A Simplified Scattering Synthesis Method for LC Ladder Filters

by

Zhiwei Zhou

A thesis
Presented to the University of Manitoba
in partial fulfillment of the
requirements for the degree of
Master of Science
in
Electrical and Computer Engineering

Winnipeg, Manitoba, 2000
Zhiwei Zhou

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A Thesis/Practicum submitted to the Faculty of Graduate Studies of The University of Manitoba in partial fulfillment of the requirements of the degree

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ABSTRACT

This thesis is devoted to the presentation of a simplified synthesis method and its application in the development of a computer program for the synthesis of LC ladder networks. The approach developed in the thesis is based on scattering transfer matrix theory which does not seem to have been applied directly to ladder synthesis before.

The most important part of this thesis is the determination of a scattering synthesis strategy for choosing the sequence of transmission zeros. A synthesis strategy was developed by W.F. Göttlicher for application to the traditional synthesis procedure. Göttlicher's method, presented in his thesis, is adapted as the basis of the scattering strategy. In addition, this thesis introduces a set of simplified scattering calculations by appropriately modifying the details of Göttlicher's strategy. The simplified scattering algorithm involves only calculation of the reflectance and the delay at a transmission zero and at zero or infinity. For multiple transmission zeros at zero or infinity, the second and third derivatives of polynomials g and h at zero or infinity are reguired in addition.

The main results are given in *Chapter 3*. In that chapter, the derivation of the scattering characterization for all of the circuits and realization details of the modified strategy are presented. In *Chapter 4*, the flowgraph of the simplified scattering *LC* ladder filter synthesis program together with several design examples is presented.

The thesis concludes with a summary and recommendations.

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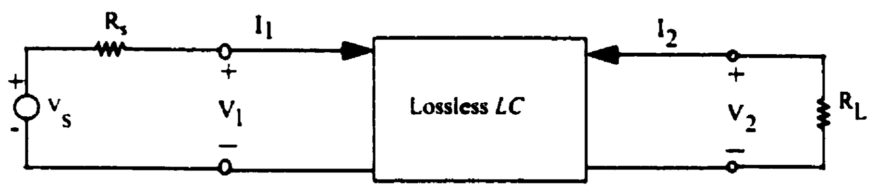
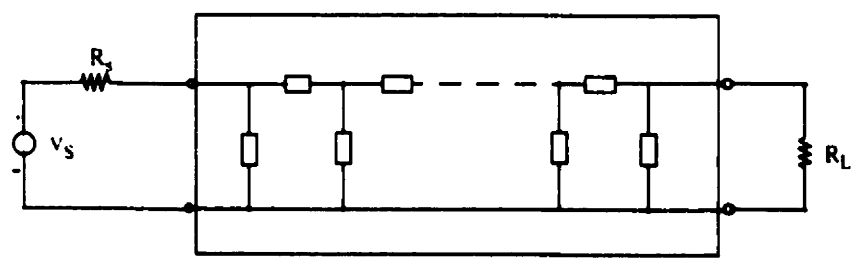
Chapter 1

INTRODUCTION

An LC filter is a *lossless transmission network* consisting of only inductors and capacitors. In normal operations, a doubly terminated two-port network as shown in Fig. 1-1(a) can be considered as an LC ladder filter. The specific *ladder* topology is shown in Fig. 1-1(b).



(a)



(b)

Fig. 1-1 (a) resistively terminated lossless two port;
(b) resistively terminated ladder structure.

Historically, *LC ladder filters* have played an important role in the design of transmission networks (filters). The availability of lossless resonance in an LC filter allows its impedance to have extremely rapid changes of magnitude and phase with changes in frequency. Accordingly with this outstanding topological property, we can construct a two-port network with very steep slopes between passbands and stopbands, and series or parallel resonance can be used to block transmission of certain frequencies completely, all in a network, which dissipates no power itself.

Lossless ladder filters are a popular structure in the field of digital signal processing. The low coefficient sensitivity and the simplicity of the structure make it suitable for constructing high quality digital filters [1],[2],[3]. For example, low sensitivity wave digital filters can be designed by using LC ladder filters as reference filters[3]. Active filters are heavily based on LC ladders for their design simulation[4]. The LC ladder concept has also been successfully applied to switched-capacitor filter networks[5]- [7] as well as to microwave impedance-matching networks[8]. Many monolithic switched capacitor LC ladder filters have been fabricated. It is obvious that *LC ladders* will continue to play an important roll in many areas of communication circuits, especially in the applications at higher frequencies ($f > 100$ kHz) where the operation of filtering devices becomes less than perfect[9].

Most of the discussions of LC ladder filters in the literature is based on the original design technique given by Darlington[10], Cauer[11-14], Bader[15-16], etc., which can be stated as follows: starting from a driving-point impedance of an LC two-port network or some other equivalent characterization, extract low-order realizable lossless subnetworks. Each subnetwork realizes a particular transmission zero. The mechanics of extracting a subnetwork generally depends on the nature of the transmission zero being extracted.

After each extraction step is completed, the remainder impedance is obtained. The whole process is repeated until the remainder impedance is exhausted.

An alternative approach to cascade network synthesis was introduced by *Belevitch* [17-21], which uses wave quantities for signal variables. The cascade decomposition of *Belevitch's* method can be accomplished by factoring the scattering transfer matrix. We should note that the representation of the scattering transfer matrix requires only three polynomials, - referred to as *Belevitch* 's representation, and the synthesis based on the scattering matrix is applicable to both the analog and digital domains. Hence, the scattering transfer matrix is a better tool for ladder network synthesis[22].

The main aim of this thesis is mainly to present a simplified scattering synthesis method and its application in the development of a computer program for synthesis of *LC* ladder networks. The new approach is based on scattering transfer matrix theory which does not seem to have been applied directly to ladder synthesis before. The first step in this synthesis procedure requires the determination of a scattering synthesis strategy for choosing the sequence of transmission zero extractions. Göttlicher's synthesis strategy [23] was developed for application to the traditional synthesis procedure. Göttlicher's method, presented in his thesis, is adapted as a basis for the scattering strategy. This thesis introduces a set of simplified scattering calculations by appropriately modifying Göttlicher's strategy in detail. The simplified scattering algorithm involves only calculation of the reflectance and the delay at a transmission zero and at zero or infinity. For multiple transmission zeros at zero or infinity, the second and third derivatives of polynomials g and h at zero or infinity are also required.

In this thesis, the zero or product of factors representation for a polynomial is used to achieve the required numerical accuracy. The synthesis equations are formulated in order to facilitate computer programming. All of the required techniques are included in the attached comprehensive *LC ladder* filter synthesis program.

In *Chapter 2*, we present the basic scattering theory, then briefly describe the properties of the reflectance and the return group delay of lossless, real, two-port networks that pertain to the synthesis problem. *Belevitch's* representation theory is very important in this chapter and the rest of thesis which forms a minimal set of necessary and sufficient conditions for a lossless scattering matrix to be realizable. Since Jarmasz's thesis [22] has made great contribution for a simplified synthesis algorithm of cascade network synthesis, his minimal characterization of the 1st- and 2nd- order elementary reciprocal sections is adopted as part of the basis of our simplified ladder scattering algorithm.

The main results are given in *Chapter 3*. In that chapter, the rationale for using a specific synthesis strategy as the back-bone for the simplified scattering ladder synthesis strategy is described. The derivation of the scattering characterization for all of the circuits and realization details in Göttlicher's strategy are presented.

In *Chapter 4*, the flowgraph of the simplified scattering *LC* ladder filter synthesis program together with several design examples is presented. Finally, in Chapter 5, the final conclusions and recommendations are given.

Chapter 2

BASIC THEORY OF CASCADE NETWORK SCATTERING SYNTHESIS

2.1 Introduction

As is known, the scattering matrix, whose entries are the scattering coefficients, exist for any passive two-port network. They are particularly useful in the description of power transfer under practical terminating conditions, therefore they are used exclusively in filter approximation theory.

This chapter deals mainly with the basic scattering theory of lossless and real two-port networks, which involves *Belevitch's* representation and the scattering properties of lossless two-port networks. A very useful synthesis algorithm called the simplified scattering algorithm[22] is described briefly in the following section.

Finally, we use tables to describe a set of scattering characteristics of the elementary reciprocal sections.

2.2. Basic scattering theory

A lossless two-port ladder network with port references R_1, R_2 is shown in Fig. 2.1.(a). A natural way of characterizing such a two-port is the use of normalized scattering variables

$$A_i = \frac{V_i + R_i I_i}{2\sqrt{R_i}} \qquad B_i = \frac{V_i - R_i I_i}{2\sqrt{R_i}} \qquad i = 1, 2 \qquad (2.1.1)$$

which are known as the incident and reflected power waves, respectively[24].

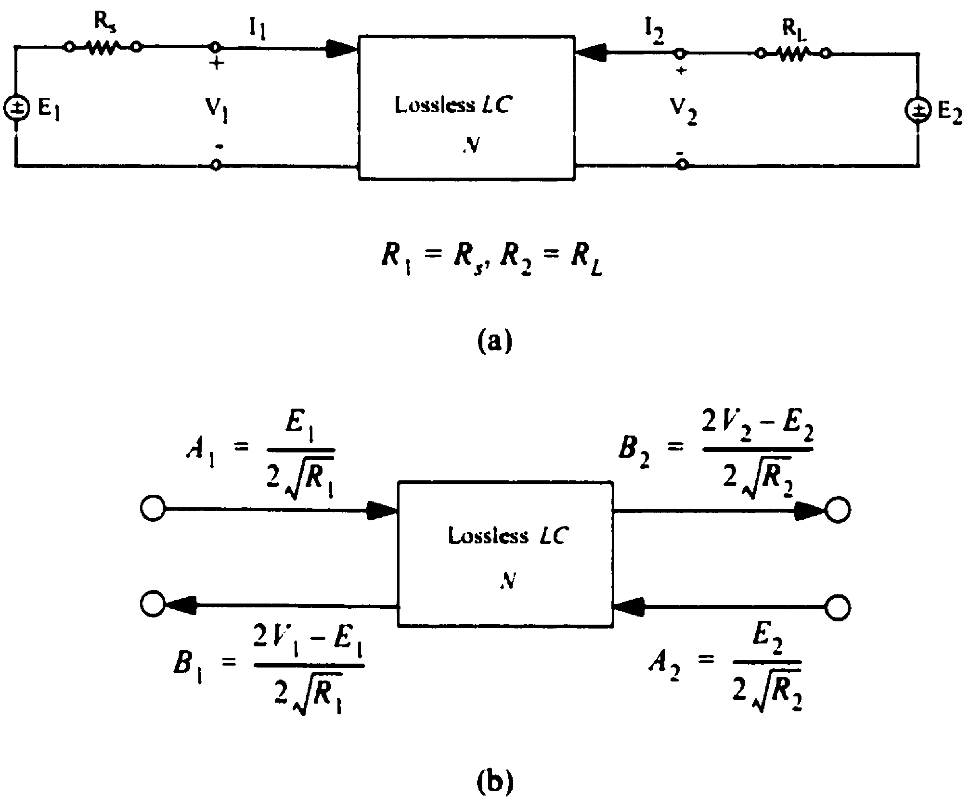


Fig. 2.1(a) A lossless two-port network inserted between resistive terminations
(b) its wave variable equivalent

There are two useful groupings of the scattering variables:

$$\begin{bmatrix} B_1 \\ B_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = S \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} \qquad (2.1.2\ a)$$

$$\begin{bmatrix} B_1 \\ A_1 \end{bmatrix} = T \begin{bmatrix} A_2 \\ B_2 \end{bmatrix} \qquad (2.1.2\ b)$$

where S, T are 2×2 matrices referred to as the *scattering* and *transfer* matrix, respectively. For the resistive terminations shown in Fig. 2.1, the entries of the S matrix- called the *scattering coefficients* – can be defined as the ratio of a reflected signal to an incident signal under the condition of zero incident signal at the other port. Specifically, using (2.1.1) and Fig. 2.1, we have

$$S_{11} = \left.\frac{B_1}{A_1}\right|_{A_2 = 0} = \frac{Z_1 - R_1}{Z_1 + R_1}$$

$$S_{22} = \left.\frac{B_2}{A_2}\right|_{A_1 = 0} = \frac{Z_2 - R_2}{Z_2 + R_2}$$

$$S_{21} = \left.\frac{B_2}{A_1}\right|_{A_2 = 0} = 2\sqrt{\frac{R_1}{R_2}}\frac{V_2}{E_1}$$

$$S_{12} = \left.\frac{B_1}{A_2}\right|_{A_1 = 0} = 2\sqrt{\frac{R_2}{R_1}}\frac{V_1}{E_2} \qquad (2.1.3)$$

where Z_1 and Z_2 are the input driving- point impedances at ports 1 and 2, respectively.

As denoted by (2.1.3), $S_{11}(S_{22})$ is called the input (output) reflectance, and it is the ratio of reflected to incident signals at the input (output) port, when the output(input) port is terminated in its reference resistance. Similarly, S_{21} (S_{12}) is called a forward (reverse) transmittance and it is the ratio of the power delivered to the load to the maximum power available from the source at the input (output) port, under the reference terminating condition.

A significant and practical simplification in the characterization of lossless two-port networks was achieved by *Belevitch* who showed that the scattering coefficients can be expressed using only three polynomials and a unimodular constant. He proved that for real, realizable and lossless two-port networks, matrices in (2.1.2) necessarily take on the following forms:

$$S = \frac{1}{g}\begin{bmatrix} h & \sigma f_* \\ f & -\sigma h_* \end{bmatrix}, \qquad T = \frac{1}{f}\begin{bmatrix} \sigma g_* & h \\ \sigma h_* & g \end{bmatrix}, \qquad (2.1.4\text{ a,b})$$

where the polynomials f, g and h satisfy the following necessary and sufficient conditions:

1. Polynomials f, g and h are real polynomials in some complex frequency variable, i.e., each of them satisfies '$P(s)$ is real for s real', and the subscript asterisk denotes paraconjugation, i.e., for a real polynomial $f_*(s) = f(-s)$, which is also referred to as *Hurwitz* conjugation.

2. $g(s)$ is a *Hurwitz* polynomial, i.e. all its zeros lie in the open left-hand plane ($\mathrm{Re}\, s < 0$).

3. σ is a unimodular constant (either +1 or -1) for real two-ports. For reciprocal two-ports, σ is specified by the ratio f/f_*, whereas for nonreciprocal two-ports it can take on either value independently.

4. The polynomials g, h and f are related by

$$gg_* = hh_* + ff_* \qquad (2.1.5)$$

which is the analytic continuation of the *Feldtkeller* equation.

Note the *Feldtkeller* equation can be written as $1 = \left|\frac{h}{g}\right|^2 + \left|\frac{f}{g}\right|^2$, if $s = j\omega$.

Clearly, the *Belevitch's* representation is not violated if polynomials f, g, h are multiplied by a same real constant K. It is convenient to choose K in such a way that, e.g. either

g or f is monic (leading coefficient equal to unity), in which case the polynomials f, g and h satisfying *Belevitch's* Representation become unique; the resulting representations (2.1.4 a, b) are referred to as *canonic* forms.

Given the *canonic* forms (2.1.4 a, b), the *Feldtkeller* equation can clearly display the complementary nature of the scattering coefficients as follow:

a) Both the transmittance f/g and the reflectance h/g are bounded real functions, because these functions satisfy the real bounded necessary and sufficient conditions:

(i) $\frac{f}{g}(s)$ and $\frac{h}{g}(s)$ is real for s real;

(ii) g is *Hurwitz*;

(iii) following from the *Feldtkeller* equation, it is obvious that

$$\left|\frac{f(j\omega)}{g(j\omega)}\right| \le 1, \qquad \left|\frac{h(j\omega)}{g(j\omega)}\right| \le 1 \qquad -\infty < \omega < \infty \qquad (2.1.6),(2.1.7)$$

b) Zeros of polynomial f are called transmission zeros. In addition, if $\frac{f}{g} \to 0$ with order n as $s \to \infty$, there is said to be an n^{th} order transmission zero at ∞. It follows from the *Feldtkeller* equation and the fact that all polynomials are real that

$$1 = \left|\frac{h(j\omega)}{g(j\omega)}\right|^2 \qquad (2.1.8)$$

which is equivalent to $\frac{h(j\omega)}{g(j\omega)} = e^{j\alpha} \to \frac{h(j\omega)}{g(j\omega)} = \pm 1$ (2.1.9)

for $\alpha = 0$ or $\alpha = \pi$

c). A function defined by

$$d(s) := \left[\ln\left(\frac{g}{h}\right)\right]' = \frac{g'}{g}(s) - \frac{h'}{h}(s) \qquad (2.1.10)$$

will be referred to as the delay. Now, we present the important characteristic of delay as follows, which was proved by Jarmasz[22]:

The delay evaluated at $s = j\omega$ such that $f(j\omega) = 0$, is real, positive, and equal to the return group delay defined by $\tau(\omega) := Ev\left\{\left[\ln\left(\frac{g}{h}(s)\right)\right]'\right\}_{s = j\omega} =$ $Ev\{d(s)\}_{s = j\omega}$.

For a function which is the quotient of two real polynomials the even part of the function, when evaluated at $s = j\omega$, is the same as the real part of the value of the function at $s = j\omega$. It follows that $d(j\omega) = \tau(\omega)$.

2.3 Scattering Synthesis of Cascade Networks

The main goal of filter synthesis is to split up an overall network characterization into a sequence of low-order sections. The problem of *cascade* network synthesis amounts to factorizing the transfer matrix T into a product $T_a T_b$, with each factor corresponding to a realizable transfer matrix. The following discussion follows the derivations given by Jarmasz[22].

The transfer matrices of lossless two-port networks N_a and N_b are given by

$$\begin{bmatrix} B_1 \\ A_1 \end{bmatrix} = \frac{1}{f_a}\begin{bmatrix} \sigma_a g_{a^*} & h_a \\ \sigma_a h_{a^*} & g_a \end{bmatrix}\begin{bmatrix} A_{2a} \\ B_{2a} \end{bmatrix} = T_a\begin{bmatrix} A_{2a} \\ B_{2a} \end{bmatrix},$$

$$\begin{bmatrix} B_{1b} \\ A_{1b} \end{bmatrix} = \frac{1}{f_b} \begin{bmatrix} \sigma_b g_{b^*} & h_b \\ \sigma_b h_{b^*} & gb \end{bmatrix} \begin{bmatrix} A_2 \\ B_2 \end{bmatrix} = T_b \begin{bmatrix} A_2 \\ B_2 \end{bmatrix} \tag{2.3.1}$$

A cascade connection of two lossless two-ports N_a and N_b is shown in Fig. 2.2.

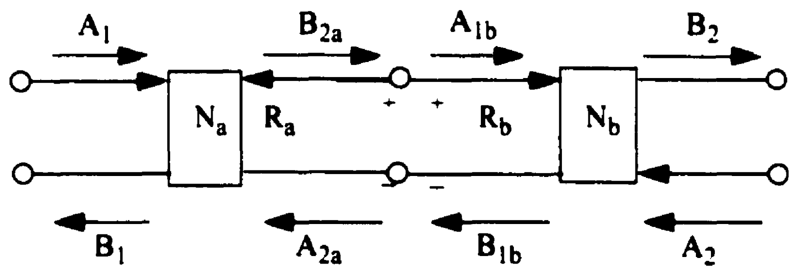


Fig. 2.2 Cascade connection of N_a and N_b

At a direct interconnecting port $V_{2a} = V_{1b}$, $I_{2a} = -I_{1b}$ and with $R_a = R_b$, it follows that $A_{2a} = B_{1b}$ and $B_{2a} = A_{1b}$, which implies that the transfer matrix for the combined network is given by

$$T = T_a T_b = \frac{1}{f} \begin{bmatrix} \sigma g_* & h \\ \sigma h_* & g \end{bmatrix} \tag{2.3.2}$$

where $\quad \sigma = \sigma_a \sigma_b, \quad f = f_a f_b, \quad g = g_a g_b + \sigma_a h_{a^*} h_b, \quad h = h_a g_b + \sigma_a g_{a^*} h_b \quad$ (2.3.3)

An important property of two-port networks arranged in a cascade can be seen by examining the signal flowgraph representation of the cascade connection as shown in Fig.2.3.

At a transmission zero $s = s_a$ of N_a, we have $f_a(s_a) = 0$ which, together with the assumption that $A_2 = 0$, means that the only path from the input terminal A_1 to B_1 is through the branch with the multiplier $\frac{h_a}{g_a}$. It follows that for $A_1 = e^{s_a t}$, we have

$\frac{B_1}{A_1}(s_a) = \frac{h}{g}(s_a) = \frac{h_a}{g_a}(s_a)$. For a reciprocal two-port N_a we also have $f_{a*}(s_a) = 0$, and both transmittances that couple to N_b are zero, thus leaving N_b completely decoupled from N_a. In this case, we can show that we also have $d(s_a) = d_a(s_a)$ [22]. We state this property in the form of Fig. 2.3:

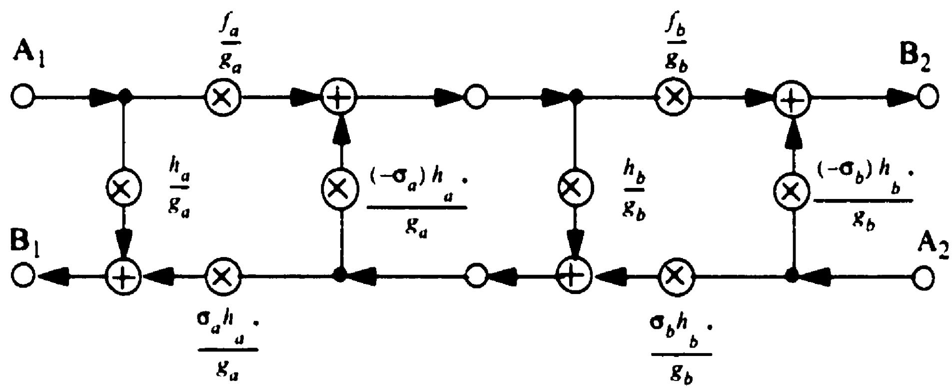


Fig.2.3 Cascade signal flowgraph representation

The values of the reflectance ρ and delay d functions of a lossless two-port network evaluated at a transmission zero of the first member of a cascade are equal to the corresponding values of that member, i.e.

$$\rho(s_a) := \frac{h}{g}(s_a) = \frac{h_a}{g_a}(s_a) =: \rho_a(s_a) \quad \text{where} \quad f_a(s_a) = 0 \quad (2.3.5)$$

and

$$d(s_a) := \frac{g'}{g}(s_a) - \frac{h'}{h}(s_a) = \frac{g_a'}{g_a}(s_a) - \frac{h_a'}{h_a}(s_a) =: d_a(s_a) \quad (2.3.6)$$

where $f_{a^*}(s_a) = f_a(s_a) = 0$

From(2.3.2) we have $T_b = T_a^{-1} T$ which yields

$$h_b = \frac{g_a h - h_a g}{\sigma_a f_a f_{a^*}}, \quad g_b = \frac{g_{a^*} g - h_{a^*} h}{f_a f_{a^*}} \qquad (2.3.7a,b)$$

For partial removal of a transmission zero at 0, where f, f_a are monic.

$$h_b = \frac{g_a h - h_a g}{\sigma_a f_{a^*}}, \quad g_b = \frac{g_{a^*} g - h_{a^*} h}{f_{a^*}} \qquad (2.3.7\ c,d)$$

Therefore, the cascade decomposition problem essentially reduces to the operation of forcing the factor $f_a f_{a^*}$ to appear in both numerators of expressions in (2.3.7a,b). More formally: given $\{\sigma, f, g, h\}$, find $\{\sigma_a, f_a, g_a, h_a\}$ such that (2.2.3) holds, h_b and g_b given by (2.3.7a,b) are polynomials, g_b is realizable (Hurwitz), and $deg\ h_b \leq deg\ g_b$. Fettweis has demonstrated that a canonic decomposition solution always exists [25]. Moreover, the solution is unique and can be performed minimally, i.e $deg\, g = deg\, g_a + deg\, g_b$.

2.4 Cascade Synthesis of Lossless Two-Ports with $j\omega$-- axis Transmission Zeros

There are three cases of transmission zeros on the $j\omega$-axis that must be considered:

1. $s = 0$. In this case,

$$f_a = d_a s, \quad f_a f_{a^*} = -d_a^2 s^2 \text{ and } \sigma_a = -1 ;$$

2. $s = \pm j\omega_0$, In this case,

$$f_a = s^2 + \omega_0^2, \quad f_a f_{a^*} = \left(s^2 + \omega_0^2\right)^2 \text{ and } \sigma_a = 1 ;$$

3. $s = \infty$, In this case,

$$f_a = d_a, \text{ and } f_a f_{a^*} = d_a^2 .$$

In the above cases, case 3 is treated as case 1 by using the mapping $s \to 1/s$ which maps $s = \infty$ to $s = 0$.

