DYNAMIC ADJUSTMENT MODELS OF THE ALBERTA BEEF INDUSTRY UNDER RISK AND UNCERTAINTY

By

Msafiri Daudi Mbaga

A Dissertation
Submitted to the University of Manitoba
in partial fulfillment of the
requirements for the degree of
Doctor of Philosophy
in the
Department of Agricultural Economics and Farm management

Winnipeg, Manitoba.

0-612-53069-8

THE UNIVERSITY OF MANITOBA

FACULTY OF GRADUATE STUDIES
*****
COPYRIGHT PERMISSION PAGE

Dynamic Adjustment Models of the Alberta Beef Industry Under Risk and Uncertainty

BY

Msafiri Daudi Mbaga

A Thesis/Practicum submitted to the Faculty of Graduate Studies of The University of Manitoba in partial fulfillment of the requirements of the degree

of

Doctor of Philosophy

Abstract

Dynamic Adjustment Models of the Alberta Industry under Risk and Uncertainty
By: Msafiri Daudi Mbaga

Major Advisor:
Professor Barry T. Coyle, Ph.D
Advisory Committee Members:
Professor Michael Popp, Ph.D
Professor Wayne Simpson, Ph.D
External Examiner
Professor John M. Marsh, Ph.D

The purpose of this thesis is to develop and estimate dynamic models of cow-calf and feedlot production decisions in Alberta under risk aversion and output price uncertainty. The thesis consists of two studies.

The first study specifies and estimates reduced form Autoregressive Distributed Lag (ADL) and Polynomial Distributed Lag (PDL) models incorporating price uncertainty. ADL and PDL models are estimated assuming distributed lags for variance of output price. The sum of lagged coefficients for output price variance is negative and significant, as expected. The elasticity is much smaller than for the (positive) sum of lagged coefficients for expected price, as anticipated.

The second study specifies and estimates dynamic Euler equation models of beef supply and investment under risk aversion and uncertainty. A beef output supply equation and an Euler equation for investment in breeding herd were specified assuming both linear and nonlinear mean-variance risk preferences.

Results for the structural cow-calf models are consistent with economic theory.

Output supply and investment are increasing in expected output price and decreasing in price variance, and the shadow price of capital is increasing in expected price and decreasing in price variance.

There are indications that dynamics is less important in feedlot production than in cow-calf production, simply because biological lags are much shorter in feedlot production. Results for Euler equations suggest that feedlot investment decisions are influenced by expected output price variance, consistent with economic theory.

To my knowledge, this is the first study of beef supply response to attempt to incorporate risk aversion. As a result, represents a significant departure from previous studies in the same area that have exclusively assumed risk neutrality by excluding the influence of uncertainty on decisions.

Acknowledgements

First, I wish to express my profound gratitude to my major advisor, Dr Barry T. Coyle for his sincere encouragements and his diplomatic way of getting me back on the right track when I was digressing. Thank you very much Dr. Coyle for introducing me to this new and very exciting area.

I am also very grateful to Dr. Mike Popp and Dr. Wayne Simpson for participating on my advisory committee. Their critical suggestions and sound technical comments were very instrumental to the improvement of this dissertation. I also thank my external examiner, Dr. John M. Marsh for his invaluable feedback and comments.

I would like to extend my deep gratitude to the Canadian Commonwealth Scholarship Agency for providing financial support. I also extend my appreciation to my office mate Michelle Bielik, my fellow graduate students and the support staff in the department for their unconditional support throughout my program.

Special thanks goes to my family, the Mbaga family, for the encouragement and support which helped to keep me going. To my dear wife Gloria, whom whatever words of thanks I say, they would not be enough to do justice to her, sweetheart we made it. Without you I could not have done this. To my son, Daudi, whom just remembering him makes me know what I want to achieve in life.

Table of Contents.

List of Tables

Appendix A

Appendix B.

Appendix C.

CHAPTER ONE

INTRODUCTION

Overview of the Canadian Beef Industry .

The beef industry is an important component of the Canadian economy. The industry generates between $4 and $5 billion in farm gate sales annually (CANFAX). This is about one fifth of total farm gate sales for all agricultural commodities. The beef industry has undergone extensive structural change during the past ten years (Agriculture and Agri-Food Canada, 1997). In the process, there has been a significant westward shift of production, and producing units have become larger in size and fewer in number. Beef production and processing are concentrated in the Prairie provinces (Appendix A, Table 1 and 2).
Overall, Western Canada accounted for about 68 to 73 percent of the Canadian cattle inventory (Table 3).

The Prairie region is well suited to the production of grains (wheat and barley), oilseeds (especially canola) and forages including alfalfa, making the region a low cost source of feed. The Prairies accounted for about two thirds of the Canadian July 1 cattle inventory during 1991-1997 with Alberta accounting for 36 to 38 percent (Table 3). The region accounted for an even larger share of the beef cattle sector: between 1991 and 1997 the Prairie provinces accounted for 83 to 85 percent of the beef cow inventory, with Alberta accounting for 42 to 44 percent (Table 4). Alberta stands as a low cost producer among the provinces in Canada. According to the Canadian Cattlemen's Association (CCA), large supplies and relatively low costs of feed grain in Alberta have contributed to increased cattle feeding there. It is estimated that 80 percent of Alberta's cow-calf growers are also grain farmers, and many of them have expanded their cattle feeding operations in recent years.

Saskatchewan is second after Alberta in terms of beef production in Canada. Beef production has always been an integral part of Saskatchewan agriculture, where a significant portion of farms are involved in beef production. Saskatchewan accounted for 18 to 19 percent of the total Canadian cattle inventory, and about 23 to 25 percent of the beef cow inventory between 1991 and 1997 (Tables 3,4). Manitoba accounted for 8 to 10 percent of the total Canadian cattle inventory and about 11 to 12 percent of the beef cow inventory (Tables 3,4). In Ontario, cattle are commonly kept as part of a diversified farming enterprise. Ontario accounted for 15 to 16 percent of the total Canadian cattle inventory, and about 9 to 10 percent of the beef cow inventory (Tables 3,4).

Cattle slaughter in Canada appears to be expanding in the Prairie provinces, especially in Alberta. The Prairie provinces accounted for about 60 to 66 percent (Table 5) of Canadian cattle slaughter, although the total number of cattle slaughtered has remained rather stable (Table 2). The expansion of the two major cattle slaughtering facilities in Alberta in 1996 (the Cargil plant at High River and the IBP plant at Lakeside, Alberta), has significantly increased the slaughtering capacity.

Changes in cattle inventories in Canada, as in the U.S., follow a cyclical pattern traditionally referred to as the "Cattle Cycle". The cattle cycle is characterized by the accumulation and liquidation of cattle inventories, generally occurring in response to changes, or anticipated changes, in profits, i.e prices received for cattle and prices paid for feed. A typical cattle cycle in North America occurs every 9 to 11 years. Larger cattle and beef supplies from the recent expansion/accumulation phase (1987 - 1995) caused cattle prices to decline in 1996. Cattle and calves were recorded at 14.9 million head in July last year (Table 1), down 1 percent from 1996. This was the turning point of the current cattle cycle. Beef cows were recorded at 4.72 million head in July last year (Table 6), down from the record herd of 4.76 million in 1996. Unlike the liquidation phase the national herd is experiencing, the Manitoba cattle herd increased 4 percent (55,000 head) from 1996 (Table 1). This suggests that Manitoba has the potential to increase significantly its cattle herd, as a result of added advantage created by recent changes in the WGTA.

The price of feeder steers in Alberta declined significantly beginning in April - June of 1995 when prices were $90.34 per 100 pounds down from $104.20 in the corresponding quarter of 1994 (Table 7). Prices remained relatively low for the rest of 1995 and throughout 1996, corresponding to increasing beef production in Canada, as the Canadian cattle cycle appeared to be in its contraction phase. The price of slaughter steers in Alberta generally declined from the second quarter of 1993, when prices were about $92.98 per 100 pounds through the second quarter of 1996, when steer prices reached $72.62 per 100 pounds (Table 8). However, prices recovered in the last half of 1996, averaging $82.70 in July - September compared with $78.71 in the corresponding quarter of 1995, and $83.96 in October - December compared with $81.42 in the corresponding quarter of 1995.

Beef Cattle Production: Cow-calf Enterprise

Beef cattle production may be considered to consist of three distinct phases of operation, namely, cow-calf, stocker-yearling/ backgrounding and the feedlot (finishing) operation. Fundamental to all phases of beef production is the cow-calf herd, the end product of which are the weaned calves, basic to the other phases. The cow-calf herd is continuously replaced by selectively introducing new and young heifers each season as old and less productive cows are culled. Replacement heifers represent a significant investment in the future. The established practice of most cow-calf operations in Canada is to breed the cows in June and July. Calves are born in March and April of the following year. The calves graze with their mothers on pastures and grassland throughout the spring, summer, and fall seasons. The average weight of calves at weaning in the fall (October or November) is about 250 kilograms (550 pounds), but weights can range from 160 to 320 kilograms (352 to 704 pounds), depending on age at weaning, the genetic background of the calf and grass condition during the summer grazing season.

Lighter calves (160 - 225 kilograms/352 - 495 pounds) are left on pasture for an extra 120 to 150 days, before they enter backgrounding and high energy feeding programs for slaughter between 18 and 24 months of age. Medium weight calves (225 - 275 kilograms/495

- 605 pounds) at weaning are normally placed on a lower energy backgrounding feeding program before being placed on a high energy grain feeding program for slaughter between 14 and 18 months of age. Heavier calves (275 - 320 kilograms/605 - 704 pounds) are normally placed on a high energy grain feeding program after weaning for up to 225 days, and are ready for slaughter between 12 and 14 months of age.

Stocker-yearling/backgrounding Enterprise

Backgrounding is the process of feeding high forage (alfalfa hay and straw) feeds to increase the weight of smaller calves up to 350 kilograms (770 pounds). After weaning, the light calves that are to be backgrounded are fed forages and grain through the winter in order to gain weight at a rate of 250 grams to 500 grams per day. In the spring, the smaller of these calves remain on pasture or are put into feedlots to gain weight at an average of 750 grams per day. The larger calves are fed high energy and high grain feed rations. Backgrounding is an alternative for farmers who have good quality roughage available, extra time during the year to work cattle, and the desire to have a flexible cattle business. Backgrounding can be undertaken by a cow-calf operation as an extension of the existing enterprise.

Feedlot/finishing Enterprise

Along with the trend towards larger and more specialized cow-calf operations, feedlots in Canada have been transformed into larger and more highly mechanized operations over the past fifteen years (Beef Export Federation). Historically, most cattle were fed in small feedlots on diversified farms that also grew feed grains and wheat for human consumption. Now feedlots range in size from a few hundred head capacity to very modern operations feeding over 40,000 animals at one time. It is estimated that over 70 percent of the cattle grain fed in Canada are produced in feedlots with capacities over 1,000 head (Beef Export Federation). In 1996, Canadian feedlots finished 2.3 million steers and heifers for slaughter in Canada.

In the feedlot/finishing operation, the feedlot purchases calves or feeder cattle from either cow-calf or backgrounding operations. Normally there are two basic types of feeding systems in the feedlot operation. The system employed depends on the weights of the animals when they are placed on the finishing program. A multi-stage feeding system is used for those steers and heifers that enter the feedlot at lighter weights. These cattle are started on a higher forage-lower grain feed ration to initially gain weight at about one kilogram per day. They are fed at this level for a few weeks, then the proportion of grain in the feed ration is gradually increased to 85 to 90 percent.

Heavier feeder cattle are directly fed high percentage grain feed rations. With these high energy rations, cattle will gain weight at about 1.7 kilograms per day. Virtually all cattle in feedlots are fed high energy grain feed rations for a maximum of 120 days, which ensures that sufficient marbling is produced and the fat is firm and white. The average live weight at slaughter for steers is about 590 kilograms (1298 pounds), and the average weight for heifers is about 550 kilograms(1210 pounds).

Thus the live cattle production process involve three stages: cow-calf, stocker-yearling/backgrounding and feedlot. The second stage seems to be less significant compared to the first and the third stage, and a close look at the second stage suggests that, it is more or less a continuation of the first stage. Although the second stage (backgrounding) is important and ideally should be modeled as a separate stage, unfortunately we are unable to do so due to the absence of time series data on the number of animals in backgrounding. As a result, for the purpose of this research we intend to model beef production as involving two main production stages, the cow-calf and the feedlot operations. Previous studies related to beef cattle production (e.g. Buhr and Kim) adopted the same approach.

Problem Statement

It is well known that dynamics plays a particularly important role in farm-level beef production decisions, due in large part to long biological lags in production. Since beef investment decisions must consider a long time horizon and uncertainty increases the further we try to predict into the future, uncertainty and in turn risk aversion are particularly important in modeling beef production decisions. Nevertheless there appear to be no published studies of dynamic beef supply response incorporating risk aversion, i.e. all studies have essentially assumed risk neutrality. Given the substantial variation in cattle prices that is often observed (e.g. consider the cattle cycle), the assumption of risk neutrality is a serious limitation in the empirical literature.

Objectives

The purpose of this study is to develop and estimate dynamic models of cow-calf and feedlot production decisions in Alberta under risk aversion and output price uncertainty. Since there has been no previous research on this topic, the first objective must be to specify and estimate a reduced form dynamic model with price uncertainty. An autoregressive distributed lag (ADL) model provides a general and parsimonius approximation to a reduced form model, so an ADL approach will be employed here. Then the second objective is to specify and estimate a particular structural dynamic model under risk aversion and uncertainty. The structural model combines recent extensions of static duality models under risk aversion with a discrete time calculus of variations Euler equation to model investment decisions under risk aversion and uncertainty.

Organization of Thesis

This thesis consists of two studies. Chapter two specifies and estimates reduced form autoregressive distributed lag and polynomial distributed lag models incorporating price uncertainty. This chapter provides a step by step identification of PDL and ADL models and the econometrics involved. Chapter three specifies and estimates dynamic Euler equation models of beef supply and investment under risk aversion and uncertainty. Chapter four concludes the thesis and provides suggestions for future research.

CHAPTER TWO

BEEF SUPPLY RESPONSE UNDER UNCERTAINTY: AN ADL MODEL

Introduction

It has long been recognized that dynamics plays a particularly important role in beef production decisions. Cattle are simultaneously capital and consumption goods, so output supply response is closely connected to investment decisions (Yver; Jarvis; Rosen; Nerlove and Fornari). Given this close connection and a typical effective reproductive life of 8-10 years for beef cows, a dynamic model of output and investment decisions has a long horizon. Since uncertainty generally increases over a planning horizon and farmers are generally considered to be risk averse, price uncertainty and risk aversion play an important role in beef production decisions.

Empirical studies of beef production have focussed on the modeling of dynamics and expected prices. These studies include models of adaptive expectations/partial supply response (Askari and Cummings), polynomial distributed lags (Kulshreshtha), more general distributed lag and time series models (Rucker, Burt and LaFrance; Shonkwiler and Hinckley), and models explicitly derived from a dynamic optimization (Nerlove, Grether and Carvalho). Newer approaches are illustrated in recent econometric studies of beef supply response and the cattle cycle (Buhr and Kim; Diebold, Ohanian and Berkowitz; Marsh 1999; Mundlak and Huang; Nerlove and Fornari; Rosen, Murphy and Scheinkman; Schmitz). These recent studies attest to the continued importance of improving models of beef supply response. However it appears that these studies have generally assumed risk neutrality by excluding the influence of uncertainty on decisions. One exception is Antonovitz and Green, who estimate static models of fed beef supply response incorporating price variance.

Apparently this is the first econometric study of dynamic beef supply response that attempts to incorporate risk aversion or more specifically uncertainty as measured by output price variance. Here we specify an autoregressive distributed lag (ADL) model, which provides a general distributed lag structure without explicitly specifying a dynamic optimization. An ADL model is adopted because little is known about the specific forms of dynamic adjustment and this approach can provide a relatively parsimonius approximation to a general dynamic process (Davidson and MacKinnon; Hendry, Pagan and Sargan). Moreover dynamic optimization models with risk aversion are not yet developed. For example risk aversion has been incorporated into static duality models of supply response (Coyle) but not yet into dynamic duality models (Coyle and Arnade provide a preliminary approach). The methodology is applied to the estimation of beef supply responses for cow-calf and feedlot operations using aggregate time series data for Alberta.

Methodology

An ADL(m,n) dynamic model relating a dependent variable y to independent variable(s) x is :

$$(1)\quad y_t = \alpha_0 + \sum_{i=1}^{m} \alpha_i y_{t-i} + \sum_{i=0}^{n} \beta_i x_{t-i} + e_t$$

where $e_t \sim \mathrm{IID}(0,\sigma^2)$. This model can be rewritten in different ways by linear transformation without changing the ability to explain data or least squares estimates of coefficients. For example an ADL(1,1) is equivalent to a standard error correction model (ECM), and model (1) can be rewritten as a generalized ECM (Bannerjee, Dolado, Galbraith and Hendry). Thus the choice between an ADL or ECM model is largely a matter of convenience in interpreting results. Here we adopt an ADL rather than an ECM approach because we are more interested in relating the model to more restrictive dynamic models, in particular polynomial distributed lag models, than in interpreting deviations from a hypothetical long-run equilibrium.

An important property of an ADL model with risk aversion is that it can be rationalized in terms of a dynamic optimization, i.e. it can be interpreted essentially as a reduced form for a structural dynamic optimization model. This is similar to the case of ADL models under risk neutrality, and the argument can be sketched as follows. It is well known that, under risk neutrality, dynamic optimization with quadratic costs of adjustment and a linear equation of motion rationalizes the ECM or equivalently ADL(1,1) model (Hendry and von

Ungern-Sternberg; Salmon; Nickell). Similarly consider the following simple dynamic optimization problem with risk aversion (for simplicity we assume atemporal uncertainty (Machina)):

$$(2) \quad \max_{\{I\}} \int_{t=0}^{\infty} U^{*}(Ep_t, w_t, w_t^k, Vp_t, K_t, I_t)\, e^{-rt}\, dt$$

$$\text{s.t.} \quad \dot{K} = A\,K + B\,I \qquad K_0 = \overline{K}$$

where $U^{*}(.)$ is the dual indirect utility function for a single period maximization problem, e.g. a mean-variance problem

$$(3) \quad \max_{x}\ Ep\ f(x,K,I) - w\,x - w^k\,I - \alpha(.)/2\ Vp\ f(x,K,I)^2 = U^{*}(Ep,w,w^k,Vp,K,I)$$

(Coyle 1999) or an expected utility maximization problem. (Ep_t, Vp_t) are the mean and variance for price p of output y at time t, w_t is the price for variable inputs x at t, w_t^k is the purchase (asset) price for capital K at t, I is gross investment, $y = f(x,K,I)$ is a production function incorporating convex costs of adjustment, $\alpha(.)$ is a nonlinear coefficient of risk aversion function $\alpha(Ep\ y - w\ x - w^k\ I,\ Vp\ y^2)$, and r is an intertemporal discount rate (as in most dynamic models, the agent's utility function is assumed to be separable over time). The dynamic maximization hypothesis places second order restrictions on the single period dual $U^{*}(.)$ with respect to K and I (Kamien and Schwartz), so assuming that $U^{*}(.)$ is quadratic in (K,I) is consistent with this hypothesis. Then a quadratic $U^{*}(.)$ and linear equation of motion implies a linear decision rule for investment I (Anderson and Moore).

[1]Furthermore the closed form solution of the Euler equation for the dynamic optimization (2) (and a standard terminal condition) imply an ECM or ADL(1,1) model (1) where x includes price variance Vp.

Data

Supply response models were constructed for cow-calf and feedlot operations using biannual and quarterly data, respectively, for Alberta over 1976-1997 (data on replacement heifers on-farm is unavailable prior to 1976). Cow-calf output (at weaning) is defined as the number of light feeder calves (400-500 lbs) on-farm Jan. 1 and July 1 in Alberta (Statistics Canada b). This series closely approximates calf production over the year, as calves grow from birth to a weight of 4-500 pounds in six months on average. Biannual inventory figures would therefore capture cow-calf output. A similar series has been used as a measure of cow-calf output in the U.S. (Buhr and Kim). The output price is in $/cwt for Alberta light feeders (400-500 lbs) (Agriculture and Agri-Food Canada). Input prices are a feed price index and hired labor wage index for Western Canada (Statistics Canada a), and price ($/cwt) for Alberta replacement heifers (700 lbs) (Agriculture and Agri-Food Canada). Investment in the cow-calf operation is measured as the number of replacement heifers (of all weights) on-farm Jan. 1 and July 1 in Alberta (Statistics Canada b). Investment decisions presumably depend on size of breeding herd, output price, price of replacement heifers, and farm input prices. These variables are measured as the number of cows on-farm Jan. 1 and

July 1 (Statistics Canada b), price ($/cwt) for Alberta feeder steers (700 lbs) (Agriculture and Agri- Food Canada), price for replacement heifers, feed price index and hired labor wage, respectively. Feedlot output is defined as the number of fed cattle slaughtered in Alberta plus net exports for slaughter from Alberta to the U.S., and the output price is measured as the price ($/cwt) for Alberta feeder steers (> 900 lbs) (Agriculture and Agri-Food Canada). Input prices are the feed price index, hired labour wage, and the price for Alberta feeder steers (700 lbs).

