

AN INVESTIGATION OF DIAGONAL TENSIONS  
IN REINFORCED CONCRETE BEAMS

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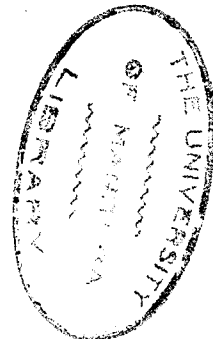
A Thesis  
Presented to  
the Faculty of Graduate Studies and Research  
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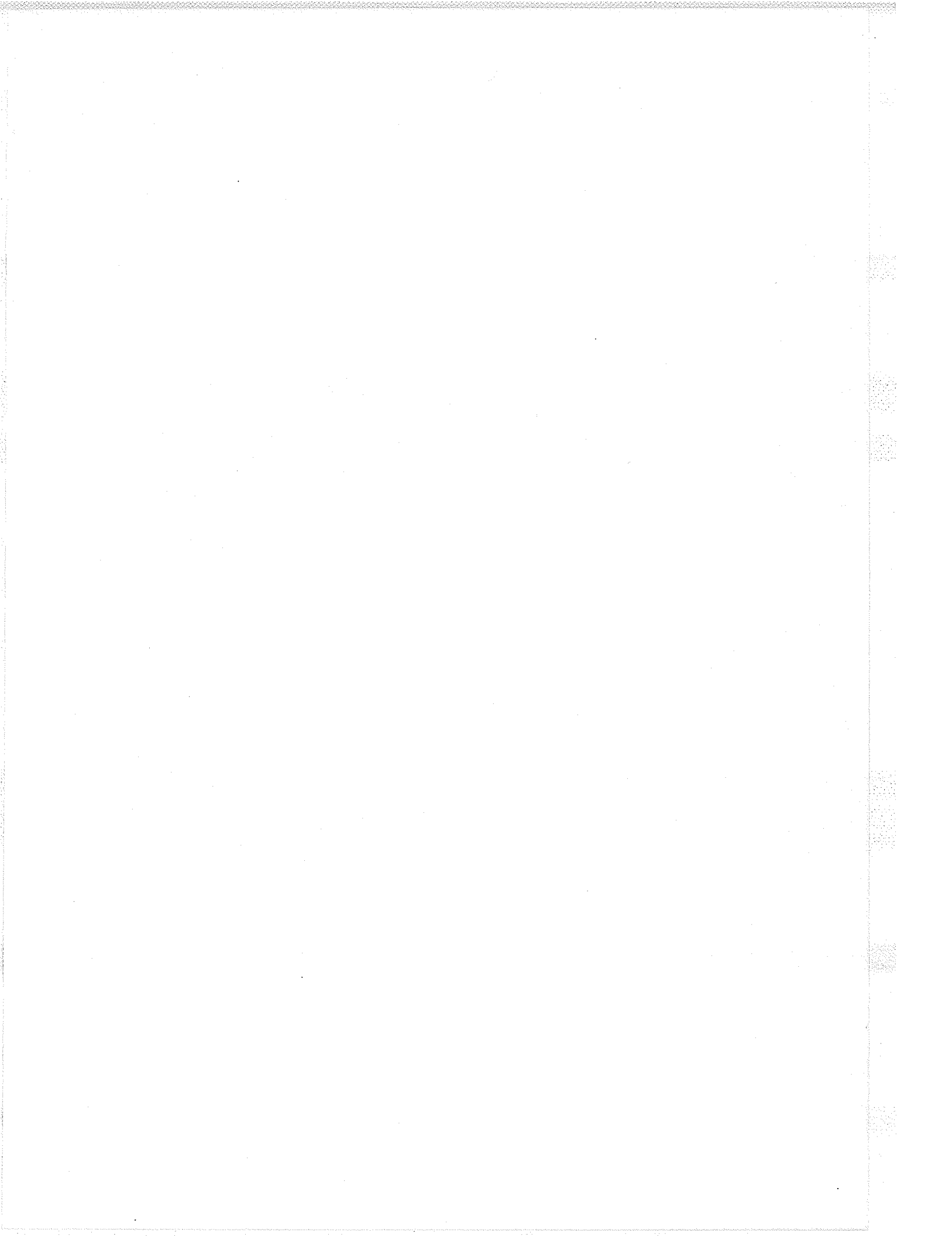
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In Partial Fulfillment  
of the Requirements for the Degree  
Master of Science in Civil Engineering

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by  
King Hau Wu  
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AN INVESTIGATION OF DIAGONAL TENSION  
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ABSTRACT

An investigation of diagonal tension of twenty-one reinforced concrete beams is presented in this thesis. Tests were carried out in three series of simply supported beams. A nest of rollers was inserted in each beam of Series I and II in order to eliminate the vertical shear stress of concrete in the compression zone. The beams of Series I were reinforced with tensile steel only while those of the other two series were provided with tensile and web reinforcement. The size of all beams was 8"x14"x6'-0". All beams except one were tested for three different positions of concentrated loads with a span of 5'-0". The compressive concrete strengths ranged from 1,800 to 5,500 psi. Deflections of the beams under load were measured and the crack patterns were observed and photographed. No strain gages were used to measure strains in the reinforcement.

All beams of Series I failed in an unexpected way other than diagonal tension failure. The test results indicate that the shear capacity of the beams of Series II seems to be affected by web and tensile reinforcement only and independent of concrete strength and shear span. The shear resistances contributed by concrete, web and tensile reinforcement in beams of Series III have been evaluated in the analysis. Due to the insufficient



number of beams tested, only a tentative ultimate shear equation was developed for computing shear capacity of reinforced concrete beams. Further, the application of the equation may be limited to beams similar to those of Series III and its accuracy will depend on the correctness of the estimation of the effective stirrups.

King Hau Wu

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## NOTATION

The following notation is used throughout this thesis.

- $A_s$  = area of tension reinforcement  
 $A_v$  = area of web reinforcement  
 $a$  = shear span which is the distance between a support and the nearest interior load point  
 $\alpha$  = angle of web reinforcement to a horizontal line  
 $b$  = width of a rectangular beam  
 $C$  and  $C'$  = resultants of compressive stresses in concrete  
 $C_1$ ,  $C_2$ , and  $C_3$  = resultants of compressive stresses in concrete at diagonal tension failure for beams of Series I, II, and III respectively  
 $\Delta C$  = increment of compressive force  
 $c$  = principal compressive stress  
 $d$  = effective depth of tension reinforcement  
 $f$  = horizontal fiber stress  
 $f_s$  = stress in tension reinforcement at failure  
 $f_v$  = critical tensile stress of web reinforcement  
 $f_y$  = yield point stress of reinforcement  
 $f'_c$  = compressive strength of 6 x 12 in. concrete control cylinders  
 $I$  = moment of inertia of cross section about neutral axis  
 $jd$  = internal moment arm  
 $K$  =  $(\sin \alpha + \cos \alpha) \sin \alpha$  and  $K = 1$  for beams with vertical stirrups  
 $k_1$  and  $k_3$  = coefficients  
 $M_s$  = moment at the critical section of a beam at shear failure

- $m$  = number of stirrups across a diagonal crack  
 $O$  = perimeter of reinforcing bar  
 $P_I, P_{II}$  and  $P_{III}$  = ultimate loads at diagonal tension failure in beams of Series I, II, and III respectively  
 $P_c$  = initial cracking load  
 $P_u$  = ultimate load at diagonal tension failure  
 $P$  = ratio of  $A_s/bd$   
 $Q$  = statical moment about neutral axis of that portion of cross section beyond the layer containing a particle  
 $q$  = moment arm between reaction of beam and resultant of tension for beams of Series I  
 $R_I, R_{II}$  and  $R_{III}$  = reactions at diagonal tension failure in beams of Series I, II, and III respectively  
 $r$  = ratio of  $A_v/b_s$   
 $S$  = shearing force along the plane of tension reinforcement  
 $S_c$  = shearing force of uncracked concrete  
 $S_t$  = shearing force of tension reinforcement  
 $s$  = spacing of stirrup  
 $s_t$  = unit shearing stress of tension reinforcement  
 $T$  and  $T'$  = tension forces in tension reinforcement  
 $T_1, T_2,$  and  $T_3$  = tension forces in tension reinforcement at diagonal tension failure for beams of Series I, II, and III respectively  
 $T_v$  = total tension force in web reinforcement crossing a diagonal tension crack  
 $\Delta T$  = increment of tension in tensile reinforcement  
 $t$  = principal tensile stress or diagonal tension

$u$  = bond stress of reinforcement

$V$  = shearing force

$V_{\text{I}}$  and  $V_{\text{II}}$  = shearing forces at diagonal tension failure for beams  
of Series I and II respectively

$v$  = nominal shearing stress =  $\frac{V}{bjd}$

$v_{\text{I}}$  and  $v_{\text{II}}$  =  $\frac{V_{\text{I}}}{bjd}$  and  $\frac{V_{\text{II}}}{bjd}$  respectively

$v_{\text{II}}$  = nominal shearing stress at diagonal tension failure

$\bar{x}$  = longitudinal distance defining the location of  $T_v$

$\phi$  = inclination of diagonal tension

CHAPTER I  
INTRODUCTION

There are many different modes of failure of reinforced concrete beams. Their cause of failure may be due to either bending or shear exceeding its resisting capacity. There have been comparatively more research work and more theories developed in ultimate flexural strength of beams than those in ultimate shearing strength. Therefore the flexural stresses are better understood and the design criterias for flexure in building codes are generally safe. However, due to inadequate information from laboratories, building codes may not have sufficiently stringent requirements for beams subjected to shear. This may be a factor leading to inadequate structural designs and consequent failure.

On 17 August 1955, one span of a rigid frame warehouse at Wilkins Air Force Base, Shelby, U.S.A. collapsed. Serious cracks opened in several other warehouses of the same design. Various studies about this matter urged The American Concrete Institute to revise the diagonal tension requirement of the 1951 ACI Building code. Increased web reinforcement for diagonal tension and specified amounts of anchorage of negative tension reinforcement have been added in the Building Code Requirements for Reinforced Concrete (ACI 318-56) published in the ACI Journal, May 1956.

Because of the importance of the shearing stresses in reinforced concrete beams more investigations should be devoted to

this field. The purpose of this study was to investigate the contribution of longitudinal tensile reinforcement, web reinforcement, and concrete strength to the ultimate shearing strength of reinforced concrete beams.

In this investigation only simply supported reinforced concrete beams were tested and the web reinforcement was limited to one type of two-legged vertical stirrups. The nominal concrete strengths varied from 2,000 psi to 5,000 psi. The ratios of the area of the tensile steel to the effective cross sectional area of the concrete varied from 0.0256 to 0.0412. The beams were rather heavily reinforced in tension steel since flexural failure was to be avoided in this investigation.

CHAPTER II  
EARLY DEVELOPMENT OF SHEAR AND DIAGONAL TENSION  
IN REINFORCED CONCRETE BEAMS

It is of interest to describe the early development of shear and diagonal tension in reinforced concrete beams. Some of the early correct theories developed in this field were rejected and soon forgotten by others until they were found again and verified to be right. The early developments were mainly in Europe and in the United States.

Early development in Europe In 1899, W. Ritter presented the first published study of web reinforcement. He suggested Eq. (1) be used in designing vertical stirrups based on truss analogy.

$$v = \frac{A_v f_v j d}{s} \quad (1)$$

However it was not accepted because the basic concepts were not understood by some other well-known engineers of that time.

E. Mörsch, a great German engineer, established the first laboratory for testing shear in reinforced concrete beams. In 1902 he presented a pattern of diagonal cracks which indicated diagonal tension as the cause of shear failures in reinforced concrete beams.

Still some engineers in Europe did not agree with Mörsch. They believed that horizontal shear is the cause of shearing failures in reinforced concrete beams. Even an authority of that day considered the resistance of stirrups to be derived from their horizontal shearing strength instead of tensile strength.

In 1907 Mörsch presented an important paper pertaining to further developments in diagonal tension. He pointed out the following items:

(a) Diagonal tension is the cause of shear failures in reinforced concrete beams according to a study of stress trajectories.

(b) The nominal shearing stress should be used as a measure of diagonal tension and it is computed by Eq. (2)

$$v = \frac{V}{bjd} \quad (2)$$

(c) Stirrups act in tension but not in horizontal shear.

(d) Failure of reinforced concrete beams in diagonal tension was explained by means of redistribution of internal stresses after the formation of diagonal tension crack. This will be discussed in detail in Chapter V. The final failure of the beam may be caused by a crushing failure in the compression zone or by a bond failure in tensile reinforcement. He supported Eq. (1) by assuming that the vertical shear is entirely taken by stirrups and diagonal crack inclined at an angle of 45 degrees.

Eq. (2) is still employed in current design as a measure of diagonal tension. Modified Eq. (1) is also used in design of vertical stirrups in the ACI Building Code. The total vertical shear  $V$  in Eq. (2) has changed to  $V^*$  which is the difference of the total vertical shear and the allowable vertical shear resisted by concrete.

Early development in the United States In the United States diagonal tension in reinforced concrete beams were reported by E. A. Moritz and A. N. Talbot in 1906. M. O. Withey introduced Eq. (1) into

the American concrete literature. He indicated that in addition to the tension in the vertical stirrups, the uncracked concrete in the compression zone, and a possible dowel action of the longitudinal tensile reinforcement may carry part of the shear. In 1909 Talbot reported a study of web stresses from tests of 188 reinforced concrete beams.

By 1910, diagonal tension was well established as the cause of the shear failures in normal reinforced concrete beams in Europe and the United States. The theories concerning shear, bond and diagonal tension will be presented in Chapter III.



## CHAPTER III

## SHEAR, BOND, AND DIAGONAL TENSION

This chapter contains the derivations of formulas for nominal shearing and bond stresses, the distinction of shearing stresses from diagonal tension, and the explanation of diagonal tension cracks by means of stress trajectories in reinforced concrete beams. It was felt that a review of these would lead to a better understanding of the investigation. The subsequent paragraphs of this chapter are devoted to this purpose.

Shearing Stresses These stresses are those which tend to make one vertical or horizontal plane of concrete sliding along its adjacent plane. For homogeneous or plain concrete beams, unit shearing stress can be computed by Eq. (3) from principles of mechanics

$$v = \frac{VQ}{Ib} \quad (3)$$

Reinforced concrete beams are not homogeneous. When they are subjected to shearing forces, they behave in quite a different manner from that of plain concrete beams. It is usually assumed that no longitudinal tension is carried by the concrete and that there is no slipping between concrete and steel in calculating shearing and bond stresses.

Consider an element with a short length  $dx$  of the beam as a free body in Fig. 1 (a). The forces acting on this element are the resultants of compressive forces  $C$  and  $C'$ , the tensile forces  $T$  and  $T'$ , and the vertical shear  $V$ . The increment of tension,  $\Delta T$ , tends to pull

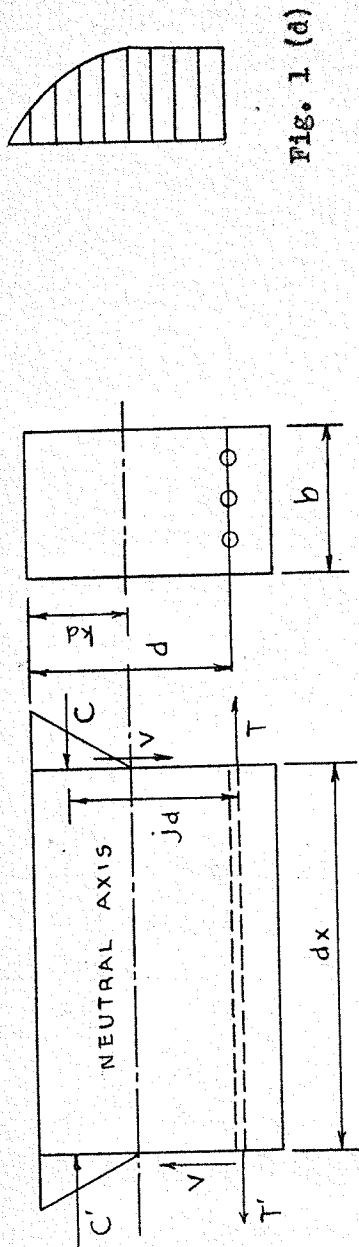


Fig. 1 (a)

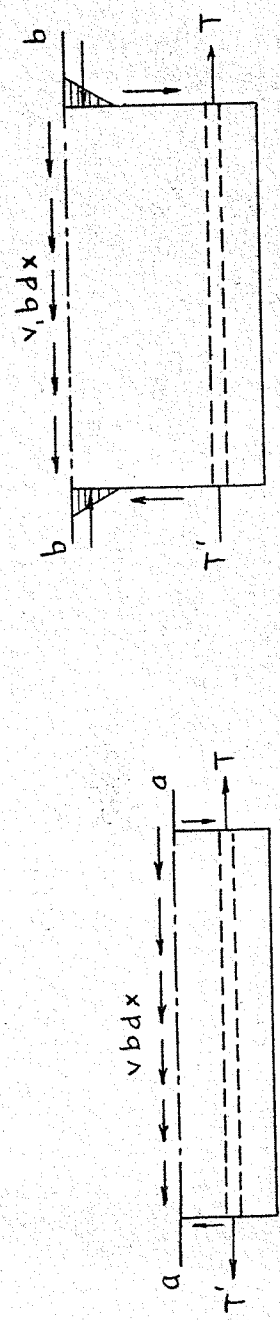


Fig. 1 (c)

Fig. 1 (b)

Figure 1

Element of beam subjected to shearing stress and its shearing stress diagram.

the lower part of the element to the right while the increment of compression,  $\Delta C$ , tends to push the upper part to the left. However, these external forces must be in equilibrium. Therefore, the increment of tension,  $\Delta T$ , in the steel can be computed by taking moments at A.

$$\begin{aligned} \Sigma M_A = 0 \quad \Delta T \cdot jd &= Vdx \\ \therefore \Delta T &= \frac{V dx}{jd} \end{aligned} \quad (4)$$

The free body in Fig. 1 (b) is a portion of the element of Fig. 1 (a) cut by the horizontal section line a - a between the neutral axis and the tension steel. The increment of tension,  $\Delta T$ , is resisted by the horizontal shearing force  $v b dx$  along the section line a-a.

Equilibrium gives

$$\Delta T = v b dx \quad (5)$$

$$\text{From Esq. (4) and (5) } v = \frac{V}{bjd} \quad (2)$$

Eq. (2) is the equation of nominal shearing stress which gives a constant unit shearing stress between the tension reinforcement and the neutral axis. This equation is applicable to reinforced concrete beams subjected to moment and shear only. If a direct thrust or a direct tension also acts upon the element, some modification to the equation is necessary.

The shearing stress in the compression zone is smaller than the nominal shearing stress because the increment of compression exists in the free body diagram. Fig. 1 (c) shows a portion of the element of Fig. 1 (a) cut by a horizontal line b-b above the neutral axis. The shearing stress at any point in the compression zone of reinforced concrete beams may be computed by Eq. (3). A typical distribution of unit

shearing stress in a cross section of a reinforced concrete beam is shown in Fig. 1 (d). Shearing stresses in compression zone are usually not calculated, only the nominal shearing stress is significant in designs and studies of reinforced concrete beams.

Bond Stresses Bond stress is the intensity of an adhesive force between the surfaces of the steel reinforcement and the concrete. The transmission of the increment in total tensile stress to the shearing stress is dependant on the bond between the concrete and the steel. Therefore in Fig. 1 (a) the bond force  $u (\Sigma 0) (dx)$  must be equal to the increment of tension,  $\Delta T$ .

$$u (\Sigma 0) dx = \Delta T \quad (6)$$

$$\text{Combine Eqs. (4) and (6)} \quad u (\Sigma 0) (dx) = \frac{V dx}{jd}$$

$$u = \frac{V}{\Sigma o jd} \quad (7)$$

Eq. (7) is the nominal bond stress in reinforced concrete beams. In this investigation, failure of reinforced concrete beams due to bond failure was very undesirable. Hence special anchorage with washers welded at the ends of steel bars was provided to prevent beams failing in this mode, although it was realized that with modern deformed bars bond failures are not probable in simple beams of normal proportion.

Diagonal tension in Plain Concrete Beams Diagonal tension is the inclined principal tension owing to the action of fiber stress and the horizontal and vertical shearing stresses in concrete beams. The simply supported plain concrete beam in Fig. 2 (a) is subjected to uniform loads which cause shearing and bending stresses. Let

Figs. 2 (b) and (c) represent infinitely small cubes of concrete below and above the neutral axis respectively. The forces acting on these elements are the vertical and horizontal shearing stresses of equal intensity  $v$  and the horizontal fiber stress  $f$  normal to the vertical axis. The horizontal stress is in tension or in compression depending on whether it is below or above the neutral axis. Because of the combined effect of these stresses, principal stresses result as shown in Figs. 2 (b) and 2 (c). The principal tensile stress  $t$  and the principal compressive stress  $c$  can be computed by Eqs. (8) and (9). The principal tensile stress in concrete is known as diagonal tension.

$$t = \frac{f}{2} + \sqrt{\frac{f^2}{4} + v^2} \quad (8)$$

$$c = \frac{f}{2} - \sqrt{\frac{f^2}{4} + v^2} \quad (9)$$

Where  $f$  is positive for a tensile fiber stress, and negative for a compressive fiber stress. The directions of principal stresses are given in Eq. (10)

$$\tan \phi = -\frac{2v}{f} \quad (10)$$

Two values of  $\phi$ , differing by 90 deg., will satisfy Eq. (10). This corresponds with the fact that principal stresses occur orthogonally at each point. According to a principle of mechanics the principal tensile stress will always lie within the principal 45 deg. angle which is the angle subtended by the algebraically greater normal stress and the shear diagonal. Thus, the inclination of diagonal tension of any particle below the neutral axis is less

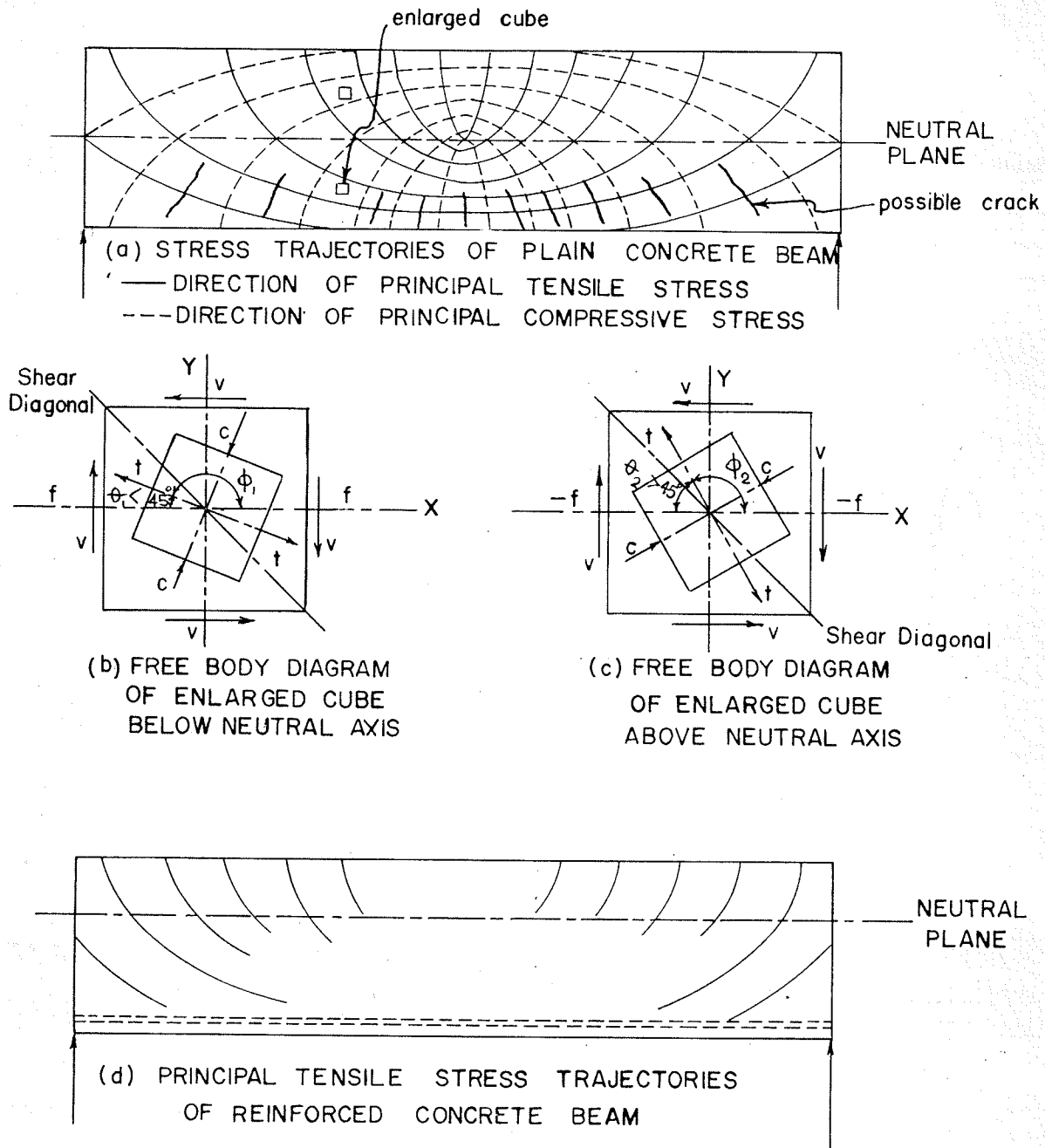


FIGURE 2  
 STRESS TRAJECTORIES OF PLAIN & REINFORCED  
 CONCRETE BEAMS

than 45 deg. and that above the neutral axis is more than 45 deg. as shown in Figs. 2 (b) and (c). For any particle along the neutral axis the direction of its diagonal tension must be 45 deg. with the horizontal since the fiber stress is equal to zero in Eq. (10). With the use of Eq. (10) two families of stress trajectories at right angles to each other can be drawn for the whole beam. Stress trajectories are those curves whose tangents and normals at all points on the curves are the directions of principal stresses. In Fig. 2 (a) the family of solid curves represents the directions of the diagonal tensions and the family of the dotted curves indicates the directions of the principal compressive stresses.

