatest loss, followed by division, addition and subtraction.

In the same year Bruene[25] investigated the effects

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[25]Elizabeth Bruene, "Effects of the Summer Vacation on the Achievement of Pupils in the Fourth, Fifth, and Sixth Grades," Journal of Educational Research, XVIII (November, 1928), 309-314.

of summer vacation on achievement in arithmetic. Sixty-nine students in grades four, five and six were administered a battery of Stanford Achievement Tests in May and again in September. Although the sample was small, the results showed that a decided summer loss was evident in computational skills at all levels. In both grades four and five, the loss amounted to about a half year's work while the loss in grade six amounted to a year's work.

During the 1930's Keys and Lawson[26] compared the gains and losses in mathematical skills during the five summer months, which included three months of summer vacation, to that of the previous seven months of schooling. Spring tests were administered in early May and fall tests three or four weeks after the resumption of school in September. The study involved 164 students from grades four to eight for a period of three years. The fall tests were postponed until the children had become acquainted with their new teachers and classrooms and readapted to school tasks in order to provide a fairer measure of actual retention of previous learn-

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[26]Noel Keys and J. V. Lawson, "Summer Versus Winter Gains in School Achievement," School and Society, XLVI (October, 1937), 541-544.

ing. This allowed for considerable relearning of skills which had suffered from the twelve to fourteen weeks out of school. Achievement was measured by means of Unit Scales of Attainment in which arithmetic was one of the eleven sub-tests. The results showed that loss of skills in the fundamental operations in arithmetic amounted to 58 per cent or 4.2 months of the previous winter's gain. Since the fall tests were administered several weeks after the resumption of school the loss over the summer months vacation might have been greater.

A study determining the extent of forgetting of arithmetic fundamentals during the summer vacation was reported by Bender.[27] The testing program involved 1592 students from grades three to eight. In each grade a standardized arithmetic test was given in the last week of May, 1936. The same test was given to the same children when they returned in the first week of school in September. The results showed that 11 per cent of the students neither gained or lost, 53 per cent of the students lost and 36 per cent of the students gained in arithmetic fundamentals.

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[27]John F. Bender, "Retention of Experience by School Children," Proceedings of the Oklahoma Academy of Science, XXV (1945), 59-61.

In the late 1950's Sister Josephina[28] conducted a study in which a non-standardized arithmetic test, consisting of thirty computational and twenty problem solving items was administered to 122 fifth-grade students. This test was administered at the end of the school term in June and again at the beginning of the new school term following the summer vacation. The mean score of the thirty computational questions dropped from 20.91 in June to 15.09 in September. The loss was found to be statistically significant beyond the one per cent level of confidence.

The most extensive study conducted on retention in recent years was undertaken by Parsley and Powell.[29] The purpose of their study was to investigate achievement gains or losses during the academic year and over the summer vacation. Investigation of achievement in gains or losses was made between males and females and between adjacent grade levels. The California Achievement Test

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[28] Sister Josephina, "Differences in Arithmetic Performances," The Arithmetic Teacher, VI (April, 1959), 152-153.

[29] Kenneth M. Parsley, Jr., and Marvin Powell, "Achievement Gains and Losses During the Academic Year and Over the Summer Vacation Period: A Study of Trends in Achievement by Sex and Grade Level Among Students of Average Intelligence," Genetic Psychology Monographs, LXVI (August, 1962), 285-342.

Batteries were administered to 180 students from grades two to seven. Different forms of the test batteries were administered to each student in the spring, at the end of the academic year, and again in the fall following the summer vacation period. Six subject areas were measured, one of which was the arithmetic fundamentals test. The California Test of Mental Maturity was used to select students with I.Q.'s from 90-110 for the study. Students in this intellectual range were used because the investigators were interested in the performance of these students, and because the conclusions drawn from the study could be more widely applied. The data revealed that losses in arithmetic fundamantals occurred at most levels. Losses did not occur for the grade seven males and females, and for the grade five females. The greatest losses occurred at the lower grade levels.

Another more recent study was one conducted by Scott[30]. He compared the effects of summer vacation on the retention of mathematical concepts taught to the modern mathematics groups with those taught to the traditional program groups. This study was conducted first

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[30] Lloyd F. Scott, "Summer Loss in Modern and Traditional Elementary School Mathematics Programs," California Journal of Educational Research, XXVIII (May, 1967), 145-151.

with students in grades one to three, and later repeated with other students in grades three to six. A test stressing the four basic operations was administered in June and again in the first week of September. Some summer loss in mathematics achievement occurred for both programs. These losses were not analyzed statistically.

Some studies reviewed were not concerned directly with loss of mathematical skills during the summer vacation. Nelson's study[31] investigated the amount of time required for pupils to regain the knowledge and skills to the level of achievement of the previous spring. The Courtis Standard Arithmetic Tests, Series B was used to test 40 grade five and 40 grade seven students. The first form of the test was administered in May just before the closing of school; the second form in September following the opening of school. A study of the mean scores showed that the loss during the summer vacation for grade seven students was negligible. However, the grade five students scored 25.3 in June and 19.1 in September, a loss almost equal to two year's

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[31]M. J. Nelson, "How Much Time is Required in the Fall for Pupils of Elementary School to Reach the Spring Level of Achievement?" Journal of Educational Research, XVIII (November, 1928), 305-308.

work.

In the fall the same test was administered three times at intervals of two weeks each, and a final test was administered just prior to Christmas holidays, approximately fifteen weeks after the commencement of school. The results showed that grade seven students regained the spring level of achievement six weeks after the opening of school. Grade five students, however, needed fifteen weeks to regain the summer's loss. It was expected that the grade five students who lost much more in arithmetic fundamentals would take a longer time for recovery than the grade seven students. No special drill was provided for the students because the teachers were not aware that successive tests were to be given after the initial test.

In another study Morgan[32] investigated the effectiveness of special training in preventing loss of mathematical skills due to the summer vacation. The study was conducted with two grade six classes designated as X and Y groups. Both groups were administered the Compass Survey Test in Arithmetic, Form A on May 11,

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[32] L. D. Morgan, "How Effective is Specific Training in Preventing Loss Due to Summer Vacation?" The Journal of Educational Psychology, XX (September, 1920), 466-471.

May 25, and September 4. The training of the Y group consisted of administering four tests; on addition, on subtraction, on multiplication and on division. Once the weaknesses were diagnosed the Y group received special training for a period of two weeks prior to summer vacation. The X group did not receive any special training. The results showed that group X lost 3.69 items while group Y lost only 2.56 items, or a difference of 1.13 items in favor of group Y. In the Y group, thirty-five pupils lost efficiency during the vacation, three remained the same, while two gained. For the X group, thirty-five lost in efficiency, one gained and one remained the same.

In other studies reviewed emphasis was placed on a specific skill area as shown by Brueckner's study.[33] Brueckner studied the loss of basic facts in addition and subtraction during the summer vacation for 224 Grade IIA pupils, fifteen of whom attended summer school, and 194 Grade IIB pupils, seventeen of whom attended summer school.

The pupils in Grade IIA were tested on 100 addition and 100 subtraction facts, the basic work for the

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[33]Brueckner, loc. cit.

year. Three tests in the basic addition and subtraction facts were given; the first was given at the close of school in June, the second on the first day of school in September, and the third two weeks later. The purpose of the third test was to measure the improvement made during the first two weeks of school. The results showed that there was very little loss in ability for the regular group in addition during the summer vacation but a considerable loss in subtraction. The scores of the summer school group were much below that of the regular group in both addition and subtraction, the loss being greater in subtraction.

The pupils in Grade IIB were tested on 64 addition facts with sums of 10 or less, and 64 subtraction facts with minuends of 10 or less, the basic work for the year. For these pupils, there was almost no loss in ability during the summer vacation for addition and subtraction. The summer school group showed a slight loss in ability for addition, which was regained by the time of the third test, but suffered substantial loss for subtraction. Even after the third test, the summer school pupils scored considerably lower than they did in June. Marked differences in the scores for individual students in both groups were recorded on the three tests. Because of this tendency, Brueckner suggested the use of pre-tests as a basis for grouping children according to ability in arithmetic.

In a similar type of study, Cook[34] tested a small group of grade two students on 100 addition facts before and after the summer recess. Students were given drill sheets and fact cards to practise addition skills during the summer vacation. The results showed that students who practised addition skills retained them, but students who practised less then three weeks during the summer vacation lost considerably.

An experiment was conducted by Evans[35] to determine the effects of an arithmetic enrichment program at summer camp as a means of counteracting the considerable drop in arithmetical abilities during the summer vacation. Thirty boys in the experimental group which received six weeks of this program were paired individually with a control group which did not. The boys in the two groups were matched on the basis of age, intelligence and arithmetic scores. The arithmetic scores were based on the Stanford Achievement Test in Arithmetic administered in

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[34]Cook, loc. cit.

[35]Forrest Furman Evans, "The Effects of a Summer Camp Arithmetic Enrichment Program" Dissertation Abstracts, 28:163, January 1958.

May. The boys in the experimental group participated in 115 different arithmetic enrichment activities at camp. A daily log was kept by each boy showing the type and number of activities and the computational skills used. It also indicated whether paper and pencil were used in solving the problem. At the end of six weeks, an alternate form of the pre-test was administered to both groups as a post-test. The experimental group lost three and three-tenths months in arithmetic ability, whereas the control group lost approximately nine months in arithmetic ability.

Two factors were obvious in the research of these retention studies. First, all studies were conducted in the United States where the summer vacation is approximately one month longer than in Canada. This investigator was unable to locate Canadian studies related to the retention of mathematical skills during the summer vacation. Second, a variety of approaches - statistical and non-statistical were used to analyze the data. Because of this inconsistency, definite conclusions could not be drawn. Better experimental designs and more sophisticated measures are needed to examine the effects of summer vacation on the retention of mathematical skills.

In summation, the following conclusions were drawn from the above review of research studies.

First, an analysis of the retention studies reviewed reveals that mathematical skills are forgotten during the summer vacation. A summary of these studies is shown in Table 2.01. Only in studies by Brueckner[36] and Kramer[37] were summer losses so slight that they were considered insignificant. Other studies, such as Nelson[38], revealed that up to two years in mathematical ability were lost during the summer vacation.

Second, several studies analyzed the retention of mathematical skills for several grade levels in which comparisons between grade levels could be made. Garfinkle[39] and Bruene[40] found that losses in arithmetical ability increased with the grade level. On the other hand, Nelson[41], Saelinger[42] and Parsley and Powell[43]

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[36] Brueckner, loc. cit.

[37] Kramer, loc. cit.

[38] Nelson, loc. cit.

[39] Garfinkle, loc. cit.

[40] Bruene, loc. cit.

[41] Nelson, loc. cit.

[42] Saelinger, loc. cit.

[43] Parsley and Powell, loc. cit.