Empirical Models

ADL models are expressed in terms of normalized prices as follows:

$$(4)\quad y_t = \alpha_0 + \sum_{i=1}^{m} \alpha_i\, y_{t-i} + \sum_{i=0}^{n} (\beta_{1i}\, Ep_{t-i} / w_{t-i}^0 + \beta_{2i}\, Vp_{t-i} / w_{t-i}^{0\,2} + \beta_{3i}\, w_{t-i} / w_{t-i}^0) + e_t$$

where w^0 is designated as the numeraire input price (in our case this will be a feed price index). This is related to the normalization implied by constant relative risk aversion (CRRA): assuming CRRA and utility maximization under risk, decisions y given parameter values (Ep, w, w^0, Vp, W_0) are unchanged under new values ($\lambda\, Ep$, $\lambda\, w$, $\lambda\, w^0$, $\lambda^2\, Vp$, $\lambda\, W_0$) for all $\lambda > 0$, including $\lambda \equiv 1 / w^0$, where W_0 is initial wealth (Pope 1988; Coyle 1999). CRRA is a common assumption in the empirical literature on asset pricing and is considered the benchmark case by Arrow. Since an adequate proxy for initial wealth specific to beef producers is unavailable, lags on normalized initial wealth are not included in the ADL. Expected output prices are proxied as a one period lag on market prices, and variances of output prices are proxied as the weighted sum of squares of prediction errors $(p_t - p_{t-1})^2$ of the previous three years, with declining weights of 0.50, 0.33, and 0.17. This particular formula for price variance has been used in other studies (Chavas and Holt; Coyle). Expected prices and price variances were also calculated from ARIMA and GARCH models expressing market prices as a distributed lag of prices, but these measures were insignificant in ADL models of output supply and were rejected for the simpler measures. These results are similar to other studies of Western Canadian agriculture under risk aversion (Coyle) that rejected proxies from ARIMA and GARCH models. Similarly a study of crop price expectations for a group of Sasketchewan farmers concluded that these reported expectations are less adequately explained as time series forecasts (Sulewski, Spriggs and Schoney). [2]

Output quantity data, price ratios (Ep/w^0, $Vp/(w^0)^2$,w/w^0) replacement heifers and herd size were tested for unit roots by standard methods (Dickey- Fuller and Phillips-Perron, with and without allowing for trend stationarity in the alternative). In all cases the unit root hypothesis was rejected at the .05 level. Since these tests are biased in favor of the unit root hypothesis in the sense that they have low power (Kwiatkowski et. al.), we assume that it is not necessary to transform data due to unit roots. This conclusion was also supported by alternative tests (Kwiatkowski et. al.).

Two of the input price variables specified for the ADL models were found to be insignificant and were dropped from the models. Hired labor wage and replacement heifer price were jointly insignificant for cow-calf output and investment equations, whereas hired labor wage was insignificant for feedlot output response. These results are not surprising since labor cost is a relatively small proportion of total costs for both cow-calf and feedlot sectors (labor costs are also relatively fixed in the short-run), and investment in breeding stock is primarily internal to the firm (relatively few replacement heifers are purchased by cow-calf producers). Based on our results, the ADL models for cow-calf and feedlot operations are specified as

$$(5)a)\quad y_{It} = \alpha_0 + \sum_{i=1}^{m} \alpha_i\, y_{It-i} + \sum_{i=0}^{n} (\beta_{1i}\, Ep_{It-i}/w_{t-i} + \beta_{2i}\, Vp_{It-i}/w_{t-i}^{2}) + e_t$$

$$b)\quad I_t = \alpha_0 + \sum_{i=1}^{m} \alpha_i\, I_{t-i} + \sum_{i=0}^{n} (\beta_{1i}\, Ep_{t-i}^{I}/w_{t-i} + \beta_{2i}\, Vp_{t-i}^{I}/w_{t-i}^{2}) + \gamma\, C_{t-1} + e_t$$

$$c)\quad y_{IIt} = \alpha_0 + \sum_{i=1}^{m} \alpha_i\, y_{IIt-i} + \sum_{i=0}^{n} (\beta_{1i}\, Ep_{IIt-i}/w_{t-i} + \beta_{2i}\, Vp_{IIt-i}/w_{t-i}^{2} + \beta_{3i}\, w_{t-i}^{c}/w_{t-i}) + e_t$$

where (y_I, Ep_I, Vp_I) are output supply, expected output price and variance of output price for cow-calf operations, $(y_{II}, Ep_{II}, Vp_{II})$ are output supply, expected output price and variance of output price for feedlot operations, w is a feed price index, and w^c is a price for beef input into feedlots. I is cow-calf investment (replacement heifers), C is stock of cows, and (Ep^I,Vp^I) are mean and variance of price for beef purchased by feedlots. In all empirical models, variables are specified in logarithms, so coefficients can be interpreted as elasticities.

Results for Cow-calf Output Supply Response

Dynamics and uncertainty presumably are particularly important in modeling cow-calf supply response due to long biological lags in production. Replacement heifers are typically bred at 15 to 27 months of age and give birth in another 9 months, so the lag in births for the cow-calf operation is 24 to 36 months (with larger numbers bred on either end of this interval in order to maintain short calving seasons). Similarly there is a biological lag of 24 to 36 months between the breeding of a replacement heifer and the production of an offspring ready for breeding.

We begin by estimating a polynomial distributed lag (PDL) model of supply response. In principle, distributed lags can reflect either the formation of expectations or lags in supply response (although, as noted above, we rejected ARIMA and GARCH models of rational expectations distributed lags). Assuming that price expectations are to some extent measured by our proxies for (Ep,Vp), we assume that distributed lags reflect lags in supply response. Then changes in prices do not influence output until after a biological lag of 24 to 36 months, i.e. an average of 5 periods using biannual data. [3]

A PDL(10,4), i.e. a 10 period lag length and 4th degree polynomial, was selected (see below for a discussion of PDL selection procedures). Results were generally as anticipated: the sum of lag coefficients for both expected price Ep and price variance Vp were significant and with anticipated signs, and the elasticity was larger for Ep (1.01) than for Vp (-0.06) (these estimates were obtained by an iterative Cochrane-Orcutt procedure). Static studies of agricultural production have also estimated considerably smaller elasticities of response for Vp than for Ep (e.g. Coyle). On the other hand, there was substantial serial correlation in residuals, and a standard test for the common factor restrictions implied by an AR(1) model rejected these restrictions (Davidson and MacKinnon, p.365). Thus the PDL model appears to be seriously mis-specified, and so results are not reported here.

The serial correlation due to mis-specification in the PDL model suggests that an ADL model is more appropriate. A serious criticism of the PDL approach is that the dependent variable depends on lagged values of the included independent variables but not on lagged values of the omitted variables reflected in the error term. Rather than respecifying the PDL model with a disturbance following an ARMA process, it is often more appropriate to specify an ADL model (Davidson and MacKinnon, p. 679).

An ADL(m,n) model is specified as

$$(6)\quad y_{lt} = \alpha_0 + \sum_{i=1}^{m} \alpha_i\, y_{lt-i} + \sum_{i=5}^{n+5} (\beta_{1i}\, Ep_{lt-i}/w_{t-i} + \beta_{2i}\, Vp_{lt-i}/w_{t-i}^{2}) + e_t.$$

In order to select m and n, they were initially set at 5 and 10 (respectively) and simple nested tests (F-tests and Schwarz Criterion) were used to reduce the lag length. Models were estimated by OLS or (if autocorrelation) a grid search maximum likelihood procedure. In this manner an ADL(1,5) model was selected, so that (relative to the PDL) the lag length on Ep and Vp is reduced from 10 to 5. In the selected ADL model, y_t depends on the lagged values $Ep_{t-5},..,Ep_{t-10}$ and $Vp_{t-5},..,Vp_{t-10}$ of Ep and Vp (earlier and later lags are insignificant). A time trend and seasonal dummy were insignificant.

Table 1A presents OLS estimates for the ADL(1,5) model. Variables in all models are specified in logarithmic form, so coefficients can be interpreted as elasticities. The sum of lag coefficients for Ep and Vp are significant and with anticipated signs. [4] [5]The coefficient of the lagged dependent variable can be interpreted somewhat similarly to Nerlove partial response models, i.e. approximately 35% of the gap between current and steady state output is closed in a single six month period. The long-run impacts of Ep and Vp on output are similar to the sum of lag coefficients for the PDL model, and are calculated as (respectively) 0.9275 (= 0.3313/(1-0.6428)) and -0.0479. Since the lagged dependent variable is significant, we conclude that this model does not reduce to a PDL model. Another study (Buhr and Kim) estimates elasticities of expected output price on U.S. calf crop output as 0.45 in the long-run and 0.05 in the short-run.

In contrast to the PDL model, there is no sign of autocorrelation. The Durbin h statistic (asymptotically distributed as a standard normal under no autocorrelation) is insignificant, and a grid search maximum likelihood procedure assuming an AR(1) yielded an insignificant value for the first order autocorrelation coefficient rho; so the hypothesis of no autocorrelation is not rejected for AR(1). [6] Since the 5 period lag length is longer than in many other reported ADL models, it is interesting to consider the effects of incorporating PDL restrictions into the ADL model. Given an ADL(1,5), a third degree polynomial was selected for the distributed lags in Ep and Vp. Results are reported in Table 1B. Since OLS led to serial correlation in the residuals, this model was estimated by a grid search maximum likelihood procedure for an AR(1) model (Beach and MacKinnon) as programmed in Shazam. [7]Results for the sum of lag coefficients are similar to Table 1A, but there is considerable variation for individual coefficients. A test of common factor restrictions implied by AR(1) rejected the AR(1) model.

These results suggest that adding PDL restrictions to the ADL model does not substantially reduce standard errors of estimates but does lead to significant model mis-specification. Consequently, in our case, the ADL model is preferred to the ADL model with PDL restrictions.

Results for Replacement Heifer Investment Response

Cow-calf output (measured as calves on-farm) is essentially linearly related to the number of beef cows, so cow-calf output response is essentially an accumulated impact of beef cow investment decisions. Nevertheless it is of interest to model directly replacement heifer investment response, since this is not easily unscrambled from output supply response. The investment decision is modelled as depending on expectations for output and input prices for the cow-calf enterprise. In addition the current investment decision obviously depends upon the accumulated stock of beef cows. In principle the rate of investment also depends on the firm's marginal rate of time preference or discount rate, which may be proxied loosely by a market interest rate. However, the effects of variable interest rates have not been incorporated into any econometric studies of beef production decisions or into any dynamic duality models, so we do not consider this here.

Given the current number of beef cows or equivalently heifers of the appropriate age on-farm, the immediate effect of an investment decision is on the allocation between replacement heifers and fed heifers. Then the changes in replacement heifers eventually leads to a change in herd size, which has a longer-run feedback effect on investment decisions. This suggests that, if we specify a (dynamic) investment equation as conditional on herd size, i.e. if we control for herd size (and hence control for the longer-run indirect feedback effects of herd size on investment decisions), lags in response may be shorter than otherwise. Alternatively an investment equation can be specified independently of number of beef cows. This can be interpreted as a reduced form investment equation incorporating effects of longer-run induced changes in herd size on investment. In this case longer lags are likely: coefficients for zero or immediate lags may reflect allocation decisions between replacement and fed heifers, and longer lags reflect interactions between investment and herd size.

A PDL model for investment conditional on beef cows can be specified as

$$(7)\quad I_t = \alpha_0 + \sum_{i=0}^{n} (\beta_{1i}\, Ep_{t-i} / w_{t-i} + \beta_{2i}\, Vp_{t-i} / w_{t-i}^2) + \gamma_1\, C_{t-1} + \gamma_2\, D_t + e_t$$

where C is number of beef cows, D is a seasonal dummy variable (D = 1 for Jan. - June and 0 otherwise), and w is a feed price index. The output price p is the feeder input price to feedlots, which is proxied by the price ($/cwt) for Alberta feeder steers (700 lbs), and (Ep,Vp) are calculated as above. A hired labor wage and replacement heifer price were also considered, but these were insignificant. This is not surprising, since labor costs are a small proportion of cow-calf total costs and relatively few replacement heifers are purchased. A time trend was also insignificant.

A recommended approach to selecting the lag length and degree of polynomial is to (a) estimate unrestricted distributed lag models with long lag lengths and use simple nested tests for reducing the lag length, and (b) (given the selected lag length) use nested tests to select the degree of the polynomial (e.g. Davidson and MacKinnon, pp. 673-6; Sargan). For simplicity, we assumed that the PDL's for Ep and Vp are polynomials of the same degree as well as being identical in lag length. It is well known that test statistics must be interpreted with caution after such model selection or pretesting procedures, and so we do not compound the problem by testing for differences in lag structures between Ep and Vp. As long as lag length is not overstated by more than the degree of the polynomial, i.e. so long as the difference between specified and true lag length is less than the degree of the polynomial, biases are not necessarily introduced into a PDL estimator (Trivedi and Pagan; Hendry, Pagan and Sargan).

A PDL(8,3) model (7) was selected following this procedure. In contrast to the PDL cow-calf output supply response model, the lag process becomes insignificant after 4 years rather than 7 years into the past. This difference in lag length is not surprising since (7) controls for the longer-run feedback effects of herd size on investment. OLS results are reported in Table 2A. The sum of lag coefficients for both expected price Ep ($\sum_i \beta_{1i}$) and price variance Vp ($\sum_i \beta_{2i}$) are significant and with anticipated signs. The restrictions on the distributed lags implied by the PDL model are not rejected (an F statistic of 0.7816, 10 and 15 df, probability = 0.646) and there is no serial correlation in the residuals. Nevertheless a one period lag in investment is significant when added to this model, i.e. this model is rejected for an ADL model (with PDL restrictions).

An ADL(m,n) model for investment conditional on beef cows is

$$(8) \quad I_t = \alpha_0 + \sum_{i=1}^{m} \alpha_i I_{t-i} + \sum_{i=0}^{n} (\beta_{1i} Ep_{t-i}/w_{t-i} + \beta_{2i} Vp_{t-i}/w_{t-i}^2) + \gamma_1 C_{t-1} + \gamma_2 D_t + e_t.$$

OLS results for the selected ADL(1,4) model are presented in Table 2B. The sums of lag coefficients for Ep and Vp are significant with anticipated signs. Long-run elasticities for Ep and Vp (conditional on herd size) are 1.2589 and -0.0806, respectively, which are similar to long-run elasticities for the PDL model (0.9317, -0.0604). The Durbin-h statistic suggests that there is no serial correlation. For comparison OLS results for an

ADL(1,4) model with lags restricted to conform to a second order poynomial are presented in Table 2C. The polynomial restrictions are not rejected (an F statistic of 0.6407, 4 and 26 df, probability = 0.638) and the Durbin-h statistic suggests no serial correlation. Results are similar to Table 2B.

In contrast, reduced form investment models cannot be estimated directly with our data set. The PDL and ADL models excluding herd size are

$$(9)\quad I_t = \alpha_0 + \sum_{i=0}^{n} (\beta_{1i}\, Ep_{t-i} / w_{t-i} + \beta_{2i}\, Vp_{t-i} / w_{t-i}^2) + \gamma_2\, D_t + e_t$$

$$I_t = \alpha_0 + \sum_{i=1}^{m} \alpha_i\, I_{t-i} + \sum_{i=0}^{n} (\beta_{1i}\, Ep_{t-i}/w_{t-i} + \beta_{2i}\, Vp_{t-i}/w_{t-i}^2) + \gamma_2\, D_t + e_t.$$

Cow-calf output supply response results include lags to 14 and 10 periods for (Ep,Vp) in PDL and ADL models, respectively (see Table 1 for ADL models). Consequently the lag lengths n for (9) should exceed 14 and 10, respectively. However the PDL model cannot be estimated with our data set, and there are insufficient degrees of freedom to obtain reasonable estimates of the ADL model.

Long-run Equilibrium Impacts on Investment for Cow-Calf Model

Estimates of long-run equilibrium impacts of Ep and Vp on reduced form investment (9) can be obtained from estimates of the calf output model and investment model conditional on beef cows. ADL model (8) of investment conditional on beef cows and results in Table 2B imply the following relation between long-run equilibrium levels I^*, Ep^*, Vp^*, C^*:

$$(10)\quad I^*(1-\alpha_i) = \Sigma_i \beta_{1i} Ep^* + \Sigma_i \beta_{2i} Vp^* + \gamma_1 C^* + \ldots\ldots \Rightarrow$$

$$I^* = 1.259\ Ep^* - 0.081\ Vp^* + 2.455\ C^* + \ldots\ldots\ldots$$

in logarithms. Similarly ADL model (6) of cow-calf output (calves on-farm) and results in Table 1A imply the following long-run equilibrium relation:

$$(11)\quad y_l(1-\alpha_1) = \Sigma_i \beta_{1i} Ep_l + \Sigma_i \beta_{2i} Vp_l + \ldots\ldots\ldots \Rightarrow$$

$$y_l = 0.928\ Ep_l - 0.048\ Vp_l + \ldots\ldots\ldots$$

where y_l is long-run equilibrium level of calves on-farm. Then impacts can be calculated assuming a relation between prices p and p_l. For example assuming that these prices move together (denoted as p), the long-run equilibrium impacts of Ep,Vp on reduced form investment can be calculated (in elasticities) as (assuming an elasticity of 1.0 for beef cows with respect to calves, i.e. weaning rate does not change with herd size)

$$(12)\quad \partial I^*/\partial Ep^* = 1.259 + 2.455\ (1.0)\ 0.928$$

$$= 3.537$$

$$\partial I^*/\partial Vp^* = -0.081 + 2.455\ (1.0)\ (-0.048)$$

$$= -0.199\ .$$

These are more than double the estimated long-run elasticities conditional on herd size (1.259, -0.081, respectively), i.e. feedback effects of changes in herd size on investment have more than doubled the calculated long-run elasticities. Another study (Buhr and Kim) estimates elasticities of expected output price on U.S. beef cow inventory as 1.11 in the long-run.

Results for Feedlot Output Supply Response

Output supply response models were also estimated in a similar manner for Alberta feedlots. After weaning (typically at 7- 8 months), calves may be backgrounded or sold to feedlots with a grain feed ration resulting in feeding periods between 6 to 10 months (2 to 3 quarters) before slaughter. Alternatively, the producer can hold back calves and place them on pasture until sold as heavy yearlings to feedlots in the following year. This involves a feeding period of fourteen to twenty weeks, depending on the ration.

A PDL model for feedlot supply response was estimated first. Using quarterly data, the lag in explanatory variables (Ep/w, Vp/w^2, w^c/w) was initially assumed to begin in 2 (alternatively 0) periods, but (surprisingly) coefficients for lags of less than 5 periods were almost always jointly insignificant for various PDL and ADL models considered. Since lag lengths appeared to be quite long, the lag in explanatory variables was respecified as beginning in 5 periods. Then the following PDL(16,5) model was selected:

$$(13)\quad y_{IIt} = \alpha_0 + \sum_{i=5}^{20} \left(\beta_{1i}\, Ep_{IIt-i}/w_{t-i} + \beta_{2i}\, Vp_{IIt-i}/w_{t-i}^{2} + \beta_{3i}\, w^{c}_{t-i}/w_{t-i}\right) + \sum_{i=1}^{3} \gamma_i D_{it} + e_t$$

where D_1, D_2, D_3 are quarterly dummies. There was significant serial correlation (a 1.29 Durbin-Watson statistic for the OLS model and a significant estimate 0.47 of rho for the maximum likelihood AR(1) model) and a test of common factor restrictions rejected the AR(1) model. Therefore the PDL model appears to be mis-specified, and in turn detailed results are not presented here. Grid search maximum likelihood estimation of the AR(1) model (13) led to the following estimates of the sums of lag coefficients for Ep,Vp,w^c: 4.524, - 0.4506,-1.526 with t-ratios 2.19,2.83,1.26, respectively.

An ADL(m,n) model is specified as

$$(14)\quad y_{IIt} = \alpha_0 + \sum_{i=1}^{m} \alpha_i\, y_{IIt-i} + \sum_{i=5}^{n+5} \left(\beta_{1i}\, Ep_{IIt-i}/w_{t-i} + \beta_{2i}\, Vp_{IIt-i}/w_{t-i}^{2}\right.$$

$$\left. + \beta_{3i}\; w^{c}_{t-i}\,{}_{w_{t-i}} \right) + \sum_{i=1}^{3} \gamma_i\, D_{it} + e_t$$

and an ADL(1,13) was selected. This implies lags of up to 4 1/2 years, which is (surprisingly) similar to the cow-calf output supply response model. It is not clear if there is serial correlation in the model: the Durbin-h statistic (2.145) for OLS results imply that zero autocorrelation is rejected at the .05 level, and a grid search maximum likelihood procedure estimated rho as 0.19 and insignificant (a t-ratio of 1.62). Nevertheless results are similar for both OLS and a grid search maximum procedure for AR(1). OLS results are reported in Table 3A. The sum of lag coefficients for Ep and Vp are again significant and with anticipated signs, whereas the sum of lags for the feeder input / feed price ratio is again less significant. As a comparison, AR(1) estimates of the sums of lag coefficients for Ep,Vp,w^c are 2.2969,-0.2216,- 0.9640 with t-ratios 3.47,4.07,2.58, respectively. The estimated coefficient of the lagged dependent variable suggests that approximately 28% of the gap between current and steady state output is closed in a single 3 month period. In this sense, speed of adjustment may be somewhat faster for feedlots than for cow-calf operations, as anticipated. The estimated long-run impact elasticities for Ep and Vp on feedlot output are 7.45 and -0.71, respectively, in contrast to 4.52 and -0.45 in the PDL feedlot model. Many studies have reported elasticities of cattle slaughter with respect to output price. For example, the long-run elasticity is estimated as 3.24 (Marsh 1994, for U.S.), 0.90 (Buhr and Kim, for U.S.), 1.30 (Kulshreshtha, for Western Canada).