Because concrete is strong in compression and weak in tension, plain concrete beams may fail suddenly without warning due to diagonal tension. Careful attention should be given particularly to diagonal tension. The first term on the left hand side of Eq. (8) will become negative for compressive fiber stress, so it is obvious that in any section of a simply supported homogeneous beam the intensities of diagonal tensions below the neutral axis are much larger than those above the neutral axis. Therefore probable cracks would usually occur below the neutral axis and in the directions normal to the principal tensile stress trajectories as shown in Fig. 1 (a). The cracks normal or nearly normal to the horizontal edge of the beam are tension cracks while the inclined cracks are called diagonal tension cracks.

Diagonal tension in reinforced concrete beams Due to the concentration of tension in steel reinforcement the magnitude of tensile

fiber stresses are quite uncertain in reinforced concrete beams. Although it is impossible to compute diagonal tension accurately by Eqs. (8) and (10), yet an approximate picture of stress trajectories may be obtained. Fig. 2 (d) shows the approximate principal tensile stress trajectories for a uniformly loaded reinforced concrete beam without web reinforcement. These curves are somewhat similar to those in the plain concrete beam in Fig. 2 (a). However, the stress trajectories in the central portion of the beam below the neutral axis are quite indeterminate and therefore are not shown in the figure. All the curves cross the neutral axis at nearly 45 deg. and approach the top edge of the beam at 90 deg. The possible cracks are also shown in the figure. In order to simplify the conceptions of these inclined stresses and for convenience in designing web reinforcement, the nominal shearing stress is arbitrarily used as a measure of the diagonal tension in common practice. It must be realized that the shearing stress is only one of the two factors affecting the diagonal tension and there is no definite relation between the diagonal tension and the shearing stress alone. For instance, the shearing stresses in a simple beam and an identical restrained beam with the same loading condition may be equal, yet the diagonal tensions at the supports of the simple beam may be much smaller than those of the restrained beam because there is no fiber stress existing at the supports of the simple beam.

Since theoretical analysis can give only very rough estimation as to diagonal tension of reinforced concrete beams, it is necessary to conduct extensive experimental tests to determine the



shearing strengths of the beams having various influencing factors. The failure by diagonal tension may be presented if the shearing stress is kept below the ultimate by a certain factor of safety of the order of two or three.

## CHAPTER IV

## REVIEW OF THE CURRENT INVESTIGATIONS OF SHEARING

## STRENGTH IN REINFORCED CONCRETE BEAMS

It was considered beneficial in the preparation of this study to make a review of current investigations being carried on by others on the shearing strength of reinforced concrete beams.

This chapter is devoted to reviewing these studies. Current studies include the papers presented by Oreste Moretto in 1945, by Arthur P. Clark in 1951, and by Moody-Viest in 1955. The beams tested by Moretto and Clark were simply supported beams while those tested by Moody-Viest consisted of both simple and restrained beams. In their tests, the strains in concrete and steel reinforcement were measured by electric strain gages to determine the corresponding stresses developed under testing. The deflections of beams were also recorded and the cracks were observed. A brief description of their individual studies will be presented in the following paragraphs.

The investigation by Oreste Moretto In his paper forty four reinforced concrete beams with stirrups welded to the longitudinal reinforcement were presented. The tests were made on simple beams and loaded at third points. It was concluded that a beam with welded stirrups might increase resistance to diagonal tension by 20 per cent over one with similar loose stirrups. Finally Eq. (11) for predicting the ultimate load on the basis of ultimate nominal shearing stress was proposed, taking into account the effect of the web reinforcement, the strength of concrete, and the amount of the tensile longitudinal reinforcement.

$$v_u = K_r f_y + 0.10 f'_c + 5,000p \quad (11)$$

where  $K = (\sin \lambda + \cos \lambda) \cdot \sin \lambda$  and  $K = 1$  for beams with vertical stirrups.

The investigation by Arthur P. Clark Clark's tests included simple beams with two different cross sections, four different spans, five different loadings and concrete strengths varying from 2,000 to 6,900 psi. He believed that in addition to the factors considered by Moretto the ratio of shear span to the effective depth would also affect the ultimate shearing strength of reinforced concrete. Eq. (12) is an empirical equation suggested by Clark.

$$v_u = 2,500 \sqrt{f'_c} + 0.12 f'_c \cdot \frac{d}{a} + 7,000 p \quad (12)$$

The investigation by Moody-Viest From the studies of 136 simple and restrained rectangular beams with and without web reinforcement, general equations applicable to both kinds of beams for predicting the initial diagonal tension cracking load and ultimate shearing load were developed. The equation for the initial cracking load was obtained from test data as an empirical expression. For the case of simple beams it was found that the initial diagonal cracking is affected by the strength of concrete and the ratio of shear span to the effective depth only.

It was stated that if a beam can sustain a load greater than the cracking load, a redistribution of internal stress will take place after the formation of diagonal crack. As a result of destruction of concrete in the compression zone above or below the diagonal tension crack, shear failure occurs. With the aid of several assumptions, a

general ultimate moment equation was developed by a semi-rational approach. Eq. (13) is the ultimate moment equation for simple beams with vertical stirrups suggested by Moody-Viest.

$$\frac{M_s}{k_1 k_3 f_c' b d^2} = \frac{p f_s}{k_1 k_3 f_c'} \left( 1 - K_2 \frac{p f_s}{k_1 k_3 f_c'} \right) + \frac{T_v \cdot \bar{x}}{k_1 k_3 f_c' b d^2} \quad (13)$$

Discussion of the current investigations In both the investigations by Moretto and Clark only simply supported beam specimens were tested. The equations of the ultimate shearing load were obtained empirically from the ultimate nominal shearing stresses with the factors known to affect the shearing load. Eqs. (11) and (12) are of similar form except that the ratio of shear span to the effective depth is introduced into Eq. (12). A comparison between the actual nominal shearing stresses of the beams tested by the investigators mentioned in this chapter and the computed shearing stresses by Eqs. (11) and (12) shows that Eq. (12) gives much better results. This indicates that the ratio of shear span to the effective depth plays an important role on the shearing strength of reinforced concrete beams.

For the tested beams without web reinforcement Eq. (13) predicts shear strengths of the test beams closer than Eqs. (11) and (12). But for the most of the tested beams with web reinforcement both Eqs. (12) and (13) can predict the shearing strengths within 10 per cent error. Because Eq. (13) is somewhat complex in its form, and usually reinforced concrete beams in actual practice have web reinforcement it is preferable to use Eq. (12) or similar form of equation for estimating the shearing strength of beams. On the other hand Eq. (12) is not applicable to restrained beams. Eq. (13) is therefore valuable for this kind of beams.

## CHAPTER V

THEORIES AND METHODS OF EVALUATION OF THREE IMPORTANT  
FACTORS IN DIAGONAL TENSION STRENGTH

As stated in the previous chapter, all the current investigations of diagonal tension in reinforced concrete beams required strain gages to determine the internal stresses. In this investigation a new approach was attempted without using strain gages. The time and labour in setting up the strain gages was saved and the testing procedures were much simplified.

From the studies of the current investigations shearing strength of a reinforced concrete beam is composed of three important factors, namely: (1) the shearing strength of the longitudinal tension steel; (2) the shearing strength developed by tension in web reinforcement, and (3) the shearing strength of uncracked concrete above the neutral axis. The theories and methods of evaluation these three factors will be presented in the succeeding paragraphs of this chapter.

Consider a reinforced concrete beam with vertical stirrups in Fig. 5 failing due to diagonal tension. Assume the diagonal tension cracks incline from the end of the support bearing plates to the neutral axis. Then, the free body diagram of the left part of the beam ~~are~~ is shown in Fig. 5A. The vertical forces acting on the free body are as follows:

1. The vertical reaction  $R_{III}$  at the support.
2. The vertical shearing force  $S_T$  of the longitudinal tension steel.

3. The resultant of the tensile force of the stirrups  $T_v$ .
4. The vertical concrete shearing force  $S_c$  above the neutral axis.

For equilibrium,  $\Sigma V = 0$

$$R_{III} = S_T + T_v + S_c \quad (14)$$

Seven ordinary reinforced concrete beams of this kind were made in this investigation. They were called beams of Series III. The details of these beams will be described in Chapter VI.

Method of evaluating the shearing strength of the longitudinal tension steel Any factor in resisting diagonal tension may be evaluated easily by testing a reinforced concrete beam with the other two factors isolated. The beam in Fig. 3 is a beam similar to the one in Fig. 5 except that no web reinforcement is provided and a portion of the compression concrete is replaced by a set of bearing plates and rollers. This kind of beams was called beams of Series I in this investigation. This beam offers no shearing resistances by uncracked concrete and stirrups, yet the compressive forces can be transmitted through the nest of rollers. After the formation of the diagonal tension crack, no stresses can exist in the concrete across the crack. The only forces to resist the reaction is the vertical shearing force of the longitudinal steel. If the diagonal tension crack is similar to the one in Fig. 5, the free body diagram for this beam at failure may be shown in Fig. 3A.

$$\text{For equilibrium, } \Sigma V = 0 \quad R_i = S_T \quad (15)$$

$$\text{But} \quad R_i = \frac{P_i}{2} \quad \text{and} \quad S_T = A_s \cdot s_T$$

$$\text{Therefore} \quad S_T = \frac{P_i}{2A_s} \quad (16)$$

where  $R_1$  and  $P_1$  are the reaction and applied load respectively of the beam in Fig. 4 at failure. Hence the shearing resistance offered by the longitudinal tension bars of a reinforced concrete beam may be obtained by testing a similar beam without stirrups but with rollers in place of a portion of concrete. Its magnitude may be equal to the ultimate reaction of the latter beam if their diagonal tension cracks are similar. Moreover, its shearing stress may be equal to the ultimate load divided by twice the area of its longitudinal reinforcement.

Method of evaluating the shearing strengths developed by tension in vertical stirrups and uncracked concrete The shear force taken by vertical stirrups may be evaluated indirectly by testing similar beams with gaps in the compression zone. The beam in Fig. 4 is a beam similar to that in Fig. 5 except that a nest of rollers is inserted. After the formation of the diagonal tension crack the reaction at the support will be resisted by the shearing strength developed by the longitudinal steel and the vertical stirrups. If the diagonal tension crack is similar to the one in Fig. 5, the free body diagram for this beam at failure may be shown in Fig. 4A. The vertical forces in this diagram are the reaction  $R_{II}$  at the support, the resultant  $T_v$  of the tensile forces in the stirrups, and the vertical shearing force  $S_T$  of the longitudinal tensile steel.

$$\text{For equilibrium} \quad \Sigma V = 0 \quad R_{II} = S_T + T_v \quad (17)$$

$$\text{But} \quad S_T = R_I$$

$$\therefore T_v = R_{II} - R_I \quad (18)$$

$$\text{Ave.} \quad f_v = \frac{R_{II} - R_I}{m A_v} \quad (19)$$

$$m A_v$$

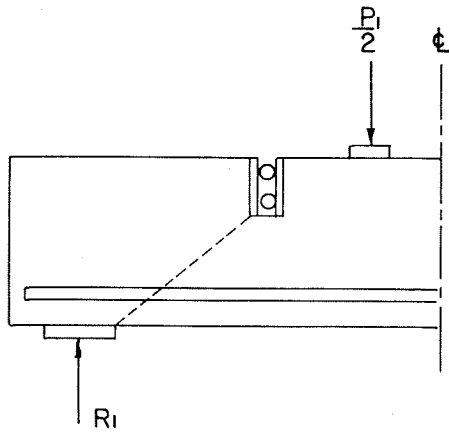


FIG. 3 A BEAM OF SERIES I WITH HYPOTHETICAL CRACK

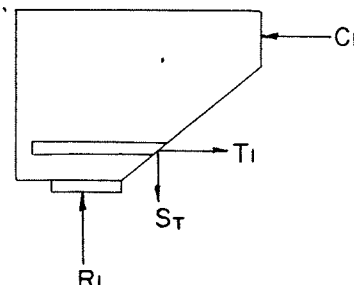


FIG. 3A THE FREE BODY DIAGRAM OF BEAM IN FIG. 3

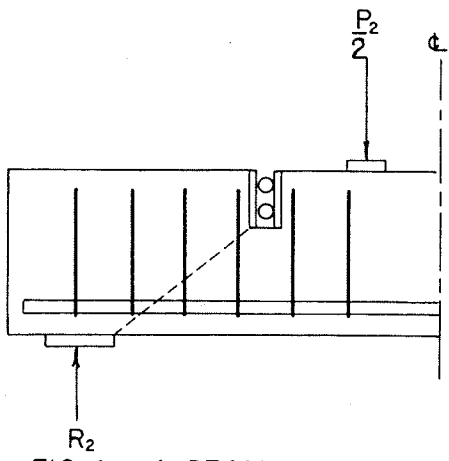


FIG. 4 A BEAM OF SERIES II WITH HYPOTHETICAL CRACK

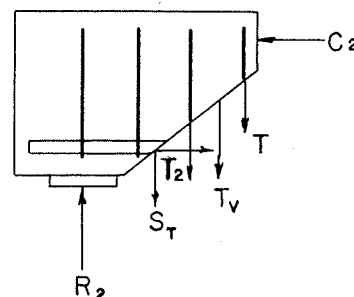


FIG. 4A THE FREE BODY DIAGRAM OF THE BEAM IN FIG. 4

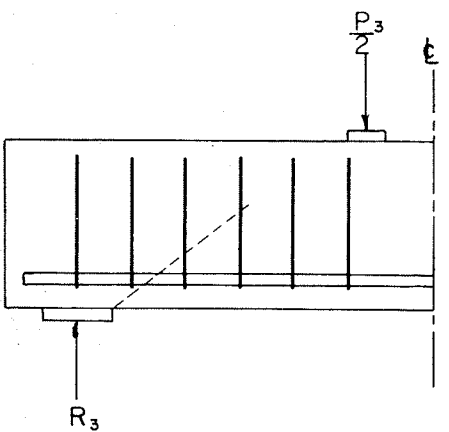


FIG. 5 A BEAM OF SERIES III WITH HYPOTHETICAL CRACK

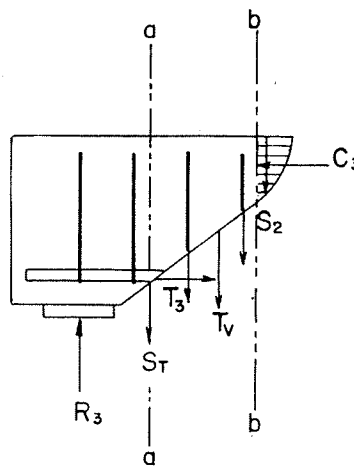


FIG. 5A THE FREE BODY DIAGRAM OF THE BEAM IN FIG. 5



where  $f_v$  is the critical tensile stress in the stirrups and  $n$  is the number of the stirrups across the diagonal tension crack.

Therefore the shearing force taken by the stirrups of a reinforced concrete beam may be equal to the difference of the end shears of a beam in Series I and its corresponding beam in Series II provided that their diagonal tension cracks are similar. Furthermore, the average vertical tensile stress in the stirrups may be obtained by dividing the shearing resistance of the stirrups by the total area of the stirrups across the diagonal tension crack.

After the shear resistances of the longitudinal tension bars and the stirrups have been evaluated, the shear force taken by the uncracked concrete above the neutral axis may be determined indirectly from the test results of the corresponding beams in Series II and III.

$$\text{From Eqs. (14) and (17)} \quad S_c = R_{III} - R_{II} \quad (20)$$

Therefore the shear resistance of the uncracked concrete above the neutral axis may be equal to the difference of end shears of the corresponding beams in Series III and II.

Redistribution of internal stresses In the free body diagram of Fig. 5.A, the moment caused by the vertical forces must be balanced by the couple of the horizontal forces. The horizontal forces acting on the free body are the horizontal tension of the longitudinal tension steel and compressive stresses above the neutral axis. The magnitude of the horizontal tension cannot be computed from the bending moment at Section a-a as in the usual manner. However, it may be determined by taking moments about Section b-b if the reaction at the

support and the forces taken by the stirrups and the tension reinforcement are known. The changes of relationship of usual tensile stresses due to the formation of diagonal tension are known as redistribution of internal stresses.

## CHAPTER VI

## SPECIMENS, EQUIPMENT, AND TEST PROCEDURES

Specimens There were twenty-one simply supported rectangular beams in three series, Series I, II and III, in this investigation. Each series contained seven beams. They were of same size with a cross section 8 X 14 inches and a length of six feet long. The nominal concrete strengths varied from 2,000 psi to 5,000 psi. The majority of the beams had a nominal concrete strength of 3,500 psi. The effective depth of the beams was  $11\frac{1}{2}$  inches approximately. Three different amounts of longitudinal reinforcement were used; namely, 3 - No. 8, 3 - No. 9, and 3 - No. 10 deformed bars. Consequently, the ratios of the area of the tensile steel bars to the effective cross sectional area of the concrete were 0.0256, 0.0326, and 0.0412 respectively. All the beams were designed to fail in diagonal tension.

Beams of Series I The beams of Series I were planned to investigate the shear strength of the longitudinal tension reinforcement. No web reinforcement was provided. A portion of the compression concrete two and one half inches wide and five inches deep was replaced by two bearing plates and two rollers. Figs. 20 - 26 show the detail of each beam of this series. The nominal concrete strengths and the amount of tension steel are listed in Table I.

Beams of Series II These beams were designed to study the contribution of the web reinforcement to the shearing strength. They were similar to their corresponding beams in Series I except that vertical stirrups were provided. All the web reinforcement was of No. 3

deformed bars. Three different spacings of stirrups were used: three inches, four and one half inches, and six inches. The details of beams of Series II are shown in Figs. 27 - 33 and in Table 2.

Beams of Series III These beams were designed to study the shearing strength offered by the concrete. The beams in this series were identical to those of Series II without gap. The details of these beams are shown in Figs. 34 - 40 and listed in Table 2.

Types of Loading Since shear span is also one of the factors influencing the shearing strength of reinforced concrete beams, different loading conditions were used in order to study the effects of various shearing spans on shearing strength. All the beams except Beam IIIA1 were tested in three different types of loading. The loading for Beam IIIA1 was a special loading which is shown in Fig. 34.

(1) "A" Loading -- the load was applied at the center of the span.

(2) "B" Loading -- the load was applied at the points six inches from the center line of the span.

(3) "C" Loading -- the load was applied at the third points of the span.

Designation of beams In the designation of beams the Roman numeral was used to designate the series, the following letter identified the type of loading and the last Arabic numerals was the specimen number in each series. The beams in the three series having the same Arabic numerals were corresponding beams which formed a combination for evaluating the three factors of shearing strengths. Of course, Beam IIIA1 was an exceptional beam which had no relation to any other beam at all.

TABLE 1  
DETAILS OF SPECIMENS OF SERIES I

Beam No.	$\frac{a}{d}$ (ratio)	Nominal $f_c'$ (psi)	Tensile Steel	
			Pieces & Bar No.	$p = A_s/bd$
IA1	2.61	3,500	3 - No. 8	0.0256
IA2	2.61	3,500	3 - No. 9	0.0326
IB3	2.09	2,000	3 - No. 8	0.0256
IB4	2.09	3,500	3 - No. 9	0.0326
IB5	2.09	5,000	3 - No. 10	0.0412
IC6	1.75	3,500	3 - No. 9	0.0326
IC7	1.75	5,000	3 - No. 9	0.0326

TABLE 2  
DETAILS OF SPECIMENS OF SERIES II & III

Beam No.	$\frac{a}{d}$ (ratio)	Nominal $f_c'$ (psi)	Tensile Steel		No. 3 Stirrups	
			Pieces & Bar No.	$p = A_s/bd$	s (in)	$r = A_v/bs$
IIA1	2.61	3,500	3 - No. 8	0.0256	6	0.00458
IIIA1	1.305	3,500	3 - No. 8	0.0256	6	0.00458
II & IIIA2	2.61	3,500	3 - No. 9	0.0326	$4\frac{1}{2}$	0.00610
II & IIIB3	2.09	2,000	3 - No. 8	0.0256	6	0.00458
II & IIIB4	2.09	3,500	3 - No. 9	0.0326	$4\frac{1}{2}$	0.00610
II & IIIB5	2.09	5,000	3 - No. 10	0.0412	3	0.00916
II & IIIC6	1.75	3,500	3 - No. 9	0.0326	$4\frac{1}{2}$	0.00610
II & IIIC7	1.75	5,000	3 - No. 9	0.0326	$4\frac{1}{2}$	0.00610

Design of beams Since the failure of the specimens by other than diagonal tension was not desirable, all beams were designed to fail in diagonal tension. Each beam of Series III was designed in such a way that its flexural capacity was greater than its shear capacity. The estimated ultimate moments due to flexural failure were based on the formulas in the article "The Plasticity Ratio of Concrete and Its Effect on the Ultimate Strength of Beams" by V. P. Jensen, while those due to shear failures were based on Moody-Viest formulas. The estimated loads for shear failures were also checked by Clark's formula. It was understood that the shear capacities for the beams of Series I and II would be lower than their corresponding beams in Series III and yet the flexural capacities would not decrease if the cement mortar for grouting the steel plates were strong enough to resist the compression. Therefore, computations for the design of these two series of beams were not necessary.

Materials All beams were made with ordinary portland cement. The specific gravity of the cement was 3.13. Both the specific gravity of fine and coarse aggregate were 2.65. The average fineness modulus of the sand was 3.02. The maximum size of the limestone aggregate was 3/4 inch. The surface moisture of the sand was 2.5%. The surface moisture of the lime stones was 0.4%.

Deformed bars of structural steel were used for the reinforcement of the beams. Control tests of two tensile coupons of No. 3 bars were made. The physical properties of Nos. 8, 9, and 10 bars were obtained from the analysis and physical report of Manitoba Rolling Mill Co. Ltd.

TABLE 3  
PHYSICAL PROPERTIES OF STEEL BARS

Bar No.	Yield Point strength (psi)	Tensile strength (psi)	Elongation % in 8" long
3	63,000	84,500	18.3
8	54,000	80,500	20.7
9	47,400	76,700	23.0
10	44,200	73,800	25.8

Design of concrete mixtures The concrete mixtures of the beams were designed to attain their corresponding nominal concrete strengths 2,000 psi, 3,500 psi, and 5,000 psi. The average mixture ratios and water cement ratios are shown in Table 4. Trial mixes were used so that the slumps were within the limit of one inch to two inches. Stiff concrete was required because of the use of vibrators.

TABLE 4  
RATIO OF CONCRETE MIXTURES

Nominal $f'_c$ (psi)	Ave. Mixtures Ratios by weight Cement: Sand: Gravel	Cement water Ratio by Weight
2,000	1 : 4.4 : 3.22	1.35
3,500	1 : 2.85 : 2.64	1.76
5,000	1 : 1.76 : 1.83	2.93

The actual concrete strengths are listed in Table 5, Chapter VII.

Fabrication of specimens The reinforcement for each beam of Series II and III was arc welded into a rigid cage as shown in Figs. 6 and 7. Two short No. 3 bars were welded at the quarter points of the longitudinal tension bars for each beam of Series I for the purpose of connecting them in proper spacing. In order to avoid possible failure of bond and anchorage, 1", 1-1/8", and 1 1/4" heavy plain washers were welded to both ends of the Nos. 8, 9, and 10 steel bars respectively. All reinforcement was supported by two inches high steel chairs in steel forms.

All the beams were cast in well oiled steel forms which mounted on the external vibrators as shown in Fig. 8. The vibrators were driven by an electric motor between the ends of the steel forms. The concrete was mixed in a tilting rotary mixer by the side of the steel forms. All batching was weighed by scale. The concrete was chuted into the forms. After each batch had been placed in the form, the concrete mixture was vibrated for about one minute for beams of Series I and II. Since two and one half batches were required to fill up one form, the total time of vibration required was about three minutes for these two series. No definite time was kept for the vibration of Series III. The time of vibration was judged only by the appearance of the concrete. Because this series of beams were the beginning few beams to be poured in the newly manufactured vibrators, steel hangers 3/8" D. and 12" deep were inserted in both ends of the beams to facilitate transportation. Great care should be taken that they would not cross the cracks and act as stirrups.

Forms were removed two days after casting. The beams were painted with a coat of Horncure. It is a liquid for curing concrete.



