TESTING FOR STATIC RESOURCE AND PRODUCT EQUILIBRIUM UNDER OUTPUT PRICE RISK: WESTERN CANADIAN AGRICULTURE, 1961-84

by

GABRIEL TOICHOA BUAHA

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in Partial Fulfilment of the Requirements
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ABSTRACT

This thesis provides econometric tests of the hypothesis of static competitive farm resource and output equilibrium in Western Canadian agriculture over the period 1961-84. These tests are conducted with and without risk aversion and price uncertainty. The theoretical models assumed for this sector included a cost function approach, an indirect utility framework, and a stochastic profit function model. Two different tests are conducted for the proposed hypotheses: (1) Wald chi-square tests of the symmetry restrictions implied by cost minimization and/or utility maximization and/or profit maximization and (2) Wald chi-square, Hauseman specification, and likelihood ratio tests of the first order conditions for static equilibrium for quasi-fixed inputs and outputs.

The symmetry restrictions implied by cost minimization are not rejected given a short run Translog cost function for Western Canadian agriculture. Given these restrictions, static competitive equilibrium is not rejected for dairy, poultry, farm produced capital, and farm land. For farm machinery the outcome of this hypothesis depends on the econometric test conducted. For crop and livestock outputs, similar hypothesis tests are inconclusive due to the significant impact of output price risk and uncertainty on the results of these tests. Overall, static farm resource and product equilibrium is rejected for the whole agricultural sector of Western Canada over the period 1961-84.

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Chapter 1. Introduction

1.1 Statement of problem

A question that has seldom been asked is that given the structure of Canadian agriculture, are resources employed and outputs produced in this sector in static competitive equilibrium? Due to the cyclical nature of agriculture, it is argued that farm resources are seldom in equilibrium since producers are unable to anticipate product prices. The high costs of adjusting quasi-fixed farm inputs also contribute to prolonged periods of farm resource disequilibrium. In the Canadian context, a primary objective in the price pooling system of the Canadian Wheat Board (CWB) is to reduce the impact in price fluctuations on producers' optimizing decisions. A second empirical question that has barely been investigated in Canadian agriculture is whether risk aversion and price uncertainty affect resource and product equilibrium. This study provides tests for the questions raised above.

1.2 Objectives and hypotheses of the study

The objective of this research is to test the following hypotheses given the characteristics of the Canadian agricultural sector over the period 1961-84:

(1.a) were Western Canadian farmers cost minimizers and/or profit maximizers?;

(1.b) were farm resources and outputs in static competitive equilibrium in Western Canadian agriculture? And did farmers' response to price risk and uncertainty affect these conditions?

To evaluate the above hypotheses three different tests are conducted: (1) Wald chi-square tests of the symmetry restrictions implied by static cost minimization and/or profit maximization; (2) Wald chi-square, Hauseman specification, and Likelihood ratio tests of first order conditions for static equilibrium for quasi-fixed inputs and/or outputs under risk neutrality; (3) Wald chi-square test of first order conditions under a simple model of risk aversion and output price uncertainty. These hypothesis tests are conducted under a cost function approach, an indirect utility framework, and a deterministic profit function approach.

1.3 Importance of this research

One of the intended goals of most agricultural policies is to increase the efficient allocation of resources among different enterprises. Analyzing the impacts of risk aversion and output price uncertainty on farm resource and output equilibrium is of upmost importance for the Canadian agricultural sector due to their potential impact on resource allocation. This thesis provides tests of the hypotheses of farm resource and product equilibrium in Western Canadian agriculture.

This study departs from previous research in that here, the hypotheses of static resource and output equilibrium are conducted under a technology that allows for risk aversion and output price uncertainty and incorporates products where quantities are exogenously determined (supply managed products).

1.4 Literature review

The dual profit function has been used to estimate the characteristics (e.g., substitution and expansion effects) of the underlying production technology of an industry. Earlier applications of the profit function approach to agriculture, Lau (1972); Lau and Yotopoulos (1972); and Yotopoulos, Lau, and Lin (1976) have assumed a Cobb-Douglas functional form which is more restrictive than other available forms.

Recent studies have significantly improved on the choice of flexible functional forms. However, a large number of these implicitly assume the equality between shadow and market prices of quasi-fixed factors, Lopez (1980, 1984); Antle (1984); and Dupont (1991). This assumption has led to the estimation of the derived demand, supply, and shadow price equations jointly without any attempt to individually test for the long-run static equilibrium conditions for each quasi-fixed factor. Other studies have assumed long-run static equilibrium that is, all inputs are variable, and proceeded to estimate a multi-product profit function, McKay, Lawrence, and Vlastuin (1983). The down side of this model is the high probability of misspecifying the actual behaviour of producers, i.e., agriculture is unlikely to exhibit long-run equilibrium.

Dynamic equilibrium models have also been employed to model the behaviour of producers, Lopez (1985). In this case, the costs of adjustment are explicitly incorporated in the model and the firm is assumed to be in dynamic rather than static equilibrium. An intertemporal profit maximization or cost minimization problem is solved for equations of motion for the fixed factors. These dynamic equilibrium models have several short-comings. They are empirically difficult to apply and have demanding data requirements, Squires (1987). Furthermore, these models also run the risk of misspecifying the equations of motion for the fixed factors. The restricted profit function framework is an alternative which sidesteps these difficulties but does not provide any explanation on the way fixed factors adjust from one equilibrium point to another.

Few studies have rigorously tested for the validity of static competitive profit-maximising models in the agricultural sector. Junankar (1980a, 1980b) tested for these restrictions but used a nonflexible flexible functional form. Kulatilaka (1985) and, Schankerman and Nadiri (1986) provide a rigorous framework for testing the validity of static equilibrium models and rates of return to fixed factors under a model of short-run cost minimization.[1] They do not specifically test for the restrictions implied by this behavioral assumption. A broader set of tests for static equilibrium and profit maximization in agriculture are conducted by Coyle (1991). This thesis expands upon these tests by allowing for risk aversion and price uncertainty and incorporating restricted outputs.

1.5 Organization of the Study

This thesis consists of five chapters. Chapter 1 has discussed the problem of static resource equilibrium and profit maximization given supply managed policies and price uncertainty in Western Canadian agriculture. This chapter also has described the objectives of the study, the hypotheses to be tested, and the importance of this research. Chapter 2 will provide the theoretical background of the study. This chapter will include three different theoretical models: cost function, indirect utility function, and nonstochastic profit function. Chapter 3 will provide a brief discussion of the data set employed in the study. Chapter 4 will apply the theoretical models described in Chapter II to the

[1] In this model, producers are postulated to minimize costs by choosing their levels of variable factors conditional not only on the level of output, but also on the level of fixed inputs (i.e., factors not easily adjusted from one period to the next.)

econometric study of production decisions in Western Canadian agriculture. Chapter 5 will provide a summary and overall conclusions and limitations of the study. The final data set used in this thesis is provided in the appendix.

Chapter 2. Theoretical Background

The advantages of the duality approach to modelling production decisions are well known (e.g. Fuss and MacFadden, Chambers 1988). Duality allows for the direct specification of the cost or profit functions and by simple differentiation of these with respect to input and/or output prices, factor demand and product supply equations can be derived. Testing for the conditions of static profit maximization is important particularly when a dual approach is chosen. If these conditions are not satisfied but imposed, econometric estimates of the dual function (e.g., profit, cost) or derived equations (e.g., supply, demand) can substantially misspecify the actual behaviour of the agents and should not be used for policy recommendations or other purposes.

To evaluate the hypotheses proposed in Chapter 1, a production process where some of the output prices are assumed stochastic is described next. A dual cost function is considered first, followed by an indirect utility approach. This Chapter concludes with a nonstochastic profit function approach.

2.1 A Short-Run Cost Function Approach

The agricultural industry is characterised by multioutput firms. Suppose that the outputs of such a firm can be partitioned in two groups: restricted and nonrestricted. Now, consider the production decision of this firm which produces M nonrestricted outputs by combining N variable inputs given R quasi-fixed factors and L output restricted products.

Assuming producers are price takers in the factor markets, the variable cost function for a multiproduct firm can be represented as

$$c(w,k,y,q) = \min_{x} \sum_{i=1}^{N} w^i x^i : x \in T(k,y,q) \qquad (2.1)$$

where y is an M-dimensional vector of nonrestricted outputs; x is an N-dimensional vector of variable inputs; k is an R-dimensional vector of quasi-fixed factors; q is an L-dimensional quantity vector of supply managed commodities; w is an N-dimensional vector of strictly positive input prices; $T(k,y,q)$ is the set of feasible x that combined with k can produce y,q. To ensure correspondence with a production possibility set or transformation function it is sufficient for the cost function $c(w,k,y,q)$ to be:

(2.1a) non-negative real valued for positive (w);

(2.1b) homogeneous of degree one in (w);

(2.1c) concave and continuous in (w);

(2.1d) non-decreasing in w;

(2.1e) differentiable with respect to all its arguments.

Assuming a constant returns to scale (CRTS) production function for the industry, then the cost function in (2.1) is linearly homogeneous in (k,y,q), i.e., $\lambda c(w,k,y,q) = c(w,\lambda k,\lambda y,\lambda q)$. The variable cost function embodies sufficient information to completely describe the production technology and thus the production possibility set if the restrictions defined in equations (2.1a)-(2.1e) hold. Differentiating the variable cost function (2.1) with respect to variable input prices gives

$$\partial c(w,k,y,q)/\partial w^i = x^i(w,k,y,q) \qquad i=1,\ldots,N \tag{2.2}$$

(Shephard's Lemma), where $x^i(w,k,y,q)$ are the optimum levels of input demand. From properties (2.1a)-(2.1e) of the cost function (2.1), it follows that the derived conditional demand equations (2.2) are homogeneous of degree zero in (w) and the Hessian matrix of second derivatives $c_{w^i w^j}$ of the cost function (2.1) is symmetric negative semidefinite (nsd).[2] It can be shown that these conditions (i.e., homogeneity of degree zero and nsd matrix) represent all the local properties that are imposed on the factor demand equations (2.2) by the hypothesis of competitive cost minimization. If the behavioral model (2.1) describes the actual decision process of the multiproduct firm, then the following symmetry or reciprocity conditions

$$\partial x^i(w,k,y,q)/\partial w^j = \partial x^j(w,k,y,q)/\partial w^i \qquad i,j=1,\ldots,N \tag{2.3}$$

hold. Restrictions (2.3) imply a joint test of the first order conditions for cost minimization and the existence of a parent cost function from which the factor demand equations (2.2) are derived.

To analyze the static competitive equilibrium levels of quasi-fixed factors, nonrestricted outputs, and supply managed products, define the long run profit maximization problem

[2] Homogeneity of degree zero of these functions follows from Euler's Theorem. The symmetric negative semidefiniteness of the Hessian matrix is due to the concavity in (w) of the cost function (2.1).

$$\max_{k,y,q} \ \Sigma_{j=1}^{M} p^j y^j + \Sigma_{i=1}^{L} p^{qi} q^i - c(w,k,y,q) - \Sigma_{i=1}^{R} w^{ki} k^i - (\alpha/2) \Sigma_{i=1}^{M} \Sigma_{j=1}^{M} y^i y^j Vp^{ij} \quad (2.4)$$

where p^j, p^{qi}, and w^{ki} are the prices for nonrestricted outputs, supply managed outputs, and quasi-fixed inputs respectively, $\alpha > 0$ is the coefficient of risk aversion assuming price uncertainty for nonrestricted outputs, and Vp represents the price variance and covariance matrix of these outputs. Model (2.4) assumes a linear mean-variance utility function for the producer defined as

$$u = E\pi - (\alpha/2) V\pi \quad (2.5)$$

where $E\pi$ is expected profits, $V\pi$ is the variance of profits. This model assumes that outputs y, q, inputs x, restricted output prices p^q, and input prices w, w^k are all nonstochastic. The first order conditions for an interior solution to (2.4) are

$$\partial c(w,k^*,y^*,q^*)/\partial k^i + w^{ki} = 0 \qquad i = 1,...,R \quad (2.6)$$

$$\partial c(w,k^*,y^*,q^*)/\partial y^j - p^j + \alpha \Sigma_{i=1}^{M} y^j Vp^{ij} = 0 \qquad j = 1,...,M \quad (2.7)$$

$$\partial c(w,k^*,y^*,q^*)/\partial q^i - p^{qi} = 0 \qquad i = 1,...,L \quad (2.8)$$

where asterisks denote the optimum levels of the choice variables. Note that if prices (w^{ki},p^j,p^{qi}) are identical across firms and if $\alpha = 0$ (risk neutrality), then the cost functions c(w,k,y,q) for individual firms satisfy the conditions for consistent linear aggregation over static competitive profit maximizing combinations of k,y,q, Coyle (1991, p.6). Conditions (2.6)-(2.8) represent the shadow price equals market price principle and if the industry is in full static equilibrium with profit maximization for all inputs and outputs, conditions

(2.6)-(2.8) are satisfied jointly with (2.3). Particularly, (2.8) provides a direct way of testing whether the ex post or fixed prices of supply managed products are equal to the ex ante or cost of producing a unit of these outputs. The above derivatives of the cost function can be interpreted as follows: $\partial c(w,k^*,y^*,q^*)/\partial k^f$ is the shadow price of quasi-fixed input k^f, that is, the impact of a marginal increase in k^f on variable cost; $\partial c(w,k^*,y^*,q^*)/\partial y^j$ is the marginal variable cost of producing output y^j; and $\partial c(w,k^*,y^*,q^*)/\partial q^e$ is the marginal variable cost of producing one unit of supply managed output q^e. Model (2.1) is useful in testing the hypotheses of static profit maximization over outputs and of static resource equilibrium for quasi-fixed factors.

For the empirical analysis of the proposed hypotheses of static resource and product equilibrium, a Translog functional form is postulated for the industry short-run cost function

$$\ln c = a_0 + \Sigma_e a_e D^e + 1/2\Sigma_e \Sigma_s a_{es} D^e D^s \tag{2.9}$$

where: D=(ln w^1,...,ln w^N, ln k^1,...,ln k^R, ln y^1,...,ln y^M, ln q^1,...,ln q^L, t); c is total variable cost; and t denotes a time trend intended as a proxy for technological change. Assuming a homothetic production function, Hicks neutral technical change coincides with share neutrality, Chambers (1988). Thus, the share neutrality condition $\partial \log s^i(w,k,y,q)/\partial t = 0$, where s^i is the share of input i in short run variable cost, is equivalent to Hicks neutrality. Furthermore, share-using and share-saving inputs can be defined depending on whether $\partial \log s^i/\partial t$

is positive or negative respectively. Short run cost minimization implies, by Shephard's lemma (2.2), the following conditional cost share equations for variable inputs

$$sx^i = a_i + \textstyle\sum_s a_{is} D^s \qquad i = 1,\ldots,N \tag{2.10}$$

where $sx^i = w^i x^i/c$. For quasi-fixed input k^f, (2.6) and static competitive equilibrium imply the cost share equations

$$\begin{aligned} -sk^f &= \partial \ln c(w,k,y,q,t)/\partial \ln k^f \\ &= a_{N+f} + \textstyle\sum_s a_{N+f,s} D^s \\ & \qquad f = 1,\ldots,R \end{aligned} \tag{2.11}$$

where $sk^f = w^{kf}k^f/c$. Also, competitive equilibrium for nonrestricted outputs y^j and supply managed products q^e imply, using (2.7) and (2.8), the following cost share equations

$$\begin{aligned} sy^j &= \partial \ln c(w,k,y,q,t)/\partial \ln y^j \\ &= a_{N+R+j} + \textstyle\sum_s a_{N+R+j,s} D^s + \alpha \sum_{i=1}^{M} y^i y^j Vp^{ij}/c \\ & \qquad j = 1,\ldots,M \end{aligned} \tag{2.12}$$

$$\begin{aligned} sq^e &= \partial \ln c(w,k,y,q,t)/\partial \ln q^e \\ &= a_{N+R+M+e} + \textstyle\sum_s a_{N+R+M+e,s} D^s \\ & \qquad e = 1,\ldots,L \end{aligned} \tag{2.13}$$

where $sy^j = p^j y^j/c$, $sq^e = p^{qe} q^e/c$ respectively.