For case 1 and case 3, note that Eq.(2.3.7) becomes

$$h_b = \frac{g_a h - h_a g}{d_a^2 s^2}, \qquad g_b = \frac{g_{a*} g - h_{a*} h}{-d^2 s^2}$$

Therefore, for the denominator to divide the numerator, one must have

$$\rho_a(0) = \frac{h_a}{g_a}(0) = \frac{h}{g}(0) = \rho(0) \text{ and } \frac{g_{a^*}}{h_{a^*}}(0) = \frac{h}{g}(0)$$

But from the *Feldtkeller* equation $\frac{g_{a^*}}{h_{a^*}}(0) = \frac{h_a}{g_a}(0)$ at the transmission zero $s = 0$.

In addition, one must have

$$d_a(0) = \left(\frac{g_a^{\cdot}}{g_a} - \frac{h_a^{\cdot}}{h_a}\right)\Bigg|_{s=0} = \left(\frac{g'}{g} - \frac{h'}{h}\right)\Bigg|_{s=0} = d(0)$$

The *Feldtkeller* equation is used to obtain the same result from a consideration of the equation for g_b.

Similarly for case 2,

$$h_b = \frac{g_a h - h_a g}{\left(s^2 + \omega_0^2\right)^2}, \qquad g_b = \frac{g_{a*} g - h_{a*} h}{\left(s^2 + \omega_0^2\right)^2}$$

and, consequently,

$$\rho_a(\pm j\omega_0) = \frac{h_a}{g_a}(\pm j\omega_0) = \frac{h}{g}(\pm j\omega_0) = \rho(\pm j\omega_0)\text{, and}$$

$$d_a(\pm j\omega_0) = \left(\frac{g_a'}{g_a} - \frac{h_a'}{h_a}\right)\bigg|_{s = \pm j\omega_0} = \left(\frac{g'}{g} - \frac{h'}{h}\right)\bigg|_{s = \pm j\omega_0} = d(\pm j\omega_0)$$

The *Feldtkeller* equation is again used to obtain the same result from a consideration of the equation for g_b. Note: also, as can easily be verified, the values for $s = j\omega_0$ and $s = -j\omega_0$ are the same.

Formulas for the calculations of delay using polynomials in factored form (product representation) are derived in Appendix I.

The extraction and recomputation steps for cascade synthesis of lossless two-ports with $j\omega$ - axis transmission zeros is depicted in Fig. 2.4.

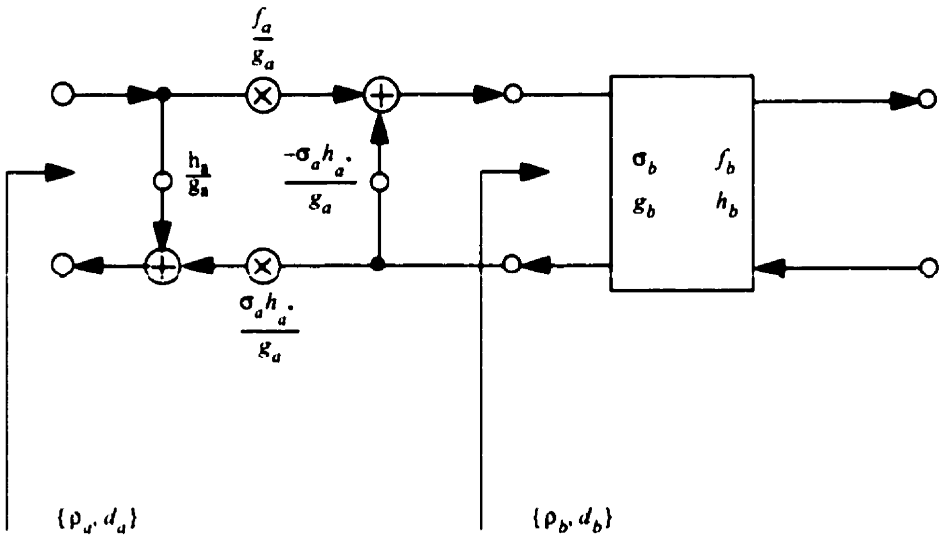


Fig.2.4 Flowgraph representation of the basic extraction step for cascade synthesis of lossless two-ports with $j\omega$ - axis transmission zeros

2.5 The minimal characterization of the elementary reciprocal sections

In this section, all of the minimal characterizations of the 1st- and 2nd- order elementary reciprocal sections were originally derived by Jarmasz [22]. The minimal scattering characterizations are described in tables, the scattering polynomials *f*, *g* and *h* use only a minimal (canonic) set of parameters. The minimal set for all cases includes the location of the transmission zero (ω_0) that the section realizes, the value of the reflectance (ρ)at the transmission zero, and, for reciprocal sections, the value of the delay (*d*) at the transmission zero.

Table 1: Section Type 1

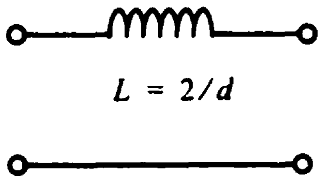
$f = d$ $g = s + d$ $h = s$ $\sigma = 1$ $d > 0$ $\rho = 1$	

Table 2: Section Type 2

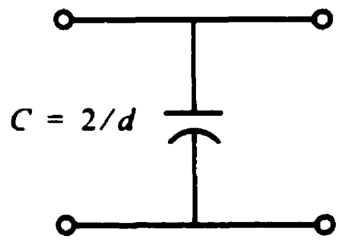
$f = d$ $g = s + d$ $h = -s$ $\sigma = 1$ $d > 0$ $\rho = -1$	

Table 3: Section Type 3

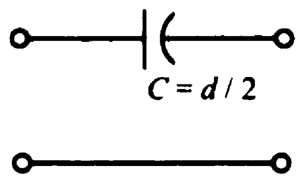
$f = ds$ $g = ds + 1$ $h = 1$ $\sigma = -1$ $d > 0$ $\rho = 1$	

Table 4: Section Type 4

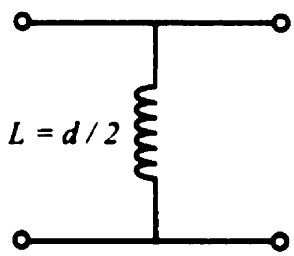
$f = ds$ $g = ds + 1$ $h = -1$ $\sigma = -1$ $d > 0$ $\rho(0) = -1$	

Table 5: Section Type 5

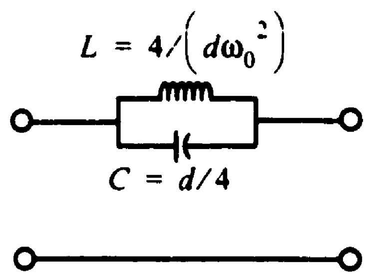
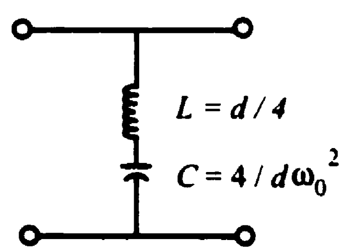
$f = s^2 + \omega_0^2$ $g = s^2 + \frac{2}{d}s + \omega_0^2$ $h = \frac{2}{d}s$ $\sigma = 1$ $d > 0$ $\rho = 1$	

Table 6: Section Type 6

$f = s^2 + \omega_0^2$ $g = s^2 + \frac{2}{d}s + \omega_0^2$ $h = -\frac{2}{d}s$ $\sigma = 1$ $d > 0$ $\rho = -1$	

Chapter 3

THE SIMPLIFIED SCATTERING SYNTHESIS STRATEGY

3.1 Introduction

A ladder network is composed of canonic sections of degrees one and two; we will call them elementary sections. That the realizablity as a ladder depends on the transmission zero sequence will be illustrated with an example. The necessary and sufficient conditions for the realization sequence are not known. Göttlicher [23] developed a strategy for choosing a sequence which has proven to be successful in a wide range of filter designs. For this purpose he combined elementary sections into 24 circuits in order to check the realizability conditions for two successive transmission zeros. The transmission zero with the maximum number of potential successors is then chosen for realization. If several transmission zeros have the same number of successors, a realizability measure is introduced to decide which transmission zero should be realized first. The realizability measure determines the margin by which the realizability condition at zero or infinity for the successor transmission zero is satisfied. The successor transmission zero with the smallest margin is chosen. The realizability measure will be described more precisely in the following. The equations for determining the potential transmission zero successors and the corresponding realizability measures for the 24 circuits will be presented in the form of tables. The 24 circuits are subdivided into eight classes, A to H.

In addition, four circuits not included by Göttlicher, in his tables, because the realizability of the first section is not affected by the second section are included in Tables 31- 34.

These four circuits are included in this thesis because of the way in which the computer program is structured.

3.2 An example to illustrate the necessity for a synthesis strategy

The following example illustrates that the realizabilty of a ladder network with positive elements depends on the sequence of transmission zero extractions.

Given

$$f = s\left(s^2+1\right)\left(s^2+4\right) = s^5+5s^3+4s$$

$$g = s^5+2s^4+\frac{13}{2}s^3+\frac{13}{2}s^2+6s+2$$

$$h = s^4-\frac{1}{2}s^3+\frac{5}{2}s^2+2s+2\,.$$

It is readily verified that these polynomials satisfy the condition for a lossless, passive two-port. There are six potential transmission zero sequences:

$(0, j1, j2)$, $(0, j2, j1)$, $(j1, 0, j2)$, $(j1, j2, 0)$, $(j2, 0, j1)$, $(j2, j1, 0)$ ..

There is a transmission zero at $s = 0$ and $\rho(0) = 1$, $d(0) = 2$, implying that a series capacitor (see Table 3) can be removed. Realizability requires that

$C \geq C_0 = \frac{d(0)}{2} = 1$. The values of the reflectance at the transmission zeros $s = j1$ and $s = j2$ are -j and +j, respectively. Thus to realize $s = j1$, a capacitance $C = 1$ (see Eq.

(3.3.11)) must be removed while for $s = j2$, $C = -\frac{1}{2}$. Therefore the sequence $(0, j1)$ is realizable while $(0, j2)$ is not. In fact, it can verified that $(0, j1, j2)$ is realizable and yields the circuit shown.

The remaining sequences begin either with j1 or j2 and $\rho(j1) = -j$, $\rho(j2) = j \neq \pm 1$. It follows that neither the circuit of Table 5 nor that of Table 6 can be removed. Therefore the only sequence that can be realized as a ladder is $(0, j1, j2)$.

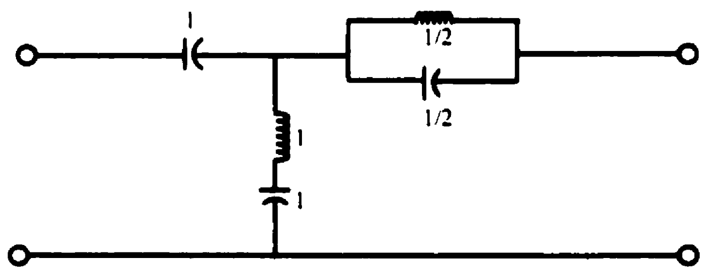


Fig. 3.1 Example circuit

It is known that the number of transmission zero sequences is m! where m is the number of transmission zeros. When m is large, checking all the sequences is impractical. We require a strategy for choosing a sequence as there are no necessary and sufficient conditions for choosing an appropriate sequence. Many researchers[10],[17],[22],[26] tried to develop simple solutions for ladder realization. The famous *Fujisawa condition* [26] has given a method, but it can only be applied to mid-series or mid-shunt low-pass ladder filter designs. The strategy from Göttlicher's thesis[23] appears to be best available for a range of filter designs, such as lowpass, highpass, bandpass and double bandpass filters.

3.3 Derivation of the scattering synthesis strategy

From Jarmasz's thesis[22], we know that if $\rho\ (j\omega_i) = \pm 1$, for $j\omega_i$ equal to ∞, 0 or $j\omega_0$, we can completely remove the transmission zero at ∞, 0, or $j\omega_0$. The section type will be one of the types from the tables in chapter 2. However if $\rho\ (j\omega_i) \neq \pm 1$ at a transmission zero $j\omega_i$ $(\omega_i \neq 0, \infty)$ a partial removal of a transmission zero at 0 or ∞ is necessary.

In Göttlicher's strategy, eight circuit classes make up the basic structure for ladder synthesis. Among the eight classes, four of them include a partial removal of a transmission zero at 0 or ∞ and a complete removal of a transmission zero at $j\omega_0$. These are designated classes A, C, E, G. In order to develop the scattering synthesis strategy, the necessary equations are derived in this section.

As is known, the reflectance has a close relation with the driving-point impedance. Therefore the derivation starts from the basic driving-point impedance as well as the driving-point admittance.

The driving-point impedance, admittance and their derivatives in term of scattering parameters can be written as follows:

$$Z = \frac{g+h}{g-h} = \frac{1+\rho}{1-\rho}$$

$$\frac{dZ}{ds} = \frac{(g-h)\left(g'+h'\right)-(g+h)\left(g'-h'\right)}{(g-h)^2} = \frac{-2\,(h/g)\left(\frac{g'}{g}-\frac{h'}{h}\right)}{(1-h/g)^2}$$

(3.3.1)

At a transmission zero, $s = j\omega_0$

$$\left.\frac{dZ}{ds}\right|_{s=j\omega_0} = \frac{\left.(-2)\,e^{j\alpha}\left(\frac{g'}{g}-\frac{h'}{h}\right)\right|_{s=j\omega_0}}{\left(1-e^{j\alpha}\right)^2} = \frac{d\,(j\omega_0)}{2\sin^2(\alpha/2)} \tag{3.3.2}$$

where $\alpha = \angle\rho\,(j\omega_0)$

$$Y = \frac{g-h}{g+h} = \frac{1-\rho}{1+\rho}$$

$$\frac{dY}{ds} = \frac{(g+h)\left(g'-h'\right)-(g-h)\left(g'+h'\right)}{(g+h)^2} = \frac{2\frac{h}{g}\left(\frac{g'}{g}-\frac{h'}{h}\right)}{(1+h/g)^2}$$

and

$$\left.\frac{dY}{ds}\right|_{s=j\omega_0} = \frac{\left.2e^{j\alpha}\left(\frac{g'}{g}-\frac{h'}{h}\right)\right|_{s=j\omega_0}}{\left(1-e^{j\alpha}\right)^2} = \frac{d\,(j\omega_0)}{2\cos^2(\alpha/2)} \tag{3.3.3}$$

The single elements, an inductor L or a capacitor C, required for the partial removal of a transmission zero, $s = j\omega_i$, can be determined as follows:

$$L = \frac{Z\,(j\omega_i)}{j\omega_i} \tag{3.3.4}$$

$$C = \frac{Y\,(j\omega_i)}{j\omega_i} \tag{3.3.5}$$

Since $\rho = \frac{Z\,(j\omega_i)-1}{Z\,(j\omega_i)+1} = \frac{j\omega_i L-1}{j\omega_i L+1} \Rightarrow |\rho| = 1 \Rightarrow \rho = e^{j\alpha_i}$, (3.3.6)

$$Z\,(j\omega) = \frac{1+\rho\,(j\omega_i)}{1-\rho\,(j\omega_i)} = \frac{1+e^{j\alpha_i}}{1-e^{j\alpha_i}} = \frac{j}{\tan(\alpha_i/2)} \tag{3.3.7}$$

where $\alpha_i = \angle\rho\,(j\omega_i)$

Substituting equation (3.3.7) into equation(3.3.4), we have

$$L = \frac{1}{\omega_i \tan(\alpha_i/2)} \tag{3.3.8}$$

Similarly, $\rho = \dfrac{1 - Y(j\omega_i)}{1 + Y(j\omega_i)} = \dfrac{1 - j\omega_i C}{1 + j\omega_i C} \Rightarrow |\rho| = 1 \Rightarrow \rho = e^{j\alpha_i}$ (3.3.9)

then

$$Y(j\omega_i) = \frac{1 - \rho(j\omega_i)}{1 + \rho(j\omega_i)} = \frac{1 - e^{j\alpha_i}}{1 + e^{j\alpha_i}} = -j\tan(\alpha_i/2) \tag{3.3.10}$$

and substituting equation (3.3.10) into equation (3.3.5), we have

$$C = \frac{-\tan(\alpha_i/2)}{\omega_i} \tag{3.3.11}$$

Next, scattering characteristics are derived for four circuits which are the first sections of Göttlicher's circuits which have been subdivided into 8 classes: A,B,C, D, E, F, G, H.

Class A: a partial removal of a transmission zero at 0 ($\rho(0) = 1$) and complete removal of a transmission zero at $j\omega_i$

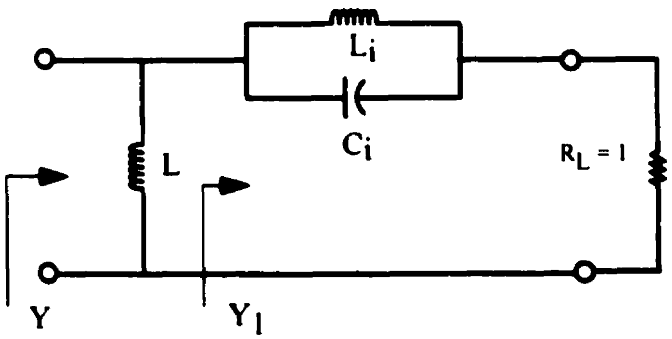


Fig. 3.2 The topology of the first section of a circuit class A

Since $\rho_a(j\omega_i) = \rho(j\omega_i) = e^{j\alpha_i}$, according to equations (3.3.4) and (3.3.7), a partial removal of a transmission zero at $s = 0$ with an inductance $L = \frac{1}{\omega_i \tan(\alpha_i/2)}$,

where $\alpha_i = \angle\rho(j\omega_i)$, yields $Y(j\omega_i) = Y_1(j\omega_i) = 0$ and the corresponding reflectance $\rho(j\omega_i) = +1$.

For admittance Y, we have

$$Y = \frac{1}{sL} + \frac{1}{\frac{1}{Y_i} + 1} = \frac{1}{sL} + \frac{Y_i}{1 + Y_i} \qquad \text{where } Y_i = sC_i + \frac{1}{sL_i}$$

Since $\frac{dY}{ds} = \frac{-1}{s^2 L} + \frac{(1+Y_i)\frac{dY_i}{ds} - Y_i\frac{dY_i}{ds}}{(1+Y_i)^2} = \frac{-1}{s^2 L} + \frac{\frac{dY_i}{ds}}{(1+Y_i)^2}$ and with $\omega_i^2 = \frac{1}{L_i C_i}$

$$\left.\frac{dY_i}{ds}\right|_{s = j\omega_i} = \frac{1}{\omega_i^2 L_i} - 2C_i, \qquad C_i = \frac{1}{2}\left.\frac{dY}{ds}\right|_{s = j\omega_i} - \frac{1}{2\omega_i^2 L} \tag{3.3.12.a}$$

Substituting equations (3.3.3) and (3.3.8) into (3.3.12), gives a set of formulas for the complete removal of a section type 5, that is capacitor C_i and inductor L_i.

$$C_i = \frac{d(j\omega_i)}{4\cos^2(\alpha_i/2)} - \frac{\tan(\alpha_i/2)}{2\omega_i}, \quad L_i = \frac{1}{\omega_i^2 C_i} \tag{3.3.12 b, c}$$

$$\text{where } d(j\omega_i) = \left.\left(\frac{g'}{g} - \frac{h'}{h}\right)\right|_{s = j\omega_i} \quad \text{and } \alpha_i = \angle\rho(j\omega_i)$$

The realizability condition for class A is $L \geq L_0 = \frac{d(0)}{2}$, where L_0 corresponds to a complete removal of a transmission zero at 0 with a shunt inductor.

Class C: a partial removal of a transmission zero at 0 ($\rho(0) = -1$) and complete removal of a transmission zero at $j\omega_i$

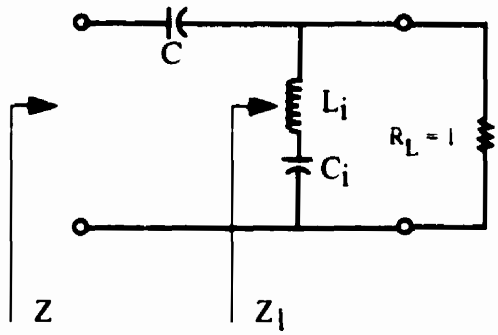


Fig. 3.3 The topology of the first section of class C

Since $\rho_a(j\omega_i) = \rho(j\omega_i) = e^{j\alpha_i}$, according to equations (3.3.9) and (3.3.11), the partial removal of a transmission zero at $s = 0$ with a capacitance $C = \dfrac{-\tan(\alpha_i/2)}{\omega_i}$, where $\alpha_i = \angle\rho(j\omega_i)$, yields $Z(j\omega_i) = Z_1(j\omega_i) = 0$. and the corresponding $\rho(\omega_i) = -1$.

For the impedance Z, we have

$$Z = \frac{1}{sC} + \frac{1}{Y_i + 1} = \frac{1}{sC} + \frac{Z_i}{1 + Z_i}$$

$$\frac{dZ}{ds} = \frac{-1}{s^2C} + \frac{(1+Z_i)\frac{dZ_i}{ds} - Z_i\frac{dZ_i}{ds}}{(1+Z_i)^2}$$

$$\left.\frac{dZ}{ds}\right|_{s=j\omega_i} = \frac{1}{\omega_i^2 C} + \left.\frac{dZ_i}{ds}\right|_{s=j\omega_i}$$

where $Z_i = sL_i + \frac{1}{sC_i}$ and $\omega_i^2 = \frac{1}{L_i C_i}$

$$\frac{dZ_i}{ds} = L_i - \frac{1}{s^2 C_i} \Rightarrow \left.\frac{dZ_i}{ds}\right|_{s=j\omega_i} = L_i + \frac{1}{\omega_i^2 C} = 2L_i$$

$$L_i = \left.\frac{1}{2}\frac{dZ}{ds}\right|_{s=j\omega_i} - \frac{1}{2\omega_i^2 C} \qquad \text{(3.3.13 a)}$$

Substituting equations (3.3.2) and (3.3.11) into equation (3.3.13 a), we get a complete removal of section 6, that is inductor L_i and capacitor C_i.

$$L_i = \frac{d(j\omega_i)}{4\sin^2\left(\frac{\alpha_i}{2}\right)} + \frac{1}{2\omega_i \tan\left(\frac{\alpha_i}{2}\right)}, \quad C_i = \frac{1}{\omega_i^2 L_i} \qquad \text{(3.3.13 b,c)}$$

The realizability condition is $C \geq C_0 = \frac{d(0)}{2}$, where C_0 corresponds to a complete removal of a transmission zero at 0 with a series capacitor.

Since the derivation for Class E is similar to class A, and that for Class G is similar to Class C, we omit them and only present the results.

Class E: partial removal of a transmission zero at ∞ $(\rho(\infty) = -1)$ and complete removal of transmission zero at $j\omega_i$

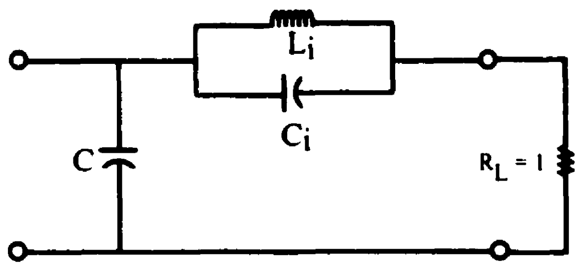


Fig. 3.4 The topology of the first section of a circuit class E

The capacitance $C = \dfrac{-\tan(\alpha_i/2)}{\omega_i}$ where $\alpha_i = \angle\rho(j\omega_i)$

The complete removal of a section type 5, capacitor

$$C_i = \frac{d(j\omega_i)}{4\cos^2(\alpha_i/2)} + \frac{\tan(\alpha_i/2)}{2\omega_i}, \text{ inductance } L_i = \frac{1}{\omega_i^2 C_i}$$

The realizability condition is $0 < C \le C_\infty = \dfrac{2}{d(\infty)}$, where C_∞ corresponds to a complete removal of a transmission zero at ∞ with a shunt capacitor.