Given the long lag length, the ADL(1,13) model was also estimated under PDL restrictions. However, in contrast to other PDL models, a high order polynomial (of degree 8) was accepted. This PDL(13,8) places relatively few restrictions on the 13 period distributed lag, but these restrictions led to greater serial correlation than in the ADL model (as indicated by a Durbin-h statistic of -3.79 for the OLS model and the maximum likelihood estimate of rho for the AR(1) model). Grid search maximum likelihood estimates for an AR(1) model are reported in Table 3B. A test of common factor restrictions rejected the AR(1) model, so the PDL restrictions apparently mis-specify the ADL model.

Conclusion

We have estimated dynamic models of beef supply response for cow-calf and feedlot operations in Alberta allowing for price uncertainty and risk aversion. Apparently this is the first study of dynamic beef supply response to incorporate price uncertainty or more specifically output price variance. As in several other studies of Western Canadian agriculture, expected output price and price variance are more effectively modeled as simple lags and weighted sums of squared prediction errors rather than as rational expectations or GARCH models. ADL and PDL models are estimated assuming distributed lags for variance of output price as well as for expected output price.

ADL models are estimated for cow-calf output (calves) and investment (replacement heifers) and for feedlot slaughter output. In all three cases the sum of lagged coefficients for output price variance is negative and significant, as anticipated. The elasticity is much smaller than for the (positive) sum of lagged coefficients for expected price, as anticipated. The distributed lags for PDL models extend back 5-7 years. The selected ADL models show shorter but still substantial lags in explanatory variables, so PDL restrictions are also considered for distributed lags in the ADL models. However these PDL restrictions introduce substantial serial correlation in residuals, which suggests that these restrictions mis-specify the distributed lags in the ADL models.

The results of this study suggest that it is feasible to incorporate price uncertainty and risk aversion into ADL or ECM models of dynamic beef supply response. Of course these are reduced form rather than structural dynamic models. The next step in such research should be to formulate and estimate structural dynamic models with price uncertainty and risk aversion, e.g. dynamic duality models with risk aversion, in an effort to obtain more understanding of the dynamic processes and the role of uncertainty and risk preferences.

FOOTNOTES

1. This also essentially implies that the Simon-Theil conditions for first period certainty equivalence are met, so the argument can be extended to temporal uncertainty in a manner somewhat similar to standard models (Salmon).

2. A somewhat similar approach to incorporating risk into a distributed lag model was followed by Lin, who estimated a PDL model for wheat acreage response including a distributed lag on a risk variable (defined as a 3 year moving average standard deviation of past returns per acre).

3. The importance of biological lags was also checked by estimating various PDL and ADL models assuming that the distributed lags begin at 0 rather than 5 periods. However lags prior to 5 periods were almost always jointly insignificant.

4. Results for the ADL model (Table 1A) indicate that magnitudes and significance of coefficients for lagged Ep and Vp do not decline as the lag length increases (this pattern is not apparent in Table 1B, but apparently this model is mis-specified due to the PDL restrictions). In contrast, estimates of ARIMA (and GARCH) models for Ep and Vp do show such a decline as lag length increases. These results suggest that distributed lags in our models may primarily reflect lags in supply response rather than in expectations (results in Table 3A also suggest this conclusion).

5. Temporal risk implies that price uncertainty influences decisions under risk neutrality (e.g. Dixit and Pindyck), so significance of price variance does not necessarily imply

rejection of risk neutrality. On the other hand, insignificance of price variance would imply rejection of risk aversion.

6. A portmanteau Lagrange multiplier test of white noise against MA(1) (Harvey, p. 278) did not suggest an MA process.

7. In the presence of lagged dependent variables, the error sum of squares criterion ESS(β,ρ) (after the model is transformed for AR(1) errors) generally has multiple solutions, and estimates of cov(b) conditional on an estimate of ρ (as in most applications of the Cochrane-Orcutt and Hildreth-Liu) are inconsistent (Betancourt and Kelejian; Davidson and MacKinnon, pp. 334-40). This suggests a combined grid search nonlinear least squares or maximum likelihood approach with β and ρ estimated jointly rather than sequentially. Nevertheless in our case similar results were obtained by an iterative Cochrane-Orcutt.

CHAPTER THREE

A DYNAMIC EULER EQUATION MODEL OF BEEF SUPPLY RESPONSE UNDER RISK AVERSION

Introduction

It has long been recognized that dynamics plays a particularly important role in beef production decisions. Cattle are simultaneously capital and consumption goods, so output supply decisions are particularly closely connected to investment decisions (Yver; Jarvis; Rosen; Nerlove and Fornari). Given this close connection and an effective reproductive life of 8-10 years for beef cows, a dynamic model of output and investment decisions has a long horizon. Since uncertainty generally increases over a planning horizon and farmers are generally considered to be risk averse, price uncertainty and risk aversion presumably play a particularly important role in beef production decisions. Empirical studies of beef production have focussed on the modeling of dynamics. These studies range from models of adaptive expectations/partial supply response (Askari and Cummings) and polynomial distributed lags (Kulshreshtha) to more general distributed lag and time series models (Rucker, Burt and LaFrance; Shonkwiler and Hinckley) and to models explicitly derived from a dynamic optimization (Nerlove, Grether and Carvalho). Newer approaches are illustrated in recent econometric studies of beef supply response and the cattle cycle (Buhr and Kim; Diebold, Ohanian and Berkowitz; Marsh 1999; Mundlak and Huang; Nerlove and Fornari; Rosen, Murphy and Scheinkman;

Schmitz). These recent studies attest to the continued importance of improving models of beef supply response. However it appears that all of these studies have assumed risk neutrality by excluding the influence of uncertainty on decisions. Two exceptions are studies of feedlot supply response under risk aversion (Antonovitz and Green) and of autoregressive distributed lag (ADL) model for Alberta beef supply (Mbaga and Coyle). However the first study assumes a static model and the second study estimates a reduced form model, which cannot identify the structure of dynamic response.

This paper presents the first econometric study of a structural dynamic model of beef supply response under risk aversion and uncertainty. We adopt the standard practice of specifying a dynamic structural model in terms of a discrete time Euler equation. However in contrast to other studies, the Euler equation model incorporates risk aversion and output price uncertainty. The methodology is applied to the estimation of beef supply responses for cow-calf and feedlot operations using aggregate time series data for Alberta.

Theoretical Models

Suppose a cow-calf beef ranch produces calf output y (measured as total weight) using variable inputs x and a breeding herd K, and denote the production function at time t by $y_t = f(x_t, K_{t-a}, K_{t-a} - K_{t-a-1})$ assuming convex adjustment costs associated with changes in the size of breeding herd (for simplicity we assume that K is the only quasi-fixed input). Calves are weaned at 7 - 8 months of age, and heifers are typically bred at 15 or

27 months of age (depending on breed) and give birth in another nine months. Thus output y_t at time t depends on the size of breeding herd 9 months earlier, and in turn the corresponding lag length "a" in the production function equals 1 or 2 periods using biannual data. In addition the lag "s" before replacement heifers reach maturity (i.e. can breed) is 7 to 17 months (depending on breed), i.e. s is between 1 and 3 periods using biannual data. Calves are weaned at 8 months of age and sold to feedlots after additional grazing and backgrounding. Calf output will either be marketed or retained as replacement heifers to augment the breeding herd. Market prices for output, variable inputs and replacement heifers are p, w and w^k, respectively, and for simplicity we assume that only output price p is uncertain (the model can easily be extended to uncertainty in all prices).

The mean and variance of output price p are denoted as Ep and Vp, respectively. p_t y_t is the market value of the calf crop at time t (including calves to be retained as replacement heifers), and the replacement heifer market price w^k is the opportunity cost for calves retained as replacement heifers and the cost of purchasing replacement heifers at t. Assuming a mortality rate δ for the breeding herd and an s period lag before replacement heifers reach maturity (i.e. enter the breeding herd K), then $K_{t-a}-(1-\delta)K_{t-a-1}$ is the number of replacement heifers (from calf crop or purchase) in period t-a-s reaching maturity in the breeding herd at time t-a, and this cost is incurred in period t-a-s.

Consider the following dynamic optimization problem over periods $t = 0,..,T$ for a cow-calf producer under constant absolute risk aversion (CARA): [1]

$$(1)\quad J(K,Ep,Vp,w,w^k) \equiv \max_{\{K\}} \; \Sigma_{t=0} V(Ep_t,Vp_t,w_t,K_{t-a},K_{t-a}-K_{t-a-1}) /(1+r)^t$$

$$- \Sigma_{t=a} w^k (K_{t-a} - (1-\delta)K_{t-a-1})) /(1+r)^{t-a-a}$$

$$\text{s.t. } K_q = \overline{K}_q \qquad \text{for } q < a$$

$$K_t - (1-\delta)K_{t-1} \geq 0 \quad \text{for all } t$$

where

$$(2)\quad V(Ep_t,Vp_t,w_t,K_{t-a},K_{t-a}-K_{t-a-1}) = \max_{x} \; Ep_t\, f(x,K_{t-a},K_{t-a}-K_{t-a-1}) - w_t\, x$$

$$- (\alpha/2)\, Vp_t\, f(x,K_{t-a},K_{t-a}-K_{t-a-1})^2 .$$

$\alpha > 0$ is a constant coefficient of absolute risk aversion. V(.) is a static dual indirect utility function under CARA and is similar to Coyle (1992). V(.) is linear homogeneous and convex in (Ep,Vp,w) and satisfies Hotelling's lemma:

$$(3)\quad \partial V(.)/\partial Ep = y \qquad \partial V(.)/\partial w = -x \; .$$

Assuming an interior solution $\{K\}^* \equiv (K_0,...,K_T) >> 0$ to the discrete time calculus of variations problem (1), and assuming that the constraint $K_t - (1-\delta)K_{t-1} \geq 0$ is not binding, problem (1) has standard first and second order conditions $\partial J(.)/\partial K_t = 0$ (t=0,..,T) and $[J_{KK}(.)]$ negative semi-definite.

The first order conditions are discrete time Euler equations (evaluating $\partial J(.)/\partial K_t = 0$):

$$\text{(4a)} \quad \partial V_t(.)/\partial K + \partial V_t(.)/\partial \dot{K} - \partial V_{t-1}(.)/\partial \dot{K} /(1+r)$$

$$- w^k (1+r)^{a-s} + w^k (1-\delta) (1+r)^{a-s-1} = 0$$

where $\dot{K}_t \equiv K_t - K_{t-1}$. The second order Legendre Clebsch condition (e.g. Stengel, p. 213) is (evaluating $\partial^2 J(.)/\partial K_t \leq 0$)

$$\text{(4b)} \quad \partial^2 V_t(.)/\partial K^2 + 2\, \partial^2 V_t(.)/\partial K \partial \dot{K} + \partial^2 V_{t-1}(.)/\partial \dot{K}^2 /(1+r) \leq 0 .$$

The above model formally assumes atemporal risk rather than temporal risk (Machina), i.e. the above planning problem assumes that no additional information about prices or the probability distribution for prices will become available over the planning horizon. However the model can be generalized to temporal risk as follows. First, temporal risk does not influence the specification of the static one period maximization problem (2) (which is conditional on capital levels) to the extent that there is a relatively small change in price information over the short one period horizon (6 months for biannual data or 3 months for quarterly data) or variable input decisions x_t for the period must largely be made before there is a substantial change in information. On the other hand, temporal risk should certainly influence investment decisions for durable capital (K), given the long productive life of beef cows and the substantial changes in information that will occur over this period. Thus the dynamic problem (1) should be respecified as a stochastic expected utility maximization problem by changing the objective function to E_0 $\{\Sigma_{t=0}$ $V(Ep_t,Vp_t,w_t,K_{t-a},K_{t-a}-K_{t-a-1})$ $/(1+r)^t$ - $\Sigma_{t=a}$ w^k $(K_{t-a}$ - $(1-\delta)K_{t-a-1}))$ $/(1+r)^{t-a-s}\}$.

Since the dynamic maximization hypothesis places only second order restrictions on the single period dual V(.) with respect to K and $\dot{K}$ (Kamien and Schwartz), we can assume that the dual V(.) is quadratic in K and $\dot{K}$. This essentially implies certainty equivalence (Simon; Theil), i.e. solution values can be substituted into first order conditions as in the Euler equation (4). Thus equations (3)-(4) generalize approximately to temporal risk.

The system of equations (3)-(4) can be specified given a functional form for the dual. For example, assuming a normalized quadratic form similar to Coyle (1992), the following derivatives of the dual are specified:

$\partial V^*_t(.)/\partial v^i_t = \beta_{i0} + \beta_{i1}\, Ep^*_t + \beta_{i2}\, w^*_t + \beta_{i3}\, K_{t-a} + \beta_{i4}\, (K_{t-a}-K_{t-a-1}) + \beta_{i5}\ Vp^*_t\, (v^i_t \equiv Ep^*_t, w^*_t, K_{t-a}, K_{t-a}-K_{t-a-1})$. Here $V^*_t \equiv (V/w^0)_t$, $Ep^*_t \equiv (Ep/w^0)_t$, $Vp^*_t \equiv (Vp/w^0)_t$, $w^*_t \equiv (w/w^0)_t$, i.e. V, Ep, Vp, w are normalized by a numeraire variable input price w^0 assuming linear homogeneity of the dual V(.) in (Ep,Vp,w,w^0). Note that this normalization implies

$\partial V(.)/\partial K = w^0\ \partial V^*(.)/\partial K$ and $\partial V(.)/\partial \dot{K} = w^0\, \partial V^*(.)/\partial \dot{K}$.

Then (3)-(4) can be specified as:

(5)a) $y_t = \beta_{10} + \beta_{11}\, Ep^*_t + \beta_{12}\, w^*_t + \beta_{13}\, K_{t-a} + \beta_{14}\, (K_{t-a}-K_{t-a-1}) + \beta_{15}\, Vp^*_t$

b) $-x_t = \beta_{20} + \beta_{21}\, Ep^*_t + \beta_{22}\, w^*_t + \beta_{23}\, K_{t-a} + \beta_{24}\, (K_{t-a}-K_{t-a-1}) + \beta_{25}\, Vp^*_t$

(6) $(\beta_{30} + \beta_{40}) + (\beta_{31} + \beta_{41})\, Ep^*_t + (\beta_{32} + \beta_{42})\, w^*_t + (\beta_{33} + \beta_{43})\, K_{t-a}$

$$+ (\beta_{34} + \beta_{44}) (K_{t-a}-K_{t-a-1}) + (\beta_{35} + \beta_{45}) Vp^{*}_{t}$$

$$- \{\beta_{40} + \beta_{41} Ep^{*}_{t+1} + \beta_{42} w^{*}_{t+1} + \beta_{43} K_{t-1-a} + \beta_{44} (K_{t+1-a}-K_{t-a})$$

$$+ \beta_{45} Vp^{*}_{t+1}\}(w^{0}_{t+1}/w^{0}_{t})/(1+r)$$

$$- \{w^{k}_{t-a-s} (1+r)^{a-s} + w^{k}_{t+1-a-s} (1-\delta) (1+r)^{a-s-1}\}/w^{0}_{t} = 0$$

with symmetry (integrability) restrictions $\beta_{13} = \beta_{31}$, $\beta_{14} = \beta_{41}$ relating output supply and Euler equations, and similar restrictions regarding input demand ($\beta_{21} = \beta_{12}$, $\beta_{23} = \beta_{32}$, $\beta_{24} = \beta_{42}$). In addition, $\partial^2 V(.)/\partial K \partial \dot{K} = \partial^2 V(.)/\partial \dot{K} \partial K$ implies the restriction $\beta_{34} = \beta_{43}$ for coefficients in the Euler equation. [2]

A similar dynamic model can be specified for feedlot production. A feedlot produces fed cattle for slaughter y using feeder cattle K and other inputs x according to a production function $y_t = f(x_t, K_{t-b}, K_{t-b}-K_{t-b-1})$. Here adjustment costs are proxied in terms of the change in feeder inputs, as in other studies (Buhr and Kim, who cite Marsh 1994 for justification). Feeder cattle are fed grain in feedlots over a period of approximately 6 to 10 months before slaughter, so the lag length "b" in the production function equals 2 or 3 using quarterly data. Market prices for fed cattle output, variable inputs and feeder cattle input are p, w and w^k, respectively. $p_t\ y_t$ is the market value of feeder output for slaughter at time t, and $w^k\ K_t$ is the cost of feeder inputs purchased at time t.

The dynamic optimization for the feedlot under CARA is

$$(7) \quad \max_{\{K\}} J(K,Ep,Vp,w,w^k) \equiv \sum_{t=0}^{T} V(Ep_t,Vp_t,w_t,K_{t-b},K_{t-b}-K_{t-b-1}) /(1+r)^t$$

$$- \sum_{t=b}^{T} w^k_{t-b} K_{t-b} /(1+r)^{t-b}$$

s.t. $K_q = \dot{K}_q$ for $q < 0$ $\qquad K_t - (1-\delta)K_{t-1} \geq 0$ for all t

where the dual indirect utility function V(.) is similar to (2). The feedlot output supply and variable input demand equations are similar to (3), and the discrete time Euler equation is:

$$(8) \quad \partial V_t(.)/\partial K + \partial V_t(.)/\partial \dot{K} - \partial V_{t-1}(.)/\partial \dot{K} /(1+r) - w^k (1+r)^b = 0 \ .$$

Assuming a normalized quadratic functional form for the dual V(.), equations (3) and (8) for the feedlot are

$$(9)a) \quad y_t = \beta_{10} + \beta_{11} Ep^*_t + \beta_{12} w^*_t + \beta_{13} K_{t-b} + \beta_{14} (K_{t-b}-K_{t-b-1}) + \beta_{15} Vp^*_t$$

$$b) \quad -x_t = \beta_{20} + \beta_{21} Ep^*_t + \beta_{22} w^*_t + \beta_{23} K_{t-b} + \beta_{24} (K_{t-b}-K_{t-b-1}) + \beta_{25} Vp^*_t$$

$$(10) \quad (\beta_{30} + \beta_{40}) + (\beta_{31} + \beta_{41}) Ep^*_t + (\beta_{32} + \beta_{42}) w^*_t + (\beta_{33} + \beta_{43}) K_{t-b}$$

$$+ (\beta_{34} + \beta_{44}) (K_{t-b}-K_{t-b-1}) + (\beta_{35} + \beta_{45}) Vp^*_t$$

$$- \{\beta_{40} + \beta_{41} Ep^*_{t-1} + \beta_{42} w^*_{t-1} + \beta_{43} K_{t-1-b} + \beta_{44} (K_{t-1-b}-K_{t-b})$$

$$+ \beta_{45} Vp^*_{t-1}\}(w^0_{t-1}/w^0_t)/(1+r) - w^k_{t-b} (1+r)^b /w^0_t \quad = 0 \ .$$

However there may be a disadvantage to this choice of functional form for feedlots: a doubling in animals fed (K_{t-b}) leads to an approximate doubling in output by total weight (y_t), so (by Euler's theorem) equation (9a) may be independent of prices, i.e. linear homogeneity of y in K implies (by Euler's theorem) that (9a) reduces to $y_t = \beta_{13} K_{t-b}$. This problem also applies to (e.g.) a feedlot supply response study by Ospina and Shumway. A similar problem also arises for the cow-calf output supply model, but we do not have the data to address the problem in this case.. In order to circumvent the problem in the feedlot model, the dual can be specified in terms of two functions a(Ep,w, $\dot{K}$,Vp) K + b(w,K, $\dot{K}$,Vp) where (e.g.) a(.) is quadratic in Ep (linear in w, $\dot{K}$,Vp) and b(.) is quadratic in (w,K, $\dot{K}$,Vp). Then the output supply and Euler equations can be respecified as

$$(9a')\quad y_t/K_{t-b} = \beta_{10} + \beta_{11}\, Ep^*_t + \beta_{12}\, w^*_t + \beta_{13}\, K_{t-b} + \beta_{14}\,(K_{t-b}-K_{t-b-1}) + \beta_{15}\, Vp^*_t$$

$$\begin{aligned}(10')\quad &(\beta_{30} + \beta_{40}) + (\beta_{31} + \beta_{41})\, Ep^*_t + (\beta_{32} + \beta_{42})\, w^*_t + (\beta_{33} + \beta_{43})\, K_{t-b} \\ &+ (\beta_{34} + \beta_{44})\,(K_{t-b}-K_{t-b-1}) + (\beta_{35} + \beta_{45})\, Vp^*_t + \beta_{36}\,(Ep^*_t)^2 \\ &- \{\beta_{40} + \beta_{41}\, Ep^*_{t-1} + \beta_{42}\, w^*_{t-1} + \beta_{43}\, K_{t-1-b} + \beta_{44}\,(K_{t-1-b}-K_{t-b}) \\ &+ \beta_{45}\, Vp^*_{t-1}\}(w^0_{t-1}/w^0_t)/(1+r) - w^k_{t-b}\,(1+r)^b/w^0_t = 0\ .\end{aligned}$$

Here the left hand side variable for output supply is weight per animal, and the term $\beta_{36}\,(Ep^*_t)^2$ is added to the Euler equation.