Parametric restrictions can be defined for equations (2.10)-(2.13) depending on the behavioral assumption specified. The existence of a translog cost function (2.9) and short run cost minimization imply the following symmetry restrictions

$$a_{ij} = a_{ji} \qquad i,j = 1,...,N \tag{2.14}$$

on coefficients for variable cost share equations (2.10). In addition, if the quasi-fixed inputs k are at their static equilibrium levels, then the additional reciprocity restrictions

$$a_{ij} = a_{ji} \qquad i,j = 1,...,N+R \tag{2.15}$$

are satisfied for equations (2.10)-(2.11). In general, if the industry is at static competitive equilibrium for quasi-fixed inputs and profit maximization over all outputs (nonrestricted and supply managed) then the full set of symmetry restrictions

$$a_{ij} = a_{ji} \qquad i,j = 1,...,N+R+M+L \tag{2.16}$$

holds for equations (2.10)-(2.13). Such restrictions where constrained outputs are explicitly incorporated in the static equilibrium conditions of the industry are not recognized in either Moschini or Lopez.

2.1.1 Hypothesis Testing

Two tests are conducted: (1) Wald chi-square tests of symmetry restrictions across share equations that are implied by static resource and product equilibrium for quasi-fixed inputs and outputs; (2) Hausman specification tests of augmenting the share equations (2.10) for variable inputs by appropriate first order conditions (2.11)-(2.13) for static equilibrium for quasi-fixed inputs and outputs. The parametric restrictions (2.14)-(2.16) are useful in testing hypotheses of static resource and output equilibrium. The reciprocity restrictions (2.14) implied by the existence of a short-run translog cost function (2.9) are tested as a necessary condition for further analysis. Then, the additional symmetry restrictions (2.15)-(2.16) are parametrically tested for each quasi-fixed input, nonrestricted output, and supply managed product. The null hypothesis in this case is that the share equations for variable inputs (2.10) and particular first order conditions for quasi-fixed factors (2.11)/nonrestricted outputs (2.12)/supply managed outputs (2.13) are consistent with a short run Translog cost function and static competitive equilibrium for the quasi-fixed inputs/nonrestricted outputs/supply managed products. The alternative proposition is that equations (2.10) are consistent with a short run Translog cost function, but coefficients of equations (2.11)-(2.13) for quasi-fixed inputs, nonrestricted outputs, and supply managed products are independent of coefficients for all other equations in the system.

This general framework provides a direct way of testing hypotheses (1.a)-(1.c) described in Chapter 1. To further evaluate these propositions, we next derive Hausman specification tests similar to those introduced by Hausman, (1978); Schankerman and Nadiri, (1986);[3]. To begin, suppose that $\hat{\beta}_0, \hat{\beta}_1$ are two estimators of the coefficients (a_i, a_{is}) in equations (2.10) for variable inputs, and that $\hat{\beta}_0$ is asymptotically efficient and $\hat{\beta}_1$ is consistent under an hypothesis H_0 of cost minimization for certain quasi-fixed inputs and/or profit maximization for some outputs; otherwise, only $\hat{\beta}_1$ is consistent under both H_0 and H_1. This in turn implies that $[var(\hat{\beta}_1) - var(\hat{\beta}_0)]$ is positive semidefinite under H_0. The asymptotic covariance of $\hat{\beta}_0$ and $\hat{q} \equiv \hat{\beta}_1 - \hat{\beta}_0$ is equal to zero under H_0 and

[3] These tests have mostly been applied to manufacturing related sectors. The only application to agriculture is provided by Coyle. These procedures have not been applied to Western Canadian agriculture.

$var(\hat{q}) = var(\hat{\beta}_1) - var(\hat{\beta}_0)$,(Hausman). This lead to a direct test of H_0 against a wide set of alternative hypotheses using the test statistic

$$M = \hat{q}^T var(\hat{q})^{-1} \hat{q} \tag{2.17}$$

which is asymptotically distributed as chi-square under H_0. An efficient estimator $\hat{\beta}_0$ of the coefficients of (2.10) under H_0 can be obtained by estimating equations (2.10) jointly with the appropriate static competitive conditions (2.11) for quasi-fixed inputs and/or (2.12)-(2.13) for all outputs. An estimator $\hat{\beta}_1$ that is consistent under both hypotheses can be obtained by estimating equations (2.10) alone.

2.2 An Indirect Short-Run Utility Function with Stochastic Output Prices Approach

The hypotheses put forward in Chapter 1 can also be evaluated under a linear mean-variance utility function framework with stochastic output prices. The properties of this model are well known (e.g., Dhrymes; Robinson and Barry; Chavas and Pope). The incorporation of stochastic output prices into duality models was first introduced by Coyle. A version of this model incorporating restricted outputs is used to test the propositions of static resource and output equilibrium for quasi-fixed inputs and outputs given stochastic prices for nonrestricted products.

Let us start by assuming that all multiproduct firms that produce M nonrestricted outputs by employing N variable inputs given R quasi-fixed inputs and L restricted outputs face stochastic prices for their variable outputs. All firms are price takers in both output and input markets. Then a particular firm's short-run utility maximization problem can be defined as

$$u(p,w,vp,k,q) = \max_{(x,y \in T)} \Sigma_{j=1}^{M} p^j y^j - \Sigma_{i=1}^{N} w^i x^i - (\alpha/2)\Sigma_{i=1}^{M}\Sigma_{j=1}^{M} y^i y^j VP^{ij} \qquad (2.18)$$

where u(p,w,vp,k,q) represents the dual utility function; that is, the maximum feasible utility given the exogenous variables p,w,vp,k,q. Utility is define here as in (2.5). It can be shown that the function (2.18) satisfies the following standard properties (see Coyle for proofs)

2.2a u(.) is increasing in p, decreasing in w and in elements of vp;

2.2b u(.) is linear homogeneous in (p,w,vp);

2.2c u(.) is convex in (p,w,vp); and

2.2d assuming u(.) is differentiable, then the first order conditions for an interior solution to (2.18) imply

$$\partial u(p,w,vp,k,q)/\partial p^j = y^{j*}(p,w,vp,k,q) \qquad j = 1,...,M \qquad (2.19)$$

$$\partial u(p,w,vp,k,q)/\partial w^i = -x^{i*}(p,w,vp,k,q) \qquad i = 1,...,N \qquad (2.20)$$

(Hotelling's lemma) where $y^{j*}(.)$ and $x^{i*}(.)$ are the optimum levels of nonrestricted outputs and variable inputs given k,q.[4] The properties of the Hessian matrix of u(.) and linear homogeneity in (p,w,vp) imply the following symmetry restrictions for utility maximization

[4] The full set of first order conditions of (2.18) include the partial derivatives of u(.) with respect to the elements of vp. These are omitted for simplicity of presentation.

$$\begin{aligned}
&\partial y^{j*}(.)/\partial w^{i} = -\partial x^{i*}(.)/\partial p^{j} \quad &&j=1,\ldots,M \\
& &&i=1,\ldots,N \\
&\partial y^{j*}(.)/\partial p^{i} = \partial y^{i*}(.)/\partial p^{j} \quad &&i,j=1,\ldots,M \\
&\partial x^{i*}(.)/\partial w^{j} = \partial x^{j*}(.)/\partial w^{i} \quad &&i,j=1,\ldots,N
\end{aligned} \tag{2.21}$$

for a multiproduct firm. This set of conditions should hold simultaneously as well as homogeneity of degree zero in (p,w,vp) of the derived supply and variable demand equations (2.19)-(2.20). To evaluate the static competitive equilibrium levels of quasi-fixed inputs k and supply managed outputs q, define the long run optimization problem

$$\begin{aligned}
u^{*}(p,w,vp,w^{k},p^{q}) = \max_{k,q} u(p,w,vp,k,q) &+ \textstyle\sum_{i=1}^{L} p^{qi}q^{i} - \sum_{i=1}^{R} w^{ki}k^{i} \\
&- (\alpha/2)\textstyle\sum_{i=1}^{M}\sum_{j=1}^{M} y^{i}y^{j}VP^{ij}
\end{aligned} \tag{2.22}$$

where $u^{*}(p,w,vp,w^{k},p^{q})$ denotes the long-run optimum utility of a multiproduct firm given the exogenous variables p,w,vp,w^{k},p^{q}. The first order conditions for an interior solution to (2.22) are

$$\partial u(p,w,vp,k^{*},q^{*})/\partial k^{i} - w^{ki} = 0 \qquad i=1,\ldots,R \tag{2.23}$$

$$\partial u(p,w,vp,k^{*},q^{*})/\partial q^{j} + p^{qj} = 0 \qquad j=1,\ldots,L \tag{2.24}$$

where asterisks denote the optimum levels of the choice variables. The first order conditions (2.23)-(2.24) can be defined as follows: $\partial u(p,w,vp,k^{*},q^{*})/\partial k$ denotes the shadow value of quasi-fixed input k and indicates a one-period increase in utility attainable if, holding restricted output quantities, prices, and variance-covariance of prices constant, the quantity of quasi-fixed factor k is increased by one unit;

$\partial u(p,w,vp,k^*,q^*)/\partial q$ denotes the shadow price of supply managed commodity q and indicates a one-period decrease in utility if, holding quasi-fixed inputs, prices, and variance-covariance of prices constant, the quantity of restricted output is increased by one unit. If the industry is in static equilibrium with utility maximization over all inputs and outputs as in (2.2), conditions (2.23)-(2.24) are satisfied jointly with (2.21). In particular, (2.24) provides a direct way of testing whether the ex post long-run prices of supply managed products are equal to their ex ante or market prices. A Normalized Quadratic functional form is adopted for the industry short-run dual utility function to evaluate the hypothesis of static resource and product equilibrium for quasi-fixed inputs and supply managed outputs. This function is defined as

$$u = a_0 + \Sigma_e a_e D^e + 1/2 \Sigma_e \Sigma_s a_{es} D^e D^s \tag{2.25}$$

where $D = (p^1/w^4,...,p^M/w^4;\, w^1/w^4,...,w^{N-1}/w^4;\, var(p^1)/w^4,...,var(p^M)/w^4;$ $cov(p^i,p^j)/w^4,\, i,j = 1,...,M;\, k^1,...,k^R;\, q^1,...,q^L;\, t)$. Short-run utility maximization and Hotelling's lemma (2.19)-(2.20) imply the following conditional supply equations for nonrestricted output and variable input demand equations

$$\begin{aligned} y^j &= a_j + \Sigma_s a_{js} D^s \qquad j = 1,...,M \\ -x^i &= a_{M+i} + \Sigma_s a_{M+i,s} D^s \qquad i = 1,...,N \end{aligned} \tag{2.26}$$

The first order condition (2.23) and static competitive equilibrium for quasi-fixed inputs k yield the inverse demand equations

$$-w^{kf} = \partial u(p,w,vp,k,q)/\partial k^{f} \qquad f=1,\ldots,R \qquad (2.27)$$
$$= a_{M+N+f} + \sum_{s} a_{M+N+f,s} D^{s}$$

For supply managed outputs, (2.24) and static utility maximization imply the inverse supply equations

$$p^{qe} = \partial u(p,w,vp,k,q)/\partial q^{e} \qquad e=1,\ldots,L \qquad (2.28)$$
$$= a_{M+N+R+e} + \sum_{s} a_{M+N+R+e,s} D^{s}$$

Given certain behavioral assumptions, parametric restrictions can be defined for equations (2.26)-(2.28). For instance, the existence of a Normalized Quadratic function (2.25) and short run utility maximization imply the following reciprocity restrictions

$$a_{ij} = a_{ji} \qquad i,j=1,\ldots,M+N \qquad (2.29)$$

on coefficients for variable supply and input demand equations (2.26). Furthermore, if the quasi-fixed inputs k are at their static equilibrium levels, then the additional symmetry restrictions

$$a_{ij} = a_{ji} \qquad i,j=1,\ldots,M+N+R \qquad (2.30)$$

apply to coefficients of (2.26)-(2.27). Finally, if the industry is at static competitive equilibrium for quasi-fixed inputs and utility maximization over supply managed outputs, then the full set of symmetry restrictions

$$a_{ij} = a_{ji} \qquad i,j = 1,...,M+N+R+L \tag{2.31}$$

are satisfied for equations (2.26)-(2.28).

2.2.1 Hypothesis Testing

Two different tests are conducted: (1) Wald chi-square tests of the additional symmetry restrictions (2.30)-(2.31) implied by static resource equilibrium for quasi-fixed inputs and utility maximization over supply managed outputs; (2) Likelihood ratio tests of the same propositions. The above parametric restrictions (2.30)-(2.31) provide direct procedures for testing the propositions of static resource and output equilibrium . First, the symmetry restrictions (2.29) implied by the existence of a short-run Normalized Quadratic utility function (2.25) are initially tested as a necessary condition for further hypothesis testing. Second, the additional symmetry restrictions (2.30)-(2.31) are parametrically tested for each quasi-fixed input and supply managed product. Under this framework, the null hypothesis is that the variable output supply and input demand equations (2.26) and particular first order conditions for quasi-fixed factors (2.27) and restricted outputs (2.28) are consistent with a short-run Normalized Quadratic utility function and static competitive equilibrium. The alternative proposition is that equations (2.26) are consistent with a short-run Normalized Quadratic utility function, however coefficients of (2.27) for quasi-fixed inputs and (2.28) for restricted outputs are independent of all other coefficients in the system.

To further evaluate the above hypotheses we next derive Likelihood Ratio tests. We start by noting that under H_0, the symmetry restrictions (2.30)-(2.31) are satisfied, and that under H_1 (2.30)-(2.31) do not apply. Now, suppose that L_1 is the log of the likelihood function obtained by maximum likelihood (ML) estimation of equations (2.26) for variable output supplies and input demands jointly with particular first order conditions from (2.27)-(2.28), and that the symmetry restrictions (2.30)-(2.31) are not imposed. L_0 is the log of the likelihood function derived by ML estimation of the same system but imposing the symmetry restrictions (2.30)-(2.31). Then, the likelihood ratio statistic

$$LR = 2 * [\ln(L_1) - \ln(L_0)] \tag{2.32}$$

is asymptotically distributed as chi-square under H_0 with degrees of freedom equal to the difference in free parameters between the two models. L_1 and L_0 can be obtained by Iterative Zellner estimation which yields parameter estimates that are numerically equivalent to those of the maximum likelihood estimator, Berndt (1991, p.463).

The hypotheses of static resource and output equilibrium given the utility function (2.25) can also be tested by Hausman specification tests similar to those derived in section 2.1.1.

