

ECONOMIC CONSIDERATIONS IN THE  
SELECTION OF ALTERNATIVE CROP AND  
LIVESTOCK ENTERPRISES



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by

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## CHAPTER I

### INTRODUCTION

There are three fundamental economic problems in society (1):

1. What commodities should be produced and in what quantities?
  2. How should goods be produced?
  3. For whom should goods be produced?
- What to produce, how, and for whom are problems if, and only if, resources are limited. Ends are numerous; means are scarce. In order to satisfy the numerous ends with the scarce means, choices must be made (2).

Production economics is concerned with choosing or decision-making in the use of the capital, labour, land and management resources in the farming industry (3). The goals of production economics are twofold: (1) to provide guidance to individual farmers in using their resources efficiently, and (2) to facilitate the efficient use of resources from the standpoint of the consuming economy (4, p. 3).

In agriculture, land, labour and capital are productive resources through which products, primary and secondary, are transformed. The farm firm is the production unit in agricultural industry (4, p. 28); its objectives are to maximize the profit and to minimize the cost. In the production process, a primary product (feed crops) is derived from productive resources and from this product a secondary product, livestock, is produced. An optimum combination of resources for such a firm demands that both the best crop program and the best livestock program be selected. Usually, each crop-rotation (primary product) includes several types of cash crops, feed grains and forage. Hogs, poultry and turkeys depend primarily on grains in the crop

rotation; dairy cattle, beef and sheep, on the other hand, depend on a crop-rotation which includes forages (5). The optimum condition of resource combination is achieved when the marginal rate of substitution for the primary product is equal to the marginal rate of substitution for the secondary product (4, pp. 260-262).

In order to illustrate the determination of an optimum resource combination, a specific farm with low economic efficiency in Central Manitoba was chosen for the present study (this specific farm will, hereafter, be called the studied farm). This farm is composed of 320 acres of land, consisting of the  $S\frac{1}{2}$ -36-4-4W in the Roland district, on mainly fine black loam soils of the Altona soil association.

Data are available for study based on (1) the studied farm record of business from 1957 to 1959 and (2) Annual Reports of the Carman District Farm Business Association for the same three years.

In this studied farm two situations are assumed:

1. The present situation: it is assumed that the owner of the farm will continue to operate his farm business.

2. The adjusted situation: it is assumed that the owner's son will take part in the farm business; a father-son arrangement will be made.

In this analysis, a general production function is used to deal with the relationship between products. In analyzing this relationship, quantities of resources are assumed to hold constant, while the competing products are varied with constant rates of substitution. The linear programming method is applied, which assumes a linear profit function subject to linear set of equations with

respect to the resource restriction.

The details of this study are included in the following chapters.

Chapter II will describe the specific objectives of this study.

Some economic theory and background relevant to this present study will be discussed in chapter III.

The methodology with respect to the logic and technique of linear programming methods will be presented in chapter IV.

A detailed analysis of the farm business used in the present study will be made in chapter V.

Chapter VI will deal with the alternative plans both in the present situation and in the father-son arrangement situation. These plans will be computed by means of the simplex and continuous methods of linear programming.

The final chapter will draw conclusions and suggest possibilities for the reorganization of the farm business.

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## CHAPTER II

### OBJECTIVES

A farm firm maximizes its net profit by allocating its limited resources in such a way that the ratio of marginal rate of substitution for the primary product divided by the marginal rate of substitution for the secondary product is equal to one. The main objective of this study is to illustrate the economic problems of integrating alternative hog and cattle enterprises. However, the specific purposes are stated as follows:

1. To compare the farm size (including the acreage of land, the quantity of capital and the total units of labour) of the studied farm with the group average.
2. To analyze the crop and livestock records for the studied farm with respect to expenses and income in order to find out its weaknesses.
3. To compare the total receipts, total cost, and other items of economic efficiency in the studied farm with the group average.
4. To apply the simplex and continuous method of linear programming to determine the optimum resource combinations.
5. To discuss the comparative advantages among the alternative plans.
6. To suggest how to reorganize the farm business in the studied farm.

A fundamental consideration in decision-making is the opportunity cost principle. This principle implies that a farmer should, if he

wishes to maximize his profit, allocate each unit of scarce resources in those enterprises yielding the greatest return.

In this studied farm there are ten alternative plans with ten competing enterprises and eight types of resources; the one plan which maximizes profits can be determined -- subject to the techniques considered, the supply level of resources available and the prices for the products and the resources -- by a mathematical procedure, linear programming. It allows the limitations of each resource to be considered in specifying the optimum plan.

Different optimum plans will result from different levels of resources supplied. If both labour and building space are available in abundant supply, the enterprises will compete for use of land and capital. However, if capital is unlimited, then the enterprises will compete for use of land, labour and building space. The highest profit combination of enterprises is dependent upon the marginal rate of substitution and the price ratio.

## CHAPTER III

### THEORETICAL BACKGROUND

#### I. PRODUCTION FUNCTION

A production function deals with the physical relationship between output and inputs (1). It indicates how much output we can obtain if we have so much labour and so much capital and so much land, etc. Thus, three primary production economic relationships are involved in this study: factor-product relationship, factor-factor relationship and product-product relationship (2, Chaps. 2-9).

##### 1. Input-output or factor-product relationship.

Input-output or factor-product relationship deals with the input of a resource and the resulting yield or output of a product. It may be expressed algebraically:

$$Y = f (x_1 | x_2 \ x_3 \ x_4 \ \dots \ x_n)$$

Where Y is denoted as output of the enterprises;  $x_1$  is the variable input and  $x_2 \ \dots \ x_n$  are fixed inputs. This equation means that Y is a function of  $x_1 \ x_2 \ x_3 \ x_4 \ \dots \ x_n$  but inputs  $x_2 \ x_3 \ x_4 \ \dots \ x_n$  are held constant in quantity, while only  $x_1$  can be varied in amount. The criteria for economic optimum of this relationship is:

$$\frac{dY}{dx_1} = \frac{P_{x_1}}{P_Y}$$

## 2. Input-input or factor-factor relationship.

The input-input or factor-factor relationship considers a constant output which can be produced by different combinations of two or more variable factors. It may be expressed algebraically:

$$Y = f(x_1, x_2, x_3, x_4, \dots, x_n)$$

where Y is denoted as output or product;  $x_1, x_2$  are two variable factors. This equation tells us that Y or output is dependent upon two variable factors,  $x_1$  and  $x_2$  and other fixed factors,  $x_3, x_4, \dots, x_n$ . A change in production of Y results as either or both factors are varied. A particular concern is with the possibilities of substituting  $x_1$  for  $x_2$ , as Y is held constant at a particular level. Under this relationship factors can be substituted for each other until they reach an equilibrium point of least-cost combination of resources; that is:

$$\frac{dx_2}{dx_1} = \frac{P_{x_1}}{P_{x_2}}$$

where  $P_{x_1}$  and  $P_{x_2}$  are denoted as the market prices of resources.

The economic criteria requires that the marginal rate of substitution between resources be equal to their price ratio.

## 3. Output-output or product-product relationship (3, pp. 3-10).

The output-output or product-product relationship refers to the relationship between two or more commodities or enterprises competing for a given amount of resources. It may be expressed algebraically:

$$(Y_1, Y_2, \dots, Y_n) = f(x_1, x_2, x_3, \dots, x_n)$$



Where  $Y_1, Y_2, \dots, Y_n$  are different outputs or products,  
 $x_1, x_2, x_3, \dots, x_n$  are given inputs of resources. Choice is now  
 among many competing products in a manner paralleling selection  
 among factors. The given resources will be allocated among the  
 competing enterprises so as to maximize profit, when

$$\frac{dY_2}{dY_1} = \frac{P_{Y_1}}{P_{Y_2}}$$

## II. CONSTANT RATE OF SUBSTITUTION BETWEEN COMPETITIVE PRODUCTS

Competitive enterprises are those which compete for use of  
 the farmer's limited resources. Use of resources to produce more  
 of one product necessitates a sacrifice in the amount of the other  
 produced. Usually, the competitive products take the following  
 two forms: (1) the products are competitive in the short run, and  
 (2) the products are competitive in the long run. The former is  
 assumed to substitute at a constant rate of substitution, while the  
 latter is assumed to exhibit either an increasing or decreasing rate  
 of substitution (4).

Figure I shows that a given resource  $x_1$  can produce different  
 quantities of two competitive products  $Y_1$  and  $Y_2$ . In figure II,  
 two enterprises  $Y_1$  and  $Y_2$  substitute for each other at a constant  
 rate (an increase in one enterprise necessitates a constant unit  
 of the other sacrificed). Any two competitive products can be sub-  
 stituted for each other but the marginal rate of substitution between

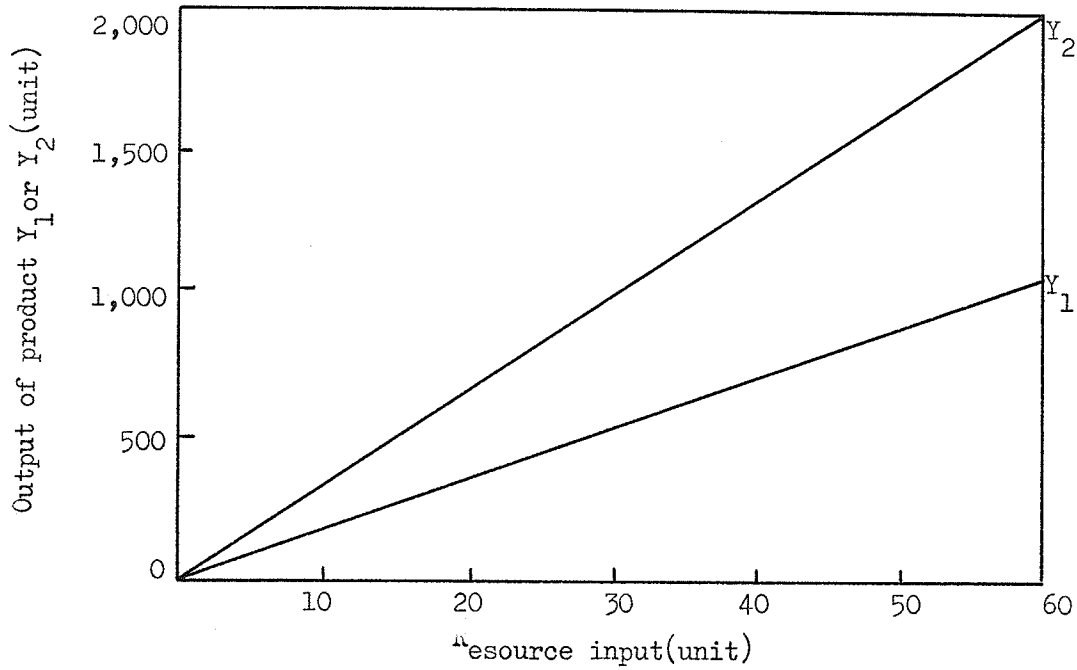


FIG. 1. PRODUCTION FUNCTION

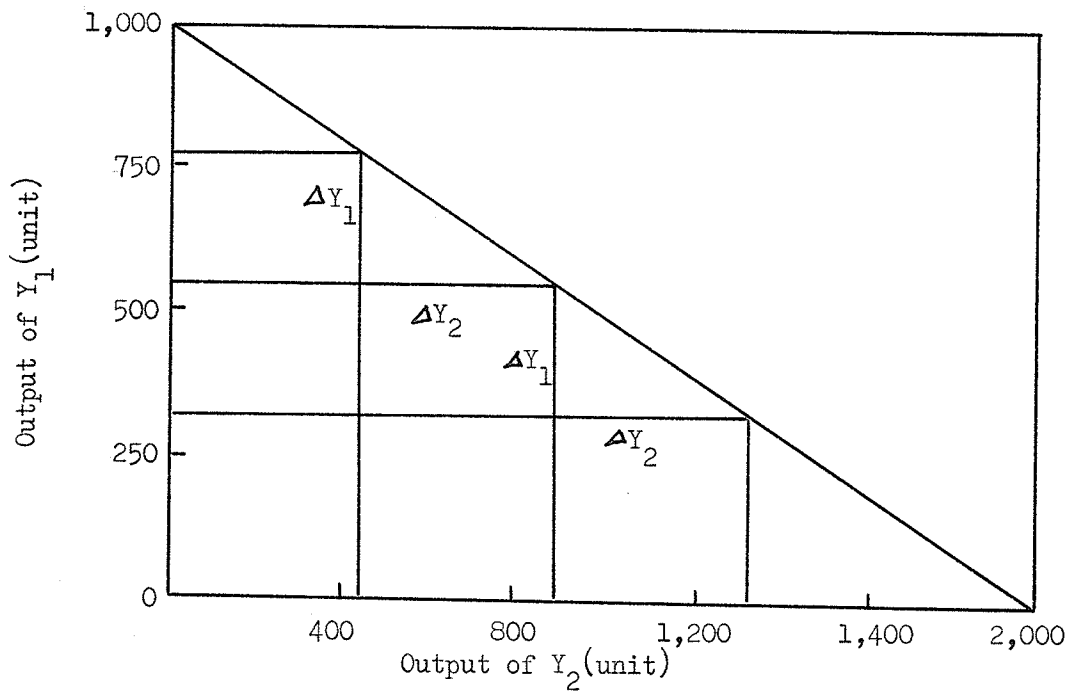


FIG. 1. LINEAR TRANSFORMATION FUNCTION

them is constant (2, pp. 204-221).