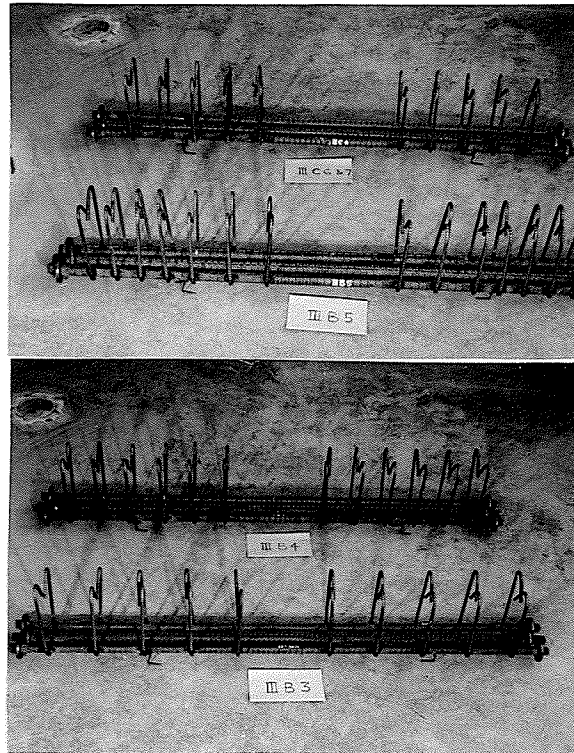


FIGURE 6  
REINFORCEMENT OF BEAMS OF SERIES II & III  
SUBJECTED TO "B" & "C" LOADINGS

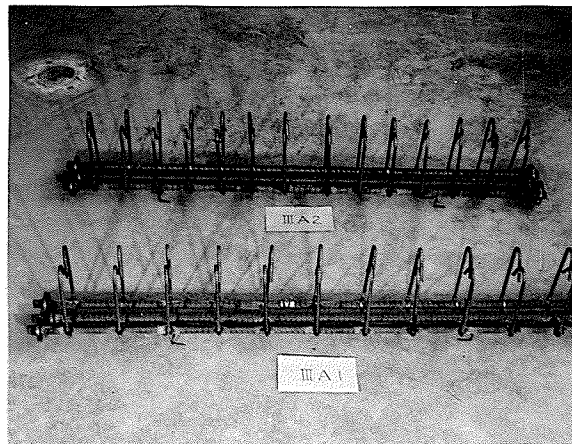


FIGURE 7  
REINFORCEMENT OF BEAMS OF SERIES II & III  
SUBJECTED TO "A" LOADING

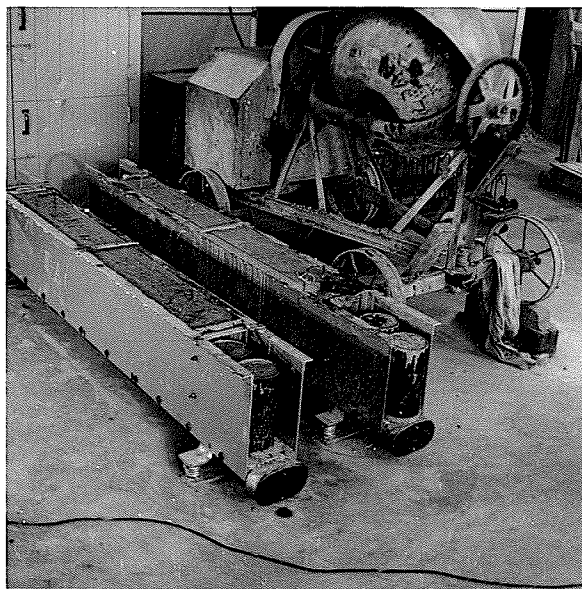


FIGURE 8

EQUIPMENT FOR FABRICATING CONCRETE BEAMS & CYLINDERS



FIGURE 9

CONCRETE BEAMS & CYLINDERS PILED UP  
IN THE LABORATORY BEFORE TESTS

It hardens to form a thin membrane which prevents the concrete from the action of rapid drying. All the beams were stored in the air of the laboratory and tested at the age of twenty eight days. The picture in Fig. 9 shows the concrete beams piled up before tests. All the beams were white washed prior to tests so that cracks would become readily visible during the tests.

In pouring the beams of Series I and II a wooden block  $3\frac{1}{4}$ " at the top, 3" at the bottom, 5" deep, and 8" long was inserted in each beam in proper position. The distances from the center of the blocks to the ends of the beams were  $2'-3"$ ,  $1'-9\frac{1}{2}"$ , and  $1'-7\frac{1}{2}"$  for the beams having the "A", "B", and "C" Loadings respectively. All the wooden blocks had been well soaked with water and then oiled before they were used, thus facilitated their removal from concrete. Since wooden blocks swell when they are soaked with water and shrink when they dry. The blocks were hammered out to form gaps where the nest of rollers were grouted in. Each nest of rollers consisted of two steel plates,  $10" \times 5" \times \frac{1}{2}"$ , and two steel rollers  $\frac{1}{2}" \text{D.} \times 10"$  long. In grouting the nest of rollers both ends of the steel plates were clamped in order to hold the rollers in the desired position. Cement mortar of 1:1 mixed was used for grouting.

Two standard  $6" \times 12"$  control cylinders for each beam were cast in paraffined cardboard molds in the same way and at the same time as the beam by putting them at the end of the steel form as shown in Fig. 8. The concrete cylinders were cured and stored in the same manner as the beams and were tested at the same age as the corresponding beams to determine the actual concrete strengths. The cylinders for beams

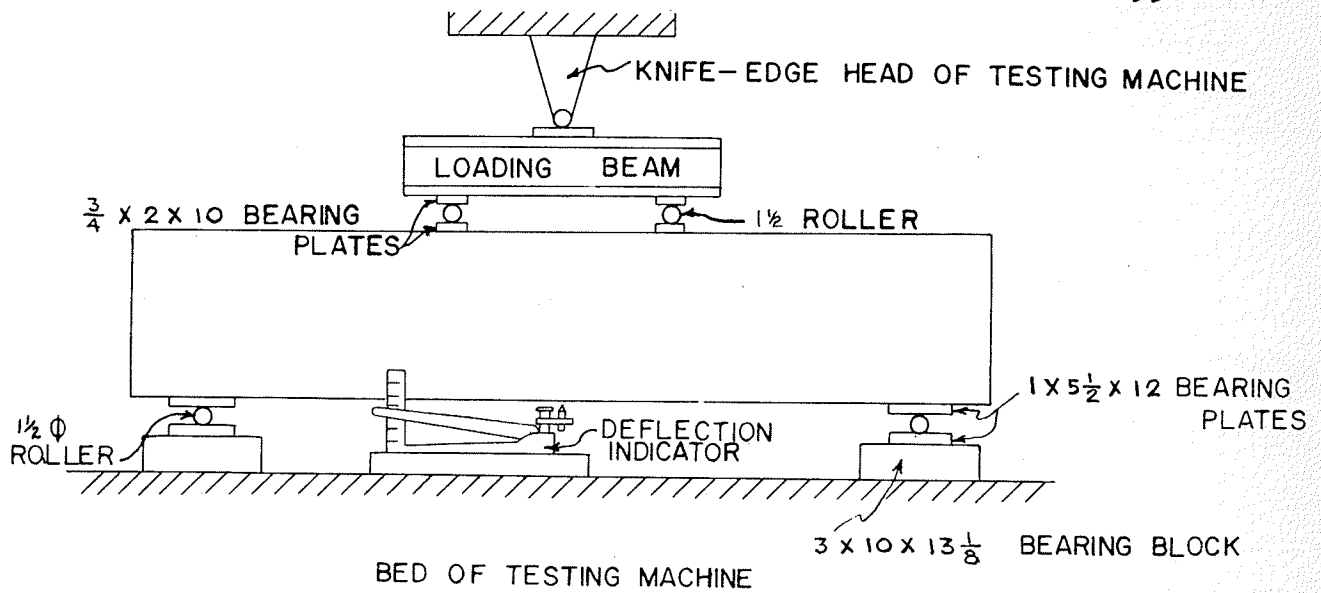


FIGURE 10  
 SKETCH SHOWING TESTING ARRANGEMENT  
 FOR BEAM SPECIMENS

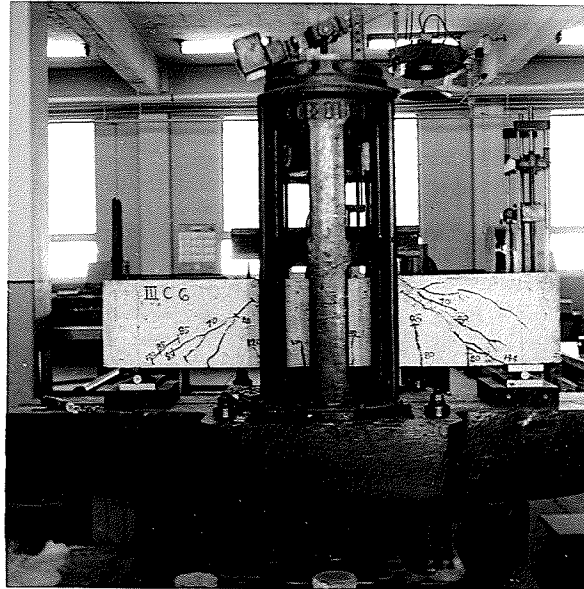


FIGURE 11  
 PICTURE SHOWING TESTING ARRANGEMENT  
 FOR BEAM SPECIMENS

of Series I were capped with plaster of Paris while those of Series II and III were capped with Vitro Bond which is a material made with tar and sulphur.

Test equipment and procedure The beams were tested with a five foot span in three different types of loadings except Beam IIIA1. A 200,000 pound capacity Riehle testing machine of the beam type was used. Details of the testing arrangement were shown in Figs. 10 and 11.

The load was applied in increments until failure. For most beams of Series I the increment was 5,000 pounds from zero load up to a total load of 20,000 pounds, then the increment was decreased to 2,000 pounds until failure occurred. For beams of Series II and III all increment of load was 5,000 pounds. After each increment the deflection at the center of each beam was measured by the Riehle cantilever type deflection indicator. The cracks were also traced and recorded at the same time. The progress of cracking and phenomenom of failure were observed visually and recorded by camera.

## CHAPTER VII

### TEST RESULTS

This chapter is devoted to presenting all the test results in tables, graphs, and photographs in connection with this investigation. The behaviors of the specimen beams will be discussed in detail in the latter part.

The test results of the three series of reinforced concrete beams are tabulated in Table 7. Both the cracking loads and the failure loads were measured by the testing machine. The cracking loads are the loads at the formation of initial diagonal tension crack and they were based on visual observations in the tests. The compressive strengths of the concrete beams in Table 7 are the average values determined from the tests of control cylinders. The individual strengths of the concrete cylinders are listed in Table 5. The yield point strengths, ultimate tensile strengths and elongations of two No. 3 steel bars are shown in Table 6.

The load deflection curves for the corresponding beams in the three series except Beam IIIA1 were plotted on the same sheets for comparison. They are presented in Figs. 12-19. Since Beam IIIA1 was tested under special loading condition, its load deflection curve is shown on a separate sheet in Fig. 12.

The progress of cracking and the crack patterns of the beams are shown in the photographs of Figs. 20-40. The detail of each beam is also presented between the photographs of the crack patterns on both sides

of the beam for reference. The unit of the loads recorded in the pictures is given in kips.

In testing reinforced concrete beams three different stages may be distinguished: (1) Behavior prior to formation of diagonal tension cracks, (2) Behavior during diagonal tension cracking, and (3) Behavior after formation of diagonal cracks. The behaviors of the beams specimens will be discussed in the following paragraphs.

TABLE 5  
CYLINDER TESTS

Beam No.	Nominal $f_c$ (psi)	Total Load (pounds)	Comp. - stress (psi)	Ave $f_c$ (psi)
IA1	3500	121,410	4,300	4,180
		114,870	4,060	
IA2	3500	133,610	4,720	4,580
		125,170	4,430	
IB3	2000	68,760	2,430	2,270
		59,650	2,110	
IB4	3500	108,710	3,840	3,780
		105,180	3,720	
IB5	5000	144,200	5,100	5,260
		153,230	5,420	
IC6	3500	89,360	3,160	3,650
		117,030	4,130	
IC7	5000	156,020	5,510	5,420
		151,010	5,330	
IIA1	3500	119,260	4,200	4,200
		80,500	2,840*	
IIA2	3500	123,000	4,350	4,270
		118,180	4,180	
IIB3	2000	67,540	2,380	2,420
		69,620	2,460	
IIB4	3500	109,090	3,860	3,910
		111,710	3,950	
IIB5	5000	152,460	5,380	5,540
		162,080	5,700	
IIC6	3500	103,980	3,680	3,730
		107,130	3,780	

TABLE 5 - continued

Beam No.	Nominal $f'_c$ (psi)	Total Load (pounds)	Comp.- stress (psi)	Ave $f'_c$ (psi)
IIC7	5000	93,680	3,300* <sup>2</sup>	4,980
		141,580	4,980	
IIIA1	3500	847,130	2,990	2,910
		801,200	2,830	
IIIA2	3500	114,080	4,040	4,150
		120,550	4,260	
IIIB3	2000	59,610	2,100	1,810
		42,720	1,510	
IIIB4	3500	76,110	2,700	3,670
		131,000	4,630	
IIIB5	5000	64,870	2,290	2,740
		90,090	3,180	
IIIC6	3500	131,020	4,630	4,630
		-	-	
IIIC7	5000	184,660	6,520	5,490
		126,010	4,450	

Note - The area of cylinder = 28.3 sq.in.

\*<sup>1</sup>The strength of this cylinder was believed to be unreliable since it was so much different from the strengths of the three other cylinders of IIA1 and IIA2 which were made with the same mix, cured in the same way and tested on the same date.

\*<sup>2</sup>The failure of this cylinder might be due to the uneven loading from the hemispherical loading block where some Vitrobond material adhered to it. Therefore the strength of this cylinder was not considered to be representative.

TABLE 6

## TEST OF #3 STEEL BARS

	Area (sq.in)	Yield Pt. Load (lb)	Ult. Load (lb)	Yield Pt. Strength (psi)	Ult. Tensile Strength (psi)	Elongation % in 8" Long
1st bar	0.111	7,000	9,375	63,000	84,500	17.5
2nd bar	0.111	7,000	9,410	63,000	84,800	19.0
Mean	0.111	7,000	9,394	63,000	84,650	18.25



TABLE 7  
RESULTS OF TESTS OF THREE DIFFERENT SERIES  
OF BEAM SPECIMENS

Beam No.	Nominal $f'_c$ (psi)	Actual $f'_c$ (psi)	Initial D.T. Cracking Load (Kips)	Ultimate Load (Kips)	$v_u = \frac{V}{bjd}$ (psi)	$\frac{P_u}{P_c}$	Mode of Failure
IA1	3500	4180	30	32.00	198	1.07	T.
IA2	3500	4430	25	27.00	168	1.08	T.
IB3	2000	2270	34	35.00	217	1.03	T.
IB4	3500	3780	40	48.00	298	1.20	T.
IB5	5000	5260	40	41.57	258	1.04	T.
IC6	3500	3650	40	72.00	447	1.80	T.
IC7	5000	5420	44	81.98	510	1.87	T.
IIA1	3500	4200	30	76.00	472	2.53	D.T.
IIA2	3500	4270	40	94.00	583	2.35	D.T.
IIB3	2000	2420	35	76.00	472	2.17	D.T.
IIB4	3500	3910	40	103.45	643	2.58	D.T.
IIB5	5000	5540	55	128.85	800	2.34	D.T.
IIC6	3500	3730	40	95.00	590	2.38	D.T.
IIC7	5000	4980	45	115.00	714	2.55	D.T.
IIIA1	3500	2910	75	138.86	865	1.85	D.T.
IIIA2	3500	4150	65	91.36	567	1.52	T.
IIIB3	2000	1810	35	73.89	458	2.11	D.T.
IIIB4	3500	3670	55	121.27	754	2.21	T.
IIIB5	5000	2740	50	113.89	708	2.28	D.T.
IIIC6	3500	4630	70	144.00	895	2.06	D.T.
IIIC7	5000	5490	70	145.00	900	2.07	D.T.

Note: T - tension failure      D.T. - diagonal tension failure

APPLIED LOAD IN KIPS

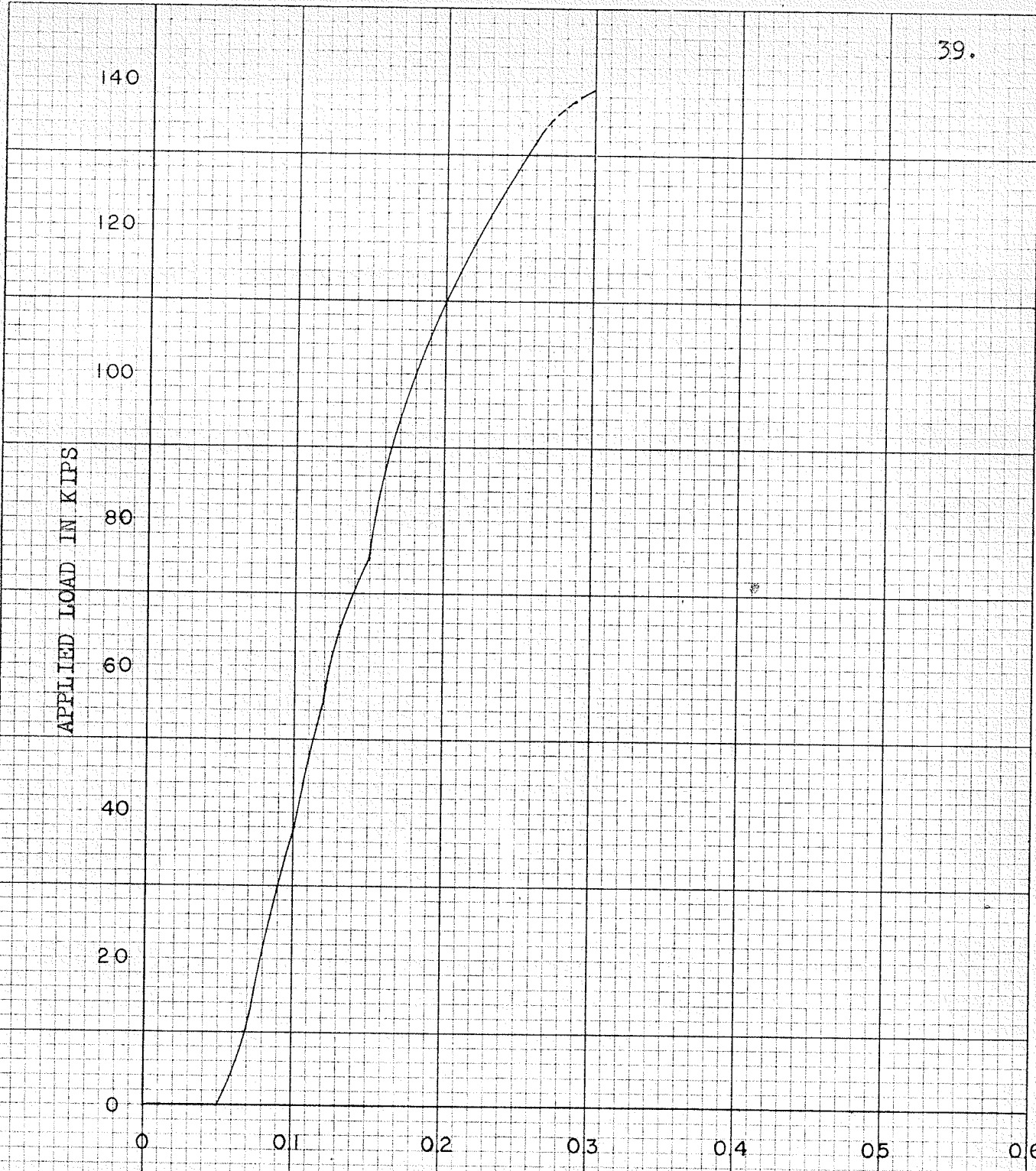
140  
120  
100  
80  
60  
40  
20  
0

0 0.1 0.2 0.3 0.4 0.5 0.6

DEFLECTION AT CENTER OF SPAN IN INCHES

FIGURE 12

LOAD DEFLECTION CURVE FOR BEAM IIIA1



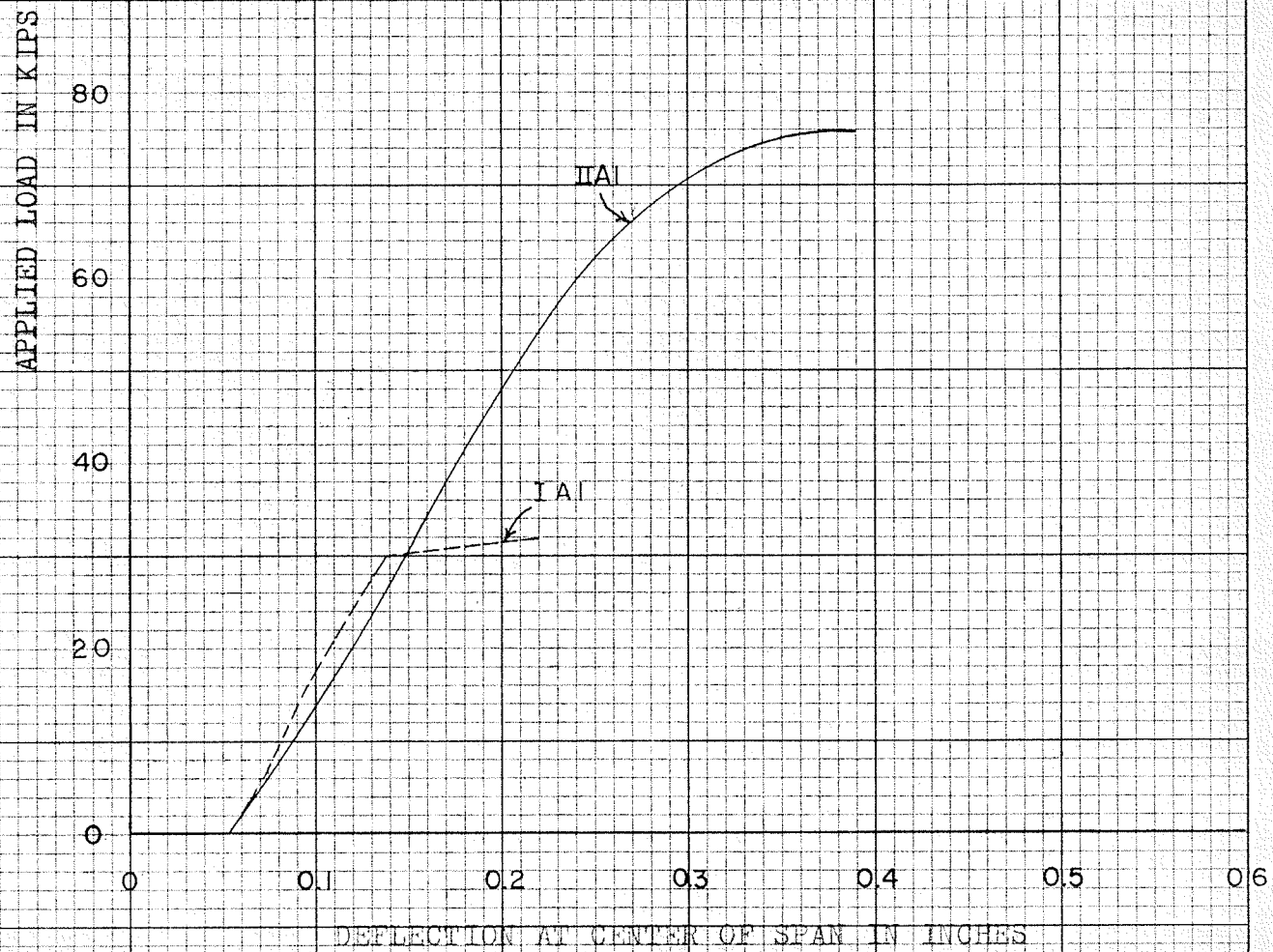
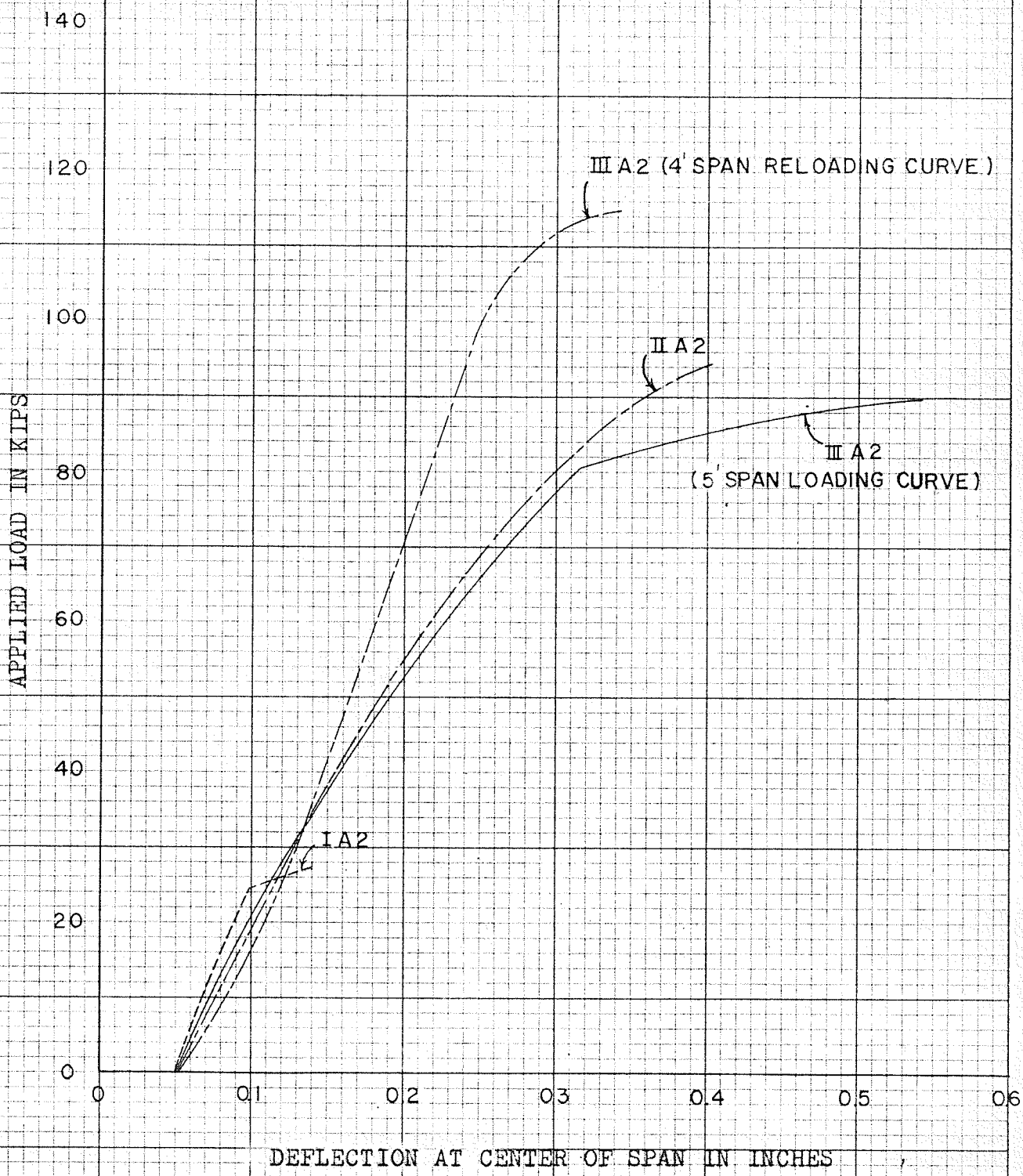