TABLE 2.01

SUMMARY OF RETENTION STUDIES

Study	Grade Range	Sample Size	Test Administered	Other Considerations	Results
Bender (1945)	3 - 8	1592	Standardized arithmetic tests	None	53% students lost; 11% neither lost or gained; 36% gained
Brueckner (1927)	2	177	Basic addition and subtraction facts	Two groups of students; summer school and non-summer school grade IIA and grade IIB	Grade IIA-slight loss in addition and greater loss in subtraction; Grade IIB-no loss in addition and subtraction; summer school students showed losses in both tests
Bruene (1928)	4 - 6	69	Stanford Achievement	None	Loss about half year's for grades 4 & 5, and about a year's work for grade 6
Cook (1942)	2	**	Basic addition facts	Students given drill sheets and fact cards to practise during summer	Students who practised during summer--no loss. Students who practised less than three months lost considerably

TABLE 2.01 (continued)

Study	Grade Range	Sample Size	Test Administered	Other Considerations	Results
Dix (1929)	5	150	Woody-McCall Mixed Fundamentals	None	Loss equivalent to approximately 4 months in arithmetic skills
Evans (1958)	**	60	Stanford Achievement Test in Arithmetic	Arithmetic enrichment summer camp	Camp students lost 3.3 months in ability. Non-camp students lost 9.0 months in ability
Garfinkle (1919)	5 - 7	747	Courtis Test in Fundamentals	None	Loss for all groups-up to two year's work for grade seven students
Sister Josephina (1959)	5	122	Non-standardized test	None	Mean score in June was 20.91 and in September was 15.09. Loss was statistically significant at .01 level
Keys and Lawson (1937)	4 - 8	164	Unit Scales of Attainment	None	Loss about 4.2 months of previous winter's gain.
Kramer (1927)	5	150	Illinois Examination	3 ability groups X - I.Q. 110 or over Y - I.Q. 88-109 Z - I.Q. less than 88	Loss of only 7% for all students; Group X lost the most and Group Z - no loss

TABLE 2.01 (continued)

Study	Grade Range	Sample Size	Test Administered	Other Considerations	Results
Morgan (1920)	6	**	Compass Survey Test in Arithmetic, Form A	One group received special training prior to summer vacation	Group without special training lost 1.13 problems more than group with training. Most students lost.
Nelson (1928)	5 - 7	40 for each grade	Courtis Standard Arithmetic Tests, Series B	None	Grade 7-loss negligible. Grade 5-loss about two year's work
Parsley and Powell (1962)	2 - 7	180	The California Achievement Batteries	None	Loss at most levels except for grade 5 females and for grade seven students.
Patterson (1925)	4 - 8	147	Woody-McCall Mixed Fundamentals (4 to 6) Woody Multiplication (grades 7 and 8)	3 ability groups - Supernormal, Normal and Subnormal	All groups showed losses except supernormal group in grades 7 and 8
Sister Irmina Saelinger (1928)	2 - 7	approx. 1000	Monroe Diagnostic Test, Diagnostic Computational Scale, Stanford Achievement, Arithmetic Computation Test 4	None	Losses for all groups. Largest losses at the lower grade levels
Scott (1967)	1 - 6	**	Non-standardized tests	Modern mathematics and traditional groups	Losses occurred for both groups

found that the greatest losses occurred at the lower grade levels. Meanwhile, Patterson[44] could not distinguish any pattern in losses between grade levels. There does not appear to be a clear pattern as to whether retention of mathematical skills increase or decrease with grade levels.

Third, only Patterson[45] and Kramer[46] catagorized the samples into groups based on intelligence tests. Although evidence is limited, it appears that the more intelligent student tends to suffer greater summer loss, especially if he or she is in grades four to six.

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[44]Patterson, loc. cit.

[45]Patterson, loc. cit.

[46]Kramer, loc. cit.

CHAPTER 3

THE INVESTIGATION

THE PROBLEM

The purpose of this study is to investigate the effects of summer vacation on the retention of learned basic facts in addition, in subtraction, in multiplication and in division. In so doing, this study is designed to answer the following questions:

1. Do students retain the learned basic mathematical facts at the grade three, grade four and grade five levels during the summer vacation?

2. Are there differences in retention of basic mathematical facts with respect to students at the grade three, grade four and grade five levels?

DEFINITION OF TERMS

Certain terms will be used throughout the study and are defined.

Achievement:-For the purpose of this study, achievement will be the score obtained by a student on a basic facts test.

Basic Facts Tests:-The tests used in this study were recorded on cassette tapes with a uniform time interval between each item. The tapes for addition, subtraction, multiplication and division were given in two parts. The first part consisted of easier facts, the second of more difficult facts. Each test is described under the section called "Description of Tests" on page number 45.

Summer Vacation:-This is a period during which school is closed for the summer. This varies slightly in different localities. Summer vacation, for this study, is the two-month period of July and August, 1973. This same term when used in the retention studies researched is approximately a three-month period.

Grade Three, Grade Four, Grade Five:-Although all the students will have moved to the next grade level during the course of this study, any reference to a student's grade level will be the grade level of that student in June rather than his grade level in September.

Meaningfully significant:-This term will be used to describe a difference in scores which are both educationally and statistically significant.

ASSUMPTIONS

Although controlling the type of mathematical experiences that a student may encounter during the summer vacation is not possible, it will be assumed that whatever experiences students receive related to the basic facts will be randomly balanced for all grade levels.

The sample of students in this study will have received instruction from teachers with varying mathematical experiences and backgrounds as well as differing teaching abilities. If these factors affect a student's development in mathematical skills, it will be assumed that these differences will be randomly balanced for all grade levels.

STATEMENT OF HYPOTHESES

This study investigates the following hypotheses under four headings: Addition, Subtraction, Multiplication, and Division. The hypotheses are stated in the "null hypotheses" form.

Addition

1. The groups will be equal in achievement between June scores and September scores as measured by the *Addition Facts Test* for students in grades three, four and five. Each grade will be treated as a discrete group.

2. The groups will be equal in the retention of the basic addition facts between the grade three, grade

four and grade five levels.

Subtraction

3. The groups will be equal in achievement between June scores and September scores as measured by the _Subtraction Facts Test_ for students in grades three, four and five. Each grade will be treated as a discrete group.

4. The groups will be equal in the retention of the basic subtraction facts between the grade three, grade four and grade five levels.

Multiplication

5. The groups will be equal in achievement between June scores and September scores as measured by the _Multiplication Facts Test_ for students in grades three, four and five. Each grade will be treated as a discrete group.

6. The groups will be equal in the retention of basic multiplication facts between the grade three, grade four and grade five levels.

Division

7. The groups will be equal in achievement between June scores and September scores as measured by the _Division Facts Test_ for students in grades three, four and

five. Each grade will be treated as a discrete group.

8. The groups will be equal in the retention of basic division facts between the grade three, grade four and grade five levels.

SAMPLE

The students in the sample attended Wayoata School which is an urban elementary school situated in Transcona, Manitoba. This school, with a population of approximately 750 students from kindergarten to grade six, has self-contained classrooms as well as open area spaces. Of the 329 students in grades three, four and five, the sample used in the study consisted of 274 students from both the self-contained classrooms and open area spaces. The remaining 55 students were excluded from the study because their scores were not available for all the basic facts tests in June and September due to absences or moving to other schools during the summer vacation. The breakdown of the sample according to grade levels and sex is shown in Table 3.01.

LIMITATION OF THE SAMPLE

The sample was limited to one school within the school division, and might not be representative of the

TABLE 3.01

BREAKDOWN OF SAMPLE ACCORDING TO GRADE LEVELS AND SEX

Grade Levels	Males	Females	Total
3	43	45	88
4	42	43	85
5	57	44	101

total population. The variables of socio-economic status and intelligence have not been controlled. Because of these restrictions any generalizations that are made or any conclusions that can be drawn must only be confined to the students of Wayoata School.

DESCRIPTION OF TESTS

Since the tests were to be administered on two different occasions to each of the eleven classes, the possibility arose of variations in scores due to change in delivery (for example, voice inflexion) and timing. To avoid this source of variance a tape recording of the tests was used. A description of each test follows:

Addition Facts Test:-This test, as shown in Appendix A, involved the oral presentation of fifty-five items in the form "a and b", where "a" and "b" are whole numbers from 0 to 9. The thirty easier addition facts contained addends from 0 to 5, whereas the twenty-five more difficult addition facts contained addends from 6 to 9. The facts were arranged in random order, each separated from the next by a response interval of four and one-half seconds. The oral presentation consisted of a number of the item followed by the item. For example, the test began in the following manner: "Number one, zero and zero (4 sec.), number two, one and three, . . ."

Subtraction Facts Test:-This test, as shown in Appendix B, involved the oral presentation of fifty-five items in the form "a from b", where "a" and "b" are whole numbers. The thirty easier subtraction facts had minuends no greater than nine, whereas the twenty-five more difficult facts had minuends from 10 to 19. The facts were arranged in random order, each separated from the next by a response interval of six seconds. The oral presentation consisted of the number of the item followed by the item. For example, the test began in the following manner: "Number one, three from nine (6 sec.), number two, zero from eight, . . ."

Multiplication Facts Test:-This test, as shown in Appendix C, involved the oral presentation of fifty-five items with the form "a b's", where "a" and "b" are whole numbers. The twenty-eight easier multiplication facts had factors no greater than five, whereas the twenty-seven more difficult multiplication facts had factors no greater than nine. The facts were arranged in random order, each separated from the next by a response interval of six seconds. The oral presentation consisted of the number of the item followed by the item. For example, the test began in the following manner: "Number one, three twos (6 sec.), number two, zero sixes, . . ."

Division Facts Test:-This test, as shown in Appendix D, involved the oral presentation of fifty-four items of the form "a's in b", where "a" and "b" are whole numbers and "b" is not zero. The twenty-seven easier division facts had divisors no greater than four, while the twenty-seven more difficult division facts had divisors from 5 to 9. The facts were arranged in random order, each separated from the next by a response interval of six seconds. The oral presentation consisted of the number of the item followed by the item. For example, the test began in the following manner: "Number one, threes in three (6 sec.), number two, fives in fifteen, . . ."

ADMINISTRATION OF TESTS

Tests in basic addition, subtraction, multiplication and division facts were administered to all students in grades three, four and five during the last two weeks of June, 1973. The same tests were administered to the same students during the first two weeks of September, 1973. Tests in June and September were given in two sittings: Addition Facts Test and Subtraction Facts Test in the first sitting; Multiplication Facts Test and Division Facts Test in the second sitting. The length of each sitting was approximately twenty-five minutes. All testing was held in self-contained classrooms.

Prior to the first set of tests, the purpose of the study was outlined to the students. Each student was provided with an answer sheet similar to the one shown in Appendix E on which to record his responses, and was told to respond to each problem by recording only the answer. An example of how the problems would be presented to the students was given before each test. Care was taken to ensure that directions accompanying the tests were closely followed. When the tests were completed, the answer sheets were collected and scored by the investigator. The testing procedure and test administrator were the same for both June and September tests.

TREATMENT OF SAMPLE

The first area investigated was to determine if there were significant differences in achievement between the June scores and the September scores of the basic facts tests for students in grades three, four and five. The t-test for the differences between correlated means was used to analyze the mean June scores and the mean September scores at the .05 level of significance. The data was analyzed with the use of a computer program called the <u>Paired Student's t-Test</u>.[1] This program was used for all the tests.

The second area investigated was the retention of basic facts between grade levels. Previous studies researched[2] showed that difference scores were utilized to determine retention after the summer vacation. However, Kerlinger states that

> One of the most difficult problems that has plagued and intrigued researchers, measurement specialists, and statisticians is how to study and analyze such difference, or change scores . . . difference scores . . . are usually less reliable than the scores from which they are calculated.
>
> The generally recommended procedure is to use so-called residualized or regressed gain scores, which are scores calculated by predicting the post-test scores from the pre-test scores on the basis of the correlation between pre-test and post-test, and then subtracting these predicted scores from the post-test scores to obtain the residual gain scores. . . . . The effect of the pre-test scores is removed from the post-test scores that is, the residual scores are post-test scores purged of the pre-test influence. Then the significance of the difference between the

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[1]"Statistical Package: Part B of the Computer Guide", (Winnipeg: University of Manitoba Computer Centre, 1972). (Computer print out), pp. 12-13.