### III. THE CRITERIA OF ECONOMIC EFFICIENCY FOR THE FARM FIRM IN EQUILIBRIUM

Linear programming is used to develop alternative plans based on different assumptions with respect to rotations, livestock enterprises, rations, prices and available resources. Under linear programming, the condition of optimum condition of resources is at the point of maximizing profit and minimizing cost. It has been stated that  $dx_2/dx_1 = P_{x_1}/P_{x_2}$  is regarded as the criteria for minimizing cost for a given level of output and  $dY_2/dY_1 = P_{Y_1}/P_{Y_2}$  as the criteria for maximizing profit for a given quantity of resources. If the limited resources such as land, capital and labour are used to produce primary products such as wheat, oats, barley and hay with which to feed the livestock such as sheep, hogs and cattle, then the problems will be: (1) what pattern of primary production will allow a maximum output of the secondary product? (2) what quantity of primary product shall be sold or purchased, if returns through the secondary product are to be maximized? These problems can be solved in terms of the following criterion of economic efficiency.

$$\left( \frac{dx_2}{dx_1} = \frac{P_{x_1}}{P_{x_2}} \right) = \left( \frac{dY_2}{dY_1} = \frac{P_{Y_1}}{P_{Y_2}} \right)$$

This criterion equation indicates that the equilibrium (least-cost) point in combining the resources must be coincident with the equilibrium (maximum profit) point in combining the enterprises. That is, the marginal rate of substitution between resources must equal their

price ratio at the same time as the marginal rate of substitution between products equals that price ratio (3, pp. 11-16).

Figure III shows the interrelationship among resources, primary products and secondary products with a constant rate of substitution. GF represents a production transformation curve which comes from the given resources land, capital and labour. CC' is an iso-revenue line for the primary product or an iso-cost line for the secondary product.  $G_1'F_1'$ ,  $G_2'F_2'$ , are denoted as different levels of secondary products (iso-quant). At  $Y_1$  of output  $G_1'F_1'$  intersects the transformation curve at two points. At point  $R_1$ , the slope of  $G_1'F_1'$  is greater than that of GF. Accordingly,  $\Delta G/\Delta F$ , the marginal rate of substitution of forage for grain in the crop rotation, is less than  $\Delta G'/\Delta F'$ , the marginal rate of substitution of forage for grain in the livestock ration. Adjustment is needed to substitute forage for grain in both the crop rotation and the livestock ration. At point R, the situation is opposite to the case mentioned above. Adjustment is necessary to substitute grain for forage in the crop rotation and livestock ration. For iso-quant  $G_2'F_2'$  the production transformation curve is tangent to iso-revenue line and to the iso-quant curve at point E. At this tangent point, the slopes of these three curves are identical and  $\Delta G/\Delta F$ , the marginal rate of feed substitution in the crop rotation, is exactly equal to  $\Delta G'/\Delta F'$ , the marginal rate of feed substitution in the livestock ration. No adjustment is needed because output of the secondary product is at a maximum condition from given resources

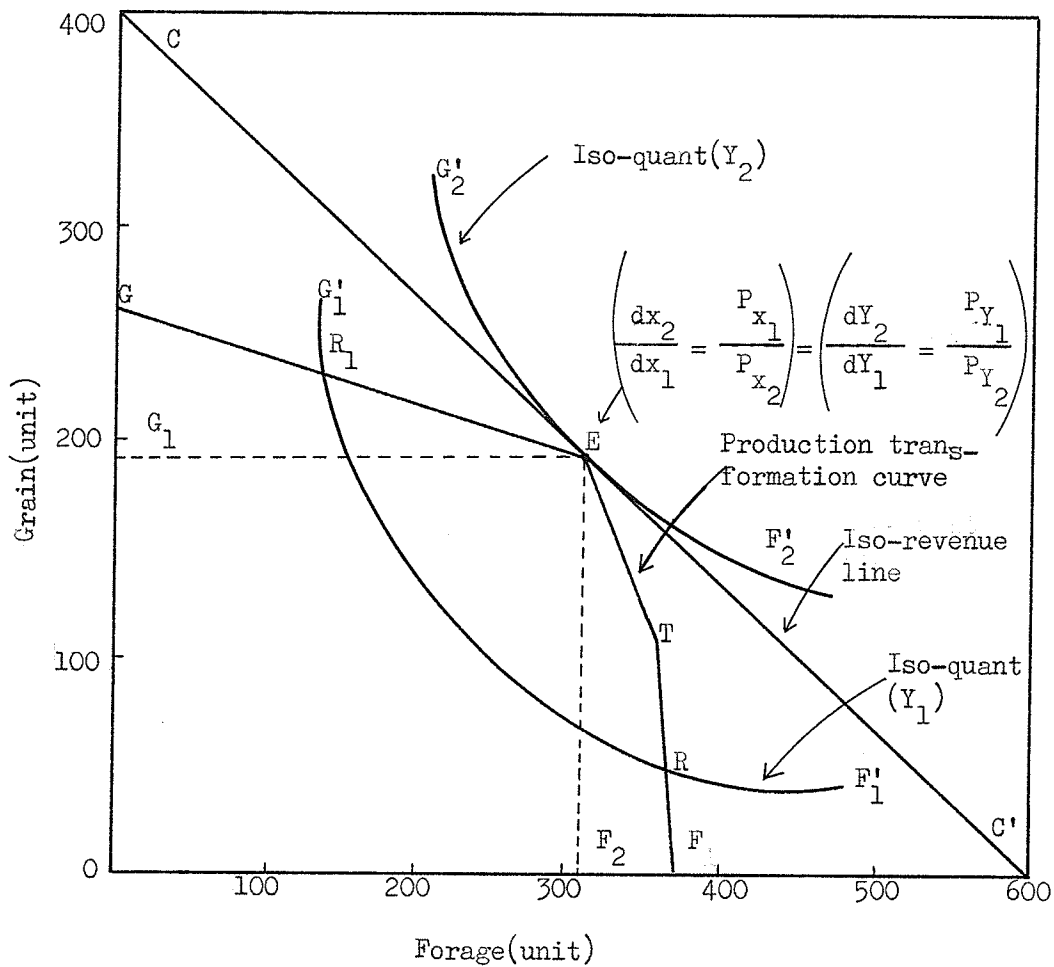


FIG. 3. INTERRELATIONSHIPS AMONG RESOURCES, PRIMARY PRODUCTS AND SECONDARY PRODUCTS, WITH CONSTANT RATE OF SUBSTITUTION (FARM FIRM EQUILIBRIUM)

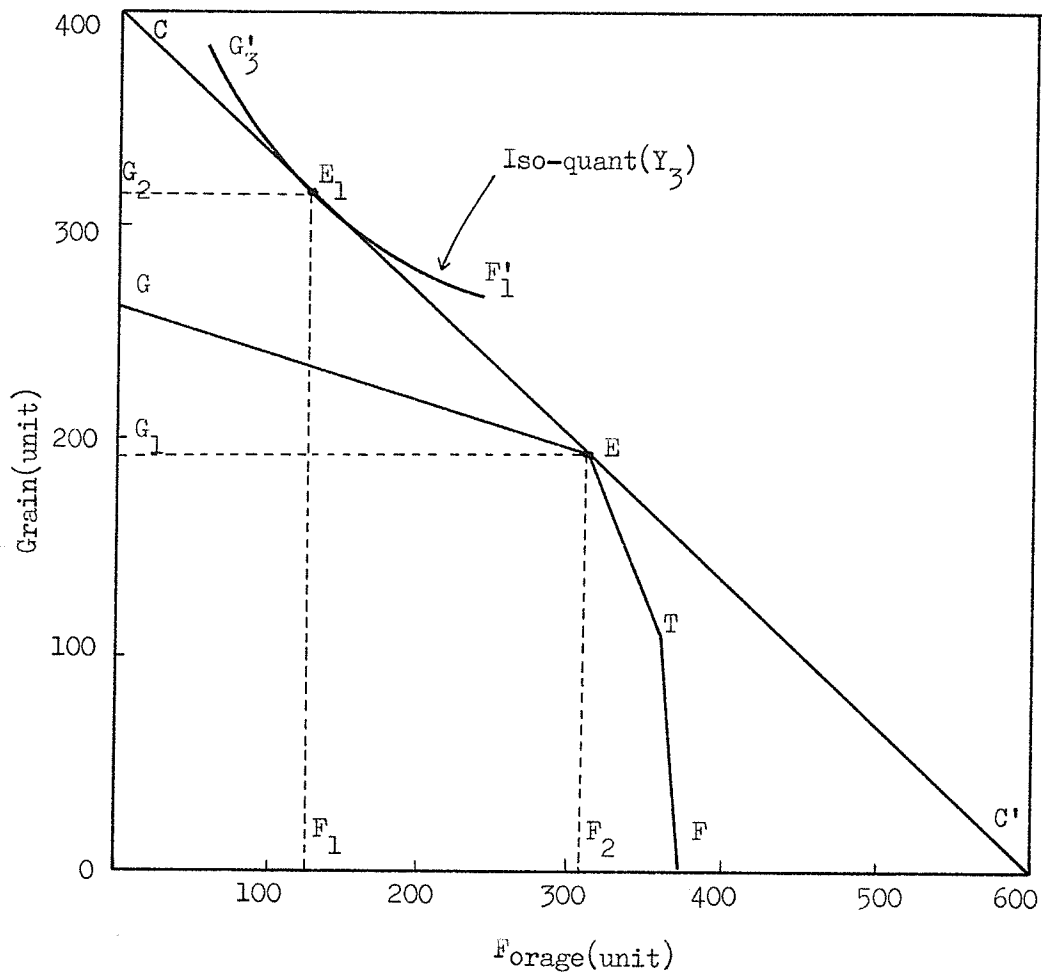


FIG. 4. INTERRELATIONSHIPS AMONG RESOURCES, PRIMARY PRODUCTS AND SECONDARY PRODUCTS ALLOWING BUYING AND SELLING GRAIN AND FORAGE

devoted to primary products (2, p. 260). Again, at point E, it represents the conditions of maximizing profit and minimizing cost, or  $(dx_2/dx_1 = P_{x_1}/P_{x_2}) = (dY_2/dY_1 = P_{Y_1}/P_{Y_2})$ , since the primary product is the input of secondary product and the iso-revenue of primary product is the iso-cost of secondary product.

Figure 4 shows the interrelationship among resources, primary products and secondary products, allowing buying and selling activities in the program.  $G_3^1 F_3^1$  is the highest livestock output (secondary product) attainable from the given cost outlay represented by  $CC'$ . As shown in Figure 4, the iso-revenue and the iso-cost lines are identical.  $E_1$  indicates another equilibrium point obtained by selling  $F_1 F_2$  units of forage and purchasing  $G_2 G_1$  units of grain.

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## CHAPTER IV

### LOGIC AND TECHNIQUE OF LINEAR PROGRAMMING

#### I. LOGIC

Linear programming assumes that the production coefficients are constant (i.e., the input-output curve or production function is linear) (1).

(A) Problems where this programming method applies.

A linear programming problem has three quantitative components: an objective, alternative activities or processes for obtaining the objective, and resource or other restrictions.

(1) The objective: The objective of linear programming is to allocate one's resources up to a point of maximizing profit and minimizing cost. The linear profit function is expressed as follows (2, p. 414):

$$Z = \sum_{i=1}^n \sum_{j=1}^k X_i C_j$$

where Z denotes maximum profit, C the unit of price, X the level of activities. This equation is subject to a linear set of restrictions; that is:

$$P X \leq S$$

where P indicates the resource requirement, S the resource restriction.

(2) Alternative activities or processes: Oats, wheat, barley, crop-rotations, hogs, beef-cattle or poultry are regarded

as enterprises or activities. In linear programming, one activity or process can be distinguished from another on the basis of the production coefficients. Given several activities or process -- different enterprises and different methods or techniques -- by which the product can be attained, we choose from them the methods or processes which are most efficient in converting resources into the product.

(3) Resources restriction: For a producing firm, restrictions are defined by the fixed quantities of certain resources. The acres of land, the dollars of capital, the hours of labour, and the square feet of building space are restricted by the available quantities and numbers. If they all are free goods, there will be no problems in linear programming.

(B) Important concepts in linear programming.

(1) Linear relationship: The term linear refers to the fact that "straight line" relationships are assumed in linear programming. For example, a linear relationship is reflected in the equation  $Z = 2x$ . The variable  $Z$  increases in direct proportion to the magnitude of the variable  $x$ . The equation  $Z = .5x^2$  is not a linear relationship (2, p. 4), as the magnitude of  $Z$  does not increase in proportion to the magnitude of  $x$ . Thus, straight line relationships, equations with variables in the first power, are the type employed in linear programming. The three economic linear relationships with which we are concerned in linear programming are described as follows (3):

The linear factor-product relationship is expressed as a linear

function  $Y = f(x)$ . This relationship shows that output,  $Y$ , is constantly proportional to input,  $x$ ; doubling the input will double the output or trebling the input will treble the output and so on. There is a constant production coefficient.

The linear factor-factor relationship: In linear programming the factor-factor relationship is a linear function,  $x_1 = f(x_2)$ . This relationship shows that the two factors have a constant rate of substitution. If  $x_1$  is increased by one unit,  $x_2$  will be decreased by a constant amount; if  $x_1$  is increased by two units,  $x_2$  will be decreased twice as much as before. Both  $x_1$  and  $x_2$  can be shifted back and forth, but the marginal rate of substitution of  $x_1$  for  $x_2$  or  $x_2$  for  $x_1$  is unchanged.