FIGURE 13

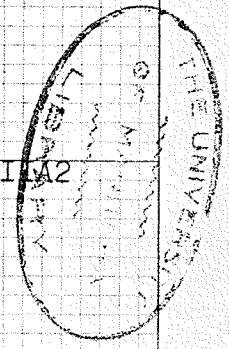
LOAD DEFLECTION CURVES FOR BEAMS IAI & IIAI



DEFLECTION AT CENTER OF SPAN IN INCHES

FIGURE 14

LOAD DEFLECTION CURVES FOR BEAMS IA2, IIA2, & IIIA2



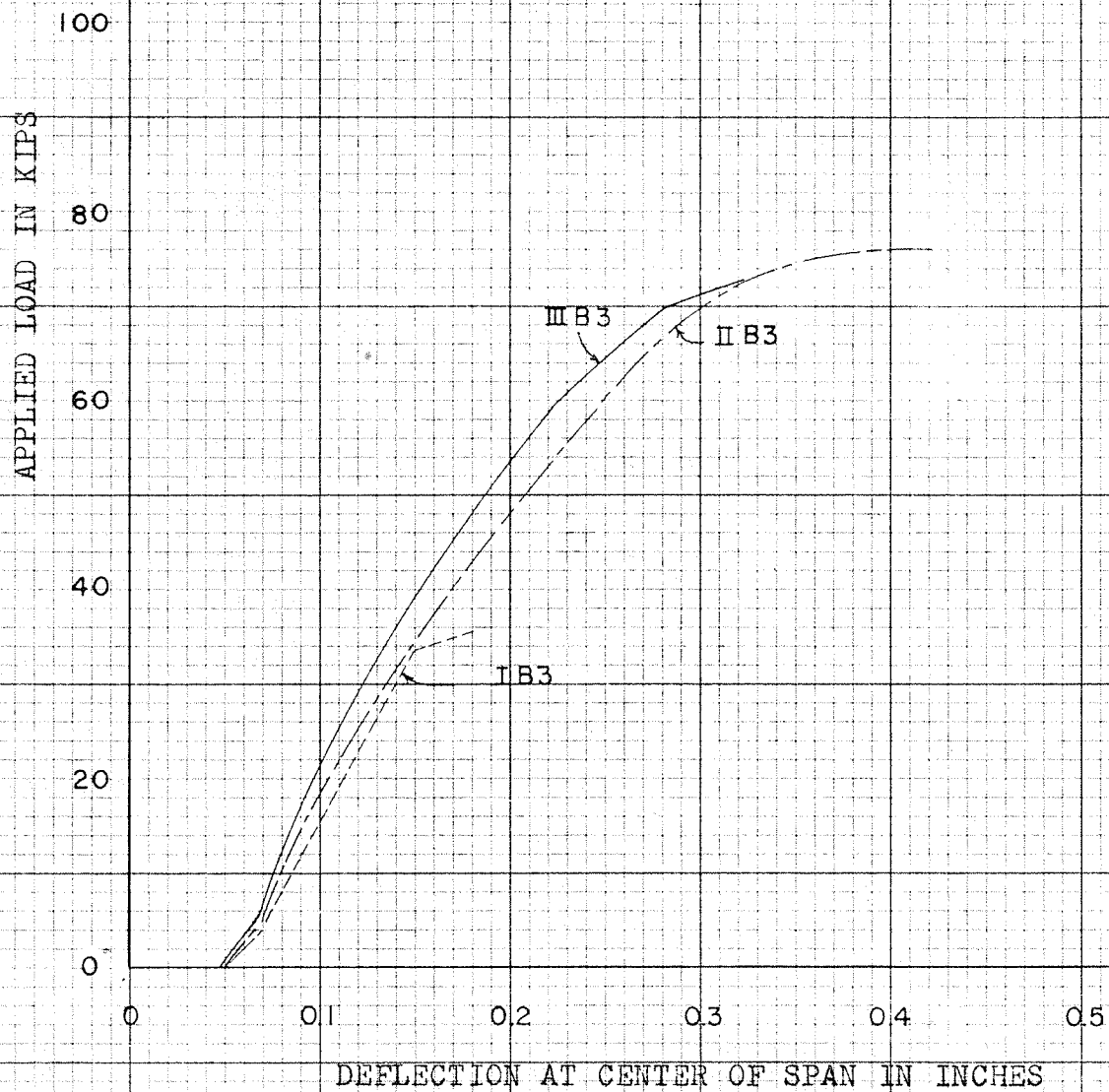


FIGURE 15

LOAD DEFLECTION CURVES FOR BEAMS IB3, IIB3, &amp; IIIB3

APPLIED LOAD IN KIPS

140

120

100

80

60

40

20

0

0

0.1

0.2

0.3

0.4

0.5

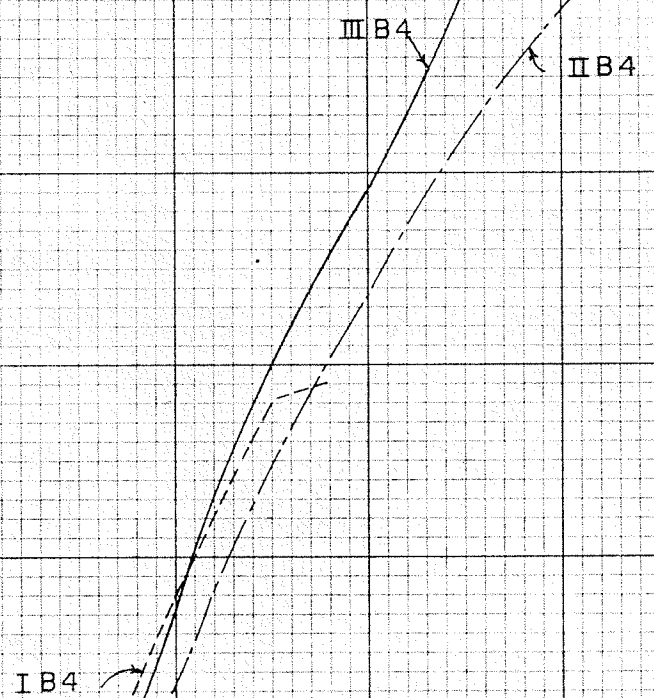
0.6

6

DEFLECTION AT CENTER OF SPAN IN INCHES

FIGURE 16

LOAD DEFLECTION CURVES FOR BEAMS IB4, IIB4, & IIIB4





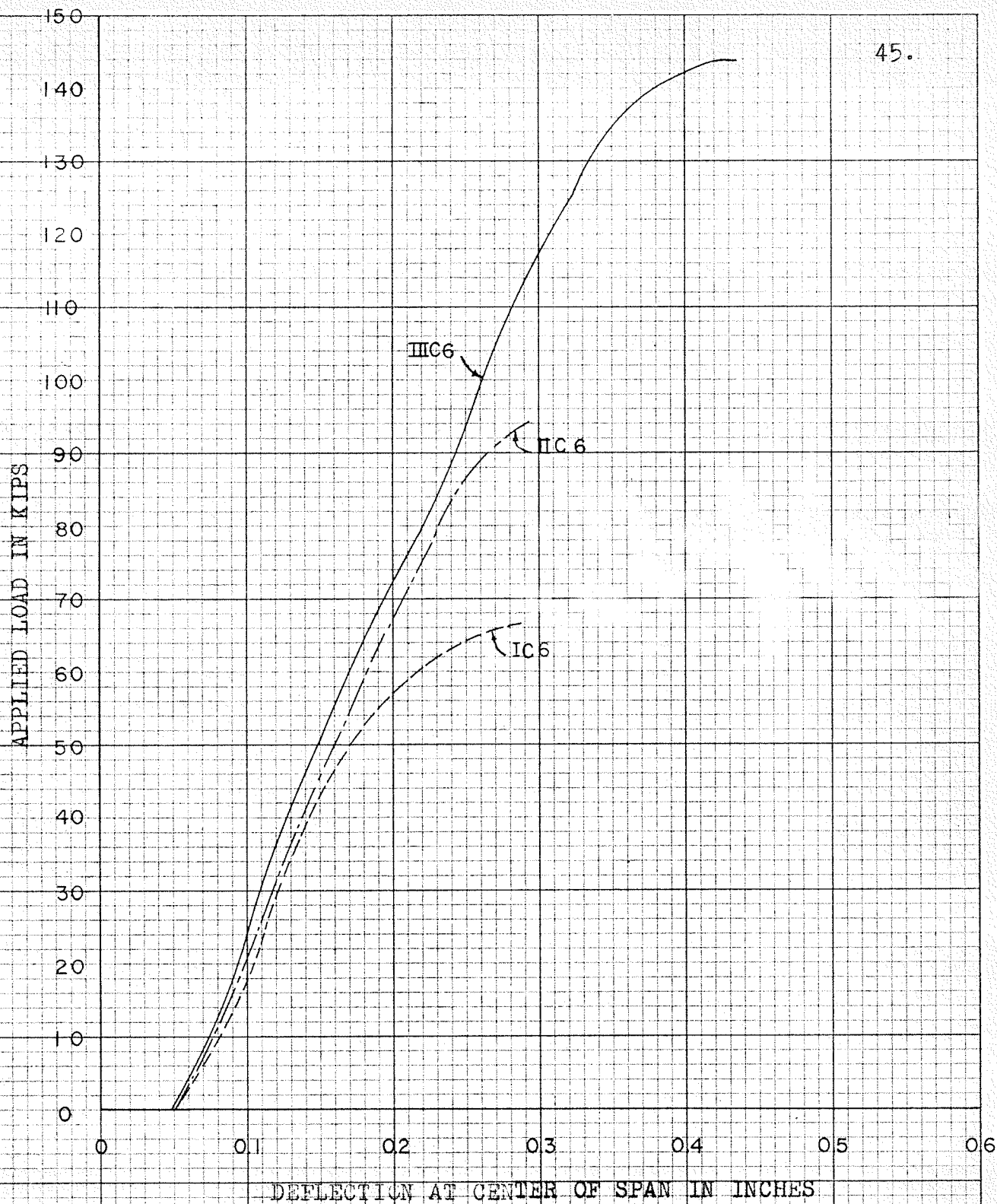


FIGURE 18

LOAD DEFLECTION CURVES FOR BEAMS IC6, IIC6, & IIIC6

140

120

100

80

60

40

20

0

APPLIED LOAD IN KIIPS

0

0.1

0.2

0.3

0.4

0.5

0.6

DEFLECTION AT CENTER OF SPAN IN INCHES

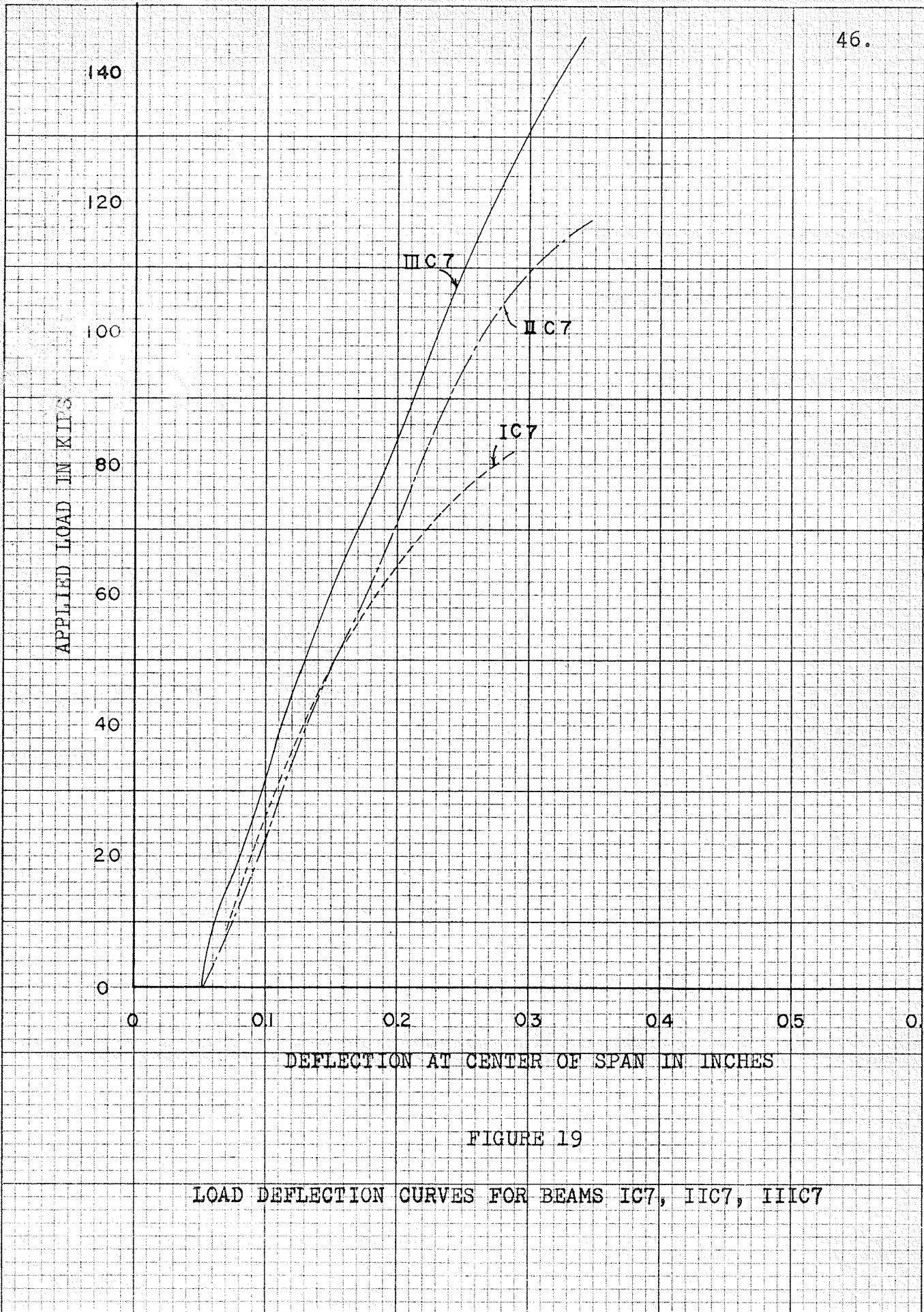
IIIC7

IIC7

IC7

FIGURE 19

LOAD DEFLECTION CURVES FOR BEAMS IC7, IIC7, IIIC7





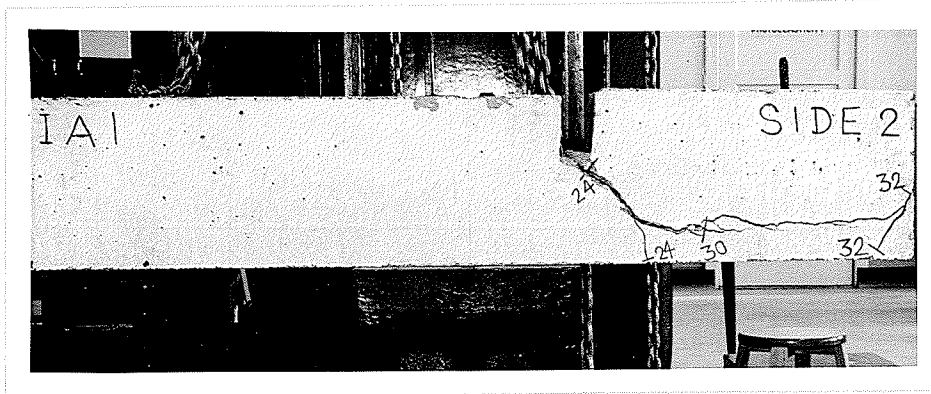
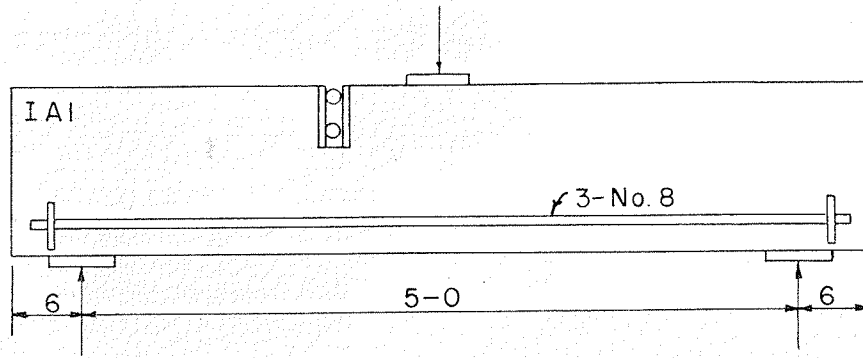
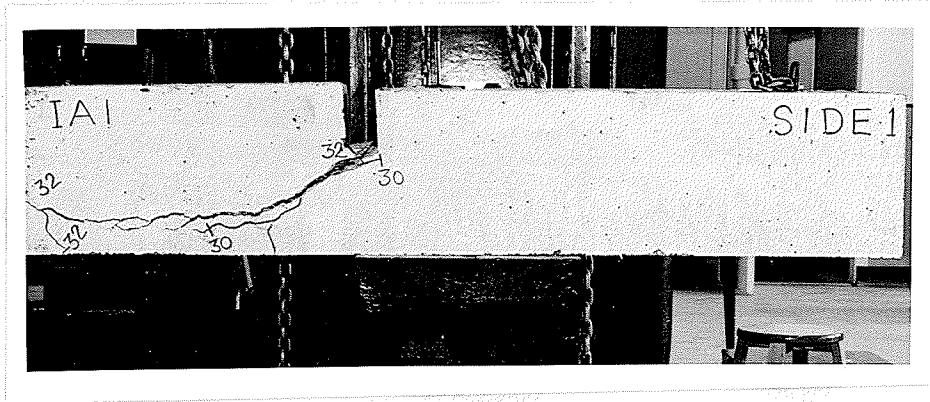


FIGURE 20

DETAIL AND CRACK PATTERNS OF BEAM IAI

SCALE  $\frac{3}{4}$ "=1'-0"

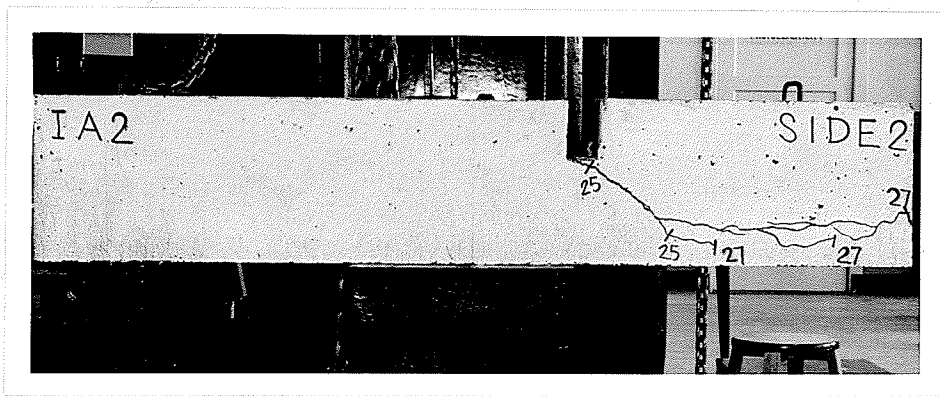
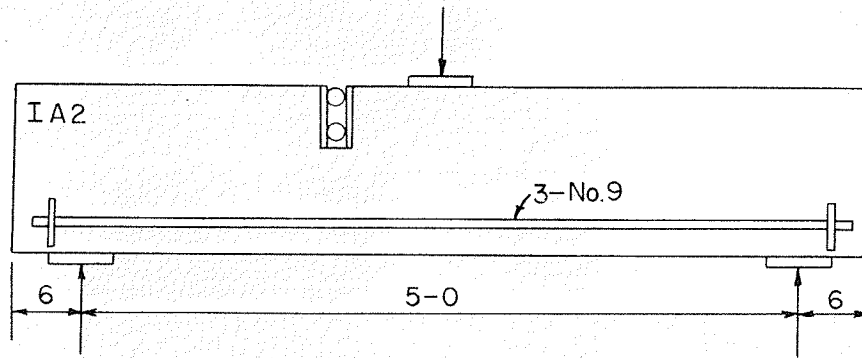
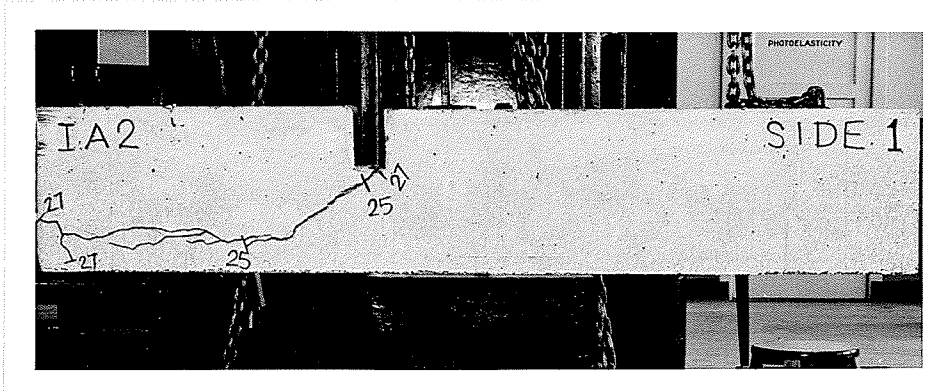


FIGURE 21

DETAIL AND CRACK PATTERNS OF BEAM IA2

SCALE  $\frac{3}{4}$ "=1'-0"

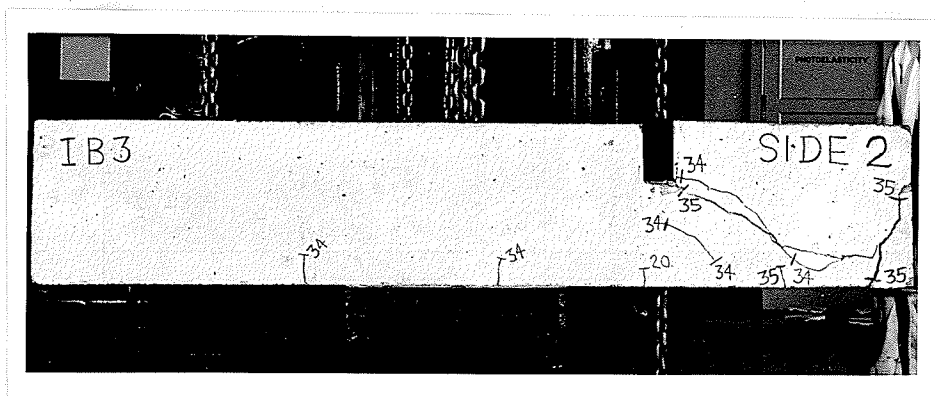
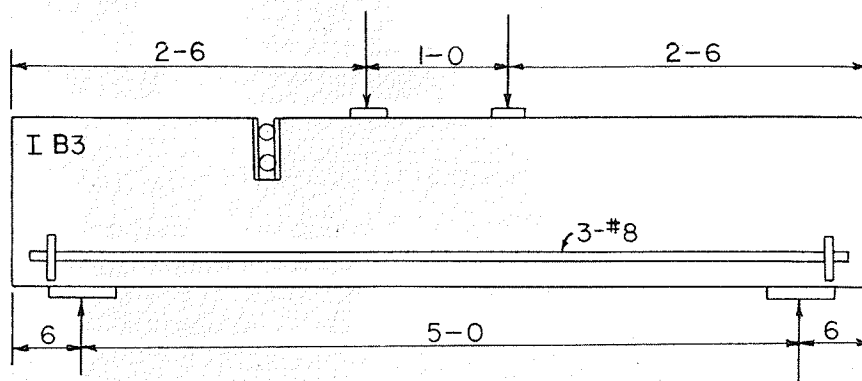
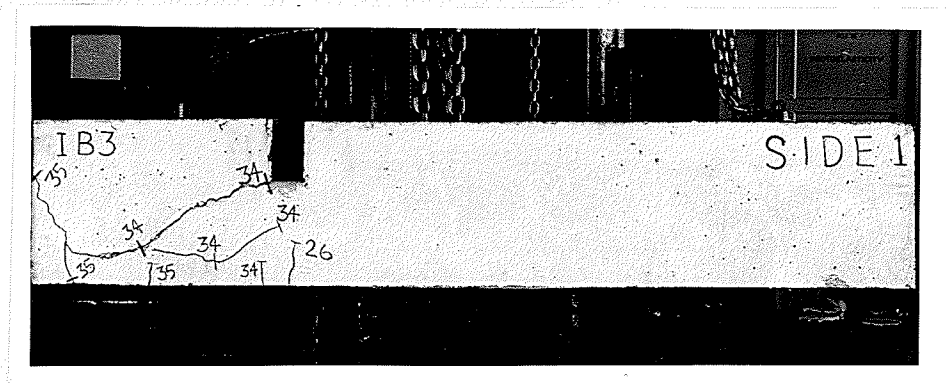


FIGURE 22

DETAIL AND CRACK PATTERNS OF BEAM IB3

SCALE  $\frac{3}{4}$ " = 1'-0"

