LONGSHORE TRANSPORT IN THE SOUTH BASIN OF LAKE WINNIPEG

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ABSTRACT

In the past six years, the shoreline of the Southern Basin of Lake Winnipeg has been receding at an alarming rate, causing property loss to local dwellers and cottage owners. It is the objective of this study to evaluate the mechanism causing this sudden recession of shoreline.

A quick beach survey was conducted during the summer of 1970. It records the beach profiles and sand sizes of beaches in the Southern Basin. A resurvey of the lines previously surveyed by Veldman at Winnipeg Beach in 1968 showed that the quantity of sand loss from the area was not significant.

The wave spectrum approach was used to compute wave energy. The combination of Liley's atmospheric pressure spectrum, Miles-Phillips' wave generation mechanism, and a wave refraction model yields a wave generation model which can be suitably applied to the Southern Basin. Wave energy distribution along the shoreline of Southern Basin was then computed for the year 1968 for wind data recorded at Gimli Meteorological Station. (Plates II and IV).

The underlying cause of the sudden recession of shoreline is not the 10 percent increase of wave energy due to the increase in lake depth by about 5 feet, rather the fact that the waves can now attack areas of the bluff which were not reached by the waves before lake stage increase. The tendency of the beach to return to the equilibrium beach profile, and to a lesser extent, the capacity of the littoral current to transport sediment, efficiently disperse the material eroded from the bluff and allow the waves to continue eroding inland. The process continues until an equilibrium beach profile is again established. The extent of erosion is inversely proportional to the slope of the equilibrium beach profile. Using the values of beach slope measured in the survey, estimates were made on the limit of shoreline recession. It is felt that shoreline recession can be predicted in qualitative terms if these estimates, in conjunction with the estimates of beach erosion calculated from the rates of longshore transport, were used with caution, provided that all the physical characteristics of the beaches are taken into consideration.

ACKNOWLEDGEMENT

In my research work for this thesis, I received valuable assistance from a number of sources. I wish to thank the Parks Branch for their help in the survey program, the Department of Highways for the permission to use their echo-sounder, the Water Resources Commission for their assistance in analyzing beach sand samples, and the Prairie Weather Centre for providing me with meteorological data.

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To my parents, I am thankful for their constant encouragement and their personal sacrifice which indirectly made it possible for me to undertake my Master's Degree program. To my wife, Jeannie, I am most grateful not only for her valuable technical aid in computation and the typing and retyping of the draft copies of the thesis, but also for her patience, tolerance, encouragement and understanding which were totally essential to my concentration at work while she spent long evenings in solitude.

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CHAPTER I

INTRODUCTION

1.1 GENERAL

Lake Winnipeg is located in the middle of southern Manitoba. It is roughly bounded by Longitude 96° W and Longitude 98° W, Latitude 50.5° N and Latitude 54° N. It is composed of a north basin and a south basin interconnected by the Narrows. Throughout this thesis these basins will be referred to as the Northern Basin and the Southern Basin respectively. The lakes are unique in that for lakes of their size, they are extremely shallow. (See Fig. 1-1).

The present study concerns only the Southern Basin. This basin measures about thirty-five to forty miles in the north-south direction and twenty miles wide. The Red River drains into the lake from the south, and the Winnipeg River from the east. For convenience, the west coast of the basin between Drunken Point in the north and San Souci in the south will be denoted as the West Coast in this thesis. Similarly, the coastline between Victoria Beach and Patricia Beach in the east will be denoted as the East Coast. The present study investigates only the coastal processes along these two reaches of coast.

The Southern Basin is presently used mainly for recreation. Rural settlements such as the towns of Matlock, Winnipeg Beach and Gimli are located along the West Coast of the lake. One of the chief industries in this area is tourism. Summer cottages crowd along both the East Coast and West Coast, concentrating at locations such as Matlock, Ponemah, Winnipeg Beach, Balsam Bay, Grand Marais and

Victoria Beach. Attractive bathing beaches include Grand Beach, beaches at Victoria Beach, Patricia Beach, Winnipeg Beach and Loni Beach. The majority of cottage owners and bathers come from the City of Winnipeg about forty miles south of the lake.

1.2 STATEMENT OF PROBLEM

Since 1965, the shoreline of Southern Basin started to recede very rapidly. In the summer of 1970, during casual surveys, local property owners around the lake reported land losses varying from 20 feet to over 100 feet of beach front. Losses of 50 feet to 70 feet were not uncommon. A number of cottages have already collapsed into the water, with many others presently being seriously threatened by more recession of the shoreline in the coming years. Furthermore, the phenomenon of property loss to the lake is not a localized one - that is, erosion is reported all along the coast; there are, as yet, few or no reports, verbal or written, of widespread and "significant" beach front increases due to sand accumulation. If there is, in fact, any noticable accretion, it is small in magnitude compared with the erosion taking place at adjacent beaches.

Evidence of erosion such as flooded tree areas and chewed-out bluffs are well documented in Veldman's thesis (1968). It does not appear that the rate of erosion has decreased for the last few years. A large number of cottage owners have undertaken individual efforts to construct beach protection structures along the shore.

Coincidental with the drastic recession of shoreline is the rise of lake level. From a historic mean elevation of 713 feet above sea level, it reached about 717 feet above sea level in 1970. There is the possibility that this increase in mean lake level is the underlying cause of the observed phenomenon of shore erosion.

This thesis sets out to investigate this erosion phenomenon. The study is particularly relevant to the Lake Winnipeg Regulation Program which is a hydro-electric power generation scheme presently being seriously considered by the authorities concerned.

1.3 SCOPE OF STUDY

The following three questions form the basis of this study:

(i) What will be the extent of erosion or shoreline recession?

(ii) What, roughly, is the pattern of sand movement in Southern Basin? And relatively, what is the rate of transport?

(iii) At present, what should be the philosophy of beach protection (while the lake level is still high)?

As far as possible, the findings of the study will be presented in quantitative terms.

1.4 APPROACH OF STUDY

1.4.1 Possible Causes of Shoreline Recession

There are two major coastal processes responsible for shoreline recession.

(A) Erosion mechanisms associated with longshore sand transport due to waves. Longshore transport, or littoral transport, as it is called, is well known to students of coastal process. It is the common erosional process causing shoreline recession, particularly on the sea coasts. Studies investigating longshore transport are too numerous to mention. The Bibliography lists some of these (especially Reference No. 135). Coastal engineers studying Lake Winnipeg in the past have also indicated longshore transport as the culprit of land loss.

(B) Effects of increase in lake level. The effects of longshore sand transport and lake level rise on the geomorphology of Southern Basin shoreline will be studied in detail.

The following sections briefly review some of the past studies and the methodology of studying these two mechanisms.

1.4.2 Longshore Transport - Previous Works

The number of studies conducted on longshore transport is high; only those in connection with studies on Lake Winnipeg are mentioned here.

(i) Veldman (1968) evaluated wave energy at different coastal locations of the Southern Basin. He also constructed a laboratory model of Winnipeg Beach. By setting up the proper wave conditions, he studied the modes of longshore movement of beach sand in the area.

(ii) Prior to 1970, Parks Branch had dumped the following quantities of sand at Winnipeg Beach, about 1,000 feet to 2,000 feet off-shore of the breakwater:

1968	June	704 cu. yd.
1968	Sept.	1,000 cu. yd.
1969	March	6,490 cu. yd.

From beach survey results, which will be covered in detail in Section 3.3, it is apparent that the sand was quickly dispersed from the dump area.

It can be concluded that there has been no in-depth study on longshore transport in Southern Basin.

1.4.3 Methods of Evaluating Longshore Transport

Longshore movement of sand could be evaluated using one or more of the approaches below.

(i) Field survey - by checking the beach profile of a beach location constantly. A change in profile will indicate the volume of sand eroded or accumulated during the time lapse. Although the extent of erosion or accretion may not be directly correlated to longshore transport, it is an index of either change in sand transport capacity (for a period of time or between updrift and downdrift beaches) or change in availability of sand supply. It is a reliable method of keeping track of beach history. Survey of beaches was started during the summer of 1970 and will likely be kept up in future years.

(ii) Model studies can be used to simulate sand transport. One difficulty here is relating transport in the model scale to prototype scale because the model time scale of sand movement cannot be evaluated, (Savage, 1962). Nevertheless, using results of studies conducted elsewhere, such as at Santa Barbara (after Wiegel, 1964), this obstacle may be overcome. In any case, qualitative and comparative results are always guaranteed. Model study was also initiated for this study, but was not completed due to time limitations. The results are not reported.

(iii) Using tracer sand to study beach sand movement is a relatively new technique in North America. It is documented by Ingle (1966). Despite Ingle's emphasis on the economy of tracer sand technique, it is doubtful that this approach is practical for the present study. To evaluate the amount of sand moved in one year, not one, but a large number of tests have to be conducted, under all weather conditions.

(iv) The installation of a wave recording gauge is another practical but comparatively expensive means of quantitatively studying coastal processes. If wave properties can be recorded, established relationships (Inman and Komar, 1968) are available for evaluating longshore transport. Because of shortage of funds, a wave gauge was not available.

(v) Theoretical Approach - In using a theoretical approach the wave energy is first evaluated. For this study the wave spectrum approach (rather than significant wave approach) will be used. There are certain advantages in using the wave spectrum and they will be discussed in the appropriate sections. Wave energy is then used to calculate "theoretical sand transport capacities" using longshore sand transport formulae. The results are for qualitative comparisons only and are rewarding to the investigation even for this purpose. Theoretical evaluation of longshore transport forms a large part of the work of this thesis.

1.4.4 Effect of Lake Level Increase

Per Bruun (1962) derived a formula for evaluating shore erosion due to sea level increase. Application of this formula to lake Winnipeg yields erosion values which are much too high compared to actual reported values. Per Bruun's model is applicable only to sea coasts under long term sea level rise. It appears that the mechanism due to sudden water level increase in a lake would be rather different. Particular attention is paid to this point in the present study.

1.5 ACCURACY

In adopting the spectrum approach of wave generating it is expected that errors may arise from a number of sources. They may be due to assumptions and limitations in the use of certain hydrometeorological or hydrodynamic formulae. They will be noted where they occur in the development of the study. The objective is to arrive at answers which yield values by no more than half an order of magnitude and preferably less than a factor of 2. As long as the errors are consistent, the results computed will at least give a good comparative picture, provided the answers are derived from valid considerations of theories.

1.6 OUTLINE OF THIS REPORT

The characteristic physical and limnological features of Southern Basin and its coasts, including a brief review of Pleistocene geomorphology, are discussed in Chapter II. This chapter provides a general background knowledge of the lake from a physical point of view. Chapter III describes the beach survey for Southern Basin, conducted in the summer of 1970. Chapter IV brings in the theory of wave generation and describes how it is adapted for generating shallow water waves for Lake Winnipeg. Wind data is then modified before being used as input data to the wave generation model as described in Chapter V. Chapter VI gives a detailed account on how the theoretical model is used to calculate wave energy for the year 1968 for various locations of Southern Basin. Chapters VII and VIII are the two important chapters. Chapter VII describes how, using the wave energy calculated, one can derive the longshore transport rate, along the coast, and where to expect erosion or accretion of beach. However, it particularly warns against placing too much confidence in these theoretical results. Chapter VIII is devoted to explaining the mechanism of the equilibrium beach restoring tendency of a rising lake (-- not longshore transport) is the actual cause of abrupt shoreline recession and shore erosion. Estimates of shoreline recession are also made. Chapter IX is reserved for miscellaneous topics that need to be reported: model studies and existing beach protection structures encountered in the Southern Basin are reported. Finally, Chapter X summarizes and concludes the study with some thoughts on further studies that are urgently needed.

This report is set up with diagrams attached to the end of the Chapter or Appendix to which they are referred. Appendices are included to provide a background reference for explaining some of the topics mentioned or the concepts used in the main context. Five larger plates are to precede the bibliography which precedes the Appendices.

LAKE WINNIPEG DRAINAGE BASIN

CHAPTER II

SOME CHARACTERISTICS OF THE SOUTHERN BASIN OF LAKE WINNIPEG

2.1 INTRODUCTION

Studies of coastal processes necessarily involve examination of the behaviour not only of the water phase of the lake itself but also of the land (or coastal) phase. It is the interaction of the sculpturing power of the waves and the resistance of the coast that determines the forms and sizes of coastal features. In this chapter the physical characteristics of the coastal zones of the Southern Basin of Lake Winnipeg will first be discussed. The geomorphic development and the coastal geology of Southern Basin will be described, to be followed by a brief discussion on some limnological aspects of the lake.

2.2 UNDERLYING BEDROCKS

The present Lake Winnipeg lies in the centre of the former glacial Lake Agassiz. It has been modified by direct ice action for a long period of time (at least during the Pleistocene Epoch). Therefore, the surface deposits of the areas around Lake Winnipeg are characteristic of continental glaciation. The bedrock formation of this area will first be discussed.

The bedrock boundaries underlying surface deposits of Southern Basin and its surrounding areas generally run in the north-south direction. (See Figure 2-1). Three types of bedrocks are distinguished: Precambrian rock, Winnipeg Formation and Red River Formation. These latter two formations were formed in the earlier periods of the Ordovician Epoch. The (igneous) Precambrian bedrock extends the area east of the Winnipeg River Delta. West of the delta is a narrow band of bedrock of the Winnipeg Formation whose westerly limit does not extend into Southern Basin except at Victoria Beach and Elk Island. Otherwise, the bedrock underlying most of the shores of Southern Basin is the Red River Formation.

As will be explained in the paragraphs to follow, the bedrocks in this region seldom outcrop above the surface glacial deposits. The Winnipeg Formation, which is chiefly sandstone, outcrops at Elk Island and also on the east shore of Victoria Beach. On the East Coast the Red River Formation bedrock (mainly of dolomite and limestone) outcrops subaqueaceously (that is, below water) offshore of Grand Beach as well as about a mile north of it, (Solohub, 1968). Outcroppings of these bedrocks are not common along the coasts of Southern Beach, (at least not in the regions covered in this study) and therefore have little direct influence on the general coastal processes and the morphological developments of the coastal features.

However, the glacial deposits, which were derived from bedrocks, now form most of the coasts of the lake and indirectly affect the coastal processes. For instance, large size rocks and boulders, whose parent rock is probably the igneous Precambrian rock to the east of the lake, have been transported to Southern Basin by glaciers. A lot of them can now be found at headlands along the coast, with their presence serving as wave energy dissipators, protecting the headlands behind them from undergoing rapid erosion.

Also, on the East Coast, south of Elk Island, the surface deposits being derived from the Winnipeg Formation bedrock are mainly sandy and silty moraines. Because of the presence of these sandy soil materials there is an abundance of sand being transported along the East Coast (as compared to the West Coast) in the littoral zones. As a consequence, clean sandy beaches are often found where this sand movement slows down, resulting in deposition. Examples are the Albert Beach area, Patricia Beach and Grand Beach. In order to be able to appreciate the influence of these physical features on coastal processes, it is necessary to briefly review the recent history (the last 10,000 years or so) of the Lake Winnipeg area.

2.3 BRIEF REVIEW OF PLEISTOCENE EVENTS

During the last ice age, continental glaciers extended into the Lake Winnipeg area from two principal directions: the Keewatin Centre from northeast and the Patricia Centre from northwest, in the respective time sequence. The tracks of the Patricia glaciers are still left on the topography of the areas around the lake, as flutings running in northwest-southeast direction. On the East Coast, tills of calcareous origin are common and are likely derived from the Ordovician formations to the west.

The location of the Lake as it now lies may have been an area of lower elevation in pre-Pleistocene times. The topography is that of a trough running in the general direction of north-south. Thus the glacier spreading over Western Canada from a predominantly northern direction found this trough a convenient entrance to the south and gouged the area to a more notable depression. (From lecture notes: Dr. R. Newbury, University of Manitoba).

During the later stages of glaciation, Lake Winnipeg was part of the Lake Agassiz when ice fronts receded to the north and the area came under water. Abandoned beaches of sand and gravel lying along the 800-foot contour of both the West Coast and the East Coast marked the coastline at one time (see Figs. 2-2 and 2-3); the water level was in the proximity of this elevation (disregarding post-glacial uplift effect) for a relatively long time. The areas above this elevation are now composed mainly of glacial till deposits, but below it, glacial lacustrine (lake) deposits appear to have subdued the till terrain.

Before proceeding to the next subsection, it is to be noted that the purpose of the present thesis is not to analyze and question the origin of interesting physical features of the area, but to point out their existence so that their influence on the coastal processes of the region may be more fully appreciated.

2.4.1 Physiology of the West Coast

The coastal region adjacent to the west shore of the lake is relatively low and gently sloped or flat. The 800-foot contour is usually far inland from the present water-line. As described above, the belt of coastal zone bound by this contour and the water-line is composed of glacial till subdued by lacustrine deposits of clay. This belt is rather gentle and wide, except for the rounded spur north of the town of Gimli. It is also to be noticed that the 725-foot contour encroaches very closely to the shoreline both at this spur and in the neighbourhood of Sandy Hook, north of Winnipeg Beach and south of Willow Point. Consequently, the bluffs are higher in these reaches than along other parts of the West Coast. The significance of high bluffs is that they tend to slow down shoreline recession as will be further discussed later.

Another interesting point is the location of large boulders along the coast. Naturally occurring boulders are likely derived from sub-layers of glacial tills. Occurring subaqueaceously or semi-subaquaceously, these boulders (some of them in the order of five to ten feet in diameter) appear to be concentrated at the headlands or protruding points of the shoreline. This pattern of distribution appears to provide some degree of protection to the headland behind them. At the same time man-deposited boulders are grossly abundant along the shorelines of both the East Coast and West Coast due to a recent "all out" effort to protect the shorelines. No doubt these boulders also fulfil the role of shore protection to some extent. Their significance on the evolution of shoreline will be mentioned elsewhere.

2.4.2 Physiology of the East Coast

The region of study on the east shore of the lake, that is, from Patricia Beach in the south to Victoria Beach in the north, including the west shore of Traverse Bay, is totally different from the West Coast in terms of soil properties and land topography. In fact, the terrain between the water's edge and about eight miles inshore is a complex of glacial till (morainic), fluvial and lacustrine deposits. It is part of the line that delimits the advance and stagnation of an ancient glacial lobe at one time. The massive till deposits that produce the high local relief are mainly of end-morainic origin. As mentioned earlier, the tills are much more sandy and silty than those on the West Coast, as well as possessing more stones and large boulders. Near the 800-foot contour (and usually also above it) the occurrence of sand and gravel marks the elevation of the ancient lake stage. Below this elevation, the till is muffled by a surface layer of lacustrine or fluvial deposits, varying from sand and gravel to clay and peat, in varying thickness.

From south to north, the general description of coastal bluffs for East Coast are as follows. The jut-land at the south-western end of Patricia Beach is part of a large till deposit located inland. From Beaconia to the township of Balsam Bay; the bluff varying from 12 to 15 feet is mainly glacial till overlain with sandy outwash. The till is generally silty, and becomes somewhat stoney at certain locations between the two points. North of Balsam Bay the soil tends to be more clayey and stoney. At Grand Marais, the bluffs are generally stratified deposits of sandy till. The bluffs vary in height from about eight to ten feet at Grand Marais to the greater height of Grand Marais Point. For the reach between north of Grand Beach and the headland north of Iron Wood Point, perusal of airphotos indicates that it is similar to the Beaconia-Balsam Bay reach. The bluffs on Victoria Beach are of a clean sandy-silt deposit, in the order of 15 to 20 feet high on the west side and 50 to 60 feet high on the east side of the island. This high cliff is the outcropping of the Ordovician sandstone of the Winnipeg Formation.

As mentioned earlier, at Grand Beach, Patricia Beach, Hillside Beach and Albert Beach, heavy sand accumulations have occurred, so that sand bars or beaches extend far out into the water, sometimes enclosing lagoons behind them. Sandy bluffs are usually found on these beaches and they may be quite high where exposed to wind action, such as at Grand Beach and Patricia Beach.

2.5 SOIL GRAIN SIZES

No detailed survey of bluff soils has yet been conducted. The information supplied by the Manitoba Soil Survey (Pratt, et al, 1961 and Smith, et al, 1967; Ref. Nos. 104 and 124) only concerns the top few feet of soil of the coastal areas and does not provide a complete picture of the soil properties of the bluffs. Nevertheless, the breakdown of soil types given by this source will at least indicate the relative distribution of bluff soil sizes. As will become apparent later, bluff soil size and bluff height are important considerations in evaluating shoreline recessions.

The soil size analysis of the "C" horizon (generally about 3 to 4 feet below ground surface) as given by Ref. Nos. 104 and 124, are indicated in Fig. 2-4. In this figure, the values given by Veldman (1968) have been revised.

2.6 CHARACTERISTIC LIMNOLOGICAL FEATURES OF SOUTHERN BASIN

Lake Winnipeg is to be distinguished from other lakes of its size by its shallowness. With a normal lake level of 713 feet above sea level, the deepest part of Southern Basin is no more than 40 feet deep. Moreover, the lake bottom is also very flat and even. Unlike other lakes, such as the Great Lakes, which continue to increase in depth towards some central part of the lake, a cross-section of Southern Basin is rather like a shallow even bottom basin. (See Fig. 2-5). It can be noticed that the increase in depth is abrupt only near the shore, with the exception of the south shore. It is suspected that the gentle lake bottom gradient in the southern end of the basin is due to accumulation of sediment derived from the Red River Basin as well as from the erosional processes of the lake which have been going on slowly but steadily since the last continental glacier disappeared.

Because of its shallowness, certain unique properties are expected to follow. Heat energy storage capacity per unit of surface area is lower in Southern Basin than in most other lakes. Consequently, it responds to climatic temperature changes very quickly. In winter it is completely ice covered. Summer thermal stratification probably is not well established when compared to deeper lakes. In this regard, the lake is probably more readily subject to wind mixing during certain times of the year, especially in spring and early summer, as well as in the fall.

Also, values of eddy viscosity and diffusivity are likely quite different from values for other lakes. In all likelihood, the lake is well mixed. For this reason, the probability of finding dissolved oxygen et cetera in all parts of the lake, as compared to other lakes, is higher. This, along with other factors, has significant influence on the water chemistry and bio-lifeforms of the lake.

The shallowness of the lake also influences its circulation pattern. Because of the small area of the lake, it is doubtful that Ekman's elementary current system (or modifications of it) will become well established. In studies conducted in the Great Lakes, it is evident that this current system is much reduced in lakes, when compared to evidence found in the oceans. In the Southern Basin of Lake Winnipeg, the Ekman current system would be even less dominant. So far there is little known of circulation patterns in Lake Winnipeg. It would be rewarding to conduct some mathematical modeling on Lake Winnipeg using existing techniques currently in use, (Liggett, 1970, Lee and Liggett, 1970). Seiches in Lake Winnipeg have been well documented by Einarsson and Lowe, (1968). It is found that the period of seich between East Coast and West Coast is about 1.7 hours, but that in the long direction of the lake it is about five hours. Furthermore, there is resonance between the seich of the Northern Basin and that of the Southern Basin. Wind set-up in the Southern Basin is found to be in the order of 1 meter.

Because of the size of Southern Basin, swells (or decaying waves passing out of the wave generation area and receiving no more energy from wind) are relatively unimportant. Only "seas" of waves (waves that are still receiving energy from wind) are of concern in this study. The presence of swells in the summer and only "seas" in the winter are known to produce summer and winter profiles on the beaches on certain sea coasts. Due to the weak influence of swells on Lake Winnipeg and the fact that the lake is ice covered in winter, there is no differentiation of summer and winter beach profiles as they are recognized in sea coast studies. If there is any significant difference in beach profiles around the year it is due to ice effects: for instance, ice thrusting and jacking in the winter.

In general, Lake Winnipeg, with regard to Southern Basin, is rather unique as a lake mainly because of its size, its shallowness and its uniform lake bottom topography. There is little known about many limnological properties of the lake due to lack of study. It appears that this kind of study is urgently needed. Furthermore, it is felt that parameters that cannot be well defined in larger or deeper lakes, and in lakes that have been studied in more detail, are dominant in Lake Winnipeg and can therefore be defined and studied more easily.

With regard to the present study, the shallowness of the Southern Basin also complicates the simulation of waves for the study of shore erosion. Not much is known about waves generated in moderately shallow water bodies (30-40 feet deep). A good portion of this study investigates this aspect of wave generation and will be discussed in detail in the chapters to follow.

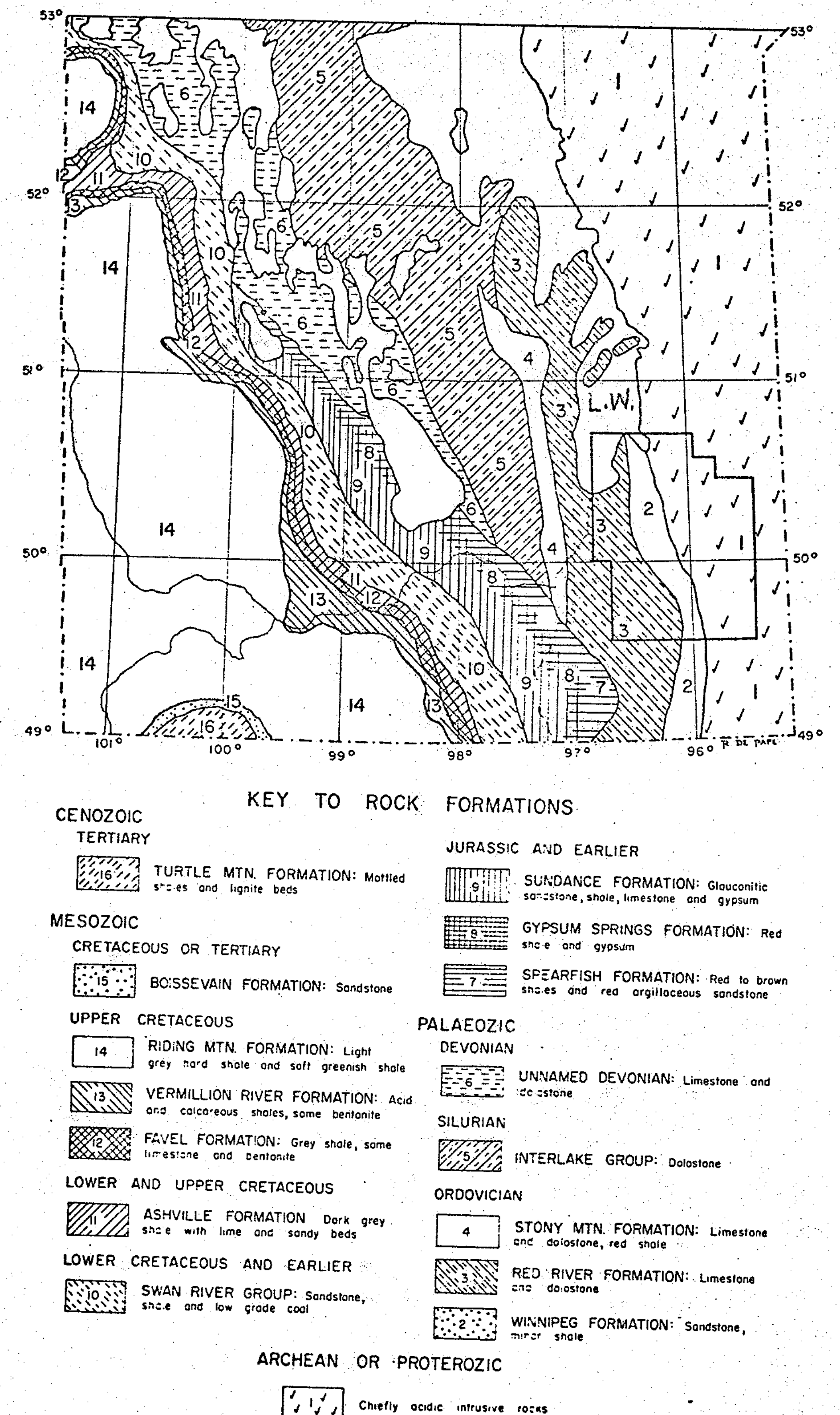


FIGURE 2.1

(After References 104 and 124).

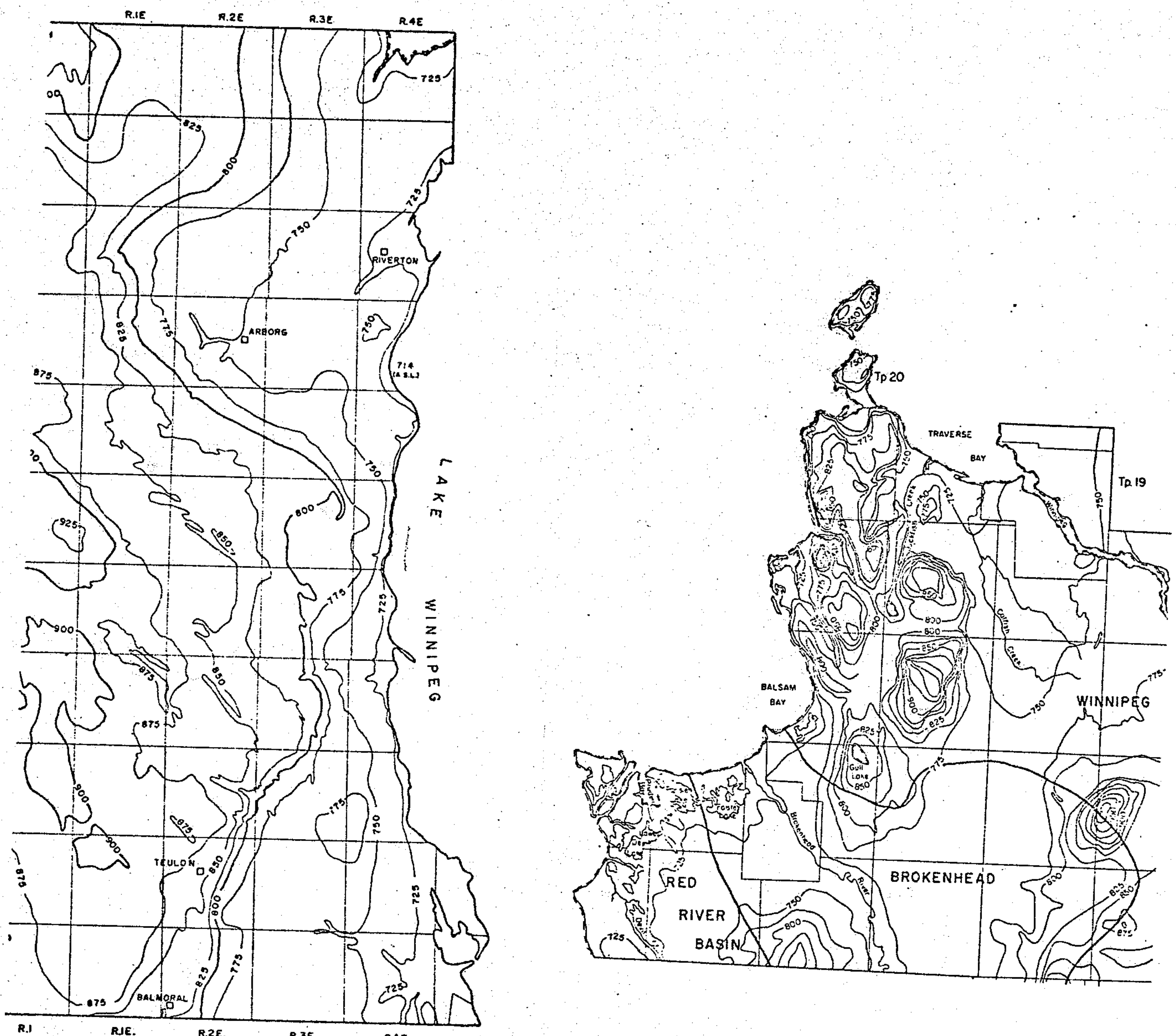
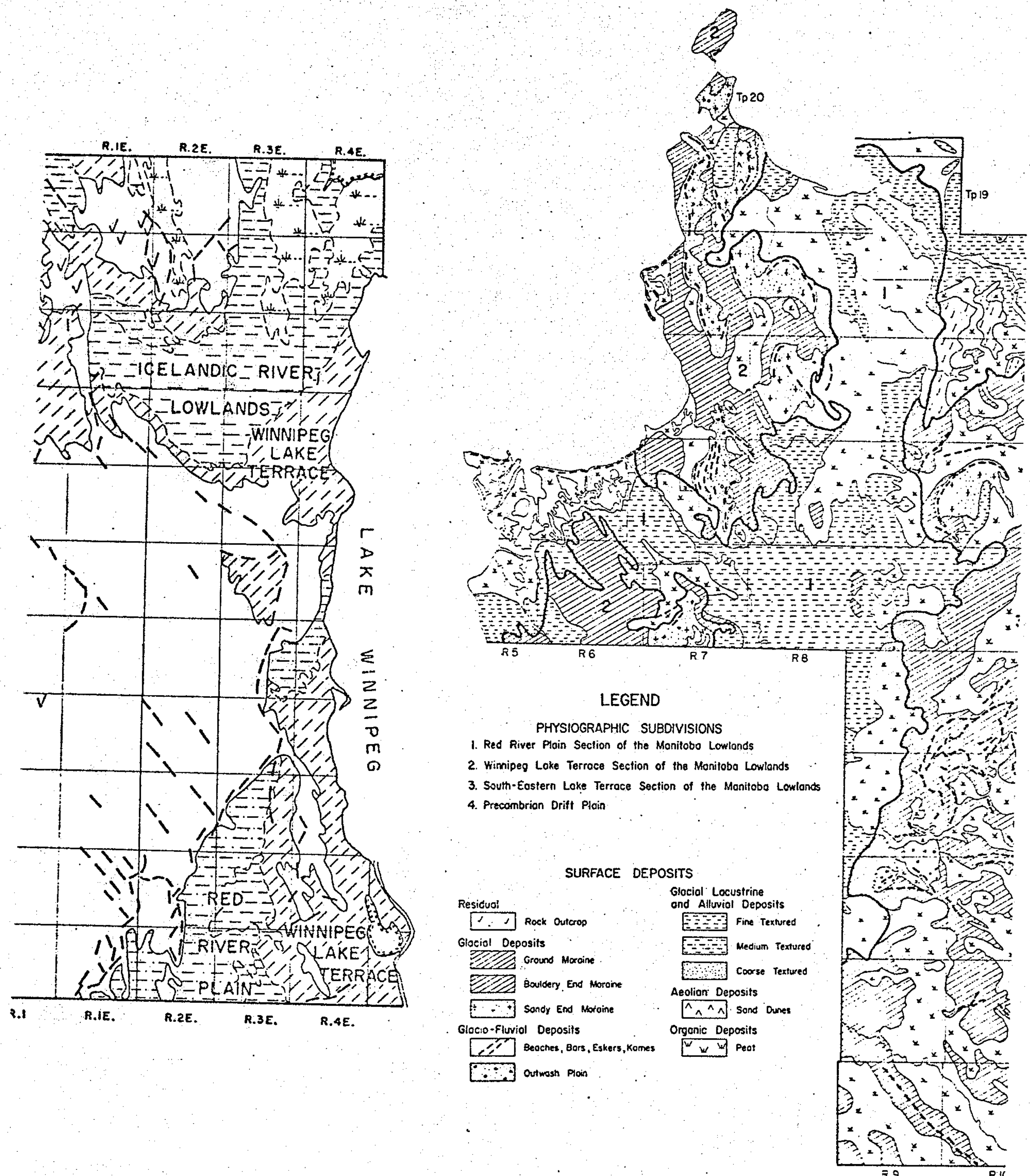
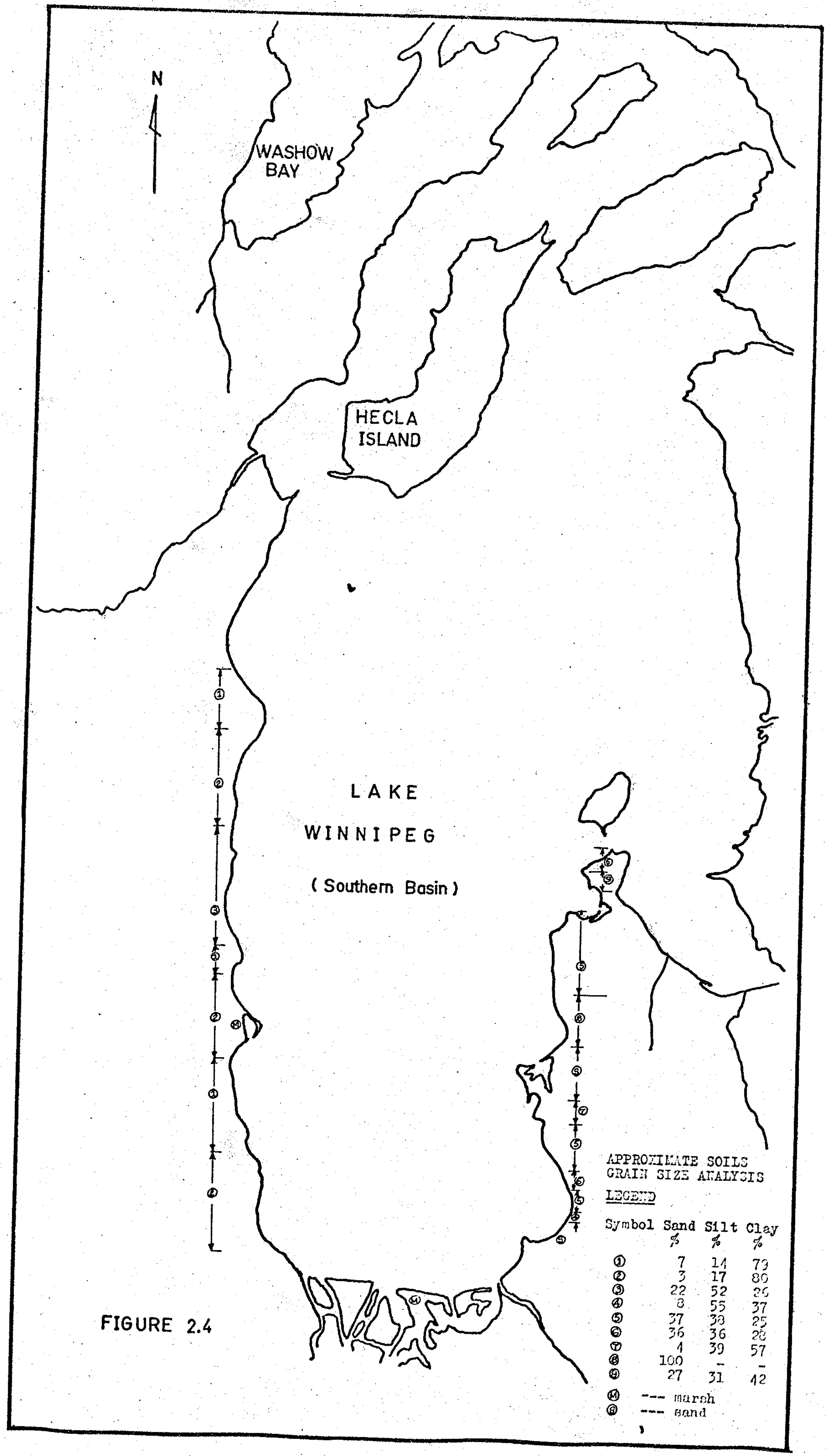


FIGURE 2.2 TOPOGRAPHY OF L. WINNIPEG AREA.

(After References 104 and 124)



<u>FIGURE 2.3</u> GLACIAL GEOLOGY OF L. WINNIPEG AREA.
(After References 104 and 124)



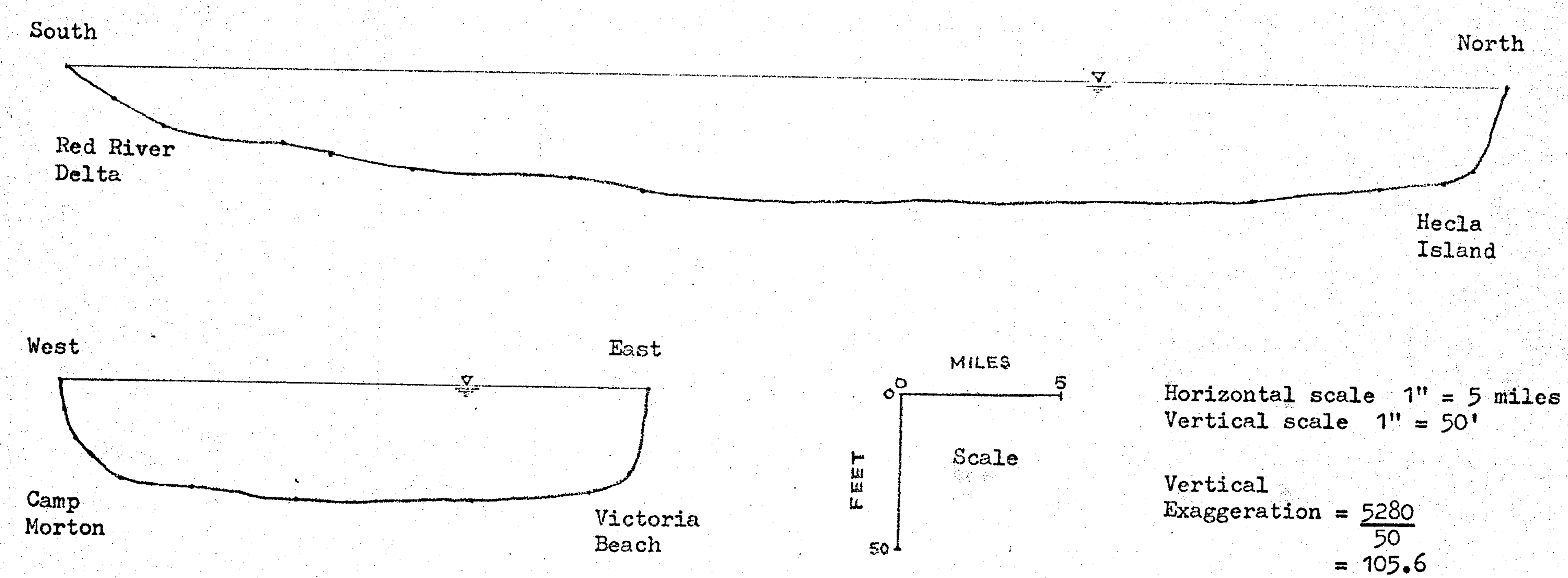
APPROXIMATE SOILS GRAIN SIZE ANALYSIS

LEGEND

Symbol	Sand %	Silt %	Clay %
①	7	14	79
②	3	17	80
③	22	52	26
④	8	55	37
⑤	37	38	25
⑥	36	36	28
⑦	4	39	57
⑧	100	-	-
⑨	27	31	42

Ⓜ --- marsh
Ⓢ --- sand

FIGURE 2.4



Note: Reference Lake Stage = 712.0' above S.L.

FIGURE 2·5 TWO CROSS-SECTIONS OF SOUTHERN BASIN

CHAPTER III

SURVEY OF BEACHES OF SOUTHERN BASIN

3.1 GENERAL

A series of preliminary beach surveys of the Southern Basin of Lake Winnipeg was conducted in the summer of 1970. The surveys were carried out for the following purposes:

(i) To establish a record of beach profiles so that future surveys may reveal erosional and depositional processes along the coastline

(ii) To try to find out from the survey results some characteristics of the beaches which may be helpful in evaluating shoreline recession. It turned out that sand sizes and beach slopes are the major factors influencing the degree of shoreline recession as a consequence of the rise of lake level, as will be discussed in Chapter VIII.

(iii) To record photographically features of the beaches, such that when this record is compared to a similar record taken in the future, it would be possible to discern features of erosion that are not revealed in (i) above. It may be the recent collapse of trees, or the failure of a seawall or other evidence.

(iv) To study the different types of existing protection structures on the coast with regard to their performance using the photographic record in (iii). It is hoped that a study of this nature will lead to better and more economical designs of coastal structures in the future.

3.2 METHOD OF SURVEY

3.2.1 Personnel and Equipment

Since the survey was operated on a limited budget, it was necessary to derive a quick way of surveying. A staff of two members was found to be adequate. Equipment used included: (1) a chest wader (because it was cold in April and May); (2) a hand level; (3) a range-finder; (4) a survey rod and other regular accessories, including stakes and tapes.

3.2.2 Method

A specially designed 10-foot extendable survey rod was used. Fig. 3-1 shows how it is used. The main feature of this rod is that the instrument-man can read off the elevation (from the yellow section) or the depression (from the red section) directly with reference to the ground he is standing on. If he is standing at the edge of the lake, all readings are automatically related to mean lake level on the date of survey. Depth beneath water level can be conveniently read off from a regular scale on the rod. The distances are either measured with the rod when they are short or otherwise read off with the range finder.

Values read off are directly plotted on graph papers prepared before hand. It is felt that by eliminating the intermediate steps of numerical recording, certain sources of human error are thereby eliminated. In particular, the advantage is that the surveyors can instantly realize whether the plotted profile "looks reasonable", so that a line may be resurveyed on the spot if necessary.

Other general information regarding wind ("strong" or "mild", and direction), waves, breakers, and physical features of the beach are also recorded. These parameters are usually only estimated, subjectively.

At least two pictures are taken at each beach location, one looking towards the left and the other towards the right, with reference to a photographer directly facing the lake. Usually more than two pictures are taken, depending on the features of interest on the beach. Coastal Protection structures are usually photographed in detail.

A samd sample is also taken at each beach location. It would be ideal of course, if more samples could be taken at each site since size distribution varies significantly from one point of a beach profile to another; but this would involve much more work in grain size analysis. The sample is usually taken near the water's edge. It is hoped that since a bimodal distribution is anticipated at this location, if the beach material is composed of coarse as well as fine materials (Ingle, 1966), the sample would at least show not only the mean sand size, but also the range of sizes one could expect to find on the beach. Sampling from a different point may fail to indicate the range because of the natural sorting effect of waves. The size of sample is in the order of 300 grams.

During the months of May, June and July, weather permitting, the crew of two was covering the East Coast and the West Coast. Since different parts of the lake were surveyed on different days it should be recognized that the results do not represent conditions of different beaches at the same time - not synoptic, so to speak. The efficiency of the team was commendable. Discounting travelling time from the City of Winnipeg to the lake, and the time spent waiting for poor weather to subside, the average time taken to survey a line was twenty minutes, including time of travel between lines.

5.3 RESULTS OF SURVEY

Altogether, seventy lines were surveyed. Their locations as well as their mean sand sizes and mean foreshore-inshore slopes are presented as in Plate III, (e) and (f) and Plate V, (e) and (f). The following comments pertain to these results.

(i) At some locations the results clearly indicate a bimodal distribution of sand size. The "mean sand size" is only used here for the sake of giving at least a rough idea of the average size of beach sand, although the two characteristic modal sizes are also plotted in the figures. The actual effective size of beach sand may be rather different from the "mean size" value, which is the 50 percent passing size.

(ii) There are locations where two slope values are detected: one extends from the backshore to some depth (say two to three feet) below water; the second slope, which is more gentle continues from where the first one stops to the offshore limit of the survey line. The steeper portion is generally associated with coarser grain size material; the more gentle portion, associated with finer material. Where two such slopes occur, they are both indicated in Plates III and V.

(iii) It can be seen from Plates III and V that there is a definite correlation between sand size and beach slope; the coarser the sand, the steeper the slope and vice versa. This aspect is further discussed in a later chapter regarding equilibrium beach and wave energy (Appendix E and Chapter VIII).

(iv) It was rather unfortunate that due to shortage of labour, sieve analysis could not be performed on all the samples gathered. Instead, samples which were said to "feel" alike were combined and treated as one sample. Because of this, many of the interesting features of sand size distribution along the coastline have been lost.

(v) Because the rodman was dressed in a chest wader during the survey, he was limited from advancing into water deeper than 4 feet. This is a limitation to the survey program because the breakline zone in deeper water is not surveyed.

(vi) A special survey of Winnipeg Beach and Stephen's Point was completed by the Parks Branch of Winnipeg. An echo-sounder was used. The lines surveyed were the same as those used by Veldman in 1968. A comparison of results is shown in Fig. 3-3.

3.4 ANALYSIS OF DATA

(i) Fig. 3-2 is a reproduction of Wiegel's Fig. 14-17 (1964) in log-log plot. It shows sand size and foreshore slope relationship for typical sea beaches. An attempt was made to plot data from the present survey onto this graph. However, it was found that the data did not conform to the relationship established by Wiegel. Furthermore, the scatter of the plot was so large that no conclusive comments could be drawn except that there is a general trend that coarser particle size corresponds to steeper slope, as mentioned earlier, and that above a certain size, the coarse grain particles seem to maintain a more gentle slope than Wiegel's chart suggests. Four factors may be responsible for the observed discrepancies.

(1) The data on Wiegel's graphs are mainly taken from studies on sea coasts, where beaches are usually more "ideal": being long, straight and with relatively uniform sand sizes. Sand on beaches in the Southern Basin is usually composed of all sizes, including gravels.

(2) Sand samples were taken close to the water's edge during the survey. This is near the surf zone where the beach material assumes a bimodal size distribution. It is rather meaningless and misleading to assume the mean sand size of the beach (foreshore) equal to the mean sand size of the sample. In fact, upon examination of the plotted data (Fig. 3-2) it is found that, whereas some data do fall into the region defined by Wiegel's curves, over half of the data cluster in a different region of the graph representing much coarser grain size. In view of this fact, it is clear that the swash zone material is not compatible with Wiegel's curves, and this is the main reason for the discrepancy. For the purpose of plotting sand size against slope, as Wiegel did, foreshore sand size should be used. This would require the sample to be taken on the beach farther away from the water-line. It is recommended that future surveys should incorporate a more detailed sampling scheme. It would be desirable to have at least three samples taken: one for the fine particles (foreshore), one for the coarser particles and one from the swash zone.

(3) Beaches on Lake Winnipeg are also different from those on the sea coast because of the absence of tide. The tidal zone also forms part of the foreshore on the sea coast. The tidal regime no doubt has a definite influence on the distribution of beach particles.

(4) Wiegel's curves in Fig. 3-2 give only the results of specific cases, all from sea coasts where waves responsible for forming the beach may be different in magnitude and characteristics from waves in the lake. For example waves in Lake Winnipeg are absent in the winter (freeze-over) months, and swells are also not important.

(ii) It is often felt that beaches of Southern Basin are relatively steeper than those on the sea coast. A major reason for this can be easily realized by comparing grain sizes and slopes. Beach materials are coarser on beaches of Southern Basin and thus tend to establish steeper slopes.

(iii) Examination of Fig. 3-3 indicates that although there are some minor changes in the beach profiles of Winnipeg Beach between 1968 and 1970, there is no noticeable accumulation or erosion of sand on the whole. It does not show any significant trend of shore erosion anywhere on the beach notwithstanding reports of severe shoreline recession in the region. This implies that the rate of erosion is slow (at least, not noticeable for over two years) and that if there was noticeable shoreline recession during the two years, it was not a result of beach sand erosion. The relationship between shoreline recession, beach sand erosion, longshore transport and other factors will be described in Chapters VII and VIII.

It should be noticed that the sand dumped by Parks Branch (8,194 cubic yards) in the same period of time would cover the beach (underwater as well as above water areas) by a mere thickness of roughly one inch or less, if spread out evenly. Thus it is not a significant quantity in the analysis and is too small to be measured with confidence in the survey.

3.5 OTHER COMMENTS

A number of the survey lines were located on sites rather atypical of their neighbouring beaches. These include lines that were run adjacent to beach protection structures in the backyards of private cottages. The choice of site depends on many factors, one of which is that the survey line can be conveniently used as a reference for evaluation of beach erosion or deposition by future survey.

It is strongly recommended that these stations should be resurveyed in the future in order to gain better insight into the coastal processes of Lake Winnipeg.

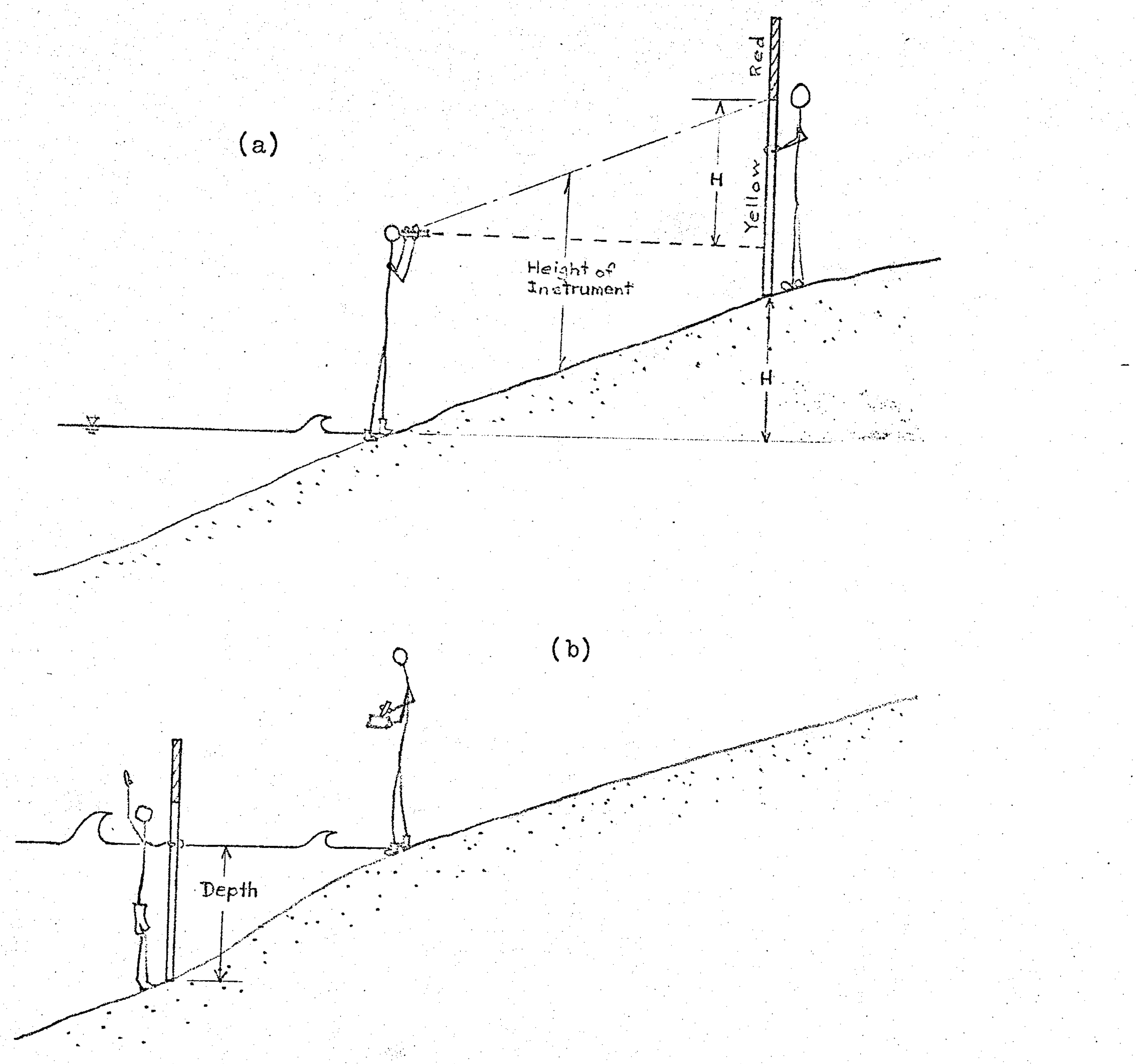


FIGURE 3.1 BEACH SURVEY

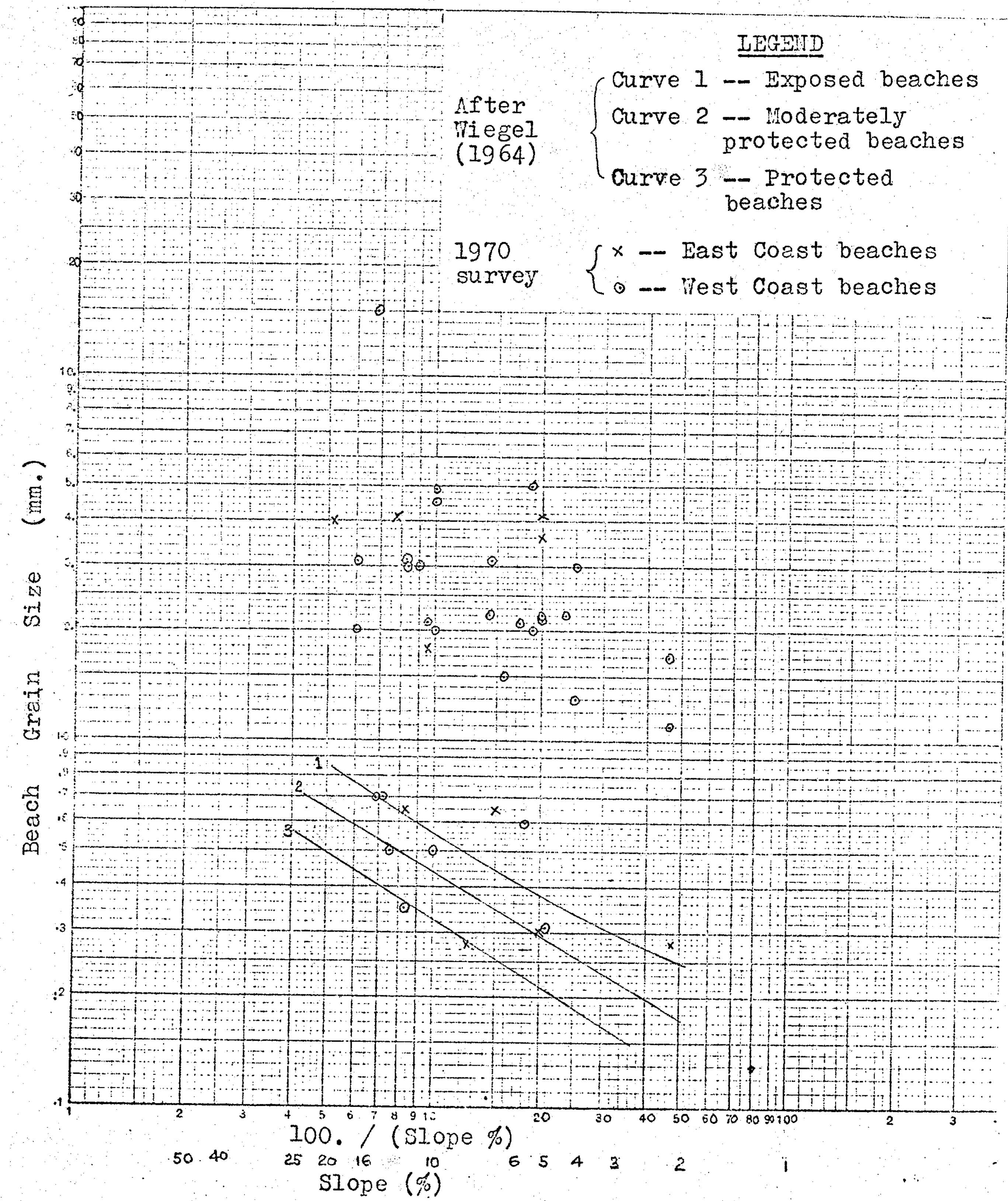


FIGURE 3.2 BEACH SURVEY DATA
- Beach grain size and slope.

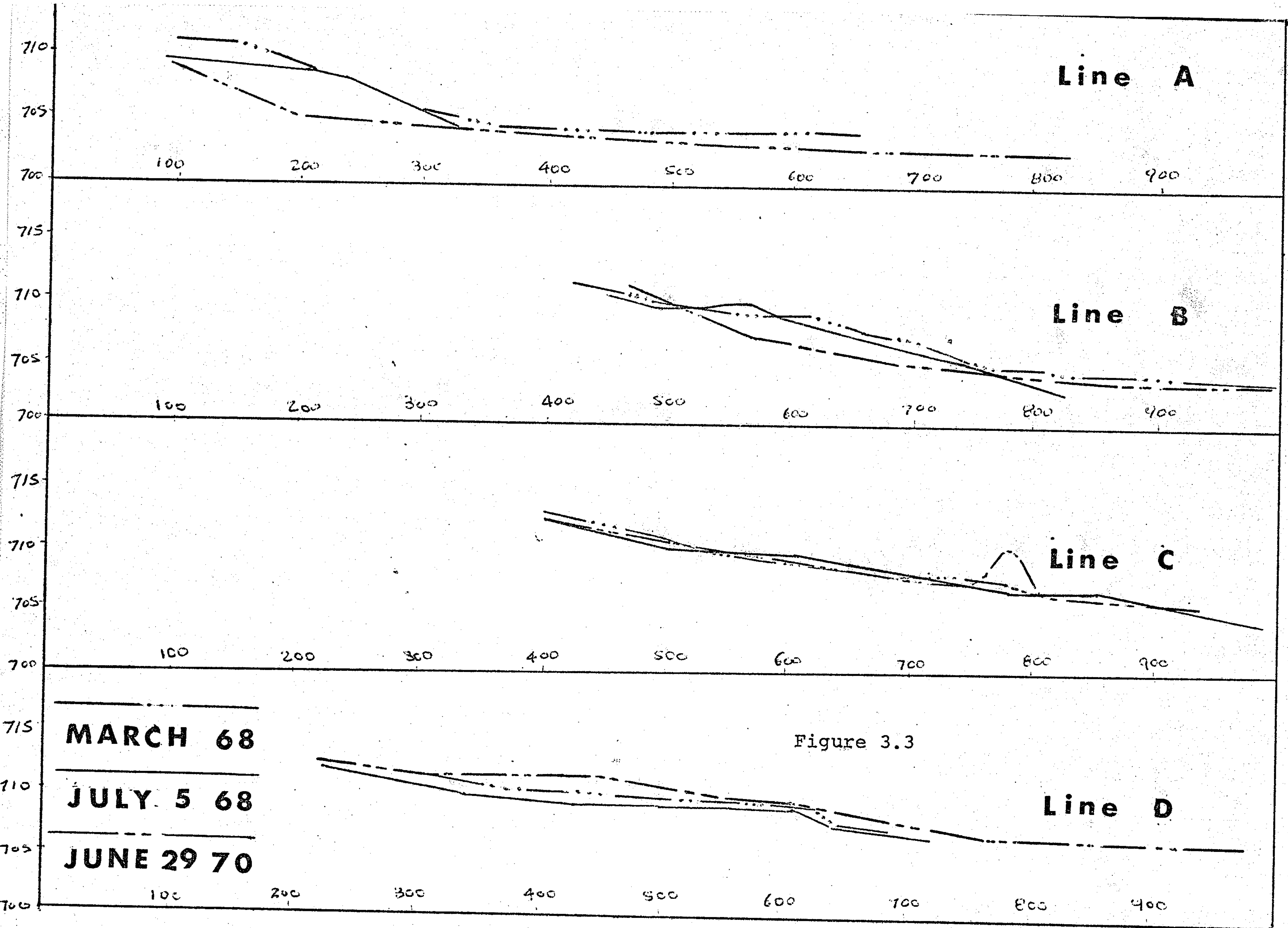


Figure 3.3

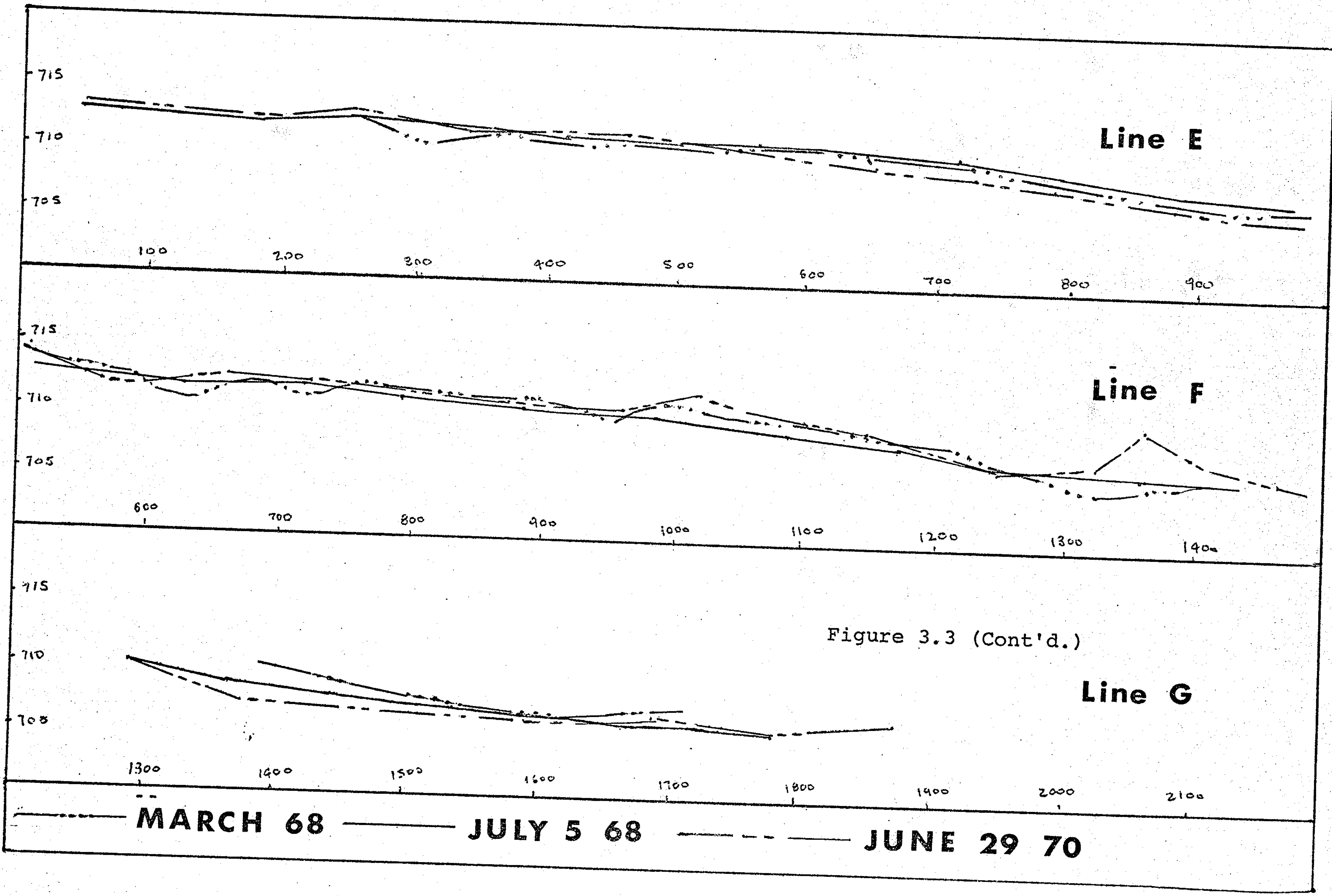


Figure 3.3 (Cont'd.)

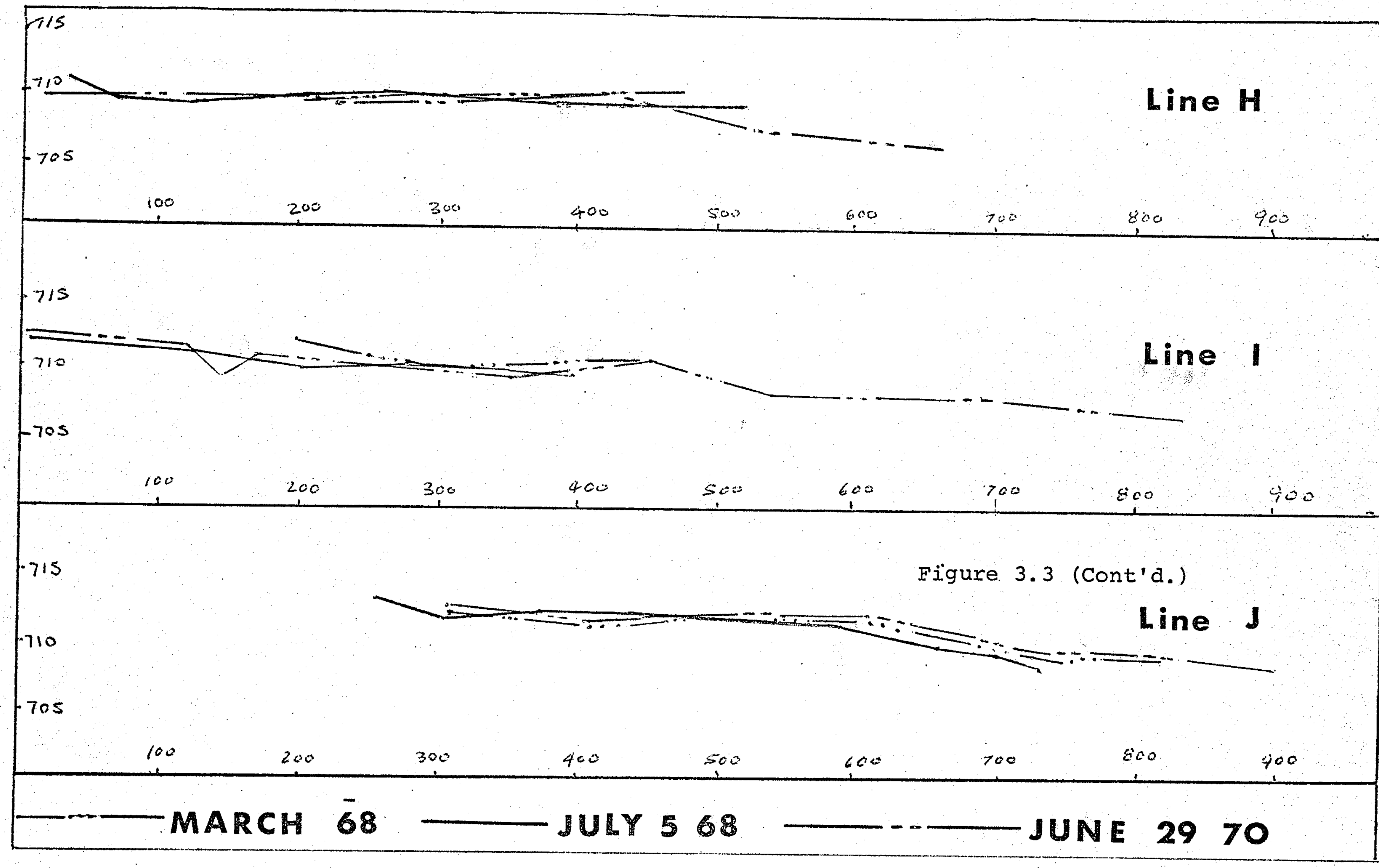


Figure 3.3 (Cont'd.)

CHAPTER IV

GENERATION OF WIND WAVES I: THEORY

4.1 GENERAL

It is common knowledge that wind generates waves on a water surface, but an exact analysis of the mechanism involved is by no means simple. This chapter describes the adaptation of existing theories for evaluating wind wave characteristics of Lake Winnipeg.

The "significant wave" is often used to represent the state of water wave generated by wind. It assumes that the mean of the highest third of the wave height population is representative of the sea of waves in all the properties of wave motion. A large number of important works by outstanding researchers on this topic are available in the literature. Among these are standard texts on oceanographical engineering, such as Ref. No. 29, 56, 91 and 135, and a number of other papers listed in the Bibliography. This thesis does not emphasize the application of significant waves.

In the past decade or so the validity of the significant wave approach has been questioned. Two of the doubts which appear to be obvious are as follows:

(i) In light of the concept of a wave spectrum the significant wave is a rather poor "approximation" for most practical applications. (Inman and Komar, 1968). It is also important to note that a wave spectrum is a realistic and measurable physical property.

(ii) When considering wave refraction in shallow water lakes, or when waves approach the coasts from offshore deep water, as in the case of a sea coast, the significant wave approach fails to give the total effect of bathymetry on wave propagation. (Pierson et al, 1955).

Prediction of a wave generated by wind using the significant wave approach is usually an empirical or semi-empirical process. The most popular method is by the use of fetch graphs (see Wiegel, 1964, pp. 195-247) which are derived from field data. It is difficult to derive an exact, analytical expression (or a number of expressions) which can predict the random and irregular water surface caused by wind when the predicting model does not take into account the random elements. It is not here to discount the virtues of classical wave prediction methods, (indeed they are very useful in many situations), but to indicate that they are valid only when conditions under which they are applied are similar to conditions under which these empirical curves (or expressions) are derived. For most sea coast applications, this requirement is satisfied, because the curves (or expressions) derived for deep water conditions are applied to deep water conditions. For shallow water wave generation, however, the situation is quite different. Due to the shallowness and the variation of lake bottom configuration from lake to lake, empirical relationships derived under a given set of conditions are probably not applicable to another lake. Certain frequency components of the wave spectrum may be excited more than usual because of the shallowness or because some components may be concentrated at, or directed away, from the measuring station due to the differential refraction of wave components.

Moreover, there are only a few documentations of waves in shallow water. The principal contribution is due to Bretschneider at Lake Okeechobee (1954). It is not known how well these results will apply to Lake Winnipeg since Lake Okeechobee is very shallow (four to five feet deep).

In the absence of reliable relationships, wind generation of shallow water waves using the wave spectrum approach is adopted in this study. It is felt that in comparison with the significant wave approach, the wave spectrum approach offers a more exact and realistic analysis. In particular the effects of wave refraction can be accounted for.

4.2 THE SPECTRUM APPROACH OF WAVE GENERATION

The concept of a spectrum and its application is not new - it has been a well known tool in electronics and many other fields. It was pioneered for application to water wave study by Eckart (1952), Pierson, Newmann and James, (1955) and others in the early fifties. The term "wave spectrum" generally refers to the variance of deformation of sea surface about its mean water level, as a function of a wave property. (This wave property can be wave number, wave length, wave period or wave frequency; but wave number is generally preferred.) The wave components referred to are the Fourier components of the random sea surface. (Or, as one may imagine, that a sea surface is actually the summation of an infinite number of components of "simple" wave trains, each of which may differ from the rest in direction of travel, wave velocity, period, amplitude et cetera). (See Fig. A-1). Appendix A through Appendix C provides a more detailed review of the concepts of wave spectrum and the developments of mathematical wave generation models.

The most rational and tested formulation of wave generation to date is the Miles-Phillips' model. Appendix C presents a more detailed description of this mechanism and may be referred to for background material on the theory involved.

The Miles-Phillips' model is as follows:

$$\Phi(k,t) \sim \frac{1}{2\rho\omega^2 c^2} \times F(mt) \times t \times \int_0^\infty \Pi(k,\tau) \times \cos[k(V\cos\alpha - c)\tau]\, d\tau \qquad (4\text{-}1)$$

where Φ is the two-dimensional wave spectrum,

k is the wave number: vector quantity if underlined, scalar otherwise,

t is the duration of energy transfer, given by $t = \text{Fetch}/C_g$.

$$\int_0^\infty \Pi(k,t) \times \cos[k(V\cos\alpha - c)\tau]\, d\tau \qquad (4\text{-}2)$$

is the atmospheric pressure spectrum, in which Π is the "spectral density" and $[\cos\ k(V\cos\alpha - c)\tau]$ is the integral time scale, τ being the time interval variable for integration.

$$F(mt) = \frac{e^{2mt} - 1}{2mt} \qquad (4\text{-}3)$$

Equation (4-3) is the modifying factor given by Miles (1960) where

$$m = (1/2)\zeta kc \qquad (4\text{-}4)$$

in which ζkc is the rate of energy increase per radian such that

$$\zeta kc = \frac{1}{\bar{E}}\frac{\partial \bar{E}}{\partial t}$$

$$= \frac{\rho_a}{\rho_w}\beta\left(\mu\frac{U_1\cos\alpha}{c}\right)^2 - \frac{4g\nu_w}{c^3} - \frac{\rho_a}{\rho_w}\left(\frac{g\nu_a}{2c^3}\right)^{1/2}$$

$$\times\left[1 + 2(\alpha+\beta)\left(\frac{U_1\cos\alpha}{c}\right)^2 + (\alpha^2-\beta^2+2\alpha\beta)\left(\frac{U_1\cos\alpha}{c}\right)^4\right] \qquad (4\text{-}5)$$

where μ is a constant; (after Hino, 1966). β will be defined below and all other notations are according to Appendix C and Appendix H. It is to be noted that the first term on the right of the equation represents positive energy transfer from wind to wave and the second and third terms represent energy loss due to viscous dissipation of water and air respectively during the process, and are generally negligible. In the computations, the second term has been included.

The variable β in Equation 4-5 is given as in Fig. C-2. The mathematical approximation is also given in Appendix C.

The atmospheric pressure spectrum as per Equation 4-2 was given by Phillips in the following form.

$$\Pi(k,0) \times \Theta(k, c\sec\alpha - U_c) \qquad (4\text{-}6)$$

The first term in this expression is the simultaneous, two-dimensional, atmospheric pressure spectrum and $\Theta(k, \frac{c}{\cos\alpha} - U_c)$ is the integral time scale for the moving reference frame. Phillips gave approximate expressions for both terms, but recently, Hino (1966) derived a more appropriate expression such that

$$\int_0^{\infty} \Pi(k,\tau) \times \cos\left[k(V\cos\alpha - c)\tau\right] d\tau = \Pi(k,0)\Theta(k, c\sec\alpha - V)$$

where

$$\Theta = \int_0^{\infty} \{\Pi(k,\tau)/\Pi(k,0)\} \cos\left[k(U_c\cos\alpha - c)\tau\right] d\tau$$

$$= \frac{a_0 U_c \cos\alpha}{2k}\left[\frac{1}{(a_0 U_c\cos\alpha)^2 + \{2U_c\cos\alpha - c(k)\}^2} + \frac{1}{(a_0 U_c \cos\alpha)^2 + c^2(k)}\right] \quad (4\text{-}6a)$$

The pressure spectrum $\Pi(k,o)$ is approximated by Equation (37) of Lilley's paper (1960 b). Thus the pressure spectrum is given by

$$\Pi(k,0) = \frac{\rho_a^2 (\overline{w'^2}/U_*^2)(\tau_1\delta_1/U_*)^2}{\pi(\sigma\delta_1)^2} \times \frac{U_*^4 \delta_1^4 (k\cos\alpha)^2 e^{-k^2/(4\sigma^2)}}{\{(k\delta_1)^2 + 2(b\delta_1)(k\delta_1)\}} \quad (4\text{-}7)$$

where the notations are as defined in Appendix H, and the following approximations (compiled by Lilley from Grant, Laufer and others) are to be applied.

$$\sqrt{\frac{\overline{w'^2}}{U_*}} = 0.8\,; \quad \frac{\tau_1\delta_1}{U_*} = 3.7\,; \quad b\delta_1 = .31$$

$$\delta_1\sigma = \frac{1}{2}\,; \quad \delta_1 l_3 = \frac{1}{2} \quad (4\text{-}8)$$

The wave spectrum at any point in the sea is therefore given by the intersection of Equation 4-1 and Phillips' spectrum for the equilibrium range (Phillips, 1958, Phillips and Kartz, 1961) which is as follows:

$$\Phi(\omega) = \lambda g^2 \omega^{-5} \tag{4-9}$$

It is to be noted that $\Phi(\omega)$ is a one-dimensional spectrum and that it is a function of radian frequency only. The equivalent relation in the two-dimensional version is not presented here, but can be found in Ref. No. 44 (Hino, 1966).

4.3 INDETERMINATE TERMS

In generating wind waves, one thus performs the task of finding the intersection of Equation 4-1 and 4-9. However, two terms in these equations must be evaluated by measurement or by other means: U_* , the shear velocity, per Equation (C-25) and (C-26), of Appendix C, and δ_1, the displacement thickness of the atmospheric boundary layer per Equation (4-7). The difficulties presented by these two quantities are described below.