The linear product-product relationship: In linear programming the product-product relationship is a linear function  $Y_1 = f(Y_2)$ . This relationship shows that two enterprises,  $Y_1$  and  $Y_2$  have a constant rate of substitution. If one unit of product  $Y_1$  is given up, a certain constant unit of the other product,  $Y_2$ , will be gained; if a second or a third unit of  $Y_1$  is shifted to  $Y_2$ , an identical amount of  $Y_2$  will be gained for each unit of  $Y_1$  given up -- the marginal rate of substitution between them remains unchanged.

(2) Linear inequality: The term inequality arises from the fact that a plan does not require using the supply of all available resources, and that the extent of an activity or amount of a commodity produced may be equal to or greater than zero.

(C) Assumptions of linear programming (2, pp. 17-18).

The assumptions of linear programming are based on the following points.

Additivity: The activities must be additive in the sense that when two or more are used, their total product must be the sum of their individual products. Furthermore, it is assumed that any two processes can be used simultaneously, within the limitations of available resources.

Divisibility: It is assumed that any process can be used to any positive extent so long as sufficient resources are available.

Linearity: Each process is characterized by certain ratios of the quantities of each of the input to the quantities of each of the outputs. The ratios are constant and independent to the extent to which the process is used.

Finiteness: It is assumed that the number of processes available is finite.

## II. ALGEBRAIC TECHNIQUE

### (A) Real activities and inequality.

Using the crop example and letting the quantities of wheat, oats, barley and hay produced be represented respectively by  $x_1$ ,  $x_2$ ,  $x_3$ , and  $x_4$ , the production possibilities of real activities for the farm can be represented by the three linear inequalities below. With supplies of land, labour and capital represented respectively as  $S_1$ ,  $S_2$  and  $S_3$ , the requirement coefficient,  $P_{ij}$ , indicates the amount of the  $i$ -th resource required to produce one unit of the  $j$ -th crop activity. The production possibilities for the crop activities then can be derived from the equations of resource requirements in the following system of equations (4).

Wheat	Oats	Barley	Hay	
$P_{11}X_1$	$+ P_{12}X_2$	$+ P_{13}X_3$	$+ P_{14}X_4$	$\leq S_1$ (land)
$P_{21}X_1$	$+ P_{22}X_2$	$+ P_{23}X_3$	$+ P_{24}X_4$	$\leq S_2$ (labour)
$P_{31}X_1$	$+ P_{32}X_2$	$+ P_{33}X_3$	$+ P_{34}X_4$	$\leq S_3$ (capital)

The framework of linear programming can be restated in terms of matrices (2, pp. 378-444). The following three matrices represented in the system of linear inequalities then are:

$$\begin{pmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ P_{21} & P_{22} & P_{23} & P_{24} \\ P_{31} & P_{32} & P_{33} & P_{34} \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{pmatrix} \leq \begin{pmatrix} S_1 \\ S_2 \\ S_3 \end{pmatrix}$$

The net prices can be represented as a transposed column vector,  $C' = C_1 C_2 C_3 C_4$ . Hence the profit equation can be stated as a matrix product.  $Z$  is denoted as profit and the profit function can be abbreviated to  $Z = C' X$ ; that is:

$$Z = \begin{pmatrix} C_1 & C_2 & C_3 & C_4 \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{pmatrix}$$

The programming now can be stated compactly in matrix form as

$$\text{Maximized profit: } f(X) = C'X$$

subject to the restrictions

$$P X \leq S$$

$$X \geq 0$$

where  $P$  is a constant coefficient,  $X$  is activity and  $S$  is the total resource supplied.

## (B) Disposal activities and equality.

Converting the linear inequality into equality,  $AX = S$  is accomplished by adding  $m$  disposal activities to the original matrix  $X$ . The total number of activities is increased to  $n + m$ . The original matrix,  $P$ , which has  $n$  columns, has been expanded to the matrix  $A$  which is  $n + m$  columns and may be expressed:  $A = (P, I)$  where  $I$  is the identity matrix of  $m$  rows and columns. The equality equations can be stated as follows:

$$P_{11}X_1 + P_{12}X_2 + P_{13}X_3 + P_{14}X_4 + 1X_5 + 0X_6 + 0X_7 = S_1$$

$$P_{21}X_1 + P_{22}X_2 + P_{23}X_3 + P_{24}X_4 + 0X_5 + 1X_6 + 0X_7 = S_2$$

$$P_{31}X_1 + P_{32}X_2 + P_{33}X_3 + P_{34}X_4 + 0X_5 + 0X_6 + 1X_7 = S_3$$

If these equations are changed into matrix form, they become:

$$\begin{pmatrix} P_{11} & P_{12} & P_{13} & P_{14} & 1 & 0 & 0 \\ P_{21} & P_{22} & P_{23} & P_{24} & 0 & 1 & 0 \\ P_{31} & P_{32} & P_{33} & P_{34} & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \\ X_6 \\ X_7 \end{pmatrix} = \begin{pmatrix} S_1 \\ S_2 \\ S_3 \end{pmatrix}$$

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## CHAPTER V

### ANALYSIS OF THE STUDIED FARM BUSINESS

The situation used for the present study had been mentioned, in detail, in chapter I. The studied farm consists of 320 acres; 5-8 acres is used for the farmstead and 312-315 acres for crop land. It is classified as a mixed farm -- grain and livestock. Generally, a five-year-crop-rotation system was followed and a small number of beef cattle was fed. The major part of the farm income was derived from the sale of beef cattle. For the sake of understanding the farm business, the following analyses are employed.

#### I. LAND USE AND CROP-LIVESTOCK PRODUCTION SITUATION

(A) Land use pattern: In the five-year-crop-rotation system, the main crops were wheat, oats and barley, and the cash crops were millet, peas and sunflower. The land use pattern of the studied farm is shown in table I for the years 1957 to 1959.



TABLE I  
LAND USE PATTERN

Land Use	1957 -----	1958 Acres	1959 -----
Wheat	46	50	46
Oats	38	11	12
Barley	55	62	58
Peas	11	15	21
Millet	--	13	20
Sunflowers	30	34	32
Hay	40	--	53
Improved pasture	45	85	43
Farmstead	5	8	5
Summerfallow	50	42	30
Total acres in farm	320	320	320

In table I, it is shown that crop acreages were slightly changed in wheat and barley from 1957 to 1959, but varied greatly for oats, sunflowers and improved pasture. The number of acres devoted to oats was reduced from 38 acres in 1957 to 11 acres in 1958 and 12 acres in 1959. In contrast, the acreage of improved pasture was increased from 45 to 85 in the same two years. Between the crop years 1958 and 1959 the acreage in improved pasture was greatly reduced from 85 to 43 acres, but the acreage of land in hay was increased in 1959. The land use pattern was changed from year to year indicating that this farm probably did not have a

definite plan for livestock.

(B) Crop production situation.

Crop yield records are shown in table II; the yield per acre of wheat was higher in 1957 and 1959 than that in 1958; but the yield per acre of oats and barley were higher in 1958 than that in the years of 1957 and 1959.

TABLE II

CROP YIELD RECORDS

Crops	Unit	Total Yield			Yield Per Acre		
		1957	1958	1959	1957	1958	1959
Wheat	Bushel	1,725	1,225	1,625	37.50	24.50	35.30
Oats	Bushel	1,610	650	200	40.30	59.10	58.30
Barley	Bushel	970	1,400	1,475	21.10	38.70	25.40
Peas	Bushel	200	15	162	18.20	21.70	12.73
Millet	Pound	---	12,000	---	---	923.00	---
Sunflowers	Pound	21,575	325	---	719.20	21.70	---
Hay	Bale	2,200	---	---	100.00	---	---

(C) Livestock production situation.

In the studied farm the livestock program is given in table III. In 1957 there were 28 head of cattle and 80 chickens; in 1958 and 1959 there were 26 and 25 head of cattle and 55 hens and 43 chickens respectively. There were no hogs. This probably was its weak point in business, which will be discussed later.

TABLE III  
LIVESTOCK ENTERPRISE AND NUMBERS

Types	1957	1958	1959
Beef-cows	11	8	8
Bulls	1	1	1
Heifers	1	3	2
Steers	1	0	-
Calves	10	14	15
Hens	-	55	-
Chickens	80	-	43

## II. CAPITAL SITUATION

### (A) Inventory, net worth and financial progress.

From 1957 to 1959 the assets, liabilities, net worth and financial progress are shown in table IV. The owner of the studied farm had a total investment in farm capital of \$33,567.95 in 1957, which was reduced to \$33,103.80 in 1958 and \$33,016.50 in 1959. His total liabilities were reduced by \$1,485.50 in 1958 in comparison with 1957. His financial progress increased from \$589.57 in 1957 to \$895.37 in 1958.

TABLE IV

## INVENTORY, NET WORTH AND FINANCIAL PROGRESS

Items	1957	1958	1959
	-----	\$ -----	-----
Real estate	20,800.00	20,800.00	20,800.00
Change in inventory	-0-	-0-	-0-
Cattle	2,685.00	2,495.00	2,800.00
Chickens	46.75	43.00	-0-
Grains, feeds and supplies	5,624.10	5,425.30	5,167.50
Machinery and equipment	4,412.10	4,340.50	4,069.00
Total farm capital	33,567.95	33,103.80	33,016.50
Personal assets	4,462.06	4,835.93	5,051.79
Account receivable	-0-	30.00	60.00
Total assets	38,030.01	37,969.73	38,128.29
Total liabilities	8,373.00	6,887.50	7,273.50
Net worth	29,657.01	31,082.23	30,854.79
Financial progress	589.57	895.37	207.84

## (B) Financial statement.

A comparison of the studied farm income is made for the years 1957 to 1959, and is summarized in table V. The total receipts were increased by \$800.19 in 1958 and by \$106.27 in 1959 and the net current income was increased by \$1,281.55 in 1958 and \$548.11 in 1959, respectively, in comparison with 1957. Farm income decreased from 1957 to 1959, with the smallest figure being in 1958.

TABLE V  
FINANCIAL SUMMARY STATEMENT

Items	1957	1958 \$	1959
Total receipts	5,325.52	6,125.71	6,385.79
Total expenses	3,927.51	3,446.15	4,439.79
Net inventory change	904.10	-994.00	-167.70
Net current income	1,398.01	2,679.56	1,946.12
Farm income	2,302.11	1,685.56	1,778.42
Farm prerequisites	408.66	535.85	133.35
Return to operator and family for labour and capital	2,710.77	2,221.41	2,144.27
Interest on capital at 5%	1,247.69	1,298.53	1,298.49
Return to operator and family labour	1,463.08	922.88	845.78
Value of family labour	440.00	-0-	-0-
Operator's labour earnings	1,023.08	922.88	845.78

III. A COMPARISON OF THE STUDIED FARM BUSINESS  
WITH ITS GROUP AVERAGE IN CARMAN AREA

(A) The situation of farm business compared with its group average.

The owner of the studied farm was a member of a farm management association in the Carman district. If a comparison of the results of his farm business for 1957, 1958 and 1959 is

made with the average for his group farm on a similar soil type in the same years, it would help to discover some of the weaknesses and strengths of his farm business. The following items which are listed in table VI are used as indicators.

In table VI it will be noted that the amount of improved land and capital invested in the studied farm were 312 acres and \$33,103.80, which were smaller than the corresponding items of the group average. Farm income is dependent upon the farm size. Smaller amounts of capital investment and acres of improved land indicates a smaller farm size; a smaller farm size is generally associated with a smaller farm income. This was the main reason why the farm income of studied farm was below that of the group average income. Again, in the studied farm the intensity of land use was 0.59 in 1957, 0.50 in 1958 and 0.59 in 1959, which were smaller than that of the group average which was 0.80, 0.86 and 0.89 in the same three years. Intensity of land use is regarded as a barometer to measure the degree to which the variable resources such as labour and capital were applied to the given fixed resource, land. If we assume that the intensity of land use given for the group farms was optimum, then the studied farm in the same three years was below optimum in terms of its production intensity. This indicates that the capacity of fixed factor, land, had not yet been fully used. If one more dollar were invested in the farm, its return would be more than its cost.