Class G: partial removal of a transmission zero $(\rho(\infty) = 1)$ at ∞ and complete removal of transmission zero at $j\omega_i$

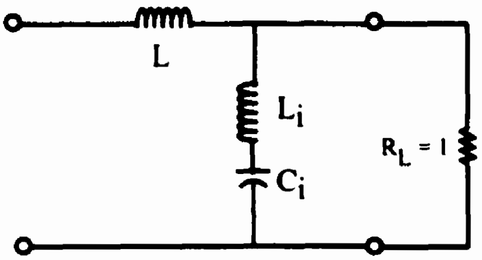


Fig. 3.5 The topology of the first section of a circuit class G

The series inductance $L = \frac{1}{\omega_i \tan(\alpha_i/2)}$, where $\alpha_i = \angle\rho(j\omega_i)$.

The complete removal of a section type 6, inductance

$$L_i = \frac{d(j\omega_i)}{4\sin^2(\alpha_i/2)} - \frac{1}{2\omega_i \tan(\alpha_i/2)}$$

capacitance $C_i = \frac{1}{\omega_i^2 L_i}$.

The realizability condition is $0 < L \le L_\infty = \frac{2}{d(\infty)}$, L_∞ corresponds to a complete removal of a transmission zero at ∞ with a series inductor.

Moreover, Göttlicher[23] has shown that $L_0 > L_\infty$ and $C_0 > C_\infty$ implying that for a given ω_i not both Classes A and G are realizable and not both Classes C and E are realizable.Furthermore, Göttlicher introduced a realizability measure, RM, which measures the margin of realizability; there are four cases:

$$RM = \frac{L}{L_0} \ge 1,\ RM = \frac{L_\infty}{L} \ge 1,\ RM = \frac{C}{C_0} \ge 1,\ RM = \frac{C_\infty}{C} \ge 1.$$

3.4 Realizablity conditions of Göttlicher circuits in term of scattering parameters

In this section the realizability condition for Göttlicher's circuits are present in the form of tables, and the circuit sections are designed by the type numbers from Chapter II. For example, 4_5_4_5 circuit denotes the connection of elementary section types 4,5,4,5. The realizability conditions are for a sequence of two successive transmission zeros, ω_i, ω_τ. Thus for a given ω_i the number of potential successor ω_τ can be determined with these conditions. Then, the ω_i with the maximum number of successors is chosen. If several transmission zeros have the same number of potential successors, the one with the smallest realizability measure, RM, is chosen. After the chosen transmission zero is removed, the process is repeated for the remaining transmission zeros until all the transmission zeros have been realized.

The subscript R is used to indicate the remaining circuit after the first transmission zero has been removed.

A sample calculation of the realizability conditions using circuit 4_5_1_6 will now be given. The results for the remaining circuits are presented in Table 7-34.

Circuit 4_5_1_6 has a transmission zero at 0 and $\rho(0) = -1$. Thus a shunt inductance can be removed. $L_0 = \frac{d(0)}{2}$ (see [22]).

To realize the finite transmission zero ω_i,

$$L_1 = \frac{1}{\omega_i \tan(\alpha_i/2)} \qquad \text{where } \alpha_i = \angle\rho(j\omega_i) \qquad \text{(see (3.3.8))}$$

must be extracted first.

For realizability $L_1 \geq L_0 > 0$ so that the remaining inductance is nonnegative. The measure of realizability, RM, is defined as $RM = \frac{L_1}{L_0} \geq 1$.

According to Eqs.(3.3.12 b, c)

$$C_i = \frac{d(j\omega_i)}{4\cos^2(\alpha_i/2)} - \frac{1}{2\omega_i^2 L_1}, \quad L_i = \frac{1}{\omega_i^2 C_i}$$

For the second section a partial removal of the transmission zero at ∞ is used.

$$L_\infty = \frac{2}{d(\infty)} \qquad \text{(see [22])}$$

At $s = \infty$ the circuit behaves as shown:

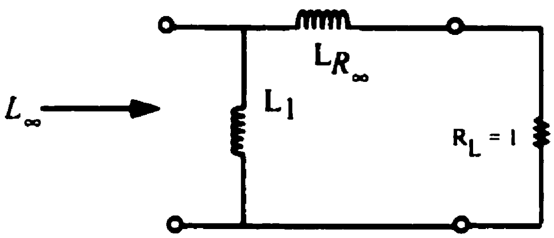


Fig. 3.6

Therefore $L_{R_\infty} = \frac{1}{1/L_\infty - 1/L_1}$.

Define $L_{1\tau} = \frac{1}{\omega_\tau \tan(\alpha_\tau/2)}$, where $\alpha_\tau = \angle\rho(j\omega_\tau)$ (see Eqs. (3.3.4) and (3.3.8))

At the transmission zero $s = j\omega_\tau$, the input admittance (see Table 9)

$$Y(j\omega_\tau) = \frac{1}{Z(j\omega_\tau)} = \frac{1}{j\omega_\tau L_1} + \frac{1}{Z_i(j\omega_\tau) + j\omega_\tau L_2} \quad \text{where}$$

Z_i is the impedance of the parallel connection of L_i and C_i.

Then

$$L_2 = \frac{1}{1/\left(\frac{Z(j\omega_\tau)}{j\omega_\tau}\right) - 1/L_1} - \frac{Z_i(j\omega_\tau)}{j\omega_\tau}$$

$$= \frac{1}{1/L_{1\tau} - 1/L_1} - \frac{L_i}{1 - \left(\frac{\omega_\tau}{\omega_i}\right)^2}$$

Realizability requires that $0 \le L_2 \le L_{R\infty}$ so that L_2 is nonnegative and so that the remaining inductance after the removal of L_2 is also nonnegative.The realizability measure, RM, is defined as $RM = \frac{L_{R\infty}}{L_2} \ge 1$.

The calculation of $d_R(0)$ and $d_R(\infty)$ in the case of double zeros at 0 and ∞ is given in Appendix II.

Table 7: Circuit 4_5_4_5

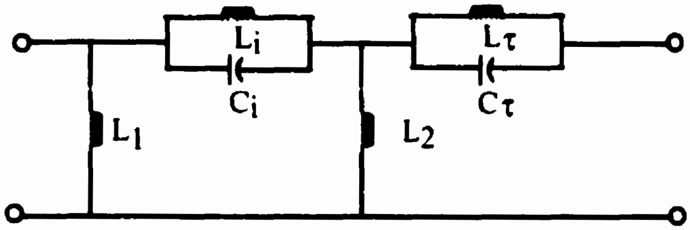
Class A	$\rho(0) = -1$, $Tr.Zeros = (\omega_i, \omega_\tau)$
Circuit_1	
Section_i	$L_0 = \dfrac{d(0)}{2}$ $L_1 = \dfrac{1}{\omega_i \tan(\alpha_i/2)}$ $\quad \alpha_i = \angle\rho(j\omega_i)$ $C_i = \dfrac{1}{4}\dfrac{d(j\omega_i)}{\cos^2(\alpha_i/2)} - \dfrac{1}{2\omega_i^2 L_1}$ $\quad L_i = \dfrac{1}{\omega_i^2 C_i}$ Realizability: $L_1 \geq L_0 > 0$ $\quad RM = \dfrac{L_1}{L_0} \geq 1$
Section_τ	$L_{Ro} = \dfrac{1}{1/L_0 - 1/L_1} - L_i$ $L_{1\tau} = \dfrac{1}{\omega_\tau \tan(\alpha_\tau/2)}$ $\quad \alpha_\tau = \angle\rho(j\omega_\tau)$ $L_2 = \dfrac{1}{1/L_{1\tau} - 1/L_1} - \dfrac{L_i}{1-(\omega_\tau/\omega_i)^2}$ Realizablity: $L_2 \geq L_{Ro} > 0$, $RM = \dfrac{L_2}{L_{R_0}} \geq 1$

Table 8: Circuit 4_5_3_5

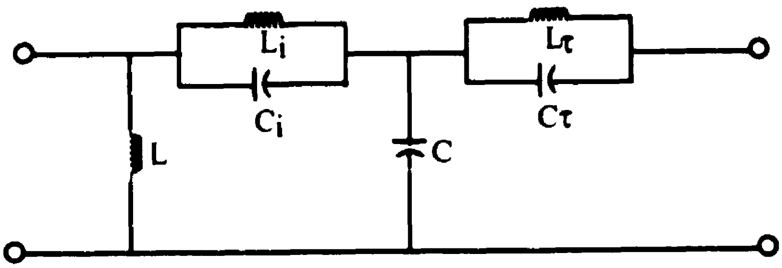
Class A	$\rho(0) = -1, \rho(\infty) = -1, Tr.Zeros = (\omega_i, \omega_\tau)$
Circuit_2	
Section_i	$L_0 = \dfrac{d(0)}{2}$ $L = \dfrac{1}{\omega_i \tan(\alpha_i/2)}$ $\alpha_i = \angle\rho(j\omega_i)$ $C_i = \dfrac{1}{4}\dfrac{d(j\omega_i)}{\cos^2(\alpha_i/2)} - \dfrac{1}{2\omega_i^2 L}$ $L_i = \dfrac{1}{\omega_i^2 C_i}$ Realizablity: $L \geq L_0 > 0$, $RM = \dfrac{L}{L_0} \geq 1$
Section_τ	$C_\infty = \dfrac{2}{d(\infty)}$ $C_{R\infty} = \dfrac{1}{1/C_\infty - 1/C_i}$ $C_{1\tau} = \dfrac{-\tan(\alpha_\tau/2)}{\omega_\tau}$, $\alpha_\tau = \angle\rho(j\omega_\tau)$ $C = \dfrac{1}{\dfrac{1}{\omega_\tau^2 L} + C_{1\tau} - \dfrac{1}{C_i - 1/\left(\omega_\tau^2 L_i\right)^2}}$ Realizability: $0 < C \leq C_{R\infty}$, $RM = \dfrac{C_{R\infty}}{C} \geq 1$

Table 9: Circuit 4_5_1_6

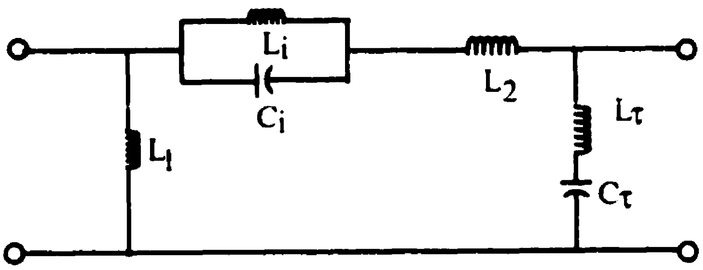
Class A	$\rho(0) = -1,\ \rho(\infty) = +1,\ Tr.Zeros = (\omega_i, \omega_\tau)$
Circuit_3	
Section_i	$L_0 = \frac{d(0)}{2}\quad L_1 = \frac{1}{\omega_i \tan(\alpha_i/2)}\quad \alpha_i = \angle\rho(j\omega_i)$ $C_i = \frac{1}{4}\frac{d(j\omega_i)}{\cos^2(\alpha_i/2)} - \frac{1}{2\omega_i^2 L_1}\quad L_i = \frac{1}{\omega_i^2 C_i}$ Realizability: $L_1 \geq L_0 > 0$, $RM = \frac{L_1}{L_0} \geq 1$
Section_τ	$L_\infty = \frac{2}{d(\infty)}$ $L_{R\infty} = \frac{1}{1/L_\infty - 1/L_1}\quad L_{1\tau} = \frac{1}{\omega_\tau \tan(\alpha_\tau/2)}\quad \alpha_\tau = \angle\rho(j\omega_\tau)$ $L_2 = \frac{1}{(1/L_{1\tau}) - (1/L_1)} - \frac{L_i}{1 - \left(\frac{\omega_\tau}{\omega_i}\right)^2}$ Realizability: $0 < L_2 \leq L_{R\infty}$, $RM = \frac{L_{R\infty}}{L_2} \geq 1$

Table 10: Circuit 4_3_6

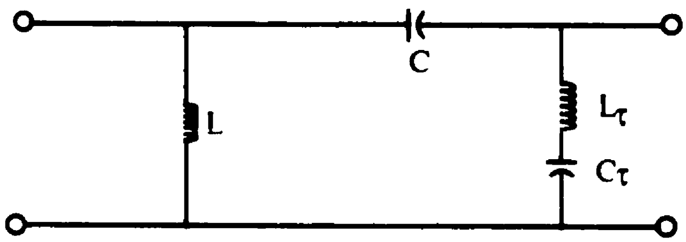
Class B	$\rho(0) = -1, Tr.Zeros = (0, \omega_\tau)$
Circuit_4	
Section_i	$L = L_0 = \frac{d(0)}{2}$ Realizability: $0 < L_0 \le L \quad RM = \frac{L}{L_0} \ge 1$
Section_τ	$d_R(0) = \frac{1}{d(0)}\left(\frac{g''(0)}{g(0)} + \frac{h''(0)}{h(0)} - \frac{2}{3d(0)}\left(\frac{g'''(0)}{g(0)} - \frac{h'''(0)}{h(0)}\right)\right)$ $C_{R0} = \frac{d_R(0)}{2}$ $C = \frac{-\tan(\alpha_\tau/2)}{\omega_\tau} + \frac{1}{\omega_\tau^2 L} \quad \alpha_\tau = \angle\rho(j\omega_\tau)$ Realizability: $0 < C_{R0} \le C, RM = \frac{C}{C_{R0}} \ge 1$

Table 11: Circuit 4_2_5

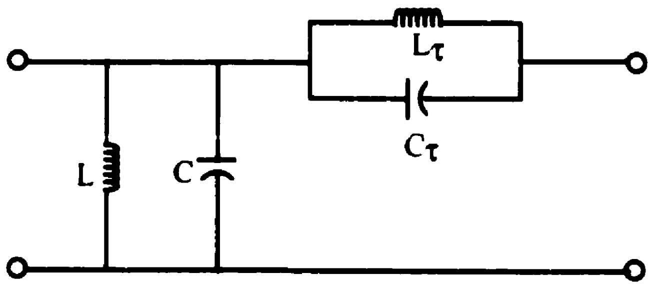
Class B	$\rho(0) = -1, \rho(\infty) = -1, Tr.Zeros = (0, \omega_\tau)$
Circuit_5	
Section_i	$L = L_0 = \frac{d(0)}{2}$ Realizability: $0 < L_0 \le L$, $RM = \frac{L}{L_0} \ge 1$
Section_τ	$C_\infty = \frac{2}{d(\infty)}$ $C = \frac{-\tan(\alpha_\tau/2)}{\omega_\tau} + \frac{1}{\omega^2_\tau L}$ $\alpha_\tau = \angle\rho(j\omega_\tau)$ Realizability: $0 < C \le C_\infty$, $RM = \frac{C_\infty}{C} \ge 1$

Table 12: Circuit 4_1_6

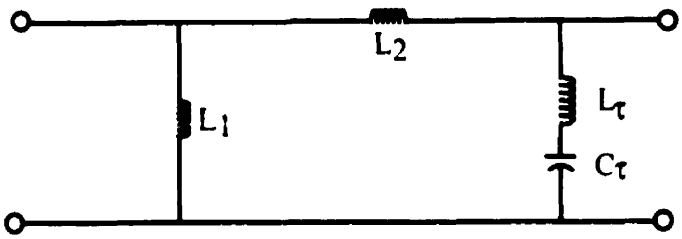
Class B	$\rho(0) = -1, \rho(\infty) = 1, Tr.Zeros = (0, \omega_\tau)$
Circuit_6	
Section_i	$L_1 = L_0 = \frac{d(0)}{2}$ Realizability: $0 < L_0 \le L_1$, $RM = \frac{L_1}{L_0} \ge 1$
Section_τ	$L_\infty = \frac{2}{d(\infty)}$ $L_{R\infty} = \frac{1}{1/L_\infty - 1/L_1}$ $L_{1\tau} = \frac{1}{\omega_\tau \tan(\alpha_\tau/2)}$, $\alpha_\tau = \angle\rho(j\omega_\tau)$ $L_2 = \frac{1}{\frac{1}{L_{1\tau}} - \frac{1}{L_1}}$ Realizability: $0 < L_2 \le L_{R\infty}$, $RM = \frac{L_{R\infty}}{L_2} \ge 1$

Table 13: Circuit 3_6_3_6

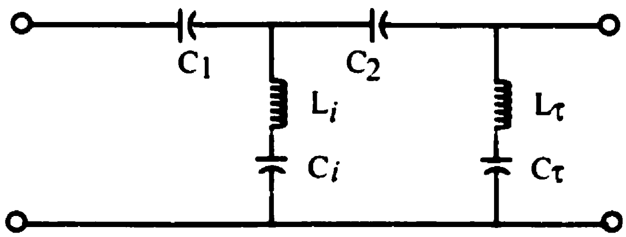
Class C	$\rho(0) = 1,\ Tr.Zeros = (\omega_i, \omega_\tau)$
Circuit_7	
Section_i	$C_0 = \frac{d(0)}{2} \quad C_1 = \frac{-\tan\left(\frac{\alpha_i}{2}\right)}{\omega_i}$ $L_i = \frac{1}{4}\frac{d(j\omega_i)}{\sin^2(\alpha_i/2)} - \frac{1}{2\omega_i C_1} \quad C_i = \frac{1}{\omega_i^2 L_i}$ Realizability: $0 < C_0 \le C_1$, $RM = \frac{C_1}{C_n} \ge 1$
Section_τ	$C_{R0} = \frac{1}{1/C_0 - 1/C_1} - C_i$ $C_{1\tau} = \frac{-\tan(\alpha_i/2)}{\omega_i}$ $C_2 = \frac{1}{1/C_{1\tau} - 1/C_1} - \frac{C_i}{\left(1 - \left(\frac{\omega_\tau}{\omega_i}\right)\right)^2}$ Realizability: $0 < C_{R0} \le C_2$, $RM = \frac{C_2}{C_{R0}} \ge 1$

Table 14: Circuit 3_6_2_5

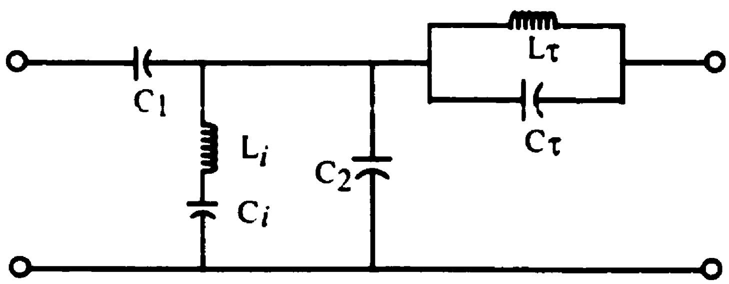
Class C	$\rho(0) = 1, \rho(\infty) = -1, Tr.Zeros = (\omega_i, \omega_\tau)$
Circuit_8	
Section_i	$C_0 = \frac{d(0)}{2}$ $\quad C_1 = \frac{-\tan(\alpha_i/2)}{\omega_i}$ $\quad \alpha_i = \angle\rho(j\omega_i)$ $L_i = \frac{1}{4}\frac{d(j\omega_i)}{\sin^2(\alpha_i/2)} - \frac{1}{2\omega_i^2 C_1}$ $\quad C_i = \frac{1}{\omega_i^2 L_i}$ Realizability: $0 < C_0 \le C_1$, $RM = \frac{C_1}{C_0} \ge 1$
Section_τ	$C_{R\infty} = \frac{1}{1/C_\infty - 1/C_1}$ $\quad C_{1\tau} = \frac{-\tan(\alpha_\tau/2)}{\omega_\tau}$ $\quad \alpha_\tau = \angle\rho(j\omega_\tau)$ $C_2 = \frac{1}{1/C_{1\tau} - 1/C_1} - \frac{C_i}{1 - \left(\frac{\omega_\tau}{\omega_i}\right)^2}$ Realizability: $0 < C_2 \le C_{R\infty}$ $RM = \frac{C_{R\infty}}{C_2} \ge 1$

Table 15: Circuit 3_6_1_6

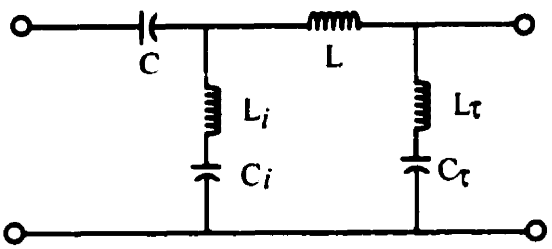
Class C	$\rho(0) = 1, \rho(\infty) = 1, Tr.Zeros = (\omega_i, \omega_\tau)$
Circuit_9	
Section_i	$C_0 = \frac{d(0)}{2}$ $C = \frac{-\tan(\alpha_i/2)}{\omega_i}, \ \alpha_i = \angle\rho(j\omega_i)$ $L_i = \frac{1}{4}\frac{d(j\omega_i)}{\sin^2(\alpha_i/2)} - \frac{1}{2\omega_i^2 C} \quad C_i = \frac{1}{\omega_i^2 L_i}$ Realizability: $0 < C_0 \le C$, $RM = \frac{C}{C_0} \ge 1$
Section_τ	$L_{R\infty} = \frac{1}{1/L_\infty - 1/L_i}$ $L_{1\tau} = \frac{1}{\omega_\tau \tan(\alpha_\tau/2)}, \quad \alpha_\tau = \angle\rho(j\omega_\tau)$ $L = \frac{1}{1/\left(L_{1\tau} + 1/\left(\omega_\tau^2 C\right)\right) - 1/\left(L_i\left(1 - (\omega_i/\omega_\tau)^2\right)\right)}$ Realizability: $0 < L \le L_{R\infty}$, $RM = \frac{L_{R\infty}}{L} \ge 1$

Table 16: Circuit 3_4_5

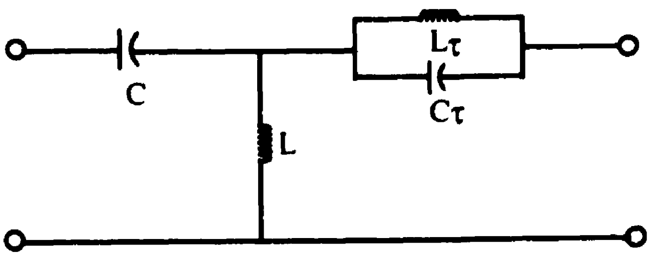
Class D	$\rho(0) = 1,\ Tr.Zeros = (0, \omega_\tau)$
Circuit_10	
Section_i	$C = C_0 = \dfrac{d(0)}{2}$ Realizability: $C \geq C_0 > 0,\ RM = \dfrac{C}{C_0} \geq 1$
Section_τ	$d_R(0) = \dfrac{1}{d(0)}\left(\dfrac{g''(0)}{g(0)} + \dfrac{h''(0)}{h(0)} - \dfrac{2}{3d(0)}\left(\dfrac{g'''(0)}{g(0)} - \dfrac{h'''(0)}{h(0)}\right)\right)$ $L_{R0} = \dfrac{d_R(0)}{2}$ $L = \dfrac{1}{\omega_\tau \tan(\alpha_\tau/2)} + \dfrac{1}{\omega_\tau^2 C},\ \alpha_\tau = \angle\rho(j\omega_\tau)$ Realizability: $0 < L_{R0} \leq L,\ RM = \dfrac{L}{L_{R0}} \geq 1$

Table 17: Circuit 3_2_5

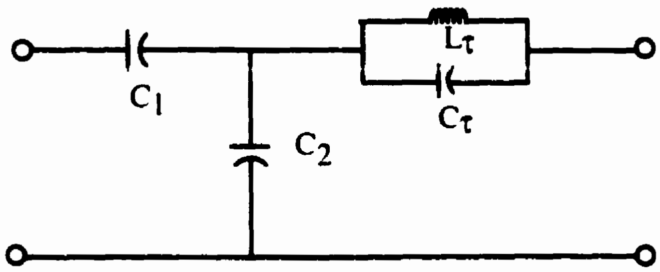
Class D	$\rho(0) = 1, \rho(\infty) = -1, Tr.Zeros = (0, \omega_\tau)$
Circuit_11	
Section_i	$C_1 = C_0 = \frac{d(0)}{2}$ Realizability: $C_1 \geq C_0 > 0$, $RM = \frac{C_1}{C_0} \geq 1$
Section_τ	$C_{R\infty} = \frac{1}{1/C_\infty - 1/C_1}$ $C_{1\tau} = -\frac{\tan(\alpha_\tau/2)}{\omega_\tau}$, $\alpha_\tau = \angle\rho(j\omega_\tau)$ $C_2 = \frac{1}{\frac{1}{C_{1\tau}} - \frac{1}{C_1}}$ Realizability: $0 < C_2 \leq C_{R\infty}$, $RM = \frac{C_{R\infty}}{C_2} \geq 1$