The dynamic maximization hypothesis and CARA imply that total weight of output supply y, conditional on capital stock K and investment ΔK, are increasing in Ep and (assuming risk aversion) decreasing in Vp. This is because the dynamic optimization problem (7) implies a static maximization problem conditional on (K,ΔK) which is similar to (2). This static problem implies that (conditional on K,ΔK) total weight y is increasing in Ep and decreasing in Vp (Coyle 1992). Since K is proxied by total number of feeder cattle purchased by feedlots (and assuming mortality rates are independent of Ep and Vp), it follows that (conditional on K,ΔK) weight per animal (y/K) is increasing in Ep and decreasing in Vp. Thus for equation (9a') we anticipate that $\beta_{11} > 0$ and $\beta_{15} < 0$.[3] A more general assumption regarding risk preferences is the nonlinear mean-variance model where the coefficient of absolute risk aversion α varies with the mean and variance of wealth. Assuming as in standard models that utility is separable over time, the coefficient of risk aversion at time t depends on (nonstochastic) initial wealth W^o plus the mean and variance of current profits π_t, i.e. $\alpha_t = \alpha(W^o + E\pi_t, V\pi_t)$ where $E\pi$,$V\pi$ are the mean and variance of profits, respectively. The mean and variance of wealth $W = W^o + \pi$ are $EW = W^o + E\pi$ and $VW = V\pi$. The dynamic models for cow-calf and feedlot producers are similar to (1) and (7) except for modifications in the dual V(.).

The dual V(.) for a cow-calf producer with nonlinear mean-variance risk preferences is

$$(11)\ V(Ep_t,Vp_t,w_t,W^o,K_{t-a},K_{t-a}-K_{t-a-1}) = \max_x \quad Ep_t\, f(x,K_{t-a},K_{t-a}-K_{t-a-1}) - w_t\, x$$

$$- (\alpha(.)/2)\ Vp_t\, f(x,K_{t-a},K_{t-a}-K_{t-a-1})^2$$

where $\alpha(.) \equiv \alpha(W^o + Ep_t\, f(.) - w_t\, x - w^k K_{t-a}, Vp_t\, f(.)^2)$. This dual is similar to Coyle (1999). A similar dual $V(Ep_t,Vp_t,w_t,W^o,K_{t-b},K_{t-b}-K_{t-b-1})$ can be defined for a feedlot producer. Properties of the dual V(.) include: $V(\lambda\ Ep, \lambda^2 Vp,\lambda\ w, \lambda\ W^o,..) = V(Ep,Vp,w,W^o,..)$ for $\lambda>0$ assuming constant relative risk aversion (CRRA), V(.) is quasiconvex in (Ep,w,W^o) assuming decreasing absolute risk aversion (DARA), and

(12) $y = \partial V(.)/\partial Ep \,/\, \partial V(.)/\partial W^o$

$-x = \partial V(.)/\partial w \,/\, \partial V(.)/\partial W^o$ (Roy's theorem)

(13) $\partial V(.)/\partial W^o = 1 - \alpha_1 \, Vp \, y^2/2$

where $\alpha_1 \equiv \partial\alpha(EW,VW)/\partial EW$. Equations (12) (Roy's theorem) are highly nonlinear in coefficients of the dual, but this problem can be simplified by substituting (13) into (12) to obtain

(14) $y = \partial V(.)/\partial Ep \,/\, (1 - \alpha_1 \, Vp \, y^2/2)$

$-x = \partial V(.)/\partial w \,/\, (1 - \alpha_1 \, Vp \, y^2/2)$.

Functional forms for V(.) and α(.) can be specified without contradiction, and these determine implicitly the technology. In addition $\alpha(\lambda \, EW, \lambda^2 \, VW) = \lambda^{-1} \, \alpha(EW,VW)$ for $\lambda>0$ assuming CRRA (Coyle 1999). Substituting $\lambda = VW^{-1/2}$ into this CRRA restriction yields $\alpha(EW \, VW^{-1/2},1) = VW^{-1/2} \alpha(EW,VW)$, i.e. $\alpha(EW,VW) = VW^{-1/2} g(EW \, VW^{-1/2})$, and assume a quadratic approximation to g(.): $g = c_0 \,(EW \; VW^{-1/2}) + c_2 \,(EW \; VW^{-1/2})^2$. Then $\alpha(.) = VW^{-1/2} g(.)$ implies $\alpha_1 = VW^{-1/2} \; \partial g(.)/\partial EW$, i.e.

(15) $\alpha_1 = c_1 \,/\, VW \; + 2 \, c_2 \, EW/(VW^{3/2})$

$= c_1 \,/\, (Vp \, y^2) \; + 2 \, c_2 \, (W^0 + E\pi)/((Vp \, y^2)^{3/2})$.

Given functional forms for V(.) and α(.), a dynamic model with nonlinear mean-variance risk aversion can be estimated using equations (13)-(14) and the Euler equation (4) or (8). For example, assuming a normalized quadratic form for the dual under CRRA similar to Coyle (1999), the following derivatives of the dual are specified for the cow-calf producer:

$\partial V^*_t(.)/\partial v^i_t = \beta_{i0} + \beta_{i1}\ Ep^*_t + \beta_{i2}\ w^*_t + \beta_{i3}\ K_{t-a} + \beta_{i4}\ (K_{t-a}-K_{t-a-1}) + \beta_{i5}\ W^{o*}_t + \beta_{i6}\ Vp^{**}_t$ ($v^i_t \equiv Ep^*_t, w^*_t, K_{t-a}, K_{t-a}-K_{t-a-1}, W^{o*}_t$). Here $V^*_t \equiv (V/w^0)_t$, $Ep^*_t \equiv (Ep/w^0)_t$, $Vp^{**}_t \equiv (Vp/(w^0)^2)_t$, $w^*_t \equiv (w/w^0)_t$ assuming CRRA. Also assume the above second order approximation to $\alpha(.)$ under CRRA. Then equations (14), Euler equation (4) and equation (13) can be specified as

$$
\begin{aligned}
(16)\ y_t = \{&\beta_{10} + \beta_{11}\ Ep^*_t + \beta_{12}\ w^*_t + \beta_{13}\ K_{t-a} + \beta_{14}\ (K_{t-a}-K_{t-a-1}) + \beta_{15}\ W^{o*}_t \\
&+ \beta_{16}\ Vp^{**}_t\} \ /\ \{1 - c_1/2 - c_2\ (W^0+E\pi)/((Vp\ y^2)^{1/2})\} \\
-x_t = \{&\beta_{20} + \beta_{21}\ Ep^*_t + \beta_{22}\ w^*_t + \beta_{23}\ K_{t-a} + \beta_{24}\ (K_{t-a}-K_{t-a-1}) + \beta_{25}\ W^{o*}_t \\
&+ \beta_{26}\ Vp^{**}_t\} \ /\ \{1 - c_1/2 - c_2\ (W^0+E\pi)/((Vp\ y^2)^{1/2})\}
\end{aligned}
$$

$$
\begin{aligned}
(17)\quad &(\beta_{30} + \beta_{40}) + (\beta_{31} + \beta_{41})\ Ep^*_t + (\beta_{32} + \beta_{42})\ w^*_t + (\beta_{33} + \beta_{43})\ K_{t-a} \\
&+ (\beta_{34} + \beta_{44})\ (K_{t-a}-K_{t-a-1}) + (\beta_{35} + \beta_{45})\ W^{o*}_t + (\beta_{36} + \beta_{46})\ Vp^{**}_t \\
&- \{\beta_{40} + \beta_{41}\ Ep^*_{t-1} + \beta_{42}\ w^*_{t-1} + \beta_{43}\ K_{t-1-a} + \beta_{44}\ (K_{t-1-a}-K_{t-a}) \\
&+ \beta_{45}\ W^{o*}_{t-1} + \beta_{46}\ Vp^{**}_{t-1}\}(w^0_{t-1}/w^0_t)/(1+r) \\
&- \{w^k_{t-a-s}\ (1+r)^{a+s} + w^k_{t-1-a-s}\ (1-\delta)\ (1+r)^{a+s-1}\}/w^0_t = 0
\end{aligned}
$$

$$
\begin{aligned}
(18)\ (W^o_t+E\pi_t)/(Vp\ y^2)^{1/2}) = 1/c_2 - 2c_1/c_2 - \{&\beta_{50} + \beta_{51}\ Ep^*_t + \beta_{52}\ w^*_t \\
&+ \beta_{53}\ K_{t-a} + \beta_{54}\ (K_{t-a}-K_{t-a-1}) + \beta_{55}\ W^{o*}_t + \beta_{56}\ Vp^{**}_t\}/c_2
\end{aligned}
$$

with symmetry restrictions $\beta_{ij} = \beta_{ji}$ (i,j = 1,..,5).

A dynamic model for a feedlot with nonlinear mean-variance risk preferences can be specified in a similar manner. Given a normalized quadratic dual V(.) and a second order approximation to $\alpha(.)$ under CRRA as above, equations (14), the Euler equation (8) and equation (13) can be specified as

$$(19)\ y_t = \{\beta_{10} + \beta_{11}\, Ep^*_t + \beta_{12}\, w^*_t + \beta_{13}\, K_{t-b} + \beta_{14}\, (K_{t-b}-K_{t-b-1}) + \beta_{15}\, W^{o*}_t + \beta_{16}\, Vp^{**}_t\} \,/\, \{1 - c_1/2 - c_2\, (W^0+E\pi)/((Vp\, y^2)^{1/2})\}$$

$$-x_t = \{\beta_{20} + \beta_{21}\, Ep^*_t + \beta_{22}\, w^*_t + \beta_{23}\, K_{t-b} + \beta_{24}\, (K_{t-b}-K_{t-b-1}) + \beta_{25}\, W^{o*}_t + \beta_{26}\, Vp^{**}_t\} \,/\, \{1 - c_1/2 - c_2\, (W^0+E\pi)/((Vp\, y^2)^{1/2})\}$$

$$(20)\quad (\beta_{30} + \beta_{40}) + (\beta_{31} + \beta_{41})\, Ep^*_t + (\beta_{32} + \beta_{42})\, w^*_t + (\beta_{33} + \beta_{43})\, K_{t-b} + (\beta_{34} + \beta_{44})\, (K_{t-b}-K_{t-b-1}) + (\beta_{35} + \beta_{45})\, W^{o*}_t + (\beta_{36} + \beta_{46})\, Vp^{**}_t - \{\beta_{40} + \beta_{41}\, Ep^*_{t-1} + \beta_{42}\, w^*_{t-1} + \beta_{43}\, K_{t-1-b} + \beta_{44}\, (K_{t-1-b}-K_{t-b}) + \beta_{45}\, W^{o*}_{t-1} + \beta_{46}\, Vp^{**}_{t-1}\}(w^0_{t-1}/w^0_t)/(1+r) - w^k_{t-b}\, (1+r)^b / w^0_t = 0$$

$$(21)\ (W^o_t+E\pi_t)/(Vp\, y^2)^{1/2}) = 1/c_2 - 2c_1/c_2 - \{\beta_{50} + \beta_{51}\, Ep^*_t + \beta_{52}\, w^*_t + \beta_{53}\, K_{t-b} + \beta_{54}\, (K_{t-b}-K_{t-b-1}) + \beta_{55}\, W^{o*}_t + \beta_{56}\, Vp^{**}_t\}/c_2$$

with symmetry restrictions $\beta_{ij} = \beta_{ji}$ (i,j = 1,..,5).

Data

Dynamic models were constructed for cow-calf and feedlot operations using biannual and quarterly data, respectively, for Alberta over 1976-1997 (data on replacement heifers on-farm is unavailable prior to 1976). Cow-calf output (at weaning) is defined as the number of light feeder calves (400-500 lbs) on-farm Jan. 1 and July 1 in Alberta (Statistics Canada b). This series closely approximates light feeder calves on-farm over the year, and a similar proxy for cow-calf output has been used in a U.S. study that has also attempted to differentiate between cow-calf and feedlot supply response (Buhr and Kim). Unfortunately data on cow-calf output is not available by weight. The output price is in $/cwt for Alberta light feeders (400-500 lbs) (Agriculture and Agri-Food Canada). Input prices are a feed price index and hired labor wage index for Western Canada (Statistics Canada a), and price ($/cwt) for Alberta replacement heifers (700 lbs) (Agriculture and Agri-Food Canada).

Investment in the cow-calf operation is defined as the number of replacement heifers on- farm Jan. 1 and July 1 in Alberta (Statistics Canada b), and investment is specified as conditional on number of cows on-farm Jan. 1 and July 1 (Statistics Canada b), price ($/cwt) for Alberta feeder steers (700 lbs) (Agriculture and Agri-Food Canada), price for replacement heifers, feed price index and hired labor wage. Feedlot output is defined as the total weight (cwt) of fed cattle slaughtered in Alberta plus exports for slaughter from Alberta to the U.S. (number of animals is multiplied by cwt per animal), and the output price is measured as the price ($/cwt) for Alberta feeder steers (> 900 lbs) (Agriculture and Agri-Food Canada).

Input prices are the feed price index, hired labor wage, and the price for Alberta feeder steers (700 lbs). Initial stock of wealth is proxied by the value of land and buildings plus machinery and equipment (Statistics Canada c).

Empirical Models

Expected output prices are proxied as a one period lag on market prices, and variances of output prices are proxied as the weighted sum of squares of prediction errors of the previous three years, with declining weights of 0.50, 0.33, and 0.17. This particular formula for price variance has been used in other studies (Chavas and Holt; Coyle). Expected prices and price variances were also calculated from ARIMA and GARCH models expressing market prices as a distributed lag of prices, but these measures were insignificant in all models and were rejected for the simpler measures.

These results are similar to other studies of Western Canadian agriculture under risk aversion (Coyle) that rejected proxies from ARIMA and GARCH models. Similarly a study of crop price expectations for a group of Sasketchewan farmers concluded that these reported expectations are less adequately explained as time series forecasts (Sulewski, Spriggs and Schoney).

Output quantity data, prices, replacement heifers and herd size were tested for unit roots by standard methods (Dickey-Fuller and Phillips-Perron, with and without allowing for trend stationarity in the alternative). In all cases the unit root hypothesis was rejected at the .05 level. Since these tests are biased in favor of the unit root hypothesis in the sense that they have low power (Kwiatkowski et. al.), we assume that it is not necessary to transform data due to unit roots. This conclusion was also supported by alternative tests (Kwiatkowski et. al.).

Hired labor wage was found to be insignificant and was dropped from all models. This result is not surprising since labor cost is a relatively small proportion of total costs for both cow-calf and feedlot sectors.

The output supply equation for the cow-calf CARA model (5)-(6) is specified as

(22) $y_t = \beta_{10} + \beta_{11}\, Ep^*_t + \beta_{13}\, K_{t-a} + \beta_{14}\, (K_{t-a} - K_{t-a-1}) + \beta_{15}\, Vp^*_t$

with a = 2. A correct specification of this equation requires that calf output y is measured as total weight. Unfortunately data is available only on total number of calves on-farm, so we must use this as a proxy for output y_t (Buhr and Kim adopt a similar approach). Obviously this proxy is particularly closely related to the size of breeding herd K_{t-a}. Nevertheless, to the extent that cow-calf producers have some flexibility in marketing their calf crop in different biannual periods through backgrounding, producers will try to sell in periods when prices are expected to be relatively high and price uncertainty is relatively low. Thus calves may still tend to be on-farm at the beginning of such periods. Accordingly we hypothesize that $\beta_{11} > 0$ and $\beta_{15} < 0$, but both elasticities should be relatively small. Various parameter normalizations are possible for the Euler equation. We choose to solve (6) for K_{t-a}-K_{t-a-1}, [4]

(23) $K_{t-a} - K_{t-a-1} = -(\beta_{34} + \beta_{44})^{-1}$

$$[(\beta_{30} + \beta_{40}) + (\beta_{31} + \beta_{41})\, Ep^*_t + (\beta_{33} + \beta_{43} + \beta_{44}/(1+r))\, K_{t-a}$$

$$+ (\beta_{35} + \beta_{45})\, Vp^*_t - \{\beta_{40} + \beta_{41}\, Ep^*_{t-1} + (\beta_{43} + \beta_{44})\, K_{t-1-a}$$

$$+ \beta_{45}\, Vp^*_{t-1}\}/(1+r)$$

$$- \{w^k_{t-a-s}\,(1+r)^{a+s} + w^k_{t+1-a-s}\,(1-\delta)\,(1+r)^{a+s-1}\}/w^0_t]$$

$$= -\{\gamma_{30} + (\gamma_{31} + \gamma_{41})\,Ep^*_t + \gamma_1\,K_{t-a} + (\gamma_{35} + \gamma_{45})\,Vp^*_t$$

$$- (\gamma_{41}\,Ep^*_{t-1} + \gamma_2\,K_{t+1-a} + \gamma_{45}\,Vp^*_{t-1})/(1+r)$$

$$- \gamma_0\,\{w^k_{t-a-s}\,(1+r)^{a+s} + w^k_{t+1-a-s}\,(1-\delta)\,(1+r)^{a+s-1}\}/w^0_t\}\ .$$

Here the Euler equation is specified as linear in coefficients γ, which are related to structural coefficients as follows: $\gamma_0 = 1/(\beta_{34} + \beta_{44})$, $\gamma_1 = \{\beta_{33} + \beta_{43} + \beta_{44}/(1+r)\}/(\beta_{34} + \beta_{44})$, $\gamma_2 = (\beta_{43} + \beta_{44})/(\beta_{34} + \beta_{44})$, $\gamma_{ij} = \beta_{ij} / (\beta_{34} + \beta_{44})$ except for the intercept $\gamma_{30} = (\beta_{30} + \beta_{40}\ r/(1+r))/(\beta_{34} + \beta_{44})$. This identifies all structural coefficients of Ep and Vp in the Euler equation. The numeraire price w^0 is the feed price index, $a = 2$, $s = 3$, $r = .05$, $\delta = .01$, and it is assumed in the Euler equation that $E_t\,(w^0_{t+1}/w^0_t) = 1$. The symmetry restrictions $\beta_{13} = \beta_{31}$, $\beta_{14} = \beta_{41}$ relating output supply and Euler equations imply $\gamma_{31} = \beta_{13} / (\beta_{34} + \beta_{44})$ and $\gamma_{41} = \beta_{14} / (\beta_{34} + \beta_{44})$, i.e.

(24) $\beta_{13} = \gamma_{31} / \gamma_0 \qquad \beta_{14} = \gamma_{41} / \gamma_0$.

Substituting these restrictions into the output supply equation yields a model that is nonlinear in coefficients. In addition, the symmetry restrictions $\beta_{34} = \beta_{43}$ imply the following restriction within the Euler equation:

(25) $\gamma_2 = 1$. [5]

Similarly the feedlot CARA model (9)-(10) is specified as

(26) $y_t = \beta_{10} + \beta_{11}\,Ep^*_t + \beta_{13}\,K_{t-b} + \beta_{14}\,(K_{t-b}-K_{t-b-1}) + \beta_{15}\,Vp^*_t$

(27) $(K_{t-b}-K_{t-b-1}) =$

$$-\{\gamma_{30} + (\gamma_{31} + \gamma_{41})\, Ep^{*}{}_{t} + \gamma_{1}\, K_{t-b} + (\gamma_{35} + \gamma_{45})\, Vp^{*}{}_{t}$$

$$- (\gamma_{41}\, Ep^{*}{}_{t-1} + \gamma_{2}\, K_{t-1-b} + \gamma_{45}\, Vp^{*}{}_{t-1})/(1+r)$$

$$- \gamma_{0}\, w^{k}{}_{t-b}\, (1+r)^{b} / w^{0}{}_{t}\}$$

with b = 3, and the symmetry restrictions are similar to the previous model.

The cow-calf CRRA model (16)-(17) is specified as

$$(28)\ y_t = \{\beta_{10} + \beta_{11}\, Ep^{*}{}_{t} + \beta_{13}\, K_{t-a} + \beta_{14}\, (K_{t-a}-K_{t-a-1}) + \beta_{15}\, W^{o*}{}_{t}$$

$$+ \beta_{16}\, Vp^{**}{}_{t}\} \,/\, \{1 - c_1/2 - c_2\, (W^{0}+E\pi)/((Vp\, y^{2})^{1/2})\}$$

$$(29)\ \ (K_{t-a}-K_{t-a-1}) =$$

$$-\{\gamma_{30} + (\gamma_{31} + \gamma_{41})\, Ep^{*}{}_{t} + (\gamma_{33} + \gamma_{2})\, K_{t-a} + (\gamma_{35} + \gamma_{45})\, W^{o*}{}_{t}$$

$$+ (\gamma_{36} + \gamma_{46})\, Vp^{**}{}_{t} - (\gamma_{41}\, Ep^{*}{}_{t-1} + \gamma_{443}\, K_{t-1-a} + \gamma_{45}\, W^{o*}{}_{t-1} + \gamma_{46}\, Vp^{**}{}_{t-1})/(1+r)$$

$$- \gamma_{0}\, \{w^{k}{}_{t-a-s}\, (1+r)^{a-s} + w^{k}{}_{t-1-a-s}\, (1-\delta)\, (1+r)^{a-s-1}\}/w^{0}{}_{t}\}$$

$$(30)\ (W^{o}{}_{t}+E\pi_{t})/(Vp\, y^{2})^{1/2}) = 1/c_2 - 2c_1/c_2 - \{\beta_{50} + \beta_{51}\, Ep^{*}{}_{t} + \beta_{53}\, K_{t-a}$$

$$+ \beta_{54}\, (K_{t-a}-K_{t-a-1}) + \beta_{55}\, W^{o*}{}_{t} + \beta_{56}\, Vp^{**}{}_{t}\}/c_2$$

with additive disturbances. The symmetry restrictions are (24-25) and (31) $\beta_{51} = \beta_{15}$

$\beta_{53} = \gamma_{35} / \gamma_0 \quad \beta_{54} = \gamma_{45} / \gamma_0$.