(A) Atmospheric mean wind profile can be given by the well known log-law. (Hinze, 1959; Sutton, 1952).

$$\frac{U_z}{U_*} = \frac{1}{K} \ln \frac{z}{z_0}$$

or

$$\frac{U_z}{U_1} = \frac{z}{z_0} \tag{4-10}$$

such that $U_1 = U_* K$ (4-11)

where k is von Karman's constant, usually taken as 0.4,

z_o is a fictitious "wall roughness thickness" of the ground or sea surface over which the air flows, and U_z is the mean wind velocity as measured at height z above the mean ground or sea level. Over land, z_o is usually known or can be determined empirically by actual measurement of wind profile. Thus, if U_* is known, relationship (4-10) can be used to determine any U_z. But over an ever-changing water surface, such is not the case; no definite measurement of either z_o or U_* has yet been possible. (The difficulties to be encountered in taking such measurements can be imagined: think of the hazards and inaccuracies in taking measurements in a boat on a stormy sea!) Even if such measurements are possible, it must also be remembered that the values of z_o and U_* are not only functions of wind velocity but also that of surface roughness, i.e. the wave conditions of the sea, which is the item to be predicted.

Hino, based on his philosophy that Nature prefers the principle of maximum or minimum, puts forward the hypothesis that the total distribution and partition of energy is such that the atmosphere would adjust itself to transfer the maximum amount of energy to water with minimal energy loss within itself. Thus working "backwards", using Miles-Phillips' model, he was able to arrive at values of U_* for various wind speeds and fetches. Summarized in graphical form, these are given in Fig. 4-1. Since there seems to be little variation due to different fetches, the relationship for a fetch of 10 Km is adopted in the present study. It may be debatable whether the validity of this relationship can be justified; yet, to date, there appears to be none to prove it otherwise. In mathematical terms it is:

$$U_* = [(U_{10} - 5) \times 0.83 / 15.0] + 0.17 \qquad (4\text{-}12)$$

(B) Another atmospheric parameter, δ_1, the displacement thickness of turbulent boundary layer, is also an elusive term to evaluate. By definition, the displacement layer is given by

$$\delta_1 = \frac{1}{U_{ob}} \int_0^{\delta} (U_{ob} - U_z)\, dz \qquad (4\text{-}13)$$

where U_z can be assumed to be given by Equation (4-10), U_{ob} is the mean wind velocity outside the boundary layer, and δ is the boundary layer thickness. Thus, δ_1 is defined by δ or U_{ob}, assuming that the same log-profile (viz. Equation (4-10)) for mean wind is valid up to the limit of the boundary layer. In most experiments where surface roughness is in the order of millimeters, boundary layer thickness is generally in the order of centimeters, (Grant, 1958). But over a sea of waves, where the wave heights are in the order of meters, (thus roughness of the same order) the boundary layer must be remarkably thicker. Moreover, it is probably a function of the free stream wind speed. (It is also a function of the wind speed as measured by meteorological stations at a height of 10 meters above ground). On the other hand, geostrophic wind, which is by definition the atmospheric free stream wind, blows at an altitude of roughly 500 meters or higher. A calculation of this wind velocity (via the equation on page 248, Sutton, 1952) yields a geostrophic wind velocity of roughly 30 m/sec. or higher at this altitude for a recorded wind speed of 10 meters/sec. at land stations. This value for U_{ob} appears to be unrealistically high.

There has been no report of the actual thickness of the boundary layer over Lake Winnipeg. However, by adopting U_* mentioned above and Hino's suggested value of μ = 4.0 for Equation (4-5), and by amplifying the pressure spectrum by four orders of magnitude (10^4), it was found that Miles-Phillips' spectrum yields remarkable agreement with the data collected by Wiegel, (1964) for fetches of 1 Km and longer, for a constant value of δ = 40 m. (See Fig. (4-2)).

A number of the assumptions made above have little theoretical background for justification. It is found that the value of significant wave height thus computed is actually very insensitive to the incorporation of 10^4 in computing spectral density and also rather insensitive to whether δ is 10, or 40, or 60, or 100 meters. It is wise to bear in mind that the objective of this study is not to prove the validity of a theorem, but to generate a reasonable wave spectrum and apply it to evaluate longshore transport phenomena.

There was difficulty in using the same model to fit Wiegel's collected data for fetches of 10 meters and shorter, which Hino so successfully did. In addition, the frequency of the spectral peaks deviate from Hino's result on the dimensionless graph of Fig. (4-3). This second discrepancy particularly indicates that if Hino's computations are correct, then the lower wave number waves (or longer waves) of this model have been over-excited at low wind speed, but under-excited at high wind speed. The error is likely the consequence of the assumption that the turbulence boundary layer is a constant, equalling 40 meters. Again these discrepancies may not be vital enough to render the results of this study invalid. As long as the trend and the relative quantities of error are consistent, (as they are), a failure to generate a precise model is in all likelihood not serious.

4.4 ADAPTATION OF MODEL TO SHALLOW WATER CONDITION

Modification of the spectrum approach for deep water wind-wave generation (deep water condition) for shallow water condition does not appear to be exceedingly complicated, at least as demonstrated by Wiegel (1966).

In generating shallow water waves one must be aware of the following four points:

(i) The characteristics of refracted waves assuming all other quantities to remain constant, are functions of depth and frequency (or period). Frequency (or period) remains approximately constant throughout the refraction of a wave ray. It follows that wave length, L, or the reciprocal quantity, wave number, k, is also a function of depth and wave frequency only.

(ii) According to Equation C-5, Phillips' model of wave excitation for a given set of meteorological conditions is a function of wave number and phase velocity only, which for waves in shallow water, can be adequately defined, (Equation (4-15) below).

(iii) Likewise, according to Equation C-25, Miles' model of wave growth for a given set of meteorological conditions, depends on wave number and phase velocity only.

(iv) The equilibrium range one-dimensional wave spectrum, according to Equation C-28, is a function of water particle acceleration and therefore a function of wave frequency (or period).

Thus, since the wave frequency remains constant regardless of depth (approximately), it is a useful parameter for defining wave conditions. Once frequency (or period) and water depth are defined, all other wave characteristics can be found via the following equations. (See Appendix A).

$$\omega^2 = gk \tanh kd \quad (3\text{-}14)$$

$$c = \frac{\omega}{k} = \frac{gT}{2\pi} \tanh kd = \left[\frac{g}{k} \tanh kd\right] \quad (3\text{-}15)$$

$$L = \frac{2\pi}{k} = \frac{gT^2}{2\pi} \tanh kd \quad (3\text{-}16)$$

$$\frac{c_g}{c} = \frac{1}{2}\left[1 + \frac{2kd}{\sinh 2kd}\right] = n \quad (3\text{-}17)$$

By substituting the appropriate expressions into the corresponding terms of Equations (4-1) through (4-9), the desired shallow wave spectrum can be computed. In this manner, generation of shallow water waves is not much more complicated than that for deep water waves.

4.5 APPLICATION TO LAKE WINNIPEG

The Southern Basin of Lake Winnipeg is considerably shallower than most lakes of its size. The average depth in the central portion of the lake is about 37 feet at a lake level of 712 feet. In applying the wave generation process described above to Southern Basin, the following procedures are carried out.

(I) Assume a given wave period T; thus the frequency is also known.

(II) From a given starting position, a wave ray is constructed with the predominant direction of travel in the direction of wind blow until it reaches shore.

(III) Of course, all wave properties, (wave number, phase velocity, group velocity, ray width, and refraction function) except wave period itself, change as the wave proceeds across the lake, but their weighted mean values are computed for the entire fetch.

These values are to be applied to the fictitious fetch F' such that the total effect is to separate out an idealized wave generation process over a straight, flat bottom fetch, rather than tackling the more complex problem of solving for refraction and wave generation simultaneously. The situation is as illustrated in Fig. 4-4. In this figure, the actual fetch ABC as in Figs. (a) and (b) is approximated by the fictitious, straight and uniform fetch AB'C'.

(IV) For a given wind velocity and direction, the wave spectrum is computed as discussed in Section 4-4 above for fetch F' and the mean values are as calculated in (III) above. This spectrum would be approximately identical to the wave spectra at B' and at B under actual field conditions. The validity of this assumption will be discussed later.

(V) The next assumption is that the spectrum as obtained per (IV) above is already present at location B, Fig. 4-4, and from here it travels to C by path ABC, over a perfectly calm lake with no wind blowing and no energy loss of any kind. It is to be noticed that ABC is the actual wave ray. At C, the value of spectral density is approximately the same as that at B, discussed in (IV), multiplied by the refraction function, as computed from Equations D-5 and D-6:

$$[K^2] = \frac{(C_g)_{AB}}{(C_g)_C} \times \frac{(b)_{AB}}{(b)_C} \quad ; \quad \langle \eta^2 \rangle_C = \langle \eta^2 \rangle_B \times [K^2]_C \tag{4-18}$$

where $(C_g)_{AB}$, $(b)_{AB}$ and $(C_g)_{AB}$, $(b)_{AB}$ are the group velocities and ray widths over fetch AB and at C respectively; $\langle\bar{\eta}^2\rangle_B$ and $\langle\eta^2\rangle_c$ are the mean square elevations at B and C respectively; $[K^2]_c$ is the refraction function at C.

Further discussion on the wave refraction function $[K^2]$ can be found in Appendix D.

(VI) The power due to this spectral density at C is given by

$$P = \text{Wave Power} = \frac{\text{Energy}}{\text{Unit area}} \times (C_g)_c = \rho g \langle\eta^2\rangle_c (C_g)_c$$

$$= \frac{1}{8} \rho g H_{rms}^2 C_g \qquad (4\text{-}19)$$

It should be noted that it is meaningless to find the spectral density at Station C measured at the water-line (the value is nil), because the depth there is zero. As an approximation, the shoreline, which is generally quite close to the 9-foot contour is assumed to have a depth of 9 feet. The choice of 9 feet is rather arbitrary. It is partly based on the observation that the depth at the breakline on Lake Winnipeg beaches is generally of this order of magnitude.

(VII) Steps (I) to (VI) above are repeated for selected values of wave frequency to obtain the corresponding values of spectral density of mean square elevation and of wave power via Equation 4-19. This set of values is then summed up (i.e. integrated to yield the power spectrum for Station C.)

(VIII) Finally, the total energy received by C due to this particular wave ray during a storm is the power obtained per (VII) above, multiplied by the duration of storm. Thus

$$\text{ENERGY} = \text{P} \times \text{DURATION OF STORM} \qquad (4\text{-}20)$$

This relationship is obvious. Further discussion can be found in Appendix A.

(IX) If it had been desired to compute the longshore and normal wave energy components at Station C, steps (I) to (VIII) can be repeated with modifications to Equation 4-19 as follows:

$$\text{Longshore power} = \langle \eta^2 \rangle_c \rho g (c_g)_c \times \cos\alpha \sin\alpha \qquad (4\text{-}19a)$$

$$\text{Normal power} = \langle \eta^2 \rangle_c \rho g (c_g)_c \times \cos^2\alpha \qquad (4\text{-}19b)$$

Where α is the angle the wave ray makes with the normal of the coastline.

4.6 REMARKS AND COMMENTS

Two other important factors must also be considered. Firstly, for the same wave period and the same storm, a point on the coast, C, may receive energy from more than one single wave ray, due to refraction effect. Thus a bundle of wave rays starting from locations close to A must be examined. Likewise, it is also possible that none of a certain wave component would reach point C at all. See Fig. 4-5(a) and (b).

The second factor is that the refraction of a finite number of wave rays of a given frequency across a lake gives rise to the situation as shown in Fig. 4-5(c). Assuming no irregularities in lake bottom configuration, it can be seen that the effects of these wave rays are confined in the shaded areas, leaving a considerable section of coastline not covered by wave action. This, of course, is not true in the real situation; the inadequacy arises only because a finite number of rays could be used for approximation.

There would be no such discrepancy if an infinite number of rays were to be used; but this is of course an impossible situation.

To by-pass this difficulty, it is assumed that the width of a wave ray may be greater than computed (or smaller), such that any location on the coast would receive a full spectrum of wave components unless this location is significantly far away from the closest point of the coast where a wave component would attack. (That is, in Fig. 4-5(c), wave condition at L due to the particular wave component is assumed to be the same at L" or L', whichever is closer). This approximation has the effect of smoothing energy distribution along the coastline. Furthermore, it neglects the situation when more than one wave ray of the same frequency attacks the same beach location. It is felt that this is one drawback of the model. To analytically compute the exact energy distribution function along a coastline would be a very laborious task. It may be desirable therefore, to adopt a simplification which would at worst give a rough indication of the actual energy distribution. As it turns out, despite this simplification, the final results to be detailed later are quite satisfactory and reasonable.

The fictitious fetch F' will approximate the actual fetch (traced by the path) of the refracted wave ray very closely on most occasions. Thus, despite the fact that the resultant error may not be easily evaluated, it is expected to be very small.

In using the one-dimensional spectrum, the fetch is not merely a narrow basin of finite width, small compared to the length, but an area (an approximate half circle) that "fans out" from B' in Fig. 5-4(d), with radial dimensions comparable to the length of the basin itself. So at B' (See Fig. 5-4(d)), the waves received would come from sources deep inland as well. Since this is not possible, it is evident that one-dimensional spectrum does not strictly apply to this analysis. The exact analysis would be to use a two-dimensional spectrum with varying fetches for wave components from different directions. However, the application of one-dimensional spectrum in this case would incur little error because of the following arguments.

It is important to notice that although theoretically, wave components take on directions of travel from 90° to the left to 90° to the right of the mean wind direction, the actual distribution of energy is concentrated in a narrow band of small angular values on both sides of the mean wind direction.

Because of this, one can then view the process as a bundle of wave components travelling in almost similar directions. At B' (Fig. 4-4(b)) the major portion of energy received is from components starting in the close proximity of A. Thus one may expect that this bundle of waves tends to undergo quite similar refraction patterns, and would end up, if not at C, very close to it.

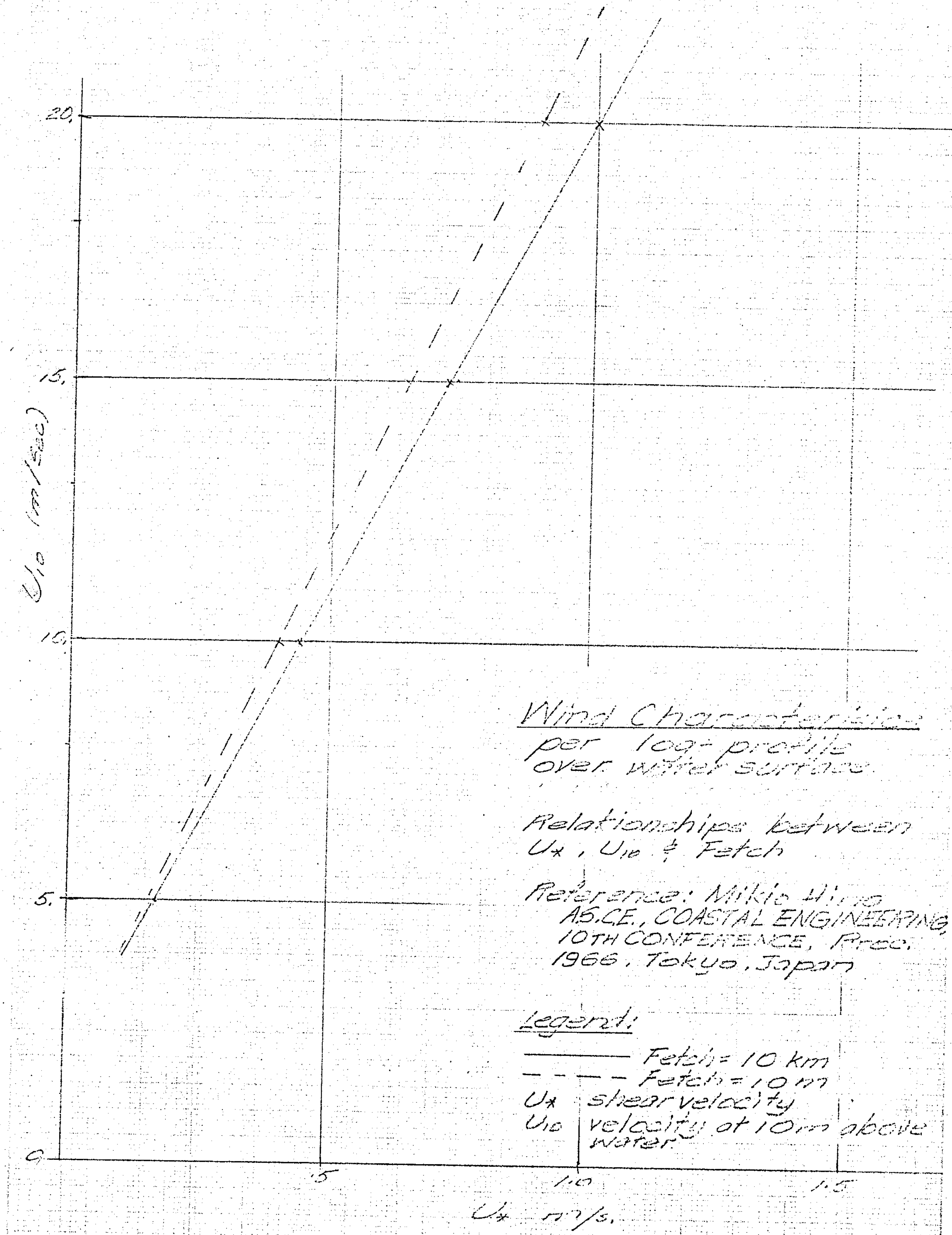
4.7 COMPUTING PROGRAMS

In applying the wave generation model in this study, two computer programs were written for the IBM 360/65 computer in Fortran IV language. They are Programs (A) and (B) as follows.

(i) Program (A) - This is a wave refraction program. It was first designed to compute the refraction of a single wave-ray when given the initial conditions (viz. starting direction and location and wave period) and lake bottom contours. The program also computes mean values of the characteristic properties of the wave ray, such as fetch, depth, wave period, phase velocity, group velocity and so forth. The program is described in detail in Appendix D. At a later stage in the study the program was adapted for generating a large number of wave rays over Southern Basin. The core of the program was not changed. Only certain parts were retouched, (mainly the first part of the main source and the format of the input and output).

(ii) Program (B) - This is a rather straight-forward program for computing wave energy values by the wave spectrum method as described in Sections 4.4 and 4.5. Input information includes wind and wave characteristics. Although the program is adapted for shallow wave generation, it is equally applicable to deep water waves. Integration is performed by the trapezoidal method.

The layout of these two programs, and their relationship is illustrated in Fig. 6-2, with the listings of the programs given in Appendix F. The use of these programs will be further described in Chapter VI.



20.
15.
10.
5.
0
.5
1.0
1.5
U10 (m/sec)
U* m/s.
Wind Characteristics
per log-profile
over water surface.
Relationships between
U*, U10 & Fetch
Reference: Mikio Hino
ASCE, COASTAL ENGINEERING
10TH CONFERENCE, Proc.
1966, Tokyo, Japan
Legend:
Fetch = 10 km
Fetch = 10 m
U* shear velocity
U10 velocity at 10 m above water

FIG. 4-1

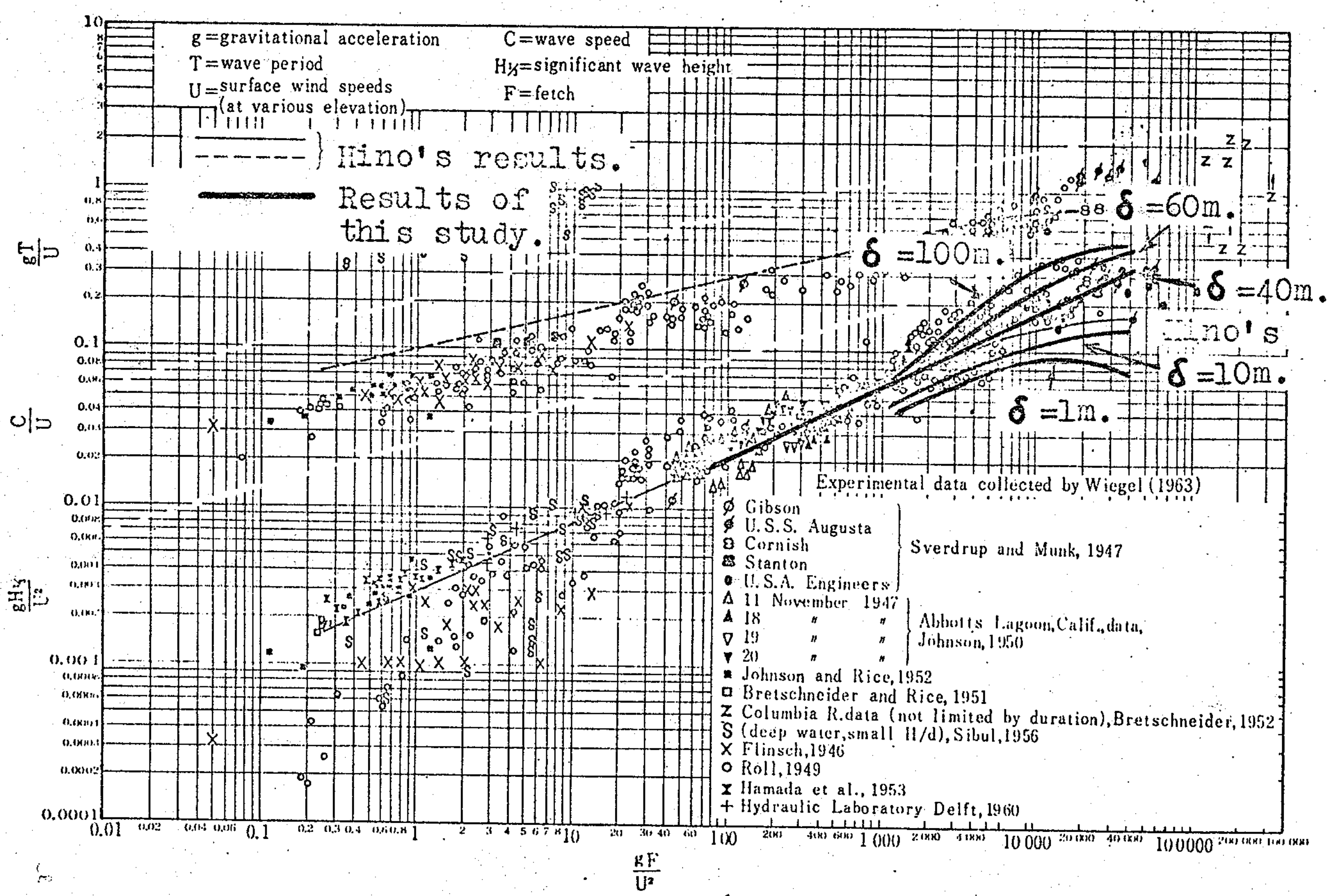


FIGURE 4.2 COMPARISON OF THEORETICAL RESULTS WITH PUBLISHED DATA (BY WIEGEL) ON DEEP WATER WAVE.

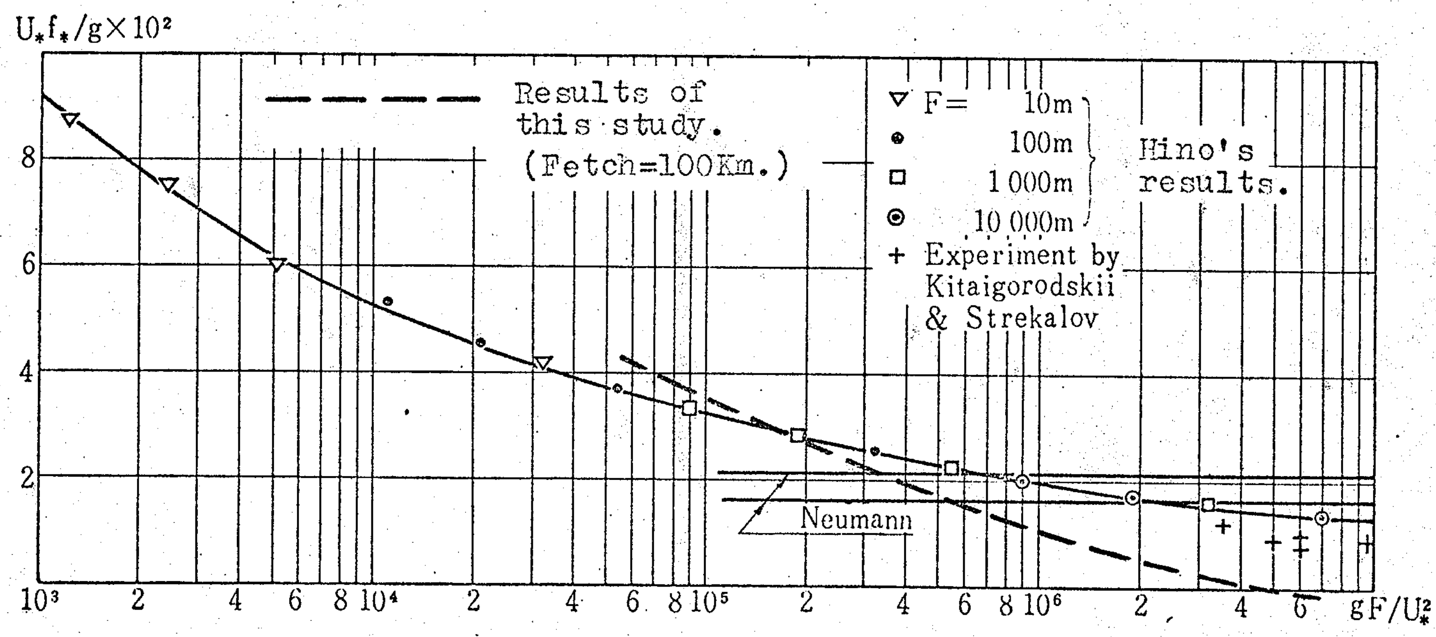


FIGURE 4.3 NON-DIMENSIONAL REPRESENTATION OF THE RELATION BETWEEN THE SPECTRAL PEAK FREQUENCY AND THE FETCH.

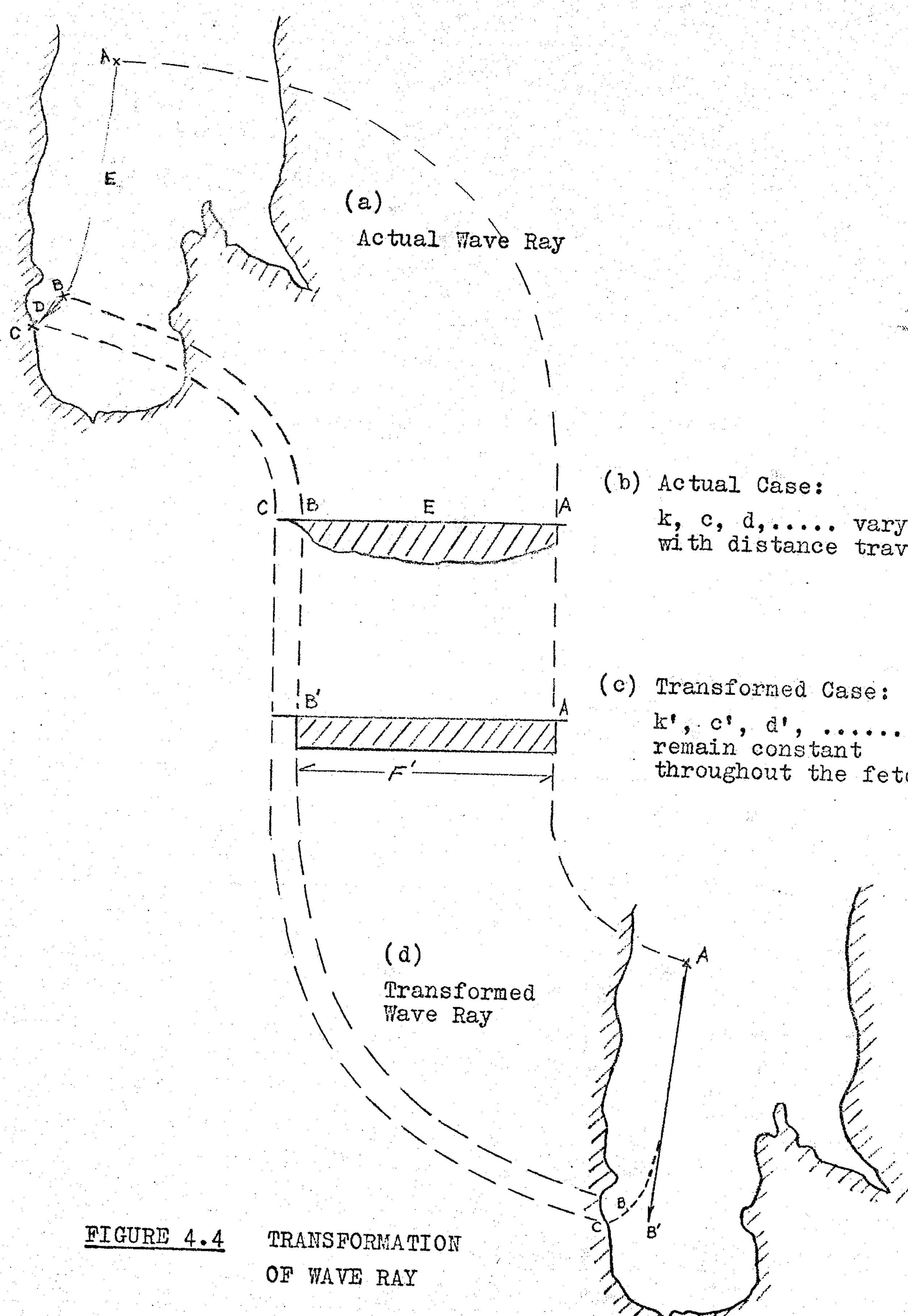


FIGURE 4.4 TRANSFORMATION OF WAVE RAY

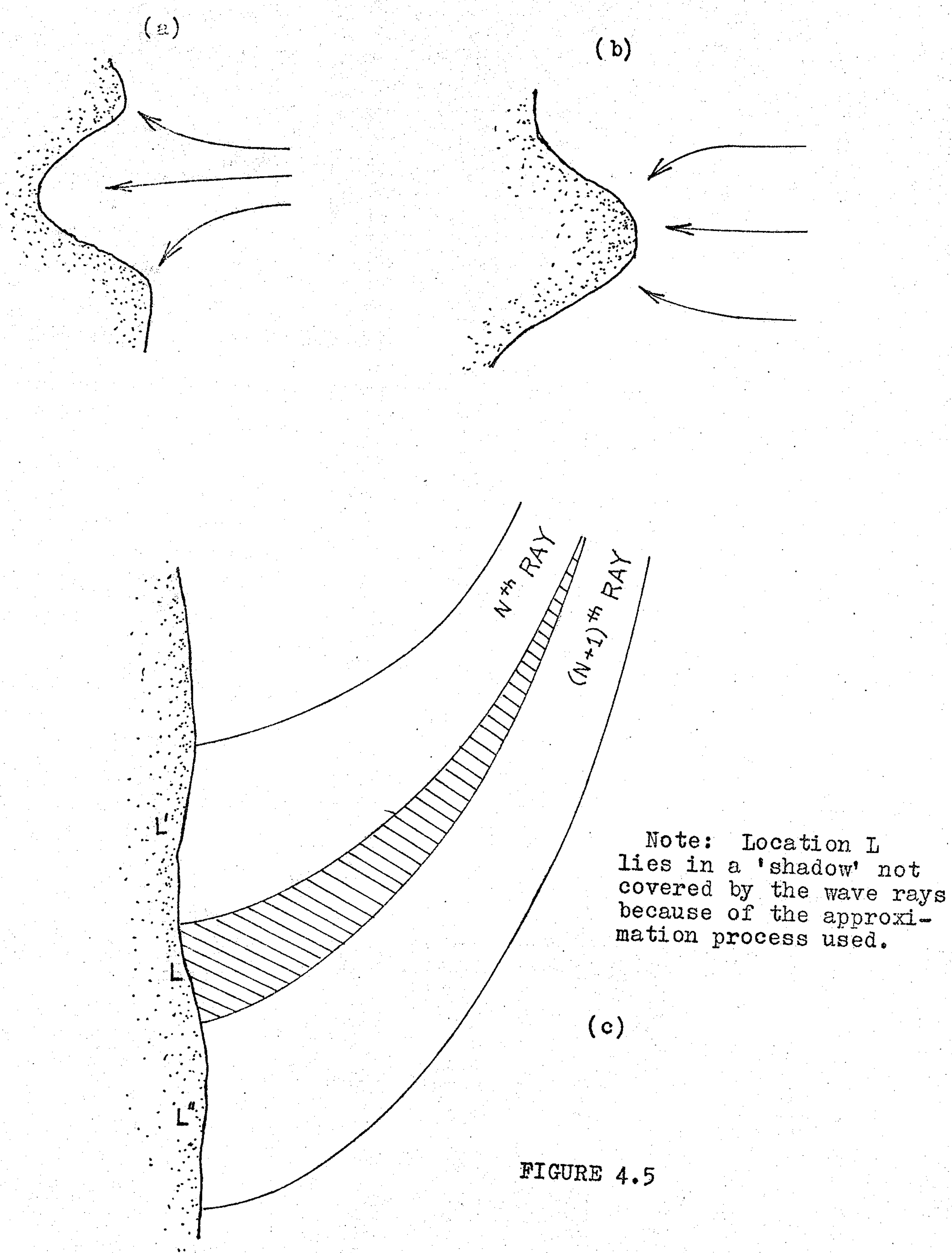

(a)
(b)
N th RAY
(N+1) th RAY
L'
L
L"
(c)
Note: Location L lies in a 'shadow' not covered by the wave rays because of the approximation process used.

FIGURE 4.5

CHAPTER V

ANALYSIS OF WIND DATA

5.1 CHOICE OF WIND RECORDS

For the purpose of wind wave simulation for Southern Basin, wind information is a necessary input. Two sets of hourly wind records are available: one is recorded at the weather station at Gimli, located one mile inland from the shore; the other is recorded on a dredging barge in operation at the Red River Delta at the southern end of Southern Basin.

Veldman (1968) used the barge data for his computations whereas the Gimli record has been used in the present study. It does not appear that one set of wind data is superior to another. It is more important to appreciate the limitations involved in using a certain set of wind data. The choice of a particular record depends on its availability, whether it is in readily usable form and other factors. The following lists some of the factors influencing the choice of the Gimli wind record.

(i) Hourly wind at Gimli Station is recorded consistently all year round.

(ii) Wind at Gimli is recorded at 10 meters above ground (standard elevation) whereas the barge data like all ship data is recorded at 19 meters. The 10 meter record can be readily utilized for the computations involved in this study while the barge record would require necessary corrections.

(iii) It is commonly understood that wind blowing over a valley would likely be modified by a "channelling and funnelling effect". This is probably true of the barge data at the Red River Delta, which is part of a very gently-sloped valley. Thus north and south winds may have been intensified. It can be expected that funnelling also somewhat distorts the Gimli wind record but the distortion will be less. This funnelling effect cannot be readily estimated, but is not expected to be overly significant. The error thus incurred is not as serious as one may anticipate because wind over a significant portion of Southern Basin would also be influenced by this channelling effect, as would wind at Gimli. On the other hand, it would be somewhat incorrect to eliminate all channelling effects from the Gimli record. The important point is that the directional distortion is not serious, constituting the principal reason that the Gimli data is chosen.

5.2 OVERWATER TO OVERLAND WIND RATIOS

It is well known that for a given set of meteorological conditions, wind measurement recorded over a water body such as Lake Winnipeg is generally higher than that measured overland. A commonly quoted wind overwater to wind overland ratio is 1.35; (see Ref. No. 14, Tech. Report No. 4, 1956). In their study of waves for the Great Lakes, Richards and Phillips (1970) used a ratio of 1.66 for annual average. Hunt (1958, after Richards and Phillips, 1970) used a ratio of 1.35 for spring, but 1.82 for fall. Hence it can be seen that it is rather difficult to assign any particular value to wind over-water to wind overland ratio. The ratio varies according to locale; the following factors are influential in this regard.

(a) Resistance to wind due to land topography and vegetation. Clearly a steep and rough coast and/or rough, luxurious vegetation would slow down overland wind.

(b) The size of the lake or sea.

(c) Location of land anemometer. A recorder deep inland would record lower wind speed than one that is located right on the coast.

(d) Similarly, for the location of an overwater recording station, wind speed being higher farther away from the coast.

(e) Meteorological conditions including the type of wind. Obviously regional cyclonic winds would behave differently from winds due to local atmospheric convection.

(f) Wind direction would also likely be influential.

(g) Atmospheric stability is actually the most important factor in considering wind over-water to wind overland ratio. If a warm wind is blowing off-shore, a colder, relatively calm layer of air would remain over the lake so that the lake surface is not extensively affected by wind. This is known as a stable condition and lake wind to land wind ratio would be smaller than otherwise. On the other hand, an unstable situation occurs when air over the lake is warmer than the air over land. Rodgers (1964) in his study of Lake Ontario, used the temperature recorded by a land meteorological station and lake water temperature as indicators of atmospheric stability, and obtained the following relationships:

(1) if $T_{al} - T_w < 0$ $\quad W_w = 0.844\ W_e + 6.97$

(2) if $T_{al} - T_w = 0$ $\quad W_w = 0.594\ W_e + 6.87$

(3) if $T_{al} - T_w > 0$ $\quad W_w = 0.343\ W_e + 6.86$

where T_{al} - air temperature over land, °F

T_w - lake water temperature, °F

W_w - wind over water, mph

W_e - wind over land, mph

Obviously, Rodgers' formulation, which yields good correlations for Lake Ontario, cannot be applied to Lake Winnipeg for reasons mentioned above. Furthermore, water temperature of Lake Winnipeg is not available.

In this study, the barge data would be less affected on the foregoing account. To use the Gimli data, it would, of course, be desirable to use some kind of wind correction factor. However, in the absence of land wind and lake wind studies on Lake Winnipeg, and considering that wind overland to wind overwater ratio in the Great Lakes varies from 1.16 in July to 2.09 in November, (Richards and Phillips, 1970), it is not easy to decide what value should be used for the ratio. It would be erroneous to apply values from other regions since Lake Winnipeg, which has very different lake thermal regime, lakewater circulation patterns, related meteorological regime and coastal topography, is different from other lakes where intensive studies are conducted. With no pertinent information available, the Gimli wind data is applied to wave simulation without modification. It is not felt that this approach is superior to using a doubtful set of wind correction factors, but it is hoped that when such correction factors become available through future studies, results from this study can be corrected by a constant of multiplication or by some simple mathematical procedure. Assuming annual effective wind overwater to wind overland ratio is 1.35, and according to the general concept that wave energy is proportional to the square of wind velocity (after Wiegel, 1964), the computed energy and littoral transport values would have to be corrected by a value of $(1.35)^2 = 2.0$. It must also be noted that Lake Winnipeg is located on a flatter terrain and would conceivably have lower corresponding ratios compared to other lakes.

5.3 DATA ANALYSIS

5.3.1 Concept of Design Wind

In his analysis, Veldman (1968) plotted the frequency curves of annual hourly wind (that is by extracting maximum hourly wind from each year and finding its distribution). With this he predicted wave heights using a wind velocity with a given return period. This is an engineer's normal approach to statistical treatment of raw data, especially in the field of water resources design where annual peak value is an important parameter. When applied to wind data, this approach tends to overlook certain aspects regarding random occurrences of high wind speed. For instance several unusually big storms occurring in one year are only represented by one entry. Furthermore it tends to neglect the significance of storm duration.

Veldman's approach is useful in designing piers and seawalls et cetera where the maximum wave force exerted per service life is to be estimated. Since the present study involves the examination of sediment transport, a process which goes on in more or less the same manner every year, the wind data for a typical year is required. To obtain this, it would be necessary to apply statistical analysis (Blackman and Tukey, 1959, Ref. No. 149) to the wind record of a number of years for extracting a mean distribution of wind from the frequency domain. The process, though apparently straight-forward, could be rather tedious. For the present purpose, no such analysis was attempted. Instead, wind data for 1968 was used as though it were a typical annual wind record of Lake Winnipeg. No detailed study, except for a brief and general comparison, was made to compare the wind record of 1968 with those of other years. It does not appear that the 1968 wind data as recorded at Gimli station is "atypical" in any sense.

5.3.2 Averaging Wind Effects

It would be desirable to compute the state of waves of the lake at the end of each hour of wind blow by applying the wave generation technique, described in Chapter IV, to every hour of the year. In this way, the exact history of lake surface disturbance can be traced. However, such an approach would not be practical because the present study does not demand such a high degree of accuracy. It seems therefore rewarding to reduce the labour of computation by simplifying and reducing the complexity of the wind data to be used as input to the computing model. Averaging is used to filter out an extensive amount of unnecessary detail. A period of wind blow, say of four hours or longer, with hourly wind velocities from the same direction and of roughly the same magnitude is represented by the number of hours and the mean (or slightly higher) wind velocity. For example, Hour 1 to Hour 10 in the set of data listed in the next section is represented by 10 hours of steady wind at 23 mph. More will be said regarding the averaging method in Section 5.4. Using this simple method, the wind record of Gimli Meteorological Station from April 1, 1968 to November 1968 inclusive, is analyzed. The results are summarized in Figures 5-1 to 5-8.

5.4 FETCH CONSIDERATIONS

Wind velocity seldom remains steady with time. For example, in a typical cyclonic storm, it is common to find wind from the north as follows.

Hour	Wind Velocity mph.
1	22
2	21
3	24
4	22
5	28
6	22
7	21
8	20
9	22
10	22
11	10

The wind record indicates that for the first 10 hours, except for Hour 5, wind stays fairly constant at about 22 mph. from the north. There are two different ways in which one can use this piece of data.

(1) Generate waves for 22 mph. wind for 9 hours, (Hour 1 to Hour 4, and Hour 6 to Hour 10) and generate waves for 28 mph. wind for 1 hour (Hour 5).

(2) Or, smooth out the wind data and assume 10 hours (Hour 1 to Hour 10) of wind at velocity of approximately 23 mph. (Since wave growth is not a linear or a simple function of wind speed, (see Chapter IV), there is no need to take the arithmetic mean of the ten wind velocity entries).

It is often convenient but somewhat misleading to think of wind wave generation on a lake such as Lake Winnipeg as a steady-state process. By this misconception, at the end of Hour 4, lake state would switch instantly from a steady-state wave state for 22 mph. to a steady-state wave state for 28 mph. No doubt, this is physically impossible. Upon examination of the results obtained from the wave simulation model, which will be described in the chapters to follow, it is found that waves carrying the largest portion of energy travels at a velocity of approximately 10 mph. With the length of the lake as 35 miles, under a north wind, these waves will take about 3 to 4 hours to reach the south shore. This means that the sea of waves will not attain a steady-state (referred to in the literature as fetch-limited) level until this period of time has elapsed. Thus between the first moment of Hour 1 and the end of Hour 3 wave energy on the south shore continues to increase. In the second half of Hour 4 or thereabout, the steady-state energy for wind speed of 22 mph. is reached, and the energy would remain at this level if the wind had remained constant for the balance of the period.

At the beginning of Hour 5, assuming the wind changed to 28 mph. abruptly, blowing over the 22 mph.-steady-state waves, and adding more energy to it. However, since it takes another 3-4 hours before steady-state for the new wind system can be attained, the south shore or any part of the lake would never experience a 28 mph.-steady-state sea.

At the beginning of Hour 6, the wind switches back to 22 mph. again. The wave state of the lake which is presently at a higher energy level (higher waves) will gradually change back to the 22 mph.-steady-state. The transition process may last a further 3-4 hours. Immediately after 10, the wind drops and remains constant at 10 mph., but the high energy sea of waves still continues to migrate southward. The south shore would register a continuous decrease of wave energy (and wave heights) until the steady-state for 10 mph. wind is reached after another 3-4 hours.

5.5 MOVEMENT OF WIND SYSTEM

In the discussions of Section 5.4 it was assumed that wind system, that is, the weather system generating wind, remains stationary. This is often not the case. The pressure system would often tend to move one way or another across the lake, usually in the general east-west direction in the Lake Winnipeg region. If the wind system is moving in the same direction as the direction of wind blow, the effect is to increase the effective fetch of wind-waves, provided that the duration of wind action is not long enough to attain steady-state wave condition. In a simple illustration, consider that in the example of Section 5.4, the wind system is also moving southward at 10 mph. For the hour (Hour 5) when the wind velocity is 28 mph., one would find that the wind system stays with the belt of waves migrating southward across the lake. In the process, energy continues to be imparted to the energy carrying waves. The result is that waves arriving at the south shore have an effective fetch equal to the length of the lake, giving rise to waves that are much higher and more powerful. Similarly, it follows that the waves will be much smaller when the pressure system travels in the opposite direction.

Information on weather system movements is not readily available, thus their effects are neglected in the present study. The error is not deemed to be significant on a year round basis.

5.6 LOCAL WINDS

Because of the difference of thermal characteristics (specific heat, for instance) of land and water, atmosphere over water and atmosphere over land are differentially heated. This induces local wind regimes. Land and sea breezes are familiar examples.

Local winds are generally lower velocity winds, but their directions at different locations of the coast of a lake may be different at the same instant. For instance, wind at Winnipeg Beach may be from the east while wind at Balsam Bay may be from northwest. It is also likely that local wind systems are not as well developed over Southern Basin as on sea coasts partly because lake temperature is much more sensitive to atmospheric temperature changes and partly because of the smaller size of the lake itself.

The results of computation, to be described in later chapters, Fig. (6-4), show that along the coast of Southern Basin, wave energy delivered by winds of 8 mph. and lower is not significant compared to that delivered by winds of 10 mph. and higher, even assuming that the fetch is at least as long as the width of the Southern Basin. (Local winds would normally be blowing over a fetch shorter than half the width of the lake, due to the configuration of convective cells over a lake). For this reason, it is felt that even though the effect of local wind is not separated from the Gimli wind record, the error subsequently induced cannot be large.

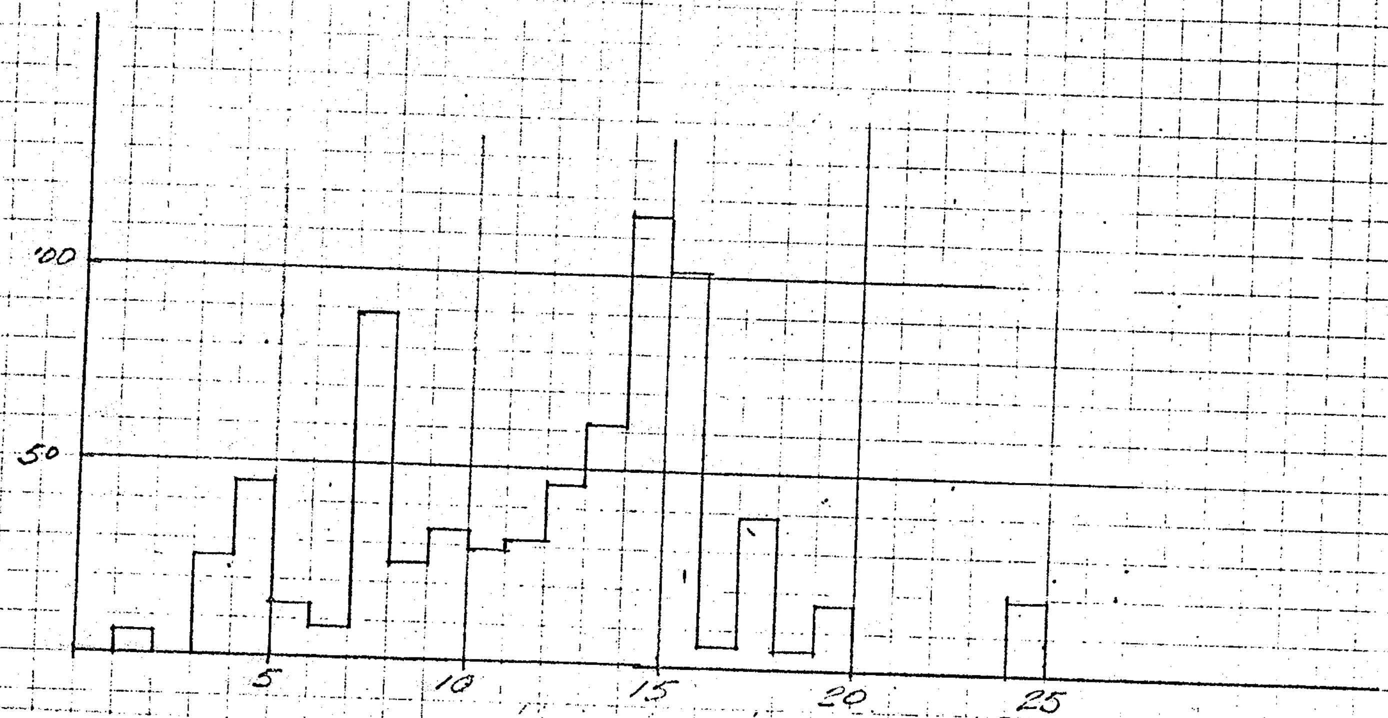


FIGURE 5.1 GIMLI WIND DATA --- North Wind
1 May to 31 November, 1968.
(Abscissa in mph.; ordinate in Hours.)

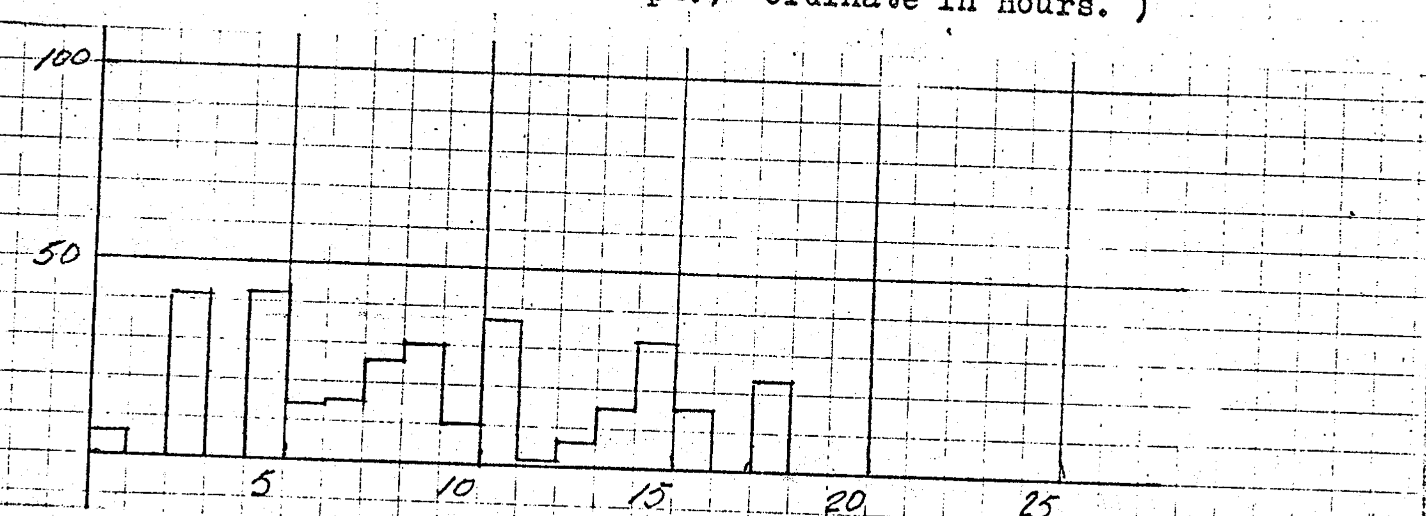


FIGURE 5.2 GIMLI WIND DATA --- North East Wind.
1 May to 31 November, 1968.
(Abscissa in mph.; ordinate in Hours.)

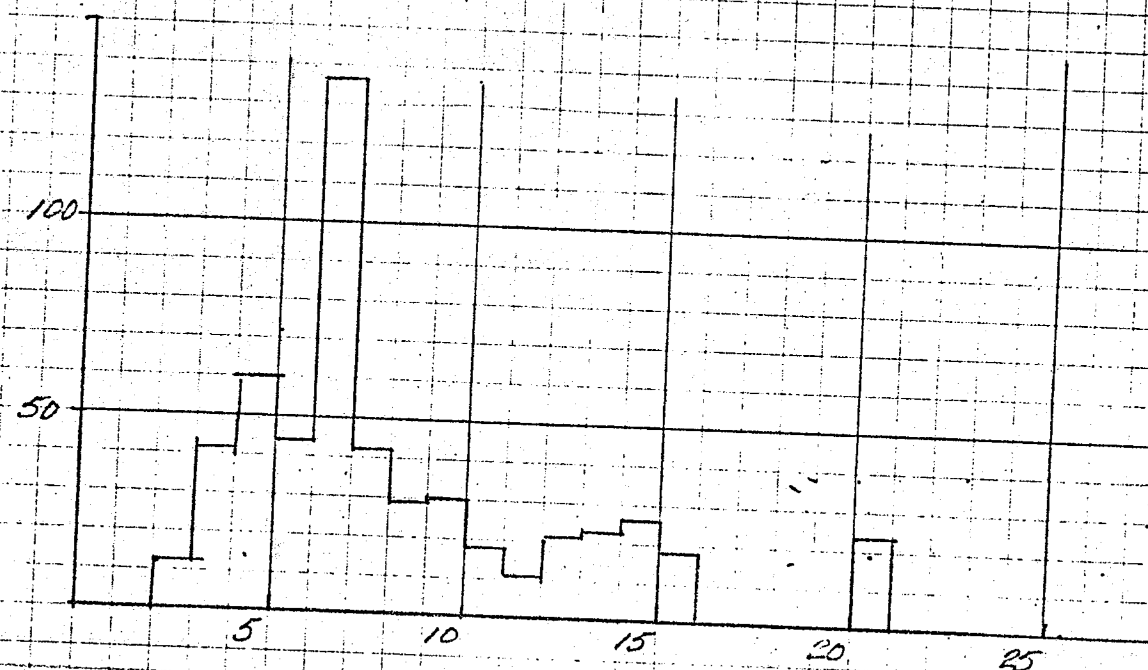


FIGURE 5.3 GIMLI WIND DATA ---- East Wind
1 May to 31 November, 1968.
(Abscissa in mph.; ordinate in Hours.)

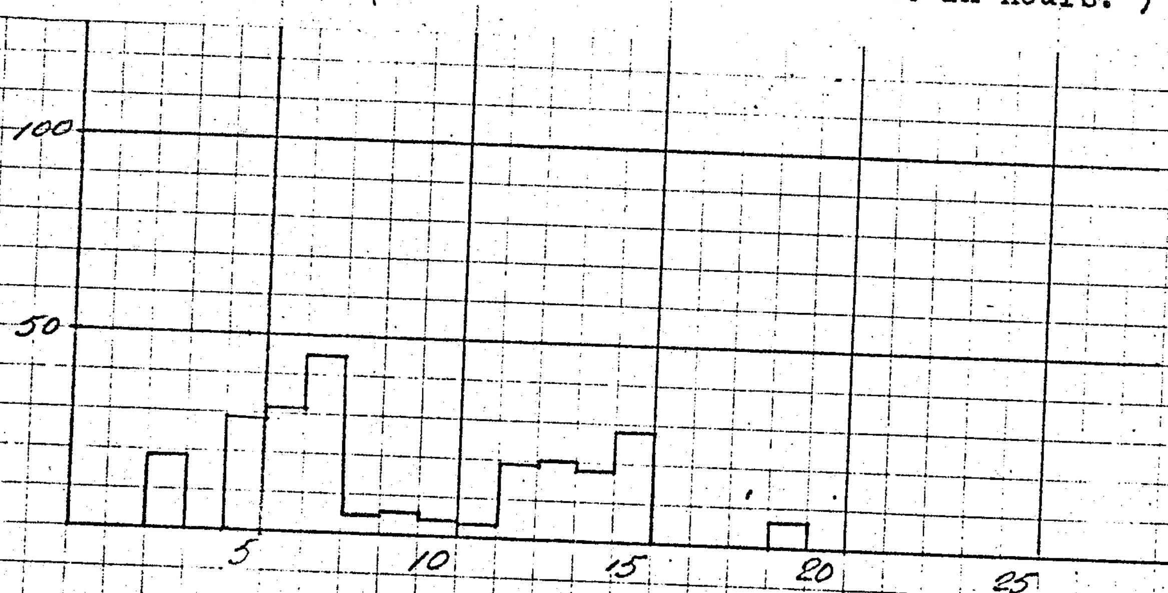


FIGURE 5.4 GIMLI WIND DATA --- Southeast Wind
1 May to 31 November, 1968.
(Abscissa in mph.; ordinate in Hours.)

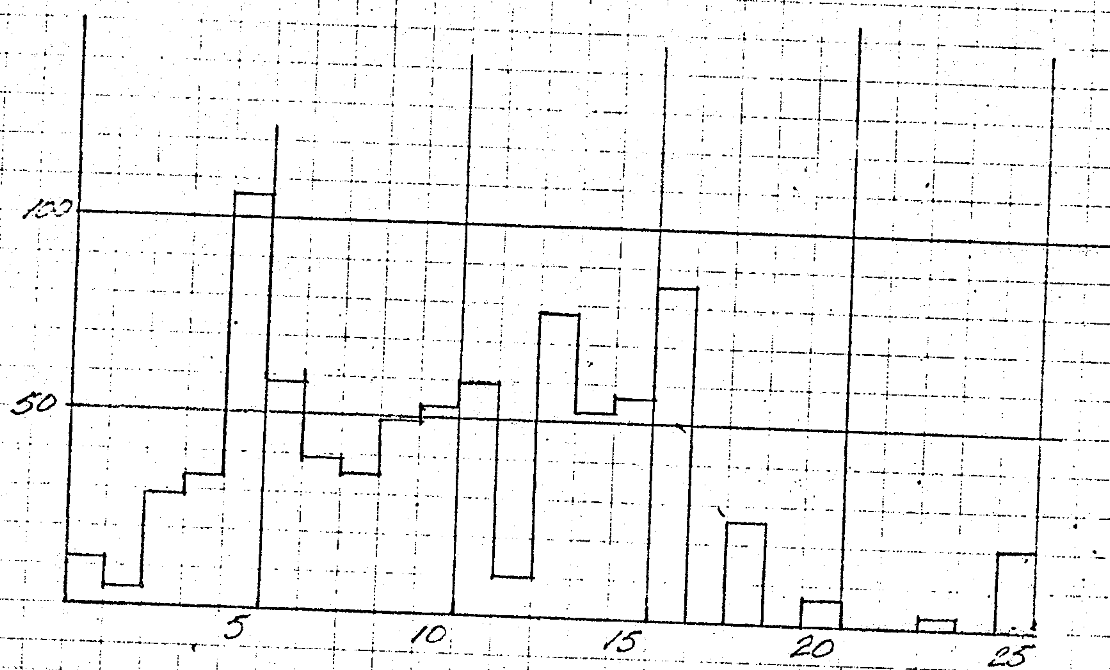


FIGURE 5.5 GIMLI WIND DATA --- South Wind
1 May to 31 November, 1968.
(Abscissa in mph.; ordinate in Hours.)

FIGURE 5.6 GIMLI WIND DATA --- Southwest Wind
1 May to 31 November, 1968.
(Abscissa in mph.; ordinate in Hours.)

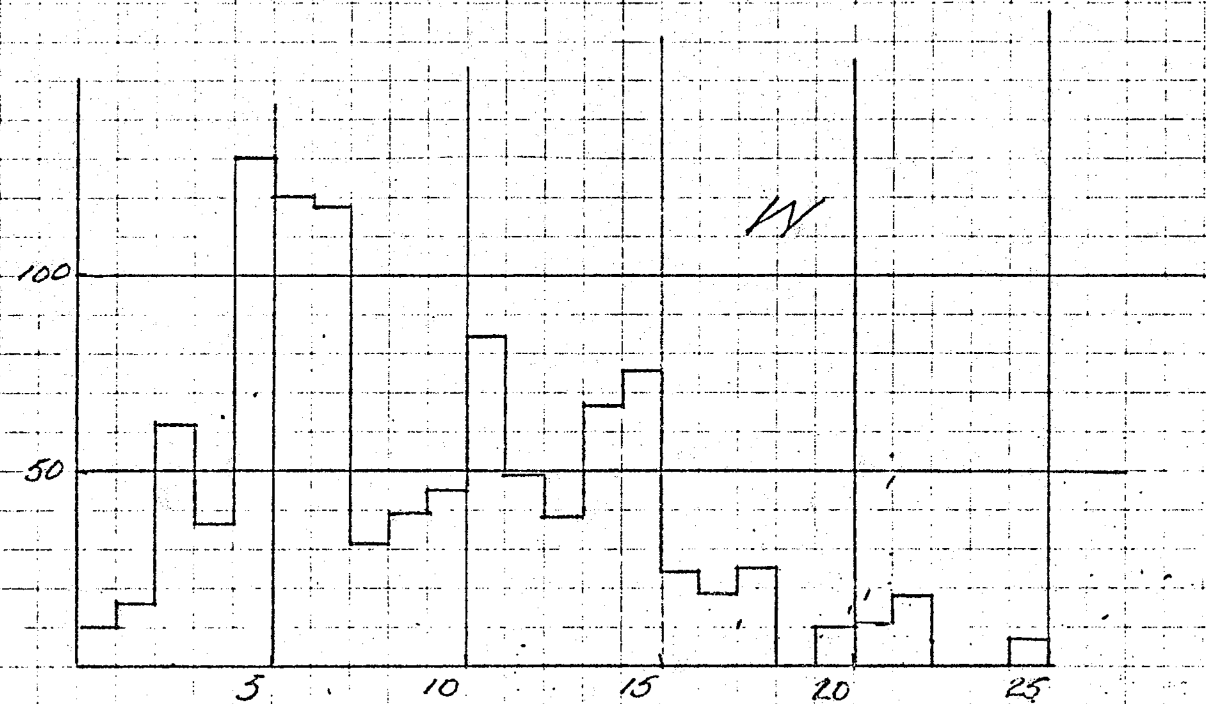


FIGURE 5.7 GIMLI WIND DATA --- West Wind
1 May to 31 November, 1968.

(Abscissa in mph.; ordinate in Hours.)

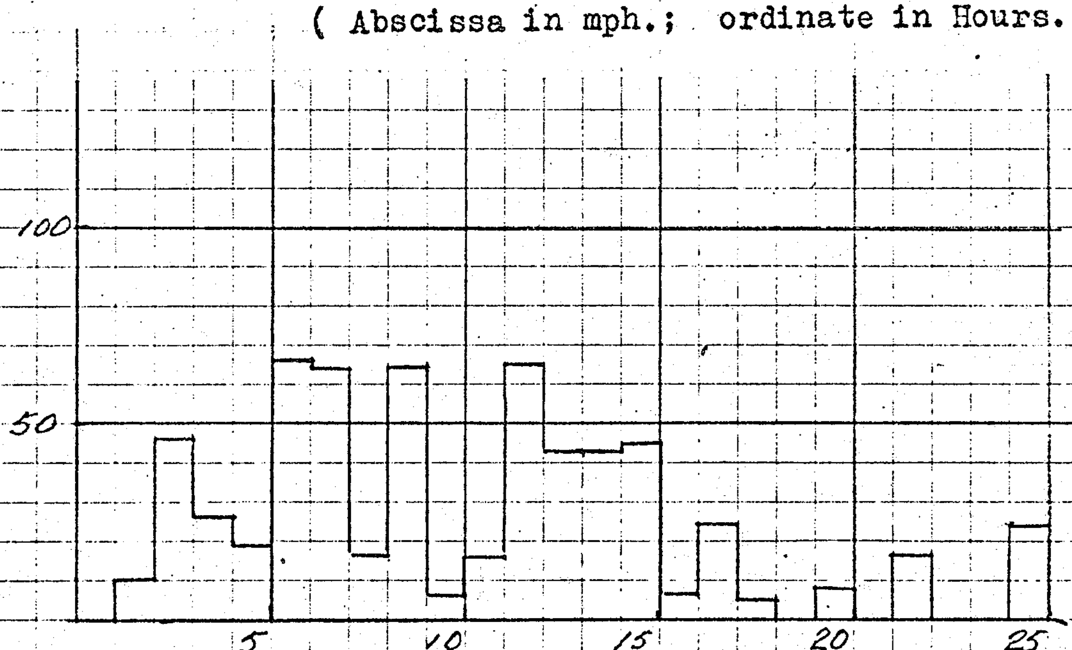


FIGURE 5.8 GIMLI WIND DATA --- Northwest Wind
1 May to 31 November, 1968.

(Abscissa in mph.; ordinate in Hours.)

CHAPTER VI

GENERATION OF WIND WAVES II: APPLICATION OF COMPUTING MODEL TO LAKE WINNIPEG AND ANALYSIS OF RESULTS

6.1 INTRODUCTION

In order to evaluate the existing processes which are affecting the coastlines of the Southern Basin of Lake Winnipeg and to gain some insight into the forces which shaped the coastal features into the forms they exist today, it is necessary to obtain some measures of the energy delivered by the waves to each unit width of shoreline per unit time (in erg. cm^{-1} $hr.^{-1}$, or erg/cm-hr - erg/cm-hr = 2.25×10^{-6} foot-pound/foot-hour). The net energy received per unit width of shoreline in a year can then be computed.

Since energy-carrying waves can arrive at the shoreline from any angle, it is convenient to resolve the energy vector into the perpendicular component and the parallel component, with respect to the shoreline. The parallel component may be positive or negative depending on whether it is in the positive x-direction when the y-axis is positive towards the lake. In other words, if one stands on the water's edge facing the lake, the parallel energy component of a wave is positive if it moves towards the right; negative if it moves towards the left. Thus on West Coast the general positive direction is towards the south, whereas on the East Coast it is towards the north.

The procedure of wave energy computation is somewhat involved as is described in detail in Appendix G. The principles are briefly summarized here.

The eight sided frame ABCDEFGHA in Fig. 6-1 is used to approximate the starting locations for generating waves. Utilizing Computer Program (A) of Chapter IV and the bathymetric chart of Plate I, wave refraction patterns for selected wave components were obtained for 220 starting locations on the frame for each of the eight wind directions: W, SW, S, SE, E, NE, N and NW. The computed information regarding refraction function, fetch et cetera, is stored on disk. Computer Program (B) retrieves the necessary information from this disk and, with input of information on wind, it calculates the normal and parallel wave energy components for any given point on East Coast or West Coast. The total wave energy is the vector sum of these components. The flow diagram of the computation tasks is as shown in Fig. 6-2.

6.2 RESULTS OF GENERATED WAVE PROPERTIES AND COMMENTS

6.2.1 Properties of the Wave Spectrum

A typical wave spectrum is plotted in Fig. 6-5 for the northeast wind of 10.0 m/s (approximately 22.5 mph) for Station WO 27, which is slightly north of the Winnipeg Beach Breakwater. At least two points are to be mentioned regarding this spectrum.

(i) The spectral density curve appears to have a saddle shape having a high peak value around $k = .15\ m^{-1}$ and a less distinguished peak closer to the y-axis. This chracteristic appears to be typical for most of the spectra generated. Two explanations may be possible for this phenomenon:

(a) The assumption that the turbulent boundary layer remains constant at 40 meters may tend to excite the low frequency components slightly too much for lower wind velocities. However, since the value of 40 meters was arrived at after fitting computed results to field data, it is not expected that the error induced in the computed values of wave height and wave energy will be large, although it may tend to distort the computed spectrum. This error, if it is significant, will also tend to remain consistent for the range of wind velocity used.

(b) The phenomenon may be due to shallow water effect. There is yet no theoretical justification to support such a hypothesis. However, a recent study (Kuo, 1970) shows that shallow wave spectrum does indeed assume the shape as shown in Fig. 6-5.

(ii) By inspection, the mean square elevation represented by the spectrum of Fig. (6-5), is about .10m^2, giving a significant wave height of 1.25m or 4.15 ft. In comparison, Veldman (1968), using the significant wave approach, computed significant wave heights of the order of 6.0 to 8.0 feet for wind velocity of about 40 mph. to 50 mph. Thus both results are in agreement with each other in order of magnitude, but it is not known which prediction is more reliable until wave measurements have been carried out in the lake.

It should be noted that since wave height predicted by the model is for waves at 9-foot deep water, the computed wave height (4.15 ft.) is somewhat larger than it should be in deeper water (say 30 to 40 feet deep). Furthermore, Veldman used fetch-limited values, whereas a wind of over 40 mph. blowing steadily over Lake Winnipeg for more than 2.5 hours over the shortest fetch (say 20 miles) can be conceived to be an extremely rare occasion; in this period of time, steady wave condition may not develop. At the same time, certain previous assumptions in the computer model, such as neglecting friction and percolation, may over-estimate wave height and wave energy.

6.2.2 Re: Energy-Wind Velocity Curves

The general trend of the Energy-Wind Velocity curves (Fig. 6-3) seems to indicate a steep fast-rising initial portion followed by a flatter portion which appears to assume a straight line relationship in the semi-log plot. The slope of this latter portion appears to be almost constant for all stations and all wind directions whenever a complete spectrum is generated. Whether this is a characteristic of shallow water wave is not known and must await future investigations for explanation.

(Herein, a "complete spectrum" means a spectrum that has all the wave components contributing to it. An "incomplete spectrum" means that some wave components are missing partly because they are refracted away from the location).

Not at all times, however, do the plotted points clearly delineate a straight line for the portion of the graph for wind velocity greater than 10 mph. Although there is no reason to presume this segment should be straight, there are at least two factors which would induce lower energy values, as compared to situations when these factors are not influential. As a result they may cause the actual curve to assume a less regular shape. They are as follows:

(i) When certain wave components are missing from the spectrum, the result is that at the wind velocity when these components should be most readily excited, the energy value will only rise very slowly.

(ii) When certain components have very low refraction functions, their influence is only slightly felt and the total effect is similar to (i) above.

6.2.3 Sources of Error

There are two sources of error which may also cause the computed values to deviate from actual field values in the energy-wind velocity diagram.

(i) A source of error is the assumption that the turbulent boundary layer remains constant at 40m. As explained in (i) of Section 6.2.1, the error involved is consistent and is quite small or negligible.

(ii) The points generated by the computer program are plotted in Fig. (6-5) as crosses (x). It can be seen that to integrate the spectrum by using the Trapezoidal Method may induce some inaccuracy. In particular, the value of the peak spectral density is not obvious. The computed values do not indicate whether it is 1 m^2/m^{-1} or .5 m^2/m^{-1} or higher. Because of this fact, the value given by Trapezoidal Integration may sometimes yield slightly inaccurate values than will actually be predicted by theory, had exact integration procedure been followed. Other intervals of integration were tested, yet the computed values remained rather consistent. This indicates that the error may not be significant.

Also, due to the fact that the computing program computes the value of mean square deformation as though the spectral density of each wave component is due to only one refracted wave ray, as discussed in Section 4.6, the spectrum calculated is sometimes a lower estimate of the actual spectrum in the case when more than one wave ray of the same k value is refracted to the same beach location; and vice versa.

However, on the whole, the results generated appear to be reasonable. As far as the present study is concerned, it is important to note that the results are very consistent, thus permitting the final results to clearly depict the relative intensity of coastal processes between different beach locations along the coastline.

6.2.4 Inadequacies Due to Scale and Diffraction Effects

Because of the scale of the bathymetric chart used, at least two inadequacies arise.

(i) Small local features are neglected and the wave characteristics due to these cannot be truly represented by the computed results. For instance, small angular headland projections at Sandy Hook, and Grand Marais Point have been neglected.

(ii) Man-made coastal features are too small to be incorporated for the same reason. These include the breakwaters at Gimli, Victoria Beach, Drunken Point and others.

Since the interest of the present study is the sand movement in the "macro-scale", inability to reproduce the influences of these smaller local features is not considered a fault of the computer program.

Another consideration is that wave diffraction was not included in the construction of the mathematical model. This effect, again, is significant only in the study of sand movement in the macro-scale and need not be considered here.

6.2.5 Energy Distribution Along Coastline

The annual energy distributions along the West Coast and East Coast as computed are shown in Plates II and IV respectively and are tabulated in Table 7.1. It is seen that East Coast receives much higher energy than West Coast mainly because the wind from the west is more frequent. Whereas on West Coast the average normal wave energy component is in the order of 300×10^{12} erg/cm and the parallel component is in the order of 80×10^{12} erg/cm, the corresponding values for East Coast are 750 x 10^{12} erg/cm and 300 x 10^{12} erg/cm respectively. In this regard the important role of the directional frequency of wind in distributing wave energy along the shore of a lake should be noted.

On the West Coast, stations receiving high energy due to the normal wave component are W002 - W003 (just south of Drunken Point); W011 - W012-W013 (at Gimli); W020 (Willow Point); and the long reach between Sandy Hook and Sans Souci. It can be noticed that along the northern half of West Coast, say at Drunken Point, south-facing shores receive higher energy from the normal wave component as compared to north-facing shores. Similarly north-facing shores receive higher wave energy along the southern half of West Coast. This phenomenon is expected since these shores are exposed to waves with much longer fetches than adjacent shores. Along the reach in the south (between Sandy Hook and Sans Souci) even east-facing shores receive high wave energy due to the normal wave component because the off-shore lake bottom topography in this region causes the wave rays to be refracted along the general direction of the normal to the shoreline.

The parallel wave component (Plate II(d)) tends to generate a distinct southward longshore drift along almost the entire West Coast except the coastal area north of Drunken Point. This is indicative of the strong influence of the predominant north wind in the Lake Winnipeg region. Locations which receive higher longshore energy than their neighbouring beaches are: Drunken Point, Camp Morton, Gimli, Sandy Hook and Winnipeg Beach.

On the East Coast the situation is quite similar to that on the West Coast. Shores receiving high normal wave energy are north-facing shores, including: Patricia Beach, Grand Beach and Grand Marais, and the reach north of Ironwood Point. Again, the longshore energy is generally directed southward, except at Hillside Beach and Grand Marais Point. Locations receiving higher longshore energy than their neighbouring beaches are: the beach south of the breakwater on Victoria Beach, Hillside Beach, Grand Beach near Grand Marais, Balsam Bay and the beach south of Grand Marais.

From the foregoing discussion, it is obvious that the location and orientation of the shore and the directional distribution of wind are the important factors which interplay to determine the magnitude of wave energy attacking the shore and the direction of longshore transport. For East Coast and West Coast, which can be considered to be the long sides of an approximately rectangular Southern Basin, the longshore transport is predominantly southward in the southern half of the basin. Similarly it follows that flow is northward in the north half of the basin. This is a natural phenomenon in lakes, as described by Zenkovitch (1967), and it will tend to transform a "long" lake into a "round" lake in the long run.

Re: Winnipeg Beach

Fig. (6-6) shows the wave refraction pattern for the north wind at Winnipeg Beach (not drawn to scale) as obtained from the computed results. This is a most interesting case which is also described in Appendix G.

From the results of wave refraction it is found that a bundle of wave rays of wave number k = .075 arrive at the neighbourhood of station W027 but none arrives at Winnipeg Beach; they reappear again south of Stephen's Point. A number of wave rays of wave number k = .075 - 0.175, the range in which the peak spectral density most frequently occurs, arrive at Winnipeg Beach, but none reaches the shore between Stephen's Point and Ponemah; they reappear again south of Ponemah.

This means that a bundle of high energy waves arrives at Winnipeg Beach whenever the north wind is blowing whereas the reach between Stephen's Point and Ponemah would receive much lower energy. South of Ponemah the wave energy increases abruptly again and remains relatively constant thereafter. This phenomenon has a unique role in shaping the coastline of this area, as will be explained in the next chapter.

