ION OPTICS CALCULATIONS FOR THE UNIVERSITY OF MANITOBA CYCLOTRON

1968-1

A thesis submitted to the Department of
Physics of the University of Manitoba
in conformity with the requirements
for the degree of Master of Science

by

Frank J. S. Wilson

August 1968

ABSTRACT

A feasibility study was undertaken with the computer code "MANCOL" to insure that satisfactory beam transport would be available after moving the swithching magnet into the vault area. Redesign of the 45° left, 45° right, 30° left, and common beamlines was made using the information gained. With the aid of a much more sophisticated beam transport program "TRANSPORT" all beamlines were studied at various energies.

The scope of the study was extended to include the problem of beam extraction or stripping. Using recalibrated magnetic field data in the computer code "ORBCAL", the radii and radial momenta of the stable orbits were determined at certain azimuths for a number of energies. From this data one was able to make an energy versus stripping foil radius plot.

A computer code "CBEJ" is also presented which calculates trajectories of extracted particles and the five dimensional matrix of the associated transition.

ACKNOWLEDGEMENTS

The writer wishes to express his thanks to Dr. D. O. Wells and Dr. W. T. H. van Oers for their constant encouragement and valuable advice during the period of this study. Thanks also are due to Dr. Moss, Mr. C. Kost and Mr. A. Falk for their assistance in various phases of the computing.

The writer also owes a special debt to Miss J. Ursel for her encouragement and for her many hours of typing.

Finally, the writer wishes to express his appreciation of the financial support given him by the University of Manitoba.

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CHAPTER 1 INITIAL BEAM TRANSPORT STUDY

In the fall of 1967 it was felt that the existing cyclotron laboratory (see fig. 1) could be greatly improved by relocating the switching magnet and the slits in the vault area. There are two primary advantages of such a change. Most important, having all slits in the vault area would reduce the background of nuclear particles in the experimental area. Another advantage is that the experimental area could be divided into two target rooms. This means that work could be done in the area in which an experiment was not being performed; therefore much preparatory work would not constitute cyclotron down time. A feasibility study was therefore undertaken to insure that satisfactory beam transport would be available to all beam lines after such a move. It was to this end that the following work was done.

At the time that this study was initiated, the only beam transport computer code that was available at this laboratory was "MANCOL",(1) a translated version of a program(2) written at the University of Colorado. MANCOL is a "first order" study which does not take into account the effects due to the finite energy spread of the beam. A first order study is concerned with charged particle beams that have small cross-sectional areas and small angular divergences. The program requires knowledge of the horizontal and verticle size and angular divergences at some starting location, along with the positions of the various components of the beam line relative to that starting location. Thereafter first order matrix theory is applied to calculate the size and divergence at other points of interest.*

The input data for the program was derived from the emittance measurements made by van Oers, Moss, and Kost.(3) For these measurements a double waist condition existed at slits "1". This study involves the beam transport from slits 1 of the common beam line to the various scattering chambers. The emittance data used is presented in Table I.

Beam transport was considered feasible for cases in which the following situation could be depicted in the computer code:

(1) a horizontal waist could be formed at the slits downstream from the switching magnet; and

(2) a horizontal and vertical waist could be formed at the position of the target.

A certain number of constraints also had to be satisfied. These were as follows:

(1) the beam had to pass easily through the switching magnet which has a 1.15 inch vertical gap and 2.0 inch diameter

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* Appendix I contains a listing of MANCOL with all information needed for preparation of the data input.

entrance and exit ports;

(2) the beam spot size and the divergences at the target had to be compatible with the experimenter's needs;

(3) the beam did not blow up beyond the extent of the beam pipe upon passing through the quadrupoles;

(4) the field gradients used in the quadrupoles had to be readily obtainable.

Beam transport was considered feasible only if the preceding conditions were satisfied for all the energies investigated (see Table I).

Calculations using the beam layout of fig. 2* indicated that solutions for all beam lines could not be found unless the system was made more flexible. To this end a quadrupole pair was proposed to be located just in front of the switching magnet, (see fig. 2). A study done with MANCOL showed that having a quadrupole singlet at this position would be insufficient. More specifically the calculations indicated that a horizontal waist could not be located at the second pair of slits irregardless of the field gradient in the quadrupole singlet. Fig. 3 shows a plot of the position of the horizontal waist relative to the centre of the switching magnet for a fifteen degree bend and 50.0 MeV. (nominal) beam spot data as input. The two curves in fig. 3 represent studies where a horizontally focussing quadrupole singlet

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* without the second pair of Varian quadrupoles,

was tried at two positions between slits "1" and the switching magnet. The curve (+) has the singlet at the position marked with an "+" on fig. 2; the curve (.) has the singlet at the position marked with a "⊙" on fig. 2. In order for a quadrupole singlet to be used, the horizontal waist must occur 96.0 inches downstream, i.e. at the position of slits "2" in fig. 2. The problem was overcome by using a quadrupole doublet as shown in fig. 2.

The final proposed system for which the MANCOL calculations were done is shown on fig. 2. The beam lines at forty-five degree right, fifteen degree right, thirty degree left, and forty-five degree left were included in the study because they were intended for use in initial plans for the experimental program. The thirty degree left line was studied only at 50.0 MeV (nominal) as this was the only energy included in the plans for experments on this line.

Graphs of the MANCOL solutions are presented in figs. 4 to 19. The horizontal axes are the so-called optic axes or reference trajectories of the beam; above and below these axes are plotted the horizontal and vertical half-extents of the beam. The zeros of the optic axes are located at the first pair of slits. All units are in inches. The labeling "VARIAN" and

"LINTOTT" refer to the names of the companies that manufactured the quadrupoles. Horizontal and vertical waists are generally labeled as "H WAIST" and "V WAIST". The angles of bend are labeled on the switching magnet and are included in the titles. The final double waists at the end of the beam profiles represent the positions of the targets.

Above and below the reference trajectory nine points are plotted which represent the half-extent of the beam at the slits, the entrance and the exit of the quadrupole pairs, the entrance and exit of the switching magnet, and the target. The beam profiles are approximated by straight lines between these points.

FIG. 1 CYCLOTRON BEAMLINE SYSTEM
(JULY 1967)

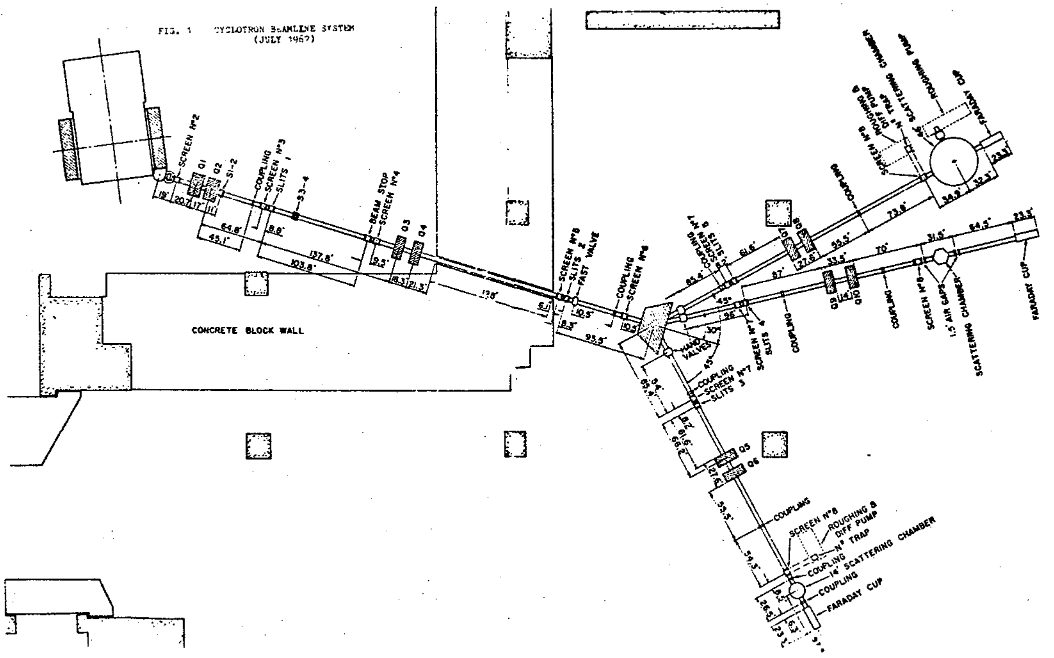
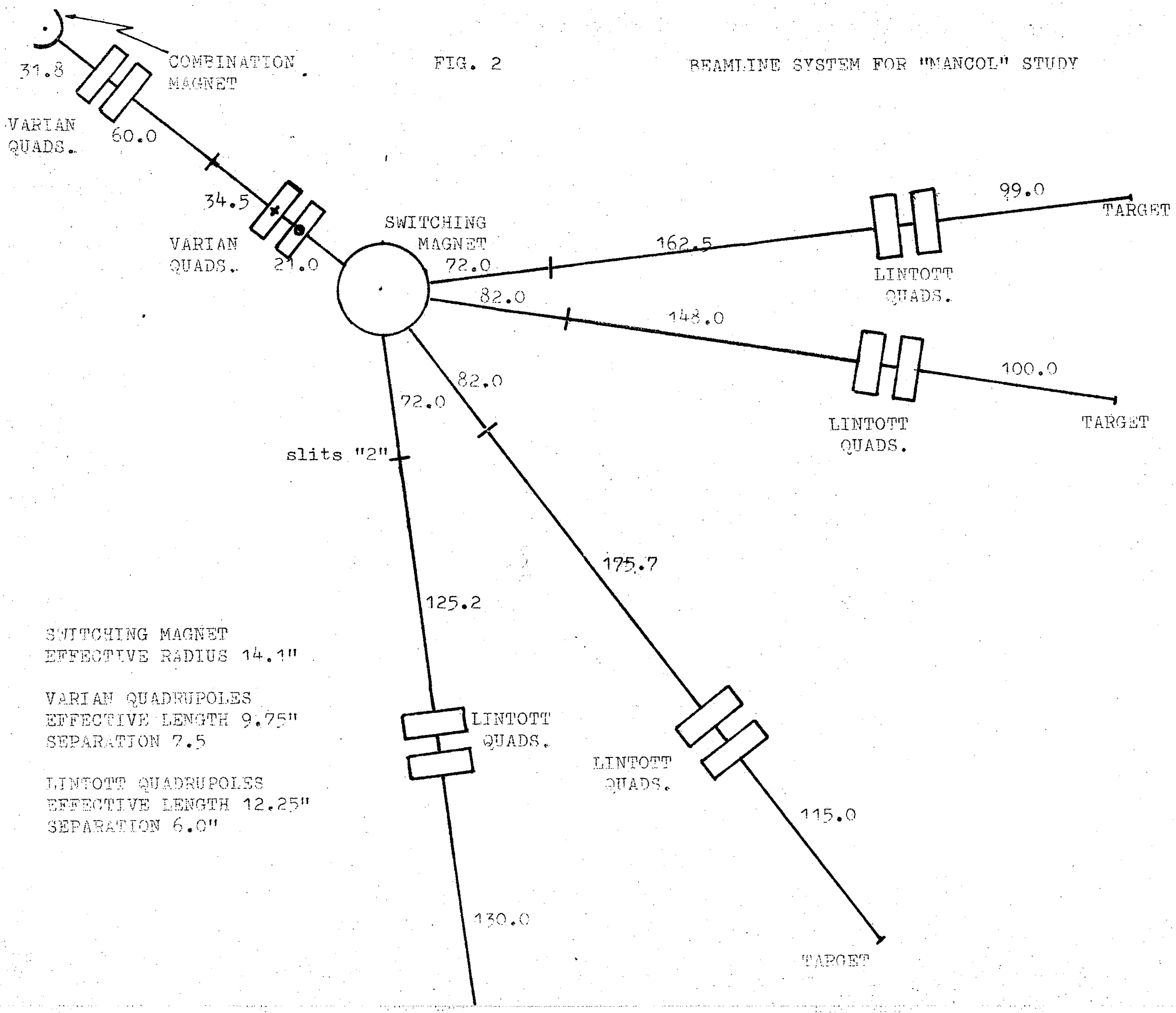


FIG. 2

BEAMLINE SYSTEM FOR "MANCOL" STUDY



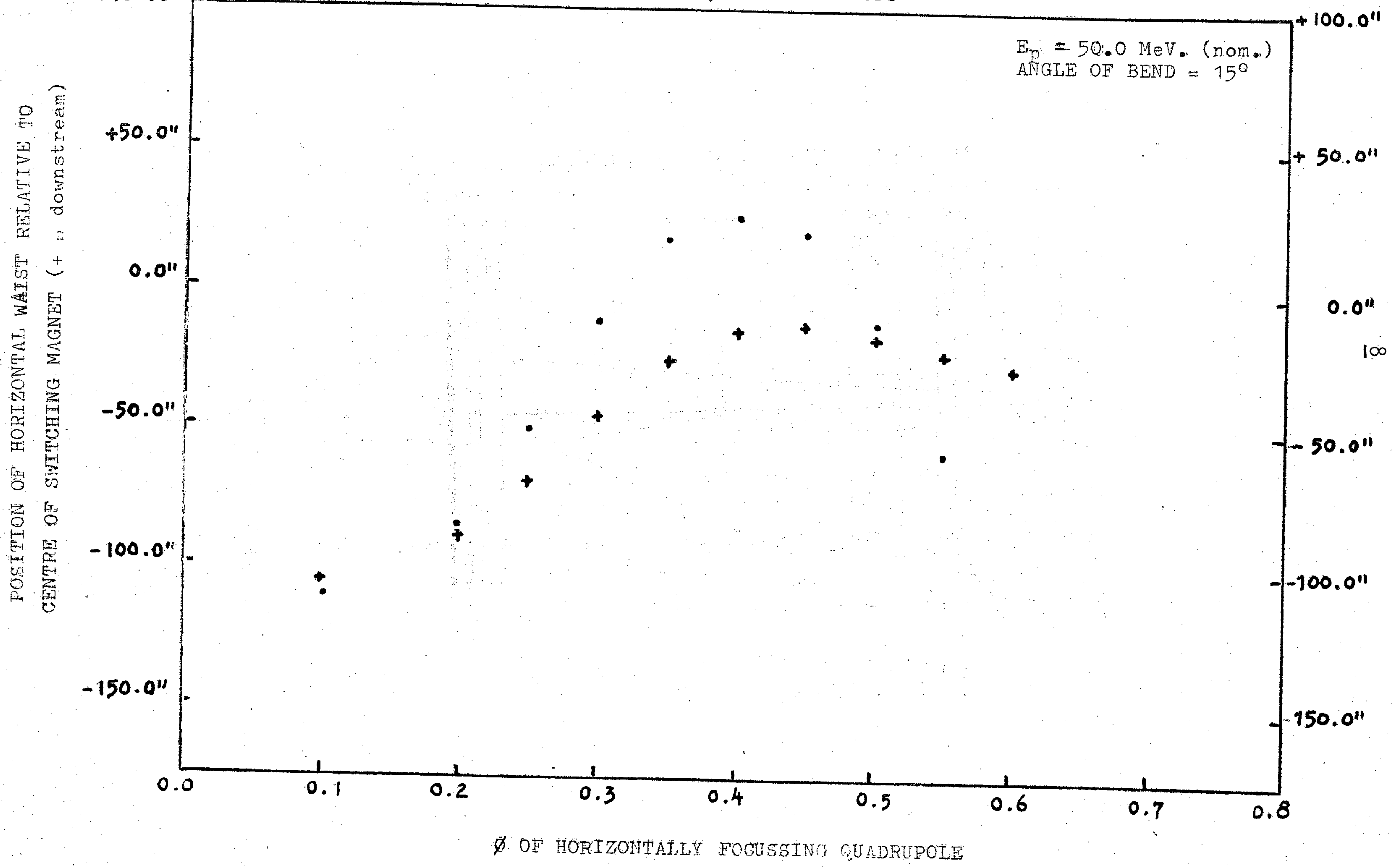
SWITCHING MAGNET
EFFECTIVE RADIUS 14.1"

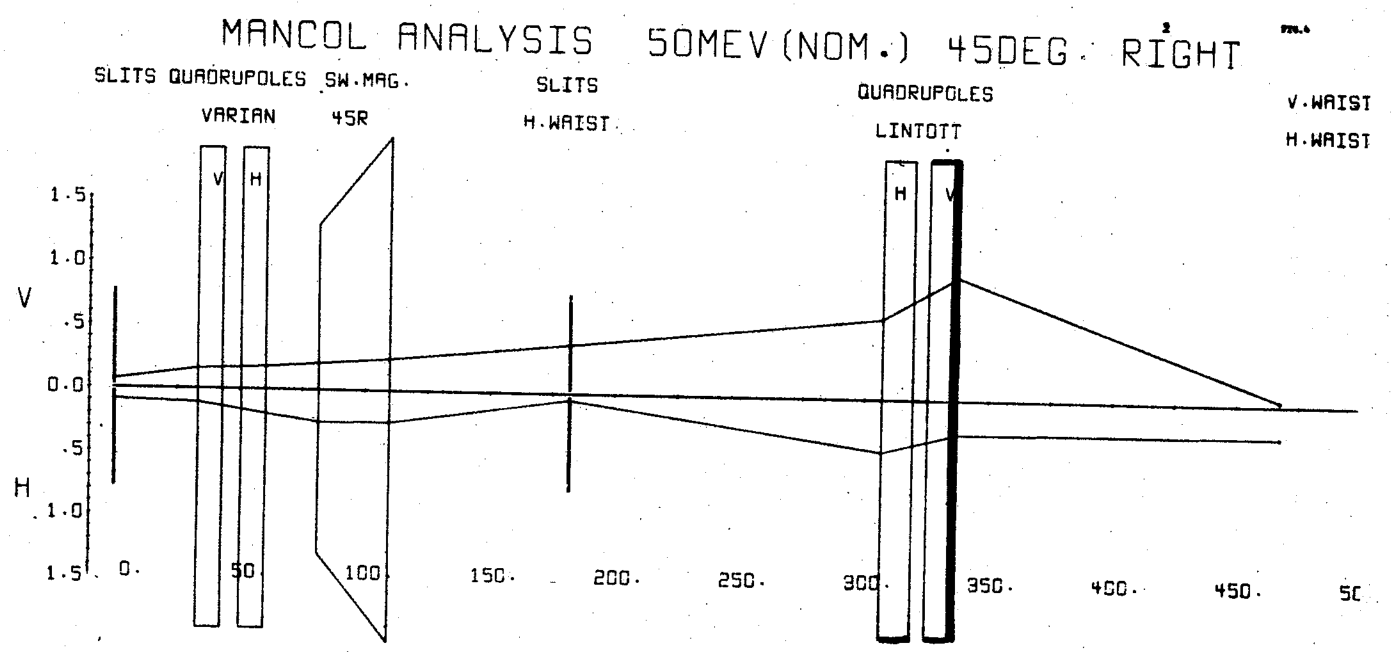
VARIAN QUADRUPOLES
EFFECTIVE LENGTH 9.75"
SEPARATION 7.5

LINTOTT QUADRUPOLES
EFFECTIVE LENGTH 12.25"
SEPARATION 6.0"

FIG. 3

SINGLE QUADRUPOLE STUDY




MANCOL ANALYSIS 50MEV (NOM.) 45DEG. RIGHT
SLITS
QUADRUPOLES
VARIAN
SW.MAG.
45R
SLITS
H.WAIST
QUADRUPOLES
LINTOTT
V.WAIST
H.WAIST
V
H
H
V
V
H
1.5
1.0
.5
0.0
.5
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1.5
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150.
200.
250.
300.
350.
400.
450.
50

Fig. 4

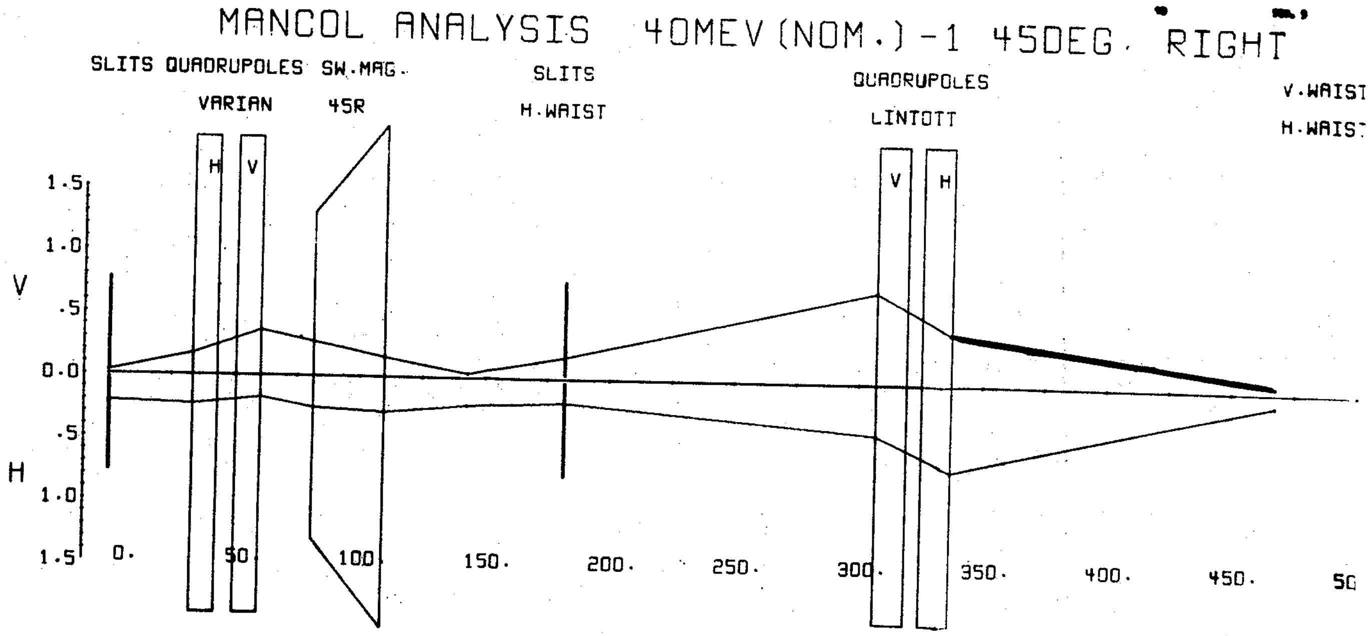
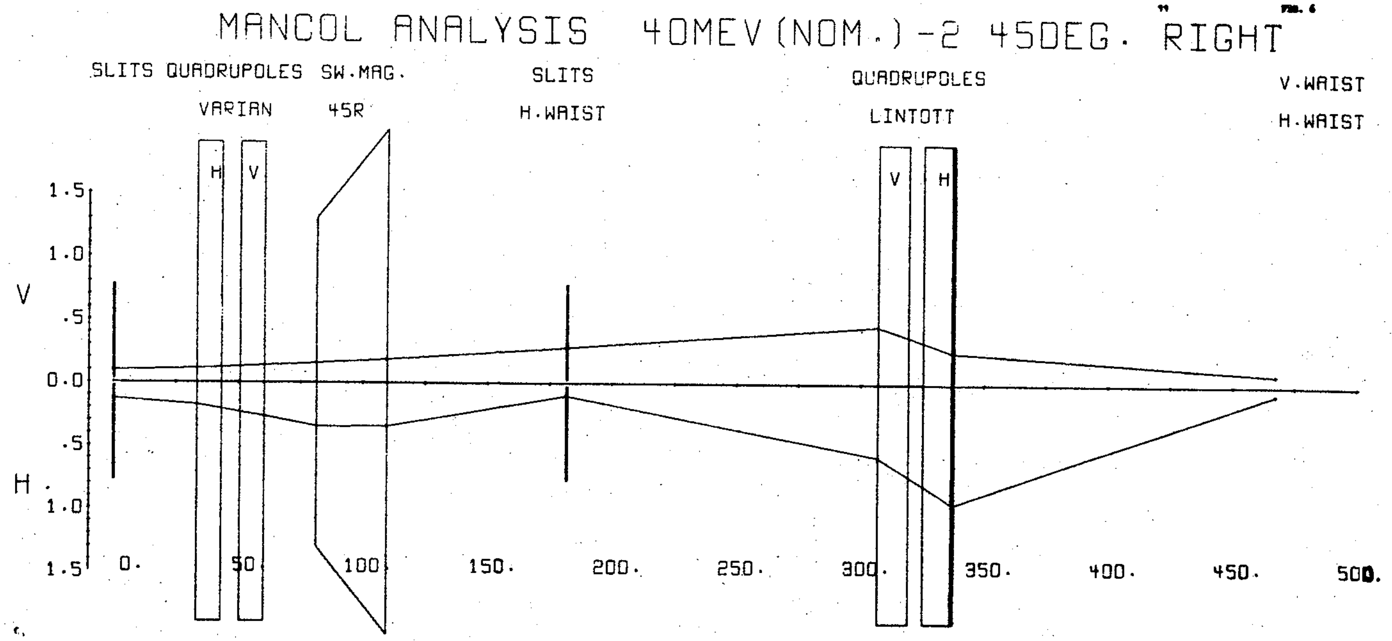

MANCOL ANALYSIS 40MEV(NOM.)-1 45DEG. RIGHT
SLITS
QUADRUPOLES
VARIAN
SW.MAG.
45R
SLITS
H.WAIST
QUADRUPOLES
LINTOTT
V.WAIST
H.WAIST
H
V
V
H
V
H
1.5
1.0
.5
0.0
.5
1.0
1.5
0.
50
100.
150.
200.
250.
300.
350.
400.
450.
50

Fig. 5



MANCOL ANALYSIS 40MEV(NOM.)-2 45DEG. RIGHT

Fig. 6

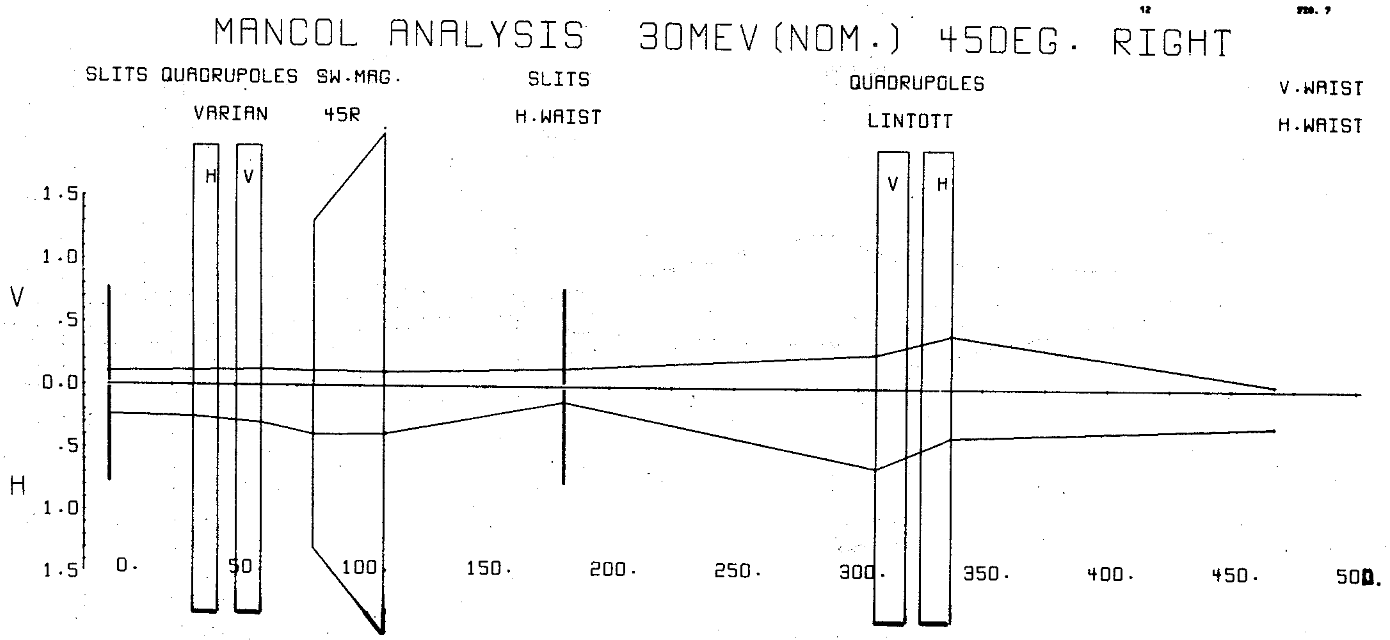

MANCOL ANALYSIS 30MEV(NOM.) 45DEG. RIGHT
SLITS
QUADRUPOLES
VARIAN
SW.MAG.
45R
SLITS
H.WAIST
QUADRUPOLES
LINTOTT
V.WAIST
H.WAIST
H
V
V
H
V
H
1.5
1.0
.5
0.0
.5
1.0
1.5
0.
50
100
150.
200.
250.
300.
350.
400.
450.
500.

Fig. 7

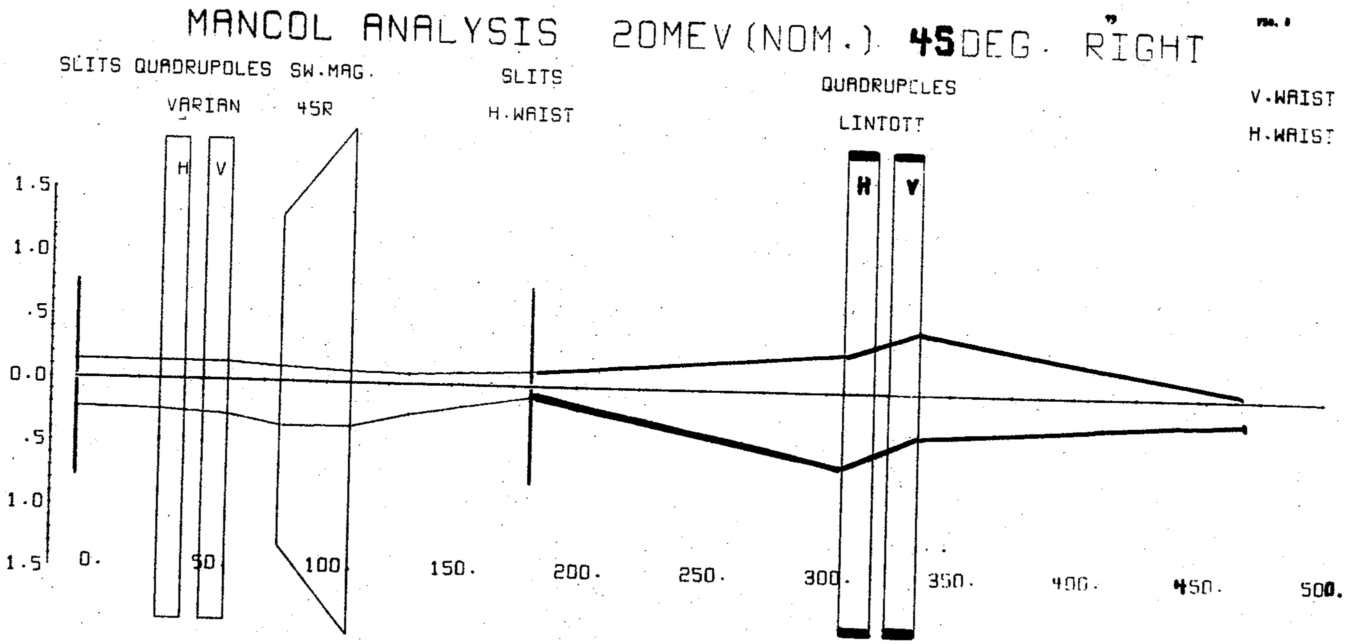
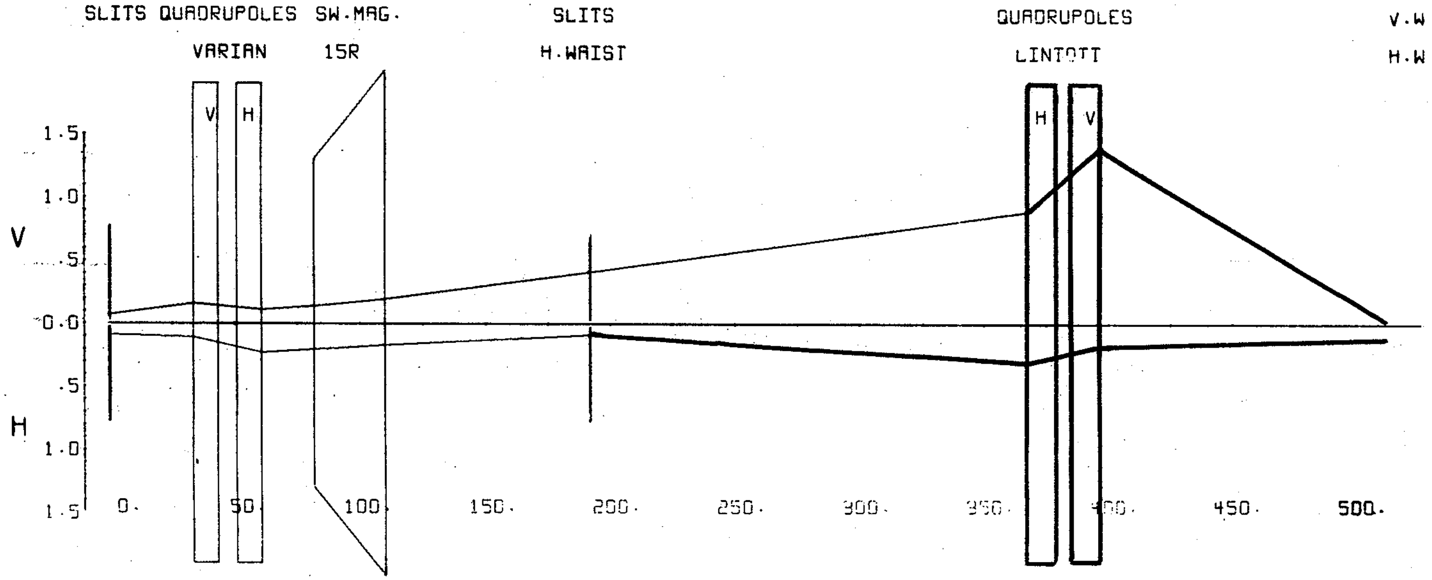

MANCOL ANALYSIS 20MEV(NOM.) 45DEG. RIGHT
SLITS QUADRUPOLES SW.MAG.
VARIAN
45R
SLITS
H.WAIST
QUADRUPCLES
LINTOTT
V.WAIST
H.WAIST
H
V
H
V
1.5
1.0
.5
0.0
.5
1.0
1.5
0.
50
100
150.
200.
250.
300
350.
400.
450.
500.