Similarly the feedlot CRRA model (19)-(20) is specified as

$$(32)\ \ y_t = \{\beta_{10} + \beta_{11}\, Ep^{*}{}_{t} + \beta_{13}\, K_{t-b} + \beta_{14}\, (K_{t-b}-K_{t-b-1}) + \beta_{15}\, W^{o*}{}_{t}$$

$$+ \beta_{16}\, Vp^{**}{}_{t}\} \,/\, \{1 - c_1/2 - c_2\, (W^{0}+E\pi)/((Vp\, y^{2})^{1/2})\}$$

(33) $(K_{t-b}-K_{t-b-1}) =$

$$-\{\gamma_{30} + (\gamma_{31} + \gamma_{41})\, Ep^*_t + (\gamma_{33} + \gamma_{443})\, K_{t-b}$$

$$+ (\gamma_{35} + \gamma_{45})\, W^{o*}_t + (\gamma_{36} + \gamma_{46})\, Vp^{**}_t - (\gamma_{41}\, Ep^*_{t+1} + \gamma_{443}\, K_{t+1-b}$$

$$+ \gamma_{45}\, W^{o*}_{t+1} + \gamma_{46}\, Vp^{**}_{t+1})/(1+r) - \gamma_0\, w^k_{t-b}\, (1+r)^b / w^0_t\}$$

(34) $(W^o_t+E\pi_t)/(Vp\, y^2)^{1/2}) = 1/c_2 - 2c_1/c_2 - \{\beta_{50} + \beta_{51}\, Ep^*_t + \beta_{53}\, K_{t-b}$

$$+ \beta_{54}\, (K_{t-b}-K_{t-b-1}) + \beta_{55}\, W^{o*}_t + \beta_{56}\, Vp^{**}_t\}/c_2$$

with symmetry restrictions (24-25) and (31).

A well known difficulty in estimating Euler equations such as (4) is that the derivative $\partial V_{t+1}(.)/\partial\ K$ depends on unobserved plans at t for next period decisions, which are usually proxied by observed next period decisions assuming rational expectations. This difficulty also applies here: the Euler equation specified at time t defines the first order condition for the decision K_{t-a}, which depends upon the unobserved plan K_{t+1-a}. Consistent estimates of such Euler equations under rational expectations can be obtained by standard instrumental variable methods such as two stage least squares (McCallum; Kennan). However rational expectations generally implies serial correlation, and then standard corrections to these methods lead to inconsistent coefficient estimates (Flood and Garber; Cumby, Huizinga and Obstfeld).

Accordingly we follow the by now standard convention of estimating a dynamic model by generalized methods of moments (GMM) (Hansen; Hansen and Singleton). GMM estimation leads to consistent and asymptotically efficient estimates for linear or nonlinear models based only on (sufficient) moment conditions, in this case the rational expectations assumption that errors in expectations are independent of the current information set. Of course there are substantial difficulties in selecting a weighting matrix for small samples and in statistical inference (Davidson and MacKinnon; Newey and West 1987b; Ghysels and Hall 1990a,b; Hall and Horowitz; Smith). In addition, recent empirical studies have suggested that GMM has not led to stable estimates of structural parameters of Euler equations (Garber and King; Oliner, Rudebusch and Sichel (1996)).

Most applications of Euler equations have assumed rational expectations for both prices and unobserved plans. However, as we argued earlier, other empirical studies have suggested that rational expectations models provide poor proxies for price expectations of Western Canadian farmers. Instead it seems to be more realistic to adopt simple backward-looking models of price expectations: expected price Ep at time t is the most recently observed price p_{t-1}, and price variance Vp is a simple weighted sum of squared prediction errors in the most recently observed periods.

This simple model of backward-looking price expectations implies further simplifications of the above Euler equations. In the Euler equation (4) corresponding to the first order condition $\partial J(.)/\partial K_t = 0$, all derivatives $\partial V_t(.)/\partial K$, $\partial V_t(.)/\partial \dot{K}$ and $\partial V_{t-1}(.)/\partial \dot{K}$ must be evaluated based on information available at time t. Thus the backward-looking expectation at time t for price p in periods t and t+1 are both p_{t-1}, so Ep_t in $\partial V_t(.)/\partial K$ and $\partial V_t(.)/\partial \dot{K}$ and Ep_{t-1} in $\partial V_{t-1}(.)/\partial \dot{K}$ are equal. Similarly Vp_t in $\partial V_t(.)/\partial K$ and $\partial V_t(.)/\partial \dot{K}$ and Vp_{t-1} in $\partial V_{t-1}(.)/\partial \dot{K}$ are equal.

Then the separate coeffients $\gamma_{31},\gamma_{41},\gamma_{35},\gamma_{45}$ are not identified (in the absence of across-equation restrictions), so (e.g.) the terms $(\gamma_{31} + \gamma_{41})\ Ep^*_t - \gamma_{41}\ Ep^*_{t-1}/(1+r)$ in (23) reduces to $\psi_{31}\ Ep^*_t$ and the terms $(\gamma_{35} + \gamma_{45})\ Vp^*_t - \gamma_{45}\ Vp^*_{t-1}/(1+r)$ in (23) reduce to $\psi_{35}\ Vp^*_t$.

Results for CARA Cow-calf Model

As indicated above, the CARA cow-calf model (22)-(23) further reduces to the (nonrecursive) system

(35) $y_t = \beta_{10} + \beta_{11}\ Ep^*_t + \beta_{13}\ K_{t-a} + \beta_{14}\ (K_{t-a}-K_{t-a-1}) + \beta_{15}\ Vp^*_t$

(36) $(K_{t-a}-K_{t-a-1}) =$

$$-\{\gamma_{30} + \psi_{31}\ Ep^*_t + \gamma_1\ K_{t-a} + \psi_{35}\ Vp^*_t - \gamma_2\ K_{t-1-a}/(1+r)$$

$$- \gamma_0\ \{w^k_{t-a-s}\ (1+r)^{a-s} + w^k_{t-1-a-s}\ (1-\delta)\ (1+r)^{a-s-1}\}/w^0_t\}$$

with a = 2, s = 3, r = .05, δ = .01, and additive disturbances. Here $\psi_{31} = (\beta_{31} + \beta_{41}\ r/(1+r))\ /(\beta_{34} + \beta_{44})$, $\psi_{35} = (\beta_{35} + \beta_{45}\ r/(1+r))\ /(\beta_{34} + \beta_{44})$, $\gamma_0 = 1/(\beta_{34} + \beta_{44})$, $\gamma_1 = \{\beta_{33} + \beta_{43} + \beta_{44}/(1+r)\}/(\beta_{34} + \beta_{44})$, $\gamma_2 = (\beta_{43} + \beta_{44})/(\beta_{34} + \beta_{44})$, $\gamma_{30} = (\beta_{30} + \beta_{40}\ r/(1+r))/(\beta_{34} + \beta_{44})$, and $\gamma_2 = (\beta_{43} + \beta_{44})/(\beta_{34} + \beta_{44})$. The restrictions $\beta_{13} = \beta_{31}$, $\beta_{14} = \beta_{41}$ relating output supply and Euler equations imply

(37) $\psi_{31} = (\beta_{13} + \beta_{14}\ r/(1+r))\ /\gamma_0$

which can be substituted into the Euler equation. The additional symmetry restriction $\beta_{34} = \beta_{43}$ corresponding to $\partial^2 V(.)/\partial K \partial \dot{K} = \partial^2 V(.)/\partial \dot{K}\ \partial K$ again implies the following restriction on the Euler equation:

(38) $\gamma_2 = 1$.

Imposing this restriction, the left hand side of the Euler equation can be transformed to

$(K_{t-a}-K_{t-a-1}) - K_{t+1-a}/(1+r)$ ..

In (36) K_{t-a} and K_{t+1-a} are treated as endogenous, and additional instruments are K lagged an additional 3 to 8 periods. Since all these lags in K are in the firm's information set, the rational expectations hypothesis for K and $\dot{K}$ suggests that these lags can define valid moment conditions. Regressing K_{t-a} and K_{t+1-a} on these additional instruments led to R^2 of approximately 0.80, which indicate that these instruments covary reasonably highly with the endogenous variables. [6]In principle the assumption of independence between instruments and disturbances can be addressed using Hansen's J-test for GMM models. Nevertheless GMM theory apparently provides little guidance in the selection of instruments for finite samples: the small sample behavior of GMM estimators may worsen as the number of instruments becomes large (Ferson and Foerster; Kocherlakota; Smith), and asymptotic efficiency may sometimes decrease as the number of instruments increases (Imbens), and the J-test apparently has low power and uncertain finite sample properties. Consistent moment selection procedures have recently been devised (Andrews 1999), but it is not yet clear how useful these are for small samples. Accordingly the selection of instruments here is essentially ad hoc.

The model was first estimated by linear two stage least squares (2SLS) using Shazam 8.0. 2SLS results are reported in Table 1 (a seasonal biannual dummy and a time trend were insignificant). Variables in (35)-(36) are normalized by 1997 levels, so that coefficient estimates can be interpreted as elasticities circa 1997. In the output supply equation, expected price Ep has a significant positive coefficient and price variance Vp has a significant negative coefficient, as is expected under risk aversion. The elasticity is smaller for Vp than for Ep, as in other studies of production decisions under risk. Both elasticities are small. In the Euler equation, Ep has a positive but insignificant impact and Vp has a significant negative impact on investment K_{t-a}-K_{t-a-1}. The symmetry restriction $\gamma_2 = 1$ on the Euler equation implied by $\partial^2V(.)/\partial K\partial \dot{K} = \partial^2V(.)/\partial \dot{K}\, \partial K$ translates into the following restriction when variables are normalized by 1997 values: $\gamma_2 = 7.61$. [7]However the corresponding estimate is -6.3967, and the symmetry restriction is rejected. Since γ_0 (the coefficient of Z in the investment equation) is insignificant, we cannot infer signs for β_{31} and β_{35} (impacts of Ep and Vp on the shadow price of capital) from the 2SLS estimates. Homoskedasticity is accepted using Breusch-Pagan tests. However there is substantial (positive) autocorrelation in both equations, and (as noted above) standard methods to correct for autocorrelation would lead to inconsistent coefficient estimates.

The output supply and Euler equations were estimated jointly by three stage least squares (3SLS). Given a matrix W of valid instruments for a linear model $y = X\beta + e$, the 3SLS criterion function $(y-X\beta)^T W\ B\ W^T(y-X\beta)$ is a quadratic form in the empirical moments $m \equiv W^T(y-X\beta)$ and the weighting matrix $B = (\Sigma \otimes W^T W)^{-1}$, Σ is the matrix of disturbance contemporaneous covariances. In principle 3SLS estimates are consistent and asymptotically normal. The asymptotic covariance matrix of 3SLS estimators of β for this model is $\text{cov}(b) = (G^T B G)^{-1}$, $G \equiv \partial m(.)/\partial\beta$.

Seemingly unrelated regression (SUR) and three stage least squares (3SLS) results are reported in Tables 2 and 3, respectively. The hypothesis of no contemporaneous covariance in the SUR model is rejected at the .01 level using the Breusch-Pagan test (although the hypothesis is only rejected at the .10 level in the 3SLS model). For the output supply equation, coefficient estimates are very similar to 2SLS, and t-ratios are somewhat higher for SUR (instrumental variable methods lose efficiency relative to OLS and SUR). Changes in coefficient estimates are larger for the Euler equation. Of most interest, both Ep and Vp are significant in the Euler equation using 3SLS. Ep and Vp have much larger elasticities in the investment equation than in the output supply equation, as we anticipated. [8]The estimated elasticity of output supply with respect to Ep, 0.12, is similar to the short-run elasticity in Buhr and Kim (0.05).

One important difference from 2SLS results is that the coefficient of Z in the Euler equation, γ_0, is now negative and significant in both SUR and 3SLS results in Tables 2 and 3. Since $\psi_{31} = (\beta_{31} + \beta_{41}\, r/(1+r)) / (\beta_{34} + \beta_{44})$, $\gamma_0 = 1/(\beta_{34} + \beta_{44})$ and $r = .05$, negative coefficient estimates for ψ_{31} and γ_0 essentially imply a positive estimate for β_{31}, which is the derivative $\partial V_K(.)/\partial Ep$. Thus the estimated impact of expected output price

Ep on the shadow price V_K for beef cows is positive, as expected. Similarly a positive coefficient estimate for $\psi_{35} = (\beta_{35} + \beta_{45}\, r/(1+r)) / (\beta_{34} + \beta_{44})$ essentially implies a negative estimate for $\beta_{35,}$ i.e. the impact of price variance Vp on the shadow price V_K is negative, as expected under risk aversion.

The output supply and Euler equation (35)-(36) were also estimated by GMM. Given a positive definite weighting matrix A, unique estimates of β are calculated from the first order conditions for the GMM criterion function $(y- X\beta)^T W\, A\, W^T(y-X\beta)$. Given certain regularity conditions, this estimator is consistent and has an asymptotic normal distribution with covariance matrix $\text{cov}(b) = (G^TAG)^{-1}\, G^T\, A\Phi A\, G\, (G^TAG)^{-1})$, where G $\equiv \partial m(.)/\partial\beta$ and Φ is the covariance matrix of the empirical moments m, i.e. $\Phi = \text{cov}(W^Te) = W^T \text{cov}(e)\, W$. Assuming that A is a consistent estimator of Φ^{-1}, substituting Φ^{-1} for A above yields an asymptotic covariance matrix

$$\text{cov}(b) = (G^T\, \Phi^{-1}\, G)^{-1}$$

$$= (G^T\, A\, G)^{-1}. \tag{39}$$

Any other choice of A leads to a covariance matrix for b that exceeds this by a positive semidefinite matrix, i.e. this choice of A leads to asymptotic efficiency in the class of GMM estimators (including 3SLS). The asymptotic covariance matrix of b is calculated as in (39), but if the weighting matrix A is not a consistent estimator of Φ^{-1} then this underestimates the asymptotic variances of the particular GMM estimator. Assuming A is a consistent estimator of Φ^{-1} also implies that the minimized value of the GMM criterion function (the J statistic) is asymptotically chi-square (Hansen). The Newey-West (1987a) approach to constructing a weighting matrix was followed using the Bartlett option in Shazam 8.0. In principle this provides a consistent estimate of Φ^{-1} under both serial correlation and heteroskedasticity, essentially so long as the number of sample autocovariances is large enough that autocorrelations at longer lags are negligible (see Newey and West, and Andrews 1991), and by construction this estimate is positive semi-definite. [9]Given a first order autocorrelation coefficient of 0.5, correlations between error terms 4 and 8 periods apart are 0.0625 and 0.0039, respectively. Alternative weighting matrices were also considered (a Quadratic Spectral (Andrews 1991) and a Heteroskedastic- consistent Covariance matrix (White 1980)), but these matrices were singular and estimation was unsuccessful.

GMM results assuming 8 autocovariances for the Newey-West matrix are reported in Table 4. The algorithm uses 3SLS estimates to form the Newey-West weighting matrix and then estimates the GMM model. [10]In Table 4, the overidentifying restrictions for the GMM model are not rejected using the J- test. Coefficient estimates are similar to 3SLS results, but there is a substantial reduction in standard errors.
The substantial reduction in standard errors for GMM relative to 3SLS reflects either (1) a substantial improvement in precision of estimates, (2) the weighting matrix used here is not a consistent estimator of the moment covariance matrix, or (3) asymptotic theory has no relevance to our data set. Unfortunately empirical applications of GMM seldom

report 3SLS results, so it is difficult to assess the first possibility. Asymptotic standard errors should be lower for GMM than for 3SLS (as noted above), and there is anecdotal evidence that 3SLS often leads to large standard errors in dynamic models. Regarding the second possibility, an 8 period autocovariance with autocorrelations of 0.5 suggest consistent estimation of the moment covariance matrix. Nevertheless if the first-step 3SLS estimates are inconsistent, then the resulting weighting matrix estimates are not consistent.

Monte Carlo results on the relevance of GMM asymptotic distributions for finite samples are mixed. The Monte Carlo study by Andrews (1991) calculates the relation between true and nominal confidence intervals for various GMM estimators assuming 64, 128 and 256 observations. None of the GMM estimators are reliable if disturbances follow an AR(1) process with an autocorrelation parameter ρ equal to 0.9 (approximating a unit root), but this is not our case. Otherwise the Quadratic Spectral (QS) estimator may be most reliable. For an AR(1) process ($\rho = 0.5$) with heteroskedasticity and 128 observations, the true confidence intervals for a nominal 95% confidence interval are reported as 87% for QS, 83% for the White Heteroskedastic-consistent Covariance matrix, and 59% for the standard LS variance estimator for iid errors. Results are not reported for the Newey-West (Bartlett) estimator, but apparently it is at least somewhat less reliable than the QS estimator.

A Monte Carlo study by Newey and West (1994) presents additional results. For an AR(4) error process ($\rho = 0.18, 0.05, 0.10, 0.12$), an AR(4) process for a (distributed lag) explanatory variable, homoskedasticity and 100 observations, a nominal 95% confidence interval is reported as 87% for the Newey-West (Bartlett) and QS. Allowing for heteroskedasticity (Garch(1,1)) in a different model with 300 observations, a nominal 95% confidence interval is only a 69% interval for Newey-West and 73% for QS. [11]

Results for CRRA Cow-calf Model

Given our assumption of backward-looking expectations, the cow-calf CRRA model (28)-(30) reduces to:

$$(40)\ y_t = \{\beta_{10} + \beta_{11}\,Ep^*_{\ t} + \beta_{13}\,K_{t-a} + \beta_{14}\,(K_{t-a}-K_{t-a-1}) + \beta_{15}\,W^{o*}_{\ t} + \beta_{16}\,Vp^{**}_{\ t}\} / \{1 - c_1/2 - c_2\,(W^0+E\pi)/((Vp\ y^2)^{1/2})\}$$

$$(41)\ (K_{t-a}-K_{t-a-1}) = -\{\gamma_{30} + \psi_{31}\,Ep^*_{\ t} + \gamma_1\,K_{t-a} + \psi_{35}\,W^{o*}_{\ t} + \psi_{36}\,Vp^{**}_{\ t} - \gamma_2\,K_{t-1-a}/(1+r) - \gamma_0\,\{w^k_{\ t-a-s}\,(1+r)^{a-s} + w^k_{\ t-1-a-s}\,(1-\delta)\,(1+r)^{a-s-1}\}/w^0_{\ t}\}$$

$$(42)\ (W^o_{\ t}+E\pi_t)/(Vp\ y^2)^{1/2}) = 1/c_2 - 2c_1/c_2 - \{\beta_{50} + \beta_{51}\,Ep^*_{\ t} + \beta_{53}\,K_{t-a} + \beta_{54}\,(K_{t-a}-K_{t-a-1}) + \beta_{55}\,W^{o*}_{\ t} + \beta_{56}\,Vp^{**}_{\ t}\}/c_2$$

with coefficients defined similarly to the CARA model. The symmetry restrictions are similar to the CARA model (37-38) plus (31).

The output supply equation is now nonlinear in coefficients, and this substantially complicates estimation of the model (42 can be estimated as linear in coefficients, at least in the absence of symmetry restrictions 31). The output supply equation can be estimated by nonlinear least squares (NLS) but not directly by the nonlinear two stage least squares (NL2SLS) algorithms in Shazam. The following procedure was followed: (1) endogenous right hand side variables of (40) (K_{t-a}, K_{t-a}-K_{t-a-1}, $(W^0+E\pi)/((Vp\ y^2)^{1/2})$) were regressed against the specified instruments (including a two period lag on the dependent variable of 42) to obtain predicted values for the endogenous variables; (2) NL2SLS estimates of coefficients were obtained by estimating (40) by NLS using predicted values for the endogenous variables; and (3) the output supply equation (40) was estimated by NL2SLS using coefficient estimates from step (2) as starting values. NLS in step 2 calculates the NL2SLS estimates of coefficients but not of standard errors. However the NL2SLS estimation in step 3 was unsuccessful even though the correct solution was given as the starting values for coefficients. Apparently the NL2SLS algorithms in Shazam are inappropriate for our model.

Table 5 presents nonlinear estimates of the output supply equation (40) using an iterative Davidon-Fletcher-Powell algorithm. Column A presents NLS results, and column B presents NLS results using predicted values for endogenous variables as in step (2) above. In both cases the coefficient estimates of Ep and Vp are similar to the linear model. CARA implies that coefficients β_{15}, c_1 and c_2 are irrelevant variables, which does not appear to be the case. The negative estimate for coefficient c_2 in the denominator (c_1 is insignificant) implies decreasing absolute risk aversion (DARA) for all observations (see (15)), as is expected.