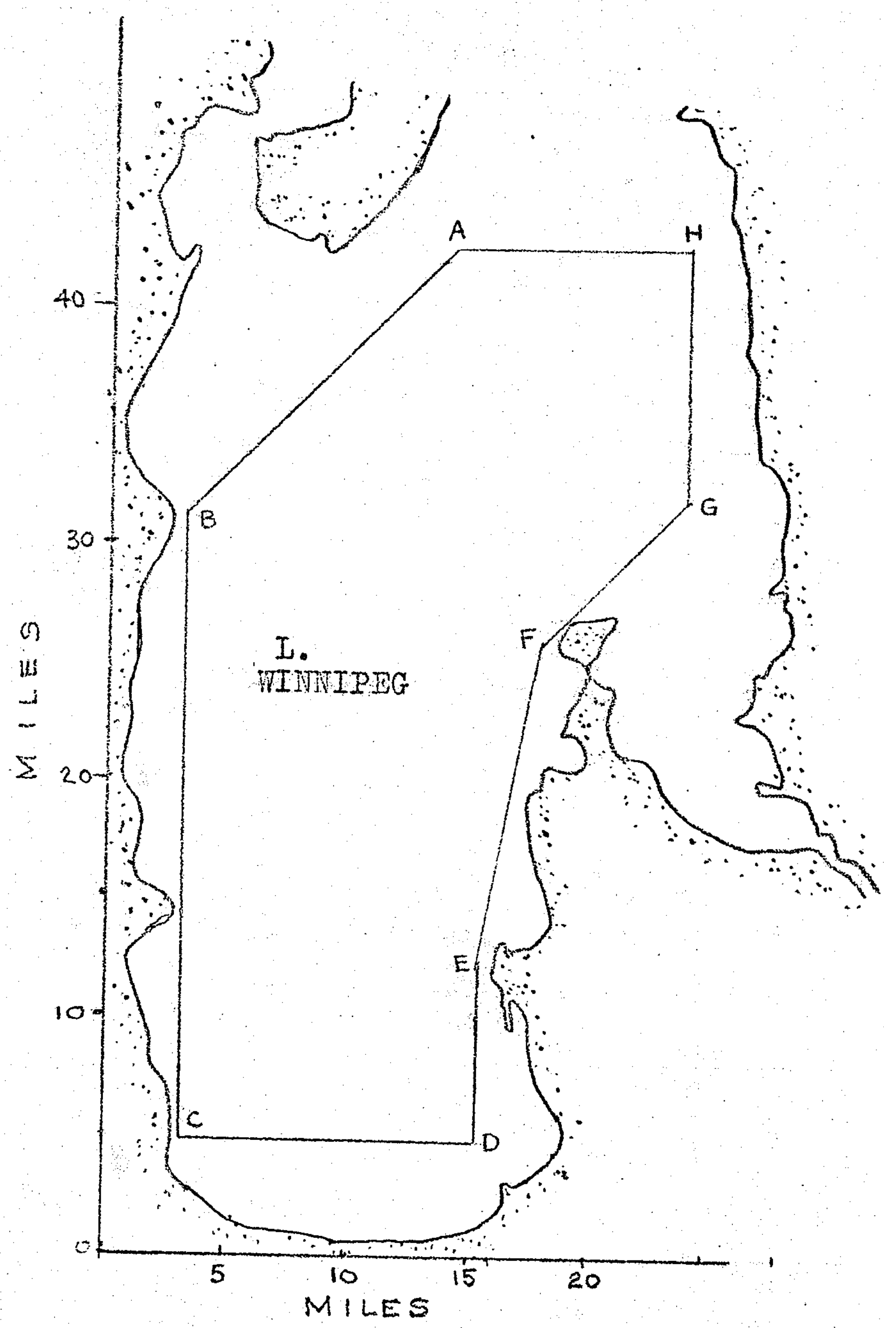
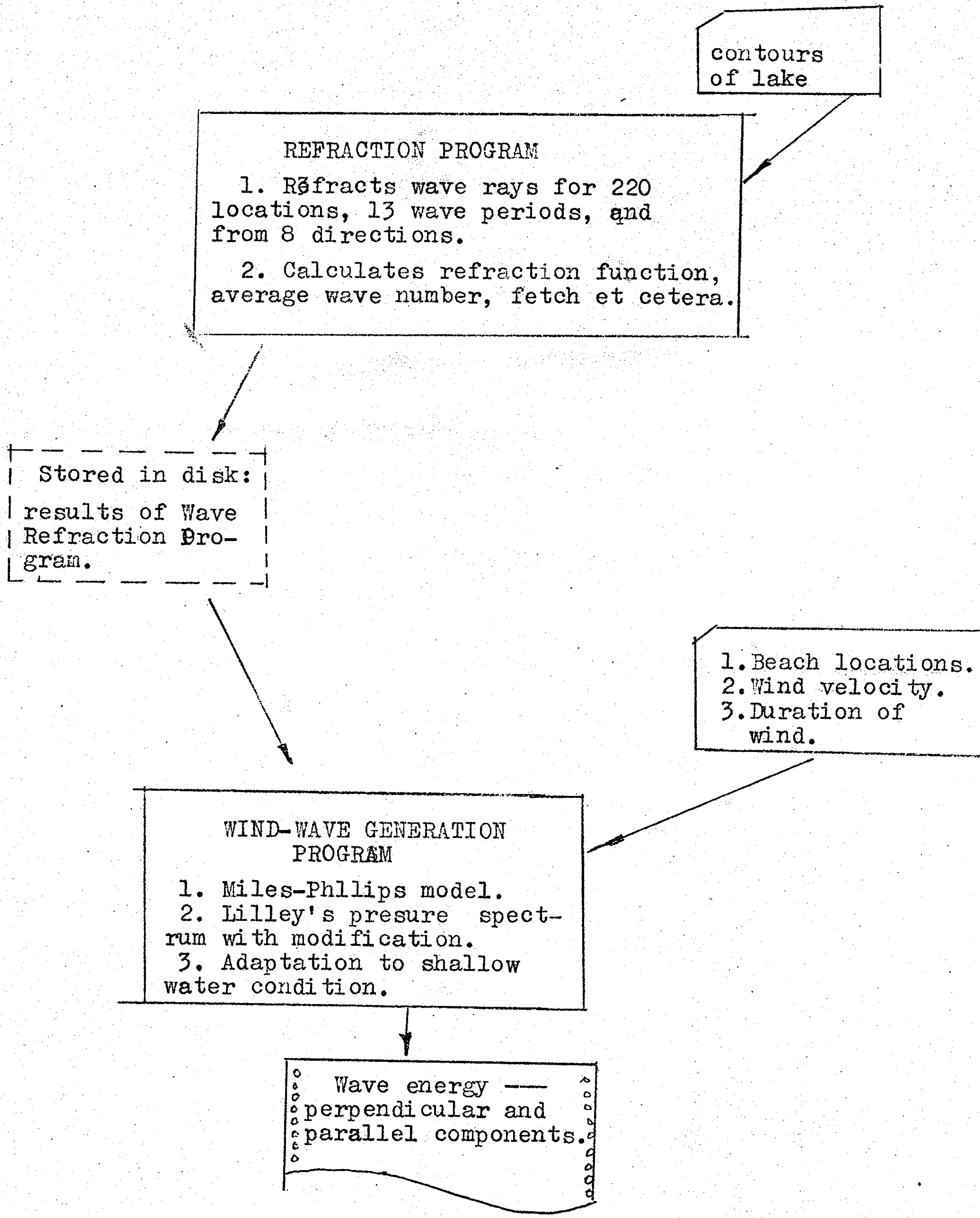
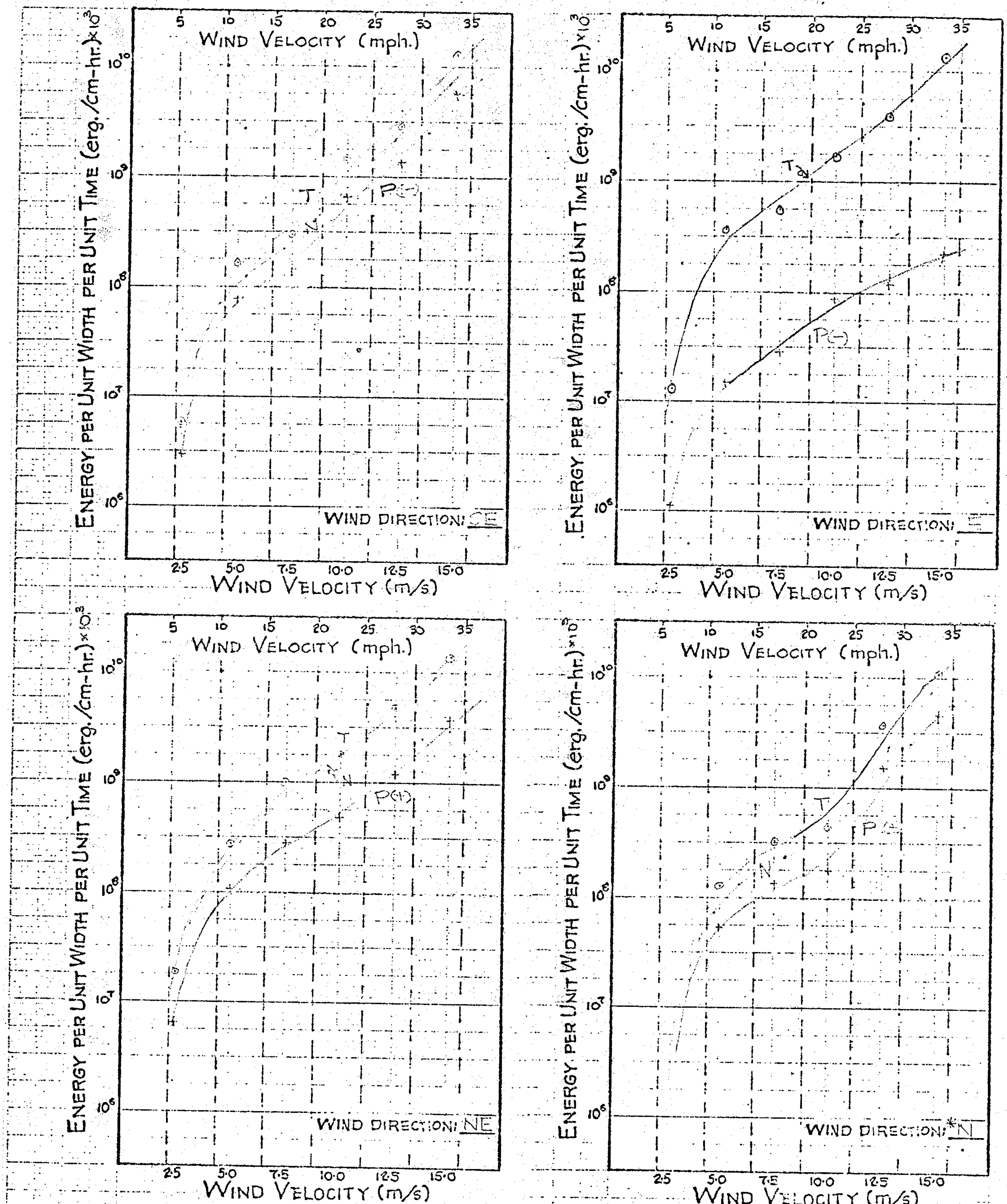


FIGURE 6.1 FRAME ABCDEFGH AND GRID USED FOR COMPUTATION MODEL.



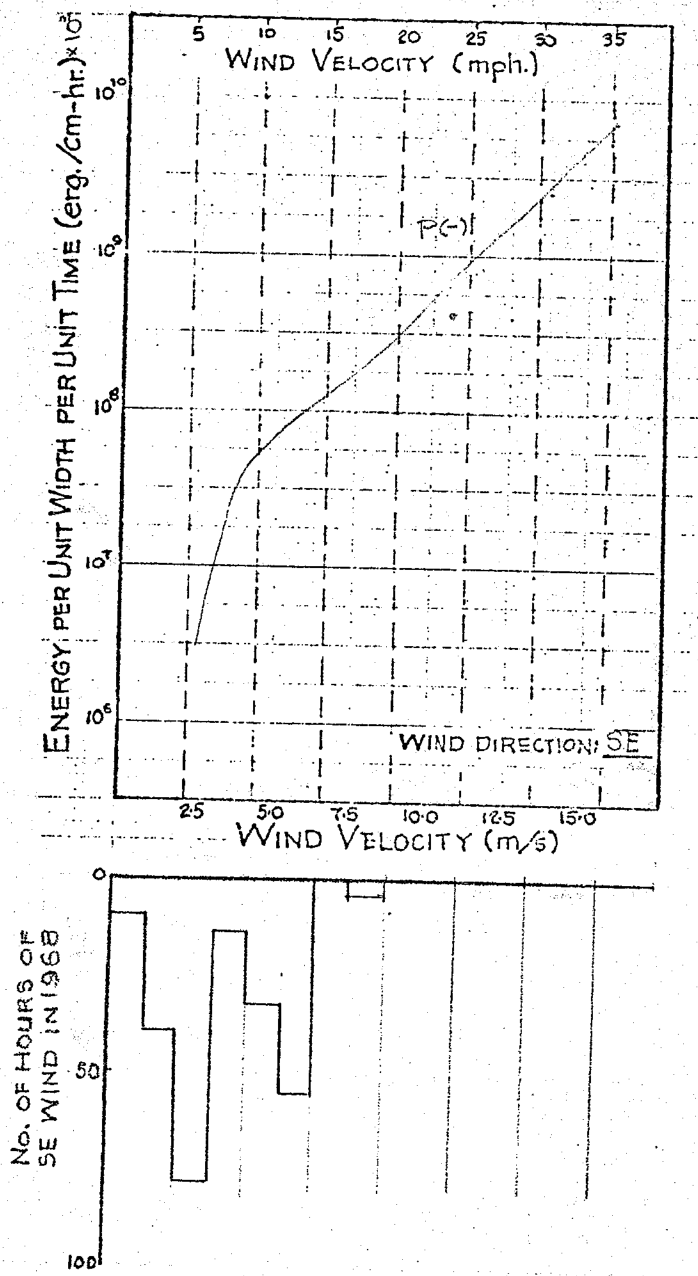
FLOW DIAGRAM OF LAKESHORE WAVE ENERGY COMPUTATIONS

FIGURE 6.2



NOTES: (i) Description of symbols:
T – Total wave energy.
P – Parallel energy component. (+) or (-) following denote direction wrt. beach.
N – Perpendicular energy component.
* – Partially complete spectrum because some components are retracted away.
(ii) Prediction is by Miles-Phillips' model.
(iii) Wind pressure spectrum is after Lilley (1960 b).

FIGURE 6·3
(LAKE WINNIPEG SHORE EROSION STUDY)
semi-log plot.
WAVE ENERGY AS FUNCTION OF WIND VELOCITY & DIRECTION FOR STATION W027



WIND VEL. mph.	WAVE POWER $x10^{12}$ erg/cm-hr.	#HOURS 1968	ENERGY $x10^{12}$ erg/cm.
0- 2.5	0	9	0
2.5- 5.0	0	39	0
5.0- 7.5	.007	78	1
7.5-10.0	.034	13	0
10.0-12.5	.07	32	2
12.5-15.0	.105	55	6
15.0-17.5	.16	0	0
17.5-20.0	.23	4	1
20.0-22.5	.45	0	0
22.5-25.0	.75	0	0
			10

Therefore total longshore energy due to SE wind at Station W027 for 1968 is (-) 10 x 10^{12} erg/cm.

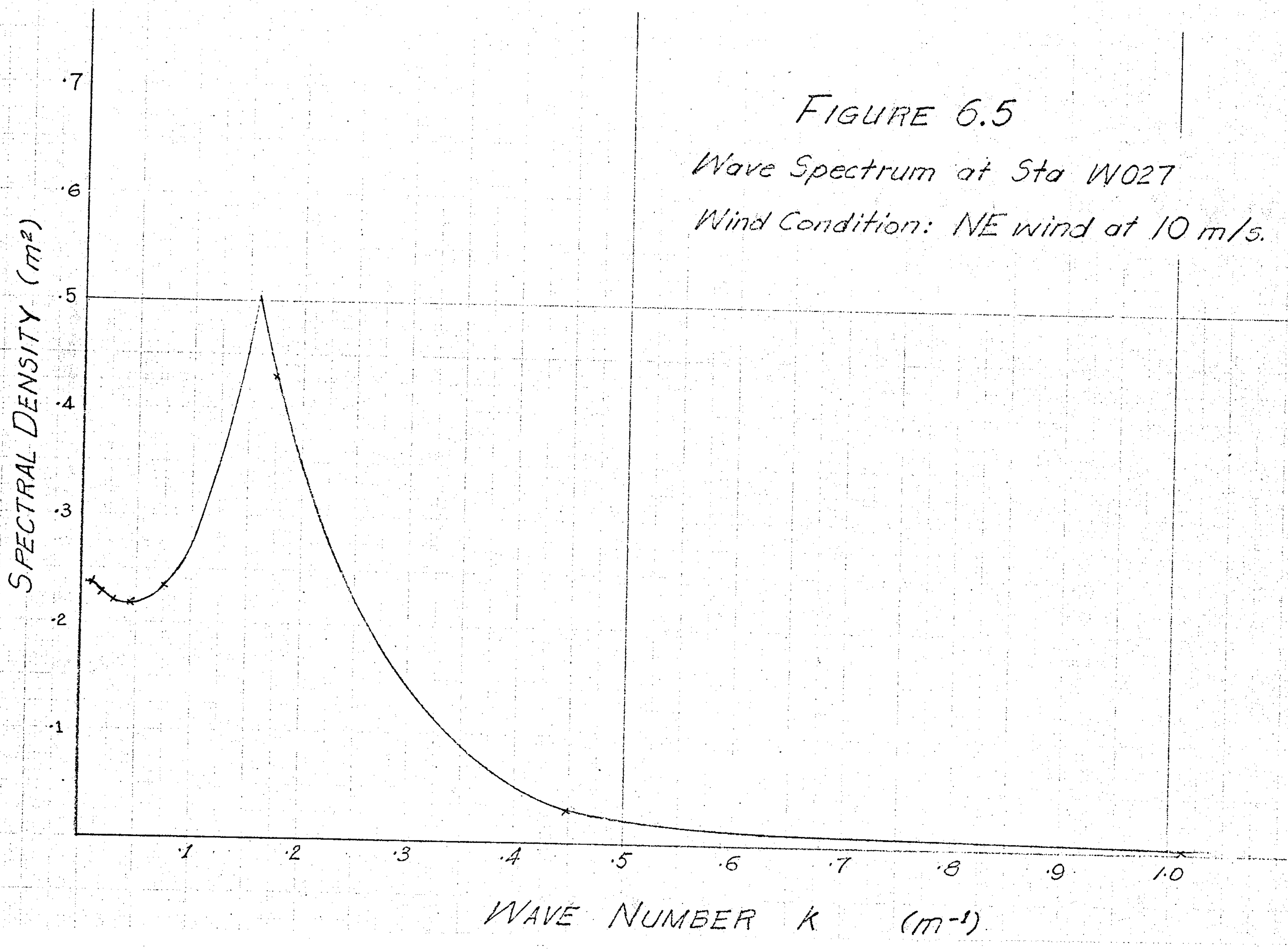
FIGURE 6.4 - Sample of Wave Energy Computation.

- Computing Longshore Wave Energy due to SE Wind at Station W027 for 1968.

FIGURE 6.5

Wave Spectrum at Sta W027

Wind Condition: NE wind at 10 m/s.



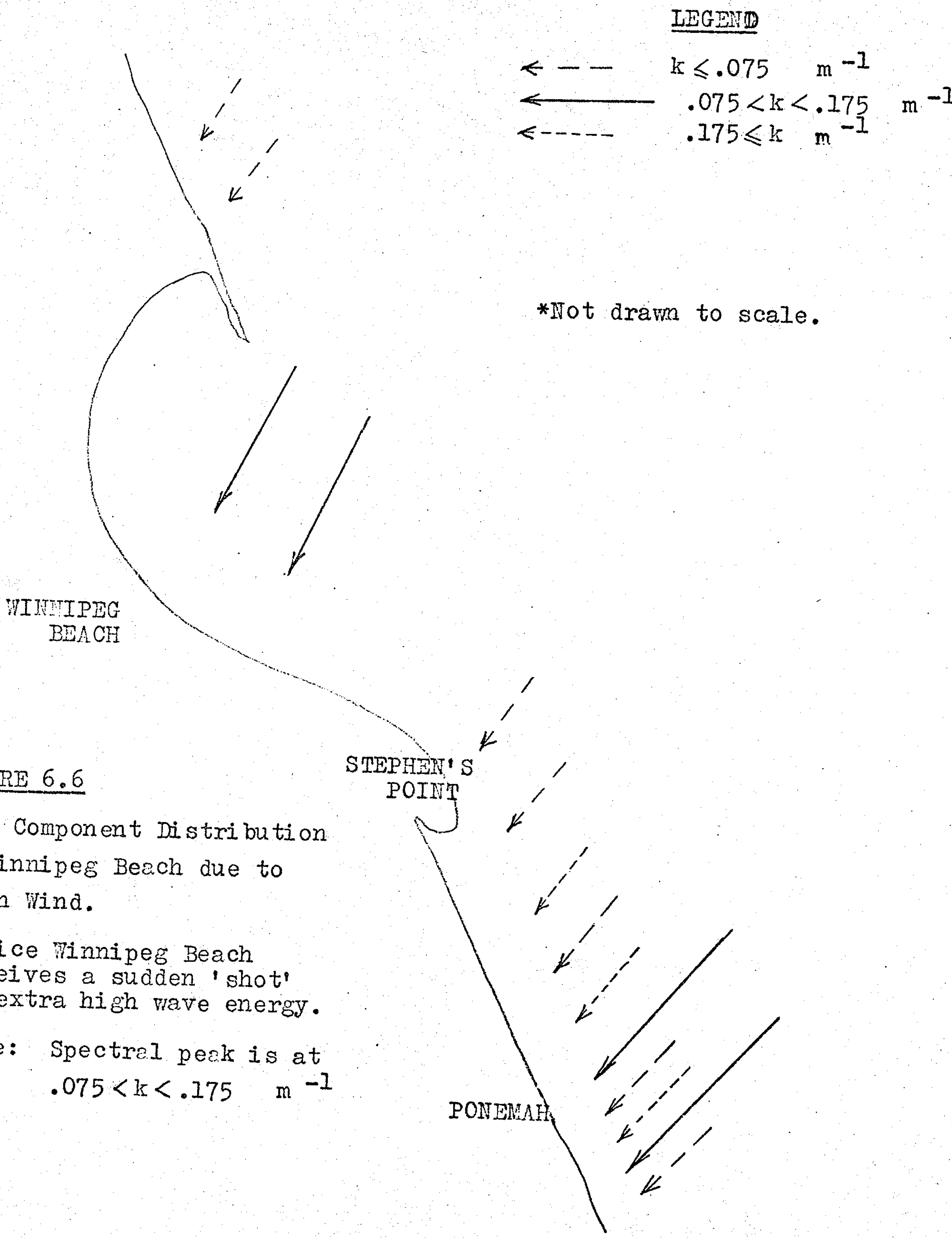


FIGURE 6.6

Wave Component Distribution At Winnipeg Beach due to North Wind.

-Notice Winnipeg Beach receives a sudden 'shot' of extra high wave energy.

Note: Spectral peak is at $.075 < k < .175 \ m^{-1}$

CHAPTER VII

MOVEMENTS OF BEACH SAND IN LAKE WINNIPEG

7.1 INTRODUCTION

There are two factors causing erosion of coastlines on Lake Winnipeg. The first one is the driving forces generated by wind wave action and responsible for moving beach sand. Nature must either be able to supply enough sand to satisfy these driving forces or be able to dissipate them through the protection of large size beach materials, or both. The second factor is the effect of increase in lake elevation. This chapter examines the movements of beach sand due to wave forces. Effects due to the second factor will be examined in the next chapter.

In general two modes of beach sand transport can be distinguished: longshore transport and transverse transport. Longshore transport or littoral transport is when the sand is influenced by littoral current and is moving in the general direction parallel to the shoreline. Transverse movement is when sand moves mainly on-shore or off-shore along the inshore and foreshore regions of the beach. The different manners in which sand can be moved by waves in either the longshore or transverse direction are discussed in Appendix E and will not be repeated here.

Ingle (1966) showed that sand seldom moves off-shore of the breaker line. This means that sand on beaches of Lake Winnipeg is not lost to deeper waters. Therefore the emphasis here is to evaluate only the longshore movements of beach sand, and show how they relate to shore erosion.

7.2 SAND TRANSPORT

7.2.1 Littoral Current Affecting Sand Transport

It is important to realize that when a wave strikes the shoreline at an oblique angle, it produces an energy component parallel to shore. This energy component generates a longshore current moving between the breaker line and the shoreline. Much like water moving in a river channel, the longshore current transports sand and dissipates its energy by friction as it moves along. This analogy, despite its simplicity, is helpful in understanding some of the features of longshore transport, in particular:

(i) the current is most efficient when it does not have to bend and turn corners. It loses energy in doing so.

(ii) when the current is carrying sufficient sand to satisfy its capacity for sand transport, it will not pick up (erode) any more beach material in subsequent reaches or flow; otherwise, it will.

(iii) particle size also influences the transport load. For instance, boulders are not moved by weaker currents.

The modes by which sand is moved on the beach by waves are actually quite different from those exhibited by channel flow under gravity, and are more complicated. (For details see Appendix E). Nevertheless, the concept of the existence of littoral current is important and will be frequently referred to in subsequent discussions.

7.2.2 Empirical Formulae for Longshore Transport of Sand

The relationship that sand transported along shore in the inshore-foreshore zone is a function of the parallel energy component of wave is supported by many investigators, (Johnson, 1952, Savage, 1962, Saville, 1950, Ingle, 1966, Inman and Komar, 1968, and others). Most of these investigators use the significant wave approach, so their observations are not compatible with the present study. Only the recent investigation by Inman and Komar, (1968), adopted wave spectral measurements and showed that

$$I_l = KE_p \qquad (7\text{-}1)$$

where I_l is the immersed weight of sand transport (dynes/hour)

E_p is the longshore wave energy component per unit shore width per unit time (erg/cm-hr)

K is a constant, equals 0.7 - twice the value used for significant wave.

7.2.3 Limitations to Sand Transport Relationship

(i) In using relationships such as Equation (7-1) one must be aware of the fact that beaches from which they are derived are long and straight. This condition is certainly not met by most of the beaches under the present study. In this case, extra energy loss will result from bending of flow. Thus sand transported will actually be less than Equation (7-1) indicates.

(ii) Another condition for Equation (7-1) is that the littoral current must have completely saturated its capacity for sand transport. This applies only if the current flowing into the beach area is already in a "saturated" state. Furthermore, longshore energy which is received throughout the downdrift stretch of beach remains constant and equals the energy dissipated by friction so that the longshore current remains saturated with sand. It is doubtful too whether this ideal situation applies to beaches of Lake Winnipeg.

(iii) Inman and Komar stated that their equation (Equation (7-1)) holds regardless of sand sizes. This observation appears to be questionable. These researchers measured sands of 180 - 360 micron. Intuitively it is conceivable that if the beach material was large gravel, or, for that matter, boulders, the rate of sediment transport would have been lower. In other words, threshold energy varies with particle size and density, and is conceivably higher for gravel than for sand. Ingle (1966) has demonstrated that sand transport is most efficient for sizes of 0.14 mm to 1.0 mm. It takes more energy to transport the same amount of material for sizes larger or smaller than this range. See Fig. 7-1.

Owing to these limitations Equation (7-1) does not really apply to most of the beaches under study. However, to be able to arrive at some sort of qualitative comparisons between beaches, it is at least necessary to quantify the effects of wave on longshore transport at this stage. For this purpose, Equation (7-1) is used for computation with the assumption that all of the conditions (i), (ii) and (iii) are satisfied. (That is, long straight beaches of sand sizes of 180 - 360 micron with saturated littoral current). The resulting estimates, of course, are exaggerations of existing situations. Calculated sand transport rates (dynes per year) are shown in Plates III(a) and V(a), for 1968. As will be explained later, it is most important to realize that the values shown do not represent actual transport rates in Lake Winnipeg but they are important in that they indicate the potential of waves to erode a beach or to deposit sand. For this reason the ordinate values of Plates III(a) and V(a) will be denoted by the term "theoretical transport capacity", and the ordinates in Plates III(b) and V(b) by "theoretical erosion or accumulation due to longshore transport". The directions of movements of beach material are also indicated in the same plates.

7.3 EROSION AND ACCUMULATION OF BEACH SAND DUE TO LONGSHORE TRANSPORT

Referring to Plates III(a) and V(a), (Linear Longshore Transport Graphs) it is possible to locate areas of deposition and serious erosion. Before discussing this aspect, it is necessary to examine the processes of erosion and deposition with regard to longshore transport.

Fig. 7-2a shows a long straight sand beach. The length scale of AF is probably 10 to 20 miles long. The arrow vector indicates waves attacking different segments of the shoreline, with the relative lengths of arrows indicating the relative amount of energy carried by waves. The case is strictly hypothetical.

Fig. 7-2b shows the rate of sand transport, in full lines, within the segments AB, BC, CD, DE and EF if condition (ii) of Section 7.2.3 is satisfied - i.e. when the littoral current enters a beach reach, its capacity of sand transport is already satisfied. (Notation: positive means moving to the right; negative, to the left). Thus the sediment load remains constant over the remaining areas of the reach. The field condition, of course, deviates from the ideal situation. The following cases can be distinguished.

(i) At B, the difference in transport capacity is $T_b - T_a$ (in weight per unit time), where T_a and T_b are the theoretical transport rates at A and B respectively. It is likely that the rate of increase of the amount actually transported is proportional to the deviation of the realized load from the theoretical load T_b; i.e.

$$\frac{dT}{dx} = k_1 \Delta T = k_1 (T_b - T)$$

$$\ln C_1 - \ln (T_b - T) = k_1 x \qquad (7\text{-}2)$$

where C_1 is the constant of integration, thus

$$C_1 (T_b - T) = e^{-k_1 x}$$

@ $x = 0, \quad T = T_a, \quad C_1 = \frac{1}{T_b - T_a}$;

$$\therefore \quad (T_b - T) = (T_b - T_a) e^{-k_1 x} \qquad (7\text{-}3)$$

where k_1 is the constant, and T is the instantaneous rate of transport.

It is not known what the value of k_1 is, but adjusted transport rate is depicted by the dot-dash line. Obviously, if the transport capacity downdrift of B, T_b, is to be satisfied, erosion must occur around the down-drift neighbourhood of B in order to supply the required quantity of sand. If k_1 is large (for loose sand), the area of erosion will be very small, and the degree of erosion is severe. If k_1 is small (for cohesive material or large boulders) erosion is less severe, but will extend over a longer stretch of the coastline.

(ii) At C, the situation is the reverse of B: deposition will occur. Accordingly

$$(T - T_c) = (T_b - T_c)\ e^{-k_2 x} \qquad (7\text{-}4)$$

where k_2 is the corresponding constant associated with deposition. (Large k_2 corresponds to coarse sand and granular materials; small k_2 corresponds to fine sand and suspended materials).

(iii) At D, two counter-flowing currents meet. This will cause the longshore transport components to cancel out each other, and total deposition will take place. The transport rate is defined by the dot-dash curve which is the algebraic sum of the two deposition curves (as predicted by Equation (7-4) at D' and D"' (dotted lines). A rip current may also form which carries the sand into deeper water.

(iv) At E, flow in both the positive and negative directions originates. This will cause erosion on both the left and right of E.

Although cases (i) to (iv) are strictly hypothetical a probable and realistic situation will be as shown in Fig. 7-2c. In fact, field situations closely follow the cases analyzed above. In particular, wherever there is an increased supply of parallel energy there is the tendency to erode; wherever there is decrease in energy or where there are counter-flowing currents, deposition occurs. Assuming both k_1 and k_2 approach infinity (that is instantaneous erosion and deposition) the quantity of erosion or deposition for any location is:

K x (Updrift longshore energy - Downdrift longshore energy) (7-5)

where K is as defined in Equation 7-1. Positive value means erosion; negative value means deposition.

In this regard, the estimated quantities of erosion and deposition due to longshore transport for locations in Lake Winnipeg's Southern Basin are as indicated in Plates III(b) and V(b) and tabulated in Table 7.1. It is important to recall that besides being exaggerations these figures may not represent the state of a lake in pseudo-equilibrium, because of the fact that a beach may be mantled by a layer of materials of sizes large enough to resist gross movements induced by waves (which tend to move them out of the beach area). This aspect will be discussed further in the following chapter. In this regard, the deviation of computed results from field conditions constitutes a major reason why the term "Theoretical erosion or accumulation due to Longshore Transport" is used.

7.4 SOME COASTLINES OF LAKE WINNIPEG THAT INDICATE CHARACTERISTIC FEATURES OF MORPHOLOGIC PROCESSES

The preceding section pointed out some concepts helpful in visualizing processes of coastline morphology. However, like all other branches of engineering, a good prediction is one that utilizes both theory and field observations. Here, a few coastal areas of particularly interesting characteristics are discussed. Because Veldman (1968) has already described most of the coastal areas of Southern Basin in detail, it is not necessary to repeat these descriptions once more. The aim is to emphasize on the finer details of coastal process and to supplement Veldman's observations which appear to be derived principally from field inspections and air-photo interpretation.

The Winnipeg Beach-Stephen's Point-Ponemah reach turns out to be one of the more exciting elements of this study and will be reserved for discussion in a separate section to follow.

7.4.1 Grand Beach

This is one example in which the annual longshore energy (Plate III(a) and IV(a)) as computed, may be misleading. According to the graph, the energy component at Station E007 has a positive component (to the right) while E008 has a weaker negative component (to the left). According to the previous section, sand is deposited between these stations and no sand is moved out the beach area past Grand Marais. However, this may not be the case.

(i) Under the action of the northwest wind, E007 indeed has a positive component and E008 has a weaker negative component. Thus no sand moves out of Grand Beach past Grand Marais. When the north wind is blowing, however, both E007 and E008 have negative components. If the littoral current is sufficiently strong, sand will move past Grand Marais Point. Still under the influence of the north wind, the sand is swept to the sand spit south of Grand Marais where it is sheltered from severe wave actions and likely will not move back to Grand Beach again, even under the action of the strongest southwest wind. The process is similar to pushing a rock over an inclined ledge and dropping it into a canyon. It suggests that sand can move out of Grand Beach as Veldman conjectured, but under the influence of a slightly different mechanism; however, field measurement is needed for confirmation.

(ii) Even under the influence of northwest wind, sand may be able to move out of the bay of Grand Beach past Grand Marais Point. This mechanism may be set up such that the littoral current due to the negative component of E008 is stronger than the counter-flowing current due to the positive component at E007 such that the former not only nulifies the effect of the latter, but also has enough residual energy to move sand out of the bay. However, it may not seem that this process is likely because the negative energy component at Station E008 is not strong, and also because Grand Marais Point acts as a barrier to the movement.

Thus, if sand moves south past Grand Marais Point at all, it is due to north wind. The quantity may or may not be significant, but once past the point, it seldom moves back into the reach again.

7.4.2 Reach Between Sand-Spit South of Grand Marais to Balsam Bay

Here the observation is that sand is being deposited at the sand spit. This implies that there is a lower value of longshore energy component (negative) here than that observed at Balsam Bay where severe erosion occurs as described below. Interpreted in this manner, an arbitrarily low value is assumed for the theoretical longshore transport capacity. It can be seen that there is a large jump in longshore negative energy between the end of the spit and Balsam Bay, therefore resulting in a high theoretical transport capacity. This may lead to severe erosion at Balsam Bay.

7.4.3 Patricia Beach

The pair of mechanisms (i) and (ii) of Section 7.4.1 which influence Grand Beach, also influence this bay in a similar manner. It is obvious from considerations of Plate V(a), that sand moves south past the jut-land of Patricia Beach.

7.4.4 Bay of Albert Beach to Breakwater of Victoria Beach

Consideration of energy distribution (Plate V(a)) indicates that sand does not move south and west past Ironwood Point out of Albert Beach. Nor does it move past the barrier of the breakwater on the north side of the bay. (There is a stronger negative energy component at station E017 to "attempt" to move sand into the bay past the breakwater). The movement of sand inside the bay is likely identical to mechanism (iii) of Section 7.3 (counterflow and rip-current) under west wind. It may also undergo a kind of "see-saw" motion between stations E016 and E015, moving south with northwest wind and north with southwest wind. For this reason, the corresponding values of "theoretical erosion and accretion" depicted in Plate III(b) must be interpreted in a different light. Certainly the realized erosion and accretion will be considerably less than as indicated. There may even be no noticeable erosion or accretion at all.

Under the influence of strong north wind and other favourable conditions, it is conceivable that sand moving south from Victoria Beach may by-pass the bay altogether. Thus from Station E018 sand will move straight across to Ironwood Point and proceed southward without entering the bay at all. This mechanism has been observed by Zenkovitch, (1968) and other investigators.

7.4.5 Drunken Point - Station W006 Reach

At station W006 wave energy is generally negative to the north and positive to the south. It acts as the dividing point of northward and southward flow.

Nevertheless, the degree of erosion it encounters, all others being equal, is likely not as serious as that of some other locations on the lake because of the small rate of energy change in this area, (see Equation 7-5).

Sand that has moved north of Drunken Point will not likely move south again because of a lack of significant fetch for the north or northeast wind to generate a strong southward flowing littoral current.

7.4.6 Gimli - Willow Point Beach

According to Plates III(a) and (b), the sand derived possibly from erosion at Loni Beach is to be deposited at Gimli Harbour. An attractive beach does exist at Gimli, probably due to the barrier effect of the harbour breakwater. It is doubtful, however, that enough material has been eroded from the short reach between Loni Beach and Gimli to cause significant deposition at Gimli on an annual basis (k_1 value may be too small). There is also a steady flow of sand in the positive direction from Gimli to Willow Island along the straight sand bar at Willow Point. On an annual basis, there should be no severe erosion along this bar. However, similar to sand moving south past Grand Marais Point, any sand moving south past Willow Island will not move back into the reach again but is accumulated on the bars south of it. Consequently, once the sand supply is cut off at Gimli, or if not enough sand is supplied, erosion may occur in this reach. But as one may appreciate, the degree of erosion may not be severe. (As will be explained in the next chapter, the apparent shore erosion here is probably due to the rise in lake level alone - not to littoral transport per se.)

7.5 WINNIPEG BEACH

At Winnipeg Beach, a very distinct crenulated bay (half-heart-shaped) can be noticed from the topographic map. This type of coastal feature was noticed by various investigators in the past (Yasso, 1965, Silvester, 1970). Mathematical formulations of its formation have also been derived, (Silvester, 1970). However, none of these investigators appears to have correlated the forms of such bays with wave energy characteristics. Grijm (1968) theorized some hypothetical shoreline shapes under the influence of wind, but he did not consider the case of crenulated bays; moreover, his model ignores physical features such as wave refraction and beach material variations.

Examination of the energy distribution curve of Plate II(d) shows a local peak of longshore energy at Winnipeg Beach, principally due to the influence of north wind. Referring to the previous chapter, the jump in energy by about 56×10^{12} Erg. cm., is not computed analytically, but logically derived from valid sources of information. It represents the additional energy the coastline would have received had it been straight instead of being an "indented" crenulated bay: this situation corresponds to the original state of the coastline when the lake was first formed. Since then, erosion has taken place, resulting in the present half-heart-shaped bay, which in turn modifies longshore energy distribution and smooths off the original energy peak somewhat. (The sequence of process is as shown by the approximate curves in Fig. 7-3). The new energy distribution decreases the rate of erosion from Winnipeg Beach, rendering a more stable condition with regard to erosion.

The correlation of half-heart-shaped bays with abrupt increase of longshore energy is observed at other locations of the West Coast as well. These are:

(i) between Loni Beach and Gimli Harbour;

(ii) just north of (i) at stations W010- W011-W012-W013-W014; and the southeast facing shore of Drunken Point between stations W005 and W001.

The half-heart shapes of these bays are not as well established as the bay at Winnipeg Beach. The reason may be that they do not have longshore energy increase comparable to the sudden energy increase at Winnipeg Beach. It is also interesting to note that no crenulated bay of similar scale can be distinguished on the East Coast. From these observations, the following conclusion can be drawn.

Crenulated bays are formed when the following conditions are satisfied:

(i) there is a marked increase in longshore energy at the location, preferably due to wind from a predominant direction.

(ii) the coastal bluffs of the area are low.

(iii) there are no obstructions to the erosional processes caused by waves. The obstructions include large boulders or other geophysical or man-made features.

(iv) the coast must be composed of easily erodible materials, and of fairly homogeneous composition.

For the locations indicated above on West Coast, all four conditions are more or less satisfied. Considering the lack of studies on the subject, these conditions must be regarded as necessary but not sufficient criteria at the present moment.

The reason that crenulated bays are not formed on East Coast is that unlike the West Coast, which is principally a lacustrine terrain, this part of the lake is a morainic and lacustrine complex. As described in Chapter II, condition (ii), (iii) and (iv) are seldom simultaneously satisfied although abrupt increase in longshore energy along the shoreline occurs at several locations.

Another interesting feature regarding the energy distribution of the Winnipeg Beach area is that between Stephen's Point and Ponemah there is a sudden drop of longshore energy because under the action of north wind, the wave components carrying most energy have "skipped" this area, as explained in Chapter V. The result is an accumulation of the sand that eroded from the beach updrift at Winnipeg Beach.

It should also be noticed that due to increase in lake level, as will be explained in Chapter VIII, the longshore energy at Winnipeg Beach also increases somewhat, in addition to corresponding increase of turbulence (also due to lake level increase) due to waves breaking against seawalls. This may cause the longshore transport of sand from Winnipeg Beach to increase somewhat in the past few years. An exact theoretical estimate of this increment is not possible without laborious and profound mathematical manipulations.

On the other hand, as discussed in Section 3.4, within a reasonable degree of accuracy, (and also owing to the fact that the survey data is still rather scanty for calculating the quantity of erosion or accretion) it is not possible to draw any positive conclusion regarding the amount of erosion or accretion at Winnipeg Beach. The only location where there appears to be some erosion is at Lines A and B in Fig. 3-3, but everywhere else except for some random variations, the three sets of data do not differ significantly from one another. It was also mentioned that sand dumped by Parks Branch would be hardly noticeable from the survey data.

The 1970 survey program also includes the surveying of the shores at Stephen's Point. (The results are not presented herein). Although it was apparent from observations during the survey that there was deposition of sand at this location sometime in the past, it was not clear at all whether any beach accretion took place within the last few years, judging only from one set of data. It can be concluded therefore, despite the theoretical deduction that longshore transport has increased at Winnipeg Beach, the erosion rate, (if erosion does occur) is so slow that it cannot be detected in beach surveys covering the short period of two years.

7.6 OTHER COMMENTS

From Plates III(a) and V(a) it can be seen that the theoretical longshore sand transport rate in Southern Basin is usually in the order of 50,000 cu. yds. per year for East Coast and 20,000 cu. yds. per year for West Coast (for 1968). This is much lower than 600×10^3 cubic yards per two years for Santa Barbara, (after Wiegel, 1964, pp. 478), but consideration must be given to the fact that Lake Winnipeg is ice-covered in winter and is therefore not subject to wave action. Also, wave energy in Lake Winnipeg is likely less than that at Santa Barbara. The figures, however, compare well with reported values for the Great Lakes (Ref. No. 151), bearing in mind that in adopting Inman and Komar's formulation, (Equation 7-1) and using the spectrum approach, the computed values are expected to be twice the values obtained by conventional methods.

It is important to stress again that the "theoretical transport capacity" is only theoretical per se. Actual sediment transport rate could be quite different from the theoretical value, due to two major limitations. Firstly, because as per Chapter V, wind overland to wind overlake ratio was not applied, the theoretical value is too low. Taking a mean ratio of 1.40, it is felt that the error induced would be in the order of 40 percent to 100 percent. Secondly, because of the restrictions in Section 7.2.3, the transport formula (Equation 7-1) is not totally valid for beaches in Southern Basin, and a significant reduction factor should be used. There exists no method of evaluating this reduction factor, but it is intuitively felt that the computed theoretical value should be reduced by at least 50 percent or more.

Subjectively, I feel that the second factor is much more dominant. The combined effects of these two limitations would therefore lead to a net exaggeration of the actual transport rate in the field. In particular, where the coast is well protected with large size beach materials or where the coastline becomes extremely irregular, theoretical values may be reduced by one order of magnitude (or two!) Nevertheless the values given in Plates III(a), (b) and V(a), (b) will indicate the potential capacity of longshore transport, and will also give a meaningful comparison between the degrees of influence it exerts on different beaches.

TABLE 7.1

THEORETICAL ESTIMATES OF LITTORAL DRIFT AND SHORE EROSION DUE TO COMPUTED LONGSHORE WAVE ENERGY FOR SOUTHERN BASIN

WEST COAST

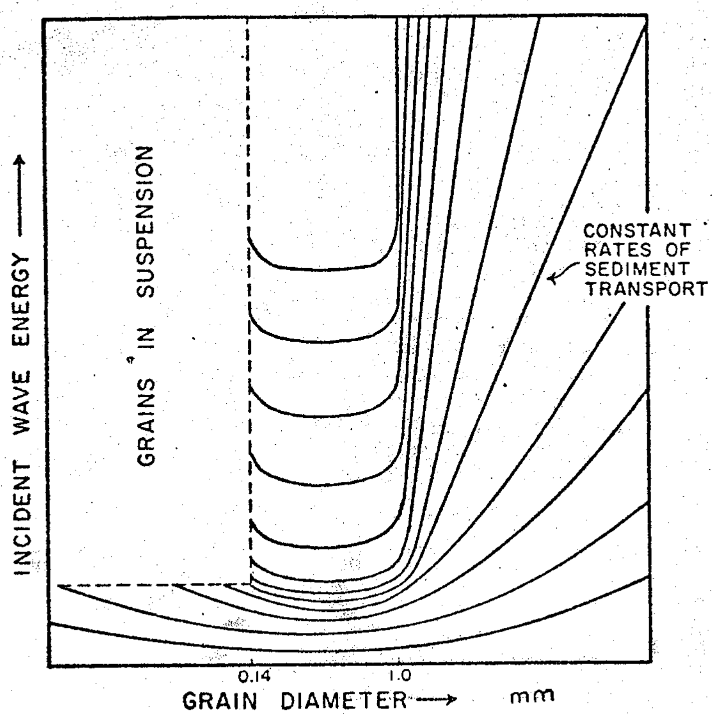
Station No.	Longshore Energy Erg/cm. (x10^{12})	Shore Erosion $ft.^3$/ linear ft.
1	- 62	
2	- 50	-19
3	-39	- 158
4	12	- 115
5	11	15
6	0	31
7	26	-71
8	21	10
9	24	-29
10	54	-33
11	24	114 & -110
12	26	-29
13		80
14	4	
15	8	-15
16	51	-198
17	51	0
18	51	0
19	77	158
20	26	-29
21	44	
22	38	31
23	66	-141
24		79
25		79
26	36	79
27	60	-46
Wpg. Beach	116	202
28		
29		
30		
31		
32		
Sans Souci	71	

TABLE 7.1 (continue)

THEORETICAL ESTIMATES LITTORAL DRIFT AND EROSION DUE TO COMPUTED LONGSHORE WAVE ENERGY

EAST COAST

Station No.	Longshore Energy Erg/cm. (x10^{12})	Shore Erosion Ft^3/linear ft.
1	-30	
		109
2	-139	
		228
3	-258	
		-18
4	-252	
5	-201	
6		
7	116	
		1001
8	-23	
		90
9	-70	
		2
10		
		2
11	-115	
		9
12	-118	
		-211
13	-41	
		-1200
14	146	
		21
15	141	
		468
16	-118	
		-87
17	-148	
		-543
18	-36	
		-73
19	-31	
		-116
20	-23	



A hypothetical energy-sediment transport envelope for the beach environment. The results of this study, together with previous theoretical, field, and laboratory evidence, suggest rapidly increasing values of wave energy (i.e., fluid velocity and turbulence) are necessary to sustain a given rate of transport when grain diameter is larger than 1.00 mm or smaller than 0.200–0.140 mm. Grains smaller than 0.140 mm are principally suspensates on most open coast beaches. Particle weight should cause slopes of transport curves to decrease when particle diameter exceeds 1.00 mm. Consequently, only sand-size sediment will exhibit ever increasing rates of bed-load transport with increasing wave energy. Dispersive grain stress (Bagnold, 1956, 1963) may well cause the critical grain diameter of 1.00 mm to be adjusted upward.

FIGURE 7.1

(After Ingle, 1966.)

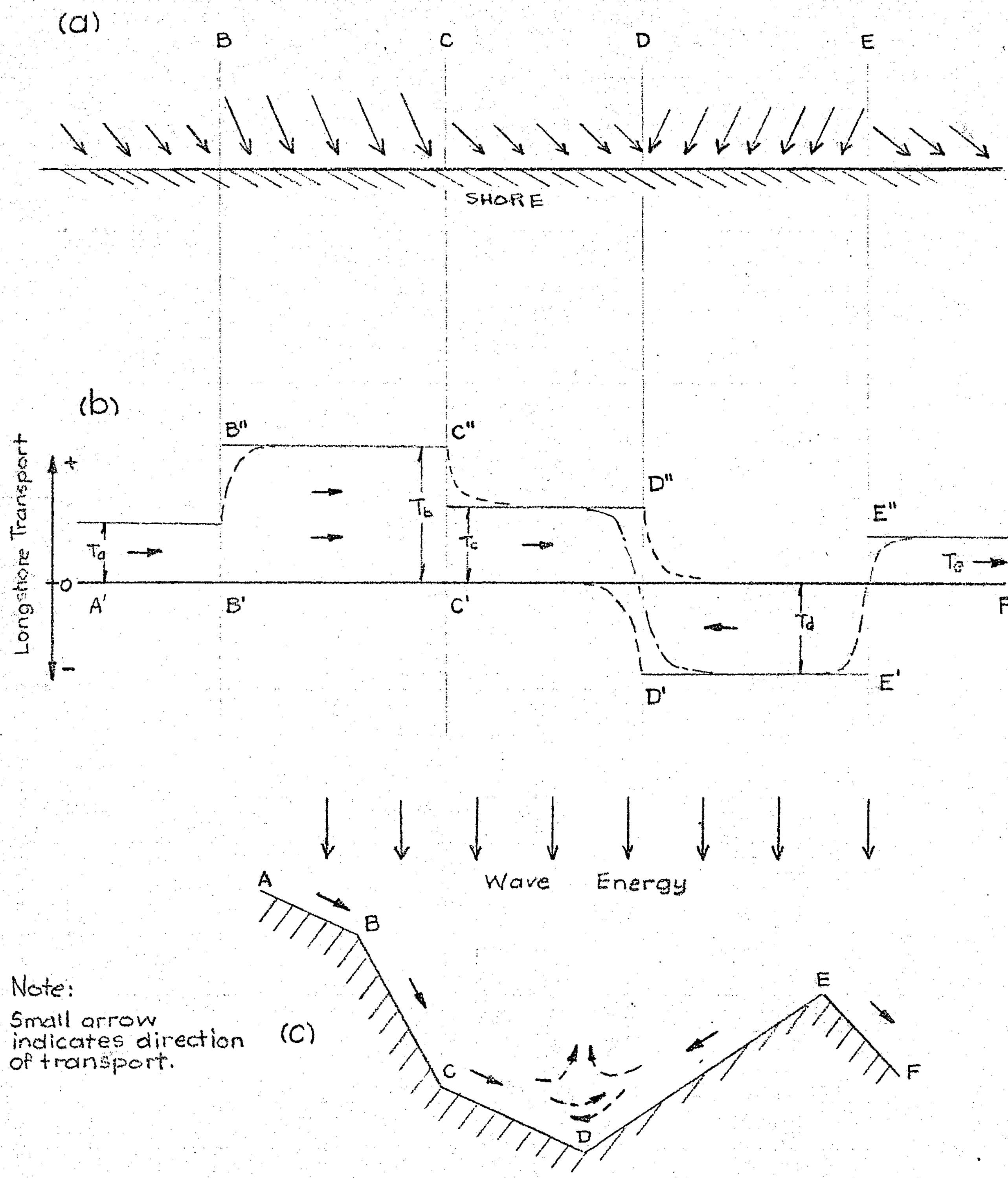


FIGURE 7-2 LONGSHORE TRANSPORT

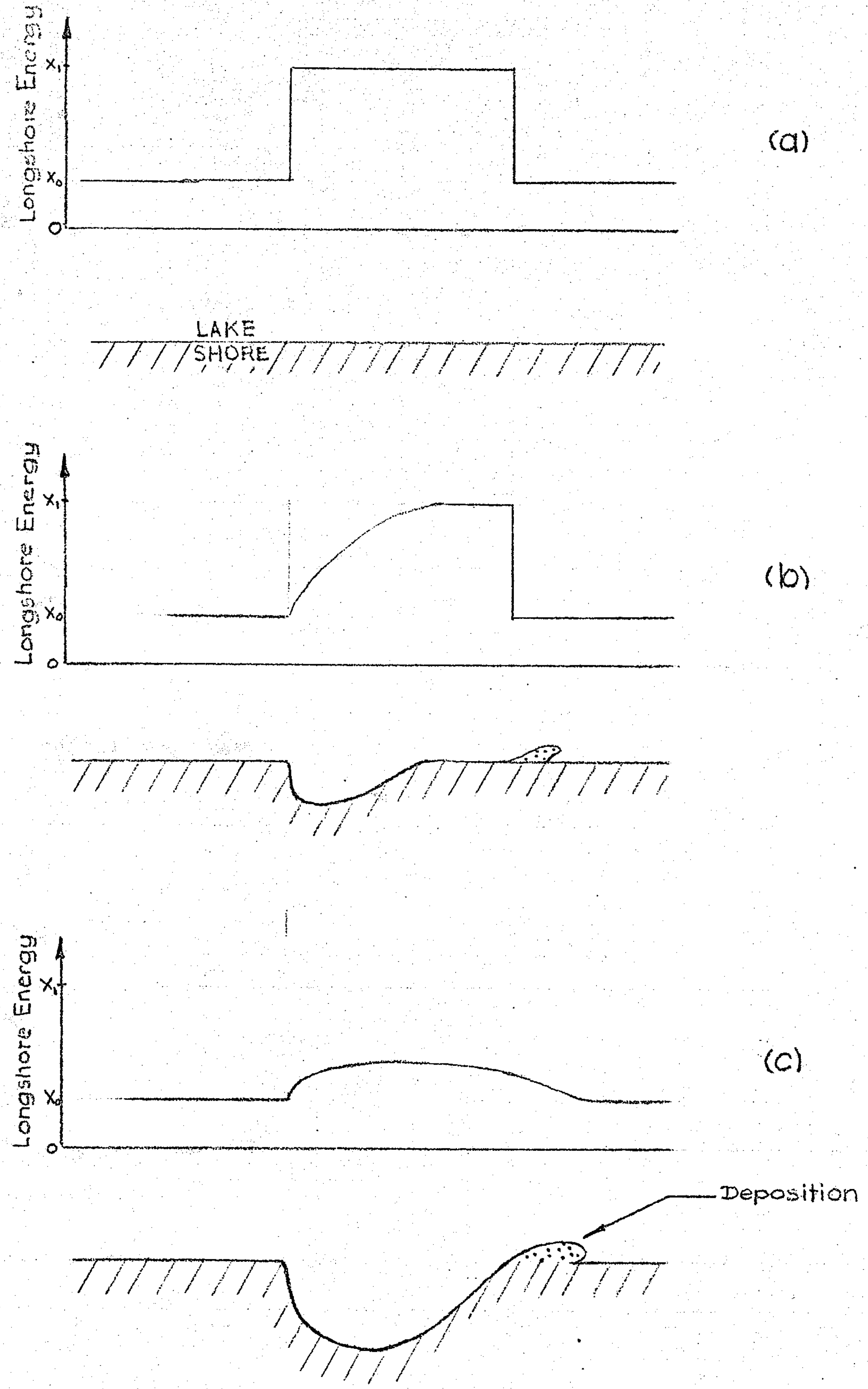


FIGURE 7-3 STAGES OF CRENULATED BAY FORMATION

CHAPTER VIII

SHORELINE RECESSION DUE TO INCREASE IN MEAN LAKE LEVEL

GENERAL

One of the objectives of this study is to provide an explanation to the phenomenon of drastic shoreline recession of Lake Winnipeg during the past five or six years. Many acres of land have been lost and lake cottages destroyed. It is reported that the shoreline has receded in the order of 20 to 70 feet or more varying from place to place, according to interviews with local people.

8.1 MECHANISMS CAUSING RECENT EROSION

Coincidental with shoreline retreat the mean water level of the lake has also been steadily increasing. It rises from a mean (summer) elevation of 712 feet in 1965 to a historical high of 717 feet in 1970. It is therefore suspected that change in water level may have caused the shoreline recession phenomenon.

A suggested answer is that the change in lake level of 5.0 feet (from elevation 712 to 717 feet) could have markedly affected the mechanism discussed in the last chapter, such that much higher "theoretical transport capacities" along the coasts of the lake are generated and shore erosion becomes more severe. Two mechanisms are affected by the increase in lake depth.

(i) In general, wave rays are striking the shore at more oblique angles. Longshore energy component, therefore the theoretical transport capacity, generally increases. (The word "generally" here is necessary, because in the case when the angle of attack before increase in lake level is greater than 45°, the longshore energy component is actually diminished). This increase, however, may not be significant because for waves with long wave lengths, the amount of bending of the wave ray from one contour to another depends more on the ratio of depth, than the absolute depth.

(ii) It has been suggested that wave-produced longshore drift is responsible for the drastic shoreline recession in the Southern Basin. With a lake level increment of 5.0 feet, the waves generated over the lake are bigger and carry more energy. Using the data collected by Wiegel, (p.231, 1964), it can be shown that for a wind velocity of, say, 30 mph, a lake 35 feet deep will have a significant wave such that, H_s = 6.35 ft., and H_s^2 = 40.5 ft.^{2} For a lake 40 feet deep, H_s = 6.65 ft. and H_s^2 = 44.2 ft.^{2} Thus there is a 10 percent increase in wave energy, since H_s^2 is proportional to wave energy. It is found that 5 foot increase in lake elevation corresponds to approximately 10 percent increase in wave energy at other significant wind velocities as well.

Upon examination of Plate III(b), the average theoretical value of shore erosion on East Coast is in the order of 100 ft.^{2} per year (1968). Assuming, rather conservatively, the eroded material is to be taken from a total height of 10 feet (which includes the total horizontal projection of the bluff, the inshore and the foreshore) the extent of theoretical recession of the shoreline is 10 ft. Due to the increase in lake level, the additional energy increment of 10 percent would mean an additional recession of about 1.0 foot per year from an original (before 1966) rate of recession of 11 feet per year.

This means that if the shore had been eroding away at 10 feet per year prior to lake level increase, it is now 11 feet per year, an increase of 10 percent or 1 foot per year. It is easy to appreciate that this additional 1 foot per year would hardly be noticeable to any proprietor, unless he has been making extremely accurate measurements.

Regardless of the original recession rate (whether 1 ft. per year or 100 ft. per year) an additional increase of 10 percent would very likely have gone undetected, and certainly does not form a likely basis for complaints.

It is important to realize that in this consideration, increase in wave energy is singled out as a mechanism for shoreline recession. This is necessary in order to demonstrate that although longshore transport may interact with other mechanisms to produce sudden shoreline recession, yet merely increasing wave energy, thereby increasing the intensity of longshore transport by raising lake level

5 feet, is by itself ineffective in causing significant erosion. Moreover, other factors could also have caused a 10 percent increase in wave energy; for instance, a year with extra strong wind could induce more than 10 percent increase in wave energy, without increasing excessive erosion as noticed in this case.

It remains to be shown that there exists a different mechanism which is responsible for rapid shoreline recession. This is the tendency of a beach to restore its equilibrium profile after the lake level increases, but before discussing this mechanism, two aspects of coastal processes have to be considered.

8.2 TWO ASPECTS OF COASTAL PROCESS

The concepts of the equilibrium beach profile and the longshore distribution of beach sands of different sizes are essential in understanding the rapid shoreline recession in Lake Winnipeg. They are described in this section, and both have been studied in detail by Zenkovitch (1966).

8.2.1 Longshore Distribution of Sand Sizes Along a Coastline

The function of sand on a beach is Nature's way of protecting the coast behind it from suffering severe erosion due to wave attacks; or at least to slow down the process. The fact that sand dissipates wave energy to attain an equilibrium profile on a beach which does not have longshore transport has already been discussed in Appendix E. The distribution of sand size along a transverse profile of the beach is also discussed in the same appendix. Here the phenomenon of sand size distribution along a coastline due to longshore transport effects will be studied. It is apparent from the ensuing discussions that sand acts as a protective mantle for the coast.

Consider a stretch of coast that is composed mainly of fine uniform materials of about 0.20 mm., which according to Ingle (1967), is the size requiring least energy to be transported. Waves of considerable magnitude attack the coastline at an angle such that a longshore sand transport is sustained. Sand is continually lost from this beach and the bluffs are cut back at the same rate to compensate for the loss. There are four possible cases:

(i) If there is a littoral drift of sand driven by a strong current flowing into this beach from the updrift end, and if the material supplied by the drift is composed of various sizes of sand, a sorting action would take place. The smaller size sands are moved out of the beach; particles that are large enough to resist swift movements due to wave energy would move more slowly while the largest particles would stay behind. As time goes on enough large size materials would have accumulated to form a beach of foreign shingles and protect the sandy bluffs from further erosion.

(ii) Consider now the other extreme case where the bluff is composed of, say, some glacial deposits of a mixture of sizes, but there is no updrift input of littoral drift. All other conditions remain the same. In this situation, the (longshore and normal) energy of waves would first erode the bluffs, but the similar sorting action again takes place; only the finer particles are transported out of the beach reach. In the end, the shore is again mantled with a cover of shingles, which are derived from the coast itself.

(iii) In nature, of course, one would generally find a combination of both situations described. It may be possible, however, that there is neither updrift input of sand nor large size grains in the composition of the bluff. In this case the coast would be left unprotected and erosion would keep cutting the shoreline back.

(iv) Another situation is that there is a supply of sand from updrift just sifficient to keep the longshore current at the beach saturated. No erosion or accumulation would take place, and the beach may or may not be covered with foreign materials, depending on whether the particles supplied are larger or smaller than that of the parent soil of the bluff.

From these four situations, it is important to notice the following.

(i) Provided sand grains are large enough, they will remain on the beach. (This is contradictory to Inman and Komar's findings that longshore transport is irrespective of sand size, (1968). Or if they do move, their motion is slow enough not to cause significant perception of erosion.

(ii) These sediments will assume an angle of repose which is characteristic of the wave energy received as well as the friction angle of sand. For a given size and type of sand, and a given magnitude and direction of energy input, the angle of repose is constant.

(iii) As the wave action continues, sand particles may collide with each other often enough that smaller grains are produced. These are transported downdrift, while a new supply of coarser grains must be moved in to replace them.

(iv) It is important to note that even if there is longshore wave energy, it does not necessarily imply that there would exist a corresponding longshore transport of sand. For this reason, along with discussions in Chapter VI, the term "theoretical longshore transport capacity" is used in this thesis. However, the longshore current will likely be able to move sand of any size smaller than those covering the beach.

(v) Longshore transport of sand is due to both the normal component of wave energy (which lowers the threshold velocity of sediments or to hold them in suspension by creating turbulence) and the longshore component (which then "moves" them along). For this reason normal wave energy component is also plotted in Plates III(d) and V(d).

The situations considered in the foregoing discussion are, of course, very much simplified versions of the actual beach, which deviates from the idealized model in the following respects.

(1) It has been implicitly assumed that the stable beach will be composed of materials of one size. On the actual beach, of course, there is also distribution if sizes

with regard to the energy distribution in the transverse section. (See Appendix E). Thus beach particles have to seek stable "equilibrium" with respect to both directions: longshore and transverse.

(2) For this variation of sizes, there is generally a smaller degree of longshore transport of the smaller particles, no matter how stable the beach is.

(3) An "equilibrium" or a "stable" beach in the strict sense of the words probably does not exist in coastal engineering. Wind direction, wind magnitude, lake elevation, and beach profile vary widely from one instant to another. The words here are used very loosely.

Having made the above observations, it is to be noted how they apply to existing concepts or how to modify these concepts for the Lake Winnipeg condition, which deserves quite a different set of considerations as compared to deep lakes, as explained below.

8.2.2 <u>Equilibrium Beach Profile or Lake Winnipeg</u>

The above discussion of transverse and alongshore movements of beach material was under the assumption that lake level or sea level remains constant during the morphological processes. However, if the sea or lake level changes, the movement of beach material is markedly affected by an additional dimension: the tendency of the beach profile to return to equilibrium.

If one considers the case of a sea coast where waves approach the beach from deep offshore water, a typical equilibrium beach profile is depicted by the full line (in Fig. 8-1(a)) extending from the edge of the sea to the deeper limit of the continental shelf. Assuming that the profile has already attained a steady state, further rise in sea level by the amount Δe, would induce further adjustment by the beach profile to attain a new equilibrium. In essence, the old profile is "jacked" up by the same vertical distance. Restrained by the definition of an equilibrium beach that the alongshore transport of sand into and out of the beach reach remains "approximately" equal, the "jacking" up of the equilibrium beach profile is only possible through the eroding of bank materials and transporting them onto the shelf. See Fig. 8-1(a). By so theorizing, Per Bruun (1962) arrived at an estimate of the distance the waterline would invade shoreward. Veldman (1968) applied this relationship to Lake Winnipeg, but obtained totally incongruous answers. It appears that Per Bruun's relationship gives shoreline recessions that are too large by an order of magnitude.

It should be noted that Per Bruun's formulation applies only to long term sea level increase, and does not include the effects of beach slope and sand sizes. It is doubtful that fine clay particles eroded from a bluff are not carried away from the beach area by wave actions in the short term case for Lake Winnipeg.

The discrepancy, however, is not surprising if one examines the basic difference between Lake Winnipeg and a seashore. One of the principal characteristics of the lake is its shallowness.

Whereas a typical beach profile extending to 1.25 miles off-shore would have a depth of 50 feet for a sea coast (Per Bruun, 1962); Lake Winnipeg, in the same consideration usually has only a depth of 16 - 18 feet. Furthermore, a "continental depth" is absent, the lake being at its deepest part no more than 40 feet deep.

It would be of interest to compare the equilibrium beach profile for Lake Winnipeg to that for a sea coast. From the above consideration, an approximate profile for a typical beach in Lake Winnipeg is depicted by Profile "A" of Fig. 8-1(b). It can be assumed that the offshore lake bottom is immobile, (or composed of non-erodible material).

The nearshore section AB is the zone comprising foreshore and inshore, and is composed of loose beach material such as sand. Survey results from Chapter III indicate that this section of the beach profile has a more gentle slope than its counterpart on a sea-coast beach profile, AB'. Yet, from theoretical consideration below, it would appear that the opposite is to be expected. Since the lake bottom, (section CD in Fig. 8-1(b)) is shallow, a portion of the wave energy must have been dissipated through friction while the waves are travelling towards shore. By the time the waves reach the inshore, there is less energy to be dissipated, so that a steeper slope should result (for beaches on the lake similar to those on the sea coast with regard to sand sizes and the degree of protection).

The discrepancy between measured results and those expected from theoretical consideration could be due to a number of factors, mainly because the mechanisms active in a shallow water lake are not necessarily the same as those in the sea. Beside being shallow, Lake Winnipeg (Southern Basin) is free from diurnal tidal effects, although wind set-up and seiche effects become much more important. In this regard, as will be explained later, the foreshore zone actually becomes the inshore zone when submerged during a storm blowing onshore. Furthermore, "swells" (decayed storm waves) is almost totally absent in the lake. In this regard, beaches do not have winter and summer profiles. Other factors such as lake bottom material (creating friction and percolation) and meteorological patterns are also influential. It is also important to bear in mind that the sand samples were taken from the surf zone and therefore tend to be coarser. This alone would render the plotting of Lake Winnipeg sand size versus beach slope incompatible with results obtained from the sea coast.

8.3 RECESSION OF SHORELINE DUE TO INCREASE IN MEAN WATER LEVEL OF LAKE WINNIPEG

Having considered the distribution of beach material and the equilibrium beach, it is in order to analyze the phenomenon of shoreline recession due to increase of lake stage. In this section, three cases are discussed. It is felt that these three examples may help to clarify some of the ideas of coastal processes in Lake Winnipeg due to increase in mean water level. To simplify theoretical developments, the assumptions below apply.

(i) Grain size distributions are perfectly uniform with regard to bluff, inshore-foreshore, offshore and littoral drift soil sizes.

(ii) Breaker line is at the intersection of foreshore and offshore.

(iii) Foreshore and inshore assume the same slope, s_1, and are composed of sands of similar sizes.

(iv) Offshore slope, s_2, has a much smaller gradient than s_1.

(v) The bluff stands at a very steep angle of repose relative to that of the inshore-foreshore. For the present purpose, it assumes an angle of 90°.

(vi) Lake elevation remains constant before and after increment of lake level.

(vii) Wave (wind) direction and magnitude remain constant.

(viii) Energy delivered to the inshore-foreshore section is not affected by the rise in lake level. This is, of course, not true because given a certain amount of wave energy, a smaller proportion will be lost by friction over the offshore lake bottom. In addition, wave energy will increase by about 10 percent due to the two mechanisms mentioned in Section 7.1.2. For the present consideration, these effects will be disregarded but will be further qualified later.

(ix) The beach is in "stable" condition before lake level increase.

A typical beach profile is as shown in Figure (8-2a).

No doubt these assumptions are very severe especially regarding sand size distributions, beach slopes, and constancy of wave properties. But only by such simplifications of the real situation can one examine the mechanism to be discussed below. The initial coastal profile for each case is FEABCD as in Fig. (8-2a) where FEA represents the bluff, AB represents the inshore-foreshore profile and BCD represents the offshore section. The lake level increment is Δe. Also assume that the sand sizes of ABC are in equilibrium with wave energy, such that no erosion is taking place.

8.3.1 CASE I

The bluff material is finer than the inshore-foreshore material, but there is a littoral drift supply of sand with grain size similar to the beach grain size.

Before the change in lake elevation, the situation is as shown in Fig. (8-2a). When lake level rises by Δe, the situation is as shown in Fig. (8-2b). The waves are now attacking point G instead of A. Since this area is not protected, erosion quickly takes place, and the bluff face GE will keep on collapsing accordingly. Since the collapsed bluff material is not in equilibrium with wave energy, it will be moved out of the beach allowing waves to attack the bluffs further inland. Meanwhile, however, sand of the same size as on AB moves in by littoral drift and is deposited on the beach such that section AA' will eventually be formed comprising of this sand and will assume the same angle of repose as AB. The wave energy from offshore can now be adequately dissipated over the slope A'AB without causing further instability with respect to sand particles, and thus causing erosion. Thus the coast is stable again. The final profile is therefore FE'A'ABC.

The recession of shoreline is the distance E'E given by:

Rise in lake level ÷ slope of foreshore-inshore or $\Delta e/s_1$ (8-1)

8.3.2 CASE II

Bluff material is the same as inshore-foreshore material. Littoral drift of sand may or may not exist.

Apparently, when waves attack the bluff at point G, (Fig. 8-3b), the bluff material there is not able to attain its present angle of repose. It will slip into water over section AB. After a sufficient volume of the bluff has been eroded such that the point of intersection of the bluff slope and the inshore-foreshore slope delimits the highest point of wave attack, a stable condition is again attained. The actual final profile may be as FE'A'B'C, but if the extra amount of sand over AB is not too thick (i.e., if the distance between AB and A'B' is not large) it may be approximated by FE'A'ABCD. In this case shoreline recession can again be given as:

$$\Delta e/s_1 \qquad (8\text{-}2)$$

8.3.3 CASE III

Bluff material is similar to or finer than offshore material (Fig. 8-4).

When waves attack G with the rise of lake level, the bluff material will move off the beach by littoral current. Unless it is very easily suspendible, it will not move offshore of the breaker line. This follows from Ingle's observations (1966).

The shoreline will keep on retreating, but coarser grains initially on AB remain within the beach reach and stay on the similar relative position with respect to the water line. The angle of repose as well as the vertical projection of AB (in the "moving" state) also remain the same. The final stable situation is reached only after the vertical projection of B"B, which is the newly exposed "offshore" due to erosion of the section E"A"B"BAEE", and which has the slope s_2, is equal to Δe. If bluff material is similar to offshore material the recession is given by

$$\Delta e/s_2 \qquad (8-3)$$

The final profile is FE'B'C' as per Fig.8-4-(c). But if the bluff material is finer than offshore material, the slope of B"B, unless it becomes mantled by foreign soils, is less than s_2 and,

$$\text{recession} > \Delta e/s_2 \qquad (8-4)$$

It is likely, however, that B"B may become mantled by foreign material of larger size than that of the parent soil. In this case, the actual recession may be between that given by equation (8-3) and Inequality (8-4), thus,

$$\text{recession} \geq \Delta e/s_2 \qquad (8-5)$$

8.3.4 Intermediate Situations

In reality, the size of bluff soil is seldom equal to soil size of inshore-foreshore or offshore. The following three cases may be distinguished.

(i) Bluff soil size finer than off-shore size, recession is given by Equation (8-5) above; i.e.

$$\text{recession} \geqslant \Delta e / s_2 \qquad (8\text{-}5)$$

(ii) Bluff soil size finer than or equal to that of inshore-foreshore zone but greater than or equal to that of offshore;

$$\Delta e / s_2 \geqslant \text{recession} \geqslant \Delta e / s_1 \qquad (8\text{-}6)$$

(iii) Bluff grain size greater than or equal to that of foreshore-inshore;

$$\text{recession} \leqslant \Delta e / s_1 \qquad (8\text{-}7)$$

8.3.5 Comments

There were a number of assumptions which made the foregoing derivations possible. Here it is in order to review some of these assumptions.

(i) Sand from the bluff, shore or littoral drift was assumed to be of uniform size. This is similar to assuming an effective sand size for each part of the shore profile. The real situation is much more complex, but the use of effective sand size should not affect the underlying logic of the derivation.

(ii) That wave energy received by the inshore-foreshore portion of the profile is higher due to increase in lake level has already been explained. A further complication is that as the length of inshore-foreshore section increases due to increase in lake level as in Case I and Case II energy dissipated per unit area in this zone will be less, all others being equal. These two effects seem to be compensating, but, in general, it is felt (since no mathematical solution or field measurement is yet possible) that energy dissipation per unit area of inshore-foreshore actually increases. This will cause the sand there to assume a more gentle slope. In the limit, if the depth of the lake is comparable to that of oceanic coasts, and a large amount of sand is available for the wave to carve out its own beach, the slope of beaches on Lake Winnipeg will approach that of the sea coast.

(iii) It has been assumed that the inshore-foreshore region assumes a slope of s_1 extending far out to the breaker line. On normal days this is not true for the majority of beaches. The beach front of the backshore above waterline on a typical beach profile slopes down into water at a constant slope for some fifty feet or so, and then flattens off into a more gentle slope, which continues for some distance until the breaker line or perhaps even farther. (See Fig. 8-4). The breaker line is usually in the order of four hundred feet away from the water's edge. Thus it would appear that inshore-foreshore actually breaks down into two sections of different slopes. This realization would nto induce too much complication in calculating shoreline recession. The logic of the theory remains the same; only an additional slope has to be considered.

Moreover, it is not known where the offshore zone begins on the beach profile when a storm is blowing over the lake. Strong wind will cause wind set-up and seiches over the lake, to the effect that an increase of water level of the order of a meter may be induced on the windward side of the lake (Einarsson and Lowe, 1968). Waves will now break closer to shore than on normal days. It is likely that the effective breaker line on such occasions is in the proximity of the intersection of the two slopes as surveyed. If this is the case, no complication has actually been introduced by the fact that two slopes have been detected on the part of beach profile which is the inshore-foreshore zone on normal days. It is likely that during a storm, the backshore on normal days has become part of the inshore-foreshore zone and the breaker line is located inside the inshore-foreshore zone for normal days. Because of the fact that storms are the most important factor in carving beach profiles, what have been measured in our surveys during normal days are indirect measurements of storm conditions, when the most effective sculpturing mechanism of the morphological processes are taking place. In this light, the two slopes are identified as s_1 and s_2 for Inequalities (8-5) through (8-7).

On beaches where only one slope could be measured, because the rodman was not able to wade far into deep water, the measured slope is identified as s_1. s_2 will be some unknown value until further surveying is possible.

(iv) The backshore has been left out from the considerations of Fig. (8-2) through Fig. (8-4) for the reason explained above: that backshore does not exist on Lake Winnipeg beaches under storm conditions.

(v) The assumption that the beaches were stable before increase of lake level appears to be questionable. Some beaches must definitely be eroding to supply some form of longshore transport, prior to the high water period. The assumption is used here for a primary (first) evaluation of shoreline recession.

8.4 SHORELINE RECESSION

8.4.1 Computed Results and Qualifications

Using inequalities (8-5), (8-6) and (8-7) extents of shoreline recession due to the rise of lake level are shown in Plates III(c) and V(c). However, the values indicated must not be heavily relied upon without applying the following qualifications.

(i) These values are "order of magnitude" values.

(ii) The extent of shoreline recession depends on the bluff material of the coast. Studies regarding bluff grain size distribution are necessary to permit proper interpretation. In this regard, not only the mean sizes are important, but attention must also be paid to whether there are irregularities in the composition of the bluff (such as layers of clay and sand) and the distribution of sizes such as whether there are extra large stones or boulders.

(iii) Plates III(c) and V(c) are derived from the survey results of 1970. Locations surveyed may not be typical of the beaches of their neighbourhood. For instance, the survey line may have been run out from the backyard of a cottage beside a small rock groin. In this regard, the estimates shown in the plates may be rather inaccurate.

(iv) At locations where cottages along the coast have erected fairly solid protection structures, estimates are unrealistic.

(v) Estimates of Plate III(c) and V(c), indicate only the limiting values of shoreline recession. For the case where the bluff material is finer than offshore material, the value given is a lower estimate.

(vi) The height of bluff is also an important consideration. It is clear that the higher the bluff, given the same soil type, it takes longer to erode away one linear foot (along the beach profile) of the bluff. This factor would seriously influence the rate of erosion.

8.4.2 Combined Mechanism Causing Shoreline Recession

Because waves are now able to dislodge the soils of the bluffs, which have smaller grain size than the soils on the inshore-foreshore, the littoral current is able to carry off the dislodged particles. The rate of recession of shoreline, therefore, depends considerably upon the magnitude of longshore energy as well. If transport capacity is low, the soil dislodged from the bluff will tend to remain on the beach profile and probably impede further erosion. Thus Plates III(c) and V(c), and Plates III(a), (b) and V(a) and (b) should be used together. Where both figures indicate high erosion values, it is likely that a high rate of erosion does occur. Where longshore transport graph indicates deposition, it does not mean there will not be recession of shoreline; but only a slower rate of recession may be expected, and that the combined effect may not be as severe as Plates III(c) or V(c) alone would indicate.

It is definitely true that by combining the figures of Plates III and V one may arrive at the rate of shoreline recession, say, in terms of feet per 25 years. (That is, of course, assuming all bluffs are of the same characteristics. In the complete analysis, effects of (vi) above must also be incorporated). However, it is not known what method to use for evaluating the combined effects of longshore transport and lake level increase quantitatively. Furthermore, it is felt that in the uncombined state, the use of these two graphs will tend to promote better understanding of the physical processes involved.

Another reason is that both graphs involve some drastic assumptions in their derivations, hence, the dependability after combination cannot be easily estimated. Furthermore, both predict only the limiting values even if sufficient information (re: bluff height and soil sizes for instance) is supplied - such information is presently lacking. Nevertheless, in the present case of high lake level, I believe that the emphasis is on Plates III(c) and V(c).

It has commonly been accepted in the past that longshore wave energy is the dominant factor responsible for beach erosion on beaches of Lake Winnipeg. From the present study, it appears that although longshore energy plays an important part in causing shoreline recession, it is the tendency of a beach to return to equilibrium that really plays the dominant role. To illustrate this point, consider a beach which is exposed to a steady but low supply of longshore energy along the shoreline. Extensive shoreline recession would still occur provided the equilibrium beach slope is small. Such is the case of, say, Patricia Beach, at the south end of Southern Basin.

8.4.3 Whether Erosion or Recession?

It is interesting to note that in certain, if not all, situations what the local people term as "shore erosion" is not erosion at all. Fig. (8-5) shows such a situation. Due to the increase in lake level, the waterline has invaded the shore by $\Delta e/s_1$ units of distance. For a practical example, say, $\Delta e = 5$ ft., $s_1 = .07$ and $\Delta e/s_1 = 70$ ft. If there is a steep bluff, it may take a period of time (say 25 years) for the shore to be eroded back 70 feet. If there is no steep bluff as in Fig. (8-5), it is easy to see the shoreline would recede 70 feet instantly without erosion. In this case, the term "beach erosion" appears to be a misnomer: the beach profile has changed little, only the waterline has risen!

8.4.4 Qualitative Evaluations

Despite the difficulty in calculating quantitative shoreline recession due to the combined mechanism mentioned above, it is possible to make some qualitative assessments here. Extent of shoreline recession depends on the following factors.

(a) Amount of wave energy - Increase of lake level by 5.0 feet increases wave energy by 10 percent. Also, because wind from the west is more frequent, the East Coast receives more energy than the West Coast. All others being equal, bluffs on East Coast would be eroded more rapidly.

(b) Bluff height - Bluffs on East Coast are significantly higher than bluffs on West Coast. All others being equal, West Coast would recede more rapidly and severely on this account.

(c) Bluff material grain size - Bluffs on East Coast have more sand and larger size materials. The West Coast has more clay bluffs, which although are resistant to wave attack, would eventually yield. Thus all others being equal, bluffs on West Coast would be eroded slowly, but more extensively. At the present moment, bluffs at, say, Camp Morton are yielding slowly.

(d) Longshore transport capacity - If the longshore current from the updrift reach is already saturated with sediment in transport, erosion at downdrift reach will not be severe. Whether the longshore transport capacity is saturated (with sediment in transport) depends on the availability of sand of proper grain size from updrift. Thus the West Coast, having poorer supplies of sand, would probably recede slightly farther at places than the East Coast, all others being equal.

(e) Equilibrium Beach Slope - This slope depends on the grain size of material available and wave energy. The slope may vary significantly from one beach to another. As detailed in Sections 8.4 and 8.5 estimated shoreline recession calculated by using beach slope is depicted by Plates III(c) and V(c).