Table 18: Circuit 3_1_6

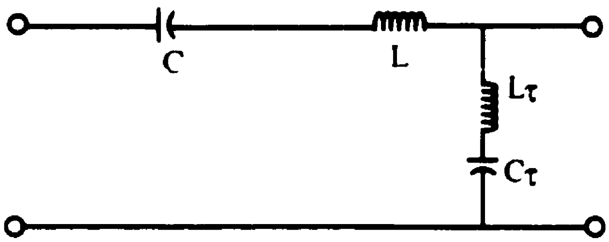
Class D	$\rho(0) = 1, \rho(\infty) = 1, Tr.Zeros = (0, \omega_\tau)$
Circuit_ 12	
Section_i	$C = C_0 = \dfrac{d(0)}{2}$ Realizability: $0 < C_0 \le C$, $RM = \dfrac{C}{C_0} \ge 1$
Section_τ	$L_{R\infty} = L_\infty = \dfrac{2}{d(\infty)}$ $L = \dfrac{1}{\omega_\tau \tan(\alpha_\tau/2)} + \dfrac{1}{\omega_\tau^2 C}, \alpha_\tau = \angle\rho(j\omega_\tau)$ Realizability: $0 < L \le L_{R\infty}$, $RM = \dfrac{L_{R\infty}}{L} \ge 1$

Table 19: Circuit 2_5_1_5

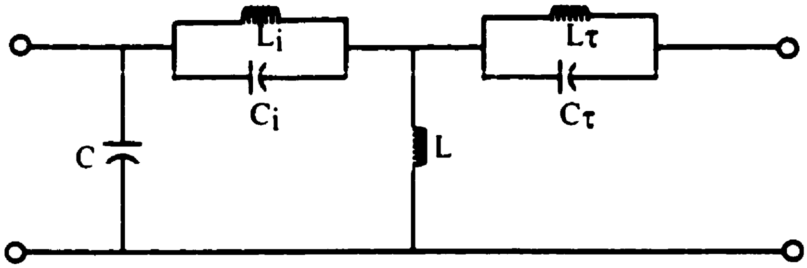
Class E	$\rho(0) = -1, \rho(\infty) = -1, Tr.Zeros = (\omega_i, \omega_\tau)$
Circuit_13	
Section_i	$C_\infty = \frac{2}{d(\infty)}$ $C = \frac{-\tan(\alpha_i/2)}{\omega_i}$ $\alpha_i = \angle\rho(j\omega_i)$ $C_i = \frac{1}{4}\frac{d(j\omega_i)}{\cos^2(\alpha_i/2)} - \frac{C}{2}$ $L_i = \frac{1}{\omega_i^2 C_i}$ Realizability: $0 < C \le C_\infty$, $RM = \frac{C_\infty}{C} \ge 1$
Section_τ	$L_0 = \frac{d(0)}{2}$ $L_{R0} = L_0 - L_i$ $L = \frac{1}{1/L_{1\tau} + \omega_\tau^2 C} - \frac{L_i}{1 - \left(\frac{\omega_i}{\omega_\tau}\right)^2}$ $L_{1\tau} = \frac{1}{\omega_\tau \tan(\alpha_\tau/2)}$, $\alpha_\tau = \angle\rho(j\omega_\tau)$ Realizability: $0 < L_{R0} \le L$, $RM = \frac{L}{L_{R0}} \ge 1$

Table 20: Circuit 2_5_1_6

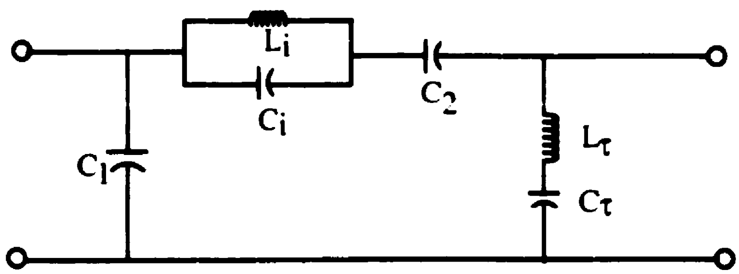
Class E	$\rho(0) = 1, \rho(\infty) = -1, Tr.Zeros = (\omega_i, \omega_\tau)$
Circuit_14	
Section_i	$C_\infty = \frac{2}{d(\infty)}$ $C_1 = \frac{-\tan(\alpha_i/2)}{\omega_i}$, $\alpha_i = \angle\rho(j\omega_i)$ $C_i = \frac{1}{4}\frac{d(j\omega_i)}{\cos^2(\alpha_i/2)} - \frac{C_1}{2}$ $L_i = \frac{1}{\omega_i^2 C_i}$ Realizability: $0 < C_1 \le C_\infty$, $RM = \frac{C_\infty}{C_1} \ge 1$
Section_τ	$C_0 = \frac{d(0)}{2}$ $C_{R0} = C_0 - C_1$ $C_2 = \cfrac{1}{\cfrac{1}{C_{1\tau} - C_1} - \cfrac{1}{C_i\left(1 - \left(\frac{\omega_i}{\omega_\tau}\right)^2\right)}}$ $C_{1\tau} = \frac{-\tan(\alpha_\tau/2)}{\omega_\tau}$, $\alpha_\tau = \angle\rho(j\omega_\tau)$ Realizability: $0 < C_{R0} \le C_2$, $RM = \frac{C_2}{C_{R0}} \ge 1$

Table 21: Circuit 2_5_2_5

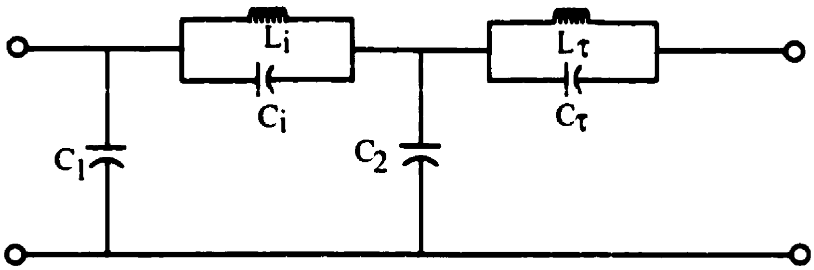
Class E	$(\rho)\ (\infty) = -1$, $Tr.Zeros = (\omega_i, \omega_\tau)$
Circuit_ 15	
Section_i	$C_\infty = \frac{2}{d(\infty)}$ $C_1 = \frac{-\tan(\alpha_i/2)}{\omega_i}$, $\alpha_i = \angle\rho(j\omega_i)$ $C_i = \frac{1}{4}\frac{d(j\omega_i)}{\cos^2(\alpha_i/2)} - \frac{C_1}{2}$ $L_i = \frac{1}{\omega_i^2 C_i}$ Realizability: $0 < C_1 \le C_\infty$, $RM = \frac{C_\infty}{C_1} \ge 1$
Section_τ	$C_{R\infty} = \frac{1}{1/(C_\infty - C_1) - 1/C_i}$ $C_{1\tau} = \frac{-\tan(\alpha_\tau/2)}{\omega_\tau}$, $\alpha_\tau = \angle\rho(j\omega_\tau)$ $C_2 = \frac{1}{1/(C_{1\tau} - C_1) - 1/\left(C_i\left(1 - \left(\frac{\omega_i}{\omega_\tau}\right)^2\right)\right)}$ Realizability: $0 < C_2 \le C_{R\infty}$, $RM = \frac{C_{R\infty}}{C_2} \ge 1$

Table 22: Circuit 2_4_5

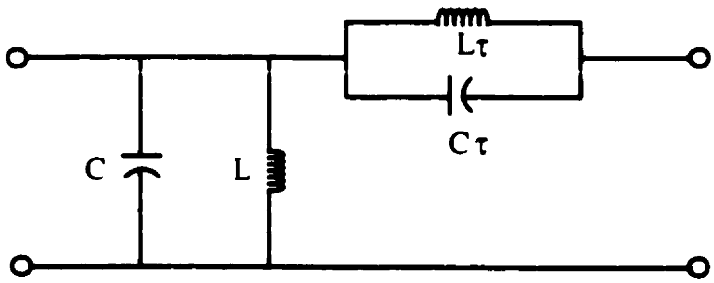
Class F	$\rho(0) = -1, \rho(\infty) = -1, Tr.Zeros = (\infty, \omega_\tau)$
Circuit_16	
Section_i	$C = C_\infty = \dfrac{2}{d(\infty)}$ Realizability: $0 < C \le C_\infty$, $RM = \dfrac{C_\infty}{C} \ge 1$
Section_τ	$L_0 = \dfrac{d(0)}{2}$ $L = \dfrac{1}{\omega_\tau \tan(\alpha_\tau/2) + \omega_\tau^2 C}$, $\alpha_\tau = \angle\rho(j\omega_\tau)$ Realizability: $L \ge L_0$, $RM = \dfrac{L}{L_0} \ge 1$

Table 23: Circuit 2_3_6

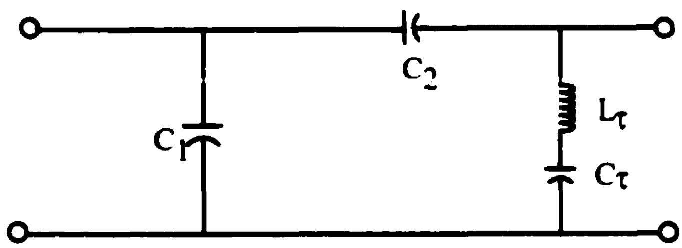
Class F	$\rho(0) = 1, \rho(\infty) = -1, Tr.Zeros = (\infty, \omega_\tau)$
Circuit_17	
Section_i	$C_\infty = C_1 = \frac{2}{d(\infty)}$ Realizability: $0 < C_1 \le C_\infty$, $RM = \frac{C_\infty}{C_1} \ge 1$
Section_τ	$C_0 = \frac{d(0)}{2}$ $C_{R0} = C_0 - C_1$ $C_2 = \frac{-\tan(\alpha_\tau/2)}{\omega_\tau} - C_1$ Realizability: $0 < C_{R0} \le C_2$, $RM = \frac{C_2}{C_{R0}} \ge 1$

Table 24: Circuit 2_1_6

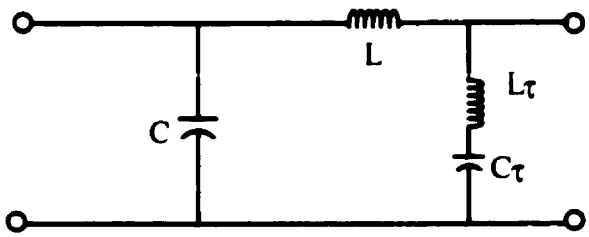
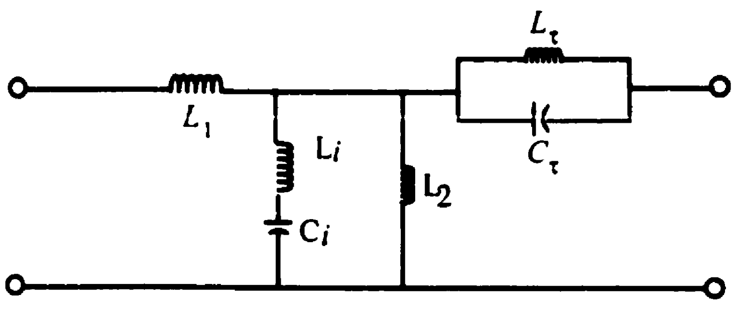
Class F	$\rho(\infty) = -1,\ Tr.Zeros = (\infty, \omega_\tau)$
Circuit_18	
Section_i	$C = C_\infty = \dfrac{2}{d(\infty)}$ Realizability: $0 < C \le C_\infty$, $RM = \dfrac{C_\infty}{C} \ge 1$
Section_τ	$d_R(\infty) = \dfrac{1}{d(\infty)}\left(\dfrac{g''(\infty)}{g(\infty)} + \dfrac{h''(\infty)}{h(\infty)} - \dfrac{2}{3d(\infty)}\left(\dfrac{g'''(\infty)}{g(\infty)} - \dfrac{h'''(\infty)}{h(\infty)}\right)\right)$ $L_{R\infty} = \dfrac{2}{d_R(\infty)}$ $L = \dfrac{1}{\omega_\tau \tan(\alpha_\tau/2) + \omega_\tau^2 C},\ \alpha_\tau = \angle\rho(j\omega_\tau)$ Realizability: $0 < L \le L_{R\infty}$, $RM = \dfrac{L_{R\infty}}{L} \ge 1$

Table 25: Circuit 1_6_4_5

<table>
<tr><td>Class G</td><td>$\rho(\infty) = 1, \rho(0) = -1, Tr.Zeros = (\omega_i, \omega_\tau)$</td></tr>
<tr><td>Circuit_19</td><td>



</td></tr>
<tr><td>Section_i</td><td>

$L_\infty = \frac{2}{d(\infty)} \quad L_1 = \frac{1}{\omega_i \tan(\alpha_i/2)}, \alpha_i = \angle\rho(j\omega_i)$

$L_i = \frac{1}{4}\frac{d(j\omega_i)}{\sin^2(\alpha_i/2)} - L_1/2 \qquad C_i = \frac{1}{\omega_i^2 L_i}$

Realizability: $0 < L_1 \le L_\infty,\ RM = \frac{L_\infty}{L_1} \ge 1$

</td></tr>
<tr><td>Section_τ</td><td>

$L_0 = \frac{d(0)}{2}$

$L_{R0} = L_0 - L_1$

$L_{1\tau} = \frac{1}{\omega_\tau \tan(\alpha_\tau/2)}, \alpha_\tau = \angle\rho(j\omega_\tau)$

$L_2 = \frac{1}{1/(L_{1\tau} - L_1) - 1/\left(L_i\left(1 - \left(\frac{\omega_i}{\omega_\tau}\right)^2\right)\right)}$

Realizability: $0 < L_{R0} \le L_2, RM = \frac{L_2}{L_{R0}} \ge 1$

</td></tr>
</table>

Table 26: Circuit 1_6_3_6

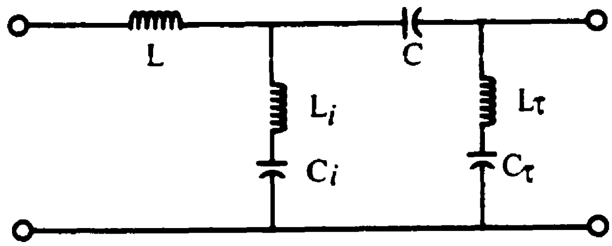
Class G	$\rho(\infty) = 1, \rho(0) = 1, Tr.Zeros = (\omega_i, \omega_\tau)$
Circuit_20	
Section_i	$L_\infty = \dfrac{2}{d(\infty)}$ $L = \dfrac{1}{\omega_i \tan(\alpha_i/2)}, \ \alpha_i = \angle\rho(j\omega_i)$ $L_i = \dfrac{1}{4}\dfrac{d(j\omega_i)}{\sin^2(\alpha_i/2)} - L/2 \qquad C_i = \dfrac{1}{\omega_i^2 L_i}$ Realizability: $0 < L \le L_\infty$, $RM = \dfrac{L_\infty}{L} \ge 1$
Section_τ	$C_0 = \dfrac{d(0)}{2} \qquad C_{R0} = C_0 - C_i$ $C = \dfrac{1}{\omega_\tau^2 L - \omega_\tau/(\tan(\alpha_\tau/2))} - \dfrac{C_i}{1 - \left(\dfrac{\omega_\tau}{\omega_i}\right)^2}$ Realizability: $0 < C_{R0} \le C$, $RM = \dfrac{C}{C_{R0}} \ge 1$

Table 27: Circuit 1_6_1_6

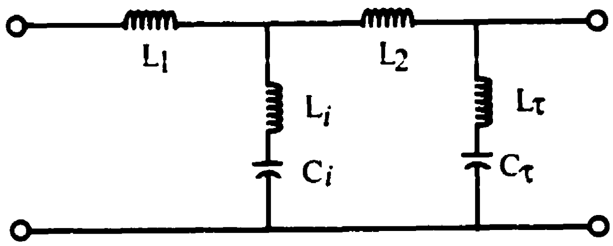
Class G	$\rho(\infty) = 1,\ Tr.Zeros = (\omega_i, \omega_\tau)$
Circuit 21	
Section_i	$L_\infty = \dfrac{2}{d(\infty)}$ $L_1 = \dfrac{1}{\omega_i \tan(\alpha_i/2)}$ $\alpha_i = \angle\rho(j\omega_i)$ $L_i = \dfrac{1}{4}\dfrac{d(j\omega_i)}{\sin^2(\alpha_i/2)} - L_1/2$ $C_i = \dfrac{1}{\omega_i^2 L_i}$ Realizability: $0 < L_1 \le L_\infty$, $RM = \dfrac{L_\infty}{L_1} \ge 1$
Section_τ	$L_{R\infty} = \dfrac{1}{1/(L_\infty - L_1) - 1/L_i}$ $L_2 = \dfrac{1}{1/(L_{1\tau} - L_1) - 1/\left(L_i\left(1 - \left(\dfrac{\omega_i}{\omega_\tau}\right)^2\right)\right)}$ $L_{1\tau} = \dfrac{1}{\omega_\tau \tan(\alpha_\tau/2)}$ $\alpha_\tau = \angle\rho(j\omega_\tau)$ Realizability: $0 < L_2 \le L_{R\infty}$, $RM = \dfrac{L_{R\infty}}{L_2} \ge 1$

Table 28: Circuit 1_4_5

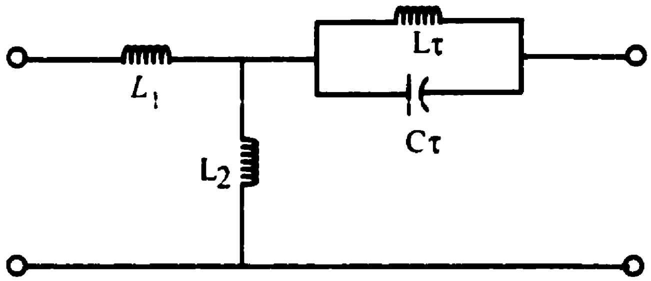
Class H	$\rho(\infty) = 1, \rho(0) = -1, Tr.Zeros = (\infty, \omega_\tau)$
Circuit_22	
Section_i	$L_1 = L_\infty = \frac{2}{d(\infty)}$ Realizability: $0 < L_1 \le L_\infty$, $RM = \frac{L_\infty}{L_1} \ge 1$
Section_τ	$L_0 = \frac{d(0)}{2}$, $L_{R0} = L_0 - L_1$ $L_2 = \frac{1}{\omega_\tau \tan(\alpha_\tau/2)} - L_1$, $\alpha_\tau = \angle\rho(j\omega_\tau)$ Realizability: $0 < L_{R0} \le L_2$, $RM = \frac{L_2}{L_{R0}} \ge 1$

Table 29: Circuit 1_3_6

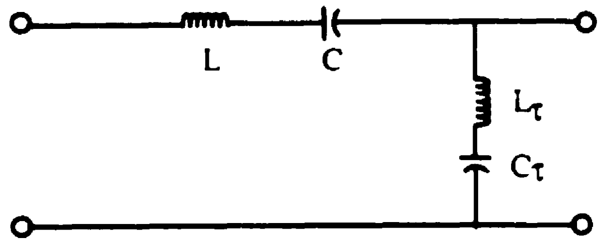
Class H	$\rho(\infty) = 1, \rho(0) = 1, Tr.Zeros = (\infty, \omega_\tau)$
Circuit_23	
Section_i	$L = L_\infty = \dfrac{2}{d(\infty)}$ Realizability: $0 < L \le L_\infty$, $RM = \dfrac{L_\infty}{L} \ge 1$
Section_τ	$C_{R0} = \dfrac{d(0)}{2}$ $C = \dfrac{1}{\omega_\tau^2 L - \omega_\tau / (\tan(\alpha_\tau/2))}$, $\alpha_\tau = \angle\rho(j\omega_\tau)$ Realizability: $0 < C_{R0} \le C$, $RM = \dfrac{C}{C_{R0}} \ge 1$

Table 30: Circuit 1_2_5

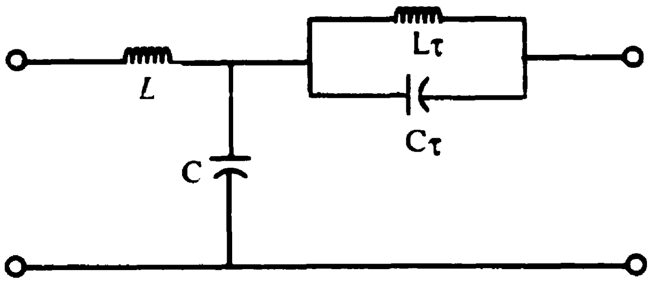
Class H	$\rho(\infty) = 1,\ Tr.Zeros = (\infty, \omega_\tau)$
Circuit_24	
Section_i	$L = L_\infty = \dfrac{2}{d(\infty)}$ Realizability: $0 < L \le L_\infty,\ RM = \dfrac{L_\infty}{L} \ge 1$
Section_τ	$d_R(\infty) = \dfrac{1}{d(\infty)}\left(\dfrac{g''(\infty)}{g(\infty)} + \dfrac{h''(\infty)}{h(\infty)} - \dfrac{2}{3d(\infty)}\left(\dfrac{g'''(\infty)}{g(\infty)} - \dfrac{h'''(\infty)}{h(\infty)}\right)\right)$ $C_{R\infty} = \dfrac{2}{d_R(\infty)}$ $C = \dfrac{1}{\omega_\tau^2 L - \omega_\tau / (\tan(\alpha_\tau/2))},\ \alpha_\tau = \angle\rho(j\omega_\tau)$ Realizability: $0 < C \le C_{R\infty},\ RM = \dfrac{C_{R\infty}}{C} \ge 1$

Table 31: Circuit 4_5_3_6

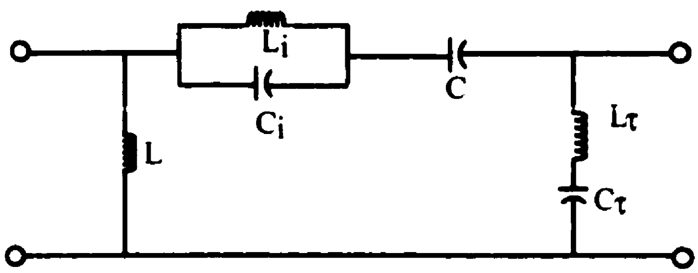
Class A	$\rho(0) = -1$, $Tr.Zeros = (\omega_i, \omega_\tau)$
Circuit_25	
Section_i	$L = L_0 = \frac{d(0)}{2}$ $\alpha_i = \angle\rho(j\omega_i)$ $C_i = \frac{1}{4}\frac{d(j\omega_i)}{\cos^2(\alpha_i/2)} - \frac{1}{2\omega_i^2 L}$ $L_i = \frac{1}{\omega_i^2 C_i}$ Realizability: $0 < L_0 \le L$, $RM = \frac{L}{L_0} \ge 1$
Section_τ	$d_R(0) = \frac{1}{d(0)}\left(\frac{g''(0)}{g(0)} + \frac{h''(0)}{h(0)} - \frac{2}{3d(0)}\left(\frac{g'''(0)}{g(0)} - \frac{h'''(0)}{h(0)}\right)\right)$ $C_{R0} = \frac{d_R(0)}{2}$ $C_{1\tau} = \frac{-\tan(\alpha_\tau/2)}{\omega_\tau}$, $\alpha_\tau = \angle\rho(j\omega_\tau)$ $C = \frac{1}{\frac{1}{C+\frac{1}{\omega_\tau^2 L}} + \frac{L_i\omega_\tau^2}{1-\left(\frac{\omega_\tau}{\omega_i}\right)^2}}$ Realizability: $0 < C_{R_0} \le C$, $RM = \frac{C}{C_{R0}} \ge 1$

Table 32: Circuit 3_6_4_5

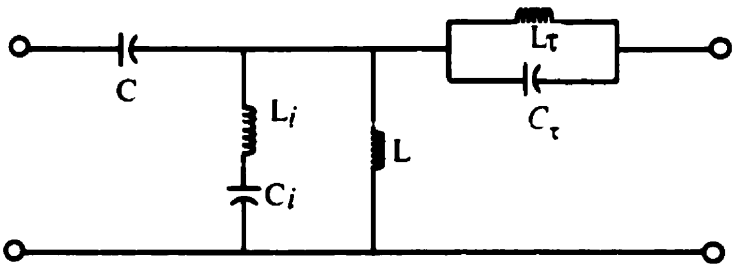
Class C	$\rho(0) = 1,\ Tr.Zeros = (\omega_i, \omega_\tau)$
Circuit_26	
Section_i	$C = C_0 = \frac{d(0)}{2} \quad \alpha_i = \angle\rho(j\omega_i)$ $L_i = \frac{1}{4}\frac{d(j\omega_i)}{\sin^2(\alpha_i/2)} - \frac{1}{2\omega_i^2 C} \quad C_i = \frac{1}{\omega_i^2 L_i}$ Realizability: $0 < C_0 \le C$, $RM = \frac{C}{C_0} \ge 1$
Section_τ	$d_R(0) = \frac{1}{d(0)}\left(\frac{g''(0)}{g(0)} + \frac{h''(0)}{h(0)} - \frac{2}{3d(0)}\left(\frac{g'''(0)}{g(0)} - \frac{h'''(0)}{h(0)}\right)\right)$ $L_{R0} = \frac{d_R(0)}{2} \quad L_{1\tau} = \frac{1}{\omega_\tau \tan(\alpha_\tau/2)} \quad \alpha_\tau = \angle\rho(j\omega_\tau)$ $L = \frac{1}{\frac{1}{L_{1\tau} + \frac{1}{\omega_\tau^2 C}} + \frac{C_i\omega_\tau^2}{1 - \left(\frac{\omega_\tau}{\omega_i}\right)^2}}$ Realizability: $0 < L_{R0} \le L \quad RM = \frac{L}{L_{R0}} \ge 1$