TABLE VI

A COMPARISON OF GENERAL AGRICULTURAL SITUATIONS FOR  
STUDIED FARM WITH ITS GROUP AVERAGE FROM 1957 TO 1959

Items	Unit	1957		1958		1959	
		Studied farm	Group average	Studied farm	Group average	Studied farm	Group average
Average capital invested	\$	33,115.90	39,055.57	33,567.95	47,359.36	33,100.35	48,654.54
Rate of capital turnover	years	6.55	4.20	5.60	3.60	5.20	3.30
Rate of return to capital	%	1.40	6.00	1.30	9.60	1.00	5.80
Financial progress	\$	589.57	1,406.16	895.37	3,834.21	-207.84	1,764.57
Labour earnings	\$	1,023.08	2,496.43	922.88	4,478.83	845.75	2,711.72
Productive man work units per man	number	140.91	239.86	156.90	263.20	183.80	266.40

(continued)

TABLE VI - continued

Items	Unit	1957		1958		1959	
		Studied farm	Group average	Studied farm	Group average	Studied farm	Group average
Crop yield index		110.90	100.00	120.60	100.00	76.00	100.00
Number of animal units		18.30	28.01	16.80	34.80	16.60	29.80
Value of live-stock products	\$	1,331.97	3,933.98	1,973.48	6,544.95	1,667.61	5,823.15
Value of live-stock products per animal unit	\$	72.79	144.49	117.47	188.94	98.70	195.29
Livestock production index		65.80	100.00	72.10	100.00	67.00	100.00
Assets-liabilities ratio		4.06	5.82	5.51	5.68	4.54	4.82
Intensity of land use		0.59	0.80	0.50	0.86	0.59	0.89



TABLE VI - continued

Items	Unit	1957		1958		1959	
		Studied farm	Group average	Studied farm	Group average	Studied farm	Group average
Machinery and equipment investment	\$	4,465.98	8,727.66	4,376.80	10,669.72	4,208.50	11,203.66
Machinery and equipment costs	\$	2,058.48	2,788.79	2,132.34	3,145.98	1,475.35	3,186.57
Machinery and equipment costs per acre of improved land	\$	6.53	6.20	6.83	6.50	6.73	6.54
Improved acres	number	315.00	453.00	312.00	484.00	312.00	487.00
Value of crop production	\$	6,354.72	8,286.79	5,475.50	9,939.61	4,063.25	9,780.36
Value of crop production per acre of improved land	\$	20.14	18.53	17.55	20.54	13.02	20.07

(continued)

In the studied farm, machinery and equipment investment and associated cost were \$4,465.98 and \$2,058.48 respectively in 1957, \$4,376.80 and \$2,132.34 in 1958, and \$4,208.50 and \$1,475.35 in 1959; but they were \$8,727.66 and \$2,788.78 respectively in 1957, \$10,669.72 and \$3,145.98 in 1958, and \$11,203.66 and \$3,186.57 in 1959 in its group average. In the studied farm, the ratio of machinery and equipment costs to their investment was 46.09% in 1957, 48.72% in 1958 and 35.06% in 1959; while the percentage of machinery and equipment costs in terms of their investment was 31.95% in 1957, 29.49% in 1958 and 28.44% in 1959 in the group average. Furthermore, machinery and equipment costs per improved acre of land, being \$6.52 in 1957, \$6.83 in 1958 and \$6.73 in 1959 in the studied farm, were larger than those in its group average which were \$6.20, \$6.50 and \$6.54 in the same three years. These results indicate that the cost of machinery per acre of improved land in the studied farm was high relative to the group average, because the latter group could spread out its machinery cost over a larger number of improved acres of land.

(B) The weaknesses of the studied farm business.

Besides many weak points such as smaller farm size, low income and higher machinery cost, the livestock investment and the value of livestock production were also below the level of its group average. The weaknesses of its livestock enterprises were found as follows:

1. Hog-enterprises had not been included in the business; only a small number of cattle were fed. Consequently, the rate

of capital turnover was longer than that of its group average.

2. The available building space was seriously under-utilized. There were 2,160 square feet of space available in which there were only 24 head of cattle fed in 1957 and 26 in both 1958 and 1959. A large part of the building space was unused. If a hog enterprise had been included in the business instead of cattle, about 400 head of hogs could have been raised; the total value of their production would be conservatively estimated to have been approximately \$1,600 per four months (we assume that each hog would have been marketed at 200 pounds and each valued at \$40.).

3. Winter labour had not been fully used for feeding livestock and a part of it was left idle.

The owner of the studied farm was prepared to carry the hog enterprise in the old cattle building and to construct a new building for his cattle. If this plan had been carried out, about 400-500 head of hogs and 100 head of cattle could have been raised.

## CHAPTER VI

### RESULTS FOR TEN ALTERNATIVE PLANS FOR THE STUDIED FARM BUSINESS

In the studied farm two situations are assumed for study -- in the present situation it is assumed that the present amount of productive resources are available and that the present owner operates the farm by himself, while in the adjusted situation it is assumed that the operator's son will also be involved in the farm business; that there will be a father-son arrangement. Besides land and building space, the labour hours and the input of capital in the adjusted situation will exceed that of the present situation. Each situation will involve five plans of reorganization.

(A) In the present situation

Plan 1: A consideration of raising hog enterprises under present crop rotation system with the present amount of resources.

The resources available to the farm business are listed in table VII. The total value of the resources was equal to \$4,865.

In this plan there were six activities as listed in table VIII. These included: raised hogs ( $P_1$ ), feeder hogs ( $P_2$ ), selling wheat ( $P_3$ ), selling oats ( $P_4$ ), selling barley ( $P_5$ ), buying oats ( $P_6$ ). The input-output coefficients for the two hog activities are listed in table IX.

TABLE VII

## TOTAL RESOURCES AND VALUE AT HAND IN STUDIED FARM

Items	Unit	Quantity	Net Prices (\$)	Values (\$)
Capital	\$	1,881	---	1,881
Summer labour	hour	249	---	---
Winter labour	hour	1,100	---	---
Building space	square feet	2,100	---	---
Wheat	bushel	1,525	1.036484	1,580
Oats	bushel	867	0.278277	241
Barley	bushel	1,500	0.468855	703
Hay	pound	72,000	0.002353	169
Sunflowers	pound	11,000	0.025558	290
Total value	\$			4,865

TABLE VIII

## REAL ACTIVITIES IN PLAN 1

Designation	Unit of activity	Type of activity
P <sub>1</sub>	head (190 lbs.)	Raised hogs
P <sub>2</sub>	head (190 lbs.)	Feeder hogs
P <sub>3</sub>	bushel	Selling wheat
P <sub>4</sub>	bushel	Selling oats
P <sub>5</sub>	bushel	Selling barley
P <sub>6</sub>	bushel	Buying oats

TABLE IX

## INPUT-OUTPUT COEFFICIENTS IN PLAN 1

Type of resources	Unit	Coefficients of raised hogs	Coefficients of feeder hogs
Capital	\$	14.670000	19.290000
Summer labour	hour	1.302000	0.781000
Winter labour	hour	1.823000	1.094000
Building space	square feet	4.628000	2.284000
Wheat	bushel	0.496823	0.486670
Oats	bushel	7.061029	4.335882
Barley	bushel	8.207187	6.319791
Hay	pound	---	---

TABLE X

## RESOURCE SITUATION IN PLAN 1

Type of resources	Unit	Designation	Amount of resources	Amount used	Amount left over
Capital	\$	P <sub>7</sub>	1,881	1,881	-0-
Summer labour	hour	P <sub>8</sub>	249	165	83
Winter labour	hour	P <sub>9</sub>	1,100	231	868
Building space	square feet	P <sub>10</sub>	2,160	587	1,572
Wheat	bushel	P <sub>11</sub>	1,525	63	1,461
Oats	bushel	P <sub>12</sub>	867	896	-29
Barley	bushel	P <sub>13</sub>	1,500	1,041	458

Activities included in the final program are shown in table XI. They were raised hogs, buying oats, selling wheat and selling barley. Raised hogs and feeder hogs were competitive enterprises which competed for use of the most limiting resource, capital. The raised hog activity was the more profitable enterprise; its total yield was 127 head and its total profit was \$2,903. This can be explained by means of the following equations:

$$1. \frac{\Delta Y_1}{\Delta Y_2} = \frac{\text{Resource required per unit for } Y_2}{\text{Resource required per unit for } Y_1} = \frac{P_{Y_2}}{P_{Y_1}}$$

$$2. \Delta Z_0 = P_{Y_2} - P_{Y_1} \frac{\Delta Y_1}{\Delta Y_2}$$

Where  $\frac{\Delta Y_1}{\Delta Y_2}$  is marginal rate of substitution of  $Y_2$  for  $Y_1$  ( $Y_1$  and  $Y_2$  are denoted as two enterprises),  $\frac{P_{Y_2}}{P_{Y_1}}$  is their price ratio,  $\Delta Z_0$

is the marginal profit. If the first equation is applied to this plan, then the marginal rate of substitution of raised hogs for feeder hogs and their price ratio are:

$$\frac{14.67}{19.29} < \frac{23.01}{16.69}$$

$$\text{or } (14.67) (16.69) < (19.29) (23.01)$$

This result indicates that the marginal rate of substitution is less than the price ratio; therefore, raised hogs should be substituted for feeder hogs in the production plan. Again, the marginal profit can be computed by means of the second equation mentioned above.

The result is:

$$\begin{aligned} \Delta Z_0 &= 23.01 - 16.69 (14.67 / 19.29) \\ &= 10.317305 \end{aligned}$$

TABLE XI

## ACTIVITIES INCLUDED IN THE FINAL PROGRAM IN PLAN 1

Types of activities	Unit	Quantity
Raised hogs	number	461
Buying oats	bushel	39
Selling wheat	bushel	1,461
Selling barley	bushel	458

TABLE XII

## THE TOTAL FARM RECEIPTS IN PLAN 1

Items	Quantity Unit	Total amount of crop sold	Cost or net price per unit (\$)	Profit or cost (\$)
Receipts in final pro- gram		---	---	5,630
Wheat	bushel	1,461	0.534510	-780
Barley	bushel	458	0.473145	-216
Hay	pound	72,000	0.000235	169
Sunflowers	pound	11,358	0.025580	290
Total net receipts		---	---	5,092



That is, each unit increase in raised hogs will increase profit by \$10.32.

The total expenses in plan 1: The resource situation is shown in table X. It indicates that the total amount of resources used included capital at \$1,881, summer labour at 165 hours, winter labour at 231 hours, building space at 587 square feet, wheat at 63 bushels, oats at 896 bushels and barley at 1,041 bushels. Excluding the labour and building space, the total value of these resources consumed by the livestock was equal to \$2,684. In other words, the studied farm had a net profit of \$2,903 after expenses of \$2,684 were deducted.

The total farm receipts: In the final program, the total gross receipts were \$5,630. If the cost of wheat and barley is deducted from this sum and the result added to the net profit for hay and sunflower, the total net farm receipts were \$5,092 as shown in table XII.

Plan 2: A consideration of feeding cattle enterprise under present crop-rotation system with the present amount of resources.

In this plan there were eight real activities as listed in table XIII. These activities included: cow calves ( $P_1$ ), steer calves ( $P_2$ ), 2 year steers ( $P_3$ ), selling wheat ( $P_4$ ), selling oats ( $P_5$ ), selling barley ( $P_6$ ), selling hay ( $P_7$ ) and buying hay ( $P_8$ ). The input-output coefficients for the activities of cow calves, steer calves and 2 year steers are listed in table XIV.

Table XV indicates the resource situation. Besides 72,000 pounds of hay, the other quantities of each of the available limit-

TABLE XIII

## REAL ACTIVITIES

Designation of activities	Unit	Type
P <sub>1</sub>	head	Cow-calves
P <sub>2</sub>	head	Steer-calves
P <sub>3</sub>	head	2 year steers
P <sub>4</sub>	bushel	Selling wheat
P <sub>5</sub>	bushel	Selling oats
P <sub>6</sub>	bushel	Selling barley
P <sub>7</sub>	pound	Selling hay
P <sub>8</sub>	pound	Buying hay

TABLE XIV

## INPUT-OUTPUT COEFFICIENTS IN PLAN 2

Type of resources	Unit	Coefficients of cow-calves	Coefficients of steer calves	Coefficients of 2 year steer
Capital	\$	34.985000	94.990000	182.100000
Summer labour	hour	7.292000	5.205000	3.253000
Winter labour	hour	19.281000	7.288000	4.555000
Building space	square feet	73.041000	20.822000	13.014000
Wheat	bushel	---	---	---
Oats	bushel	18.471567	15.941176	13.249117
Barley	bushel	22.513083	24.437500	20.416666
Hay	pound	9,522.483000	1,862.000000	1,703.000000



TABLE XV

## RESOURCE SITUATION IN PLAN 2

Resources	Unit	Designation	Available amount of resources	Amount used	Amount left over
Capital	\$	P <sub>9</sub>	1,881	1,881	-0-
Summer labour	hour	P <sub>10</sub>	249	124	124
Winter labour	hour	P <sub>11</sub>	1,100	210	889
Building space	square feet	P <sub>12</sub>	2,160	672	1,487
Wheat	bushel	P <sub>13</sub>	1,525	-0-	1,525
Oats	bushel	P <sub>14</sub>	867	365	501
Barley	bushel	P <sub>15</sub>	1,500	537	962
Hay	bushel	P <sub>16</sub>	72,000	72,000	-0-

TABLE XVI

ACTIVITIES INCLUDED IN THE FINAL  
PROGRAM IN PLAN 2

Type of activities	Unit	Quantities
Cow-calves	head	3.97
Steer-calves	head	18.33
Selling wheat	bushel	1,525.00
Selling oats	bushel	501.24
Selling barley	bushel	962.37

ing resources were identical to those in plan 1. The total input of capital and hay were used up, while some of the other inputs were left over.

Activities included in the final program are shown in table XVI. The optimum condition of resource combination in the plan included four head of cow-calves and 18 head of steer calves.

In this plan the most limiting resources were capital and hay. The most profitable enterprises were not only dependent upon the marginal rate of substitution of capital and the price ratio but were also dependent upon the marginal rate of substitution of hay. As the coefficients of capital for 2 year steers was too high and its net price too low, the enterprise of 2 year steers was an expensive activity in capital which could not compete with such activities as cow calves and steer calves which need less capital. The optimum enterprises including four head of cow-calves and 18 head of steer calves can be achieved simply by solving the simultaneous equations as follows:

$$93.910 S + 47.910 C = \$ 1,881$$

$$1,862.000 S + 9,522.000 C = 72,000 \text{ lbs.}$$

Where S denoted steer calves; C is cow-calves; \$1,881 is the most limiting resource of capital; and 72,000 is the most limiting resource of hay.