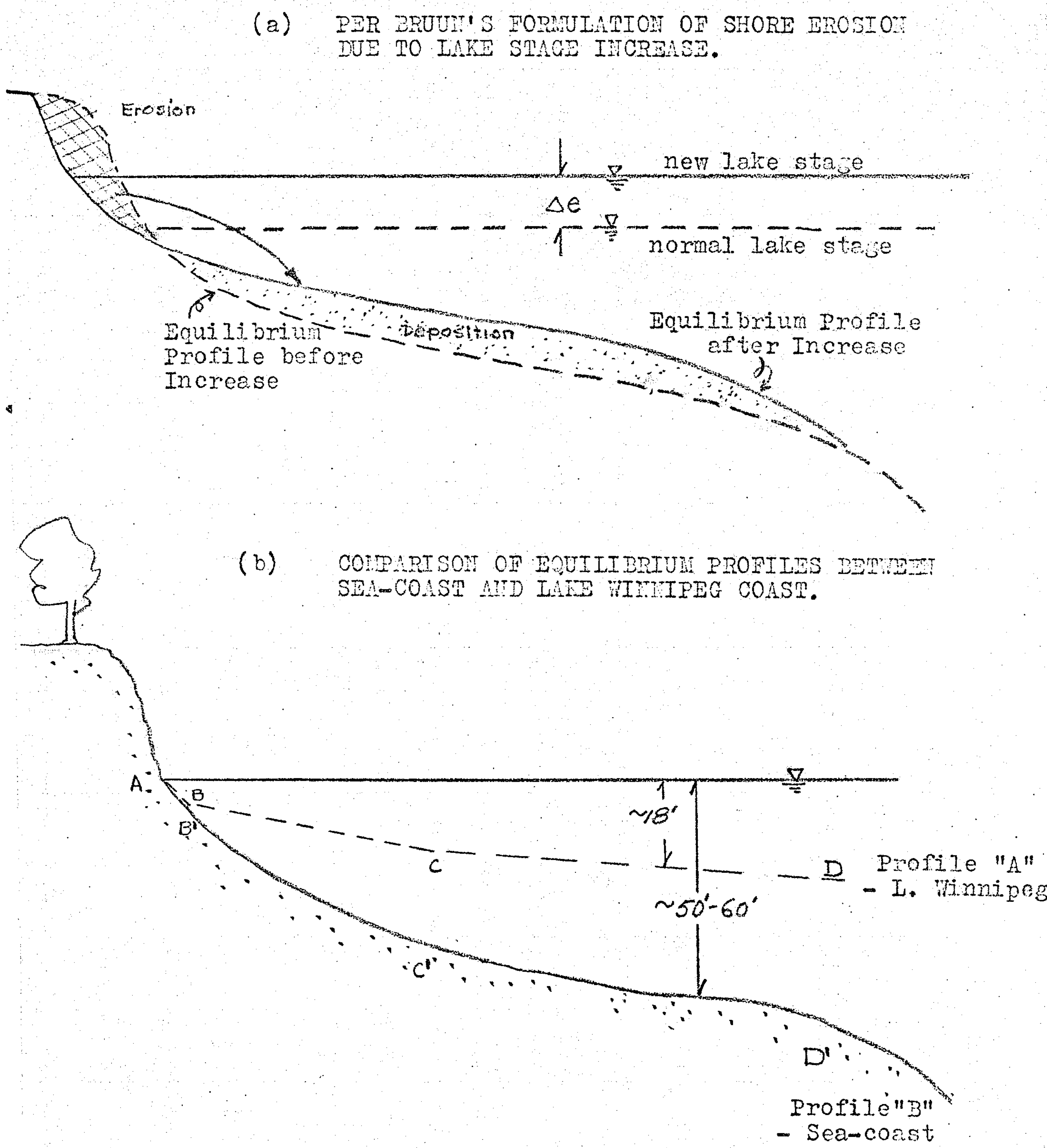


FIGURE 8.1 EQUILIBRIUM PROFILES.

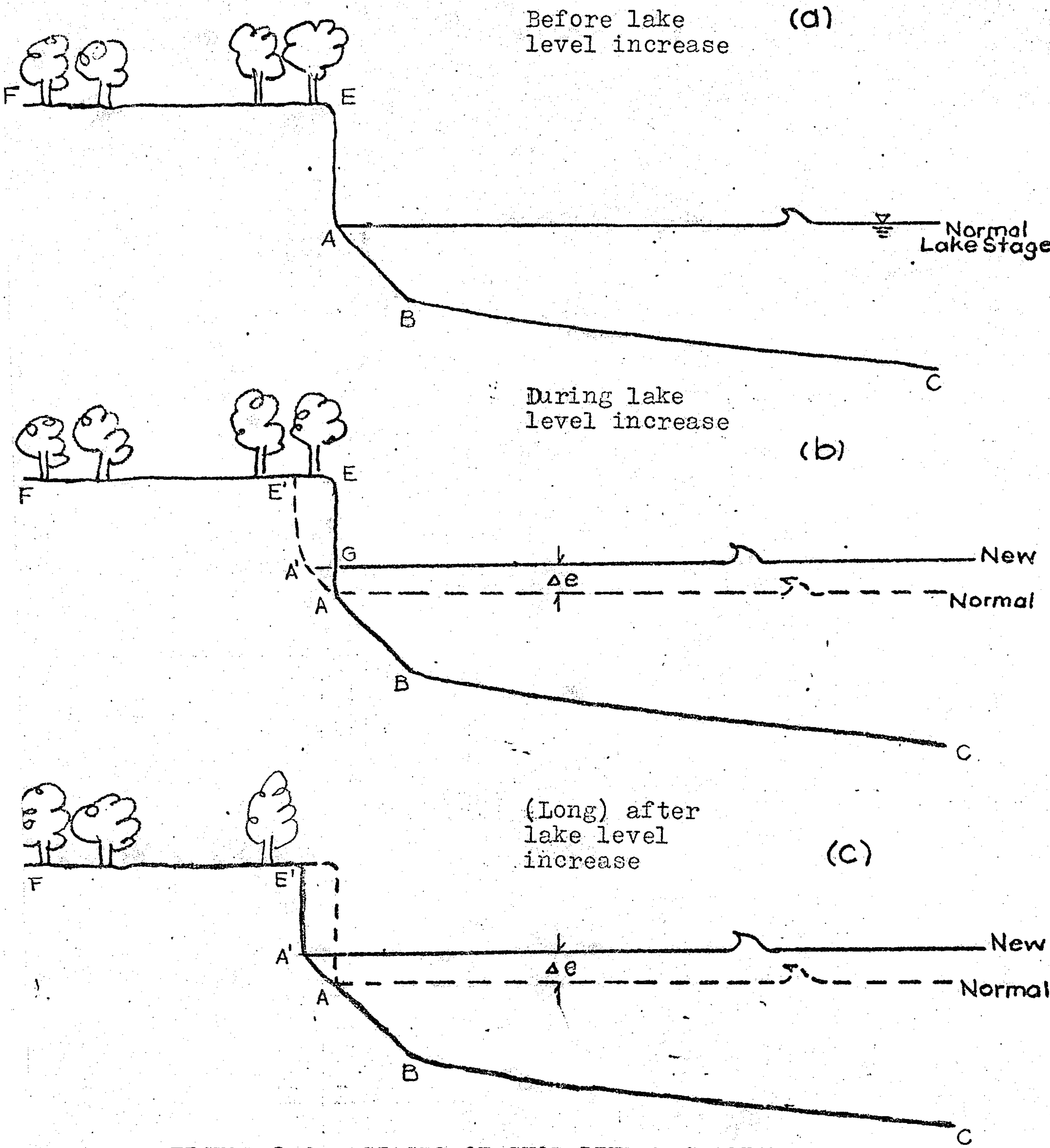


FIGURE 8.2 STAGES OF SHORELINE RECESSION DUE TO LAKE LEVEL INCREASE.

CASE I: Bluff material is finer than inshore-foreshore material; there may be a littoral drift supply of sand of size similar to the inshore-foreshore sand grain size. Recession = $\Delta e/s_1$.

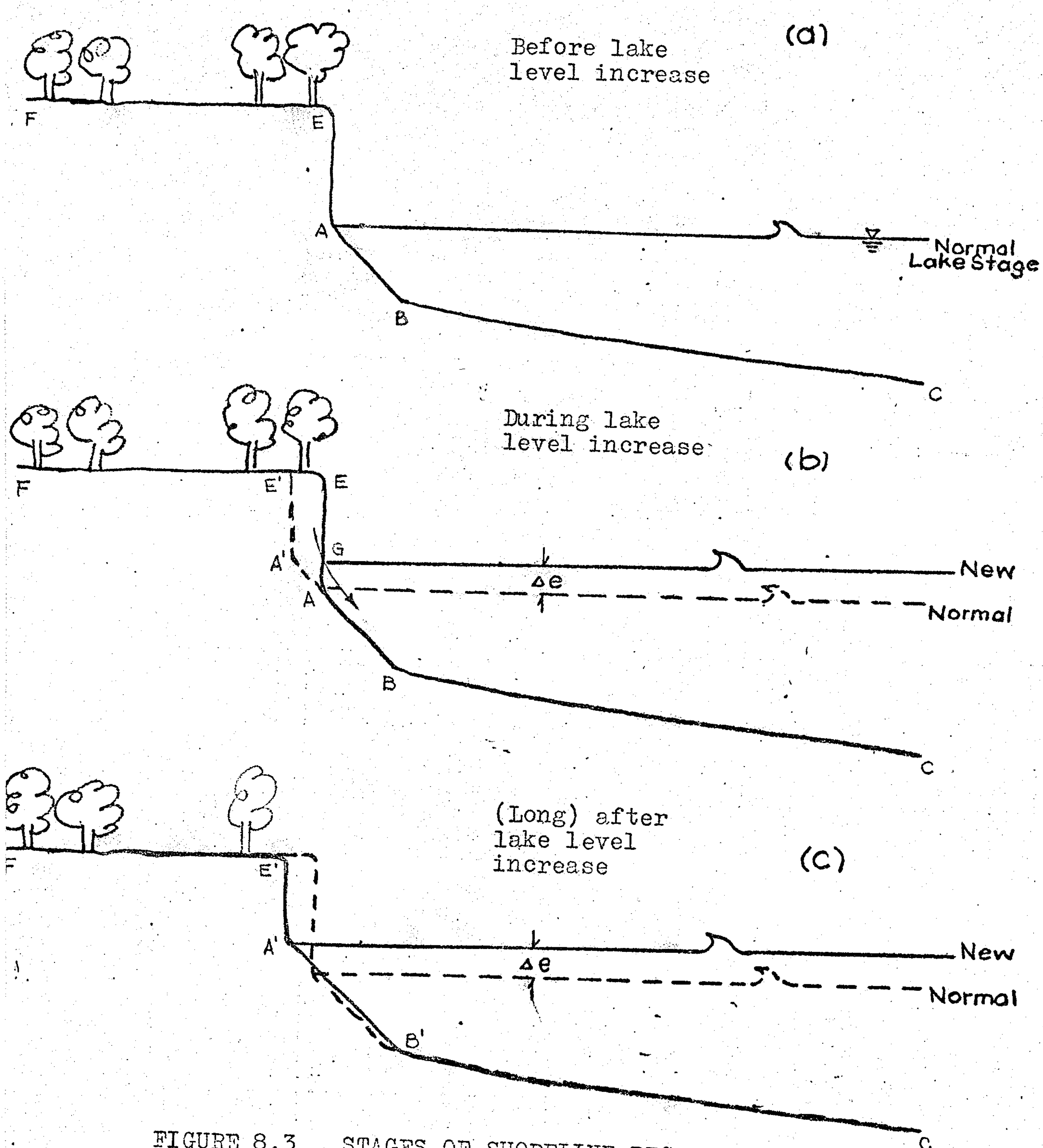


FIGURE 8.3 STAGES OF SHORELINE RECESSION DUE TO LAKE LEVEL INCREASE.

CASE II: Bluff material is same as inshore-foreshore material. Littoral drift of sand may or may not exist. Recession $\doteqdot \Delta e / S_1$.

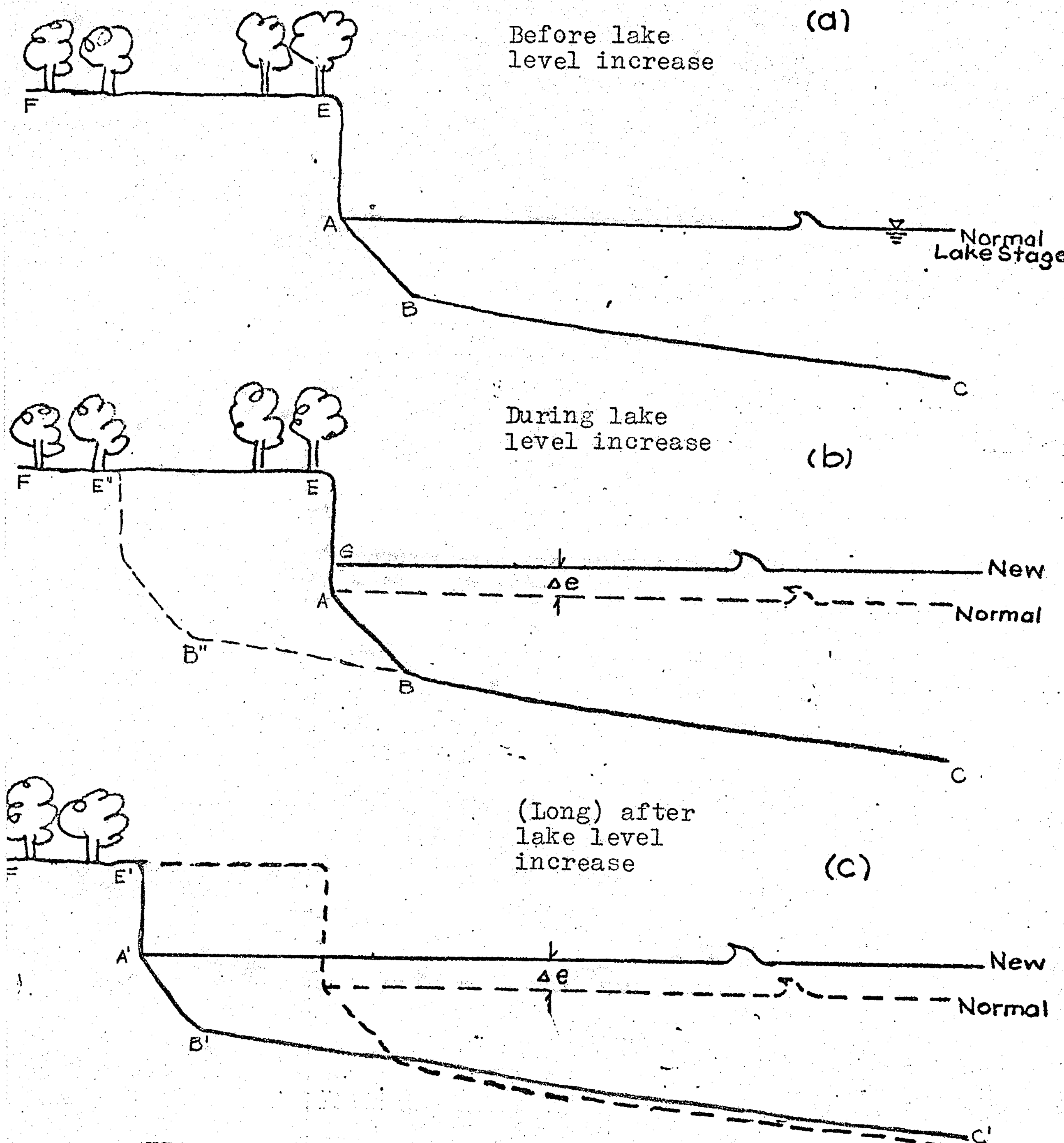


FIGURE 8.4 STAGES OF SHORELINE RECESSION DUE TO LAKE LEVEL INCREASE.

CASE III: Bluff material similar to of finer than off-shore material. Recession $\geqslant \Delta e/S_2$.

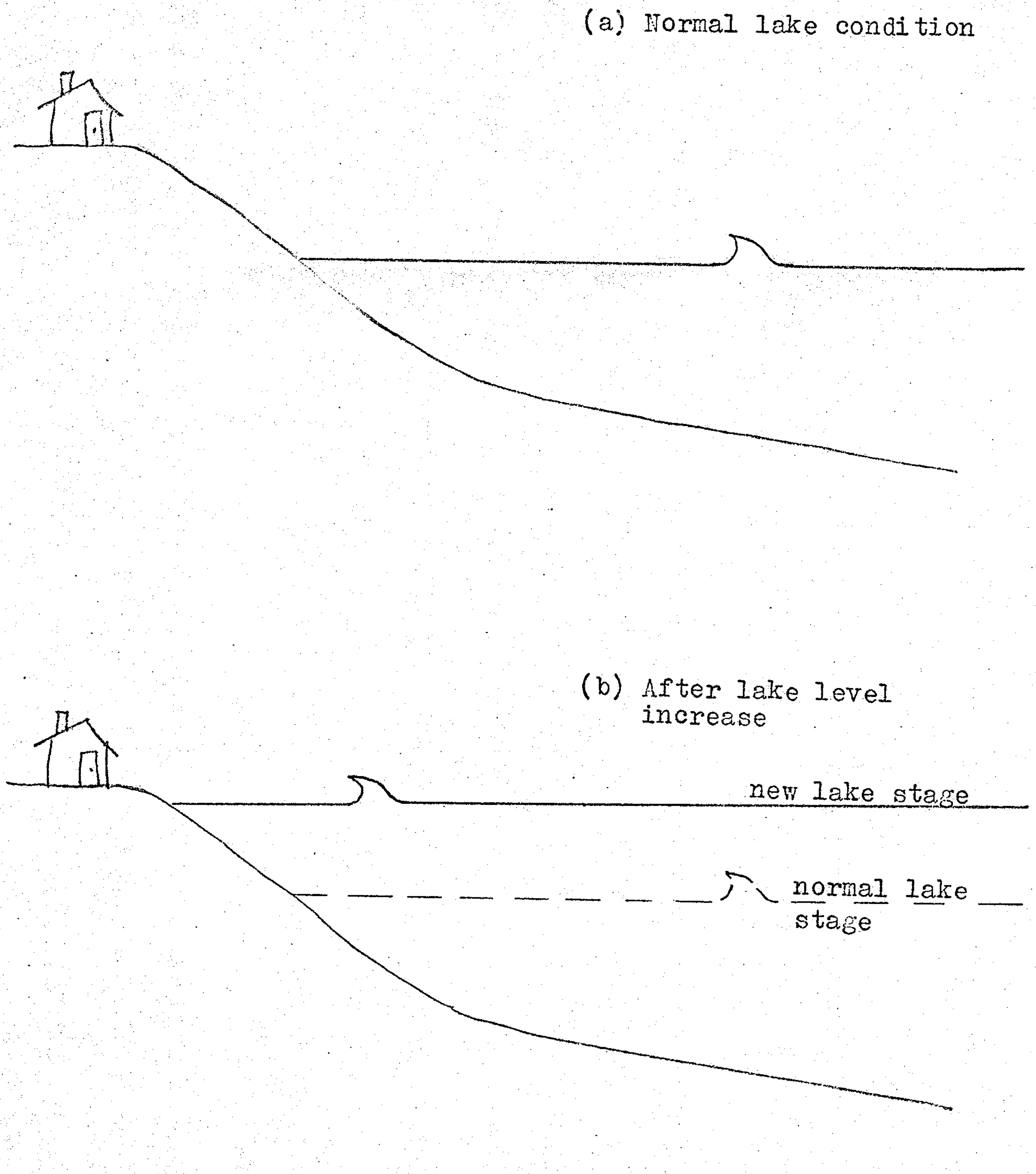


FIGURE 8.5 EFFECT OF LAKE STAGE INCREASE WITHOUT EROSION.

(b) may create the impression that erosion has taken place.

CHAPTER IX

SOME COMMENTS ON BEACH PROTECTION METHODS AND MODEL STUDIES

9.1 GENERAL

In this chapter three aspects related to shore protection and coastal processes in Southern Basin will be described. One of the aspects is the study of existing shore protection structures; the second aspect concerns the effects of these beach protection structures on coastal processes, and the approach for evaluating methods of shore protection; the last aspect is the study of coastal processes and shoreline protection by using the model of Winnipeg Beach built by Veldman, (1968). (In this chapter "Winnipeg Beach" refers to the beach rather than the town of Winnipeg Beach).

9.2 SHORELINE PROTECTION STRUCTURES

While surveying the coastline of the Southern Basin, various types of beach protection structures erected by private cottage owners were encountered. A photographic record of these structures was collected. Due to limitation of space and time, a detailed analysis of these coastal structures is not presented herein, but it may suffice to point out a few interesting aspects regarding their limitations and functions.

The existing coastal protection structures on Southern Basin belong to either one of the two types. One type is the extension of a structure into the water. (See Fig. 9-1). The function of such a structure is to hold back the littoral drift rather than to interfere with the wave energy before it attacks the shore behind it. In most cases, these structures, often in the form of rock groins, are generally located at the mean waterline; this would render them rather ineffective where the set-up due to wind is significant. In storm conditions they contribute little to dissipate the wave energy which attacks the bluffs above and behind them.

The second type of shore protection structure is the "wall" type, as in Figs. 9-2 and 9-3. These are generally located above the mean water line, and may be permeable or impermeable. Their primary function appears to be the dissipation of wave energy. Some of these sea walls appear to be performing quite efficiently while others may not have been adequately designed. Too low a wall would allow the waves to attack the bluff above it. An impermeable and vertical structure generally causes underscouring at its toe due to the intense turbulence created, eventually leading to its (partial) failure. A permeable wall, composed principally of large boulders, may allow the waves to induce a pumping action and scour the bluffs behind it.

While there is no sure answer to the problem of beach protection, it does appear that a relatively economical and effective design is a permeable seawall composed of large rocks and boulders on the outside, (facing the lake) but gradually grading into gravels and stones of smaller sizes on the bluff-side. This would ensure effective dissipation of energy without developing intense turbulence. In fact, it is similar in principle to the rip-rap on a dam. For an impermeable type of protection structure, there are many variations of tested designs recommended by the Coastal Engineering Research Centre, (1956).

9.3 RE: BEACH PROTECTION

With increase in lake level, waves are now able to attack the bluffs which they could not reach under normal lake conditions; or the waterline will now extend far inland at locations where bluffs are low and bluff slopes are gentle. This creates an apparent threat of property loss to local people, who consequently erect beach protection structures of various sizes and shapes along the coast.

The presence of these protection structures appears to create two effects. Firstly where these structures are not functioning properly, they tend to generate more turbulence, thus enhancing sand transport and beach erosion. Secondly, where these structures are successful in retaining beach material, the longshore transport current will be starved of sand supply. When this starved current reaches an unprotected downdrift beach where longshore wave energy has not decreased, excessive erosion will take place.

(As an illustrative example, before lake level increase, the sand transport capacity of the littoral current is, say, 50 percent saturated, noting that from discussions of Section 7.6, 100 percent saturation is not necessary. After lake stage increase and shore protection structures constructed, there is enough sand to effect, say, only a 10 percent saturation. This is referred to herein as a state of "starvation". This starved current, therefore, has a strong "urge" to pick up sand, thus causing erosion, due to the additional deficiency of sand in transport).

Thus along a reach where there is a significant deficiency of sand in transport, severe local erosion would take place if the entire shoreline is not fully lined with effective protection structures. In fact, at locations in

Southern Basin where erosion is severe, sporadic efforts by individuals have led to the aggregated effect that the shoreline is practically 100 percent armoured with seawalls or groins. The rate of shoreline recession is consequently slowed down to an extent, but at the same time any sand on the beach is more likely to be removed by the starved longshore current. Furthermore, the sand that is left behind is generally coarse. To what degree this process is manifested in the lake is not known, but the process is detrimental to recreation and resort areas where the quality and quantity of beach sand are important attractions to tourists. In the long run, if the lake level continues to remain high, artificial intervention may be needed to ensure the presence of sandy beach at a given location by erecting large groins or offshore breakwaters, or by sand nourishment.

For setting up a list of priority for beach protection, Plates III(b), (c) and Plates V(b), (c) could be used. Attention must be focused upon those areas where the graphs predict severe shoreline recession. On the other hand, at locations where "theoretical erosion or accretion" graphs, Plates III(b) and V(b), indicate accretion, and where bluffs are low, and land property value is not high (that is, no houses or cottages) it may not be necessary to construct expensive protection structures. Once the high lake stage subsides, lost land and wide beaches would re-emerge.

It is possible to protect shores and maintain beaches by erecting structures such as seawalls and groins mentioned above, or by means of beach sand nourishment. The last method of beach sand nourishment has been successfully applied to beaches on the sea coast. Its effective application to Southern Basin is expected to involve a trial-and-error approach, but a (rough) first estimate of the ideal points of application and the quantity of sand required could probably be derived from Plates III(a), (b) and V(a), (b), provided they are used with caution. For instance, sand may be dumped in large quantities at the breakwater of Winnipeg Beach. This sand may eventually come to rest at Stephen's Point where it can be dredged and hauled back to the breakwater again for re-deposition. The economics of this method, however, cannot be readily evaluated without further studies.

9.4 MODEL STUDY

Since Veldman used his model to investigate certain aspects of coastal processes taking place in Winnipeg Beach, the model has been utilized for studies by Huggins and Edgehill (1969) and by Bray and Burgess, (1970). The results of these studies are summarized in Table 9-1. Huggins and Edgehill studied the accumulation of sand at Winnipeg Beach Harbour while Bray and Burgess investigated the effectiveness of groins for beach protection. It is to be noticed that, of these three studies, only Bray and Burgess ran their tests under consistent lake conditions (that is constant sand feed and constant wave energy) and also carried each test long enough to obtain the steady state. Their results showed that the use of groins is indeed one of the answers to the problem of shore erosion at Winnipeg Beach.

During the summer of 1970 the author intended to extend Bray and Burgess' study to investigate the best arrangement of groins with regard to different testing conditions. However, due to time limitation, only a smaller number of the series of tests planned were carried out. The partial results, nevertheless, confirm the findings of Bray and Burgess and also indicate a slightly better arrangement of groins. These results are presently on file at the University of Manitoba, and are not published in this thesis because of lack of completeness. It is hoped that the remaining tests in the series will be completed in the near future.

Another series of tests was also planned for the summer of 1970 and also came short of completion for the similar reason. This series of tests was designed to confirm the effects of lake level increase as hypothesized in the previous chapter. It would appear that continuation of this series of experiments should be implemented as soon as possible.

From the conclusions of Chapter VIII, it also appears that using the sand accumulated at Stephen's Point to nourish Winnipeg Beach is another likely consideration of beach protection. Thus future model studies should also include studies for sand nourishment.

TABLE 9.1

WINNIPEG BEACH MODEL STUDY
Tests performed prior to May 1970 — SUMMARY

ABBREVIATIONS: WMV — Results from Ref. No. 131
EH — " " Ref. No. 49
BB — " " Ref. No. 8

By	Test No.	Lake Stage (Ft)	Wave Height (Ft)	Wave Length (Ft)	Wave Period (Sec)	Rate of Feed (lb/hr)	Length of Test (Hr)	MAN-MADE STRUCTURES (other than seawall as per 1968): With or without Breakwater	Other Structures	Type	Length (Ft)	Orientation	Location	Initial Beach Form	Final Beach Form	Remarks
WMV	1	715	4.3	128	5	NIL	1	Yes	NIL	—	—	—	—	As per March 1968	(1) Minor Bar at Breakwater; (2) Erosion updrift of rip rap (3) Even contours along seawall.	
WMV	2	712	3.0	"	"	20lb after 1hr. 5lb after 1.5hr	2	"	NIL	—	—	—	—	Ditto	(1) Bar in harbour moves inshore.	
WMV	3(a)	717	4.9	Ditto		10lb after 1¼hr	2	"	NIL	—	—	—	—	Ditto	(1) Bar in harbour (2) Erosion updrift of riprap	
	(b)	714	3.8	"	"	20lb before test 20lb after 1hr.	2	" "	NIL	—	—	—	—	As per final beach form of Test 3(a)	(1) Bar built up continues to form beach (rate slower than in test 3(a)) (2) Scour offshore of riprap.	
WMV	4	714	3.8	"	"	NIL	1	"	NIL	—	—	—	—	As per March 1968 Sand dumped 1000' off ℄ breakwater	(1) Sand moves S.W. (2) Some sand redeposits in harbour	
WMV	5	714	3.8	"	"	1lb after ¾hr	1	"	Off-shore breakwater	submerged concrete blocks	600'	SSW	Off-shore of rip-rap	As per March 1968	(1) No erosion at rip-rap area	Breakwater appears to hold back erosion at rip-rap
WM	6	714	3.8	"	"	1lb periodically	2	"	Off-shore breakwater	submerged concrete blocks	600'	N-S	Off-shore of old seawall	Ditto	Result claimed to be similar to that of Test 3(b)	Breakwater appears ineffective
WMV	7	713	3.5	"	"	NIL	1	"	Off-shore breakwater	"solid"	500'	SSW	off-shore of riprap	Ditto	Result similar to that of Test 5	
WMV	8(a)	717	4.9	"	"	1lb periodically	2	"	Breakwater	Impermeable similar to old	160'	20°R of ℄ of old breakwater into harbour	attached to old breakwater	Ditto	(1) Bar built up from breakwater towards seawall	
	(b)	714	3.8	"	"	Ditto	2	"	Breakwater	"	"	"	"	As per final beach form of Test 8(a)	(1) Bar built-up in harbour (2) Erosion offshore of riprap	
WMV	9(a)	717	4.9	"	"	Ditto	2	"	Breakwater	Ditto	Ditto	20°L of ℄ old breakwater	Ditto	As per March '68	(1) Heavy bar built-up in harbour	
	(b)	714	3.8	"	"	Ditto	2	"	Breakwater	"	"	"	"	As per final beach form of Test 9(a)	(1) Bar moves shoreward (2) Serious erosion offshore of riprap	
WMV	10(a)	717	4.9	"	"	Ditto	2	"	Breakwater	Impermeable Crest El of 712	160'	20°R of ℄ of Old Breakwater	Ditto	As per March 1968	(1) Small bar in harbour (2) Bar moves farther inshore	(1) Movement around breakwater reduced
	(b)	714	3.8	"	"		3	"	Breakwater				"	As per final beach form of Test 10(a)	Results same as for Test 10(a)	

WINNIPEG BEACH MODEL STUDY

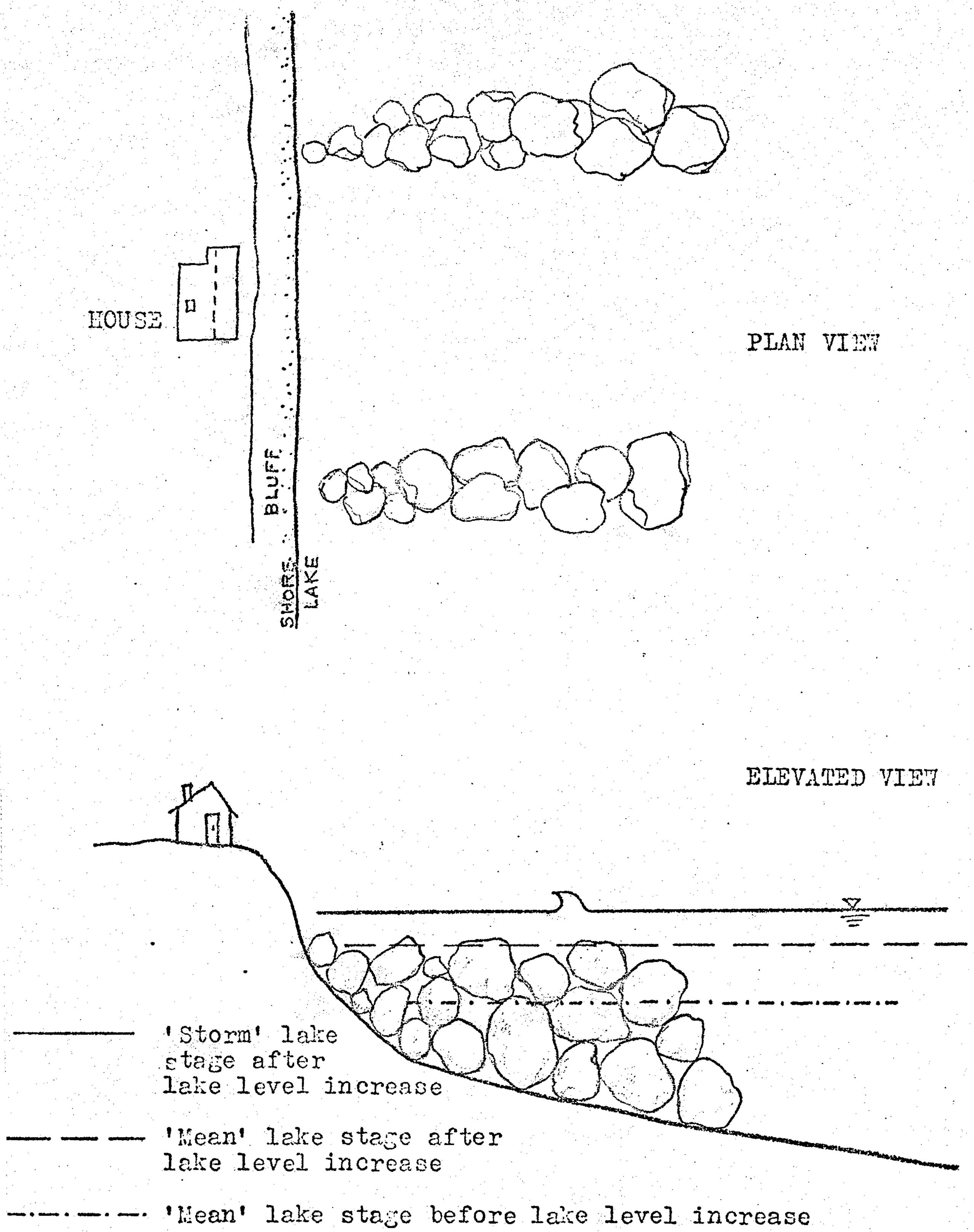
TABLE 5-1 (cont'd)

TEST NO	LAKE STAGE (FT)	WAVE HEIGHT (FT)	WAVE LENGTH (FT)	PERIOD (SEC)	RATE OF FEED (lb/hr)	LENGTH OF TEST (HR)	MAN-MADE STRUCTURES (OTHER THAN SEA WALL AS PER 1968) WITH OR WITHOUT BREAKWATER	OTHER STRUCTURES	TYPE	LENGTH (FT)	ORIENTATION	LOCATION	INITIAL BEACH FORM	FINAL BEACH FORM	REMARKS
BB 1	714	4		5	Nil	2	Yes	—	—	—	—	—	As per March 1968	(1) Accumulation in harbour (2) erosion of beach	
BB 2	714	4		5	25 for first 2 hrs. None after	5	Yes	—	—	—	—	—	Ditto	(1) Bar forms in harbour and then parallel the beach	Wave direction is not uniform along crest during test.
BB 3	715	4'		5	9	7½	Yes	—	—	—	—	—	Ditto	Sand moves offshore, partly due to undesirable wave effects	Ditto
BB 4	715	4		5	10	1¼	Yes	—	—	—	—	—	Ditto	(1) Accretion in harbour (2) Erosion at beach	Wave effects appear to be desirable. Approximate field conditions similated.
BB 5-1	715	4'		5	9	1¼	Yes	2 Groins ~~Groins~~	Impermeable not submerged	300'	$\alpha=105^\circ$	1 @ riprap 1 @ 640' N.	Ditto	(1) Erosion between groins at riprap (2) Erosion at seawall lessened	
BB 5-2	715	4'		5	9	2	Yes	Ditto	Ditto	Ditto		—	As per final Beach form of Test BB-5-1	(1) North groin starts accumulation (2) S groin has not caused accretion	Should try different arrangement.
BB-6-1	715	4		5	9	1½	Yes	2 Groins	Ditto	300' both	N. groin $\alpha=100^\circ$ S. groin $\alpha=110^\circ$	N. & S. of riprap	As per March 1968	(1) Beach material maintained b'twn groins (2) S of S. groin erosion at foot of seawall	B & B concludes design is desirable. Adjustment may be further required
BB 6-2	←					Ditto						→	As per Final Beach Form of Test BB-6-1 with harbour bar removed	(1) Slightly more sand accumulates at N. groin (2) No other apparent change in beach form	(1) Dredging has little effect on beach form with present groin design (2) Groins as designed appears to stablize beach
BB-7	715	4		5	9	1½	Yes	2 Groins 1 Groin	Ditto Ditto	200' 100'	Ditto	Ditto N.S. end of Seawall	As per March 68	(1) Heavy build along beach except S. of SeaWall. (2) Reduction between two north groins	(1) 2 North groins should be left at 300' long.

WINNIPEG BEACH MODEL STUDY

TABLE 9.1 (cont'd)

TEST NO.	LAKE STAGE (FT)	WAVE HEIGHT (FT)	WAVE LENGTH (FT)	WAVE PERIOD (SEC)	RATE OF FEED (lb/hr)	LENGTH OF TEST (HR)	MAN-MADE STRUCTURES (OTHER THAN SEA WALL AS PER 1968): WITH OR WITHOUT BREAKWATER	OTHER STRUCTURES	TYPE	LENGTH (FT)	ORIENT-ATION	LOCATION	INITIAL BEACH FORM	FINAL BEACH FORM	REMARKS
EH-1					180	3/4	Yes	—	—	—	—	—	As per March 1968	Large bar built up that migrates around the breakwater into the harbour and tends to seal it off	
EH-2					?	2		Breakwater	Stones & Impermeable slope = 2:1	320	60° L of Forms ⊥ of 1968 extension	Breakwater of 1968 Breakwater	As per March 1968	① Stable beach enclosed by interior & formed by breakwaters (← N. of 1968 Breakwater) ② Thence, bar (thinner than Test EH-1) migrates into harbour, tends to seal it	
EH-3					?	1					60° R		Ditto	① Bar wider than Test EH-1 migrates into harbour and then curves towards shoreline. ② Bar tends to block off harbour	
EH-4					?			Off-shore Breakwater	Impermeable	320	NW-SE ⊥ wave front	120' N of 1968 Breakwater at closest pt.	Ditto	① Sand accumulate between breakerwaters ② Beach N of offshore breakwater heavily built up ③ Double tombolo between breakwaters ④ Later bar builds up and migrates into harbour	
EH-5					?			Ditto	Ditto			320' & Ditto above	Ditto	① Results similar to that of Test EH-4 except that it takes longer to attain the similar result.	E & H. recommends this configuration suitable for prevention of siltation in harbour



<u>FIGURE 9.1</u> COMMON SHORE PROTECTION STRUCTURE.
- Rock Groins.

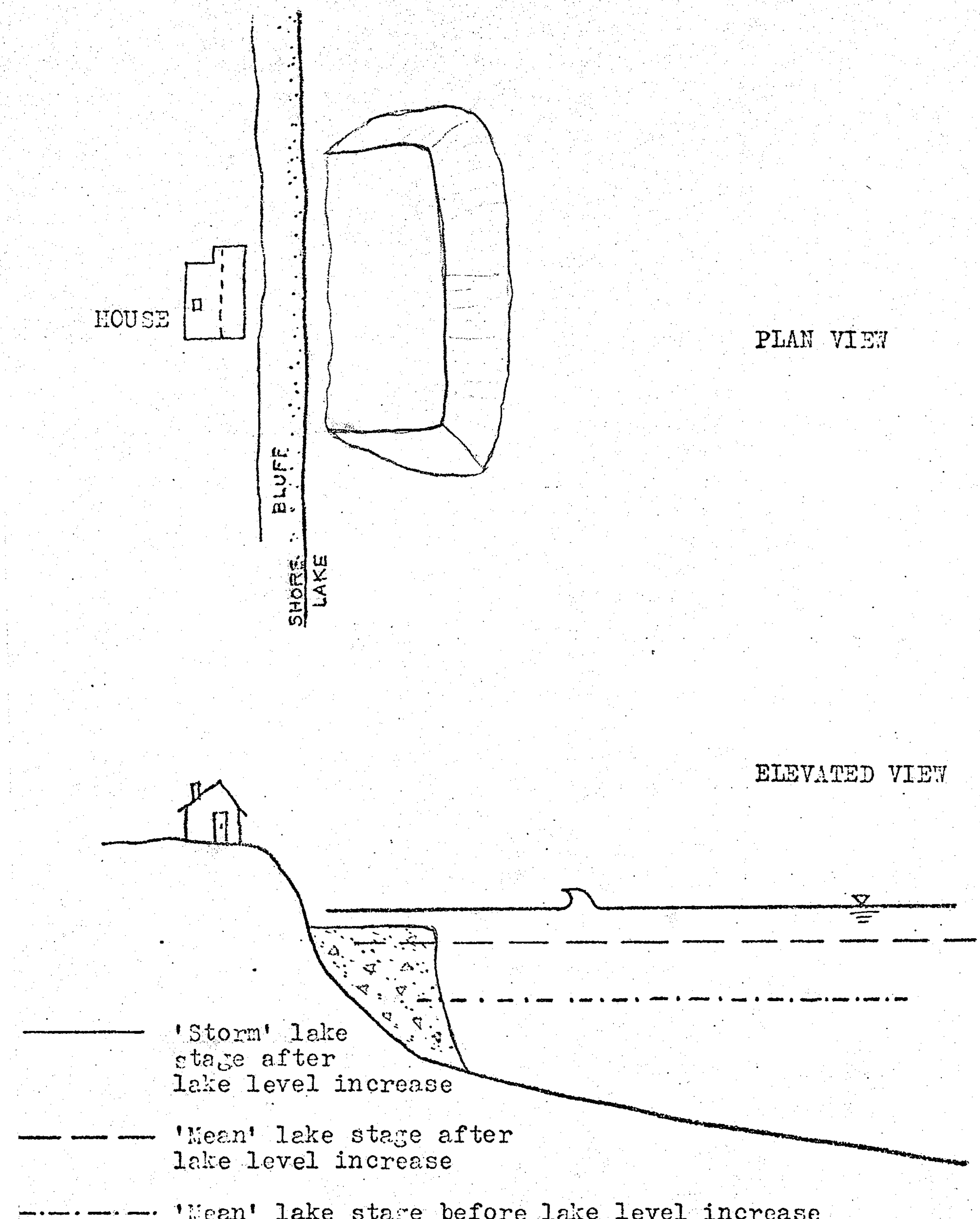


FIGURE 9.2 COMMON SHORE PROTECTION STRUCTURE.
- Impermeable Seawall.

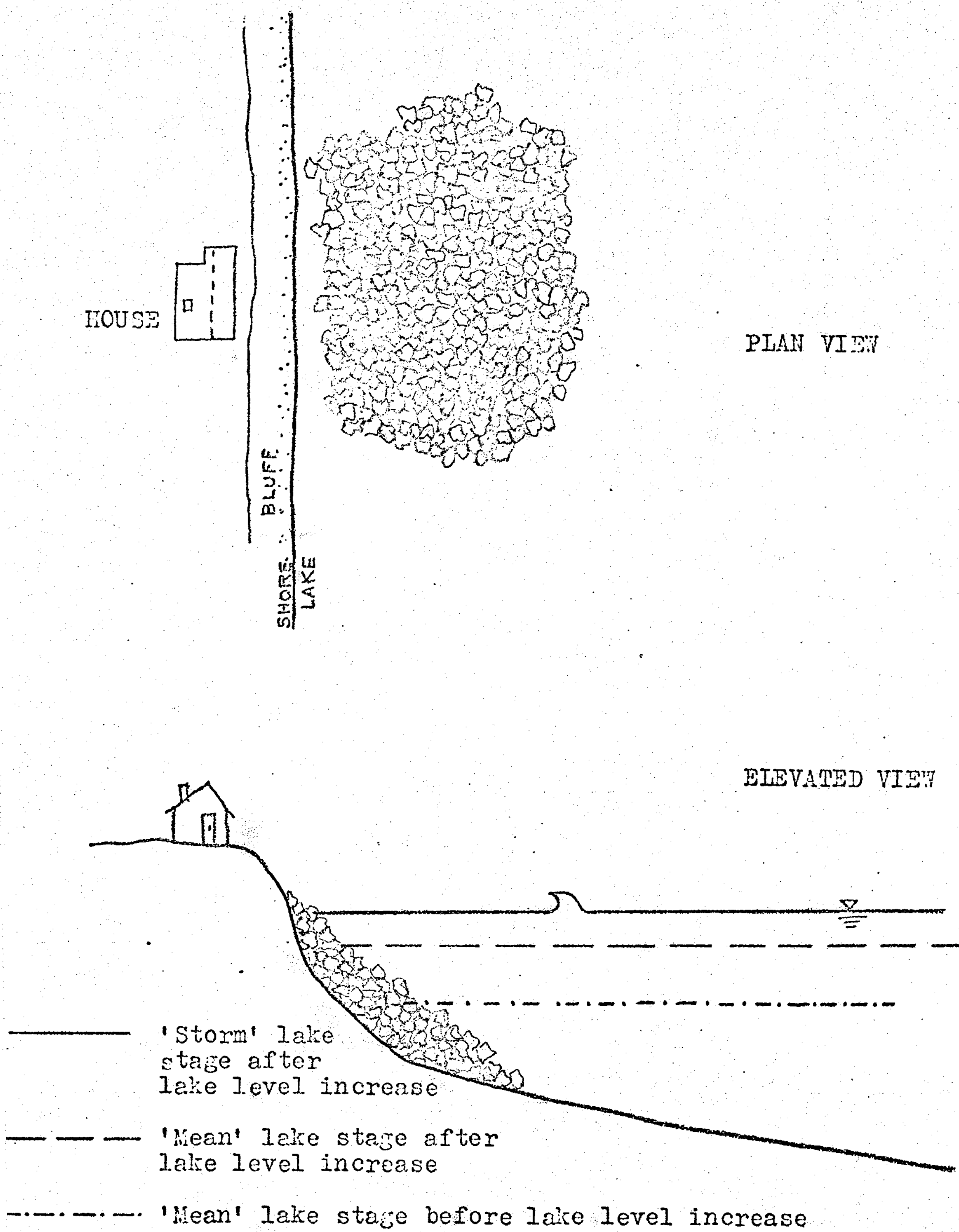


FIGURE 93 COMMON SHORE PROTECTION STRUCTURE.
- Permeable Seawall.

CHAPTER X

CONCLUSION AND RECOMMENDATIONS FOR FUTURE STUDIES

10.1 CONCLUSION

RE: COASTAL PROCESSES & SHORELINE RECESSION

(i) The results of wave characteristics for waves generated over shallow water (40 feet deep) using Miles-Phillips' model appears to be correct, or at least, in good agreement with the results of studies elsewhere. Nevertheless, there is the need to clearly define turbulent boundary layer thickness which was assumed to be 40 meters.

(ii) Computed theoretical longshore transport capacity for Southern Basin is in good agreement with measured values elsewhere but it is doubtful that this capacity for sand transport is fully satisfied at any location in the Southern Basin at normal lake level, since there was no noticeable drastic beach erosion prior to 1960 even at beaches where the theory would predict erosion before and after lake rise. Irregularities of coastline and large size beach particles would have drastically reduced this transport capacity. Actual sand transport rates in Southern Basin, although likely to be proportional to but less than the theoretical transport capacity, cannot be evaluated theoretically. Only actual field measurements such as obtained by tracer sand studies can provide an answer.

(iii) Increase of lake level by 5.0 feet would increase wave energy by approximately 10 percent. As discussed in Section 8.1, this may not be enough to cause the sudden shoreline recession in recent years.

(iv) Sudden recession of shoreline is due to the tendency for a beach to restore its equilibrium beach profile, such that

Recession = (lake stage increase)/(Slope of inshore-foreshore or offshore)

The computed values for recession are in the correct order of magnitude bearing in mind that they represent the long term limit of recession. Assuming no longshore wave energy, this mechanism of shoreline recession is a function of normal wave energy, grain size of bluff material, height of bluff and the amount and the grain size of beach materials supplied or removed. Longshore wave energy, if present, combines with this mechanism, resulting in enhanced erosion and causing even more extensive shoreline recession.

(v) To obtain a qualitative estimate of the extent of shoreline recession in Southern Basin, Plates III(b) and V(b), which depict the degree of shore erosion due to longshore transport, and Plates III(c) and V(c), which give the extent of recession due to lake level increase, could be used. Although I believe Plates III(c) and V(c) bear the most important results, all these graphs should be used conjunctively. Furthermore, the physical characteristics of the coast, the influence of existing beach protection structures and other important factors should be taken into account in the assessment of shoreline recession.

(vi) Silvester (1970) suggests that the formation of crenulated bays is due to longshore wave energy sweeping a coast which is protected in the updrift reach but is not protected elsewhere.

In this study it is found that crenulated bays in the Southern Basin are due to a sudden increase followed closely by a sudden decrease, of longshore wave energy in a region with low homogeneous bluffs of fine grain soils. Winnipeg Beach is an example.

RE: BEACH PROTECTION

(vii) Groins composed of large boulders are common on the West Coast and East Coast. However, it is doubtful that they are often effective in preventing shore erosion. They are often located too far below the storm lake stage.

(viii) A more effective beach protection structure is a permeable wall type constructed with loose gravel and/or large boulders. Although a permeable seawall of boulders only is not as effective as one with gravel, it is much better than having no protection at all.

(ix) Increase in lake level will lead to extensive shoreline recession as a result of either real or apparent erosion as explained in Section 8.5. This will lead to an "all out" effort to protect beaches by the erection of protection structures. The long term result is that the longshore current may become starved of sand, and wide, fine sandy beaches will become a rarity along the coastline, unless artificially protected or created.

(x) Since the amount of longshore sand transport cannot be evaluated theoretically for Southern Basin, no realiable figure can be quoted as to how much sand is required for beach nourishment. Again, Plates III(a) and V(a) give only theoretical values that could be taken as the upper limit. The method of beach nourishment would be most attractive, however, for important tourist attraction beaches, such as Winnipeg Beach. Sand can be picked up at the downdrift end of an erosion-deposition reach and dumped at the updrift end; for example, picking up at Stephen's Point and dumping at Winnipeg Beach. Plates III(a)

III(b), V(a) and V(b) would provide the guide as to where to pick up sand and where to dump it. The amount to be handled annually in this manner must be determined either by tracer sand analysis or by trial and error.

10.3 FUTURE STUDIES

Through the present study on coastal processes in the Southern Basin, it is a small surprise to discover the lack of work in its regard. Studies in several areas require immediate attention. Without the knowledge that can be made available through such studies, it would be impossible to study shore erosion or other physical processes of the lake in the proper light.

(i) Wave measurements are needed to fully appreciate the magnitudes and directions of forces that cause the continual coastal processes. Careful spectral analysis of waves would reveal some characteristics of the so-called "littoral currents". (It is to mention that it is not possible to measure littoral currents directly with regular current meter, since they are slower than the threshold velocity of the meter).

(ii) Tracer sand study would provide a most direct method of measuring longshore sand transport. It would provide a basis for evaluating beach nourishment. It is definitely important for studying coastal processes per se.

(iii) Due to the limited budget, the 1970 beach survey was rather inadequate in some respects, but could form a basis for future studies. It is recommended that in future surveys, more soil samples should be taken not only from the beach (including underwater samples) but also from the bluff. Slope measurement should also include more of the foreshore and offshore areas. Also, typical beach profiles should be distinguished from those that are not, such as those beside a pier. Constant comparison should be made between profiles taken at different times to assess erosion or accretion.

(iv) Model studies (using the model built by Veldman or other models) should be conducted to test the validity of the formulation of shoreline recession due to equilibrium beach restoring tendency as discussed in Chapter VIII.

(v) Since all physical studies of lakes are in one way or another related to meteorological data, it appears imperative that certain basic atmospheric characteristics should be further studied. These include in relation to the present study, the wind overwater to wind overland ratio.

(vi) Various forms of beach protection structures can be found in the literature. In this study, it is suggested that permeable wall would be most effective. In related studies (not published) on Winnipeg Beach protection, groins (fairly large in size) are also found to be effective. It would be most beneficial to carefully evaluate the effectiveness of different types of structure in withholding shoreline recession due to lake level increase. The aspect of economics is important in this regard. A detailed catalogue of existing structures and their history in the Southern Basin would be most rewarding.

(vii) Of course, an economic study of the region is of prime importance. Such a study would examine the details of land use, social and economic significance of different areas, and so forth. It is only with relevance to such a study that studies such as this thesis can find justifiable application.

CHAPTER XI

BIBLIOGRAPHY

ABBREVIATIONS

ASCE: American Society of Civil Engineers

WWH: Journal, Waterways, Harbours and Coastal Engineering

CERC: Coastal Engineering Research Centre

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APPENDIX A

PROPERTIES AND CHARACTERISTICS OF WATER WAVES

A.1 INTRODUCTION

"Ocean Waves are complex phenomena, difficult, if not impossible, to describe correctly in mathematical terms". (Wiegel, 1964). In fact, waves can be better described as statistical processes governed by the physical properties of air and water. More complicated yet is the phenomenon of shallow water waves, whether they travel from deep water into shallow water or whether they have actually been generated over shallow water.

A concise description of waves can be found in Pierson et al's illustration, (1954) which is reproduced in Fig. A-1. Before dealing with the more complicated aspects of water waves, it is necessary to examine the simple wave, and its properties with respect to time and space. The shallow water effect on such a wave form will also be studied.

A.2 A SIMPLE WAVE

It has been observed that the simplest wave form (such as the kind generated by a paddle-type wave generator in a beach model) is very similar to the sine function. (See Fig. A-2(a)). Although it is convenient and acceptable to approximate water waves by sine functions, care should be exercised where the approximation is to be employed. For example, sine functions cannot be used to represent waves when a very strong wind is blowing over water, or when the waves are in shallow water. In both cases, the waves do not conform to the roundedness and the symmetry of the sine function and may form white caps due to breaking. The front slope of the wave form is steeper than the back slope and the crest is no longer as high above the mean water line as the trough is below it.

Fig. A-2(a) shows a "time frozen" simple sine wave, i.e. as in a snap shot. y is the ordinate axis measuring the vertical distance from mean water level (MWL); abscissa x is horizontal distance in the direction of wave propagation. The distance between two consecutive wave crests or wave troughs is constant, and is called the wave length. The vertical distance from crest to trough is termed the wave height, half of which is called the amplitude. In this illustration, the wave height is also a constant. The wave form can be described as

$$y_s = a_o \sin(kx + \epsilon_x) \quad \text{(A-1)}$$

where y_s is the ordinate of surface elevation at a given value of x ; a_o is the wave amplitude; k, is a constant; ϵ_x , is a constant called the phase angle.

Fig. A-2(b) shows a record of the same wave as recorded at a given station in the sea. In this case, the instantaneous water surface elevation is shown as a function of time elapsed since measurement began. The constant T is the wave period, such that the cycle repeats itself in T units of time. Correspondingly,

$$y_s = a_o \sin(\omega t + \epsilon_t) \quad \text{(A-2)}$$

where t is the time elapsed, and the remaining quantities correspond to the similar ones in Equation (A-1).

It is apparent that y_s is a function of both the distance from the measuring station and the time elapsed. Combining Equations (A-1) and (A-2) yields

$$y_s = f(x,t) = a_o \sin(kx + \omega t + \epsilon) \quad \text{(A-3)}$$

Shifting the origin to use cosine function (only as a matter of convenience) and removing phase angle :

$$y_s = a_o \cos(kx + \omega t) \quad \text{(A-4a)}$$

$$= \frac{H}{2} \cos\left(2\pi\left(\frac{x}{L} + \frac{t}{T}\right)\right) \quad \text{(A-4b)}$$

where H is the wave height,

$k = 2\pi/L$, often known as the wave number,

$\omega = 2\pi/T$, is the angular velocity or frequency in radian.

Other terms which are commonly used as parameters in the wave equations are:

$f = 1/T$ (in cycles/sec.) - the frequency.

c - the velocity of wave (or wave crest), more usually referred to as the phase velocity such that,

$$L = cT \quad \text{(A-5)}$$

It is found that a group of wave crests would travel at a group velocity $C_g = C/2$, in deep water. Waves at the head of the group would disappear while new waves are formed at the rear of the wave train. In this manner the wave train is able to travel at a velocity slower than the phase velocity.

The energy of a wave is given by

$$E = E_{KE} + E_{PE} = \rho g \frac{H^2 L}{8} \quad \text{(A-6)}$$

per unit width, where ρg is the unit weight of water E_{KE} and E_{PE} are the kinetic and potential energy per unit area of water surface. It is obvious that the energy of a disturbance travels with the velocity of the disturbance, C_g.

Another interesting feature of wave motion is that water particles in the water perform circular paths, while the net mass transfer is in the direction of wave advance at a usually unnoticeable rate. (See Fig. A-3). The particle under the crest of a wave will be at the top of the circular locus, and vice versa. The size of the circle decreases with depth and becomes practically nil at a depth equal to half the wave length; thus the water below this depth is almost not disturbed by the wave motion.

A.3 SHALLOW WATER EFFECT ON THE SINE WAVE

When a simple sine wave such as the one discussed above, travels into shallow water, it is subject to the boundary conditions imposed upon it by the lake or ocean floor. The wave is said to "feel" the bottom when the depth, d, is less than L/2, where L is the wave length, although for many practical purposes, shallowness refers to the situation when $d/L < \frac{1}{25}$. The orbital locii performed by water particles are no longer circular, but are "squashed" to become somewhat elliptic, being more elliptic as depth decreases. For an analytical solution the wave equation (the cosine function of Equation (A-4)) is to be solved for the boundary conditions that zero pressure exists at the free surface and that the water particles remain in contact with the bottom. In addition, the conservation of mass and energy have to be satisfied. The solution of the problem is facilitated by choosing the appropriate potential function. After mathematical manipulations which include neglecting the second and higher order terms of approximation, the relationships (A-7) through (A-17) below are derived. The steps in these mathematical derivations are not given here but extensive treatment of this subject can be found in textbooks on hydrodynamics, such as Ref. 135 (Eiegel, 1964), and Ref. 56 (Ippen, 1962).

$$\omega^2 = gk \tanh kd \qquad \text{(A-7)}$$

$$C = \omega/k = \frac{gT}{2\pi} \tanh \frac{2\pi}{L} d$$

$$= \left[\frac{gL}{2\pi} \tanh \frac{2\pi}{L} d\right]^{1/2} \qquad \text{(A-8)}$$

$$C_o^2 = \frac{gL_o}{2\pi} = gk_o; \quad d > \frac{L}{2} \qquad \text{(A-9)}$$

$$L = \frac{gT^2}{2\pi} \tanh \frac{2\pi d}{L} \qquad \text{(A-10)}$$

$$L_o = \frac{gT^2}{2\pi}; \qquad d > \frac{L}{2} \qquad \text{(A-11)}$$

$$C/C_o = \tanh kd \qquad \text{(A-12)}$$

$$L/L_o = \tanh kd \qquad \text{(A-13)}$$

$$d/L_o = \frac{d}{L} \tanh kd \qquad \text{(A-14)}$$

where subscript "o" denotes deep water values. Also

$$C_g/C = \frac{1}{2}\left[1 + \frac{2kd}{\sinh 2kd}\right] = n$$

$$P = \text{wave power} = \frac{nE}{T}$$

where E = wave energy $= \rho g \frac{H^2 L}{8}$

It should be mentioned that the foregoing is the first-order approximation solution. To date, many sets of solutions have been derived by a number of researchers, with some solutions including terms up to the fifth-order approximation. (Dean, 1970; Wiegel, 1964). However, it is felt that the first-order solution (generally known as The Linear (Airy) Theory) is sufficiently adequate for the present study in the range of accuracy required.

A.4 THE ACTUAL WAVES

The foregoing paragraphs illustrated the characteristics of a simple sine wave. However, a sea of waves never approaches such a condition. If one carefully observes the waves at sea or on a lake, one would find waves of different amplitudes, periods and wave lengths follow one another. In fact, it becomes difficult to apply the terms 'period', 'wave height' and 'wave length' to this situation. A wave train may be as shown in Fig. 4-2(c). Generally a wave period may be taken to be the time lapse between two consecutive wave crests, and a wave height is the vertical distance from the trough of one crest to the trough of the next crest. To further complicate the picture, different sets of waves may be travelling in (slightly) different directions. It becomes impossible to apply the simple sine wave to the real situation.

To be able to analyze an actual sea surface one would imagine the actual waves as the sum of numerous sets of simple sine waves, as illustrated in Fig. A-1. (After Pierson et al, 1954). The recorded waves in Fig. (A-2(c)) can thus be visualized as the resultant of infinite sets of simple sine waves acting together. Each set of waves has its particular wave amplitude, wave period, wave length et cetera. It also has its particular direction of travel, which may or may not be different from the wind direction or the general direction of wave propagation. Since the classical formula (Equations (A-7) to (A-17)) applies to each set of sine wave, it is therefore possible to describe the behaviour of the complex combination of wave trains.

A.5 WAVE SPECTRUM

It is convenient to apply the concept of spectrum to water waves. In the past two decades, there have been many studies on water wave spectrum. In fact, spectral analysis has become an almost indispensable tool in parameterizing waves and many other phenomena in oceans and lakes. In the past, the significant wave was taken to be a sufficient description for a sea of waves, but it is now known that the concept of spectrum provides a better understanding and a more precise wave description of the actual state of the sea or lake.

The mathematical derivation of a (wave) spectrum may appear confusing to those who are not familiar with the subject. The foregoing section described how a complex wave pattern on the water surface can be broken down into simple individual components of wave trains. This is the basic idea of spectral analysis of waves. Perhaps, one may draw an analogy between a wave spectrum and the sieve analysis of sand sizes. Just as the weight of sand of size, say, between 1 mm to 2 mm can be represented by an ordinate (or a bar) on the grain size distribution graph, so can the spectral density (of square of surface deformation) of frequency, say, between 1 cps to 2 cps be represented on a spectral curve. The total weight of the sand sample is an integration of the area under the sand size distribution curve; similarly, the total mean square deformation of a unit area of the sea can be obtained by integrating the spectral curve. The mean square deformation wave spectrum is therefore one with the square of surface elevation plotted on the ordinate and the frequency plotted on the abscissa. (See Fig. A-4).

A spectrum is a function that mathematically describes the distribution of a certain dependent variable with respect to an independent variable. This could be the distribution of mean wave height with respect to wave period, or mean wave slope with respect to wave number. Typical of such in the study of water waves is the distribution of mean-square deformation (or amplitude) with respect to T, k, f, or ω. (If any one of T, k, f, or ω is defined, the remaining three are determined). A wave spectrum is shown in Figure (A-4).

To describe a wave record in terms of a wave spectrum, say, as the mean square deformation of water surface, it is necessary to separate out wave components of approximately the same frequency, say, within the range of $f + \Delta f/2$ and

$f - \Delta f/2$, where Δf is a very narrow frequency band, and find the mean value of wave height square. This would give the spectral density of the wave components in the frequency band; that is, the ordinate of the spectral curve at frequency f. The total area under the curve, obtained by integration, represents the mean square deformation for all wave components, $\langle \eta^2 \rangle$, per unit area of the water surface. Since, by a former relationship, (Equation (A-6)), $\langle \eta^2 \rangle$, is directly proportional to energy, E, this quantity represents the available energy of the sea or the lake, at the particular locations and at the particular time, per unit area.

Another property of the wave spectrum is that if the spectral density of the wave component at frequency f is multiplied by its group velocity, one obtains the wave power for the frequency component, or the rate at which energy is delivered across a vertical plane perpendicular to the direction of wave propagation. Thus, it is possible to measure the energy state of the sea by the wave power delivered. The total wave power is the summation of the power delivered by all the frequency bands in the wave spectrum.

A.6 ENERGY DELIVERED BY WAVES IN A STORM

Suppose a storm lasts for t hours, and it takes t_1 hours for the waves to attain a relatively steady state, as at a measuring station near the shore, provided the wind speed remains constant at U m/sec. At the end of t hours, the wind stops abruptly. See Fig. (A-5). Waves from offshore are still travelling shoreward, so that the wave record shows diminishing wave height, (therefore, wave energy) for say, t_2 hours. Thus the coast receives a constant level of wave energy for $t - t_1$ hours, plus energy of increasing and decreasing intensity for t_1 and t_2 hours respectively. For all practical purposes, it is convenient to consider the total effect as the summation of the three periods viz, t_1, $t - t_1$, and t_2, by the approximation that the same U blowing over the fetch of water for t* hours would have produced the level of wave energy actually measured. An unrealistic but simpligying qualification is that the steady state wave energy is delivered abruptly to the coastal station at the beginning of t* and lasted until the end of t*. (See Fig. (A-5(c)). Of necessity, t* is greater than or equal to $t - t_1$, but less than or equal to t. For most practical purposes, t* can be taken as an approximation of t.

To find the energy delivered by wave components within a small frequency band between $f - \Delta f/2$ and $f + \Delta f/2$ (such that f is the average frequency) one sums the mean square deformation by multiplying the spectral density by the band interval Δf. By converting the product to energy per unit area and multiplying it by the group velocity, the wave power due to wave components of the small frequency band between $f + \frac{1}{2}\Delta f$ and $f - \frac{1}{2}\Delta f$ can be obtained. The quantity of energy delivered is the product of wave power and t*, as follows

$$\text{Energy} = t^* \int_0^\infty \Phi(f) \times C_g df \qquad \text{(A-18)}$$

A.7 OTHER PROPERTIES OF WAVE SPECTRUM

The foregoing discussion showed how a spectrum can be expressed as a function of frequency. However, in many applications it is more convenient to express a wave spectrum with wave number, k, or angular frequency, ω, as the independent variable. The following notations and terminology are commonly used for water wave spectra.

(i) Spectral density - the ordinate under the spectral curve as shown in Fig. A-4(a). It has the dimension of ($Length^2$ - time) when frequency is used as an independent variable, and the square of surface deformation is plotted on the ordinate.

(ii) Co-cumulative Spectra - This is a term used by Pierson et al. (1955); also abbreviated as C.C.S. It is given as:

Ordinate of Co-cumulative spectra at frequency $f_1 = \int_{f_1}^{\infty}$ spectral density d (frequency).

(A-19)

The ordinate of the C.C.S. at f = 0.0 is the mean-square-wave-height of the sea, which is of course directly proportional to the energy in the sea; <u>this value is defined as E</u>. (Pierson et al.,1955). See Fig. (A-4(b)).

(iii) $S(\omega)$, $S(f)$ -- et cetera - spectral density as functions of ω , f et cetera.

(iv) $S(f)$, $S(\omega)$ -- et cetera - cumulative spectra as functions of ω, f, et cetera.

(v) $\bar{\xi}^2$, $\bar{\eta}^2$ ....$\langle\xi^2\rangle$,$\langle\eta^2\rangle$ - mean-square of water surface elevation deviating from the MWL, or mean square deformation.

(vi) $\Phi(\omega)$, $\Phi(f)$, $\Phi(k)$.....etc. - spectral density functions of square water surface elevation as functions of ω, f, k, et cetera, such that

$$\bar{\eta}^2 = \int_0^{\infty} \Phi(\omega)\, d\omega \qquad \text{(A-20)}$$

This definition of spectrum is called <u>the one dimensional spectrum</u>.

(vii) $\Phi(\underline{k})$ - is mean-square surface deformation spectrum in vector form, where $\underline{k}$ is a vectorial wave number.

(viii) $\Phi(k,\alpha)$ - an equivalent expression for $\Phi(\underline{k})$. Both $\Phi(k,\alpha)$ and $\Phi(\underline{k})$ are called the <u>two-dimensional</u> spectra density such that

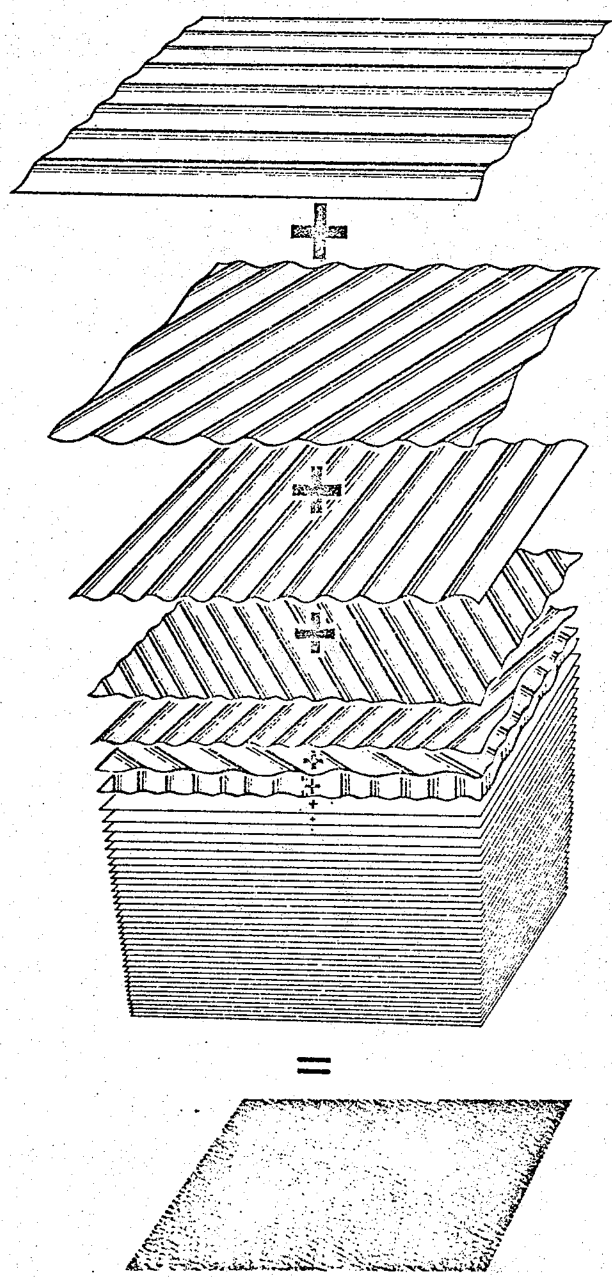
$$\bar{\eta}^2 = \int_0^\infty \int_{-\pi/2}^{\pi/2} \Phi(k,\alpha)\, dk\, d\alpha = \int_0^\infty \Phi(\underline{k})\, d\underline{k} \qquad \text{(A-21)}$$

(ix) $\Phi(\underline{k}, t)$, $\Phi(k, \alpha, t)$ - two-dimensional spectrum with time (duration of wind action) as an additional independent variable, such that

$$\bar{\eta}^2(t) = \int_0^\infty \int_{-\pi/2}^{\pi/2} \Phi(k,\alpha,t)\, d\alpha\, dk = \int_0^\infty \Phi(\underline{k},t)\, d\underline{k} \qquad \text{(A-22)}$$

In a sea, waves can approach a station from any direction. Generally they come from a 180° sector bound by 90° to the left and right of the mean wind direction. Waves from outside this sector are negligible, unless there are notable shore effects et cetera. A wave spectrum can be conveniently represented by definition (vii) or (viii) above. When the wave components within the 180° sector are summed one obtains the one-dimensional spectrum as defined in (vi). Thus, one-dimensional and two-dimensional spectrum are related as follows.

$$\int_0^\infty \int_{-\pi/2}^{\pi/2} \Phi(k,\alpha)\, dk\, d\alpha = \int_0^\infty \Phi(\underline{k})\, d\underline{k} = \int_0^\infty \Phi(k)\, dk \qquad \text{(A-23)}$$



A sea can be represented by the sum of many simple wave trains moving in different directions. [After W. J. Pierson, Jr., G. Neumann, and R. W. James, 1955, *Practical Methods for Observing and Forecasting Ocean Waves by Means of Wave Spectra and Statistics*, U.S. Navy Hydrographic Office Publication 603.]

FIGURE A.1

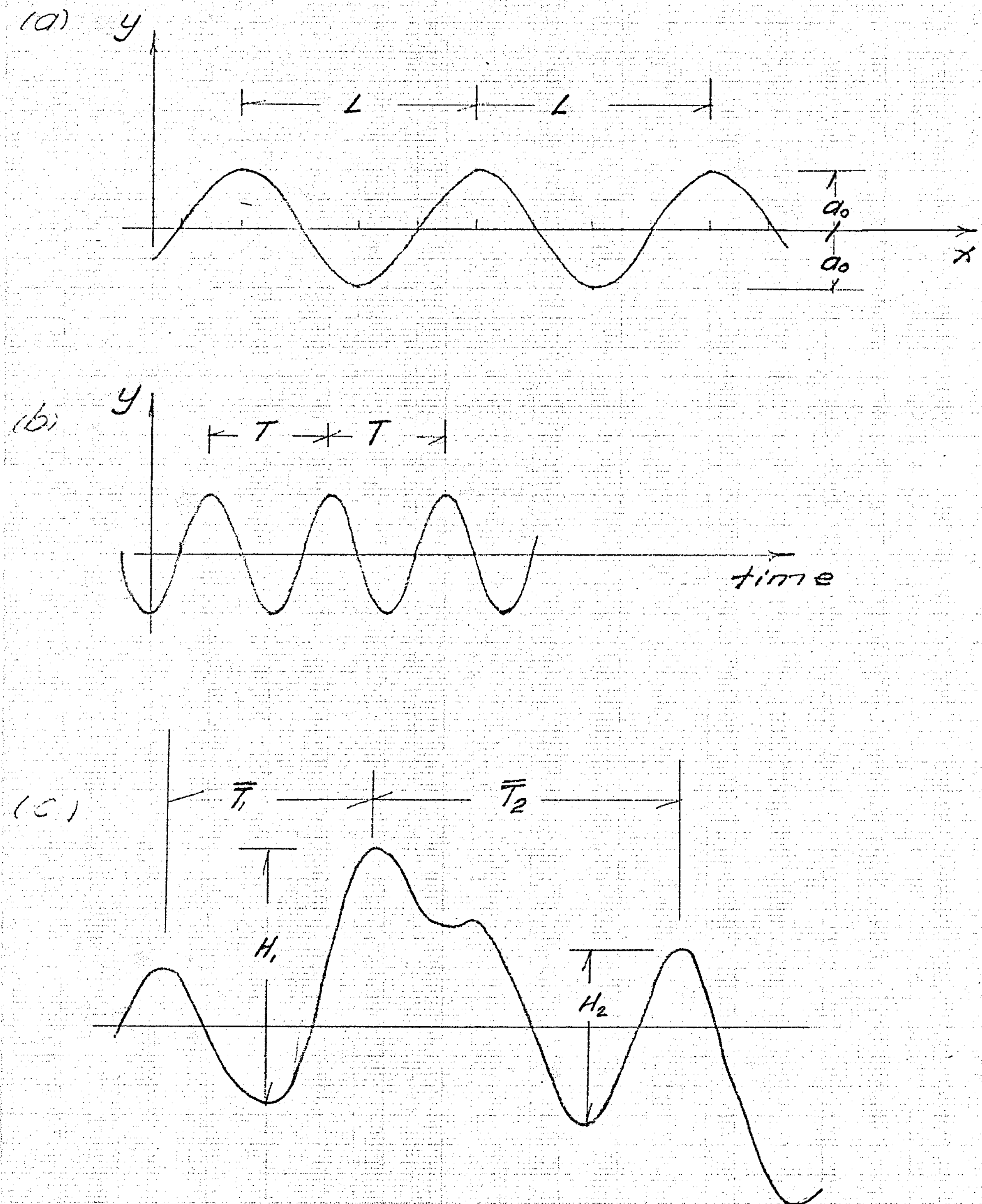
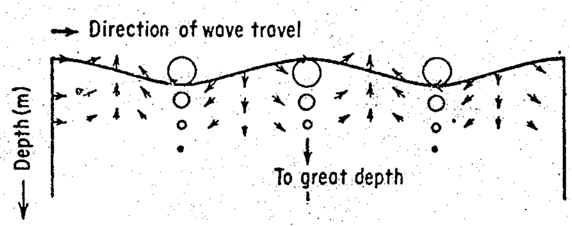
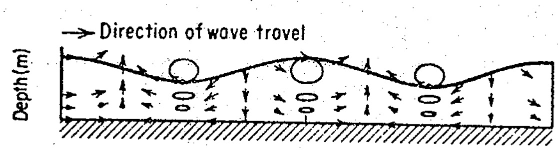


FIGURE A2



(a) Instantaneous velocity vectors at time shown and orbital paths of fluid particles in a wave motion in deep water. Velocities are very small at a depth equal to one-half the wavelength.



(b) Instantaneous velocity vectors at time shown and orbital paths for one cycle of fluid particles in a wave motion in water of constant depth.

<u>FIGURE</u> A.3

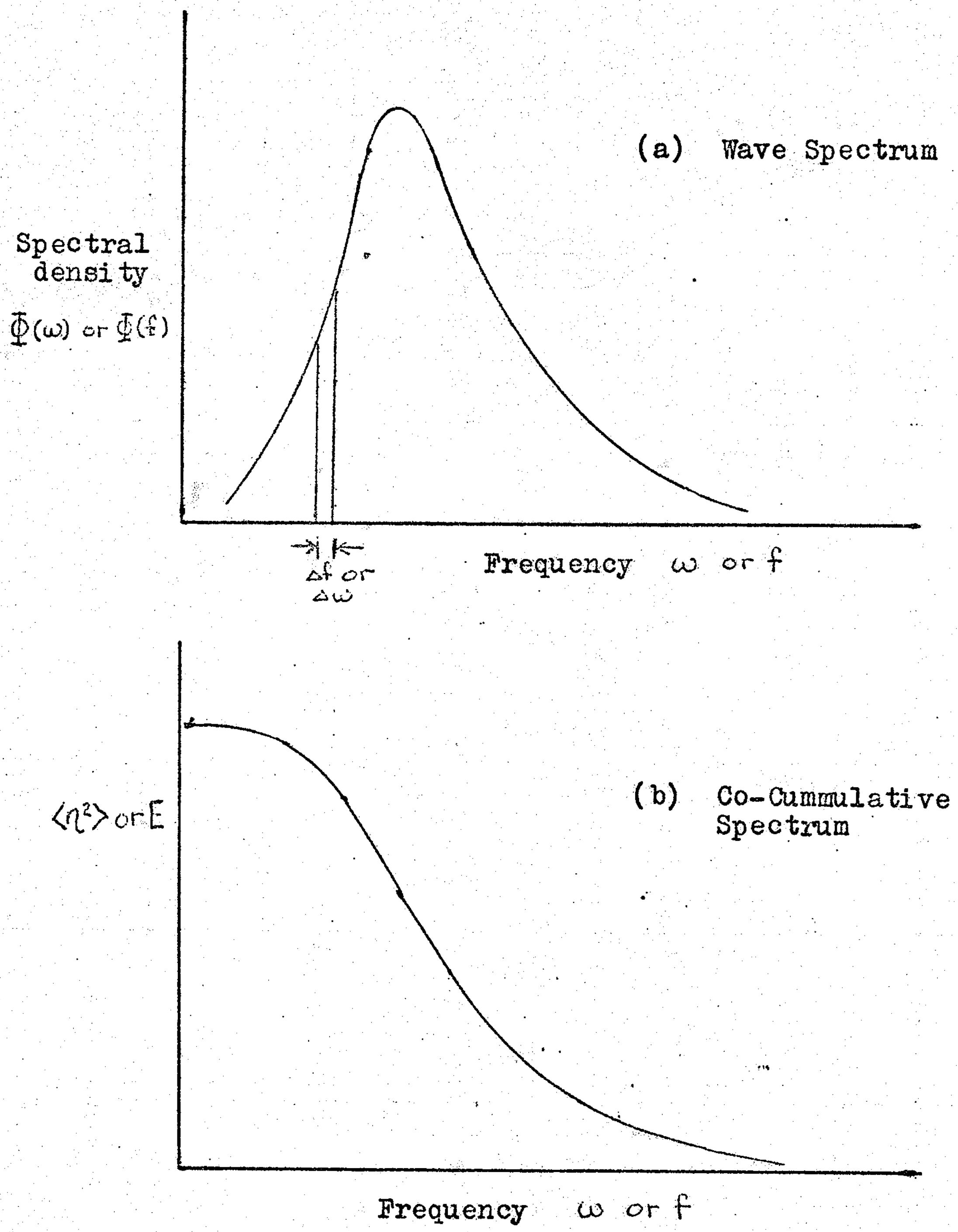


FIGURE A-4. ONE-DIMENSIONAL WAVE SPECTRUM AND CO-CUMMULATIVE WAVE SPECTRUM.

(Note: not drawn to scale.)