2.3 A Nonstochastic Profit Function Approach

The restricted dual profit function for a competitive multiproduct firm can be specified as

$$\pi(p,w,;k,q) \equiv \max_{x,y \in T} \Sigma_{j=1}^{M} p^j y^j - \Sigma_{i=1}^{N} w^i x^i : F(y,x;k,q) = 0 \tag{2.33}$$

where $\pi(p,w;k,q)$ denotes short run economic profits i.e., total revenue less cost of variable inputs, F(y,x;k,q) is a continuous output or transformation function. Note that model (2.33) assumes that expected prices are perfectly forecasted by farmers. The properties of the profit function $\pi(.)$ are well known (e.g., Varian; Chambers (1988)) given certain properties of the underlying technology. Application of Hotelling's lemma yields the following optimum supply and demand functions

$$\partial\pi(p,w;k,q)/\partial p^j = y^{j*}(p,w;k,q) \qquad j=1,\ldots,M \tag{2.34}$$

$$\partial\pi(p,w;k,q)/\partial w^i = -x^{i*}(p,w;k,q) \qquad i=1,\ldots,N \tag{2.35}$$

For quasi-fixed inputs and supply managed outputs, the Envelope theorem provides

$$\begin{aligned} \partial\pi(p,w;k,q)/\partial k^i &= w^{ki} \qquad i=1,\ldots,R \\ \partial\pi(p,w;k,q)/\partial q^j &= -p^{qj} \qquad j=1,\ldots,L \end{aligned} \tag{2.36}$$

where w^k is the vector of rental prices for quasi-fixed inputs, p^q is the vector of fixed prices for restricted outputs. The derived conditional supply and demand equations (2.34), (2.35) can be employed to test for static profit maximizing behaviour. Linear homogeneity of the profit function $\pi(.)$ in prices and the properties of its Hessian matrix imply the following symmetry restrictions for static profit maximization

$$\partial y^{j*}(.)/\partial w^{i} = -\partial x^{i*}(.)/\partial p^{j} \qquad j=1,...,M \quad i=1,...,N$$

$$\partial y^{j*}(.)/\partial p^{i} = \partial y^{i*}(.)/\partial p^{j} \qquad i,j=1,...,M \tag{2.37}$$

$$\partial x^{i*}(.)/\partial w^{j} = \partial x^{j*}(.)/\partial w^{i} \qquad i,j=1,...,N.$$

This set of conditions should hold simultaneously as well as homogeneity of degree zero in prices for output supply and factor demand equations (2.34), (2.35).

At the individual firm level, long run competitive profit maximization implies a solution to the following problem

$$\pi^{*}(p,w,w^{k},p^{q}) = \max_{k,q\geq 0} \pi(p,w;k,q) + \sum_{i=1}^{L} p^{qi}q^{i} - \sum_{i=1}^{R} w^{ki}k^{i} \tag{2.38}$$

where $\pi^{*}(p,w,w^{k},p^{q})$ is the maximum long run profits attainable. The first order conditions for an interior solution to (2.38) are

$$\partial\pi(p,w;k^{*},q^{*})/\partial k^{i} - w^{ki} = 0 \qquad i=1,...,R \tag{2.39}$$

$$\partial\pi(p,w;k^{*},q^{*})/\partial q^{j} + p^{qj} = 0 \qquad j=1,...,L \tag{2.40}$$

where asterisks indicate the optimum solutions. These derivatives of the profit function can be interpreted as follows: $\partial\pi(p,w,k^{*},q^{*})/\partial k^{i}$ is the shadow value of quasi-fixed input k^i i.e., the impact of a marginal increase in k^i on variable profits;

$\partial\pi(p,w,k^*,q^*)/\partial q^j$ is the shadow price of supply managed output q^j or the marginal impact of a one unit increase in q^j on variable profits.

To evaluate the hypotheses of static resource equilibrium and profit maximization, we assume a restricted Translog functional form for the industry short run profit function specified as a second-order Taylor's series approximation

$$\ln\pi = a_0 + \Sigma_e a_e D^e + 1/2\Sigma_e\Sigma_s a_{es} D^e D^s \tag{2.41}$$

where: $D = (\ln w^1,...,\ln w^N; \ln p^1,...,\ln p^M; \ln k^1,...,\ln k^R; \ln q^1,...,\ln q^L, t)$, π is short run variable profits, and t denotes a time trend intended as a proxy for technological change. Short run profit maximization implies, by Hotelling's lemma (2.34)-(2.35), the conditional revenue and cost share equations for variable outputs and inputs respectively

$$\begin{aligned} ssy^j &= a_j + \Sigma_s a_{js} D^s \qquad j = 1,...,M \\ ssx^i &= -(a_{M+i} + \Sigma_s a_{M+i,s} D^s) \qquad i = 1,...,N \end{aligned} \tag{2.42}$$

where $ssy^j = p^j y^j/\pi$, $ssx^i = w^i x^i/\pi$. For quasi-fixed input k^f and supply managed output q^e, theorem (2.36) and static competitive equilibrium imply the share equations

$$\begin{aligned} ssk^f &= \partial\ln\pi(p,w,k,q)/\partial\ln k^f \\ &= a_{M+N+f} + \Sigma_s a_{M+N+f,s} D^s \qquad f = 1,...,R \end{aligned} \tag{2.43}$$

$$-ssq^{e} = \partial \ln \pi(p,w,k,q) / \partial \ln q^{e}$$
$$= a_{M+N+R+e} + \Sigma_{s} a_{M+N+R+e,s} D^{s} \qquad e = 1,\ldots,L \tag{2.44}$$

where $ssk^{f}=w^{kf}k^{f}/\pi$, $ssq^{e}=p^{qe}q^{e}/\pi$. Various cross restrictions can be defined for equations (2.42)-(2.44) depending on the behavioral assumptions. The existence of a short run Translog profit function (2.41) and static profit maximization imply the symmetry restrictions

$$a_{ij} = a_{ji} \qquad i,j = 1,\ldots,M+N \tag{2.45}$$

on coefficients in equations (2.42). In addition, if the industry is at static competitive equilibrium for quasi-fixed inputs and there is profit maximization over supply managed outputs, then the full set of symmetry restrictions

$$a_{ij} = a_{ji} \qquad i,j = 1,\ldots,M+N+R+L \tag{2.46}$$

apply to coefficients for the complete system (2.42)-(2.44).

2.3.1 Hypothesis Testing

Two types of tests are conducted: (1) Wald chi-square tests of the additional symmetry restrictions that are implied by static resource equilibrium and profit maximization; (2) Likelihood ratio tests of augmenting the share equations for variable outputs and inputs (2.42) by appropriate first order conditions (2.43)-(2.44) for static resource equilibrium and profit maximization. The structural equations (2.42)-(2.44) provide useful parametric tests for a variety of hypotheses. A particulary important test is whether a restricted Translog profit function (2.41) is consistent with static equilibrium and profit maximization. Also of interest is the hypothesis of independence between the coefficients of the variable output and input equations (2.42) and those of quasi-fixed inputs (2.43) or supply managed outputs (2.44). This is a valuable test because of the assumption of static equilibrium with respect to some variable inputs conditional upon the observed levels of quasi-fixed factors, Squires (1987, p.559). We note that acceptance of the first hypothesis should be interpreted as acceptance of a short run Translog profit function, static equilibrium and profit maximizing but not as acceptance of profit maximizing behaviour as opposed to a broader set of propositions. The second hypothesis can only test for the internal consistency of quasi-fixed inputs and supply managed outputs in the context of static competitive profit maximization. It is important to keep these points in mind when interpreting the results of these tests. Also, note that all of the tests in this section are conducted assuming all prices (output and input) are nonstochastic as opposed to those tests in section 2.2.1.

Evaluation of our hypotheses against a broader set of alternatives requires the formulation of likelihood ratio tests of these propositions. Suppose that L_1 is the log of the likelihood function obtained by maximum likelihood (ML) estimation of equations (2.42) for variable output supplies and input demands jointly with particular first order conditions (2.43)-(2.44) for quasi-fixed inputs and restricted outputs. Symmetry restrictions (2.46) are not imposed. L_0 is the log of the likelihood function derived by ML estimation of the same system but imposing the additional symmetry restrictions (2.46). Then, under H_0 the likelihood ratio statistic

$$LR = 2 * [\ln(L_1) - \ln(L_0)] \tag{2.47}$$

is asymptotically distributed as chi-square with degrees of freedom equal to the difference in free parameters between the two models. As in section 2.2.1, L_1 and L_0 can be obtained by Iterative Zellner estimation.

Chapter 3. Data Source and Transformations

A brief discussion of the data source and generation is provided in this Chapter. The empirical illustration of models (2.1), (2.18), and (2.33) is conducted with annual data, 1961-84 for Western Canadian agriculture, obtained from Agriculture Canada and Coyle. This data include price and quantity for all crops and livestock and Divisia indexes for four input categories (crop inputs, energy, capital services, and farm produced durables). Other inputs include prices and quantities for farm hired labour and farm land and quantities for farm family labour. The following Divisia quantity indexes were constructed: (1) for dairy output (industrial milk, home dairy products); (2) for poultry output (chicken, turkey, eggs); and (3) for livestock input (feed, veterinary services, feeder cattle and hog). Implicit Tornqvist price indexes were calculated for these commodities using the following formula:

$$\begin{aligned} \overline{P} &= TR/\overline{Y} \\ \overline{W} &= TC/\overline{X} \end{aligned} \tag{3.1}$$

where:

$\overline{P}$ = implicit output price index.

TR = total revenue (only outputs included in the index).

$\overline{Y}$ = quantity index of output.

$\overline{W}$ = implicit input price index.

TC = total cost (only inputs included in the index.

$\overline{X}$ = quantity index of input.

Capital services, farm produced durable, land, and family labour are assumed to be quasi-fixed; that is, these inputs are not easily adjusted from one period to the next. The other inputs (e.g., crop input, livestock input, hired labour, and energy) are considered variable in the short run. It is important to note that this classification apply for the macro industry (i.e., Western Canadian Agriculture) as opposed to individual farms. The outputs are grouped into two major categories: variable outputs (crops and livestock) and supply managed products (poultry and dairy).

Time series approximations of the price variances for crop and livestock outputs were calculated as in Chavas and Holt; Coyle (1992):

$$\begin{aligned} Var_t(p^i) &= 0.50(p^i_{t-1} - p^i_{t-2})^2 + 0.33(p^i_{t-2} - p^i_{t-3})^2 \\ &+ 0.17(p^i_{t-3} - p^i_{t-4})^2 \\ & i = 1,2\ (crops, livestock) \end{aligned} \tag{3.2}$$

that is current variance equals the sum of squares of predictions errors of the previous three years, with declining weights 0.50, 0.33, and 0.17. The covariance of crop and livestock prices was calculated in a similar manner:

$$\begin{aligned} Cov_t(p^i,p^j) &= 0.50(p^i_{t-1} - p^i_{t-2})(p^j_{t-1} - p^j_{t-2}) \\ &+ 0.33(p^i_{t-2} - p^i_{t-3})(p^j_{t-2} - p^j_{t-3}) \\ &+ 0.17(p^i_{t-3} - p^i_{t-4})(p^j_{t-3} - p^j_{t-4}) \\ & i = crops,\ \ j = livestock \end{aligned} \tag{3.3}$$

The final data set utilized in the empirical application is provided in the appendix.

Chapter 4. Empirical Analysis

In this chapter, the theoretical models (2.1), (2.18), and (2.33) are applied to the agricultural sector of Western Canada. The empirical application and results of the cost function approach are presented first, followed by the indirect utility function framework. This chapter ends with the empirical application and results of the deterministic profit function approach.

4.1 Short-Run Cost Model

For empirical implementation of model (2.1), three different specifications are assumed for Western Canadian agriculture. These include a Translog, Generalized Leontief, and Normalized Quadratic functional forms.

4.1.1 A Translog Cost Model

Given the short-run translog cost function (2.9), homogeneity of degree one of the cost function in prices is imposed by normalizing the price of energy, i.e., dividing the prices of crop inputs, livestock inputs, and hired labour by the price of energy. Shephard's lemma and CRTS imply the following variable factor demand equations for this sector

$$
\begin{aligned}
sx1 = {} & a_1 + a_{11}\ln(w1/w4) + a_{12}\ln(w2/w4) + a_{13}\ln(w3/w4) \\
& + a_{14}\ln(k1/k4) + a_{15}\ln(k2/k4) + a_{16}\ln(k3/k4) \\
& + a_{17}\ln(y1/k4) + a_{18}\ln(y2/k4) + a_{19}t \\
& + b_{11}\ln(q1/k4) + b_{12}\ln(q2/k4) + b_{13}\ln k4 \\
sx2 = {} & a_2 + a_{21}\ln(w1/w4) + a_{22}\ln(w2/w4) + a_{23}\ln(w3/w4) \\
& + a_{24}\ln(k1/k4) + a_{25}\ln(k2/k4) + a_{26}\ln(k3/k4) \\
& + a_{27}\ln(y1/k4) + a_{28}\ln(y2/k4) + a_{29}t \\
& + b_{21}\ln(q1/k4) + b_{22}\ln(q2/k4) + b_{23}\ln k4 \\
sx3 = {} & a_3 + a_{31}\ln(w1/w4) + a_{32}\ln(w2/w4) + a_{33}\ln(w3/w4) \\
& + a_{34}\ln(k1/k4) + a_{35}\ln(k2/k4) + a_{36}\ln(k3/k4) \\
& + a_{37}\ln(y1/k4) + a_{38}\ln(y2/k4) + a_{39}t \\
& + b_{31}\ln(q1/k4) + b_{32}\ln(q2/k4) + b_{33}\ln k4
\end{aligned}
\tag{4.1}
$$

where:

sx1 = w1x1/c, share of crop inputs in variable cost.

sx2 = w2x2/c, share of livestock inputs in variable cost.

sx3 = w3x3/c, share of hired labour in variable cost.

w1 = price index for crop inputs.

w2 = price index for livestock inputs.

w3 = wages of farm hired labour.

w4 = price of energy.

k1 = farm capital machinery.

k2 = farm produced capital.

k3 = farm land.

k4 = farm family labour.

y1 = quantity of crop output.

y2 = quantity of livestock output.

q1 = dairy products.

q2 = poultry products.

Constant returns to scale can be evaluated by testing the parametric restrictions $b_{13}=b_{23}=b_{33}=0$ in equations (4.1). The symmetry restrictions implied by the integrability of equations (4.1) are defined here as

$$\begin{aligned} a_{12} &= a_{21} \\ a_{13} &= a_{31} \\ a_{23} &= a_{32} \end{aligned} \tag{4.2}$$

The cost share equations for quasi-fixed inputs given (2.11) and the first order conditions (2.6) are

$$\begin{aligned} sk^i = -(&a_{i+3} + a_{i+3,1}\ln(w^1/w^4) + a_{i+3,2}\ln(w^2/w^4) + a_{i+3,3}\ln(w^3/w^4) \\ &+ \textstyle\sum_{j=1}^3 a_{i+3,j+3}\ln(k^j/k^4) + a_{i+3,7}\ln(y^1/k^4) + a_{i+3,8}\ln(y^2/k^4) \\ &+ a_{i+3,9}t + b_{i+3,1}\ln(q^1/k^4) + b_{i+3,2}\ln(q^2/k^4) + c_{i+3}\ln k^4) \end{aligned} \tag{4.3}$$

where:

i = 1,2,3 (farm capital machinery, farm produced durables, farm land).

sk^i = $w^{ki}k^i/c$.

For crop and livestock outputs, (2.12) and the first order conditions (2.7) for static equilibrium imply the following cost share equations

$$\begin{aligned} sy^i = &a_{i+6} + a_{i+6,1}\ln(w^1/w^4) + a_{i+6,2}\ln(w^2/w^4) + a_{i+6,3}\ln(w^3/w^4) \\ &+ \textstyle\sum_{f=1}^3 a_{i+6,f+3}\ln(k^f/k^4) + a_{i+6,7}\ln(y^1/k^4) + a_{i+6,8}\ln(y^2/k^4) \\ &+ a_{i+6,9}t + b_{i+6,1}\ln(q^1/k^4) + b_{i+6,2}\ln(q^2/k^4) + c_{i+6}\ln k^4 \\ &+ \alpha[(y^i)var(p^i) + \underset{j \neq i}{y^j cov(p^i,p^j)}] * p^i/c \end{aligned} \tag{4.4}$$

where:

i = 1,2 (crop output, livestock output).

sy^i = $p^i y^i / c$.

p^1 = price index for crop output.

p^2 = price index for livestock output.

$var(p^i)$ = variance of nonrestricted output prices (crops, livestock).

$cov(p^1,p^2)$ = covariance of crop and livestock output prices.