Fig. 8

Fig. 9

MANCOL ANALYSIS 50MEV(NOM.) 15DEG. RIGHT



MANCOL ANALYSIS 40MEV(NOM.)-1 15DEG RIGHT

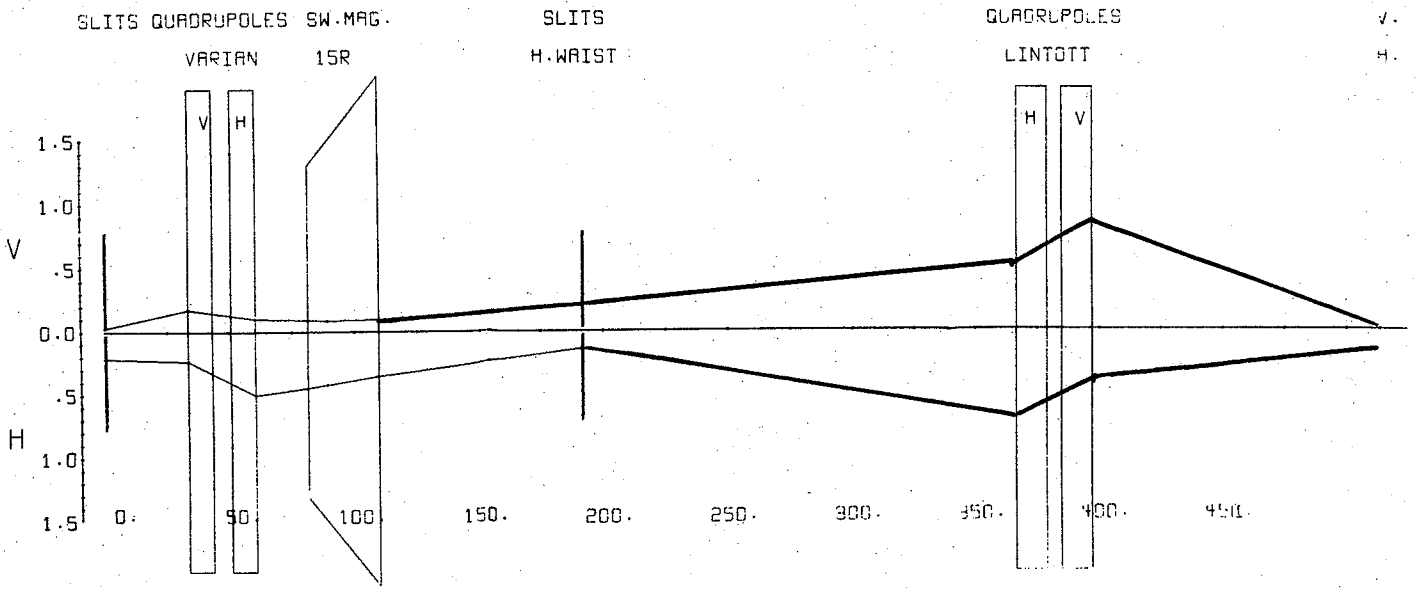
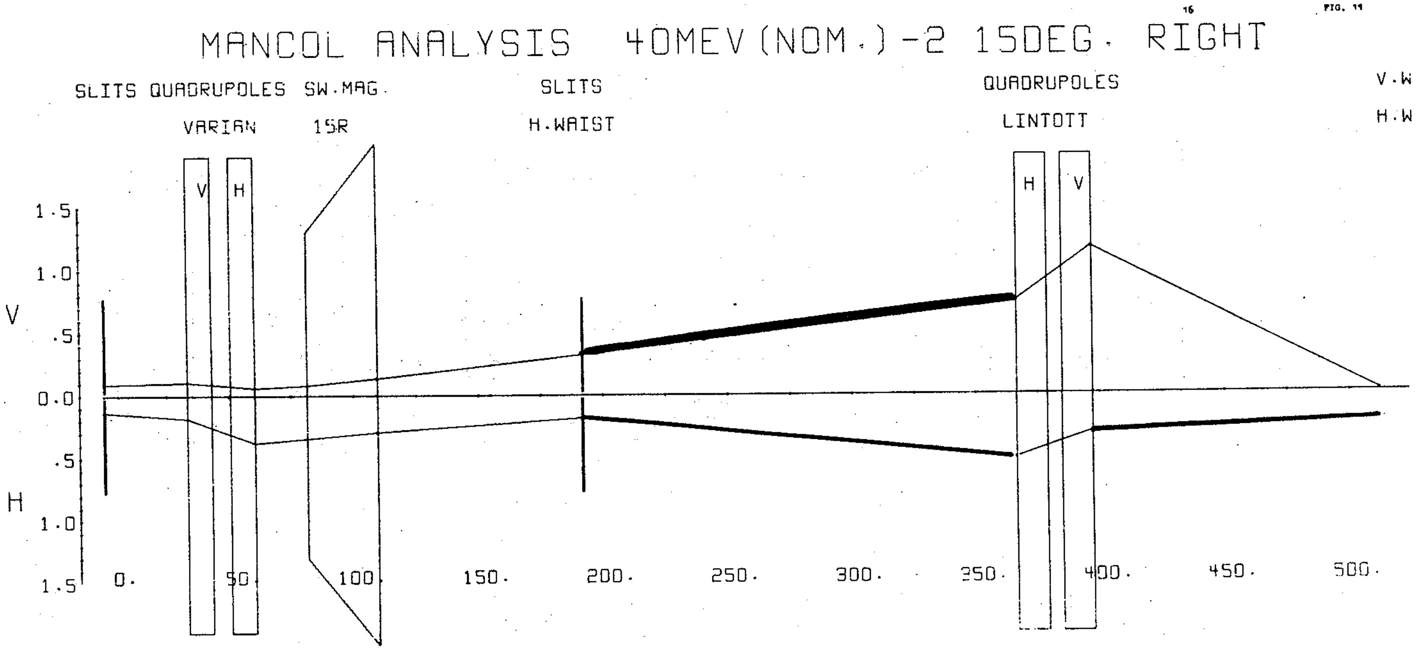


Fig. 10


16
FIG. 11
MANCOL ANALYSIS 40MEV(NOM.)-2 15DEG. RIGHT
SLITS
QUADRUPOLES
VARIAN
V
H
SW.MAG.
15R
SLITS
H.WAIST
QUADRUPOLES
LINTOTT
H
V
V.W
H.W
V
H
1.5
1.0
.5
0.0
.5
1.0
1.5
0.
50
100
150.
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300.
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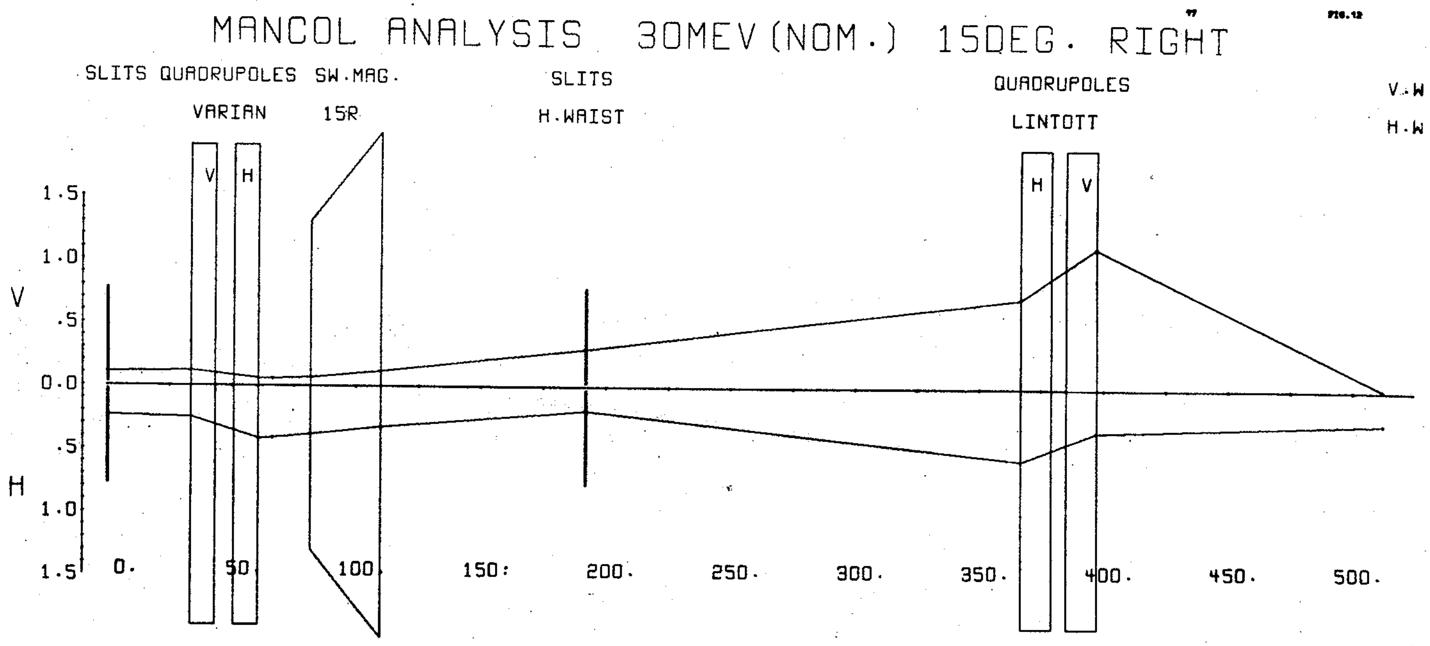
400.
450.
500.

Fig. 11

Fig. 12


MANCOL ANALYSIS 30MEV(NOM.) 15DEG. RIGHT
SLITS
QUADRUPOLES
VARIAN
SW.MAG.
15R
SLITS
H.WAIST
QUADRUPOLES
LINTOTT
V.W
H.W
V
H
H
V
V
H
1.5
1.0
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0.0
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1.5
0.
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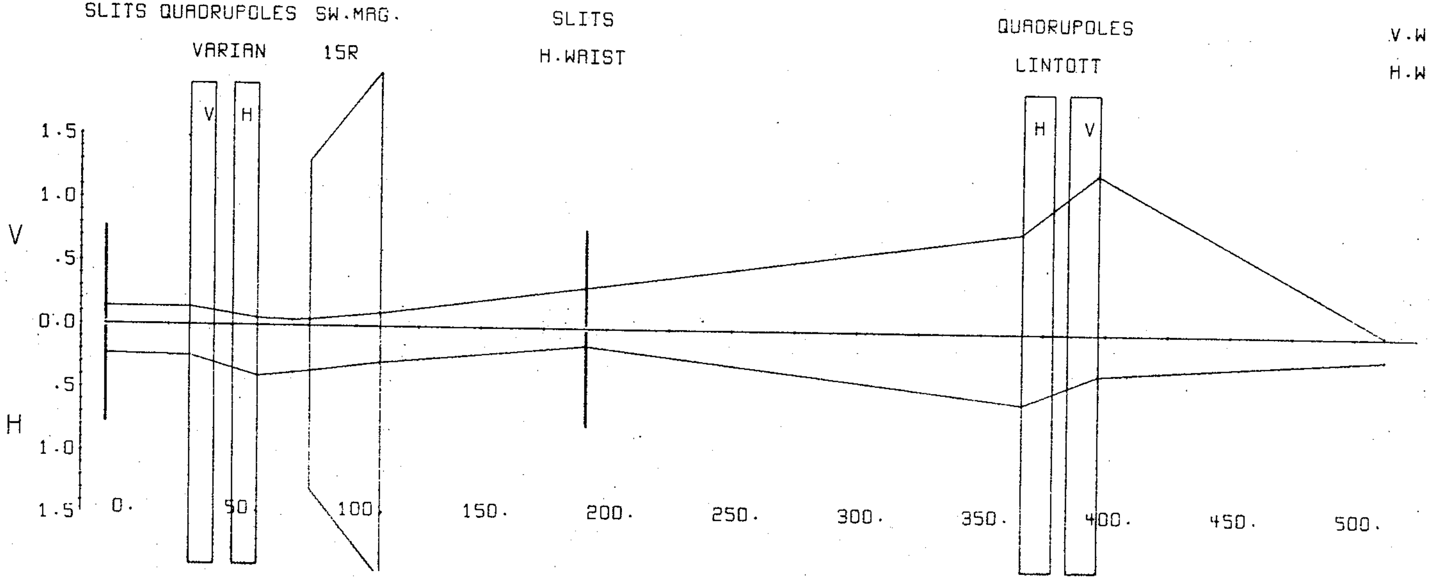
100
150
200
250
300
350
400
450
500

Fig. 13

FIG. 13

MANCOL ANALYSIS 20MEV(NOM.) 15DEG. RIGHT



MANCOL ANALYSIS 50MEV(NOM.) 30DEG. LEFT

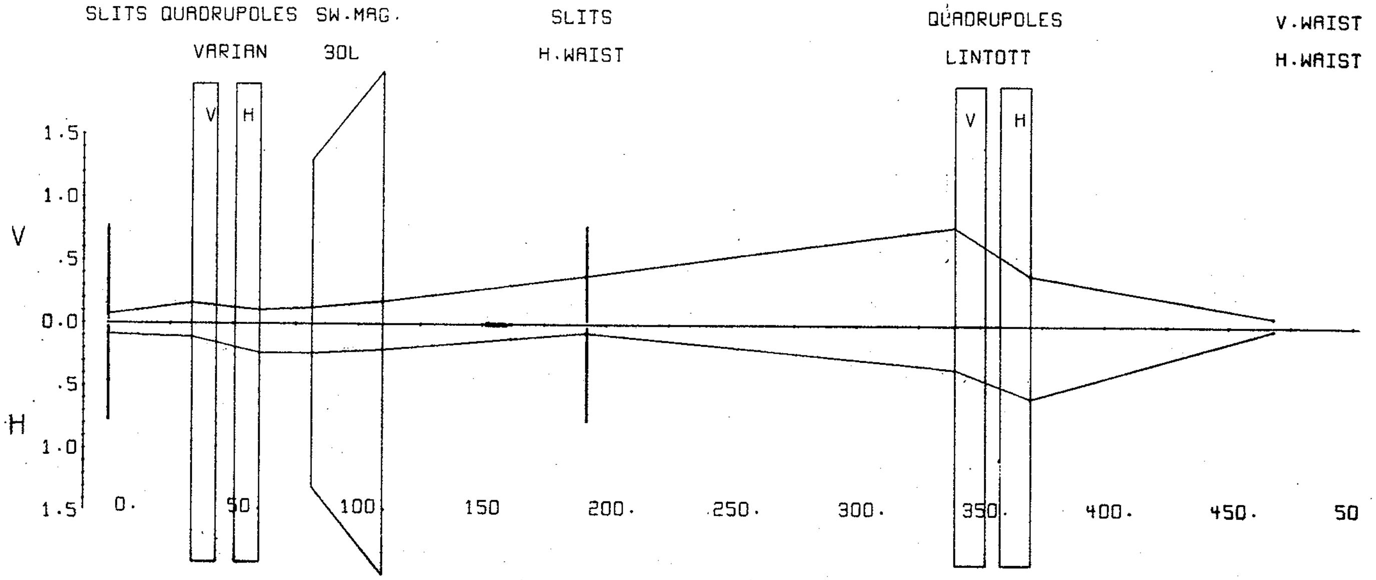
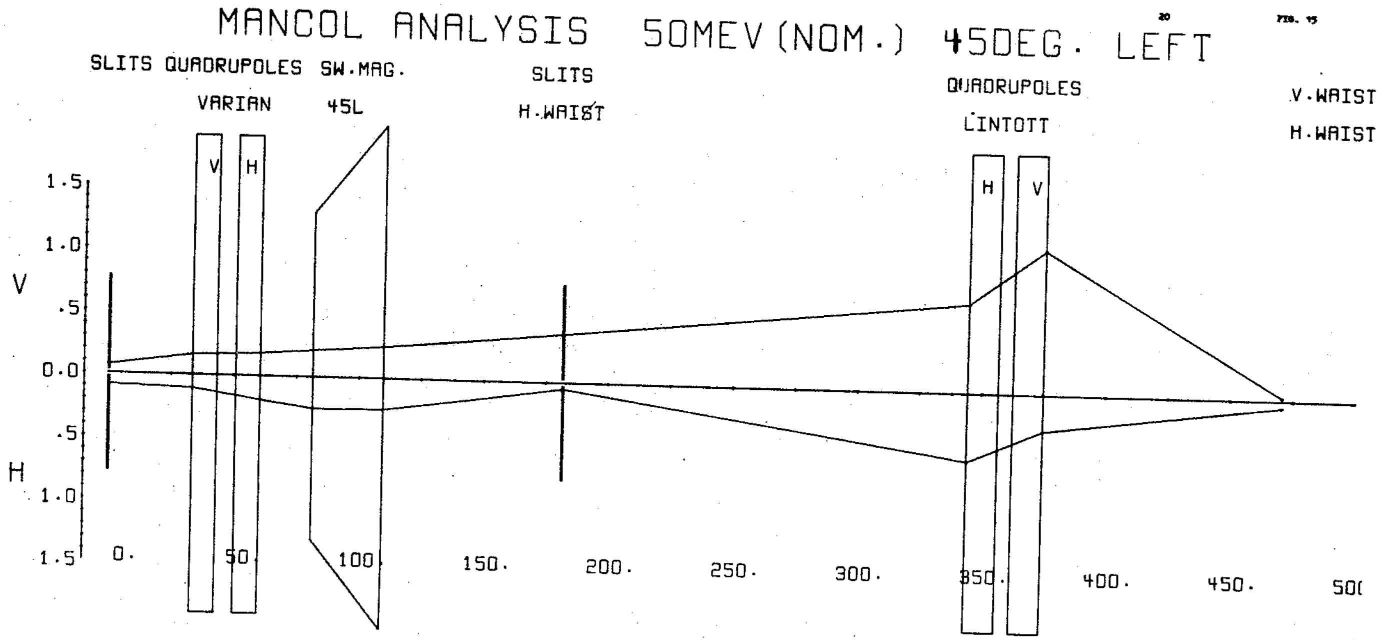


Fig. 14

Fig. 15


MANCOL ANALYSIS 50MEV(NOM.) 45DEG. LEFT
SLITS
QUADRUPOLES
VARIAN
SW.MAG.
45L
SLITS
H.WAIST
QUADRUPOLES
LINTOTT
V.WAIST
H.WAIST
V
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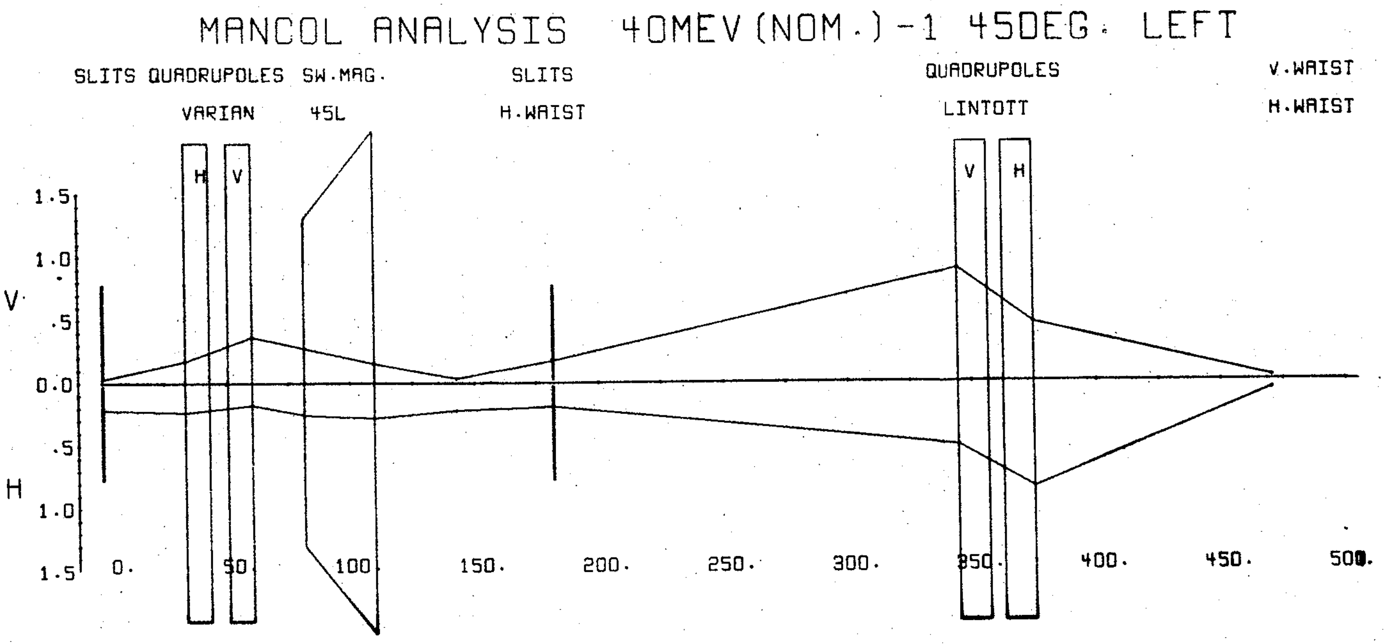
400.
450.
50(

Fig. 16


MANCOL ANALYSIS 40MEV(NOM.)-1 45DEG. LEFT
SLITS
QUADRUPOLES
VARIAN
SW.MAG.
45L
SLITS
H.WAIST
QUADRUPOLES
LINTOTT
V.WAIST
H.WAIST
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1.0
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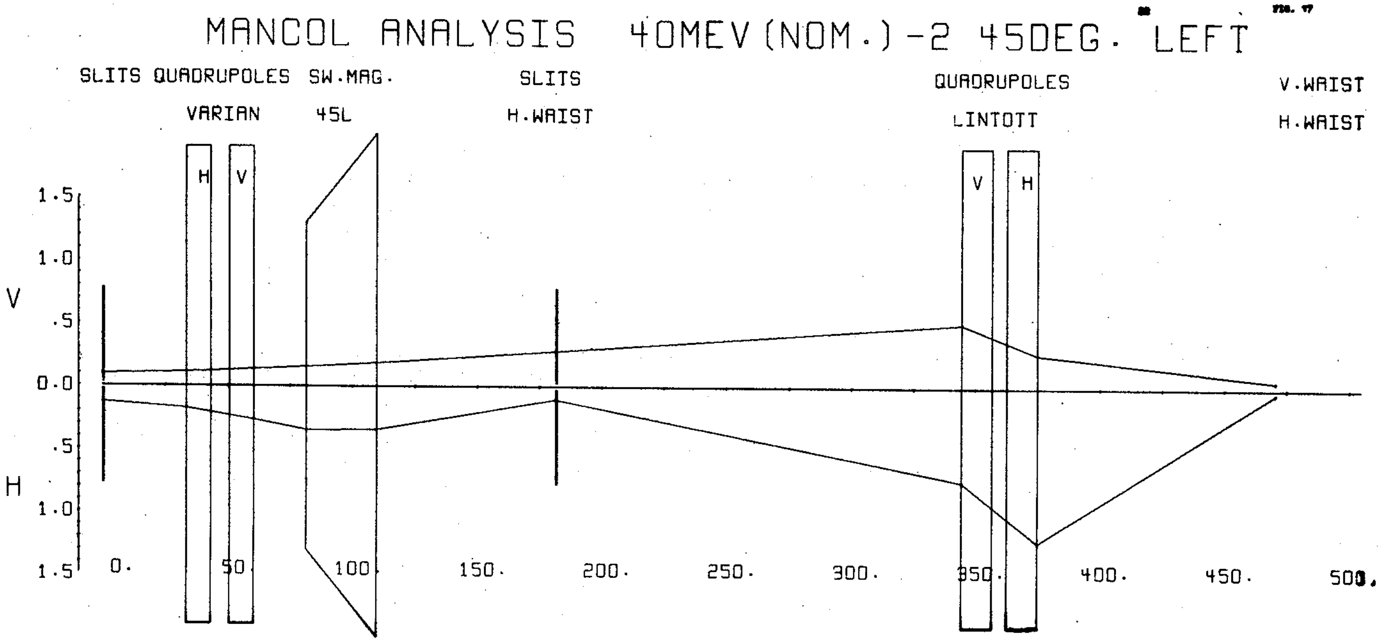
500.


MANCOL ANALYSIS 40MEV(NOM.)-2 45DEG. LEFT
FIG. 17
SLITS
QUADRUPOLES
VARIAN
SW.MAG.
45L
SLITS
H.WAIST
QUADRUPOLES
LINTOTT
V.WAIST
H.WAIST
H
V
V
H
V
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1.5
1.0
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0.0
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1.5
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300.
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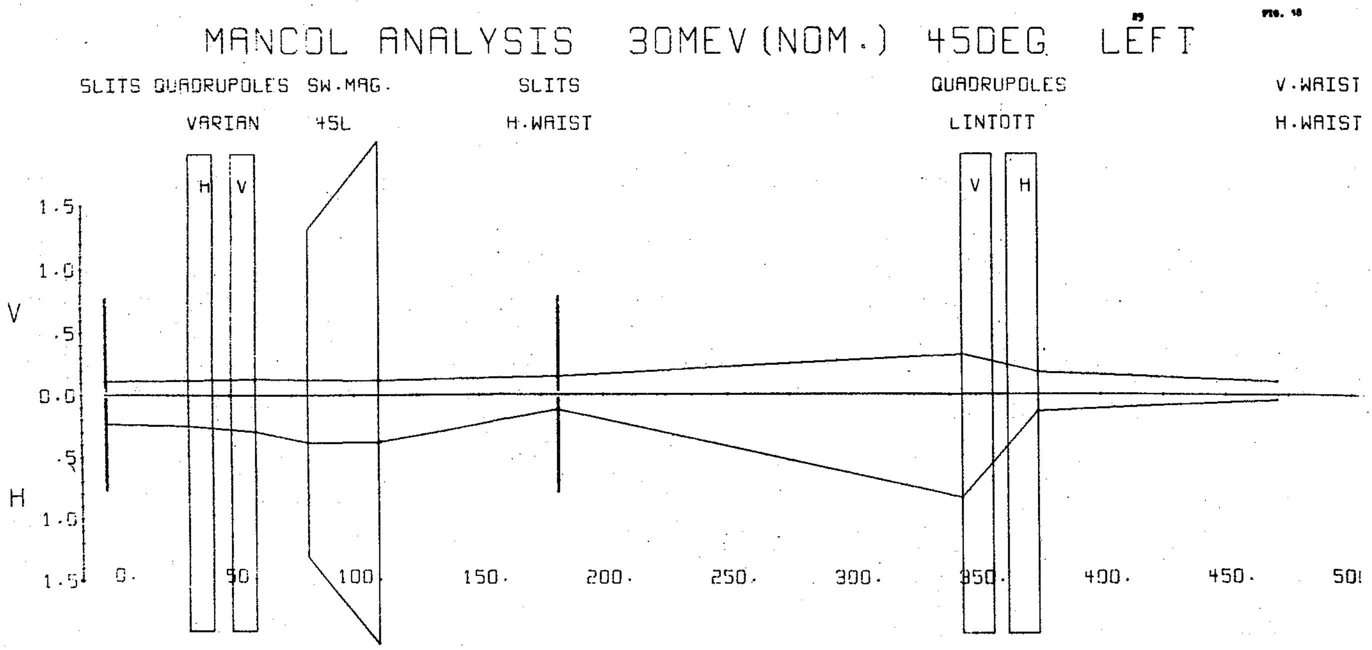
400.
450.
500.

Fig. 17


MANCOL ANALYSIS 30MEV(NOM.) 45DEG LEFT
SLITS
QUADRUPOLES
VARIAN
SW.MAG.
45L
SLITS
H.WAIST
QUADRUPOLES
LINTOTT
V.WAIST
H.WAIST
H
V
V
H
1.5
1.0
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0.0
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1.0
1.5
V
H
0.
50
100
150.
200.
250.
300.
350.
400.
450.
50

Fig. 18

MANCOL ANALYSIS 20MEV(NOM.) 45DEG. LEFT

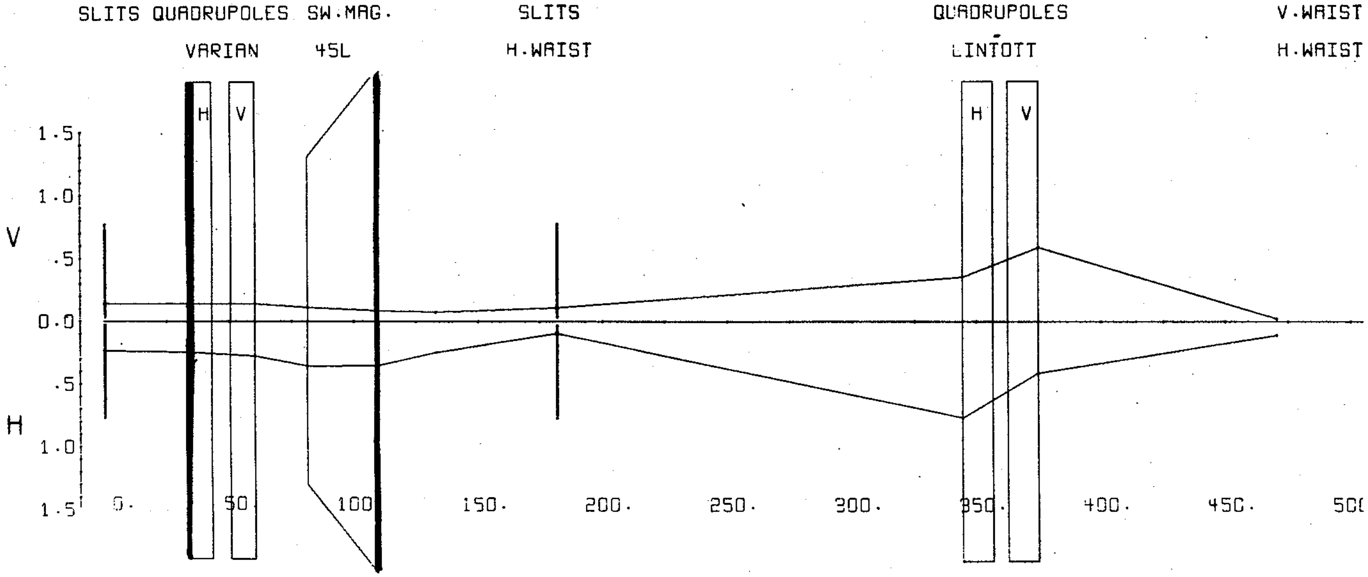

Fig. 19

TABLE I **EMITTANCE DATA**

NOMINAL ENERGY (MeV.)	ACTUAL ENERGY (MeV.)	HORIZONTAL HALF SIZE (in.)	HORIZONTAL HALF DIV. (rad.)	VERTICAL HALF SIZE (in.)	VERTICAL HALF DIV. (rad.)
50.0	45.0	.089	.0017	.070	.0043
40.0 #1	36.0	.215	.0025	.032	.0052
40.0 #2	36.0	.126	.0038	.099	.0016
30.0	27.0	.232	.0025	.114	.0013
20.0	18.0	.233	.0018	.140	.0009

CHAPTER 2 "TRANSPORT" ANALYSIS

The computer program MANCOL has two major disadvantages:

(1) effects due to finite momentum spread are not considered, and

(2) convergence of its parameter search is slow and unreliable.

These problems have been overcome by using a more sophisticated program "TRANSPORT"(4) (FORTRAN - IV, G-LEVEL), a computer program for designing beam transport systems. The TRANSPORT version that has been used in this study was obtained from A. C. Paul(5).

TRANSPORT requires that the following data be provided:

(1) the beam to be injected at the beginning of the system,

(2) the elements of the magnet system (bending magnets, quadrupoles, drift spaces, slits, etc.) through which the beam is to pass, and

(3) the requirements to be imposed upon the transformation matrix(6) and/or the beam matrix such as the size of the beam and the tilt of the phase space ellipse.

The initial beam is specified by the parameters

(i) the horizontal half extent x,

(ii) the horizontal half divergence θ,

(iii) the vertical half extent y,

(iv) the vertical half divergence ϕ,

(v) the pulsed beam length z (z=0 if the beam is dc), and

(vi) the half momentum spread δ.

In specifying the elements of the magnet system one must include the field strengths, lengths and gradients.

The program contains a general least-squares parameter fitting routine to adjust the parameters properly indicated in the instruction cards. The other parameters are held fixed at their original values. First order corrections to the variables are computed using the fitting routine. When an excess number of variables is indicated, a set* is automatically selected. The correction process is repeated until the system is adjusted to given tolerances, or until the computer indicates that it cannot find a solution. Convergence is not guaranteed, but for reasonable initial guesses and meaningful system constraints, it is usually obtained. It has been found that the program takes approximately twelve seconds on the University of Manitoba IBM-360-65 in fitting a system of ten constraints.

TRANSPORT is now available in source form on a library

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* which is most effective in satisfying the constraints

magnetic tape at the University of Manitoba Computer Center. Calling instructions for this tape and formation of the data input are to be found in Appendices II and III, respectively.

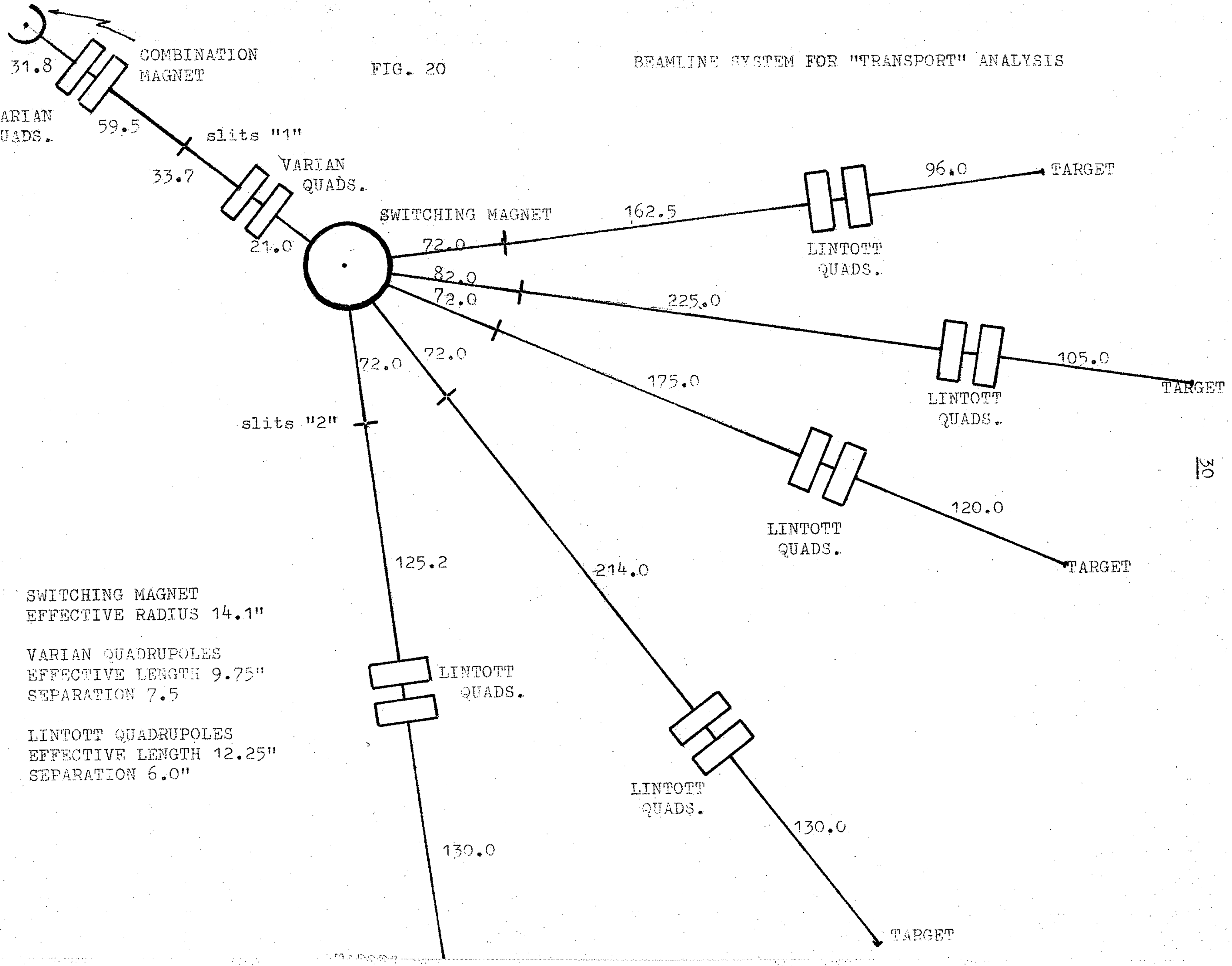
All seven beam lines have been studied with this program, but at the time of this writing analysis of the thirty degree right and zero degree lines has not been completed. The complete beam transport system studied differed slightly from the system studied using MANCOL. As one may note in the following figures, the main difference in the system lies in the thirty degree left line. The drift space from slits "2" to the entrance of the quadrupole doublet was increased from 148.0" to 225.0" and the drift space from the exit of the quadrupole doublet to the target was increased from 100.0" to 105.0". Minor changes have also been made in the positioning of various elements of the common beam line. The complete beam line system studied by TRANSPORT is shown in fig. 20.