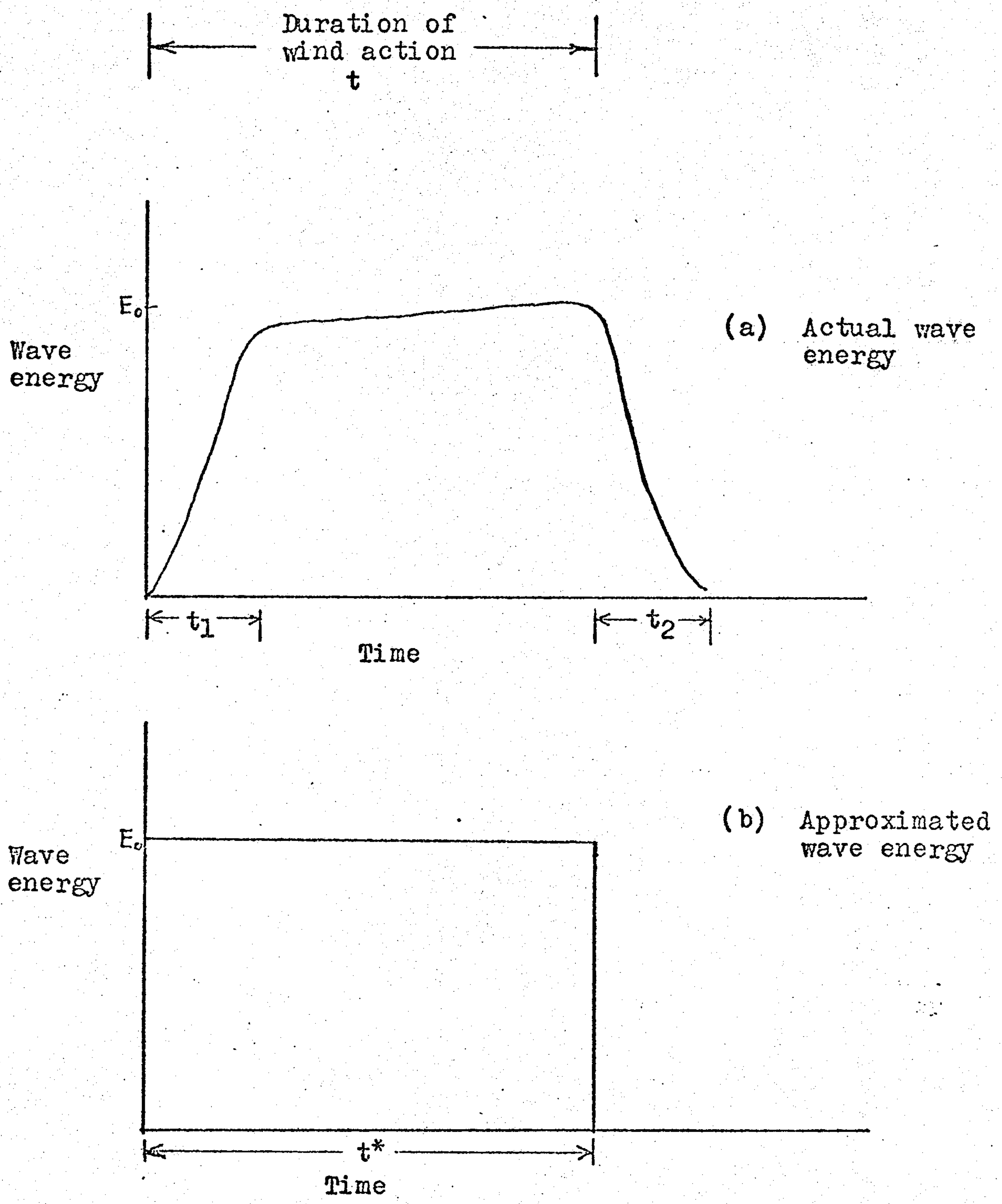


FIGURE A-5. APPROXIMATING WAVE ENERGY DUE TO STORM OF DURATION t.

(Note: not drawn to scale.)

APPENDIX B

WAVE STATISTICS

B.1 INTRODUCTION

An important contributor to wave statistics is M.S. Longuet-Higgins. In this appendix the essence of his 1952 paper will be presented. The objective here is to appreciate the statistical properties inherent in a sea of waves. In the final analysis, it is to suggest that waves cannot be adequately described with anything less than the idea of wave spectrum.

Assuming any arbitrary distribution of wave heights, such as the one shown in Fig. B-1(a), $H^{(p)}$ is the wave height of (p x 100) percent highest wave in the entire wave population. "p" for a given wave height is the probability that the wave height will be equalled or exceeded. As defined, $H^{(1/3)}$ or $H^{(.333)}$ is the generally used significant wave height - the mean wave height of the top third highest waves. The similar notation applies to wave amplitude, $a^{(p)}$.

B.2 TWO SIMPLE CASES

(i) In a simplified picture there exists only one sinusoidal wave train in the sea, say $\eta(t) = a_o \sin(\omega t)$ as in Fig. B-1(b). The distribution of wave amplitude (in this case taken as half of the wave height), is a constant ($a_{max.} = a^{(1)} = a^{(1/3)} = a_{min}$). The mean-square amplitude is defined as:

$$\overline{a}^2 = \frac{1}{N}\left(a_1^2 + a_2^2 + a_3^2 + \cdots\cdots + a_N^2\right) \tag{B-1}$$

where N is the number of wave amplitudes measured.

(ii) If there are two wave trains in the sea rather than one, with angular frequencies of ω_1 and ω_2 but with similar wave amplitude and phase angle, the resultant surface elevation is given by:

$$\eta(t) = a_o \cos \omega_1 t + a_o \cos \omega_2 t \tag{B-2}$$

By trigonometric identity:

$$\eta(t) = 2 a_o \cos\left(\frac{\omega_1 + \omega_2}{2}\right)t \cos\left(\frac{\omega_1 - \omega_2}{2}\right)t \tag{B-3}$$

It is to be noticed that the argument of the first cosine gives the effect of a wave with a short oscillation period, while the argument of the second cosine function produces the effect of much more slowly oscillating wave train with angular frequence of $(\omega_1 - \omega_2)/2$. The resultant water surface is therefore as illustrated in Fig. B-1(c), as though a shorter sinusoidal wave train of the type cos-$(\omega_1 + \omega_2)t/2$ has been modulated by $2a_o \cos(\omega_1 - \omega_2)t/2$. The closer ω_1 approaches ω_2 the longer the modulating wave length will be. Or one can also say that a wave train of $\eta(t) = 2a_o \cos(\omega_1 + \omega_2)t/2$ is modified by the function $\cos(\omega_1 - \omega_2)t/2$.

Since $(\omega_1 + \omega_2)$ is constant, each cycle of the longer wave contains a constant number of cycles of the shorter wave. In other words, in each quarter cycle of the longer wave, say, $0 < t < \pi/(\omega_1 - \omega_2)$, the number of smaller scale sea surface undulation N, is a constant. Now if one designates these small wave scale amplitudes by a, it is evident that the distribution of the N waves, which is a representative sample of the wave population, is identical to the distribution function defined by the quarter cycle sine function.

$$\eta(t) = 2a_o \cos\frac{(\omega_1-\omega_2)}{2}t \quad ; \quad o < t < \pi/(\omega_1-\omega_2) \tag{B-4}$$

The mean amplitude of pN largest wave amplitudes $a^{(p)}$, is thus

$$a^{(p)}\frac{p\pi}{\omega_1-\omega_2} = \int_o^{p\left(\frac{\pi}{\omega_1-\omega_2}\right)} 2a_o \cos(\omega_1-\omega_2)t/2 \, dt \tag{B-5}$$

or

$$a^{(p)} = 2a_o \frac{2}{p\pi} \sin\frac{p\pi}{2} \tag{B-6}$$

where p is a specified probability. From this, it also follows that

$$\bar{a} = (\bar{a}^2)^{\frac{1}{2}} = a_o\sqrt{2} \tag{B-7}$$

$$a^{(p)} = \sqrt{2}\cdot\frac{2}{p\pi} \sin\frac{p\pi}{2}\bar{a} \tag{B-8}$$

therefore,

$$\begin{aligned} a^{(1/10)} &= 1.408\ \bar{a} \\ a^{(1/3)} &= 1.350\ \bar{a} \\ a^{(1.0)} &= .901\ \bar{a} \end{aligned} \tag{B-9}$$

The probability distribution function of wave amplitude is therefore

$$P(a) = \begin{cases} \dfrac{2}{\pi}\dfrac{1}{(2\bar{a}^2 - a^2)^{1/2}} & ; (a < \sqrt{2}\,\bar{a}) \\ 0 & ; (a > \sqrt{2}\,\bar{a}) \end{cases} \tag{B-10}$$

B.3 A RANDOM SEA

Using the similar approach, but adopting the more realistic view that a sea of waves is composed of many different components, (not just two components as in the foregoing example), and that the process by which they are generated is random, Longuet-Higgins arrived at some very interesting results. One of these, regarding mean probable wave amplitudes, is reproduced in Table B-1. In particular, it is to be noticed that the significant wave amplitude is $\sqrt{2}$ of the root-mean-square amplitude; similarly, therefore, for wave height. This value is not far from 1.350 given by the simplified model of two simple sine waves, as in Section B.2.

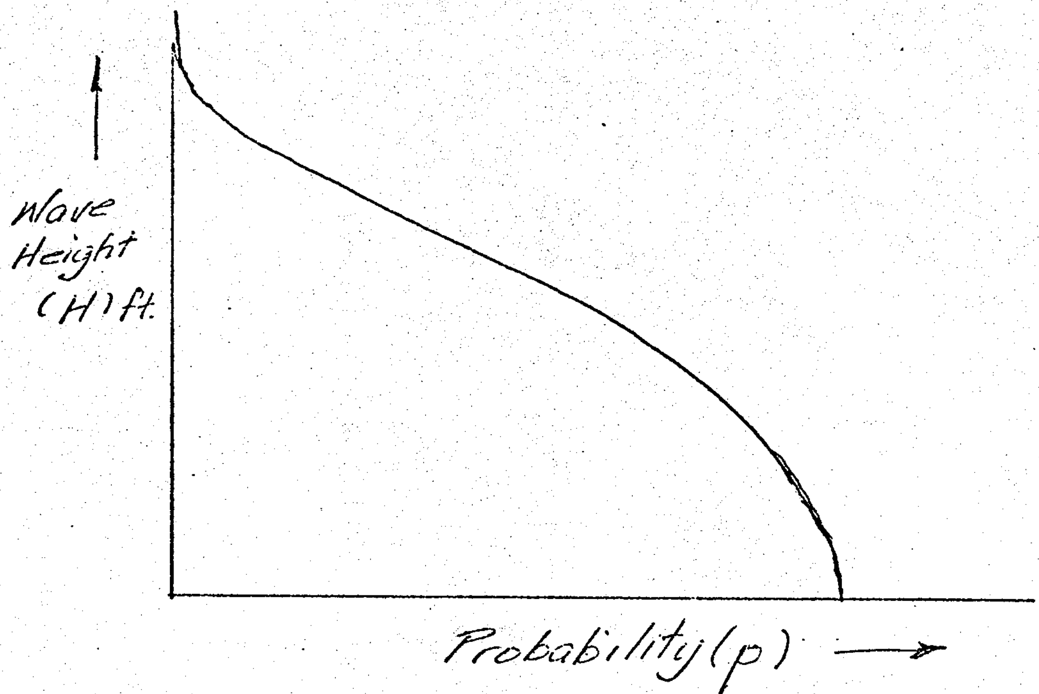
The details of Longuet-Higgins derivation is not reproduced here, but one can appreciate from the simplified model that the basic concept that a sea surface is the random interaction of a multitude of different wave components. The use of root-mean-square value as a parameter is also very convenient with special reference to wave spectrum analysis. Thus knowing the spectrum of a sea, one knows the mean-square surface elevation of the sea, and since for sinusoidal wave $\bar{\zeta}^2 = \frac{1}{2}\bar{a}^2$, one can therefore directly find properties such as significant wave height.

TABLE B-1

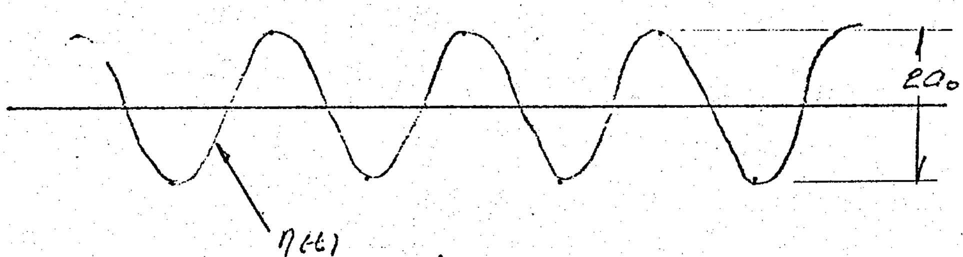
REPRESENTATIVE VALUES OF $\frac{a^{(p)}}{\bar{a}}$

IN THE CASE OF A NARROW WAVE SPECTRUM

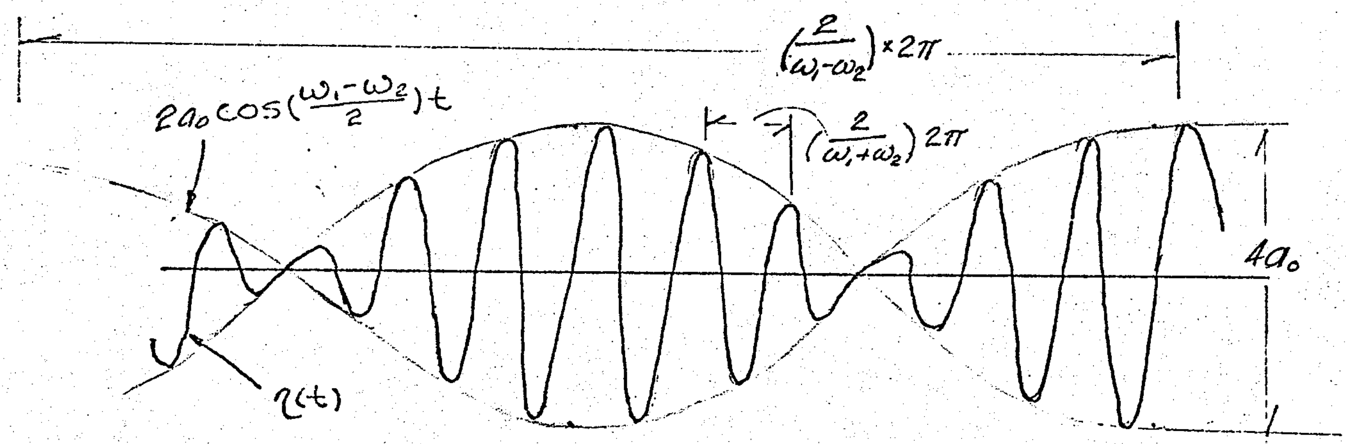
p	$a^{(p)}/\bar{a}$	p	$a^{(p)}/\bar{a}$
0.01	2.359	0.4	1.347
0.05	1.986	0.5	1.256
0.1	1.800	0.6	1.176
0.2	1.591	0.7	1.102
0.25	1.517	0.8	1.031
0.3	1.454	0.9	0.961
0.3333	1.416	1.0	0.886



(a) Wave Height Distribution



(b) A Simple Sine Wave



(c) Resultant sea surface of the combination of two waves

FIGURE B-1

APPENDIX C

THEORIES OF WIND WAVE GENERATION

C.1 INTRODUCTION

The study of wave generation by wind has always been a subject of much interest to many researchers in coastal engineering and fluid dynamics. Of major practical importance is the problem of predicting wave magnitudes when given a certain set of meteorological conditions, so that a designer would know what forces a coastal structure would have to withstand, or what is the required freeboard for a certain dam; and so forth. But owing to the complex nature of wind wave generation, there is no simple method of predicting waves without expending a minimal amount of energy in measuring waves in the field or toying with long equations. There are two different approaches to study waves. The first approach, which is frequently referred to as the significant wave approach in this thesis, treats a sea of waves as though it can be represented by a single train of (simple sinusoidal) "effective" wave. This is the so called "significant wave" approach in which the significant wave height (trough to crest) is chosen to be the mean of the largest one-third wave height in the wave height population. Various investigators have spent considerable time and energy collecting field data and plotted them on a dimensionless plot, as in Fig. 4-2. A clearly defined relationship for wave prediction therefore exists.

The second approach is more theoretical. Whereas the significant wave approach is basically semi-empirical, and measures only the effective wave (a fictitious situation), this theoretical approach attempts to follow and explain the exact mechanism of wind wave generation. With the introduction of the concept of wave spectrum by theoreticians such as Eckart (1952), Phillips (1957) and Miles (1957), it is felt that the exact process of wind wave generation has been rather satisfactorily explained. Pieces of supporting evidence (such as that gathered by Wiegel (1966) and Hino (1968)) of the theories proposed is increasing in number. Another very important fact is that the wave spectrum approach provides a much more realistic and concise description of the complex nature of a sea of waves. This appendix will attempt to follow through the development of wave generation theories, starting with Jeffery, who along with the earlier theoreticians accepts the use of an effective (or significant) wave train.

C.2 DEVELOPMENT OF THEORY

(I) Jeffery

Jeffery studied wind wave generation as early as 1924. His sheltering hypothesis says that as a wind passes over a wave there is a redistribution of pressure. Due to eddies on the leeward side of the wave form, there is a reduction of pressure there, thus causing a transfer of energy to the waves. The hypothesis involves a sheltering coefficient "s". However, measurement from experiments indicates that the actual value of "s" is much too low for significant contribution to wind wave generation, (Phillips, 1957; Wiegel, 1964); particularly for waves of small amplitude. It is no doubt that Jeffery's effect exists, but probably only of minor importance in the usual field conditions.

(II) Sverdrup and Munk

By introducing the effects of tangential stresses, Sverdrup and Munk took up where Jeffery left off.

They proposed the hypothesis that the transfer of energy from wind to wave is due to both the normal stress (pressure distribution as Jeffery suggested) and the tangential stress. Their formulations of the rate of energy transfer due to the normal stress R_n and tangential stress R_t are as follows.

$$R_n = \frac{1}{L}\int_0^L p_z \, w_o \, dx \qquad \text{(C-1)}$$

where $w_o = -kaC \cos k(x-ct)$ is the particle vertical velocity at the surface. p_z is the normal tension (-pressure) on the surface. And,

$$R_t = \int_0^L \tau \, u_o \, dx \qquad \text{(C-2)}$$

where $u_o = ka\,C \sin k(x-ct)$ is the horizontal component of particle velocity at the surface, and where

$\tau = k \rho_o U^2$ is the wind stress.

All other symbols are as previously defined or as defined in Appendix H. k, the resistance coefficient was taken by Sverdrup and Munk to be 2.6×10^{-3}, but other researchers found that its value varies from experiment to experiment.

Thus if R_u is the rate of loss of energy due to dissipation by viscosity, a wave would gain energy and increase if $(R_t \pm R_n) > R_u$. Notice that R_n can take on both positive as well as negative values. At the early stages of wave generation, energy transfer is predominantly due to normal stress, whereas, in the latter stage (when c/U 0.37) tangential stress becomes the important means of energy transfer while the energy transfer due to normal stress may even become negative. Bretschneider (Ref. 56) later also added that the "fully developed sea" is generally said of a water surface which, for a given wind velocity, has reached some sort of a steady state condition, in which energy gained is balanced by energy lost (through viscous dissipation of wave breaking et cetera). In the light of recent research, however, the concept of a "fully developed sea" is found to be misleading, although it is still in common use, (Wiegel, 1964).

Although Sverdrup and Munk's model seems to be adequate for explaining the processes of wind-wave generation, later measurements by Rolls and Hamada, Metsuyasu and Hase (in 1953) reveal conflicts of data which question the validity of the theory. (After Wiegel, 1964, pp.227-229). In addition, (in 1966) Wiegel also showed that transfer of energy due to tangential stress plays a minor, if not insignificant role in the total mechanism of wind wave generation.

(III) Kelvin and Helmholtz

Another hypothesis put forward belongs to studies of classical fluid mechanics by Kelvin and Helmholtz as early as the late nineteenth century (1887, 1888). These investigators (independently) applied the model of oscillations at an interface of two different fluid media, say, for air and water with densities ρ_a and ρ_w respectively. (See Fig. C-1). The subscripts "a" and "w" denote air and water, respectively.

It is found that for a free gravity wave travelling with a phase velocity of c_o on the interface of air with wind speed of U_a in the same direction as the wind and water velocity U_w a critical state is reached at a value of $(U_a - U_w)$, above which the system becomes unstable and the amplitude of the wave train will continue to grow, but below which, the system will remain stable. Since U_w is usually small compared to U_a, the differential velocity $(U_a - U_w)$ is generally taken as U_a and the critical value of U_a is given by

$$U_a > \frac{1 + \rho_a/\rho_w}{\sqrt{\rho_a/\rho_w}} C_o$$ (after Ippen, 1966; Wiegel, 1964).

Nevertheless, not only is Kelvin-Helmholtz theory incapable of explaining how an oscillation at the interface is initiated but that it requires unreasonably high wind speed to produce instability of water waves of a given phase velocity. Thus in accordance with the equation, for a 10 ft. wave (L = 10 ft.) to grow, the minimum wind speed should be 123 knots. (After Wiegel 1964, pp.222). Miles (1958) also showed that Kelvin-Helmholtz's model only becomes effective at wind speeds much higher than usually recorded.

It is important to bear in mind the following comments pertaining to the models discussed above: Jeffery's sheltering hypothesis; Sverdrup and Munk (and Bretschneider's) normal stress and tangential stress model and Kelvin-Helmholtz interface instability model.

(1) Each (and all) of the three hypothesis is (are) unable to account for the major portion of energy transferred from wind to wave but everyone of these models is able to account for a small proportion of energy transferred.

(2) These hypothesis generally assume that a sea of waves can be represented by a significant wave train. The validity of the concept of a significant wave appears to be unsatisfactory as discussed elsewhere in this thesis.

(3) Each of these models, in one way or another, fails to explain how water waves are initially formed by wind.

(IV) Eckart

It was not until two decades ago that spectral analysis was applied to the study of wind-waves. The concept of a spectrum is discussed in Appendix A and will not be repeated here. Among those who used spectral analysis in the field of coastal engineering were Pierson, Neumann, James, Moskowitz and others. These researchers took measurements from actual ocean conditions and arrived at certain semi-empirical formulations which proved to be extremely convenient for general applications, (Pierson et al 1955; Pierson and Moskowitz, 1964; Moskowitz, 1964; Pierson, 1952), although minor objections have been raised to the validity of some of the findings of Pierson-Neumann-James (also known as PNJ) method, with particular regard to the -6th power law of their wave spectrum, (Phillips, 1959). Nevertheless, these and other investigators pioneered coastal engineering into a new dimension.

One of the first to apply wave spectral analysis theoretically was Carl Eckart. He also introduced a more concise and realistic concept of the mechanism of wave generation by considering atmospheric turbulence. He considers a simplified model of wind blowing over water, such that the wind is idealized by a moving system of eddies of identical size but with different locations of gust centres. These eddies are responsible for "gusts", and therefore, pressure fluctuations, in a random way at any location. The pressure fluctuation can be represented in a spectral form (as a function of wave number and frequency) at any spatial location by the use of Fourier analysis. It is because of this phenomenon that water waves are induced. The waves can also be represented by a corresponding spectrum such that the pressure fluctuation component of a certain wave number k and frequency ω, (therefore, of a certain phase velocity ω/k), is responsible for inducing the water wave component of the corresponding wave number or frequency. Thus, contrary to Jeffery's hypothesis in which a variable pressure distribution in the atmosphere is induced by a train or trains of water waves already present, Eckart theorized that the atmospheric pressure variability is an inherent statistical property of the atmosphere, and it is the cause of initial wave formation on a water surface - the reverse of Jeffery's basic argument. For if the pressure is initially uniform over a water body no wave of any kind can be induced.

Furthermore, Eckart also gave the reasoning to a commonly observed phenomenon. Because the gust centres travel with the mean wind velocity U, a wave with phase velocity component equal to U will continue to receive energy from the eddies. Thus, for a wave travelling at an angle to the wind such that $c \cos \alpha = U$, where α is the angle between wind direction and the preferred direction of wave travel, the amplitude will continue to grow. The result is the commonly observed "fish-tail" wave pattern.

(V) Phillips

Phillips argued that the simplified model of atmospheric turbulence due to eddies of identical size is not realistic. This is why Eckart's results tend to be low. The atmospheric turbulence is indeed most complicated to analyze. Not only are eddy centres (gust centres) randomly distributed in space, but the length scales of these eddies are also random functions, describable only in statistical terms. The pressure distribution due to the eddies can be broken down into many components of different wave numbers $\underline{k}_a$, where subscript 'a' denotes 'air'. Each component is borne forth with or convected by the wind at different velocities. The larger eddies occupy larger space, therefore, having larger length scales, and are convected by the wind at a higher location above the water surface while smaller eddies lie closer to the surface. Phillips associates the convection velocity of a water wave component of wave number k with mean wind velocity at a distance 1/k above the surface. Thus for most applications, assuming the log-law wind velocity profile to be valid, most wave components with wave length greater than 1.7 cm are convected at a velocity very close to the mean recorded wind velocity. Serafini (after Hino, 1966) shows that the approximation $U_c(k) = U_\infty$ is a reasonable assumption for most practical applications, where $U_c(k)$ denotes the convection velocity of the wave component and U_∞ is the free stream velocity.

Eckart suggested that water waves are first generated by identical atmospheric eddies in a wave generating region, and then the waves grow as they travel under wind action over a fetch of water. Phillips' eddies, however, develop, interact and decay throughout the duration of the storm and over the entire fetch. He also considered the process of wave generation in two separate stages. The initial stage of development is one in which the time of wave generation, t, is very short. (In fluid mechanics terms: $t \ll \Theta(k)$; where $\Theta(k)$ is the "time scale", or the development time of pressure fluctuations). During this time the wave spectrum is related to the pressure spectrum as follows:

$$\Phi(k,t) \approx \Gamma k^2 \Pi(k) t^4 / \rho_w^2 \qquad \text{(C-4)}$$

in which Φ (k, t) is the wave spectrum; $\Pi(k)$ is the pressure spectrum of the atmosphere, Γ is the response factor (dimensionless) which is a correlation factor between Φ and Π ; t is the development time variable and all other variables are as previously defined.

The response factor is also a function of the following two conditions. (i) A wave travelling at a phase velocity c, and an angle α from the direction of mean wind travel will respond most intensely if $c = U_c \cos\alpha$, such that $\alpha = \cos^{-1}(c/U_c)$ yields the condition for resonance. (ii) Due to the physical property of μ_a, the kinematic viscosity of air, components of $\Pi(k)$ with large wave number, k, can be neglected.

For the principal stage of wave development, Phillips gave the following expression for the wave spectrum

$$\Phi(k,t) \sim \frac{k^2}{2} \frac{\Pi(k)\,t}{\sqrt{2}\;\rho_w^2 n_2^2}\;\Theta(k,v) \qquad \text{(C-5)}$$

where $V = c(k)/\cos\alpha$ and $n_2 = (gk + Tk^3/p)^{1/2}$.

It is important to note that Φ is a linear function of time. Furthermore, the shape of the wave spectrum with respect to k, the wave number, remains constant until the slopes of the waves become so large that effects of dissipation become important. Subsequently, the mean square deformation of a sea can be known; $\bar{\eta}^2 = \iint \Phi(k,t)\,dt\,dk$. Phillips further showed that

$$\Theta(k,V) = 1/(k(U_c - c(k)/\cos\alpha)) \qquad \text{(C-6)}$$

and thus

$$\bar{\eta}^2 \sim \frac{\Pi(k)\,t}{2\sqrt{2}\;\rho_w^2\,U_c\,g} \qquad \text{(C-7)}$$

It was discovered later that the expression is too small by a factor of $\sqrt{2}$. (Longuet-Higgins, 1962).

(V) Miles

Miles approached the problem of wave generation with a line of thought quite similar to Jeffery's. He assumed that wave forms are already in existence. Furthermore, the pressure distribution of the atmosphere is affected by the perturbation of the water waves. The water surface deformation (deviation from mean water level) is given by the complex form

$$\eta(x,t) = a e^{ik(x-ct)} \qquad \text{(C-8)}$$

where a is the wave amplitude and all notations are as previously defined. This representation is in essence identical to that of the sine function as in Equation (A-3), $\eta(x,t) = a_0 \cos(kx + \omega t + \epsilon)$. Subsequently, since the surface deformation is a function of air pressure, p_a, the following relationship can be assumed.

$$p_a = ((\alpha + i\beta)\rho_a U_1^2)\, k \times \eta \qquad \text{(C-9)}$$

where U_1 is an arbitrary reference air velocity; $\alpha + i\beta$ is a complex dimensionless coefficient for pressure, in which both α and β are functions of c and k; all other notations are as previously defined. It may be noted that $k\eta = \frac{2\pi}{L}\eta$ is a form of wave slope.

The surface deformation and atmospheric pressure are related to each other by the equation of motion such that the resistance stress due to deformation plus change in inertia equals the force applied due to pressure variation. The mathematical form for this balance for a unit area is:

$$L\eta + m\eta_{tt} = -p_a \qquad \text{(C-10)}$$

or

$$L\eta + mk^2c^2\eta = -(\alpha + i\beta)\rho_a U_1^2 k\eta \qquad \text{(C-11)}$$

where m is the mass per unit area; η_{tt} is an acceleration with suffix 't' denoting differentiation with respect to time; L is a linear operator (a transformation function) such that $L\eta$ yields the resisting stress. Assuming for resistance stresses, free surface wave condition approximates the enforced wave condition,

$$L\eta = m k^2 c_f^2 \eta$$

leading to the following equation

$$c^2 = c_f^2 + s(\alpha + i\beta) U_1^2 \qquad \text{(C-12)}$$

where c_f is the phase velocity of the enforced wave; $s = \rho_a/\rho_w$; U_1 is the wind velocity at a reference level. As an approximation, however,

$$c = c_f \left[1 + \frac{1}{2} s(\alpha + i\beta)\left(\frac{U_1}{c_f}\right)^2 \right] \qquad \text{(C-13)}$$

The rate of growth per radian, (or the negative damping ratio) by virtue of some operations of mathematics is

$$\zeta_a = 2 \times \frac{\text{Imaginary part of c}}{\text{Real part of c}} \qquad \text{(C-14)}$$

such that $\zeta_a = s\beta\left(\frac{U_1}{c}\right)^2$ (C-15)

where ζ_a is the negative damping ratio for transference of energy from air; c can be assumed to approximate c_f . Loss of energy due to viscosity, on the other hand, is given by the following damping ratio,

$$\zeta_w = -4\,\nu_w \frac{k}{c} \qquad \text{(C-16)}$$

so that the net damping ratio is

$$\zeta = s\beta\left(\frac{U_1}{c}\right) - 4\,\nu_w \frac{k}{c} \qquad \text{(C-17)}$$

The effect of tangential stress due to wind as suggested by Sverdrup and Munk can similarly be included. This effect, however, has been shown to be insignificant and can be disregarded for most practical applications. (Wiegel, 1966, Hino, 1966).

By definition,

$$\mathscr{E} = \frac{1}{kc\overline{E}} = \frac{\partial \overline{E}}{\partial t}, \qquad \text{(C-18)}$$

where $\overline{E}$ denotes the mean energy. Thus

$$\mathscr{E}\, kc\, \partial t = \frac{\partial \overline{E}}{\overline{E}} \qquad \text{(C-19)}$$

Integrating,

$$\ln \frac{\overline{E}}{\overline{E}_o} = \mathscr{E} kct, \quad \text{or} \quad \overline{E} = \overline{E}_o e^{\mathscr{E} kct} \qquad \text{(C-20)}$$

where $\overline{E}$ is the energy at the initial state. Since $\overline{E}$ is proportional to H^2 and the mean square elevation of the sea, the wave spectrum is,

$$\Phi = \Phi_o\, e^{\mathscr{E} kct} \qquad \text{(C-21)}$$

Thus it is a simple matter to evaluate sea surface conditions, such as the mean square deformation, once an initial state and the duration t is known. The difficult problem, however, is the evaluation of β, the complex component of the pressure coefficient in Equation (C-9). The solution to this problem was, indeed, the thesis of Miles first papers on wave generation.

Adopting the dimensionless parameters,

$$\xi = ky; \quad U - c = U_1 \omega(\xi); \quad \psi = U_1 \phi(\xi)\eta(x,t) \qquad \text{(C-22)}$$

where ψ is the atmospheric stream function and ϕ is a dimensionless stream function; all other variables are as previously defined. (It is easy to see that ξ is a dimensionless height, and $\omega(\xi)$ is a dimensionless velocity). Imposing the proper boundary conditions on the stream function at $y = y_o$ and $y = \infty$, the expression for β is

$$\beta = -\pi\, |\phi_c|^2 \left(\frac{\omega_c''}{\omega_c'}\right); \qquad \omega_c = 0 \qquad \text{(C-23)}$$

where the subscript 'c' denotes evaluation at $\xi = \xi_c$ where $U = c$. Thus the problem being solved is the atmospheric functions: $\phi(\xi)$ and $\omega(\xi)$. This, Miles did by assuming the log-profile for mean wind velocity, and approximating a solution to the Orr-Sommerfield equation (an equation that describes motion of perturbed fluids - the details of which will not be attempted in this thesis). Two years later, after a solution to the equation was made possible, Miles re-evaluated β. The values of β have been supported by experimentation to date, (after Hino, 1966; Wiegel, 1966 and others) and the original graphical form is reproduced in Fig. C-2. The arithmetical approximation of β is as follows: (after Hino, 1966).

$$\left.\begin{array}{ll}
\beta = 3.39 - 0.9406\{\log(8.61 \times 10^{-3}/\xi_c)\}^{1.860} & \\
\qquad (\xi_c \leq 8.61 \times 10^{-3}) & \\
\beta = 3.39 - 1.294\{\log(\xi_c/8.61 \times 10^{-3})\}^{2.323} & \\
\qquad (8.61 \times 10^{-3} < \xi_c \leq 5.48 \times 10^{-2}) & \\
\beta = -0.1402 - 2.181 \log \xi_c & \\
\qquad (5.48 \times 10^{-2} < \xi_c \leq 3 \times 10^{-1}) & \\
\beta = \{\log(3/\xi_c)\}^{2.362} & \\
\qquad (3 \times 10^{-1} < \xi_c \leq 2) & \\
\beta = 0.017 \exp\{2(2 - \xi_c)\} & \\
\qquad (2 < \xi_c) &
\end{array}\right\} \qquad \text{(C-24)}$$

where $\xi_c = \Omega\left(\frac{U_1 \cos\varphi}{c}\right)^2 \exp\{c/U_1 \cos\varphi\}$, $\Omega = gz_0/(U_1 \cos\varphi)^2$ and z_0 denotes the roughness parameter. The value of β becomes almost independent of Ω when $\xi_c < 2$.

It is of interest to note the following.

(1) β, as per Equation C-23, is proportional to the negative derivative of the slope of the shear velocity profile, $-U_c$. This is in agreement with Taylor's findings.. (After Miles, 1957; after Lighthill, 1962, Ref. No. 143).

(2) With regard to the same equation, where $-U''$ is larger, the growth of wave is faster. Thus smaller

waves, corresponding to larger k, responding to eddies in the lower portion of the wind profile where -U" is large, tend to gain energy more readily.

(3) The transference of energy to waves with phase velocity c is due only to the thin layer of the atmosphere where U = c.

(4) Whereas Phillips' spectrum tends to gain energy in linear proportion with time, Miles' mechanism rules that the growth rate of wave spectrum is an exponential function of time to the base e.

(5) Phillips' mechanism of energy transfer is one of random atmospheric fluctuation; Miles' is the consequence of shear flow (thus the log-profile) of the atmospheric structure. Furthermore, by Miles' process, the pressure fluctuation and surface deformation are "coupled" and are 180° out of phase (that is, a "low" of water surface corresponds to a "high" of air pressure) - a fact that has been verified by many (such as Longuet-Higgins, 1962; Longuet-Higgins Cartwright and Smith, 1963; Wiegel, 1966; Hamada, 1968, (Ref. No. 150), etc.)

(6) Lighthill also gave a different interpretation of Miles' results in a later paper. (Lighthill, 1962, Ref. 143).

(7) Miles assumed that waves are already in existence when the wind blows over them but did not account for how waves are initially produced.

C.3 MILES-PHILLIPS' MODEL

Noting the last point (7) mentioned above, it is possible to simulate the entire process of wind-wave generation by combining Phillips' and Miles' models. At the initial stage of development, the water surface is smooth and waves can only be produced by random atmospheric pressure fluctuations as described by Phillips. At later stages when the wave forms are well established, the shear flow of air above them is perturbed, and Miles' mechanism becomes dominant. Miles (1960) advanced the following combination,

$$\Phi(k,t) = \frac{1}{2\rho_w^2 c^2} F(mt) \times t \times \int_0^\infty \Pi(k,t) \cos\left[k(V\cos\alpha - c)\Upsilon\right] d\Upsilon \qquad \text{(C-25)}$$

where $$F(mt) \approx (e^{2mt} - 1)/2mt\,; \qquad \text{(C-26)}$$

$$m = \frac{1}{2}\xi kc\,; \qquad \text{(C-27)}$$

Υ is a time scale of atmospheric fluctuation; all other notations are as defined previously. When the value of t is small F(mt) approaches unity and Equation C-25 becomes essentially identical to equation C-7. When t is large, the F(mt) grows exponentially with time, so does the spectrum, consequently. Phillips also investigated the occurrence of the transition frequency, above which Miles' effect becomes important, below which Phillips' mechanism is predominant (Phillips and Katz, 1961).

C.4 THE EQUILIBRIUM RANGE

A wave, obviously, cannot continue to grow exponentially without limit as per Equation C-25. The physical limitations to this growing process are essentially the loss of energy due to viscous dissipation and the transference of energy from wave components of a higher wave number to one of lower wave number. The resulting wave spectrum is called the equilibrium spectrum.

When a wave has grown to such a state (such a magnitude of amplitude and steepness) that the acceleration near the crest is larger than g, the gravitational acceleration, the particles at and near the crest are in a state of tension. Consequently, the surface would tend to break and become detached from the rest of the wave form.

Thereafter, a viscous dissipation of energy will be involved, which is commonly observed as the formation of white cap waves in a stormy sea. Thus a wave component with a given frequency (or angular velocity) can only attain a certain maximum wave height. By dimensional analysis, Phillips arrives at the following relationship (1958).

$$\Phi(\omega) = \alpha g \omega^{-5} \tag{C-28}$$

where $\alpha = 7.4 \times 10^{-3}$, a dimensionless constant. This relationship has been tested and proved valid by many researchers. (Longuet-Higgins, 1962; Cartwright, Longuet-Higgins and Smith, 1963 (Ref. No. 144); Snyder and Cox, 1966; Hess, Hidy and Plate, 1969; Pierson and Moskowitz, 1962; and others). It also finds fault in the Neumann Spectrum (which is $\Phi(\omega) \sim C_1 \omega^{-6}$ where C_1 is a constant).

The second mechanism is the transferring of energy from higher wave number components to lower wave number components. The process is one of non-linear combination satisfying the condition of $\omega^2 = g|\underline{k}|$ where $\underline{k}$ is a wave number vector. This energy transferring mechanism does not play an important role in the applications involved in this thesis and is neglected herein. Descriptions of this process can be found in Phillips' original paper (1960) or Longuet-Higgins' interpretation (1962).

C.5 SUMMARY OF WAVE GENERATION MODELS

Some of the important developments of wave generation have been reviewed in this appendix. The following points may serve to reiterate the main ideas.

(1) Phillips' mechanism of random pressure fluctuation, much like Eckart's, is responsible for "wave induction" in the initial stage of wave generation.

(2) It is not until wave forms are well established that the atmospheric pressure becomes "perturbed". During the later stages of wave generation, the wave growth is in accordance with Miles' model.

(3) In an actual situation, Kelvin-Helmholtz model appears to be negligible for wind speed of normal experience. It only becomes noticeable at much higher wind velocities.

(4) Sverdrup and Munk's tangential stress has been found to contribute only a small portion of the energy transferred - no more than 10 percent at any time (Wiegel,1966). In most applications it is negligible.

(5) The total mechanism of wave generation, therefore, is a combination of Phillips' and Miles' models as expressed by Equation C-25.

(6) The process of wave growth, however, is limited by wave breaking and energy transfer between components. The former is the more important one of the two mechanisms and is given by Equation C-28.

C.6 WINDWAVE SPECTRUM

The water wave spectrum due to any storm can be defined as the intersection of the spectrum of wave growth due to Equation C-25, and the equilibrium spectrum. Diagrammatically, this is represented in Fig. C-3. The curve A is the limit due to the equilibrium spectrum; curves B_1 and B_2 are due to Equation (C-25). B_1 defines the spectrum for a storm of a shorter duration while B_2 is for one of a longer duration and so on. The spectrum for any storm is the triangle bounded by the A curve and the corresponding

B curve, and the x-axis.

The physical significance of Fig. C-3, is, again, that when a storm starts blowing, the higher wave number waves are more easily excited and are readily generated. But soon these waves have grown to the state such that they will break and dissipate all the additional energy transferred to them from the wind. Meanwhile, the longer waves take longer time and more energy to excite, though they tend to carry a larger amount of energy when fully developed. While the shorter waves are breaking, the longer waves are still in the growing stages. If the storm duration and the fetch are sufficiently long, the longer waves will also reach the breaking state. Now energy will be transferred to waves with yet longer wave lengths and the process repeats itself.

The mean square deformation of the sea is the area under the spectrum (bound by curve A and B) as in Fig. C-3. This is easily obtained by an integration operation, thus

$$\bar{\eta}^2 = \int_0^\infty \Phi(k)\, dk \qquad \text{(C-29)}$$

It must be remembered, however, that the spectrum is one-dimensional. Clearly, the operation with a two-dimensional spectrum is similar in principle. Thus

$$\bar{\eta}^2 = \int_0^\infty \int_{-\pi/2}^{\pi/2} \Phi(k,\alpha)\, dk\, d\alpha \qquad \text{(C-30)}$$

The mean-square wave amplitude is,

$$\bar{a}^2 = 2\,\bar{\eta}^2 \qquad \text{(C-31)}$$

And from Longuet-Higgins' findings (1952

$$a^{(1/3)} = \sqrt{2}\,\sqrt{\bar{a}^2} \qquad \text{(C-32)}$$

(See Appendix B). Thus for significant wave height,

$$\begin{aligned} H^{(1/3)} &= 2\times\sqrt{2}\,\sqrt{\bar{a}^2} = 2\times\sqrt{2}\times\sqrt{2\,\bar{\eta}^2} = 4\sqrt{\bar{\eta}^2} \\ &= 4\left[\int_0^\infty\int_{-\pi/2}^{\pi/2} \Phi(k,\alpha)\, dk\, d\alpha\right] \end{aligned} \qquad \text{(C-33)}$$

C.7 "FULLY DEVELOPED WAVE SPECTRUM"

In the light of Phillips' and Miles' contribution, it is evident that a sea may not reach a "steady-state" at all, regardless of how long the wind has been blowing, provided the fetch is sufficiently long, and the water is sufficiently deep. For a given wind velocity, waves greater than a certain wave length continue to receive energy directly from the wind as well as from wave components of shorter wave length via non-linear interactions as mentioned earlier. (Also see Wiegel, 1964). However, the growth of long waves may be rather slow and may escape notice by observers. In this context, the "fully developed sea" is taken to mean the state of the sea, when changes in wave condition is not noticeable.

Pierson and Moskowitz, 1964; (also Moskowitz, 1964,) gave the following relationship for the "fully developed" deep water wave spectrum.

$$S(\omega)\, d\omega = (\alpha g^2/\omega^5)\, e^{-\beta(\omega_0/\omega)^4}\, d\omega \qquad \text{(C-34)}$$

where $S(\omega)$ is the spectral density as a function of frequency in radians/sec., (or angular velocity); $\alpha = 8.10 \times 10^{-3}$; $\beta = 0.74$; $\omega_0 = g/U$ is the spectral peak frequency and U is the mean wind velocity reported by weather ships, i.e. at the height of 19.5 meters. This relationship is the result of dimensional analysis aided by interpretations from selected samples from a large collection of data. It is important to note the fact that Pierson and Moskowitz reported the difficulties of obtaining reliable data. Other similar useful relationships concerning well developed wave spectrum are given by a number of investigators including Bretschneider, Neumann, Rolls et cetera. (Ocean Wave Spectra, 1963; also after Snyder and Cox, 1966). A sample of Pierson and Moskowitz' fully developed wave spectrum is given in Fig. C-4.

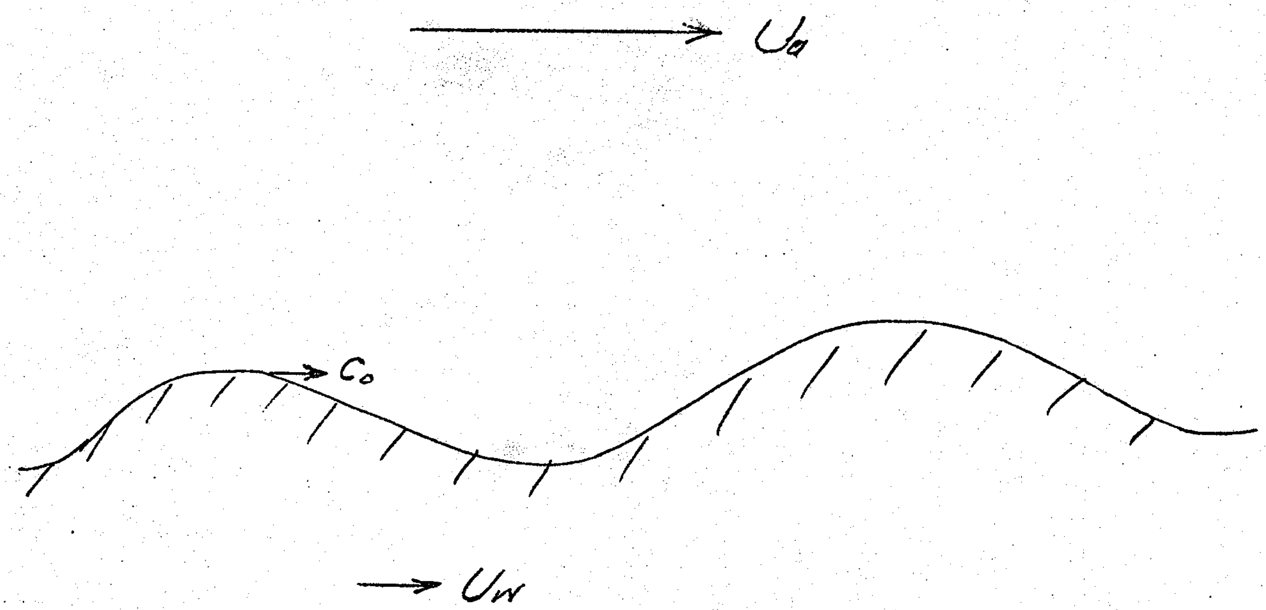
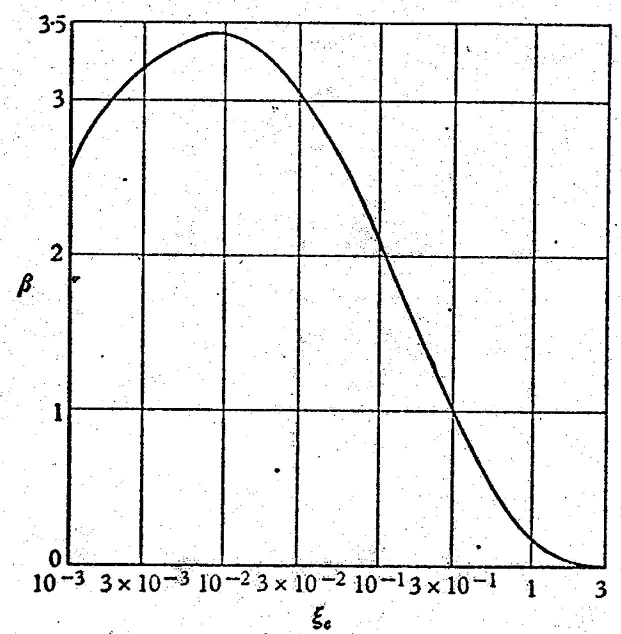
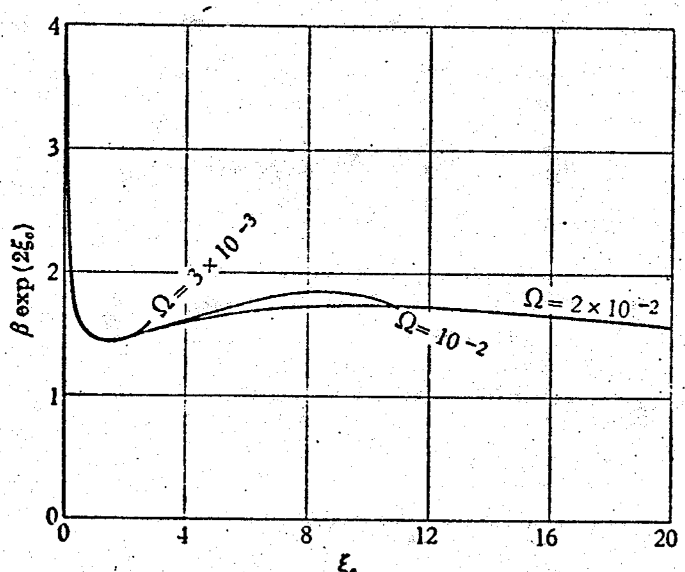


FIGURE C-1. KELVIN-HELMHOLTZ'S AIR-WATER INTERFACE WAVE GROWTH BY INSTABILITY.



(a) β vs ξ_c.



(b) $\beta \exp(2\xi_c)$ vs ξ_c.

FIGURE C-2. VALUE OF β FOR MILES' MODEL. (After Miles, 1959.)

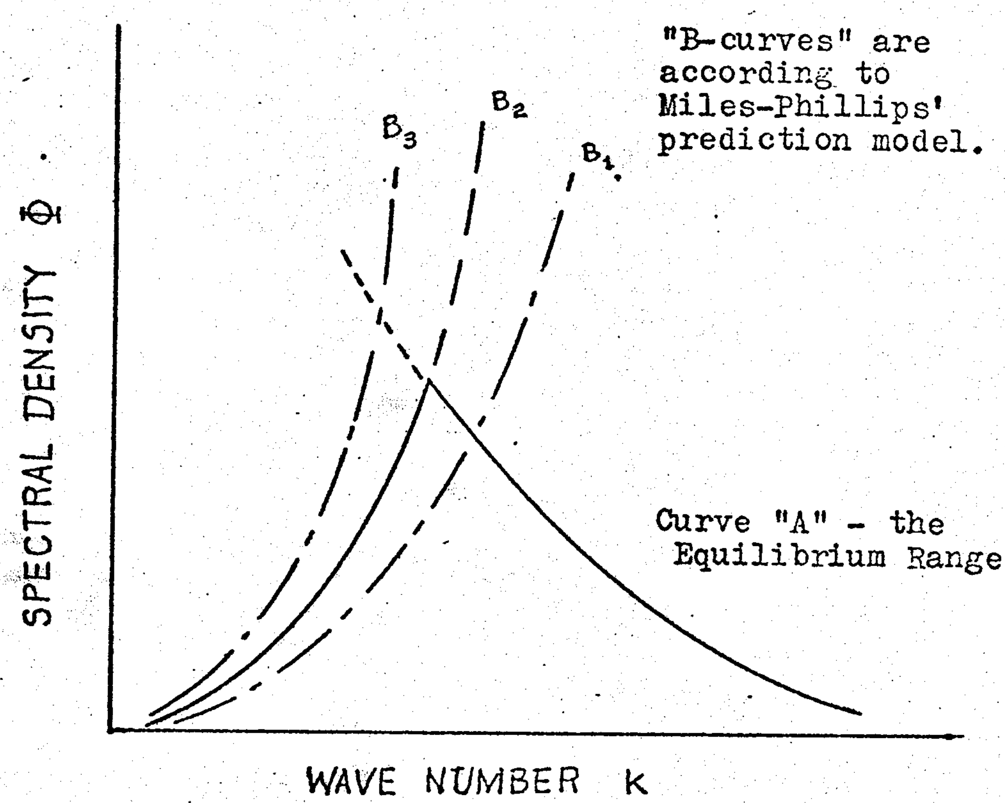


FIGURE C-3. GROWTH OF WAVE SPECTRUM.
(Note: Not drawn to scale.)

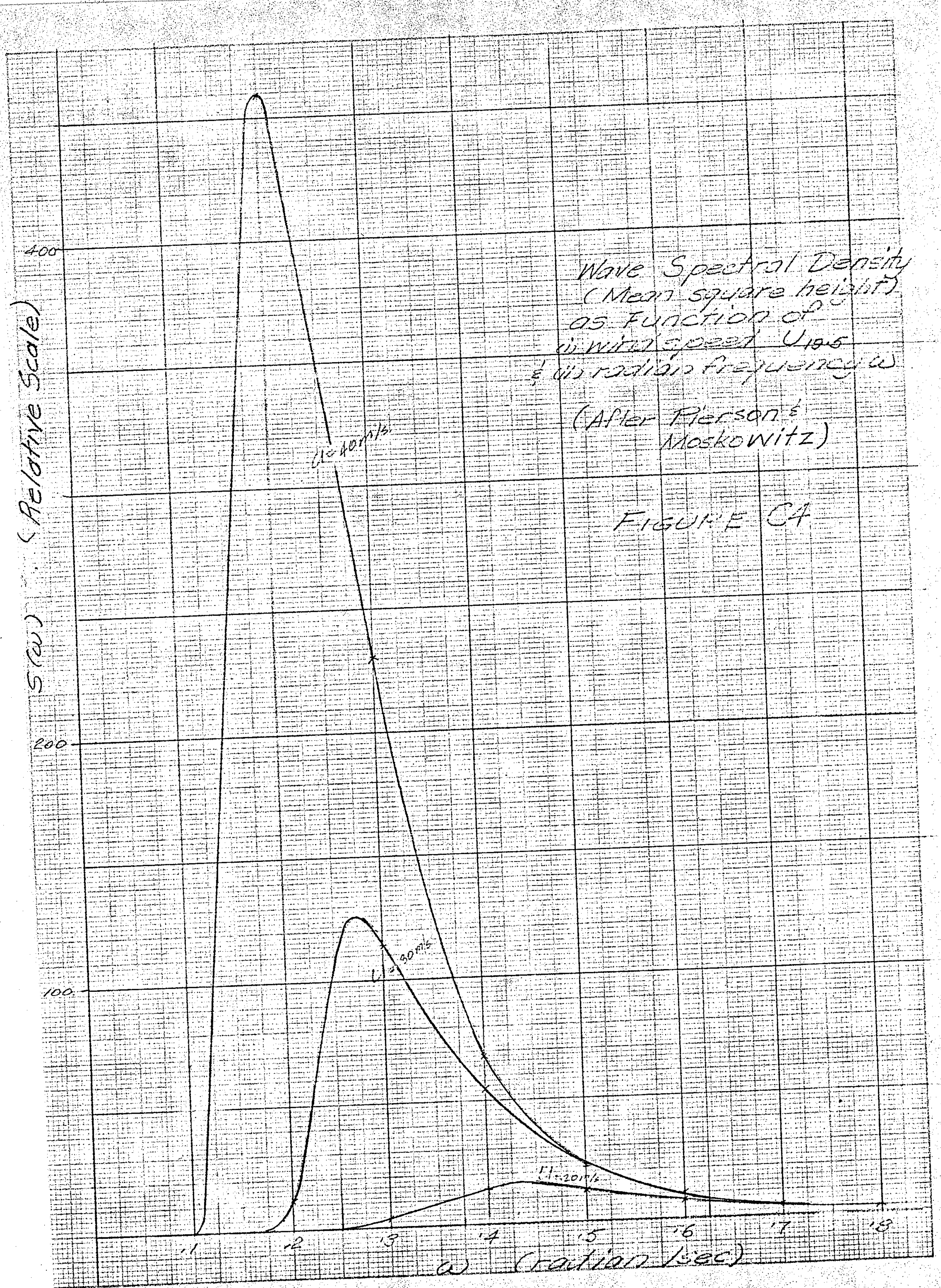

Wave Spectral Density
(Mean square height)
as Function of
(i) wind speed U19.5
& (ii) radian frequency ω
(After Pierson &
Moskowitz)
S(ω) (Relative Scale)
400
200
100
U=40m/s
U=30m/s
U=20m/s
.1
.2
.3
.4
.5
.6
.7
.8
ω (radian/sec)

FIGURE C4

APPENDIX D

WAVE REFRACTION

D.1 INTRODUCTION

Given a simple sine wave train in a lake or sea, in the plan view, the term wave ray refers to the line drawn everywhere perpendicular to the wave crests, and the continuous line depicting the location of a continuous wave crest is called the wave front. If a series of sine waves is travelling in a deep sea or lake, the wave fronts are parallel to one another, and the wave rays remain straight. But as the waves approach shallow water where the depth is less than half wave length, the waves begin to "feel" the shallowness effect: the wave rays slowly bend towards the normal of the shoreline and the wave front tends to conform to the configuration of the shoreline as it progresses towards the shore. This phenomenon is called wave refraction, and either directly or indirectly, it is responsible for the formation of most, if not all, existing coastal features in regard to their shape, size, orientation and other physical properties. The significance of wave refraction has in recent years been noticed by many coastal engineers. The following are some discussions on the mechanism of wave refraction and methods for constructing refraction patterns, with particular reference to the approach adopted in this thesis.

D.2 SNELL'S LAW

The basic theory of wave refraction is taken from Snell's law of optic refraction, viz.,

$$\frac{i_2}{i_1} = \frac{\sin\alpha_2}{\sin\alpha_1} \qquad \text{(D-1)}$$

where $\dot{\iota}_1$ and $\dot{\iota}_2$ are speeds of light and α_1 and α_2 are the angles between the light path and the perpendicular to the interface of the two media: Medium 1 and Medium 2 respectively. (See Fig. D-1(a)). This relationship can be rewritten for water wave travel as

$$\frac{c_2}{c_1} = \frac{\sin\alpha_2}{\sin\alpha_1} \tag{D-2}$$

for the situation where a wave of phase velocity c_1 suffers a change in velocity of travel due to change in depth. The new phase velocity is c_2. If the wave ray approaches the normal of the depth contour at an angle of α_1 with phase velocity c_1, it will leave the depth contour at an angle of α_2 with its normal. It must be borne in mind, however, that whereas the interface of two media in light refraction is often, if not always, well defined, (such as air-glass interface), the change of depth of an ocean floor or a lake floor is usually continuous and very gradual. Nevertheless, the same principle applies. Furthermore, by way of geometry, if the width of the wave front is b_1 before refraction, the width b_2 after refraction is given by relationship:

$$\frac{b_2}{b_1} = \frac{\cos\alpha_2}{\cos\alpha_1} \tag{D-3}$$

It is in order to recall certain equations from Appendix A:

$$\frac{c}{c_o} = \tanh kd \qquad\qquad c_o^2 = gk_o$$

$$\frac{c_g}{c} = \frac{1}{2}\left[1 + \frac{2kd}{\sinh 2kd}\right] = n \qquad\qquad E = \gamma_w \frac{H^2 L}{8}$$

$$c = \left[\frac{gL}{2\pi}\tanh kd\right]^{\frac{1}{2}} \qquad\qquad P = \frac{nE}{T}$$

where the notations are as defined in the same Appendix, and in Appendix H.

Assuming no energy loss due to bottom friction and viscosity et cetera, the power carried by a wave ray remains constant through the refraction process. Thus,

$$\text{POWER AT ANY LOCATION} = P = P_o \qquad \text{(D-4)}$$

where subscript 'o' denotes deep water condition.

By manipulating Equation D-2 with the previous equations it can be shown for a particular wave ray that:

$$\left[\frac{H}{H_o}\right]^2 = \left[\frac{C_{go}}{C_g}\right] \times \left[\frac{b}{b_o}\right]$$

$$= \frac{b_o/b}{(\tanh kd)^{1/2}\,(1 + 2kd/\sinh 2kd)} \qquad \text{(D-5)}$$

The first expression in equation (D-5) is termed the refraction function (denoted by $[K^2]$ - Pierson et al., 1955). It indicates the energy per unit width of wave front as a ratio to that at deep water, since energy is directly proportional to wave height square.

Similarly the ratio of energy per unit width of wave front as recorded at two different stations is expressed as:

$$\left[\frac{H_A}{H_B}\right]^2 = \frac{C_{gB}}{C_{gA}} \times \frac{b_B}{b_A} \qquad \text{(D-6)}$$

where subscripts A and B denote conditions at locations A and B respectively which lie in the path of the wave ray; and

$$C_{gA} = \frac{1}{2}\left[1 + 2k_A d_A/\sinh 2k_A d_A\right] C_o (\tanh k_A d_A)^{\frac{1}{2}}$$

$$C_{gB} = \frac{1}{2}\left[1 + 2k_B d_B/\sinh 2k_B d_B\right] C_o (\tanh k_B d_B)^{\frac{1}{2}} \qquad \text{(D-7)}$$

Equation (D-5) is generally applicable to oceans where deep water wave conditions are known and can be used as a reference. The use of equation (D-6) appears to be more suitable for Lake Winnipeg, which is relatively shallow, as will be explained later.

It is to be noticed that in wave refraction, wave period T is assumed to remain constant - a condition which is valid for all practical purposes.

D.3 NUMERICAL METHOD

Another method for wave refraction has been advanced by a group of researchers: Griswold, Nagle, Mehr, Harrison and Wilson. This method, though its basic concept is similar to Snell's Law, is most suitable for numerical computation, whereas the approach given by Pierson et al., (1955) as described in Section D.2 is suitable for graphical construction. However, if both are solved mathematically and the increment of interval for approximation is suitably chosen in each case, it can be said that both approaches would yield the similar degree of accuracy, given the similar set of data.

Graphical construction has been in common use for charting wave refraction patterns. In fact, before the new method (often referred to as the "improved method") was evolved by Griswold and others, and before the adaptation of the electronic computer became an everyday affair, the graphical method was the only solution to wave refraction problems. For the graphical approach there are two commonly used methods: the wave-ray method and the wave front method. Since these methods are not used in this study, their description will not be further pursued herein. (See pp. 163f, Wiegel, 1964, for a comprehensive description of these methods).

In this study, the method described in Reference No. 101 (Pierson et al., 1955), is applied mathematically, and further adopted for computerization. It is felt that if the depth contours, say for Lake Winnipeg, are properly drawn and selected the results would be comparable to those computed by using the numerical model due to Griswold, Wilson and others. In passing, it is mentioned of course, that graphical methods are not suitable for application to the present study of wave spectrum, chiefly because of the immense amount of labour that would be involved.

D.4 REFRACTION OF A SPECTRUM

In the past when water wave spectrum was still relatively unknown to coastal engineers, the significant wave with significant wave length (L_s), wave period (T_s), and wave height (H_s) was generally used to represent a disturbed sea. Refraction diagrams would be constructed manually (Wiegel, 1964) for a significant wave only. Such practice, although producing desirable results, in many applications, has been found to be only a very crude approach, and tends to give erroneous answers in situations where wave refraction plays a significant role. (Pierson et al., 1955).

A more realistic approach is to consider the disturbed sea as composed of many (infinite in number) trains of wave components differing in amplitude, period, direction et cetera, that is, as a spectrum, (see Appendix A on spectrum) and refract each component train separately. Then add these components together again at the beach location of interest. It is often found that a particular area of the coastline may tend to receive more wave rays of longer wave-length components while another area may tend to receive more wave rays of shorter wave-length. This fact is important because a large portion of the total energy may be concentrated in wave components of a certain range of wave-length, and the beach locations receiving a large share of these components may consequently receive a significant proportion of the total wave energy. The importance of the refraction of wave spectrum can thus be appreciated.

D.5 PRESENT APPLICATION

The task of manually constructing a refraction diagram by either calculation or graphical construction for one wave frequency is a lengthy and laborious task. To obtain a refraction function chart (such as the one shown in Pierson et al., 1955, pp.209) by refracting waves of different frequencies (say, 20 different frequencies) for waves approaching a section of coastline from various different directions (say, 15 different directions at 10° intervals) would require a good deal of patience and endurance (Pierson et al., 1955). With the aid of a modern computer, the labour for achieving the same results can be reduced. For this thesis a computer program is written for wave refraction. This program is later used as a subprogram for wave generation over Lake Winnipeg, as is explained in Chapters IV and VI.

Notwithstanding that the numerical method has already been developed for wave refraction, by Griswold, Harrison, Wilson, Mehr and others, and that, in particular, a computer program written by Wilson (Wilson, 1966) was readily available, there was the need to derive a separate program which can be suitably applied to Lake Winnipeg and conveniently adapted for wave generation. It is not felt that this computer model is superior to the one formulated by Wilson, but the results should be close, if not similar. It is also felt that whereas Wilson's model is more efficient where bathymetric lines are more or less evenly spaced, the model developed here is more suitable for irregular bottom configurations, particularly for lakes.

In using it, effects of small but important details are not lost due to limitations of the scale of a grid system as in Wilson's model. In particular, wave refraction over the entire lake can be done in "one-shot", rather than having to transfer from a larger co-ordinate system to a smaller one. The input data and therefore the memory storage required are less and can be handled easily and conveniently.

D.6 COMPUTER PROGRAM FOR WAVE REFRACTION

The computer program is designed to yield output that will give sufficient information for the plotting of wave rays, if desired. The program can be conveniently modified for plotting wave rays by an electronic plotter if necessary. Outputs from the program also consist of refraction function and the width of the wave front as well as other information. This program is written in the Fortran IV language for machine IBM 360/65 located at the University of Manitoba. A listing of the program is given in Appendix F as Computer Program (A).

The following is a brief introduction regarding the inputs and outputs and the various parts of the program itself.

D.6.1 Outputs

For the present study, the following items are computed and printed as output in E-format.

(i) The wave period.

(ii) The fetch of the wave ray in the direction of the wind.

(iii) The mean width of the fetch.

(iv) The mean depth of the fetch.

(v) The mean wave number along the fetch.

(vi) The refraction function $[K^2]$ of

the wave ray as it approaches the coast as a ratio to that along the major portion of the wave ray.

(vii) The angle at which the wave ray attacks the shore.

(viii) The width of wave front at the end of the wave ray travel.

(ix) &
(x) The x-, y- co-ordinates of the final position of the wave ray.

D.6.2 Inputs

Input information to the program requires four items:

(i) Initial direction of wave ray, (THETA), in radians.

(ii) Wave period (T), in seconds.

(iii) The co-ordinate (XO, YO) of the starting point of the wave ray. These can be expressed in units of the co-ordinate system used. Thus, if a co-ordinate system with 1 mile as the unit distance is used, 1.5 miles will simply be entered as 1.5.

(iv) The depth contour information is to be arranged as follows. Each depth contour is approximated by straight line sections, hence defined by the co-ordinates of the number of points the program user selects. (Hereafter in this appendix unless otherwise specified, the word "point" will be used in this sense). The co-ordinate unit used is similar to that described in (iii). It would be most desirable if the depth contour is a complete loop (as opposed to an open-end contour line) or can be approximated by such. If, however, this is not the case, it would be necessary to add one more point to the number of points already defining the contour line such that the effect is to add a fictitious segment which would lead the "tail" of the contour to go around the starting point of the wave ray by going behind it (referring to "front" meaning in the advancing direction of the ray), and then join the first point of the contour.

The data cards containing the co-ordinates of the points of each depth contour are to be preceded by a card containing the depth (DEEP (l)) of the contour. If the last point of a contour does not fill in the last format fields of a data card, the remaining fields of the card should be filled in with "1's". If it does, it should be followed by an additional card fully punched with "1's".

After all the depth contour data have been entered, the data deck is to be ended with a card punched with "-1" according to the format prescribed for depth.

With regard to (i) above, the angle is to be measured as in trigonometric convention. (i.e. anticlockwise as positive, and East is "zero radian").

The formats for data input used in the present case are the same (F 10.4). The program user, of course, may choose for himself any other arrangement.

The output formats are as defined in WAVRAY and in SHALOW; but again, it can be adjusted to the preference of the user.

D.6.3 The Main Program

This part of the program functions no more than reading in information, organizing it and then assigning the computing tasks to the subprograms. The route through which the assignments are passed down the subprograms is as follows:

MAIN → SHALOW → WAVRAY → SEARCH → LOCAT → REFRAT → and others.

These subprograms will be reviewed in the reverse order.

D.6.4 Subprogram REFRAT

This is a relatively straight-forward subroutine performing the function of evaluating the refracted direction of travel after a wave ray has undergone a change of phase velocity. The Snell law (described earlier) is used.

D.6.5 Subprogram LOCAT

To test for which section of a particular contour line the wave ray will intersect, the first step is to see whether the contour line comes into a "focus area" or an "area of interest". This is accomplished by, say, setting the focus area as an area of twenty miles square surrounding the previously defined point in the path of the wave ray. This point is generally the intersection of the wave ray with a contour in the last computation step. It is not necessary to test every point defining the contour; for this study, every tenth point on the contour is tested. Care must be taken, however, that within an area of the size of the focus area, there should be no less than ten consecutive points of the depth contour being tested. As soon as the computer finds out that a contour line enters the focus area, it will then set out to discover which segments of the contour are in the area, so that it can recall them at a later time to test whether they will be intersected by the wave ray. All these steps are performed in the subprogram LOCAT.

The adjustable parameters are as follows:

(i) PSCOP2 and PSCOP1 define the half length of the sides of the focus area.

(ii) INSTEP used here has a value of 10. It defines the interval between points which will be tested for a depth contour.

D.6.6 Subprogram SEARCH

The subprogram SEARCH then determines which segment of the contour line the wave ray would intersect. It is found that this task can be more efficiently performed by using the rotating co-ordinate system, so that one only has to compare the rotating cartesian ordinates. A contour may be intersected at more than one location, but only the location in the direction of the advancing ray, closest to the last intersection (of the ray with the depth contour) is retained. If it is found that the depth contour is not intersected by the wave ray at all, then a fictitious location of intersection is assumed to be at some unreasonably larger distance away so that when compared with other locations of intersections with other contours, this point will be disregarded.

The information regarding the point of intersection on the contour is returned to the calling program; in this case, subprogram WAVRAY.

D.6.7 Subprogram WAVRAY

This subroutine is responsible for a number of functions. The more important of these are listed below.

(i) A request for a wave ray to be computed is passed to this subprogram by the main program. The information given consists of the location of the starting point of the ray, its direction and its wave characteristics such as period. The first operation of SEARCH is to find out which depth contour to start working from (in finding which depth contour is closest to starting location). The process is similar to locating probable intersections as will be described below.

(ii) When a starting contour (which will be referred to as "base contour" hereafter) is selected, it will be used as some kind of a base, and a number of contours neighbouring it together with itself are selected for further inspection. (Seven contours are used in this case). The main objective of WAVRAY is to find out which one of these selected contours will be intersected by the wave ray, and at what location. Then the subprogram will assume the intersected contour as the new base contour and the location of intersection will be the new starting point for the wave ray. In this fashion the process reiterates itself until the wave ray arrives at the shore, or at least very close to the shoreline.

(iii) To find which depth contour a wave ray would intersect, all the possible locations of intersection of the depth contours selected for examination determined by subprograms LOCAT and SEARCH are compared. The location closest to the starting point is most likely the next location of the wave ray. But the computation for charting the wave ray is not yet complete, since refraction has not been accounted for. The procedure described in the next paragraph, in addition to the calling of subprogram REFRAT, completes the process of charting.

(iv) The wave refraction computer program is based on the principle of Snell's Law as discussed earlier.

(See Fig. D-1(b)). Assume that the wave ray starts at A1 on contour A and is refracted due to a rise of lake bottom until it reaches B1 on contour BB. It has travelled along the curve dotted path A1B1, such that its phase velocity is c_1 at A1 and c_2 at A2. If the mean contour MM is the mean of

AA and BB, on account of both depth and orientation, the wave before refraction making an angle α_1 with the normal of MM, will make an angle of α_2 with it after refraction. If the curve A1B1 is an approximate arc of a circle, the actual gradual process of refraction can be approximated by assuming the whole process to take place at the point M1 such that A1M1 and B1M1 are of the same length. Since the location of A1 and the values of c_1, c_2, α_1 and α_2 are known, M1 and B1 can be easily located.

(v) As an appending note to (i) above, the search for the starting depth contour at the commencement of charting a ray is almost identical to procedure (iii) above, with the only exception being that all depth contours are examined rather than only seven of them.

(vi) Other tasks performed by WAVRAY include calculating the width of the wave ray, its refraction function $[K^2]$, its phase velocity, group velocity and its angle of intersection with depth contours. The information is relayed back to the calling subprogram SHALOW.

D.6.8 Subprogram SHALOW

This subprogram is mainly responsible for allotting assignments to WAVRAY and compiling the results received. It also computes for a ray: the length of fetch, the mean depth of the fetch, the mean wave number during the travel of the wave ray, and the refraction function close to the shoreline.

D.6.9 Other Subprogram

There are also a number of other subroutines and function subprograms attending to the tasks of finding trigonometric values, rotating co-ordinates, locations of intersection and so on.

Subroutines ANGLE1, ANGLE2

-- evaluates in which "quadrant" of the trigonometric plane a line lies.

Subroutines TRIG1, TRIG2

-- evaluate trigonometric values.

Functions XRØT, YRØT, XXRØT, YYRØT

-- rotates the co-ordinate system.

Function ASLØP

-- evaluates the slope of a given segment of the depth contour.

Function XTRI

-- locates the point of intersection of a wave ray with a depth contour in the rotated cartesian system.

D.6.10 Restrictions

If the program is to be used as it is written (that is, without altering any part of the object deck), the following restrictionsmust be applied.

(a) The co-ordinate system must be in units of one mile.

(b) The number of contour lines allowed is no more than 20, with no more than 150 points on each line.

(c) The data input for depth contour as per (iv) above, must be arranged in such a way that a set of data for one depth contour is to be placed next to that for an adjacent contour.

(d) The first contour should preferably be the one closest to the shore. In general, not much accuracy is lost by either approximating the 5-foot-depth contour as the shoreline.

(e) No section of any contour is to have less than 10 points per 10 linear miles in the X- or Y- direction.

(f) Outputs are in foot-pound-second units.

None of these restrictions applies, however, with the exception of (c) and (d) if the following parameters are adjusted.

CONVER - This is a conversion factor expressing the co-ordinate unit in either feet or meters. The parameter GEE, which is the gravitational constant 'g', is to be specified accordingly either in meter-gram-sec. units (i.e. 9.8) or in f-p-s units (i.e. 32.2). The output will be in the corresponding units and conditions (a) and (f) are relaxed. In this study CONVER = 5280; GEE = 32.2.

PSCOP1, PSCOP2, PSCOPE - These are parameters units of length of the co-ordinate system, specifying the length scale of the "area of interest" in subprogram LOCAT described above. In the present study, PSCOP1 = 10, PSCOP2 = 10, PSCOPE = 40, for a co-ordinate system that is approximates 40 units by 20 units. By adjusting these and IPSTEP below, restriction (e) may be relaxed.

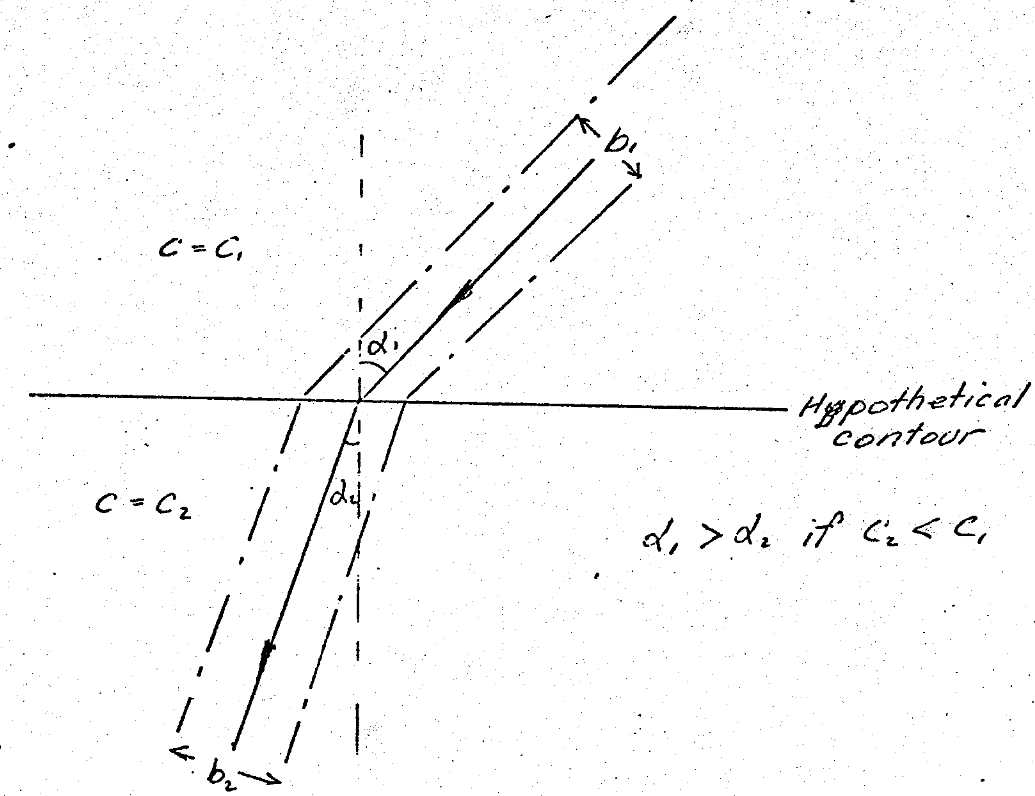
IPSTEP - In the subprogram LOCAT, this parameter performs the function of specifying at what interval of points the contour should be tested with regard to whether it traverses the "focus area". For the present application a value IPSTEP = 10 is assigned. A larger value may cause a depth contour to "skip" the "focus area". Too small a value will incur excessive computation time.

By adjusting IPSTEP, PSCOP1, PSCOP2, and PSCOPE, restriction (e) may be eased.

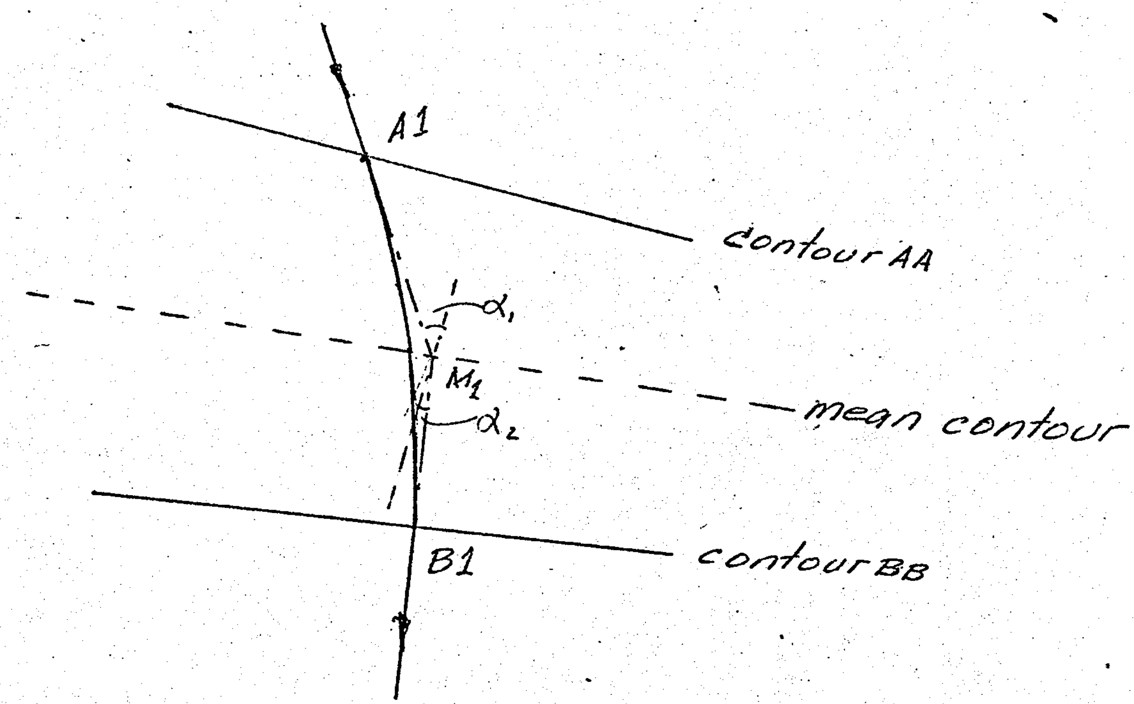
Finally, depending on the computer storage available one may wish to store in more depth contour information. This can be conveniently achieved by respecifying the dimensions in COMMON/AREA 2. For instance, in X(l,j) defining the abscissa of each point, "l" is the dimension for the number of contour lines allowed, and j, the number of points per contour.