[2]Sister Irmina, Bruene, Kramer, Keys and Lawson, Parsley and Powell, op. cit.

> means of these scores is tested. All this can be accomplished by using . . . the analysis of covariance. . . .[3]

Recent studies[4] show that the analysis of covariance has been used in statistical designs involving pre-test and post-test scores. For this study the June scores would be the pre-test scores or covariate while the September scores would be the post-test scores.

To test differences in retention the one-way analysis of covariance design was employed with three levels and a single covariate. The data were analyzed by a computer program, namely the Simple Covariance Program.[5] An aposteriori procedure, Scheffé test was applied to the adjusted mean scores in order to locate significant contrasts. The analysis of covariance and

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[3]Fred Kerlinger, Foundations of Behavioral Research, (Toronto: Holt, Rinehart and Winston, 1973), pp. 337-338.

[4]Two studies cited where the analysis of covariance design was employed involving pre-test and post-test scores.

Fletcher R. Norris, "Student Mathematical Achievement as Related to Teaching Inservice Work," Mathematics Teacher, LXII (April, 1969), 321-327.

Roland F. Gray and Donald E. Allison, "An Experimental Study of the Relationship of Homework to Pupil Success in Computation with Fractions," School Science and Mathematics, LXXI (April, 1971), 339-346.

[5]Statistical Package, op. cit. pp. 39-41.

Scheffé test using the .05 level of significance were used each time for addition, subtraction, multiplication and division.

The final statistical treatment was the correlational analysis of the data. The main purpose of this treatment was to determine whether the June score was a true predictor of the September score. Thus, the greater the correlation coefficient the more accurate is the prediction of the September scores from the June scores. The scores from the tests were analyzed by a computer program, namely the Simple Correlations Program.[6] A matrix for the sample and one for each grade level were constructed. Each matrix was based on the eight variables involving the June and September scores for the addition, subtraction, multiplication and division tests.

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[6]Ibid., pp. 45-46.

CHAPTER 4

ANALYSIS OF DATA

The purpose of this chapter is to present detailed results of the study described in the preceeding chapter. More specifically, the hypotheses set out in Chapter 3 were either accepted or rejected. To facilitate reporting, the results are presented under five headings: Addition, Subtraction, Multiplication, Division, and Correlation Matrices.

ADDITION

Hypothesis 1 stated that there would be no difference in achievement between June scores and September scores as measured by the Addition Facts Test for students at each grade level. The paired t-test was used to examine the difference between the mean June score and mean September score. The tabled critical value needed for significance at .05 level was 2.00 for 60 df. Results for each grade level are shown in Table 4.01. The differences in means for grade three and for grade four were significant at the .05 and .01 levels, rejecting the null hypotheses at these grade levels. However, the difference in means at grade five level was

TABLE 4.01

COMPARISON OF MEAN JUNE SCORES AND MEAN SEPTEMBER SCORES IN ADDITION FACTS TEST

Test	Mean	S.D.	Difference	t Value
GRADE 3				
June	52.76	3.17		t=7.72**
September	49.18	5.57	3.58	df=87
GRADE 4				
June	53.31	2.65		t=3.87**
September	51.54	4.91	1.77	df=84
GRADE 5				
June	53.97	2.13		t=1.21
September	53.72	2.16	.25	df=100

**Significant at .01 level ($t_{.01}$=2.66)
*Significant at .05 level ($t_{.05}$=2.00)

not significant at .05 level, and the null hypothesis was accepted.

Hypothesis 2 stated that there would be no significant difference in the retention of the basic addition facts between the three grade levels. The summary of the analysis of covariance treatment is shown in Table 4.02. Because the tabled F value needed for significance at .05 level was 3.04, the F value shown in the table was significant at .05, rejecting the null hypothesis. Also, the F value exceeded the .01 level of confidence. The computed F value showed that there were significant differences between grade levels, but it does not indicate where these differences occurred.

An aposteriori procedure, the Scheffé method, was applied to locate significant differences in retention between pairs of grade levels. For the Scheffé method, the value F' was derived from the formula $F' = (k-1)F^{o}$ where F^{o} was 3.04 for 200 df at the .05 level of confidence. Therefore, the critical F' value needed for significance at the .05 level is 6.08 where k = 3. Then, the F values for each pair of grade levels were derived from the following formula:[1]

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[1]George A. Ferguson, Statistical Analysis in Psychology and Education (3rd ed., Toronto: McGraw-Hill Book Company, 1966), p. 296.

TABLE 4.02

ANALYSIS OF COVARIANCE SUMMARY TABLE
FOR ADDITION FACTS TEST

Source of Variance	Sum of Squares	df	Mean Square	F Value
Between	531.53	2	265.76	20.24**
Within	3544.97	270	13.13	
Total	4076.50	272	14.19	

**Significant at .01 level ($F_{.01}$=4.71)
*Significant at .05 level ($F_{.05}$=3.04)

$$F = t^2 = \frac{(\bar{X}_1 - \bar{X}_2)^2}{S_W'^2 \quad (n_1+n_2)/\ n_1 n_2}$$

In this formula, $S_W'^2$ is the adjustment of the adjusted within group variance estimate[2], $\bar{X}_1$ and $\bar{X}_2$ are adjusted mean scores and n_1 and n_2 are the size of each group. The June and September test scores and the adjusted mean scores for each grade level and for each basic facts test are shown in Table 4.03. If the F value is greater than or equal to F', then the mean difference is significant.

The results of Scheffé tests are shown in Table 4.04, and significant differences are indicated by asterisks. Significant differences at the .05 level were found between all pairs of grade levels with retention differences between grades three and five, and between grades three and four exceeding the .01 level of confidence.

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[2]In using an aposteriori procedure, the Scheffé test with the analysis of covariance design, adjustments to the adjusted within group variance must be made. This adjustment is given by the formula for average experimental error per unit shown on page 786 of B. J. Winer's book Statistical Principles in Experimental Design.

TABLE 4.03

JUNE AND SEPTEMBER TEST SCORES
AND THE ADJUSTED MEAN SCORES

	Grade	June Test Scores	September Test Scores	Adjusted Mean Scores
Addition	3	52.76	49.18	49.75
	4	53.31	51.54	51.60
	5	53.97	53.72	53.17
Subtraction	3	49.63	46.23	47.88
	4	50.85	50.08	50.42
	5	52.76	52.07	50.35
Multiplication	3	49.14	39.95	40.20
	4	47.75	44.66	46.10
	5	51.08	49.81	48.39
Division	3	45.47	38.44	39.29
	4	44.86	40.72	42.09
	5	48.65	48.30	46.40

TABLE 4.04

SCHEFFÉ TESTS FOR ADDITION FACTS

Comparison	Adjusted Mean	Difference	F Value
Grade 3 and Grade 4	49.75 51.60	1.85	11.06**
Grade 4 and Grade 5	51.60 53.17	1.57	8.39*
Grade 3 and Grade 5	49.75 53.17	3.42	41.15**

*Significant at .05 level ($F'_{.05}=6.08$)

**Significant at .01 level ($F'_{.01}=9.42$)

SUBTRACTION

Hypothesis 3 stated that there would be no significant difference in achievement between the June scores and the September scores as measured by the Subtraction Facts Test. The results of the paired t-test for grades three, four and five are shown in Table 4.05. The tabled critical value needed for significance at the .05 level was 2.00 for 60 df. The differences in means were significant at the .05 level for all grade levels, with the difference in means for grade three exceeding the .01 level. The null hypothesis for each grade level was rejected.

Hypothesis 4 stated that there would be no significant difference in the retention of the basic subtraction facts between grade levels. The tabled critical F value needed for significance at the .05 level was 3.04. The summary of the analysis of covariance treatment shown in Table 4.06 reveals that the F value was significant at both the .05 and .01 levels, rejecting the null hypothesis.

Using the adjusted mean scores, the Scheffé method was applied to locate significant differences in retention at the .05 level between pairs of grade levels. The F values were computed in the same manner as outlined for

TABLE 4.05

COMPARISON OF MEAN JUNE SCORES AND MEAN SEPTEMBER SCORES IN SUBTRACTION FACTS TEST

Test	Mean	S.D.	Difference	t Value
GRADE 3				
June	49.63	5.28		t=6.46**
September	46.23	7.76	3.40	df=87
GRADE 4				
June	50.85	3.56		t=2.13*
September	50.08	4.67	.77	df=84
GRADE 5				
June	52.76	2.92		t=2.55*
September	52.07	4.10	.69	df=100

**Significant at the .01 level ($t_{.01}$=2.63)

*Significant at the .05 level ($t_{.05}$=2.00)

TABLE 4.06

ANALYSIS OF COVARIANCE SUMMARY TABLE
FOR SUBTRACTION FACTS TEST

Source of Variance	Sum of Squares	df	Mean Square	F Value
Between	353.25	2	176.63	12.69**
Within	3758.40	270	13.63	
Total	4111.65	272	15.12	

**Significant at .01 level (F=4.71)
*Significant at .05 level (F=3.04)

addition. The results of the Scheffé method are shown in Table 4.07, and significant differences are indicated with asterisks. Significant differences in retention between grades three and five, and between grades three and four at the .05 and .01 levels are indicated in the table. However, no significant difference was found between grades four and five.

MULTIPLICATION

Hypothesis 5 stated that there would be no significant difference in achievement between the June scores and the September scores as measured by the Multiplication Facts Test. The results of the paired t-test for grades three, four and five are shown in Table 4.08. The tabled critical value needed for significance at the .05 level was 2.00 for 60 df. The differences in the means were significant at both the .05 and .01 levels for all grades. The null hypothesis for each grade level was rejected.

Hypothesis 6 stated that there would be no significant difference in the retention of the basic multiplication facts between grade levels. The tabled critical F value needed for significance at the .05 level was 3.04. The summary of the analysis of covariance treatment shown in Table 4.09 reveals that the F value

TABLE 4.07

SCHEFFÉ TESTS FOR SUBTRACTION FACTS

Comparison	Adjusted Mean	Difference	F Value
Grade 3 and Grade 4	47.88 50.42	2.54	19.00**
Grade 4 and Grade 5	50.42 50.35	0.07	.02
Grade 3 and Grade 5	47.88 50.35	2.47	19.71**

**Significant at .01 level ($F'_{.01}$=9.42)

*Significant at .05 level ($F'_{.05}$=6.08)

TABLE 4.08

COMPARISON OF MEAN JUNE SCORES AND MEAN SEPTEMBER SCORES IN MULTIPLICATION FACTS TEST

Test	Mean	S.D.	Difference	t Value
GRADE 3				
June	49.14	7.86		t=11.92**
September	39.95	10.73	9.19	df=87
GRADE 4				
June	47.75	7.37		t=5.54**
September	44.66	6.16	3.09	df=84
GRADE 5				
June	51.08	6.09		t=3.41**
September	49.81	6.92	1.27	df=100

**Significant at .01 level ($t_{.01}$=2.66)
*Significant at .05 level ($t_{.05}$=2.00)

TABLE 4.09

ANALYSIS OF COVARIANCE SUMMARY TABLE
FOR MULTIPLICATION FACTS TEST

Source of Variance	Sum of Squares	df	Mean Square	F Value
Between	3266.26	2	1633.13	55.86**
Within	7893.46	270	29.24	
Total	11159.72	272	41.03	

**Significant at .01 level ($F_{.01}$=4.71)
*Significant at .05 level ($F_{.05}$=3.04)

was significant at both the .05 and .01 levels, rejecting the null hypothesis.