The net profit <sup>1/</sup> in this plan was \$1,640. The total amount of resources used included \$1,881 of capital, 124 hours of summer labour, 210 hours of winter labour, 672 square feet of cattle

<sup>1/</sup> The net profit gross receipts in final program minus the value of selling activities and the costs of wheat, oats, and barley.

TABLE XVII

## THE TOTAL FARM RECEIPTS IN PLAN 2

Items	Quantity unit	Total amount of crop sold	Cost or net price per unit (\$)	Profit (+) or cost (-) (\$)
Total gross farm receipts				5,257
Wheat	bushel	1,525	0.534510	-815
Oats	bushel	501	0.350723	-175
Barley	bushel	962	0.443145	-416
Sunflowers	pound	11,358	0.025580	290
Total net farm receipts				4,130

space, 366 bushels of oats, 538 bushels of barley, and 72,000 pounds of hay. The total value of these resources was equal to \$2,404. These results indicate that plan 2 would have a net profit of \$1,640 after the expenses of \$2,404 are deducted.

The total farm receipts: In the final program, the total gross farm receipts were \$5,257. If the cost of wheat, oats and barley are deducted from the total gross profit and the result added to the net profit for sunflowers, the total net farm receipts would be \$4,131 as shown in table XVII.

Plan 3: A consideration of selecting alternative hog and cattle enterprises under present crop-rotation system with the present amount of resources.

In this plan there were 13 activities as listed in table XVIII. These activities included: cow-calves ( $P_1$ ), steer calves ( $P_2$ ), 2 year steers ( $P_3$ ), raised hogs ( $P_4$ ), feeder hogs ( $P_5$ ) and selling and buying activities of wheat, oats, barley and hay ( $P_6$ ,  $P_7$ ,  $P_8$  .....  $P_{13}$  respectively.

The resource situation is shown in table XIX. None of the hay was used but the total available input of capital was used up. In addition, there was a shortage of oats and 32 bushels were bought from the market. Some of all the other resources were left over.

Activities included in the final program are indicated in table XX. The major object of this plan was to select the highest profit among these five competing enterprises. The total net profit of the hog-enterprise in plan 1 was greater than that of

TABLE XVIII

## REAL ACTIVITIES IN PLAN 3

Designation of activities	Unit of activities	Type of activities
P <sub>1</sub>	head	Cow-calves
P <sub>2</sub>	head	Steer calves
P <sub>3</sub>	head	2 year steers
P <sub>4</sub>	head	Raised hogs
P <sub>5</sub>	head	Feeder hogs
P <sub>6</sub>	bushel	Selling wheat
P <sub>7</sub>	bushel	Selling oats
P <sub>8</sub>	bushel	Selling barley
P <sub>9</sub>	pound	Selling hay
P <sub>10</sub>	bushel	Buying wheat
P <sub>11</sub>	bushel	Buying barley
P <sub>12</sub>	bushel	Buying oats
P <sub>13</sub>	pound	Buying hay

TABLE XIX  
RESOURCE SITUATION IN PLAN 3

Type of resource	Unit	Designation	Amount of resources	Amount used	Amount left over
Capital	\$	P <sub>9</sub>	1,881	1,881	-0-
Summer labour	hour	P <sub>10</sub>	249	165	84
Winter labour	hour	P <sub>11</sub>	1,100	231	869
Building space	square feet	P <sub>12</sub>	2,160	596	1,574
Wheat	bushel	P <sub>13</sub>	1,525	64	1,463
Oats	bushel	P <sub>14</sub>	867	899	-32
Barley	bushel	P <sub>15</sub>	1,500	1,040	458
Hay	pound	P <sub>16</sub>	72,000	-0-	72,000

TABLE XX  
ACTIVITIES INCLUDED IN THE FINAL  
PROGRAM IN PLAN 3

Type of activities	Unit	Quantity
Raised hogs	head	125
Buying oats	bushel	32
Selling wheat	bushel	146
Selling oats	bushel	460
Selling hay	pound	72,000



TABLE XXI

## THE TOTAL FARM RECEIPTS IN PLAN 3

Items	Quantity unit	Total amount of crop sold	Cost or net price per unit (\$)	Profit or cost (\$)
Gross total farm receipts				6,060
Wheat	bushel	1,463	0.534510	-781
Barley	bushel	460	0.443145	-203
Hay	pound	72,000	0.003647	-262
Sunflower	pound	11,358	0.025580	290
Total net farm receipts				5,102

the cattle enterprise in plan 2. Obviously, the enterprise of raised hogs in plan 3 could also have the highest profit on account of the supply of resources, the coefficients of enterprises and their net prices from plan 1 to plan 3 are in identity.

Again, table XIX indicates that the total amount of resources used included capital at \$1,881, summer labour at 165 hours, winter labour at 231 hours, building space at 586 square feet, wheat at 64 bushels, oats at 899 bushels and barley at 1,040 bushels. The total value of these resources was equal to \$2,685. The results in plan 3 indicate that this studied farm could obtain \$2,905 of net profit after the expenses of \$2,685 are deducted.

The total receipts: In the final program, the total gross

farm receipts were \$6,060. If the costs of wheat, barley and hay are deducted from the total gross farm receipts and the results added to the net profit for sunflowers, the total net farm receipts are \$5,102 as shown in table XXI.

Plan 4: A consideration of alternative crop-rotation systems and livestock enterprises with limited capital and the present amount of resources.

In this plan, there were 16 real activities as listed in table XXII. These activities included: 3-year-crop-rotation ( $P_1$ ), 4-year-crop-rotation ( $P_2$ ), 5-year-crop-rotation ( $P_3$ ), 6-year-crop-rotation ( $P_4$ ), 8-year-crop-rotation ( $P_5$ ), raised hogs ( $P_6$ ), feeder hogs ( $P_7$ ), cow-calves ( $P_8$ ), steer calves ( $P_9$ ), 2-year-steers ( $P_{10}$ ), four selling activities (wheat, oats, barley and hay) and two buying activities (oats and hay). The input-output coefficients for these activities are listed in table XXIII.

The resource situation is shown in table XXIV. Winter labour and building space were not fully used; the available capital was used up and there were 8 acres of land left over.

Activities included in the final program are indicated in table XXV. The three-year-crop-rotation was the highest profit enterprise including 2,045 bushels of wheat, 1,636 bushels of oats and 1,227 bushels of barley. There was no livestock activity in the program. The total net profit was \$3,809.

In this plan, the most limiting resource was capital of \$4,439. Ten enterprises including five in livestock and five in crop rotation

TABLE XXII

## REAL ACTIVITIES IN PLAN 4

Designation of activities	Unit of activities	Type of activities
P <sub>1</sub>	number	3-year-crop-rotation
P <sub>2</sub>	number	4-year-crop-rotation
P <sub>3</sub>	number	5-year-crop-rotation
P <sub>4</sub>	number	6-year-crop-rotation
P <sub>5</sub>	number	8-year-crop-rotation
P <sub>6</sub>	head	Raised hogs
P <sub>7</sub>	head	Feeder hogs
P <sub>8</sub>	head	Cow-calves
P <sub>9</sub>	head	Steer calves
P <sub>10</sub>	head	2-year-steers
P <sub>11</sub>	bushel	Selling wheat
P <sub>12</sub>	bushel	Selling oats
P <sub>13</sub>	bushel	Selling barley
P <sub>14</sub>	bushel	Selling hay
P <sub>15</sub>	bushel	Buying hay
P <sub>16</sub>	bushel	Buying oats

TABLE XXIII

## INPUT-OUTPUT COEFFICIENTS IN PLAN 4

Type of resource	Unit	Coefficients of 3-year-rotation	Coefficients of 4-year-rotation	Coefficients of 5-year-rotation	Coefficients of 6-year-rotation	Coefficients of 8-year-rotation
		3	4	5	6	8
Land	acre					
Capital	\$	43.4115	57.9505	75.9505	88.803	106.988
Summer labour	hour	6.82	10.43	12.85	16.30	19.06
Winter labour	hour	---	---	---	---	---
Building space	square feet	---	---	---	---	---
Wheat	bushel	-20	-20	-20	-40	-40
Oats	bushel	-16	-16	-16	-16	-32
Barley	bushel	-12	-12	-12	-12	-24
Hay	pound	---	-3,800	---	-5,600	-5,600
Sunflowers	pound	---	---	-543	---	---
Flax	pound	---	---	-8.9	---	---

TABLE XXIV

## RESOURCE SITUATION IN PLAN 4

Type of resources	Unit	Designation of activities	Available amount of resources	Amount used	Amount left
Land	acre	P <sub>17</sub>	315	306	8
Capital	\$	P <sub>18</sub>	4,439	4,439	-0-
Summer labour	hour	P <sub>19</sub>	1,484	697	786
Winter labour	hour	P <sub>20</sub>	1,100	---	1,100
Building space	square feet	P <sub>21</sub>	2,160	---	2,160

TABLE XXV

ACTIVITIES INCLUDED IN FINAL  
PROGRAM IN PLAN 4

Type of activities	Unit	Amount
3-year-rotation		102
Selling wheat	bushel	2,045
Selling oats	bushel	1,636
Selling barley	bushel	1,227

competed for use of a given amount of capital. Why did the 3-year-crop-rotation have the highest profit among these ten competing enterprises in this plan? A detailed discussion of this question follows:

(1) A comparison of net profits of the five alternative crop rotations.

Based on the results of the computation of the linear program, the total capital was \$4,439 of which \$2,972 was competed for use by five enterprises in crop rotation and the remainder of \$1,467 was competed for use by the enterprises of livestock and 3-year-crop-rotation. The net profit per unit of crop rotation is listed in table XXVI and a comparison of total profits of the five alternative crop-rotations with capital \$2,972 are shown in table XXVII. The total net profit of the 3-year-crop-rotation was \$2,556 which was higher than those of the other enterprises.

(2) A comparison of profits of the 3-year-crop-rotation and raised hogs on the results of the final program in the computed table of linear programming.

Based on the results of computed table, one hundred head of raised hogs and 3 $\frac{1}{4}$  units of 3-year-crop-rotation competed for entering the final section in the program. In terms of the principle of opportunity cost, if 100 head of raised hogs were gained, 3 $\frac{1}{4}$  units of 3-year-crop-rotation would be sacrificed; and vice versa (this is obtained by dividing the remaining capital of \$1,467 by the coefficients of raised hogs and 3-year-crop-rotation respectively,

TABLE XXVI

NET PROFITS PER UNIT OF CROP  
ROTATIONS IN PLAN 4

Rotation Crops	Unit	3-year-rotation Quantity	3-year-rotation Profit	4-year-rotation Quantity	4-year-rotation Profit	5-year-rotation Quantity	5-year-rotation Profit
Wheat	bushel	20	31.42	20	20.73	20	31.42
Oats	bushel	16	10.30	16	10.00	16	10.06
Barley	bushel	12	11.30	12	11.30	12	11.30
Hay	bushel	-	---	3,800	8.94	-	---
Sunflowers	pound	-	---	-	---	543	24.16
Flax	bushel	-	---	-	---	1.5	3.05
Total profits per unit	\$	-	52.78	-	61.72	-	---
Total costs per unit	\$	-	15.44	-	17.66	-	30.38
Net profits per unit	\$	-	37.34	-	44.66	-	49.62

(continued)

TABLE XXVI - continued

Rotation Crops	Unit	6-year-rotation		8-year-rotation	
		Quantity	Profit	Quantity	Profit
Wheat	bushel	40	62.84	40	62.84
Oats	bushel	16	10.06	32	20.06
Barley	bushel	12	11.30	24	22.60
Hay	bushel	5,600	17.88	5,600	17.88
Sunflowers	pound	-	---	-	---
Flax	bushel	-	---	-	---
Total profits per unit	\$	-	102.09	-	123.39
Total costs per unit	\$	-	39.03	-	47.14
Net profits per unit	\$	-	63.05	-	76.25



TABLE XXVII

A COMPARISON OF PROFITS OF THE 5 ALTERNATIVE CROP-ROTATIONS WITH CAPITAL OF \$2,972 (\$4,439 - \$1,467)

Crop-rotation	3-year-rotation	4-year-rotation	5-year-rotation	6-year-rotation	8-year-rotation
Capital	\$ 2,972.00	\$ 2,972.00	\$ 2,972.00	\$ 2,972.00	\$ 2,972.00
Coefficients	43.41	57.95	75.95	88.80	106.98
Units of enterprise	68.46	51.28	39.13	33.46	27.77
Net profits per unit	37.34	44.07	49.98	63.05	76.25
Total net profits	2,556.47	2,260.18	1,955.79	2,110.38	2,118.18

giving 100 head of raised hogs and 34 units of 3-year-crop-rotation). If 100 head of raised hogs fed and 68 units of 3-year-crop-rotation is carried out, then the total profit would be \$3,576; on the other hand, if 102 units ( $34 + 68 = 102$ ) of the 3-year-crop-rotation is carried out, the total profit would be \$3,809.

Plan 5: A consideration of alternative crop rotation systems and livestock enterprises with unlimited capital and present amount of other resources.

From plan 1 to plan 4, capital was assumed to be limited and the standard simplex solution for linear programming problems was used. The optimum plan for a given situation depends upon the available resources, the input-output coefficients, and the net prices employed in the programming.