The shadow price conditions (2.8) implied by static competitive equilibrium over supply managed products (dairy, poultry) and equations (2.13) give the following cost share equations for restricted commodities

$$\begin{aligned} sq^i = a_{i+8} &+ a_{i+8,1}\ln(w^1/w^4) + a_{i+8,2}\ln(w^2/w^4) + a_{i+8,3}\ln(w^3/w^4) \\ &+ \textstyle\sum_{j=1}^{3} a_{i+8,j+3}\ln(k^j/k^4) + a_{i+8,7}\ln(y^1/k^4) + a_{i+8,8}\ln(y^2/k^4) \\ &+ a_{i+8,9}t + b_{i+8,1}\ln(q^1/k^4) + b_{i+8,2}\ln(q^2/k^4) + c_{i+8}\ln k^4 \end{aligned} \tag{4.5}$$

where:

i = 1,2 (dairy, poultry).

sq^i = $p^{qi} q^i / c$.

p^{q1} = price index for dairy output.

p^{q2} = price index for poultry output.

4.1.2 A Generalized Leontief Cost Model

If a short-run Generalized Leontief cost function is assumed for the agricultural sector of Western Canada, then the following variable input demand equations are derived given Shephard's lemma and CRTS

$$
\begin{aligned}
x1/k4 = {} & a_{11} + a_{12}(w2/w1)^{1/2} + a_{13}(w3/w1)^{1/2} + a_{14}(w4/w1)^{1/2} \\
& + a_{15}(k1/k4) + a_{16}(k2/k4) + a_{17}(k3/k4) \\
& + a_{18}(y1/k4) + a_{19}(y2/k4) + b_{11}(q1/k4) \\
& + b_{12}(q2/k4) + b_{13}t \\
x2/k4 = {} & a_{21}(w1/w2)^{1/2} + a_{22} + a_{23}(w3/w2)^{1/2} + a_{24}(w4/w2)^{1/2} \\
& + a_{25}(k1/k4) + a_{26}(k2/k4) + a_{27}(k3/k4) \\
& + a_{28}(y1/k4) + a_{29}(y2/k4) + b_{21}(q1/k4) \\
& + b_{22}(q2/k4) + + b_{23}t \\
x3/k4 = {} & a_{31}(w1/w3)^{1/2} + a_{32}(w2/w3)^{1/2} + a_{33} + a_{34}(w4/w3)^{1/2} \\
& + a_{35}(k1/k4) + a_{36}(k2/k4) + a_{37}(k3/k4) \\
& + a_{38}(y1/k4) + a_{39}(y2/k4) + b_{31}(q1/k4) \\
& + b_{32}(q2/k4) + b_{33}t \\
x4/k4 = {} & a_{41}(w1/w4)^{1/2} + a_{42}(w2/w4)^{1/2} + a_{43}(w3/w4)^{1/2} + a_{44} \\
& + a_{45}(k1/k4) + a_{46}(k2/k4) + a_{47}(k3/k4) \\
& + a_{48}(y1/k4) + a_{49}(y2/k4) + b_{41}(q1/k4) \\
& + b_{42}(q2/k4) + + b_{43}t
\end{aligned}
\tag{4.6}
$$

where:

x1/k4 = normalized demand for crop inputs.

x2/k4 = normalized demand for livestock inputs.

x3/k4 = normalized demand for hired labour.

x4/k4 = normalized demand for energy.

Note that (4.6) is consistent with a short-run cost function where **c/k4** is Generalized Leontief in **w** and **c** is linear homogeneous in (k, y, q). The normalized demands for quasi-fixed inputs k, variable outputs y, and supply managed products q can be derived by similar procedures as in section 4.1.1.

4.1.3 A Normalized Quadratic Cost Model

Alternatively, a short-run Normalized Quadratic cost function can be adopted for this industry. Shephard's lemma and CRTS yield the following variable input demand equations for this sector

$$
\begin{aligned}
x1/k4 = a_1 &+ a_{11}(w1/w4) + a_{12}(w2/w4) + a_{13}(w3/w4) \\
&+ a_{14}(k1/k4) + a_{15}(k2/k4) + a_{16}(k3/k4) \\
&+ a_{17}(y1/k4) + a_{18}(y2/k4) + a_{19}t \\
&+ b_{11}(q1/k4) + b_{12}(q2/k4) \\
x2/k4 = a_2 &+ a_{21}(w1/w4) + a_{22}(w2/w4) + a_{23}(w3/w4) \\
&+ a_{24}(k1/k4) + a_{25}(k2/k4) + a_{26}(k3/k4) \\
&+ a_{27}(y1/k4) + a_{28}(y2/k4) + a_{29}t \\
&+ b_{21}(q1/k4) + b_{22}(q2/k4) \\
x3/k4 = a_3 &+ a_{31}(w1/w4) + a_{32}(w2/w4) + a_{33}(w3/w4) \\
&+ a_{34}(k1/k4) + a_{35}(k2/k4) + a_{36}(k3/k4) \\
&+ a_{37}(y1/k4) + a_{38}(y2/k4) + a_{39}t \\
&+ b_{31}(q1/k4) + b_{32}(q2/k4)
\end{aligned}
\tag{4.7}
$$

where:

x1/k4 = normalized demand for crop inputs.

x2/k4 = normalized demand for livestock inputs.

x3/k4 = normalized demand for hired labour.

System (4.7) is in accordance with a short-run cost function where **c/w4k4** is quadratic in (w^{N-1}/w4) and **c** is linear homogeneous in (k, y, q). The full set of demand and supply equations for this sector are derived as in the above Translog model.

4.1.4 Empirical Results

For econometric purposes, stochastic random errors are attached to all equations in the three specifications. These errors are assumed to be additive and normally distributed with zero means and positive semidefinite and contemporaneous variance-covariance matrix Ω. These disturbances could simply reflect optimization errors on the part of producers or, producers could be envisaged as differing from each other according to parameters that are known by the manager of the firm but not observable in the aggregate. These firm effects can manifest themselves as random parameters in the Translog, Generalized Leontief, and Normalized Quadratic cost functions and as additive disturbances in the variable demand systems (4.1), (4.6), and (4.7).

Tables 4.1-4.3 report results obtained by iterative linear three stage least squares (I3SLS) using Shazam 6.2. Nonrestricted outputs (crops and livestock) and quasi-fixed inputs are treated as endogenous variables in the above cost share equations. That is, producers simultaneously choose the amount of inputs (variable and quasi-fixed) used and the level of nonrestricted outputs produced in each time period. The set of instruments include logarithms of an index of Canadian farm input prices, a time trend, current quantities of supply managed outputs, lagged quantities of quasi-fixed inputs as well as normalized prices of variable inputs, quasi-fixed inputs, and nonrestricted outputs. Even though the symmetry restrictions (4.2) implied by the integrability of the derived demand equations for variable inputs and static cost minimization are not rejected in the Translog and Normalized Quadratic specifications (see Tables 4.1-4.3) at the 99 percent significant level, the Translog provides a better fit of the data. This is indicated by the significance of the estimated own-price parameters (a11, a22, a33) in all three equations in Table 4.1 as compared to those in Tables 4.2 and 4.3. Also, the Durbin-Watson statistic for the Translog equations do not show any autocorrelation problems while those of the livestock demand equation for the Generalized Leontief (GL) and Normalized Quadratic (NQ) functions show some potential autocorrealtion problems (Tables 4.1,4.2). Due to this potential lack of fit and/or specification problem shown by the GL and NQ forms, further analysis and hypothesis testing is conducted with the Translog specification.

Table 4.1 reports estimates of the normalized cost share equations for variable inputs (4.1) obtained by I3SLS using Shazam 6.2. In the restricted model, the symmetry restrictions a12=a21, a13=a31, a23=a32 are imposed. Wald chi-square test of these restrictions are conducted. These are not rejected at the 99 percent level of significance. Acceptance of the symmetry restrictions (4.2) as noted earlier, implies both existence of a short-run Translog cost function; that is, the derived demand equations for variable inputs (4.1) can be integrated up to a short-run Translog cost function (Hurwicz and Uzawa) and first order conditions for static cost minimization. The restrictions a12=a21, a13=a31, a23=a32 are maintained in subsequent estimations and tests in this section so that all test results in Tables 4.4-4.6 are subject to the existence of a short-run Translog cost function.

Table 4.1 I3SLS Parameter Estimates for Translog Cost Model: Derived Cost Share Equations for Variable Inputs (Crop, Livestock, and Hired Labour) in Western Canadian Agriculture, 1961-84.

	No Symmetry		Symmetry	
Parameter	Estimate	T-Ratio	Estimate	T-Ratio
a1	-2.6533	1.67	0.4281	0.27
a11	0.1255	2.30	0.1697	4.30
a12	0.0801	2.05	--	--
a13	-0.1022	2.38	--	--
a14	0.0693	1.79	0.0323	0.91
a15	-0.0198	0.38	0.0280	0.45
a16	-0.1927	1.98	-0.0394	0.39
a17	-0.0018	0.08	0.0165	0.64
a18	-0.0912	0.74	-0.0975	0.70
a19	0.2258	2.38	0.0804	0.79
b11	0.1293	1.61	-0.0401	0.59
b12	0.1102	1.69	0.1275	1.87
a2	-0.3219	2.74	-0.2790	2.38
a21	-0.0129	3.19	-0.0104	2.74
a22	0.0251	8.68	0.0227	8.56
a23	0.0025	0.77	--	--
a24	-0.0014	0.47	-0.0026	0.95
a25	0.0056	1.45	0.0057	1.47
a26	-0.0244	3.39	-0.0207	2.94
a27	0.0022	1.35	0.0027	1.72
a28	0.0001	0.01	0.0001	0.01
a29	0.0136	1.92	0.0102	1.47
b21	0.0138	2.32	0.0108	1.86
b22	0.0019	0.39	0.0034	0.72
a3	4.9604	1.85	-0.0646	0.03
a31	-0.1316	1.42	-0.1217	2.38
a32	-0.0932	1.41	0.0016	0.51
a33	0.2121	2.92	0.2180	2.57
a34	-0.0336	0.51	-0.0066	0.11
a35	0.0303	0.34	-0.0710	0.76
a36	0.2474	1.51	0.0631	0.40
a37	-0.0151	0.41	-0.0308	0.73
a38	0.2145	1.03	0.2285	0.99
a39	-0.2662	1.66	-0.0856	0.50
b31	-0.1784	1.31	0.0712	0.64
b32	-0.3064	2.77	-0.2798	2.65
Equation	Durbin-Watson		Durbin-Watson	
SX1	2.30		1.91	
SX2	2.07		2.08	
SX3	2.14		2.10	
Test of Symmetry Restriction: $x^2 = 10.91$			$x^2(3)_{.99} = 11.34$	

SX1 = Crop Inputs; SX2 = Livestock Inputs; SX3 = Hired Labour

Table 4.2 I3SLS Parameter Estimates for Generalized Leontief Cost Model: Western Canadian Agriculture, 1961-84.

Parameter	No Symmetry		Symmetry	
	Estimate	T-Ratio	Estimate	T-Ratio
a11	-0.2112	2.68	-0.0883	1.95
a12	0.0411	2.33	--	--
a13	-0.1452	3.62	--	--
a14	0.1030	2.57	--	--
a15	0.8264	1.70	0.2060	0.50
a16	-1.6133	1.52	-1.9054	1.78
a17	0.9685	1.66	1.3326	2.42
a18	-0.0001	1.76	-0.0089E-02	1.51
a19	-0.0034E-03	1.58	-0.0052E-03	2.18
b11	0.0030E-05	0.33	0.0054E-05	0.77
b12	0.0017E-02	1.88	0.0028E-02	3.62
b13	0.0083	3.44	0.0075	3.45
a21	0.0007	0.23	0.0027	1.06
a22	-0.0019E-02	0.02	-0.0012	1.58
a23	0.0004	0.30	--	--
a24	0.0004	0.29	--	--
a25	-0.0087	1.30	-0.0145	2.13
a26	0.0240	1.65	0.0205	1.34
a27	0.0109	1.33	0.0131	1.52
a28	0.0011E-03	1.41	0.0011E-03	1.39
a29	-0.0066E-06	0.21	-0.0026E-05	0.79
b21	-0.0018E-06	1.45	-0.0012E-06	0.93
b22	0.0021E-04	1.65	0.0032E-04	2.49
b23	-0.0041E-03	0.12	-0.0038E-03	0.11
a31	-0.0387	0.19	-0.1378	3.77
a32	-0.0521	1.08	0.0008	0.55
a33	-0.0637	0.57	0.0031	0.03
a34	0.2062	1.99	--	--
a35	-1.7635	1.39	-0.8482	0.75
a36	1.9608	0.76	2.9738	1.27
a37	6.3146	4.16	3.9704	2.43
a38	-0.0003	2.67	-0.0005	3.47
a39	-0.0094E-03	1.64	-0.0015E-02	2.14
b31	-0.0098E-05	0.46	0.0049E-04	2.83
b32	-0.0018E-02	0.76	-0.0022E-02	1.02
b33	0.0092	1.61	0.0206	3.48
a41	0.1114	2.52	0.0568	2.64
a42	-0.0156	1.44	-0.0012	0.88
a43	-0.0791	3.18	-0.0766	3.31
a44	0.0359	0.88	0.0532	1.34
a45	-0.5307	1.71	-0.2630	1.08
a46	-0.6880	1.02	-0.5139	0.82
a47	2.2446	5.73	2.0605	5.83
a48	0.0073E-02	1.95	0.0061E-02	1.72
a49	-0.0018E-03	1.23	-0.0012E-03	0.78

Table 4.2 Continued

	No Symmetry		Symmetry	
Parameter	Estimate	T-Ratio	Estimate	T-Ratio
b41	-0.0025E-04	4.56	-0.0025E-04	4.74
b42	0.0036E-02	5.95	0.0031E-02	6.20
b43	-0.0042	2.78	-0.0036	2.61
Equation	Durbin-Watson		Durbin-Watson	
x1/k4	1.96		1.73	
x2/k4	2.79		2.62	
x3/k4	1.79		1.32	
x4/k4	2.03		2.10	
Test of Symmetry Restriction: $x^2 = 28.22$	$x^2(6)_{.95} = 12.58$; $x^2(6)_{.99} = 16.81$			

x1/k4 = Crop Inputs; x2/k4 = Livestock Inputs; x3/k4 = Hired Labour;
x4/k4 = Energy

Table 4.3 I3SLS Parameter Estimates for Normalized Quadratic Cost Model: Western Canadian Agriculture, 1961-84.