An aposteriori Scheffé method, using the adjusted mean scores, was employed to locate significant differences between pairs of grade levels. The procedure outlined for addition was used in computing the F value. In Table 4.10, significant differences in retention are indicated with asterisks. Significant differences between all pairs of grade levels were found at .05 level; differences exceeding the .01 level were found between grades three and five, and between grades three and four.

DIVISION

Hypothesis 7 stated that there would be no significant difference in achievement between the June scores and the September scores as measured by the _Division Facts Test_. The results of the paired t-test for each grade level are presented in Table 4.11. The tabled critical value needed for significance at the .05 level was 2.00 for 60 df. The null hypothesis was rejected for grade three and for grade four because the differences between the means were significant at the .05 and .01 levels. No significant difference was found for the grade five students, thus the null hypothesis was ac-

TABLE 4.10

SCHEFFÉ TESTS FOR MULTIPLICATION FACTS

Comparison	Adjusted Mean	Difference	F Value
Grade 3 and Grade 4	40.20 46.10	5.90	50.50**
Grade 4 and Grade 5	46.10 48.39	2.29	8.12*
Grade 3 and Grade 5	40.20 48.39	8.19	106.02**

**Significant at the .01 level ($F'_{.01}$=9.42)
*Significant at the .05 level ($F'_{.05}$=6.08)

TABLE 4.11

COMPARISON OF MEAN JUNE SCORES AND MEAN SEPTEMBER SCORES IN DIVISION FACTS TEST

Test	Mean	S.D.	Difference	t Value
GRADE 3				
June	45.47	8.99		t=8.42**
September	38.44	10.93	7.03	df=87
GRADE 4				
June	44.86	7.24		t=7.33**
September	40.72	8.30	4.14	df=84
GRADE 5				
June	48.65	5.95		t=0.69
September	48.30	7.02	.35	df=100

**Significant at .01 level ($t_{.01}$=2.63)

*Significant at .05 level ($t_{.05}$=2.00)

cepted.

Hypothesis 8 stated that there would be no significant difference in the retention of the basic division facts between grade levels. The tabled critical F value needed for significance at the .05 level was 3.04. The summary of the analysis of covariance treatment shown in Table 4.12 reveals that the F value was significant at both the .05 and .01 levels. The null hypothesis was rejected.

Using the adjusted mean scores, an aposteriori Scheffé method was applied to locate significant differences in retention between pairs of grade levels. The procedure outlined for addition was used in computing the F value. In Table 4.13 significant differences in retention are indicated by asterisks. Significant differences between all pairs of grade levels were found at .05 level; differences exceeding the .01 level were found between grades three and five, and between grades four and five.

CORRELATION MATRICES

Correlation matrices representing the total sample and also each grade level appear in Tables 4.14, 4.15, 4.16 and 4.17. Eight variables from the study

TABLE 4.12

ANALYSIS OF COVARIANCE SUMMARY TABLE
FOR DIVISION FACTS TEST

Source of Variance	Sum of Squares	df	Mean Square	F Value
Between	2343.60	2	1171.80	31.63**
Within	10001.39	270	37.04	
Total	12344.99	272	45.39	

**Significant at .01 level (F=4.71)
*Significant at .05 level (F=3.04)

TABLE 4.13

SCHEFFÉ TESTS FOR DIVISION FACTS

Comparison	Adjusted Mean	Difference	F Value
Grade 3 and Grade 4	39.29 42.09	2.80	8.91*
Grade 4 and Grade 5	42.09 46.40	4.31	22.56**
Grade 3 and Grade 5	39.29 46.40	7.11	62.52**

**Significant at .01 level ($F'_{.01}=9.42$)

*Significant at .05 level ($F'_{.05}=6.08$)

TABLE 4.14

CORRELATION MATRIX FOR GRADES THREE, FOUR AND FIVE

Variable	June Addition	Sept. Addition	June Subt.	Sept. Subt.	June Mult.	Sept. Mult.	June Division	Sept. Division
June Addition	1.00							
Sept. Addition	.58	1.00						
June Subt.	.58	.63	1.00					
Sept. Subt.	.55	.71	.78	1.00				
June Mult.	.47	.41	.49	.40	1.00			
Sept. Mult.	.57	.61	.60	.60	.71	1.00		
June Division	.51	.40	.54	.46	.85	.66	1.00	
Sept. Division	.55	.63	.61	.66	.68	.81	.73	1.00

N=274

TABLE 4.15

GRADE THREE CORRELATION MATRIX

Variable	June Addition	Sept. Addition	June Subt.	Sept. Subt.	June Mult.	Sept. Mult.	June Division	Sept. Division
June Addition	1.00							
Sept. Addition	.63	1.00						
June Subt.	.63	.59	1.00					
Sept. Subt.	.57	.70	.78	1.00				
June Mult.	.52	.43	.52	.40	1.00			
Sept. Mult.	.63	.60	.53	.54	.74	1.00		
June Division	.50	.38	.56	.44	.88	.64	1.00	
Sept. Division	.64	.67	.63	.68	.73	.83	.71	1.00

N=88

TABLE 4.16

GRADE FOUR CORRELATION MATRIX

Variable	June Addition	Sept. Addition	June Subt.	Sept. Subt.	June Mult.	Sept. Mult.	June Division	Sept. Division
June Addition	1.00							
Sept. Addition	.52	1.00						
June Subt.	.53	.63	1.00					
Sept. Subt.	.45	.63	.71	1.00				
June Mult.	.30	.36	.32	.30	1.00			
Sept. Mult.	.35	.46	.42	.42	.72	1.00		
June Division	.47	.35	.44	.41	.84	.70	1.00	
Sept. Division	.38	.47	.51	.56	.70	.71	.78	1.00

N=85

TABLE 4.17

GRADE FIVE CORRELATION MATRIX

Variable	June Addition	Sept. Addition	June Subt.	Sept. Subt.	June Mult.	Sept. Mult.	June Division	Sept. Division
June Addition	1.00							
Sept. Addition	.54	1.00						
June Subt.	.41	.54	1.00					
Sept. Subt.	.53	.63	.75	1.00				
June Mult.	.59	.54	.65	.60	1.00			
Sept. Mult.	.61	.51	.66	.58	.84	1.00		
June Division	.54	.44	.54	.55	.79	.71	1.00	
Sept. Division	.51	.48	.46	.49	.64	.74	.69	1.00

N=101

were intercorrelated in order to examine relationships between the June and September scores at the different grade levels and also between the skill areas for June and for September. These variables are the June *Addition Facts Test* scores, the September *Addition Facts Test* scores, the June *Subtraction Facts Test* scores, the September *Subtraction Facts Test* scores, the June *Multiplication Facts Test* scores, the September *Multiplication Facts Test* scores, the June *Division Facts Test* scores, and the September *Division Facts Test* scores. All correlations were significant at the .05 level because the obtained value of r exceeded .217 for 80 df. The correlation coefficients relevant to this study are circled.

CHAPTER 5

DISCUSSION AND CONCLUSIONS

The purpose of this chapter is to state the limitations of this study, to discuss the statistical findings and to interpret the results. Implications of this study are presented and considerations for further research are offered. The chapter ends with a summary and conclusions.

Although the findings of this study may be statistically significant, further exploration of the data is necessary. Because of the large sample, the statistical procedures may reveal a significant difference even though this difference may not substantially affect a change in a student's behavior. For example, the difference between the mean June scores and the mean September scores may be statistically significant at a prescribed level of significance, but the actual difference in the mean scores may be so small that it would not affect the student's competency when the use of the basic facts is involved. A loss of this degree may be attributed to chance errors, and in the opinion of this investigator would not be considered meaningfully significant. Each hypothesis will be considered for its statistical, meaningful and educational inferences.

LIMITATIONS OF THE STUDY

Generalizations from the results of this study are limited by the testing instrument used. The test required the student to respond to each problem within a given time limit and did not provide any opportunity to attempt missed problems. Also, the test was presented audibly so that students were not given a visual representation of the problem thereby making the student totally dependent on his auditory skills. The format and organization of the test did not consider conceptual complexities of the various skills.

A further limitation was that other independent variables were not used. It may have been that listening skills, attention span or even reaction time in responding to each problem may be a contributing factor to a student's achievement.

DISCUSSION OF FINDINGS

In this section the findings will be discussed under four headings: Addition, Subtraction, Multiplication and Division.

Addition

Because students begin learning addition skills in grade one, studies by Brueckner[1] and Cook[2] showed that retention losses during the summer vacation by the end of grade two were slight. However, in this study the results of the paired t-test showed that significant differences between the mean June scores and mean September scores were found not only in grade three, but also in grade four. From the results of previous studies, it would be expected that significant losses in retention would not occur by the end of grade three and grade four. In comparing the results of previous studies the testing instrument that is used in gathering the data must be considered. The students in Brueckner's and Cook's studies were not expected to complete each item on the test within a given time interval. In this study a student was expected to answer each item on the test within the alloted time interval, and because the items were presented orally, the student was unable to go back and complete any of the missed items. Although it is quite possible that some students knew some of the facts they missed, they were prevented from scoring higher because of the time restriction. This may have been responsible for the significant

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[1]Brueckner, op. cit.

[2]Cook, op. cit.

difference in achievement especially at the grade three level.

The significant difference in achievement found at the grade four level needs to be examined further. The actual difference in the means between the June scores and September scores was 1.77. A difference of 1.77 represents a retention loss of approximately four per cent of the June addition facts. In a practical application of the addition facts, a loss of this degree would not significantly affect the efficiency of a student's computational skill in addition. Therefore, the loss at the grade four level cannot be considered meaningfully significant

Because the Addition Facts Test was the same for both the June and September sittings, a high correlation between the June scores and September scores could be expected. Surprisingly, the correlation matrices for each level revealed that the correlation coefficients in addition of .63 for grade three, .52 for grade four and .54 for grade five were the lowest of all the operations tested. The lower correlation coefficients imply that only thirty to forty per cent of the information needed for the perfect prediction of the September scores was known. Because the remaining sixty to seventy per cent remained unexplained, factors other than summer vacation might have influenced the September scores. The *Addition Facts Test* was the first test administered both in June and September. The unfamiliarity of the test procedures might have had an effect on the correlation coefficient at all grade levels, and thus could have lowered the correlation coefficient.

Significant difference in retention of the addition facts was experienced between students in grades three and five. This result was consistent with expectations because a grade five student would have the advantage of two years experience in using addition skills, and this would assist in the retention of the basic facts. Although significant differences in retention were found between grades three and four, and between grades four and five, the data showed that the actual differences in the adjusted mean scores were 1.85 and 1.57 respectively, representing retention differences of approximately four per cent between pairs of grade levels. This need not result in a grade three student performing educationally differently in addition skills from a grade four student, or a grade four student performing meaningfully differently from a grade five student. Therefore, the differences in retention between grades three and four, and between grades four and five were not meaningfully significant.

Subtraction

Statistically significant differences in achievement between the mean June scores and mean September scores were found at all three grade levels. However, the data shows that the actual mean differences between the June and September scores were .77 for the grade four students and .69 for the grade five students. These differences represent retention losses of less than two per cent of the June subtraction facts, or differences too small to have any significant effect on a student's computational skill in subtraction. The losses at the grade four and the grade five levels were not meaningfully significant.