Linear 2SLS estimates of the Euler equation (41) and the third equation (42) are presented in Table 6. Results for the Euler equation are broadly similar to the CARA model (Table 1). The only differences in specification are addition of initial wealth and normalization of Vp by the square of w^0 (feed price index) rather than by w^0. Results for the third equation indicate that only Vp is significant, and the R^2 is quite small. The general insignificance of coefficients in this equation may reflect model mis-specification. On the other hand, note that $\alpha_1 = 0$ under CARA and in turn $\partial V/\partial W_0 = 1$ (see 13), i.e. all coefficients for parameters of the dual derivative $\partial V/\partial W_0$ would equal zero. Although the assumption of CARA is quite restrictive, this line of reasoning does suggest that the poor results for this equation may be partly explained by risk preferences.

Table 7 presents linear 3SLS results for the Euler equation and the third equation. Results are similar to 2SLS, and the hypothesis of zero contemporaneous covariance is only rejected at the 0.10 level using the Breusch-Pagan test. The output supply equation could not be estimated jointly with either equation. Table 8 presents GMM results for the Euler equation and GMM results for third equation using the Bartlett option and 8 autocovariances (the output supply equation could not be estimated by GMM). As in most of the CARA models, the coefficient of Z (γ_0) is negative and significant in the Euler equation. Thus the estimated impact of expected output price Ep and price variance Vp on the shadow price $\partial V(.)/\partial K$ for beef cows are positive and negative, respectively, as expected. Coefficient estimates of equation (42) are similar to 3SLS. GMM leads to a substantial reduction in standard errors for both equations.

Results for CARA Feedlot Model

As in the cow-calf models, our assumption of backward-looking price expectations implies that the feedlot CARA model (9a')-(10') reduces to

(43) $y_t/K_{t-b} = \beta_{10} + \beta_{11}\,Ep^*_{\ t} + \beta_{13}\,K_{t-b} + \beta_{14}\,(K_{t-b}-K_{t-b-1}) + \beta_{15}\,Vp^*_{\ t} + \beta_{16}\,t$

(44) $(K_{t-b}-K_{t-b-1}) =$

$$-\{\gamma_{30} + \psi_{31}\,Ep^*_{\ t} + \gamma_1\,K_{t-b} + \psi_{35}\,Vp^*_{\ t} + \gamma_{36}\,t + \gamma_{37}\,(Ep^*_{\ t})^2$$

$$-\ \gamma_2\,K_{t-1-b}/(1+r) - \gamma_0\,w^k_{\ t-b}\,(1+r)^b/w^0_{\ t}\}$$

with b = 3, and the symmetry restrictions are similar to the CARA cow-calf model. This model treats weight per animal (y/K) as the output supply variable and feeder cattle input (K) as a proxy for capital input in feedlots. Thus changes in total weight are decomposed into an output effect (y/K) and an input effect (K). A time trend (t) is added as a proxy for impacts of technical change on weight per animal and also on investment. [12]

However changes in feeder cattle input may provide a poor proxy for dynamic costs of adjustment, and changes in weight per animal or total weight may provide better proxies. Euler equations for weight per animal and total weight can be defined as, respectively,

(45a) $(y_t/K_{t-b}-y_{t-1}/K_{t-b-1}) =$

$$-\{\gamma_{30} + \psi_{31}\, Ep^*_{\ t} + \gamma_1\, y_t/K_{t-b} + \psi_{35}\, Vp^*_{\ t} + \gamma_{36}\, t$$

$$- \gamma_2\, y_{t-1}/K_{t-1-b}\, /(1+r)$$

(45b) $(y_t-y_{t-1}) =$

$$-\{\gamma_{30} + \psi_{31}\, Ep^*_{\ t} + \gamma_1\, y_t + \psi_{35}\, Vp^*_{\ t} + \gamma_{36}\, t$$

$$- \gamma_2\, y_{t-1}\, /(1+r) - \gamma_0\, w^k_{\ t-b}\, (1+r)^b\, /w^0_{\ t}\} \ .$$

The first ad hoc Euler equation (a) cannot be specified jointly with the output supply (44) in terms of a common functional form (y/K cannot meaningfully be specified as a function of itself in 44), so the $(Ep^*_{\ t})^2$ term may as well be dropped from (a). A similar conclusion holds for the second Euler equation ((b) could be specified jointly with a modified (44) that is conditional on y rather than K, but then this (44) would explain K^{-1} rather than y/K). In addition, since the price of feeder cattle input is not a direct cost associated with weight per animal, this price is dropped from Euler equation (a). Finally, (44) and (45a) can be viewed as a decomposition of (45b), so that Euler equations (44) and (45a) can hold jointly.

2SLS estimates of the output supply equation (43) are presented in Table 9. The choice of instruments is similar to the cow-calf model. The equation is estimated with and without seasonal (quarterly) dummy variables. Here expected price has a small but statistically significant positive impact on weight per animal (elasticities circa 1997 are 0.039 and 0.049 for the two equations). The time trend, which is included as a proxy for improvements in productivity, also has a significant positive impact. The measure of price variance Vp is insignificant. This may be because biological lags in production are much shorter at feedlots than at the cow-calf level, so sale prices can be forecasted more accurately by feedlot producers, i.e. there is less price uncertainty than is proxied by a time series variance in market prices. One of the three dummy variables (D3, for the third quarter) is significant. Note that R^2 (0.887, 0.865) is similar to the cow-calf output supply (see Table 1).

GMM results for output supply are reported in Table 10 (with 8 autocovariances). Results are somewhat similar to 2SLS (K is now significant in the equation with dummies). The over-identifying restrictions are not rejected using the J-test. The above results can be contrasted to a recent study by Marsh (1999b) of beef slaughter weights for the U.S. Assuming risk neutrality, weight per animal is regressed on expected prices, seasonal dummies, time trend, lagged weight per animal, and number of fed cattle produced. Expected output price is estimated to have a small but statistically significant negative impact on weight per animal, in contrast to the above Tables. Marsh notes that this impact is ambiguous in a static long-run equilibrium model. However his dynamic model perhaps is more similar to the short-run equilibrium model (conditional on K and ΔK) estimated here, and in this case weight per animal should be increasing in Ep (see the earlier discussion after equations 9a'- 10'). Another study estimates the elasticity of slaughter weight with respect to output price as 0.034 for Canada (Kulshreshtha and Wilson).

2SLS estimates of the Euler equation (44) for feeder cattle input K are presented in Tables 11 and 12 in the absence and presence of seasonal dummy variables, respectively. Euler equations are estimated with and without the $(Ep)^2$ term. In all cases expected output price Ep is insignificant. On the other hand, price variance Vp has a significant negative impact (in Table 11) on investment as proxied by changes in K. The coefficient of Z is insignificant. Coefficient estimates cannot meaningfully be interpreted as elasticities since the normalizing value of the dependent variable (ΔK) is not representative for the data set (the magnitude of ΔK in the fourth period of 1997 is (-) 13657, whereas the average magnitude for the four 1997 quarters is 47569). Thus coefficient estimates can be divided by approximately 4 to obtain elasticity estimates representative of 1997. One dummy variable (D2, for quarter 2) is significant. R^2 is extremely low in all cases, particularly without the dummy variables.

GMM estimates of (44) for feeder cattle input K are reported in Tables 13 and 14. Results are broadly similar to 2SLS. The over-identifying restrictions are not rejected. The above results for Euler equations suggest that changes in feeder input K provide a poor proxy for feedlot investment decisions, or that the investment process is not dynamic as modelled by the Euler equation. Consequently the ad hoc approximations (45a-b) to an Euler equation were also estimated. However results suggested that these approximations were no better (and perhaps worse) than the above model, so results for these alternative Euler equations are not reported here.

Results for CRRA Feedlot Model

The feedlot CRRA model can be specified as:

$$(46)\ y_t/K_{t-b} = \{\beta_{10} + \beta_{11}\, Ep^{*}_{\ t} + \beta_{13}\, K_{t-b} + \beta_{14}\, (K_{t-b}-K_{t-b-1}) + \beta_{15}\, W^{o*}_{\ t}$$

$$+ \beta_{16}\, Vp^{**}_{\ t} + \beta_{17}\, t\} \,/\, \{1 - c_1/2 - c_2\, (W^0+E\pi)/((Vp\ y^2)^{1/2})\}$$

$$(47)\ (K_{t-b}-K_{t-b-1}) =$$

$$-\{\gamma_{30} + \psi_{31}\, Ep^{*}_{\ t} + \gamma_1\, K_{t-b} + \psi_{35}\, W^{o*}_{\ t} + \psi_{36}\, Vp^{**}_{\ t} + \gamma_{37}\, t + \gamma_{38}\, (Ep^{*}_{\ t})^2$$

$$- \gamma_2\, K_{t-1-b}/(1+r) - \gamma_0\, w^k_{\ t-b}\, (1+r)^b/w^0_{\ t}\}$$

$$(48)\ (W^o_{\ t}+E\pi_t)/(Vp\ y^2)^{1/2}) = 1/c_2 - 2c_1/c_2 - \{\beta_{50} + \beta_{51}\, Ep^{*}_{\ t} + \beta_{53}\, K_{t-a}$$

$$+ \beta_{54}\, (K_{t-a}-K_{t-a-1}) + \beta_{55}\, W^{o*}_{\ t} + \beta_{56}\, Vp^{**}_{\ t} + \beta_{57}\, t\}/c_2$$

along with seasonal dummies (D1,D2,D3). Nonlinear OLS and 2SLS estimates of coefficients of the output supply equation (46) are presented in Table 15. These results were obtained in the same manner as for the output supply equation of the CRRA cow-calf model. Coefficient estimates of expected price Ep are less significant than in the CARA output supply equation (Table 9). This is not surprising given the difficulties in nonlinear estimation.

2SLS and GMM estimates of the Euler equation (47) with dummy variables are presented in Tables 16 and 17. Expected price is more significant here than in the CARA Euler equations. Indeed both expected price Ep and price variance Vp are significant (and with anticipated signs) in the GMM results. The over-identifying restrictions for the GMM models are not rejected. These results presumably are of more interest than the CARA Euler equation results, since the CRRA Euler equation is less restrictive and is as simple to estimate as the CARA Euler equation. Estimates of the wealth equation (48) are presented in Table 18. As in the case of the cow-calf wealth equation, many coefficients are insignificant. However the estimated coefficient of Ep as well as of Vp is significant for GMM.

Conclusion

This study is the first attempt to estimate a structural dynamic model of cow-calf production allowing for risk aversion. Risk aversion and biological lags have been incorporated into a discrete time calculus of variations dynamic model. A beef output supply equation and an Euler equation for investment in breeding herd have been specified assuming both linear and nonlinear mean-variance risk preferences. Unfortunately cow-calf output must be proxied by number of animals rather than by total weight, due to data limitations, and this seriously mis-specifies the short-run output supply equation. Models are estimated primarily by two stage least squares (2SLS) and generalized method of moments (GMM). At a general level results for the cow-calf models are consistent with theory. Output supply and investment are increasing in expected output price and decreasing in price variance, and the shadow price of capital is interpreted as increasing in expected price and decreasing in price variance. On the other hand, a symmetry restriction on Euler equations implied by dynamic maximization is rejected (we are unaware of other studies reporting this test).

This study also attempts to estimate a structural dynamic model of feedlot production under risk aversion. Since biological lags are much shorter in feedlot production than in cow-calf production, it is anticipated that dynamics is less important in feedlot production than in cow-calf production. Due to data limitations, feeder cattle input is used as a proxy for capital stock in feedlots, and so change in feeder cattle input is used as a proxy for feedlot investment. The short-run output supply (conditional on the proxies for capital stock and investment) is defined as slaughter weight per animal. The corresponding output supply and Euler equations are specified assuming linear and nonlinear mean-variance risk preferences, and are estimated primarily by 2SLS and GMM. Results for the feedlot output supply equation suggest that expected output price has a small positive impact on slaughter weight in the short-run, as is implied by theory. Price variance does not appear to have a significant impact on slaughter weight. Results for Euler equations suggest that feedlot investment decisions may be influenced by expected output price and price variance, but there are also indications that feeder cattle input level is a poor proxy for feedlot capital stocks.

FOOTNOTES.

1. The biological lags incorporated into this dynamic model can be viewed as an example of "time-to-build" lags in investment (Kydland and Prescott). These lags have typically been ignored in Euler equation models (see Oliner, Rudebusch and Sichel (1995) for an exception).

2. An Euler equation implies a much more complex decision rule or closed form investment equation (e.g. Blanchard; Fuhrer, Moore and Schuh), so it is standard procedure to estimate the Euler equation.

3. On the other hand, for a static long-run equilibrium problem (CARA, risk aversion) where K is endogenous, total weight y is increasing in Ep and decreasing in Vp, but weight per animal y/K is not necessarily increasing in Ep and decreasing in Vp. This ambiguity is mentioned in the risk-neutral case by Marsh (1999b).

4. See Fuhrer, Moore and Schuh for a summary of alternative normalization used in linear-quadratic inventory models. Obvious alternatives here are to solve (6) for K_{t-a} or K_{t-a-1}.

5. Note that the left hand side variable of the Euler equation (under this or alternative normalization) is closely related to right hand side variables that are alternative functions of K. Thus the R^2 for an Euler equation presumably reflects primarily this relation rather than the impact of other variables (Ep, Vp, w^k) on investment.

6. A low covariance between the endogenous variables and additional instruments would imply, in addition to low asymptotic efficiency, poor small sample properties for standard instrumental variable estimators (Nelson and Startz).

7. The symmetry restriction $\gamma_2 = 1$ in the non-normalized model is $\gamma_2 = (1+r)^{-1}\ \partial I/\partial K_1 = 1$. The normalized data is $\hat{I} = I / I_{97}$, $K|_1 = K_1 / K_{1,97}$, where $I_{97} = 229000$ and $K_{1,97} = 1742000$, and denote the coefficient in the corresponding regression model as $\gamma|_2 = (1+r)^{-1}\ \partial\hat{I}/\partial K|_1$. Then $\gamma|_2 = (1+r)^{-1}\ \partial I/\partial K_1\ (K_{1,97}/I_{97}) = \gamma_2\ 7.61$.

8. Limited information maximum likelihood (LIML) was also considered as an alternative to least squares methods. In contrast to least squares, LIML estimators are invariant to parameter normalization (i.e. choice of left hand side variable) and may have better finite sample properties. Although Shazam does not provide an algorithm for LIML, LIML estimates can be obtained by applying iterative SUR to a five equation model. The model consists of the output supply and Euler equations (35)-(36) and the reduced form equations for $K_{t-a}, K_{t-1-a}, K_{t-a}-K_{t-a-1}$ as specified by the choice of instruments (Davidson and MacKinnon; Pagan). LIML results are not reported here since the iterative process did not converge. However, at the last reported iteration, coefficient estimates were similar to 3SLS results.

9. One Monte Carlo study (Andrews 1991) suggests that, in the neighborhood of the optimal lag truncation, changes in the number of autocovariances has little effect on performance. Newey and West (1994) discuss a procedure for selecting autocovariances.

10. Normalization of variables (here by 1997 values) was necessary in order to estimate the GMM model using Shazam 8.0. Even though the model is linear in coefficients, the nonlinear algorithms in Shazam (which must be used with GMM) were unable to estimate the model using non-normalized data.

11. In terms of inference assuming a standard normal distribution, this would imply that the GMM standard errors in this case should be interpreted as double their nominal levels, i.e. nominal t-ratios should be halved.

12. Buhr and Kim also use feeder cattle input as a proxy for capital input in feedlots. However output is specified as total number of animals slaughtered in a quarter and is conditional on feeder cattle input, so Buhr and Kim model the extent to which feeder cattle input can be marketed in different quarters rather than modelling slaughter weights.

CHAPTER FOUR

CONCLUSION

The purpose of this thesis is to develop and estimate dynamic models of beef supply response, allowing for price uncertainty and risk aversion. Apparently this is the first study of dynamic beef supply response to attempt to incorporate risk aversion. Beef production is modeled as involving two main production stages, cow-calf and the feedlot, and risk aversion is incorporated into the beef supply response and investment models.

The first part of the thesis specified simple reduced form dynamic models, where autoregressive distributed lag (ADL) and polynomial distributed lag (PDL) models are estimated assuming distributed lags for variance of output price as well as for expected output price. ADL models are estimated for cow-calf output and investment, and for feedlot slaughter output. Consistent with economic theory, in all three cases, the sum of lagged coefficients for output price variance is negative and significant. As expected, the elasticity is much smaller than for the (positive) sum of lagged coefficients for expected price. These results seem to suggest that it is feasible and perhaps appropriate to incorporate price uncertainty and risk aversion into dynamic models.

The second part of the thesis develops and estimates structural dynamic models with price uncertainty and risk aversion. Risk aversion and comprehensive biological production lags were incorporated into a discrete time calculus of variations dynamic model. A beef output supply equation and an Euler equation for investment in breeding herd were specified assuming both linear and nonlinear mean-variance risk preferences. Results for the structural cow-calf models are consistent with economic theory. Output supply and investment are increasing in expected output price and decreasing in price variance, and the shadow price of capital is increasing in expected price and decreasing in price variance.

There are indications that dynamics is less important in feedlot production than in cow-calf production, simply because biological lags are much shorter in feedlot production. Results for the feedlot output supply equation suggest that expected output price has a positive impact on slaughter weight in the short-run, although price variance is not significant here. Results for Euler equations suggest that feedlot investment decisions are influenced by expected output price and price variance, consistent with economic theory.

Nevertheless this study has serious data limitations. Cow-calf output is correctly defined as total weight of calves, but data is only available for number of calves on-farm. Data on capital stocks for feedlots is unavailable, so feeder cattle placements are used as a proxy. Specification of models under nonlinear mean-variance risk preferences are limited by poor proxies for initial wealth and farm income. Quite simple measures of price expectations and uncertainty are used here, although experience for Western Canada suggests that more sophisticated rational expectations measures are less appropriate. Data on labor and feed use is unavailable, so demand equations for these inputs cannot be estimated.

There are obvious important extensions of this research that should be considered. First, the cow-calf and feedlot production decisions can be specified as a system rather than as separable, as in this study. Since the extent of backgrounding on-farm presumably varies with price expectations, the duration of the cow-calf and feedlot stages should be endogenous to the model. This suggests that cow-calf and feedlot production should be modelled jointly. Within this system, the primary investment decision can be modelled as an Euler equation for replacement heifers/breeding herd, and the primary output decision can be modelled as (an envelope/Hotelling's lemma relation for) a short-run slaughter weight output supply equation. Feeder cattle placements can be modelled as a short-run variable input decision (or by an Euler equation, to the extent that there is significant dynamics specific to feedlot production and capital stocks are adequately proxied by placements). A short-run output supply equation for calves on-farm would reflect, at most, part of backgrounding decisions.

Second, the dynamic specification in this and most other Euler equation models relies heavily upon the theory of dynamic costs of adjustment. However in recent years investment theory has been improved substantially by considerations of temporal uncertainty and irreversibility, which incorporate the option value of delaying investments in a world where information evolves over time. It is important to try to incorporate these advances into empirical Euler equation models, or into dynamic duality models based on optimal control.

Third, as the empirical specification of the dynamic model of beef production improves, the model should be applied to simulate effects of various policies. For example, application to the Canadian National Tripartite Stabilization Program should be relatively straightforward.

REFERENCES

Agriculture and Agri-Food Canada. Livestock Market Review. Red Meat Section, Market

Information Service, Animal Industry Division, Agriculture and Agri- Food Canada, Ottawa.

Andrews, D.W.K. "Heteroskedasticity and Autocorrelation Consistent Covariance Matrix Estimation." Econometrica 59(1991):817-58.

Andrews, D.W.K. "Consistent Moment Selection Procedures for Generalized Method of Moments Estimation." Econometrica 67(1999):543-64.

Antonovitz, F. and R. Green. "Alternative Estimates of Fed Beef Supply Response to Risk." Amer. J. Agr. Econ. 72(1990):475-87.

Askari, H. and J.T. Cummings. Agricultural Supply Response: A Survey of the Econometric Evidence. Praeger: New York, 1976.

Askari, H. and J.T. Cummings. "Estimating Agricultural Supply Response with the Nerlove Model: A Survey." Intern. Econ. Rev. 18(1977):257-92.

Berndt, E.R., M. Fuss and L. Waverman. "A Dynamic Model of Cost of Adjustment and Interrelated Factor Demands." Institute for Policy Analysis Working Paper 7925, University of Toronto, 1979.

Blanchard, O.J. "The Production and Inventory Behavior of the American Automobile Industry." J. Political Economy 91(1983):365-400.

Buhr, B.L. and H. Kim. "Dynamic Adjustment in Vertically Linked Markets: The Case of the U.S. Beef Industry." Amer. J. Agr. Econ. 79(1997):126-38.

Chavas, J.-P. and M.T. Holt. "Acreage Decisions Under Risk: The Case of Corn and Soybeans." Amer. J. Agr. Econ. 72(1990):529-38.

Coyle, B.T. "Risk Aversion and Price Risk in Duality Models of Production: A Linear Mean-Variance Approach." Amer. J. Agr. Econ. 74(1992):849-59.

Coyle, B.T. "Risk Aversion and Yield Uncertainty in Duality Models of Production: A Mean-Variance Approach." Amer. J. Agr. Econ. 81(1999): 553-67.

Cumby, R.E., J. Huizinga and M. Obstfeld. "Two-Step Two-Stage Least Squares Estimation in Models with Rational Expectations." J. Econometrics 21(1983):333-55.