As in Chapter 1, the beam was traced from slits "1" to the position of the target. The size and divergence of the beam at slits "1" were again obtained from previously mentioned emittance measurements, (see Table II). For all cases studied a momentum spread of ± 0.5% was assumed. The energies used for input were 90% of the nominal values, since these values are within 1 MeV of the actual values as determined from calibration data. On each beam line a horizontal half width of 0.2 inches and vertical half width of 0.5 inches was required at the first slits after the switching magnet.

Beam profiles for forty-five degree left, thirty degree left, fifteen degree left, fifteen degree right and forty-five degree right presented in figs. 21 to 45. The axes and labeling for these beam profiles are identical to those mentioned in Chapter 1. The TRANSPORT beam profiles are more complete than those of MANCOL in that the beam extent is calculated at the entrance and exit of each quadrupole rather than at the entrance and exit of each quadrupole doublet. Thus one has greater insurance against beam blow up.

FIG. 20

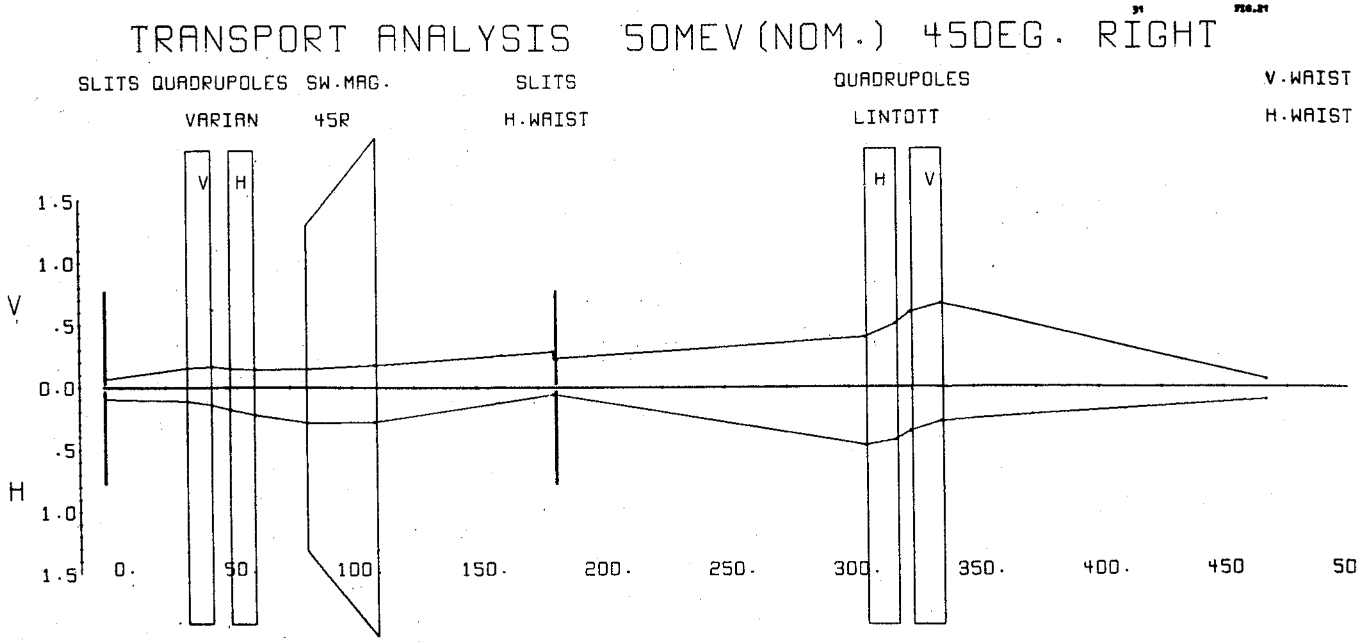
BEAMLINE SYSTEM FOR "TRANSPORT" ANALYSIS



SWITCHING MAGNET
EFFECTIVE RADIUS 14.1"

VARIAN QUADRUPOLES
EFFECTIVE LENGTH 9.75"
SEPARATION 7.5

LINTOTT QUADRUPOLES
EFFECTIVE LENGTH 12.25"
SEPARATION 6.0"


TRANSPORT ANALYSIS 50MEV(NOM.) 45DEG. RIGHT
SLITS
QUADRUPOLES
VARIAN
SW.MAG.
45R
SLITS
H.WAIST
QUADRUPOLES
LINTOTT
V.WAIST
H.WAIST
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H
H
V
V
H
1.5
1.0
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300.
350.
400.
450
50

Fig. 21

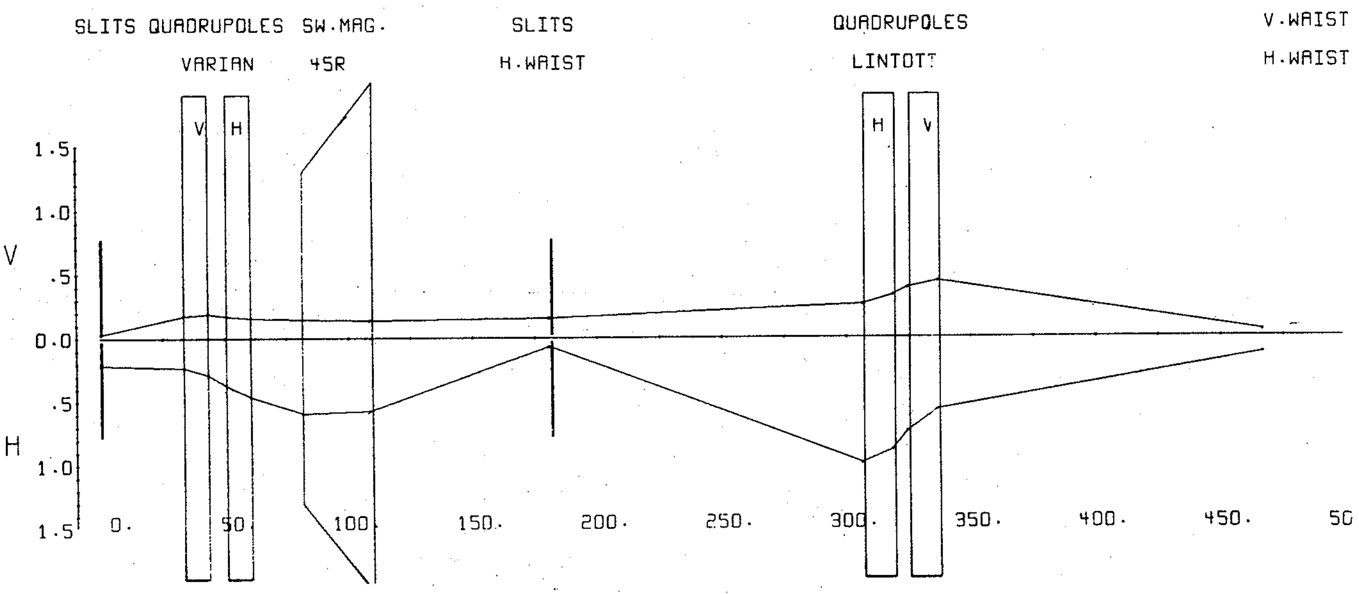
TRANSPORT ANALYSIS 40MEV(NOM.)-1 45DEG. RIGHT

Fig. 22

TRANSPORT ANALYSIS 40MEV(NOM.)-2 45DEG "RIGHT"

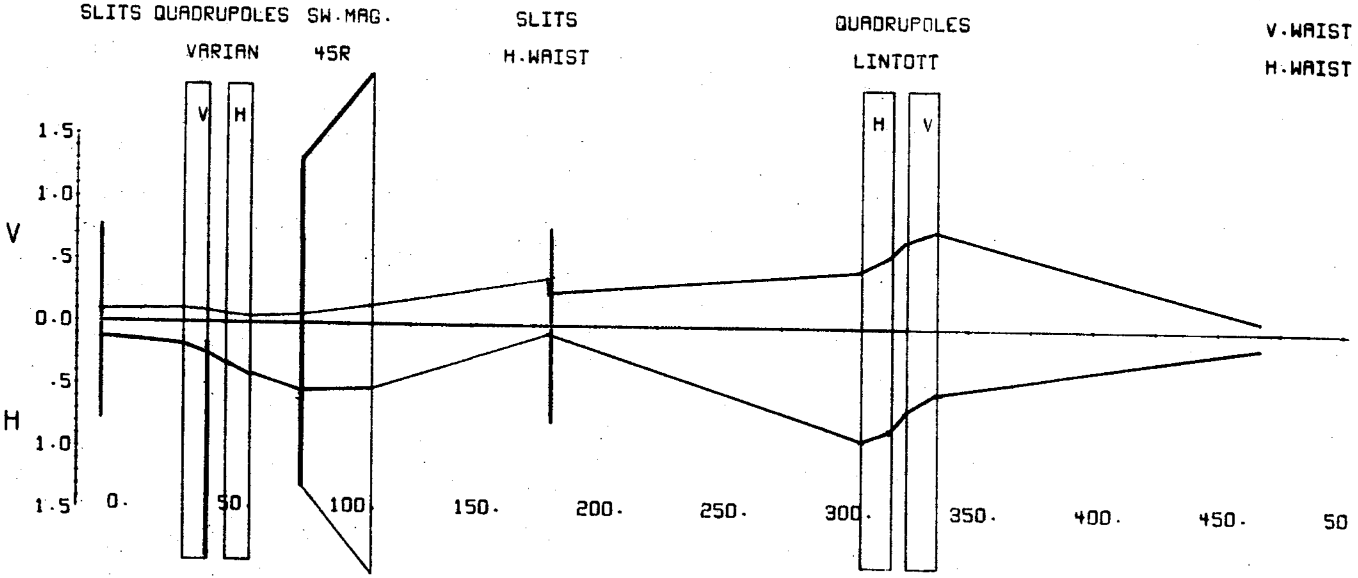

Fig. 23

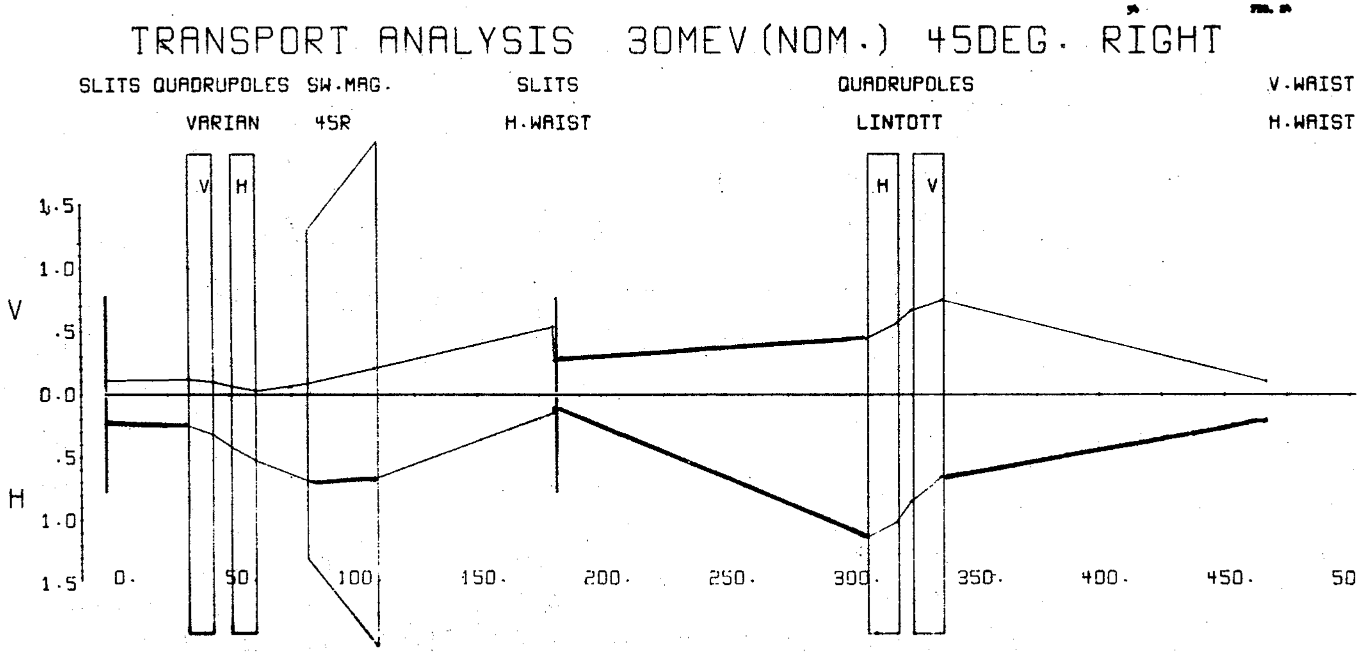
TRANSPORT ANALYSIS 30MEV(NOM.) 45DEG. RIGHT
SLITS
QUADRUPOLES
VARIAN
SW.MAG.
45R
SLITS
H.WAIST
QUADRUPOLES
LINTOTT
V.WAIST
H.WAIST
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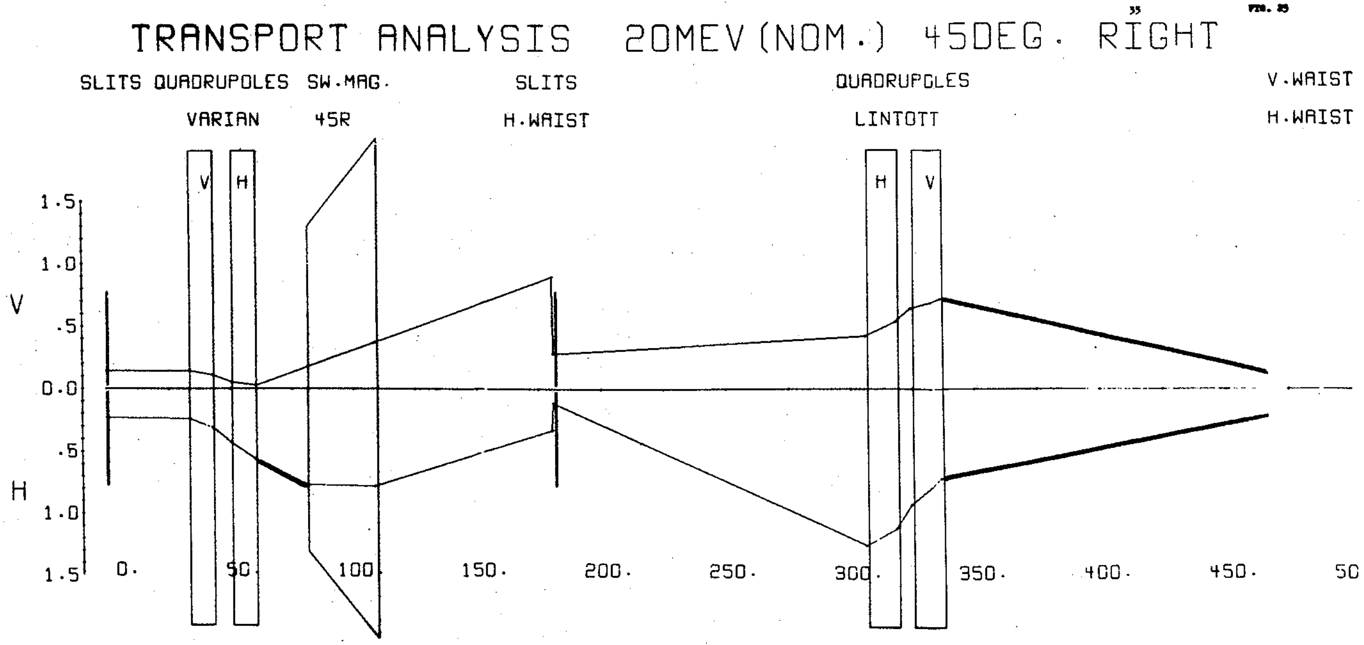
Fig. 24


TRANSPORT ANALYSIS 20MEV(NOM.) 45DEG. RIGHT
SLITS
QUADRUPOLES
VARIAN
SW.MAG.
45R
SLITS
H.WAIST
QUADRUPOLES
LINTOTT
V.WAIST
H.WAIST
V
H
H
V
V
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1.5
1.0
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0.0
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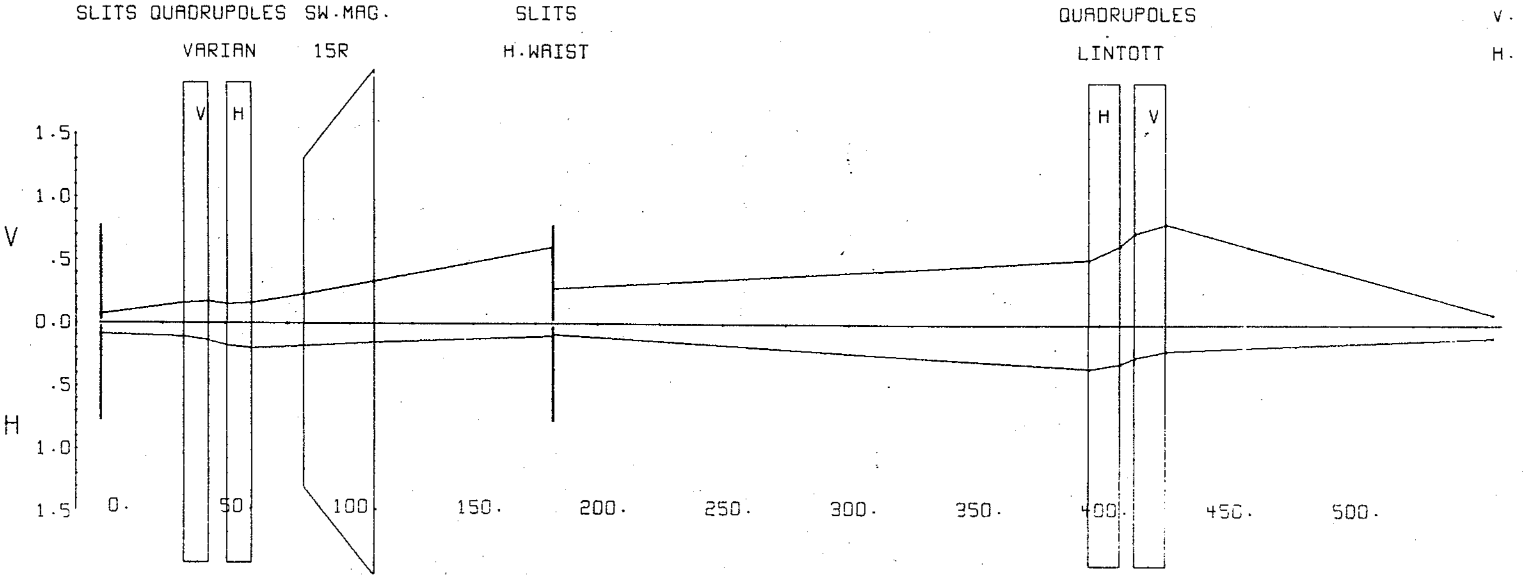
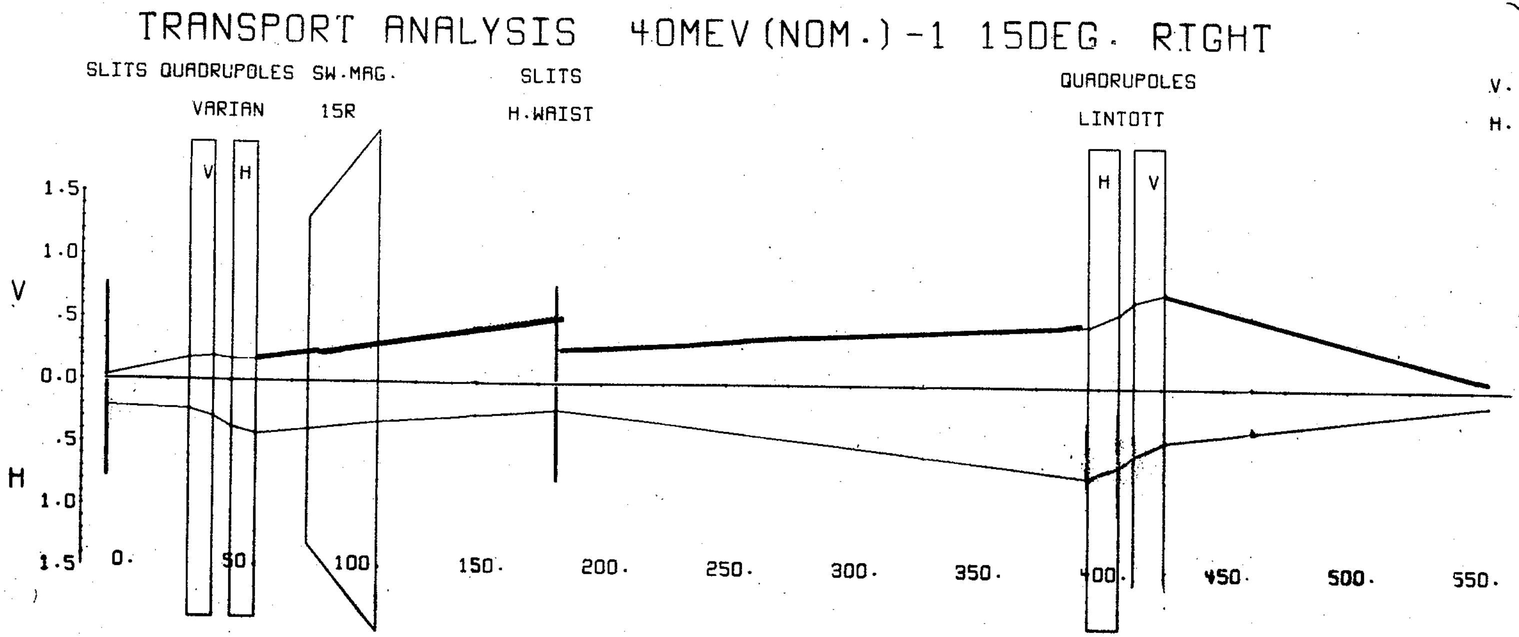
1.0
1.5
0.
50
100
150.
200.
250.
300.
350.
400.
450.
50

Fig. 25

Fig. 26

TRANSPORT ANALYSIS 50MEV(NOM.) 15DEG. RIGHT

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SLITS
QUADRUPOLES
VARIAN
SW.MAG.
15R
SLITS
H.WAIST
QUADRUPOLES
LINTOTT
V.
H.
V
H
H
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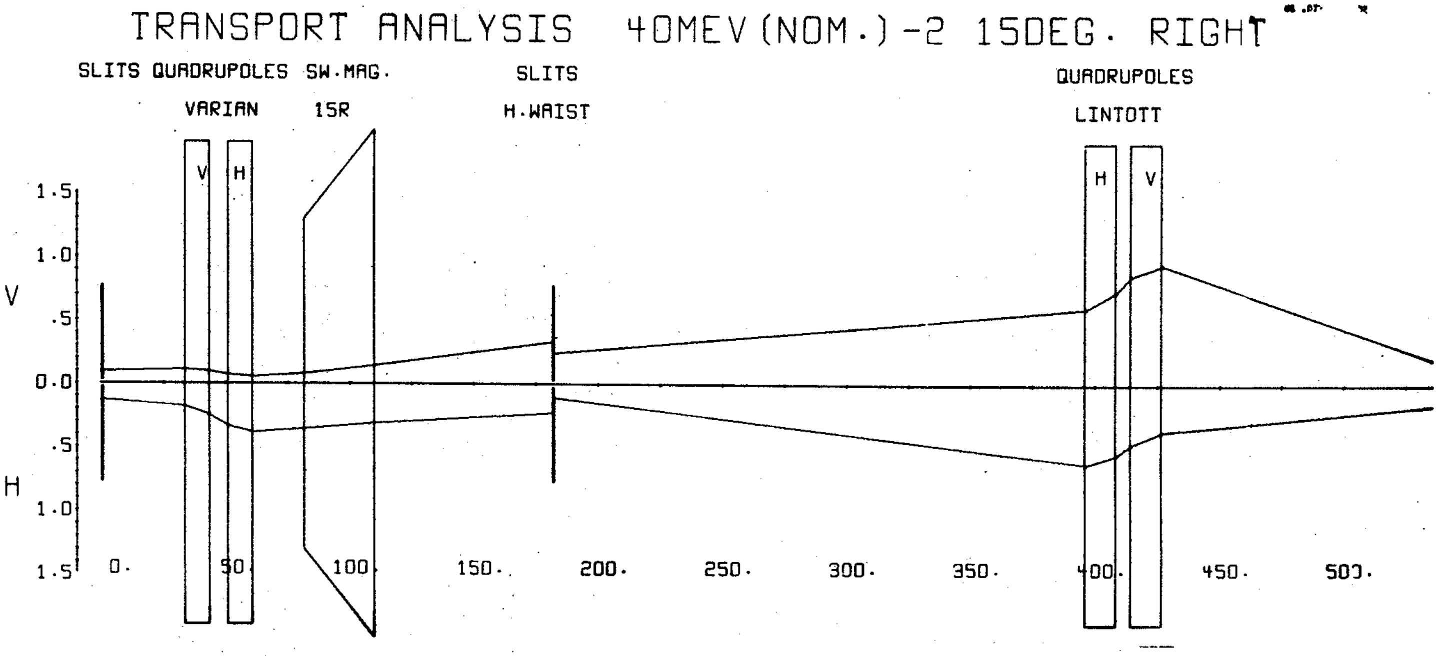
350.
400.
450.
500.
550.

Fig. 27

Fig. 28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QUADRUPOLES
VARIAN
SW.MAG.
15R
SLITS
H.WAIST
QUADRUPOLES
LINTOTT
V
H
H
V
V
H
1.5
1.0
.5
0.0
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1.0
1.5
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50
100
150.
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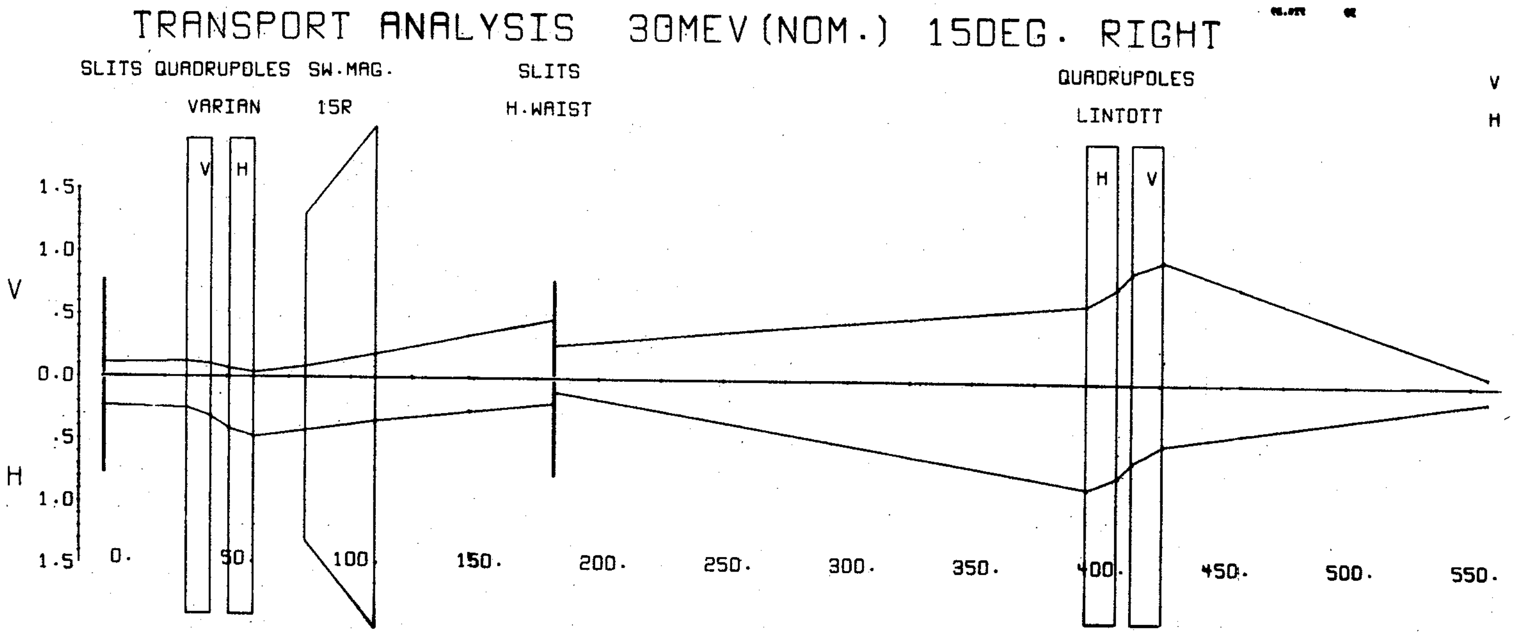
250.
300.
350.
400.
450.
500.


TRANSPORT ANALYSIS 30MEV(NOM.) 15DEG. RIGHT
SLITS
QUADRUPOLES
VARIAN
SW.MAG.
15R
SLITS
H.WAIST
QUADRUPOLES
LINTOTT
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H
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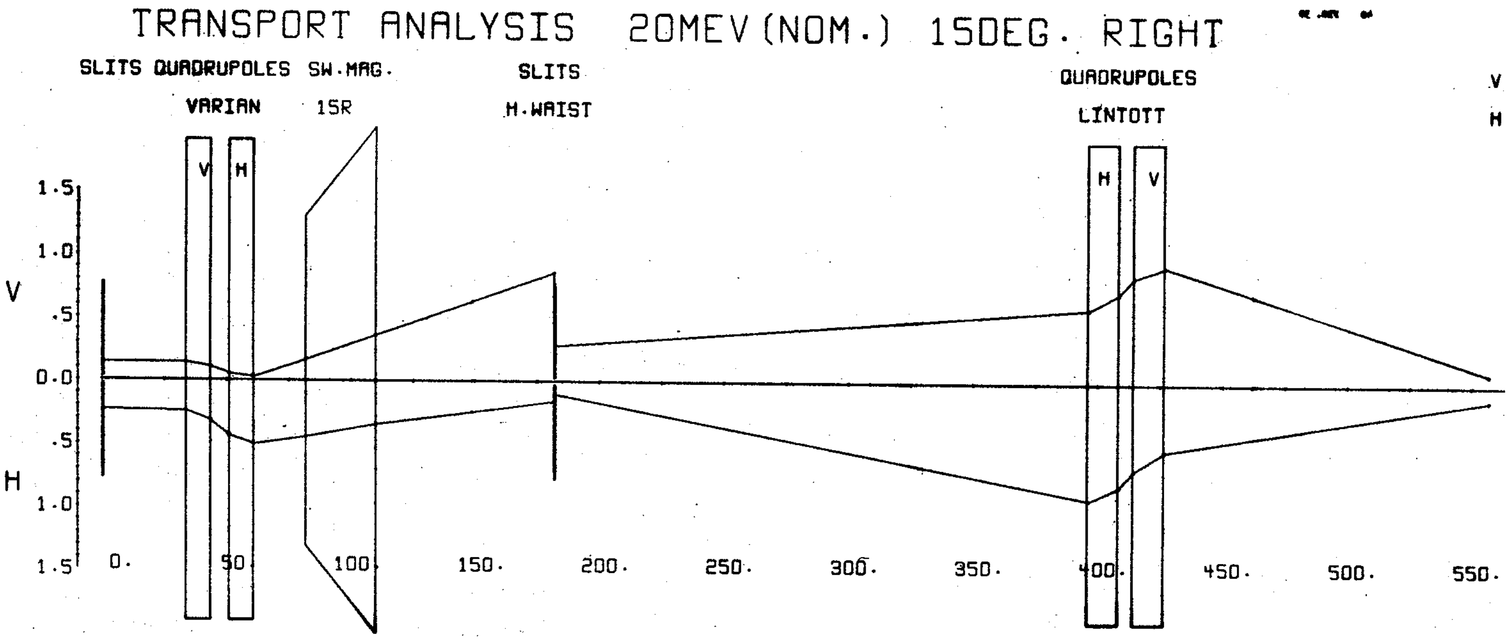
450.
500.
550.

Fig. 29


TRANSPORT ANALYSIS 20MEV(NOM.) 15DEG. RIGHT
SLITS
QUADRUPOLES
VARIAN
SW.MAG.
15R
SLITS
H.WAIST
QUADRUPOLES
LINTOTT
V
H
V
H
H
V
1.5
1.0
V
.5
0.0
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H
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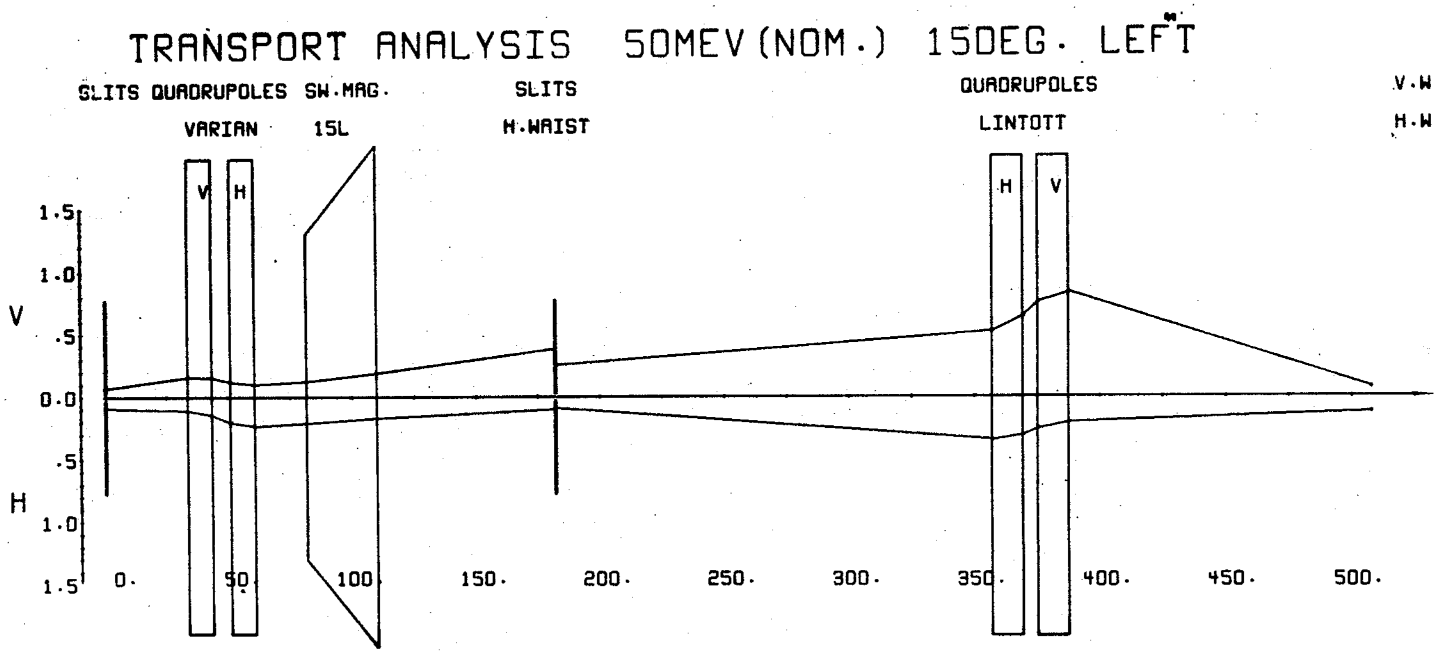
450.
500.
550.