The correlation coefficient as shown by the correlation matrices were higher for subtraction than addition for all levels. A correlation coefficient of .78 for the total sample indicates that sixty per cent of the information needed to make a perfect prediction of the September scores was known. This implies that summer vacation had more influence on the September scores than other factors. Therefore, the retention loss at the grade three level might have been, in all likelihood, affected by the summer vacation.

In determining retention differences between grade levels, significant differences were found between grades three and five, and between grades three and four. But the difference of 2.47 in the adjusted mean scores between grades three and five, and a difference of 2.54 between grades three and four signify that there was less than five per cent difference in the adjusted scores between the two grade levels. A difference of this degree would not substantially favor one grade level over the other in solving subtraction questions. Although statistical differences were found between grade levels, for educational relevancy there are no meaningful differences in retention between any of the grade levels.

Observation of the data shows that the adjusted mean score of 50.42 for grade four was slightly higher than the adjusted mean score of 50.35 for the grade five student. As shown by the subtraction content in the curriculum, it is expected that grade five students will have attained mastery of the subtraction process, and that less time is allotted for this skill. The higher mean score at the grade four level is the reflection of the degree of emphasis placed on subtraction in the curriculum.

Multiplication

Significant differences in achievement between the mean June scores and the mean September scores occurred at all grade levels. Correlation coefficients of .74 for grade three and .72 for grade four were obtained. These correlation coefficients imply that more than fifty per cent of the information needed for the perfect prediction of the September scores was known, whereas less than fifty per cent remained unexplained and may be attributed to other factors. Therefore, summer vacation appears to have influenced the differences in achievement more than other factors.

For the grade five students, the actual difference between the means of the June and September scores was 1.27, or a retention loss of about three per cent of the June scores. A loss of this degree would not have detrimental effect on a student's ability to compute multiplication questions. Although the difference was statistically significant, for its practical application, the difference is not meaningfully significant.

It should be noted that the mean June achievement score was 49.14 for grade three students as compared to 47.75 for the grade four students. Multiplication is usually introduced in grade three, and much emphasis is placed upon learning the basic facts during the latter part of the school term. It is thus understandable that the June scores for the grade three students were higher than the scores obtained by the grade four students. This margin of 1.39 was short-lived, as the mean score of 39.95 for the grade three students in September was considerably lower than the mean score of 44.66 for the grade four students. This resulted in a difference of 9.19 for grade three as compared to 3.09 for grade four. The concentrated practice on learning the multiplication facts at the end of grade three was not sufficient for students to retain the facts. Meanwhile, students in grade four derived practice by applying the basic facts to other multiplication processes, and as a result retained more facts than the grade three students. The results imply that retention may be more effective when facts are applied rather than just memorized or that the facts may take longer to assimilate.

As was mentioned, the basic multiplication facts are learned in grade three, while in grades four and five emphasis is placed on the application of the basic facts in developing multiplication processes. The curriculum allots greater time for multiplication in grades four and five than in grade three with the result that significant differences in retention were found between grades three and four, and between grades three and five. The difference in retention between grades four and five was not meaningfully significant, and might imply that the most effective learning period for multiplication facts is in grade four in our present curriculum structure.

Division

As in multiplication, significant differences in achievement between the mean June scores and mean September scores for the Division Facts Test were found for students in grade three and grade four. The correlation coefficients of .71 for grade three and .78 for grade four imply that more than fifty per cent of the variance in the September scores may be attributed to the June scores. Thus, differences in achievement appear to have been influenced more by summer vacation than by other factors.

As in multiplication, the grade three students achieved a higher mean score in June on the Division Facts Test than the grade four students. The mean score was 45.47 for the grade three students as compared to 44.86 for the grade four students. However, in September the mean score of 38.44 for grade three and 40.72 for grade four resulted in mean differences of 7.03 and 4.14 respectively. Like the multiplication facts, division is usually introduced in grade three, and receives concentrated attention near the end of the school term. This effort produces a short-lived retention as shown by the large drop evidenced by the grade three students. The grade four students, on the other hand, experience more practice in using the basic facts to solve easier division questions. The application of the facts in solving computational questions in division seems to assist students in mastering the facts so that the loss during the summer vacation is less than the loss experienced by grade three students.

The difference in retention between grade levels occurred between all pairs of grade levels. Unlike multiplication, the least difference in the adjusted mean scores was found between grades three and four. This suggests that greater growth of division facts for retention occurs at the grade five level rather than the grade four level. Also, this might imply that the understanding of the division process may be more complex than the multiplication process, and that the mastering of division facts is dependent on the multiplication facts.

GENERAL COMMENTS

Although the hypotheses were tested under specific headings, many relational trends were also explored. Figure 4 shows a graph of the adjusted mean scores for addition, subtraction, multiplication and division. As was expected, the data shows that as the grade level increases, the achievement level or the number of basic facts that a student has learned increases. Furthermore, the graph in Figure 5 shows the difference scores between the mean June scores and mean September scores for the four skill areas which indicates that the degree of retention increases as the grade level increases.

Also, the data reveals that a trend exists in the adjusted mean scores for the different skills. Figure 6 shows in a graph that the adjusted mean scores decrease from addition to subtraction to multiplication and to division for all grade levels. Generally, in grade one, the learning of addition is followed by the learning of subtraction because the two skills are interrelated. However, the difficulty of the process must be considered. The concept of addition is based upon a single model, the union of disjoint sets; subtraction is based upon three models, the missing addend, the removal of a subset from a given set and the difference between two sets. In

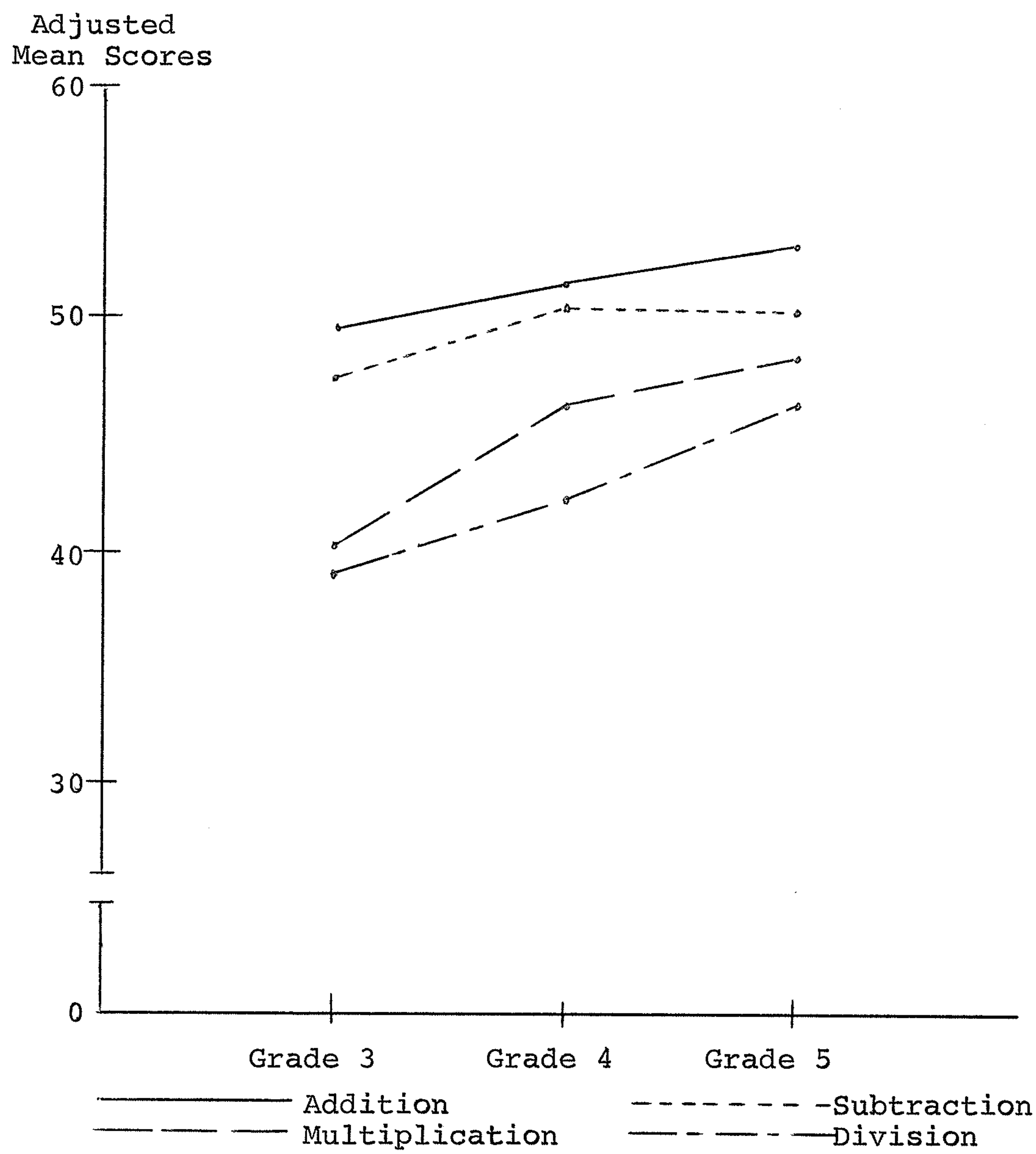

Figure 4 The Adjusted Mean Scores for Addition, Subtraction, Multiplication and Division Facts by Grade Levels