Davidson, R. and J.G. MacKinnon. Estimation and Inference in Econometrics.Oxford Univ.Press: Oxford, 1993.

Diebold, F.X., L.E. Ohanian and J. Berkowitz. "Dynamic Equilibrium Economies: A Framework for Comparing Models and Data." Rev. Econ. Stud. 65(1998):433-51.

Epstein, L.G. "Duality Theory and Functional Forms for Dynamic Factor Demands." Rev. Econ. Stud. 48(1981):81-95.

Ferson, W.E. and S.R. Foerster. "Finite Sample Properties of the Generalized Method of Moments in Tests of Conditional Asset Pricing Models." J. Financial Econ. 36(1994):29-55.

Fleischman, C.A. "The GMM Parameter Normalization Puzzle." Division of Research and Statistics, Federal Reserve Board, Washington D.C., 1997.

Flood, R.P. and P.M. Garber. "A Pitfall in Estimation with Rational Expectations." J.Monetary Econ. 6(1980):433-35.

Fuhrer, J.C., G.R. Moore and S.D. Schuh. "Estimating the Linear-quadratic Inventory Model:Maximum Likelihood versus Generalized Method of Moments." J. Monetary Econ.35(1995):115-57.

Garber, P.M. and R.G. King. "Deep Structural Excavation? A Critique of Euler Equation Methods." NBER Technical Working Paper No. 31, 1983.

Ghysels, E. and A. Hall. "A Test for Structural Stability of Euler Conditions Parameters Estimated via the Generalized Method of Moments Estimator." Intern. Econ. Rev., 31(1990a):355-64.

Ghysels, E. and A. Hall. "Are Consumption-based Intertemporal Capital Asset Pricing Models Structural?" J. Econometrics 45(1990b):121-39.

Ghysels, E. and A. Hall. "Testing Non-Nested Euler Conditions with Quadrature- Based Methods of Approximation." J. Econometrics 46 (1990c): 273-308.

Hall, P. and J.L. Horowitz. "Bootstrap Critical Values for Tests Based on Generalized-Method-of-Moments Estimators." Econometrica 64(1996):891-916.

Hansen, L.P. "Large Sample Properties of Generalized Method of Moments Estimators." Econometrica 50(1982):1029-54.

Hansen, L.P., J. Heaton and A. Yaron. "Finite-Sample Properties of Some Alternative GMM Estimators." J. Bus. Econ. Stat. 14(1996):262-80.

Hansen, L.P. and K.J. Singleton. "Generalized Instrumental Variables Estimation of Nonlinear Rational Expectations Models." Econometrica 50(1982):1269-86.

Imbens, G.W. "One-Step Estimators for Over-Identified Generalized Method of Moments Models." Rev. Econ. Stud. 64(1997):359-83.

Jarvis, L.S. "Cattle as Capital Goods and Ranchers as Portfolio Managers: An Application to the Argentine Cattle Sector." J. Polit. Econ. 82(1974):489-520.

Kamien, M.I. and N.L. Schwartz. Dynamic Optimization. North-Holland: New York, 1981.

Kennan, J. "The Estimation of Partial Adjustment Models with Rational Expectations." Econometrica 47(1979):1441-55.

Kocherlakota, N.R. "On Tests of Representative Consumer Asset Pricing Models." J.Monetary Econ. 26(1990):285-304.

Kulshreshtha, S.N. "An Analysis of the Canadian Cattle Supply Using Polynomial Distributed Lags." Can. J. Agr. Econ. 24(1976):1-14.

Kulshreshtha, S.N. and A.G. Wilson. "An Open Econometric Model of the Canadian Beef Cattle Sector." Amer. J. Agr. Econ. 59(1972):84-91.

Kwiatkowski, D., P.C.B. Phillips, P. Schmidt and Y. Shin. "Testing the Null Hypothesis of Stationarity against the Alternative of a Unit Root: How Sure Are We that Economic Time Series Have a Unit Root?" J. Econometrics 54(1992):159-78.

Kydland, F.E. and E.C. Prescott. "Time to Build and Aggregate Fluctuations." Econometrica 50(1982):1345-70.

Lopez, R.E. "Supply Response and Investment in the Canadian Food Processing Industry."Amer. J. Agr. Econ. 67(1985):40-48.

Machina, M.J. "Temporal Risk and the Nature of Induced Preferences." J. Econ. Theory 33(1984):199-231.

Marsh, J.M. "Estimating Intertemporal Supply Response in the Fed Beef Market." Amer. J. Agr. Econ. 76(1994):444-53.

Marsh, J.M. "The Effects of Breeding Stock Productivity on the U.S. Beef Cattle Cycle."

Amer. J. Agr. Econ. 81(1999a):335-46.

Marsh, J.M. "Economic Factors Determining Changes in Dressed Weights of Live Cattle and Hogs". J Agr. Resource Econ. 24(1999b):313-26.

Mbaga, M. and B.T. Coyle. "Beef Supply Response Under Uncertainty: An ADL Model." University of Manitoba, 1999.

McCallum, B.T. "Rational Expectations and the Natural Rate Hypothesis: Some Consistent Estimates." Econometrica 44(1976):43-52.

Mortenson, D.T. "Generalized Costs of Adjustment and Dynamic Factor Demand Theory." Econometrica 41(1973):657-665.

Mundlak, Y. and H. Huang. "International Comparisons of Cattle Cycles." Amer. J. Agr. Econ. 78(1996):855-68.

Nerlove, M. and I. Fornari. "Quasi-rational Expectations, an Alternative to Fully Rational Expectations: An Application to U.S. Beef Cattle Supply." J. Econometrics 83(1998):129-61.

Nerlove, M., D.M. Grether and J.L. Carvalho. Analysis of Economic Time Series: A Synthesis. Academic Press: New York, 1979.

Nelson, C.R. and R. Startz. "Some Further Results on the Exact Small Sample Properties of the Instrumental Variable Estimator." Econometrica 58(1990a):967-76.

Nelson, C.R. and R. Startz. "The Distribution of the Instrumental Variables Estimator and Its t-Ratio When the Instrument is a Poor One." J. Business 63(1990b):S125-S140.

Newey, W.K. and K.D. West. "A Simple, Positive Semi-definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix." Econometrica 55(1987a):703-08.

Newey, W.K. and K.D. West. "Hypothesis Testing with Efficient Method of Moments Estimation." Intern. Econ. Rev. 28(1987b):777-87.

Newey, W.K. and K.D. West. "Automatic Lag Selection in Covariance Estimation." Rev. Econ. Studies. 61(1994):631-35.

Oliner, S.O., G.D. Rudebusch and D. Sichel. "New and Old Models of Business Investment: A Comparison of Forecasting Performance." J. Money, Credit and Banking 27(1995):806-26.

Oliner, S.O., G.D. Rudebusch and D. Sichel. "The Lucas Critique Revisited: Assessing the Stability of Empirical Euler Equations for Investment." J. Econometrics 70(1996):291-316.

Ospina, E. and C.R. Shumway. "Disaggregated Analysis of Short-run Beef Supply Response."Western J. Agr. Econ. 3(1979):43-59.

Pagan, A.R. "Some Consequences of Viewing LIML as an Iterated Aitken Estimator." Economics Letters 3(1979):369-72.

Rosen, S. "Dynamic Animal Economics." Amer. J. Agr. Econ. 69(1987):547-57.

Rosen, S., K.M. Murphy and J.A. Scheinkman. "Cattle Cycles." J. Polit. Econ. 102(1994):468-92.

Rucker, R.R., O.R. Burt and J.T. LaFrance. "An Econometric Model of Cattle Inventories."Amer. J. Agr. Econ. 66(1984):131-44.

Schmitz, J.D. "Dynamics of Beef Cow Herd Size: An Inventory Approach." Amer. J. Agr. Econ. 79(1997):532-42.

Shonkwiler, J.S. and S. Hinckley. "A Generalized Supply Response/Factor Demand Model and its Application to the Feeder Cattle Market." Western J. Agr. Econ. 10(1985):245-53.

Simon, H.A.. "Dynamic Programming under Uncertainty with a Quadratic Criterion Function." Econometrica 24(1956):74-81.

Smith, D.C. "Finite Sample Properties of Tests of the Epstein-Zin Asset Pricing Model." J. Econometrics 93(1999):113-48.

Statistics Canada a. Farm Input Price Indexes 62-004. Farm Income and Prices Div., Statistics Canada, Ottawa.

Statistics Canada b. Livestock and Animal Products Statistics 23-203. Agriculture Div., Livestock and Animal Products Section, Statistics Canada, Ottawa.

Theil, H. "A Note on Certainty Equivalence in Dynamic Planning." Econometrica 25(1957):346-49.

Statistics Canada c. catalogue #21-603 UPE.

Treadway, A.B. "The Globally Optimal Flexible Accelerator." J. Econ. Theory 7(1974):17-39.

White, H. "A Heteroskedasticity-consistent Covariance Matrix Estimator and a direct Test for Heteroskedasticity." Econometrica 48(1980):817-38.

White, K.J. "The Durbin-Watson Test for Autocorrelation in Nonlinear Models." Rev. Econ. Stat., 74(1992):370-73.

White, K.J. SHAZAM User's Reference Manual Version 8.0. McGraw-Hill, 1997.

Yver, R. "The Investment Behavior and the Supply Response of the Cattle Industry in Argentina." Ph.D. Dissertation, University of Chicago, 1971.

Appendix A.
Tables for chapter 1.

Table1. Cattle and Calf Inventory in Canada, by Provinces
July 1991 97.
(1,000 animals)

Province	1991	1992	1993	1994	1995	1996	1997
Alberta	4671	4811	4941	5316	5608	5666	5605
Saskatchewan	2279	2382	2484	2607	2838	2952	2885
Manitoba	1095	1167	1169	1236	1342	1424	1479
Prairie Provinces	8045	8360	8594	9159	9788	10042	9969
British Columbia	748	773	766	803	846	867	865
Western Cnd Total	8793	9133	9360	9962	10634	10909	10834
Ontario	2275	2172	2151	2188	2295	2299	2259
Quebec	1438	1385	1383	1443	1469	1486	1488
Central Provinces	3713	3557	3534	3631	3764	3785	3747
Maritime Provinces	337	335	333	331	332	332	332
Eastern Cnd Total	4050	3892	3867	3962	4096	4117	4079
Canada Total	12843	13025	13227	13924	14730	15026	14913

Source: Statistics Canada, Agriculture Division, Cat. No. 23-603.

Table 2: Cattle Slaughter in Canada, by Provinces and Regions 1991- 1997. (1,000 animals)

Province	1991	1992	1993	1994	1995	1996
Alberta	1246	1372	1435	1488	1537	1795
Saskatchewan & Manitoba	309	266	158	164	194	209
Prairie Provinces	1555	1638	1593	1652	1731	2004
British Columbia	65	65	59	51	51	52
Western Cnd Total	1620	1703	1652	1703	1782	2056
Ontario	579	720	648	635	632	684
Quebec	242	233	215	217	202	218
Central Provinces	821	953	863	852	834	902
Maritime Provinces	86	93	86	86	88	90
Eastern Cnd Total	907	1046	949	938	922	992
Canada Total	2527	2749	2601	2641	2704	3048

Source: Statistics Canada, Agriculture Division, Cat. No.23-603.

Table 3: Distribution of Cattle Inventories, by Provinces and Regions
July 1, 1991-1997
(Percent)

Province	1991	1992	1993	1994	1995	1996	1997
Alberta	36	37	37	38	38	38	38
Saskatchewan	18	18	19	19	19	19	19
Manitoba	8	9	9	9	9	9	10
Prairie Provinces	62	64	65	66	66	66	67
British Columbia	6	6	6	6	6	6	6
Western Cnd Total	68	70	71	72	72	72	73
Ontario	18	17	16	16	16	16	15
Quebec	11	11	11	10	10	10	10
Central Provinces	29	28	27	26	26	26	25
Maritime Provinces	3	2	2	2	2	2	2
Eastern Cnd Total	32	30	29	28	28	28	27
Canada Total	100	100	100	100	100	100	100

Source: Statistics Canada, Agriculture Division, Cat. No.23-603.

Table 4: Distribution of Beef Cows in Canada, by Provinces and Regions
July 1, 1991-97.
(Percent)

Province	1991	1992	1993	1994	1995	1996	1997
Alberta	43	42	42	43	44	42	42
Saskatchewan	23	24	24	23	23	25	25
Manitoba	11	11	11	11	11	11	12
Prairie Provinces	77	77	77	77	78	78	79
British Columbia	6	6	6	6	6	6	6
Western Cnd Total	83	83	83	83	84	84	85
Ontario	10	10	10	10	10	9	9
Quebec	5	5	5	5	5	5	5
Central Provinces	15	15	15	15	15	14	14
Maritime Provinces	2	2	2	2	1	2	1
Eastern Cnd Total	17	17	17	17	16	16	15
Canada Total	100	100	100	100	100	100	100

Source: Statistics Canada, Agriculture Division, Cat. No.23-603.

Table 5: Cattle Slaughter Distribution, in Canada, by Provinces and regions 1991- 1997. (Percent)

Province	1991	1992	1993	1994	1995	1996
Alberta	49	50	55	56	57	59
Saskatchewan & Manitoba	12	10	6	6	7	7
Prairie Provinces	61	60	61	62	64	66
British Columbia	3	2	2	2	2	2
Western Cnd Total	64	62	63	64	66	68
Ontario	23	26	25	24	23	22
Quebec	10	9	8	8	7	7
Central Provinces	33	35	33	32	31	29
Maritime Provinces	3	3	3	4	3	3
Eastern Cnd Total	36	38	36	36	34	32
Canada Total	100	100	100	100	100	100

Source: Statistics Canada, Agriculture Division, Cat. No.23-603.

Table 6: Beef Cow Inventories in Canada, by Provinces and Regions
July 1991-97.
(1,000 animals)

Province	1991	1992	1993	1994	1995	1996	1997
Alberta	1635	1667	1760	1917	2050	2023	1970
Saskatchewan	897	960	990	1035	1105	1180	1170
Manitoba	411	436	439	476	500	540	560
Prairie Provinces	2943	3063	3189	3428	3655	3743	3700
British Columbia	243	254	250	264	270	278	270
Western Cnd Total	3186	3317	3439	3692	3925	4021	3970
Ontario	390	413	425	460	469	440	435
Quebec	188	198	220	223	225	230	240
Central Provinces	578	611	645	683	694	670	675
Maritime Provinces	64	65	67	68	71	72	72
Eastern Cnd Total	642	676	712	751	765	742	747
Canada Total	3828	3993	4151	4443	4690	4763	4717

Source: Statistics Canada, Agriculture Division, Cat. No.23-603.

Table 7: Feeder Steers: Prices in Alberta, by quarter 1992-1996. (Canadian dollars per 100 pounds)

Year	Jan. - Mar.	Apr. - Jun.	Jul. - Sept.	Oct. - Dec.
1992	86.74	88.52	93.52	95.50
1993	101.97	102.58	108.52	106.97
1994	109.02	104.20	107.84	100.51
1995	99.34	90.34	88.76	86.30
1996	70.22	70.99	83.21	81.89

Source: Livestock Market Review, 1996, Table 12.

Table 8: Slaughter Steers: Direct Sales Prices in Alberta, by quarter 1992-1996 (Canadian dollars per 100 pounds)

Year	Jan. - Mar.	Apr. - Jun.	Jul. - Sept.	Oct. - Dec.
1992	78.69	80.81	80.69	85.78
1993	95.93	92.98	89.35	87.88
1994	90.55	86.07	83.74	86.00
1995	92.64	81.32	78.71	81.42
1996	76.69	72.62	82.70	83.96

Source: Livestock Market Review, 1996, Table 12

Appendix B.

Tables for chapter 2.

Table 1. Cow - calf Output Supply Response

		A. ADL (1,5): OLS		B. ADL(1,5) + PDL(5,3): Auto (GS,ML)	
variable	lag	coeff	t-ratio [a]	coeff	t-ratio [b]
y	1	.6428	4.78	.8065	10.27
Ep	5	.1172	1.10	.1042	2.05
	6	.1884	1.83	.0287	0.92
	7	-.2172	2.05	.0013	0.06
	8	.3040	3.07	.0040	0.29
	9	-.2088	2.56	.0190	0.80
	10	.1476	2.62	.0284	0.92
Vp	5	.0016	0.26	-.0017	0.49
	6	.0186	2.78	-.0072	2.39
	7	.0089	1.16	-.0035	1.56
	8	-.0189	2.50	.0025	1.10
	9	.0262	3.77	.0039	1.45
	10	-.0163	2.88	-.0059	1.79
constant		5.03	2.64	2.71	2.44
R^2		.9803		.9755	
rho(GS,ML)		-.23	1.37	-.63	4.73
Durbin-h(OLS)		-1.59		-.3.06	
Sum of lag coefficients:					
$\sum$ EP		.3313	2.52	.1857	2.45
$\sum$ Vp		-.0171	3.16	-.0120	3.96

a. 20 degrees of freedom

b. 23 degrees of freedom

Table 2. Cow-calf Investment (Replacement Heifers) Equation

A. PDL (8,3): OLS B. ADL (1, 4): OLS C.ADL(1,4)+PDL(4,2): OLS

variable	lag	coeff	t-ratio[a]	coeff	t-ratio[b]	coeff	t-ratio[c]
y	1	----		.6112	4.54	.6318	5.37
Ep	0	.4561	4.06	.3809	2.18	.4845	3.98
	1	.2869	6.52	.2783	1.27	.0702	1.82
	2	.1071	1.76	-.2127	0.96	-.1248	1.81
	3	.0250	0.43	-.1767	0.81	-.1005	2.49
	4	-.0109	0.28	.2197	1.46	.1431	1.32
	5	-.0126	0.31	----		----	
	6.	.0084	0.18	----		----	
	7	.0403	1.11	----		----	
	8	.0714	0.75	----		----	
Vp	0	-.0006	0.06	-.0152	0.96	-.0270	2.39
	1	-.0076	1.73	-.0180	0.96	-.0015	0.30
	2	-.0081	1.55	.0105	0.51	.0087	1.22
	3	-.0049	0.99	.0144	0.73	.0037	0.83
	4	-.0007	0.16	-.0235	1.70	-.0165	1.83
	5	.0017	0.36	----		----	
	6	-.0005	0.11	----		----	
	7	-.0101	3.09	----		----	
	8	-.0297	3.45	----		----	
C		1.853	12.87	.9549	3.44	.9291	3.93
constant		-13.98	6.76	-8.583	3.18	-8.479	3.60
D		-.4393	19.13	-.5946	13.60	-.5996	15.01
R^2		.9633		.9548		.9503	
rho (GS,ML)		-.16	0.97	-.10	0.63	-.24	1.56
Durbin-Watson		2.15		----		----	
Durbin-h		----		-0.167		-0.627	
Sum of lag coefficients:							
$\sum$ Ep		0.9317	4.14	0.4895	3.28	0.4724	3.41
$\sum$ Vp		-0.0604	4.93	-0.0313	3.08	-0.0325	3.37

a. 25 degrees of freedom b. 26 degrees of freedom. c. 30 degrees of freedom

Table 3. Feedlot Output Supply Response

		A. ADL (1,13): OLS		B. ADL (1, 13) + PDL (13, 8): Auto (GS, ML)	
variable	lag	coeff	t-ratio[a]	coeff	t-ratio[b]
y	1	.7213	6.93	.7685	11.88
Ep	5	-.0263	0.11	-.1187	0.82
	6	.0753	0.29	.2044	1.28
	7	.1838	0.75	.1849	1.44
	8	.2143	0.84	.0112	0.11
	9	-.1710	0.68	-.0419	0.37
	10	.3305	1.22	.0697	0.80
	11	.0851	0.28	.2127	2.10
	12	.0401	0.14	.2477	2.41
	13	.6312	2.14	.1571	1.50
	14	.0714	0.24	.0479	0.38
	15	-.0611	0.21	.0369	0.30
	16	.1834	0.59	.1088	0.69
	17	.0471	0.17	.1251	0.76
	18	.4731	2.05	.2437	1.74
Vp	5	-.0358	1.38	-.0053	0.32
	6	-.0004	0.01	-.0336	2.46
	7	-.0225	0.80	-.0076	0.96
	8	-.0154	0.66	.0042	0.62
	9	.0109	0.52	-.0045	0.77
	10	-.0228	1.13	-.0165	3.01
	11	-.0327	1.51	-.0184	2.86
	12	-.0012	0.06	-.0116	1.77
	13	-.0243	1.17	-.0079	1.28
	14	-.0074	0.34	-.0156	2.17
	15	-.0625	2.58	-.0254	2.77
	16	.0304	1.27	-.0107	1.17
	17	.0022	0.09	.0305	2.37
	18	-.0148	0.83	-.0213	1.97

Table 3 (concluded)

w^c	5	-.5107	2.28	-.3235	2.48
	6	.7486	2.49	.3775	2.01
	7	-.4005	1.37	-.1321	1.22
	8	.0463	0.15	-.0343	0.34
	9	.0038	0.01	.1581	1.59
	10	.1499	0.49	.0481	0.50
	11	-.1195	0.42	-.1954	2.08
	12	-.3236	1.14	-.2338	2.68
	13	-.1055	0.38	-.0096	0.13
	14	.1870	0.66	.1752	1.86
	15	-.1090	0.38	.0156	0.16
	16	.0193	0.07	-.3031	2.44
	17	-.5008	2.12	-.1405	1.02
	18	.0342	0.17	.0125	0.09
constant		2.721	2.22	2.349	3.13
D1		.1516	3.35	.0968	2.82
D2		.2177	4.76	.1954	7.26
D3		.1206	2.92	.0587	1.95
R^2		.9671		.9359	
rho (GS, ML)		.19	1.62	-.45	4.21
Durbin-h (OLS)		2.145		-1.572	
Sum of lag coefficients:					
$\sum$ Ep		2.0768	1.97	1.4894	2.26
$\sum$ Vp		-0.1962	2.18	-0.1432	2.52
$\sum w^c$		-0.8805	1.48	-0.5853	1.61

a. 23 degrees of freedomb. 37 degrees of freedom

Appendix C.