Fig. 30


TRANSPORT ANALYSIS 50MEV(NOM.) 15DEG. LEFT
SLITS
QUADRUPOLES
VARIAN
SW.MAG.
15L
SLITS
H.WAIST
QUADRUPOLES
LINTOTT
V.W
H.W
V
H
H
V
V
H
1.5
1.0
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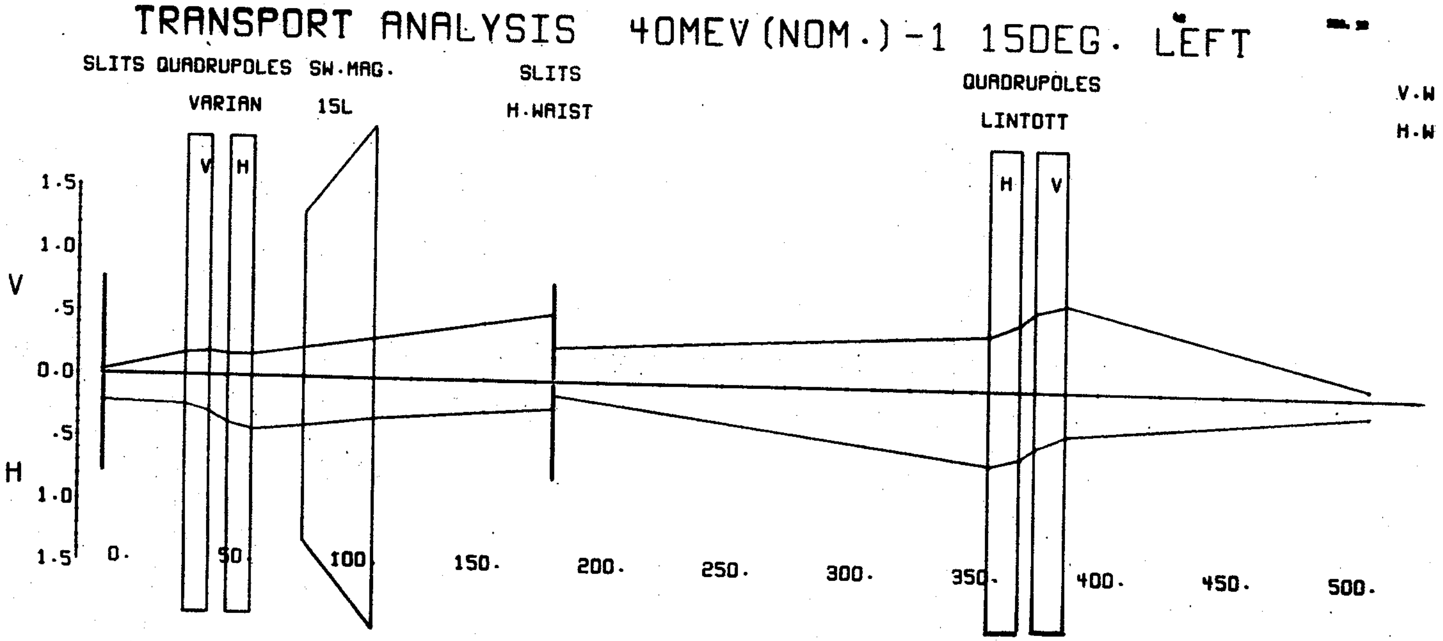
400.
450.
500.

Fig. 31


TRANSPORT ANALYSIS 40MEV(NOM.)-1 15DEG. LEFT
SLITS
QUADRUPOLES
VARIAN
SW.MAG.
15L
SLITS
H.WAIST
QUADRUPOLES
LINTOTT
V.W
H.W
V
H
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50.
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150.
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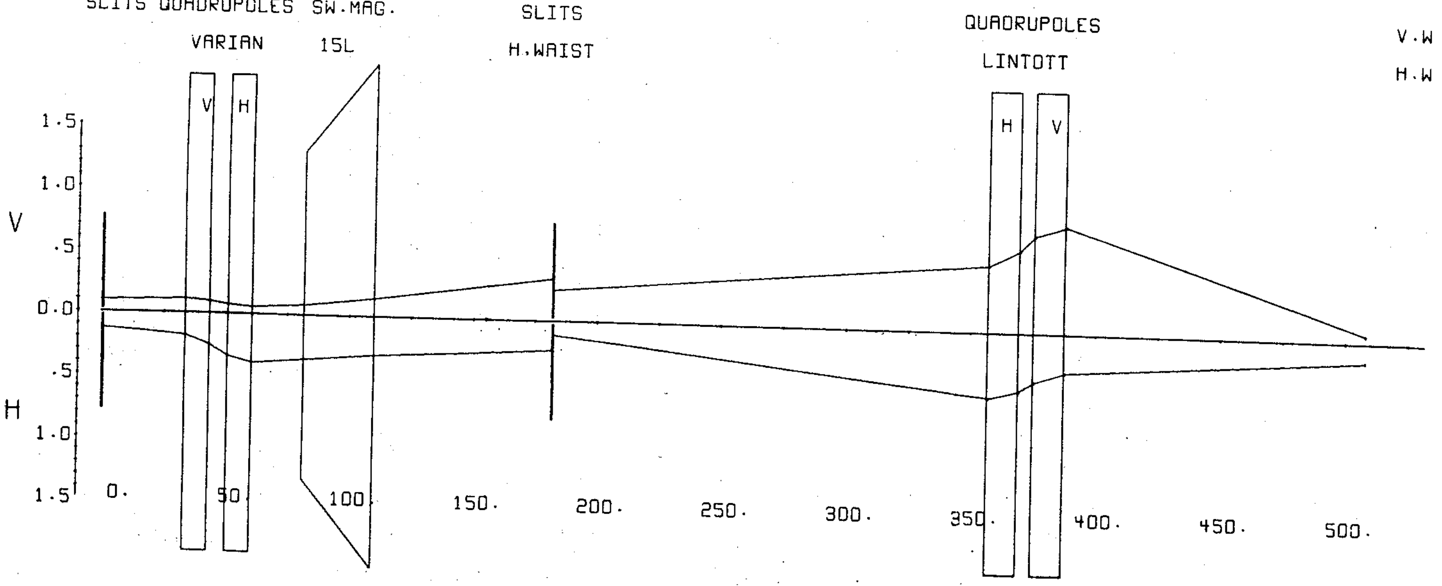
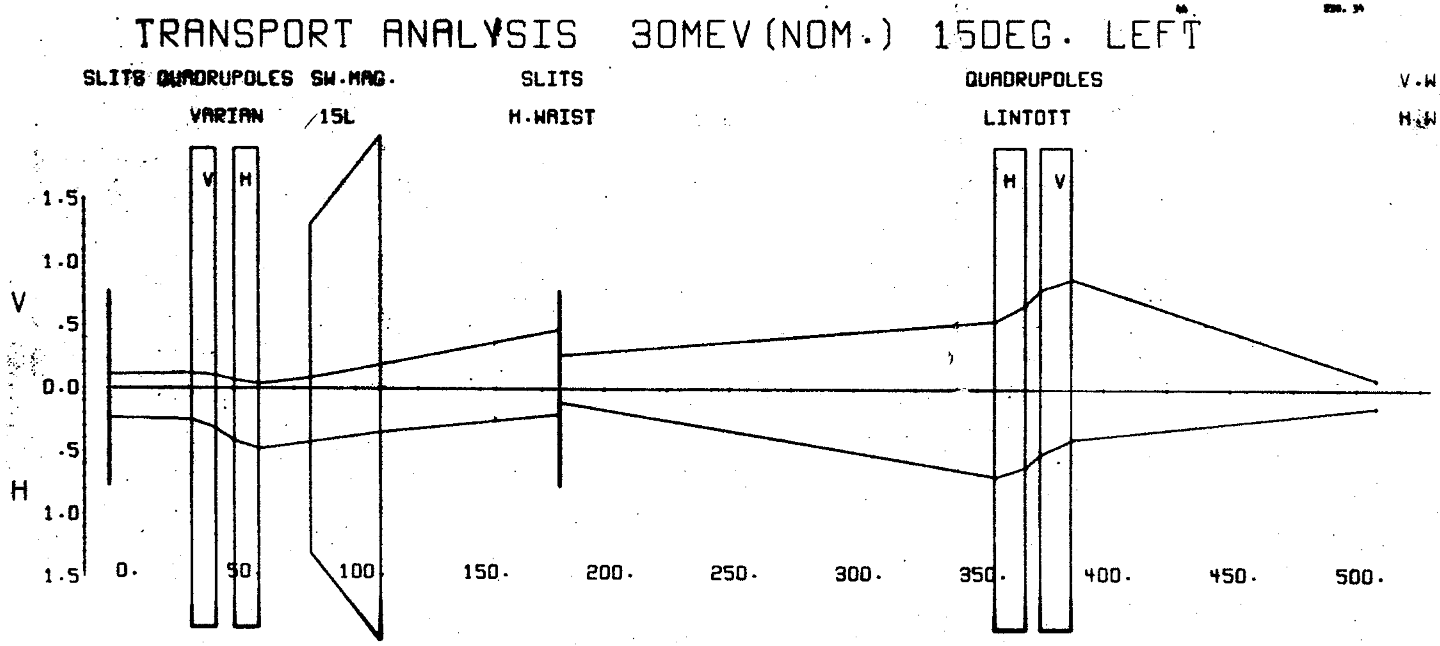
300.
350.
400.
450.
500.

Fig. 32

Fig. 33

TRANSPORT ANALYSIS 40MEV(NOM.)-2 15DEG. LEFT




TRANSPORT ANALYSIS 30MEV(NOM.) 15DEG. LEFT
SLITS
QUADRUPOLES
VARIAN
SW.MAG.
15L
SLITS
H.WAIST
QUADRUPOLES
LINTOTT
V.W
H.W
V
H
H
V
V
H
1.5
1.0
.5
0.0
.5
1.0
1.5
0.
50
100
150.
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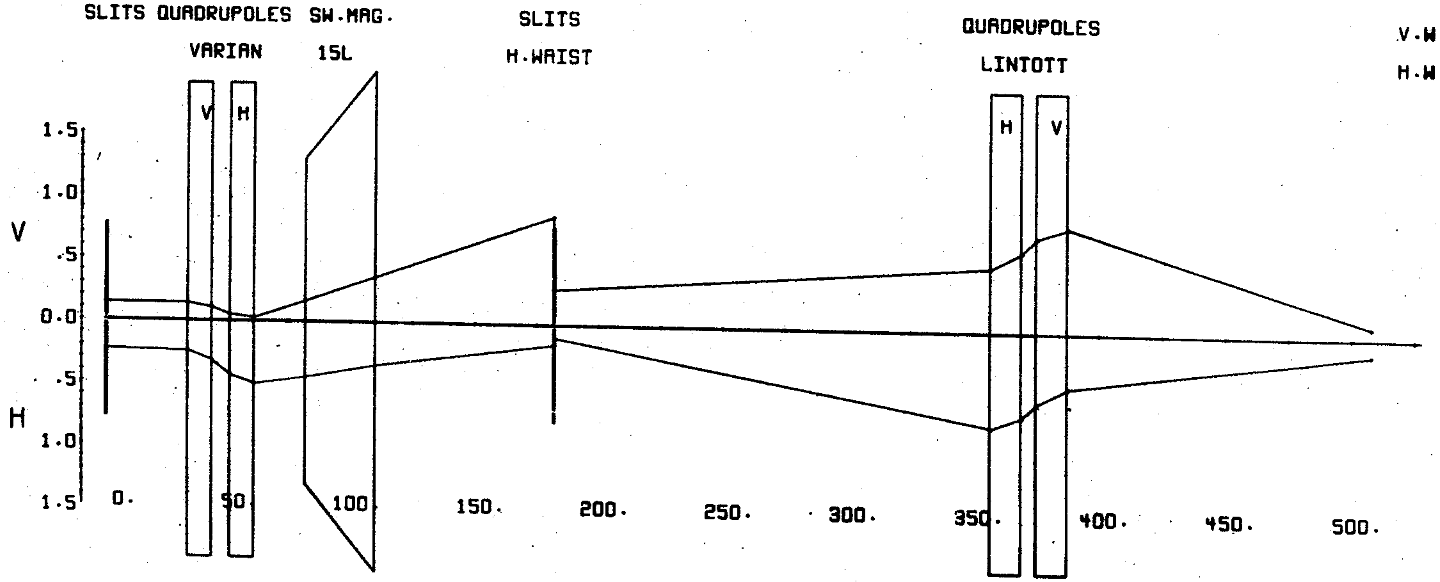
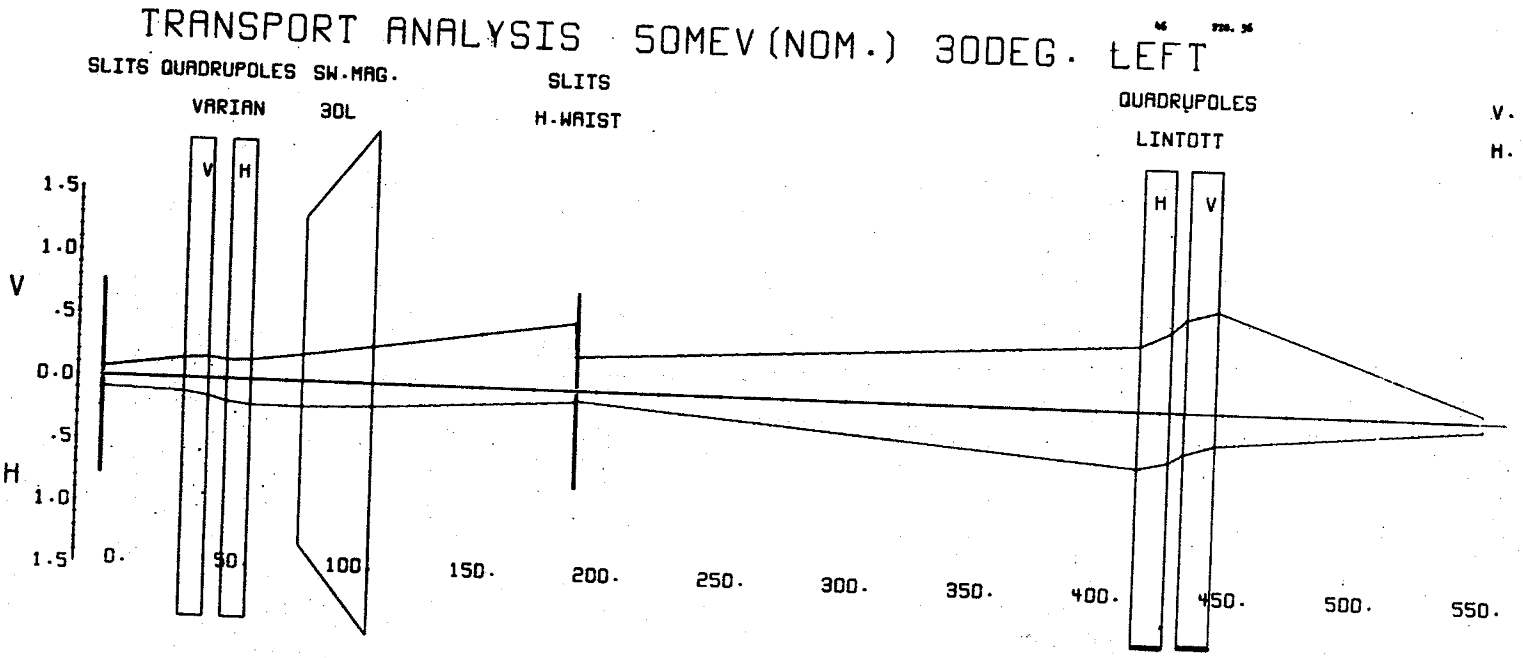
300.
350.
400.
450.
500.

Fig. 34

Fig. 35

TRANSPORT ANALYSIS 20MEV (NOM.) 15DEG. LEFT




TRANSPORT ANALYSIS 50MEV (NOM.) 30DEG. LEFT
SLITS
QUADRUPOLES
VARIAN
SW.MAG.
30L
SLITS
H.WAIST
QUADRUPOLES
LINTOTT
V
H
V
H
H
V
V.
H.
V
H
1.5
1.0
.5
0.0
.5
1.0
1.5
0.
50.
100.
150.
200.
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300.
350.
400.
450.
500.
550.

Fig. 36

TRANSPORT ANALYSIS 40MEV (NOM.) -1 30DEG. LEFT

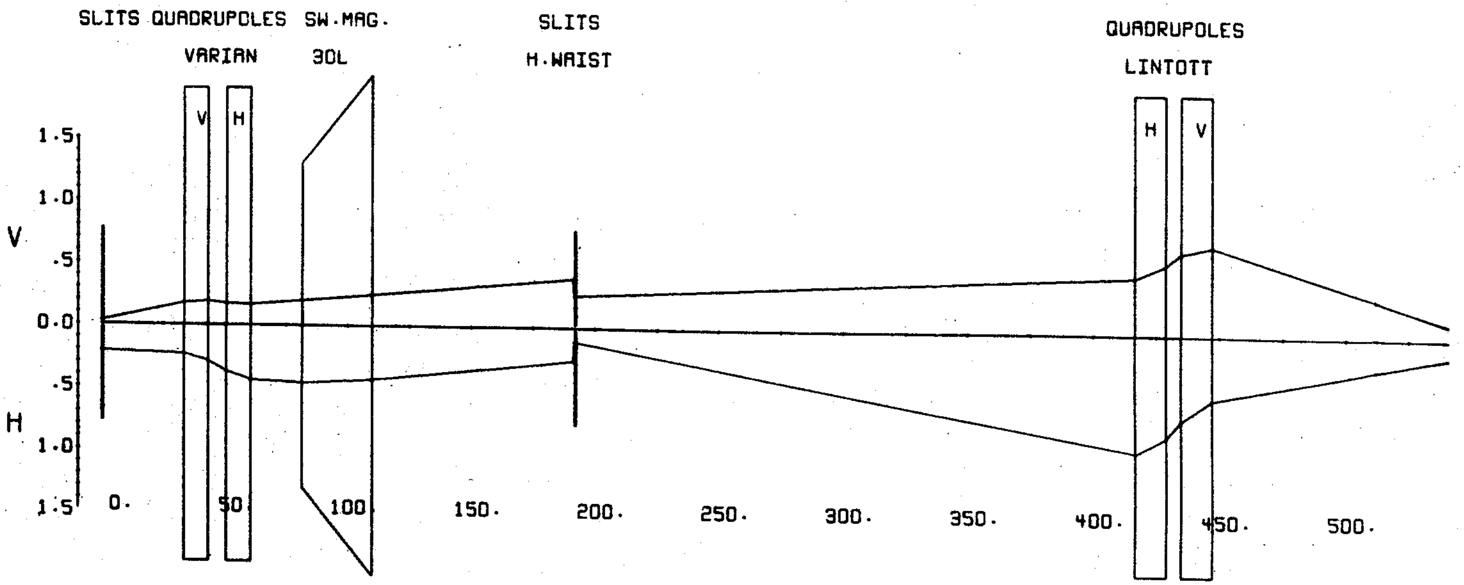
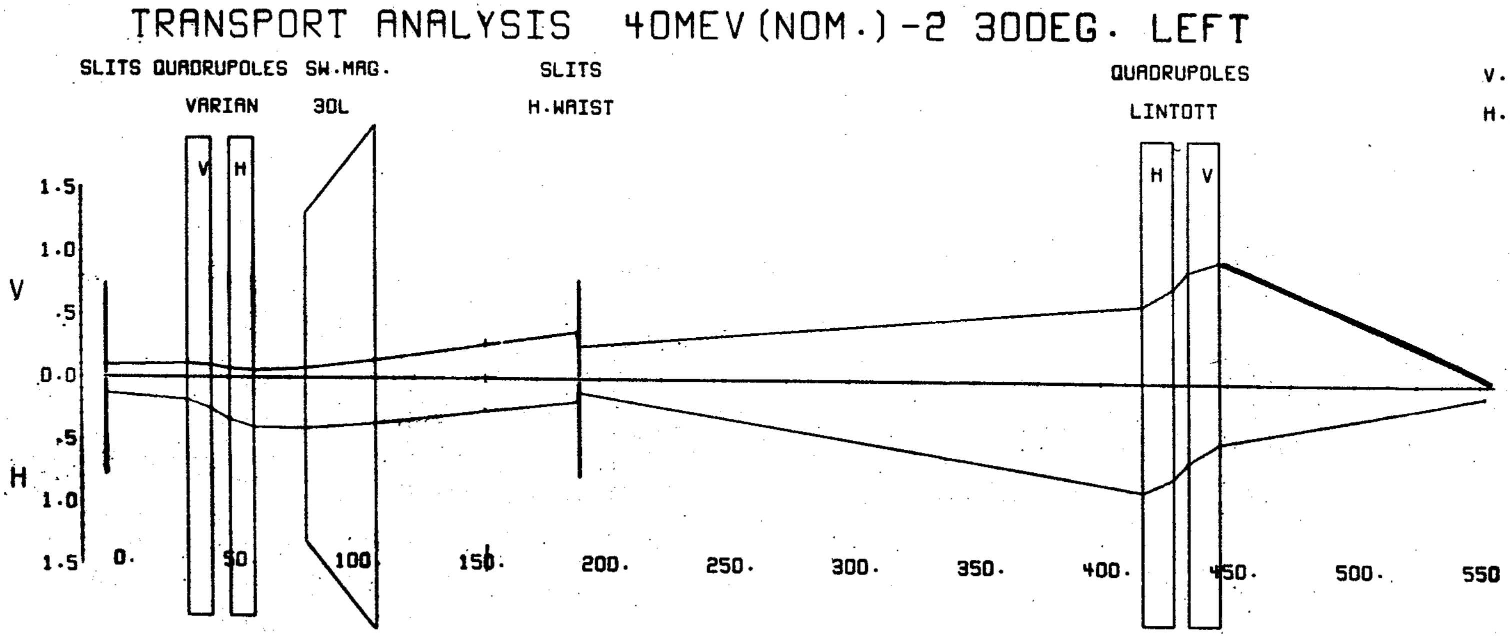


Fig. 37


TRANSPORT ANALYSIS 40MEV(NOM.)-2 30DEG. LEFT
SLITS
QUADRUPOLES
VARIAN
SW.MAG.
30L
SLITS
H.WAIST
QUADRUPOLES
LINTOTT
V.
H.
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H
H
V
V
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1.5
1.0
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0.0
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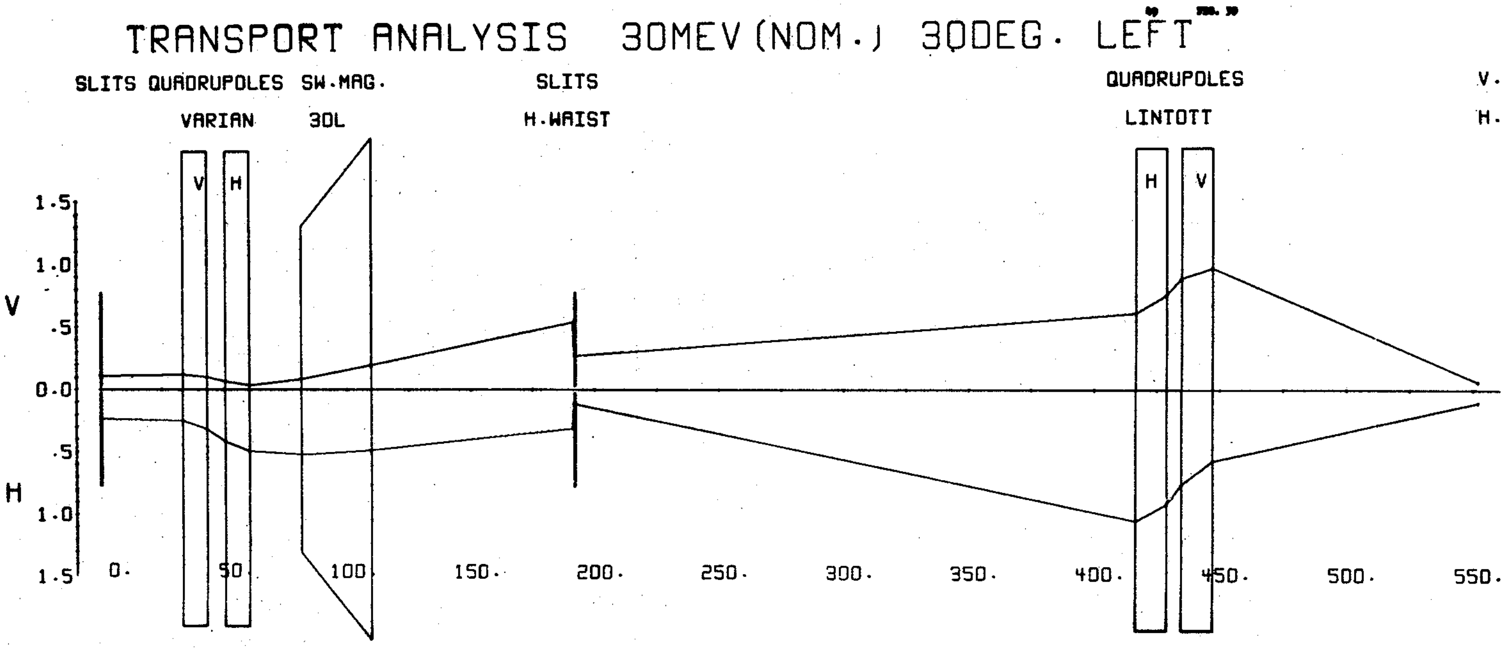
400.
450.
500.
550

Fig. 38


TRANSPORT ANALYSIS 30MEV(NOM.) 30DEG. LEFT
SLITS
QUADRUPOLES
VARIAN
SW.MAG.
30L
SLITS
H.WAIST
QUADRUPOLES
LINTOTT
V.
H.
V
H
H
V
V
H
1.5
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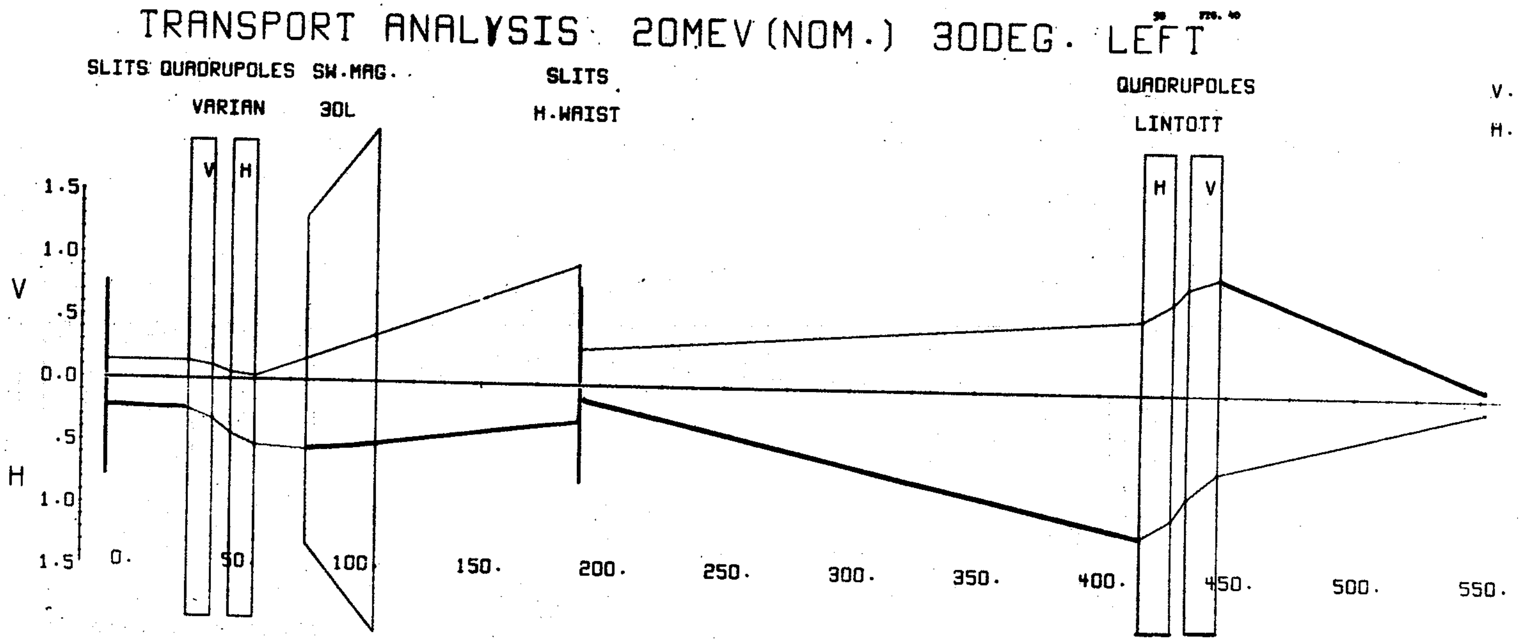
350.
400.
450.
500.
550.

Fig. 39

Fig. 40


TRANSPORT ANALYSIS 20MEV(NOM.) 30DEG. LEFT
SLITS
QUADRUPOLES
VARIAN
SW.MAG.
30L
SLITS
H.WAIST
QUADRUPOLES
LINTOTT
V.
H.
V
H
H
V
V
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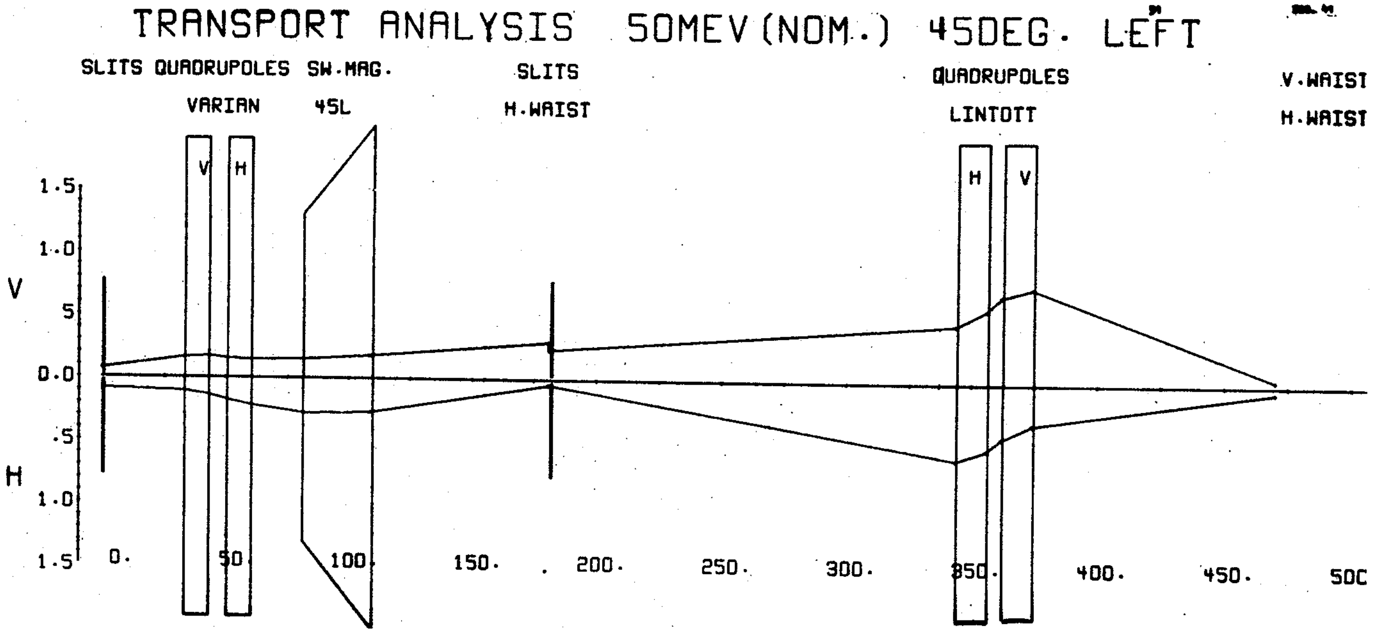
TRANSPORT ANALYSIS 50MEV(NOM.) 45DEG. LEFT
SLITS
QUADRUPOLES
VARIAN
SW.MAG.
45L
SLITS
H.WAIST
QUADRUPOLES
LINTOTT
V.WAIST
H.WAIST
V
H
H
V
V
H
1.5
1.0
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1.0
1.5
0.
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150.
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250.
300.
350.
400.
450.
50C

Fig. 41

TRANSPORT ANALYSIS 40MEV(NOM.)-1 45DEG. LEFT

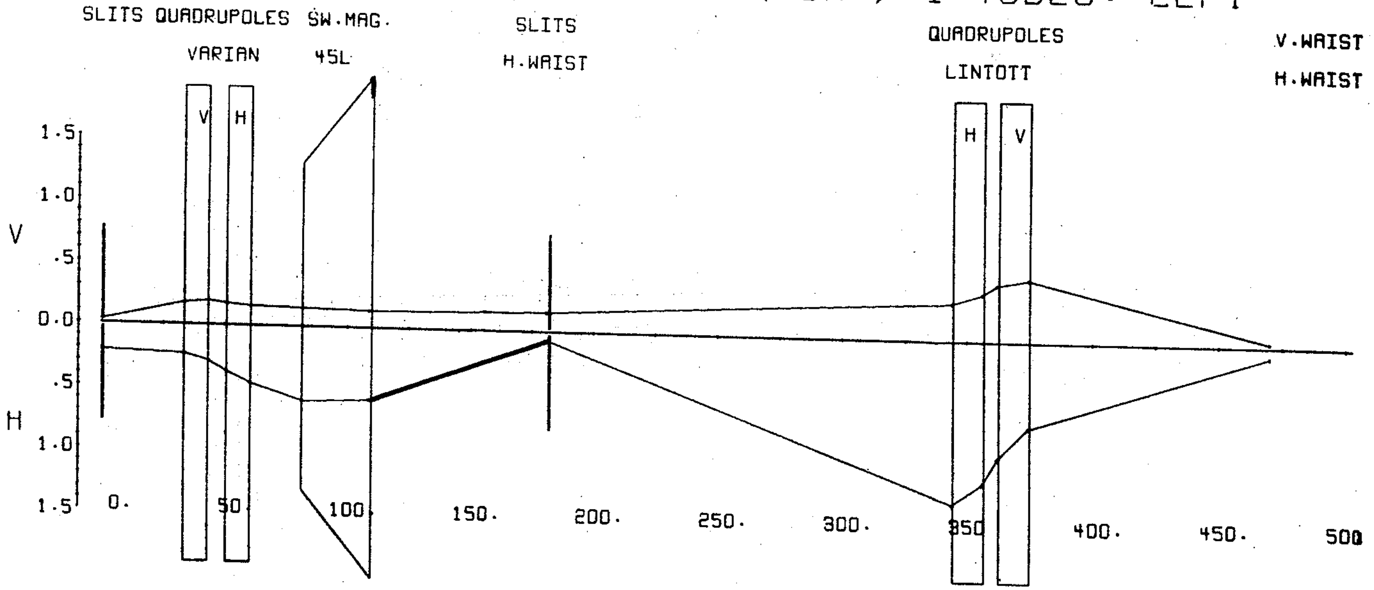
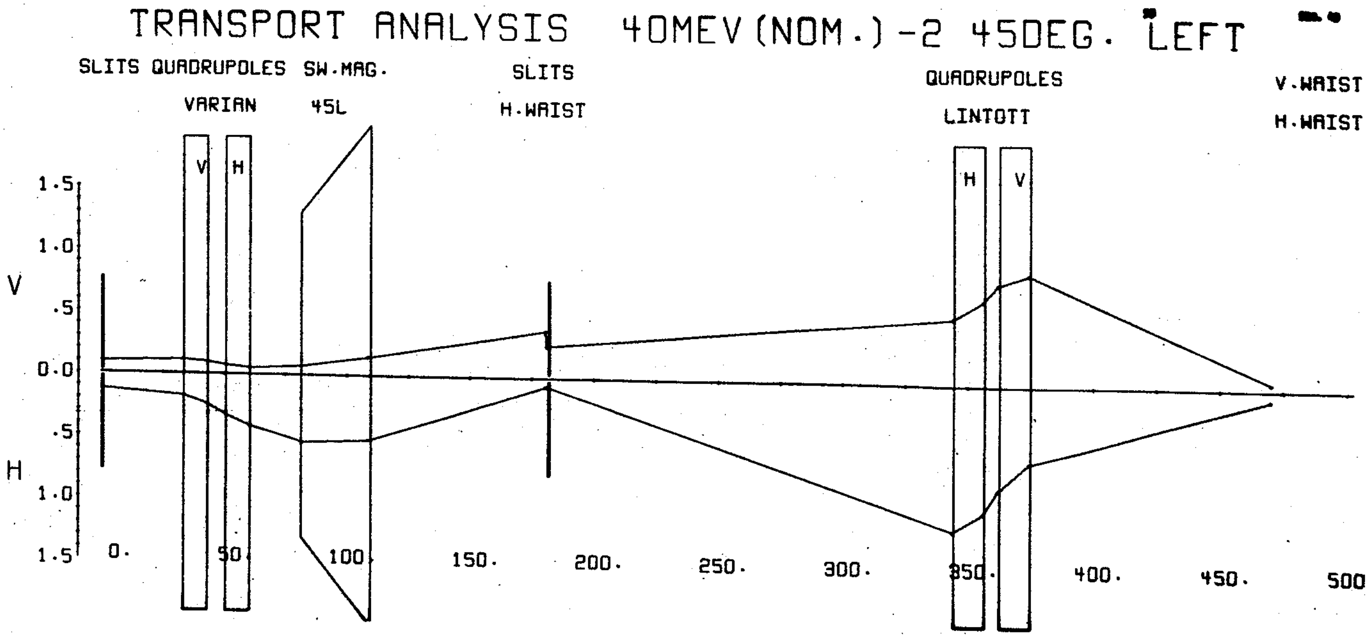