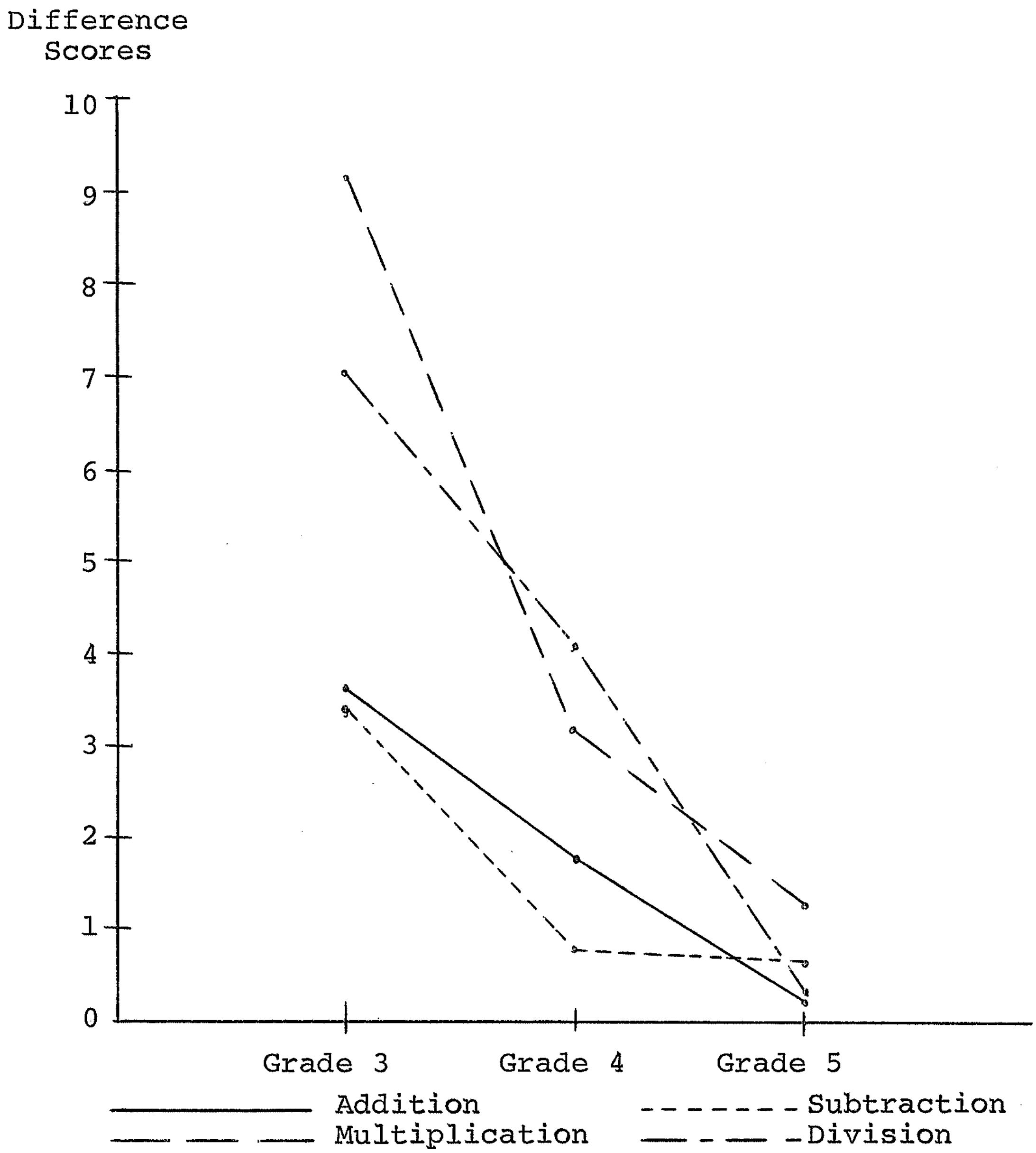

Figure 5 The Difference Scores between the Mean June Scores and the Mean September Scores for Addition, Subtraction, Multiplication and Division Facts Tests by Grade Levels

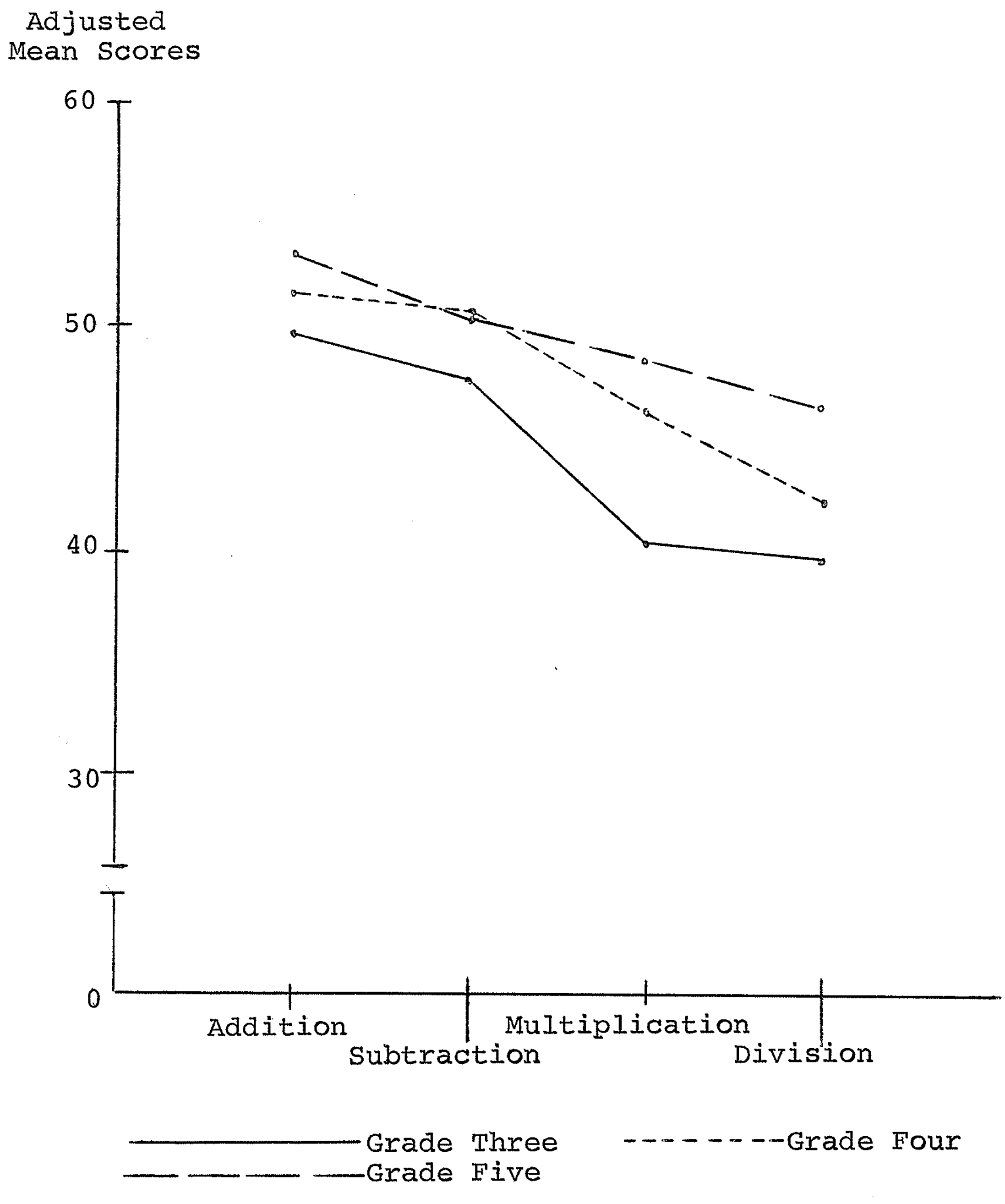

Figure 6 The Basic Facts Adjusted Mean Scores for Grade Three, Grade Four and Grade Five

considering the complexity of the two skills, it was expected and was shown by this study that students achieve a higher mean score in addition.

Similar to addition, the concept of multiplication is based upon a single model, the union of equivalent sets. On the other hand, division is based upon two models: the quotitive and the partitive. In the quotitive model, the student finds the number of equivalent subsets in a given set whereas in the partitive model, the student finds the number of elements in each equivalent subset of a given set. Thus, the complexity of the division concept is reflected in lower achievement scores.

Because multiplication is related to addition, and division is related to subtraction, both multiplication and division are introduced to the students after learning addition and subtraction facts, usually in grade three. The time difference in the introduction of the different skills has an effect on the achievement scores. The advantage of two years' experience in addition and subtraction is a benefit to the student when comparing the achievement scores in addition, subtraction, multiplication and division. This is reflected in the adjusted mean scores. Therefore, the differences in the adjusted mean scores may be attributed to the complexity of learning a skill and to the experience a student has with a skill.

An interesting observation is the high correlation coefficient for all grade levels between the Multiplication Facts Test scores and the Division Facts Test scores for both the June and September sittings. The correlation coefficients between the June multiplication and division scores were .88 for grade three, .84 for grade four and .79 for grade five whereas the September scores showed correlation coefficients of .83 for grade three, .71 for grade four and .74 for grade five. Because multiplication and division tests were administered in the same sitting, the interrelatedness of the two skills may have contributed to the high correlation coefficients. This suggests that a strong relationship exists between multiplication and division scores, and that the knowledge of one skill assists the other in the tests.

A high correlation coefficient would be expected between the addition test scores and subtraction test scores because the two skills are also interrelated and both tests were administered at the same sitting. The correlation coefficients between the June addition and subtraction test scores were .63 for grade three, .53 for grade four and .41 for grade five. This represents a weak relationship between the two test scores. However, as was stated earlier, the addition facts test was the first test administered in this study. Because the procedure and the test were new to the students, this might have affected the correlation coefficients. The correlation coefficients for September were somewhat higher with .71 for grade three, .70 for grade four and .63 for grade five. However, this does not compare to results for multiplication and division. Again, these tests were the first administered after the return to school in September. The adjustment to school routines after the summer vacation might have affected the correlation coefficients.

A further observation of the data reveals that there is an overlap of the achievement scores among the grade levels in the various tests. Using the results of the June and September Multiplication Facts Test, the graphs in Figure 7 and Figure 8 show the overlap in the test scores for students in grade three, grade four and grade five. The data for the graphs are found in Appendix G and Appendix H respectively.

Figure 7 shows that .56 of the students in grade three, .45 in grade four and .73 in grade five had scores in the 51 to 55 range for the June multiplication test. Similar results were found for scores in other ranges.

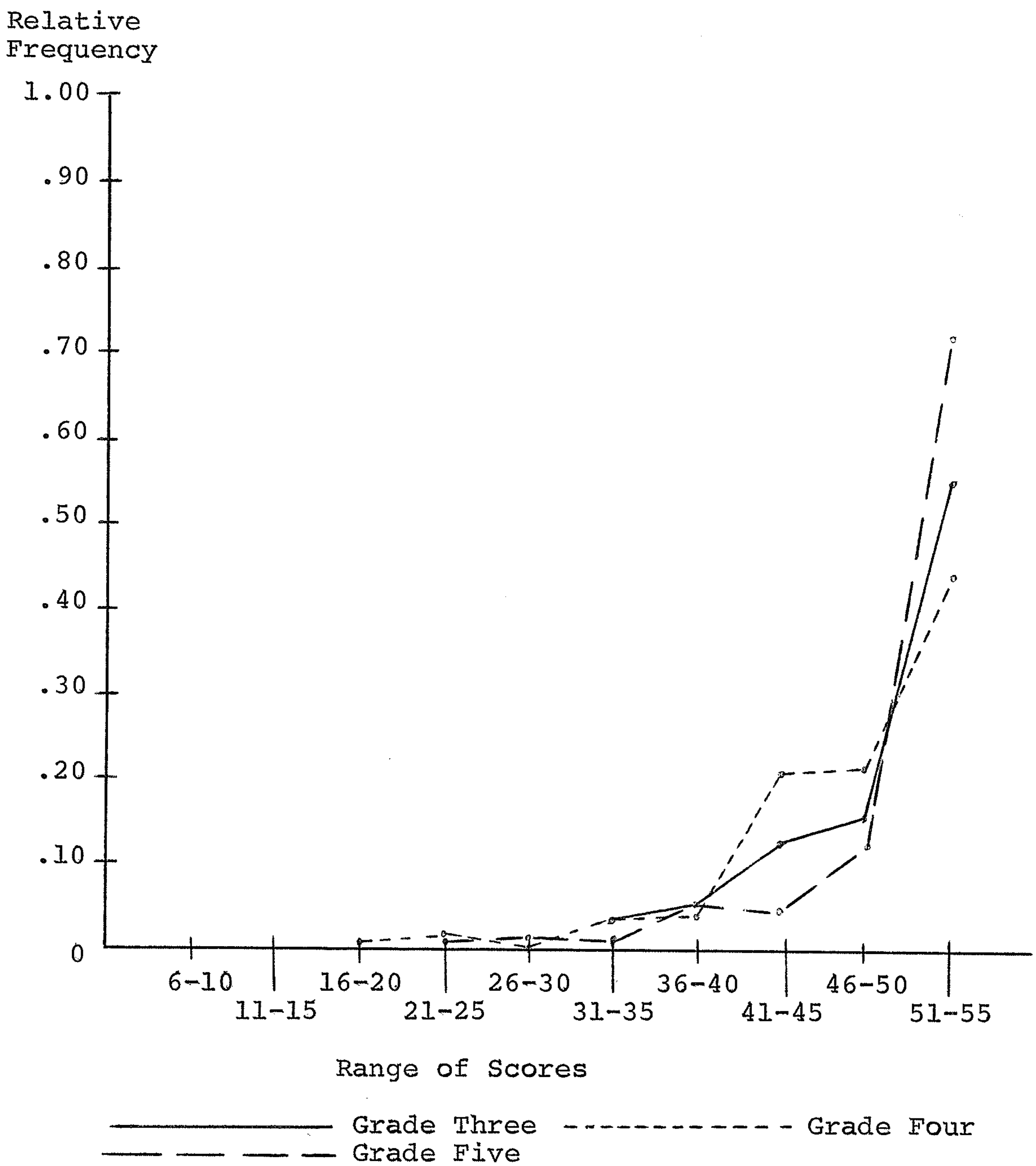

Figure 7 Relative Frequency Polygon Showing the June Multiplication Facts Test Scores for Grade Three, Grade Four and Grade Five