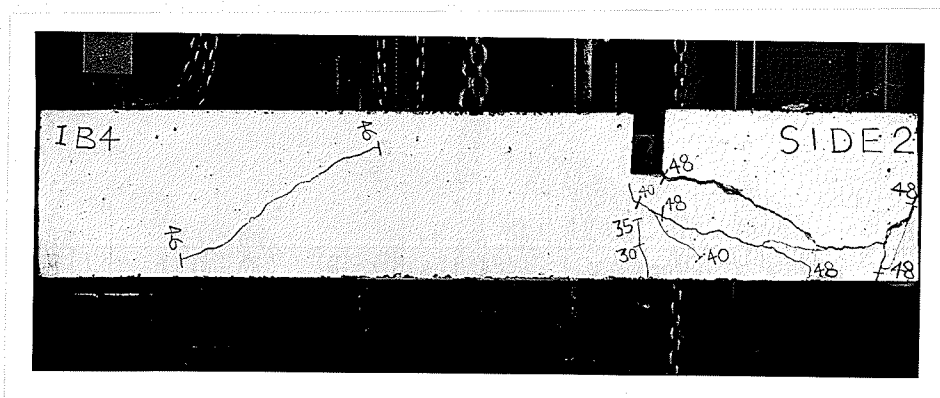
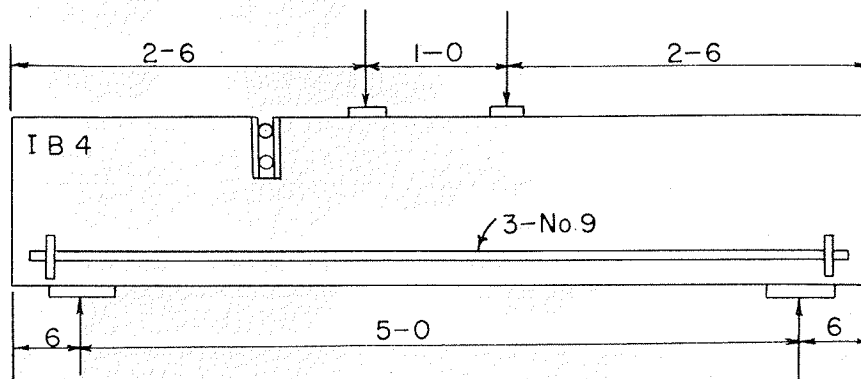
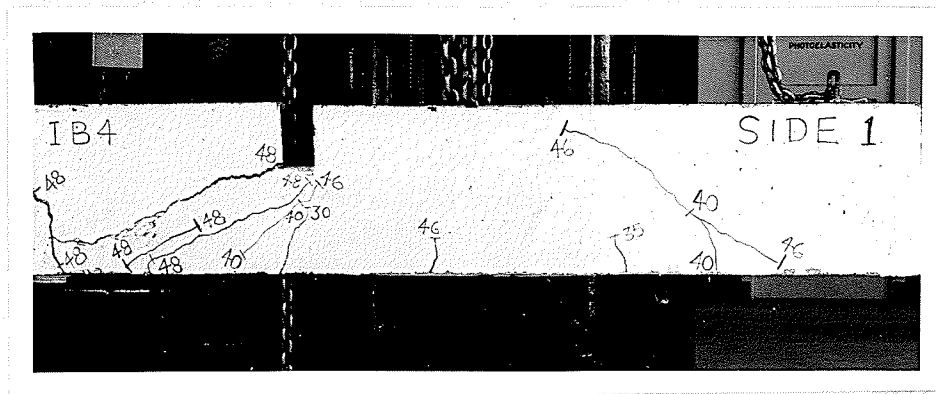


FIGURE 23

DETAIL AND CRACK PATTERNS OF BEAM IB4

SCALE  $\frac{3}{4}''=1'-0''$

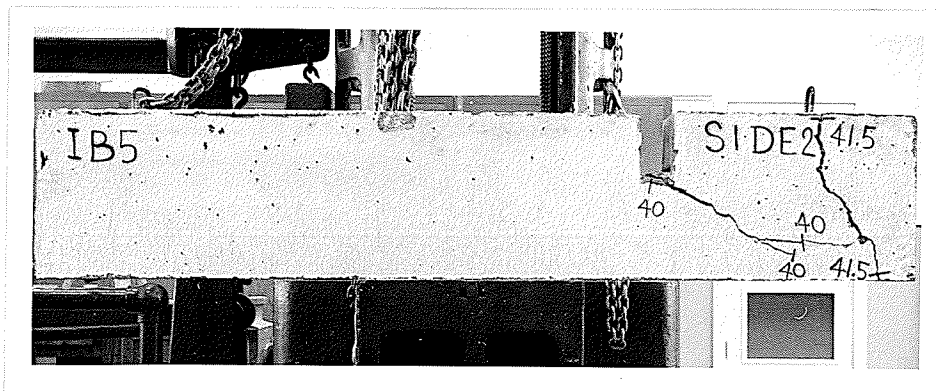
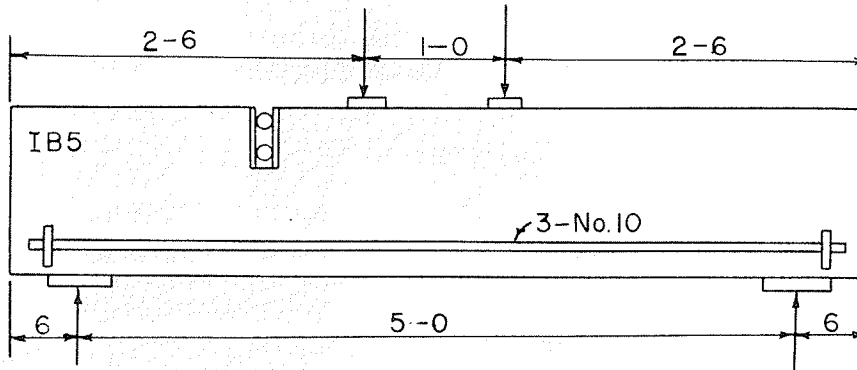
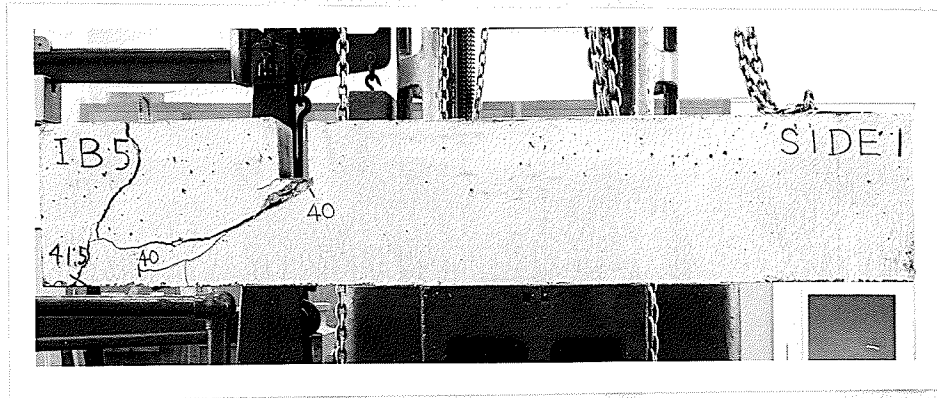


FIGURE 24

DETAIL AND CRACK PATTERNS OF BEAM IB5

SCALE  $\frac{3}{4}$ "=1'-0"

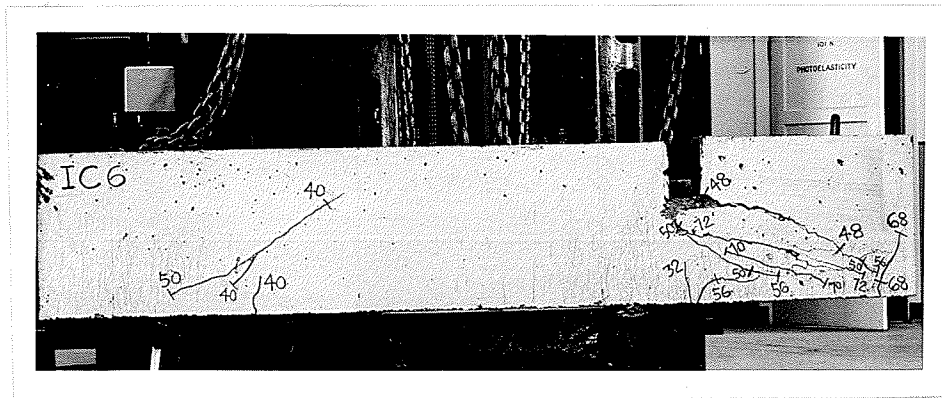
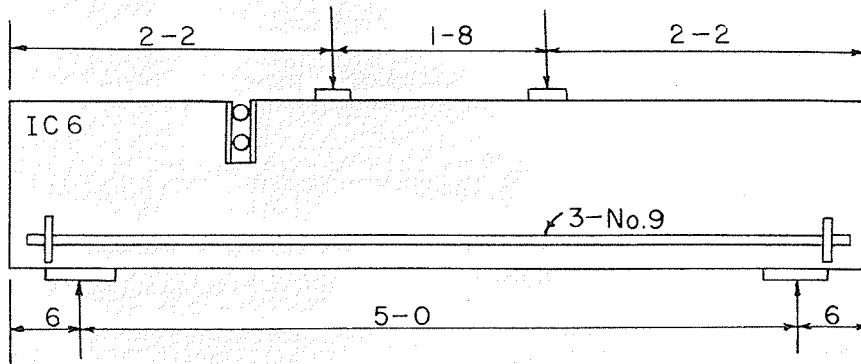
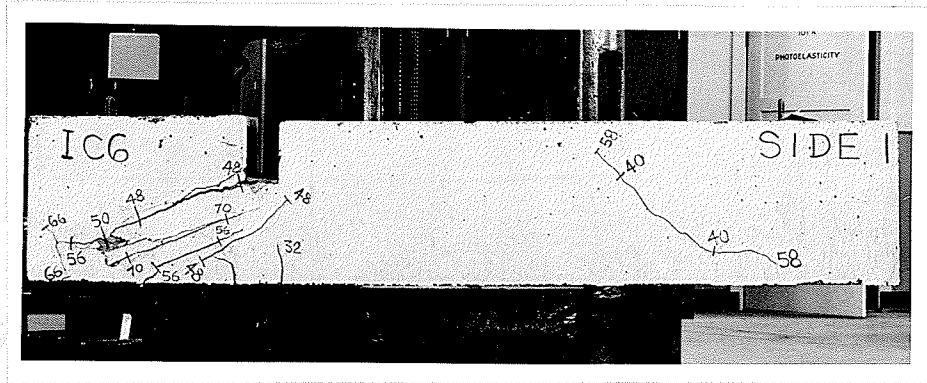


FIGURE 25

DETAIL AND CRACK PATTERNS OF BEAM IC6

SCALE  $\frac{3}{4}''=1'-0''$

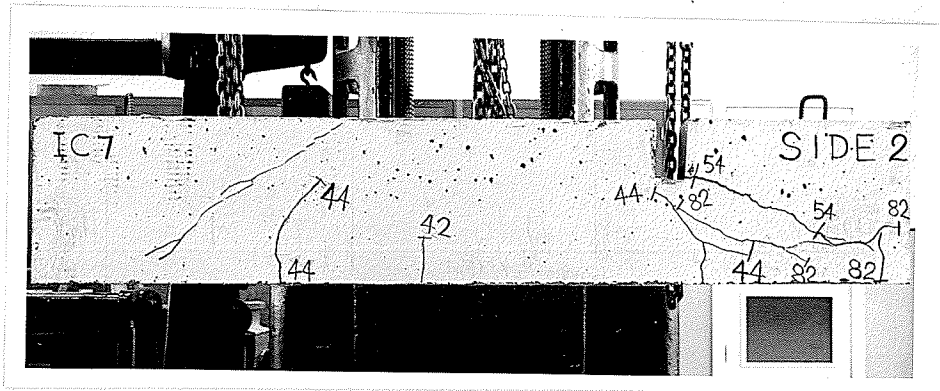
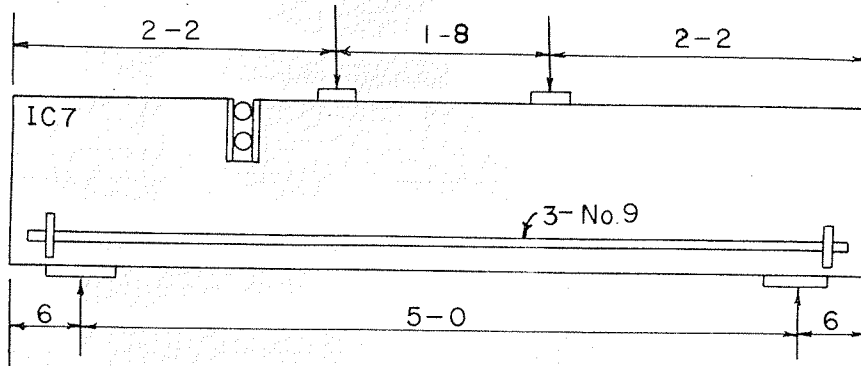
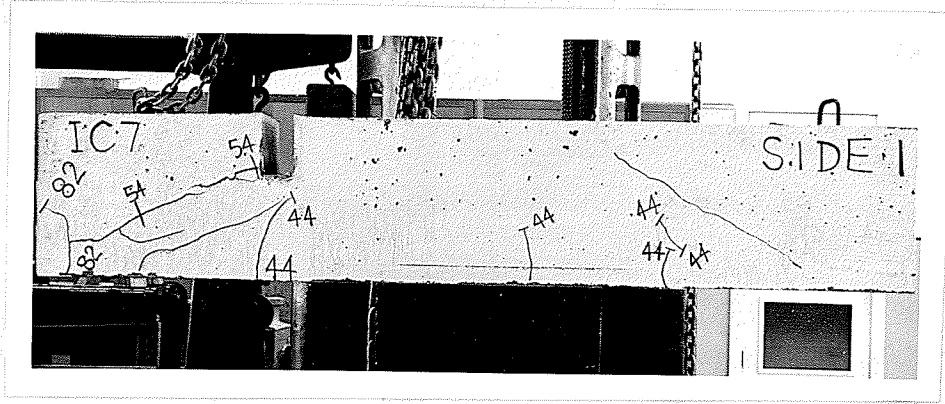


FIGURE 26

DETAIL AND CRACK PATTERNS OF BEAM IC7

SCALE  $\frac{3}{4}$ "=1'-0"

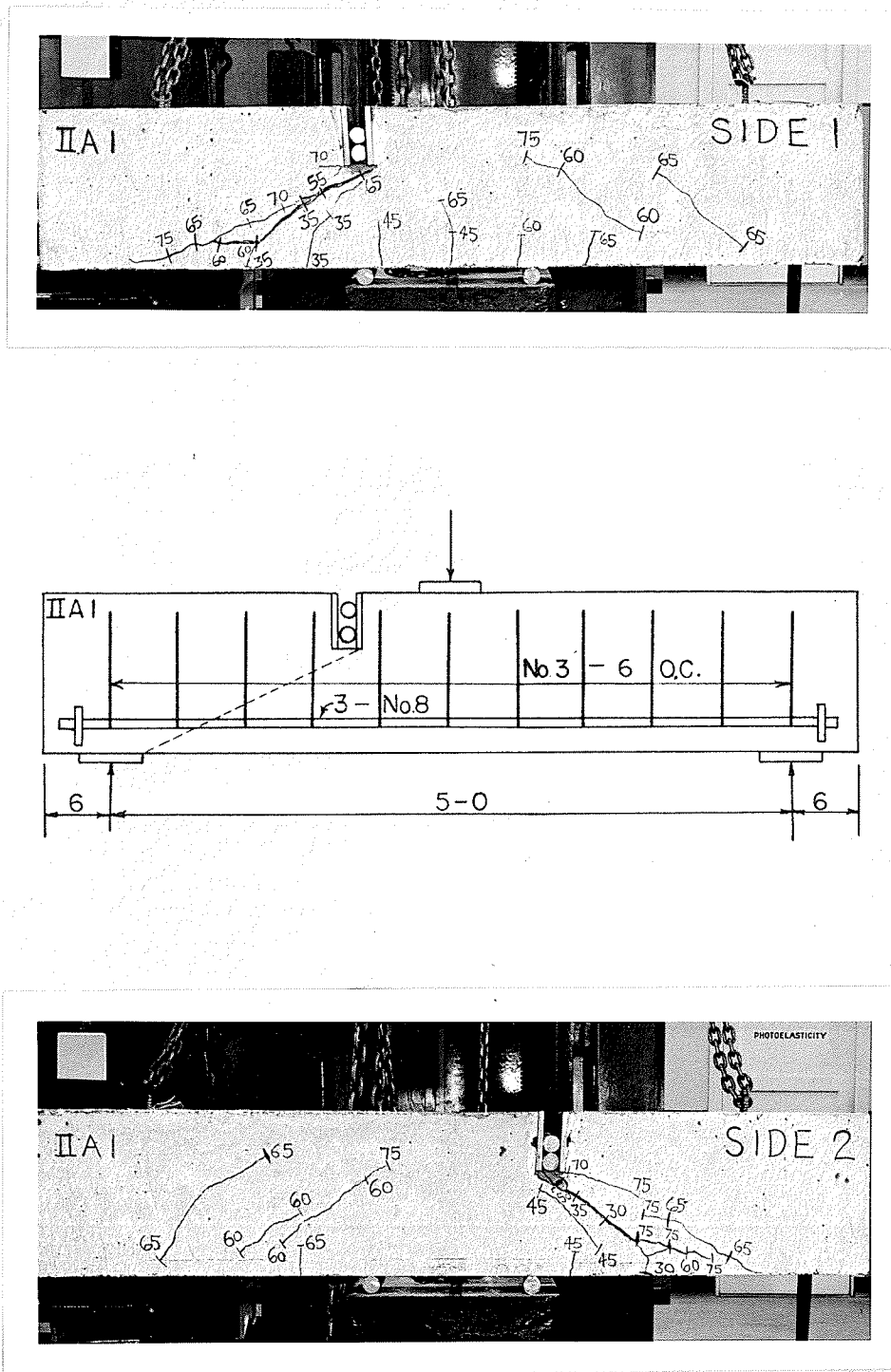


FIGURE 27  
 DETAIL AND CRACK PATTERNS OF BEAM IIA1

SCALE  $\frac{3}{4}$ " = 1'-0"



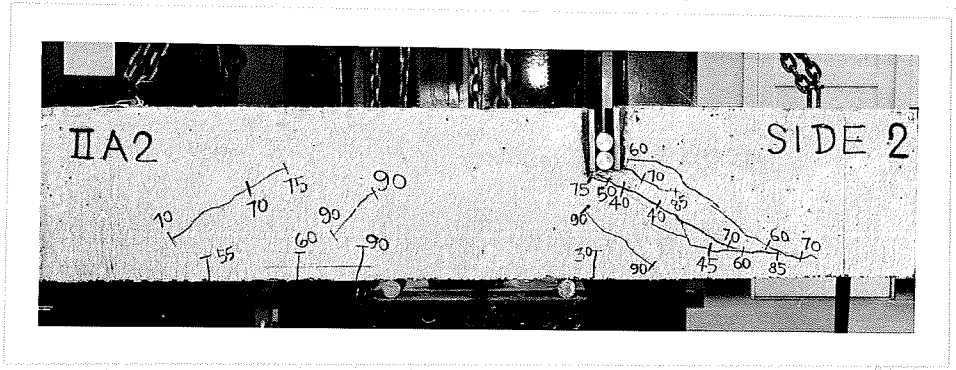
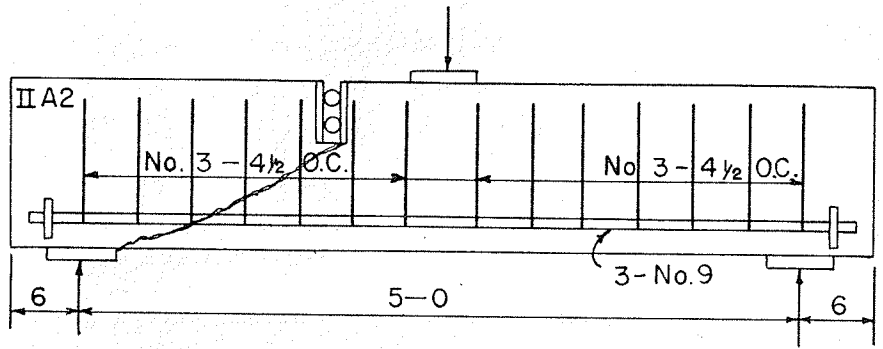
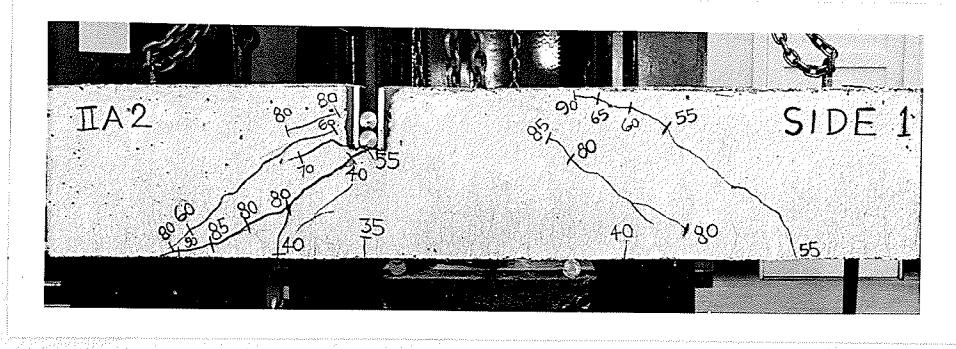


FIGURE 28  
DETAIL AND CRACK PATTERNS OF BEAM IIA2  
SCALE  $\frac{3}{4}$ "=1'-0"

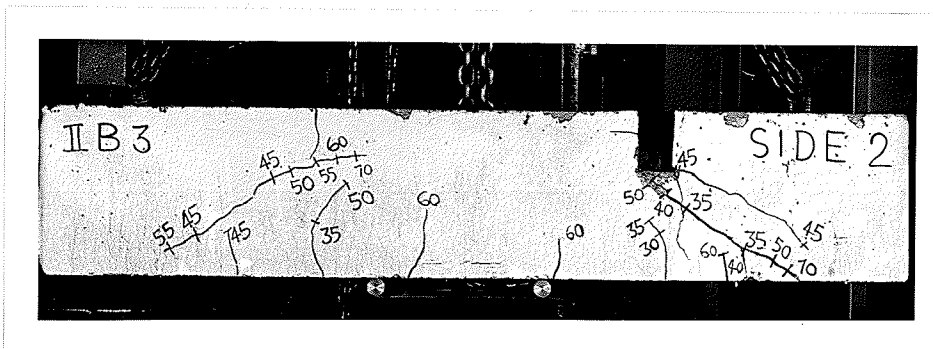
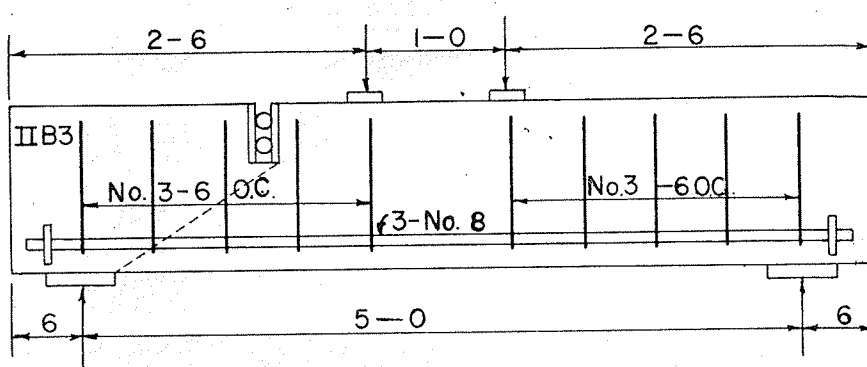
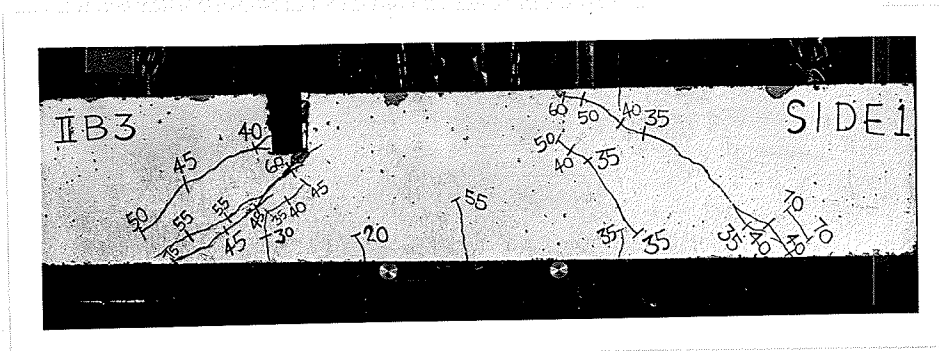


FIGURE 29

DETAIL AND CRACK PATTERNS OF BEAM IIB3

SCALE  $\frac{3}{4}$ "=1'-0"