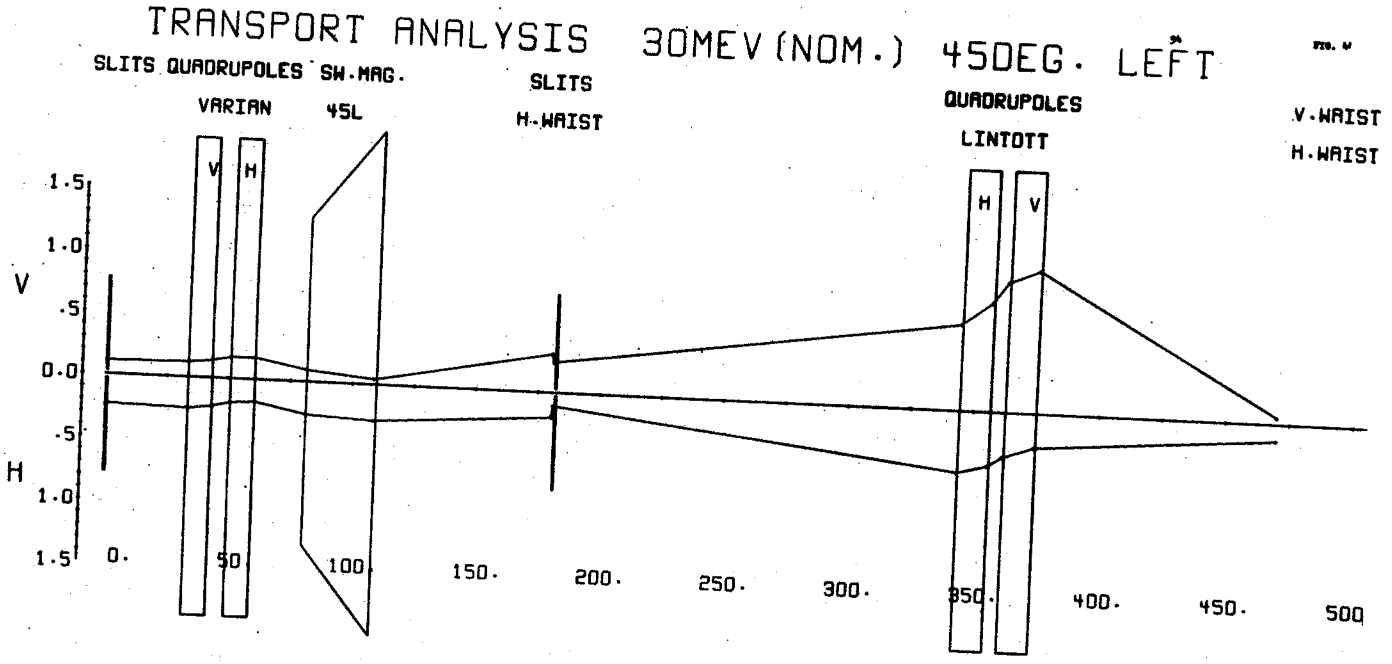
Fig. 42



TRANSPORT ANALYSIS 40MEV(NOM.)-2 45DEG. LEFT

Fig. 43

Fig. 44


TRANSPORT ANALYSIS 30MEV(NOM.) 45DEG. LEFT
SLITS
QUADRUPOLES
VARIAN
SW.MAG.
45L
SLITS
H.WAIST
QUADRUPOLES
LINTOTT
V.WAIST
H.WAIST
V
H
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V
V
H
1.5
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1.5
0.
50
100
150.
200.
250.
300.
350.
400.
450.
500

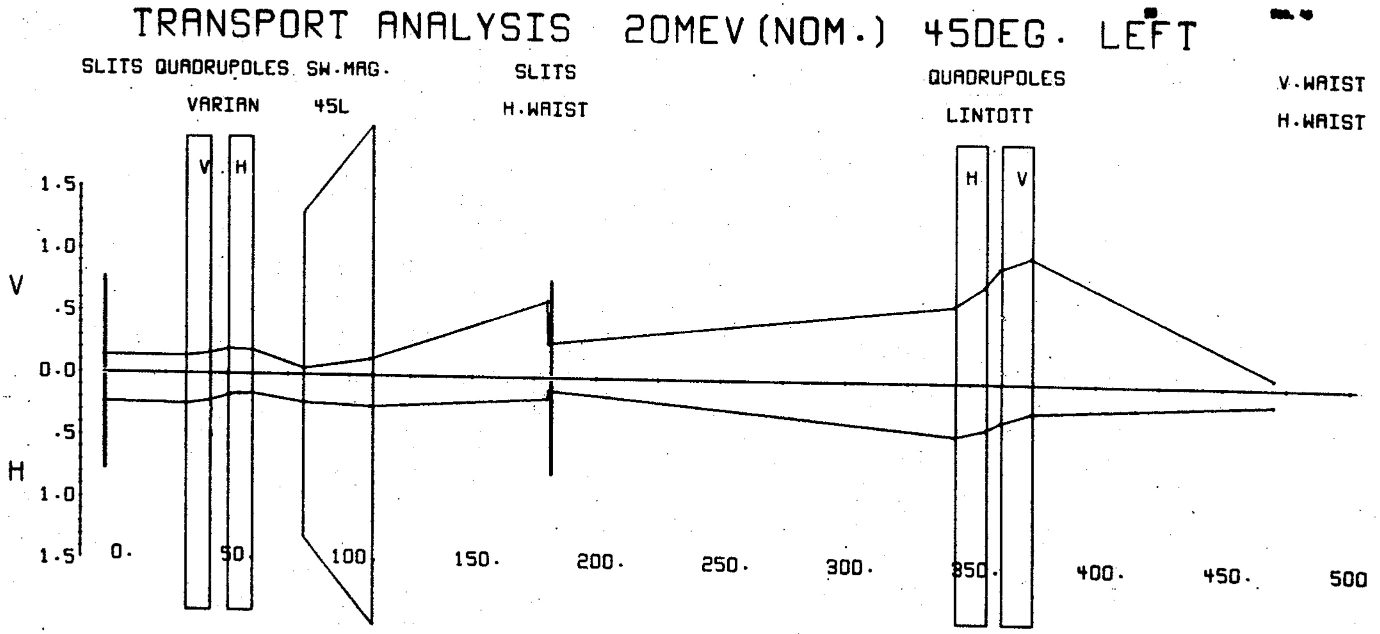
TRANSPORT ANALYSIS 20MEV(NOM.) 45DEG. LEFT
SLITS
QUADRUPOLES
VARIAN
SW.MAG.
45L
SLITS
H.WAIST
QUADRUPOLES
LINTOTT
V.WAIST
H.WAIST
V
H
H
V
V
H
1.5
1.0
.5
0.0
.5
1.0
1.5
0.
50.
100.
150.
200.
250.
300.
350.
400.
450.
500

Fig. 45

Suggestions for Future TRANSPORT Studies

The results of this chapter indicate that beam transport is indeed feasible on the beam lines studied. However, a great deal can be done to improve the rigour, quality, and completeness of the analysis presented in this thesis. In this light, some suggestions for future work are given here:

(1) Emittance measurements: The validity of the results is limited by the quality of the input data. Thus there is need for more extensive information on the emittance from the cyclotron for use in specifying the initial beam.

Furthermore, the emittance data used (see Table I) were obtained under poor operating conditions. More specifically, the emittance measurements were made for a beam having a double waist at slits "1". This may be an unnecessarily difficult input to the code because in general practice one prefers to have a vertical waist near the centre of the switching magnet. Having a vertical waist at slits "1" therefore implies that the beam has a larger vertical divergence, which may lead to beam blow up in the switching magnet and quadrupoles.

Rigourously speaking, the emittance measurements were not applicable for another reason. In normal operation foci are required at slits "1" and "2" for the following reason. A focus condition is represented by the transformation matrix element,(6) M_{12}, equal to zero. Thus the horizontal position of any particle at the image is equal to a constant, M_{11}, times the horizontal position of the particle at the object; i.e., the horizontal position at the focus does not depend upon the particle's divergence at the object. Then the distribution (energy vs. horizontal position) which leaves the cyclotron is best preserved by placing slits "1" at the first focus after quadrupole pair 1 and energy selection is best made by placing slits "2" at the first focus after the switching magnet.

(2) Dispersion studies: Study of dispersion in the beam transport system requires a knowledge of correlation between the horizontal position and the total momentum. The code has been prepared in a manner that allows easy utilization of this information, if available. Therefore the dispersion introduced by the fringing field, the combination magnet, and switching magnet should be analyzed and a better indication of the energy selection, performed by the slits, obtained.

(3) Second order effects: The rigour of the calculations can be improved by including the following second order effects:

(i) chromatic aberrations of quadrupoles - the change in focal length of a quadrupole with energy,

(ii) geometric effects in the bending magnets,

(iii) non-uniformity of fields of bending magnets in the radial plane, and

(iv) the spread in longitudinal extent of a pulse due to rays not being parallel to the central trajectory.

CHAPTER 3 STABLE ORBITS

Stable orbits for the University of Manitoba cyclotron have been found by means of the computer code "ORBCAL"(7) (FORTRAN IV, G-LEVEL), obtained from A. C. Paul. A stable or equilibrium orbit is defined as having $z = p_z = 0$, a constant total momentum p, and initial and final values of the radius (r_i, r_f) and radial momentum (p_{ri}, p_{rf}), such that they satisfy the following inequality.

$$|r_i - r_f| + |p_{ri} - p_{rf}| < \varepsilon \quad \cdots\cdots\cdots\cdots \quad (3.1),$$

where ε is arbitrarily small.

Details of the calculation can be found in ref. 8 . It will suffice to say here that the equations of motion were linearized, and the equilibrium orbits were found and analyzed on the basis of Floquet's theory of coupled first order differential equations with periodic coefficients. ORBCAL is now available on magnetic tape in source form at the University of Manitoba Computer Center. Calling instructions for the tape are furnished in Appendix II; a guide to preparation of the data input can be found in Appendix IV .

The purpose of these calculations was threefold:

(1) To obtain a plot of the energy versus stripping

foil radius;

(2) To obtain values for the component of the radial momentum at the point of stripping; and

(3) To obtain the flutter and the average magnetic field as a function of radius.

Stable orbits have been sought for the following energies: 50.0, 45.0, 40.0, 35.0, 30.0, 25.0, and 20.0 MeV. The radii and radial momenta of protons describing stable orbits have been determined at azimuths from 0.3 degrees to 27.1 degrees in steps of 1.8 degrees. Azimuths were measured in a manner described in fig. 46.

A magnetic field grid in the medium plane for a quadrant of the cyclotron must be supplied as input for the code. The quadrant grid used ranged from azimuth 16.5 degrees to 106.5 degrees in steps of 1.8 degrees, and from radius 1.0 inches to 50.0 inches in steps of 1.0 inches.

ORBCAL was run with three different magnetic field grids as input. These differed by a constant factor. The first field tried, henceforth to be refered to as "data set #1", was derived from the latest complete field measurements(9) multiplied by a factor of 1.020507. The calibration factor was determined by means of an NMR measurement.(10) "Data set #2" refers to an earlier set of field measurements, and "data set #3" refers to the earlier values of the field multiplied by a factor of 1.0/1.0253.

The ORBCAL calculations for stable orbits using data sets 1, 2, and 3 are to be found in Tables II, III, and IV respectively.

Seven points on the locus of the stripping foil have been determined.(11) They are listed in Table V. By interpolating the** ORBCAL calculations at these positions of the stripping foil, one may find the energy of stable proton orbits. The ORBCAL output of Tables II, III, and IV is interpreted in the three theoretical plots of energy vs. stripping foil radius, (corrected for non-zero azimuth), of fig. 47.

Experimental measurements(12) using the "cross-over" technique are plotted as + marks. They seem to be in good agreement with the lower curve. Furthermore, since the experimental results are at most in error by ± 0.3 MeV, it is tentatively concluded that the postion of the ion source relative to the centre of the machine is significantly off. Fluctuations in the INVAR may also contribute to the lack of agreement. Conversely the problems may lie in the code due to the assumption of 4-fold

** results of the

symmetry in the magnetic field.

Because of the uncertainty associated with the E vs R plots, one cannot readily say which values of p_r are correct. The values of p_r consistent with the magnetic field data sets are included in Tables II, III, and IV.

(8)

The flutter of the magnetic field as a function of radius is shown in Table VI, along with the average magnetic field for that radius.

FIG. 46 GEOMETRY USED IN ORBCAL AND CBEJ (TOP VIEW)

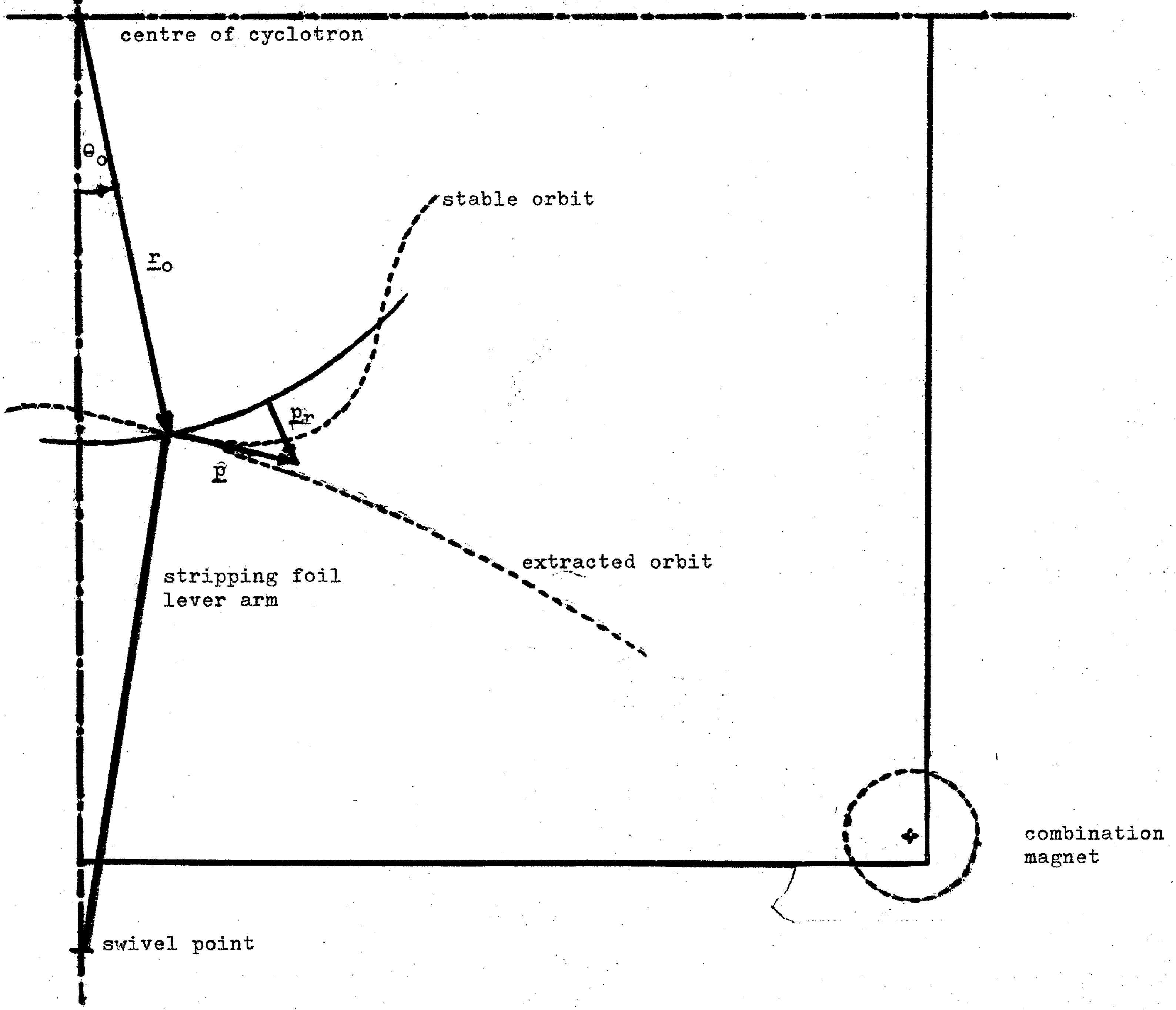

TABLE II STABLE ORBIT DATA - FIELD SET # 1

AZIMUTH (deg.)	ENERGY (MeV.)	RADIUS (in.)	RADIAL MOMENTUM ($m_o c$)
0.3	50.0	20.37	-.008449
	45.0	19.38	-.011361
	40.0	18.36	-.013424
	35.0	17.27	-.014908
	30.0	16.10	-.015801
	25.0	14.81	-.016130
	20.0	13.37	-.015477
2.1	50.0	20.35	-.007218
	45.0	19.36	-.009915
	40.0	18.34	-.011844
	35.0	17.24	-.013398
	30.0	16.07	-.014453
	25.0	14.78	-.015013
	20.0	13.34	-.014788
3.9	50.0	20.34	-.005969
	45.0	19.34	-.008455
	40.0	18.31	-.010277
	35.0	17.21	-.011883
	30.0	16.04	-.013105
	25.0	14.75	-.013872
	20.0	13.31	-.013939
5.7	50.0	20.33	-.004727
	45.0	19.33	-.006994
	40.0	18.30	-.008696
	35.0	17.19	-.010380
	30.0	16.01	-.011756
	25.0	14.73	-.012712
	20.0	13.28	-.013022
7.5	50.0	20.32	-.003441
	45.0	19.31	-.005492
	40.0	18.28	-.007113
	35.0	17.17	-.008867
	30.0	15.99	-.010406
	25.0	14.70	-.011552
	20.0	13.26	-.012068

TABLE II CON'T

AZIMUTH (deg.)	ENERGY (MeV.)	RADIUS (in.)	RADIAL MOMENTUM (m_oc)
9.3	50.0	20.32	-.002130
	45.0	19.30	-.003972
	40.0	18.27	-.005513
	35.0	17.16	-.007359
	30.0	15.97	-.009055
	25.0	14.68	-.010380
	20.0	13.24	-.011101
11.1	50.0	20.31	-.000749
	45.0	19.30	-.002401
	40.0	18.25	-.003902
	35.0	17.15	-.005842
	30.0	15.96	-.007706
	25.0	14.66	-.009211
	20.0	13.22	-.010118
12.9	50.0	20.31	+.000691
	45.0	19.30	-.000790
	40.0	18.25	-.002275
	35.0	17.14	-.004327
	30.0	15.94	-.006353
	25.0	14.64	-.008034
	20.0	13.20	-.009134
14.7	50.0	20.32	+.002222
	45.0	19.30	+.000878
	40.0	18.25	-.000627
	35.0	17.13	.002801
	30.0	15.93	.004996
	25.0	14.63	.006857
	20.0	13.18	.008140
16.5	50.0	20.32	+.003832
	45.0	19.30	+.002587
	40.0	18.25	+.001036
	35.0	17.12	-.001277
	30.0	15.92	-.003638
	25.0	14.62	-.005675
	20.0	13.16	-.007145
18.3	50.0	20.33	+.005545
	45.0	19.31	+.004348
	40.0	18.25	+.002709
	35.0	17.12	+.000253
	30.0	15.92	-.002280
	25.0	14.61	-.004495
	20.0	13.15	-.006145

TABLE II CON'T

AZIMUTH (deg.)	ENERGY (MeV.)	RADIUS (in.)	RADIAL MOMENTUM (m_oc)
20.1	50.0	20.34	+.007366
	45.0	19.32	+.006156
	40.0	18.26	+.004399
	35.0	17.13	+.001784
	30.0	15.91	-.000917
	25.0	14.50	-.003309
	20.0	13.14	-.005143
21.9	50.0	20.36	.009305
	45.0	19.33	.008018
	40.0	18.27	.006104
	35.0	17.13	.003323
	30.0	15.91	.000445
	25.0	14.59	-.002122
	20.0	13.13	-.004133
23.7	50.0	20.38	.011364
	45.0	19.35	.009926
	40.0	18.28	.007827
	35.0	17.14	.004866
	30.0	15.92	.001812
	25.0	14.59	-.000930
	20.0	13.12	-.003124
25.4	50.0	20.41	.013540
	45.0	19.37	.011881
	40.0	18.30	.009563
	35.0	17.15	.006415
	30.0	15.92	.003180
	25.0	14.59	.000261
	20.0	13.12	-.002108
27.3	50.0	20.43	.015821
	45.0	19.39	.013883
	40.0	18.32	.011322
	35.0	17.16	.007971
	30.0	15.92	.004554
	25.0	14.59	.001458
	20.0	13.11	-.001095

TABLE III STABLE ORBIT DATA - FIELD SET # 2

AZIMUTH (deg.)	ENERGY (MeV.)	RADIUS (in.)	RADIAL MOMENTUM (m_oc)
0.3	50.0	20.89	-.006444
	45.0	19.80	-.009967
	40.0	18.76	-.012335
	35.0	17.64	-.014076
	30.0	16.44	-.015162
	25.0	15.14	-.015660
	20.0	13.66	-.015288
2.1	50.0	20.88	-.005266
	45.0	19.78	-.008667
	40.0	18.74	-.010813
	35.0	17.61	-.012580
	30.0	16.42	-.013792
	25.0	15.11	-.014518
	20.0	13.63	-.014505
3.9	50.0	20.87	-.004127
	45.0	19.77	-.007327
	40.0	18.72	-.009317
	35.0	17.59	-.011060
	30.0	16.39	-.012435
	25.0	15.08	-.013354
	20.0	13.60	-.013610
5.7	50.0	20.87	-.002956
	45.0	19.75	-.006007
	40.0	18.70	-.007793
	35.0	17.57	-.009568
	30.0	16.37	-.011070
	25.0	15.05	-.012184
	20.0	13.58	-.012662
7.5	50.0	20.86	-.001794
	45.0	19.74	-.004624
	40.0	18.68	-.006269
	35.0	17.55	-.008049
	30.0	16.34	-.009715
	25.0	15.03	-.011007
	20.0	13.55	-.011693

TABLE III CON'T

AZIMUTH (deg.)	ENERGY (MeV.)	RADIUS (in.)	RADIAL MOMENTUM (m_oc)
9.3	50.0	20.86	-.000580
	45.0	19.73	-.003234
	40.0	18.68	-.004714
	35.0	17.54	-.006550
	30.0	16.33	-.008348
	25.0	15.01	-.009830
	20.0	13.53	-.010709
11.1	50.0	20.86	.000659
	45.0	19.73	-.001767
	40.0	18.67	-.003149
	35.0	17.52	-.005021
	30.0	16.31	-.006992
	25.0	14.99	-.008645
	20.0	13.51	-.009719
12.9	50.0	20.86	.001976
	45.0	19.73	-.000262
	40.0	18.66	-.001548
	35.0	17.51	-.003512
	30.0	16.30	-.005623
	25.0	14.97	-.007461
	20.0	13.49	-.008723
14.7	50.0	20.87	.003355
	45.0	19.73	.001326
	40.0	18.66	.000074
	35.0	17.51	-.001970
	30.0	16.29	-.004261
	25.0	14.96	-.006271
	20.0	13.47	-.007722
16.5	50.0	20.88	.004840
	45.0	19.73	.002964
	40.0	18.66	.001726
	35.0	17.51	-.000445
	30.0	16.28	-.002887
	25.0	14.95	-.005083
	20.0	13.46	-.006717
18.3	50.0	20.89	.006423
	45.0	19.74	.004681
	40.0	18.67	.003392
	35.0	17.51	.001103
	30.0	16.28	-.001523
	25.0	14.94	-.003892
	20.0	13.45	-.005710

TABLE III CON'T

AZIMUTH (deg.)	ENERGY (MeV.)	RADIUS (in.)	RADIAL MOMENTUM $(m_o c)$
20.1	50.0	20.90	.008150
	45.0	19.75	.006461
	40.0	18,68	.005088
	35.0	17.51	.002639
	30.0	16.28	-.000146
	25.0	14.93	-.002699
	20.0	13.44	-.004699
21.9	50.0	20.92	.010023
	45.0	19.77	.008323
	40.0	18.69	.006807
	35.0	17.52	.004199
	30.0	16.28	.001224
	25.0	14.93	-.001501
	20.0	13.43	-.003684
23.7	50.0	20.94	.012070
	45.0	19.78	.010248
	40.0	18.70	.008551
	35.0	17.53	.005752
	30.0	16.28	.002606
	25.0	14.93	-.000302
	20.0	13.42	-.002666
25.4	50.0	20.97	.014281
	45.0	19.81	.012248
	40.0	18.72	.010319
	35.0	17.54	.007325
	30.0	16.29	.003983
	25.0	14.93	.000899
	20.0	13.42	-.001645
27.3	50.0	21.00	.016634
	45.0	19.83	.014308
	40.0	18.74	.012113
	35.0	17.56	.008896
	30.0	16.30	.005371
	25.0	14.93	.002102
	20.0	13.41	-.000624

TABLE IV STABLE ORBIT DATA - FIELD SET # 3

AZIMUTH (deg.)	ENERGY (MeV)	RADIUS (in.)	RADIAL MOMENTUM (m_oc)
0.3	45.0	20.27	-.008339
	40.0	19.19	-.011210
	35.0	18.04	-.013204
	30.0	16.82	-.014522
	25.0	15.48	-.015217
	20.0	13.98	-.015081
2.1	45.0	20.25	-.007159
	40.0	19.16	-.009795
	35.0	18.02	-.011714
	30.0	16.79	-.013137
	25.0	15.45	-.014042
	20.0	13.95	-.014221
3.9	45.0	20.24	-.005956
	40.0	19.15	-.008382
	35.0	18.00	-.010218
	30.0	16.76	-.011763
	25.0	15.42	-.012851
	20.0	13.92	-.013288
5.7	45.0	20.23	-.004763
	40.0	19.13	-.006955
	35.0	17.98	-.008729
	30.0	16.74	-.010384
	25.0	15.40	-.011662
	20.0	13.89	-.012310
7.5	45.0	20.22	-.003522
	40.0	19.12	-.005503
	35.0	17.96	-.007228
	30.0	16.72	-.009013
	25.0	15.37	-.010464
	20.0	13.87	-.011325
9.3	45.0	20.21	-.002260
	40.0	19.11	-.004028
	35.0	17.95	-.005728
	30.0	16.70	-.007632
	25.0	15.35	-.009270
	20.0	13.84	-.010321
11.1	45.0	20.21	-.000930
	40.0	19.10	-.002518
	35.0	17.94	-.004211
	30.0	16.69	-.006259
	25.0	15.34	-.008067
	20.0	13.82	-.009320

TABLE IV CON'T

AZIMUTH (deg.)	ENERGY (MeV.)	RADIUS (in.)	RADIAL MOMENTUM (m_oc)
12.9	45.0	20.21	.000455
	40.0	19.10	-.000969
	35.0	17.93	-.002696
	30.0	16.68	-.004875
	25.0	15.32	-.006869
	20.0	13.80	-.008308
14.7	45.0	20.21	.001927
	40.0	19.10	.000619
	35.0	17.93	-.001155
	30.0	16.67	-.003495
	25.0	15.31	-.005661
	20.0	13.79	-.007296
16.5	45.0	20.22	.003470
	40.0	19.10	.002246
	35.0	17.93	.000383
	30.0	16.66	-.002105
	25.0	15.30	-.004459
	20.0	13.77	-.006276
18.3	45.0	20.23	.005109
	40.0	19.11	.003905
	35.0	17.93	.001938
	30.0	16.66	-.000723
	25.0	15.29	-.003250
	20.0	13.76	-.005260
20.1	45.0	20.24	.006843
	40.0	19.12	.005604
	35.0	17.93	.003494
	30.0	16.66	.000670
	25.0	15.28	-.002043
	20.0	13.75	-.004234
21.9	45.0	20.25	.008681
	40.0	19.13	.007343
	35.0	17.94	.005068
	30.0	16.66	.002060
	25.0	15.28	-.000829
	20.0	13.74	-.003210
23.7	45.0	20.27	.010623
	40.0	19.15	.009119
	35.0	17.95	.006647
	30.0	16.67	.003460
	25.0	15.28	.000384
	20.0	13.74	-.002179

TABLE IV CON'T

AZIMUTH (deg.)	ENERGY (MeV.)	RADIUS (in.)	RADIAL MOMENTUM (m_oc)
25.4	45.0	20.30	.012667
	40.0	19.17	.010931
	35.0	17.97	.008241
	30.0	16.68	.004858
	25.0	15.28	.001601
	20.0	13.73	-.001149
27.3	45.0	20.32	.014805
	40.0	19.19	.012781
	35.0	17.99	.009846
	30.0	16.69	.006266
	25.0	15.29	.002818
	20.0	13.73	-.000114

TABLE V STRIPPING FOIL LOCUS

NOMINAL ENERGY (MeV.)	RADIUS (in.)	AZIMUTH (deg.)
50.0	21.162	1.47
45.0	20.164	3.22
40.0	19.419	3.76
35.0	18.003	7.85
30.0	16.677	13.04
25.0	15.397	19.27
20.0	14.974	27.12

FIGURE 47 ORBCAL CALCULATIONS

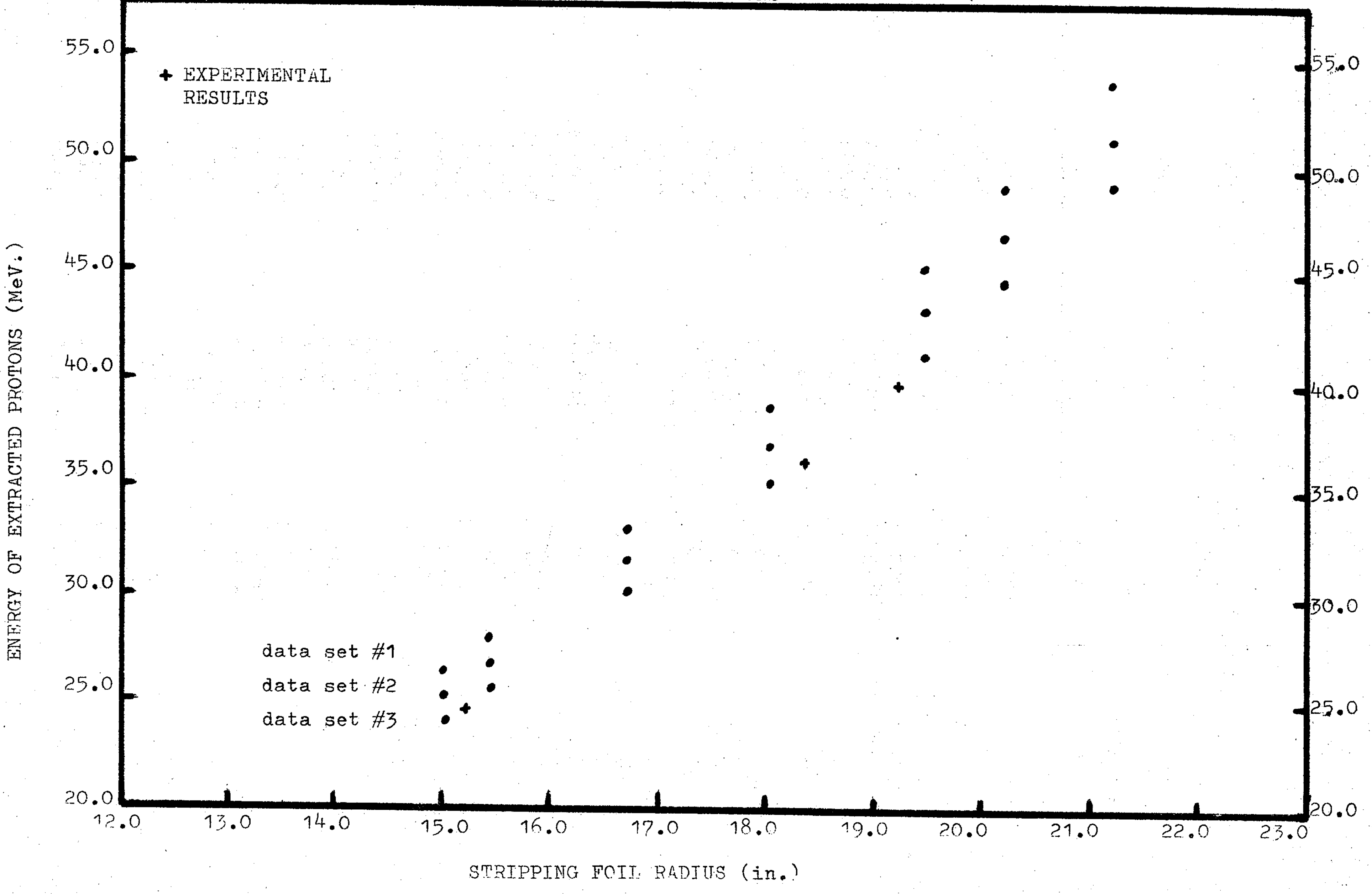

TABLE VI FLUTTER CALCULATIONS

RADIUS (in.)	AVERAGE MAGNETIC FIELD (gauss)			FLUTTER SQUARED
	Data Set #1	Data Set #2	Data Set #3	
1.0	18890.	18518.0	18061.0	0.0000
2.0	19054.	18679.0	18218.0	0.0000
3.0	18921.	18547.8	18090.1	0.0001
4.0	18795.	18425.1	17970.5	0.0035
5.0	18803.	18432.1	17977.3	0.0141
6.0	18822.	18450.8	17995.5	0.0232
7.0	18838.	18466.5	18010.8	0.0287
8.0	18868.	18496.0	18039.6	0.0332
9.0	18868.	18495.9	18039.5	0.0361
10.0	18913.	18540.0	18082.5	0.0385
11.0	18955.	18581.1	18122.6	0.0405
12.0	19010.	18635.2	18175.3	0.0421
13.0	19054.	18678.9	18218.0	0.0433
14.0	19126.	18749.5	18286.8	0.0445
15.0	19186.	18807.7	18343.6	0.0455
16.0	19260.	18880.2	18414.3	0.0464
17.0	19334.	18953.4	18485.7	0.0470
18.0	19428.	19045.2	18575.2	0.0465
19.0	19552.	19166.4	18693.5	0.0432
20.0	19719.	19330.6	18853.6	0.0368
21.0	19730.	19340.7	18863.5	0.0308
22.0	19088.	18711.9	18250.1	0.0296
23.0	16357.	16034.5	15638.8	0.0185
24.0	11798.	11565.1	11279.7	0.0005
25.0	9171.	8990.2	8768.3	0.0016
26.0	7319.	7174.7	6997.6	0.0014
27.0	5955.	5838.1	5694.0	0.0013
28.0	4830.	4735.0	4618.2	0.0011
29.0	3899.	3822.2	3727.9	0.0010
30.0	3111.	3049.5	2974.2	0.0016
31.0	2409.	2361.9	2303.6	0.0017
32.0	1828.	1792.0	1747.8	0.0036
33.0	1313.	1287.4	1255.7	0.0065
34.0	880.	862.6	841.3	0.0155
35.0	513.	503.1	490.7	0.0458
36.0	216.	211.3	206.1	0.2628
37.0	-16.	-15.3	-14.9	36.6459
38.0	-170.	-167.1	-162.9	0.3804
39.0	-287.	-281.3	-274.4	0.1294
40.0	-357.	-350.0	-341.4	0.0993
41.0	-415.	-406.5	-396.4	0.0645
42.0	-454.	-445.4	-434.4	0.0448
43.0	-477.	-467.9	-456.4	0.0330
44.0	-489.	-479.3	-467.4	0.0300
45.0	-492.	-482.8	-470.9	0.0286
46.0	-483.	-473.2	-461.5	0.0286
47.0	-467.	-458.3	-446.9	0.0300
48.0	-442.	-432.9	-422.2	0.0360
49.0	-428.	-419.5	-409.1	0.0427
50.0	-434.	-425.3	-414.8	0.0300

CHAPTER 4 CYCLOTRON FRINGING FIELD

The fringing field of the cyclotron is sufficiently complicated in its radial and azimuthal dependence to require the use of numerical procedures for calculating charged particle trajectories. The magnetic field for a sector of angle $\pi/2$ is generally given as a grid of field points in polar coordinates. A computer code, "CBEJ", (FORTRAN IV, G-LEVEL) has been written to accept an arbitrary grid of field points and to calculate the trajectory of the stripped particle and the matrix(6) of the fringing field along that trajectory. A brief outline of the theory of the program and the approximations made therein follows.