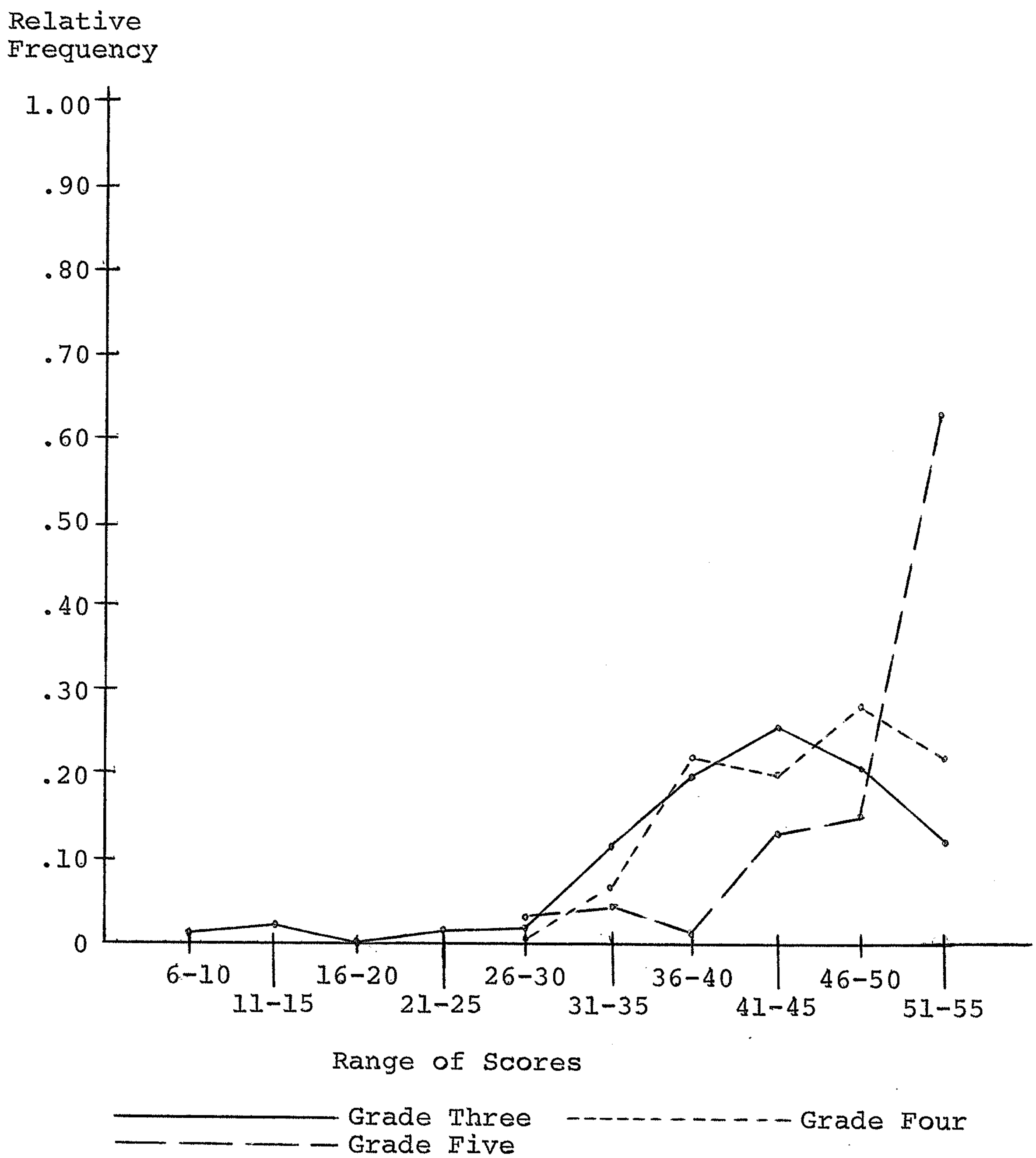

Figure 8 Relative Frequency Polygon Showing the September Multiplication Facts Test Scores for Grade Three, Grade Four and Grade Five

Also, Figure 8 shows that there is an overlap of the scores among the grade levels for the various ranges of scores. This suggests that there are students at each grade level capable of handling multiplication facts to the same level of competency. A more effective approach of teaching mathematical skills might be to group students according to their mathematical abilities, and thereby reduce the large range of abilities found in a graded structure.

Also, the graph in Figure 7 shows that the greatest number of the students in each grade level for June was in the 51 to 55 range. However, in Figure 8 the peak representing the greatest number of students had shifted to a lower range of scores for both the grade three and grade four students whereas the peak for the grade five students remained stationary. This suggests that the students in grade three and grade four forgot more of the multiplication facts than the students in grade five.

Finally, the graph in Figure 7 shows that the range in scores of the difference between the highest and lowest scores was the least for the grade three in June. It appears that a more intense concentration on the multiplication facts during the latter part of the school term might have limited the range whereas in other grades emphasis was likely placed upon other skills. However, in

September the range in scores for the grade three was the largest among the grade levels. It suggests that the grade three students were probably more affected by the summer vacation than the other grades, and thus a larger range in scores may have resulted.

IMPLICATIONS AND FURTHER CONSIDERATIONS

Although summer vacation affected the retention of the basic facts at the grade three level, and to some degree at the grade four level, the study shows that grade five students retain the learned basic facts for all skills. As expected the retention differences for all tests except subtraction were significant between grades three and five, while the least differences in retention occurred between grades four and five for all tests except division. Several implications for elementary teachers of mathematics arise from these findings.

The study suggests that a steady growth in the learning of the basic facts occurs as the grade level increases. With the increase in grade levels, there is a trend towards a greater level of achievement of the tests as well as greater retention during the summer vacation. This implies that growth may be enhanced through applying the basic facts in solving computational questions, rather than mastering the facts and then learning the computational processes. Learning the basic facts through application seems more relevant to students and will probably be more easily retained.

Also, the study suggests that the difference scores or the retention levels improve as the grade level increases for all basic facts tests irregardless of the achievement level. This trend is shown by a graph in Figure 6.

According to Piaget[3], the child's perceptual development may be identified by four distinct stages; the sensory-motor stage, pre-operational stage, the concrete operations stage, and the formal operations stage. It is not until the ages of eleven or twelve that an individual enters the formal operations stage, a stage in which a child can think in abstract terms. This is the stage that a student may begin to adequately perform mental operations.

Because the test used in this study presented the basic facts abstractly - the student had to perform mental operations within a given time limit - the intellectual development of the student would have an affect on the retention of the basic facts. The results of this study showed that grade five students (ages ten and eleven) displayed a high retention level, and that retention level

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[3]Bärbel Inhelder and John Piaget, The Growth of Logical Thinking, (New York: Basic Books Inc., 1958).

increased with grade level and age. The implication of this would suggest to teachers that the mental process involved in learning the basic facts is not fully developed by grade three and even grade four, and would account for the higher difference scores. These students, according to Piaget, are in the concrete stage. Because their ability to think abstractly is not fully developed, the teacher, at this stage, should make the greatest use of manipulative or concrete materials.

A further implication is that teachers should not assume that what a student knows in June is what he will know in September. The data shows that there is a substantial drop in achievement for the grade three students in both multiplication and division facts tests from the June scores to the September scores. Nelson[4] recommends that a teacher-made diagnostic test be administered at the beginning of each school year to assess the student's mathematical strengths and weaknesses, and to determine the nature and extent of remedial work. This would give the teacher some criteria to group students for continuous development, and to provide a more effective approach to meeting individual needs. Attempting to teach

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[4] Nelson, op. cit.

more advanced skills on a poor basic facts foundation will lead to confusion.

Although the grade five students retained the learned basic facts during the summer vacation, the levels of achievement for multiplication and division were not comparable to that of addition and subtraction. As was indicated before, the knowledge of the basic facts improves as students gain more experience in utilizing the facts to solve other computational questions. A diagnostic test may reveal which basic facts are weak and programs may be devised to strengthen these facts.

In future studies the nature of the testing instrument must be more closely analyzed. In this study the basic facts were presented audibly to the students with an uniform time interval between each item, and only answers were recorded. This did not allow students a second opportunity to answer any missed items or allow some students sufficient time to answer all the questions.

Another factor that was not considered in this study was the differences in conceptual complexities related to the different processes. Addition, as was mentioned previously, is based upon a single model, and may be represented by the equation forms:

$$3 + 2 = \square \qquad\qquad 2 + 3 = \square$$

A student would respond by finding the sum of 3 and 2. With the knowledge of the commutative property of addition, a student would realize that the second equation form would have the same sum as the first form.

However, subtraction, which is based on three models, may be represented by different equation forms for the same set of numerals. Some of these forms are:

$2 + \square = 5 \qquad 5 - 3 = \square$

$\square + 3 = 5 \qquad 5 - 2 = \square$

$5 - \square = 3 \qquad \square - 2 = 3$

$5 - \square = 2 \qquad \square - 3 = 2$

The complexities of the subtraction equations require greater mental flexibility, as is shown by the variety of locations that a missing numeral may be placed. The testing instrument in this study utilized only the simplest of the subtraction equation forms:

$5 - 3 = \square \qquad 5 - 2 = \square$

Thus, to measure a conceptual understanding of subtraction facts a testing device using the various forms of the subtraction equation should be included. Such a test would allow for greater differences in retention between grade levels.

Multiplication, like addition, is based upon a single model and has two equation forms:

$$2 \times 3 = \square \qquad 3 \times 2 = \square$$

The response to both equations is the same because of the commutative property in multiplication.

Division may be represented by many different equation forms for the same set of numerals. These forms are:

$2 \times \square = 6$	$6 \div 3 = \square$	$3\overline{)6}$
$\square \times 3 = 6$	$6 \div 2 = \square$	$2\overline{)6}$
$6 \div \square = 2$	$\square \div 2 = 3$	$\frac{6}{2} = \square$
$6 \div \square = 3$	$\square \div 3 = 2$	$\frac{6}{3} = \square$

Because the various equations require greater flexibility in the mental process, division is more complex than multiplication. In order to assess the conceptual understanding in division facts, the various equations should be represented in the testing instrument.

The testing instrument used in this study was limited in scope and did not evaluate the conceptual understanding of subtraction and division facts. In future studies, the differences in the complexities of the processes should be considered.

Another method of measuring the retention of the basic mathematical facts would be through the application of these facts involving more complex computational questions. Specific facts may be isolated from the test problems without knowledge of the students and providing a score reflecting their knowledge of the facts. This would enable the investigator to determine the effects of summer vacation under more realistic conditions. Although there may be more difficulty in developing an adequate measuring device, the results would be more relevant to educators where mastery of computational skills in addition, subtraction, multiplication and division are dependent on the application of these facts.