In contrast, capital was assumed to be unlimited and a modified simplex method was used in plan 5. This modified method may be described as continuous programming and allows that several plans can be arranged along with the corresponding level of the scarce resource (capital).

Sixteen activities were involved in plan 5 as listed in table XXII. These activities were the same as those in plan 4.

The resource situation is indicated in table XXVII. The amount of capital needed in this plan was \$12,325. Land and building space were the most limited resources and were used up. The amount of resources left over were summer labour at 461 hours and winter labour at 676 hours.

TABLE XXVII

## RESOURCE SITUATION IN PLAN 5

Type of resources	Unit	Designation	Amount of resource	Amount used	Amount left over
Land	acre	P <sub>17</sub>	315	315	-0-
Capital	\$	P <sub>18</sub>	unlimited	12,325	-0-
Summer labour	hour	P <sub>19</sub>	1,484	1,023	461
Winter labour	hour	P <sub>20</sub>	1,100	424	676
Building space	square feet	P <sub>21</sub>	2,160	2,160	-0-

The summary of the final program which was derived from plan 5 is shown in table XXVIII. This result indicates that the selection of the different combination of enterprises was dependent upon the amount of available capital. If the studied farm should have \$5,381 of capital, the profitable enterprises would be 39 units of 8-year-crop-rotation and 24 head of cow-calves as presented in section 3 of table XXVIII. If available capital were \$8,081, the profitable enterprises would be 33 units of 3-year-crop-rotation, 27 units of 8-year-crop-rotation, 21 head of cow-calves and 146 head of feeder hogs as presented in section 6. The total income and the marginal value productivity of capital are indicated in columns 12 and 13 respectively in table XXVIII. The highest total income coincided with the largest amount of capital

TABLE XXVIII

## A SUMMARY OF THE PROGRAMS OBTAINED IN PLAN 5

Sections (1)	Capital needed (\$) (2)	(P <sub>1</sub> ) 3-year-crop- rotation (3)	(P <sub>2</sub> ) 4-year-crop- rotation (4)	(P <sub>5</sub> ) 8-year-crop- rotation (5)	(P <sub>8</sub> ) Cow-calves (head) (6)	(P <sub>9</sub> ) Steer calves (head) (7)
3	5,381	-	-	39	24	-
4	7,287	-	-	39	19	23
5	7,574	18	-	33	18	26
6	8,081	33	-	27	21	-
7	8,322	33	54	-	21	-
8 and 9	9,881	40	49	-	19	-
10	12,325	105	-	-	-	-

(continued)

TABLE XXVIII - continued

Sections	(1)	(2)	(8)	(9)	(10)	(11)	(12)	(13)
		Capital needed (\$)	Feeder hogs (head)	Buying barley (bushel)	Selling wheat (bushel)	Cost of crop rotation (\$)	Total net income (\$)	Marginal value of productivity of capital (\$)
3		5,381	-	-	-	1,856	354	0.065787
4		7,287	-	-	-	1,856	1,432	0.566682
5		7,574	-	-	-	1,833	1,601	0.566333
6		8,081	146	-	-	1,782	2,552	1.000000
7		8,322	146	-	-	1,463	2,872	1.327800
8 and 9		9,881	219	260	1,670	1,483	6,312	2.206542
10		12,325	387	414	1,918	1,622	7,562	0.511456

used. But, the highest marginal value productivity of capital did not necessarily do so. Column 13 in table XXVIII indicates that as the amount of capital increased, the marginal value productivity of capital first rose; then it fell off after a point F (figure 5) was reached.

A graphic representation of the data in table XXVIII is shown in figure 5. The horizontal axis refers to the amount of capital from zero to \$12,325, while the vertical axis records the marginal value productivity of capital, total income and amount of real activities, respectively, corresponding to capital levels of \$0, \$5,381, \$7,287, \$7,574, \$8,081, \$8,322, \$9,881 and \$12,325. Line AA' indicates the capital level at \$5,381 which intersects the total net income at \$354, the 8-year-crop-rotation ( $P_5$ ) at 39 units and cow-calves ( $P_8$ ) at 24 units. Moving on to BB', it shows that total capital level at \$7,287 which intersects the 8-year-crop-rotation ( $P_5$ ) at 39 units, the steer calves ( $P_9$ ) at 23 head, cow calves ( $P_8$ ) at 19 head and the total net income at \$1,432. The marginal value productivity of capital was \$0.566333. Similarly, the same meaning applies to lines CC', DD', EE', FF' and GG'. The corresponding level of capital needed was at \$7,574, \$8,081, \$8,322, \$9,881 and \$12,325, respectively.

Furthermore, figure 5 can show the successive steps of the continuous solution for the studied farm. At first, zero level of capital gave no production and zero amount income. When the capital level was at \$5,381, land became the most limiting resource

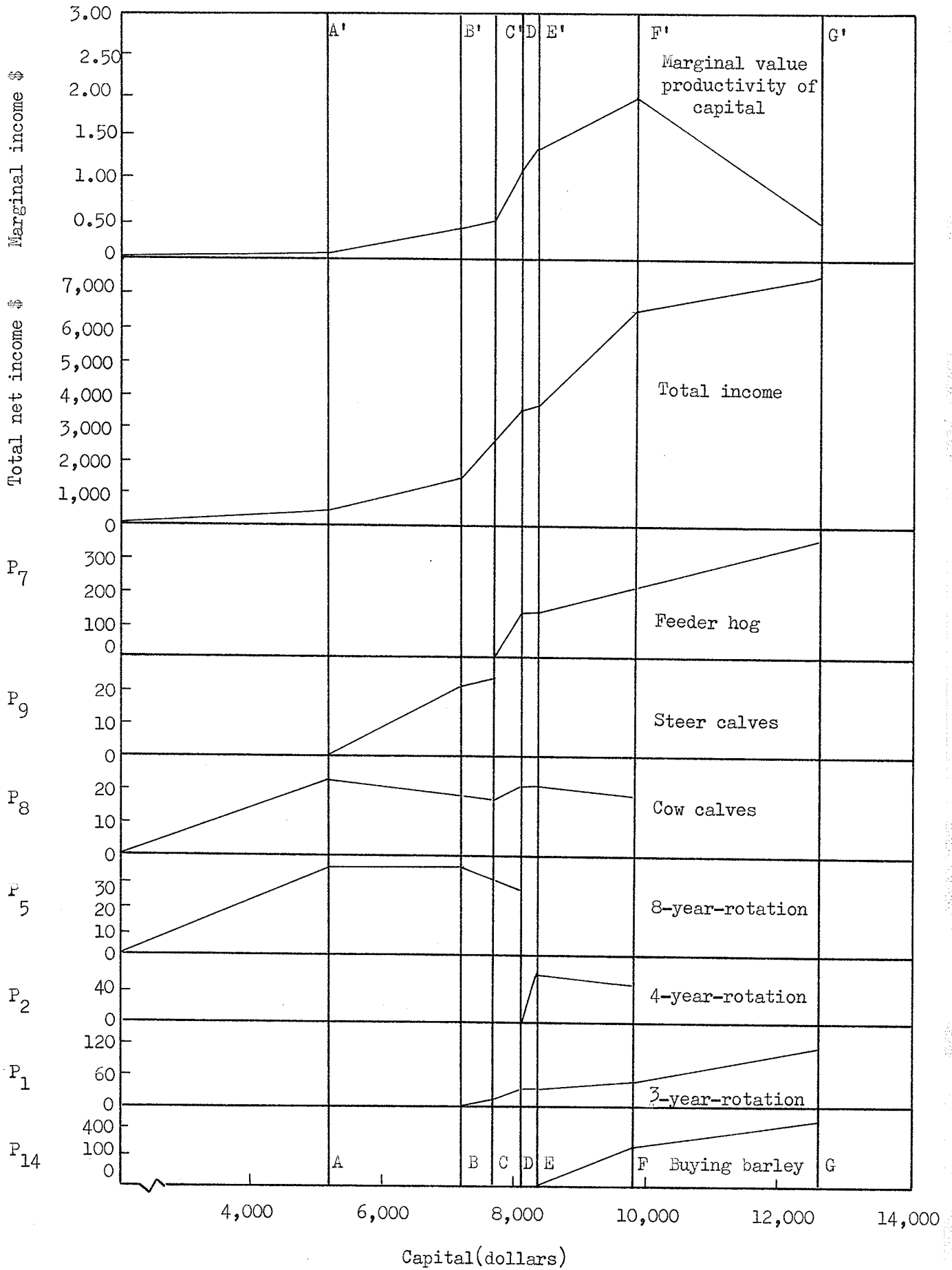


Fig. 5. Graphic representation of table 28

and the 8-year-crop-rotation was the most profitable enterprise. When the capital level was at \$7,287, steer calves and cow-calves are the most profitable enterprises and the building space became the most limiting resource. When capital level was increased to \$7,574, the cow-calves dropped from 24 to 18 head and steer calves increased from 23 to 26 head. When capital level was at \$8,081, the 3-year-crop-rotation increased from 18 to 33 units, while the 8-year-crop-rotation decreased to 27 units. When the capital level was raised from \$8,081 to \$8,322, the 8-year-crop-rotation was substituted by the 4-year-crop-rotation. In the final program, capital level was at \$12,325, the most profitable enterprises were feeder hogs at 387 head and the 3-year-crop-rotation at 105 units.

In short, land and building space in this plan were the most limiting resources; the enterprises of rotation systems competed for land while the livestock enterprises competed for building space. The optimum combination of resources was 105 units of the 3-year-crop-rotation, 387 head of feeder hogs and \$12,325 of capital in the final program.

(B) In the adjusted situation.

In the adjusted situation it is assumed: (1) that the owner's son takes part in the farm business, (2) that the supply level of resources such as capital, labour hours and building space are greater than those of the present situation, and (3) that a father-son agreement has been arranged and five plans (from plan 6 to plan 10) for farm business have been designed. The simplex method of linear programming was applied from plan 6 to plan 9, and a modified simplex method was used in plan 10.



Plan 6: A consideration of hog-enterprises under present crop-rotation system adjusted to a father-son arrangement.

In this plan, the real activities and their coefficients are the same as those in plan 1 in the present situation. The resource situation is shown in table XXX in which capital is \$3,762, summer labour 1,733 hours, winter labour 2,200 hours, building space 4,320 square feet, and wheat, oats, and barley, 1,525, 867 and 1,500 bushels, respectively. Four hundred and twenty-three bushels of oats had to be purchased. Some of the other resources are left over.

Raised hogs and feeder hogs are two competitive enterprises which compete for the use of the most limiting resource, barley. If the total quantity of barley, 1,500 bushels, is invested in feed for the raised hogs, then  $182 (1,500 / 8.207187)$  head of raised hogs can be produced with a resulting net profit of \$4,187. If the same quantity of barley is used for the enterprise of feeder hogs, then  $237 (1,500 / 6.319791)$  head can be produced with a resulting net profit of \$3,955. The difference between the profits of these two hog enterprises is \$232 indicating that the profit of raised hogs is greater than that of feeder hogs.

Activities included in the final program are shown in table XXXI. It indicates that the optimum use of resources in this situation would be to feed 182 head of raised hogs, to sell 1,434 bushel of wheat and to buy 423 bushel of oats.

Table XXXII shows the total farm receipts. Gross profit in final program is \$6,191, wheat cost is \$766, net income of hay and sunflower are \$169 and \$290 respectively. The total farm receipts of \$5,884 can be obtained by deducting the wheat cost and adding the net income of hay and sunflower to the gross profit.

TABLE XXX

## RESOURCE SITUATION IN PLAN 6

Type of resources	Unit	Designation	Amount of resources	Amount used	Amount left over
Capital	\$	P <sub>7</sub>	3,762	2,948	814
Summer labour	hour	P <sub>8</sub>	1,733	238	1,495
Winter labour	hour	P <sub>9</sub>	2,200	334	1,866
Building space	square feet	P <sub>10</sub>	4,320	846	3,474
Wheat	bushel	P <sub>11</sub>	1,525	917	1,434
Oats	bushel	P <sub>12</sub>	867	1,290	-423
Barley	bushel	P <sub>13</sub>	1,500	1,500	-0-

TABLE XXXI

ACTIVITIES INCLUDED IN THE FINAL  
PROGRAM IN PLAN 6

Type of activities	Unit	Quantity
Raised hogs	head	182
Selling wheat	bushel	1,434
Buying oats	bushel	423

TABLE XXXII

TOTAL FARM RECEIPTS IN PLAN 6

Items	Total amount of crop sold (bushel)	Cost per unit (\$)	Profit or cost (\$)
Gross profit in final program	---	---	6,191
Wheat	1,434	0.534510	-766
Hay	---	---	169
Sunflowers	---	---	290
Total net farm receipts	---	---	5,884

Plan 7: A consideration of feeding cattle enterprises with present crop-rotation system adjusted to a father-son arrangement.

In plan 7, the types of activities and their coefficients are identical with plan 2.

The resource situation is indicated in table XXXIII. Besides 72,000 pounds of hay, the other quantities of each of the available limiting resources are the same as those in plan 6. The total inputs of capital and hay are consumed; wheat has not been used and some of each of the other resources are left unused.