Table 33: Circuit 2_5_1_6

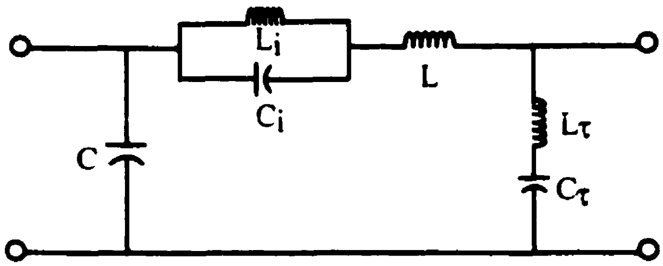
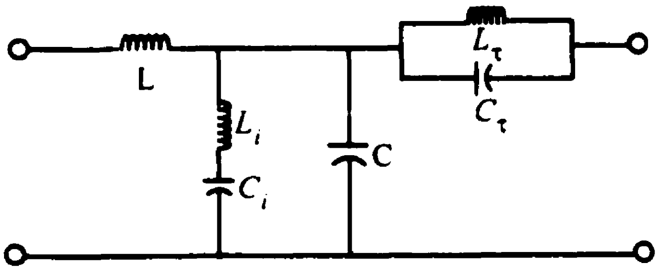
Class E	$\rho(\infty) = -1,\ Tr.Zeros = (\omega_i, \omega_\tau)$
Circuit_27	
Section_i	$C = C_\infty = \frac{2}{d(\infty)}$ $C_i = \frac{1}{4}\frac{d(j\omega_i)}{\cos^2(\alpha_i/2)} - \frac{C}{2}\quad L_i = \frac{1}{\omega_i^2 C_i}\quad \alpha_i = \angle\rho(j\omega_i)$ Realizability: $0 < C \le C_\infty$, $RM = \frac{C_\infty}{C_0} \ge 1$
Section_τ	$d_R(\infty) = \frac{1}{d(\infty)}\left(\frac{g''(\infty)}{g(\infty)} + \frac{h''(\infty)}{h(\infty)} - \frac{2}{3d(\infty)}\left(\frac{g'''(\infty)}{g(\infty)} - \frac{h'''(\infty)}{h(\infty)}\right)\right)$ $L_{R\infty} = \frac{1}{d_R(\infty)}\quad L_{1\tau} = \frac{1}{\omega_\tau \tan(\alpha_\tau/2)}\quad \alpha_\tau = \angle\rho(j\omega_\tau)$ $L = \frac{1}{\frac{1}{L_{1\tau}} + \omega_\tau^2 C} - \frac{L_i}{1 - \left(\frac{\omega_\tau}{\omega_i}\right)^2}$ Realizability: $0 < L \le L_{R\infty}$ $RM = \frac{L_{R\infty}}{L} \ge 1$

Table 34: Circuit 1_6_2_5

Class G	$\rho(\infty) = 1,\ Tr.Zeros = (\omega_i, \omega_\tau)$
Circuit_28	
Section_i	$L = L_\infty = \dfrac{2}{d(\infty)} \qquad \alpha_i = \angle\rho(j\omega_i)$ $L_i = \dfrac{1}{4}\dfrac{d(j\omega_i)}{\sin^2(\alpha_i/2)} - L/2 \qquad C_i = \dfrac{1}{\omega_i^2 L_i}$ Realizability: $0 < L \le L_\infty$, $RM = \dfrac{L_\infty}{L} \ge 1$
Section_τ	$d_R(\infty) = \dfrac{1}{d(\infty)}\left(\dfrac{g''(\infty)}{g(\infty)} + \dfrac{h''(\infty)}{h(\infty)} - \dfrac{2}{3d(\infty)}\left(\dfrac{g'''(\infty)}{g(\infty)} - \dfrac{h'''(\infty)}{h(\infty)}\right)\right)$ $C_{R\infty} = \dfrac{2}{d_R(\infty)} \qquad C_{1\tau} = -\dfrac{\tan(\alpha_\tau/2)}{\omega_\tau} \qquad \alpha_\tau = \angle\rho(j\omega_\tau)$ $C = \dfrac{1}{\dfrac{1}{C_{1\tau}} + \omega_\tau^2 L} - \dfrac{C_i}{1 - \left(\dfrac{\omega_\tau}{\omega_i}\right)^2}$ Realizability: $0 < C \le C_{R\infty}$ $RM = \dfrac{C_{R\infty}}{C} \ge 1$

Chapter 4

SYNTHESIS PROCEDURE AND EXAMPLES

4.1 Introduction

In this chapter, an operational flowgraph of the ladder scattering synthesis program is presented to illustrate the operational procedure of the simplified synthesis strategy. The details of the synthesis program are in Appendix(IV):Scarttering Synthesis Program Listing.

In the remaining section, four examples, which are a 7th_order lowpass, a 14th_order very narrow bandpass, a 10th_order highpass and a 13th_order double narrow bandpass filter are presented, to demonstrate the effectiveness of the scattering synthesis algorithm and the simplified synthesis strategy. Finally, there are additional examples in Appendix(III):Additional Examples.

4.2 Flowgraph of synthesis operation

Figure 5 below illustrates the flow of operations in the scattering synthesis program. Note that Tr.zero in figure 5 is a sequence of sorted transmission zeros, which lists transmission zeros as a sequence of 0 first, followed by the positive imaginary parts in order of increasing magnitude and then finally infinity. The successor checking accounts for the 24

Göttlicher circuits plus 4 additional circuits in Tables 31-34. If two sections of any circuit satisfy Realizability_τ >= 1 and realizabilty_i >= 1, then a successor for the circuit is counted. The additional successors checking accounts for four additional circuits corresponding to transmission zeros at 0 and ∞. Since Göttlicher ignored them for successor checking, an additional checking algorithm to make the strategy applicable to a wider range of circuits is developed .

Göttlicher didn't present his synthesis program in any papers nor did he give a program listing in his thesis. The new scattering program was developed independently. For the extraction process; the implementation of Eqs. (2.3.7a,b,c,d) used a zero finding program, which was developed by Dr. G.O. Martens, as the part of the procedure of calculating polynomials f_b, g_b, h_b at each step.

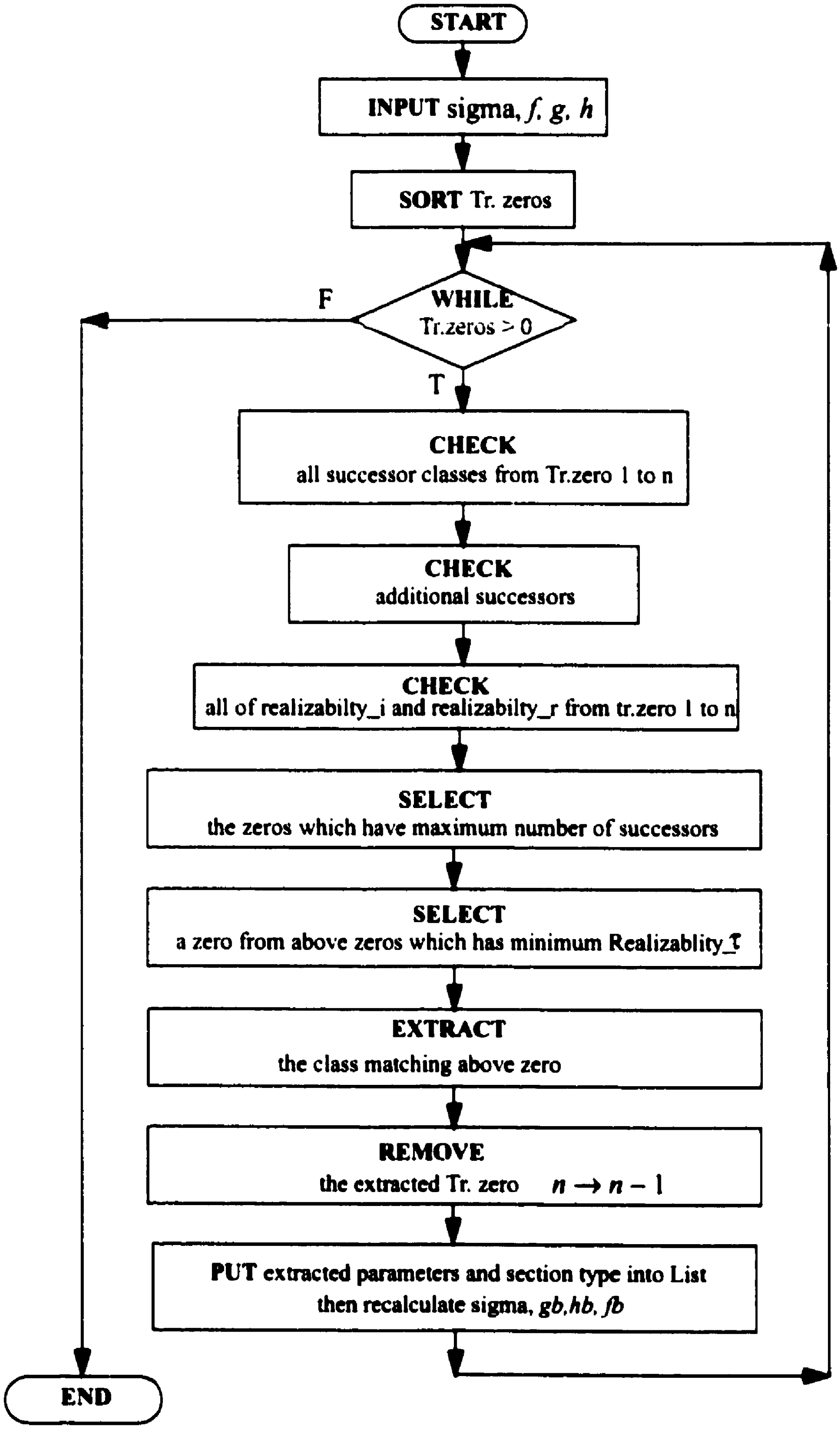

START
INPUT sigma, f, g, h
SORT Tr. zeros
WHILE
Tr.zeros > 0
F
T
CHECK
all successor classes from Tr.zero 1 to n
CHECK
additional successors
CHECK
all of realizabilty_i and realizabilty_r from tr.zero 1 to n
SELECT
the zeros which have maximum number of successors
SELECT
a zero from above zeros which has minimum Realizablity_T
EXTRACT
the class matching above zero
REMOVE
the extracted Tr. zero n → n − 1
PUT extracted parameters and section type into List
then recalculate sigma, gb, hb, fb
END

Fig. 5

4.3 Example of a 7th - order lowpass filter

The first design example is taken from 'Handbook of filter synthesis'[27]. The input scattering polynomials are the results of cascade analysis. The sub_program for cascade analysis was written by G.O. Martens; it multiplies the transfer matrices of the elementary sections together. This example shows that the simplified scattering synthesized circuit is almost exactly the same as the Initial circuit.

Zeros of f	Zeros of h
	0
- 2.857407769783873e+00i	8.115548680831247e-07 - 4.460089180626319e-01i
+2.857407769783873e+00i	8.115548681134951e-07 + 4.460089180626319e-01i
- 3.522738278781446e+00i	1.649129595881117e-06 - 7.921507705628069e-01i
+3.522738278781446e+00i	1.649129595575480e-06 + 7.921507705628071e-01i
- 6.256515729188977e+00i	-6.682035666221775e-07 - 9.765437183518240e-01i
+ 6.256515729188977e+00i	-6.682035665827198e-07 + 9.765437183518240e-01i
Constant Multiplier = 1.0	Constant Multiplier = -2.265517815209827e+04

Zeros of g
-4.591209891487081e-01
-4.034702590631772e-01 - 4.947063833824538e-01i
-4.034702590631772e-01 + 4.947063833824538e-01i
-2.642020156420269e-01 - 8.686097136900197e-01i
-2.642020156420273e-01 + 8.686097136900197e-01i
-9.027018114299958e-02 - 1.061594504814765e+00i
-9.027018114299956e-02 + 1.061594504814765e+00i
Constant Multiple = 2.265517815209827e+04

Table 35: Initial Circuit Parameters of the First Example

Extracted Section Type	Transmission Zero	Initial Parameters
2	2	C1=9.550700000000000e-01
5	2	L1=1.357060000000000e+00 C2=5.938000000000000e-02
2	1	C3=1.829240000000000e+00
5	1	L2=1.505930000000000e-02 C4=8.133000000000000e-02
2	3	C5=1.859160000000000e+00
5	3	L3=1.415330000000000e+00 C6=1.805000000000000e-02
2	4	C7=9.947700000000000e-01
0		n = 1

Table 36: Synthesis Results of the First Example

Extracted Section Type	Extracted transmission Zero	Synthesized Parameters
2	2	C1=9.550663508781705e-01
5	2	L1=1.357060685661019e+00 C2=5.937996999798778e-02
2	1	C3=1.829241170131920e+00
5	1	L2=1.505929963621951e+00 C4=8.133000196465101e-02
2	3	C5=1.859158810744358e+00
5	3	L3=1.415329350716419e+00 C6=1.805000828045333e-02
2	4	C7=9.947736662902097e-01
0		n=9.999999803145 69e-0

The errors between the computer synthesized element values and initial circuit values are in the range 1e-06 to 1e-08. These differences are quit small.

The Figures below show the attenuation responses and the circuit diagram. The transformer terminated in a 1Ω resistor can be replaced by a resistor with $R = n^2$.

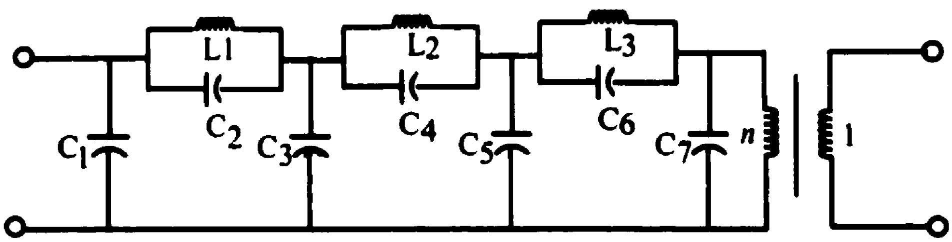


Fig 4.3.1 Circuit of the 7th - order lowpass filter

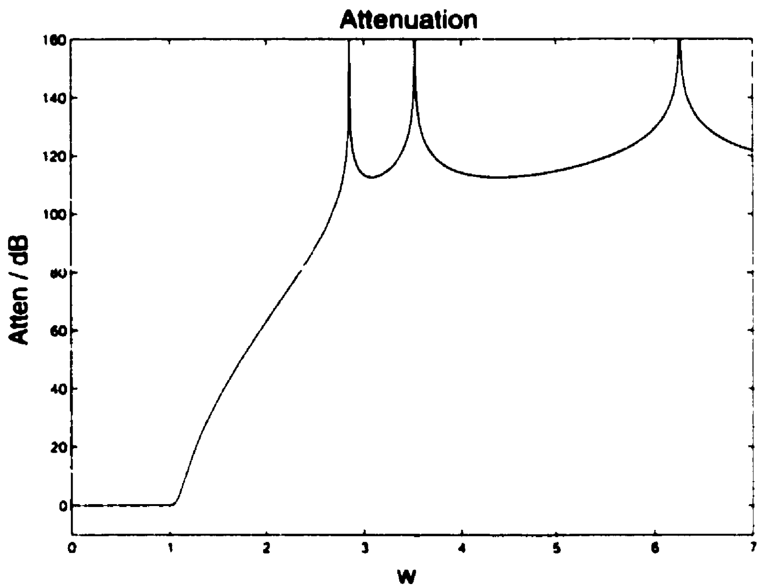


Fig 4.3.2

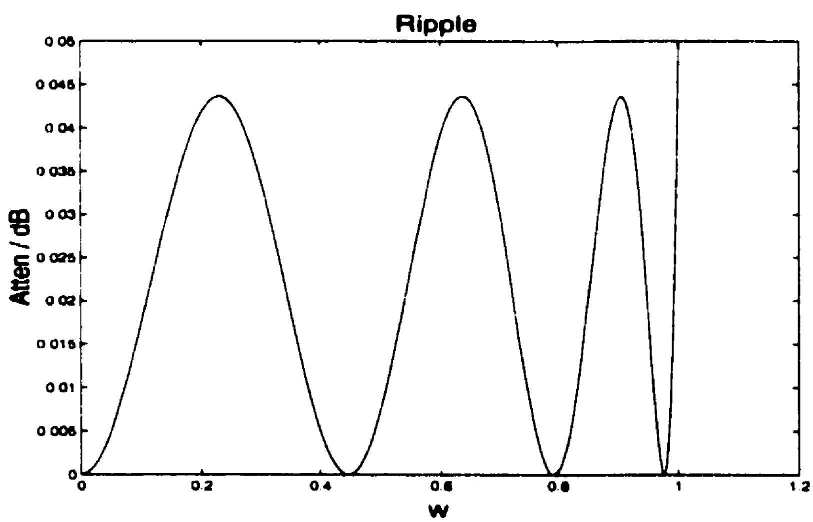


Fig 4.3.3

4.4 Example of a 10th-order double bandpass filter

The second example was taken from Göttlicher's thesis[23] p.76, where it was used as an example for Göttlicher's synthesis technique. Here it is used to demonstrate that the synthesis results of the new syntheses program are very close to Göttlicher's results.

Note: for this example, the network polynomial are defined in the following three tables. Figure 4.4.2 and Figure 4.4.3 present the attenuation and ripple of the synthesized network.

Zeros of f	Zeros of h
+j3.7976519521633 -j3.7976519521633 +j4.1823250304382 -j4.1823250304382 +j8.1072461790375 -j8.1072461790375 +j9.5956464786104 -j9.5956464786104	+j1.1101147226189 -j1.1101147226189 +j2.5149508819034 -j2.5149508819034 +j5.4340982707213 -j5.4340982707213 +j6.4975498890500 -j6.4975498890500. +j7.1733180423423 -j7.1733180423423
Constant Multiplier = 1.0	Constant Multiplier = -0.4985701650666650e-01

Zeros of g
-1.78761512448885e+0+j1.6159314309630 -1.78761512448885e+0-j1.6159314309630 -3.67638237598864e-1+j2.8650486112076 -3.67638237598864e-1-j2.8650486112076 -3.00738516648895e-1+j5.2869317126282 -3.00738516648895e-1-j5.2869317126282 -5.32735762232205e-1+j6.5654852839711 -5.32735762232205e-1-j6.5654852839711 -1.48233261064441e-1+j7.2533696252984 -1.48233261064441e-1-j7.2533696252984
Constant Multiplier = 4.985701650666650e-01

Table 37: Synthesis Result of Second Example

Extracted Section Type	Extracted TransmissionZero	Synthesized Parameters
2	3	C1 = 1.7401361614531640e-01
5	3	L1 = 5.1208014485421500e-02 C2 = 2.9710868116009740e-01
2	1	C3 = 2.0524640274146570e-01
5	1	L2 = 2.1183223830470130e-01 C4 = 3.2732383058402090e-01
2	6	C5 = 1.0083364305702720e-01
1	2	L3 = 7.1299216536388900e-02
6	2	L4 = 2.9360038145926840e-01 C6 = 1.9471877213520810e-01
1	4	L5 = 1.4572400833600810e-01
6	4	L6 = 8.5584501536431920e-02 C7 = 1.2689847285634850e-01
1	5	L7 = 1.7054689275901020e-01
0		n = 8.4995576484896020e-01

Table 38: Göttlicher's results

Extracted Section Type	Extracted Transmission Zero	Initial Parameters
2	3	C1 = 1.7401361614532064e-1,
5	3	L1 = 5.1208014485419314e-2, C2 = 2.9710868116011217e-1,
2	1	C3 = 2.0524640274146579e-1,
5	1	L2 = 2.1183223830470498e-1, C4 = 3.2732383058401975e-1,
2	6	C5 = 1.0083364305702435e-1,
1	2	L3 = 7.1299216536390986e-2,
6	2	L4 = 1.9471877213520460e-1, C6 = 2.9360038145927915e-1,
1	4	L5 = 1.4572400833600455e-1,
6	4	L6 = 8.5584501536433197e-2, C7 = 1.2689847285634740e-1,
1	5	L7 = 1.7054689275901 02e-01
0		n = 8.4995576484898 13e-01,

The errors between the scattering synthesized element values and Göttlicher' technique are in the range 1e-015 to 1e-016. Note that a transformer terminated in a 1Ω resistor can be replaced by a resistor with $R = n^2$.

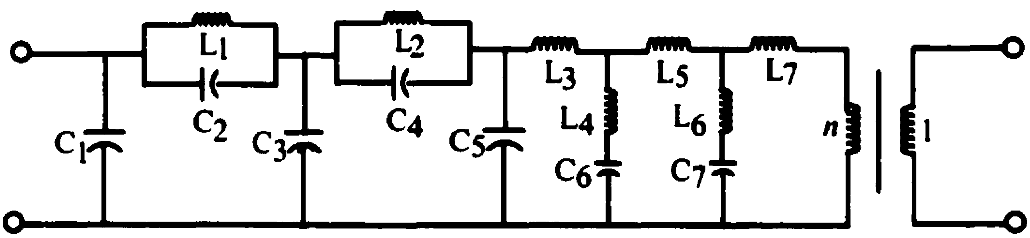


Fig. 4.4.1 Circuit of 10th-order double bandpass filter

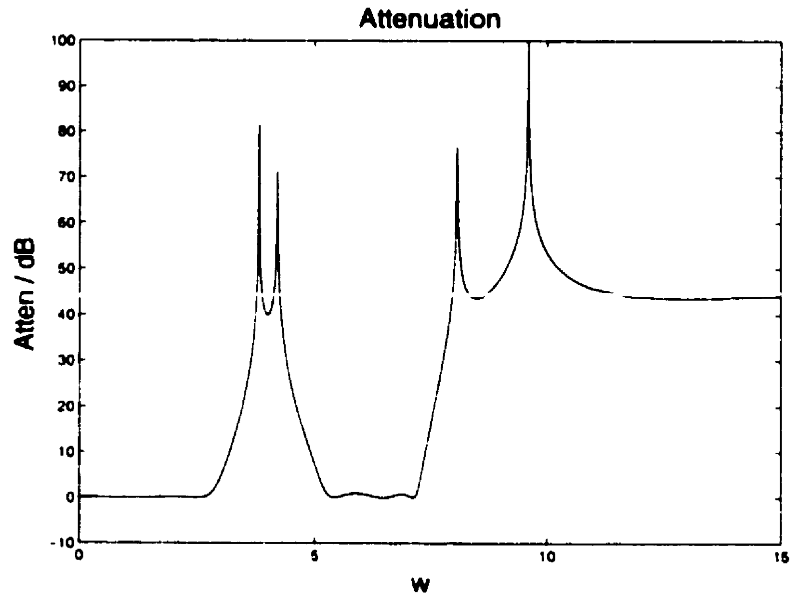


Fig. 4.4.2

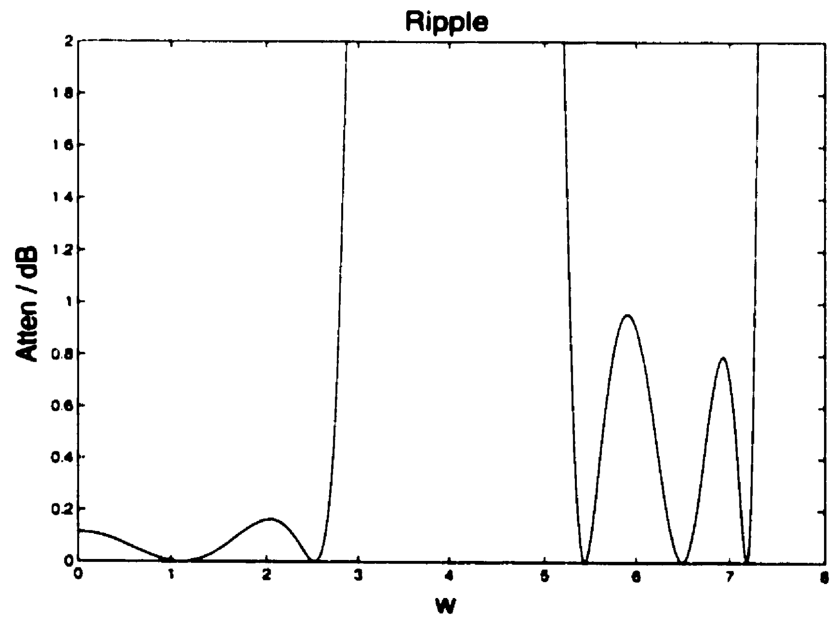


Fig 4.4.3

4.5 Example of a 9th-order highpass filter

The third example is taken from Göttlicher's thesis [23] p.71. The example shows that the simplified scattering synthesis method can be used not only to synthesize lowpass and bandpass filters, but also highpass filters, i.e. it can be used for any type of ladder filters. The example also shows that the synthesis result is very close to the initial circuit. The error between the two of them is in the range of 1e-011 to 1e-012.