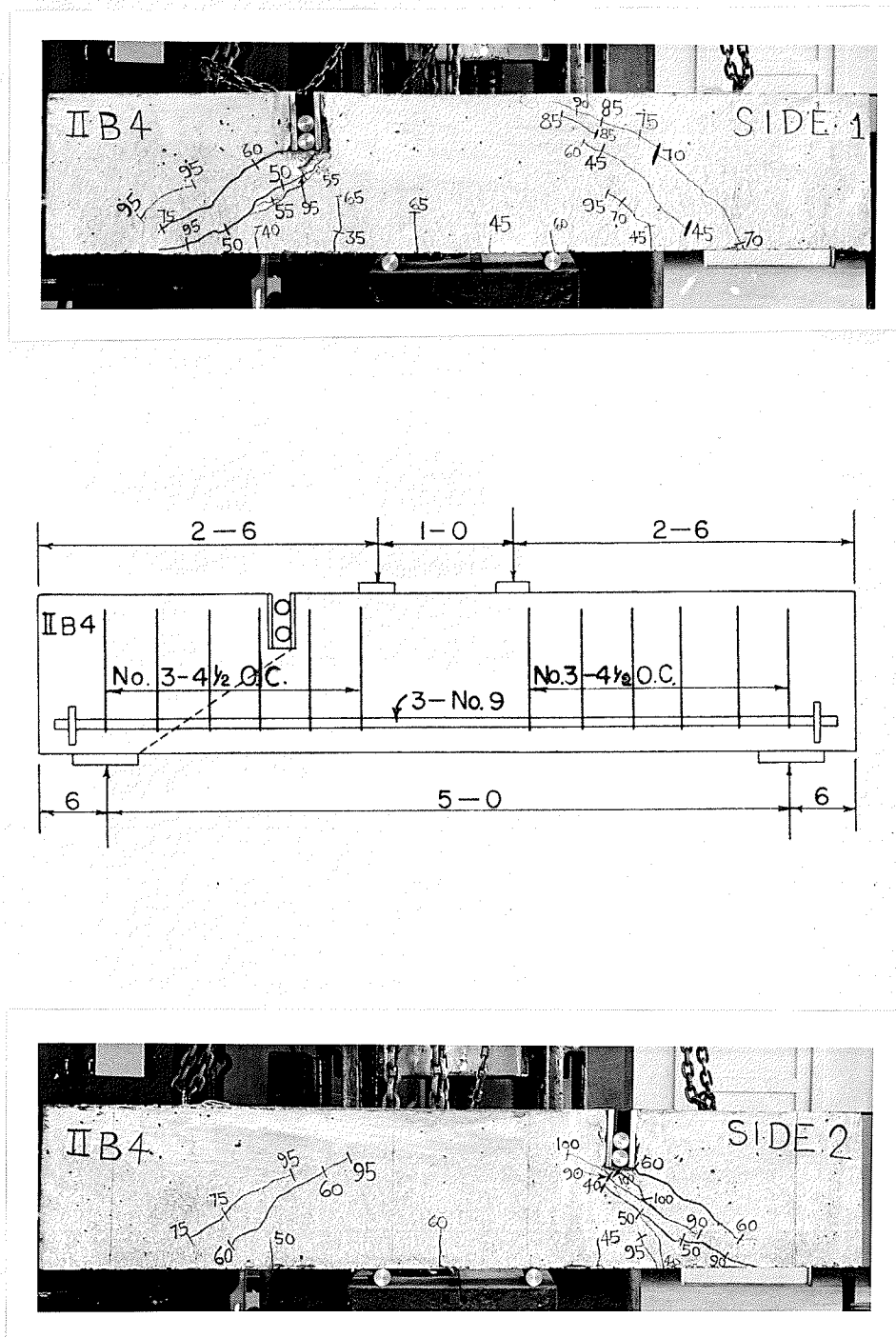


FIGURE 30

DETAIL AND CRACK PATTERNS OF BEAM IIB4

SCALE  $\frac{3}{4}$ "=1'-0"



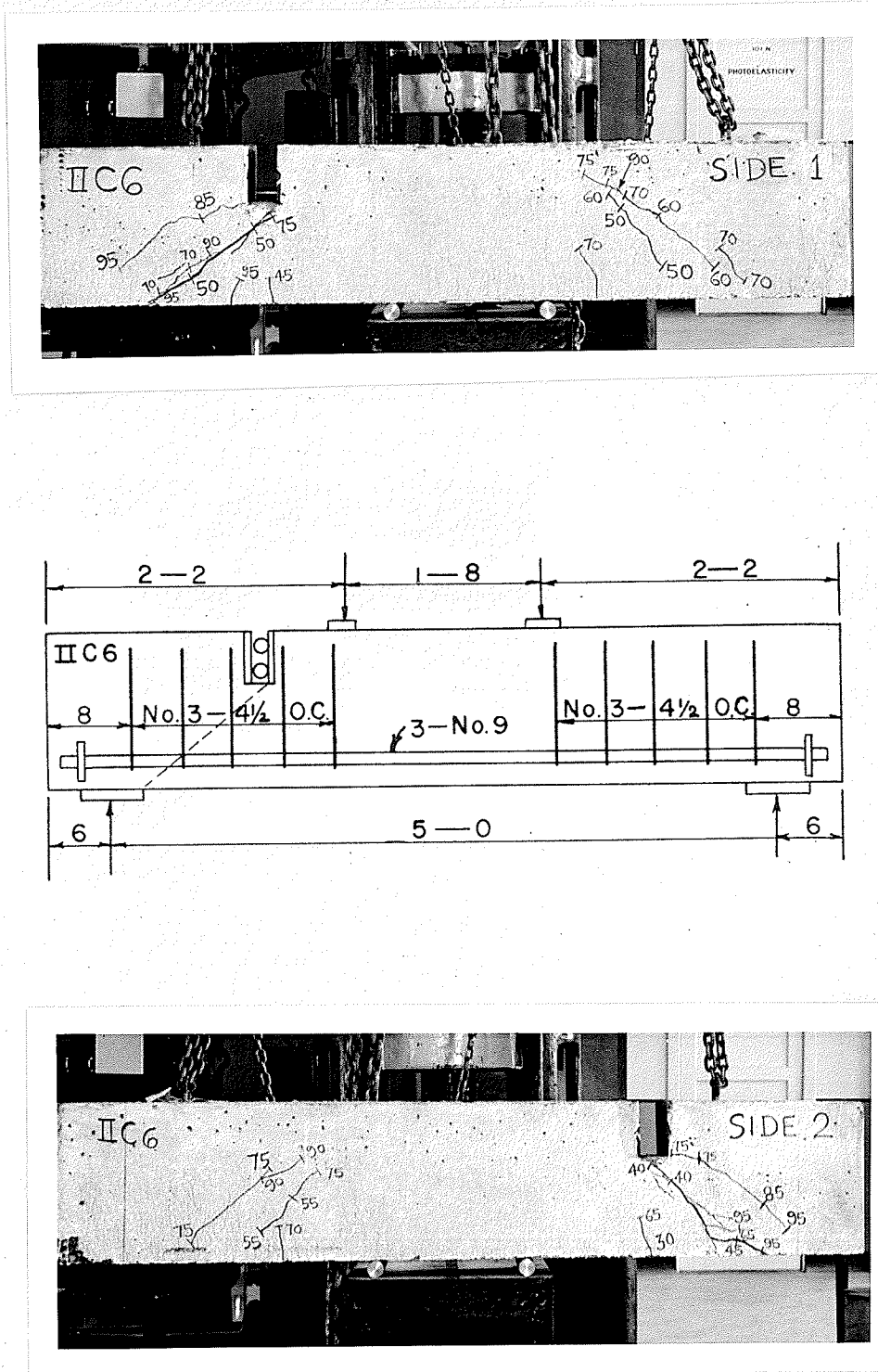


FIGURE 32

DETAIL AND CRACK PATTERNS OF BEAM IIC6

SCALE  $\frac{3}{4}'' = 1'-0''$

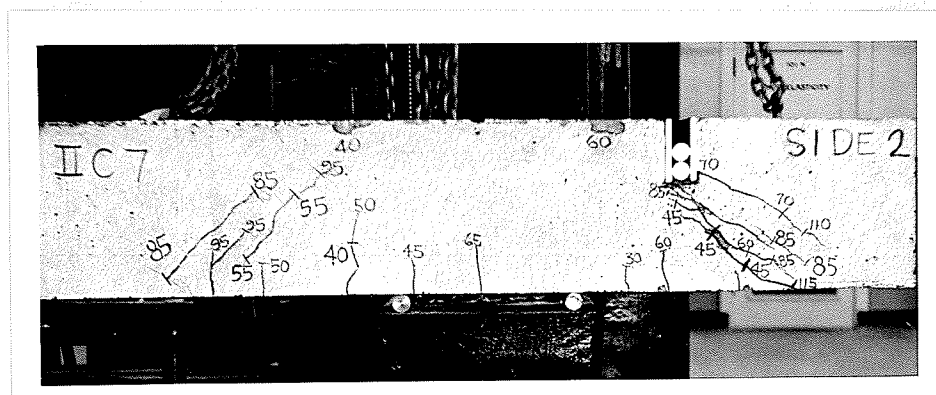
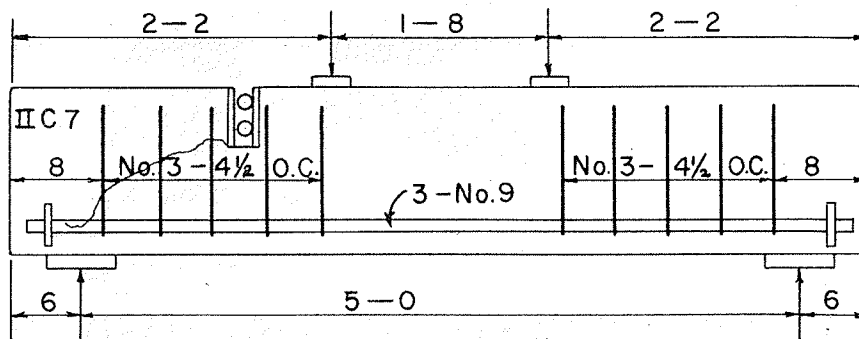
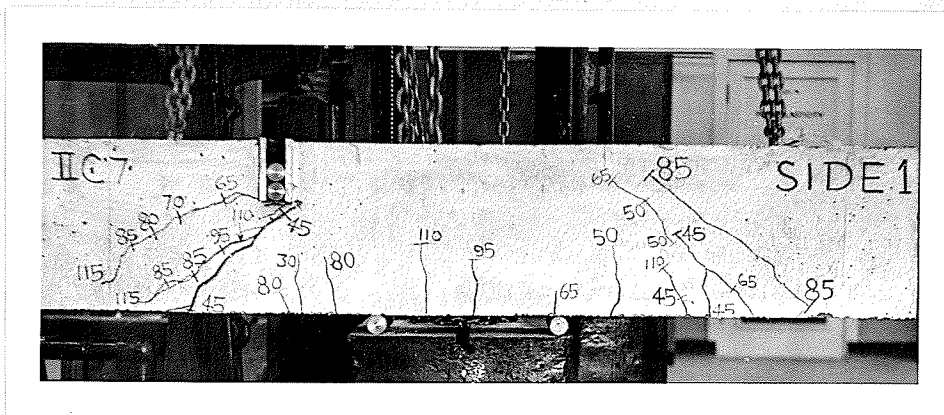


FIGURE 33  
 DETAIL AND CRACK PATTERNS OF BEAM IIC7  
 SCALE  $\frac{3}{4}$ "=1'-0"

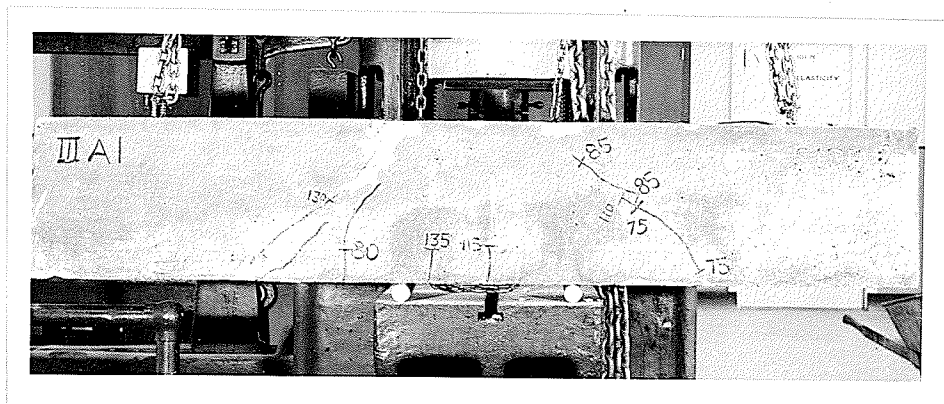
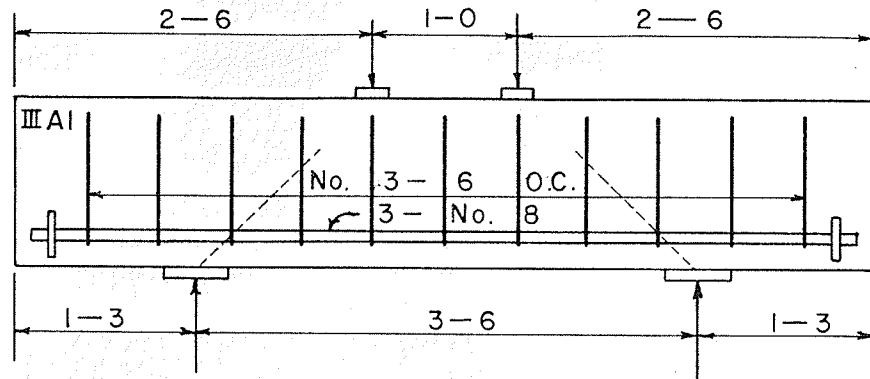
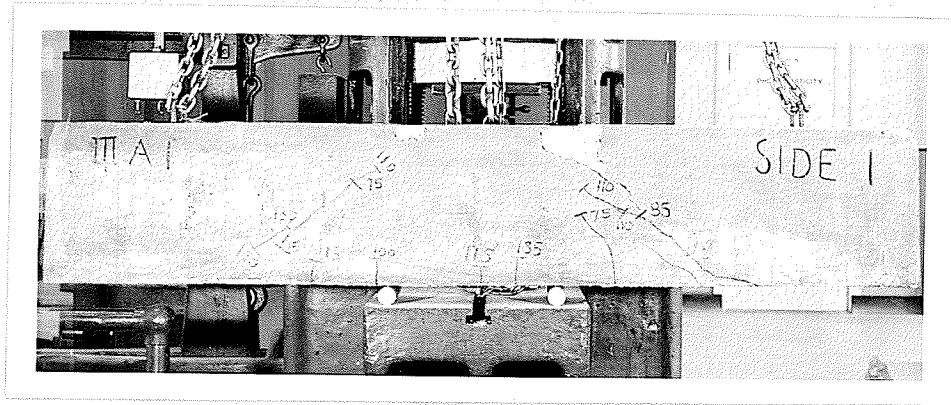


FIGURE 34

DETAIL AND CRACK PATTERNS OF BEAM IIIA1

SCALE  $\frac{3}{4}$ "=1'-0"





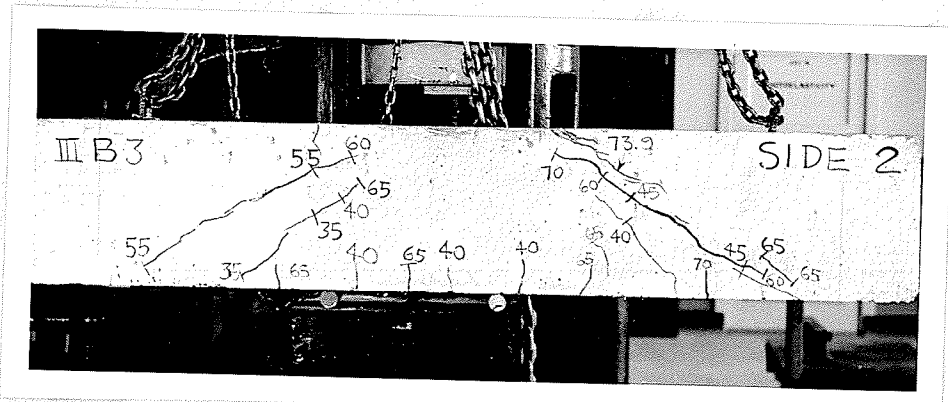
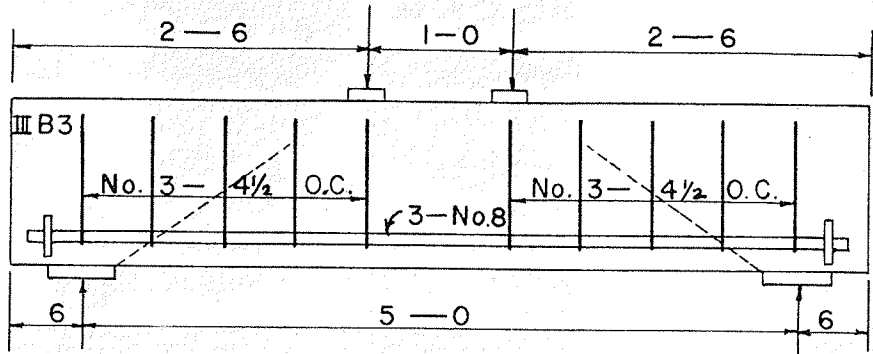
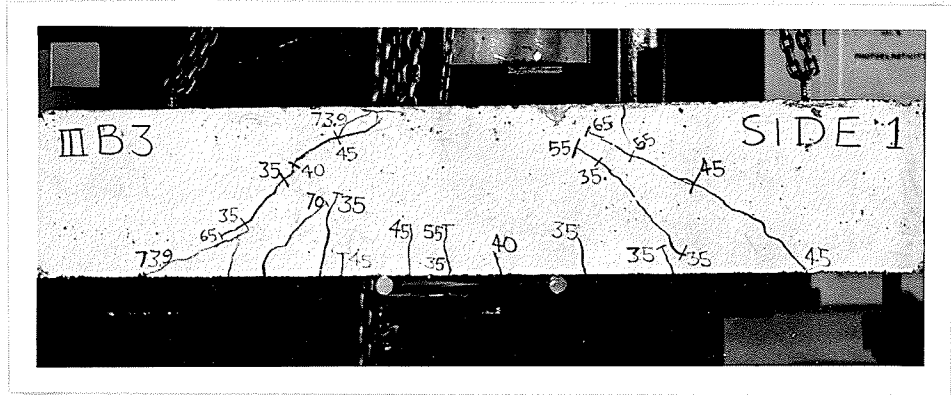


FIGURE 36

DETAIL AND CRACK PATTERNS OF BEAM III B3

SCALE  $\frac{3}{4}$ " = 1'-0"

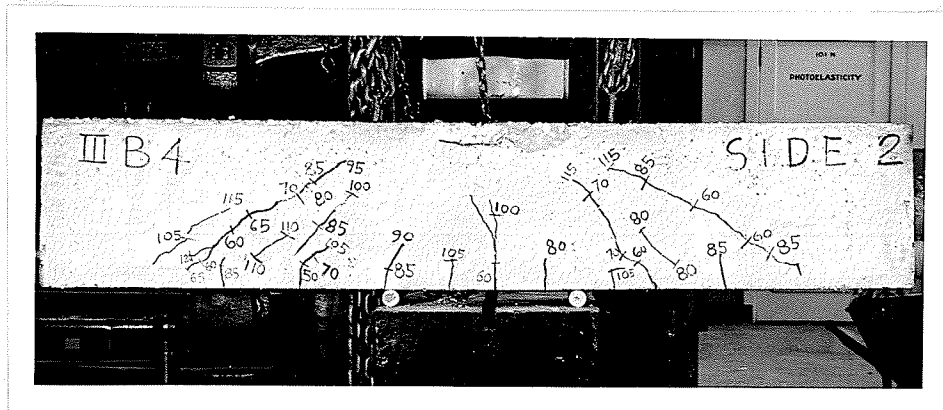
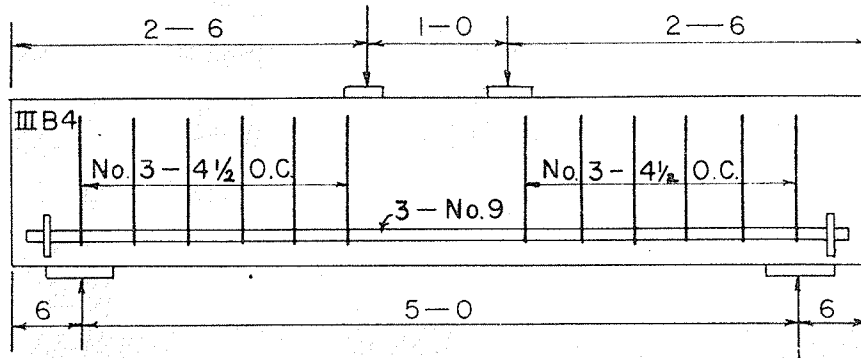
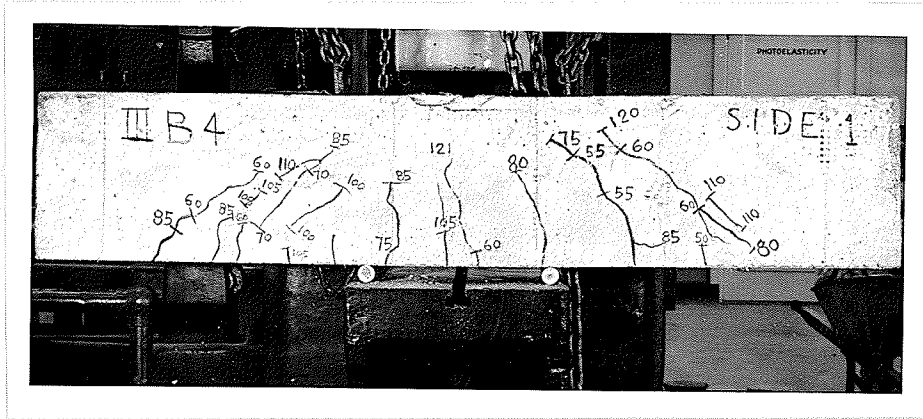


FIGURE 37

DETAIL AND CRACK PATTERNS OF BEAM III B4

SCALE  $\frac{3}{4}" = 1'-0"$

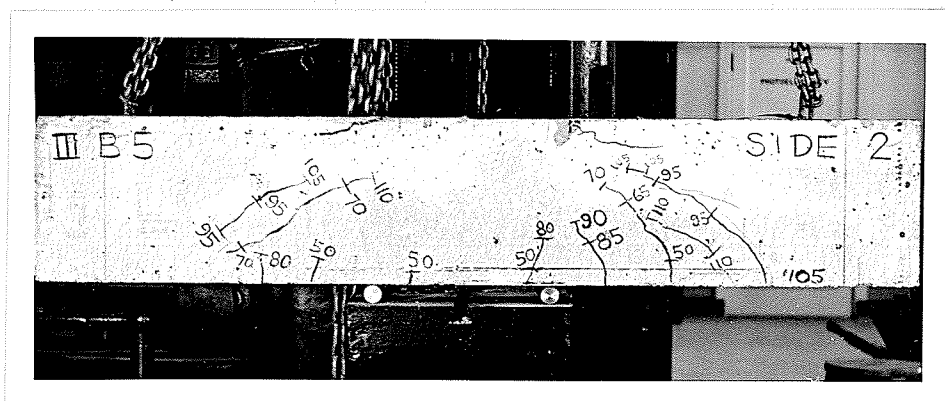
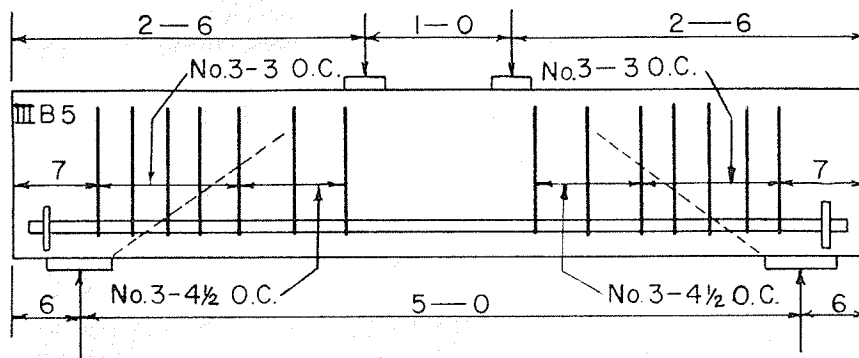
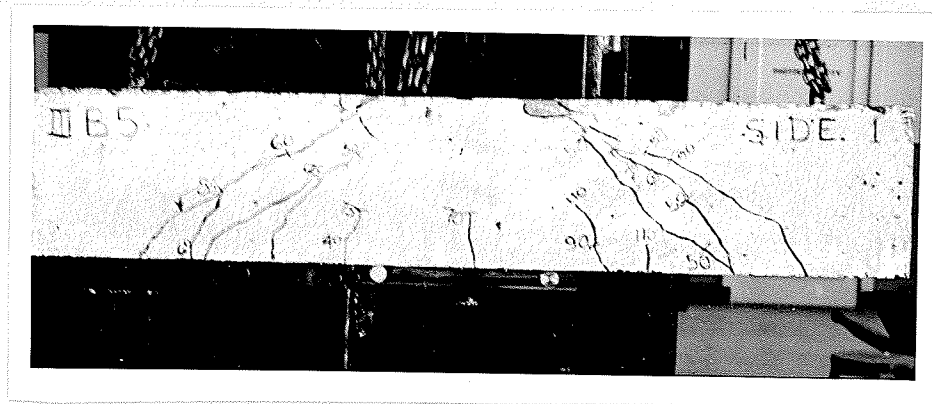