Although differences in retention between grade levels were compared in this study, other factors could have been considered. Studies similar to Patterson[5] and Kramer[6] in which students were grouped according to ability or intelligence would be relevant and useful. Such groupings may be determined by mathematical achievement tests, intelligence tests or a combination of both tests. Groupings related to sex differences as in the study by Parsley and Powell[7] would be another possibility. Future

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[5]Patterson, op. cit.

[6]Kramer, op. cit.

[7]Parsley and Powell, op. cit.

studies involving ability groups, intelligence groups and sex differences would provide additional information into the effects of summer vacation on the basic mathematical facts. A Factorial Analysis of Covariance design may be used to analyze several variables.

In conclusion, there are many facets of mathematical development that need to be explored and re-explored. The mastery of the basic mathematical facts is only a small portion of mathematical development, and yet essential for the growth of other related skills. Further studies with greater emphases in the testing procedures, instruments and grouping arrangements would add to the information generated by this study. Thus, further research is needed before definitive statements can be made about the effects of summer vacation on the basic mathematical facts and ways to prevent loss during the summer by adjusting curriculum content and methods of presentation.

SUMMARY AND CONCLUSIONS

The administration of the Basic Facts Tests in addition, subtraction, multiplication and division in June, 1973 and September, 1973 enabled this investigator to determine the effects of summer vacation on the retention of the basic mathematical facts.

The paired t-test was used to determine significance differences in achievement at all grade levels between the June scores and September scores for all tests. When the data was analyzed and found to be significantly different, the actual differences in the mean scores were considered in terms of its meaningful and realistic effects.

For addition and subtraction facts, differences were meaningfully significant only at the grade three level. For multiplication and division facts, differences were meaningfully significant at the grade three and grade four levels. However, students in grade five showed no meaningful or statistical difference in achievement between the June scores and September scores for any of the basic facts tests.

The analysis of covariance technique was used to determine whether differences in retention existed between grade levels. In regard to the analysis of covariance results, the null hypotheses for addition, subtraction, multiplication and division were rejected.

The aposteriori Scheffé test showed that differences in retention for addition facts were meaningfully significant between grades three and five only. For subtraction facts, meaningful differences were not found between any of the grade levels. Differences in retention for the multiplication facts were meaningfully significant between grades three and five, and between grades three and four, while differences were meaningfully significant between all pairs of grade levels for division facts.

Although this investigator deliberately studied the effects of the summer vacation for addition, subtraction, multiplication and division separately, the conclusions are presented in more generalized terms. The study found that achievement and retention of the basic facts during the summer vacation increased for all skills as the grade level increased. Also, the level of achievement and retention varied with each skill. Addition showed the highest achievement and retention levels followed by subtraction, multiplication and division. This result was consistent for all grade levels, and could be attributed to the experience of a student with a skill and to the varying conceptual complexities within a skill. Although loss in retention occurred at the grade three level, this declined as the students moved up in grades.

The statistical analysis of the data indicated that the retention of the basic mathematical facts were seriously affected by the summer vacation. However, assessing the educational impliclations of the data, the results were not always found to be relevant.

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APPENDICES

APPENDIX A

BASIC ADDITION FACTS TEST

<u>Easy</u>

1. 0 and 0
2. 1 and 3
3. 2 and 5
4. 1 and 8
5. 8 and 0
6. 1 and 6
7. 2 and 2
8. 4 and 3
9. 5 and 0
10. 2 and 1
11. 5 and 1
12. 3 and 5
13. 0 and 9
14. 2 and 6
15. 7 and 0
16. 4 and 1
17. 0 and 6
18. 7 and 2
19. 1 and 1
20. 2 and 3
21. 4 and 4
22. 0 and 1
23. 4 and 2
24. 7 and 1
25. 3 and 3
26. 0 and 4
27. 6 and 3
28. 2 and 0
29. 0 and 3
30. 4 and 5

<u>Hard</u>

1. 6 and 8
2. 7 and 5
3. 9 and 8
4. 7 and 4
5. 9 and 9
6. 8 and 2
7. 9 and 7
8. 5 and 8
9. 3 and 9
10. 8 and 8
11. 5 and 9
12. 7 and 6
13. 5 and 5
14. 9 and 6
15. 6 and 6
16. 9 and 1
17. 8 and 3
18. 4 and 6
19. 8 and 4
20. 5 and 6
21. 3 and 7
22. 4 and 9
23. 7 and 8
24. 2 and 9
25. 7 and 7

APPENDIX B

BASIC SUBTRACTION FACTS TEST

Easy

1. 3 from 9
2. 0 from 8
3. 7 from 7
4. 5 from 8
5. 9 from 9
6. 7 from 8
7. 1 from 7
8. 0 from 4
9. 2 from 8
10. 3 from 5
11. 5 from 6
12. 7 from 9
13. 2 from 7
14. 0 from 2
15. 1 from 5
16. 3 from 3
17. 2 from 6
18. 3 from 4
19. 5 from 5
20. 1 from 2
21. 4 from 8
22. 0 from 6
23. 1 from 1
24. 2 from 4
25. 0 from 0
26. 3 from 6
27. 5 from 9
28. 1 from 3
29. 4 from 7
30. 1 from 9

Hard

1. 7 from 11
2. 5 from 12
3. 9 from 17
4. 8 from 14
5. 1 from 10
6. 8 from 13
7. 6 from 15
8. 5 from 10
9. 3 from 11
10. 9 from 12
11. 4 from 13
12. 8 from 16
13. 7 from 14
14. 6 from 12
15. 2 from 10
16. 9 from 18
17. 6 from 11
18. 9 from 16
19. 6 from 10
20. 7 from 15
21. 5 from 14
22. 8 from 12
23. 6 from 13
24. 7 from 10
25. 9 from 11

APPENDIX C

BASIC MULTIPLICATION FACTS TEST

Easy

1. 3 twos
2. 0 sixes
3. 4 ones
4. 1 seven
5. 4 zeros
6. 2 sixes
7. 5 twos
8. 9 zeros
9. 2 ones
10. 1 five
11. 0 twos
12. 1 three
13. 6 ones
14. 1 zero
15. 2 fours
16. 3 zeros
17. 1 nine
18. 2 twos
19. 8 ones
20. 7 zeros
21. 2 sevens
22. 4 threes
23. 0 zeros
24. 3 fives
25. 1 one
26. 0 eights
27. 3 threes
28. 5 zeros

Hard

1. 2 eights
2. 7 threes
3. 4 fives
4. 5 nines
5. 3 eights
6. 5 sevens
7. 3 sixes
8. 4 sevens
9. 6 fours
10. 5 fives
11. 4 nines
12. 7 sevens
13. 6 fives
14. 9 threes
15. 7 sixes
16. 9 twos
17. 6 eights
18. 9 sixes
19. 8 fours
20. 9 sevens
21. 8 nines
22. 4 fours
23. 7 eights
24. 6 sixes
25. 8 fives
26. 9 nines
27. 8 eights

APPENDIX D

BASIC DIVISION FACTS TEST

<u>Easy</u>

1. threes in 3
2. fives in 15
3. nines in 0
4. ones in 1
5. twos in 8
6. twos in 6
7. ones in 0
8. sevens in 7
9. threes in 0
10. sixes in 0
11. twos in 10
12. sevens in 14
13. fives in 0
14. ones in 2
15. ones in 6
16. twos in 12
17. nines in 9
18. twos in 0
19. threes in 9
20. ones in 8
21. threes in 12
22. eights in 0
23. fives in 5
24. sevens in 0
25. twos in 4
26. fours in 0
27. ones in 4

<u>Hard</u>

1. fives in 20
2. sevens in 35
3. nines in 72
4. sevens in 49
5. nines in 81
6. eights in 64
7. nines in 36
8. threes in 21
9. eights in 48
10. fours in 32
11. eights in 16
12. fives in 30
13. eights in 56
14. nines in 45
15. sixes in 54
16. eights in 24
17. sixes in 42
18. sevens in 63
19. fives in 40
20. threes in 27
21. fours in 24
22. sevens in 28
23. sixes in 36
24. twos in 18
25. fours in 16
26. fives in 25
27. sixes in 18

APPENDIX E

Date ____________ Name ______________________________

Sitting __________

score	score	score	score	score
1 ______	1 ______	1 ______	1 ______	1 ______
2 ______	2 ______	2 ______	2 ______	2 ______
3 ______	3 ______	3 ______	3 ______	3 ______
4 ______	4 ______	4 ______	4 ______	4 ______
5 ______	5 ______	5 ______	5 ______	5 ______
6 ______	6 ______	6 ______	6 ______	6 ______
7 ______	7 ______	7 ______	7 ______	7 ______
8 ______	8 ______	8 ______	8 ______	8 ______
9 ______	9 ______	9 ______	9 ______	9 ______
10 ______	10 ______	10 ______	10 ______	10 ______
11 ______	11 ______	11 ______	11 ______	11 ______
12 ______	12 ______	12 ______	12 ______	12 ______
13 ______	13 ______	13 ______	13 ______	13 ______
14 ______	14 ______	14 ______	14 ______	14 ______
15 ______	15 ______	15 ______	15 ______	15 ______
16 ______	16 ______	16 ______	16 ______	16 ______
17 ______	17 ______	17 ______	17 ______	17 ______
18 ______	18 ______	18 ______	18 ______	18 ______
19 ______	19 ______	19 ______	19 ______	19 ______
20 ______	20 ______	20 ______	20 ______	20 ______
21 ______	21 ______	21 ______	21 ______	21 ______
22 ______	22 ______	22 ______	22 ______	22 ______
23 ______	23 ______	23 ______	23 ______	23 ______
24 ______	24 ______	24 ______	24 ______	24 ______
25 ______	25 ______	25 ______	25 ______	25 ______
26 ______	26 ______	26 ______	26 ______	26 ______
27 ______	27 ______	27 ______	27 ______	27 ______
28 ______	28 ______	28 ______	28 ______	28 ______
29 ______	29 ______	29 ______	29 ______	29 ______
30 ______	30 ______	30 ______	30 ______	30 ______

APPENDIX F

JUNE MULTIPLICATION TEST SCORES FOR GRADES THREE, FOUR AND FIVE

Range of Scores	Grade Three		Grade Four		Grade Five	
	Frequency	Relative frequency	Frequency	Relative frequency	Frequency	Relative frequency
51-55	49	.60	38	.45	74	.73
46-50	14	.17	19	.22	13	.13
41-45	11	.13	18	.21	5	.05
36-40	5	.06	4	.05	6	.06
31-35	3	.04	3	.04	1	.01
26-30					1	.01
21-25			2	.02	1	.01
16-20			1	.01		
11-15						
6-10						

APPENDIX G

SEPTEMBER MULTIPLICATION TEST SCORES FOR GRADES THREE, FOUR AND FIVE

Range of Scores	Grade Three		Grade Four		Grade Five	
	Frequency	Relative Frequency	Frequency	Relative Frequency	Frequency	Relative Frequency
51-55	10	.12	19	.22	64	.63
46-50	18	.21	23	.28	15	.15
41-45	23	.26	17	.20	13	.13
36-40	17	.20	19	.22	1	.01
31-35	10	.12	6	.07	5	.05
26-30	2	.02	1	.01	3	.03
21-25	2	.02				
16-20						
11-15	3	.03				
6-10	2	.02				