D.7 CLOSING REMARKS

Due to the limitation of time available for this project, the computer program despite its size, is by no means a model of computer sophistication and efficiency. It should be recalled that the primary objective is to construct a wave refraction program.



(a) SNELL'S LAW OF REFRACTION



(b) WAVE REFRACTION PROCESS

FIGURE D1

APPENDIX E

MOVEMENT OF BEACH MATERIAL

E.1 INTRODUCTION

It is a commonly known fact that beach materials are moved along the shoreline by currents and waves coming from a certain predominant direction. Nevertheless, the simulation of such sand movement by a simple mathematical model is still awaited. It was not until recently that results, accumulated from a large number of research studies, have led to better understandings of the dynamics involved. In this section a general discussion on the topic is presented; details can be found in the excellent texts by Ingle (Ref. 54, 1966) and Zenkovitch (Ref. 141, 1967). The terminology adopted in this appendix as well as in other parts of this thesis is according to Ingle (1966, pp. 12, 13), whose definitive diagrams are reproduced in Fig. E-2. In addition, the term "downdrift" implies in the direction of current (longshore) flow; "updrift", in the opposite direction.

E.2 THE HYDRODYNAMICS OF WAVES IN SHALLOW WATER

It was mentioned in an earlier appendix that in deep water, water particles in wave motion travel in circular locii. As the wave train approaches a shore, in water of depth less than half of the wave length, these orbits become "squashed" into elliptical forms, and the smaller the depth the less circular these locii. Under such conditions, the wave is said to have "felt the bottom", and the water surface deformation starts to deviate from that of a perfect sine wave. The bottom also interferes with the wave motion through percolation and friction, (Bretschneider, 1955).

However, until the elliptic orbits of water particles are totally disrupted, the use of a sine function for an approximate description of the dynamics involved is still acceptable, and the effects of friction and percolation for the present consideration, at least before the waves arrive at the breaking zone, can be assumed to be negligible in this study.

Given a sea surface disturbed by a single wave train (such as "significant wave"), the location where this wave will break is governed by the laws of hydrodynamics, subject to boundary conditions of the physical characteristics of the coast. The details of such an analysis can be found in existing literature on breaking wave conditions and will not be described herein, but in general a wave breaks at a depth approximately 1.3 times the wave height, which on most beaches (on Lake Winnipeg) corresponds to the order of hundreds of feet from the water line. Quite often, but not as a rule, the wave would break over the crest of an offshore bar.

When the wave breaks, the elliptic orbits of water particles are disrupted, and turbulence occurs, making this breaker zone a region of remarkably high energy dissipation compared to other regions of the beach profile. After breaking, if the inshore is deep enough and long enough, the disturbance at the breaker zone would be transmitted forward as reformed waves and a secondary breaker zone will occur further inshore. If, however, the depth inshore of the breaker line is sufficiently shallow, a bore would be formed and would travel up the beach profile, with some energy being dissipated in the process. The second zone of high energy dissipation, however, is the swash zone where the final thrust of the bore is broken and dissipated due to bed friction and turbulent viscous dissipation.

An illustrative sketch of the distribution of wave energy dissipation is therefore as shown in Fig. E-3. (Combined from Ingle, 1966, pp. 63, 181 and Zenkovitch, 1967, pp.327). It can be seen that the two high energy zones are located at the break-line and the transition zone.

A wave arriving at the breaker line at an angle can be considered to contribute two (vector) energy components to the coast: perpendicular or normal and parallel to the shoreline. The first component, the "on-shore" component, is responsible for creating much turbulence and energy dissipation in the proximity of the breaker zone and surf zone. The effect is, of course, churning up of the bed materials, and perhaps holding them in suspension, thus allowing them to be more readily removed once the longshore energy is present. The second energy component, more conveniently referred to as the "longshore" component, generally undergoes less energy dissipation as compared to that of the "on-shore" component. It is responsible for the longshore transport of sediment. It tends to generate a form of current flow between the two high energy zones at the breaker line and the transition zone. (It is not clear how this current travels; it is suggested that it performs helical motion about its mean direction of travel (Zenkovitch, 1967); in any case, such detail is not of significant importance to this study.) Near the offshore boundary of this current, sand motion is that of a mixed turbulence; on the inshore side, water motion is more of a zig-zag onshore-offshore trace in accordance with the thrust and recession of the breaking bores amid much turbulence.

E.3 RETURN FLOW

As the waves move onshore from the offshore there is a mean onshore transport of water mass. This transport rate is very slow but it is important in its role on sediment movement. Because of the transfer of mass, there is a "pile-up" of water close to the shoreline, shoreward of the breaker zone. This sets up a hydraulic gradient, which in turn causes a seaward return flow which may assume either or both of the following modes of movement.

(i) Rip current - This travels as a relatively fast and narrow jet current which cuts across the foreshore and inshore, breaking through any traversing bars to return water to the deeper areas shoreward of the breaker line. The velocity attained by this current partly accounts for the fact that coarse particles are sometimes found in relatively deep water.

(ii) Seaward sheet flow - The water may travel seaward in a thin layer immediately adjacent to the bed irrespective of the motion of water immediately above it. (Ingle, 1966). Such motion may account for the existence of ripples on the bed and a portion of the sediment in transport. In this manner the sheet of seaward underflow is actually moving in a direction opposite to that of the bores above it.

Besides these two principal mechanisms the water may percolate into the beach floor and return via diffusion through soil grains. Or if structures exist at the shoreline such that wave reflection becomes important, return flow may even take the form of reflected wave mass transport. In any case, items (i) and (ii) generally play the major role of return flow on natural beaches.

E.4 MOVEMENT OF BEACH MATERIALS

Movement of beach sediments is by no means easily described. The following is a list of important influencing parameters:

(1) Wave conditions, (magnitude et cetera)
(2) Angle of wave attack
(3) Existing beach slope
(4) Existing beach materials (commonly designated by mean grain diameter)
(5) Wind direction
(6) Water level
(7) Density of beach material
(8) Gravity

There has been considerable time and effort spent in the search for a general model for coastal sediment transport. Nevertheless, before discussing the findings obtained from experimentations and measurements made in field, it is in order to describe the modes of sediment transport of a coastline. These can generally be distinguished primarily by two associated directions: onshore-offshore and alongshore (parallel to shoreline). It is often convenient to associate the perpendicular wave energy component with the first (onshore-offshore, or transverse) mode of sand movement and the parallel wave energy component with the second (alongshore) mode.

E.5 THE EQUILIBRIUM BEACH PROFILE

One concept which is very useful in visualizing the causes and effects of beach erosion or accretion is that of the adjustment of a beach profile to effect an energy distribution pattern which is characteristic of the energy dissipated per unit area of beach floor, the beach material on the beach and the slope of the beach. The beach profile that finally reaches equilibrium is theoretically concave for gravel beaches, being more concave for shingle and less concave for sand, if all grains on the beach are of uniform size. (Zenkovitch, 1967, pp. 103, ff.). The equilibrium profile therefore is a function of the rate of energy dissipation of the waves (bores) and the "resistivity to move", or, the threshold velocity, of the sediment particles. Little is known of an exact formulation of the energy distribution on a beach, but provided in a prolonged period of time the beach material and the wave condition remains the same, and that the beach material supply is constant, the beach will eventually adjust to its equilibrium profile. An application of this concept is given in Chapter VIII regarding water level effects. In the sections to follow the following characteristics of a theoretically equilibrium beach profile should be noted, all others being equal.

(i) The more wave energy arrives at a beach, the more energy dissipation is required so that all the energy is totally exhausted by the time the wave reaches the waterline. Nature can accomplish this by the two subsequent adjustments:-

(ii) By having a more flat foreshore and inshore, thus decreasing energy to be dissipated per unit area; and/or

(iii) By removing the finer sediments and retaining the coarser beach materials which tend to dissipate more energy per a shorter transverse distance of the beach profile.

(iv) For a constant rate of energy supplied to the shore, (ii) and (iii) will interplay such that the energy can be adequately dissipated by the beach material per unit area.

E.6 TRANSVERSE MOVEMENT OF BEACH MATERIAL

(A) The modes of onshore and offshore transports of beach material shoreward of breaker line have been briefly mentioned in Section E.3 on seaward return flows. Offshore of the breaker line, the interacting dynamics of water and sediment movements may not be as complicated as that shoreward of it; notwithstanding this fact, there is even less known about sediment movement in these deeper waters. Ingle (1966) conducted tracer sand experiments on the coast of California and found that sand seldom moves offshore beyond the breaker line. Although there may be a high concentration of churned-up sand at the breaker zone, and that suspended materials tend to "diffuse" into the offshore zone due to the unbalanced concentrations, the net motion, because of the more intense sand movement in the lower depths, is usually onshore. This observation is consistent with those of other researchers (Zenkovitch, 1967). Therefore, for the case of Lake Winnipeg, where recently beach sands are suspected to have been lost from the beaches to deeper water, it is likely that the sands are lost to the deeper offshore parts of the lake. Rather, they are likely lying somewhere between the shoreline and the breaker line close to, but probably downdrift of the locations from where they are lost.

(B) Shoreward of the breaker line, the transverse movement of beach materials results in erosion and accretion of the beaches, causing much annoyance to the beach users and landowners at times, depending on the situation. Erosion is usually a very fast process, in the order of feet per hour during a storm. Accretion, on the other hand, is much slower, in terms of days, weeks or even months. (Thompson and Harlett, 1968, Ref. No. 145).

The transverse movement of sediment is usually correlated with the steepness (H_o/L_o ratio) of the waves prior to breaking. It has been suggested the steeper waves tend to move beach material offshore, whereas the more gentle waves will restore the sand to the beach, the critical condition being $H_o/L_o = 0.025$. (Thompson and Harlett, 1968; Johnson, 1952). The wave power is also commonly used as a parameter, but then for the same beach, steeper waves generally are associated with higher wave power; it is therefore not surprising that severe erosion is also associated with higher wave power. That the waves with more power also set up more turbulence, thus lowering the threshold velocity for beach sand must not be overlooked. Moreover, the offshore transport of beach material also appears to be a function of mass transport of water due to wave motion.

E.7 LONGSHORE TRANSPORT (LITTORAL DRIFT)

The longshore transport of beach material has since times of old interested many coastal residents and coastal engineers, and has always been suggested as the culprit of land loss. This mode of sand movement, as mentioned earlier, is associated with the longshore component of wave energy. It appears that the greater the longshore energy, the more transport results.

Littoral drift is suspected to take on several different modes of transportation. The mechanism of transport can be suspension or saltation. The following lists a number of the modes of movement:

(i) Ingle (1966) found that sand grains of a certain size will occupy a certain belt of the surf zone. It constantly moves obliquely onshore and offshore within this belt but the principal movement is in the direction of the littoral drift.

(ii) It is found that most longshore sediment transport takes place slightly shoreward of the breaker line. Ingle also found that over 50 percent of sediment transport occurs within 35-40 percent of this region of the inshore-foreshore. The sand may be greatly disturbed by the existing turbulence in the region thus becoming more ready to move alongshore when the appropriate longshore energy is available.

(iii) In the swash zone where turbulence is again predominant, longshore transport of material is also important. Here, the particle follows zig-zag paths.

(iv) Between the two higher energy dissipation zones, the water flows in a manner which as suggested by some investigators, is probably not dissimilar to that of open channel flow. Hence it is not surprising at all to find some features of river-bed regimes, such as transverse dunes, in the surf zone, as revealed by Russian studies. (Zenkovitch, 1967).

(v) Over bars and ridges, the water motion may tend to be more turbulent, resulting in higher rates of transport. In the runnels and deeper ruts, the transport rate decreases. (Ingle, 1966; Zenkovitch, 1967).

(vi) Per Bruun found that beach material also travels in large mass (Zenkovitch, 1967).

Thus, sand may accumulate in the form of a small bar which slowly migrates downdrift. Relationships governing the movement is given by Per Bruun (1968).

The discussions listed above are relevant to the conditions in Lake Winnipeg, where beach materials are generally of sand size gradation and the widths of the surf zones are appreciable. Extremely exceptional cases are rare.

To analytically quantify the sediment transport as described by (i) to (vi) above, individually, and then to sum them up is an impossible task to date, although approximations to (iv) (Bijker, 1968, Ref. No. 146) and (vi) are now possible. A more convenient and simplified solution is to relate the total longshore mass transport to the total longshore energy available. Although it is not known how this energy is partitioned between the different modes of transport or what is the variability of the proportions, the correlations obtained from field and model measurements show that longshore sediment transport is directly proportional to longshore wave energy.

The first such relationship was given by Munch-Peterson as early as 1936 (in the form of $M = KS^2F\sqrt{D}\cos\alpha$; for details, see Zenkovitch, 1967). Since then, considerable field and model studies have been carried out. One of the more rational formulations is as given by Saville (1962):

$$E_T = E \cos\alpha \sin\alpha \qquad \text{(E-1)}$$

where E is the total energy of the wave; E_T is the longshore energy component; α is the angle of wave approach. And Savage (1962), combining the observations of many researchers, (including Sauvage, Johnson, Krumbein and others) suggested that

$$Q = c_1 E_T^{c_2} \tag{E-2}$$

where Q is the rate of longshore transport; c_1 and c_2 are constants.

Most researchers, however, have committed themselves to the use of the significant wave: i.e., representing the condition of a disturbed sea by a selected simple wave train. Such an approach fails to relate the true nature of wave action. In the light of spectral analysis (See Appendix C), it is felt that all coastal phenomena can now be more accurately represented. Of the recent investigators, Thornton (1968), Inman and Komar (1968), adopt the spectral approach. Their measurements show that

$$I_1 = KP_1$$

where I_1 is the transport rate of the immersed weight of sand per unit time and P_1 is the longshore power per unit width. K is an empirical coefficient and is found to be 0.7, twice the value found for the significant wave analysis. The resultant relationship of Inman and Komar is reproduced in Fig. E-4. These researchers also suggest that the relationship is valid regardless of sand size, but this view appears to be questionable.

In the applications of the present thesis, Inman and Komar's formulation is followed while spectral analysis is used to obtain wave power. It is not suitable to use the classical significant wave approach, especially in the present case of the shallow Lake Winnipeg. (Inman and Komar, 1968). (See Appendix C for the comparisons between these two approaches.)

In using Equation (E-3), however, one must be aware of the fact that it is governed by the following assumptions:

(i) Beach material is generally of sand-size gradation, or at most gravel. Its application to clay or large shingle beaches is doubtful.

(ii) Locations where Equation (E-3) has been tested are generally long straight beaches with moderately wide surf zones.

(iii) Equation (E-3) is applicable to a beach reach where the quantity of sand transported by the incoming longshore current at the updrift end and the out-going current at the downdrift end have both reached the capacity of the currents to transport sand (a condition which will hereafter be referred to as "saturation"). In other words, if the littoral current is "saturated" at both the updrift and downdrift ends of the beach reach, erosion will only occur if the parallel energy component at the downdrift end is greater than that at the updrift end.

For the case of Lake Winnipeg, conditions (i) and (ii) are occasionally satisfied; whereas condition (iii) is likely not satisfied for many locations. Even so, or even if none of the three conditions are satisfied for any beach, the amount of wave power (alongshore) received, is an indication of the intensity of the "capacity to erode".

A comment regarding (i) above is that sand is the easiest to move. This conclusion follows from Ingle's measurements, (1966).

Besides wave power, one of the other major parameters in littoral drift is known to be wave steepness. This fact has been proven in model tests, in particular, by Johnson (1952). Field measurements, on the other hand, indicate such correlation is not of major importance. It is to be admitted, however, that wave power or energy, perpendicular to the shore may serve as a secondary indicator of some sort, since the more intense the perpendicular wave component, the more turbulence there is and the more the sediment is likely to be moved. In the present study, wave steepness is neglected, but the perpendicular wave component is plotted in the final results. (See Plates III(d) and V(d)).

APPENDIX F

LISTING OF COMPUTER PROGRAMS

Computer Programs (A) and (B) described in Chapters IV and VI and Appendix D are listed in this appendix.

COMPUTER PROGRAM (A): WAVE REFRACTION

```
C MAIN-LINE
      COMMON /AREA1/LD,JD
      COMMON /AREA3/VLUSIN,VLUCOS,VLUTAN
      COMMON /AREA2/X(12,140),Y(12,140),DEEP(12),T
      COMMON /AREA4/THETA,DELTA,SLOP2
      COMMON /AREA5/XO,YO,XOP,YOP
      COMMON /AREA6/ILIMT(25),LNUM,L,LO
      COMMON /AREA7/JFIRST(25),JNUM(25),KINDEX
      COMMON /AREA8/XI(25),YI(25),JCOUNT(25),XDIF,JCNTO,KLO,JCNTO2,LO2,
     XKSCOP
      COMMON /AREA9/YDUM1,YDUM2
      COMMON /AREA10/C1,C2,CO,WLO,WLTRY,DEPTH,K,WLC,WKC,BRETHO,BRETH1,
     XBRETH2,CG2,SPEKSQ,DDEL1,DDEL2,ATNH
      COMMON /AREA15/IFETH,FETHS(50),DEPS(50),WKS(50),IFETCH,REFUNC(50)
     XWIDTH(50)
      COMMON /AREA16/PI,SCOPE1,SCOPE2,GEE,CONVER,SCOPE
      CONVER=4500.
      GEE=32.2
      SCOPE1=5.*CONVER
      SCOPE2=5.*CONVER
      PI=3.1416
      T=14.
      DO 7 J=1,20
      L=J
      READ (5,10) DEEP(L)
      IF (DEEP(L).LT.(-1.)) GO TO 8
      I1=1
    1 I2=I1+4
      READ (5,10) (X(L,I),Y(L,I),I=I1,I2)
      IF (X(L,I2).GT.1000.) GO TO 2
      I1=I2+1
      GO TO 1
    2 CONTINUE
      DO 3 I=1,5
      I2=I2-1
      IF (X(L,I2).LT.1000.) GO TO 4
    3 CONTINUE
    4 CONTINUE
      IPI3I=25
      I3I=IPI3I
      IPI2=19
      IF (I2.LE.IPI2) I3I=1
      IF (I2.LE.IPI3I.AND.I2.GT.IPI2) I3I=IPI3I/2
      DO 5 I=1,I3I
      I3=I2+I
      X(L,I3)=X(L,I)
      Y(L,I3)=Y(L,I)
    5 CONTINUE
      ILIMT(L)=I3
      WRITE (6,11) L,DEEP(L)
      WRITE (6,12) (X(L,MM),Y(L,MM),MM=1,I3)
      DO 6 M=1,I3
      X(L,M)=X(L,M)*CONVER
      Y(L,M)=Y(L,M)*CONVER
    6 CONTINUE
    7 CONTINUE
    8 LNUM=L-1
      THETA=-(PI/2.)
      READ (5,13) THETA
      READ (5,13) AXO,AYO
```

```
      XO=AXO*CCNVER
      YO=AYO*CCNVER
      WRITE (6,14)
      WRITE (6,15) AXO,AYO,THETA,T
      CALL SHALOW
    9 CONTINUE
      CALL EXIT
   10 FORMAT (20F8.1)
   11 FORMAT ('0','LINE NO. ',I2,'          DEPTH, IN FEET = ',F8.2)
   12 FORMAT (' ',5(2F8.2,4X))
   13 FORMAT (2F10.3)
   14 FORMAT ('0',T4,'AXO',T14,'AYO',T24,'THETA',T34,'T')
   15 FORMAT (' ',10F10.4)
      END
```

```
C
      SUBROUTINE SHALOW
      COMMON /AREA1/LD,JD
      COMMON /AREA2/X(12,140),Y(12,140),DEEP(12),T
      COMMON /AREA3/VLUSIN,VLUCOS,VLUTAN
      COMMON /AREA4/THETA,DELTA,SLOP2
      COMMON /AREA5/XO,YO,XOP,YOP
      COMMON /AREA6/ILIMT(25),LNUM,L,LO
      COMMON /AREA7/JFIRST(25),JNUM(25),KINDEX
      COMMON /AREA8/XI(25),YI(25),JCOUNT(25),XDIF,JCNTO,KLO,JCNTO2,LO2,
     XKSCOP
      COMMON /AREA9/YDUM1,YDUM2
      COMMON /AREA10/C1,C2,CO,WLO,WLTRY,DEPTH,K,WLC,WKC,BRETHO,BRETH1,
     XBRETH2,CG2,SPEKSQ,DDEL1,DDEL2,ATNH
      COMMON /AREA15/IFETH,FETHS(50),DEPS(50),WKS(50),IFETCH,REFUNC(50),
     XWIDTH(50)
      COMMON /AREA16/PI,SCOPE1,SCOPE2,GEE,CONVER,SCOPE
      CALL WAVPAY
      IF ((ABS(ABS(DDEL2)-PI/2.)).LT..00001) GO TO 5
      FETCH=0.0
      WIDGEN=0.0
      DEPGEN=0.0
      WKGEN=0.0
      DO 1 IGEN=1,IFETCH
      FETCH=FETHS(IGEN)+FETCH
      WKGEN=WKGEN+WKS(IGEN)*FETHS(IGEN)
      WIDGEN=WIDGEN+WIDTH(IGEN)*FETHS(IGEN)
    1 CONTINUE
      DESALO=DEPGEN/FETCH
      WKSALO=WKGEN/FETCH
      WDSALO=WIDGEN/FETCH
      IWIDTH=1
      DO 3 IGEN=1,IFETCH
      IF (WIDTH(IGEN).GE.WDSALO) GO TO 2
      GO TO 3
    2 IWIDTH=IGEN
      GO TO 4
    3 CONTINUE
    4 FNSALO=REFUNC(IWIDTH)/REFUNC(IFETH)
      PRLENF=FNSALO*SIN(DDEL2)*CG2
      PNDENF=FNSALO*COS(DDEL2)*CG2
    5 CONTINUE
      XOF=XO/CONVER
      YOF=YO/CONVER
      WRITE (6,6)
      WRITE (6,7) XOF,YOF,FETCH,WIDTH(IFETCH),DESALO,WKSALO,WDSALO,T
      RETURN
    6 FORMAT ('0',T5,'XOF',T16,'YOF',T27,'FETCH',T38,'WIDFIN',T49,'DESAL
     XO',T60,'WKSALO',T71,'WDSALO',T82,'T')
    7 FORMAT (' ',11E11.4)
      END
```

```
C
C
      SUBROUTINE WAVRAY
C MUST KNOW XO,YO,T,THETA,X(-,-),Y(-,-),DEEP(-)
      COMMON /AREA1/LD,JD
      COMMON /AREA2/X(12,140),Y(12,140),DEEP(12),T
      COMMON /AREA3/VLUSIN,VLUCOS,VLUTAN
      COMMON /AREA4/THETA,DELTA,SLOP2
      COMMON /AREA5/XO,YO,XOP,YOP
      COMMON /AREA6/ILIMT(25),LNUM,L,LO
      COMMON /AREA7/JFIRST(25),JNUM(25),KINDEX
      COMMON /AREA8/XI(25),YI(25),JCOUNT(25),XDIF,JCNTO,KLO,JCNTO2,LO2,
     XKSCOP
      COMMON /AREA9/YDUM1,YDUM2
      COMMON /AREA1C/C1,C2,CO,WLO,WLTRY,DEPTH,K,WLC,WKC,BRETHO,BRETH1,
     XBRETH2,CG2,SPEKSQ,DDEL1,DDEL2,ATNH
      COMMON /AREA15/IFETH,FETHS(50),DEPS(50),WKS(50),IFETCH,REFUNC(50),
     XWIDTH(50)
      COMMON /AREA16/PI,SCOPE1,SCOPE2,GEE,CONVER,SCOPE
      LD=25
      JD=50
      THETAO=THETA
      KLAN=-4
      LO2=2
      JCNTO2=1
      JCNTO2=0
      JCNTO3=0
      JCNTO4=0
      JCNTO5=0
      LO3=0
      LO4=0
      LO5=0
      JCNTO=1
      LL2=300
      JCNT2=300
      LOOP=0
      IFETCH=0
      IFETH=0
      BRETHO=1.
      BRETH1=BRETHO
      PI=3.1416
      CO=(32.2*T)/(2.*PI)
      WLO=CO*T
      C1=CO
      WLTRY=WLO
      KTHET=0
      THETLO=THETAO
    1 THETA=THETLO
      LO=1
      SCOPE=SCOPE1
      DELTA=THETA
      CALL TRIG2(DELTA)
      XOP=XXROT(XO,YO)
      YOP=YYROT(XO,YO)
      L=1
      PARA=900.
      XDIF1=PARA*CONVER
      KIND=0
      KLO=0
    2 CALL SEARCH
```

```
      IF (KINDEX.EQ.1) GO TO 3
      XDIF2=XDIF
      IF (XDIF1.LT.XDIF2) GO TO 3
      XDIF1=XDIF2
      KIND=KIND+1
      LOTRY=L
    3 CONTINUE
      IF (L.GE.LNUM) GO TO 4
      L=L+1
      GO TO 2
    4 CONTINUE
      IF (KIND.EQ.0) GO TO 5
      GO TO 7
    5 THETLO=THETLO+PI/2.-.0001
      KTHET=KTHET+1
      IF (KTHET.GT.4) GO TO 6
      GO TO 1
    6 WRITE (6,49)
      GO TO 47
    7 LO=LOTRY
      JCNTO=JCOUNT(LO)
      DEPTH=DEEP(LO)
      DELTA=PI/2.
      K=5
      CALL REFRAT
      C1=C2
      THETA=THETAO
      DELTA=THETA
      CALL TRIG2(DELTA)
      XOP=XXROT(XO,YO)
      YOP=YYROT(XO,YO)
      JCNT=0
      LCNT=0
      SCOPE=SCOPE2
      WRITE (6,50)
    8 L1=LO-2
      L5=LO+2
      KSCOP=0
      JCNAN3=-3
      JCNAN2=-2
      JCNAN1=-1
      LOOP=LOOP+1
      IF (L1.LT.1) L1=1
      IF (L5.GT.LNUM) L5=LNUM
      PXMMIN=1000.
      XMMIN=PXMMIN*CONVER
      KIND=0
      L=L1
      KL1=1
      KL=0
    9 CONTINUE
      IF (KL1.EQ.2) L=L1
      IF (KL1.EQ.2) KL1=3
      IF (KL1.EQ.1) L=1
      IF (KL1.EQ.1) KL1=2
      IF (KL.EQ.1) GO TO 12
      IF ((LO3.EQ.LO).AND.(JCNTO3.EQ.JCNTO)) GO TO 10
      IF ((LO4.EQ.LO).AND.(JCNTO4.EQ.JCNTO)) GO TO 11
      IF ((LO5.EQ.LO).AND.(JCNTO5.EQ.JCNTO)) GO TO 47
      GO TO 12
```

```
   10 CONTINUE
      L=LO-2
      IF (LO4.GT.LO3) KL=1
      IF (LO4.LT.LO3) L=LO+2
      GO TO 12
   11 CONTINUE
      L=LO-3
      IF (LO5.GT.LO4) KL=1
      IF (LO5.LT.LO4) L=LO+3
      GO TO 12
   12 CONTINUE
      IF (L.GT.LNUM) GO TO 47
      IF (L.LT.1) GO TO 47
      KLO=1
      CALL SEARCH
      IF (KINDEX.EQ.1) GO TO 13
      GO TO 15
   13 IF ((KIND.EQ.C).AND.L.GE.L5) GO TO 14
      L=L+1
      IF (KL.EQ.1) L=L-2
      IF (L.GT.L5) GO TO 20
      GO TO 9
   14 WRITE (6,51)
      GO TO 47
   15 KIND=KIND+1
      JCNANT=JCOUNT(L)
      IF ((KLAN.EQ.L).AND.(IABS(JCNANT-JCNAN3).LT.3)) GO TO 17
      IF ((XDIF.GE.0.0).AND.(XDIF.LE.XMMIN)) GO TO 16
      PRINT ,'W 8','     L=',L,'     XDIF=',XDIF,'     XMMIN=',XMMIN
      IF ((LO2.EQ.0).AND.(JCNTO2.EQ.0).AND.(L.EQ.L5)) GO TO 18
      GO TO 17
   16 IF (L.EQ.LO.AND.XDIF.LT.(0.01*CONVER)) GO TO 17
      LCNT=L
      JCNT=JCOUNT(L)
      XMMIN=ABS(XDIF)
      PRINT ,'W10','     L=',L,'     XDIF=',XDIF,'     XMMIN=',XMMIN
   17 IF (JCNT.NE.0.OR.LCNT.NE.0) GO TO 19
   18 CONTINUE
      LCNT=LO
      JCNT=JCNTO
      ATHETA=THETA
      GO TO 45
   19 IF (L.GE.L5) GO TO 20
      L=L+1
      IF (KL.EQ.1) L=L-2
      GO TO 9
   20 IACNT=0
      SLOP1=ASLOP(LO,JCNTO)
      THETA1=THETA
      XOPO1=XOP
      YOPO1=YOP
   21 SLOP2=ASLOP(LCNT,JCNT)
      SLODEL=THETA1
      CALL ANGLE2(SLODEL,KSLOPE)
   22 CONTINUE
      STOTAL=SLOP1+SLOP2
      IF (ABS(STOTAL).LT.ABS(SLOP1-SLOP2)) GO TO 23
      GO TO 28
   23 CONTINUE
      IF ((SLOP2.LT.0.0).AND.(KSLOPE.EQ.2.OR.KSLOPE.EQ.4).AND.(SLODEL
```

```
     X.LE.ABS(SLOP2))) GO TO 25
      IF ((SLOP2.LT.0.0).AND.(KSLOPE.EQ.1.OR.KSLOPE.EQ.3).AND.(SLODEL
     X.LE.ABS(SLOP1))) GO TO 25
   24 CONTINUE
      IF ((SLOP2.GE.0.0).AND.(KSLOPE.EQ.1.OR.KSLOPE.EQ.3).AND.(SLODEL
     X.LE.ABS(SLOP2))) GO TO 25
      IF ((SLOP2.GE.0.0).AND.(KSLOPE.EQ.2.OR.KSLOPE.EQ.4).AND.(SLODEL
     X.LE.ABS(SLOP1))) GO TO 25
      GO TO 28
   25 CONTINUE
      IF (STOTAL) 26,27,27
   26 SLOPE=PI/2.+STOTAL/2.
      GO TO 29
   27 SLOPE=-PI/2.+STOTAL/2.
      GO TO 29
   28 SLOPE=STOTAL/2.
   29 CONTINUE
      DEPTH=DEEP(LCNT)
      THETA=THETA1
      CALL ANGLE1(THETA,SLOPE,DELTA,K)
      DELTAR=DELTA
      KR=K
      SIN1=VLUSIN
      COS1=VLUCOS
      THETA3=2.*PI-THETA1
      CALL TRIG2(THETA3)
      SIN3=VLUSIN
      COS3=VLUCOS
      DELTA=DELTAR
      K=KR
      FTHET1=THETAO-THETA
      CALL REFRAT
      IF ((ABS(ABS(DDEL2)-PI/2.)).LT..00001) GO TO 30
      GO TO 31
   30 XO=10000.*CONVER
      YO=10000.*CONVER
      BRETH2=1.
      IFETCH=1
      WIDTH(1)=1.
      GO TO 44
   31 CONTINUE
      FTHET2=THETAO-THETA
      IF (ABS(THETAO-THETA).GE..5) IFETCH=IFETH
      THETA2=THETA
      CALL TRIG2(THETA2)
      SIN2=VLUSIN
      COS2=VLUCOS
      YDUM1=YROT(LCNT,JCNT)
      YDUM2=YROT(LCNT,JCNT+1)
      XOP=XOPO1
      YOP=YOPO1
      XOPO=XOP
      XOPO1=XOPO
      YOPO=YOP
      XDEL1=ABS(XOPO-XOP)
   32 CONTINUE
      XOP=XXROT(XO,YO)
      YOP=YYROT(XO,YO)
      XTRY=XTRI(LCNT,JCNT)
      XDEL2=XTRY-XOP
```

```
      XDEL=XDEL1-XDEL2
      IF (ABS(XDEL).LT.(.001*CCNVER)) GO TO 33
      XDEL1=XDEL1-XDEL/2.
      XOPO=XOPO1+XDEL1
      VLUSIN=SIN3
      VLUCOS=COS3
      XO=XXROT(XOPO,YOPO)
      YO=YYROT(XOPO,YOPO)
      VLUSIN=SIN2
      VLUCOS=COS2
      GO TO 32
   33 CONTINUE
      PRINT ,'W132','     XDEL=',XDEL
      IF (YOP.GE.YDUM1) GO TO 34
      IF (YOP.GE.YDUM2) GO TO 43
      GO TO 36
   34 IF (YOP.GE.YDUM2) GO TO 35
      GO TO 43
   35 IF (YDUM1.GE.YDUM2) GO TO 37
      GO TO 38
   36 IF (YDUM1.GE.YDUM2) GO TO 38
   37 JCNT=JCNT-1
      GO TO 39
   38 JCNT=JCNT+1
   39 CONTINUE
      IF (IACNT.GE.4) GO TO 42
      PRINT ,'LCNT=',LCNT,'   JCNT=',JCNT,'   IACNT=',IACNT,'   XO=',XO,'
     XYO=',YO,'   THETA=',THETA
      JCNAN3=JCNAN2
      JCNAN2=JCNAN1
      JCNAN1=JCNT
      IF (JCNAN3.EQ.JCNAN1) GO TO 40
      GO TO 41
   40 KLAN=LCNT
      LOOP=0
      GO TO 8
   41 CONTINUE
      IACNT=IACNT+1
      GO TO 21
   42 WRITE (6,52)
      GO TO 47
   43 CONTINUE
      IFETH=IFETH+1
      DEPS(IFETH)=DEPTH
      CCOS1=COS(FTHET1)
      CCOS2=COS(FTHET2)
      FETHS(IFETH)=XDEL1*(CCOS1+CCOS2)
      WKS(IFETH)=WKC
      WKC=(2.*PI)/WLC
      FACWKC=2.*WKC*DEPTH
      SSINH=SINH(FACWKC)
      BRETH2=(COS(DDEL2)/COS(DDEL1))*BRETH1
      CG2C2=.5*(1.+FACWKC/SSINH)
      SPEKSQ=(BRETHO/BRETH2)/((SQRT(ATNH))*CG2C2)
      CG2=CG2C2*C2
      BRETH1=BRETH2
      WIDTH(IFETH)=BRETH2
      REFUNC(IFETH)=SPEKSQ
      XTR=XTRI(LCNT,JCNT)
      PRINT ,'W16','     LCNT=',LCNT,'     JCNT=',JCNT
```

```
      YOP=YYROT(XC,YO)
      ATHETA=THETA
      THETA=2.*PI-ATHETA
      DELTA=THETA
      CALL TRIG2(DELTA)
      XO=XXROT(XTR,YOP)
      YO=YYROT(XTR,YOP)
      THETA=ATHETA
   44 CONTINUE
      XO1=XO
      YO1=YO
      XO=XO/CONVER
      YO=YO/CONVER
      WRITE (6,53) XO,YO,WLC,WKC,BRETH2,CG2,C2,SPEKSQ
      XO=XO1
      YO=YO1
      IF ((ABS(ABS(DDEL2)-PI/2.)).LT..00001) GO TO 48
      IF (LCNT.GT.1.AND.LCNT.LE.LNUM) GO TO 45
      GO TO 48
   45 C1=C2
      IF ((LCNT.EQ.LO).AND.((ABS(X(LO,JCNT)-X(LO,JCNTO))).LT.(.0001*
     XCONVER)).AND.((ABS(Y(LO,JCNT)-Y(LO,JCNTO))).LT.(.0001*CONVER)))
     XJCNT=JCNTO
      IF (KSCOP.EQ.1) GO TO 46
      LO5=LO4
      JCNTO5=JCNTO4
      LO4=LO3
      JCNTO4=JCNTO3
      LO3=LO2
      JCNTO3=JCNTO2
      IF (LOOP.EQ.1) LO=0
      IF (LOOP.EQ.1) JCNTO=0
      LO2=LO
      JCNTO2=JCNTO
   46 CONTINUE
      LO=LCNT
      JCNTO=JCNT
      IF ((JCNT2.EQ.JCNT).AND.(LL2.EQ.LCNT).AND.(JCNTO2.NE.JCNT)) JCNT=
     XJCNTO2
      JCNT2=JCNT
      LL2=LCNT
      PRINT ,'W17','    LO=',LO,'    JCNTO=',JCNTO,'    LO2=',LO2,'    JCNTO
     X2=',JCNTO2
      CALL TRIG2(ATHETA)
      GO TO 8
   47 CONTINUE
   48 WRITE (6,54)
      IF (IFETCH.EQ.0) IFETCH=IFETH
      RETURN
   49 FORMAT (1H0,'CANNOT LOCATE LO')
   50 FORMAT (' ',T5,'XO',T16,'YO',T27,'WLC',T38,'WKC',T49,'BRETH2',T60,
     X'CG2',T71,'C2',T82,'SPEKSQ')
   51 FORMAT (1H0,'5 IS NOT ENOUGH FOR SEARCH')
   52 FORMAT (1H0,'IACNT GE 4')
   53 FORMAT (' ',2F11.3,10E11.4)
   54 FORMAT (1H  ,'SEARCH IS COMPLETE.')
      END
```

```
C
C
      SUBROUTINE SEARCH
      COMMON /AREA1/LD,JD
      COMMON /AREA2/X(12,140),Y(12,140),DEEP(12),T
      COMMON /AREA3/VLUSIN,VLUCOS,VLUTAN
      COMMON /AREA4/THETA,DELTA,SLOP2
      COMMON /AREA5/XO,YO,XOP,YOP
      COMMON /AREA6/ILIMT(25),LNUM,L,LO
      COMMON /AREA7/JFIRST(25),JNUM(25),KINDEX
      COMMON /AREA8/XI(25),YI(25),JCOUNT(25),XDIF,JCNTO,KLO,JCNTO2,LO2,
     XKSCOP
      COMMON /AREA9/YDUM1,YDUM2
      COMMON /AREA16/PI,SCOPE1,SCOPE2,GEE,CONVER,SCOPE
C VALVE OF L MUST BE KNOWN
      CALL LOCATE
      IF (KINDEX.EQ.1) GO TO 15
      KIS=0
      PXDIF=1000.
      XDIF=PXDIF*CONVER
      KOUNT=0
      XOP=XXROT(XO,YO)
      YOP=YYROT(XO,YO)
      JEAN=JFIRST(L)
      YDUM1=YROT(L,JEAN)
      JINDEX=JNUM(L)-1
    1 CONTINUE
      DO 13 J=1,JINDEX
      JTR=JFIRST(L)+J
      JCNT=JTR-1
      YDUM2=YROT(L,JTR)
      IF (YOP-YDUM1) 3,2,2
    2 CONTINUE
      IF (YOP-YDUM2) 4,4,12
    3 CONTINUE
      IF (YOP-YDUM2) 12,5,5
    4 KS=1
      GO TO 6
    5 KS=0
    6 KOUNT=KOUNT+1
      XTRY=XTRI(L,JCNT)
      XMIN=XTRY-XOP
    7 CONTINUE
      IF ((L.EQ.LO).AND.(KLO.EQ.0).AND.(ABS(XMIN).LT.ABS(XDIF))) GO TO 9
      IF ((L.EQ.LO).AND.(KLO.EQ.1).AND.(ABS(XMIN).LT.(0.01*CONVER)))
     XGO TO 9
      IF ((L.EQ.1).AND.(KLO.EQ.1).AND.(ABS(XMIN).LT.(0.01*CONVER)))
     XGO TO 8
      IF ((XMIN.GT.0.0).AND.(XMIN.LT.XDIF)) GO TO 10
      KOUNT=KOUNT-1
      GO TO 12
    8 XDIF=.0001*CONVER
      GO TO 11
    9 JCNTO=JCNT
      KOUNT=KOUNT-1
      GO TO 12
   10 XDIF=XMIN
   11 XI(L)=XTRY
      JCOUNT(L)=JCNT
      KIS=1
```

```
   12 YDUM1=YDUM2
   13 CONTINUE
      IF (KIS.EQ.1) GO TO 14
      PXI=5000.
      PYI=5000.
      XI(L)=PXI*CONVER
      YI(L)=PYI*CONVER
      PXDIF2=50000.
      XDIF=PXDIF2*CONVER
      JCOUNT(L)=500
   14 CONTINUE
      IF (KOUNT.GE.1) GO TO 16
   15 CONTINUE
      PXI2=10000.
      PYI2=10000.
      XI(L)=PXI2*CONVER
      YI(L)=PYI2*CONVER
      JCOUNT(L)=JFIRST(L)
      GO TO 17
   16 YI(L)=YOP
   17 CONTINUE
      PRINT ,'L=',L,'   JFIRST(L)=',JFIRST(L),'    JNUM(L)=',JNUM(L),'
     XXDIF=',XDIF,'    JCOUNT(L)=',JCOUNT(L)
      RETURN
      END
```

```
C
C
      SUBROUTINE LOCATE
      COMMON /AREA1/LD,JD
      COMMON /AREA2/X(12,140),Y(12,140),DEEP(12),T
      COMMON /AREA5/XO,YO,XOP,YOP
      COMMON /AREA6/ILIMT(25),LNUM,L,LO
      COMMON /AREA7/JFIRST(25),JNUM(25),KINDEX
      COMMON /AREA8/XI(25),YI(25),JCOUNT(25),XDIF,JCNTO,KLO,JCNTO2,LO2,
     XKSCOP
      COMMON /AREA16/PI,SCOPE1,SCOPE2,GEE,CONVER,SCOPE
      KINDEX=100
      IPSTP1=4
      IPSTP2=2
      IPSTP3=IPSTP1-IPSTP2
      I=1
      JFIRST(L)=2
      PSCOPT=40.
      SCOPET=SCOPE
      IF ((KLO.EQ.1).AND.(JCNTO2.EQ.JCNTO).AND.(LO2.EQ.LO)) GO TO 1
      GO TO 2
    1 SCOPET=PSCOPT*CONVER
      KSCOP=1
    2 KK=0
      KKK=0
      SCOPET=40.*CONVER
    3 K=0
    4 CONTINUE
      IPI2=19
      IF (ILIMT(L).LT.(IPI2+1)) GO TO 5
      GO TO 6
    5 K=1
      KKK=1
    6 CONTINUE
      IF (I.GT.ILIMT(L)) GO TO 7
      GO TO 13
    7 CONTINUE
      IF (KK.EQ.0) GO TO 8
      GO TO 10
    8 CONTINUE
      IF (K.EQ.1) GO TO 9
      GO TO 12
    9 JFIRST(L)=1000
      JNUM(L)=1
      KINDEX=1
      GO TO 30
   10 CONTINUE
      IF (K.EQ.1) GO TO 11
      GO TO 12
   11 JLAST=ILIMT(L)
      JNUM(L)=JLAST-JFIRST(L)+1
      WRITE (6,31) L
C THIS WRITE STATEMENT MAY NOT BE REQUIRED.
      KINDEX=0
      GO TO 30
   12 K=1
      I=I-IPSTP3
      GO TO 4
   13 DX=X(L,I)-XO
      DY=Y(L,I)-YO
```

```
      EXPX=ABS(DX)-SCOPET
      EXPY=ABS(DY)-SCOPET
      IF (KK.EQ.0) GO TO 14
      EXPX=-EXPX
      EXPY=-EXPY
      IF ((EXPX.LT.0.0).OR.(EXPY.LT.0.0)) GO TO 23
      GO TO 15
   14 CONTINUE
      IF ((EXPX.LT.0.0).AND.(EXPY.LT.0.0)) GO TO 23
      GO TO 15
   15 CONTINUE
      IF (KKK.EQ.1) GO TO 16
      GO TO 17
   16 JFIRST(L)=1
      JNUM(L)=ILIMT(L)
      KINDEX=0
      GO TO 30
   17 CONTINUE
      IF ((ILIMT(L).GT.50).AND.(JFIRST(L).EQ.1).AND.(SCOPET.GE.((PSCOPT-
     X1.)*CONVER))) GO TO 19
      IF ((ILIMT(L).GT.50).AND.(JFIRST(L).EQ.1)) GO TO 18
      GO TO 19
   18 XL1=XROT(L,1)
      IF ((XOP-XL1).GT.(SCOPE/2.)) GO TO 19
      I=ILIMT(L)/2
      JFIRST(L)=1000
      GO TO 2
   19 CONTINUE
      IF (K.EQ.0) GO TO 22
      GO TO 21
   20 CONTINUE
      IF (KK.EQ.0) GO TO 22
   21 I=I+IPSTP2
      GO TO 4
   22 I=I+IPSTP1
      GO TO 4
   23 CONTINUE
      IF (K.EQ.0) GO TO 27
      GO TO 24
   24 IF (KK.EQ.0) GO TO 26
   25 GO TO 29
   26 JFIRST(L)=I
      KK=1
      GO TO 3
   27 K=1
      IF (KK.EQ.0) GO TO 28
      GO TO 28
   28 CONTINUE
      IF (I.EQ.1) GO TO 4
      I=I-IPSTP3
      GO TO 4
   29 JLAST=I
      JNUM(L)=JLAST-JFIRST(L)+1
      KINDEX=0
   30 CONTINUE
      RETURN
   31 FORMAT (' ','WARNING: LIMIT OF LINE',I5,'IS EXCEEDED.')
      END
```

```
C
C
      SUBROUTINE REFRAT
      COMMON /AREA4/THETA,DELTA,SLOP2
      COMMON /AREA6/ILIMT(25),LNUM,L,LO
      COMMON /AREA10/C1,C2,C0,WLO,WLTRY,DEPTH,K,WLC,WKC,BRETH0,BRETH1,
     XBRETH2,CG2,SPEKSQ,DDEL1,DDEL2,ATNH
      COMMON /AREA15/IFETH,FETHS(50),DEPS(50),WKS(50),IFETCH,REFUNC(50),
     XWIDTH(50)
      COMMON /AREA16/PI,SCOPE1,SCOPE2,GEE,CCNVER,SCOPE
      PI=3.1416
      DELTA=PI/2.-DELTA
      DO 1 M=1,80
      ATNH=TANH((2.*PI*DEPTH)/WLTRY)
      WLC=WLO*ATNH
      IF (ABS(WLTRY-WLC).LE.0.05) GO TO 2
    1 WLTRY=(WLC+WLTRY)/2.
      WRITE (6,6) L
      CALL EXIT
    2 C2=C0*ATNH
      WKC=2.*PI/WLC
      SSIN2=(C2*SIN(DELTA)/C1)
      IF (SSIN2.GE.1.) GO TO 3
      DELTA2=ARSIN(SSIN2)
      GO TO 4
    3 DELTA2=PI/2.
    4 DDEL1=DELTA
      DDEL2=DELTA2
      IF (IFETH.LE.5.AND.(ABS(DDEL2).GT.ABS(DDEL1))) DDEL2=DDEL1
      DDEL=DELTA-DELTA2
      IF ((K.EQ.1).OR.(K.EQ.3)) GO TO 5
      DDEL=-DDEL
    5 THETA=THETA+DDEL
      RETURN
    6 FORMAT (1H0,'REFRAT OF ',I2,'DOES NOT APPROX.')
      END
```

```
C
      SUBROUTINE ANGLE1(THETA,SLOP2,DELTA,K)
      DELTA=THETA-SLOP2
      ENTRY ANGLE2(DELTA,K)
      PI=3.1416
      IF (DELTA.LT.0.0) DELTA=2.*PI+DELTA
    1 CONTINUE
      IF (DELTA.LT.(PI/2.)) GO TO 2
      IF (DELTA.LT.PI) GO TO 3
      IF (DELTA.LT.(PI*3./2.)) GO TO 4
      IF (DELTA.LT.(PI*2.)) GO TO 5
      DELTA=DELTA-2.*PI
      GO TO 1
    2 K=1
      GO TO 6
    3 K=2
      DELTA=PI-DELTA
      GO TO 6
    4 K=3
      DELTA=DELTA-PI
      GO TO 6
    5 K=4
      DELTA=2.*PI-DELTA
    6 CONTINUE
      RETURN
      END
```

```
C
      SUBROUTINE TRIG1(THETA,SLOP2)
      COMMON /AREA1/LD,JD
      COMMON /AREA3/VLUSIN,VLUCOS,VLUTAN
      DELTA=THETA-SLOP2
      CALL TRIG2(DELTA)
      RETURN
      END
```

```
C
      SUBROUTINE TRIG2(DELTA)
      COMMON /AREA1/LD,JD
      COMMON /AREA2/X(12,140),Y(12,140),DEEP(12),T
      COMMON /AREA3/VLUSIN,VLUCOS,VLUTAN
      CALL ANGLE2(DELTA,K)
      IF (K.EQ.1.OR.K.EQ.2) GO TO 1
      VLUSIN=-SIN(DELTA)
      GO TO 2
    1 VLUSIN=SIN(DELTA)
    2 IF (K.EQ.1.OR.K.EQ.4) GO TO 3
      VLUCOS=-COS(DELTA)
      GO TO 4
    3 VLUCOS=COS(DELTA)
    4 VLUTAN=VLUSIN/VLUCOS
      RETURN
      END
```

```
C
      FUNCTION ASLOP(LCNT,JCNT)
      COMMON /AREA1/LD,JD
      COMMON /AREA2/X(12,140),Y(12,140),DEEP(12),T
      PI=3.1416
      XJ=X(LCNT,JCNT)
      XJ1=X(LCNT,JCNT+1)
      IF (ABS(XJ-XJ1).LT..000001) GO TO 2
      ASLOPE=(Y(LCNT,JCNT+1)-Y(LCNT,JCNT))/(X(LCNT,JCNT+1)-X(LCNT,JCNT))
      IF (ASLOPE.GE.0.0) GO TO 1
      ASLOP=-ATAN(ABS(ASLOPE))
      GO TO 3
    1 ASLOP=ATAN(ASLOPE)
      GO TO 3
    2 ASLOP=PI/2.
    3 CONTINUE
      RETURN
      END

C
      FUNCTION YROT(L,J)
      COMMON /AREA1/LD,JD
      COMMON /AREA2/X(12,140),Y(12,140),DEEP(12),T
      COMMON /AREA3/VLUSIN,VLUCOS,VLUTAN
      YROT=-X(L,J)*VLUSIN+Y(L,J)*VLUCOS
      RETURN
      END

C
      FUNCTION XROT(L,J)
      COMMON /AREA1/LD,JD
      COMMON /AREA2/X(12,140),Y(12,140),DEEP(12),T
      COMMON /AREA3/VLUSIN,VLUCOS,VLUTAN
      XROT=X(L,J)*VLUCOS+Y(L,J)*VLUSIN
      RETURN
      END

C
      FUNCTION XXROT(XO,YO)
      COMMON /AREA3/VLUSIN,VLUCOS,VLUTAN
      XXROT=XO*VLUCOS+YO*VLUSIN
      RETURN
      END

C
      FUNCTION YYROT(XO,YO)
      COMMON /AREA3/VLUSIN,VLUCOS,VLUTAN
      YYROT=-XO*VLUSIN+YO*VLUCOS
      RETURN
      END
```

```
C
      FUNCTION XTRI(L,JCNT)
      COMMON /AREA1/LD,JD
      COMMON /AREA2/X(12,140),Y(12,140),DEEP(12),T
      COMMON /AREA3/VLUSIN,VLUCOS,VLUTAN
      COMMON /AREA5/XO,YO,XOP,YOP
      COMMON /AREA9/YDUM1,YDUM2
      XDUM1=XROT(L,JCNT)
      JCNT1=JCNT+1
      XDUM2=XROT(L,JCNT1)
      IF (ABS(XDUM1-XDUM2).LT..00001) GO TO 1
      SLOPE=(YDUM1-YDUM2)/(XDUM1-XDUM2)
      IF (ABS(SLOPE).LT..0001) GO TO 1
      XTRI=((YOP-YDUM1)/SLOPE)+XDUM1
      GO TO 2
    1 XTRI=(XDUM1+XDUM2)/2.
    2 CONTINUE
      RETURN
      END
```

COMPUTER PROGRAM (B): WAVE GENERATION

```
      DEFINE FILE 11(120,715,U,IREC),14(120,715,U,IREC),15(120,715,U,
     XIREC),16(120,715,U,IREC),18(120,715,U,IREC),12(120,715,U,IREC),13(
     X120,715,U,IREC),17(120,715,U,IREC)
      DIMENSION AVWK(300),AVDEL(300),PWRDN(300),AVOME(300)
      DIMENSION POS(220,13)
      COMMON /AREA20/XBH,YBH,DDEL
      INTEGER STATN
C
      DATA AVWK/300*0.0/,AVDEL/300*0.0/,PWRDN/300*0.0/,AVOME/300*0.0/
      GEE=9.8
      PI=3.1416
      FPSMGS=1./0.305
      NLOC=220
      NWK=10
C     NWK=13
      NMDPOS=4
      NAB=NLOC
      NMPOSD=NAB/NMDPOS
      U=20.
    1 CONTINUE
      READ (5,23,END=22) STATN,XBH,YBH
C     DO 34 INDFIN = 1,3
      WINFIN=1.05
      WINFIN=1.1
      WINFIN=1.2
      WINFIN=1.4
      WINFIN=1.3
      DO 20 IU=2,12,2
      U=(15./12.)*IU
      WRITE (6,24)
C     DO 31 IFL1 = 1,8
      DO 19 IFL1=1,8
      IFL=11
      IFL=IFL1+10
      WRITE (6,25) U,STATN,XBH,YBH,IFL1
      N2WK=NWK/2
      IREC=N2WK*NMDPOS+1
      IPOS1=0
      IPOS2=0
      DUMSPF=0.0
      DO 2 INDPOS=1,NMDPOS
      IPOS1=IPOS2+1
      IPOS2=IPOS2+NMPOSD
      READ (IFL'IREC) ((POS(IPOS,JPOS),JPOS=1,13),IPOS=IPOS1,IPOS2)
    2 CONTINUE
      JLOC1=0
      JLOC2=0
      DO 4 INDLOC=1,NLOC
      IPOS=INDLOC
      XOF=POS(IPOS,1)
      YOF=POS(IPOS,2)
      WIDFIN=POS(IPOS,7)
      DDEL=POS(IPOS,8)
      WID11=WIDFIN*10.
      DDIS=DIS(XOF,YOF,WID11)
      IF (DDIS.LT.0.0) GO TO 3
      JLOC1=INDLOC-30
      JLOC2=INDLOC+35
      JLOCO=INDLOC
      IF (JLOC1.LT.1) JLOC1=1
```

```
      IF (JLOC2.GT.NLOC) JLOC2=NLOC
      GO TO 5
    3 CONTINUE
    4 CONTINUE
    5 CONTINUE
      IF ((JLOC1.EQ.0).AND.(JLOC2.EQ.0)) GO TO 14
      IREC=1
      WRITE (6,26)
C     DO 23 JWK=1,NWK
      DO 13 JWK=1,NWK
      DUMSPF=0.0
      DUMCGF=0.0
      DUMPLL=0.0
      DUMPND=0.0
      DUMWK=0.0
      IPOS1=0
      IPOS2=0
      DO 6 INDPOS=1,NMDPOS
      IPOS1=IPOS2+1
      IPOS2=IPOS2+NMPOSD
      READ (IFL'IREC) ((POS(IPOS,JPOS),JPOS=1,13),IPOS=IPOS1,IPOS2)
    6 CONTINUE
      DO 9 INDFIN=1,80
      WINFIN=1.+(INDFIN-1)*.1
      DO 8 JLOC=JLOC1,JLOC2
      IPOS=JLOC
      XOF=POS(IPOS,1)
      YOF=POS(IPOS,2)
      WIDFIN=POS(IPOS,7)
      DDEL=POS(IPOS,8)
      T=POS(IPOS,3)
      OMERAD=2.*PI/T
      WID11=WIDFIN*WINFIN
      DDIS=DIS(XOF,YOF,WID11)
      WKSALO=POS(IPOS,4)*FPSMGS
      IF (DDIS.LT.0.0) GO TO 7
      WDSALO=POS(IPOS,5)/FPSMGS
      DESALO=POS(IPOS,6)/FPSMGS
      FETCH=POS(IPOS,12)/FPSMGS
      CGF=POS(IPOS,13)/FPSMGS
      THETA=POS(IPOS,9)
      REFUN=POS(IPOS,10)
      REFUN=1./REFUN
      AKR=POS(IPOS,11)
      KR=AKR
      CALL WAVSPE(U,FETCH,DESALO,WKSALO,FOMRAD,SPDNWK)
      SPERF=SPDNWK*REFUN
      DUMSPF=DUMSPF+SPERF
      DUMWK=DUMWK+WKSALO*SPERF
      DUMCGF=DUMCGF+CGF*SPERF
      DDEL=ABS(DDEL)
      IF (((KR/2)*2).NE.KR) DDEL=-DDEL
      DUMPLL=DUMPLL+SPERF*(SIN(DDEL))*CGF
      DUMPND=DUMPND+SPERF*(COS(DDEL))*CGF
      WRITE (6,28) JWK,REFUN,SPDNWK,CGF,DUMPLL,DUMPND,XOF,YOF,OMERAD
      GO TO 10
    7 CONTINUE
    8 CONTINUE
    9 CONTINUE
   10 CONTINUE
```

```
      OMERAD=2.*PI/T
      AVOME(JWK)=OMERAD
      AVWK(JWK)=WKSALO
      IF (ABS(DUMSPF).LT..00001) GO TO 11
      IF (ABS(DUMPND).LT..00001) GO TO 11
      AVWK(JWK)=DUMWK/DUMSPF
      AVCG=DUMCGF/DUMSPF
      AVDEL(JWK)=ATAN(DUMPLL/DUMPND)
      POPWR1=SQRT(DUMPLL**2+DUMPND**2)
      GO TO 12
   11 CONTINUE
      AVCG=0.0
      AVDEL(JWK)=0.0
      POPWR1=0.0
   12 CONTINUE
C     WRITE (6,207 )
      POPWR2=1.48*((1./10.)**2)*(GEE**2)*((1./OMERAD)**5.)*AVCG
      IF (POPWR1.GT.POPWR2) POPWR1=POPWR2
      PWRDN(JWK)=POPWR1
   13 CONTINUE
      GO TO 16
   14 CONTINUE
      DO 15 INWK=1,NWK
      AVWK(INWK)=0
      AVDEL(INWK)=0.0
      PWRDN(INWK)=0.0
   15 CONTINUE
   16 CONTINUE
      WRITE (6,30)
      WRITE (6,31) ((AVWK(II),AVDEL(II),PWRDN(II),AVOME(II)),II=1,NWK
      NWK1=NWK-1
      BHPWR=0.0
      BHPLL=0.0
      BHPND=0.0
      PWRDN1=PWRDN(1)
      COSWK1=COS(AVDEL(1))
      SINWK1=SIN(AVDEL(1))
      DO 17 INWK=1,NWK
      PWRDN2=PWRDN(INWK+1)
      COSWK2=COS(AVDEL(INWK+1))
      SINWK2=SIN(AVDEL(INWK+1))
      WDTHWK=AVWK(INWK)-AVWK(INWK+1)
      COMPWR=(PWRDN(INWK)+PWRDN(INWK+1))*WDTHWK/2.
      COMPLL=(PWRDN1*COSWK1*SINWK1+PWRDN2*COSWK2*SINWK2)*WDTHWK/2.
      COMPND=(PWRDN1*COSWK1*COSWK1+PWRDN2*COSWK2*COSWK2)*WDTHWK/2.
      BHPWR=BHPWR+COMPWR
      BHPLL=BHPLL+COMPLL
      BHPND=BHPND+COMPND
      PWRDN1=PWRDN2
      COSWK1=COSWK2
      SINWK1=SINWK2
   17 CONTINUE
      SPCONV=(10.**4.)*3600.
      BHPWR=BHPWR*SPCONV
      BHPLL=BHPLL*SPCONV
      BHPND=BHPND*SPCONV
      WRITE (6,32)
      WRITE (6,33) STATN,IFL1,XBH,YBH,BHPWR,BHPLL,BHPND
      WRITE (6,34)
   18 CONTINUE
```

```
 19 CONTINUE
 20 CONTINUE
 21 CONTINUE
    GO TO 1
 22 CONTINUE
    STOP
 23 FORMAT (A4,2F8.2)
 24 FORMAT ('1')
 25 FORMAT (/' ','WIND VEL. = ',F8.3,4X,A4,2F8.3,'     DRET=',I5//)
 26 FORMAT ('0',T6,'JWK ',T16,'REFUN',T26,'SPDNWK',T36,'CGF',T46,'DUMP
   XLL',T56,'DUMPND',T66,'XOF',T76,'YOF'//)
 27 FORMAT (' ',13E10.3)
 28 FORMAT (' ',I10,8E10.3)
 29 FORMAT ('0')
 30 FORMAT ('0',T6,'AVWK',T16,'AVDEL',T26,'PWRDN',T36,'AVOME')
 31 FORMAT (' ',4E10.3)
 32 FORMAT ('0',T4,'STATN',T14,'DRET',T24,'XBH',T35,'YBH',T46,'BHPWR',
   XT57,'BHPLL',T68,'BHPND')
 33 FORMAT (' ',A10,I5,5X,10E11.4)
 34 FORMAT (//' ','**************************************************')
    END
```

```
C
      SUBROUTINE WAVSPE(FU,FFETCH,FDEPTH,FWK,FOMRAD,THEINT)
      COMMON /AREA11/GEOS,USTR,CARMAN,ELNZO
      COMMON /AREA12/FETCH,PI,RHOA,WUS,BDEL,TAUS,SIGDEL,DELL3,GEE,RHOW,
     XVSCUSW,AMU,T,U,Z,ALFA,WL,C,CG,WK,U1,ZO,ZDELTA,DEL1,DURATN,OMERAD
      COMMON /AREA13/THETA
      COMMON /AREA14/SPEC
      EXTERNAL AINT
      FETCH=FFETCH
      DEPTH=FDEPTH
      WK=FWK
      U=FU
      THETA=0.0
      CARMAN=.4
      PI=3.1416
      RHOA=1276.
      WUS=0.8
      BDEL=0.31
      TAUS=3.7
      SIGDEL=.5
      DELL3=.5
      GEE=9.8
      RHOW=1.*(10.**6.)
      VSCUSW=(1.217/(10.**5.))*((.305)**2.)
      USTR=((U-5.)*.83)/15.+.17
      IF (U.LT.5.) USTR=U*.17/5.
      AMU=4.
      Z=10.
      U1=USTR/CARMAN
    1 ELNZO1=U*CARMAN/USTR-ALOG(Z)
      ZO1=EXP(ELNZO1)
      ZO=1./ZO1
      ELNZO=ALOG(ZO)
      ZDELTA=40.
      GEOS=(USTR/CARMAN)*(ALOG(ZDELTA)-ELNZO)
    2 DEL1=DEL11(ZDELTA)-DEL11(ZO)+DEL12(ZDELTA)-DEL12(ZO)+ZO/2.
      WL=2.*PI/WK
      OMERAD=SQRT(GEE*WK*TANH(WK*DEPTH))
      FREQ=OMERAD/(2.*PI)
      T=1./FREQ
      C=(GEE*T/(2.*PI))*TANH(WK*DEPTH)
      WKFAC=2.*WK*DEPTH
      CG=C*(.5*(1.+WKFAC/SINH(WKFAC)))
    3 DURATN=FETCH/CG
      A=-(PI/4.)
      B=(PI/4.)
      N=10
      THEINT=AINT(A,B,N)
      FOMRAD=OMERAD
      RETURN
    4 FORMAT ('0','U=',F6.3,'      USTR=',F6.3,'      ZO=',F7.4,'       ELNZ
     XO1=',E11.4)
    5 FORMAT ('0',T5,'ZDELTA',T16,'GEOS',T27,'DEL1')
    6 FORMAT (' '83E11.4)
    7 FORMAT ('0',T5,'T',T16,'WK',T27,'OMERAD',T38,'FREQ',T49,'C',T60,'W
     XL',T71,'FETCH',T82,'DURATN')
    8 FORMAT (' ',11E11.4)
    9 FORMAT ('0','THETA',T14,'PIE',T19,'AUCCOS',T25,'TSCALE',T31,'OMEGA
     X',T40,'EC',T50,'BETA',T57,'AMT',T78,'SPEC')
      END
```

```
      REAL FUNCTION AINT(A,B,N)
      COMMON /AREA11/CEOS,USTP,CARMAN,ELNZO
      COMMON /AREA12/FETCH,PI,RHOA,WUS,BDEL,TAUS,SIGDEL,DELL3,GEE,RHOW,
     XVSCUSW,AMU,T,U,Z,ALFA,WL,C,CG,WK,U1,ZO,ZDELTA,DEL1,DURATN,OMERAD
      COMMON /AREA13/THETA
      COMMON /AREA14/SPEC
      AINT1=1.48*((1./10.)**2.)*(GEE**2.)*((1./OMERAD)**5.)
      XN=N
      H=(B-A)/XN
      X=A
      AINT=0.
      CALL SPETHE(A,KSPEC)
      DUMFA=SPEC
      IF (KSPEC.EQ.1) GO TO 2
      DO 1 I=1,N
      X=X+H
      CALL SPETHE(X,KSPEC)
      DUMFX=SPEC
      IF (KSPEC.EQ.1) GO TO 2
    1 AINT=AINT+DUMFX
      CALL SPETHE(B,KSPEC)
      DUMFB=SPEC
      IF (KSPEC.EQ.1) GO TO 2
      AINT=.5*H*(DUMFA+2.*AINT-DUMFB)
      IF (AINT1.LT.AINT) AINT=AINT1
      GO TO 3
    2 AINT=AINT1
    3 CONTINUE
      RETURN
      END
```

```
C
      SUBROUTINE SPETHE(THETA,KSPEC)
      COMMON /AREA11/GEOS,USTR,CARMAN,ELNZO
      COMMON /AREA12/FETCH,PI,RHOA,WUS,BDEL,TAUS,SIGDEL,DELL3,GEE,RHOW,
     XVSCUSW,AMU,T,U,Z,ALFA,WL,C,CG,WK,U1,ZO,ZDELTA,DEL1,DURATN,OMERAD
      COMMON /AREA14/SPEC
      KSPEC=0
      THETAC=THETA
      IF (THETAC.LT.0.0) THETAC=-THETAC
      COSTHE=COS(THETAC)
    1 SIGMA=(1./2.)/DEL1
      IF ((-(WK**2.)/(4.*SIGMA**2.)).LT.(-45.)) GO TO 2
      PIE1=((RHOA**2.)*(WUS**2.)*(TAUS**2.))/(PI*(SIGDEL**2.))
      PIE2=((USTR**4.)*(DEL1**4.)*((WK*COSTHE)**2.)*EXP(-(WK**2.)/(4.*
     XSIGMA**2.)))/((WK*DEL1)**2.+(2.*BDEL*WK*DEL1))
      PIE=PIE1*PIE2*(10.**4)
      GO TO 3
    2 PIE=0.0
    3 CONTINUE
      IF (PIE.LT.(1./(10.**50.))) PIE=0.0
    4 UC=GEOS*.8
      AO=0.7/(2.*PI)
      AUCCOS=AO*UC*COSTHE
      TS1=AUCCOS/(2.*WK)
      TS2=1./(AUCCOS**2.+(2.*UC*COSTHE-C)**2)
      TS3=1./(AUCCOS**2.+C**2.)
      TSCALE=TS1*(TS2+TS3)
    5 U1COS=U1*COSTHE
      U1COSC=U1COS/C
      OMEGA=GEE*ZO/((U1COS)**2.)
      IF ((1./U1COSC).GT.50.) GO TO 6
      EC=OMEGA*(U1COSC**2.)*EXP(1./U1COSC)
      GO TO 7
    6 CONTINUE
      EC=1000.
    7 CONTINUE
      IF (EC.LE..00861) GO TO 8
      IF (EC.LE..0548) GO TO 9
      IF (EC.LE..3) GO TO 10
      IF (EC.LE.2.) GO TO 11
      BEDUM=2.*(2.-EC)
      IF (BEDUM.LT.(-20.)) BEDUM=-20.
      BETA=0.017*EXP(BEDUM)
      GO TO 12
    8 BETA=3.39-0.9406*((ALOG10(.00861/EC))**1.860)
      GO TO 12
    9 BETA=3.39-1.294*((ALOG10(EC/.00861))**2.323)
      GO TO 12
   10 BETA=-0.1402-2.181*ALOG10(EC)
      GO TO 12
   11 BETA=(ALOG10(3./EC))**2.362
   12 IF (BETA.LT.(1./(10.**30.))) BETA=0.0
      CI1=((RHOA/RHOW)*(BETA)*((AMU*(U1*COSTHE/C))**2.))
      CI2=4.*GEE*VSCUSW/(C**3.)
      CI=CI1-CI2
   13 AMT=CI*WK*(C/2.)*DURATN
      IF (AMT.LE.0.0) GO TO 14
      IF (AMT.GE.75.) GO TO 16
      FMT=(EXP(2.*AMT)-1.)/(2.*AMT)
      GO TO 15
```

```
14 FMT=1.
15 SPEC=(1./(2.*(RHOW**2.)*(C**2.)))*DURATN*FMT*PIE*TSCALE
   GO TO 17
16 SPEC=-1000.
   KSPEC=1
17 CONTINUE
   RETURN
18 FORMAT (' ',F6.3,E13.6,F6.3,F6.3,F8.5,E13.6,F6.3,F6.2,E14.6,I4)
   END
```

```
C
      FUNCTION DIS(XOF,YOF,WIDFIN)
      COMMON /AREA20/XBH,YBH,DDEL
      DISBH=SQRT((XOF-XBH)**2+(YOF-YBH)**2)
      DISBHC=WIDFIN/(2.*COS(DDEL))
      DIS=DISBHC-DISBH
      RETURN
      END


      FUNCTION DEL12(Z)
      COMMON /AREA11/GEOS,USTR,CARMAN,ELNZO
      DEL12=((USTR/CARMAN)/GEOS)*((ELNZO*Z)-(Z*ALOG(Z))+Z)
      RETURN
      END


      FUNCTION DEL11(Z)
      COMMON /AREA11/GEOS,USTR,CARMAN,ELNZO
      DEL11=(1./GEOS)*(GEOS*Z)
      RETURN
      END
```

APPENDIX G

COMPUTATION OF WAVE ENERGY

G.1 PROCEDURE OF WAVE ENERGY COMPUTATION

To assist computation, the two computer programs described in Chapter IV - Computer Program (A) (Wave Refraction) and Computer Program (B) (Spectrum of Shallow Water Wind Wave) are utilized. The procedures of obtaining wave energy distribution along the shoreline of Southern Basin are described in the following paragraphs.

(i) A bathymetric chart of Southern Basin is needed for Computer Program (A). Since no such map is available a detailed bathymetric graph is drawn up from the data of Map #6251 (Ref. No. 142) with contour intervals of 1.0 foot where possible. Of these contours, only eight are selected for further use: 4', 10', 19', 24', 27', 31', 34', and 37' - with reference to a mean lake elevation of 712.0 feet. The 5-foot-contour is taken to be the coastline as an approximation. This bathymetric map is shown in Plate I.

(ii) A grid system is then laid down using a unit distance of one mile with the origin located at the southwestern corner of the map; all locations of the Southern Basin lie within the first quadrant so that only positive co-ordinate values are used, for the convenience of numerical manipulation. (Fig. 6-1).

(iii) The contours of (i) above are then approximated by multiple linear segments. This is done by defining the contour lines with points. The co-ordinates of these points are used as input data for Computer Program (A).

(iv) Computer Program (A) still requires two more input items: the starting locations of the wave rays and the mean wind direction. For this purpose an "r shape" frame is used such that it closely approximates the general shape of the Southern Basin, but lies inside the water-covered areas of the lake. This is shown as the 8-sided polygon ABCDEFGHA in Fig. 6-1. The use of this frame is such that when generating refractions for west wind, the computer will take the western sides ABC and use the points lying on these lines (AB and BC) as the starting points of the wave rays. For a specified wave frequency 220 wave rays are generated for each wind direction, so that for the west wind 110 rays with starting points equally spaced are generated from AB and BC respectively. In the same manner, the most southerly side CD is used for south wind so that CD serves as the starting point of 220 rays. The use of the frame for the remaining directions is tabulated as follows.

ind ection	Reference* Direction	Sides of Frame ABCDEFGHA Used	Total Number of Rays Generated	Number of Rays Per Side
t	1	ABC	220	110
thwest	2	BCD	220	110
th	3	CD	220	220
theast	4	CDEF	220	73
t	5	DEFGH	220	55
theast	6	EFGHA	220	55
th	7	HAB	220	110
thwest	8	GABC	220	73

ı the computer program the wind directions are denoted by numbers. example, west wind is referred to as direction "1" and southwest l is direction "2", and so forth.

(v) Refraction is then performed for all eight wind directions (W, SW, S, SE, E, NE, N, NW) at 220 rays for each direction and for 10 values of wave frequencies. (T = 6.3, 3.1, 2.1, 1.3, 0.7, .42, .29, 0.19, 0.13, 0.10 sec.) For each ray the pertinent information such as average depth and refraction function, is computed and stored in a magnetic disc.

(vi) Computer Program (B) is then used to extract the required information from the above disc and calculate the wave spectrum for a specific location of the coast - such as Patricia Beach. Apart from information stored on disc, the only data input required by this program are the wind velocity and direction, and location of the coast for which wave energy is to be calculated. For instance, for Patricia Beach (Station E002) only the co-ordinates (17.0, 3.0), (in miles) are needed. The computer program would then calculate the wave spectrum, total energy per unit width of wave front, the normal and longshore energy per beach width and the average angle of attack for all eight directions and six values of wind velocity (2.5, 5, 7.5, 10, 12.5, and 15 meters/sec.) as measured at 10 meters above ground.

When the station is not influenced by a certain wind direction, such as South wind on Patricia Beach, the value of the spectrum, and the wave energy would of course be nil. A schematic illustration of the programing procedures is as shown in Fig. 6-2.

(vii) Finally, the results of (vi) are graphed as functions of wind velocity for each wind direction. A sample of these graphs, that for Station W027 (North of Winnipeg Beach), is as shown in Fig. 6-3. The ordinate measures the total energy per unit wave front, normal wave energy component and long-shore wave energy component per unit beach front in energy per unit width per unit time, (erg/cm-hr.).

Because of the large number of stations used for computations, (60 stations), the graphs for the remaining stations cannot be conveniently presented herein.

It should be noticed that these graphs represent the <u>fetch-limited</u> energy characteristics of wave at the respective coastal location and their use simplifies the exact computing procedures.

(viii) To obtain an annual wave energy for a particular wind velocity from a specified direction for a coast location, the procedure is to multiply the energy per unit width per unit time from the energy-wind velocity graphs in step (vii) by the corresponding number of hours of the year the wind has been blowing in this direction at the velocity. Only the ice-free period is relevant. The total energy due to wind from this direction is the sum of the product of the ordinates of the energy-wind velocity graph and the wind velocity distribution graph. In the present study the integration is done by numerical calculation using wind velocity intervals of 2.5 mph. A sample of this computation is shown in Fig. 6-4.

(ix) The last step is to add all the normal energy components and the parallel energy components together. This represents the total wave energy at the beach location for the year.

These series of steps were used to generate wave energy characteristics for certain locations on the East Coast and the West Coast. In most cases, these locations are the same as the stations that have been surveyed in the summer of 1970. (See Chapter III). They include Stations E001 to E021 and W001 to W038. However, due to various reasons, results for Stations E010 and W028 - W038 could not be obtained . The wind data used is as measured at Gimli meteorological station for the period March 1, 1968 to November 31, 1968. An analysis of this wind information has been discussed in Chapter IV.

G.2 COMPUTATION RESULTS

The wind wave energy characteristics thus computed for the year 1968 are plotted against station location on the linear graphs of Plates II and IV, and are joined by dotted lines unless where the full lines (representing adjusted values) coincide with dotted lines - in which case they are joined by full lines. It is to be noticed that although the values of two adjacent stations are herein linked by a straight line, the energy distribution generally is not a linear function of distance. Straight line portions are used on these graphs only for convenience.

G.3 INCOMPLETENESS OF RESULTS AND THEIR CORRECTIONS

There are a few places in the computed results where details are missing or incomplete. In the paragraphs to follow, attempts to remedy the completeness will be described.

(i) Because the frame ABCDEFGHA (see Section 6-1) does not extend far south enough, waves due to west wind are not computed for stations E001, E002, E003, and E004. Since for these stations, waves coming from the west would have travelled over water of approximately the same depth, and fetch of approximately the same distance as for Station E005, the total energy per unit width of wave front can be assumed to be the same as that of Station E005. Knowing the approximate angle the waves will attack the shore at, these stations (E001, E002, E003, and E004), the longshore and normal wave energy components can be computed.

(ii) North and South winds blowing over the narrow stretch of water bounded by the coastline of West Coast on the west, the side AB of the polygon ABCDEFGHA on the east, by Drunken Point on the north and by Willow Point on the south certainly also produce wave energy, but this has not been considered in the computer program. The waves produced by the south wind will affect the coastline between stations W001 and W005. The north wind will affect the north-facing shores of Willow Point, and Loni Beach. Here the wave characteristics produced by the north and south winds are assumed to be similar to that generated by west wind at Station E005 again. (Similar fetch and similar depth). But since the strip of water is adjacent to the coast, the actual spectrum cannot be a truly normal one-dimensional spectrum; arbitrarily, a discount factor of 20 percent was used. For Loni Beach, this discount factor is 40 percent because the fetch considered is shorter. The remaining steps are similar to that followed for stations considered in (i) above.

(iii) Results for stations south of Winnipeg Beach are not available and are supplemented as below.

Throughout the reach from Winnipeg Beach to Sans Souci, wave energy due to the east wind changes little from one location to another and is generally perpendicular to the shoreline. Therefore, for most stations it can be approximated by the wave energy due to the east wind at Station W027, which is slightly north of Winnipeg Beach.

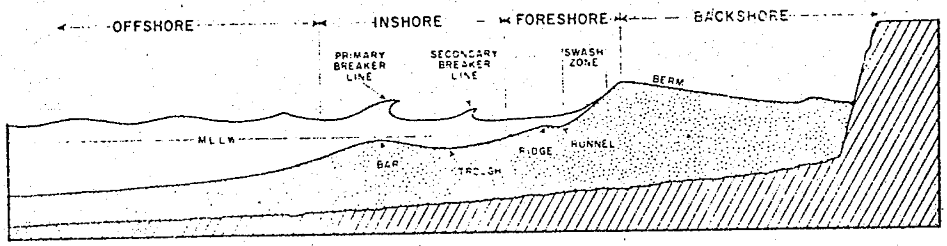
The energy due to the northeast wind per unit wave front throughout the reach can also be approximated by that for station W027, only that the angle of attack decreases southward - south of San Souci, it is practically zero. Thus it can be assumed the parallel energy component decreases from Station W027 until it reaches zero somewhere in the southern neighbourhood of San Souci.

The effect of southeast wind also decreases from Station W027 to nil at Sans Souci, due to the diminishing fetch.