	No Symmetry		Symmetry	
Parameter	Estimate	T-Ratio	Estimate	T-Ratio
a1	-0.0994	1.91	-0.0653	1,16
a11	-0.0415	1.26	0.0136	0.54
a12	0.0073	2.27	--	--
a13	-0.0599	2.59	--	--
a14	0.8008	1.38	-0.0068	0.01
a15	-0.9363	0.73	-0.9981	0.68
a16	1.4501	1.61	2.6825	3.30
a17	-0.0002	2.77	-0.0002	2.18
a18	-0.0062E-03	1.88	-0.0082E-03	2.21
a19	0.0103	3.77	0.0076	2.76
b11	0.0024E-05	0.22	-0.0097E-05	0.95
b12	0.0016E-02	1.43	0.0027E-02	2.42
a2	-0.0001	0.19	-0.0043E-02	0.05
a21	-0.0069E-02	0.15	0.0002	0.37
a22	0.0011E-03	0.02	-0.0024E-02	0.54
a23	0.0001	0.33	--	--
a24	-0.0095	1.15	-0.0127	1.54
a25	0.0192	1.06	0.0169	0.92
a26	0.0096	0.75	0.0140	1.09
a27	0.0012E-03	1.35	0.0013E-03	1.49
a28	-0.0015E-05	0.32	-0.0021E-05	0.43
a29	0.0039E-03	0.09	-0.0082E-03	0.21
b21	-0.0014E-06	0.92	-0.0018E-06	1.19
b22	0.0025E-04	1.62	0.0030E-04	1.94
a3	0.0932	0.73	0.0549	0.45
a31	0.0065	0.08	-0.0782	3.35
a32	-0.0093	1.18	0.0015E-02	0.05
a33	-0.0409	0.73	-0.0089	0.17
a34	-1.7778	1.25	-0.6194	0.55
a35	1.3103	0.42	2.0123	0.82
a36	5.8212	2.65	4.1815	2.27
a37	-0.0003	1.83	-0.0003	2.59
a38	-0.0046E-03	0.57	-0.0024E-03	0.29
a39	0.0062	0.93	0.0105	1.75
b31	-0.0018E-04	0.68	-0.0015E-05	0.06
b32	-0.0019E-02	0.72	-0.0038E-02	1.82
Equation	Durbin-Watson		Durbin-Watson	
X1/k4	2.09		1.99	
X2/k4	2.77		2.78	
X3/k4	1.92		1.88	
Test of Symmetry Restriction:			$x^2(3)_{.95} = 7.81$	
$x^2 = 6.85$			$x^2(3)_{.99} = 11.34$	

X1/k4 = Crop Inputs; X2/k4 = Livestock Inputs; X3/k4 = Hired Labour

Table 4.4 reports similar chi-square test results of the symmetry restrictions implied by long-run cost minimization and static equilibrium for various combinations of quasi-fixed inputs and outputs. For example for crop output y1, variable input equations (4.1) are estimated jointly with the equation for share sy1 (4.4). These tests are conducted with and without output price uncertainty. Given the Translog cost function (2.9), static competitive equilibrium behaviour for y1 implies the reciprocity restrictions a18=a81, a28=a82, a38=a83. Hence the results in Table 4.4 imply rejection of static equilibrium for crop output at both the 95 and 99 percent significance levels. However for livestock output, rejection or acceptance of this hypothesis depends on the absence or presence of price risk. Results of Table 4.4 also suggest acceptance of static profit maximization for dairy and poultry outputs respectively. Static resource equilibrium is rejected for farm machinery at the 95 percent significance level but accepted for farm produced capital and land. Even though these tests are accepted for individual quasi-fixed inputs and outputs overall, the hypothesis of static resource and output equilibrium is rejected for Western Canadian agriculture over the period 1961-84.

Table 4.4 Wald Chi-Square Tests of Symmetry Restrictions for Long-Run Cost Minimization and Static Equilibrium in Western Canadian Agriculture, 1961-84.

Input/Output in Equilibrium	Number of Symmetry Restriction(s)	x^2 Statistic		Critical Region	
		CRA	No CRA	$x^2(s)_{.95}$	$x2(s)_{.99}$
k1	3	--	8.95[a]	7.81	11.34
k2	3	--	0.71	7.81	11.34
k3	3	--	7.61	7.81	11.34
k1,k2	7	--	31.24[a,b]	14.06	18.47
k2,k3	7	--	24.09[a,b]	14.06	18.47
k1,k2,k3	12	--	127.6[a,b]	21.02	26.21
y1	3	20.7[a,b]	51.19[a,b]	7.81	11.34
y2	3	23.3[a,b]	0.07	7.81	11.34
q1	3	--	4.76	7.81	11.34
q2	3	--	1.99	7.81	11.34
k1,y1	7	136[a,b]	119[a,b]	14.06	18.47
k1,y2	7	19.1[a,b]	34.12[a,b]	14.06	18.47
k2,y1	7	79.9[a,b]	200.1[a,b]	14.06	18.47
k2,y2	7	24.5[a,b]	9.86	14.06	18.47
y1,y2	7	101[a,b]	134.5[a,b]	14.06	18.47
y1,y2,q1	12	122[a,b]	193.5[a,b]	21.02	26.21
y1,y2,q1,q2	18	139[a,b]	231.3[a,b]	28.86	34.80
y1,y2,q1,q2 k1	25	205[a,b]	375.8[a,b]	37.65	44.31
y1,y2,q1,q2 k1,k2,k3	42	1762[a,b]	1285[a,b]	58.12	66.20

[a]Statistically significant (equilibrium rejected) at 5 percent.

[b]Statistically significant (equilibrium rejected) at 1 percent.

k1 = farm machinery; k2 = farm produced capital; k3 = farm land; y1 = crop output; y2 = livestock output; q1 = dairy products; q2 = poultry products.

Hauseman specification tests of static farm resource and product equilibrium given a short-run Translog cost function are reported in Table 4.5. These chi-square statistics are calculated by comparing the iterative three stage least squares estimates of the coefficients for variable inputs (4.1) in two models. In the first model, equations (4.1) are estimated independently of other equations and in model two equations (4.1) are estimated jointly with various first order conditions from (4.3)-(4.5). All symmetry restrictions are imposed in both models. Results from Table 4.5 again indicate acceptance of static resource and product equilibrium for individual quasi-fixed inputs and outputs. However the overall acceptance or rejection of this hypothesis for the whole industry depend entirely on the assumption of output price risk.

The hypothesis of risk neutrality was evaluated by testing whether $\alpha = 0$ in the equations for crop and livestock outputs (4.4). The additional symmetry restrictions implied by static competitive equilibrium for these outputs were imposed. A positive estimate was obtained for alpha with a t-ratio of 2.02. Thus, risk neutrality is rejected in favour of risk aversion at both the 95 percent significance level.

Table 4.5 Hausman Specification Tests of First Order Conditions for Long-Run Cost Minimization and Static Equilibrium in Western Canadian Agriculture, 1961-84.

Inputs/Outputs in Equilibrium	M	
	CRA	No CRA
k1	--	4.97
k2	--	0.55
k3	--	1.89
k1,k2	--	24.18
k2,k3	--	7.52
k1,k2,k3	--	27.80
y1	6.10	38.15
y2	1.24	0.87
q1	--	6.84
q2	--	0.68
k1,y1	-58.03	2.89
k1,y2	-0.02	22.15
k2,y1	2.55	24.11
k2,y2	2.41	8.92
y1,y2	6.41	56.43
y1,y2,q2	2.45	34.06
y1,y2,q1,q2	36.73	158.42
y1,y2,q1,q2,k1	72.60	44.28
y1,y2,q1,q2,k1,k2,k3	116.00	-436.32

Critical region: $x^2(33)_{.95} = 46.19$; $x^2(33)_{.99} = 53.48$

k1 = farm machinery; k2 = farm produced capital; k3 = farm land;
y1 = crop output; y2 = livestock output; q1 = dairy products;
q2 = poultry products.

Tests of CRTS and Hicks neutral technical change are also conducted. Constant returns to scale was evaluated by testing whether the coefficients b13, b23, b33 in equations (4.1) are all equal to zero. This hypothesis is not rejected at either the 95 or 99 percent significance levels in Western Canadian agriculture for the sample period. Then a chi-square test of the hypothesis of Hicks neutrality (interpreted as neutrality of cost shares with respect to time trend) was also conducted for this sector by testing the parametric restrictions a19 = a29 = a39 = 0 in equations (4.1). Hicks neutral technical change is not rejected at both the 95 and 99 percent significance levels. This implies that in Western Canadian agriculture, changes in technology did not affect substitution possibilities among farm variable inputs during the sample period. It was also found that technical change was labour using and crop and livestock inputs saving. This implies an efficiency problem in terms of resource allocation. In other words, labour was not used at cost minimizing levels in Western Canadian agriculture during 1961 to 1984.

4.1.5 Conclusion

The following conclusions are drawn given the results obtained in this section. First, the Translog functional form was found to fit the data set better than the Generalized Leontief and Normalized Quadratic specifications. Second, rejection or acceptance of the standard hypothesis of static resource and output equilibrium for all inputs and outputs over the period 1961-84 critically depends on the absence or presence of output price uncertainty. Third, the hypothesis of static equilibrium (conditional on the levels of quasi-fixed inputs) is accepted (Tables 4.4,4.5) for individual supply managed products (dairy, poultry). For nonrestricted outputs (crop, livestock) there is no conclusive evidence for either rejecting or accepting this hypothesis due to the significant impact of price uncertainty on the outcome of this test (Table 4.4). Fourth, static resource equilibrium is not rejected for farm produced capital and farm land. For farm machinery, the Wald chi-square test and Hauseman specification test provide contradictory results. Fifth, the hypothesis of risk neutrality for outputs is rejected for Western Canadian agriculture over the period 1961-84.

4.2 Short-Run Indirect Utility Model

In this section, model (2.18) is applied to Western Canadian Agriculture. A Generalized Leontief and Normalized Quadratic functional forms are employed to model this sector. These two alternative specifications are derived next.

4.2.1 A Generalized Leontief Utility Model

The functional form described here can be viewed as a generalization of a Generalized Leontief dual profit function. Assuming constant returns to scale for the production function of this industry and Hotelling's lemma (2.19),(2.20) we obtain the following variable output supply and input demand equations

$$
\begin{aligned}
y1/k4 = {} & a_{11} + a_{12}(p2/p1)^{1/2} + a_{13}(w1/p1)^{1/2} + a_{14}(w2/p1)^{1/2} \\
& + a_{15}(w3/p1)^{1/2} + a_{16}(w4/p1)^{1/2} + a_{17}(k1/k4) \\
& + a_{18}(k2/k4) + a_{19}(k3/k4) + b_{11}(q1/k4) \\
& + b_{12}(q2/k4) + b_{13}t + c_{11}(var(p1)/p1) \\
& + c_{12}(var(p2)/p1) + c_{13}(cov(p1,p2)/p1) \\
y2/k4 = {} & a_{21}(p1/p2)^{1/2} + a_{22} + a_{23}(w1/p2)^{1/2} + a_{24}(w2/p2)^{1/2} \\
& + a_{25}(w3/p2)^{1/2} + a_{26}(w4/p2)^{1/2} + a_{27}(k1/k4) \\
& + a_{28}(k2/k4) + a_{29}(k3/k4) + b_{21}(q1/k4) \\
& + b_{22}(q2/k4) + b_{23}t + c_{21}(var(p1)/p2) \\
& + c_{22}(var(p2)/p2) + c_{23}(cov(p1,p2)/p2) \\
x1/k4 = {} & a_{31}(p1/w1)^{1/2} + a_{32}(p2/w1)^{1/2} + a_{33} + a_{34}(w2/w1)^{1/2} \\
& + a_{35}(w3/w1)^{1/2} + a_{36}(w4/w1)^{1/2} + a_{37}(k1/k4) \\
& + a_{38}(k2/k4) + a_{39}(k3/k4) + b_{31}(q1/k4) \\
& + b_{32}(q2/k4) + b_{33}t + c_{31}(var(p1)/w1) \\
& + c_{32}(var(p2)/w1) + c_{33}(cov(p1,p2)/w1) \\
x2/k4 = {} & a_{41}(p1/w2)^{1/2} + a_{42}(p2/w2)^{1/2} + a_{43}(w1/w2)^{1/2} + a_{44} \\
& + a_{45}(w3/w2)^{1/2} + a_{46}(w4/w2)^{1/2} + a_{47}(k1/k4) \\
& + a_{48}(k2/k4) + a_{49}(k3/k4) + b_{41}(q1/k4) \\
& + b_{42}(q2/k4) + b_{43}t + c_{41}(var(p1)/w2) \\
& + c_{42}(var(p2)/w2) + c_{43}(cov(p1,p2)/w2) \\
x3/k4 = {} & a_{51}(p1/w3)^{1/2} + a_{52}(p2/w3)^{1/2} + a_{53}(w1/w3)^{1/2} \\
& + a_{54}(w2/w3)^{1/2} + a_{55} + a_{56}(w4/w3)^{1/2} + a_{57}(k1/k4) \\
& + a_{58}(k2/k4) + a_{59}(k3/k4) + b_{51}(q1/k4) \\
& + b_{52}(q2/k4) + b_{53}t + c_{51}(var(p1)/w3) \\
& + c_{52}(var(p2)/w3) + c_{53}(cov(p1,p2)/w3) \\
x4/k4 = {} & a_{61}(p1/w4)^{1/2} + a_{62}(p2/w4)^{1/2} + a_{63}(w1/w4)^{1/2} \\
& + a_{64}(w2/w4)^{1/2} + a_{65}(w3/w4)^{1/2} a_{66} + a_{67}(k1/k4) \\
& + a_{68}(k2/k4) + a_{69}(k3/k4) + b_{61}(q1/k4) \\
& + b_{62}(q2/k4) + b_{63}t + c_{61}(var(p1)/w4) \\
& + c_{62}(var(p2)/w4) + c_{63}(cov(p1,p2)/w4)
\end{aligned}
\tag{4.8}
$$

where:

y1/k4 = supply of crop outputs.

y2/k4 = supply of livestock outputs.

x4/k4 = demand for energy.

Note that if all coefficients c_{ij} (i=1,2,3,4,5,6 and j=1,2,3) equal zero, then (4.8) is consistent with a short-run profit function where $\pi/k4$ is Generalized Leontief in (p,w) and π is linear homogeneous in (k,q). This specification permits the addition of one more equation to the system to be estimated which provides further information that can aid in obtaining consistent and efficient estimators.

The symmetry restrictions implied by the integrability of equations (4.8) and static utility maximization are defined here as

$$\begin{aligned} &a12 = a21; a13 = -a31; a14 = -a41;\\ &a15 = -a51; a16 = -a61; a23 = -a32;\\ &a24 = -a42; a25 = -a52; a26 = -a62;\\ &a34 = a43; a35 = a53; a36 = a63;\\ &a45 = a54; a46 = a64; a56 = a65 \end{aligned} \tag{4.9}$$

4.2.2 A Normalized Quadratic Utility Model

Alternatively, a Normalized Quadratic functional form can be adopted for the indirect utility function of Western Canadian agriculture. Assuming constant returns to scale and Hotelling's lemma (2.19),(2.20) the following system of variable output supply and input demand equations can be derived for this sector

$$
\begin{aligned}
y1/k4 &= a_1 + a_{11}(p1/w4) + a_{12}(p2/w4) + a_{13}(w1/w4) \\
&\quad + a_{14}(w2/w4) + a_{15}(w3/w4) + a_{16}(k1/k4) \\
&\quad + a_{17}(k2/k4) + a_{18}(k3/k4) + a_{19}t \\
&\quad + b_{11}(q1/k4) + b_{12}(q2/k4) + c_{11}(var(p1)/w4) \\
&\quad + c_{12}(var(p2)/w4) + c_{13}(cov(p1,p2)/w4) \\
y2/k4 &= a_2 + a_{21}(p1/w4) + a_{22}(p2/w4) + a_{23}(w1/w4) \\
&\quad + a_{24}(w2/w4) + a_{25}(w3/w4) + a_{26}(k1/k4) \\
&\quad + a_{27}(k2/k4) + a_{28}(k3/k4) + a_{29}t \\
&\quad + b_{21}(q1/k4) + b_{22}(q2/k4) + c_{21}(var(p1)/w4) \\
&\quad + c_{22}(var(p2)/w4) + c_{23}(cov(p1,p2)/w4) \\
x1/k4 &= a_3 + a_{31}(p1/w4) + a_{32}(p2/w4) + a_{33}(w1/w4) \\
&\quad + a_{34}(w2/w4) + a_{35}(w3/w4) + a_{36}(k1/k4) \\
&\quad + a_{37}(k2/k4) + a_{38}(k3/k4) + a_{39}t \\
&\quad + b_{31}(q1/k4) + b_{32}(q2/k4) + c_{31}(var(p1)/w4) \\
&\quad + c_{32}(var(p2)/w4) + c_{33}(cov(p1,p2)/w4) \\
x2/k4 &= a_4 + a_{41}(p1/w4) + a_{42}(p2/w4) + a_{43}(w1/w4) \\
&\quad + a_{44} + a_{45}(w3/w4) + a_{46}(k1/k4) \\
&\quad + a_{47}(k2/k4) + a_{48}(k3/k4) + a_{49}t \\
&\quad + b_{41}(q1/k4) + b_{42}(q2/k4) + c_{41}(var(p1)/w4) \\
&\quad + c_{42}(var(p2)/w4) + c_{43}(cov(p1,p2)/w4) \\
x3/k4 &= a_5 + a_{51}(p1/w4) + a_{52}(p2/w4) + a_{53}(w1/w4) \\
&\quad + a_{54}(w2/w4) + a_{55}(w3/w4) + a_{56}(k1/k4) \\
&\quad + a_{57}(k2/k4) + a_{58}(k3/k4) + a_{59}t \\
&\quad + b_{51}(q1/k4) + b_{52}(q2/k4) + c_{51}(var(p1)/w4) \\
&\quad + c_{52}(var(p2)/w4) + c_{53}(cov(p1,p2)/w4)
\end{aligned} \tag{4.10}
$$

where all variables are defined as before. If all coefficients c_{ij} (i=1,2,3,4,5 and j=1,2,3) are equal to zero, then (4.9) is consistent with a short-run profit function where $\pi/w4k4$ is quadratic in (p/w4, w^{N-1}/w4). In this case, the reciprocity restrictions implied by a parent Normalized Quadratic utility function and static utility maximization are

$$
\begin{aligned}
&a12 = a21;\ a13 = -a31;\\
&a14 = -a41;\ a15 = -a51;\\
&a23 = -a32;\ a24 = -a42;\\
&a25 = -a52;\ a34 = a43;\\
&a35 = a53;\ a45 = a54
\end{aligned}
\tag{4.11}
$$

For quasi-fixed inputs, static competitive equilibrium and (2.27) provide the following inverse demand equations

$$
\begin{aligned}
w^{ki} = -(&a_{i+5} + a_{i+5,1}(p1/w4) + a_{i+5,2}(p2/w4) + a_{i+5,3}(w1/w4)\\
&+ a_{i+5,4}(w2/w4) + a_{i+5,5}(w3/w4) + a_{i+5,6}(k1/k4)\\
&+ a_{i+5,7}(k2/k4) + a_{i+5,8}(k3/k4) + a_{i+5,9}t\\
&+ b_{i+5,1}(q1/k4) + b_{i+5,2}(q2/k4) + c_{i+5,1}(var(p1)/w4)\\
&+ c_{i+5,2}(var(p2)/w4) + c_{i+5,3}(cov(p1,p2)/w4))
\end{aligned}
\tag{4.12}
$$

where:

i = 1,2,3 (farm capital machinery, farm produced durables, farm labour).