Consider the motion of a charged particle in a magnetic field in Cartesian coordinates and cgs units. (see fig. 46) Relativistically,

$$\frac{d(\vec{p})}{dt} = \underline{F} \quad \cdots\cdots\cdots\cdots \quad (4.1)$$

$$\frac{d(m\dot{\underline{r}})}{dt} = \frac{q}{c}(\dot{\underline{r}} \times \underline{B}) \quad \cdots\cdots\cdots\cdots \quad (4.2)$$

It is convenient to introduce v_x, v_y, v_z by the following equations:

$$\frac{dx}{dt} = v_x \quad \cdots\cdots\cdots\cdots \quad (4.3)$$

$$\frac{dy}{dt} = v_y \quad \cdots\cdots\cdots\cdots \quad (4.4)$$

$$\frac{dz}{dt} = v_z \quad \cdots\cdots\cdots\cdots \quad (4.5)$$

Then (4.2) becomes, (since $\frac{d|\vec{v}|}{dt}$ and therefore $\frac{dm}{dt}$ is zero in a magnetic field)

$$m\frac{dv_x}{dt} = \frac{q}{c}(v_yB_z - v_zB_y) \quad .... \quad (4.6)$$

$$m\frac{dv_y}{dt} = \frac{q}{c}(v_zB_x - v_xB_z) \quad .... \quad (4.7)$$

$$m\frac{dv_z}{dt} = \frac{q}{c}(v_xB_y - v_yB_x) \quad .... \quad (4.8)$$

$$B_z = B_z(z=0) + z\left.\frac{\partial B_z}{\partial z}\right|_{z=0} + 0(z^2) \quad ... \quad (4.9)$$

But $\left.\frac{\partial B_z}{\partial z}\right|_{z=0} = 0$; then for small z, $B_z \approx B_z(z=0)$

Also for z=0, $B_x=B_y=0$; and for small z, $B_z \gg B_x$ or B_y. Then we may approximate (4.5) and (4.6) by

$$\frac{dv_x}{dt} = \frac{qv_yB_z}{cm}(z=0) \quad .... \quad (4.10)$$

$$\frac{dv_y}{dt} = \frac{-qv_xB_z}{cm}(z=0) \quad .... \quad (4.11)$$

Equations (4.3), (4.4), (4.10), (4.11) are four first order diffenential equations which may be simultaneously integrated to give the trajectory in the medium plane. The integration is carried out using a fifth order Runge-Kutta technique.(13) The trajectory is segmented into many short intervals of finite length, Δs. The step size used in the integration is therefore $\Delta t = \Delta s/v$.

Initial conditions for the particle are specified by giving its position (r_o, θ_o) in polar coordinates and its initial

Unfortunately, there does not exist a field map of the combination magnet area. One is therefore limited in the scope of the work which can be done with "CBEJ", since the ion optic matrix can not be extended beyond that area. If this were available one would be able to do dispersion studies in the beamline system. Similarly, if a more complete programme of beam transport is to be pursued, one must know data on the beam extent at the position of the stripping foil.

TABLE VII FRINGING FIELD MATRIX FROM POINT OF STRIPPING TO 35 INCHES DOWNSTREAM

45 MeV.	.86697	35.33116	9.70962	0.0	0.0
	-.00204	1.07034	.57891	0.0	0.0
	0.0	0.0	1.0	0.0	0.0
	0.0	0.0	0.0	.99193	35.26987
	0.0	0.0	0.0	-.00015	.98842
40 MeV.	.77212	40.48989	13.69166	0.0	0.0
	-.00356	1.10100	.66937	0.0	0.0
	0.0	0.0	1.0	0.0	0.0
	0.0	0.0	0.0	.99234	40.24444
	0.0	0.0	0.0	-.00017	.98472
35 MeV.	.83736	40.79016	15.23880	0.0	0.0
	-.00137	1.12727	.79631	0.0	0.0
	0.0	0.0	1.0	0.0	0.0
	0.0	0.0	0.0	.97870	39.87917
	0.0	0.0	0.0	-.00056	.96964
30 MeV.	.69100	42.76483	21.11717	0.0	0.0
	-.00831	.93305	.89464	0.0	0.0
	0.0	0.0	1.0	0.0	0.0
	0.0	0.0	0.0	.97972	44.97849
	0.0	0.0	0.0	-.00052	.97367
25 MeV.	.49697	37.82250	23.34375	0.0	0.0
	-.01834	.61635	.97396	0.0	0.0
	0.0	0.0	1.0	0.0	0.0
	0.0	0.0	0.0	.98509	45.24023
	0.0	0.0	0.0	-.00041	.98270

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APPENDIX I

LISTING OF MANCOL (DOCUMENTED)

```
ZJOB 5                      1600008R
ZFORX52
PSTSN4
POBJP4
   PROGRAM BEAM OPTICS TRACE     REVISED MAY 10, 1965  PWA                 MANC   40
   WRITTEN BY                              ,UNIVERSITY OF COLORADO         MANC   50
          CONVERTED TO FORTRAN 2-D WITH SENSE SWITCH CONDITIONS            MANC   60
           REPLACED BY FIXED POINT INDICES BY D. PETERSON,                 MANC   70
           UNIVERSITY OF MANITOBA.  PROGRAMME CAN BE CONVERTED             MANC   80
           TO FORTRAN 4-E BY REPLACEMENT OF INPUT/OUTPUT STATEMENTMANC     90
           AND LIBRARY SUBROUTINE NAMES                                    MANC  100
                                                                           MANC  110
      PROGRAMME AND SUBROUTINE LOADINGS --------                           MANC  120
          MAIN 14000 12552  /  15 26552 00538  /  12 27090 00940
          03 28030 00510  /  02 28540 01158  / NEXT COMMON 39355
                                                                           MANC  150
           TO BRANCH TO READ XR,XPR, ETC, INSERT  4914542                  MANC  160
           TO BRANCH TO READ M, MN          INSERT  4915016                MANC  170
           TO BRANCH TO READ RM,TH,C        INSERT  4915900                MANC  180
                                                                           MANC  190
                                                                           MANC  200
   THIS PROGRAM CALCULATES PHASE-SPACE ELLIPSE TRAJECTORIES TO FIRST MANC  210
   ORDER FOR PASSAGE THROUGH ANY COMBINATION OF MAGNETIC QUADRUPOLES,MANC  220
   CONSTANT-FIELD BENDING MAGNETS, AND FIELD-FREE REGIONS. THE BEAM  MANC  230
   RESOLUTION IS DEFINED (AT A WAIST) AS THE RATIO OF THE IMAGE      MANC  240
   SIZE OF THE MAIN MOMENTUM COMPONENT OF THE BEAM TO THE            MANC  250
   DISPLACEMENT FROM THE BEAM AXIS OF A ZERO-MOMENTUM COMPONENT.     MANC  260
   THE PROGRAM ACCEPTS THE PARAMETERS M, MN TO SELECT THE MAGNETIC   MANC  270
   ELEMENTS OF INTEREST, AND IF ISEQ = 1, THE NEW BEAM               MANC  280
   PARAMETERS ARE ENTERED AS THE INITIAL CONDITIONS SO THAT THE      MANC  290
   PROCESS MAY BE REPEATED INDEFINITELY.BEAM DIVERGENCES ARE HALF    MANC  300
   ANGLES IN RADIANS AND BEAM SIZES ARE HALF WIDTHS IN INCHES.       MANC  310
   THE BENDING MAGNET ENTRY AND EXIT ANGLES ARE ZERO AND THE BENDING MANC  320
   ANGLE IS IS DEGREES.                                              MANC  330
                                                                     MANC  340
       FOR Q-POLES THE PROGRAMME ACCEPTS INPUT VALUES FOR DNR (RO)   MANC  350
      AND DNV (VO) FOR WHICH IT WILL SEARCH FOR QK1, QK2.            MANC  360
      IN ORDER TO FINISH IN A FINITE AMOUNT TO TIME, SEARCH          MANC  370
      MANUALLY BY USING IQSTR = 1 IN GROUPS OF CARDS CONTAINING THE  MANC  380
      FOLLOWING --- ( LOOPM,IQSTR,ISEQ / QL,T / QK1,QK2 /            MANC  390
      RO,VO / LOOPM, ETC., ETC. ) UNTIL YOU GET WITHIN A FACTOR      MANC  400
      OF TWO OR SO OF THE DESIRED VALUE.                             MANC  410
    FOR Q-POLES THE PROGRAMME WILL DO AN AUTOMATIC SEARCH UP TO A    MANC  420
    MAXIMUM NUMBER OF LOOPS SPECIFIED BY LOOPM, OR UNTIL DISTANCES   MANC  430
   WITHIN 0.1 INCHES OF THE DESIRED VALUE ARE FOUND.   USE INTER = 1 MANC  440
   TO CHECK THE RATE OF CONVERGENCE OF THE SEARCH.                   MANC  450
                                                                     MANC  460
      WHEN RO AND VO ARE GREATER THAN ABOUT 150 INCHES, THE          MANC  470
      CONVERGENCE IS QUITE SLOW AND THE INITIAL GUESS AT QK1,QK2     MANC  480
      MUST BE QUITE CLOSE.  THE PROGRAM WORKS BETTER FROM A HIGH     MANC  490
      GUESS THAN FROM A LOW ONE.                                     MANC  500
                                                                     MANC  510
                                                                     MANC  520
    IQSTR = 1 TO ACCEPT NEW VALUES FOR (LOOPM,IQSTR,ISEQ/QL,T/       MANC  530
        QK1,QK2/RO,VO) AFTER OUTPUT PUNCHED                          MANC  540
   ISEQ = 1 TO SET INITIAL CONDITIONS EQUAL TO FINAL CONDITIONS      MANC  550
      IN ORDER TO TRACE OUT OPTICS FOR MANY ELEMENTS.                MANC  560
   INTER = 1 TO PROVIDE OUTPUT AT END OF EACH SEARCH LOOP            MANC  570
                                                                     MANC  580
                                                                     MANC  590
```

```
C     XR= RADIAL SOURCE WIDTH.                                          MANC 600
C     XPR= RADIAL SOURCE DIVERGENCE.                                    MANC 610
C     XPS= DISPERSION (IN INCHES) FOR A ZERO-MOMENTUM BEAM COMPONENT.   MANC 620
C     AR= DISTANCE FROM RADIAL SOURCE TO FIRST PROGRAM ELEMENT.         MANC 630
C     Z = DISTANCE BETWEEN ELEMENTS WHEN ISEQ = 1 FOR MANY              MANC 640
C          ELEMENT CALCULATION.                                         MANC 650
C     M= +1 FOR VERTICAL FOCUSSING FIRST, -1 FOR RADIAL, 0 FOR NO       MANC 660
C          QUADRUPOLE.                                                  MANC 670
C     MN= +1 FOR BEND TO THE RIGHT, -1 FOR LEFT, 0 FOR NO B-MAG.        MANC 680
C     QL= Q-POLE LENGTH.                                                MANC 690
C      M AND MN CANNOT BOTH BE NON-ZERO                                 MANC 700
C     T= Q-POLE SPACING.                                                MANC 710
C     QK1, QK2= STRENGTH OF FIRST AND SECOND ELEMENTS OF Q-POLE.        MANC 720
C         RM = B-MAG RADIUS.  -- RADIUS OF EQUIVALENT CIRCULAR MAGNET   MANC 730
C     TH= ANGLE OF BEND.                                                MANC 740
C     C= DISTANCE FROM Q1 TO B-MAG (=0 FOR NO Q-POLE).                  MANC 750
C     DNRF= DISTANCE FROM LAST ELEMENT TO RADIAL FOCUS.                 MANC 760
C     DNR= DISTANCE FROM LAST ELEMENT TO RADIAL WAIST.                  MANC 770
C     WR= RADIAL SIZE AT WAIST.                                         MANC 780
C     WPR= RADIAL DIVERGENCE AT THE WAIST.                              MANC 790
C     WPS= MOMENTUM DISPERSION AT THE WAIST IN INCHES.                  MANC 800
C           NOTE THAT WPS IS POSITIVE IF THE HIGHER ENERGY              MANC 810
C           PARTICLES ARE TO THE LEFT OF THE BEAM, NEGATIVE IF TO THE   MANC 820
C           RIGHT. LEFT IS LEFT WHEN THE BEAM IS HITTING YOU IN THE BACKMANC 830
C           AND YOU ARE LOOKING IN THE DIRECTION OF THE BEAM.           MANC 840
C     WRF= RADIAL SIZE AT FOCUS.                                        MANC 850
C     WRE= RADIAL SIZE AT THE EXIT OF THE LAST ELEMENT.                 MANC 860
C     SMR= RADIAL MAGNIFICATION AT THE FOCUS.                           MANC 870
C          GREATER THAN 0 - VIRTUAL IMAGE / LESS THAN 0 - REAL IMAGE    MANC 880
C     RO= DESIRED VALUE FOR DNR FOR SEARCH.                             MANC 890
C     RES = RESOLUTION = 2*(WR/WPS)   (WPS = DISPERSION AT WAIST)       MANC 900
      DIMENSION E(2,2),B(2,2),F(2,2)                                    MANC 910
C                                                                       MANC 920
C          ALL OUTPUT FORMATS HAVE A 407-PRINTER CONTROL IN COL. 1.     MANC 930
C           1-NEW PAGE / 0-DOUBLE SPACE / BLANK-SINGLE SPACE            MANC 940
C                                                                       MANC 950
      COMMON XR,XPR,XV,XPV,XPS,AR,AV,A,M,MN,BC,JZZ,KZ,QL,T,QK1,QK2,L,E, MANC 960
     1RM,TH,C,F,B,FC,AN,BN,SM,DNF,FR,SMR,DNRF,ANR,BNR,FV,SMV,DNVF,ANV,  MANC 970
     2BNV,JVR,X,DFW,DN,W,WP,WE,WF,DNV,DVPV,WC,WVE,WVF,DNR,WPR,WR,       MANC 980
     3WRE,WRF,Q1,Q2                                                     MANC 990
C                                                                       MANC1000
C         SET INITIAL VALUES FOR BENDING MAGNET.  THIS AVOIDS UNDEFINED MANC1010
C           VARIABLES IN STATE. 960 WHEN A Q-POLE IS INVESTIGATED FIRST.MANC1020
      RM = 0.                                                           MANC1030
      TH = 0.                                                           MANC1040
      C  = 0.                                                           MANC1050
      CT = 0.                                                           MANC1060
      ST = 0.                                                           MANC1070
C                                                                       MANC1080
C                                                                       MANC1090
C         READ INITIAL PARAMETERS                                       MANC1100
    1 READ 608,          XR,XPR,XV,XPV,XPS                              MANC1110
  608 FORMAT(5F10.4)                                                    MANC1120
      PUNCH 609,         XR,XPR,XV,XPV,XPS                              MANC1130
  609 FORMAT(1H1/20H0XR,XPR,XV,XPV,XPS = ,5F10.4)                       MANC1140
      READ 608,         AR,AV                                           MANC1150
      PUNCH 613,        AR,AV                                           MANC1160
  613 FORMAT        ( 8H AR,AV = ,2F10.3/)                              MANC1170
    2 READ 624,          M,MN                                           MANC1180
  624 FORMAT(4I4)                                                       MANC1190
```

```
      PUNCH 628,         M,MN
  628 FORMAT(        7H M,MN = ,2I4)
  724 READ 624,              LOOPM,IQSTR,ISEQ,INTER
      PUNCH 625,             LOOPM,IQSTR,ISEQ,INTER
  625 FORMAT(25H0LOOPM,IQSTR,ISEQ,INTER = ,4I4)
      BC= 1.
      KZ=0

          IS IT Q-POLE OR B-MAG
      IF(M) 4,103,4
  103 IF(MN)95,73,95

          BENDING MAGNET
   95 L=1
      QK1= 0.
      QK2= 0.
      JZZ = 1
   80 E(1,1)=1.0
      E(1,2)=0.0
      E(2,1)=0.0
      E(2,2)=1.0
   56 GO TO (8,57),L
    8 IF(KZ)73, 20, 21



          MAGNET PARAMETERS  (RM = RADIUS OF CURVATURE OF PARTICLE)
   20 READ 608,          RM,TH,C
      PUNCH 640,           RM,TH,C
  640 FORMAT(10H RM,TH,C =,3F10.4)
      TH= TH/57.3
      RM= RM* COS(TH/2.)/SIN(TH/2.)
      KZ=1
   21 ST=SIN(TH)
      CT=COS(TH)

      SETTING UP OF THE MAGNET MATRICES AND MULTIPLICATION BY THE
      QPOLE MATRICES
   57 DO 96 J=1,2
   96 E(1,J)=E(1,J)+C*E(2,J)
      GO TO (50,51),L
   50 A=AR

          BENDING-MAGNET MATRIX    HORIZONTAL
      F(1,1)= CT
      F(1,2)=RM*ST
      F(2,1)= -ST/RM
      F(2,2)= CT
      GO TO 52
          BENDING-MAGNET MATRIX    VERTICAL
   51 IF(MN)104,73,105
  104 BC=+1.0
      GO TO 106
  105 BC=-1.0
  106 A=AV
      F(1,1) = 1.
      F(1,2)=RM*TH
      F(2,1)= 0.
      F(2,2)= 1.
      GO TO 52
```

```
C                                                                       MANC180
C                                                                       MANC181
C          QUADRUPOLE PARAMETERS                                        MANC182
    4 READ 608,              QL,T                                       MANC183
      PUNCH 632,              QL,T                                      MANC184
  632 FORMAT(10H QL,T      = , 2F10.4)                                  MANC185
    5 READ 608,              QK1,QK2                                    MANC186
      PRINT 1010
 1010 FORMAT (7H SET SW    )
      PAUSE
      IF (SENSE SWITCH 1) 1001,1002
 1001 ACCEPT  1020 , QK1,QK2
 1020 FORMAT (F6.0,F20.0)
```

see note at end.

```
 1002 PUNCH 636,  QK1,QK2
  636 FORMAT(10H QK1,QK2 = 2F10.4)                                      MANC188
      LOOP = 0                                                          MANC189
      JZZ = 1                                                           MANC190
  842 LOOP = LOOP + 1                                                   MANC191
  843 L= 1                                                              MANC192
C                                                                       MANC193
C     IN THE NEXT GROUP OF CARDS THE E MATRIX IS CALCULATED FOR         MANC194
C     CONVERGING AND DIVERGING QPOLE ELEMENTS                           MANC195
    6 S1=SIN(QK1)                                                       MANC196
      S2=SIN(QK2)                                                       MANC197
      C1=COS(QK1)                                                       MANC198
      C2=COS(QK2)                                                       MANC199
      SH1=(EXP(QK1)-EXP(-QK1))*0.5                                      MANC200
      SH2=(EXP(QK2)-EXP(-QK2))*0.5                                      MANC201
      CH1=(EXP(QK1)+EXP(-QK1))*0.5                                      MANC202
      CH2=(EXP(QK2)+EXP(-QK2))*0.5                                      MANC203
C     QPOLE MATRICES                                                    MANC204
   99 IF(M)9,73,97                                                      MANC205
C                                                                       MANC206
C         Q-POLE MATRIX    VERT. FOCUS.                                 MANC207
   97 E(1,1)=C2*CH1+T*QK1*SH1*C2/QL+QK1*SH1*S2/QK2                      MANC208
      E(1,2)=QL*(T*CH1*C2/QL+SH1*C2/QK1+CH1*S2/QK2)                     MANC209
      E(2,1)=-(T*QK1*QK2*SH1*S2/QL-QK1*SH1*C2+QK2*CH1*S2)/QL            MANC210
      E(2,2)=CH1*C2-T*QK2*S2*CH1/QL-QK2*S2*SH1/QK1                      MANC211
        GO TO 55
C                                                                       MANC213
C         Q-POLE MATRIX    RADIAL FOCUSING                              MANC214
    9 E(1,1)=CH2*C1-T*QK1*CH2*S1/QL-QK1*SH2*S1/QK2                      MANC215
      E(2,1)=-(T*QK1*QK2*SH2*S1/QL-QK2*SH2*C1+QK1*CH2*S1)/QL            MANC216
      E(1,2)=QL*(T*CH2*C1/QL+CH2*S1/QK1+SH2*C1/QK2)                     MANC217
      E(2,2)=CH2*C1+QK2*T*SH2*C1/QL+QK2*SH2*S1/QK1                      MANC218
   55   IF (MN) 56,137,56
  137 BC= 0.                                                            MANC219
  100 GO TO (101,102),L                                                 MANC220
  101 A=AR                                                              MANC221
      GO TO 82                                                          MANC222
  102 A=AV                                                              MANC223
C                                                                       MANC224
C          BENDING MATRIX == IDENTITY FOR Q-POLE                        MANC225
   82 F(1,1)=1.0                                                        MANC226
      F(1,2)=0.0                                                        MANC227
      F(2,1)=0.0                                                        MANC228
      F(2,2)=1.0                                                        MANC229
C          MULTIPLY BENDING MATRIX (F) X Q-POLE MATRIX (E)              MANC230
   52 DO 81 I=1,2                                                       MANC231
      DO 81 J=1,2                                                       MANC232
```

```
      B(I,J)= 0.
      DO 81 II=1,2
   81 B(I,J)=F(I,II)*E(II,J)+B(I,J)
      TEST FOR VERTICAL FOCUSSING
   62 IF(B(2,1))69, 502, 69
  502 FC= 1.0E+10
      AN= FC
      BN= FC
      SM = 1.
      DNF= -A- RM*TH
      GO TO 503
   69 FC= -1./B(2,1)
   85 AN= B(2,2)*FC
      BN= B(1,1)*FC
      SM= FC/(AN-A)
      DNF= BN- FC*SM
  503 GO TO(59, 60, 59, 14),L
   59 FR= FC
      SMR= SM
      DNRF= DNF
      ANR= AN
      BNR= BN
   60 L= L+ 1
      FV= FC
      SMV= SM
      DNVF= DNF
      ANV= AN
      BNV= BN
      GO TO (73, 90, 14, 73),L
   90 IF(M)97, 80, 9
   14 JVR= 1
      X= XV
      XP= XPV
  140 DFW= FC*SM/(1.+ (XP*FC/(X*SM))**2)
      DN= DNF+ DFW
      W = XP*SQRTF (FC*DFW/SM)
      WP= X*XP/W
      WE = SQRTF (W**2+ (WP*DN)**2)
      WF = SQRTF (W**2+(WP*DFW)**2)
      GO TO (371, 372),JVR
  371 JVR = 2
      X= XR
      XP= XPR
      DNF= DNRF
      DNV = DN
      WPV= WP
      WV= W
      WVE= WE
      WVF= WF
      FC= FR
      SM= SMR
      GO TO 140
  372 DNR= DN
      WPR= WP
      WR= W
      WRE= WE
      WRF= WF

          TRANSFER (JZZ) FOR Q-POLE SEARCH
  660 GO TO (990, 991, 992, 993), JZZ
```

```
  993 JZZ= 2                                                            MANC2930
  990 WPS= XPS*WR/XR                                                    MANC2940
      IF(SMR)950, 950, 960                                              MANC2950
  950 WPS= -WPS                                                         MANC2960
C         BC = -MN                                                      MANC2970
C         FINAL DISPERSION = DISPERSION DUE TO MAGNIFICATION            MANC2980
C            AND MAGNET DISPERSION                                      MANC2990
  960 WPS= WPS- BC*(DNR*ST+ RM*(1.- CI))                                MANC3000
      RES= 2.*WR/WPS                                                    MANC3010
      IF(M)989,112,989                                                  MANC3020
  989 GO TO (987, 985), JZZ                                             MANC3030
C                                                                       MANC3040
C                                                                       MANC3050
C                                                                       MANC3060
C      SEARCH ROUTINE FORM HERE TO STATEMENT 800.  LINEAR APPROXIMATIONSMANC3070
C     RELATE WAIST DISTANCES TO THE QPOLE STRENGTHS AND THE CORRECT     MANC3080
C     QPOLE STRENGTHS ARE EXTRAPOLATED.                                 MANC3090
C      SEARCH NOT USED FOR BENDING MAGNETS.                             MANC3100
C                                                                       MANC3110
C         IF Q-POLE STRENGTH SEARCH TAKES OVER FIVE MINUTES,            MANC3120
C            IT PROBABLY IS NOT POSSIBLE TO BRING THE RADIAL AND        MANC3130
C          VERTICAL WAISTS TO THE DESIRED DISTANCE.  THIS CAN BE TOLD   MANC3140
C          BY THE LARGE DIFFERENCE BETWEEN FOCAL DISTANCE AND WIAST     MANC3150
C          DISTANCE.  NORMALLY THE SEARCH SHOULD TAKE ONLY A            MANC3160
C          FEW MINUTES.                                                 MANC3170
C                                                                       MANC3180
  987 READ 608,           RO,VO                                         MANC3190
      PUNCH 664,          RO,VO                                         MANC3200
  664 FORMAT(10H RO,VO    = 2F10.4)                                     MANC3210
      ZZ= 1.0E+08                                                       MANC3220
  985 ZZZ= ((RO- DNR)**2+ (VO- DNV)**2)*100.- 100.0
      IF(ZZZ)845,845,958                                                MANC3240
  845 PUNCH 645,          LOOP                                          MANC3250
  645 FORMAT(21H0SEARCH REQUIRED ONLY ,I5,7H LOOPS.)                    MANC3260
      GO TO 112                                                         MANC3270
  958 IF(LOOPM-LOOP)849,849,959                                         MANC3280
C                                                                       MANC3290
C          PUNCH OUTPUT - 407 PRINTER SWITCH 4 ON DURING LISTING         MANC3300
  849 PUNCH 649,          LOOP                                          MANC3310
  649 FORMAT(19H0SEARCH ENDED AFTER, I5,7H LOOPS. )                     MANC3320
  112 INTER = 0                                                         MANC3330
  110 PUNCH 644,          DNR,DNV,DNRF,DNVF                             MANC3340
      PRINT 1333, DNR,DNV,WRE,WVE
      PRINT 1333, WR,WPR,WV,WPV
 1333 FORMAT (4E10.4)
  644 FORMAT(20H0DNR,DNV,DNRF,DNVF = 4(2XE10.4))                        MANC3350
  109 PUNCH 646,          WR,WPR,WV,WPV                                 MANC3360
  646 FORMAT(20H WR,WPR,WV,WPV      =4(2XE10.4))                        MANC3370
      PUNCH 648,          WRE,WVE,WRF,WVF                               MANC3380
  648 FORMAT(20H WRE,WVE,WRF,WVF    = 4(2XE10.4))                       MANC3390
      PUNCH 652,          SMR,SMV,WPS,RES                               MANC3400
  652 FORMAT(20H SMR,SMV,WPS,RES    =4(2XE10.4))                        MANC3410
      PUNCH 656,          QK1,QK2                                       MANC3420
  656 FORMAT(8H QK1,QK2,11X1H=2(2XE10.4))                               MANC3430
      IF(INTER-1)956,842,956                                            MANC3440
  956 IF(M)800,330,800                                                  MANC3450
C                                                                       MANC3460
C                                                                       MANC3470
C        TEST WHETHER IMAGE WITHIN 0.1 INCHES OF RO AND VO              MANC3480
  959 IF(ZZZ- ZZ)910, 910, 911                                          MANC3490
```

```
 911 JZZ= 4
     QK1= (QK1+Q1)/2.
     QK2= (QK2+Q2)/2.
     GO TO 957
 910 JZZ= 2
     Q1= QK1
     Q2= QK2
     ZZ= ZZZ
     QK1= QK1 + .001
     R1= DNR
     V1= DNV

        PUNCH INTERMEDIATE SEARCH OUTPUT IF DESIRED (INTER = 1)
 957 IF(INTER-1)842,858,842
 858 PUNCH 658,              LOOP
 658 FORMAT(29H0INTERMEDIATE OUTPUT.  LOOP = ,I4)
     GO TO 110
 991 JZZ = 3
     QK1 = QK1-.001
     QK2= QK2+ .001
     AL= 1000.*(DNR- R1)
     GM= 1000.*(DNV- V1)
      GO TO  842
 992 JZZ =4
     BT = 1000.*(DNR-R1)
     DL= 1000.*(DNV- V1)
     DEN= AL*DL- BT*GM
     QK2= QK2- .001
     QK1= QK1+ (DL*(RO- R1)- BT*(VO- V1))/DEN
     QK2= QK2- (GM*(RO- R1)- AL*(VO- V1))/DEN
      GO TO 842
 800 IF(IQSTR-1)330,2,330

     THE INITIAL VALUES ARE SET EQUAL TO THE COMPUTED  VALUES SO THAT
     THE TRACE MAY BE CONTINUED WITH NEW ELEMENTS
 330 XPS = WPS
 906 XR= WR
     XPR=WPR
     XV= WV
     XPV=WPV
     PUNCH 614,          XR,XPR,XV,XPV,XPS
 614 FORMAT(1H0/1H0,F9.4,4F10.4,21H  = XR,XPR,XV,XPV,XPS )
     IF(ISEQ-1)1,400,1
 400 READ 608,           Z
     PUNCH 612,          Z
 612 FORMAT(4H Z = ,F10.4)
     AR= Z- DNR
     AV= Z- DNV
     PUNCH 613,          AR,AV
     KZ= 0
     JZZ= 1
     GO TO 2
  73 CALL EXIT


        ORGANIZATION OF INPUT DATA IS AS FOLLOWS------

       FOR ALL RUNS THE FIRST FOUR CARDS MUST BE
               (XR,XPR,XV,XPV,XPS)           (5F10.4)
               (AR,AV)                        (2F10.4)
```

```
C                 (M,MN)                                (3I4)              MANC4100
C                 (LOOPM,IQSTR,ISEQ)                    (3I4)              MANC4110
C                                                                          MANC4120
C                                                                          MANC4130
C     M = +/- 1          QUADRUPOLE (VH/HV)    MN = 0                      MANC4140
C         LOOPM EXCEEDS ZERO                                               MANC4150
C                                                                          MANC4160
C         IQSTR = 0,    ISEQ = 0                                           MANC4170
C             (QL,T)                              (2F10.4)                 MANC4180
C             (QK1,QK2)                           (2F10.4)                 MANC4190
C             (RO,VO)                             (2F10.4)                 MANC4200
C             (XR,XPR,XV,XPV,XPS)                (5F10.4)                  MANC4210
C             (AR,AV)                             (2F10.4)                 MANC4220
C             (M,MN)                              (3I4)                    MANC4230
C             (LOOPM,IQSTR,ISEQ)                  (3I4)                    MANC4240
C                --- ETC. DEPENDING ON M AND MN                            MANC4250
C                                                                          MANC4260
C         IQSTR = 0,    ISEQ = 1                                           MANC4270
C             (QL,T)                              (2F10.4)                 MANC4280
C             (QK1,QK2)                           (2F10.4)                 MANC4290
C             (RO,VO)                             (2F10.4)                 MANC4300
C             (Z)                                 (F10.4)                  MANC4310
C             (M,MN)                              (3I4)                    MANC4320
C             (LOOPM,IQSTR,ISEQ)                  (3I4)                    MANC4330
C                --- ETC. DEPENDING ON M AND MN                            MANC4340
C                                                                          MANC4350
C         IQSTR = 1,                                                       MANC4360
C             (QL,T)                              (2F10.4)                 MANC4370
C             (QK1,QK2)                           (2F10.4)                 MANC4380
C             (RO,VO)                             (2F10.4)                 MANC4390
C             (M,MN)                              (3I4)                    MANC4400
C             (LOOPM,IQSTR,ISEQ)                  (3I4)                    MANC4410
C             (QL,T)                              (2F10.4)                 MANC4420
C             (QK1,QK2)                           (2F10.4)                 MANC4430
C             (RO,VO)                             (2F10.4)                 MANC4440
C                 --- ETC. DEPENDING ON IQSTR                              MANC4450
C                                                                          MANC4460
C                                                                          MANC4470
C     MN = +/- 1          BENDING MAGNET(R/L)       M = 0                  MANC4480
C     LOOPM = IQSTR = 0                                                    MANC4490
C                                                                          MANC4500
C     ISEQ = 1                                                             MANC4510
C             (RM,TH,C)                           (5F10.4)                 MANC4520
C             (Z)                                 (F10.4)                  MANC4530
C             (M,MN)                              (3I4)                    MANC4540
C             (LOOPM,IQSTR,ISEQ)                  (3I4)                    MANC4550
C                --- ETC. DEPENDING ON M AND MN                            MANC4560
C                                                                          MANC4570
C     ISEQ = 0                                                             MANC4580
C             (RM,TH,C)                           (5F10.4)                 MANC4590
C             (XR,XPR,XV,XPV,XPS)                (5F10.4)                  MANC4600
C             (AR,AV)                             (2F10.4)                 MANC4610
C             (M,MN)                              (3I4)                    MANC4620
C             (LOOPM,IQSTR,ISEQ)                  (3I4)                    MANC4630
C                --- ETC. DEPENDING ON M AND MN                            MANC4640
      END                                                                  MANC4650
```

Note: if SW #1 "up", QK1, QK2 may be read in from typewriter in (F6.0, F20.0) to facilitate a manual search.

APPENDIX II

GUIDE TO OPERATION OF "TRANSPORT" AND "ORBCAL" MAGNETIC TAPES

The code "TRANSPORT" is now availible on magnetic tape #000191 in source form at the U. of Man. Computer Center. To execute insert the following cards before the data input;

```
"job card"
//  EXEC  FORTGCLG,PARM.FORT=NOMAP,PARM.LKED=MAP
//FORT.SYSIN DD UNIT=2400,VOLUME=SER=000191,DSNAME=TRNSPT
/*
//G0.FT05F001  DD  UNIT=SYSDA,DISP=NEW,SPACE=(80,(500,300))
//G0.FT06F001  DD  SYSOUT=A
//GO.FT10F001  DD  *
"data input"
/*
```

The code "ORBCAL" is in source form on magnetic tape #000192. To execute insert the following cards before the data input.

```
"job card"
//  EXEC  FORTGCLG,PARM.FORT=(BCD,NOSOURCE)
//FORT.SYSIN DD UNIT=2400,VOLUME=SER=000192,DSNAME=ORBCAL
/*
//GO.SYSIN DD *
"data input"
/*
```

APPENDIX III

GUIDE TO PREPARATION OF DATA INPUT FOR "TRANSPORT"

SECTION 1. INPUT FORMAT

Since TRANSPORT was developed for application to a wide variety of problems encountered in designing beam transport systems, it requires a flexible, consequently complex, set of input conventions. The basis of these conventions is the element, the building block from which systems are constructed. An element is a component of, or condition upon, the system--a magnet, a focus condition, an alignment tolerance, or any other set of parameters that help describe the system under study. Each element is usually (though not necessarily) punched on a separate card to minimize the difficulty of changing the input.