Zeros of f
0
0 - 4.445259318396255e-01i
0 + 4.445259318396255e-01i
0 - 7.675067431476281e-01i
0 + 7.675067431476281e-01i
0 - 9.327984702028416e-01i
0 + 9.327984702028416e-01i
0 - 9.934899672309181e-01i
0 + 9.934899672309181e-01i

Zeros of h
-9.999999999999842e-01
0 - 1.107453247258636e+00i
0 + 1.107453247258636e+00i
0 - 1.183857348269596e+00i
0 + 1.183857348269596e+00i
0 - 1.483843704828418e+00i
0 + 1.483843704828418e+00i
0 - 3.478900709899011e+00i
0 + 3.478900709899011e+00i
Constant Multiplier = 4.972110896877390e-02

Zeros of g
-8.989380085020487e-01 -2.638866145524699e-02 - 1.066729187133896e+00i -2.638866145524699e-02 + 1.066729187133896e+00i -1.132820520451316e-01 - 1.082133830649153e+00i -1.132820520451316e-01 + 1.082133830649153e+00i -3.524680991850715e-01 - 1.090990909372980e+00i -3.524680991850715e-01 + 1.090990909372980e+00i -9.240297112440859e-01 - 7.578008332506821e-01i -9.240297112440859e-01 + 7.578008332506821e-01i
Constant Multiplier = 1.001235331316811e+00

Table 39: The Synthesis Result of Third Example

Extracted Section Type	Extracted Transmission Zero	Synthesized Parameters
3	3	C1 = 2.238970563476569e+00
6	3	L1 = 1.083889063387l2e+00 C2 =1.566211572103117e+00
3	4	C3 = 8.396973284224715e_01
6	4	L2 = 1.087913229964595e+00 C4 =1.056404132604849e+00
3	5	C5 =1.071553811228406e+00
6	5	L3 = 1.745991282969497e+00 C6 = 5.802711214231960e-01
3	2	C7 = 9.578308585564244e-01
6	2	L5 = 9.149947990371583e-01 C8 = 5.530790044950940e+00
3	1	C9 = 2.749368332142l5e+00
0		n = 1.050956440285585e+00

Table 40: Initial Circuit of Third Example

Section Type	Transmission Zero	Circuit Parameters
3	3	C = 2.2397056347662290e+00
6	3	L = 1.0838899063387248e+00 C = 1.5662115721030977e+00
3	4	C = 8.3969732842247713e-01
6	4	L = 1.0879132299645808e+00 C = 1.0564041326048628 0e+00
3	5	C = 1.0715538112283772e-01
6	5	L = 1.7459912829694775e+00 C = 5.8027112142320258e-01
3	2	C = 9.5783085855643435 0e-01
6	2	L= 9.1499479903714853e-01 C = 5.5307900449509995e+00
3	1	C = 2.749368332137 82e+00
0		R = 1.1045094393777478e+00

Note: a transformer terminated in a 1 Ω resistor can be replaced by a resistor with $R = n^2$.

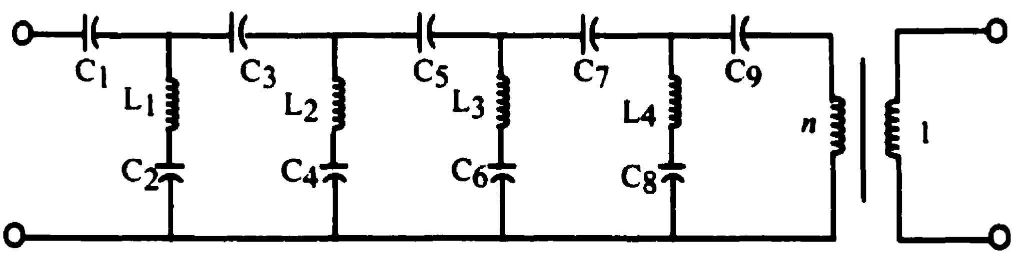


Fig. 4.5.1 Circuit of 9th-order highpass filter

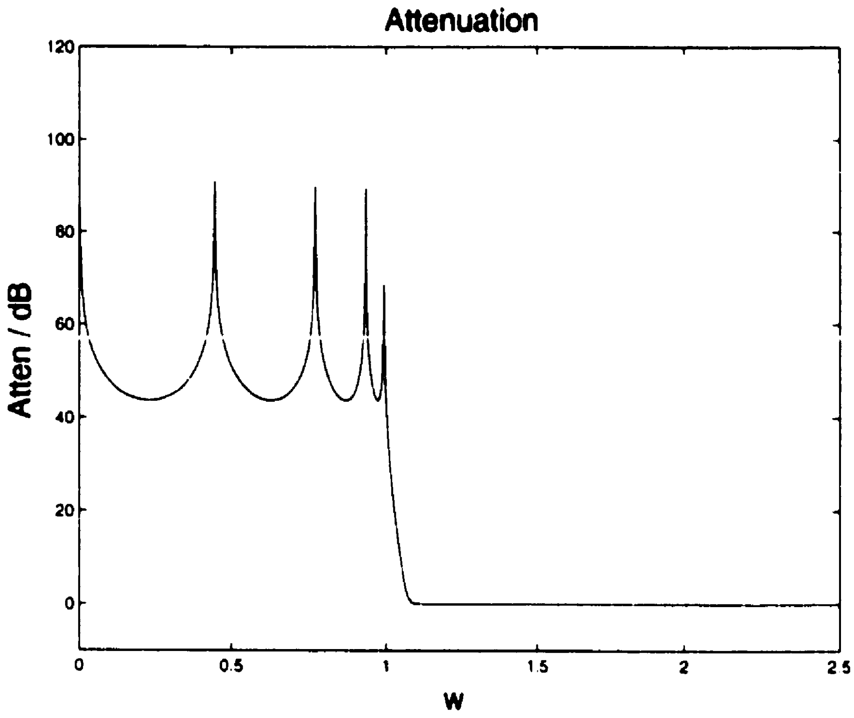


Fig. 4.5.2

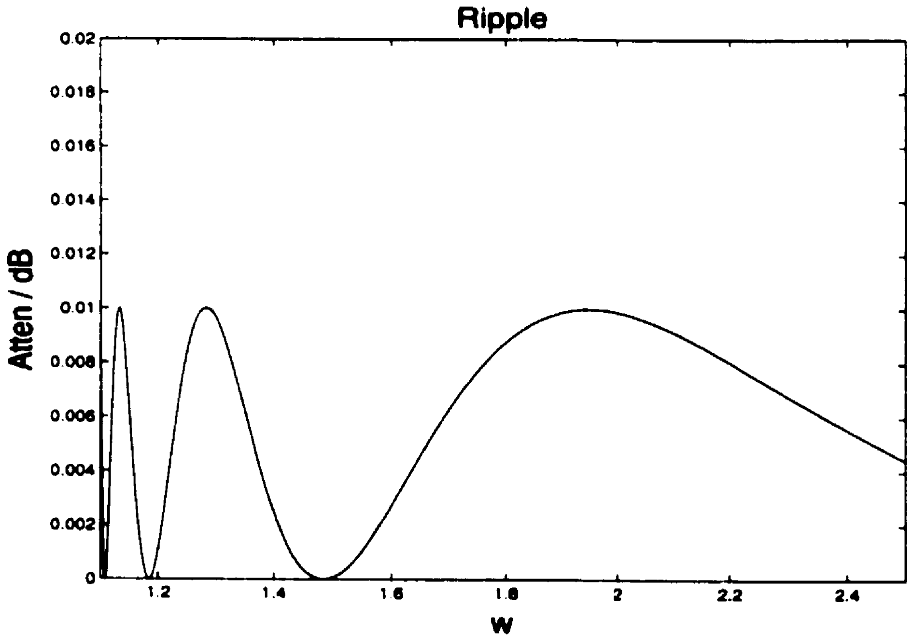


Fig. 4.5.3

4.6 Example of a 13th_order Double bandpass filter

This example was chosen to show the numerical robustness of the synthesis algorithm, since the example contains two narrow passbands very close together which makes this design very sensitive to roundoff error accumulation during synthesis.

Zeros of f
0.0000000000000000000c+0
+8.5417326628242841110e-1i
-8.5417326628242841110e-1i
+7.5514740727158266110e-1i
-7.5514740727158266110e-1i
+1.0550017410860102390e+0i
-1.0550017410860102390e+0i
+6.5390664331829375810e-1i
-6.5390664331829375810e-1i
+1.1000007197667314510e+0i
-1.1000007197667314510e+0i
Constant Multiplier = 9.9999999999999999990e-1

Zeros of h
1.0393603099708103560e-2+9.7020960853994034160e-1i
1.0393603099708103560e-2-9.7020960853994034160e-1i
-6.9313688630879318330e-4+8.9347815387796735330e-1i
-6.9313688630879318330e-4-8.9347815387796735330e-1i
4.6647193568119437060e-3+1.0053725224416203320e+0i
4.6647193568119437060e-3-1.0053725224416203320e+0i
1.2370427303532437690e-1+9.7364119532050673720e-1i
1.2370427303532437690e-1-9.7364119532050673720e-1i
-3.0010052987973729090e-3+8.2831707667499913750e-1i
-3.0010052987973729090e-3-8.2831707667499913750e-1i
-1.1964530378812893150e-3+7.9292778070077049510e-1i
-1.1964530378812893150e-3-7.9292778070077049510e-1i
2.0062483958225394260e+0
Constant Multiplier = -1.2452084287204495370e+2

Zeros of g
-6.377890375185803443 0e-3+1.006781174304437012 0e+0i
-6.377890375185803443 0e-3-1.006781174304437012 0e+0i
-1.883719091835757915 0e-2+9.706321819970877643 0e-1i
-1.883719091835757915 0e-2-9.706321819970877643 0e-1i
-8.060332080210103377 0e-3+8.917421282905717704 0e-1i
-8.060332080210103377 0e-3-8.917421282905717704 0e-1i
-1.237795163203485247 0e-1+9.734672245407543147 0e-1i
-1.237795163203485247 0e-1-9.734672245407543147 0e-1i
-4.176859266219868906 0e-3+8.285107874915994527 0e-1i
-4.176859266219868906 0e-3-8.285107874915994527 0e-1i
-1.803936266610482955 0e-3+7.928119711324544944 0e-1i
-1.803936266610482955 0e-3-7.928119711324544944 0e-1i
-2.006245825048490324 0e+0
Constant Multiplier = 1.245208428720449537 0e+2

Table 41: Synthesis Results of Third Example

Extracted Section Types	Extracted Transmission Zero	Synthesized Parameters
3	2	C1 = 1.226745607243308e+00
6	2	L1 = 1.408970076061800e+00 C2=1.659841929912941e+00
2	5	C3 = 4.381434440642106e-01
5	5	L2 = 4.651907237340155e-01 C4 = 1.931357192689231e+00
3	3	C5 = 3.424255480954451e-01
6	3	L3 = 6.942119446285880e+00 C6 = 2.526064565875787e-01

Table 41: Synthesis Results of Third Example

Extracted Section Types	Extracted Transmission Zero	Synthesized Parameters
3	4	C7 = 1.736345147370902e+00
6	4	L4 = 7.183946830752446e+00 C8 = 1.907853138817627e-01
2	7	C9 = 3.828189542526473e+00
1	6	L5 = 7.026259267737013e-01
6	6	L6 = 1.126650151161485e+00 C10 = 7.335419949111991e-01
3	1	C11 = 2.336879787985904e-01
1	8	L7 = 5.238472705081596e+00
0		n = 4.066775133853233e-01

Note: the transformer terminated in a 1Ω resistor may be replaced by a resistor with $R = n^2$.

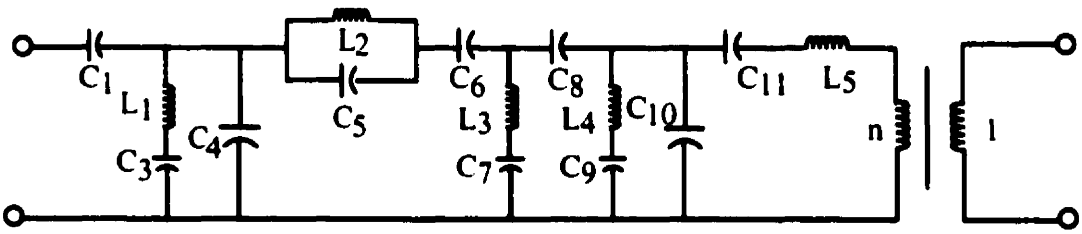


Fig. 4.6.1 13th_order double bandpass filter

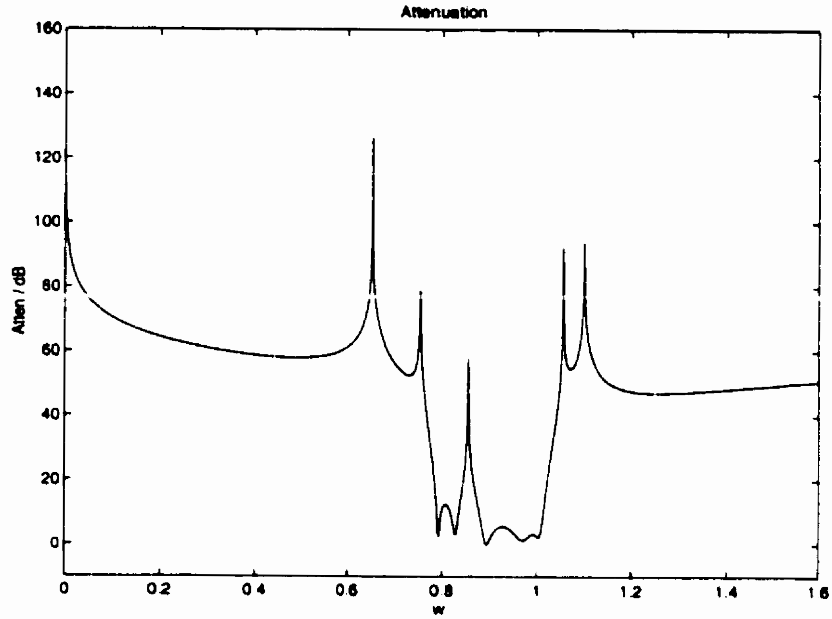


Fig. 4.6.2

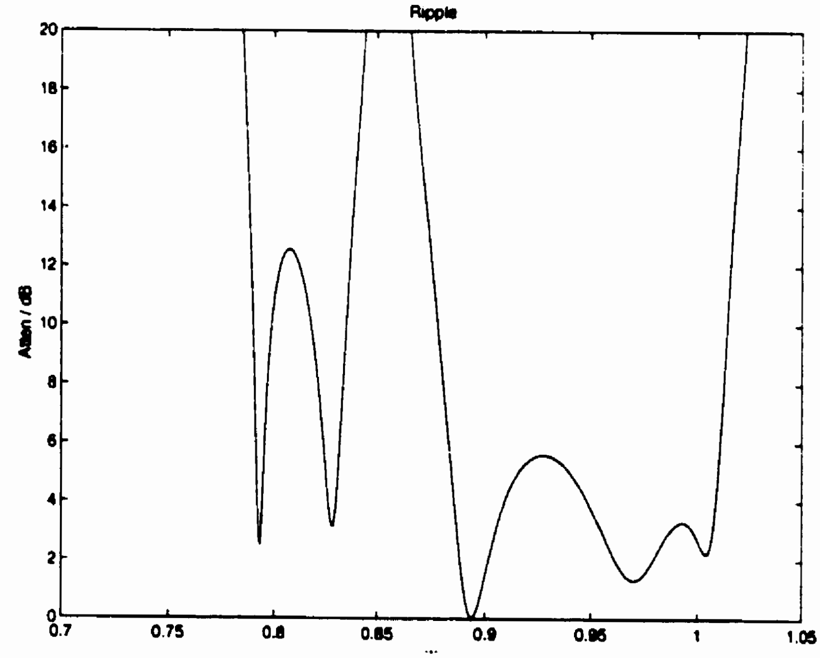


Fig. 4.6.3

Chapter 5

CONCLUSIONS

The synthesizing of *lossless ladder filters* has been studied for more than 50 years. The most compact and efficient method of cascade synthesis is based on the transfer scattering matrix. The most difficult part of any method is the selection of the transmission zero sequence, if the number of zero sequences is large. Göttlicher[22] proposed a strategy to solve the problem a few years ago. However, he only applied his strategy to a traditional synthesis procedure, and did not publish his program for implementing it.

In this thesis, a simplified scattering synthesis algorithm was developed for the realizability calculations of a *ladder* circuit (including the determination of circuit elements and sections). A program which combined a simplified scattering synthesis algorithm and Göttlicher's synthesis strategy to perform synthesis of lossless two-port ladder networks was written. It has been shown that the new scattering algorithm is able to determine the network element values with a high degree of accuracy. The new program has also been found to be numerically robust and relatively immune to roundoff error accumulation.

The strategy adapted from Göttlicher's thesis has proven to be effective for a range of filter designs from lowpass, very narrow band bandpass to double bandpass filters and highpass filters.

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APPENDIX_I[28]

DERIVATIVES OF POLYNOMIALS IN PRODUCT REPRESENTATION

Let y be polynomial in product represention:

where K is the constant factor and z_i are the zeros.

$$y = k\prod_{i=1}^{n}(p - z_i)$$

$$\ln y = \ln k + \sum_{i=1}^{n}\ln(p - z_i)$$

$$\frac{y'}{y} = \sum_{i=1}^{n}\frac{1}{p - z_i}$$

$$y'' = y'\sum_{i=1}^{n}\frac{1}{p - z_i} + y\sum_{i=1}^{n}\frac{-1}{(p - z_i)^2}$$

$$\frac{y''}{y} = \frac{y'}{y}\sum_{i=1}^{n}\frac{1}{p - z_i} + \sum_{i=1}^{n}\frac{-1}{(p - z_i)^2}$$

$$= \left(\sum_{i=1}^{n}\frac{1}{p - z_i}\right)^2 - \sum_{i=1}^{n}\frac{1}{(p - z_i)^2}$$

$$= \sum_{i=1}^{n}\sum_{\substack{j=1 \\ i \neq j}}^{n}\frac{1}{(p - z_i)(p - z_j)}$$

$$y'' = y\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n}\frac{1}{(p-z_i)(p-z_j)}$$

$$y''' = y'\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n}\frac{1}{((p-z_i)(p-z_j))} - 2y\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n}\frac{\left(p-\frac{(z_i+z_j)}{2}\right)}{(p-z_i)^2(p-z_j)^2}$$

$$\frac{y'''}{y} = \frac{y'}{y}\frac{y''}{y} - 2\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n}\frac{\left(p-\frac{(z_i+z_j)}{2}\right)}{(p-z_i)^2(p-z_j)^2}$$

At $p = 0$, with $z_i \neq 0$,

$$\frac{y'(0)}{y(0)} = -\sum_{i=1}^{n}\frac{1}{z_i}$$

$$\frac{y''(0)}{y(0)} = \sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n}\frac{1}{z_i z_j}$$

$$\frac{y'''(0)}{y(0)} = \frac{y'(0)}{y(0)}\frac{y''(0)}{y(0)} + \sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n}\left(\frac{1}{z_i z_j^2}+\frac{1}{z_j z_i^2}\right)$$

The guantities $\frac{\hat{y}'(0)}{\hat{y}(0)}, \frac{\hat{y}''(0)}{\hat{y}(0)}, \frac{\hat{y}'''(0)}{\hat{y}(0)}$ will be defined in terms of the mapping

$\hat{y}(s) = s^n y\left(\frac{1}{s}\right)$, and evaluated at $s = 0$.

$$\hat{y}(s) = s^n y\left(\frac{1}{s}\right) = k\prod_{i=1}^{n}(1-sz_i)$$

$$\frac{\hat{y}'(s)}{\hat{y}(s)} = -\sum_{i=1}^{n} \frac{z_i}{1 - sz_i}$$

$$\frac{\hat{y}'(0)}{\hat{y}(0)} = -\sum_{i=1}^{n} z_i$$

Denoting $\dfrac{\hat{y}'(0)}{\hat{y}(0)}$ by $\dfrac{y'(\infty)}{y(\infty)}$ obtain

$$\frac{y'(\infty)}{y(\infty)} = -\sum_{i=1}^{n} z_i$$

Similarly

$$\frac{y''(\infty)}{y(\infty)} = \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j \neq i}}^{n} z_i z_j = \sum_{i=1}^{n} z_i \sum_{\substack{j=1 \\ j \neq i}}^{n} z_j$$

and

$$\frac{y'''(\infty)}{y(\infty)} = \frac{y'(\infty)}{y(\infty)} \frac{y''(\infty)}{y(\infty)} + \sum_{i=1}^{n} z_i \sum_{\substack{j=1 \\ j \neq i}}^{n} z_j (z_i + z_j)$$

APPENDIX_II [28]

DERIVATION OF THE DELAY OF THE SECOND SECTION

The delay of the second section of two at a double transmission zero, $s = 0$, of a circuit such as circuit 3_4_5 is obtained as follows:

The polynomials g_b and h_b are determined by Eqs. (2.2.7 a, b). The polynomials f_a, g_a, h_a of the first section are of the form [22]

$$f_a = s, \sigma_a = -1, g_a = s + a, h_a = \varepsilon a, \varepsilon = \pm 1, a = \frac{1}{d}$$

Let

$$\hat{g}_b = \sigma_a g_{a*} g - \sigma_a h_{a*} h = (s-a) g + \varepsilon a h \tag{1}$$

$$\text{then } g_b = \frac{\hat{g}_b}{s^2}, \tag{2}$$

Therefore, since g_b must be a polynomial,

$$\hat{g}_b(0) = 0, \hat{g}_b'(0) = 0, \text{and } g_b(0) = \frac{1}{2}\hat{g}_b''(0), \tag{3}$$

$$g_b'(0) = \frac{1}{6}\hat{g}_b'''(0). \tag{4}$$

Similarly, let $\hat{h}_b = g_a h - h_a g = (s + a) h - \varepsilon a g$, (5)

$$\text{then} \quad h_a = \frac{\hat{h}_b}{s^2}, \tag{6}$$

and since h_b must be a polynomial,

$$\hat{h}_b(0) = 0,\ \hat{h}_b'(0) = 0, \text{ and } h_b(0) = \frac{1}{2}\hat{h}_b''(0), \tag{7}$$

$$h_b'(0) = \frac{1}{6}\hat{h}_b'''(0). \tag{8}$$

Also, $g_a(0) = \varepsilon h_a(0)$ and $g(0) = \varepsilon h(0)$; and,

for the second section at $s = 0$,

$$g_b(0) = -\varepsilon h_b(0). \tag{9}$$

Then substituting (3) and (7) into (9)

obtain

$$\hat{g}_b''(0) = -\varepsilon \hat{h}_b(0) \tag{10}$$

Using (1) and (5) in (10) get

$$a\left(g''(0) - \varepsilon h''(0)\right) = g'(0) + \varepsilon h'(0) \tag{11}$$

$$d_R(0) = \frac{g_b'(0)}{g_b(0)} - \frac{h_b'(0)}{h_b(0)} = \frac{g_b'(0) + \varepsilon h_b'(0)}{g_b(0)} \tag{12}$$

From (3) and (1)

$$g_b(0) = \frac{1}{2}\hat{g}_b''(0) = g'(0) - \frac{a}{2}\left(g''(0) - \varepsilon h''(0)\right)$$

and using (11)

$$g_b(0) = \frac{1}{2}\left(g'(0) - \varepsilon h'(0)\right)$$

also from (4) and (1)

$$g_b'(0) = \frac{1}{6}\hat{g}_b'''(0) = \frac{1}{2}g''(0) - \frac{a}{6}\left(g'''(0) - \varepsilon h'''(0)\right)$$