For supply managed commodities, (2.28) and the first order conditions (2.24) imply the inverse supply equations

$$
\begin{aligned}
p^{qi} = &a_{i+8} + a_{i+8,1}(p1/w4) + a_{i+8,2}(p2/w4) + a_{i+8,3}(w1/w4)\\
&+ a_{i+8,4}(w2/w4) + a_{i+8,5}(w3/w4) + a_{i+8,6}(k1/k4)\\
&+ a_{i+8,7}(k2/k4) + a_{i+8,8}(k3/k4) + a_{i+8,9}t\\
&+ b_{i+8,1}(q1/k4) + b_{i+8,2}(q2/k4) + c_{i+8,1}(var(p1)/w4)\\
&+ c_{i+8,2}(var(p2)/w4) + c_{i+8,3}(cov(p1,p2)/w4)
\end{aligned}
\tag{4.13}
$$

where:

i = 1,2 (dairy, poultry).

4.2.3 Empirical Results

Econometric estimation of the Generalized Leontief and Normalized Quadratic models requires appending stochastic random errors to each equation in systems (4.8) and (4.10). These errors are assumed to be spherical and additive with positive semidefinite and contemporaneous variance-covariance matrix Ω. In this case, these disturbances can be viewed as errors in optimization due to firm specific circumstances (i.e., managerial skills, physical and environmental differences, and other internal characteristics of particular firms).

Tables 4.6 and 4.7 report parameter estimates for the Generalized Leontief and Normalized Quadratic models obtained by the iterative Zellner's technique using Shazam 6.2. Neither specification shows evidence of autocorrelation as indicated by the Durbin Watson statistics.

Tests of the symmetry restrictions (4.9) and (4.11) implied by a Generalized Leontief or a Normalized Quadratic utility functions and static utility maximization for both models (4.8) and (4.10) are reported in Table 4.8.

Table 4.6 ITSUR Parameter Estimates for Generalized Leontief Utility Model: Variable Output Supply and Input Demand Equations in Western Canadian Agriculture, 1961-84.

	No Symmetry		Symmetry	
Parameter	Estimate	T-Ratio	Estimate	T-Ratio
a11	180.85	0.50	-316.57	2.00
a12	-17.05	0.09	--	--
a13	-200.93	2.88	--	--
a14	3.6407	0.16	--	--
a15	106.21	2.89	--	--
a16	32.936	0.87	--	--
a17	8123.5	4.63	2918.6	2.22
a18	9342.1	1.76	7777.5	1.34
a19	-7833.2	2.44	289.48	0.14
b11	0.0015	3.73	0.0033E-01	1.09
b12	-0.1841	4.02	-0.0435	1.19
b13	42.793	4.75	9.9977	1.13
c11	0.0193	1.29	0.0221	1.66
c12	646.4	0.85	-923.27	1.74
c13	3.596	0.55	1.3817	0.19
a21	19199	3.04	143.5	1.85
a22	-9821.5	2.78	-18080	3.25
a23	-835.88	0.66	--	--
a24	-1163.5	2.35	--	--
a25	2788.8	3.55	--	--
a26	716.48	0.95	--	--
a27	-118540	3.78	-127950	2.88
a28	-97477	0.88	-622530	3.22
a29	47275	0.86	211520	2.48
b21	0.0048	0.66	0.0262	2.40
b22	3.4936	5.06	-1.0234	0.80
b23	91.047	0.59	1503.6	4.33
c21	0.0630	0.18	-2.8316	9.88
c22	25735	1.46	76076	6.19
c23	181.07	1.06	337.08	1.69
a31	0.1183	0.25	1.0137	4.10
a32	0.6078	1.68	-3.4172	11.62
a33	0.0494	0.46	0.5684	3.46
a34	-0.0341	0.94	--	--
a35	-0.2287	3.29	--	--
a36	0.0583	0.82	--	--
a37	1.2193	3.69	-1.2539	0.97
a38	-3.8962	3.20	7.7574	1.55
a39	0.3778	0.63	-1.3376	0.53
b31	-0.0035E-05	0.42	-0.0073E-04	2.49
b32	0.0029E-02	3.29	0.0017E-01	4.87
b33	0.0047	2.72	-0.0363	3.67
c31	-0.0059E-01	2.68	0.0032	13.93
c32	24.635	2.03	-96.096	8.18
c33	0.2526	2.74	0.1760	1.81

Table 4.6 Continued

	No Symmetry		Symmetry	
Parameter	Estimate	T-Ratio	Estimate	T-Ratio
a41	0.0378	2.31	0.0198	0.18
a42	-0.0142	1.38	1.7760	12.87
a43	-0.0065	1.97	-0.0607	5.27
a44	-0.0018	1.78	-0.0770	3.08
a45	-0.0055	2.65	--	--
a46	0.0039	2.09	--	--
a47	0.0047	1.23	0.5701	2.79
a48	0.0059	0.46	-1.4770	1.89
a49	-0.0091E-01	0.13	0.0033	0.00
b41	0.0029E-07	0.32	0.0013E-04	2.97
b42	-0.0077E-05	0.82	-0.0013E-02	4.44
b43	0.0065E-02	3.41	0.0071	4.56
c41	-0.0034E-02	2.17	-0.0038	21.19
c42	1.5187	1.81	68.551	6.65
c43	0.0085	1.22	-0.1098	1.61
a51	-1.3847	1.76	1.1627	1.40
a52	1.3610	2.38	9.5370	10.97
a53	1.0775	5.91	0.2294	9.29
a54	-0.1225	2.14	-0.1634	16.94
a55	-0.4837	3.63	-2.6409	6.21
a56	-0.3204	2.58	--	--
a57	-3.1299	5.56	3.4227	1.01
a58	1.3667	0.59	-29.461	2.20
a59	5.8876	6.76	6.1835	0.94
b51	-0.0051E-04	4.16	0.0023E-03	2.92
b52	0.0042E-02	3.24	-0.0045E-01	4.66
b53	-0.0049	1.79	0.1311	5.03
c51	-0.0022	5.57	-0.0126	18.14
c52	-65.349	3.32	329.51	10.41
c53	-0.3226	1.93	-1.2393	5.82
a61	0.4236	1.19	0.4587	2.65
a62	-0.3017	1.26	2.3282	8.93
a63	-0.1712	2.13	0.1175	4.22
a64	-0.0065	0.24	-0.0515	6.37
a65	0.0186	0.39	-0.2001	6.71
a66	0.1622	2.56	-0.4332	5.21
a67	0.4000	1.54	2.1829	3.38
a68	-1.3789	1.54	-4.7432	1.94
a69	1.5684	3.52	0.9124	0.72
b61	-0.0074E-05	1.16	0.0034E-04	2.27
b62	0.0028E-03	0.45	-0.0061E-02	3.31
b63	0.0011	0.79	0.0218	4.45
c61	0.0022E-01	1.44	-0.0016	12.41
c62	19.464	2.68	28.178	4.63
c63	0.0616	1.12	-0.1240	1.74

Table 4.6 Continued

Equation	Durbin-Watson	Durbin-Watson
y1/k4	2.34	2.15
y2/k4	2.53	2.09
x1/k4	2.12	1.86
x2/k4	2.44	1.88
x3/k4	2.06	1.83
x4/k4	2.65	1.87

y1/k4 = Crop Outputs; y2/k4 = Livestock Outputs; x1/k4 = Crop Inputs;
x2/k4 = Livestock Inputs; x3/k4 = Hired Labour; x4/k4 = Energy Inputs.

Table 4.7 ITSUR Parameter Estimates for Normalized Quadratic Utility Model: Variable Output Supply and Input Demand Equations in Western Canadian Agriculture, 1961-84.

Parameter	No Symmetry		Symmetry	
	Estimate	T-Ratio	Estimate	T-Ratio
a1	-160.17	0.58	-476.98	1.97
a11	6590.6	0.54	20506	2.94
a12	1760.4	0.25	--	--
a13	-480.98	1.59	--	--
a14	-7.5730	0.18	--	--
a15	270.28	1.36	--	--
a16	6530.3	3.22	8440.0	4.31
a17	10836	1.39	1322.7	0.16
a18	-5134.6	1.45	-9384.4	2.74
a19	36.869	2.87	36.171	3.35
b11	0.0012	2.02	0.0017	3.41
b12	-0.1688	3.09	-0.0963	2.51
c11	1.2790	1.05	-1.1139	0.99
c12	21581	0.31	-16818	0.43
c13	-35.536	0.08	-48.983	0.10
a2	5586.8	1.09	4927.3	1.38
a21	575710	2.52	1084.8	0.26
a22	-396440	2.99	-127030	2.04
a23	-5710.6	1.01	--	--
a24	-1520.9	1.97	--	--
a25	10452	2.82	--	--
a26	-108230	2.84	-195850	6.67
a27	-130460	0.89	107990	0.83
a28	38134	0.57	222970	5.26
a29	359.81	1.49	-86.286	0.46
b21	0.0141	1.23	-0.0171	2.51
b22	2.3165	2.26	3.2020	4.69
c21	5.9699	0.26	24.531	1.29
c22	211859	1.65	303270	0.46
c23	1868.0	0.23	-784.39	0.09
a3	-0.0661	1.31	-0.0416	0.93
a31	1.4793	0.65	0.0335	1.26
a32	1.2179	0.93	1.4356	2.56
a33	0.0521	0.93	0.0048	0.27
a34	-0.0070	0.92	--	--
a35	-0.0866	2.37	--	--
a36	1.4919	3.98	1.4391	4.79
a37	-5.1208	3.58	-3.8752	3.21
a38	0.3238	0.49	0.4500	0.81
a39	0.0060	2.54	0.0063	3.64
b31	0.0030E-07	0.00	-0.0023E-05	0.29
b32	0.0021E-02	2.14	0.0013E-02	1.89
c31	-0.0056E-01	2.51	-0.0004	1.93
c32	12.277	0.97	-10.552	1.64
c33	0.2307	2.95	0.2656	3.41

Table 4.7 Continued

	No Symmetry		Symmetry	
Parameter	Estimate	T-Ratio	Estimate	T-Ratio
a4	0.0005	0.85	0.0004	0.71
a41	0.0429	1.54	0.0335	1.26
a42	-0.0191	1.18	-0.0122	0.84
a43	-0.0009	1.44	-0.0007	1.28
a44	-0.0001	1.34	-0.0001	1.27
a45	0.0008	1.77	--	--
a46	0.0019	0.42	0.0011	0.23
a47	0.0008	0.04	0.0050	0.28
a48	0.0029	0.37	0.0049	0.62
a49	0.0080E-02	2.72	0.0071E-02	2.59
b41	0.0056E-07	0.40	0.0086E-08	0.06
b42	-0.0012E-04	1.02	-0.0092E-05	0.82
c41	-0.0041E-03	1.48	-0.0045E-03	1.69
c42	0.2570	1.64	0.1958	1.42
c43	0.0003	0.31	0.0004	0.41
a5	-0.0183	0.22	0.2960	2.67
a51	-4.4719	1.24	0.0335	1.26
a52	4.4908	2.15	-2.9808	1.96
a53	0.5061	5.70	-0.0841	5.94
a54	-0.0232	1.92	0.0006	1.52
a55	-0.2239	3.84	-0.0329	0.87
a56	-2.6088	4.36	-2.0576	2.51
a57	-1.9462	0.84	-1.1075	0.34
a58	6.8250	6.54	5.9034	3.90
a59	-0.0123	3.25	0.0007	0.15
b51	-0.0081E-04	4.46	-0.0044E-04	2.01
b52	0.0065E-02	4.05	-0.0023E-02	1.35
c51	-0.0020	5.65	-0.0004	0.76
c52	-66.089	3.28	24.837	1.48
c53	-0.0456	0.36	-0.0341	0.16
Equation	Durbin-Watson		Durbin-Watson	
y1/k4	2.43		1.99	
y2/k4	2.70		2.42	
x1/k4	2.30		1.93	
x2/k4	2.45		2.47	
x3/k4	2.41		1.59	

y1/k4 = Crop Outputs; y2/k4 = Livestock Outputs; x1/k4 = Crop inputs;
x2/k4 = Livestock Inputs; x3/k4 = Hired Labour.

Table 4.8 Wald Chi-Square and Likelihood Ratio Tests of Symmetry Restrictions for Indirect Utility Model.

Functional Form[a]	All $c_{ij} = 0$		All c_{ij} Included	
	x^2	LR	x^2	LR
GL	58.75[b]	41.80[b]	142.13[b]	167.49[b]
NQ	35.91[b]	27.66[c]	92.38[b]	48.62[b]

Critical Region: $x^2(15)_{.99} = 30.57$ $x^2(15)_{.95} = 24.99$

[a] Generalized Leontief (GL) and Normalized Quadratic (NQ).

[b] Statistically significant (symmetry rejected at 5 and 1 percent).

[c] Statistically significant (symmetry rejected at 5 percent).

This joint hypothesis is generally rejected at both the 95 and 99 percent levels of significance for both functional forms. Results of this test are invariant to output price uncertainty. However, to further evaluate the hypotheses of static resource and output equilibrium for quasi-fixed inputs and supply managed outputs, these symmetry restrictions are imposed as a necessary condition. The Normalized Quadratic specification (4.10) is employed for further analysis and hypothesis testing.

Table 4.9 reports chi-square test results of additional symmetry restrictions implied by static resource equilibrium and utility maximization for particular quasi-fixed factors and restricted outputs. For example for quasi-fixed input k1, variable output supply and input demand equations (4.10) are estimated jointly with the inverse demand equation wk1 (4.12). Given the short run Normalized Quadratic utility function (2.25), static resource equilibrium for k1 implies the symmetry restrictions a16=-a61, a26=-a62, a36=a63, a46=a64, a56=a65. Results in Table 4.9 therefore suggest rejection of static competitive equilibrium for farm machinery, farm produced capital, and farm land. Static utility maximization is also rejected for dairy and poultry products at both the 95 and 99 percent significance levels. Table 4.9 also indicate that test results of these hypotheses are invariant to specifications of risk neutrality or aversion.