Most of the input data appears in the output listing for verification purposes; that portion of the output should be self-evident. The additional output associated with each element will be described below; the more important portions will be found marked on the examples in Section 6.

The bulk of this description concerns the various elements that may be used. They will be described essentially in the order in which they would be encountered in using the program for the first time.

A group of elements that describe a single system is termed an input deck. All input conforms to standard SUBALGOL free-format conventions. Specifically, input parameters are punched in cols. 2 - 72. (Col. 1 and cols. 73 - 80 are blank.) The individual parameters must be separated by at least one blank. Each parameter must contain a decimal point, and a zero must be punched for a parameter whose value is zero. In general, only the <u>number</u> of parameters and the <u>order</u> in which they appear on the card are important.

Each input deck must contain the following cards:

1. Title card - labels the card deck and the printed output. Any alphameric characters save ' (apostrophe: 8-4 punch) may appear in cols. 3 - 71.

col.	1	2	3 - 71	72	73 - 80
entry		'	title	'	blank

2. Second card - input option (described in Section 3.4.).

col.	1	2	3 - 80
entry		0	blank

3. Last card - marks end of system.

col.	1	2 - 9	10 - 80
entry		SENTINEL	blank

Between the second and last cards may be any combination of the data cards containing the elements described below. The only restriction upon the number of elements in a deck is that the total number of parameters may not exceed 300. (Only the first 300 will be used.)

Any number of these input decks may be stacked for successive consideration.

SECTION 2. BASIC ELEMENTS

This section is intended to provide the reader with a minimum capability for using TRANSPORT. It describes the elements that seem to be common to most transport systems, and some of the facilities provided to manipulate them. These elements are introduced in the order they will most likely be required and it is suggested that the reader glance through them and consider in detail only those he will be needing. This suggestion also applies to the remainder of the manual which is devoted to a description of the more specialized elements.

2.1. First Order Beam Trace

The elements provided by TRANSPORT must be assembled to form a reasonable model for the magnet system being considered. The following elements will enable the designer to construct a magnet system and follow the beam through it. The system should be simplified as much as possible and its natural complexity gradually re-introduced as the designer gains familiarity with the program and his system. For instance, use wedge magnets (unless pole focusing is essential) initially and switch later to rectangular magnets.

2.1.1. Beam

This element specifies the initial beam including the average beam energy. Normally, it is the third card in an input deck.

There are eight parameters:

1 - <u>type code</u> 1. (specifying a beam).
2 - horizontal extent (cm).
3 - horizontal divergence (in units of 10^{-4} radians).
4 - vertical extent (cm).
5 - vertical divergence (10^{-4} radians).
6 - longitudinal extent (cm).
7 - momentum spread (in units of 0.1%).
8 - design momentum of beam (Gev/c).
9 - IF = 0 Then Beam will add to beam

Each of the six beam parameters (2 - 7 above) is positive, but should be thought of as ±x. They specify the semi-axes of a 6-dimensional ellipsoid, usually termed the phase ellipsoid or, alternatively, the second moment ellipsoid of the particle distribution.

The longitudinal extent is useful for pulsed beams. It indicates the spread in time of a pulse. It does not interact with any other component and may be set to zero if the pulse length is not important.

The beam is normally printed as output after each element. The projection of the ellipsoid upon each of its six coordinate axes is printed in a vertical array. The correlations among these components are printed in a lower triangular array (see Appendix IV). The input and output units are the same, but may be changed if desired (see Section 3.3).

2.1.2. Drift

A drift space is a field-free region through which the beam passes. There are two parameters:

1 - type code 3. (specifying a drift length).
2 - (effective) drift length (meters).

2.1.3. Bending Magnet (Wedge)

A bending magnet serves primarily to deflect the beam through a certain angle and secondarily to disperse the beam as a function of particle energy. Deflection is concealed in the output since the reference coordinate system is deflected the same amount. Dispersion is indicated by a non-zero correlation between momentum spread and beam width in the plane of the bend.

A wedge magnet implies the beam enters and exits perpendicular to the pole face. It acts like a drift space in the plane parallel to the magnetic field. Non-wedge magnets will be described later (see Section 4.4).

There are four parameters:

1 - type code 4. (specifying a bending magnet).

2 - (effective) magnet length (meters). Length is measured along the reference trajectory. A negative length is a convention that indicates a vertical bend; a positive length indicates a horizontal bend.

3 - bending field (kg). The field may be positive (upward) or negative. A positive field bends electrons to the left as one looks in the direction of beam motion; negative fields bend the beam in the opposite direction.

4 - field gradient, n (dimensionless). The parameter n is defined by the equation

$$B = B_o \left[1 - n(x/\rho_o)\right]$$

which gives the radial dependence of the bending field. For non-zero n, the magnet is presumed to be curved so that the field is constant along the reference trajectory. Straight 'n-magnets' will be added to the program when a need arises.

The angle of bend, as computed from the length and field of the magnet and the energy of the beam, is printed on the output listing.

2.1.4. Quadrupole

A quadrupole provides focusing in one plane at the cost of defocusing in the other.

There are four parameters:

1 - type code 5. (specifying a quadrupole).
2 - (effective) magnet length (meters).
3 - field at pole tip (kg). For electrons a − field produces horizontal focusing; a + field, vertical focusing.
4 - aperture (cm). Radius of the circle tangent to the pole tips.

The strength of the quadrupole is computed from field and aperture. The horizontal focal length is printed as output; a negative focal length indicates defocusing.

2.2. Parameter Fitting

Quadrupoles, bending magnets and drifts form the basic building blocks of a magnet system. Once the desired system has been synthesized from these elements and described qualitatively, the exact values of the fields and of the lengths required to meet the design criteria must be determined.

TRANSPORT will adjust these parameters to fulfill specified conditions upon the beam or the magnets. To accomplish this, the program requires a statement of which parameters it may vary and the conditions imposed on the system that defines them. The following sections (2.2.1. and 2.2.2.) describe the necessary conventions.

TRANSPORT employs a differential fitting scheme. This implies that the initial estimate of the variable parameters must be reasonably close. It is difficult to assign a quantitative definition of "reasonably" but an error of 50% can usually be tolerated. The appearance of large corrections with no solution being obtained suggests either that the initial guesses were unreasonable or that the designer has specified conditions which cannot be satisfied by the system. More often than not, the latter explanation is correct. Experience has shown that when TRANSPORT is given a well-posed problem, it rarely fails to find a solution. (See Section 6.1.)

At each step of the correction procedure, TRANSPORT prints an estimate of the closeness of the fit (standard deviation) together with the corrections made to improve the fit (listed in the order in which the variables were encountered). If the fit deteriorates on two consecutive steps or if it fails to converge after ten iterations, the program prints FAILED. When the program converges it prints the covariance matrix for the variable parameters. The diagonal components are the standard deviation of the variables as defined by the tolerances on the constraints. The off-diagonal components are the correlation coefficients. A row of zeros indicates the variable was not changed (the last iteration).

2.2.1. Vary Codes

Associated with certain elements of a system is a vary code which specifies the parameters that may be varied. This code occupies the fraction portion of the type code of the element. It has one digit for each parameter, the digits having the same order in the code as the parameters have on the card. A '0' indicates the parameter may not be varied; a '1' that it may be. For instance, 3.0 is the combined type and vary code for a drift length which is to remain fixed; 3.1 indicates a drift length that may be varied. The type code 4.010 indicates a bending magnet with a variable field. In punching the code 3.0, the zero need not be punched. In punching the 4.010 code, the first zero must be punched but the second zero need not be.

The following parameters may be variable (0 or 1 may be placed in positions marked 1; only 0 may be placed in positions marked 0):

beam.... 1.111111 - all components of the input beam may be varied.
drift... 3.1 - the drift length may be varied.
bend.... 4.011 - the length may not be varied; the field (first 1) and/or the n-value (second 1) may be varied.
quad.... 5.010 - the length may not be varied; the field may be, the aperture may not be.

The use of the permissive 'may' rather than the imperative 'will' in discussing variables is meaningful. The program will choose the parameters it will vary from among those that it may vary. In general it chooses to vary those parameters that have the greatest influence upon the conditions to be fit.

In Section 5.1. an extension of the vary code convention is described.

2.2.2. Constraint

A number of constraints are available to specify the conditions to be fit. One type specifies the value of an element of the product transformation matrix (R) from some specified point in the system to the current position; another, the size of a component of the beam. (Other types of constraints are discussed in Section 5.)

There are five parameters:

1 - type code 10. (specifying a constraint).
2 - code digit (i).
3 - code digit (j).
4 - desired value of function (Δ).
5 - desired accuracy of fit (standard deviation) (σ).

The correct interpretation of the last four above (2 - 5) is: "The quantity specified by digits i and j should have the value $\Delta \pm \sigma$."

To specify the i,j element of the product matrix (R_{ij}), the code digits are -i and j. Thus (10. -2. 1. 0. .1) will specify that $R_{21} = 0 \pm 0.1$.

To specify the i component of the beam (σ_i) the code digits are i and i. Thus (10. 3. 3. 1. .1) will specify the vertical extent of the beam: $\sigma_3 = 1. \pm 0.1$ cm.

To specify the i,j covariance of the beam (σ_{ij}), the code digits are i and j (with $i > j$). Thus (10. 2. 1. 0. .01) will specify that $\sigma_{21} = 0 \pm 0.01$. It is difficult to choose the accuracy in this case. In general if $\sigma = 0.01\ \sigma_2\sigma_1$, the requirement that $\sigma_{21} = 0 \pm \sigma$ corresponds to a 1% tolerance in the fit.

2.2.2.(cont'd)

Some commonly used conditions and the appropriate constraints are:

'parallel' beam to focus:

horizontal plane $R_{11} = 0$
vertical plane $R_{33} = 0$

focus to focus:

horizontal plane $R_{12} = 0$
vertical plane $R_{34} = 0$

waist:

horizontal $\sigma_{21} = 0$
vertical $\sigma_{43} = 0$

parallel beam:

horizontal σ_{22} = small

achromatic beam:

horizontal $\sigma_{61} = \sigma_{62} = 0$ or $R_{16} = R_{26} = 0$

A constraint upon the beam (σ_i or σ_{ij}) has an important side effect. It 'updates' the beam. The only important consequence of the update is that the origin of the transformation matrix is redefined as starting from the point of the update. Therefore, any constraint upon the transformation matrix (R_{ij}) concerns the transformation from the last preceding update, not necessarily from the start of the system. See Section 4.2. for other elements which cause an update and Section 4.3. for a means of circumventing it.

The actual value of the specified function is printed beneath the desired value to facilitate comparison.

2.3. Resolution

To determine the resolution of a spectrometer and in general the behavior of the beam after passing through a slit, a special element has been provided. It behaves in the following fashion: The beam shape is updated to a point immediately preceding the slit. This shape is supposed to define the second moment ellipsoid of a Gaussian distribution. It is transformed to the second moment of the distribution (no longer Gaussian) immediately following the slit, which may then be transformed in the usual manner through succeeding elements. In particular, the energy spread of the resolved beam is represented by σ_6.

The slit acts like any other element (with zero length) save for several properties: It presumes a Gaussian ellipsoid, whereas other elements make no assumption about the structure of the ellipsoid. It updates the beam, so that the transformation matrix is cleared and accumulation of a new matrix begins from the point immediately following the slit. (In this last respect it acts similarly to a beam constraint.)

There are three parameters:

1 - type code 6. (specifying a slit).
2 - component of the beam upon which slit acts.
3 - half-width of slit (cm).

For example, (6. 1. .5) specifies a slit that acts upon the horizontal component (σ_1) and restricts it to ±0.5 cm. The other components are affected through their correlations with σ_1.

The fraction of the beam that is transmitted is printed as output. It provides a measure of the relative beam intensity in the remainder of the system.

2.4. Second Order Beam Trace

A second order beam trace may be obtained provided neither parameter fitting nor alignment elements are employed. A special element instructs the program to employ second order perturbations. It must be inserted immediately following the beam (1. element).

There are four parameters:

1 - type code 17.

2 - second moment of energy distribution (1. for Gaussian distribution).

3 - third moment of energy distribution (0. for Gaussian distribution).

4 - fourth moment of energy distribution (3. for Gaussian distribution).

The distributions in the other coordinates are assumed to be Gaussian.

The output from this trace will have the same format as the first order trace but the values of the beam components may be perturbed from their first order value by the second order aberrations. To locate the sources of large aberrations it is possible to print out the second order terms. See element (13. 4.) of Section 3.1.3.

At present the following second order effects are simulated:

1. Chromatic aberrations of quadrupoles – the change in focal length of a quadrupole with energy.

2. Geometric effects in bending magnets.

3. Non-uniformity of fields of bending magnets in the radial plane.

B is assumed to have the form:

$$B = B_o \left[1 - n(x/\rho_o) + \beta(x/\rho_o)^2 \right]$$

where

x is radial displacement,

ρ_o is the nominal radius of curvature in the bending magnet, and

n and β determine the linear and quadratic approximation of the field B. Note that n is the field gradient for $x = 0$.

4. The spread in longitudinal extent of a pulse due to rays not being parallel to the central trajectory.

The value of β is read into the program via the parameter input element. (See Section 4.1.)

2.5. Magnet Alignment Tolerances

The misalignment of a magnet to first order affects only the first moment of the beam--the position of the central trajectory. Two situations are commonly encountered: the magnet is displaced and/or rotated by a known amount; the actual position of the magnet is uncertain within a given tolerance. Both these effects may be simulated through use of the 'align' element.

There are eight parameters:

1 - type code 8. (specifying alignment).
2 - magnet displacement in horizontal direction (cm).
3 - rotation about horizontal axis (in units of 10^{-4} radians).
4 - displacement in vertical direction (cm).
5 - rotation about vertical axis (10^{-4} radians).
6 - displacement in beam direction (cm).
7 - rotation about beam direction (10^{-4} radians).
8 - code number (defined below).

The coordinate system employed is that to which the beam is referred at the point it enters the magnet. For example, a rotation of a bending magnet about the beam direction (parameter 7 above) is referred to the direction of the beam where it enters the magnet. In this case the axis of rotation does not coincide with a principal axis of the bending magnet. The units employed are identical with the units defined for the beam.

Only the misalignments of bending magnets or quadrupoles may be simulated. The align element must directly follow the magnet element with no more than a pole face rotation or other align elements intervening; otherwise the align element will have no effect on the magnet.

An align element will update the beam.

2.5.(cont'd)

The code number provides several alternatives:

The units position distinguishes between a single magnet and a set of magnets:

0. In this case, the information on the align card refers only to the <u>single</u> magnet preceding it. This is the only option currently usable with a bending magnet.
1. With this option all of the elements since the last update (front of system, constraint upon beam, or slit) are treated as a unit and the misalignment information on the card is applied to the unit as a whole. By successive application of align elements, for example, the elements of a doublet could be misaligned relative to each other and then the doublet as a whole could be misaligned.
2. Same as 1, except that the transformation matrix, R_2, from the last (6. 0. 2.) card is employed. (See Section 4.3.) This code digit is necessary for studying misalignment of triplets and other magnet combinations involving more than two magnets. It makes use of the fact that this matrix remains unaffected by the usual update codes.

The tens position defines the system axis along which the succeeding magnets are positioned:

0. They are positioned along the same axis as if the magnet were not displaced.
10. They are positioned along the axis defined by the magnetic axis of the displaced magnet.

For instance, if a quadrupole is rotated, the remainder of the system may be left alone (0.) or rotated with it (10.).

The hundreds position distinguishes between an uncertainty in position (0.) or a deliberate displacement (100.).

Any combination of digits may be used to define the exact circumstances intended. Thus, code 111. (= 1. + 10. + 100.) indicates the <u>displacement</u> of a <u>set</u> of magnets <u>and</u> the <u>remainder</u> of the system (referred to the point the beam enters the set). Code 101. would leave the remainder of the system alone.

The tolerances may be varied. Thus, type-vary code 8.111111 permits any of the six parameters (2 - 7 above) to be adjusted to satisfy whatever constraints may follow.

SECTION 3. INPUT-OUTPUT OPTIONS

TRANSPORT is essentially an interpreter that is programmed with coded decimal input. The power of a program of this type may be greatly enhanced by providing increased control over input-output operations. Most of the conventions described in this section are unnecessary for cursory examination of simple systems. However, they are extremely useful for large systems for which reduction of output and flexibility of input become essential.

3.1. Type 13. Element

A number of control codes which transmit input/output instructions to the program have been consolidated into a single element:

There are two parameters:

1 - type code 13.
2 - code number

The effects of the various code numbers will be described below (not in numerical order). Code numbers that are not discussed have not been defined; if they are inadvertently used, they have no effect.

3.1.1. Envelope Trace (13. 11.)

This output option can only be used if the designer has a CalComp Digital Plotter available.

The envelope of the beam is defined as the plot of beam size vs. position along the system. The (13. 11.) code instructs the program to prepare such a plot. Actually the program writes a magnetic tape (A6). This plot tape is then mounted on the CalComp Digital Plotter which draws the envelope trace.

Interpretation of the trace is fairly obvious. A rectangle is drawn to represent a magnet, the size of the rectangle being defined by the magnet length and aperture. The beam size is interpolated linearly in drift spaces. (The program has information defining the beam size only at element interfaces.) Both the horizontal and vertical planes are plotted--the vertical plane above the center line, the horizontal plane below. Axes are drawn and labeled in the input units.

Several comments may be helpful.

Quadrupole apertures are defined on the quadrupole card. Bending magnet apertures are arbitrarily set at one input unit. They may be redefined via a 16. element. (See Section 4.1.)

If greater detail is desired, for instance over a long drift length or inside a magnet, the element may be divided into several elements of smaller length. In this way the program will have the beam size available for plotting at each interface.

Only that portion of the system following the (13. 11.) card will be plotted. Thus the control card is usually near the front of the deck. Provision has been made for turning off the plot once started; use a (13. 2.) element (see Section 3.1.3.).

At the Stanford Computation Center one need only write "PLOT" boldly on the No. 1 run request card in order to obtain a plot. The plot will subsequently appear in the output box some finite time later.

3.1.2. Layout Trace (13. 12.)

This output option can only be used if the designer has a CalComp Plotter at his disposal.

The layout of a system refers to the position of the magnets and the deflections they introduce. It is literally a floor plan of the system. This code instructs the program to prepare a plot tape of the layout. As with the envelope trace, the plot tape is mounted on the CalComp Plotter which actually sketches the layout.

Magnets are drawn as rectangles of the proper length with coded shapes to indicate magnet type. Between magnets the reference trajectory is drawn. The layout is intended to be an aid in visualizing the system spatially. It eliminates the recurring task of drawing the layout by hand.

In addition to the sketch, the data from which the plot is prepared are printed in a tabular form. This table serves as a definition of the system suitable for transmission to mechanical engineers, draftsmen, etc.

3.1.3. Printed Output Controls 1., 2., 3., 4., 24.

Several codes are available to control various aspects of the printed output. Most cards produce a line of output that advertises their existence. Those that do not, usually have an obvious effect upon the remainder of the output and thus make their presence clear.

(13. 2.): Many elements cause the beam matrix to be printed. This code instructs the program to suppress the printing of the beam matrix. Using this code suppresses the envelope trace but not the layout trace.

(13. 1.): This code temporarily overrides the suppress code. It causes a single beam matrix to be printed at that point in the system.

(13. 3.): This code permanently overrides the suppress code and restores the normal mode in which every beam matrix is printed.

(13. 4.): When fitting a focus condition, it is often desirable to print out the transformation matrix, R. This code instructs the program to print the current transformation matrix. (If the program is computing a second order trace, the second order transformation matrix will be included.) This matrix is cumulative from the last beam update with the following exception. Immediately after an update, the matrix is still available for printing, but <u>not</u> for fitting. In any event, a matrix is always printed. Its proper interpretation is up to the designer. The units of the elements of the matrix are consistent with the input units. That is, if the ray vector is expressed in the input units, the transformation matrix is correct as it stands to yield the transformed vector by standard matrix-vector multiplication.

(13. 24.) This code causes the secondary transformation matrix, R_2, to be printed. (See Section 4.3.) The format and units of R_2 are identical with those of R which is produced by a (13. 4.) code.

3.2. Correlation Input

To allow the output beam from some point in a system to become the input beam of some succeeding system, provision has been made for re-entering the correlation matrix which appears as a triangular matrix in the output.

There are 16 parameters:

1 - type code 12.

2-16 - the 15 correlations among the 6 beam components - in the order printed (by rows).

Several cards may be used to transmit the 15 correlations, if necessary.

Since this element is solely an extension of the beam input, a 12. element, if used, must follow a 1. (beam) element.

3.3. Input - Output Units

TRANSPORT was originally designed with a standard set of units that have been used throughout this manual. However, to accommodate other units conveniently and to relieve certain parameter-fitting problems, provision has been made for redefining the units to be employed. This is accomplished with the following element.

There are 4 parameters:

1 - type code 15.
2 - code digit.
3 - the BCD abbreviation of the unit. This will be printed on the output listing. It must be enclosed in quotes (8 - 4 punch) and is a maximum of 3 characters long (4 for energy). The first character should be a blank.
4 - the scale factor that multiplies a parameter expressed in the new units to convert it to the appropriate reference unit. The example below will clarify this.

The various units that may be changed are:

Code	Quantity	Reference Unit
1.	horizontal and vertical beam extent	cm
2.	Horizontal and vertical beam divergence	10^{-3} radian
3.	vertical beam extent (only)	cm
4.	vertical beam divergence (only)	10^{-3} radian
5.	pulsed beam length	cm
6.	energy spread	%
7.	undefined	
8.	length	meters
9.	magnetic fields	kg
10.	mass	electron mass
11.	momentum	GeV/c

The reference units are not the same as the standard units used in this manual. Reference units differ from standard units only for the divergence (10^{-3} vs 10^{-4} radians) and the momentum spread (1% vs 0.1%). In the absence of any unit changes, the standard units will be used. However, the reference units determine the required scale factors for unit changes.

Units are not restored at the start of a run. Once changed, they remain the same for all succeeding runs in an input deck, unless specifically changed. The 15. elements are normally the first cards in a deck, and perhaps the last cards--in the event units are being restored to avoid disturbing succeeding runs.

Example: To change length to feet, width to inches, and momentum to Mev/c, add to the front of the deck the elements

```
15.  8.  'FT'* 0.3048
15.  1.  'IN'  2.54
15.  11. 'MEV' 0.001
```

The scale factor, 0.3048, multiplies a length expressed in the new unit, feet, to convert it to the reference unit, meters.

* Omit ' marks with this version

3.4. Type Code List

Upon occasion it is useful to be able to make several consecutive runs with the same data. For instance, it is not unusual to fit system parameters to first order and then make a second order trace with the optimized parameters. To avoid the delay of repunching the input cards with the new values, conventions have been established for manipulating the array of parameters within the machine.

Two conventions are required:

(1) If the type code for an element is negative, the element will be ignored. It is available in memory as described below, but will be skipped whenever encountered by the program.

(2) If the second card in the input deck (which usually contains only a 0) contains a 1, a special input list is expected immediately following this card. This list consists solely of type-vary codes, the same type codes that appeared in the immediately preceding input deck. However, the type codes may have different vary codes associated with them, or may have minus signs added so that the element will be ignored, or minus signs deleted so that the element will be introduced. These type-vary codes will replace the codes already in memory, but none of the parameters will be changed.

After the type code list has been exhausted, input returns to its normal mode and additional elements are read until a SENTINEL is encountered. The type code list is normally terminated after the complete set of type codes in memory has been read. However, if the last type codes are to be suppressed anyway, they may be deleted (erased from memory) by replacing them with a SENTINEL. In this case, a single input deck would contain two SENTINEL cards (see Section 3.4.2. below).

The flexibility permitted by these conventions provides added opportunity for input errors and extra care should be taken in preparing the input deck. All possible checks are made by the program to catch input errors, but there is a limited number that can be made.

The type code list has proved invaluable at SLAC. However, the individual designer will have to experiment with the option in order to evaluate its difficulty, reliability and relative advantage for him.

The following are two examples of the flexibility provided by this option. In the input decks below, most of the parameter values have been deleted for simplicity.

3.4.1. Example 1

Consider the problem mentioned earlier, that of a first order fit followed by a second order trace.

```
'FIRST ORDER FIT                                                 '
0
1.    ___  ___  ___  ___  ___  ___  ___
-17.  ___  ___  ___
3.    ___
.
.
.
5.01  ___  ___  ___
5.01  ___  ___  ___
.
.
.
10.   ___  ___  ___  ___
10.   ___  ___  ___  ___
SENTINEL
```

```
'SECOND ORDER TRACE                                              '
1
1.    17.   3.    ...   5.    5.    ...   -10.   -10.
SENTINEL
```

In these runs the 17. card is ignored during the first run while the quadrupoles (5.01) are being fit to the contraints (10.). In the second run the 17. element is activated while the constraint elements (10.) are suppressed. The quadrupole fields calculated in the first run are held constant (5.01 → 5.) in the second.

3.4.2. Example 2

As a second example, consider the problem of designing an achromatic bending system.

```
'FIT DRIFT AND QUAD FOR ACHROMATIC SYSTEM                        '
0
-16.   4.   ___
-16.   5.   ___
-13.   11.            1.   ___  ___  ___  ___  ___  ___  ___
4.
3.     ___  ___  ___
5.01   ___
3.1    ___  ___  ___
4.     ___
10.    -1.  6.   0.   .001
10.    -2.  6.   0.   .001
SENTINEL

'FORM WAIST WITH ACHROMATIC BEAM AND PLOT                   '
1
16.    16.  13.1. 4.   3.    5.    3.  4.
SENTINEL
3.     ___
5.01   ___  ___  ___
5.01   ___  ___  ___
3.     ___
10.    2.   1.   0.   .01
10.    4.   3.   0.   .01
SENTINEL
```

In the first run the field of a quadrupole (5.01) and a drift length (3.1) are varied to produce an achromatic system, that is, one whose (x,δ) and (θ,δ) matrix elements are zero. In the second run a pair of quadrupoles are added to the first system and their fields are adjusted to produce a double waist at a proposed target. The quad field and drift length fit in the first run are held fixed in the second run by deleting the vary code for these elements from the type code list (5.01 → 5., 3.1 → 3.). Following the fitting procedure in the second run, an envelope trace of the resulting beam will be drawn. In the first run this trace was suppressed by the (-) sign in front of the 13. The 16. elements transmit, to the program, values for the horizontal and vertical apertures of the bending magnets for the envelope trace. The achromatic constraints of the first run are deleted in the second by the first SENTINEL card.

SECTION 4. ADDITIONAL ELEMENTS

This section describes a number of additional elements that have proven convenient or essential for certain systems. It is anticipated that this section will be subject to the greatest expansion as the program evolves in order to fit different and more elaborate systems.

The order of the following elements is arbitrary and the casual reader need observe only the facilities available without bothering with the details associated with each.

4.1. Parameter Input

A number of constants are used by the program which do not appear as parameters on any of the other element cards. A special element has been provided to allow the designer to set their values.

There are three parameters:

1 - type code 16.
2 - code digit
3 - value of constant

A number of such constants have been defined in this manner. All have a normal value that is initialized at the beginning of each run.

Code Digit	Description
1.	β - quadratic term of bending field; normally 0. This parameter is used for second order only - see Section 2.4.
2.	ΔB - error or uncertainty of bending field; normally 0. (Currently unused.)
3.	Mass of the particles comprising the beam, in units of the electron mass; normally 0. A non-zero mass introduces the dependence of pulse length on velocity, an important effect in low energy pulsed beams.
4.	Horizontal aperture of bending magnet, in units of horizontal beam width; normally 1. This parameter is used in drawing the envelope trace.
5.	Vertical aperture of bending magnet, in units of vertical beam width; normally 1.
6.	Cumulative length of system, in units of length. It is set to zero initially, then increased by the length of each element, and finally printed at end of system. This element allows the cumulative length to be reset as desired (for the envelope trace).

Undefined codes used inadvertently have no effect.

4.2. Update

To provide a facility for updating the beam there is an extension of the code for a slit: If the component number is 0, the beam is updated, but no transformation is effected. Thus (6. 0. 1.) forces an update and initiates the accumulation of a new transformation matrix from the point of the update. This facility is useful in conjunction with misaligning a set of magnets or fitting only a portion of a system.

The complete list of elements which update the beam is then:

A slit (6. element)
A constraint upon the beam (subset of possible 10. elements)
A misalignment (8. element)
A beam code (1. element)

4.3. Auxiliary Transformation Matrix

To circumvent the difficulties caused by frequent beam updates and consequent redefinition of the cumulative transformation matrix, R, provision has been made for an independent transformation matrix, the R_2 matrix.

This matrix is not normally accumulated, but must be introduced by defining its starting point with a (6. 0. 2.) card. This variant of the slit code also serves to redefine the starting point of the R_2 matrix (update it) when desired.

R_2 has no effect upon the beam as does R nor do any changes in the beam react upon it. It is solely a bookkeeping device to keep track of the transformations induced by the linear elements of a system. It may be printed by a (13. 24.) card. Any components of R_2 may be constrained by a 10. card with code digits: - (i + 20.) and j.

For example, (10. -1. 2. 0. .01) and (10. -21. 2. 0. .01) are identical constraints applied to the R and R_2 matrices, respectively.

R_2 is also updated by a beam (1.) element, provided R_2 has been introduced, but by <u>no</u> other element. Its use slows the speed of the program, so it should be avoided where not actually required. No provision has been made for suppressing it once it is introduced.

One typical use of R_2 is to allow overlapping focus constraints. Another is to permit the misalignment of a triplet. For example, an <u>uncertainty</u> in the position of the following triplet

```
5.  1.  -8.  10.
5.  2.  +7.  10.
5.  1.  -8.  10.
```

may be induced by appropriate 8. elements as noted.

```
6.  0.   2.
5.  1.  -8.   10.
6.  0.   1.
5.  2.  +7.   10.
5.  1.  -8.   10.
8.  ——  ——   ——   ——   ——   ——   0.
8.  ——  ——   ——   ——   ——   ——   1.
8.  ——  ——   ——   ——   ——   ——   2.
```

The first 8. card in the list refers to misalignment of the third magnet <u>only</u>. The second 8. refers to misalignment of the second and third magnets as a unit (via R). The last 8. refers to misalignment of the whole triplet as a unit (via R_2).

4.4. Pole-Face Rotation

To provide non-wedge bending magnets, an element is available for imposing a pole-face rotation upon a standard wedge magnet.

There are two parameters:

1 - type code 2.

2 - angle of rotation (degrees).

A 2. element must immediately precede a bending magnet (in which case it indicates an entrance face rotation) or immediately follow one (exit face rotation). The sign of the angle is such that a rectangular magnet has both angles with the same sign as the bending field (to provide vertical focusing).

For example, a rectangular bending magnet whose total bend is 10 degrees would be represented by the three cards: (2. 5.), (4. ——), (2. 5.). Should it be desired to misalign such a magnet, an update must be forced prior to the first 2. and the convention appropriate to misalignment of a set of elements applied, since, indeed, three separate transformations are involved.

The angle of rotation may be varied. For example, the element (2.1 5.) would allow the angle to vary from an initial guess of 5 degrees to a final value which would, say, satisfy a vertical focus constraint later in the system.

A rotation that follows one bending magnet and precedes another will be associated with the latter.

4.5. Accelerator

An energy gain is reflected in both the divergence and the width of the beam. This element provides a simulation of an energy gain over a drift length.

There are three parameters:

1 - type code 11.
2 - drift length (meters)
3 - energy gain (Gev)

The new beam energy is printed as output.

The energy is assumed to increase linearly over the entire drift length. If this is not the case, an appropriate model may be constructed by combining separate 11. elements. An 11. element with a zero energy gain is identical to a drift length.

None of the parameters may be varied.

4.6. Arbitrary Matrix

To allow for fringe fields and to provide a model for beam steering and other specific (perhaps non-phase-space-conserving) transformations, provision has been made for reading in a particular transfer matrix. The 6 × 6 matrix is read in row by row.

There are eight parameters:

1 - type code 14.

2-7-the six numbers comprising the row. The units must be those used to print the transfer matrix; in other words, consistent with the beam input/output.

8 - row number. (1.- 6.)

9 $N = 0$ will read 30 2nd order coefficients.

For example, (14. -.1 .9 0. 0. 0. 0. 2.) introduces a transformation matrix whose second row is given but which is otherwise a unit matrix. Note that this transformation does not conserve phase space because $R_{22} = 0.9$.

A complete matrix must be read and applied one row at a time; of course, rows that do not differ from the unit transformation need not be read.

Any of the components of a row may be varied, however there are several restrictions.

Normally, 14. elements that immediately follow one another will all be used to form a single transformation matrix. If distinct matrices are desired, another element must be inserted to separate the 14. cards. Several do-nothing elements are available, (13. 0.) is a convenient one.

If components of a 14. card are to be varied it must be the last 14. card in its matrix. This will force a matrix to be split into factors if more than one row has variable components.

4.7. Repetition

Many systems include a set of elements that are repeated several times. To minimize the chore of input preparation, a 'repeat' facility has been added.

There are two parameters:

1 - type code 9.

2 - code digit. If non-zero, it states the number of repetitions desired from the point it appears (open parentheses). If zero, it marks the end of a repeating unit (close parentheses).

For example, a total bend of 12 degrees composed of four 3 degree bending magnets each separated by 0.5 meters could be represented by (9. 4.),(4. ——), (3. .5), (9. 0.). Those elements (in this case a bend and drift) between the (9. 4.) and (9. 0.) would be employed four times.

There is no indication of the 9. cards in the output, save the repeated listing of the elements they control. Vary codes may be used within a repeating unit in the usual fashion.

Repeat cards may be nested four deep.

4.10. Beam Rotation

This beam co-ordinates x and y may be rotated through an angle α about the z axis (The axis or central trajectory). Thus a tilted quadrupole or bending magnet may be inserted into the system (see example below). Tilting a quadrupole 90° is equivalent to changing its polarity.

There are two parameters:

1 - type code 20.

2 - angle of beam co-ordinate rotation, in degrees.

The angle of beam co-ordinate rotation may be varied.

Example 1: a converging horizontally quadrupole rotated + 60°:

```
20.     60.
 5.      1.     +10.     10.
20.    -60.
```

or

```
20.    -30.
 5.      1.     -10.     10.
20.     30.
```

Example 2: a vertical bending magnet:

```
20.     90.
 4.      3.      10.      0.
20.    -90.
```

NOTE - This transformation assumes that the vertical and horizontal units are consistent (i.e., the selected units of x and θ must agree with those of y and ϕ respectively). This is always true unless a 15. 3. or 15. 4. card is used.

4.9. Sextupole

Sextupole magnets are sometimes used to correct second-order aberrations in a beam transport system. The action of a sextupole on beam particles is purely a second order effect, so on first order runs (absence of the 17. card) this element will act as a drift space.

There are 4 parameters:

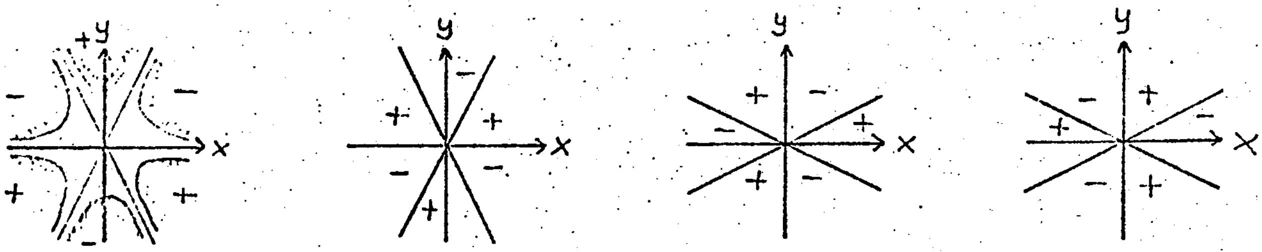
1 - type code 18.