Similarly from (8) and (5)

$$h_b'(0) = \frac{1}{6}\hat{h}_b'''(0) = \frac{1}{2}h''(0) + \frac{a}{6}\left(h'''(0) - \varepsilon g'''(0)\right)$$

Substituting the above into (12) the expression for $d_R(0)$ yields

$$d_R(0) = \frac{g''(0) + \varepsilon h''(0) - \frac{2a}{3}\left(g'''(0) - \varepsilon h'''(0)\right)}{g'(0) - \varepsilon h'(0)}$$

$$= \frac{1}{d(0)}\left(\frac{g''(0)}{g(0)} + \frac{h''(0)}{h(0)} - \frac{2}{3d(0)}\left(\frac{g'''(0)}{g(0)} - \frac{h'''(0)}{h(0)}\right)\right) \tag{13}$$

where $d(0) = \frac{g'(0)}{g(0)} - \frac{h'(0)}{h(0)} = \frac{g_a'(0)}{g_a(0)} - \frac{h_a'(0)}{h_a(0)} = d = \frac{1}{a}$

The corresponding result for the delay of a second section with a double transmission zero at $s = \infty$ is obtained by using the mapping $s \rightarrow 1/s$ (see Appendix I). Then

$$d_R(\infty) = \frac{g_b'(\infty)}{g_b(\infty)} - \frac{h_b'(\infty)}{h_b(\infty)}$$

$$= \frac{1}{d(\infty)}\left(\frac{g''(\infty)}{g(\infty)} + \frac{h''(\infty)}{h(\infty)} - \frac{2}{3d(\infty)}\left(\frac{g'''(\infty)}{g(\infty)} - \frac{h'''(\infty)}{h(\infty)}\right)\right) \tag{14}$$

where $d(\infty) = \frac{g'(\infty)}{g(\infty)} - \frac{h'(\infty)}{h(\infty)} = \frac{g_a'(\infty)}{g_a(\infty)} - \frac{h_a'(\infty)}{h_a(\infty)} = d_a(\infty)$

APPENDIX_III

ADDITIONAL EXAMPLES

3.1 Additional example of a 10 degree narrow bandpass filter[29, p.387]

Zeros of f
0
0
0
0 - 7.650520500976998e+01i
0 + 7.650520500976998e+01i
0 - 8.278395310845262e+01i
0 + 8.278395310845262e+01i
0 - 1.466443398429018e+02i
0 + 1.466443398429018e+02i
Constant Multiplier = 1.000000000000000e+00

Zeros of g
-1.305635283009075e+00 - 8.857486966284060e+01i
-1.305635283008548e+00 + 8.857486966284121e+01i
-5.171259714118013e+00 - 9.171247862133175e+01i
-5.171259714120635e+00 + 9.171247862133356e+01i
-1.154464095851323e+01 - 1.017389940775243e+02i
-1.154464095851688e+01 + 1.017389940775251e+02i
-1.321527139456090e+01 + 1.212757790850019e+02i
-1.321527139456280e+01 - 1.212757790850039e+02i
-4.155982758248053e+00 + 1.334035248550387e+02i
-4.155982758249002e+00 - 1.334035248550406e+02i
Constant Multiplier = 3.622380046693072e-01

Zeros of h
2.392342793163219e-02 - 8.977054805964822e+01i 2.392342792997965e-02 + 8.977054805964920e+01i -3.634134037965850e-02 + 9.381252535585607e+01i -3.634134037677358e-02 - 9.381252535586262e+01i 1.237961726745460e-02 + 1.040165844324301e+02i 1.237961725173895e-02 - 1.040165844324377e+02i 6.319825149501135e-03 - 1.193211466853977e+02i 6.319825146139234e-03 + 1.193211466854135e+02i -7.365216812956254e-03 + 1.297900952753172e+02i -7.365216807858941e-03 - 1.297900952753203e+02i];
Constant Multiplier = -3.622380046693063e-01

Extracted Section Types	Extracted Transmission Zero	Synthesized Parameters
4	5	L1 = 1.152169023847305e-02
5	5	L2 = 2.580144206266234e-03 C1 = 5.655404827591382e-02
2	6	C2 = 1.984798761641732e-02
5	6	L3 = 9.662716992380181e-04 C3 = 4.812493218695149e-02
4	4	L4 =1.067943442242621e-03
5	4	L5 =8.569568045495478e-04 C5 = 1.993696754526694e-01
4	3	L6 = 1.293810962856785e-04
3	2	C6 =7.217677857655482e-01
4	1	L7 = 1.081953767866160e-05
2	7	C7 = 7.825526922564519e+00
0		n = 6.003737526020636e-02

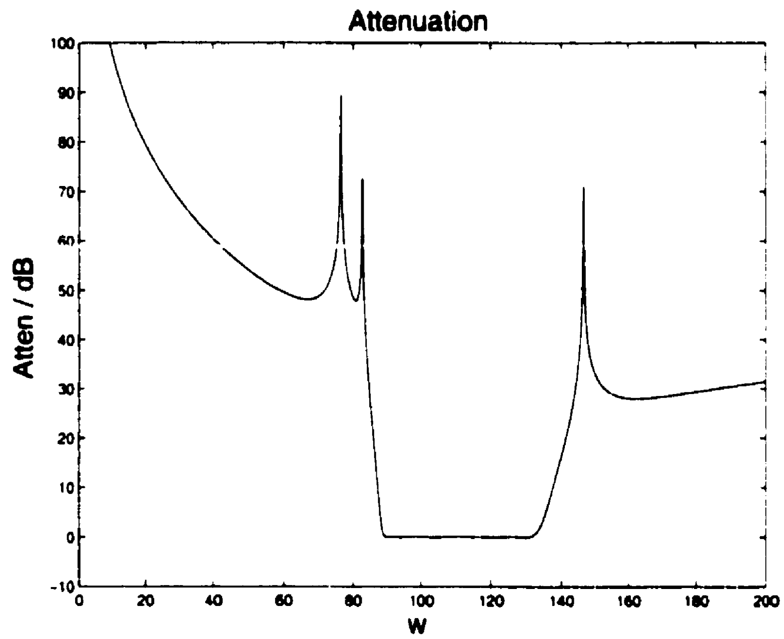

Fig.A.3.1.1

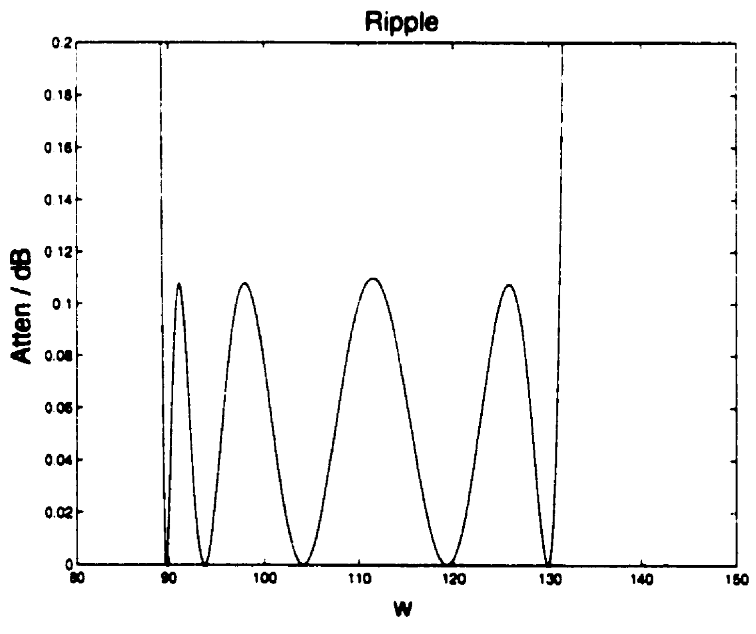

Fig.A.3.1.2

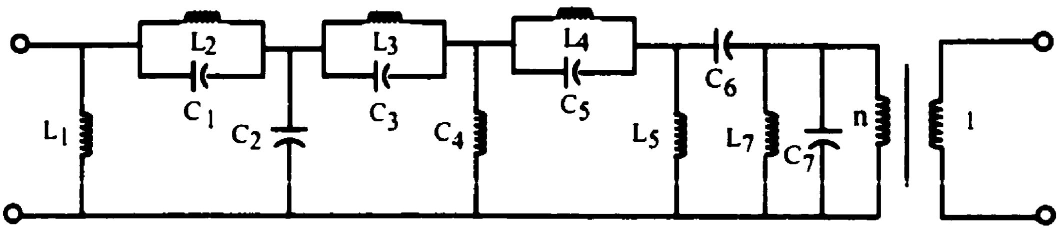

Fig.A.3.1.3 Circuit of a 10 degree narrow bandpass filter

3.2 Additional example of a 14 degree very narrow bandpass filter from Göttlicher [23, p.167]

Zeros of f	zeros of h
+j*8.992424181e-1; -j*8.992424181e-1; +j*9.004825819e-1; -j*9.004825819e-1; +j*8.996210097e-1; -j*8.996210097e-1; +j*9.001039903e-1; -j*9.001039903e-1; +j*8.996751863e-1; -j*8.996751863e-1; +j*9.000498137e-1; -j*9.000498137e-1;	+8.998624919e-1; -8.998624919e-1; +j*8.997450966e-1; -j*8.997450966e-1; +j*8.997714358e-1; -j*8.997714358e-1; +j*8.998270474e-1; -j*8.998270474e-1; +j*8.998979526e-1; -j*8.998979526e-1; +j*8.999535642e-1; -j*8.999535642e-1; +j*8.999799034e-1; -j*8.999799034e-1
Constant Multiplier = 1.0	Constant Multiplier = -3.0356223658419e+3

Zeros of g
-1.1552342827076e-5+j*8.9977021150841e-1; -1.1552342827076e-5-j*8.9977021150841e-1; -1.9199339335190e-5+j*8.9982635148176e-1; -1.9199339335190e-5-j*8.9982635148176e-1; -1.9199339335362e-5+j*8.9989864851824e-1; -1.9199339335362e-5-j*8.9989864851824e-1; -1.1552342827346e-5+j*8.9995478849159e-1; -1.1552342827346e-5-j*8.9995478849159e-1; -3.4936618244783e-6+j*8.9998092104884e-1; -3.4936618244783e-6-j*8.9998092104884e-1; -3.4936618243715e-6+j*8.9974407895116e-1; -3.4936618243715e-6-j*8.9974407895116e-1 -8.9969779610465e-1; -9.0002721784407e-1
Constant Multiplier = 3.0356223658419e+3

Table 42: Göttlicher's Example Result

Extracted Section Types	Extracted Transmission Zero	Synthesized Parameters
3	2	C1=1.243109386213119e+00
6	2	L1=8.339440764800440e+01 C2=1.482891931645905e-02
2	7	C3=1.031680570710744e+01
5	7	L2=1.081403245069809e-02 C4=1.140411789152826e+02
2	6	C5=1.503764263763107e-01
5	6	L3=1.230049253982286e-01 C6=1.003441651568025e+01
3	3	C7=4.971255378129980e-03

Table 42: Göttlicher's Example Result

Extracted Section Types	Extracted Transmission Zero	Synthesized Parameters
6	3	L4= 3.892655802479963e+04 C8=3.174203872551133e-05
3	4	C9= 7.212175170884430e-02
6	4	L5= 7.686259777041075e+03 C10=1.607361110579601e-04
2	5	C11=4.781154172707520e-01
5	5	L6=4.243800714668578e-03 C12=2.908787034256589e+02
3	1	C13=1.238011120145088e-01
2	8	C14=1.728893365246930e-01
0		n=2.023483129611001e+00

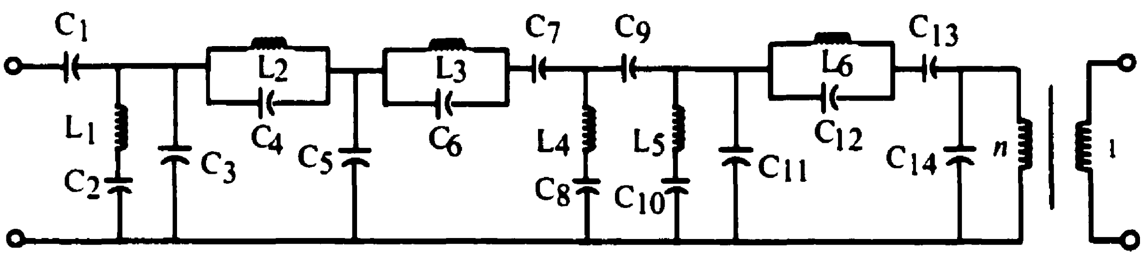

Fig. A.3.2.1 Circuit of a 14 degree very narrow bandpass filter

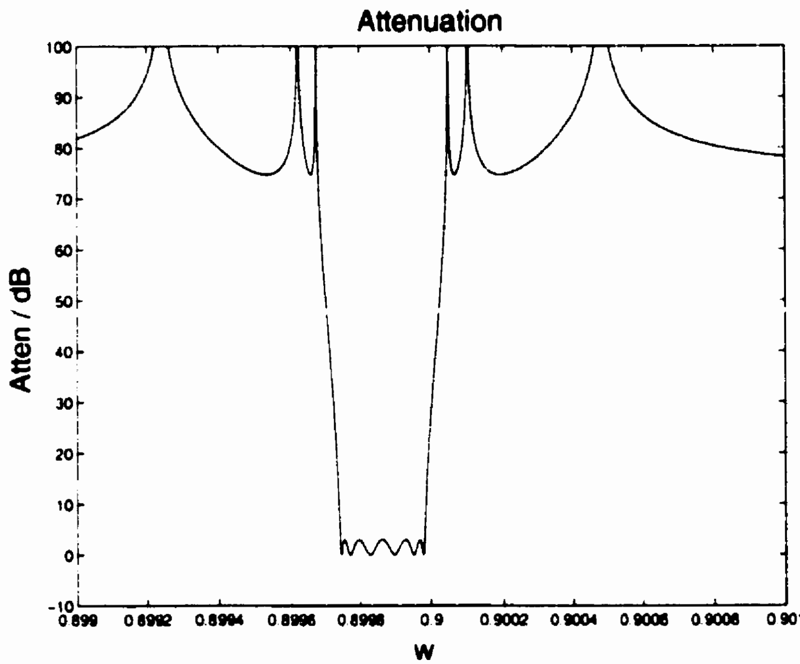

Fig. A.3.2.2

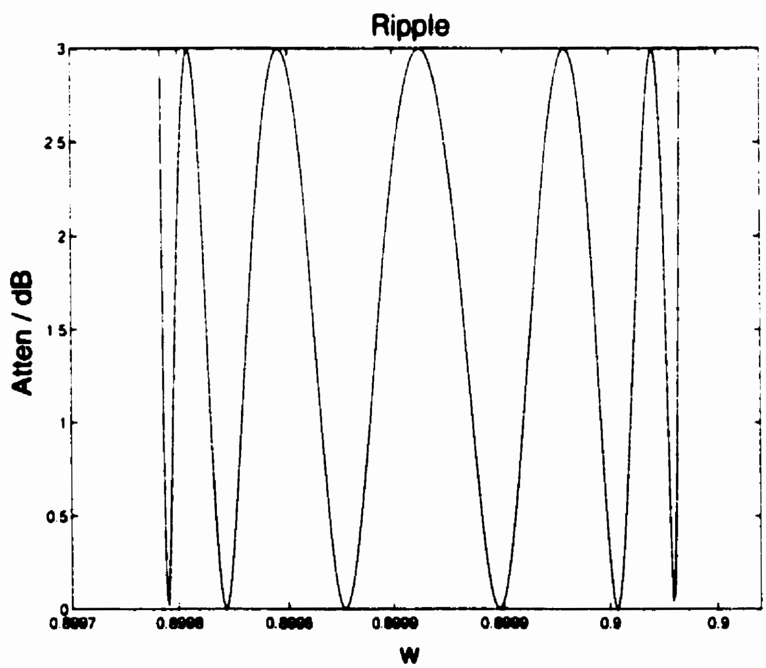

Fig. A.3.2.3

APPENDIX_IV

SCATTERING SYNTHEISIS PROGRAM LISTING

Main implementation program:

```
format long e
[g0,f,g,h] = CancelCommonZeros(f,g,h);
[No0,NoZeros,Noinf,Tr_zeros] = sortzeros(f,g,h);
n = length(Tr_zeros);
circuit_list = [];
Parameters_list = [];
while n > 0
disp('No0 = '), disp(No0);
disp('Noinf = '), disp(Noinf);
disp('Tr_zeros='), disp(Tr_zeros);
n = length(Tr_zeros);
successor_list = [];
P.r_m_i  = [];
NoOfSuccessors_i = [];
Rr_m_i  = [];
zero_i = [];
for i = 1:n
wi = Tr_zeros(i)
disp('i-loop');
disp('i='), disp(i);
wi = Tr_zeros(i)
successor_list = [i];
Rr_m_j  = [];
[par1, par2, Li, Ci,class, C_R] = classrithm(wi, No0, Noinf, g, h)
successor_list1 = AddationSuccCheck_class(C_R,class, No0, Noinf);
successor_list = [successor_list, successor_list1];
Rr_m_j = additionalR_iCheck(successor_list)
if class ~= 'I'
for j=1:n
wr = Tr_zeros(j)
if wr ~= wi & wr ~= 0 & wr ~= inf
[realizability_i,realizability_r] = circuitrithm(par1,par2,Li,Ci,class,wi,wr,No0,Noinf,g,h)
[successor_list2,Rr_m_j1] = SuccessorCheck(realizability_i,realizability_r,j)
successor_list = [successor_list,successor_list2];
Rr_m_j = [Rr_m_j, Rr_m_j1]
end
end     % j_loop finished
```

```
disp('successor_list = '); disp(successor_list)
x = length(successor_list)-1;
if x > 0
NoOfSuccessors_i = [NoOfSuccessors_i, x]
[mini_Rr_j,y] = min( Rr_m_j);
Rr_m_i = [Rr_m_i ; mini_Rr_j]
zero_i = [ zero_i;Tr_zeros(i)]
else
Rr_m_j = additionalR_iCheck(successor_list)
Rr_m_i = [Rr_m_i ; Rr_m_j ]
zero_i = [ zero_i;Tr_zeros(i)]
end
else
disp(' no macth circuit')
end
end
% i_loop finished
[ mini_Rr_i, order]= Select_maxinum( NoOfSuccessors_i,Rr_m_i);
disp('Final_ zero_i='), disp(zero_i(order));
wi = zero_i(order);
[par1, par2, Li, Ci, class]= classrithm(wi, No0, Noinf, g, h)
sigma_b = sigma;
fb = f;
gb = g;
hb = h;
Li = real(Li);
Ci = real(Ci);
[SecTypeNos, Parameters, sigma_b, fb, gb, hb]=class_remove(par1, par2, Li, Ci, class,
wi, sigma_b, fb, gb, hb);
circuit_list = [circuit_list , SecTypeNos]
Parameters_list = [Parameters_list; Parameters]
sigma = sigma_b;
f = fb;
g = gb;
h = hb;
norm_diff=Feldtkeller_prodRep(f,g,h);
disp('Feldtkeller check norm_diff='),disp(norm_diff)
[No0, NoZeros, Noinf, Tr_zeros] = sortzeros(f, g, h)
n = length(Tr_zeros)
if n <=  2
break
end
end
if n == 2 & (No0 == 1 | Noinf == 1 )
disp( 'This is stage n = 2');
```

```
wi = selectzero(No0,Noinf,Tr_zeros, n);
[par1, par2, Li, Ci, class] = classrithm(wi, No0, Noinf, g, h)
sigma_b = sigma;
fb = f;
gb = g;
hb = h;
[SecTypeNos, Parameters, sigma_b, fb, gb, hb] = class_remove(par1, par2, Li, Ci, class,
wi, sigma_b, fb, gb, hb);
end
circuit_list = [circuit_list, SecTypeNos];
Parameters_list = [Parameters_list; Parameters];
sigma_b = sigma;
f = fb;
g = gb;
h = hb;
norm_diff = Feldtkeller_prodRep(f,g,h);
disp('Feldtkeller check norm_diff='),disp(norm_diff)
[No0, NoZeros, Noinf, Tr_zeros] = sortzeros(f, g, h);
n = length(Tr_zeros);
end
if n == 1
wi = Tr_zeros(1);
[par1, par2, Li, Ci, circuit] = Ladderithm(wi, No0, Noinf, g, h);
sigma_b = sigma;
fb = f;
gb = g;
hb = h;
[SecTypeNos, Parameters, sigma_b, fb, gb, hb] = Ladder_Remove(par1, par2, Li, Ci, cir-
cuit, wi, sigma_b, fb, gb, hb)
circuit_list = [circuit_list, SecTypeNos];
Parameters_list = [Parameters_list; Parameters];
sigma_b = sigma;
f = fb;
g = gb;
h = hb;
norm_diff = Feldtkeller_prodRep(f, g, h);
disp('Feldtkeller check norm_diff='),disp(norm_diff);
[No0, NoZeros, Noinf, Tr_zeros] = sortzeros(f, g, h );
n = length(Tr_zeros);
end
if n == 0
n = transformer(g, h)
circuit_list = [circuit_list , 0];
Parameters_list = [Parameters_list; n];
end
```

```
disp('final circuit Type ='), disp(circuit_list)
disp('final Parameters = '), disp(Parameters_list)
save SecParFile1 circuit_list Parameters_list
```

Addational Succseor Checking

```
function successor = AddationSuccCheck(C_R, circuit, No0, Noinf)
if (circuit == 19 | circuit == 20 | circuit == 21)
if (C_R == 1 & ((Noinf > 2) | ( (Noinf == 2) & ( No0>0 ) ) ))
successor = inf;
elseif (C_R == 0 & ((Noinf >= 1) | ( (Noinf == 1) & ( No0>0 ) ) ))
successor = inf;
elseif (C_R == 1 & ((No0 == 1) | ( (Noinf > 1) & ( No0 > 1 ) )))
successor = 0;
elseif (C_R == 0 & (No0 >= 1))
successor = 0;
else
successor = 999;
end
elseif (circuit == 1 | circuit == 2 | circuit == 3)
if (C_R == 1 & ((No0 > 2) | ( (No0 == 2) & ( Noinf > 0 ) )))
successor = 0;
elseif (C_R == 0 & ((No0 > 1) | ( (No0 == 1) & ( No0 > 0 ) )))
successor = 0;
elseif (C_R == 1 & ((Noinf == 1) | ( (No0 > 1) & ( Noinf > 1 ) )))
successor = inf;
elseif (C_R == 0 & (Noinf > 0))
successor = inf;
else
successor = 999;
end
elseif (circuit == 13 | circuit == 14 | circuit == 15)
if (C_R == 1 & ((Noinf > 2) | ((Noinf == 2) & ( No0 > 0 ))))
successor = inf;
elseif (C_R == 0 & ((Noinf >= 1) | ((Noinf == 1) & ( No0 > 0 ))))
successor =  inf;
elseif (C_R == 1 & ((No0 == 1) | ( (Noinf > 1) & ( No0 > 1 ) )))
successor = 0;
elseif C_R == 0 & (No0 > 0 )
successor = 0;
else
successor = 999;
end
elseif (circuit == 7 | circuit == 8 | circuit == 9)
if (C_R == 1 & ((No0 > 2) | ( (No0 == 2) & ( Noinf > 0 ) )))
successor = 0;
```

```
elseif (C_R == 0 & ((No0 > 1) | ( (No0 == 1) & ( Noinf > 0 ) ) ))
successor = 0;
elseif (C_R == 1 & ((Noinf == 1) | ( (No0 > 1) & ( Noinf > 1 ) )))
successor = inf;
elseif (C_R == 0 & (Noinf > 0))
successor = inf;
else
successor = 999;
end
else
successor = 999;
end
```