Tables for chapter 3.

Table 1. Two Stage Least Squares (2SLS) Estimates of Linear Mean-Variance Cow-Calf Model

	Output Supply (y_t)		Investment ($K_{t-a} - K_{t-a-1}$)	
variable	coef	t-ratio	coef	t-ratio
constant	-0.4573	5.94	-0.9643	3.18
Ep^*_t	0.1164	4.74	0.1680	1.46
Vp^*_t	-0.0378	4.00	-0.1105	2.53
K_{t-a}	1.5323	16.58	7.9632	7.40
$K_{t-a} - K_{t-a-1}$	-0.0507	1.78	-----	
$K_{t-a-1}/(1+r)$	-----		-6.3967	8.28
Z_t	-----		0.2202	1.55
DW	1.27		1.01	
rho	0.326		0.492	
R^2	0.8872		0.8646	

$$Z_t = \{w^k_{t-a-s}(1+r)^{a-s} + w^k_{t-a-s+1}(1-\delta)(1+r)^{a-s-1}\}/w^o_t$$

All variables are normalized by 1997 (second half) values.

Table 2. Seemingly Unrelated Regressions (SUR) Estimates of Linear Mean-Variance Cow-Calf Model

	Output Supply (y_t)		Investment ($K_{t-a} - K_{t-a-1}$)	
variable	coef	t-ratio	coef	t-ratio
constant	-0.4850	6.58	-1.2186	4.49
Ep^*_t	0.1246	5.07	0.3881	3.80
Vp^*_t	-0.0432	4.64	-0.1913	4.94
K_{t-a}	1.5632	17.90	8.5155	15.68
$K_{t-a} - K_{t-a-1}$	-0.1259	8.12	-----	
$K_{t-a-1}/(1+r)$	-----		-6.3082	14.79
Z_t	-----		-0.2176	2.63
DW	0.692		0.789	
rho	0.619		0.566	
R^2	0.8864		0.8378	

Breusch - Pagan LM test for diagonal covariance matrix: $\chi^2 = 13.57$ (1 df)

$$Z_t = \{w^k_{t-a-s}(1+r)^{a-s} + w^k_{t-a-s-1}(1-\delta)(1+r)^{a-s-1}\}/w^o_t$$

All variables are normalized by 1997 (second half) values.

Table 3. Three Stage Least Squares (3SLS) Estimates of Linear Mean-Variance Cow-Calf Model

	Output Supply (y_t)		Investment ($K_{t-a} - K_{t-a-1}$)	
variable	coef	t-ratio	coef	t-ratio
constant	0.4592	6.30	-1.3648	3.78
Ep^*_t	0.1190	5.15	0.4439	3.70
Vp^*_t	-0.0397	4.48	-0.2164	4.76
K_{t-a}	1.533	17.51	10.518	9.58
$K_{t-a} - K_{t-a-1}$	-0.0768	3.38	-----	
$K_{t-a-1}/(1+r)$	-----		-7.9635	10.14
Z_t	-----		-0.2424	2.32
DW	0.772		1.002	
rho	0.574		0.452	
R^2	0.8985		0.8398	

Breusch - Pagan LM test for diagonal covariance matrix: $\chi^2 = 3.35$ (1 df)

$Z_t = \{w^k_{t-a-s}(1+r)^{a-s} + w^k_{t-a-s-1}(1-\delta)(1+r)^{a-s-1}\}/w^o_t$

All variables are normalized by 1997 (second half) values.

Table 4. Generalized Methods of Moments (GMM) Estimates of Linear Mean-Variance Cow-Calf Model

	Output Supply (y_t)		Investment (K_{t-a}- K_{t-a-1})	
variable	coef	t-ratio	coef	t-ratio
constant	-0.4554	16.43	-1.2670	8.59
Ep^*_t	0.1213	12.15	0.3650	9.18
Vp^*_t	-0.0404	19.72	-0.1838	22.99
K_{t-a}	1.5236	40.92	9.8190	54.47
K_{t-a}- K_{t-a-1}	-0.0926	39.68	-----	
$K_{t-a-1}/(1+r)$	------		-7.5460	43.64
Z_t	-----		-0.0979	5.08
DW	0.605		0.888	
rho	0.658		0.535	
R^2	0.8991		0.8530	

J - test of overidentifying restrictions: $\chi^2 = 4.849$ (17 df)

$$Z_t = \{w^k_{t-a-s}(1+r)^{a-s} + w^k_{t-a-s+1}(1-\delta)(1+r)^{a-s-1}\}/w^o_t$$

All variables are normalized by 1997 (second half) values.

Table 5. Coefficient Estimates of Output Supply for Nonlinear Mean-Variance Cow-Calf Model

	A NLS		B NL2SLS	
coefficients	estimate	t-ratio	estimate	t-ratio
B_{10}(constant)	-0.1252	1.30	-0.0831	0.94
$B_{11}(Ep^{*}_{t})$	0.1195	2.54	0.1173	3.18
$B_{16}(Vp^{*}_{t})$	-0.0306	2.69	-0.0334	3.71
$B_{15}(W_{o}{}^{*}_{t})$	-0.2030	2.25	-0.2012	2.65
$B_{13}(K_{t-a})$	1.3954	2.64	1.3075	3.41
$B_{14}(K_{t-a}-K_{t-a-1})$	-0.0971	2.56	-0.0942	3.22
C_1	-0.0357	0.05	0.0579	0.10
C_2	-0.0048	2.26	-0.0088	3.98
DW	1.17		1.42	
rho	0.245		0.052	
R^2	0.9496		0.9555	

All variables are normalized by 1997 (second half) values.

Table 6. 2SLS Estimates of Investment and Wealth Equations for Nonlinear Mean-Variance Cow-Calf Model

	Investment (K_{t-a} - K_{t-a-1})		Wealth ($((W_{ot}+E\pi_t)/V\pi_t ½)$)	
variable	coef	t-ratio	coef	t-ratio
constant	-0.4523	1.18	14.826	1.81
Ep^*_t	0.2205	1.59	-0.1610	0.09
Vp^{**}_t	-0.0740	1.80	-1.2068	2.64
$W^*_{o\,t}$	-0.4292	1.71	-2.0830	0.39
K_{t-a}	8.0962	16.25	-10.7550	1.77
K_{t-a}- K_{t-a-1}	-----		-0.0232	0.02
$K_{t-a-1}/(1+r)$	-6.6860	15.46	-----	
Z_t	0.1989	0.76	-----	
DW	0.81		1.32	
rho	0.570		0.335	
R^2	0.8895		0.2460	

$$Z_t = \{w^k_{t-a-s}(1+r)^{a+s} + w^k_{t-a-s-1}(1-\delta)(1+r)^{a-s-1}\}/w^o_t$$

All variables are normalized by 1997 (second half) values.

Table 7. 3SLS Estimates of Investment and Wealth Equations for Nonlinear Mean-Variance Cow-Calf Model

	Investment $(K_{t\text{-}a} - K_{t\text{-}a\text{-}1})$		Wealth $((W_{ot}+E\pi_t)/V\pi_t½)$	
variable	coef	t-ratio	coef	t-ratio
constant	-0.4476	1.17	15.408	1.89
Ep^{*}_{t}	0.2428	1.79	-0.2028	0.11
Vp^{**}_{t}	-0.0813	2.02	-1.1911	2.61
$W^{*}_{o\,t}$	-0.4244	1.69	-1.9227	0.36
$K_{t\text{-}a}$	8.1742	16.50	-11.738	1.93
$K_{t\text{-}a} - K_{t\text{-}a\text{-}1}$	-----		0.4750	0.37
$K_{t\text{-}a\text{-}1}/(1+r)$	-6.7299	15.63	-----	
Z_t	0.1488	0.59	-----	
DW	0.79		1.31	
rho	0.577		0.344	
R^2	0.8895		0.2430	

Breusch - Pagan LM test for diagonal covariance matrix: $\chi^2 = 2.83$ (1 df)

$Z_t = \{w^k_{t\text{-}a\text{-}s}(1+r)^{a-s} + w^k_{t\text{-}a\text{-}s\text{-}1}(1\text{-}\delta)(1+r)^{a-s-1}\}/w^o_t$

All variables are normalized by 1997 (second half) values.

Table 8. GMM Estimates of (Separate) Investment and Wealth Equations for Nonlinear Mean-Variance Cow-Calf Model

	Investment $(K_{t-a} - K_{t-a-1})$		Wealth $((W_{ot}+E\pi_t)/V\pi_t^{1/2})$	
variable	coef	t-ratio	coef	t-ratio
constant	-0.3116	0.86	14.548	4.25
Ep^{*}_{t}	0.4396	6.27	0.1167	0.29
Vp^{**}_{t}	-0.1477	5.24	-1.2426	7.97
$W^{*}_{o\,t}$	-0.4002	2.91	-1.5331	0.79
K_{t-a}	8.1441	36.24	-11.643	4.85
$K_{t-a} - K_{t-a-1}$	-----		0.5003	0.86
$K_{t-a-1}/(1+r)$	-6.4763	31.71	-----	
Z_t	-0.3578	1.71	-----	
DW	0.87		1.31	
rho	0.511		0.344	
R^2	0.8776		0.2421	

J-test of overidentifying restrictions: $\chi^2 =$ 4.789 (10 df) 3.529 (11 df)

$$Z_t = \{w^k_{t-a-s}(1+r)^{a-s} + w^k_{t-a-s-1}(1-\delta)(1+r)^{a-s-1}\}/w^o_t$$

All variables are normalized by 1997 (second half) values.

Table 9. 2SLS Estimates of Output Supply (Weight/Animal) Equation for Linear Mean-Variance Feedlot Model

	no seasonal dummies		seasonal dummies	
variable	coef	t-ratio	coef	t-ratio
Constant	0.6844	23.75	0.6643	17.12
Ep^*_t	0.0388	2.46	0.0491	2.88
Vp^*_t	0.0002	0.64	-0.0001	0.60
K_{t-b}	-0.0065	0.14	0.0523	1.11
$K_{t-b} \cdot K_{t,b-1}$	-0.0009	0.67	-0.0030	2.23
t	0.0026	7.26	0.0022	7.05
D1	-----		-0.0000	0.13
D2	-----		0.0049	0.72
D3	-----		-0.0278	4.82
DW	1.64		1.53	
rho	0.151		0.218	
R^2	0.8522		0.8903	

All variables are normalized by 1997 (fourth quarter) values.

Table 10. GMM Estimates of Output Supply (Weight/Animal) Equation for Linear Mean-Variance Feedlot Model

	no seasonal dummies		seasonal dummies	
variable	coef	t-ratio	coef	t-ratio
constant	0.6814	32.87	0.6976	18.64
Ep^*_t	0.0389	2.96	0.0336	2.15
Vp^*_t	0.0002	1.39	-0.0002	1.00
K_{t-b}	-0.0121	0.38	0.0644	2.24
$K_{t-b} - K_{t-b-1}$	-0.0006	0.83	-0.0029	4.76
t	0.0027	9.64	0.0021	10.85
D1	-----		-0.0000	1.06
D2	-----		0.0059	1.41
D3	-----		-0.0287	8.08
DW	1.57		1.51	
rho	0.187		0.227	
R^2	0.8518		0.8908	

J-test of over-identifying restrictions χ^2 = 7.692 (6 df) 6.543 (6 df)

All variables are normalized by 1997 (fourth quarter) values.

Table 11. 2SLS Estimates of Investment (Feeder Cattle Input) Equations for Linear Mean-Variance Feedlot Model: No Seasonal Dummy Variables

	With Ep^2		Without Ep^2	
variable	coef	t-ratio	coef	t-ratio
constant	-30.403	1.72	-8.5382	1.86
Ep^*_t	37.352	1.44	2.9454	0.86
Ep^{*2}_t	-14.19	1.37	-----	
Vp^*_t	-0.0652	1.97	-0.0772	2.10
K_{t-b}	21.179	2.67	20.215	2.49
$K_{t-b-1}/(1+r)$	1.2744	1.08	1.1540	0.96
Z_t	-0.3066	0.29	-0.4976	0.46
t	-0.1312	2.31	-0.1368	2.19
DW	1.47		1.43	
rho	0.253		0.276	
R^2	0.1194		0.0957	

$$Z_t \equiv w^k_{t-b}(1+r)^b / w^o_t$$

All variables are normalized by 1997 (fourth quarter) values.

Table 12. 2SLS Estimates of Investment (Feeder Cattle Input) Equations for Linear Mean-Variance Feedlot Model: Seasonal Dummy Variables

	With Ep^2		Without Ep^2	
variable	coef	t-ratio	coef	t-ratio
constant	-7.1681	0.49	1.2489	0.25
Ep^{*}_{t}	14.429	0.62	1.3241	0.47
Ep^{*2}_{t}	-5.4411	0.57	-----	
Vp^{*}_{t}	-0.0507	1.68	-0.0584	1.71
K_{t-b}	13.704	1.83	14.005	1.86
$K_{t-b-1}/(1+r)$	0.7362	0.66	0.7174	0.66
Z_t	-1.1218	1.21	-1.2288	1.30
t	-0.0752	1.60	-0.0826	1.59
D1	-0.0000	1.30	-0.0000	1.32
D2	2.1690	2.81	2.1467	2.74
D3	-0.8476	1.06	-0.9596	1.24
DW	2.01		1.95	
rho	-0.021		0.012	
R^2	0.3162		0.3022	

$$Z_t \equiv w^k_{t-b}(1+r)^b/w^o_t$$

All variables are normalized by 1997 (fourth quarter) values.

Table 13. GMM Estimates of Investment (Feeder Cattle Input) Equations for Nonlinear Mean-Variance Cow-Calf Model: No Seasonal Dummy Variables

	With Ep^2		Without Ep^2	
variable	coef	t-ratio	coef	t-ratio
constant	-23.824	1.87	-7.4951	1.68
Ep^*_t	29.186	1.79	3.4941	1.23
Ep^{*2}_t	-10.369	1.62	-----	
Vp^*_t	-0.0447	2.09	-0.0495	1.60
K_{t-b}	15.000	1.62	14.166	1.51
$K_{t-b-1}/(1+r)$	0.4662	0.47	0.3204	0.31
Z_t	-0.0848	0.54	-0.8454	0.57
t	0.0854	1.32	-0.0867	1.20
DW	1.76		1.75	
rho	0.112		0.118	
R^2	0.1339		0.1145	

J-test of overidentifying restrictions: $\chi^2 =$ 6.231 (4 df) 6.166 (4 df)

$Z_t \equiv w^k_{t-b}(1+r)^b/w^o_t$

All variables are normalized by 1997 (fourth quarter) values.

Table 14. GMM Estimates of Investment (Feeder Cattle Input) Equations for Linear Mean-Variance Feedlot Model: Seasonal Dummy Variables

	With Ep^2		Without Ep^2	
variable	coef	t-ratio	coef	t-ratio
constant	-4.5452	0.54	1.2659	0.39
Ep^{*}_{t}	9.7842	0.81	1.0300	0.61
Ep^{*2}_{t}	-3.6243	0.68	-----	
Vp^{*}_{t}	-0.0483	3.38	-0.0539	3.20
K_{t-b}	13.446	2.49	13.718	2.70
$K_{t-b-1}/(1+r)$	0.4753	0.67	0.4552	0.66
Z_t	-0.7156	0.90	-0.7904	0.99
t	-0.0792	2.53	-0.0838	2.54
D1	-0.0000	1.69	-0.0000	1.70
D2	2.0025	2.40	1.972	2.48
D3	0.8898	2.21	-0.9578	2.74
DW	1.93		1.89	
rho	0.021		0.045	
R^2	0.2971		0.2869	

J-test of overidentifying restrictions: χ^2 = 7.145 (4 df) 7.066 (4 df)

$Z_t \equiv w^k_{t-b}(1+r)^b/w^o_t$

All variables are normalized by 1997 (fourth quarter) values.

Table 15. Coefficient Estimates of Output Supply (Weight/Animal) for Nonlinear Mean-Variance Feedlot Model: Seasonal Dummy Variables

	A		B	
	NLS		NL2SLS	
coefficients	estimate	t-ratio	estimate	t-ratio
B_{10} (constant)	0.5112	2.50	0.4131	1.92
B_{11} (Ep^{*}_{t})	0.0314	1.52	0.0176	1.03
B_{16} (Vp^{**}_{t})	0.0022	0.36	0.0069	1.10
B_{15} ($W_{o}{}^{*}_{t}$)	0.0137	0.50	0.0102	0.46
B_{13} (K_{t-a})	0.0210	0.68	0.0035	0.14
B_{14} (K_{t-b} - K_{t-b-1})	0.0003	0.27	0.0002	0.18
B_{17}(t)	0.0017	2.42	0.0014	1.85
d_{11} (D1)	0.0018	0.28	0.0039	0.88
d_{12} (D2)	-0.0166	1.73	-0.0114	1.43
d_{13} (D3)	0.0038	0.58	0.0024	0.48
C_1	0.4445	0.72	0.7727	1.19
C_2	0.0153	2.27	0.0295	1.85
DW	1.32		1.20	
rho	0.294		0.3657	
R^2	0.8965		0.8985	

All variables are normalized by 1997 (fourth quarter) values.

Table 16. 2SLS Estimates of Investment (Feeder Cattle Input) Equation for Nonlinear Mean-Variance Feedlot Model: Seasonal Dummy Variables

	With Ep^2		Without Ep^2	
variable	coef	t-ratio	coef	t-ratio
constant	-22.489	1.32	1.1753	0.30
Ep^{*}_{t}	50.073	1.72	8.4174	2.60
Ep^{*2}_{t}	-16.492	1.44	-----	
Vp^{**}_{t}	-1.077	1.19	-1.23	1.49
$W^{*}_{o\,t}$	-20.378	2.90	-14.885	2.76
K_{t-b}	15.117	1.74	12.812	1.65
$K_{t-b-1}/(1+r)$	19.402	1.95	12.217	1.55
Z_t	-1.0287	1.06	-1.1863	1.34
t	-0.2414	3.15	-0.1814	3.09
D1	1.7859	1.61	1.6652	1.65
D2	-0.9342	0.75	-1.6582	1.61
D3	-1.3272	1.36	-1.8030	2.15
DW	1.58		1.78	
rho	0.192		0.085	
R^2	0.3496		0.3967	

$Z_t \equiv w^{k}_{t-b}(1+r)^{b}/w^{o}_{t}.$

All variables are normalized by 1997 (fourth quarter) values.

Table 17. GMM Estimates of Investment (Feeder Cattle Input) Equation for Nonlinear Mean-Variance Feedlot Model: Seasonal Dummy Variables

	With Ep^2		Without Ep^2	
variable	coef	t-ratio	coef	t-ratio
constant	-19.013	2.04	2.0145	1.21
Ep^*_t	46.100	2.96	9.4420	4.22
Ep^{*2}_t	-14.403	2.34	-----	
Vp^{**}_t	-1.4387	2.72	-1.4074	3.27
$W^*_{o\,t}$	-21.454	6.47	16.879	4.82
K_{t-b}	17.594	2.63	14.534	2.99
$K_{t-b-1}/(1+r)$	18.432	3.61	12.678	3.01
Z_t	-1.2839	1.41	-1.2768	1.79
t	-0.2614	7.38	-0.2046	5.94
D1	1.6208	2.01	1.6939	2.54
D2	-1.3538	1.85	-1.8661	3.93
D3	-1.4658	2.86	-1.850	5.09
DW	1.40		1.62	
rho	0.284		0.164	
R^2	0.3443		0.3835	

J-test of over-identifying restrictions: $\chi^2 = 7.061$ (10 df) 7.320 (11 df)

$Z_t \equiv w^k_{t-b}(1+r)^b/w^o_t.$

All variables are normalized by 1997 (fourth quarter) values.

Table 18. Estimates of Wealth Equation for Nonlinear Mean-Variance Feedlot Model: No Seasonal Dummies

	2SLS		GMM	
variable	coef	t-ratio	coef	t-ratio
constant	-0.4967	1.35	-0.3217	0.95
Ep^*_t	0.2456	1.18	0.3199	2.96
Vp^*_t	-0.3119	4.25	-0.2712	4.70
$W_o{}^*_t$	0.3573	0.70	0.0843	0.24
K_{t-b}	0.1590	0.24	0.2707	0.92
K_{t-b}- K_{t-b-1}	0.0291	2.05	0.0209	2.29
t	0.0043	0.81	0.0025	0.96
DW	1.48		1.43	
rho	0.243		0.265	
R^2	0.3555		0.3745	

J-test of over-identifying restrictions: $\chi^2 = 5.200$ (9 df)

The dependent variable is $(W_{ot}+E\pi_t)/V\pi_t{}^{1/2}$

All variables are normalized by 1997 (fourth quarter) values.