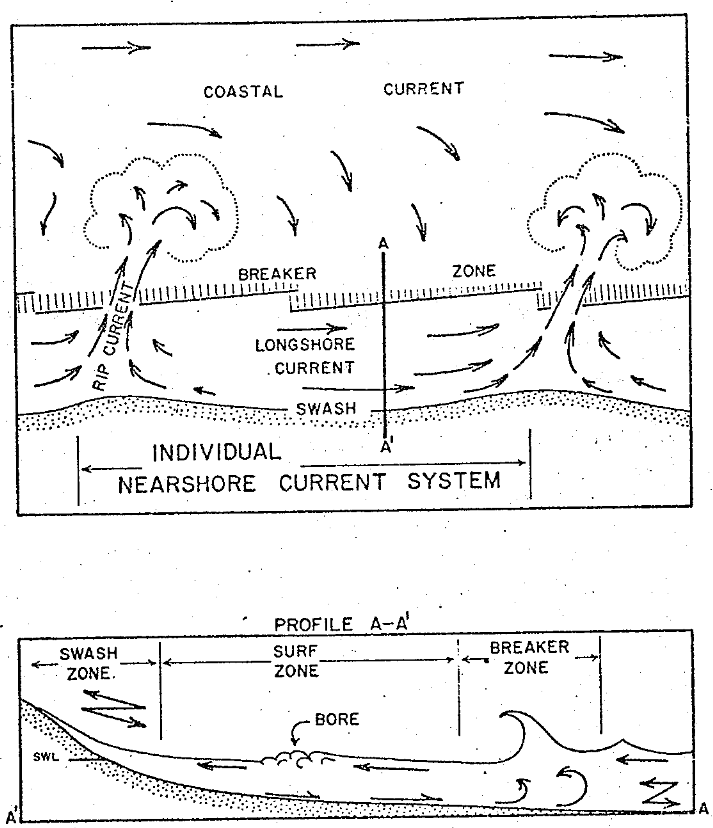
From Fig. (6-6) it can be seen that the north wind exhibits the most interesting case. From the results of wave refraction it is found that a bundle of wave rays of wave number k = .075 arrive at the neighbourhood of station WO27, none arrive at Winnipeg Beach; and then reappear again south of Stephen's Point. A number of wave rays of wave number, K = .075 to k = 0.175, where the range in which the peak spectral density most frequently occurs strike at Winnipeg Beach, none appearing between Stephen's Point and Ponemah. These reappear again south of Ponemah. This means that a bundle of high energy waves arrive at Winnipeg Beach whenever the north wind is blowing whereas the reach between Stephen's Point and Ponemah would receive much less energy. South of Ponemah the wave energy increases abruptly again and remains relatively constant thereafter. This phenomenon has a unique role in shaping the coastline of this area, as will be explained in Chapter VIII.

For a rough estimate, the full spectrum due to north wind at Winnipeg Beach (from k = 0 to k = 1.5) is assumed to be three times that of the part of the spectrum between k = 0 and k = 0.075 for Station WO27 (by inspection of Fig. (6-4)). The normal and parallel components are then calculated as before (assuming Winnipeg Beach is not an indented coastline). Between Stephen's Point and Ponemah the values drop to those of Station WO27. The energy gradually increases to that of Winnipeg Beach south of Ponemah.

It should be noticed however, that the values at Winnipeg Beach are underestimates since wave rays are concentrated in this region. Moreover, the approximation method is very rough. These results should not be used for important practical application without further justification.

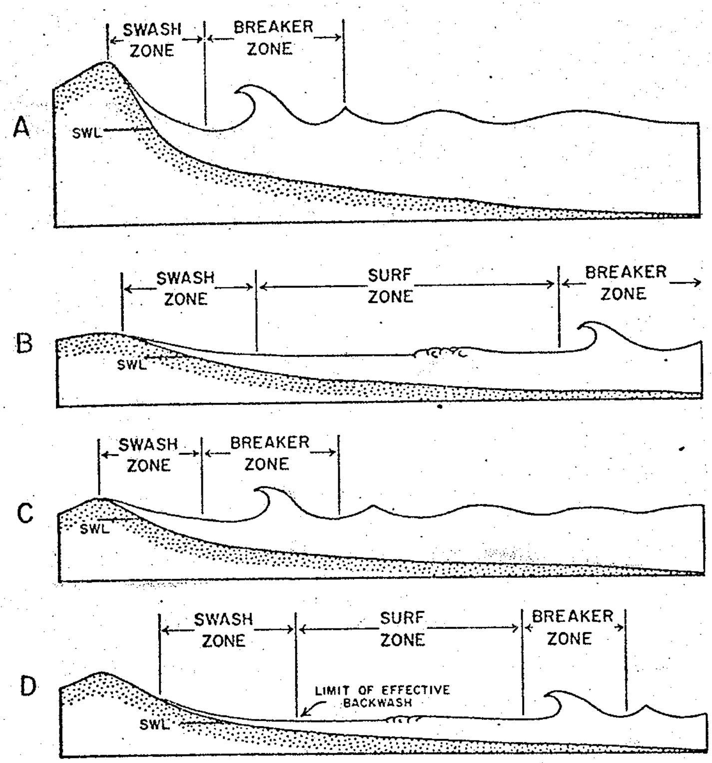


Terminology associated with the beach environment. M.L.L.W. = mean lower low water.



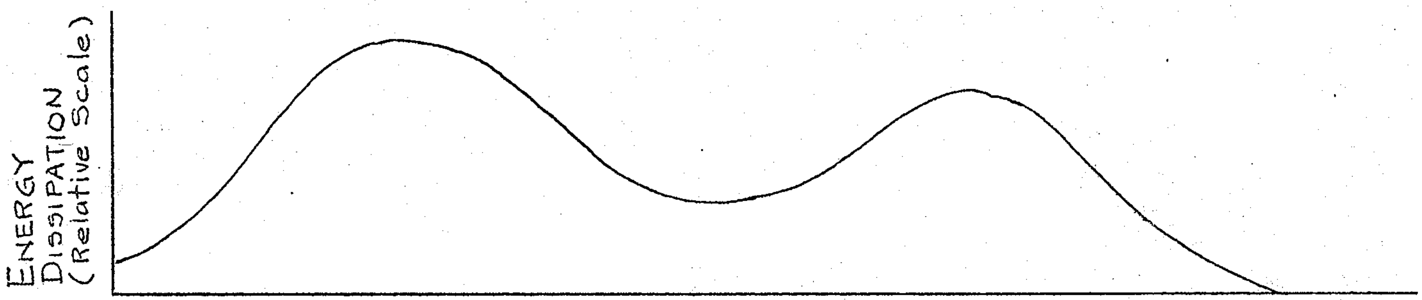
Terminology of nearshore current systems.
Each individual system begins and terminates with a seaward flowing rip current. Arrows indicate direction of water movement in plan and profile. Existence of the controversial seaward return flow along the foreshore–inshore bottom (profile A–A') has recently been confirmed by electromechanical measurements in the surf zone These measurements ndicated a seaward bottom flow exceeding 1.2 ft./sec often occurs at the same time surface flow is shoreward. The surf zone is here defined as the area between the seaward edge of the swash zone and the breaker zone. S.W.L. = still water line.

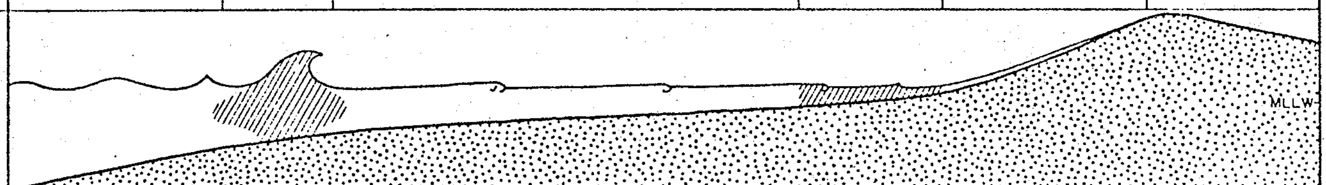
FIGURE E-1. BEACH AND BEACH ENVIRONMENT TERMINOLOGY. (After Ingle, 1966.)



Character of the surf zone. Existence and width of a surf zone is governed by beach slope and tidal phase. Steep-sloped beaches seldom possess a surf zone as relatively deep water allows waves to break close to shore and thus swash zone meets the breaker zone (*A*). Gentle beaches possess a surf zone under almost all conditions as waves must break at some distance from the seaward edge of the swash zone (*B*). Moderately sloping beaches commonly lack a surf zone during high tide (*C*), but exhibit one during all other phases of the tidal cycle (*D*). SWL = still water line.

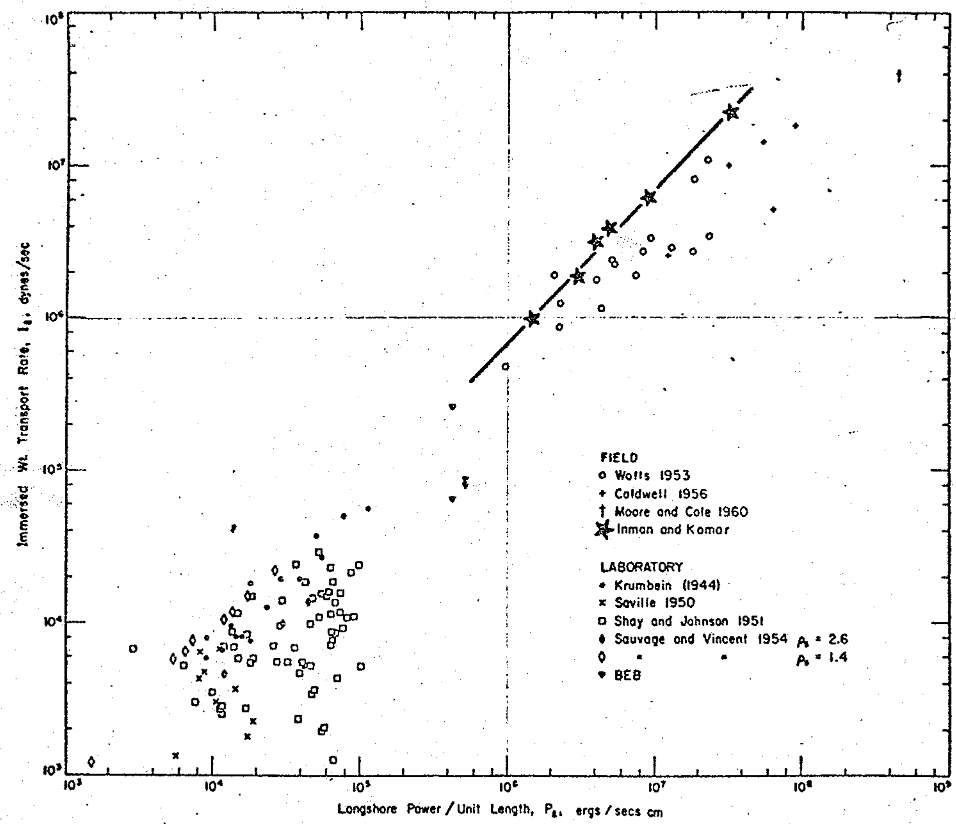
FIGURE E-2. BEACH TERMINOLOGY. (After Ingle, 1966.)



WATER MOTION	OSCILLATORY WAVES	WAVE COLLAPSE	WAVES OF TRANSLATION (BORES); LONGSHORE CURRENTS; SEAWARD RETURN FLOW; RIP CURRENTS	COLLISION	SWASH; BACKWASH	WIND
DYNAMIC ZONE	OFFSHORE	BREAKER	SURF	TRANSITION	SWASH	BERM CREST
PROFILE						
SEDIMENT SIZE TRENDS	COARSER →	COARSEST GRAINS	← COARSER →	BI-MODAL LAG DEPOSIT	← COARSER	WIND - WINNOWED LAG DEPOSIT
PREDOMINANT ACTION	ACCRETION	EROSION	TRANSPORTATION	EROSION	ACCRETION AND EROSION	
SORTING	← BETTER	POOR	MIXED	POOR	BETTER →	
ENERGY	INCREASE →	HIGH	GRADIENT →	HIGH	→ ←	

Summary diagram schematically illustrating the effect of the four major dynamic zones in the beach environment. Hatchered areas represent zones of high concentrations of suspended grains. Dispersion of fluorescent sand and electromechanical measurements (SCHIFFMAN, 1963, 1965) indicate that the surf zone is bounded by two high-energy zones; the breaker zone and the transition zone. MLLW = mean lower low water.

FIGURE E-3 CHARACTERISTICS OF A BEACH
(Reference: Ingle, 1966.)



Relation between the immersed weight longshore transport rate and the longshore component of wave power per unit length of beach. The various sources of plotted data are listed in the references; the "BEB" data is given in Savage (1962).

FIGURE E-4. LONGSHORE TRANSPORT RATE.
(After Inman et.al., 1968.)

APPENDIX H

List of Symbols

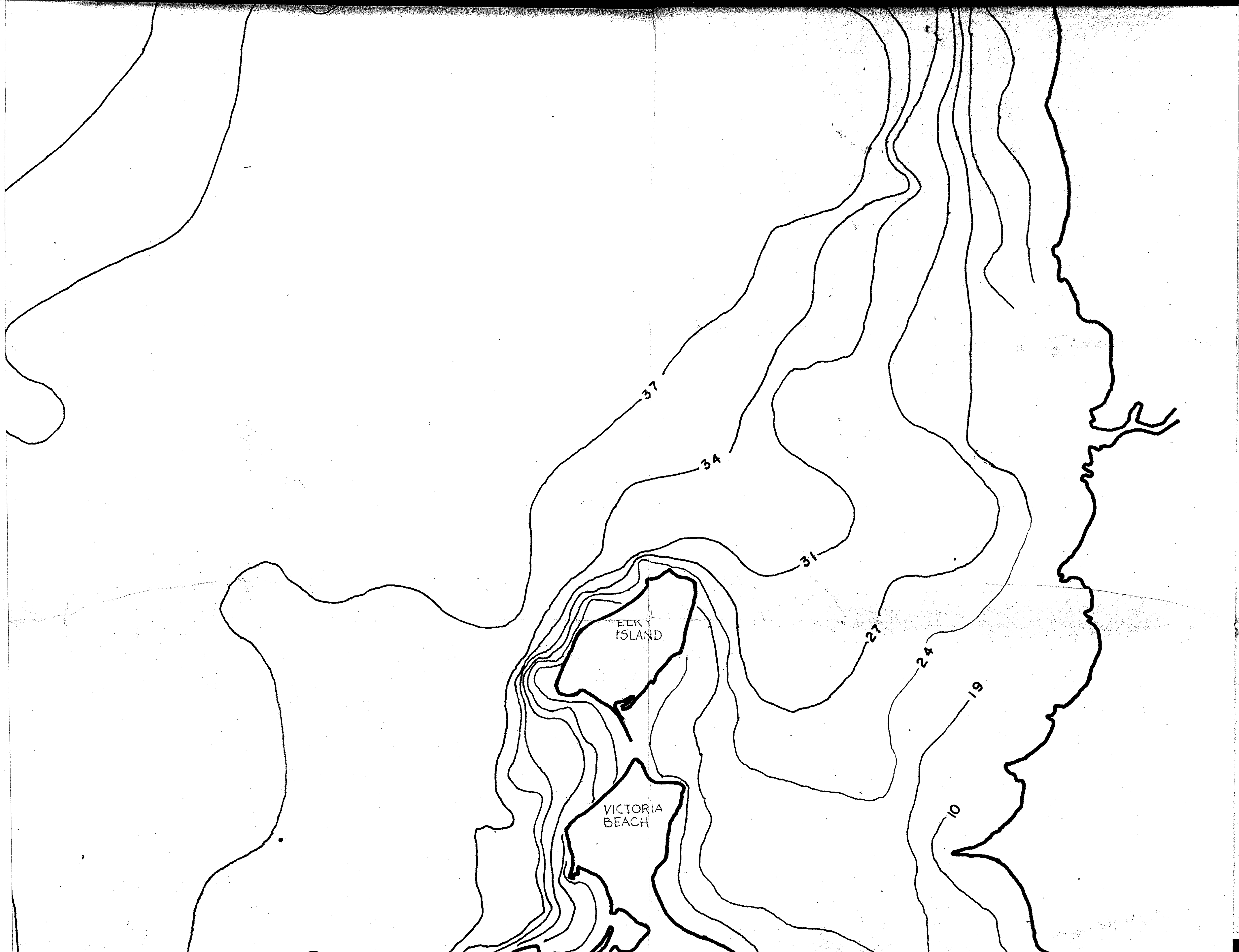
a	(subscript) - property of air
a_o	a constant; wave amplitude
b	width of a wave ray
b	mean shear parameter
c	a constant
c	wave (phase) velocity
c_f	phase velocity of enforced wave
c_g	group velocity of waves
C_1, C_2	constants
d	depth of water
Δe	increase in lake level
E	energy
E_{KE}	kinetic energy of wave
E_{PE}	potential energy of wave
E_T	longshore energy
f	frequency
F(mt)	modifying factor in Miles - Phillips' model
g	gravitation constant
i	velocity of light
I_1	transport rate of the immersed weight of sand
H	wave height
H_{rms}	root mean square wave height
k	a constant, wave number
k	von Karman's constant
k_1, k_2	constants governing the rate of erosion and deposition
K	constant
$[K^2]$	wave refraction function
l_3	vertical scale of turbulence
L	a linear operator

L	wave length
m	mass per unit area
m	$(1/2) \rho kc$, in Miles - Phillips' model
n	$(gk + Tk^3/p)^{1/2}$ as in Phillips' model
n	the ratio of group velocity to phase velocity
N	number of wave crests
o	(subscript) deep water condition, initial condition
p	probability
p_z	normal tension on water surface
P	wave power
P_1	parallel component of wave power
Q	rate of longshore transport
R_u	rate of energy loss due to viscosity
R_t	rate of energy transfer to wave due to tangential stress
R_n	rate of energy transfer to wave due to normal stress
s	spectral density
s	ρ_a/ρ_w
s	(subscript) property of significant wave
s_1	slope of inshore - foreshore
s_2	slope of offshore
S	co-cumulative spectrum
t	time
t*	duration of storm
T	wave period
T_{al}	atmospheric temperature over land
T, T_a, - T_b, T_c	sediment transport load
T_w	atmospheric temperature over water (lake)
U	wind velocity
U_c	convection velocity of wave component

U_{ob}	wind velocity outside boundary layer
U_z	wind velocity at height z
U_{10}	mean wind velocity at a height of 10 meters
U_*	shear velocity
U_∞	free stream wind velocity
w'	instantaneous velocity component in the z direction
w	(subscript) - property of water
W_e	wind velocity over land
W_w	wind velocity over water (lake)
x	x-co-ordinate defining wave form
y_s	elevation of water surface
z	height above ground or water surface
z_o	roughness layer thickness
α	angle between wave component and wind direction; angle between wave ray and the normal of shoreline; real part of complex dimensionless coefficient in Miles' model; angle between direction of light and the normal of an interface; a constant
β	a constant
β	imaginary part of the complex dimensionless coefficient in Miles' model
δ	boundary layer thickness
δ_1	displacement thickness
$\epsilon, \epsilon_t, \epsilon_x$	phase in wave equation
ζ	resistance coefficient
ζ	damping ratio in Miles' model
ξ	dimensionless height
γ	density
λ	a constant
$\langle\eta^2\rangle, \langle\xi^2\rangle$	mean square surface elevation
μ	constant in Miles-Phillips' model, has a value of 4.0

ν	viscosity
ψ	atmospheric stream function
Π	atmospheric pressure spectrum
ϕ	angle between wave ray and wind direction
ϕ	dimensionless atmospheric stream function
Φ	spectral density function
ρ	mass of water
σ	inverse turbulent scale
τ	time
τ_1	mean shear
θ	time scale, or development time of pressure fluctuation
Γ	response factor between wave spectrum and atmospheric pressure spectrum in Phillips' equation
ω	dimensionless atmospheric stream function
ω	angular velocity; radian frequency in wave equation


HECLA
ISLAND


37
34
31
27
24
19
10
ELK ISLAND
VICTORIA BEACH

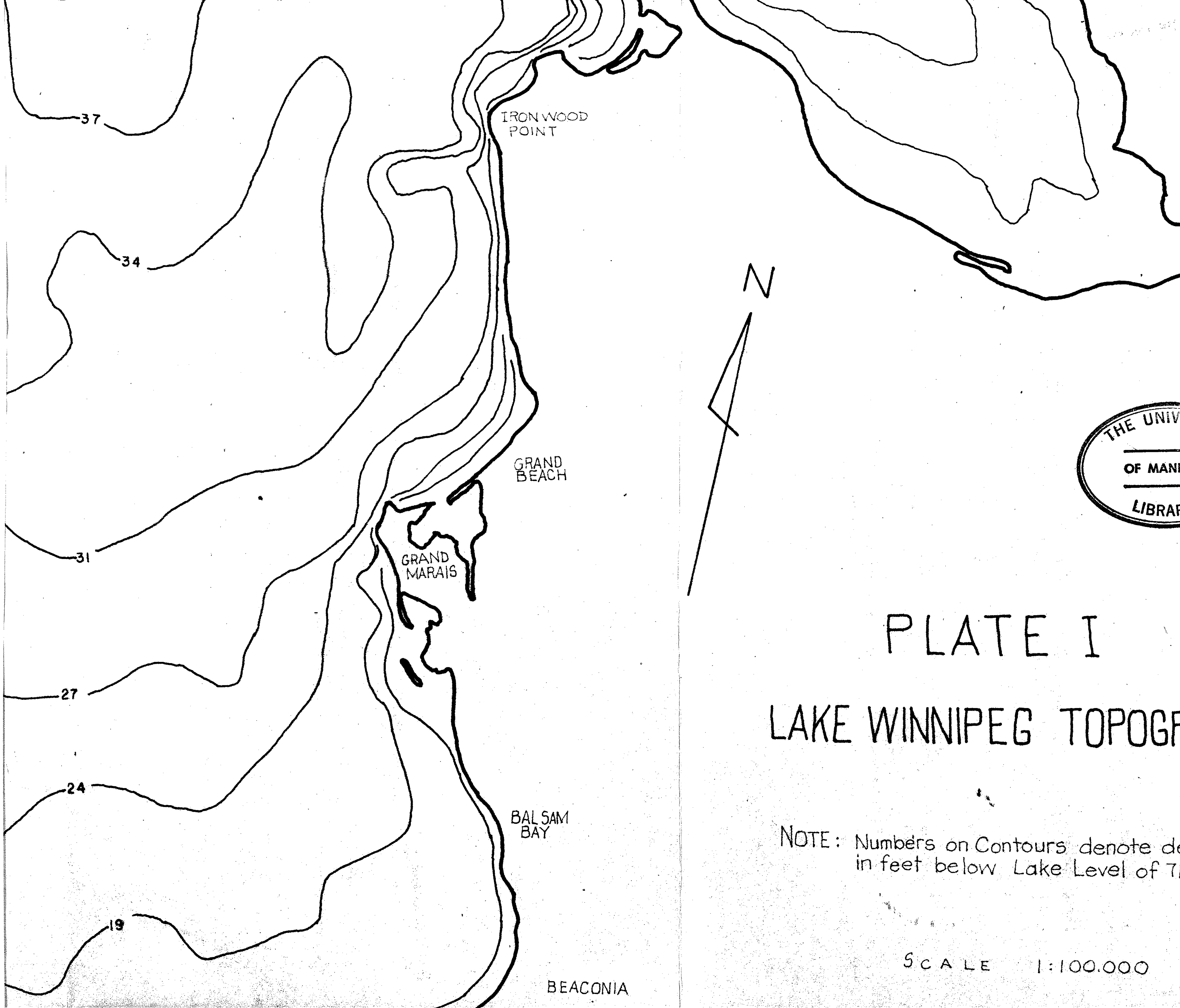


PLATE I

LAKE WINNIPEG TOPOGRAPHY

NOTE: Numbers on Contours denote depth in feet below Lake Level of 712' above s.

SCALE 1:100,000

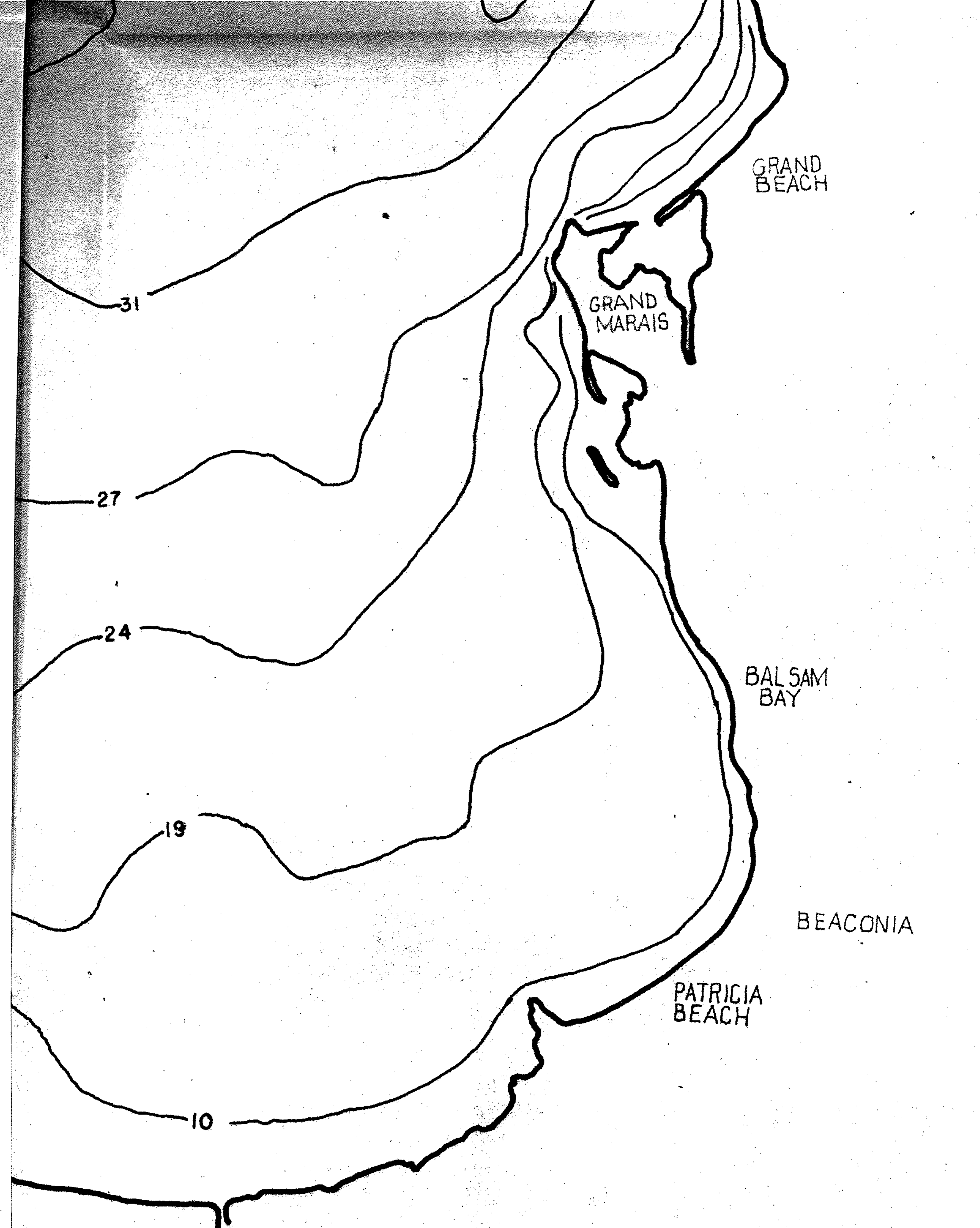
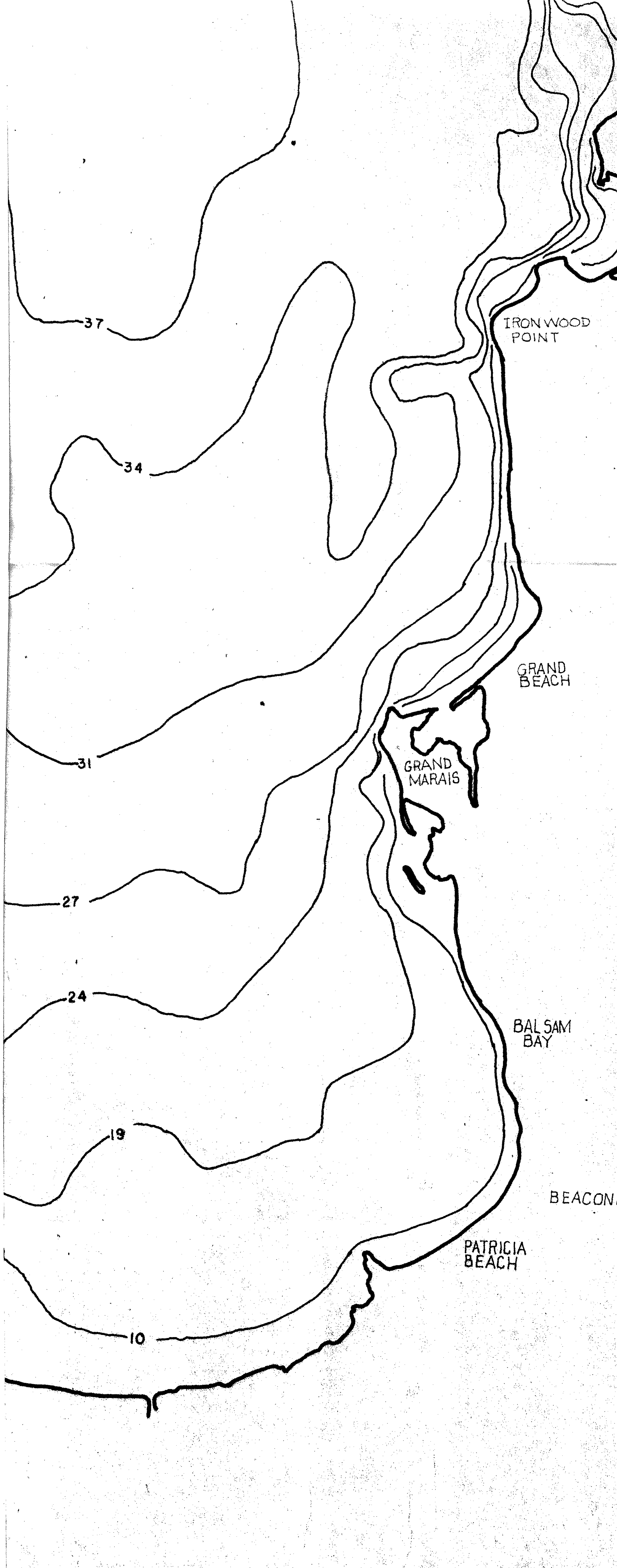


PLATE I

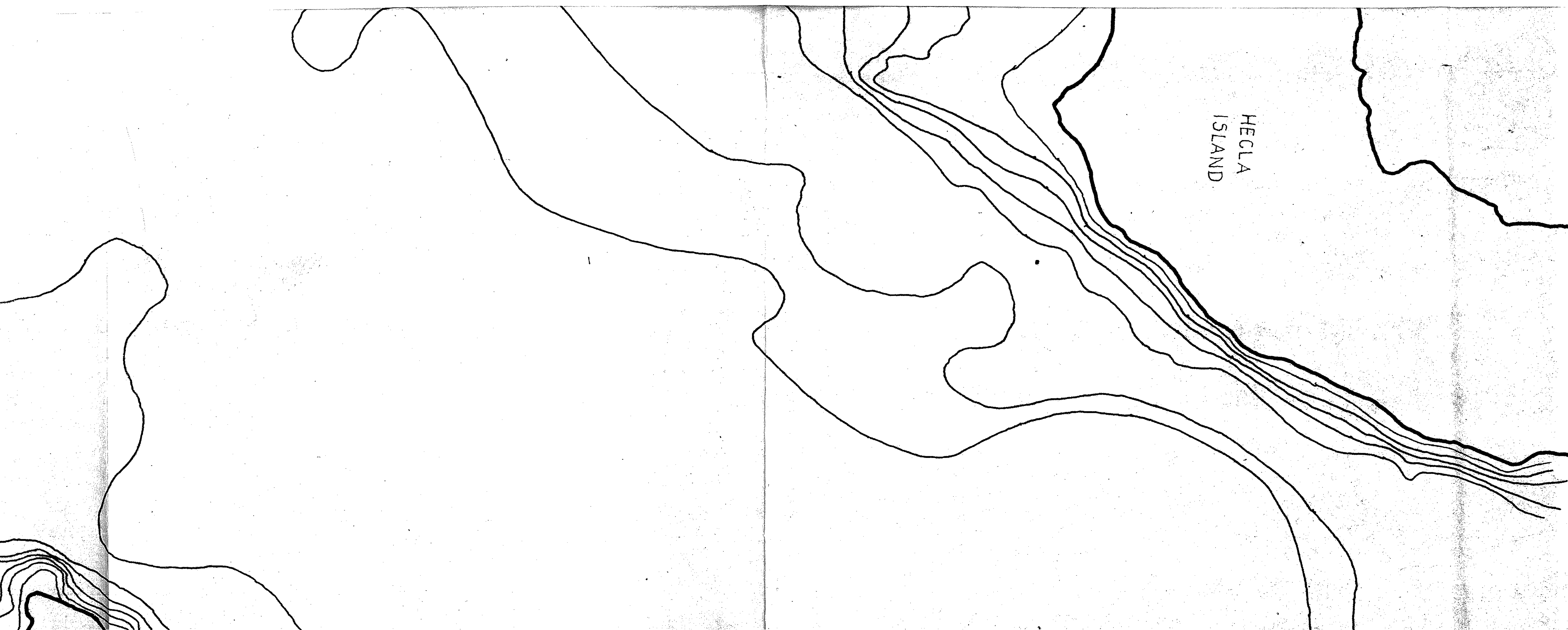
LAKE WINNIPEG TOPOG

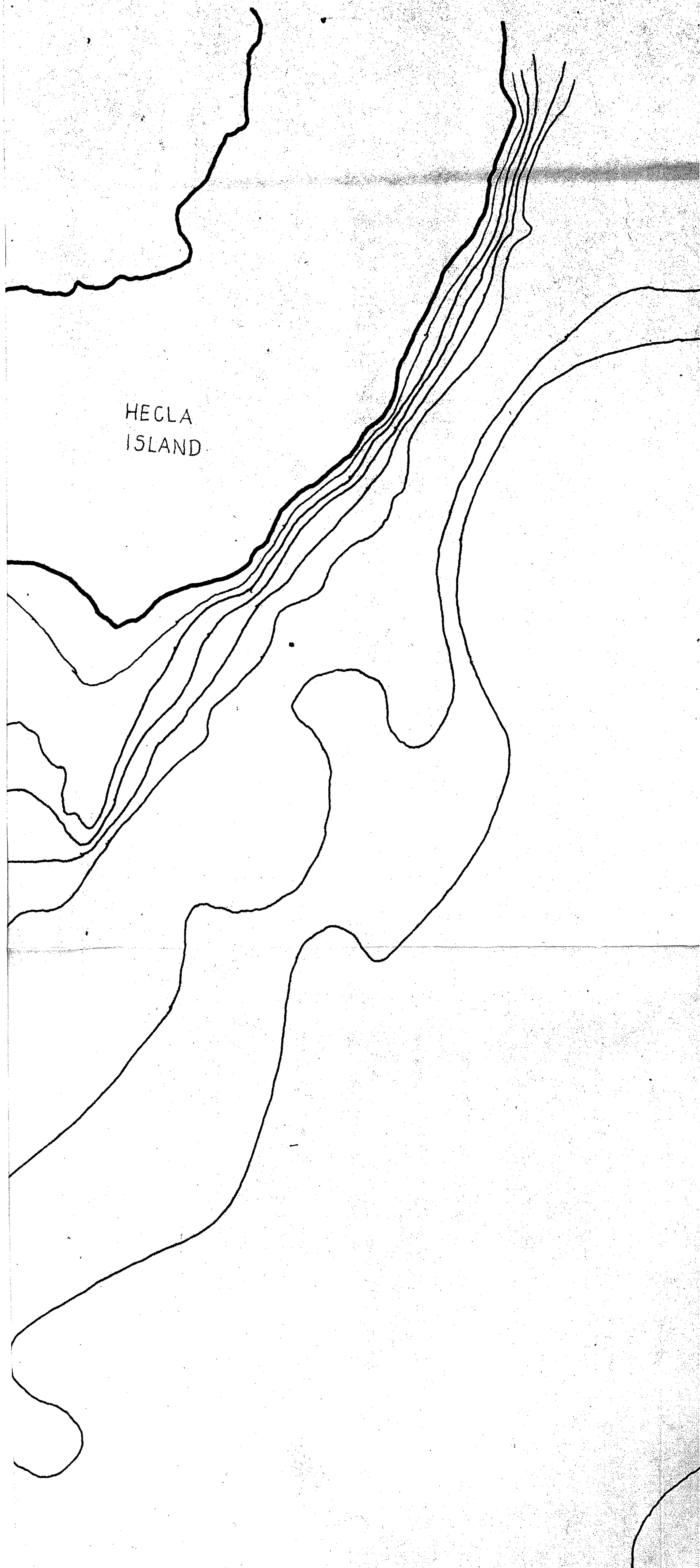
NOTE: Numbers on Contours denote
in feet below Lake Level of

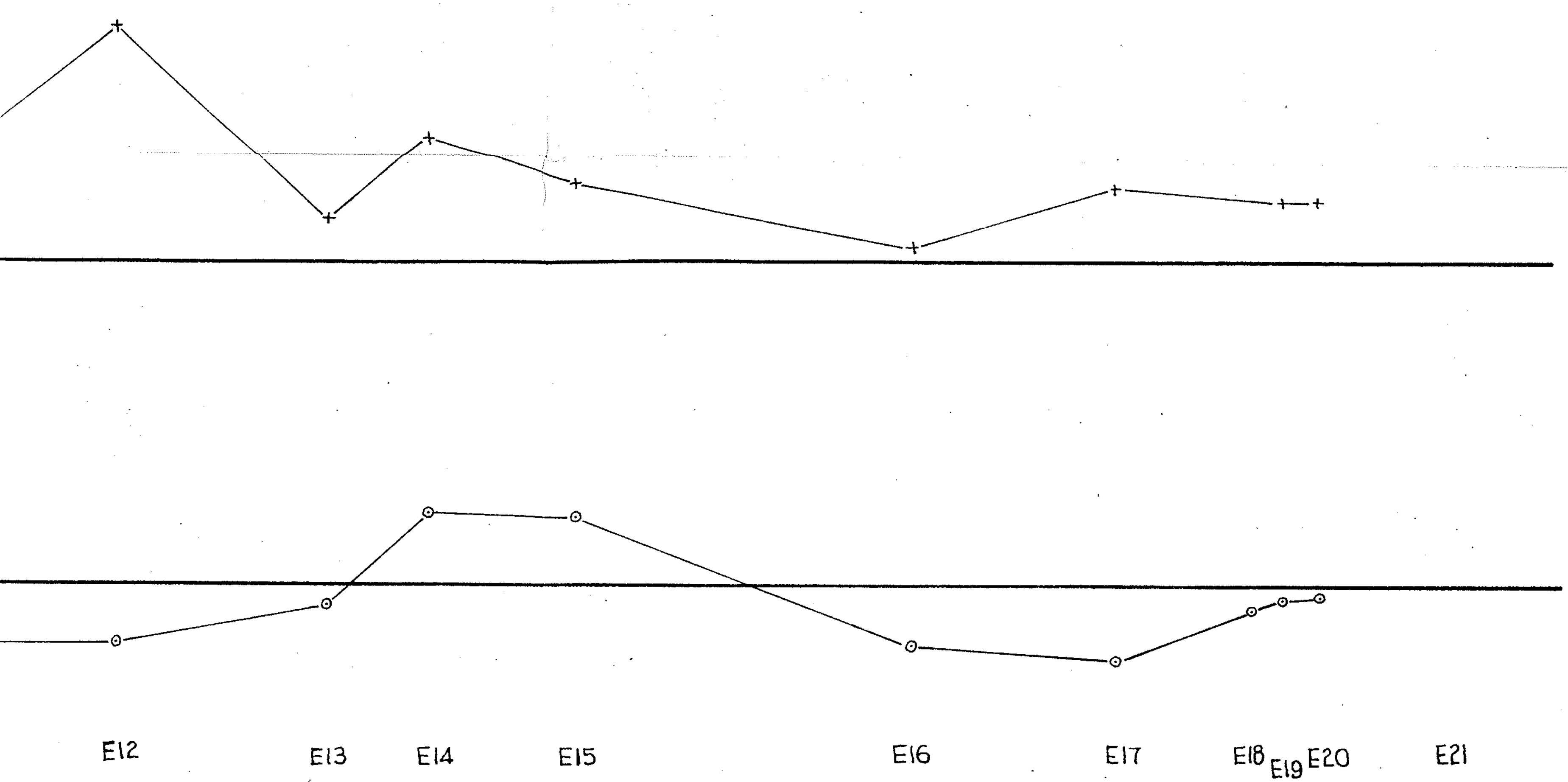
SCALE 1:100.000



IRONWOOD POINT
37
34
GRAND BEACH
31
GRAND MARAIS
27
24
BALSAM BAY
19
BEACON
PATRICIA BEACH
10


HECLA
ISLAND

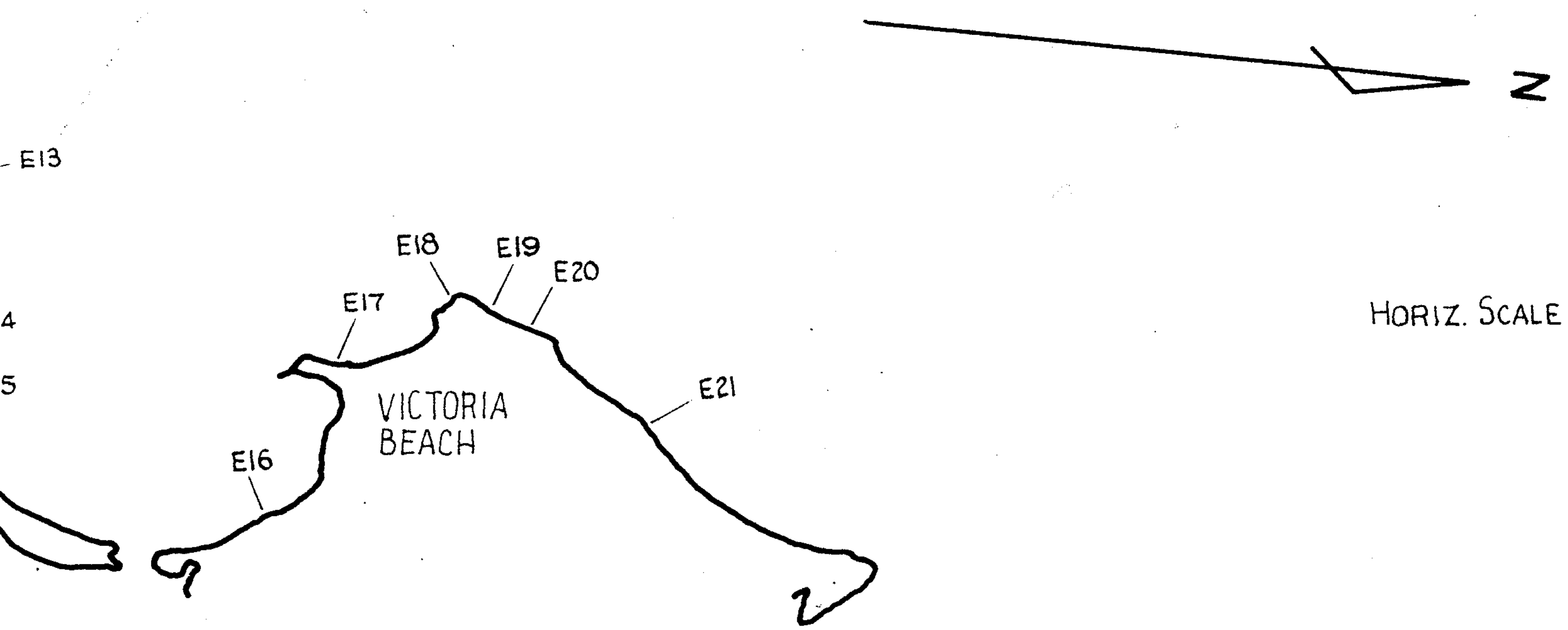

HECLA
ISLAND



IN FIGS. (c) & (d):-

+	Normal Energy Component
⊙	Longshore " "
*	Estimated Value

NOTE
In Figs. (b) & (d), positive value denotes direction is to the right; negative value indicates direction is to the left.



HORIZ. SCALE 2" = 1.58 mi
(1:50,000)

UNIVERSITY OF MANITOBA

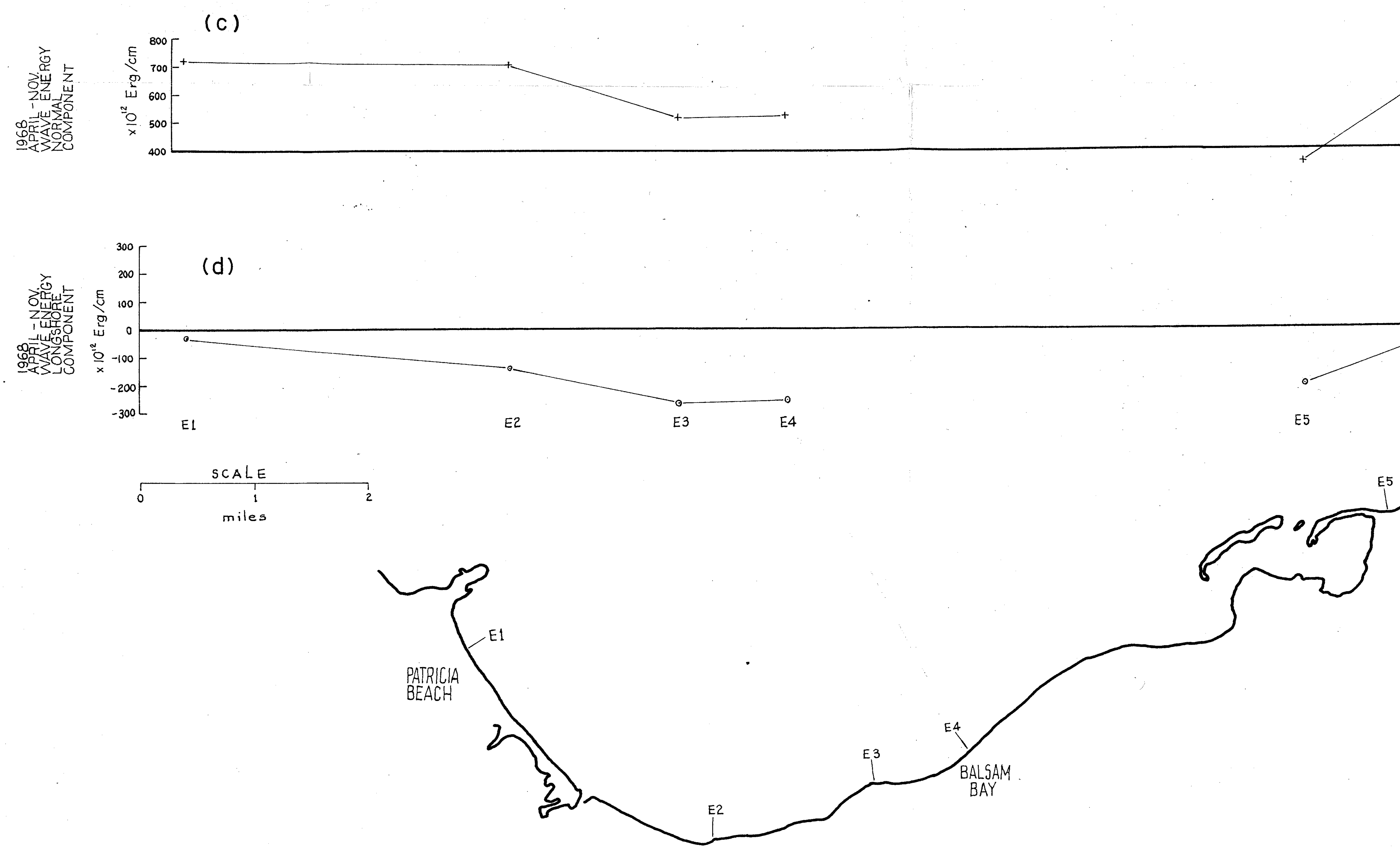
LAKE WINNIPEG SHORE EROSION STUDY

PLATE IV EAST COAST

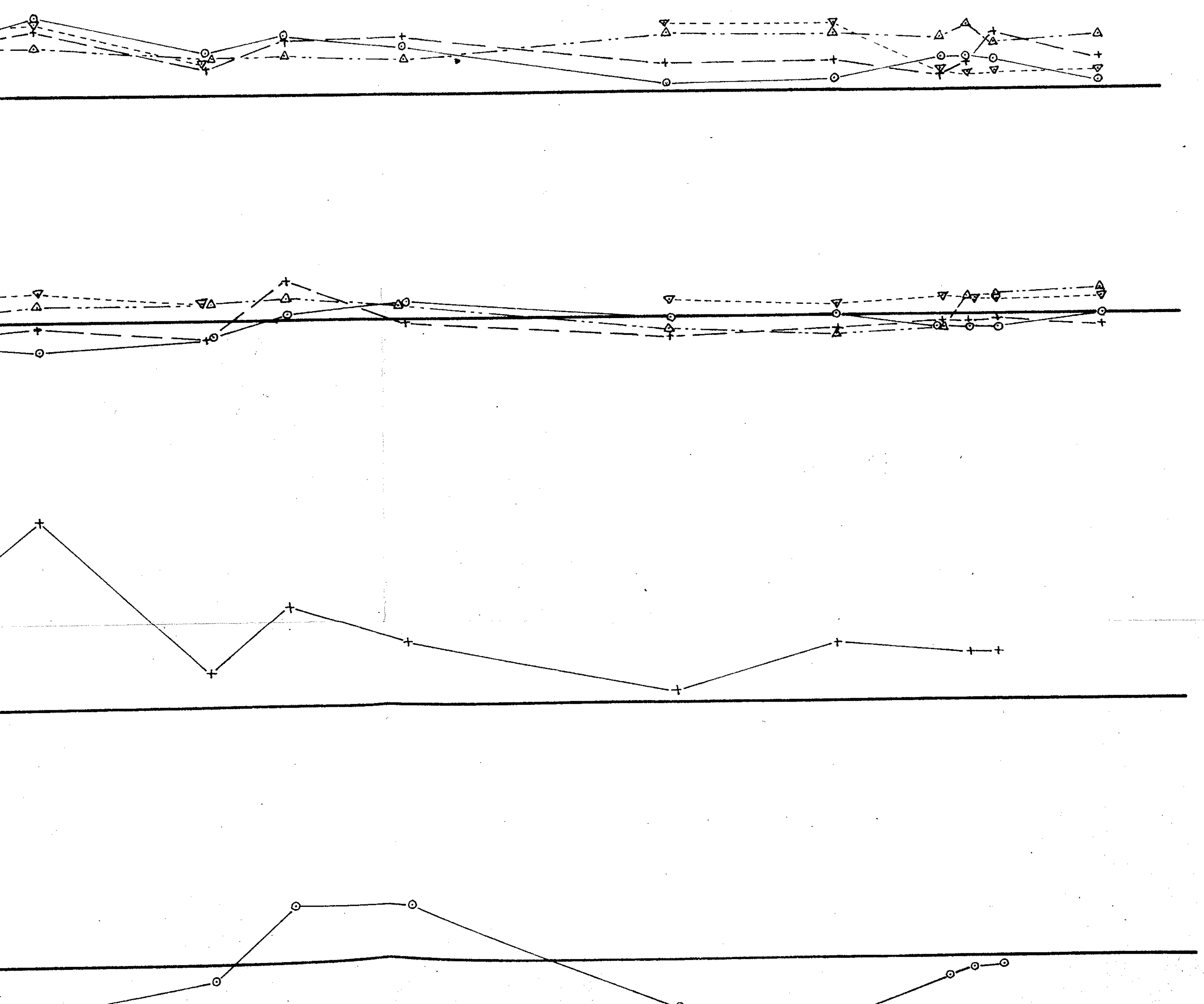
CALCULATED BY: wkc
DRAWN BY: wkc
DATE: March, 1971

(a) Wave Power, Normal Component.
(b) Wave Power, Longshore "
(c) Wave Energy, Normal "
(d) Wave Energy, Longshore "

E5
E6
E7
E8
E9
E10
E11
GRAND MARAIS
IRONWOOD POINT
BALSAM BAY
E12
E14
E15



1968
APRIL-NOV.
WAVE ENERGY
NORMAL
COMPONENT
(c)
x 10^{12} Erg/cm
800
700
600
500
400
1968
APRIL-NOV.
WAVE ENERGY
LONGSHORE
COMPONENT
(d)
x 10^{12} Erg/cm
300
200
100
0
-100
-200
-300
E1
E2
E3
E4
E5
SCALE
0
1
2
miles
PATRICIA
BEACH
BALSAM
BAY



LEGEND and REMARKS

IN FIGS. (a) & (b):–

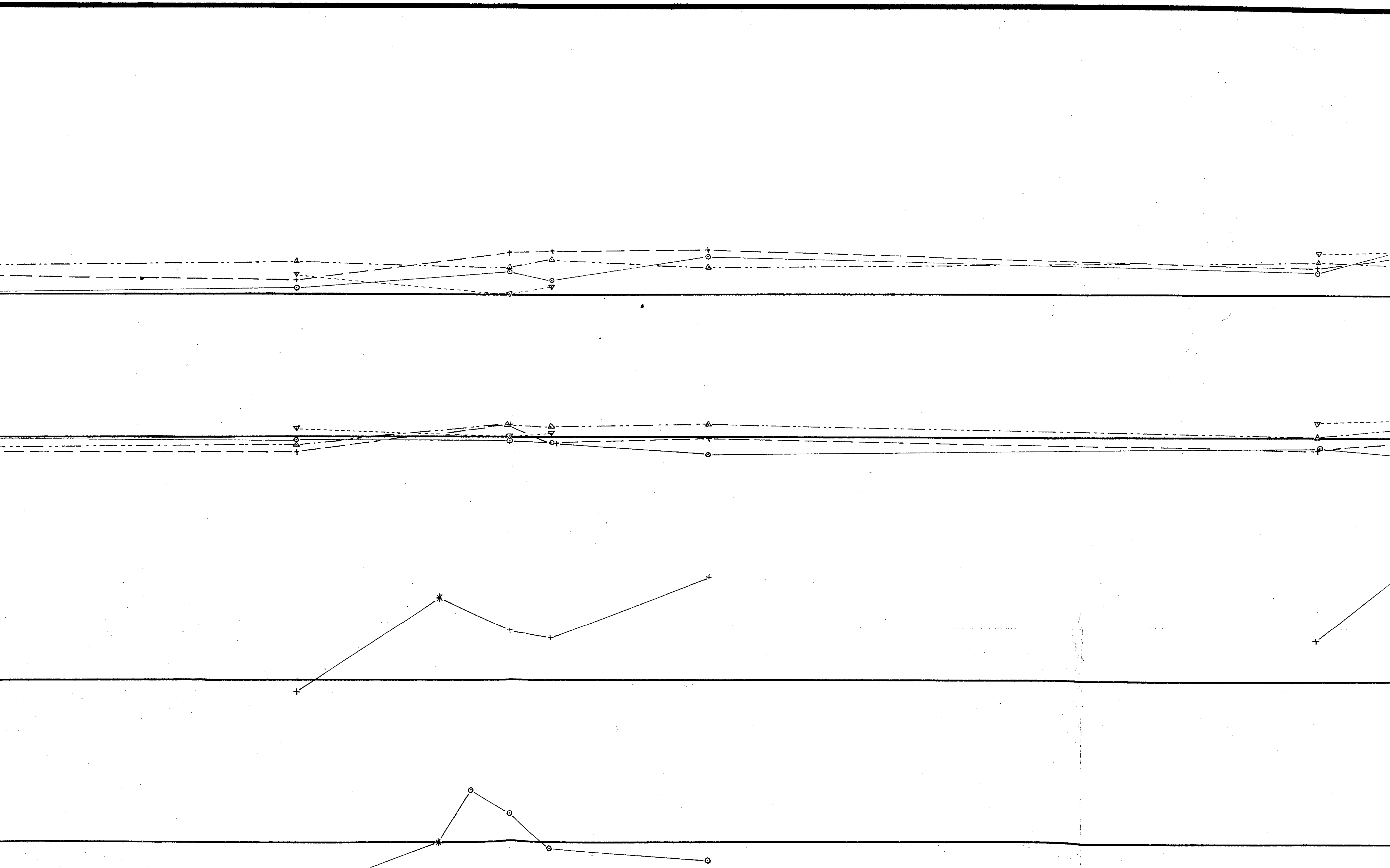
△	△	W wind
▽	▽	SE "
⊙	⊙	N "
+	+	NW "

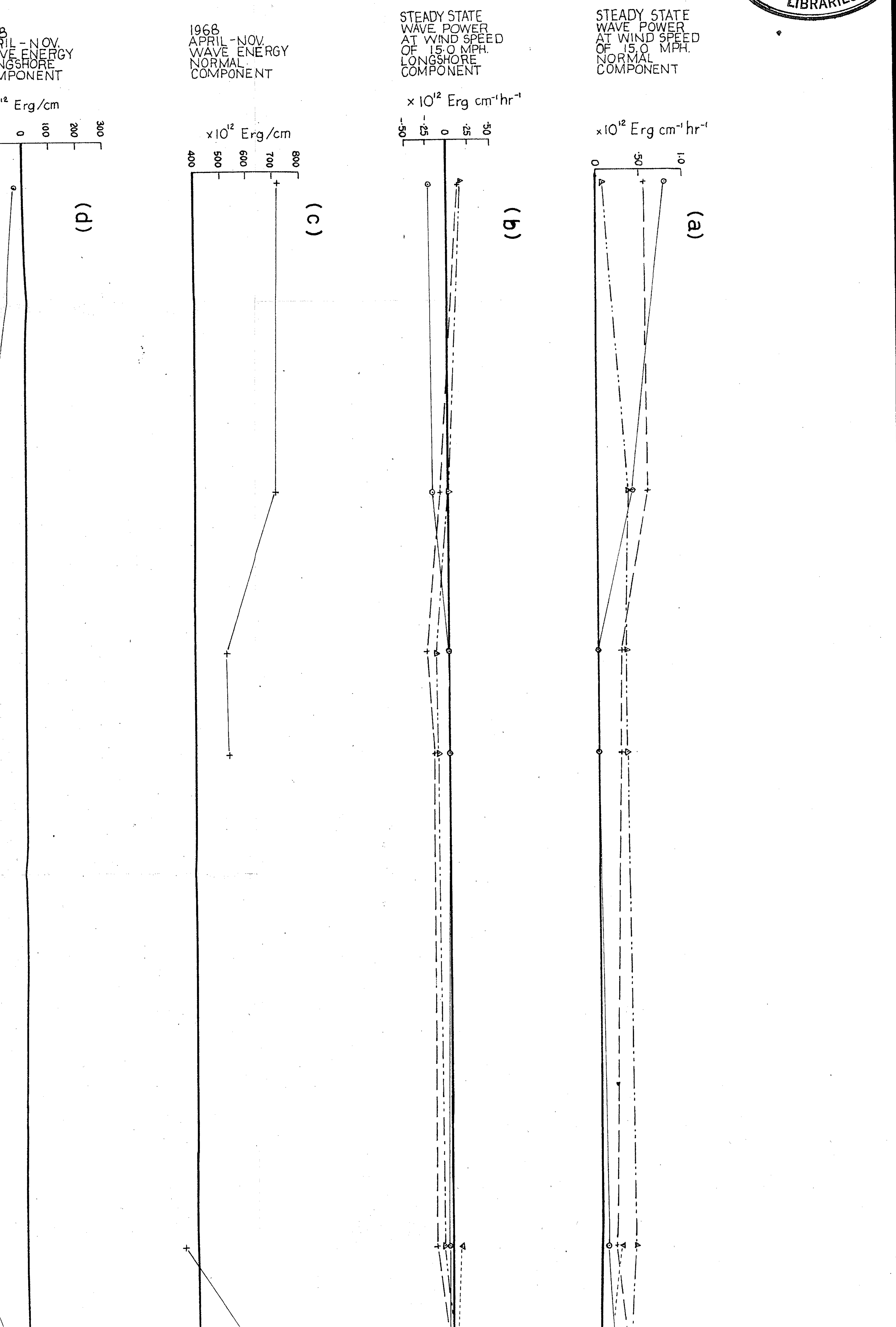
IN FIGS. (c) & (d):–

+	Normal Energy Component
⊙	Longshore " "
*	Estimated Value

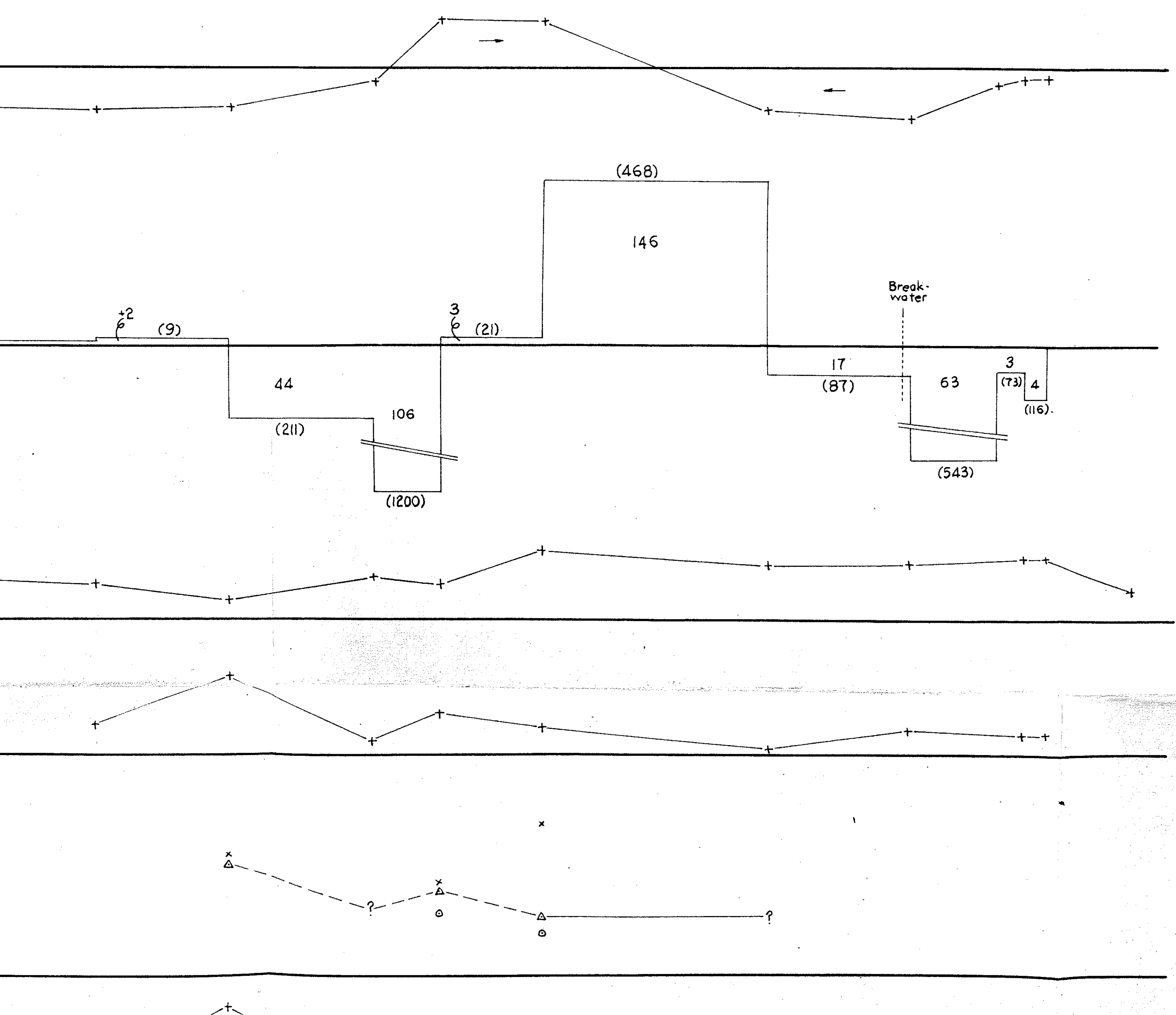
NOTE

In Figs. (b) & (d), positive value denotes direction is to the right; negative value indicates direction is to the left.





STEADY STATE WAVE POWER AT WIND SPEED OF 15.0 MPH. NORMAL COMPONENT
x10^12 Erg cm⁻¹ hr⁻¹
0
.50
1.0
(a)
STEADY STATE WAVE POWER AT WIND SPEED OF 15.0 MPH. LONGSHORE COMPONENT
x 10^12 Erg cm⁻¹ hr⁻¹
-.50
-.25
0
.25
.50
(b)
1968 APRIL-NOV. WAVE ENERGY NORMAL COMPONENT
x10^12 Erg/cm
400
500
600
700
800
(c)
Erg/cm
0
100
200
300
(d)



LEGEND and REMARKS

+ Computed value

? / ___ } Value not computed; for qualitative indication only. (— not to scale.)

? Value is not computed; for qualitative indication only.

Notes: (i) Number not in brackets indicates volume of erosion/accretion represented by rectangular block — $\times 10^3$ cu. yd.

(ii) Number in brackets indicates extent of erosion/accretion in cubic feet per foot beach front.

+ Computed Value

? / ___ } Value not computed; estimate shown for qualitative comparison only. (— not to scale.)

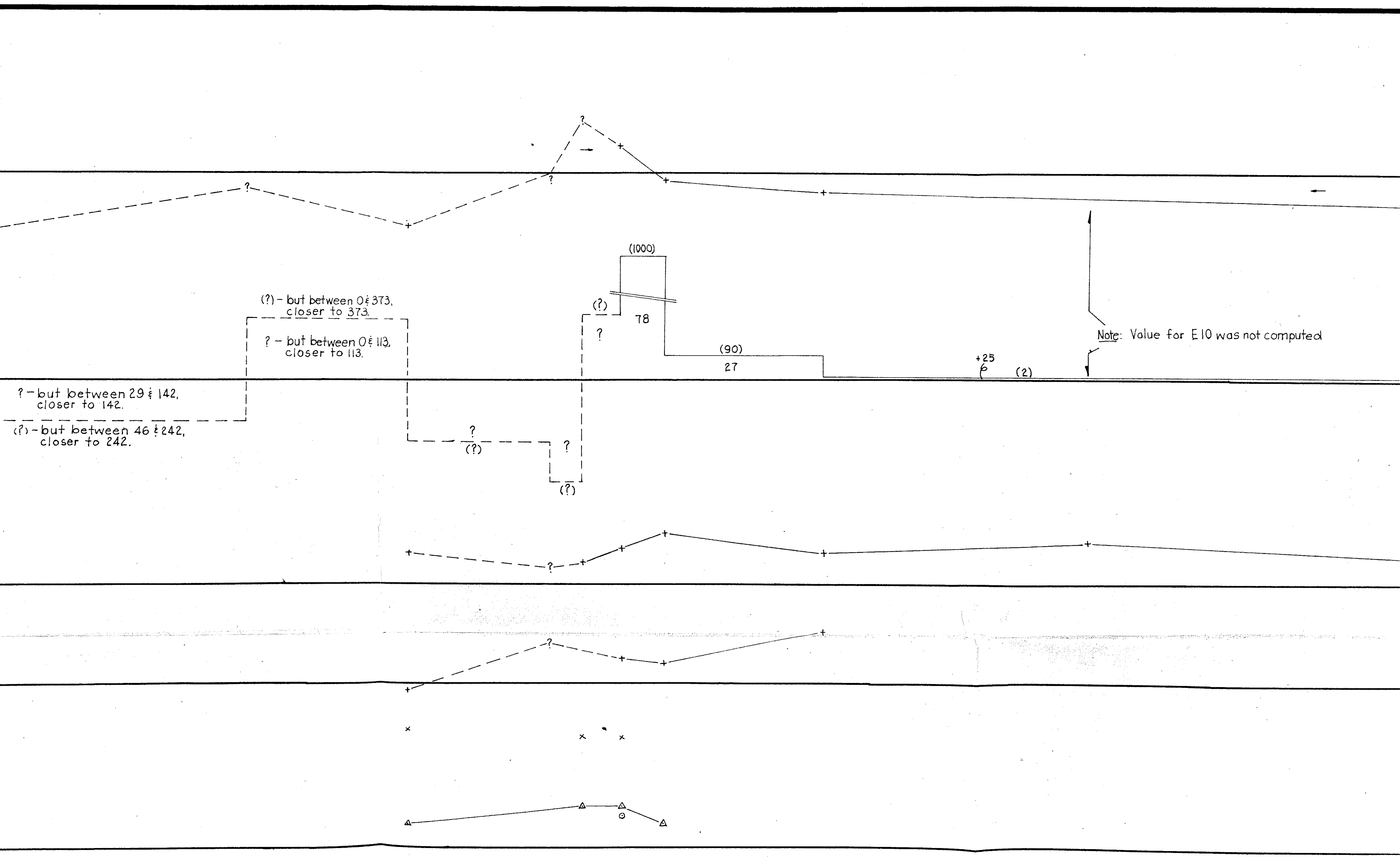
Ditto above.

× Coarse fraction

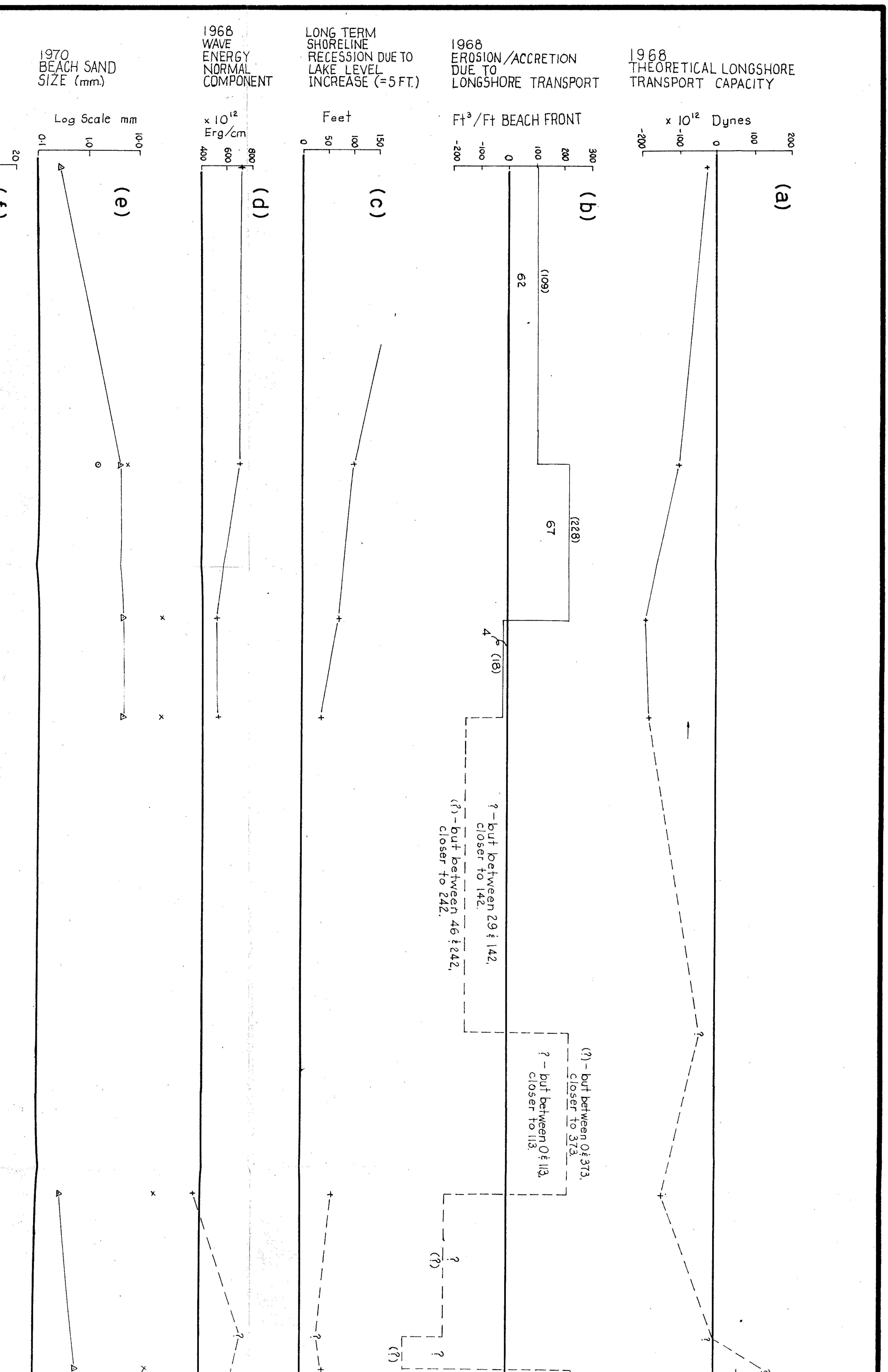
⊙ Fine fraction

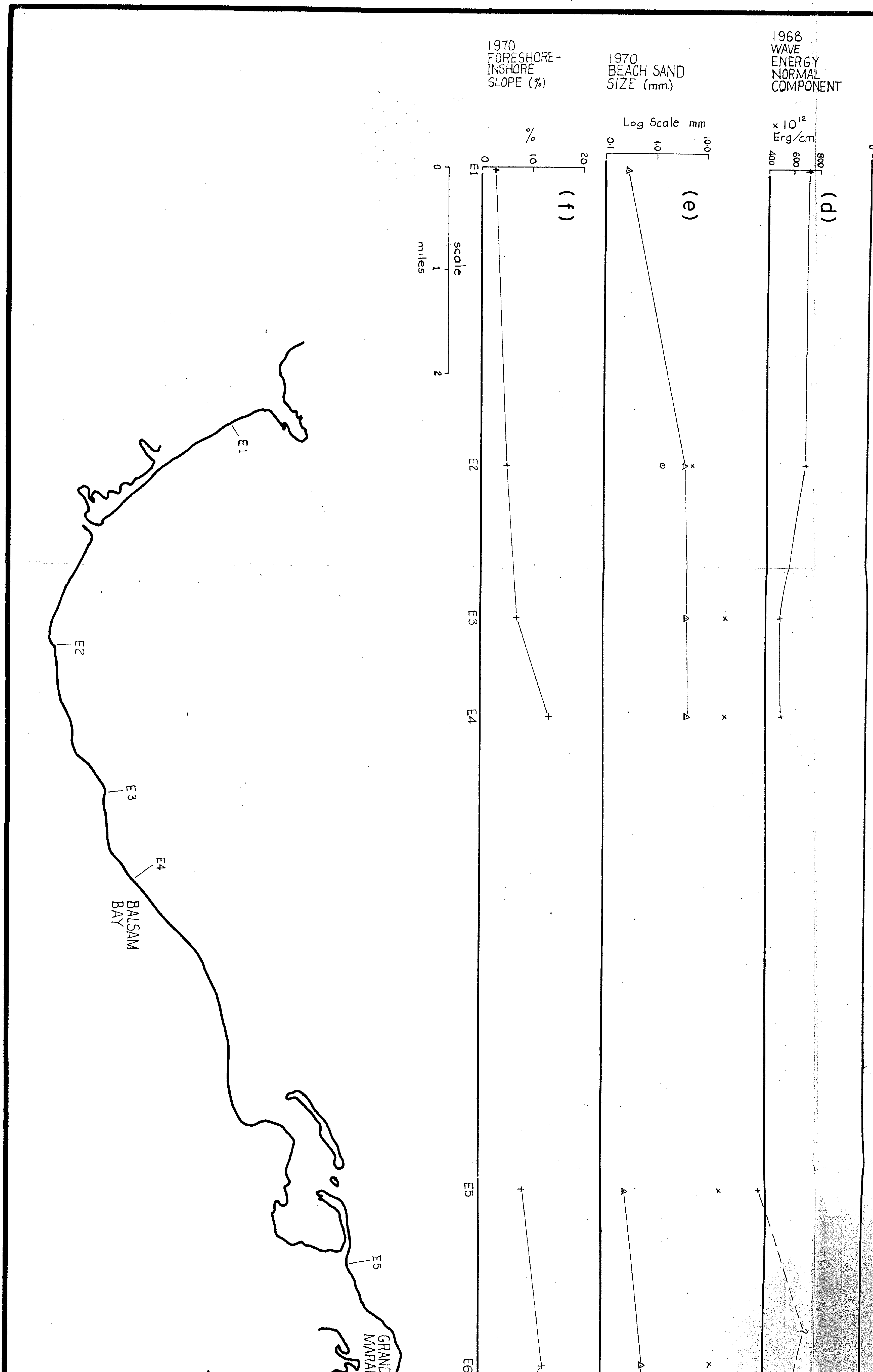
△ Mean Value

(Note: From survey, summer 1970.)

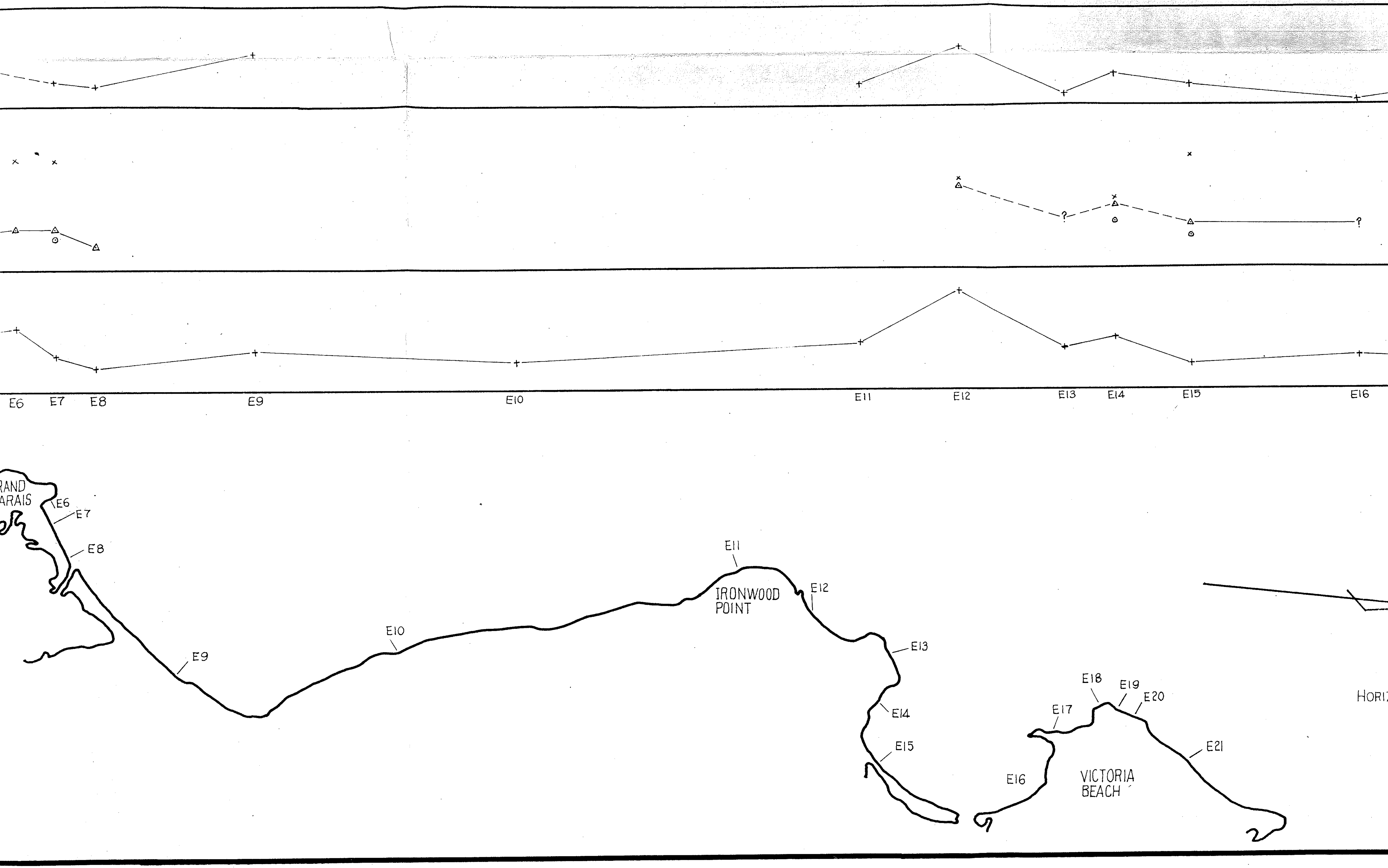


(1000)
(?) - but between 0 & 373, closer to 373.
? - but between 0 & 113, closer to 113.
(?)
78
?
(90)
27
+25
(2)
Note: Value for E10 was not computed
? - but between 29 & 142, closer to 142.
(?) - but between 46 & 242, closer to 242.
?
(?)
?
(?)