FIGURE 38

DETAIL AND CRACK PATTERNS OF BEAM III B5

SCALE  $\frac{3}{4}''=1'-0''$

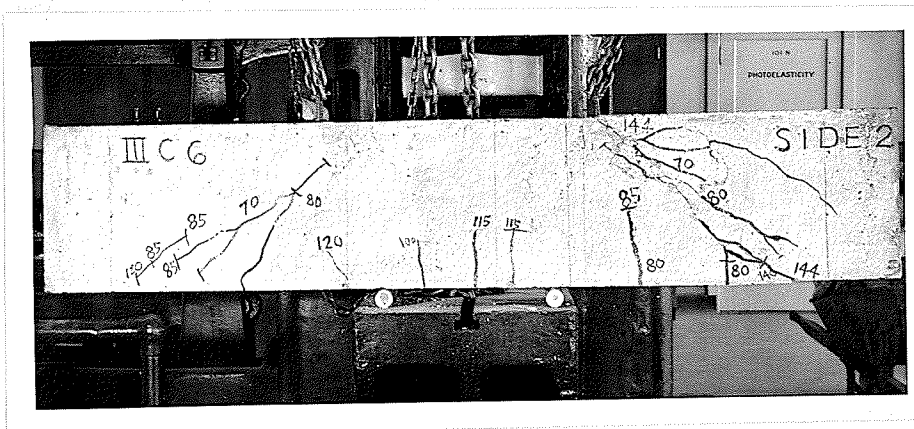
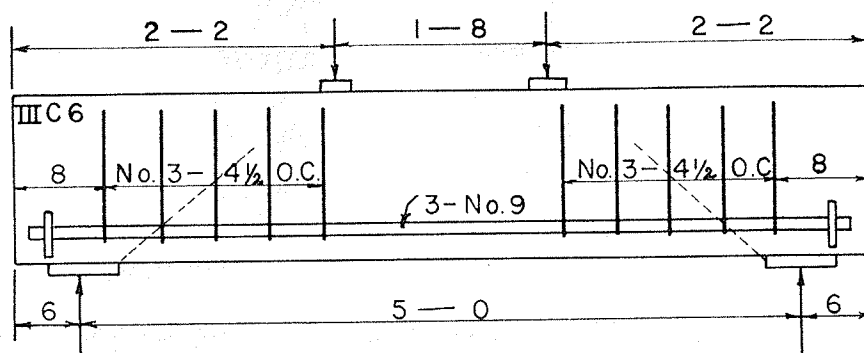
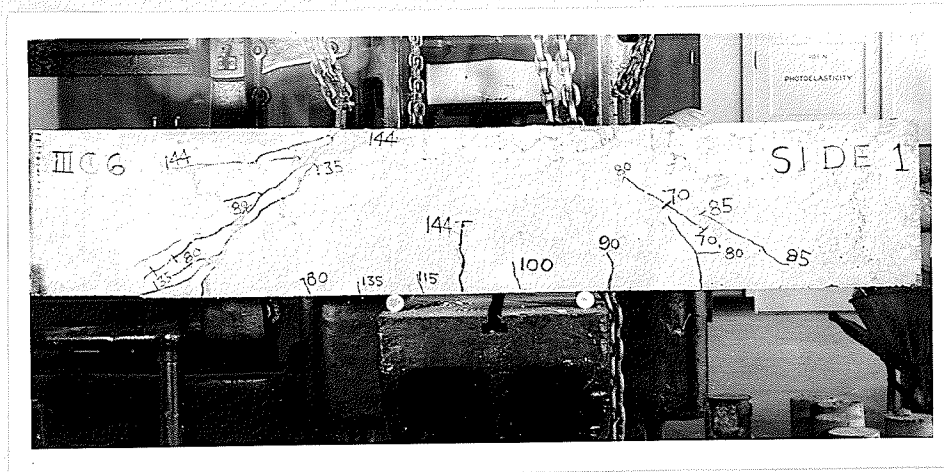


FIGURE 39

DETAIL AND CRACK PATTERNS OF BEAM III C 6

SCALE  $\frac{3}{4}$ " = 1'-0"

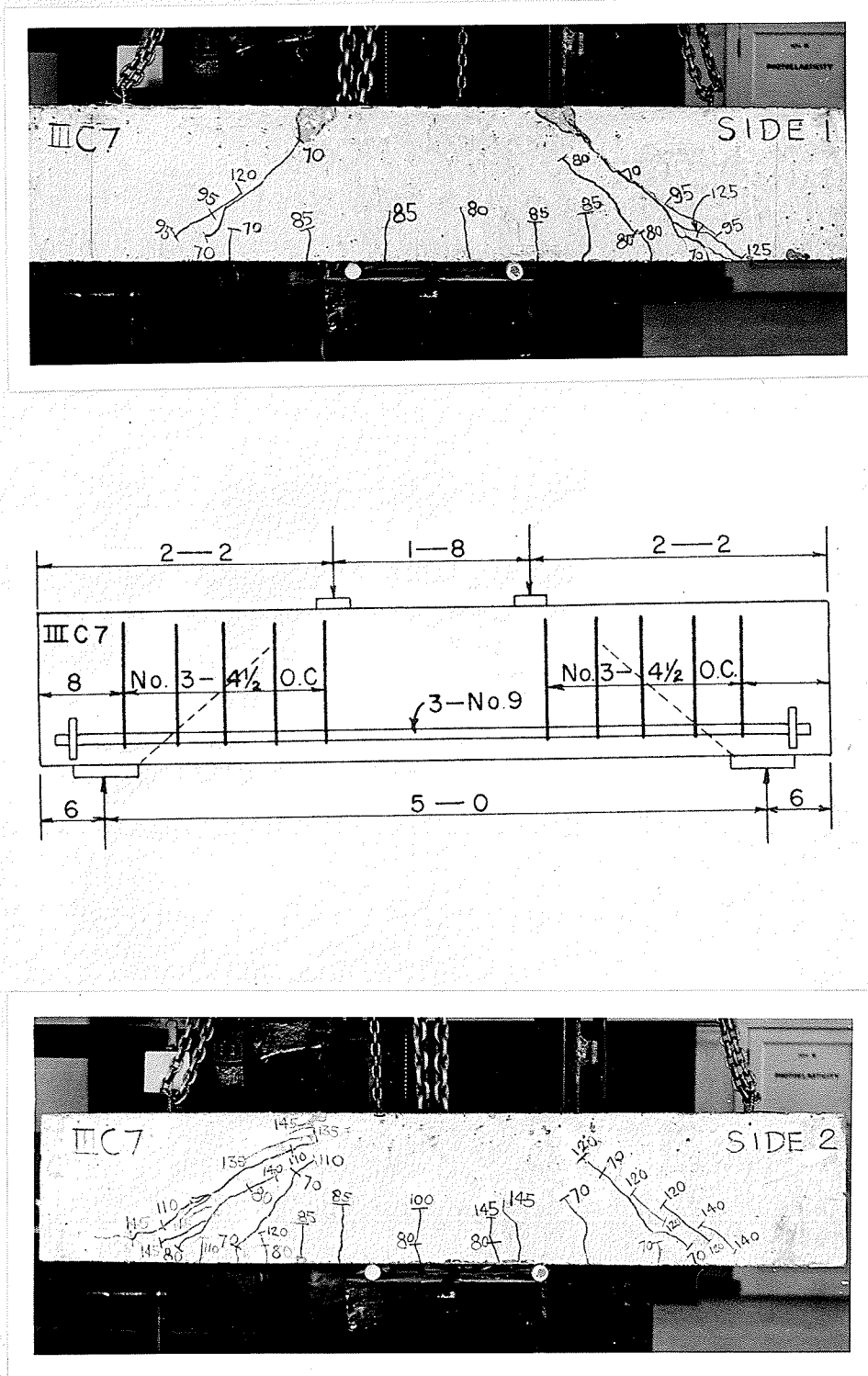


FIGURE 40

DETAIL AND CRACK PATTERNS OF BEAM IIIC7

SCALE  $\frac{3}{4}$ "=1'-0"

(1) Behavior prior to formation of diagonal tension cracks

In Figs. 12-19 it will be noted that the portions of the load deflection curves below the cracking load are rather straight and they are more or less parallel for the corresponding beams in the three series. These indicate that prior to the formation of initial diagonal tension cracks the behavior of all beams was essentially elastic and the presence of the web reinforcement had practically no effect on the behavior of the beams. In a few beams some tension cracks formed from the bottom surface of the beams and spread vertically upward.

(2) Initial diagonal tension cracking In Series I the initial diagonal tension cracks usually occurred from the bottom of the gap inclined to the tensile reinforcement. In some beams these cracks were curved down from the gap to the bottom surface of the beams. Again some beams had initial cracks starting above the tensile reinforcement and below the gap. Beam IC6 had an initial crack formed in the shearing span not containing the gap and Beam IC7 had initial cracks in both shearing spans of the beam at the same time.

In most beams of Series II the cracks were initiated at the bottom of the beam and inclined toward the gap. In the two other beams the diagonal tension cracks initiated at the bottom of the gap and were inclined downward. None of the beams in this series had initial crack on the shear spans not containing the gaps. The magnitudes of the cracking loads of the corresponding beams in Series I and II were nearly the same. This suggests that the initial cracking loads were immaterial of the presence of the web reinforcement.

In all beams of Series III the diagonal tension cracks started at or above the tensile steel bars and were inclined upward. The cracking loads were comparatively higher than the other two series of beams. It was believed that the concentrated shearing stress increased the intensity of diagonal tension in the beams of Series I and II. Consequently, these two series of beams cracked at lower loads.

(3) Behavior after the formation of diagonal tension cracks

In five beams of Series I under "A" and "B" Loadings the beams failed suddenly with little increase of load after the formation of initial diagonal cracks. The ratios of the ultimate loads to the cracking loads were from 1.03 to 1.20. After failure the cracks extended along the longitudinal steel and along the anchored washers. In Beam IB4 a second set of diagonal cracks formed above the original cracks simultaneously with failure. The second set of diagonal cracks were the cracks which caused the failure of the beam. In all beams the major cracks were wide open and the portions of the beams above the cracks were completely or almost completely separated from the beams. The wide opening of the cracks can be visible in the photographs of these beams in Figs. 20-24 and are indicated in the flat portions at the ends of the load deflection curves in Figs. 12-17. No crack on the opposite half of the beam was found except Beam IB4.

For the other two beams of Series I under "C" Loading the major cracks developed following the formation of initial cracks. These cracks were curved down from the corner of the gap and opened wide as soon as they appeared. However, the beams could still resist the increase of loads, but greater deflections resulted. More cracks

formed when the loads were close to the failure loads. At failure the major cracks extended in the shapes as the other beams in this series and opened wider than when they began to form. The crack patterns of these two beams are shown in Figs. 25 and 26. The failure loads of these two beams were comparatively higher in this series. The ratios of ultimate loads to the cracking loads were 1.80 and 1.87.

All the beams in Series I failed in the same mode. The broken parts of Beam IB3 are shown in Fig. 41. In this picture it is evident that the inclined curved plane was opened up by diagonal tension. However, the prints of the steel bars on the concrete of the broken part were very clear and these indicate the opening up of the horizontal plane along the reinforcement was due to tension. These prints could have been sheared off if the horizontal cracking plane was opened up by shear force. Therefore the mode of failure may be considered to be tension failure.

For the beams of Series II the upper end of the initial crack extended closer to the inner corner at the bottom of the gap and the lower end extended or split into a new crack toward the support as the applied load increased. Then, one or two diagonal cracks formed in the opposite half of the beams. As further load was applied on the beams, another diagonal crack developed from the outer corner of the gap in most beams and extended down to the longitudinal steel. At higher load some more closely spaced diagonal cracks formed in some beams yet the major crack was usually the one from the inner corner of the gap inclined to the edge of the support bearing plate. At failure these major cracks were open wide and were



quite distinct from the rest of the cracks. Most of them can be easily seen in the photographs in Figs. 27-33 as they were accentuated in ink. Because of the presence of the web reinforcement the width of these cracks were much narrower in comparison with those in Series I. The failure loads were much higher than those in Series I. The ratio of the ultimate loads to the cracking loads were from 2.17 to 2.58 and the nominal shearing stresses ranged from 472 to 800 psi. All the beams in Series II failed in diagonal tension.

In the beams of Series III five out of seven beams failed in diagonal tension and the remaining two, Beams IIIA2 and IIIB4, failed in tension. The behaviors of the beams were different for different modes of failure. For the beams which failed in diagonal tension in this series, the diagonal tension cracks extended further from both ends of the initial cracks and similar cracks formed in the opposite half of the beams as the load increased. Then several diagonal tension cracks formed near cracks which had already formed. With further increase of load the upper ends of the diagonal cracks extended toward the load. The lower end of at least one crack approached the supports while the others inclined down to the bottom surface of the beam at about the mid way of the shear spans. At failure one of the diagonal cracks reached the top edge of the beams or a new diagonal crack formed there. In some of these beams several diagonal cracks were open to the same width. But in the other beams major cracks could still be distinguished because only these cracks were open wide at failure. These major cracks usually occurred from the inner edge of the bearing plates of the supports to those of the loads. The crack patterns of

the beams which failed in diagonal tension are shown in Figs. 34, 36, 38, 39, and 40.

The ratios of the failure loads to the cracking loads varied from 1.85 to 2.28 and the nominal shearing stresses were from 458 to 900 psi.

For Beam IIIA2 the diagonal cracks were very short as shown in Fig. 35. The initial diagonal crack did not develop as the load increased. The tension cracks formed at the central part of the beams and extended more or less vertically upward. With further increase of load some more short diagonal cracks formed. They were approximately parallel to the initial diagonal cracks. At failure the tension cracks developed up to the mid depth of the beam and these cracks were open instead of the diagonal cracks. The beam failed in tension.

For Beam IIIB4 several diagonal cracks formed in both the shear spans. These cracks extended much longer than those in Beam IIIA2. However, tension cracks in the middle part of the beam developed at the same time. At failure the tension cracks were open wide but not the diagonal tension cracks. It was doubtful whether the beam would fail in diagonal tension if the beam were reloaded with a four foot span and a center load. In reloading the beam in this way there was no further development in the diagonal tension cracks nor in the tension cracks. The beam did resist a higher load. At failure some cracks formed in the compression zone of the central part of the beam. This indicated that the beam failed in compression under reloading condition. Both the

loading and reloading curve are presented in Fig. 14 and the crack pattern of the beam is shown in Fig. 37.

Originally Beam IIIA1 was planned to be loaded as the usual "A" Loading, having a center load and a five foot span. The shearing capacity of Beam IIIA1 was less than that of Beam IIIA2 because of the weaker concrete strength and smaller amounts of both tensile and web reinforcement in the former beam. In the testing program Beam IIIA2 was tested before that of Beam IIIA1. After having understood that Beam IIIA2 failed in tension, it was decided to test Beam IIIA1 to a special loading with 3'-6" span and with loads six inches from the center of the beam in order to avoid tension failure. As expected, it failed in diagonal tension under the special loading condition. The behavior of this beam was similar to the other beams in Series III which failed in diagonal tension.

CHAPTER VIII  
ANALYSIS OF TEST RESULTS

Since the mode of failure for the beams of Series I was not in diagonal tension and the crack patterns of the beams in the other two series did not entirely follow the cracks as assumed in Chapter V, some modifications to the method of analysis which had been set up in that chapter had to be made. However, the general principles were still believed to be applicable. The purpose of this chapter is to discuss the probable reasons for the failure of beams in Series I and to present an analysis of the test results. From the results and the information of other investigations in this field, a tentative conclusion is possible for the magnitudes of shear resistances offered by web reinforcement, tension steel, and uncracked concrete together with the effect of ratio of beam depth to shear span.

Study of failure for beams in Series I As already mentioned in the previous chapter the final failure of all beams in Series I was due to the opening up of cracks along the longitudinal steel by tension following the formation of diagonal tension cracks. The sketch in Fig. 42 shows a typical crack pattern of a beam in this series. Fig. 42a represents a free body diagram above the center plane of the tension steel in the left part of the beam just before the opening of the horizontal cracks. It is obvious that the compressive force  $C_1$  and the reaction  $R_1$  are two of the forces acting on the free body. No stress can exist across the diagonal crack. In order to maintain equilibrium it seems logical to consider that there is a

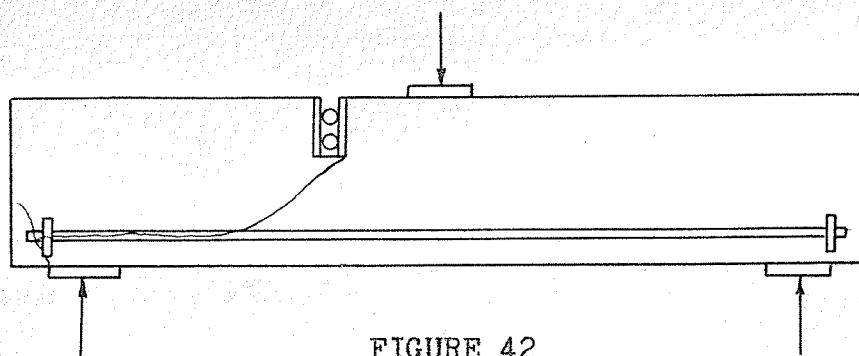


FIGURE 42

TYPICAL FAILURE FOR BEAMS OF SERIES I

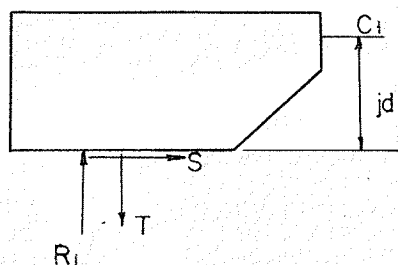


FIGURE 42A

FREE BODY DIAGRAM FOR BEAMS OF SERIES I  
AFTER THE FORMATION OF DIAGONAL CRACK

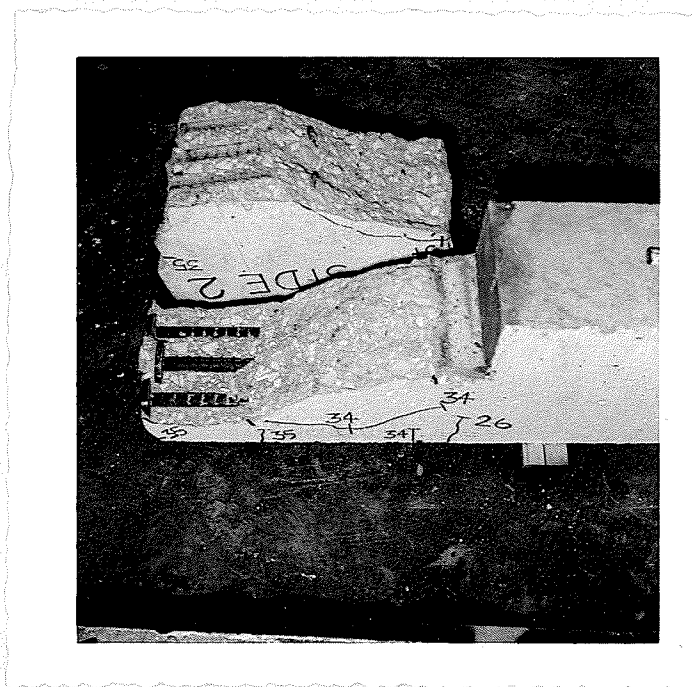


FIGURE 41

BROKEN PARTS OF BEAM IB3 SHOWING TYPICAL VIEW  
OF BEAMS OF SERIES I AFTER FAILURE

horizontal shear force  $S$  acting along the center plane of the longitudinal steel in the opposite direction of the compressive force  $C$  and that there is a vertical tension force  $T$  against the reaction  $R_1$ . It is believed that this tension force is the force causing failure in Series I. The couples formed by the horizontal forces and the vertical forces must be equal in magnitude and opposite in direction. So Eqs. (21), (22) and (23) may be obtained.

$$\sum F_x = 0 \quad C_1 = S \quad (21)$$

$$\sum F_y = 0 \quad T = R_1 = \frac{P_1}{2} \quad (22)$$

$$\sum M_o = 0 \quad C_1 \cdot jd = \frac{P_1}{2} \cdot q \quad (23)$$

Eq. (23) may be written as in Eq. (24)

$$P_1 = \frac{2 C_1 \cdot jd}{q} \quad (24)$$

From Eq. (24) it will be noted that the distance  $q$  between the vertical forces is one of the factors affecting the ultimate load  $P_1$ . The distance  $q$  seems to be governed by shear span. It is therefore likely that the longer the shear span, the lower the ultimate load required to break the beam. This is consistent with the test results which indicated that the ultimate loads increase as the shear spans in beams decreased.

#### Derivation of ultimate shear equation for beams of Series

II Because tension failure occurred in beams of Series I, the test results of these beams were of no practical value in evaluating the important factors resisting diagonal tension. Therefore the analyses of diagonal tension had to be started from beams in Series II.

It will be noted in the results of tests for the beams of Series II in Table 7 that both the ultimate loads of Beams IIA1 and IIB3 were equal to seventy six kips. Each of these two beams had three No. 8 bars as the tension steel and had No. 3 stirrups spacing at six inches in their shear spans. However, the concrete strength of Beam IIA1 was 4,200 psi while that of Beam IIB3 was 2,420 psi. Further the lengths of their shear spans were different. Again, the failure loads of Beams IIA2 and IIC6 were nearly equal. The former failed at a load of ninety-four kips and the latter failed at a load of ninety-five kips. The failure load of Beam IIB4 was about 103 kips, which was only 9 per cents higher than Beams IIA2 and IIC6. These three beams had the same amounts of tension steel, three No. 9 steel bars, and the same amounts of stirrups, No. 3 bar spacing at  $4\frac{1}{2}$  inches, in their shear spans. However, their concrete strengths and shear spans were different. The strengths of Beams IIA2, IIB4, and IIC6 were 4,270 psi, 3,910 psi, and 3,730 psi respectively. Their ratios of shear span to effective depth were 2.61, 2.09 and 1.75. The characteristics of these beams indicate that the ultimate loads for the beams of Series II seemed to be independent of the concrete strengths and the shear spans. Thus it agrees with the principle presented in Chapter V, that the applied loads for beams of Series II had to be resisted by stirrups and tension reinforcement after the formation of diagonal crack.

From the above studies it was believed that the free body diagrams for the beams of this series must be somewhat similar to the one as shown in Fig. 4A. The slight difference would be that portion

of the diagonal cracks because the actual cracks were not exactly the same as the assumed crack. It was realized that the stresses in the stirrups at points crossing a major diagonal crack would be very close to the yield point stress of the web reinforcement. Therefore in each beam the total approximate tension taken by the stirrups can be determined by multiplying the yield point stress of stirrup, 63,000 psi, by the total area of all the stirrups crossing the major crack. Then, the shear force taken by the tension reinforcement must be equal to the difference of the ultimate end shear and the tension of the stirrups. It is very likely that the shear resistance offered by tension steel is directly proportional to the area of steel. By plotting the shear forces of the tension reinforcement against the areas of the tension steel for all seven beams, a straight line which passes through the origin of the co-ordinates and fit the points best should represent the equation of ultimate shear for the beams of Series II. X

Although the details of beams and the pictures of crack patterns in Figs. 27 - 33 can be used as guides to estimate roughly the numbers of stirrups across the major cracks of the beams, yet the numbers so determined are not clear enough for some beams such as Beams IIB3 and IIB5. In Fig. 31 it is seen that the major crack of Beam IIB5 was not distinguishable. If the crack from the minor corner of the gap inclined to the inner edge of the support bearing plate was the major crack, it is obvious that the fourth and the fifth stirrups from the left end of the beam were crossed by this major crack. However, the third stirrup from the end was cut by this crack in the vicinity of the tension reinforcement. It was



doubtful whether this stirrup should be involved in the free body to evaluate the shear force taken by the web reinforcement. Similarly, in Beam IIB3 it was possible to consider that either two or four No. 3 steel bars of the web reinforcement were stressed nearly to the yield point stress. It was felt that the best way to solve this problem was to plot all the possible shear values resisted by the tension steel of Beams IIB3 and IIB5 together with the more definite shear values of the tension steel from the rest of the beams in Series II against their corresponding areas of tension steel. The group of points defining a best straight line through the origin was adopted. Such a plot is shown in Fig. 43 and the calculations are listed in Table 8. Four No. 3 steel bars in web reinforcement of Beam IIB3 and six No. 3 steel bars in Beam IIB5 were found to be in action in their free body diagrams. From the straight line in Fig. 43 the unit shearing stress of tension reinforcement was evaluated at 6,000 psi. The ultimate equation for beams of Series II failing in diagonal tension could be expressed in Eq. (25) which is an equation of ultimate end shear in terms of the area of tension and web reinforcement.

$$V_{II} = 63 A_v + 6 A_s \quad (25)$$

where  $V_{II}$  = Ultimate shear force in kips at diagonal tension failure for beams of Series II.

$A_v$  = Area of stirrups in square inches crossed by a major crack in the shear span with gap.

$A_s$  = Area of tensile steel in square inches.

Beams IIC6 and IIC7 were very similar although they had different concrete strengths. Each one had three No.9 tension

reinforcement bars and No. 3 vertical stirrups spaced at  $4\frac{1}{2}$  inches in their shear spans. Both were loaded at the third points. The concrete strength for Beam IIC6 was 3,730 psi and that for Beam IIC7 was 4,980 psi. The ultimate end shears were 47.5 kips for the former and 57.5 kips for the latter one. At first glance, it would appear that this was contradictory to the principle that the ultimate load for beams of Series II failing in diagonal tension depends upon their reinforcement alone. However, the test results might be explained by the following argument. In the pictures of Beam IIC6 in Fig. 32, it will be observed that the major crack was distinguished from the other cracks and it cut the second and third stirrups from the end of the beam. Therefore, four No. 3 steel bars of web reinforcement were included in its free body. In the pictures of Beam IIC7 in Fig. 33, approximate parallel cracks occurred at the shear span and it was difficult to distinguish which was the major crack. If that part of this beam above the uppermost crack starting from the outer corner of the gap was taken as a free body, then the first, second, and the third stirrups from the left end of the beam would be involved. In other words in Beam IIC7 a total number of six No. 3 steel bars or two more than that of Beam IIC5 should be counted in for the evaluation of the ultimate end shear. In this way the computed ultimate end shears based on Eq. (25) were figured to be 45.7 kips for Beam IIC6 and 59.6 kips for Beam IIC7. Their percent errors compared with the actual shears are less than 4 per cent.

A comparison of the actual and computed ultimate shear loads for the seven beams of Series II are presented in Table 9. The maximum computed ultimate shear based on Eq. (25) was found to be 11.7

TABLE 8

EVALUATION OF THE SHEAR FORCES TAKEN BY TENSION STEELFOR BEAMS OF SERIES II

(1) Beam No.	(2) Ult. Shear $V_{II}$ (kips)	(3) $A_V$ (sq.in.)	(4) $63 A_V$ (kips)	(5) $V_{II} - 63 A_V$ (kips)	(6) $A_S$ (sq.in.)
IIA1	38.00	0.44	27.70	10.30	2.35
IIA2	47.00	0.44	27.70	19.30	3.00
IIB3	38.00	0.44	27.70	10.30	2.35
IIB4	51.73	0.44	27.70	24.03	3.00
IIB5	64.43	0.66	41.60	22.83	3.79
IIC6	47.50	0.44	27.70	19.80	3.00
IIC7	57.50	0.66	41.60	15.90	3.00

TABLE 9

COMPARISON OF ACTUAL AND COMPUTED SHEAR LOADSFOR BEAMS OF SERIES II

(1) Beam No.	(2) Cal. $V_{II}$ (kips)	(3) Actual $V_{II}$ (kips)	(4) Cal. $V_{II} - V_{II}$ (kips)	(5) Percentage error
IIA1	41.8	38.00	+3.80	+10.0
IIA2	45.7	47.00	-1.30	- 2.8
IIB3	41.8	38.00	+3.80	+10.0
IIB4	45.7	51.73	-6.03	-11.7
IIB5	64.4	64.43	0	0
IIC6	45.7	47.50	-1.80	- 3.8
IIC7	59.6	57.50	+2.1	+ 3.7

per cent less than the actual shear in Beam IIB4. For the rest of the beams their ultimate shear loads could be computed by means of Eq. (25) to within 10 per cent accuracy. This ultimate equation was therefore considered to be satisfactory for these seven beams in Series II. A graphical presentation of the comparison is also available and is shown in Fig. 44. The actual ultimate shears were plotted as abscissas while their corresponding computed shears by Eq. (25) were plotted as ordinates. If the computed shear value is equal to its actual shear then this point should lie on the 45 deg. line such as the point for Beam IIB5 on the graph. Therefore the closer the points to the 45 deg. line, the more accurate the computed shear would be.