Three competitive enterprises, cow-calves, steer calves, and 2-year old steers compete for the use of the two most limiting resources, hay and capital. The 2-year steer enterprise is a relatively unprofitable activity in comparison with the other activities because of its low net price and its relatively high capital coefficient. The cow-calves and steer calves activities are relatively more profitable. The net price of steer calves is slightly smaller than that of cow-calves but the coefficients of hay for cow-calves is four times as much as that for steer calves. Under these conditions the enterprise of steer calves is in a favourable situation. An optimum combination of resources in the final program, therefore, is 39 head of steer calves and zero units of cow-calves. This can be explained by means of the following simultaneous equations.

$$9,522.483 C + 1,862.000 S = 73,762.000$$

$$34.985 C + 94.990 S = 3,763.000$$

Where 73,762 is the most limited resource of hay, 3,763 is the most limited resource of capital, C denotes cow-calves, S is steer calves, 9,522.483 and 34.985 are the coefficients of hay and capital for cow-calves, 1,862.000 and 94.990 are the coefficients of hay and capital for steer calves.

The results derived from the above equations are: the number of cow-calves is zero and the number of steer calves is 39 head.

All activities included in the final program are indicated in table XXXIV. In addition to the 39 head of steer calves, there are 1,525 bushels of wheat, 237 bushels of oats and 575 bushels of barley sold. This plan calls for the purchase of 1,562 pounds of hay.

The net value of 39 head of steer calves is equal to \$2,610. The resources which are used by the activity of steer calves include 3,762 units of capital, 823 square feet of building space, 206 hours of summer labour, 288 hours of winter labour, 73,562 pounds of hay, 639 bushels of oats and 925 bushels of barley. Besides labour hours and building space, the total value of these resources is equal to \$4,542. Therefore, plan 7 yields net profit of \$2,610 from an expenditure of \$4,542.

Table XXXV indicates the total farm receipts. The gross profit in the final program is \$5,683. The total net farm receipts of \$4,820 can be obtained by deducting the costs of wheat, oats and barley and adding the net profit of the sunflower enterprise.

Plan 8: A consideration of selecting hog-cattle enterprises with present crop-rotation system adjusted to a father-son arrangement.

This plan is composed of 13 activities which are identical with those in plan 3 in the present situation. The coefficients of hog-enterprises are the same as those in plan 1 and the coefficients of cattle-enterprises as those in plan 2.

TABLE XXXIII

## RESOURCE SITUATION IN PLAN 7

Type of resources	Unit	Designation	Amount of resources	Amount used	Amount left over
Capital	\$	P <sub>9</sub>	3,762	3,762	-0-
Summer labour	hour	P <sub>10</sub>	1,733	206	1,527
Winter labour	hour	P <sub>11</sub>	2,200	288	1,912
Building space	square feet	P <sub>12</sub>	4,320	823	3,497
Wheat	bushel	P <sub>13</sub>	1,525	-0-	1,525
Oats	bushel	P <sub>14</sub>	876	639	237
Barley	bushel	P <sub>15</sub>	1,500	925	575
Hay	pound	P <sub>16</sub>	72,000	73,562	-1,762

TABLE XXXIV

ACTIVITIES INCLUDED IN THE FINAL  
PROGRAM IN PLAN 7

Type of activities	Unit	Quantity
Steer calves	head	39
Selling wheat	bushel	1,525
Selling oats	bushel	237
Selling barley	bushel	575
Buying hay	pound	1,562

TABLE XXXV

THE TOTAL FARM RECEIPTS IN PLAN 7

Items	Total amount of crop sold (bushel)	Cost per unit (\$)	Profit (+) or cost (-) (\$)
Gross profit in final program			5,683
Wheat	1,525	0.534510	-815
Oats	237	0.350723	-83
Barley	575	0.443145	-255
Sunflowers			290
Total net farm receipts			4,820

Resource situation in plan 8 is indicated in table XXXVI. The most limiting resources are oats and hay, while some of each of the other resources are left over.

In the light of the results obtained from linear programming, raised hogs, feeder hogs, cow-calves and steer calves enter the sections of the program and compete for use of the most limiting resources of oats and hay. Hay is necessary for cattle but not for hogs. Hence, raised hogs and feeder hogs compete for use of oats, while the enterprises of cow-calves and steer calves compete for use of hay. Activities included in final program in this plan are shown in table XXXVIII indicating that the optimum condition of resource combination includes 8 head of cow-calves and 169 head of feeder hogs. This can be expressed in terms of simultaneous equations.

$$9,522.522 C + 0 F = 72,000.000$$

$$18.472 C + 4.335 F = 876.000$$

Where C denotes cow-calves, F is feeder hogs; 9,522.483 is the coefficient of hay and 18.472 is the coefficient of oats for cow-calves; 4.335 is the coefficient of oats for feeder hogs.

Plan 9: A consideration of alternative crop-rotation systems and livestock enterprises with limited capital adjusted to a father-son arrangement.

In this plan there are 18 activities as listed in table XXXIX. The input-output coefficients of these activities are identical with those of tables IX, XIV and XIX.



TABLE XXXVI

## RESOURCE SITUATION IN PLAN 8

Type of resource	Unit	Designation	Amount of resource	Amount used	Amount left over
Capital	\$	P <sub>14</sub>	3,762	3,501	261
Summer labour	hour	P <sub>15</sub>	1,733	187	1,546
Winter labour	hour	P <sub>16</sub>	2,200	330	1,870
Building space	square feet	P <sub>17</sub>	4,320	936	3,384
Wheat	bushel	P <sub>18</sub>	1,525	79	1,446
Oats	bushel	P <sub>19</sub>	876	876	-0-
Barley	bushel	P <sub>20</sub>	1,500	1,231	269
Hay	pound	P <sub>21</sub>	72,000	72,000	-0-

TABLE XXXVII

## THE TOTAL FARM RECEIPTS IN PLAN 8

Items	The amount of crop sold (bushel)	Cost per unit (\$)	Profit (+) or cost (-) (\$)
Gross profit in final program			6,151
Wheat	1,446	0.534510	-772
Barley	269	0.443145	-119
Sunflowers			290
Total net farm receipts			5,550

TABLE XXXVIII  
 ACTIVITIES INCLUDED IN THE FINAL  
 PROGRAM IN PLAN 8

Type of activities	Unit	Quantity
Cow-calves	head	8
Feeder hogs	head	169
Selling wheat	bushel	1,446
Selling barley	bushel	269

The resource situation is shown in table XL. The most limiting resources are land and the intermediate product, barley. Five enterprises of crop-rotation compete for land while five enterprises of livestock compete for barley.

Activities in the final program in plan 9 are shown in table XLI indicating that there are 92 head of raised hogs and 63 units of 5-year-crop-rotation including 1,260 bushels of wheat, 1,008 bushels of oats, 756 bushels of barley, 34,209 pounds of sunflower and 560 bushels of flax. Barley is all used but some of each of the other intermediate products are left over; these situations are shown in table XLII.

The total farm receipts: In the final program, the total gross farm receipts are \$9,012. If the costs of wheat, oats, sunflower, and flax are deducted from the total farm receipts, the total net farm receipts are \$7,043 as shown in table XLIII.

TABLE XXXIX

## REAL ACTIVITIES IN PLAN 9

Types of activities	Unit	Designation
3-year-rotation	number	P <sub>1</sub>
4-year-rotation	number	P <sub>2</sub>
5-year-rotation	number	P <sub>3</sub>
6-year-rotation	number	P <sub>4</sub>
8-year-rotation	number	P <sub>5</sub>
Raised hogs	head	P <sub>6</sub>
Feeder hogs	head	P <sub>7</sub>
Cow-calves	head	P <sub>8</sub>
Steer calves	head	P <sub>9</sub>
2-year-steers	head	P <sub>10</sub>
Selling wheat	bushel	P <sub>11</sub>
Selling oats	bushel	P <sub>12</sub>
Selling barley	bushel	P <sub>13</sub>
Selling hay	pound	P <sub>14</sub>
Selling sunflowers	pound	P <sub>15</sub>
Selling flax	bushel	P <sub>16</sub>
Buying oats	bushel	P <sub>17</sub>
Buying hay	pound	P <sub>18</sub>

TABLE XL

## RESOURCE SITUATION IN PLAN 9

Type of resource	Unit	Designation	Amount of resources	Amount used	Amount left over
Land	acre	P <sub>19</sub>	315	315	-0-
Capital	\$	P <sub>20</sub>	6,320	6,100	220
Summer labour	hour	P <sub>21</sub>	2,968	930	2,038
Winter labour	hour	P <sub>22</sub>	2,200	168	2,032
Building space	square feet	P <sub>23</sub>	4,320	427	3,893

TABLE XLI

ACTIVITIES INCLUDED IN THE FINAL  
PROGRAM IN PLAN 9

Types of activities	Unit	Quantity
5-year-rotation	number	63
Raised hogs	head	92
Selling wheat	bushel	1,214
Selling oats	bushel	357
Selling sunflowers	pound	34,209
Selling flax	bushel	560

TABLE XLII

## INTERMEDIATE PRODUCTS IN PLAN 9

Types of intermediate products	Unit	Amount produced	Amount used	Amount left over
Wheat	bushel	1,260	46	1,214
Oats	bushel	1,008	651	357
Barley	bushel	756	756	-0-
Sunflowers	pound	34,209	-0-	34,209
Flax	bushel	560	-0-	560

TABLE XLIII

## THE TOTAL FARM RECEIPTS IN PLAN 9

Items	Quantity unit	Total amount of crop sold	Cost per unit (\$)	Profit (+) or cost (-) (\$)
Gross profit				9,012
Wheat	bushel	1,214	0.534516	-634
Oats	bushel	357	0.350723	-122
Sunflower	pound	34,209	0.022000	-753
Flax	bushel	561	0.820000	-460
Total net farm receipts				7,043

Plan 10: A consideration of alternative crop-rotation systems and livestock enterprises with unlimited capital adjusted to a father-son arrangement.

In this plan there are 19 activities, besides buying barley, the other activities are identical with those of table XXXIX in plan 9. The input-output coefficients of these activities are identical with those of tables IX, XIV and XIX.

The resource situation is shown in table XLIV. The amount of capital needed in this plan is \$28,789. Land and building space are the most limiting resources which are fully used. The amount of resources left over are summer labour at 1,009 hours and winter labour at 607 hours.

A summary of the programs which are derived from plan 10 is shown in table XLV. The result indicates that the selection of different combinations of enterprises is dependent upon the amount of available capital. The maximum output which can be found in the final program are 4-year-crop-rotation at 78.75 units, raised hogs 400 head and steer-cow 118 head. Maximum output cannot represent maximum profit. An optimum combination of resources depends on the marginal value productivity of capital and the marginal cost function.

Figure 6 is a graphic representation of table XLIII. The optimum combination of enterprises is indicated for various levels of capital input by the lines AA', BB', CC', DD', EE', FF', and GG'. If the studied farm has \$5,023 of capital, the line AA' indicates that 39.375 units of 8-year-crop-rotation and 23 head

TABLE XLIV

## RESOURCE SITUATION IN PLAN 10

Type of resources	Unit	Designation	Amount of resources	Amount used	Amount left over
Land	acre	P <sub>20</sub>	315	315	-0-
Capital	\$	P <sub>21</sub>	unlimited	28,789	-0-
Summer labour	hour	P <sub>22</sub>	2,968	1,959	1,009
Winter labour	hour	P <sub>23</sub>	2,200	1,593	607
Building space	square feet	P <sub>24</sub>	4,320	4,320	-0-

of cow-calves are to be considered as the most profitable enterprises. If capital level is at \$6,886, the line of BB' indicates that 39.375 units of 8-year-crop-rotation, 19 head of cow-calves and 21 head of steer calves are to be considered as the most profitable enterprises. Similarly, the lines CC', DD', EE', FF', and GG' correspond to the capital levels of \$7,237, \$7,615, \$12,199, \$18,044 and \$28,789 respectively. The optimum combination of enterprises can be found at the intersections of each line.