Likelihood ratio tests of the same hypotheses given a short run Normalized Quadratic utility function are reported in Table 4.10. These statistics are calculated by comparing the log of the likelihood function in two models. In model 1, equations (4.10) are estimated jointly with particular first order conditions from (4.12)-(4.13) and the additional symmetry restrictions implied by static equilibrium for quasi-fixed inputs and supply managed outputs are not imposed. In model 2, the same system is jointly estimated and the additional symmetry restrictions are imposed. Both models are estimated by the iterative Zellner's technique. Results from Table 4.10 also indicate rejection of static resource and product equilibrium for all quasi-fixed inputs and supply managed outputs.

Table 4.9 Wald Chi-Square Test of Symmetry Restrictions for Long Run Static Resource Equilibrium and Utility maximization in Western Canadian Agriculture, 1961-84.*

Input/Output in Equilibrium	Number of Symmetry Restriction(s)	x^2 Statistic		Critical Region	
		c_{ij}	$c_{ij}=0$	$x^2(s)_{.95}$	$x2(s)_{.99}$
k1	5	84.92[a]	84.81[a]	11.07	15.08
k2	5	31.75[a]	25.37[a]	11.07	15.08
k3	5	29.99[a]	49.83[a]	11.07	15.08
k1,k2	11	146.05[a]	144.68[a]	19.67	24.72
k2,k3	11	142.39[a]	146.64[a]	19.67	24.72
k1,k2,k3	18	414.07[a]	489.24[a]	28.86	34.81
q1	5	178.16[a]	283.01[a]	11.07	15.08
q2	5	149.30[a]	56.33[a]	11.07	15.08
q1,q2	11	490.42[a]	367.65[a]	19.67	24.72

* Tests conducted under Normalized Quadratic utility model (4.10).

[a] Statistically significant (equilibrium rejected) at 5 and 1 percents.

k1 = farm machinery; k2 = farm produced capital; k3 = farm land;
q1 = dairy products; q2 = poultry products.

Table 4.10 Likelihood Ratio Test of Symmetry Restrictions for Long Run Static Resource Equilibrium and Utility maximization in Western Canadian Agriculture, 1961-84.*

Input/Output in Equilibrium	Number of Symmetry Restriction(s)	LR		Critical Region	
		c_{ij}	$c_{ij}=0$	$x^2(s)_{.95}$	$x2(s)_{.99}$
k1	5	37.66[a]	38.34[a]	11.07	15.08
k2	5	20.10[a]	18.54[a]	11.07	15.08
k3	5	19.96[a]	30.50[a]	11.07	15.08
k1,k2	11	62.76[a]	61.46[a]	19.67	24.72
k2,k3	11	61.84[a]	70.94[a]	19.67	24.72
k1,k2,k3	18	196.68[a]	118.94[a]	28.86	34.81
q1	5	40.41[a]	51.22[a]	11.07	15.08
q2	5	41.20[a]	30.26[a]	11.07	15.08
q1,q2	11	109.42[a]	95.22[a]	19.67	24.72

* Tests conducted under Normalized Quadratic utility model (4.10).

[a] Statistically significant (equilibrium rejected) at 5 and 1 percents.

k1 = farm machinery; k2 = farm produced capital; k3 = farm land;
q1 = dairy products; q2 = poultry products.

A chi-square test of variable output price uncertainty was conducted. This proposition was evaluated by testing the parametric restrictions $c_{ij} = 0$ (i=1,2,3,4,5; j=1,2,3) in model (4.10). This hypothesis is rejected ($x^2(5) = 40.93$) at both the 95 and 99 percent significance levels.

4.2.4 Conclusion

The hypotheses of static farm resource and output equilibrium are tested in this section under a stochastic indirect utility function framework where output price uncertainty is directly incorporated in the objective function. Generalized Leontief (GL) and Normalized Quadratic (NQ) functional forms are estimated for the agricultural sector of Western Canada. The following conclusions can be drawn given the results in this section. First, both the GL and NQ functional forms reasonably fit the data. The Durbin-Watson statistics for both GL and NQ equations do not indicate serious autocorrelation problems. Second, the symmetry restrictions implied by the integrability of the derived supply and demand equations and static competitive equilibrium are rejected in both specifications in spite of output price uncertainty (Table 4.8). Third, the hypothesis of (linear) risk aversion is rejected for Western Canadian Agriculture over the period 1961-84. Fourth, the proposition of static resource equilibrium for quasi-fixed inputs (farm capital machinery, farm produced durables, farm land) and competitive equilibrium over supply managed commodities (dairy, poultry) is rejected for individual and particular combinations of quasi-fixed inputs and restricted outputs under both output price uncertainty and certainty (Tables 4.9,4.10).

4.3 A Translog Profit Model

As indicated in Chapter 2, a deterministic profit function approach is added as an intermediate case between the cost function approach where output price risk is indirectly incorporated via a long run optimization problem (2.4) and the stochastic indirect utility function approach. In this section, the restricted Translog profit function (2.41) is assumed for the agricultural sector of Western Canada. Linear homogeneity in prices is imposed on the profit function by normalizing the price of energy. Hotelling's lemma and CRTS imply the following variable revenue and cost share equations for outputs and inputs in this sector

$$
\begin{aligned}
ssy1 = {} & a_1 + a_{11}\ln(p1/w4) + a_{12}\ln(p2/w4) + a_{13}\ln(w1/w4) \\
& + a_{14}\ln(w2/w4) + a_{15}\ln(w3/w4) + a_{16}\ln(k1/k4) \\
& + a_{17}\ln(k2/k4) + a_{18}\ln(k3/k4) + a_{19}t \\
& + b_{11}\ln(q1/k4) + b_{12}\ln(q2/k4) \\
ssy2 = {} & a_2 + a_{21}\ln(p1/w4) + a_{22}\ln(p2/w4) + a_{23}\ln(w1/w4) \\
& + a_{24}\ln(w2/w4) + a_{25}\ln(w3/w4) + a_{26}\ln(k1/k4) \\
& + a_{27}\ln(k2/k4) + a_{28}\ln(k3/k4) + a_{29}t \\
& + b_{21}\ln(q1/k4) + b_{22}\ln(q2/k4) \\
ssx1 = {} & a_3 + a_{31}\ln(p1/w4) + a_{32}\ln(p2/w4) + a_{33}\ln(w1/w4) \\
& + a_{34}\ln(w2/w4) + a_{35}\ln(w3/w4) + a_{36}\ln(k1/k4) \\
& + a_{37}\ln(k2/k4) + a_{38}\ln(k3/k4) + a_{39}t \\
& + b_{31}\ln(q1/k4) + b_{32}\ln(q2/k4) \\
ssx2 = {} & a_4 + a_{41}\ln(p1/w4) + a_{42}\ln(p2/w4) + a_{43}\ln(w1/w4) \\
& + a_{44}\ln(w2/w4) + a_{45}\ln(w3/w4) + a_{46}\ln(k1/k4) \\
& + a_{47}lm(k2/k4) + a_{48}\ln(k3/k4) + a_{49}t \\
& + b_{41}\ln(q1/k4) + b_{42}\ln(q2/k4) \\
ssx3 = {} & a_5 + a_{51}\ln(p1/w4) + a_{52}\ln(p2/w4) + a_{53}\ln(w1/w4) \\
& + a_{54}\ln(w2/w4) + a_{55}\ln(w3/w4) + a_{56}\ln(k1/k4) \\
& + a_{57}\ln(k2/k4) + a_{58}\ln(k3/k4) + a_{59}t \\
& + b_{51}\ln(q1/k4) + b_{52}\ln(q2/k4)
\end{aligned}
\tag{4.14}
$$

where:

ssy1 $= \boldsymbol{p1y1/\pi}$, share of crop output in variable profits.

ssy2 $= \boldsymbol{p2y2/\pi}$, share of livestock output in variable profits.

ssx1 $= \boldsymbol{w1x1/\pi}$, share of crop input in variable profits.

ssx2 $= \boldsymbol{w2x2/\pi}$, share of livestock input in variable profits.

ssx3 $= \boldsymbol{w3x3/\pi}$, share of hired labour in variable profits.

Integrability of equations (4.14) and static competitive profit maximization imply the following symmetry restrictions

$$\begin{aligned}
&a12 = a21;\ a13 = -a31;\\
&a14 = -a41;\ a15 = -a51;\\
&a23 = -a32;\ a24 = -a42;\\
&a25 = -a52;\ a34 = a43;\\
&a35 = a53;\ a45 = a54
\end{aligned} \tag{4.15}$$

For quasi-fixed inputs and supply managed outputs, the Envelope theorem (2.36) and static equilibrium imply the share equations

$$\begin{aligned}
ssk^{i} = -(&a_{i+5} + a_{i+5,1}\ln(p1/w4) + a_{i+5,2}\ln(p2/w4) + a_{i+5,3}\ln(w1/w4)\\
&+ a_{i+5,4}\ln(w2/w4) + a_{i+5,5}\ln(w3/w4) + a_{i+5,6}\ln(k1/k4)\\
&+ a_{i+5,7}\ln(k2/k4) + a_{i+5,8}\ln(k3/k4) + a_{i+5,9}t\\
&+ b_{i+5,1}\ln(q1/k4) + b_{i+5,2}\ln(q2/k4))
\end{aligned} \tag{4.16}$$

where:

i = 1,2,3 (farm capital machinery, farm produced durable, farm land),

ssk^{i} $= w^{ki}k/\pi$, and

where:

$$
\begin{aligned}
ssq^{i} = {} & a_{i+8} + a_{i+8,1}\ln(p1/w4) + a_{i+8,2}\ln(p2/w4) + a_{i+8,3}\ln(w1/w4) \\
& + a_{i+8,4}\ln(w2/w4) + a_{i+8,5}\ln(w3/w4) + a_{i+8,6}\ln(k1/k4) \\
& + a_{i+8,7}\ln(k2/k4) + a_{i+8,8}\ln(k3/k4) + a_{i+8,9}t \\
& + b_{i+8,1}\ln(q1/k4) + b_{i+8,2}\ln(q2/k4)
\end{aligned}
\tag{4.17}
$$

i = 1,2 (dairy, poultry),

$ssq^{i} = p^{qi}q^{i}/\pi$,

for quasi-fixed inputs and restricted outputs respectively.

4.3.1 Empirical Results

Econometric estimation requires attaching random error terms to equations (4.14), (4.16)-(4.17). These disturbances are assumed to have similar properties as in section 4.1.4 and 4.2.3 of this Chapter.

Table 4.11 reports iterative linear SUR parameter estimates of system (4.14) using Shazam 6.2. In the second case, the set of symmetry restrictions (4.15) implied by a short run Translog profit function and static profit maximization are imposed. This joint test is rejected at both the 95 and 99 percent levels of significance. However, restrictions (4.15) are necessary conditions for the first order conditions (2.39)-(2.40) for quasi-fixed inputs and supply managed outputs respectively; therefore, they are maintained in subsequent tests in this section. We should note that results obtained in this section as in the Normalized Quadratic utility model (4.10), section 4.2.2, should be interpreted keeping in mind that the symmetry restrictions implied by a Translog profit function (under consideration in this section) or Normalized Quadratic utility function (considered in section 4.2.2) are rejected.

Table 4.11 ITSUR Parameter Estimates for Translog Profit Model: Variable Output Supply and Input Demand Equations in Western Canadian Agriculture, 1961-84.

Parameter	No Symmetry		Symmetry	
	Estimate	T-Ratio	Estimate	T-Ratio
a1	0.0145	0.06	0.2022	1.82
a11	0.0066	1.03	0.0029	1.17
a12	-0.0034	0.98	--	--
a13	-0.0104	1.32	--	--
a14	0.0013	0.11	--	--
a15	0.0135	1.58	--	--
a16	0.0089	2.19	0.0043	1.57
a17	0.0075	1.34	0.0038	0.76
a18	-0.0160	0.92	0.0096	1.09
a19	0.0011	2.23	0.0004	1.54
b11	0.0208	1.46	0.0030	0.78
b12	-0.0319	3.57	-0.0221	2.81
a2	1.0045	4.73	0.7922	7.13
a21	-0.0068	1.08	-0.0028	1.15
a22	0.0023	0.67	0.0018	0.72
a23	0.0102	1.31	--	--
a24	-0.0006	0.05	--	--
a25	-0.0138	1.62	--	--
a26	-0.0023	2.17	-0.0041	1.51
a27	-0.0077	1.38	-0.0042	0.83
a28	0.0180	1.04	-0.0092	1.05
a29	-0.0011	2.28	-0.0004	1.41
b21	-0.0220	1.55	-0.0023	0.61
b22	0.0314	3.54	0.0214	2.72
a3	0.0005	0.36	-0.0015	1.35
a31	-0.0027E-02	0.67	-0.0027E-02	1.27
a32	-0.0002	8.27	0.0002	7.94
a33	0.0080E-02	1.59	0.0002	3.70
a34	0.0001	1.70	--	--
a35	-0.0001	1.91	--	--
a36	0.0001	4.78	0.0001	4.83
a37	-0.0017E-02	0.48	-0.0044E-02	1.18
a38	0.0066E-02	0.59	-0.0062E-02	0.65
a39	0.0011E-02	3.55	0.0014E-02	5.23
b31	0.0015E-03	0.02	0.0002	2.26
b32	-0.0001	2.29	-0.0001	2.06
a4	0.0002	0.99	-0.0013E-02	0.09
a41	-0.0010E-02	1.79	0.0071E-03	2.03
a42	-0.0015E-02	5.08	0.0015E-02	5.28
a43	-0.0014E-02	1.99	-0.0047E-03	0.99
a44	0.0037E-02	3.47	0.0027E-02	5.66
a45	0.0019E-03	0.25	--	--
a46	0.0013E-03	0.35	-0.0012E-04	0.03
a47	0.0011E-02	2.27	0.0078E-03	1.60
a48	0.0016E-02	0.98	0.0078E-03	0.67

Table 4.11 Continued

	No Symmetry		Symmetry	
Parameter	Estimate	T-Ratio	Estimate	T-Ratio
a49	0.0027E-04	0.59	0.0056E-04	1.59
b41	-0.0096E-03	0.74	0.0041E-03	0.47
b42	-0.0017E-02	2.10	-0.0014E-02	1.93
a5	0.0153	2.92	-0.0024	0.69
a51	-0.0001	0.73	-0.0052E-02	0.70
a52	-0.0006	7.46	0.0006	5.87
a53	-0.0002	0.97	-0.0031E-02	0.73
a54	0.0003	0.91	0.0064E-03	1.33
a55	0.0064E-02	0.30	0.0007	7.74
a56	-0.0022E-02	0.22	0.0093E-03	0.11
a57	-0.0001	0.81	-0.0002	1.34
a58	0.0014	3.36	0.0003	1.10
a59	-0.0029E-02	2.39	0.0011E-02	1.24
b51	-0.0009	2.67	0.0004	2.37
b52	-0.0003	1.31	-0.0005	2.09
Equation	Durbin-Watson		Durbin-Watson	
sy1	2.23		2.14	
sy2	2.23		2.12	
sx1	2.01		2.09	
sx2	2.16		2.26	
sx3	2.42		2.27	

Test of Symmetry Restriction: $x^2(10)_{.95} = 18.31$; $x^2(10)_{.99} = 23.20$
$x^2 = 33.58$

sy1 = Crop Outputs; sy2 = Livestock Outputs; sx1 = Crop Inputs;
sx2 = Livestock Inputs; sx3 = Hired Labour

Chi-square test results of the additional symmetry restrictions implied by the first order conditions (2.39)-(2.40) for particular quasi-fixed inputs and supply managed outputs are reported in Table 4.12. In the case of restricted output q1 for example, system (4.14) is estimated jointly with the equation for share ssq1 (4.17). Given the Translog profit function (2.41), static competitive profit maximization behaviour for q1 implies the symmetry restrictions a18=a81, a28=a82, -a38=a83, -a48=a84, -a58=a85. Thus, results in Table 4.12 indicate rejection of static profit maximization for dairy and poultry products at both the 95 and 99 percent significance levels. Static resource equilibrium is rejected for farm machinery but not rejected for farm produced capital and farm land at the 99 percent significance level.