2 - effective length (meters); a negative length is a convention to orient the sextupole with respect to x axes (see figures below)

3 - field at pole tips (kg). Both positive and negative fields are possible (see figures below)

4 - aperture (cm) radius of circle tangent to pole tips

Orientation of sextupole:



B > 0
L > 0

B < 0
L > 0

B > 0
L < 0

B < 0
L < 0

Other orientations of the sextupole may be had using the beam rotation element. None of the parameters of the sextupole may be varied.

4.8. Solenoid

The solenoid is sometimes used as a focusing element in systems passing low energy particles. Particles in a solenoidal field travel helical trajectories. The fringe effects necessary to produce the focusing are included.

There are three parameters:

1 - type code 19.

2 - effective length of the solenoid

3 - the field (kg) a positive field by convention points in the direction of positive z.

The length and the field may be varied by using 19.1 or 19.01 respectively. See section 2.2.

SECTION 5. ADDITIONAL CONSTRAINTS

In Section 2.2.2 the basic parameter fitting constraints were presented. In general they dealt with fitting an element of the transformation matrix or of the beam matrix. In the course of using this program, it has proven convenient to provide additional constraints and conventions of a different nature. These are discussed in this section. As is the case with Section 4, the casual reader should simply scan the following pages to become aware of these added facilities.

5.1. Coupled Vary Code

Vary codes may follow element types 1., 2., 3., 4., 5., 6., 8., and 14.

It is possible to apply the same correction to each of several variables. This may be done by replacing the digit 1 in the vary code with one of the digits 2 through 8. All variables whose vary digit has one of these values (and appears in the same position within the vary code) will receive the same correction. For example, the three type-vary codes (5.02, 5.01, 5.02) might represent a symmetric triplet. The same correction will be made to the first and third quadrupoles, guaranteeing that the triplet will remain symmetric. Variables whose vary digits appear in different positions will not be so tied together (the vary digit positions are completely independent). Thus, the codes (8.666666, 3.6) apply the same correction to the drift length as to the horizontal misalignment, but do not restrict the six misalignment parameters with respect to each other. (Such a combination would never arise in practice.)

If the vary digit 9 is used, the correction associated with the vary digit 4 will be subtracted rather than added to this variable. Thus, the type-vary codes (3.4, 5.01, 3.9) will slide the quadrupole back and forth within a prescribed drift length, but will not change the total drift length. This option may not be used with non-linear elements (types 1., 6., 8.).

The total number of variables in a run is limited to 15 by reasons of memory allocation and running time. So far as this limit is concerned, variables that are tied together count as one variable.

5.2. Correlation Constraint

It is possible to fit the correlation between i and j as distinct from the covariance between i and j (σ_{ij}). This is non-linear and consequently difficult to fit; the corresponding covariance is usually to be preferred. The usual 10. constraint element is used. The code digits are i + 10 and j. Thus (10. 12. 1. .1 .01) specifies that the (2,1) correlation is to be made 0.1 ± 0.01.

5.3. Conditional Constraints

A constraint card has the general format: (10. i, j, Δ, σ,). If, instead of 10., a type code 10.1 is used (not a vary code in this case) the constraint will be considered a lower limit. That is, if the value of the computed function is less than Δ, the constraint will be employed as usual. If it is greater than Δ, the constraint will be ignored. Similarly, type code 10.2 indicates an upper limit.

The upper limit might be used to keep a beam with magnet apertures. Thus (10.2 1. 1. 6. .1) will produce a constraint on the horizontal beam extent to keep it less than 6 cm. If the beam is already less than 6 cm this element will be ignored.

5.4. Aperture Constraint

One dominant consideration in designing transport systems is that the beam fit within the apertures of the magnets. The element (13. 10.) instructs the program to monitor the apertures. Following this element, an upper limit constraint on both the vertical and horizontal beam size will be applied at the entrance and exit face of each quadrupole.

This code does nothing that could not be done with the conditional constraint (10.2) described in Section 5.3. It is equivalent to the following cards at the entrance and exit of each quadrupole:

```
10.2   1.   1.   a   .1a
10.2   3.   3.   a   .1a
```

where a is the quadrupole aperture. However, it does provide a significant reduction in the number of input cards to be punched.

This is not completely satisfactory, since the maximum beam size usually occurs inside a quadrupole. However, where necessary, quadrupoles can be split, whereupon the constraint will be applied at the interface.

Since this is a constraint upon the beam, it will force a beam update at each quadrupole. Thus any focus conditions upon the transformation matrix must use the R_2 matrix of Section 4.3.

At some future date the program may be changed to include bending magnet apertures. This is not being pressed since quadrupoles are usually the limiting elements.

To use this constraint, the quadrupole aperture should be defined as the usable region of the quadrupole or as the region over which a first order approximation of the quadrupole transformation is valid. In any case, the field given must correspond to the aperture specified. If the aperture is not the pole-tip radius, the field is correspondingly smaller in direct ratio to the pole-tip field.

5.5. System Length Constraint

A running total of the lengths of the various elements encountered is kept by the program and may be fit. The code digits are i = 0., j = 0.

Thus the element (10. 0. 0. 150. 5.) would make the length of the system prior to this element equal to 150 ± 5 meters. Presumably there would be a variable drift length somewhere in the system. By redefining the cumulative length via the 16. element, partial system lengths may be accumulated and fit.

5.6. Beam Intensity Constraint

It is possible to fit parameters of a system to obtain a given beam intensity through a slit. The 10. constraint code is used with code digits i = 0., j = 1.

As an example consider the following input list.

```
4.    ——    ——    ——
3.1   ——
6.    1.    1.
10.   0.    1.    .2    .01
```

The program will adjust the drift length (3.1) until the beam intensity through a ± 1 cm slit in the horizontal plane is 20% ± 1% of the full beam intensity.

One might also fit the width of a slit in the vertical plane to intercept, say, 10% ± 1% of the beam. The necessary elements would be:

```
6.1   3.    ——
10.   0.    1.    .1    .01
```

5.7 AGS Machine Constraint

Provision has been made in the program for fitting the betatron phase shift angle, μ, associated with usual AGS treatment of magnet systems.

In the horizontal plane: use code digits $i = -11.$, $j = 2.$ and specify:

$$\Delta = \frac{1}{2\pi} \cos^{-1}\left[0.5\ (R_{11} + R_{22})\right] = \frac{\mu}{2\pi}\ \text{(horiz)}$$

$$= \text{freq}/(\text{no. of periods}).$$

In the vertical plane: $i = -13.$, $j = 4.$, and

$$\Delta = \frac{1}{2\pi} \cos^{-1}\left[0.5\ (R_{33} + R_{44})\right] = \frac{\mu}{2\pi}\ \text{(vert)}$$

For example, if there are 16 identical sectors to a proposed AGS machine and the betatron frequencies per revolution are to be 3.04 and 2.14 for the horizontal and vertical planes respectively, then the last element of the sector should be followed by the constraints:

```
10.   -11.   2.   .190   .0001
10.   -13.   4.   .134   .0001
```

5.8. Internal Constraints

A set of upper and lower bounds on the value of each type of parameter is in memory. If a correction is computed for a parameter which would take its value outside this range, it is reset to the limit of the range, and a constraint constructed to keep it within range. The current limits are:

$0.5 <$ drift < 100 (m)
$-14 <$ quad field < 14 (kg)
$-16 <$ bend field < 16 (kg)
$-100 <$ bend gradient < 100
$0.1 <$ slit width < 100 (cm)
$-45 <$ pole rotation < 45 (deg)
$0.002 <$ alignment displacement < 1 (cm)
$0.1 <$ alignment rotation < 50 (10^{-4} rad)
$0.01 <$ input beam < 1000 cm or 10^{-4} rad or 0.1%

These limits apply only when a parameter is being varied. Fixed values that exceed this range may be used as desired.

These constraints were included to avoid physically meaningless solutions. However, they are rather ineffective since systems that require values outside these limits usually have some basic difficulty calling for redesign.

The limits are not adjusted for different input units. So if drift lengths are expressed in inches, the effective limits are $0.5 <$ drift < 100 (in).

5.9. First Moment Constraint

Misalignments and second order effects cause the center of the phase ellipsoid to be shifted from the reference trajectory, i.e., they cause the beam to have a non-zero first moment. The first moments appear in a vertically array to the left of the vertical array which give the σ_i's. The units of corresponding quantities are the same.

It is perhaps helpful to emphasize that the origin always lies on the reference trajectory. First moments refer to this origin. However, the ellipsoid is defined with respect to its center, so the covariance matrix, as printed, defines the second moment about the mean.

First moments may be fitted. The code digits are $i = 7.$ and j, where j is the index of the quantity being fit. Thus (10. 7. 1. .1 .01) constrains the horizontal (1.) displacement of the ellipsoid to be 0.1 ±0.01 cm.

This constraint is useful in deriving the alignment tolerances of a system. For an example, see Section 6.4.

APPENDIX IV

A GUIDE TO PREPARATION OF DATA INPUT FOR "ORBCAL"

INPUT PARAMETERS TØ ORBCAL BY CARD

Card No.	Parameter	Format	Card No.	Parameter	Format
0	JTAPE	I5	3B	ØRB	F5.0
1	RMIN	E15.4		IO	I5
	RMAX	E15.4		IRMAX	I5
	DR	E15.4		IRDEL	I5
	N	I5		IFIX	I5
	IMAXQ	I5		JØUT	L5
2	RESTM	F10.3		LMAX	I5
	NCHARG	I5	4B	FINDEQ	L5
	BUNIT	F10.2		KMAX	I5
1A	(CØMMENT)	72H		TD	F5.0
2A	IØUT	L5		IARC	L5
	KTAPE	I5		INUR	L5
3A*	FMT1	(72H)		ØUTSUP	F5.0
4A*	(DATA)	FMT1	5B	EPS	E8.1
1B	(CØMMENT)	72H		DX	F8.4
2B	E1	F11.6		DPX	F8.4
	ERF	F9.6		RID	F10.6
	IRF1	I5		PRID	F10.6
	IRFDEL	I5		ØUTEQ	L3
	TAURF	F10.5	6B	DRIA1	F10.6
	IACCEL	I5		DPRIA1	F10.6

*These cards may be present only if certain options are chosen (See Notes #1, 10, and 11).

Card No.	Parameter	Format
6B(con't.)		
	DRIA	F10.6
	DPRIA	F10.6
	NLINES	I5
7B	ZIA1	F10.6
	PZIA1	F10.6
	VERSTA	L5
8B	KKMAXU	I5
	KKMAXD	I5
	DE	F10.5
9B	RFALL	F10.4
10B	KWRITE	L5
	KTAPE	I5
11B*	RIA	F10.6
	PRIA	F10.6
	ZIA	F10.6
	PZIA	F10.6
LAST	IG0	I5

*These cards may be present only if certain options are chosen. (See Notes #1, 10, and 11).

INTERPRETATION OF INPUT PARAMETERS (BY CARD)

Card	Parameter	Dimension	Notes	
1	JTAPE	---		The output tape (logical) number
2	RMIN	INCHES		Initial radius of data
	RMAX	INCHES		Final radius of data
	DR	INCHES		Radial increment of data
	N	---		Number of sectors (may be 1)
	IMAXQ	---		Number of (azimuthal) data supplied at each radius
3	RESTM	MeV		Rest energy of particle
	NCHARG	e		Particle charge (absolute value)
	BUNIT	GAUSS		Central magnetic field
1A	(CØMMENT)	---		Any comment
2A	IØUT	---		TRUE: Input field printed out FALSE: Input field not printed out
	KTAPE	---	1	= 0: Input data read from cards ≠ 0: Input data read from tape #KTAPE

Card	Parameter	Dimension	Notes		
3A*	FMT1	---	1	Only if KTAPE $\neq 0$	Format for Input Data
4A*		BUNIT			Input Data
⋮					
1B	(CØMMENT)	---		Any Comment	
2B	E1	MeV (or m_oc)	2	Initial energy (or momentum) for calculations	
	ERF	MeV	3	RF voltage	
	IRF1	---	3	Ørientation azimuth for RF structure	
	IRFDEL	---	3	= 120: Triants = 180: Dees	
	TAURF	---	3	Ratio RF frequency to revolution frequency	
	IACCEL	---	3	Type of acceleration	
3B	ØRB	---	4	Controls frequency of printout	
	IO	---	5	Initial azimuth	
	IRMAX	---		Maximum number of revolutions (per particle)	
	IRDEL	---	4	Used with IO	
	IFIX	---	6	= 0,1: N unchanged > 1: N replaced by IFIX	

*These cards may be present only if certain options are chosen.

Card	Parameter	Dimension	Notes	
3B(con't.)				
	J0UT	---		TRUE: Input data printed cut and Fourier analysis FALSE: No printout
	LMAX	---		Maximum harmonic number to be calculated
4B	FINDEQ	---		TRUE: Eq. orbs to be found FALSE: Eq.orbs not to be found
	KMAX	---	7	Number of particles to be traced
	TD	---		= -1: Particles move in direction defined by input data = +1: Particles mcve in opposite direction to above
	IARC	---		TRUE: Arc length (in Med. Plane) calculated FALSE: Arc length (in Med. Plane) not calculated
	IN.UR	---		TRUE: Betatron freqs. calculated for E.0. particles FALSE: Not calculated

Card	Parameter	Dimension	Notes	
4B(con't.)				
	ØUTSUP	---	8	Controls amount of Øutput when FINDEQ TRUE
5B	EPS	---	9	Used in calculation of Eq. Ørbs
	DX	c.u.	9	Used in calculation of Eq. Ørbs
	DPX	$m_o c$	9	Used in calculation of Eq. Ørbs
	RID	c.u.	9	Initial R guess for E.Ø. (energy E1)
	PRID	$m_o c$	9	Initial PR guess for E.Ø. (energy E1)
	ØUTEQ	---	9	TRUE: Details of Bet.Freq. Calc. printed out FALSE: Not printed
6B	DRIA1	c.u.	10	Particles to be tracked after each E.Ø. is found
	DPRIA1	$m_o c$	10	
	DRIA	c.u.	10	
	DPRIA	$m_o c$	10	
	NLINES	---	10	
7B	ZIA	c.u.	10	
	PZIA	$m_o c$	10	
	VERSTA	---	10	

Card	Parameter	Dimension	Notes	
8B	KKMAXU	---	2	Number of E.Ø.'s to be found with $E \geq E1$
	KKMAXD	---	2	Number of E.Ø.'s to be found with $E < E1$
	DE	MeV (or $m_o c$)	2	Energy (or momentum) increment
9B	RFALL	INCHES		Maximum radius of Isochronous field (May be 0)
10B	KWRITE	---		TRUE: E.Ø. properties summarized on tape #KTAPE FALSE: No summary
	KTAPE	---		Number of summary tape (may be same as input tape)
11B*	RIA	c.u.	11	Particles to be tracked when no E.Ø.'s are found
.	PRIA	$(m_o c)$	11	
24B*	ZIA	c.u.	11	
	PZIA	$(m_o c)$	11	
LAST	IGØ	---		= 0: To be followed by card 1B, etc. =11111: To be followed by card 1, etc. = 33333: Indicates conclusion of computations

*These cards may be present only if certain options are chosen.

NOTES

#1 The magnetic field may be read either from cards or tape.

A) From tape (field in units of BUNIT)

KTAPE ≠ 0 (may 7, 8, 9, 10 in present system)

Data must be presented in following form:

1 BCD record of IMAXQ words followed by

1 BCD record with 1 word (value 11111 causes reading to continue; value 33333 causes reading to stop -- only to be used to halt before all radii have been read in)

Repeat above pattern until all radii have been read in

B) From cards (field in units of BUNIT)

KTAPE = 0

Data is read in according to format input on card 3A, but each radius must be preceded by a blank card.

Card 3A (and following cards of group A) must not be present if KTAPE ≠ 0.

IMPORTANT: Subroutine FIELD contains all these statements, and may be changed at the users convenience, but the data must be entered in memory according to the way shown in the present version of this subroutine.

#2. The energies to be investigated

(This excludes acceleration, for which see Note 3.)

The parameters of interest are

E1, KKMAXU, KKMAXD, DE

A) Setting either (or both) KKMAXU or KKMAXD to be > 0 amounts to putting (KKMAXU + KKMAXD) data packs in series, vis., suppose that we have stipulated a set S of parameters, where S excludes KKMAXU, KKMAXD and DE. Then the scheme on Page 7 will be followed. The energy E is that at which every particle described by S will start.

B) If E1 is input as < 0 it will be transformed into a momentum (units $m_c c$), as will DE, i.e.,

$$P1 = -E1$$

$$DP = DE$$

Section A is followed, with "momentum" everywhere replacing "energy".

LOCATION: MAIN; EQ0RB IFN ~ 30; 0RBCAL IFN ~ 50

#3 Acceleration

Parameters of interest are E1, ERF, IRF1, IRFDEL, TAURF, IACCEL

A) If ERF = 0, there is no acceleration

B) If ER $\neq$ 0, the following is true

1) TAURF: The frequency of the RF is (1/TAURF) times the natural frequency of the particles ($\omega_{NAT} = B_o c/m_o c$).

#3-B) (Con't.)

2) IACCEL: This may have one of three values

= 1: at each of the $\frac{N \cdot IMAXQ}{2}$ R-K steps

$$E \Rightarrow E + 2 \cdot ERF \cdot \left(\frac{2}{N \cdot IMAXQ} \right)$$

= 2: at each edge (see IRFDEL) of the accelerator structure we have $E \Rightarrow E + ERF$

= 3: At each edge $E \Rightarrow E + ERF^*$ where ERF^* now takes the sinusoidal shape, and the frequency, of the accelerator voltage into account.

3) IRFDEL: This may have one of two values

= 120: TRIANTS assumed to be present

= 180: DEES assumed to be present.

Edges, in each case, are radial.

4) IRF1: The azimuth of one of the edges. The angle is taken with respect to the zero defined by the way in which the data was read in, and is self-adjusting for whatever value of IO that is being used. It is in units of $\Delta\Theta$, i.e., say

$$IMAXQ = 120$$

$$N = 6$$

$$\therefore$$

$$\Delta\Theta = 0.5°$$

and

$$I(\Theta) = \frac{\Theta}{0.5} + 1$$

or

$$I(53^{\circ}) + 107.$$

LOCATION: PREP2, ØRBCAL IFN ~ 40 and ~ 295.

#4 Frequency of Output

We refer to how many lines of output there will be.

Parameters of interest are

ØRB, IRDEL.

1) ØRB: This may have several values

= 0: Output once each sector

> 0: Output every ØRB R-K steps, e.g.,

ØRB = 5

Output every 5th R-K step

2) IRDEL: This must be ≥ 1. It suppresses output except on the IRDEL'th revolution, e.g.,

IRDEL = 5

Output only on 5th, 10th, etc. revolutions.

LOCATION: ØRBCAL IFN ~ 185.

#5 Internal Arrangement of Data

Parameters of interest are

IO, IRF1

The data is arranged as it was read in, but according to the scheme (I,J) where J is the radius

#5 (Con't.)

$$J = \frac{R - RMIN}{DR} + 1$$

so

$$J(RMIN) = 1$$

$$J(RMAX) = JMAX$$

and I is the azimuth

$$I = \frac{\Theta}{\Delta\Theta} + 1$$

$$I(0) = 1$$

$$I(360°) = IMAX = N \cdot IMAXQ,$$

$$\Delta\Theta = \frac{N \cdot IMAXQ}{360}$$

1) IO: This may have any value from 1 to IMAXQ. The data is shifted so that IO is the azimuth at which all particles will be started; it is the new zero degrees.

2) IRF1: This has been discussed in Note 3.

LOCATION: PREP1, PREP2.

#6 IFIX

In the case that a fixed point with periodicity different from N is to be found, the parameter IFIX is set according to:

The frequency of interest is

$$\nu_r \approx p/q$$

p,q reduced integers,

q revolutions are Nq sectors

#6 (Con't.)

∴ There are p oscillations every Nq sectors and the new periodicity is

$$\left(\frac{p}{Nq} \right)^{-1} .$$

This is the desired value for IFIX, it must be integral.

Example:

$$\nu_r = \frac{3}{2}$$

$$N = 6$$

$$\therefore$$

$$IFIX = \left(\frac{3}{2 \times 6} \right)^{-1} = 4$$

All computations for that particular data pack will be based upon N $\Longrightarrow$ IFIX

LOCATION: INPUT IFN ~ 95.

#7 KMAX

This parameter is used in two different ways.

If FINDEQ is TRUE, see Note 10.

If FINDEQ is FALSE, see Note 11.

#8 ØUTSUP

If FINDEQ is FALSE, ØUTSUP has no effect.

IF FINDEQ is TRUE, and if E.Ø.'s are only being found and nothing else is being done (see sections "Operations") then ØUTSUP is used to suppress unneeded ØUTPUT.

#8 (Con't.)

ØUTSUP = 1: No suppression

ØUTPUT = 2: "Moderate" suppression

ØUTPUT = 3: Maximum suppression

LOCATION: ØRBCAL IFN ~ 80, 240, 280

#9 Computation of E.Ø. and Betatron frequencies.

Paramters of interest

EPS, DX, DPX, RID, PRID, ØUTEQ

If FINDEQ = FALSE, these parameters are not used

If FINDEQ = TRUE:

1) EPS: Defines "how closely" E.Ø. is desired:

at azimuth IO

Initial $R = R_o$, $PR = PR_o$

After one revolution

Final $R = R_f$, $PR = PR_f$

Define

$$\varepsilon = \left| R_o - R_f \right| + \left| PR_o - PR_f \right|$$

The computations halt if

$$\varepsilon \leqslant \text{EPS},$$

Continue (for at most ten tries) otherwise.

NOTE: EPS must be $> 10^{-8}$.

2) DX, DPX: These are displacement of the "off axis" particles needed to compute ν_r and ν_z according to Floquets's method. Usually

$$DX = DPX = 0.0001.$$

#9 (Con't.)

3. RID, PRID: These are the initial guesses for R_{eq} and PR_{eq} for the first particle

(E = E1).

If

RID = 0.0

it is replaced by

RID = β = v/c

4. ØUTEQ: This may have one of two values

= TRUE: Details of 2) above printed out.

= FALSE: No output.

LOCATION: EQØRB

#10 Starting Conditions (FINDEQ = TRUE)

Parameters of Interest

DRIA1, DPRIA1, DRIA, DPRIA, NLINES, ZIA, PZIA, VERSTA, KMAX.

If FINDEQ = FALSE, these are ignored.

If FINDEQ = TRUE:

I) Suppose an E.Ø. has been found, with R = REQ, PR = PREQ. Then if KMAX > 1, a group of particles will be tracked.

1a) NLINES = 0: (KMAX -1) particles will be followed - see Figure 10.1.

1b) NLINES > 0: (KMAX - 1) * NLINES particles will be tracked. See Figure 10.2.

#10 (Con't.)

1c) NLINES < 0: (KMAX - 1) * NLINES particles will be tracked. See Figure 10.3.

2. VERSTA: may have one of two values

= FALSE: Each particle described in 1) has vertical coordinates

$Z = ZIA1$

$PZ = PZIA1$

= TRUE: Vertical stability will be investigated:

IRMAX = 1

Each particle is run twice, once with

$(Z_1)_o = 1$

$(PZ_1)_o = 0$

and once with

$(Z_2)_o = 0$

$(PZ_2)_o = 1$

and then the trace of the Z-matrix is computed

$TRACE = (Z_1)_f + (PZ_2)_f$

II) If KMAX = 1, only the E.Ø. particle is tracked, with

$Z = ZIA1$

$PZ = PZIA1$

LOCATION: 3CØRB, ØRBCAL

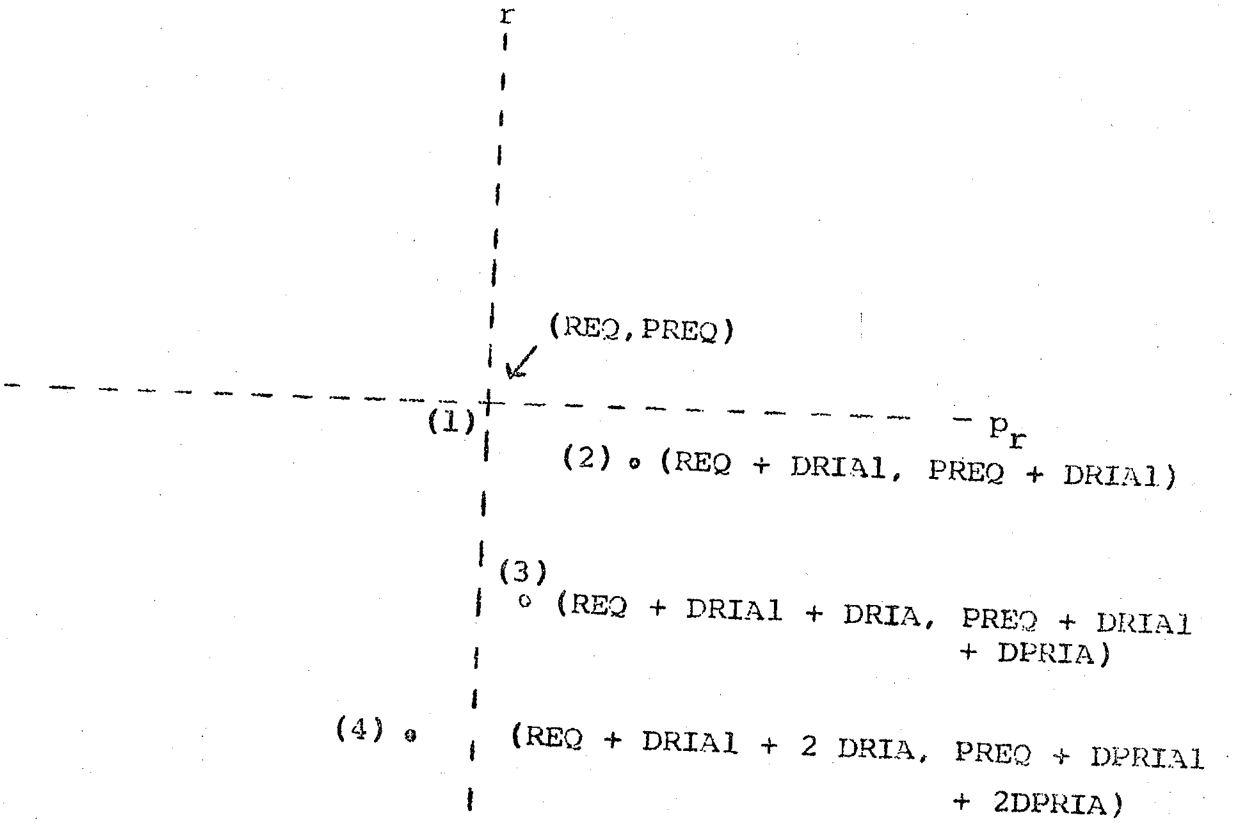


FIGURE 10.1

KMAX = 4

NLINES = 0

DRIA1 < 0

DPRIA1 > 0

DRIA < 0

DPRIA < 0

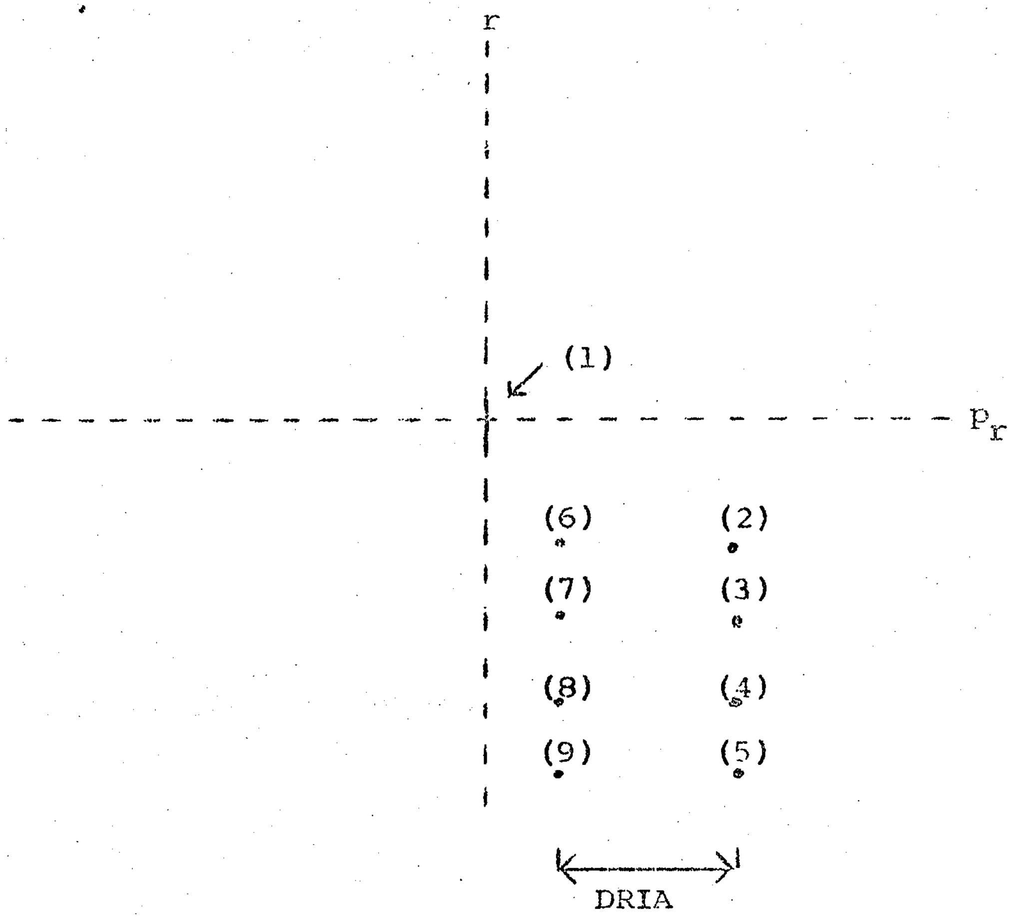


FIGURE 10.2

KMAX = 5

NLINES = 2

DRIA1 < 0

DPRIA1 > 0

DRIA < 0

DPRIA < 0

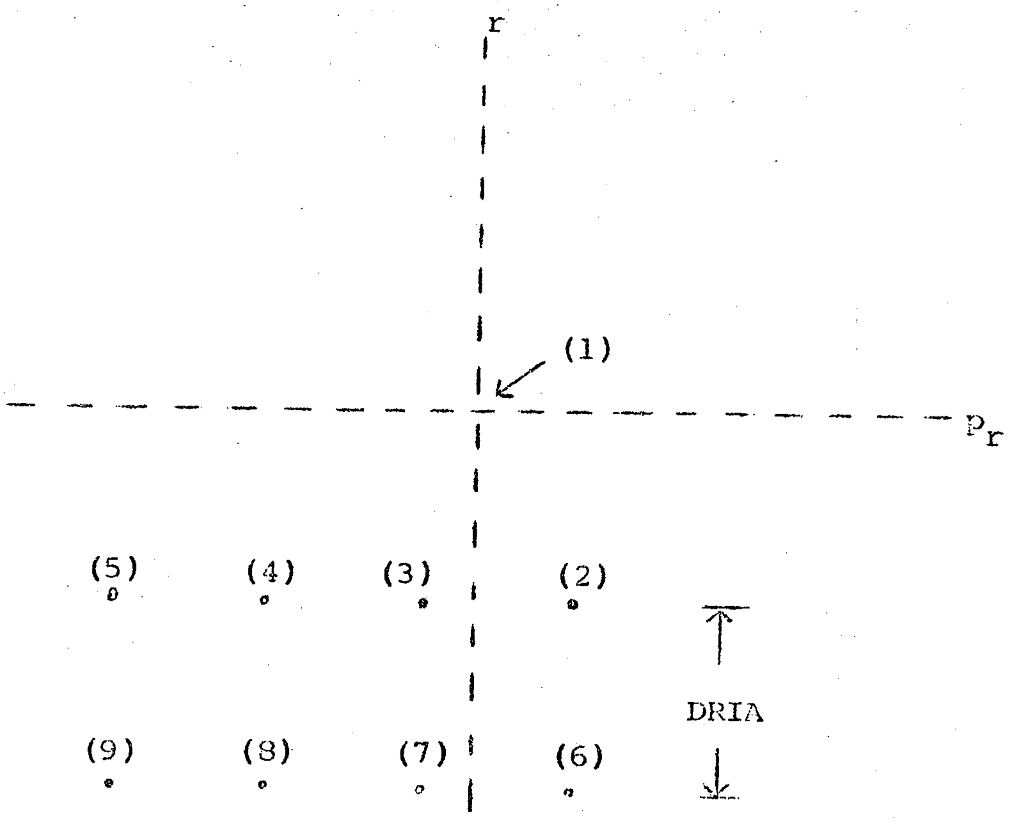


FIGURE 10.3

KMAX = 5

NLINES = -2

DRIA1 < 0

DPRIA1 > 0

DRIA < 0

DPRIA < 0

#11 Starting Conditions (FINDEQ = FALSE)

Parameters of Interest

RIA, PRIA, ZIA, PZIA, KMAX

The particles described by the KMAX particles

RIA_1, $PRIA_1$, ZIA_1, $PZIA_1$

.
.
.

RIA_{KMAX}, etc. will be followed ($KMAX \leq 50$)

NOTE: KKMAXU and KKMAXD modifies this also, so usually

KKMAXU = 1

KKMAXD = 0

The card bearing RIA, etc. must not be in the deck

if FINDEQ = TRUE.

APPENDIX V

A GUIDE TO THE PREPARATION OF DATA INPUT FOR "CBEJ"

A GUIDE TO PREPARATION OF DATA INPUT FOR CBEJ

The variables used must be in standard cgs units. All angles are in degrees. Scale factors may be used to facilitate preparation of the input. For example, R0 may be read in in inches but SCAFAC(1) must be 2.54 to convert it to cm. To study a new particle, a card with a zero in column two is inserted and following that, the new particle is defined starting with the "R0" card. Any variables not redefined will be assumed to be constant. The variables are defined consistent with figure 46.

CARD #	VARIABLE(S)	FORMAT	COMMENTS
1	NUM	I2	number of particles tracked
2	NR	I3	number of radii in magnetic field grid
3	NA	I3	number of azimuths in magnetic field grid
"4"	B(I,J)	1X,10F7.4	magnetic field data in gauss for a grid*-
5	SFACT	F10.0	scale factor for magnetic field
6	1,R0,SCAFAC(1)	I2,2E20.8	r_0
7	2,AZM0,SCAFAC(2)	"	θ_0
8	3,AZM00,SCAFAC(3)	"	initial azimuth of field grid
9	4,PHI0,SCAFAC(4)	"	p_r/p
10	5,AM0,SCAFAC(5)	"	rest mass
11	6,VC,SCAFAC(6)	"	velocity of light
12	7,ACHARG,SCAFAC(7)	"	charge of particle
13	8,E0,SCAFAC(8)	"	energy of particle
14	9,DS,SCAFAC(9)	"	integration step size
15	10,DR,SCAFAC(10)	"	radial separation of grid points
16	11,DA,SCAFAC(11)	"	azimuthal separation of grid points

* the magnetic field is read in starting from the smallest radius and cycling through all azimuths for that radius.

CARD#	VARIABLES	FORMAT	COMMENTS
17	12,RMIN,SCAFAC(12)	I2,2E20.8	min. radius of grid
18	13,RMAX,SCAFAC(13)	"	max. radius of grid
19	14,Z0,SCAFAC(14)	"	initial z
20	15,THZ0,SCAFAC(15)	"	initial 0_z
21	16,I0UT,SCAFAC(16)	"	ion optic matrix printed out every I0UT steps
22	17,IMAX,SCAFAC(17)	"	max. # of int. steps
23	0	I2	last card

END OF THESIS