Realizability Checking

```
function [realizability_i,realizability_r]=circuit-
rithm(par1,par2,Li,Ci,class,wi,wr,No0,Noinf,g,h)
reflectance = refl_at_s_prodRep(wr, g, h);
delay  = DelayRefl_prodRep(wr, g, h);
alpha = angle(reflectance);
omiga_r = imag(wr);
omiga_i = imag(wi);
C_R = 1;
if Noinf > 0
rho_at_inf = refl_at_inf(g, h);
delay_at_inf = delay_inf(g, h);
Cinf = 2/ delay_at_inf;
Linf = delay_at_inf/2;
end
if No0 > 0
rho_at_0 = rho_at_zero(h, g);
delay_0 = DelayAt_0(g,h);
C0 = delay_0 /2;
L0 = 2/delay_0 ;
end
number = class;
if number == class
switch number
case {'A'}
Lro = 1/((1/par1)-(1/par2))-Li;
Lr = 1/( omiga_r*tan(alpha/2))
L2 = 1/((1/Lr)-(1/par2)) - (Li/(1-(omiga_r/omiga_i)^2))
realizability_i= par2/par1 ;
realizability_r = L2/Lro;
if realizability_i >= (1-1e-8) & realizability_r >= (1-1e-8)
disp(' circuit = 1')
```

```
realizability_i= par2/par1 ;
realizability_r = L2/Lro;
else
C_R = 0;
end
if (C_R == 0)
if (rho_at_inf < 0) & (Noinf > 0)
disp(' circuit = 2')
Cinf = 2/ delay_at_inf;
Cr = -tan(alpha/2)/omiga_r;
Crinf = 1/((1/Cinf)-(1/Ci));
C1= 1/(omiga_r^2*par2);
C2= 1/(Ci-1/(omiga_r^2*Li));
C3 = 1/(C1+Cr);
C= 1/(C3 - C2);
realizability_i= par2/par1 ;
realizability_r = Crinf/C;
elseif (C_R == 0) & (rho_at_inf > 0) & (Noinf > 0)
disp(' circuit = 3')
Linf = 2/ delay_at_inf;
Lr = 1/tan(alpha/2)/omiga_r;
Lrinf = 1/((1/Lro)-(1/par2));
L1= 1/(1/Lr - 1/par2);
L2= 1/(1/Li-(omiga_r^2*Ci));
Ls= L1-L2;
realizability_i= par2/par1 ;
realizability_r = Lrinf/Ls;
end
end
case {'B'}
Cro = delay_0/2;
Cr = -tan(alpha/2)/omiga_r;
C2 = 1/(omiga_r^2*par2);
Cs= Cr+C2;
realizability_i= par2/par1 ;
realizability_r = Cs/Cro;
if realizability_i >=(1-1e-8) & realizability_r >= (1-1e-8)
disp(' circuit = 4')
realizability_i= par2/par1 ;
realizability_r = Cs/Cro;
else
C_R = 0;
end
if C_R == 0 & Noinf > 0 & rho_at_inf < 0
disp(' circuit = 5')
```

```
delay_at_inf = delay_inf(g,h);
Cinf = 2/delay_at_inf;
C1= 1/(omiga_r^2*par2);
C2= tan(alpha/2)/omiga_r;
C = C1-C2;
realizability_i= par2/par1 ;
realizability_r = Cinf/C;
elseif C_R == 0 & Noinf > 0 & rho_at_inf > 0
disp(' circuit = 6')
delay_at_inf = delay_inf(g,h);
Linf = 2/ delay_at_inf;
Lrinf = 1/((1/Linf)-(1/par2));
Lr= 1/tan(alpha/2)/omiga_r;
Ls = 1/(1/Lr - 1/par1);
realizability_i= par2/par1 ;
realizability_r = Lrinf/Ls;
end
case {'C'}
Cro = 1/((1/par1)-(1/par2))-Ci;
Cr = -tan(alpha/2)/omiga_r;
C1 = 1/Cr - 1/par2;
C2 = Ci/(1- (omiga_r/omiga_i)^2);
Cs = 1/C1 - C2;
realizability_i= par2/par1 ;
realizability_r = Cs/Cro;
if realizability_i >= (1-1e-8) & realizability_r >= (1-1e-8)
disp(' circuit = 7')
realizability_i= par2/par1 ;
realizability_r = Cs/Cro;
else
C_R = 0;
end
if C_R == 0 & Noinf > 0 & rho_at_inf < 0
disp(' circuit = 8')
Cinf = 2/ delay_at_inf;
Crinf = 1/((1/Cinf)-(1/par2));
Cr = -tan(alpha/2)/omiga_r;
C1 = 1/Cr - 1/par2;
C2 = Ci/(1- (omiga_r/omiga_i)^2);
C = 1/C1 - C2;
realizability_i= par2/par1 ;
realizability_r = Crinf/C;
elseif C_R == 0 & Noinf > 0 & rho_at_inf > 0
disp(' circuit = 9')
Linf = 2/delay_at_inf;
```

```
Lrinf = 1/((1/Linf)-(1/Li));
Lr = 1/tan(alpha/2)/omiga_r;
L1 = 1/(Lr + 1/(omiga_r^2*par2));
L11 = 1/((omiga_r^2)*Ci);
L2 = 1/(Li-L11);
Ls = 1/(L1 -L2);
realizability_i= par2/par1 ;
realizability_r = Lrinf/Ls;
end
case {'D'}
db_At_0=db_At_0_prodRep(g,h);
Lro = db_At_0/2;
Lr = 1/tan(alpha/2)/omiga_r;
Lp = Lr + 1/(omiga_r^2*par2);
realizability_i= par2/par1 ;
realizability_r = Lp/Lro;
if realizability_i >=(1-1e-8) & realizability_r >= (1-1e-8)
disp(' circuit = 10')
realizability_i= par2/par1 ;
realizability_r = Lp/Lro;
else
C_R = 0;
end
if C_R == 0 & Noinf > 0 & rho_at_inf < 0
disp(' circuit = 11')
Cinf = 2/delay_at_inf
Crinf = 1/((1/Cinf)-(1/par1))
Cr = -tan(alpha/2)/omiga_r
Cp = 1/(1/Cr - 1/par1)
realizability_i= par2/par1
realizability_r = Crinf/Cp
elseif C_R == 0 & Noinf > 0 & rho_at_inf > 0
disp(' circuit = 12')
Linf = 2/delay_at_inf;
Lrinf =Linf;
Lr = 1/(omiga_r*tan(alpha/2));
Ls = Lr +1/(omiga_r^2*par1);
realizability_i= par2/par1 ;
realizability_r = Lrinf/Ls;
end
case {'E'}
Cinf = 2/delay_at_inf;
Crinf = 1/( 1/(Cinf - par2) - 1/Ci);
Cr = -tan(alpha/2)/omiga_r;
C1 = 1/(Cr - par2);
```

```
C11 = 1/(Ci*(1- (omiga_i/omiga_r)^2));
Cp = 1/(C1 - C11);
realizability_i= par1/par2 ;
realizability_r = Crinf/Cp;
if realizability_i >=(1-1e-8) & realizability_r >= (1-1e-8)
disp(' circuit = 15')
realizability_i= par1/par2 ;
realizability_r = Crinf/Cp;
else
C_R = 0;
end
if C_R == 0
 if rho_at_inf < 0 & No0 > 0 & rho_at_0 < 0
disp(' circuit = 13')
L0 = delay_0/2;
Lr0 =L0 - Li;
Lr = 1/(omiga_r*tan(alpha/2));
Lp = 1/(1/Lr + (omiga_r^2*par2)) + 1/Ci/(omiga_r^2 - omiga_i^2);
realizability_i= par1/par2 ;
realizability_r = Lp/Lr0;
elseif rho_at_inf < 0 & No0 > 0 & rho_at_0 > 0
disp(' circuit = 14')
Cinf = 2/delay_at_inf;
C0 = delay_0/2;
Cr0 = C0 - par2;
Cr = -tan(alpha/2)/omiga_r;
C1 = 1/(Cr - par2);
C11 = 1/Ci/(1- (omiga_i^2)/(omiga_r^2));
Cs = 1/(C1 - C11);
realizability_i= par1/par2 ;
realizability_r = Cs/Cr0;
end
end
case {'F'}
Cinf = 2/delay_at_inf;
db_AtInf = db_AtInf_prodRep(g,h);
Lrinf = 2/db_AtInf;
L1 = omiga_r^2*par1;
Lr = 1/omiga_r/tan(alpha/2);
Ls = 1/(L1+ 1/Lr);
realizability_i= par1/par2 ;
realizability_r = Lrinf/Ls;
if realizability_i >=(1-1e-8) & realizability_r >= (1-1e-8)
disp(' circuit = 18')
realizability_i= par1/par2 ;
```

```
realizability_r = Lrinf/Ls;
else
C_R = 0;
end
if C_R == 0
if No0 > 0 & rho_at_0 < 0
disp(' circuit = 16')
delay_at_inf = delay_inf(g,h);
delay_0 = DelayAt_0(g,h);
Cinf = 2/delay_at_inf;
L0 = delay_0/2;
Lr = 1/(omiga_r*tan(alpha/2));
Lp = 1/((1/Lr)+omiga_r^2*par1);
realizability_i= par1/par2 ;
realizability_r = Lp/L0;
elseif C_R == 0 & No0 > 0 & rho_at_0 > 0
disp(' circuit = 17')
Cinf = 2/delay_at_inf;
C0 = delay_0/2;
Cr = -tan(alpha/2)/omiga_r;
Cs = Cr - par1;
realizability_i= par1/par2 ;
realizability_r = Cs/C0;
end
end
case {'G'}
Linf = 2/delay_at_inf;
Lr = 1/(omiga_r*tan(alpha/2));
Lr1 = 1/(Linf - par2);
Lr11 = 1/Li;
Lrinf = 1/(Lr1 -Lr11);
L1 = 1/(Lr - par2);
L11 = 1/Li/(1- (omiga_i/omiga_r)^2);
Ls = 1/(L1 - L11);
realizability_i= par1/par2 ;
realizability_r = Lrinf/Ls;
if realizability_i >=(1-1e-8) & realizability_r >= (1-1e-8)
disp(' circuit = 21')
realizability_i= par1/par2 ;
realizability_r = Lrinf/Ls;
else
C_R = 0;
end
if C_R == 0
if No0 > 0 & rho_at_0 < 0
```

```
disp(' circuit = 19')
L0 = delay_0/2;
Lro = L0 - par2;
Lr = 1/(omiga_r*tan(alpha/2));
L1 = 1/(Lr - par2);
L11 = 1/Li/(1- (omiga_i/omiga_r)^2);
Lp = 1/( L1 - L11);
realizability_i= par1/par2 ;
realizability_r = Lp/Lro;
elseif C_R == 0 & No0 > 0 & rho_at_0 > 0
disp(' circuit = 20')
C0 =  delay_0/2;
Cro = C0 - Ci;
Cr = -tan(alpha/2)/omiga_r;
C1 = 1/(omiga_r^2*par2 + 1/Cr);
C11 = Ci/(1- (omiga_r/omiga_i)^2);
Cs = C1 - C11;
realizability_i= par1/par2 ;
realizability_r = Cs/Cro;
end
end
case {'H'}
db_AtInf = db_AtInf_prodRep(g,h)
Crinf = 2/db_AtInf;
Cr = -tan(alpha/2)/omiga_r;
C1 = (omiga_r^2)*par1;
Cp = 1/(C1 + 1/Cr);
realizability_i= par1/par2 ;
realizability_r = Crinf/Cp;
if realizability_i >=(1-1e-8) & realizability_r >= (1-1e-8)
disp(' circuit = 21')
realizability_i= par1/par2 ;
realizability_r = Crinf/Cp;
else
C_R = 0;
end
if C_R == 0
if No0 > 0 & rho_at_0 < 0
disp(' circuit = 22')
delay_0 = DelayAt_0(g,h);
L0 = delay_0/2;
Lro = L0 - par1;
Lr = 1/(omiga_r*tan(alpha/2));
Lp = Lr - par1;
realizability_i= par1/par2 ;
```

```
realizability_r = Lp/Lro;
elseif C_R == 0 & No0 > 0 & rho_at_0 > 0
disp(' circuit = 23')
Cr0 = delay_0/2;
Cr = -tan(alpha/2)/omiga_r;
C1 = omiga_r^2*par1;
Cs = 1/(C1 + 1/Cr);
realizability_i= par1/par2 ;
realizability_r = Cs/Cr0;
end
end
case {'I'}
realizability_i= 999;
realizability_r = 999;
end
end
function [f, g, h, sigma] = chain(fa, ga, ha, sigma_a, fb, gb, hb, sigma_b)
% [f,g,h, sigma] = chain returns the polynomials
% f, g, h and the constant sigma for a chain connection of
% two-ports Na and Nb
% the polynomials are row vectors of coefficients in descending order
sigma = sigma_a*sigma_b;
f = conv(fa,fb);
h = sigma_a*lower_star(ga);
h = polyadd(conv(h,hb),conv(ha,gb));
g = sigma_a*lower_star(ha);
g = polyadd(conv(g,hb),conv(ga,gb));
```

Remove circuit class

```
function [SecTypeNos,Parame-
ters,sigma_b,fb,gb,hb]=class_remove(par1,par2,Li,Ci,class,wi,sigma_b,fb,gb,hb)
switch class
case {'A'}
C_R = 0;
Tr_zeros = wi;
L = par2;
[SecTypeNos1,Parameters1,sigma_b,fb,gb,hb] =
RemoveShunt_L_2(L,Tr_zeros,C_R,sigma_b,fb,gb,hb)
L = Li;
C = Ci;
Tr_zeros = wi;
C_R = 1;
[SecTypeNos2,Parameters2,sigma_b,fb,gb,hb] =
Remove_Series_Section2(L,C,Tr_zeros,C_R,sigma_b,fb,gb,hb)
Parameters = [Parameters1;Parameters2],
```

```
SecTypeNos = [SecTypeNos1,SecTypeNos2]
case {'B'}
C_R = 1;
Tr_zeros = wi;
L = par1;
[SecTypeNos,Parameters,sigma_b,fb,gb,hb] =
RemoveShunt_L_2(L,Tr_zeros,C_R,sigma_b,fb,gb,hb)
case {'C'}
C_R = 0;
Tr_zeros = wi;
C = par2;
[SecTypeNos1,Parameters1,sigma_b,fb,gb,hb] =
RemoveSeries_C_2(C,Tr_zeros,C_R,sigma_b,fb,gb,hb)
L = Li;
C = Ci;
Tr_zeros = wi;
C_R = 1;
[SecTypeNos2,Parameters2,sigma_b,fb,gb,hb] =
Remove_Shunt_Section2(L,C,Tr_zeros,C_R,sigma_b,fb,gb,hb)
Parameters = [Parameters1;Parameters2],
SecTypeNos = [SecTypeNos1,SecTypeNos2]
case {'D'}
C_R = 1;
Tr_zeros = wi;
C = par1;
[SecTypeNos,Parameters,sigma_b,fb,gb,hb] =
RemoveSeries_C_2(C,Tr_zeros,C_R,sigma_b,fb,gb,hb)
case {'E'}
C_R = 1;
Tr_zeros = wi;
C = par2;
[SecTypeNos1,Parameters1,sigma_b,fb,gb,hb] =
RemoveShunt_C_2(C,Tr_zeros,C_R,sigma_b,fb,gb,hb)
C_R = 1;
Tr_zeros = wi;
L = Li;
C = Ci;
[SecTypeNos2,Parameters2,sigma_b,fb,gb,hb] =
Remove_Series_Section2(L,C,Tr_zeros,C_R,sigma_b,fb,gb,hb)
Parameters = [Parameters1;Parameters2],
SecTypeNos = [SecTypeNos1,SecTypeNos2]
case {'F'}
C_R = 1;
Tr_zeros = wi;
C = par1;
```

```
[SecTypeNos,Parameters,sigma_b,fb,gb,hb] =
RemoveShunt_C_2(C,Tr_zeros,C_R,sigma_b,fb,gb,hb)
case {'G'}
C_R = 1;
Tr_zeros = wi;
L = par2;
[SecTypeNos1,Parameters1,sigma_b,fb,gb,hb] =
RemoveSeries_L_2(L,Tr_zeros,C_R,sigma_b,fb,gb,hb)
C_R = 1;
Tr_zeros = wi;
L = Li;
C = Ci;
[SecTypeNos2,Parameters2,sigma_b,fb,gb,hb] =
Remove_Shunt_Section2(L,C,Tr_zeros,C_R,sigma_b,fb,gb,hb)
Parameters = [Parameters1;Parameters2],
SecTypeNos = [SecTypeNos1,SecTypeNos2]
case {'H'}
C_R = 1;
 Tr_zeros = wi;
L = par1;
[SecTypeNos,Parameters,sigma_b,fb,gb,hb] =
RemoveSeries_L_2(L,Tr_zeros,C_R,sigma_b,fb,gb,hb)
end

function [SecTypeNos,Parameters,sigma_b,fb,gb,hb] = Remove_Series_Section2
(L,C,Tr_zeros,C_R,sigma_b,fb,gb,hb
[fa,ga,ha,sigma_a]=series_ShuntSection2(C, L)  % prodRep version
if C_R==1
[sigma_b,fb,gb,hb]=RemoveSection_a(sigma_a,fa,ga,ha,sigma_b,fb,gb,hb);
%remove section a
end
SecTypeNos=[5];
Parameters=[L;C];

function [SecTypeNos,Parameters,sigma_b,fb,gb,hb] =
Remove_Shunt_Section2(L,C,Tr_zeros,C_R,sigma_b,fb,gb,hb)
[fa,ga,ha, sigma_a]=series_ShuntSection2(C, L)  % prodRep version
if C_R==1
[sigma_b,fb,gb,hb]=RemoveSection_a(sigma_a,fa,ga,ha,sigma_b,fb,gb,hb);
%remove section a
SecTypeNos=[SecTypeNos,5];
Parameters=[Parameters;L;C];

function [SecTypeNos,Parameters,sigma_b,fb,gb,hb] =
Remove_Shunt_Section2(L,C,Tr_zeros,C_R,sigma_b,fb,gb,hb)
```

```
[fa,ga,ha, sigma_a]=shunt_SeriesSection2(C, L)  % prodRep version
%disp('*********');
%disp('sigma_a='), disp(sigma_a);
%disp('fa='), disp(fa);
%disp('ga='), disp(ga);
%fb=f_FromTrZeros(Tr_zeros);
if C_R==1
[sigma_b,fb,gb,hb]=RemoveSection_a(sigma_a,fa,ga,ha,sigma_b,fb,gb,hb);
%remove section a
end
SecTypeNos=[6];
Parameters=[L;C];

function [sigma_b,fb,gb,hb]=RemoveSection_a(sigma_a,fa,ga,ha,sigma,f,g,h)
% remove section a
sigma_b=sigma*sigma_a;
% gb=(ga*g-ha*h)/fafa*
a=lower_star_prodRep(ga); % prodRep version
a=polyMul_prodRep(a,g);
b=lower_star_prodRep(ha);
b=polyMul_prodRep(b,h);
b(1)=-b(1);
c=lower_star_prodRep(fa);
c=polyMul_prodRep(c,fa);
%a,b,c,
%p1=polyAdd_prodRep(a,b)
gb=Zeros_a_Plus_b_c(a,b,c);
% determines zeros of p=(a+b)/c given:c|(a+b)d=real(d);
% hb=(gah-hag)/sigma_afafa*
a=polyMul_prodRep(ga,h);
b=polyMul_prodRep(ha,g);
b(1)=-b(1);
c=lower_star_prodRep(fa);
c=polyMul_prodRep(c,fa);
c(1)=c(1)*sigma_a;
%a,b,c,
% if length(c)==3
% save TestFile1 a b c
% end
% p2=polyAdd_prodRep(a,b)
hb=Zeros_a_Plus_b_c(a,b,c);
% determines zeros of p=(a+b)/c given:c|(a+b)d=real(d);
tol=1e-12;
[g,a,b] = CancelCommonElems(fa,f,tol); % removes the common elements
```

```
fb=[1;b];

function [SecTypeNos,Parameters,sigma_b,fb,gb,hb] =
RemoveSeries_C_2(C,Tr_zeros,C_R,sigma_b,fb,gb,hb)
[fa,ga,ha, sigma_a]= series_C_Section2(C); % prodRep version
%fb=f_FromTrZeros(Tr_zeros);
if C_R==1
[sigma_b,fb,gb,hb]=RemoveSection_a(sigma_a,fa,ga,ha,sigma_b,fb,gb,hb);
%remove section a
else
[sigma_b,fb,gb,hb]=PartiallyRemoveSection_a(sigma_a,fa,ga,ha,sigma_b,fb,gb,hb);
% partially remove section a
end
SecTypeNos=[3];
Parameters=[C];

function [SecTypeNos,Parameters,sigma_b,fb,gb,hb] =
RemoveSeries_L_2(L,Tr_zeros,C_R,sigma_b,fb,gb,hb)
[fa,ga,ha, sigma_a]=series_L_Section2(L); % prodRep version
%fb=f_FromTrZeros(Tr_zeros);
[sigma_b,fb,gb,hb]=RemoveSection_a(sigma_a,fa,ga,ha,sigma_b,fb,gb,hb);
%remove section a
SecTypeNos=[1];
Parameters=[L];

function [SecTypeNos,Parameters,sigma_b,fb,gb,hb] =
RemoveShunt_C_2(C,Tr_zeros,C_R,sigma_b,fb,gb,hb)
[fa,ga,ha, sigma_a]=parallel_C_Section2(C); % prodRep version
%fb=f_FromTrZeros(Tr_zeros);
[sigma_b,fb,gb,hb]=RemoveSection_a(sigma_a,fa,ga,ha,sigma_b,fb,gb,hb);
%remove section a
SecTypeNos=[2];
Parameters=[C];

function [SecTypeNos,Parameters,sigma_b,fb,gb,hb] =
RemoveShunt_L_2(L,Tr_zeros,C_R,sigma_b,fb,gb,hb)
[fa,ga,ha, sigma_a]=parallel_L_Section2(L); % prodRep version
%fb=f_FromTrZeros(Tr_zeros);
if C_R==1
[sigma_b,fb,gb,hb]=RemoveSection_a(sigma_a,fa,ga,ha,sigma_b,fb,gb,hb);
%remove section a
else
[sigma_b,fb,gb,hb]=PartiallyRemoveSection_a(sigma_a,fa,ga,ha,sigma_b,fb,gb,hb);
% partially remove section a
end
```

```
SecTypeNos=[4];
Parameters=[L];

function [f,g,h, sigma] = series_C_Section(C)
% [f,g,h, sigma] = series_C_Section(C) returns the polynomials
% f, g, h and the constant sigma for a series C-Section
% the polynomials are row vectors of coefficients in descending order
% the input is the parameter C
f=[1 0];
g=[1 1/(2*C)];
h=[1/(2*C)];
sigma=-1;

function [f,g,h, sigma] = series_C_Section2(C)
% [f,g,h, sigma] = series_C_Section(C) returns the polynomials
% f, g, h and the constant sigma for a series C-Section
% the polynomials are column vectors of constant factor and followed by zeros
% the input is the parameter C
f=[1; 0];
g=[1; -1/(2*C)];
h=[1/(2*C)];
sigma=-1;

function [f,g,h, sigma] = series_L_Section(L)
% [f,g,h, sigma] = series_C_Section(L) returns the polynomials
% f, g, h and the constant sigma for a series L-Section
% the polynomials are row vectors of coefficients in descending order
% the input is the parameter L

f=[1];
g=[L/2 1];
h=[L/2 0];
sigma=1;

function [f,g,h, sigma] = series_L_Section2(L)  % prodRep version
% [f,g,h, sigma] = series_L_Section(L) returns the polynomials
% f, g, h and the constant sigma for a series L Section
% the polynomials are column vectors of constant factor and followed by zeros
% the input is the parameter L
d=2/L;
f=[1];
g=[1/d; -d];
h=[1/d; 0];
sigma=1;
```

```
function [f,g,h, sigma] = series_ShuntSection(C, L)
% [f,g,h, sigma] = series_ShuntSection(C, L) returns the polynomials
% f, g, h and the constant sigma for a series ShuntSection (parallel resonance)
% the polynomials are row vectors of coefficients in descending order
% the input is the parameters C and L which are in parallel
f=[1 0 1/(L*C)];
g=[1 1/(2*C) 1/(L*C)];
h=[1/(2*C) 0];
sigma=1;

function [successor_list1,Rr_m_j1] = SuccessorCheck2(realizability_i,realizability_r, circuit, No0,Noinf, j)
successor_list1 = [];
Rr_m_j1 = [];
if realizability_i == 1 | realizability_i == (1-1e-10)
C_R = 1;
elseif realizability_i ~= 1 | realizability_i ~= (1-1e-10)
C_R = 0;
end
successor=AddationSuccCheck(C_R,circuit, No0,Noinf);
disp('successor='), disp(successor)
if successor == 0 | successor == inf
successor_list1 = [successor_list1, successor];
else
successor_list1 = [successor_list1];
end
if realizability_i >= (1-1e-10 ) & realizability_r >= (1-1e-10 )
successor_list1 = [successor_list1, j];
Rr_m_j1 = [ Rr_m_j1 ;realizability_r];
else
successor_list1 = [successor_list1];
Rr_m_j1  = [ Rr_m_j1; 99 ];
end

function [f,g,h,sigma]=Transformer(n)
% f, g, h sigma are constants
% the input is the parameter n
f=[1];
g=[(n^2+1)/n/2];
h=[(n^2-1)/n/2];
sigma=1;
```