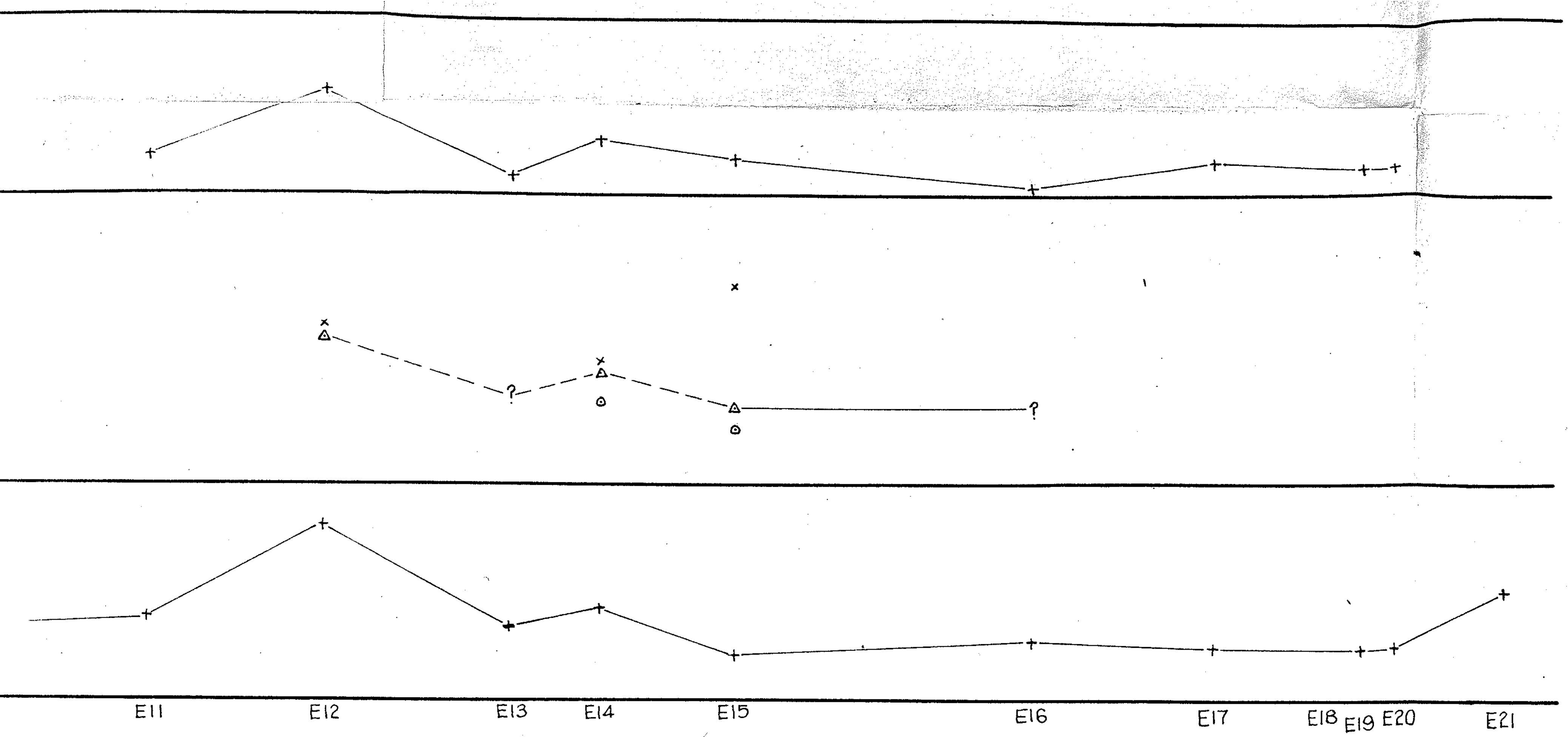

1970
BEACH SAND
SIZE (mm.)
Log Scale mm
(e)
1968
WAVE
ENERGY
NORMAL
COMPONENT
x 10^12
Erg/cm
400
600
800
(d)
LONG TERM
SHORELINE
RECESSION DUE TO
LAKE LEVEL
INCREASE (=5 FT.)
Feet
0
50
100
150
(c)
1968
EROSION/ACCRETION
DUE TO
LONGSHORE TRANSPORT
Ft³/Ft BEACH FRONT
-200
-100
0
100
200
300
(b)
(109)
62
(228)
67
4
(18)
? - but between 29 & 142,
closer to 142.
(?) - but between 46 & 242,
closer to 242.
(?) - but between 0 & 373,
closer to 373.
? - but between 0 & 113,
closer to 113.
1968
THEORETICAL LONGSHORE
TRANSPORT CAPACITY
x 10^12 Dynes
-200
-100
0
100
200
(a)



1970 FORESHORE-INSHORE SLOPE (%)
1970 BEACH SAND SIZE (mm.)
1968 WAVE ENERGY NORMAL COMPONENT
Log Scale mm
x 10^12 Erg/cm
%
0
10
20
0·1
1·0
10·0
400
600
800
(f)
(e)
(d)
E1
E2
E3
E4
E5
E6
scale
miles
0
1
2
BALSAM BAY
GRAND MARAI



E6
E7
E8
E9
E10
E11
E12
E13
E14
E15
E16
RAND
ARAIS
IRONWOOD
POINT
E17
E18
E19
E20
E21
VICTORIA
BEACH
HORI



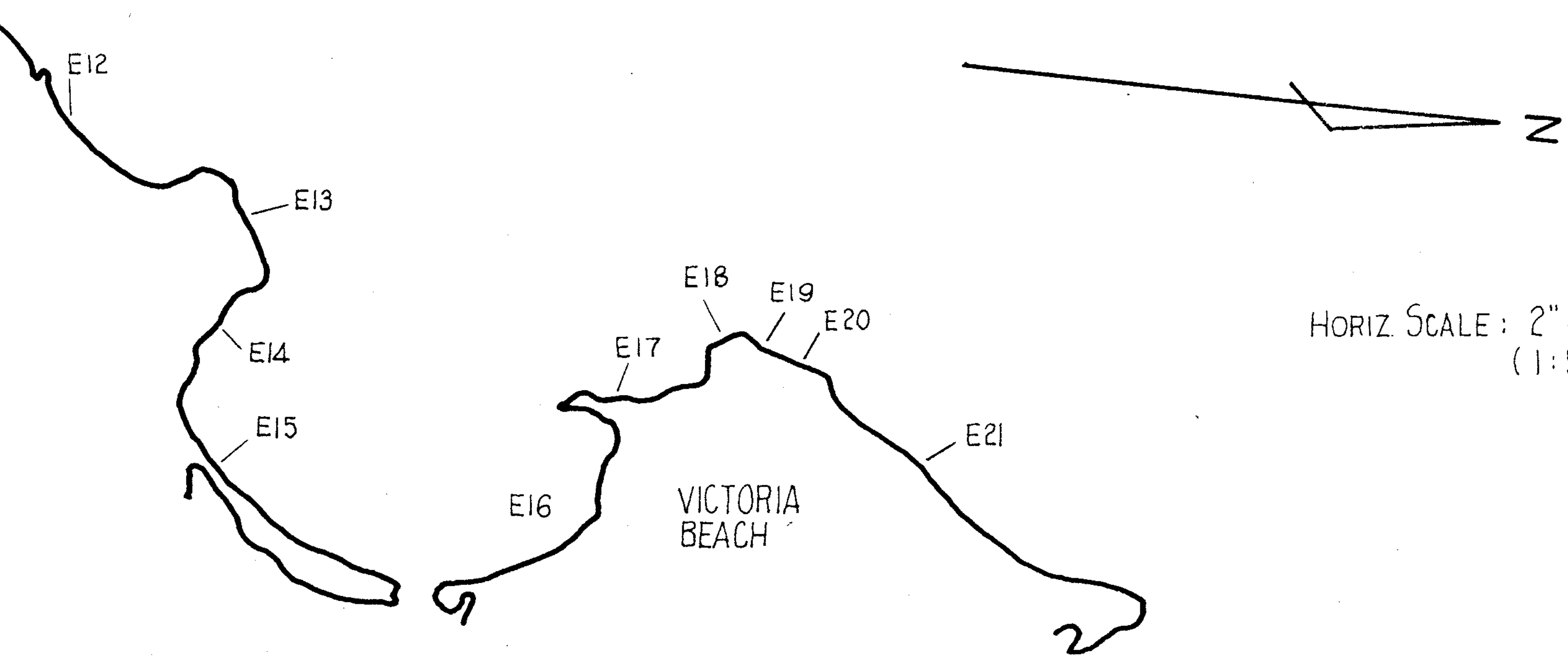
--- for qualitative comparison only.
(— not to scale.)

Ditto above.

× Coarse fraction
⊙ Fine fraction
△ Mean Value

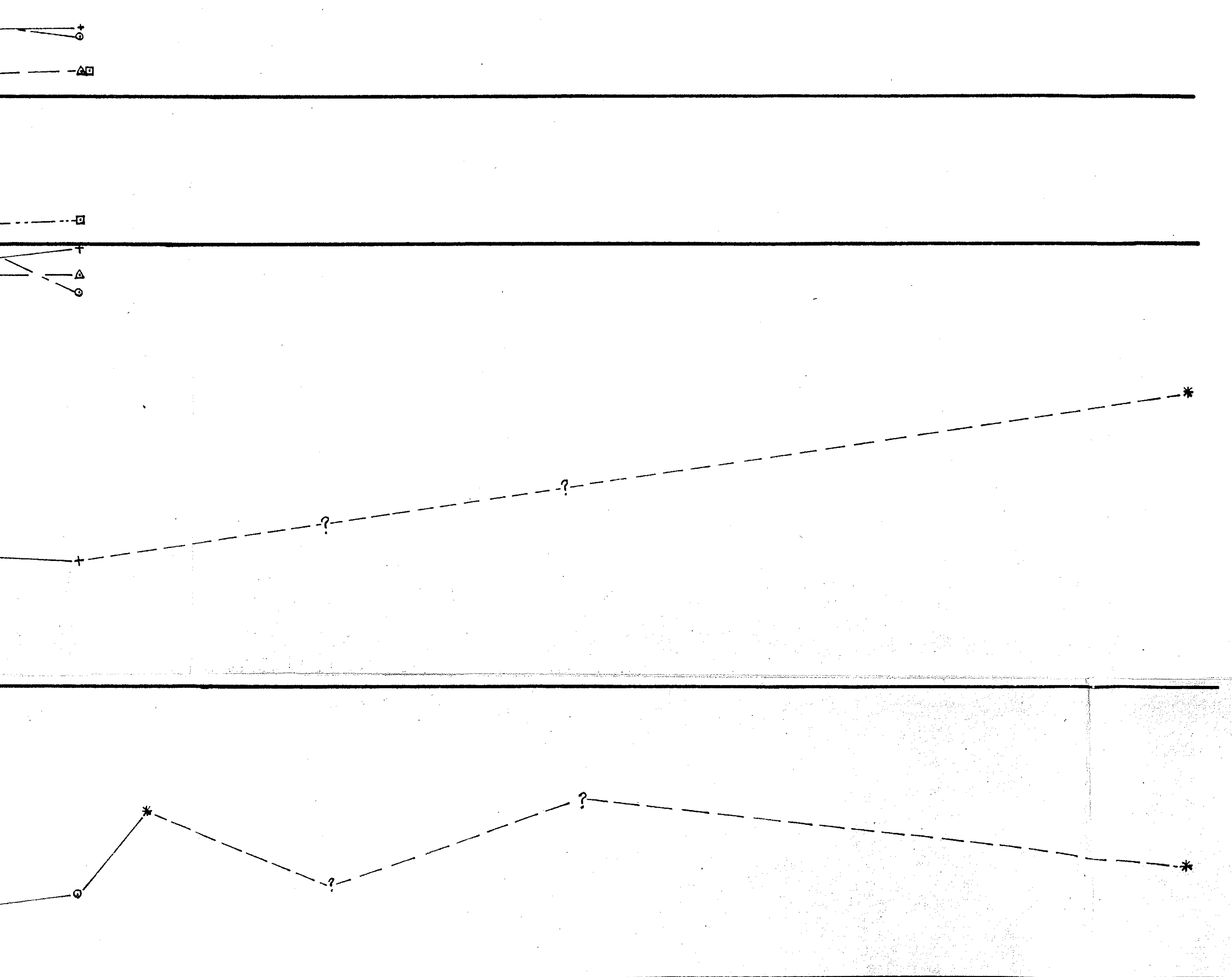
(Note: From survey, summer 1970.)

\+ Value obtained from survey, summer 1970.



HORIZ. SCALE: 2" = 1.58 mi
(1:50,000)

UNIVERSITY OF MANITOBA	
LAKE WINNIPEG EROSION STUDY	
PLATE V	EAST COAST
CALCULATED BY: wkc	(a) Longshore Transport (b) Erosion & Accretion (c) Shoreline Recession
DRAWN BY: wkc	(d) Wave Energy (e) Beach Sand Size
DATE: March 1971	(f) Beach Slope



LEGEND and REMARKS

IN FIGS. (a) & (b):

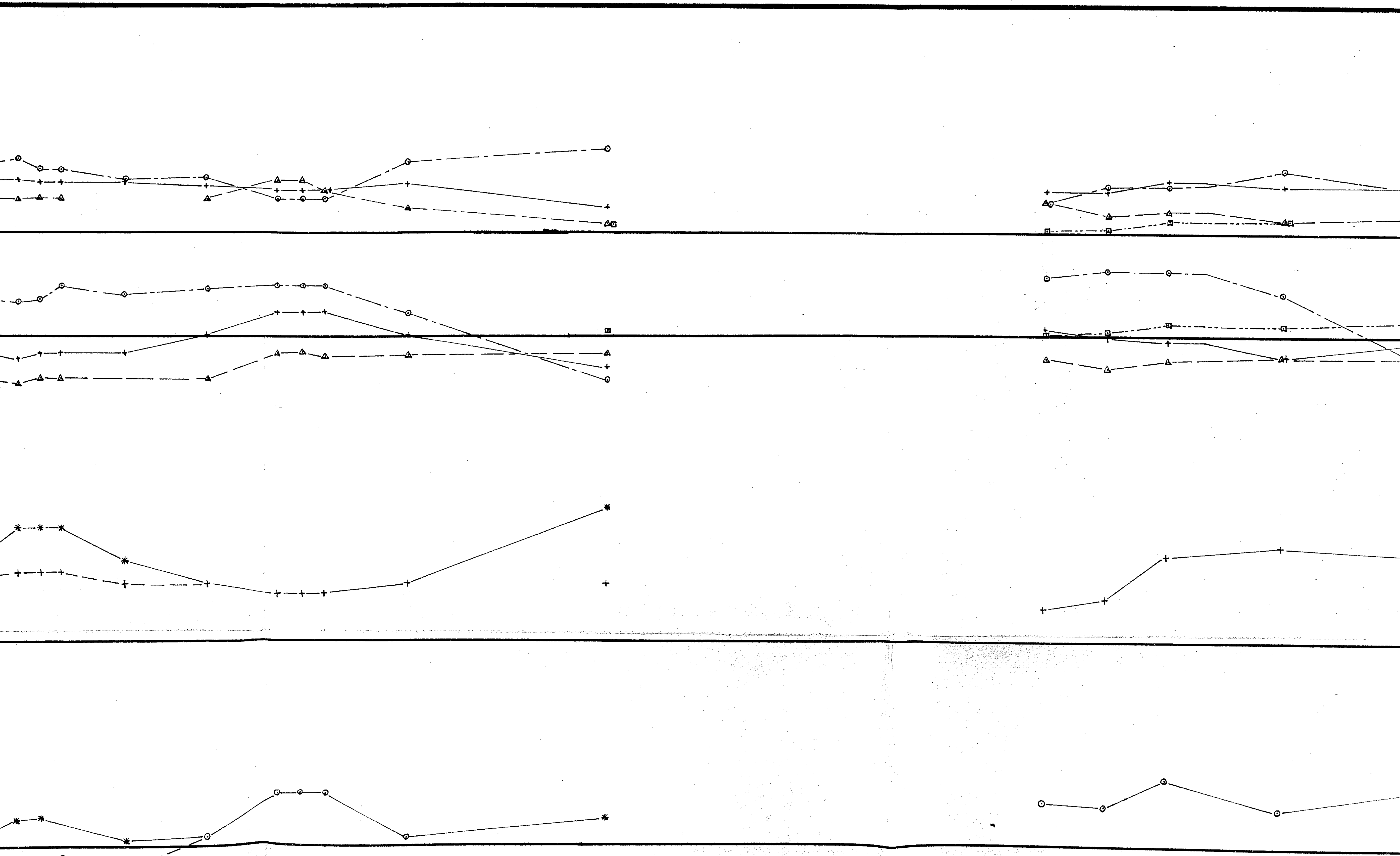
SYMBOL	WIND DIRECTION
▽- - - - - - - - -▽	S
△— — — — —△	SE
+————————+	E
⊙———— - ———⊙	N
⊡ - - ———- · ———- -⊡	NE

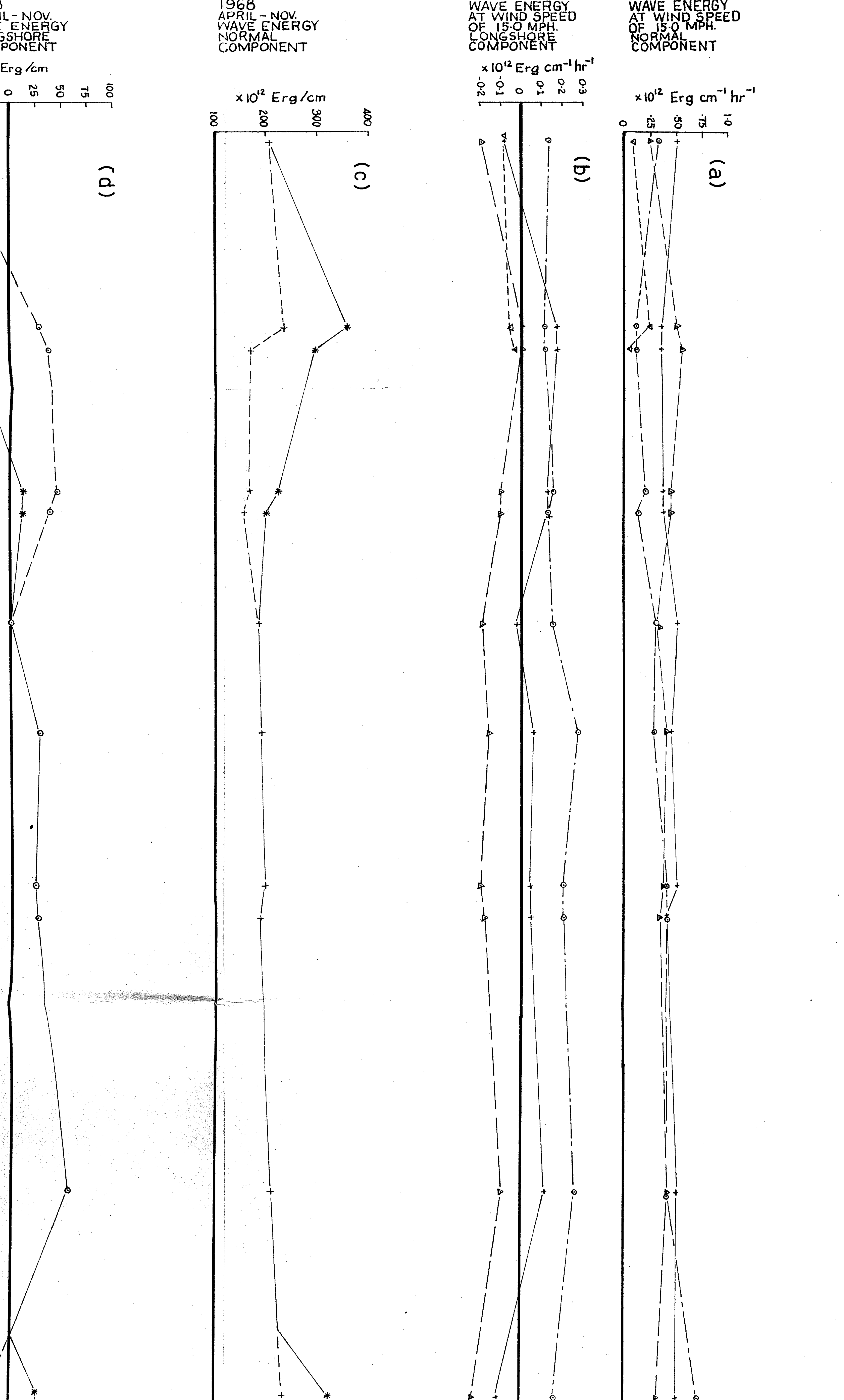
IN FIGS. (c) & (d) :

+	Normal energy component, computed.
⊙	Longshore energy component, computed.
*	Estimated value.
?	Value shown is for qualitative indication only — not calculated.

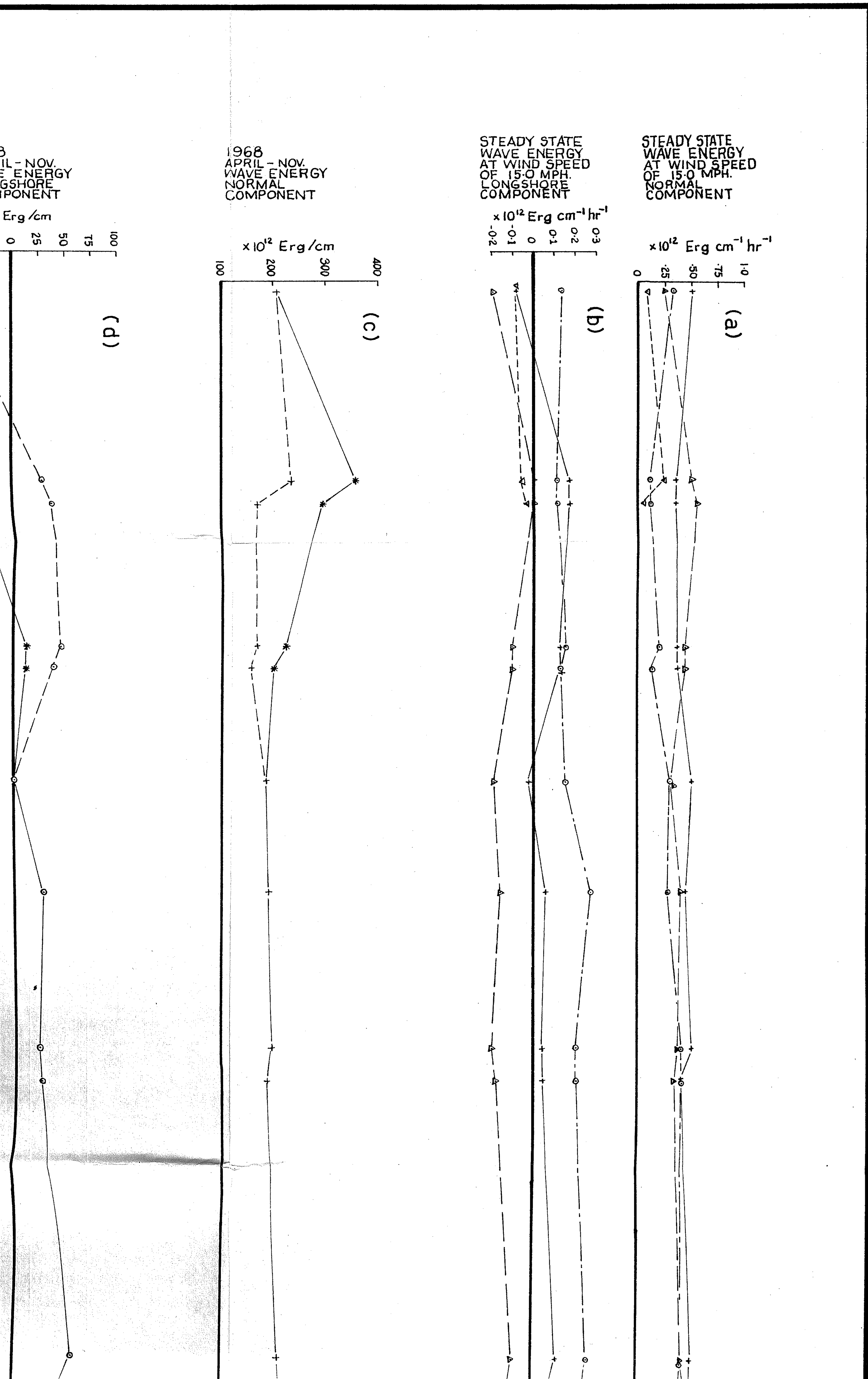
NOTE :

In Figs. (b) & (d), positive value denotes direction towards right; negative value iddicates direction towards left.

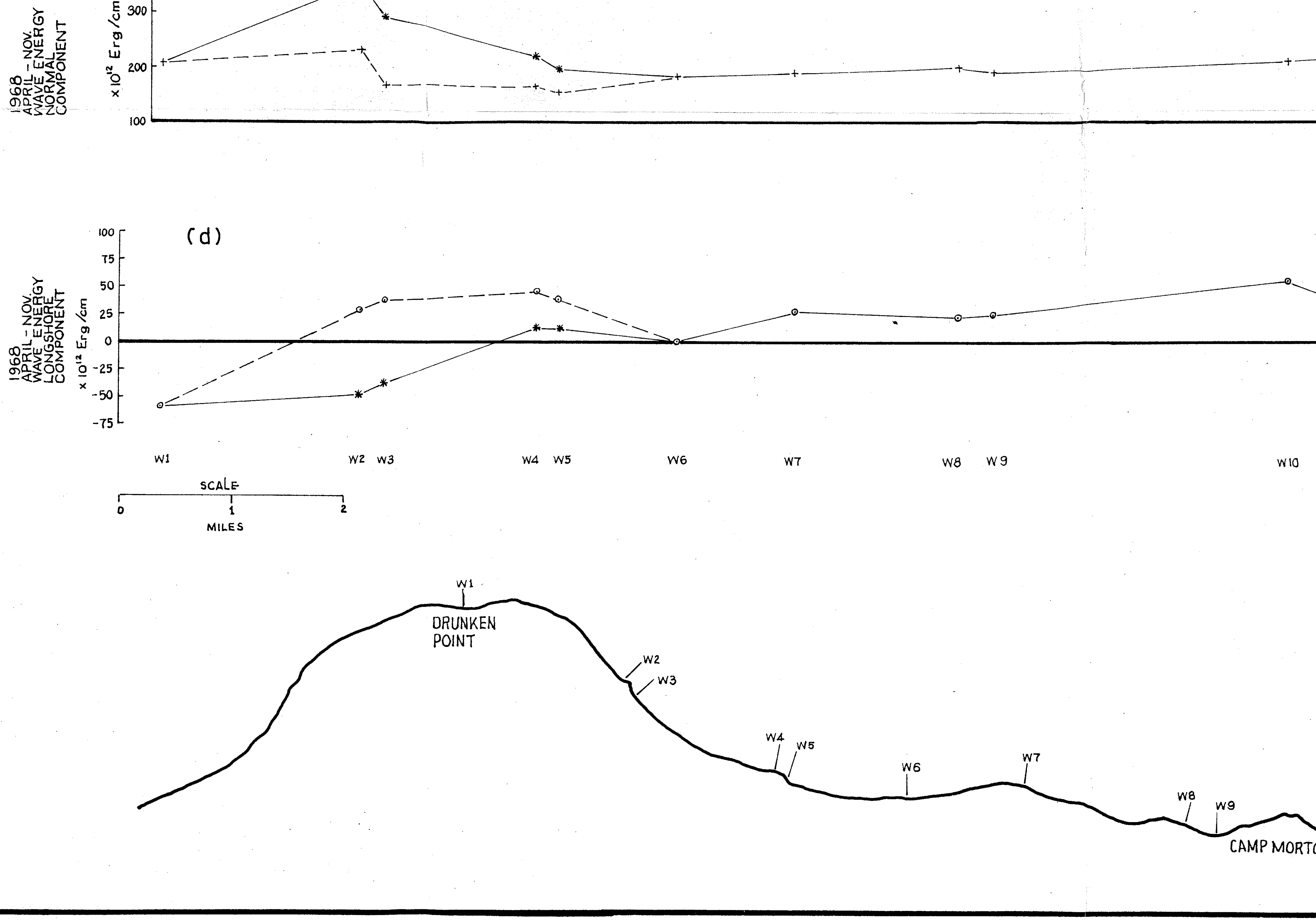




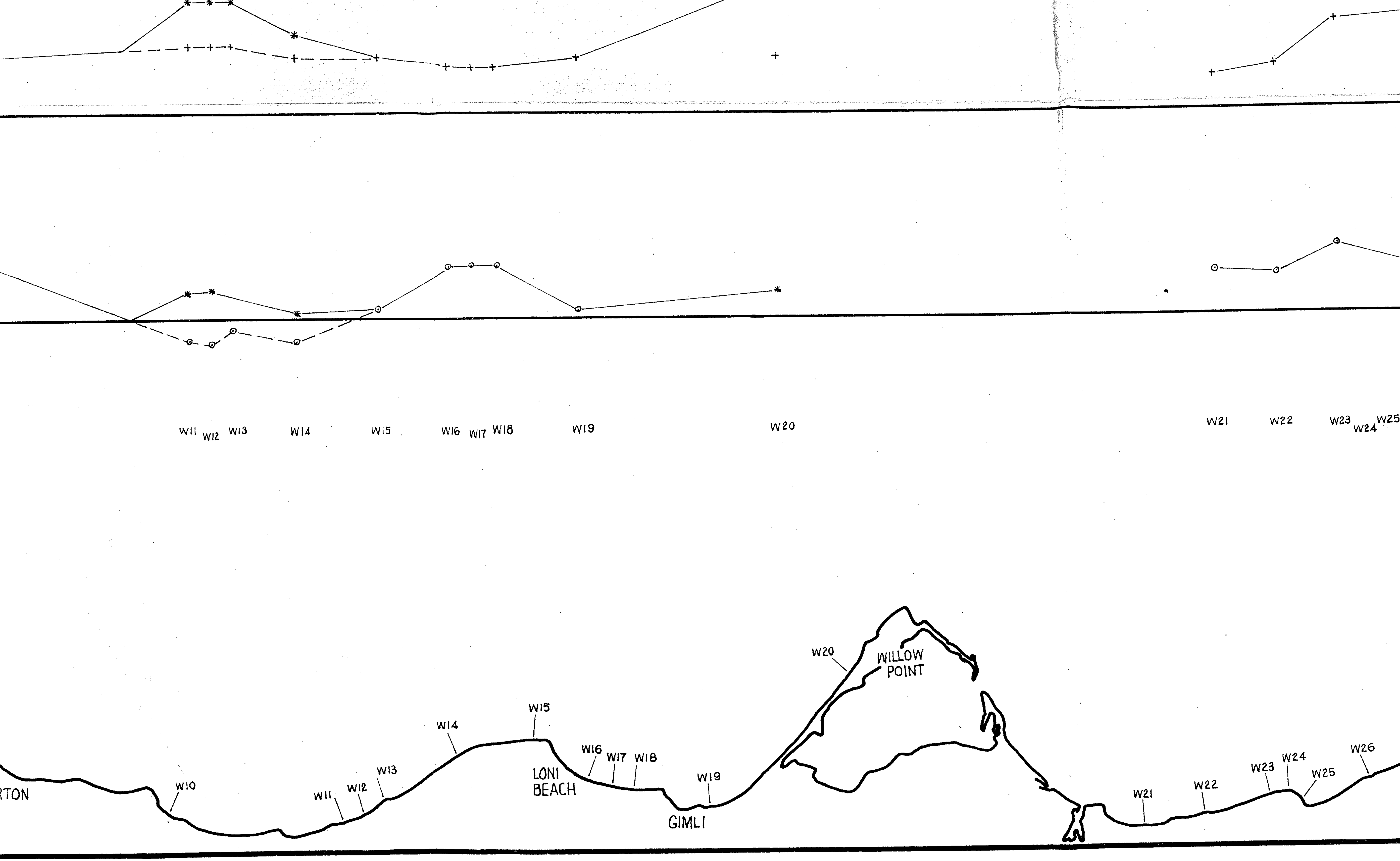
IL - NOV.
ENERGY
GSHORE
PONENT
Erg /cm
0 25 50 75 100
(d)
1968
APRIL - NOV.
WAVE ENERGY
NORMAL
COMPONENT
x10^12 Erg /cm
100 200 300 400
(c)
WAVE ENERGY
AT WIND SPEED
OF 15·0 MPH.
LONGSHORE
COMPONENT
x10^12 Erg cm^-1 hr^-1
-0·2 -0·1 0 0·1 0·2 0·3
(b)
WAVE ENERGY
AT WIND SPEED
OF 15·0 MPH.
NORMAL
COMPONENT
x10^12 Erg cm^-1 hr^-1
0 ·25 ·50 ·75 1·0
(a)



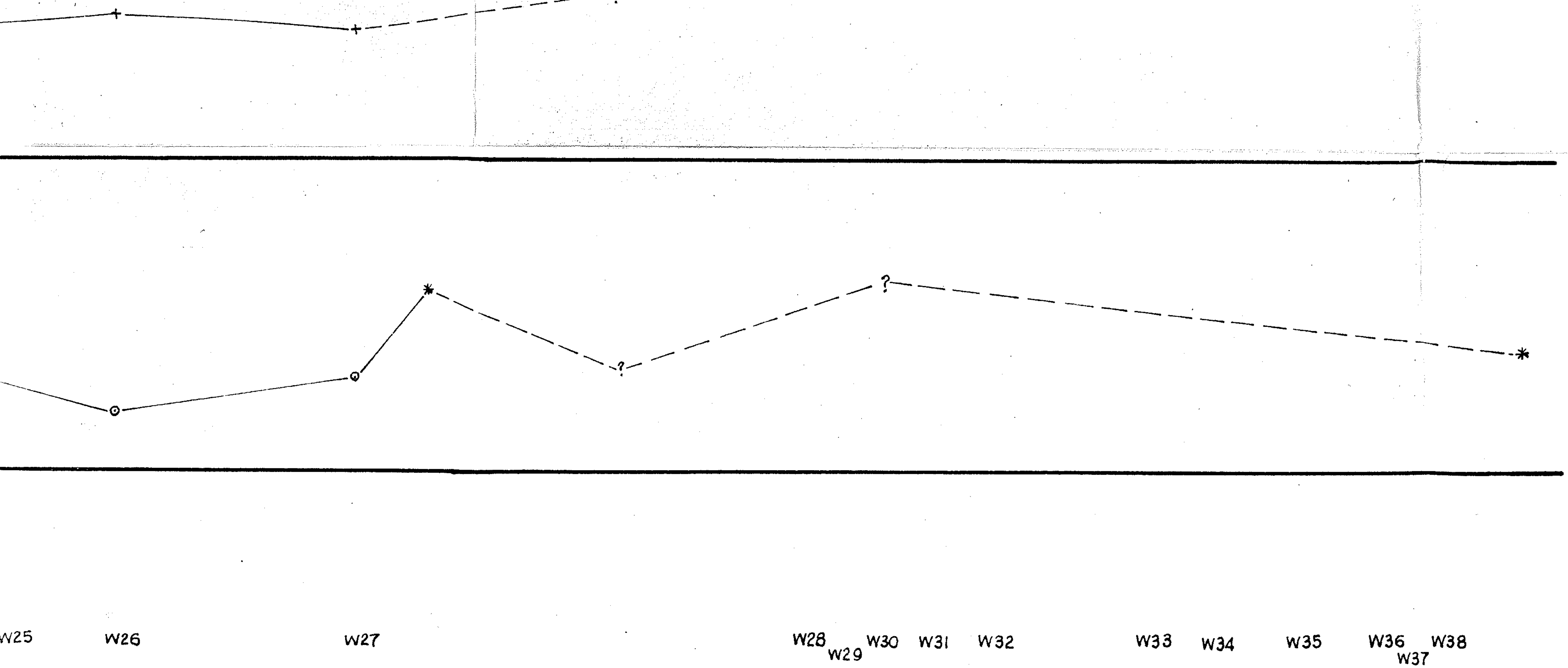
STEADY STATE WAVE ENERGY AT WIND SPEED OF 15·0 MPH. NORMAL COMPONENT
x10¹² Erg cm⁻¹ hr⁻¹
0
·25
·50
·75
1·0
(a)
STEADY STATE WAVE ENERGY AT WIND SPEED OF 15·0 MPH. LONGSHORE COMPONENT
x 10¹² Erg cm⁻¹ hr⁻¹
-0·2
-0·1
0
0·1
0·2
0·3
(b)
1968 APRIL – NOV. WAVE ENERGY NORMAL COMPONENT
x10¹² Erg/cm
100
200
300
400
(c)
IL – NOV.
E ENERGY
GSHORE
PONENT
Erg/cm
0
25
50
75
100
(d)



1968
APRIL - NOV.
WAVE ENERGY
NORMAL
COMPONENT
x 10¹² Erg/cm
300
200
100
(d)
1968
APRIL - NOV.
WAVE ENERGY
LONGSHORE
COMPONENT
x 10¹² Erg/cm
100
75
50
25
0
-25
-50
-75
W1
W2
W3
W4
W5
W6
W7
W8
W9
W10
SCALE
0
1
2
MILES
DRUNKEN
POINT
CAMP MORTO



W11
W12
W13
W14
W15
W16
W17
W18
W19
W20
W21
W22
W23
W24
W25
RTON
W10
W11
W12
W13
W14
W15
LONI
BEACH
W16
W17
W18
W19
GIMLI
W20
WILLOW
POINT
W21
W22
W23
W24
W25
W26

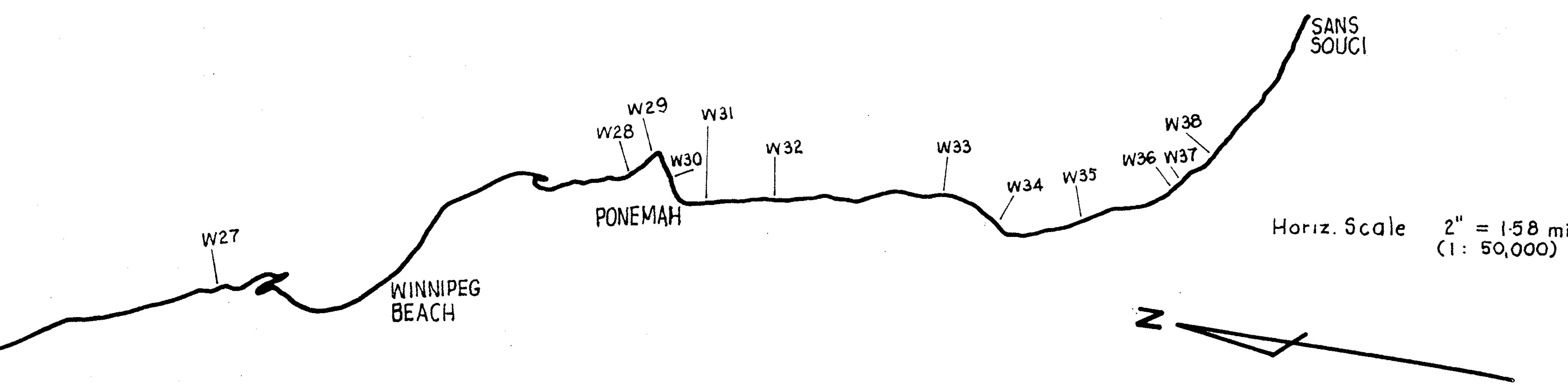


IN FIGS. (c) & (d) :

- \+ Normal energy component, computed.
- ⊙ Longshore energy component, computed.
- * Estimated value.
- ? Value shown is for qualitative indication only — not calculated.

NOTE :

In Figs. (b) & (d), positive value denotes direction towards right; negative value iddicates direction towards left.



UNIVERSITY OF MANITOBA

LAKE WINNIPEG SHORE EROSION STUD

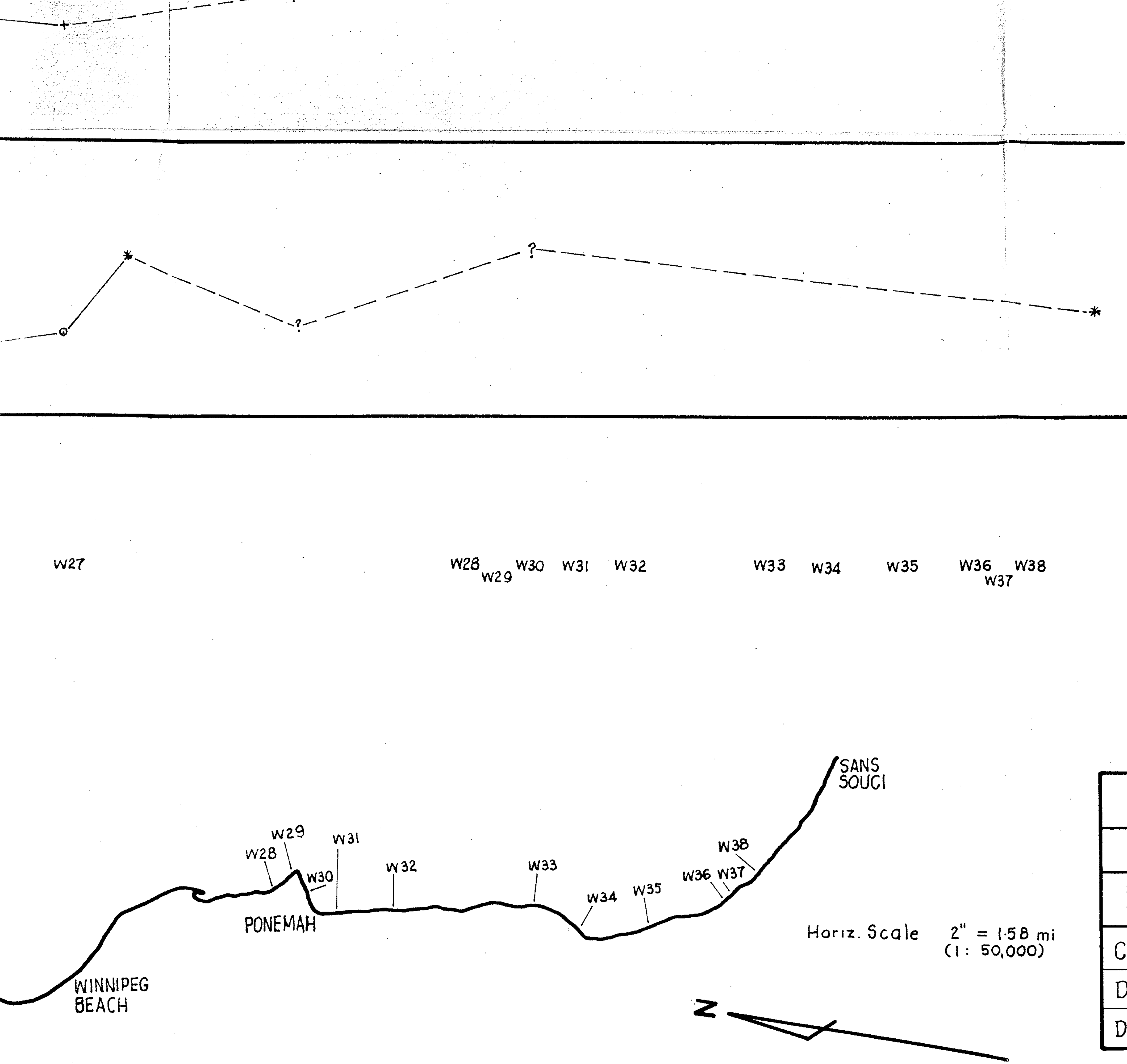
PLATE II WEST COAST

CALCULATED BY: wkc

DRAWN BY: wkc

DATE: March, 1971.

(a) Wave Power, Normal Com
(b) Wave Power, Longshore
(c) Wave Energy, Normal Com
(d) Wave Energy, Longshore



IN FIGS. (c) & (d) :

- + Normal energy component, computed.
- ⊙ Longshore energy component, computed.
- ✱ Estimated value.
- ? Value shown is for qualitative indication only — not calculated.

NOTE :

In Figs. (b) & (d), positive value denotes direction towards right; negative value iddicates direction towards left.

UNIVERSITY OF MANITOBA

LAKE WINNIPEG SHORE EROSION STUDY

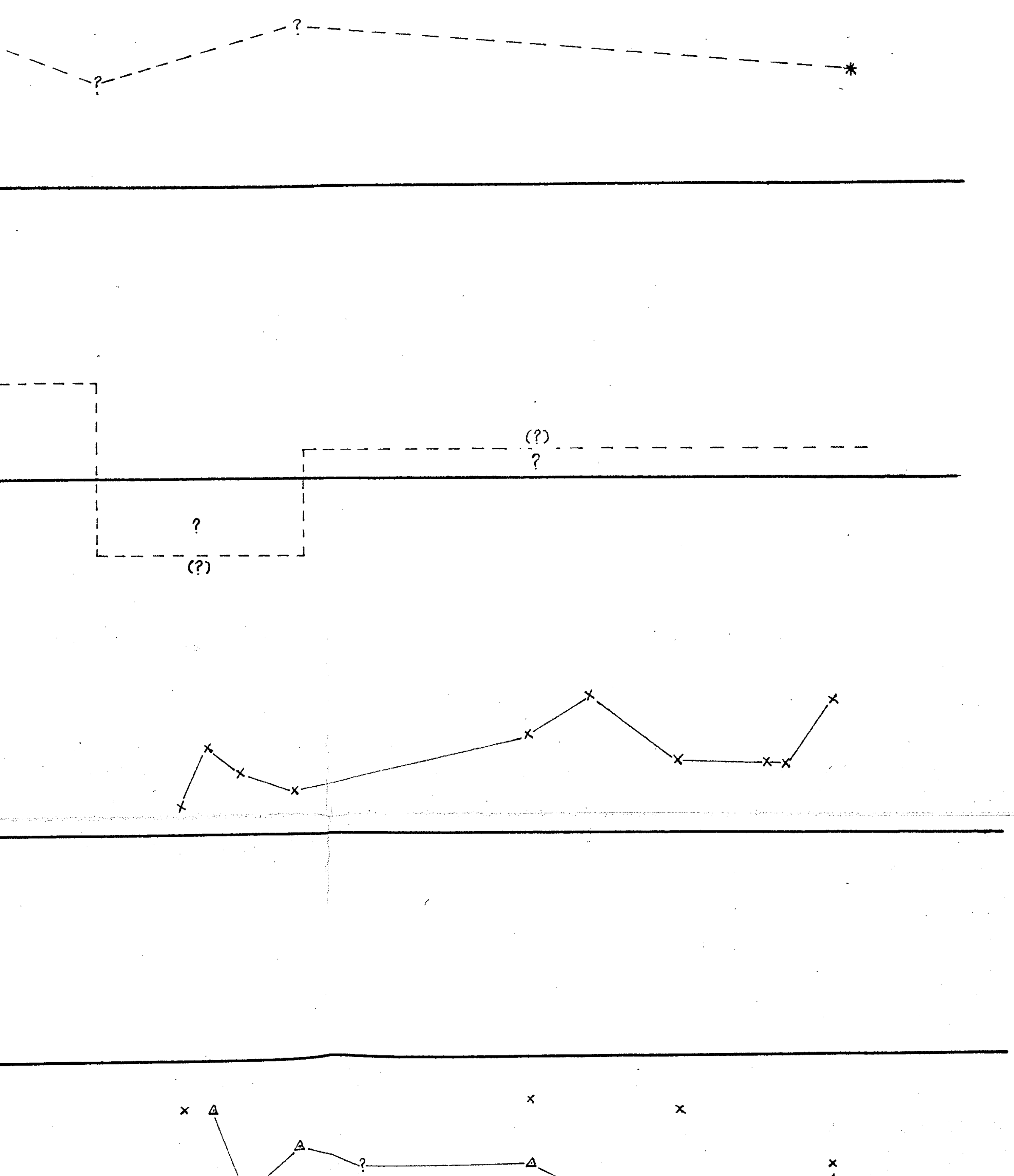
PLATE II — WEST COAST

CALCULATED BY: wkc

DRAWN BY: wkc

DATE: March, 1971.

(a) Wave Power, Normal Component
(b) Wave Power, Longshore "
(c) Wave Energy, Normal Component
(d) Wave Energy, Longshore "



LEGEND and REMARKS

* Estimated value
\+ Computed value
----?} Value not computed — for qualitative indication only (not to scale).

NOTE: Positive value designates movement towards right; negative designates movement to the left.

----?} Value indicated is not computed; for qualitative indication only.

Notes : (i) Number not in brackets indicates volume of accretion/erosion represented by retangular block — in 10^3 cu. yd.

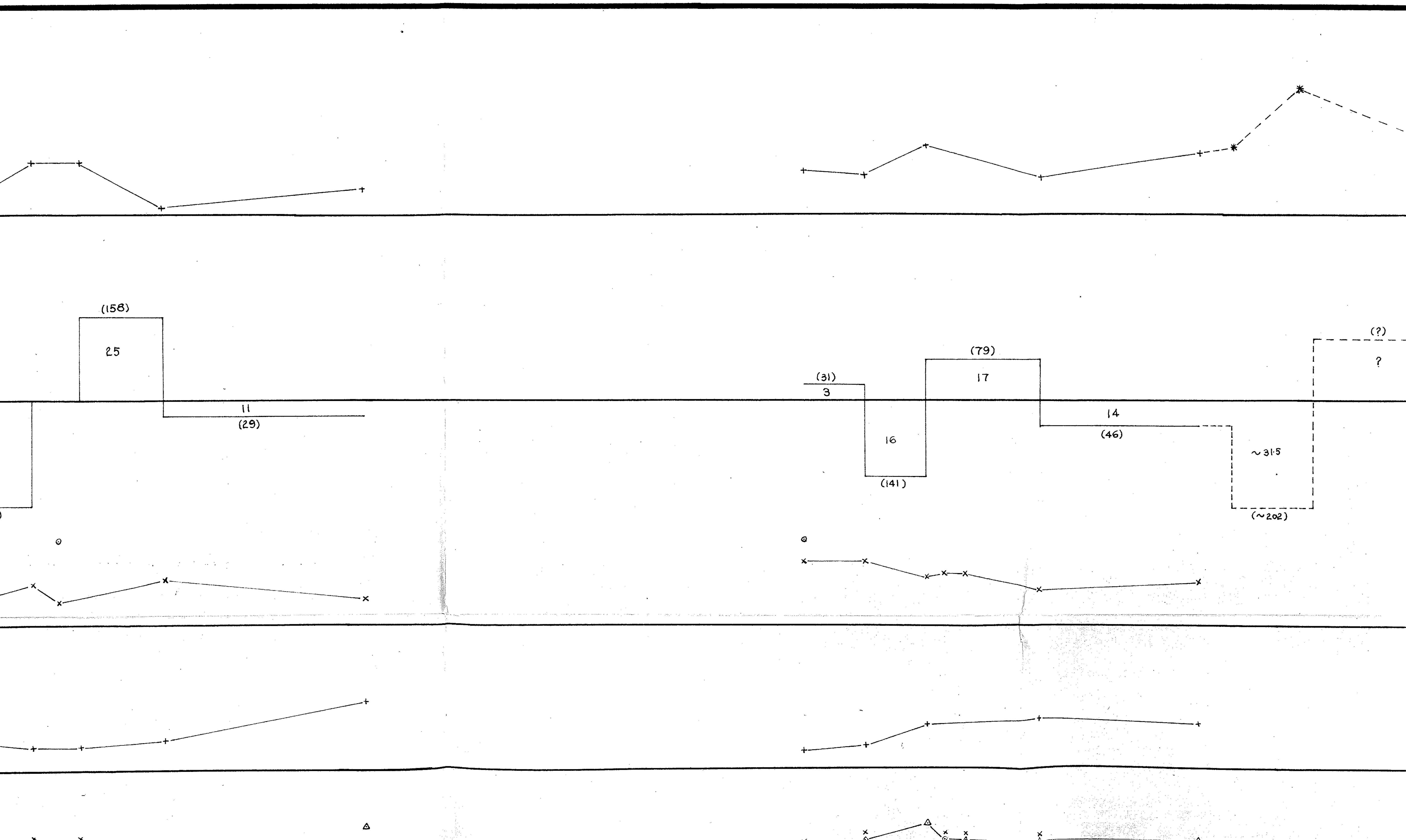
(ii) Number in brackets indicates extent of erosion/accretion in cu. ft. per linear ft. of beach front.

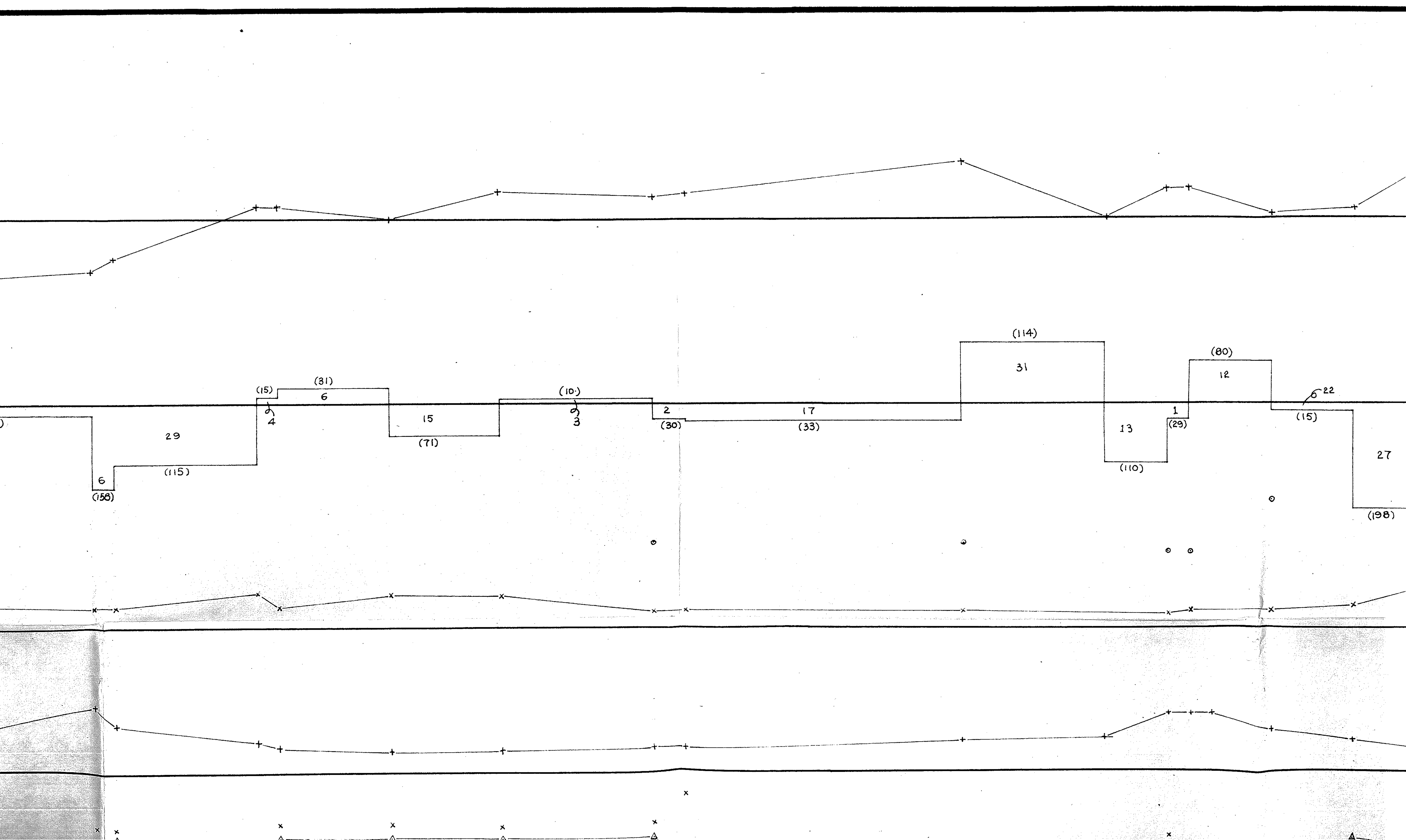
(iii) Positive value indicates accretion; vice versa.

× Lower limit } Estimated from 1970 data
⊙ Upper limit }

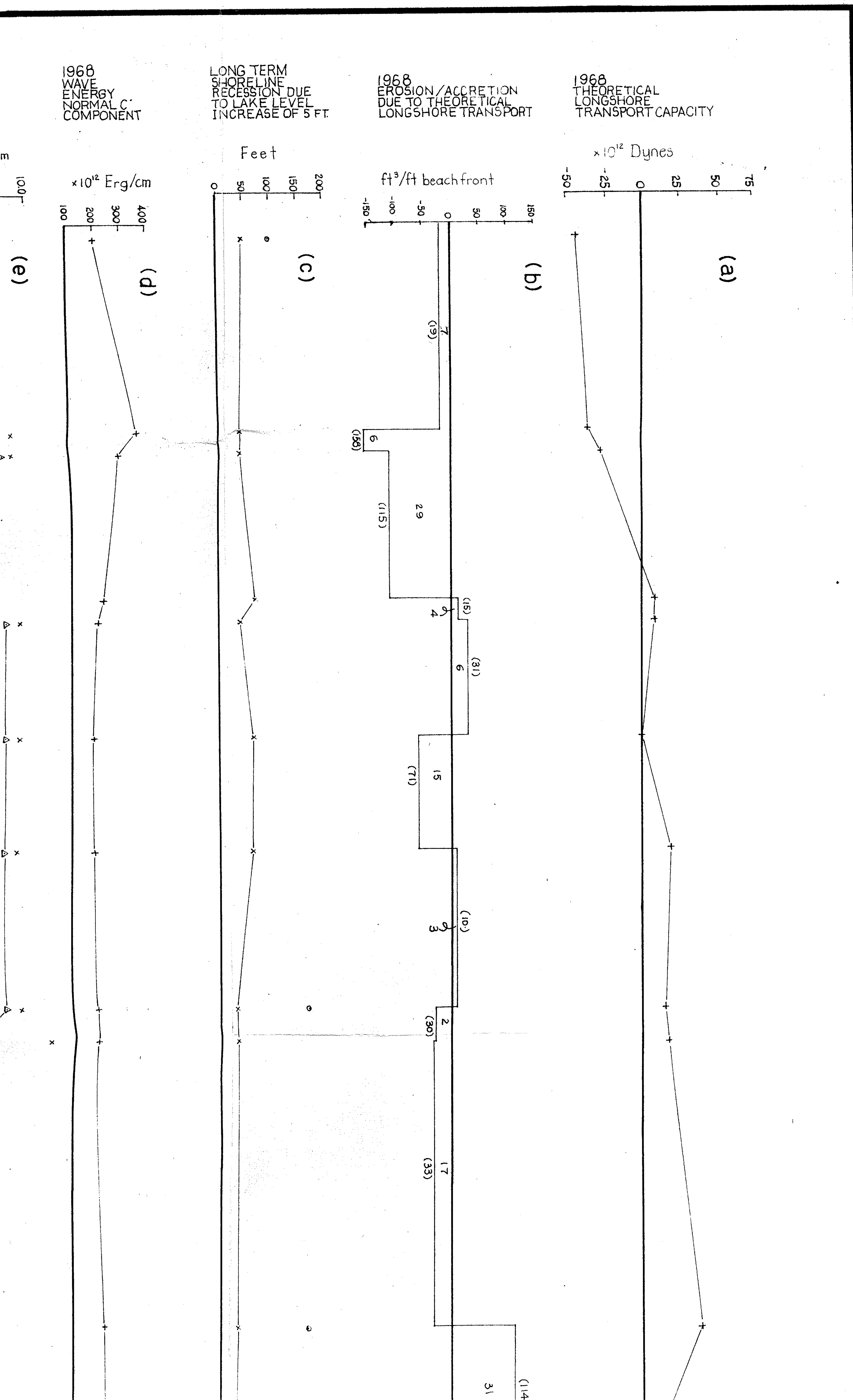
\+ Computed value
----?} Value not computed; estimate shown is for qualitative comparison only.

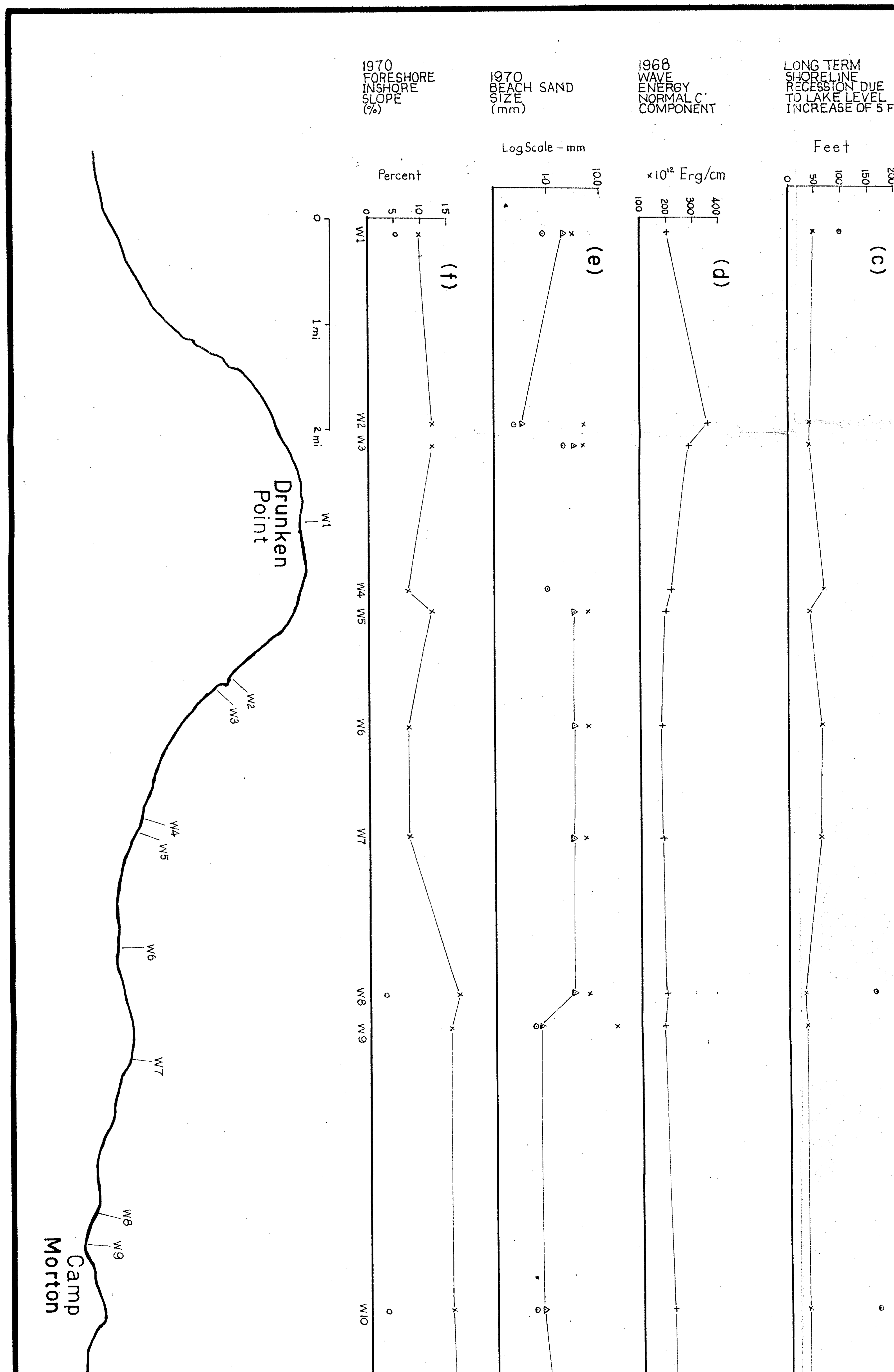
× Fine fraction }


(158)
25
11
(29)
(31)
3
16
(141)
(79)
17
14
(46)
~31·5
(~202)
(?)
?

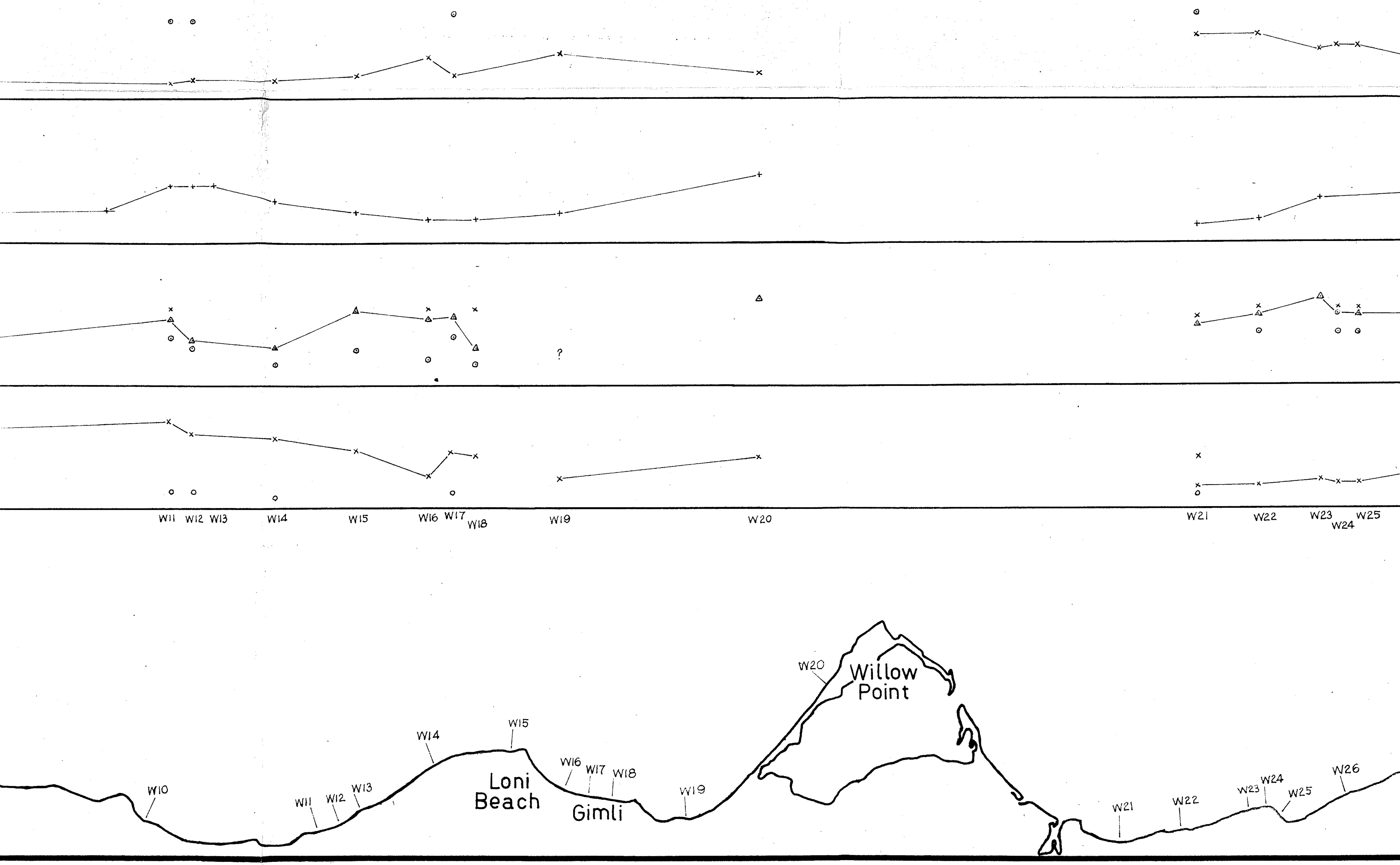


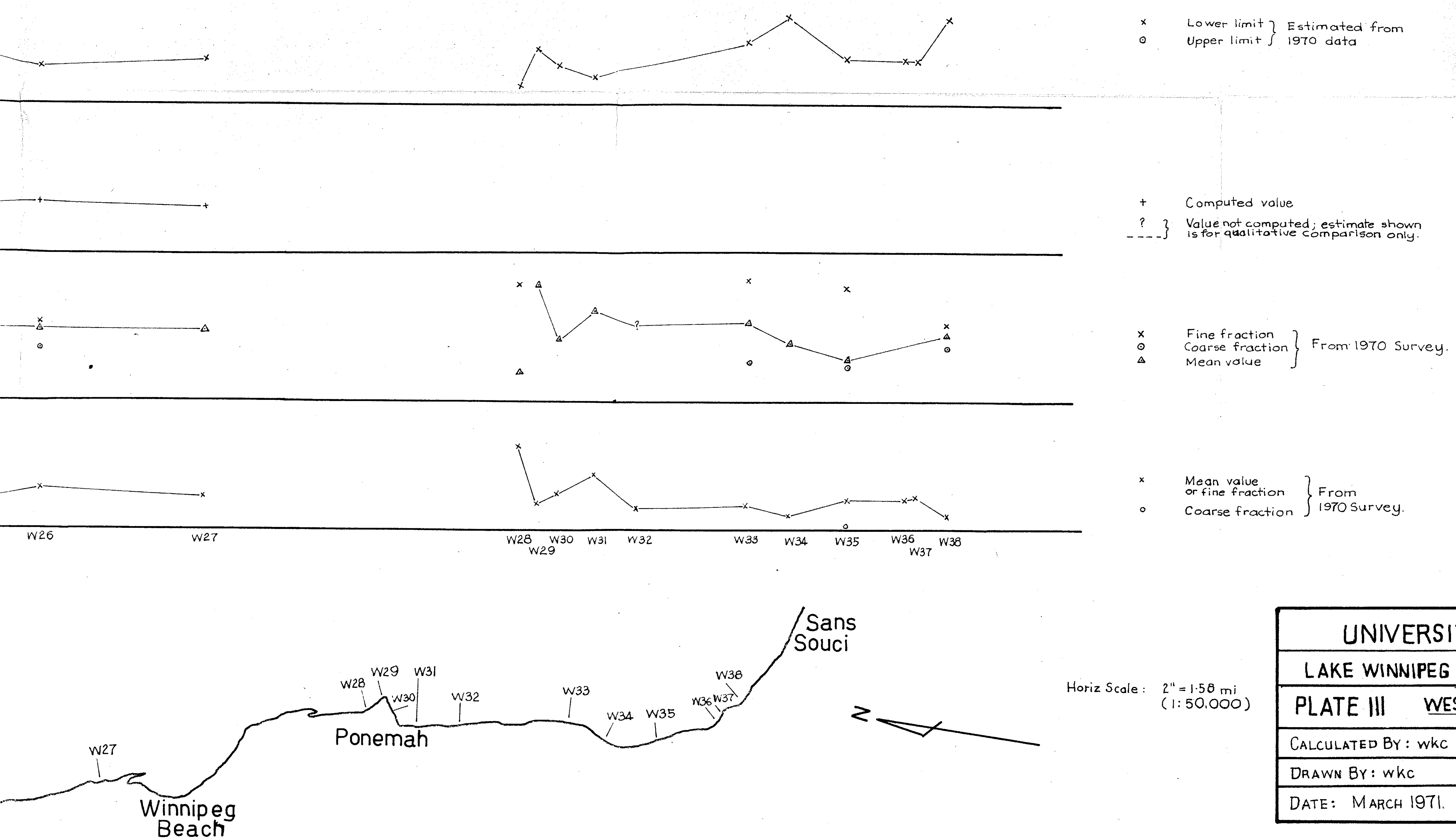
(114)
31
(80)
12
(15)
(31)
6
(10·)
2
3
2
17
22
15
29
4
(30)
(33)
1
(29)
(15)
13
27
(71)
(115)
(110)
6
(158)
(198)


1968 WAVE ENERGY NORMAL C' COMPONENT
LONG TERM SHORELINE RECESSION DUE TO LAKE LEVEL INCREASE OF 5 FT.
1968 EROSION/ACCRETION DUE TO THEORETICAL LONGSHORE TRANSPORT
1968 THEORETICAL LONGSHORE TRANSPORT CAPACITY
×10¹² Erg/cm
Feet
ft³/ft beachfront
×10¹² Dynes
(a)
(b)
(c)
(d)
(e)



1970 FORESHORE INSHORE SLOPE (%)
Percent
(f)
1970 BEACH SAND SIZE (mm)
Log Scale – mm
(e)
1968 WAVE ENERGY NORMAL C COMPONENT
×10¹² Erg/cm
(d)
LONG TERM SHORELINE RECESSION DUE TO LAKE LEVEL INCREASE OF 5 F
Feet
(c)
0
1 mi
2 mi
Drunken Point
Camp Morton
W1
W2
W3
W4
W5
W6
W7
W8
W9
W10


?
W11 W12 W13 W14 W15 W16 W17 W18 W19 W20 W21 W22 W23 W24 W25
W10
W11
W12
W13
W14
W15
Loni Beach
W16
W17
W18
Gimli
W19
W20
Willow Point
W21
W22
W23
W24
W25
W26



UNIVERSI

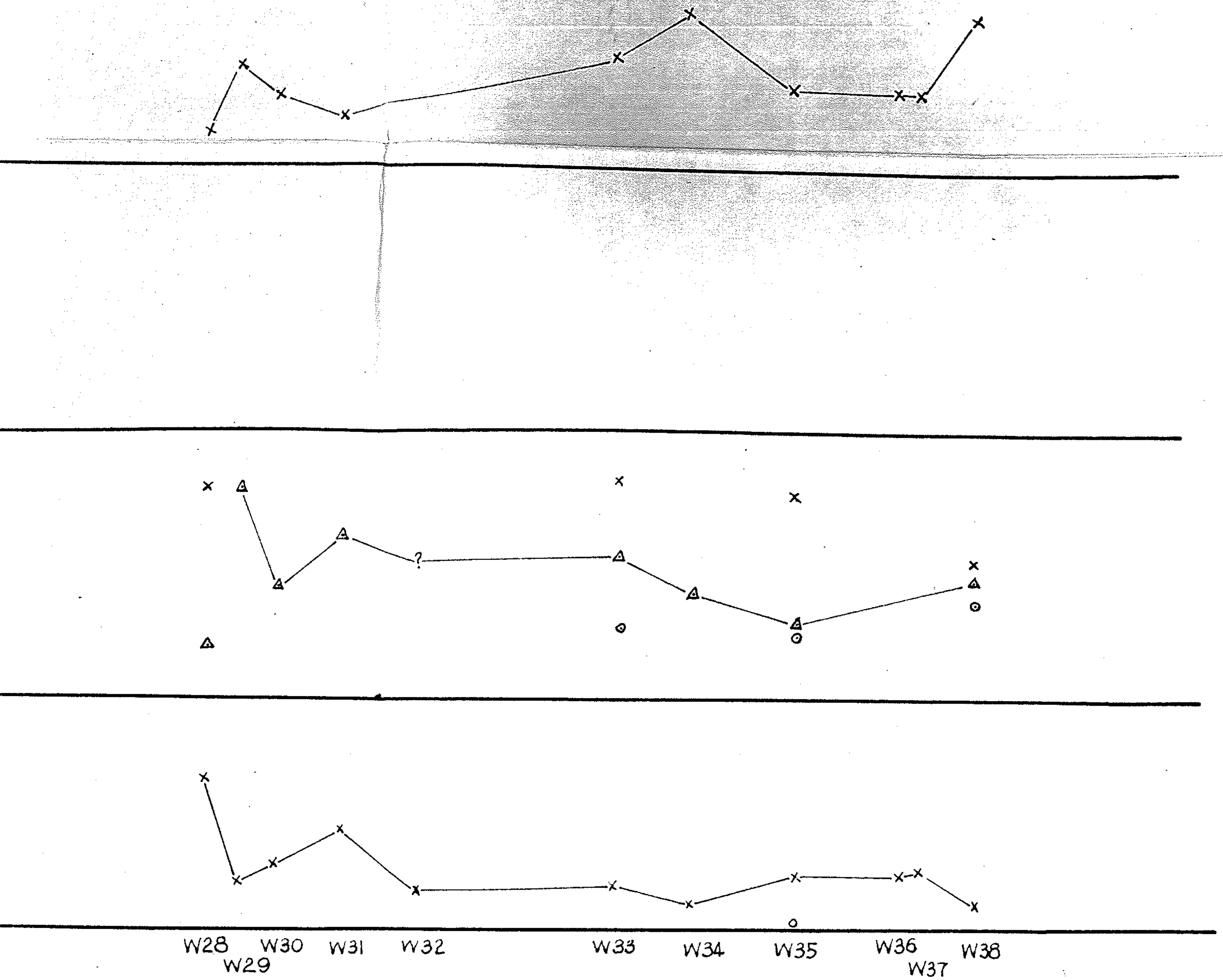
LAKE WINNIPEG

PLATE III WES

CALCULATED BY: wkc

DRAWN BY: wkc

DATE: MARCH 1971.



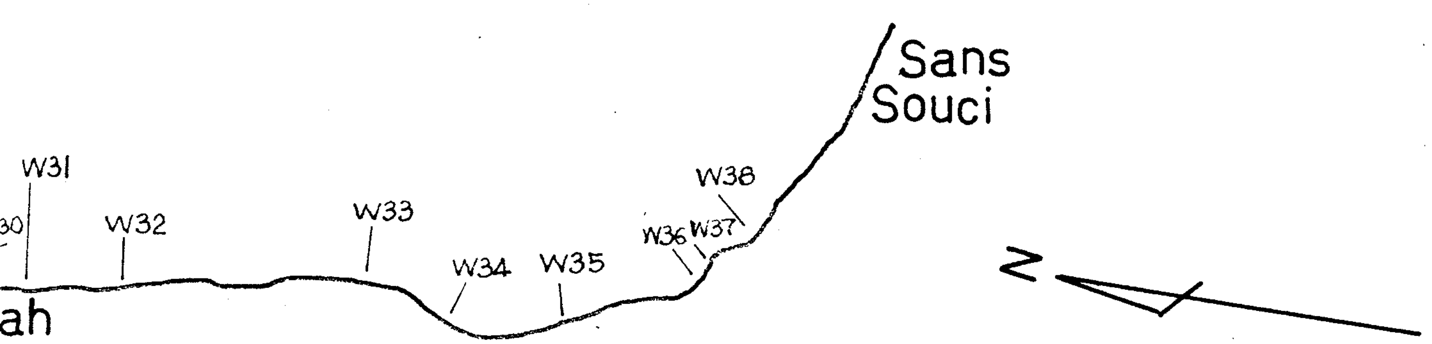
× Lower limit } Estimated from
⊙ Upper limit } 1970 data

\+ Computed value

? } Value not computed; estimate shown
\- - - -} is for qualitative comparison only.

× Fine fraction }
⊙ Coarse fraction } From 1970 Survey.
△ Mean value }

× Mean value or fine fraction } From
○ Coarse fraction } 1970 Survey.



Horiz Scale: 2" = 1·58 mi
(1:50,000)

UNIVERSITY OF MANITOBA

LAKE WINNIPEG SHORE EROSION STUDY

PLATE III WEST COAST

(a) Longshore Transport
(b) Erosion & Accretion
(c) Shoreline Recession
(d) Wave Energy
(e) Beach Sand Size
(f) Beach Slope

CALCULATED BY: wkc

DRAWN BY: wkc

DATE: MARCH 1971.