Table 4.13 reports Likelihood Ratio tests for similar hypotheses. These statistics are calculated by comparing the natural log of the likelihood functions in two models. In the first model, the variable revenue and cost share equations (4.14) are estimated jointly with particular first order conditions from (4.16) and/or (4.17), but not imposing the additional symmetry restrictions implied by static resource equilibrium and profit maximization. In the second model, the same system is jointly estimated but imposing these additional symmetry restrictions. Model 1 and 2 are estimated by iterative SUR using Shazam 6.2. Results in Table 4.13 hence suggest rejection of static profit maximization for supply managed commodities (dairy, poultry). Static resource equilibrium is rejected for farm machinery but not rejected for farm produced durables and farm land.

Table 4.12 Wald Chi-Square Test of Symmetry Restrictiions for Long Run Static Resource Equilibrium and Profit Maximization in Western Canadian Agriculture, 1961-84.*

Input/Output in Equilibrium	Number of Symmetry Restriction(s)	x^2 Statistic	Critical Region $x^2(s)_{.95}$	Critical Region $x^2(s)_{.99}$
k1	5	39.26[a]	11.07	15.08
k2	5	11.31[b]	11.07	15.08
k3	5	14.29[b]	11.07	15.08
k1,k2	11	65.34[a]	19.67	24.72
k2,k3	11	31.61[a]	19.67	24.72
k1,k2,k3	18	310.92[a]	28.86	34.81
q1	5	84.88[a]	11.07	15.08
q2	5	67.48[a]	11.07	15.08
q1,q2	11	272.75[a]	19.67	24.72

* Tests conducted under Translog Profit model (4.14)

[a] Statistically significant (equilibrium rejected) at 5 and 1 percents.
[b] Statistically significant (equilibrium rejected) at 5 percent.

k1 = farm machinery; k2 = farm produced capital; k3 = farm land;
q1 = dairy products; q2 = poultry products.

Table 4.13 Likelihood Ratio Test of Symmetry Restrictiions for Long Run Static Resource Equilibrium and Profit Maximization in Western Canadian Agriculture, 1961-84.*

			Critical Region	
Input/Output in Equilibrium	Number of Symmetry Restriction(s)	LR	$x^2(s)_{.95}$	$x^2(s)_{.99}$
k1	5	23.12[a]	11.07	15.08
k2	5	7.66	11.07	15.08
k3	5	8.28	11.07	15.08
k1,k2	11	37.86[a]	19.67	24.72
k2,k3	11	22.14[b]	19.67	24.72
k1,k2,k3	18	85.32[a]	28.86	34.81
q1	5	40.18[a]	11.07	15.08
q2	5	28.70[a]	11.07	15.08
q1,q2	11	73.60[a]	19.67	24.72

* Tests conducted under Translog Profit model (4.11)

[a]Statistically significant (equilibrium rejected) at 5 and 1 percents.
[b]Statistically significant (equilibrium rejected) at 5 percent.

k1 = farm machinery; k2 = farm produced capital; k3 = farm land;
q1 = dairy products; q2 = poultry products.

4.3.2 Conclusion

The following conclusions can be drawn given a deterministic Translog profit function and the results obtained in this section. First, the estimated equations derived from the short run Translog profit function do not show autocorrelation problems as indicated by the Durbin-Watson statistics (Table 4.11). Second, the symmetry restrictions implied by the integrability of the variable revenue and cost share equations and static profit maximization are rejected. Third, the hypothesis of static competitive profit maximization is rejected for dairy and poultry products. And fourth, static resource equilibrium is rejected for farm machinery, but rejection or acceptance of this proposition for farm produced capital and farm land depends on the statistical test conducted (Wald chi-square or Likelihood ratio, Tables 4.12,4.13).

Chapter 5. Conclusions

5.1 Summary of Methodology

The primary objectives of this thesis were to test the hypotheses of static cost minimization/profit maximization, farm resource and output equilibrium, and risk neutrality under price uncertainty in Western Canadian agriculture over the period 1961-84. To evaluate these propositions three different theoretical models were considered: a short run cost function approach where output price risk is indirectly incorporated via a long run optimization problem; a stochastic indirect utility function; and a nonstochastic restricted profit function. The empirical application of these models included a Translog, Generalized Leontief, and Normalized Quadratic specifications for the cost model; a Generalized Leontief and Normalized Quadratic functional forms for the stochastic indirect utility model; and a Translog specification for the nonstochastic profit model. All these specifications were applied to Western Canadian agriculture with annual data, 1961-84. Iterative Three Stage Least Squares and Seemingly Unrelated Regression techniques in Shazam 6.2 were employed to estimate the empirical specifications of all models.

5.2 Summary of Results

First, first order conditions for static cost minimization over the period 1961-84 were not rejected given a short run Translog cost function for Western Canadian agriculture. The hypothesis of static competitive equilibrium (conditional on the levels of quasi-fixed inputs) was not rejected for individual supply managed products (dairy, poultry). That is, the prices of these commodities were on the average equal to the opportunity cost of producing them. For crop and livestock outputs, similar hypothesis tests were inconclusive due to the significant impact of output price risk and uncertainty on the outcome of these tests. For quasi-fixed inputs, static resource equilibrium was not rejected for farm produced capital and land. In the case of farm machinery, the outcome of this proposition depended on the econometric test conducted. Within the Translog cost function framework, the hypothesis of risk neutrality in terms of nonrestricted outputs was rejected for Western Canadian agriculture over the period 1961-84.

Second, the symmetry restrictions implied by a Normalized Quadratic utility function and static competitive equilibrium were rejected for Western Canadian agriculture. These restrictions are necessary for application of the first order conditions implied by static resource and product equilibrium; so these restrictions were imposed for further hypothesis tests. Within this framework, the hypothesis of static product market equilibrium for supply managed products (dairy, poultry) and static resource equilibrium for quasi-fixed inputs (farm machinery, farm produced durables, farm land) were rejected under both output price uncertainty and certainty. The proposition of output price certainty was also rejected for Western Canadian agriculture in this model over the period 1961-84.

Third, the behavioral assumption of static competitive profit maximization was rejected given a short run Translog profit function for Western Canadian agriculture over the period 1961-84. The symmetry restrictions implied by this proposition were imposed as necessary conditions for analyzing the competitive levels of quasi-fixed inputs and supply managed commodities. Under this approach, the proposition of static profit maximization over dairy and poultry products was rejected. Static resource equilibrium was rejected for farm machinery, but rejection or acceptance of this hypothesis for farm produced capital and farm land depended on the statistical test conducted.

Fourth, the joint hypothesis of static farm resource and product equilibrium for the whole agricultural sector of Western Canada over the period 1961-84 was rejected regardless of risk aversion and price uncertainty and theoretical model employed.

5.3 Limitations of the Study

It is important to note that all three theoretical models used in this thesis have some inherent drawbacks. For example, the linear mean-variance assumption in both the cost and indirect utility approaches is very restrictive. Furthermore, these models as presented in this study do not allow for yield uncertainty. The assumptions implied by a stochastic profit function are more restrictive and generally not satisfied in empirical applications. Another limitation of this study is the level of aggregation in the data set employed. The results and conclusions drawn in this study should therefore be interpreted given the properties of the theoretical models and the characteristics of the data set.

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Appendix: Final Data Set

APPENDIX: FINAL DATA SET

OBS	YEAR	T	X1	W1	X2	W2	X4	W4	X3	W3	K1	WK1	K2	WK2	K3	WK3	K4	Y1
1	1961	11	24.47	26.7	0.38506	192.174	53.34	24.9	147	18.3	15.9600	100	10.1900	100	49.3299	97	834	68537.44
2	1962	12	27.93	27.2	0.46096	208.256	53.83	25.1	143	18.7	15.9515	103	9.0973	113	48.9903	103	830	132124.86
3	1963	13	29.81	27.7	0.47006	202.099	55.60	25.2	133	19.3	16.1981	106	10.0459	109	49.0439	114	810	159164.15
4	1964	14	33.08	28.5	0.51400	200.384	57.13	25.3	130	20.2	16.7706	109	11.8247	97	49.2969	128	795	127144.48
5	1965	15	34.99	28.8	0.54771	199.007	59.41	25.3	148	21.8	18.6518	112	11.6598	97	49.9320	147	693	150858.43
6	1966	16	42.02	28.1	0.63223	202.456	61.84	25.5	139	24.3	19.9052	116	10.8707	116	49.5329	167	638	180783.50
7	1967	17	46.01	28.4	0.68834	210.648	64.99	26.3	142	27.0	20.3866	119	10.6694	124	50.1376	189	637	134121.50
8	1968	18	47.24	28.8	0.64606	210.504	67.09	27.7	118	29.1	20.6393	122	10.3740	123	49.9567	208	572	157375.05
9	1969	19	46.06	28.7	0.73533	198.547	67.56	29.0	130	31.3	20.1111	126	10.5667	150	50.0400	200	642	171557.66
10	1970	20	47.00	27.0	0.79887	190.267	70.06	29.5	130	32.4	19.3643	129	10.8634	161	50.0050	200	573	142563.10
11	1971	21	52.12	27.9	0.84950	195.407	79.73	30.5	140	33.9	18.6970	132	10.8110	164	50.3300	200	589	190161.02
12	1972	22	57.01	28.9	0.92891	200.233	82.08	31.5	162	36.9	18.9485	136	11.3196	194	55.1818	198	551	166616.79
13	1973	23	63.25	33.4	0.93818	295.249	85.23	33.2	141	42.0	19.9504	141	11.7012	251	54.9417	240	540	174089.69
14	1974	24	66.58	53.4	0.91980	405.521	90.70	36.6	147	50.7	22.9494	158	14.0991	212	55.1683	315	520	142022.38
15	1975	25	66.68	66.0	0.85912	419.029	85.84	44.1	147	61.8	26.4541	185	11.1823	203	55.2481	399	557	175752.63
16	1976	26	70.25	64.0	0.91638	420.124	87.60	50.9	150	70.5	30.4112	197	12.0673	208	55.4746	472	518	203402.34
17	1977	27	74.18	61.2	0.94397	408.904	91.49	55.4	144	78.2	32.0237	211	11.1596	213	55.4098	549	466	202893.95
18	1978	28	87.06	63.4	0.90861	410.514	96.23	58.1	127	82.5	33.3319	232	10.4018	341	55.3589	666	482	218697.94
19	1979	29	95.24	70.0	0.98226	471.359	104.42	60.6	139	87.2	34.6475	261	11.0195	461	55.3625	858	471	184860.42
20	1980	30	92.61	87.4	0.99999	566.000	103.14	72.2	128	93.9	35.5510	294	13.5516	397	55.3455	1120	423	192187.98
21	1981	31	100.00	100.0	1.06306	626.486	100.00	100.0	127	100.0	36.0734	327	14.0663	377	55.3760	1266	442	230138.43
22	1982	32	101.94	98.2	1.04674	598.041	99.86	117.3	134	106.6	35.8357	347	13.6000	385	55.4277	1279	415	250386.85
23	1983	33	103.77	95.6	0.94279	601.402	101.23	123.6	137	110.9	35.0496	363	13.0232	388	55.5879	1228	408	227615.65
24	1984	34	115.91	98.7	1.07230	637.878	98.84	128.3	149	114.4	34.1340	373	13.0541	388	55.4541	1165	413	207510.80

OBS	P1	Y2	P2	Y3	P3	Y4	P4	VCOST	VARP1W	VARP2W	COV12W
1	0.41526	10437124.85	0.48133	385290056.67	0.20832	2222321.60	0.42969	4745.61	1.02	0.00089	0.00722
2	0.43907	9377869.61	0.54662	374397382.72	0.20529	2016613.38	0.43795	4880.93	0.56	0.00030	0.00757
3	0.43169	10215567.91	0.51723	368862039.88	0.20553	2103551.60	0.46959	4888.75	62.99	0.00229	0.36829
4	0.43915	11393265.53	0.47745	364410374.18	0.20665	2206030.85	0.41291	5117.17	41.63	0.00188	0.23273
5	0.44225	11057629.36	0.51379	334122338.31	0.21949	2197993.25	0.45104	5846.18	22.35	0.00180	0.14545
6	0.46210	10898031.86	0.58004	314674492.68	0.24405	2325849.26	0.48807	6263.38	7.41	0.00133	0.08153
7	0.44836	11420152.46	0.58418	295185203.51	0.28209	2382552.60	0.43364	6994.92	9.89	0.00290	-0.05287
8	0.39508	11073357.86	0.58725	296968374.83	0.29229	2379119.78	0.46295	6788.70	70.43	0.00168	-0.02367
9	0.34346	12047051.35	0.66337	284409293.92	0.27805	2579895.59	0.50489	7496.16	44.55	0.00076	-0.02106
10	0.35077	13803563.36	0.65812	284148020.61	0.27089	2866617.77	0.45480	7699.77	243.40	0.00290	-0.79315
11	0.35216	12913614.69	0.66972	269818017.64	0.28966	2841596.03	0.44296	8797.91	147.25	0.00193	-0.52475
12	0.44135	13709535.31	0.77994	265132073.26	0.31731	2709555.54	0.49249	10396.91	76.03	0.00106	-0.27093
13	0.83168	14457720.14	1.04944	252407839.22	0.39741	2950277.27	0.70110	11141.18	3.16	0.00612	0.13001
14	1.16458	14252412.51	1.03129	224493701.73	0.57072	2803854.03	0.79861	14700.89	752.61	0.04035	5.30660
15	1.00000	13930273.51	1.00000	224196132.78	0.70040	2684754.96	0.78213	17631.02	1931.86	0.02620	3.00403
16	0.88439	13089315.42	1.01696	213845780.51	0.72132	2767587.89	0.83987	19914.84	1329.84	0.01295	1.20488
17	0.80335	13329019.51	0.98764	214404912.75	0.75697	2822698.43	0.84738	21255.16	853.79	0.00052	-0.12870
18	0.86396	13337950.20	1.36828	204903568.30	0.81860	2958486.47	0.86973	21961.06	2910.55	0.00069	1.12158
19	1.03566	13426670.90	1.79031	199069039.71	0.88359	3244990.65	0.95589	25578.45	1866.71	0.07278	0.47819
20	1.22998	13668950.06	1.72942	211302641.68	1.00000	3172992.28	0.99999	28126.02	1708.07	0.13701	8.58519
21	1.34550	13919921.19	1.69461	222627533.66	1.09953	3237224.91	1.15979	33365.99	2243.09	0.08526	3.66950
22	1.18219	13072195.70	1.72557	238287266.41	1.19867	3214754.03	1.15020	36634.48	2182.65	0.03211	0.99742
23	1.15625	13631952.58	1.67758	231415122.93	1.19392	3281471.56	1.14899	38192.73	2609.76	0.00151	-1.91122
24	1.24820	14336064.12	1.74754	229596653.64	1.26419	3368285.42	1.24385	41851.08	1275.09	0.00167	-0.67536

T	= time trend
x1	= crop inputs
w1	= price of crop inputs
x2	= livestock inputs
w2	= price of livestock inputs
x3	= hired labour
w3	= price of hired labour
x4	= energy
w4	= price of energy
k1	= capital machinery
wk1	= price of machinery
k2	= farm produced durables
wk2	= price of farm produced durables
k3	= farm land
wk3	= price of farm land
k4	= family labour
y1	= crop outputs
p1	= price of crop outputs
y2	= livestock outputs
p2	= price of livestock outputs
y3	= dairy products
p3	= price of dairy products
y4	= poultry products
p4	= price of poultry products

Note: These variables are Divisia quantity indexes. Prices are implicitly derived as indicated in equation (3.1).