TABLE XLV

A SUMMARY OF THE PROGRAMS OBTAINED IN PLAN 10

Sections (1)	Capital needed (\$) (2)	4-year-crop- rotation (P <sub>2</sub> ) (3)	8-year-crop- rotation (P <sub>5</sub> ) (4)	Cow-calves (head) (P <sub>8</sub> ) (5)	Steer calves (head) (P <sub>9</sub> ) (6)
3	5,023	-	39,375	23	-
4	6,886	-	39,375	19	21
5	7,237	78.75	--	19	21
6	7,615	78.75	--	23	-
7	12,199	78.75	--	10	67
8	18,044	78.75	--	-	118
9	28,789	78.75	--	-	118

(continued)



TABLE XLV - continued

Sections	(1)	(2)	(7)	(8)	(9)	(10)
	Capital needed (\$)	Raised hogs (head)	Selling wheat (bushel)	Buying oats (bushel)	Buying barley (bushel)	
	(P6)	(P11)	(P17)	(P19)		
3	5,023	--	--	--	--	--
4	6,886	--	--	--	--	--
5	7,237	--	--	--	--	--
6	7,615	118	1,516	--	544	
7	12,199	-	1,574	--	929	
8	18,044	-	1,574	428	1,949	
9	28,789	400	1,375	3,452	5,232	

(continued)

TABLE XLV - continued

Sections	Capital needed (\$)	Cost of rotation (\$)	Total net income (\$)	Marginal value productivity of capital (\$)
(1)	(2)	(11)	(12)	(13)
3	5,023	1,852	547	0.108899
4	6,886	1,852	1,534	0.529790
5	7,237	1,390	1,999	1.324786
6	7,615	1,390	3,213	3.211640
7	12,199	1,390	5,760	0.556628
8	18,044	1,390	7,779	0.345423
9	28,789	1,390	10,801	0.281247

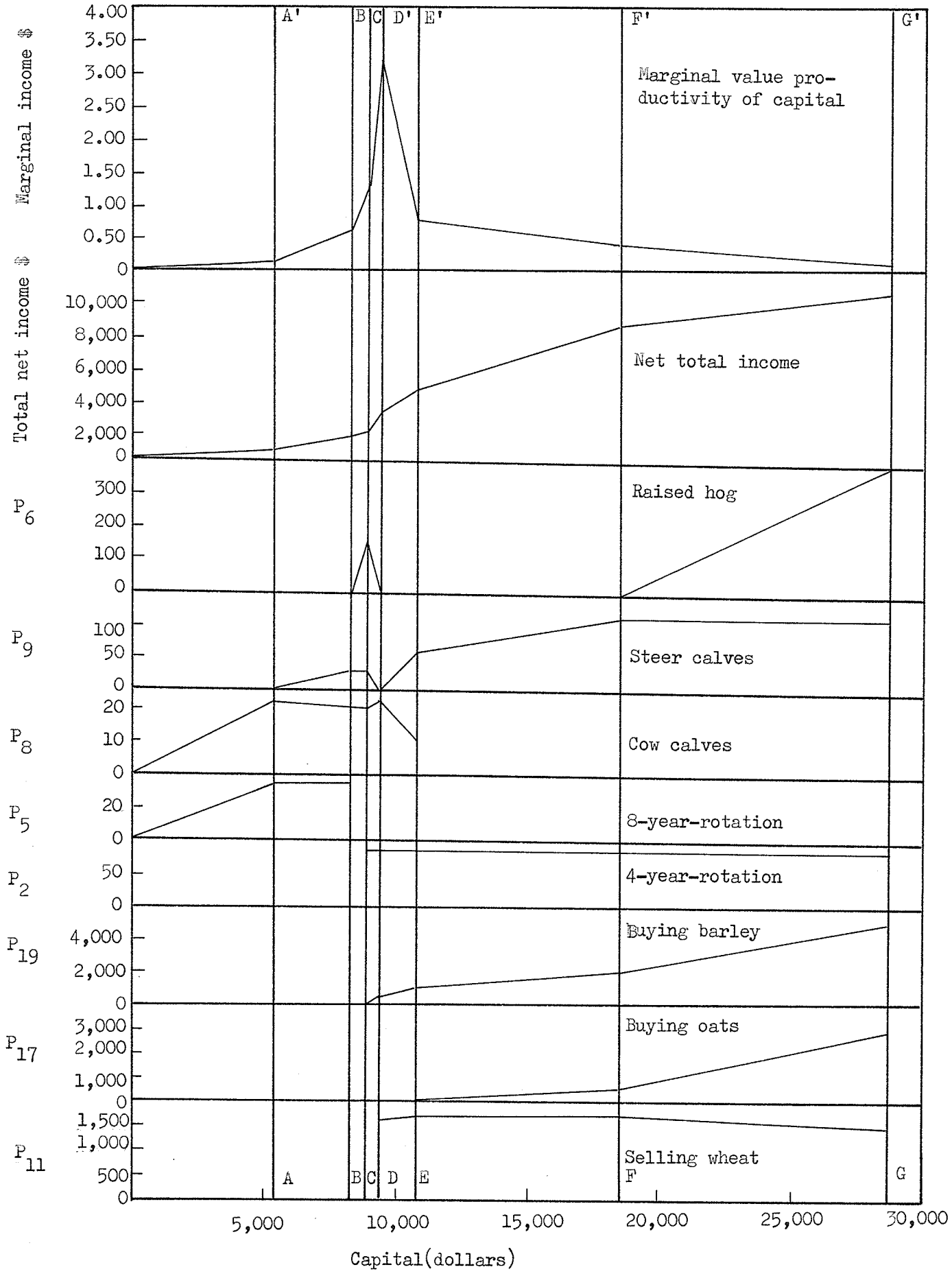


Fig. 6. Graphic representation of table 43

(C) Comparison of the ten alternative plans in the studied farm.

The optimum economic efficiency for each of the ten alternative plans for the studied farm is summarized in table XLVI. The advantages and disadvantages of these ten alternative plans discussed in the following section are based on the numerical figure of total profit.

In plan 1 and plan 3, the most profitable enterprise was raised hogs. Capital needed was \$1,881. The resources left over were building space at 1,572 square feet, summer labour at 83 hours and winter labour at 866 hours. The total profit was \$2,902 and marginal value productivity of capital was \$1.54. If 5% of interest rate is regarded as marginal cost, then plan 1 was far from the optimum condition and more capital should be invested because the marginal value productivity of capital was greater than the price of capital.

In plan 2, the most profitable enterprises were steer calves and cow-calves. The capital needed was the same as that in plan 1, but the quantities of resources left over were greater and the marginal value productivity of capital and the total profit were smaller in comparison to plan 1. These results indicate that the economic efficiency of plan 2 was lower than that of plan 1.

In plan 4, the most profitable enterprise was the 3-year-crop-rotation. Capital needed was \$4,439, total profit \$3,809 and marginal value productivity of capital \$0.858079. There were no livestock enterprises. The quantities of resources left over

were greater than those in plan 1.

In plan 5, capital needed was \$12,325. The most profitable enterprises were feeder hogs and the 3-year-crop-rotation. The total available quantities of land and building space were used up. The idle resource of winter labour was 676 hours which was the smallest amount left over as compared to other plans mentioned above. The marginal value productivity of capital was \$0.505076 and the total profit was \$7,561. That is, the economic efficiency in plan 5 was greater than those of other plans in the present situation in terms of its total profit.

In plan 6, the most profitable enterprise was raised hogs. Capital needed was \$3,678. Oats and barley were the most limiting resources. The total profit and the marginal value productivity of capital were \$6,181 and \$1.68 respectively. The resources left over are summer labour at 1,495 hours, winter labour at 1,866 hours and building space at 3,474 square feet.

In plan 7, the most profitable enterprise was steer calves. Capital and hay were the most limiting resources and were completely used. Marginal value productivity of capital was \$1.51. The quantities of resources left over were greater than those in plan 6.

In plan 8, the most profitable enterprises were cow-calves and feeder hogs. Capital needed was \$3,501. Oats and hay were the most limiting resources. The quantities of resources left over and the marginal value productivity of capital were quite close to those in plan 6.

In plan 9, the most profitable enterprises were raised hogs and 5-year-crop-rotation. Capital needed was \$6,100. The most limiting resources were land and barley. The total profit was \$7,043 and marginal value productivity of capital was \$1.15. The resources left over were summer labour at 2,038 hours, winter labour at 2,032 hours and building space at 3,893 square feet.

In plan 10, the most profitable enterprises were steer calves, raised hogs and the 4-year-crop-rotation. Capital needed was \$28,789. The total quantities of land, oats, barley and building space were used up. Six hundred and nine hours of winter labour left over was the smallest amount as compared with other plans. Marginal value productivity of capital was \$0.28 and total profit was \$10,810. That is to say, plan 10 is the most profitable plan in the adjusted situation in terms of its total profit.

TABLE XLVI

## SUMMARY OF THE RESULTS OF THE TEN ALTERNATIVE PLANS

Plans	Capital needed (\$)	The most limiting resource	Profitable enterprises	Land left over (acre)	Summer labour left over (hour)
*1	1,881	Capital	Raised hogs	-0-	83
*2	1,881	Capital and hay	Steer calves, cow-calves	-0-	124
*3	1,881	Capital	Raised hogs	-0-	83
4	4,439	Capital	3-year-crop-rotation	8	783
5	12,325	Land and building space	3-year-crop-rotation and feeder hogs	-0-	461
*6	3,678	Oats and barley	Raised hogs	-0-	1,495

(continued)

\* Intermediate products are based on inventory.

TABLE XLVI - continued

Plans	Capital needed (\$)	The most limiting resource	Profitable enterprises	Land left over (acre)	Summer labour left over (hour)
*7	3,762	Capital and hay	Steer calves	-0-	1,527
*8	3,501	Oats and hay	Cow-calves and feeder hogs	-0-	1,546
9	6,100	Land and barley	5-year-crop-rotation and raised hogs	-0-	2,038
10	28,789	Land, oats, barley and building space	4-year-crop-rotation, steer calves and raised hogs	-0-	1,009

\* Intermediate products are based on inventory.

(continued)



TABLE XLVI - continued

Plans	Capital needed (\$)	Winter labour left over (hour)	Building space left over (square feet)	Total profit (\$)	Marginal value productivity of capital (\$)
*1	1,881	866	1,572	2,902	1.542796
*2	1,881	889	1,487	1,640	0.871876
*3	1,881	866	1,572	2,902	1.542796
4	4,439	1,099	2,160	3,809	0.858076
5	12,325	676	-0-	7,561	0.505076
*6	3,678	1,866	3,474	6,191	1.683252
*7	3,762	1,912	3,497	5,683	1.510632
*8	3,501	1,870	3,384	6,151	1.756927
9	6,100	2,032	3,893	7,043	1.154590
10	28,789	609	-0-	10,810	0.281248

\* Intermediate products are based on inventory.

## CHAPTER VII

### SUMMARY AND CONCLUSION

This study is based on survey data collected from 1957 to 1959 from the studied farm.

The analysis of this study substantiates the hypothesis that the optimum farm-plan will differ for different patterns of enterprises and for different levels of the supply of resources. Its objective is to select a best plan in order to maximize profit and to minimize cost.

In this study two situations are considered: the first one is the present situation (plan 1 to plan 5) where it is assumed that the owner of the studied farm continued to operate his farm business; while the other is the adjusted situation (plan 6 to plan 10) where it is assumed that the owner's son will take part in the farm business.

With the exception of plan 5 and plan 10, calculated by means of the continuous form of linear programming method, the other eight plans were computed by applying the simplex linear programming method. The results of this analysis and suggestions are stated as follows:

1. Plan 5 is the most profitable plan in the present situation, while plan 10 is the most profitable plan in adjusted situation.

2. If the owner of this studied farm continued to operate his farm business, it is suggested: (1) that plan 5 should be adopted, (2) that capital of \$12,325 should be applied, (3) that 105 units of the 4-year-crop-rotation should be carried out, and (4) that 387 head of feeder hogs should be fed. The total output produced and the profit gained are estimated in plan 5 as shown in table XLVII. There are no cattle. Three hundred and fifteen acres of land, 1,680 bushels of oats and 2,160 square feet of building space are used up. Barley is bought to the extent of 414 bushels and 1,918 bushels of wheat are sold. The resources left over are summer labour at 461 hours and winter labour at 676 hours. Total profit is \$7,562 and marginal value productivity of capital is \$0.511.

3. If the owner's son takes part in the farm business, it is suggested: (1) that plan 10 should be adopted, (2) that \$28,789 of capital should be applied, (3) that 78.75 units of the 4-year-crop-rotation should be carried out, and (4) 118 head of steer calves and 400 head of raised hogs should be fed. The total output produced and the profit gained are estimated in plan 10 as shown in table XLVII. Land, hay and building space are used up. Oats bought, barley bought and wheat sold are 3,452, 5,232 and 1,375 bushels respectively. The resources left over are summer labour at 1,009 hours and winter labour at 607 hours. Total profit is \$10,801 and marginal value productivity of capital is \$0.28.

4. If plan 10 is carried out, the following points are stressed:

- a. Land should be increased to the point where summer labour will be used up.
- b. Any profitable supplementary enterprise should be carried out and extended until the winter labour will be used up.

5. If land cannot be extended, it is suggested that the owner of this studied farm should adopt plan 5, because the total profit of each share in plan 10 is \$5,403 (the total profit of \$10,801 is divided between father and son) which is less than that of \$7,562 in plan 5 (see table XLVII).

TABLE XLVII

## A COMPARISON OF THE RESULTS FROM PLAN 5 TO PLAN 10

Plans	Capital needed (\$)	Land (acre)		Summer labour (hour)		Winter labour (hour)			
		Total amount used	Amount left over	Total amount used	Amount left over	Total amount used	Amount left over		
5	12,325	315	-0-	1,484	1,023	467	1,100	424	676
10	28,789	315	-0-	2,968	1,959	1,009	2,200	1,593	607

(continued)

TABLE XLVII - continued

Plans	Capital needed (\$)	Building space (square feet)		The most profitable enterprises				
		Total amount used	Amount left over	3-year- crop- rotation	4-year- crop- rotation	Steer calves (head)	Raised hogs (head)	Feeder hogs (head)
5	12,325	2,160	2,160	-0-	105	---	---	---
10	28,789	4,320	4,320	-0-	---	78.75	118	400

(continued)

TABLE XLVII - continued

Plans	Capital needed (\$)	Intermediate products								
		Wheat (bushel)		Oats (bushel)		Barley (bushel)				
		Total amount used	Amount left over	Total amount used	Amount left over	Total amount used	Amount left over			
5	12,325	2,100	182	1,918	1,680	1,680	-0-	1,260	1,674	-414
				(the amount sold)						(the amount bought)
10	28,789	1,575	200	1,375	1,260	4,712	-3,452	945	6,177	-5,232
				(the amount sold)			(the amount bought)			(the amount bought)

(continued)

TABLE XLVII - continued

Plans	Capital needed (\$)	Intermediate products		Total profit (\$)	Marginal value productivity of capital (\$)
		Total amount	Hay (pound)		
		amount used	Amount left over		
5	12,325	---	-0-	7,562	0.5111456
10	28,789	299,250	299,250	10,801	0.281247