Derivation of ultimate shear equation for beams of Series

III As already stated in the previous chapter two beams of Series III failed in tension, so the test results of only five beams were available for the analysis. Since the major cracks in most of these beams were not distinguished, the free bodies of these beams at diagonal tension failure were arbitrarily taken to be similar to that in Fig. 5A with the major crack inclined up to the neutral axis. Although it was impossible to evaluate the location of the neutral axes of these beams from the available data in this investigation, yet it was reasonable to assume the depths of the neutral axes of these beams to be within the range of three to five inches. Based on these assumptions the major cracks of these beams may be sketched as shown in the details of Figs. 34, 36, 38, 39, and 40. Under these circumstances the stirrup immediately adjacent to the applied load in the span between the inner edge of the support bearing plate and the outer edge of the load bearing

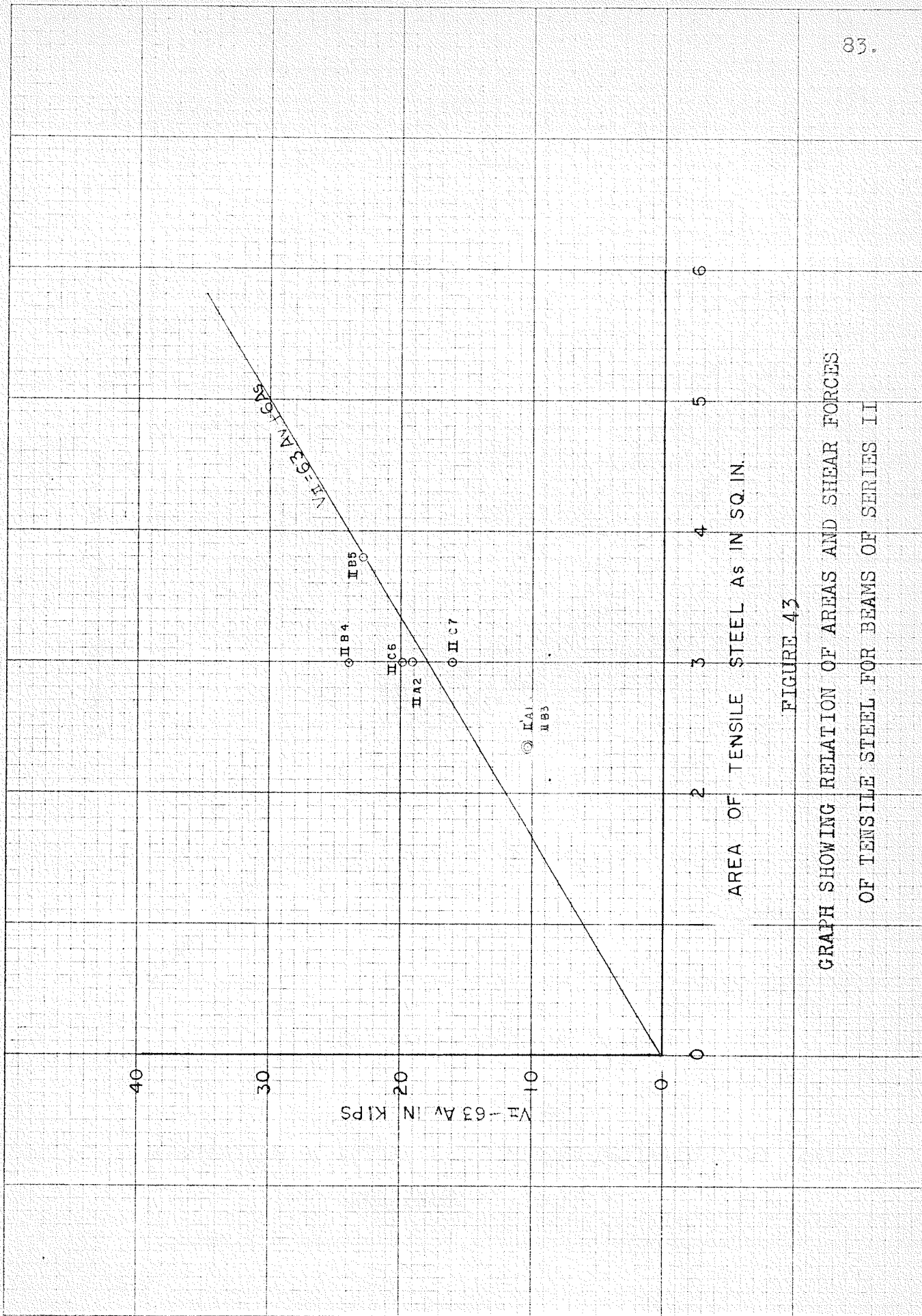


FIGURE 43  
 GRAPH SHOWING RELATION OF AREAS AND SHEAR FORCES  
 OF TENSILE STEEL FOR BEAMS OF SERIES II

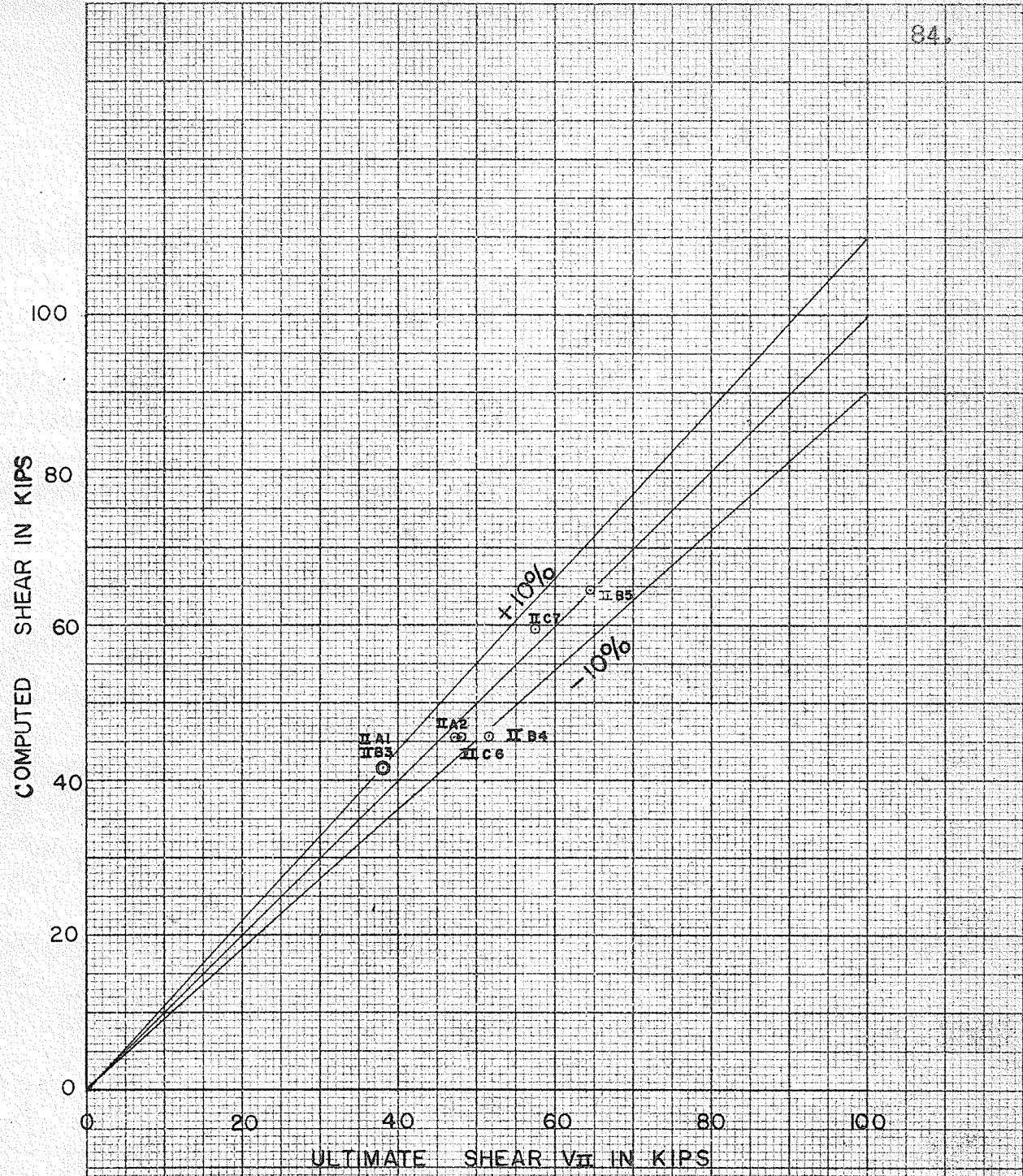


FIGURE 44  
COMPARISON OF ACTUAL AND COMPUTED ULTIMATE SHEAR LOADS  
FOR BEAMS OF SERIES II

plate was not included in the free body diagram of each beam except Beam IIIA1. It may be argued that this stirrup would be in action, since actually the major diagonal crack of each beam crossed it. Of course, this is quite true. However, it has been measured by some investigators in this field that the strains are always largest in the stirrups located near the middle of the shear span and decreases toward the support and load points. Hence the effect of neglecting this particular stirrup near the load might be compensated by assuming the rest of the stirrups in the shear span stressed to the yield point stresses.

According to the principle set forth in Chapter V the shear loads for the beams of Series III should be taken by the shear resistance of the tension reinforcement, the tension of the stirrups and the shearing strength of the concrete above the neutral axis. All the ultimate loads of the beams in Series III were expected to be considerably higher than those of their corresponding beams in Series II. However, two beams in Series III had comparatively lower loads than their corresponding beams in Series II. The ultimate loads of Beams IIIB3 was 73.89 kips while that of Beam IIB3 was 76 kips. The ultimate loads for Beams IIIB5 and IIB5 were 113.89 kips and 128.85 kips respectively. This might be explained by the reason that the numbers of stirrups corssing the major cracks might not necessarily be the same for the corresponding beams in these two series. It was decided to assume that two No. 3 steel bars of the web reinforcement for Beam IIIB3 and four No. 3 steel bars of that for Beam IIIB5 in the middle of the shear spans were considered to be effective, and stressed up to the yield point stress of 63,000 psi. Consequently, the direct

application of Eq. (20) for the evaluation of the shear resistances of concrete above neutral axes was prevented. Fortunately, it was realized that the portion of the nominal shearing stress contributed from concrete for beams failing in diagonal tension would be directly proportional to the compressive concrete strength and the ratio of effective depth to shear span. Based on this and Eq. (25) an ultimate load equation for the beams of Series III failing in diagonal tension was derived.

The necessary computations for the derivation of the ultimate load equation are shown in Table 10. Most of the columns contained in this table are self explanatory. The effective areas of the stirrups crossed by the major cracks of the beams are listed in Col. (3) of Table 10. It should be noted that these areas for Beams IIIB3, IIIB5, and IIIC7 are different from those for their corresponding beams in Series II. With the areas of both tension and effective web reinforcement known, that part of the nominal shearing stresses taken by the reinforcement can be computed and are listed in Col. (4). Now the remaining part of the nominal shearing stress resisted by concrete in Col. (5) must be equal to the difference of the total nominal shearing stresses in Col. (2) and those in Col. (4). The values for the products of compressive concrete strength and ratio of effective depth to shear span for the beams are presented in Col. (7). By plotting these values as abscissas and the nominal shearing stresses taken by concrete as ordinates as shown in Fig. 45, a straight line fit on these points through the origin of the graph was obtained. The straight line determines the amount of the contribution of the concrete



TABLE 10  
COMPUTATIONS FOR THE DERIVATION OF ULTIMATE EQUATION  
FOR THOSE BEAMS OF SERIES III WHICH FAILED  
IN DIAGONAL TENSION

(1) Beam No.	(2) $v_{II} = \frac{V_{II}}{b_j d}$ (psi)	(3) $Av$ (sq.in.)	(4) $Cal v_{II} = 1,000 \times \frac{63Av + A_s}{b_j d}$ (psi)	(5) $v_{II} - Cal v_{II}$ (psi)	(6) $\frac{d}{a}$ (ratio)	(7) $f_c \frac{d}{a}$ (psi)
IIIA1	865	0.44	520	345	0.765	2220
A2						
B3	458	0.22	348	110	0.478	865
B4						
B5	708	0.44	627	81	0.478	1310
C6	895	0.44	568	327	0.572	2650
C7	900	0.44	568	332	0.572	3140

TABLE 11  
COMPARISON OF NOMINAL AND CALCULATED ULTIMATE SHEARING STRESSES  
FOR THOSE BEAMS OF SERIES III WHICH FAILED IN DIAGONAL TENSION

(1) Beam No.	(2) $v_{II} = \frac{V_{II}}{b_j d}$ (psi)	(3) $Cal v_{II}$ (psi)	(4) $Cal v_{II} - v_{II}$ (psi)	(5) Percentage error
IIIA1	865	786	-79	-9.1
A2				
B3	458	452	-4	-0.8
B4				
B5	708	784	+76	+10.7
C6	895	886	-9	-1.0
C7	900	945	+45	+5.0

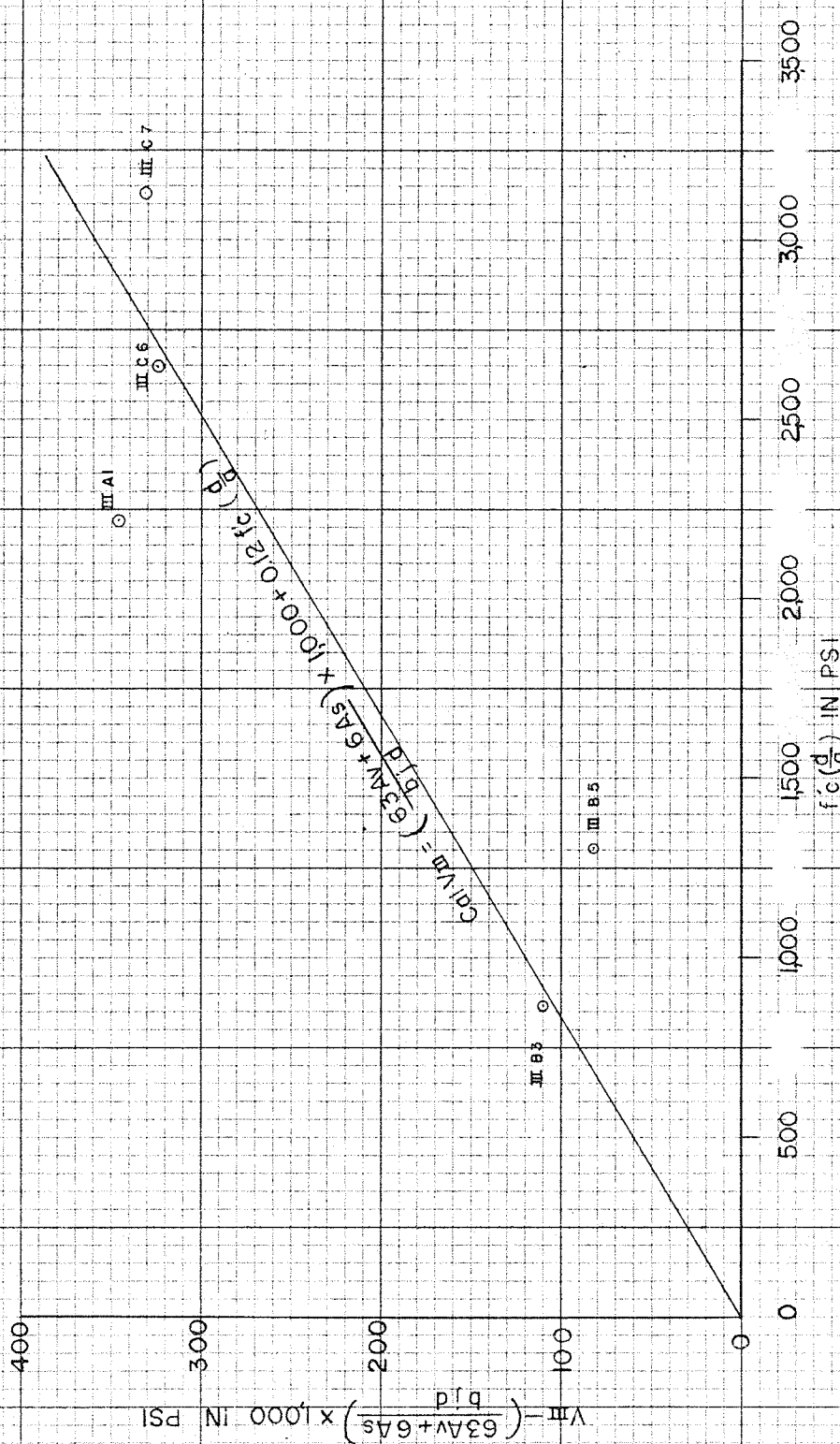


FIGURE 45  
 RELATION BETWEEN NOMINAL SHEARING STRESSES CONTRIBUTED  
 BY CONCRETE IN COMPRESSION ZONE AND  $f'_c (d/a)$

strength to shear resistance. Furthermore, it represents the ultimate shear equation, Eq. (26), for the beams of Series III which failed in diagonal tension.

$$v_{III} = \left( \frac{63A_V + 6A_S}{bjd} \right) \times 1,000 + 0.12 f_c' \frac{d}{a}$$

or, 
$$v_{III} = \frac{63,000A_V}{bjd} + \frac{6,000A_S}{bjd} + 0.12 f_c' \frac{d}{a} \quad (26)$$

This ultimate equation is expressed by the ultimate nominal shearing stress in pound per square inch. The first and the second terms in the right hand side of the equation are nothing more than that of Eq. (25) expressed in nominal shearing stress and the last term is the shear resistance contributed by the concrete of the compression zone. This term agrees exactly with the term representing the contribution of concrete and the effect of shear span in Eq. (12), Clark's empirical ultimate equation. Moreover, the second term of Eq. (26) may be rewritten as  $6,000 p/j$ . If  $j$  is taken as  $7/8$  then this term becomes very close to  $7,000p$  which agrees again with the shear resistance of tension steel in Clark's formula.

The comparison of the nominal and computed ultimate shearing stresses of the beams in Series III failing in diagonal tension are presented in Table 11. The maximum error is in Beam IIIB5 and the computed value is 10.7 per cent more than its nominal stress. For the rest of the beams the percentage errors are within 10 per cent accuracy. The graphical presentation for the comparisons are shown in Fig. 46.

## CHAPTER IX

## SUMMARY, DISCUSSIONS AND RECOMMENDATIONS

Summary This investigation of diagonal tension of reinforced concrete beams was based on the experimental study of twenty-one simply supported rectangular beams. These beams were eight inches wide, fourteen inches deep, and six feet long. Three different types of loading were used: center load, load at points six inches from center of span, and third point load. The loading span was five feet for all beams except one beam. The web reinforcement was limited to vertical two-legged stirrups. The reinforcing skeletons were arc welded before being put in the steel forms for pouring concrete. The concrete was consolidated with external vibrators.

Three different series of beams were made and each series included seven beams. Each beam of Series I and II had a nest of rollers and bearing plates inserted in the compression zone of the concrete. Web reinforcement was provided for beams of Series II and III. It was attempted to evaluate the shearing resistances of the tension steel, the web reinforcement and the portion of concrete above the neutral axis from the beams of Series I, II, and III respectively.

The beams of Series I failed in an unexpected way. The beams cracked and opened up along the longitudinal steel in the region of the support after the formation of diagonal tension cracks. No information was obtained for the analysis. All the beams of Series II failed in diagonal tension. The major cracks were distinguishable for most

of these beams. From the studies of the test results it was tentatively concluded that the shear capacity of this type of beams is governed by reinforcing steel and independent of the concrete strength. Based on the assumption that the stirrups crossed by the major cracks were stressed up to the yield point strength, the shear resistance of the tension steel was evaluated to be 6,000psi. Eq. (25) was developed to be the ultimate shear equation for the beams of Series II.

Two beams of Series III failed by ordinary tension failure. The rest of them failed in diagonal tension. For most of the beams failing in diagonal tension, the major cracks were not distinguishable. In some of these beams several approximately parallel cracks opened wide. From the test results it revealed that the failure of the beams in Series III might occur before the yielding of all stirrups which were crossed by the major cracks. However, the number of the effective stirrups for the beams of Series III could be judged from the test results. The nominal shearing stress contributed by concrete in the compression zone was evaluated to be  $0.12f_c^{\prime}d/a$  and the ultimate shear equation for the beams of Series III failing in diagonal tension was obtained and expressed in Eq. (26). Both the ultimate equations were satisfactory for computing the ultimate shears for the beams of Series II and III failing in diagonal tension.

Discussions The amount of the contribution of tension steel and uncracked concrete to shear strength evaluated in this investigation agree with those in Clark's empirical formula. However, the evaluated shearing resistance of the tension steel seems to be considerably high if it is offered by the vertical shear of tension

steel alone. For a very short element of round longitudinal steel bar which is acted on by a couple of vertical forces at both ends and pulled by tension forces along the axis of the bar, the bond stress required to maintain equilibrium can be computed from the formula  $u = v \pi / 4$ . By this formula the bond stress has been computed as high as 4,720 psi for a vertical shearing stress of 6,000 psi. This is an exceptional high value for bond stress. In a study of the distribution of tensile and bond stress R.M. Mains measured the maximum bond stress to be 3,900 psi. It seems possible that the shearing resistance of tensile steel is contributed by the combined action of the vertical shearing stress and the vertical component of the tensile force of the longitudinal reinforcement. The latter might be a result due to that portion of steel reinforcement which would be bent into an inclined position within the major diagonal crack.

It must be emphasized that both Eqs. (25) and (26) are just tentative equations since they were derived from a few beam specimens. The application of Eq. (26) is limited to beams similar to those of Series III and its accuracy is depended on the correctness of the estimation of the numbers of effective stirrups. Therefore further investigation is necessary to cover more types of reinforced concrete beams and to determine the effectiveness of web reinforcement.

The control of the concrete strengths of the beam specimens was not very satisfactory especially for those in Series III. The descrepancy of the designed and the actual concrete strengths are

listed in Table 5, Chapter VII. The actual strengths for most of the beams were comparatively higher than the designed strengths. "Design and Control of Concrete Mixtures" published by Portland Cement Association states: "Vibration of itself does not make concrete stronger, ..." Therefore, it was clear that the increase of strengths was not due to vibration. Probably, it was caused by the better curing method of painting the sealing curing compound on the surfaces of the beams. However, two beams in Series III had concrete strengths much lower than their designed strengths. A possible cause might be due to the high water content of the sand used in mixing the concrete for these two beams. It was not realized at that time that a new supply of sand having a high moisture content was added to the stockpile.

Recommendations The following recommendations are made for further investigation in the future.

1. The failure of beams of Series I due to the longitudinal crack may be prevented by providing stirrups in the over-hanging ends beyond the supports as shown in Fig. 47.
2. If the same concrete strength is required for any three corresponding beams in different series, the concrete may be poured as layers of same depth into the three beam forms from one mixing batch. If possible, support them on one vibrator so that they will be subject to same vibration.
3. Beams with T section and other kind of web reinforcement can be investigated by the similar theories and methods.
4. If necessary, use strain gages to check the effectiveness of web reinforcement.

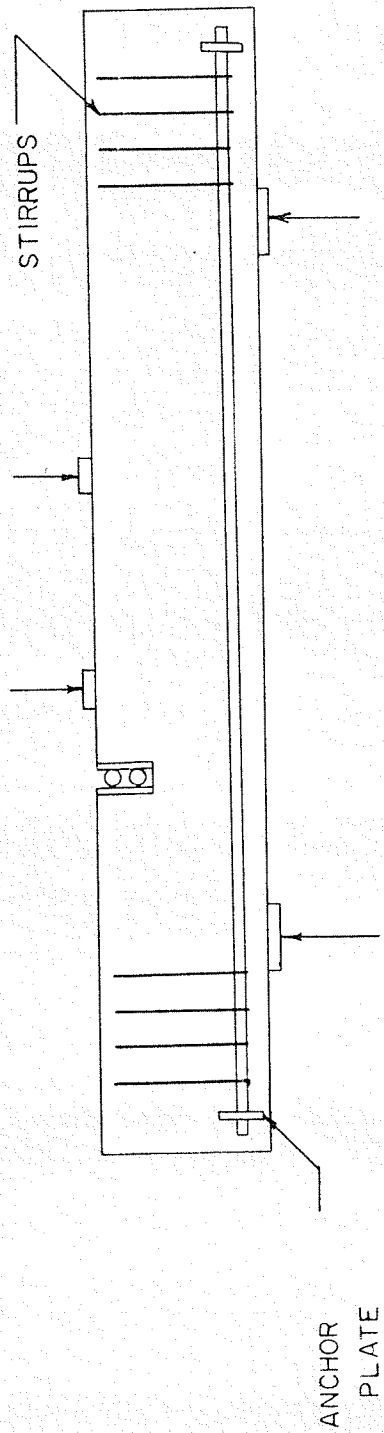


FIGURE 47  
RECOMMENDED BEAMS IN PLACE OF THE BEAMS OF SERIES I  
FOR FURTHER INVESTIGATION



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